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Essays in Optimal Pricing

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To Giannis and his fight

Thesis Abstract

The thesis comprises four essays, the first is the relevant literature review and the other three use theoretical methods to study the optimal pricing strategy of firms in economic environments where the strategy is affected more by the consumers' behavior than the market structure and the type of competition. Moreover, it focuses on complex pricing schemes that are hard to explain in standard models of non-linear pricing.

The first chapter of the thesis is a review of the literature in optimal non-linear pricing with a focus on market where there are consumers with bounded rationality. It provides a comprehensive survey of the different predictions and implications in the optimal pricing strategy of the firm of the theories of individual decision making most frequently used in behavioral economics.

The second chapter focuses on the well established habit forming behavior, namely the behavior when the valuation of the good in each period is affected by whether consumption occurred in the preceding periods. More specifically, it focuses on the market for access services, like communication and subscription services and considers how consumers' habit formation affects the pricing policy of firms. It analyses a two periods model where consumers purchase 1 unit at most in each period and are ex-ante uncertain about their per-period evaluation of the service. Consumers solve a dynamic programming problem and buy all units valued above a critical threshold, which is a function of the present evaluation and past consumption. Two types of consumers are considered, sophisticated and naive. The latter do not realize that their current consumption is affecting future consumption. Our main result is that under naive habit formation, the optimal pricing pattern is a three part tariff, namely a fixed fee, an amount of units priced below cost and after their end pricing above marginal cost. This a pricing scheme mostly observed in these kind of markets. Different from Grubb (2009), we claim that only one mistake, in our case underestimation of future demand, combined with a forward looking consumer that updates her consumption strategy during the contract period, is sufficient for three part tariff to be optimal.

The third chapter studies a market that consists of one firm and habit forming

consumers of different degrees of sophistication. The firm knows that all the consumers are habit forming but cannot observe if they are aware of it or not, so it needs a screening mechanism in order to screen between the different types of consumers and maximize its profits handling the market failure due to asymmetric information. Two types of consumers are considered, sophisticated and naive. The latter, as in the previous chapter, do not realize that their current consumption is affecting future consumption. Our main result is that the menu of contract offered consist of a two-part tariff and a three-part tariff. Moreover, the naive consumer is ex post worst off in the presence of sophisticated consumers with respect to the full information case, even if her naivety cannot be exploited. By way of contrast, the sophisticated consumer is better off.

In the forth chapter of the thesis, we propose a new explanation of three part tariffs, based on the assumption that consumers are forward-looking but impatient. In a dynamic stochastic setting, prices that apply to large volumes tend to be paid towards the end of the contracting period and so are more heavily discounted by consumers. As a result, high prices for large volumes represent an efficient way of extracting surplus. Low (or even vanishing) prices for small volumes, on the other hand, serve to stimulate early consumption, making it more likely that high marginal prices will indeed apply later. Although firms design contracts so as to take advantage of consumers' impatience, impatience in fact benefits consumers as it prevents full extraction of their surplus. However, when both patient and impatient consumers coexist in the market, patient consumers gain more than impatient ones.

Chapter 1

Behavioral Optimal Pricing

Eleftheria Triviza

Abstract

This chapter is a review of the literature in optimal non-linear pricing with a focus on markets where there are consumers with bounded rationality. It provides a comprehensive survey of the different predictions and implications in the optimal pricing strategy of the firm, of the theories of individual decision making most frequently used in behavioral economics. There are many occasions where market structure or and the characteristic of the firms cannot explain the observed optimal pricing policy. In many of these occasions the observable pricing policy, that could not be explained by the presence of rational consumers, could be explained by the presence of bounded rational consumers.

1.1 Introduction

In the last decades behavioral economics has experienced significant development. Behavioral economics studies the effects of psychological, cognitive, and emotional factors on the economic decisions of the individuals. There is a growing body of laboratory and field evidence that documents individuals who do not optimize according to the standard preference but they experience persistent and systematic biases in decision-making.

The firms, on the other hand, have as well monitored this persistent and systematic deviation from the standard preferences and they have incorporated it into their own decision making. Behavioral industrial economics is the stream of the literature that studies exactly this, namely how these deviations are affecting the optimal strategies of the firms in terms of optimal pricing, product differentiation, market structure, intensity of competition e.t.c.

The goal of this paper is to provide a survey of a part of the behavioral industrial economics literature and more specifically the part that studies the optimal pricing policies of the firm as a response to the biased preferences of the consumers. The introduction of menu of contracts that cannot be explained by standard preference is well documented and for this reason alternative explanations have been studied. Thus, our objective is to make a survey on studies that analyze the implications that consumers' bounded rationality and the extent to which they are aware of it have for the firms' pricing decisions.

The structure of this review is such as to summarize, organize and make a link between the pricing contracts that we observe in several markets and the behaviors that could explain the introduction of such a contract. Thus, I do not consider papers that discuss psychology and economics analysis on classical contract theoretic topics in general, namely I do not discuss for example moral hazard issues and incomplete contracts[1]. Moreover, it is beyond the scope of this survey the implications that the

[1] For a comprehensive survey in behavioral contract theory that covers all the topic of contract theory see Kőszegi (2014)

bounded rationality of the consumer has for the market complexity, market competition, market structure or product differentiation[2]. Thus, the focus is on organizing and categorizing the different pricing policies that we frequently observe in the markets and on summarizing the potential explanations that the literature has provided for each of them.

In traditional economics with standard preference and rational consumers, the optimal contract would be a two part tariff that consists of a fixed fee and a marginal price equal to the marginal cost. Though, this is not what we observe in several markets. To the contrary we observe much different and complex contracts. Each section of the survey is devoted to a different type of contract that we frequently observe but traditional economics cannot justify.

More specifically, in section 2, I discuss the explanations that the behavioral literature proposes for the introduction of flat rates and contracts with marginal prices below marginal cost. There are three potential causes because of which the firms find it optimal to charge such a contract. First, the taxi meter effect, namely the effect created when marginal price enters directly into the utility and the consumer feels a stress-disutility as she consumes. Second, the overestimation of demand when the consumer mistakenly believe that he will consume more than she actually consumes when her demand type is revealed. Finally, the insurance effect which comes from the need of loss averse consumers to insure themselves against a loss. A loss that cannot be avoided otherwise since they sign their contracts ex ante being uncertain about their level of demand.

Section 3 considers contracts that have marginal prices above marginal cost. In this case, there are mainly two potential explanations. On the one hand, a cause could

[2]See Spiegler (2011) for a detailed, comprehensive and educational textbook that cover many industrial organization models. Moreover, Armstrong (2016) provides an interesting survey of the use of nonlinear pricing as a method of price discrimination, with a focus on environments where it is profitable to offer quantity discounts and bundle discounts. Rabin (1998) provides a detailed survey on studies that provide evidence for departure from standard preferences. Finally, DellaVigna (2009) surveys the empirical evidence from the field on three classes of deviations, namely nonstandard preference, nonstandard beliefs and nonstandard decision making.

be the presence in the market of consumers who do not understand or foresee specific features of a product, like prices and fees. This could happen due to like myopia (Gabaix and Laibson, 2006; Armstrong and Vickers, 2012). On the other hand, the presence of consumers who mispredict their own behavior with respect to the product, namely dynamically inconsistent consumers (DellaVigna and Malmendier, 2004).

Finally, section 4 refers to a more complex contract which is called "three part tariff". The "three part tariff" consist of a fixed fee, a number of units, the allowance, that are charged with zero marginal price and after the end of the allowance a marginal price that is positive and above marginal cost. The main explanation for its introduction is overconfidence (Grubb, 2009), namely individuals that underestimate their uncertainty. There is a discussion both of papers that explain the introduction of three part tariff but also of papers that provide evidence of its presence as a behavior.

1.2 Flat rate - Marginal price below marginal cost

In several markets, the contracts offered charge prices below marginal cost or even flat rates with marginal prices equal to zero and a fixed fee. This marginal pricing would have as a result the quantity demanded to be above the efficient level, namely overconsumption for the consumers and profits below the maximum for the firms. Though this would be the case if we consider that the consumers participating in the market are rational. The literature has offered explanations for the introduction and optimality of this kind of contracts by considering the presence of bounded rational consumers.

An additional observation in this kind of markets is that the consumers often do not select the tariff option that minimizes their expenditure for observed consumption patterns. To the contrary, consumers often prefer a flat-rate tariff, where they pay a high fixed fee, to a measured tariff where they would pay depending on the amount of quantity consumed, even though they would save money with the later. Train et al. (1991) referred to this phenomenon as the "flat-rate bias"; a bias that seems to make consumers willing to pay a "flat-rate premium", thus an additional cost in order to avoid pay per use charges. Lambrecht and Skiera (2006) conducts empirical analyses in order to provide evidence of the "flat-rate bias" and provides an overview of the work on it. It classifies three main potential causes for this kind of bias the taxi meter effect, demand overestimation and insurance motives. In the next subsections there is a discussion of each of them.

1.2.1 Taxi meter effect

The "taxi meter effect" belongs to the concept of mental accounting which was first established by Thaler (1999). Mental accounting describes the process that people use in order to evaluate economic outcomes. Consumers have been documented that often experience an immediate disutility of paying, which decreases the utility derived from consumption. More specifically, the marginal price enters directly to the utility function and the consumers experience disutility from units of consumption that have marginal charges. A common example is that of a using the service of a taxi. The consumer, during a taxi ride, suffers disutility from observing the meter running. According to mental accounting theory, the taxi-meter effect can be avoided if the payments of the services is not counted by the meter but it is a fixed amount agreed with the driver before the ride gets started.

Prelec and Loewenstein (1998) proposes a "double-entry" mental accounting theory, that it calls prospective accounting, and develops a model that describes the implications and the combination of the pleasure of consumption and the pain of paying. Main prediction of this model is that the consumers, instead of having as their goal the minimization of the present value of payments, prefer to prepay their consumption. This is because the consumption is decoupled[3] from the payment and the consumer makes decisions without thinking about the need to pay for it in the future. Moreover, it predicts that the consumer should prefer to pay a fixed fee and zero

[3]Soman and Gourville (2001) predict that price bundling, namely charging a fixed fee for more than one services or quantity, leads to a disassociation or "decoupling" of transaction costs and benefits. Consumers offered such contracts pay less attention to sunk costs and decreases their likelihood of consuming a paid-for service.

marginal price, namely a fixed monthly fee, to a payment at the margin even if the later would be less costly for the same level of consumption.

The critic for using such an explanation for the introduction of flat rate in several markets, like internet service, is that this bias in order to be present needs that the consumer while consumes the good is feeling a strong link between the payment and the consumption. Though this is not the case in the markets where the consumer first consumes the good during the whole contract period and she pays the bill at its end. In this instance, there is long time distance between the good consumed, that could even in the beginning of the contract period and the payment at its end (Herweg and Mierendorff, 2013).

1.2.2 Overestimation of demand

It is well documented that consumers tend to overestimate their demand. This could happen either because they are influenced by the advertising policy of the firm (Mitchell and Vogelsang, 1991) or because they are dynamically inconsistent in their consumption of specific kind of goods (Malmendier and Della Vigna, 2006; DellaVigna and Malmendier, 2004).

A consumer overestimates her demand when she is naive of discounting in quasi-hyperbolic way investment goods. Investiment goods are the goods which have immediate costs and delayed benefits. A commonly used example of investment good is the gym attendance. In this case there is an immediate cost for exercising and a benefit observed with some delay in the health and the appearance of the consumer. The fact that there is this time delay between the cost and the benefit from the consumption of the good makes the consumers to overestimate their future demand at the contractual period and thus overvalue a contract with unlimited usage. DellaVigna and Malmendier (2004) show that with naive quasi-hyperbolic discounters, a profit maximizing firm is charging this kind of goods with a two-part tariff consisting of a marginal price below marginal costs and a fixed fee. Interestingly, this kind of contract is optimal regardless of the degree of sophistication. In the case of a sophisticated consumer a marginal price below marginal cost is optimal and works as a commitment device in order to increase the consumption of the investment good in the future. On the other hand, in the case of a naive consumer, DellaVigna and Malmendier (2004) was the first to point out that firms might fine-tune contracts to exacerbate consumer's mistakes.

Overestimation of demand is a well documented behavior. Malmendier and Della Vigna (2006) provides evidence that many customers of health clubs overpredict their future usage. It uses data from US health clubs consisting of the type of membership and the day-to-day attendance. They claim that this mis-prediction could be caused by naive quasi-hyperbolic discounting. They find that the consumers mainly choose contracts that ex post appear to be suboptimal given their attendance frequency. Interestingly, the consumers who choose monthly membership would pay less with pay as you go contract, which is a example of "flat rate bias". Moreover, the consumers who choose monthly contracts are more likely to continue renewing their contract after one year than the consumer who has chosen an annual contract. Main explanation for their findings is overconfidence about future self control or about future efficiency. Interestingly, overconfident agents overestimate attendance as well as the cancellation probability of automatically renewed contracts.

1.2.3 Insurance Effect

A flat rate could also be used as a mean of insurance [4] (Kridel et al., 1993; Lambrecht and Skiera, 2006; Lambrecht et al., 2007). The consumers at the contractual period are uncertain about their future demand and they are afraid of experiencing a high demand shock. For this reason they find it optimal to choose a flat rate instead of a pay-as-you-go contract, in order to avoid unexpected high charges.

[4] Train et al. (1989), point out that customers do not choose tariffs with complete knowledge of their demand, but rather choose tariffs [...] on the basis of the insurance provided by the tariff in the face of uncertain consumption patterns (p. 63).

Risk aversion

Risk aversion is the behavior of individuals when exposed to uncertainty, to attempt to reduce the risk from that uncertainty. It is a rational rather than a bounded rational behavior but it interesting at this point to discuss also this behavior since could be thought as a reason for the existence of "flat rate bias" but based on the literarure it does not really explain the optimalith of such a contract.

Miravete (2002) mentions the importance of the time lag between signing a contract and actually consuming the good. It studies the implications of consumers' uncertainty concerning their future consumption when a monopolist offers them the possibility to choose in advance from a menu of optional two-part tariffs[5]. As it is pointed out the optimal pricing becomes a two-stage nature of decision under optional tariffs, namely tariffs that consist of a fixed fee and unlimited usage or a smaller fixed fee and an allowance. They show that there is significant assymmetry of information between consumers and the monopolist and that this assymmetry is really relevant for the design of the tariffs.

Though, even if this risk aversion could explain why the consumers would prefer flat rate tariff to measured one,it cannot really explain why the monopolist would find optimal to offer such a pricing scheme. Moreover, it is not sufficient to explain in a realistic way the motive that the consumer has for insurance, since the unexpected charges are usually relatively small compared to the total consumer's income.

Loss aversion

The behavioral explanation that could make relevant the introducion of a flat rate for insurance motives is loss aversion. An individual is loss averse when she evaluates outcomes relative to reference points and weight losses more heavily than gains. Loss aversion could affect tariff choice and optimal pricing if the negative value attributed to losses relative to the price of the flat rate is higher than the positive value attributed

[5]In the paper there is an analysis also of a unique, mandatory non linear tariff, which they call "ex post" tariff

to a gain of the same amount (Kahneman and Tversky, 1979; Tversky and Kahneman, 1991).

There are two main streams in the literature of optimal pricing in markets where there are loss averse consumers. On the one hand, there are theories that the firms through their pricing policy find it optimal to create an environment so as to minimize the losses. This is achieved by decreasing the uncertainty in the market. On the other hand, there are theories like Heidhues and Kőszegi (2014) where the opposite is the attempted. In this case the firm optimally introduces random “sale” into an environment where there is no other source of uncertainty and risk, in order to create an environment that feels like risky for the consumer[6].

Herweg and Mierendorff (2013) argues that the prevalence of flat rate contracts can be due to consumers’ loss aversion. It considers a monopolist who offers a two part tariff to an ex ante homogeneous group of consumers. It claims that the optimal two part tariff is a tariff with a fixed fee and zero marginal price if the market is characterized by three main features. Firstly, the marginal costs of the firm are not too high, this means the marginal cost is not much bigger than zero so a marginal price equal to zero is not detrimental for the firm. Secondly, the loss aversion of the individuals is relatively intense, namely the consumer suffers relatively a lot when experiences losses. Finally, there should be strong variations in demand, so the probability of having a high demand shock and thus a loss is quite high.

An other paper that is in line with the literature that the firms find it optimal create a less risky environment is Heidhues and Kőszegi (2008). It introduces consumer loss aversion into a model of horizontally differentiated firms. They show that in equilibrium, nonidentical but asymmetric competitors charge identical prices for differentiated products. Loss averse consumers are more responsive to increase of the prices since they weight monetary losses heavily. This behavior intensifies competition

[6]There are also theories of screening loss averse consumers where the main implication of the presence of consumers heterogeneous in their willingness to pay is that they are discriminated with just few products (“coarse screening”), namely they are offered less different qualities (Hahn et al., 2012; Carbajal and Ely, 2012).

and reduces price variation both within and between products. In the same line of creating a less uncertain environment, Sibly (2002) shows that consumer loss aversion leads the optimal prices of the firms to respond less in demand and cost shocks.

Considering moral hazard with loss averse consumers, Herweg et al. (2010) provide an explanation for the frequent usage of lump-sum bonus contracts. A repeated moral hazard model with a loss-averse agent is analyzed by Macera (2012). Both of these papers demonstrate that the optimal incentive contract features less variation in the wage than would be expected based on classical models.

On the other hand, the optimality of a more risky environment is proposed by Heidhues and Kőszegi (2014). It claims that for any degree of consumer loss aversion, the monopolist's optimal price distribution consists of a low, variable and random "sale" prices and a high and atomic "regular" price. The monopolist announces a price distribution, and the consumer forms her expectations after observing the price distribution. The consumer's reference point is her recent rational expectations about the purchase. Then, a price is drawn form the distribution and the consumer decides whether to buy a single item of the good or not. Realizing that she will buy at the sales prices and hence that she will purchase with positive probability, the consumer chooses to avoid the painful uncertainty in whether she will get the product by buying also at the regular price.

1.3 Marginal price above marginal cost

There are several markets where we observe that the firms charge inefficiently high prices, namely prices above marginal cost. This pricing policy of high charges would lead to underconsumption and thus profits less than the maximum if we assume that in the market there are rational consumer. In the literature, the underestimation of demand[7] is the main reason for marginal prices above marginal cost. There are several behavioral instances that have as a result the underestimation of future demand. In

[7] Lambrecht and Skiera (2006) provide evidence that underestimation of usage is a major cause of the pay-per-use bias.

this section, there is a review of these kind of consumer behaviors.

1.3.1 Dynamic Inconsistency

In the literature there is a significant discussion about dynamic inconsistency or time inconsistency, namely a behavior where the individual's preferences change over time in such a way that her preference today can become inconsistent at another point in time. This behavior can be explained as the decision makers having many different "selves" within them, with each "self" representing the decision-maker at a different point in time. Thus, the inconsistency occurs when all preferences are not aligned.

The hyperbolic discounting is an example of dynamic inconsistent behavior. More specifically models that use the concept of quasi-hyperbolic discounting (Strotz, 1955; Phelps and Pollak, 1968; Laibson, 1997; O'donoghue and Rabin, 1999), are time-inconsistent models of discounting. Individuals using hyperbolic discounting make choices today that their future self would prefer not to have made, despite using the same reasoning. Moreover, this dynamic inconsistency happens because the value of future payoff is much lower under hyperbolic discounting than under the commonly used exponential discounting. This kind of behavior could be seen as having "present bias" in the sense that the consumer values more the present payoffs than the future ones.

DellaVigna and Malmendier (2004), as I discussed before, is studying how rational firms respond to consumer biases and more specifically to hyperbolic discounting consumers. In the case of leisure goods, which are goods with immediate benefits and delayed costs such as credit card-financed consumption, the consumer at the contractual period is underestimating her demand in the future[8]. This happens because they discount the costs more than the benefits and thus at the period when they make their decisions they think that they will consume more often that they will actually do

[8]See DellaVigna (2009) for a detailed survey on the empirical evidence from the field. It discusses the existence of consumers that have a preference for immediate gratification. Moreover, it is a survey that discusses the three main respects in which individuals deviate, namely non standard preferences, non standard beliefs and nonstandard decision making.

during the contract period. In order to capture this kind of discounting they develop a model where at period zero the firm offers a contract and the consumer decides whether to accept it or not. Then the contract period consists of two periods. In the first period the consumer pays the fixed fee and experience the benefit minus the marginal price if she consumes and in the second period she experiences the cost of consumption. This cost of consumption is irrelevant to the contract, it is a characteristic of the consumer which is not deterministic and the agent learns at the end of period zero. They show that the firm finds optimal to charge a positive fixed fee and a high marginal price above marginal cost.

This kind of contract occurs either as a commitment device or because of overconfidence depending on the level of sophistication of the consumer. If the consumer is sophisticated, namely aware of being time inconsistent and having mistaken expectations for here future consumption, the high per-unit cost is a commitment device designed to solve the overconsumption problem. On the other hand, if the consumer is naive which means that she does not realize that has "present bias" then above-marginal-cost pricing is aimed at exploiting the underestimation of the probability of a purchase.

A perfectly competitive market with quasi-hyperbolic discounters who have a taste for immediate gratification is analyzed by Heidhues and Kőszegi (2010). They focus on credit contracts only, and the welfare implications of the presence of quasi-hyperbolic discounters. Moreover, it proposes possible welfare-improving interventions. It considers two types of consumers the sophisticated ones who are aware of their time inconsistency and non-sophisticated who are either partially aware of being quasi-hyperbolic or not at all. They develop a model following O'Donoghue and Rabin (2001) in order to introduce a consumer who is overoptimistic regarding her future self-control during the contractual period. This contract has two kinds of implications on non-sophisticated consumers. On the one hand, the consumers being naive of their taste for immediate gratification the pay both the penalties and they repay their credit more often than they prefer. On the other hand, the same mistaken expectations lead naive consumers to underestimate the cost of credit and borrow too much. Moreover, the optimal penalties are so high that even if the consumers are failing only by a bit to predict their future taste and the firms do not have complete information about their preference and beliefs, there are significant welfare implications for the non sophisticated consumers. Heidhues and Kőszegi (2010) claims that the policy intervention that could raise the level of welfare, even if in the market there are relatively few non sophisticated consumers, would be to forbid charging high penalties when the agent fails to repay or postpones the payment of small amounts of money. This welfare improvement happens because in this way the borrowers, who are not too naive, do not drastically mispredict their future tastes.

There is also literature that studies a different type of time inconsistency. For example, Esteban et al. (2007) also analyze the optimal nonlinear pricing scheme for a monopolist who sells to consumers with self-control problems. Instead of assuming hyperbolic discounting and a consumer that would like to commit on her future decision because she will have different tastes in the future, they model self-control problems using the concept of Gul and Pesendorfer (2001). In this concept, the agent would like to commit in order to lower temptation in the future. They consider two types of temptation the "upward" temptation and the "downward" temptation and they show that the optimal menu is different for each type. In the case of "upward" temptation, the consumer is more tempted by high consumption than low, e.g. cigarettes, then the optimal menu is small, actually a singleton and makes self-control unnecessary. On the other hand, if the consumers have "downward" temptation, thus she is more tempted by low consumption than high, then the optimal menu is large, actually infinite, with a price ceiling and consumers incur positive self-control costs. The latter contract which is tailored for consumers with "downward" temptation, is identical to that with standard preferences equal to ex post preferences. Moreover, they show that the existence of temptation does not mean that the firm can exploit it and make bigger profits. The intuition for this is that the fact that the consumers are aware of their tendency to be tempted and they prefer to commit, even by not participating in the market, outweighs the advantage that the firm would have to exploit this temptation.

It would be interesting at this point to mention that the time inconsistent behavior has as implication the introduction of a high penalty in the future also in other environments of mechanism design. O'Donoghue and Rabin (1999) study optimal contracts for motivating a worker who is present biased and tends to procrastinate to complete a task. The firm wants to incentivize the worker so as not to procrastinate and complete the task earlier than later leaving the worker freely choose when to work. If the workers are naive about their preference for procrastination, the optimal contract for the firm may be to charge a high penalty after a period. This contract could be viewed as a "deadline".

Another instance is Gilpatric (2008) which develops a model of a worker who is present bias and he has to do a task at a fixed point in time. The timing of the game is such that the worker is receiving her reward the next period after she has completed the task. This time lag between the cost and the benefit leads the worker to procrastinate. In this case the firm may finds it optimal to offer a contract with a high penalty such that a sophisticated present biased consumer would not participate in the market in order to avoid the this penalty after shirking the task.

1.3.2 Myopia - Adds on pricing

In many industries, it is common to sell high-priced add-ons. For example, hotels charge high prices for additional services other than accommodation like telephone calls, minibar items etc. Ink cartridges are typically more expensive than consumers might expect, sometimes a substantial fraction of the cost of the printer. Credit cards have high late-payment fees. A justification that has been provided for such pricing scheme is myopia. Myopic is the consumer that is not aware or cannot take into consideration future attributes of the good when she buys at the primary market. This means that the willingness-to-pay of such a consumers for a good is not affected by the expected future cost of using this good.

Ellison (2005) discusses the optimality of high add-on prices and notes a couple of ways that could explain its introduction. Firstly, it may be simply seen as the outcome of a standard multi-good price discrimination model where the add-ons are better quality characteristics for the good very price sensitive consumers are present in the market. In this case, firms will charge higher markups on higher quality goods. On the other hand , Lal and Matutes (1994) claims that if add-on prices are not observed by consumers when choosing between firms, then add-ons will always be priced at the ex post monopoly price. Though, as Ellison (2005) points out this effect would vanish if in the market were present really price sensitive consumers. This is because in such a market the firms encounter a severe adverse selection problem. The firms with the price cuts for the initial good attract the price sensitive consumers that are less likely to buy add-ons, namely the part of the sale from which the firm is expecting to make profits.

Armstrong and Vickers (2012) discusses contingent charges, namely charges that are triggered only if particular contingencies arise. Contigencies that often catch consumers unaware, either because they were not aware of the fee or the fact that the triggering event would happen[9]. The most charges are contingent on a consumer choice, but key features are that the supplier can usually take payment without further agreement from the consumer and the perception that many consumers choose supplier in these markets without taking adequate account of the level of that supplier's contingent charges. After-market prices often appear to be high, and resistant to competitive challenge. They show that contingent charges are above marginal cost. Sophisticated consumers obtain better deals than naive consumers. Moreover, they study the economics of contingent charges in a stylized setting with naive and sophisticated consumers. Two kind of situation arise in the one the naive consumers benefit from the presence of sophisticated and in the other the sophisticated are subsidized at the expense of the naive consumers.

[9]see Shapiro (1995) for a critical discussion of a number of theories of after-market power, where that fact that seller cannot and does not precommit on after-market pricing has as a result a pricing that is inefficiently low in the begining and then high. Moreover, shrouding cannot survive, arguing that competitive firms should educate other firms' customers by offering to those customers efficient pricing schemes, and consequently win their business.

Gabaix and Laibson (2006) develops a boundedly rational explanation for why add-on prices often are not advertised. They claim that if the consumers are rational the optimal hidden add-on prices will be high and since they will anticipate it, firms will not shroud information in equilibrium. If consumers are all rational or aware, shrouding should actually hurt the firm, since it would hurt its reputation. Therefore, shrouding may occur in an economy where there are some myopic or unaware of hidden add-on prices consumers who incompletely analyze the future. Such shrouding creates an inefficiency, which firms may have an incentive to eliminate by educating their competitors' customers, which would hurt their competitors reputation, as mentioned before, wining over customers. In this kind of environment there are two ways of exploitation. Firstly, the firm is exploiting the naivety of the consumer by shrouding through marketing policies the highly charged adds-on. Moreover, the sophisticated consumers, namely the ones that are aware of the hidden features of the good in the future, end up with a subsidy from policies designed for myopic customers. This kind of marketing policy is profitable even in highly competitive markets or in markets with costless advertising.

Miao (2010) develops as well a model with myopic consumers who optimize period per period and it studies the optimal pricing of the aftermarkets, namely the add-ons. It is an overlapping generation model of consumers in a market with two firms which offer simultaneous a homogeneous product in the primary market and add-ons. It finds that firms charge add-ons with monopolistic prices through the strategic use of incompatibility. Moreover, it shows that the Bertrand competition result do not apply and the duopoly firms earn positive profits even if there is competition with homogeneous good. The monopolistic pricing of add-ons persist since neither firm has an incentive to compete by educating myopic consumers. This comes in contrast to views as Shapiro (1995) who claims that if there is significant competition in the primary market, then firms cannot make profits out of consumers myopia on the market of add-ons since the severity of the competition will affect the aftermarket as well. Therefore, Shapiro (1995) claims that competition can protect myopic consumers and there is no need for policy intervention.

There is a number of empirical papers that studies whether there are myopic consumers. Hausman (1979) was the first to study such a behavior when purchasing durable goods that differ in their consumption of energy. Moreover, there is literature that finds evidence of myopia using the market for cars[10]. These empirical papers exploit the link between the price of the car and the discounted value of the expected future fuel costs of that car. Though, they have different predictions. For example, Busse et al. (2013) studies whether car buyers are myopic about future fuel cost. They have a two step analysis. Firstly, they estimate how the equilibrium prices of cars are affected by the prices of gasoline. Then they use the estimated effect to calculate a range of implicit discount rates which result being similar to the range of interest rates paid by car buyers who borrow. They interpret this results as showing little evidence of consumer myopia. Thus, the presence of myopia depends on the type market, the marketing policies and the significance and the magnitude of the expenditure with respect to total income.

1.4 Three part tariff

Three part tariff is a complicated contract that consist of a fixed fee, an amount of free units, namely an allowance, and after the end of the allowance a marginal price which is positive and above marginal cost. This kind of tariff, namely a pricing scheme with increasing marginal prices, is difficult to be explain by models of rational consumers which in general predict that the marginal prices should be decreasing and non increasing[11].

A behavior that may explain the introduction of three part tariff is overconfidence. An overconfident consumer place overly narrow confidence intervals around forecasts, namely underestimate the variance of the valuation of the good in the future. Grubb (2009) shows that three features lead to the introduction of three part part tariff[12]. As

[10] Busse et al. (2013) has also a review of the related literature.

[11] For models that predict decreasing marginal prices see Mussa and Rosen (1978) and Maskin and Riley (1984)

[12] In Grubb (2009) the focus is on telecommunication markets, there is literature that studies this

mentioned already the main feature is over-confidence about the precision of the prediction when making difficult forecasts. Moreover, overconfidence combined with free disposal and relatively small marginal cost of production would explain its introduction. He claims that three part tariff is the optimal pricing scheme when necessarily the behavior of the consumer is characterized by overestimation of the demand given the demand being low and underestimation of the demand given it being high.

Grubb (2009) develops a model of a market consisting of a profit maximizing monopolist[13] and consumers that are ex ante uncertain about their future demand type, but they have information about the probability distribution of their type. Though, the consumers have mistaken beliefs about this distribution, more specifically they underestimate the variance of their future type. On the other hand, the firm being longer in the market and observing consumers behavior knows both that the consumer has mistaken beliefs and what their true distribution is[14]. The monopolist that faces such a consumer finds optimal to charge three part tariff.

The intuition for the optimality of the introduction of such a contract has two aspects. First, the consumer underestimates the probability of consuming low quantities and thus she does not find too costly to pay the fixed fee since the average price does not seem that high given her expectations. Second, the consumer learns her demand type during the contract period when the fixed fee is considered a sunk cost. Thus, if she is high demand type then she would accept to pay the high marginal price for high levels of demand. Moreover, considering the welfare implications, the consumer has losses if she consumes less than expected since she pays a relatively big fixed fee. However, if she consumes more than she would expect, she remains with zero ex post consumer surplus. Thus, the consumer is exploited only in the case she is optimistic and she expects that she will not consume too much.

Grubb (2014) shows that inattentive behavior, namely inattention to past usage

kind of behavior in other type of markets like the insurance market [Sandroni and Squintani (2010), Spinnewijn (2013)]

[13]It shows that also perfect competition has the same implications

[14]See Spiegler (2011) for a simplified and comprehensible example, in which he turns the model of monopoly pricing into a model with many states and overconfident consumers.

combined with awareness of the contract terms that leads to uncertainty about the marginal price of the next unit, has similar features to overconfidence. Thus, again it claims that both mistakes overestimation of the demand at the beginning of the contract period and underestimation in the end is needed for three part tariff to be optimal. However, the focus of Grubb (2014) is on the evaluation of the implications of the bill-shock regulation. A regulation that forces the firms to provide information to their clients that restores attention. He develops a model where the consumer consider the consumption dynamically within the contract period and not statically as a total quantity like in Grubb (2009). Except for the interesting implication from the optimal pricing point of view, it has interesting welfare implications as well. It claims that if the consumers in the market are sophisticated inattentive, namely they are aware of being inattentive, but heterogeneous in their expected demand, then a bill-shock regulation would reduce the social welfare in fairly-competitive markets.

An other environment that could explain the introduction of three part tariff is the one discussed by Eliaz and Spiegler (2006). It is developing a screening model where the agents that participate in the market are heterogeneous in terms of their sophistication about their dynamically inconsistent preferences. Thus, the agent differ only in their ability to forecast the change in their future tastes, namely she is aware that the tastes may change but she has mistaken beliefs about the probability with which it will change. On the other hand, the firm is informed about the dynamic inconsistency of the consumer but it does not observe the level of sophistication of each consumer. Eliaz and Spiegler (2006) fully characterize the menu of contracts which the principal offers in order to screen the agent's sophistication and maximize its expected profits. It claims that this menu of contracts can be implemented by a menu of three part tariffs, but it cannot be implemented by a menu of two part tariff.

The optimality for the introduction of a three part tariff stems from then non-common prior assumption which they interpreted as "a situation in which the agents have a systematic bias in forecasting their future tastes, whereas the principal has an unbiased forecast". This assumption seems to be really important in the case they study, where the consumer is uncertain as to whether his preference will change, but he knows exactly into what they could change. The firm takes advantage of its superior information and contracts also the event that the consumer thinks unlikely to happen. Even if the consumers are dynamic inconsistent and they evaluate their future actions according to their first period utility function the fact that they know in what their taste could change into gives space for exploitation. This feature becomes important because the contract is singed before the consumer experiences the change in her utility and she cannot renegotiate the contract after she signs it.

Eliaz and Spiegler (2008) consider a similar with Eliaz and Spiegler (2006) model though instead of screening dynamic inconsistent consumers with different cognitive ability, the firm offer a menu of contracts to optimistic consumers whose private information is their degree of optimism. A consumer is overoptimistic if she systematically assigns a bigger probability to the good state. Moreover, the good state is the state that the consumer would choose if she has such a possibility. The consumers have biased priors and two types of ex post demand high or low. A higher demand type would be a more optimistic outcome. They develop a model of price discrimination in the presence of incomplete information with similar feature to Eliaz and Spiegler (2006) .

At this point, for completeness it is useful to discuss that overconfidence is a well documented behavior both in the form of overoptimism and overprecision. First, there are several studies on field data that provide such evidence. These data consist of contract choices that provide information about the consumer beliefs and usage during the contract period. The comparison of the two leads to conclusions on consumer biases. Malmendier and Della Vigna (2006), using data of contract choice and attendance in health clubs, finds evidence of overoptimism about self control. Grubb and Osborne (2015) show that consumer are overconfident about the precision of their future forecasts, using data of cellular phone service. Moreover, there are studies that use data from the credit card market on changes in the interest rate and the likelihood of repaying debt that have findings consistent with overoptimism about self-control (Ausubel, 1999; Shui and Ausubel, 2005; Heidhues and Kőszegi, 2015). A common characteristic of all these markets is that the consumers are offered menu of complex contracts and they are asked to sign them ex ante before observing their type. This kind of environment makes more probable the emergence of such behavior where the consumer make mistakes about their ability or precision of forecasts.

Moreover, there are experiments that provide evidence of overconfidence. For example, Ericson (2011) develops an experiment where the participants are asked to choose between a small payment that they would receive automatically or a large payment that they should remember to claim after a period. They show that the majority of them chose the large payment but only 53 percent actually claimed it. This failure to maximize their payoff could be explained by overconfidence for their ability to remember an action even if this action is quite profitable.

1.5 Conclusion

In the last decades, there is a growing body of laboratory and field evidence that document individuals who do not optimize according to the standard preference but they experience persistent and systematic biases in their decision-making. The firms being longer in the markets and having the possibility to analyze big data, monitor this persistent and systematic deviation from standard preferences and they take it into consideration in their own decision-making.

This paper is a survey of the part of behavioral industrial economics literature that analyzes the optimal pricing policies of firms when in the market there are bounded rational consumers. There are several types of contracts that cannot be explained by standard preference and alternative, psychological explanations have been proposed.

In traditional economics with standard preference and rational consumers, the optimal contract would be a two part tariff that consists of a fixed fee and a marginal price equal to the marginal cost. Though, the last decades this is not what we observe in several markets. To the contrary we observe much different and more complex contracts. Each section of the survey is devoted to a different type of contract that we frequently observe but traditional economics cannot justify.

Moreover, this survey summarizes not only the implications that consumers' bounded rationality has but also the extent to which the consumers are aware of it. The different degree of sophistication of the consumers, namely whether they are informed of their biases or not is a parameter that affects significantly optimal pricing and it is part of this review as well.

The structure of this review is such as to summarize, organize and make a link between the pricing contracts that we observe in several markets and the behaviors that could explain the introduction of such a contract. Thus, it is beyond the scope of this survey the implications that bounded rationality has for the market complexity, market competition, market structure or product differentiation. Thus, the focus is on organizing and categorizing the different pricing policies that we frequently observe in the markets and on summarizing the potential explanations that the literature has provided for each of them.

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Chapter 2

The Optimal Pricing Scheme when Consumer is Naive Habit Forming

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Abstract

It is well established that consumption is often habit forming, namely the valuation of the good in each period is affected by whether consumption occurred in the preceding periods. This paper focuses on the market for access services, like communication services and subscription services and considers how consumers' habit formation affects the pricing policy of firms. Two types of consumers are considered, sophisticated and naive. The latter do not realize that their current consumption is affecting future consumption. Our main result is that under naive habit formation, the optimal pricing pattern is a three part tariff, namely a fixed fee, an amount of units priced below cost and after their end pricing above marginal cost, which is the observed pricing scheme in these markets. Different from Grubb (2009), we claim that only one mistake, in our case underestimation of future demand, is sufficient for three part tariff to be optimal.

2.1 Introduction

In several markets, a prevalent pricing pattern is that of three-part tariff, which includes a fixed fee, an allowance of free units, and a positive price for additional units beyond the allowance. Examples of this kind of markets are *communication services*, i.e. wireless phone services, internet access, and *subscription services*, i.e. on-line music download, on-line newspaper, data center hosting e.t.c. Such a pricing scheme, namely increasing prices, is hard to explain in standard models of non-linear pricing, which tend to predict that marginal prices should be decreasing.

These markets[1] have attracted the attention of psychologists, who have found that the stock of past consumption typically affects the consumption today. That is, preferences exhibit habit formation. Some researchers have suspected that consumption may be even addictive (Bianchi and Phillips, 2005; Park, 2005; Hooper and Zhou, 2007) for which there is not yet a clear answer.

Moreover, the existence and the implications of habit forming behavior has been studied in a number of different economic environments. There are two types of habit forming consumers that have been studied and we consider as well. On the one hand, the *Sophisticated (rational) Habit Forming* consumer who is aware that the today consumption affects future consumption (Becker and Murphy, 1988; Constantinides, 1990; Abel, 1990; Campbell and Cochrane, 1999; Jermann, 1998; Boldrin et al., 2001; Carroll et al., 2000; Fuhrer, 2000). On the other hand, the *Naive (myopic) Habit Forming* consumer who recognizes that her current satisfaction depends on past habits, but she neglects the impact of current decisions on her future preferences (Pollak, 1970; Loewenstein et al., 2003). Interestingly, Muellbauer (1988) provides an excellent overview of the two extremes, and concludes that the empirical evidence seems to be in favor of the presence of myopic habits.

This paper claims that the habit forming behavior of the consumers may explain why three part tariff is the optimal pricing policy for the firm. We show that with sophisticated habit forming consumers it is optimal to charge a two part tariff. How-

[1]Mostly, the markets of internet, cell phone calling and messaging.

ever, naive habit formation makes it optimal to charge three part tariff. We show that if the consumption choice is considered sequentially within the contract period, the consumer undervaluates the offered contract at the contracting period and underestimates the demand conditional on it being high, which are the main characteristics of a naive habit forming behavior, it is optimal for the firm to offer a three part tariff.

The consumption choice is considered sequentially means that the consumer does not form her expectation for future consumption statically as a total consumption in the beginning of the contract period but she considers dynamically each consumption opportunity. The consumer knows that she will update her consumption strategy for every consumption opportunity given her past consumption and the opportunity cost of foregone future purchases during the contract period. Furthermore, the fact that the consumer is unaware of her habit forming behavior at the contract period has two basic effects on the optimal pricing. First, she undervalues the offered contract since she cannot foresee that she will value the good more the more she consumes. Therefore, the firm cannot absorb all the consumer surplus at the contract period and thus finds it optimal to distort the marginal pricing. Second, she underestimates the probability of consuming in the future, since her past consumption makes her more prompt than she expects to consume in the future.

Naive habit formation could be viewed as an alternative channel other than overconfidence of Grubb (2009) that induces this kind of pricing scheme. Grubb (2009), after considering a number of alternative explanations, as for example the flat-rate bias, hyperbolic discounting e.t.c, claims that in order for such a pricing scheme to be optimal it is necessary that the consumer overestimates her demand conditional on it being low and underestimates the demand conditional on it being high, behavior which is actually captured by overconfidence. We show that both mistakes are not necessary if the elements mentioned before are taken into consideration and it is enough that she underestimates the demand conditional on it being high.

We develop a model where the consumer has two consumption opportunities. We begin with the benchmark model which consists of a not habit forming consumer and a monopolist. The optimal pricing scheme in the benchmark model is two part tariffs.

Then, we enrich the model by considering the sophisticated habit forming consumer. In this case, both the consumer and the monopolist have the same beliefs about the behavior of the consumer at the contractual stage, when she is called to accept or reject the contract. The optimal pricing scheme in this case is still *two part tariffs*. The habit forming behavior and the fact that the consumer is forward looking makes it more probable for this type of consumer than the non habit forming one to consume in both periods.

Moreover, we consider a naive habit forming consumer, namely a consumer who realizes that she is habit forming only after she has consumed. In this case, the monopolist has different prior beliefs from the consumer. The firm being longer in the market can recognize the type of the consumer, thus her habit forming behavior and her naivety. Given the naive habit forming behavior of the consumer, the optimal contract offered by the firm resembles a three part tariff. For low volumes, the price is smaller than the marginal cost, and then for high volumes it becomes bigger than the marginal cost. Moreover, when the marginal cost is low enough the optimal pricing scheme resembles even more the observed one, since low volumes are free of charge and high volumes are charged above marginal cost. Interestingly, when we enrich the model with Hotelling competition, which is an important element since this kind of market are characterized by tough competition, the optimal pricing scheme continues being three part tariffs.

The firm cannot absorb all the consumer surplus at the contracting period, because the consumer undervalues it, thus it has the incentive to distorts the optimal marginal prices in order to mitigate this undervaluation. The more habit forming is the consumer, namely the more she underestimates the utility gained from the consumption of the good, the more distorted are the optimal marginal prices. This is because the need for mitigation of the undervaluation is bigger. Therefore, the bigger the distortion the smaller is the optimal marginal price for low volumes and the bigger for high ones.

Interestingly, even if the optimal marginal price for low volumes is smaller than the marginal cost or even zero, the naive consumer underconsumes compared to the sophisticated one. This is because, the consumer being forward looking but naive, she takes into consideration the price change at the second unit, but not the future benefit of consuming. The firm finds it optimal to charge such a pricing scheme because even if the second period is the period when the firm could take advantage of the mistaken expectations of the consumer for the probability of consumption; at the same time, it is the period when the consumer cannot foresee the real value of the good. Thus, in order to mitigate the undervaluation, the firm finds it optimal to charge prices such that would decrease the probability of consumption for the naive consumer relative to the one that would be optimal for the sophisticated. In this way, it is less probable to consume at the first period and thus less probable for the firm to be at a situation where the consumer undervalues. Similarly, for high volumes the consumer underconsumes. Thus, the firm by charging this kind of contract exacerbate the mistake the consumer does because of her naivety in order to mitigate the profit losses it has.

The paper proceeds as follows. Section 2 discusses related literature. Section 3 is dedicated to the introduction of the benchmark model of a non habit forming consumer. Section 4 discusses the case of the sophisticated habit forming consumer and Section 5 the case of the naive habit forming consumer both for a monopolistic and an oligopolistic market.

2.2 Related Literature

This paper is related to different streams of the literature. First, it is clearly related to models that try to explain the introduction of three part tariffs. Grubb (2009) shows that over-confidence about the precision of the prediction when making difficult forecasts, free disposal and relatively small marginal cost would explain the use of three part tariff. He claims that three part tariff is the optimal pricing scheme when necessarily the behavior of the consumer is characterized by overestimation of the demand given the demand being low and underestimation of the demand given it being high. In our case, we propose a different behavior that could explain this pricing scheme without necessarily both mistakes being present. Moreover, we study an environment where the firm observes the amount actually consumed by the consumer in each period[2] and not only the amount the consumer has bought.

Grubb (2014) shows that inattentive behavior, having similar features to overconfidence, could explain the introduction of three part tariff. Thus, again it claims that both the mistake of overestimation of the demand at the beginning of the contract period and the mistake of underestimation in the end is needed for such a pricing scheme to be optimal. The common element between our model and Grubb (2014) is that we both consider the consumption dynamically within the contract period but we propose different type of behavior.

Eliaz and Spiegler (2008) consider a model where consumers have biased priors, that we do as well, but only two types of ex post demand high or low. The consumers are optimistic and think that the good state is more probable to happen. They describe a situation where consumers are dynamically inconsistent and she under or overestimate average demand. Thus, Eliaz and Spiegler (2008) studies a completely different behavioral bias.

Moreover, it is related to models of non linear pricing. Papers like Mussa and Rosen (1978) and Maskin and Riley (1984) explain contracts with high marginal prices for early units and marginal cost pricing for late units consumed; though, they cannot predict the inverse which is marginal prices below marginal cost at the early stage and an increase in the marginal prices later on.

In particular, we study the optimal pricing scheme when the good is habit forming thus papers that discuss the optimal pricing of habit goods (Nakamura and Steinsson, 2011; Fethke and Jagannathan, 1996) or even addictive (Becker et al., 1991; Driskill and McCafferty, 2001) are connected to our study. But unlike to this kind of literature we consider habit formation and optimal pricing within a contract period when a contract is signed at a zero period and there is no possibility for the firm to renegotiate the price during the contract period.

Moreover, the section that discusses the case of a naive habit forming consumer

[2]Though, we assume that the firm cannot observe all the consumption opportunities of the consumer

is closely related to a number of papers that consider the optimal non linear pricing induced by different consumer biases or nonstandard preferences. On the one hand, there are papers discussing biased beliefs like naive quasi-hyperbolic discounting for leisure good (DellaVigna and Malmendier, 2004; Eliaz and Spiegler, 2006), naivety about self-control (Esteban et al., 2007; Heidhues and Kőszegi, 2010) and myopia (Gabaix and Laibson, 2006; Miao, 2010). A common result of these papers is that due to this kind of biases and preferences there is underestimation of the demand which leads to high marginal prices above marginal cost. Thus, they cannot explain why the marginal prices are below marginal cost for low volumes of quantity.

On the other hand, biases like naive quasi-hyperbolic discounting for investment goods (DellaVigna and Malmendier, 2004) and flat rate bias (Herweg and Mierendorff, 2013; Lambrecht and Skiera, 2006) could explain optimal pricing below marginal cost but fail to explain prices above marginal cost.

Also, it is somehow related to literature in marketing that studies the effect of three part tariff. For example, Lambrecht et al. (2007) claims that it is a price discrimination tool with respect to the variation.

2.3 Benchmark

This section presents the basic structure of the model and the definition of the benchmark optimal pricing policy in the absence of habit forming behaviour.

The model follows Grubb (2012) in modeling a consumer who has two consumption opportunities and in each period purchases at most 1 unit. Moreover, the consumer is ex-ante uncertain about her per-period evaluation of the service.

2.3.1 The Model

Consider a model where there is mass 1 of consumers and 1 firm. The consumer is uncertain about her valuation of the good in each period.

The contract period is $T = 2$, at each period $t \in \{1, 2\}$ the consumer learns the realization of a taste shock υ_t, randomly drawn from a cumulative distribution function $F(\upsilon)$ with support [0,1]. This is the valuation that a unit of good has in period t. Then, given her valuation, she makes a binary quantity choice $q_t = \{0, 1\}$, considering whether or not to purchase the good.

The total payment $p(\mathbf{p}, \mathbf{q})$

$$p(\mathbf{q}) = p_1 q_1 + p_2 q_2 + F,$$

is a function of quantity choices $\mathbf{q} = (q_1, q_2)$ and the pricing scheme $\mathbf{p} = (F, p_1, p_2)$. The pricing scheme $\mathbf{p}$ consists of p_1 (the price of the first unit consumed), p_2 (the price of the second unit consumed), and F (a fixed payment)[3]. The timing of the game is described from Figure 3.1.

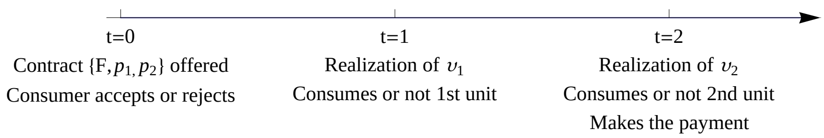

Figure 2.1: Timing of the game

The optimal consumption strategy, for given prices, is a function mapping valua-

[3]Contrary to Grubb (2012), we assume that the firm cannot distinguish the period in which the consumer is consuming the unit. The firm, when sells the first quantity does not know whether the consumer did not have before a consumption opportunity or she did have one but she preferred her outside option because the taste shock was not big enough. For this reason, the first unit has the same price irrespectively of the period t that is consumed. This kind of pricing would not be possible if the firm could observe and record the opportunities to consume, if for example the consumer had a direct communication with the firm in every opportunity to consume. Though, it is a relative assumption to assume that the firm cannot observe every time that the consumer is thinking to consume or not.

tions to quantities:

$$\mathbf{q}(\boldsymbol{v};\mathbf{p}) : \boldsymbol{v} \rightarrow \mathbf{q}$$

Moreover, the ex ante expected gross utility of the consumer from making optimal consumption choices is:

$$U = E[u(\mathbf{q}(\boldsymbol{v};\mathbf{p}), \boldsymbol{v}, \mathbf{p})]$$

The expected profits per consumer equal the revenues less the variable cost with marginal cost $c \geq 0$ per unit produced. The fixed cost is normalized to zero. Thus, the profit function is:

$$\Pi = E[p(\mathbf{q}(\boldsymbol{v};\mathbf{p}), \mathbf{p}) - c(q_1(\boldsymbol{v};\mathbf{p}) + q_2(\boldsymbol{v};\mathbf{p}))$$

Finally, the expected social surplus is:

$$S = E[\sum_{t=1}^{2}(v_t - c)\mathbf{q}(\boldsymbol{v};\mathbf{p})]$$

2.3.2 Consumer's optimization problem

The consumer compares the valuation of the unit with the actual price, every time that she is faced with a consumption decision. The actual price in each period is the optimal threshold above which the valuation should be in order for the consumer to consume the unit. Since the valuation of the unit is random for each unit and the consumer does not know it ex ante, she calculates the optimal threshold, as an optimal consumption rule different for each potential consumption decision. Thus, these thresholds are the consumption strategy of the consumer and thus the argument of maximization of her expected utility for the respective unit. In this way, she maximizes her ex ante utility.

The consumer maximizes its expected utility by solving backwards. For the second unit the optimal threshold is:

$$v^*_{2NH} = \begin{cases} p_1 & \text{if} \quad q_1 = 0 \\ p_2 & \text{if} \quad q_1 = 1 \end{cases}$$

This means that if the consumer has consumed the first unit, she will consume the second one if the randomly drawn valuation of the good is greater or equal to p_2. On the other hand, if she has not consumed before, then she faces the price of the first unit p_1 and consumes only if her valuation for the good is greater or equal than p_1.

Given v_2^*, the expected first period gross utility of the consumer is:

$$U_1 = -F + \int_{v_1^*}^{1} \left(v_1 - p_1 + \int_{p_2}^{1} (v_2 - p_2) f(v_2) dv_2 \right) f(v_1) dv_1 + F(v_1^*) \int_{p_1}^{1} (v_2 - p_1) f(v_2) dv_2 \tag{2.1}$$

The consumer maximizes her utility by choosing the optimal threshold v_1^*. The first part is the fixed fee, the second the expected utility if both units are consumed and the third if only the second unit is consumed.

The first order condition is:

$$\frac{dU_1}{dv_1^*} = \left(-v_1^* + p_1 - \int_{p_2}^{1} (v_2 - p_2) f(v_2) dv_2 + \int_{p_1}^{1} (v_2 - p_1) f(v_2) dv_2 \right) f(v_1^*) = 0$$

Thus, the first period optimal threshold is:

$$v_1^* = p_1 + \int_{p_1}^{p_2} (1 - F(v_2)) dv_2$$

The consumer is forward looking, so she takes into account the price change of future consumption opportunities. Thus, she considers that the cost of consuming the first unit is not only its marginal price but also the cost from not encountering p_1 but p_2 for the second unit. In particular, if $p_1 < p_2$ consuming the first unit raises the future price and thus there is an opportunity cost [4]. If $p_1 = p_2$ then the optimal threshold is just the marginal price p_1.

2.3.3 Firm's optimization problem

The firm is choosing the pricing scheme $\{p_1, p_2\}$ that maximizes its profits given the consumer optimal behaviour. The profits of the firm are equal to the social surplus since with the fixed fee F absorbs all the consumer surplus.

The expected social surplus is:

$$S = \int_{v_1^*}^{1} \left(v_1 - c + \int_{p_2}^{1} (v_2 - c) f(v_2) dv_2 \right) f(v_1) dv_1 + F(v_1^*) \int_{p_1}^{1} (v_2 - c) f(v_2) dv_2$$

[4] see Grubb (2012) equation (5). The intuition is that v_1^* equals the expected marginal price conditional on purchase plus the expected opportunity cost of foregone second-period purchase.

The maximization problem of the firm is:

$$\max_{p_1,p_2} S - U \quad s.t. \quad U \geq 0$$

The optimal pricing scheme is *two part tariff* with marginal cost pricing, namely $p_1 = p_2 = c$. The firm in this way is achieving the maximum possible social surplus and with a fixed fee F equal to the consumer surplus $U_1(c, c)$ calculated at the optimum, is absorbing all the produced economic surplus. Thus, it is not optimal for a monopolist to charge a three part tariff to a rational forward looking consumer.

2.4 Sophisticated Habit Formation

Let us now consider a consumer who is sophisticated habit forming. The consumption behavior of such a consumer at period t is affected by the quantity consumed at period $t-1$ and she is aware of it. This means that the consumer knows that her consumption today is affected by her consumption in the past, and that the consumption today will affect her consumption tomorrow.

Definition 2.1. *A sophisticated habit forming consumer is one who is aware that her current consumption will affect her consumption in the future.*

The valuation of the good for a sophisticated consumer at period t is:

$$\tilde{\upsilon}_t = \tilde{\upsilon}_t(q_{t-1}, \boldsymbol{\upsilon}) = \upsilon_t q_t + \beta q_{t-1} q_t \tag{2.2}$$

Now the consumer valuation for the unit does not depend only on its actual realized valuation at period t. Namely, as described by equation (2.2) the valuation of the unit at period t is a function not only of the realization of the valuation of the current unit but also of having consumed at the previous period or not. The habit formation coefficient β indicates the significance of the previous unit at the valuation of the current unit, and $\beta \in (0, 1]$. If the consumer has consumed before, she acquires a habit that increases her probability to consume and thus decreases her optimal threshold.

2.4.1 Consumer's optimization problem

As before the consumer maximizes her expected utility for each unit with respect to the optimal threshold for this unit. Solving backwards the consumer optimization problem, the *second period* maximization problem of the consumer depends on whether she has consumed the first unit or not.

If $q_1 = 1$, namely the first unit has been consumed, the consumer maximizes her expected utility from the consumption of the second unit as below:

$$\begin{aligned} \max_{v_{2S}^*} U_{2S} &= \int_{v_{2S}^*}^{1} (\tilde{v}_2 - p_2) f(v_2) dv_2 \\ &= \int_{v_{2S}^*}^{1} (v_2 + \beta - p_2) f(v_2) dv_2 \end{aligned}$$

, where the second period valuation is affected by the first period one and since the first unit has already been consumed the marginal price is p_2.

The first order condition is:

$$\begin{aligned} \frac{dU_{2S}}{dv_{2S}^*} = 0 &\Rightarrow (-v_{2S}^* - \beta + p_2) f(v_{2S}^*) = 0 \\ &\Rightarrow v_{2S}^* = p_2 - \beta \end{aligned}$$

Thus, the consumer compares the realized valuation v_2 with the optimal threshold $v_{2S}^* = p_2 - \beta$. If $v_2 > p_2 - \beta$ then she consumes the second unit otherwise she does not. Her optimal threshold consists of two parts the marginal price of the second unit since the first unit has been consumed, and the habit forming coefficient given that it has consumed and has acquire a habit. The more habit forming is the consumer the bigger is β and the more probable is that she will consume the second unit.

If $q_1 = 0$, namely the first unit has not been consumed, the consumer maximizes her expected utility from the second unit as follows:

$$\max_{v_{2S}^*} U_{2S} = \int_{v_{2S}^*}^{1} (v_2 - p_1) f(v_2) dv_2$$

,the marginal price is of the first quantity and the valuation is not affected by previous consumption, since there is none.

The first order condition is:

$$\frac{dU_{2S}}{dv_{2S}^*} = 0 \Rightarrow (-v_{2S}^* + p_1)f(v_{2S}^*) = 0 \Rightarrow v_{2S}^* = p_1$$

The optimal threshold of the consumer given that she did not consumed at period 1 equals to the marginal price of the first unit since the firm cannot charge depending on the consumption opportunities but the quantity consumed.

Summarizing, the second period threshold depends on the first period consumption as follows:

$$v_{2S}^* = \begin{cases} p_1 & \text{if} \quad q_1 = 0 \\ p_2 - \beta & \text{if} \quad q_1 = 1 \end{cases}$$

Given this optimal thresholds, the first period expected utility of the consumer is:

$$\begin{aligned} U_{1S} = & - F^S + \int_{v_{1S}^*}^{1} \left(v_1 - p_1 + \int_{p_2-\beta}^{1} (v_2 + \beta - p_2) dF(v_2) \right) dF(v_1) \\ & + F(v_{1S}^*) \int_{p_1}^{1} (v_2 - p_2) dF(v_2) \end{aligned} \tag{2.3}$$

, which is the ex ante utility of the consumer for the whole contract period. The first part is the fixed fee, the second the expected utility if both units have been consumed and the third if only the second unit has been consumed.

Consider now the choice of the first period threshold. The first order condition is:

$$\begin{aligned} \frac{dU_{1S}}{dv_{1S}^*} &= -v_{1S}^* + p_1 - \int_{p_2-\beta}^{1} (v_2 + \beta - p_2) f(v_2) dv_2 + \int_{p_1}^{1} (v_2 - p_1) f(v_2) dv_2 \\ &= -v_{1S}^* + p_1 - \int_{p_2-\beta}^{1} (1 - F(v_2)) dv_2 + \int_{p_1}^{1} (1 - F(v_2)) dv_2 = 0 \end{aligned}$$

Thus, the optimal first period threshold is:

$$v_{1S}^* = p_1 - \int_{p_2-\beta}^{p_1} (1 - F(v_2)) dv_2 \tag{2.4}$$

Again as before, the consumer being forward looking, does not compare her valuation just with p_1. Contrary, she takes into consideration both the fact that she is habit forming, thus that the consumption of the first unit affects and increases the valuation of the second one, and the fact that there is a difference in the marginal prices between the units. The threshold in this case is smaller than the marginal price of the first unit p_1[5].The habit forming consumer, given that she expects to experience a bigger utility in the future, because of consuming the first unit, finds it optimal to increase the probability of consuming the first unit and thus decrease the optimal threshold.

Comparing the optimal threshold of the non habit forming consumer with the one of the habit forming consumer, it is evidents that the latter is consuming more often. The optimal threshold of a non habit forming consumer is greater than the one of a sophisticated habit forming consumer for the first unit $v^*_{1NH} > v^*_{1S}$ and for the second unit when she has already consumed the first one. For example, in the case that $p_1 = p_2$, the non habit consumer would have as an optimal first unit threshold just the marginal price p_1 since there is neither additional expected cost from consuming the first unit nor second period foregone utility because of the difference of the prices when the threshold for the habit forming consumer would be smaller.

Moreover, the more habit forming is the consumer, namely the more significant is for her the past consumption and thus the bigger is the habit forming coefficient β, the smaller is the optimal threshold in both periods and the more probable is to consume both units. This could easily be seen and summarized by the derivatives below:

$$\frac{dv^*_{1S}}{d\beta} = -v^*_{1S}(1 - F(p_2 - \beta)) < 0 \quad \text{and} \quad \frac{dv^*_{2S}}{d\beta} = -1 < 0$$

2.4.2 Firm's optimization problem

In the case of the sophisticated habit forming consumer, as before the firm is maximizing its profits by choosing the marginal prices that maximize the expected gross surplus subject to the participation constraint of the consumer. Thus, the maximization problem of the firm is:

$$\max_{p_1,p_2} S^S - U_{1S} \quad s.t. \quad U \geq 0$$

,where S^S is the expected gross surplus:

$$S^S = \int_{v^*_{1S}}^{1} (v_1 - c) dF(v_1) + F(v^*_{1S}) \int_{p_1}^{1} (v_2 - c) dF(v_2) + \int_{v^*_{1S}}^{1} \int_{p_2-\beta}^{1} (v_2 + \beta - c) f(v_2) dv_2 dF(v_1)$$

[5]This holds when $p_1 > p_2 - \beta$, which on its turn holds at the optimal when the consumer is sophisticated habit forming.

and U_{1S} the expected utility of the consumer from the contract.

The optimal pricing scheme for the firm is, as before, *two part tariff* where $p_1 = p_2 = c$, thus marginal cost pricing and the fixed fee F^S, which is:

$$F^S = \int_{v_{1S}^*}^{1} \left(v_1 - p_1 + \int_{p_2-\beta}^{1} (v_2 + \beta - p_2) f(v_2) dv_2 \right) f(v_1) dv_1 + F(v_{1S}^*) \int_{p_1}^{1} (v_2 - p_2) f(v_2) dv_2$$

, calculated at the optimal prices.

Lemma 2.1. *If the consumer is sophisticated habit forming, the equilibrium allocation is the first best allocation. There is marginal cost pricing, namely the prices that maximize the profits of the firm are* $(p_1, p_2) = (c, c)$

Proof: The firm maximizes its profit by charging marginal prices that induce the first best allocation and then with the fixed fee F^S it absorbs all the consumer surplus. The firm does the rent extraction in such a way that is balanced with the participation as in a basic monopoly pricing problem.

As in the benchmark case, the firm finds it optimal to charge marginal cost prices produce the maximum social surplus and then with the fixed fee to absorb all the consumer surplus. Therefore, it makes profits equal to the economic surplus. The firm has no incentive to distort this pricing scheme since with this scheme makes the maximum possible profits and thus three part tariff cannot be explained by such a behavior.

2.5 Naive Habit Formation

In this section, we discuss the case of a naive habit forming consumer. This is a consumer who is habit forming but is not aware of being it. Moreover, we relax the common prior assumptions and we assume that while the consumer is naive the firm correctly anticipates her dynamic preference function.

Definition 2.2. *Naive habit forming consumer is the consumer that is able to realize that the consumption today is affected by yesterday consumption but cannot realize that the today consumption can affect the consumption tomorrow.*

2.5.1 Consumer's optimization Problem

The consumer that has this kind of behavior is equivalent to one that does not know that she is habit forming at the first period. This means that ex ante has the mistaken belief that her consumption of first unit will not influence the consumption of the second one.

Thus, the second unit threshold is the same as for the non habit forming consumer.

$$v_{2N}^* = v_{2NH}^* = \begin{cases} p_1 & \text{if} \quad q_1 = 0 \\ p_2 & \text{if} \quad q_1 = 1 \end{cases}$$

Comparing this with the second unit optimal threshold of the sophisticated consumer v_{2S}^*, it is evident that she makes a mistake in the case that she does consume at the first period. Moreover, from the perspective of the contracting period 0 she expects that her preference will not change but if $q_1 = 1$ then her actual valuation will be v_{2S}^* and thus she underestimate her demand.

Given the second period optimal thresholds, the naive habit forming consumer maximizes the perceived first period utility:

$$\begin{aligned} \max_{v_{1N}^*} U^N = & - F^N + \int_{v_{1N}^*}^1 (v_1 - p_1) dF(v_1) + (1 - F(v_{1N}^*)) \int_{p_2}^1 (1 - F(v_2)) dv_2 \\ & + F(v_{1N}^*) \int_{p_1}^1 (1 - F(v_2)) dv_2 \end{aligned} \tag{2.5}$$

, thus the optimal threshold v_{1N}^* is the same as the one of the non habit forming consumer:

$$v_{1N}^* = v_{1NH}^* = p_1 + \int_{p_1}^{p_2} (1 - F(v_2)) dv_2 \tag{2.6}$$

In fact, the true ex-ante utility of the consumer is the one of the sophisticated habit forming consumer with the difference that the first period optimal threshold is the one of the non habit forming consumer:

$$\begin{aligned}\tilde{U} = &- F^N + \int_{v_{1N}^*}^{1} (v_1 - p_1) dF(v_1) + (1 - F(v_{1N}^*)) \int_{p_2-\beta}^{1} (1 - F(v_2)) dv_2 \\ &+ F(v_{1N}^*) \int_{p_1}^{1} (1 - F(v_2)) dv_2\end{aligned} \tag{2.7}$$

Thus, the consumer at the contractual period, when she is choosing the optimal for her contract, she perceives herself as non habit forming[6] and considers these as her optimal consumption rules since she believes that her expected utility is U^N. Though, her actual expected utility and the one that the firm expects that she will have is $\tilde{U}$.

The consumer consumes the first unit less often than she should if she was sophisticated. Though, at the first period she does not do any mistake due to her naivety. She uses the same threshold she planed to use when she chose her contract at period 0. The probability of consuming in the first period is $1 - F(v_{1N}^*)$ as it is expected at the contracting period. Thus, there is no inconsistency between her expectation about her future self and how actually acts, namely no mistake that the firm could take advantage of. The only implication that the first threshold has is that affects the magnitude of the consumer surplus and consecutively the social surplus. Due to the naivety of the consumer, the expected consumer surplus is smaller than the one that could be produced if the consumer was sophisticated. Though, it has no implication on the pricing scheme since the consumer does not consume more or less than expected.

In the second period, given that the consumer has not consumed before ($q_1 = 0$), she does not realize that she is habit forming and thus she consumes as much as she was expecting to consume at the contract period. The probability of consuming is $F(v_{1N}^*)(1 - F(p_1))$ and it is not different from what the consumer would expect. The consumer does not overestimate the probability of buying only one unit, actually does not make any mistake given that her consumption is low.

[6]This whole analysis hold also when the consumer is partially naive, namely she knows that she is habit forming but she believes that she is less habit forming than she actually is. In this case, the perceived valuation of the good is $\tilde{v}_t = v_t q_t + \hat{\beta} q_{t-1} q_t$ and $\hat{\beta} < \beta$. As in the case of the naive consumer the partially naive consumer has not mistaken beliefs given the demand being low but she underestimates her demand given it being high. See the Appendix.

On the other hand, given that the consumer has consumed before ($q_1 = 1$), she underestimates the probability of consuming two units. She would expect that her optimal threshold in this case would be p_2 but she realizes that it is $p_2 - \beta$. Thus, the probability of consuming at the second period is expected to be $(1-F(v_{1N}^*))(1-F(p_2))$ but given she has consumed at period 1 she realizes that it is $(1-F(v_{1N}^*))(1-F(p_2-\beta))$. This follows from the fact that the consumer believes that she is not habit forming and she realizes only after she has consumed. This means that at the contracting period the consumer underestimates her demand given she has high demand, namely underestimates the probability of consuming the second unit.

Lemma 2.2. *The naive habit forming consumer is a consumer that makes no mistake given that her demand is low and underestimates her demand given it is high.*

This mistake of the consumer is also one of the elements that make the consumer to undervaluate the offered contract at the contracting period. Firstly, the consumer does not anticipate that consuming in the first period will increase the valuation of her second unit so she does not expect the β additional valuation. Secondly, since she does not anticipate that she is habit forming she underestimates the probability of consuming $(1 - F(v_{1N}^*))(1 - F(p_2)) < (1 - F(v_{1N}^*))(1 - F(p_2 - \beta))$ the second unit and thus acquiring this extra utility.

2.5.2 Firm's optimization Problem

Monopoly

The firm recognizes that it faces a naive habit forming consumer and that her participation to the market will depend on her mistaken expected utility. Moreover, it knows that the social surplus that is produced is given by:

$$S^N = \int_{v_{1N}^*}^1 (v_1-c)dF(v_1)+(1-F(v_{1N}^*))\int_{p_2-\beta}^1 (v_2+\beta-c)dF(v_2)+F(v_{1N}^*)\int_{p_1}^1 (v_2-c)dF(v_2)$$

The firm considers that in the first period the consumer does not know that she is habit forming and consumes only if the valuation of the unit is greater than v_{1N}^*.

Moreover, it takes into account that given that she has consumed in the first period she realizes that she is habit forming. Therefore, it considers that she will update her second unit threshold and her valuation for the second unit, if she has consumed in the first period.

The firm maximizes its profits as the difference between the social surplus and the consumer surplus subject to the participation constraint of the consumer. In the case though of the naive habit forming consumer, the true consumer surplus produced $\tilde{U}$ (equation 2.7) is different from the one the consumer expects at contracting period U^N (equation 2.5). Thus, the optimization problem of the firm is:

$$\begin{aligned}\max_{U^*,p_1,p_2} \Pi &= S^N - \tilde{U} \\ &= S^N - U^N - (\tilde{U} - U^N) \\ &= S^N - U^N - \Delta \\ \text{s.t.} \quad & U^N \geq 0\end{aligned}$$

where

$$\Delta = \tilde{U} - U^N =$$

$$\int_{v_{1N}^*}^1 \big(v_1 - p_1 + \int_{p_2-\beta}^1 (v_2 + \beta - p_2) dF(v_2)\big) dF(v_1) + F(v_{1N}^*) \int_{p_1}^1 (v_2 - p_1) dF(v_2) - F^N$$
$$-\left(\int_{v_{1N}^*}^1 \big(v_1 - p_1 + \int_{p_2}^1 (v_2 - p_2) dF(v_2)\big) dF(v_1) + F(v_{1N}^*) \int_{p_1}^1 (v_2 - p_1) dF(v_2) - F^N\right)$$

is the difference between the true expected utility of the contract $\tilde{U}$ and the mistaken expected utility U^N, given the respective optimal consumption rules, as shown by equation (2.6). The firm cannot absorb all the consumer surplus, it chooses a pricing scheme that makes the participation constraint bidding, $U^N = 0$. Though, there is the part Δ which is the expected utility that the firm knows and expect for the consumer to have. After some simplifications, Δ is:

$$\Delta = (1 - F(v_{1N}^*)) \left(\int_{p_2-\beta}^{p_2} (1 - F(v_2)) dv_2\right)$$

Then, the maximization problem of the firm becomes:

$$\max_{p_1,p_2} \Pi = S^N - U^N - (\tilde{U} - U^N) = S^N - \Delta$$

calculating the marginal prices that maximize the above expression we are lead to Proposition 1.

Proposition 2.1. *Monopoly: If the consumer is naive habit forming the optimal pricing scheme is:*

$$\begin{aligned} c = 0: &\quad p_1^N = 0, \quad p_2^N > c, \quad F^N = U^N(\mathbf{p^N}) \\ c > 0: &\quad p_1^N < c, \quad p_2^N > c, \quad F^N = U^N(\mathbf{p^N}) \end{aligned} \qquad \textit{“three part tariff”}$$

Proof: See Appendix A

The firm that faces a naive habit forming consumer has an incentive to distort the efficient allocation in order to maximize its profits. Since the consumer has a mistaken belief about her expected utility which leads to a mistaken participation constraint, the firm cannot maximize its profits by maximizing the social surplus and charging a fixed fee equal to the consumer surplus. This is because the expected from the consumer surplus is smaller than the surplus actually produced and thus the firm cannot ask as a fixed fee the one that maximizes its profits. Therefore, the firm needs to distort the marginal prices by choosing the ones that maximize $S^N - \Delta$ and not just S^N. Thus, the undervaluation of the contract by the consumer explains why the firm charges prices different than the marginal cost.

The reason why the marginal charges are distorted in this way is explained by two characteristics of the consumer's behavior, firstly that she is forward looking and secondly that she underestimates the probability of consuming the second unit. Since the consumer underestimates the probability of consuming the second unit the firm has an incentive to charge a price bigger than the marginal cost as it has become evident also by the literature on hyperbolic discounting and myopia. On the other hand, given that the consumer is forward looking and takes into consideration the opportunity cost of consuming the first unit and she would face an augmented marginal price for the second unit, the firm finds it optimal to decrease below cost the marginal charge of the first unit in order not to become extremely costly for the consumer its consumption. But more importantly, because in this way the second unit mistake is exacerbated. A distorted price below marginal cost makes it more probable the consumption of the first unit and thus also the consumption of the second unit.

For the above reasons, the optimal pricing scheme when the consumer is naive habit forming resembles to the scheme we observe at several markets, namely *three part tariff*, which is a fixed fee, an included allowance of units for which the marginal price equals to zero and a positive marginal price for units beyond the allowance. When the marginal cost is equal to zero the marginal price of the first unit is equal to zero and the marginal price of the second unit is bigger that the marginal cost and it becomes bigger the bigger is the habit forming coefficient β.

Interestingly, even if it seems that for the first unit there would be overconsumption, the fact that the consumer is forwards looking and naive of his habit forming behavior produces the inverse result. For example, when the marginal cost is zero, $c = 0$, even if the marginal price of the first unit is zero its optimal threshold is positive thus there is underconsumption with respect to the efficient allocation of the sophisticated habit forming consumer. Moreover, the bigger is the habit formation coefficient β the bigger is the first unit threshold v_1^* since the bigger is the difference between the first and second unit optimal marginal price.

The reason why the firm finds it optimal to charge such a pricing scheme that induce underconsumption at the first period contrary to our initial intuition is because in this way mitigates the undervaluation of the contract and thus decreases Δ. In the profit function of the firm there are two opposites effects. On the one hand, the firm wants to charge such a price that would inflate the second period mistake, namely a price that would make more probable the consumption of the first unit and consequently of the second unit whose consumption could take advantage of. On the other hand though, the firm have the incentive to minimize Δ in order to maximize its profits. The firm even if can overcharge the second unit because of the mistaken beliefs, it cannot absorb ex ante all the consumer surplus from this period. Therefore the firm chooses such a pricing scheme that makes it less probable to arrive to the second period.

Similarly, there is underconsumption of the second unit. The optimal second unit threshold for the naive consumer at the optimal is always bigger than the one of the sophisticated consumer, $v_2^N > v_2^S$. The consumer consumes more often than if the optimal price was equal to the marginal cost but always consumes less than the efficient allocation of the sophisticated habit forming consumer, consequently there is underconsumption.

As it has already been mentioned, the consumer even if always consumes less than the optimal, she is left with consumer surplus, the firm cannot absorb it all. This would give an incentive to the consumer to remain naive and not pay the cost of getting sophisticated and learning her true type. To the contrary, remaining naive is beneficial for her and she avoids any information cost.

Oligopoly-Hotelling Competition

In this subsection we introduce competition in order to examine if the pricing structured that is optimal under monopoly would be also optimal in an oligopolistic environment.

Let a market with a continuum of naive habit forming consumers uniformly distributed on a uniform Hotelling line and two firms $i = \{A, B\}$, positioned at the extremes of the line.

The maximization problem of the firm i is:

$$\max_{U_i} \Pi_i = D(U_i, U_{-i})(S_i^N - U_i^N + \Delta_i)$$
$$\text{s.t.} \quad U_i^N \geq 0$$

,where $D(U_i, U_{-i}) = \dfrac{U_i - U_{-i} + \tau}{2\tau}$ is the market share function[7]. The competition is in the utility space and τ is the transportation cost. Moreover, S_i^N is the social surplus, U_i^N is the consumers surplus and Δ_i the difference between the actual and the mistaken expected consumer surplus that are created by firm i. These functions

[7]The consumer who is indifferent between the two firms is given by t$U_i - \tau x = U_{-i} - \tau(1-x) \Rightarrow x = \frac{U_i - U_{-i} + \tau}{2\tau}$

are as follows:

$$S_i^N = \int_{v_{1N}^*}^1 (v_1 - c)dF(v_1) + (1 - F(v_{1N}^*)) \int_{p_{2i}-\beta}^1 (v_2 + \beta - c)dF(v_2)$$
$$+ F(v_{1N}^*) \int_{p_{1i}}^1 (v_2 - c)dF(v_2)$$

$$U_i^N = \int_{v_{1N}^*}^1 (v_1 - p_{1i})dF(v_1) + (1 - F(v_{1N}^*)) \int_{p_{2i}}^1 (v_2 - p_{2i})dF(v_2)$$
$$+ F(v_{1N}^*) \int_{p_{1i}}^1 (v_2 - p_{1i})dF(v_2) - F_i$$
$$\Delta_i = (1 - F(v_{1N}^*)) \left(\int_{p_{2i}-\beta}^{p_{2i}} (1 - F(v_2))dv_2 \right)$$

If there is strict full market coverage when firms set marginal prices optimally and charge markup τ, then this is the equilibrium. [Armstrong-Vickers (2001)]

In this case, there is strict full market coverage when:

$$\frac{2}{3}(S_i + \Delta_i) \geq \tau$$

If we make the assumption that the above inequality holds and thus there is full market coverage in this market, then we could claim Proposition 2.2.

Proposition 2.2. *Hotelling Duopoly: Let τ be sufficiently small for strict full market coverage and the consumer be naive habit forming then the optimal pricing scheme is:*

$$c = 0: \quad p_1^{N*} = 0, \quad p_2^{N*} > c, \quad F_i^{N*} = \tau$$
$$c > 0: \quad p_1^{N*} < c, \quad p_2^{N*} > c, \quad F_i^{N*} = \tau$$

The prices equal to the monopolists one, the competition among the firms is being made in the utility space and thus it affects only the fixed fee charged comparing to the contract offered by the monopolist. The pricing scheme is the exactly the same in both cases since there is full market coverage. Interestingly, three part tariff contract is still the optimal pricing scheme when there is competition.

As we would expect, the more intensive the competition, namely the smaller τ, the less scope for price discrimination.

2.6 Conclusion

There is evidence that the consumption of communication services like cell phones and internet are habit forming, in the sense that past consumption affects current consumption. Moreover, in this kind of markets, "three part tariff" contracts, namely a fixed fee, an allowance of free units, and a positive price for additional units beyond the allowance, are becoming increasingly popular.

This paper claims that habit forming behavior is an important characteristic of this kind of markets and plays a significant role to their pricing policy. Moreover, naive habit formation makes it optimal for the firm to charge "three part tariff". We show that if the consumption choice is considered sequentially within the contract period, the consumer undervaluates the offered contract at the contracting period and underestimates the demand conditional on it being high, which are the main characteristics of a naive habit forming behavior, it is optimal for the firm to offer a three part tariff.

The monopolist has different prior beliefs from the consumer. The firm being longer in the market can recognize the type of the consumer, thus her habit forming behavior and her naivety. Interestingly, when we enrich the model with Hotelling competition, which is an important element since this kind of markets are characterized by tough competition, the optimal pricing scheme continues being three part tariffs.

It could be viewed as an alternative channel other than overconfidence of Grubb (2009) that induces this kind of pricing scheme. We show that both mistakes are not necessary if the elements mentioned before are taken into consideration and it is enough that she underestimates the demand conditional on it being high.

The firm cannot take advantage of the naivety of its client, to the contrary it is worse off when it encounters a naive habit forming consumer with respect to a sophisticated one. This is because the firm cannot ask for a fixed fee that absorbs all the consumer surplus at the contractual period. The firm would have an incentive to inform the consumer about their naivety but this would make the consumer worst off. On the other hand the consumer has no incentive to pay the cost of getting informed about her own type.

Interestingly, even if the optimal marginal price for low volumes is smaller than the marginal cost or even zero, the naive consumer underconsumes compared to the sophisticated one. This is because, the consumer being forward looking but naive, she takes into consideration the price change at the second unit, but not the future benefit of consuming. The firm finds it optimal to charge such a pricing scheme because even if the second period is the period when the firm could take advantage of the mistaken expectations of the consumer for the probability of consumption. At the same time, it is the period when the consumer cannot foresee the real value of the good. Thus, in order to mitigate the undervaluation, the firm finds it optimal to charge prices that decrease the probability of consumption relative to the one of the sophisticated. In this way, it is less probable to consume at the first period and thus less probable for the firm to be at a situation where the consumer undervalues. Similarly, for high volumes the consumer underconsumes. Thus, the firm by charging this kind of contract exacerbate the mistake the consumer does because of her naivety in order to mitigate the profit losses it has.

This kind of model is not only applicable in the telecommunication market but also in other type of markets where the goods could be viewed as habitual and we observe this pricing pattern. For example, on-line music download, on-line newspaper and data center hosting could be viewed as alternative type of services.

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Appendix A

Proof of Proposition 1 Thus the optimization problem of the firm is:

$$\max_{U^*,p_1,p_2} \Pi = S^S - U^N + (U^N - U^S) = S - U^N + \Delta \quad \text{s.t.} \quad U^N \geq 0$$

and optimal consumption rule is:

$$v_{1N}^* = p_1 + \int_{p_1}^{p_2} (1 - F(v_2))dv_2$$

The expected gross surplus is the one produced in a market with a habit forming consumer.

$$\begin{aligned} S = &\int_{v_{1N}^*}^1 (v_1 - c)dF(v_1) + F(v_{1N}^*) \int_{p_1}^1 (v_2 - c)dF(v_2) + \int_{v_{1N}^*}^1 \int_{p_2-\beta}^1 (v_2 + \beta - c)f(v_2)dv_2dF(v_1) \\ = &\int_{v_{1N}^*}^1 (v_1 - c)dF(v_1) + F(v_{1N}^*) \int_{p_1}^1 (v_2 - c)dF(v_2) + \int_{v_{1N}^*}^1 \int_{p_2-\beta}^1 (v_2 + \beta)f(v_2)dv_2dF(v_1) \\ &- c\int_{v_{1N}^*}^1 f(v_1)(1 - F(p_2 - \beta)) \end{aligned}$$

Moreover, Δ is the difference between the perceived and the optimal utility of the consumer.

$$\Delta = U^N - U^S = (1 - F(v_{1N}^*)) \int_{p_2}^1 (1 - F(v_2))dv_2 + p_2 \int_{v_{1N}^*}^1 f(v_1)(1 - F(p_2 - \beta))dv_1$$

Simplifying and deleting $\int_{v_{1N}^*}^1 \int_{p_2-\beta}^1 (v_2+\beta)f(v_2)dv_2dF(v_1)$ from S and Δ then the first order conditions are:

with respect to p_1:

$$\frac{\partial \Pi}{\partial p_1} = \frac{\partial S}{\partial v_{1N}^*}\frac{\partial v_{1N}^*}{\partial p_1} + \frac{\partial \Delta}{\partial v_{1N}^*}\frac{\partial v_{1N}^*}{\partial p_1} + \frac{\partial \Delta}{\partial p_1}$$

$$\begin{aligned} \frac{\partial S}{\partial v_{1N}^*} &= \left(-v_{1N}^* + c + \int_{p_1}^1 (v_2 - c)dF(v_2) + c(1 - F(p_2 - \beta))\right) f(v_{1N}^*) \\ \frac{\partial \Delta}{\partial v_{1N}^*} &= \left(-\int_{p_2}^1 (1 - F(v_2))dv_2 - p_2(1 - F(p_2 - \beta))\right) f(v_{1N}^*) \\ \frac{\partial v_{1N}^*}{\partial p_1} &= 1 - (1 - F(p_1)) = F(p_1) \\ \frac{\partial S}{\partial p_1} &= -F(v_{1N}^*)(p_1 - c)f(p_1) \end{aligned}$$

Then the first order condition is:

$$\frac{\partial \Pi}{\partial p_1} = \left(-v_{1N}^* + c + \int_{p_1}^{1} (v_2 - c) dF(v_2) + c(1 - F(p_2 - \beta)) \right) F(p_1) - \frac{F(v_{1N}^*) f(p_1)}{f(v_{1N}^*)} (p_1 - c)$$
$$- F(p_1) \int_{p_2}^{1} (1 - F(v_2)) dv_2 - p_2(1 - F(p_2 - \beta)) F(p_1) =$$
$$= -v_{1N}^* + c + \int_{p_1}^{1} (v_2 - c) dF(v_2) + c(1 - F(p_2 - \beta) - \frac{F(v_{1N}^*) f(p_1)}{f(v_{1N}^*) F(p_1)} (p_1 - c)$$
$$- \int_{p_2}^{1} (1 - F(v_2)) dv_2 - p_2(1 - F(p_2 - \beta)) =$$
$$= -p_1 - \int_{p_1}^{p_2} (1 - F(v_2)) dv_2 + c + \int_{p_1}^{1} (v_2 - c) dF(v_2) + c(1 - F(p_2 - \beta))$$
$$- \frac{F(v_{1N}^*) f(p_1)}{f(v_{1N}^*) F(p_1)} (p_1 - c) - \int_{p_2}^{1} (1 - F(v_2)) dv_2 - p_2(1 - F(p_2 - \beta)) =$$
$$= p_1 + c - 1 + p_1 + 1 - p_1 F(p_1) - c(1 - F(p_1)) + c(1 - F(p_2 - \beta))$$
$$- \frac{F(v_{1N}^*) f(p_1)}{f(v_{1N}^*) F(p_1)} (p_1 - c) - p_2(1 - F(p_2 - \beta))$$

Then

$$p_1 \left(F(p_1) + \frac{F(v_{1N}^*) f(p_1)}{f(v_{1N}^*) F(p_1)} \right) = c \left(F(p_1) + 1 - F(p_2 - \beta) + \frac{F(v_{1N}^*) f(p_1)}{f(v_{1N}^*) F(p_1)} \right)$$
$$- p_2(1 - F(p_2 - \beta))$$
$$p_1 \left(\frac{F(p_1)^2 f(v_{1N}^*) + F(v_{1N}^*) f(p_1)}{F(p_1) f(v_{1N}^*)} \right) = c \left(\frac{F(p_1)^2 f(v_N) + F(v_{1N}^*) f(p_1)}{F(p_1) f(v_{1N}^*)} \right) + c(1 - F(p_2 - \beta))$$
$$- p_2(1 - F(p_2 - \beta))$$

Thus

$$p_1 = c - (p_2 - c) \left(\frac{F(p_1) f(v_{1N}^*)(1 - F(p_2 - \beta))}{F(p_1)^2 f(v_{1N}^*) + F(v_{1N}^*) f(p_1)} \right) \tag{2.8}$$

with respect to p_2:

$$\frac{\partial \Pi}{\partial p_2} = \frac{\partial S}{\partial v_{1N}^*} \frac{\partial v_{1N}^*}{\partial p_2} + \frac{\partial \Delta}{\partial v_{1N}^*} \frac{\partial v_{1N}^*}{\partial p_2} + \frac{\partial \Delta}{\partial p_2}$$

$$\frac{\partial S}{\partial p_2} = c \int_{v_{1N}^*}^{1} f(v_1) f(p_2 - \beta) dv_1$$
$$\frac{\partial \Delta}{\partial p_2} = -(1 - F(v_{1N}^*))(1 - F(p_2)) + \int_{v_{1N}^*}^{1} f(v_1)(1 - F(p_2 - \beta)) dv_1 - p_2 \int_{v_{1N}^*}^{1} f(v_1) f(p_2 - \beta) dv_1$$
$$\frac{v_{1N}^*}{p_2} = 1 - F(p_2)$$

$$
\begin{aligned}
\frac{\partial \Pi}{\partial p_2} &= \left(-v_{1N}^* + c + \int_{p_1}^1 (v_2 - c) dF(v_2) + c(1 - F(p_2 - \beta)) \right) f(v_{1N}^*)(1 - F(p_2)) \\
&+ c \int_{v_{1N}^*}^1 f(v_1) f(p_2 - \beta) dv_1 - (1 - F(v_{1N}^*))(1 - F(p_2)) \\
&+ \left(- \int_{p_2}^1 (1 - F(v_2)) dv_2 - p_2(1 - F(p_2 - \beta)) \right) f(v_{1N}^*)(1 - F(p_2)) \\
&+ \int_{v_{1N}^*}^1 f(v_1)(1 - F(p_2 - \beta v_1)) dv_1 - p_2 \int_{v_{1N}^*}^1 f(v_1) f(p_2 - \beta) dv_1 \\
&= -v_{1N}^* + c + \int_{p_1}^1 (v_2 - c) dF(v_2) + c(1 - F(p_2 - \beta)) \\
&+ c \left(\frac{\int_{v_{1N}^*}^1 f(v_1) f(p_2 - \beta) dv_1}{f(v_{1N}^*)(1 - F(p_2))} \right) - \int_{p_2}^1 (1 - F(v_2)) dv_2 - p_2(1 - F(p_2 - \beta)) \\
&- \frac{1 - F(v_{1N}^*)}{f(v_{1N}^*)} + \frac{\int_{v_{1N}^*}^1 f(v_1)(1 - F(p_2 - \beta)) dv_1}{f(v_{1N}^*)(1 - F(p_2))} - p_2 \left(\frac{\int_{v_{1N}^*}^1 f(v_1) f(p_2 - \beta) dv_1}{f(v_{1N}^*)(1 - F(p_2))} \right) = 0
\end{aligned}
$$

Substituting for the optimal threshold and after some algebra [8]

$$
\begin{aligned}
\frac{\partial \Pi}{\partial p_2} =& c - p_1 F(p_1) - c(1 - F(p_1)) + c(1 - F(p_2 - \beta)) + c \left(\frac{\int_{v_{1N}^*}^1 f(v_1) f(p_2 - \beta) dv_1}{f(v_{1N}^*)(1 - F(p_2))} \right) \\
& - p_2(1 - F(p_2 - \beta)) - \frac{1 - F(v_{1N}^*)}{f(v_{1N}^*)} + \frac{\int_{v_{1N}^*}^1 f(v_1)(1 - F(p_2 - \beta)) dv_1}{f(v_{1N}^*)(1 - F(p_2))} \\
& - p_2 \left(\frac{\int_{v_{1N}^*}^1 f(v_1) f(p_2 - \beta) dv_1}{f(v_{1N}^*)(1 - F(p_2))} \right) = 0
\end{aligned}
$$

[8] Let $p_1 < p_2$

$$
- \int_{p_1}^{p_2} (1 - F(v_2)) dv_2 - \int_{p_2}^1 (1 - F(v_2)) dv_2 - \int_{p_1}^1 F(v_2) dv_2 = -1 + p_1
$$

$$
\int_{p_1}^1 (v_2 - c) dF(v_2) = 1 - p_1 F(p_1) - \int_{p_1}^1 F(v_2) dv_2
$$

Then:

$$p_2\left(1-F(p_2-\beta)+\left(\frac{\int_{v_{1N}^*}^1 f(v_1)f(p_2-\beta)dv_1}{f(v_{1N}^*)(1-F(p_2))}\right)\right)=$$

$$-p_1F(p_1)+\frac{\int_{v_{1N}^*}^1 f(v_1)(1-F(p_2-\beta))dv_1-(1-F(v_{1N}^*))(1-F(p_2))}{f(v_{1N}^*)(1-F(p_2))}$$

$$+c\left(1-1+F(p_1)+1-F(p_2-\beta)+\left(\frac{\int_{v_{1N}^*}^1 f(v_1)f(p_2-\beta)dv_1}{f(v_{1N}^*)(1-F(p_2))}\right)\right)$$

Substituting (3.1) and rearranging:

$$p_2\left(1-F(p_2-\beta)+\left(\frac{\int_{v_{1N}^*}^1 f(v_1)f(p_2-\beta)dv_1}{f(v_{1N}^*)(1-F(p_2))}\right)-\frac{F(p_1)^2f(v_{1N}^*)(1-F(p_2-\beta))}{F(p_1)^2f(v_{1N}^*)+F(v_{1N}^*)f(p_1)}\right)=$$

$$+c\left(1-F(p_2-\beta)+\left(\frac{\int_{v_{1N}^*}^1 f(v_1)f(p_2-\beta))dv_1}{f(v_{1N}^*)(1-F(p_2))}\right)-\frac{F(p_1)^2f(v_{1N}^*)(1-F(p_2-\beta))}{F(p_1)^2f(v_{1N}^*)+F(v_{1N}^*)f(p_1)}\right)$$

$$+\frac{\int_{v_{1N}^*}^1 f(v_1)(1-F(p_2-\beta))dv_1-(1-F(v_{1N}^*))(1-F(p_2))}{f(v_{1N}^*)(1-F(p_2))}$$

Moreover, let for simplicity

$$A=1-F(p_2-\beta)+\left(\frac{\int_{v_{1N}^*}^1 f(v_1)f(p_2-\beta)dv_1}{f(v_{1N}^*)(1-F(p_2))}\right)-\frac{F(p_1)^2f(v_{1N}^*)(1-F(p_2-\beta))}{F(p_1)^2f(v_{1N}^*)+F(v_{1N}^*)f(p_1)}$$

$$=\left(\frac{\int_{v_{1N}^*}^1 f(v_1)f(p_2-\beta)dv_1}{f(v_{1N}^*)(1-F(p_2))}\right)+(1-F(p_2-\beta))\left(1-\frac{F(p_1)^2f(v_{1N}^*)}{F(p_1)^2f(v_{1N}^*)+F(v_{1N}^*)f(p_1)}\right)$$

$$=\left(\frac{\int_{v_{1N}^*}^1 f(v_1)f(p_2-\beta)dv_1}{f(v_{1N}^*)(1-F(p_2))}\right)+(1-F(p_2-\beta))\left(\frac{F(p_1)^2f(v_{1N}^*)+F(v_{1N}^*)f(p_1)}{F(p_1)^2f(v_{1N}^*)+F(v_{1N}^*)f(p_1)}\right)$$

$$+(1-F(p_2-\beta))\left(\frac{-F(p_1)^2f(v_{1N}^*)}{F(p_1)^2f(v_{1N}^*)+F(v_{1N}^*)f(p_1)}\right)$$

$$=\frac{\int_{v_{1N}^*}^1 f(v_1)f(p_2-\beta)dv_1}{f(v_{1N}^*)(1-F(p_2))}+\frac{(1-F(p_2-\beta))F(v_{1N}^*)f(p_1)}{F(p_1)^2f(v_{1N}^*)+F(v_{1N}^*)f(p_1)}>0$$

and

$$B=\frac{\int_{v_{1N}^*}^1 f(v_1)(1-F(p_2-\beta))dv_1-(1-F(v_{1N}^*))(1-F(p_2))}{(f(v_{1N}^*)(1-F(p_2)))}>0$$

Then the optimal price for the second quantity is:

$$p_2=c+\frac{B}{A} \tag{2.9}$$

Finally, substituting (3.2) back to (3.1), we get:

$$\begin{aligned}
p_1 &= c - (p_2 - c)\left(\frac{F(p_1)f(v_{1N}^*)(1-F(p_2-\beta))}{F(p_1)^2 f(v_{1N}^*) + F(v_{1N}^*)f(p_1)}\right) \\
&= c - (c + \frac{B}{A} - c)\left(\frac{F(p_1)f(v_{1N}^*)(1-F(p_2-\beta))}{F(p_1)^2 f(v_{1N}^*) + F(v_{1N}^*)f(p_1)}\right) \\
&= c - \frac{B}{A}\left(\frac{F(p_1)f(v_{1N}^*)(1-F(p_2-\beta))}{F(p_1)^2 f(v_{1N}^*) + F(v_{1N}^*)f(p_1)}\right)
\end{aligned}$$

Moreover if c=0 since p_1 cannot be negative since $F(p_1)$ cannot be negative then $p_1 = 0$

Partially Naive Consumers

The optimization problem of the firm is:

$$\max_{U^*, p_1, p_2} \Pi = S^S - U^P + (U^P - \tilde{U}) = S - U^P - \Delta \quad \text{s.t.} \quad U^N \geq 0$$

and optimal consumption rule is:

$$v_{1P}^* = p_1 + \int_{p_1}^{p_2 - \tilde{\beta}} (1 - F(v_2))dv_2$$

The expected gross surplus is the one produced in a market with a habit forming consumer.

$$S = \int_{v_{1P}^*}^{1} (v_1 - c)dF(v_1) + F(v_{1P}^*)\int_{p_1}^{1}(v_2 - c)dF(v_2) + \int_{v_{1P}^*}^{1}\int_{p_2-\beta}^{1}(v_2 + \beta - c)f(v_2)dv_2 dF(v_1)$$

Moreover, Δ is the difference between the true utility and the perceived of the consumer.

$$\begin{aligned}
\Delta &= \tilde{U}(\mathbf{p^N}) - U^N(\mathbf{p^N}) = \\
&= \int_{v_{1P}^*}^{1}\left(v_1 - p_1 + \int_{p_2-\beta}^{1}(v_2 + \beta - p_2)dF(v_2)\right)dF(v_1) \\
&+ F(v_{1P}^*)\int_{p_1}^{1}(v_2 - p_1)dF(v_2) - F^N \\
&- \left(\int_{v_{1P}^*}^{1}\left(v_1 - p_1 + \int_{p_2-\tilde{\beta}}^{1}(v_2 + \tilde{\beta}k - p_2)dF(v_2)\right)dF(v_1)\right. \\
&\left. + F(v_{1P}^*)\int_{p_1}^{1}(v_2 - p_1)dF(v_2) - F^N\right)
\end{aligned}$$

Thus, Δ is:

$$\Delta = (1 - F(v_{1P}^*))\left(\int_{p_2-\beta}^{p_2-\tilde{\beta}k}(1 - F(v_2))dv_2\right)$$

Then the first order conditions with respect to p_1 is:

$$\frac{\partial \Pi}{\partial p_1} = \frac{\partial S}{\partial v_{1P}^*}\frac{\partial v_{1P}^*}{\partial p_1} + \frac{\partial \Delta}{\partial v_{1P}^*}\frac{\partial v_{1P}^*}{\partial p_1} + \frac{\partial \Delta}{\partial p_1} + \frac{\partial S}{\partial p_1}$$

$$\begin{aligned}
\frac{\partial S}{\partial v_{1P}^*} &= \left(-v_{1P}^* + c - \int_{p_2-\beta}^{1}(v_2 + \beta - c)dF(v_2)) + \int_{p_1}^{1}(v_2 - c)dF(v_2))\right) f(v_{1P}^*)\\
\frac{\partial \Delta}{\partial v_{1P}^*} &= -\left(\int_{p_2-\beta}^{p_2-\tilde{\beta}k}(1 - F(v_2))dv_2\right) f(v_{1P}^*)\\
\frac{\partial v_{1P}^*}{\partial p_1} &= 1 - (1 - F(p_1)) = F(p_1)\\
\frac{\partial S}{\partial p_1} &= -F(v_{1P}^*)(p_1 - c)f(p_1)
\end{aligned}$$

Then the first order condition is:

$$\begin{aligned}
\frac{\partial \Pi}{\partial p_1} &= \left(-v_{1P}^* + c - \int_{p_2-\beta}^{1}(v_2 + \beta - c)dF(v_2)) + \int_{p_1}^{1}(v_2 - c)dF(v_2))\right) f(v_{1P}^*)F(p_1)\\
&+ \left(\int_{p_2-\beta}^{p_2-\tilde{\beta}k}(1 - F(v_2))dv_2\right) f(v_{1P}^*)F(p_1) - F(v_{1P}^*)(p_1 - c)f(p_1) =\\
&= \Big(- p_1 + (p_1 - p_2 + \tilde{\beta}) + c + c(1 - F(p_2 - \beta)) - c(1 - F(p_1))\Big) f(v_{1P}^*)F(p_1)\\
&+ \Big(- (1 - (p_2 - \beta)F(p_2 - \beta)) - \beta(1 - F(p_2 - \beta))\Big) f(v_{1P}^*)F(p_1)\\
&+ \Big(1 - p_1F(p_1) + (p_2 - \tilde{\beta} - p_2 + \beta)\Big) f(v_{1P}^*)F(p_1)\\
&- F(v_{1P}^*)(p1 - c)f(p_1) = 0
\end{aligned}$$

Then

$$p_1 = c - (p_2 - c)\frac{f(v_{1P}^*)F(p_1)(1 - F(p_2 - \beta))}{(f(v_{1P}^*)F(p_1)^2 + f(p_1)F(v_{1P}^*))}$$

The first order condition with respect to p_2 is:

$$\frac{\partial \Pi}{\partial p_2} = \frac{\partial S}{\partial v_{1P}^*}\frac{\partial v_{1P}^*}{\partial p_2} + \frac{\partial \Delta}{\partial v_{1P}^*}\frac{\partial v_{1P}^*}{\partial p_2} + \frac{\partial \Delta}{\partial p_2} + \frac{\partial S}{\partial p_2}$$

then the respective derivatives are:

$$\frac{\partial S}{\partial p_2} = (1 - F(v_{1P}^*))(-1)(p_2 - c)f(p_2 - \beta)$$
$$\frac{\partial \Delta}{\partial p_2} = 1 - F(p_2 - \tilde{\beta}) - (1 - F(p_2 - \beta)) = F(p_2 - \beta) - F(p_2 - \tilde{\beta})$$
$$\frac{\partial v_{1P}^*}{\partial p_2} = 1 - F(p_2 - \tilde{\beta})$$

Thus the first order condition with respect to p_2 becomes:

$$\begin{aligned}
\frac{\partial \Pi}{\partial p_2} &= \Big(- v_{1P}^* + c - \int_{p_2-\beta}^{1} (v_2 + \beta - c) dF(v_2)) + \int_{p_1}^{1} (v_2 - c) dF(v_2)) \\
&+ \int_{p_2-\beta}^{p_2-\tilde{\beta}} (1 - F(v_2)) dv_2 \Big) f(v_{1P}^*)(1 - F(p_2 - \tilde{\beta})) + F(p_2 - \beta) - F(p_2 - \tilde{\beta}) \\
&- (1 - F(v_{1P}^*))(p_2 - c) f(p_2 - \beta) = \\
&= (-p_1 + (p_1 - p_2 + \tilde{\beta}) + c + c(1 - F(p_2 - \beta)) - c(1 - F(p_1)) \\
&- (1 - (p_2 - \beta)F(p_2 - \beta)) - \beta(1 - (p_2 - \beta)) + 1 - p_1 F(p_1) + \\
&+ (p_2 - \tilde{\beta} - p_2 + \beta))(v_{1P}^*)(1 - F(p_2 - \tilde{\beta})) - (1 - F(v_{1P}^*))(p_2 - c) f(p_2 - \beta) \\
&- (F(p_2 - \beta) - F(p_2 - \tilde{\beta})) = 0
\end{aligned}$$

[9]

$$\begin{aligned}
p_2 = c + (c - p_1) &\frac{f(v_{1P}^*)F(p_1)(1 - F(p_2 - \tilde{\beta})}{(f(v_{1P}^*)(1 - F(p_2 - \beta))(1 - F(p_2 - \tilde{\beta})) + f(p_2 - \beta)(-1 + F(v_{1P}^*)))} \\
+ &\frac{F(p_2 - \tilde{\beta})) - (F(p_2 - \beta)}{f(v_{1P}^*)(1 - F(p_2 - \beta))(1 - F(p_2 - \tilde{\beta})) + f(p_2 - \beta)(1 - F(v_{1P}^*))}
\end{aligned}$$

Finally solving the above system of equations we get:

$$p_1 = c - \frac{f(v_{1P}^*)F(p_1)(1 - F(p_2 - \beta))(F(p_2 - \tilde{\beta}) - F(p_2 - \beta))}{f(p_1)f(v_{1P}^*)(1 - F(p_2 - \beta))(1 - F(p_2 - \tilde{\beta}))F(v_{1P}^*) + f(p_2 - \beta)(1 - F(v_{1P}^*))(f(v_{1P}^*)F(p_1)^2 + f(p_1)F(v_{1P}^*)))}$$

$$p_2 = c + \frac{(F(p_2 - \tilde{\beta}) - F(p_2 - \beta))(f(v_{1P}^*)F(p_1)^2 + f(p_1)F(v_{1P}^*))}{f(p_1)f(v_{1P}^*)(1 - F(p_2 - \beta))(1 - F(p_2 - \tilde{\beta}))F(v_{1P}^*) + f(p_2 - \beta)(1 - F(v_{1P}^*))(f(v_{1P}^*)F(p_1)^2 + f(p_1)F(v_{1P}^*))}$$

---

[9]

$$- \int_{p_1}^{p_2} (1 - F(v_2)) dv_2 - \int_{p_2}^{1} (1 - F(v_2)) dv_2 - \int_{p_1}^{1} F(v_2) dv_2 = -1 + p_1$$
$$\int_{p_1}^{1} (v_2 - c) dF(v_2) = 1 - p_1 F(p_1) - \int_{p_1}^{1} F(v_2) dv_2$$

Chapter 3

Screening Habit Forming Consumers

Eleftheria Triviza

Abstract

It is well documented that consumption is often habit forming, namely the current valuation of a good may be affected by whether consumption occurred in the preceding periods. This paper focuses on a market that consists of one firm and habit forming consumers of different degrees of sophistication. The firm knows that all the consumers are habit forming but cannot observe if they are aware of it or not. Two types of consumers are considered, sophisticated and naive. The latter do not realize that their current consumption is affecting future consumption. Our main result is that the menu of contract offered consist of a two-part tariff and a three-part tariff. Moreover, the naive consumer is ex post worst off in the presence of sophisticated consumers with respect to the full information, even if her naivety cannot be exploited. By way of contrast, the sophisticated consumer is better off.

3.1 Introduction

During the recent years, the provision of menus of contracts consisting of two part tariff and three part tariff is prevalent in a number of markets. While, two part tariff consist of a fixed fee and a marginal charge per call, three part tariff consist of a fixed fee, an amount of free calls and a marginal charge for any usage in excess of the free calls.

Moreover, it is well studied the existence and the implications of habit forming behavior in a number of different applications. There are two types of habit forming consumers that have been studied. On the one hand, the Sophisticated (rational) Habit Forming consumer who is aware that the today consumption affects future consumption (Becker and Murphy (1988), Constantinides (1990), Abel (1990), Campbell and Cochrane (1999), Jermann (1998), Boldrin et al. (2001), Carroll et al. (2000), and Fuhrer (2000)). On the other hand, the Naive (myopic) Habit Forming consumer who recognizes that her current satisfaction depends on past habits, but she neglects the impact of current decisions on her future preferences (Pollak, 1970; Loewenstein et al., 2003). Interestingly, Muellbauer (1988) provides an excellent overview of the two extremes, and concludes that the empirical evidence seems to favor myopic habits[1].

The aim of this paper is to study the implications of screening habit forming consumers of different sophistication. Main assumption is that there is a firm with capacity and willingness to collect and analyze tremendous amounts of data about consumers, and the agent is an individual consumer. The firm knows that all the consumers in the market are habit forming but cannot observe the type of the consumer, which is the level of her naivety. Thus, the consumer's private information is actually her level of sophistication.

This paper claims that the observed offered menu of contracts could be explained by the existence of consumers of diverse sophistication in the market. We show that

[1]Studies in markets like cell phone where "three part tariff" is prevalent provide mixed evidence on whether the consumer become addicted, dependent or compulsive (Hooper and Zhou, 2007). Moreover, psycological literature has developed psychological predictors of problematic mobile phone use (Bianchi and Phillips, 2005; Park, 2005).

the firm offers two part tariff as incentive compatible to sophisticated consumers and three part tariff as incentive compatible to naive consumers.

We develop a model where the consumer has two opportunities of consuming. We begin, using as benchmark the case of full information based on previous work of ours. Then, we consider the model where there is asymmetric information and the firm cannot observe the type of the consumer which is actually her level of sophistication. We are using the taxation principle, i.e. screening with respect to the pricing scheme and not the type of the consumer which could be viewed as a realistic assumption since it resembles what we observe in the market.

Interestingly, both types are left with a rent and still the firm cannot exploit[2] the naivety of the consumer. The presence of naive consumers in the market has a positive externality to the sophisticated consumer. The sophisticated consumer is left with the Information rent taking advantage of his information superiority. By way of contrast, naive consumers are ex post worst off in the presence of sophisticated, since her Mis-perception rent decreases with respect to the full information case. As Mis-perception rent we call the difference between her true expected utility and her perceived expected utility, and thus the rent with which she is left ex post. We show that Mis-perception rent is decreasing with respect to the marginal prices, and thus the increase in the marginal price at the contract of the naive consumer, due to asymmetric information and screening, is affecting her negatively ex post.

We would expect that the sophisticated consumer would not have any incentive to mimic the naive and choose a contract that penalties high consumption. This type of consumer knows that she is more likely to consume higher levels of consumption and thus she would not have any incentive to choose a contract that distorts her consumption. Thought, the fact that the firm cannot exploit consumers naivety and offers a less expensive contract to her, leaving her the Mis-perception rent, incentivizes the sophisticated consumer to mimic the naive one. Thus, the firm in order to avoid the mimicking is offering the Information rent and makes her contract incentive compati-

[2]Exploitative in the sense of Eliaz and Spiegler (2006) where "An exploitative contract extracts more than the agent's willingness to pay, from his first-period perspective"

ble.

Moreover, we would expect that the firm could exploit the naivety of the consumers by introducing contracts that charge the high levels of consumption above marginal cost and that the presence of a sophisticated consumer would mitigate the exploitation. The expectation for exploitation is because the consumer does not expect the increase at her valuation of the good because of previous consumption, and that is more probable, than she would expect, to consume when the marginal price is high. Interestingly, we show that even though the firm has superior information with respect to her being habit forming, it cannot use it to its own benefit. This is due to the fact that the firm cannot absorb all her consumer surplus, since she underestimates her expected utility at the contracting period. Nevertheless, the distortion at the optimal marginal pricing due to asymmetric information makes the naive worst off. Her Mis-perception rent decreases with respect to the full information case because of the distortion of her allocation. Moreover, even if the naive cannot be exploited the marginal pricing exacerbates the mistake of underconsumption that she does due to her naivety.

The paper proceeds as follows. Section 2 discusses related literature. Section 3 is dedicated to the introduction of the benchmark model of full information. Section 4 discusses the case of the asymmetric information and Section 5 has the comparison between the full and asymmetric information case and comparative statics. Finally, Section 6 summarizes and concludes.

3.2 Literature

This paper is related to different streams of the literature. First, it is clearly related to behavioral screening literature where the principal is screening the agents with respect to cognitive features (i.e. loss aversion [Hahn et al. (2012), Carbajal and Ely (2012)], present bias, temptation disutility (Esteban et al., 2007), overconfidence in the insurance market [Sandroni and Squintani (2010), Spinnewijn (2013)]

Rubinstein (1993) studies for the first time the problem of a principal who wishes to discriminate between consumer types according to their cognitive features. In this paper, consumers have bounded ability to categorize realizations of a random variable. Different consumer types have different categorization abilities, and the principal's optimal contract is designed to screen their type. Piccione and Rubinstein (2003) perform a similar exercise, when different consumer types different in their ability to perceive temporal patterns.

DellaVigna and Malmendier (2004) were the first to point out that firms might fine-tune contracts to exacerbate consumer's mistakes. A number of papers explore the specific feature as a mean to exploit consumer naivety. In our case, the firm offers a contract that exacerbates consumer's mistake but it cannot absorb all the consumer surplus produced.

This paper is related to the literature of exploitative contracting where the firms are designing their contracts with central consideration to profit from the agent's mistake, and other considerations or constraints are non-existent, not binding, or not central. There are two kind of consumers' mistakes that more often are exploited in the literature. Firstly, the consumer does not understand all features of a product (all prices and fees) [Gabaix and Laibson (2006), Armstrong and Vickers (2012)]. For example, as at Gabaix and Laibson (2006), she underestimates the probability of needing an add-on after buying the good. The other kind of mistake is to mispredict her own behavior with respect to the product (DellaVigna and Malmendier, 2004). This kind of mistake is more close to the case we study here since the consumer mispredicts that her valuation for the good will change if she has consumed before and thus her behavior will change.

Eliaz and Spiegler (2006) claims that the motive to speculate stems from then non-common prior assumption which they interpreted as "a situation in which the agents have a systematic bias in forecasting their future tastes, whereas the principal has an unbiased forecast". This assumption seems to be really important in the case they study where the consumer is uncertain as to whether his preference will change, but she knows exactly what they could change into. The firm takes advantage of his superior information and contracts also the event that the consumer thinks unlikely to happen. Even if the consumers, they consider, are dynamic inconsistent and they evaluate their future actions according to their first period utility function the fact that they know in what their taste could change into gives space for exploitation. In our case that the consumer does not know that her utility function will change after consuming in the first period, the firm cannot exploit his superior information that the consumer is habit forming. This feature becomes important because in both cases the contract is singed before the consumer experiences the change in her utility and she cannot renegotiate the contract after she experiences.

Moreover, it is clearly related to models that try to explain the introduction of three part tariffs. Grubb (2009) shows that over-confidence about the precision of the prediction when making difficult forecasts, free disposal and relatively small marginal cost would explain the use of three part tariff.

Finally, this adds to the literature of optimal non linear pricing induced by different consumer biases or nonstandard preferences. Biased beliefs like optimism (Eliaz and Spiegler, 2008) and overconfidence (Grubb, 2009), naive quasi-hyperbolic discounting (DellaVigna and Malmendier, 2004; Eliaz and Spiegler, 2006) and myopia (Gabaix and Laibson, 2006; Miao, 2010). A common result of these papers is that due to this kind of biases and preferences there is underestimation of the demand which on its turn leads to high marginal prices above marginal cost.

3.3 Benchmark

This section presents the basic structure of the model and the definition of the benchmark optimal pricing policy when there is full information.

The model follows Grubb (2012) in modeling a consumer who has two consumption opportunities and in each period purchases at most 1 unit. Moreover, the consumer is ex-ante uncertain about her per-period evaluation of the service.

3.3.1 The Model

Consider a model where there habit forming consumers of different sophistication and 1 firm. The consumers are uncertain about their valuation of the good in each period.

The model consists of three periods, $T = 3$. At period 0, the firm offers a menu of contracts

$$\mathbf{p}^{\theta} = \{F^{\theta}, p_1^{\theta}, p_2^{\theta}\}$$

The contract $\mathbf{p}^{\theta}$ consist of p_1^{θ} (the price of the first unit consumed), p_2^{θ} (the price of the second unit consumed), and F^{θ} (a fixed payment). At each consecutive period $t \in \{1, 2\}$, the consumer learns the realization of a taste shock υ_t, randomly drawn from a cumulative distribution function $F(\upsilon)$ with support [0,1], the same for all types of consumers. This is the valuation that a unit of good has in period t. Then, given her valuation, she makes a binary quantity choice $q_t = \{0, 1\}$, considering whether or not to purchase the good.

The total payment $p^{\theta}(\mathbf{p}^{\theta}, \mathbf{q})$

$$p^{\theta}(\mathbf{q}) = p_1^{\theta} q_1 + p_2^{\theta} q_2 + F^{\theta},$$

is a function of quantity choices $\mathbf{q} = (q_1, q_2)$ and the pricing scheme $\mathbf{p}^{\theta} = (F, p_1^{\theta}, p_2^{\theta})$. The timing of the game is described from Figure 3.1.

The optimal consumption strategy, for given prices, is a function mapping valuations to quantities:

$$\mathbf{q}(\boldsymbol{\upsilon}; \mathbf{p}^{\theta}) : \boldsymbol{\upsilon} \to \mathbf{q}$$

Moreover, the ex ante expected gross utility of the consumer from making optimal consumption choices is:

$$U = E[u(\mathbf{q}(\boldsymbol{\upsilon}; \mathbf{p}^{\theta}), \boldsymbol{\upsilon}, \mathbf{p}^{\theta})]$$

The expected profits per consumer equal the revenues less the variable cost with marginal cost $c \geq 0$ per unit produced. The fixed cost is normalized to zero. Thus, the profit function is:

$$\Pi = E[p^{\theta}(\mathbf{q}(\boldsymbol{\upsilon}; \mathbf{p}), \mathbf{p}^{\theta}) - c(q_1(\boldsymbol{\upsilon}; \mathbf{p}^{\theta}) + q_2(\boldsymbol{\upsilon}; \mathbf{p}^{\theta}))$$

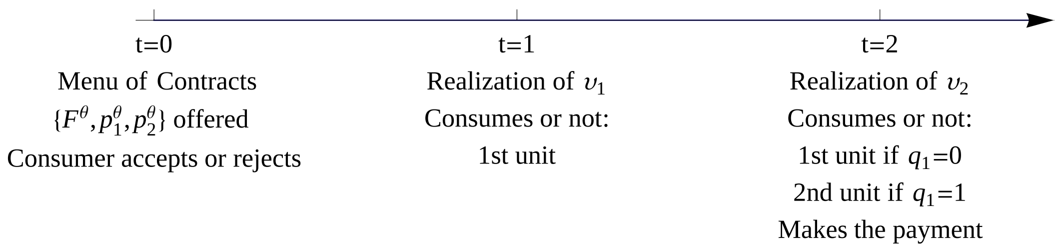


Figure 3.1: Timing of the game

Finally, the expected social surplus is:

$$S = E[\sum_{t=1}^{2}(v_t - c)\mathbf{q}(\boldsymbol{v}; \mathbf{p}^\theta)]$$

3.3.2 Consumer side

Consider a consumer who is habit forming in the sense that her consumption today is affected by her consumption in previous periods. Her valuation for the service at period t is:

$$\tilde{v}_t = v_t q_t + \theta\beta k q_{t-1} q_t$$

This means that if she consumed in the previous period her valuation for the service today increases by $\theta\beta k$, where $0 \leq k \leq 1$ is the extra valuation that the consumer could have because of previous consumption. Moreover, $0 \leq \beta \leq 1$ is the habit formation coefficient, namely it defines how much habit forming is the consumer, how much she is affected by a k increase in the valuation created by previous consumption. Finally, $0 \leq \theta \leq 1$ is the type of the consumer, it is a measure of her naivety and of how much she realizes that she is habit forming. The bigger is θ the less naive is the consumer and the more she realizes that she is affected by her previous consumption. Thus,

$\theta = 1$ means that the consumer is sophisticated habit forming, $0 < \theta < 1$ means that she partially naive, and $\theta = 0$ that she is completely naive.

The expected utility of the consumer at the contracting period is:

$$U(\mathbf{p}) = \int_{v_1^*}^{1} \left(v_1 - p_1 + \int_{p_2-\theta\beta k}^{1} (v_2 + \theta\beta k - p_2) dF(v_2) \right) dF(v_1) + F(v_1^*) \int_{p_1}^{1} (v_2 - p_1) dF(v_2) - F$$

Let us now consider two types of consumer, a sophisticated habit forming with $\theta = 1$ and a naive one with $\theta = 0$. The consumers at each period are choosing the optimal threshold above which is optimal for them to consume. At the contracting period, the consumer does not know the realization of her valuation of the call, thus in order to maximize her expected utility she is choosing in each period, by solving backwards, the optimal threshold.

Sophisticated Habit Forming Consumer ($\theta = 1$): Solving backwards the second period optimal threshold is:

$$v_{2S}^* = \begin{cases} p_1 & \text{if} \quad q_1 = 0 \\ p_2 - \beta k & \text{if} \quad q_1 = 1 \end{cases}$$

Thus, if she has not consumed before the first unit is still available and she consumes if the valuation of the call v_2 is bigger than the first unit marginal price, p_1. Respectively, if she has consumed before then she consumes if her valuation is bigger than the second unit marginal price taking into consideration how previous consumption is affecting her valuation for the call, $p_2 - \beta k$.

Given v_{2S}^*, the first period maximization problem of the sophisticated habit forming consumer is:

$$\max_{v_{1S}^*} U^S(\mathbf{p^S}) = \int_{v_{1S}^*}^{1} \left(v_1 - p_1 + \int_{p_2-\beta k}^{1} (v_2 + \beta k - p_2) dF(v_2) \right) dF(v_1)$$
$$+ F(v_{1S}^*) \int_{p_1}^{1} (v_2 - p_1) dF(v_2) - F^S$$

maximizing with respect to v_{1S}^* the optimal first period threshold is:

$$v_{1S}^* = p_1 + \int_{p_1}^{p_2-\beta k} (1 - F(v_2)) dv_2$$

The consumer being forward looking and aware of being habit forming, she taking into consideration the price change for the second unit and the increase in her valuation due to the habit. Thus, the first period threshold increases if the second unit marginal price increases and decreases the more habit forming is the consumer.

Naive Habit Forming Consumer ($\theta = 0$): Solving backwards the second period optimal threshold is:

$$v_{2N}^* = \begin{cases} p_1 & \text{if} \quad q_1 = 0 \\ p_2 & \text{if} \quad q_1 = 1 \end{cases}$$

In this case the consumer does not know that the previous consumption would have an effect on his valuation of the good and she takes into consideration only the change in the marginal price.

Given v_{2N}^*, the first period maximization problem of the naive habit forming consumer is:

$$\max_{v_{1N}^*} U^N(\mathbf{p^N}) = \int_{v_{1N}^*}^1 \left(v_1 - p_1 + \int_{p_2}^1 (v_2 - p_2) dF(v_2) \right) dF(v_1) + F(v_{1N}^*) \int_{p_1}^1 (v_2 - p_1) dF(v_2) - F^N$$

maximizing with respect to v_{1N}^*, the optimal first period threshold is:

$$v_{1N}^* = p_1 + \int_{p_1}^{p_2} (1 - F(v_2)) dv_2$$

The consumer being forward looking is taking into consideration the change in the marginal prices, she anticipates it and the first period threshold is decreasing if $p_1 < p_2$. This optimal threshold is the same as that of a non habit forming consumer. Moreover, $v_{1N}^* > v_{1S}^*$ and $v_{2N}^* > v_{2S}^*$ thus the naive consumer under-consumes in both periods for given marginal prices.

The two types of consumers, sophisticated and naive, are ex post identical the only difference is the ex ante perception of how much habit forming they are.

3.3.3 Firm Side

Consider a market with a monopolist and two types of consumers: a sophisticated habit forming and a naive habit forming consumer. The cost of production of one unit of good is $0 \leq c \leq 1$.

Let consider the case that the firm can observe the type of the consumer and can offer a type specific contract.

Sophisticated Consumer: The maximization problem of the firm is:

$$\max_{\mathbf{p^S}} \Pi^S = S^S(\mathbf{p^S}) - U^S(\mathbf{p^S}) \quad \text{s.t} \quad U^S(\mathbf{p^S}) \geq 0$$

It is the difference between the expected gross surplus produced minus the expected consumer surplus. Maximizing with respect to $\mathbf{p^S}$ the optimal contract is

Lemma 3.1. *If the consumer is sophisticated habit forming, the equilibrium allocation is the first best allocation. There is marginal cost pricing, namely the prices that maximize the profits of the firm are* $(p_1, p_2) = (c, c)$ *and the fixed fee,* F^S*, equals to the consumer surplus.*

Proof: The firm maximizes its profit by charging marginal prices that induce the first best allocation and then with the fixed fee F^S it absorbs all the consumer surplus. The firm does the rent extraction in such a way that is balanced with the participation as in a basic monopoly pricing problem.

Naive Consumer: The maximization problem of the firm is:

$$\max_{\mathbf{p^N}} \Pi^N = S^N(\mathbf{p^N}) - U^N(\mathbf{p^N}) - \Delta \quad \text{s.t} \quad U^N(\mathbf{p^N}) \geq 0$$

where $\Delta = U^N(\mathbf{p^N}) - \tilde{U}(\mathbf{p^N})$ and $\tilde{U}(\mathbf{p^N})$ is the expected utility of the consumer when at the second period realizes that she is habit forming.

$$\Delta = \tilde{U}(\mathbf{p^N}) - U^N(\mathbf{p^N}) =$$
$$= \int_{v_{1N}^*}^1 \Big(v_1 - p_1 + \int_{p_2-\beta k}^1 (v_2 + \beta k - p_2) dF(v_2) \Big) dF(v_1) + F(v_{1N}^*) \int_{p_1}^1 (v_2 - p_1) dF(v_2)$$
$$- \Big(\int_{v_{1N}^*}^1 \big(v_1 - p_1 + \int_{p_2}^1 (v_2 - p_2) dF(v_2) \big) dF(v_1) + F(v_{1N}^*) \int_{p_1}^1 (v_2 - p_1) dF(v_2) \Big)$$

The firm cannot absorb all the consumer surplus at the contracting period since the consumer is unaware of her habit. Thus, the profits of the firm is the difference between the expected gross surplus, minus the consumer surplus $U^N(\mathbf{p^N})$ that the firm can absorb through the fixed fee, minus Δ which is the rent that remains to the naive consumer due to her unawareness.

Lemma 3.2. *If the consumer is naive habit forming the optimal pricing scheme is "three part tariff", namely*

- $c = 0$: $\quad p_1^N = 0, \quad p_2^N > c, \quad F^N = U^N(\mathbf{p^N})$

- $c > 0$: $\quad p_1^N < c, \quad p_2^N > c, \quad F^N = U^N(\mathbf{p^N})$
  when βk is relatively small

Proof: See Appendix

The above pricing scheme captures the main features of the observed, in reality, "three part tariff", since for the first unit the price is below cost, and for relative small cost the first unit price tends to be zero as observed. The second unit and thus for high levels of quantity the marginal price is increasing above the marginal cost.

3.4 Asymmetric Information

The firm cannot observe the type of the consumer but it is common knowledge that the probability of her being sophisticated is γ, $Pr(S) = \gamma$.

We use the taxation principle, thus the screening is done with respect to the pricing scheme.[3]. The firm offers a menu of contracts but, without any loss of generality, we restrict the analysis to the pair of optimal choices made by the two types of buyers that exist in the market, namely $\mathbf{p^N} = \{F^N, p_1^N, p_2^N\}$, $\mathbf{p^S} = \{F^S, p_1^S, p_2^S\}$ the contract for the naive and the sophisticated consumer respectively. Moreover, imposing this kind of pricing scheme completely defines the allocation, since there is a price for each unit[4] and the fixed fee that absorbs the consumer surplus.

The maximization problem of the firm is:

$$\max_{\mathbf{p^S},\mathbf{p^N}} \quad \gamma(S^S(\mathbf{p^S}) - U^S(\mathbf{p^S})) + (1-\gamma)(S^N(\mathbf{p^N}) - U^N(\mathbf{p^N}) + \Delta)$$

$$\text{s.t.} \quad \begin{array}{ll} U^N(\mathbf{p^N}) \geq 0 & IR_N \\ U^S(\mathbf{p^S}) \geq 0 & IR_S \\ U^N(\mathbf{p^N}) \geq U^N(\mathbf{p^S}) & IC_N \\ U^S(\mathbf{p^S}) \geq U^S(\mathbf{p^N}) & IC_S \end{array}$$

$U^N(\mathbf{p^N}) \geq 0$ and $U^S(\mathbf{p^S}) \geq 0$ are the *participation constraints* of the naive and sophisticated consumer respectively. Moreover, $U^N(\mathbf{p^N}) \geq U^N(\mathbf{p^S})$ and $U^S(\mathbf{p^S}) \geq U^S(\mathbf{p^N})$ are the *incentive compatibility constraints*: that each type should not have any incentive to mimic of the other. Note that there is no third period participation constraint. Ones the consumer has signed the contract, she is obliged to keep it for the whole contract period even if she would have an incentive to deviate.

First the firm choose the marginal prices, $\{p_1^S, p_2^S\}, \{p_1^N, p_2^N\}$, which determine the optimal allocation and thus expected social surplus from serving each type. Then, the firm chooses the fixed fees, $\{F^S, F^N\}$, which on their turn determine the utilities offered to each type.

As we saw before, in the case of the full information, i.e. when the type is observable, the profit of the firm is bigger when in the market there is only a sophisticated

[3]The use of the taxation principle is more realistic and more close to what we observe. Moreover, the nature of the problem, i.e. the multi-dimensional uncertainty makes the problem not tractable. Multi-dimensional uncertainty for the firm because, first the firm is uncertain of the type of consumer at the contracting period, and second the valuation of the good is unknown to both parties at the contracting period but it is known to the consumer in each period before consuming and not the firm.

[4]We assume that the good is indivisible.

consumer. The profit in the case of the sophisticated consumer is the first best, since there is marginal cost pricing, first best allocation and with the fixed fee all the consumer surplus is extracted which in this case equals the social surplus. In the case of the naive habit forming consumer the firm finds it optimal to distort the allocation, since it cannot extract all the consumer surplus, charging $p_1 < c$, $p_2 > c$ and a fixed fee equal to her ex ante expected utility as she perceives it at period zero.[5]

The above discussion indicates that in order to relax the problem, the incentive constraint that we expect to be satisfied at the optimum is the one of the naive consumer. This is because the naive consumer at the contract period does not know that she will acquire a habit and that her utility will be bigger than the one she expects to be. Marginal cost pricing is creating bigger expected utility to the sophisticated than the naive consumer, thus the firm is charging a fixed fee that the naive consumer would not be willing to pay. On the other hand the optimal contract of third degree price discrimination is not incentive compatible for the sophisticated consumer because she would prefer the contract of the naive consumer rather than her own first best allocation. Even if the marginal pricing is distorting her allocation, it allows her to enjoy a strictly positive surplus equal to $U^S(\mathbf{p^N}) - U^N(\mathbf{p^N})$.

The fact that we expect the incentive compatibility constraint of the naive consumer to be satisfied (i.e. actually slack at the optimum) implies that there will be marginal cost pricing and first best allocation for the sophisticated consumer. If this was not true then setting $\{p_1^S, p_2^S\}$ equal to $\{c, c\}$ while keeping U^S constant would keep the incentive compatibility and the participation constraint of the sophisticated unaffected and it would not violate the incentive constraint of the naive since it is relaxed. But this would increase the surplus and the profits of the firm, a contradiction.

The incentive compatibility constraint of the sophisticated consumer is binding $U^S(\mathbf{p^S}) = U^S(\mathbf{p^N})$. Suppose not and $U^S(\mathbf{p^S}) > U^S(\mathbf{p^N})$ then $\mathbf{p^N}$ and $\mathbf{p^S}$ would be the same of the third degree price discrimination since the incentive constraint of the naive is relaxed and the one of the sophisticated slack. The firm could increase the

[5] The relative ranking of optimal profit is important because it determines which market segment the firm would like to offer a discounted markup to.

fixed fee of the sophisticated consumer without violating the incentive compatibility constraint of the sophisticated consumer and increase its profits. Thus, we expect that it binds at the optimum.

Since we expect the incentive compatibility constraint of the sophisticated consumer to bind at the optimum, it could be written as:

$$U^S(\mathbf{p^S}) = U^S(\mathbf{p^N}) \Rightarrow U^S(\mathbf{p^S}) = U^S(\mathbf{p^N}) - U^N(\mathbf{p^N}) + U^N$$

Lemma 3 is summarizing the above discussion and how we expect to be the constraints at the optimum.

Lemma 3.3. *Relaxing the maximization problem:*

$$\begin{aligned}
&U^N(\mathbf{p^N}) = 0 && IR_N \quad \textit{binding}\\
&U^S(\mathbf{p^S}) > 0 && IR_S \quad \textit{slack}\\
&U^N(\mathbf{p^N}) > U^N(\mathbf{p^S}) && IC_N \quad \textit{slack}\\
&U^S(\mathbf{p^S}) = U^S(\mathbf{p^N}) && IC_S \quad \textit{binding}\\
&\Rightarrow U^S(\mathbf{p^S}) = U^S(\mathbf{p^N}) - U^N(\mathbf{p^N}) + U^N &&
\end{aligned}$$

Thus, taking into consideration Lemma 3 the relaxed problem is:

$$\max_{\mathbf{p^N}} \quad \Pi = -\gamma \underbrace{\Big(U^S(\mathbf{p^N}) - U^N(\mathbf{p^N})\Big)}_{\substack{\text{Information rent of}\\ \text{Sophisticated}}} + (1-\gamma)\Big(S^N(\mathbf{p^N}) + \underbrace{U^N(\mathbf{p^N}) - \tilde{U}(\mathbf{p^N})}_{\substack{\text{Mis-perception rent}\\ \text{of Naive}}}\Big)$$

The first part is the relative to the maximization problem part of the profits that the firm makes from the sophisticated consumer and the second part is the one from the naive consumer. Interestingly, both types of consumers are left with a rent and the firm cannot extract all their surplus. The sophisticated consumer has an information rent due to the asymmetry of information, she has an incentive to deviate and choose the contract tailored for the naive consumer and with this rent the firm avoids the deviation. On the other hand, the naive consumer, even if has no incentive to deviate, she is left with the *Mis-perception rent*. This type of consumer is left with this rent because of her naivety and the fact that she does not know what her true level of utility will be after consuming the first unit. Thus, since she cannot accept a more expensive contract at the contract period, she is left ex post with the *Mis-perception rent.*

Proposition 3.1. *The optimal contract that the firm offers is:*

- ***Sophisticated consumer:***

  $p_1^S = c, \quad p_2^S = c, \quad F^S = U^S(\mathbf{p^S}) - U^S(\mathbf{p^N})$

  "two part tariff"

- ***Naive Consumer:***

  - $c = 0$: $p_1^N = 0, \quad p_2^N > c, \quad F^N = U^N(\mathbf{p^N})$

  - $c > 0$: $p_1^N < c, \quad p_2^N > c, \quad F^N = U^N(\mathbf{p^N})$

    when βk is relatively small

    "three part tariff"

Proof: See Appendix

The firm offers a menu of contract consisting of a two part tariff for the sophisticated consumer and a three part tariff for the naive consumer. The pricing patterns that are optimal under full information are still optimal under asymmetric information.

The naive consumer is offered a seemingly expensive contract. At the contract period, she believes that she is choosing a contract more expensive than it actually is ex post. She is left with the Mis-perception rent Δ that is bigger that her expected utility at the contract period, $\Delta > U_N(\mathbf{p_N}) = 0$

It remains to be shown that the constraints and actually the incentive compatibility constraint of the naive consumer slacks at the optimum that is proved at the Appendix.

3.5 Assuming Uniform Distribution

3.5.1 Comparing Asymmetric Information vs Full Information Case

Assuming that the distribution of the valuation of the service is uniform allows us to make clear comparisons between the results of the full information case and the asymmetric information case[6].

The marginal prices of the contract of the sophisticated consumer $\{p_1^S, p_2^S\}$ remain equal to the marginal cost when the fixed fee, F^S, is decreasing. The sophisticated consumer is better off from the presence of the naive consumer in the market. Thus, the naive consumer has a positive externality to the sophisticated consumer, even if she is less profitable. On the other hand, the marginal prices of the naive, $\{p_1^N, p_2^N\}$, are distorted upwards and the fixed fee, F^N is decreasing. This type of consumer is worst off from the presence of the sophisticated (Figure 3.2)[7]. This is because in this way the firm makes the contract of the naive less attractive to the sophisticated consumer. Moreover, $\Delta^F > \Delta^S$ the decrease of the expected utility of the naive consumer, U_N, is less than her true expected utility, $\tilde{U}$ thus the distortion of the allocation has a bigger effect on the true one. More specifically, we see that

$$\begin{aligned}
\Delta &= \tilde{U}(\mathbf{p^N}) - U^N(\mathbf{p^N}) = \\
&= (1 - F(v_{1N}^*))\left(\int_{p_2-\beta k}^{1} (1 - F(v_2))dv_2 - \int_{p_2}^{1} (1 - F(v_2))dv_2\right) = \\
&= (1 - F(v_{1N}^*))\left(\int_{p_2-\beta k}^{p_2} (1 - F(v_2))dv_2\right)
\end{aligned}$$

The derivatives with respect to the marginal prices are:

$$\frac{d\Delta}{dp_1} = -\frac{dF(v_{1N}^*)}{dv_{1N}^*}\frac{dv_{1N}^*}{dp_1}\left(\int_{p_2-\beta k}^{p_2} (1 - F(v_2))dv_2\right) = -\frac{dF(v_{1N}^*)}{dv_{1N}^*}\left(\int_{p_2-\beta k}^{p_2} (1 - F(v_2))dv_2\right)$$

[6] See Appendix for detailed calculations

[7] Moreover, for the graphical representation we have assumed that $c = 0.10$, $\gamma = 0.10$ and $k = 0.10$. This holds for all the Figures presented at this section

$$\frac{d\Delta}{dp_2} = -\frac{dF(v_{1N}^*)}{dv_{1N}^*}\frac{dv_{1N}^*}{dp_2}\left(\int_{p_2-\beta k}^{p_2}(1-F(v_2))dv_2\right) + (1-F(v_{1N}^*))\Big(F(p_2-\beta k)-F(p_2)\Big)$$

since

$$\frac{dF(v_{1N}^*)}{dv_{1N}^*} > 0, \frac{dv_{1N}^*}{dp_1} > 0, \frac{dv_{1N}^*}{dp_2} > 0 \quad \text{and} \quad F(p_2-\beta k) - F(p_2) < 0$$

Thus,

$$\frac{d\Delta}{dp_1} < 0 \quad \text{and} \quad \frac{d\Delta}{dp_2} < 0$$

This means that an increase marginal prices is decreasing the Mis-perception Rent. The marginal prices are increasing with respect to the full information case and thus the naive consumer is worst off.

Moreover, as we see at Figure 3.2, the more habit forming are the consumers the worst off they are from the presence of the sophisticated. The rent left to the naive consumer when she is more habit forming is greater, thus contract of the naive is even more attractive to the sophisticated and the firm has a greater incentive to distort the contract of the naive.

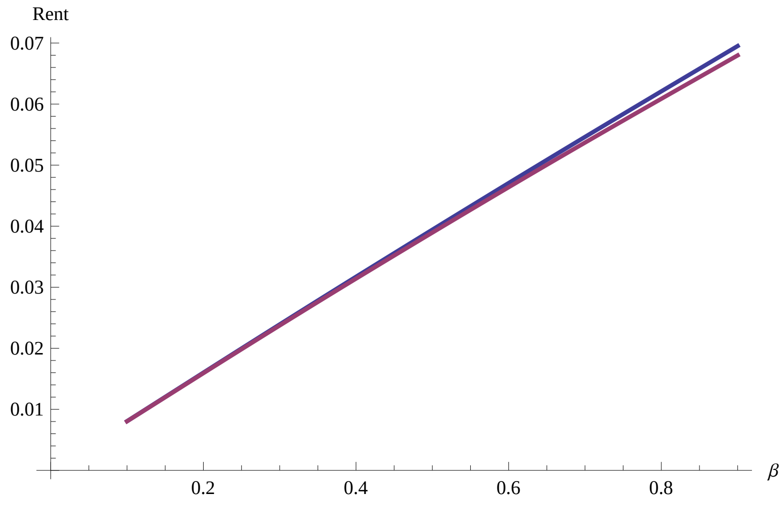


Figure 3.2: Rent of Naive Full Information > Rent of Naive Screening

The naive consumer, as has been discussed before, is less probable to consume than the sophisticated at the first period because she mistakenly believes that she is not habit forming. Comparing the three part tariff with the marginal cost pricing we see that the contract offered to the naive consumer exacerbates this mistake. The decrease at the first unit marginal price is not enough to correct this mistake due to the increase of the second period marginal price and the fact that the consumer is forward looking. This means that the optimal first period threshold given $\mathbf{p^N}$ is greater than the one given marginal cost pricing $v^*_{1N}(\mathbf{p^N}) > v^*_{1N}(\mathbf{p^S}) = v^*_{1N}(\mathbf{c})$ (Figure 3.3). As expected, the sophisticated consumer is more probable to consume when she is choosing the contract tailored for the naive consumer, $v^*_{1N}(\mathbf{p^S}) > v^*_{1S}(\mathbf{p^N})$, and even more probable when she is choosing the contract tailored for her, $v^*_{1S}(\mathbf{p^N}) > v^*_{1S}(\mathbf{p^S})$. Moreover, as we see at Figure (3.3), the more habit forming is the consumer the bigger the exacerbation of the mistake.

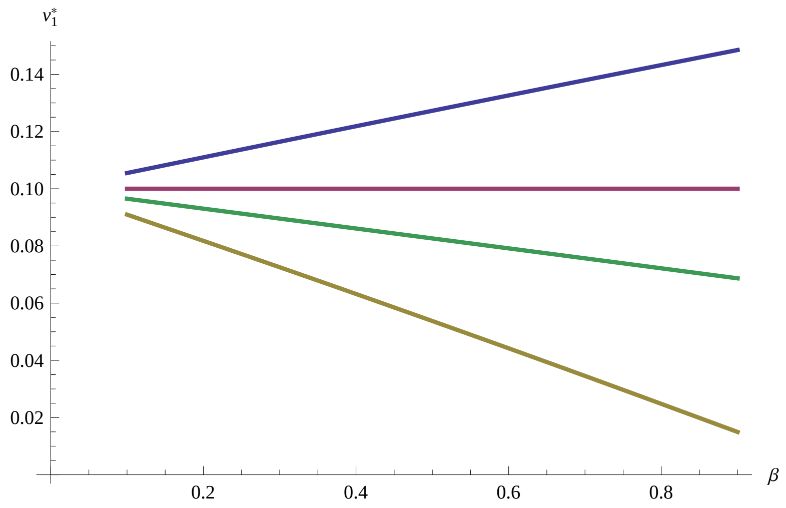


Figure 3.3: $v^*_{1N}(\mathbf{p^N}) > v^*_{1N}(\mathbf{p^S}) > v^*_{1S}(\mathbf{p^N}) > v^*_{1S}(\mathbf{p^S})$

Comparing now the optimal first period threshold in the case of the full information with respect to the one of the asymmetric information, we see that there is an increase and thus the under-consumption is greater. The firm is distorting the marginal pricing for the naive consumer in order to make it less attractive to the sophisticated and in this way since both marginal prices are increasing the mistake amplifies even more (Figure 3.4). Moreover, the more habit forming is the consumer, i.e. the greater the habit forming coefficient β the bigger the mistake exacerbation.

The profits of the firms decrease with respect to the full information case both for

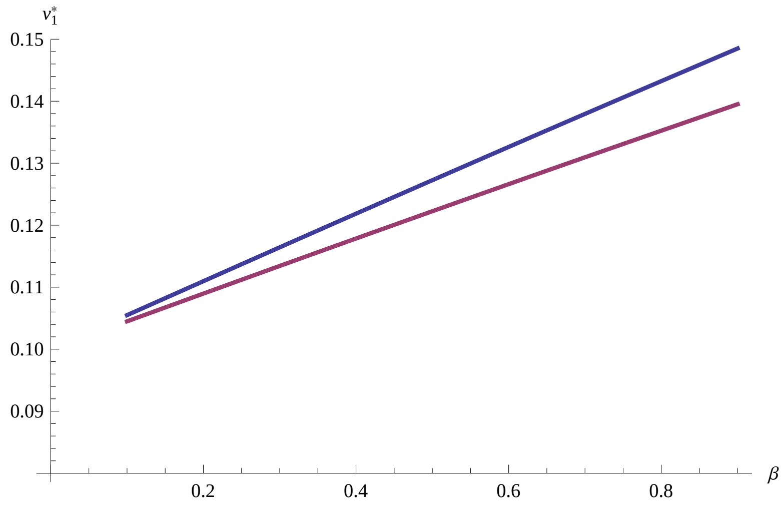


Figure 3.4: $v^*_{1NS}(\mathbf{p_1^{NS}}, \mathbf{p_2^{NS}}) > v^*_{1NF}(\mathbf{p_1^{NF}}, \mathbf{p_2^{NF}})$

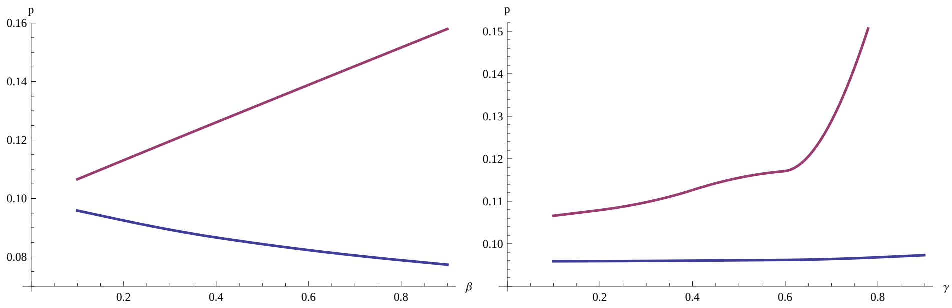
the sophisticate and the naive consumer. The firm since cannot exploit the naivety of the consumer and at the same time cannot observe the her type, it decreases its profits in order to offer a menu of contracts that is incentive compatible. If the portion of sophisticated consumers is relatively small then the firm finds optimal to offer only one contract.

3.5.2 Comparative Statics

Let now consider how the optimal contracts would change when parameters of the model would change. The marginal prices of the sophisticated consumer would remain unchanged equal to the marginal cost and what changes is the fixed fee of the sophisticated, F^S, and the whole contract of the naive, $\mathbf{p^N}$.

The more habit forming the consumer, i.e. as the habit forming coefficient β increases the first unit price, p_1^N, decreases and the second unit price, p_2^N, increases (Figure 3.5a). Thus, the more habit forming she is, the greater is the difference between the marginal prices of the two units.

Finally, the more sophisticated consumers there are in the market, i.e. as γ increases both the first unit marginal price, p_1^N, and the second unit marginal price, p_2^N, increase (Figure 3.5b).



(a) $\frac{dp_{1N}}{d\beta} < 0 \quad \frac{dp_{2N}}{d\beta} > 0$ (b) $\frac{dp_{1N}}{d\gamma} > 0 \quad \frac{dp_{2N}}{d\gamma} > 0$

Figure 3.5: Comparative Statics

3.6 Conclusion

During the recent years, the provision of menu of contracts consisting of two part tariff and three part tariff is prevalent in a number of markets. Moreover, it is well studied the existence and the implications of habit forming behavior in a number of different applications. There are two types of habit forming consumers that have been studied. On the one hand, the Sophisticated (rational) Habit Forming consumer and the Naive (myopic) Habit Forming consumer who recognizes that her current satisfaction depends on past habits, but she neglects the impact of current decisions on her future preferences.

This paper claims that the observed offered menu of contracts could be explained by the existence of consumers of diverse sophistication in the market. We show that the firm offers two part tariff as incentive compatible to sophisticated consumers and three part tariff as incentive compatible to naive consumers.

Interestingly, both types are left with a rent and still the firm cannot exploit the naivety of the consumer. The presence of naive consumers in the market has a positive externality to the sophisticated consumer. The sophisticated consumer is left with the Information rent taking advantage of his information superiority. By way of contrast, naive consumers are ex post worst off in the presence of sophisticated, since her misperception rent decreases with respect to the full information case. As Mis-perception rent we call the difference between her true expected utility and her perceived expected utility, and thus the rent with which she left ex post.

We would expect that the sophisticated consumer would not have any incentive to mimic the naive and choose a contract that penalties high consumption. This type of consumer knows that she is more likely to consume higher levels of consumption and thus she would not have any incentive to choose a contract that distorts her consumption. Thought, the fact that the firm cannot exploit consumers naivety and offers a less expensive contract to her leaving her the Mis-perception rent, incentivizes the sophisticated consumer to mimic the naive one. Thus, the firm in order to avoid the mimicking is offering the Information rent and makes her contract incentive compatible.

Interestingly, we show that even though the firm has superior information with respect to her being habit forming, it cannot use it to its own benefit. This is due to the fact that the firm cannot absorb all her consumer surplus, since she underestimates her expected utility at the contracting period. Nevertheless, the distortion at the optimal marginal pricing due to asymmetric information makes the naive worst off. Her Mis-perception rent decreases with respect to the full information case because of the distortion of her allocation. Moreover, even if the naive cannot be exploited the marginal pricing exacerbates the mistake of underconsumption that she does due to her naivety.

Moreover, the model allows for further research in several directions. The introduction of competition would be an interesting extension, since this kind of market are characterized by intense competition and maybe market forces would change the equilibrium outcome. Moreover, the introduction of partially naive consumers in the market could be an other extension.

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Appendix

Proof of Lemma 1

Thus the optimization problem of the firm is:

$$\max_{U^*,p_1,p_2} \Pi = S^S - U^N + (U^N - U^S) = S - U^N + \Delta \quad \text{s.t.} \quad U^N \geq 0$$

and optimal consumption rule is:

$$v_{1N}^* = p_1 + \int_{p_1}^{p_2} (1 - F(v_2))dv_2$$

The expected gross surplus is the one produced in a market with a habit forming consumer.

$$
\begin{aligned}
S = &\int_{v_{1N}^*}^{1} (v_1 - c)dF(v_1) + F(v_{1N}^*)\int_{p_1}^{1}(v_2 - c)dF(v_2) \\
&+ \int_{v_{1N}^*}^{1}\int_{p_2-\beta k}^{1}(v_2 + \beta k - c)f(v_2)dv_2 dF(v_1) \\
&= \int_{v_{1N}^*}^{1} (v_1 - c)dF(v_1) + F(v_{1N}^*)\int_{p_1}^{1}(v_2 - c)dF(v_2) + \int_{v_{1N}^*}^{1}\int_{p_2-\beta k}^{1}(v_2 + \beta k)f(v_2)dv_2 dF(v_1) \\
&- c\int_{v_{1N}^*}^{1} f(v_1)(1 - F(p_2 - \beta k))
\end{aligned}
$$

Moreover, Δ is the difference between the perceived and the optimal utility of the consumer.

$$
\Delta = U^N - U^S = (1 - F(v_{1N}^*))\int_{p_2}^{1}(1 - F(v_2))dv_2 + p_2\int_{v_{1N}^*}^{1} f(v_1)(1 - F(p_2 - \beta k))dv_1
$$

Simplifying and deleting $\int_{v_{1N}^*}^{1}\int_{p_2-\beta k}^{1}(v_2 + \beta k)f(v_2)dv_2 dF(v_1)$ from S and Δ then the first order conditions are:

with respect to p_1:

$$
\frac{\partial \Pi}{\partial p_1} = \frac{\partial S}{\partial v_{1N}^*}\frac{\partial v_{1N}^*}{\partial p_1} + \frac{\partial \Delta}{\partial v_{1N}^*}\frac{\partial v_{1N}^*}{\partial p_1} + \frac{\partial \Delta}{\partial p_1}
$$

$$
\begin{aligned}
\frac{\partial S}{\partial v_{1N}^*} &= \left(-v_{1N}^* + c + \int_{p_1}^{1}(v_2 - c)dF(v_2) + c(1 - F(p_2 - \beta k))\right) f(v_{1N}^*) \\
\frac{\partial \Delta}{\partial v_{1N}^*} &= \left(-\int_{p_2}^{1}(1 - F(v_2))dv_2 - p_2(1 - F(p_2 - \beta k))\right) f(v_{1N}^*) \\
\frac{\partial v_{1N}^*}{\partial p_1} &= 1 - (1 - F(p_1)) = F(p_1) \\
\frac{\partial S}{\partial p_1} &= -F(v_{1N}^*)(p_1 - c)f(p_1)
\end{aligned}
$$

Then the first order condition is:

$$\frac{\partial \Pi}{\partial p_1} = \left(-v_{1N}^* + c + \int_{p_1}^1 (v_2 - c)dF(v_2) + c(1 - F(p_2 - \beta k))\right) F(p_1) - \frac{F(v_{1N}^*)f(p_1)}{f(v_{1N}^*)}(p_1 - c)$$
$$- F(p_1)\int_{p_2}^1 (1 - F(v_2))dv_2 - p_2(1 - F(p_2 - \beta k))F(p_1) =$$
$$= -v_{1N}^* + c + \int_{p_1}^1 (v_2 - c)dF(v_2) + c(1 - F(p_2 - \beta k) - \frac{F(v_{1N}^*)f(p_1)}{f(v_{1N}^*)F(p_1)}(p_1 - c)$$
$$- \int_{p_2}^1 (1 - F(v_2))dv_2 - p_2(1 - F(p_2 - \beta k)) =$$
$$= -p_1 - \int_{p_1}^{p_2} (1 - F(v_2))dv_2 + c + \int_{p_1}^1 (v_2 - c)dF(v_2) + c(1 - F(p_2 - \beta k))$$
$$- \frac{F(v_{1N}^*)f(p_1)}{f(v_{1N}^*)F(p_1)}(p_1 - c) - \int_{p_2}^1 (1 - F(v_2))dv_2 - p_2(1 - F(p_2 - \beta k)) =$$
$$= p_1 + c - 1 + p_1 + 1 - p_1F(p_1) - c(1 - F(p_1)) + c(1 - F(p_2 - \beta k))$$
$$- \frac{F(v_{1N}^*)f(p_1)}{f(v_{1N}^*)F(p_1)}(p_1 - c) - p_2(1 - F(p_2 - \beta k))$$

Then

$$p_1\left(F(p_1) + \frac{F(v_{1N}^*)f(p_1)}{f(v_{1N}^*)F(p_1)}\right) = c\left(F(p_1) + 1 - F(p_2 - \beta k) + \frac{F(v_{1N}^*)f(p_1)}{f(v_{1N}^*)F(p_1)}\right)$$
$$- p_2(1 - F(p_2 - \beta k))$$
$$p_1\left(\frac{F(p_1)^2 f(v_{1N}^*) + F(v_{1N}^*)f(p_1)}{F(p_1)f(v_{1N}^*)}\right) = c\left(\frac{F(p_1)^2 f(v_N) + F(v_{1N}^*)f(p_1)}{F(p_1)f(v_{1N}^*)}\right) + c(1 - F(p_2 - \beta k))$$
$$- p_2(1 - F(p_2 - \beta k))$$

Thus

$$p_1 = c - (p_2 - c)\left(\frac{F(p_1)f(v_{1N}^*)(1 - F(p_2 - \beta k))}{F(p_1)^2 f(v_{1N}^*) + F(v_{1N}^*)f(p_1)}\right) \tag{3.1}$$

with respect to p_2:

$$\frac{\partial \Pi}{\partial p_2} = \frac{\partial S}{\partial v_{1N}^*}\frac{\partial v_{1N}^*}{\partial p_2} + \frac{\partial \Delta}{\partial v_{1N}^*}\frac{\partial v_{1N}^*}{\partial p_2} + \frac{\partial \Delta}{\partial p_2}$$

$$\frac{\partial S}{\partial p_2} = c\int_{v_{1N}^*}^1 f(v_1)f(p_2 - \beta k)dv_1$$
$$\frac{\partial \Delta}{\partial p_2} = -(1 - F(v_{1N}^*))(1 - F(p_2)) + \int_{v_{1N}^*}^1 f(v_1)(1 - F(p_2 - \beta k))dv_1 - p_2\int_{v_{1N}^*}^1 f(v_1)f(p_2 - \beta k)dv_1$$
$$\frac{v_{1N}^*}{p_2} = 1 - F(p_2)$$

$$
\begin{aligned}
\frac{\partial \Pi}{\partial p_2} = & \left(-v_{1N}^* + c + \int_{p_1}^{1} (v_2 - c) dF(v_2) + c(1 - F(p_2 - \beta k))\right) f(v_{1N}^*)(1 - F(p_2)) \\
& + c \int_{v_{1N}^*}^{1} f(v_1) f(p_2 - \beta k) dv_1 + \left(- \int_{p_2}^{1} (1 - F(v_2)) dv_2 - p_2(1 - F(p_2 - \beta k))\right) f(v_{1N}^*)(1 - F(p_2)) \\
& - (1 - F(v_{1N}^*))(1 - F(p_2)) + \int_{v_{1N}^*}^{1} f(v_1)(1 - F(p_2 - \beta v_1)) dv_1 - p_2 \int_{v_{1N}^*}^{1} f(v_1) f(p_2 - \beta k) dv_1 \\
& = -v_{1N}^* + c + \int_{p_1}^{1} (v_2 - c) dF(v_2) + c(1 - F(p_2 - \beta k)) + c \left(\frac{\int_{v_{1N}^*}^{1} f(v_1) f(p_2 - \beta k) dv_1}{f(v_{1N}^*)(1 - F(p_2))} \right) \\
& - \int_{p_2}^{1} (1 - F(v_2)) dv_2 - p_2(1 - F(p_2 - \beta k)) - \frac{1 - F(v_{1N}^*)}{f(v_{1N}^*)} + \frac{\int_{v_{1N}^*}^{1} f(v_1)(1 - F(p_2 - \beta k)) dv_1}{f(v_{1N}^*)(1 - F(p_2))} \\
& - p_2 \left(\frac{\int_{v_{1N}^*}^{1} f(v_1) f(p_2 - \beta k) dv_1}{f(v_{1N}^*)(1 - F(p_2))} \right) = 0
\end{aligned}
$$

Substituting for the optimal threshold and after some algebra [8]

$$
\begin{aligned}
\frac{\partial \Pi}{\partial p_2} = & c - p_1 F(p_1) - c(1 - F(p_1)) + c(1 - F(p_2 - \beta k)) + c \left(\frac{\int_{v_{1N}^*}^{1} f(v_1) f(p_2 - \beta k) dv_1}{f(v_{1N}^*)(1 - F(p_2))} \right) \\
& - p_2(1 - F(p_2 - \beta k)) - \frac{1 - F(v_{1N}^*)}{f(v_{1N}^*)} + \frac{\int_{v_{1N}^*}^{1} f(v_1)(1 - F(p_2 - \beta k)) dv_1}{f(v_{1N}^*)(1 - F(p_2))} \\
& - p_2 \left(\frac{\int_{v_{1N}^*}^{1} f(v_1) f(p_2 - \beta k) dv_1}{f(v_{1N}^*)(1 - F(p_2))} \right) = 0
\end{aligned}
$$

[8] Let $p_1 < p_2$

$$
- \int_{p_1}^{p_2} (1 - F(v_2)) dv_2 - \int_{p_2}^{1} (1 - F(v_2)) dv_2 - \int_{p_1}^{1} F(v_2) dv_2 = -1 + p_1
$$

$$
\int_{p_1}^{1} (v_2 - c) dF(v_2) = 1 - p_1 F(p_1) - \int_{p_1}^{1} F(v_2) dv_2
$$

Then:

$$p_2\left(1-F(p_2-\beta k)+\left(\frac{\int_{v_{1N}^*}^1 f(v_1)f(p_2-\beta k)dv_1}{f(v_{1N}^*)(1-F(p_2))}\right)\right)=$$

$$-p_1F(p_1)+\frac{\int_{v_{1N}^*}^1 f(v_1)(1-F(p_2-\beta k))dv_1-(1-F(v_{1N}^*))(1-F(p_2))}{f(v_{1N}^*)(1-F(p_2))}$$

$$+c\left(1-1+F(p_1)+1-F(p_2-\beta k)+\left(\frac{\int_{v_{1N}^*}^1 f(v_1)f(p_2-\beta k)dv_1}{f(v_{1N}^*)(1-F(p_2))}\right)\right)$$

Substituting (3.1) and rearranging:

$$p_2\left(1-F(p_2-\beta k)+\left(\frac{\int_{v_{1N}^*}^1 f(v_1)f(p_2-\beta k)dv_1}{f(v_{1N}^*)(1-F(p_2))}\right)-\frac{F(p_1)^2f(v_{1N}^*)(1-F(p_2-\beta k))}{F(p_1)^2f(v_{1N}^*)+F(v_{1N}^*)f(p_1)}\right)=$$

$$+c\left(1-F(p_2-\beta k)+\left(\frac{\int_{v_{1N}^*}^1 f(v_1)f(p_2-\beta k))dv_1}{f(v_{1N}^*)(1-F(p_2))}\right)-\frac{F(p_1)^2f(v_{1N}^*)(1-F(p_2-\beta k))}{F(p_1)^2f(v_{1N}^*)+F(v_{1N}^*)f(p_1)}\right)$$

$$+\frac{\int_{v_{1N}^*}^1 f(v_1)(1-F(p_2-\beta k))dv_1-(1-F(v_{1N}^*))(1-F(p_2))}{f(v_{1N}^*)(1-F(p_2))}$$

Moreover, let for simplicity

$$A=1-F(p_2-\beta k)+\left(\frac{\int_{v_{1N}^*}^1 f(v_1)f(p_2-\beta k)dv_1}{f(v_{1N}^*)(1-F(p_2))}\right)-\frac{F(p_1)^2f(v_{1N}^*)(1-F(p_2-\beta k))}{F(p_1)^2f(v_{1N}^*)+F(v_{1N}^*)f(p_1)}$$

$$=\left(\frac{\int_{v_{1N}^*}^1 f(v_1)f(p_2-\beta k)dv_1}{f(v_{1N}^*)(1-F(p_2))}\right)+(1-F(p_2-\beta k))\left(1-\frac{F(p_1)^2f(v_{1N}^*)}{F(p_1)^2f(v_{1N}^*)+F(v_{1N}^*)f(p_1)}\right)$$

$$=\left(\frac{\int_{v_{1N}^*}^1 f(v_1)f(p_2-\beta k)dv_1}{f(v_{1N}^*)(1-F(p_2))}\right)+(1-F(p_2-\beta k))\left(\frac{F(p_1)^2f(v_{1N}^*)+F(v_{1N}^*)f(p_1)}{F(p_1)^2f(v_{1N}^*)+F(v_{1N}^*)f(p_1)}\right)$$

$$-(1-F(p_2-\beta k))\left(\frac{F(p_1)^2f(v_{1N}^*)}{F(p_1)^2f(v_{1N}^*)+F(v_{1N}^*)f(p_1)}\right)$$

$$=\frac{\int_{v_{1N}^*}^1 f(v_1)f(p_2-\beta k)dv_1}{f(v_{1N}^*)(1-F(p_2))}+\frac{(1-F(p_2-\beta k))F(v_{1N}^*)f(p_1)}{F(p_1)^2f(v_{1N}^*)+F(v_{1N}^*)f(p_1)}>0$$

and

$$B=\frac{\int_{v_{1N}^*}^1 f(v_1)(1-F(p_2-\beta k))dv_1-(1-F(v_{1N}^*))(1-F(p_2))}{(f(v_{1N}^*)(1-F(p_2)))}>0$$

Then the optimal price for the second quantity is:

$$p_2=c+\frac{B}{A} \tag{3.2}$$

Finally, substituting (3.2) back to (3.1), we get:

$$\begin{aligned}
p_1 &= c - (p_2 - c)\left(\frac{F(p_1)f(v_{1N}^*)(1-F(p_2-\beta k))}{F(p_1)^2 f(v_{1N}^*) + F(v_{1N}^*)f(p_1)}\right) \\
&= c - (c + \frac{B}{A} - c)\left(\frac{F(p_1)f(v_{1N}^*)(1-F(p_2-\beta k))}{F(p_1)^2 f(v_{1N}^*) + F(v_{1N}^*)f(p_1)}\right) \\
&= c - \frac{B}{A}\left(\frac{F(p_1)f(v_{1N}^*)(1-F(p_2-\beta k))}{F(p_1)^2 f(v_{1N}^*) + F(v_{1N}^*)f(p_1)}\right)
\end{aligned}$$

Proof of Proposition 1

Profits of Naive:

$$\begin{aligned}
\Pi^N &= S^N(\mathbf{p^N}) - U^N + \Delta \\
&= \int_{v_{1N}^*}^1 (v_1 - c)f(v_1)dv_1 + F(v_{1N}^*)\int_{p_1}^1 (v_2 - c)f(v_2)dv_2 \\
&+ (p_2 - c)(1 - F(v_{1N}^*))(1 - F(p_2 - \beta k)) + (1 - F(v_{1N}^*))\int_{p_2}^1 (1 - F(v_2))dv_2
\end{aligned}$$

$$\begin{aligned}
S^N(\mathbf{p^N}) &= \int_{v_{1N}^*}^1 \big(v_1 - c + \int_{p_{2N}-\beta k}^1 (v_2 + \beta k - c)dv_2\big)f(v_1)dv_1 + F(v_{1N}^*)\int_{p_{1N}}^1 (v_2 - c)f(v_2)dv_2 = \\
&= \int_{v_{1N}^*}^1 \big(v_1 - c)f(v_1)dv_1 + (1 - F(v_{1N}^*))\int_{p_{2N}-\beta k}^1 (v_2 + \beta k - c)dv_2\big) \\
&+ F(v_{1N}^*)\int_{p_{1N}}^1 (v_2 - c)f(v_2)dv_2
\end{aligned}$$

$$\begin{aligned}
U^S(\mathbf{p^N}) &= \int_{v_{1S}^*}^1 \big(v_1 - p_{1N} + \int_{p_{2N}-\beta k}^1 (v_2 + \beta k - p_{2N})dv_2\big)f(v_1)dv_1 \\
&+ F(v_{1S}^*)\int_{p_{1N}}^1 (v_2 - p_{1N})f(v_2)dv_2 - F^N = \\
&= \int_{v_{1S}^*}^1 \big(v_1 - p_{1N})f(v_1)dv_1 + (1 - F(v_{1S}^*))\int_{p_{2N}-\beta k}^1 (1 - F(v_2))dv_2 \\
&+ F(v_{1S}^*)\int_{p_{1N}}^1 (1 - F(v_2))dv_2 - F^N
\end{aligned}$$

$$
\begin{aligned}
U^N(\mathbf{p^N})) = &\int_{v_{1N}^*}^1 \big(v_1 - p_{1N} + \int_{p_{2N}}^1 (v_2 - p_{2N}) f(v_2) dv_2\big) f(v_1) dv_1 \\
&+ F(v_{1N}^*) \int_{p_{1N}}^1 (v_2 - p_{1N}) f(v_2) dv_2 - F^N = \\
&= \int_{v_{1N}^*}^1 \big(v_1 - p_{1N}) f(v_1) dv_1 + (1 - F(v_{1N}^*)) \int_{p_{2N}}^1 (1 - F(v_2)) dv_2 \\
&+ F(v_{1N}^*) \int_{p_{1N}}^1 (1 - F(v_2)) dv_2 - F^N
\end{aligned}
$$

$$
\begin{aligned}
\Delta = &U^N(\mathbf{p^N}) - U_n^S(\mathbf{p^N}) = \\
&= (1 - F(v_{1N}^*)) \left(\int_{p_2}^1 (1 - F(v_2)) dv_2 - \int_{p_2 - \beta k}^1 (1 - F(v_2)) dv_2 \right)
\end{aligned}
$$

$$
\begin{aligned}
U_n^S(\mathbf{p^N}) = &\int_{v_{1N}^*}^1 \big(v_1 - p_{1N} + \int_{p_{2N} - \beta k}^1 (v_2 + \beta k - p_{2N}) f(v_2) dv_2\big) f(v_1) dv_1 \\
&+ F(v_{1N}^*) \int_{p_{1N}}^1 (v_2 - p_{1N}) f(v_2) dv_2 - F^N \\
&= \int_{v_{1N}^*}^1 \big(v_1 - p_{1N}) f(v_1) dv_1 + (1 - F(v_{1N}^*)) \int_{p_{2N} - \beta k}^1 (1 - F(v_2)) f(v_2) dv_2 \\
&+ F(v_{1N}^*) \int_{p_{1N}}^1 (1 - F(v_2)) dv_2 - F^N
\end{aligned}
$$

$$
v_{1N}^* = p1 + \int_{p_{1N}}^{p_{2N}} (1 - F(v_2)) dv_2
$$

$$
v_{1S}^* = p1 + \int_{p_{1N}}^{p_{2N} - \beta k} (1 - F(v_2)) dv_2
$$

The profits of the Sophisticated are:

$$
\Pi^S = S^S(\mathbf{p^S}) - U^S(\mathbf{p^S})
$$

$$
\begin{aligned}
U^S(\mathbf{p^S}) = &U^S(\mathbf{p^N}) - U^N(\mathbf{p^N}) = \\
&= \int_{v_{1S}^*}^{v_{1N}^*} (v_1 - p_1) f(v_1) dv_1 - (F(v_{1N}^*) - F(v_{1S}^*)) \int_{p_1}^1 (1 - F(v_2)) dv_2 \\
&- (1 - F(v_{1N}^*)) \int_{p_2}^1 (1 - F(v_2)) dv_2 + (1 - F(v_{1S}^*)) \int_{p_2 - \beta k}^1 (1 - F(v_2)) dv_2
\end{aligned}
$$

Thus, the profit function for the screening model is:

$$
\Pi = \gamma \Pi^S + (1 - \gamma) \Pi^N
$$

$$
\begin{aligned}
\frac{d\Pi}{dv_{1N}^*} &= f(v_{1N}^*)\Bigg((\gamma-1)(c-p_2)F(p_2-\beta k)-(\gamma-1)\left(\int_{p_1}^{1}(v_2-c)f(v_2)dv_2\right)\\
&\quad -2c(\gamma-1)+g\int_{p_1}^{1}(1-F(v_2))dv_2-\int_{p_2}^{1}(1-F(v_2))d\mathrm{v}2+\gamma(p_1+p_2)-p_2-v_{1N}^*\Bigg)\\
\frac{d\Pi}{dp_1} &= (1-\gamma)(p_1-c)f(p_1)F(v_{1N}^*)+\gamma\Bigg((F(p_1)-1)(F(v_{1N}^*)-F(v_{1S}^*))\Bigg)\\
&\quad +\gamma\int_{v_{1S}^*}^{v_{1N}^*}f(v_{1N}^*)dv_1\\
\frac{d\Pi}{dp_2} &= F(p_2-\beta k)\Bigg((1-\gamma)F(v_{1N}^*)+\gamma F(v_{1S}^*)-1\Bigg)\\
&\quad +\gamma(F(v_{1N}^*)-F(v_{1S}^*))-F(p_2)(F(v_{1N}^*)-1)\\
&\quad -(1-\gamma)(p_2-c)(F(v_{1N}^*)-1)f(p_2-\beta k)\\
\frac{dv_{1N}^*}{dp_1} &= F(p_1)\\
\frac{dv_{1N}^*}{dp_2} &= 1-F(p_2)
\end{aligned}
$$

Then, the first order condition with respect to p_1 is:

$$
\begin{aligned}
\frac{d\Pi}{dp_1} &= \frac{d\Pi}{dv_{1N}^*}\frac{dv_{1N}^*}{dp_1}+\frac{d\Pi}{dp_1}=\\
&= f(v_{1N}^*)F(p_1)\Bigg((\gamma-1)(c-p_2)F(p_2-\beta k)-(\gamma-1)\left(\int_{p_1}^{1}(v_2-c)f(v_2)dv_2\right)\\
&\quad -2c(\gamma-1)+\gamma\int_{p_1}^{1}(1-F(v_2))dv_2-\int_{p_2}^{1}(1-F(v_2))dv_2+\gamma(p_1+p_2)-p_2-v_{1N}^*\Bigg)\\
&\quad +(1-\gamma)(c-p_1)f(p_1)F(v_{1N}^*)\\
&\quad +\gamma\left((F(p_1)-1)(F(v_{1N}^*)-F(v_{1S}^*))+\int_{v_{1S}^*}^{v_{1N}^*}f(v_1)dv_1\right)=\\
&= f(v_{1N}^*)F(p_1)\Bigg((\gamma-1)(c-p_2)F(p_2-\beta k)-(\gamma-1)\left(\int_{p_1}^{1}(v_2-c)f(v_2)dv_2\right)\\
&\quad -2c(\gamma-1)+\gamma\int_{p_1}^{1}(1-F(v_2))dv_2-\int_{p_2}^{1}(1-F(v_2))dv_2+\gamma(p_1+p_2)-p_2-v_{1N}^*\Bigg)\\
&\quad +(1-\gamma)(c-p_1)f(p_1)F(v_{1N}^*)\\
&\quad +\gamma\left((F(p_1)-1)(F(v_{1N}^*)-F(v_{1S}^*))+(F(v_{1N}^*)-F(v_{1S}^*))\right)
\end{aligned}
$$

after some algebra[9] and substituting v_{1N}^*:

$$\frac{d\Pi}{dp_1} = F(p_1)(1-\gamma)f(v_{1N}^*)\Big((p_2-c)F(p_2-\beta k) + F(p_1)(c-p_1) + c - p_2\Big)$$
$$+ f(p_1)(1-\gamma)(c-p_1)F(v_{1N}^*) + F(p_1)\gamma(F(v_{1N}^*) - F(v_{1S}^*)) = 0$$

The first order condition with respect to p_2 is:

$$\frac{d\Pi}{dp_2} = \frac{d\Pi}{dv_{1N}^*}\frac{dv_{1N}^*}{dp_2} + \frac{d\Pi}{dp_2} =$$
$$= f(v_{1N}^*)(1-F(p_2))\Bigg((g-1)(c-p_2)F(p_2-\beta k) + (1-\gamma)\left(\int_{p_1}^1 (v_2-c)f(v_2)dv_2\right)$$
$$- 2c(\gamma-1) + g\int_{p_1}^1 (1-F(v_2))dv_2 - \int_{p_2}^1 (1-F(v_2))dv_2 + \gamma(p_1+p_2) - p_2 - v_{1N}^*\Bigg)$$
$$+ (\gamma-1)(c-p_2)(F(v_{1N}^*)-1)f(p_2-\beta k) + F(p_2-\beta k)((1-\gamma)F(v_{1N}^*) + \gamma F(v_{1S}^*) - 1) = 0$$

then again after some algebra and substituting v_{1N}^*:

$$\frac{d\Pi}{dp_2} = (F(p_2)-1)(\gamma-1)f(v_{1N}^*)((p_2-c)F(p_2-\beta k) + F(p_1)(c-p_1) + c - p_2)$$
$$+ (\gamma-1)(c-p_2)(F(v_{1N}^*)-1)f(p_2-\beta k) + F(p_2-\beta k)(\gamma F(v_{1N}^*) + \gamma F(v_{1S}^*) + F(v_{1N}^*) - 1)$$
$$+ \gamma(F(v_{1N}^*) - F(v_{1S}^*)) - F(p_2)(F(v_{1N}^*) - 1) = 0$$

Solving the system of the first order conditions:

$$p_2 = c + \frac{F(p_1)^2 f(v_{1N}^*)(F(p_2)-F(p_2-\beta k))((\gamma(F(v_{1N}^*)-F(v_{1S}^*))+1-F(v_{1N}^*))}{(1-\gamma)f(p_1)f(v_{1N}^*)(1-F(p_2))\Big(1-F(p_2-\beta k)F(v_{1N}^*)+f(p_2-\beta k)(1-F(v_{1N}^*))(f(v_{1N}^*)F(p_1)^2+f(p_1)F(v_{1N}^*))\Big)}$$
$$+ \frac{f(p_1)F(v_{1N}^*)\Big(\gamma(1-F(p_2-\beta k))(F(v_{1N}^*)-F(v_{1S}^*))+(1-F(v_{1N}^*))(F(p_2)-F(p_2-\beta k))\Big)}{(1-\gamma)f(p_1)f(v_{1N}^*)(1-F(p_2))\Big(1-F(p_2-\beta k)F(v_{1N}^*)+f(p_2-\beta k)(1-F(v_{1N}^*))(f(v_{1N}^*)F(p_1)^2+f(p_1)F(v_{1N}^*))\Big)}$$
$$p_1 = c + \frac{F(p_1)\Big(\gamma(1-F(v_{1N}^*))f(p_2-\beta k)(F(v_{1N}^*)-F(v_{1S}^*))\Big)}{(1-\gamma)\Big((1-F(v_{1N}^*))f(p_2-\beta k)\Big(F(p_1)^2 f(v_{1N}^*)+f(p_1)F(v_{1N}^*)\Big)+f(p_1)(1-F(p_2))f(v_{1N}^*)F(v_{1N}^*)(1-F(p_2-\beta k))\Big)}$$
$$- \frac{F(p_1)\Big(f(v_{1N}^*)(F(p_2)-F(p_2-\beta k))(1-F(p_2-\beta k))((\gamma(F(v_{1N}^*)-F(v_{1S}^*))+1-F(v_{1N}^*))\Big)}{(1-\gamma)\Big((1-F(v_{1N}^*))f(p_2-\beta k)\Big(F(p_1)^2 f(v_{1N}^*)+f(p_1)F(v_{1N}^*)\Big)+f(p_1)(1-F(p_2))f(v_{1N}^*)F(v_{1N}^*)(1-F(p_2-\beta k))\Big)}$$

---

[9]

$$(1-\gamma)\int_{p_1}^1 (v_2-c)f(v_2)dv_2 + \gamma\int_{p_1}^1 (1-F(v_2))dv_2 - \int_{p_2}^1 (1-F(v_2))dv_2$$
$$= -(1-\gamma)(1-F(p_1))c + (1-\gamma)(1-p_1F(p_1)) + \gamma(1-p_1) - (1-p_2) - \int_{p_1}^{p_2} F(v_2)dv_2$$

Then $p_2 > c$ since $F(p_2) < F(p_2 - \beta k)$, $F(v_{1N}^*) > F(v_{1S}^*)$

Moreover, $p_1 < c$ if:

$$F(p_2) - F(p_2 - \beta k) < \frac{\gamma f(p_2 - \beta k)(1 - F(v_{1N}^*))(F(v_{1N}^*) - F(v_{1S}^*))}{f(v_{1N}^*)(1 - F(p_2 - \beta k)(1 - F(v_{1N}^*) + \gamma(F(v_{1N}^*) - F(v_{1N}^*)))}$$

or

$$F(p_2) - F(p_2 - \beta k) < \frac{f(p_2 - \beta k)(1 - F(v_{1N}^*))(F(v_{1N}^*) - F(v_{1S}^*))}{f(v_{1N}^*)(1 - F(p_2 - \beta k))(1 - F(v_{1S}^*))}$$

and

$$\gamma < \frac{(F(p_2) - F(p_2 - \beta k))f(v_{1N}^*)(1 - F(v_{1N}^*))(1 - F(p_2 - \beta k))}{(F(v_{1N}^*) - F(v_{1S}^*))((1 - F(v_{1N}^*))f(p_2 - \beta k) - (F(p_2) - F(p_2 - \beta k))f(v_{1N}^*)(1 - F(p_2 - \beta k)))}$$

Assuming Uniform Distribution

The maximization problem of the consumer becomes:

$$\max_{\mathbf{p^N}} \quad \Pi = \gamma\Big(- (U^S(\mathbf{p^N}) - U^N(\mathbf{p^N}))\Big) + (1-\gamma)\Big(S^N(\mathbf{p^N}) + \Delta\Big)$$

where

$$\begin{aligned}
S^N(\mathbf{p^N}) &= \int_{v_{1N}^*}^1 \big(v_1 - c + \int_{p_{2N}-\beta k}^1 (v_2 + \beta k - c)dv_2\big)dv_1 + v_{1N}^* \int_{p_{1N}}^1 (v_2 - c)dv_2 \\
U^S(\mathbf{p^N}) &= \int_{v_{1S}^*}^1 \big(v_1 - p_{1N} + \int_{p_{2N}-\beta k}^1 (v_2 + \beta k - p_{2N})dv_2\big)dv_1 + v_{1S}^* \int_{p_{1N}}^1 (v_2 - p_{1N})dv_2 - F^N \\
U^N(\mathbf{p^N})) &= \int_{v_{1N}^*}^1 \big(v_1 - p_{1N} + \int_{p_{2N}}^1 (v_2 - p_{2N})dv_2\big)dv_1 + v_{1N}^* \int_{p_{1N}}^1 (v_2 - p_{1N})dv_2 - F^N \\
\Delta &= U^N(\mathbf{p^N}) - \tilde{U}(\mathbf{p^N}) \\
\tilde{U}(\mathbf{p^N}) &= \int_{v_{1N}^*}^1 \big(v_1 - p_{1N} + \int_{p_{2N}-\beta k}^1 (v_2 + \beta k - p_{2N})dv_2\big)dv_1 + v_{1N}^* \int_{p_{1N}}^1 (v_2 - p_{1N})dv_2 - F^N \\
v_{1N}^* &= p1 + \int_{p_{1N}}^{p_{2N}} (1 - v_2)dv_2 \\
v_{1S}^* &= p1 + \int_{p_{1N}}^{p_{2N}-\beta k} (1 - v_2)dv_2
\end{aligned}$$

Making the calculation we take:

$$
\begin{aligned}
\max_{p_1,p_2} \Pi = &\frac{1}{8}(8-8\gamma-(\beta k)^4\gamma+4(\beta k)^3\gamma(p_2-1)+2(\beta k)^2\gamma(p_1^2-3p_2^2+2p_2-4)\\
&+4\beta k(\gamma-p_2)(p_1^2-2-p_2^2+2p_2)-4c(\gamma-1)(4(p_2-1)\\
&+(p_1-p_2)(p_1+p_1^2-p_2^2-3p_2)+\beta k(-2+p_1^2-(-2+p_2)p_2))\\
&+(\gamma-1)(3p_1^4+2p_1^2(4-3p_2)p_2+p_2^2(8+p_2(-8+3p_2))))
\end{aligned}
$$

The derivative with respect to p_1:

$$
\begin{aligned}
\frac{d\Pi}{dp_1} = \frac{1}{2}\Big(&p_1\left(\gamma\beta^2k^2+2\beta k(\gamma-p_2)+(\gamma-1)\left(3p_1^2+(4-3p_2)p_2\right)\right)\\
&-c(\gamma-1)\left(2p_1(\beta k-p_2+1)+3p_1^2-(p_2-2)p_2\right)\Big)
\end{aligned}
$$

with respect to p_2:

$$
\begin{aligned}
\frac{d\Pi}{dp_2} = \frac{1}{2}\Big(&\gamma\beta^3k^3-3\gamma\beta^2k^2(p_2-1)+c(\gamma-1)\big(2\beta k(p_2-1)+p_1^2\\
&+2p_1(p_2-1)-3(p_2-2)p_2-4\big)-\beta k\left(2\gamma(p_2-1)+p_1^2+(4-3p_2)p_2-2\right)\\
&-(\gamma-1)\left(p_1^2(3p_2-2)+p_2(-3(p_2-2)p_2-4)\right)\Big)
\end{aligned}
$$

Cost equals to zero

Let assume that the cost is zero, $c=0$, then the first order conditions are:

$$
\begin{aligned}
\frac{d\Pi}{dp_{1N}} &= \frac{1}{2}p_{1N}\left(\beta^2\gamma k^2+2\beta k(\gamma-p_{2N})+(\gamma-1)\left(3p_{1N}^2+(4-3p_{2N})p_{2N}\right)\right)\\
\frac{d\Pi}{dp_{2N}} &= \frac{1}{2}\big(\beta^3\gamma k^3-3\beta^2\gamma k^2(p_{2N}-1)-\beta k\left(2\gamma(p_{2N}-1)+p_{1N}^2+(4-3p_{2N})p_{2N}-2\right)\\
&\quad-(\gamma-1)\left(p_{1N}^2(3p_{2N}-2)+p_{2N}(-3(p_{2N}-2)p_{2N}-4)\right)\big)
\end{aligned}
$$

then, the optimal price for the naive consumer of the first and the second unit are

:

$$
\begin{aligned}
p_{1N} &= 0\\
p_{2N} &= \frac{1}{6(\gamma-1)}\left(2^{2/3}A+\frac{2\sqrt[3]{2}\gamma\beta k((\gamma-1)(3\beta k+2)+\beta k)}{A}-2\beta k+4\gamma-4\right)
\end{aligned}
$$

,where A is:

$$A = \sqrt[3]{-2\beta^3 k^3 + \sqrt{\left(\beta^3\left(9(g-1)g^2+2\right)k^3 + 3\beta^2\gamma(3\gamma-1)(\gamma-1)k^2 + 6\beta(\gamma+1)(\gamma-1)^2 k + 8(\gamma-1)^3\right)^2 - 4\beta^3 k^3((\gamma-1)\gamma(3\beta k+2)+\beta k)^3} - \gamma^3(3\beta k(3\beta k(\beta k+1)+2)+8) + 3\gamma^2(\beta k(\beta k(3\beta k+4)+2)+8) - 3\gamma(\beta k(\beta k-2)+8) - 6\beta k + 8}$$

Examining the first order conditions at $p_{1N}, p_{2N} = \{0, 0\}$ we see that:

$$\left.\frac{d\Pi}{dp_{1N}}\right|_{\{0,0\}} = 0, \quad \left.\frac{d\Pi}{dp_{1N}}\right|_{\{p_{1N}=0\}} = 0$$
$$\left.\frac{d\Pi}{dp_{2N}}\right|_{\{0,0\}} = \frac{1}{2}\left(\beta k(2+2\gamma) + 3(\beta k)^2\gamma + (\beta k)^3\gamma\right) \geq 0$$

Thus the equilibrium is $\mathbf{p_N} = \{p_{1N} = 0, p_{2N} > c, F^N = U^N(\mathbf{p_N})\}$ and $\mathbf{p_S} = \{p_{1S} = 0, p_{2S} = 0, F^S = U^S(\mathbf{p_S}) - F^S - (U^S(\mathbf{p_N}) - U^N(\mathbf{p_N}))\}$

Fixed Fee of Sophisticated :

The fixed fee of the Sophisticated consumer can be derived from her incentive compatibility constraint thus it is:

$$F^S = \int_{v_{1S}^*}^1 \left(v_1 - p_{1S} + \int_{p_{2S}-\beta k}^1 (v_2 + \beta k - p_{2S}) dv_2\right) dv_1 + v_{1S}^* \int_{p_{1S}}^1 (v_2 - p_{1S}) dv_2$$
$$- \left(\int_{v_{1S}^*}^1 \left(v_1 - p_{1N} + \int_{p_{2N}-\beta k}^1 (v_2 + \beta k - p_{2N}) dv_2\right) dv_1 + v_{1S}^* \int_{p_{1N}}^1 (v_2 - p_{1N}) dv_2 - F^N\right)$$

,where

$$F^N = \int_{v_{1N}^*}^1 \left(v_1 - p_{1N} + \int_{p_{2N}}^1 (v_2 - p_{2N}) dv_2\right) dv_1 + v_{1S}^* \int_{p_{1N}}^1 (v_2 - p_{1N}) dv_2$$

thus since $p_{1S} = p_{2S} = c = 0$, $p_{1N} = 0$ and $p_{2N} > 0$ then the above equation becomes:

$$F^S = \frac{1}{4}\left(4 + \beta k(2 + \beta k + 2(4 + \beta k(3 + \beta k))p_{2N} - 3(2 + \beta k)p_{2N}^2 + 2p_{2N}^3)\right)$$

checking numerically substituting the prices it is reasonable expect for the parameter levels that there is not real root.

Incentive Compatibility Constraint of the Naive : In order to show that the constraint that was relaxed, it really slacks at the optimum, it is needed to show that:

$$U^N(\mathbf{p^N}) > U^N(\mathbf{p^S})$$

thus since at the equilibrium $U^N(\mathbf{p^N}) = 0$ and the expected utility of the naive consumer at $\mathbf{p^S}$ equals 1 then it needs to be shown that:

$$0 > 1 - F^S \Rightarrow F^S > 1$$

which is true for $0 \leq k \leq 1$, $0 \leq \beta \leq 1$ and $p_2 > 0$.

Chapter 4

Impatience and three-part tariffs

Eleftheria Triviza Vincenzo Denicoló

Abstract

We propose a new explanation of three part tariffs, based on the assumption that consumers are forward-looking but impatient. In a dynamic stochastic setting, prices that apply to large volumes tend to be paid towards the end of the contracting period and so are more heavily discounted by consumers. As a result, high prices for large volumes represent an efficient way of extracting surplus. Low (or even vanishing) prices for small volumes, on the other hand, serve to stimulate early consumption, making it more likely that high marginal prices will indeed apply later. Although firms design contracts so as to take advantage of consumers' impatience, impatience in fact benefits consumers as it prevents full extraction of their surplus. However, when both patient and impatient consumers coexist in the market, patient consumers gain more than impatient ones.

4.1 Introduction

Information technologies have greatly facilitated the monitoring of individual consumption, spurring the use of non-linear pricing in many markets. In particular, three-part tariffs – where the customer pays a fixed fee, obtains a free allowance, and is charged a constant price for each unit in excess of the allowance – have become increasingly popular. Examples include credit cards, telephone services, internet access, on-line music download, on-line newspapers, data center hosting, and many more.

The prevalence of three-part tariffs is a challenge for economic theory. Under complete information, standard models predict that the price should be equal to the marginal cost and hence that the surplus from trade should be maximized. This outcome is obtained both under perfect competition, where the surplus accrues to consumers, and under monopoly, where the surplus is extracted by the firm through a fixed fee – e.g. with a two-part tariff if the marginal cost is constant. Under asymmetric information, marginal prices should exceed marginal costs if firms have market power. Furthermore, standard models typically predict that marginal prices should decrease with the quantity bought, whereas under three-part tariffs they increase.

Given these difficulties, in order to explain the prevalence of three-part tariffs economists have turned to behavioural theories in which consumers deviate from the standard model of rational decision making. There is no doubt that in reality consumers depart from rationality in all sorts of ways. However, this paper argues that the optimality of three-part tariffs requires only that consumers are sufficiently more impatient than the firms.

The intuition is as follows. Consider to fix ideas the case of monopoly and suppose, as we shall do throughout the paper, that marginal costs are constant. Take as a starting point the two-part tariff that would be optimal if consumers had the same discount factor as the firm. With overdiscounting, the firm can no longer use the fixed fee to fully extract the surplus, since consumers underestimate the benefits from future trade. Therefore, the firm will raise marginal prices for large volumes in order to extract more surplus from those trades that occur later and hence are more heavily discounted by consumers.

This conclusion, in itself, does not come as a surprise. It is well known that prices may be distorted when consumers misperceive (from the firms' viewpoint) the benefits from trade: see, for instance, DellaVigna and Malmendier (2004), Eliaz and Spiegler (2008) and Grubb (2009). The novel part of our argument rests on the fact that consumption decisions are not made once and for all at the beginning of the contracting period. Rather, they are made sequentially as new consumption opportunities stochastically arise over time.

In this dynamic stochastic framework, forward looking consumers who face higher marginal prices for large volumes will realise that the first units bought in a contracting period have an opportunity cost in addition to the direct cost. This implies a tendency to underconsume early units, which is bad for profits, as the firm can extract most of the surplus precisely from such early trades. Therefore, the firm has an incentive to stimulate early consumption by reducing the marginal price for small volumes. As a result, it will set the price below the marginal cost for low volumes and above the marginal cost for large volumes. The same pattern is obtained when firms compete, even though competition will generally decrease the fixed fee that the firms can charge.

The above argument can therefore explain the combination of below-cost pricing for low volumes and above-cost pricing for large volumes, which is the quintessence of three-part tariffs. This suggested explanation assumes that consumers are impatient but strategic. Both assumptions are well grounded empirically.

To begin with, consumers' impatience is well documented. Of special relevance for our purposes is the study of Yao et al. (2012), which focuses on the Chinese market for mobile phone services where three-part tariffs are indeed employed. Yao et al. (2012) estimate that Chinese mobile phone users have a weekly discount rate of approximately 10%. To be precise, they estimate that users value 1 minute of calls at the beginning of the contracting period (a month) as much as 1 minute and 34 seconds of calls at the end of the period.

At the same time, Yao et al. (2012) and many other empirical papers document that consumers are strategic in their usage of allowance minutes. For example, consumers begin to conserve minutes as the allowance starts getting exhausted, whereas they accelerate usage if they are still below the quota towards the end of the contracting period. This strategic behaviour is also confirmed by experimental evidence. In a cleverly designed experiment, Leider and Sahin (2014) face subjects with three-part tariffs in an artificial market for phone calls. A major finding is that only a small minority of subjects are completely myopic. Most subjects do not make every call that has a positive value, even if they have not exhausted their allowance yet. Evidently these subjects internalize, at least partially, the opportunity cost of consuming the stock of free calls.[1]

While the main ingredients of our suggested explanation are realistic, it should be pointed out that the theory may also be consistent with several possible deviations from the standard model of rational decision making. For example, the theory does not assume that discounting is exponential. In fact, under exponential discounting weekly discount rates of around 10% would compound into exorbitant yearly rates. The empirical evidence of Yao et al. (2012) may therefore be taken to suggest the presence of hyperbolic discounting. Our theory is agnostic about the type of discounting, a fact that we emphasize by casting the analysis in a two-period framework in which all forms of discounting are equivalent.

It should also be noted that consumers' impatience is analogous to their underestimating future demand in a model where firms and consumers have different priors. This latter approach is adopted in an influential paper by Grubb (2009), who however assumes that consumption choices are made once and for all at the beginning of the contracting period. In his static framework, the optimality of three-part tariffs requires that consumers underestimate both the possibility that demand is very high and that it is very low. Allowing for sequential choices not only adds to realism but

[1] While it is crucial for our results that consumers internalize the opportunity cost, such internalization need not be complete or perfect. The experimental data of Leider and Sahin (2014) in fact suggests that subjects tend to approximate the fully optimal policy, which may be quite complex when demand is uncertain, with simple heuristics. Although we do not model this possibility explicitly, our results should be robust to such boundedly rational behaviours.

also provides a more parsimonious explanation.

Finally, observe that impatience is also analogous to loss aversion. In a stochastic environment such as ours, fixed fees entail losses when consumption opportunities are unfavourable and gains when they are favourable. If consumers value gains less than losses, fixed fees will not allow efficient surplus extraction, and the same mechanisms as under consumers' impatience will operate.

The rest of the paper proceeds as follows. Section 2 sets out the model. Section 3 analyzes the case of complete information. Section 4 extends the analysis to the case in which some consumers are patient and others are impatient, and the degree of impatience is private information. Section 5 summarizes the main arguments and concludes the paper.

4.2 The model

In this section, we provide a description of the elements of the model that are common to all specifications considered in the paper.

We adapt the sequential consumption choice model of Grubb (2014). Each contracting period is divided into two sub-periods, $t = 1, 2$. Our insights apply also to the case of more than two sub-periods, but this extension would complicate the analysis and require specific assumptions about the form of discounting that we prefer to eschew here.

In each sub-period, the consumer can purchase one unit of the good. *Ex ante*, the value of consumption is uncertain. The consumer's willingness to pay in sub-period t, $v_t \geq 0$, is randomly and independently drawn from a stationary cumulative distribution function $G(v_t)$. For simplicity, we assume that $G(v_t)$ is atomless and has a density function $g(v_t)$. We also assume that the support of $G(v_t)$ is finite, and without any further loss of generality we normalize it to $[0, 1]$.

At the beginning of the contracting period, and before the realization of uncertainty, firms offer contracts. Consumers then either choose a contract or decline the offers. In the latter case, they obtain a reservation utility that is normalized at zero.

A contract $\mathcal{C} = \{F, p_1, p_2\}$ comprises a fixed payment F, a price p_1 for the first unit bought, and a price p_2 for the second unit bought. In our setting, this is a fully general non-linear pricing scheme that encompasses as special cases linear pricing ($F = 0$ and $p_1 = p_2$), two-part tariffs ($p_1 = p_2$), and three-part tariffs ($p_1 = 0$). The fixed fee is paid in period 1, whereas the variable payments are made as consumption occurs.[2]

In each sub-period t, the consumer learns the true value of v_t and makes the consumption choice $q_t \in \{0, 1\}$. The consumer's utility, in monetary terms, is:

$$u = q_1 v_1 + \delta q_2 v_2, \tag{4.1}$$

where δ is the discount factor. The consumer's net utility is therefore

$$U = u - F - q_1 p_1 - \delta[q_1 p_2 + (1 - q_1)p_1]q_2, \tag{4.2}$$

where the term inside square brackets captures the fact that the second-period price depends on first-period consumption.

Unlike consumers, firms do not discount future profits.[3] Assuming a constant marginal cost of c, a firm's profit is

$$\pi = F + q_1(p_1 - c) + [q_1 p_2 + (1 - q_1)p_1 - c]q_2. \tag{4.3}$$

In the baseline model, we assume that both firms and consumers are risk neutral.

Summarizing, the timing of the model is as follows. At the beginning of the contracting period, contracts are offered. Consumers either sign a contract or receive their reservation utility. If a consumer signs a contract, he pays the fixed fee and the game proceeds to sub-periods 1 and 2. In each sub-period, the consumers learns the realization of demand, chooses whether or not to buy, and makes any associated payments.

In this game, a strategy for a firm is a contract $\mathcal{C} \in R^3$, or a menu of such contracts. A strategy for a consumer is a triple comprising (i) an acceptance decision, (ii) a

[2]The results do not change if the consumer deposits $F + p_1 + p_2$ at the beginning of the contracting period and any unused deposit is refunded at the end of the period.

[3]All the results continue to hold as long as firms' discount rate is higher than the consumers'.

function $\chi_1 : [0,1] \to \{0,1\}$ mapping first-period valuations to first-period quantity, and (iii) a function $\chi_2 : [0,1] \times \{0,1\} \to \{0,1\}$ mapping second-period valuations and first-period quantities to second-period quantity.

The functions χ_1 and χ_2 can be characterized easily by noting that the optimal consumption strategy is given by cut-off rules. That is, a consumer who has signed a contract $\mathcal{C} = \{F, p_1, p_2\}$ should consume if and only if the willingness to pay exceeds critical thresholds. The second-period optimal threshold is obviously:

$$v_2^* = \begin{cases} p_1 & \text{if} \quad q_1 = 0 \\ p_2 & \text{if} \quad q_1 = 1 \end{cases} \tag{4.4}$$

Consider now the first period. Given that the consumer follows the cut-off rule v_2^*, the first-period net expected utility is:

$$\begin{aligned} E(U) \quad = \quad & -F + \int_{v_1^*}^{1} \left[v_1 - p_1 + \delta \int_{p_2}^{1} (v_2 - p_2) g(v_2) dv_2 \right] g(v_1) dv_1 + \\ & + \delta G_1(v_1^*) \int_{p_1}^{1} (v_2 - p_1) g(v_2) dv_2 \end{aligned} \tag{4.5}$$

The first term in this expression is the fixed fee; the second is the net expected utility if first-period consumption is positive; and the third is the net expected utility if first-period consumption is nil. The optimal first-period cutoff v_1^* maximises $E(U)$. This yields:

$$v_1^* = p_1 + \omega, \tag{4.6}$$

where the term

$$\omega = \delta \left[\int_{p_1}^{1} (v_2 - p_1) g(v_2) dv_2 - \int_{p_2}^{1} (v_2 - p_2) g(v_2) dv_2 \right]$$

reflects the opportunity cost of first-period consumption. This is the expected cost (or benefit, if $p_2 < p_1$) of turning the second-period marginal price from p_1 to p_2. After an integration by parts, one can write the opportunity cost as:

$$\omega = \delta \int_{p_1}^{p_2} (1 - G(v_2)) dv_2 \tag{4.7}$$

These conditions rest on the assumption that forward looking consumers take into account the opportunity cost of consumption, in addition to the direct cost. As discussed in the introduction, there is considerable empirical evidence for such forward-looking behaviour.

4.3 Complete information

We now proceed to characterize the model's equilibrium. We start our analysis in this section from the case of complete information. In this case, all parameters of the model are common knowledge, except the consumer's willingness to pay.

The structure of equilibrium contracts is largely independent of the degree of product market competition. Initially we focus on the two polar cases of monopoly on one hand and perfect competition on the other hand. We show that marginal prices p_1 and p_2 are the same in both cases, the only difference being the level of the fixed fee. After analyzing these cases, we shall argue that this conclusion extends also to certain models of oligopoly.

Under complete information, firms can offer personalized contracts. Therefore, we may focus on a single consumer without any loss of generality. For any given contract $\mathcal{C} = \{F, p_1, p_2\}$, the consumer anticipates a surplus of

$$\begin{aligned} E(U) &= -F + \int_{v_1^*}^1 \left[v_1 - p_1 + \delta \int_{p_2}^1 (v_2 - p_2) g(v_2) dv_2 \right] g(v_1) dv_1 + \\ &\quad + \delta G(v_1^*) \int_{p_1}^1 (v_2 - p_1) g(v_2) dv_2, \end{aligned} \tag{4.8}$$

and accepts the contract if and only if this is non negative. On the other hand, the firm anticipates a profit of

$$\begin{aligned} E(\pi) &= F + \int_{v_1^*}^1 \left[p_1 - c + \int_{p_2}^1 (p_2 - c) g(v_2) dv_2 \right] g(v_1) dv_1 + \\ &\quad + G(v_1^*) \int_{p_1}^1 (p_1 - c) g(v_2) dv_2. \end{aligned} \tag{4.9}$$

Under perfect competition, expected profits must be driven to zero. The competition among the firms ensures that the equilibrium contract maximizes the expected surplus (7) under the constraint that $E(\pi) = 0$. Defining the expected "social surplus" as

$$
\begin{aligned}
E(S) &= E(U) + E(\pi) = \\
&= \int_{v_1^*}^{1} \left\{ v_1 - c + \int_{p_2}^{1} \left[\delta v_2 + (1-\delta) p_2 - c\right] g(v_2) dv_2 \right\} g(v_1) dv_1 + \\
&\quad + G(v_1^*) \int_{p_1}^{1} \left[\delta v_2 + (1-\delta) p_1 - c\right] g(v_2) dv_2, \qquad (4.10)
\end{aligned}
$$

it then appears that the equilibrium marginal prices p_1 and p_2 must maximize S. The reason for this is that the fixed fee F cancels out in the definition of the social surplus S, but for any given choice of p_1 and p_2 it can be freely adjusted so as to satisfy the constraint $E(\pi) = 0$.

Under monopoly, on the other hand, the consumer's expected surplus is set equal to the reservation level, which is nil. The equilibrium marginal prices p_1 and p_2 must then maximize the expected profit (8) under the constraint that $E(U) = 0$. Once again, this problem coincides with the maximization of S, and the constraint is then met by appropriate choice of the fixed fee. Thus, the equilibrium marginal prices are the same as under perfect competition, as was claimed above.

Before proceeding, it may be worth noting that S is the social surplus as it is perceived by the agents. If one regards discounting as irrational, it is tempting to define the "true" consumer surplus as $\tilde{U} = u - F - q_1 p_1 - [q_1 p_2 + (1 - q_1) p_1] q_2$, and the "true" social surplus as $\tilde{S} = \tilde{U} + \pi$. It is also tempting to use $\tilde{U}$ and $\tilde{S}$, rather than U and S, for the purposes of welfare analysis, even though such a paternalistic approach is not unanimously endorsed in the economics literature.

4.3.1 Social surplus maximization

Going back to equilibrium analysis, consider the problem of social surplus maximization. For simplicity, assume that the function $E(S)$ is strictly concave in p_1 and p_2. This assumption guarantees that the solution is unique. The condition imposes restrictions on the distribution function G, which are not easily interpreted economically. However, the condition is met in many specific examples, such as for instance the case of a uniform distribution.

The solution is then characterized by the following first-order conditions:

$$\frac{dE(S)}{dp_1} = \frac{\partial E(S)}{\partial v_1^*} - G(v_1^*)\,(p_1 - c)\,g(p_1) + (1-\delta)G(v_1^*)\,[1 - G(p_1)] + \frac{d\omega}{dp_1}\frac{\partial E(S)}{\partial v_1^*} = 0 \tag{4.11}$$

and

$$\frac{dE(S)}{dp_2} = -\,[1 - G(v_1^*)]\,(p_2 - c)\,g(p_2) + (1-\delta)\,[1 - G(v_1^*)]\,[1 - G(p_2)] + \frac{d\omega}{dp_2}\frac{\partial E(S)}{\partial v_1^*} = 0, \tag{4.12}$$

where

$$\begin{aligned}\frac{\partial E(S)}{\partial v_1^*} = g(v_1^*)\Big\{ & -(v_1^* - c) - \int_{p_2}^{1} (1-\delta)(p_2 - p_1) g(v_2) dv_2 \\ & + \int_{p_1}^{p_2} [\delta v_2 + (1-\delta)p_1 - c]\, g(v_2) dv_2 \Big\}, \\ \frac{d\omega}{dp_1} & = -\delta(1 - G(p_1)),\end{aligned}$$

and

$$\frac{d\omega}{dp_2} = \delta(1 - G(p_2)).$$

The first two terms of the first-order condition for p_1 capture the standard welfare effect of raising prices: that is, higher prices reduce consumption, and this impacts negatively social welfare so long as the value of the good v_t (which equals p_t for the marginal consumer) exceeds the unit cost c. In particular, an increase in p_1 always reduces first-period consumption, and it also reduces second-period consumption with a positive probability, $G(v_1^*)$.

In the absence of additional terms, expected social surplus would be maximized by marginal cost pricing. However, there are two more terms in the first-order condition. The third term, i.e. $(1-\delta)G(v_1^*)\,[1 - G(p_1)]$, reflects the fact that an increase in the price of second-period consumption (which is p_1 with probability $G(v_1^*)$) raises the expected profit more that it reduces the consumer's rent, because of consumer's overdiscounting. This term captures the "misperception effect" analyzed by Della Vigna and Malmendier (2004) and others, and tends to push prices above marginal cost. Finally, the last term is an "opportunity cost effect" that arises as non-constant prices create an opportunity cost of first-period consumption.

The interpretation of the first-order condition for p_2 is similar. The only difference is that a change in p_2 directly affects only second-period consumption, and it does so with probability $1 - G(v_1^*)$.

When $\delta = 1$, the misperception effect vanishes. Furthermore, there is no incentive to artificially create any opportunity cost, positive or negative, of first-period consumption: formally, $\frac{\partial E(S)}{\partial v_1^*} = 0$ at $p_1 = p_2 = c$. This leads to the following well known result:

Lemma 4.1. *When $\delta = 1$, the equilibrium contract is a two-part tariff with $p_1^* = p_2^* = c$.*

Proof. When $\delta = 1$ the social surplus as perceived by the agents, S, coincides with the "true" social surplus and hence

$$E(S) = \int_{v_1^*}^{1} \left[(v_1 - c) + \int_{p_2}^{1} (v_2 - c)\, g(v_2) dv_2 \right] g(v_1) dv_1 + G(v_1^*) \int_{p_1}^{1} (v_2 - c)\, g_2(v_2) dv_2.$$

Inspection of the first-order conditions reveals that this is maximized by marginal cost pricing. This implies that both cut-offs are optimally set at the marginal cost, i.e. $v_1^* = \bar{v}_2^* = c$. ■

A monopolist would then extract, *via* the fixed fee, all the expected surplus from trade. Under perfect competition, in contrast, the fixed fee would be set at zero, leaving all the surplus to the consumer.

As soon as $\delta < 1$, however, the misperception effect kicks in, and marginal cost pricing is no longer optimal. In particular, there is an incentive to raise the price that applies to second-period consumption. Since this may be either p_2 (with probability $1 - G(v_1^*)$) or p_1 (with probability $G(v_1^*)$), the misperception effect creates an incentive to raise both prices.

In fact, starting from $p_1 = p_2 = c$ there may be a stronger incentive to raise p_1 than p_2.[4] However, an increase in p_1 reduces not only second-period consumption, but

[4]Formally, this point may be seen by noting that when $p_1 = p_2 = c$ we have

$$\frac{dE(S)}{dp_1} = (1 - \delta) G(c) \left[1 - G(c)\right] > 0$$

also the first period one. Any distortion in first-period consumption is however costly, because first-period surplus can be extracted fully. This implies that prices should be set with an eye towards limiting the distortion in first-period consumption, which implies that p_1 will be set below p_2.

Therefore, the misperception effect by itself may explain the increasing pattern of marginal prices that is typical of three-part tariffs. However, the misperception effect by itself would imply that both prices should exceed the marginal cost. But it cannot explain below-cost pricing for small volumes, nor, *a fortiori*, the possibility that p_1 may be optimally set to zero. Therefore, something else is needed to explain three-part tariffs.

In our model, this extra ingredient is that consumers are forward looking. This point can be seen most clearly in a model in which consumers are not only impatient but also myopic. In that case, consumers would not perceive the opportunity cost of first period consumption, and so v_1^* would coincide with p_1. In other words, ω would be identically equal to zero. The last terms of the first-order conditions would therefore vanish. As a result, in equilibrium one would have $p_2 > p_1 > c$. When consumers are forward looking, however, an additional effect comes into play. The fact that $p_2 > p_1$ creates an opportunity cost that further distorts first-period consumption. This creates an incentive to reduce p_1 so as to alleviate the distortion.

The main result of this section is that this additional effect may make it optimal to set prices below-cost, or even at zero, for small volumes. First of all, we prove that with impatient consumers marginal prices must indeed be increasing.

Proposition 4.1. *When $\delta < 1$, marginal prices are increasing: $p_1 < p_2$.*

Proof. Let p_2^* denote the optimal value of p_2, and let us evaluate $\frac{dE(S)}{dp_1}$ at $p_1 = p_2^*$. The

and

$$\frac{dE(S)}{dp_2} = (1-\delta)\left[1-G(c)\right]^2 > 0.$$

While these formulas show that there is an incentive to raise both p_1 and p_2, it appears that the incentive to raise p_1 is stronger when $G(c) > \frac{1}{2}$.

derivative is

$$\left.\frac{dE(S)}{dp_1}\right|_{p_1=p_2^*} = \frac{\partial E(S)}{\partial v_1^*}\frac{dv_1^*}{dp_1} + G(p_2^*)\left[(1-\delta)(1-G(p_2^*)) - (p_2^*-c)\,g(p_2^*)\right].$$

From the first-order condition for p_2, which must hold at the optimum, we have

$$(1-\delta)(1-G(p_2^*)) - (p_2^*-c)\,g(p_2^*) = -\frac{\dfrac{\partial E(S)}{\partial v_1^*}\dfrac{dv_1^*}{dp_2}}{[1-G(p_2^*)]}.$$

Substituting into the previous expression one gets

$$\left.\frac{dE(S)}{dp_1}\right|_{p_1=p_2^*} = \frac{\partial E(S)}{\partial v_1^*}\left[\frac{dv_1^*}{dp_1} - \frac{G(p_2^*)}{1-G(p_2^*)}\frac{dv_1^*}{dp_2}\right].$$

Using the first-order conditions, the term inside square brackets reduces to $(1-\delta)$. On the other hand, since at $p_1 = p_2^*$ one has $\left.\frac{\partial S}{\partial v_1^*}\right|_{p_1=p_2^*} = -v_1^* + c$ and $v_1^* = p_2^*$, one gets

$$\frac{\partial E(S)}{\partial v_1^*} = -(p_2^*-c).$$

Therefore

$$\left.\frac{dE(S)}{dp_1}\right|_{p_1=p_2^*} = -(p_2^*-c)(1-\delta) < 0$$

where the negative sign follows as $p_2^* > c$. By the concavity of $E(S)$, this implies that at the optimum $p_1^* < p_2^*$. ■

As mentioned above, increasing marginal prices would also be obtained in a model in which consumers are myopic. With forward-looking consumers, however, it may be optimal to set p_1 below the marginal cost c. The solution to the system of first-order conditions (11)-(12) may even entail a negative value of p_1. If one imposes a non-negativity constraint on prices, a corner solution would then arise in which $p_1 = 0$. This corresponds precisely to a three-part tariff.

To demonstrate these possibilities, we set p_1 at c, or at 0, and p_2 at the corresponding optimal value (that is, such that $\frac{dE(S)}{dp_2}$ vanishes.) We then evaluate the derivative $\frac{dE(S)}{dp_1}$ at these prices. If the derivative is negative, then by the concavity of $E(S)$ the optimal price p_1^* must be lower than c, or negative (in which case a non-negativity constraint on prices would imply that $p_1^* = 0$).

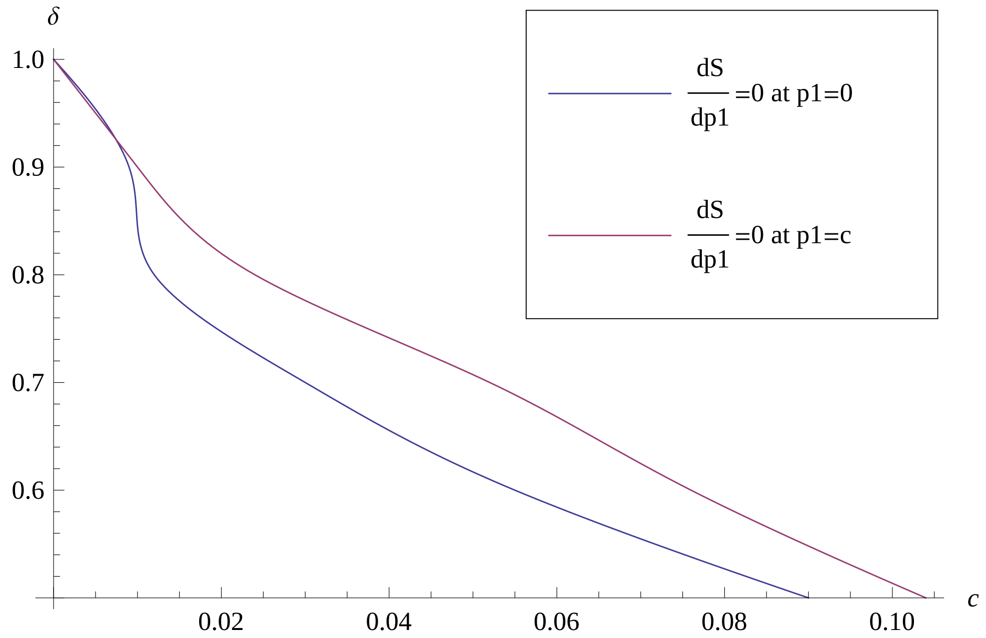

Figure 4.1: Region of parameter values when three part tariffs is optimal

We have performed the evaluation for the case of a uniform distribution $G(v_t) = v_t$. Figure 4.1 illustrates the region of parameter values where it is optimal to price below cost for small volumes, and the region where a three-part tariff is indeed optimal when negative prices are ruled out. Both possibilities arise as soon as $\delta < 1$, provided that the marginal cost c is small enough. As the discount factor δ decreases, below cost or zero prices are optimal for larger and larger marginal cost. Unsurprisingly, three-part tariffs are less likely to be optimal when marginal costs are large – a pattern that seems consistent with the prevalence of three-part tariffs in industries where marginal costs tend to be small.

4.3.2 Oligopoly

We have noted above that the first-order conditions (11) and (12) must hold, at an interior solution, under both monopoly and perfect competition. In fact, the equilibrium contracts must maximize the expected social surplus under more general conditions. For example, $E(S)$ must be maximized also under oligopoly, in models of one-stop shopping in which consumers buy only from one firm. The reason for this is that in such models firms compete in utility space, and therefore the competitive pressure from rivals affects only the level of utility that must be left to consumers. For any given expected utility level $E(U)$ that must be left to consumers, firms will then maximize profits. Since the utility level can be adjusted using the fixed fee, the solution must maximize the expected surplus $E(S)$.

To be more specific, consider an Hotelling model in which two firms are located at the opposite ends of a unit segment, and consumers are distributed along the segment. Consumers must pay a transportation cost to reach the firm they buy from, so their net utility is $E(U)$ minus the transportation cost. Consumers patronize the firm that guarantees the highest net utility. In this model, firms compete for market shares. The competition at the extensive margin will determine the equilibrium level of $E(U)$. However, the net utility level can be controlled through the fixed fee that firms charge. For any given fixed fee, and hence for any given set of consumers that will patronize a firm, the firm has an incentive to offer a contract that maximizes $E(S)$, as it can fully extract any extra surplus that it creates by a more efficient choice of the contract.

4.3.3 Welfare effects

We end this section with a few remarks on the agents' equilibrium payoffs. A common theme of the literature on behavioural contracts is that irrationality exposes consumers to the risk of being exploited by rational firms. Regarding impatience as a form of irrationality, one could expect that firms take advantage of consumers' impatience, and that as a results impatient consumers must be worse off than patient ones.

However, things are more complicated than this intuition may suggest. To begin with, the impact of impatience on agents' payoffs depends on market structure. Under perfect competition, firms profits vanish anyway. The pricing distortions increase consumers' expected utility $E(U)$ as it is perceived by consumers themselves, but reduce their "true" utility $E(\tilde{U})$. All of this is as expected.

Under monopoly, on the other hand, consumers' impatience reduces the firm's profit as compared to the case $\delta = 1$. The reason for this is twofold. First, the pricing distortions reduce the true social surplus $E(\tilde{S})$ below the efficient level. Second, while the surplus of patient consumers $E(\tilde{U})$ would be extracted fully, impatient consumers now get a positive share of the true social surplus $E(\tilde{S})$. This follows from the fact that the fixed fee is set at a level such that $E(U) = 0$, but this implies that $E(\tilde{U}) > 0$. Intuitively, since impatient consumers underestimate the value of future consumption, in the second period they can obtain a positive rent. In this sense, impatience is good for consumers. If one regards overdiscounting as a form of irrationality, one may conclude that in this case irrationality actually prevents consumers from being exploited by the firms.

4.4 Incomplete information

The fact that impatience can be good for consumers raises the issue of what happens when consumers have different degrees of impatience. Do consumers still gain as compared to the case in which all consumers are patient? And, if so, who gains most, patient or impatient consumers?

To address these issues, in this section we extend the model to account for the possibility that there may be two groups of consumers: patient consumers with $\delta = 1$, and impatient consumers with $\delta = \underline{\delta} < 1$. We normalize the total number of consumers to one, and let μ denote the fraction of impatient consumers. The degree of impatience is consumers' private knowledge, so all that firms know is that the fraction of impatient consumers is μ.

Firms will now offer menus of tariffs as a screening device. Given that there are two types only, only two tariffs need to be offered in equilibrium. We shall denote with an upper bar the contract intended for patient consumers, $\bar{\mathcal{C}} = \{\bar{F}, \bar{p}_1, \bar{p}_2\}$, and with a lower bar that intended for impatient consumers, $\underline{\mathcal{C}} = \{\underline{F}, \underline{p}_1, \underline{p}_2\}$. As under complete information, some results do not depend on the degree of product market competition. However, to fix ideas we focus on the case of monopoly.

If patient consumers indeed sign the contract intended for them, the firm obtains from each of these consumers a profit of

$$
\begin{aligned}
E(\bar{\pi}) \quad = \quad & \bar{F} + \int_{\bar{v}_1^*}^{1} \left[\bar{p}_1 - c + \int_{\bar{p}_2}^{1} (\bar{p}_2 - c) g(v_2) dv_2 \right] g(v_1) dv_1 + \\
& + G(\bar{v}_1^*) \int_{\bar{p}_1}^{1} (\bar{p}_1 - c) g(v_2) dv_2, \qquad (4.13)
\end{aligned}
$$

where

$$
\bar{v}_1^* = \bar{p}_1 + \int_{\bar{p}_1}^{\bar{p}_2} (1 - G(v_2)) dv_2.
$$

Similarly, from each impatient consumer who signs a contract intended for him the firm gets a profit of

$$
\begin{aligned}
E(\underline{\pi}) \quad = \quad & \underline{F} + \int_{\underline{v}_1^*}^{1} \left[\underline{p}_1 - c + \int_{\underline{p}_2}^{1} (\underline{p}_2 - c) g(v_2) dv_2 \right] g(v_1) dv_1 + \\
& + G(\underline{v}_1^*) \int_{\underline{p}_1}^{1} (\underline{p}_1 - c) g(v_2) dv_2, \qquad (4.14)
\end{aligned}
$$

where

$$
\underline{v}_1^* = \underline{p}_1 + \underline{\delta} \int_{\underline{p}_1}^{\underline{p}_2} (1 - G(v_2)) dv_2.
$$

Assuming that it is optimal to serve both types,[5] the firm maximizes

$$
E(\pi) = (1 - \mu) E(\bar{\pi}) + \mu E(\underline{\pi}). \qquad (4.15)
$$

The equilibrium contracts must satisfy the incentive compatibility constraints that each consumer prefers the contract intended for him to the contract intended for the other type. This requires that

$$
\begin{aligned}
& -\bar{F} + \int_{\bar{v}_1^*}^{1} \left[v_1 - \bar{p}_1 + \int_{\bar{p}_2}^{1} (v_2 - \bar{p}_2) g(v_2) dv_2 \right] g(v_1) dv_1 + G(\bar{v}_1^*) \int_{\bar{p}_1}^{1} (v_2 - \bar{p}_1) g(v_2) dv_2 \quad \geq \\
& -\underline{F} + \int_{\bar{v}_1^d}^{1} \left[v_1 - \underline{p}_1 + \int_{\underline{p}_2}^{1} (v_2 - \underline{p}_2) g(v_2) dv_2 \right] g(v_1) dv_1 + G(\bar{v}_1^d) \int_{\underline{p}_1}^{1} (v_2 - \underline{p}_1) g(v_2) dv_2 \qquad (4.16)
\end{aligned}
$$

where

$$
\bar{v}_1^d = \underline{p}_1 + \int_{\underline{p}_1}^{\underline{p}_2} (1 - G(v_2)) dv_2,
$$

[5]This requires that the fraction of impatient consumers is large enough.

and that

$$-\underline{F}+\int_{\underline{v}_1^*}^{1}\left[v_1-\underline{p}_1+\underline{\delta}\int_{\underline{p}_2}^{1}(v_2-\underline{p}_2)g(v_2)dv_2\right]g(v_1)dv_1+\underline{\delta}G(\underline{v}_1^*)\int_{\underline{p}_1}^{1}(v_2-\underline{p}_1)g(v_2)dv_2 \ \geq$$
$$-\bar{F}+\int_{\underline{v}_1^d}^{1}\left[v_1-\bar{p}_1+\underline{\delta}\int_{\bar{p}_2}^{1}(v_2-\bar{p}_2)g(v_2)dv_2\right]g(v_1)dv_1+\underline{\delta}G(\underline{v}_1^d)\int_{\bar{p}_1}^{1}(v_2-\bar{p}_1)g(v_2)dv_2 \quad (4.17)$$

where

$$\underline{v}_1^d=\bar{p}_1+\underline{\delta}\int_{\bar{p}_1}^{\bar{p}_2}(1-G(v_2))dv_2.$$

In addition, the following participation constraints must hold (again, assuming that it is optimal to serve both types):

$$\begin{aligned} E(\bar{U}) &= -\bar{F}+\int_{\bar{v}_1^*}^{1}\left[v_1-\bar{p}_1+\int_{\bar{p}_2}^{1}(v_2-\bar{p}_2)g(v_2)dv_2\right]g(v_1)dv_1+ \\ &\quad +G(\bar{v}_1^*)\int_{\bar{p}_1}^{1}(v_2-\bar{p}_1)g(v_2)dv_2\geq 0, \end{aligned} \tag{4.18}$$

and

$$\begin{aligned} E(\underline{U}) &= -\underline{F}+\int_{\underline{v}_1^*}^{1}\left[v_1-\underline{p}_1+\underline{\delta}\int_{\underline{p}_2}^{1}(v_2-\underline{p}_2)g(v_2)dv_2\right]g(v_1)dv_1+ \\ &\quad +\underline{\delta}G(\underline{v}_1^*)\int_{\underline{p}_1}^{1}(v_2-\underline{p}_1)g(v_2)dv_2\geq 0, \end{aligned} \tag{4.19}$$

As is well known, only one incentive compatibility constraint and one participation constraint will generally bind in equilibrium. If a single crossing condition holds, the binding participation constraint is that of low types whereas the binding incentive compatibility constraint is that of high types. The next Lemma shows that in our model the binding participation constraint is that of impatient consumers, whereas the binding incentive compatibility constraint is that of the patient consumers.

Lemma 4.2. *In equilibrium, only the incentive compatibility constraint (4.16) and the participation constraint (4.19) bind.*

Proof. See the Appendix. ■

Lemma 2 effectively shows that in our model impatient consumers are the low types and patient consumers are the high types.

With this insight, the characterization of the equilibrium is simple. The firm will offer two contracts. The no-distortion-at-the-top property implies that the contract intended for patient consumers must be efficient. The fixed fee $\bar{F}$ is no longer set so as to extract the full surplus, however, as the incentive compatibility constraint implies that patient consumers will enjoy an information rent. The contract intended for impatient consumers, on the other hand, is similar to the one characterized in the previous section. However, there are additional distortions that serve to make this contract less attractive to patient consumers, thereby increasing the fixed fee $\bar{F}$ that is charged to them.

Proposition 4.2. *With incomplete information, the firm will offer a menu of two contracts:*

With incomplete information, the firm offers a menu of two contracts:

$$\bar{\mathcal{C}} = \{\bar{F}, c, c\}, \tag{4.20}$$

and

$$\underline{\mathcal{C}} = \{\underline{F}, \underline{p}_1^*, \underline{p}_2^*\}, \tag{4.21}$$

where $\underline{p}_1^* < \underline{p}_2^*$. *The price* $\underline{p}_1^*$ *may be lower than c, and may even be negative.*

Proof. See the Appendix. ■

The additional distortions that serve to reduce the information rent obtained by patient consumers go in the same direction as the distortions analyzed in the previous section. Intuitively, raising the price that applies to second-period consumption is more costly for patient consumers than for impatient ones, and thus is the most efficient way to create the additional distortion. This implies that both p_1 and p_2 should be increased as compared to the complete information prices. However, p_1 should be increased less, or may be even decreased, because pricing first-period consumption above marginal cost distorts also first-period consumption. Since the surplus from first-period consumption can be extracted fully, this distortion is especially costly and thus should be kept to a minimum.

Now consider the consumers' equilibrium payoffs. The impatient consumers' participation constraints bind at equilibrium, implying that $E(U) = 0$. However, the "true" expected rent of impatient consumers $E(\tilde{U})$ is positive, for the same reasons as under complete information. As for patient consumers, their rent is the same as if they chose the contract intended for impatient consumers. Therefore, they get a positive rent. This is actually greater than the "true" rent of impatient consumers, as patient consumers who face the same contractual conditions make more efficient consumption choices (from the point of view of patient consumers) than impatient consumers. Thus, patient consumers are better off than impatient consumers, regardless of the welfare criterion adopted.

4.5 Conclusion

In this paper we have proposed a new explanation of three-part tariffs. The explanation is based on the assumption that consumers are forward-looking, but are more impatient than the firms. This creates a misperception effect, in that consumers undervalue future trades. As a result, firms have an incentive to raise the prices that tend to apply to such future trades – that is, in any given contracting period, prices that apply to large volumes. But this creates an opportunity cost effect: forward looking consumers, that is to say, will realize that early consumption increases the expected cost of late consumption. This distorts early consumption, which is bad because the surplus from early consumption can be extracted efficiently by means of fixed fee. To alleviate this distortion, one has to reduce marginal prices for low volumes. This effect can be sufficiently strong that marginal prices fall below cost, or may even become negative.

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