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EPISTEMOLOGICAL ACTIVATORS AND STUDENTS' EPISTEMOLOGIES IN LEARNING MODERN STEM TOPICS

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ABSTRACT

This dissertation is a collection of works and ideas I developed during my Ph.D. program within the Physics Education Research (PER) group of the University of Bologna. I hope, with this work, to contribute to the research in this fascinating field, that exists as a mediator between educational practice and ideas from science, philosophy, history, linguistics, pedagogy and sociology. Indeed, the core object of this research field is learning, and in my specific case, learning physics and STEM topics in High School. The extremely complex nature of this phenomenon, involving both the cultural richness of science and the complexity of human thinking, requires researchers to walk on the edges of disciplines, models of learning, and classrooms. In these three years, my interest has been driven by role of *epistemology* in science as a mean to orient learning, understanding, and identity construction. This focus has been informed both by the tradition of the PER group in which I worked, that has a long story of research along this perspective, and by the nature of the challenges I observed during my master dissertation about very interesting aspects of students' understanding of Quantum Physics (Ravaioli, 2016). In fact, I found evidence that, in that context, the shaping of students' personal stances happened mainly on an epistemological level. When I speak about epistemology, I refer to two fundamental nuances of it. The first is tied to the classical philosophical notion of epistemology as a theory of knowledge, that in the last two centuries century has been increasingly identified with theories about scientific knowledge. This first account of epistemology studies the foundations, the validity, the limits and the forms of scientific knowledge as understood by scientists and philosophers. The second refers instead to the beliefs of an individual with respect to knowledge and learning. Up to now, this second account is also referred to as *personal epistemology* (Elby & Hammer, 2016). In the context of this dissertation, I use both the notions of epistemology to pursue different goals. Specifically, my intent is (i) to characterize epistemologically the design of teaching modules for High School on two main modern STEM topics: Artificial Intelligence (AI) and Quantum Physics (QP), and (ii) to study students' epistemologies dynamics in the context of learning QP.

In Part I, I describe the work I did on the design of teaching modules for High School about two modern STEM topics: respectively, Artificial Intelligence (AI), and Quantum Physics (QP). Both the courses have been developed following the Design Based Research methodologies, where the design process is recurrently informed both by theoretical design principles and by the analyses of implementations of the course. The use that I do of epistemology, in this context, involves the individuation of transversal themes, activities, and ideas that can structure students' knowledge on a meta-level and foster them to reflect on the nature of disciplines and knowledge in general.

At the end of Chapter 1, in the context of the AI teaching module, I will define these kinds of themes / activities / ideas as 'epistemological activators', in the sense that they can foster the activation of epistemological reflections on the nature of knowledge and science itself. The design of both the courses revolves around a few design principles that attempt to pursue the outlined direction, and a set of conceptual, epistemological, and social/emotional goals has been pointed out for each activity of the teaching modules. In the context of AI teaching (Ravaioli et al., in preparation), I pointed out the role of ideas such as the ones of 'complexity', 'programming paradigms', 'symbolism / sub-symbolism' and 'induction / deduction', in activating an epistemological level of reflection that subsume and values technical details of the field. Furthermore, the use of the encompassing theme of 'future', structured upon specifically designed thinking tools, was highlighted as a resource in raising specific technical concepts to bear on socially important issues.

In the context of QP teaching, I exploit the notion of 'productive complexity' (Levrini & Fantini, 2013) to inform the design of the course, trying to create an environment in which the students can reflect on their own personal position with respect to the scientific content. Specifically, I propose epistemological insights on the relationship between 'models', 'representations', and 'narratives' (Ravaioli, 2019) throughout all the treated topics as a valuable teaching tool to foster students to think about their understanding of specific QP topics and of science in general. Also, I propose the use of 'epistemological talks' (along the 'quantum talk' of Bungum, Bøe and Henriksen, 2018), as a mean to widen and deepen their understanding through the exchange of views and perspectives.

In Part II, I conduct a qualitative study on students' epistemologies in learning QP. During my master dissertation (Ravaioli, 2016), I analyzed the stances of some students who explicitly did not accept QP as a personally reliable explanation of the physical reality. Through the analysis, I found evidence of the emergence of three specific requirements that can trigger the stance of acceptance or non-acceptance, which I referred to as *epistemic needs*: the needs of visualization, comparability and 'reification' (Ravaioli & Levrini, 2018). Along these initial results, I decided to conduct a more precise study to find out the nature of the factors that trigger students' stances in learning QP, building on the research literature on personal epistemologies and their entanglement with affect-related aspects. The aim of this study is also to find evidence that can suggest whether students' acceptance can be due to factors that are intrinsic to QP theory or if they are just due to didactical choices. To this extent, I collected written and recorded data of High School students participating in a teaching course on QP I developed (the same described in Part II of this dissertation), and ended up analyzing extensively the cases of three students that seemed particularly interesting from this perspective. The analysis highlighted evidence of (i) a possible entanglement between specific students' epistemologies and their meta-affective stances towards challenges in learning QP, and of (ii) expectations about the role of 'visual modeling' and 'mathematics' as two personally reliable means to bridge classical and quantum domains.

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INDEX

PART I

DESIGN OF TEACHING MODULES ON TWO MODERN STEM TOPICS

INTRODUCTION TO PART I – GENERAL DESIGN PRINCIPLES

In in this first Part of the dissertation I describe the design of two teaching modules for High School, focused on two modern STEM topics: Artificial Intelligence (AI) and Quantum Physics (QP). The choice of these two themes comes, on one side, from the focus of my Ph.D. project, that is on teaching/learning issues of Quantum Physics, being these also at the center of my master dissertation. On the other side, in these three years I have been directly involved in the European Erasmus+ Project I SEE (Inclusive STEM Educating to Enhance the capacity to aspire and imagine future careers), coordinated by prof. Olivia Levrini and carried out with partners from Italy, Finland, Island and United Kingdom I SEE project (Branchetti, Cutler, Laherto, Levrini, Palmgren, Tasquier & Wilson, 2018; www.iseeproject.eu)[1]. The main goal of the project, in fact, is to design innovative teaching-learning modules on advanced scientific issues (e.g. climate change, artificial intelligence, quantum computing) to foster students' capacities to imagine the future and aspire to STEM careers.

In this paragraph, I outline the general principles upon which the design of both the modules has been carried out. The criteria upon which they have been built, indeed, evolved from the initial design phase throughout all the implementations. However, I identify here three transversal Design Principles (DP) that are common to both the modules, and that informed all the other specific choices, largely described in Chapter 1 and Chapter 2. These three DPs emerged as results of the experience gained through the years by the research group in STEM education of the University of Bologna, and can be considered a distinctive trait of the approach of the group to STEM teaching.

Forms of productive complexity (DP1)

With 'productive complexity', I refer to a criterion with which the learning environment of the modules has been thought, following what described by Levrini & Fantini (2013) in the context of a course on Quantum Physics for High School. The authors claim that some forms of hyper-simplification "are at risk of dangerously distorting the content as well as the process of learning physics", and they bring out in opposition examples of productive forms of complexity within a learning environment; also, they point out three guiding criteria for the design of teaching proposals, chosen to "problematize knowledge and enhance its cultural significance" (Levrini & Fantini, 2013). In the article, these are described with respect to Quantum Physics teaching, but the research group largely used them as guiding criteria also for the design of other teaching modules, as in the case of the ones described in this dissertation. Thus, I report them here without reference to a specific topic, but as general dimensions upon which any STEM teaching module can be designed:

- multi-perspectiveness: the same contents are analyzed from different perspectives so to encourage multiple connections among the content and conceptual routes

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[1] For the citations in this introduction, see the references at the end of Part I

- multi-dimensionality: the different perspectives and multiple connections are analyzed and compared for their philosophical-epistemological peculiarities, as well as for their relations with experiments and formalism
- longitudinality: the 'game' of modelling is systematically analyzed and compared with the models already encountered by the students during the study of other topics

Epistemology as a learning dimension (DP2)

This second DP can be expressed as the conviction that an epistemological perspective on scientific content is a powerful tool to understand, compare and appropriate STEM disciplines. With the word epistemology, I refer here to its philosophical meaning, as a body of knowledge on scientific knowledge itself, as understood by scientists and philosophers. Several studies from the research literature, and some also conducted from the research group in Physics Education of the University of Bologna, highlighted the role of epistemology in teaching physics. Within diverse physics domains, such as Thermodynamics, Quantum Physics and Relativity it was pointed out that epistemology can be an effective tool for coordinating and organizing knowledge, that is, a tool that can support students to make sense of apparently disconnected chunks of scientific information and appropriate their meaning. The epistemological dimension is a structural element in the design of the activities, and it informs also most of the contextual and domain-specific choices.

Keeping an eye on future(s) (DP3)

The teaching modules have been designed also to foster students' capacities to imagine the impact of scientific developments in the present and in the future, and to begin to consider themselves as active agents of the evolvement of the society. The attention on this dimension comes from the experience gained by the research group in physics education of the University of Bologna within the I SEE project. The project took up the challenge of 'futurizing' STEM education and gathered it as an opportunity to transform the role of education into a lab to prepare the young generation to manage uncertainty (Levrini, Tasquier, Branchetti & Barelli, 2019). The philosophy of the project is grounded on the conviction that science owns tools and reasoning criteria to look towards the future and to interpret the evolvement of the changing society. In fact, starting from the consideration that future is intrinsic to science, given that 'prediction' stands at the core of science modelling, the project aims to make explicit the shift from the linear, deterministic and univocal future of classical Newtonian physics to other paradigms of prediction, as the non-linear one of complex systems and the probabilistic one of quantum mechanics; both these models entail a specific idea of uncertainty and probability, that has to become part of the conceptual tool with which we look at the future and at the evolvement of the society. The plural character of the future is represented by the Voros' cone (Voros, 2003) in Figure 1.1, in which different kinds of futures are introduced: possible, plausible, probable and desirable. Starting from the conceptual and epistemological core of the disciplines, some activities in the teaching modules have been explicitly designed to build this awareness and to engage the students' in future-oriented reasoning.

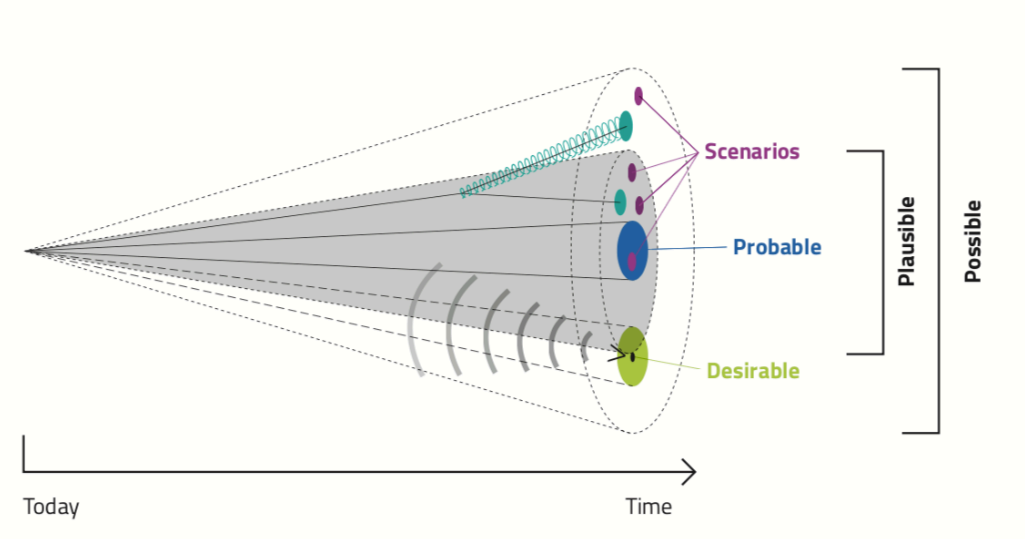


Figure 1.1 Voros' cone of futures

CHAPTER 1

DESIGN OF A TEACHING MODULE ON ARTIFICIAL INTELLIGENCE

In this Chapter, I present the analysis of a teaching module on Artificial Intelligence (AI) designed and implemented within the EU Erasmus+ project I SEE. In this context, following DP1, we tried to build a learning environment where students could be introduced to this new field by bridging and integrating the perspectives of the traditional disciplines (Science-Technology-Engineering-Mathematics). Within the process of refining the module during and after the implementations, we realized that we were defining, more or less explicitly, a specific approach to STEM teaching, and in particular to the teaching of AI and Machine Learning (ML). In this perspective, we decided to re-analyze the module a posteriori, so as to make a step towards the definition of this emergent STEM teaching approach. The analysis has been oriented by the following questions (criteria of analysis):

(1) can we draw a theoretical characterization of the relationship between the different contents, themes and ideas of the teaching module on AI?

(2) what kind of integration between STEM disciplines was realized?

Following these guiding criteria, I point out the role of ideas such as the ones of ‘complexity’, ‘programming paradigms’, ‘symbolism / sub-symbolism’ and ‘induction / deduction’, in activating an epistemological level of reflection that subsume and values the technical details of the field (DP2). Furthermore, the use of the encompassing theme of the ‘future’, structured upon specifically designed activities and thinking tools, was highlighted as a resource in raising specific technical concepts to bear on socially important issues (DP3). At the end of the Chapter, I define these kinds of themes / activities / ideas as ‘epistemological activators’, in the sense that they can foster the activation of epistemological reflections on the nature of knowledge and science itself by either (i) organizing knowledge on a higher abstractive level or (ii) setting a context where specific ideas can become key concepts.

The module design revolves around the general DPs outlined in the introduction, and a set of conceptual, epistemological, and social/emotional goals has been pointed out for each activity.

1.1 CONTEXT AND RESEARCH LITERATURE

The module on AI has been developed by the I SEE research group in physics education of the University of Bologna with academic experts of the AI field and High School teachers. The structure and the contents have been initially designed on the basis of the consolidated structure of all the I SEE teaching modules, that foresees different parts, schematically represented in Figure 1.2: i) encountering the focal topic and futures thinking, ii) a phase of interplay among conceptual knowledge, epistemological knowledge and practice, and inquiry practice about the focal topic, iii) future-oriented and action competence activities. The connection between the first two phases – mainly oriented to disciplinary STEM teaching – and the third one – more devoted to transversal activities – is provided by the "<" arrow in the diagram and consists in bridging activities that link the disciplinary topics to the future studies through the concepts of the science of complex systems. Each part of the module will be described in detail in the next section by referring to the specific activities carried out within the module on AI.

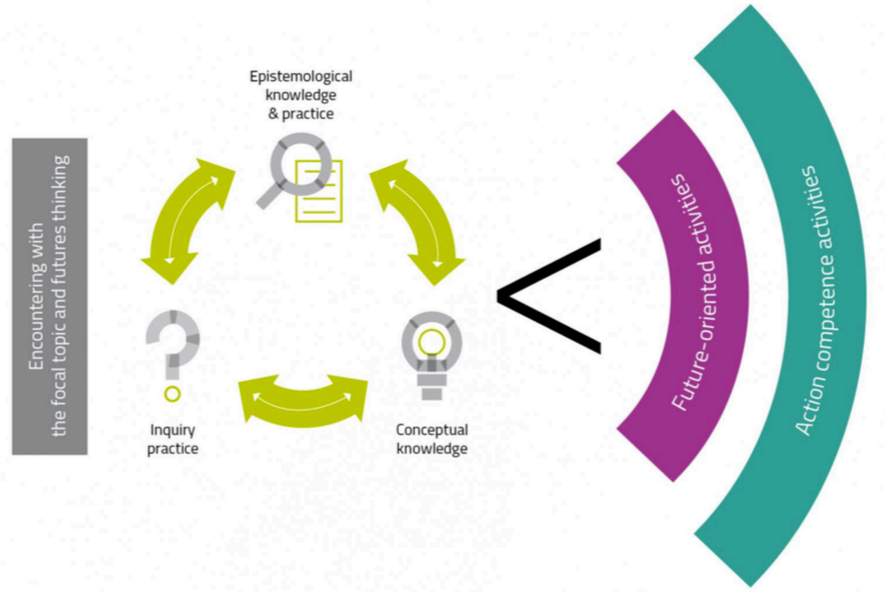


Figure 1.2 Structure of the I SEE modules

The design process was oriented by the subsequent implementations intertwining the development of a theoretical approach to teaching AI with the local and contextual experimentations, following the typical processes of the Design-based research (Barab & Squire, 2004). The module has been implemented and tested in five different contexts with classes of 16-17 years old students, all co-held by High School teachers, University professors and Ph.D. students, but with a clear identification of a leading figure that could give a unitary perspective on the module. In total, the module reached about 120 students. The first two implementations, 20 hours each, ran in the same period (January-February 2018) at the 'Liceo Einstein' in Rimini (25 students) and at the Department of Physics and Astronomy of the University of Bologna (25 students). In June 2018, the module was implemented in an extended version of 36 hours during a summer school for 40 students. In November-December 2018, the module of 20 hours was replicated at the 'Liceo Einstein' with 20 students. In February 2019, the module was adapted to be implemented in a Finnish context for 9 students. The common trait of all the implementations is the extra-curricular and after-school contexts in which they have been carried out. In particular, the Italian ones were part of the proposal offered to upper secondary-school students within the '*Alternanza Scuola-Lavoro*' project, a national program, mandatory for 16-19 years old students, that is supposed to join together the knowledge acquired at school with the competences required by the job market. In addition to that, the experimentations at the Department of Physics and Astronomy of the University of Bologna have been proposed also in collaboration with another national program '*Piano Lauree Scientifiche*' aimed to foster students' interest towards scientific careers and research.

To contextualize the design choices of the teaching module, in what follows I give a brief account of the current research literature about AI teaching / learning issues and about the integration of STEM disciplines in educational contexts.

Teaching AI in High Schools

With the advent of Big Data and Machine Learning (ML), the scientific development of Artificial Intelligence (AI) systems and algorithms is assuming more and more importance both for science and society. In fact, rich data sources and data analysis methods are by now an essential part of our everyday lives (Brynjolfsson, McAffee, 2014) and are transforming the way in which science proceeds and evolves (Kitchin, 2014). In the last years, massive attention has been devoted, in the context of Computer Science Education, to teaching *computational thinking* (Grover & Pea, 2013). Computational thinking was informally defined by Wing (2006) as "thinking like a computer scientist" to solve problems and, more formally, as "the thought processes involved in formulating problems and their solutions so that the solutions are represented in a form that can be effectively carried out by an information-processing agent" (Wing, 2011). Zeng (2013) noted this approach shows especially "logic- and algorithm-based perspectives". He proposed to also consider what he called "AI thinking" to "leverage knowledge bases and case bases in problem solving, capture and reason about commonsense, enable processing of semantics and contexts, and deal with unstructured data, among others". In facts, the need of professionals able to tackle with AI problems is ever-growing and there are many claims for introducing it also in lower grades and in Secondary Schools, also considering that High School students generally express great interest towards the topic of AI, although expressing little confidence with ML (Evangelista, Blesio & Benatti, 2018).

So far, most of the educational efforts (including online courses and MOOCs) in this direction have been focused on University-level instruction (Torrey 2012; McGovern, Tidwell, and Rushing 2011), aimed at preparing computer and data scientists to deal with technical problems and advanced methods. In fact, teaching fundamental concepts of AI in School is quite rare. Moreover, some authors recognize there is a deficiency of research literature about teaching ML problem-solving techniques (Burgsteiner, Kandlhofer, & Steinbauer, 2016; Ko, 2017; Evangelista, Blesio & Benatti, 2018). How and Hung (2019) underline that "ideally, in order for students to learn about these AI-related concepts, they would need to first master a computer programming language such as C++, Python or Julia, and subsequently learn to write programs to translate algorithms from mathematical symbols into computer code. It could be assumed that educators and students in pre-university levels neither have the time nor the pre-requisite skills to learn how to write programs within the precious class timeslots”. The problem is thus to find an approach to teach ML that: (1) allows to not get stuck in the technicalities of a specific programming language, which are currently out of the skills of a secondary school student, (2) can be integrated in the school curricular disciplines, and (3) provides horizontal criteria to understand its role in the change of the society.

To this extent, following DP2, the teaching module has been designed integrating epistemological reflections on the scientific content, so to compare and appropriate the content on a higher level of organization of knowledge. In fact, the current scientific evolvements of AI own a special epistemological importance; as Kitchin (2014) points out, the advent of Big Data and ML opens up the possibility to reframe the “epistemology of science, social science and humanities”. The emergence of what someone claim to be a new scientific paradigm lies on a “data intensive exploration, that challenges the established scientific deductive approach” (Kitchin, 2014). The specific choices will be extensively described in the description of the module (par. 1.2).

AI as a case of STEM integration

Being AI a typical STEM topic, the debate on teaching AI is strictly intertwined with the debate about the integration among disciplines that is needed in order to value the educational potential of AI. Since the 1950s, in USA, a new field of research and practice has arisen with the purpose to meet urgent economic and market needs: the STEM (Science, Technology, Engineering and Mathematics) education (Gonzalez & Kuenzi, 2012). In the last decade, great attention has been paid to this field by institutions, policy makers, entrepreneurial world and business organizations. In 2011, the Obama Administration in the United States of America established the Committee on Science, Technology, Engineering and Maths Education (CoSTEM), in order to coordinate federal programs in support of STEM education. The European Commission, updating its recommendation on key competences for lifelong learning, has mentioned the importance of developing STEM competences to nurture scientific understanding and increase the attractiveness to follow a career in this field (European Commission, 2018).

Accordingly, also educational organizations have been moving towards the implementation of specific STEM educational models. “STEM” is an acronym coined in 2001 at the U.S. National Science Foundation for indicating scientific subjects in education (Hallinen, 2017), but there is not a univocal and uniformly accepted definition of the STEM domain within the research community. The claims for STEM education are based on the assumption that efforts in fostering students to apply science, technology, engineering, and mathematics in real contexts, “not only better reflect the multidisciplinary and interdisciplinary nature of the work of most current STEM professionals (Lantz, 2009; Wang, Moore, Roehrig & Park, 2011), but also make connections between school, community, and work (Tsupros, Kohler & Hallinen, 2009)”. In fact, STEM programs mainly pursue the creation of relationships between the disciplines as well as the development of people’s critical and creative thinking skills (Siekmann, 2016). A literature review shows that these goals are reached in few cases; despite the abundance of school reforms and measures to realize an effective STEM education, the implementation of these recommendations has remained at an early stage. Indeed, often, technology and engineering are superimposed to standard science and mathematics curricula; furthermore in many cases science, technology, engineering, and mathematics often are still being compartmentalized (Lantz, 2009; Moore, Stohlmann, Wang, Tank, Glancy & Roehrig, 2014).

Many authors stressed the need of pursuing integration in STEM curriculum (Education Council, 2015; Honey, Pearson & Schweingruber, 2014; Johnson, Peters-Burton & Moore, 2016). In fact, sometimes the emphasis is implicitly posed on only one or two of the four disciplines, in other cases the four disciplines are presumed to be separate but equal, and other authors refer to an integration of the four (Bybee, 2013, Vasquez, 2015). The blurred meaning of the term is reflected also in the attempts to define and pursue STEM teaching/learning. In part this can be ascribed to the difficulties in interpreting the issue of integration between disciplines, on which there is no consensus (Tibauth et al., 2018). Some have argued that "STEM education is an approach to learning that removes the traditional barriers separating the four disciplines, and integrates them into real-world, rigorous, relevant learning experiences" (Vasquez, Sneider & Comer, 2013), and again that the four disciplines "cannot and should not be taught in isolation, just as they do not exist in isolation in the real world or the workforce" (STEM Task Force Report, 2014; p. 9). In this perspective, Vasquez and colleagues have proposed that the integration level can be represented by an inclined plane (Vasquez, 2015). The lowest level is a *multidisciplinary* integration, where concepts and skills are taught in separate courses but linked through a common theme. Moving up, the *interdisciplinary* integration foresee an organization of the curriculum itself around common learning across the disciplines, where the lines between the disciplines become more blurred. The highest level is *transdisciplinary* integration, where interdisciplinary integration is carried out with students through a real project. On the opposite, other authors claim that "more integration is not necessarily better" (Pearson, 2017) and, although a form of integration is necessary, the integrity of each discipline has to be preserved by highlighting and valuing the core disciplinary contents and processes (English, 2017; Pearson, 2017).

These theoretical perspectives have delivered a plurality of design criteria to orient the revision of curricula. One of the most recent proposals is to base STEM teaching on *big ideas* (Chalmers, Carter, Cooper & Nason, 2017). Among these, the authors distinguish between three main types: i) *within-disciplines* big ideas that have application in other disciplines; ii) *cross-discipline* big ideas; iii) *encompassing* big ideas. The first are concept, notions, ideas typical of a specific disciplinary context; they are STEM big ideas since they find applications to other disciplines (e.g. the proportional reasoning, developed by mathematics but applied to the formulation of physical laws and to the design of engineering and technological artefacts). The second category comprehends ideas that belong natively to two or more disciplines (e.g. the concepts of model and the practice of proof that are both mathematical and scientific ideas). The last ones are superordinate concepts, principles or models shared across the STEM disciplines that not only subsume but also enable one to integrate and build upon sets of more localized disciplinary ideas (e.g. the issue of representation).

The teaching module described in this Chapter has been designed upon the belief that traditional disciplines can still play a decisive role in interdisciplinary STEM contexts, as indeed AI is. In fact, it is a matter of fact that, up to now, all the scholastic systems do have to deal with programs founded on disciplines. The problem is then to find an approach to guide students to navigate through interdisciplinary spaces starting from the disciplines themselves, also allowing them to find perspectives to compare and understand the nature of the disciplines (Branchetti & Levrini, 2019). Moreover, as Pearson (2017) suggests, “integration should be made explicit”, as “students do not spontaneously integrate concepts across different representations and materials on their own”, and educators should provide “intentional and explicit support to help students build knowledge and skills within and across disciplines.

1.2 MODULE DESCRIPTION

The module is articulated in four main parts, plus a bridge between the second and the third, according to the structure common to all the I SEE modules. In part 1, students are introduced to the history of AI in general and the contexts of application of AI in the present, taking from the beginning the perspective of complex systems as a breakthrough in its evolution. This is exploited through two experts’ lectures and a group activity. In part 2, through lectures and interactive class activities, students are guided to compare three different approaches to AI for teaching a machine to learn playing tic-tac-toe: two symbolic approaches (imperative/procedural and logical) and one sub-symbolic (machine learning with neural networks). In the *bridge* part, the disciplinary contents introduced until this moment are linked to the futures studies *via* the science of complex systems; here a lecture and a group activity are conducted. In part 3 future-oriented and action competence activities are proposed, aimed to (i) explore the future scenarios triggered by different exploitations of AI, (ii) individuate a desirable future scenario, and (iii) through a back-casting, plan actions in the present to make it plausible in the future.

Table 1.1 Chronological structure of the teaching module

Part 1 Encountering with the focal topic and futures thinking	1. Overview lectures on AI and the perspective of complex systems	Lectures
	2. The words of complexity	Group activity
	3. Where can AI be encountered today?	Group activity
Part 2 Interplay among conceptual knowledge, epistemological knowledge, and inquiry practice about the focal topic	4. AI - *Imperative paradigm* TIC-TAC-TOE & imperative paradigm (Python)	Lecture + Class Activity
	5. AI - *Declarative paradigm* TIC-TAC-TOE & logical paradigm (Prolog)	Lecture + Class activity
	6. AI – *Machine Learning paradigm* TIC-TAC-TOE & neural networks (Matlab)	Lecture + Class activity

Bridge From STEM to futures studies	7. Predict, hypothesize and imagine the futures: from physics to futures studies	Lecture
	8. The town of ADA 1: analysis of a complex citizenship context of urban planning	Group activity
Part 3 Future-oriented and action competence activities	9. The town of ADA 2: possible future scenarios	Group activity
	10. The town of ADA 3: desirable future, back-casting and action planning	Group activity

PART 1 - ENCOUNTERING AI

This first part is constituted by two overview lectures and two group activity. The lectures, held by an expert in Artificial Intelligence and an expert in science of complex systems, aim to introduce the conceptual and epistemological knowledge that will be developed and deepened throughout the teaching module. The first group activity is aimed at reinforcing the concepts of complexity introduced in the lectures, while the second is meant to build an overall picture of where AI can be encountered today.

AI and the perspective of complex systems (overview lectures)

The first lecture begins with a question: can machines think? After a recollection of the diverse definitions of intelligence, the Turing test is introduced, with its power and limitations, leading to the notions of *weak* and *strong* AI. Among the approaches used to bring the machines to learn, there are two main strands: the top-down or symbolic approach, that starts from a theoretical knowledge (‘from above’), and the bottom-up or connectionist, which starts from the statistical analysis of examples (‘from below’). As an example, if we want the machine to build a circle, a top-down approach would either describe its mathematical properties (declarative language) or set a method to draw it with a compass (procedural language). The bottom-up approach, instead, requires that the concept of circumference is learned implicitly starting from examples. This kind of method was born in the attempt of emulating the cognitive and perceptive human brain processes, leading to the early rise of Neural networks, Machine Learning and Pattern Recogniction. In this perspective, some famous examples of AI games can be shown: in 1997 Deep Blue defeats the world champion Kasparov at chess; in 2011 the supercomputer Watson beats the human adversaries at Jeopardy; in 2014 AlphaGo wins against the world champion at Go. Thus, from a conceptual point of view, the first lecture aims to point out that:

1 - c1 there can be different approaches to teach a machine to ‘reason’ and to solve a problem;

1 - c2 the main distinction between them can be expressed in terms of top-down/symbolic approaches and bottom-up/sub-symbolic approaches.

The second lecture on the role of complexity in AI is introduced through an historical account. In the historical evolution of the conception of the world, from Lucrezio to Einstein, passing through Galileo and Copernicus, the world itself has often been considered a machine, giving body and structure to an intrinsically mathematical world. Along with this perspective were the classical notions of order and causality, that remained unchanged until the middle of the 20th century, when the advent of computers led to the creations and exploration of complex virtual worlds, impossible to explain in terms of simple causality. An example of a machine of this type is the cellular automaton, a system that creates structures unknown to individual components but at the same time undoubtedly dependent on the evolution of the components themselves. The innovative power of these machines consists in the possibility of creating approximate models of real phenomena in their very complexity. The traffic in a roundabout, the flocks of birds, the neat colonies of ants are all examples of complex systems that exist in nature and that can be virtually simulated. Indeed, the most studied complex system in nature is the human brain, and there are up to now many attempts to simulate its processes. From these latter, mathematical models of artificial neural networks were born, simulating the adaptive exchange of information between neurons. Thus, synthetically, second lecture aims to introduce:

1 - c3 the perspective of complexity in studying a problem;
1 - c4 the most significant aspects that characterize a complex system: i) the presence of individual agents, ii) a high sensitivity to initial conditions, iii) the occurrence of emergent properties of the system, and iv) feedbacks and circular causality.

The ‘words’ of complexity

In order to consolidate the main concepts introduced by the lecture on complex systems, an activity titled "the words of complexity" is carried out. The activity is realized through teaching materials, consisting of two *worksheets* and two *simulations* in Net Logo - the flight of a flock of birds and the spread of a virus. The students are given a claim including the main features of complex systems and the first worksheet guides them to: (i) read the sentence very carefully and try to identify the words/phrases that you think characterize a complex system; also, highlight the words/phrases that are not clear to you or whose meaning you do not know; (ii) write down your observations highlighting those that, from your discussion, have emerged as fundamental features of a complex system. The second worksheet includes the following steps about the two simulations: (i) explore the two simulations; (ii) for each one, highlight the typical characteristics of the complex systems as discussed in the first teamwork task. The stnthetic conceptual goals of the ‘words of complexity’ group activity are:

1 - c5 to understand what it means to study a problem from the point of view of complexity
1 - c6 to understand the most significant aspects that characterise a complex system
1 - c7 to understand the necessity of a new lexicon, *the words of complexity*

Where can AI be encountered today?

For this further encountering activity, six cards have been prepared regarding various areas of application of AI: art, archeology, autonomous vehicles, scientific research, astronomical observations, services. Each card has the same format; the description is articulated in three sections: definitions, examples, and links to external resources. Together with the cards students are provided with worksheets that schedule the activity: every group (4/5 people) is required to decide which applications and/or aspects are more interesting/preferable for them, and which applications and/or aspects are more worrying/frightful, specifying why. In a second moment, the group is required to focus on one application (following their own criterion of choice) and to indicate a) both the potential and the risks related to the application, b) what possible changes can be produced by a large-scale dissemination of that application in different dimensions: political, social, economic, ethical, environmental, professional, and c) to think about new professions and which professions can be replaced by the applications. The conceptual goals of the group activity about the AI applications are;

1 - c8 to explore the implications of AI applications along different dimensions (political, social, economic, ethical, environmental, professional, etc)
1 - c9 to understand that AI opens new opportunities and perspectives in the job market
1 - c10 to be introduced to the AI specific language

Goals of part 1

Part1 - epistemological goals

From an epistemological point of view, the activities of the first part aim at building the basements for making students aware of the importance of:

1 - e1 finding criteria to compare the different forms of AI
1 - e2 assuming the perspective of the science of complexity to understand the evolution of artificial systems and modern science in general
1 - e3 adopting a new lexicon*, the words of complexity*
1 - e4 taking in account different dimensions (political, social, economic, ethical, environmental, professional, etc) when evaluating the impact of an AI application

Part 1 - social/emotional goals

Alongside with conceptual and epistemological goals, the activities aimed to foster students' social and emotional competences, namely:

1 - s1 to begin to reflect on the risks and potentialities of different A.I. applications according to their own world view and values
1 - s2 to enlarge imagination about possible future STEM careers
1 - s3 to get personally involved in group discussion according to their own interests
1 - s4 to learn to share different points of view, and acquire the ability to mediate between different perspectives

PART 2 - AI: CONCEPTS, EPISTEMOLOGY AND INQUIRY

The second part of the module is aimed at addressing and deepening the conceptual and epistemological knowledge introduced in the first part. Three main programming paradigms are introduced: procedural/imperative, logical/declarative, and machine learning. To do this, the problem of coding a Tic-Tac-Toe player is addressed with each paradigm, exploited in three different programming languages (Python, Prolog, Matlab). Thus, this part consists of three theoretical lectures (one for each programming paradigm), each followed by a class activity where the execution of a code is showed and commented together.

AI – Imperative / Procedural paradigm

After an introduction on problem-solving as an interplay of 'basis of knowledge' and 'strategy', the students are asked to play some Tic-Tac-Toe games in couples, paying attention at their own strategies, and in a second moment to try to write down in natural language a sequence of simple instructions to teach a machine to play. The code is written in Python, following Newell and Simon's algorithm (Newell & Simon, 1972). During the execution, the teacher shows that the data (the board) are systematically checked by the machine and the instructions are executed step by step. In this way, the teacher can highlight the very reasoning structure of this paradigm; for example, given a configuration of the board 1) the machine checks if there is a move that allows to win; in this case, it makes that move; 2) if not, it checks if the opponent can win and, in this case, prevents her/him to win; 3) and so on. The actions are carried out following the characteristic structures of the imperative paradigm: *sequence* (one instruction is executed sequentially after another), *cycle* (a set of instructions is executed several times under condition), and *selection* (if a certain configuration occurs, there is a choice between alternatives). In this case, the machine is a simple performer exploiting its memory and computational power to quickly evaluate all the possibilities, and the programmer needs to know and express the knowledge about all the possible situations of the board. Synthetizing, the lecture (+ class activity) on the procedural/imperative paradigm follows the introduction, pursuing the following conceptual goals:

2 - c1 to point out that an AI system/solution exploited with an imperative approach is comprised of *data* (variables, constants, etc.) and an *algorithm* (or instructions; a finite and unambiguous sequence of steps that indicate exactly what actions the machine executes)

2 - c2 to underline that with an *imperative* approach, the actions on the data are executed in a rigid and pre-established order, where an action "cause" the next following the sequence of the algorithm (actions "unroll" in time in a known way)

2 - c3 to understand and/or to consolidate the concept of linear causality and deterministic paradigm of prediction

AI – Logical / Declarative paradigm

The Prolog language is characterized by a positive reasoning (*modus ponens*): from true premises, the set of facts that constitute the knowledge base, it deduces true conclusions, through the application of declared rules. Before the Tic-Tac-Toe, a first simple example is presented and implemented in Prolog, explicating the *facts* ("Giovanni is Anna's father", "Carlo is Antonio's father", "Andrea is Carlo's father", "Andrea is Giovanni's father"), the *rules* (implications that considered to be correct, e.g. "If it is true that X is the father of Y and Y is the father of Z, then X is the grandfather of Z"), the *question* ("Is Andrea Anna's grandfather?") and the *conclusions* (T / F answers to questions related to 'being a grandfather of'). A Tic-Tac-Toe player coded in Prolog is then discussed step by step with the class. The students are guided to recognize what the programmer needs to know - true facts and correct rules - and what is hidden, but part of the Prolog language implementation - a deterministic algorithm (inference procedure) that acts on facts and rules to generate the decision tree. Although the logical approach exploits a higher-level reasoning engine that allows not to write down every move specific for a given situation of the board, both the procedural and logical approaches are symbolic approaches, insomuch as the strategy is explicitly expressed and coded in the program. Thus, the second lecture (+ class activity), introducing the *logical/declarative* paradigm, aims to:

2 - c4 introduce the concepts of logical proposition, formula, tables of truth, rules of inference (modus ponens, modus tollens), deductive mechanism and decision tree

2 - c5 understand that with a logical-declarative approach it is necessary to establish a-priori only *facts* (knowledge base; propositions assumed as true) and *rules* (strategy; formulas including material conditional assumed to be correct), without specifying every action to be executed (as with the imperative approach)

2 - c6 underline that the actions that generate the decision tree are performed in a pre-established order based on correct rules, where the application of a rule is activated when pre-established conditions are verified (rules "unroll" the deductions in time in a known way)

2 - c7 understand that the deductive mechanism (i.e. inference procedure) is a deterministic algorithm and is 'upstream', independent from applications

AI – Machine Learning paradigm

After a theoretical introduction of some main concepts concerning the machine learning approach in general (see c8), the focus moved into one particular machine learning algorithm: the feed-forward artificial neural network (NN). Its main trait were introduced, then the analogy of a NN with a complex system was stressed, by highlighting its non-deterministic prediction paradigm, the circular causality between the inputs and the outputs in the training process, the sensitivity of the output to the initial conditions, and the 'learning' of the system as an emergent property (Barelli, 2019). A feed-forward NN Tic-Tac-Toe player has been coded in Matlab, specifically written to be trained from different databases and to let it play against different opponents (a human, a perfect artificial 'imperative' player, and a random artificial player). The students are guided in the construction of an examples dataset (making play against each other artificial random and imperative players n-times). The winner moves are to be taken as target examples. The feed-forward NN player is then trained in different ways to point out some main correlations: (i) the NN player wins more games increasing the *number* of games examples, also when the database is built with games between random players, and (ii) the efficacy of the

NN player increases with the *variety* of games examples: when trained upon a database of games between only perfect imperative algorithms, the NN player does not know what to do against a random player (overfitting). The dependence on dimension and variety of a database allows to explain the role of Big Data in making NNs powerful analysis tools. The NN player ability to win against player from which he/she has learnt shows a form of ‘creativity’. But, yet, it is a sub-symbolic form of creativity; in fact, any strategy can be read looking through the connections of the network, as instead would be the case with a logical or procedural algorithm. Summing up, the third lecture (+ class activity), introducing the *Machine Learning* approach, aims to:

2 - c8 introduce the following concepts: supervised and unsupervised learning, feature, target value, hypothesis function (and its mathematical formulation for linear regression and binary classification), cost function (and the minimization procudeure to tune the parameters of the hypothesis function), feed-forward NN, training, validation, test and efficiency of a NN

2 - c9 understand that the problems that are more frequently addressed using neural networks are the perceptive ones, most of the times not approachable with symbolic algorithms (e.g. multiclass images classification)

2 - c10 understand qualitatively the training process as comprising a ‘forward-propagation’ of the input data through the layers of the network, and an ‘error back-propagation’ for the minimization of the cost function and the assessment of the connections weights

2 - c11 understand that the process of decision making of a machine learning system results in generating predictions that have a *probabilistic nature*, and it’s strongly influenced by the dataset itself

Goals of part 2

Part 2 - epistemological goals

From an epistemological point of view, the second part of the module aims to guide students:

2 - e1 to recognize that each reasoning approach implies a structure of causal reasoning and a paradigm of prediction

2 - e2 to recognize that the type of intelligence displayed by neural networks (i.e. learning from a database of examples) is different from the ones displayed by machines in the imperative (i.e. execution of a pre-ordered set of instructions to perform a task) and logical/declarative paradigms (i.e. inference from facts and rules)

2 - e3 to compare the different approaches by means of the programmer’s role and of their epistemological traits (top-down / bottom-up, symbolic / sub-symbolic, deterministic / non-determinist prediction paradigm):

Table 1.2: distinctive traits of the AI programming paradigms

	Imperative paradigm	**Declarative paradigm**	**ML Paradigm**
programmer role	code a non-ambiguous sequence of instructions (organized in procedures) that solve the specific problem for every possible situation	declare the properties of the desired result through logical statements (facts and rules). The machine will infer the output through an inference engine.	collect an example dataset from which to extract information through a learning algorithm.
epistemological traits	▪ top-down approach ▪ symbolic approach ▪ deterministic prediction paradigm		▪ bottom-up approach ▪ sub-symbolic approach ▪ non-deterministic prediction paradigm

2 - e4 to individuate the role of specific concepts to address the issue of AI creativity

2 - e5 to better understand what typologies of problems are better solved with each approach and what are their limits

Part 2 - social/emotional goals

The social and emotional goals of the second part of the module are to guide students:

2 - s1 to get involved personally in discussion

2 - s2 to feedback on one’s own personal approach to problem solving, learning and reasoning (in and out of school)

2 - s3 to develop a meta-reflection about what means to learn and to develop a deductive reasoning to solve a problem

BRIDGE – FROM STEM TO FUTURES STUDIES

In the bridge part, the disciplinary contents introduced until this moment are linked to the following part of the module, more explicitly related to future thinking. This connection is exploited through the science of complex systems, already introduced in the activities n° 1, 2 and 6. In this part, consisting of a lecture and a group work, the concepts of complexity are addressed in more details and exemplified with the use of simulations, then are used as tools to reason about a complex citizenship problem. During the lecture, a parallelism is shown between the new ideas of complexity and the perspective of futures studies.

From physics to futures studies

The first activity is a lecture about complex systems and the common traits of this new way of reasoning with the perspective of futures studies. Because the main ideas of complexity were presented since the beginning of the modules, this lecture aims to clarify them using examples and simulations. The lecture starts with an introduction about the conception of time and future in classical physics. The choice of a simple problem of classical mechanics allows to highlight the assumptions behind it (e.g. superposition of effects, linearity of relations, reductionism, …) and to focus on the concepts of determinism, uncertainty, system decomposition and space-time scales in classical physics. Then, some examples of complex dynamics are presented with the use of simulations: the Schelling's model of racial segregation (emergent properties and non-linearity), the predator-prey Lotka and Volterra's model (non-linearity, feedback and circularity between causes and effects), the Lorenz' model for meteorological predictions (non-linearity and deterministic chaos). The transition from the concept of prediction, as a univocal result of the application of a model, to that of projection, as a range of possibilities, is the *bridge* element that allows to introduce the discipline of futures studies as a new way to face the challenge of imagining what the future could be. So, the key concepts are introduced: futures' cone, the differences between probable, plausible possible and desirable futures, scenario and the procedures of backcasting and anticipation. Hence, from a conceptual point of view, this lecture aims to guide the students:

B - c1	to recognise the conceptual assumptions behind a common problem of classical mechanics and how the perspective of complexity challenges them
B - c2	to "see in action" the main features of foundational models of complexity (the Schelling's, Lotka-Volterra's and Lorenz' models) with the use of simulations
B - c3	to recognise the similarities between the ideas of complexity and the concepts of futures studies (e.g. the plurality of possible futures, the non-determinism of predictions)
B - c4	to understand the concept of scenario and backcasting, as the process carried out after having defined the desirable future, retracing the temporal path to identify policies, programs, actions that can lead from the present to that future

The town of ADA 1: analysis of a complex citizenship context of urban planning

The second activity represents a bridge between scientific ideas - developed in the previous activities - and future-oriented practices. The activity opens up a series of team-works on "the town of Ada", aimed to guide students to reflect critically on the social, political, ethical, economic implications of a decision concerning AI, as well as on the values, interests implied in any citizenship decision or plan. A sheet is provided to the students with a detailed description of Ada, a small imaginary city, living an extraordinary season in terms of opportunities for future development. The description includes the city urban structure, the people who live there, and the operating companies, the most important of which is "Babbage", an emerging company that produces hardware for AI systems. The improvements in the AI field can give new impetus both to the company and the city; in this perspective Babbage makes some proposals to the city administration. The Mayor has to take decisions that interweave both private and collective interests. The students are required at first to recognize the stakeholders involved in any possible decision, the needs and interests of the different stakeholders, and the interactions between them. In a second moment, they have to assume the role of Ada's Mayor and take a decision about Babbage proposals. Synthetically, the conceptual goals of this activity are:

B - c5 to turn typical concepts of complex reasoning (linear or circular causality, feedback…) into skills to analyze a citizenship context where complex dynamics are involved

B - c6 to reflect critically on the concept of *multidimensionality* and turn it into skills to recognize the different dimensions (political, social, economic, scientific, ethical…) involved in a complex citizenship context

B - c7 to turn an in-depth analysis of a citizenship context into skills to imagine possible future implications of a decision

Bridge goals

From an epistemological perspective, the bridge phase of the module aims to guide students:

B - e1 to recognise that in classical physics: i) the future is deterministically and univocally predictable, starting from initial conditions; ii) the uncertainty of prediction is as small as the accuracy in the measurement of initial conditions is high; iii) the system can be decomposed as sum of its minimal elements and the total effect is the sum of the effects on its minimal components; iv) a change of scale from inside-the-system (internal eye) to outside-the-system (external eye) is not necessary to understand what is happening, since the same laws hold for the agents and the system

B - e2 to recognize that science, throughout its history, has developed alternative ways to think about the future and to "predict" it in terms of plural possibilities

B - e3 to recognize modelling, schematizing, arguing, explaining, posing questions, formulating hypotheses as important processes and epistemic practices that, borrowed from science, can be used to analyze any complex context

B - e4 to recognize the concept of "*dimension*" (political, social, economic, scientific, ethical, environmental, professional ...) as important for unpacking the relationships among the different components of a complex context and among the stakeholders

B - e5 to recognize that the linear model of causal explanation and the deterministic prediction somehow do not work in complex settings

Bridge - social/emotional goals

The social and emotional goals of the bridge activities are to guide students:

B - s1 to understand that the uncertainty that characterizes complex systems can be interpreted not only in a negative way (impossibility of prediction) but also in a productive one (uncertainty that opens up a range of possibilities)

B - s2 to enrich the perception of the future with the dimension of imagination and choice provided by the preferable future

B - s3 to unveil the main assumptions of classical physics studied at school so as to encourage more engagement with the science discourse, through the lens of futures issues
B - s4 to get involved personally in group or collective discussion

PART 3 - FUTURE-ORIENTED ACTIVITIES

In the last part of the teaching module, students are involved in group activities about the imagination of a future ideal city, more or less influenced by AI, and are asked to design actions to take in the present with an eye to desirable futures, also thinking about possible future careers. This part is constituted of two activities, both based on the previous activity about the town of Ada.

The town of ADA 2: possible future scenarios

The first activity is focused on the concept of *future scenarios* and guides the students to reflect about events that may have caused a possible and/or a *desirable scenario* and which values are involved. Three different possible future scenarios are proposed to the students. The first one is the '*hyper technological scenario'*, in which the citizens of Ada entrust Deep Thinking (AI) to administer the town and to organize the life of everyone. The third one is the '*rural scenario'*, in which citizens decided to bid on a return to a natural life, good food, good relationships, handicraft, culture. Innovation is considered cause of isolation and irresponsibility. Between the two extremes there is a '*balanced, creative and plural scenario*', in which the quantity and variety of activities that co-exist allow the citizens to keep together tradition, innovation, cultural differences and social inclusion. The students are required (i) to analyse the three scenarios, identifying both interesting/positive and worrying/negative aspects, choose the preferred scenario and explain the reasons for their own personal choice (individual work), and (ii) to share their reasoning and to identify at least two events that may have caused the emergence of the preferred scenario and which values are expressed (team work). The conceptual goals of this second activity are:

3 - c1 to consolidate concepts like *projection, space of possibilities, scenarios* (already introduced during the bridg parts) and turn them into skills for thinking of a *plurality of futures*
3 - c2 to turn the concept of *scenarios* into skills for thinking about *different ways* to realize *possible futures*
3 - c3 to consolidate the concept that different future scenarios are *not values-neutral*
3 - c4 to turn the concept that "the different future scenarios are not values-neutral" into skills for thinking about one's own *desirable future*

The town of ADA 3: desirable future, back-casting and action planning

The second activity is focused on the concept of *desirable future* and, through action competence strategies, the students are guided to play with *forecasting* and *back-casting* activities and to plan actions that can contribute to realising the desirable future. During the activities the students are pushed to imagine possible future careers and to exploit their creativity. The students are required (i) to image a "desirable scenario", finding out a meaningful and original slogan that characterizes Ada as the ideal town to live in or visit in 2040, and to arrange a brief description of this ideal town; and (ii) to plan actions, by *moving forward to the future* – to identify a problem considered to be significant and that, in the desirable town in 2040, has been solved – and *coming back in the present* – to find an original idea (a leverage point) to solve the problem and plan the actions that can be undertaken in the present time in order to solve the problem in 2040. Conceptually, this final activity aims:

3 - c5	to consolidate the idea that an individual – in order to be able to choose among alternative futures – has to be exposed to the sense of *alternative futures*
3 - c6	to consolidate the concept of *multi-dimensionality* and turn it into skills to plan actions
3 - c7	to consolidate the concept of *agent* in a complex system and turn it into a skill to plan actions
3 - c8	to consolidate the concept of *complex system* (and its characteristic concepts of *circular causality, feedback, non-linearity…*) and turn it into skills to analyze the effects of an action by implementing local-global strategies of thinking
3 - c9	to get acquainted with the concepts of *back-casting* and *forecasting* and turn them into skills to activate back and forth dynamics between present and future

Goals of part 3

Part 3 - epistemological goals

From an epistemological perspective, the last phase of the module aims to guide students:

3 - e1	to recognize that accurate predictions are rarely possible and, usually, not necessary
3 - e2	to move from the idea of a unique future to the ideas of plurality of futures (so that 'scenario' becomes a keyword)

Part 3 - social/emotional goals

The social and emotional goals of the last part of the modules are to guide students:

3 - s1	to get involved personally in group or collective discussion
3 - s2	to get aware of the values implied in specific choices and to recognize that the different possible decisions are not values-neutral
3 - s3	to think about personal objectives, wishes, aspirations, values
3 - s4	to learn to find a mediation between different points of view and reach consensus
3 - s5	to recognize, in a discussion, what is possible/valuable to negotiate (positions to be revised, re-conceptualized or differently situated in a global shared view) and

what is not possible to negotiate (since it refers to values conceived as irreducible for a person or a culture)

3 - s6 to recognize that is socially, economically and personally relevant to adopt a way of thinking in terms of “possibilities” (as opposed to “necessity”) and to explore different ways to realize possible futures

3 - s7 to recognize that desirable futures are mainly emotional and ethical rather than cognitive

3 - s8 to get involved personally in the exploration of the dimensions (social, economic, personal…) that are involved in a context

3 - s9 to get acquainted that each individual can become agent in a complex society and, hence, can play an active and responsible role to create one’s own future

3 - s10 to learn to cope rationally, emotionally, creatively and responsively with their own future

3 - s11 to enlarge the imagination about possible future STEM careers

All the goals of the whole teaching module are synthetically reported in tables 1.3 and 1.4.

Table 1.3. Conceptual, epistemological and social/emotional goals of Parts 1 and 2.

	PART 1			PART 2		
	Overview lectures	**The words of complexity**	**Where can AI be encountered today?**	**AI - imperative paradigm**	**AI- declarative paradigm**	**AI – Machine Learning paradigm**
CONCEPTUAL GOALS	c1 there can be different approaches to teach a machine to solve a problem c2 top-down/symbolic approaches and bottom-up/sub-symbolic approaches c3 perspective of complexity in studying a problem c4 complex system: i) individual agents, ii) high sensitivity to initial conditions, iii) emergent properties, and iv) feedbacks and circular causality	c5 what it means to study a problem from the point of view of complexity c6 complex system: i) individual agents, ii) high sensitivity to initial conditions, iii) emergent properties, and iv) feedbacks and circular causality c7 new lexicon, the words of complexity	c8 to explore the implications of AI applications along different dimensions (political, social, economic, ethical, environmental, professional, etc) c9 to understand that AI opens new opportunities and perspectives in the job market c10 to be introduced to the AI specific language	c11 imperative approach: *data* and an *algorithm* c12 the actions on the data are executed in a rigid and pre-established order c13 linear causality and deterministic paradigm of prediction	c14 concepts of: logical proposition, formula, tables of truth, rules of inference (modus ponens, modus tollens), deductive mechanism and decision tree c15 logical-declarative approach: *facts* and *rules* c16 the actions are performed in a pre-established order based on correct rules c17 the deductive mechanism is a deterministic algorithm and is 'upstream', independent from applications	c18 concepts of: supervised and unsupervised learning, feature, target value, hypothesis function, cost function, feed-forward NN, training, validation, test and efficiency of a NN c19 the problems that are more frequently addressed using neural networks are the perceptive ones, c20 'forward-propagation' of the input data through the layers of the NN, and an 'error back-propagation' c21 the process of decision making of a ML system have a *probabilistic nature*, and it's strongly influenced by the dataset itself
EPISTEMOLOGICAL GOALS	e1 to recognize the importance of finding criteria to compare the different forms of AI e2 to assume the perspective of the science of complexity to understand the evolution of artificial systems and modern science in general e3 to recognize the necessity of a new lexicon, i.e. the words of complexity e4 to recognize the importance of taking in account different dimensions (political, social, economic, ethical, environmental, professional, etc) when evaluating the impact of an AI application			e5 to recognize that each reasoning approach implies a structure of causal reasoning and a paradigm of prediction e6 to recognize that the type of intelligence displayed by neural networks (i.e. learning from a database of examples) is different from the ones displayed by machines in the imperative (i.e. execution of a pre-ordered set of instructions to perform a task) and logical/declarative paradigms (i.e. inference from facts and rules) e7 to compare the different approaches by means of the programmer's role and of their epistemological traits (top-down / bottom-up, symbolic / sub-symbolic, deterministic / non-determinist prediction paradigm): e8 to individuate the role of specific concepts to address the issue of AI creativity e9 to better understand what typologies of problems are better solved with each approach and what are their limits		
SOCIAL / EMOTIONAL GOALS	s1 to begin to reflect on the risks and potentialities of different A.I. applications according to their own world view and values s2 to enlarge imagination about possible future STEM careers s3 to get involved in group discussion according to their own interests s4 to learn to share different points of view, and acquire the ability to mediate between different perspectives			s5 to get involved personally in discussion s6 to feedback on one's own personal approach to problem solving, learning and reasoning (in and out of school) s7 to develop a meta-reflection about what means to learn and to develop a deductive reasoning to solve a problem		

Table 1.4. Conceptual, epistemological and social/emotional goals of the 'bridge' part and of part 3.

	BRIDGE		PART 3	
	From physics to future studies	**The town of ADA 1: analysis of a complex citizenship context**	**The town of ADA 2: possible future scenarios**	**The town of ADA 3: desirable future, back-casting and action planning**
CONCEPTUAL GOALS	c22 Problems of classical mechanics VS complexity c23 Schelling's, Lotka-Volterra's and Lorenz' models c24 complexity ideas in the futures studies c25 concepts of: scenario, desirable future, back-casting	c26 concepts of complex reasoning (linear or circular causality, feedback…) to analyze a citizenship complex problem c27 different dimensions (political, social, economic, scientific, ethical…) involved in a citizenship complex problem c28 possible implications of a decision on the future of a city	c29 to consolidate concepts like *projection, space of possibilities, scenarios* (already introduced during the bridg parts) and turn them into skills for thinking of a *plurality of futures* c30 to turn the concept of *scenarios* into skills for thinking about *different ways* to realize *possible futures* c31 to consolidate the concept that different future scenarios are *not values-neutral* c32 to turn the concept that "the different future scenarios are not values-neutral" into skills for thinking about one's own *desirable future*	c33 to consolidate the idea that an individual – in order to be able to choose among alternative futures – has to be exposed to the sense of *alternative futures* c34 to consolidate the concept of *multi-dimensionality* and turn it into skills to plan actions c35 to consolidate the concept of *agent* in a complex system and turn it into a skill to plan actions c36 to consolidate the concept of *complex system* (and its characteristic concepts of *circular causality, feedback, non-linearity*…) and turn it into skills to analyze the effects of an action by implementing local-global strategies of thinking c37 to get acquainted with the concepts of *back-casting* and *forecasting* and turn them into skills to activate back and forth dynamics between present and future
EPISTEMOLOGICAL GOALS	e10 to characterize the paradigm of prediction that holds in classical physics e11 to recognize that science has developed alternative ways to think about the future and to "predict" it in terms of plural possibilities e12 to recognize modelling, schematazing, arguing, explaining, posing questions, formulating hypotheses as important epistemic practices that, borrowed from science, can be used to analyze any complex context e13 to recognize the importance of *multi-dimensionality* (political, social, economic, scientific, ethical…) to analyze complex contexts e14 to recognize the inadequacy of linear causality and deterministic prediction for complex settings		e15 to recognize that accurate predictions are rarely possible and, usually, not necessary e16 to move from the idea of a unique future to the ideas of plurality of futures (so that 'scenario' becomes a keyword)	
SOCIAL / EMOTIONAL GOALS	s8 to understand that the uncertainty of complex systems can be interpreted not only in a negative way (impossibility of prediction) but also in a productive one (uncertainty that opens up a range of possibilities) s9 to enrich the perception of the future with the dimension of imagination and choice provided by the preferable future s10 to unveil the main assumptions of classical physics studied at school, so to encourage more engagement with the science discourse, through the lens of futures issues s11 to get involved personally in group or collective discussion s12 to enlarge imagination about possible future STEM careers s13 to get involved in group discussion according to their own interests		s14 to get involved personally in group or collective discussion s15 to get aware that the different possible decisions are not values-neutral s16 to think about personal objectives, wishes, aspirations, values s17 to learn to find a mediation between different points of view and reach consensus s18 to recognize, in a discussion, what is possible/valuable to negotiate and what is not s19 to recognize that is socially, economically and personally relevant to think in terms of "possibilities" (opposed to "necessity") and to explore different ways to realize possible futures s20 to recognize that desirable futures are mainly emotional and ethical rather than cognitive s21 to get involved personally in the exploration of the dimensions (social, economic, personal…) that are involved in a context s22 to get acquainted that each individual can become agent in a complex society and, hence, can play an active and responsible role to create one's own future s23 to learn to cope rationally, emotionally, creatively and responsively with their own future s24 to enlarge the imagination about possible future STEM careers	

1.3 RE-ANALYSIS A POSTERIORI: EPISTEMOLOGICAL ACTIVATORS

I draw here an analysis *a posteriori* of the teaching module, following the questions chosen as analysis criteria (that I report here for convenience):

(1) can we draw a theoretical characterization of the relationship between the different contents, themes and ideas of the teaching module on AI?

(2) what kind of integration between STEM disciplines was realized?

Pieces of knowledge – map of the concepts

The first phase of the analysis is aimed to recognize and map the technical and conceptual pieces of knowledge introduced in the module in terms of disciplinary S-T-E-M elements (RQ1). In table 2.1, a classification of the main S-T-E-M elements belonging to each specific activity is proposed. We found this simple categorization to be a good starting point to draw a theoretical characterization of the integration between the disciplines.

Given that when we speak about Computer Science, and there is not yet a sharp accepted boundary between S, E and T, we decided to consider every theoretical aspect of Computer Science contents (i.e. Turing test, definitions of weak/strong AI, definition of Neural Network ect.) as belonging to Sciences (S), leaving to Engineering (E) the design of hardware/software and the optimization/data-management practices, and to Technology (T) the technological applications of AI systems (table 1.4).

Table 1.4: map of the concepts introduced in the teaching module, divided in STEM categories

	PART 1 Encountering with the focal topic			**PART 2** Epistemological knowledge and practice		
	1. History of AI (lecture)	**2. AI & applications (group act.)**	**3. AI & complex syst. (lecture)**	***4/5.procedural* approach (lect. + class act.)**	***6/7. Logical* approach (lect. + class act.)**	***8/9. Machine Learning* approach (lect. + class act.)**
S ELEMENTS	- definitions of ‘intelligence’ - Turing test - weak and strong AI - top-down / bottom-up approaches		- complex systems properties - computers as complex systems simulators	- Processor / memory / programs - knowledge base - computational cost definition of algorithm		- neural network NN - training and test of a NN - overfitting / underfitting - database dimension and variance
T ELEMENTS	- AI games - robots	- art - archelogy - autonomous vehicles - scientific research - astronomical observations - services				- technological availability of Big Data - ML art and design applications
E ELEMENTS				- TicTacToe implementation in Python - optimization	- TicTacToe implementation in Prolog	- TicTacToe in Matlab - pattern recognition - data selection / cleaning - ML *good practices*
M ELEMENTS					- Truth tables - Modus ponens / modus Tollens - Implication and inference	- weights and bias - logistic function - output as a linear combination - min. squared error - statistical operations on the database - results accuracy

Big ideas – interdisciplinary lenses and themes

Although the given categorization comprises many of the STEM concepts and processes used in the module, it does not give a picture of the connections between them. As mentioned, there exist many theoretical frameworks to orient, and thus describe, the design of integrated STEM teaching modules. We use here the work of Chalmers and colleagues (Chalmers et al., 2017) to individuate which big ideas (and how) allowed to cross the S-T-E-M boundaries, in terms of *within-discipline*, *cross-discipline* and *encompassing* ideas.

The within-discipline big ideas are concept, notions, ideas typical of a specific disciplinary context; they are STEM big ideas since they find applications to other disciplines (e.g. the proportional reasoning, developed by mathematics but applied to the formulation of physical laws and to the design of engineering and technological artefacts). The cross-discipline category comprehends ideas that belong natively to two or more disciplines (e.g. the concepts of model and the practice of proof that are both mathematical and scientific ideas). The encompassing big ideas can be distinguished in *conceptual* and *content* encompassing: the first ones are "superordinate concepts, principles or models shared across the STEM disciplines that not only subsume but also enable one to integrate and build upon sets of more localized/specific STEM big ideas" (e.g. the issue of representation). The *content encompassing big ideas* are "based around a theme that enables interdisciplinary lenses from science, technology, engineering, and mathematics to be brought to bear on important problems. The major motivation behind the use of these themes is to improve students' engagement in STEM by (a) situating the study of STEM in contexts that are familiar and relevant to the students, and (b) studying global challenges addressed by STEM".

According to the definitions given by the authors, we individuate in our teaching module five big ideas that mainly allowed to create connections through the S-T-E-M content of the module.

Big idea 1: AI programming paradigms

As described, with 'programming paradigms' we refer to a classification of the possible ways in which a AI algorithm can be exploited. In the teaching module three paradigms were introduced: imperative/procedural, the logical/declarative, and Machine Learning.

According to the classification given by Chalmers and colleagues (Chalmers et al., 2017), we propose this big idea to be *cross-discipline* between Science and Mathematics (S,M), as the distinction between the paradigms naturally belongs to the Computer Science domain, but the paradigms themselves were formed around mathematical concepts (especially the logical/declarative, and the Machine Learning in its theoretical development). All the paradigms require an engineering work in the way the algorithms are exploited and optimized, and they also lead to different technological applications in terms of what specific problems they are best suited for.

Big idea 2: inductive / deductive form of reasoning

The distinction between induction and deduction reasoning belongs to the mathematical domain, but comes to assume an important role in this module when applied to re-interpret the approaches to AI. Furthermore, it owns a strong and established connection with the philosophical and psychological studies on human thought (Dewey, 1910), and it constitutes a powerful tool of self-reflection. It can thus be considered a Mathematical (M) *within-discipline* big idea with applications in Computer Science (S) and Philosophy / Psychology (not included in STEM, but recently proposed to be integrated unique acronym STEAM, where A is for 'all the others' (SiS.net, 2016)). The nature of procedural and logical approaches is of course deductive, where the solution has to be formally expressed and communicated to the machine.

Big idea 3: symbolic / sub-symbolic approach to AI

The distinction between symbolic and sub-symbolic approaches belongs to the computer science domain, but it finds its roots in the mathematical one. We can thus consider this Big idea to be a *cross-discipline* one, that has applications also in the Engineering and Technological domains, as the choice of one or the other approach leads to a different implementation of the algorithms and to a different choice of the technological tools.

Big idea 4: complexity and non-determinism

The big idea of complexity can be considered *cross-discipline*. In fact, the science of complexity was born between mathematical and physical domains (M,S), and had a great impact in the technological and engineering one because of the increasing role of computer simulations in approaching non-linear problems. Within the teaching it allows us to point out some characteristics that assume a special importance in the context of AI and ML.

Big idea 5: future(s)

The ideas and the activities around the theme of future, introduced with the specific terms and categories from the futures studies research field and from science models of foresight, can be considered a content encompassing big idea. In fact, they are ingrained in all the phases of the teaching module, enabling interdisciplinary STEM lenses – such as the previous big ideas of complexity, AI approaches and paradigms, inductive/deductive forms of reasoning – to be brought to bear on important problems, orienting the group discussions and the presentation of conceptual themes. At first (activity 2), they serve to orient the exploration of AI applications along different dimensions (political, social, economic, ethical, environmental, professional, etc). In the second part of the module, the attention on the different scientific models of deterministic and non-deterministic prevision (in terms of linear and complex causality) allow to compare the different approaches to AI. In the third part the concepts of projection, scenarios, and back-casting explicitly orient the whole analysis of the ADA city. The future is enriched with a personal dimension - in which the involvement of the individual is central - absent in the traditional vision of forecasting.

Big Ideas – interactions with the knowledge pieces

Up to now, we described the various ideas in a static manner, essentially setting their belonging to and their influence on the S-T-E-M disciplines. We can say that indeed the big ideas have a role in crossing the S-T-E-M knowledge pieces mapped in table 2.1, and it could be an option to set a hierarchy of ideas in terms of their broadness and interdisciplinary character. However, in order to answer RQ1 and RQ2, what we found to be particularly interesting is the *dynamical* interaction of these big ideas with the S-T-E-M disciplinary aspects.

Big idea of 'complexity' to re-interpret Neural Networks

The big idea of complexity becomes particularly powerful when connected to the inner structure of a NN. During the NN activity we introduced many technical concepts coming from the specific S-T-E-M disciplinary fields (see conceptual goal c8 in part 2). Yet, the interesting discovery was the role of the big idea of complexity to trigger the relationships among these knowledge elements, giving them structure and meaning. As mentioned, the encompassing idea of *complexity* was presented at the beginning of the module, underlying the essential features of complex systems: i) the presence of *individual agents*, ii) *high sensitivity to initial conditions*, iii) the occurrence of *emergent properties* of the system, and iv) *feedbacks* and *circular causality*. This became a lens to re-organize and value all the knowledge pieces about neural networks (e.g. neuron, weight, bias, logistic function, target value, hypothesis function, forward and back propagation, etc.) in a consistent manner, by interpreting the NN as a complex system. In fact:

- the rules of a single neuron (agent) of the network are very simple and specific, fixed by a weight, a bias and a logistic function;
- a little variation of a critical connection weight can lead to a big unpredictable difference in the final prevision;
- the prediction ability of the network is shaped in the training process in a 'bottom-up' way, and comes to be an emergent property of the system interactions;
- the network training is carried out in a circular process, where the output obtained with the forward-propagation of a new input has a feedback on the structure of the network itself, through the back-propagation of the error.

Thus, the big idea of complexity allowed to organize the information about neural networks, and at the same time, the NN became a contextual and concrete example of complex system, so that the very idea of complexity is filled up with a new meaning.

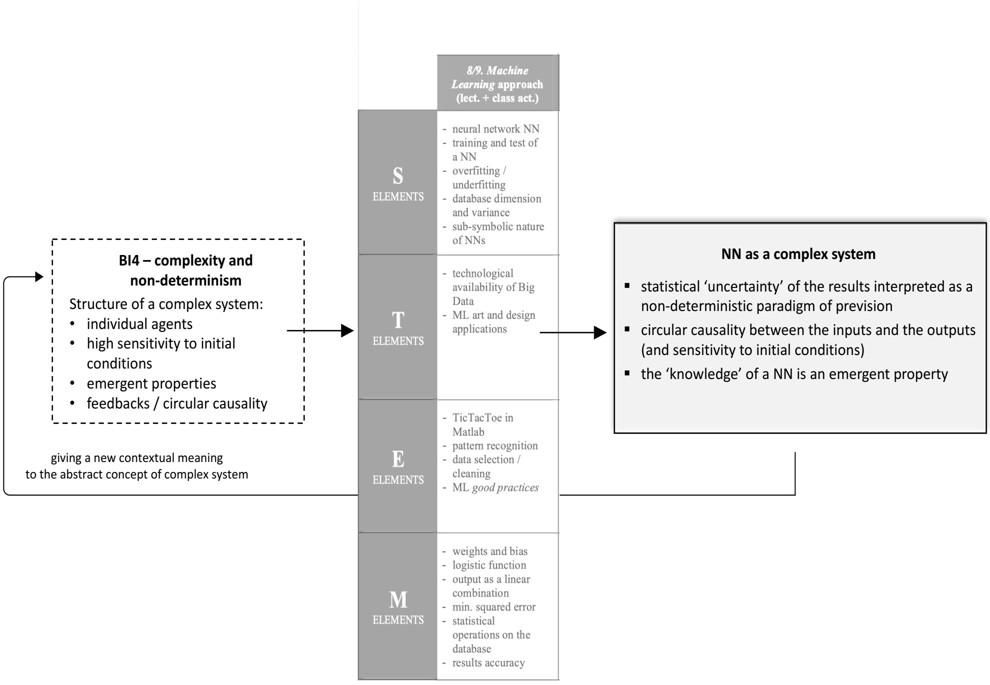


Figure 1.3: Complexity as a conceptual lens.
The big idea of 'complexity' (BI4) comes to be a lens to re-interpret the knowledge pieces concerning Neural Networks

Big idea of 'complexity' to compare the AI programming paradigms

Yet, the interaction between these first four big ideas allowed to draw up a comparative epistemology between the different AI programming paradigms, characterizing the nature of ML approach with respect to the others. In fact, the big idea of AI programming paradigms (BI1) assumes a deeper epistemological value when enriched with the distinction between inductive and deductive forms of reasoning (BI2), the distinction between symbolic / sub-symbolic approaches to AI (BI3), and with the model of complexity (BI4). In a few words, BI2, BI3 and BI4 come to shape the epistemological traits of each programming paradigm, as described in the epistemological goals of parts 3 and 4 of the teaching module. The enriched distinction between AI programming paradigms of course becomes a scheme to organize and re-read many of the conceptual elements introduced in the second part of the module (table 3).

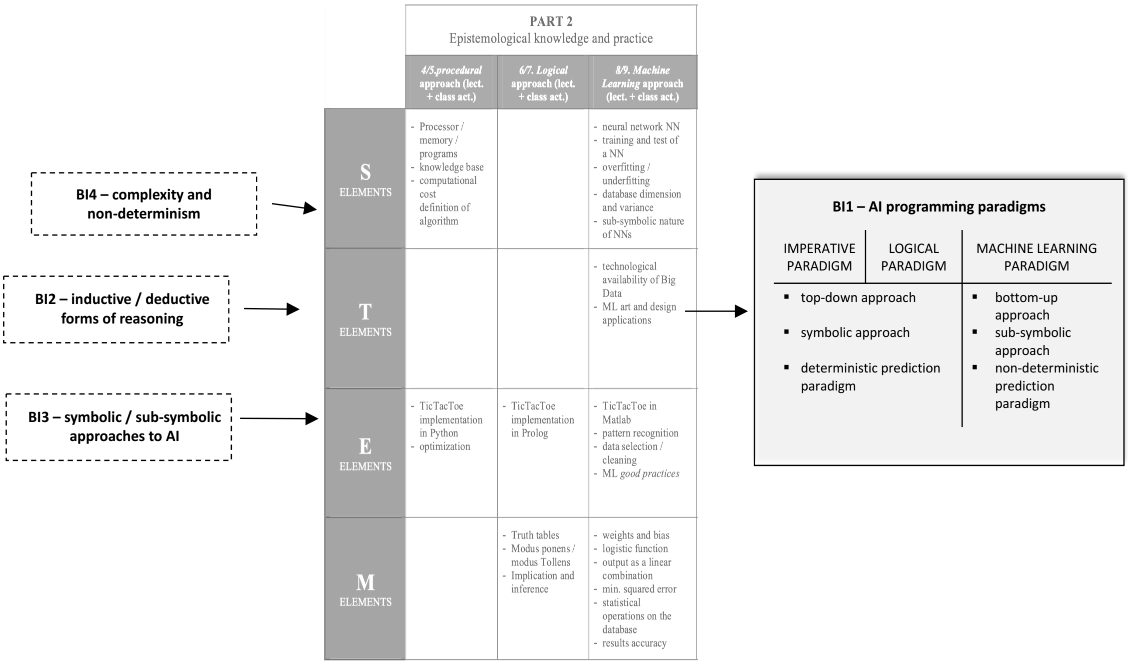


Figure 1.4: complexity and other BIs as organizers.
The interaction between the big ideas BI2 (inductive/deductive form of reasoning), BI3 (symbolic/sub-symbolic for of reasoning) and BI4 (complexity) allows to compare the AI programming paradigms (BI1) and re-organize the knowledge pieces

Future(s) as a context to enable S-T-E-M ideas to bear on important problems

As introduced, the encompassing theme of the future became a context where other S-T-E-M ideas of the teaching module were recognized as playing a key role with respect to specific issues. In other words, it allowed to raise the need for criteria to face important problems. For instance:

- one of the issues raised (and frequently emerged in the class discussions) is the common feeling of suspicion and fear towards the machine learning systems and towards their possible future improvements. A S-T-E-M idea that shed light on this social emergence is the sub-symbolic nature of the neural networks, that does not allow the programmer to read into the connections and explain the output with any specific symbolic expression. We don't claim that this idea removed the feeling of fear towards AI, but for sure it set a new vocabulary to talk and reason about it.
- the dependence of the accuracy of the networks to the quality of the database (introduced in the Tic-Tac-Toe activity implemented with neural networks) became a key scientific concept to talk about issues of social trust and policy decisions about the reliability of ML automation systems;
- the ability of a NN in finding unpredicted solutions to known problems (shown with the Tic-Tac-Toe game and with other applications introduced in activity 2) became a

standpoint to consider when reasoning about the changing role of humans in the labor market.

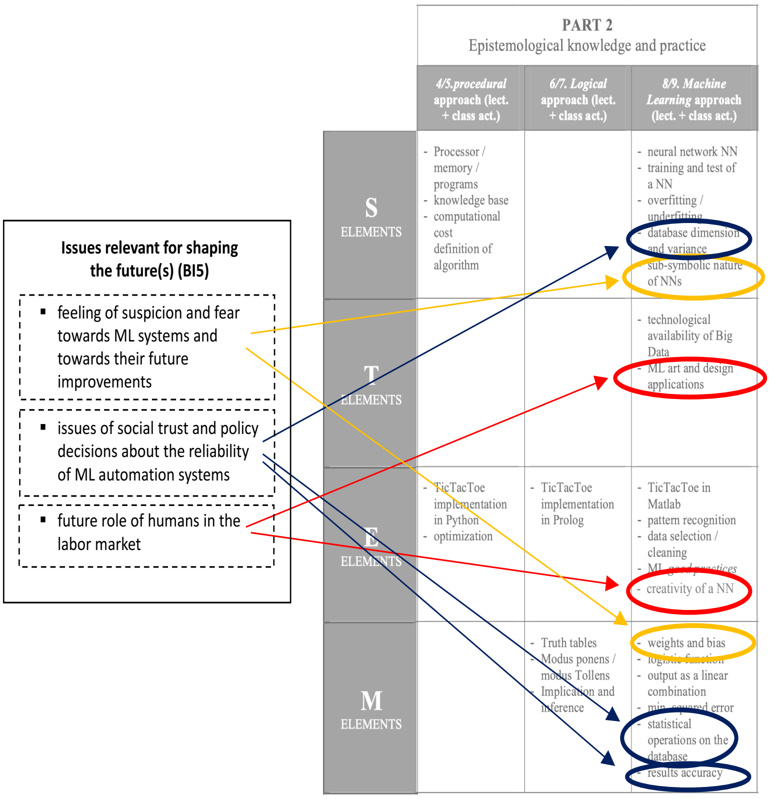


Figure 1.5: big idea of future(s) as a context to value STEM ideas.
S-T-E-M ideas recognized to be important in light of future-oriented problems

Epistemological activators

Even though all the big ideas contributed to build an interdisciplinary texture within the teaching module, we want to focus here specifically on the big idea of 'complexity' (BI4) and on the big idea of future(s) (BI5). In fact, we believe that these two ideas played a special role with respect to the others, that we call *epistemological activation*. To define operationally what an epistemological activator is, it should be worthy to synthetize the roles that, as pointed out in the last section, these two big ideas took in valuing and re-organizing the knowledge elements.

The big idea of 'complexity' played three distinct roles: (i) a *conceptual lens* to reinterpret the structure and the principles of the neural networks, (ii) a *criterion* to distinguish and compare the AI approaches, and (iii) a new *paradigm* to think about the future and the role of science/technology in the society. While the first two roles could stay under the category of *knowledge organization*, in the sense that they helped to build relationships between the knowledge elements by elevating them on a higher abstractive level, the third one owns a different nature. In fact, in this case complexity is used as a paradigm, comparing it with the classical paradigms of science; this is not only a form of organization of knowledge, but it comes to be a tool to raise questions about the nature of science itself. We are aware that this should be supported also with an analysis of students discourses, and not only claimed as a property 'a priori'; however, to the purpose of this study it will suffice to state that some big ideas, as the one of complexity, can eventually and contextually own this power of raising questions about the nature of science. Of course, this will be the starting point for future student-oriented studies.

For what concerns the role of the given perspective on 'future(s)' (BI5), it could be summarized saying that (i) it *gave a context* where specific knowledge elements can become 'leverage concepts' and (ii) it *raised the need* to find interpretive lenses and epistemological criteria to think about science itself, and about its role in the society. In this case, the first role could be considered as a form a *contextualization*/*valorization* of scientific knowledge, and the second, as well as it was for 'complexity', could be thought as a catalyst of questions about the nature of science and about its role within the society.

At this point we could attempt for an operative definition of what we call *epistemological activator:*

> An *epistemological activator* is an idea, a theme, or an activity, that has the potential either (a) to organize knowledge on a higher abstractive level, or (b) to set a new context where specific ideas can become key concepts. Because and by means of this potential, an epistemological activator also owns the power (c) to raise questions about the nature and the role of science itself.

The listed properties can eventually become markers to identify, or to design, epistemological activators in other contexts. We choose the term 'epistemological' because these ideas both provide tools and raise the need of forms of epistemology, in terms of knowledge organization and knowledge about scientific knowledge.

Analysis conclusions

The analysis led to characterize the relationship between the different content, themes and ideas of the teaching module (analysis criterion 1) in terms of big ideas: a wise choice of one or more big ideas can set a fruitful interaction with the disciplinary elements, activating epistemological reasoning and comparisons. The big ideas give meaning and organization to the disciplinary knowledge, and at the same time the details enrich and fill with new contextual meanings and perspectives these big encompassing themes. Furthermore, some big ideas can raise questions about the nature of science itself, becoming what we call epistemological activator. Analyzing the role of these special big ideas (in this case the ones of 'complexity' and 'future') we give an operative definition of epistemological activator. I find the role of epistemological activation to be particularly important in the context of the present hyper-specialization of scientific research. Now, more than before, science community and the labor market require younger generations to develop highly specific skills that can allow them to be competitive and productive from the very beginning of their carrier. In this context, as educators we need to find ways and tools to make science personally and socially relevant.

The characterization of the epistemological activation mechanism gives us clues also about the kind of integration that was exploited between the disciplines (analysis criterion 2). In fact, given that the disciplines themselves are well-established forms of knowledge organization, the activation of epistemological perspectives is precisely the creation of higher-level spaces where disciplinary knowledge can be re-organized and structured. This kind of integration goes in the direction of what Person suggests (Pearson, 2017) about searching for integrations that preserve the integrity of the disciplines, and, to some extent it is also a form of *explicit* integration, in the sense that students are explicitly brought to build themselves comparisons and connections across the boundaries.

CHAPTER 2

DESIGN OF A TEACHING MODULE ON QUANTUM PHYSICS

In this Chapter, I present the design of a teaching module on QP for High School, realized at the University of Bologna in January / February 2019. Following the general design principles outlined in the introduction, I exploit the notion of 'productive complexity' (DP1) to inform the design of the course, trying to create an environment in which the students can reflect on their own personal position with respect to the scientific content. The way in which I use epistemology (DP2) to inform the design of the activities, is by proposing insights on the relationship between 'models', 'representations', and 'narratives' (Ravaioli, 2019) throughout all the treated topics; on this basis, I built a grid of guidelines for teachers, specifying both the conceptual and the epistemological goals embedded in each activity. The perspective on the future (DP3) is accomplished here by introducing the students to the main quantum applications - as for example cryptography, computing and teleportation – and to an analysis of the Quantum Manifesto, so to find the possible impact of Quantum Technologies on different dimensions of our society (economical, political, social, scientific, environmental, ethical).

The specific design choices that characterize the teaching module are outlined in the Chapter; among them, I want note here the use of what I refer to as 'epistemological talks' (along the 'quantum talk' of Bungum, Bøe and Henriksen, 2018) - non-authoritative open discussions with the students on epistemological themes - as a valuable teaching tool to widen and deepen their understanding through the exchange of perspectives and (also contradictive) viewpoints.

2.1 UNIVERSITY OF BOLOGNA TEACHING PROPOSALS

The research group in physics education of the University of Bologna has a rather consolidated tradition of research in teaching QP for High Schools, and designed a teaching/learning path that underwent under contextual and structural changes through the years; however, the core of the original design principles (productive complexity) has remained unchanged and has demonstrated to be successful along different contexts and constraints.

The first path was developed in 2005, and it is described in Tarozzi (2005) and in Levrini & Fantini (2013). This first course is born, as it will be described, to make the learning environment rich and complex, navigable by the students in different ways. It has been implemented with a class at the fifth year of an Italian Scientific *Liceo* (High School), with a specific address on scientific and informatics subjects.

The second (2009-2013) was developed in collaboration with the CNR-IMM of Bologna in order to offer volunteer students, as part of an Italian program for scientific degrees (PLS: Piano Lauree Scientifiche) activities, the opportunity to grasp the essential elements of QP, starting from "The most beautiful experiment in Physics" (Levrini et al., 2014; Merli, Missiroli & Pozzi, 1976; Lulli, 2013; Stefanini, 2013; http://l-esperimento-piu-bello-della-fisica.bo.imm.cnr.it/).

The two previous paths converged into a third one (2013-2016) that was designed after the reform of Italian *Licei Scientifici* (2010), introducing topics of QP in the last year of High School. Thus, this path has been thought to be implemented in real classes of scientific *Liceo*. A detailed description is reported in Lodovico (2016).

Productive complexity

The first path has been implemented by the teacher Paola Fantini in a class at the fifth year of a Scientific *Liceo* with a specific address on science and informatics, in the 2004/05 scholastic year. The approach adopted is characterized by three guiding criteria, chosen to "problematize knowledge and enhance its cultural significance" (Levrini & Fantini, 2013):

- multi-perspectiveness: the same physical contents (phenomenology) are analyzed from different perspectives so to encourage multiple connections among the content and conceptual routes
- multi-dimensionality: the different perspectives and multiple connections are analyzed and compared for their philosophical-epistemological peculiarities, as well as for their relations with experiments and formalism
- longitudinality: the 'game' of modelling quantum phenomena is systematically analyzed and compared with the models already encountered by the students during the study of other physics topics (classical mechanics, electromagnetism and thermodynamics)

These three criteria have been chosen consistently with a precise image of science where there are more than one legitimate (and possible) points of view and where not only previously interested students can find their place. To apply the first two criteria, two main choices have been done: (i) analyses of historical-epistemological debates were carried out, in order to present different visions of the physicists involved in QP developments (multi-perspective), and (ii) an epistemological perspective on the contents was developed (multi-dimensionality). The systematic comparison between QP and classical theories (longitudinality) was explicitly realized through the choice of the following question as a common thread: "How does the concept of 'object' changes from classical physics to quantum physics?"

> "This thread led to articulate the educational path into two parts, each of them divided in two phases. Each of the four phases is characterized by a strongly different perspective (historical and epistemological in the first two, phenomenological-descriptive and formal in the second ones) in order to be able to tackle with different needs of the students. The underlying theme of the first part is an historical-epistemological analysis of how the concept of 'object' changed in the transition from classical physics to quantum physics, carried out through the introduction of the "old quantum theory", of the concepts of indeterminacy and complementarity and through the analysis of passages and debates of the protagonists of that period. There second part goes is focused on the formal systematization of the concepts already introduced, developed through a phenomenological analysis of the experiments of Stern and Gerlach. The path foresees both a general formal introduction using the Dirac notation, and an insight on matrix representation using the specific example of Pauli matrices". (Tarozzi, 2005; translated by the author of this dissertation)

The analysis of the data collected during the realization of the path revealed that the difficulties of the students had been transformed into cultural challenges that engaged them even more. Specifically, the data show how the students, immersed in a rich and complex learning environment, are generally very willing to get involved and gather the opportunity to reflect on the complexity of physics. Their words, during collective discussions, showed a profoundly authentic involvement (Levrini & Fantini, 2013). However, no individual interviews were planned and it was not possible to analyze in detail the dynamics of individual learning. Although the experimentation gave positive results, there were different issues related to the use of this path in a generic Scientific High School. First, the scheduled time for the teaching module: the students of the specific address (PNI) where the path was tested therefore had four hours per week of physics, against the three hours dedicated in the other sections. This allowed to introduce more topics and to dwell more on each of them, favoring connections and collective discussions between students. Another aspect was the mathematical formalism. In fact, those students had been previously introduced to matrices in general, and it was thus possible to use Pauli's matrices. Finally, more generally, the PNI classes naturally select students with an interest in physics and mathematics.

The most beautiful experiment (MBE)

The second proposal has been designed and carried out in collaboration with Elisa Ercolessi (University of Bologna), Giorgio Lulli (CNR-IMM of Bologna), and Vittorio Monzoni (University of Ferrara), and became, starting from 2009, a course-laboratory proposed in Bologna in the context of the '*Piano Lauree Scientifiche'* (an Italian program for encouraging students to enroll in scientific degrees) (Levrini et al., 2014). The course was entitled: 'The most beautiful experiment', since it is focused on the single-electron double-slit interference experiment. This experiment, according to Feynman, reveals the very 'heart' of quantum physics, and realized for the first time by three professors of the University of Bologna, Pier Giorgio Merli, Gian Franco Missiroli, Giulio Pozzi (Merli, Missiroli & Pozzi, 1976), obtaining from 'Physics World' journal's reader the recognition of 'the most beautiful physics experiment of all time', in a 2002 survey. Since it was an extra-curricular course, it was also free from the scholastic programs constraints. It consisted of six afternoon lectures, 3 hours each, and was generally attended by a group of 10-15 volunteer students from the last year of the *Liceo Scientifico*, physics enthusiasts and already interested in QP. As Stefanini (2013) points out:

> "The course was designed to show how the analysis of the most beautiful experiment and of its possible variants, can be an occasion to face concepts and topics of quantum physics and, at the same time, provide a modern account of the various dimensions of physical knowledge (conceptual, experimental, historical, formal, logical-interpretative level). As suggested by Feynman, from the experiment of interference of single electrons it is possible to raise the main contradictions and the interpretative limits of classical mechanics, which impose the elaboration of a new logic to overcome the inconsistencies. In fact, the course foresees to develop from the experiment a minimal formalism, able to describe and interpret what is of 'apparently' incomprehensible in quantum phenomena. Therefore, in the course-laboratory, a specific educational approach is pursued, in which several perspectives are intertwined: experimental, logical-conceptual, applicative, historical-philosophical. The path, in particular, was built to introduce genuinely quantum concepts, needed to build the 'new lenses' with which to look at the world and interpret the experiments results. The concepts are those of state, quantum superposition, probability amplitude, entanglement, introduced and discussed using a specific formal language and basic mathematical tools (among which, not Pauli the matrices). To underline how the formalism has been treated with the explicit purpose of showing the new logic, the 'new lenses' with which quantum physics requires us to look at reality, we chose to call the approach 'conceptual approach to the formalism', in strong opposition to an attitude that views the mathematics of quantum physics as a pure tool for fitting the counts" (Stefanini, 2013; translated by the author of this dissertation)

A further element of characterization of the course was the choice to focus on the use of QP concepts and tools to introduce some technological applications described, in order to show the practical utility and innovative potential of physics (Levrini et al, 2014). From the realization of the path analyzed in Stefanini (2013), it emerged how:

> "The students were able to accept the mathematics introduced and didn't perceived it beyond their capabilities. Also, the mathematical formalism has been observed to be recognized as a useful tool for the interpretation and understanding of the experiments. In different answers to the questionnaires, […] a great variety of interests was noted among students, highlighting that the course, due to its multi-dimensionality, has been able to stimulate different types of curiosities and to enhance different approaches to scientific knowledge. The students also understood the language of the formalism within the discussion of quantum applications, and often exploited it in their learning re-elaborations. The PLS project has therefore obtained a very positive feedback, both from students and from teachers, and showed a great potential for its possible use as an educational path in the Scientific High School scene". (Stefanini, 2013; translated by the author of this dissertation)

Three-layered structure

The two previous paths converged into a third one that was designed by a working group of people that involved researchers in physics education, four physics and mathematics teachers, post-doc students and undergraduate students. The group met every three weeks from December 2014 to May 2015 in order to analyze the previous paths and adapt them to the new school contexts. The challenge was to account both for the National Indications and for the results in physics education. The core idea developed by the group was to join up a destructive part belonging to the 'old quantum physics' (the *pars destruens*) with a constructive framework (*pars construens*) by using the MBE as an epistemological, experimental and conceptual *junction*. As suggested by Feynman, in fact, this experiment touches the very core of quantum physics, leading to face directly with some contradictions and interpretative limits of classical paradigms.

The *pars destruens* revolved around the four fundamental phenomena related to the "old quantum theory" and foreseen in the National Indications for scientific High Schools: black body, photoelectric effect, Compton effect and Bohr's atomic model. Even if the choice of dealing with these issues was somehow obliged by ministerial guidelines, the attempt was to strongly bet on this part, in order to foster the discrete-continuous debate. The latter was chosen as a leading thread to connect in a sensible way the various phenomena and situate them into a "significance framework".

The *bridge* part had the role of leading students towards the *pars costruens* by presenting the first steps that led to the search for a new comprehensive theoretical framework that could account for all those phenomena that challenged and put in crisis the classical paradigms. The topics treated are the uncertainty relations, complementarity and the MBE, in the ways that we deeply describe in chapter 3 and in chapter 4. A special role was played by the contribution of Giorgio Lulli, senior researcher at IMM-CNR, and by his line for presenting the experimental and interpretative challenges opened with the MBE (Lulli, 2013).

As far as the *pars construens* is concerned, the group chose to follow the path developed for the PLS context, focused on Stern-Gerlach experiments, so as to build a constructive framework not linked to classical-like properties and to avoid any semi-classical misconception. Following Pospiech (2000), the researchers decided to focus the construction of the genuine interpretative apparatus on something completely new, as the *spin* of Ag atom.

2.2 REVISION: RESEARCH LITERATURE AND DESIGN CHOICES

The results of the previous proposal have been an important starting point to orient the design of the fourth revision of the teaching module, discussed in what follows. The teaching module is built both following the general design principles outlined in the introduction (DP1: productive complexity, DP2: epistemology as a learning dimension, DP3: keeping an eye on the future), and from a critical analysis of the research literature on quantum physics education. In what follows, I propose an overview of the accounted issues by dividing them in paragraphs (only for reasons of order) and specifying the resulting design choices.

Models and measurements

The research in physics education has shown that not all the practices used to design lab activities reveal to be useful to enforce student's conceptual understanding (Hodson, 1996). The two extremes are, on one side, the use of laboratory as a simple confirmation of the already owned knowledge and, on the other side, the so-called 'discovery learning', based on an oversimplified inductive use of experiments (Koponen and Mäntylä, 2006). To overcome the previous extremes, recent papers discuss lab strategies based on *inquiry* (Sandoval and Reiser, 2004) and/or on the development of epistemological knowledge about the design of experiments and the interconnections of experimental data and theoretical inferences (Sandoval and Reiser, 2004; Etkina et al., 2002), also through explicit modelling activities (Zwickl et al., 2015). Etkina et al. (2002) have proposed a classification of the different typologies of experiments that can be used in instruction on the base of their goals: *observational, testing* and *classification* experiments, distinguishing between *qualitative* and *quantitative* for each category; the researchers argue that being aware of a taxonomy of experiments can be helpful for teachers, and for students as well. Zwickl et al. (2015) have instead recently revised the consolidated use of *models* and *modelling* in labs for including the upper-division labs, which commonly "do not seek to inductively develop new fundamental principles, but more commonly to apply known principles to explain observable phenomena or test predictions"; for this purpose, they have proposed to explicitly include activities where students are guided to model also the *measurement process* and they noticed that students ended up to spend productively even more time in analyzing and modeling the used measurement tools than in modeling the physical systems. The framework for modeling in the laboratory became an iterative and aware process of construction and refinement of the models of both the physical system and the measurement tools. On the basis of the previous results, we decide to take the following design choice:

design choice 1. to structure the LAB activities so to provide ways and time to let students to explicitly model the physical systems and the measurement processes, so as to enable them to analyze consciously the passage from the theoretical design of the experiment to the reality of instruments.

Physical systems and processes VS physical objects

Quantum physics laboratories are not so common in Italian high school programs, even though the national guidelines require the *Old Quantum Physics* to be taught in the fifth year of the scientific *licei*. The previous remarks take a crucial role in the context of quantum physics, whose teaching/learning issues have been deeply debated within the community of physics education. Independently of the conceptual approach chosen, quantum physics raises up epistemic and cognitive requirements that often produce deep skepticism, and sometimes even a difficulty to accept the theory (Ravaioli, 2016); in fact, it has been shown that also at graduate and undergraduate levels, students' epistemic and cognitive needs are often not satisfied by the only confidence with mathematical formalism (Levrini and Fantini, 2013; Baily and Finkelstein, 2010). Baily and Finkelstein (2010) pointed out the relevance of teachers' choices about interpretative issues, founding that an 'agnostic' stance can produce naïve *realist* interpretations in students and that addressing epistemological issues seems to play a crucial role for students' understanding. The *photon* concept, for example, can be seen from a realist perspective (as Einstein's one) as being localized, even we do not know its position, or can be considered only as "a click on the detector" (Zeilinger et al., 2005), taking an instrumentalist stance. As Klassen (2009) suggests, "the dominant picture of photons as particles of light is misleading, […] [and] what should be emphasized, rather, is the quantum-mechanical nature of the interaction of light with matter". As Mannila and Koponen (2001) point out, students "are used to direct their attention to properties of entities (particles, bodies, etc.), create images and draw pictures, where illustrations concentrate on the behaviour of entities. A similar approach is very difficult in QP where the properties of basic entities are difficult to approach and one should really concentrate on properties of phenomena". All these remarks led us to point out the second design choice:

design choice 2. to focus the attention on the models of *interaction processes* and on narratives focused on *systems* more than on the physical *objects*, so to avoid inappropriate hyper-simplistic object-based interpretations of quantum phenomena.

Representations in physics education

The use of visual representations (pictures, drawings, schemes, applets, verbal descriptions, graphs) in science education is known to be, on one side, a tool of great didactical power, and, on the other side, a very delicate point in which models, narratives and representations interact in a complex fashion. In the past fifty years, there has been a large amount of studies about their effectiveness in educational contexts (Mandl & Levin, 1989; Eilam & Gilbert, 2014), but I want to focus here on their relationship with modeling in physics. Learning quantum physics, in fact, brings to the extremes the relationship between phenomena and abstract models, and it requires the learner to gain an epistemological awareness of this complex relationship. Indeed, abstractness is characteristic of every model, and every scientific representation can be considered as a cognitive hybrid (Giardino & Piazza, 2008), a mixture of pictorial and verbal elements built upon implicit models of phenomena. In this sense, representations always hide theoretical or personal perspectives on physical reality, and must be interpreted. Knowledge about the so-called of 'visual language' involves the awareness of semantic and interpretative processes at the basis of communication through visual representations (Schnotz, 2002), and specifically how information is encoded in scientific representations through the representational syntax (Ainsworth, 2006; Galano et al., 2018). Podolefski and Finkelstein (2006), for example, studied the influence of different representations on students' understanding of oscillating phenomena, showing that when the represented phenomenon has a high degree of *abstraction* (in the article, sound propagation vs oscillation of a string), they tend to read the representations at hand quite literally. With 'abstract', the authors mean that students do not have a 'phenomenological grounded knowledge' about the phenomenon. In other words, the more the experiential knowledge ground is not stable, the more representations tend to cue students to focus their attention on details, or particular aspects, of the represented phenomena (Podolefski and Finkelstein, 2006), assuming a bi-univocal relationship between representations and reality. Elby (2000) has described this as *What-You-See-Is-What-You-Get* (WYSIWYG), a read-out strategy (diSessa, 2004) eventually involved and activated.

These claims coming from the literature resonate with some results emerging from a *questionnaire* we built to look at students' attitude towards different representations of quantum phenomena. The questionnaire, regarding the photoelectric effect and Bohr's atomic model, was submitted to a class of 25 high school students, who had been briefly exposed to the arguments in the classroom, but only verbally, without any use of representations. It consisted of two ranking exercises, one with 4 different representations of the photoelectric effect, or of some of its features, and the other with 4 regarding the Bohr's atomic model - followed by two open questions: (1) "Which image do you prefer as a representation of the phenomenon? Please rank the images and justify your answer"; (2) "What image wouldn't you choose as a representation of the phenomenon? Please rank the images and justify your answer".

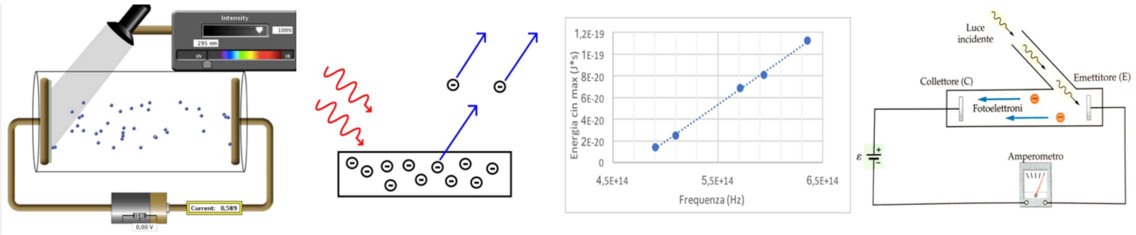


Figure 1.6 the four representations of the photoelectric effect used in the questionnaire, from left to right: 1.A (Phet, 2018), 1.B (CommonsWikimedia.org, 2018), 1.C, 1.D (Walker, 2012)

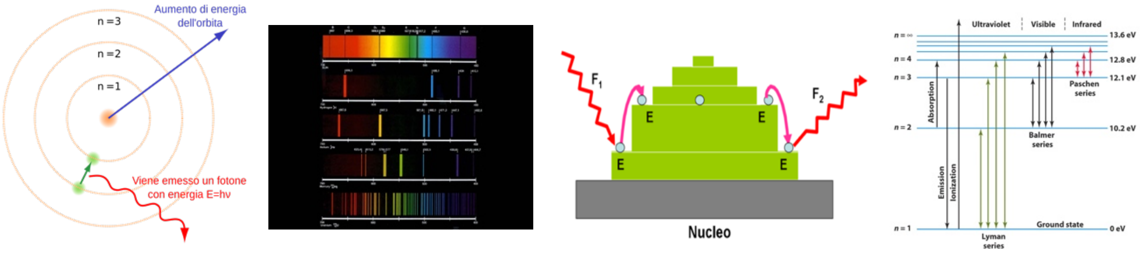


Figure 1.7 the four representations of Bohr's atomic model used in the questionnaire, from left to right: 2.A (Commons.Wikimedia.org, 2018), 2.B, 2.C (infinitoteatrodelcosmo.it, 2018), 2.D (chimichiamo.org, 2018)

In the wake of the literature, students' preferences and open answers highlighted two main expectations:

- the *language* of a representation must be *immediate*: students seem to expect representations to be immediate in the sense of 'time of comprehension', and to be immediate in the sense of 'without the need of any mediation', being it of knowledge or of an analogical structure. For the photoelectric effect, in fact, the most preferred representations are the 1.A and the 1.B (figure 1.6), and some of the explanations are that "it is all clear at a glance", "it's clear and immediate", "it is the most complete", or "comprehensible for its essentiality". The least scored one for representing the photoelectric effect is quite unanimously the 1.C, because "it doesn't represent the process", "it requires a knowledge ground to be interpreted", and to someone "it's less interesting, because it's a graph". As for what concerns Bohr's atomic model, the most chosen representation is the 2.A (figure 1.7), on the basis of explanations like "it's intuitive and detailed, it's easy to be visually remembered", "it's immediate" or "it is the most concrete representation".
- the content of a representation must be focused on physical objects: students seem to look at the objects involved in the phenomena, more than at the processes and at the experimental data, by searching for their positions, movements, and shapes. For example, someone describes her/his choice of not choosing the metaphorical representation 2.C as follows: "it is not effective, because it doesn't recall the shape of an atom". This is a well-known issue in the literature (Knight and Burciaga, 2004) and we think it to be decisive also for what concerns the photoelectric effect, and for all those physical phenomena that require a high degree of abstraction.

The research literature shows that representations can be powerful tools, but these results, together with the other cited above, claim for a special attention in using them for educational purposes. Furthermore, some research has shown that the understating of physical phenomena can be enhanced by improving students understanding of representations (Podolefski and Finkelstein, 2006), developing what diSessa and Sherin (2000) term as *meta-representational competences*. Ainsworth claims for the importance of using multiple representations in physics instruction, inasmuch they can assume different roles and help to (i) gain complementary scientific information, (ii) let one representation to constraint interpretations of another one, and (iii) to construct a deeper understanding in terms of abstraction, relation and extension of knowledge between representations (Ainsworth, 1999; 2006; 2008). Along with these results we fleshed out the third design choice:

design choice 3. to explicitly compare and discuss with the students different representations and different representational forms to develop students' meta-representational competences and to individuate the main constituting elements and hidden models upon which the representations are built (along DP2).

A 'reasoned jump' from classical to quantum

As Mannila & Koponen (2001) point out, when learning QP, "for students the main difficulty lies in the *conceptual shift* needed in order to form a new ontology". An analysis of the research literature led us to identify three main didactical approaches to QP ontological shift (Ravaioli, 2016):

- a clear-cut *refusal of any reflection* about the ontology, being a controversial point. This particular stance about the ontological problem, aligns with Copenhagen interpretation of QP. A famous statement by D. Mermin captures the mood: "If I were forced to sum up in one sentence what the Copenhagen interpretation says to me, it would be: shut up and calculate!". Some teaching proposals taking up this perspective are the ones which faithfully follow the Feynman's model for explaining QP, based upon the Path's integrals' theory (Feynman, 1965). An example is the teaching proposal reported in Dobson (2000) for British schools, that completely entails Feynman's reflections and builds up the phasor's methodology explicitly avoiding any interpretative question:

> "The gamma photons arrive randomly. How do they get from A to B? We have no way of answering this question. This fact is strongly emphasized to students. All we can observe is what happens at A and what happens at B. We don't try to talk about wave–particle duality—this confuses the issue. When students studied waves they saw 'interference' effects, which are fairly easily explained on a wave model—but how can photons get into this act? Or, how can a 'wave' pack itself into a space small enough to trigger a GM tube? We don't know. We don't really care" (Dobson, 2000)

- a progressive *cleaning-up process* from a classical to a quantum ontology, to build up a new vocabulary to deal with quantum objects; this approach entails a process of increasing refinement of the ontological descriptions of the object. Along this line are the proposals of the research group of Pavia University (Malgieri, Onorato, De Ambrosis, 2014), which is based on Feynman's approach, but it however deals with interpretative issues, and the one of Udine University (Michelini, Santi, Stefanel, 2010) grounded on the proposal of Ghirardi (Ghirardi, 1997) about polarisation of light. Taking up Pavia's proposal, the refinement is done, mostly, through the analysis of modern quantum optics experiments so as to outline photons' quantum features step by step.

- a *'reasoned' jump* to the quantum ontology through the introduction of a new interpretative scheme, for describing a wide range of phenomena. A third approach chosen for facing off the ontological shift is to 'jump' directly to the quantum formal and philosophical description of the phenomena, as to not get trapped in semi-classical views, and only after compare this picture with the classical paradigms. An example of this stance is the didactical proposal of Pospiech, who choose a logical-philosophical approach for teaching quantum physics (Pospiech, 2010). In some of her works, the author points out that "most difficulties in understanding quantum theory arise from trying to develop quantum theory starting from classical concepts and then explaining the differences", "but it is just these classical concepts borne from daily experiences that have to be thrown away" (Pospiech, 2000). In order to reach an appropriate understanding of QP concepts, the proposal is to develop a formal framework starting from *spin*, which is indeed not classical and allows for getting inside simple formal tools as Pauli's matrices, which have been demonstrated to be well greeted by students. Next, some core concepts of quantum physics can be consequently introduced, as uncertainty principle and entanglement, which form a logical structure to be compared with classical paradigms. Such an approach could be read as a sort of accomplishment to the ontological shift, but quite opposite from the phenomenological one, as it starts giving a synthetic quantum picture to be next compared with the classical ones from a formal, epistemological and philosophical point of view.

design choice 4. within the diversity of choices, we choose to adapt this latter line ('reasoned jump') to the structure of our module, trying to bring out transversal threads form the 'Old QP' part to raise the need for a 'jump' to the quantum logic, and only then compare the two domains on an epistemological level. The structure according will thus follow this sequence:

I. single-electrons interferometer to pose the problem of classical categories inadequacy

II. Levy-Leblond's platypus / quanton metaphor to set the need for a new ontology and to introduce a new grammar (DP2)

III. two-state spin approach to introduce the formalism and detach from classical quantities

IV. formal comparison between two-way systems (single-electrons interferometer, Stern-Gerlach magnets, Mach-Zender interferometer) to foster the transfer of quantum state descriptive power to a broader phenomenological range, and to shift the narrative from objects to systems (along also dc2)

V. entanglement as an only-quantum property

Quantum applications and 'Quantum Manifesto'

One of the goals of the I SEE Erasmus+ project, from where the attention on the dimension of the future has its origin, has been to build teaching modules on modern science topics, as Artificial Intelligence (see Chapter 1 of this dissertation), Climate Change (Branchetti et al., 2018; Tasquier, Branchetti, Levrini, 2019) and Quantum Computing (QC) (Satanassi, 2018; Spada, 2109). The reflections brought out for this latter module on QC, indeed, influenced also the choices of the present course; in fact, they have been experimented in parallel, and part of the choice of the introduction of QP applications as Quantum Computers and Teleportation has been informed by the design of the I SEE teaching module.

design choice 5. along DP3, we choose to foresee an introduction of the basics of quantum cryptography, teleportation, and computing, and give an account of the 'second quantum revolution' impact on the society. To this extent, we also designed an activity to introduce the 'Quantum Manifesto' (de Touzalin et al., 2016) and to let the students become aware of the dimensions on which a new technology can impact.

Epistemological talk

The research literature in physics education has shown that the use of explicit discussions with students about epistemological issues can have a positive influence both on their understanding and on their attitude towards learning. For example, Bungum, Bøe and Henriksen (2018), in the context of a teaching course on QP where 'Quantum talks' where proposed to students, provide evidence that small-group discussions about the unresolved dilemmas, like the wave-particle duality and the Schrödinger's cat, "have potential for enhancing students' understanding and philosophical reflections in quantum physics", and that they allowed the students to better "articulate conceptual difficulties, deepen their understanding through exchange of views, and formulate new questions". As the authors point out, the research literature on the so-called 'dialogic teaching' identifies its root in Vygotsky, Bruner, and Bakhtin (Alexander, 2006; Mortimer & Scott, 2003). Following Alexander (2006), dialogic teaching should be *collective* (all the students should be able to participate), *reciprocal* (students should share their viewpoints), *supportive* (students should share their ideas without having the fear of making mistakes), *cumulative* (each step of the dialog should build on the previous one), and *purposeful* (the teacher should have specific educational goals in mind).

design choice 6. we propose moments in the teaching module to discuss with the whole class, extending the arguments not only to themes related to QP dilemmas, but also to general epistemological issues, such as (i) the interplay between modeling, experiments and mathematics, (ii) the role of visualization and representations in physics, and (iii) the comprehensibility of QP. The discussions are held by one (or two, in one case) professor with the whole class, fostering as much as possible students' engagement and freedom by posing the problems in non-authoritative ways, exploring students' ideas and allowing contradictive viewpoints (Mortimer & Scott, 2003).

2.3 MODULE DESCRIPTION

The module is articulated in three main phases, according to the structure of the previous module developed by the UNIBO research group. The *pars destruens* is aimed at introducing the students to two critical phenomenologies that led the development of QP theory: the photoelectric effect and the atomic spectra. This part foresees group laboratory activities, lectures and class discussions about the role of modeling in physics. The *bridge* part consists of the presentation of the double-slit single electron experiment and it is aimed at raising the need for a new theoretical description. This presentation is followed by a discussion of the main protagonists of QP development, their backgrounds and perspectives, so as to build a multi-perspective dimension on the contents. The *pars construens* is specifically aimed at introducing the new logic of quantum physics, by building the formalism of quantum states superposition, the entanglement and exploring some QP applications. The scheme of the structure is the following:

Table 1.5 Structure of the teaching module on QP

PARS DESTRUENS	**1. Lab activity 1: photoelectric effect** - Hallwachs' experiments (qualitative) - Lenard's experiment (quantitative) - Einstein's interpretation Epistemological talk: models and representations	group LAB + lecture
	2. Lab activity 2: atomic spectra - Rydberg constant and atomic spectra (quantitative) - Bohr's atomic model Epistemological talk: models and representations	group LAB + lecture
BRIDGE	**3. The most beautiful experiment** - Davisson-Germer experiment (demonstrative) - conceptual / experimental / linguistic challenges of the most beautiful experiment - Levy-Leblond's quanton	demonstrative LAB + 2 lectures
PARS CONSTRUENS	**4. New logic of QP (1): superposition states** - Stern-Gerlach experiments: superposition of quantum states - two-state systems: same formalism for different physical systems	lecture + activities

	Epistemological talk: QP comprehensibility **5. New logic of QP (2): entanglement** - QBIT & entanglement - Quantum Cryptography	lecture + activities
	6. New logic of QP (3): applications - ‘Quantum Manifesto’ - Computing and teleportation	lecture + activities

PARS DESTRUENS

The *pars destruens* consists of two lab activities about the radiation-matter interaction phenomenology: (1) the photoelectric effect and Einstein’s interpretation, and (2) atomic spectra (or Frack-Hertz experiment) and Bohr’s atomic model. From a disciplinary point of view these activities were built upon the main results in physics education research about photoelectric effect, atomic spectra, Franck-Hertz experiment, and Bohr’s model, as described in the following paragraphs. Great part of these considerations were published in (Ravaioli, 2019).

The experimental activities on the photoelectric effect, the atomic spectra and the Franck-Hertz experiment share a common basic structure, comprising the following parts, not necessarily in the same order:

- *alignment of students’ knowledge*: the phenomenon is contextualized through an historical introduction, creating a common framing for the whole group of students. The reflections are built on the line of dc1 and dc2, underlining the role of models in scientific research development and using words focused on interaction processes more than on the models of physical objects.
- *qualitative investigation*: analysis of qualitative the evidence coming from the experiment’s realization. This choice, in coherence with dc1, aims at setting the variables of the physical system model, building up an accessible imaginary upon which the quantitative part of the module can be modeled discussing with students.
- *explicit modeling activities*: following dc1, this part is aimed to address the connection between two kinds of modeling: (i) modeling of the physical system (modeling as a mechanistic explanation, microscopic or macroscopic) and (ii) modeling of the measurement process (modeling as the individuation of salient and measurable macro-variables), as informed by the first one. Collective discussions are carried out to let students imagine themselves a model both of the physical system and of the measurement tools needed to make a quantitative analysis.

- *quantitative investigation*: analysis of the data obtained with the new experimental setup. The results of this part are systematically commented by recalling the imaginary the qualitative observations.
- *meta-reflection*: this part is aims to (i) interpret the quantitative results, (ii) look at the phenomenon from different perspectives (historical, physical, mathematical, experimental and technological), (iii) summarize the whole activity and (iv) open new other questions. In this part, following dc2 and dc3, a final reflection is proposed about the difference between the representations that can be used for presenting and describing the phenomenon, with a special focus on the implicit physical models used to build them.

Lab activity (1): Photoelectric effect

In this section, we describe in detail the design of the lab activity on the photoelectric effect, in order to give an idea of how the design principles have been translated into practice.

Historical/theoretical introduction

The module starts with a review of the models of light and a meta-reflection about what we can infer from observations of geometrical and physical optics phenomena. Following dp2, the discussion is held so that students can reflect on the difference between the questions “what do you think the light is?” and “what do you know about light behavior?”, explicitly highlighting them the role of modeling in physics. After a brief historical introduction, electromagnetic waves are recalled in their principal features, supporting the students to take confidence with the parameters that identify a plane wave propagating – amplitude and frequency – playing with dimmable lights and lasers of different colors; this distinction is known to be not always resolved in students’ conceptions (Steinberg, Oberem and McDermott, 1996). The proportionality of the energy transfer to $|A|^2$ is recalled, pointing out that this is what the classical electromagnetic theory requires; attention is paid to avoid talking about light with high frequencies as ‘more energetic’, as this is a linguistic inference coming from quantum physics. Finally, Hertz’ experiment on the detection of electromagnetic waves, and his posthumous observations about the effects of interposing plates of different materials are briefly introduced to build a historical-like bridge between the electromagnetic theory and the first encountering with the phenomenon next recognized as the photoelectric effect.

Hallwachs’ experiments (qualitative investigation)

Hallwachs’ experiments, performed with a leaf-electroscope, a zinc plate and different light sources (dimmable white lights, and a UV-C neon tube), serve to provide students with an accessible imaginary upon which the quantitative part of the module can be modeled and grounded, in coherence with dp1. The simplicity of the experimental setup allows the discussion to be moved onto an interpretive plane, and serves to recognize the main variables and parameters of the *physical system model* (i.e. the classical electromagnetic model of light), upon which the modeling activity is based; the main aim, in fact, is to individuate the critical points in which the photoelectric effect puts in crisis the electromagnetic theory, problematizing the common belief that the existence of the photoelectric effect *per se* requires the development of a new theory (Budde et al., 2002).

The *qualitative* observations that usually emerge discussing with the students are the following: (1) the variation of white light intensity does not affect the discharge of the electroscope leaves, (2) the variation of the frequency can affect the discharge of the electroscope leaves (it can be suggested that it probably occurs with a threshold frequency), (3) when negatively charged, the electroscope discharge is almost immediate and (4) when positively charged it discharges more slowly, so probably what is *emitted* in the interaction with light are the negative charges (this last observation would be clear if performed in vacuum conditions, where the positively charged leaves would be opening at a greater angle). It should be noted that, generally, students associate the concept of charge directly with electrons by themselves, probably because of their previous studies on electromagnetism and because the corpuscular perspective it's the most common imagery; it is crucial here to point out with them that they are already implicitly taking a microscopic perspective and that, at the time of Hallwachs investigations, the model of electron was still developing. This highlight allows to recognize the relationship between a microscopic model and the individuation of macroscopic measurement variables (next part). All these observations are crucial to understand the photoelectric effect, and will be re-stressed in the quantitative part of the lab; with this experimental setup, the only 'invisible' phenomenon is the dependence of the discharge to the light intensity variation when the frequency is greater than the threshold.

Modeling the physical system and the measurement process

This intermediate part aims at figuring out with students an experimental setup to investigate quantitatively the photoelectric effect encountered with the Hallwachs' experiment; this *modeling activity* reveals to be crucial also for the theoretical understanding of the Lenard experiment results, and explicitly requires the students to model both the *physical system* and the *measurement process*.

The discussion is conducted to let the students think freely about what more could be experimentally investigated, by asking questions like: "what would you like to investigate better about this phenomenon?". This is the part concerning the physical system modeling and students tend to individuate the *number*, the *velocity* and the *direction* of the emitted charges as investigable quantities; when the hypothesis that electrons may have a velocity is introduced, we chose to write down together an equation as general as possible that can hold for the entire lab:

$$E_{light} = W_{bond} + \text{(eventual) } E_{kinetic} \qquad [1]$$

where the symbol "=" stands for *have to be equal to*, so that it is clear that what we want to investigate is the energy of light. We notice that by doing so we are implicitly taking Einstein's perspective about the necessity of a revision of the physical laws describing the process of light energy transfer; historically, this is not the only reasonable perspective (Klassen, 2009), but at this stage we chose to take it with students to let them enter slowly into Einstein's proposal (only in the last part of the activity this point will be restressed).

The design task now turns to the *measurement tools*, by finding out an experimental setup that could let to study and measure how the variation of light intensity and frequency affects the direction, the number and the velocity of the emitted electrons. The crucial point is to associate the number of electrons (in time) with their knowledge of electrical currents, so as to spread the way for introducing circuits; with a circuit, we could measure the current with a simple ammeter, and it would also provide a way to collect the electrons emitted in all the directions. This connection also opens the way to talk about the relationship between the variables of a microscopic model with macroscopic measurable variables. The Lenard experiment is hence introduced.

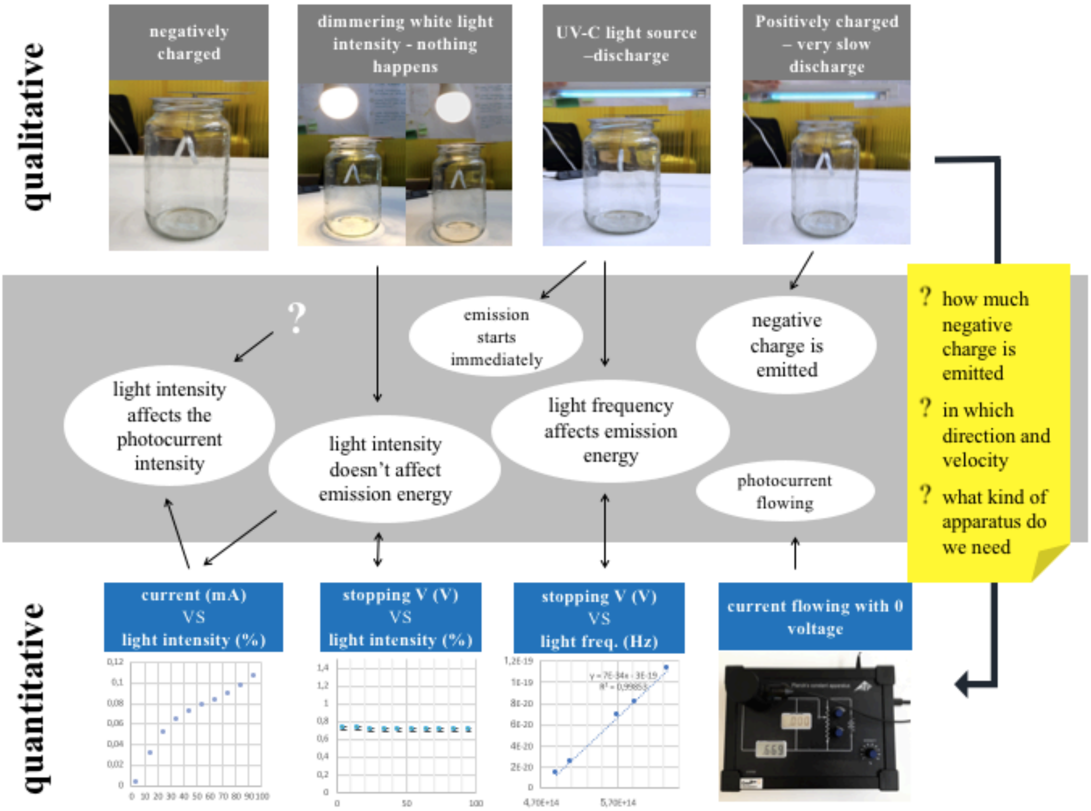


Figure 1.8 schematic image of the conceptual passage from Hallwachs' experiment to Lenard's experiment (Ravaioli, 2019)

Lenard's experiment (quantitative investigation)

The quantitative investigation is structured to follow Hallwachs' experiment's conceptual steps, and the results are systematically commented by recalling its imaginary, as schematically resumed in Fig. 1.8. The experiments are performed with a didactical apparatus built with a Cs photocathode, that physically focuses the attention on the role of the involved variables, light intensity, and control voltage. Firstly, the aims of the experiment are recalled to set down the sense of the procedures: to measure the current (the number of emitted electrons, indirectly) with an ammeter, and the kinetic energy of the emitted electrons, variating the intensity and the frequency of light. By inspecting the kinetic energy, in fact, we have a direct information about the energy of light, thanks to the equation [1]. The procedure to measure the kinetic energy is not trivial to be invented, so it is introduced following some steps: (i) setting the light intensity to be ≠ 0, and V = 0, current is flowing even though we would be tempted to use the Ohm's law V=RI, thus the cathode is emitting electrons due to the photoelectric effect, with a certain kinetic energy, and (ii) setting the potential as the measured current is = 0 (stopping potential) we can deduce the kinetic energy, by the expression

$$E_{kin} = eV_{stop} \qquad [2]$$

Next, the students follow a 2-step tutorial to take the experimental data, investigating the observations accounted with the Hallwachs' experiment: (1) does the light intensity affect the current and the energy of the emitted electrons? This is an important issue, as the literature shows that students acquire the concept that the intensity does not affect the energy, but it comes to be extended to the idea that it doesn't affect anything at all (McKagan et al., 2009). (2) How (i.e. with which type of dependence) does the frequency of light affect the energy of the emitted electrons? Physically speaking, and again accounting implicitly Einstein's point of view, what is Elight a function of? The quantitative results and graphs are schematically resumed in Fig. 3.

Einstein's model and representations

In this part, the experimental results are commented from different perspectives: historical, physical, mathematical, experimental and technological. The comments and the discussion have been built following dc2.

The relationship between the kinetic energy and the frequency is identified with a straight line with a certain inclination and a discussion is held about the physical meaning of the intercept with the frequency-axis (threshold frequency) and of the angular coefficient (Planck's constant). The results are commented by pointing out all their incompatibilities with the electromagnetic theory. Finally, Einstein's 1905 article is introduced by reading the whole introduction to give an expressive idea of his point of view. This is aimed to highlight the character of the two pictures of the world that he describes to introduce his theoretical proposal: the electromagnetic continuous perspective and the corpuscular discrete one. To highlight that the origin of Einstein's proposal is based on a theoretical idea, and not on a mathematical constraint as it was for Planck, the formal analogy that he sets down between the entropy of an electromagnetic radiation in cavity and the entropy of a gas is presented in qualitative terms. The explanation of the photoelectric effect is underlined to be an application of Einstein's broader theoretical derivation, presented at the end of his paper. He claimed that it was possible for one light quantum to be completely absorbed by a single electron. If the electron is near the surface of the metal, it can be emitted, requiring an amount of energy characteristic of the metal itself. The remaining energy, can be observed in the form of a kinetic energy. The energies of the electrons so ejected will have a maximum value, as expressed in the simple relationship:

$$E_{cin}^{max} = h\nu - W_0 \qquad [3]$$

where W_0 is a complessive binding energy characteristic of the metal itself. Finally, we propose a simple synthesis of the account that Einstein's proposal provides for the experimental results of Lenard's experiment (table 1.6).

Table 1.6 Synthesis of Lenard's results, electro-magnetic expectations and Einstein's model accounts

Lenard's experimental results	**Maxwell's electromagnetism expectations**	**Einstein's proposal**
1. occurrence of a threshold frequency ν_0	1. emission for every frequency, proportional to the light intensity	1. given a frequency, if f(ν) < W_{bond}, an electron cannot be emitted
2. the stopping voltage V_0 varies linearly with the light frequency	2. V_0 should vary with the intensity and not with the frequency	2. the energy of light 'quanta' depends only on the frequency, and so does V_0
3. varying the intensity *i*, only the current *I* varies, and not the stopping voltage V_0	3. same as 2	3. the number of electrons is proportional to number of light 'quanta' → $i \propto I$
4. the emission is immediate, despite of the light intensity	4. energy transferred in time → delay in the emission	4. the emission of one electron depends only on one light 'quanton'

Next (along dc3) a discussion is proposed about some representations concerning the photoelectric effect, reasoning with students about the different elements constituting them to individuate the hidden implicit models and discuss their plausibility, potentialities, possible risks, and limitations. For example, the representation of Lenard's experiment in Fig. 1.9 is discussed, to recognize (i) elements of experimental reality, as the ammeter and the generator, (ii) representations of models of physical objects, as the little balls for the electrons and the light spots with waves inside for the photons, and (iii) models of physical processes, as the arrows indicating the emission direction.

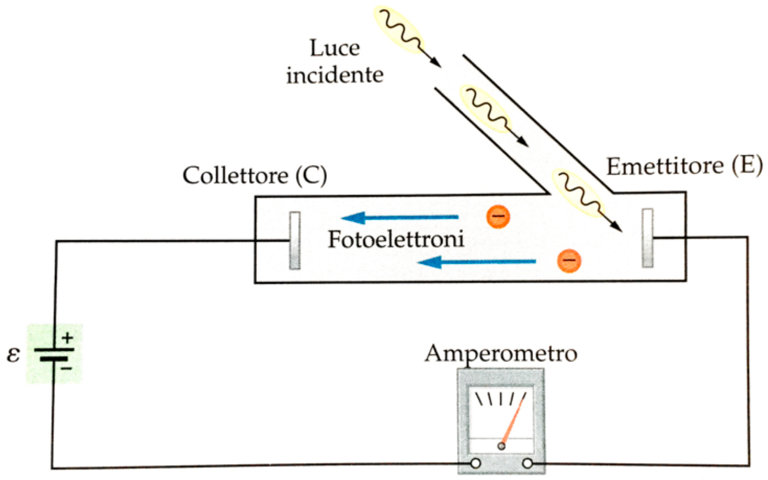


Figure 1.9 schematic figure representing Lenard's experiment,
from a high-school physics textbook (Walker, 2012)

This representation is then discussed to stress some of its implicit assumptions, and at least one important simplification. For instance, the spatial division of photons, represented as oscillating spots, is not a principle upon which the quantum model of photon is built, but only an inference to visualize their characteristic of transferring discrete amounts energy; furthermore, it is neither required by the experimental results obtained in the quantitative part of the module. Even historically, in fact, Einstein's proposal about the nature of light was not the only reasonable way to explain the photoelectric effect, and for a lot of years it has not been fully accepted, also after Millikan's results in 1916 (Klassen, 2009). Moreover, the straight-oriented direction of the emitted electrons and the equal length of the two arrows are an implicit and dangerous simplifications in the representation of the phenomenon; these details hide the assumption that all electrons are emitted with the same energy and in the same direction, which is in contrast with the experimental data obtained plotting the current I in function of the potential V (McKagan et al., 2009).

This discussion, carried out interactively, is designed to improve meta-representational competences (diSessa and Sherin, 2000), turning the cited request of *immediacy* (see dc3) in a more sophisticated request of sense, to individuate the different accounted models, with their specific limitations, and to fix the distinction between model principles and model inferences. It moreover helps also to stress the idea that what we experimentally observe does not immediately imply a certain *object ontology*, as the *photon* for example, but forces us to focus on interactions processes (dp2). This insight is crucial also for the pars construens, where QP formalism comes to challenge classical categories on an ontological and epistemological level. A very similar discussion can be carried out on Bohr's atomic model spatial representations, that can lead students to implicitly consider the atomic spatial structure as a planetary system with electrons moving on circular orbits, although the experimental results and the theoretical model do not require that inference. Finally, some of the actual applications of the photoelectric effect are mentioned.

Goals of Lab activity 1

Summing up, from a conceptual point of view, the goals of the lab on the photoelectric effect are the following, divided by activities

Introduction:

1 - c1 to briefly recall geometrical optics and physical optics models of light
1 - c2 to recall the concepts of amplitude and frequency of an electromagnetic wave
1 - c3 to recall that electromagnetic theory requires the energy transfer to be proportional to $|A|^2$
1 - c4 to recall Hertz' experiment on the detection of electromagnetic waves, and his posthumous observations of the phenomenon after recognized as the photoelectric effect

Hallwachs' experiments:

1 - c5 to observe that the variation of white light intensity does not affect the discharge of the electroscope leaves
1 - c6 to observe that the variation of the frequency can affect the discharge of the electroscope leaves, probably with a threshold frequency
1 - c7 to observe that, when negatively charged, the electroscope discharge is almost immediate
1 - c8 to understand that what is emitted in the interaction with light is negatively charged

The modeling activity:

1 - c9 to write down a simple but general equation ([1]) from the observations with the Hallwachs effect
1 - c10 to individuate the number, the velocity and the direction of the emitted charges as crucial variables of the microscopic model to take in account for more accurate investigations
1 - c11 to recognize that a circuit gives access to measurable macroscopic quantities salient to derive the microscopic ones: the number (current) and the velocity (stopping voltage) of the emitted charges.

Lenard's experiment:

1 - c12 to confirm that the light intensity does not affect the energy of the emitted charges, by measuring and plotting the stopping voltages in function of the light intensity
1 - c13 to confirm that the light frequency affects the energy of the emitted charges, and to find the type of function by plotting the stopping voltages in function of the frequencies of the lights
1 - c14 to observe that, for lights able to emit a photocurrent, the intensity influence the amplitude of the current itself

Einstein's model and representations:

1 - c15 identify the relationship of the stopping voltage (and thus of the kinetic energy) and the frequency of the light with a linear dependence
1 - c16 understand that the intercept of the line with the frequency axis is the threshold frequency below of which the photoelectric effect doesn't occur

1 - c17 understand that the angular coefficient is the equivalent of the Planck's constant, even though the experimental results wouldn't require it to be a constant (it's necessary to use different metals to prove this, as Millikan did in 1916)

1 - c18 introduce Einstein's hypothesis and to show how it goes along with the experimental results

1 - c19 understand that Einstein's proposal was not the only plausible explanation for the photoelectric effect

1 - c20 understand that Einstein's proposal about quantization of light energy does not come directly from the interpretation of Lenard's results, but is instead a theoretical hypothesis coming from a formal analogy (it's not necessary to go in detail, it will suffice to say the it is an analogy between the expressions of the entropy of a gas and the entropy of radiation in a cavity, suggesting the light can behave like a gas, in specific situations)

From an epistemological point of view, instead, the goals of the whole activity on the photoelectric effect are, synthetically:

1 - e1 to recognize that there is not a constraining relationship between theoretical models, experiments and physical phenomena (along multi-dimensionality in DP1). As an example, Einstein's model was not the only plausible account for the photoelectric effect experimental evidences (1 - c19)

1 - e2 to recognize that physical models can be part art of a creative process that brings with itself a specific picture of the world (along multi-perspective in DP1). For instance, Einstein's proposal was born in light of a formal analogy between the entropy of an electromagnetic system and the entropy of gas, and not primarily due to a mathematical constraint. This is clear also in Einstein's introduction to his paper

1 - e3 to recognize that the design of a measurement apparatus is informed by the model of the physical system itself (it is theory-laden)

1 - e4 to recognize the existence of microscopic and macroscopic perspectives on physical phenomena, and to become familiar with the characteristics of both (along multi-perspective in DP1)

1 - e5 to recognize that the relationship between mathematics and physical models is not always univocal (along multi-dimensionality in DP1). As an example, even if Millikan, in 1916, provided experimental evidence for Einstein's relation [3] to be right, he and great part of the scientific community didn't accept its physical model

1 - e6 to begin to shift the focus of the investigation from physical objects to physical processes, or at least to recognize the difference (along dc2)

1 - e7 to recognize that science development is a complex process, with many different actors and contextual influencing factors (along multi-perspective dimension in DP1)

1 - e8 to recognize the potentialities and the implicit assumptions that can underlie a representation (along dc3)

Lab activity (2): Atomic spectra and Bohr's model

In this section, I describe in detail the design of the lab activity on atomic spectra and Bohr's model, highlighting the main design principles and choices. Bohr's model is known to raise very delicate didactical issues in learning quantum physics. In fact, even though it sets a clear-cutting limit with respect to classical physics, its usual imagery is intuitive and easily visible (Müller & Wiesner, 2002) and is thus often tied to classical categories, as material points and deterministic trajectories (Petri & Niedderer, 1998; Kalkanis, Hadzidaki & Stavrou, 2003): the quantum object comes to be associated with classically measurable properties, as position and velocity (Budde, Niedderer, Scott & Leach, 2002). McKagan and colleagues (2008) claimed to be useful to teach it in a tight comparison with previous and following models of the atom; even though in High School teaching is not always possible to compare it with Schröedinger wavefunction model, I find the idea of building criteria to move back and forth from a model to another to be very effective and on the line of our design principles. In this perspective, I designed this activity with moments aimed to (i) investigate experimentally a phenomenology that led to the development of Bohr's model (atomic spectra), (ii) explicitly discuss and deepen the role of modeling in physics as well as the interplay of mathematics and physics, (iii) focus the attention on the energetic picture of Bohr's model and provide another basis other than the intuitive mental picture of the planetary atom, and (iv) reflect about their representations.

The order of the parts is slightly different from the photoelectric effect activity; in this case, the lab begins with a qualitative investigation of light diffraction phenomena, followed by the other parts.

Diffraction of different light sources (qualitative investigation)

The module starts with a qualitative experiment conducted with the whole class. Each student is provided with 3 diffraction gratings (100, 300 and 600 lines/mm) and they are asked to look at different sources of light - incandescent bulbs, LEDs and discharge tubes. The aim of this part is to discuss interactively with them the diffraction phenomenon, highlighting its main features and variables: the mechanism of light diffraction, the change of the spectrum width with the grating step, the occurrence of multiple orders of diffraction, the difference between continuous and discrete spectra, and the occurrence of areas of the spectrum with higher/lower intensity. The emission mechanism of each kind of light source is then briefly explained.

Historical/theoretical introduction

After the qualitative investigation, the history of spectra is presented; Newton prisms, the discovery that the atomic spectrum extends over the region of visible light, and the successive use of spectroscopy technology as an investigative tool during the nineteenth century. The lecturer can introduce the results of Stokes, Ångström, Kirchoff and Bunsen about the relationship between chemical elements and spectra and the reciprocity between emission and absorption spectra. After this overview, the main theoretical expressions to describe the occurrence of atomic spectra lines are outlined: Balmer formula for Hydrogen [4], Rydberg-

Ritz formula for lines out of the visible region [5], and its generalization for atoms different from the Hydrogen [6]:

$$\lambda_n = \lambda_\infty \left(\frac{n^2}{n^2-2^2}\right) \qquad [4]$$

where n is the number of the line, and $\lambda_\infty$is a known wavelength obtained as the limit of the series when n tends to infinite;

$$\frac{1}{\lambda} = R\left[\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right] \qquad [5]$$

$$\frac{1}{\lambda} = R\left[\frac{1}{(n_1+a)^2} - \frac{1}{(n_2+b)^2}\right] \qquad [6]$$

where n_1 and n_2 are integers identifying the spectral series ($n_2 \geq n_1 + 1$), and a and b are constants characteristic for each atom.

It is worthy to note that, at this stage, no assumption on or explicit modeling of the atomic structure has been carried out with the students, and no references to the quantum interpretation of spectra have been done. The exploited terminology, following DP2, is always belonging to the light-matter interaction processes vocabulary. It can be objected that (i) the students still will have previous personal or instructed models about atomic energy levels, and that these should not be avoided, and (ii) that the Rydberg-Ritz formula comes directly from Ritz model of the atom. However, in my opinion, in order to let the students to be able to criticize semi-classical interpretations of Bohr's model based on trajectories and shapes, it is better to not use an atom-based vocabulary and to provide another perspective from the beginning; in fact, as introduced, students' conceptions about the atomic structure more than often bring with themselves a classical and deterministic trajectory-based imagery, that comes to be very difficult to overcome when passing to the quantum model (Petri & Niedderer, 1998; Budde, Niedderer, Scott & Leach, 2002; Müller & Wiesner, 2002; Kalkanis, Hadzidaki & Stavrou, 2003).

Rydberg constant with a spectro-goniometer (quantitative investigation)

The quantitative part of the activity has two main goals: (i) to find an experimental value for the Rydberg constant by measuring the wavelengths of a Hydrogen discharge tube light (Plücker tube), and (ii), given an unknown gas, to recognize it by comparing the measured wavelengths of its spectrum with known spectra.

The students are introduced to the experimental setup, consisting of a discharge lamp (Frederiksen, 230 V AC /50 Hz), several spectral tubes (H_2, Ar, Ne, Br, H_2O) and a spectro-goniometer to measure the wavelengths following the expression

$$d \cdot sin\theta = m\lambda \qquad [7]$$

where d is the grating step, θ the diffraction angle, m the order of diffraction and λ the wavelength to be measured. The light passes through a width-adjustable slit and then trough an optical collimator. The diffraction grating is in the middle of the spectro-goniometer and, by turning the telescope on the goniometer and measuring the precise angle of diffraction, the various light components can be detected.

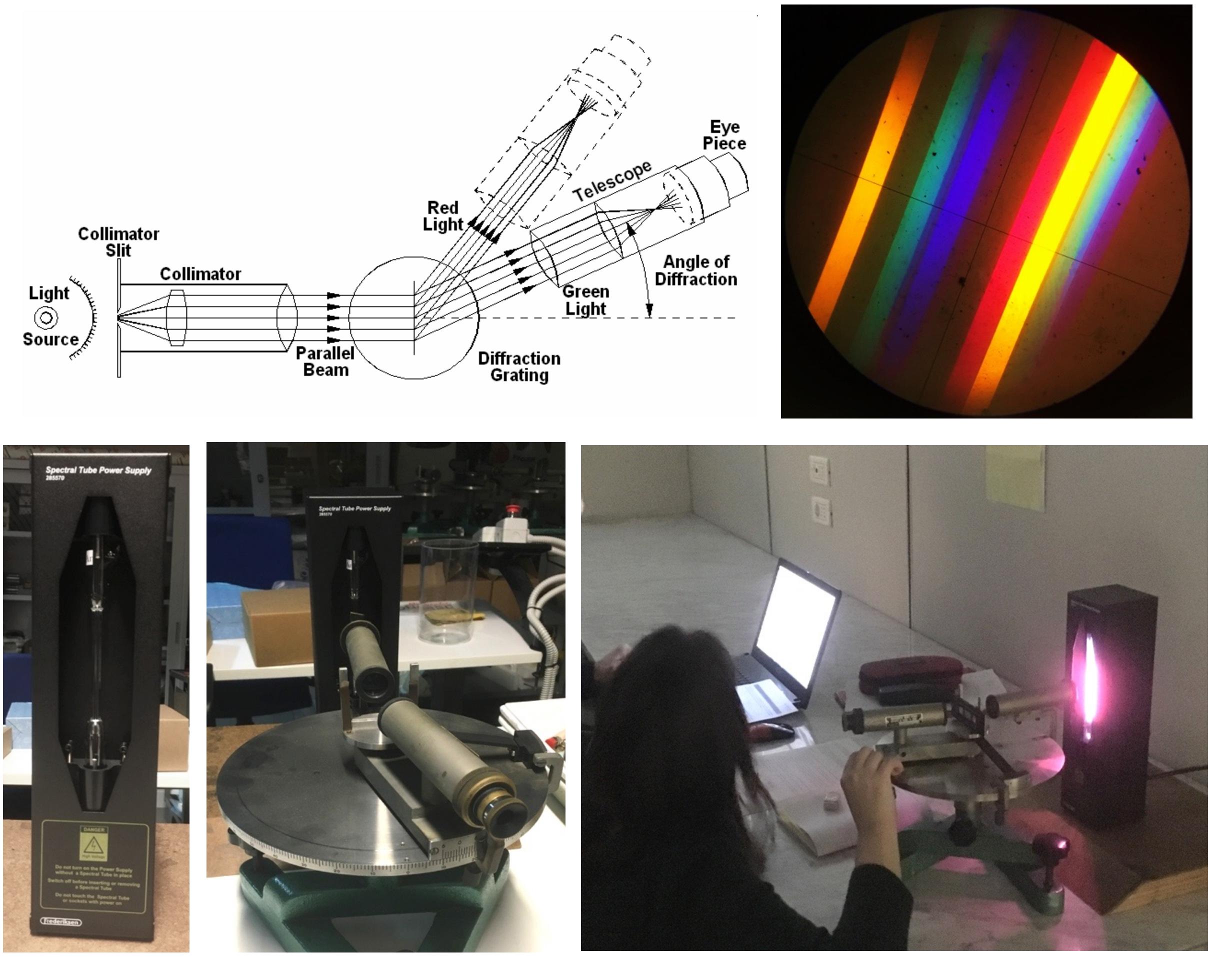
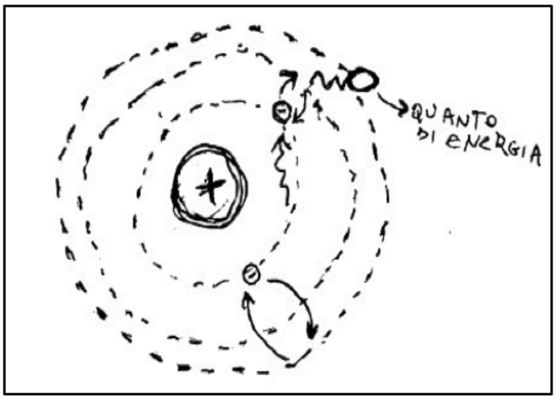


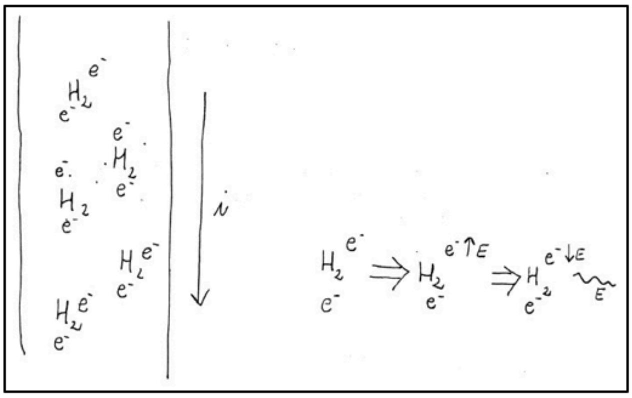
Figure 1.10 On the top, from left to right: scheme of the spectro-goniometer; typical image of the spectrum in the eye piece of the telescope (the width of the lines can be adjusted by narrowing the slit); on the bottom, pictures from the lab.

The first part of the lab consists of a measurement of the Rydberg constant, following Rydberg-Ritz formula [5] for the Hydrogen ($n_1 = 2; a, b = 0$). The students use a H_2 spectral tube. One issue to be tackled is the occurrence of more than the expected 4 Balmer lines ($\alpha, \beta, \gamma, \delta$: red, turquoise, violet, dark violet), due to the excitement of H_2 molecules, and not of H atoms; however, the Balmer lines occur with a higher intensity. The measurement of the wavelengths is repeated both for the 300 lines/mm and 600 lines/mm gratings, comparing the experimental values with the theoretical Rydberg constant value (in the SI system of units $R = 1.097\ x\ 10^{7}\ m^{-1}$). The goal of the second measurement is a comparison of an unknown gas spectrum wavelengths with those of known spectra. This part aims also to give the idea the spectroscopy is a very powerful tool to recognize the chemical composition of elements both from the earth and from astronomical objects.

After the quantitative experimental part, the students are asked to answer the following open question: “Give a personal explanation of the observed phenomenon and provide a graphical representation”. This exercise is aimed to explicit students’ personal models and ontologies, so that in the final discussion of the Bohr’s model they can compare it with their own mental pictures. I didn’t carry out a whole analysis of students’ answers, that goes beyond the scope of this section; however, what stands clear is that, even is such a small class, the knowledge used to make sense of the phenomenon is very diverse and rich. Generally, students’ representations (some of them are reported in figure 1.11) and explanations of this phenomenon highlighted that:

- the students blend their knowledge coming from physics and chemistry;
- the most of them have already been exposed to a quantum-like explanation of atomic dis-excitation in terms of photons, quanta of energy and scattering;
- the most of them think about the atoms in terms of Rutherford’s model (planetary-like system);
- some of them can associate the emitted light with the increase of the vibrational kinetic energy of the atoms (due to the increase of the temperature);
- some of them struggle to interpret the spectrum as a ‘non-spatial’ representation of the light and dark (a spectrum is of course a spatial figure, but it also owns a highly abstract character)





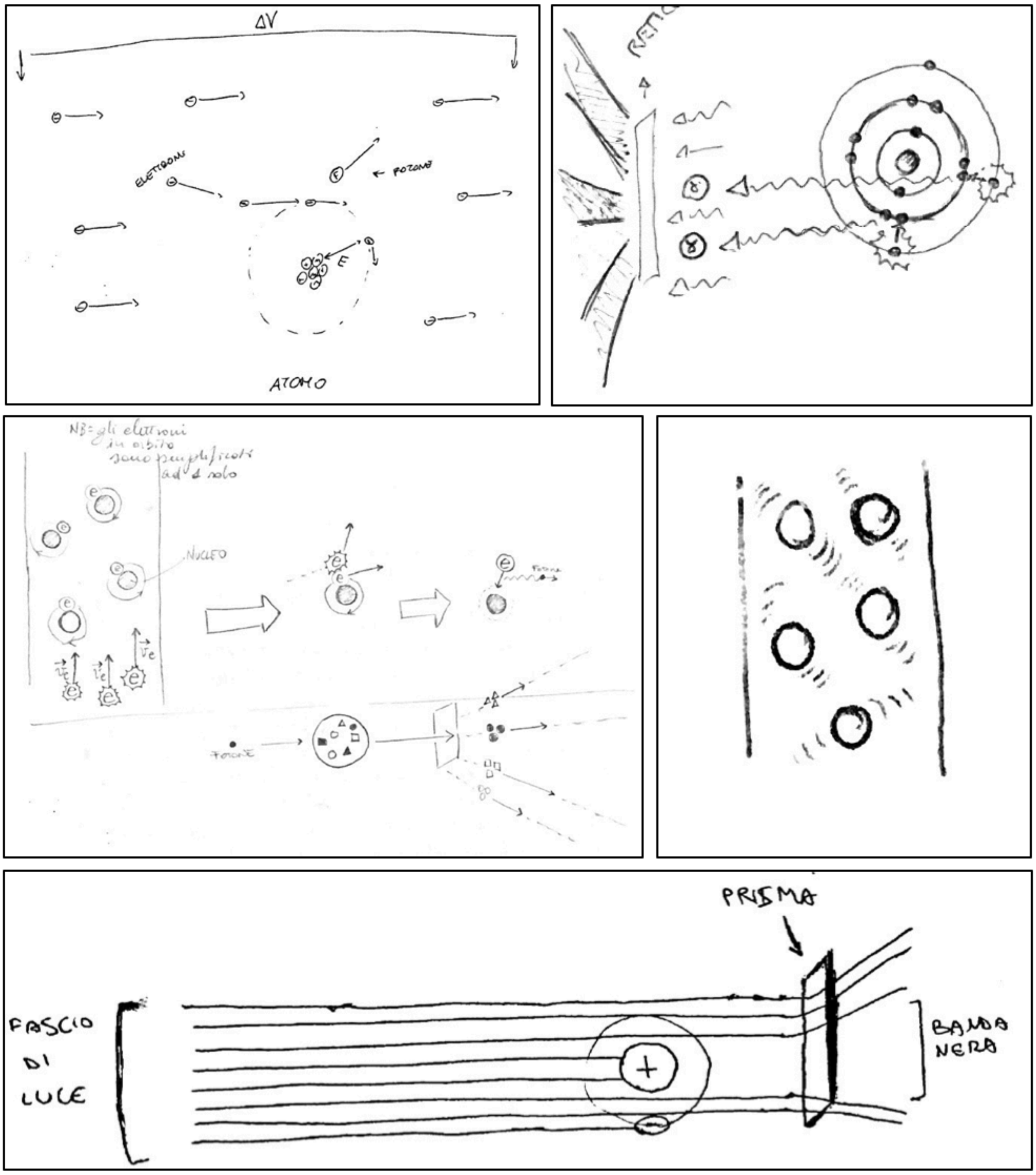


Figure 1.11 Some students' representations of light emission in discharge tubes and spectral decomposition.

Balmer's formula: intertwinement of physics and math

After this activity, the lecturer holds a discussion around Balmer's work to find his expression of the relationship between the spectral lines of the Hydrogen atoms. The aim of this brief insight is to show an example of how mathematical structures and physics modeling can intertwine with each other in the development of a theory, or of a formula, in this case. The main ideas to be communicated in this insight are the followings (it is not necessary here to show the whole process that led Balmer to his expression, but just to comment it on a meta-level)

(i) The analogy between spectra lines and harmonic vibrations had been set both on physical and geometrical considerations. In fact, the idea that spectral lines could include a quantitative relationship similar to the harmonic overtones had been introduced by Mascart in 1863 (Banet, 1966), both based on the evident regularity of the spaces between the lines and on the established way of thinking about light as a vibration of the ether.

(ii) The reason of Balmer's success over all the others contemporary physicists lies on his ability with geometrical reasoning. Balmer's formula is often introduced in textbooks as phenomenological formula based on simple mathematical steps, but the real process that led to is more nuanced. All the attempts of finding a suitable ratio and a fundamental tone had previously failed. Balmer was a math high school teacher and an occasional lecturer at the University in Basel; his background was in mathematics and architecture. He took this idea of the harmonic tones but, as Banet points out, "Balmer's success in the field where physicists with much more training in physics, background information and experimental facilities had failed, was greatly due to the fact that the visible pattern of spectral lines was to him – specialist in projective and descriptive geometry – a meaningful geometrical pattern which just *had to have* a specific numerical or quantitative interpretation".

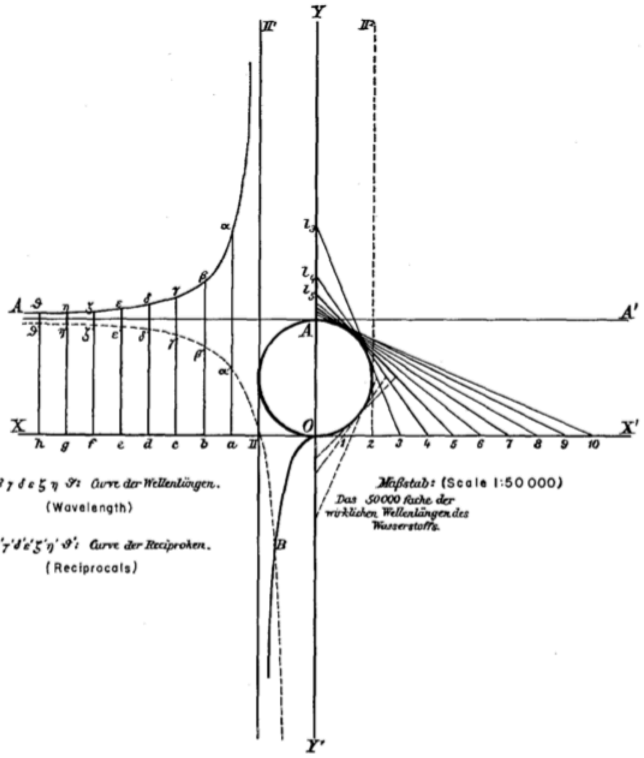


Figure 1.12 Balmer's geometrical construction of hydrogen line series (Balmer, 1897)

Bohr's model and representations

Finally, Bohr's model is introduced. A discussion with the class is conducted to recall the development of the main atomic models, following the classical narrative (see for example Haendler, 1982): ancient Greeks' idea of atom (Democrito), Thomson's model (following the discovery of negative charges: Faraday electrolysis, Crookes tubes, Thomson measurement of e/m ratio and Millikan measurement of e), and Rutherford's planetary model (following Rutherford's gold foil experiment that led to the discovery of a positive nucleus in 1911). The two main limits of the latter are then pointed out: (i) the stability of the atoms, that, unlike planetary systems, can be stressed with terribly violent perturbations but will always go back to their initial configuration, and (ii) the occurrence of discrete spectra lines, unique for each chemical element.

As introduced, in order to do a contrastive comparison with the planetary picture of the atom, which seems to be almost always predominant and yet present from previous classes, we choose to introduce Bohr's model using the energy levels terminology, and to compare it afterwards with the semi-classical picture. To this extent, two exercises are proposed to the whole class, concerning (i) the differential character of energy-levels diagrams, and (ii) the difference of planetary-like and evergy-levels representations. The first is well expressed in the following exercise (figure 1.13, adapted from an Italian High School textbook):

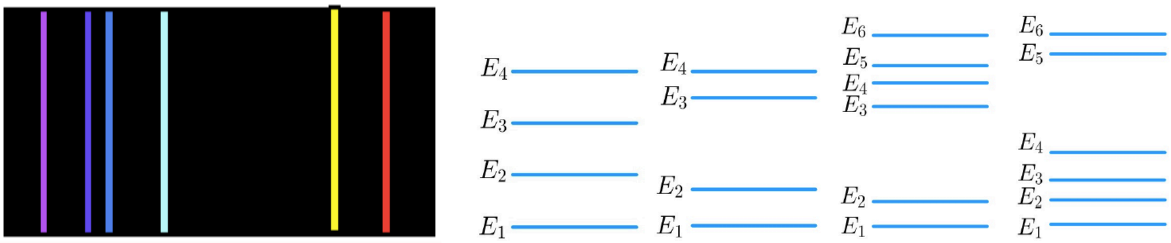


Figure 1.13 Spectrum and energy levels: the exercise asks to identify the series of energy levels corresponding to the spectrum

The students are asked to associate the spectrum with one of the four energetic diagrams on the right. This allows to recognize that the color of the lines depends on the energy gap between two levels, and that there is not a direct association between them (namely, the number of lines is not the number of energy levels). In a second moment, a discussion (epistemological talk, DP2) is led with the students about the risks and the potentialities of both the planetary-like and the energy levels representations, trying to highlight the hidden assumptions upon which either one or the other have been implicitly built. The aim of this discussion is also to highlight how an analogy or a metaphor can be a research tools, inasmuch they own a programmatic role, a productive openness (Petruccioli, 1995): they set a comparison to explore the limits and the differences. Bohr himself did not consider the association of a frequency to the energy of an electron to be a 'real' representation, but it was indeed necessary, because, "to gain a reasonable idea of the stationary states, we don't have, at least for now, other means than classical mechanics" (Bohr, 1913). The idea of revolution frequency set a research program that went on till the discovery of the Zeeman effect, and ended with the foundation of quantum mechanics in 1927 (Petruccioli, 1995).

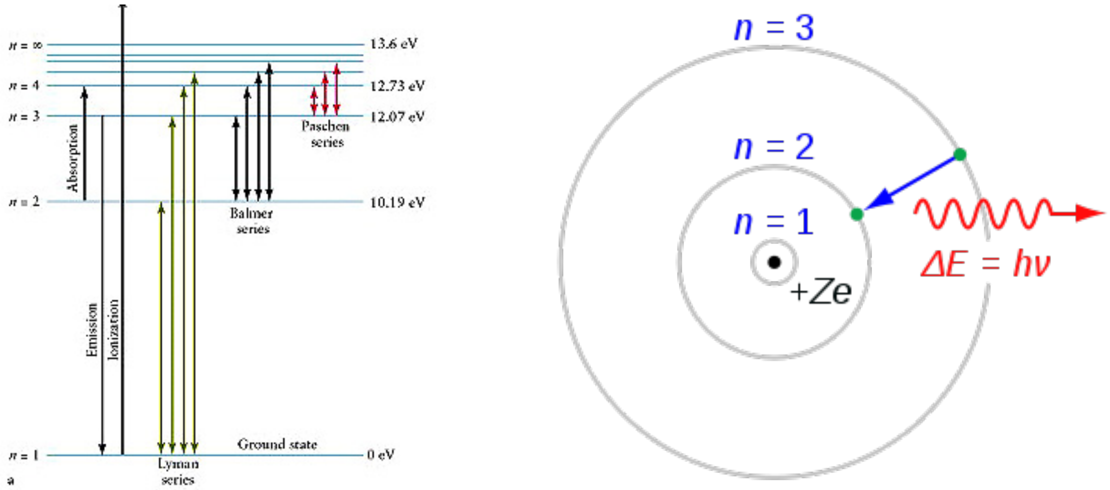


Figure 1.14 Comparison between two representations: a diagram of energetic levels (chimichiamo.org) and a planetary-like Bohr atom (scienceabc.com)

Goals of Lab activity 2

Thus, from a conceptual point of view, this lab aims to (divided in activities):

Diffraction of different light sources (qualitative investigation)

2 - c1 recall the main features of light diffraction and decomposition
2 - c2 observe that some sources emit light with a continuous spectrum and others with a discrete one
2 - c3 observe that the lines of a spectrum have different intensities

Historical introduction

2 - c4 to go through the main historical steps of that brought to the science of atomic spectra
2 - c5 to introduce Balmer's formula [4] and Rydberg-Ritz formula [5]

Rydberg constant experiment

2 - c6 to understand how a discharge lamp works
2 - c7 to understand how a spectro-goniometer works
2 - c8 to appropriate the Rydberg-Ritz formula [5] and the formula [7] for light diffraction in a grating
2 - c9 to enforce the connection of light color with its wavelength
2 - c10 to understand that some lines are due to molecular excitement and other to atomic excitement

Balmer's formula

2 - c11 to show an example of how mathematical structures and physics modeling can intertwine with each other in the development of a theory, or of a formula, in this case.

2 - c12 the analogy between spectra lines and harmonic vibrations had been set both on physical and geometrical considerations.

2 - c13 the reason of Balmer's success over all the others contemporary physicists lies on his ability with geometrical reasoning.

Bohr's model and representations

2 - c14 Bohr's theoretical account in terms of 'stationary states' and discrete atomic 'energy levels'

2 - c15 to understand the difference between spectra and energy levels

The epistemological goals of the whole activity are:

2 - e1 to recognize the different roles that physics and math play in building an explanation of a phenomenon, and to appreciate their complex intertwinement (along multi-dimensionality in DP1). For example, Balmer's work has been guided by the physical analogy of emission lines with harmonics of a fundamental vibration, a model previous to his efforts. But, along this idea, his results have been achieved due to his (apparently disconnected) ability with prospective geometry.

2 - e2 to characterize the role of analogies in scientific research, recognizing (i) their power in setting a research program, as for example happened with the frequency of revolution in the Bohr's atomic model, and (ii) their risks, when they are not critically considered as thinking tools but as direct representations of reality

2 - e3 to begin to shift the focus of the investigation from physical objects to physical processes, or at least to recognize the difference (along dc2)

2 - e4 to recognize the potentialities and the implicit assumptions that can underlie a representation (along dc3)

2 - e5 to recognize that science development is a complex process, with many different actors and contextual influencing factors (along multi-perspective in DP1)

BRIDGE - THE MOST BEAUTIFUL EXPERIMENT OF PHYSICS

At the end of the *pars destruens* the issue is to elaborate a theoretical basis capable of interpreting, in a unified vision, the picture of somewhat disconnected pieces gained from the phenomenology that have undermined the classical paradigm. We chose the presentation of the double-slit experiment (DSE) performed with a single-electron source to pose the problem of the inadequacy of classical categories in a consistent and surprising way, that could resume in one experiment the very 'heart' of quantum physics, as Feynman was used to say (Feynman, Leighton & Sands, 1963). This choice is also due to a contextual factor: the thought experiment imagined at first by Einstein, and then used by Feynman in his quantum physics lectures, was actually realized for the first time in 1976 by three professors of the University of Bologna, Pier Giorgio Merli, Gian Franco Missiroli and Giulio Pozzi (Lulli, 2013; Merli, Missiroli & Pozzi, 1976; Rosa, 2012), and recently performed again at the department of Physics and Astronomy of the university of Bologna with physical slits instead of the electronic bi-prism (Matteucci et al., 2013). The experiment, whose realization had been said by Feynman himself to be impossible due to the technical difficulties, received the recognition of "the most beautiful physics experiment of all time" by the readers of Physics World magazine in a 2002 survey (Crease, 2002); the research group in physics education, in collaboration with the CNR-IMM of Bologna, put a big effort in developing a didactical transposition of its theoretical and experimental challenges, along with the production of a website [1] and a DVD ("Electron interference - the most beautiful experiment"). The presentation, besides describing and explaining the experiment in its mental version, also details its practical realization. The main purposes of this part are: (i) to introduce the superposition principle as an abstract mathematical structure, (ii) to make explicit the 'micro' and 'macro' narratives and models used to describe the same physical phenomena (Davisson-Germer electronic diffraction and the most beautiful experiment), and (iii) to raise the need for a 'new physics'.

Davisson-Germer electronic diffraction (demonstrative)

When Feynman introduces the 'thought' experiment performed with electrons, he imagines to use directly a single-electron source, so as to reduce eventual collective effects and analyze directly the oddity of the pattern obtained with single electrons hitting the screen one by one. I chose instead, at this point, to do a qualitative demonstration of Davisson's and Germer's experiment of electrons diffraction on a crystal, to give a concrete account of the ondulatory properties of a charged beam. This can be a beautiful example of the process of blending different theoretical models to describe an experimental situation.

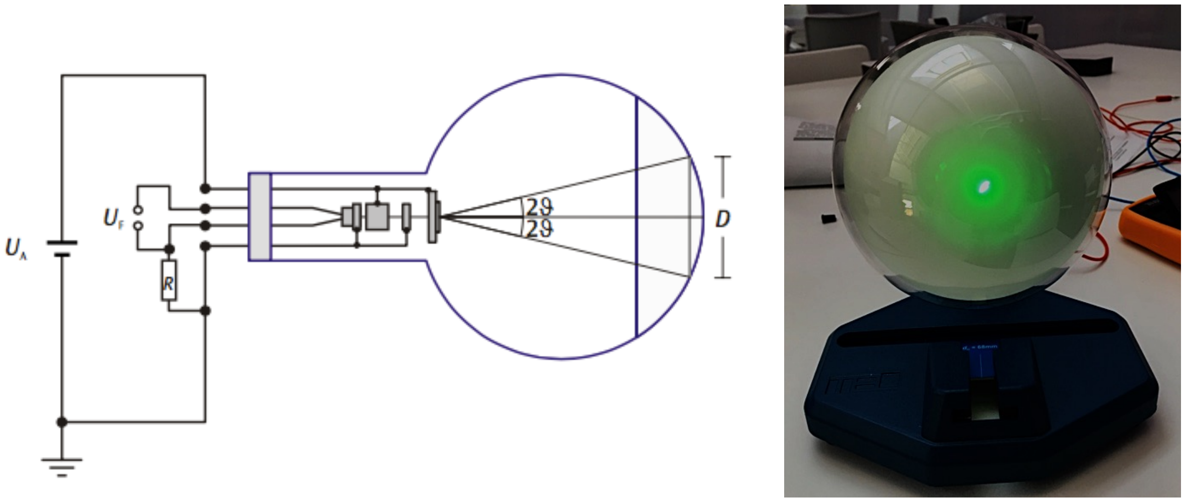


Figure 1.15 Schematic representation of the vacuum tube used for the electronic diffraction experiment (from the manual on the factory website [2]), and picture of the diffraction rings obtained during the lab.

Attention should be paid to the vocabulary used to present the experiment, that can implicitly be grounded in a specific model; I propose to explicit with the students the use of the two different terminologies. The macroscopic wave-based view, sets the narration on the production of a charged beam from a cathode filament in the vacuum tube (the electric charge can be verified by approaching external magnets) diffracting on a sample of polycrystalline graphite. The diffracted beam hits a fluorescent screen, generating bright interference rings (Figure 1.15). This phenomenon is perfectly analogous to Bragg's experiments with X-rays diffraction, and a value for the beam wavelength can be measured from the diffraction rings, using Bragg's condition:

$$2 \cdot d \cdot \sin\theta = n \cdot \lambda \qquad [8]$$

where θ is the Bragg angle (deducible from the diameter of each ring), n the diffraction order, and d the distance between the reticular planes. It is important to note that this value has been measured only using consideration typical of the physical optics domain.

The microscopic particle-based perspective, allows a narration of the phenomenon based on electrons, or negative charges. Emitted from the cathode filament, they are accelerated on the graphite sample of polycrystalline graphite. This perspective allows to think about the circuit tension U_A as an accelerating apparatus, connected to electrons kinetic energy. Seen that the pattern on the screen is typical diffraction pattern, a possible theoretical choice could be the one rejecting the particle description. De Broglie proposal of associating a wavelength to particles can be introduced here. A value for the de Broglie wavelength $\lambda = h/p$ can be estimated. In fact, for electrons with mass m_e and charge e, undergoing through an acceleration tension U_A:

$$\lambda = \frac{h}{\sqrt{2 \cdot m \cdot e \cdot U_A}} \qquad [9]$$

This value can be compared with one obtained from the Bragg condition. The eventual, and easily verified, match of the results obtained starting from the two different models carries with itself deep theoretical implications on the nature of electrons (or more aseptically, on the nature of electricity). The blending of the two models is of course surprising if one takes the microscopic perspective and builds on the ontology of a single electron a sort of extension to include the wavelength as a property of the particle. However, the opposite logic has an eventual escape way: a 'beam' can interfere, by definition, and if one wants to think about it in terms of electrons, the occurrence of the interference can be due to a collective effect.

Goals of Davisson-Germer experiment

Thus, the conceptual goals of this part are:

3 - c1 to understand the characteristics of microscopic and macroscopic models of electronic diffraction
3 - c2 to understand the argumentative logic, that uses measurements gained within different theoretical models to set a physical analogy
3 - c3 to understand that macroscopically, the phenomenon is analogous to Bragg's X rays diffraction, and that Bragg's condition [8] can be used to measure the wavelength
3 - c4 to understand the use of the momentum of an electron and of de Broglie equation to set a theoretical value of the wavelength

The epistemological goals of this demonstration are:

3 - e1 to recognize that different models (in this case microscopic and macroscopic) can account for the same phenomenology (along multi-perspective in DP1)
3 - e2 to recognize that a picture of the world (a model) brings with itself different formal expressions and equations
3 - e3 to recognize the role of measurements and theoretical previsions to build a scientific argument

The most beautiful experiment of physics

Conceptual challenges

At first, the main characteristics of the concept of 'wave' are re-called (they were introduced for the activity on the photoelectric effect, furthermore High School students encounter waves in their fourth school year): amplitude, wavelength, frequency and diffraction/interference patterns. Also, it is made clear the one of 'wave' is an abstract concept describing a diverse range of phenomena, like mechanical, electromagnetic and gravitational waves. In introducing the interference of waves, attention is given to the main transversal thread of this part: the *superposition principle*, that is purposely meant to become an abstract tool to reflect on the nature of classical and quantum interference. More specifically, the main message here is that the occurrence of an interference pattern implies the superposition principle at the ground of its mathematical description. Also, it is pointed out that, in the classical domain, interference implies the presence of (at least) two wave sources.

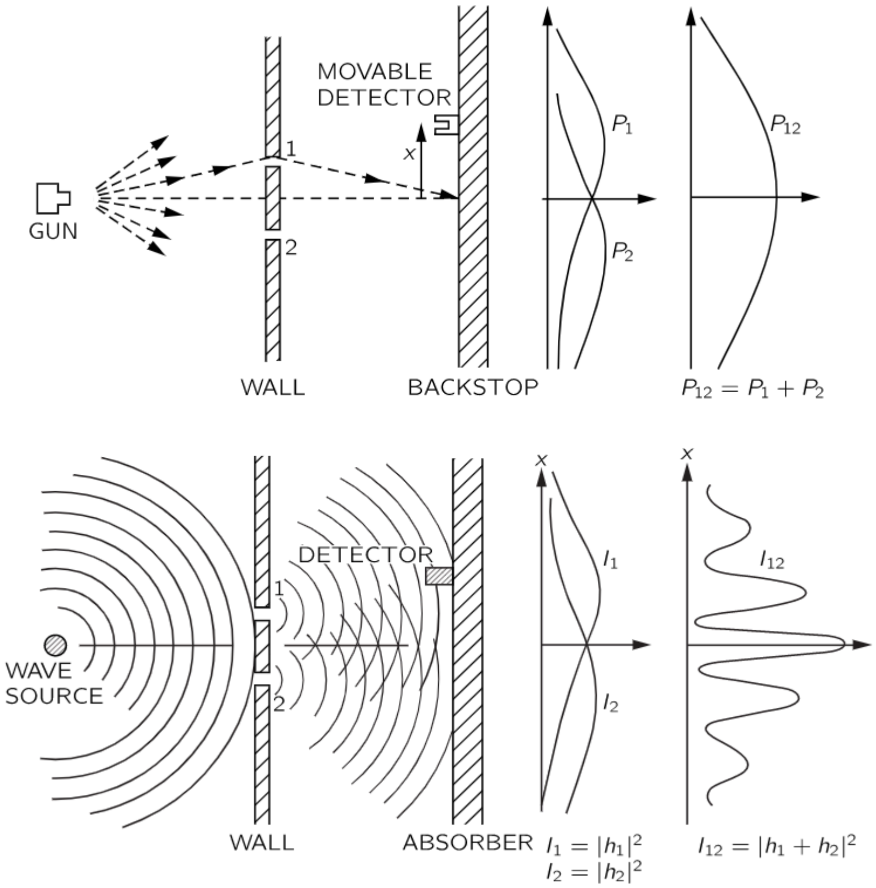


Figure 1.16 DSE with bullets (on the top) and DSE with waves (on the bottom). From Feynman lectures (Feynman, Leighton & Sands, 1963).

The thought experiment part essentially follows the structure of Feynman's presentation (Feynman, Leighton & Sands, 1963): it begins with the two-sided pattern hypothetically obtained sending 'bullets' through the two slits (top of figure 1.16), where the resulting total probability distribution is actually the sum of the probabilities for an electron to pass through each of the two slits 1 and 2 ($P_{12} = P_1 + P_2$), compared with the interference pattern obtained with waves water (bottom of figure 1.16), whose intensity distribution owns a cross-term typical of an interference phenomenon ($I_{12} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta$, where θ is the phase difference). An equivalent situation, Young's experiment (1801) with light, producing interference through the two slits, can be recalled (it is usually known to the students at this stage). Einstein forced this exact logic proposing an experiment performed with single-electrons, so that the interference could not be addressed to a macro-effect, and this is same path that Feynman follows in his lectures. The surprising aspect is that, even though single electrons are sent one after another, an interference pattern is formed on the screen (figure 1.17), spot after spot. This result suggests that neither the wave theory nor the corpuscular one, taken individually, are able to explain all the observed phenomena. The students are therefore brought to the need for a radical change from classical physics categories.

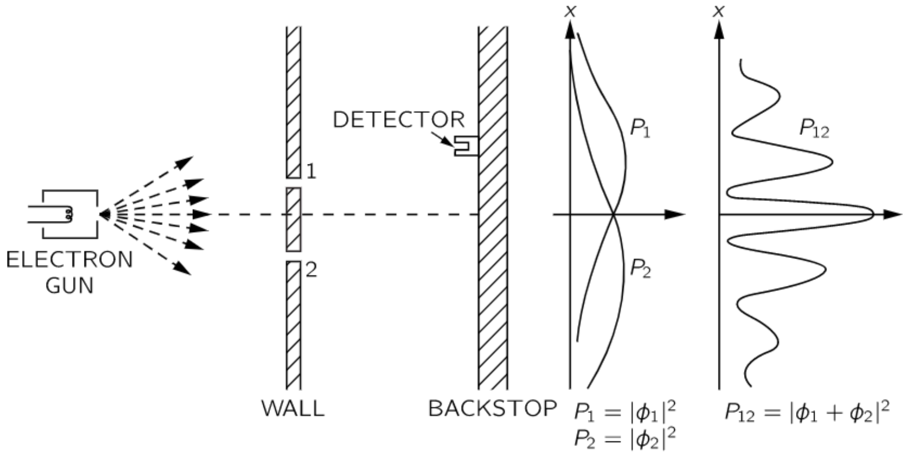


Figure 1.17 DSE with single electrons (Feynman, Leighton & Sands, 1963)

Feynman's argument reaches its best oddity with a "Which way?" (WW) set-up, set to detect the path followed by the electrons. When a detector is set to gather information about the path, the interference pattern disappears. In the actual realization of these experimental set-up, the apparatuses have been designed to not classically disturb the system; as an example of a non-disturbing measurement, the experiment with atoms proposed by Scully, Englert and Walther (1991) can be briefly described.

Linguistic challenges: between entities and systems

It should be noted that the logic of Feynman's argument is purposely (i) on an ontological level, and (ii) entity-based (microscopic). In fact, it aims to bring out the question: "what is an electron?". The occurrence of an interference pattern without having different (classical) wave sources, leads to a logical contradiction, expressed by well-known Dirac's statement: "Each photon [or electron] interferes with itself" (Dirac, 1947). In a previous analysis (Ravaioli, 2016), the implications of the use of this statement had been investigated, leading to the suggestion to present Dirac's sentence as a *linguistic act* that works to provide an apparently effective 'picture' of the quantum object, a synthetic idea, but that it cannot be read literally. This aspect is not trivial and was stressed by Dirac himself when he wrote:

> One may extend the meaning of the word 'picture' [mental model] to include any way of looking at the fundamental laws that makes their self-consistency obvious. (Dirac, 1947)

In the words of Dirac, that linguistic act doesn't provide a real physical account for the phenomenon, but it suggests a "way of looking at the fundamental laws" that can align intuition and reasoning consistency. However, it should be noted that Dirac's statement is strongly characterized by an object-focused narrative, that pushes the reader to look at the phenomenon by following a space-time evolution of the photon (or electron). At this point, besides Dirac's statement, we suggest to explicitly present another linguistic description on the production of the interference pattern: “what produces an interference patter is the superposition of possible states; ‘the *interference of alternatives*’ is the real characteristic of the quantum world” (Englert, 1999). This vocabulary is implicitly suggesting a real *gestalt* change. According to this statement, a reader that looks at the figure 1.18(a) is invited to focus her/his attention to the whole configuration of the system, and to recognize the possible alternatives; interference is said to arise from their superposition. In this case, the narrative is not a space-time story of an object travelling through an apparatus, but a systemic a-temporal story built on the recognition of symmetries and indistinguishable paths (states) (along dc2).

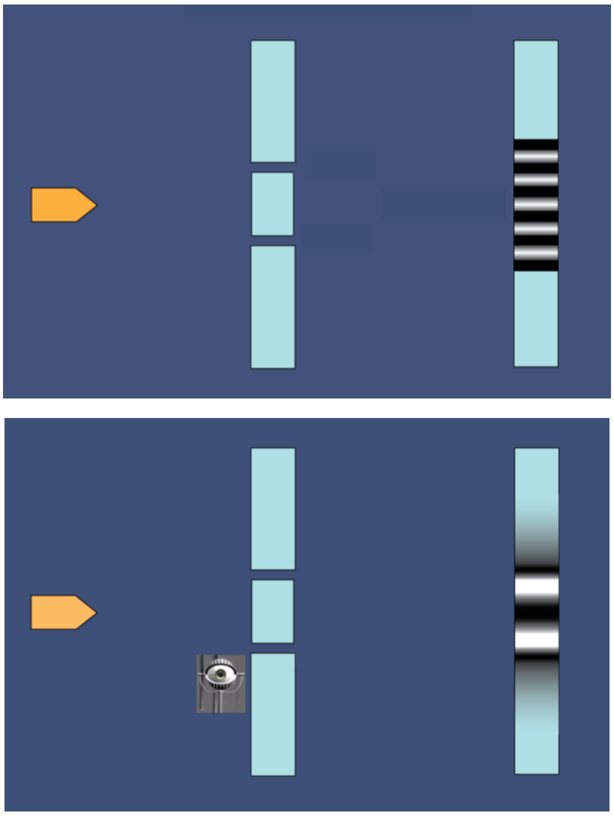


Figure 1.18 (a) Double-slit experiment set-up; (b) which way configuration. From prof. Lulli’s presentation.

The vocabulary of the ‘interference of alternatives’ allows to easily compare the set-up in Figure 11.a with the Which-Way configuration (figure 1.18(b)). Here the interference disappears, but without any assumption on the object; what changes is the set-up itself. In the ‘alternatives’ grammar, gathering information about the path destroys the interference pattern because there is no more superposition of alternatives. Of course, the expression “interference of alternatives” is indeed, again, a linguistic act, like Dirac’s statement. However, its potential lies in (i) the narrative that it suggests, focused on a systemic view and a state-ontology (along dc2), (ii) its generalizability to every quantum system and configuration, including the SG ones, and in (iii) its closeness to the quantum superposition mathematical structure. From Feynman words:

> “regardless of the quantum system, any information – recorded or not – about the

> alternative taken by a quantum process capable of following more than one alternative, destroys the interference between alternatives" (Feynman & Hibbs, 1965)

As a result, the ontology of the superposition state concerns the physical presence of indistinguishable alternatives, and not in wave properties of the object. In this sense, the object can no more be described separately to the measurement apparatus, as the measurement process has a crucial role in determining the state itself; this is a form of entanglement between the system and the object, and it will be recalled with the introduction of the entanglement in the final part of the teaching module. Other research studies already pointed out the importance of this issue, proposing different ways to deal with it; for example, Malgieri, Onorato and De Ambrosis (2017), in their realization of the 'sum over paths approach' with secondary School students, used Feynman's argument to avoid a "vague dualistic descriptions of quantum objects […]. In the sum over paths perspective, in fact, the model only contains point-like quantum objects, which simultaneously explore all possible paths" (Malgieri, Onorato & De Ambrosis, 2017, pp. 5).

A new 'thing': Levy-Leblond's quanton

On the wave-particle duality there is a still open debate and no consensus has been reached yet on its interpretations (Cheong & Song, 2014). What is sure is that the very concept suggests a picture deeply rooted in classical categories that, as Pospiech points out, risk to hinder a proper conceptual shift to quantum physics categories, as "most difficulties in understanding quantum theory arise from trying to develop quantum theory starting from classical concepts and then explaining the differences" (Pospiech, 2000). Besides the choices on the constructive part of the module, that will be explained in what follows, we suggest here the use of metaphors to set the picture of a new ontology, such as those of Lévy-Leblond's cylinder and platypus (Lévy-Leblond & Balibar, 1990; Lévy-Leblond, 2003). In the first metaphor, the quantum object is compared to a cylinder. Depending on how one looks at it, the cylinder can 'collapse' into a rectangle or into a circle, but of course the cylinder is neither a rectangle, nor a circle; and it's not the union of the two. The cylinder possesses properties that neither the rectangle nor the circle possess, just as a quantum object possesses properties which neither the wave nor the corpuscle possess.

The second metaphor is structurally similar to the first one and is based on the true story of the zoological nomenclature of the platypus, named at first duck-mole for the presence both of a duck bill and of a mole tail. A brief extract from the author article is presented:

> Neither Waves, Nor Particles, but Quantons!
>
> That the true nature of quantum objects has long been misunderstood is proved by their still all too common description in terms of an alleged "wave-particle duality". It must be remarked first of all that this formulation is at best ambiguous. For it may be understood as meaning either that a quantum object is at once a wave and a particle, or that it is sometimes a wave and sometimes a particle. Neither one of these interpretations in fact make sense.

> "Wave" and "particle" are not things but concepts, and incompatible ones; as such, they definitely cannot characterize the same entity. While it is true that quantum objects may in some cases look like waves, and in other cases like particles, it is truer still that in most situations, particularly the ones explored by the elaborate modern experiments, they resemble neither one nor the other. The situation here is reminiscent of that encountered by the first explorers of Australia, when they discovered strange animals dwell- ing in brooks. Viewed from the forefront, they exhibited a duckbill and webbed feet, while, seen from behind, they showed a furry body and tail. They were then dubbed "duckmoles". It was later discovered that this "duck-mole duality" was of limited validity, and that the zoological specificity of these beasts deserved a proper naming, which was chosen as "platypus". Much in the same way, we thus can (and must) safely assert that quantum objects are neither waves, nor particles, but are to be described by a specific and novel concept, which certainly deserve a name of its own. Bunge's proposal to call them "quantons", building on the common terminology (electrons, photons, nucleons, etc.) and extending it to a common categorization, is most to the point, and it is to be hoped that this terminology gradually gains ground. (Lévy-Leblond, 2003)

The quantum object is neither a wave nor a particle and therefore deserves a name of its own: 'quanton'. Through these aspects, it is possible to highlight the need to overcome some ideas based on the classical experience and to build a different logic, capable of grasping the much deeper relationships existing between the elements of physical reality. I want to note here that the choice of speaking in terms of quantons presents a drawback: it sets again an object-based narrative, that we want to overcome, However, this discussion is only an introduction to the pars construens, where we try to set down definitively a systemic narrative. Thus, this linguistic metaphor serves mainly here to give the idea that there a paradigm shift must occur.

Goals of the 'most beautiful experiment' part

Conceptually, the presentation of the most beautiful experiment aims to:

Conceptual challenge:

3 - c5 to understand that the concept of 'wave' is an abstract concept describing a diverse range of phenomena (mechanical, electromagnetic, gravitational etc.)
3 - c6 to understand that the occurrence of an interference pattern implies the superposition principle at the ground of its mathematical description
3 - c7 to understand that with 'bullets' the final probability distribution would be the sum $P_{12} = P_1 + P_2$
3 - c8 to understand that with waves the final distribution would follow an expression for the intensity $I_{12} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\theta$, and that there is a cross term that in the other case is absent

Linguistic challenge:

3 - c9 to understand that Feynman's narration is entity-based

3 - c10 to understand that interference is not between two classical waves, but it can be described as an interference between alternatives
3 - c11 to understand what is a which-way configuration, and in what sense the measurement is not ‘disturbing’ the physical system
3 - c12 to ‘observe’ that interference concerns the presence of indistinguishable alternatives, and to understand that a which-way configuration changes the system alternatives

The epistemological goals related to the lecture on the most beautiful experiment are:

3 - e4 to recognize that there can be different models of the same phenomenon
3 - e5 to recognize the difference of a entity-based narrative and a systemic narrative (along dc2)
3 - e6 to recognize that interference is an abstract concept, and it does not entail only classical waves
3 - e7 to recognize that naming an object presupposes a conceptualization of its properties
3 - e8 to recognize, metaphorically, that classical physics categories are not adequate for the description of QP phenomena

PARS CONSTRUENS

New logic of QP (1): quantum state and superposition

Stern-Gerlach experiments: quantum superposition state

The formal systematization of the concepts already introduced with the double-slit experiment follows the approach developed in collaboration with prof. Elisa Ercolessi of the University of Bologna. The first part resembles a spin-first approach, following mainly the logic developed by McIntyre and colleagues (McIntyre, Manogue and Tate, 2013). After the introductive part about the physical properties of Stern Gerlach magnets, where it is pointed out that a S-G magnet works exactly as a spin 'analyzer' (the atomic beam splits up in two separated spots), a phenomenological argument is built through an analysis of different experimental configurations with subsequent S-G magnets. Going through the experiment with only one S-G apparatus (either directed along the x-axis or the z-axis) and with single atoms[2], three annotations are done:

- each atom arrives either in the upper or in the lower spot;
- each atom arrives only in one spot;
- if repeated several times, half atoms will get the upper spot and half in lower. Thus, they have 50% of probability to be revealed with spin-up or spin-down.

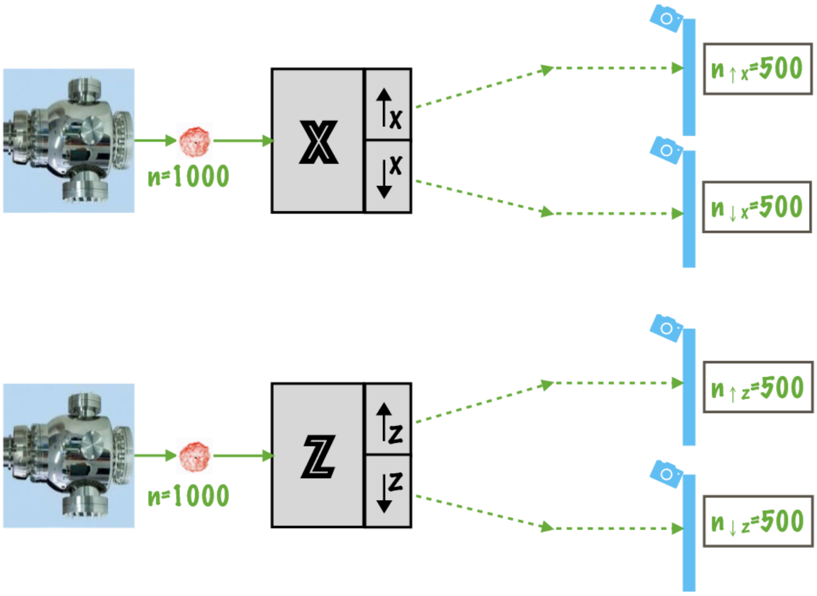


Figure 1.19 Stern-Gerlach magnets along the X and Z directions (from prof. Ercolessi's presentation)

Here the concept of state is introduced to describe atoms' spin in the moment of the measurement: $|+\rangle$ for spin-up state, and $|-\rangle$ for spin-down. It is important to note that, given

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[2] The following images are taken from the presentation developed by prof. E. Ercolessi

this situation, one could be prone to think that the atoms own a precise value of spin (spin-up or spin-down), and that the apparatus just reveals it as it was before passing through the magnet. Afterward, some more complex configurations with SG magnets are proposed and analyzed as performed in several experiments, getting the statistical results reported in Figures 1.20 and 1.21.

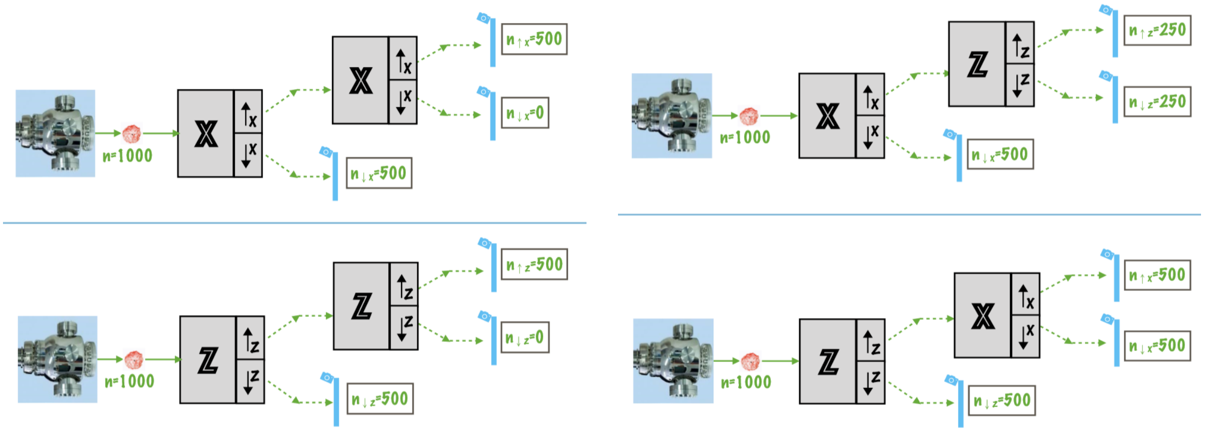

Figure 1.20 Stern-Gerlach magnets in sequence (from prof. Ercolessi's presentation)

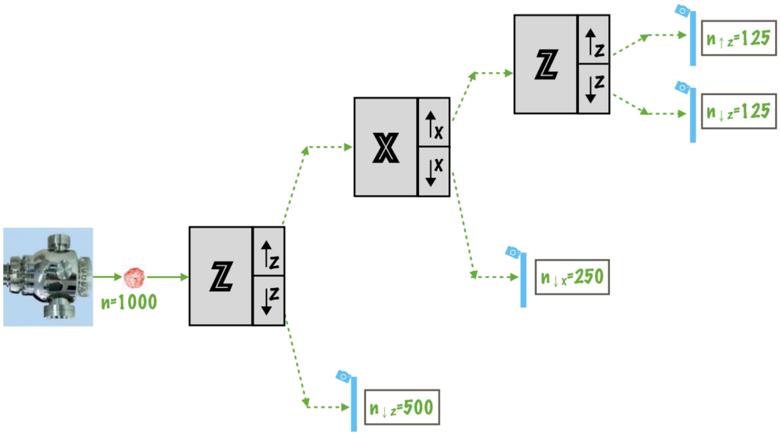

Figure 1.21 Stern-Gerlach magnets in sequence (from prof. Ercolessi's presentation)

After the configuration reported in figure 1.21, a final configuration (figure 1.22) is given and it often happens that students read it along a classical 'path logic' and foresee a result of 50% and 50%, different from what experimentally observed.

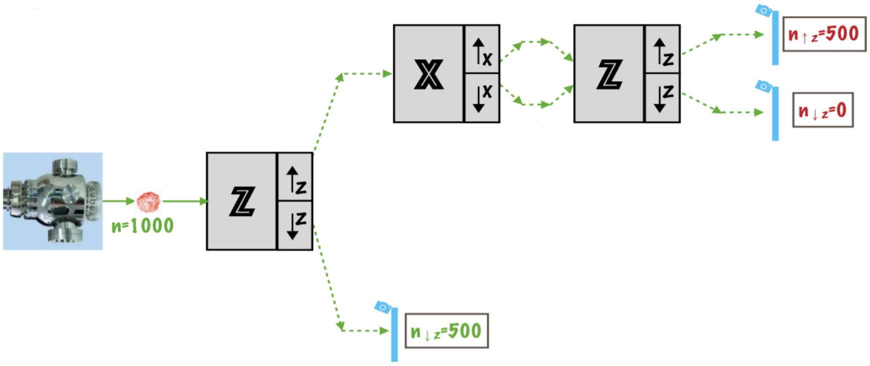

Figure 1.22 Stern-Gerlach magnets in sequence (from prof. Ercolessi's presentation)

The surprising experimental outcomes impose to re-think about our implicit conception of property, and a reflection on this point is proposed, giving the question: "according to classical physics[3], what value of spin does the atom own between the two Z magnets?"

- both $|+\rangle$ and $|-\rangle$: this is not unacceptable option, as all the experiments confirm that atoms do not split up;
- neither $|+\rangle$ nor $|-\rangle$: unacceptable, as all the experiments confirm tha atoms do not vanish;
- either $|+\rangle$ or $|-\rangle$: again, not acceptable, as the experiment in figure 1.19 gives different outcomes.

This logical 'empasse' is challenged by pointing out that we are making an implicit assumption: revealing an atom with spin-up or spin-down means that, before the measurement, the atom did own that specific value of spin. Dismissing this assumption, the *superposition state* is introduced, as describing a linear combination of the classically admitted alternatives, whose coefficients are the corresponding probability amplitudes; it is strongly underlined that this description does not belong to anyone of the previous logical option for the atom's spin. According to the new formalism, the probability amplitude follows three main rules:

- in succession, the probability amplitudes multiply;
- the final amplitude is the sum of the possible alternatives;
- the final probability is the square of the final amplitude.

The procedure, different from the classical case, shows how the concept of probability amplitude leads to give up the image of the path and allows to interpret the results of the measurement. Some notions about vectors and linear algebra are then briefly introduced. The superposition state is expressed as the state vector $|+\rangle_z$:

$$|+\rangle_z = cos\frac{\theta}{2}|+\rangle_x + sin\frac{\theta}{2}|-\rangle_x \qquad [10]$$

---

[3] Indeed, spin is not a classical property, but here the intention was to stress the implicit classical-like logic we are used to think with

where θ is the angle between the directions of the magnetic field of the two magnets (for the case presented, X and Z). In this particular case, being $\frac{\theta}{2} = 45°$, we obtain:

$$|+\rangle_z = \frac{1}{\sqrt{2}}|+\rangle_x + \frac{1}{\sqrt{2}}|-\rangle_x \qquad [11]$$

$$|-\rangle_z = \frac{1}{\sqrt{2}}|+\rangle_x - \frac{1}{\sqrt{2}}|-\rangle_x \qquad [12]$$

Assuming this new definition of the state, together with the rules for calculating probability amplitudes (multiply in sequence - add for the final amplitude - square to get probability), the outcomes are justified and predicted. Finally, to allow the description of spin states directed in any space direction, the more general complex linear combination is introduced:

$$|\theta, \phi\rangle_z = \ cos\frac{\theta}{2}|+\rangle_z + e^{i\phi} sin\frac{\theta}{2}|-\rangle_z \qquad [13]$$

After this lecture, some exercises with SG magnets are proposed to the students, to let them become familiar with the new logic and bring out calculations also with the magnets rotated at different angles.

Two state systems: same formalism for different physical systems

As shown from the results of a previous analysis (Ravaioli, 2016), students often end up with considering the formalisms of introductory QP course as a mere mathematical tool for fitting the counts, but it does not become a reliable explanation of the physical reality. These considerations led us to develop a reflection on the relationship of math and physics, based on the formal equivalence of the description of three systems: the double-slit experiment with electrons (DS), the Stern-Gerlach magnets (SG), and the Mach-Zender optical interferometer (MZ). This reflection is focused on states and systems (instead of on objects and their properties) and it is supposed to show how the mathematical structure built on the SG apparatus is effective to interpret other phenomenology.

The strategy is to reframe the SG apparatus as an *interferometer*. This passage is crucial, since it explicitly requires to overcome the idea that behind the interference there is a superposition of waves. In fact, the generic mathematical structure of the state vector $|S\rangle$,

$$|S\rangle = c_1|A_1\rangle + c_2|A_2\rangle \qquad [14]$$

where $|A_1\rangle$ and $|A_2\rangle$ are the vector states for each alternative of the system and the coefficients c_1 and c_2 are the corresponding probability amplitudes, is indeed common to all two-state systems, or, to say, to all two-way interferometers, such as DS (where the amplitudes correspond to two slits), SG (A_1 and A_2 corresponding to spin-up and spin-down), MZ (A_1 and A_2 corresponding to path-1 and path-2) or others, like 'bi-prism' interferometers, or Ramsey-Bordé interferometers for two-level atoms (Berman, 1997; Miffre et al., 2008). For symmetrical systems, the state comes to be expressed, equivalently as:

$$|S\rangle = \frac{1}{\sqrt{2}}(|A_1\rangle + |A_2\rangle) \qquad [15]$$

The MZ apparatus is thus presented to the students. The apparatus is composed of a low-energy photon source, two beam splitters (semi-reflective surfaces, going through which the photon has a 50% probability of being reflected or transmitted), two mirrors, and two single-photon detectors, arranged as in figure 1.23.

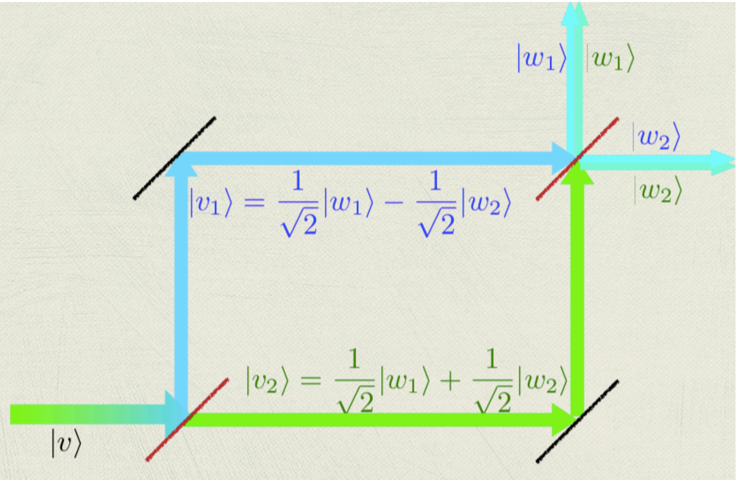

Figure 1.23 Mach-Zender experiment scheme (from prof. Ercolessi's presentation)

In the first configuration only one beam splitter is adopted, and the results (again obtained statistically) do fit classical expectations (50% e 50%); therefore, it can be perfectly explained on the basis of classical optics. However, when another beam splitter is interposed, the outcome is again not predictable with classical probabilities; it can be described only through the introduction of the superposition quantum state. A simulation from University of St. Andrews was used to let the students play with the Mach-Zender interferometer [3].

Being the mathematical structure completely equivalent, the only difference is in the physical interpretation of the state and the amplitudes; despite describing a photon or an electron, and despite being the observables the polarization degrees or the position, the formalism is exactly the same. This is the great abstractive power of the quantum formalism, gathering which, in our opinion, students could shift the focus to the physical system as a whole (object + apparatus).

More generally, as introduced to students after S-G experiments, the most general expression for the state is described in the Bloch sphere, in dependence of two angles

$$\left|S(\theta, \phi)\right\rangle = cos\frac{\theta}{2}\left|A_1\right\rangle + e^{i\phi} sin\frac{\theta}{2}\left|A_2\right\rangle \qquad [16]$$

that for symmetric interferometers comes to be in dependence of the only *interferometric phase difference* ϕ[4],

$$|S(\phi)\rangle = \frac{1}{\sqrt{2}}(|A_1\rangle + |A_2\rangle e^{i\phi}) \qquad [17]$$

The phase difference ϕ assumes different physical meanings depending on the experimental set-up: for the DS, ϕ is determined by the site where the electron hits the screen, for MZ by the difference of the optical path's lengths, and for SG set-up by the spatial orientation of the second magnet (actually, a generic orientation in the space need also the angle θ to be described, that for DS and MZ is determined by the position of electrons' source beside the slits, and by the possible variable reflectivity of the beam splitter, respectively). For the state S, the interference pattern emerges clearly in the probability P of finding the superposition state $S(\phi)$[5],

$$P(S(\phi)) = |\langle S(\phi)|S\rangle|^2 = \frac{1}{2}(1 + \cos\phi) \qquad [18]$$

where the interference fringes are clearly described by the dependence to $\cos(\phi)$. Thus, rereading the results of the basic SG and MZ set-ups (which were only of 100% or 0%), we can say that these are the maxima and the minima revealed both with a phase difference $\phi = 0°$ and $\phi = 90°$. Therefore, changing the phase difference (i.e. moving along the DS back screen, rotating the SG magnet and changing the paths' lengths, or equivalently playing with a phase shifter, in MZ) means moving along the interference pattern.

Goals of part 4

The conceptual goals of this introduction to the new logic of QP are:

Stern-Gerlach experiments

4 - c1. to understand how a S-G magnet works (each atom arrives either in the upper or in the lower spot; each atom arrives only in one spot; if repeated several times, half atoms will get the upper spot and half in lower)

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[4] The phase factor is multiplied only to one of the basis kets because of taking one the two complex amplitudes as positive and real, which we are free to choose

4 - c2. to understand the logic of different configurations of multiple S-G magnets in sequence and to get the logical 'empasse' of the mixed configuration

4 - c3. to understand that to overcome the 'empasse' an implicit assumption must be dismissed: revealing an atom spin means that, before the measurement, the atom did own that specific value of spin

4 - c4. to understand the concepts of superposition principle, probability, and amplitude, as well as the rules for calculating them

4 - c5. to become familiar with the Dirac notation for expressing the states, and to understand the role of the angles

Two-state systems

4 - c6. to get the logic of the Mach-Zender experiment, and to describe it in with quantum states

4 - c7. to understand the analogy of the MZ interferometer with the SG apparatus, and with the DS experiment, interpreting all of them as quantum interferometers

4 - c8. to understand that the parameters of the states can assume different physical meanings according to the set-up they are describing (for instance the phase difference ϕ, for the DS is determined by the site where the electron hits the screen, for MZ by the difference of the optical path's lengths, and for SG set-up by the spatial orientation of the second magnet)

The epistemological goals are:

4 - e1. to recognize the different roles (epistemic and ontological) that probability plays in classical and quantum domains

4 - e2. to recognize that there can be different models of a physical phenomenon, such as entity-based or systemic, and to recognize that the quantum state is a systemic one

4 - e3. to recognize that a formalism can describe more equivalent physical systems. For example, the quantum state provides a description of every two-way system, being it an optical interferometer or a Stern-Gerlach magnet

3 - e9 to recognize that interference is an abstract concept, and it does not entail only classical waves

New logic of QP (2): entanglement

Class discussion on QP comprehensibility

As a preparation to the introduction of the entanglement, we decided to foresee a class discussion. Along the design principle DP2, this moment is meant to be an 'epistemological talk', where students can express and discuss their own views and perceptions of QP and of their learning of QP so far. Some parts of this discussion will be analyzed in Part III of this dissertation. The focus of this epistemological talk is the 'comprehensibility' of QP. Typical question posed to the students are "Do you think that Quantum Mechanics explains phenomena?", "Is QP comprehensible to you?", or "Try to think when you have the feeling 'oh, this thing has finally been explained to me!". By choice, most of the questions were pointing at bringing out students own 'voices', by posing open-ended questions; in fact, we tried to lead the discussion so to make it as much as possible *collective, reciprocal, supportive*, *cumulative* and *purposeful* (Alexander, 2006). We are prone to think that this kind of discussions allowed the students to better "articulate conceptual difficulties, deepen their understanding through exchange of views, and formulate new questions" (Bungum, Bøe & Henriksen (2018)), and part of these results will be discussed in the qualitative analysis of some students' discourses (Part III).

QUBITs & entanglement

The concept of entanglement is crucial to consolidate the characterization of QP formalism, as it is an 'only' quantum property and it challenges classical paradigms at their very core; furthermore, it opens the way to most of the applications of QP, such as quantum cryptography and quantum teleportation.

In the teaching module, entanglement is introduced with the notion of qubit. This is both a practical and a prospective choice; in fact, the qubit is both the simplest computational element representing the core of QP, and at the same time it's building block of a whole research field of quantum computing. Starting from the classical bit, a binary element that can represent every system with a '0 or 1' alternative (whatever the physical realization of the system is), the qubit is at first simply introduced as the quantum analogous of this basic computational element, entailing the superposition principle of the states O and 1. Thus, a generic state of the qubit is expressed in the familiar way as:

$$|Q\rangle = a|0\rangle + b|1\rangle \qquad [19]$$

The qubit can potentially encode more information that the classical bit, and by means of the Bloch's sphere it is pointed out that a qubit can represent any vector in it.

As the scope of this part is to introduce the entanglement, it is not necessary to go more in detail with the physical realization of qubits or with quantum information detail. It seems to be possible to find ways to introduce High School students to quantum information issues as quantum gates, quantum algorithms and computational complexity; some attempts have been already carried out with encouraging results (see for example Satanassi, 2018). However, in this context it suffices to introduce the qubit as an abstract mathematical entity with the typical properties of a quantum superposition state. To introduce the entanglement, a system of 2 qubits must be considered. Classically, the possible combinations would be four: $|00\rangle, |01\rangle, |10\rangle, |11\rangle$. Thus, the generic state of a two qubits system is the superposition:

$$|Q_1Q_2\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle \qquad [20]$$

where $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$. The way we introduce entanglement is via the following argument. Supposing to have two particles created in a state $|Q_1Q_2\rangle$), and supposing to send each of these particles to two different persons (at whatever distance), the usual Alice and Bob. They can do measurements in their respective labs to determine with which probability their qubit is in a state $|0\rangle$ or $|1\rangle$. The eventual entanglement will be revealed in the correlation of the measurements of Alice and Bob. Her are two examples of possibilities. In the case that:

- $a = b = \frac{1}{\sqrt{2}}, \quad c = d = 0$

QBIT state before measurement	QUBIT state after measurement	result of the measurement on Q_1	result of the measurement on Q_2
$\|Q_1Q_2\rangle = a\|00\rangle + b\|01\rangle$	$\|00\rangle$ with probability $\|a\|^2 = {}^1/_2$ $\|01\rangle$ with probability $\|b\|^2 = {}^1/_2$	0 0	0 1

In this case, there is no correlation between the result of the measurement on Q_1 and the result of the measurement on Q_2, as Q_2 will always result to be in states $|0\rangle$ or $|1\rangle$ with equal probability. Supposing instead to be in the case where:

- $a = d = \frac{1}{\sqrt{2}}, \quad c = b = 0$

QBIT state before measurement	QUBIT state after measurement	result of the measurement on Q_1	result of the measurement on Q_2
$\|Q_1Q_2\rangle = a\|00\rangle + d\|11\rangle$	$\|00\rangle$ with probability $\|a\|^2 = {}^1/_2$ $\|11\rangle$ with probability $\|d\|^2 = {}^1/_2$	0 1	0 1

In this case, the measurements on the two qubits are indeed correlated, even if not predictable; two qubits in this kind state are said to be *entangled*. In other words, two quantum systems are entangled when a measurement on one of them influences the other one. After this argumentation, students are introduced to the four maximally entangled states, namely the Bell states:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \quad [21]$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \quad [22]$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \quad [23]$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \quad [24]$$

Another simulation from the University of St. Andrews is exploited to let the students 'play' with quantum correlations [4].

EPR paradox

After the introduction of the entanglement, a brief account of the historical paradoxes connected to it can be given. We chose to go briefly through the Schröedinger's cat paradox and the Einstein-Podolsky-Rosen (EPR) paradox, with the subsequent Bell inequalities theorem. This latter is worth to be introduced, time permitting, also because after Aspect's and colleagues' (1982) experiment that prove Bell's inequalities, it's possible to cite the "Big Bell test" performed in 2016 (Abellán, Acín, Alarcón, 2016), where more than 100.000 people around the world generated casual sequences of 0 and 1 to ensure the randomness of the process. Furthermore, the EPR story is a good example of how physics development is a complex interaction of different perspectives and images of the world (along multi-perspective in DP1).

Goals of part 5

The conceptual goals of this part are:

5 - c1. to understand that the qubit, being in a superposition state, represents the core of QP, and to compare is with the classical bit in terms of possible states
5 - c2. to understand that two qubits are entangled when they are in a particular superposition state for which a measurement on one influences the other
5 - c3. to understand that the Bell states express the maximal possible entanglement between two qubits
5 - c4. time permitting, to understand the logic of the EPR paradox and of Bell's theorem

The epistemological goals of this part are:

5 - e1. to recognize that there can be properties, like the entanglement, that are characteristic only of a specific domain or theory
5 - e2. to recognize, through the class discussion, that the criteria with which one judges that a theory is a good explanation of the physical reality, can be different from person to person
5 - e3. to recognize, through the class discussion and the EPR paradox, how physics development is a complex interaction of different perspectives and images of the world (along multi-perspective in DP1)

New logic of QP (3): applications and society

A good part of the following considerations has been developed within the I SEE European Erasmus+ Project (Branchetti et al., 2018) in which I was involved together with whole research group in Physics Education of the University of Bologna. These proposals have been the object of a master thesis (Satanassi, 2018) and of a bachelor thesis (Spada, 2019), in which the results are described in great detail.

'Quantum Manifesto'

Along the third design principle (DP3), we chose to 'keep an eye on the future' by introducing some of the main QP applications. However, we chose to give the breath of the ongoing quantum 'revolution', also by proposing an institutional document that is orienting efforts of the "European Quantum Technologies Flagship": the 'Quantum Manifesto' (de Touzalin et al., 2016). The document has been edited in 2016 from a team of researchers as a call to launch a European collective initiative on quantum technologies. The Flagship concerns a long-term research initiative with the allocation of a one-billion-euros budget for a period of ten years. The four 'pillars' of the Flagship are quantum communication, quantum simulators, sensors, and quantum computers.

The sense of introducing the students to this kind of issues is to give a concrete context of the development of Quantum technologies, where the concepts learned in the course can become lenses to look towards the future. As introduced, the context where this activity was firstly designed and implemented is the teaching module on Quantum Computers that was developed by the research group in STEM education od the University of Bologna, within the European Erasmus+ Project I SEE. As already introduced, I was involved in the design of the course as well, and it run in parallel to the present course with a class of nearly 40 High School students. This and other related activities have been the object of a dissertation (Spada, 2019), where the author underlines that the objectives of the activity were to:

- get acquainted with the terminology, perspectives and contents of relevant institutional documents like the Quantum Manifesto
- reflect and reason about the present and future applications and implications of quantum technologies
- get aware about the multiple dimensions (e.g. social, economic, political, research, educational, ethical, environmental, etc.) such technologies involve
- express and highlight their inner vocational, societal and personal dimensions in the discourse, by bringing together their values and opinions in a group project and to recognize where and how quantum technologies can impact their personal lives

Concretely, the quantum manifesto is presented to the students, and then some informative sheets about the main applications of QP second revolution are given to the students, underlying their essence, their role in the society, and providing links to let the students search online by themselves. Time permitting, after the introduction of Quantum Computers and Cryptography (see next paragraph), two group activity can be done, along what elaborated by Spada (2019). The first, on texts inspired by the institutional literature on quantum technologies and specifically elaborated to enable them to grasp the necessity of such an effort in our contemporary world and to develop critical thinking on the topic. The second, to let the students build a map of the possible implications on different dimension (social, economic, political, research, educational, ethical, environmental, etc.) of one of the four applications, as for example Quantum Computers.

Cryptography

After the entanglement, the introduction of quantum cryptography is indeed very straightforward, and it seems also to be also a very effective way to strengthen the conceptual basis of quantum formalism. Some examples of classical cryptography can be given, as the RSA protocol, so to explain that in almost all the cases decoding a message would be classically possible, even if extremely long; for example, for the RSA protocol this would require the computer to factorize the key in prime numbers, which is theoretically possible, but practically too long to be useful. With quantum cryptography, instead, to decode a message without interfering with the system in a measurable way, it’s impossible in principle. The same narrative of Alice and Bob can be exploited, showing how the measurements on two entangled states can serve to generate a secret key. Using again St. Andrews simulations (website [4]), it can be shown very easily that, introducing a hypothetical ‘Eve’ trying to intercept the message, her presence is visible by confronting statistically two subsets of Alice’s and Bob’s measurements.

Teleportation

A very interesting, and indeed fascinating, example of the use of the entanglement is the case of teleportation. Although the technical realization of this kind of experiments is at the edge of technological advancements, their essential logic is somehow simple and one can try to break it down in a sequence of logical steps. This has been done by Satanassi (2018) in the context of the quantum computers teaching module cited above, and it was re-proposed in this course in a shorter version. We chose to use as an example the experimental realization of a teleportation across the Danube performed by the group of Zeilinger and colleagues (Ursin et al., 2004).

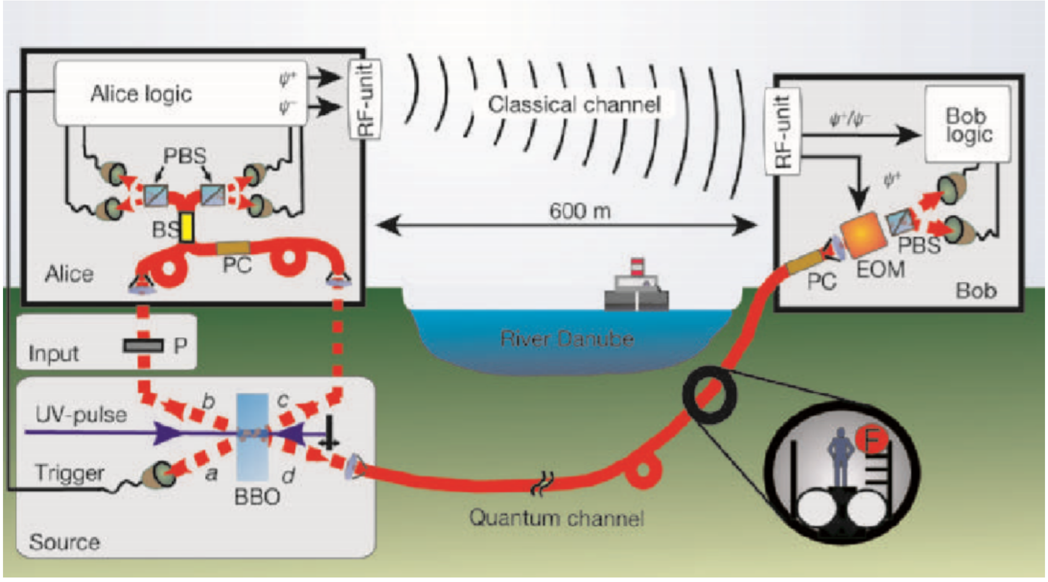

Figure 1.24 Scheme of the experimental realization of the teleportation experiment across the Danube (from Ursin et al., (2004))

Satanassi built a comparison of the experimental realization of teleportation with the equivalent quantum algorithm consisting in logical gates; indeed, this was possible because the students had been previously introduced to quantum algorithms and gates. In this course, due to time issues, we chose to present only the experimental set-up (however, in the next section I also sketch the essence of the comparison). The experiment represented in Figure 1.24, suggesting a space-time narrative, can be divided in five main steps:

1. the production of two pairs of entangled photons (*a* and *b*, *c* and *d*) through a pulsed laser, a non-linear crystal, and a mirror. Alice will have to teleport the state of *b* to the photon *d*, that goes to Bob. It's important to note that it's the *state* that must be teleported, and not the photon itself.
2. the projection of Alice's two photons (b and c), not initially entangled, in an entangled Bell state. Physically, the states are modified by a single-mode optical-fibre beam splitter (BS) connected to four polarizing beam splitters (PBS). At this point b, c and d are all entangled.
3. Alice's measurement on b and c, which is carried out through the combination of the PBS and four detectors. This measurement project the state of the whole system. Thus, Bob's photon *d* will be projected in one out of four possible (theoretically known) states. Each of these states can be easily transformed in the initial state of *b*, the one to be teleported. However, Bob cannot know how to transform *d* without knowing in which of the four states it has been projected. An information is missing.
4. The communication of Alice's measurement via classical channel with a microwave channel. This allows Bob to know in which of the four states the photon *d* has been projected. This classical communication ensures that the principle of relativity is preserved, and there is not an instant exchange of information

5. the (unitary) transformation, according to Alice's measurement, to recover the original state. In the experiment, the state is modified by applying a voltage pulse through the EOM (electro-optic modulator).

Indeed, it is not necessary that the students understand the technical details of the experimental set-up. However, this is a concrete example of the use of entanglement, and it also raises interest and fundamental questions in the students.

Computing

Even though during the present course we did not introduce quantum computation, due to time issues, I do think that teaching Quantum Computing has deep and positive conceptual / emotional consequences on students learning of QP. In what follows I trace very briefly the logic of what we developed for the Quantum Computing module (Satanassi, 2018). Quantum computation is not only on of the most promising and most funded applications among quantum technologies; it can also provide the conceptual ground to think QP in a completely different way. The qubit itself, as introduced, already expresses the very core of QP, namely, the superposition principle. But there is also another conceptual advantage in introducing quantum computation: as far as a computational perspective allows to re-read any physical experiment in terms of information (figure 1.25), in the context of QP it can be a way to give sense to the new 'odd' logic of QP experiments.

Namely, as well as any physical experiment consists of (i) a preparation of the physical 'state' (design and preparation of the experimental set-up itself), (ii) a transformation of the state, and (iii) a measurement, the same three phases can be traced for any computational process, where (i) an input information is prepared, (ii) processed, and finally (iii) 'read' as an output.

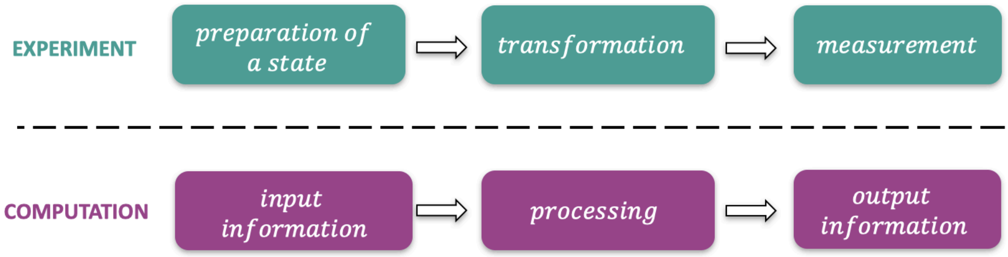

Figure 1.25 Analogy between experiments and computation

The case of valves and transistors is indeed the most representative classical example of this logic. In fact, on one side, they are experiments themselves, in which a physical state (a current) is transformed and measured; on the other side, they assume the role of computational items (logical gates), where the initial current represents an input information that can be coded in bits, that is processed through them, and finally 'read' as an output. It is interesting to note that, the informational process can be physically realized in different ways, as for examples with pulleys and ropes instead of transistors, without influencing the *logic* of the system.

This reasoning comes to be useful in QP, where a physical experiment, as for example the one of Stern and Gerlach, can be read in these terms of ‘preparation’, ‘transformation’, and ‘measure’ (Figure 1.26). Correspondingly, once having set this metaphor with classical computing, the quantum gates can be introduced as the computational elements that process the quantum information (i.e. the superposition state) through operations with qubits. The difference is in (i) the informational content of qubits, that can encode more information that the classical bit, being in a superposition state, and (ii) the role of measurement, which in QP is not only a ‘read out’ act, but it’s destructive and probabilistic.

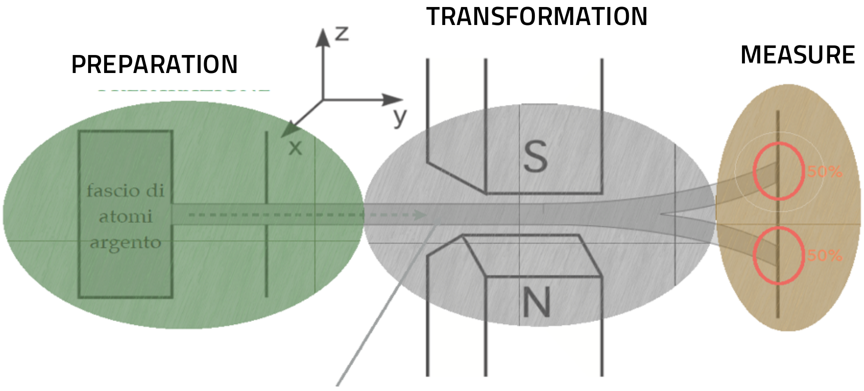

Figure 1.26 Phases of an the Stern-Gerlach experiments in terms of preparation, transformation and measure

On the basis of this analogy between experiments and computation, Satanassi (2018) built a comparison between the physical realization of the teleportation protocol (sketched in the previous paragraph) and the corresponding quantum algorithm, highlighting the connections of the experimental steps with the quantum gates of the circuit (Figure 1.27).

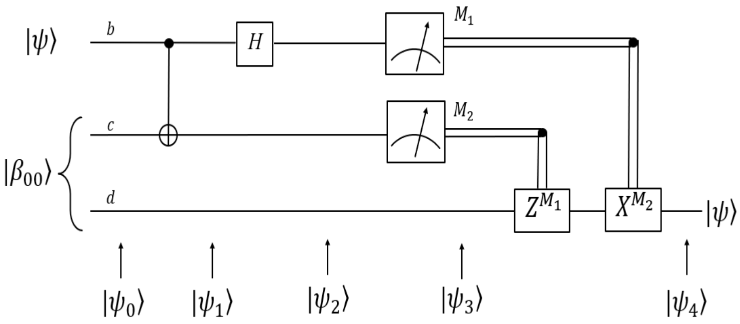

Figure 1.27 Quantum teleportation circuit

The students are led to recognize how the circuit expresses formally each step of the physical realization of the experiment, following these steps:

1. the production of two pairs of entangled photons (*a* and *b*, *c* and *d*) results in a quantum state of the system, expressed as

$$|\psi_0\rangle = |\psi_b\rangle|\beta_{11}\rangle_{cd} = (\alpha|0\rangle + \beta|1\rangle)_b \left(\frac{|01\rangle-|10\rangle}{\sqrt{2}}\right)_{cd} \qquad [25]$$

2. the projection of Alice's two photons (b and c), in an entangled Bell state is realized in the circuit through a CNOT gate and an Hadamard gate (H), ending up with

$$\begin{aligned}|\psi_2\rangle &= \frac{1}{2}[\alpha(|0\rangle_b + |1\rangle_b)(|01\rangle - |10\rangle)_{cd} + \beta(|0\rangle_b - |1\rangle_b)(|11\rangle - |00\rangle)_{cd}] \\ &= \frac{1}{2}[|00\rangle_{bc}(\alpha|1\rangle_d - \beta|0\rangle_d) - |01\rangle_{bc}(\alpha|0\rangle_d - \beta|1\rangle_d) \\ &\quad + |10\rangle_{bc}(\alpha|1\rangle_d + \beta|0\rangle_d) - |11\rangle_{bc}(\alpha|0\rangle_d + \beta|1\rangle_d)\end{aligned} \qquad [26]$$

   where every term of the sum contains Alice's qubits b and c in one of the four states $|00\rangle, |01\rangle, |10\rangle, |11\rangle$ and Bob's qubit d in a superposition of $|0\rangle$ and $|1\rangle$.

3. thus, depending on Alice's measurements on the state of b and c, in $|\psi_3\rangle$, Bob's qubit will be in one of the four possible superposition states:

$$|00\rangle_{bc} \rightarrow (\alpha|1\rangle_d - \beta|0\rangle_d) \qquad [27]$$

$$|01\rangle_{bc} \rightarrow (\alpha|0\rangle_d - \beta|1\rangle_d) \qquad [28]$$

$$|10\rangle_{bc} \rightarrow (\alpha|1\rangle_d + \beta|0\rangle_d) \qquad [29]$$

$$|11\rangle_{bc} \rightarrow (\alpha|0\rangle_d - \beta|1\rangle_d) \qquad [30]$$

   Alice's communicates the result via classical channel, allowing Bob to know in which of the four states the photon *d* has been projected.

4. depending on the state of his qubit, Bob knows which transformation is needed to bring d to the state of b (the one to be teleported), $\alpha|0\rangle + \beta|1\rangle$. In the experiment, this is executed by applying a voltage pulse through the EOM (electro-optic modulator), while in the circuit it is represented by the logical gates Z^{M_1} and X^{M_2}, eventually applied as follows:

$$|00\rangle_{bc} \quad \rightarrow \quad \hat{Z}\hat{X}\,|00\rangle_{bc} = \alpha|0\rangle + \beta|1\rangle \qquad [31]$$

$$|01\rangle_{bc} \quad \rightarrow \quad \hat{X}\,|01\rangle_{bc} = \alpha|0\rangle + \beta|1\rangle \qquad [32]$$

$$|10\rangle_{bc} \quad \rightarrow \quad \hat{Z}\,|10\rangle_{bc} = \alpha|0\rangle + \beta|1\rangle \qquad [33]$$

   If Alice measures $|11\rangle_{bc}$, Bob's qubit is already in the right state $\alpha|0\rangle_d + \beta|1\rangle_d$.

It is important to note that the circuital representation owns an implicit narrative focused on the system more than on objects, as the state is expressed at each step as a property of the whole system. This seems to be a promising way to introduce QP physics new logic, going towards the direction of building an ‘ontology’ of the quantum state as a systemic property, but at the same time not as mere tool for fitting the counts.

Goals of part 6

The conceptual goals of this part are:

6 - c1. to start to grasp the essence of entanglement as a resource for applications, through the activity on the quantum manifesto and through the introduction of cryptography, teleportation, and computing
6 - c2. to understand what cryptography is and what are the main differences from “classical” and quantum cryptography
6 - c3. to understand the effect of an observer on a system when two persons are exchanging information
6 - c4. to understand that cryptography opens new opportunities whose impact span different dimensions (political, social, economic, ethical, environmental, professional…)
6 - c5. to understand the experimental logic of teleportation in terms of the entanglement
6 - c6. time permitting, to understand the analogy among physical experiments and computation, and to re-read the teleportation in terms of a circuit

The epistemological goals of this last part are:

6 - e1. to recognize that science development is deeply intertwined with institutional pushes and technological advancements.
6 - e2. to recognize, through the activity on the ‘Quantum Manifesto’, the concept of ‘dimension’ (political, social, economic, scientific, ethical, environmental, professional ...) as important for understanding the impact of a scientific technology
6 - e3. to recognize ‘information’ as a key transversal concept that allows to see physical systems as tool to build a higher abstract logic
6 - e4. to recognize that there can be different narratives to explain a phenomenon and to recognize that quantum states provide a systemic narrative (instead of a space-temporal one)

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PART II

STUDENTS' EPISTEMOLOGIES IN LEARNING QUANTUM PHYSICS

INTRODUCTION TO PART II

In this Part, I conduct a qualitative study on students' epistemologies in learning QP. The study draws on the results I gained during my master dissertation (Ravaioli, 2016), analyzing the cases of students who explicitly did not accept QP as a personally reliable explanation of the physical reality. Through the analysis, I found evidence of the emergence of three specific requirements that can trigger the stance of acceptance or non-acceptance, which I referred to as *epistemic needs*: the needs of visualization, comparability and 'reification' (Ravaioli & Levrini, 2018). Along these initial results, I decided to conduct a more precise study to find out the nature of the factors that trigger students' stances in learning QP, building on the research literature on personal epistemologies and their entanglement with affect-related aspects. The aim of this study is to find out what epistemological expectations and beliefs trigger students' stances (for instance, acceptance or non-acceptance) in learning QP, and whether these stances are triggered by challenges intrinsic to QP. To this extent, I collected written and recorded data of High School students participating in course on QP I developed (the same described in Part II of this dissertation), and ended up analyzing extensively the cases of three students that seemed particularly interesting in this perspective. The analysis highlighted evidences of (i) of a possible entanglement between specific students' epistemologies and their meta-affective stances (term by deBellis and Goldin (2006), referring to the feelings about feelings, for example the enjoyment of otherwise undesirable emotions) towards challenges in learning QP, and (ii) evidence of expectations about the role of 'visual modeling' and 'math' as two personally reliable means to bridge classical and quantum domains.

In Chapter 3, I sketch the methods and the results of the previous analysis, pointing out also the research questions that will guide the present study.

In Chapter 4, the analysis is contextualized within the research literature, carried out extensively on three students, and discussed in light of the research questions.

CHAPTER 3

EPISTEMIC NEEDS: INITIAL PEEK

3.1 PREVIOUS ANALYSES: GOALS AND RESEARCH QUESTIONS

The first analysis (Ravaioli, 2016; Ravaioli & Levrini, 2018) has been conducted with the aim of investigating a specific phenomenon, that, even if not new in physics' education research, had never been addressed in detail: the phenomenon of students' *non-acceptance* of QP. With 'non-acceptance' I refer to a specific reaction or stance of some students who explicitly did not accept QP as an adequate and personally reliable explanation of reality. We found evidence of the emergence of this phenomenon in the data coming from different contexts (all of them were High School experimentations). Some cases were particularly interesting since they concerned students who appeared to be rather confident with the formalism and to have appropriated the basic concepts of the teaching path as it was proposed (Malgieri et al., 2018). The main motivation of the study is the impression that students' non-acceptance reaction could not always be interpreted in terms of a philosophical divergence, echoing the historical and authoritative realist positions who reacted against the Copenhagen interpretation. Instead, we felt that behind non-acceptance, some epistemic requirements could be recognized, deeply involved in the fundamental mechanisms of understanding. In light of these preliminary impressions, we decided to analyze the data so as to answer the following research questions:

RQ1 what kind of needs re not satisfied satisfy in learning QP? To what extent can they lead to non-acceptance?

RQ2 can we support students' acceptance by improving teaching or does the process of non-acceptance refer merely to QP per se and its intrinsic features?

3.2 CONTEXTS AND METHODS

To make the analysis reliable and as much as possible context-independent, the study considered materials from different teaching experimentations (some of which cited in Chapter 2 of this dissertation). Specifically, the experimentations led by the research group of the

University of Bologna in 2012 and in 2015 in Rimini (analyzed from other perspectives by Levrini & Fantini (2013) and by Lodovico (2016)), in 2015 in Bagno di Romagna and in 2016 in Castel San Pietro (Ravaioli, 2016); these data have been also contrasted with data coming from the experimentations conducted by the research group in physics education of the University of Pavia in 2014 and 2015, where the teacher followed a 'sum over path approach' (Malgieri, 2015). Semi-structured interviews carried out at the end of each teaching module. From a methodological point of view, a qualitative approach was chosen. The analysis started by focusing on three cases of evident and *explicit* non-acceptance, from three different school contexts (Marco, Cheng and Alice)[6]. Two of them had also shown a sensible degree of *appropriation* of the basic ideas of QP as it had been proposed in the learning paths (Levrini & Fantini, 2013). Students' profiles have been built so to flash out the key-words used by the students to describe their perplexities on QP, and to talk about the topics and concepts they found particularly puzzling. We then recognized the semantic fields to which students' words belonged and formulated a hypothesis about students' requirements (*needs*) underlying non-acceptance. These needs have been then used as a lens to analyze the discourses of other students, also those who accepted QP; this has been made to check if, and how, they showed similar needs to the three identified, and, in case, what ways they found to satisfy them. Hence, after the recognition of the main needs showed by the non-acceptance cases, we re-analyzed all students' interviews by exploiting the needs as an interpretive key.

3.3 THREE CASES OF EXPLICIT NON-ACCEPTANCE

In what follows, the three cases of clear non-acceptance of QP are described, two of which from Bologna's experimentations and one from Pavia's. To investigate the nature of non-acceptance, we focused on students who appeared to grasp the sense and the basic rules of the formalism, and appropriate the fundamental concepts that were addressed (Malgieri et al, 2018); this allowed us to avoid cases in which non-acceptance was due merely to a lack of preparation and hence to focus, at least at this stage, on real cognitive and epistemic needs (and, not, for example, on cases where motivational aspects were evidently the main source of non-acceptance). The first two cases (Marco and Cheng) show a typical epistemic refusal of the quantum description of reality, and still claim for a more 'realistic' theory to be found in future. Instead, in the third case (Alice), non-acceptance seems to have a slightly different – even though related - nature, as it emerges as a difficulty to understand physics without the support of space-time visualizations.

Marco: postulating 'well-defined properties'

Marco (from the experimentation carried out in Rimini 2015), is a student whose idiosyncratic idea of science is mainly founded in its utility and its possible applications: "science has to be used in the technical field […] to create, let's say, great inventions". Marco is considered by his classmates as a good and hardworking student, and the analysis reported by Lodovico (2016) shows that he appears to have generally appropriated the basic ideas of QP. Nevertheless, he

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[6] All the students' names in this dissertation are pseudonyms

consistently insists that he cannot accept QP as a complete explanation of the physical reality, and specifically, as a reliable description of quantum objects. For example, when asked about the superposition principle (in the context of Stern-Gerlach experiments), during the final interview he answers as follows:

> Marco: If I hypothetically can take a measure with a sufficiently sophisticated instrument, that object would have a well-defined property. The object itself does own a well-defined property, that's what I believe. […] As Einstein, mine postulate is that an object must embody well-defined properties.

Or, on the uncertainty principle:

> INT: […] What was the most useful way for you to comprehend the meaning of quantum uncertainty in its revolutionary holding?
>
> MARCO: So… if I have to be honest, none.
>
> INT: None of these [he had cited them previously]?
>
> MARCO: All of these partly contributed, but none gave me a thorough explanation. Namely, what I was searching for as an explanation, I haven't found it in any of these. […] [I was in] a great confusion, not mostly because of the mathematical part, [in fact] I could understand the concepts the teacher was talking about, […] they were logically comprehensible. The point is that I couldn't understand how couldn't a body have its own properties, well-defined properties…

Marco's requirement of classical-like properties plays the role of a real postulate, whose strength probably comes from his idiosyncratic idea of science. In this sense, the nature of this requirement is genuinely epistemic and it is an idea that generally produces in Marco a form of skepticism towards QP, affecting his acceptance of the uncertainty relation and superposition principle, but also of the concepts of quanton[7] and probability, as he states:

> MARCO: To me, the word 'probability' is quite an 'escamotage' [trick] that we use to…to determine the phenomenon with certainty […] But, indeed, these are the errors induced by this way of representing this fundamental issue, namely the one of non-defined properties.

Marco's search seems moreover very interested in applications and technological developments, that serve him to partially postpone the problem of accepting QP in its implications. But in postponing the problem he always specifies his concern. For example, on uncertainty relation:

> MARCO: although I don't agree with it, I understood that Heisenberg's hypothesis [of uncertainty] in necessary at this moment. […] I notice that considering the quanton as a non-defined particle, even though I don't agree, is in any event fruitful for the moment. Just like as your mother tries to convince you that black dogs are evil, and you know they're not […], but she gives you 50 euros every time you say: "yes, ok, ok".

Hence, although Marco's idiosyncratic idea of science reveals a sort of empiricism, and the very reason that leads him to not accept QP is founded on epistemological requirements and considerations; he feels the necessity to "find a more epistemologically accurate meaning".

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[7] the term *quanton*, firstly proposed by Bunge, has been re-adopted by Levy-Leblond (2003) for categorizing quantum physical objects on the basis of the common quantum properties, so as to avoid classical categories to describe them. This choice has been extensively discussed with students in classrooms experimentations (see Chapter 2 of this dissertation).

Cheng: “I would like to know more about reality”

Cheng is a student from the experimentation of the group of Pavia, whose case has been extensively investigated (Malgieri, 2015; Malgieri et al., 2018). He seems to have well understood the disciplinary contents of the course and, on the basis of the markers proposed in Levrini and colleagues (2015) and ri-elaborated by Malgieri et al. (2018), he seems to have also appropriated the basic concepts.

As it appears in Malgieri’s Ph.D. Dissertation (Malgieri, 2015), Cheng correctly talks about the main historical developments of QP, describes the wave-particle duality from the point of view of some different scientists, and explains in detail the most recent developments proposed in classroom (entanglement among the others). But, despite his confidence with all these issues, when interviewed he explicitly states that he cannot accept quantum theory as a ‘final’ explanation:

> INT: So you are not convinced by the idea of a quantum object, which is neither wave nor particle (...) You believe a better explanation exists.
>
> CHENG: I think it exists, but hasn't been found yet […] I would like to discover why it’s that way.
>
> INT: Is it my impression, or there is something that you don’t accept?
>
> CHENG: Exactly. I would like to know more about reality.
>
> INT: So you don't accept it. Sooner or later it will be discovered.
>
> CHENG: Yes. Exactly.

Cheng doesn’t face any repulsion towards mathematics; on the contrary, he firmly believes in the explicating power of formalism: “Images can help you understand, while the mathematical model simplifies everything. If we know how it works, it makes us remember everything at a glance".

This confidence with mathematics leads him to consider QP understandable, as he demonstrates when speaking about ‘Which Way’ measurements: “it is surprising because it does not follow the classical probability rule, but it’s not incomprehensible, because it follows the quantum probability rule. So, it’s surprising, but only because it’s computed in a different way”. He shows also to have a precise idea of the relationship between physics and mathematics, as a description of intrinsic laws of Nature (which he demands to be the classical physics ones):

> INT: So, you believe that Newton's formula for gravitation exists somewhere, and we just have to discover it.
>
> CHENG: It exists, in the sense that it's intrinsic. But it's not mathematics. We mathematize it.

Hence, Cheng declares to understand QP and to be able to visualize, for example, Feynman’s model; furthermore, he seems peaceful to momentarily accept QP for its results in calculations. But at the same time, confronting his idea with those of the most important scientists who developed QP, he is sure that this is not the final answer, as he explains:

> CHENG: I believe objects to have a definite position and momentum. There is something that escapes our understanding. But it is not that uncertainty is due to measurement. It is due to some other reason. Something which we still don't know.

As it was for Marco, this need for more 'realistic' properties affects his acceptance of the uncertainty principle, and of the nature of *quantons*. It is also interesting to notice that both Marco and Cheng consciously focused their attention on the formal apparatus and its relations with the experimental devices, since they both consider QP useful and very effective for its technological applications. What they seem to keep faraway is the modeling game that the formalism seems to suggest to provide a new interpretation of the world.

Alice: "The ball is round, and the state?"

Alice is a student from the experimentation held in Castel San Pietro in 2016. Her personality shows to be always curious and ready to accept the challenge with every topic proposed in classroom. She likes to dialogue both with the teachers and her classmates, even if she is not sure to have the right answers. Alice suffers of a slight linguistic fragility, which often leads her to not fully comprehend the texts, and which weakens some of her logic arguments; for instance, she does not feel comfortable with most of the metaphors proposed in the course, mainly because of her tendency to read them literally and to miss the appropriate connections. Despite this slight difficulty, Alice is considered to be quite a good student, and physics is her favorite subject matter; her final dissertation was about gravitational waves and general relativity. Alice showed a great interest towards the QP course and was the most active student during the lessons. Nevertheless, when interviewed, she expressed her difficulties in dealing with QP, some of which remained unsolved. Specifically, she felt bothered by the problem of 'imagine' the quantum state:

> ALICE: Quantum physics has been difficult to comprehend with respect to the other physics fields because…it's a kind of physics that I cannot imagine, or contextualize […]. When we talk about an electron, I know that I cannot see it but, at least, I imagine it as it is drawn in the textbook. Quantum physics instead…namely, the quantum state is much more difficult to be imagined.
>
> INT: […] So, how did you imagine the state when we were talking about it in the classroom?
>
> ALICE: …when you said that the [a specific] state comes to be defined only with a measurement…this shocked me a little, because that is not an 'intrinsic' characteristic, and so I really don't know how to visualize it…

Alice's idea of comprehension appears to be strongly influenced by her need of visualization, as the example of the electron shows. When trying to visualize the quantum state, she searches for an intrinsic property that can characterize it and let her to use the imagination. We claimed that the word 'intrinsic', as it was for Marco's 'well-defined' properties, tacitly identifies with properties held by a state or an object in a classical sense: properties that have a single, well-defined value to be revealed through measurement. In another extract, to get to the point, she enforces her argumentation through a metaphor:

> INT: So, what is your concern with the quantum state?
>
> ALICE: I would like to understand better what it is. We didn't say: the state is this, or that…we only talked about some of its features…so to speak, the ball is round, and the state?

Consistently, the role of measurements in determining the state seems to be an awkward point for her conception of science:

ALICE: I was used to think that all scientific subjects had to describe all the phenomena with certainty, but this issue of measurements changing the state…it makes me a little bit perplexed".

Even if not explicitly addressed by Alice herself, as it was instead for Marco and Cheng, we are prone to consider her case as a non-acceptance one. In fact, although she seems to have appropriated the basic concepts of the teaching proposal, she does not feel comfortable with QP's description of the world, as clearly pointed out in the following:

ALICE: I'm used to think about the world and about reality through classical physics. Sure enough, even with relativity I had some difficulties in imaging its 'curvatures'…but for me quantum physics requires even a greater effort, because it's a too small world…it's too abstract. I haven't fully grasped it yet…

Alice is trying to use the concept of state as a cognitive lens for understanding quantum phenomena, but she fails to finalize and establish such a shift due to the lack of visualization.

3.4 QUALITATIVE ANALYSIS: THE EPISTEMIC NEEDS

A comparative analysis of the three cases of explicit non-acceptance showed some main evidences: (1) all the students mention three main conceptual topics against which their acceptance clashed, namely, the concept of quantum object, the superposition state, and the uncertainty relations; (2) the words used by the students to complain their difficulties can be grouped in three semantic fields, namely *visualization/imagination*, *to know more/better*, *reality/existing*. Some key expressions that mark problems of acceptance are "to know more about reality", "to give meaning to the formulas", or "compatibility with reality", and reveal the need to strengthen or establish an interpretative and epistemological connection between the new mathematical structure and the world. In some sense, it seems that the modeling dimension, that is the hypotheses and the features of the new paradigm, is not completely grasped or accepted.

In front of these evidence we hypothesize that behind non-acceptance dynamics lie some basic cognitive requirements, which we initially termed 'cognitive needs', which emerge with strength in dealing with QP. With the evolvement of the study, we realized the nature of this needs seemed to be epistemic, in the sense that it entails students' beliefs about knowledge and knowing; consequently, we decided to refer to them as 'epistemic needs'. We pointed out three of them: the *need of visualization* (to have a comprehensive view that can guide intuition), the *need of comparability* (to have criteria to understand where and how the epistemological description/interpretation of QP is different from the classical one), and the need of ontology (to attach a reliable and "realistic" meaning to new basic elements, like states, on which reasoning has to be developed). We later begun to refer up to this last need as the *need of 'reification'*, following the concept of reification as the objectification of an abstract construction process, shaped in the work of Anna Sfard (1991). As introduced, we hypothesized that these needs do not belong only to those students who do not accept the theory, but that traces of them can be found in many students' discourses; simply, depending on other idiosyncratic or contextual factors, they can be activated with a variable strength. In what follows I provide a brief description of each of the epistemic needs, using also excerpts from other students besides the three reported above.

Need of visualization

The need of visualization, expressed by Alice, is also stressed by Anna (experimentation in Rimini, 2015), that similarly, when asked about the quantum objects, answers as follows:

> ANNA: In my head I've no ideas about the quanton […], I've not a clear image in mind. […] But I've made up the idea that this is quite a new stuff, and it seems almost unreachable, as it is not to be understood…

Although Anna seems not prevented by her need of visualization in accepting quantum theory, she clearly considers the possibility to build up an image of the *quanton* at the same level of her understanding of the latter; as she cannot reach a clear image or idea of the 'quanton', it cannot be properly understood. Another example comes from one of the two classes under study in Levrini & Fantini (2013), where the formalism was recognized from all the students as necessary to understand; the issue of *visualization* of quantum phenomena was generally recognized as a clear-cut point of detachment from classical physics. This generated a lively discussion in class, where different positions came to light. The case of Jessica is particularly interesting in this perspective.

> PIETRO: The picture of microscopic reality, in this case, is sufficiently supplied by the mathematical formalism. Therefore, in my opinion, to have a graphical representation is not important for scientific progress: What's the use of the graphical representation? It may help in explaining the object as it is to children. But mathematics already explains it. […] In my opinion anyway, the picture of microscopic reality is already described well enough by mathematics. It is enough to have the tools for comprehending it and it seems to me that everyone can do so…
>
> JESSICA: […] But for me it [visualization] is necessary in order to understand…
>
> PIETRO: Ah, but what if you can't do it…
>
> JESSICA: Because it is impossible to talk about something without trying to have a picture of what we are talking about, even unconsciously. It may help, in my opinion, also to give a meaning to formulas, because otherwise, even if we say that it is nonsense to represent the microscopic object, we make a picture anyway… I think so, although we decide not to draw it because we don't want to give a model that… […] it helps me, it helps me to remember. […] honestly I can explain the Compton effect by keeping in mind the drawing. […] we know that to be untrue but…
>
> PIETRO: Ok, but it is just an icon, you could draw a little star to make a photon.
>
> JESSICA: Yes, exactly.

Pietro is accepting the impossibility to visualize quantum phenomena, founding his confidence in the possibilities of scientific progress, and refusing any other need of description. Jessica, instead, assigns to her need of visualization a critical role for understanding: the formalism must be interpreted in terms of pictures that, being implicitly connected with the classical world, allow for the use of an ordinary language. She restates many times that pictures do not have to be a true representation of physical reality; however, for Jessica, visualization is an obliged way to travel through to face her necessity to "give meaning to the formulas". The authors of the article point out that this personal requirement someway recall the position interpreted by Schrödinger in the historical debate about formulations of Quantum Mechanics, for whom *visualizability* (Anschaulichkeit) is not only a useful way to comprehend the content of a theory but concerns the very aim of scientific research, as himself states:

> “Physics does not consist only of atomic research, science does not consist only of physics, and life does not consist only of science. The aim of the atomic research is to fit our empirical knowledge concerning it into our outer thinking. All of this other thinking, so far as it concerns the outer world, is active in space and time. If it cannot be fitted into space and time, then it fails in its whole aim and one does not know what purpose it really serves” (de Regt, 1997).

This clear-cut line of thought is not of course consciously accounted from students as a philosophical stance, and indeed neither entirely in its methodological and epistemological implications; nevertheless, their words evidence that, for someone, understanding and acceptance are tightly bound to visualization. All these remarks led us to individuate the need of visualization, that synthetically emerges in students as the need to have a comprehensive view that can guide intuition.

Need of comparability

The second epistemic need we pointed out concerns *comparability*, and emerges in students as the need to bridge, both formally and imaginatively, the quantum world to the classical one, so to allow imagination to move from one to the other. The absence of an explicit demarcation line between classical and quantum domains often leads students to perceive them as completely detached from each other, and as it is for Marco and Cheng, the quantum formalism comes to be a ‘trick’ to account for the experimental results without really interpreting the world. Federico (experimentation in Rimini, 2015), for example, when asked to compare his studies about QP to the others, answers as follows:

> FEDERICO: […] In the past two years [Federico was exposed to the experimentation about relativity in the previous year] my idea of physics has changed from the one where science had to determine everything, calculate, and tell us everything with certainty. Science has become an endless research of truths; truths that have to be proved wrong, or even made more true, by the following theory […].
>
> INT: Yes. In fact, in your essay you were claiming that it’s not clear yet how it is possible the coexistence between classical and quantum worlds, with such great differences…
>
> FEDERICO: Yes, that’s an issue I dealt with. […] What I can’t explain is how could they can coexist, but just as how could relativity and classical mechanics coexist. […] This is closer to philosophy than to physics! Or maybe this is true physics, I don’t know.

In dealing with relativity and QP, Federico’s idea of science had been enriched and enlarged from those limits that were fixed in classical domains. Science development assumed the image of a dynamical process, where ‘truths’ are always to be questioned and deepened, and Federico comes to face his need for a ‘coexistence’ of the different theories, probably making the implicit assumption that all of them are needed to explain the whole reality. It is interesting, from this perspective, what Federico states about everyday reality:

FEDERICO: The difficulty I encountered is, as I said before, that quantum concepts are so much distant from the Newtonian reality we experience every day.

What Federico is missing is an explicit connection between the daily experience, which is to him undoubtedly assumed to be Newtonian, and the new quantum concepts (like discreteness of the process or abstract spaces, as himself points out in other excerpts). Similarly, it is interesting what Silvia (Levrini & Fantini, 2013) points out during a discussion led in the classroom:

SILVIA: In relativity it was different […] there you have a demarcation line. If you apply our velocity in formulas, you re-find our formulas. [In relativity] the two things are compatible, here not. […] In relativity, in my opinion, there was a greater compatibility with reality.

As the authors highlight in their analysis,

> “without such a demarcation line and hence a comparative criterion, the quantum formalism risks becoming nothing but a “mechanism”, “a mentality” (Silvia) to jump into, lacking what she felt to be a way for making the worlds comparable. Silvia was not compelling the impossibility of projecting classical images on the quantum world. She was instead manifesting the need of making the two ‘worlds’, however different, comparable, where comparability includes also the knowledge of where one fades in the other” (Levrini & Fantini, 2013).

On the basis of these observations, the need of comparability can be defined as the need to find out criteria to understand where and how the epistemological description/interpretation of QP is different from the classical one, as to be able to move back and forth from one to the other.

Need of ‘reification’

This need emerges quite systematically when talking about the quantum object, the superposition state, and the uncertainty principle. Andrea (experimentation in Bagno di Romagna, 2015), for example, when asked about the nature of the quanton*,* answers as follows:

ANDREA: [This is] a word quite particular to describe it, but maybe it could be said to be mysterious, as up to now it’s difficult to define what it really is; we don’t know yet how to define it well, if particle, wave, or something which lies outside both natures. […] the ‘quanton’ is a totally new kind of thing, it’s difficult to tell its properties… it’s something that is not well definable.

In the attempt to find a definition of quantum objects Andrea implicitly does the assumption that the words ‘property’ and ‘definition’ are strictly linked to classical quantities. To reach a more ‘realistic’ identification of the quanton, imagination searches for those classical-like properties on which students are used to rely and, thus, considered more ‘real’. In the previous exceprts, Cheng repeats the same requirement many times, and in a way very similar to that of Marco about the uncertainty relations (“the point is that I couldn’t understand how couldn’t a body have its own properties, well-defined properties…”), and in about the superposition state (“the object itself does own a well-defined property, that’s what I believe. […] As Einstein, mine postulate is that an object has to embody well-defined properties”). Also in Alice’s interview the need of ‘intrinsic’ properties is mentioned. Another nuance of this requirement is that of determinism, raised up by Simone (Levrini & Fantini, 2013):

SIMONE: The hardest point to understand has been giving up classical determinism […] Deterministic physics was an exact science, at least at a theoretical level. Quantum mechanics is upsetting since it requires facing the knowledge problem, it makes you ask if what we observe is really what it is.

All these statements somehow recall Einstein's philosophical stance on the concept of quantum state, even though students are less conscious:

> "I am not ashamed to put the concept of «real state of a physical system» ("existing objectively, independently of any observation or measure, and that can in principle be described through the means of expression of physics") at the very center of my meditation" (Einstein, 1953).

Nevertheless, we are prone to think that these students' difficulties are not exclusively due to epistemological issues that immediately recall to mind the well-known debate about QP foundations; they can be ascribed also to a cognitive lack of a reliable *ontology*. With cognitive ontology we refer to those basic knowledge elements belonging to each theoretical formalism that allows to interpret the physical reality in a new reliable and fruitful way. With need of reification we thus refer to the need to attach a reliable meaning to new basic knowledge elements, like states, on which reasoning has to be developed.

3.5 RESULTS AND NEW RESEARCH QUESTIONS

The research questions were addressed as follows: (1) in the context of QP, students' acceptance is strongly influenced by the emergence of some requirements: the needs of *visualization*, *comparability*, and *reification*. The nature of these needs seems to be primarily not philosophical, but also epistemic (*epistemic needs*), and they can be activated in students with a variable strength, depending on a variety of factors. (2) Identified these requirements, the work is to find ways to monitor their appearance and to accomplish them.

From this initial study, advancements in many directions could be pursued. Indeed, it would be worthy to characterize the relation of acceptance with other factors, such as students' epistemologies, understanding and appropriation. In fact, acceptance is a complex issue where all these dimensions co-operate to its emergence; thus, to model its dynamics, all of them must be taken in account. Identified the emergence of this phenomenon in the specific context of QP, and highlighted the occurrence of three main epistemic needs in students discourses, I outlined two research questions that could guide the study that is focus of this Chapter:

RQ1 what epistemological expectations and beliefs trigger students' stances (for instance, acceptance or non-acceptance) in learning QP?

RQ2 are these stances triggered from challenges intrinsic to QP theory?

CHAPTER 4

ANALYSIS OF STUDENTS' EPISTEMOLOGIES

4.1 THEORETICAL FRAMEWORK

In order to address the RQs, the first step has been to search in the literature for theoretical approaches and lenses that could point out the complexity of a process of acceptance of a physical theory. The issue of acceptance is well-known and explored in relation to socio scientific issues (SSI) and to the public debates that mainly involved Darwin's theory or the climate change issues (Sinatra, 2018). The case of QP is substantially different since, nowadays, the stance of acceptance of the theory for the students is more epistemological than political. For this and other reasons, narrowing the focus on students' epistemologies seems to be appropriate to answer the RQs in this context: in fact, (i) as shown, the analysis conducted on previous courses highlighted the emergence of students' needs, or requirements, that seem to live at an epistemic level, and that must be better defined and contextualized, (ii) the course where these last data come from has been explicitly designed to discuss and reflect critically on an epistemological level (see Chapter 2 of this dissertation), and (iii) quantum physics, more than other theories, challenges habits of mind and forces to re-think the process of knowledge construction and the ontology of classical categories. Thus, the epistemological component of students' thinking will be at the very center of this analysis, and I choose the literature on students' epistemologies as the main theoretical reference. Specifically, through the present study I investigate if and how students' epistemologies about their own learning and about science orient their responses and attitudes towards Quantum Physics (QP). In this perspective, I spent three months hosted by the Physics Education Research group of the University of Maryland, that indeed represents one of the lead research teams on students' cognition and epistemologies in the world. I owe to them an important part of the inspiration and the methodologies that guided me through this analysis, and I'm grateful for the rich and nuanced feedback they gave me on my work.

In what follows I recall the main research results about students' epistemologies achieved in introductory physics and QP learning contexts. In doing so, I provide also a couple of explicit examples coming from the literature that show explicitly the role of the epistemological component in learning physics concepts and point out how an analysis based on concepts and conceptions can be insufficient to give an account of students' learning dynamics. Afterwards, I give a brief account of the development of this research strand, focusing in more detail on two specific theoretical models: the 'resource' model (Elby & Hammer, 2010; Hammer & Elby, 2002, 2003; Hammer et al., 2005; Redish, 2004) and the AIR model (AIR: Aims-Ideals-Reliability. Chinn, Buckland & Samarapungavan, 2011; Chinn, Rinehart &Buckland, 2014).

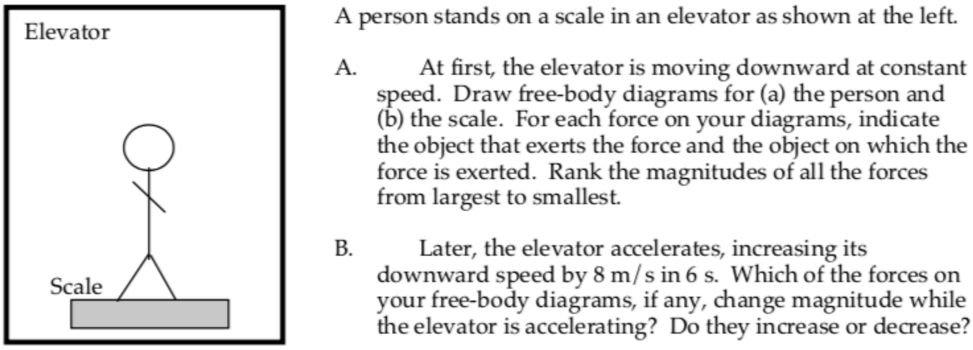
Evidences of epistemology in learning introductory physics

Learning regards not only what we know, but also how we know and how we learn. When students encounter new theories or models, or new information in general, they will have to interpret and evaluate them, whether implicitly or explicitly. For instance, learning to examine why and to what extent we accept something as a scientific truth, or why in specific contexts we prefer to use some representations instead of others, is a critical element of science itself. As Hofer (2001) points out in her review, "increasingly, educational and instructional psychologists have become interested in how a student's underlying beliefs about knowledge and knowing are a part of the process of learning, and how these beliefs affect or mediate the knowledge-acquisition and knowledge-construction process". The way we learn content, and the value we attach to it, is deeply influenced from expectations, or beliefs, about learning and knowledge itself; it has been claimed that epistemological beliefs can influence strategy use (Schommer *et al*., 1992), cognitive processing (Kardash and Howell, 2000), and conceptual change learning (Qian and Alvermann, 2000). A bunch of research studies has provided evidence for the role of an epistemological component in learning introductory physics (Hewson, 1985; Gunstone, 1992; Carey & Smith, 1993; Linn & Songer, 1993; Hammer, 1994; Roth & Roychoudhury, 1994; Halloun, 1998; Redish, Steinberg, & Saul, 1998; Elby, 1999; Smith, Maclin, Houghton, & Hennessey, 2000; Elder, 2002; Tuminaro & Redish, 2007; Lising & Elby, 2005; Ding, 2014).

Hammer (1994), for example, identified students' stable epistemological beliefs from interviews of six students of an introductory physics course. He met them several times over one semester, and the analysis of their discourses allowed him to categorize some of their beliefs in terms of:

- beliefs about the *structure of physics knowledge* as (a) a collection of isolated pieces of (b) a single coherent system.
- beliefs about the *content of physics knowledge* as (a) formulas or (b) concepts that underlie the formulas.
- beliefs about *learning physics,* whether it means (a) receiving information or (b) involves an active process of reconstructing one's understanding.

Another study from Roth and Roychoudhury (1994) examined what the authors called epistemological commitment in high school physics students, and identified differences between constructivist and objectivist beliefs. Lising and Elby (2005) developed a case study about a student whose epistemological views discouraged her to relevant experiences and intuitions, undermining her conceptual understanding. I want to report here more extensively a couple of examples that show how epistemologies are entangled with conceptual learning. I take an example from the work of Hammer and colleagues (Hammer et al., 2005), that bring out a discussion where three students, while solving a problem, negotiate between themselves what the authors refer to as an *epistemological frame* (also in Hammer et al., 2005). A frame is a term to indicate individuals' structures of expectations for a specific situation, and the authors define epistemological frames those framing situations where an individual answers the question "how should I approach knowledge?". The specific framing of a situation will determine the activation of a specific 'set' of resources to interpret that situation. The students, attending an algebra-based introductory physics course (from Tuminaro, 2004), are working on a problem in which a person is standing on a scale in an elevator (figure 2.1):



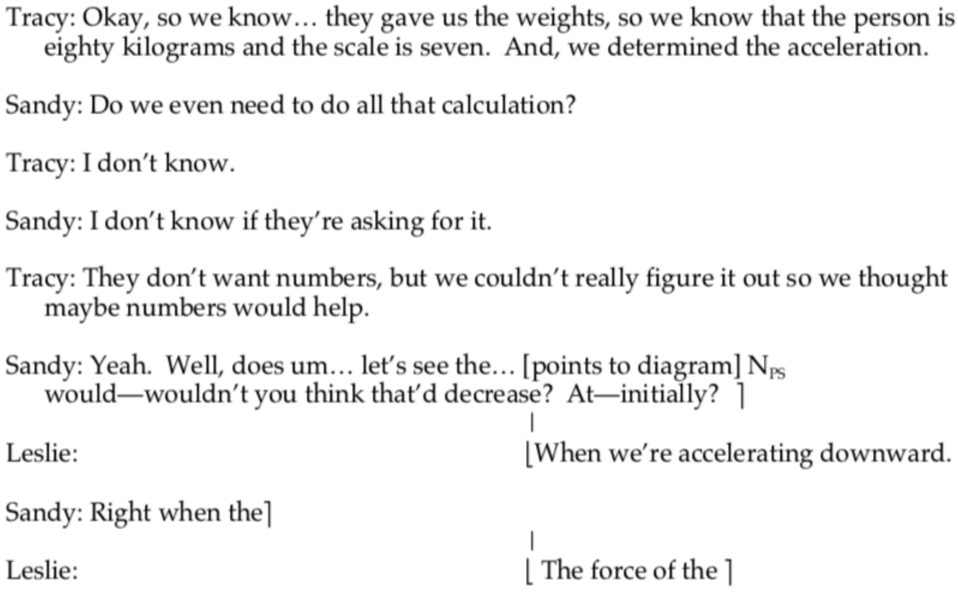
A person stands on a scale in an elevator as shown at the left.

A. At first, the elevator is moving downward at constant speed. Draw free-body diagrams for (a) the person and (b) the scale. For each force on your diagrams, indicate the object that exerts the force and the object on which the force is exerted. Rank the magnitudes of all the forces from largest to smallest.

B. Later, the elevator accelerates, increasing its downward speed by 8 m/s in 6 s. Which of the forces on your free-body diagrams, if any, change magnitude while the elevator is accelerating? Do they increase or decrease?

Figure 2.1 The elevator problem, from Hammer et al. (2005)

In the following excerpt, Tracy, Sandy and Leslie are starting to consider the question of which forces would change magnitude if the elevator begins to accelerate downward.



Tracy: Okay, so we know… they gave us the weights, so we know that the person is eighty kilograms and the scale is seven. And, we determined the acceleration.

Sandy: Do we even need to do all that calculation?

Tracy: I don't know.

Sandy: I don't know if they're asking for it.

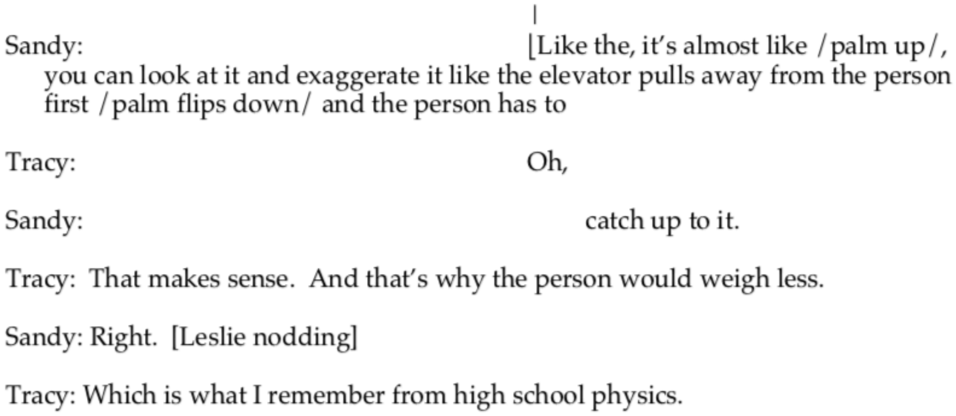
Tracy: They don't want numbers, but we couldn't really figure it out so we thought maybe numbers would help.

Sandy: Yeah. Well, does um… let's see the… [points to diagram] N_{PS} would—wouldn't you think that'd decrease? At—initially?]

Leslie: [When we're accelerating downward.

Sandy: Right when the]

Leslie: [The force of the]



Sandy: [Like the, it's almost like /palm up/, you can look at it and exaggerate it like the elevator pulls away from the person first /palm flips down/ and the person has to

Tracy: Oh,

Sandy: catch up to it.

Tracy: That makes sense. And that's why the person would weigh less.

Sandy: Right. [Leslie nodding]

Tracy: Which is what I remember from high school physics.

Figure 5.1 Excerpt from Hammer et al. (2005)

In the analysis, the authors identify a shift of framing, negotiated between the students. I report an extensive excerpt of the original article here to give another example of the kind of analysis that can be done on this data. “Tracy initially seems to be framing the problem as a quantitative one, an occasion to apply physics formalism, laying out all the numerical quantities in the problem, apparently expecting some algebraic manipulations to answer the question. Sandy, on the other hand, seems to have a different expectation, asking whether they ‘even need to do all that calculation’ and doubting that ‘they’re asking for it [calculation]’. Viewing the question as an occasion for intuitive sense-making (rather than calculation), Sandy proceeds to construct a narrative of the physical mechanism by which the so-called ‘normal’ force changes magnitude. Sandy’s narrative is informal and significantly aided by gestures, which we note in the transcript between slashes (/like this/). Algebra is conspicuously absent. The other students give indications that they understand Sandy’s argument, and the group frames the next several questions that arise in terms of this informally mechanistic, almost kinesthetic analysis. […] Word choices further indicate the presence of contrasting expectations; for example, when Sandy says ‘Do we even need to do all that calculation?’ it communicates a sense that Tracy’s approach is excessive (‘even’, ‘all that’) and possibly uncalled for (‘do we need to?’). Sandy’s question marks a bid to define the problem as an occasion for kinesthetic sense making rather than quantitative analysis. The group’s definition of the problem type will dramatically affect their next steps; the choice is between one set of resource activations and another, each set including epistemological resources for understanding what sorts of knowledge are relevant” (Hammer et al., 2005). From this and other examples, the authors outline a definition of framing as “(i) the forming of this set [of epistemological resources], and then, once it is formed, (ii) the use of those resources to interpret utterances, sensory inputs, and so on” (Hammer et al., 2005).

In a very recent article, Radoff, Jaber and Hammer (2019) offer another account of a student’s transformation in learning physics. Marya is student in a general physics course, and in the first phases she had been expressing her difficulties and feeling of anxiety about the course. The authors argue that her progress mainly involved *meta-affect*, (i.e., her feelings about feelings) and *epistemology* (i.e., her sense of what it means to know and learn). I report here an excerpt of Marya’s response to an online problem (shown in figure 2.2):

Marya: I think enemy ship 1 has the greater speed because its parabolic trajectory shows a steeper positive slope than does enemy ship 2. If we were to go back to the two time values at which the projectiles are at zero, the second value (where the projectile hits the ship) is dependent on the initial speed and the gravitational pull [(2 × initial velocity)/g]. The greater the speed in the nominator, the greater the result of the fraction meaning the greater the time. Enemy ship 2 will be hit first because it has the lower speed.

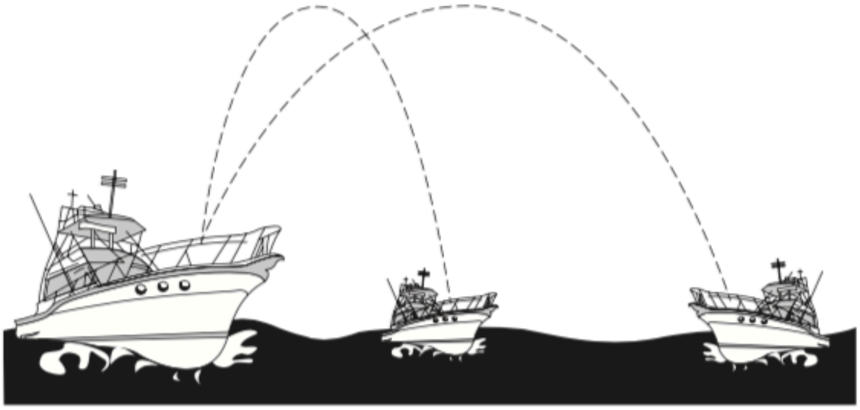


Figure 2.2 “A destroyer simultaneously fires two shells with different initial speeds at two different enemy ships. The shells follow the parabolic trajectories shown. Which ship gets hit first?” (from Hammer, 2018 - in Amin and Levrini, 2018)

In a recent book that collects and puts in dialogue different theoretical perspectives on the issue of ‘conceptual change’ (Amin and Levrini, 2018), Hammer comments this extract underlining the contextual character of the activation of Marya’s resources, deploying the use of ‘conceptions’ as good theoretical descriptors of her understanding. I again report an extensive part of the discussion: “There is a rationality to her response, which is important to appreciate. But what does it indicate about Marya? Accounts of conceptual change have traditionally focused on identifying and addressing students’ misconceptions. To me, though, her response does not indicate she has a misconception. […] In making sense of Marya’s response, I consider the dynamics of the moment. They involve resources I can attribute to Marya, including her sense of slope: I’m sure she sees slope easily, in terrain, in images, across many contexts. She has a sense of time as a duration, which she applies in reasoning that larger t for #1 means #2 hits first. There’s also evidence she has the symbolic form “prop+” (Sherin, 2001), by which she reads the expression $t = 2v_0/g$ as saying that larger v_0 means larger t”. […] Her response as a whole is a local “soft-assembly” (Thelen & Smith, 2006) of a system that extends beyond Marya herself. The set of activations and connections nests within and involves features of the situation: She’s responding to checkpoint questions after watching a formula-intensive prelecture within a required course she finds intimidating. It is part of that soft-assembly that Marya misconceives as an association of slope with speed, but only a part: It would be a mistake to attribute that to her as a misconception she has as an individual. In just about any other situation involving motion, Marya would think differently. It’s hard to imagine her on a tennis court explaining that she’d need to hit the ball more slowly to lob it a greater distance”.

In the same article (Radoff, Jaber & Hammer, 2019), the authors analyze other episodes, which I don't report here, showing a shift in Marya's epistemological frame, from "throwing symbols all over the place", to "trying to honestly have a good grasp of what was going on". On these bases, they provide evidence for an entanglement of some of her epistemologies with meta-affective stances, highlighting the co-occurrence of epistemologies about physics and feelings about being uncertain (table 2.1):

Table 2.1 Entanglement between Marya's epistemologies and meta-affect. From (Radoff, Jaber & Hammer, 2019)

Epistemology	Meta-affect
Physics is about absolute rights and wrongs	Anxiety about feeling uncertain
Physics is about the journey and the question	Comfort with feeling uncertain
Doing physics is a process of making sense of the world	Excitement about feeling uncertain

These few examples show an irreducible intertwining between epistemologies and learning, highlighting that, besides conceptions, attention must be paid to students' epistemologies as well. A big amount of work has been done, up to now, in searching for epistemologies in introductory physics courses. However, this kind of research is still a little bit under-explored in the QP domain. In what follows I provide an outline of the main results about epistemologies in QP learning.

Evidences of students' epistemology in learning quantum physics

In the context of quantum physics, most of the research is indeed focused on conceptual issues and on how to improve understanding. Even though the results gained about student epistemologies in other contexts trace a direction indeed useful also for learning QP, this subject raises also new specific epistemological issues. In fact, some authors, in line with the results I gained in previous analysis, highlight that the main difficulties in learning quantum physics often do not concern the logic of the formalism or the concepts themselves, but epistemological and ontological aspects.

Mannila & Koponen (2001) point out, that when learning QP, "for students the main difficulty lies in the conceptual shift needed in order to form a new ontology". Brookes & Etkina (2007) explored the role of metaphors in teaching QP, pointing out that "students categorize concepts into ontological categories based on the grammatical structure of physicists' language", and that their learning may be "distracted by an overly literal interpretation".

Levrini and Fantini (2013) analyzed students from a course on QP for High School, highlighting some of them arguing that the "formalism was necessary but not sufficient to have the feeling of understanding: comprehension requires the ‘formal mechanism’ to be interpreted also in terms of (smooth) links to ordinary language and classical description”. The authors pointed that instruction in QM should compare and contrast epistemologies in classical and QP.

Baily and Finkelstein (2010; 2015) studied the role of interpretations in teaching QP. In a study led in 2010 they investigated effective relevance of teachers’ choices about issues of interpretative nature. The study regards a statistical survey conducted in two university classes, dealing with courses on QP. The courses’ structure was quite the same, but the respective teachers chose two different positions about dealing with interpretative issues: one opted for an ‘agnostic’ position, and the other one chose to deal explicitly with them, often taking a ‘realist’ stance on the electron description (for example, explicitly assuming that in the double slit interference experiment the electron passes through only slit, being it a tiny particle). The results show that those students who dealt with the ‘agnostic’ teacher tended to maintain more easily a ‘realist’ and still purely classical visualization of the phenomenon and of the electron itself. In (Baily & Finkelstein, 2015) the authors drew on these findings in designing a reformed course in which, with respect to more conventional courses, “students developed more consistent interpretations of quantum phenomena, more sophisticated views of uncertainty, and greater interest in quantum physics”.

Mason and Singh (2010) found most students in their QP course did not learn from mistakes on exam problems, for example about how to manipulate bra and ket.

Bing and Redish (2012) used the notion of ‘frame’ (introduced in the previous paragraph) to analyze problem solving in QP, identifying four main types of frames among which students shifted dynamically: (i) calculation, (ii) physical mapping, (iii) invoking authority, and (iv) mathematical consistency. The authors argue that part of the ability in problem solving can be gained from the coordination of these framings.

Dreyfus, Sohr, Gupta and Elby, conducted a series of studies on students’ sense-making in QP. In (Dreyfus, 2015), they examined video recordings of a focus group reasoning about the properties of a quantum particle, documenting, within the discussion, the occurrence of “metacognitive moments when the group explicitly considers whether classical intuitions apply to the quantum system”. The authors support with these data the claim for a spontaneous ontological flexibility in students reasoning, as well as for the spontaneous engagement of some students in metacognitive reflections. In (Sohr, 2016), they focused on a students’ sense-making with the use of inscriptions, documenting that his inscriptional system shapes and is shaped by his sense-making and causal reasoning. The authors suggest that in this context “providing opportunities for flexible use of representations might involve creating an instructional environment for sense-making”. In (Dreyfus, 2017), the authors analyze the video recording of a discussion between two upper division physics students tackling with a QP problem, and from these data they claim that mathematical sense-making in QP has a continuity with mathematical sense-making in introductory physics. However, they point out, in QP “the connections between formalism, intuitive conceptual schema, and the physical world become more compound (nested) and indirect”. The authors, along Sherin’s work (2001), also identify two symbolic forms (‘transformation’ and ‘eigenvector-eigenvalue’) that seem to be present in students reasoning.

Dini & Hammer (2017), present a case study of a "successful" student's over two semesters of QP course investigating variations in personal epistemology. The authors noted the persistence of his search for a connection between mathematics and physical meaning, arguing that this was part of his success. At the same time, concerning scattering, they provide evidence for him to be to entirely stable in his framing.

Models of personal epistemology

The research on epistemology in learning has been developing upon a diversity of cultural traditions and theoretical frameworks, and has been identified under different names: epistemological beliefs, reflective judgment, ways of knowing, epistemological reflection, epistemological theories, epistemic beliefs, and epistemological resources. Up to now, a generally accepted term to comprehend all of what is related to epistemic issues is 'epistemic cognition', or 'epistemic thinking' (Kitchener, 2002; Green at al., 2008; Chinn et al., 2011; Hammer & Elby, 2016; Smith, 2018). In what follows I will give a brief account of the evolutions of the models of epistemic cognition, based on some comprehensive reviews (Hofer and Pintrich, 1997; Schommer, 1994; Elby, Macrander, Hammer, 2016; Dini, 2017). Within the description, I will go a little more in detail with two models that have inspired my research and that are part of the methodology I used to bring out the analysis: the AIR model (Chinn, Buckland & Samarapungavan, 2011; Chinn, Rinehart &Buckland, 2014) and the 'resource' model (Elby & Hammer, 2010; Hammer & Elby, 2002, 2003; Hammer et al., 2005; Redish, 2004).

Early developments

Research on personal epistemology has its origins in the research of William Perry (1968/1999), and the subsequent studies that went along the same Piagetian developmental perspective. These works modeled epistemologies in terms of subsequent stages of sophistication ("positions", with the words of Perry), essentially considering the evolution to be happening on a single dimension, going from (i) a dualistic /absolutist stage, where children search for right and wrong claims, verified through authority and/or direct experience, passing through (ii) a multiplist stage, where subjects begin to see knowledge as a subjective construction, taking in account multiple valid perspectives, and finally (iii) an 'evaluativist' stage, where competing claims can be debated and judged.

In 1990, Schommer proposed a multi-dimensional account for students' epistemologies, claiming (after the analysis of questionnaires administered to more than two-hundred students) for the need of a more complex account with respect to what had been proposed previously. She proposed epistemic cognition to consist of five independent dimensions of beliefs (Schommer, 1990; Schommer, 1994):

- source of knowledge: from knowledge is handed down by omniscient authority to knowledge is reasoned out through objective and subjective means
- certainty of knowledge: from knowledge is absolute to knowledge is constantly evolving

- organization of knowledge: from knowledge is compartmentalized to knowledge is highly integrated and interwoven
- control of learning: from ability to learn is genetically pre-determined to ability to learn is acquired through experience
- speed of learning: from learning is quick or no-to-all learning is a gradual progress

Hofer and Pintrich (1997) proposed a deep review of what had been done up until that year, organizing the theoretical dimensions of each perspective in accounts for the nature of knowledge, the nature of knowing and peripheral beliefs about learning instruction and intelligence. They proposed a two-dimensional model, divided in two dimensions for the nature of knowledge, and two others for the nature of knowing (they rejected Shommer's last two dimensions, claiming them to regard 'learning', and not 'knowing', and thus to be not epistemological):

Nature of knowledge:

- certainty of knowledge: the degree to which one sees knowledge as fixed or more fluid. At lower levels absolute truth exists with certainty, at higher levels knowledge is tentative and evolving
- simplicity of knowledge: knowledge from viewed as an accumulation of facts to knowledge as highly interrelated concepts

Nature of knowing:

- source of knowledge: at a lower level, knowledge originated outside the self and is transmitted from an authority. At higher levels, the self is conceived as a knower, with the ability to construct knowledge in the interaction with others
- justification for knowing: how individuals evaluate knowledge claims, including the use of evidence, the use they make of authority and expertise, and their evaluation of experts

The AIR model

This process of refinement of the dimensions of epistemic cognition went on until very recent years. The AIR model (aims, ideals, reliability) is one of the most notable efforts in this direction; it is the result of the reflections brought out by Chinn and colleagues (Chinn, Buckland, & Samarapungavan, 2011; Chinn, Rinehart, Buckland, 2014). In the article appeared in 2011, they built a comprehensive framework for epistemic cognition integrating philosophical insights and substantially extending the outlined framework of Hofer and Pintrich (1997) and other models of metacognition in the following ways: (i) they argue for a context-dependent nature of epistemic cognition, (ii) they include non-belief-like components such as epistemic aims, epistemic values and epistemic virtues, and (iii) they include a component regarding the reliability of processes to achieve epistemic aims, coming from the philosophical tradition of reliabism. Epistemic cognition consisted of a network of five interconnected dimensions:

- *epistemic aims and values*. Epistemic aims are a subset of the goals people adopt, specifically those goals related to inquiry and finding things out. The authors claim the epistemic aims to be context-specific. The three main types of epistemic aims are: (i) *knowledge* - beliefs that accurately represent particular aspects of the world and that are supported by adequate reasons, (ii) *acquiring true beliefs / avoiding false beliefs* – 'conservative' believers will be cautious about adopting new beliefs, at the opposite of 'liberal' believers, who will be open to adopt new beliefs even if risking to fall in false ones, and (iii) *understanding / explanations* – distinct from knowledge in the search for explanatory connections between items of information. Epistemic value refers to the worth of specific epistemic achievements. Whether learners adopt aims in line with their judgments of value is likely to depend on their judgments of the costs of pursuing the aims relative to the value of the resulting achievements

- *structure of knowledge and other epistemic achievements*. The authors propose a multi-dimensional extension of what proposed by Schommer (1990) and by Hofer and Pintrich (1997). The main structural forms are: (i) simplicity versus complexity of knowledge, (ii) universality versus particularity of knowledge, (iii) deterministic versus stochastic knowledge, (iv) structures of explanations: general law, causal explanation, etc., and (v) specific structural forms: structure of mechanisms in molecular biology, structure of models in mechanics, forms of causal knowledge

- *sources and justification of knowledge and related epistemic stances*. Extending the dimensions discussed by Hofer and Pintrich (1997), the *source of knowledge* refers to where knowledge originates. The main sources identified by philophers (Steup, 2005) are (i) perceptual, (ii) introspection, (iii) memory, (iv) reasoning, (v) testimony. *Epistemic stances* refers to the attitudes that people take with respect to an idea, such as believing it, doubting it, tentatively endorsing it, holding it as absolutely certain, or entertaining it as a possibility. *Justification* refers to people's reasons for their beliefs. The authors recall philosophers justificatory 'standards', divided in two categories:
    i. evidential standards – evidence is used to justify beliefs. Large range of data, low number of anomalies, mathematical precision, statistical tests, case studies, ect.
    ii. non-evidential standards - beliefs are justified to the degree to which they cohere with other established beliefs, simplicity of a belief system, internal logical consistency, elegance, how understandable it is to other scientists, fruitfulness in opening up new lines of research, etc.

- *epistemic virtues and vices*. Epistemic virtues are praiseworthy dispositions of character that aid the attainment of epistemic aims. In contrast, epistemic vices are those dispositions that hinder the achievement of epistemic aims. Examples of epistemic virtues are open-mindedness, conscientiousness, intellectual carefulness, perseverance, humility, vigor, flexibility, courage, thoroughness, etc. Examples of epistemic vices are dogmatism, unwillingness to give up beliefs, need for closure, etc.

- *reliable and unreliable processes for achieving epistemic aims*. This component concerns the processes are achieved. The processes fall into four broad categories:

cognitive processes, formal processes for conducting inquiry, interpersonal processes, and community processes

Chinn, Rinehart and Buckland (2014), synthetized the structure of this model in three components, discussing how this model can help to understand how people evaluate information: (A) the epistemic *aims* one might pursue and the values placed on these aims, (I) the epistemic ideals one holds which are used as standards to evaluate whether one has achieved one's epistemic aims, and (R) the knowledge of reliable processes that can be used to achieve epistemic ends. Each component includes many subcomponents (most of which are comprised in the five-layered structure in Chinn et al. (2011)) which may be applied differently in different contexts.

- *Epistemic aims.* Epistemic aims are a subset of goals that people adopt, specifically those goals related to inquiry and finding things out. The authors claim the epistemic aims to be context-specific. The three main types of epistemic aims are the same as in the previous model (Chinn, Buckland, & Samarapungavan, 2011)
- *Epistemic ideals.* The epistemic ideals specify criteria or standards that must be met for a person to judge whether his/her epistemic aims have been achieved. In other words, they are criteria used to justify the acceptance of an epistemic product, such as an explanation. The authors propose five broad categories of epistemic ideals: (i) ideals about the internal *structure of an explanation*, (ii) ideals about *connections with other knowledge* (e.g. cohering with other explanations), (iii) ideals about present and future *connection to empirical evidence*, (iv) ideals to believe someone else's *testimony* (e.g. sincerity, willingness to know), (v) ideals of *good communication* (e.g. clarity, comprehensibility)
- *Reliable processes.* This component specifies consists of schemas specifying the reliability of processes by which epistemic aims (or products) can be produced. Individuals must acquire a large number of schemas about how (the conditions under which) processes produce reliable knowledge. Processes can involve different sources of knowledge (same as in Chinn et al. (2011))

'Epistemological resources' model

The resource-based view of epistemic cognition is part of a wider model of students' reasoning, the 'resource model', a fine-grained model of students thinking based on insights from neuroscience, cognitive science and behavioral sciences (Elby & Hammer, 2010; Hammer & Elby, 2002, 2003; Hammer et al., 2005; Redish, 2004). This model challenges the ontology of epistemologies as conceived as unitary 'theories', 'beliefs' or 'positions', analogically to how the it challenges the misconception models for what concerns reasoning in general. In fact, research about students' reasoning has been largely focused on robust and coherent conceptions, or naïve theories, often assumed to resemble theories constructed by natural philosophers throughout history (Caramaza, McCloskey & Green, 1981; Vosniadou & Brewer, 1992). This theory is often referred to as the 'misconception', 'naïve conception', or 'alternative conception' theory. In this perspective, when a conception is incorrect, it can impede learning and needs to be 'replaced' with a correct one. Even though this view can be useful for specific aims and it can be appropriate to describe robust and persistent cognitive structures, it goes in contrast with a body of empirical research in learning experiences that claim for short time-scale and context-dependent changes in conceptions, where student reasoning might appear fragmented and inconsistent (McDermott, 1984; diSessa, 1993).

In the same way that conceptual change researchers have described students' prior knowledge as consisting of stable, robust misconceptions that cannot contribute to expert understanding, epistemology researchers have mostly described naive epistemologies in terms of stable, counterproductive beliefs that must be replaced in order to achieve sophistication (Hammer & Elby, 2003). Evidences of contextual variability of epistemologies had been observed in different studies (Roth and Roychoudhury, 1994; Kienhues et al, 2008; Leach et al., 2000; Muis & Geich, 2014, Porsch et al., 2011), challenging the unitary perspective. The model of epistemological resources foresees that people have metacognitive and epistemological resources for understanding knowledge in wide variety of forms:

- *source of knowledge* (Knowledge as transmitted stuff, Knowledge as fabricated stuff, Knowledge as free creation, and others)
- *forms of knowledge* (Story, Rule, Fact, Game, and others)
- *knowledge-related activities* (Accumulation, Formation, Checking, and others)
- *stances toward knowledge* (Acceptance, Understanding, Puzzlement, and others).

Within this model, Redish (2004) proposed the notion of 'epistemological framing', connecting epistemologies with the literature on frames. A 'frame' is a 'structure of expectations' about what is taking place; for what concerns epistemologies, a frame answers to the question "how should I approach knowledge?" (Hammer et al, 2005). The setting of a frame acts a choice between one set of resource activations and another, each set including epistemological resources for understanding what sorts of knowledge are relevant, metacognitive resources for forming and manipulating those kinds of knowledge, and of course conceptual resources. Framing, then, is (i) the forming of this set, and then, once it is formed, (ii) the use of those resources to interpret utterances, sensory inputs, and so on. (Hammer et al., 2005).

4.2 CONTEXT AND METHODOLOGY

Data collection and selection

The data I collected entirely come from High School students attending an introductory course on Quantum Physics we designed and implemented in January / February 2019 (described in detail in chapter 2 of this dissertation). The course aimed at tapping the main concepts of QP by (DP1) creating a productively complex learning environment where students can find their own appropriation of the contents (Levrini & Fantini, 2013; Malgieri et al., 2018), (DP2) foreseeing moments for 'epistemological talks', where the students can discuss their own understanding of controversial themes or of the nature of science itself (following Bungum, Bøe and Henriksen, 2018), and (DP3) identifying those key scientific concepts and applications of QP that allow to look towards the future in the multiplicity of its dimensions (Tasquier, Branchetti, Levrini, 2019). The teaching module consisted of 6 post-school meetings, 3 hours each, held at the Department of Physics and Astronomy of the University of Bologna (Italy). 21 students applied for and attended the course (15 males and 6 females), most of which didn't know each other previously because they came from different schools and cities in the region around Bologna. 17 students were enrolled in a scientific High School program (Liceo Scientifico), and only 3 in a 'classical' one, more focused on classical studies of humanities, like Greek and Latin cultures and literatures (Liceo Classico). 18 students were attending their last year of High School (5th year), whereas 3 of them were in their 4th. None of them had previously studied anything about Quantum Physics at school, even if, indeed, all of them were interested in the argument (attending the course was not mandatory for anyone, but a free and personal choice). I report here (table 2.2) the synthetic structure of the teaching module, described in detail in Part II of this dissertation.

Table 2.2 Structure of the teaching module on QP (see Part II of this dissertation)

PARS DESTRUENS	**1. Lab activity 1: photoelectric effect** - Hallwachs' experiments (qualitative) - Lenard's experiment (quantitative) - Einstein's interpretation Epistemological talk: models and representations	group LAB + lecture
	2. Lab activity 2: atomic spectra - Rydberg constant and atomic spectra (quantitative) - Bohr's atomic model Epistemological talk: models and representations	group LAB + lecture

BRIDGE	3. **The most beautiful experiment** - Davisson-Germer experiment (demonstrative) - conceptual / experimental / linguistic challenges of the most beautiful experiment - Levy-Leblond's quanton	demonstrative LAB + 2 lectures
PARS CONSTRUENS	4. **New logic of QP (1): superposition states** - Stern-Gerlach experiments: superposition of quantum states - two-state systems: same formalism for different physical systems	lecture + activities
	<u>Epistemological talk: QP comprehensibility</u> 5. **New logic of QP (2): entanglement** - QBIT & entanglement - Quantum Cryptography	lecture + activities
	6. **New logic of QP (3): applications** - 'Quantum Manifesto' - Computing and teleportation	lecture + activities

Given the complex focus of this study, involving different dimensions of students' learning dynamics, I designed the data collection in a way that could possibly provide the most comprehensive picture of their attitudes and personality through the whole duration of the course. The whole corpus of data consists of a diverse variety of sources, specifically:

source 1. video recordings of every lecture of the teaching module, most of which were dialogical and interactive

source 2. a personal written exercise where the students are asked to write down a free explanation of the phenomena observed in the LAB (photoelectric effect and atomic spectra) and to represent them with an image

source 3. a personal written exercise on Stern-Gelarch configurations, where the students are asked to foresee the probabilities for different set-ups

source 4. an online questionnaire, given to the students as homework after the fourth meeting, about their own perception of QP, asking them to compare it with classical physics (CP) in terms of language, formalism, and imagination.

source 5. a video recording of a discussion with the whole class about their views on the comprehensibility of QP (40 minutes long), happened at the beginning of the 5th meeting

source 6. a personal written final exam (without evaluation), made of closed and open questions, meant to be a synthesis of the main concepts introduced in the course

source 7. 9 post-course semi-structured interviews with individual students

Once the data collection had ended, at first I went through all the data to have an overview of what kind of data could be more useful for the purposes of this study. Indeed, the more the data are rich, idiosyncratic and open-ended, the more they can be useful in searching for students' epistemologies. Due to this first outlook, I chose the class discussion (source 5) and the semi-structured interviews (source 7) as the main sources where to carry out the qualitative analysis. The other sources were used for minor purposes, as counterchecking or contextualizing specific claims about the students, or to sketch out general tendencies in the class.

Both the selected sources were at first transcribed in Italian, and then translated in English. In transcribing, I occasionally wrote down explicit descriptions of students tone and gestures (in the case of the video recording), but only when it seemed to be relevant for the analysis. Indecision or thinking pauses are expressed with '…', emphasis with '!'. Overlapping voices and gestures are expressed in brackets. As an example, I report the following excerpt from the class discussion:

d56 S4: No, I mean… I have personal perspectives [*he agitates his hands as he is referring to something*
d57 *'far'*] ... I would say that physics is only a measure of reality and it's not the absolute truth… it's only
d58 a facet of reality, which can give us beliefs that can also be, let's say, exhaustive about nature, but
d59 it will never give us more complex aspects that can be interpreted with a logic that starts from
d60 different bases, and that is still valid, like art, philosophy…

d61 Professor1: Do you [all] agree on these things?

d62 S4: …and therefore there is no absolute truth.

d63 S5: In my opinion, what he says is true but, since we perceive reality for what it is, even if there is
d64 another way of measuring another reality we still must have answers about what is happening now
d65 [*hand up and down, to enforce the concept*]. I mean, in my opinion, it's ok to say that we are not able
d66 to arrive at the 'true truth' of things, but only to what we perceive. However, for me it's already a very good
d67 starting point to be able to understand [*her head is turned towards S4, and with the eyes and hand*
d68 *enforces the word 'understand'*], let's say, how the truth that we perceive works and then understand if
d69 there exists another 'true truth'... a more 'true truth'… [*class laughing*]

Source 5. The class discussion has been held by two professors (PR1 and PR2) with the whole class at the beginning of the 5th meeting. This moment, aligned with the design principle DP2, was meant to be an 'epistemological talk', where students could express their own views and perceptions of QP and of their learning of QP. The main topic of this epistemological talk was the 'comprehensibility' of QP. One of the professors (PR1) was the main conducer of the discussion, whereas the other (PR2) only intervened in specific moments. By choice, most of PR1's hints were pointing at bringing out students own 'voices', by posing open-ended questions without a pre-known answer. For example, in a moment where PR1 had asked "Do you think that Quantum Mechanics explains phenomena?", and discussion was stuck in philosophical arguments, PR1 shifts the attention to students' feeling of understanding, asking "Try to think when you have the feeling 'oh, this thing has finally been explained to me!", raising more personal answers. The active participants have been 9 students (only one female, S5), out of 21. In figure 2.3 a simple map of the succession of interventions is reported (each intervention is considered here to have the same duration).

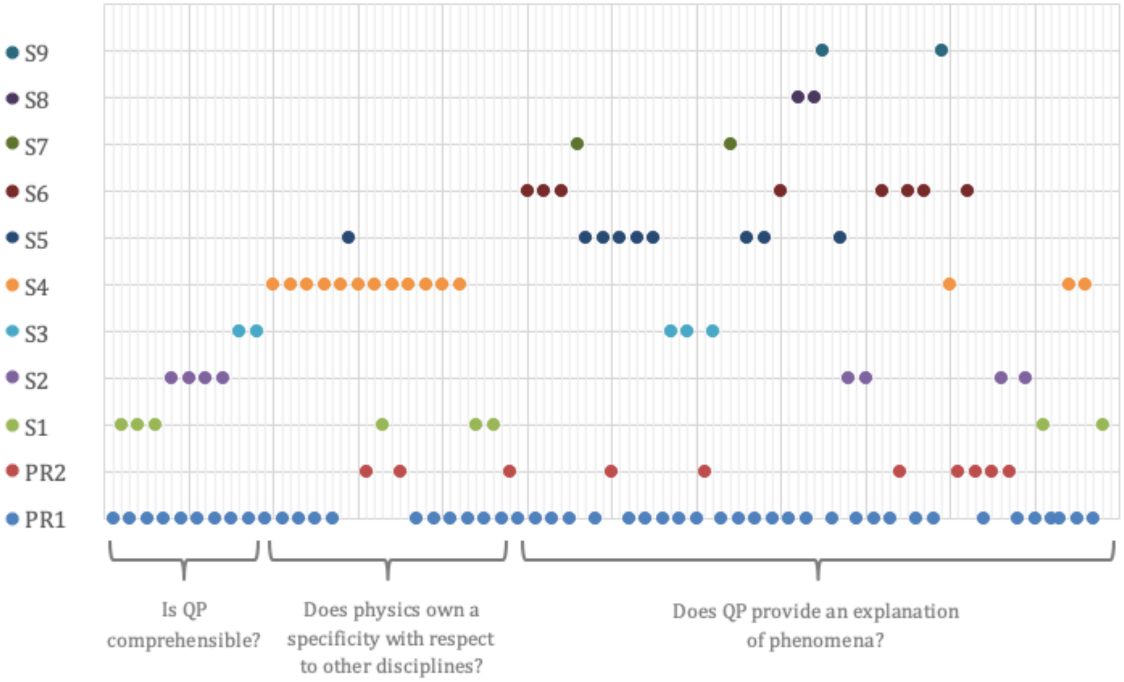


Figure 2.3 temporal sequence of the interventions in the class discussion

Source 7. The semi-structured interviews followed a specific protocol of questions. They were all recorded within 1-2 weeks after the end of the course. The kind of questions I chose do not explicitly address students' epistemologies, or at least the major part; this would probably create a biased situation. Instead, I engaged subjects mainly pointing at their experience of the course, and only on the basis of their responses. I designed a list of questions to do in the interviews, but, being semi-structured, I felt free to follow what was emerging in the moment to orient the discourse in a not-invasive way; for this reason, I sometimes also altered the order of the questions or passed over some of them. The sequence of questions was the following:

Table 2.3 Questions of the semi-structured interviews (source 7)

1. What did strike you most about the topics covered in the course? What do you take home with you?
Questions on specific concepts:
2. Thinking about the path we followed, what are the concepts that you think to have understood better, and which worse? What difficulties did you encounter? Are there any concepts that you think you could not understand at all?

3. Which words would you use to describe a quantum object? What differentiates it from a classic object?
4. We met the superposition principle in various moments of the course. What can you say about the it for each of these cases? How the quantum superposition of states does differ from the classical superposition of waves?
5. What did you understand about the entanglement? What do you think about it? Is it comprehensible for you?
6. Which tools among images, analogies, models, experiments, formalisms, exercises, ect., help you the most to understand the main aspects of QP as it was proposed in the course?

Questions on learning QP:

7. To what extent is quantum mechanics comprehensible for someone who wants to learn it?
8. How is quantum physics positioned with respect to your previous knowledge in physics?
9. What is the core of QP? Would you say that you have a synthetic image, or a mental organization, of QP contents?
10. To what extent is quantum physics revolutionary for you?

General questions about the course:

11. Do you think the course was stimulating for the class? Did it foster debates and dialogues between you and the others?
12. Did you observe any difference between the way in which the path on QP was proposed with respect to the way you have been introduced to other topics?
13. What expectations did you have about the course?

Personal questions:

14. During the course, did it happen to you to connect some concepts of QP with a specific philosopher, writer or artist? If not, can you attempt a connection now?
15. Which university would you like to enroll in the next year? Why?

Analysis methods

The analysis of students' interactions was carried out in two phases.

1. At first, I went through the class discussion (source 5) looking for evidences of the occurrence of personal stances – with a special attention on acceptance issues - and for the emergence of epistemic needs (particularly the three of visualization, comparability and reification pointed out in the previous analysis). Indeed, following the grounded-theory approach, I tried to maintain at the beginning a wide perspective and to let the epistemologies emerge from the data also beyond the three-layered framework identified previously. A complex interaction such as the one of the whole class can indeed offer a rich account of students' epistemologies and stances, inasmuch they are raised in a spontaneous and comparative environment; at the same time, this first account has been

the starting point to select those students that I considered the most interesting, for the purposes of this study, to be interviewed personally at the end of the course. I used discourse markers to identify epistemic needs coming from the previous experimentations. For example:

Table 2.4 Discourse markers for the epistemic needs

	Discourse markers examples
Need of visualization	"I usually imagine this as..." "I need to visualize" "I would represent this with…"
Need of comparability	"In classical physics it was different" "I can't find and analogous" "analogically…"
Need of reification	"A physical object must have its own defined properties" "the ball is round, and the state?"
Acceptance	"I can't accept that a quantum object does not own properties" "I can rely on this…"

To keep the analysis still open, without narrowing too much the analytical framework, I had three sessions of intensive analysis with the whole PER group of the University of Maryland, where I received rich feedbacks and insights about students epistemological and affective aspects. I repeated the same kind of analysis on the transcribed semi-structured interviews (source 7), so to sketch a characterization of students' stances and to individuate the most relevant moments in the interactions with them.

2. In a second phase, I went more in detail in the analysis of students' epistemologies, grounding the analysis on the current literature on epistemic cognition. The aim of this part was to widen the theoretical account of students' emergent needs and stances, looking at more dimensions, according to the existing literature. To this extent, I used the AIR model of epistemic cognition (Chinn, Buckland & Samarapungavan, 2011; Chinn, Rinehart & Buckland, 2014) as the main comprehensive reference; however, I borrowed definitions and descriptors of the components of students' epistemologies also from other theoretical frameworks, and mainly from the 'resource' model (Elby & Hammer, 2010; Hammer & Elby, 2002, 2003; Hammer et al., 2005; Redish, 2004). This choice is basically of a practical nature: as far as this research field is evolving and currently undergoing through a period of dramatic coordination between different models and research traditions, and given that this study does not aim to confirm or propose structural changes to the current models of epistemic cognition, it seems to be worthy to have more than one vocabulary to give account of students' epistemologies in this context. Of

course, attention needs to be paid to the differences of the models, as to not fall into inconsistencies within the analysis; but I'm prone to think that, at this stage of the research, the theoretical accounts for students' epistemic cognition that I will use for this analysis are neither mutually exclusive nor self-complete. In fact, even though they differ in the basic structural elements that characterize epistemic cognition, the two models share substantial commonalities. I want to clarify here two main theoretical riverbanks that I am implicitly applying in analyzing students' discourses, and that the two models take into account in their structures:

- epistemic cognition is context-dependent. Even if it's sometimes useful to underline stabilities and general tendencies in students' epistemologies, the kind of data we are trying to go through needs a quite fine-grained analysis;
- affection and identity-related aspects do have an influence on students' epistemologies.

The affective dimension has been claimed to be entangled with epistemologies and epistemological stances (Radoff, Jaber & Hammer, 2019); seen the origin of this study (non-acceptance of QP), I included affection in the dimensions of the analysis. On these bases, I built a schema of dimensions to analyze the interactions with the students, choosing to search for (i) references to the nature of scientific knowledge, (ii) references to the nature of knowing (distinguishing between aims, sources of knowledge, reliable processes of knowledge constructions, structure of explanations and justificatory standards), and (iii) evidences of affect-related aspects, and in particular of meta-affection evidences, so to eventually relate them with the epistemic traits. In the two following tables I report synthetically some of the theoretical accounts for each dimension (table 2.5), and some examples of related linguistic markers (table 2.6), coming both from my own analysis and from previous studies in the literature, as the one of Dini & Hammer (2017)):

Table 2.5 Examples of theoretical accounts for each dimension of the analysis scheme

Analysis dimensions		**Theoretical accounts**
Nature of knowledge	Certainty	SCHOMMER 1990, 1994 absolute opposed constantly evolving HOFER AND PINTRICH 1997 certainty opposed to tentativeness
	Structure	SCHOMMER 1990, 1994 from knowledge is compartmentalized to knowledge is highly integrated and interwoven HOFER AND PINTRICH 1997 simplicity opposed to complexity CHINN ET AL. 2011 simplicity versus complexity universality versus particularity deterministic versus probabilistic specific structural forms: structure of mechanisms in molecular biology, structure of models in mechanics, forms of causal knowledge…
Nature of knowing	Aims	AIR MODEL / CHINN ET AL. 2011 Knowledge Acquiring true beliefs / avoiding false beliefs (conservative vs liberal) Understanding / explaining / making useful models
	Sources	HAMMER 1994 Learning physics as: (i) receiving information or (ii) an active process of reconstructing one's understanding RESOURCE MODEL Knowledge as transmitted stuff, Knowledge as fabricated stuff Knowledge as free creation SHOMMER 1990, 1994 handed down by omniscient authority VS reasoned out through objective and subjective means CHINN ET AL. 2011 perceptual – through one of the five senses introspection - examination of the contents of their own minds, knowledge of one’s own internal experience memory – knowledge stored in a persons’ past reasoning – application of rules. ‘A priori’ knowledge, without accessing experience

		testimony - all social forms of sharing information and knowledge with others
	Reliable Processes	CHINN ET AL. 2011 cognitive processes formal processes for conducting inquiry interpersonal processes community processes
	Justifications	CHINN ET AL. 2011 Evidential standards: evidence is used to justify beliefs. Large range of data, low number of anomalies, mathematical precision, statistical tests, case studies, ect. Non-evidential standards: coherence with other established beliefs, simplicity of a belief system, internal logical consistency, elegance, how understandable it is to other scientists, fruitfulness in opening up new lines of research
Meta-Affect		RADOFF ET AL (2019) Anxiety about feeling uncertain Comfort with feeling uncertain Excitement about feeling uncertain

Table 2.6 Examples discourse markers for each dimension of the analysis scheme

Analysis dimensions		**Discourse markers examples**
Nature of knowledge	Certainty (that is knowledge as certain and given VS knowledge as tentative, evolving)	“it could take an entire life to create a model”
	Structure (that is, for example, simplicity vs complexity or deterministic vs probabilistic, or specific structural forms, like structure of mechanisms, models, forms of causal knowledge……)	“I need to know a fact before moving on...” “We can only know to a certain probability...” “None of these ideas is related...”
Nature of knowing	Aims	“I want to understand...” “I need to remember this idea...”

	Sources	“The teacher said so...” “I built the idea from mathematics...” “I used my intuition...” “I measured it, so I know it”
	Reliable Processes of	“you give me a mathematical formula, I put a little effort in it, and I am convinced
	Justification	“so that the argument is coherent with the axioms of mathematics” “the data show it!”
Meta-affect		“I was a little disappointed” “I’m very willing to know more”

From a theoretical point of view, the three epistemic needs we claimed for in the previous analysis can already be contextualized in terms of the AIR model. The need of visualization can be probably considered as emerging from a judgement about the reliability of visualization as a source of knowledge. The need of comparability can emerge either as an ideal about the coherence with other knowledge domains, or as a judgement of ‘comparing’ as a reliable process to build knowledge. The need of reification is more difficult to classify; even though ‘reification’ can be considered a process where mathematical considerations become properties of an object (Sfard, 1991), I think that this need emerges in students more as the search for an actor-object of causal / spatial-temporal explanations. However, the aim of this study is not to give a theoretical account of students’ needs, but instead to highlight the activation of patterns of epistemologies in the context of QP learning.

In the following sections, this analytic framework is applied to three students that, as previously stressed, I selected because of the diversity of their attitude toward QP and knowledge in general.

4.3 PIETRO (S2)

Pietro is male student attending the last year of a scientific High School. He is generally successful at school and he is strongly oriented towards beginning a degree in physics in the coming year. I want to discuss here the main epistemological traits that seem to be emerging in Pietro's reasoning about his own understanding of QP. The following excerpts come from the discussion with the whole class (marked "d" in the quotations numbering) held during the fourth meeting of the course (source 5), and from the semi-structured interview conducted with Pietro a week after the end of the course (source 7).

Pietro claims for a difficulty in learning QP that is not primarily tied to a specificity of the disciplinary content, as for example a specific aspect of the formalism itself. The challenge he feels seems to come from a dis-alignment of his expectations about knowledge and learning with the actual 'way of knowing' embedded in QP. The sources are analyzed in chronological order; the titles in italics are only meant to give a reference to orient through the analysis.

Source 5 – class discussion

"I measure it, so it is so": measurement as revealing

Pietro (student 2 here) intervenes at the very beginning and in the last part of the discussion. In this excerpt, he is answering to the question that the teacher asked at the beginning: "Do you think that QP is comprehensible? How much you think you understood what was proposed so far?". The issue Pietro claims to be struggling with in learning QP is the impossibility to "be able to see, to visualize" [line d17].

d16 S2 (M): In my opinion the most difficult aspect is that we are used in classical physics to be able to
d17 see, to visualize. That is, to have a model and to measure, above all, the things that we then can see,
d18 analyze. Instead in quantum physics for one reason or another, either they cannot be seen and
d19 visualized, as in the case of the electron through the two slits, or they cannot be measured, like the
d20 position that is not a datum but a probability. So I think it is difficult, very difficult for this...

d21 PR1: So, the difficult aspects are having to give up the visualization and this idea that there are
d22 properties that I can measure. These are two important points that break down [in QP]… what does
d23 it mean to measure in classical physics?

d24 S2: Usually in classical physics if you measure, you know

d25 PR1: 'If I measure, I know'. What does it presuppose? Why do I know?

d26 S2: Because I measure it, so it is so.

In classical physics, "to be able to see, to visualize" coincides with having "a model and to measure, above all, the things that we can see" [line d16]. Pietro is pointing out that the process of knowledge construction in QP is different, inasmuch it unravels the entanglement of visualization, measurements and modeling, that in classical physics are implicitly tied together as elements of the same act. He points out that, in classical physics, measurement is indeed a reliable source of knowledge, "in classical physics, if you measure you know" [line d24]; however, again this is not effortless to him. When the teacher asks: "Why do I know?", shifting the subject from the generic ‘you’ of his previous sentence to the first person, Pietro answers in first person and speaks about measurement as an objective means to gain evidence of the truth: “because I measure it, so it is so” [line d26]. It’s interesting that he doesn’t need to provide a justification for the reliability of measurement as a source of reliable knowledge, it’s just “because I measured it”. Thus, the tension here is due to the loss of reliability of measurement; it can no more be identified as a ‘direct’ source of evidence.

Source 7: semi-structured interview with Pietro

Meta-affection: excitement about challenges / difficulties

These are the first minutes of the interview with Luca:

1 INT: What impressed you most about the course, what do you take home with you?

2 PIETRO: Let's say that… I don’t have a single thing, a single thought. It is more like “ok, these are
3 the basics, let's see what's beyond, what can I build on this". Which is then one of the reasons that
4 will probably push me to do physics at university, rather than anything else.

5 INT: which other faculties would you choose?

6 PIETRO: ...computer engineering would have been the other. Let's say that, apart from the first two
7 years, that is mostly classical physics, so somehow already addressed... I did not compare them, but
8 the concepts are the same. From the moment that you start quantum physics onwards, it will be more
9 or less all new. Apart from these bases, very 'basic' indeed, it will all be a riot of novelties and various
10 oddities to be studied and understood. I take home a lot of desire to go and see how this new topic
11 works.

12 INT: So, what are the concepts that you think to have understood better and which ones that instead
13 put you in the most trouble… and why?

14 PIETRO: I start from the difficulties, because I like to start with difficult things. […].

Even though this is not visible from the transcript, the tone used by Luca to answer the very first questions Luca reveals enthusiasm about the course and excitement about it. When saying “let’s see what I can build on this” [line 3] he reveals that what he learnt is something reliable, he can be a basis from which he can go “beyond” it. In line 8 he is speaking about the possibility of studying QP at the University, and the possible “novelties” and “oddities” that he will eventually discover in the future are a source of curiosity, “desire to go and see how this new topic works” [line 10].

The possible oddities of QP are exciting because they have “to be studied and understood” [line 10]. In expressing his desire to “see how this topic works” [line 11], Pietro reveals that ‘understanding’ is his epistemic aim in this context. Furthermore, his excitement regards not only the future, but also the difficulties that he still feels about quantum physics after the teaching module. When the interviewer (the author of this dissertation) asks Pietro to talk about the concepts he thinks to have understood better and the difficulties he encountered during the course, Pietro decides to start from the difficulties he experienced, explicitly saying that he likes “to start from the difficult things” [line 14]. From the tone of his voice, he seems very eager to speak about the difficulty he encountered, speaking very fast and gesturing a lot. This *excitement* about difficulties / challenges in learning seems to be quite stable throughout the various data sources, and is clearly entangled with his personal epistemology. In fact, epistemologies and affection have been shown to be highly entangled, generating also meta-affective attitudes as for example 'excitement about uncertainty' (Radoff, et al, 2019).

Visualization, math and models: a demanding separation

Also in the interview, Pietro claims to be struggling with is the impossibility to "model in a visual way" [line 16].

14 PIETRO: I start from the difficulties, because I like to start with difficult things. The biggest difficulty,
15 in my opinion, is the impossibility, I don't know if it is temporary or if it is inherent to quantum
16 physics, of modeling in a visual way. I mean… it's modeled mathematically, a quantum object is
17 modeled mathematically, the wave function… but it cannot be imagined as an object. I mean, one
18 can't say "okay, this quantum object is a small sphere that has these properties, and behaves like
19 this, or otherwise based on what I do to them". I can't. Because if I make it become a sphere,
20 automatically, since it is a sphere, it has characteristics that the object does not actually own, or that
21 it has only in specific situations. An electron with a positive spin and an electron with a negative
22 spin are the same in themselves, but behave differently. I can't imagine an electron with a '+' drawn
23 above or with a '-' drawn above…

24 INT: Can't you just imagine it like that?

25 PIETRO: No, because it doesn't represent to me physically how it behaves. That is, an electron with +
26 drawn above is always an electron, but it behaves differently from one with a - drawn above.

Pietro seems to be aware that math and visualization are both involved in modeling, but that they are distinct and separable. In fact, he distinguishes the act of modeling a physical object mathematically from the possibility to imagine it, "it's modeled mathematically, a quantum object is modelled mathematically, the wave function… but it cannot be imagined as an object" [line 16]. However, this separation is not effortless to him. He shows to be aware that, in theory, a model is a model also when math is the only anchor to its meaning, but still this is "the biggest difficulty" [line 14].

In fact, for Pietro, a model and its representation must coherently coincide. In his words, "it cannot be imagined as an object" [line 17], the very term 'object' brings with itself an issue of definition: an object, to him, is something that can be seen and represented in a pictorial way. This stands out from his argument. The act of modeling the quantum object involves for Pietro the association with known representable models, as the one of a 'sphere' [line 18]. But in asking for a representation that could include the electron spin [line 21], he is claiming for a model, and thus a representation, that includes all the possible behaviors of the object. Thus, a model has to account for the physical behavior ("an electron with a positive spin and an electron with a negative spin are the same in themselves, but behave differently" [line 21]), but the representation has to be coherent with the model ("I can't imagine an electron with a '+' drawn above or with a '-' drawn above" [line 22]). When the teacher asks whether he could imagine it like that (as an electron with a + drawn above), his answer ("No, because it doesn't represent to me physically how it behaves" [line 25]) shows that he is not searching for a merely synthetic picture that could recall at once all the characteristics of the electron (as for example was found for the case of Jessica by Levrini and Fantini (2013)); he is searching for a 'visible' ontology to think about the quantum object. If the model is 'visible', that is, if its representation is the reification of all the physical properties of its behavior (the spin, for example), then it becomes a source of understanding. It seems that Pietro's needs of visualization and reification are deeply tied together, as to him an acceptable representation must be the objectified synthesis of all the physical properties.

"If I make it become a sphere…": agency in knowledge development

From the previous excerpt, another note can be done about it Pietro's attitude towards learning. Expressions like, "if I make it become a sphere" [line 19], "I can't imagine" [line 22], "it doesn't represent to me" [line 25] suggest that Pietro thinks about himself as an agent of knowledge development, and not only a receiver of knowledge. In terms of epistemological beliefs, he can probably be described in this context as conceiving learning physics not as "receiving information", but "an active process of reconstructing one's understanding" (Hammer, 1994), or as having an epistemology about the source of knowledge not "handed down by omniscient authority" but "reasoned out through objective and subjective means" (Schommer, 1994). Within the broader account of Chinn and colleagues, the main sources of his knowledge seems to be reasoning. He is the actor of his own understanding. In terms of epistemological resources (Hammer & Elby, 2002), he seems to easily and productively activate together "knowledge as fabricated stuff" as a resource about the nature of knowledge, and "formation" as resource for epistemological activities. It is impossible to measure how much this epistemological attitude is a product of the environment built in the teaching module and how much it comes from Pietro's previous learning experiences. However, in light of Pietro's explicit statements about the course, I'm prone to think that the answer has to be somewhere in the middle, and that the design choices have played a role.

Modeling: "we give characteristics […] for which it's perhaps easier to work on"

In the previous excerpt, when in line 15 he says, "I don't know if it is temporary or if it is inherent to quantum physics", he shows to be thinking about science as something that can evolve. It can't be established here if this belief has been activated only by the specific context of the course on QP or if it is a stable epistemology. Moreover, a few minutes later, Pietro claims that a model's value is tied to its 'practicality' (not his own word):

41 INT: So, what does modeling mean to you?

42 PIETRO: We give the characteristics, let's say, the ideal characteristics, for which it's easier perhaps
43 to work on. It is easier to imagine the motion of the typical projectile, ignoring the friction of the
44 air and I say 'okay, my bullet is a microscopic ball, and all the mass is there'. I imagine this dot
45 moving and following this parabolic motion. Because it's easier, because ok, that is… it's simpler, I
46 imagine this little ball, it's the same, it behaves the same way: whether it's a ball or an elephant, if I
47 don't take friction into consideration, it's the same thing. And it is easier to imagine it because - the

48 stupid example of the motion of the projectile - I can't imagine an elephant being shot, but I can
49 imagine this little ball being thrown. It is a very stupid example but it's easier to imagine.

Modeling means to "give characteristics, […] ideal characteristics, for which it's easier to work on" [line 42]. Modeling is a creative act, where 'we' is the subject, and we 'give' characteristics. Again, his attitude in developing knowledge personally and gradually stands out, but he also he seems to consider scientific knowledge to be *tentative* and *evolving*, using Schommer's words (Schommer, 1990; Schommer, 1994; Hofer & Pintrich, 1997). Considering science as something that can evolve allows Pietro to think about models not only as the final goal, but also as work tools to be used and manipulated. In this perspective, 'easiness' and 'simplicity' are, to Pietro, criteria to choose a model over another ("because it's easier […] it's simpler" [line 45]), and they take the role of non-evidential justifications (Chinn et al., 2011) for his need to find a coherent model of the quantum object: in other words, the fact that 'easiness' and 'simplicity' are embedded in scientific research practice, justifies to an extent his need for a coherent visible model of the quantum object.

Modeling: comparison with "objects that I know"

In the following excerpt from the interview, Pietro recalls Young's experiment as something that helped him in understanding the argumentation built upon the DSE with single electrons. In doing so, he explicates that building a model means often to associate something new with known models, setting down an analogy on the bases of common characteristics.

50 INT: And what did help you to understand this thing here in particular [the double slit experiment
51 with single electrons] within the course?

52 PIETRO: The part of... well let's say that the possibility of having an experiment that I already knew,
53 as a model of this new paradigm, has helped a lot, that is ...

54 INT: What experiment?

55 PIETRO: I already knew Young's experiment. Not made with electrons, but essentially made with
56 water waves in a ripple tank, [and then] made with light… indeed, it would be the same thing but
57 we had treated it as a wave at the time. Because it is one of the great demonstrations of the fact that
58 light is a wave. It's not for real, but at that time, when we were discussing whether it was
59 a wave or a particle, we said that we can see… I mean, this experience let you see that it
60 behaves like a particle, and then there's this other experience, that is Young's experiment,
61 which instead lets you see that it's a wave. So, it's both or just one? It's neither, but at the time
62 we said 'ok, so it also acts as a wave' in some cases. That was the experiment where I said 'ok, so,
63 water waves, in the case of the ripple tank, behave like this, and light behaves in the same way.
64 Thus, by association… by modeling. As something that you can understand, comprehensible,
65 visible. I say 'ok, it behaves in the same way'. It is like a wave of water, but made up of very
66 small particles that form this large wave. It's not that, but [still] it's a model. Wrong, that's the problem.
67 In quantum physics, I can't model it with objects that I know, because they neither embrace
68 the entirety, nor they are flexible enough to be modified every time something like this happens.
69 That is, there is not yet a model coherent enough to be used to describe such a strange, complex
70 object.

Understanding that light behaves like water in the ripple tank happens “by association… by modeling” [64]. To Pietro, association is a means of modeling, as it sets a common ground to understand different phenomena; it is “something that you can understand, comprehensible, visible. I say ok, it behaves in the same way” [lines 64-65]. Even previously, in line 18, he argues that the image of a ‘little sphere’ brings with itself some characteristics. Again, visualization is tied with the process of modeling itself, and is the ground for any comparison. It’s interesting to note that he seems not to have a realist expectation from a model, and he seems to be aware of its abstract and simplifying nature. For example, when describing light “like a wave of water, but made up of very small particles that form this large wave”, he comments “It’s not that, but [still] it’s a model” [line 66]. Thus, within the act of modeling, Pietro explicitly identifies *associating* and *comparing* different models as reliable processes to build knowledge. However, it should be noted that *what* he is comparing, is the ‘image’ that the models bring with themselves, and not explicitly their formal properties. In fact, in QP, where visualization and models are sometimes disentangled, “I can’t model it with objects that I know, because they neither embrace the entirety, nor they are flexible enough to be modified every time something like this happens” [lines 67-68]. The issue is with the image that classical models bring with themselves, being not “coherent enough to describe such a strange, complex object” [line 69]. This dis-alignment could be indeed expressed in terms of the ‘need of comparability’ between classical and quantum domains; however, as it was pointed out, it emerges as a need of comparison between ‘images’. Thus, visualization is also at the very center of Pietro’s need of comparability, as it is the means through which he expects to be able to compare a quantum object with a classical one.

A model works if “I can easily replace it with the object itself”

Pietro requires the model, and so its representations, to be expressive of all the relevant characteristics defining the object physical behavior**.**

71 INT: Where with ‘coherent’, you mean…?

72 PIETRO: it is a model that works, that is that ... a model of quantum object that is always valid. That
73 if I make it pass through two slits, that model is fine, if I use it through two magnets to see the spin,
74 it's okay the same way, if I use it to do other thousands of things, it works. It's a bit like the
75 material point: the material point model works wherever there is a material point, that is, wherever
76 there is a situation in which I don't care about the friction of the air, I don't care to calculate the
77 moment, and so on. That material point is a model that works, I can easily replace it with the object
78 itself and it’s the same thing.

‘Coherent’ means that the model, and so its representations, must be expressive of all the (analogue) *situations* where an object shows the same behavior. A model must to be “always valid” [line 72] in the sense that it has to imply a selection of the relevant characteristics needed to explain a behavior; for example, the model of material point works when “there is a situation in which I don't care about the friction of the air, I don't care to calculate the moment, and so on” [line 75]. Again, modeling has something of a creative act in which the scientist owns a decisive role; Pietro seems to be activating the epistemic resource ‘knowledge as a free creation’ (Hammer & Elby, 2003). However, again the focus is on the object as a representation of the model, that must be able to “replace” [line 77] the object itself. Even this can be expressed in terms of the ‘need of reification’; the case of Pietro suggests that this need can be activated not as a naïve realist philosophical perspective, but as a search for a cognitive reference, an ‘image’ for Pietro, to think about QP phenomena. It’s a matter of knowing, more than a matter of physical ontology.

In this search for connections [lines 67-68] and coherence [line 69] of models, he also shows to consider scientific knowledge itself as a coherent system, and not as a collection of isolated concepts. This recalls Hammer’s dimension of epistemic beliefs about the structure of physics knowledge (Hammer, 1994), or Schommer’s epistemic dimension of knowledge organization, going from “compartmentalized” to “highly integrated and interwoven” (Schommer, 1994). Pietro shows to consider knowledge as having an “integrated” structure, with a character of universality, “always valid” [line 72] (opposed to particularity in Chinn’s et al. (2011) framework).

“Visualizability would be fantastic… however, it can’t be the final goal”

Again, ahead in the interview, Pietro clarifies and supports his trust in visualization by underlying its role within scientific research: part of the choice of a physical model over another is based on its power of making visible a phenomenon.

99 INT: What do you think is the purpose of scientific research and science? For example, Schröedinger
100 spoke of the aim of science as ‘visualizability’. If you were to do research, what would be the purpose
101 for you?

102 PIETRO: well, let's say that the ‘visualizability’ of the concepts would be fantastic…in the sense that
103 I'm sure it would help a lot to get into the topic, to understand it better, maybe to go further. Like
104 classical physics: the introduction of the models was not accidental. It’s not that they decided one
105 day to get along all together and say: 'let's use this new model'… while studying, they realized that
106 maybe introducing a model, besides simplifying a whole series of calculations, it was also easier to
107 visualize. As I said before, if you want to study an elephant that is being shot - in addition to the fact
108 that the calculations would be too difficult in the case in which I should consider this elephant as an
109 object in itself - it is also easier to imagine it as a material point, said that it does not change anything
110 for practical purposes. However, on the other hand, it [visualization] can’t be the final
111 goal… let’s say, I mean… it could be a starting point, perhaps, or a temporary arrival point, in the
112 sense that if I arrive at a coherent model, as I said, I make life easier to all those who will come later,
113 because they will have a model on which to work. But in the end, [a coherent model] in itself, it’s
114 not what I search for. If I research on and study quantum physics, it’s because I want to know how
115 my quantum object behaves and maybe how to apply its specific properties. An unfortunate
116 example is that of the atom: once the atom was studied, they acknowledged that there was a lot of
117 energy that could be exploited there. Then it was exploited in a particular way… but already the
118 concept of fusion and fission, besides creating weapons of mass destruction, could guarantee a
119 certain source of energy that would be rather welcome at this very moment […].

‘Visualizability’ is worthy because it would “help a lot to get into the topic, to understand it better, maybe to go further” [line 103]. Pietro’s epistemic aim is again understanding, and not merely knowledge, as he is constantly seeking for explanations and connections between items of information. He justifies his position by addressing the importance of visualization for the scientists that built classical physics, “It’s not that they decided one day to get along all together and say: 'let's use this new model'… while studying, they realized that maybe introducing a model […] it was also easier to visualize” [lines 104-107]. This is a non-evidential justification (Chinn et al., 2011) for his belief about visualization importance in understanding physics, specifically pointing out what would be its role in making it more understandable to other scientists.

After having highlighted the role of visualization (and modeling), Pietro recognizes that within the process of scientific investigation “it can’t be the final goal. […] It could be a starting point, perhaps, or a temporary arrival point” [line 110]. In fact, he feels the need here to specify that the goal of studying QP is “to know how my quantum object behaves and maybe how to apply its specific properties” [line 114]. Here Pietro seems to take the perspective of science in general, claiming that the aim of scientific investigation is not to model, but to find ways to apply scientific knowledge. Taking this perspective, he shows to be aware that from a general point of view “to know how an object behaves” can eventually be independent of having a model of the object. Modeling can for sure make things “easier” for the ones that have to come [line 112], but it’s not the aim of scientific research.

But when in line 123 Pietro shifts the focus on himself, he admits that modeling could be his personal goal in doing research.

120 INT: so you’re saying: ‘visualization would be useful, fantastic, to understand, but it can also be
121 a starting point'.

122 PIETRO: Yes, I mean… if I create, me as a [hypothetical] physicist, a model, it’s not to be given for
123 granted that it’s my arrival point ... that could be so for me, personally, it could take one an entire
124 life to create this model, and it would have been worth it, from a scientific point of view. But
125 for another scientist who comes after me, it could be a starting point,
126 as it was for Bohr with the atom: Bohr arrived to his model of the atom, and then it was a
127 starting point. Other scientists came and said: “Ok. This model works”. Though, it
128 only works for hydrogen, only in particular cases, let's see how to improve it. Then it was
129 changed a lot, and we got to the theory of orbitals, which also is based on probability.
130 It seems that we cannot detach from this probability, it is always there, latent.

There seems to be a slight tension between his epistemology about the nature of science and his personal understanding needs. Pietro feels the personal need to build an imaginative and ‘visible’ model of a phenomenon (epistemology about his personal understanding), but at the same time admits the possibility that other scientists, and science in general, can pursue different goals with different investigative processes (epistemology about science).

Here Pietro shows a fairly pragmatic perspective on the evolvement of scientific investigation, where the scientist has to put himself in the simplest situation to understand phenomena. Again, one of the goals of modeling is gaining simplicity to develop models and progress theoretically. A model fruitfulness (see the example with Bohr [line 124]) in opening the way for new discoveries seems to be a justification to Pietro for his belief about the importance of visualization in developing models. This shows again his attitude to be an active agent in achieving understanding (“for me personally” [line 123]), as well as his view about the tentative and evolving nature of scientific knowledge ("it could have taken a whole life, and it would have been worthy" [line 123]).

Pietro - discussion

In Pietro's discourses, I found evidences of the occurrence of all the three needs of visualization, comparability and reification. However, as it was pointed out, both the needs of comparability and reification emerged in terms of the first: (i) the need of comparability emerged as a need of comparison between the 'images' that the models bring with themselves, (ii) the need of reification emerged as a search for a cognitive reference, an 'image', to think about QP phenomena, and not as a realist philosophical perspective. Thus, visualization is at the very center of Pietro's epistemology, being a reliable source of knowledge, through which he expects to be able to compare and identify a quantum object with respect to a classical one.

Specifically, the dis-entanglement of visualization, models, math, and measurements that occurs in QP is very demanding for him, and he expresses this discomfort many times; however, he is able to distinguish the nature of each of these components and to speak appropriately about their relations. While "in classical physics, if you measure you know" [line d24], here the measurement process can no more be identified as a 'direct' source of evidence, and the pictorial role of models is challenged due to the incoherence of models' representations with the totality of quantum physical behavior.

However, despite Pietro's constant focus on the difficulty of visualizing quantum objects, and despite he is explicit asking for a more 'complete' model ("there is not yet a model coherent enough to be used to describe such a strange, complex object" [line 69]), he seems to be (almost peacefully) accepting the theory as a reliable description of reality and the content of the teaching module as a reliable basis upon which to build in the future [line 3]. Furthermore, I found evidence of meta-affection in Pietro's excitement towards QP novelties, oddities and difficulties.

We can find hints of reasons for the occurrence of these stances in his own epistemologies (as suggested by Radoff and colleagues (2019). In the analyzed sources, I found the following traces of epistemology and epistemic stances (in table 2.7 I report the lines of reference of some examples in brackets):

Table 2.7 Evidences of epistemology and meta-affect in Pietro

Analysis dimensions		Evidences in the student's discourse
Nature of knowledge	Certainty	Physics is: evolving [15, 123] tentative [42-46, 123]
	Structure	integrated, universal [67, 69, 72]

Nature of knowing	Aims	understanding [9, 103, 123]
	Sources	he feels to be an active agent of knowledge building [19, 22, 25] knowledge (through modeling) is a free creation [75] visualization [14-26, d16, 102-110] measurement [d24-26]
	Reliable processes	association, comparison [18, 64, 66] making models [19, 45]
	Justifications	For the reliability of measurement as a process. He doesn't feel the need to justify it, "I measured it, so it is so" [d26] For his need of a visible model of the quantum object. Non-evidential: 'easiness' and 'simplicity' [42-46] For the importance of visualization in modeling. (i) Non-evidential: how understandable it is to other scientists [104] (ii) Non-evidential: fruitfulness in opening up new lines of research [124]
Meta-affect		Meta-affection: excitement about difficulties / challenges [10, 14]

First of all, Pietro thinks about himself as an agent of knowledge development, and not only as a receiver of knowledge; furthermore, his epistemic aim seems to be 'understanding', at least through the all interview. Also, the most of his justificatory standards are non-evidential in this context (coming from reasoning or introspection), underlining the engagement of his own personality. Models exist to be used, and 'easiness' and 'simplicity' are criteria to choose a model over another ("we give characteristics […] for which it's perhaps easier to work on" [line 42]); on this line, he seems to consider scientific knowledge to be *tentative*, *evolving* and *integrated*. Finally, he uses scientific research as a reference to justify his beliefs, allowing him to contextualize his needs with respect to the fruitfulness in helping science to proceed.

Pietro's epistemology seems to be quite stable throughout all the interview, and also consistent with what he states in the class discussion and in the open questionnaire. Even though my purpose here is not to make assumptions about long-term stabilities, his epistemology seems to be *structural* to a degree (Elby & Hammer, 2010), because he shows a bunch of different and rich perspectives to illustrate his beliefs, that seem to be quite consolidated. It can't be figured out if this stability is *contextual*; he shows up his epistemology in the context of this course about QP, and we can't affirm to what extent he would activate the same resources in other learning contexts.

4.4 GIACOMO (S4)

Giacomo is a male student attending the last year of a scientific High School. He is generally successful at school, and he is mainly interested in physics and philosophy. In fact, the following year he will enroll in a physics degree. I chose to analyze Giacomo's case because he was one of the most engaging and active students in the whole course. Furthermore, his attitude towards the challenges of QP seemed to be interesting and significant for the purposes of this study.

Source 5 – class discussion

"Physics is only a measure of reality": opposing to others' puzzlement

The following excerpt follows a question of PR1 about whether QP owns specificities with respect to the other disciplines; Giacomo (S4 in the discussion) supports a claim for which physics reveals only a facet of reality and it does not reveal an absolute truth [line d57]. In doing this, he strongly opposes his position to the puzzlement felt by others (for example S5).

d54 PR1:and there was a person who said "why is it true? why should we think that physics or a
d55 theory is true?" [in the answers to the questionnaire]. Let's think about this.

d56 S4: No, I mean… I have personal perspectives [*he agitates his hands as he is referring to something*
d57 *'far'*] ... I would say that physics is only a measure of reality and it's not the absolute truth… it's only
d58 a facet of reality, which can give us beliefs that can also be, let's say, exhaustive about nature, but
d59 it will never give us more complex aspects that can be interpreted with a logic that starts from
d60 different bases, and that is still valid, like art, philosophy…

d61 PR1: Do you [all] agree on these things?

d62 S4: …and therefore there is no absolute truth.

d63 S5 (F): In my opinion, what he says is true but, since we perceive reality for what it is, even if there is
d64 another way of measuring another reality we still must have answers about what is happening now
d65 [*hand up and down, to enforce the concept*]. I mean, in my opinion, it's ok to say that we are not able
d66 to arrive at the 'true truth' of things, but only to what we perceive. However, for me it's already a very good
d67 starting point to be able to understand [*her head is turned towards S4, and with the eyes and hand*
d68 *enforces the word 'understand'*], let's say, how the truth that we perceive works and then understand if
d69 there exists another 'true truth'... a more 'true truth'… [*class laughing*]

d70 S4: I mean… I have a mental example. Imagine to be on a bus, that is still, and you are still on the
d71 bus. Suddenly, the bus accelerates, and you are bounced back. If you analyze the situation from outside
d72 it's a non-inertial reference system, and so you know how to explain this with laws. Instead, if you
d73 are on the bus, you feel an apparent force that pulls you backward, and you can't explain this with
d74 an inertial reference system. The laws are not valid anymore. I mean… human condition cannot
d75 automatically see the laws of the very little and the very big, it cannot be aseptic. It's always influenced
d76 by something that is our measure, the human measure [*S5 nods with the head and whispers "eh, ok, and*
d77 *so...*]. So…

d78 PR2: But is this, according to you, a typical aspect of physics or of thought in general? Why can't I
d79 apply this to art or philosophy?

d80 S4: Precisely… therefore reducing everything to physics means to apply a human perspective that d81 takes away something in some way… that takes away ourselves.

Giacomo doesn’t expect QP to give him an 'holistic' explanation of reality, because it's *only* part of the science domain, which excludes art, philosophy and in general all the human attempts to make sense of reality (“I would say that physics is only a measure of reality and it’s not the absolute truth…” [line d57]). In line d70, he enriches his position using an analogy with reference frames: we can't know in an aseptic manner, we are always measuring from a specific point of view. From the tone he uses, he seems quite sure of what he is saying and his stance seems to be well-established, inasmuch as he can set a fairly complex metaphor very briefly. Giacomo uses this argument to affirm the incapability of science, and of every other discipline ‘per se’, of gathering the wholeness of reality. This becomes a (non-evidential) justification for his position against the counterintuitive nature of QP (see also the semi-structured interview): physics can’t provide for an absolute truth, so we must limit our expectations from it, just because scientists observes the reality from inside and from an unescapably partial point of view.

It seems that Giacomo thinks about science as something ‘consistent’, even though it is not accounting for the whole complexity of reality and knowledge in general. In fact, physics can provide for “exhaustive” [line d58] beliefs about nature, but what remains out of the physical description are “more complex aspects that can be interpreted with a logic that starts from different bases” [line d59]. Furthermore, science seems to be for him the product of a process, led by us, that starts from specific “bases” [line d60], whose validity can be discussed (“that is still valid” [line d60], “it cannot be established which is more valid” [line d98]). Thus, science and knowledge in general seem to be tentative and evolving (Hofer & Pintrich, 1997), where the subject-knower’s point of view has a strong role in the production of knowledge (in line d74 he talks about “human condition”, in line d84 about the relationship between “what we are” and science “point of view”).

d83 S4: [PR1 overlapping to S4: wait.. what did you say? The physical law...] …yes…it is a different
d84 point of view! A point of view which does not coincide with what we are [S10: but it is still a logic of
d85 person...] It is still a logic of a person, but from another point of view, how to see it from outside the
d86 bus [previous example], and see and be able to explain well what is happening.

d87 PR2: But physics does not pretend to explain why you like white instead of red

d88 S4: No, physics does not pretend… maybe one day will come to say why I like blue [laughing]. One
d89 day maybe it will come to this point, but it does not take away a measure that other things can give
d90 you. I mean, that’s it… there is no single truth.

d91 PR1: But does the physical description of the world have specificities with respect to art, philosophy
d92 or are they all equivalent?

d93 S4: What do you mean?

d94 PR1: I mean, the descriptions also regarding reliability, trustworthiness, if we do not want to talk
d95 about truth. Does the physical description of the world have characteristics that make it different from
d96 other possible descriptions of the world?

d97 S4: It starts from different bases with different logical procedures, they are different paradigms of
d98 interpretation and it cannot be established which is more valid, they are arbitrary

d99 PR1: But is there the same level of arbitrariness?

d100 S4: It can be established that… if we say we do physics, we must establish certain rules, if instead we are
d101 making art, we must establish certain rules, arbitrary, but certain rules

This belief seems not to be activated only from the specific domain of QP; the stability of this epistemological stance seems to be deliberate (Elby & Hammer, 2010) and the result of previous personal reasoning on the nature of science in general ("I have personal perspectives [*agitating his hands as he is referring to something far*]" [line d54]). Nevertheless, it's plausible to think that the 'quantum weirdness' pushes the student to take this position in this context. However, in the context of this discussion Giacomo seems to be intentionally placing himself with respect to the others' positions. His attitude in this part of the discussion seems to become almost 'ironic' (he laughs sedately) towards the objections, and the more the discussion goes on, the more he shows confidence with his position. Moreover, Giacomo's aim in setting these rules of knowledge ("we must establish certain rules" [line d100]) seems to be the one of *avoiding false beliefs* (Chinn et al, 2014). Also in line d88, he doesn't really care the point to which physics can arrive, the important thing is to recognize its limits ("one day maybe it will come to this point, but it doesn't take away a measure that other things [instead] could give you…there is no single truth" [line d88].

It is interesting to note that Giacomo's use of the concept of 'measurement', here used as a metaphor to explain a semi-philosophical idea ("physics is *only* a measure of reality" [line d57], "it is always influenced by something that is our measure, the human measure" [line d75], "maybe it will come to this point, but it does not take away a measure that other things can give you" [line d89]), reveals an analogy with the concept of measurement in QP, where measuring is an 'disturbing' act, a projection on a specific basis, or "facet" in this case ("it's *only* a facet of reality" [line d57]). Given that the one of measurement is one the most treated themes of the course, the continuous use of the word 'measure' suggests that this concept has become part of Giacomo's own metaphorical imagery.

"It's a matter of relating to something already known": math to compare different contexts

Giacomo shows also an epistemological thought about the abstract nature of mathematics itself. He refers to the possibility to apply a mathematical formalism to different contexts and describe different physical problems (for example, the vector composition), as a justification in the search for a physical meaning.

d310 S4: […] yet from last lesson, with the explanation of quantum laws,
d311 mathematical laws on vectors entered… the vector composition. It seems to me that it has been
d312 said that to talk about general relativity we use matrices, but these mathematical elements,
d313 which can be studied, are used in a field that maybe is not the one to which classical physics
d314 would have applied them. Classical physics would have applied other models to that situation, perhaps
d315 with different logics.

d316 PR1: I agree with you. The particularity is the use of a mathematics that also gives metalinguistic
d317 tools. The strangeness is that they were used in different contexts and therefore highlighted
d318 relationships or aspects that were different.

d319 S4: Yes. In the end I am applying concepts, the same object-language, assembled with a different
d320 syntax

Even though, being a high school student, he doesn't own profound knowledge of the mathematics underlying QP, Giacomo speaks about mathematics as something that can be "used" [line d313] or "applied" [line d319]; this allows him to accept the formalism (here the superposition principle and probability), considering it metaphorically as a word of a language that takes different meanings when used with different syntaxes [line d320]. The expression "in the end" [line d319] reveals that he finds in this metaphor an ideal to tackle with problems of sense-making. To a degree, to him the weirdness of this new 'quantum syntax' is justified, and judged to be apparent, by the abstractness and transferability of mathematics. In fact, this part of his epistemology is so strong here that brings him, during the class discussion, to affirm more than once that "we are not talking fundamentally about anything new" [line d334].

d334 S4: We are not talking fundamentally about anything new…we are looking at it differently

d335 PR1: Well, something is new, eh...[laughing]

d336 S4: Of course, but for me you can bring it back to…I mean, probability is a concept I've already met, I
d337 know what it means. I have not met the probability calculation in these terms.

d338 PR2: Mathematicians and physicists met it in the late '700

d339 S4: I studied it at school. In this case, the probability has an ontological value. It's only a matter of
d340 relating to something already known, which I have already encountered in some way, perhaps with a
d341 value, an importance, a context, with laws… that are different.

When saying: "we are looking at *it* differently" [line d334], "*it*", mathematics, is something that can be looked at. Mathematics itself seems to be a reliable ontology to him, and thus whatever its declination will be reliable and acceptable. In fact, he can "bring it back" [line d336] to something known, to something that he already knows "what it means" [line d337]. Of course, these statements are entangled with the context of the discussion, where Giacomo is repeatedly opposing to the positions of other students. But in force of mathematics he seems to not feel the lack of a new ontology to speak about quantum physics.

Also, it seems that, also for Giacomo, *comparisons* and *associations* are important to achieve understanding; in fact, "it's only a matter of relating to something already known, […] encountered in some way" [line d339]. As it was for Pietro, *comparing* seems to be a worthy epistemological activity (Hammer & Elby, 2002), a reliable process (Chinn et al., 2011) to build knowledge and understanding. However, there seems to be an important difference from the case of Pietro: to Giacomo, the mean that allows this comparison among contexts is mathematics, instead for Pietro the comparison works as an association of visual models of the physical objects. The coherence with something known previously, in this case mathematical formalisms, becomes a justificatory standard to claim "we are not talking fundamentally about anything new".

Source 7 – semi-structured interview with Giacomo

"Counterintuitive, but with a logic behind"

A week after the end of the course, Giacomo seems to be confident with the arguments treated, and he explicitly says to be happy for having took part to this experience. From the very beginning of the interview, he re-states his confidence in mathematics, in line with his position during the class discussion.

28 INT: what words would you use to […] the quantum object? What does distinguish it from the
29 classic object, how do you 'identify' it?

30 GIACOMO: so… I would say 'weird', very informally. More formally, I could say
31 'counterintuitive', but that still with a logic behind it. It's not that 'strange' of which one cannot
32 trace a leading thread.

33 INT: and what is the logic?

34 GIACOMO: the logic owns fundamentally mathematical bases, at least from what it seemed to me,
35 that are at least sharable.

36 INT: for example?

37 GIACOMO: for instance, the thing about superposition as a vector sum… that one is a shareable
38 mathematical concept. Perhaps, what changes is its application: in classical physics the vector sum
39 can be given to forces, velocities… or other vectors like this. Here there is a new application of
40 mathematical concepts that are not… that don't go against the founding axioms of mathematics.

The quantum object is "weird, […] counterintuitive, but still with a logic behind" [lines 30-31]. The logic is "behind", it owns a sort of priority, and takes the role of a "leading thread" [lines 31-32], something that unifies disconnected pieces. This logic is mathematical, has "mathematical bases", and this bases are "sharable" [line 34]. It must be noted here that Giacomo is not referring to a generic sharing among scientists, but with himself; in fact, he seems to conceive his knowledge not as coming directly from authority, but as a personal understanding and re-organization (Schommer, 1994; Chinnet al., 2011). As in the discussion, is aim in searching for the foundations seems again to be to avoid false beliefs. The mathematical basis is 'sharable' because "for instance, the superposition as a vector sum is a sharable mathematical concept". As well as in the class discussion, where the formalism could be "used" or transferred to other contexts, here the vector sum can be "given" to forces and velocities [line 39]. The justificatory standard for claiming that there is "fundamentally anything new", is that the new concepts "don't go against the founding axioms of mathematics" [40]. This is a non-evidential standard about mathematical consistency, probably near to what Chinn and colleagues refer to as internal logical consistency (Chinn et al., 2011).

"Mathematical formulas give you a certainty that something else doesn't"

Afetr having talked about some arguments of the course, the interviewer asks what did help him to understand. Giacomo liked the fact that in the course there were some calculations to do, because, to him, mathematics is a source of understanding.

117 INT: […] Which of the images, analogies, models, experiments, formalisms,
118 phenomena, etc… that we went through during the course, helped you more to understand and enter
119 into these concepts?

120 GIACOMO: probably I'm weird… but I understood through mathematical formulas.

121 INT: you are not strange at all! There were scientists, as for example Heisenberg, that probably
122 would have been of your same idea.

123 GIACOMO: I mean, images are indeed explicative, and so on… but the mathematical formulas
124 give you a [feeling of] certainty that something else doesn't give you.

125 INT: So, the mathematical formulas in which part of ...

126 GIACOMO: Well, even in the superposition. Basically there. Or even when in the standard
127 experiments of Stern and Gerlach we calculated the probability for it [the silver atom] to arrive on a
128 certain spot, using those key concepts of probability amplitudes that add up, multiply, and so on.
129 There, we basically led mathematical calculations.

Giacomo states that visualization and representations are not so important to his understanding as is mathematics: "images are indeed explicative, and so on… but the mathematical formulas give you a [feeling of] certainty that other things don't" [lines 123-124]. In saying this, he also positions himself with respect to his mates ("probably, I'm weird… but I understood through mathematical formulas" [line 120]). Thus, to him, making calculations and building on mathematical facts is a reliable process to achieve understanding.

"Mental flexibility" to overcome QP counter-intuitiveness

During the interview, Giacomo has the opportunity to express also his view about understanding and knowing when related to QP:

130 INT: so, in your opinion, what is needed to understand quantum physics?

131 GIACOMO: first of all, definitely mental flexibility.

132 INT: In what sense?

133 GIACOMO: In the sense that you don't have to immediately say: 'this doesn't make sense'. Wait a
134 little to say that. Don't be influenced by your own prejudices, which everyone has…

135 INT: Even scientific [prejudices]?

136 GIACOMO: Yes, even scientific. Prejudice is a human cognitive method, but don't let yourself be
137 influenced by those.

This stance ("you don't have to immediately say: this doesn't make sense" [line 133]) is a deliberate strategy to tackle with uncertainty and sense-making issues. It is a form of control on the way he thinks to be better to learn, probably also related to what Schommer (1994) refers to as 'learning is gradual'. He ascribes the need for a 'control' on his own knowledge to the novelty of QP thought categories and, mainly to control the “human cognitive methods” represented by “prejudices” [lines 134, 136]. This point is stressed also in the following of the interview (see below), when he talks about classical physics, by saying that “ [it] can bring to mental fossilizations, that maybe one wouldn’t have had if he studied quantum physics first” [line 150]. This excerpt suggests also that Giacomo doesn’t conceive in this context knowledge as a *free creation*, but as something *given*, to which the learner as to ‘bend’ (“don’t let yourself be influenced by those [prejudices]” [line 137])

146 GIACOMO: it is of course less intuitive... some doubts came up in me [at a certain point]... no, no,
147 but I had only a momentary doubt if we had to study quantum physics first and only then classical
148 physics… if it could be done, maybe it would be less 'confusing', but I don't think so.

149 INT: Why? Try to explain this concept, it's interesting

150 GIACOMO: Perhaps, studying classical physics too much can lead to mental fossilizations, which
151 maybe he would not have if he had studied quantum physics before. But I don't think so, because
152 classical physics basically comes from the world around us, so we would already be mentally
153 conditioned to not accept quantum physics even if we didn't study the classical physics first. It was
154 such a confusing moment…

[…]

170 INT: […] What would quantum physics give you more than classical physics?

171 GIACOMO: well… in my concrete reality, sincerely nothing. Perhaps you could arrive at an
172 experience of quantum physics… perhaps you can, how to say, understand that sometimes there is
173 much more behind things. Something that maybe one would easily be ready to deny, or to say: ‘it
174 can’t be so’. But if you look at it very carefully it is sometimes something that makes sense, its own
175 sense.

176 INT: [a sense] that classical physics sometimes ...

177 GIACOMO: Sometimes classical physics doesn't give it to you… I don't know. Quantum physics
178 is more, so to speak, ethereal, almost elusive, it gives you just another measure of things.

Giacomo is not sure if our classical mental fossilizations come from our everyday experience or from classical physics, but still, quantum physics categories may allow to “understand that sometimes there is much more behind things” [lines 172-173]. The initial sense-making struggle has to be patiently put aside, as the dust in an old attic [line 187], to be able to eventually understand the inner logic and “reconstruct” [line 189] a plausible thread within disconnected pieces:

184 INT: […] When you think about quantum physics, how do
185 you ‘organize’ it, how do you think about it? How do you organize your knowledge about it? We
186 have seen so many concepts, so many things…

187 GIACOMO: …maybe, like an old dusty attic. You go there and maybe the first feeling you have is
188 to go back down and not go inside anymore. Then you start removing the dust from the boxes, you
189 start removing the tape, the cobwebs. And maybe you reconstruct the story of a life there, in that
190 attic.

191 INT: what is the ‘dust’ for you?

192 GIACOMO: dust is all the difficulty, all the anti-intuitive component, which sometimes makes it
193 difficult for you to enter that world.

194 INT: so it's worth taking it off.

195 GIACOMO: it's hard, but yes.

The very term “reconstruct” [line 189] suggests that Giacomo thinks about understanding as something that must be actively reconstructed by the learner, and that this is a gradual process. However, as in line 150, he does not conceive it as a *free creation*, but more as a discovery of something that is already there and that you need to recognize by searching for the proper perspective. In fact, classical physics brings to “fossilizations” [line 150].

“QP can provide an explanation of reality, but only from a certain point of view”

As he stated during the class discussion, Giacomo re-affirms here his epistemological stance towards the role of physics in investigating reality. He considers it to be a way to explain and describe it, but among many other ways (“every human activity is a way of explaining reality, one different from another, more comprehensive, more rigorous, less rigorous, what it is…” [lines 213-214]).

211 GIACOMO: Yes, from a certain point of view. That is, as I think it is, any human activity has an
212 explanatory power over reality, because doing… being ourselves is basically an act of knowledge
213 towards reality, of relating to things. Therefore, every human activity is a way of explaining
214 reality, one different from another, more comprehensive, more rigorous, less rigorous, what it is…
215 but yes, quantum physics can provide an explanation of reality, but only from a certain point of view.

216 INT: Which one?

217 GIACOMO: the one that is not ours. The one that does not take man into consideration as a measure
218 of things.

219 INT: As a measure of things sorry, try to explain it to me…

220 GIACOMO: What I want to say is that it completely cancels human subjectivity, perhaps not
221 completely, but in large part, as a starting point to reach knowledge, which then, perhaps, is the spirit
222 that animates all physics

223 INT: The one of ...

224 GIACOMO: The one of removing oneself from the human perspective and identifying oneself with
225 a perspective that does not belong to us

226 INT: Remove the impediments, so to say. Is this good for you?

227 GIACOMO: it is so, as long as one takes into account that it is partial, as long as one takes into
228 account that it does not give us the true measure of things. Because fundamentally I can also be

229 described as a quantum object, but I do not find myself fitting there. I don't feel indeterminate, I don't
230 feel like swinging, or being a superposition of two wave functions.

When asked to specify the character of science with respect to the other disciplines, Giacomo highlights the tendency of physics of reducing subjectivity in favor of objectivity ("it completely cancels human subjectivity, , perhaps not completely, but in large part" [lines 220-221]. In his words, science tries to identify in "a perspective that doesn't belong to us" [line 225]. Interestingly, this is the only passage where Giacomo expresses a discomfort towards QP theory, claiming that "I can also be described as a quantum object, but I do not find myself fitting there. I don't feel indeterminate, I don't feel like swinging, or being a superposition of two wave functions" [line 228]. This seems a justification for his belief about the partiality of scientific knowledge.

"If you give me something mathematical, I put a little effort in it, […], and I am convinced"

When speaking about entanglement, Giacomo doesn't show to be bother by its counter-intuitive nature. He grounds his acceptance on the reasonableness of the mathematics underlying it ("if you give me something mathematical, I put a little effort in it, I understand it, and I am convinced" [line 273]).

272 INT: […] And what's counterintuitive about this for you [about the entanglement]]?

273 GIACOMO: Mathematically nothing fundamentally. I said it before… if you give me something
274 mathematical, I put a little effort in it, I understand it, and I am convinced. At a conceptual level,
275 moving from a mathematical level, speaking of the probability of obtaining a certain measure when
276 one influenced, we said, at a distance the measure of another, is something that goes against, at least
277 even according to Einstein, the traditional information transmission methods.

278 INT: Exactly. Is this acceptable for you?

279 GIACOMO: Basically yes. If you give it to me mathematically yes, I accept it. Now, if I'm honest,
280 we've done a lot of quantum physics on explanations, on computer experiments. I would like to do
281 something more, maybe see something with my eyes. Or something with my eyes or in any case
282 have the chance to be more engaged than just studying in this way.

To him, the conceptual and the mathematical levels are distinct and separable [line 274], as it is for Pietro. However, to Pietro this separation is not effortless, instead for Giacomo intuition and visualization are not as important as math is in this context ("if you give it to me mathematically yes, I accept it" [line 279]). He accepts the counterintuitive character of quantum physics, not stressing his own learning difficulties. Quantum physics is counterintuitive in the sense that it challenges the "traditional information transmission methods" [line 277]: the challenge is between the theories or between great scientists as Einstein, and not between the theory and himself.

Math is thus enough to accept; however, still remains the eager to "see something with my eyes" [line 281], to "be more engaged" [line 282]. Thus, visualization, as a resource, is activated also for Giacomo in this context, but in his case it is not triggering his acceptance.

Giacomo - discussion

Giacomo’s trust in mathematics and beliefs about the nature of knowledge and science, become tools to tackle with the issue of “counter-intuitiveness” [line 31] of QP. He seems, in fact, to accept QP, in force of a multiplicity of factors.

Emotionally, he externalizes most of the times an attitude of acceptance of the “weirdness” of QP, being calm and explicitly claiming for the need of patience and mental flexibility in front of lack of sense [lines 130-137, 187-195] when learning QP. In the class discussion, he opposes his position to the ones of others, minimizing their puzzlement in front the challenges they felt.

I found evidence of at least three justifications for this stance (minimizing the issue of QP meaning / counter-intuitiveness). Also for him most of the justifications are non-evidential, and involve beliefs about the nature of knowledge: (i) scientific knowledge, as well as other forms of knowledge, is always partial [lines d310-320, d336-341], (ii) mathematics is transferable, in the sense that a known formalism (as for example the superposition principle) can be applied to different contexts [lines d310-320, d336-341, 38-40], without loosing (iii) consistency with the foundations of mathematics [line 40].

Even though he refers to visualization [line 123] as a tool for understanding, to him it is not as important as math is in this context (“mathematics give you certainty that something else doesn’t” [line 124]; “if you give it to me mathematically yes, I accept it” [line 279]). In fact, there seems to be an important difference from the case of Pietro: for Giacomo, the mean that allows to compare QP with other theories is mathematics, instead for Pietro the comparison works as an association of visual models of the physical objects. Thus, both math and visualization are tools to enable comparisons between models and theories, but the activation of one the of two seems to be context- and subject- dependent.

Table 2.8 Evidences of epistemology and meta-affect in Giacomo

Analysis dimensions		**Evidences in the student’s discourse**
Nature of knowledge	Certainty	Physics is: objective [220-225]
	Structure	Mathematics is: transferable, context-independent [d310-320, d336-341, 38-40] Knowledge in general is: complex, multi-dimensional [d59] partial, domain-specific [d56-60, d70-77, d83-101] like a measurement, a projection on a specific basis [d57-59, d75, d89]

Nature of knowing	Aims	avoiding false beliefs, by searching for foundational ‘rules’ [d88, d100, 34, 281]
	Sources	knowledge as something given, to which one adheres [130-137, 150, 187-195, 273] understanding as a personal re-organization [34, 273]
	Reliable processes	formal process: rules and calculations [120-129, 273]
	Justifications	For minimizing the issue of QP meaning / counter-intuitiveness (i) non-evidential: partiality of knowledge [d310-320, d336-341] (ii) non-evid.: mathematics transferability [d310-320, d336-341, 38-40] (iii) non-evidential: consistency with mathematics foundations [40] (iv) because math works [273] For considering physics as only a measure on reality: Non-evidential: “I don’t feel indeterminate” [228]
Epistemic Stances		Acceptance [279]
Meta-affect		Placidness, patience and mental flexibility towards counter-intuitiveness [130-137, 187-195] discomfort about thinking QP as a ‘real’ description of reality and of himself [228]

Recalling the main traits of his epistemology about knowing, Giacomo thinks about learning as an active and gradual process of reconstructing one's understanding, and his aim seems to be the one of ‘avoiding false beliefs’. The most reliable process to consolidate his knowledge is making calculations. His views on knowledge in general, as introduced, are that it is always tentative, complex, and partial (domain-specific). Science, with respect to other domains, tries to be objective, and mathematics is context-independent, in the sense that it is transferable.

Giacomo’ epistemology and affective stances seems to be quite stable throughout all the interview, and consistent with what he states in the class discussion and in the open questionnaire. His epistemology seems to *structural* to a degree, as it seems that many thigs he speaks about are due to previous reflections.

4.5 CLARA (S5)

Clara is a female student that intervened very often in the class discussion, and almost always expressing her claims with a deep emotional involvement. Unfortunately, I did not have the chance to have an interview with her after the course, and so the discussion is the only interactive data source I have about her; however, because of her involvement, I decided to analyze her epistemologies, even if only in the context of the class.

Source 5 – class discussion

"We must still have answers about what is happening now"

In the following excerpt (already reported in Giacomo's analysis), Clara reacts to Giacomo's (S4) claim about the absence of an absolute truth. To her, knowing that physics cannot reach the 'true truth' is not an excuse, is not enough to stop seeking for understanding how things work.

d56 S4: No, I mean… I have personal perspectives [*he agitates his hands as he is referring to something*
d57 *'far'*] ... I would say that physics is only a measure of reality and it's not the absolute truth… it's only
d58 a facet of reality, which can give us beliefs that can also be, let's say, exhaustive about nature, but
d59 it will never give us more complex aspects that can be interpreted with a logic that starts from
d60 different bases, and that is still valid, like art, philosophy…

d61 PR1: Do you [all] agree on these things?

d62 S4: …and therefore there is no absolute truth.

d63 S5 (F): In my opinion, what he says is true but, since we perceive reality for what it is, even if there is
d64 another way of measuring another reality we must still have answers about what is happening now
d65 [*hand up and down, to enforce the concept*]. I mean, in my opinion, it's ok to say that we are not able
d66 to arrive at the 'true truth' of things, but only to what we perceive. However for me it's already a very good
d67 starting point to be able to understand [*her head is turned towards S4, and with the eyes and hand*
d68 *enforces the word 'understand'*], let's say, how the truth that we perceive works and then understand if
d69 there exists another 'true truth'... a more 'true truth'… [*class laughing*]

Clara considers whatever claim about the nature of scientific knowledge, as the one of Giacomo (S4), to not be an acceptable justification to renounce to her sense-making expectations ("we must still have answers about what is happening now" [line d64]). What can be gained "now" is already a "very good starting point to understand how the truth we perceive works" [lines d66-d67], and only "then" to understand "if there exists another 'true truth'" [lines d68-d69]. In these sentences, Clara shows to conceive learning as something that can built up step by step, and probably she has an attitude in expecting to gain knowledge in brief time-scales.

"The teacher gives me a balance, puts a weight on it, I read the weight, and that is the weight"

Ahead in the discussion, when the teacher shifts the focus on the explanatory power of QP [line d132], S7 questions the ability of quantum physics to explain reality ["In my opinion it limits itself to putting events in a different light" [line d134]]. After this comment Clara gains with the discussion immediately, without leaving time after S7's comment; she seems to feel this to be something important for her to point out.

d130 PR1: Yes, but compared to the kind of argument you have seen with respect to finding a justification
d131 for those experimental results, you have seen that there are experimental results and we need to build a
d132 model that allows us to interpret those data. Is it a model that seems to you an explanation?

d133 S7 (M): In my opinion quantum physics cannot explain, because, in fact, it does not depend on the
d134 logic of the traditional world. In my opinion it limits itself to putting events in a different light, that's
d135 all, not explaining them

d136 S5 (F): In my opinion we are not able, maybe, to understand the explanation it gives to us...

d137 PR1: We who?

d138 S5: We humans

d139 PR2: We invented quantum mechanics, eh [laughing]

d140 S5: I personally think of a teacher who explains something to me, he explains it to me ...

d141 PR1: What do you expect?

d142 S5: ... he gives me a balance, puts a weight on it, I read the weight, and that is the weight. [S2: Eh, you
d143 see, always the measurement process that comes back...] Yes, it is a problem of measuring... and
d144 giving something. And at the moment I say…

d145 PR1: and why does it give you the feeling of explaining?

d146 S5: I say, either I find a way to disprove his [the teacher's] explanation, or I take it as true,
d147 because he is right...

d148 PR1: I take atoms of silver, I make them pass through a shaped magnet, that happens there, I measure
d149 that they have two states of spin [S5: eh…] and why is it different from putting an object on a scale? [S5:
d150 eehm…[laughing]] Eh! Let's go into it, why is there this feeling?

In saying "in my opinion we are not able, maybe, to understand the explanation it gives to us…" [line d136], Clara seems to be accepting the authority of physics as a source of reliable knowledge; physics is considered as something 'given'. Also, when she tries to explicate her claim, she talks of her teacher trying to explain her something [line d140]. However, she doesn't describe the way the teacher is teaching, she just gives an example of a classical measurement [line d142]. The way she describes the act of measuring is like making a list of sequential steps ("he gives me a balance, puts a weight on it, I read the weight, and that is the weight" [line d142]); the 'listing' is very fast, so to show her confidence with this way of learning. She tells it this way probably because she lacks this confidence in QP learning. In line d146, Clara again refers to the authority (of the teacher in this case) as a source of reliable knowledge, in fact "either I find a way to disprove his [the teachers'] explanation, or I take it as true, because he is right" [line d146]. After her comment, PR1 compares the sequential logic of the weight measurement with the one of Stern-Gerlach magnets, to make Clara think about the difference, using the same list-like narrative to enhance the similarity; however, Clara does not catch up the provocation. It must be noted here that PR1 is expertly attending to students' epistemology and affect, and to their entanglement. In fact, she often exploits affect attitudes to get a refinement of epistemological stances.

“It can't be both, I want us to find something else, but that is just that”

The situation has another shift when S7 interprets our difficulty with quantum physics as an habitude to Aristotelian logic:

d164 S7 (M): Maybe because with Erwin's socks we use the Aristotelian logic [*S5 evidently*
d165 *expresses confusion about S7’s comments with her face*]. If this is red, it is not blue, and if it’s blue, it
d166 is not red…

d167 PR1: That is, we attribute properties that become properties of the object [S5 whispers: ah, in fact..].
d168 Once I discovered them, those are defined, right? [S7: yes]

d169 S5 (F): Yes, one thing excludes the other. We are at a level in which if it is A it is not B. Maybe it can
d170 be another thing, but it is not that [*referring to B. Gesturing to stress this point*]. And so…I was not
d171 here the last time [double-slit experiment], but saying corpuscular light or ondulatory light... I mean,
d172 either is one or the other… [*moving hands as to state an absolute certainty*] I.... I can’t accept
d173 anyone to tell me: “Yes, it’s both”... It’s not both [*smiling*]!

d174 PR2: But it is not both, it’s not both! They know... [*the class here in general engages in the discussion*,
d175 *sustaining this comment, S2 S4 and S4 smile and comment. Someone says, “it’s a platypus!”*]... it’s a
d176 platypus [laughing]!

d177 S5: …since it can't be both, I want us to find something else, but that is just that, that is not “is A and
d178 B…”

After PRI comment in line d167 some students seem to be very engaged: S7 nods, S8 raises his hand, and Clara finds PR1’s words to be enlightening, and talks immediately. She goes along the S7's comment about Aristotelian logic, and arrives to a point in which she claims that she cannot accept it to be out of this logic of excluding properties (“it can be another thing, but it is not that” [d169], “either is one or the other” [line d172], “I can’t accept anyone to tell me ‘Yes, it’s both…’. It’s not both” [line d172]). It seems that Clara feels bothered, or anxious about this point; here there is a point she is struggling with. She needs “to find something else, but that is just that” [line d177]. It’s interesting here the difference between the remissive position towards authority she showed a few minutes earlier (for example in line d146), and the clear-cutting non-acceptance of such an answer in these lines; however, I can’t find, in this excerpt, any explicit justification for her stance.

Understanding is “when I have no more questions left to do”

A few minutes later, PR1 shifts the discussion from whether QP can provide an explanation of phenomena, to what it means personally to have the “feeling” [line d209] that something has been explained.

d209 PR1: Try to think when you have the feeling ‘oh, this thing has finally been explained to me!’

d210 S5 (F): When I have no more questions left to do. That is, when someone comes to a point where I can

d211 no longer object to what he is saying, then I say you are right. I understood what you say then maybe I
d212 do not agree, but with physics, probably because I know less than the teacher usually, so I take what he
d213 says to be true. With quantum physics, getting to say I no longer have questions for me is, for now, a
d214 limit that… I don't even see yet

Clara again answers immediately. I want to note two main things here. First, to her something is explained when "I have no more questions left to do" [line d210]. The feeling of understanding is tied to not having so many unresolved / unanswered questions; this suggests a view of knowledge as something certain, and that her epistemic aim is slightly different from 'understanding', being the resolution of every question. In this sense, her case is different from the ones of Pietro and Giacomo; this point will be discussed in the general discussion. Second, Clara again considers her understanding to be achieved in a comparison with an authority, often the teacher ("I know less than the teacher usually, so I take what he says to be true" [line d212]). I say authority also because she refers to making "questions" [line d210]) as a reliable process for achieving of knowledge.

Clara - discussion

During the class discussion Clara intervenes often, and every time she seems to react to a sort of provocation, in a sort of impetuous way. She seems to feel a tension inside, and she is trying to find the words to express it. Her attitude has also strong shifts during the discussion. At first, she strongly opposes to Giacomo's claim about the partiality of physics as a justification for minimizing his expectations on QP ("we must still have answers about what is happening now!" [line d64]); in a second moment, she seems to address the problem to our understanding capabilities ("in my opinion we are not able, maybe, to understand the explanation it gives to us..." [line d136]); in line d173, speaking about the Aristotelian logic, she arrives to a point where she says she can't accept a duality of properties, she needs "to find something else, but that is just that" [line d177]; she explicitly says, "I can't accept anyone to tell me: 'Yes, it's both'… It's not both" [line d173].

This kind of requirement is well known in the context of these analyses; since she seems to feel anxious about this point, what is interesting to me is to find out which are the factors that trigger Clara's stance towards this requirement. From the analysis, I have some hints about her views on knowledge and knowing.

Table 2.9 Evidences of epistemology and meta-affect in Clara

Analysis dimensions		Evidences in the student's discourse
Nature of knowledge	Certainty	Knowledge as something certain, "resolved" [d210]
	Structure	The explanation must be simple and 'direct' [d142]

Nature of knowing	Aims	understanding, in the sense of having no more questions left to do [d66-69, d210]
	Sources	Physics as something “given” [d136] given from authority (the teacher) [d140, d146, d210-214]
	Reliable processes	To make questions to the teacher [d140-144, d210-214]
	Justification	Within the discussion, she does not provide explicit justifications for her claims and stances
Affect		Anxiety towards duality

Clara’s stance seems to be (not-exclusively) affected by all the reported factors; her epistemic aim, in this context, seems slightly different from ‘understanding’, being having “no more questions left to do” [line 210]. In this sense, her case is different from the ones of Pietro and Giacomo; in fact, as already said, even if they also felt their knowledge about QP to be counter-intuitive, unresolved, or uncertain, they both showed ‘positive’ meta-affective stances towards it, respectively excitement and ‘placidness’. Another difference is in her trust in authorities as sources of knowledge (for example the teacher, or physics itself); a reliable process to build knowledge is, to her, making questions to the teacher.

4.6 GENERAL DISCUSSION

Entanglement of epistemology and meta-affection towards QP challenges

In all the three students, I found evidences of an entanglement between affection and epistemologies.

Pietro showed excitement about difficulties, challenges and QP oddities, and specifically towards his main concern: the lack of visualization in QP. In accordance with what found from Radoff and colleagues (2019), his excitement seems to be entangled with a view of doing physics as a "process of making sense of the world". Contributing to this epistemology are (i) his view about physics as a 'tentative' and 'evolving' process, (ii) the perception of himself as an active agent of his own understanding and of the creation of knowledge (source of knowledge), and (iii) his epistemic aim, that seems to be 'understanding' in all the analyzed excerpts. Concurring to the first point, is Pietro's understanding of the role of models in physics as something to be 'used' and 'applied', like a sort of useful draft to work on ("we give characteristics, […] ideal characteristics, for which it's easier to work on" [line 42]). In fact, to be "useful", a model must be "simple" and "easy" [lines 42-46]. Moreover, his justifications (see table 5.7) support the claim for his view about the tentativeness of knowledge. Interestingly, he seems to consider also himself as a 'modeler' ("If I make it become a sphere…" [line 19], "maybe my personal goal would be that one [of making models] [line 123]), supporting his view of knowledge as a free creation. All these factors concur to letting him accept the lack of visualization as a positive challenge, and being willing to see "what's beyond".

Table 2.10 Evidences of a possible entanglement of Pietro's epistemology and meta-affection

Physics is tentative and evolving (models are 'drafts' to work on)	**Meta-affection** excitement towards the lack of visualization
Sources of knowledge self an agent of understanding and of science development, knowledge is a free creation (self as a 'modeler')	
Epistemic aims understanding, making sense of the world (building models)	

Giacomo showed to be calm in expressing how he tackled the counter-intuitiveness of QP, and he explicitly claimed for the need of patience and mental flexibility in front of lack of sense.

This attitude recalls Marya’s “comfort” about being uncertain in (Radoff et al., 2019), that was associated with “physics as a journey” in the sense that Marya understood that ‘not knowing’ is part of doing physics. From my analysis, Giacomo’s comfort with the counter-intuitiveness of QP is triggered by a bunch of factors (table 2.11): (i) his view on every form of knowledge as revealing only a facet of the truth, which is instead multi-dimensional (metaphorically, knowledge is a measurement, it ‘projects’ the truth on a specific basis), (ii) the general expectation on knowledge as something that is ‘given’ (source of knowledge), to which one can adhere (and not as a free creation, as it is instead for Pietro). A hint for this second point is Giacomo’s claim for ‘mental elasticity’ in learning QP, which is a deliberate form of control on his own requirements to adhere to something new. (iii) Maybe more importantly, his epistemic aim of avoiding false beliefs, by clarifying the rules and the foundations of every claim or form of knowledge. This latter is witnessed also by his search of mathematical consistency as a prove of the goodness of QP theory. I claim this epistemology to be tied to his meta-affection also in force of a passage in line 228 where, to justify his belief about the partiality of physics, he suddenly expresses a discomfort for the fact that “I could also be described as a quantum object, but I don’t feel fitting there. I don’t feel indeterminate”.

Table 2.11 Evidences of a possible entanglement of Giacomo’s epistemology and meta-affection

Physics (and every form of knowledge) is partial with respect to the truth	**Meta-affection** comfort and calm about QP challenges (as comprehensibility, lack of visualization and entanglement)
Sources of knowledge knowledge as something given, to which one adheres (claim for ‘mental elasticity’ in learning QP)	
Epistemic aims avoiding false beliefs, searching for foundations and basic ‘rules’	

Clara intervenes often very ‘dramatically’ in the class discussion, showing anxiety and tension towards the wave-particle duality (“I want something else, that is just that” [line d177]). Her attitude has also heavy shifts during the discussion. At first, she strongly opposes to Giacomo’s claim about the partiality of physics as a justification for minimizing his expectations on QP (“we must still have answers about what is happening now!” [line d64]); in a second moment, she seems to address the problem to our understanding capabilities (“in my opinion we are not able, maybe, to understand the explanation it gives to us...” [line d136]); in line d173, she goes back to her stance, stating that “I can’t accept anyone to tell me ‘it’s both’… it’s not both!” [line d173], even if some minutes she expressed her trust in the teacher’s authority (“either I find a way to disprove his [the teachers’] explanation, or I take it as true, because he is right” [line d146]. This variability can suggest both that she is trying find the words to express her discomfort, and that she is influenced by what is happening in the discussion. However, in her words I found hints of epistemology about (i) the source of knowledge, always mediated in the relationship with an authority [d136, d146, d173], and (ii) her epistemic aim, being to have “no more questions left to do” [d210].

Table 2.12 Evidences of a possible entanglement of Clara’s epistemology and meta-affection

Sources of knowledge knowledge as something coming from authority (physics itself, or the teacher)	**Meta-affection** anxiety towards, and non-acceptance of, wave-particle duality
Epistemic aims having “no questions left to do”	

Bridging classical and quantum to make sense: ‘math’ VS ‘visual models’

A comparison between the cases of Pietro and Giacomo shows evidence of some consistencies: (i) both are challenged by QP counter-intuitiveness, and (ii) both ground their sense-making in the search for criteria and means to compare classical and quantum domains. This latter is in accordance with the evidence for a ‘need of comparability’ highlighted in the previous analysis. However, recognized this need, the two students show to rely on different means to bridge and compare domains and theories; respectively, modeling for Pietro, and mathematics for Giacomo.

From the beginning of his interview, Pietro claimed his biggest difficulty to be the impossibility to “model in a visual way”. As pointed out, modeling to him is not necessarily the final goal of science, but is a tool to work on and think about physical phenomena (“we give characteristics, […] ideal characteristics, for which it’s easier to work on” [line 42]); this reveals part of his view of science as evolving and tentative. As part of this development, to him, understanding entails modeling new phenomena “with objects that I know” [line 67]. He brings the example of the analogy between the ripple tank and Young’s experiment with light as something enlightening, something “that you can understand, comprehensible, visible” [line 64]. However, *what* he is comparing, is the ‘image’ that the model brings with itself. In fact, in QP, where visualization and models are often disentangled, “I can’t model it with objects that I know, because they neither embrace the entirety, nor they are flexible enough to be modified every time something like this happens” [line 67]. Thus, visualization is at the very center of Pietro’s need of comparability, as it is the means through which he expects to able to compare a quantum object with a classical one. Due to this important role, the lack of visualization in QP becomes for Pietro an issue to tackle with in learning QP.

Giacomo, during the discussion, comes to a point where he states that in QP “we are not talking fundamentally about anything new…we are looking at it differently”. Speaking about probability in QP, he says that he can “bring it back” to something he already knows “what it means” [line d337], and that in general the problem of understanding is “a matter of relating to something already known, which I have already encountered in some way” [line d340]. Thus, again, comparisons with previous knowledge are reliable processes to achieve understanding. However, differently from Pietro, the means through which he brings back new concepts to something already known, is math itself. The reliability of math is justified in terms of: (i) it’s abstractness and transferability, in the sense that a formalism, such as the superposition principle, can be “used” [line d313], “given” to forces and velocities [line 39], “applied” [line d319] in different contexts. This sort of ‘warranty’, allows him to rely on the formalism (here the superposition principle and probability), considering it metaphorically as a word of a language that takes different meanings when used with different syntaxes [line d320], where math is the grammar and the physical context (the application) is the syntax.

Conclusive remarks

The analysis conducted in this study allowed to answer the research questions in the following ways.

(RQ1) it was shown evidence of a possible entanglement between specific students’ epistemologies and their meta-affective stances towards challenges in learning QP. The three cases I analyzed showed difficulties with issues as the lack of visualization in QP, the counter-intuitiveness of entanglement and superposition principle, and the wave-particle duality. Indeed, this is by no means a comprehensive list of the possible challenges, as this study comprised only three students. However, it was found evidence that epistemic aims, together with beliefs about the nature and the sources of knowledge, do trigger students’ attitudes towards these challenges in a significant way.

Furthermore, from a comparison of two students, expectations have been shown about the role of ‘visual modeling’ and ‘math’ as two personally reliable means to bridge classical and quantum domains. Indeed, (RQ2) some intrinsic characteristics of QP (as for example the minor importance given to visualization of models) deeply challenges some of these expectations, opening the way to processes of sense-making and conceptual change.

Besides the achieved results, this study exploited, along the previous research literature, a promising methodology to investigate students learning dynamics in a challenging context like the one of QP learning. Indeed, further advancements could be achieved both by increasing the number of analyzed students and comparing students coming from different contexts and grades; this kind of study would allow to recognize eventual typical patterns of epistemological stances in learning QP, which could help teachers in dealing with students’ difficulties and attitudes. Also, a factor that has not been taken in account in the present study, is students’ appropriation (Levrini et al., 2015) of the contents of the course; it would be indeed of great interest to shape the relationship between appropriation, personal epistemologies and meta-affective stances.

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