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How teachers think about the role of digital technologies in student assessment in mathematics

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Ethics Statement

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a. human research ethics approval from the Simon Fraser University Office of Research Ethics,

or

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Abstract

This study concerns teachers' use of digital technologies in student assessment, and how the learning that is developed through the use of technology in mathematics can be evaluated. Nowadays math teachers use digital technologies in their teaching, but not in student assessment. The activities carried out with technology are seen as 'extra-curricular' (by both teachers and students), thus students do not learn what they can do in mathematics with digital technologies. I was interested in knowing the reasons teachers do not use digital technology to assess students' competencies, and what they would need to be able to design innovative and appropriate tasks to assess students' learning through digital technology.

This dissertation is built on two main components: teachers and task design. I analyze teachers' practices involving digital technologies with Ruthven's *Structuring Features of Classroom Practice*, and what relation these practices have to the types of assessment they use. I study the kinds of assessment tasks teachers design with a DGE (Dynamic Geometry Environment), using Laborde's categorization of DGE tasks. I consider the competencies teachers aim to assess with these tasks, and how their goals relate to the learning outcomes of the curriculum.

This study also develops new directions in finding how to design suitable tasks for student mathematical assessment in a DGE, and it is driven by the desire to know what kinds of questions teachers might be more interested in using. I investigate the kinds of technology-based assessment tasks teachers value, and the type of feedback they give to students. Finally, I point out that the curriculum should include a range of mathematical and technological competencies that involve the use of digital technologies in mathematics, and I evaluate the possibility to take advantage of technology feedback to allow students to continue learning while they are taking a test.

Keywords: Dynamic Geometry Environments; digital technologies, formative assessment; feedback; teachers; Sketchpad

Dedication

This is more an inspiring vision than a dedication. Since the first time I watched the video of this talk by Conrad Wolfram at TED, I have tried to communicate this idea of mathematics. I was astonished to see that what I thought about math and technology in education was so well expressed by his words, and I would like to report some pieces of his talk to share with the reader one of my first sources of inspiration.

> People confuse the order of the invention of the tools with the order in which they should use them for teaching. So just because paper was invented before computers, it doesn't necessarily mean you get more to the basics of the subject by using paper instead of a computer to teach mathematics. If you were born after computers and paper, it doesn't really matter which order you're taught with them in, you just want to have the best tool.
>
> What I really am suggesting here is we have a unique opportunity to make mathematics both more practical and more conceptual, simultaneously. […] What I really think we gain from this is students getting intuition and experience in far greater quantities than they've ever got before. And experience of being able to play with the math, interact with it, feel it. We want people who can feel the math instinctively. That's what computers allow us to do.
>
> One of the roadblocks we have in moving this agenda forward is exams. In the end, if we test everyone by hand in exams, it's kind of hard to get the curricula changed to a point where they can use computers during the semesters.
>
> I believe there is critical reform we have to do in computer-based math. […] We can engage so many more students with this, and they can have a better time doing it. […] I want to see a completely renewed, changed math curriculum built from the ground up, based on computers being there, computers that are now ubiquitous almost. Calculating machines are everywhere and will be completely everywhere in a small number of years.

Conrad Wolfram, Teaching kids real math with computers (TEDGlobal, July 2010)

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Introduction

When the first digital technologies appeared in Mathematics Education, teachers focused "on the teaching of technology rather than the use of technology as a tool for mathematics and as a tool for teaching mathematics" (Zbiek and Hollebrands, 2008, p. 314). Nowadays the situation is quite different: most of the teachers have integrated technology[1] in their teaching, and they are also trying to move forward, in order to answer positively to the provocative question of Caron and Ben-El-Mechaiekh (2010): Instead of trying to teach {mathematics} with technology, could we consider teaching {**mathematics with technology**}? However, despite the widespread use of digital technologies in mathematics classrooms, they play a very small role in most teachers' assessment practices. Indeed, many researchers have acknowledged the lack of research on how digital technology can and should be used in the context of assessment. As in the teaching of mathematics, the focus should be on **students using technology *in assessment***, as opposed to the **teachers using technology *to assess***.

Since this research project is about the use of digital technologies in the assessment, it is situated in the intersection of two quite broad topics: **teachers and task design.** Teachers need to choose to include digital technologies in their teaching and consequently in the assessment, and assessment requires the design of tasks that evaluate the competencies students acquire using digital technologies in mathematics. In order to better focus of the research, this dissertation studies teachers' practice in student assessment with digital technologies in mathematics, and task design in a Dynamic Geometry Environment (DGE).

The goal of this study is to analyze teachers' practice in order to find out if they use digital technology in the assessment, the difficulties they may encounter and the resources they would need to move forward. I will use the five Ruthven's *Structuring Features of Classroom Practice*, which I modified in order to include assessment in technology-based teachers' practice. I also added a category to the five existing

[1] I need to clarify that every time I use the term 'technology' in this study, it stands for 'digital technology'.

features in order to consider the important role of feedback during and after assessment.

This research also draws on previous work on task design in Dynamic Geometry Environments to provide a framework for identifying and designing different types of assessment tasks according to the specific goals of the teacher. These types of tasks will be exemplified using tasks designed for the iPad-based *Sketchpad Explorer*. In order to do that, I adapted the work of Laborde (2001) and the results of Sinclair (2003) to the context of formative assessment.

Following the aim of the study, teachers were asked to design assessment tasks. I analyzed the tasks they created and their comments on my sketches, in order to find out which kinds of tasks teachers are likely to design, and what might be more interested in using.

If teachers want their students to learn how to effectively use digital technologies in mathematics, they should consider it as a learning goal. As a consequence, teachers should include this competence in *formal* assessment, and provide *feedback* on students' performance. Thus, students will understand that the goal is valued and they will try to improve their work. In that way teachers will "engage students in a mathematical practice that is empowering, meaningful, and coherent" (Caron & Steinke 2005, p. 4).

Chapter 1.

Literature on assessment using digital technologies

In this section I aim to provide an overview of the research related to the role of digital technologies in assessment for the mathematics classroom. I will first describe the different types of assessments currently being used in school, not necessarily in mathematics or with the use of digital technology. I then consider the barriers to the use of digital technologies in assessment, but also how they could support the evaluation of some prescribed learning outcomes of the curriculum. Finally, I will illustrate the importance of technology feedback in the teaching of mathematics, how it can be used in the assessment and how students' answers in a digital technology context could be evaluated.

Educators and teachers affirm that digital technologies could help students in some mathematics learning goals that are difficult to reach, such as problem solving, reasoning, making deductions and conjectures, and conceptual understanding (Laborde et al., 2006). As Sangwin et al. (2010) argue, if a teacher's goal is to support students to learn to use technology for purposeful mathematical activity, then they should find a way to assess this competence; otherwise students do not see how to effectively use it in mathematics as a learning goal:

> If a teacher encourages students to make extensive use of tools in a course but does not allow their use on the end-of-course test, are students being given the opportunity to show what they learned with the use of such tools? (p. 229).

There is a lot of literature on the use of digital technologies in the teaching and learning of mathematics, and on the different possibilities that technology offers to do new kinds of mathematical activities (see Ferrara et al., 2006; Laborde et al., 2006). However, it is quite difficult to find some literature on how to effectively integrate digital technologies in student assessment.

Drijvers, Mariotti, Olive and Sacristán (2010) explain that one of the themes that served to frame the 17th ICMI Study, *Mathematics Education and Technology – Rethinking the Terrain* was on assessing mathematics with and through digital technologies, and on balancing use of mental, paper-and-pencil, and digital tools in both assessment and teaching activities:

> This theme will concentrate on developing understandings of how students learn mathematics with digital technologies and the implications of the integration of technological tools into mathematics teaching for assessment practices. […] Additionally, the theme will address the challenges involved in balancing use of mental, paper-and-pencil, and digital tools in both assessment and teaching activities. (p. 82)

Moreover, in the same document, this question was formulated:

> How can the assessment of students' mathematical learning be designed to take into account the integration of digital technologies and the ways that digital technologies might have been used in the learning of mathematics? (p. 82)

However, these authors observe that nobody has presented any research that can answer that question:

> An explicit discussion of how to take into account those different uses for assessment purposes, or how to develop assessment methods that evaluate the learning that is developed through the use of digital technologies, is not included, and remains an area that requires still much research. (p. 85)

If students are accustomed to using digital technologies in the learning of mathematics, then they will be comfortable in being assessed in the same way. On the other hand, "it is difficult to integrate a given tool into assessment before having integrated it into the corresponding teaching" (Trouche, 2005, p. 31). While many institutions and teachers have introduced the use of digital technologies into the teaching and learning of mathematics, nobody seems to know how to integrate them in student assessment, and how to evaluate the learning that is developed through the use of digital technologies.

Olive et al. (2010) observe that "the literature indicates that interactions among students, teachers, tasks, and technologies can bring about a shift in empowerment from the teacher to the students as generators of mathematical knowledge and practices" (p. 133), and Zbiek and Hollebrands (2008) confirm that "technology can quickly assume, or open to students, roles in the classroom that were once the exclusive domain of teachers" (p. 336). Thus, also in the evaluation the focus should be on "students using technology in assessment, as opposed to the teacher using technology to assess" (Buteau & Sinclair, 2012, p. 96). However, students must be very comfortable in using digital technologies, otherwise teachers could evaluate students' ability of using tools rather than students' mathematical learning.

Based on the literature on assessment using technology, I identified the strongest and most recurrent points that researchers have made about the use of digital technologies in mathematics. In table 1 I list the main **affordances of digital technologies** that could be helpful for teachers and students in the mathematical assessment:

Teachers	Students
teachers can see student reasoning through the 'actions' that they do in the technology environment	technology helps students express and communicate their ideas
teachers have the opportunity to design tasks that enable certain mathematical thinking that is not accessible with paper-and-pencil tasks	technology offers students the possibility to show different kinds of abilities and knowledge
technology helps teachers recognize student misconceptions	Technology fosters students in some mathematical processes they struggle with: problem solving, finding invariance, proving...

Table 1: Teachers and Students

These affordances offer deeper insights into student mathematical understanding, and provide the students with new ways to express their mathematical knowledge and competencies. These aspects of digital technologies could be useful to assess students' learning in mathematics, since students may be facilitated in communicating their understanding, and teachers could have a wider view of students' performance.

In the next section, I will examine the most discussed kinds of assessment in the literature. I will explain the differences among the definitions of assessment, and illustrate the aim of each kind of assessment: how and when they should be used.

1.1. Different types of assessments currently being used/described

Since assessment plays a fundamental role in the learning cycle, it is defined as "all those activities undertaken by teachers – and by their students in assessing themselves - that provide information to be used as feedback to modify teaching and learning activities" (Black & Wiliam, 1998, p. 140). Taras (2010) states that assessment is a judgment, and this judgement is justified according to specific weighted set goals and parameters (standards, criteria, context…). The parameters may be implicit (in the assessor's head), or explicit and shared with others. In an educational context, the content of assessment could be processes, products or interactions. The result of a judgment could be represented in words as feedback, or in a summary judgment in the form of a grade or mark in accordance with an agreed scale. Taras' definition of assessment is based on Scriven's work (1967), but she suggests adding a further stage to Scriven's definition: the justification of the judgement against the stated goals and criteria. Moreover, Taras (2005) observes that if the parameters driving the assessment process are explicit, then it is possible to create a shared forum for assessment and therefore to allow transparency of process. If teachers explain to their students what their parameters of assessment are, then they are able to understand the criteria of the evaluation, and consequently try to modify their learning process, since "an explicit and logical process must surely improve the efficiency of the assessment system" (Taras, 2005, p. 475).

There are many types of assessment currently being described in the literature. They mainly differ in the purpose of the evaluation, and in the way or time they are used. I will discuss seven types here: *Diagnostic assessment, Summative assessment, Formative assessment, Dynamic assessment, Effective assessment, Formal assessment, Informal assessment.*

Diagnostic assessment is used prior to new learning in order to determine what students know on the notion about to be taught, or to establish the different student levels. Teachers could use it to derive course orientations based on students' answers, and to provide a benchmark to assess progress.

Summative assessment is used at the end of learning, and it's the most common kind of assessment used in schools. Usually, it is a sort of 'review' in order to see if objectives have been met in terms of results, acquisitions, and progress. It is concerned with establishing the achievement of the student, and reporting the judgment especially for purposes of validation and certification. It is also called *Assessment of Learning* to highlight the fact that "it is essentially passive and does not normally have immediate impact on learning, although it often influences decisions which may have profound educational and personal consequences for the student." (Sadler, 1989).

Formative assessment is defined by Black and Wiliam (1998) and Taras (2010) as follows:

> Assessment becomes formative when the information is used to adapt teaching and learning to meet students' needs.
>
> Summative Assessment + Feedback = Formative Assessment.

This kind of assessment is used in the course of learning, and it is also called *Assessment for Learning*, because "it is based on principles to support learners through assessment" (Taras, 2010, p. 3015). In Taras' 'equation', *summative assessment* is the evaluation of student understanding, knowledge and competence; it becomes *formative assessment* when teachers give comments (feedback) on student performance in relation to the learning goals. This feedback should be used to "update, change and improve the work" (Taras 2010, p. 3) both of the teacher and of the students. Formative assessment could evaluate both a process and a product, and it is supposed to have benefits both for students and for teachers. Students can be aware of the milestones that have been reached, and of the difficulties that they have to overcome. Teachers can see if the teaching program is achieving its goals, and find the obstacles that may impede progress. Moreover, teachers can find sources of errors and remediate, or they can adapt didactical/pedagogical interventions to the learning occurring in the classroom.

The key element in formative assessment is the **feedback:** "the information about how successfully something has been or is being done, or the knowledge of results" (Sadler, 1989, p. 120). It gives information about the gap between the actual level and the reference level of a system parameter that is used to alter the gap in some way (Ramaprasad, 1983). This feedback becomes *formative*, and consequently part of the learning cycle, when teachers and students use it to improve their performance, as Sadler (1989) asserts:

> Teachers use feedback to make programmatic decisions with respect to readiness, diagnosis and remediation. Students use it to monitor the strengths and weaknesses of their performances, so that aspects associated with success or high quality can be recognized and reinforced, and unsatisfactory aspects modified or improved. (p. 120)

Taras (2005) observes that assessment and feedback must be negotiated for understanding and take-up by learner, therefore students' self-assessment is mandatory. Taras reformulates the 'Formative Assessment Equation' in this way:

SA + feedback (negotiated, decided + used by learner) = FA

Improvement in student learning can occur if the teacher provides detailed remedial advice, and if the student understands this feedback, and follows it through after an accurate self-assessment.

In table 2, Wiliam and Thompson (2007) suggest a framework obtained crossing the learning processes with the agents involved, to indicate that formative assessment can be conceptualized as consisting of five **key strategies**:

Processes ⇨ **Agents ⇩**	**Where the learner is going**	**Where the learner is right now**	**How to get there**
Teacher	1. Clarifying learning intentions and criteria for success	2. Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding	3. Providing feedback that moves learners forward
Peer	Understanding and sharing learning intentions and criteria for success	4. Activating students as instructional resources for one another	
Learner	Understanding learning intentions and criteria for success	5. Activating students as the owners of their own learning	

Table 2: Processes and Agents

These five strategies can be enacted by the five main **types of activity** occurring in the formative assessment practice according to (Black & Wiliam, 2009):

1. Sharing success criteria with learners
2. Classroom questioning
3. Comment-only marking
4. Peer- and self-assessment
5. Formative use of summative tests

Dynamic assessment occurs in the course of learning. It is a system with an approach very similar to that of formative assessment, “the main difference is that it is based on an explicit theory for guiding and interpreting teachers’ work” (Black & William, 2009, p. 19). It requires that the teacher’s responses be guided by the aim of challenging and developing the learner’s thinking:

> The teacher should not be content with immediate interventions that resolve a particular learning obstacle, but should follow up each success in a sustained and strategic way to build up further the learner’s capacity to learn, i.e., exploit to the full the learner’s ZPD, pursuing this aim even if this focus is at the expense of the teacher’s aims in making progress with the learner’s understanding of any particular curriculum topic. (p. 20).

Effective assessment occurs at the end of learning. It is defined in (Broughton, Hernandez-Martinez, and Robinson, 2013):

> Effective assessment is, in part, defined as an assessment that enables the student to achieve his/her learning goals. However, the student does not act alone in this activity. Assessors and peers have roles as members of the learning community. They provide opportunities to assess learning and provide feedback. They also have an influence on the student when setting learning goals. (p. 116)

This type of assessment is very similar to the formative assessment, also in the aspect that effective assessment should be both a process and part of a cycle, but it involves lecturers and peers, who have an influence in setting goals, setting assessments and providing feedback. Broughton, Hernandez-Martinez, and Robinson (2013) add that, “In effective assessment, the student gains experience, knowledge and understanding so that he/she can take more responsibility for these stages of the learning process” (p. 116). Thus, an “effective assessment” tests whether students have achieved their learning goals, it is part of a learning cycle in which students set more challenging learning goals with diminishing influence from lecturers and peers, and it gives opportunities for students to receive feedback on their performance in relation to their learning goals.

Formal assessment describes any assessment that includes a grade, which would be put in a book by the teacher. This means that the assessment activity counts for the final grade of the students. Every type of assessment described above could be formal, because the teacher can always decide to put a grade on a student answer.

Informal assessment describes the kind of assessment where teachers ask questions to the students in any form, and students’ answers will not be graded. The aim of this type of assessment is having an idea of the situation of the class and of students’ understanding, so that the teacher can plan the future actions. If the teacher provides feedback to the students on their learning, it could help them understand where they are and what they need to do to improve their work. Every kind of assessment, except the summative one, could be informal.

Comparing Summative and Formative Assessment

Among these different types of assessment the most diffused and discussed are *summative* and *formative* assessment. Taras (2005) observes that, “Formative assessment is increasingly being emphasized, yet its relationship to summative assessment has been little explored” (p. 466). It is argued that many of the principles appropriate to summative assessment are not necessarily transferable to formative assessment (Sadler, 1989), mainly because summative assessment can provide a limited type of formative information, and there is no claim that just any summative assessment can support learning effectively (Bennett, 2011). Wiliam (2000) asserts that these two processes are identified by the way the information provided on the result of the assessment is used:

> The terms *formative* and *summative* do not describe assessments - the same assessment might be used both formatively and summatively - but rather are descriptions of the use to which information arising from the assessment is put. (p. 1)

However, Taras (2010) argues that, "basing summative and formative assessments on functions is the most disorientating aspect of discussions in the literature, and contributes nothing to the understanding of assessment processes" (p. 3016).

Another possible interpretation is that *formative assessment* focuses on the process of assessing and using feedback, while *summative assessment* tends to focus on the product. However, *formative assessment* could evaluate also a final product, and *summative assessment* could evaluate also a required process. Many advocates of the 'process view' appear to prefer 'assessment *for* learning' to indicate *formative assessment*, and 'assessment *of* learning' to denote *summative assessment*. From a definitional perspective, however, Bennett (2011) points out that "this substitution is potentially problematic in that it absolves summative assessment from any responsibility for supporting learning" (p. 7). From the definition point of view it seems that these two types of assessments are completely different, but in teachers' practice it is quite unpractical:

> Some have argued that formative and summative assessments are so different in purpose that they have to be kept apart. However, our teachers found it unrealistic to practice such separation and so sought to achieve a more positive relationship between the two (Black et al 2003, p. 31).

In this dissertation I will consider *summative assessment* as the evaluation of students' learning through a grade that indicates their level of competencies compared to 'where they are expected to be'; this grade is often a number. This number determinates the existence of a 'gap' between the actual level of the work being assessed and the required standard. *Summative assessment* becomes *formative* when the teacher gives *feedback* to the students, which should explain their understanding situation and give suggestions as to how their learning can be improved to reach the required outcome. *Feedback* cannot take place without the judgment of *summative* *assessment*, as Taras (2005) affirms: “It is not possible for assessment to be uniquely formative without the summative judgment having preceded it” (p. 468). Therefore, *summative assessment* must come first, because it is necessary to assess the quality of the work before *feedback* can be given for the learner to use (Taras, 2005). Sadler (1989) states that, “Strictly speaking, all methods of grading which emphasize rankings or comparisons among students are irrelevant for formative purposes” (p. 127). He also adds that a grade could actually be counterproductive for formative purposes. This fact led to a negative connotation of assessment, which devalues personal worth and future prospects, and has prompted many teachers to see *summative assessment* in a ‘dark light’ and promote *formative assessment*:

> In the educational context, the terrors evoked by the term ‘assessment’ have distorted its necessity, centrality and its potentially neutral position. […] Currently, Formative Assessment is the antiseptic version of assessment and Summative Assessment has come to represent all the negative social aspects. (Taras, 2005, p. 469)

1.2. Barriers to the use of digital technologies in assessment

In his study on the integration of technology in the educational institutions, Trouche (2005) describes how “the spread of calculators raises various questions (about assessment, for example) and provokes lively discussion within professional associations” (p. 9). After illustrating the evolutions of the computational tools in mathematics education and in the society, he considers the points of view of students and teachers regarding the integration of new computing tools in the educational institutions. Then he describes the institutional evolutions due to the integration of the technology in the schools, which involves changes in the curriculum, in the textbooks and in the assessment. In regard to the latter, he states:

> The issue of assessment is complex; it can be tackled from different points of view:
>
> - how the educational institution chooses to assess the use of tools it prescribes;
> - which choices are made by different educational systems;
> - what types of exercise are considered basic in order to assess the mastery of a given tool. (p. 27)

Trouche (2005) describes the situation in France, where the ministerial circular stipulated in 1995 that, “the utilization of calculators is provided for in numerous educational programs and they must be widely used in examinations” (p. 29). However, the institution still does not know how to integrate calculator use during the examination. In general, the possible choices of the institutions in regard to the kind of approach to technology in the assessment can be summarized in four different categories:

- technology partially allowed;
- technology allowed, but benefits avoided;
- technology recommended and useful, but no added marks;
- technology obligatory and rewarded.

However, Trouche (2005) concludes that regardless of the approach to technology in assessment, "the objective of assessment seems to be to bypass the calculator's existence" (p. 31).

Trouche (2005) makes a list of several origins that may cause teachers' mistrust of new tools, which can lead to a reluctance to use digital technologies in their teaching:

> - these tools are too crude: results, for graphic calculators, are only given approximately, and this may lead to certain errors;
> - these tools prevent some elementary learning processes, like the 'four operations';
> - these tools do not fit the conception of mathematics which teachers have, reducing mathematics to an experimental practice restricts the place of formal proof;
> - the institutional discourse concerning the importance of integrating technology has often underplayed difficulties of managing calculator environments;
> - the integration of complex tools into the classroom requires teachers to undertake deep questioning about their course, their exercises, and their professional methods. (p. 19)

Moreover, even if some teachers and institutions are strongly motivated to use digital technologies in assessment, there are still many conceptual and practical barriers to the effective implementation of this idea:

- *Ensuring fairness*: it is difficult to isolate and control an environment that involves the use of laptop or tablets during exams. Caron and Steinke (2005) observe that "ensuring fairness and honesty in exams that incorporates technology is constantly challenged by students' creativity" (p. 4), and students can use network communication in real-time. It is also easily chatting among peers: "communication among students during in-class assessment is found to be problematic" (Buteau, Jarvis, and Lavicza, 2013, p. 49).

- *Shift of responsibility*: students could give up their mathematical authority to the computing machine. Olive et al. (2010) suggest that it is necessary "to pay particular attention to the design of the mathematical tasks in order to avoid students perceiving the role of the technology as their master rather than their servant or partner" (p. 167).

- *Social equality*: not all the students have laptops or tablets at home, which they can use for exercises and homework.

- *Access to the technological tools*: nowadays almost every school has a computer laboratory, but sometimes it could be difficult to have the access to it; further, tablets are not so frequent in schools yet.

- *Number of students*: sometimes there are not enough digital tools for all the students; thus they have to work in pairs or groups.

- *Teacher training*: Zbiek and Hollebrands (2008) observe that teachers express concern about whether they have the necessary time and knowledge to deal effectively with the demands of new technology in a classroom environment, since designing tasks for student assessment with digital technologies requires very specific competencies.

- *Development of mathematics*: assessment with digital technologies requires new kinds of questions, teachers have the possibility to test what students can do, not only what they know, because digital technology does the elementary mathematical processes.

- *Curriculum change*: the goals of the mathematics curriculum should be changed in order include digital technologies in the assessment since, as Trouche (2005) points out, “the modification of tools is accompanied by significant modifications of the ‘corresponding’ mathematical field” (p. 24).

- *Time*: designing tasks for the assessment with digital technologies is challenging, it requires more time and knowledge, and also correcting the digital tests could require more time.

Overall, institutions and teachers have to face all these problems in order to effectively integrate digital technologies in student assessment in mathematics. Some of these difficulties could be solved through professional development; for example, teachers could learn how to design appropriate tasks for student assessment as well as put them into practice. Also, students need to be accustomed to using digital technologies in mathematics, in order to prevent obstacles due to the tools, and to avoid a shift of responsibility to the technology. Then, there are practical aspects that have to be taken into account, like social equality of the tools resources that every students has at home for training, and the time that it takes to prepare, and put in practice this kind of assessment. Some of the difficulties are strictly related to the assessment practice, like the problems associated to technical aspects, such as the available technological tools, and the number of students, or trying to ensure fairness during assessment. Finally, it is important to notice that the introduction of digital technologies in the assessment implies a development of the mathematics that can be done at school. Consequently, some changes in the curriculum become necessary, in order to include some competencies that can be acquired only with the use of digital technologies in mathematics.

Having considered the difficulties that might arise from the use of digital technologies in student assessment, I will move to the potential meaning that the tools can assume in the mathematical activities. Thus, in the next section, I will take into account the possible roles that teachers might assign to digital technologies in the assessment of some mathematical and technological competencies.

1.3. Roles assessment plays in supporting the use of digital technologies in the classroom

What kinds of competencies, abilities and knowledge do teachers want to assess? The emerging educational goals motivate the diversification of modes of assessment away from the traditional paper-and-pencil tests. All that I have included in the literature review so far is broadly applicable to a wide range of technologies. I will focus my future considerations on a particular digital technology: the Dynamic Geometry Environments, because assessment is differently applicable for different technologies.

Instead of looking at the affordances of DGEs, and then asking which kind of competencies can be evaluated through that features, I will try to see how a specific learning outcome that students have to achieve can be tested in a DGE. Since I was going to test my sketches and to interview teachers in Vancouver (British Columbia), I looked through the BC curriculum in order to identify both the competencies that grade 8-9 students need to achieve in mathematics and the role that digital technologies might play in the learning. According to the *BC Curriculum* (grade 8-9) students are expected to achieve the following seven mathematical processes:

1. communicate in order to learn and express their understanding
2. connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines

3. demonstrate fluency with mental mathematics and estimation
4. develop and apply new mathematical knowledge through problem solving
5. develop mathematical reasoning
6. select and use technologies as tools for learning and solving problems
7. develop visualization skills to assist in processing information, making connections, and solving problems (Ministry of Education, 2008)

Moreover, the Ministry of Education (2008) adds that, "students must encounter critical components in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics" (p. 18).

As we can see, the program goals of the *Mathematics Curriculum* already incorporate new kinds of competencies and abilities that students could acquire using technology in the classroom. How can the assessment of these learning outcomes support the use of digital technologies?

1. *Communicating in order to express their understanding*. Through the 'actions' that students do in a DGE, teachers can see their reasoning and recognize possible misconceptions:

   > Even without sophisticated constructions, a student's simple action of manipulating a dynamic figure can already be a meaningful mode to demonstrate their understanding of geometric concepts. (Sangwin et al. 2010, p. 235)

   Through the different possibilities of answer recording that digital technologies offer, students can explain their mathematical understanding using tools like screenshot, script and recording voice or video while they are working in a DGE. These instruments allow teachers to see broader aspects of student thinking: "by studying a student's script, a teacher can infer ways that the student is thinking about the object or procedure" (Wilson, 2008, p. 417).

2. *Connecting mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines*. Through the measure and the simulation tools of a DGE, students can study and model real life phenomena, since the ease with which students can represent, explore and manipulate data with these tools allows them to use technology to solve interesting problems. (Olive et al. 2010).

3. *Generalizing properties or theorems.* In a DGE the process of generalizing is fostered by the drag mode, because students can test the 'stability' of a

property as along they are working on a diagram. Anthony (2013) gives examples of 'soft constructions', which are constructions in a DGE where the action of dragging is not intended to verify some properties, but part of the construction itself:

> Through dragging, the general can emerge from the specific by searching empirically for the locus of figures fulfilling the given conditions. Soft constructions offer a transition from an empirical approach to a theoretical approach in solving a geometry problem. (p. 91)

4. *Exploring, finding invariance and properties*. In a DGE students can explore a domain in order to find invariance and relationships among objects, or the laws that drive a certain construction. Students need to know how to interpret an answer or a non-answer from a machine in order to make inferences and deductions, for example how a certain construction behaves as long as students are using the drag mode. Anthony (2013) affirms that "a task in soft construction could foster operative apprehension" (p. 95), interpreting Duval's 'operative apprehension' as the following:

> Operative apprehension of a mathematical concept or problem in DGE is the insights into the concept or the solution of the problem revealed by operating on a pre-designed figure in the environment through dragging. (Anthony, 2013, p. 91)

5. *Developing and apply new mathematical knowledge through problem solving*. In a DGE students can put forward conjectures and predictions, and test their validity or functionality. If a teacher wants to assess students' ability to use effectively digital technologies in mathematics to solve unfamiliar problems and make rational conjectures, then students can also continue learning (new mathematical concepts) while they are taking a test through technology feedback.

6. *Developing mathematical reasoning*. Mathematical reasoning does not consist in doing calculations and improving practical skills; in a DGE, students can focus on concepts, and consequences, leaving the symbolic and numerical computation to the tool: "expressive tools (such as DGEs) assist students in the move from action and visualization to conjectures and reasoning" (Olive et al. 2010, p. 167).

7. *Selecting and using technological tools for solving problems*. Students need to choose the right tools in a DGE and use them in an appropriate way to carry out the task.

8. *Developing visualization skills to assist in processing information, making connections, and solving problems*. Through the visualization affordance of a DGE, students can observe objects, graphs, and phenomena, and make deductions or inferences about them; moreover, students have the possibility to visualize abstract mathematical concepts in a DGE:

> Students can model, experiment, and test their emerging mathematical understandings using dynamic visualization software in many mathematical domains. (Olive et al. 2010, p. 166)

These learning goals that have to be assessed actually support the use of digital technologies, which suggests that teachers can take advantage of the affordances of technology to design innovative tasks that effectively evaluate students' competence and understanding. In particular, digital technologies could help teachers see the rationality leading student thinking, because they can analyze the 'hidden' students' actions. Moreover, Zbiek and Hollebrands (2008) assert that, "technology offers new ways in which teachers may capture and replay student work, thus suggesting good ways to assess students' understanding" (p. 335).

As in the *BC curriculum*, in many other countries it is possible to find similarities in curricular topics and objectives across various curricula. Every year curricula are update in order to meet the new learning outcomes expected by society and the technology advance. However, teachers may find it difficult to change their ways of teaching according to the progress of digital technologies and to put into practice innovative instruments and techniques every year.

In the next section, I will consider the role of technology feedback in assessment, and, consequently, how students' responses could be evaluated.

1.4. Technology feedback in the assessment and students' answers

Technologies such as DGEs offer timely feedback to the action of the user. Technology responses can vary from displaying text, to changing the appearance of a graph or other screen object as its parameters are changed. This kind of feedback is neutral, since any possible evaluation comes from the students themselves, as they discover which actions enable them to reach their goals (Mackrell, 2015). Laborde & Laborde (2011) observe that:

> It is commonly accepted that an important feature of technology and technology based tasks lies in their interactivity and in the possibility of providing feedback to students' actions. (p. 59)

Feedback through technology offers a great deal of opportunity for new ways of understanding mathematics, because feedback from student interactions with technology could have a strong impact on their mathematical understandings and practices. It can be used so that tasks take advantage of the computer's potential to provoke situations of validation as well as action and formulation (Joubert, 2013a, p. 2592). Moreover, Olive et al. (2010) affirm that, "feedback provided by computational tools can shift the focus of the student from micro-procedures (that the tool performs) towards macro-procedures that involve higher-level cognitive processes" (p. 167).

However, students sometimes interpret feedback from technology in unexpected ways. During the mathematics lesson the teacher can intervene to 'put the students back on the path', but in an assessment situation the teacher would not be there, students would have to figure out their way by themselves. In a classroom activity, when students' reasoning is not correct, technology feedback gives evidence that the solution is inadequate showing a sort of inconsistency, or providing a new situation, then students could refine their thinking iteratively as they design, rather than at the end of the design process (Laborde et al., 2006). Moreover, "research on the role of feedback provided by technological tools suggests that learning is most likely to occur when the feedback is unexpected" (Olive et al., 2010, p. 167).

During assessment, technology feedback is limited, just as the teachers' role, in order to avoid the possibility that students try to guess the answer or to solve the problem without thinking:

> The difficulty, perhaps, lies in a confusion related to the role of the computer, which does the mathematical work of creating the graph, and the question then is, what mathematics will the students do? (Joubert, 2013a, p. 2592)

If the teacher's goal is to assess the students' investigation competence, the digital technologies can take on the role of the teacher (Sinclair & Jackiw, 2010), because they are not supposed to intervene: the feedback of technology should be sufficient in giving students what they need to find the solution, and it should foster their exploration.

The role of the feedback depends on the kind of the task, and on the role of the technology, but primarily on the aim of the assessment. It is important that teachers establish the aim of the assessment, and the specific competencies they want to test, taking into account the use of the digital tools: they should have a clear idea of the mathematics the computer will do and the mathematics the students will do in order to avoid technology taking the role of the students.

How student answers might be evaluated?

Although teachers could have a clear idea of the intended learning outcome they want to get when they design or assign a task, it does not mean that a student who completes the task has a correct understanding of the mathematics involved:

> There is a tacit assumption that the completion of mathematical tasks chosen or designed by the teacher will result in the student learning the intended mathematics. (Ainley & Margolinas, 2013, p. 71)

In the assessment, sometimes students merely reproduce a memorized procedure or algorithm, without an assimilation of the concept. A careful task design could help minimize the gap between teacher's intentions and student mathematical activity. A students' answer could be influenced by the design of the task; teacher unconscious choices about some tools or diagrams may be the cause of some kinds of student reasoning. Deeper reflections have to be done in exploration activities, where the role of the software is fundamental to carry out the task:

> Because of the potential power of feedback from the computer, designers need to take into account the meanings students read into this feedback (Joubert, 2013b, p. 75).

In the next section, I will focus on task design in order to examine assessment practice in detail, since teachers would need to either choose or create the assessments questions to be used by their students in a DGE.

1.5. Task Design in a DGE

In recent years, there has been growing attention to the importance of task design in mathematics education, as evidenced by the 22nd ICMI Study on *Task Design in Mathematics Education*. Of particular interest to this chapter is the research on task design in the context of digital technologies, especially in terms of how it might relate to assessment. I investigate not only how task design might be affected by the use of Dynamic Geometry Environments, but also what particular constraints and affordances might be involved in developing and using such tasks in assessment situations. I will summarise some of the research around tasks, task design and DGE-based tasks. I will then adapt the constructs that emerge from the described literature to the context of student assessment in mathematics. The main aim is to explore the way that task design can be carried out in order to enable the use digital technologies as part of teachers' assessment practices.

1.5.1. On Mathematical Tasks and Task Design

The word 'task' is intended to collect a wide range of 'things to do':

> A task is anything that a teacher uses to demonstrate mathematics, to pursue interactively with students, or to ask students to do something. Task can also be anything that students decide to do for themselves in a particular situation. Tasks, therefore, are the mediating tools for teaching and learning mathematics and the central issues are how tasks relate to learning, and how tasks are used pedagogically (Watson et al., 2013, p.10).

Joubert (2013b) points out that "the literature distinguishes three main task types: *exercises (or routine problems)*, *problems* and *investigations*" (p. 69). She also observes that these types of task can be distinguished by their goals, and also by the context and the prior learning of the students. I describe these types of task, as well as their relevance to the particular context of DGEs:

- *Repetitive exercises (or routine problems)* include processes like giving results, using well-known procedures and stating concepts. These practices have little in common with most uses of DGEs. However, Thomas and Lin (2013) observe that "other tasks that do have well known by-hand techniques can quickly assume the character of procedural tasks when technology is used" (p. 110), for example *constructing objects* in a paper-and-pencil context could be considered as an exercise, but in a DGE there is a change of technique, and students have the possibility to test the validity of their construction (Laborde, 2001).

- The '*Problem'* category includes many kinds of tasks: exemplifying definitions, creating counter-examples, solving single-stage and multi-stage problems, deciding between two possibilities. A DGE is the ideal environment for reasoning and trying to solve problems, even with trial-and-error techniques, because students can 'make experiments' with the tools, test their conjectures and come up with counter-examples.

- *Investigations*: digital tasks help students reason in activities like exploring, discovering, proving properties and finding invariance. Many students struggle during this kind of activity in a paper-and-pencil context. In a DGE, processes such as guessing, pattern-seeking, making connections, predicting, hypothesising and proving, are supported by the tools and the possibilities of the software (visualisation, dragging, calculation, etc.) (Mariotti, 2006). 'Noticing', intended as developing conjectures based on observations and testing the conjectures, is seen as a key formulation activity in a DGE:

> An important implication of the computer's ability to 'do the mathematics' is in the opportunities which can be developed for the students to work inductively rather than deductively as is more usual in mathematics classrooms. (Joubert, 2013b, p. 73).

Moreover, Joubert states that in an investigation task "students should move between the pragmatic/empirical field and the mathematical/systematic field in order to reach the goal of the task" (p. 75).

As with the word 'task', the word 'design' can involve a wide range of materials (necessary instruments, task sequences, list of misconceptions that could emerge, etc.), actions (ways of working, whole lesson sequences, teachers' reactions to possible students' actions) and verbal interventions (things that the teacher might say, answers to possible students' comments or questions). Sometimes the design of the task includes pedagogic advice for the teachers on effective choices. Therefore, task design can look like a script for a drama with designers as scriptwriters, teacher and students as actors, but while the teacher knows the lines, the students have improvise (though teachers can usefully improvise as well, as Zazkis, Sinclair & Liljedahl 2013 have argued).

If digital technologies are involved, the design of the task includes also tools, colours, figures, movement, visualization and much more. Laborde (2001) states that "the context (and in particular a technological context) deeply affects the task carried out by the student […] Through the materials we may be able to improve the context in which students learn the mathematics of dynamic geometry" (p. 292), while Thomas and Lin (2013) point out that:

> One of the central issues in the use of technology is the design and implementation of tasks that will encourage the learning and understanding of mathematics, and in particular mathematical thinking. (p. 109)

Sinclair (2003) carries out a study that investigated the benefits and limitations of using pre-constructed, web-based, dynamic geometry sketches. She analyzed the relationship between the activities and the development of geometric thinking skills, and the connection between the design of the materials, and the exploration process. She highlighted particular characteristics that a task designed in a DGE should have depending on the aim of the task, and provided some hints to design the sketches in order to address the questions and the tools:

> 1. When a question aims to focus student attention, the sketch must provide the visual stimulus. It must draw attention through colour, motion, and markings. (p. 312)

Especially in an exploration task, if students should be concentrated on a particular aspect of the task, then the sketch should include some clues through movements or buttons that let the students know where the core of the problem is.

> 2. When a statement prompts action, such as asking students to drag, observe or deduce, the sketch must contain the necessary provisions. It must provide affordances so that the student can take the required steps. (p. 312)

If students need to follow some steps in order to carry out the task, the sketch should provide the crucial information to guide them along the path. The task should give students the hints to foster the mathematical reasoning through buttons, tools and dynamic objects.

> 3. Questions that invite exploration are open-ended. In order to explore uncharted territory, the student requires a sketch that allows options. Thus, when a question invites exploration, the sketch must provide alternate paths. (p. 312)

The options for the students could be provided by a choice among different tools or buttons, which offer different possibilities of action, in order to foster students' exploration.

> 4. A question can surprise – which may lead to further exploration; however, the teacher is not necessarily there to correct any misinterpretation. Thus, the sketch must support experimentation to unmask the confusion. It must be flexible enough to help students examine cases, yet constrained enough to prevent frustration. (p. 312)

In an assessment situation, students are not monitored by the teachers, thus if the sketch shows something unexpected, it should also include the information to figure out the explanation for the 'surprise'. Students need to feel comfortable enough to explore the environment, but also intrigued by some unpredicted situations. The sketch must contain the necessary information and tools to test any correct or incorrect hypothesis that could emerge.

> 5. Questions that check understanding are important parts of any learning situation. In the study tasks, the checking involved students looking together for the answer. Study results showed that the sketch aided the process of peer-interactions by providing a shared image for students to consider and discuss. (p. 312)

The multi-touch technology (like *Sketchpad Explorer*) allows the students not only to share the screen of the tablet, but also to use their fingers simultaneously on the sketch. They can work together in an exploration activity, but also check their understanding through the communication of their reasoning among peers on the same screen.

Sinclair underlines the importance of the decisions made about the design of the tasks in a DGE, because they "have the potential to support or impede the development of exploration strategies and geometric thinking skills" (p. 313). She concludes by affirming that, "through the materials we may be able to improve the context in which students learn the mathematics of dynamic geometry" (p. 313).

1.5.2. Designing assessment tasks

In an assessment situation, a task is an activity for the students, which is proposed by the teacher in order to verify the learning and the understanding of some specific mathematical concept(s) or practice(s). Both in classroom activity, and in test situations, the goal for the student is to answer questions in order to complete the task. For the teacher these two goals are deeply different: in the first situation, the aim of the task is to foster the understanding of a new mathematical concept, and promote learning, while in the second one the goal is to evaluate student knowledge and competence. Therefore, it seems reasonable to believe that the design of such tasks will be different.

In the proceedings of the 22nd ICMI Study, several articles dealt with designing tasks to support student learning, such as Job and Schneider (2013), who note that: "teachers, researchers and the mathematical community in general have an interest in designing tasks to help students acquire mathematical knowledge" (p. 203), and Thomas and Lin (2013), who points specifically to this in the concept of digital environments: "many educators promote digital technology as having a role to play in helping students to develop mathematical thinking" (p. 109). In most of the tasks teachers are a fundamental part of the activity: teachers usually introduce the task, explain the situation and give prompts while students are going along the activity. However, in assessment situations, **teachers usually play a very different role**. This will affect task design because it changes the participation of the teacher. The task should be 'self-consistent' in that students should be able to solve the task by themselves, without the teacher's intervention. In a formative assessment situation, the design of the tasks often excludes interactions among the teacher and the students (this is not always the case, as in the Ontario provincial exam initiative of 2002, where assessment occurred over a three day period and where teachers had specific roles in introducing and guiding the students' work). In this situation, the teacher acts as the designer, the students as the actors, and there is no script. Students have to show what they learnt and understood by themselves.

In such a context, therefore, some of the **characteristics of an assessment task** should look as follows (adapted from Savard, Polotskaia, Freiman, & Gervais, 2013):

- The *goal of the task* should be explicitly communicated in the instructions of the task.
- The *text of the task* should be very short, clear, and it should contain simple words and expressions that the students are familiar with.
- All the *instructions* and the *necessary information* should be provided in the text.
- The task should provide all the *instruments* and the *tools* that the teacher wants to incorporate.
- If it is an exploration task, it should include an *intriguing element*, which would foster students' investigation.

In her paper entitled "Using computers in classroom mathematical tasks: revisiting theory to develop recommendations for the design of tasks" (2013b, p. 69), Joubert highlights the importance of paying attention to the intended mathematical learning of students as they work through the task, adapting their strategies as they negotiate epistemological obstacles. In the same way, in formative assessment, teachers need to know what **mathematical and technological competencies** they want to evaluate while they are using a DGE to assess their students' understanding, because, as Joubert points out:

> The fact that software can perform some of the mathematical processes can be confusing; if the computer does the mathematics, what learning is there for the students to do? (p. 71)

The fact that digital tools carry out some mathematics procedures should not be a disadvantage: teachers have the possibility to assess particular mathematical concepts knowing that the computer does some of the mathematics while students do *some other* mathematics, which could be more meaningful than calculating a product, drawing a graph, or computing an area. Moreover, teachers have the opportunity to design tasks that enable certain mathematical thinking that is not accessible with paper-and-pencil tasks, as Olive et al. (2010) observe:

> Several researchers have focused on the importance of task design (e.g. Sinclair 2003; Laborde 2001) in technological environments. They argue for designing tasks that are transformed by the technology, leading to new mathematical practices (e.g. modeling real-life phenomena, making deductions based on observations), rather than tasks that could be just as easily completed without the technology. (p. 167)

The use of digital technologies in assessment needs a change in the mathematical concepts/competencies/abilities to be evaluated. Keeping in mind that designers should pay attention to avoid making the task too easy through the technology feedback:

> Changes that make it easier for the student to complete the task may have the effect of undermining the designers' intentions, and reinforcing students' attention of completion (of the task) as the priority. (Ainley & Margolinas, 2013, p. 151)

Some tasks lose their meaning if digital tools are used, because technology could prevent some elementary learning processes, or could reduce mathematics to an experimental practice. There is also the risk that a task in a DGE might suffer from constraints of the technology, which may limit the students' exploration space, such as requirements for input formats and styles, and pre-designed tools that may incorporate too much guidance. Careful consideration is required in order to design tasks for the purposes of assessment, in order to give students the opportunity to express their knowledge and to show their competencies.

Buteau & Sinclair (2012) stress the point of view of addressing the question of *how one can use technology instead of avoiding it*. Three **strategies for problem design** emerged from the whole group discussion while reporting and reflecting on the activity:

- Create questions requiring some mathematical modeling, i.e., in which the mathematical notation is not given, and as such, cannot be directly typed into the technology.
- Reverse one of the traditional problem formats: give properties and ask to find an example.
- Create questions involving comparing instead of identifying.

1.6. Conclusion on the literature

The literature on assessment using digital technologies is difficult to be found and the main types of assessments currently being used are differently defined. Further, there are many barriers to the use of digital technologies in assessment. Despite all this, the roles that assessment plays in supporting the use of digital technologies in the classroom are really promising.

There are many advantages in using digital technologies in the assessment, but **not so many examples**. Teachers need some help in developing and organizing a kind of assessment that takes advantage of all the innovations of the technology, and that helps students solve mathematical problems using technology. Digital technologies offer new ways of doing mathematics, however, the essential nature of the tasks has to change, in order to include questions that students are not able to solve in a paper-and-pencil context, like Caron and Steinke (2005) state:

> We must also look at what mathematical problems could now be tackled by students with the use of technology, what concepts and techniques ('new' and 'old') would be mobilized in solving these problems and how the solving of such problems could contribute to the development of a creative, powerful and rigorous mathematical practice. (p. 3)

Having assessment tasks like these ones may help students understand the importance of using digital technologies in mathematics, since according to Heidenberg and Huber (2006): "what you test is what you get" (p. 104). It means that if digital technologies are never part of assessment, students do not see the value of using them to solve problems and unfamiliar situations. If teachers consider something important, they will likely want to find a way to assess it, and the process of providing feedback to students demonstrates that the goals are valued.

As elaborated on in this chapter, the word *feedback* is used in two different contexts in the literature, with two different meanings. From now on, I will use two different labels, in order to indicate the role and the context of the *feedback*.

I will call ***Activity Feedback*** the feedback teachers or digital technologies give to students during mathematical activities in class. This kind of feedback is about the activity that students are doing, like prompts given by the teacher to make them reasoning, or technology responses to students' actions, and it is used by the students to carry on the activity they are undertaking. This kind of feedback should encourage students to focus on the goal of the task, and on the strategies to attain that goal.

I will call ***Student Feedback*** the feedback teachers or digital technologies give to students after or during the assessment (*formal* or *informal*). This kind of feedback is about students' performance and learning. It is used to make students aware of their level of competencies compared to the required achievement, and to give students advice on what they have to do to improve their work. It includes the capability to self-assess, and it is a fundamental part of *formative* assessment. *Student feedback* becomes *formative* when students use it to enhance their learning.

Chapter 2.

Theoretical Framework

After years of discussion on the advantages of using computers and other digital technologies in the teaching of mathematics, in the last few years the presence of these kinds of tools in schools has become a reality. However, as Trouche (2005) states:

> Explaining the potentialities of new tools and actually integrating them into a class are not similar tasks. […] The integration of tools into schooling requires specific strategies and deep reflection. (p. 34)

If the teachers have the instruments, but they do not know how to meaningfully use them to in mathematics education, these new promising tools do not generate any kind of benefits, because "what accounts for differences in learning is the *use* of resources, rather than just the *having* of resources" (Herbst & Chazan, 2012, p. 602). For this reason, the recent integration of digital technologies in the teaching of mathematics has required a theoretical framework to study how teachers see and use technology in their practice, in order to help them take advantage of all the possibilities offered by the new tools to gain a student better understanding of mathematical concepts.

Different theories have been proposed to better understand the way in which teachers use digital technologies. Some theories focus on the analysis of teachers' beliefs and conceptions of technology. Others study the knowledge teachers have about technology and its impact on students learning. While other theories investigate the teachers' expertise in class using technology, which includes actions that they may even be unaware of. I will examine the main theoretical frameworks in a synthetic way and compare the leading aspects of them, in order to find the best one for my data analysis.

The *Technological Pedagogical Content Knowledge (TPACK)* framework is based on the idea that teaching is an intersection of different kinds of knowledge. The interaction of these bodies of knowledge: technological, pedagogical, and content, "produces the types of flexible knowledge needed to successfully integrate technology use into teaching" (Koehler & Mishra, 2009, p. 60). Teachers are supposed to apply this specialized knowledge on many cases and contexts. The main reason of *TPACK* framework ineffectiveness is that it is unproductively focused on an epistemological demarcation of types of teacher knowledge, rather than on its functional organization. Moreover, the content knowledge of the *TPACK* framework does not focus on mathematics, it is teachers' knowledge about the subject matter to be learned or taught. The framework states that, "understanding the impact of technology on the practices and knowledge of a given discipline is critical to developing appropriate technological tools for educational purposes" (Koehler & Mishra, 2009, p. 65). However, the framework does not provide specific indications for every subject, each of which entails its own difficulties; mathematics, in particular, presents many challenges when technology is involved.

On the other hand, frameworks like the *Instrumental Orchestration* study the instrumental approach to tool use, which are supposed to mediate human actions in doing mathematical activities, where the teacher is the orchestra leader who has to coordinate all the instruments and the players to work properly together in order to achieve student understanding and learning:

> An instrumental orchestration is the teacher's intentional and systematic organisation and use of the various artefacts available in a learning environment – in this case a computerised environment – in a given mathematical task situation, in order to guide students' instrumental genesis. (Drijvers, 2014, p. 191)

In particular, this framework describes teachers' practice in teaching situations, and teachers' actions and techniques are analyzed and classified in *orchestrations*. It emphasises how the incorporation of new tools "depends on teachers adapting and developing appropriate craft knowledge to underpin their classroom work" (Ruthven, 2014, p. 384):

> Teachers are considered as crucial players in education, and their ability to exploit the opportunities technology offers determines to a high extent the success of the integration of digital technology in mathematics education. (Drijvers et al., 2014, p. 191)

This theory could help develop a classification of different ways to organize classroom activities around the use of a tool system. However, it seems to be a quite superficial description of the class situation, because the categorization consists of a detailed list of actions and techniques that teachers do during mathematical activities in class that include the use of a tool. This framework does not analyze and explain the reasons of these kinds of behaviour, teachers' expectations, the preparation of the activity, the problems about it and students' impact and reactions. The aim of the

Instrumental Orchestration theory is to "identify common characteristics related to the integration of technology by *ordinary* mathematics teachers" (Abboud-Blanchard, 2014, p. 298), where the term 'ordinary teachers' refers to teachers who are not technology-experts and who are not involved in experimental projects. However, it does not consider the different types of teachers' knowledge, although they are implied to exist, because their strategies are based on previous knowledge and experience. In a way, it is as if the *TPACK* is the hidden heart of the *Instrumental Orchestration.*

What researchers need is a practical, but motivated theory to frame teachers' practice. In this view, the *Practical Rationality* framework could be a good theory, because its goal is to "identify systemic sources of justification for actions by teachers" (Herbst & Chazan, 2012, p. 601). This framework analyzes the work of mathematics teachers in instruction and the rationality behind this work. Although it does not explicitly consider the way in which teachers use digital technologies, it studies the teaching as the management of instructional exchanges: explicit and implicit rules drive teachers' decisions on strategies, which can include *instructional situation, norms,* and *breaching experiments*. This theory investigates "what is the rationality that might (or might not) support teachers' management of authentic mathematical work by students" (Herbst & Chazan, 2011, p. 406). Moreover, it examines the justification of teachers' actions in mathematics teaching, which can be caused by the nature of the instructional activity. Herbst and Chazan (2011) state that "understanding stable systems of practices as well as understanding how those systems react to perturbations is fundamental for the design of new practices" (p. 452), thus the integration of digital technologies could be seen as a 'perturbation' of teachers' stable systems of practices, and it is meaningful to observe how these systems react to this incorporation of technological tools, in order to adapt teachers' practice in this new situations. The *Practical Rationality* framework has a descriptive approach, and its ultimate goal is to seek for ways in which "the teacher's work could conceivably contribute to the creation of opportunities for students to do authentic mathematical work" (Herbst & Chazan, 2011, p. 405).

However, the *Practical Rationality* framework is still too restricted to the explanation of teachers' actions with their knowledge, background and experience, without considering the classroom situation, the tools and the people:

> This kind of justification complements attempts to explain action in terms of individual knowledge, goals, or beliefs by providing grounds on which the wisdom of an action can be determined. (Herbst & Chazan, 2012, p. 611)

A good characteristic of the *Instrumental Orchestration* and the *Practical Rationality* frameworks is that they are focused on the expertise. Another framework that analyzes teachers' knowledge and practice involving digital technologies is the *Structuring Features* framework. This framework provides a more differentiated characterisation of several key aspects of *Instrumental Orchestration* (Ruthven, 2013). The *Instrumental Orchestration* framework describes specific activity formats that develop a particular resource system, while the *Structuring Features* framework takes into account the integration of technology in the resource system, the reshaping of the activity structures in order to support the use of this new tool, and the problem of task design in the curriculum.

For these reasons the *Structuring Features* framework seems to be the most helpful in understanding the role that assessment might play in teachers' practices involving technology. I will thus described it in more detail in the next section.

2.1. Structuring Features of Classroom Practice

This framework has been explicitly designed by Ruthven (2009) to support the identification and analysis of how teachers can adapt and develop the knowledge that they have acquired through daily practice to facilitate the integration of new technological tools in the classroom:

> The Structuring Features of Classroom Practice framework was devised by bringing a range of concepts from earlier studies of classroom organisation and interaction and of teacher craft knowledge and thinking to bear on this specific issue of technology integration. (Ruthven, 2013, p. 386)

In the 2009 paper, Ruthven develops a conceptual framework that identifies key structuring features of classroom practice. He organizes them in five categories, and he explains how they relate to the incorporation of technology use, in order to describe the professional adaptation on which digital technologies integration into classroom practice depends. Instead of focusing on the complexity and significance of craft knowledge in teaching, the *Structuring Features of Classroom Practice* framework emphasises how the classroom practice can be analyzed with these concepts:

> Understanding the challenges of incorporating new technologies into classroom practice calls for the development of naturalistic perspectives that situate their adoption and use within the everyday work of teaching. (Ruthven, 2009, p. 131)

Ruthven identifies **five key features** used to classify classroom practice: *working environment, resource system, activity structure, curriculum script,* and *time* *economy*. I will illustrate the five categories of the framework described in Ruthven (2009) and Ruthven (2013), and the respective changes I have made in order to include student assessment in the analysis of teachers' practice involving digital technologies.

2.1.1. Working Environment

> The introduction of new technologies often involves changes in the working environment of lessons in terms of *room location, physical layout, and class organisation* (Ruthven, 2013, p. 386).

In particular, this category takes into account the physical surroundings where lessons take place, the general technical infrastructure available, the layout of facilities, and the associated organisation of people, tools and materials. These alterations of the environment require modification of the classroom routines, which enable lessons to flow smoothly. If the lesson is in the computer laboratory, all the well-established routines that usually drive the time in the regular classroom have to be adapted to this new environment, students may also have many opportunities for distraction in this new situation. Teacher and students must modify their habits in order to deal with an unfamiliar working environment, which can involve some changes in the class organization, for example, if the number of workstations is smaller than the number of students, they have to work in pairs or groups, and also the way in which teachers collect student answers and contributions has to be changed. Moreover, the modifications in the working environment may introduce new demands on teachers and students: each of these disruptions or additions to normal practice may increase complexity and uncertainty, and call for significant adaptation of classroom routines. However, Ruthven (2009) observes that, "as the provision of sets of handheld devices or laptop computers for use in ordinary classrooms becomes more common in schools, these organisational issues shift rather than disappear" (p. 135). In many schools it is already possible to find computer (or calculator) projection facilities, or interactive whiteboards in ordinary classrooms.

In this section I consider the technological tools that teachers use in the assessment, how they capture student answers, and what are the technical difficulties that they have in assessing students with digital technologies.

2.1.2. Resource System

> While new technologies broaden the range of tools and materials available to support school mathematics, they present the challenge of building a coherent resource system of *compatible elements that function in a complementary manner and which participants are capable of using effectively.* (Ruthven, 2013, p. 386)

In this category it has to be considered the collection of *didactical tools* and materials in use. However, a collection of resources by itself does not constitute a coherent system. The coordination of the use of these resources towards subject activity and *curricular goals* has to be taken into account, because a 'resource system' is composed of "the combined operation of the mathematical tools and curriculum materials in classroom use, particularly on their compatibility and coherence of use, and on factors influencing this" (Ruthven, 2009, p. 136). The most recurrent problem is coordinating the use of these tools, and teachers report that they would be much more likely to use technology if *ready-to-use resources* were available, and clearly mapped to their scheme of work, in order to make effective use of them and to integrate them successfully in classroom activities. In the their initial approach to digital technologies, teachers look for some materials that would allow them an immediately productive integration of old and new tools, like lesson plans and exercises linked to each section in the textbooks, or applets providing demonstrations and interactivities.

In this feature I analyze the didactical tools teachers have, and the roles they assign to digital technologies in the assessment to achieve the curricular goals. Moreover, I look at the way teachers coordinate students' answers with the *activity feedback* of the technology, and match the digital tools with the learning outcomes of the regular curriculum. I also describe how teachers manage the double instrumentation (old and new tools), and their need for ready-to-use resources. I add to this section the reasons teachers think are valuable to make the use of digital technologies effective in student assessment.

2.1.3. Activity Structure

> Innovation may call for adaptation of the established repertoire of activity formats that frame the *action and interaction of participants* during particular types of classroom episode, and combine to create prototypical activity structures or cycles for particular styles of lesson. (Ruthven, 2013, p. 386)

Teachers' and students' actions are often recurrent in a scheme that is built through the time during the lessons. This leads to the creation of prototypical activity structures for particular types of lesson. Thus, the activity structure focuses on the templates for classroom action and interaction which frame the contributions of teachers and students to particular types of lesson segment. It involves the different kinds of lessons integrating technologies: both the use of calculators, and activity formats that imply more radical changes, and call for new classroom routines and norms for participation, like investigations of open tasks, and assessment with digital tools. This category includes the specifications of the roles of the individuals involved in the activity: teachers have to apply substantial changes to their way of managing and interacting with the class, and students could be asked to work in pairs assuming alternate responsibilities in order to carry out an established activity.

In this category, I describe the *informal* assessment activity, which is the most recurrent kind of student assessment carried out with digital technologies. I also describe the new roles of teachers, students and tools in this kind of activity. Moreover, I point out that using technology in the classroom implies a whole discussion on how the tools work, and how students should use them in mathematics.

2.1.4. Curriculum Script

> Incorporating new tools and resources into lessons requires teachers to develop their curriculum script for a mathematical topic. This 'script' is an *event-structured organisation of knowledge*, forming a loosely ordered model of *goals, resources and actions* for teaching the topic, incorporating potential emergent issues and alternative courses of action. (Ruthven, 2013, p. 386)

This *Curriculum Script* takes into account the *model of goals, resources, actions and expectancies* for teaching a curricular topic including likely difficulties and alternative paths. It also considers developing links among mathematical ideas, the use of appropriate topic-related tasks, and suitable activity formats. In this category also the *anticipation of potential student difficulties* is included, in order to guide the teacher in formulating a suitable lesson agenda, and in enacting it in a flexible and responsive way. Although the integration of new tools in the teaching and learning asks for a rethinking of the curriculum script, "teachers frequently appear to be viewing the use of new technologies in terms of the adaptation and extension of established curriculum scripts" (Ruthven, 2009, p. 138). They would use new tools to improve existing practices, and to provide more vivid and dynamic presentations. Teachers often feel unprepared: they claim for teaching resources that can help them devise and conduct lessons on an investigative model with technology, and align the new ‘technological’ practice with the school curriculum. The point is that teachers need to know how to develop a coordinated understanding of both the knowledge-building and task-effecting value of components of the classical and computer systems, and to establish a coherent teaching sequence for developing student knowledge and skill within the two systems (Ruthven, 2009).

In this section I study how teachers redefine the aim and the content of the assessment with digital technologies, and how they choose and create new tasks. I examine their expectations of students’ reactions, and consequently their possible response to them. Moreover, teachers also need to rethink their way of interpreting students answers and giving value to them.

2.1.5. Time Economy

> The introduction of new technologies may influence the time economy within which teachers operate, changing the ‘rate’ at which the physical time available for classroom activity can be converted into *a ‘didactic time’ measured in terms of the advance of knowledge* (Ruthven, 2013, p. 386)

Since “Time is a currency in which teachers calculate many of their decisions” (Ruthven, 2009, p. 138), frequently, teachers do not use digital technologies for investigative tasks, or problem solving, they prefer to use them when the processes of ‘facilitating routine’ and ‘raising attention’ serve in make the activity effective, both in terms of time, and students’ productivity. Moreover, integrating technology in the classroom requires another cost in terms of time for teachers, because they have to develop the requisite craft knowledge to include the tools in their practice, and the lessons require more detailed preparation, and they are often conducted less efficiently and flexibly. Ruthven (2009) states that “a critical issue is what teachers perceive as the return in terms of recognised mathematical learning from students using new tools” (p. 139), because they are not sure that such a big investment would actually increase rates of return in terms of student understanding.

In this feature I take into account the time teachers have to invest in the preparation, the implementation, and in the evaluation of students’ answers in an assessment activity involving digital technologies. The expectation is that, through the accumulation of experience and didactical formation, teachers would be able to change the ‘rate’ at which the time invested in carrying out the assessment is converted into a ‘didactic time’ measured in terms of having a deeper insight on students’ understanding.

2.1.6. Assessment Feedback

Since I am using Ruthven’s structuring features to analyze teachers practice and expertise in student assessment using digital technologies, I identified the need to introduce a specific category to describe the *types of assessment* teachers use to evaluate their students’ competencies, and the frequency of the assessment activities in the classroom. I also include the specific role teachers give to *feedback* during and after the assessment, the forms of feedback they use and how their students use this feedback to improve their learning. I consider which kinds of assessment could include digital technologies, which ones teachers prefer to use when technology is involved, and which ones they are more hesitant in using with tools.

2.2. Integration of technology in the design of geometry tasks

Laborde (2001) describes a case study on teachers designing tasks for a DGE, it analyzes every task considering the choices made by the teachers with respect to the following aspects:

- the place of the task in the mathematics curriculum;
- the role that teachers assigned to the DGE in the tasks;
- the creation of new tasks linked to technology, which means what degree of change has the task designed for the DGE compared to the paper-and-pencil context.

In her research, Laborde identified four different categories of tasks that were used to drive the teachers’ tasks:

- tasks in which the DGE facilitates the material aspects of the task
- tasks in which the DGE facilitates the mathematical task
- tasks modified when given in a DGE
- tasks only existing in a DGE

2.2.1. Tasks in which the DGE facilitates the material aspects of the task

This category includes the tasks in which the DGE is used mainly as facilitating material aspects of the task while not changing it conceptually. The difference lies in the *drawing facilities* offered by the DGE. The solution strategies of both tasks (DGE and paper-and-pencil) do not differ deeply. In these tasks technology could help solve the problem, but it is not part of the solution of the task.

2.2.2. Tasks in which the DGE facilitates the mathematical task

In these tasks the DGE is supposed to facilitate the mathematical task, which is considered as unchanged. The role of DGE is to help students make conjectures about the relations using the drag mode. It is not really used as a tool for solving a task, it is used as a *visual amplifier* in the task of identifying properties, thus the "visual power of technology is used, but mainly for seeing and conjecturing and not for experimenting, in order better to understand the mathematical situation" (p. 289).

2.2.3. Tasks modified when given in a DGE

The DGE is supposed to modify the solving strategies of the task due to the use of some of its tools and to the possibility that the task might be rendered more difficult. The task in the DGE requires more mathematical knowledge, which usually students find difficult to put into action.

2.2.4. Tasks only existing in a DGE

The task itself takes its meaning or its "raison d'être" from the DGE, in particular from the drag mode which preserves geometrical relations; it necessitates reasoning and knowledge. Such tasks require identifying geometrical properties as spatial invariants in the drag mode and possibly performing experiments with the tools of the DGE on the diagram. The identification of underlying properties is not easy and constitutes the question. These are tasks in which the environment allows efficient strategies that are not possible in a paper-and-pencil context, or tasks that are raised by the technology, which means tasks that can be carried out only in a DGE. Typically, these tasks appear in two forms:

- *'black box' situations*: students have to explore a certain environment, and find properties or relations in order to reconstruct a dynamic diagram; the invariance properties here become remarkable rather than routine phenomena, because they are tools for identifying the 'hidden construction'.

- *prediction tasks*: students have to predict the behaviour of a certain system/construction. As Laborde (2001) states, the DGE "allows a confrontation between what is predicted and what is observed" (p. 305).

2.2.5. Implications of Laborde's categories

Categorizing the tasks following Laborde's scheme shows the **meanings teachers give to technology, as well as to mathematics**. The role of the DGE in a task gives evidence of the aim of the tools, since they could be used to simplify a task (supporting the calculus or the visualization), to solve a task with different strategies, and to foster exploration or imagination. If a teacher wants to use technology to help students in their usual work, they would prefer to use tasks of the first and second category; if a teacher wants 'solving tasks in a DGE' to be a fundamental part of the activity, and a competence to be evaluated, then they would design tasks of the third and fourth category.

Through the tasks teachers design, also the **meaning that they give to assessment** emerges: what kinds of questions are 'typical' for formal and informal assessment, what competencies they choose to grade and what competencies they prefer to evaluate in a 'learning situation'. Therefore, teachers could pick the category for the tasks depending on the role that the DGE assumes in the task, the mathematical and technological competencies they want to evaluate, and the situation in class (formal or informal assessment).

The role of DGE in a mathematical task. If teachers do not want to change the meaning of the tasks compared to a paper-and-pencil context, then they would use tasks of the first and second category; if they would like their students to develop different strategies for problem solving, they would prefer tasks of the third category; if teachers promote prediction and exploration tasks, they would design tasks belonging to the fourth category. The *dynamic* nature of the DGE has a fundamental role in tasks of the second, third and fourth category because it gives deep insights into the mathematics, while it does not give a significant contribution to the tasks of the first one. Since there are many studies (see Caron & Steinke, 2005; Laborde, 2001; Mariotti, 2012) supporting the fact that *constructing* a figure in a DGE could give a better understanding of the properties of the figure, many teachers use the DGE to have their students creating objects, as Laborde (2001) observes:

> Technology moved from being a visual amplifier or provider of data towards being an essential constituent of the meaning of tasks and as a consequence affected the conceptions of the mathematical objects that the students might construct. (p. 283)

However, constructing is not the only valuable action available in a DGE. Anthony (2013) underscores the fact that the action of *dragging* is part of the construction itself, and Sangwin et al. (2010) affirm that students' action of *manipulating* a dynamic figure could be considered as a meaningful manifestation of their understanding.

Teachers could choose to evaluate only the mathematical competencies needed to answer the task, or to **evaluate also the technological competencies** required to carry out the task in a DGE. A teacher might thus choose tasks of the first category if she wants students to be able to use the tools in a DGE to solve the task, but their use is not part of the evaluation. Tasks of the second category would be chosen by teachers who wish to use the DGE to help their students solve problems they struggle with, problems that students do not understand, or that are difficult to be represented. Tasks belonging to the third category would be designed by teachers who expect their students to be able to use the tools to show different solving strategies. Finally, the fourth category would be preferred by teachers who want their students to feel confident to observe, solve problems and make conjectures in a DGE.

Teachers could associate tasks to situations: they would use tasks of the first and second category for formal assessment, and they would prefer tasks of the third and fourth category for informal assessment, like classroom activities. Sometimes technology would be optional; other times it would be necessary to complete the task. Sometimes digital technology would be a tool to help students, other times it would be a fundamental part of the assessment. Laborde (2001) observes that, "at the beginning, most were observation tasks for conjecturing, whereas more diverse tasks appeared in later versions" (p. 295). Teachers like to ask, "what do you notice?" when students are observing a dynamic object, or exploring a figure, but in most of the cases, this is a typical question for informal assessment. Less often, teachers design tasks with prediction situations for conjecturing and proving. Teachers might be interested in knowing how they can create 'black box' and prediction tasks, even if "the design of such tasks represents a conceptual break with the usual tasks performed in a paper-and-pencil environment" (Laborde 2001, p. 294). However, they still have some problems in including these tasks in the assessment, probably because they are convinced that it is better to give exploration tasks in a classroom activity, in order to have their students finding out something and learning something new. Teachers often think that they have to assess what students know, not their ability to explore and make conjectures in unfamiliar situations, as Madison (2006) observes:

> Mathematics faculty members are accustomed to formulate learning goals in terms of mathematical knowledge rather than in terms of student performance in using mathematics. This creates tension between testing what students know and testing for what students can do. Since judging student performance is usually far more complex than testing for specific content knowledge. (p. 6)

So far I have illustrated the literature on assessment using digital technologies, and the framework on task design in a DGE. I have also described the theoretical framework that I will use to analyze the data on teachers talking about their use of digital technologies in student assessment in mathematics. Now I proceed with the methodology of my study, which includes the research questions, the instruments that I designed to collect the data, and the participants to the study.

Chapter 3.

Methodology

Based on my review of the literature, as well as on the theoretical framework described in the previous chapter, I can now articulate my research questions. The goal of this study is to address the following questions:

1. How are teachers' practices involving digital technologies structured by Ruthven's five features? What relation to these practices have to the types of assessment they use?

2. What kinds of assessment tasks do teachers design with a DGE? What are they aiming to assess with these tasks?

3. What kinds of technology-based assessment tasks do teachers value? How do their preferences relate to the type of feedback offered in the task?

For this study I selected a curriculum topic, the software, and the digital technology to create the sketches/tasks. I designed some tasks for student assessment, and I tried them with students in two different classes. I asked some teachers to answer a questionnaire, composed of questions about their view and use of technology in student assessment. Then, I met the teachers in face-to-face semi-structured interviews, during which I asked for clarifications on their answers to the questionnaire, and comments on the tasks I designed and on students' results.

In this chapter, I describe the reasons that lead me to the choice of the software and the digital technology, and I illustrate the process of designing the assessment tasks in *Sketchpad*. Then, I introduce the participants of my study, and I explain their academic background and their familiarity with *Sketchpad*. Finally, I describe the questions I designed to collect the data in the questionnaire and in the semi-structured interview.

3.1. The choice of the software and the digital technology

I decided to test student competencies in a **Dynamic Geometry Environment** for different reasons. Firstly, DGEs are easy to use both for teachers and students. Secondly, DGEs have been used widely for over two decades, beginning with *Cabri* and *The Geometer's Sketchpad*, and have received wide ratification both in the research literature and amongst teachers (Mariotti, 2006; Becker, 2000). A DGE offers many possibilities of actions, as showed in the literature, while other technologies like graphic calculators, for example, have limited functions and do not allow exploration activities. Graphic calculators are perfect to test results and check solutions, however, students cannot drag or manipulate objects in order to find invariants or construct geometrical shapes. Moreover, teachers cannot create pre-constructed tasks, thus calculators can be additional tools, but not the main environment for the assessment. Other software, like MATLAB and *Mathematica*, are more complicated to use; they need very specific knowledge about the software, and a particular language for programming. Moreover, the competencies that can be tested in these environments are strictly connected to numerical computation. More meaningful and challenging questions can be asked in a DGE, since it allows enough options to formulate very different kinds of questions, and it provides instant feedback to students' actions.

In particular, I chose to design the pre-constructed dynamic sketches in ***Sketchpad Explorer*** (*Sketchpad* version for the iPad), because it allows teachers to give students a 'restricted environment' for the assessment, where students can explore certain diagrams/situations under set conditions, and use only the tools provided by the teacher. Thus, teachers are able to check student understanding on specific properties. Moreover, Sinclair (2013) points out that

> Pre-constructed dynamic sketches are central elements of the learning activity, and therefore, decisions about their design have the potential to support or impede the development of exploration strategies and geometric thinking skills. (p. 289)

3.2. Tasks for student assessment in Sketchpad

Mackrell, Maschietto and Soury-Lavergne (2013) state that, "both the design of tasks and the design of technology have been identified as important factors in the effective use of technology-based tasks in the classroom" (p. 79). I used the literature on technology to take advantage of the affordances that a DGE offers for the task design, (Sinclair, 2003) to design the sketches, and (Laborde, 2001) to drive the tasks.

I designed some tasks to assess student understanding on **Circle Geometry**. Clearly, some mathematical topics are more suitable to be assessed in a DGE, in particular geometry, because students can drag objects, construct figures, and explore a domain.

In the 'Student Achievements' of the ***BC curriculum*** *on Mathematics grade 9* (Ministry of Education, 2008), I found the competence on Circle Geometry that students are expected to acquire: they need to know some specific properties of the circle, and they have to achieve some learning outcomes, like providing examples, solving problems, measuring, and explaining relationships (see Appendix A).

I took some tasks from the textbooks, the web, and game-competitions in mathematics, and I adapted them for a DGE, while I invented more specific tasks that take their meaning from the DGE. I gave these tasks to the **students** of two classes: grade 9 and grade 10. I asked them to try the sketches working in pair on the tasks. They took a screenshot of the iPad to show me the solution of the task, and they explained their reasoning on a piece of paper. They were not used to working with technology in mathematics, but knowing how *Sketchpad* works was not necessary, because *Sketchpad Explorer* is very simple - the students that were using it for the first time were able to do so easily. I collected some interesting answers from them, especially in the grade 9 classroom, because they had just finished the Circle Geometry unit. Then I showed some of these results to the teachers that I interviewed, and I asked them some comments on the sketches and on students' answers.

3.3. Participants to the study

The participants in the study were eight teachers. In order to protect their privacy, I will refer to them with codes: from T1 to T8. The teachers were five females (T1, T3, T4, T6, T8) and three males (T2, T5, T7). Six of them (T1, T2, T3, T5, T6, T7) were teaching at the high school, one of them taught in an elementary school (T4), and another one was tutoring in a classroom at the high school (T8). Four participants were PhD students in Mathematics Education (T1, T2, T3, T7), and three of them (T5, T6, T8) were enrolled in a Masters course in Mathematics Education at the time of interview. The PhD students were my peers at the PhD in Mathematics Education, while the Masters students were my classmates in the Geometry Course of their program. I asked them to participate in my study, and their participation was entirely voluntary.

All of the teachers took a course that included a significant amount of attention to the use of DGEs in the teaching and learning of mathematics. The PhD students were very familiar with *Sketchpad* since they had used it a lot in their courses at the university. The Masters students had a shorter familiarity with *Sketchpad*, since they had just finished their Geometry course at the time of interview. In that course, they learnt how to use *Sketchpad* and they also used it for a project in their classroom as a requirement of the course.

3.4. Questions for the questionnaire and for the interview

In the theoretical framework I noticed that sometimes teachers themselves do not know why they act in a certain way in a specific situation, they just ‘feel’ they have to act in that way. Ruthven (2014) states that “much of the knowledge that teachers use is ‘tacit’ and resides in schemes of perception and action which they are typically unable to articulate, and may even be unaware of” (p. 390). Teachers’ choices on which kind of actions to adopt could be supported by many factors: the experience, the knowledge of their students, some competencies they think their students have, the attempt to prevent some kind of misconceptions they think students often have, etc. There could be many explanations for a teacher’s decision, and the intersection of all of them is happening in teacher’s mind, but sometimes he/she is not able to explicitly determine the content of this intersection. For this reason, asking teachers for the rationale of their behaviour in classroom seems to me quite ineffective. That is also the point of the *Instrumental Orchestration* and of the *Structuring Features of Classroom Practice*: focusing on teachers expertise rather than on their knowledge and rationality: “the organising concept for the *Structuring Features* model is one of how material-cultural factors structure the functional organisation of teaching expertise” (Ruthven, 2014, p. 391).

I thus decided to focus on teachers’ actions and choices they made or they would make in a hypothetical situation without asking for their rationale. I consider teachers’ answers in order to find commonalities in the decisions they take in student mathematical assessment involving digital technologies.

I formulated the questions for the questionnaire and for the interview following this principle, also the way questions are asked, especially in the interview, shows that I am not as interested in teachers’ believes as I am in the rationality of their practice. I did not ask teachers “what do you believe?” Rather, I invite them to imagine themselves in a certain situation, and then asked them how they would act in that situation. Moreover, in the questionnaire, I asked teachers to design a task to evaluate student understanding of one of the properties of the circle students learnt in classroom by using *Sketchpad*. Appendix B includes the questionnaire and Appendix C contains the questions of the interview. All the interviews lasted between 40 and 60 minutes each.

In my analysis, I am looking for patterns in teachers practice regarding student assessment with technology. The *Structuring Features* framework provided me with a useful tool to analyze teachers practice in class with digital technologies. I used the five features of Ruthven, plus the one I added, to frame the teachers' choices and activities about the use of technology in assessment. I analyzed teachers' answers to my questionnaire and their explanations in the interview using these features.

I use Laborde's (2001) categories to drive the tasks teachers designed, and to see which category they prefer through the analysis of the answers they gave to my questions, and the comments they made on my sketches. I wish to know which kinds of tasks teachers might be more willing or interested in using: if they would confirm Laborde's conclusion or change their approach to assessment in a DGE looking at the recent results on digital technologies in task design and at my sketches.

Chapter 4.

Identifying and designing different types of assessment tasks

In this chapter I describe the tasks I designed in *Sketchpad* and my analysis of them in terms of Sinclair (2003) and Laborde (2001).

4.1. Sample DGE-based assessment tasks

As part of my research, I designed DGE-based tasks that could be used to assess students on basic theorems in Circle Geometry (aimed at students of about 14 years old in the British Columbia curriculum) using *Sketchpad Explorer*. I wanted to design a task for each category of Laborde, it was also my intention to create sketches which include different properties of the circle, and which evaluate different mathematical and technological competencies. The intersection of these three requirements lead me to the sketches I created. This also allowed me to collect teachers' responses to different possibilities of assessment tasks during the interviews.

I followed Sinclair's (2003) guidelines to design the tasks, but I also wanted to better understand the affordances and constraints of the different types of tasks described by Laborde (2001), so I designed a task for each category of her study.

4.1.1. The Counter-Example

If you think this statement is true, make a drawing to represent it;
if you think it's not, create a counterexample.

The centre of any circle is the intersection of the perpendicular bisectors of any two chords in the circle.

This sketch is particularly subtle, because there is only one example that does not satisfy the statement: when the two chords are parallel. A deep understanding of the theorem is necessary in order to answer the question correctly.

Students know exactly what to do, because they have to choose one of the two options: drawing a diagram as example, or a diagram as counter-example. The task, following Sinclair's indications (2003), prompts students' reflection and action through the tools.

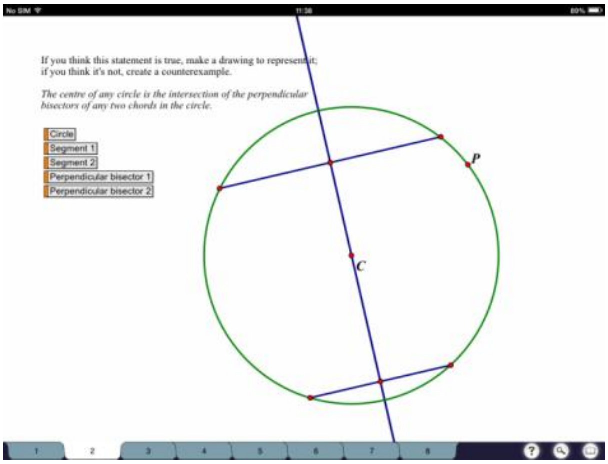


Figure 1. The Counter-Example

The sketch contains the necessary material to draw the diagram students choose to draw, it helps students reflect through the geometrical objects they can drag around the screen, trying different positions for the chords on the circle.

This task belongs to the first category of Laborde (2001), because *Sketchpad* facilitates the material aspects of the task in making the drawing. The task is not changed conceptually compared with a paper-and-pencil environment. The solution strategies of both tasks do not differ significantly: the only difference could be that students can move the chords all around the circle, and they can accidentally find the case in which the two chords are parallel.

4.1.2. The Dog

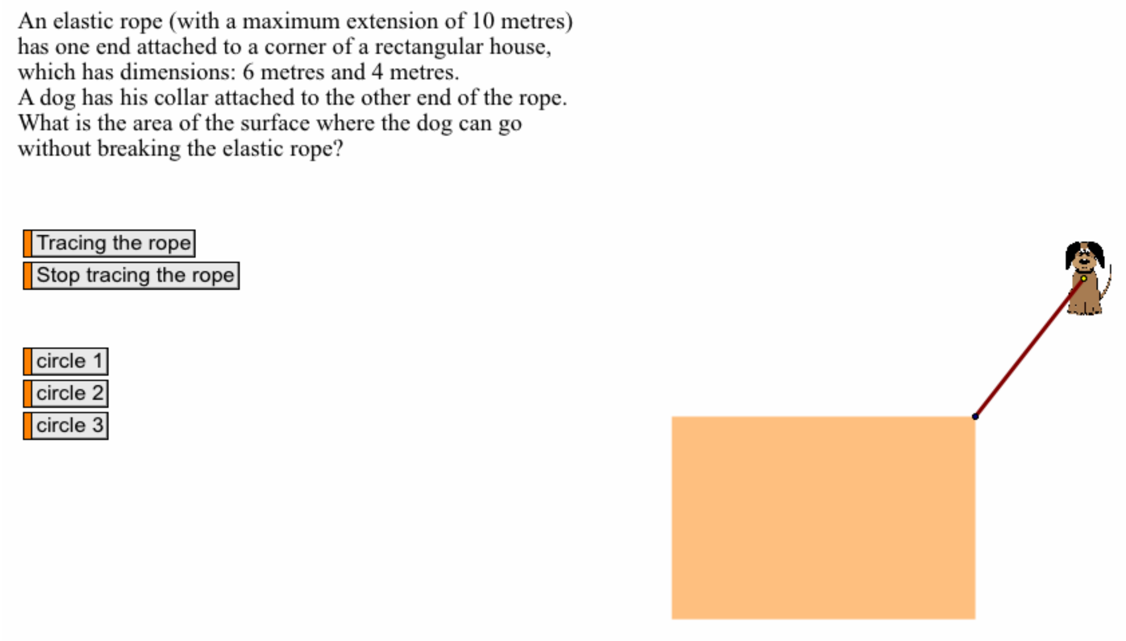


Figure 2. The Dog

In this task, *Sketchpad* acts as a visual amplifier for students as they explore the situation by dragging the dog around the screen. *Sketchpad* allows students to see where the dog can go within the constraints set by the rope and offers visual cues into the problem's solution that would be difficult to obtain through paper-and-pencil alone. Students can drag the dog around the screen and notice the limits of where it can go without the rope breaking. To gain a better sense of the bounds imposed by the rope, students can press the button 'Tracing the rope', so that the rope leaves behind a trace of all its locations as they drag the dog. Pressing the buttons 'circle1', 'circle2', and 'circle3' students view three circles, whose location and size they can change simply by dragging them. They are supposed to position the circles so that they represent the bounds imposed by the rope. Thinking about the radii of the circles and their placement may help students determine the area where the dog can roam.

Following Sinclair's (2003) suggestion, the sketch focuses students' attention on spatial reasoning trough the visual stimulus of the dog roaming around the house and the coloured trace left by the rope. The representation of the rope breaking when the dog is going too far or inside the house is a clear image for students' understanding. They see immediately where the problem is, but they still have some difficulties in representing the area with the three circles, and in finding the exact number of the area.

This task belongs to the second category of Laborde (2001), because the mathematical task is considered unchanged, but the DGE facilitates it: *Sketchpad* is used as a visual amplifier in the task of solving the problem (see Appendix D for the procedure I used to implement the sketch in *Sketchpad*).

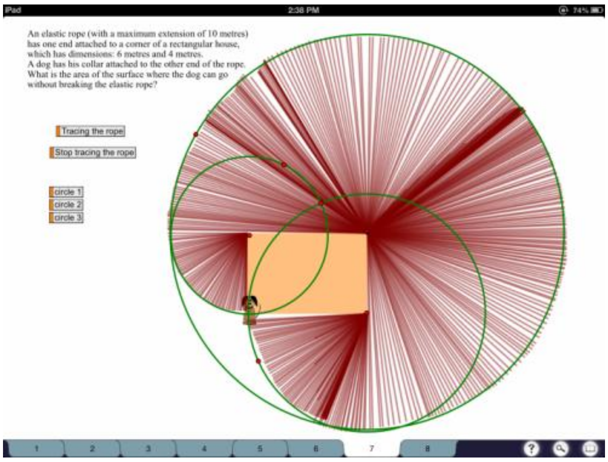


Figure 3. Tracing the rope

4.1.3. The Right Triangle

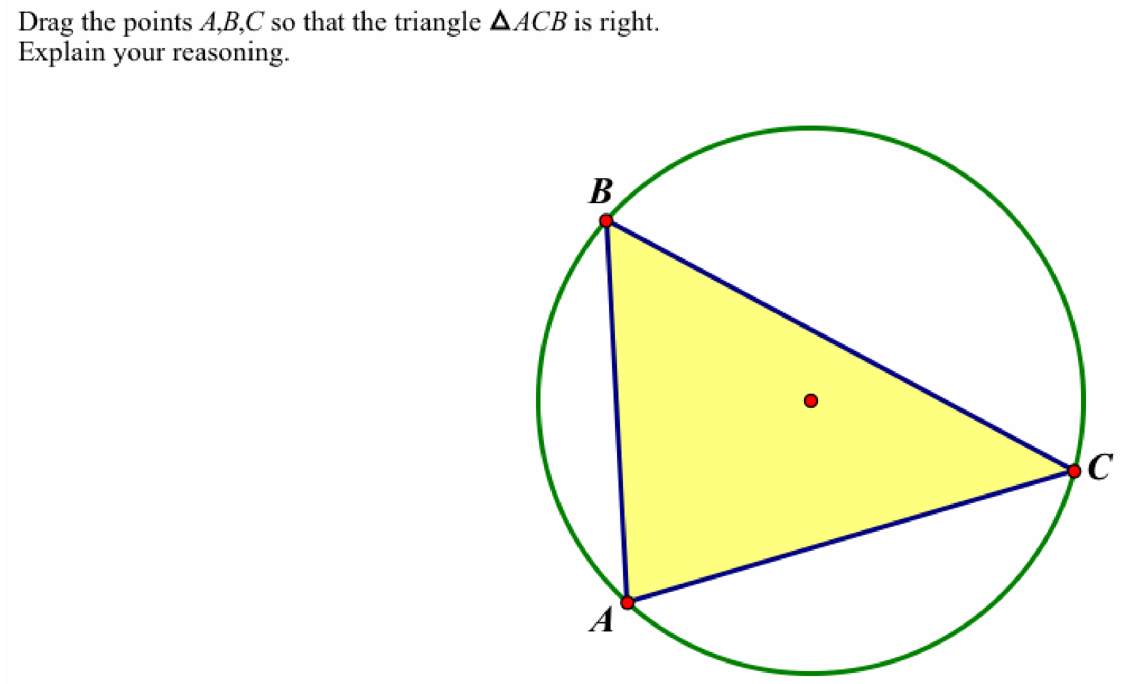


Figure 4. The Right Triangle

Students are supposed to know the property that “if a central angle and an inscribed angle of a circle are subtended by the same chord and on the same side of the chord, then the central angle is twice the inscribed angle”. In this sketch students do not have the measurement tool for the angles, so they have to explain how to obtain a right triangle inscribed in a circle depending on the position of the points. The three vertices of the triangle are constructed on the circle: students can drag the points only on the circle.

In this task, the aim is to focus student attention on the invariance of the measure of the inscribed angle when the subscribed arc is the same, thus the sketch provides the visual stimulus through the motion of the points along the circle. As suggested in Sinclair (2003), the sketch contains the necessary provisions to prompt action, since the task asks students to drag and observe.

This sketch belongs to the third category of Laborde (2001), because the task is modified in *Sketchpad*, compared to a paper-and-pencil context: the solving strategy is different, because students have to drag the points on the circle so that the triangle *ABC* is right. Laborde (2001) states that there are “two ways of using the mediating functions of the drag mode, as a test mode on the one hand and as a search mode on the other”. In this example the drag mode is used as a search mode to find the right triangle. Students need to know the property, and to notice that in a right triangle inscribed in a circle the hypotenuse is the diameter, but the task in *Sketchpad* actually requires more mathematical knowledge: students can move the third point around the circle, thus they have to know that wherever they decide to place it, the triangle will be right, and this is something that students usually find difficult to put into action.

4.1.4. The Ball

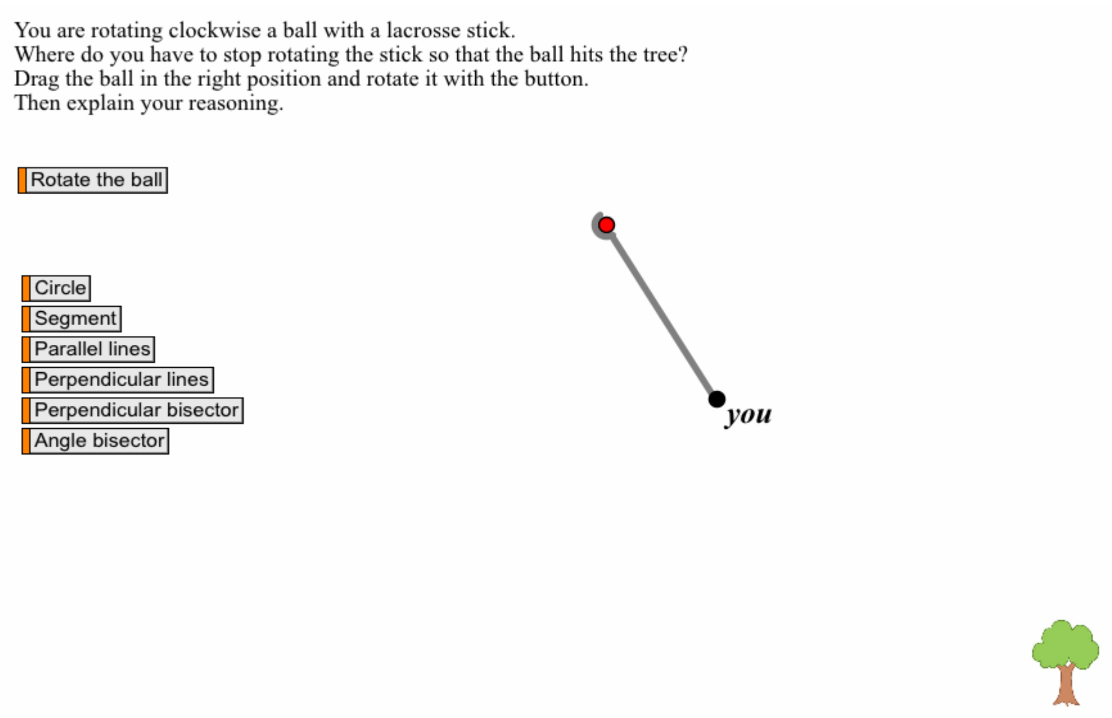


Figure 5. The Ball

In this sketch, students are supposed to explore the situation pressing the button ‘Rotate the ball’, and observe what happens. They have to find the right initial position for the stick, so that the ball hits the tree after one rotation of the stick. It is quite reasonable to expect that students will try to rotate the ball until they find the solution, but then they need to explain why that one is the correct position. At this point, they will use the tools to try to explain the solution, and they will find that the trajectory of the ball is the tangent to the circle whose centre is the point ‘you’, and radius is the length of the stick.

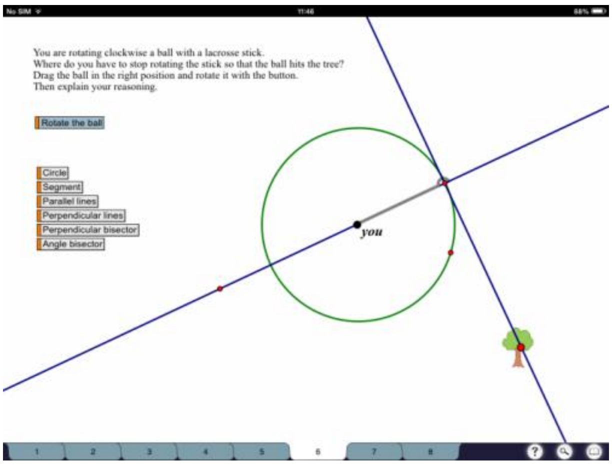


Figure 6. The tangent

In this task, students are invited to explore the situation, thus, following the hints of Sinclair (2003), the sketch provides different tools and allows options. This question can also surprise students, since some of them could expect the ball going along a circular path after the stick has stopped. Students could try different configurations for the path of the ball with the tools, and then test them. This sketch supports experimentation, since the teacher is not there to correct any misinterpretation.

This task belongs to the fourth category of Laborde (2001), because it can exist only in a DGE, and it takes its meaning from it. Students have to guess where the ball is going when the stick stops, and they can check their answer with the button. Then, they have to find the ‘hidden construction’ of the situation using the tools provided. This task requires reasoning and knowledge: students need to know that a tangent to a circle is perpendicular to the radius at the point of tangency. They have to identify the geometrical properties as spatial invariants in the drag mode, because as long as they drag the stick, the trajectory of the ball is always tangent to the radius of the circle, and they can perform experiments with the provided tools on the diagram. The identification of the underlying property that the trajectory of the ball is the tangent is not easy and constitutes the question, because they have to reconstruct the dynamic diagram of the simulation.

4.2. Comparison and Discussion

Since teachers have to be clear about the mathematical and technological competencies they want to evaluate, in order to assess their students’ learning, I summarise these two kinds of competencies in the table below for each of the assessment tasks.

Tasks	Mathematical competencies	Technological competencies
The Counter-example	Correct use of examples in proving or refuting conjectures; generalizing theorems and finding exceptions; applying mathematical reasoning; constructing mathematical representations.	Making a draw with the tools; appropriate use of the instruments; selecting and using technological tools to construct a diagram.
The Dog	Exploring; problem solving; applying mathematical reasoning; pattern-seeking; ability of converting a diagram in algebraic operations; connecting mathematical ideas to everyday experiences.	Exploring the situation with the tools; appropriate use of the instruments; ability to use effectively digital technologies in mathematics to solve unfamiliar problems and make rational conjectures; developing visualization

		skills to assist in processing information.
The Right Triangle	Reading a diagram; explaining the mathematical reasoning with own words; using a property in a particular example; stating concepts; generalizing; proving; 'noticing'.	Dragging objects to find invariance; dragging to test the 'stability' of a property as along they are working on a diagram; developing visualization skills to assist in processing information.
The Ball	Understanding what is going on (interpreting the situation); exploring; predicting; finding invariance and properties; problem solving; making assumptions; using a property in a particular context; developing and apply new mathematical knowledge through problem solving; explaining the mathematical reasoning with own words; generalizing; proving; guessing; pattern-seeking; making connections; 'noticing'; connecting mathematical ideas to everyday experiences, and to other disciplines.	Exploring the situation; 'make experiments' with the tools; trial-and-error techniques; selecting and using appropriately technological tools; interpreting the 'behaviour' of a construction to make inferences and deductions about it; using the tools to test the validity of conjectures; ability to use effectively digital technologies in mathematics to solve unfamiliar problems and make rational conjectures; developing visualization skills to assist in processing information.

Table 3: Mathematical and Technological Competencies

It is important to notice that the same question could be an exploration task for one person, and an exercise or an application of a well known property for another, depending on students' previous knowledge. The competencies that a task aims to evaluate could be different for different classes or students, only a good knowledge of the students and their 'history of learning' can allow a teacher to design a task that evaluates the competencies that he/she wants to test.

In the tasks illustrated above I described the ways in which they follow Sinclair's suggestion in the design of the sketch depending on the aim of the task. Is there something that makes a task better than another one? A good task is composed of a good question and a good design.

I contend that the task on the **counter-example** is a good question because it asks students to think about a property of the circle they are supposed to know, to see if they are able to find an exception, or if the property is always correct. The sketch aims to make students reflect through the tools, but there are not pre-constructed diagrams, students should draw the diagram that represents the property or the diagram that represents the exception to the property. In this context the technology is not very useful, because students could draw a diagram also with paper and pencil, without using the tools of the DGE.

The task with the **dog** makes students reflect on an image of a dog roaming around a house. In order to find the area of the surface where the dog can go without breaking the rope, students need firstly to figure out where the dog is allowed to go, and then to represent that surface with geometrical shapes with known area. Since all the sketches are related to circle geometry, and this task in particular offers three circles as tools to find the area, then it is easy for students to recognize that they need to use circles to represent the area. However, it is not immediately clear where to place the three circles; also, finding the exact number of the area is not a simple request, because students need to add and subtract portions of the circles. Here we have a good questions and a good design, and the aim of the design is helping students exploring the situation with a dynamic representation of the problem.

The task on the **triangle** asks students to think about a particular case of a general property. The question is quite simple, but is formulated in a way that does not imply the direct connection to the property of the central angles and the inscribed angles subtended by the same arc. The DGE has a fundamental role, since the questions prompt students to drag the points on the circle. The task could be seen also as an exploration task for students who do not remember the property - they could drag the vertex on the circle and find the invariance of the right inscribed angle when the other two points are the end points of a diameter.

The task on the **ball** is definitely a good question, and the DGE is the core of the problem: the movement and the tools are the key elements of the sketch. There may be some doubt as to whether it is an assessment question or not. It is an exploration task where students are asked to investigate the situation and explain a certain behaviour of the sketch with a property of the circle. This task evaluates the ‘noticing’, and the problem solving competence, since students need to observe and to make connections. It also evaluates the technological competence of exploring with the tools, and using them to draw the dynamic diagram that represents the hidden construction of the situation.

4.3. Associating task category to assessment purpose

In theory, teachers choose to design a task in a certain way based on the purpose of the assessment and on the competencies they want to evaluate. Firstly, the use of a DGE to assess students in mathematics depends on the mathematical content and on the competencies teachers want to evaluate. A DGE is not always the best way to test student knowledge and understanding, but it offers a wide range of possibilities that teachers do not even think about.

Caron and Steinke (2005) declare that, “technology asks for rethinking assessment in mathematics” (p. 4), because different instruments cause a different learning, as Trouche (2005) states: “the modification of tools is accompanied by significant modifications of the ‘corresponding’ mathematical field” (p. 24). The integration of digital technologies implies a change in the mathematics curriculum, and consequently the content that has to be evaluated:

> The focus of this assessment is to re-evaluate the program goals of the math core curriculum and update these goals to incorporate the ability of the laptop computer to not only explore, experiment, and discover mathematical and scientific concepts in the classroom, but also provide a useful medium to build and store a progressive library of their analytical and communicative abilities. (Heidenberg & Huber, 2006, p. 103)

Laborde (2001) suggests that it is easier for teachers to adapt paper-and-pencil tasks for a DGE, but much more difficult to create novel technological tasks that are different in nature from what one might do with paper-and-pencil. If teachers only design tasks belonging to the first and second category of Laborde, as we just said, this is a sort of paradox, because students acquire different competencies in a DGE from the competencies that they acquire in a paper-and-pencil context, thus it is impossible to test this new kind of competencies with paper-and-pencil alone.

If teachers like designing construction tasks for student assessment, rather than prediction situations for conjecturing and proving, they will prefer tasks of the third category. If they think that assessment tasks should only assess what students know, then they will not choose tasks of the fourth category, but they could use them for classroom activities and informal assessment.

Teachers that choose to use a DGE to assess students’ competencies in mathematics should try to use tasks of the third and fourth category of Laborde, since they deeply influence the nature of the mathematical competencies that are evaluated. These competencies have to be assessed, otherwise students do not give importance to this kind of activities. Since DGEs offer the opportunity to learn and test abilities like finding invariance and 'noticing', exploration or investigation questions should be part of the assessment, because "assessment should measure what is worth learning, not just what is easy to measure" (Steen, 1999, p. 3).

Now that I have described the methodology I used in this study, and the tasks I designed in *Sketchpad*, I analyze the data I collected in the questionnaires and in the interviews.

Chapter 5.

Analyzing the data

I begin by showing how the theories discussed in the theoretical framework are relevant to analyze the data in order to look at the way secondary teachers use digital technologies in student assessment in their practice.

5.1. Teachers writing and talking about technology in student assessment

I list here the first part of the questionnaire:

1. *How and when do you typically assess your students?*
2. *Where are you in this table?*

You as a Teacher	*Technology in Teaching*	*No Technology in Teaching*
Technology in Assessment		
No Technology in Assessment		

3. *What is the role of technology in your teaching and in your assessment? Give an example.*
4. *Do you see any differences in what you learn about what your students know between technology-based and paper-and-pencil assessment tasks? If so, which differences?*
5. *Do you think digital technologies are more useful for assessing and teaching some aspects of learning than others? If so, which aspects?*
6. *Do you think it is valuable to assess only the mathematical competencies, or also the technological competencies (like choosing the appropriate tools, using them in the right way..)?*

7. *What forms of feedback do you give to your students? When do you give feedback to them? What kind of feedback do you give to your students when they are working with technology?*

In the first question, I asked teachers to identify their situation on the use of technology in teaching and in assessment with a yes/no question. This first answer allowed me to divide teachers in four categories, the ones that use technology in their teaching and in student assessment, the ones who use technology in teaching, but not in the assessment, the ones that use technology in assessment, but not in teaching, and the ones that do not use technology either in teaching and in the assessment. Figure 7 represents the answers I received:

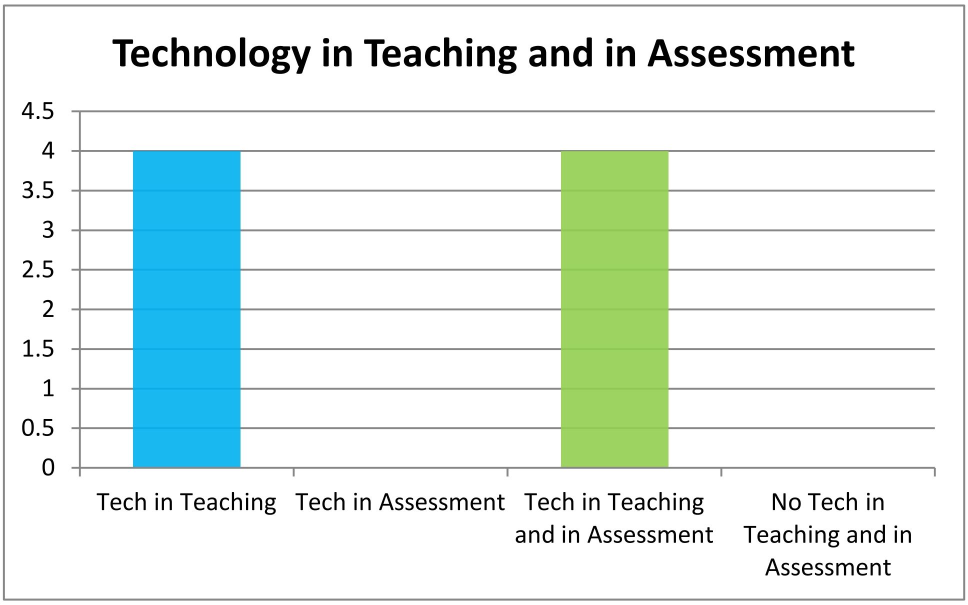


Figure 7: Technology in Teaching and in Assessment

Not surprisingly, none of the teachers reported using technology only for student assessment. On the other hand, it is interesting to see that all the teachers I interviewed use technology in their teaching. It could seem unexpected that half of the teachers (T1, T2, T3, T7) use technology in student assessment, however, I need to clarify that through the questionnaires and the interviews, I found out that the technology they use in student assessment are calculators only.

When I analyzed the questionnaires and the interviews, I used six colors to fit teachers' words in the five-plus-one *Structuring Features of Classroom Practice*:

working environment, *resource system*, *activity structure*, *curriculum script*, *time economy*, *assessment feedback*.

Going back through the data I decided that it would be useful to see what teachers were more concerned about. During the interview, I spoke very little, trying to let the teachers follow their stream of thoughts. I tried to let the conversation following its course. Moreover, both in the questionnaires and in the interviews, teachers answer differently, in the sense that different teachers answered the same question with comments that corresponded to different structuring feature of their practice. This is an example:

4. *Do you see any differences in what you learn about what your students know between technology-based and paper-and-pencil assessment tasks? If so, which differences?*

   I do know that students do not like questions on which they have to use a graphing calculator. The graphing calculator feels like memorized steps and I don't think it is clear to students how the graphing calculator mirrors what they can do algebraically. There is a breakdown between pencil and paper learning tasks and graphing calculator learning tasks. (T6)

   Yes, an app like *ShowMe* allowed me a better understanding of what the students were thinking because it required them to verbalize their thinking out loud as they showed their work. I could more easily identify their misconceptions and therefore could structure my future lessons more specifically to their individual needs. (T4)

At the same question these two teachers answer with two different *structuring features*, the first one is concerned with the gap between technology and paper-and-pencil, while the second one is underlying the fact that digital technology helps her recognize students' misconceptions.

Furthermore, teachers answered with the same *structuring feature* to different questions, for example I got the feature of *Time Economy* in two different question of the questionnaire:

6. *Do you think it is valuable to assess only the mathematical competencies, or also the technological competencies (like choosing the appropriate tools, using them in the right way..)?*

I think we are so busy with the mathematical part, that we don’t have time to think about the technology part. (T5)

3. *What is the role of technology in your teaching and in your assessment? Give an example.*

[…] In assessment there is not enough time to do these activities so usually for a test I will not ask them questions that will specifically involve technology. (T7)

Then, after the analysis of the questionnaires and the interviews, I collected all the words of the same colour, and I came up with the graph in Figure 8, representing the percentage of words for every feature. This graph shows what teachers consider most important to take into account when they talk about digital technologies in the assessment.

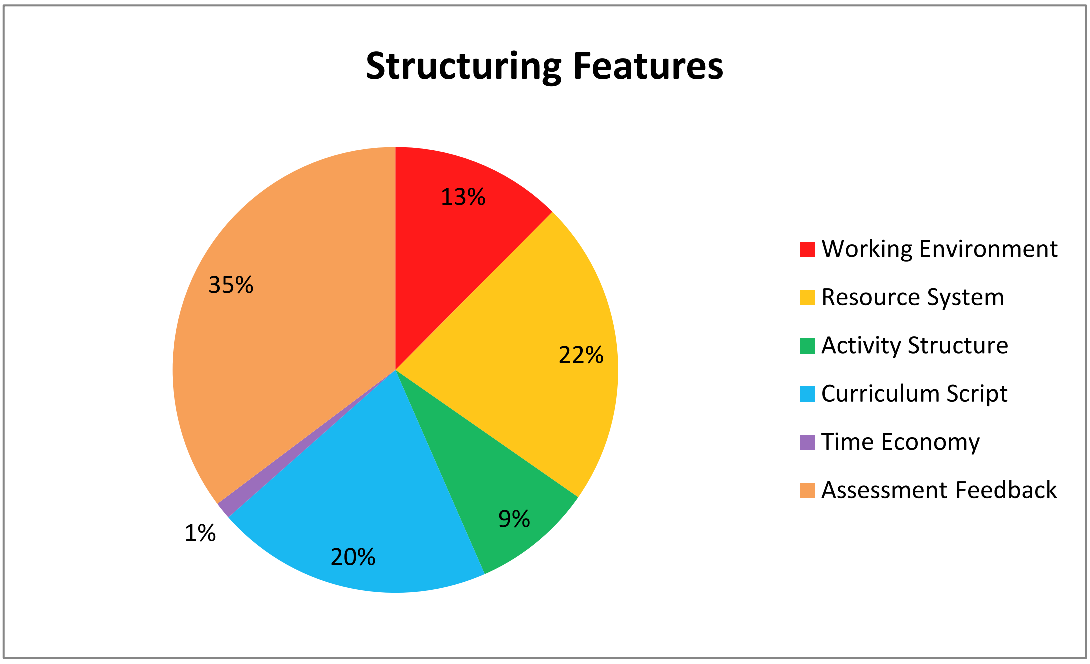


Figure 8: Structuring Features

In the next five tables (from 4 to 9), I organized the data I collected in the questionnaires and in the interviews using the five-plus-one categories of the *Structuring Features* framework. On the right column there are sentences taken from the questionnaire or the transcript of the interview. On the left column I examine teachers’ comments underling the main contents and I group them in sections depending on the topic.

Working Environment	
In terms of **technological tools**, teachers must or prefer to use calculators during the assessment.	T1: On using technology in assessment, I only use calculators and not other forms of technology for assessment. T2: Because the assessment model is fixed for IB (only using graphing calculators) other uses are limited when assessing of learning. T6: I don't use technology in my assessment, except for a few graphing calculator questions that I feel are supposed to be there. T8: None of my students use any technology, other than a scientific calculator.
Teachers use tools, like iPads or iClickers, to collect students' answers, the technology involved does not modify the mathematical task or the solving strategies required to carry out the task. The most recurrent ways of **capturing student answers** in a digital form are: - screenshot - saving file - recording voice or video - collecting multiple choice answers	T4: Had a 1 to 1 iPad classroom - technology was used by the students for both process and product. Technology was used to assess using specific apps like ShowMe or web tools like Today's Meet. […] 'ShowMe' is a free app, so as they are writing they can record their voice, talk and say what they are doing while they are doing it.. it's recording and writing at the same time. 'TodaysMeet' is a web tool, I can show something to the class, I can pose a question to the class, they type in their answer on their iPad, and it projects upon my screen, and everybody sees what everybody answers at the same time. So that is a quick assessment that I use a lot. T5: I have used iClickers in the past, but I haven't been using them much lately. The iClickers were mostly used for review (both individual and group); but, I also used them for quizzes. I liked this, because I was able to discuss questions as soon as the class finished voting on their answer.

Usually, students prefer this kind of tools to record their answer compared to paper-and-pencil, because they like **using words to explain their reasoning** rather than writing their thoughts, especially the youngest ones.	T2: People explain things a lot more expansively when they do than when they write it. T6: Ask them to record what they are doing**.** T4: In a typical classroom, a teacher may not be able to listen to every student explain their thinking/understanding, but recordable technology makes that possible. Students are typically very reluctant to write explanations for their thinking with pen and paper; they would much rather just tell you verbally. […] Kids love being able to explain in words to me what they are doing, they do not write on a piece of a paper, because they say: "Why can't I just tell you? Why do I have to write it on paper?"
Teacher perceive a lot of **barriers** in the *working environment* during the assessment with digital technologies, both in terms of physical surroundings and organization of the materials: • there are difficulties in *providing the tools, and managing them;*	T3: We don't have the *Sketchpad* licenses for the entire school, and we ask for *Sketchpad* in the iPads, but it has not been done yet. T5: But again is having this (referred to the iPad) that is hard to have. I think we are pretty close, even with the phones.
• *organising student access to, and use of equipment* is another issue;	T2: How many schools have all the technology? And often it doesn't work when you want it to work. There are so many barriers to using technology as an assessment tool. If you got a room full of kids using these devices, any given time there are two or three that don't work properly, or the kids have done something that you can't get to. So you have a trouble issue for the technology and you

	don't get the time sometimes to sit back and watch what the kids are doing with it. That can be an issue too.
• it's difficult to *assure fairness* when digital technologies are involved, because students can have the access to other applications, or because they can chat more easily among each other;	T6: I have access to ten iPads at school. […] But an iPad with *Desmos* or a computer with *Desmos*, also come along with emails, messages and again there is only ten iPads right now, so students would have to bring their own device, and I should implement a password to lock them the screen, it's a weird situation, it's also new for me anyways.. T1: Even though I want my students to be able to use tablets in assessments, I find it very difficult to implement it because students can be easily distracted by other apps on the tablet; they could even cheat by accessing the internet while being tested if they had a tablet. I feel I don't have enough resources to control more advanced technology (besides calculators) during assessment. T7: In a computer lab they are sitting right beside each other, so one person can see what the next person is doing.
• also *technical problems* are likely to occur every time technology is involved, at least at the beginning, if students are not experienced in using some specific applications, they would easily get lost in the commands of the tools.	T7: I suppose the few times I have asked them technology on a test, students had a problem with the software and got all upset, I remember a long time ago I had an online word processing test, and a student couldn't use tabs correctly and he had a bit of a breakdown and he couldn't continue the test. Although tabs was a small part of the test, this incident affected the whole exam. T6: Let's do this on a graphic calculator, and it becomes about: "Which button do I press? How do I get there? Which choice do I use?"

Table 4. Working Environment

In terms of physical surroundings where lessons take place, schools are moving to digital technology integrated classrooms. Teachers will not need to take the students to the computer laboratory in the future, because they will be able to use digital technologies in the classroom. Some schools already have iPads that teachers can use, as T1, T3, T4 and T6 assert. Four teachers out of eight affirm to use only graphic calculators in student assessment, while two others state that they use iPads and iClickers to collect students' answers in the assessment. All the teachers teaching at the high school (T1, T2, T3, T5, T6, T7) observe that there are many barriers in the *working environment* when digital technologies are involved, especially in terms of assessment, like the small number of available technical tools, the associated organisation of people, and the technical problems that are likely to occur when digital technology is involved. These alterations of the environment require modification of the assessment practice: students need to know how the tools work, and to be very comfortable in using them in mathematics. Moreover, the introductions of digital tools in the environment could be a source of student distraction, and during the assessment it could become difficult to assure fairness.

Teachers like to collect students' solutions using different modalities (screenshots, files, recording voices or videos), because it allows them to see students' different aspects of learning and understanding, like T1 and T4 observe. However, since it seems to be difficult to assess students using digital technologies, teachers usually prefer to do **informal assessment** when digital technologies are involved. I will analyze this kind of assessment in the *activity structure* section.

Resource System	
In terms of **didactical tools**, the most frequent digital technologies teachers use in student informal assessment are the DGEs (*Sketchapad, Cabri, Geogebra, Desmos*..)	T2: I use *Sketchpad* for problem solving. T3: I use *Sketchpad*, but I don't use it enough. T6: *Desmos* is beautiful, because the axes are very clear, different input come up in different colors, and then you can just tap on the intersection points, and it shows up. T7: I also use *GSP* a lot. We have made

	tessellations, created some dynamic sketches manifesting circle properties. T8: I used both *Sketchpad* and *Desmos*.
The **reasons** for using this kind of tools in informal assessment are mainly that: From the students' point of view:	
• the **visual power** of technology helps students connect algebraic with graphical and geometrical representations of concepts that are dynamic in nature, or that are difficult to be represented;	T1: Without using technology to represent a concept dynamically, most students will choose to learn the concept by memorising a set of procedures algebraically as opposed to try to understand the concept and why it works. T5: I also use technology to show relationships that may be difficult to learn, such as graphing, and 3-d shapes. T8: For example, it was easy for them to see that $\cos(x^2)$ is different than $(\cos x)^2$. T2: Learning by seeing what they can do while they explore I think it shows a lot about what they know, so I think it is valuable.
• also the **motion** has a deep impact to students' understanding, and this is a huge difference from paper-and-pencil;	T8: They are not only fascinated by seeing math, they are also interested in its movement.
• students usually are really motivated in using technology, since it offers different stimulus, and raises their **curiosity**; thus, they can do deeper exploration and spend much time, because they are engaged in the activity;	T6: I think technology is great for catching students' attention and getting them to wonder why something is happening.
• students are also prompted in doing **more tries**, until they get to the answer;	T4: They are not willing to pursue on paper in the same way they are with technology. That's what I found in my experience, that they use to **try, try and try again**. […] Students like *Sketchpad* because they don't have to redraw everything, they can just take one thing

	off and try it again without having to start way back to the beginning and erase everything. That is the affordance of this kind of technology, they students are much more able to try and retry and retry, with paper and pencil the first time I ask to erase everything, they don't want to do it again, but with technology, they will keep trying until they get it. T8: Technology allows you to make mistakes that you can **quickly delete and then try again**.
• technology helps students express and **communicate** their ideas;	T4: Technology which allows for recording orally or visually is far more useful for assessing tasks where you want to understand what the student is thinking - any task that requires them to explain their thinking. […] That's what the *ShowMe* app does, they can show me and tell me at the same time, and that just changes everything.
• digital technologies also foster **creativity**; • some actions become more meaningful, such as **constructing** a geometrical object; • technology **gives insights** to a problem, and promote contemplation;	T7: I do think tech magnifies some aspects of learning. **Creativity**, final product can be more meaningful, that is instead of just performing an algorithm to show ability, **creating a mathematical object** so that it remains on the screen allows contemplation of that object. This contemplation on paper is rare. Even in a regression line on a grid, the window settings can be changed to better view the line, the line can be thicker, etc, you can "Play" with the object.
• students looking for information on the web to **solve real-life context problems**;	T3: When students are allowed to search online for whatever they need to incorporate into their assignments, they tend to go above and beyond. When students are allowed to use whatever technology they want, they tend to do pretty good too.
• digital technologies foster **generalization**, since students have	T6: Technology can better demonstrate generalizability (for example students can

the possibility to try different dimensions and positions of the objects involved; From the teachers' point of view: • teachers can see student **reasoning through their actions** and their way of exploring and dragging objects in a DGE; • technology helps teachers recognize students' **misconceptions**.	see that a rule applies for all circles because they can change the size of a circle in sketchpad and see that the rule always holds). T3: I think as long as it allows students to show their thinking process, tech can help teachers recognize students' misconceptions. T4: An app like *ShowMe* allowed me a better understanding of what the students were thinking because it required them to verbalize their thinking out loud as they showed their work. I could more easily identify their misconceptions and therefore could structure my future lessons more specifically to their individual needs.
Beside recording students' answers, as illustrated in the previous category, **teachers use digital technologies** in informal assessment during activities in class mainly **for**: - exploring - visualizing - checking solutions - modeling - analyzing data - measuring - discovering concepts	T6: I have done a few lessons where students use *Desmos* on the iPads to explore functions. T1: Technology is a very useful tool for exploring, visualising mathematics as well as checking solutions. T8: Also, individual students were able to see if they were right or wrong by the graphs that they created. T6: They are able to check in *Desmos* and use *Desmos* as a tool for making things all right. T2: I use technology such as graphing calculators; GSP (especially in calculus but also in other areas where I want a dynamic approach to help visualize); and data logging (as a way to generate data to look for patterns etc). T2: Students use tools to measure, model

	and statistically analyze data as part of their assessment. T6: What I love technology is able to do is that it can reverse, so typically in paper-and-pencil they give name to something, "reflection means…, now let's reflect all of these points". This is how paper-and-pencil goes, while on technology you can black box it, so "Here's a point, here's another point, and you move one, and the other one moves, what's happening?" Then, "Oh we have a name for that, it's called reflection". I think technology is really powerful for that, and having students explore before giving a name is meaningful.
Sometimes digital technology can become an **obstacle or a time impediment** for teachers, because it can create confusion. Some examples are:	
• teachers feel some digital technologies are *missing functions* that they consider indispensable for a better understanding;	T1: I find that there is a problem when you are estimating with the technology. It is the same problem as the TI (Texas Instruments) calculators: when you want to find the maximum of a function with TI calculators, you have to press many buttons and it is not precise at all, while if you use other technologies, when you take a point on a function and you drag it near the maximum, it snaps on the maximum.
• in some situations the digital tool provides too much *activity feedback*, in a way that students can try to guess the answer or to solve the problem without thinking;	T2: There are some where you can get some check in there. You don't want the kids just to guess around until they get a value that seems right. [...] You can see where things come out, and at the end you can get an answer. Maybe they will take away from that. If you took in the assessment, it may not be exact enough.
• teachers observe digital technologies *prompt students in using the tools*,	T3: With the technology they need tools to answer the question.

even if they do not need them.	
Teachers often have to manage **double instrumentation** in which old technologies remain in use alongside new ones during the assessment: • teachers usually ask students to report the answer of a problem on a piece of paper, or to record their voice to explain their reasoning;	T1: For informal assessments, students use iPads to record videos or write reflections about what they learned after a lesson. T7: I would request that they write the steps in text so I could see what they did.
• lower grades usually have some tools in mathematics that they can manipulate with their hands, while digitals technologies offer virtual objects to be manipulated.	T4: Students are really comfortable in using the iPads. There were some apps I used for virtual manipulatives, but I had actual manipulatives in my classroom as well, so that was up to the kids to choose what they wanted to use, so in grade 4 someone would still go to the actual hands on manipulatives, not the ones they would use on the screen, that was their choice all the time.
Teachers feel the need to know what advantages they would have in assessing students with digital technologies. There is a strong claim for **ready-to-use resources** that can firstly explain all the benefits of using digital technologies in student assessment, and secondly help teachers design suitable tasks to evaluate student learning.	T1: I think I want to know more of just the ideas around this, I would like to hear more about how assessment and technology can be tied together, and what it does look like. Because it's pretty new to me actually. So just kind of hearing from people that do it and trying to make myself thinking about this idea, not trying to avoid it or something.

Table 5. Resource System

All of the teachers prefer to use Dynamic Environments as didactical tools for informal assessment activities, like *Sketchapad, Cabri, Geogebra,* and *Desmos*. This kind of digital technology allows teachers to represent concepts dynamically, as T1 states; and, as T6 adds, generalization of a mathematical concept is more likely to occur, since students do not have a static image, but a dynamic drawing that they can drag and stretch. The visualization of a DGE gives the possibility to show relationships among mathematical objects, like T5 and T8 observe. While the motion offered by DGEs foster curiosity, creativity, and a deeper reflection in exploration tasks, as T2, T6, T7, and T8 affirm. Moreover, T4 and T8 perceive that students are more willing to do more tries in a DGE, because they are usually engaged in the activity, and they can quickly delete and try again. Some routine mathematical exercises become more meaningful, such as constructing an object, and some real-life problems could be tackled, like T7 and T3 observe. What is more, teachers can infer students' thinking through their actions in a DGE, and they could even be able to see some hidden misconceptions, like T3 and T4 state.

However, the resource system is not only composed of the affordances of the didactical tools, In order to be coherent, it includes also *appropriate techniques and norms* for the use of new tools towards classroom activities and curricular goals. These norms depend on the **role teachers decide to give to digital technologies**, which deeply influence assessment and students' performance. Teachers T2, T6, and T1 state that the most recurrent use of a DGE for student informal assessment is as a tool for exploration; secondly, digital technology is used to check solutions, and to analyze data.

Teachers like T1, T2, and T3, point out that there are still some problems with the use of digital technologies in student assessment, which could make it difficult to **coordinate the use and interpretation of tools for the students**. For example, T1 observes that some digital technologies are not very precise, and this fact can create confusion to the students. While T2 states that the *activity feedback* of digital technology encourages students in trying to do something without thinking too much, and they can also casually obtain the correct answer. At the same way, students are prompted in using the tools of a DGE to obtain the answer, like T3 notices.

Another issue is **coordinating the use of digital technologies and previous tools**. At the beginning teachers look for some techniques that would afford a better transition between the different materials, for example still using the paper to record students answers or reasoning, as T7 does. Teachers like T1 and T7 sustain that they would be much more willing to use technology if ready-to-use resources were available, not only lesson plans and exercises, but also assessment tasks. Moreover they state that they would like to have pre-made sketches with an explanation of the benefits of the tasks and the situations when they should be used, in order to make effective use of them and to integrate them successfully in their practice.

Activity Structure	
The most recurrent situation in the teachers' use of digital technologies in student assessment is the informal assessment, where students are asked to undertake an activity of exploring or problem solving. The activity templates are usually organised around predict-test-explain sequences to take advantage of the availability of the rapid ***activity feedback*** of the technology.	T2: Technology is useful when assessing as learning or for learning as it gives the student good feedback in another modality and allows the teacher to set up situations where deeper understanding is examined. An example is in the use of technology to give instant feedback on a student's response to a question, or generate real-time graphs of functions. T6: Students can test conjectures and problem pose and adapt their thinking as thoughts come to them with the quick feedback that technology can provide. T8: Technology allows for investigation and immediate feedback.
Here ***student feedback*** is a fundamental part of the activity, because teachers help students be aware of their level of understanding, how they can improve it, and go on in the activity.	T6: My students receive a lot of verbal feedback from me. Mostly this verbal feedback is in the form of questions to get them thinking about another aspect of the task or to head in a different direction for the task. With each student (or group of students) I make choices about how much feedback to give them while they are working in class. Some groups like to use feedback as a crutch and I try to avoid being a crutch for those students. T4: My feedback is ongoing and verbal during a lesson whether it's technology based or not. T8: For example, through conversation while we write on paper or on a whiteboard, I continually run a feedback loop and often refine what they are saying

	or sketch what I think they are saying, then they reply and I assess anew. T8: It was easy to assess what they understood because we were all working and talking together.
For informal assessment teachers establish **new structures of interaction** involving the students, the teacher and the tools, and the appropriate (re)specifications of their roles:	
• **Students** usually work in pairs or small groups, with a computer/tablet. This interaction increases their ability to communicate between them using technology. They are asked to explore/solve problems using digital technologies, and then they have to explain their reasoning recording their voice or writing on a piece of paper.	T1: My students are enthusiastic about working on the informal assessments such as making a video and reflecting on iPads. These tech-based activities provide more opportunities for students to talk about mathematic compared to paper-and-pencil assessment tasks. I find that they become stronger in talking about the concepts and what they are doing mathematically as a result of working with tech-based assessments.
• The **teacher** walks around and ask students for clarification about what they are doing. Sometimes teachers give hints to prompt the students in the activity, or they try to challenge them asking something new. Teachers prefer to make students talk, because they think students tend to explain what they are doing more expansively when they can describe their actions verbally.	T2: Usually as I walk around, and someone is doing something, I get him to explain it verbally, because I'm not sure he was get the same answer. So usually if I do that type of things I'll get them to tell me what they are doing, and listen to their response in order to get a much better sense. T7: I walk around and talk to them, I ask them to show me how they got to where they are at, to show me how something works. I rarely will tell them how to do something, I may give them hints. Or if they appear to have finished I will challenge them with other aspects of the problem or software. T5: I don't like to answer questions too often, because I rather them figure it out on their own. I find this more valuable to

	them.
• There could be different **tools** in the classroom. Teachers and students should discuss on which tool is better for each kind of activity.	T4: As part of implementing a 1 to 1 iPad classroom, it was vital that I taught the students when to use technology, when an actual, physical manipulative would be more useful, or when paper and pencil would be effective. I didn't assess their choice of tool but it was a natural part of a student/teacher discussion during a task. Sometimes when I would notice that they had chosen a tool that was hindering their learning, I would redirect them to something else.
When tools are involved, the activity includes a whole **communication on the technology itself**. Students need to learn how to use digital technologies in order to carry out the task.	T6: Most of the feedback from working with technology (as this is newer in my classroom) has been about troubleshooting the technology. With graphing calculators it's often about order of buttons to press. With *Desmos* it's about finding the location of functions they want to graph. I also find I have to spend time helping students make sense of what they are viewing on screen and facilitating the connection to pencil and paper tasks.

Table 6. Activity Structure

Teachers are pretty new to the idea of using digital technologies in assessment, and they prefer not to grade students' performance with these new tools. Thus they choose to use digital technology in informal assessment. The roles of teachers and students change in this kind of activity. Students usually work in pairs or small groups, and they interact with the digital technology involved in the activity in order to carry out a specific task. The *activity feedback* of digital technologies promotes problem solving and exploration tasks. The teacher walks around providing *student feedback* in order to foster students in the activity, and to make students improve their work. Teachers ask questions to the students in order to make them explain what they are doing. Also *feedback from peers* is important, since it could foster student learning through the explanation of their ideas, and improve their communication skill. Moreover, digital technologies bring to the classroom new routines and norms on the use of the tools in mathematics.

Curriculum Script	
The **content of the assessment**. What kinds of competencies do teachers consider valuable to evaluate? If teachers use digital technologies in their assessment, then the **technological and the mathematical competencies** are correlated.	
• Some teachers think that these two competencies could be **assessed separately**.	T8: If we're assessing math, then we should be assessing the mathematical fluency and understanding through, or with the help of, technology. If we are assessing technological fluency, then we assess that.
• While some others think that you **can't distinguish them**.	T7: I think you can't distinguish the mathematical competencies from the technological competencies, the mathematics and the tool are the same. But if I accept they are distinct competencies, I suppose I would value them both the same.
Teachers do not want the 'mathematical part' being damaged from the students' incapability or inefficiency with digital technology, mainly because they think that the mathematics is more important than the technology. Since teachers are worried about testing students' competence on using technology, instead of their mathematical learning, they may even refuse to **grade the technological competencies**, even if that competencies are indispensable to carry out the mathematical task.	T3: If I'm using technology for assessment purposes, I am, in a sense, testing students' ability to use the technology, and **I'm not sure if I am completely comfortable with that**. T6: **I think it is important to assess the technological competencies** and provide feedback to students on their achievement of these competencies. […] However, **I don't think I would directly grade the technological competencies**. Especially as the students work to learn a new software or tool. If I saw a student was struggling with the technology but not the 'math' I would find alternative ways for them to show me that they know and get the grade.
The first problem is that digital technology should be an acquired element of the	T2: You can't just give someone a new situation, they need to have a culture of

class, **students need to be very comfortable in using the tools**, before they can use them successfully in mathematics.	playing around with these things, and if they have that then they can work quite comfortably. T3: I guess the idea is that **they need to be familiar with the technology**? And when they are forced to use something they are not familiar with, they need to focus both on the technology and the assignment/assessment, then it becomes difficult. […] I would say if it is a continuous thing, than it can be something valuable. T6: My classes and my students are not yet at a place where the **technology can fall to the background** and the feedback can centre around the mathematics. T8: When you assess it they have to be fluent with the program. That's part of it. Right?
The second issue is that being able to solve mathematical tasks using digital technologies could not be part of the **prescriber learning outcomes of the curriculum.** The use of new tools should support subject activity and curricular goals, but frequently, it's difficult to use digital technologies in order to evaluate the learning outcomes of the regular curriculum.	T2: One of the things there is that I can't think of anywhere in the ministry intended learning outcomes where he would ask something you learn you do with technology that is an intended outcome. It misses using technology do this or show using. But it's not explicit, so if it's not an outcome, it's probably passed on. T6: Graphic calculators are THE technology, there were questions on the provincial exams where students HAVE TO use the graphic calculators.
Digital technologies allow teachers to test some **mathematical competencies** that they would not be able to assess in any other way: • students' **capacity to explore** in a DGE, finding invariance and properties;	 T2: My own preference is to give them open ended questions, so give less things to use as tools, sometimes tools can be a little bit confusing.

• **crating objects with the tools** provided, and test the validity of their construction; • **interpreting some results** from the technology; • student **understanding of changes**, predicting and testing the conjectures;	T7: I suppose when I ask students to do something on GSP I am looking for a **well polished product with the tool** (ie, a dynamic square versus 4 segments fitted together), **creativity** within the tool (did they add some aspects that adds to the image, without me having to ask for it), do they feel comfortable with their creation, on the calculator can they **interpret results,** since the process of regression is black boxed can they then be mathematical about the result. T2: Technology allows you to assess our **understanding of how things are changing**, so usually if they have a good understanding or something, if you say: “what happens if you change this?” And they can go on in their thinking. It gives you the ability to probe what they are doing a little bit more than just the straight full a paper and pencil test, where maybe you think to know what they meant, maybe you can’t tell. While when you can see it dynamically, I think it gives you a much greater visualization of what they are doing, I think.
One of the most discussed issues is the assessment of **problem solving**.	T4: **Problem solving is difficult to assess**, because in class we work on problems together, and then also you can see some of them can't work on the problems by themselves, some kind of answer you would get is not correct.. So what I tented to do is give problems to the all class, and then as a summative give a similar problem, that they can work and try on their own. […] In our curriculum problem solving is not a specific learning outcome, so I don't have to say “they got 92% in problem solving”, so I can kind of skip around that a little bit instead of actually have a grade on problem solving, which is kind of nice..

Problem solving is **hard to teach.**	T5: I did a little bit of problem solving last year, but it wasn't really individualized in the sense of me observing the students and assessing their work. I did something with grade 12 last year, where we decided what is important about problem solving, I did have them complete that, but I didn't do much with it. It showed them that I valued it, it showed them that it was important, but it never really worked its way in any sort of record keeping. And I did it once. T6: You need to have students be really comfortable coming out a problem from different ways, to have students get comfortable not knowing the answer right away, showing their thinking and their work even if they haven't arrived to the answer even if they are not even close to it, but they are still able to logically express their thinking. That's a way you can kind of teach problem solving, sort of, teach the desired skills or competencies.
Teachers think it is difficult having a **problem solving question in a test**, since some students would reach the end of the problem, while some others wouldn't even start it. Also **designing problem solving tasks** is hard, since posing examination problems to allow a greater variety of problem solving strategies is problematic.	T7: With some exploration tasks there is no strategy, it's very difficult to do them, and they are not accessible questions, there is a huge gap between those who get it and those who don't. T6: As soon as you have done an example like the test problem, then it is not a problem solving question anymore. T7: How do you get something similar? That's the thing, the task. How do you get a task that is similar? That's hard.
Teachers need to **choose or devise curricular tasks** that exploit new tools: teachers need to develop links among mathematical ideas, to use appropriate topic-related tasks, and suitable activity formats.	T2: I mixture, some of them I invented over the years, maybe a problem that is already in the textbook, and I get them to do it, I take some from the NCTM website illuminations which has some quite good problems in there

Where do teachers take the tasks from? - Textbooks - Websites - Their mind	T5: If you are lucky you can find some sources on the web. It is quite good. […] There are some contributors that have very nice sketches. T3: Usually I just invent them. T1: Half and half, because there are so many questions in one test, I don't want to create everything. I use something standard it saves me time to take some tasks from the textbooks. Hypothetically, if I have time, I would like to create all of the questions.
What is the value teachers give to assessment with technology? How do they choose to **give a grade** to students answers? What are the parameters they use? Is the technology affecting these parameters? Teachers prefer to give more value to students' explanations of their answer and reasoning, instead of the result obtained with digital technologies. Some of them also think that they can't give more value to the problem solving part, because 'weaker students' would be penalized.	T2: Where the student is interacting with the technology directly there is more involvement and so the 'actions' of the student can be seen. This allows me to have a **better idea of the way they are thinking**. Questions can be directly asked in relation to what, and why, they are doing something. In many ways the process is less abstract and so more easily identifiable. T2: I probably **value their explanation** of what they are doing more, because sometimes it can look that what you are doing is right but you have no idea of what you are doing. T3: A third of the assessment will be about the **content**, another third will be more understanding, but it is always related to **staff that we have done in class**, that can be activities or textbook, the last third will be **things that I came up with, and they haven't seen before.** The thinking part will probably be a 10th to a fifth of the mark, because you can't put too much worth on it.

Teachers need to develop new ways of **staging such tasks** and to manage **patterns of student response** (alternative courses of action, and anticipation of potential student difficulties). There was also evidence of certain **technology- supported lines of questioning** becoming invariant elements of teachers' curriculum scripts for the topic.	T1: If a teacher asks students to record a video talking about what they learn, or asks them to represent a certain problem geometrically on iPad, the teacher will then be able to find out what misconceptions the student has. If a teacher is interested in student's thinking about a problem, she can find it out by asking questions such as "how do you know?" or "Can you justify?" That way, the teacher can see what misconceptions the student has.
Teachers have to recognise and respond to ways in which **technologies may help/hinder** specific processes and objectives involved in learning/assessing a topic:	T8: It takes time for them to understand what they are looking at whereas I thought it would be easy because they are fluent on technology.
• Students can **follow the instructions** of a task without knowing what they are doing; making them explaining what they are doing is again the best way to check their understanding.	T2: One of the issues I think with technology is that the students can go through this, **almost like a recipe**, and at the end they have done everything, but they didn't know what they've done. So I think that it's important keep going around and ask to show you, and ask what are you getting from this. I think it shows their understanding a lot more. Because they are generating their understanding as they are explaining, so I think that the process of talking to them I think really helps them to understand what they are doing in order to get some sense of what their understanding is in the conversation you are having.
• Students could also **memorize the steps** they have to follow in order to carry out a task involving certain technology.	T6: Getting them to make all these connections let's make them memorize which button they have to press, so I really found there is a total disconnection between paper and graphic calculator.
• There could be a deep **gap between technology and paper-and-pencil**. Students often struggle in seeing the connection between these two	T6: I do know that students do not like questions on which they have to use a graphing calculator. The graphing calculator feels like memorized steps and I

context. The teacher should guide them in this transition, and it could be a difficult translation to explain. It could happen that students understand a concept in a DGE, and then they are not able to transfer the concept to a paper-and-pencil context.	don't think it is clear to students how the graphing calculator mirrors what they can do algebraically. There is a breakdown between pencil and paper learning tasks and graphing calculator learning tasks. […] It came up in my classroom, they verbally expressed that they struggle with them, that they feel that they understand in *Desmos*, but they don't understand on paper.
• If digital technologies are not used correctly, they could **'hide' the concept** to the students, just giving them the solution to a task, or being a way to bypass the problem. Thus, the teacher could choose to use the paper-and-pencil before, and then go to the technology, or use it to build the concept in a different way, like predicting the graph of a function and then typing it into a tool.	T6: I did something reciprocal function, I went back to the curriculum, and it says: "to be able to graph 1/x with or without technology". My students can tap it into a calculator or into *Desmos* and have a graph come up, but they have no clue of what it meant. You say you want us to use technology, but if it isn't used correctly, well students would just graph it, and they actually have no understanding of the function. So I'm not going to go to technology first, because I want to be sure that they have an understanding of the function, and then I am going to do something with *Sketchpad*, and helping them having an understanding of the function. So even though the curriculum says to use technology, a teacher may use technology and find out that it is not helping students, so why should I use it?
• Sometimes students can **guess and check** the answer to a question; and if they do not explain their reasoning teachers can't know if they have really understood the concept. As a consequence, teachers can have some difficulties in evaluating students' answers.	T3: In this case I think they get it but I'm not sure. It is difficult to say if they got it with guess and check.
• Teachers think that in a technology context students' would be prompted in **using all the tools** they have.	T3: They want to use every single thing. How often does it happen that you give everything and they don't need to use anything? I saw a lot of them pressing everything.

• Although students today are fluent on technology, they could have some difficulties in **interpreting the mathematics** of the tools.	T8: It takes time for them to **understand what they are looking** at whereas I thought it would be easy because they are fluent on technology.

Table 7. Curriculum Script

With the introduction of digital technologies, the goals, the resources, and also the expectancies for assessing a curricular topic deeply change. Starting from the content of the assessment, teachers could decide to evaluate only the mathematical competencies, or also the technological ones, if it is possible to distinguish them. Since teachers think digital technologies should help students, not be an impediment, they have difficulties in seeing technology as a graded component of the assessment. However, students may not care to learn how to properly use technology in mathematics if they are not going to be graded on that competence.

The integration of new tools in the teaching and learning asks for a rethinking of assessment, nevertheless, and perhaps a new articulation with the prescribed learning outcomes of the curriculum. Teachers have to create new tasks for assessment with digital technologies, and they often feel unprepared. Teachers need resources to create or modify the tasks and for understanding which competencies every task evaluates, and how they should grade their students.

Moreover, many problems are likely to occur when digital technologies are involved in the assessment, thus teachers need to be ready to respond to the difficulties that they and their students could face.

Time Economy	
At the beginning, teachers need a lot of time to have their students comfortable in using digital technologies in mathematics. Designing assessment tasks with digital technologies takes a big amount of time too. Then, teachers have to spend time also in capturing, analyzing and correcting students' answers.	T5: You just need time to sit, time and will to. T2: You have to have time every week to do these kind of things, so that the technology itself doesn't become the barrier.

Then, when teachers are able to efficiently manage modes of use of tools in the assessment, they could reduce the 'time cost', and increase the 'rate of return' in terms of student learning.	T2: The problem with using the technology in assessment is the time that takes to do it. T3: I like it, but the question is a problem, creating the task is a problem, assessing them is another problem. I think it is time to move forward. T8: It takes time to mark that, you don't have a yes/no question, you have to go through their thinking.

Table 8. Time Economy

Teachers admit that using digital technologies in assessment requires a lot of time to prepare the tasks, to assess the students, and to correct the answers. Both teachers and students need to develop new knowledge in order to effectively use digital technologies in mathematics, and this takes a lot of time too. Improving the working environment, the resource system, the activity structure and the curriculum script can optimise the didactic return on time investment. However, if teachers are not really convinced in the potential of digital technologies in mathematics, they would not spend precious time in learning how to use digital technology, and in teaching their students the role of the tools.

Further, while some teachers find that there may be some benefits to using digital technologies in terms of helping students learn more quickly, the use of digital technologies in assessment is unlikely to be more economical.

Assessment feedback	
Types of assessment teachers use to evaluate their students' competencies, and how often they assess them.	
• **Informal assessment**: this is more about teachers asking questions at the beginning of the lesson in order to come back to what they did in the previous lessons, or it could be a series of questions teachers ask to foster an activity in class. It could be used in form of peer-checking, or as homework. It helps students keep up with the lessons, and making them aware of their level of competencies at that moment. It also give to the teacher a sense of the understanding of the class. It is never graded, since it is informal.	T6: Informally, I am always assessing my students. I am asking questions and watching their work to determine the pace of a lesson or unit and discover what topics need review. I am also asking questions to help students recognize what they know and don't know. […] Usually, once per class, I ask students to complete a question from a previous lesson. This may be in the form of a homework question most students didn't complete or I may put a question on the board and ask students to complete it on a separate piece of paper. Sometimes I photocopy a practice quiz that is similar in format to a quiz (for marks) that is coming up. In all of these cases, students are responsible for marking their own work. **These types of assessments are meant to help students test themselves and their knowledge.** These assessments give me a sense of how the class as a whole is getting along with a particular topic. I can also use these to watch for particular students that I know have had previous struggles. I do not have a method for recording any data from these assessments and they never count for grades.
• **Formal assessment**: it is something formally recognized as questions that the teacher asks to the students to know their competencies. Teachers grade students on the level of competencies acquired.	T5: I assess my students formally at least once every three classes. This is usually in the form of a partner's quiz. Once every couple of months, my students write an individual unit test, and then a final exam in June. I check homework for completion 4 times a term (there are 3 terms in a year). T8: I'm not using technology in formal

	assessment.
• **Formative assessment**: teachers conceive this kind of assessment as asking questions, usually on something students do not know or they are working on, and giving feedback. Then they could decide to grade students' answers or not, even if most of the teachers says formative assessment is not graded. Teachers usually use the term *formative assessment* to indicate the comments and prompts they provide to the students during an activity in class, for example when they are solving a task. Students use these suggestions to improve their work and modifying their understanding.	T7: Just when they are working on Geometer's Sketchpad and I'm walking around them asking questions, looking at the screen and talking to them, that can be formative assessment. T4: To me formative assessment **is never giving a grade**, it is just: "you have done this right, you need to think about this a little bit more, perhaps go check this", but without a grade. Once there is a grade on it, it seems to them that they can't do it anymore, so they won't go back and re-do the things that they need to re-do. So it is more just like: "oh have you thought about this?", and then they go back and work some more and then they bring it back to me. T6: So I've assessment, and I give it back to them with a lot of comments, and I say you are on this scale in terms of competency and I record that, even though it doesn't end up in a number at the end, students know that I'm watching, and I'm recording, and I'm thinking that it is valuable, and they work to improve on that, even though it is not part of the grade. So I think you can assess without giving a grade or making it count. Because the number one question is if it is going to count.
• **Summative assessment**: teachers see this kind of assessment as asking questions on something students should know, and giving a grade on their knowledge or abilities. Usually this kind of assessment is composed of quizzes and tests.	T6: The **graded assessments** in my classes are typically quizzes and tests. There are usually two quizzes per unit and one unit test. Quizzes are small and usually worth 5 marks and are on one, maybe two topics. I take the quizzes home to mark and students receive them back the next day. […] I think we often use technology in "how can we teach concepts?", and it's not always "how do I assess with technology?", because all of

	your bright technology ideas went in your teaching. There is a split there, and the split exists in my own classroom, and I'm trying to negotiate some way across that split, but I'm not there yet. I want to be there, and I'm thinking about getting there, but I'm not there yet, if that makes sense..
Teachers should give to students some **final feedback** on their performance, in order to make students aware of their 'level of competence', since they need to know what they have done wrong, and how they can improve their work. The main **role of feedback** is providing information to the students on their learning and understanding, and suggestion to keep up with the required level of competence. Also **peer feedback** is used to comment each other work, thus also the students who make the observations can improve their competencies and their communication skill.	T4: Feedback on a product is usually in the form of a short, written comment or through the use of a rubric. If the product is technology based, and uploaded to their digital portfolio, I would post the feedback directly on their site. T6: I also give written feedback on pen and paper assessments that go back to the students. I try to make the feedback clear enough for students to understand what they did not complete well. Every few quizzes I try to take extra time handing them back and talking to the students individually about their quiz. I find this the most helpful feedback because it is coming quite quickly after a low-risk assessment and the conversation ensures that the students are not confused by the written feedback. Talking to each student can easily take half (or more) of the class making it difficult to do often. T4: I also used a lot of peer feedback with students directly commenting on each other's work.

Table 9. Assessment Feedback

I have compared the most discussed kinds of assessment in the literature in teacher's practice when digital technologies are involved: informal and formal assessment, summative and formative assessment. As I stated before, *informal assessment* appears to be the most recurrent form of assessment, mainly because teachers prefer not to grade their students when digital technologies are involved, they 'do not feel ready yet'. *Formal* assessment is still difficult to be considered in a digital technology context, mainly for the reasons described in the previous category. *Formative* assessment could take place with digital technology, because teachers give a lot of feedback during an activity involving tools. Rarely we find digital technologies in *summative assessment*. However, teachers state that they would like digital technologies being part of the assessment, because they see that 'teaching how to use technology in mathematics' and not assessing their students on what they learnt in mathematics using technology does not make sense.

In terms of *student feedback*, teachers give much feedback during *informal* assessment, while the students are completing the task involving the digital technology. However, not all the teachers give detailed feedback on students' product.

5.1.1. Conclusion on teachers' comments

Using the categories of the *Structuring Features of Classroom Practice* helps me find some recurrent patterns among teachers' practice in the assessment involving digital technologies. It is sometimes difficult to obtain direct answers to certain questions, particularly when the responses involve a complex of issues - some of which are not explicitly related to the question. Teachers feel the need to communicate some findings that came out in their classroom, or certain conclusions that they deduced after years of practice at school. Through these five-plus-one features I was able to sift through teachers' answers in order to identify the main problems teachers face when they use technology in assessment, the opportunities they see, and what they need to be able to design suitable tasks to assess students' competencies.

Mainly, teachers use digital technologies for teaching instead of for assessment, as exemplified in this response:

> I teach Pythagorean theorem using sketchpad because I can use different areas of different shapes. I think *Sketchpad* is a nice tool to help them to explore and understand the concept, but I'm still not sure on how it can be used for the assessment. (T3)

When technology is part of the classroom, teachers face many challenges in every category of the *structuring features*. In the *working environment* feature, teachers assert they have difficulties in providing the tools, and managing them; moreover, fairness is difficult to be assured, and technical problems are likely to occur. In the *resource system* section, teachers point out that digital technology could become an obstacle or a time impediment, it could provide too much activity feedback, and students could be prompted in using the tools, even if they do not need them. In the *time economy* category, teachers observe that they need time to work a lot with students on mathematics with digital technologies, then they would need a lot of time to design assessment tasks with digital technologies, and to capture, analyze and correct students' answers. In the *curriculum script* feature, teachers affirm they conceive digital technologies as something that should help students, but they have difficulties in assessing students' ability to properly use the tools in mathematics - particularly when students are not completely comfortable with them. Moreover, teaches cannot give a grade on competencies that are not part of the prescribed learning outcomes of the curriculum; consequently, students would not care to learn that competencies. In the *assessment feedback* section I found out that some teachers, like T2 and T6, conceive *formative assessment* as *informal assessment*: asking questions to the students, and providing *activity feedback* while they are working on a task, without grading their competencies. However, if teachers do not give *student feedback* after an activity of *informal assessment*, as Teacher 7 states: "I'm not giving any kind of feedback to the students at the end of an activity involving technology", then students will not see the value of that activity.

In general, the teachers seemed quite new to the idea of using digital technologies as environments for assessment, as Figure 7 "Technology in Teaching and in Assessment" showed, and T3 states:

> Using iPads for assessments is definitely new to me. I'm open to the idea, but I think there's much work before it will be useful in a classroom.

In the most recurrent situation, students could use calculators and other tools to record their answers or for problem solving, as T2 states:

> I have always used what tools I can which I think helps the students to see the material from a different light, or simply to gather a lot of data (as in probability, modelling, statistics).

As I illustrated in the *activity structure* section, all the teachers of my study use digital technologies during *informal* assessment, rather than during *formal* assessment:

> I will do an assessment which would be no traditional in a sense as a unit test, but it would be an assessment as going around and watch for certain things, and at students doing certain things, like an ongoing assessment, and I may no assess all the class at the same time, so I will tell the students that the thing I'm looking for is their ability to communicate with the tool. And as I'm going around in this particular problem in this particular week it may be only be the chance to talk with them. It tends to be a more informal testing, but it gives me make sense about things they understand. (T2)

Teacher 1 prefers not to require the use of technology to solve a task; she would rather give it as an optional tool that students can use. For example, in a problem solving question students can choose to use the tools, or to solve the problem by paper and pencil, as this teacher explains:

> I've never really thought about that way, in terms of assessment I thought of technology as a generic tool for problem solving, so I would just give them the technology, and then they can use that tool to solve most of the problems anyways. (T1)

This teacher considers technology as a tool that students can use to solve problems or not, it is up to them. She does not want technology to be mandatory, neither a surplus, but an available instrument that students could use to carry out a task, like in the real life. This leads to the view of technology as something that is separate from the mathematics, it is a tool to help only the students who need it:

> I don't want to assess them using technology to solve the problem. I don't want technology to be something that is like an added thing, like: "Can you solve the problem by paper? Ok, now solve it with the technology". I don't want technology to be something like an extra. I think of a problem first, and then I think on how you can do it. (T1)

In this kind of situation, teacher are more comfortable in posing open-ended questions, exploration tasks and problem solving activities, because they think students are not concerned by the test pressure, and because they can work in pairs or small groups:

> I think if I get to the point where I'm assessing students on problem solving, I may not put it on a test, it may be like a project that you have them working in pairs, and each pair has to submit some sort of solution, so that it still is an assessment that values in the classroom, but also there is that ability to find something, and there is not necessarily the time pressure and the anxiety that are associated with the test as well. (T6)

Here we have an explicit mention that problem solving is not assessed. This connects with the idea that technology does not get used in assessment because it is part of the 'competencies' that are not formally assessed in mathematics. We can see the teachers separate 'assessable mathematics' from 'non-assessable mathematics'. Since teachers have to firstly assess the prescribed learning outcomes of the curriculum, they do not have enough time to get their students at the point they are ready to be assessed with problem solving tasks and digital technologies, as T6 observes:

> I feel like I couldn't just start doing problem solving in tests and expect my kids to come along, that needs to be a part of the class culture, and then I could include it into the assessment. I'm not there yet..

In the *BC Curriculum grade 8-9* (Ministry of Education, 2008) there is a section that indicates specific learning outcomes that can be achieved using digital technologies:

> Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems. (p. 19)

It indicates some activities that can be done with calculators and computers. However, it's not explicitly required that students have to know how to use technology to carry out those activities.

As I explained in the *resource system* section, the teachers in this study want to know the advantages they would have in assessing students with digital technologies in order to 'make the step' in their practice:

> If you are comfortable, then you don't want to think outside of your comfort zone. So I think I would like to force myself to think about something like that, and see if I want to pursue that direction. (T1)

I described the affordances teachers see of digital technologies in supporting the assessment of some specific learning goals. Teachers want to see that it would be useful and effective in terms of time and deeper insight of student understanding, before introducing a totally new way of assessing their students. This introduction implies many changes in teachers' practices, and they do not feel supported in this investment:

> I feel comfortable in teaching with technology, but I don't feel very comfortable using technology in assessments. I believe the biggest reason why I don't feel comfortable is that there are not enough ready-made technology for assessment. (T1)

Some of the teachers I interviewed, mostly the PhD students, who are accustomed to reading research articles, demand for some results in the research on assessing students in mathematics in a DGE. They feel the need to know why and how they should use digital technology in assessment, as T1 states: "I think I want to know more of just the ideas around this. I do not think I need the practical part". Other teachers of this research state the need for tasks as examples to be used in the assessment, and some indications on how they should evaluate students' answers.

Teachers need to re-build the curriculum in order to include the mathematical and technological competencies that can be assessed through digital technologies. Moreover, students need to understand how the new environment works, they have to learn how to effectively use the new tools in mathematical activities, and they have to get used to this new kind of assessment with digital technologies. This process requires time and a lot of work, thus T1, T5 and T7 ask for ready-to-use resources for designing tasks and for their actual implementation in assessment:

> Another thing is that you have to have a good task, and how do you get a good task? It's not easy, it's very difficult actually. Also you have to believe in the task, and that takes a process to get the teacher familiar with the task, so that they know what to look for and what results they may get, and I don't have the time always to do that […] Teachers need simplicity, they need something quick, easy… because right now there are some GSP tasks online, but you don't know what they are for, and how they are going to work. (T7)

Teachers use some problems from the textbooks, because they are quick and easy and reliable, but they are not always satisfied with them, as T1 observes:

> I don't want to use the textbooks too much, because sometimes I don't like it, and other times I would read it and then I would interpret. Let me understand the concept first, and then I would design something.

Nowadays, it is also easy to find mathematical tasks on-line, as T5 notices: "I think the problem is designing the tasks. If you are lucky you can find some sources on the web. It is quite good" (T5). The problem is that on-line tasks are not always reliable, and they are not inserted in a context, thus teachers find it difficult to use them:

> Another thing is that you have to have a good task and how do you get a good task? It's not easy, it very difficult actually, also you have to believe in the task and that takes a process to get the teacher familiar with the task, so that they know what to look for and what results they may get, and I don't have the time always to do that (T7).

Digital technology does not have the dependability that print resources have. Ruthven (2009) states that “well designed textbooks normally include sections which develop the techniques required in using calculators and establish some form of mathematical framing for them” (p. 136). However, even the textbooks that contain a digital technology based lesson plan in a DGE, and printed worksheets for classroom activities, do not provide examples of assessment with digital technologies. Using some resources like the categorization of Laborde (2001), and other teachers' experiences, teachers might be able to design appropriate/innovative tasks to assess students on the competencies they develop with digital technologies.

The literature on task design, as many articles of the 22nd ICMI Study, points out that designing the tasks for assessment in a DGE is probably the most complex issue, and T3 admits: “I like it, but the question is a problem, creating the task is a problem, assessing them is another problem. I think it is time to move forward”.

5.2. Teachers designing tasks for student assessment

In the questionnaire (see Appendix B) I asked teachers to design a task to assess students on their learning on one property of the Circle Geometry Curriculum (grade 9). I did not ask them explicitly to design the task in a DGE, but I said that students learnt Circle Geometry using *Sketchpad*, and that “they are accustomed to use *Sketchpad* in both learning and assessment situations”. This choice is due to the fact that I did not want teachers feeling forced to design a task in a DGE, because the ones who were less familiar with *Sketchpad* could answer that they were not able to design such a task. Thus, I just said that Max's students are comfortable in using *Sketchpad* in assessment. However, from the questions of the questionnaire teachers figured out that I was interested in the use of digital technology in the assessment, so they read into this ‘indication’ that I preferred that they designed the task in a DGE, and that it is exactly what I obtained, because all the teachers provide *Sketchpad* as mandatory or optional.

All the teachers of this study created a task, except for one of them (T8), who wrote just her idea of the assessment in a DGE:

> Presently what I would do is use *Jing* and have them answer a more traditional question by showing their work on *Sketchpad* and both hearing and seeing what they do.

I will analyze the tasks created by the teachers in order to find differences and similarities among them. Since the sketch component of their designed tasks are not provided, I do not know if the tasks are following Sinclair's suggestions, thus I'm looking for:

1. Laborde's category it belongs to (role of the technology).
2. The goal of the task, mathematical and technological competencies the task evaluates.
3. Clarity of the text and of the instructions of the task.

Teacher 1

A tunnel is shaped in a semi-circle of diameter 80 metres, two lanes are constructed in each direction (incoming and outgoing). 10 metres are not used at both ends of the lanes because the height will not fit any cars. Assuming that it is safe to drive on the edge of a lane, determine the height restrictions for both lanes in each direction.

Note: If I had being teaching and assessing with technology in my class, I would let my students choose between two types of assessment: paper-and-pencil and *Sketchpad*. In the *Sketchpad* assessment, they have to construct everything from scratch, and they can even use the 'measure' function to solve the problem.

1. The nature of this task changes deeply depending on the students' choice: if they decide to do it with paper-and-pencil, it is a 'classic' task where students have to use the Pythagorean Theorem to solve the problem. While if students choose to solve the task in a DGE, the strategy changes completely: the task belongs to the third category of Laborde (2001), because it is modified when given in a DGE. Students have to firstly draw a sketch of the problem with the tools, and secondly they can use the measure tool of *Sketchpad* in order to find the solution of the problem. In

this way, the DGE modifies the solving strategy of the task due to the use of some of the drawing and measure tools.

2. The goal of the task is being able to solve the problem, with or without the tools the technology offers. It is up to the students to choose to use the property of the chord or the measure tool of *Sketchpad*:

> That's my point, I thought that nowadays you have technology, so I certainly know that when they use technology to measure something, they don't need to use the chord property at all, and I think that is in a way what they're doing in real life anyways. Technology is supposed to help you save time, it is supposed to be a tool that is more efficient that way. So if they can solve the problem also without having to use the chord property, but using the technology, that's what I want. Because in a sense they need to know how to use the technology: they are constructing the objects, and knowing how to measure, knowing how to do it. (T1)

In order to carry out this task students do not need to know any property of the circle, it is sufficient to know the definition of the circle as the set of all points in a plane that are at a given distance from the centre, and what the diameter is. Students need to: connect mathematical ideas to everyday experiences, draw a representation of the situation of the problem (on paper or on *Sketchpad*), measure the height restrictions (with the Pythagorean Theorem or with the 'measure tool' in *Sketchpad*). The difference between the two situations (paper and *Sketchpad*) lies in the drawing, which in a paper context would be a sketch to represent the idea of the problem, while in *Sketchpad*, it would be an accurate representation of the situation, thus in the first case students need to recognize they have to use the Pythagorean Theorem to find the height restrictions, while in the second case students would rely on the appropriate construction of the situation to find the height restrictions with the measure tool.

If the student chooses to use the property of the circle, then the teacher is assessing the ability to apply the Pythagorean Theorem, while if the student chooses to solve the task in *Sketchpad*, then the teacher is assessing the competence of selecting and using technological tools for solving problems. Thus the teacher is going to assess a technological competence or a mathematical one, it seems she values both of them, since the important learning outcome is the ability to solve problems:

> As a teacher if my goal is to teach them how to solve this problem, then they are able to do it. That's good, but then they still need the skill to use *Sketchpad.* I'm testing different

> things for sure. I would like to know whether they can solve the problem or not, and then I'm happy if they use the measure tool. I give them a choice, some people will prefer to do on paper. I try to mirror in a real life you can choose strategies, where some people are more technology, and some people are not. (T1)

This teacher has a clear idea of the role technology might have in student assessment, and also about the aim of the assessment:

> I know that *Sketchpad* changes the nature of the task, but to me if I'm giving them a technology in assessment, that's what I want to be able to assess. […] I mean, for example using Sketchpad without the measure tool can't help them, if the only they can do is to draw diagrams, without the measuring tool, it doesn't change anything. I think there is no point on using technology, unless you give them the option to measure. (T1)

3. The text of the task is quite clear and it uses simple words, it is short but it contains the necessary information to solve the problem.

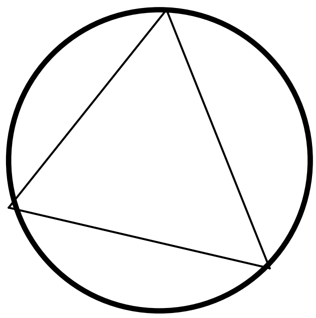
Teacher 2

1. Write down a definition of the following terms
   a. *inscribed*
   b. *Chord*

   Ask your teacher to check your answers before you proceed to part 2.

2. Davina and Mehar are working on a geometry problem when Mehar makes a claim that the angle formed by triangle inscribed in a circle is always a right angle. The following conversation occurs:

D: What do you mean, that can't be?
M: No, really, I remember we learned that before.
D: Any triangle in a circle has a right angle? I don't think so, look I'll draw one..



D: (Cont.) I don't see any right angle there……

How should Mehar respond to Davina's objection? Write down what you think Mehar would say.

Show your teacher your response before you continue…….

3. Davina requires proof that Mehar's new explanation is correct and so Mehar decides to show this using GSP. Set up a new GSP drawing that Mehar could use to prove her claim in this way. Continue the above conversation to indicate what Mehar would tell Davina as she creates the diagram.

1. In this task the paper-and-pencil context is dominant in the first two questions, but the third question requires the use of a DGE, and it belongs to the third category of Laborde (2001), because in this particular example drawing a diagram on a piece of paper to prove Mehar's claim would not be accepted as a proof. In this last question the DGE is used to support a conjecture: students have to draw a diagram using the tools of the DGE, and the diagram has to satisfy specific properties. The tools of the DGE modify the solving strategies of the task, which is rendered more difficult, because it requires more mathematical knowledge which students find difficult to put into action: drawing the diagram correctly, and use it to prove a conjecture.

2. Here the teacher evaluates both the mathematical and technological competencies. This task assesses student knowledge of the definitions of inscribed and chord, and the understanding of a property of the circle: a triangle inscribed in a semicircle is right. Designing the task that includes a conversation between peers help students expressing and communicating their ideas. In this particular example students are asked to support a thesis through a sketch in *Sketchpad*: students need to select and use technological tools in *Sketchpad* them in an appropriate way to prove their thesis. Moreover, the teacher asks to continue the conversation between Mehar and Davina while Mehar is drawing the diagram, in order to have students describing the construction step by step. Although this is supposed to be an assessment situation, the *activity feedback* of the teacher is needed two times to check students' progress before they can proceed in the task.

3. The situation described in the task is clear, even if the text is not very short. The task contains simple words and expressions that the students are familiar with. The author asks for the definition of the term 'inscribed', but this term can be intended as a polygon inscribed in a circle, or as a circle inscribed in a polygon. Since this question is the first one of the task, and the figure with the triangle inscribed in a circle comes later, the author should specify the definition he/she wants. Then the instructions and the necessary information to go on with the task are provided in the text, although students need to identify themselves with the Davina and Mehar in order to continue their conversation.

Teacher 3

Max has a fish. The fish's name is Pepsi. Pepsi lives in a spherical fish bowl. Max wants to put a small volcano in the centre of the fish bowl under the water. Help Max locate the centre of the fish bowl (can be either a sketchpad or pencil and paper assessment).

1. Students have to locate the centre of the circle using the intersection of two perpendicular bisectors of two chords in the circle. The technology in this task is supposed to facilitate the mathematical tasks, because students can use the drawing tools in order to find the centre of the circle, thus this task carried out in a DGE belongs to the second category of Laborde (2001).

2. This task is designed to assess the knowledge and the ability to apply this property: the perpendicular from the centre of a circle to a chord bisects the chord. However, in *Sketchpad* students could use different tools, like the 'circle tool' that draw a circle of any radius, thus students could use this tool to construct a circle and make it overlap with the bowl of the fish, in order to find the center. In this situation the teacher can't evaluate the property of the circle, but instead an ability to use the tools in *Sketchpad*. If the teacher gives a pre-constructed sketch using *Sketchpad Explorer* for the iPad, she could decide not to give the circle tool, so that the students cannot use it to find the centre of the circle.

   This teacher does not want to evaluate the technological competencies, but she thinks *Sketchpad* could be used to facilitate the drawing of the diagram:

   > I like using *Sketchpad* for geometry because it shows the dynamic that shows things moving around, and the drawing is more accurate. (T3)

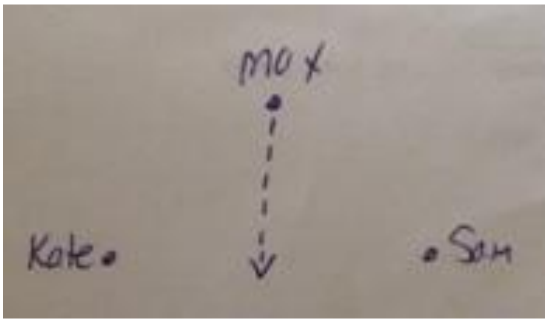
   She states that assessment should be focused on a specific mathematical competence, like the understanding of a property, and the ability to apply it in a particular context: "Assessment has to be more specific assessing a particular thing, to assess them on a particular property".

3. The text of the task is very short and clear: it draws a situation using simple words. The instructions are straightforward: students have to find the centre of the bowl. However, the bowl is spherical, and students need to find the centre of a circle in two dimensions. Thus the author should reformulate the instructions as 'Help Max

locate the centre of the bottom of the fish bowl'. The necessary information are provided in the text.

Teacher 4

Max lives equidistance from his friends Sam and Kate. He insists that if he walks along the dotted path and stops between them, then Sam and Kate would each walk the exact same distance to meet him. Is he right? How do you know?



1. This task asks students to use a property of the circle without actually having a circle. There are three points, and students should connect the situation to their knowledge that in a circle every point has the same distant to the centre of the circle (Max), and that the perpendicular from the centre of a circle to a chord bisects the chord, where the chord is the segment between Kate and Sam. This task should belong to the first category of Laborde (2001), since it could be asked also in a paper-and-pencil context, and if the same task is asked in *Sketchpad*, students could use the affordances of the tools, such the tools for drawing circles and lines. Thus, a DGE would facilitate the material aspects of the task, which does not change conceptually compared to the paper-and-pencil context.

2. The goal of the task is associating a known geometrical figure and its properties to a real situation. This is not an easy question if it is asked in a final test, but if this task is proposed at the end of the circle unit, then students would probably associate more easily the concept of the circle to this example. The tasks evaluates some mathematical competencies, like connecting mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines, developing and apply mathematical knowledge through problem solving. If the task is asked in a DGE students can 'make experiments' with the tools, and try to make conjectures on

pattern-seeking and test them, if they do not see immediately that a circle could be used to answer the question. Students also need to communicate in order to express their understanding, to prove their thinking. In this situation, also the technological competencies could be evaluated, such as dragging to test the construction of the circle with center in Max and radius equal to the distance from Max to Kate or Sam, test the 'stability' of a property as along they are working on a diagram, using the tools to test the validity of conjectures, and to show their knowledge of the property, but also develop visualization skills to assist in making connections and solving problems.

The teacher likes the use of technology as formative assessment:

> You should be tested in the way that you learnt. […] Well formative assessment also to me is really not a kind of a test, it's usually just one quick question, if we were working on an activity in mathematics, and they were working in partner or small groups, at the end of the class I would just pull a child at the side and say "show me how you did this". And that would be a quick form of assessment for me, and it is always verbal feedback. (T4)

It becomes summative assessment when students are ready to show to the teacher what they can do with the technology or on a paper:

> If it is just one quick question, then I'm sitting with them, and I would record that question in my mark book afterwards. So if they are showing me something on "show me" that would become a summative assessment, if they are showing something on a paper, that would become summative assessment. […] When I think they are ready, then I have them work with me and they show me what they know, and then it becomes summative assessment. All until that point is formative, until I'm sure that they have a mastery of the concept. So it's like a all unit test that they would be able to do with technology. (T4)

3. The text of the task is very short, and it contains simple words and expressions the students are familiar with. It is not very clear what the 'dotted path' is in the figure. It seems to be the perpendicular bisector of the segment Kate-Sam (KS), but there is no hint to justify that assumption, like a right angle between the dotted path and the KS line, and the indication that the dotted path passes through the midpoint of the segment KS. Moreover, this task could be solved without circle geometry, using the property of the perpendicular bisector that each of its points is equidistant from the endpoints of the segment. Finally, the instructions are given as questions, which are not very focused: the first one is a yes/no question, but the way the second one is posed does not require a specific answer.

Teacher 5

I would probably have them create a dynamic object that shows one of these properties, and then demonstrate it to me. Or have them work with a partner, and record a screen-cast their object's creation and dynamic properties.

1. Here the teacher does not provide the text of the task. I can only infer that the kind of task he is thinking about belongs to the third category of Laborde (2001). The property is evident in the sketch, students do not need to find it, but they need to demonstrate it, probably using the tools provided in the sketch. In this way, the solving strategy, which here correspond to the way the students choose to demonstrate the property, is deeply different compared to a paper-and-pencil context. Students have a dynamic image to refer, so they can also use a different language and specific words that involve the dynamic nature of the figure. Moreover, they can use the tools to prove their thinking, and to explain their reasoning.

2. The goal of the task should be verifying if students are able to demonstrate in a DGE, rather than see if they know the property of the circle. It seems that the teacher wants to assess the mathematical competence of proving, and the technological competence of using the tools to demonstrate something in a DGE. The teacher think that if students are able to construct a dynamic object in a DGE, which is maintaining the properties through dragging, it means that the students know the properties of the figure, and how they can construct such figure:

   > I think the nice thing about sketchpad is that they have to construct it, so if they did construct it, it shows that they know how to construct it. They cannot easily look at somebody's work and copy the construction. I think if you have a working object in sketchpad, that's working, it shows that they know what they are doing with the construction. A logical connection between the object and the properties isn't necessary with sketchpad, because it shows that they know the properties of the figure. (T5)

   Through the interview I know how the teacher evaluates the answer: he does not give comments on a test, but just a grade, if students need more feedback, they are supposed to ask the teacher for some clarifications in order to understand their mistakes and correct their learning:

   > Regrettably right now it's just a grade.. then they'll come in, and they'll ask extra questions on it and they'll get feedback on it. What I do is to retest, so they have to

understand what they have done wrong, so they usually come and ask me some clarifications. (T5)

3. Since the words of the task are not provided, I do not know if the task is short, clear and simple worded, and if all the instructions and the necessary information are provided in the text.

Teacher 6

I would provide students with a sketch that has one inscribed angle constructed. Students would be expected to construct a congruent inscribed angle.

1. This task changes the meaning and the 'solution strategy' depending on the context: a DGE or a paper. If this task is asked in *Sketchpad*, then the students have three possible strategies, as noticed by the teacher who designed the task:

   (a) Immediately construct an inscribed angle that subtends to the existing arc. Students know the property and are comfortable in sketchpad.

   (b) Measure the length of the arc, construct a second arc with the same length, and then construct an angle subtending to this new arc. Perhaps a less elegant solution, but still demonstrates understanding of the property and sketchpad.

   (c) Measure the angle and then construct a new inscribed angle and adjust it until the new angle has the same measure of the original. This student does not know the property in questions, but is comfortable in sketchpad. I would want to see students complete the assessment for this case. Students could do this and then hide the evidence of using the measure angle feature. A final product does not provide me with enough information.

   This same questions given as a pencil and paper task allow only for answers given in (a). Answers in (b) and (c) rely on *Sketchpad* features and would not be possible.

1. What the teacher found out is that in a DGE students can answer the question in three different ways, and that is quite remarkable, because knowing how to find the solution is great, but knowing additional ways to find the solution is incredibly more useful, and it helps students be flexible with the strategies. If in the future, they will not remember the specific property of the circle, they would know how to solve the same problem in at least another way. That way refers to some technological tools, but it can be generalized, in the sense that once students know when and how to

use the tools, they can solve a wider range of problems. If the teacher thinks that knowing how to use the tool for measuring the angle does not worth to be evaluated, or she wants to see a different kind of competence, then Sketchpad version for the iPad (*Sketchpad Explorer*) can solve this problem. In premade sketches the teacher chooses the tools that the students can use to solve the task. Thus, she could choose not to provide the students the measure tool for the angles. Moreover, on the iPad the option to hide objects is not provided, thus students do not have the possibility to hide their work, thus the teacher can see what strategy they have used to solve the task. This task belongs to the third category of Laborde (2001), because it could be asked also in a paper-and-pencil context, but the solution strategy would be different.

2. The goal of the task depends on how the sketch is implemented. If the teacher wants to verify if students know the property of the circle, then the sketch should not include the measure tool for the angles, but only two segments that students can use as chords should be provided. If the sketch is implemented in a way that allows students to use all the tools provided by the DGE, like the angle measure tool, then the task could evaluate the mathematical competence of knowing the property, the deep understanding of it, and the ability to apply the property to construct a congruent angle on the circle, in this situation students give evidence of their learning of the property of the circle, and the ability of using the tools in a DGE: they just need to draw two chords on the circle in order to build an inscribed angle that subtends to the existing arc. Otherwise, students could use some tools of the DGE to carry out the task, without showing the mathematics that they know (the property of the circle), but only their ability in a DGE. This choice is a students' choice: if students feel more comfortable with the mathematics, then they would use the property of the circle, if they are comfortable in a DGE, they would use the tools provided in order to carry out the task. If the sketch is designed in a way that does not allow the students to use all the tools, but only some tools chose by the teacher, than the task evaluates the mathematical competence of knowing the property of the circle, and the ability to apply it in a particular context.

The teacher explains that if students show their knowledge of the circle property (constructing an inscribed angle that subtends to the existing arc, or measuring the length of the arc, constructing a second arc with the same length, and then constructing an angle subtending to this new arc) she would be satisfied, because students are able to apply the property and are confident in the DGE. However, if students could use the measure angle feature, they would show their ability to use the tools in a DGE, but they could also decide to hide the measure tool, and the teacher wouldn't be able to see their work. If a task allows more 'solution strategies', then it's more difficult to grade it. Every strategy involves a different number of steps, different competencies, and probably also a different order of complexity. Grading is a tuff task, because teachers have to be fair and equal in assessing their students, and it is difficult to create equilibrate grading table for different strategies. The text of the task is not provided explicitly, but it is clear what the teacher is looking for. She states that:

> When considering an assessment situation using Sketchpad I want to be able to see what students are doing, not just the final product. I feel that for the assessment to be an accurate representation of the student's learning it needs to capture the whole construction process. I would want to watch students complete the assessment task or have the students create a screen capture video of their work and submit the video. (T6)

However, this accurate process would require a huge amount of time:

> An important consideration is how long it would take as a teacher to do the assessment. If every student creates a one minute video, that is now 30 minutes to grade one question. (T6)

3. Since the words of the task are not provided, I do not know if the task is short, clear and simple worded. The instructions the teacher wants to give are very simple, she just needs to reword the text, to choose the instruments for the students to solve the task, and to make explicit if the students could/have to use the tools provided in *Sketchpad*.

Teacher 7

#1 I would ask them to draw three random points on the screen. I would then ask them to find the centre of the circle that passes through all three points. I would request that they write the steps in text to the right of the points so I could see what they did.

1. This task belongs to the first category of Laborde (2001), since it can be asked also in a paper-and-pencil context, and it does not change conceptually if the same

questions are asked in a DGE. The solution strategy is the same (if the teacher does not provide the tool for the construction of the circle, otherwise students could construct the circle without using the property of the circle), and the tools in the DGE can facilitate the material aspects of the task, like the construction of the perpendicular bisectors to find the centre of the circle.

> I think that the first task it is easier to construct. They can play around a lot more and they can try different things, because if they don't know the answer, then trying to figure out the answer using Geometer's Sketchpad is much better than using a compass and a tool to draw a line and see if it actually works out. The other thing is that *Geometer's Sketchpad* is not only quicker, but also more accurate, probably.. […] Well.. they can test their answer, once they have drawn the perpendicular bisector and there is a center, they can start dragging the points and notice that there is a consistency. But that's after they have answered, that could be part of it. In reflection, and in evaluation of their answer they can look at the dragging the points, and that would be very helpful. Because that instantly helps to see as opposed to the paper, they are not quite sure.. (T7)

2. The goal of the task is finding if the students are able to apply the property in order to find the centre of the circle. The task requires a further step, because students needs to draw the chords connecting two couple of points before drawing the two perpendicular bisectors.

> They should already know the property, that's why the first question is a good one, because I wouldn't have talk them exactly that, but I would have talk them that if they have a circle and two chords, then the perpendicular bisector pass through the centre. But that is different, because now you have three points and they have to figure out how to connect the chords, and then they have an opportunity to choose the chords, there are three different chords that they can draw.

The focus of the task is evaluating the ability of problem solving, a deep knowledge of some elements (perpendicular bisectors, circle, chords), students need to have a deep understanding of the property in order to apply it to solve a particular problem. Some technological competencies the task evaluates are choosing the right tools, making a draw with the tools, and appropriate use of the instruments.

In this task the teacher wants to assess students' ability to apply a known property of the circle. The teacher wants his students write the steps of their answer, so that he would be able to see their reasoning along the task. He evaluates students' answer making some comments and drawing attention to the errors:

> Sometimes on a summative assessment I will write comments, not very in depth though, usually just to point out certain things that they have done wrong. (T7)

3. Since the words of the task are not provided, I do not know if the task is short, clear and simple worded. The instructions the teacher wants to give are straightforward, he just needs to reword the text, and to choose which instruments providing to the students to solve the task in *Sketchpad*.

#2 Now on one side of the chord, choose 3 random points on the circle. Connect these three points to the endpoints of the chord. Now hide the chord. What do you notice about the three angles just created and the remaining arc?

1. This task belongs to the second category of Laborde (2001), since it can be asked also in a paper-and-pencil context, but the DGE is used as a visual amplifier, since the difference could be that in a DGE the students have the possibility to move the points on the circle and observe the inscribed angle that is not changing. Thus the DGE facilitates the mathematical task.

2. The focus of the task is figuring out the property, because “if they already know the property than it is a stupid question” (T7). However, it is difficult to ask if the students are ‘noticing’ something about the three angles just created and the remaining arc, because as the author of the task states: “This is bad, basically I’m telling them that the three angles are equal”.

   The teacher knows that evaluating the ability to find properties or invariants in a diagram is difficult. He wants to see that the students have insights into the problem:

   > If I ask them to identify some certain properties, maybe they find something I didn't think about, you know? to get full marks, it has to be correct. I wouldn't give them 0, because it's correct. […] It's hard to evaluate problem solving questions in a test, but I would look for specific sort of insights, intelligent steps based on the material we have already learned.. If my question is vague enough, such that they find something that satisfied the question, then I would give them full mark, I have to. (T7)

3. The task is short, clear and the instructions are straightforward. The text is composed of simple words and expressions that the students are familiar with. The question “What do you notice?” is not very precise, since the teacher is looking for the invariance of the angles subtended by the same arc, but students could also ‘notice’ something that the teacher does not expect, since the questions is not

specific. The teacher also needs to choose which instruments providing to the students to solve the task in *Sketchpad*.

#3 Draw an inscribed quadrilateral in a circle. Draw the diagonals of the quadrilateral. Drag the vertices of the quadrilateral until it is a square, what do you notice? (I like this one!!)

1. This task belongs to the third category of Laborde (2001), since the DGE is modifying the solving strategies of the task: students need to drag the vertices of the quadrilateral until it is a square. In a paper-and-pencil context, the students "can't answer what you notice" (author of the task). The task requires more mathematical knowledge, because the students need the ability to "notice", in other words, the students need to develop conjectures based on observations and test the conjectures. This is seen as a key formulation activity in a DGE, and asks "the students to work inductively rather than deductively" (Joubert 2013, p. 73), because the need to figure out that when the quadrilateral is a square, then the diagonals are two diameters:

   > I think the nice thing about the last one is that they don't know what property I'm looking for, so there are a lot of things they can focus on.. it makes them attend to features or properties I suppose.. they attend to that, there is no a 90 degree angle drawn, but if they make a square then they can see that the diagonals are a perfect cross, they are 90 degrees in the center, and they are themselves a 90 degree. So they can then see, possibly, that half a square is a 90 degree angle. It makes them attend to features, I guess, of a square, and then they can see that the diameter is the diagonal of the square.

2. The aim of the task is not to figure out the property that a triangle inscribed in a semicircle is right, but noticing a particular that is constant for every square inscribed in a circle (the diagonals are two diameters):

   > I think that (finding the general property without the quadrilateral) would be too hard tough, I don't think they would get there. Because first of all there is a lot of things they can focus on..the circle, the points, the diagonals… how they are going to draw upon a 90 degree angle and then an inscribed angle subtended by the diameter. They would not get that. Maybe. I'm not saying no.. .I'm just saying that is hard. It would be very difficult..

   This is an investigation task, since students need to find an invariance, through the visualisation and the dragging. This task evaluates the mathematical competence of 'noticing', intended as developing conjectures based on observations and testing the conjectures. Here, the students should work inductively rather than deductively in order to find the invariance and generalize the property. The technological competencies the task evaluates are

In a problem-solving question like this one, the teacher evaluates the process and the correct answer, but he prefers to see that his students are able to make some steps in the right direction:

> I'm looking for process and a correct answer […]. I have already give them the opportunity at homework to show me their efforts, now I'm looking for their ability […]. When I give them a problem solving question in a test I'm looking for good problem solving evidence and not just wrong directions, and also insights into.. a connection between what they have done and their answer to the questions, again, just not right or wrong, I'm not looking for a correct answer, but insights.

3. The task is short, clear and the instructions are straightforward. The text is composed of simple words and expressions that the students are familiar with. The question "What do you notice?" again is not very precise, since students are asked to 'notice' something while their are exploring the diagram they have previously drawn, thus they could also come up with something the teacher does not expect. The teacher also need to choose which instruments providing to the students to solve the task in *Sketchpad*.

5.2.1. Conclusion on the tasks

In terms of Laborde (2001) categories, Figure 9 represents the tasks teachers designed for a DGE.

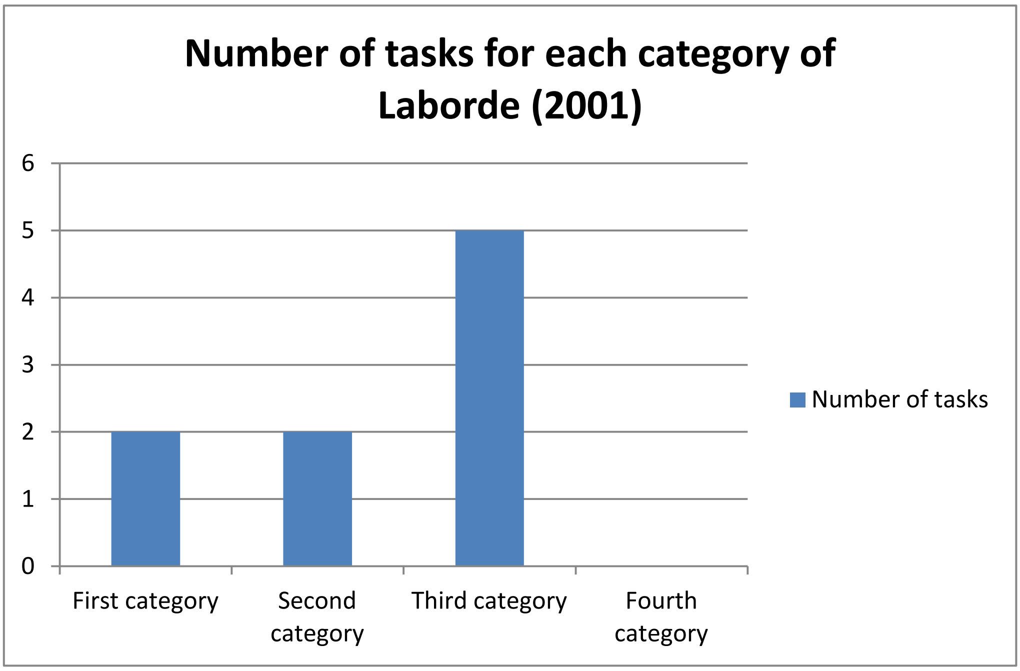


Figure 9. Number of Tasks

Six teachers out of eight design one task, T8 did not provide any task, and T7 designed three tasks (one for each category except the fourth one). We can notice that teachers designed mainly tasks belonging to the third category of Laborde (2001), where the digital technology influences the solving strategies of the task. While two teachers remained 'closer' to a paper-and-pencil context: one of them (T4) actually did not require the use of technology in the task, and the other teacher (T3) provided the digital technology as facilitator for the mathematical task.

Some of the tasks teachers designed offer the DGE as an **optional tool**, where students have the possibility to choose to use it or not. This seems to be because teachers want to assess the mathematical competencies, as stated by T1:

> If students can use the technology as long as they are able to solve the problem, it's good enough, because they have the skill, whatever it is, it could be that they know a property, but it could be that they have other skills in order to fill the missing property, so that they can still solve the problem. The technology helps them solve the problem empirically in a way that they don't need to know the property. It depends on what kind of things you are assessing, are you assessing the reasoning or the problem solving? It's a bit different.

As I showed in Table 7: Curriculum Script, some teachers like T2, T3, and T6, stated that **students must be comfortable** in using a DGE in mathematics before including it in the assessment, because they do not want the technology to be an impediment for the solution of the task. However, even if teachers would reach the point where students are able to use effectively digital technologies to solve mathematical tasks, they would still have **problems in grading the technological competencies,** as T6 explains below:

> It would have to be the very right context, does it make sense? So that, because there is always this sort of worrying thing, so assessing them on the use of technology before they are confident or fluent in the tech, knowing the point where they are competence and fluent, or should be competent and fluent. And then the other sort of worrying side is when I'm going to assess or assign a grade, the grade that I give them has to be based on their demonstration of the prescriber learning outcomes of the curriculum, so I can't grade them on "yes or no you use *Sketchpad* properly", even though it is valuable. I can say "I think you are correctly using these tools, and the correct use of these tools counts, and I'm marking you on this", and I may not actually put it in the book later on. Maybe this is an unfair thing, but the students know that it is important, so they care. (T6)

The main reasons for these impediments are that teachers struggle in conceiving the **technological competencies as part of the assessment**, as Teacher 5 affirms: "I suppose I can see value in assessing the technological competencies, but I don't", and that this kind of competence is not a **prescribed learning outcome** of the curriculum.

After trying the tasks that I designed (see Chapter 4), some teachers, like T4 and T5, actually reviewed their position in regard to the assessment in a DGE: "Absolutely, with this kind of thing. This is the kind of summative assessment that I would do. Where there is a bigger task than just answering a question" (T4). However, T5 insisted on the fact that he **does not have the necessary tools** and the **resources in terms of time and material to design the tasks**:

> I will have no problem in using this (sketches on the iPad) as assessment, I mean, I saw what you did, and thought that is the ideal way to assess a geometry unit. Would some teachers have problems with them? I think the problem is having the technology and design the tasks. I know in my school we have tablets put in teachers hand, but they don't have any will, they don't want to. There is so much potential there, I think, it's just hard to convince them. I mean even teachers that I consider to be quite technology capable are resistant to use it in the classroom. (T5)

In this statement there is a shift from task design to time management and resources. This teacher affirms that he would like to use the kind of tasks I created for the assessment, however, designing them is complicated. He also asserts that having the technology is difficult too. Then he shifts the problem to the willingness of the teachers, and the time it takes to use digital technology in student assessment. Thus, although digital technologies are available at school, teachers would need **pre-made resources** to design the sketches and save time, as T7 points out:

> If I was given a thumb-drive with a thousand GSP tasks on it, and I was told what file tests what sort of curriculum objective, I would buy it for hundred dollars.

The leading idea of the teachers is that a DGE helps students in the **construction of a diagram**, and it gives a deeper meaning to what they are drawing. All the tasks teachers designed are questions where students need to draw a sketch with the tools, in some tasks the role of the diagram is helping students reason in a problem solving question, in others the diagram is fundamental to support a process of demonstration. The mathematical competencies teachers prefer to assess in a DGE are the ability to demonstrate with a dynamic diagram, to 'notice' some invariants in a diagram drawn by the students, and to solve simple problems applying the previous knowledge.

Figure 10 represents the number of tasks teachers designed where DGE is optional or required, and the main goal of the task.

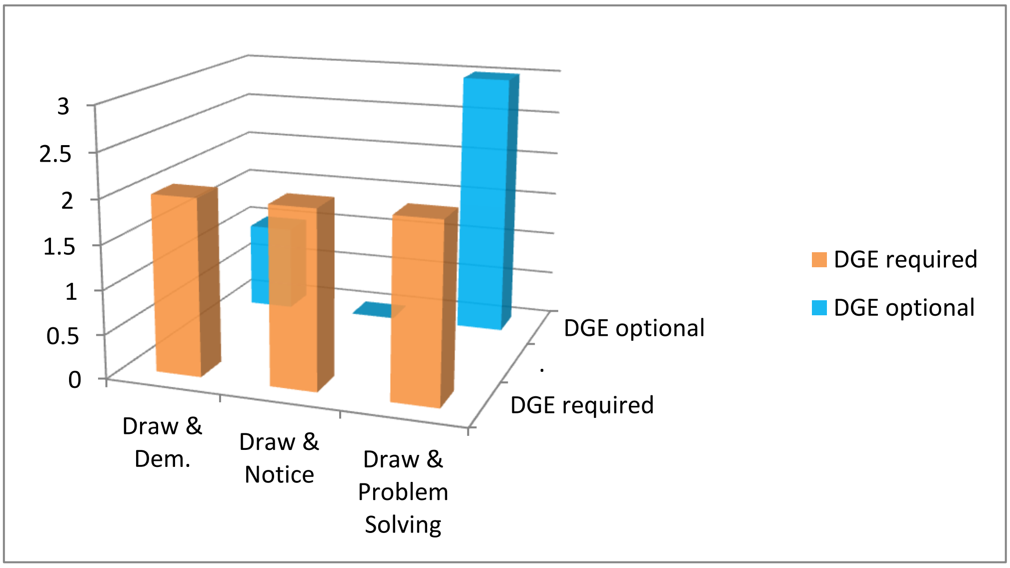


Figure 10. Tasks designed by the teachers

In Figure 10 there are ten tasks, because T6 created a task with two options: DGE required or DGE optional, T7 designed three different tasks, and T8 does not provide the task. To summarize the graphic: of the ten tasks, six provides the DGE as an optional tool, and four requires the DGE as part of the task. It is important to Three tasks ask to draw a diagram and demonstrate something, two tasks invite to draw a diagram and to 'notice' some invariants, and five tasks include problem solving questions. It is interesting to notice that, besides T1 and T2, teachers usually do not ask problem solving questions in the assessment, but here we can see that in an imaginary situation in a different classroom, five teachers chose to design a problem solving task. It probably means that they would like to do **problem solving** in a DGE in their classrooms, but in previous sections (5.1 and 5.2) we saw that they do not feel prepared enough, or their students are not ready yet to be formally assessed, or again problem solving is not a prescribed outcome of the curriculum.

Techniques and recurrent patterns that teachers use in designing their tasks for student assessment with digital technologies are:

- open tasks, like exploration tasks to demonstrate or to notice;
- convergent tasks, like finding the centre of the circle;
- using more shapes and geometrical constructions, and less numbers.

It is also important to notice that all the teachers that required the DGE as an indispensable tool to carry out the task, designed the sketch for *Sketchpad*, not for *Sketchpad Explorer*. The construction of a diagram in order to notice or to demonstrate can be done only in *Sketchpad*. *Sketchpad Explorer* needs pre-constructed sketches, where students can only use the tools provided by the teacher, and they cannot draw a diagram that maintains its properties through the dragging. In pre-constructed sketches the teacher creates the diagram with the properties he/she wants, and chooses the tools the students could use.

In the next section, I illustrate teachers' comments on the sketches I implemented in *Sketchpad*, which I have described in Chapter 4.

5.3. Teachers' comments on my sketches

In the interview I showed to the teachers the pre-constructed sketches I created in *Sketchpad Explorer* on Circle Geometry, and I asked them some questions like:

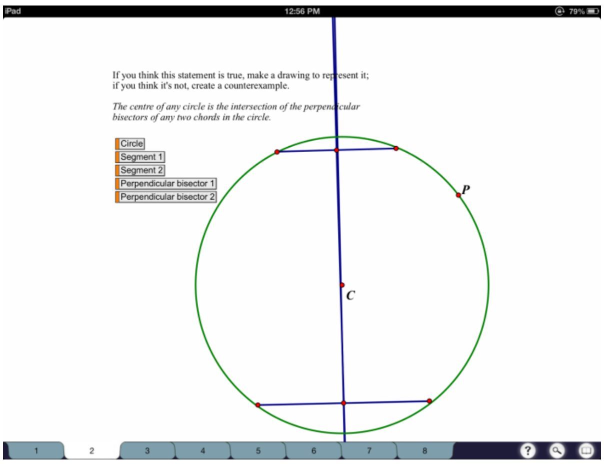
- *What is this task assessing (in terms of content and process)?*
- *In your opinion, how is the technology affecting a student's response?*
- *How might the use of technology for this task affect what you learnt about your students' understanding?*

I also showed them some students' answers to my tasks, and then I asked them some other questions, like:

- *Choose the task that you like the most. Why do you like it?*
- *Choose the task that you dislike most. Why do you dislike it?*
- *If you were Max, would you use these tasks to assess your students? Why?*

I collected their comments, and I analyzed them through the categorization of Laborde (2001).

First category



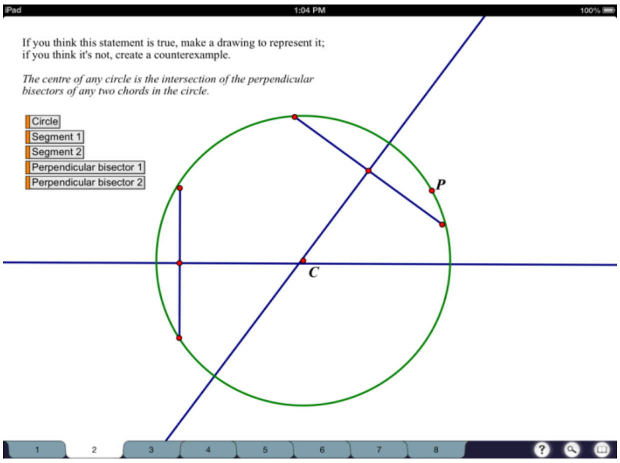
Student: "False, because the two perpendicular bisector are overlapping. See picture."

Some teachers do not like the tasks of the first category, mainly because "You don't need the technology to do it", as T3 commented in regard to the 'Counter-Example' sketch.

Moreover, they notice that *Sketchpad Explorer* is not precise in the drawing of the diagram, because teachers have to pre-construct the mathematical objects, and students can use them as tools in the drawing. The technology is not exact how students expected, and this fact can confuse them, especially in the case that they are not very sure of the concept they learnt, because they usually trust the precision of the technology.

Therefore, students could also question their understanding as a consequence of the ‘not accurate’ *activity feedback* of the technology.

> Here the technology is not precise at all, it is an approximation. There is something wired because you are not finding the centre exactly. Thus, why should students use it if it’s not helping them? (T1)



Student: “Yes, because we tried different positions and the two bisectors intersect in the centre.”

This question is pretty difficult, as T7 noticed: “That's an hard question.. Lot of kids would not get that”. This teacher actually liked this task, mainly because students are asked to think about a property of the circle they should know very well and generalize it. If students think the property is always true, for every position of the two chords, then they will draw a diagram as example, if they find the counter-example, when the two chords are parallel, then they will draw a diagram to prove it:

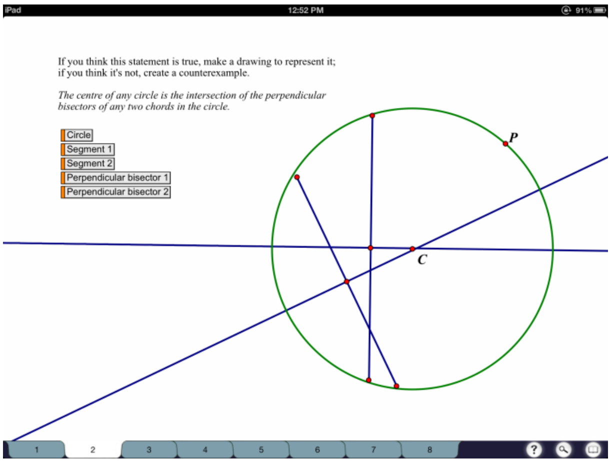
> It's nice and simple, it's very straight forward, and they have to prove something or not to prove something. They have to draw a diagram, even if they get it wrong, they have to do something and drag something. Even if they don't get the counter-example, they have to do something: draw the chords and the perpendicular bisectors.. (T7)

However, grading this kind of question could be tricky. If students do not find the counter-example, but they say that it is a well known theorem, and they draw a diagram. Would it be an ‘acceptable’ answer?

> That would be the ultimate level.. but also quite nice to say “yes, it’s true”, they still have to draw the chords, it would be still a pretty good mark, I would still be satisfied with it. Would you not to? I mean.. It's still better than a question with no graphic at all, there is some good understanding, as drawing the picture. Of course it's not as good as that, but this is exactly what we would them to do, we want them to try different things, that’s what I mean when I say that also the drawing as example would be a good answer, even though they don’t make the last connection. (T5)

Some teachers affirm that they would not grade a question like this one because not all the students would answer correctly.

That's a cruel question. I wouldn't assess it.. Because some of the students will get it, and some not.. If I would assess this question, I would give the 80% of the mark to the students who draw the circle and the property, while I would give only 20% more to the students who draw the counter example. (T7)



Student: “Yes. Where they intersect is the centre of the circle”

Second category

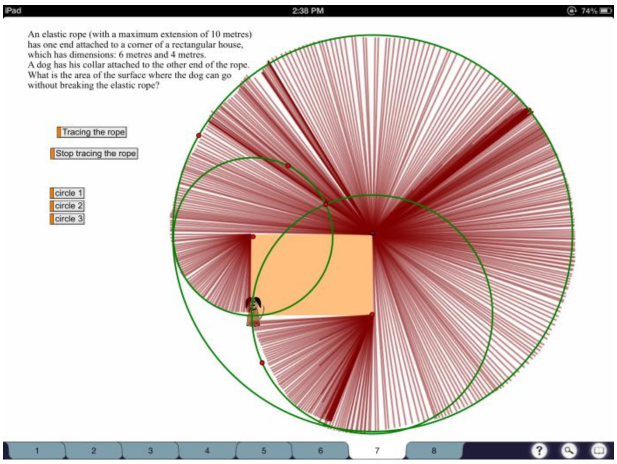
Teachers like the tasks of the second category, when the DGE acts as a “visual amplifier” and helps students understand the concept in a dynamic way, as T2 explained:

There are other situations where the technology helps visualize something that you could do otherwise, but it just makes a lot better, calculus is a good example that work.

Some teachers appreciate the fact that the sketch helps the students in ‘visualizing the situation’ of the task:

Neat, that's a good one.. I like how you did the bending.. It takes away the suffering of the problem. (T5)

The dog sketch is very classic, it is dynamic, it’s a classic geometry question. I would imagine this to be one of the most difficult question, and it’s very helpful to have technology for it. I really like the dog one. (T3)



Student: “One large circle, the elastic breaks shortly, One medium circle, One small circle”

While others think that the sketch used as a 'visual amplifier' is a tool for helping only the students who struggle with the 'visualization':

> Nice… that's cute […] It's the visualization that kids need, that most of us need… You want to give them a situation, and you want them to visualize it and be able to drive just from the…, it depends on what you want to get out of it. In my grade 4 classroom there were some kids that absolutely would have need to have me give them this part… other kids I could just given the scenario, I would probably have given the scenario to the all class as we started, and then going to the individual small groups, and some of them I would have given this..., the ones that couldn't come up with the visualization by their own, like a differentiation for some. (T4)

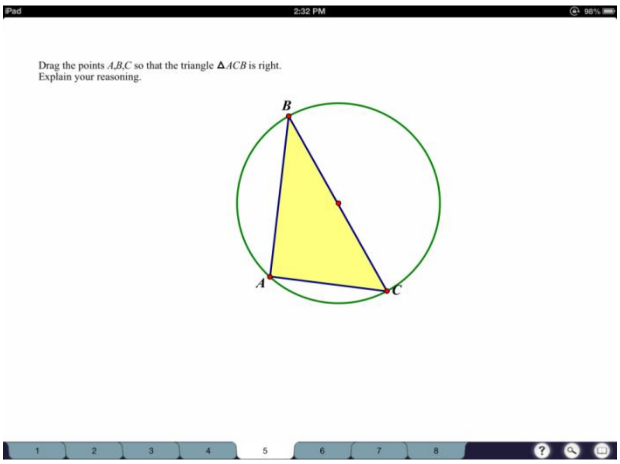
Third category

The teachers also declared that they use to design tasks belonging to the third category for informal assessment in class:

> If I give them a question which is just a purely problem-solving type question, then I've given them a question where they can only do it using *Sketchpad*, say. So they have to be able to use technology in order to explain the answer, to work out the answer or do it. So I'm trying pick situations where the technology can do something they couldn't do just with paper and pencil. (T2)

Laborde (2001) observes that "at the beginning, most were observation tasks for conjecturing, whereas more diverse tasks appeared in later versions". Although in the interview teachers affirm that they usually design exploration and problem solving tasks for informal assessment, analyzing the tasks that they created in this pilot, I've noticed that they prefer to ideate *construction* situations for student assessment, rather than *prediction* situations for conjecturing and proving.

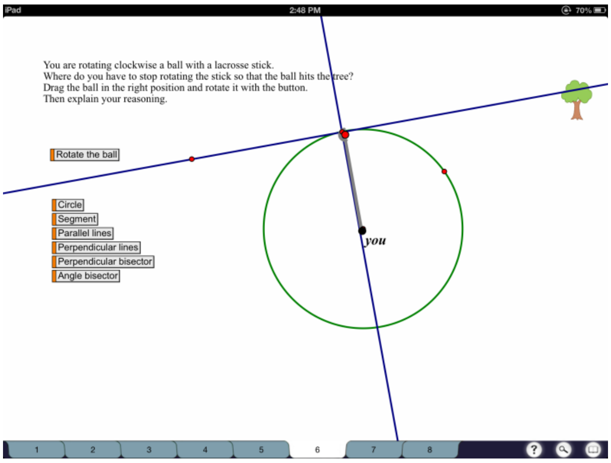
> I didn't like so much this one, because there wasn't so much construction. (T5)



Student: "180° angle is the central angle, so the inscribed angle is 90°."

Fourth category

The teachers were interested in knowing how they could create '*black box*' and *prediction* tasks, even if "the design of such tasks represents a conceptual break with the usual tasks performed in a paper-and-pencil environment." (Laborde, 2001). However, they still have some problems in conceiving these kinds of tasks as being appropriate in assessment, as Teacher 1 affirms, in relation to the "the Ball" sketch:



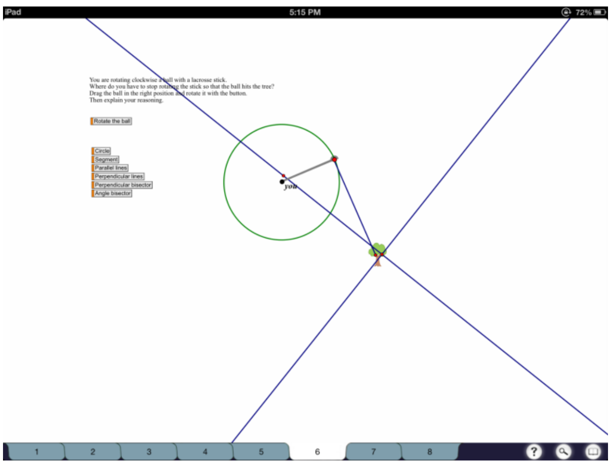
Student: "The trajectory is a tangent that is perpendicular to the radius."

> What about this as an exploratory activity, not an assessment activity? Because they find out something, they actually discover new things, but if you make this an explanatory activity, then they learn this and then the assessment can come after that.

Teacher 1 thinks that teachers have to assess what students know, not their ability to explore, and make conjectures in unfamiliar situations. While other teachers are comfortable with the idea of assessing their students with such tasks:

> Asking what is the definition of tangent is something I wouldn't have in a test, that's useless to me. Here they are exploring what tangent means, and actually doing something with it. Even if they are confident with the exact correct vocabulary, they have a better understanding doing something like this, to me. (T4)

This task could not be so clear in the way it is worded, because students could give any sort of explanation to their answer, also a 'non-mathematic' one:



Student: "See picture. Reasoning: first we used the circle and made it turn. We guessed and checked."

> I like this one, the only problem with this one is that I don't like the way it is worded, because when you say 'explain your reasoning' is too vague and open, and there is no

right or wrong answer. You have to be more clear on what you are looking for, if you say show an example or a counter example, then it's really nice, because it's specific. You can say: "Given the length of the arm, and any position of the tree, give a specific step by step process to achieve the solution to this problem, and explain how it would work every time." (T7)

5.3.1. Conclusion on teachers' comments on my sketches

Laborde (2001) observes that the teachers' experience in using the technology does not influence the category of the task they design, since tasks of the first category were found only in the scenarios written by the novice teacher expert in the use of technology:

> The teacher who was a novice in technology, but experienced in teaching, designed tasks which could not be done with paper and pencil. […] Paradoxically, the teacher who was expert in technology did not change the nature of tasks he gave, in contrast to the novice in technology, who did, at least partly. (p. 289)

In Figure 11, I grouped the answers to the question: *Choose the task that you like the most. Why do you like it? Choose the task that you dislike most. Why do you dislike it?* During the interviews the questions differ slightly, so not all the teachers answered this question.

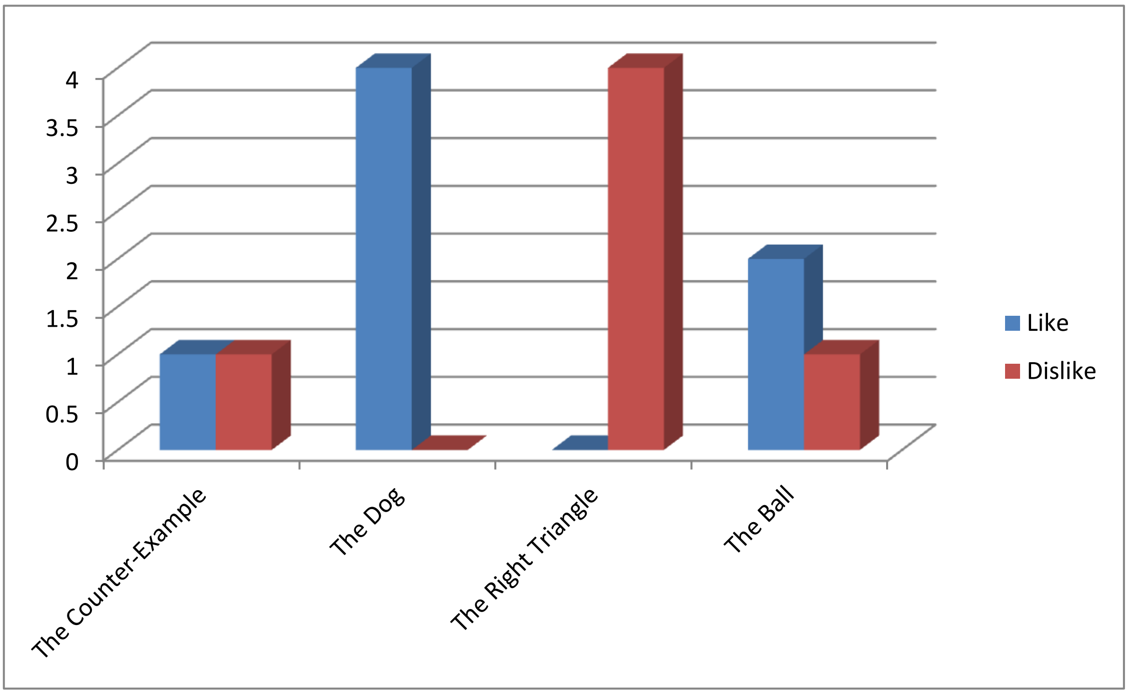


Figure 11. Like vs Dislike

From the graph in Figure 11 we can see that teachers' opinions are quite similar for the task of the Dog, which was very appreciated, and the task of the Triangle, which was not really appreciated. The Dog sketch is appreciated for the insights that it gives into the problem through the visualization and the dynamic skills, making the problem solving question more accessible to the students:

> The dog one is the sort of problem I have kids doing even in grade 11 and 12, because I think it's a good problem solving question. (T2)

The Triangle task is not valued because it looks too simple, and not meaningful, as T7 observed: "Boring, too simple". While teachers' ideas on the Counter-Example task and on the Ball sketch are quite different. For the Counter-Example task we have a positive opinion, and a negative one. The reason for the negative opinion is that asking for a counter-example is not a classic question for the assessment, as T3 stated:

> I think if I want to check their understanding I would pose the questions differently, not ask for a counter-example, assessment for me has to be a little more clear.

While the reason for the positive opinions is that it helps generalization, since it makes students think about a property they know, and teachers like T7 appreciate the way the question is formulated: "If you say show an example or a counter example then it's really nice, because it is specific". For the Ball sketch we can notice two positive views, and a negative one. Teachers who like it appreciate the idea that students should explore the situation to look for the 'hidden construction': "They are going to think that the ball will probably spiral or something, then they can find the tangent" (T5). And T1 adds:

> They can animate the rotate ball and they can visualize the problem dynamically, and then they can actually empirically find some angle, some data for them to observe, and they can even just not use this perpendicular bisector and they can do it empirically.

While the teachers who do not like it think that it is too difficult, and some students would not solve the task: "This one takes a lot for kids to figure it out. I expected only the stronger would get it" (T3). In addition, T7 does not like the way the task is worded: "The only problem with this one is that I don't like the way is worded, because when you say 'explain your reasoning' is too vague".

Driving the tasks teachers designed with the categories of Laborde (2001) and collecting teachers' comments on the sketches I designed helped me identify the type of task teachers prefer, and the kind of competencies they would like to evaluate in a DGE. In general, teachers design tasks belonging to the second and third category of (Laborde, 2001); in particular, they like construction tasks for conjecturing and proving. Teachers perceive the inefficacy of using digital technologies in the tasks of the first category, where technology is not very useful in terms of visualization and calculation. Teachers prefer to use tasks of the fourth category for class activities and *informal* assessment. Questions for assessment purpose could be 'open' in order to allow real research during examinations, and should give students the opportunity to take initiatives. However, teachers prefer to converge exploration during a test, like T3 states: "For the assessment you have to converge the exploration". Moreover, the use of the digital technology depends on the aim of the assessment: if a teacher's goal is to see if students are able to solve a problem, like in a real life context, they can use tools to solve it, but they can also decide to use paper and pencil alone, it is up to them.

Chapter 6.

Conclusions, Implications and Limitations

In the methodology I stated the research questions of this study. Firstly, I wanted to find out if teachers use technology in student assessment in mathematics, and the reasons leading their choice in their practice. Secondly, I was interested in knowing what kinds of task teachers design in a DGE, and if they value the different competencies that can be assessed in a DGE. The final intent of this research was to investigate the kinds of technology-based assessment tasks teachers value, and to understand how their preferences relate to the type of feedback offered in the task.

Through the literature and the analysis of the teachers' data with the *Structuring Features of Classroom Practice* framework, I will try to answer to the first research question in the first section of this chapter. Using Laborde (2001) I have analyzed the tasks teachers designed and their comments on my sketches, and I will try to respond to the second question in the second section of the chapter. Then, I will try to answer to the final research intent combining the conclusions of the previous questions. In the last section of this chapter, I will evaluate the theoretical and practical contributions of this work, the limitations of this research and the implications of this study for the future.

6.1. First research question

In the questionnaire I found out that teachers are pretty new to the idea of using digital technologies in student assessment in mathematics. Through the five-plus-one *structuring features of classroom practice* I have investigated the difficulties teachers encounter when they use digital technologies in the classroom, also for *informal assessment*, and the problems they think will occur if they use digital technology in *formal assessment*. Teachers explained how they conduct assessment practice in their classrooms, the competencies they want to evaluate in their students, and what kind of feedback they give to them. Based on my findings, it appears that the **goals of assessment** would need to change in order for the use of digital technologies in assessment to be warranted. Teachers would need to re-define what they want to evaluate with the technology, if they want to test only **what students know** or if they want to assess also **what students can do** with the technology, like their capacities to solve problems, explore environments and find properties. Teachers would also need to value **technological competencies,** as this teacher in my study pointed out explicitly: "I think in a technologically integrated classroom, students must be technologically competent in order to access some of the mathematical competencies" (T6).

In regard to **activity feedback**, my findings show that teachers are not ready yet to accept that digital technologies could give a constructive *activity feedback* also in assessment. There may be several reasons for this, but a major factor may be that students could try to solve the task by a guess-and-check technique using the feedback of the digital technology, as T2 observed. Moreover, students could also cheat more easily through the technology, since teachers do not feel that they have a complete control of the elements involved, like T6 and T1 noticed.

In regard to **student feedback**, teachers give a lot of value to feedback in *informal* assessment, because usually *formal* assessment is the final one, so students are not expected to improve their performance during such assessment. Teachers give a lot of *formative* feedback to their students during the learning process, but not after *summative* assessment.

If teachers do not *formally* evaluate students' use of digital technologies in mathematics, and do not provide *student feedback* after *formal* and *summative* assessment, then students would not understand how they can improve their learning. Even teachers that regularly include digital technologies in their teaching prefer to use them for *informal assessment*, like for classroom activities and group projects. Digital technologies are rarely adopted in classroom-based formal assessment. One of my points is that integrating digital technologies only in *informal* assessment is not enough, because **students do not see that they are valued**. Further, and perhaps more problematically, students develop the belief that digital technologies are merely crutches that might help support learning but that should eventually be replaced by mental activity only. So, while a teachers might think that observing students manipulating objects and making conjectures in a DGE is meaningful and can provide them with some insight into students' thinking, this value will not be perceived by students. The main reason is that students do not see that teachers are putting value on this kind of activities, because often they do not receive *student feedback* on their product, both in terms of summative assessment (a grade), and formative assessment (comments on where they are and what they have to do to improve).

6.2. Second research question

Secondly, I was interested in knowing what kinds of task teachers design in a DGE, and if they value the different competencies that can be assessed in a DGE.

In regard to **task design**, showing the teachers in my study Laborde's (2001) classification was useful to make them aware of the different kinds of tasks they can design for assessment purposes. When I showed the sketches I created in *Sketchpad* for student assessment, teachers explained their doubts on using them for *formal assessment*, but some of them, like T4 and T5, actually changed their way of thinking on using digital technologies in student assessment.

As I showed in the analysis of the tasks designed by the teachers, and in their comments to my sketches, they prefer to design tasks belonging to the *third category* of Laborde (2001). Teachers perceive the inefficacy of using digital technologies in the tasks of the *first category*, and they like the role of the technology as a visual amplifier in the tasks of the *second category*. Finally, teachers state that they usually design tasks of the *fourth category* for class activities and *informal* assessment, and they would like to know more about how to design 'black box' tasks.

The use of the technology depends on the **aim of the assessment**. The **mathematical competencies** teachers prefer to assess in a DGE are the ability to demonstrate with a dynamic diagram, to 'notice' some invariants, and to solve simple problems applying previous knowledge. They would like to design problem-solving tasks for assessment, but they find it difficult to put that into practice, because that kind of task does not require a standard procedure, and students could be stuck, as T4, T6, and T7 observed in Chapter 5.1. Teachers do not seem to value the **technological competencies**, but students need them in order to achieve some mathematical competencies, and to keep up with the evolution of technology in the society.

Teachers were interested in knowing how I designed the sketches and which students' competencies I was trying to evaluate. However, my study highlighted the teachers' insistence on the need for **ready-to-use resources** for designing tasks and for their actual implementation in student assessment, like teachers T1 and T7 pointed out. Implementing more complicated sketches is quite complex, thus I claim the need of people different from the teachers to do it. These experts could be the mathematics education researchers, who also know when the tasks should be used, and what mathematical and technological competencies they assess. Researchers could provide all the necessary information to the teachers, like how and why they should use some kinds of digital technologies and some types of tasks.

6.3. Third research question

Teachers value the tasks of the second, third and fourth category of Laborde (2001), tasks where technology has a fundamental role (modify the solving strategies or give meaning to the task). They see the potential of digital technologies in student assessment, because they are aware of the fact that if they do not assess students on the competencies they acquire while using digital technologies in mathematics, they will not value them. However, teachers are held back by the prescribed learning outcomes of the curriculum, and by the difficulties they may encounter in using digital technologies in assessment.

Firstly, I am arguing that **curriculum** should include a range of mathematical and technological competencies that involve the use of digital technologies in mathematics, as Caron & Steinke (2005) observe:

> Curriculum and Assessment need to evolve with the integration of technology in order to engage students in a mathematical practice that is empowering, meaningful, and coherent (p. 4).

Secondly, if teachers want to see whether students know how to use effectively digital technologies in mathematics to solve unfamiliar problems and make rational conjectures, then exploratory problems should be included in student assessment. Moreover, such problems are warranted if we assume that taking a test can also be an opportunity for learning, so that students **continue to learn during the assessment** through the *activity feedback* of the technology: "feedback provided through the use of different technologies can contribute to students' learning" (Olive et al., 2010, p. 133).

6.4. Contributions of this work, Limitations and Implications

Besides the answers to the research questions, I will try to sum up what I think are the main contributions, both theoretical and practical, of this work of thesis.

A theoretical and interesting observation is that teachers conceive *formative assessment* as *informal assessment*, thus they do not give **feedback** to students after *formal assessment*. While, as we saw, it is fundamental to give *student feedback* after *summative assessment* in order to make students aware of their level of understanding and of the next required steps for their learning.

If teachers value the competencies that students can acquire using digital technologies in mathematics, they firstly need to establish them as **learning goals**. Then, they should make students aware of these goals, and integrate digital technologies also in their *formal assessment* in order to give value to the established goals. Finally, they might provide students feedback on their performance, so that students have the possibility to improve their work.

Another issue is the content of the **curriculum** in relation to the content of the assessment. If the use of digital technologies in mathematics is included in the learning outcomes of the curriculum, then teachers should find a way to integrate digital technologies also in the assessment. However, what is really needed is a change of the learning outcomes of the curriculum, in order to include also different kinds of mathematical problems that cannot be solved through paper-and-pencil alone.

A good idea would be to better inform teachers of the possible kinds of tasks they could design for student assessment in a DGE, for example showing them the Laborde's categorization of DGE tasks. Moreover, teachers claim for the need of **ready to use resources** to design and implement appropriate/innovative tasks to assess the competencies that students acquire using digital technologies in mathematics.

I also found that teachers find it challenging to conceive exploration and problem solving tasks as part of the assessment. Moreover, they are reluctant to the idea that students could **continue learning during assessment** through the *activity feedback* of the technology.

I suggest that the *activity feedback* of digital technologies can take the role of the teachers during the assessment, when students are doing exploration tasks in order to find invariants or properties. Figure 12 is a representative scheme of the idea:

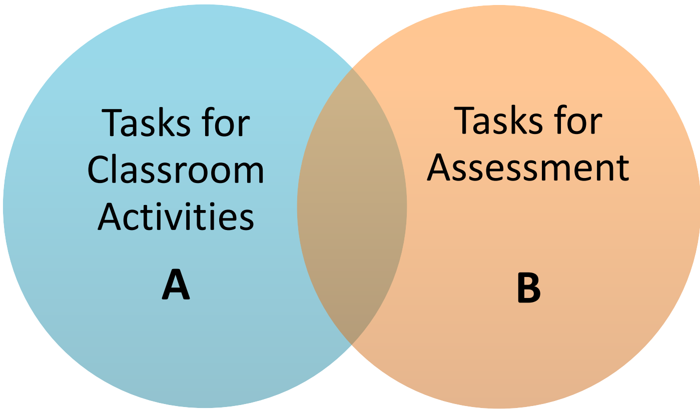


Figure 12. Digital tasks

In A – (A∩B) I include all the kinds of task where the teacher wants to introduce a new topic; these tasks actually ask for something that students have not been taught. In this situation, digital technologies can be optional tools, but the teacher's *activity feedback* is necessary to carry out the activity, and develop new learning.

In B – (A∩B) I consider the tasks in which the teacher asks for something that the students should know. In this situation, the teacher's *activity feedback* is not involved, and the *activity feedback* of digital technologies is optional, if they are used in the tasks.

In A∩B I incorporate tasks in which the teacher asks for something students are not supposed to know. In this situation, the *activity feedback* of digital technologies is needed in order to carry out the task. These tasks usually are of the '*black box'* type or of *prediction* situations, where students have to explore and find the 'hidden construction' by studying the invariance through dragging of the diagram or observation of some properties. These problem-solving situations are the most appropriate to enhance student understanding, and also the most effective for students to 'put in practice' their knowledge. If this kind of tasks is used in the classroom, teachers can help and prompt students in the activity. If it is used during the assessment, the *activity feedback* of technology should be sufficient to give students what they need in order to find the solution.

Students do not have the possibility to improve their ability to explore and to make conjectures using digital technologies because they do not know what the final goal of these activities is, nor where they are with respect to the achievement of that goal. This places pressure on teachers who choose to use digital technologies in their teaching to establish what are the learning goals of using them in mathematics, and make the students aware of these goals. Then, these teachers would have to integrate the new tools also in the assessment, in order to give value to the established goals. Finally, they would have to give *student feedback* on their performance, in order to make students aware of their level of achievement of the goal, and what they have to do to improve. The process of providing *student feedback* demonstrates the goals are valued and promotes students' learning, because, as Sangwin et al. (2010) suggest, "assessment is a fundamental part of the learning cycle, is central to learning and is also often a primary driver of students' activity" (p. 227).

As every research, this study presents some **limitations**. Firstly, **the set of participants** I chose might be un-representative of the rest of the population, since they have advanced degrees in mathematics and they are taking courses in mathematics education. This might affect the results because they are all very good teachers that spent a lot of time for their classrooms, and they are also documented on the current research in mathematics education, since they read articles in their PhD/Master courses. We can also observe that they are quite expert in the use of digital technologies, at least in the teaching of mathematics, both from the tasks they created and from the interviews. Almost all of the teachers (7 out of 8) designed tasks for a DGE, and they all use digital technology in their teaching (see Figure 7).

Secondly, I decided to focus my research on Dynamic Geometry Environments on iPads, and it might not generalize to other **software and technologies**. I also created my sketches on a very specific topic (circle geometry), and I tested them in specific grades (9 and 10). I needed these results to show to the teachers samples of student response. Different considerations should be done for other topics or grades, like the different competencies involved in the assessment, and the different types of tasks that could be created. Laborde (2001) research focused on teachers designing tasks in a DGE, thus the categories she uses are specific for that environment. If we consider other tools, grades and topics of the assessment, the results may be different in terms of *activity* and *student* feedback.

Finally, the **questions** I asked in the interview/questionnaire may provide only partial information that may be different from what could be observed in real-time teaching. Moreover, what teachers say they do in their practice and their opinions on imaginary situations might be influenced by my questions, and by the background of my study.

I would like to make a final consideration on what I learnt in terms of **task design for the future**. Through this study I have had the opportunity to read a lot of research on task design, and to design assessment tasks by myself following the literature on assessment with digital technologies. What I found out is that firstly it is important to know all the different affordance that a certain technology offers, in order to take advantage of them to design tasks that are completely different in nature from the tasks that can be designed on paper. Secondly, it is possible to design a good task only if you know which competencies you want to evaluate, taking into account also the technological competencies that can be valuable to assess in mathematics.

What I can say for the future of task design is that we are moving to **automatic assessment**. In 2010 Sangwin et al. describe "computer aided assessment of mathematics by focusing on the micro-level of automatically assessing students' answers" (p. 227). The principle of mathematical computer aided assessment is that students create mathematical objects using a computer; these objects could be algebraic expressions, graphs, or geometric figures. Then, the computer automatically establishes the mathematical properties of the objects, and it assigns outcomes, including feedback, on the basis of these properties. The aim of their research was to provide grades and feedback in student assessment through the digital technology:

> In the case of summative assessment, we saw that CAA systems permit a quick evaluation and a saving of time for the teacher. [...] In the case of formative assessment, feedback may be adapted to the student's answer especially in open tasks and may guide the student to the correct answer (Sangwin et al., 2010, p. 248).

During the last CERME (Congress of European Research in Mathematics Education) held in Prague, in February (4th-9th, 2015), Galit Nagari Haddif and Michal Yerushalmy presented a research paper on *digital interactive assessment in mathematics: the case of construction e-tasks.* Their study is focused on designing e-tasks in a dynamic MLR (multiple linked representations) environment that provides reflecting feedback through e-assessment. Construction e-tasks consist on creating examples that satisfy some given conditions, or counter-examples to prove something.

These e-tasks may also have an infinite number of possible correct solutions, which can be checked automatically by the digital technology.

These are important steps forward to assessment using digital technologies, mainly for three reason. Firstly, assessment can include tasks that ask for different kinds of answers from the classic multiple-choice ones, or from the answers that have only one correct solution. Secondly, automatic assessment would save time to teachers, because it would considerably reduce their work; teachers may check the process to assign partial credit if the solution is partially correct. Finally, students can use automatic assessment at home to have immediate feedback on their performance, and this is a huge contribution to their learning.

Sangwin et al. (2010) observe that assessment through digital technologies is motivated by the need to evaluate new mathematical competencies:

> The need to address emerging educational goals motivates the diversification of modes of assessment away from the traditional, dominant mode of timed paper-and-pencil tests. Technologies can support or even initiate such changes (p. 249).

While Haddif and Yerushalmy point out that automatic assessment may be the most appropriate method to assess students' mathematical learning in the future: "Our research arises from the challenge to design e-tasks that faithfully assess future learning and teaching".

I would also like to add a consideration on the figure of the **task designer**, this role could be assumed by the mathematics teacher, who should spend extra time on keeping up with the new digital technologies, and the different kinds of mathematical tasks that can be designed with different software and technology for student assessment. Moreover, designing the tasks and implementing the sketches in the software require time and specific knowledge. However, since teachers have so many other important things to do and think about, and the development of digital technologies is very fast, it seems unreasonable that teachers should do everything by themselves. Thus, it would be wise to have another professional figure doing that, like researchers in mathematics education qualified in mathematics, but also in education and technology, who would be able to include the newest and most efficient software in student assessment in mathematics. These people would design the tasks for student assessment, and they would provide a clear explanation for the teachers on: the goal of each task, the correspondent outcome in the curriculum, and the mathematical/technological competencies the task evaluates.

Chapter 7.

Some mathematical considerations

In this dissertation the use of *Sketchpad* is focused on the assessment of some mathematical and technological competencies. The reasons that could motivate the students in using specific software to carry out some mathematical activities are not taken into account. In this chapter, I explain some of the reasons that can lead the students to use digital technologies in mathematics by providing some historical examples. Firstly, I provide a brief history of an irrational number whose nature is strictly connected to the circle, and some problems solved through the properties of the circle in Astronomy. Then, I show some examples of the use of *Sketchpad* for the teacher to demonstrate some properties of the circle, and for the students to solve problems involving the circle.

7.1. History of π

Since the history of mathematics could provide precious hints to engage students in the discovery/construction of the mathematics, I provide a brief history of π, through a famous Greek problem: **squaring the circle**.

> It would be difficult to select another special problem, an account of the history of which would afford so good an opportunity of obtaining a glimpse of so many of the main phases of the development of general Mathematics. (Hobson,1913, p.2)

In the ancient Greece, mathematics was strictly connected with philosophy. One of the greatest problems of classical mathematics was 'squaring the circle': the problem of finding, using only a ruler and a compass, the side of a square of area equal to that of a circle. Greek geometers knew that the problems of the squaring and the rectification of the circle were equivalent: the ratio of the length of the circle to the diameter has a definite value, equal to that of the area of the circle to that of a square of which the radius is side. Since the time of Euler (1707-1783), this ratio has always been denoted by the familiar notation π (Hobson, 1913). Today we know that this problem is not solvable, because it requires the construction with a ruler and a compass of a segment of length $\sqrt{\pi}$, while only segments whose measure is the solution of a quadratic equation can be built with these tools.

Hobson (1913) divided the history of the solution of the 'squaring the circle' problem in three periods, marked out by different methods, aims and equipment in terms of intellectual tools. The first one involves the determinations of the ratio of the circumference to the diameter of a circle until the invention of the differential and integral calculus. The second period starts in the middle of the seventeenth century, and lasted for about a century, it was characterized by the application of the powerful analytical methods provided by the new Analysis to the determination of analytical expressions for the number π in the form of convergent series, products, and continued fractions. The last period, which lasted from the middle of the eighteenth century until late in the nineteenth century, attention was turned to critical investigations of the true nature of the number π itself, considered independently of mere analytical representations: it was finally established by a method which involved the use of some of the most modern devices of analytical investigations that the number π is transcendental.

Here, I consider only the first period of Hobson (1913), in order to show the first development of the problem. The earliest traces of a determination of π are to be found in the Papyrus Rhind. It states that the area of a circle is equal to that of a square whose side is the diameter diminished by one ninth: $A = \left(\frac{8}{9}\right)^2 d^2$, which compared to the formula $A = \frac{1}{4}\,\pi\, d^2$ gives that $\pi = \frac{256}{81} = 3{,}16049\ldots$ The approximation $\pi = 3$ was already known by the Babylonians, and it was current for many centuries, also in the Old Testament there is a statement assuming that constant: "*And he made a molten sea, ten cubits from the one brim to the other: it was round all about, and his height was five cubits: and a line of thirty cubits did compass it about.*" (I Kings 7, 23). The diameter of the sea is ten cubits and the circumference is thirty cubits, so their ratio is 3 (a rudimental approximation of π).

The first systematic treatment of the 'squaring the circle' problem is due to Anaxagoras of Clazomene (500 - 428 B.C.). He made an approximate construction of an equal square, however, there is not precise documentation of his construction. About the year 420 B.C., Hippias of Elis invented a curve, called 'quadratrix', which is constructed as follows: a segment translates uniformly from DC to AB, and at the same time the radius AQ rotates about point A uniformly clockwise from the position AD to AB. The locus of P, traced out by the intersection of the segment and the radius, is the ‘quadratrix’.

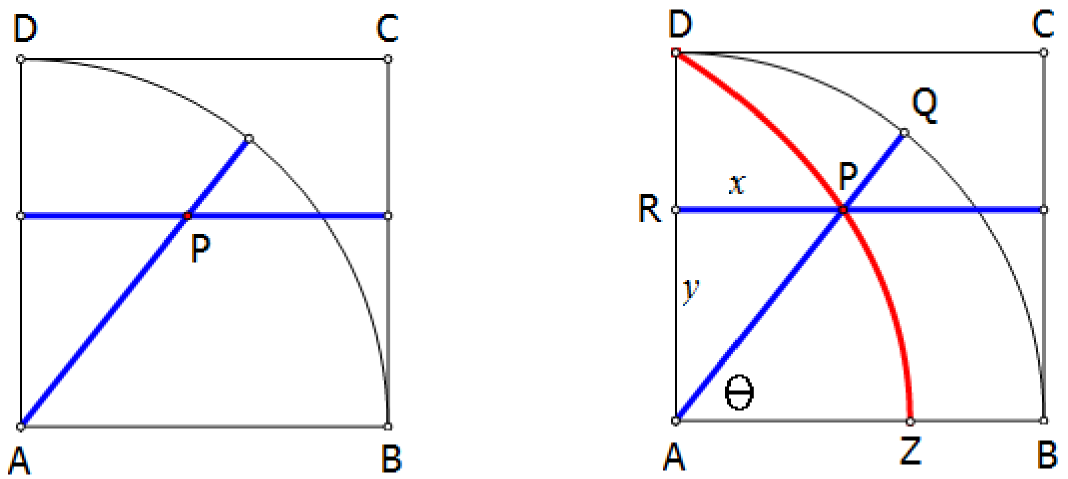


Figure 13. Quadratrix

It was proven by Dinostratus, around 335 B.C., that the quadratrix could be used to square the circle with radius a. Since $y : a = \theta : \frac{\pi}{2}$, and $\frac{y}{x} = tg(\theta)$, then we have that:

$$x = \frac{y}{tg\left(\frac{\pi}{2a}\,y\right)}$$

If we use the unit circle, then $a = 1$, and the rectangle with the same area of the unit circle, constructed by Dinostratus, will have as sides the radius 1, and $\frac{2}{AZ}$, where:

$$AZ = \lim_{y \to 0} x = \lim_{y \to 0} \frac{y}{tg\left(\frac{\pi}{2}\,y\right)} = \frac{2}{\pi}$$

Thus the area of the rectangle is π. Then the rectangle can easily be squared.

The problem of squaring the circle is that the point Z on the quadratrix cannot be constructed. In fact, Hippias’ definition of the quadratrix fails at point Z, because the intersection of the segment parallel to AB and the radius AQ is the entire segment. Thus, point Z can be found with the limit, but it cannot be constructed. The transcendence of π implies that it is impossible to solve the ancient challenge of squaring the circle with a ruler and a compass.

Another approximation of π is connected to the **method of exhaustion** (the term 'exhaustion' was introduced by Gregory De Saint Vincent in 1647). Euclid gives an example of this method in his book XII. 2, where he shows that the areas of two circle are to one another as the squares of their diameters, and his proof is due to Eudoxus, "to whom various other applications of the method of exhaustions are specifically attributed by Archimedes (287-212 B.C.)" (Hobson, 1913, p. 17). Eudoxus of Cnidus (408-355 B.C.) was the first who was able to formalize the process by introducing a lemma (axiom of continuity) that served as the basis for the method of exhaustion.

The method of exhaustion is based on contiguous classes: in order to prove that a figure A is equal to a figure B, it is necessary to prove that can neither be A < B or A > B; this is done by contradiction. Proposition 2 of Euclid's Elements states that the circles are to one another as the squares of the diameters. In the proof of Proposition 2, the method of exhaustion is used, and there is the idea of approximating the area of the circle with a succession of polygons with *n* sides, where *n* increases. The Greek Mathematicians attributed to Eudoxus the idea of inscribing and circumscribing regular polygons in the circle, and increasing the number of the sides of the polygons to get a better approximation of the area of the circle. However, he was not able to conclude the reasoning, because the concept of limit was unknown at the time. Figure 14 is a demonstration in *Sketchpad* where the number of the sides of the polygon inscribed in a circle with radius 1 can be increased in order to 'fill' the area of the circle.

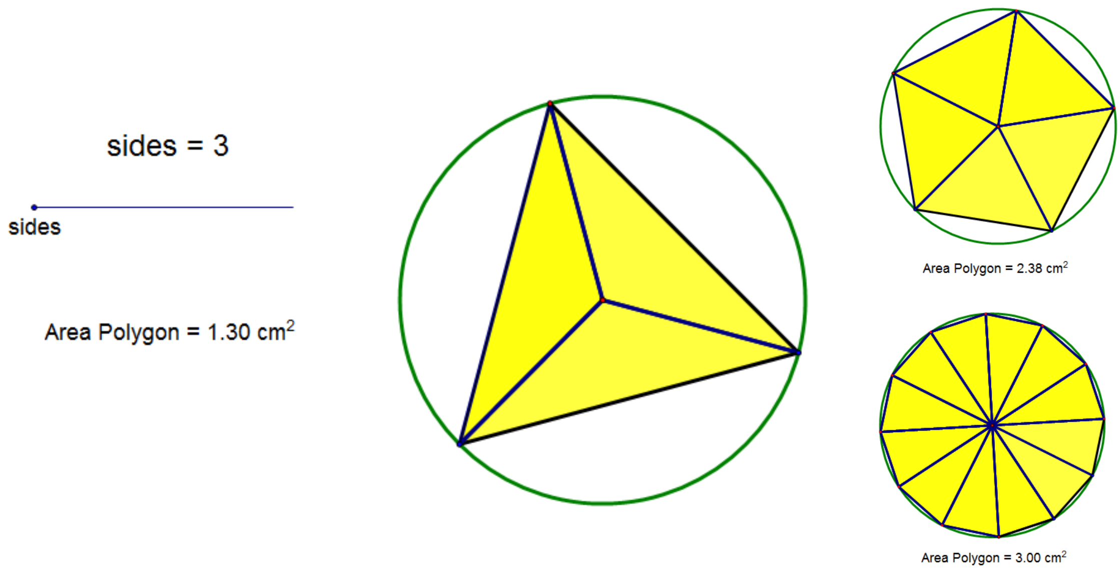


Figure 14. Method of Exhaustion

7.2. In Astronomy

7.2.1. The length of the Earth

Since the VI sec. B.C. it was known that the Earth was a sphere, but the first good approximation of its circumference is due to Eratosthenes of Cyrene in 240 B.C (we have the description of his method in *Caelestia* by Cleomede, 1990). Eratosthenes was the chief librarian at the Library of Alexandria, he knew that at local noon on the summer solstice in Syene (today it is Aswan), the Sun appeared at the zenith, directly overhead, since he could see that in the bottom of the wells. Eratosthenes used a gnomon to measure the Sun's angle of elevation at noon on the solstice in Alexandria, that angle was the 50th fraction of 360° (~ 7.12°). Since the Sun is far away from the Earth, its rays can be considered parallel lines, thus the angle that has the centre of the Earth as vertex and the lined passing through Syene and Alexandria as edges is again the 50th fraction of 360° (~ 7.12°). The distance from Syene to Alexandria was estimated at about 5000 stadia, thus the 'length of the Earth' should be 50 times that distance, which Eratosthenes estimated to be 250000 stadia. The value of the stadium used by Eratosthenes was 157.5 m, thus he estimated the length of the Earth as about 39375 kilometers, he did an error inferior to 2% compared to the actual measurement.

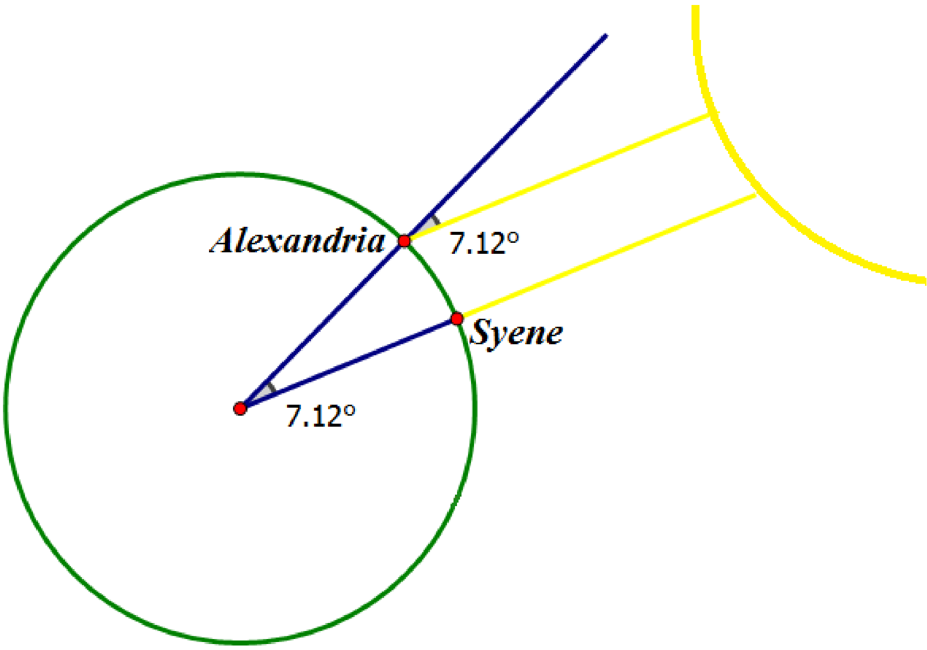


Figure 15. The length of the Earth

An error so small was always considered with great suspicion, especially because both the assumptions that Syene and Alexandria were on the same meridian, and that Syene was on the tropic are a rough approximation. The good result obtained by Eratosthenes was probably due to some errors that casually compensate each other. Firstly, the distance between Syene and Alexandria was measured in days trip, thus that distance in stadia was an estimation. Secondly, difficulties in measuring the angle. What is more, the correction of the circumference made from Eratosthenes from 250000 to 252000 stadia was probably done to obtain a number that is divisible by all the natural numbers from 1 to 10. Finally, there were different measurements for a stadium, and the value used by Eratosthenes is a factor of the result obtained as the length of the meridian (Russo, 1996).

7.2.2. Distance Venus-Sun

Using another property of the circle, in the first half of the XVI century, Nicolaus Copernicus was able to measure the distance between the Sun and the other visible Planets of our Solar system. For example, to calculate the distance between the Sun and Venus, Copernicus observed that when an inferior planet (a planet between the Sun and the Earth) is at the greatest elongation, there was the particular configuration showed in Figure 16.

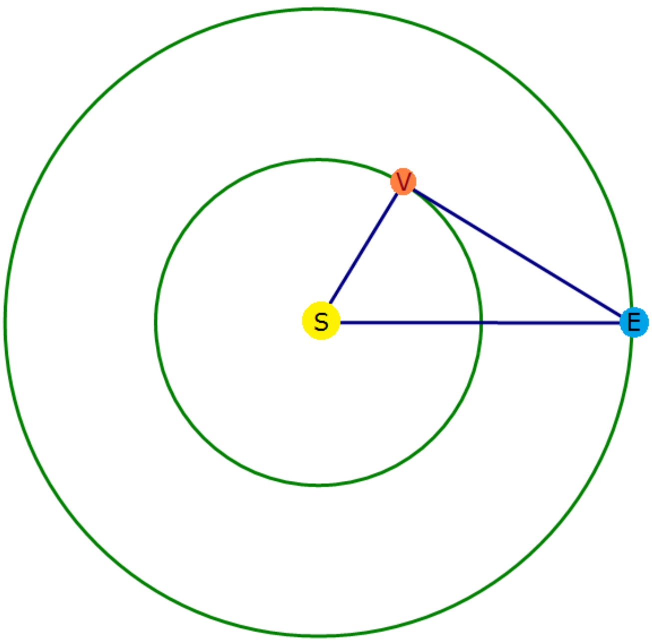


Figure 16. Distance Venus-Sun

The Venus-Earth line is tangent to the orbit of the Sun, then the Sun, Venus, and the Earth are at a right angle. From the Earth the angle *VES* could be measured, and it was about 46°, then $\sin(VES) = \frac{VS}{SE}$, and $VS = SE \sin(VES)$. The distance *SE* is one Astronomical Unit, or AU, thus the distance Venus-Sun is equal to $\sin(VES) = 0.72$ AU. At the time, Copernicus didn't know how big an AU was in everyday units to any degree of precision, but the value he obtained is very close to the actual measurement of the distance Venus-Sun, which is 108.2 millions of kilometers. Copernicus still considered the planet orbits as circles, but that assumption didn't influence the calculus of the distance Venus-Sun.

These examples aim to show students how mathematics could be used to solve real problems when people can't use other instruments. Students could use *Sketchpad* to model the problem with a diagram, and to work on that diagram with the tools of the DGE.

7.3. In the classroom

Besides the history of mathematics, it would be useful for teachers to use 'animations' to facilitate student understanding of some abstract concepts. For example, teachers could use *Sketchpad* to explain the first of Archimedes' prepositions on the 'measuring of the circle':

> The area of any circle is equal to a right-angled triangle in which one of the sides about the right angle is equal to the radius, and the other to the circumference, of the circle.

Teachers could shown a circle that is gradually 'unrolled' in a right triangle that has one side about the right angle that is equal to the radius, and the other side that is equal to the length of the circle.

Another interesting source of inspiration for an activity on the circle could be the *Apollonius' Problem*, which was posed and solved by Apollonius in his work *Tangencies*, but unfortunately it has been lost. The problem consists in constructing one or more circles that are tangent to three given circles, which can also degenerate to points or lines. This could be an interesting activity for students, because they would need to use the geometrical properties of the circle in order to find the circles that respect the given conditions. Moreover, a software like *Sketchpad* could give insights to the problem, since students have the possibility to construct the geometrical shapes using the tools provided.

The possible combinations of points, lines and circles are: PPP, LPP, LLP, LLL, CLP, CPP, CCP, CLL, CCL, CCC. Here I provide some examples of them.

PPP: *Find a circle that passes through 3 given points A, B, C.*

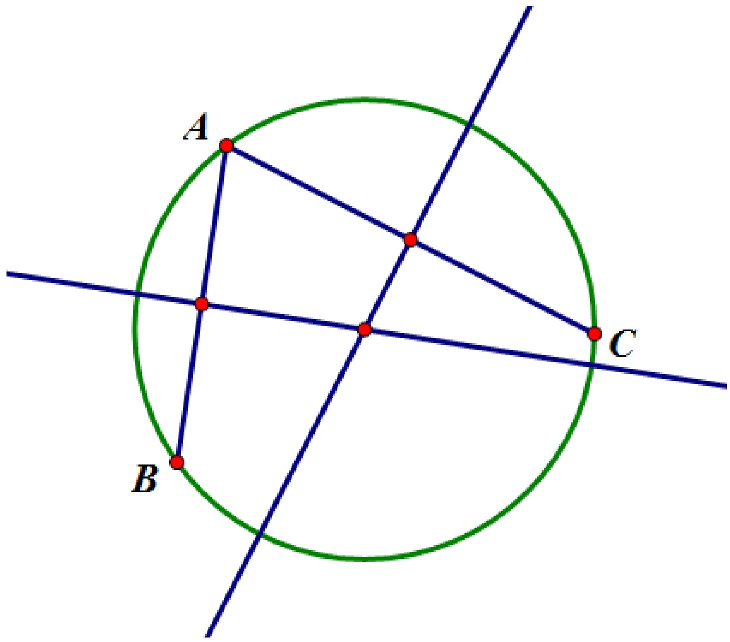


Figure 17. PPP

This is probably the simplest of Apollonius' problems, since it is easy to find out that the centre of the circle has to be the point of intersection between the perpendicular bisectors of two segments, for example *AB* and *AC*. Then, the radius of the circle is the distance between the centre and one of the point *A, B,* or *C*.

PPL: *Find a circle that passes through 2 given points A and B, and that is tangent to a given line r.*

This problem is a little more complicated, since the solution requires more steps. Firstly, it is necessary to take a point *T* on the given line *r*, and the perpendicular line to *r* in *T*. Then, the point of intersection between this new line and the perpendicular bisector of segment *AT* is the centre of the circle.

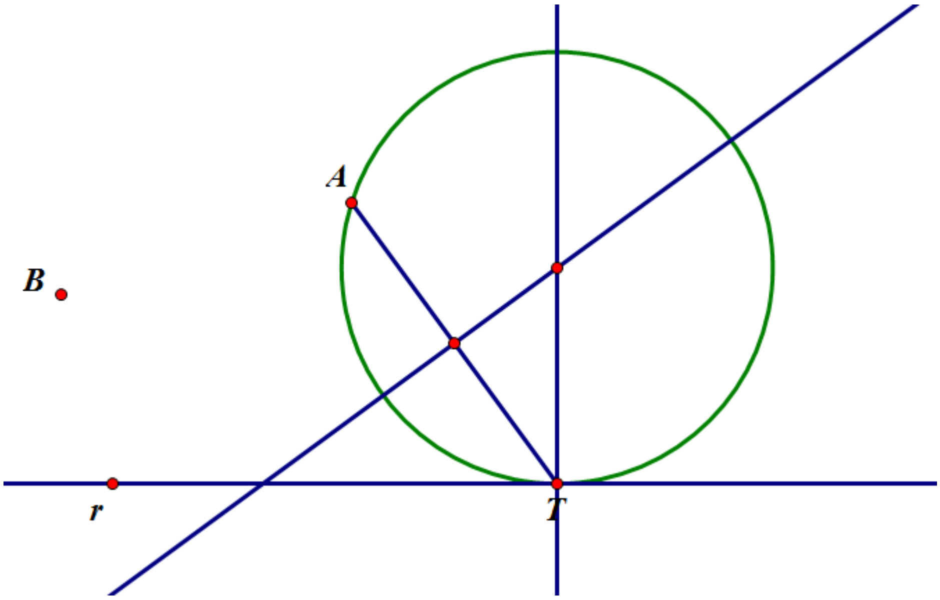


Figure 18. PPL1

However, this is not the circle of the solution, since it didn't pass through point *B*. In order to find the circle of the solution, it is sufficient to drag the point *T* on line *r* until the circle passes through the point *B*, or until the centre of the circle is on the perpendicular bisector of the segment *AB.*

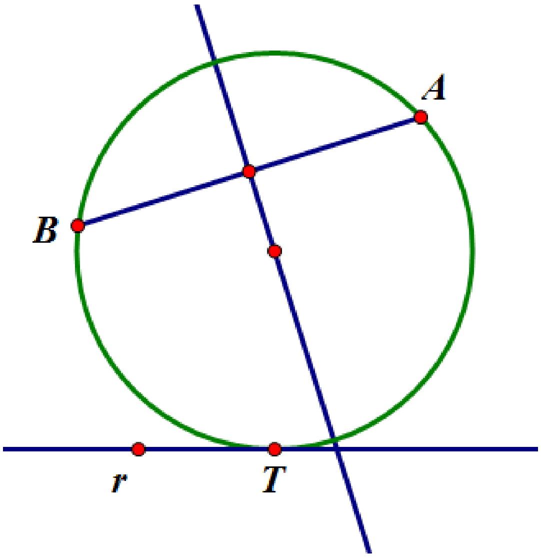


Figure 19. PPL2

There is also another circle that satisfies the given condition, and it can be found dragging the point *T* on line *r.*

LLP: *Find a circle that is tangent to 2 given lines r and s, and that passes through a given point A.*

There are two possible ways to solve the problem. The first one consists in taking a point *T* on line *s*. Then, the point *P* of intersection between the perpendicular bisector of the segment *AT* and the perpendicular line to *s* in *T* is the centre of the circle *C*.

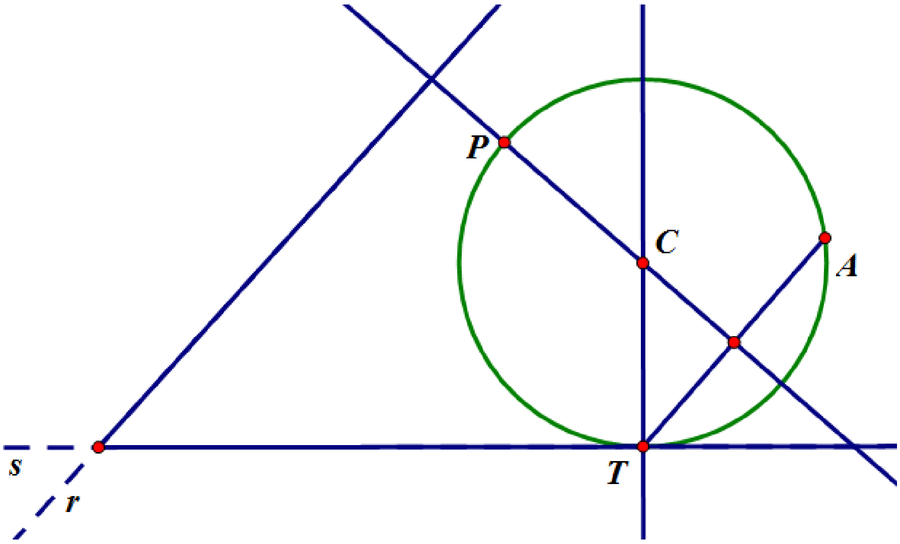


Figure 20. LLP1

Finally, it is sufficient to move the point *T* on line *s*, until the point *P* is on line *r.*

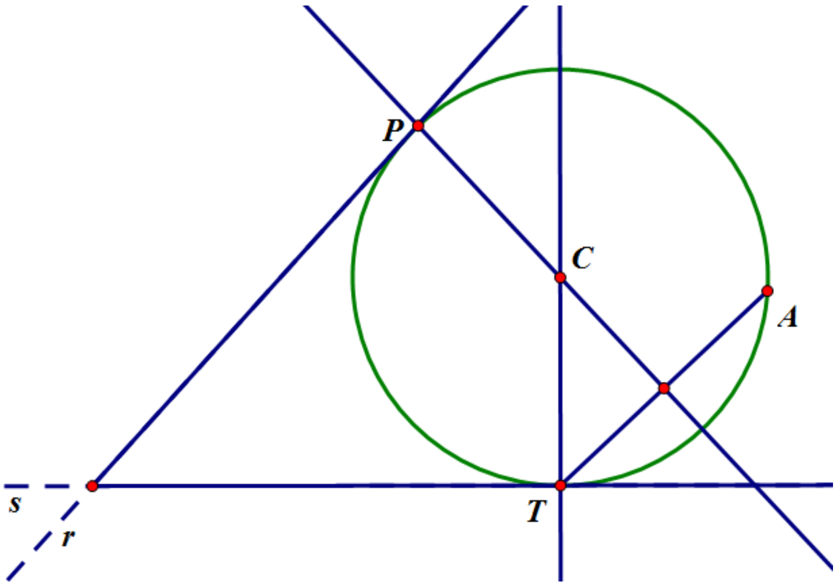


Figure 21. LLP2

Another way to solve the problem consists in taking the angle bisector of the angle between lines *r* and *s*, and a point *T* on line *s*. Then, the point *C* of intersection between the angle bisector and the perpendicular line to *s* in *T* would be the centre of the circle.

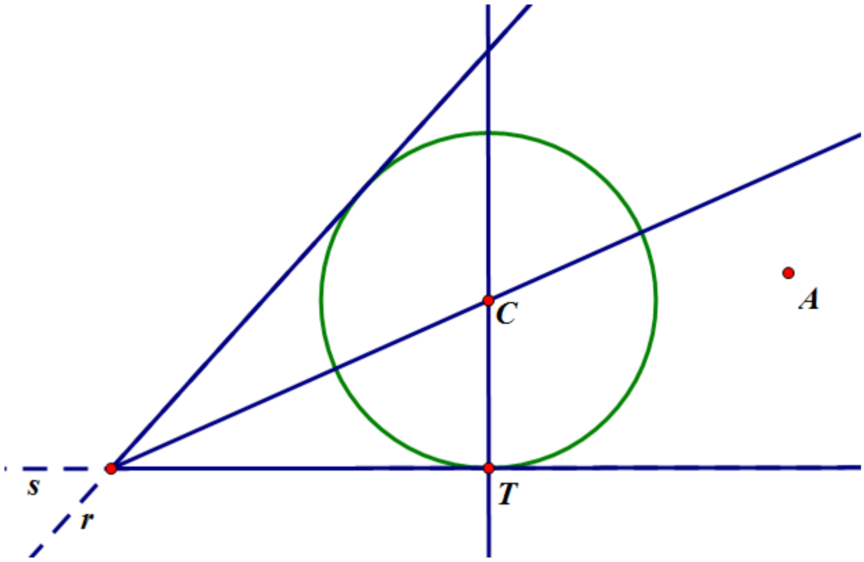


Figure 22. LLP3

Finally, it is sufficient to move the point *T* on line *s*, until the circle passes through point *A.*

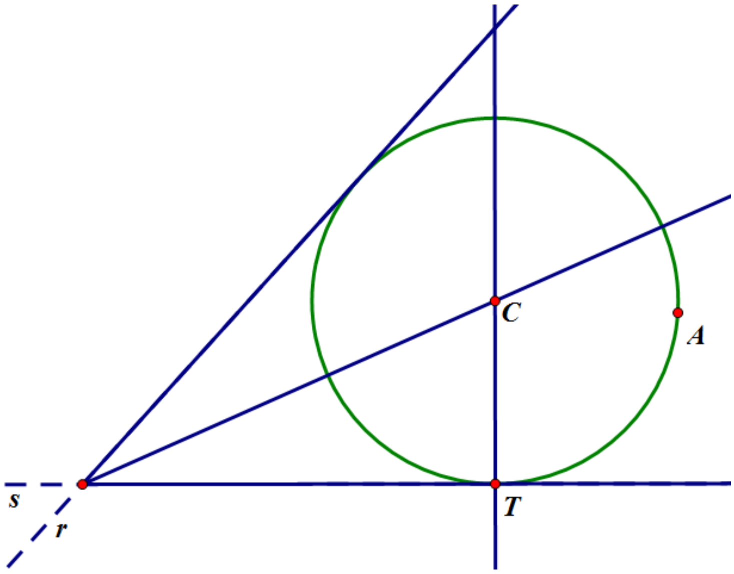


Figure 23. LLP4

There is another circle that is tangent to *r* and *s*, and that passes through *A*, it can be obtained dragging the point *T* on line *s*.

LLL: *Find a circle that is tangent to 3 given lines r, s and t.*

In order to find a circle tangent to three different lines, it is necessary to take the angle bisectors of two angles formed by the lines. Then, the point of intersection of the two angle bisectors is the centre of the circle *C*.

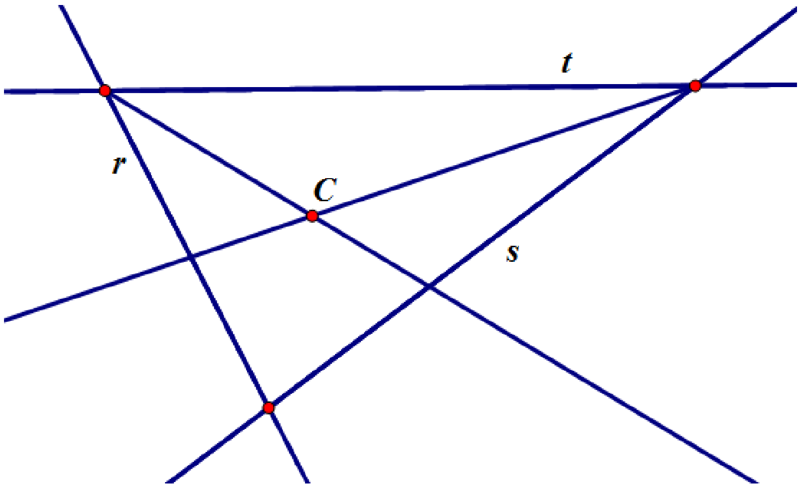


Figure 24. LLL1

Then, it is necessary to take the line that passes through the centre of the circle and that is perpendicular to one of the lines. The point of intersection between this line and its perpendicular line is point *T*, and the length of the segment *CT* is the radius of the circle.

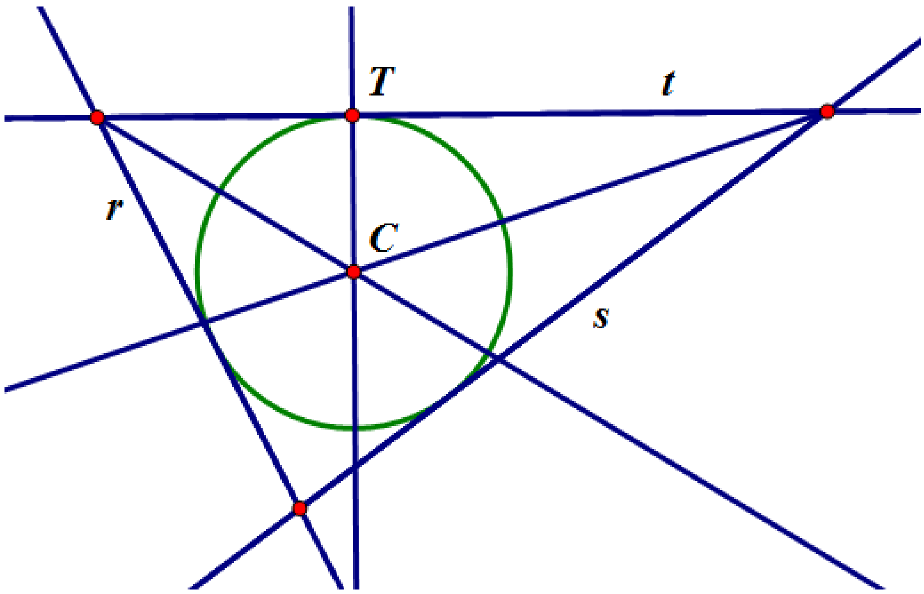


Figure 25. LLL2

However, there are three more circles that are tangent to the three given lines. They can be found in the same way: taking two angle bisectors and a perpendicular line passing through the centre of the circle to find the radius of the circle.

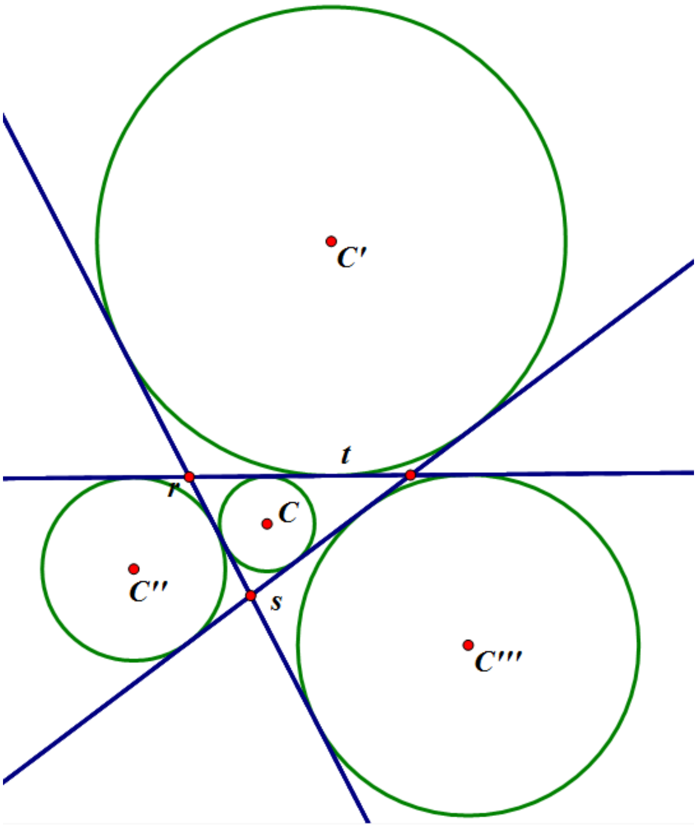


Figure 26. LLL3

The other problems involve the use of the *circle inversion*, which were pioneered by Julius Petersen in 1879. The inverse of a point *P* in a circle with center *C* and radius *r*, is a point *Q* on the radius *CP* that satisfies the condition:

$$\frac{CP}{r} = \frac{r}{CQ}$$

At the same way, it is possible to find the inverse of a line in a circle, and the inverse of a circle in a circle.

However, I do not provide the other constructions of the Apollonius' problems, since the aim of this section was just to give an example of what teachers and students can do on the circle in a DGE.

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Appendixes

Appendix A.

BC Curriculum grade 8-9 on Circle Geometry

Shape and Space (Measurement)

General Outcome: Use direct or indirect measurement to solve problems.

Prescribed Learning Outcomes	Suggested Achievement Indicators
It is expected that students will:	*The following set of indicators may be used to assess student achievement for each corresponding Prescribed Learning Outcome.* *Students who have fully met the Prescribed Learning Outcome are able to:*
C1 solve problems and justify the solution strategy using circle properties, including – the perpendicular from the centre of a circle to a chord bisects the chord – the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc – the inscribed angles subtended by the same arc are congruent – a tangent to a circle is perpendicular to the radius at the point of tangency [C, CN, PS, R, T, V]	❑ provide an example that illustrates - the perpendicular from the centre of a circle to a chord bisects the chord - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc - the inscribed angles subtended by the same arc are congruent - a tangent to a circle is perpendicular to the radius at the point of tangency ❑ solve a given problem involving application of one or more of the circle properties ❑ determine the measure of a given angle inscribed in a semicircle using the circle properties ❑ explain the relationship among the centre of a circle, a chord, and the perpendicular bisector of the chord

[C]	Communication	**[ME]**	Mental Mathematics and Estimation	**[PS]**	Problem Solving	**[T]**	Technology
[CN]	Connections			**[R]**	Reasoning	**[V]**	Visualization

Taken from the Ministry of Education (2008), p. 81.

Appendix B.

Questionnaire

1. How and when do you typically assess your students?
2. Where are you in this table?

You as a Teacher	Technology in Teaching	No Technology in Teaching
Technology in Assessment		
No Technology in Assessment		

3. What is the role of technology in your teaching and in your assessment? Give an example.
4. Do you see any differences in what you learn about what your students know between technology-based and paper-and-pencil assessment tasks? If so, which differences?
5. Do you think digital technologies are more useful for assessing and teaching some aspects of learning than others? If so, which aspects?
6. Do you think it is valuable to assess only the mathematical competencies, or also the technological competencies (like choosing the appropriate tools, using them in the right way..)?
7. What forms of feedback do you give to your students? When do you give feedback to them? What kind of feedback do you give to your students when they are working with technology?

DESIGN A TASK

Max taught the Circle Geometry unit in the BC curriculum (grade 9) by using dynamic geometry software (Sketchpad) with his classroom set of laptops. Now he has to assess students' learning. They are accustomed to use Sketchpad in both learning and assessment situations.

Design a task to evaluate students' understanding of *one of these properties of the circle* that they learnt in class:

1. the perpendicular from the centre of a circle to a chord bisects the chord
2. the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
3. the inscribed angles subtended by the same arc are congruent
4. a tangent to a circle is perpendicular to the radius at the point of tangency
5. a triangle inscribed in a semicircle is right

Appendix C.

Questions for the Interview

This list of questions is a draft, because when I actually interviewed the teachers, I asked questions in order to follow the path where the discussion was going.

Thinking about your task:

- How might students' experience in using technology affect their response?
- Usually, where do you take the ideas for your assessment tasks?

Show 'my tasks' on Circle Geometry to the teachers.

- What is this task assessing (in terms of content and process)?
- In your opinion, how is the technology affecting a student's response?
- How might the use of technology for this task affect what you learnt about your students' understanding?

While we are talking about these two last questions, show some tasks and students' answers.

- Choose the task that you like the most. Why do you like it?
- Choose the task that you dislike most. Why do you dislike it?
- If you were Max, would you use these tasks to assess your students? Why?
- What advantages and disadvantages do you see in using technology to assess student understanding?
- If you were using this task in your class, would you change something? How do you think your students would answer to this task?
- Would you consider a diagram an answer? Do you think you can learn something about student understanding from a diagram?
- If Max would use the task of the ball in class instead of using it in the assessment, what would be different?
- How do you choose the tasks for the activity in class and the tasks for the assessment?

  What's the difference between a task for the classroom and a task for the assessment?
- Do you think students can continue learning during the assessment? Why?

Show teachers Laborde (2001) categorization to make them aware of the different kinds of tasks that they can design for assessment purpose.

In regard to *student feedback*:

- What value do you give to feedback? what kind of feedback do you give to your students? (also after class activities and *informal* assessment)

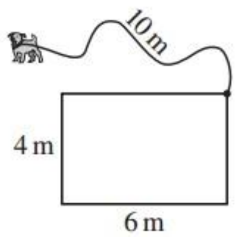
Appendix D.

The Dog Sketch

I was looking to design some tasks that would be suitable for use with *Sketchpad*, and I found the ‘dog leash task’ in a *Kangourou des Mathématiques* test (March, 2007)[2].

KANGOUROU 2007 J-3

13 Une laisse de 10 mètres est attachée à un coin d’une maison rectangulaire de côtés 6 m et 4 m.
Un chien est attaché au bout de la laisse.
Quel est le périmètre de la surface que peut parcourir le chien ?
A) 20π B) 22π
C) 40π D) 88π
E) 100π



I was drawn to it because *Sketchpad* could act as a visual amplifier for students trying to solve the problem. The sketch helps students’ imagination: they can explore the situation by dragging the dog around the screen, and they can see where the dog actually can go without breaking the leash.

Most of the teachers I interviewed use *Sketchpad* in their practice, and they design sketches for their lessons in mathematics. When they tried this sketch, they were very impressed for the ‘bending’ of the leash around the house, and they wondered how I did it. The difficulty in designing the sketch was to find a way to limit the area where the dog can go without breaking the leash: the ‘accessible area’. I tried many different unsuccessful ways, and finally I came up with a *polar translation* (a linear translation in polar coordinates) of the *dog point* into the *corner point*: when you move the dog, the *dog point* is translated into the *corner point*, which is a point situated in one of the corner of the house, its position depends on where the dog is. The translation is a *polar translation*, with angle θ and distance ρ, which are expressed in two formulas that give the correct angle and distance as a function of the position of the *dog point*. Then, I created the leash, which is the segment between the *dog point* and the *corner point*. Thus, if the dog (*dog point*) is in a position outside the ‘accessible

[2] The *Kangourou Sans Frontières* Association organizes the *Kangourou* game-competition for more than four million participants from the all over the world.

area' the translated point (*corner point*) does not exist, and neither does the segment that connects them (the leash). I provide a detailed description of the mathematical technique I used to model the problem in *Sketchpad.* Firstly, I divided the area where the dog can room in three different parts that I coloured in yellow, blue and green, as shown in Figure 29.

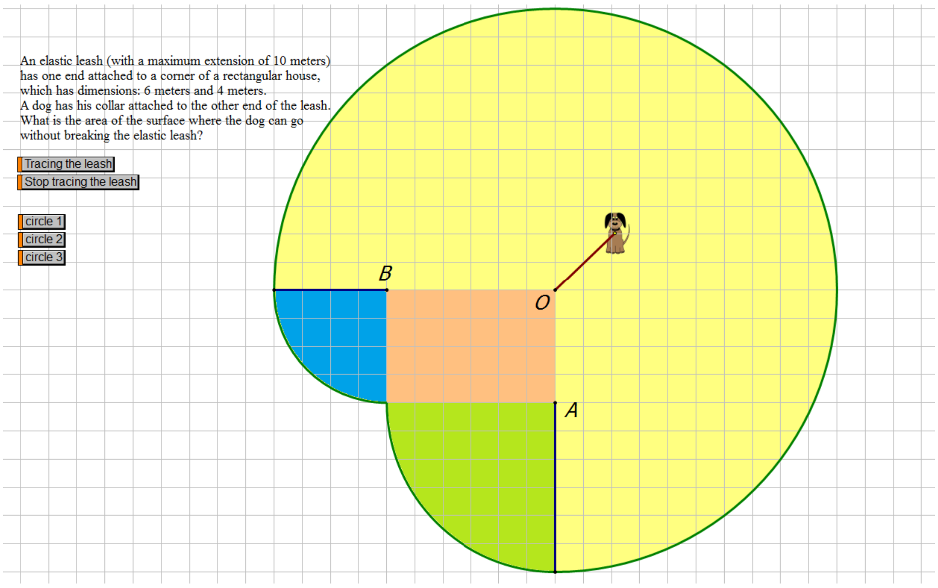


Figure 27. Accessible Area

There were two major problems:

1. the leash should bend around the house when the dog goes around the corner;
2. the leash should break when the dog goes out from the 'accessible area'.

The *polar translation* allowed me to solve both of them:

1. The *corner point* is the point obtained through the *polar translation* of the *dog point*. Thus, when the dog is inside the 'accessible area', the *corner point* is translated to the vertex of the house O, where the leash is tied. If the dog turns around the house, the *corner point* is translated to one of the vertex of the house: *A* or *B*.

2. I decided to create the formula for the angle of the *polar translation* with a fraction: if the dog goes out of the limits imposed by the leash, the denominator of the fraction goes to 0. Thus, the angle of the *polar translation* does not exist, and neither the segment connecting the point 0 with the *corner point*.

These are the conditions for limitations of the three different sections:

$$(1)\ \ OD \leq 10 \wedge [\, x \geq 0 \vee (x \leq 0 \wedge y \geq 0)]$$

$$(2)\ \ AD \leq 6\ \wedge\ [\, x < 0 \wedge y \leq -4\,]$$

$$(3)\ \ BD \leq 4\ \wedge\ [\, x \leq -6 \wedge y < 0\,]$$

Using the measure tool of *Sketchpad*, these conditions give 1 as result, if the *dog point* is in the area, and they give 0 as result, if the *dog point* is outside the area. These are the values θ and ρ for the *polar translations* for every position of the *dog point*:

$$\theta = \begin{cases} \arctan\left(\dfrac{y_D}{x_D}\right) \\ \arctan\left(\dfrac{y_D + 4}{x_D}\right) \\ \arctan\left(\dfrac{y_D}{x_D + 6}\right) \end{cases} \qquad \rho = \begin{cases} -DO & if\ x \leq 0\ \ (1a) \\ DO & if\ x \geq 0\ \ (1b) \\ BD & \\ AD & \end{cases}$$

I wanted to use all of them in a formula for the angle θ, and in a formula for the radius ρ. These are the formulas I came up with:

$$\theta = \frac{1}{[\,(1) \vee (2) \vee (3)]} \cdot [\, \theta_1 \cdot (1) + \theta_2 \cdot (2) + \theta_3 \cdot (3)\,]$$

$$\rho = \rho_{1a} \cdot (1a) + \rho_{1b} \cdot (1b) + \rho_2 \cdot (2) + \rho_3 \cdot (3)$$

Sketchpad is not a software for programming, thus the Boolean algebra was pre-constructed with the creation of new tools in *Sketchpad*. The software allows to create functions, and then save them as tools. It provides some basic functions, like the function *sign(x)* and *abs(x).* These are the constructed logical operators:

AND $(x \wedge y)$: $|sign(x \cdot y)|$

OR $(x \vee y)$: $sign(|sign(x)| + |sign(y)|)$

NOT $(\neg x)$: $1 - |sign(x)|$

Other tools were used for comparison with 0, and other values or functions. I provide the formulas I used. These formulas should give 0 or 1 as solution, for example when x = 3 and y = 2 :

$$x<0 \qquad \frac{1-\operatorname{sgn}(0.5+\operatorname{sgn}(x))}{2}=0 \qquad\qquad x<y \qquad \frac{1-\operatorname{sgn}(0.5+\operatorname{sgn}(x-y))}{2}=0$$

$$x\leq 0 \qquad \frac{1-\operatorname{sgn}(-0.5+\operatorname{sgn}(x))}{2}=0 \qquad\qquad x\leq y \qquad \frac{1-\operatorname{sgn}(-0.5+\operatorname{sgn}(x-y))}{2}=0$$

$$x>0 \qquad \frac{1+\operatorname{sgn}(-0.5+\operatorname{sgn}(x))}{2}=1 \qquad\qquad x>y \qquad \frac{1+\operatorname{sgn}(-0.5+\operatorname{sgn}(x-y))}{2}=1$$

$$x\geq 0 \qquad \frac{1+\operatorname{sgn}(0.5+\operatorname{sgn}(x))}{2}=1 \qquad\qquad x\geq y \qquad \frac{1+\operatorname{sgn}(0.5+\operatorname{sgn}(x-y))}{2}=1$$

Figure 30 shows all the formulas I used to implement the sketch that are hidden to the user.

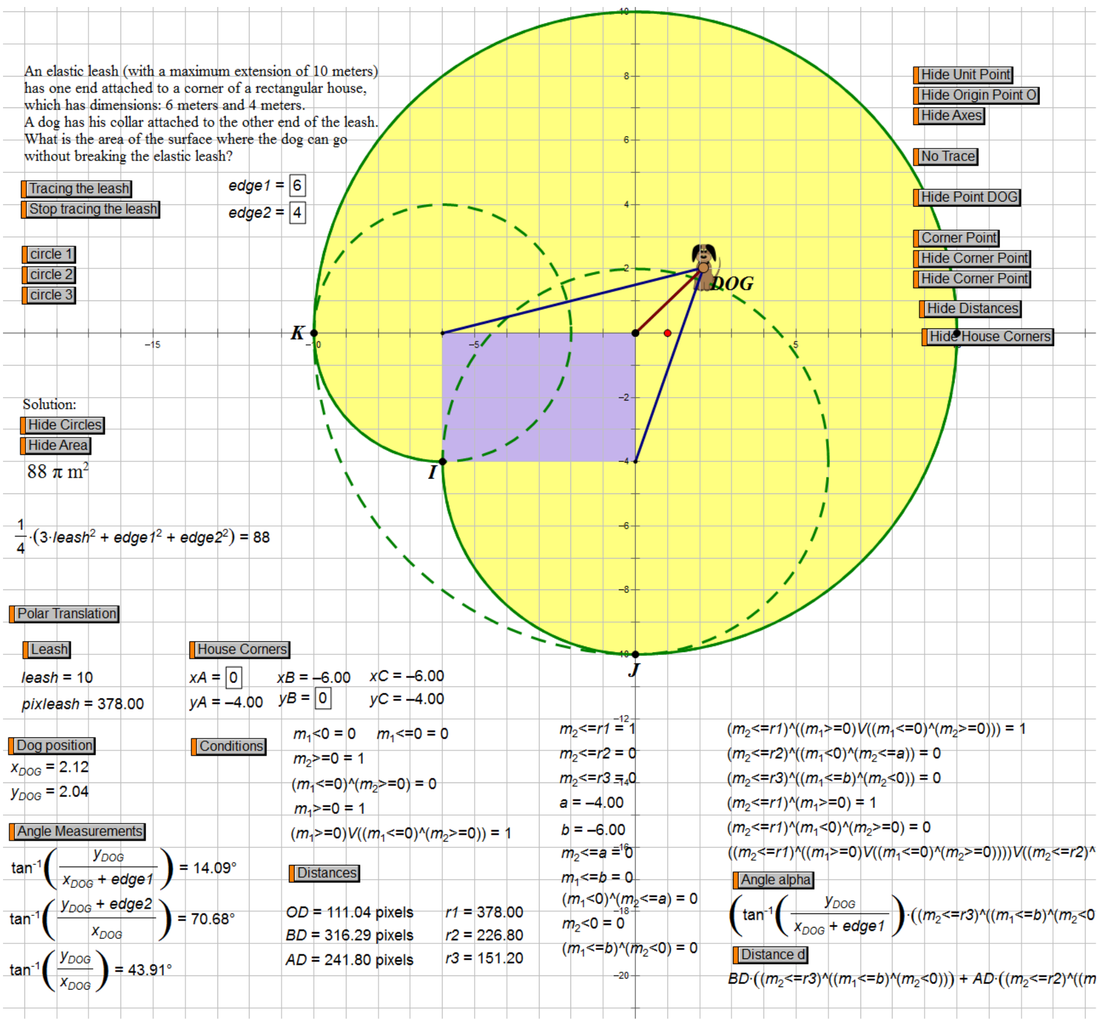


Figure 28. Show all hidden