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Abstract

This thesis consists of three essays on information economics. I explore how information is strategically communicated or designed by senders who aim to influence the decisions of a receiver. The first two chapters study cheap talk games with multiple senders while the third one studies bayesian persuasion with a privately informed sender.

In the first chapter, I study a cheap talk game between two imperfectly informed experts and an uninformed decision maker. The experts receive noisy signals about the state and sequentially communicate the relevant information to the decision maker. In environments where the experts perfectly observe the state, the information is fully transmitted to the receiver. Conversely, the presence of noise renders a fully revealing equilibrium impossible. characterise the most informative equilibrium that might arise in such environments. I refine the self-serving belief system under uncertainty and show that there are two necessary and sufficient conditions for a semi-revealing equilibrium to exist: a) the noise structure is common knowledge and b) the experts are biased in opposite directions.

In the second chapter, I consider the case where a decision maker seeks advice from a biased expert who cares also about establishing a reputation of being competent. The expert has the incentives to misreport her information but she faces a trade-off between the gain from misrepresentation and the potential reputation loss. We show that the equilibrium is fully-revealing if the expert is not too biased and not too highly reputable. The threat of reputation loss is not enough to discipline a highly reputable expert and as a result it makes the decision maker avoiding the experts with too low or too high reputation. If there is competition between two experts the information transmission is always improved. However, in cases where the experts are more than two the result is ambiguous, and it depends on the players' prior belief over states.

In the last chapter, I consider a model of strategic communication where a privately and imperfectly informed sender can persuade a receiver. The sender may receive favorable (*good type*) or unfavorable (*bad type*) private information about her preferred state. I show that considering binary action space the private information of the sender does not improve the informativeness of theequilibrium which is only pooling. I describe two ways that are adopted in real life situations and theoretically improve equilibrium informativeness given sender's private information. First, a policy that suggests symmetry constraints to the experiments' choice and leads to a separating equilibrium for a range of prior beliefs. Second, an approval strategy characterised by a low precision threshold where the receiver will accept the sender with a positive probability and a higher one where the sender will be accepted with certainty. I show that this approval strategy supports always a separating equilibrium where the high type prefers to provide less noisy information in return of certainty while the bad type will provide the lowest possible precision.

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Chapter 1

Sequential Cheap Talk with Imperfectly Informed Experts[1]

Abstract

This paper studies a cheap talk game between two imperfectly informed experts and an uninformed decision maker. The experts receive noisy signals about the state and sequentially communicate the relevant information to the decision maker. In environments where the experts perfectly observe the state, the information is fully transmitted to the receiver. Conversely, the presence of noise renders a fully revealing equilibrium impossible. We characterise the most informative equilibrium that might arise in such environments. We refine the self-serving belief system under uncertainty and we show that there are two necessary and sufficient conditions for a semi-revealing equilibrium to exist: a) the noise structure is common knowledge and b) the experts are biased in opposite directions.

[1]I am deeply grateful to Elias Carroni, Vincenzo Denicolò and Davide Dragone for providing guidance and support. I also thank Ennio Bilancini, Leonardo Boncinelli, Francesca Barigozzi and Emilio Calvano as well as seminar participants at the University of Bologna and the 14th RGS conference for providing valuable feedback and comments. All remaining errors are mine.

1.1 Introduction

Decision makers are often uninformed and must rely on experts' advice to make informed choices. For example, public funding for research is often allocated through a system of peer review in which applications are evaluated by scientists with specialised knowledge. Similarly when someone experiences pain or illness, she may consult medical experts. Tension arises when the decision-maker's preferences over the final outcome are in conflict with those of the experts. In this case, the information transmission is strategic in the sense that the objective of the experts is not to provide information of the highest possible quality but to maximise their payoff. For example, reviewers or referees can be biased when the research proposal in-hand is close to their own research interests.

In this paper, we study whether information can be fully transmitted in these setups and, if no, what is the best equilibrium outcome we can achieve and how. We consider a cheap talk game between two imperfectly informed experts and one receiver in a one-dimensional large bounded state space. The experts receive noisy signals about the state and sequentially send messages to the receiver. That is, the second expert observes the message of the first one before choosing his message.

First, we show that the level of information that the decision maker manages to extract depends on how informed is the expert who starts the game. We start from the game of Crawford and Sobel (1982) where the decision maker has access to one single biased but perfectly informed expert and the amount of transmitted information depends on the level of expert's bias. We show that by adding a second expert who is not perfectly informed we can achieve the same level of informativeness as in the situations where both experts are perfectly informed. It is well known that when the experts are perfectly informed full information transmission is possible in two-senders and one-receiver situations. Krishna and Morgan (2001b) show that if the experts are biased in opposite directions, then there is a fully revealing equilibrium which is supported by the self serving belief system. That is the receiver by taking advantage of the experts' conflict of interest makes each expert to check whether the other one sends a truthful message. The experts are biased in opposite direction, in the meaning that each expert has a bliss point to the left (negatively biased) or to the right (positively biased) of the true state. According to the self serving criterion there exists for each expert a "lying zone" based on his bias' magnitude and the message of the other. Any message that belongs to this zone is ignored by the decision maker as self-serving and adopts any message outside the zone. If the first expert deviates, for instance, to the left then the lying zone of the second expert moves also to the left by leaving room to the second expert to deviate successfully to the right. Intuitively, the decision maker knows the exact distance between the experts' bliss points and by comparing the experts' messages can infer who lied first and then punish him. Therefore, the first expert has never the incentives to send an untruthful message because he will be always detected and punished by the second expert who has the second-mover's advantage.

The result of full information transmission depends crucially on the assumption of perfect information, at least of the first expert. The decision maker can apply the self-serving criterion if first, both experts receive the exact same signal or at least share the same posterior belief over the state and second, the magnitude and direction of their bias are common knowledge. However, under the assumption of both experts being imperfectly informed there is agreement between the signals with 0-probability and therefore the experts do not share the same posterior over the state. In turn, the distance between the bliss points is not certain anymore. Therefore, a straightforward application of self-serving belief does not support an equilibrium. For this reason, we propose a refinement of the self-serving belief system proposed by Krishna and Morgan (2001b) which allows the decision maker to apply the self-serving criterion by considering the minimum and the maximum distance between the bliss points and in turn an "extended lying zone" for each expert. We take advantage of the experts' state dependent preferences and the fact that the second expert forms his belief not only based on his signal but also considering the first's expert message if and only if he believes that the message is truthful. Hence, the extended lying zone is based on the posterior beliefs that the second expert should have formed if he has updated his belief considering the second expert's message. We show that there exists a semi-revealing partition equilibrium supported by this refined self-serving belief system where the signal of the first expert is fully revealed to the decision maker. There are two necessary and sufficient conditions for this equilibrium to exist: a) the experts are biased in opposite directions and b) the noise structure is common knowledge.

Similarly to our approach Ambrus and Lu (2014) show that there exists an equilibrium of the same level of informativeness as the one that we propose but through a different protocol. [2] In line with the model of Battaglini (2004) where the experts provide information

[2]The equivalence of simultaneous and sequential protocol in terms of equilibrium informativeness has been also pointed out by Hu and Sobel (2019) in a model of information disclosure. More precisely they come to similar conclusions with us and claim that the choice of protocol can lead to the same level of information disclosure but there are crucial differences between the two procedures. The sequential protocol requires that the

on different dimensions of the problem, in Ambrus and Lu (2014) the experts are asked to provide simultaneously complementary information on a unidimensional problem. In other words, they transform the one-dimensional state space into two-dimensional and their positive result of almost full information transmission is driven by the limited power that the experts have over the final decision.

This paper contributes to the literature by suggesting a different methodology to achieve the same level of informativeness without the need of messages' complementarity in a unidimensional state space. In our model, the experts provide the same piece of information but the sequentiality and the conflict of interest works as discipline device that make any deviation non profitable.

1.1.1 Related Literature

The literature on strategic information transmission, which dates back to the seminal work of Crawford and Sobel (1982), shows that in the presence of just one single informed expert, the information transmission can be substantially reduced due to strategic motives. Several papers by extending the model of Crawford and Sobel have shown that the information loss due to the conflict of interest between the expert and the decision maker can be fully retrieved through the presence of multiple experts. This paper belongs to this strand of the literature that studies the two senders-one receiver case. In general, in situations where decision maker has the chance to ask advice by more than one experts the advising is framed in two main setups: simultaneous or sequential communication.

Starting from simultaneous multi-sender cheap talk, Battaglini (2002) defines a two-dimensional unbounded state space and analyses simultaneous communication between two experts and a decision maker. Under his belief system, the decision maker forces each sender to report only one piece of the two-dimensional private information. By aggregating both messages, the decision maker is able to infer the state. Ambrus and Takahashi (2008) consider the same problem in bounded state space and show that the full revelation result in Battaglini (2002) depends crucially on the unboundedness of the state space.

Regarding sequential multi-sender cheap talk, Miura (2014) and Kawai (2015) extended Krishna and Morgan (2001b) sequential cheap talk model to the case of multidimensional state space. Miura (2014) proposes a new belief system, called extended self-serving belief, that is more restricted than the original one such as to prevent the compromised deviations

decision maker knows with high accuracy the preferences of the experts and in turn the design of the optimal protocol depend on them. Instead, the simultaneous protocol does not depend on the preferences of the experts.

that might arise in case of not perfectly opposite biases. The extended self-serving belief system proposed by Miura (2014) can be treated as a complement to Kawai (2015) belief system. Kawai (2015) belief system is suitable for environments where the receiver is able to choose any final action independently of the experts' messages. In the original system proposed by Krishna and Morgan (2001b) and the extended self-serving belief of Miura (2014) the decision maker had to choose one of the recommended actions.

The papers listed above describe situations where experts operate under perfect information, either in simultaneous or sequential communication setups. In this paper however we study cases with sequential communication but with imperfectly informed experts.[3] [4]The closest paper to ours in the literature investigating cheap talk games with imperfectly informed experts is Ambrus and Lu (2014) that has been motivated by the negative result of Battaglini (2002). Battaglini (2002) shows that in a one-dimensional state space, it cannot exist any fully revealing equilibrium with two imperfectly informed experts. However, he does not characterise the most informative equilibrium that might arise in a one-dimensional state space. In a subsequent study, considering a multidimensional state space and two imperfectly informed experts Battaglini (2004) shows that the decision maker can fully extract the information by the experts in a specific model with continuous noise. He proves that the multidimensionality, and in turn the complementarity of the messages, are the necessary elements for equilibrium existence. Ambrus and Lu (2014) similarly prove the existence of a semi-revealing equilibrium in one-dimensional state space where the experts send, simultaneously, complementary messages to the decision maker.

For large bounded state space and bounded continuous noise our model leads to the same level of informativeness as Ambrus and Lu (2014). For this reason, in one-dimensional environments and bounded state space, where in practice it is difficult to convert the problem from unidimensional to two-dimensional problem the sequential advising seems more reasonable. Often, in real life there are decisions that cannot be divided into different dimensions. However, sequential communication requires accurate knowledge about the preferences of the experts, while simultaneous communication results hold for any level and

[3]Foerster (2019) considers a cheap talk game between an imperfectly informed expert who receives multiple binary signals about the state and he can report his information either directly or indirectly. He shows that fully informative equilibria exist if the conflict of interest is small and only indirect-transmission equilibria are partially informative for intermediate conflicts of interest.

[4]Lu (2017) considers the case where the distributions of senders' signals are not common knowledge but it is common knowledge that their observations are near the true state. He proves that the only equilibria that are robust to noise of unknown structure are not close to fully-revealing but to the one-sender game of Crawford and Sobel (1982).

direction of bias.

The rest of the paper is organised as follows. Section 2 introduces the model. Section 3 analyses and characterises the most informative equilibrium and Section 4 concludes.

1.2 Model

We consider a cheap talk game of three players: one receiver and two senders. The receiver is the decision-maker and the senders are experts denoted by E_i, where $i = \{1, 2\}$.[5] The experts receive signals $\omega_i \in \Omega$ of a random variable $\theta \in \Theta$, where θ is the *state* and $\Theta = [-\Lambda, \Lambda]$ is the state space. Let $F(\cdot)$ be the prior distribution of true state θ with density function $f(\cdot)$ which is strictly positive and continuous in state space Θ. Let $G(\theta, \omega_1, \omega_2)$ be the c.d.f of the joint distribution of $(\theta, \omega_1, \omega_2)$ and let $G_i^{\omega_1}$ be the marginal distribution of (θ, ω_2) conditional on ω_1. Similarly $G_i^{\omega_2}$ is the marginal distribution of (θ, ω_1) conditional on ω_2. Let $\Omega_i(\theta)$ be the support of ω_i conditional on θ with infimum and supremum $\underline{\omega}_i(\theta)$ and $\overline{\omega}_i(\theta)$ respectively. Analogically, $\Theta_i(\omega_i)$ is the support of θ conditional on ω_i with infimum $\underline{\theta}_i(\omega_i)$ and supremum $\overline{\theta}_i(\omega_i)$. We consider as a benchmark case the game described in Crawford and Sobel (1982) in which the expert observes the state perfectly ($\omega_1 = \theta$) but we add a second expert who receives a noisy signal about it. Then we study the case where both experts receive noisy signals. We assume that the structure of noise is common knowledge and that the observed signals follow a continuous distribution around the state. After observing their signals the experts advice sequentially and publicly the decision-maker. Let M_i be the experts' message space and $m_1(\omega_1) \in M_i$ and $m_2(\omega_2, m_1) \in M_i$ are the messages sent by the experts *sequentially* to the decision-maker. There are no restrictions on message space which coincides with state space Θ. The decision-maker is totally uninformed about θ and has to take an action based on the experts' advice. Let Y be the decision-maker's action set and let y be the decided final action.

The preferences of the players can be not perfectly aligned. For example, the experts may be biased in different directions and degrees relatively to the true state θ.[6] In other words, a biased expert prefers the decision maker to take an action which is not the optimal one given the state. Let us define the differences on preference bias by the parameter $b_i \in \mathbb{R}$. The preference bias of decision maker, denoted by b_0 is normalised to 0. We assume that,

[5] For convenience, we treat the experts as male and the decision-maker as female for the rest of the paper.

[6] The biases are assumed to be known and state-independent, b_i. However, our results hold also for state-dependent bias, $b_i(\theta)$. In that case we would just need and extra assumption about a universal bound of bias: $\forall \eta \geq 0$, there is a $\Lambda(\eta) > 0$ such as if $|y - \theta| \leq \eta$ and $|y' - \theta| \geq \Lambda(\eta)$ then $U_E(y, \theta) > U_E(y', \theta)$, for all $\theta \in \Theta$,

b_i is common knowledge and works as a measure of how expert i is biased compared to the decision-maker. We assume that $b_1 \neq b_2$ and $b_1, b_2 \neq 0$. The multiple experts problem can be divided into two cases regarding the direction of the biases. If $b_1 \cdot b_2 > 0$, then the experts are said to have like biases. If $b_1 \cdot b_2 < 0$, then the experts are said to have opposite biases. We assume a quadratic loss utility function, $U_i(y, \theta, b_i) = -U(y - (\theta + b_i))^2$, which satisfies the following assumptions and is commonly employed in most of the literature. The players' payoff function U is a twice continuously differentiable von Neumann-Morgenstern utility function strictly concave with a unique maximum in y. The ideal points for the decision-maker, Expert 1 and Expert 2 are $x_0 = \theta, x_1 = \theta + b_1, x_2 = \theta + b_2$ respectively. Moreover $U_{yb} > 0$ since if $b_i > 0$ then it must follow that $y^*(\theta, b_i) > y^*(\theta)$ and if $b_i < 0$ then it must follow that $y^*(\theta, b_i) < y^*(\theta, 0)$, where y^* is the optimal action given experts messages. We also make the following assumption:

Assumption 1.1 *The experts have opposite and small biases relatively to the state space:* $b_1 \cdot b_2 < 0$ *and* $|b_1|, |b_2| < \frac{\Lambda}{2}$.

1.3 Analysis

1.3.1 Noise in the signal of the second expert

We start our analysis with the simple case of two opposing biased experts where the first one observes perfectly the state while the second observes it with some noise. We will show that this scenario is similar to the case where both experts are perfectly informed.[7] We assume that:

Assumption 1.2 *There exists a* $\delta > 0$, *which is common knowledge, such that* $\omega_2 \in [\theta - \delta, \theta + \delta]$.

This situation can be interpreted as an extension of the basic setting of one sender-one receiver described by Crawford and Sobel (1982). The addition of the second expert even if he is imperfectly informed can lead to full information extraction similarly to the case where both experts are perfectly informed. The second expert, due to sequentiality, learns the state by the first expert even if he doesn't directly observe it and in equilibrium agrees with the first expert. The decision maker takes advantage of the conflict of interest between the two experts and by applying the self-serving criterion proposed by Krishna and Morgan (2001b)

[7] The equilibrium would coincide with the case of two perfectly informed experts also if the second expert observes either the true state or nothing (all-or-nothing scenario). This scenario has been discussed by Miura (2014) as an extension of his multidimensional model (see Section 5.5, Miura (2014)). A similar analysis could be implemented for the unidimensional scenario that we consider here and lead to the same consusion.

can discipline the experts to reveal fully the information. Before giving the intuition behind this result we should add first the definition of the self-serving belief system:

Definition 1.1 ***(Self-serving belief system)***

i) When Expert 1 sends a truthful message m_1, a message $m_2 \neq m_1$ sent by Expert 2 is self-serving if :

$$U_2\left(y^* = m_2, \Omega_2(m_1), b_2\right) > U_2(y^* = m_1, \Omega_2(m_1), b_2)$$

ii) The decision-maker has the self-serving belief if the posterior belief $\mu(\cdot\,|m_1,m_2)$ satisfies the following conditions:

1. *if m_2 is self-serving given m_1 then $\mu(m_1|m_1, m_2) = 1, \quad \forall m_i \in M_i$*
2. *if m_2 is not self-serving given m_1 then $\mu(m_2|m_1, m_2) = 1, \quad \forall m_i \in M_i$*

Self-serving messages would induce an action with higher utility to the Expert 2 given Expert 1's message. Hence, under the self-serving belief, we define an interval, "a lying zone", where the decision-maker believes m_1 and not m_2. Outside this zone, he believes Expert 2's message. The application of the self-serving criterion depends on the knowledge not only on the direction of the biases but also the magnitude. This belief system is particularly useful in setups where the experts provide their advice sequentially. Other existing alternative ways of opinion integration would not allow the receiver to fully extract the experts' information. For instance, the complementarity between the experts' messages that Battaglini (2002) proposes would not lead to a positive outcome. The second mover's advantage would give to the second expert the flexibility to tailor his report for his own benefit. The self-serving belief system relies on relatively restrictive assumptions (e.g accurate knowledge of senders' preferences) but from the other side is quite intuitive.[8] The decision maker knows nothing about the true state but she is able to compare the experts in terms of their bias and by fixing one's bliss point can infer the other's. Before stating the first result we introduce the following definition by Krishna and Morgan (2001b):

Definition 1.2 (*Extreme Preferences*) *An expert with bias $b_i > 0$ holds extreme views in state θ if $U(y^*(\theta), \theta, b_i) \leq U(y^*(\Lambda), \theta, b_i)$. Similarly, an expert with bias $b_i < 0$ holds extreme views in θ if $U(y^*(\theta), \theta, b_i) \leq U(y^*(-\Lambda), \theta, b_i)$,*

[8]We can imagine real life situations where one can apply the self serving criterion to update their belief. Assume that there are three friends A,B and C and two of them, friend A and B, have visited a restaurant. It is known that A is quite demanding and normally tends to characterise the food quality worse than it really is. Friend B instead tends to characterise it better than it is in reality. If friend C seeks for their opinion about this restaurant and friend A claims that the restaurant was good and the friend B argues instead that it was very good then the friend C will infer that the restaurant was just good. The message "very good" will be considered as self-serving since friend B characterise always the food quality a bit higher than it is.

If a right-biased expert holds extreme views in θ, then all actions that are higher than $y^*(\theta)$ are preferred by the expert. The same holds for a left-biased expert and low actions. An expert who is not an extremist is a moderate. This definition is important for the states that are close to the boundaries. In some cases, the decision maker cannot apply the self-serving criterion because there is not enough "space" for it due to boundness of state space.[9] In this case a "babbling" equilibrium arises, that is both messages are ignored by the decision maker. For the states where the second expert has no extreme preferences we state the following result:

Proposition 1.1 *If Expert 1 is perfectly informed and Expert 2 observes the state with some small noise under A1.2, then in the states for which Expert 2 does not hold extreme views the decision maker extracts fully the information by Expert 1.*

Proof. See Appendix A ■

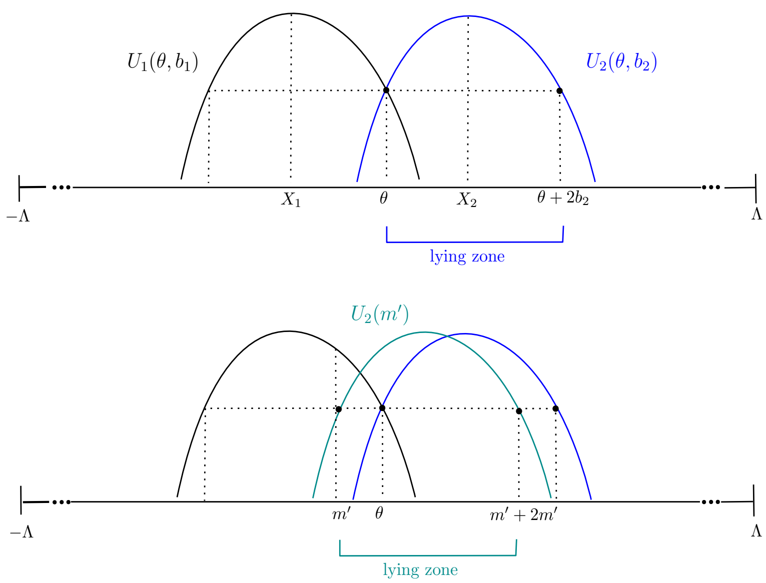


Figure 1.1 | **Expert 1 sends false message m'.** The upper graph represents the preferences of the two experts and the lying zone of the second expert when the Experts 1 sends a truthful message. The lower graph shows how the preferences and the lying zone of the second expert are perceived by the decision maker after an untruthful message m'.

Consider two experts who are biased in opposite directions. Without loss of generality, suppose that $b_1 < 0 < b_2$ (see Figure 1.1). First suppose that Expert 1 sends a truthful message $m = \theta$. Expert 2, does not observe $\omega_2 = \theta$ but something in the interval $[\theta - \delta, \theta + \delta]$. The second expert uses the message of the first expert in order to learn the state if he believes that

[9] If state space is unbounded, experts have never extreme preferences and this makes the full information extraction possible.

m_1 is truthful conditional on his own message. He considers m_1 as truthful if $|m_1 - \omega_2| < \delta$. Given that the message $m = \theta$ is truthful, Expert 2 cannot improve his utility by lying because any message that would make him better off will be considered as self-serving and the decision maker would just ignore it. Therefore, by sending the true message, expert 1 can induce the first-best action $y = \theta$. However, we have to see whether this is optimal for Expert 1.Now, suppose that Expert 1 sends a false message, $m_1 = m^{'}$, which is smaller than θ (closer to his bliss point $\theta + b_1$). With positive probability p, $|m^{'} - \omega_2| > \delta$. In this case the, the second expert does not update his belief considering m_1. He ignores the message of the first expert as untruthful and considers $\theta = \omega_2$. The lying zone of second expert is $(m', m' + 2b_2)$. However, expert 2 believes $\theta = \omega_2$ then there is a beneficial untruthful message outside (on the right hand side) of $(m', m' + 2b_2)$. Therefore, Expert 2 can always send a credible message that would lead to an action closer to his bliss point. Thus, expert 1 has no incentive to lie. es the second expert to crosscheck whether the first expert's message is credible despite the fact that he is less informed. The assumption about biases' direction is crucial for the equilibrium simply because in case of similar preferences a possible deviation of the first expert would be adopted also by the second expert as well. However, It is not possible for the decision maker to apply the self-serving criterion for $\theta > \Lambda - 2b_2$. This is because the second expert after a possible deviation of expert 1 would send a message $m_2 > \Lambda$ but this is not possible due to boundness of state space. Therefore for $\theta > \Lambda - 2b_2$, there is only a "babbling" equilibrium. In the next section, we show that by allowing Expert 1 to rebut m_2, that is by changing the order of experts, we can solve this problem and obtain a more informative equilibrium than the babbling. If the state space is unbounded then the decision maker achieves full information extraction for every state θ.

1.3.2 Noise in the signals of both experts

From now on, we assume that both experts are imperfectly informed. We will consider the following assumption about the signals' distribution:

Assumption 1.3 (Bounded continuous noise) *There exists δ_i such that $\omega_i \in [\theta - \delta_i, \theta + \delta_i]$. The support of ω_i given ω_j is common knowledge. Moreover $\forall \omega_i$ there exist a $\underline{p} > 0$ such that $g(\omega_j|\omega_i) > \underline{p}$ everywhere on $\Omega_j(\omega_i)$ where:*

$$\underline{\omega}_i(\omega_j) = \min\{\omega_i \in \Omega_i(\omega_j) \| \ \omega_j \in \Theta\} \quad (1.1)$$

$$\overline{\omega}_i(\omega_j) = \max\{\omega_i \in \Omega_i(\omega_j) \| \ \omega_j \in \Theta\} \quad (1.2)$$

$\underline{\omega}_j(\cdot)$ and $\overline{\omega_j}(\cdot)$ are continuous and strictly increasing.

Definition 1.3 (*Maximum signal support*) *We denote by d the maximum joint support of senders' signals, which is the distance between $\underline{\omega}_i(\omega_j)$ and $\overline{\omega}_i(\omega_j)$:*

$$d \equiv |\overline{\omega}_i(\omega_j) - \underline{\omega}_i(\omega_j)|$$

The maximum distance between two signals is $\frac{d}{2}$.

Definition 1.4 (*Maximum belief support*) *We denote by c the maximum support of senders' beliefs on state, which is the distance between $\underline{\theta}(\omega_i, \omega_j)$ and $\overline{\theta}(\omega_i, \omega_j)$*

$$c \equiv |\overline{\theta}(\omega_i, \omega_j) - \underline{\theta}(\omega_i, \omega_j)|$$

The maximum distance between the beliefs of senders about the state is $\frac{c}{2}$.

We assume that the signals follow a uniform distribution around the state. Similarly to section 3.1, we assume that the noise has local size which is common knowledge but not necessarily so small [10] and the preciseness of the signals can differ ($\delta_i \neq \delta_j$). As we will show in the next section it is optimal for the decision maker to ask the advice first from the expert with the higher precision (smaller δ). Everything is common knowledge except for the realisation of the signals. The solution concept we consider is a weak concept of perfect Bayesian equilibrium and we focus on the pure strategy equilibria.

Definition 1.5 *The Decision-maker's action rule $\hat{y}$, the belief rule $\hat{\mu}$ and the experts' signalling rule $\hat{m}_i$ constitutes a pure strategy weak Perfect Bayesian Nash equilibrium if:*

1. $\forall \omega_1 \in \Theta$, $\hat{m}_1(\omega_1) = \arg\max_{m_1 \in \mathcal{M}_1} \int_{\theta, \omega_2 \in \Theta} U_1(\hat{y}(m_1, m_2(m_1, \omega_2), \theta, b_1)) dG_1^{\omega_1}$

2. $\forall \omega_2 \in \Theta$, $\hat{m}_2(\omega_1, \hat{m}_1) = \arg\max_{m_2 \in \mathcal{M}_2} \int_{\theta, \omega_1 \in \Theta} U_2(\hat{y}(\hat{m}_1, m_2(m_1, \omega_2), \theta, b_2)) dG_2^{\omega_2}$

3. $\forall (m_1, m_2) \in \mathcal{M}_1 \times \mathcal{M}_2$, $\hat{y}(m_1, m_2) = \arg\max_{y \in \mathcal{Y}} \int_{\theta \in \Theta} U_0(\theta, y) d\mu(m_1, m_2)$.

4. *$\hat{\mu}(m_1, m_2)$ is derived by Bayes' rule, whenever it is possible.*

Intuitively, after receiving the messages from the experts, the policy maker correctly updates his beliefs about the possible states of the world and make his optimal decision. On the other side, each expert sends the message that maximises his payoff given the policy maker's and the other expert's optimal strategies. The decision maker aims to extract all the information from the experts. In the rest of the paper we refer to this as *fully-revealing equilibrium.*

[10] For the rest of the paper we will consider only the uniform distribution. We report that our results can be generalised for any continuous and symmetric distribution with bounded support.

1.3.3 Self-serving belief under uncertainty

Under A1.3 a straightforward application of self-serving belief does not support an equilibrium. The two experts never share the same posterior over θ because both are imperfectly informed and they observe the same signal with 0-probability. In turn, decision maker never receives $m_1 = m_2$ so she cannot directly apply the self-serving belief criterion. This makes the refinement of the self-serving belief system for the case of uncertainty necessary for equilibrium existence.

Under perfect information, the first expert is not able to deviate even slightly without being detected by the second expert and the decision maker. This is because the decision maker knows the exact distance between the experts' bliss points for every state θ and based on that she decides which recommendation to adopt. However, under imperfect information this distance is not anymore common knowledge. The certain disagreement between the two messages does not allow the existence of any separate equilibrium. In this case, there is only a "babbling" equilibrium where all messages from both experts are completely ignored by the decision maker. We propose below an extension of the self-serving belief system proposed by KM which allows to the decision maker to apply the self serving criterion under the assumption that the signals are not identical.

The decision maker can infer the minimum and the maximum distance between the most preferred actions of experts given the distribution of their signals. An important detail is that under perfect information the beliefs of the second expert are not affected by the first expert's message, but under imperfect information they do. The first expert updates his beliefs based on the signal that he receives but the second one uses also the message of the first expert if and only if he believes that this is truthful conditional on his own signal. Otherwise, the second expert ignores the first expert's signal and form his belief about the state based only on his signal. Therefore, the decision maker has to consider the maximum distance between the signals of the experts and the maximum value (if he is positively biased) or minimum value (if he is negatively biased) of his bliss point given that the first message is truthful. We define the *self serving belief under uncertainty* as follows:

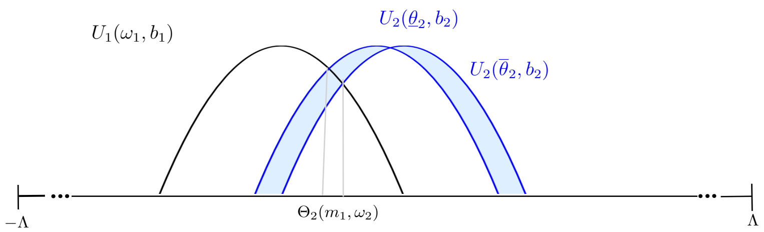
Definition 1.6 (Self-serving belief under uncertainty) *Given that Expert 1 sends a truthful message, m_1, then a message m_2 sent by Expert 2 is considered as self-serving if:*

$$U_2\left(y^*(m_2),\ \overline{x}_2\right) > U_2\left(y^*(m_1),\ \overline{x}_2\right) \quad \textit{if} \quad b_2 > 0$$

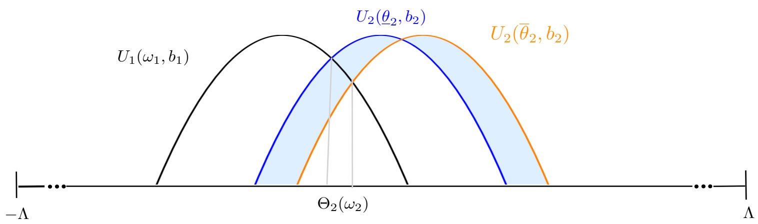
$$U_2\left(y^*(m_2),\ \underline{x}_2\right) > U_2\left(y^*(m_1),\ \underline{x}_2\right) \quad \textit{if} \quad b_2 < 0$$

where, given a truthful m_1, $\overline{x}_2$ *is the maximum value that the bliss point of a positively biased expert takes and* $\underline{x}_2$ *is the minimum value that the bliss point of a negatively biased expert takes.*

We assume an *"extended lying zone"* (see Figure A.5) where any message coming from a right-biased expert that leads to an action in $[m_1, \overline{x}_2 + b_2]$ will be considered as self-serving. Similarly, from a left-biased expert any message that leads to an action in $[\underline{x}_2 - b_2, m_1]$.



Case A: Posterior of Expert 2 considering m_1



Case B: Posterior of Expert 2 ignoring m_1

Figure 1.2 | **Self-serving belief under uncertainty.** The first graph represents the range of the posterior beliefs of E_2 considering the message of E_1. Instead, the second graph shows his posterior without considering the first expert's message.

For a right-biased expert, the decision maker has to consider the highest posterior of θ given a truthful m_1 and for a left side biased expert the lowest posterior of θ. This belief system is based on the idea that if the first expert deviates and the second expert realises it, then Expert 1 is uncertain first about Expert's 2 posterior over θ and in extension his message strategy. In Figure 1.2A, we present graphically the beliefs of decision maker about expert's preferred actions. Fixing the bliss point of Expert 1 the decision maker infers that the utility curves of the Expert 2 belongs to the blue area . In Figure 1.2B, we show the range of Expert's 2 preferences in case that Expert 1 deviates and is detected by Expert 2: the blue area expands because the second expert ignores m_1. To sum up, the difference between the original belief system and the refined one that we propose comes from the fact that we do not require agreement between the experts' *signals* but between their posterior *belief* over the state. This is an important element under imperfect information and state dependent preferences because in equilibrium the second expert will naturally update his belief based both on his signal and the other expert's message.

1.3.4 Partition Equilibrium

We propose a partition equilibrium where the partition depends on maximal belief support, as defined in D1.4, and which coincides with $\Theta(\omega_1)$. The equilibrium messages of the two experts are supposed to coincide as in the case of perfect information. The difference now, is that the experts are not asked to report the exact value of the state but the interval where the state belongs to. Intervals, i.e categories is a typical way to convey information and judgements when a fine-grained message cannot be sent. Ambrus and Lu (2014) propose a similar partition equilibrium which is based instead on maximum signal support as it is defined in D1.3.

We assume that the senders observe the state with bounded continuous noise as it is defined in A1.3 and we prove that there exists a partition equilibrium. An equilibrium is considered as a partition equilibrium if the state space Θ can be partitioned into intervals such that all the experts who observe ω_j which belongs in a given interval use the same message strategy. More formally for messages m and signals ω:

Definition 1.7 *A Bayesian Nash Equilibrium $(\hat{q}(\cdot), \hat{y}(\cdot))$ is considered as a partition equilibrium of size N, if we partition the state space $\Theta = [-\Lambda, \Lambda]$ in N intervals: $-\Lambda = a_0 < a_1 < \ldots < a_N = \Lambda$ such that $q(m|\omega) = q(m|\omega')$ if $g(\theta|\omega), g(\theta|\omega') \in (a_i, a_{i+1})$, and if $q(m|\omega) > 0$ for $g(\theta|\omega) \in (a_i, a_{i+1})$ then $q(m|\omega') = 0$ for $g(\theta|\omega') \in (a_j, a_{j+1})$ where $j \neq i$.*

Therefore, in a partition equilibrium the decision maker can infer from experts' messages in which interval the true state belongs to. We identify the messages with the right limit of the interval that refer to, i.e $m^i = [a_{i-1}, a_i)$ where $[a_{i-1}, a_i)$ [11] is the interval where ω_j lies.

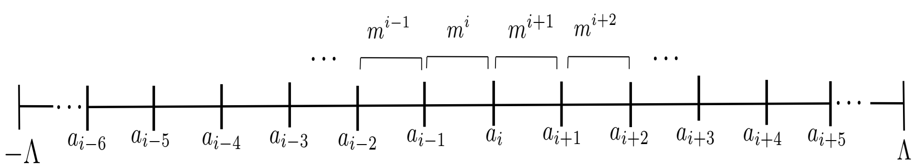


Figure 1.3 | **Space Partition.** This is the space partition announced by the first expert where the length of each interval is equal to the maximum belief support $\Theta(\omega_1)$.

[11] As a matter of notation we assume that each interval is closed on the left and open to the right except for the last one which is closed from both sides

We have to construct a state space partition (see Figure 1.3), where the decision maker can apply the self-serving belief system despite the experts' signals disagreement. In other words, we should partition Θ in such a way that experts' beliefs and in turn messages will coincide with probability-1even if their signals are not identical. We assume that the support of the signals, given the signals of the other expert, is common knowledge. Under this assumption if the experts truthfully report their belief, their messages will agree with probability-1. In equilibrium we have a finite number of intervals constructed based on the signals' distribution. Intuitively, the policy maker can accept a maximum level of disagreement between the experts' belief that could be justified by the noise. The signals are not perfectly correlated but both depend on the state θ so cannot be too far away from each other. The following Lemma and Propositions provide the conditions for the equilibrium partition.

Lemma 1.1 *Under A1.3, $m_i = [\underline{a}, \overline{a}]$ is a message sent in a partition equilibrium supported by self-serving beliefs, then $|\overline{a} - \underline{a}| \geq c$*

Proof. See Appendix A ■

Lemma 1.1 implies that the length of each interval should be at least as large as $|\overline{\theta}(\omega_j) - \underline{\theta}(\omega_j)|$ in order to allow agreement of belief with probability-1 if the experts are truthful. Therefore, for even a small amount of noise there is never a separate (non partition) equilibrium supported by the self-serving belief system:

Proposition 1.2 *Under A1.1 and A1.3, the number of intervals sent in a partition equilibrium is finite if $\delta_i > 0$.*

Proof. See Appendix A ■

Instead, for $\delta_i = 0$ we have infinite number of intervals (separate equilibrium) and our model coincides with the model of Krishna and Morgan (2001b). If $\delta_1 = 0$, we have technically a partition equilibrium but the decision maker can learn exactly the state and it is supported by the self-serving belief system of Krishna and Morgan (2001b)without the need of refinement. We consider the following definition:

Definition 1.8 *An expert with bias $b_i > 0$ holds extreme preferences about θ if given his posterior about θ he prefers the right extreme interval $[a_k, \Lambda]$.*
Similarly, an expert with bias $b_i < 0$ holds extreme preferences about θ if given his posterior about θ he prefers the left extreme interval $[-\Lambda, a_l]$.

We state the following result for $\delta_1, \delta_2 > 0$:

Proposition 1.3 *Under A1.1 and A1.3, there exists an equilibrium supported by self-serving belief under uncertainty, where if:*

1. *none of the experts has extreme preferences given $\Theta(\omega_1)$, then ω_1 is fully revealed to the decision maker*

2. *second expert has extreme preferences given $\Theta(\omega_1)$, then ω_2 is fully revealed to the decision maker*

Proof. See Appendix A ■

The Expert 1 can proceed to a *serious* or a *small* deviation which are related to disagreement probability and we define formally as follows:

Definition 1.9 (Deviations) *An Expert proceeds to a small deviation $m_i' = \omega_i + \epsilon$ if $\Omega_j(\omega_i) \cap \Omega_j(\omega_i + \epsilon) \neq \emptyset$, while to a serious deviation $m_i' = \omega_i + E$ if $\Omega_j(\omega_i) \cap \Omega_j(\omega_i + E) = \emptyset$.*

We consider as serious any deviation greater than c and a as small any deviation less than c. In case of a small deviation ω_j does not lie in m_i with a positive probability $p = g(\omega_j|\omega_i)\epsilon$. Instead, a serious deviation ω_j does not lie in m_i with probability-1.

Below we summarise the equilibrium construction of **Proposition 1.3** and we present graphically (see Figure 1.4 and Figure 1.5) [12] why a serious deviation by Expert 1 is not profitable.

Suppose that the two experts observe ω_1 and ω_2 which are really close to each other. Then Expert 1 has to send $m_1 = m^i$ such as $|a_{i-1} - a_i| = |\overline{\theta}(\omega_1) - \underline{\theta}(\omega_1)|$. Then Expert 2 has to send a message $m_2 \in \Lambda(m^i)$. Assume that the bliss point of Expert 1 lies in the interval m^{i-1} and the bliss point of expert 2 lies in the interval m^{i+1}. Expert 1 sends the true message $m_1 = m^i$. Following the same reasoning of perfect information model, given the message of expert 1, Expert 2 cannot improve his utility by lying because such messages are always self-serving. Thus, by sending the true message, expert 1 can induce the action $y^*(m^i) = \omega_1$. Next, suppose that expert 1 sends a false message, $m_1' = m^{i-1}$. Given that message m_1', the policy maker considers the bliss point of expert 2 being m^i and expert 2 can always send a credible message that gives him higher utility. The optimal message for expert 2 is m^{i+1}. Because both experts have opposing-biased preferences, $y = m^{i+1}$ is worse for expert 1 than $y = \omega_1$. Thus, expert 1 has no incentive to lie. Therefore the signal of the first expert is fully revealed and it is adapted by the decision maker.

[12] For convenience, we use graphs that appear to be extremely symmetric. However, the results hold for asymmetric biases and expertises.See Appendix B for numerical examples.

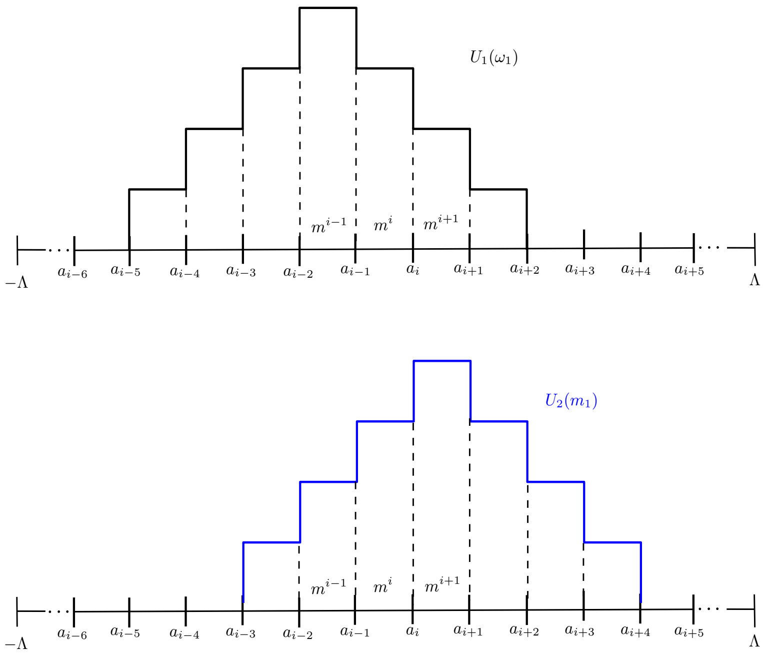


Figure 1.4 | **On the equilibrium path.** The Expert 1 sends truthfully m^i.

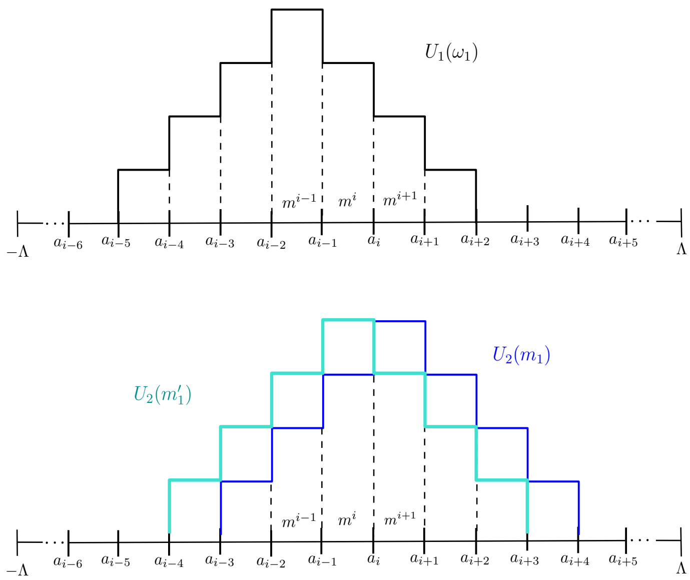


Figure 1.5 | **Off the equilibrium path.** The Expert 1 sends a false message m^{i-1}

Up to now, we gave the intuition of equilibrium existence where the signal of the first expert is fully revealed. Regarding the second part of **Proposition 1.3**, similarly to the case of perfect information we have to change the order of advising. The game should start with the second expert who have extreme preferences, and this happens if $\Theta(\omega_1)$ and $[2\Lambda - 2b_2, \Lambda]$ are two overlapping intervals.

1.3.5 Characterisation

Given **Proposition 1.3**, the next result establishes that the length of each interval should have the exact same size with $\Theta(\omega_1)$ such as if expert 1 deviates then with positive probability the signal of Expert 2 will not fall in the interval that he recommended.

Proposition 1.4 *There is a unique partition equilibrium $\Lambda^*(\omega_1)$ where the length of the intervals is exactly c.*

Proof. See Appendix A ■

In Crawford and Sobel (1982) there is a maximum number of intervals that support an equilibrium. In our case, for every combination of experts there is a unique number of intervals of length c that supports the partition equilibrium. Moreover, the length of the interval in the model of Crawford and Sobel (1982) determines the informativeness of the equilibrium. In our case only the expert who starts the game affects it.

Proposition 1.3 implies that there is not a fully revealing equilibrium but only a semi-revealing which results to an induced action close to the true state but it does not coincide with it. Therefore we would like to know how close is this action to the first best. Given that under the proposed equilibrium construction the signal of the first expert is fully revealed and the optimal action it coincides with it we state the following remark:

Remark 1.1 *There is Semi-Revealing equilibrium where, If none of the experts has extreme preferences given $\Theta(\omega_1)$, as $\delta_1 \to 0$, the optimal action of the policy maker converges to the first best regardless of δ_2. The loss of information is equal to σ_1^2, where σ_1^2 is the variance of δ_1.*

In this case informativeness of the equilibrium depends only on the preciseness of first expert's signal. [13] In other words, the distance between the true state θ and the final induced action y on the equilibrium path depends only on the preciseness of the expert's 1 signal. The preciseness of expert's 2 signal does not determine the length of the intervals as in Ambrus and Lu (2014) and in equilibrium it does not affect the final action. The decision maker prefers one equilibrium over the other based on the length of the intervals, like it happens in Crawford and Sobel (1982) , where the informativeness of the equilibrium depends on the length of intervals which are constructed based on the expert's bias. Therefore, since the experts can have, by assumption, different levels of expertise there is an optimal order of sending the messages.

Definition 1.10 (Informativeness) *An equilibrium Γ is the most informative equilibrium if the decision maker prefers ex-ante Γ over any equilibrium $\Gamma' \neq \Gamma$.*

[13] It is easy to show that with unbounded state space ω_1 is always fully revealed.

Given **Remark 1.1**, we can make the following remark:

Remark 1.2 *The most informative equilibrium is the equilibrium where the most informed expert starts the game.*

Remark 2 holds as soon as there are non extreme preferences. However, given that the space is large enough relatively to the biases and the noise, the most probable is that none of the experts has extreme preferences given $\Theta(\omega_1)$. Therefore the proof of **Remark 1.2** is straightforward because the partition will be constructed based on the signal of the most informed expert. Finally, as **Remark 1.1** states the second expert's information does not affect the final action. However, it is affect the behaviour of the first expert:

Remark 1.3 *The risk that the first expert faces after a small deviation increases as the precision of the second expert's signal increases.*

Proof. See Appendix A ■

The preciseness of the first expert's signal is decisive for the whole process. The informativeness of the equilibrium depends only on the first expert and the role of the second one is just to confirm the information that the first expert has already provided. However, there is a positive probability that the first expert deviates without being detected and this causes an informational loss. It is easy to see though that this probability decreases as the precision of second expert's signal increases. Graphically, after a deviation by the first expert, the smaller the interval, the higher the probability that the second expert observes a signal outside of it.

1.4 Conclusion and Discussion

Starting from the scenario where only the second expert receives a noisy signal while the first one observes perfectly the state, we show that the level of information that the decision maker manages to extract depends only on the expert who starts the game. The addition of a second expert who is not perfectly informed can lead to the same level of informativeness as in the sequential cheap talk games where both experts are perfectly informed. Focusing on this first result one can conclude that the value of a second opinion is not necessarily linked to the additional information that is provided but through the control that is exercised by the second expert to the first one. In our setting, the second expert has nothing to add in terms of knowledge since the first one is perfectly informed but he is able to discipline him.

Regarding the case of two perfectly informed experts, Krishna and Morgan (2001b) show that if the experts are biased in opposite directions, then there is a fully revealing equilibrium which is supported by the self serving belief system. However, we show that self-serving belief system of Krishna and Morgan (2001b) does not support an equilibrium in case that both experts are imperfectly informed. The certain disagreement between experts' reports makes a straightforward application of self-serving belief impossible. For this reason we extended the self-serving belief system and we proposed a partition equilibrium based on the distribution of signals. We showed that the most informative equilibrium that we can achieve is a semi-revealing equilibrium where the signal of the first expert is fully revealed to the decision maker and the second expert just confirms that his signal belongs to the same interval with first's expert signal. The common knowledge of the noise structure and the opposite biases are required for a semi-revealing equilibrium existence. In this paper, the expert who starts the game determines the level of equilibrium informativeness. Similarly, Ambrus and Lu (2014) who consider the same setup but simultaneous protocol show that the information the decision maker manages to extract depends on the expert who acts first and chooses the space partition. Therefore both protocols can lead to the same informativeness as soon as there are designed accordingly.

Considering this paper together with Ambrus and Lu (2014) we could not come to any definite conclusion as to which protocol is preferable and under what conditions. They both seem to be appealing in different dimensions. For instance, the sequential protocol requires precise knowledge of the experts bias which might be a restrictive assumption. Conversely, the simultaneous protocol can be successful regardless the experts' preferences but the required complementarity of the messages may not be easily applicable in a one-dimensional state space. In general, we see that in real life the two protocols are implemented and preferred in different settings. Therefore, a detailed and careful comparison of the two protocols could be very interesting and valuable given the numerous possible applications.

There are many possible other avenues for future research. The extension of Krishna and Morgan (2001b) by Miura (2014) and Kawai (2015) into n-dimensional state space was not trivial. As it has been shown by these two papers in a n-dimensional state space it is possible that the two experts have incentives for compromises so the problem becomes much more complex. More precisely, a possible deviation of first expert will be adopted also from the other. To this end a natural extension of our model would be to see what happens if the state space has n-dimensions and the experts are imperfectly informed.

Moreover, a crucial assumption is that the structure of the noise is common knowledge. Our partition equilibrium is constructed basically given the support of Expert's 2 signal given Expert's 1 signal. Without this knowledge an informative equilibrium is not possible. Lu (2017) studies simultaneous cheap talk games with multiple senders and unknown signal distributions. It would be interesting to investigate whether we could obtain a positive result in case of sequential cheap talk since the resulted equilibrium of Lu (2017) does not approach full revelation. We leave these extensions for future work.

Chapter 2

Conflict of Interest, Reputation and Competition [1]

Abstract

In a cheap-talk framework, a decision maker seeks advice from a biased expert who cares also about establishing a reputation of being competent. The expert has the incentive to misreport her information but she faces a trade-off between the gain from misrepresentation and the potential reputation loss. We show that the equilibrium is fully-revealing if the expert is not too biased and not too highly reputable. The threat of reputation loss is not enough to discipline a highly reputable expert and as a result it makes the decision maker to avoid the experts with too low or too high reputation. If there is competition between two experts the information transmission is always improved. However, in cases where the experts are more than two the result is ambiguous, and it depends on the players' prior belief over the states.

[1] I am deeply grateful to Elias Carroni, Vincenzo Denicolò and Davide Dragone for providing guidance and support. I also thank Ennio Bilancini, Leonardo Boncinelli, Emilio Calvano and Francesca Barigozzi as well as seminar participants at the University of Bologna, the University of Málaga and the 15th RGS conference for providing valuable feedback and comments. All remaining errors are mine.

2.1 Introduction

When decision-makers do not have specialised knowledge over the subjects they take decisions on, they seek advice from experts. In this case, there are two key elements one should pay attention to: the competence and the credibility of the expert. Expert's competence concerns her information accuracy while her credibility has to do with her preferences over the final decision. In many situations the preferences of the expert might not be in line with those of the decision maker. The latter might be aware of this conflict of interest and he is called to interpret her advice in the light of her bias. For instance, in politics, ministers rely on their special advisers' opinion despite the fact that they might have their own agenda. Fortunately, the expert's concern to be considered as competent or the possibility of the decision maker to seek an additional opinion can mitigate the risk of biased advice.

In this paper, we study these situations through the lens of a cheap talk game between an expert (*She*) and a decision maker (*He*). The state of the world is unknown to both of them but they share the same prior over it. The expert receives a noisy signal about the state and its precision depends on her competency. Both of them are uncertain over the accuracy of expert's information and the only knowledge is the prior over it. The decision maker's choice problem is binary which means that for extreme priors, it is optimal for him to disregard the message by an expert who is not reputable enough. For this reason we focus on the cases where the expert can provide decision relevant information (*effective messages*). We assume that the expert is biased such that she prefers the same decision for all states and this is common knowledge. She is concerned about establishing a reputation for providing valuable information but at the same time she has clear incentives to misreport her information. In other words, she faces a trade-off between misinterpretation gain and reputation loss; if she manages to "move" the decision maker's beliefs towards her preferred action by an untruthful recommendation she might have a loss in terms of reputation. Considering the above setup we show how priors over state and expert's competence determine the amount of credible and relevant information transmitted in equilibrium and characterise the most informative equilibrium that might arise out of this situation.

More precisely, we find that there is a maximum degree of conflict between the expert and the decision maker that allows the existence of a fully revealing equilibrium, meaning that the expert does not send always the same message irrespectively of her signal. Moreover, contrary to the scenario of unbiased experts with reputational concern on ability (Ottaviani and Sørensen, 2001, 2006), the information transmission is not always improved as the initial reputation of the expert increases. The biased expert misrepresents her information when the priors are polarised but at the same time she sends untruthful messages also being driven by her bias. One of the main results is that an expert can be "too reputable" to provide truthful advice. A very reputable expert might have limited reputational gains in comparison to a less reputable one. Therefore the decision maker faces a trade-off between accuracy and discipline and avoids the experts with too low or too high reputation, for different reasons respectively. The experts of low reputation are not competent enough and the ones of high reputation not credible.

Finally, we consider also the case where the decision maker has access to a second opinion. We assume the simplest case where the decision maker has at his disposal two identical experts in terms of bias and prior reputation who provide simultaneously their information. We show that the second expert facilitates the information transmission in three ways. First, the range of priors over the state where the experts together can provide effective messages expands. Second, the additional expert acts as a discipline device to the other expert since in case of disagreement an untruthful message can lead to reputation loss without any other gain. Third, it increases the level of initial reputation that maximises the information transmission. Similarly to the one expert case, there is a maximum level of conflict that allows for a separating equilibrium although higher than the respective threshold of single expert case.

Considering the scenario where the decision maker can reach out a third expert, we find that despite the fact that the overall information transmission increases, it is optimal to consult less experts for a range of low prior belief. Similarly to the two-experts case, the additional expert always increases the range of priors for which the receiver will adopt the recommendations (*effective messages provision*). For this reason, the decision maker with polarised prior belief is affected and benefit from two or more truthful recommendations that coincide and move his belief towards the true state, something that couldn't happen with only one expert. However, the biased experts make truthful but risky for their reputation recommendation when they expect a gain from it, otherwise their behaviour is only driven by their reputation. For extreme priors, the experts who are driven only by their reputation benefit from biasing their suggestions towards the prior. This is the classical *herding on the prior effect* which the bias can mitigate. The addition of more than two experts decreases significantly the gain from truthful but risky recommendations due to lower probability that the recommendation will affect the final decision. Therefore, for a certain range of priors more than two experts can harm the information transmission due to the persistence of the herding effect.

2.1.1 Related Literature

This paper is related to three main strands of the literature on cheap-talk games. The first strand studies the strategic information transmission in the case where senders' and receivers' preferences are not aligned. This strand dates back to the seminal work of Crawford and Sobel (1982) and shows that in the presence of just one single biased expert, the information transmission can be substantially reduced due to strategic motives. Krishna and Morgan (2001b) depart from the case of one sender and study situations where two senders send messages simultaneously and prove that if the the conflict of interest is not too large it is possible to achieve full information transmission. A number of subsequent studies have highlighted the importance of the second expert's opinion as a discipline device that can retrieve the full information transmission result e.g Krishna and Morgan (2001b), Battaglini (2002).

The second strand starting with Sobel (1985), considers cases in which there is uncertainty about the preferences of the expert and the expert has reputational concerns for being unbiased. In this case, the experts' reputational concern is for their integrity (*bias*) in contrast to our paper where it is common knowledge that the expert has his own interest which is not always in line with the decision maker's. Morris (2001) and Ely and Välimäki (2003) point out how reputational concerns lead an unbiased expert to engage in inefficient behavior for signaling his type. In Morris (2001), when reputational concerns are strong, information revelation completely breaks down and babbling is the only equilibrium. Ely and Välimäki (2003) consider an infinite-horizon principal-agent model, and show that principals anticipate the "bad reputation" effect and hence never hire an agent, thereby leading to the loss of all surplus. In our case, there is not such effect since the preferences are common knowledge and the experts do not differ in terms of their type but through the precision of the signals that they receive.

The third strand of literature deals with the case where experts are not biased and they care only for their reputation of competency (Ottaviani and Sørensen, 2001, 2006; Trueman, 1994; Brandenburger and Polak, 1996; Scharfstein and Stein, 1990; Holmström, 1999; Klein, Mylovanov et al., 2011; Schottmüller, 2019). In this paper we consider the same type of reputational concern for ability but assuming also that there is a *conflict of interest* between the expert and the decision maker.[2] Several papers in the literature on experts and advice, e.g. Brandenburger and Polak (1996), analyze how an expert who wants to maximize his reputation for being competent will misrepresent his information. The main result is that the adviser will then misrepresent her signal towards the prior. Our paper is closer to Ottaviani and Sørensen (2001) where they study information reporting by privately informed experts who are solely motivated by the desire to be perceived as competent, and show that the amount of information that is credibly transmitted is always increasing in the quality of the expert's information. Schottmüller (2019) develops a dynamic set up in which experts do not care directly about their reputation but they care instead about maximizing their expected bonus stream. He shows that some experts are too good to be truthful. This result is confirmed in our setting as well. However, Schottmüller (2019) leaves as an open question how the equilibrium changes with the existence of a second expert. We show that the information transmission is improved by adding an identical second expert who acts as a discipline device but by increasing the number of experts does not always improve the communication.

The rest of the paper is organised as follows. Section 2 introduces the model. Section 3 characterises the most informative equilibrium, Section 4 introduces competition between experts, and Section 5 concludes.

2.2 The Model

We consider a cheap talk game between an expert (*She*) and a decision maker (*He*). The state of the world is a binary random variable $\theta \in \Theta = \{\theta_0, \theta_1\}$. For the rest of the paper, we will refer to θ_0 as the low state and to θ_1 as the high state. We summarise the timing as follows: The game starts with the nature choosing the state of the world which is not observed by the players. Then the expert receives a private and noisy signal and she sends a message to the decision maker. The decision maker after receiving the message takes an action and the state of the world is observed by both players. Finally, the decision maker uses the state of the world to update the reputation of the expert in conjunction with her recommendation and players' payoff is realised .

Both players hold the same prior belief about the state $\mu \equiv Pr(\theta_1)$. The expert receives a

[2] Andina-Díaz and García-Martínez (2020) consider experts (judges) with both types of reputational concerns: ability and bias. Their focus is the role of transparency and how it affects the quality of the decision making process.

private and non-verifiable signal $s \in \mathbb{S} = \{0, 1\}$ about the state whose precision is exogenous and can be either high (*good*) or low (*bad*), $p = \{g, b\}$. An expert who receives signals of high precision is considered competent and has signal distribution: $Pr(s = 1|\theta_1, g) = Pr(s = 0|\theta_0, g) = g$ and $Pr(s = 0|\theta_1, g) = Pr(s = 1|\theta_0, g) = 1 - g$. Instead, she is considered incompetent if she receives signals of low precision and has respectively signal distribution: $Pr(s = 1|\theta_1, b) = Pr(s = 0|\theta_0, b) = b$ and $Pr(s = 0|\theta_1, b) = Pr(s = 1|\theta_0, b) = 1 - b$. We assume $\frac{1}{2} \leq b < g < 1$.

Assumption 2.1 *The expert and the decision maker share the same prior beliefs about the expert's signal precision. They assign probability $Pr(p = g) = \pi$ to the expert receiving signals of high precision and probability $Pr(p = b) = 1 - \pi$ to the expert receiving signals of low precision. We denote by ρ the expected precision of the expert's signal: $\rho = \pi g + (1 - \pi)b$.*

We will refer to this probability π as the initial reputation of the expert. Following most papers in the literature of reputational concerns, we employ **Assumption 2.1** as it facilitates the analysis (Ottaviani and Sørensen, 2001; Prat, 2005). The relaxation of this assumption would make the analysis much more complicated because the expert's message would be interpreted by the decision maker as a signal of the state but also as a signal of her ability (type). In case that an expert knows her own type she could use her message as a costly signal of her ability (see Trueman (1994)). In our analysis we are not interested in distinguishing between experts' types but between signals. An equilibrium is considered informative if the decision maker manages to extract truthful information by the expert even if he will never learn the exact precision of her signal. Then, given the received signal, she forms a posterior belief about the state, which we denote by $\hat{\mu} \equiv Pr(\theta_1|s = i)$, with $i \in \{0, 1\}$ and in turn sends a message $m \in \mathbb{M} = \{0, 1\}$ to the decision maker who observes the message sent by the expert and the realisation of the state of the world. Given the information that he has at his disposal, he has to choose a binary action $a \in \{0, 1\}$. Consequently, comparing the expert's suggestion and the true state he updates his beliefs about the competency of the expert. The expert is concerned about her reputation for providing valuable information. We denote by $\hat{\pi} \equiv Pr(p = g|\theta, m)$ the posterior belief of decision maker on expert's signal precision. In case that the expert's recommendation coincides with the true state ($m = \theta$) the posterior belief of decision maker is:

$$\hat{\pi} = Pr(p = g|\theta, m) = \frac{\pi g}{\pi g + (1 - \pi)b} \equiv \pi^+ \tag{2.1}$$

In case that the expert's recommendation does not coincide with the true state ($m \neq \theta$) his posterior belief is:

$$\hat{\pi} = Pr(p = g|\theta, m) = \frac{\pi(1-g)}{\pi(1-g) + (1-\pi)(1-b)} \equiv \pi^{-} \tag{2.2}$$

The objective function of the expert is given by:

$$U_E = U(\beta, a) + Pr(t = g|m, \theta) \tag{2.3}$$

Moreover the preferences of expert are not aligned with those of decision maker. We assume that $U(\beta, a) = \beta a$. She is positively biased and receives a benefit $\beta \in [0, 1]$ when the decision $a = 1$ is reached. We assume that the bias is known and independent of competency. The payoff of the decision maker depends on the chosen action and the state θ. We assume symmetric payoff: $U(a = 0, \theta_0) = U(a = 1, \theta_1) = 0$ and $U(a = 0, \theta_1) = U(a = 1, \theta_0) = -1$.

2.3 Equilibrium Analysis

In this section we characterise the most informative equilibrium. More precisely, we study how the prior belief over the state and the expert's reputation determine the quantity of credible information that can be transmitted by a biased expert in equilibrium. We use the concept of perfect Bayesian equilibrium and focus on informative equilibria defined as equilibria in which, the decision maker receives advice that is decision-relevant. In the next subsection we will provide the conditions under which the experts provide relevant and truthful information.

Definition 2.1 *(Perfect Bayesian Nash Equilibrium). The strategy $\sigma^*(m_i|s_i)$ constitutes a Perfect Bayesian Nash Equilibrium if :*

1. *the expert chooses m^* to maximise his expected payoff:*

$$\sigma^*(m_i|s_i) = arg \max_{m \in \{0,1\}} \beta a^*(\mu, m_i) + E[Pr(t = g|m_i, \theta)] \forall i \in \{0, 1\} \forall s \in \{0, 1\}$$

   where $a^*(\mu, m_i)$ is the optimal action of decision maker. [3]

2. *the decision maker updates its belief about the expert's competency by applying the Bayes' rule*

3. *the decision maker correctly forecasts the expert's strategy*

[3]To make notation lighter, in the rest of the paper we use m_i instead of $m = i$ and s_i instead of $s = i$, where $i \in \{0, 1\}$.

2.3.1 Effective messages

In our setting where the action space is binary and the prior beliefs are not balanced, $\mu \in (0,1)$, the decision maker's optimal action is the following :

$$a^*(m_i,\mu) = \begin{cases} 1 & \text{if} \quad Pr(\theta_1|m_i,\mu) \geq \frac{1}{2} \;\; \forall i \in \{0,1\} \\ 0 & \text{if} \quad Pr(\theta_1|m_i,\mu) \leq \frac{1}{2} \;\; \forall i \in \{0,1\} \end{cases}$$

We notice that with discrete actions not all truthful recommendations are adopted by the decision maker. A message can be truthful (*informative*) but not effective. More precisely, if the prior on the state is extreme, a message by the expert may induce the decision maker to revise her beliefs over the state, but this revision may not be sufficient to induce the decision maker to choose the recommended action. For extreme priors, only a highly reputable expert can move the beliefs of the decision maker and implement an action potentially opposite to the status quo. Therefore, a biased expert can also benefit by an untruthful message only if this is *effective*. For instance, consider an editor who has to decide whether to accept or reject a paper and he asks the opinion of a referee. If the author is well known the prior of the editor is high. If the referee is a junior researcher with limited experience and proposes rejection even if the decision maker believes that this is his honest opinion, still he will probably not adopt his recommendation. A very reputable referee can affect the decision regarding a very good author. Formally, we define an effective message as follows:

Definition 2.2 ***(Effective message)*** *An expert's message m_i is effective if:*

$$Pr(\theta_i|m_i,\mu) > Pr(\theta_j|m_i,\mu) \forall i \neq j \text{ and } \mu \in (0,1)$$

A biased expert has incentives to misrepresent her information when her messages are effective. If she cannot affect the final decision then she is driven only by her reputational concerns. At the same time the decision maker needs the effective messages to benefit the most by the expert. An expert who cannot send effective messages, even if she is truthful, she cannot strictly improve the welfare of the decision maker. [4][5] **Remark 2.1** states that there is always a combination of μ and π where the decision maker can seek for effective advice. As the initial reputation π increases the range of μ where the messages are effective increases as well (see Figure 2.1).

[4]For example, in a setting where the decision maker pays a cost for receiving the advice of the expert, asking advice from people that are not able to send effective messages entails only costs without any additional benefit.

[5]For balanced priors ($\mu = 0.5$) and/or continuous action space any truthful message is effective.

Remark 2.1 *There is always a non-empty set* $EMR \equiv [1-\rho, \rho]$ *where the expert with expected signal precision* ρ *sends effective messages.*

Proof. See Appendix B ■

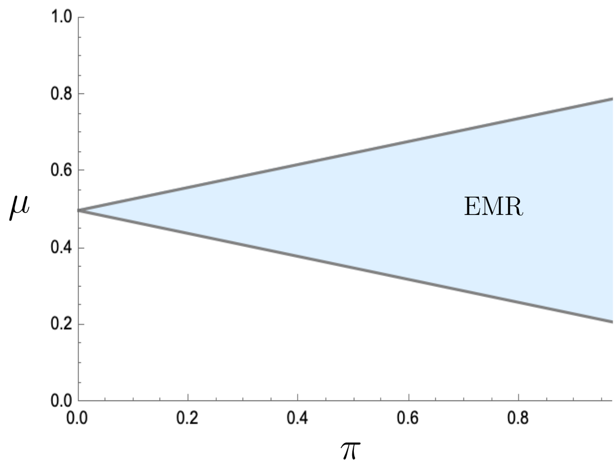


Figure 2.1 | **Effective Messages Region (EMR) .** Horizontal axis: initial reputation π; Vertical axis: prior belief over the state μ. The shaded area is the Effective Messages Region. The upper boundary it's given by $\mu = \rho$ and the lower boundary by $\mu = 1-\rho$, where $\rho = \pi g + (1-\pi)b$; Parameters: $b = 0.5, g = 0.8$

In the next subsection we provide the conditions under which an expert sends both truthful and effective messages.

2.3.2 Fully Revealing Equilibrium

We will analyse the conditions under which the most informative equilibrium is fully-revealing. We are interested in the priors of state and reputation for which the experts sends credible information. In a *pooling* (uninformative or babbling) equilibrium the expert sends the same massage irrespectively of her signal. Given the decision makers's correct conjecture of her strategy, if the equilibrium is pooling then he does not draw any meaningful inference about the state. In a *separating* (fully-revealing) equilibrium the expert sends a message depending on the signal received such as $m_j = s_j$. The objective of the expert is to maximise her payoff. The message that gives the highest payoff, is the message that induces an action $a = 1$ and at the same time maximises her reputation. The expected effect that the message s_j has in expert's reputation is given by:

$$E[Pr(p = g|m, \theta)] = Pr(\theta_j|s_j)\hat{\pi}(\theta = m_j|s_j) + Pr(\theta_i|s_j)\hat{\pi}(\theta \neq m_j|s_j) \quad (2.4)$$

We denote by $\hat{\pi}(m_i|s_j)$ the expected posterior reputation of the expert after receiving

signal s_j, and sending message m_j. In case that her recommendation is proved correct, which happens with probability $Pr(\theta_j|s_j)$, the expected reputation gain is: $\hat{\pi}(\theta = m_j|s_j) - \pi$. Instead, if her recommendation proved wrong, which happens with probability $Pr(\theta_i|s_j)$, her reputation loss will be: $\pi - \hat{\pi}(\theta \neq m_j|s_j)$.
More analytically, we derive the conditions under which the expert sends $m_j = s_j \forall j \in 0, 1$. The expert will send $m = 0$ after a signal $s = 0$ if:

$$\mathbb{E}U(m_0|s_0)) \geq \mathbb{E}U(m_1|s_0)) \Rightarrow \beta \leq (1 - 2Pr(\theta_1|m_0, \mu)\Delta\pi \tag{2.5}$$

Similarly, an expert will truthfully send $m = 1$ after a signal $s = 1$ if

$$\mathbb{E}U(m_1|s_1) \geq \mathbb{E}U(m_0|s_1)) \Rightarrow \beta \geq (1 - 2Pr(\theta_1|m_1, \mu)\Delta\pi \tag{2.6}$$

where $\Delta\pi \equiv (\pi^+ - \pi^-)$.
We will refer to the conditions (2.5) and (2.6) as the truth-telling conditions. The trade-off that an expert faces is quite clear from these conditions. The left-hand side is the misinterpretation gain of sending an untruthful message while the right-hand side refers to the reputational gain of sending a truthful message. By solving for the truth telling conditions we obtain the incentive compatibility region, that is any pair of (μ, π) for which a separating equilibrium exists. If the action space is binary, then relations (2.5) and (2.6) are valid inside Effective Messages Region (EMR).

Starting from the EMR region, we will describe analytically the behaviour of the expert. Inside the Effective Messages Region, the expert is driven by both her bias and her reputational concern. The intersection of the EMR region and the region formed by the truth-telling conditions, which is the Incentive Compatibility Region, demonstrates that the expert is truthful and sends $m_i = s_i$. Inside the EMR region but above $\overline{\mu}$, she always sends $m = 1$. This is because the expected gain from misinterpretation is higher than the expected reputation gain. Similarly, above the upper bound of EMR she always sends a high message irrespectively of her signal but she is driven only by her reputation. The decision-maker is interested in the effective advices, however one would wonder whether the experts are willing to provide truthful information even if this would not be decision-relevant. As the following Lemma 1 states, the expert is truthful only inside the EMR . Outside of it, similarly to the behaviour an unbiased expert, she tries to maximise her reputation. Therefore, she sends messages that are most probably correct. In other words, the conformist behaviour (*herding effect*) that is observed in the case of unbiased experts (Ottaviani and Sørensen, 2001) is still present, such as she sends always $m = 0$ for every $\mu < 1 - \rho$ and $m = 1$ for every $\mu > \rho$.[6]

Formally, we define Incentive Compatibility Region (shaded area of Figure 2.2) as follows.

Definition 2.3 *The Incentive Compatibility Region, is defined as the region inside EM:*

$$ICR \equiv \{\mu \in (0,1) |\ \beta a^*(m_i, \mu) + \mathbb{E}\hat{\pi}(m_i|s_i)) \geq \beta a^*(m_j, \mu) + \mathbb{E}\hat{\pi}(m_j|s_i))\},$$

$$\forall i \neq j \in \{0, 1\}$$

which is:

$$ICR = [\max\{\underline{\mu}, 1 - \rho\}, \min\{\overline{\mu}, \rho\}]$$

where

$$\underline{\mu} = \mu |\ \ \beta a^*(m_1, \mu) + \mathbb{E}(\hat{\pi}(m_1|s_1)) = \beta a^*(m_0, \mu) + \mathbb{E}\hat{\pi}(m_0|s_1)) \tag{2.7}$$

and

$$\overline{\mu} = \mu |\ \ \beta a^*(m_0, \mu) + \mathbb{E}\hat{\pi}(m_0|s_0) = \beta a^*(m_1, \mu) + \mathbb{E}\hat{\pi}(m_1|s_0) \tag{2.8}$$

Figure 2.2 presents graphically the Incentive Compatibility Region, illustrated by the shaded area. The lower boundary of the ICR coincides with the lower boundary of the Effective Messages Region. By measuring the shaded are we can obtain the probability with which the decision maker will get a truthful recommendation for a random combination of (π, μ). This is what we call information transmission. If this probability is high (low) then we will refer to high(low) degree of information transmission. Therefore, in what follows, when we refer to improvement of information transmission we mean that the probability of full revelation increases. Formally, we define Information Transmission as follows:

Definition 2.4 *The Information Transmission is calculated by the area defined by ICR. This is the probability with which the decision maker will receive a truthful message for a random combination of (π, μ).*

Lemma 2.1 *The set ICR is the only informative equilibrium.*

Proof. See Appendix B ■

Lemma 2.1 states that there is not any other equilibrium where the experts may send truthful but not effective messages. As we discussed above, outside the EMR, even if the expert is not driven by her bias, still her reputational concerns makes her untruthful since

[6] The truth-telling conditions outside the EMR are given by:

$$0 \leq (1 - 2Pr(\theta_1|m_0, \mu)\Delta\pi$$

and

$$0 \geq (1 - 2Pr(\theta_1|m_1, \mu)\Delta\pi$$

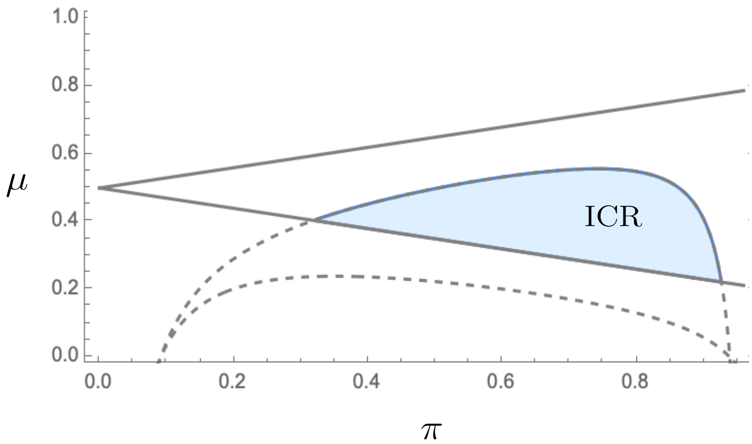


Figure 2.2 | **Incentive Compatibility Region (ICR) .** Horizontal axis: initial reputation π; Vertical axis: priors over the state μ. The dashed grey curves are given by relations (2.7) and (2.8) and represent the truth-telling conditions. The grey solid lines are the lower and the upper boundary of the Effective Messages region, $\underline{\mu} = 1 - \rho$ and $\overline{\mu} = \rho$ respectively. The intersection of the orange and the grey area is the Incentive Compatibility Region (ICR), where the expert truthfully reports her signal. (Parameters: $\beta = 0.1, b = 0.5, g = 0.8$)

she behaves in a conformist way. For this reason the equilibrium described above is the only informative equilibrium.

Regarding the loss of information due to bias, it can be calculated by subtracting the shaded area (ICR) from the Effective Messages Region. For example, given D 2.4 and the parameterisation of Figure 2.2, we can calculate the loss of information due to bias. The probability with which the decision maker would receive a truthful information if the expert was unbiased is $p_u = 0.3$ whereas for the biased expert $p_b = 0.113$. Therefore the information transmission has decreased by 0.187.

Lemma 2.2 *There is a separating equilibrium, for any combination of μ and π within the Incentive Compatibility Region. There is a pooling equilibrium for any combination of μ and π outside the Incentive Compatible Region.*

Proof. See Appendix B ■

Lemma 2, states there is always a range of prior beliefs where the expert has the mixed strategy $q = (m_0, m_1) \in [0, 1]^2$ which represents the probability that the expert sends $m = 1$, given the two possible realisations of the signal. Outside of ICR, the expert either sends always $m = 0$ or $m = 1$ irrespectively of her signal.

Proposition 2.1 *For every level of bias β which is smaller than a maximal level of bias $\overline{\beta}$ there exists a separating equilibrium.*

Proof. See Appendix B ■

Proposition 1 states that there exists a maximum degree of conflict that allows the existence of an informative equilibrium. Figure 2.2 represents this equilibrium: The grey area is the Effective Messages Region (EMR) , while the orange curves represent $\underline{\mu}$ and $\overline{\mu}$ as decribed by equations (2.8) and (2.9). It holds that $\forall \beta < \overline{\beta}$ then $\overline{\mu} \geq 1 - \rho$, which means that Incentive Compatibility Region is not an empty set. The maximum value of β is determined by the point in which the equation (2.8) is tangent to the lower boundary of EM. Figure B.3 and Figure B.4 in Appendix B.2 show how the equilibrium is affected as we increase the level of bias.

Proposition 2.2 *For every $\beta < \overline{\beta}$, there is a level of prior reputation π^* such that any prior reputation π greater than this reduces the information transmission*

Proof. See Appendix B ■

Let us assume that the decision maker can choose among a pool of experts with different levels of prior reputation. Proposition 2, states that there is a non-monotonic effect of reputation on information transmission. There is always a level of reputation after which the information transmission decreases. Therefore the decision maker shouldn't choose the most reputable expert in order to maximise the information transmission. This is because the reputation loss is not enough to discipline experts of very high reputation.

In **Proposition 2.1** and **Proposition 2.2**, the maximal level of $\overline{\beta}$ and prior reputation π^* depends also on exogenous precision parameters b and g. In Appendix B.2, Figure B.5 shows that the higher precision of good type has a positive effect on information transmission, keeping the rest fixed. On the other hand, the negative effect of higher precision of bad type is presented by Figure B.6. The information transmission is positively related to the difference between these parameters, because the reputation gain (loss) after a correct (wrong) recommendation depends positively on the difference between good and bad types, on how much the two types differ in terms of ability.

2.3.3 Continuous Action Space

In many situations we can imagine that the decision maker can choose among finitely many options and not just two. The scenarios where the decision maker has many options can be described by assuming continuous action space. For example, an investor who has to decide his assets portfolio seems that his action space can be described better by a continuum. In this section we maintain the same assumption about the expert's signal but we assume that the decision maker can choose an action $a \in \mathbb{R}$.[7] The decision maker's utility still depends on the state of the world θ and his action a. As is standard in the literature, we assume that the decision maker's utility is given by the quadratic loss function $U = -(a-\theta)^2$. This implies that the action that maximises the expected utility of the decision maker is equal to the probability he assigns to the state of the world being 1. More precisely, the decision maker's optimal action is the following:

$$a^*(m_i, \mu) = \begin{cases} Pr(\theta_1|m_1, \mu) = \frac{\mu\rho}{\mu\rho+(1-\mu)(1-\rho)} \equiv a^+ & \text{if} \quad m = 1 \\ Pr(\theta_1|m_0, \mu) = \frac{\mu(1-\rho)}{\mu(1-\rho)+(1-\mu)\rho} \equiv a^- & \text{if} \quad m = 0 \end{cases}$$

In section 3.1, we discussed which messages can be relevant for the decision maker. In this case, where the decision maker can perfectly adjust his action according to his posterior belief, any truthful message is effective. We maintain the assumption where the expert always prefers the highest action and we assume that her utility function is: $U_E = a + Pr(t = g|m, \theta)$. Regarding the bias, we assume that her payoff increases as the implemented action gets closer to one. This is different from the bias we assumed in previous subsection which was a fixed bias parameter $\beta \in [0, 1]$ and it was achieved only if the adopted action was the high one. The incentives of the experts depend on the trade-off between the first element of the utility function, gain from misinterpretation, and the second element, the reputation gain, which is always bounded to $[0, 1]$. For this reason, we need to bound also the gain from misinterpretation, otherwise a truthful equilibrium would never exist. In the continuous case, it is bounded by construction since it depends on the implemented action while in binary case we have to assume it through the bias parameter $\beta \in [0, 1]$. [8]

We provide below the condition under which a fully revealing equilibrium exists.[9] The expert will send $m = 0$ after a signal $s = 0$ if:

$$\mathbb{E}U(m_0|s_0)) \geq \mathbb{E}U(m_1|s_0)) \Rightarrow \Delta a \leq (1 - 2Pr(\theta_1|m_0, \mu)\Delta\pi \tag{2.9}$$

Similarly, an expert will truthfully send $m = 1$ after a signal $s = 1$ if:

$$\mathbb{E}U(m_1|s_1) \geq \mathbb{E}U(m_0|s_1)) \Rightarrow \Delta a \geq (1 - 2Pr(\theta_1|m_1, \mu)\Delta\pi \tag{2.10}$$

[7] Morris (2001) in his seminal paper assumes that the experts receive binary signals, but the decision maker chooses an action from a continuum. His optimal action is a continuous increasing function of the probability he attaches to state

[8] We could consider $U_E = \beta a + Pr(t = g|m, \theta)$ but this wouldn't change qualitatively our results.

[9] See Appendix B for analytical expressions of the truthtelling conditions.

where $\Delta a = a^+ - a^-$ and $\Delta\pi = \pi^+ - \pi^-$. [10]

Similarly to **Definition 3**, we define below the incentive compatibility region for the continuous action space which is defined only by the two truth-telling conditions (2.5) and (2.6).

Definition 2.5 *The Incentive Compatible Region under competition (ICR′), is defines as:*

$$ICR'_c \equiv \{\mu \in [0,1] | Pr(1|m_i, \mu) + \mathbb{E}\hat{\pi}(m_i|s_i) \geq Pr(1|m_j, \mu) + \mathbb{E}\hat{\pi}(m_j|s_i)\}, \forall i, j \in \{0,1\}$$

which is:

$$ICR' = [\underline{\mu}', \overline{\mu}']$$

Where $\underline{\mu}'$ ($\overline{\mu}'$) is the lowest (highest) prior belief on the high state for which there exists a separating equilibrium. They are given by the following relations:

$$\underline{\mu}' = \mu | \;\; Pr(\theta_1|m_1, \mu) + \mathbb{E}\hat{\pi}(m_1|s_1) = Pr(1|m_0, \mu) + \mathbb{E}\hat{\pi}(m_0|s_1) \tag{2.11}$$

and

$$\overline{\mu}' = \mu | \;\; Pr(\theta_1|m_0, \mu) + \mathbb{E}\hat{\pi}(m_0|s_0) = Pr(1|m_1, \mu) + \mathbb{E}\hat{\pi}(m_1|s_0) \tag{2.12}$$

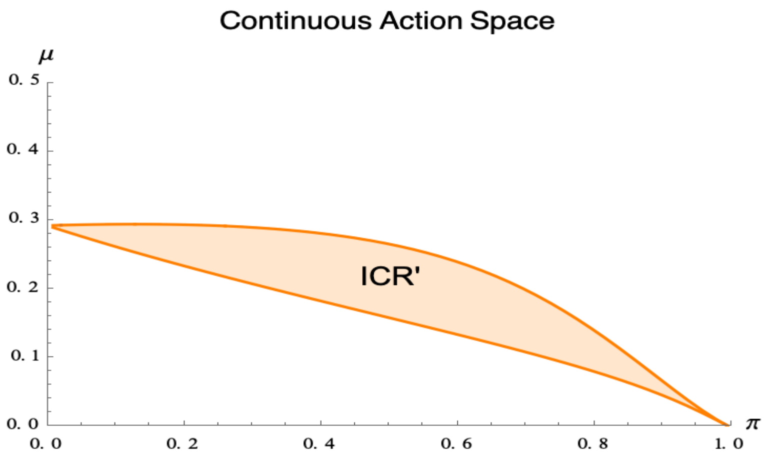


Figure 2.3 | **Incentive Compatibility Region for continuous actions (ICR′).** Horizontal axis: initial reputation π; Vertical axis: priors over the state μ. The orange area is the Incentive Compatibility Region and is is given by relations (11) and (12) which are the truth-telling conditions for the one-expert case and continuous action space.(Parameters: $b = 0.5, g = 0.8$)

Lemma 2.3 *There is a separating equilibrium, for any combination of μ and π within the ICR'. There is a pooling equilibrium for any combination of μ and π outside the ICR'.*

Proof. See Appendix B ■

Similarly to the previous subsection, in **Lemma 2.3** we state the existence of a fully revealing equilibrium. In this case the ICR depends only on the lower and upper boundaries

[10] See Appendix B for explicit expression of (2.9) and (2.10).

of truth-telling conditions since we do not need anymore the effective messages region. The results are very similar to those of binary actions. The ICR lies in the lower range of values of prior beliefs. However, a direct comparison of the ICR and ICR' would be problematic due to different utility functions. As we mentioned before, in the binary actions case the gain from misinterpretation is fixed and equal to β in case that the decision maker chooses the high action. In the continuous action case, the expert's messages always affect his payoff and increase as the final action increases. While we can draw conclusions about the overall incentives we cannot make a direct comparison of Information transmission since this depends heavily on experts' bias and differs between the two cases.

Proposition 2.3 *There is a level of prior reputation π' such that any π greater than that, reduces the information transmission.*

Proof. See Appendix B ■

The non-monotonic effect of reputation on information transmission is confirmed also in this case. However, it is even stronger because the gain from misinterpretation depends heavily on prior reputation. In the binary case, any expert that is able to send effective messages gains the same, β. In the continuous case, the most reputable experts gain more because they can direct the beliefs of the decision maker towards the high action easier. The non monotonic effect in the previous case comes for the limited reputational gain or loss that a reputable expert can have after a wrong recommendation while in this case by both higher misinterpretation gain and lower reputational loss. We also notice that in case of continuous action the bias allows for equilibrium existence that is not possible if the expert is unbiased. The gain allows them to make more risky recommendations.

2.4 Multiple Experts

In this section, we examine the impact of competition among experts. For exposition purposes we will start with the continuous action space. We consider the case where the decision maker has access to opinions of multiple experts who are identical in terms of initial reputation, π. Experts observe neither their ability nor the ability of the other experts. They provide their advice simultaneously [11] to the decision maker and we assume that experts' signals are

[11] We consider the simultaneous protocol with identical experts to highlight the effect of uncertainty on experts' behavior. The experts even if they know that they have the same preferences, the uncertainty about the other's expert signal and message discipline them to some extent. It would be a natural extension to consider also a sequential protocol. In this case, considering the same assumptions about the experts' preferences and reputational concerns, we expect lower information transmission since a deviation by the first expert would be adopted also by the second one. However, the introduction of relative reputational concern would mitigate the

conditionally independent:

$$Pr(s^i = 1|\theta, s^-i) = P(s^i = 1|\theta)$$

$$Pr(s^i = 0|\theta, s^-i) = P(s^i = 0|\theta)$$

We start our analysis by considering two experts. As mentioned before any message is effective and there is no reason to consider separate intervals of priors. The action rule of decision maker after truthful m_l^1 and m_k^2:

$$a_c^*(m_l^1, m_k^2, \mu) = \begin{cases} \mu \quad \text{if} \quad l \neq k \\ Pr(\theta_1|m_1^1, m_1^2) = \frac{Pr(\theta_1|m_1,\mu)\rho}{Pr(\theta_1|m_1,\mu)\rho+(1-Pr(\theta_1|m_1,\mu))(1-\rho)} \equiv a_c^+ \quad \text{if} \quad l = k = 1 \\ Pr(\theta_1|m_0^1, m_0^2) = \frac{Pr(\theta_1|m_0,\mu)(1-\rho)}{Pr(\theta_1|m_0,\mu)(1-\rho)+(1-Pr\theta_1(|m_0,\mu)\rho} \equiv a_c^- \quad \text{if} \quad l = k = 0 \end{cases}$$

We derive the following truth-telling conditions for the expert i, given the strategies of the other expert. We have assumed that relative reputational concerns do not exist, in the sense that the experts do not care to be perceived as more informed than the other expert. For this reason, the right-hand side of the conditions is identical to the one-expert case. The difference comes from the left-hand side where each expert has to consider the probabilities that the other expert receives a low or high signal. The expert will send $m = 0$ after a signal $s = 0$ if:

$$\mathbb{E}U(m_0^i|s_0, m^{-i})) \geq \mathbb{E}U(m_1^i|s_0, m^{-i})) \Rightarrow (\lambda - \kappa)(\mu - \Delta a_c) \leq (1 - 2Pr(\theta_1|m_0, \mu)\Delta\pi \tag{2.13}$$

Similarly, an expert will truthfully send $m = 1$ after a signal $s = 1$ if

$$\mathbb{E}U(m_1^i|s_1, m^{-i})) \geq \mathbb{E}U(m_0^i|s_1, m^{-i}) \Rightarrow (\lambda - \kappa)(\mu - \Delta a_c) \geq (1 - 2Pr(\theta_1|m_1, \mu)\Delta\pi \tag{2.14}$$

where $\Delta a_c = a_c^+ - a_c^-$, $\Delta\pi = \pi^+ - \pi^-$, $\kappa \equiv Pr(s^i = 1|\theta) = \mu\rho + (1 - \mu)(1 - \rho)$, and $\lambda \equiv Pr(s^i = 0|\theta) = \mu(1 - \rho) + (1 - \mu)\rho$.

Similarly, we can derive the condition for more than two experts.[12] Formally, we define the Incentive Compatibility Region for multiple experts and continuous action space as follows:

Definition 2.6 *(ICR' under competition). Assuming truth-telling for both experts, the set of prior*

problem. We leave this as an open question and a possible extension of the model.

[12]see Appendix B) for the cases of three and four experts

beliefs under which expert i has no incentive to deviate from truth-telling is:

$$ICR'_C \equiv \{\mu \in [0,1] | Pr(\theta_1|m_k, m^{-i}) + \mathbb{E}(\hat{\pi}(m_{ik}|s_k)) \geq Pr(\theta_1|m_l, m^{-i}) + \mathbb{E}(\hat{\pi}(m_l|s_k)), \forall l, k \in \{0,1\}$$

which is:

$$ICR'_C = [\underline{\mu^c}, \overline{\mu^c}]$$

Where $\underline{\mu^C}$ ($\overline{\mu^C}$) *is the lowest (highest) prior belief on the high state for which there exists a separating equilibrium under competition. They are given by the following relations:*

$$\underline{\mu^C} = \mu | \ Pr(\theta_1|m_1, m^{-i}) + \mathbb{E}(\hat{\pi}(m_1|s_1)) = Pr(\theta_1|m_0, m^{-i}) + \mathbb{E}(\hat{\pi}(m_0|s_1)) \tag{2.15}$$

and

$$\overline{\mu^C} = \mu | \ Pr(\theta_1|m_0, m^{-i}) + \mathbb{E}(\hat{\pi}(m_0|s_0)) = Pr(\theta_1|m_1, m^{-i}) + \mathbb{E}(\hat{\pi}(m_1|s_0)) \tag{2.16}$$

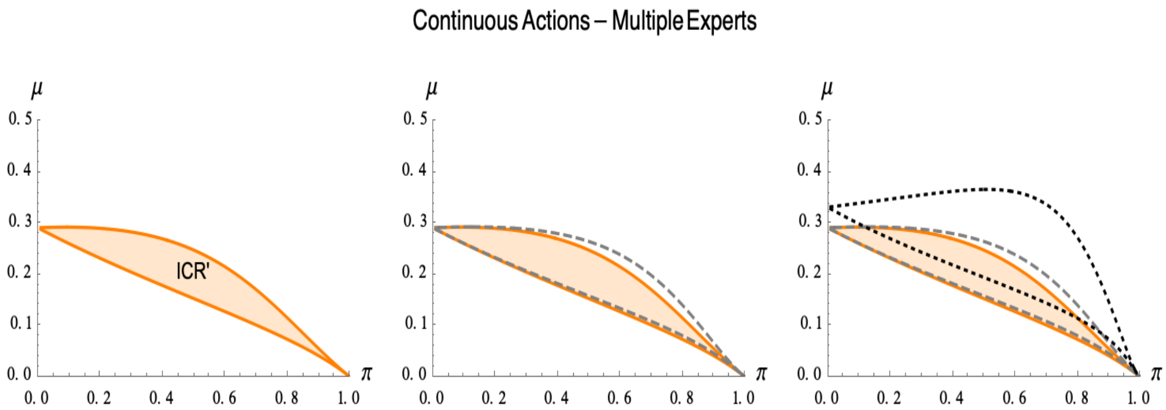


Figure 2.4 | **Incentive Compatibility Region for continuous actions (ICR').** Horizontal axis: initial reputation π; Vertical axis: priors over the state μ. From the left to the right: Incentive Compatibility Regions for one, two and three experts. The orange curves are given by conditions (2.9) and (2.10) and represent the ICR for a single expert case. The grey dashed curves are given by relations (2.13) and (2.14) and correspond to the two-experts scenario. The **black** dotted curves are given by relation (2.42) and (2.43)(see Appendix B)) and correspond to the three-experts case. . (Parameters: $b = 0.5, g = 0.8$))

Proposition 2.4 *If the experts have the same prior reputation and they provide simultaneously their information then there is an equilibrium where as we increase the number of experts:*

1. *the ICR′ always expands*
2. *the ICR′ always moves upwards*
3. *there is always a combination of* (π, μ) *for which there is a separating equilibrium for* n *but not for* $n+1$ *experts, where* $n \geq 1$.

Proof. See Appendix B ■

The left side plot in Figure 2.4, shows the ICR with one single expert. As we discussed in the previous section, qualitatively the result is similar to the binary case. There is a non monotonic effect of prior reputation and the expert is truthful for the low priors. In the central plot, we present the competition between two experts where the ICR expands relatively to one expert case and the maximum prior reputation is going to the right. Competition softens the incentives of highly reputable experts to misinterpret their information. On the right side plot, we present the three experts scenario where we notice a significant improvement of information transmission. The existence of a second expert limits the power that the experts have on the decision maker's decision. The gain from misinterpretation is less and less as the number of experts increases so that the experts are more and more driven by their reputation. The fact that the ICR moves upward and expands as the number of experts increases implies that the overall probability of receiving truthful information increases but there are combinations of low μ and π where it is strictly better to use fewer experts. For example, considering the combination of $(\pi, \mu) = (0.8, 0.1)$ there is a fully revealing equilibrium if there is only one expert but not if there are three experts. For low states, the experts' bias allows for equilibrium. Let us consider $b = 0.1$, $g = 0.8$ and $b = 0.5$ (see Figure 2.4). If the expert is unbiased, then the probability that the decision maker receives truthful information (*information transmission*) over the whole range of priors is $p_u = 0.3$. In case that the expert is biased this probability drops to $p_1 = 0.057$. Assuming that the decision maker has access to more than one experts, he can achieve $p_2 = 0.065$ by consulting two experts and $p_3 = 0.117$ by consulting three experts. The addition of the second expert improves the information transmission by 0.008 while the third one by 0.06. In the next sections, after analysing how the additional experts affect the information transmission when the decision maker faces binary action space, we will discuss further the intuition behind this result.

2.4.1 Binary Actions

In this subsection, we keep studying the behaviour of the experts under competition but we make the assumption of binary actions. Similarly to the case of a single expert, we have to consider the region where the messages of the experts would be relevant for the decision. We have to consider the Effective Message Regions for two, three or more experts. **Remark 2.2** states that there is always a combination of μ and π where the decision maker can seek for effective advices. As π increases the range of μ where the messages are effective increases as well and it expands as we increase the number of experts.

Remark 2.2 *There is always a non-empty set* $EMR_c[(\underline{\mu}_e, \overline{\mu}_e)]$*where the experts with expected signal* *precision ρ can send effective messages.* [13]

Proof. See Appendix B ■

Assuming N Experts, we derived $\underline{\mu}_e$ and $\overline{\mu}_e$ by solving the following relations:

$$Pr(\theta_1|s_1^1,...,s_1^n) = \frac{P(\theta_1|s_1^1,...,s_1^{n-1})\rho}{P(\theta_1||s_1^1,...,s_1^{n-1})\rho + (1 - P(\theta_1||s_1^1,...,s_1^{n-1}))(1-\rho)} \geq \frac{1}{2} \tag{2.17}$$

$$Pr(\theta_1|s_0^1,...,s_0^n) = \frac{P(\theta_1|s_0^1,...,s_0^{n-1})(1-\rho)}{P(\theta_1|s_0^1,...,s_0^{n-1})(1-\rho) + (1 - P(\theta_1|s_0^1,...,s_0^{n-1})\rho} \leq \frac{1}{2} \tag{2.18}$$

Given the region EMR_c where the messages are effective, the decision maker updates his belief about the state considering both messages if they are truthful. In case of disagreement his posterior belief coincides with his prior belief. Since the action space is binary, in order to study the reporting incentives of the experts, we partition the prior space into two intervals:

(A) For $\mu > \frac{1}{2}$ the conditions under which the experts report truthfully their information is: An expert after a signal $s = 0$ will truthfully send $m = 0$ if

$$E(U(m_0|s_0, m^{-i})) \geq E(U(m_1|s_0, m^{-i})) \Rightarrow \lambda\beta \leq (1 - 2Pr(\theta_1|m_0, \mu)(\pi^+ - \pi^-) \tag{2.19}$$

Similarly, an expert will truthfully send $m = 1$ after a signal $s = 1$ if

$$E(U(m_1|s_1, m^{-i})) \geq E(U(m_0|s_1, m^{-i})) \Rightarrow \lambda\beta \geq (1 - 2Pr(\theta_1|m_1, \mu)(\pi^+ - \pi^-) \tag{2.20}$$

(B) For $\mu < \frac{1}{2}$ the conditions under which the experts report trutfully their information is: An expert after a signal $s = 0$ will truthfully send $m = 0$ if

$$E(U(m_0|s_0, m^{-i})) \geq E(U(m_1|s_0, m^{-i})) \Rightarrow \kappa\beta \leq (1 - 2Pr(\theta_1|m_0, \mu)(\pi^+ - \pi^-) \tag{2.21}$$

[13] For the two-experts case the upper and lower bound are the following:

$$\underline{\mu_e} = \frac{(1+\rho)^2}{(1 - 2b + 2b^2 + 2b\pi - 4b^2\pi - 2g\pi + 4bg\pi + 2b^2\pi^2 - 4bg\pi^2 + 2g^2\pi^2)}$$

$$\overline{\mu_e} = \frac{(-\rho)^2}{(1 - 2b + 2b^2 + 2b\pi - 4b^2\pi - 2g\pi + 4bg\pi + 2b^2\pi^2 - 4bg\pi^2 + 2g^2\pi^2)}$$

Similarly, an expert will truthfully send $m = 1$ after a signal $s = 1$ if

$$E(U(m_1|s_1, m^{-i})) \geq E(U(m_0|s_1, m^{-i})) \Rightarrow \kappa\beta \geq (1 - 2Pr(\theta_1|m_1, \mu)(\pi^+ - \pi^-) \quad (2.22)$$

where

$$\kappa \equiv Pr(s^i = 1|\theta) = \mu\rho + (1-\mu)(1-\rho)$$

$$\lambda \equiv Pr(s^i = 0|\theta) = \mu(1-\rho) + (1-\mu)\rho$$

The definition of ICR_c is similar to definition of ICR but in this case we consider the EMR_c.

Definition 2.7 *The Incentive Compatibility Region under Competition, is defined as the region inside EMR_c:*

$$ICR_c \equiv \{\mu \in (0,1)|\ \beta a^*(m_i, m_n) + \mathbb{E}\hat{\pi}(m_i|s_i)) \geq \beta a^*(m_j, m_n\mu) + \mathbb{E}\hat{\pi}(m_j|s_i))\}, \forall i \neq j \in \{0,1\}$$

$$\forall i \neq j \in \{0,1\}$$

which is:

$$ICR = [\max\{\underline{\mu}_c, \underline{\mu}_e\}, \min\{\overline{\mu}_c, \overline{\mu}_e\}]$$

where

$$\underline{\mu_c} = \mu|\ \ \beta a^*(m_1, \mu) + \mathbb{E}(\hat{\pi}(m_1|s_1)) = \beta a^*(m_0, \mu) + \mathbb{E}\hat{\pi}(m_0|s_1))$$

and

$$\overline{\mu_c} = \mu|\ \ \beta a^*(m_0, \mu) + \mathbb{E}\hat{\pi}(m_0|s_0) = \beta a^*(m_1, \mu) + \mathbb{E}\hat{\pi}(m_1|s_0)$$

The following proposition states that, similarly to previous section, there exists a maximum level of bias but lower than that of the single expert case. In other words, the existence of the second expert allows for equilibrium existence under larger conflict of interest. In extension, for some level of conflict of interest we might not have any informative equilibrium but a quite informative one by adding a second expert.

Proposition 2.5 *For every level bias β smaller than a maximal level bias $\overline{\beta_c}$, there exists a separating equilibrium under competition, where the maximum level of bias $\overline{\beta_c}$ is greater than the maximum level of bias $\overline{\beta}$ of single expert scenario.*

Proof. See Appendix B ■

Moreover, with the following propositions we state that the non-monotonic effect of the prior reputation on information transmission is still persistent. However, the competition increases the level of initial reputation that maximises the information transmission.

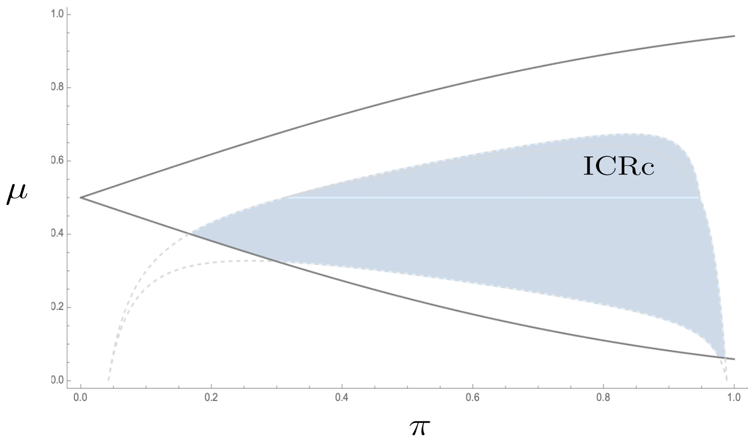


Figure 2.5 | **Incentive Compatibility Region - Competition (ICR_c).** Horizontal axis: initial reputation π; Vertical axis: priors over the state μ. The grey lines are the upper and lower bound of Effective Messages Region under competition among two experts, given by (17) and (18), see footnote 8. The shaded are is the Incentive Compatibility Region. The grey curves are given by the truth-telling conditions (19),(20) and (21). For $\mu \leq 0.5$, the lower bound is determined by relation (20) and the higher by the relation (19). For $\mu \geq 0.5$, the higher bound by the relation (21). (Parameters $\beta = 0.1, g = 0.8, b = 0.5$)

Proposition 2.6 *Given $\beta < \overline{\beta}_c$, there is a level of prior reputation π^{**} such as any level of prior reputation π greater than π^{**} reduces the information transmission, where this maximum π^{**} is greater that the corresponding level π^{*} of single expert scenario.*

Proof. See Appendix B ■

Proposition 2.7 *For $\beta < \overline{\beta_c}$, the second expert improves the information transmission ($ICR_1 \subset ICR_2$).*

Proof. See Appendix B ■

Propositions 6 and **7** state that for a given bias β the second expert facilitates the information transmission. This happens in three ways. First, the range of priors over the state where the experts together can provide effective messages expands. Second, the ICR expands because the second expert disciplines the other expert even if he has the same preferences with him. The experts send the messages simultaneously to the decision maker. The uncertainty over the message of the other due to simultaneity makes the gain by a distorting message uncertain. In case of disagreement an untruthful message can lead to reputation loss without any other gain. Technically, the existence of the second expert decreases the expected manipulation gain from β to $\lambda\beta$ or $\kappa\beta$. Third, the maximum value of initial reputation that maximises the probability of a fully revealing equilibrium is higher (see Figure 2.6). Figure 2.6 is the combination of Figure 2.2 and Figure 2.52 and allows for direct comparison of the information transmission between the two scenarios. Starting from the left: the two curves that begin from $\mu = 0.5$ are the lower boundaries of $EMR's$. As we can notice the black colour which corresponds to the competition scenario is lower than the orange that corresponds to the single expert case. This means that the range of μ in which the experts provide together effective messages is broader. This is one way of improving the information transmission. Then the grey region above the lower boundary of EMR_c is the ICR_c. While the region that is is formed by the orange dashed curves between the lower boundary of EMR is the ICR. We have proved that $ICR_1 \subset ICR_c$, the Incentive Compatibility Region under competition is always greater than the Incentive Compatibility Region with a single expert. Moreover, we can notice that under competition the experts are truthful for some $\mu < 1 - \rho$, which is an improvement relatively to the single and unbiased expert case. This means that the existent bias can facilitate the information transmission for the priors opposite to her preferred state. In other words, experts take more risky for their reputation decisions regarding their message strategy because they have less incentives to behave in a conformist way in the states opposite to their bias.

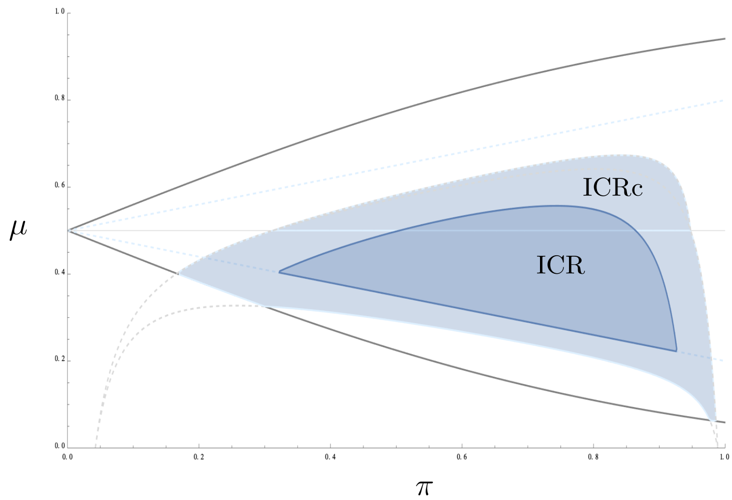


Figure 2.6 | **Incentive Compatibility Region - Comparison.** Combination of Figure 2.2 and Figure 2.5: Comparison between a single expert and two experts

In order to show the generalisation of the result, we study explicitly the scenario where the decision maker has access to three identical experts , E_1, E_2, E_3, who provide their information simultaneously. It is impossible to consider immediately N experts but following the same reasoning one can easily prove that our conclusion for the case where we go from two to three experts holds also for the case where we go from three to four experts and so on. We maintain the assumption that the signals are conditionally independent. Therefore, without loss of generality E_1 faces the following probabilities regarding the other two experts:

$$Pr(s_2 = 1, s_3 = 1|\theta) = Pr(s_2 = 1|\theta) \cdot Pr(s_3 = 1|\theta) = (\mu\rho + (1-\mu)(1-\rho))^2 = \kappa^2$$

$$Pr(s_2 = 0, s_3 = 0|\theta) = Pr(s_2 = 0|\theta) \cdot Pr(s_3 = 0|\theta) = (\mu(1-\rho) + (1-\mu)\rho)^2 = \lambda^2$$

$$Pr(s_2 = 0, s_3 = 1|\theta) = Pr(s_2 = 0|\theta) \cdot Pr(s_3 = 1|\theta) = [\mu\rho+(1-\mu)(1-\rho)][(\mu(1-\rho)+(1-\mu)\rho)] = \lambda\kappa$$

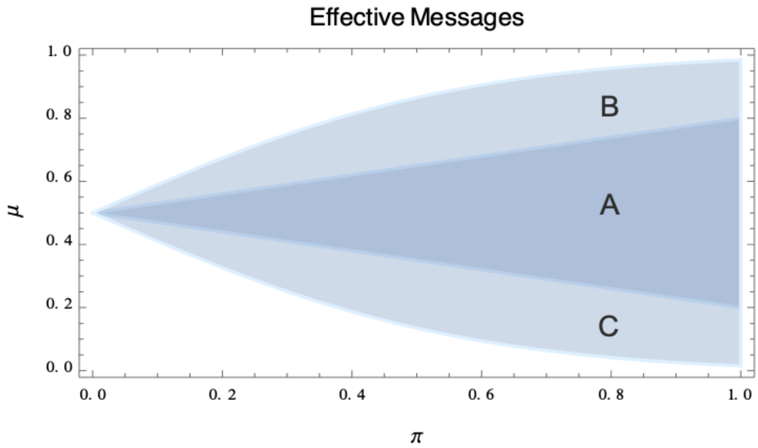


Figure 2.7 | **Effective Messages Region for three experts.** A+B+C: Effective messages region for three experts. The region A is given by relations $\underline{\mu} = 1 - \rho$ and $\overline{\mu} = \rho$ while the boundaries or the whole region A+B+C is given by solving relations (17) and (18) for three experts. Parameters: $g = 0.8$ and $b = 0.5$

Similarly to the previous case, the experts behave differently in each region. We will partition the prior space in three regions as they are shown in the graph. We have assumed that the experts are identical in terms of prior reputation. This means that they have equal power on the decision. As we discussed in previous subsection if the decision maker receives two opposite messages, they cancel out each other and his posterior belief is equal to his prior belief. Therefore it is important to consider whether the behaviour of expert is affected by his ability to provide effective messages by himself in case that the three experts do not agree. We will consider three regions (Figure 2.7): A. The region where in case of disagreement the expert can provide effective messages by his own; B. The region where only three equal messages can move the beliefs of the decision maker while his priors suggest the high state;

C. The region where, as in region B., only if the three experts agree then decision maker can revise sufficiently his priors and adopt a high action, otherwise he follows his priors.[14]

(A) In this region a single expert would be able to provide effective messages, it is the region that we have considered in section 3.1. In this case the decision maker has the following action rule:

$$a^*(m_i, m_{-i}, \mu) = \begin{cases} 1 & \text{if} \quad \text{at least two experts send } m = 1 \\ 0 & \text{if} \quad \text{at least two experts sent } m = 0 \end{cases}$$

Given the action rule of decision maker, the truth-telling conditions after a signal $s = 0$ and $s = 1$ of Expert i are respectively the following:

$$E(U(m_0|s_0, m^{-i})) \geq E(U(m_1|s_0, m^{-i})) \Rightarrow \kappa\lambda\beta \leq (1 - 2Pr(\theta_1|m_0, \mu)(\pi^+ - \pi^-) \quad (2.23)$$

$$E(U(m_1|s_1, m^{-i})) \geq E(U(m_0|s_1, m^{-i})) \Rightarrow \kappa\lambda\beta \geq (1 - 2Pr(\theta_1|m_1, \mu)(\pi^+ - \pi^-) \quad (2.24)$$

(B) In this region we have to consider two things. First, the decision maker can ask the advice of experts that wouldn't be able to provide effective messages by their own. However, by combining the expertise and knowledge of three experts can extract decision-relevant information but only if they agree. Second, we have to consider the case that the experts do not agree. In this case, the two messages cancel out each other and the third one is ineffective. This leads the decision maker to adopt the action which agrees with his prior beliefs which in this case suggest the high state. The action rule of the decision maker and the corresponding truth-telling conditions can be written as follows:

$$a^*(m_i, m_{-i}, \mu) = \begin{cases} 0 & \text{if} \quad \text{three experts send } m = 0 \\ 1 & \text{if} \quad \text{otherwise} \end{cases}$$

$$E(U(m_0|s_0, m^{-i})) \geq E(U(m_1|s_0, m^{-i})) \Rightarrow \lambda^2\beta \leq (1 - 2Pr(\theta_1|m_0, \mu)(\pi^+ - \pi^-) \quad (2.25)$$

$$E(U(m_1|s_1, m^{-i})) \geq E(U(m_0|s_1, m^{-i})) \Rightarrow \lambda^2\beta \geq (1 - 2Pr(\theta_1|m_1, \mu)(\pi^+ - \pi^-) \quad (2.26)$$

[14]Upper boundary of region B. and lower boundary of region C. are given by solving :

$$Pr(\theta_1|s_1^1, s_1^2, s_1^3) = \frac{P(\theta_1|s_1^1, s_1^2)\rho}{P(\theta_1|s_1^1, s_1^2)\rho + (1 - P(\theta_1|s_1^1, s_1^2))(1-\rho)} \geq \frac{1}{2}$$

$$Pr(\theta_1|s_0^1, s_0^2, s_0^3) = \frac{P(\theta_1|s_0^1, s_0^2)(1-\rho)}{P(\theta_1|s_0^1, s_0^2)(1-\rho) + (1 - P(\theta_1|s_0^1, s_0^2)\rho} \leq \frac{1}{2}$$

(C) Similarly to the previous region, in region C we have to consider what happens if all of the experts agree and what changes if we have a disagreement. The action rule is similar to the one of region B but in this case if there is disagreement between reports the decision maker always chooses the low action which is suggested by his prior belief. The action rule can be written as follows:

$$a^*(m_i, m_{-i}, \mu) = \begin{cases} 0 & \text{if} \quad \text{three experts send } m = 0 \\ 1 & \text{if} \quad \text{otherwise} \end{cases}$$

The truth-telling conditions of expert i for low and high signal respectively are the following:

$$E(U(m_0|s_0, m^{-i})) \geq E(U(m_1|s_0, m^{-i})) \Rightarrow \kappa^2\beta \leq (1 - 2Pr(\theta_1|m_0, \mu)(\pi^+ - \pi^-) \quad (2.27)$$

Similarly, an expert will truthfully send $m = 1$ after a signal $s = 1$ if

$$E(U(m_1|s_1, m^{-i})) \geq E(U(m_0|s_1, m^{-i})) \Rightarrow \kappa^2\beta \geq (1 - 2Pr(\theta_1|m_1, \mu)(\pi^+ - \pi^-) \quad (2.28)$$

Similarly to **Proposition 4** we prove that as we increase the number of experts the information transmission is overall improved. The ICR moves upward and it expands. The degree of improvement depends also on the level of bias. As bias increases the improvement is higher as well (see Figure B.7 in Appendix B.2) and in turn there is always a level of bias for which there is a fully revealing equilibrium when the experts are three but not if there are two. Qualitatively, the results are in line with with the cases where we go from one expert to two experts under both binary and continuous actions.

Proposition 2.8 *If the experts have the same prior reputation and they provide simultaneously their information then for a fixed bias β there is an equilibrium where as we increase the number of experts*

1. *the EMR_n always expands*
2. *the ICR' always expands*
3. *the boundaries of ICR' always move upwards*
4. *there is always a combination of (π, μ) for which there is a separating equilibrium for n but not for $n+1$ experts, where $n \geq 2$*
5. *there is a number of experts $\overline{n}$ such as $ICR_{\overline{n}} = EMR_1$*

Proof. See Appendix B ■

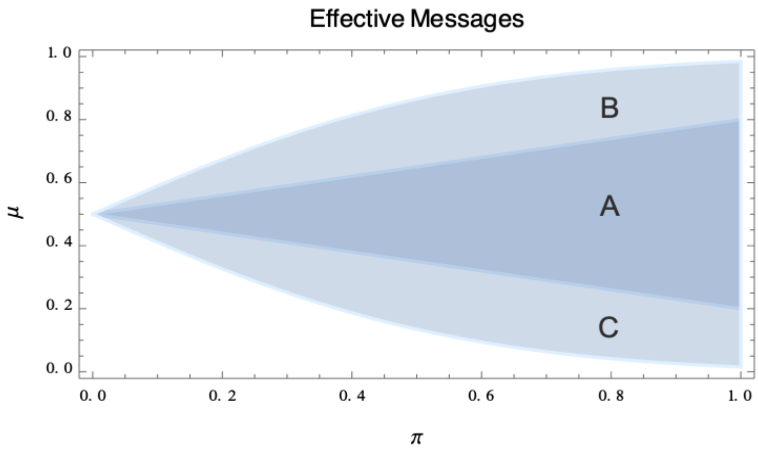


Figure 2.8 | **Incentive Compatibility Regions for Two and Three Experts.** ICR_c: Incentive Compatibility Set with Two Experts; ICR_c': Incentive Compatibility Set with Three Experts which is described by relations (25)-(28). The dashed lines form the Effective Messages Region for two (inner curves) and three experts (outer curves). (Parameters $\beta = 0.1, g = 0.8, b = 0.5$)

The following final remark concerns both the case of binary and continuous actions. It states formally what we implied through our previous results. From **Proposition 4** and **Proposition 5** we know that the addition of experts moves the ICR upwards. This practically means that as the number of experts increases the region where the experts are truthful converges to the ICR of unbiased experts which is cantered around $\mu = 0.5$. Therefore, equilibrium does not exist for very high or very low priors expect for the case where the experts are very reputable. The bias of the experts, harms the information transmission overall but locally for the low priors actually allows for equilibrium existence. This is because, due to bias, the expected gain from truthful but risky suggestion is high enough to mitigate the herding on the prior effect. Therefore, for low states the decision maker can benefit by the bias of the expert. This is exactly the channel through the information transmission is facilitated by a lower number of experts under low priors. The high competition practically reduces the gain from misinterpretation by making the expert more and more uncertain about the final decision.

Remark 2.3 *If the decision maker consult more than one experts, the overall information transmission is weakly improved as we increase the number of experts, but for some low values of prior beliefs is better to consult fewer experts.*

Proof. See Appendix B ■

In Figure 2.8, we observe that the ICR_c (darkest region), which corresponds to two-experts scenario, expands to ICR_c' (lighter region), which describes the three-experts case. **Remark 2.3**, refers to these combination of (π,μ) for which a fully revealing equilibrium exists if the experts are not more than two. In Figure 2.8, points (0.8, 0.22), (0.6, 0.28) and (0.35, 0.35) are some combinations of priors for which is optimal to ask the advice by two experts instead of three. While for combinations (0.8, 0.7), (0.6, 0.65) and (0.35, 0.55), it is optimal to ask advice by a third expert. This conclusion refers to cases where the decision maker has access to multiple experts and has to decide if he will consult two or more than two experts. In cases where the decision maker has to decide whether to consult one or more than one experts, then the additional expert always improves the information transmission. The difference between the continuous and the binary action space scenarios comes for the effective messages concept. In binary case more experts are always better than one because even if the expert would be truthful for low priors still he wouldn't be able to affect the final decision. Instead, with continuous action space where the decision maker can adjust his actions according to his belief, the expert has potential gain from any recommendation.

2.5 Conclusion and Discussion

This paper analysed the behaviour of biased experts who care about their reputation. The decision maker's choice problem is binary which means that for extreme priors, it is optimal for the decision maker to disregard the message by an expert who is not reputable enough. For this reason we focus on the cases where the expert can provide decision relevant information (*effective messages*). We show that given the expert is able to send effective messages, there is a maximum degree of conflict between the expert and the decision maker that allows the existence of a fully revealing equilibrium. Moreover, we find that some experts can be too reputable to provide truthful advice. This is because, in comparison to a less reputable expert, an expert with an established reputation might have limited reputational gain (loss) in case of a correct (wrong) recommendation. She could afford to make a recommendation that might be proven wrong while he has managed to induce her preferred action. To this end, the decision maker faces a trade-off between accuracy and discipline. Ideally, he would consult the most reputable expert but since his recommendation is untruthful with high probability, it is beneficial for him to sacrifice a bit of accuracy in exchange of a credible advice.

Finally, we consider the case where the decision maker has access to more than one experts which is something we observe in many situations. We show that the additional experts facilitates the information transmission in three ways. First the range of priors where the experts together can provide effective messages expands. This means that when the prior beliefs are extreme the experts can provide relevant information easier. Second the second expert acts as a discipline device to the other expert. In case of disagreement an untruthful message can lead to reputation loss without any other gain. Technically, the expected gain from misrepresentation decreases due to the additional expert. Third, it increases the level of initial reputation that maximises the information transmission. However, we find that despite the fact that the overall information transmission increases with the number of experts, for the low prior belief it is optimal to consult less experts.

We can think of two possible extensions of the model. First, we can modify **Assumption 2.1** by adding another dimension of asymmetric information and we study the case where the expert knows his type (if he will receive with high or low probability the correct signal) but the receiver does not. This means that the decision maker will update his belief about expert's ability not only by observing the state and comparing it with her recommendation but also given her strategy. Therefore, the game obtains a signalling dimension.

Second, we can consider a scenario where the receiver does not always receive feedback on the realisation of the state.[15] Particularly, we can assume that in case of inaction ($\alpha = 0$) the receiver is not able to update his belief about expert's ability. Consider, a situation where a referee proposes to an editor to reject a paper and the editor adopts this recommendation. Then, she will not learn the true quality of the paper and she will fail to update her belief about referee's ability. We leave these extensions for future research.

[15]See Gentzkow and Shapiro (2006), Mariano (2012), Rüdiger and Vigier (2019)

Chapter 3

Bayesian Persuasion with Private Information and Binary Actions[1]

Abstract

We consider a model of strategic communication where a privately and imperfectly informed sender can persuade a receiver. The sender may receive favorable (*good type*) or unfavorable (*bad type*) private information about her preferred state. We show that considering binary action space the private information of the sender does not improve the informativeness of equilibrium which is only pooling. We describe two ways that are adopted in real life situations and theoretically improve equilibrium informativeness given sender's private information. First, a policy that suggests symmetry constraints to the experiments' choice and leads to a separating equilibrium for a range of prior beliefs. Second, an approval strategy characterised by a low precision threshold where the receiver will accept the sender with a positive probability and a higher one where the sender will be accepted with certainty. We show that this approval strategy supports always a separating equilibrium where the high type prefers to provide less noisy information in return of certainty while the bad type will provide the lowest possible precision.

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3.1 Introduction

The recent literature on *Bayesian persuasion* has attracted interest in economics due to its myriads applications. We can refer to examples that vary from university admissions and advertising to medical procedures. Consider for example a seller who aims to convince through advertising the buyers that they need his product, or a software company that provides trial periods to the consumers before purchase. The informativeness of the persuasion procedure is a strategic choice: the advertising or the trial version of the software are designed strategically in order to minimise the likelihood that the buyer decides not to buy it. An important element that we focus on in this work is that senders are better informed than the receiver. For instance, sellers are typically more informed about the features of their product and how they fit the consumer's needs than the consumer himself.

In this paper, we study these situations by considering a model à la Kamenica and Gentzkow (2011) where a sender (female) strategically designs a Blackwell experiment aiming to influence a receiver's (male) belief and binary decision. We assume that initially the sender and the receiver have the same prior belief about the state but before designing her experiment the sender receives a private and imperfect signal about the state and update her belief accordingly. By introducing this information asymmetry between the players, the receiver does not take an action only based on his prior belief and the signal from the experiment but also based on the choice of the experiment. This means that the communication between the sender and the receiver includes also a signalling game (Spence, 1978). The sender's type is defined only by the private signal she receives before the experiment's choice and the senders differ only in this dimension. I assume that the receiver's action is of accept/reject form and his payoff is state dependent. Respectively, the payoff of sender is state independent, binary and depends only on the final action.

There is a strand of the literature which combines Bayesian persuasion with other forms of strategic communication and more precisely, those where the persuasion problem becomes a signalling game (Perez-Richet, 2014; Hedlund, 2017; Koessler and Skreta, 2021). This paper can be considered as complementary to Hedlund (2017) who considers the same game but assuming continuous action space and continuous and monotonic preferences. He shows that in these situations the equilibrium is either separating (i.e., the sender's choice of signal reveals his type to the receiver) or fully disclosing (i.e., the sender's chosen signal fully reveals the state). However, we show that for the binary action space this result does not hold because the high type sender does not have any incentive to signal her type to the receiver. Therefore, the receiver cannot benefit by the information asymmetry anymore. This paper studies ways to improve the informativeness of equilibrium by taking advantage of senders' private information.[2] To this end, we describe two ways that are implemented in different situations. The first is a policy that we will refer to it as *symmetry constraints* while the second is an approval strategy adopted by the decision maker, from now on a *double cutoff rule*.

First, we show that a more informative equilibrium can be achieved by introducing some constraints on the experiments choice, in the sense that the probability of observing the correct signal is the same for both states and this is what we call *symmetry constraints*. This set up is motivated by the medical procedures where these symmetry-constraints can be implemented through the medical guidelines[3] aiming to minimise and balance false positives and false negative results[4] and in turn the overtreatment.[5] In the medical procedures, similarly to our model's assumption, the physicians appear to be better informed (through the diagnosis procedure) about the probability that a patient has a specific disease than the patient himself even before any other medical exam. A doctor who has the incentives to maximise the probability of disease detection would perform the medical exams that would give with higher probability a false positive result, or in other words the exams with the lowest possible *specificity*. On the other side a doctor has incentives to choose a medical exam with the lowest possible probability of false negative, meaning highest possible *sensitivity*. Despite the fact that the *sensitivity* of the medical procedures, is very important for timely diagnosis and treatment which in

[2]There is a number of papers that introduce ways to increase the informativeness of the signals Gentzkow and Kamenica (2017) and Kamenica and Gentzkow (2017) introduce competition between the senders and this leads to more informative equilibrium. Kolotilin et al. (2017) considers a privately informed decision maker and Kolotilin (2015) assumes that the receiver obtains information not only by the sender but also public information. Similarly, Bizzotto and Vigier (2020) considering a dynamic model, allows the decision maker to accumulate information beyond the sender's control.

[3]A medical guideline or clinical practice line is a document with the aim of guiding decisions and setting criteria regarding diagnosisand treatment in specific areas of healthcare and any physician is obliged to know the medical guidelines of her field.

[4]In general, the balance of sensitivity and the specificity of medical devices is extensively discussed. See for instance:
https://www.healthnewsreview.org/toolkit/tips-for-understanding-studies/understanding-medical-tests-sensitivity-specificity-and-positive-predictive-value/, https://www.healthnewsreview.org/2017/09/most-news-coverage-on-new-psa-testing-study-acknowledged-costly-trade-offs/

[5]Overtreatment is an extensively discussed issue for a wide range of clinical activities, see for instance Armstrong (2018) for a discussion on it.

serious situations is also translated to higher probability of survival, the low specificity can lead to stress, continuous testing or even unnecessary treatments.[6] For this reason medical guidelines to reduce false positives are beneficial.[7] Carroni and Pignataro (2021) through a similar setting suggest a minimal information standard in order to reduce the *false positives* and in turn the overtreatment. We discuss instead a strict balance between the *sensitivity* and the *specificity* of medical procedure in the sense that the doctor is allowed to use any medical test/ procedure and make a recommendation based on it if the probability of false negatives is the same with the probability of *false positive*.[8]. We show that in our model the introduction of this constraint leads to a separating equilibrium but only for a range of prior belief. In the separating equilibrium the choice of the experiment can be used by the receiver to make inference about the belief of the sender and then assess the validity of the experiment's signal accordingly. In our example the choice of medical exam can be taken into account by the patient and make inference about the truthful diagnosis of the doctor before the medical exams and then trust the results or perform additional testing.[9] The intuition behind the result comes from the non-monotonic preferences that the senders have on the experiment precision in case of symmetry constraints. Depending on their private information, if they have favorable private information then they weigh more the probability of true positives while in opposite case they weigh more the probability of false positives. Therefore by observing the experiment choice the receiver can make inference about the private information of the senders.

[6]For instance, Marco et al. (2006), citing various examples and studies, claim that the interaction of physicians with the pharmaceutical industry has detrimental effects on patients, including unnecessary treatment

[7]For instance, false positives in cancer screening are common: " approximately 50% of women and 10% to 12% of men receive a false-positive outcome for mammography and prostate-specific antigen (PSA) tests, respectively, and 23% of patients have a negative result from confirmatory colonoscopies after a false-positive fecal occult blood test (FOBT)." See https://www.oncologynurseadvisor.com/home/cancer-types/general-oncology/false-positive-cancer-screening-results-may-increase-likelihood-of-future-screenings/. European Commission presented the Europe's Beating Cancer Plan in February 2021 and one of its aims is to provide guidelines about the screening, the diagnosis and the treatment of cancer through its council. Therefore European Commission even if it doesn't have the legislative power to directly implement such constraints, through its guidelines it could propose a balance of sensitivity and specificity of medical exams for cancer screening.

[8]Medical tests that characterised by very similar *sensitivity* and *spesificity* levels are not rare.For instance, nuclear cardiac stress test can perform at equally high specificity and sensitivity level:https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3366298/Another example is, Saliva and Nasopharyngeal Swab Nucleic Acid Amplification Testing for Detection of SARS-CoV-2 which both have high and balanced sensitivity and specificity levels:https://jamanetwork.com/journals/jamainternalmedicine/fullarticle/2775397

[9]We can also refer to the classic trial process example, as in Kamenica and Gentzkow (2011), and claim that the judges are concerned about convicting innocents (false positive). Blackstone had stated in 1765 "Better that ten guilty persons escape, than that one innocent suffer". This message is known as Blackstone's ratio and is still a key element of legal thinking. Therefore the prosecutor is called to present the results of his investigation and he will be persuasive if his investigation is designed such a way that the probability of false positives is low enough.

Second we describe a strategy of a double cutoff rule by the receiver.[10] Consider an example related to the PhD admissions. The students in order to be considered eligible for admission, have to provide some compulsory (necessary) certificates, e.g their BSc and MSc degree. However, there are other documents that are not compulsory but strongly recommended (sufficient), for example the GRE score. Having all the certificates and documents, both compulsory and recommended with good marks and scores, will lead to acceptance with very high probability (or even with certainty). But if you choose to provide only the necessary documents you become eligible but accepted with a lower probability. We will show that the bad types (for this example students of lower quality) will prefer to submit only the necessary documents because the more information might reveal their true quality (imagine for example a bad score in GRE). Instead, the good type (a high quality student) will prefer to submit all the documents (compulsory and recommended) in return for certainty since she does not worry about the information that can be revealed. Therefore, we allow the decision maker to ask for both *sufficient* and *necessary* information. This means that, up to some minimum level of information precision you can be considered eligible for acceptance and then after a higher level you get acceptance with certainty. The decision maker who adopts a standard binary decision rule is not able to give enough incentives to the high type to signal her type. To this end, one way to create some separation is to reward the high type with certainty in return for a more precise signal. This strategy can be considered related to the resistance strategy proposed by Tsakas, Tsakas and Xefteris (2021) where the decision maker commits to bear a cost if he picks the sender's preferred action. In Tsakas, Tsakas and Xefteris (2021) the receiver increases his cutoff point, through the cost that he commits to bear, and in this way he resists to persuasion. Similarly, the double cutoff can be perceived as the resistance of the decision maker to accept a low and a high type with the same probability. The decision maker commits to follow the experiment's signal with probability-1 if and only if he is certain that the sender is a good type. In the opposite case he will accept with probability less than one. This can be translated as a commitment to follow the signal for a specific signal precision and to follow the signal for a lower precision with a positive probability but not with certainty. Another way to see this rule is that in one case you adopt totally the suggestion of the sender if the precision is high enough and you partially accept it for

[10] Richet-Perez and Skreta (2021) study optimal test design under falsification and characterise a similar receiver's approval strategy.

a lower precision. In contrast to the symmetry constraints, the adaptation of double cutoff supports a separating equilibrium along the full range of prior beliefs.

Related Literature. This paper contributes to the literature of Bayesian persuasion pioneered by Kamenica and Gentzkow (2011) (henceforth, "KG"), which has been developed in several and different directions.[11] However, few papers in the literature investigate how the private information of the sender may affect the informativeness of equilibrium.

A pioneering paper is Perez-Richet (2014) who studies a similar model to ours but with perfectly informed senders. In contrast to our case where the experts have private but imperfect information, in his model the type of the sender coincides with the actual state of the world. Akin to our approach, he considers sender's preferences that are discontinuous in the receiver's updated beliefs and for this reason he focus on pooling equilibrium. Several refinements concepts that he applies lead to the selection of the high type sender optimal equilibria. Degan and Li (2015), who also assumes perfectly informed sender, considers costly communication by the sender and studies two different possible sender's strategies : commitment to an experiment before learning his type, or choosing the experiment after the privately get informed. They show that the sender's preferred strategy depends on prior beliefs of the receiver.

Hedlund (2017) and Kosenko (2020) are very closely related works. Both consider imperfectly informed sender but they differ at some crucial assumptions. Hedlund (2017) works with a similar model but with continuous action space and continuous and monotonic sender's preferences. He shows that in these situations the equilibrium is either separating (i.e., the sender's choice of signal reveals his type to the receiver) or fully disclosing (i.e., the sender's chosen signal fully reveals the state). However, I show that for the binary action space and discontinuous preferences this result does not hold and the equilibrium is identical to Kamenica and Gentzkow (2011) .This is because, without the continuity and the monotonicity of the preferences the high type sender does not have any incentive to signal her type to the receiver. Therefore, the receiver cannot benefit by the information asymmetry and the equilibrium is uninformative relatively to Hedlund (2017). Kosenko (2020) explores the role of some key assumption of Hedlund (2017) such as the availability of signals and the actions available to

[11] For a recent review and discussion see Bergemann and Morris (2019), Kamenica (2019) and Kamenica, Kim and Zapechelnyuk (2021)

the receiver. He shows that the informativeness of equilibria relies heavily on the availability of a fully revealing experiment and the compact action space. Therefore, the private information may not lead to more informative equilibria. Koessler and Skreta (2021) study also situations where the sender has private information before the communication. They consider a general interim information design framework and they identify a set of incentive compatible information disclosure mechanisms.

Our paper is also related to Alonso and Câmara (2016) and Alonso and Câmara (2018). Alonso and Câmara (2018) investigate whether or not the sender can benefit from having private information prior to designing the experiment and show that sender has nothing to gain from learning his type before persuasion. Alonso and Câmara (2016) consider a situation where the sender and receiver have different about the state of the world. This paper studies a similar situation, since the type of the the sender identify his belief about the state after receiving private information. The difference in Alonso and Câmara (2016) is that the beliefs are common knowledge.

The rest of the paper is organised as follows. Section 2 introduces the model, Section 3 contains the equilibrium analysis and some preliminary results, Section 4 introduces symmetry constraints on the experiments, Section 5 analyses the double cut off rule and Section 6 concludes.

3.2 Model

We consider two players: a sender (S) and a receiver (R). The payoff-relevant states of the world are $\omega \in \Omega = \{\omega_b, \omega_g\}$. Initially, the players share a common prior $\mu_0 \in (0,1)$ that the state is ω_g, $Pr(\omega_g) = \mu_0$. The expert receives external signal $\theta = \{\theta_b, \theta_g\}$ and given these signals updates μ_0 to $\mu_t = \{\mu_1, \mu_2\}$. We will refer to the sender's private information as her type. The exogenous signals are generated as follows:

$$Pr(\theta_g|\omega_g) = Pr(\theta_b|\omega_b) = \psi \ ; \quad Pr(\theta_g|\omega_b) = Pr(\theta_b|\omega_g) = 1 - \psi$$

for $\psi \geq 0.5$. The sender after observing her private signal, her objective is to persuade the decision maker to choose her preferred action by modifying his belief μ_0. After observing θ_b, the sender's posterior is μ_1 and after θ_g it is μ_2:

$$\mu_1 = \frac{\mu_0(1-\psi)}{\mu_0(1-\psi) + (1-\mu_0)\psi} \ ; \quad \mu_2 = \frac{\mu_0 \cdot \psi}{\mu_0 \cdot \psi + (1-\mu_0)(1-\psi)}$$

The sender is imperfectly but more informed than the receiver. She chooses an experiment (information structure) without any constraint. Each experiment is denoted by $\Pi = (\pi(\cdot|\omega_g), \pi(\cdot|\omega_b))$. The conditional probability distributions $\pi(\cdot|\omega_b)$ and $\pi(\cdot|\omega_g))$ over a finite set of outcomes $s = \{g, b\}$ are, $\pi(s = g|\omega_g) = \pi_g$ and $\pi(s = b|\omega_b) = \pi_b$ where $\pi_g, \pi_b \in [\frac{1}{2}, 1]$. We consider the following posteriors $\mu_{t,s} = Pr(\omega_g|\theta, s)$ of senders given signal θ and π.

$$\mu_{1g} = \frac{\mu_1 \pi_g}{Pr_1(g)}; \quad \mu_{1b} = \frac{\mu_1(1-\pi_g)}{1-Pr_1(g)}; \quad \mu_{2g} = \frac{\mu_2 \pi_g}{Pr_2(g)}; \quad \mu_{2b} = \frac{\mu_2(1-\pi_g)}{1-Pr_2(g)}$$

where $Pr_1(g) = \mu_1\pi_g + (1-\mu_1)(1-\pi_b)$ and $Pr_2(g) = \mu_2\pi_g + (1-\mu_2)(1-\pi_b)$. The choice of the experiments and the realisation of the signals are observed by all the players. The receiver observes the signals π chosen by the sender and makes an interim update of his belief, $\nu(\theta|\pi) = \{\mu_0, \mu_1, \mu_2\}$, that the payoff-relevant state is ω_g. That is, the receiver can make inference about the senders' types from the senders' choices of experiment. The receiver next observes the realisation of the experiment and updates to his final belief to

$$\beta(\omega_g|\pi, s, \mu) = \frac{\pi_g \mu_t}{\pi_g \mu_t + \pi_b(1-\mu_t)}$$

which is the mapping of an experiment π, a signal s_i and an interim belief μ_t to a posterior probability that the state is high. At the end, the receiver chooses an action $a \in \{0, 1\}$. The sender has state-independent preferences, with utility $V(a|\omega) = a$. On the other hand, the receiver has state-dependent preferences such as $U(a = 1, \omega = 1) > U(a = 0, \omega = 1) = 0$ and $U(a = 0, \omega = 0) > U(a = 1, \omega = 0)$.[12]

The timing of the game is summarised as follows:

1. Nature chooses ω
2. Sender receives privately exogenous signal θ about ω
3. Sender chooses an experiment π

4 The receiver forms interim belief about ω given the sender's experiment choice.

5. The receiver forms his posterior belief given the signal of the experiment.
6. The receiver takes and action and the payoffs are realised.

[12]We consider a general form of receiver's payoff because the aim of this chapter is to focus on sender's behavior. In Section 6, we discuss how the payoff of the receiver should be adjusted so as he commits to adopt the double cutoff rule proposed in Section 5.

3.3 Equilibrium Analysis

The solution concept that we consider is perfect Bayesian Equilibrium (PBE).

Definition 3.1 *A Perfect Bayesian equilibrium is a sender strategy π, a receiver strategy a and receiver interim and final belief (ν, β) such as :*

1. $\forall \mu_t \;\; \pi^* \in arg \max V_t(\pi, \mu_t, a)$
2. $a^* \in arg \max U(\beta, a, \omega)$
3. *interim and final belief (ν, β) are computed using Bayes rule whenever possible.*

We start our analysis with the equilibrium behaviour of the senders first for symmetry constraints and second for the double cut-off rule.

3.3.1 Preliminary Results

Starting with the case where the senders have no constraints on the experiment's choice, we show that the private information of the senders do not lead to more precise experiments like in Hedlund (2017). This difference comes from the actions available to the decision maker. On equilibrium, the receiver follows the realised signal with certainty if the signal is persuasive.[13] Therefore the senders, irrespectively of their type, maximise the expected probability of the high signal. In Hedlund (2017), the utility of the senders is continuous and strictly increasing in receiver's belief. Instead, with binary action space the high type would like to signal his type to the decision maker in order to be able to persuade him with a less precise signal. Hence, she has nothing to gain by signalling her type with a more precise signal since this comes in contrast with her benefit.

We show that both types pool in the optimal signal precision as in the benchmark case of Kamenica and Gentzkow (2011)- hereafter we denoted it by KG and the corresponding experiment $\Pi_{KG} = (\pi_g^{KG}, \pi_b^{KG})$- and the decision maker cannot make any inference about the sender's type, $\nu(\theta|\pi) = \mu_0$.

Proposition 3.1 *With unconstrained experiments there is only a pooling equilibrium where* $\Pi_1 = \Pi_2 = \Pi_{KG}$. **Proof.** See Appendix C ■

Therefore, the high type does not have any incentive to signal her type and prefers to pool in the π^{KG}. The receiver neither find out the type of sender nor the state of

[13] I will refer to persuasive signals, as the signals that satisfies: $\mu_{2g} \geq \underline{\beta}$ and $\mu_{2b} < \underline{\beta}$, where $\underline{\beta}$ is the cutoff rule adopted by the decision maker.

the world. Therefore considering Hedlund (2017) as our benchmark the equilibrium is uninformative. Regarding the two types and whether they would prefer to reveal their private information, the low type benefits by the private information but the high type does not. The good type would prefer to make her private signal as public and persuade easier the receiver. From the side of the decision maker, he would not follow the signal of precision π^{KG} if he knew that the sender is a low type. This is important especially for the case that the senders could choose their types. If the senders could be for example good or bad types and this decision was associated with a higher or lower cost respectively, then in some cases it would be beneficial for the sender to be a bad type. This result is in line with Alonso and Câmara (2018) where if there are no restrictions on experiments' choice the sender does not gain from obtaining private information. At the next sections, I characterise constraints and decision rules that the decision maker is make use of in order to achieve some separation.

3.4 Symmetric experiments

The aim of the decision maker is to achieve some separation between the good and the bad type and gain from the sender's private information. We show that one way is to constraint the experiments' choice and that will lead to different preferences for the senders but only for a range of priors. We consider the scenario where an external authority impose some symmetry constraints on the information structure such as $Pr(s = g|\omega_g) = Pr(s = b|\omega_b)$. The information structure can be summarised by the following table:

	Pr(s=g | ω)	Pr(s=b | ω)
ω_g	π_i	1-π_i
ω_b	1-π_i	π_i

where $\pi_i \in [\frac{1}{2}, 1]$. Similarly to the previous section, we consider the following posterior beliefs $\mu_{t,s} = Pr(\omega_g|\theta, s)$ of senders given signal θ and π.

$$\mu_{1g} = \frac{\mu_1\pi_1}{Pr_1(g)}; \quad \mu_{1b} = \frac{\mu_1(1-\pi_1)}{1-Pr_1(g)}; \quad \mu_{2g} = \frac{\mu_2\pi_2}{Pr_2(g)}; \quad \mu_{2b} = \frac{\mu_2(1-\pi_2)}{1-Pr_2(g)}$$

where $Pr_1(g) = \mu_1\pi_1 + (1-\mu_1)(1-\pi_1)$ and $Pr_2(g) = \mu_2\pi_2 + (1-\mu_2)(1-\pi_2)$.

We start the analysis by considering symmetric information in order to show how the symmetry constraint would affect the KG set up. The designer's problem is read as:

$$\max_{\pi^C} \mathbb{E}(V_C) = \mu\pi^C + (1-\mu)(1-\pi^C) \qquad \text{s.t} \qquad \mu_g \geq \underline{\beta}, \quad \mu_b < \underline{\beta}$$

with

$$\Delta = \frac{\partial V_C(\cdot)}{\partial \pi^C} = 2\mu_0 - 1 \lesseqgtr 0 \ \text{ if } \ \mu_0 \lesseqgtr \frac{1}{2}$$

Lemma 3.1 *Under symmetric information and symmetry constraints the equilibrium level of experiments precision* $\pi^C = \frac{\underline{\beta}(1-\mu_0)}{\underline{\beta}+\mu_0-2\underline{\beta}\mu_0}$ *for* $\mu_0 < \frac{1}{2}$ *and* $\pi^C = 1$ *if* $\mu_0 > \frac{1}{2}$.

It is trivial to show that the decision maker has nothing to gain from such constraints in absence of private information ($U(\Pi^C) = U(\Pi^{KG})$).[14] However, the senders strictly worse off under the symmetry restriction. We now turn to the scenario of sender's private information.

Under asymmetric information, the senders' maximisation problems are similar to the previous section. We start by the low type:

$$\max_{\pi_1} \mathbb{E}(V_i) = \mu_1\pi_1 + (1-\mu_1)(1-\pi_1) \qquad \text{s.t} \qquad \mu_{1g} \geq \underline{\beta}, \quad \mu_{1b} < \underline{\beta}$$

In the previous scenario, the designers had no constraint so they would like to maximise the probability of correct signal when the state is high and minimise the corresponding probability when the state is low. Instead, under symmetry constraints the senders face a trade-off. Therefore their preference of the probability of generating the correct signals depends on the precision of their private information and their prior beliefs. Consider the following comparative statics:

$$\Delta_1 = \frac{\partial V_1(\cdot)}{\partial \pi_1} = \frac{2(1-\psi)\mu_0}{\psi(1-\mu_0)+(1-\psi)\mu_0} - 1 \lesseqgtr 0 \ \text{ if } \ \psi \lesseqgtr \mu_0 \tag{3.1}$$

Similarly, the maximisation problem of the high type is read as follows:

$$\max_{\pi_2} \mathbb{E}(V_2) = \mu_2\pi_2 + (1-\mu_2)(1-\pi_2) \qquad \text{s.t} \qquad \mu_{2g} \geq \underline{\beta}, \quad \mu_{2b} < \underline{\beta}$$

[14] The decision maker for $\mu_0 < \frac{1}{2}$ which is the range of the prior beliefs that the solution is not no trivial gains nothing from constraints.

While the corresponding comparative static is as follows:

$$\Delta_2 = \frac{\partial V_2(\cdot)}{\partial \pi_2} = \frac{2\psi\mu_0}{(1-\psi)(1-\mu_0)+\psi\mu_0} - 1 \lesseqgtr 0 \quad \text{if} \quad \psi \lesseqgtr 1-\mu_0 \tag{3.2}$$

Therefore we see from Equation 3.1 and Equation 3.2 that the preferences of the two types do not coincide for $\mu_0 \in (1-\psi, \psi)$ and there is a separating equilibrium for this range of prior beliefs. While for any other μ_0 there is only a pooling equilibrium either with fully informative experiments or poorly informative ones. This is visualised by Figure 3.1, where we plot Δ_i versus prior beliefs μ_0. The grey solid line corresponds to Δ_2 while the blue dashed one to Δ_1. One should notice that the figure is split in three regions: **a.** $\Delta_1, \Delta_2 < 0$, **b.** $\Delta_1 < 0$ and $\Delta_2 < 0$, **c.** $\Delta_1, \Delta_2 > 0$

We first consider the case that both experts would prefer the highest possible π which is the fully informative signal. They both prefer the highest possible signal when $\Delta_1, \Delta_2 > 0$ which holds for $\mu_0 > \psi$. Proposition 3.2 describes the pooling equilibrium where the decision maker does not learn the types of the designers but learns the true state with probability-1.

Proposition 3.2 *There is a* ***pooling equilibrium*** *where* $\pi_1^* = \pi_2^* = 1$ *if* $\psi \leq \mu_0$.

Proof. See Appendix C ■

In Figure 3.1, at the central area the preferences of the two types differ. The high type chooses the highest possible precision but the low type aims at the lowest possible one. This means that the good type chooses on equilibrium the fully informative signal while the best that the low type can achieve is:

$$\pi_1^* = \frac{\underline{\beta}\psi(1-\mu_0)}{\underline{\beta}(\psi-\mu_0)+\mu_0(1-\psi)}$$

This is because if the receiver observes something smaller than $\pi = 1$ he believes that $Pr(\theta_g|\pi < 1) = 0$. This is the only case that the decision maker can make some inference about the types by observing their experiments. This means that the low type chooses the most noisy persuasive experiment. Formally:

Proposition 3.3 *There is a* ***separating equilibrium*** *where* $\pi_1^* = \frac{\underline{\beta}\psi(1-\mu_0)}{\underline{\beta}(\psi-\mu_0)+\mu_0(1-\psi)}$ *and* $\pi_2^* = 1$ *if* $\psi \geq 1-\mu_0$.

Proof. See Appendix C ■

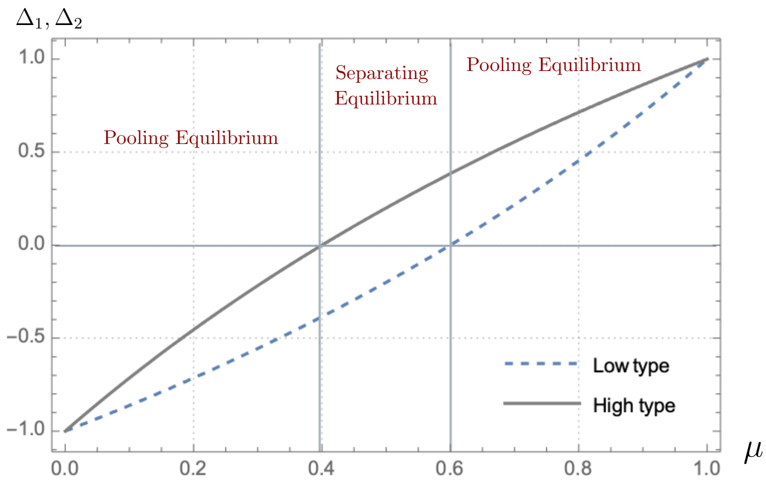


Figure 3.1 | **Constrained experiments.** Horizontal axis: prior beliefs μ_0; Vertical axis: change of each type's payoff as the experiments precision increases,$\Delta_i = \frac{\partial V(\cdot)}{\partial \pi_i}$. **Parameters:** private information's precision $\psi = 0.6$ and cut-off decision rule $\underline{\beta} = 0.5$. The blue dashed line represents change in utility of the low type as the precision of the experiment increases. The grey solid line represents the change in utility of the hight type as the precision of the experiment increases. We see that for $\mu \leq 0.4$ both types prefer the lowest possible precision $(\Delta_1, \Delta_2 < 0)$ and there is only a pooling equilibrium where $\pi_1 = \pi_2 = \pi^{KG}$. For $0.4 < \mu_0 < 0.6$ there is separating equilibrium because $\Delta_1 < 0$ while $\Delta_2 > 0$. Therefore the low type chooses the lowest persuasive experiment while the high type the fully informative one. For $\mu_0 > \psi$ there is again only a pooling equilibrium but since $(\Delta_1, \Delta_2 > 0)$, it is fully informative.

We notice that there is a separating equilibrium for a specific range of prior beliefs and this range depends on the precision of the senders' private information. As the precision of the exogenous signals increases, also the range of prior beliefs that support separating equilibrium increases.[15] This means that the decision maker benefits from the private and precise information.

Last, when both types aim to the lowest possible precision $Pr(\theta_g|\pi = \hat{\pi}) = \mu_0$. In this case both choose $\pi = \pi^C$ which is the best that they can achieve.

Proposition 3.4 *There is a **pooling equilibrium** where* $\pi_i^* = \pi^C$ *if* $\psi \leq 1 - \mu_0$.

Proof. See Appendix C ■

The receiver is not able to update his belief about the state by observing the experiment choice and the two types pool in π^C which is the optimal level of precision with symmetry constraints but with symmetric information and $\pi^C < \pi_1^*$, where π_1^* is the equilibrium precision in the separating equilibrium. Moreover, the utility of the decision maker is identical to the previous section.

To sum up, the achieved separation is only for a range of prior beliefs and then for high priors we have full revelation. So in a way we come to a similar conclusion with

[15] See, for example, Figure C.1 in Appendix C.2 that considers a higher level of precision and the region of separation significantly increases.

Hedlund (2017) for priors $\mu_0 < 1 - \psi$: the decision maker learns either the types or the state. However, this separation is different from the one of Hedlund (2017). The separation is not coming from the high type who wants to signal her type to the receiver but from the non-monotonic preferences imposed by the symmetric experiments. In the next section, we describe a strategy that can be adopted by the decision maker so as to achieve separation along the full range of prior beliefs.

3.5 Double cutoff rule

Up to now we assumed that the decision maker follows a two step decision rule: If the precision of the experiment is at least as high as a certain threshold $\underline{\beta}$, then the decision maker follows the realised signal, otherwise he follows his prior belief. This means that if the sender provides the *necessary and sufficient* amount of information then the decision maker commits to follow the signal with probability-1. However, the decision maker could allow the sender to provide the *sufficient* or only the *necessary* information. In other words, up to some minimum level of information precision the sender can be considered eligible for acceptance (providing *necessary* information) and after a higher level she get accepted (providing *sufficient* information). As we showed before the decision maker who adopts a standard binary decision rule is not able to give enough incentives to the high type to signal her type. We show that one way to create some separation is to reward the high type with certainty in return for a more precise signal. The decision maker could commit to follow the experiment's signal with probability-1 if and only if he is certain that the sender is a good type. In the opposite case will accept the sender with probability $\hat{p} < 1$. This can be translated as a full commitment to follow the signal of a specific precision and to follow the signal of a lower precision with a positive probability but not with certainty. Another way to see this rule is that in one case you adopt totally the suggestion of the sender if the precision is high enough and you partially accept it for a lower precision. The high type will prefer to provide less noisy information in return of certainty while the bad type will provide the lower possible precision even if this comes with some uncertainty over the final decision Formally, this can be seen as a three-step approval rule where $\hat{p}$

is the probability of acceptance after a high signal:

$$\hat{p} = \begin{cases} 1 & \text{if} \quad \mu_g \geq \alpha \\ [\frac{1}{2}, 1) & \text{if} \quad \beta \leq \mu_g < \alpha \\ 0 & \text{if} \quad \mu_g < \beta \end{cases}$$

where $\frac{1}{2} \leq \beta < \alpha \leq 1$

The timeline is modified as follows:

1. Nature chooses ω
2. The receiver commits to the cutoff rules and the probability $\hat{p}$.
3. Sender receives privately exogenous signal θ about ω
4. Sender chooses an experiment π
5. The receiver forms interim belief about ω given the sender's experiment choice.
6. The receiver forms his posterior belief given the signal of the experiment.
7. The receiver takes and action and the payoffs are realised.

The only difference in the timing is the stage 2 where the decision maker has to commit to a combination of α, β and $\hat{p}$. Proposition 3.5 states that for some α, β and $\hat{p} \geq \frac{\beta(\alpha+\psi-2\alpha\psi)}{\alpha(\beta+\psi-2\beta\psi)}$ there is a separating equilibrium where the decision maker learns the types of the senders. We know $\pi_g = 1$ for both senders in equilibrium and we consider a low cutoff rule β and a high cutoff rule α. There is a combination of α, β and $\hat{p}$ such as the following incentive compatibility conditions hold simultaneously:

$$V_2(\alpha) \geq V_2(\beta) \Rightarrow m_2 + (1 - m_2)(1 - \pi_b^\alpha) \geq \hat{p}(m_2 + (1 - m_2)(1 - \pi_b^\beta)) \qquad (IC_H)$$

$$V_1(\alpha) \leq V_1(\beta) \Rightarrow m_1 + (1 - m_1)(1 - \pi_b^\alpha) \leq \hat{p}(m_1 + (1 - m_1)(1 - \pi_b^\beta)) \qquad (IC_L)$$

We denote by $\Pi_h = (\pi_g^h, \pi_b^h)$ the experiment chosen by the high type while by $\Pi_l = (\pi_g^l, \pi_b^l)$ the low type's choice.

Proposition 3.5 *There is a triple* $(\alpha, \beta, \hat{p})$ *satisfying* $\hat{p}(\alpha, \beta, \psi) \geq \frac{\beta(\alpha+\psi-2\alpha\psi)}{\alpha(\beta+\psi-2\beta\psi)}$ *that supports a separating equilibrium where* $\Pi_h = \left(1, \frac{(\mu_0-\alpha)}{\alpha(\mu_0-1)}\right)$ *and* $\Pi_l = \left(1, \frac{(\mu_0-\beta)}{\beta(\mu_0-1)}\right)$ *for all* μ_0.

Proof. See Appendix C ■

Example 1. Assume that the prior probability that a sender is of a high type is $\mu_0 = 0.3$. The decision maker has to adopt and announce the following strategy. Following a high signal, If the sender is willing to provide a minimum necessary level of information, ($\beta = 0.5$) will be eligible for approval with probability $\hat{p} = 0.6$. Instead if she is willing to provide more precise information ($\alpha = 0.8$) will be admitted with probability-1. The precision of private information is $\psi = 0.8$. In this case, given the prior probability $\mu_0 = 0.3$, the good type knows with probability $\mu_2 = 0.63$ that the state is high while the bad type with probability $\mu_1 = 0.01$. The senders have to choose between two level of precisions: $\Pi_L = (1, 0.57)$ and admission with probability $p = 0.6$ or $\Pi_H = (1, 0.89)$ with $p = 1$. Low type: $V_1(\beta) = 0.29 > V_1(\alpha) = 0.19$. High type: $V_2(\beta) = 0.47 < V_2(\alpha) = 0.67$. Therefore the high type prefers the high precision while the low type the opposite.

Figure 3.2 visualises the above result considering Example 1. The graph a. on the left represents for specific parameter values the expected utility of the high type for choosing the high precision/ high cutoff (grey curve) or low precision/ low cutoff (red curve). The graph b. on the right represents the corresponding expected utility of the low type. In this case the the high type prefers the high precision for every μ while the low type the opposite. It is easy to show that if the decision maker increases the probability of acceptance with low precision $\hat{p}$ both senders will prefer to choose the low precision. Same holds if the high cutoff α increases after a point and the opposite if the low cutoff decreases enough keeping the rest constant.

As Proposition 3.5 states the main condition that has to be satisfied in order to have separation is $\hat{p}(\alpha, \beta, \psi) \geq \frac{\beta(\alpha+\psi-2\alpha\psi)}{\alpha(\beta+\psi-2\beta\psi)}$. We can consider the following comparative statics:

$$\frac{\partial \hat{p}(\alpha, \beta, \psi)}{\partial \beta} = \frac{\psi(\alpha + \psi - 2\alpha\psi)}{\alpha(\beta + \psi - 2\beta\psi)^2} > 0 \quad \forall\ \beta, \alpha, \psi \in [0.5, 1]$$

$$\frac{\partial \hat{p}(\alpha, \beta, \psi)}{\partial \alpha} = \frac{\beta\psi}{\alpha^2(\beta(2\psi - 1) - \psi)} < 0 \quad \forall\ \beta, \alpha, \psi \in [0.5, 1]$$

Intuitively, the probability of acceptance $\hat{p}$ that comes with the low cutoff rule can decrease but still supports a separating equilibrium as long as the required precision decreases as well, holding fixed the hight cutoff and the precision ψ. The low type accepts the uncertainty if the required precision is low enough. Instead, the probability of acceptance can decrease, holding fixed the rest, if the high cutoff increases. The low type prefers the uncertainty over certainty if the high cutoff is high enough.

The high type prefers he lowest possible α which comes with the lowest possible β and $\hat{p}$. Consider the following example.

Example 2. Assume that the prior belief that a sender is high type is $\mu_0 = 0.3$. The decision maker has to adopt and announce the following strategy. If the sender is willing to provide a minimum level of information, ($\beta = 0.5$) will be eligible for approval with probability $\hat{p} = 0.5$ (lower than the previous result). Instead if she is willing to provide more precise information ($\alpha = 0.7$, which comes from Proposition 3.5 and $\hat{p}(\alpha, \beta, \psi) > \frac{\beta(\alpha+\psi-2\alpha\psi)}{\alpha(\beta+\psi-2\beta\psi)}$) will be admitted with probability-1. The precision of private information is $\psi = 0.8$. In this case, given the prior probability $\mu_0 = 0.3$, the good type knows with probability $\mu_2 = 0.63$ that the state is high while the bad type with probability $\mu_1 = 0.01$. The senders have to choose between two level precisions: $\Pi_L = (1, 0.57)$ and admission with probability $\hat{p} = 0.5$ or $\Pi_H = (1, 0.84)$ with $\hat{p} = 1$. Low type: $V_1(\beta) = 0.24 > V_1(\alpha) = 0.239$. High type: $V_2(\beta) = 0.39 < V_2(\alpha) = 0.69$. Similarly to the previous example the high type prefers the high precision while the low type the opposite. However the utility of the high type has been improved.

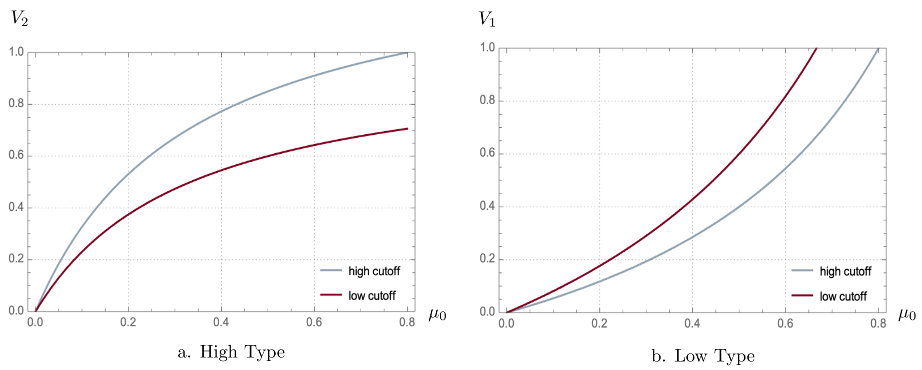


Figure 3.2 | **Ex-ante expected utility of senders with double cutoff rule.** Horizontal axis: prior beliefs μ_0; Vertical axis: expected utility of senders V_1, V_2. Graph a. represents the expected payoff of the high type while graph b. the expected payoff of the low type. The grey curves correspond to the expected utility of both types if they choose the high cutoff α. The dark red curves correspond to low cutoff rule. We have considers the parameter values and the values of β, α and p of Example 1: low cutoff $\beta = 0.5$ corresponding probability of acceptance $\hat{p} = 0.6$, high cutoff $\alpha = 0.8$ and precision of private information $\psi = 0.8$.The high type prefers the high cutoff while the low type the low cutoff.

Proposition 3.6 *The equilibrium preferred by the high type is supported by the following strategy:*

$$\hat{p} = \begin{cases} 1 & if \;\; \mu_g \geq \frac{c}{2c-0.5} \\ 0.5 & if \;\; 0.5 \leq \mu_g < \frac{c}{2c-0.5} \\ 0 & if \;\; \mu_g < 0.5 \end{cases}$$

Proof. See Appendix C ■

The optimal strategy that leads to the most preferred equilibrium of the high type is the one that given that there is separation he is as much reluctant as he can to accept the low type while as less reluctant as he can for the high type. The lowest possible $\beta = 0.5$ and $\hat{p} = 0.5$ with the lowest possible α which is equal to $a = \frac{c}{2c-0.5}$. That way both decision maker and high type better off. The decision maker also better off by accepting the low type with the lowest possible probability. Similarly to the previous session the decision maker benefits from the higher precision of private information. Considering

$$\frac{\partial \hat{p}(\alpha,\beta,\psi)}{\partial \beta} = \frac{\beta(\beta-\alpha)}{\alpha(\beta+\psi-2\beta\psi)^2} < 0 \quad \forall\ \beta,\alpha,\psi \in [0.5,1]$$

we see that as ψ increases the low type avoids the high cutoff rule and she is willing to be accepted with even lower probability.

3.6 Conclusion and Discussion

In this paper, we consider a model à la Kamenica and Gentzkow (2011) where a sender strategically designs an experiment aiming to influence the decision of a receiver. We assume that the sender and the receiver have the same prior belief about the state but the sender before designing her experiment receives a private and imperfect signal about it.

First, we show that a more informative equilibrium can be achieved by introducing some constraints on the experiments choice, in the sense that the probability of observing the correct signal is the same for both states. In this case, we can obtain a separating equilibrium for a specific range of prior beliefs. However, this separation is different from the one of Hedlund (2017) and it exists only for a range of prior belies. The separation is not coming from the high type who wants to signal his type to the receiver but from the non-monotonic preferences imposed by the symmetric experiments.

Second we describe a strategy of a double cutoff rule by the receiver. We allow the decision maker to ask for both *sufficient* and *necessary* information. This means that, up to some minimum level of information precision you can be considered eligible for acceptance but above a higher level you get acceptance with certainty. The decision maker who adopts a standard binary decision rule is not able to give the incentive to the high type to signal her type. To this end, one way to create some separation is to reward the high type with certainty in return for a more precise signal. The decision maker commits to follow the experiment's signal with probability-1 if and only if he is certain that the sender is a good type. In the opposite case he will accept with probability less than one. This can be translated as a commitment to follow the signal for a specific signal precision and to follow the signal for a lower precision with a positive probability but not with certainty. Another way to see this rule is that in one case you adopt totally the suggestion of the sender if the precision is high enough and you partially accept it for a lower precision.

We plan to delve into the analysis of this asymmetric information problem between the persuaders and the receivers. We observe that in reality the decision and policy makers adopt strategies that allow them to infer at least to some extend the private information of the persuaders or limit their persuasion power. Our initial aim is to understand and analyse the gain from extracting sender's private information, the cost that the decision makers face in opposite situation and how they manage to commit to these strategies. For instance, the receiver can commit to the double cutoff rule by bearing a cost in case of approving with certainty and no cost in case of approving with the nominal probability. In other words, we can use a slightly modified version of the resistance strategy proposed by Tsakas, Tsakas and Xefteris (2021). In this sense the double cutoff rule could be seen as receiver's resistance either before the experiment's signal (high threshold with certainty) or after the experiment's signal (low threshold with some uncertainty).Then a natural step would be to investigate whether there are other strategies or policies that would improve even more the welfare of the receiver.

Appendix A

A.1 Proofs

The following will be used throughout the Appendix:

Posterior beliefs of experts.

Assume $m_1 = [\overline{a}, \underline{a}]$ is the message sent by expert 1. We consider that θ is uniformly distributed on Θ and the signals ω_i are also uniformly distributed around θ.

The first expert's posterior distribution of θ after receiving ω_1 is denoted by $g(\theta|\omega_1)$. Analogously, the second expert's posterior distribution of θ after observing m_1 and receiving $\omega_2 \in m_1$ is denoted by $g(\theta|\omega_2, \overline{a}, \underline{a})$. Therefore, we can write for the second expert that his posterior distribution of θ is uniform on the interval $[max\{\underline{a} + \delta_2, \omega_2 - \delta_2\}, min\{\overline{a} - \delta_2, \omega_2 + \delta_2\}]$. If $\omega_2 \notin [\overline{a}, \underline{a}]$ the expert's 2 posterior distribution of θ after receiving ω_2 is denoted by $g(\theta|\omega_2)$. Again, we can write that posterior distribution of θ is uniform on $[\omega_2 - \delta_2, \omega_2 + \delta_2]$.

From the point of view of the first expert, after sending truthfully m_1, he knows with certainty that $\omega_2 \in m_1$. In that case the posterior of expert 2 is $g(\theta|\omega_2, \overline{a}, \underline{a}]$ where $\omega_1 = \frac{\overline{a}+\underline{a}}{2}$.

On the other hand, If expert 1 deviates by sending $m_1' = [\underline{a}-\epsilon, \overline{a}-\epsilon]$, where $\omega_1' = \frac{\overline{a}-\epsilon+\underline{a}-\epsilon}{2}$, such as $\Omega_2(\omega_1) \cap \Omega_2(\omega_1') \neq \emptyset$, with probability $Pr(\omega_2 \notin m_1'|\omega_1) = g(\omega_2|\omega_1)\epsilon$,where $g(\omega_2|\omega_1)$ is the conditional pdf of ω_2 given ω_1, the expert's 2 posterior distribution of θ is uniform on $[\omega_2 - \delta_2, \omega_2 - \delta_2]$ where $\omega_2 \in [(\overline{a} - \epsilon, \overline{a}]$. The second's expert posterior is $Pr(\omega_2 \notin m_1'|\omega_1) = g(\theta|\omega_2, \overline{a} - \epsilon, \underline{a} - \epsilon]$ with probability $1 - g(\omega_2|\omega_1)\epsilon$. If expert 1 sends m'$_1 = [\underline{a'}, \overline{a'}]$ such as $\Omega_2(\omega_1) \cap \Omega_2(\omega_1') = \emptyset$ then the posterior of expert 2 is $\theta \in [\omega_2 - \delta_2, \omega_2 - \delta_2]$ where $\omega_2 \in [(\overline{\omega_2}(\omega_1) - \epsilon, \overline{\omega_2}(\omega_1)]$.

Posterior beliefs of decision maker and optimal action: If $m_1 = m_2$ then the posterior of decision maker coincides with the posterior of expert 1,$g(\theta|\underline{a}, \overline{a})$:
The optimal action is then: $y^*(m_1) = \arg\max_y \int_{\underline{a}}^{\overline{a}} U_0(y, \theta)g(\theta|\underline{a}, \overline{a})d\theta$.
If $m_1 \neq m_2 \equiv [\overline{a}, \underline{a}]$ and m_2 is not self-serving, then the posterior distribution of θ is $g(\theta|\underline{a'}, \overline{a'})$
The optimal action is then: $y^*(m_1) = \arg\max_y \int_{\underline{a'}}^{\overline{a'}} U_0(y, \theta)g(\theta|\underline{a'}, \overline{a'})d\theta$

Proof Proposition 1. We assume Expert 1 receives signal $\omega_1 = \theta$ and Expert 2 receives ω_2 such as $\omega_2 = [\theta - \delta, \theta + \delta]$ We have to prove that the $(\hat{\mu_1}, \hat{\mu_2}, \hat{y}, \hat{\mu})$ defined as follows is a PBE.

- $\mu_1(\theta) = \theta$
- $\mu_2(\omega_2, m_1) = m_1$ if $|m_1 - \omega_1| \geq \delta$
- $\mu_2(\omega_2, m_1) = \omega_2$ if $|m_1 - \omega_1| > \delta$ & m_1 is self-serving given
- If $m_1 = m_2$ then $\hat{y}(m_1, m_2) = m_1$
- If $m_1 \neq m_2$ and m_2 given m_1 is perceived as self-serving then $\hat{y}(m_1) = m_1$
- If $m_1 \neq m_2$ and m_2 given m_1 is not perceived as self-serving then $\hat{y}(m_2) = m_2$

We consider expert 2's decision given his belief μ_2. If $|s_1 - m_1| \leq \delta$, then this scenario is identical to the noiseless case. Hence, $m_2 = m_1$. If $|s_1 - m_1| > \delta$ If $|s_1 - m_1| > \epsilon$ then expert 2 believes that expert 1 has deviated if m_1 is self-serving. In this case, $m_2 = m_1 + 2b_2$ is optimal. Given the above beliefs, it is always optimal for Expert 1 to send $m_1 = \theta$ since with probability equal to 1 for serious deviation and for small deviations equal to $g(\omega_2|\omega_1)\epsilon$ will be detected. ■

Proof Lemma 1. Suppose by way of contradiction that we could find a partition equilibrium in which message $m_1 = [\underline{a}, \overline{a}]$ with $\overline{a} - \underline{a} < c$ was sent.
Expert 1 observes ω_1, therefore we know that $\theta \in \Theta(\omega_1) \equiv [\underline{\theta_1}(\omega_1), \overline{\theta_1}(\omega_1)]$. Expert 2 observes $\omega_2 = \omega_1 + e$ such as therefore $\theta \in \Theta(\omega_2) \equiv [\underline{\theta_2}(\omega_2), \overline{\theta_2}(\omega_2)]$ but $\omega_2 \in \Omega_2(\omega_1) \equiv [\underline{\omega}_2(\omega_1), \overline{\omega}_2(\omega_1)]$ where $\Theta(\omega_1) \subset \Omega_2(\omega_1)$. Since $\overline{a} - \underline{a} < c$ there is no zero probability $\omega_2 \notin m_1$ since $\Theta(\omega_2) \not\subset \Omega_2(\omega_1)$. This means that miscoordination will happen with positive probability even if expert's 1 message is truthful. For this reason there is not a partition equilibrium with $\overline{a} - \underline{a} < c$ supported by self-serving beliefs. ■

Proof of Proposition 2. The proposition follows as a corollary of Lemma 1 and the fact that $\Theta \equiv [-\Lambda, \Lambda]$ is bounded. ■

Proof of Proposition 3.

Without loss of generality we assume $b_1 < 0 < b_2$.

First we prove the following lemma:

Lemma A.1 *Any message that could give the second expert higher utility than the first expert's message is self-serving according to self-serving criterion under uncertainty.*

Proof. This result is necessary in order to assure that it is enough to define the extended lying zone considering only the possible signals that an expert will receive towards his bliss point relative to the first expert's message. We have to prove that the only self serving message that Expert 2 can send without being detected by the decision maker is $m_1 = m_2$. This will happen if he receives a signal ω_1^-. Let us assume that Expert 1 sends m_1 with mid point z and Expert 2 observes $\omega_2 = z - \frac{\delta_2}{2}$. The extended lying zone is $(z, z + \delta_1 + 2b_2]$. The actual lying zone of Expert 2 is $(z - \frac{\delta_2}{2}, z - \frac{\delta_2}{2} + 2b_2]$ where $z \in (z - \frac{\delta_2}{2}, z - \frac{\delta_2}{2} + 2b_2]$. In this case the optimal strategy of the second expert is to coordinate with the first expert by sending $m_2 = m_1$ which will lead to $y = z$. In this case the message m_2 is self serving given the signal ω_2 but any other message greater than z would be ignored since it would belong to extended lying zone. ■

Now we proceed with the proof of Proposition 3.

A. Strategy profiles

Stage 1: E_1 sends $m_1 = m^i \in \mathcal{M}_1$, where $m^i \equiv [a_{i-1}, a_i]$. He chooses the equilibrium partition based on ω_1 from an infinite number of possible partitions such that:

1. State space, $\Theta \equiv [-\Lambda, \Lambda]$ is partitioned in N intervals such that $-\Lambda = a_0 < a_1 < a_2 < ... < a_{j-1} < a_j < ..... < a_N = \Lambda$.
2. The space partition $\Lambda^*(\omega_1)$ starts from the interval where ω_1 belongs to. Say that ω_1 belongs to the interval $[a_{i-1}, a_i]$ and $\frac{a_i + a_{i-1}}{2} = \omega_1$. This interval has length c and at least $N - 2$ intervals have the same length.[1]

[1] If $\frac{2\Lambda}{c}$ is not an integer number and $-\Lambda < a_{i-1} < a_i < \Lambda$ then at least one of the extreme (left or right) intervals has length different than c. If $a_{i-1} = -\Lambda$ then the extreme right interval has length smaller or bigger than c. If $a_i = \Lambda$ then the extreme left interval has length smaller or bigger than c.

3. The partition is constructed considering the possible mistakes of expert 2 given the possible mistakes of expert 1. This means that under $\Lambda^*(m_1)$ partition, ω_2 belongs to m^i with probability-1.

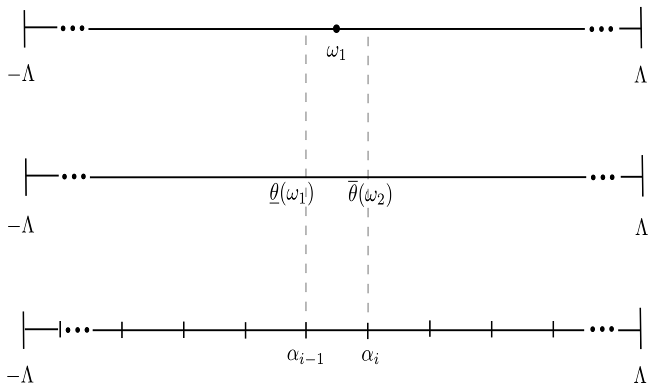


Figure A.1 | **Space Partition** $\Lambda^*(\omega_1)$**.**

Stage 2: E_2 has to send $m_2 \in \Lambda^*(m_1)$ and we distinguish two cases:

a. On the equilibrium path: E_1, did not deviate in stage 1; for E_2, ω_2 lies in the same interval as ω_1 with probability-1, E_2 sends $m_2 \equiv [a_{i-1}, a_i]$

b. Off the equilibrium path: E_2 following deviation in stage 1, let's say $m_1' = [a_{i-1} - \beta c, a_i - \beta c]$, where $\omega_1' = \frac{a_{i-1}-\beta c+a_i-\beta c}{2} = \omega_1 - \beta c$, faces two cases:

   1. Small deviation by E_1: a small deviation means that $\Omega_2(\omega_1) \cap \Omega_2(\omega_1') \neq \emptyset$. In this case ω_2 does not lie in m_1' with a positive probability equal to $g(\omega_2|\omega_1)\beta c$.

   2. Serious deviation by E_1: a serious deviation means that $\Omega_2(\omega_1) \cap \Omega_2(\omega_1') = \emptyset$. In this case ω_2 does not lie in m_1' with probability 1.

Stage 3: The decision maker aggregates the experts' messages and take an optimal action $y^*(m_1, m_2)$: [2]

a. If $m_1 = m_2$ then $y^*(m_1, m_2) = \frac{a_i+a_{i-1}}{2}$.

b. If $m_1 \neq m_2$ and m_2 given m_1 is perceived as self-serving then $y^*(m_1) = \frac{a_i+a_{i-1}}{2}$

c. If $m_1 \neq m_2$ and m_2 given m_1 is not perceived as self-serving then $y^*(m_2) = \frac{a_j+a_{j-1}}{2}$

[2] The $y^*(m_1, m_2)$ is the optimal action and it depends on the prior distribution of θ and the signal distributions. Assuming that θ is uniformly distributed on state space and the signals follow a uniform distribution around θ then given an equilibrium message m^i the optimal action would be the midpoint of $[a_{i-1}, a_i]$.

B. Optimality

I. Optimality for the Decision Maker on the equilibrium path

On the equilibrium path, the Policy Maker has learned whether ω_1 and ω_2 lies in $[a_{i-1}, a_i]$. It is optimal for the policy maker to to choose $y^*(m_1, m_2) \in m_1 = [a_{i-1}, a_i]$

II. Optimality for the Decision Maker off the equilibrium path

Off the equilibrium path where $m_1 = [a_{i-1}, a_i] \neq m_2 = [a_{j-1}, a_j]$:

- If m_2 is self-serving given $\omega_2(m_1)$ then $y^*(m_1) \in m_1 = [a_{i-1}, a_i)]$. This means that,

$$U_2(y^* = \frac{\underline{a}' + \overline{a}'}{2}, b_2, \overline{a} - \delta_2) > U_2(y^* = \frac{\underline{a} + \overline{a}}{2}, b_2, \overline{a} - \delta_2)$$

- If m_2 is not self-serving given $\omega_2(m_1)$ then $y^*(m_2) \in m_2 = [a_{j-1}, a_j]$. This means that,

$$U_2(y^* = \frac{\underline{a}' + \overline{a}'}{2}, b_2, \overline{a} - \delta_2) \leq U_2(y^* = \frac{\underline{a} + \overline{a}}{2}, b_2, \overline{a} - \delta_2)$$

III. Optimality for Expert 2 on the equilibrium path

On the equilibrium path ω_2 lies with probability-1 in $m_1 = [a_{i-1}, a_i]$. Expert 2 has no profitable deviation such that she sends a credible message $m_2 \neq m_1$. There is not m_2 that gives higher utility to Expert 2 and perceived as credible message by the policy maker. Experts' payoff depend on the induced action and the true state, neither their message nor their signal. This means that after receiving a signal ω_2 that belongs to the same interval with ω_1 then she wants to maximise his expected utility that depends on the the final action, the true state and his bias. His posterior distribution of θ is uniform on $[max\{\underline{a} + \delta_2, \omega_2 - \delta_2\}, min\{\overline{a} - \delta_2, \omega_2 + \delta_2\}]$. Given the self-serving criterion under uncertainty, any message that would give him higher utility will be disregarded by the decision maker. There he sends $m_2 = m_1$.

IV. Optimality for Expert 2 off the equilibrium path

Off the equilibrium path ,$m_1' \neq m_1(\omega_1)$ the signal ω_2 does not lie in $m_1' = [a_{i-1}, a_i]$ with positive probability. If ω_2 does not lie in $[a_{i-1}, a_i]$, expert 2 can deviate and gain higher utility. There exist given m_1', $m_2' \equiv [a_{i-1} + \beta c, a_i + \beta c]$ [3] that is credible and influential.[4].

[3]We have assumed that $b_2 > 0$ therefore Expert 2 has incentives to send a message on the right of his signal
[4]By influential we mean that it affects the induced action

His posterior distribution of θ is uniform on $[\omega_2 - \delta_2, \omega_2 + \delta_2]$. After a deviation by expert 1, $\omega_2 > a_i - \delta_2$ therefore $\omega_2 + b_2 > a_i - \delta_2 + b_2$. The expert can send a message that would induce an action close to $\omega_2 + b_2$ without being considered as self-serving. Therefore the sender sends m_2' such as $U_2(y(m_2'), \theta, b_2) > U_2(y(m_1'), \theta, b_2)$.

V. Optimality for Expert 1 on the equilibrium path

It is not profitable for Expert 1 to *seriously* deviate and send m_1' that leads to a partition $\Lambda'(m_1')$ where $\Omega_2(\omega_1) \cap \Omega_2(\omega_1') = \emptyset$. Consider $m_1 \equiv [a_{i-1} - \gamma c, a_i - \gamma c]$ with $\gamma > 1$. As a respond to this message, E_2 will send with probability-1 $m_2' \equiv [a_{i-1} - \gamma c + \beta c, a_i - \gamma c + \beta c]$ which will be adapted by the decision maker.

Expert 1 could *slightly* deviate (we consider as small a deviation smaller than c) and send m_1' that leads to a partition $\Lambda'(m_1')$ where $\Omega_1(\omega_1) \cap \Omega_2(\omega_1') \neq \emptyset$. The announcement of $\Lambda'(m_1')$ would make expert 1 uncertain about the message of expert 2. Specifically we have two scenarios: a) the induced action will be slightly closer to his bliss point or b) at least an interval away from the opposite direction. It is easy to see that the following inequality always holds:

$$U_1(y = \omega_1, b_1, \omega_1) > Pr(\omega_2 \in m_1'|\omega_1)U_1(y = \omega_1', b_1, \omega_1) + Pr(\omega_2 \notin m_1'|\omega_1)U_1(y = \omega_2', b_1, \omega_1)$$

Therefore, it is not optimal for expert 1 to deviate.

Extra round of communication: Rebuttal

For $\overline{\omega}_1(\theta) \in [2\Lambda - 2b_2, \Lambda]$ the decision maker cannot credibly apply the self-serving criterion under uncertainty because there is no "space" for the expert 2 to punish a possible deviation by the first expert because of the boundness of the space.

Therefore, we have to modify the game for $\overline{\omega}_2(\omega_1) \in [2\Lambda - 2b_2, \Lambda]$ where the second experts holds extreme preferences. Expert 1 should "babbles" at the first stage and let the second expert to start the game first. The procedure is the same as before, but the order of experts changes.

Stage 1: E_1 "passes".

Stage 2: E_2 sends $m_2 = m^i \in \mathcal{M}_2$, where $m^i =\equiv [a_{i-1}, a_i]$. He chooses the equilibrium partition based on ω_2 from an infinite number of possible partitions like before such as under $\Lambda^*(m_2)$ partition, ω_1 belongs to m^i with probability-1.

Stage 3: E_1 has to send $m_1 \in \Lambda^*(m_2)$.

Stage 4: The decision maker aggregates the experts' messages and take an optimal action $y^*(m_1, m_2)$

a. If $m_2 = m_1$ then $y^*(m_1, m_2) = \frac{a_i + a_{i-1}}{2}$.

b. If $m_2 \neq m_1$ and m_1 given m_2 is perceived as self-serving then $y^*(m_2) = \frac{a_i + a_{i-1}}{2}$

c. If $m_2 \neq m_1$ and m_1 given m_2 is not perceived as self-serving then $y^*(m_1) = \frac{a_j + a_{j-1}}{2}$

The optimality of strategies are the same as before and this finishes the proof. ■

Proof of Proposition 4. This lemma follows immediately from Lemma 1 and Proposition 2. According to Lemma 2 the length of intervals cannot be smaller than c. But also partition into larger than c intervals would allow expert 1 to deviating but still send a credible message. ■

Proof of Remark 3. Let us consider the following deviation $m_1 = \omega_1^-$ where ω_1 follows a continuous distribution around θ.[5]. Since $m_1 = \omega_1^-$ then the interval where m_1 belongs to is not $[\underline{\omega_2}(\omega_1), \overline{\omega_2}(\omega_1)]$ but $[\underline{\omega_2}(\omega_1)^-, \overline{\omega_2}(\omega_1)^-]$. There is a probability ω_2 belongs to $[\overline{\omega_2}(\omega_1)^-, \overline{\omega_2}(\omega_1)]$ which is $P([\overline{\omega_2}(\omega_1)^-, \overline{\omega_2}(\omega_1)]) \cong f_\omega(\cdot)(\overline{\omega_2}(\omega_1) - \overline{\omega_2}(\omega_1)^-)$ where $f_\omega(\cdot)$ is the PDF of ω_2. This probability increases as variance of ω_2 decreases. ■

A.2 Numerical Examples

In this section we provide numerical examples of equilibria considering different scenarios regarding the expertise and the bias of the experts. For each case we show what happens if the Expert 1 a. sends a truthful message, b. proceeds to a small deviation and c. proceeds to a serious deviation.

For the rest of the section we suppose the following. The state θ follows a uniform distribution on $\Theta \equiv [-50, 50]$ and both experts observe ω_i uniformly distributed on $U[\theta - \delta_i, \theta + \delta_i]$ whenever $[\theta - \delta_i, \theta + \delta_i] \subseteq \Theta$. Given the information structure we know that : $\Theta(\omega_1) \equiv [\omega_1 - \delta_1, \omega_1 + \delta_1]$, $\Theta(\omega_2) \equiv [\omega_2 - \delta_2, \omega_2 + \delta_2]$, $\Omega_1(\omega_2) \equiv [\omega_1 - \delta_1 - \delta_2, \omega_1 + \delta_1 + \delta_2]$, $\Omega_2(\omega_1) \equiv [\omega_2 - \delta_1 - \delta_2, \omega_2 + \delta_1 + \delta_2]$, posterior of Expert 2 if $\omega_2 \in \Omega_2(\omega_1)$ is $\Theta(\omega_2, m_1) \equiv [\omega_1 - \delta_1, \omega_1 + \delta_1]$ and posterior of Expert 2 if $\omega_2 \notin \Omega_2(\omega_1)$ is $\Theta(\omega_2) \equiv [\omega_2 - \delta_2, \omega_2 + \delta_2]$.

[5] Let's denote a small deviation as ω_1^- when m_1 does not coincide to ω_1 but slightly left to it.

Example 1. Symmetric biases - Same Expertise

Let us assume that the experts have the same expertise and are equally biased but in opposite directions. Let us consider the case of $\delta_1 = \delta_2 = 0.2$, $b_1 = -1$, $b_2 = 1$. We start this example by assuming that Expert 1 observes $\omega_1 = 3$. In this case the following hold:

i. $\Theta(\omega_1) = [3 - 0.1, 3 + 0.1] = [2.8, 3.2]$

ii. $\Theta(\omega_2) = [\omega_2 - 0.2, \omega_2 + 0.2]$,

iii. $\Omega_1(\omega_2) = [3 - 0.2 - 0.2, 3 + 0.2 + 0.2] = [2.6, 3.4]$,

iv. $\Omega_2(\omega_1) = [\omega_2 - 0.2 - 0.2, \omega_2 + 0.2 + 0.2] = [\omega_2 - 0.4, \omega_2 + 0.4]$

a. Truthful message.

Suppose the Expert 1 sends *truthfully* $m_1 = [2.8, 3.2]$. Expert's 2:

a. signal ω_2 falls in m_1 with probability-1

b. posterior is $\Theta(\omega_2, m_1) = [2.8, 3.2]$,

c. *maximum bliss point* is $\overline{x_2} = 4.2$ and

d. *extended lying zone* is $(3, 5.2]$.

The decision maker will consider as self serving the messages coming from the Expert 2 that belong to the partition $\Lambda(3)$ (Figure A.2) and give him an expected utility greater than $U_2 = -(3 - 4.2)^2 = -1.44$. [6] Any message up to interval $[4.8, 5.2]$ sent by expert 2 will be considered by the decision maker as self-serving (Figure A.3):

$U_2 = -(3.4 - 4.2)^2 = -0.64 > -1.44$,

$U_2 = -(3.8 - 4.2)^2 = -0.16 > -1.44$,

$U_2 = -(4.2 - 4.2)^2 = -0 > -1.44$,

$U_2 = -(4.6 - 4.2)^2 = -0.16 > -1.44$,

$U_2 = -(5 - 4.2)^2 = -0.64 > -1.44$,

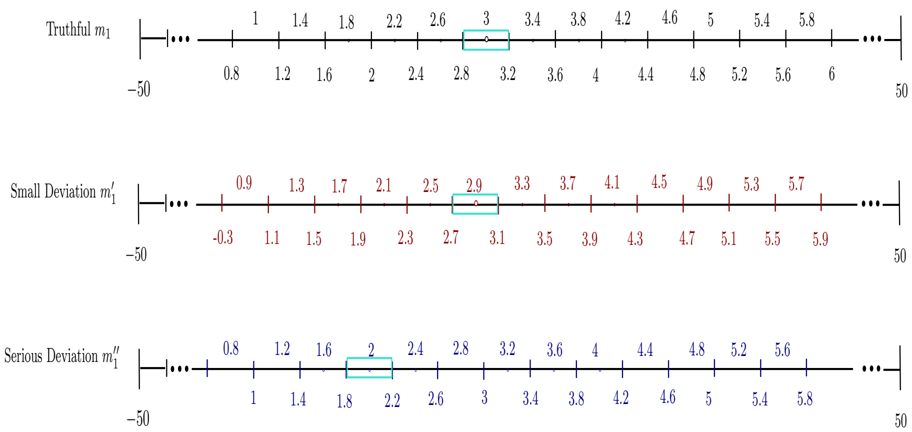
$U_2 = -(5.4 - 4.2)^2 = -1.44 = -1.44$.

$U_2 = -(5.8 - 4.2)^2 = -2.56 < -1.44$.

Therefore for any $\omega_2 \in [2.8, 3.2]$, Expert 2 cannot proceed to any profitable and successful deviation .

[6]Recall that on equilibrium, if $m_1 = [\underline{a}, \overline{a}]$ then $\frac{\underline{a}+\overline{a}}{2} = \omega_1$)

Figure A.2 | **Equilibrium Partition for $\omega_1 = 3$ and $\delta_1 = \delta_2 = 0.2$.**



b. Small deviation

We maintain the assumption that $\omega_1 = 3$. However, we assume that Expert 1 slightly deviates by sending $m_1' = [2.7, 3.1]$.

With positive probability ω_2 falls in $m_1' = [2.7, 3.1]$. In this case:

a. the posterior of Expert 2 is $\Theta(\omega_2, m_1) = [2.7, 3.1]$

b. his *lying zone* is $(2.9, 5.1]$.

The decision maker will consider as self serving the messages that belongs to the partition $\Lambda(2.9)$ and that would give expected utility to the Expert 2 greater than $U_2 = -(2.9 - 4.1)^2 = -1.44$. As we can see any message up to interval $[4.7, 5.1]$ sent by Expert 2 will be considered by the decision maker as self-serving:

$U_2 = -(3.3 - 4.1)^2 = -0.64 > -1.44$,

$U_2 = -(3.7 - 4.1)^2 = -0.16 > -1.44$,

$U_2 = -(4.1 - 4.1)^2 = -0 > -1.44$,

$U_2 = -(4.5 - 4.1)^2 = -0.16 > -1.44$,

$U_2 = -(4.9 - 4.1)^2 = -0.64 > -1.44$,

$U_2 = -(5.3 - 4.1)^2 = -2.56 < -1.44$.

Following the reasoning of the case where Expert 1 sends truthfully his message, Expert 2 sends $m_2 = m_1$. With positive probability Expert 1 manages to deviate without being detected. However, the Expert 2 observes $\omega_2 \notin m_1' = [2.7, 3.1]$ with positive probability $p = 0.375$. In this case, the posterior of Expert 2: $\Theta(\omega_2) = [\omega_2 - 0.2, \omega_2 + 0.2]$ and Expert's 2 *lying zone* is $(\omega_2, \omega_2 + 2 \cdot 1]$. Let us assume that Expert 2 observes $\omega_2 = 3.2$. In this case the Expert's 2 *lying zone* is $(3.2, 5.2]$. More analytically:

$U_2 = -(2.9 - 4.2)^2 = -1.69$

$U_2 = -(3.3 - 4.2)^2 = -0.81 > -1.69,$

$U_2 = -(3.7 - 4.2)^2 = -0.25 > -1.69,$

$U_2 = -(4.1 - 4.2)^2 = -0.01 > -1.69,$

$U_2 = -(4.5 - 4.2)^2 = -0.09 > -1.69,$

$U_2 = -(4.9 - 4.2)^2 = -0.49 > -1.69,$

$U_2 = -(5.3 - 4.2)^2 = -1.21 > -1.69,$

$U_2 = -(5.7 - 4.2)^2 = -2.25 < -1.69.$

Under the assumption that m_1 is truthful the message $m_2 = [5.1, 5.5]$ wouldn't be considered as self-serving. However, we see that $U_2 = -(5.3 - 4.2)^2 = -1.21 > -1.69$. Therefore following Expert's 1 deviation, Expert 2 finds it optimal to send $m_2 = [5.1, 5.5]$ that will lead to $y^* = 5.3$.[7].See graph.[8]

c. Serious deviation

We maintain the assumption that $\omega_1 = 3$. However, we assume that Expert 1 seriously deviates by sending $m_1'' = [1.8, 2.2]$. With probability-1, $\omega_2 \notin m_1'' = [1.8, 2.2]$. Given m_2 the lying zone of Expert 2 is $[2.1, 4.1]$, therefore any message up to interval $[3.8, 4.2]$ will be considered as self-serving. Let as assume that Expert 2 observes, $\underline{\omega_2}(\omega_1) = 2.8$. Then:

$U_2 = -(2 - 3.8)^2 = -3.24,$

$U_2 = -(2.4 - 3.8)^2 = -1.96 > -3.24,$

$U_2 = -(2.8 - 3.8)^2$=-1>-3.24,

$U_2 = -(3.2 - 3.8)^2 = -0.36 > -3.24,$

$U_2 = -(3.6 - 3.8)^2 = -0.36 > -3.24,$

$U_2 = -(4 - 3.8)^2 = -0.04 > -3.24,$

$U_2 = -(4.4 - 3.8)^2 = -0.04 > -3.24,$

$U_2 = -(4.8 - 3.8)^2 = -0.36 > -3.24.$

The Expert 2 sends the optimal $m_2'' = [4.2, 4.6]$ that leads to $y^* = 4.4$. Therefore there is a

[7]Expected Payoffs of Expert 1: $Truthful > Deviation * NotDetected + Deviation * Detected$; $-(2-3)^2 > -0.75(2-2.9)^2 - 0.25(2-5.3)^2 \Rightarrow -1 > 1.255$

[8]Let us now assume that the Expert 2 observes a signal really close to the proposed right interval, for instance $\omega_2 = 3.12$. In this case the Expert's 2 *lying zone* is $(3.12, 5.12]$. More analytically, $U_2 = -(2.9 - 4.12)^2 = -1.48$, $U_2 = -(3.3 - 4.12)^2 = -0.67 > -1.48$, $U_2 = -(3.7 - 4.12)^2 = -0.17 > -1.48$, $U_2 = -(4.1 - 4.12)^2 = 0 > -1.48$, $U_2 = -(4.5 - 4.12)^2 = -0.14 > -1.48$, $U_2 = -(4.9 - 4.12)^2 = -0.61 > -1.48$, $U_2 = -(5.3 - 4.12)^2 = -1.39 > -1.48$, $U_2 = -(5.7 - 4.12)^2 = -2.49 < -1.48$. Again, under the assumption that m_1 is truthful the message $m_2 = [5.1, 5.5]$ wouldn't be considered as self-serving. However, we see that $U_2 = -(5.3 - 4.12)^2 = -1.39 > -1.48$. Therefore following Expert's 1 deviation, Expert 2 finds it optimal to send $m_2 = [5.1, 5.5]$ that will lead to $y^* = 5.3$.

profitable deviation for Expert 2 if he receives any signal $[2.8, 3.2]$. For this reason it's not optimal for Expert 1 to deviate.

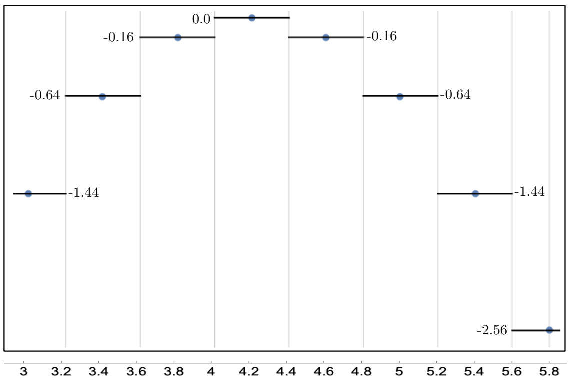


Figure A.3 | **Truthful message by Expert 1 - Example 1a .** This figure represents the level of utility that Expert 2 gains at each interval of partition $\Lambda(3)$ when $\omega_1 = 3$ (truthful message by E_1). According to self-serving criterion under uncertainty the extended lying zone for the Expert 2 with bias $b_2 = 1$ is $(3, 5.2)$. Indeed, any message up to interval $[4.8, 5.2]$ is considered as self-serving relatively to the interval $[2.8, 3.2]$. Therefore Expert 2 cannot proceed to any profitable and effective deviation.

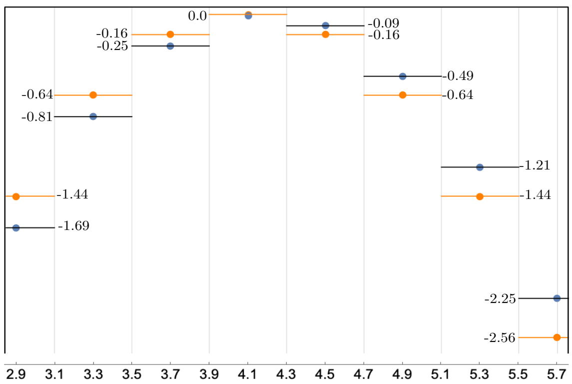


Figure A.4 | **Small deviation by Expert 1- Example 1b.** The orange lines represent the utility of the Expert 2 if $\omega_2 \in [2.7, 3.1]$. These are the utility levels that from the decision's maker point of view determine which messages are self-serving and which not. If both experts agree that the state belongs to interval $[2.8, 3.2]$ - which should be the case if the first expert does not deviate- then any message up to interval $[4.7, 5.1]$ are self serving while message $m_2 = [5.1, 5.5]$ is not. We consider the case that the Expert 1 has deviated and a miscoordination happens: the second expert receives a signal outside of $[2.7, 3.1]$, let's say $\omega_2 = 3.2$, then the utility levels are the blue lines. In this case the second expert can deviate successfully by sending $m_2 = [5.1, 5.5]$.

Example 2. Asymmetric biases-Same Expertise

For this example we assume the experts are of the same expertise but they differ with respect to their bias. Let us consider the case of $\delta_1 = 0.2$, $\delta_2 = 0.2$, $b_1 = -1$, $b_2 = 0.1$. We suppose again that $\omega_1 = 3$.

a. Truthful message

We start with the Expert 1 sends truthfully $m_1 = [2.8, 3.2]$. From the point of view of Expert 2:

a. Expert's 2 signal ω_2 falls in m_1 with probability-1,

b. Expert's 2 posterior belief is $\Theta(\omega_2, m_1) = [3 - 0.2, 3 + 0.2] = [2.8, 3.2]$ and

c. Expert's 2 Lying zone is $(3, 3.4]$.

From the point of view of the decision maker holds the same.The decision maker will consider as self serving the messages that belongs to the partition $\Lambda(3)$ and that would give expected utility to the expert 2 greater than $U_2 = (3 - 3.3)^2 = -0.09$. There is. no message sent by Expert 2 that would be considered by the decision maker as self-serving since: $U_2 = -(3.4 - 3.3)^2 = -0.01 > -0.09$

$U_2 = -(3.8 - 3.3)^2 = -0.25 < -0.09$

$U_2 = -(4.2 - 3.3)^2 = -0.81 < -0.09$

$U_2 = -(4.6 - 3.3)^2 = -1.69 < -0.09$

$U_2 = -(5 - 3.3)^2 = -2.89 < -0.09$

Therefore for any $\omega_2 \in [2.8, 3.2]$, there is not any profitable deviation by Expert 2.

b. Small deviation

We assume that Expert 1 slightly deviates by sending $m_1' = [2.7, 3.1]$.

If $\omega_2 \in m_1' = [2.7, 3.1]$ then the posterior of Expert 2 is $\Theta(\omega_2, m_1) = [2.9 - 0.2, 2.9 + 0.2] = [2.7, 3.1]$ and Expert's 2 *lying zone* is $(2.9, 3.3]$. The decision maker will consider as self serving the messages that belongs to the partition $\Lambda(2.9)$ and that would give expected utility to the expert 2 greater than $U_2 = -(2.9 - 3.2)^2 = -0.09$. Considering the lying zone with see that there is no any profitable deviation for Expert 1:

$U_2 = -(3.3 - 3.2)^2 = -0.01 > -0.09$

$U_2 = -(3.7 - 3.2)^2 = -0.25 < -0.09$

However, If the Expert 2 observes $\omega_2 \notin m_1' = [2.7, 3.1]$ then:

a. the posterior of Expert 2 $\Theta(\omega_2) = [\omega_2 - 0.2, \omega_2 + 0.2]$ and

b. Expert's 2 *lying zone* is $(\omega_2, \omega_2 + 2 \cdot 0.1]$.

Let us assume that Expert 2 observes $\omega_2 = 3.2$. The lying zone now is $(3.2, 3.4]$ and any message that would give higher than $U_2 = -(2.9 - 3.3)^2 = -0.16$ will be self-serving. Expert

2 by sending message $m_2' = [3.1, 3.5]$ will receive $U_2 = -(3.4 - 3.3)^2 = -0.01 > -0.16$. Under the assumption of truthful $m_1 = 2.7, 3.1]$ this message is not be considered as self-serving by the decision maker. Therefore following Expert's 1 deviation, Expert 2 finds it optimal to send $m_2 = [3.1, 3.5]$ that will lead to $y^* = 3.3$.

c. Serious deviation

We assume that Expert 1 seriously deviates by sending $m_1'' = [1.8, 2.2]$. With probability-1, $\omega_2 \notin m_1'' = [1.8, 2.2]$. Given m_2 the lying zone of Expert 2 is $[2, 2.2]$.

Let us assume that Expert 2 observes $\underline{\omega_2}(\omega_1) = 2.8$. Then, the following hold:

$U_2 = -(2 - 2.9)^2 = -0.81$,

$U_2 = -(2.4 - 2.9)^2 = -0.25 > -0.81$,

$U_2 = -(2.8 - 2.9)^2 = -0.01 > -0.81$.

$U_2 = -(3.2 - 2.9)^2 = -0.09 > -0.81$.

Therefore there are profitable deviation for Expert 2 who optimally chooses $m_2 = [2.6, 3]$. For this reason it's not optimal for Expert 1 to deviate.

A.3 Graphs

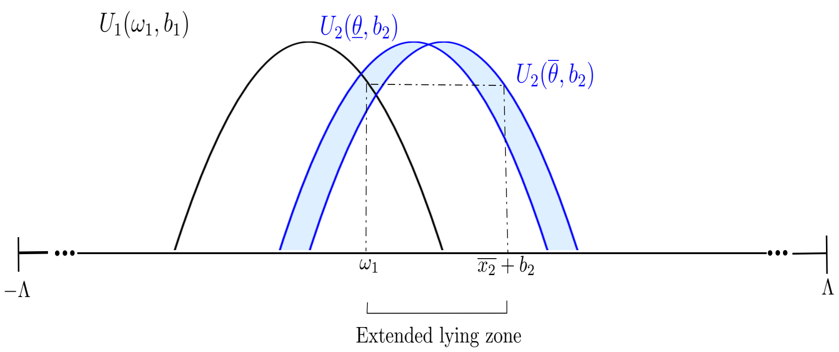


Figure A.5 | **Extended Lying Zone.** For this figure we consider $b_1 < 0 < b_2$. The blue shadowed area is the range of the second expert's preferences given the maximum belief support. Assuming that the first expert truthfully reports his signal ω_1, the extended lying zone is the interval $(\omega_1, \overline{x_2} + b_2)$, where $\overline{x_2}$ is the maximum bliss point of Expert 2 given the maximum belief support. The extended lying zone is the modified version of the lying zone as it is described in Figure 1.1.

Appendix B

B.1 Proofs

Proof of Remark 1. Assuming that the experts are truthful, the decision maker updates his belief by using the Bayes' rule:

$$Pr(\theta_1|s_1,\mu) = Pr(\theta_1|m_1,\mu) = \frac{\mu\rho}{\mu\rho + (1-\mu)(1-\rho)}$$

$$Pr(\theta_1|s_0,\mu) = Pr(\theta_1|m_0,\mu) = \frac{\mu(1-\rho)}{\mu(1-\rho) + (1-\mu)\rho}$$

$$Pr(\theta_0|s_1,\mu) = Pr(\theta_0|m_1,\mu) = 1 - \frac{\mu\rho}{\mu\rho + (1-\mu)(1-\rho)}$$

$$Pr(\theta_0|s_0,\mu) = Pr(\theta_0|m_0,\mu) = 1 - \frac{\mu(1-\rho)}{\mu_h(1-\rho) + (1-\mu)\rho}$$

An expert can send ex-ante effective messages $m_i = \{0,1\}$ if the following conditions hold:

$$Pr(\theta_1|m_1,\mu) > Pr(\theta_1|m_0,\mu) \Rightarrow \frac{\mu\rho}{\mu\rho + (1-\mu)(1-\rho)} > \frac{1}{2} \tag{B.1}$$

$$Pr(\theta_0|m_0,\mu) > Pr(\theta_0|m_1,\mu) \Rightarrow \frac{\mu(1-\rho)}{\mu(1-\rho) + (1-\mu)\rho} \leq \frac{1}{2} \tag{B.2}$$

By solving (4) and (5) we obtain:

$$\mu \geq 1-\rho \tag{B.3}$$

$$\mu \leq \rho \tag{B.4}$$

where $\rho = \pi g + (1-\pi)b$. We have assume that $b < g < 1$.Therefore, $\rho > 1-\rho$ and the set $EM = [1-\rho, \rho]$ is aways non-empty. ∎

Proof of Lemma 1.

We have to prove that the experts report truthfully their signals only for $[\underline{\mu}, \overline{\mu}]$. We start by proving the existence of $\overline{\mu}$ and $\underline{\mu}$ and then with the proof of non-emptiness of the set $[\underline{\mu}, \overline{\mu}]$. The analytical conditions that have to be satisfied for the separating equilibrium are the following:

- If $s = 0$ then the expert will truthfully send $m = 0$ if

$$\mathbb{E}U(m_0) \geq \mathbb{E}U(m_1) \Rightarrow$$

$$\beta a^*(m_0, \mu) + \mathbb{E}\hat{\pi}(m_0|s_0) \geq \beta a^*(m_1, \mu) + \mathbb{E}\hat{\pi}(m_1|s_0) \Rightarrow$$

$$\beta a^*(m_1, \mu) \leq \mathbb{E}\hat{\pi}(m_0|s_0) - \mathbb{E}\hat{\pi}(m_1|s_0)$$

  where

$$\mathbb{E}\hat{\pi}(m_0|s_0) - \mathbb{E}\hat{\pi}(m_1|s_0) =$$

$$P(\theta_0|s_0)\hat{\pi}(m_0|s_0) + Pr(\theta_1|s_0)\hat{\pi}(m_0|s_0) - Pr(\theta_0|s_0)\hat{\pi}(m_1|s_0) - Pr(\theta_1|s_0)\hat{\pi}(m_1|s_0) =$$

$$(1-P(\theta_1|s_0))\hat{\pi}(m_0|s_0)+P(\theta_1|s_0)\hat{\pi}(m_0|s_0)-(1-P(\theta_1|s_0))\hat{\pi}(\theta, m_1|s_0)-P(\theta_1|s_0)\hat{\pi}(\theta, m_1|s_0)) =$$

$$(1-P(\theta_1|s_0))\hat{\pi}(m_0|s_0)+P(\theta_1|s_0)\hat{\pi}(m_0|s_0)-(1-P(\theta_1|s_0))\hat{\pi}(m_1|s_0)-P(\theta_1|s_0)\hat{\pi}(m_1|s_0)) =$$

$$(1 - P(\theta_1|s_0))[\hat{\pi}(m_0|s_0) - \hat{\pi}(m_1|s_0)] - P(\theta_1|s_0)[\hat{\pi}(m_1|s_1) - \hat{\pi}(m_0|s_0)] =$$

$$(1 - P(\theta_1|s_0))[\hat{\pi}^+ - \hat{\pi}^-] - P(\theta_1|s_0)[\hat{\pi}^+ - \hat{\pi}^-)] =$$

$$(1 - 2P(\theta_1|s_0))[\hat{\pi}^+ - \hat{\pi}^-] =$$

$$(1 - \frac{2\mu(1-\rho)}{\mu(1-\rho) + (1-\mu)\rho})(\frac{\pi g}{\pi g + (1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g) + (1-\pi)(1-b)})$$

  Therefore the condition can be written as:

$$\beta \leq [(1 - \frac{2\mu(1-\rho)}{\mu(1-\rho) + (1-\mu)\rho})(\frac{\pi g}{\pi g + (1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g) + (1-\pi)(1-b)})] \tag{B.5}$$

- If $s = 1$ then the expert will truthfully send $m = 1$ if

$$\mathbb{E}U(m_1) \geq \mathbb{E}U(m_0) \Rightarrow$$

$$\beta a^*(m_1, \mu) + \mathbb{E}(\hat{\pi}(m_1|s_1)) \geq \beta a^*(m_0, \mu) + \mathbb{E}\hat{\pi}(m_0|s_1) \Rightarrow$$

$$\beta a^*(m_1, \mu) \geq \mathbb{E}\hat{\pi}(m_0|s_1) - \mathbb{E}\hat{\pi}(m_0|s_1)) \Rightarrow \beta \geq -\mathbb{E}\hat{\pi}(m_1|s_1) - \mathbb{E}\hat{\pi}(m_0|s_1))$$

  where

$$\mathbb{E}\hat{\pi}(m_1|s_1) - \mathbb{E}\hat{\pi}(m_0|s_1)) =$$

$$Pr(\theta_1|s_1)\hat{\pi}(m_0|s_1) + Pr(\theta_0|s_1)\hat{\pi}(m_0|s_1) - P(\theta_1|s_1)\hat{\pi}(m_1|s_1) - Pr(\theta_0|s_1)\hat{\pi}(m_1|s_1) =$$

$$Pr(\theta_1|s_1)\hat{\pi}(m_0|s_1)+(1-Pr(\theta_1|s_1))\hat{\pi}(m_0|s_1)-P(\theta_1|s_1)\hat{\pi}(m_1|s_1)-(1-Pr(\theta_1|s_1))\hat{\pi}(m_1|s_1) =$$

$$Pr(\theta_1|s_1)\pi^- + (1 - Pr(\theta_1|s_1))\pi^+ - P(\theta_1|s_1)\pi^+ - (1 - Pr(\theta_1|s_1))\pi^- =$$

$$(1 - P(\theta_1|s_1))[\hat{\pi}^+ - \hat{\pi}^-] - P(\theta_1|s_1)[\hat{\pi}^+ - \hat{\pi}^-)] =$$

$$(1 - 2P(\theta_1|s_1))[\hat{\pi}^+ - \hat{\pi}^-] =$$

$$(1 - \frac{2\mu\rho}{\mu\rho + (1-\mu)(1-\rho)})[\frac{\pi g}{\pi g + (1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g) + (1-\pi)(1-b)})]$$

Therefore the condition can be written as:

$$\beta \geq (1 - \frac{2\mu\rho}{\mu\rho + (1-\mu)(1-\rho)})[\frac{\pi g}{\pi g + (1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g) + (1-\pi)(1-b)})] \tag{B.6}$$

The upper boundary of the ICR is given by the condition (34): We consider separately the LHS and the RHS of the relation (28).The $LHS = F_b = \beta$ is the gain from manipuatation and assumed to be constant.

Therefore $F_b(\mu) = \beta = F_b(0) = \beta = F_b(1) = \beta \in [0,1]$

The $RHS = F_l(\mu) = (1 - \frac{2\mu(1-\rho)}{\mu(1-\rho)+(1-\mu)\rho})(\frac{\pi g}{\pi g+(1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g)+(1-\pi)(1-b)})$ is the reputational loss by a wrong recommendation. $F_l(0) = 1$ and $F(1) \leq 0$

By solving RHS=0 in term of μ, we obtain $f_1(\pi) = \pi g + (1-\pi)b$. which is continuous and strictly increasing in π with $f_1(0) = b$

We see that $F_l(0) \gtrless F_b(0)$ and $F_l(1)$<$F_b(1)$. Therefore there is a maximum $\overline{\mu} \in [0,1]$ but always $\overline{\mu} \leq \rho$ given a maximum $\overline{\beta}$ for which relation (28) is satisfied.

Analogously, by considering :

$$\beta \geq (1 - \frac{2\mu\rho}{\mu\rho + (1-\mu)(1-\rho)})[\frac{\pi g}{\pi g + (1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g) + (1-\pi)(1-b)})] \tag{B.7}$$

Therefore there is a maximum $\underline{\mu} \in [0,1]$ but always $\overline{\mu} \leq 1 - \rho$ given a maximum $\overline{\beta}$ for which relation (29) is satisfied.

For the non-emptiness of the region it is sufficient to prove that the LHS of relation (34) is always greater than LHS of relation (35). The rest is identical. We know:

$$\frac{2\mu(1-\rho)}{\mu(1-\rho) + (1-\mu)} < \frac{2\mu\rho}{\mu\rho + (1-\mu)(1-\rho)} \forall b \in [0.5, g), g \in (b, 1), \pi \in (0, 1) \tag{B.8}$$

Therefore the set $[\underline{\mu}, \overline{\mu}]$ is non empty.

To sum up:

$$ICR = [1-\rho, \overline{\mu}]$$

where $\overline{\mu}$ obtained solving (34) equal to zero in terms of μ.

Outside $[\underline{\mu}, \overline{\mu}]$:

$$\beta > (1 - \frac{2\mu(1-\rho)}{\mu(1-\rho)+(1-\mu)\rho})(\frac{\pi g}{\pi g+(1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g)+(1-\pi)(1-b)}) \tag{B.9}$$

$$\beta < (1 - \frac{2\mu\rho}{\mu\rho+(1-\mu)(1-\rho)})[\frac{\pi g}{\pi g+(1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g)+(1-\pi)(1-b)})] \tag{B.10}$$

This means that there is only a pooling equilibrium, where for $\mu < \underline{\mu}$ experts send only $m = 0$ and for $\mu > \overline{\mu}$, experts send always $m = 1$ ■

Proof of Proposition 1. The conditions (34) and (35) determine the ICR. The relation (34) gives the upper boundary of ICR and $1 - \rho$ gives the lower boundary.

Therefore we need to prove that there is a $\overline{\beta}$ such as $EM \cap ICR \neq \emptyset$

$$\overline{\mu} = \frac{((\rho(\pi - g^2\pi^2 - (\beta + b\beta(\pi-1))\rho + g\pi(1 + b(\pi-1) + \rho + \beta\rho)}{(g^2\pi^2 + \rho(\pi + \beta(1 + b(\pi-1))(2\rho-1)) - g\pi(1 + b(\pi-1) + \rho + \beta\rho(2\rho-1)}$$

$$\overline{\mu} > 1-\rho \Rightarrow \beta < \frac{\pi(2\rho-1)(bg - g - bg + g^2\pi + \rho - g\rho)}{(b - b\pi + g\pi - 1)(\rho - 2\rho^2 + 2\rho^3)} \equiv \overline{\beta} \in [0,1)$$

$\forall \in (0,1), b \in [0.5, g), g \in (b,1)$

$F(\cdot) = \overline{\beta}$ Then $F'(g) > 0, F''(g) = 0, F'(\rho) > 0, F''(\rho) = 0, F''(\pi) < 0.$ ■

Proof of Proposition 2.

Condition (34) and (35) coincide ($\mu = 0$) for :

$$\pi_1 = -\frac{((b(-1+\beta-2b\beta) + g + (-1+2b)\beta g + \sqrt{(b-g)(b(1+\beta)^2 - (1+\beta(-2+4b+\beta))g)})}{(2(b-g)(1+b\beta-\beta g)))}$$

$$\pi_2 = -\frac{((b(-1+\beta-2b\beta) + g + (-1+2b)\beta g - \sqrt{(b-g)(b(1+\beta)^2 - (1+\beta(-2+4b+\beta))g)})}{(2(b-g)(1+b\beta-\beta g)))}$$

where $\pi_1, \pi_2 \in [0,1]$ and $\pi_2 < \pi_1$.

Therefore by continuity there is a π^* between π_1 and π_2 where ICR is decreasing in π. ∎

Proof of lemma 2. We have to prove that $EM \equiv ICR_u$ where ICR_u is the Incentive Compatibility Region for an unbiased expert. Conditions (34) and (35) for $\beta = 0$ are :

$$0 \geq (1 - \frac{2\mu\rho}{\mu\rho + (1-\mu)(1-\rho)})[\frac{\pi g}{\pi g + (1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g) + (1-\pi)(1-b)})] \tag{B.11}$$

$$0 \leq [(1 - \frac{2\mu(1-\rho)}{\mu(1-\rho) + (1-\mu)\rho})(\frac{\pi g}{\pi g + (1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g) + (1-\pi)(1-b)})] \tag{B.12}$$

By solving (40), (41) in terms of μ we obtain:

$$\mu \geq 1 - (\pi g + (1-)b) = 1 - \rho$$

and

$$\mu \leq \pi g + (1-)b = \rho$$

Therefore $ICR_u = [1-\rho, \rho] = EM$ ∎

Proof of Lemma 3.

We state analytically the truth-telling conditions for the single expert case.

If $s = 0$ then the expert will truthfully send $m = 0$ if $E[U(m = 0)] \geq E[U(m = 1)]$. Therefore the truth-telling condition can be written as:

$$\beta(\frac{\mu\rho}{\mu\rho + (1-\mu)(1-\rho)} - \frac{\mu(1-\rho)}{\mu(1-\rho) + (1-\mu)\rho}) \leq$$

$$[(1 - \frac{2\mu(1-\rho)}{\mu(1-\rho) + (1-\mu)\rho})(\frac{\pi g}{\pi g + (1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g) + (1-\pi)(1-b)})] \tag{B.13}$$

If $s = 1$ then the expert will truthfully send $m = 1$ if $E[U(m = 1)] \geq E[U(m = 0)]$. Therefore the truth-telling condition can be written as:

$$\beta(\frac{\mu\rho}{\mu\rho + (1-\mu)(1-\rho)} - \frac{\mu(1-\rho)}{\mu(1-\rho) + (1-\mu)\rho}) \geq$$

$$(1 - \frac{2\mu\rho}{\mu\rho + (1-\mu)(1-\rho)})[\frac{\pi g}{\pi g + (1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g) + (1-\pi)(1-b)})] \tag{B.14}$$

We have to prove that relations (42) and (43) are the upper and the lower boundary of ICR which is always a non empty set. Let us denote the left-hand side part of the relations (42) and (43) by $L_1(\mu)$ and $L_2(\mu)$ respectively while similarly we denote the right-hand side by $R_1(\mu)$ and $R_2(\mu)$.

1 $L_1(0) = 0$ while $L_1(1) = 0$

2 $R_1(0) = [\frac{\pi g}{\pi g+(1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g)+(1-\pi)(1-b)})]$, $R_1(1) = -[\frac{\pi g}{\pi g+(1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g)+(1-\pi)(1-b)})]$

As we mention in **Proof of Lemma 1**, $L_1 = L_2$ and $R1 < R2$. So (42) is the upper bound and (43) the lower one of ICR and ICR is a non empty set. ■

Proof of Proposition 3. Similar to **Proof of Proposition 2** ■

Multiple Experts-Preliminaries

We start with analytical expressions of truth telling conditions for **continuous action space**.

Two Experts

When expert i receives $s = 0$ will truthfully send $m = 0$ if $E(U(m = 0|m^{-i})) \geq E(U(m = 1|m^{-i}))$:

$$\beta Pr(\theta_1|m_0, m^{-i}) + \delta E(\hat{\pi}(m_0|s_0)) \geq \beta Pr(\theta_1|m_1, m^{-i}) + \delta E(\hat{\pi}(m_h|s_l)) \Rightarrow \tag{B.15}$$

$$\beta[Pr(\theta_1|m_0, m^{-i}) - Pr(\theta_1|m_0, m^{-i})] \leq [E(\hat{\pi}(m_0|s_0)) - E(\hat{\pi}(m_1|s_0)] \Rightarrow \tag{B.16}$$

$$(\lambda\mu + \kappa\frac{\mu\rho^2}{\mu\rho^2 + (1-\mu)(1-\rho)^2}) - (\kappa\mu + \lambda\frac{\mu(1-\rho)^2}{\mu(1-\rho^2) + (1-\mu)\rho^2}] \leq (E(\hat{\pi}(m_0|s_0)) - E(\hat{\pi}(m_1|s_0)) \Rightarrow \tag{B.17}$$

The truthtelling condition for $s^i = 0$ can be written as:

$$[(\lambda\mu + \kappa\frac{\mu\rho^2}{\mu\rho^2 + (1-\mu)(1-\rho)^2}) - (\kappa\mu_h + \lambda\frac{\mu(1-\rho)^2}{\mu(1-\rho^2) + (1-\mu)\rho^2}] \leq (1 - \frac{2\mu(1-\rho)}{\mu(1-\rho) + (1-\mu)\rho})(\frac{\pi g}{\pi g + (1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g) + (1-\pi)(1-b)}) \tag{B.18}$$

Similarly, the truth-telling condition for $s^i = 1$ can be written as:

$$(\lambda\mu + \kappa\frac{\mu\rho^2}{\mu\rho^2 + (1-\mu)(1-\rho)^2}) - (\kappa\mu_h + \lambda\frac{\mu(1-\rho)^2}{\mu(1-\rho^2) + (1-\mu)\rho^2} \leq (1 - \frac{2\mu\rho}{\mu\rho + (1-\mu)(1-\rho)})(\frac{\pi g}{\pi g + (1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g) + (1-\pi)(1-b)}) \tag{B.19}$$

Three Experts

The truthtelling condition for $s^i = 0$ can be written as:

$$\kappa^2(\frac{\mu\rho^3}{\mu\rho^3+(1-\mu)(1-\rho)^3}-\frac{\mu\rho}{\mu\rho+(1-\mu)(1-\rho)})+\kappa\lambda(\frac{\mu\rho}{\mu\rho+(1-\mu)(1-\rho)}-\frac{\mu(1-\rho)}{\mu(1-\rho+(1-\mu)\rho})+...$$
$$...+\lambda^2(\frac{\mu(1-\rho)}{\mu(1-\rho+(1-\mu)\rho}-\frac{\mu(1-\rho)^3}{\mu(1-\rho)^3+(1-\mu)\rho^3})\leq$$
$$(1-\frac{2\mu(1-\rho)}{\mu(1-\rho)+(1-\mu)\rho})(\frac{\pi g}{\pi g+(1-\pi)b}-\frac{\pi(1-g)}{\pi(1-g)+(1-\pi)(1-b)}) \tag{B.20}$$

Similarly, the truth-telling condition for $s^i = 1$ can be written as:

$$\kappa^2(\frac{\mu\rho^3}{\mu\rho^3+(1-\mu)(1-\rho)^3}-\frac{\mu\rho}{\mu\rho+(1-\mu)(1-\rho)})+\kappa\lambda(\frac{\mu\rho}{\mu\rho+(1-\mu)(1-\rho)}-\frac{\mu(1-\rho)}{\mu(1-\rho+(1-\mu)\rho})+...$$
$$...+\lambda^2(\frac{\mu(1-\rho)}{\mu(1-\rho+(1-\mu)\rho}-\frac{\mu(1-\rho)^3}{\mu(1-\rho)^3+(1-\mu)\rho^3})\leq$$
$$(1-\frac{2\mu\rho}{\mu\rho+(1-\mu)(1-\rho)})(\frac{\pi g}{\pi g+(1-\pi)b}-\frac{\pi(1-g)}{\pi(1-g)+(1-\pi)(1-b)}) \tag{B.21}$$

Four Experts

The truthtelling condition for $s^i = 0$ can be written as:

$$\kappa^3(\frac{\mu\rho^4}{\mu\rho^4+(1-\mu)(1-\rho)^4}-\frac{\mu\rho^2}{\mu\rho^2+(1-\mu)(1-\rho)^2})+\kappa^2\lambda(\frac{\mu\rho^2}{\mu\rho^2+(1-\mu)(1-\rho)^2}-\mu)+\kappa\lambda^2(\mu-\frac{\mu(1-\rho)^2}{\mu(1-\rho)^2+(1-\mu)\rho^2}+$$
$$+\lambda^3(\frac{\mu(1-\rho)^2}{\mu(1-\rho)^2+(1-\mu)\rho^2}-\frac{\mu(1-\rho)^4}{\mu(1-\rho)^4+(1-\mu)\rho^4}))\leq$$
$$(1-\frac{2\mu(1-\rho)}{\mu(1-\rho)+(1-\mu)\rho})(\frac{\pi g}{\pi g+(1-\pi)b}-\frac{\pi(1-g)}{\pi(1-g)+(1-\pi)(1-b)}) \tag{B.22}$$

Similarly, the truth-telling condition for $s^i = 1$ can be written as:

$$\kappa^3(\frac{\mu\rho^4}{\mu\rho^4+(1-\mu)(1-\rho)^4}-\frac{\mu\rho^2}{\mu\rho^2+(1-\mu)(1-\rho)^2})+\kappa^2\lambda(\frac{\mu\rho^2}{\mu\rho^2+(1-\mu)(1-\rho)^2}-\mu)+\kappa\lambda^2(\mu-\frac{\mu(1-\rho)^2}{\mu(1-\rho)^2+(1-\mu)\rho^2}+$$
$$+\lambda^3(\frac{\mu(1-\rho)^2}{\mu(1-\rho)^2+(1-\mu)\rho^2}-\frac{\mu(1-\rho)^4}{\mu(1-\rho)^4+(1-\mu)\rho^4}))\leq$$
$$(1-\frac{2\mu\rho}{\mu\rho+(1-\mu)(1-\rho)})(\frac{\pi g}{\pi g+(1-\pi)b}-\frac{\pi(1-g)}{\pi(1-g)+(1-\pi)(1-b)}) \tag{B.23}$$

Proof of Proposition 4. For N Experts we can write the truthtelling condition for $s^i = 0$ as follows:

$$\kappa^{n-1}(\frac{\mu\rho^n}{\mu\rho^n+(1-\mu)(1-\rho)^n}-\frac{\mu\rho^{n-2}}{\mu\rho^{n-2}+(1-\mu)(1-\rho)^{n-1}})+\kappa^{n-2}\lambda(\frac{\mu\rho^{n-2}}{\mu\rho^{n-2}+(1-\mu)(1-\rho)^{n-2}}-\frac{\mu\rho^{n-4}}{\mu\rho^{n-4}+(1-\mu)(1-\rho)^{n-4}})+...$$
$$...+\kappa\lambda^{n-2}(\frac{\mu\rho^{n-4}}{\mu\rho^{n-4}+(1-\mu)(1-\rho)^{n-4}}-\frac{\mu(1-\rho)^{n-2}}{\mu(1-\rho)^{n-2}+(1-\mu)\rho^{n-2}}+\lambda^{n-1}(\frac{\mu(1-\rho)^{n-2}}{\mu(1-\rho)^{n-2}+(1-\mu)\rho^{n-2}}-\frac{\mu(1-\rho)^n}{\mu(1-\rho)^n+(1-\mu)\rho^n}))\leq$$
$$(1-\frac{2\mu(1-\rho)}{\mu(1-\rho)+(1-\mu)\rho})(\frac{\pi g}{\pi g+(1-\pi)b}-\frac{\pi(1-g)}{\pi(1-g)+(1-\pi)(1-b)}) \tag{B.24}$$

Similarly, the truth-telling condition for $s^i = 1$ can be written as:

$$\kappa^{n-1}\left(\frac{\mu\rho^n}{\mu\rho^n+(1-\mu)(1-\rho)^n}-\frac{\mu\rho^{n-2}}{\mu\rho^{n-2}+(1-\mu)(1-\rho)^{n-1}}\right)+\kappa^{n-2}\lambda\left(\frac{\mu\rho^{n-2}}{\mu\rho^{n-2}+(1-\mu)(1-\rho)^{n-2}}-\frac{\mu\rho^{n-4}}{\mu\rho^{n-4}+(1-\mu)(1-\rho)^{n-4}}\right)+\ldots$$

$$\ldots+\kappa\lambda^{n-2}\left(\frac{\mu\rho^{n-4}}{\mu\rho^{n-4}+(1-\mu)(1-\rho)^{n-4}}-\frac{\mu(1-\rho)^{n-2}}{\mu(1-\rho)^{n-2}+(1-\mu)\rho^{n-2}}\right)+\lambda^{n-1}\left(\frac{\mu(1-\rho)^{n-2}}{\mu(1-\rho)^{n-2}+(1-\mu)\rho^{n-2}}-\frac{\mu(1-\rho)^{n}}{\mu(1-\rho)^{n}+(1-\mu)\rho^{n}}\right)\leq$$

$$\left(1-\frac{2\mu\rho}{\mu\rho+(1-\mu)(1-\rho)}\right)\left(\frac{\pi g}{\pi g+(1-\pi)b}-\frac{\pi(1-g)}{\pi(1-g)+(1-\pi)(1-b)}\right) \tag{B.25}$$

From (52) and (53) we see that as $\uparrow n$ the LHS decreases up to 0 because $\kappa, \lambda < 0$. And when it reaches a $\overline{n}$ where increasing the experts has zero effect since the experts face the following truth telling conditions:

$$0 \leq \left(1-\frac{2\mu(1-\rho)}{\mu(1-\rho)+(1-\mu)\rho}\right)\left(\frac{\pi g}{\pi g+(1-\pi)b}-\frac{\pi(1-g)}{\pi(1-g)+(1-\pi)(1-b)}\right)$$

$$0 \geq \left(1-\frac{2\mu\rho}{\mu\rho+(1-\mu)(1-\rho)}\right)\left(\frac{\pi g}{\pi g+(1-\pi)b}-\frac{\pi(1-g)}{\pi(1-g)+(1-\pi)(1-b)}\right)$$

Which are the truth-telling conditions of unbiased experts. Therefore $ICR_n = EMR_1$ and we proved the expansion and the upward movement of ICR. ■

Proof of Remark 2.

The expected posterior of the decision maker after observing m_n and m_k:

$$Pr(\theta_i|s_n^1, s_k^2)\begin{cases}\mu & \text{if} \quad n \neq k \\ \frac{P(\theta_1|s_1)\rho}{P(\theta_1|s_1)\rho+(1-P(\theta_1|s_1))(1-\rho)} & \text{if} \quad n = k = 1 \\ \frac{P(\theta_1|s_0)(1-\rho)}{P(\theta_1|s_0)(1-\rho)+(1-P(\theta_1|s_0)\rho} & \text{if} \quad n = k = 0\end{cases}$$

$$\frac{P(\theta_1|s_1)\rho}{P(\theta_1|s_1)\rho+(1-P(\theta_1|s_1))(1-\rho)} \geq \frac{1}{2} \tag{B.26}$$

$$\frac{P(\theta_1|s_0)(1-\rho)}{P(\theta_1|s_0)(1-\rho)+(1-P(\theta_1|s_0)\rho} \leq \frac{1}{2} \tag{B.27}$$

$$\underline{\mu_e} = \frac{(1-b+b\pi-g\pi)^2}{(1-2b+2b^2+2b\pi-4b^2\pi-2g\pi+4bg\pi+2b^2\pi^2-4bg\pi^2+2g^2\pi^2)} \Rightarrow \tag{B.28}$$

$$\underline{\mu_e} = \frac{(1+\rho)^2}{(1-2b+2b^2+2b\pi-4b^2\pi-2g\pi+4bg\pi+2b^2\pi^2-4bg\pi^2+2g^2\pi^2)} \tag{B.29}$$

$$\overline{\mu_e} = \frac{(-b+b\pi-g\pi)^2}{(1-2b+2b^2+2b\pi-4b^2\pi-2g\pi+4bg\pi+2b^2\pi^2-4bg\pi^2+2g^2\pi^2)} \Rightarrow \tag{B.30}$$

$$\overline{\mu_e} = \frac{(-\rho)^2}{(1-2b+2b^2+2b\pi-4b^2\pi-2g\pi+4bg\pi+2b^2\pi^2-4bg\pi^2+2g^2\pi^2)} \tag{B.31}$$

Let us denote by D the denominator of the relation (45) and (47). We know that

$\overline{\mu_c}, \underline{\mu_c} \in [0,1]$. Therefore:

$$D > (1+\rho)^2$$

$$(1+\rho)^2 > (1-\rho)D$$

$$(-\rho)^2 > \rho D$$

Therefore $EM \subset EM_c$. ■

Proof Proposition5. The proof is similar to **Proposition 1**.

The conditions for separating equilibrium with multiple experts are:

(A) For $\mu > \frac{1}{2}$ the conditions under which the experts report trutfully their information is: An expert after a signal $s = 0$ will truthfully send $m = 0$ if

$$E(U(m_0|s_0, m^{-i})) \geq E(U(m_1|s_0, m^{-i})) \Rightarrow$$

$$\lambda\beta \leq [(1 - \frac{2\mu(1-\rho)}{\mu(1-\rho)+(1-\mu)\rho})(\frac{\pi g}{\pi g+(1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g)+(1-\pi)(1-b)})] \quad \text{(B.32)}$$

Similarly, an expert will truthfully send $m = 1$ after a signal $s = 1$ if

$$E(U(m_1|s_1, m^{-i})) \geq E(U(m_0|s_1, m^{-i})) \Rightarrow$$

$$\lambda\beta \geq (1 - \frac{2\mu\rho}{\mu\rho+(1-\mu)(1-\rho})[\frac{\pi g}{\pi g+(1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g)+(1-\pi)(1-b)})] \quad \text{(B.33)}$$

(B) For $\mu < \frac{1}{2}$ the conditions under which the experts report trutfully their information is: An expert after a signal $s = 0$ will truthfully send $m = 0$ if

$$E(U(m_0|s_0, m^{-i})) \geq E(U(m_1|s_0, m^{-i})) \Rightarrow$$

$$\kappa\beta \leq [(1 - \frac{2\mu(1-\rho)}{\mu(1-\rho)+(1-\mu)\rho})(\frac{\pi g}{\pi g+(1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g)+(1-\pi)(1-b)})] \quad \text{(B.34)}$$

Similarly, an expert will truthfully send $m = 1$ after a signal $s = 1$ if

$$E(U(m_1|s_1, m^{-i})) \geq E(U(m_0|s_1, m^{-i})) \Rightarrow$$

$$\kappa\beta \geq (1 - \frac{2\mu\rho}{\mu\rho+(1-\mu)(1-\rho)})[\frac{\pi g}{\pi g+(1-\pi)b} - \frac{\pi(1-g)}{\pi(1-g)+(1-\pi)(1-b)})] \quad \text{(B.35)}$$

where

$$\kappa \equiv Pr(s^i = 1|\theta) = \mu\rho + (1-\mu)(1-\rho)$$

$$\lambda \equiv Pr(s^i = 0|\theta) = \mu(1-\rho) + (1-\mu)\rho$$

The LHS_c are identical to LHS of the single expert case. The difference is the RHS. We know:

a. From proof of **Remark 2**: $EM \subset EM_c$

b. From proof of **Proposition 1**: the maximum value$\overline{\beta}$ is given by $\overline{\mu} = 1 - \rho$ for the single expert case and $\overline{\mu_c} = \underline{\mu_e}$

c. Given $\kappa, \lambda \in [0, 1]$ then $(\kappa\beta) < \beta$ and $(\lambda\beta) < \beta$.

Given (a.),(b.) and (c.) $\overline{\mu_c} > \overline{\mu}$ and $\underline{\mu_e} < 1 - \rho$. Therefore, $\overline{\beta_c} < \overline{\beta}$ such as $ICR_c \neq \emptyset$ ■

Proof Proposition 6. The first part of the proof is identical to **Proposition 2**. The only difference are the parameters κ and λ but do not affect qualitatively the results of separating equilibrium existence. We have to show that $\pi^{**} > \pi^*$. We can easily show that $\frac{\pi^*}{\partial\beta} < 0$, where π^* is the solution of $\frac{\partial\overline{\mu}(\cdot)}{\partial pie} = 0$.

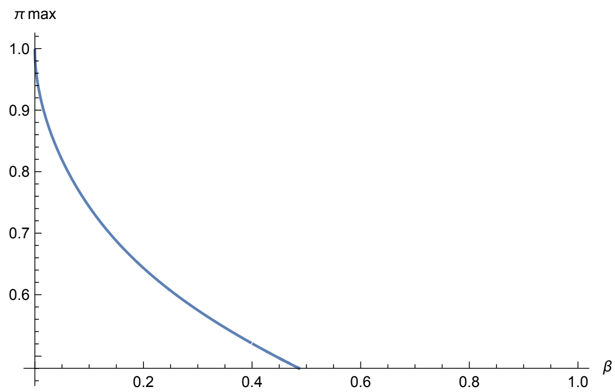


Figure B.1 π^* as a function of bias for fixed $b = 0.5$ and $g = 0.8$.

Since $\beta_c = \lambda\beta$ or $\beta_c = \kappa\beta$ we know that $\pi^{**} > \pi^*$. ■

Proposition 7. Directly from **Proposition 3** and **Proposition 4**. ■

B.2 Graphs

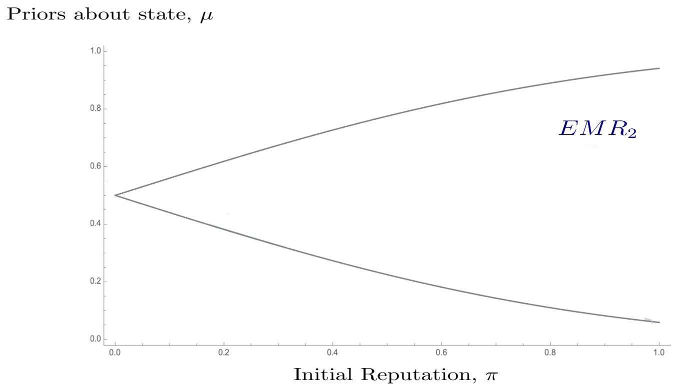


Figure B.2 | **Effective messages region under competition - Two Experts.** Parameters: $\beta = 0.1$, $g = 0.8$ and $b = 0.5$

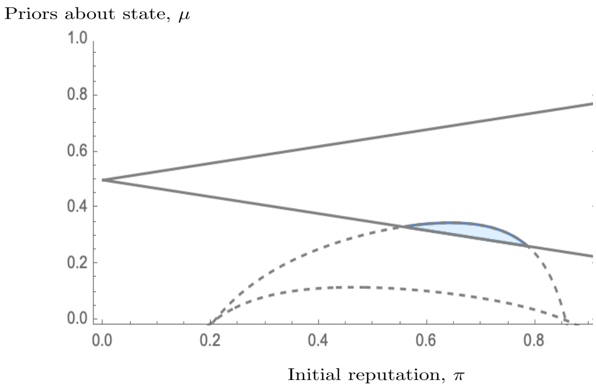


Figure B.3 | **Incentive Compatibility Region - High Bias.** Parameters: $\beta = 0.2$, $g = 0.8$and $b = 0.5$

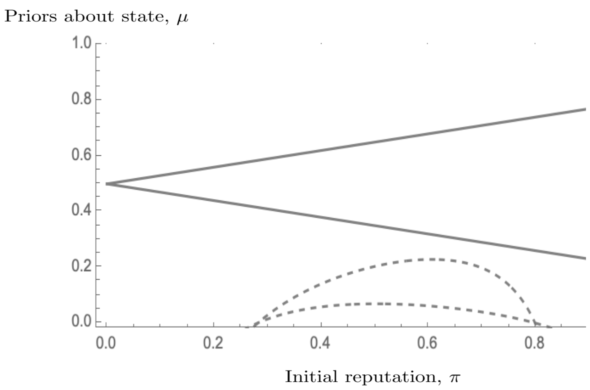


Figure B.4 | **Incentive Compatibility Region - High Bias- No Truthtelling Equiilibrium.** Parameters: $\beta = 0.3$, $g = 0.8$and $b = 0.5$

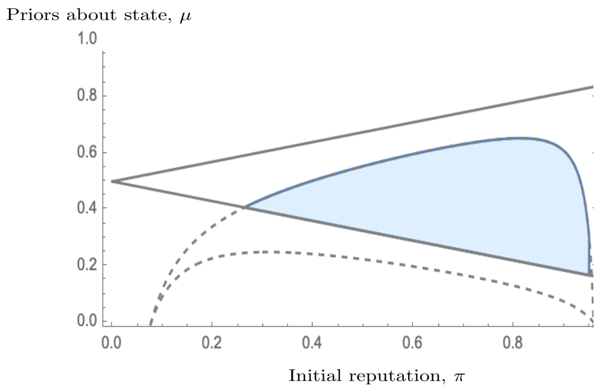


Figure B.5 | **Incentive Compatibility Region - Higher probability the good type receives the correct signal .** Parameters: $\beta = 0.1$, $g = 0.9$and $b = 0.5$

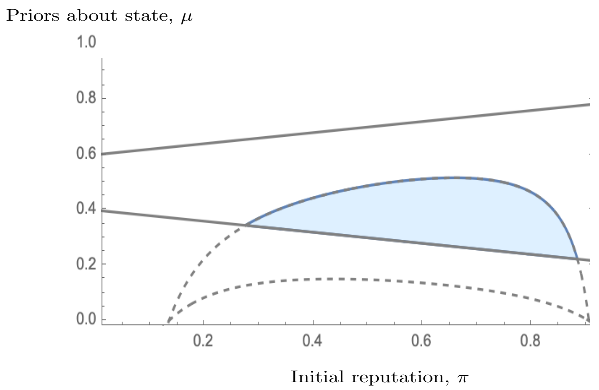


Figure B.6 | **Incentive Compatibility Region - Higher probability the bad type receives the correct signal .** Parameters: $\beta = 0.1$, $g = 0.8$and $b = 0.6$

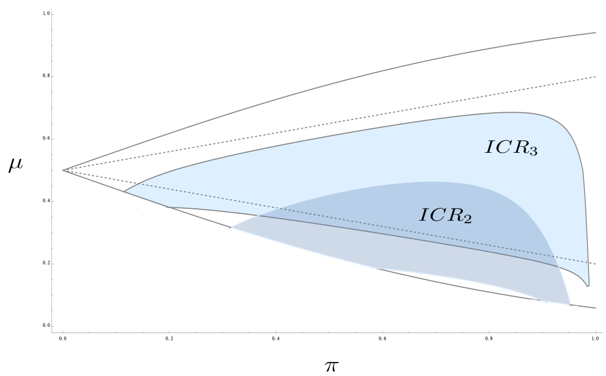


Figure B.7 | **Incentive Compatibility Region - Two and Three Experts .** Figure 8 with parameters: $\beta = 0.3, b = 0.5, g = 0.8$

Appendix C

C.1 Proofs

Proof of Proposition 1.
We start by considering that the receiver observes the types. In this case the high type's optimisation is as follows :

$$\max_{\pi_{2g};\pi_{2b};} \mathbb{E}(V_i) = \mu_2\pi_g + (1-\mu_2)(1-\pi_b) \qquad \text{s.t} \qquad \mu_{2g} \geq \underline{\beta}, \qquad \mu_{2b} < \underline{\beta}$$

Given the following comparative statics:

$$\frac{\partial V_2}{\partial \pi_2 g} > 0; \quad \frac{\partial V_2}{\partial \pi_2 b} < 0$$

We have the following optimal signals:

$$\pi^*_{2g} = 1; \quad \pi^*_{2b} = \frac{\underline{\beta} - \underline{\beta}\psi - \underline{\beta}\mu_0 - \psi\mu_0 + 2\underline{\beta}\psi\mu_0}{\underline{\beta}(\psi-1)(\mu_0-1)}$$

$$V^*_2 = \frac{\psi\mu_0}{\underline{\beta} - \underline{\beta}\psi - \underline{\beta}\mu_0 + 2\underline{\beta}\psi\mu_0}$$

V^*_2 is the maximum level of utility that the high type can reach and corresponds to the symmetric information scenario.We consider now the optimisation problem of the low type:

$$\max_{\pi_{1g};\pi_{1b}} \mathbb{E}(V_i) = \mu_1\pi_g + (1-\mu_1)(1-\pi_b) \qquad \text{s.t} \qquad \mu_{1g} \geq \underline{\beta}, \qquad \mu_{1b} < \underline{\beta}$$

Given the following comparative statics:

$$\frac{\partial V_1}{\partial \pi_1 g} > 0; \quad \frac{\partial V_1}{\partial \pi_1 b} < 0$$

$$\pi^*_{1g} = 1; \quad \pi^*_{1b} = \frac{\mu_0 - \underline{\beta}\psi - \underline{\beta}\mu_0 - \psi\mu_0 + 2\underline{\beta}\psi\mu_0}{\underline{\beta}\psi(\mu_0-1)}$$

$$V^*_1 = \frac{\mu_0 - \psi\mu_0}{\underline{\beta}(\psi + \mu_0 - 2\psi\mu_0)}$$

We now turn to the scenario where the type of the sender is private information. The receiver observes the experiment and he would like to be able to make some inference about the type of the sender. The low type has incentives to mimic the high type because $\pi^*_{2b} \leq \pi^*_{1b}$ and in turn $V_1(\pi^*_{2b}) > V^*_1$. The high type would like to signal his type in order to be able to persuade the receiver with $\pi^*_h = (\pi_{2g}, \pi_{2b})$. However, by choosing π^*_h the low type will be able to mimic him and sending as well π^*_h. In this case the receiver will not be able to make any inference on sender's type; he doesn't update his prior belief that remain μ_0. The only way to signal successfully his type is to choose $\pi^{'}_h \geq \pi^*_h$. Formally we write this problem as:

$$\max_{\pi_{2g};\pi_{2b};} \mathbb{E}(V_i) = \mu_2\pi_g + (1-\mu_2)(1-\pi_b)$$

such that

$$\mu_{2g} \geq \underline{\beta} \tag{C.1}$$

$$\mu_{2b} < \underline{\beta} \tag{C.2}$$

$$V_1(\pi_1, m_1, \mu_1) \geq V_1(\pi_2, m_2, \mu_1) \tag{C.3}$$

where m_t is the interim belief of the receiver. Therefore $\pi^*_{2b} \geq \pi^*_{1b}$. In this case the payoff in a separating equilibrium with π^*_{2b} would be:

$$V_2^S = \frac{(\psi-1)^2 + \underline{\beta}\mu_0(2\psi-1)}{\underline{\beta}\psi(1-\mu_0+\psi(2\mu_0-1))} < V_2^*$$

In any pooling equilibrium, the receiver's interim belief is $m_t = \mu_0$. Given these beliefs the senders can pool on the optimal signal for these belief which is π^{KG} for which it holds $\pi_{2b} < \pi^{KG} < \pi_{1b}$. In this case:

$$V_2^{KG} = \frac{\psi + \underline{\beta}(1-2\psi)\mu_0 - 1}{(\underline{\beta}(\psi+\mu_0-2\psi\mu_0-1)} > V_2^S$$

Therefore it is optimal for the high type to pool in π^{KG}. ∎

Proof Lemma 1. The receiver follows the signal if:

$$\frac{\mu_0\pi^C}{\mu_0\pi^C + (1-\mu_0)(1-\pi^C)} \geq \underline{\beta}$$

$$\frac{\mu_0(1-\pi^C)}{1-(\mu_0\pi^C + (1-\mu_0)(1-\pi^C))} < \underline{\beta}$$

For $\mu_0 < \frac{1}{2}$ the senders aims at the lowest possible precision therefore from above

$$\pi^C = \frac{\underline{\beta}(1-\mu_0)}{\underline{\beta}+\mu_0-2\underline{\beta}\mu_0}$$

For $\mu_0 \geq \frac{1}{2}$ the senders aims at the highest possible precision therefore $\pi^C = 1$. ∎
Proof of Proposition 2.
For $\pi_t = 1$ the payoff of both senders is :

$$V_t = \mu_t$$

For a $\pi'_t = 1 - \epsilon$ the corresponding payoff is: $V'_t = \mu_t(1-\epsilon) + (1-\mu_t)(1-(1-\epsilon))$
We consider first the low type:

$V_1 = \mu_1$ and $V'_1 = \mu_1(1-\epsilon) + (1-\mu_1)(1-(1-\epsilon))$. We can show that:

$$\mu_1 \leq \mu_1(1-\epsilon) + (1-\mu_1)(1-(1-\epsilon)) \Rightarrow$$

$$\frac{\mu_0(1-\psi)}{\mu_0(1-\psi)+(1-\mu_0)\psi} \geq \epsilon + \frac{((1-\psi)\mu)}{(-\psi(1-\mu_0)+(1-\psi)\mu_0)} - \frac{(2(1-\psi)\epsilon\mu_0)}{(-\psi(1-\mu_0)+(1-\psi)\mu_0)} \Rightarrow$$

$$\psi \leq \mu_0$$

We can proceed similarly for the high type. $V_2 = \mu_2$ and $V'_2 = \mu_2(1-\epsilon) + (1-\mu_2)(1-(1-\epsilon))$.
We can show that:

$$\mu_2 \leq \mu_2(1-\epsilon) + (1-\mu_2)(1-(1-\epsilon)) \Rightarrow$$

$$\frac{\mu_0 \cdot \psi}{\mu_0 \cdot \psi + (1-\mu_0)(1-\psi)} \geq \epsilon + \frac{\mu_0 \cdot \psi}{\mu_0 \cdot \psi + (1-\mu_0)(1-\psi)} - 2\epsilon \frac{\mu_0 \cdot \psi}{\mu_0 \cdot \psi + (1-\mu_0)(1-\psi)} \Rightarrow$$

$$\psi \geq 1 - \mu_0$$

Therefore $\pi_1 = \pi_2 = 1$ for $\mu_0 > \psi$. ■

Proof of Proposition 3.
We consider $1 - \psi \leq \mu_0 \leq \psi$. For high type problem see **Proof of Proposition 2**.
Given that $\pi_2^* = 1$, the decision maker is able to update his interim belief regarding sender's type by observing the experiment's choice. $\Delta_1 < 0$, therefore we are looking for the lowest possible precision of π_1^* which can be found by :

$$\frac{\mu_1\pi_1}{\mu_1\pi_1 + (1-\mu_1)(1-\pi)} \geq \underline{\beta} \tag{C.4}$$

$$\frac{\mu_1(1-\pi_1)}{\mu_1(1-\pi_1) + (1-\mu_1)\pi_1} < \underline{\beta} \tag{C.5}$$

By (6) we obtain :

$$\pi_1^* = \frac{\underline{\beta}\psi(1-\mu_0)}{\underline{\beta}(\psi-\mu_0) + \mu_0(1-\psi)}$$

which satisfies also (7) and $\pi_1^* > \pi^C$ ■
Proof of Proposition 4. See **Lemma 1** for $\mu_0 < \frac{1}{2}$. ■
Proof of Proposition 5.
We have to show that there are combinations of α, β and $\hat{p}$ such that first the low type is

at least indifferent between low and high cutoff rule. This holds if :

$$V_1(\alpha) = V_1(\beta) \Rightarrow m_1 + (1 - m_1)(1 - \pi_b^\alpha) = \hat{p}(m_1 + (1 - m_1)(1 - \pi_b^\beta))$$

$$\frac{(\beta\psi + \alpha\beta(2\psi - 1)(\hat{p} - 1) - \alpha\psi\hat{p})\mu_0}{\alpha\beta(\psi(2\mu_0 - 1) - \mu_0)} = 0 \Rightarrow$$

$$\hat{p} = \frac{\beta(\alpha + \psi - 2\alpha\psi)}{\alpha(\beta + \psi - 2\beta\psi)}$$

The low type strictly prefers the low cutoff if $\hat{p} > \frac{\beta(\alpha+\psi-2\alpha\psi)}{\alpha(\beta+\psi-2\beta\psi)}$. Then for $\hat{p} = \frac{\beta(\alpha+\psi-2\alpha\psi)}{\alpha(\beta+\psi-2\beta\psi)}$ we can prove that the high type always strictly prefer the high cutoff rule $\forall\mu_0$ and $0.5 \geq \beta < \alpha \leq 1$. The high type prefers the high cutoff if:

$$V_2(\alpha) \geq V_2(\beta) \Rightarrow m_2 + (1 - m_2)(1 - \pi_b^\alpha) \geq \hat{p}(m_2 + (1 - m_2)(1 - \pi_b^\beta))$$

$$-\frac{(\alpha - \beta)(2\psi - 1)\mu_0}{\alpha(\beta + \psi - 2\beta\psi)(\psi + \mu_0 - 2\psi\mu_0 - 1)} \geq 0$$

$$(\alpha - \beta)(2\psi - 1)\mu_0 \geq 0$$

$$\alpha(\beta + \psi - 2\beta\psi) \geq 0$$

$$(\psi + \mu_0 - 2\psi\mu_0 - 1) < 0$$

This means that:

$$-\frac{(\alpha - \beta)(2\psi - 1)\mu_0}{\alpha(\beta + \psi - 2\beta\psi)(\psi + \mu_0 - 2\psi\mu_0 - 1)} \geq 0$$

Therefore for $\hat{p} = \frac{\beta(\alpha+\psi-2\alpha\psi)}{\alpha(\beta+\psi-2\beta\psi)}$, $V_2(\alpha) \geq V_2(\beta)$. ∎

C.2 Graphs

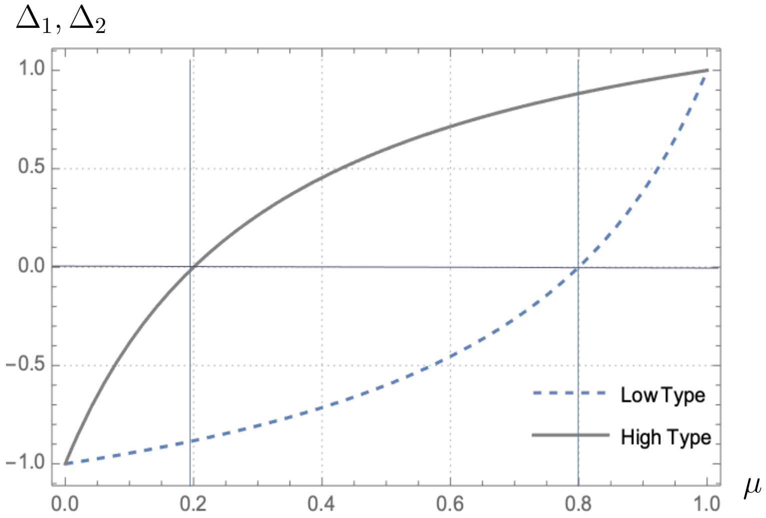


Figure C.1 | **Constrained experiments.** Horizontal axis: prior beliefs μ_0; Vertical axis: change of each type's payoff as the experiments precision increases,$\Delta_i = \frac{\partial V(\cdot)}{\partial \pi_i}$. **Parameters:** private information's precision $\psi = 0.8$ and cut-off decision rule $\underline{\beta} = 0.5$. The blue dashed line represents change in utility of the low type as the precision of the experiment increases. The grey solid line represents the change in utility of the hight type as the precision of the experiment increases. We see that for $\mu \leq 0.2$ both types prefer the lower possible precision $(\Delta_1, \Delta_2 < 0)$ and there is only a pooling equilibrium where $\pi_1 = \pi_2 = \pi^{KG}$. For $0.2 < \mu_0 < 0.8$ there is separating equilibrium because ($\Delta_1 < 0$ while $\Delta_2 > 0$). Therefore the low chooses the lowest persuasive experiment while the high type the fully informative one. For $\mu_0 > 0.8$ there is again only a pooling equilibrium but since $(\Delta_1, \Delta_2 > 0)$, it is fully informative.

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