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#### ESSAYS ON ASSET TRADE

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# Essays on Asset Trade

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## Contents

1	Efficient Asset Trade - A Model with Asymmetric Informa-									
	tion and Asymmetric Liquidity Needs									
	1.1	Introduction	1							
	1.2	Model	9							
	1.3	Perfect Bayesian Equilibrium	12							
		1.3.1 Separating equilibrium	13							
		1.3.2 Pooling equilibrium	17							
		1.3.3 Equilibrium characterization	22							
	1.4	Welfare Analysis	23							
		1.4.1 Profits	23							
		1.4.2 Welfare	24							
	1.5	Conclusion	26							
<b>2</b>	(In)Efficient Asset Trade and a rationale for a Tobin Tax 28									
	2.1	Introduction	28							
	2.2	Model	32							
	2.3	Perfect Bayesian Equilibrium	35							
		2.3.1 Equilibrium existence	36							
		2.3.2 Equilibrium characterization	41							
	2.4	Welfare	42							
		2.4.1 Profits	42							
		2.4.2 Welfare	43							
		2.4.3 Welfare analysis	44							
	2.5	Conclusion	46							

3 Op	timal Timing of Asset Purchases	48					
3.1 Introduction							
3.2	Model	52					
3.3	Perfect Bayesian Equilibrium	54					
	3.3.1 Separating equilibrium	55					
	3.3.2 Pooling equilibrium	62					
3.4	Discussion	66					
3.5	Conclusion	67					
Apper	ndices	68					
Apper	ndix A Efficient Asset Trade - A Model with Asymmetric	;					
Infe	ormation and Asymmetric Liquidity Needs	68					
A.1 Conditions for the ranking of liquidity thresholds							
A.2	Proof of proposition 3	69					
Apper	$\operatorname{Mix} \mathbf{B} \hspace{0.1 in} (\operatorname{In})  ext{efficient} \hspace{0.1 in}  ext{asset} \hspace{0.1 in}  ext{trade} \hspace{0.1 in}  ext{and} \hspace{0.1 in}  ext{a} \hspace{0.1 in}  ext{trade}  ext{and}  ext{a} \hspace{0.1 in}  ext{trade}  ext{trade}  ext{a}  ext{trade}  ext{a}  ext{trade}  e$	L					
Tax	Σ.	72					
B.1	Proof of proposition 7	72					
Apper	ndix C Optimal Timing of Asset Purchases	75					
C.1	Incentive Compatibility in the separating equilibrium in T=1 $$	75					
C.2 Optimality in the separating equilibrium in $T=1$							
C.3	Incentive Compatibility in the pooling equilibrium in T=1	77					
Biblio	graphy	78					

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## Chapter 1

# Efficient Asset Trade - A Model with Asymmetric Information and Asymmetric Liquidity Needs

#### 1.1 Introduction

Do asset prices efficiently guide the allocation of investment (Hayek, 1945)?

The manager of a firm takes into account his firm's asset price on the secondary stock market when taking an investment decision. The manager takes into account his firm's asset price, because there are traders on the stock market who have, in addition to the manager's information, information about the perspective of the investment opportunity which are displayed in the stock price.<sup>1</sup> The information may concern appropriate capital cost of the investment, the competitive situation of the firm after the investment or future demand of the economy. Consider the following example for superior information about competitiveness with the two following char-

<sup>&</sup>lt;sup>1</sup>Empirical evidence supporting the hypothesis that corporate investment is guided by the stock price is provided by Durnev, Morck and Yeung (2004); Luo (2005) and Chen, Goldstein and Jiang (2007).

acteristics. Innovation is incremental rather than radical and the firm is relatively small. A public firm, listed in a minor stock index, develops a new version of its product. This firm is not big enough yet to have a marketing department providing a worldwide market analysis for the new version of the product. A big investment firm has the facilities to perform a such market analysis and evaluate the future demand. Clearly, with a radical innovation, also the investment firm would not be able to evaluate future demand.

What is the nature of traders who have the kind of aforementioned information? These are large traders. Three examples of Financial Markets' traders for which large traders are a prominent phenomenon nowadays are Investment Management Firms, Hedge Funds and Mutual Funds. Being large has three major features. First, a large trader has the capacity to acquire information. He employs regional or topical specialists in order to evaluate the prospective demand and the prospective competitive situation. Moreover, banks often have business ties to public firms so that they are able to evaluate the firm's financial situation in comparison to its competitors. Second, a large trader's transaction moves the market.<sup>2</sup> And third, a large trader is likely to have different liquidity needs with respect to small traders.

Liquidity needs are reflected by borrowing rates. In the US, there are different borrowing rates. There is a small number of traders who have access to the FED Funds Rate. Those are the Primary Dealers<sup>3</sup> who are eligible to engage in repurchase agreements (REPOs) with the FED. Essentially, the FED provides a collateralized debt to the Primary Dealers. Currently, there are 22 Primary Dealers. Among them only the biggest financial institutions in the world in terms of assets under management (AUM). For example BNP Paribas, Barclays, Credit Suisse, Deutsche Bank, Goldman Sachs, J.P. Morgan, Morgan Stanley, Nomura and UBS. Most of the other investment institutions face the Bank Prime Loan Rate which is offered by banks to

 $<sup>^2 {\</sup>rm These}$  first two aspects are also standard assumptions in the literature following Kyle (1985).

<sup>&</sup>lt;sup>3</sup>The list of current and historic Primary Dealers can be found on New York's Fed website: http://www.ny.frb.org/markets/pridealers\_current.html\#tabs-1.

their most favorable clients. The Bank Prime Loan Rate and the FED Funds Rate are depicted in figure 1.1. There is a systematic difference

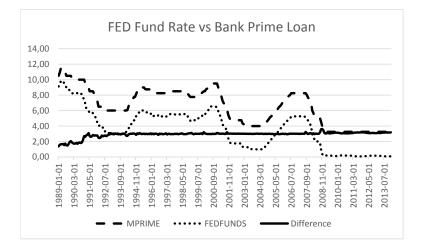


Figure 1.1: Asymmetric borrowing rates

in the borrowing rate of the Primary Dealers, i.e. the FED Funds Rate, and the borrowing rate of most of the other Financial Market participants, i.e. the Bank Prime Loan Rate. Facing a relatively high borrowing rate, Financial Market participants who hold assets can, instead of borrowing, sell their assets. Since Primary Dealers face a lower borrowing rate, they can borrow money and buy the assets from the other Financial Market participants. The latter are willing to decrease the asset price, at which they sell, proportionally to their liquidity needs. This creates a motive for trade.

There is a growing literature studying the question of whether asset prices efficiently guide the allocation of investment. The debate in the literature evolves around the question, whether asset prices reveal informed traders' information about the investment opportunity.

This paper introduces asymmetric liquidity needs between the informed trader and the uninformed traders which allows to provide a complete welfare analysis. From Tirole (1982) and Milgrom and Stokey (1982), it is well understood that there is no trade if traders have asymmetric informa-

tion only. Trade requires a second asymmetry. Asymmetric liquidity needs are one natural asymmetry serving the purpose of generating trade. In a related paper, Biais, Foucault and Moinas (2014) consider fast and slow traders which is another asymmetry in the context of High Frequency Trading. Under asymmetric liquidity needs, I provide conditions under which information revelation yields greater welfare than no revelation. This is the intuitive result. Laffont and Maskin (1990) however show that if the variance of the investment opportunity is sufficiently small, welfare without information revelation is larger. Their result is driven by the fact that private information has no social value. By adding the firm with its investment opportunity, I give social value to private information. Still, no information revelation can yield greater welfare since welfare depends on both information and the amount of trade. Since the expected amount of trade is larger with no information revelation, welfare can be larger if the gain from information revelation is sufficiently small, i.e. the variance of the investment opportunity is small.

The model considered goes as follows. There is a manager of a firm facing an investment opportunity with uncertain outcome, either good or bad. When the manager decides upon the investment, abstracting from any moral-hazard issues<sup>4</sup>, he takes into account the firm's asset price on the secondary stock market, i.e. the manager updates his prior beliefs about the outcome of the investment opportunity. He does so because there is an informed trader who has information about the perspective of the investment opportunity. Depending on whether the firm's asset price (does not) reveals available information, the manager takes an (in)efficient investment decision. In the case in which the asset price does not reveal information, the manager over invests (under invests) in the bad (good) state. Information revelation, and thus the inefficiency, is determined by the interaction of asset traders.

Asset trade takes place between an informed trader and uninformed traders in a model à la Laffont and Maskin (1990). The informed trader

<sup>&</sup>lt;sup>4</sup>Moral hazard of the firm manager would create an additional inefficiency. Essentially, the investment level would be further decreased due to shirking of the manager. This would change the level of investment but not the qualitative results of the paper.

observes either good or bad information about the investment prospect of the firm. The uninformed traders are holding the assets of the firm, i.e. they are the owners of the firm. The uninformed traders are a large amount of stock holders with little asset holding each. This is to say they do not communicate directly with the management. Furthermore, the management has no superior information to the uninformed traders so that even if there was communication between owner and management, the owner would not learn any inside information. By observing the demand of the informed trader, the uninformed traders update their beliefs about the quality of the asset and decide whether to sell<sup>5</sup>. Notice, the informed trader takes into account the effect of his purchase on the asset price. Notice however that the market clearing mechanism is not explicitly modeled. Instead, P satisfies the break even condition of the uninformed trader. And B is determined such that it satisfies incentive compatibility and participation of the informed trader.

There are two types of pure strategy equilibria. First, a separating equilibrium in which the informed trader reveals private information by demanding a larger quantity when he has good information than when he has bad information<sup>6</sup>. The equilibrium price hence is either high or low. In order for trade to take place, in either state, the uninformed trader has to be more liquidity constrained than the informed trader. The equilibrium asset prices depends on the liquidity needs of the uninformed trader. Or differently, when the uninformed trader needs liquidity, he is willing to decrease the price at which he sells the assets proportional to his borrowing costs. Trade occurs for an infinitesimal small difference in liquidity needs. Since the asset prices reveal available information, the firm's manager takes an efficient investment decision and therefore the firm value is maximized given the observed information.

Second, there exists a pooling equilibrium in which the informed trader does not reveal private information by demanding the same quantity no matter whether he has good or bad information. Then the uninformed trader

 $<sup>^5{\</sup>rm The}$  same results carry through if markets are anonymous and risk-neutral, competitive market makers clear demand and supply.

<sup>&</sup>lt;sup>6</sup>Either quantity is observed by the uninformed trader.

cannot infer information from the informed trader's demand and hence stays with the prior beliefs. In the pooling equilibrium, the asset price reflects the expected value of the asset which is below the prospect of the informed trader with good information and above the prospect of the informed trader with bad information. Just like in the separating equilibrium, also in the pooling equilibrium, the asset price depend on the liquidity needs of the uninformed trader. Since for the informed trader with good information the pooling equilibrium price is relatively low in comparison to his prospect, he is always willing to buy. The informed trader with bad information however is only willing to buy if the negative difference between his prospect and the expected value of the asset is outweighed by the uninformed trader's liquidity needs. In other words, in the pooling equilibrium, the uninformed trader needs to be more liquidity constrained than in the separating equilibrium for trade to take place between the uninformed trader and the informed trader with bad information. With an uninformative asset price, the firm's manager over (under) invests in case of bad (good) information. Given available information, the inefficient investment leads to a lower firm value than in the separating equilibrium.

I show that separating equilibrium and pooling equilibrium co-exist if the variance of the investment's outcome is relatively small and the difference in liquidity needs is intermediate. More generally, this characterizes a situation in which gains from asset trade for the informed trader are moderate. The welfare analysis is carried out for the set of parameters for which separating equilibrium and pooling equilibrium co-exist.

Welfare is defined as the ex-ante profit of the informed trader and the uninformed trader. The profits of the traders increase in the quantity traded and for the ones owning assets of the firm, in information revelation. Since traders are risk neutral, there exist equilibria in which all possible assets are traded, i.e. there are corner solutions. In the pooling equilibria the informed trader purchases always the maximal amount of assets. In order for a separating equilibrium to exist, the informed trader with bad information has to trade less than the informed trader with good information. Therefore, in expectation, the pooling equilibrium exhibits more trade than the separting equilibrium. Therefore, there is a trade-off for the traders between quantities traded and information revelation. In fact, an equilibrium in which information is not revealed (pooling equilibrium) can yield higher welfare if the gain from information revelation (separating equilibrium) is small. The gain from information revelation is small if the variance of the prospective outcome of the investment is small. The welfare analysis therefore provides conditions on the quantities traded and the variance of the prospective outcome of the investment.

The allocational role of asset prices (=feedback effect) has been studied in papers by Leland (1992), Dow and Gorton (1997), Subrahmanyam and Titman (2001), Dow and Rahi (2003), Goldstein and Guembel (2008) and Edmans, Goldstein and Jiang (2014).<sup>7</sup> These papers have in common, asset prices do not always reveal available information and hence lead to inefficient investment allocation. There are two interrelated drawbacks of these kind of models.

First, the non-informativeness of the prices is exogenous. The reason why prices are non-informative is that there are traders aside the informed trader and the asset owners, who are trading for non-asset related motives. Since uninformed traders cannot distinguish between information trading and other non-asset related trading, the asset price not only reflects asset related information but also other non-asset related information. In Goldstein and Guembel (2008), Subrahmanyam and Titman (2001) and Edmans, Goldstein and Jiang (2014) these are passive noise traders. In Dow and Rahi (2003), though active, they are uninformed traders buying or selling for exogenous endowment shocks. The fact that prices do not reveal available information in these models is exogenous and thus the inefficient investment allocation. As such, noise traders are just a technical issue. In fact, the price in the pooling equilibrium resembles the price in an equilibrium with a lot of noise trading and the prices in the separating equilibrium approximately occur in models with noise trading when there is almost no noise trading. In

<sup>&</sup>lt;sup>7</sup>The feedback effect literature differs from e.g. Medrano and Vives (2004) insofar as asset trade affects the investment level, whereas in Medrano and Vives (2004) investment takes place before private information is observed and assets are traded.

the noise trader models however, there cannot exist an equilibrium exhibiting full information revelation unless, on top of asymmetric information, there is an additional asymmetry between informed trader and uninformed traders. The never full information revelation result is somewhat artificial. Models à la Laffont and Maskin (1990) exhibit equilibria in which both information revelation and no information revelation occur. The major drawback of noise traders is that their preferences are not specified and hence welfare cannot be analyzed. Although for a slightly different setup, Medrano and Vives (2004) show that welfare analysis with noise traders is often mislead.

Which brings about the second drawback. The overall welfare analysis is unclear and therefore it is not possible to evaluate the effect of asset prices on the real economy. Goldstein and Guembel (2008), Subrahmanyam and Titman (2001) and Edmans, Goldstein and Jiang (2014) refrain from a welfare analysis altogether since the motive, and hence the profit, of the noise traders is, at best, unclear. Indeed, most of the time, noise traders make negative profits. Dow and Rahi (2003) provide an incomplete welfare analysis insofar as they cannot define whether the informed trader's profit from non-information revelation outweighs the loss of the uninformed trader.

Considering a model with perfect competition among informed traders such as Subrahmanyam and Titman (2001) and Dow and Rahi (2003) but without noise trading, would always yield information revelation through asset prices (Grossman and Stiglitz (1980)) and hence will never create an inefficiency. Therefore, models with perfect competition among informed traders should not be subject of concern in discussions of inefficient Financial Markets. A model which creates endogenous inefficiency is a model with a monopolistically informed trader<sup>8</sup>. Laffont and Maskin (1990) propose such a model. Differently from the aforementioned literature, the model of Laffont and Maskin (1990) exhibits an equilibrium in which information is not revealed in the price by choice of the traders. Laffont and Maskin (1990) do not study however which effect the asset price has on real investment decisions. This paper builds on Laffont and Maskin (1990) and adds a

<sup>&</sup>lt;sup>8</sup>Another model creating endogenous inefficiencies considers career concerns of investment managers (Dasgupta and Prat (2006)).

welfare analysis which incorporates both, the secondary financial market and the real economy.

I suggest a model which alleviates the two aforementioned issues in the existing literature, i.e. a model in which (i) the inefficiency is driven by preferences and (ii) a complete welfare analysis is carried out.

In the remainder of this paper, section 1.2 presents the model set-up. In section 1.3, I derive both separating equilibrium and pooling equilibrium. The welfare comparison between the two types of equilibria is carried out in section 1.4 and section 1.5 concludes.

#### 1.2 Model

The model has five dates  $t \in \{0, 1, 2, 3, 4\}$  and a firm whose stock is traded in the financial market. There are two types of risk-neutral traders  $i \in \{I, U\}$ . An informed trader I and uniformed traders U of measure E. Each of the uninformed traders holds one unit of the entire stock of the asset. In line with their little asset holding, the uninformed traders are assumed to be in perfect competition and thus price takers. Throughout the model they are treated as one representative agent with an asset holding of E. Informed trader and uninformed trader have different liquidity needs. Liquidity needs are modeled with discount factors  $1 > \delta_i > 0$ . The higher  $\delta_i$  the less liquidity constrained is the trader. Assume, the informed trader is less liquidity constrained than the uninformed trader,  $\delta_I > \delta_U$ . This is in line with the stylized fact depicted in figure 1.1. The uninformed traders own assets of a firm which faces an uncertain investment opportunity  $V \in \{V_H, V_L\}$ .

In t = 0, the informed trader observes private information  $\omega \in \{H, L\}$ about the profitability of the firm's investment opportunity. With probability  $0 \le \beta \le 1$ , the firm's investment opportunity yields a payoff  $V_H$  and with probability  $1 - \beta$ ,  $V_L$ . Where  $V_H > V_L$ . Alternatively, the informed trader can invest in a riskless asset of which the revenue is normalized to 0, i.e. both the riskless rate and the revenue of the asset are 0. In t = 1, the informed trader decides to buy  $E \ge B \ge 0$  assets from the uninformed trader. In t = 2, the uninformed trader observes the informed trader's demand and decides to sell or to keep the assets. In t = 3, the firm observes the asset price P and takes its investment decision k. Eventually, in t = 4, either the high payoff  $V_H$  or the low payoff  $V_L$  realizes. The timeline is depicted in figure 1.2

Informed trader	Informed trader	Uninformed trader observes <i>B</i> and	Manager observes P and decides to	$V_H$ or $V_L$ realizes	
observes $\omega$	buys B assets	decides to sell at P	invest k	VH OF VL TEANZES	
t = 0	t = 1	t = 2	t = 3	t = 4	► t

#### Figure 1.2: Timeline

After observing the quantity chosen by the informed trader B, the uninformed trader updates the prior belief and form the conditional belief  $q = Pr(V_H|B)$ . Similarly the firm's manager updates his belief about the quality of the investment after observing the asset price P and form the conditional belief  $r = Pr(V_H|P)$ . Since there is not other private or public information besides the information about the outcome of the investment opportunity, in equilibrium, the price will reflect the demand of the informed trader only, and thus P conveys the same information as B. Therefore I can write r = q. For ease of notation, beliefs of both, the uninformed trader and the firm will be denoted by  $q = Pr(V_H|B)$ .

The firm value F increases in investment k at a decreasing rate  $\forall k \leq k^*$ , where  $k^*$  is the optimal investment level. c is a fixed marginal cost of investment. The firm's manager maximizes the firm value by choosing the investment level k given the price he observes on the stock market. The firm value increases in the prospect of the investment  $V_{\omega}$ . The manager's objective function is written as

$$F(k) = kV_{\omega} - \frac{c}{2}k^2 \tag{1.1}$$

so that the expected firm value becomes

$$E(F|B) = kE(V_H|B) - \frac{c}{2}k^2.$$
 (1.2)

The firm value function is adopted from Dow and Rahi (2003). The concavity of the firm value function in k implies that private information has social value even ex-ante. This will become clearer once the optimal k for both types of equilibria is derived. I postpone this discussion therefore to section 1.3.

After observing information in t = 0, the informed trader decides to buy a quantity B at a price P in period t = 1. When choosing B, the informed trader not only conditions on his private information  $\omega$  but also takes into account the signal his choice is sending to the uninformed trader and the firm. In t = 4, when the investment value  $V_{\omega}$  realizes and thus the firm value F, the informed trader cashes in on the assets bought. The informed trader evaluates the cash-flow from the perspective of period t = 1, i.e. when deciding on the purchase. The informed trader discounts the payoff of period t = 4 by  $\delta_I$ . By how much he discounts depends on how liquidity constrained he is. If, for example, the borrowing rate is zero, the informed trader is indifferent between a payoff today and tomorrow such that  $\delta_I = 1$ . The higher the borrowing rate, the lower the discount factor and the less willing is the informed trader to give up a payoff today for a payoff tomorrow. The informed trader's cash flow from buying the risky asset at date t = 1 is

$$U_I(V_\omega, k) = -PB + \delta_I BF. \tag{1.3}$$

Instead of buying the risky asset, the informed trader can also buy the riskless asset and obtain 0 payoff.

In t = 2, when selling an amount *B* of the total endowment *E*, the uninformed trader receives a revenue *PB* from the sale. In t = 4, after the realization of the investment value, just like the informed trader, the uninformed trader cashes in on the assets held. Evaluating the cash-flow from period t = 1, the uninformed trader discounts the payoff from period t = 4 by  $\delta_U$ . The uninformed trader's net present value (NPV) at t = 2 is

$$U_U(B,k) = PB + \delta_U(E-B)F.$$
(1.4)

In order to state the expected value of the uninformed trader's NPV, I have to specify the beliefs. Therefore, the introduction of the expected NPV is deferred to section 1.3. Instead of selling assets, the uninformed trader can also keep all the assets, i.e. B = 0, and receive a NPV at time t = 2 of  $\delta_U EF$ .

#### **1.3** Perfect Bayesian Equilibrium

The informed trader strategy is a mapping  $B : \{V_{\omega}\} \to \Re_0^+$  that prescribes a quantity  $B(V_{\omega})$  on the basis of the trader's private information  $\omega$ . The uninformed trader strategy is a mapping  $P : \Re_0^+ \to \Re_0^+$ . The firm manager strategy is a mapping  $k : \Re_0^+ \to \Re_0^+$ . Conditional beliefs for the uninformed trader and the firm manager are represented by a mapping that associates to each quantity B a probability function  $Pr(\cdot|B)$  on  $\{V_H, V_L\}$ , where  $Pr(V_{\omega}|B)$  is the probability that the uninformed trader and the firm manager attach to a value  $V_{\omega}$  given quantity B.

The perfect Bayesian equilibrium is defined by a triple of strategies  $(B(\cdot), P(\cdot), k(\cdot))$  and a family of conditional beliefs  $Pr(\cdot|\cdot)$  such that (i) for all B in the range of  $B(\cdot)$ ,  $Pr(\cdot|B)$  is the conditional probability of  $V_{\omega}$  obtained by updating the prior  $(\beta, (1 - \beta))$ , using  $B(\cdot)$  in Bayesian fashion; (ii) for all  $B(\cdot)$ ,  $P \ argmax_P \in E(U_U(\cdot))$ , (iii) for all  $B(\cdot)$   $k^* \in argmax_k \ E(F|B)$  and (iv) for all  $\omega \ B \in argmax_B \ E(U_I(\cdot))$ . Condition (i) stipulates that the uninformed trader and the firm's manager have rational expectations. Conditions (ii) to (iv) require that traders be optimizing. In particular, they imply participation constraints and incentive compatibility constraints.

Market clearing takes place through the adjustment of the price P to the quantity demanded B. I.e. the informed trader submits a market order. Observing the market order, the uninformed trader, acting as a market maker, updates the belief about the quality of the asset and sets the price. In equilibrium, there has to be a unique price-quantity bundle  $\{P, B\}$ .

#### 1.3.1 Separating equilibrium

In a separating equilibrium, the informed trader buys different quantities in either state. Therefore, the purchase reveals private information. Suppose, the informed trader buys  $B^H$  after observing H and  $B^L$  after observing L, then I invoke the uninformed trader's and the firm's conditional beliefs as:

$$q = Pr(H|B) = \begin{cases} 1 & \text{if } B = B^{H} \\ 0 & \text{if } B = B^{L} \\ 1 & B' \neq B^{H} \wedge B' \neq B^{L} \end{cases}$$
(1.5)

The conditional beliefs imply that the uninformed trader and the firm update their priors such that if they observe  $B^H$ , they are sure to face the informed trader with good information and if they observe  $B^L$ , they know the informed trader with bad information is buying the asset. The intuition for the off-equilibrium belief is the informed trader with good information Hwants to mimic the informed trader with bad information L in order to get a low price<sup>9</sup>. Discontinuity of the conditional beliefs is a natural consequence of the binomial distribution of the random variable  $V_{\omega}$ . This is different from Laffont and Maskin (1990). They can potentially obtain continuous, monotonic beliefs since they consider a general distribution function. I will have to show that the conditional beliefs satisfy incentive compatibility of the informed trader and ensure participation of the uninformed trader in equilibrium.

Observing the asset price P from t = 2, the firm's manager forms the belief q. Since asset trade reveals information, there will be two prices  $P^{\omega}$ depending on the private information or, equivalently, the demand from the informed trader. In t = 3, based on the beliefs, the firm takes the decision on the investment level k in order to maximize the conditional, expected firm value

$$max_k E(F|B) = F_{\omega} = k(qV_H + (1-q)V_L) - \frac{c}{2}k^2.$$
(1.6)

<sup>&</sup>lt;sup>9</sup>The off-equilibrium beliefs stipulated here are not the only possible ones.

Depending on the price observed, the optimal investment choice  $k^*$  is  $k^{\omega} = \frac{V_{\omega}}{c}$  and therefore the firm's equilibrium value is  $E(F|B^{\omega}) = F_{\omega} = \frac{V_{\omega}^2}{2c}$ .

The uninformed trader infers the private information  $\omega$  from the demand B of the informed trader. As price taker, the uninformed trader decides whether to sell or to retain the assets for a given price  $P^{\omega}$ . The equilibrium price therefore has to satisfy the following participation constraint:

$$P^{\omega}B^{\omega} + \delta_U(E - B^{\omega})F_{\omega} \ge \delta_U E F_{\omega}.$$
(1.7)

Due to perfect competition, the uninformed trader breaks even. Then, the equilibrium price is  $P^{\omega} = \delta_U F_{\omega}$ , depending on the demand observed. It is intuitive that the uninformed trader wants to sell the asset at the price which reflects the value of the firm discounted by his liquidity need. The more liquidity constrained the uninformed trader, the more he is willing to decrease the price.

The informed trader is willing to buy the risky asset if the NPV of buying the risky asset outweighs the NPV of the riskless asset. Given the price  $P^{\omega}$ , the informed trader's participation constraint, depending on private information, is

$$-\delta_U F_\omega B^\omega + \delta_I B^\omega F_\omega \ge 0. \tag{1.8}$$

No matter which information the informed trader observed, the participation constraint is satisfied since  $\delta_I \geq \delta_U$ . Since the uninformed trader can observe the quality of the investment, the only gain from trade is the asymmetry in liquidity needs. Namely that the uninformed trader is eager to sell assets because of the tight liquidity constraint. Given the tight liquidity needs of the uninformed trader, the informed trader can buy the asset relatively cheap if he is less liquidity constrained than the uninformed trader.

In order to show that  $B^{\omega}$  is the optimal choice for the informed trader given the observed information  $\omega$ , consider the incentive compatibility constraints. As shown before, the informed trader faces a price  $P^{\omega} = \delta_U F_{\omega}$  when choosing  $B^{\omega}$ . Incentive compatibility is satisfied if

$$-\delta_U F_\omega B^\omega + \delta_I B^\omega F_\omega \ge -\delta_U F_{-\omega} B^{-\omega} + \delta_I B^{-\omega} F'_\omega$$
(1.9)  
where  $-\omega \ne \omega$ .

with  $F'_H = \frac{V_L}{c}(V_H - \frac{V_L}{2})$  and  $F'_L = \frac{V_H}{c}(V_L - \frac{V_H}{2})$ . These are the firm values in which the manager chooses  $k^{-\omega} = \frac{V_{-\omega}}{c}$  although the real value was  $V_{\omega}$ . It is straightforward to show that the ranking of the firm value is  $F_H > F'_H > F_L > F'_L$  and  $F'_L > 0$  if and only if  $V_H < 2V_L$ . This implies that the informed trader with information  $\omega$  decreases, not only the others' payoffs, but also his own payoff by mimicking the other type  $-\omega$ .

If  $\delta_U \frac{F_H}{F_L} > \delta_I > \delta_U$  and  $V_H < 2V_L$  (from  $F_L' > 0$ ), the off-equilibrium payoff of the informed trader with bad information is negative for any  $B^H$ . His incentive compatibility is therefore satisfied  $\forall B^L > 0$ . The informed trader with good information has a postive off-equilibrium payoff. He wants to mimic the informed trader with bad information in order to obtain a lower purchase price. Therefore restrictions on  $B^H$  in comparison to  $B^L$  are required. The conditions satisfying the informed trader incentive compatibility if  $\delta_U \frac{F_H}{F_T} > \delta_I$  are summarized as follows

$$\frac{(-\delta_U + \delta_I)F_H}{-\delta_U F_L + \delta_I F'_H} B^H \ge B^L \ge 0.$$
(1.10)

In order for  $B^L$  strictly different from  $B^H$ ,  $\frac{(-\delta_U + \delta_I)F_H}{-\delta_U F_L + \delta_I F'_H} < 1$ . The latter inequality can be rewritten as  $\delta_U \frac{F_H - F_L}{F_H - F'_H} > \delta_I$ . Therfore,  $min\{\delta_U \frac{F_H - F_L}{F_H - F'_H}, \delta_U \frac{F_H}{F'_L}\} > \delta_I$ . Make the following two observations:  $\frac{\partial \frac{F_H - F_L}{F_H - F'_H}}{\partial V_H} < 0$  and  $\frac{F_H - F_L}{F_H - F'_H} < \frac{F_H}{F'_L}$  if and only if  $V_H > \frac{1}{2}(1 + \sqrt{5})V_L$ . This implies for increasing  $V_H$ , the liquidity difference  $\delta_I - \delta_U$  has to decrease. Intuitively, the informed trader with good information is more inclined to mimic the low type the higher  $V_H$ , given  $V_L$ . In order for him to refrain from doing so, liquidity asymmetry has to decrease, i.e. the gain from trading on liquidity difference decreases.

If instead  $\delta_I > \delta_U \frac{F_H}{F'_L}$  given  $2V_L > V_H > V_L$ , the liquidity difference is so large that both the informed trader with good information and the informed trader with bad information has an incentive to mimic the other type. The reason for the informed trader with good information wanting to mimic is still the lower price. The reason for the informed trader with bad information is that the loss from facing a higher price is outweighed by the gain from purchasing a larger quantity. The following condition, obtained from the incentive compatibility constraints from inequality 1.9, guarantees that neither of the two type mimics the other one,

$$\frac{(-\delta_U + \delta_I)F_H}{-\delta_U F_L + \delta_I F'_H}B^H > B^L > \frac{-\delta_U F_H + \delta_I F'_L}{(-\delta_U + \delta_I)F_L}B^H.$$
(1.11)

Observe  $\frac{(-\delta_U + \delta_I)F_H}{-\delta_U F_L + \delta_I F'_H} > \frac{-\delta_U F_H + \delta_I F'_L}{(-\delta_U + \delta_I)F_L}$  so that there exists  $B^L > 0$ . Recall that in order for the informed trader with bad information not be able to mimic the informed trader with good information, not only the lower bound has to be satisfied, but also the upper bound has to be smaller than one,  $\frac{(-\delta_U + \delta_I)F_H}{-\delta_U F_L + \delta_I F'_H} < 1$ . In order for a separating equilibrium with  $\delta_I > \delta_U \frac{F_H}{F'_L}$  to exist,  $\frac{F_H - F_L}{F_H - F'_H} > \frac{F_H}{F'_L}$ . If and only if  $\frac{1}{2}(1 + \sqrt{5})V_L > V_H > V_L$ ,  $\frac{F_H - F_L}{F_H - F'_H} > \frac{F_H}{F'_L}$ . Consequently, there exists a candidate separating equilibrium if  $\frac{1}{2}(1 + \sqrt{5})V_L > V_H > V_L$  and  $\delta_U \frac{F_H - F_L}{F_H - F'_H} > \delta_I > \delta_U \frac{F_H}{F'_L}$  with trade as specified in condition 1.11.

Besides choosing  $B^{-\omega}$ , the informed trader with private information  $\omega$  can also choose any other quantity  $B' \neq B^{\omega}$ . In order to ensure optimality of  $B^{\omega}$  consider also the following incentive compatibility constraints for either type  $\omega$ :

$$(-P^H + \delta_I F_H)B^H \ge (-P' + \delta_I F_H)B' \quad \forall B' \tag{1.12}$$

$$(-P^L + \delta_I F_L)B^L \ge (-P' + \delta_I F'_L)B' \quad \forall B' \tag{1.13}$$

As specified by the off-equilibrium belief in equation 1.5, the uninformed trader believes that he is facing a high type after observing B'. When breaking even, the uninformed trader asks an off-equilibrium price  $P' = \delta_U F_H$ .

After reformulating conditions 1.12 and 1.13, it is straightforward to

show that  $B^{\omega}$  are optimal if,

$$B^H = E \tag{1.14}$$

$$B^{L} \ge max\{0, \frac{-\delta_{U}F_{H} + \delta_{I}F_{L}'}{(-\delta_{U} + \delta_{I})F_{L}}E\}.$$
(1.15)

Together with condition 1.10 and 1.11, the equilibrium quantities are given by

$$B^H = E, (1.16)$$

$$\frac{(-\delta_U + \delta_I)F_H}{-\delta_U F_L + \delta_I F'_H} E \ge B^L \ge max\{0, \frac{-\delta_U F_H + \delta_I F'_L}{(-\delta_U + \delta_I)F_L}E\}.$$
(1.17)

I hereby have described a perfect Bayesian equilibrium in which private information is revealed. There exist multiple separating equilibria, depending on both, the difference in liquidity needs and the range of admissible  $B^L$ . This summarizes the following proposition.

**Proposition 1.** Separating equilibrium. There exist separating equilibria with price  $P^{\omega} = \delta_U F_{\omega}$ ,  $\omega \in \{H, L\}$  and quantities as specified in 1.16 and 1.17 if

•  $\delta_U \frac{F_H - F_L}{F_H - F'_H} > \delta_I > \delta_U$  and  $V_H > \frac{1}{2}(1 + \sqrt{5})V_L$  or •  $\delta_U \frac{F_H - F_L}{F_H - F'_H} > \delta_I > \delta_U \frac{F_H}{F'_L}$  and  $\frac{1}{2}(1 + \sqrt{5})V_L > V_H > V_L$ .

#### 1.3.2 Pooling equilibrium

Next, I characterize the conditions under which a pooling equilibrium exists. In a pooling equilibrium, the informed trader buys identical quantities in either state  $\omega$ . Therefore, the uninformed trader and the firm cannot infer the informed trader's private information. Suppose the informed trader chooses  $B^P$  in either state, then the uninformed trader's and the firm's conditional belief is equal to their priors. If they observe and  $B' \neq B^P$ , their conditional belief implies that the informed trader has good information:

$$q = Pr(H|B) = \begin{cases} \beta & \text{if } B = B^P \\ 1 & B' \neq B^P \end{cases}.$$
 (1.18)

The intuition for the belief "off-equilibrium" is, that the informed trader with good information is more inclined to deviate from the equilibrium quantity  $B^P$  since the price in the pooling equilibrium is relatively low. These conditional beliefs will have to satisfy the uninformed trader's participation constraint as well as the informed trader's incentive compatibility.

Observing the asset price in t = 2, the firm's manager updates the belief according to equation 1.18. Given these beliefs, he takes the investment decision k in order to maximize the conditional, expected firm value in the pooling equilibrium  $F_P$ :

$$max_k E(F|B) = F_P = k(\beta V_H + (1-\beta)V_L) - \frac{c}{2}k^2.$$
(1.19)

Denote  $(\beta V_H + (1-\beta)V_L) = E(V)$ . The optimal choice of the firm's manager in the pooling equilibrium is  $k^P = \frac{E(V)}{c}$  so that the firm value becomes  $F_P = \frac{E(V)^2}{2c}$ . Now, I can readily comment on the social value of private information. Therefore, observe that  $\beta F_H + (1-\beta)F_L > F_P$  for any  $\beta > 0$ . It is this relationship which gives social value to private information even from an ex-ante perspective. The concave firm value function is driving this relationship. If it was linear instead, the expected value of the firm in a separating equilibrium would be identical to the expected value of the firm in the pooling equilibrium.

After observing the informed trader's demand, the uninformed trader forms the beliefs given in equation 1.18. Then he decides whether to sell or to keep the asset. Since the informed trader purchases the same quantity  $B^P$  regardless of the state, there is just one price P for both states. The uninformed trader's participation constraint becomes

$$\beta(PB^P + \delta_U(E - B^P)F_P) + (1 - \beta)(PB^P + \delta_U(E - B^P)F_P) \ge \delta_U E(\beta F_P + (1 - \beta)F_P).$$
(1.20)

Recall, the uninformed trader is price taker and in competition for the sale with the other small, uninformed traders. The equilibrium price P has to satisfy inequality 1.20 when it is binding, i.e. the uninformed trader breaks even. If inequality 1.20 is binding,  $P = \delta_U F_P$ .

Just like in the separating equilibrium, the price decreases the more liquidity constrained the uninformed trader. For a given  $\delta_U$  and  $\beta > 0$ , the equilibrium price in the pooling equilibrium lays between the prices in the separating equilibrium in the good state and in the bad state,  $P^H > P > P^L$ . The price in the pooling equilibrium does not reflect available information and consequently will lead to the inefficient level of investment. It does not reflect available information because the informed trader chooses the same demand in either state.

Given the price  $P = \delta_U F_P$ , the informed trader decides whether to buy  $B^P$  of the risky asset or the riskless asset which gives a return of 0. His participation constraints in either state is

$$-\delta_U F_P B^P + \delta_I B^P F_P^\omega \ge 0. \tag{1.21}$$

The firm value from the perspective of the informed trader takes into account the investment decision of the manager,  $k^P = \frac{E(V)}{c}$ , given the privately observed information  $\omega$ . Therefore, the firm value from the informed trader's perspective are  $F_P^{\omega} = \frac{E(V)}{c}(V_{\omega} - \frac{E(V)}{2})$  with  $F_P^L > 0$  if and only if  $\frac{V_L}{V_H - V_L} > \beta$ . Observe,  $F_P^H > F_P > F_P^L$ . Implying that the informed trader with good information faces a higher return from the risky asset than the informed trader with bad information, given the uninformed choice of the manager. The participation constraint for the informed trader with bad information is more binding. In fact, if  $\omega = L$ , the left hand side of the inequality is smaller than if  $\omega = H$ . Solving the participation of the informed trader with bad information for  $\delta_I$  yields for any  $B^P \geq 0$ :  $\delta_I \geq \delta_U \frac{F_P}{F_P^L}$ . Since  $\frac{F_P}{F_P^L} > 1$ , the informed trader needs to be considerably less liquidity constrained than the uninformed trader, and in particular less than in the separating equilibrium where participation was ensured if  $\delta_I > \delta_U$ . Observe that  $\frac{F_P}{F_P^L}$  increases in  $\beta$ . That is the more likely the good outcome, the larger needs to be the difference between the informed trader's liquidity needs and the uninformed trader's liquidity needs.

The mechanism behind the equilibrium condition  $\delta_I \geq \delta_U \frac{F_P}{F_P^L}$  is driven by the prospect of the informed trader with bad information,  $\delta_I F_P^L$ . Given bad information, the informed trader does not want to buy the asset at a high price. For a given  $\delta_U$ , the pooling equilibrium price  $P = \delta_U F_P$  is high relative to the prospect. So the informed trader is only willing to buy if the uninformed trader is sufficiently liquidity constrained, i.e.  $\delta_U$  is small relative to  $\delta_I$ .

It is left to be shown that choosing  $B^P$  in either state is optimal over choosing any other quantity B'. So far, I have only characterized the uninformed trader's and the firm's best response after they observe  $B^P$ , i.e. the price  $P = \delta_U F_P$  and the optimal investment level  $k^P = \frac{E(V)}{2c}$ . So what happens if uninformed trader and firm respectively observe B'? Consider first the firm. The belief invoked by equality 1.18 implies that if B', or equivalently P', is observed, the firm believes that the informed trader has good information and therefore chooses  $k' = \frac{V_H^2}{2c}$ . Same thing for the uninformed trader, when observing B', he believes, according to equality 1.18, to face an informed trader with good information. Just like on the equilibrium path, uninformed traders are assumed to be price takers and to be in perfect competition. So that when they break even, the off-equilibrium price becomes  $P' = \delta_U F_H$ . Now, I can study incentive compatibility. If the informed trader chooses the equilibrium quantity  $B^P$ , he is facing a price  $P = \delta_U F_P$ . If instead he chooses the off-equilibrium quantity B', he faces the off-equilibrium price  $P' = \delta_U F_H$ . The last two statements are formalized

for either type  $\omega$  in the following inequalities:

$$(-\delta_U F_P + \delta_I F_P^H) B^P \ge (-P' + \delta_I F_H) B' \quad \forall B' \tag{1.22}$$

$$(-\delta_U F_P + \delta_I F_P^L) B^P \ge (-P' + \delta_I F_L') B' \quad \forall B' \tag{1.23}$$

For the informed trader with good information not wanting to deviate,  $-\delta_U F_P + \delta_I F_P^H > -\delta_U F_H + \delta_I F_H$ . Otherwise, the informed trader would always prefer to choose  $B' < B^P$ , even for  $B^P = E$ , and obtain the higher off-equilibrium payoff. Therefore,  $-\delta_U F_P + \delta_I F_P^H > -\delta_U F_H + \delta_I F_H$  or equivalently,  $\delta_I < \delta_U \frac{F_H - F_P}{F_H - F_P^H}$ . Recall, that for the informed trader to participate,  $\delta_I > \delta_U \frac{F_P}{F_P^L}$ . Together with the condition from the incentive compatibility constraint,  $\delta_U \frac{F_H - F_P}{F_H - F_P^H} > \delta_I > \delta_U \frac{F_P}{F_P^L}$ . There exists a  $\delta_I \in [0, 1]$  satisfying the latter condition if  $\frac{F_H - F_P}{F_H - F_P^H} > \frac{F_P}{F_P^L}$ , i.e.  $\beta < \frac{V_L^2}{(V_H - V_L)^2}$ . Recall for  $F_P^L > 0$ ,  $\beta < \frac{V_L}{V_H - V_L}$  and hence  $\beta < min\{\frac{V_L}{V_H - V_L}, \frac{V_L^2}{(V_H - V_L)^2}\}$ .

From 1.22 we observe, the informed trader with good information is indifferent between the equilibrium payoff and the off-equilibrium payoff if  $E > B^P = \frac{(-\delta_U + \delta_I)F_H}{-\delta_U F_P + \delta_I F_P^H} E$ . The equilibrium payoff however is maximized for  $B^P = E$ .

Next, consider incentive compatibility for the informed trader with bad information in condition 1.23. Since  $\frac{-\delta_U F_H + \delta_I F'_L}{-\delta_U F_P + \delta_I F_P^L} < 1$  it becomes clear that  $B^P = E$  also satisfies incentive compatibility of the informed trader with bad information. Since  $\frac{-\delta_U F_H + \delta_I F'_L}{-\delta_U F_P + \delta_I F_P^L} < \frac{(-\delta_U + \delta_I)F_H}{-\delta_U F_P + \delta_I F_P^L}$  for  $\delta_I > \delta_U \frac{F_P}{F_P^L}$ , incentive compatibility is indeed more binding for the high type than for the low type.

This completes the characterization of the pooling equilibrium. Therein the price  $P = \delta_U F_P$  does not reveal private information. There exist multiple pooling equilibria depending on the difference in liquidity needs. This is summarized in the following proposition.

**Proposition 2.** Pooling equilibrium. If  $\delta_U \frac{F_H - F_P}{F_H - F_P} > \delta_I > \delta_U \frac{F_P}{F_P}$  and  $\beta < min\{\frac{V_L}{V_H - V_L}, \frac{V_L^2}{(V_H - V_L)^2}\}$ , there exists a pooling equilibrium with a price  $P = \delta_U F_P$  and equilibrium trade  $B^P = E$ .

The fact that trade in a the pooling equilibrium is always maximal,  $B^P =$ 

E, implies that the expected quantity in the pooling equilibrium  $B^P$  is larger than the expected quantity in the separating equilibrium  $\beta E + (1 - \beta)B^L$ . This will be crucial for the welfare analysis in section 1.4.

#### 1.3.3 Equilibrium characterization

After stating the existence conditions for each type of equilibrium, I can now characterize all possible equilibria given the beliefs in 1.5 and 1.18. The objective is to characterize equilibria depending on the liquidity difference,  $\delta_I - \delta_U$ . Observe from the two previous propositions that the existence of equilibria depends on the following three thresholds which characterize the liquidity difference:  $\frac{F_H - F_P}{F_H - F_P^H}$ ,  $\frac{F_P}{F_P^L}$  and  $\frac{F_H - F_L}{F_H - F_H^H}$ . In order to rank them, I need to derive conditions on  $\beta$  and the difference  $V_H - V_L$ , the parameters on which the firm values F depend. The derivation of the condition is relagated to the appendix A.1. The equilibrium characterization is a preparatory step for the welfare analysis. It provides hence the areas of parameters  $\beta$ ,  $V_{\omega}$  and  $\delta_I - \delta_U$  for which pooling equilibrium and separating equilibrium co-exist, for which they exist in adjacent parameter areas and for which they exist in distant areas.

**Lemma 1.** Separating equilibrium only. For  $1 \ge \beta > \min\{\frac{V_L}{V_H - V_L}, \frac{V_L^2}{(V_H - V_L)^2}\}$ , there exists a separating equilibrium only (if  $\delta_U \frac{F_H - F_L}{F_H - F'_H} > \delta_I > \max\{\delta_U, \delta_U \frac{F_H}{F'_L}\}$ ).

Lemma 1 tells, if the difference between the high outcome and the low outcome is very small, only the separating occurs.

**Lemma 2.** Separating equilibrium and pooling equilibrium do not overlap. For  $min\{1, \frac{V_L}{V_H - V_L}, \frac{V_L^2}{(V_H - V_L)^2}\} > \beta > \frac{V_L^2}{(V_H - V_L)V_H}$ , there exists

- a separating equilibrium only (if  $\delta_U \frac{F_H F_L}{F_H F'_H} > \delta_I > max\{\delta_U, \delta_U \frac{F_H}{F'_L}\})$ and
- a pooling equilibrium only (if  $\delta_U \frac{F_H F_P}{F_H F_P^H} > \delta_I > \delta_U \frac{F_P}{F_P^L}$ ).

For a given  $V_H - V_L$  and  $\beta$ , the separating equilibrium exists for a small liquidity difference and the pooling equilibrium for a large liquidity difference since  $\frac{F_P}{F_P} > \frac{F_H - F_L}{F_H - F'_H}$ .

**Lemma 3.** Separating equilibrium and pooling equilibrium overlap. For  $min\{1, \frac{V_L^2}{(V_H - V_L)V_H}\} > \beta$ , there exists

- a pooling equilibrium only if  $\delta_U \frac{F_H F_P}{F_H F_P^H} > \delta_I > \delta_U \frac{F_H F_L}{F_H F_H'}$ ,
- both a pooling equilibrium and a separating equilibrium if  $\delta_U \frac{F_H F_L}{F_H F'_H} > \delta_I > max \{ \delta_U \frac{F_P}{F_D^L}, \delta_U \frac{F_H}{F'_L} \}$  and
- a separating equilibrium only if  $\delta_U \frac{F_P}{F_P^L} > \delta_I > max\{\delta_U, \delta_U \frac{F_H}{F_L'}\}.$

Pooling equilibrium and separating equilibrium overlap if for an increasing difference in the investment's outcome  $V_H - V_L$ , the probability of observing the high outcome  $\beta$  decreases and the liquidity difference is intermediate. This characterizes a situation in which the informed trader can make moderate gains from trade since both the gain from the liquidity asymmetry and the gain from information asymmetry are moderate.

#### 1.4 Welfare Analysis

Recall, the purpose of this paper is to study how asset trading in the presence of a large, monopolistically informed trader affects the investment in the real economy. The firm representing the real economy is entirely owned by the traders. In order to study the welfare, it is therefore sufficient to add the traders' profits. And I will do so from an ex-ante perspective<sup>10</sup>.

#### 1.4.1 Profits

Denote by  $\psi \in \{S, P\}$  either type of equilibrium, i.e. separating equilibrium or pooling equilibrium. The equilibrium profit of an informed trader I in a separating equilibrium with  $P^{\omega} = \delta_U F_{\omega}$  and  $B^{\omega}$  is

$$\Pi^{\omega SI} = (-\delta_U + \delta_I) F_\omega B^\omega. \tag{1.24}$$

 $<sup>^{10}</sup>$ As it is done in Dow and Rahi (2003) and Laffont and Maskin (1990).

For the uninformed trader U the equilibrium profit becomes

$$\Pi^{\omega SU} = \delta_U EF_\omega. \tag{1.25}$$

Analogously, I obtain the profit of the informed trader I in a pooling equilibrium with the equilibrium price  $P = \delta_U F_P$  and the equilibrium quantity  $B^P$ 

$$\Pi^{\omega PI} = (-\delta_U F_P + \delta_I F_P^{\omega}) B^P.$$
(1.26)

For the uninformed trader U the equilibrium profit becomes

$$\Pi^{\omega SU} = \delta_U F_P E. \tag{1.27}$$

Observe that the profit of the informed trader is increasing in the quantities  $B^{\omega}$  and  $B^{P}$  while the profit of the uninformed trader is constant in the quantities traded. Moreover, profits are increasing in the firm value F. These two observations will guide the following welfare analysis.

#### 1.4.2 Welfare

Summing the profits in the separating equilibrium yields the following exante welfare  $W_S$ :

$$W_{S} = \delta_{U} E(\beta F_{H} + (1 - \beta) F_{L}) + (\delta_{I} - \delta_{U})(\beta F_{H} B^{H} + (1 - \beta) F_{L} B^{L}).$$
(1.28)

The first summand is the expected gain of the owner of the firm and the second summand is the expected gain from asset trade. Recall,  $B^H = E$  but  $B^L(\tau)$ . Welfare increases in the amount of trade and in the firm value.

Similarly for the pooling equilibrium, ex-ante welfare  $W_P$  is the sum of the expected profits:

$$W_P = \delta_U E F_P + (\delta_I (\beta F_P^H + (1 - \beta) F_P^L) - \delta_U F_P) B^P.$$
(1.29)

Again, the first summand is the expected gain of the firm's owner and the second summand reflects the expected gain from asset trade. With  $B^P = E$ ,

in comparison to welfare in the separating equilibrium, expected trade is always maximal. Firm values however are lower. Recall,  $(\beta F_H + (1-\beta)F_L) > F_P$ ,  $F_H > F_P^H$  and  $F_L > F_P^L$ .

With the equilibrium quantities  $B^P = B^H = E$ , the separating equilibrium yields greater welfare if

$$\Delta W = W_S - W_P = \delta_U E(\beta F_H + (1 - \beta) F_L) + (\delta_I - \delta_U)(\beta F_H E + (1 - \beta) F_L B^L) - \delta_I (\beta F_P^H + (1 - \beta) F_P^L) E > 0.$$
(1.30)

Whether welfare in the separating equilibrium is larger than in the pooling equilibrium depends on how much the informed trader with bad information is trading, i.e.  $B^L$ . It makes sense to compare the two types of equilibria if they exist at the same time. Lemma 3 establishes, separating equilibrium and pooling equilibrium exist at the same time if the following conditions are satisfied

$$\delta_U \frac{F_H - F_L}{F_H - F'_H} > \delta_I > \delta_U \frac{F_P}{F_P^L},\tag{1.31}$$

$$\delta_U < \frac{F_P^L}{F_P}$$
 and (1.32)

$$\beta < \min\{1, \frac{V_L^2}{V_H(V_H - V_L)}\}.$$
(1.33)

The pooling equilibrium can yield greater welfare because the expected quantity traded is larger than the expected quantity traded in the separating equilibrium as obvious from the equilibrium conditions in subsection 1.3.3. The increase in welfare from higher expected trade can outweigh the worse investment decision in the pooling equilibrium with respect to the separating equilibrium. For  $\Delta W > 0$ , conditions have to specify the quantity of trade of the informed trader with bad information  $B^L$ , the difference in liquidity needs  $\delta_I - \delta_U$  and the informational wedge characterized by  $\beta$ ,  $V_H$  and  $V_L$ .

**Proposition 3.** Welfare comparison. The separating equilibrium yields greater welfare than the pooling equilibrium if  $\delta_U \frac{F_H - F_L}{F_H - F'_H} > \delta_I > \delta_U \frac{F_P}{F_P^L}$ ,  $\frac{V_L^2}{V_H(V_H - V_L)} > \beta$  and

• either  

$$\frac{(\delta_I - \delta_U)F_H}{\delta_I F'_H - \delta_U F_L} E > B^L \ge 0, \quad \frac{V_L^2}{V_H (V_H - V_L)} > \beta > \frac{3V_L - V_H}{(V_H - V_L)^2} \quad and \quad V_H > (1 + \sqrt{2})V_L$$

• or

$$B^{L}(V_{H}) = \frac{2\beta\delta_{I}(V_{H}V_{L} - \frac{V_{H}^{2}}{2})}{(\delta_{I} - \delta_{U})V_{L}^{2}}E, \min\{\delta_{U}\frac{F_{H} - F_{L}}{F_{H} - F_{H}'}, \delta_{U}\frac{1}{1 - \beta}\} > \delta_{I}, \beta > \frac{2V_{H} - 3V_{L}}{V_{H} - V_{L}}$$
  
and either  $\frac{3}{2}V_{L} > V_{H} > V_{L}$  or  $2V_{L} > V_{H} > \frac{1}{4}(3 + \sqrt{17})V_{L}.$ 

The proof is relegated to the appendix.

Proposition 3 implies that the larger the difference between  $V_H$  and  $V_L$  (the first bullet point of the proposition), the greater the inefficiency from over investment (under investment) in the pooling equilibrium and therefore even for no trade  $B^L$ , the separating equilibrium yields greater welfare. As the difference between  $V_H$  and  $V_L$  is decreasing (second bullet point of the proposition), the gain from information revelation is decreasing and therefore trade  $B^L$  has to be strictly positive.

#### 1.5 Conclusion

In order to answer the question whether asset prices efficiently guide the allocation of investment, a welfare analysis, including all players, has to be carried out. The existing literature has two drawbacks. First, it considers models in which the non-informativeness of the price is exogenous and thus the inefficient investment decision of the firm. And second, if any, it does not provide a complete welfare analysis.

In order to improve on the two drawbacks, this paper studies an asset trade model in which an informed trader buys assets from uninformed traders. The uninformed traders observe the informed traders demand and infer the quality of the asset. Trade takes place due to asymmetric liquidity needs. There exist two types of equilibria. One in which the asset prices reveal private information and another in which the asset price is uninformative.

The informed trader has private information about the investment opportunity of a firm. The firm's assets are traded on the secondary asset market. Basing its investment decision on the its asset price, the firm takes the efficient investment decision, in case prices reveal information, and an inefficient investment decision in case the price is uninformative. The existence of multiple equilibria establishes an endogenous inefficiency. I provide testable conditions for the equilibrium existence.

Summing up the informed trader's profit and the uninformed traders' profits, I obtain a measure for welfare. The welfare analysis provides testable conditions for which the equilibrium exhibiting information revelation yields greater welfare.

This paper integrates asset trade and real economy investment in a simple model. This allows future research to study regulatory measures.

## Chapter 2

# (In)Efficient Asset Trade and a rationale for a Tobin Tax

#### 2.1 Introduction

The Financial Transaction Tax (FTT), also known as Tobin tax or securities transaction tax dates back to the article of James Tobin in 1978. Since then it has been introduced in a lot of states. In some of those states it has also been abolished afterwards. Matheson (2012) gives an overview of the countries in which a FTT is currently active. Most recently, the FTT has been introduced in France (August 2012) and Italy (March 2013). In these two countries a tax between 0.1% and 0.22% is levied on purchases of stocks. In the UK, since the early 90s', there exists a so called "Stamp Duty" on equity purchases which amounts to 0.5%. Discussions about the introduction of a FTT have restarted in the wake of the Financial crisis in 2008, in most Western countries<sup>1</sup>. Currently ongoing is a debate among EU-countries about the introduction of a FTT by  $2016^2$ .

Most of the time, governments introduce a FTT to raise money. Tobin (1978), Stiglitz (1989) and Summers and Summers (1989) argue that the

<sup>&</sup>lt;sup>1</sup>The discussion is followed, for example, by a theme-page of the Financial Times: http://www.ft.com/intl/in-depth/financial-transaction-tax

<sup>&</sup>lt;sup>2</sup>The current proposition can be found here (found on May 8th 2014):http://ec. europa.eu/taxation\_customs/taxation/other\_taxes/financial\_sector/

tax affects mostly short-term speculation.

In this paper, I ask a more general question. Is a FTT able to improve welfare?

Therefore, I setup the following model. There is a manager of a firm facing an investment opportunity with uncertain outcome, either good or bad. When the manager decides upon the investment, he takes into account the firm's asset price on the secondary stock market, i.e. the manager updates his prior beliefs about the outcome of the investment opportunity.

Asset trade takes place between an informed trader and uninformed traders in a model à la Laffont and Maskin (1990). The informed trader observes either good or bad information about the investment prospect of the firm. The uninformed traders are holding the assets of the firm, i.e. they are the owners of the firm. By observing the demand of the informed trader, the uninformed traders update their beliefs about the quality of the asset and decide whether to sell. Notice, the informed trader takes into account the effect of his purchase on the asset price. Trade takes place due to asymmetric liquidity needs. The uninformed traders are more liquidity constrained than the informed trader and hence the uninformed traders want to sell the assets to the informed trader<sup>3</sup>. The FTT is levied on every purchase. This is the case for most countries.

There are two types of pure strategy equilibria. First, a separating equilibrium in which the informed trader reveals private information by demanding a larger quantity when he has good information than when he has bad information. The equilibrium price hence is either high or low. In order for trade to take place, in either state, the uninformed trader has to be more liquidity constrained than the informed trader. The equilibrium asset prices depends on the liquidity needs of the uninformed trader. Or differently, when the uninformed trader needs liquidity, he is willing to decrease the price at which he sells the assets proportional to his borrowing costs. Trade occurs for an infinitesimal small difference in liquidity needs. Since the asset prices reveal available information, the firm's manager takes an efficient investment decision and therefore the firm value is maximized given

<sup>&</sup>lt;sup>3</sup>For an extensive discussion of asymmetric liquidity needs, refer to chapter 1.

the observed information.

Second, there exists a pooling equilibrium in which the informed trader does not reveal private information by demanding the same quantity no matter whether he has good or bad information. Then the uninformed trader cannot infer information from the informed trader's demand and hence stays with the prior beliefs. In the pooling equilibrium, the asset price reflects the expected value of the asset which is below the prospect of the informed trader with good information and above the prospect of the informed trader with bad information. Just like in the separating equilibrium, also in the pooling equilibrium, the asset price depend on the liquidity needs of the uninformed trader. Since for the informed trader with good information the pooling equilibrium price is relatively low in comparison to his prospect, he is always willing to buy. The informed trader with bad information however is only willing to buy if the negative difference between his prospect and the expected value of the asset is outweighed by the uninformed trader's liquidity needs. In other words, in the pooling equilibrium, the uninformed trader needs to be more liquidity constrained than in the separating equilibrium for trade to take place between the uninformed trader and the informed trader with bad information. With an uninformative asset price, the firm's manager over (under) invests in case of bad (good) information. Given available information, the inefficient investment leads to a lower firm value than in the separating equilibrium.

I show that separating equilibrium and pooling equilibrium co-exist if the variance of the investment's outcome is relatively small and the difference in liquidity needs is intermediate. More generally, this characterizes a situation in which gains from asset trade for the informed trader are moderate. The welfare analysis is carried out for the set of parameters for which separating equilibrium and pooling equilibrium co-exist.

Since the firm is owned by traders, welfare in this economy is the ex-ante joint profit of traders. The expected firm value increases in information revelation. Hence, the firm owners prefer the separating equilibrium. Welfare on the asset trade market increases in quantities traded. Since expected trade in a pooling equilibrium is larger than in a separating equilibrium and agents are both traders and owners, the welfare trade-off is information vs. trade. I provide conditions on the quantities traded such that the separating equilibrium yields greater welfare.

Then, I show that if the economy is in a pooling equilibrium, there exists an optimal tax which coordinates the economy on a separating equilibrium. While the results are cast in terms of FTT, one can also interpret the tax as any other transaction cost specific to the purchaser<sup>4</sup>. The mechanism works as follows. For a pooling equilibrium to exist, the gains from liquidity asymmetry must outweigh the loss from information asymmetry of the informed trader with bad information. The Pareto optimal tax reduces the gains from liquidity needs such that the loss from information asymmetry of the informed trader with bad information is no longer outweighed.

There are few analytical analysis of the FTT, most notably Subrahmanyam (1998), Dow and Rahi (2000), Dupont and Lee (2007) and Davila (2013). These models have two common shortcomings.

First, information has no social value and therefore the notion of economic welfare is restricted to the asset market. Therefore, the FTT in their models can at best mitigate inefficiencies on the asset trade market. In this paper, I extend the definition of welfare to the real economy and can thus evaluate the FTT more holistically.

Second, the inefficiency in their models, i.e. the non-informativeness of the prices, arises by assumption. In Subrahmanyam (1998) as in Dupont and Lee (2007), there are passive noise traders "blurring" the informational content of the prices. Dow and Rahi (2000) consider on the buying side uninformed liquidity traders in addition to the informed trader. Whether prices reveal information depends on the share of uninformed traders and is hence exogenous. Davila (2013), the closest in spirit to this analysis, adopts a different asymmetry among traders' preferences. He characterizes an optimal FTT when traders disagree in beliefs. How much information

 $<sup>^{4}</sup>$ The same result could not be achieved through variation of the Central Bank rate. Recall from chapter 1, the interest rate difference (=liquidity difference) remains constant for changes in the FED's rate. But what would is needed is a cost/subsidy which affects either buyer or seller.

the price reveals depends on the degree of disagreement.

In order to alleviate the two drawbacks, I use a real investment function as in Dow and Rahi (2003) and introduce it in an asset trade model à la Laffont and Maskin (1990). They provide a signaling model in which non-information revelation occurs by choice. In Laffont and Maskin (1990), trade takes place for asymmetric risk aptitudes. This leads to non-linear profit functions which are hardly summable for welfare analysis. To obtain linear equilibrium profits, I introduce asymmetric liquidity needs.

The remainder of this paper is organized as follows. In section 2.2, I lay out the model setup. Section 2.3 characterizes the pooling and separating equilibrium. The welfare analysis is carried out in section 2.4 and section 2.5 concludes.

# 2.2 Model

Although I described the model setup already at length in chapter 1, I restate the setup again for better readability. The reader aware of the model setup can immediately skip to the next section where I provide the conditions for the equilibrium existence.

The model has five dates  $t \in \{0, 1, 2, 3, 4\}$  and a firm whose stock is traded in the Financial Market. There are two types of risk-neutral traders  $i \in \{I, U\}$ . An informed trader I and uniformed traders U of measure E. Each of the uninformed traders holds one unit of the entire stock of the asset. In line with their little asset holding, the uninformed traders are assumed to be in perfect competition and thus price takers. Throughout the model they are treated as one representative agent with an asset holding of E. Informed trader and uninformed trader have different liquidity needs. Liquidity needs are modeled with discount factors  $1 > \delta_i > 0$ . The higher  $\delta_i$  the less liquidity constrained is the trader. Assume, the informed trader is less liquidity constrained than the uninformed trader,  $\delta_I > \delta_U$ . The uninformed traders own assets of a firm which faces an uncertain investment opportunity  $V \in \{V_H, V_L\}$ .

In t = 0, the informed trader observes private information  $\omega \in \{H, L\}$ 

about the profitability of the firm's investment opportunity. With probability  $0 \leq \beta \leq 1$ , the firm's investment opportunity yields a payoff  $V_H$  and with probability  $1 - \beta$ ,  $V_L$ . Where  $V_H > V_L$ . Alternatively, the informed trader can invest in a riskless asset of which the revenue is normalized to 0, i.e. both the riskless rate and the revenue of the asset are 0. In t = 1, the informed trader decides to buy  $E \geq B \geq 0$  assets from the uninformed trader. In t = 2, the uninformed trader observes the informed trader's demand and decides to sell or to keep the assets. In t = 3, the firm observes the asset price P and takes its investment decision k. Eventually, in t = 4, either the high payoff  $V_H$  or the low payoff  $V_L$  realizes. The timeline is depicted in figure 2.1

Informed trader observes $\omega$	Informed trader buys <i>B</i> assets	Uninformed trader observes $B$ and decides to sell at $P$	Manager observes P and decides to invest k	$V_H$ or $V_L$ realizes	
t = 0	t = 1	t = 2	t = 3	t = 4	► t

Figure 2.1: Timeline

After observing the quantity chosen by the informed trader B, the uninformed trader updates the prior belief and form the conditional belief  $q = Pr(V_H|B)$ . Similarly the firm's manager updates his belief about the quality of the investment after observing the asset price P and form the conditional belief  $r = Pr(V_H|P)$ . Since there is not other private or public information besides the information about the outcome of the investment opportunity, in equilibrium, the price will reflect the demand of the informed trader only, and thus P conveys the same information as B. Therefore I can write r = q. For ease of notation, beliefs of both, the uninformed trader and the firm will be denoted by  $q = Pr(V_H|B)$ .

The firm value F increases in investment k at a decreasing rate  $\forall k \leq k^*$ , where  $k^*$  is the optimal investment level. c is a fixed marginal cost of investment. The firm's manager maximizes the firm value by choosing the investment level k given the price he observes on the stock market. The firm value increases in the prospect of the investment  $V_{\omega}$ . The manager's optimization problem is written as

$$F(k) = kV_{\omega} - \frac{c}{2}k^2 \tag{2.1}$$

so that the expected firm value becomes

$$E(F|B) = kE(V_H|B) - \frac{c}{2}k^2.$$
 (2.2)

The firm value function is adopted from Dow and Rahi (2003). The concavity of the firm value function in k implies that private information has social value even ex-ante.

After observing information in t = 0, the informed trader decides to buy a quantity B at a price P in period t = 1. On the purchase, he pays a tax  $\tau$ . When choosing B, the informed trader not only conditions on his private information  $\omega$  but also takes into account the signal his choice is sending to the uninformed trader and the firm. In t = 4, when the investment value  $V_{\omega}$  realizes and thus the firm value F, the informed trader cashes in on the assets bought. The informed trader evaluates the cash-flow from the perspective of period t = 1, i.e. when deciding on the purchase. The informed trader discounts the payoff of period t = 4 by  $\delta_I$ . By how much he discounts depends on how liquidity constrained he is. If, for example, the borrowing rate is zero, the informed trader is indifferent between a payoff today and tomorrow such that  $\delta_I = 1$ . The higher the borrowing rate, the lower the discount factor and the less willing is the informed trader to give up a payoff today for a payoff tomorrow. The informed trader's cash flow from buying the risky asset at date t = 1 is

$$-(1+\tau)PB + \delta_I BF. \tag{2.3}$$

Instead of buying the risky asset, the informed trader can also buy the riskless asset and obtain 0 payoff.

In t = 2, when selling an amount B of the total endowment E, the uninformed trader receives a revenue PB from the sale. In t = 4, after the realization of the investment value, just like the informed trader, the uninformed trader cashes in on the assets held. Evaluating the cash-flow from period t = 1, the uninformed trader discounts the payoff from period t = 4 by  $\delta_U$ . The uninformed trader's net present value (NPV) at t = 2 is

$$PB + \delta_U (E - B)F \tag{2.4}$$

In order to state the expected value of the uninformed trader's NPV, I have to specify the beliefs. Therefore, the introduction of the expected NPV is deferred to section 2.3. Instead of selling assets, the uninformed trader can also keep all the assets and receive a NPV at time t = 2 of  $\delta_U EF$ .

The government receives all the tax revenues, i.e.

$$\tau PB.$$
 (2.5)

# 2.3 Perfect Bayesian Equilibrium

The informed trader strategy is a mapping  $B : \{V_{\omega}\} \to \Re_0^+$  that prescribes a quantity  $B(V_{\omega})$  on the basis of the trader's private information  $\omega$ . The uninformed trader strategy is a mapping  $P : \Re_0^+ \to \Re_0^+$ . The firm manager strategy is a mapping  $k : \Re_0^+ \to \Re_0^+$ . Conditional beliefs for the uninformed trader and the firm manager are represented by a mapping that associates to each quantity B a probability function  $Pr(\cdot|B)$  on  $\{V_H, V_L\}$ , where  $Pr(V_{\omega}|B)$  is the probability that the uninformed trader and the firm manager attach to a value  $V_{\omega}$  given quantity B.

The perfect Bayesian equilibrium is defined by a triple of strategies  $(B(\cdot), P(\cdot), k(\cdot))$  and a family of conditional beliefs  $Pr(\cdot|\cdot)$  such that (i) for all B in the range of  $B(\cdot)$ ,  $Pr(\cdot|B)$  is the conditional probability of  $V_{\omega}$  obtained by updating the prior  $(\beta, (1 - \beta))$ , using  $B(\cdot)$  in Bayesian fashion; (ii) for all  $B(\cdot)$ ,  $P \ argmax_P \in E(U_U(\cdot))$ , (iii) for all  $B(\cdot)$   $k^* \in argmax_k \ E(F|B)$  and (iv) for all  $\omega \ B \in argmax_B \ E(U_I(\cdot))$ . Condition (i) stipulates that the uninformed trader and the firm's manager have rational expectations. Conditions (ii) to (iv) require that traders be optimizing. In particular, they imply participation constraints and incentive compatibility constraints.

Market clearing takes place through the adjustment of the price P to the quantity demanded B. I.e. the informed trader submits a market order. Observing the market order, the uninformed trader, acting as a market maker, updates the belief about the quality of the asset and sets the price. In equilibrium, there has to be a unique price-quantity bundle  $\{P, B\}$ .

I derive the equilibria as in chapter 1. To avoid repetition, I remark which optimization problems remain unchanged and just state the resulting conditions as derived in chapter 1. For those optimization problems affected by the introduction of the tax  $\tau$ , I will briefly describe the effect of  $\tau$  on the optimization problem and state the resulting condition.

#### 2.3.1 Equilibrium existence

#### Separating equilibrium

In the separating equilibrium, depending on the private information  $\omega \in \{H, L\}$ , the informed trader buys different quantities  $B^H$  and  $B^L$ .

The firm manager's beliefs and the uninformed trader's beliefs are not affected by the introduction of a tax.

$$q = Pr(H|B) = \begin{cases} 1 & \text{if } B = B^{H} \\ 0 & \text{if } B = B^{L} \\ 1 & B' \neq B^{H} \land B' \neq B^{L} \end{cases}$$
(2.6)

Also the firm manager's optimal choice remains,  $k^{\omega} = \frac{V_{\omega}}{c}$ . Given the optimal choice, the firm value from the manager's perspective (and the uninformed trader's perspective) is  $F_{\omega} = \frac{V_{\omega}^2}{2c}$ .

The tax  $\tau$  is levied on purchases. The uninformed trader is only selling assets. Therefore, participation of the uninformed trader is unaffected by the tax. Prices remain hence unchanged, i.e.  $P^{\omega} = \delta_U F_{\omega}$ . Notice, the price decreases the more liquidity constrained the uninformed trader.

Whether the informed trader buys the risky asset or the risk-less asset is affected by the tax since he has to pay the tax on the value purchased.

The equilibrium firm value from the informed trader's perspective after

observing information  $\omega$  is  $F_{\omega} = \frac{V_{\omega}^2}{2c}$ . With the equilibrium price  $P^{\omega} = \delta_U F_{\omega}$ , the participation constraint of the informed trader with information  $\omega$  becomes:

$$-(1+\tau)\delta_U F_\omega B^\omega + \delta_I B^\omega F_\omega \ge 0, \qquad (2.7)$$

with the revenue of the risk-less asset normalized to 0. For any positive trade  $B^{\omega} \geq 0$  both types' participation constraints are satisfied if  $\frac{\delta_I}{(1+\tau)} > \delta_U$ . The informed trader buys the risky asset whenever he is less liquidity constrained than the uninformed trader. The informed trader has to pay a proportion relative to  $\tau$  of the purchase value to the government. For the informed trader to be willing to purchase the risky asset, the difference in liquidity needs, needs to be larger than without a tax.

In the separating equilibrium, I have to show, given beliefs q as specified in 2.6, prices  $P^{\omega} = \delta_U F_{\omega}$  and firm value  $F_{\omega} = \frac{V_{\omega}^2}{2c}$ , that  $B^H$  and  $B^L$  are optimal choices for the respective type of informed trader. In particular that the informed trader with good information H does not want to mimic the informed trader with bad information L and vice versa:

$$-(1+\tau)\delta_U F_{\omega}B^{\omega} + \delta_I B^{\omega} F_{\omega} \ge -(1+\tau)\delta_U F_{-\omega}B^{-\omega} + \delta_I B^{-\omega} F'_{\omega} \qquad (2.8)$$
  
where  $-\omega \neq \omega$ .

If an informed trader with information  $\omega$  chooses  $B^{-\omega}$ , from his perspective the firm's value  $F'_{\omega} = \frac{V_{-\omega}}{c} (V_{\omega} - \frac{V_{-\omega}}{2})$ . Moreover, I have to show that the informed trader with information  $\omega$  chooses  $B^{\omega}$  and not any other quantity  $B' \neq B^{\omega} \forall \omega$ , i.e.

$$(-P^H + \delta_I F_H)B^H \ge (-P' + \delta_I F_H)B' \quad \forall B' \text{ and}$$
 (2.9)

$$(-P^L + \delta_I F_L)B^L \ge (-P' + \delta_I F'_L)B' \quad \forall B'.$$

$$(2.10)$$

With the off-equilibrium price  $P' = \delta_U F_H$ . The reasoning for finding  $B^H$ 

and  $B^L$  is identical to chapter 1 and yields the following conditions:

$$B^{H} = E,$$

$$\frac{(-(1+\tau)\delta_{U} + \delta_{I})F_{H}}{-(1+\tau)\delta_{U}F_{L} + \delta_{I}F'_{H}}E \ge B^{L} \ge max\{0, \frac{-(1+\tau)\delta_{U}F_{H} + \delta_{I}F'_{L}}{(-(1+\tau)\delta_{U} + \delta_{I})F_{L}}E\}.$$
(2.12)

In the following proposition, I report for which parameters of the model, there exists a separating equilibrium.

**Proposition 4.** Separating equilibrium. There exist separating equilibria with price  $P^{\omega} = \delta_U F_{\omega}$ ,  $\omega \in \{H, L\}$  and quantities as specified in 2.11 and 2.12 if

•  $\delta_U \frac{F_H - F_L}{F_H - F'_H} > \frac{\delta_I}{(1+\tau)} > \delta_U$  and  $V_H > \frac{1}{2}(1+\sqrt{5})V_L$  or •  $\delta_U \frac{F_H - F_L}{F_H - F'_H} > \frac{\delta_I}{(1+\tau)} > \delta_U \frac{F_H}{F'_L}$  and  $\frac{1}{2}(1+\sqrt{5})V_L > V_H > V_L$ .

In the separating equilibrium prices reveal available information. Trade is maximal given good information. For the informed trader with good information not to mimic the informed trader with bad information, trade must be less than maximal in the case of bad information. Moreover, the maximal amount of trade, refer to the left hand side of inequality 2.12, in the presence of bad information is decreasing the higher the tax  $\tau$ . The intuition is that the tax increases the potential gain of the informed trader with bad information from mimicking the informed trader with bad information and thus pay a lower price.

Also, existence of the separating equilibrium depends on the tax  $\tau$ . An increase in the tax reduces the difference in liquidity needs and hence the scope for trade. In the separating equilibrium, trade takes place due to asymmetric liquidity needs, i.e. the informed trader buys assets from the uninformed trader if the uninformed trader is asking a lower price than the informational value in order to satisfy liquidity needs. If however the informed trader has to pay a tax on his purchases, the gain from the liquidity difference shrinks.

#### Pooling equilibrium

In the pooling equilibrium, the informed trader buys the same quantity  $B^P$  in either state.

Again, beliefs of the manager and the uninformed trader are unaffected by the tax  $\tau$ :

$$q = Pr(H|B) = \begin{cases} \beta & \text{if } B = B^P \\ 1 & B' \neq B^P \end{cases}.$$
 (2.13)

Also the firm manager's optimal choice remains,  $k^P = \frac{E(V)}{c}$  with  $E(V) = \beta V_H + (1-\beta)V_L$ . Given the optimal choice, the firm value from the manager's perspective (and the uninformed trader's perspective) is  $F_P = \frac{E(V)^2}{2c}$ .

Participation of the uninformed trader is unaffected by the tax. Prices remain hence unchanged, i.e.  $P = \delta_U F_P$ .

The informed trader's participation however is affected by the tax. Given the price  $P = \delta_U F_P$ , the informed trader decides whether to buy  $B^P$  of the risky asset or the riskless asset which gives a return of 0. His participation constraints in either state is

$$-(1+\tau)\delta_U F_P B^P + \delta_I B^P F_P^\omega \ge 0. \tag{2.14}$$

The firm value from the perspective of the informed trader takes into account the investment decision of the manager,  $k^P = \frac{E(V)}{c}$ , given the privately observed information  $\omega$ . Therefore, the firm value from the informed trader's perspective are  $F_P^{\omega} = \frac{E(V)}{c} (V_{\omega} - \frac{E(V)}{2})$  with  $F_P^L > 0$  if and only if  $\frac{V_L}{V_H - V_L} > \beta$ . Observe,  $F_P^H > F_P > F_P^L$ . Implying that the informed trader with good information faces a higher return from the risky asset than the informed trader with bad information, given the uninformed choice of the manager. Both, the informed trader with good information and the informed trader with bad information however faces a lower payoff than if the manager made an informed choice,  $F_P^{\omega} < F^{\omega}$ . The participation constraint for the informed trader with bad information is more binding. In fact, if  $\omega = L$ , the left hand side of the inequality is smaller than if  $\omega = H$ . Solving the participation of the informed trader with bad information for  $\delta_I$  yields for any  $B^P \ge 0$ :  $\frac{\delta_I}{1+\tau} \geq \delta_U \frac{F_P}{F_P^L}$ . Since  $\frac{F_P}{F_P^L} > 1$ , the informed trader needs to be considerably less liquidity constrained than the uninformed trader, and in particular less than in the separating equilibrium where participation was ensured if  $\delta_I > \delta_U$ . Observe that  $\frac{F_P}{F_P^L}$  increases in  $\beta$ . That is the more likely the good outcome, the larger needs to be the difference between the informed trader's liquidity needs and the uninformed trader's liquidity needs.

In the pooling equilibrium, I have to show that there exists a single  $B^P$  for which both types of informed traders are willing to buy the risky asset. In particular, that there is no  $B' \neq B^P$  that either of the two types would prefer over buying  $B^P$ . This is formalized in the following incentive compatibility constraint:

$$(-(1+\tau)\delta_U F_P + \delta_I F_P^H)B^P \ge (-(1+\tau)P' + \delta_I F_H)B' \quad \forall B'$$
(2.15)

$$(-(1+\tau)\delta_U F_P + \delta_I F_P^L)B^P \ge (-(1+\tau)P' + \delta_I F_L')B' \quad \forall B'$$

$$(2.16)$$

with  $P' = \delta_U F_H$ . The off-equilibrium price follows with the off-equilibrium beliefs and the uninformed trader breaking even. Incentive compatibility is satisfied if  $B^P = E$ . The reasoning for this result

From 2.15 we observe, the informed trader with good information is indifferent between the equilibrium payoff and the off-equilibrium payoff if  $E > B^P = \frac{(-(1+\tau)\delta_U + \delta_I)F_H}{-(1+\tau)\delta_U F_P + \delta_I F_P^H} E$ . The equilibrium payoff however is maximized for  $B^P = E$ . This is a sufficient condition for incentive compatibility of the informed trader with bad information.

The lower bound for  $B^P$ ,  $\frac{(-(1+\tau)\delta_U+\delta_I)F_H}{-(1+\tau)\delta_UF_P+\delta_IF_P^H}$  is decreasing in  $\tau$ .  $B^P = E$ hence remains an optimal choice for the informed trader for any  $1 > \tau > 0$ .

The existence of a pooling equilibrium is summarized in the following proposition.

**Proposition 5.** Pooling equilibrium. If  $\delta_U \frac{F_H - F_P}{F_H - F_P^H} > \frac{\delta_I}{1 + \tau} > \delta_U \frac{F_P}{F_P^L}$  and  $\beta < \min\{\frac{V_L}{V_H - V_L}, \frac{V_L^2}{(V_H - V_L)^2}\}$ , there exists a pooling equilibrium with a price  $P = \delta_U F_P$  and equilibrium trade  $B^P = E$ .

The pooling equilibrium depends on the tax through the participation constraint of the informed trader and the incentive compatibility constrained of the informed trader. As for the separating equilibrium, the gain from trade due to the difference in liquidity needs decreases and thus the range of the existence of the pooling equilibrium. The optimal amount of trade is not affected by the tax. This is a crucial observation for the welfare analysis.

#### 2.3.2 Equilibrium characterization

After stating the existence conditions for each type of equilibrium, I can now characterize all possible equilibria given the beliefs in 2.6 and 2.13. The objective is to characterize equilibria depending on the liquidity difference,  $\delta_I - \delta_U$ . Observe from the two previous propositions that the existence of equilibria depends on the following three thresholds which characterize the liquidity difference:  $\frac{F_H - F_P}{F_H - F_P}$ ,  $\frac{F_P}{F_P}$  and  $\frac{F_H - F_L}{F_H - F_H}$ . In order to rank them, I need to derive conditions on  $\beta$  and the difference  $V_H - V_L$ , the parameters on which the firm values F depend. The derivation of the condition is identical to chapter 1. The equilibrium characterization is a preparatory step for the welfare analysis. It shows when the parameter areas for which separating equilibrium and pooling equilibrium exist respectively, overlap and when they do not.

**Lemma 4.** Separating equilibrium only. For  $1 \ge \beta > \min\{\frac{V_L}{V_H - V_L}, \frac{V_L^2}{(V_H - V_L)^2}\}$ , there exists a separating equilibrium only (if  $\delta_U \frac{F_H - F_L}{F_H - F'_H} > \frac{\delta_I}{1 + \tau} > \max\{\delta_U, \delta_U \frac{F_H}{F'_L}\}$ ).

Lemma 4 tells, if the difference between the high outcome and the low outcome is very small, only the separating occurs.

**Lemma 5.** Separating equilibrium and pooling equilibrium do not overlap. For  $min\{1, \frac{V_L}{V_H - V_L}, \frac{V_L^2}{(V_H - V_L)^2}\} > \beta > \frac{V_L^2}{(V_H - V_L)V_H}$ , there exists

- a separating equilibrium only (if  $\delta_U \frac{F_H F_L}{F_H F'_H} > \frac{\delta_I}{1 + \tau} > max\{\delta_U, \delta_U \frac{F_H}{F'_L}\})$ and
- a pooling equilibrium only (if  $\delta_U \frac{F_H F_P}{F_H F_P^H} > \frac{\delta_I}{1 + \tau} > \delta_U \frac{F_P}{F_P^L}$ ).

For a given  $V_H - V_L$  and  $\beta$ , the separating equilibrium exists for a small liquidity difference and the pooling equilibrium for a large liquidity difference since  $\frac{F_P}{F_P} > \frac{F_H - F_L}{F_H - F'_H}$ .

**Lemma 6.** Separating equilibrium and pooling equilibrium overlap. For  $min\{1, \frac{V_L^2}{(V_H - V_L)V_H}\} > \beta$ , there exists

- a pooling equilibrium only if  $\delta_U \frac{F_H F_P}{F_H F_P^H} > \frac{\delta_I}{1 + \tau} > \delta_U \frac{F_H F_L}{F_H F_H'}$ ,
- both a pooling equilibrium and a separating equilibrium if  $\delta_U \frac{F_H F_L}{F_H F'_H} > \frac{\delta_I}{1 + \tau} > max \{ \delta_U \frac{F_P}{F_P}, \delta_U \frac{F_H}{F'_L} \}$  and
- a separating equilibrium only if  $\delta_U \frac{F_P}{F_P^L} > \frac{\delta_I}{1+\tau} > max\{\delta_U, \delta_U \frac{F_H}{F_L'}\}.$

Pooling equilibrium and separating equilibrium overlap if for an increasing difference in the investment's outcome  $V_H - V_L$ , the probability of observing the high outcome  $\beta$  decreases and the liquidity difference is intermediate. This characterizes a situation in which the informed trader can make moderate gains from trade since both the gain from the liquidity asymmetry and the gain from information asymmetry are moderate.

# 2.4 Welfare

The firm representing the real economy is entirely owned by the traders. Therefore, welfare is captured by the sum of the traders' profits and the government's revenue from the tax. Consider ex-ante profits as in Dow and Rahi (2003) and Laffont and Maskin (1990).

#### 2.4.1 Profits

Denote by  $\psi \in \{S, P\}$  either type of equilibrium, i.e. separating equilibrium or pooling equilibrium. The equilibrium profit of an informed trader I in a separating equilibrium with  $P^{\omega} = \delta_U F_{\omega}$  and  $B^{\omega}$  is

$$\Pi^{\omega SI} = (-(1+\tau)\delta_U + \delta_I)F_{\omega}B^{\omega}.$$
(2.17)

For the uninformed trader U the equilibrium profit becomes

$$\Pi^{\omega SU} = \delta_U E F_{\omega}. \tag{2.18}$$

The government's revenue is

$$\Pi^{\omega SG} = \tau \delta_U F_\omega B^\omega. \tag{2.19}$$

Analogously, I obtain the profit of the informed trader I in a pooling equilibrium with the equilibrium price  $P = \delta_U F_P$  and the equilibrium quantity  $B^P$ 

$$\Pi^{\omega PI} = (-(1+\tau)\delta_U F_P + \delta_I F_P^{\omega})B^P.$$
(2.20)

For the uninformed trader U the equilibrium profit becomes

$$\Pi^{\omega PU} = \delta_U F_P E. \tag{2.21}$$

And the government's revenue is

$$\Pi^{\omega PG} = \tau \delta_U F_P B^P. \tag{2.22}$$

Observe that the profit of the informed trader is increasing in the quantities  $B^{\omega}$  and  $B^{P}$ . Furthermore, profits are increasing in the firm value F. These two observations will guide the following welfare analysis.

#### 2.4.2 Welfare

The sum of the expected profits in the separating equilibrium is:

$$W_{S} = \delta_{U} E(\beta F_{H} + (1 - \beta) F_{L}) + (\delta_{I} - \delta_{U})(\beta F_{H} B^{H} + (1 - \beta) F_{L} B^{L}).$$
(2.23)

And welfare in the pooling equilibrium becomes:

$$W_P = \delta_U E F_P + (\delta_I (\beta F_P^H + (1 - \beta) F_P^L) - \delta_U F_P) B^P.$$
 (2.24)

Observe, the tax  $\tau$  does not enter in either of the two welfare functions, which implies there is no primary tax effect. This is due to the fact that all agents, traders and government, are risk neutral and thus have a linear utility function. Therefore, the tax payment of the trader and the tax revenue of the government cancel out. The tax affects welfare through the participation constraints and incentive compatibility constraints of the informed trader.

#### 2.4.3 Welfare analysis

The objective of the analysis is to find a tax  $\tau$  such that it improves welfare. Therefore I first Pareto rank separating equilibrium and pooling equilibrium and then show for which tax the Pareto optimal equilibrium is attained.

It makes sense to compare equilibrium if they exist for the same values of parameters. From lemma 6, we know pooling equilibrium and separating equilibrium overlap if  $min\{1, \frac{V_L^2}{(V_H - V_L)V_H}\} > \beta$  and  $\delta_U \frac{F_H - F_L}{F_H - F'_H} > \frac{\delta_I}{1 + \tau} > max\{\delta_U \frac{F_P}{F_P^L}, \delta_U \frac{F_H}{F'_L}\}$ . Moreover, observe adjacent to the area of multiple equilibria, there is the area  $\delta_U \frac{F_P}{F_P^L} > \frac{\delta_I}{1 + \tau} > max\{\delta_U, \delta_U \frac{F_H}{F'_L}\}$  in which only the separating equilibrium exists.

Consider the following situation. The economy is in a pooling equilibrium with  $\delta_U \frac{F_H - F_L}{F_H - F_H} > \delta_I > max \{ \delta_U \frac{F_P}{F_P^L}, \delta_U \frac{F_H}{F_L^\prime} \}$ . The introduction of a tax  $\tau$  changes the liquidity difference to  $\delta_U \frac{F_P}{F_P^L} > \frac{\delta_I}{1 + \tau} > max \{ \delta_U, \delta_U \frac{F_H}{F_L^\prime} \}$  such that only a separating equilibrium exists. Therefore I have to show that welfare in the separating equilibrium is larger when equilibria overlap and that welfare in the separating equilibrium is still larger when only the separating equilibrium exists and the informed trader has to pay a tax.

Given the equilibrium quantities  $B^P = B^H = E$ , welfare in the separating equilibrium is larger than in the pooling equilibrium if

$$\Delta W = W_S - W_P = \delta_U E(\beta F_H + (1 - \beta) F_L) + (\delta_I - \delta_U)(\beta F_H E + (1 - \beta) F_L B^L) - \delta_I (\beta F_P^H + (1 - \beta) F_P^L) E > 0.$$
(2.25)

If separating equilibrium and pooling equilibrium co-exist, from chapter 1, we know for which conditions the separating equilibrium yields larger welfare than the pooling equilibrium. For readability, the proposition is reported here  $again^5$ .

**Proposition 6.** Welfare comparison. The separating equilibrium yields  $5^{5}$  For the proof, refer to chapter 1.

greater welfare than the pooling equilibrium if  $\delta_U \frac{F_H - F_L}{F_H - F'_H} > \delta_I > \delta_U \frac{F_P}{F_P^L}$ ,  $\frac{V_L^2}{V_H(V_H - V_L)} > \beta$  and

- either  $\frac{(\delta_{I}-\delta_{U})F_{H}}{\delta_{I}F'_{H}-\delta_{U}F_{L}}E > B^{L} \ge 0, \quad \frac{V_{L}^{2}}{V_{H}(V_{H}-V_{L})} > \beta > \frac{3V_{L}-V_{H}}{(V_{H}-V_{L})^{2}} \text{ and } V_{H} > (1 + \sqrt{2})V_{L}$
- or

$$B^{L}(V_{H}) = \frac{2\beta\delta_{I}(V_{H}V_{L} - \frac{V_{H}^{2}}{2})}{(\delta_{I} - \delta_{U})V_{L}^{2}}E, \min\{\delta_{U}\frac{F_{H} - F_{L}}{F_{H} - F_{H}'}, \delta_{U}\frac{1}{1 - \beta}\} > \delta_{I}, \beta > \frac{2V_{H} - 3V_{L}}{V_{H} - V_{L}}$$
  
and either  $\frac{3}{2}V_{L} > V_{H} > V_{L}$  or  $2V_{L} > V_{H} > \frac{1}{4}(3 + \sqrt{17})V_{L}.$ 

After the introduction of the tax, the separating equilibrium yields larger welfare for the following tax.

#### Proposition 7. Pareto optimal FTT. There is a Pareto optimal tax

- With  $B^L \ge 0$ ,  $\min\{\frac{V_L^2}{V_L^2 \beta(V_H V_L)^2} 1, \frac{\delta_I}{\delta_U} 1\} > \tau > \frac{\delta_I}{\delta_U} \frac{F_P^L}{F_P} 1$  if  $\min\{\delta_U \frac{F_P}{F_P^L} \frac{V_L^2}{V_L^2 \beta(V_H V_L)^2}, \delta_U \frac{F_H F_L}{F_H F_H'}\} > \delta_I > \delta_U \frac{F_P}{F_P^L}.$
- And a Pareto optimal tax with  $B^L > 0$ ,  $min\{\frac{\delta_I}{\delta_U} 1, \frac{\beta}{1-\beta}\} > \tau > 0$ , if  $\frac{\delta_U}{1-\beta} > \delta_I > \delta_U \frac{F_P}{F_P^L}$  and  $\beta > \frac{2V_H 3V_L}{V_H V_L}$ .

The proof is relegated to appendix B.1.

As we know from lemma 6, pooling equilibrium and separating equilibrium overlap if gains for the informed trader from both information asymmetry  $^{6}$  and liquidity asymmetry are intermediate. For given information asymmetry, for a small liquidity asymmetry there exists a separating equilibrium only. The idea of proposition 7 is to decrease liquidity asymmetry such that the pooling equilibrium ceases to exist and the separating equilibrium exists only.

In the following, I try to shed some light on the underlying mechanism. Recall, prices have two components, the informational component,  $E(V_{\omega}|B)$ and the liquidity component,  $\delta_U$ . In a pooling equilibrium, the informational component of the price, E(V), is higher than private information

<sup>&</sup>lt;sup>6</sup>For a given  $\beta$ , information asymmetry is large if  $V_H - V_L$  is large. Or, for a given  $V_H - V_L$ , information asymmetry is large if  $\beta = \frac{1}{2}$ .

of the informed trader with bad information  $V_{\omega}$ . In order for him to buy the asset nevertheless, the uninformed trader must be sufficiently liquidity constrained such that the informational loss is at least outweighed by the liquidity gain. When pooling equilibrium and separating equilibrium co-exist, liquidity gains outweigh informational loss by few. The introduction of the tax decreases further the liquidity gain up to the point that the informed trader's informational loss is no longer outweighed by the liquidity gain and thus the informed trader with bad information no longer wants to buy the asset.

# 2.5 Conclusion

I identify a "case" in which asset trade leads to economic inefficiency. I show that in this "case" the inefficiency can be alleviated by the introduction of a FTT. The "case" is an economy consisting of an asset trade market and a firm representing the real economy. The firm has an investment opportunity with an uncertain return. Before deciding on the investment level, the firm's manager consults the firm's asset price. It does so because the informed trader has superior information about the investment's return. The asset trade market is modeled in a simple signaling setup in which a monopolistically informed trader, with either good or bad information, buys assets from uninformed traders. On every purchase, a FTT is levied. The uninformed trader sells the assets for liquidity needs. I find a separating equilibrium in which the informed trader chooses different quantities depending on the information observed. Hence, prices reveal available information and the firm takes the efficient investment decision. There exists also a pooling equilibrium in which the informed trader chooses the same quantity regardless of his information. Therefore the price does not reveal available information and the firm takes an inefficient investment decision. The welfare ranking of the two types of equilibria depends on both the amount of trade and information revelation. For the trader(s) who own(s) the firm, information revelation (separating equilibrium) is always better. The pooling equilibrium features more trade than the separating equilibrium and thus increases welfare of the pooling equilibrium. The trade-off for total welfare is hence information revelation vs. trade. I provide conditions on the traded quantities for which the separating equilibrium yields greater welfare. If the economy is in a pooling equilibrium, the government can introduce a tax which coordinates the economy on the Pareto optimal separating equilibrium.

The main contribution of this article is to identify an endogenous inefficiency which can be corrected by a FTT.

It remains a partial equilibrium model and hence intrinsically features the typical shortcomings. First, it still does not allow to study the effect of the tax on a more complex economy with different countries for example. The paper also does not provide any insights on inter-temporal effects of the tax.

# Chapter 3

# Optimal Timing of Asset Purchases

# 3.1 Introduction

The idea that Financial Markets aggregate and reveal dispersed information is an important part of economic thinking. The seminal paper by Kyle (1985) and the more recent papers by Back and Baruch (2004) and Ostrovsky (2012) find that if a monopolistically trader with long-lived private information buys assets in finitely many periods, information is always revealed. To obtain information revelation, this literature requires that trade takes place in every period and the presence of noise traders.

This paper shows that in the presence of rational traders, instead of nonrational noise traders, it is not optimal to trade in every period and at the same time information is not revealed.

The model considered goes as follows. Asset trade takes place between an informed trader and uninformed traders in a twice repeated model à la Laffont and Maskin (1990). The informed trader observes either good or bad information about the prospect of the asset. The uninformed traders are holding the assets. By observing the demand of the informed trader, the uninformed traders update their beliefs about the quality of the asset and decide whether to sell. Notice, the informed trader takes into account the effect of his purchase on the beliefs and thus the asset price. Trade can take place in two periods. The informed trader is less liquidity constrained than the uninformed traders. The liquidity needs can vary over time but the the informed trader is always required to be less liquidity constrained than the uninformed trader. Liquidity needs are modeled with different discount factors. The assumptions on the liquidity needs are supported by the stylized fact in figure 1.1. It shows the liquidity needs by different types of traders.

Liquidity needs are reflected by borrowing rates. In the US, there are different borrowing rates. There is a small number of traders who have access to the FED Funds Rate. Those are the Primary Dealers<sup>1</sup> who are eligible to engage in repurchase agreements (REPOs) with the FED. Essentially, the FED provides a collateralized debt to the Primary Dealers. Currently, there are 22 Primary Dealers. Among them only the biggest financial institutions in the world in terms of assets under management (AUM). For example BNP Paribas, Barclays, Credit Suisse, Deutsche Bank, Goldman Sachs, J.P. Morgan, Morgan Stanley, Nomura and UBS. Most of the other investment institutions face the Bank Prime Loan Rate which is offered by banks to their most favorable clients. The Bank Prime Loan Rate and the FED Funds Rate are depicted in figure 1.1.

There is a systematic difference in the borrowing rate of the Primary Dealers, i.e. the FED Funds Rate, and the borrowing rate of most of the other Financial Market participants, i.e. the Bank Prime Loan Rate. Facing a relatively high borrowing rate, Financial Market participants who hold assets can, instead of borrowing, sell their assets. Since Primary Dealers face a lower borrowing rate, they can borrow money and buy the assets from the other Financial Market participants. The latter are willing to decrease the asset price, at which they sell, proportionally to their liquidity needs. This creates a motive for trade. Over time, figure 1.1 depicts an almost constant positive difference between the Bank Prime Loan Rate and the FED Funds Rate.

<sup>&</sup>lt;sup>1</sup>The list of current and historic Primary Dealers can be found on New York's Fed website: http://www.ny.frb.org/markets/pridealers\_current.html\#tabs-1.

The main result is that it is not optimal to trade in both periods in the presence of rational traders. Moreover, with one period trade, there exist two types of equilibria. A separating equilibrium in which the price reveals available information. And a pooling equilibrium exhibiting no information revelation.

In order to provide the intuition, I first lay out the drivers of a one period equilibrium. In pooling equilibria, the informed trader does not reveal private information by demanding the same quantity no matter whether he has good or bad information. Then the uninformed trader cannot infer information from the informed trader's demand and hence stays with the prior beliefs. In the pooling equilibrium, the asset price reflects the expected value of the asset which is below the prospect of the informed trader with good information and above the prospect of the informed trader with bad information. The equilibrium price is decreasing in the liquidity needs of the uninformed trader. The asset price consists of an informational component and a liquidity component. Given the informed trader is at most as liquidity constrained as the uninformed trader, for the informed trader with good information the pooling equilibrium price is relatively low in comparison to his prospect and thus he is always willing to buy. The informed trader with bad information however is only willing to buy if the negative difference between his prospect and the expected value of the asset is outweighed by the uninformed trader's liquidity needs. In other words, in the pooling equilibrium, in addition to the liquidity wedge, there is an information wedge. The information wedge is to the detriment of the uninformed trader with bad information and to the benefit of the informed trader with good information. Therefore, the liquidity wedge has to outweigh, i.e. the informed trader has to be strictly less liquidity constrained than the uninformed trader, the information wedge for the uninformed trader with bad information to participate.

In the one-period separating equilibrium, the informed trader reveals private information by purchasing more after observing good information than after observing bad information. Trade takes only place if the uninformed trader is at least as liquidity constrained as the informed trader<sup>2</sup>. The asset price, again, has an informational component and a liquidity component. The informational component reflects the information of the informed trader. Due to the liquidity component, the asset price decreases the more the uninformed trader is liquidity constrained.

Long-lived information imply that the information observed at the beginning persist over the two rounds of trade until the outcome of the asset eventually realizes. Traders discount future revenue from the asset when they trade. Therefore the future value of the asset in the first round of trade is smaller than in the second round of trade. Having clarified the roles of liquidity needs and long-lived information, I can proceed to the presentation of the main result of the paper.

Trade takes place in the first period only in both the separating equilibrium and the pooling equilibrium. This is due to the fact that the informed trader is less liquidity constrained than the uninformed trader in both periods. Comparing the price of the first and the second period, from the perspective of the first period, the informed trader faces a lower price in the first period than in the second period. More broadly speaking, the relative (to the informed trader) urgency of cash of the uninformed traders, makes the uninformed traders decrease the price for the assets from the informed trader and the more so, the earlier the uninformed traders can get the cash.

The result of information revelation in the literature is an artifact of the selection of equilibrium beliefs and noise traders. The separating equilibrium in this paper in fact features information revelation. This is the result obtained by the literature (Kyle (1985), Bach and Baruck (2004) and Ostrovsky (2012)). It however features also no information revelation in the pooling equilibrium. Which is different from Kyle (1985), Bach and Baruck (2004) and Ostrovsky (2012).

The introduction of asymmetric liquidity needs, helps to circumvent the use of non-rational noise traders. Morevover, it simplifies the analysis such that I can consider beliefs, as suggested by Laffont and Maskin (1990), which

<sup>&</sup>lt;sup>2</sup>Note, the difference. For the pooling equilibrium to exist, the uninformed trader needs to be strictly more liquidity constrained than the informed trader.

generate pooling equilibria in which information is not revealed. The literature following Kyle (1985), and in particular most recent publications by Ostrovsky  $(2012)^3$  and Back and Baruch (2004), finds that in a model with a monopolistically informed trader and finitely repeated trade, information is revealed through asset prices. This is due to the uncertain demand of noise traders. Noise traders demand/supply follows a random distribution. Substituting noise traders by optimizing, uninformed traders yields that it is not optimal to trade in every period and information is not always revealed.

In the remainder of this paper, section 3.2 presents the model set-up. In section 3.3, I derive both separating equilibrium and pooling equilibrium. In section 3.4, the assumption of asymmetric liquidity needs is discussed. Section 3.5 offers some concluding remarks.

# 3.2 Model

There are four periods  $T \in \{0, 1, 2, 3\}$ . In periods T = 1 and T = 2 trade takes place on two dates  $t \in \{1, 2\}$ . There are two types of risk-neutral traders  $i \in \{I, U\}$ . An informed trader I and uniformed traders U of measure E. Each of the uninformed traders holds one unit of the entire stock of the asset. In line with their little asset holding, the uninformed traders are assumed to be in perfect competition and thus price takers. Throughout the model they are treated as one representative agent with an asset holding of E. Informed trader and uninformed trader have different liquidity needs. Liquidity needs are modeled with discount factors which may be different for each period T,  $1 > \delta_{iT} > 0$ . The higher  $\delta_{iT}$  the less liquidity constrained is the trader. According to the stylized fact, the informed trader is less liquidity constrained than the uninformed traders but the size of the liquidity wedge is determined endogenously. Moreover, assume that traders know each others liquidity needs now and in the future period<sup>4</sup>.

 $<sup>^{3}</sup>$ Theorem 5.

<sup>&</sup>lt;sup>4</sup>It is a strong assumption that there is certainty about future liquidity constraints. This assumption can be relaxed by having  $\delta_{i2}$  follow some random distribution  $f(\delta_{i2})$ . As long as  $f(\delta_{i2})$  is independent of the distribution of the asset's return, the results of the model are unaffected.

In T = 0, the informed trader observes a private information  $\omega \in \{H, L\}$ about the profitability of the risky asset. With probability  $0 \leq \beta \leq 1$ , the risky asset yields a payoff  $V_H$  and with probability  $1 - \beta$ ,  $V_L$ . Where  $V_H > V_L$ . Alternatively, the informed trader can invest in a riskless asset of which the revenue is normalized to 0, i.e. both the riskless rate and the revenue of the asset are 0. In T = 1 and T = 2, asset trade takes place as follows: In t = 1, the informed trader decides to buy  $E \geq B_T \geq 0$  assets from the uninformed trader. In t = 2, the uninformed trader observes the informed trader's demand  $B_T$ , updates the prior belief according to Bayes' rule  $q_T = Pr(V_H|B_T)$  and decides to sell or to keep the assets. In T = 3, either the high payoff  $V_H$  or the low payoff  $V_L$  realizes.

In t = 1 of T = 1 and T = 2, the informed trader decides to buy a quantity  $B_T$  at a price  $P_T$ . When choosing  $B_T$ , the informed trader not only conditions on his private information  $\omega$  but also takes into account the beliefs of the uninformed trader. In T = 3, when the asset's value  $V_{\omega}$ realizes, the informed trader cashes in on the assets bought. The informed trader evaluates the cash-flow from the perspective of period T = 1 and T = 2 respectively, i.e. when deciding on the purchases. The informed trader discounts the payoff of period T = 3 by  $\delta_{I2}$  when buying in T = 2and by  $\delta_{I1}\delta_{I2}$  when buying in T = 1. By how much he discounts depends on how liquidity constrained he is. If, for example, the borrowing rate is zero, the informed trader is indifferent between a payoff today and tomorrow such that  $\delta_{IT} = 1$ . The higher the borrowing rate, the lower the discount factor and the less willing is the informed trader to give up a payoff today for a payoff tomorrow. The informed trader's cash flow from buying risky assets in period T = 2 is

$$-P_2B_2 + \delta_{I2}(B_1 + B_2)V_{\omega}.$$
 (3.1)

If the informed trader bought assets in T = 1, he will continue to hold them in T = 2 regardless of whether he purchases further assets or not so that his outside option, i.e. the NPV from buying the riskless asset and not the risky asset becomes  $\delta_{I2}B_1V_{\omega}$ . When purchasing assets in period T = 1, the informed trader takes into account the effect of his purchase  $B_1$  on revenues in T = 2 and T = 3.

$$-P_1B_1 + \delta_{I1}(-P_2B_2 + \delta_{I2}(B_1 + B_2)V_{\omega}). \tag{3.2}$$

Instead of buying the risky asset in T = 1, the informed trader can also buy the riskless asset and obtain the payoff  $\delta_{I1}(-P_2B_2 + \delta_{I2}B_2V_{\omega})$ .

In t = 2 of periods T = 1 and T = 2, when selling an amount  $B_T$  of the total endowment E, the uninformed trader receives a revenue  $P_T B_T$ from the sale. In T = 3, after the realization of the asset's value, just like the informed trader, the uninformed trader cashes in on the assets held. Evaluating the cash-flow from period T = 1 and T = 2 respectively, the uninformed trader discounts the payoff from period T = 3 by  $\delta_{U1}\delta_{U2}$ , in T = 1 and by  $\delta_{U2}$ , in T = 2. I delay the introduction of the uninformed traders' uncertainty about the asset's value until section 3.3 and instead express the uninformed traders' payoffs as net present value (NPV) given the state  $\omega$ . The uninformed trader's NPV in state  $\omega$  at T = 2 is

$$P_2 B_2 + \delta_{U2} (E - B_1 - B_2) V_{\omega}. \tag{3.3}$$

The NPV of the uninformed trader from not selling assets in T = 2 is  $\delta_{U2}(E - B_1)V_{\omega}$ . When selling assets in period T = 1, the uninformed trader takes into account the effect of his purchase  $B_1$  on revenues in T = 2 and T = 3. The uninformed trader's NPV in T = 1 is

$$P_1B_1 + \delta_{U1}(P_2B_2 + \delta_{U2}(E - B_1 - B_2)V_{\omega}). \tag{3.4}$$

Instead of selling assets, the uninformed trader can also keep the assets, i.e.  $B_1 = 0$ , and receive a NPV of  $\delta_{U1}(P_2B_2 + \delta_{U2}(E - B_2)V_{\omega})$  in period T = 1.

### 3.3 Perfect Bayesian Equilibrium

The informed trader strategy is a mapping  $B_T : \{V_\omega\} \to \Re_0^+$  that prescribes a quantity  $B_T(V_\omega)$  on the basis of the trader's private information  $\omega$ . The uninformed trader strategy is a mapping  $P_T : \Re_0^+ \to \Re_0^+$ . Conditional beliefs for the uninformed trader are represented by a mapping that associates to each quantity  $B_T$  a probability function  $Pr(\cdot|B_T)$  on  $\{V_H, V_L\}$ , where  $Pr(V_{\omega}|B_T)$  is the probability that the uninformed trader and the firm manager attach to a value  $V_{\omega}$  given quantity  $B_T$ .

The perfect Bayesian equilibrium is defined by a bundle of strategies  $(B_T(\cdot), P_T(\cdot))$  and a family of conditional beliefs  $Pr(\cdot|\cdot)$  such that (i) for all  $B_T$  in the range of  $B_T(\cdot)$ ,  $Pr(\cdot|B_T)$  is the conditional probability of  $V_{\omega}$  obtained by updating the prior  $(\beta, (1 - \beta))$ , using  $B_T(\cdot)$  in Bayesian fashion; (ii) for all  $B_T(\cdot)$ ,  $P_T \ argmax_{P_T} \in E(U_U(\cdot))$ , and (iii) for all  $\omega$ ,  $B_T \in argmax_{B_T} E(U_I(\cdot))$ . Condition (i) stipulates that the uninformed trader and the firm's manager have rational expectations. Conditions (ii) to (iii) require that traders be optimizing. In particular, they imply participation constraints and incentive compatibility constraints.

Market clearing takes place through the adjustment of the price  $P_T$  to the quantity demanded  $B_T$ . I.e. the informed trader submits a market order. Observing the market order, the uninformed trader, acting as a market maker, updates the belief about the quality of the asset and sets the price. In equilibrium, there has to be a unique price-quantity bundle  $\{P_T, B_T\}$ .

#### 3.3.1 Separating equilibrium

In a separating equilibrium, the informed trader buys different quantities in either state. Therefore, the purchase reveals private information. Suppose, the informed trader, in period T, buys  $B_T^H$  after observing H and  $B_T^L$  after observing L, then I invoke the uninformed trader's conditional beliefs as:

$$q_{1} = Pr(H|B_{1}) = \begin{cases} 1 & \text{if } B_{1} = B_{1}^{H} \\ 0 & \text{if } B_{1} = B_{1}^{L} \\ 1 & \text{otherwise} \end{cases}$$
(3.5)

The conditional beliefs in T = 1 imply that the uninformed trader and the firm update their priors such that if they observe  $B_1^H$ , they are sure to face the informed trader with good information and if they observe  $B^L$ , they know the informed trader with bad information is buying the asset. The intuition for the off-equilibrium belief is the informed trader with good information H wants to mimic the informed trader with bad information L in order to get a low price<sup>5</sup>. Discontinuity of the conditional beliefs is a natural consequence of the binomial distribution of the random variable  $V_{\omega}$ . This is different from Laffont and Maskin (1990). They can potentially obtain continuous, monotonic beliefs since the signal in their model is noisy.

In T = 2, the uninformed trader's beliefs depend on the trades observed in both periods. I.e. if the informed trader observed H, he has to buy  $B_1^H$  and  $B_2^H$  for the uninformed trader to believe that he observed good information. The same reasoning applies for the informed trader with bad information L.

$$q_{2} = Pr(H|B_{2}) = \begin{cases} 1 & \text{if } B_{2} = B_{2}^{H} & \wedge & B_{1} = B_{1}^{H} \\ 0 & \text{if } B_{2} = B_{2}^{L} & \wedge & B_{1} = B_{1}^{L} \\ 1 & \text{otherwise} \end{cases}$$
(3.6)

The intuition for the off-equilibrium belief from T = 1 applies here as well, the informed trader with good information wants to mimic the informed trader with bad information in order to get a lower price. In addition, the offequilibrium belief implies after observing an off-equilibrium quantity  $B'_1$  in T = 1, the uninformed trader, no matter whether observing an equilibrium quantity  $B_2^H/B_2^L$  or off-equilibrium quantity  $B'_2$ , the uninformed trader will hold the off-equilibrium belief. I will have to show that the conditional beliefs specified in 3.5 and 3.6 satisfy incentive compatibility of the informed trader and ensure participation of the uninformed trader in equilibrium.

The twice repeated dynamic game is solved by backward induction. Consider first, period T = 2.

**Period** T = 2 In t = 2, the uninformed trader infers the private information  $\omega$  from the demand  $B_2^{\omega}$  of the informed trader. As price taker, the

<sup>&</sup>lt;sup>5</sup>The off-equilibrium beliefs stipulated here are not the only possible ones.

uninformed trader decides whether to sell or to retain the assets for a given price  $P_2^{\omega}$ . If the uninformed trader retains the assets, he has a holding of assets of  $E - B_1^{\omega}$ . The equilibrium price therefore has to satisfy the following participation constraint:

$$q_2 W_{H2} + (1 - q_2) W_{L2} \ge q_2 (\delta_{U2} V_H (E - B_1^H)) + (1 - q_2) (\delta_{U2} V_L (E - B_1^L))$$
(3.7)
with  $W_{\omega 2} = P_2^{\omega} B_2^{\omega} + \delta_{U2} (E - B_1^{\omega} - B_2^{\omega}) V_{\omega}.$ 

Due to perfect competition, the uninformed trader breaks even. Then, the equilibrium price is  $P_2^{\omega} = \delta_{U2}V_{\omega}$ , depending on the demand observed. It is intuitive that the uninformed trader wants to sell the asset at the price which reflects the value of the firm discounted by his liquidity need. The more liquidity constrained the uninformed trader, the more he is willing to decrease the price. These are the two components of the price, the information component and the liquidity component.

In t = 1, the informed trader is willing to buy the risky asset if the NPV of buying the risky asset outweighs the NPV of the riskless asset. Given the price  $P_2^{\omega}$ , the informed trader's participation constraint, depending on private information, is

$$-P_2^{\omega} B_2^{\omega} + \delta_{I2} V_{\omega} (B_1^{\omega} + B_2^{\omega}) \ge \delta_{I2} V_{\omega} B_1^{\omega}.$$
(3.8)

No matter which information the informed trader observed, the participation constraint is satisfied if  $\delta_{I2} \geq \delta_{U2}$ . Since the uninformed trader can observe the quality of the investment, the only gain from trade stems from the asymmetry in liquidity needs, the liquidity wedge. Intuitively, the uninformed trader wants to sell assets because he needs liquidity. Given his tight liquidity needs, the uninformed trader is willing to reduce the price below its informational value. As long as the informed trader is less liquidity constrained, the price of the asset is relatively cheap with respect to its prospective.

In order to show that  $B_2^{\omega}$  is the optimal choice for the informed trader

given the observed information  $\omega$ , consider the incentive compatibility constraints. As shown before, the informed trader faces a price  $P_2^{\omega} = \delta_{U2}V_{\omega}$ when choosing  $B_2^{\omega}$ . First, I show that neither type wants to mimic the other type:

$$(-\delta_{U2}+\delta_{I2})V_{\omega}B_{2}^{\omega} \ge (-\delta_{U2}V_{-\omega}+\delta_{I2}V_{\omega})B_{2}^{-\omega}$$
with  $\omega \ne -\omega$ .
$$(3.9)$$

Notice that in the latter inequality,  $\delta_{I2}V_{\omega}B_1^{\omega}$  enters both sides and hence cancels each other out. It enters also the right hand side because at T = 2the choice of T = 1 cannot be altered any more.

For both types  $\omega$ , condition 3.9 is satisfied if

$$\frac{(\delta_{I2} - \delta_{U2})V_H}{\delta_{I2}V_H - \delta_{U2}V_L}B_2^H \ge B_2^L \ge max \left\{ 0, \frac{\delta_{I2}V_L - \delta_{U2}V_H}{(\delta_{I2} - \delta_{U2})V_L}B_2^H \right\}.$$
 (3.10)

The upper limit ensures incentive compatibility for the informed trader who observed H and the lower limit ensures incentive compatibility for the informed trader with bad information L. Notice,  $\frac{\delta_{I2}V_L - \delta_{U2}V_H}{(\delta_{I2} - \delta_{U2})V_L}B_2^H > 0$  if  $\delta_{I2} > \delta_{U2}\frac{V_H}{V_L}$ . Furthermore, observe  $1 > \frac{(\delta_{I2} - \delta_{U2})V_H}{\delta_{I2}V_H - \delta_{U2}V_L} > \frac{\delta_{I2}V_L - \delta_{U2}V_H}{(\delta_{I2} - \delta_{U2})V_L}$ . This implies that  $B_2^H > B_2^L$ , meaning that the informed trader with good information H has to buy more than the informed trader with information L for a separating equilibrium to exist.

Besides choosing  $B_2^{\omega}$ , the informed trader with private information  $\omega$  can also choose any other quantity, i.e. a quantity  $B'_2$  which satisfies  $B'_2 \neq B_2^{\omega}$ and  $B'_2 \neq B_2^{-\omega}$ . After observing  $B'_2$  and the beliefs as specified in 3.6, the uninformed trader believes to face a trader with good information. Then from the uninformed trader's participation constraint the off-equilibrium price is  $P'_2 = \delta_{U2}V_H$ . In order to ensure optimality of  $B_2^{\omega}$  consider the following incentive compatibility constraint for each state  $\omega$ :

$$(-\delta_{U2} + \delta_{I2})V_{\omega}B_2^{\omega} \ge -P_2'B_2' + \delta_{I2}V_{\omega}B_2' \quad \forall \quad B_2'.$$
(3.11)

 $B_2^{\omega}$  is indeed optimal if

$$B_2^H = max\{B_2'\}$$
 and (3.12)

$$B_{2}^{L} \ge max \left\{ 0, \frac{\delta_{I2}V_{L} - \delta_{U2}V_{H}}{(\delta_{I2} - \delta_{U2})V_{L}} max\{B_{2}'\} \right\}$$
(3.13)

with  $max\{B'_2\} = E - B_1^{\omega}$ .

Conditions 3.10 and 3.13 combined yield

$$\frac{(\delta_{I2} - \delta_{U2})V_H}{\delta_{I2}V_H - \delta_{U2}V_L} (E - B_1^H) \ge B_2^L \ge max\{0, \frac{\delta_{I2}V_L - \delta_{U2}V_H}{(\delta_{I2} - \delta_{U2})V_L} (E - min\{B_1^L, B_1^H\})\}$$
(3.14)

**Period** T = 1 Period T = 1 has the same structure as T = 2, i.e. in t = 2, the uninformed trader observes the informed trader's demand  $B_1^{\omega}$  and decides whether to sell at a given price  $P_1^{\omega}$  or retain the assets and in t = 1, the informed trader decides whether to purchase  $B_1^{\omega}$  units of the risky asset or the riskless asset. In T = 1 however, traders take into account their actions from T = 2.

In t = 2, the uninformed trader infers the private information  $\omega$  from the demand  $B_1^{\omega}$  of the informed trader. As price taker, the uninformed trader decides whether to sell or to retain the assets for a given price  $P_1^{\omega}$ . The equilibrium price therefore has to satisfy the following participation constraint:

$$q_{1}W_{H1} + (1 - q_{1})W_{L1} \ge q_{1}O_{H1} + (1 - q_{1})O_{L1}$$
with  $W_{\omega 1} = P_{1}^{\omega}B_{1}^{\omega} + \delta_{U1}(P_{2}^{\omega}B_{2}^{\omega} + \delta_{U2}(E - B_{1}^{\omega} - B_{2}^{\omega})V_{\omega}),$ 

$$O_{\omega 1} = \delta_{U1}(P_{2}^{\omega}B_{2}^{\omega} + \delta_{U2}(E - B_{2}^{\omega})V_{\omega})$$
and  $P_{2}^{\omega} = \delta_{U2}V_{\omega}.$ 
(3.15)

Due to perfect competition, the uninformed trader breaks even. Then, the equilibrium price is  $P_1^{\omega} = \delta_{U1} \delta_{U2} V_{\omega}$ , depending on the demand observed.

In t = 1, the informed trader is willing to buy the risky asset if the NPV of buying the risky asset outweight the NPV of the riskless asset. Given the

prices  $P_1^{\omega}$  and  $P_2^{\omega}$ , the informed trader's participation constraint, depending on private information, is

$$-P_1^{\omega}B_1^{\omega} + \delta_{I1}(-P_2^{\omega}B_2^{\omega} + \delta_{I2}(B_1^{\omega} + B_2^{\omega})V_{\omega}) \ge \delta_{I1}(-P_2^{\omega}B_2^{\omega} + \delta_{I2}B_2^{\omega}V_{\omega}).$$
(3.16)

With  $P_2^{\omega} = \delta_{U2} V_{\omega}$  and  $P_1^{\omega} = \delta_{U1} \delta_{U2} V_{\omega}$ , condition 3.16 yields

$$\delta_{I1}\delta_{I2} \ge \delta_{U1}\delta_{U2}.\tag{3.17}$$

I have to demonstrate now, that  $B_1^{\omega}$  is the optimal choice. First, I show that each type  $\omega$  of informed trader does not want to mimic the other type  $-\omega \neq \omega$ . Write the informed traders' incentive compatibility constraints as:

$$-P_{1}^{\omega}B_{1}^{\omega} + \delta_{I1}(-P_{2}^{\omega}B_{2}^{\omega} + \delta_{I2}(B_{1}^{\omega} + B_{2}^{\omega})V_{\omega}) \geq -P_{1}^{-\omega}B_{1}^{-\omega} + \delta_{I1}(-P_{2}^{-\omega}B_{2}^{-\omega} + \delta_{I2}(B_{1}^{-\omega} + B_{2}^{-\omega})V_{\omega})$$
(3.18)

According to the beliefs specified in 3.5 and 3.6, the informed trader can only mimic the other type in T = 1 if he has already done so in T = 2.

The following is a sufficient condition for incentive compatibility as in inequality 3.18

$$\frac{(-\delta_{U1}\delta_{U2} + \delta_{I1}\delta_{I2})V_H}{(-\delta_{U1}\delta_{U2}V_L + \delta_{I1}\delta_{I2}V_H)}B_1^H \ge B_1^L \ge max\{0, \frac{(-\delta_{U1}\delta_{U2}V_H + \delta_{I1}\delta_{I2}V_L)}{(-\delta_{U1}\delta_{U2} + \delta_{I1}\delta_{I2})V_L}B_1^H\}$$
(3.19)

The proof is relegated to appendix C.1. The upper bound ensures incentive compatibility of the high type and the lower bound ensures incentive compatibility of the low type. Since  $1 > \frac{(-\delta_{U1}\delta_{U2} + \delta_{I1}\delta_{I2})V_H}{(-\delta_{U1}\delta_{U2}V_L + \delta_{I1}\delta_{I2}V_H)}$ ,  $B_1^L < B_1^H$ . Next, I have to show that either informed type does not want to choose

any other quantity  $B'_1 \neq B^{\omega}_1$ .

$$-P_{1}^{\omega}B_{1}^{\omega} + \delta_{I1}(-P_{2}^{\omega}B_{2}^{\omega} + \delta_{I2}(B_{1}^{\omega} + B_{2}^{\omega})V_{\omega}) \geq -P_{1}^{\prime}B_{1}^{\prime} + \delta_{I1}(-P_{2}^{\prime}B_{2}^{\prime} + \delta_{I2}(B_{1}^{\prime} + B_{2}^{\prime})V_{\omega})$$
(3.20)

With beliefs as specified in 3.5 and 3.6, it is clear that if the informed trader chooses any quantity different from  $B_1^{\omega}$ , the uninformed trader will believe he is facing the high type.

In appendix C.2, it is shown that the optimality condition 3.20 is satisfied if

$$B_1^H \ge B_1' \tag{3.21}$$

$$B_1^L \ge max\{0, \frac{\delta_{I1}\delta_{I2}V_L - \delta_{U1}\delta_{U2}V_H}{(\delta_{I1}\delta_{I2} - \delta_{U1}\delta_{U2})V_L}B_1'\}$$
(3.22)

Inequality 3.21 yields  $B_1^H = E$ . With  $B_1^H = E$  and conditions 3.19 and 3.22,  $B^L$  has to satisfy

$$\frac{(-\delta_{U1}\delta_{U2} + \delta_{I1}\delta_{I2})V_H}{(-\delta_{U1}\delta_{U2}V_L + \delta_{I1}\delta_{I2}V_H)}E \ge B_1^L \ge max\{0, \frac{(-\delta_{U1}\delta_{U2}V_H + \delta_{I1}\delta_{I2}V_L)}{(-\delta_{U1}\delta_{U2} + \delta_{I1}\delta_{I2})V_L}E\}$$

We can now state existence of a separating equilibrium.

**Proposition 8.** Separating Equilibrium. There exists a separating equilibrium with trade in period T = 1 only,  $B_1^H = E$  and  $B_1^L = \frac{(-\delta_{U1}\delta_{U2} + \delta_{I1}\delta_{I2})V_H}{(-\delta_{U1}\delta_{U2}V_L + \delta_{I1}\delta_{I2}V_H)}E$ , since  $\delta_{I1} > \delta_{U1}$  and  $\delta_{I2} > \delta_{U2}$ . Prices are  $P_1^{\omega} = \delta_{U1}\delta_{U2}V_{\omega}$ .

The proof is straightforward. Given that the informed trader with good information buys all assets in T = 1,  $B_1^H = E$ , the optimality condition for the high type in T = 2,  $B_2^H = E - B_1^\omega = 0$ . By the same token, the optimality condition for the low type yields,  $0 \ge B_2^L$ . This completes the proof.

That trade takes only place in period T = 1 stems from the fact that, from the perspective of T = 1, the informed trader faces a lower price in T = 1 than in T = 2. Observe from the left-hand side of inequality 3.20 the price in T = 1 is  $P_1^{\omega} = \delta_{U1}\delta_{U2}V_{\omega}$  and the price in T = 2 is  $\delta_{I1}P_2^{\omega} = \delta_{I1}\delta_{U2}V_{\omega}$ . Since  $\delta_{I1} > \delta_{U1}$ , the price in T = 1 is lower than the price in T = 2. At the same time the uninformed trader is indifferent between selling everything in the first period and selling everything in two periods.

#### 3.3.2 Pooling equilibrium

Next, I characterize the conditions under which a pooling equilibrium exists. In a pooling equilibrium, the informed trader buys identical quantities in either state  $\omega$ . Therefore, the uninformed trader cannot infer the informed trader's private information. Suppose the informed trader, in period T, chooses  $B_T^P$  in either state, then the uninformed trader's and the firm's conditional belief is equal to their priors. If they observe and  $B'_T \neq B_T^P$ , their conditional belief implies that the informed trader has good information:

$$q_1 = Pr(H|B_1) = \begin{cases} \beta & \text{if } B_1 = B_1^P \\ 1 & \text{if } B_1 = B_1' \end{cases}.$$
 (3.23)

The intuition for the belief off-equilibrium<sup>6</sup> is, that the informed trader with good information is more inclined to deviate from the equilibrium quantity  $B_T^P$  since the price in the pooling equilibrium is relatively low.

In T = 2, the uninformed trader's beliefs depend on the trades observed in both periods. I.e. both types of informed trader  $\omega$  have to buy  $B_1^P$  and  $B_2^P$  for the uninformed trader not to be able to infer private information:

$$q_2 = Pr(H|B_2) = \begin{cases} \beta & \text{if } B_2 = B_2^P & \wedge & B_1 = B_1^P \\ 1 & \text{otherwise.} \end{cases}$$
(3.24)

The off-equilibrium belief implies, after observing an off-equilibrium quantity  $B'_1$  in T = 1, the uninformed trader, no matter whether observing the equilibrium quantity  $B'_2$  or off-equilibrium quantity  $B'_2$ , the uninformed trader will hold the off-equilibrium belief. I will have to show that the conditional beliefs specified in 3.23 and 3.24 satisfy incentive compatibility of the informed trader and ensure participation of the uninformed trader in equilibrium.

The twice repeated dynamic game is solved by backward induction. Consider first, period T = 2.

<sup>&</sup>lt;sup>6</sup>The off-equilibrium beliefs stipulated here are not the only possible ones.

**Period** T = 2 In t = 2, after observing the informed trader's demand  $B_2^P$ , the uninformed trader forms the beliefs given in equation 3.24. Then he decides whether to sell or to keep the asset. Since the informed trader purchases the same quantity  $B_2^P$  regardless of the state, there is just one price  $P_2$  for both states. The uninformed trader's participation constraint becomes

$$q_2 W_{H2} + (1 - q_2) W_{L2} \ge q_2 (\delta_{U2} V_H (E - B_1^P)) + (1 - q_2) (\delta_{U2} V_L (E - B_1^P))$$
(3.25)  
with  $W_{\omega 2} = P_2 B_2^P + \delta_{U2} (E - B_1^P - B_2^P) V_{\omega}.$ 

Recall, the uninformed trader is price taker and in competition for the sale with the other small, uninformed traders. The equilibrium price  $P_2$  has to satisfy inequality 3.25 when it is binding, i.e. the uninformed trader breaks even. If inequality 3.25 is binding,  $P_2 = \delta_{U2}E(V)$  with  $E(V) = \beta V_H + (1 - \beta)V_L$ .

Just like in the separating equilibrium, the price decreases the more liquidity constrained the uninformed trader. For a given  $\delta_{U2}$  and  $\beta > 0$ , the equilibrium price in the pooling equilibrium lays between the prices in the separating equilibrium in the good state and in the bad state,  $P_2^H > P_2 > P_2^L$ .

In t = 1, the informed trader is willing to buy the risky asset if the NPV of buying the risky asset outweighs the NPV of the riskless asset. Given the price  $P_2$ , the informed trader's participation constraint, depending on private information, is

$$-P_2 B_2^P + \delta_{I2} V_\omega (B_1^P + B_2^P) \ge \delta_{I2} V_\omega B_1^P.$$
(3.26)

The participation constraint of the informed trader with bad information L, is more difficult to satisfy. The informed trader's participation is therefore ensured if  $\delta_{I2} \geq \delta_{U2} \frac{E(V)}{V_L}$ . Since  $\frac{E(V)}{V_L} > 1$ , the informed trader needs to be considerably less liquidity constrained than the uninformed trader, and in particular less than in the separating equilibrium where participation in

T = 2 was ensured if  $\delta_{I2} > \delta_{U2}$ . Observe that  $\frac{E(V)}{V_L}$  increases in  $\beta$ . That is the more likely the good outcome, the larger needs to be the difference between the informed trader's liquidity needs and the uninformed trader's liquidity needs.

It is the inequality  $\delta_{I2} \geq \delta_{U2} \frac{E(V)}{V_L}$  which drives non-information revelation. The fact that  $\frac{E(V)}{V_L} > 1$  is due to the non-linear equilibrium beliefs. The intuition is, the uninformed trader has to be so liquidity constrained that the price decrease outweighs the low prospect of the uninformed trader with bad information.

In order to show that  $B_2^P$  is the optimal choice for the informed trader given the observed information  $\omega$  and beliefs 3.24, consider the incentive compatibility constraints. As shown before, the informed trader faces a price  $P_2 = \delta_{U2}E(V)$  when choosing  $B_2^P$ . Then the incentive compatibility constraints become:

$$(-\delta_{U2}E(V) + \delta_{I2}V_{\omega})B_2^P \ge (-\delta_{U2}V_H + \delta_{I2}V_{\omega})B_2'.$$
 (3.27)

Notice that in the latter inequality,  $\delta_{I2}V_{\omega}B_1^P$  enters both sides and hence cancels each other out. It enters also the right hand side because at T = 2the choice of T = 1 cannot be altered any more.

I find that informed trader with good information has a more binding incentive compatibility constraint which is satisfied if

$$B_{2}^{P} \geq \frac{(\delta_{I2} - \delta_{U2})V_{H}}{\delta_{I2}V_{H} - \delta_{U2}E(V)}B_{2}'$$
with  $B_{2}' = E - B_{1}^{P}$ 
(3.28)

Since  $1 > \frac{(\delta_{I2} - \delta_{U2})V_H}{\delta_{I2}V_H - \delta_{U2}E(V)}$ , inequality 3.28 is satisfied  $\forall B'_2$  if  $B_2^P = E - B_1^P$ .

I hereby have described a perfect Bayesian equilibrium in T = 2 in which private information is not revealed. I proceed to describe the pooling equilibrium of T = 1.

**Period** T = 1 Period T = 1 has the same structure as T = 2, i.e. in t = 2, the uninformed trader observes the informed trader's demand  $B_1^P$ 

and decides whether to sell at a given price  $P_1$  or retain the assets and in t = 1, the informed trader decides whether to purchase  $B_1^P$  units of the risky asset or the riskless asset. In T = 1 however, traders take into account their actions from T = 2.

In t = 2, the uninformed trader observes the uninformative demand  $B_1^P$ and decides whether to sell or to retain the assets for a given price  $P_1$ . The equilibrium price therefore has to satisfy the following participation constraint:

$$q_1 W_{H1} + (1 - q_1) W_{L1} \ge q_1 O_{H1} + (1 - q_1) O_{L1}$$
with  $W_{\omega 1} = P_1 B_1^P + \delta_{U1} (P_2 B_2^P + \delta_{U2} (E - B_1^P - B_2^P) V_{\omega})$ 
and  $O_{\omega 1} = (\delta_{U1} (P_2 B_2^P + \delta_{U2} (E - B_2^P) V_{\omega})).$ 

$$(3.29)$$

With  $P_2 = \delta_{U2} E(V)$  and perfect competition among uninformed traders,  $P_1 = \delta_{U1} \delta_{U2} E(V)$ .

Solving backwards, in t = 1, the informed trader is willing to buy the risky asset if the NPV of buying the risky asset outweights the NPV of the riskless asset. Given the prices  $P_1$  and  $P_2$ , and the uninformed trader's beliefs  $q_1$  as specified in 3.23, the informed trader's participation constraint, depending on private information, is

$$-P_1B_1^P + \delta_{I1}(-P_2B_2^P + \delta_{I2}(B_1^P + B_2^P)V_{\omega}) \ge \delta_{I1}(-P_2B_2^P + \delta_{I2}B_2^PV_{\omega}).$$
(3.30)

After simplifying inequality 3.30 for both types  $\omega$ , it is clear that the informed trader with bad information is less willing to participate. He participates only if

$$\delta_{I1}\delta_{I2} \ge \delta_{U1}\delta_{U2}\frac{E(V)}{V_L}.$$
(3.31)

I have to demonstrate now, that  $B_1^P$  is the optimal choice. Write I's ICs

$$-P_1B_1^P + \delta_{I1}(-P_2B_2^P + \delta_{I2}(B_1^P + B_2^P)V_{\omega}) \ge -P_1'B_1' + \delta_{I1}(-P_2'B_2' + \delta_{I2}(B_1' + B_2')V_{\omega})$$

In accordance with the beliefs specified in 3.23 and 3.24, once the informed trader deviated from the equilibrium quantity  $B_1^P$ , the uninformed trader believes to face the high type ever after. The demonstration of incentive compatibility in T = 1 is relegated to appendix C.3.

I can readily state the following proposition now.

**Proposition 9.** Pooling Equilibrium. There exists a pooling equilibrium with trade in T = 1 only, with price  $P_1 = \delta_{U1}\delta_{U2}E(V)$  and trade  $B_2^P = E$  since  $\delta_{I1} > \delta_{U1}$  and  $\delta_{I2} > \delta_{U2}$ 

The intuition for trade in T = 1 only is analogous to the one in the separating equilibrium. From the perspective of T = 1, the informed trader faces a lower price for T = 1 than for T = 2. Since he knows with certainty the asset's payoff, he is facing no risk from holding the asset. Observe that the price does not reveal information.

This provides a contrasting example to the literature (Kyle (1985), Back and Baruch (2004) and Ostrovsky (2012)) which finds that it is optimal to trade in every period such that in the limit information is revealed.

#### **3.4** Discussion

Trade in one period only may be a specifity of linear payoffs and asymmetric liquidity needs. Linear payoffs naturally yield corner solutions. In the model in the previous sections, one possible critique could be, the informed trader buys all assets in the first period such that he cannot buy any more assets in the second period<sup>7</sup>. In addition, the price in the first period is lower than in the second, which is driven by the asymmetric liquidity needs, and

as:

<sup>&</sup>lt;sup>7</sup>This is true for both separating equilibrium and pooling equilibrium. In the separating equilibrium however, the informed trader with good information buys all assets in the first period. Through incentive compatibility constraints, this causes the informed trader with bad information not to buy anything in the second period neither.

hence there is only trade in the first period. In order to show that the of trade in one period only is not an artifact of the assumptions, I have to vary the assumptions in question. Namely, I could change the trade motive from liquidity asymmetry to risk-sharing. Then, the informed trader buys assets from the uninformed trader because the uninformed trader is more riskaverse. Risk-aversion is not only a different trade motive but also generates non-linear utility functions and thus interior solutions. The introduction of risk-aversion would allow to address both issues, i.e. linear preferences and a trade motive favoring early trade.

Even if trade in every period is restored with risk-aversion, it is not obvious why the pooling equilibrium, i.e. no information revelation, should vanish if trade is repeated finitely. This would still prove a counter-example to the literature (Kyle (1985), Back and Baruch (2004) and Ostrovsky (2012)) in which information is always revealed.

#### 3.5 Conclusion

Models in which a monopolistically informed trader buys assets from noise traders through competitive market makers over finitely many periods, yield that it is optimal to trade in every period and that the price reveals private information.

By introducing rational, liquidity constrained traders instead of noise traders, I can provide a simple two period example, in which it is optimal to trade in the first period only. Moreover, there exist equilibria in which private information is not revealed.

Whether the one-period-trade-only result is robust to different preferences of the uninformed trader remains a task for future research. Appendix A

# Efficient Asset Trade - A Model with Asymmetric Information and Asymmetric Liquidity Needs

A.1 Conditions for the ranking of liquidity thresholds

$$\begin{split} & \text{If } V_H > 2V_L \\ & 1 > \frac{V_L}{V_H - V_L} > \frac{V_L^2}{(V_H - V_L)^2} > \frac{V_L^2}{V_H(V_H - V_L)}, \\ & \text{if } 2V_L > V_H > \frac{1}{2}(1 + \sqrt{5})V_L \\ & \frac{V_L^2}{(V_H - V_L)^2} > \frac{V_L}{V_H - V_L} > 1 > \frac{V_L^2}{V_H(V_H - V_L)} \text{ and} \\ & \text{if } \frac{1}{2}(1 + \sqrt{5})V_L > V_H > V_L \\ & \frac{V_L^2}{(V_H - V_L)^2} > \frac{V_L}{V_H - V_L} > \frac{V_L^2}{V_H(V_H - V_L)} > 1. \end{split}$$

These conditions allow to rank the thresholds on the liquidity difference. Given  $1 > \beta > 0$ ,

$$\begin{array}{l} \text{for any } \displaystyle \frac{V_L}{V_H - V_L} > \beta > \displaystyle \frac{V_L^2}{(V_H - V_L)^2}, \\ \displaystyle \frac{F_P}{F_P^L} > \displaystyle \frac{F_H - F_P}{F_H - F_P^H} > \displaystyle \frac{F_H - F_L}{F_H - F_H'}, \\ \text{for any } \displaystyle \min\{ \displaystyle \frac{V_L}{V_H - V_L}, \displaystyle \frac{V_L^2}{(V_H - V_L)^2}\} > \beta > \displaystyle \frac{V_L^2}{V_H(V_H - V_L)}, \\ \displaystyle \frac{F_H - F_P}{F_H - F_P^H} > \displaystyle \frac{F_P}{F_P^L} > \displaystyle \frac{F_H - F_L}{F_H - F_H'} \text{ and} \\ \text{for any } \displaystyle \frac{V_L^2}{V_H(V_H - V_L)} > \beta, \\ \displaystyle \frac{F_H - F_P}{F_H - F_P^H} > \displaystyle \frac{F_H - F_L}{F_H - F_H'} > \displaystyle \frac{F_P}{F_P^L}. \end{array}$$

#### A.2 Proof of proposition 3

Using the terms for  $F_H$ ,  $F_L$ ,  $F_P^H$  and  $F_P^L$ , the welfare comparison of inequality 1.30 can be rewritten as

$$\Delta W = \frac{(1-\beta)(\beta \delta_I E (V_H - V_L)^2 + (\delta_I - \delta_U)(B^L - E)V_L^2)}{2c} > 0$$
  
$$\Rightarrow (\delta_I (\beta (V_H - V_L)^2 - V_L^2) + \delta_U V_L^2)E + (\delta_I - \delta_U)V_L^2 B^L > 0$$

If  $(\delta_I(\beta(V_H - V_L)^2 - V_L^2) + \delta_U V_L^2) \ge 0$ ,  $B^L$  can be as small as 0. Recall, welfare of separating equilibrium and pooling equilibrium is compared for a parameter space such that both types of equilibria exist:  $\delta_U \frac{F_H - F_L}{F_H - F_H'} > \delta_I > \delta_U \frac{F_P}{F_P'}$ ,  $\delta_U < \frac{F_P}{F_P}$  and,  $\beta < min\{1, \frac{V_L^2}{V_H(V_H - V_L)}\}$ . Together with these conditions,  $(\delta_I(\beta(V_H - V_L)^2 - V_L^2) + \delta_U V_L^2) \ge 0$  if the following conditions are simultaneously satisfied

$$\delta_I < \delta_U \frac{V_L^2}{V_L^2 - \beta (V_H - V_L)^2},$$
  
$$\beta > \frac{3V_L - V_H}{(V_H - V_L)^2} \text{ and }$$
  
$$V_H > (1 + \sqrt{2})V_L$$

Otherwise,  $B^L$  has to be strictly larger than 0. In the following, a such  $B^L$  is derived. Observe, that with  $V_H = V_L$ , the pooling equilibrium yields greater welfare for any  $E > B^L \ge 0$ . Therefore, I characterize a  $B^L(V_H)$ , that yields a non-negative difference  $W_S - W_P$  for any  $V_H$ , given  $V_L$ . Conjecture, the larger  $V_H$ , the smaller can be  $B^L$ . A change in  $V_H$  affects the welfare difference

$$\frac{\partial \Delta W}{\partial V_H} = \frac{(1-\beta)(2\beta\delta_I E(V_H - V_L)^2 + (\delta_I - \delta_U)\frac{\partial B^L(V_H)}{\partial V_H}V_L^2)}{2c}$$

In order to derive the conditions for which  $\Delta W > 0$ , I study the minimum of  $\Delta W$ . A candidate minimum of  $\Delta W$  will have to satisfy  $\frac{\partial \Delta W}{\partial V_H} = 0$  and  $\frac{\partial^2 \Delta W}{\partial V_H^2} \ge 0$ . From the necessary condition, an ordinary differential equation is obtained

$$\frac{\partial B^L(V_H)}{\partial V_H} = -\frac{2\beta\delta_I(V_H - V_L)}{(\delta_I - \delta_U)V_L^2}E$$

The function for  $B^L(V_H)$  satisfying the necessary condition is  $B^L(V_H) = \frac{2\beta\delta_I(V_HV_L - \frac{V_H^2}{2})}{(\delta_I - \delta_U)V_L^2}E$ . In order for  $B^L(V_H) > 0$ ,  $V_H < 2V_L$ . From the existence condition of the separating equilibrium,  $\frac{(\delta_I - \delta_U)F_H}{\delta_I F'_H - \delta_U F_L}E > B^L$ . In order to show that there exists a  $B^L > 0$  satisfying both the existence condition of the separating equilibrium and the minimum condition of the welfare difference,  $\frac{(\delta_I - \delta_U)F_H}{\delta_I F'_H - \delta_U F_L} > \frac{2\beta\delta_I(V_HV_L - \frac{V_H^2}{2})}{(\delta_I - \delta_U)V_L^2}$ . This is satisfied for  $2V_L > V_H > V_L$ .

The sufficient condition for a minimum to exist is

$$\frac{\partial^2 \Delta W}{\partial V_H^2} = \frac{(1-\beta)(2\beta\delta_I E + (\delta_I - \delta_U)\frac{\partial^2 B^L(V_H)}{\partial V_H^2}V_L^2)}{2c}$$

For  $B^L(V_H) = \frac{2\beta\delta_I(V_HV_L - \frac{V_H^2}{2})}{(\delta_I - \delta_U)V_L^2}E$ , the sufficient condition equals 0. The candidate minimum is hence local.

With  $B^L(V_H)$ ,

$$\Delta W = \frac{(1-\beta)(\delta_U - (1-\beta)\delta_I)V_L^2}{2c}E$$

$$\begin{split} \Delta W &> 0 \text{ if and only if } \frac{1}{1-\beta}\delta_U > \delta_I. \text{ Recall from the existence conditions, } \\ \delta_U \frac{F_H - F_L}{F_H - F_H'} > \delta_I > \delta_U \frac{F_P}{F_P'}. \text{ For a } 1 > \delta_I > 0 \text{ which satisfies } \frac{1}{1-\beta}\delta_U > \delta_I \\ \text{and } \delta_U \frac{F_H - F_H}{F_H - F_H'} > \delta_I > \delta_U \frac{F_P}{F_P'}, \frac{1}{1-\beta} > \frac{F_P}{F_P'}. \text{ The latter condition is satisfied} \\ \text{if } \frac{2V_H - 3V_L}{V_H - V_L} < \beta. \text{ For the pooling equilibrium and the separating equilibrium to exist for the same } \delta_I, \ \beta < \frac{V_L^2}{V_H(V_H - V_L)}. \text{ There exists a } 1 > \beta > 0 \\ \text{which satisfies } \frac{2V_H - 3V_L}{V_H - V_L} < \beta < \frac{V_L^2}{V_H(V_H - V_L)} \text{ for either } \frac{3}{2}V_L > V_H > V_L \text{ or } \\ 2V_L > V_H > \frac{1}{4}(3 + \sqrt{17})V_L. \text{ This completes the characterization of the conditions under which the separating equilibrium yields greater welfare than the pooling equilibrium. \end{split}$$

## Appendix B

## (In)efficient asset trade and a rationale for a Tobin Tax

#### B.1 Proof of proposition 7

Using the terms for  $F_H$ ,  $F_L$ ,  $F_P^H$  and  $F_P^L$ , the welfare comparison of inequality 2.25 can be rewritten as

$$\Delta W = \frac{(1-\beta)(\beta \delta_I E (V_H - V_L)^2 + (\delta_I - \delta_U)(B^L - E)V_L^2)}{2c} > 0$$
  
$$\Rightarrow (\delta_I (\beta (V_H - V_L)^2 - V_L^2) + \delta_U V_L^2)E + (\delta_I - \delta_U)V_L^2 B^L > 0$$

Recall that  $\delta_U \frac{F_H - F_L}{F_H - F'_H} > \delta_I > max \{ \delta_U \frac{F_P}{F_P^L}, \delta_U \frac{F_H}{F'_L} \}$  and that  $\delta_U \frac{F_P}{F_P^L} > \frac{\delta_I}{1 + \tau} > max \{ \delta_U, \delta_U \frac{F_H}{F'_L} \}$ . The last inequality can be rewritten in terms of the tax  $\tau$ ,  $\frac{\delta_I}{\delta_U} - 1 > \tau > \frac{\delta_I}{\delta_U} \frac{F_P}{F_P} - 1$  if  $V_H < 2V_L$ .

$$\begin{split} \frac{\delta_{I}}{\delta_{U}} &-1 > \tau > \frac{\delta_{I}}{\delta_{U}} \frac{F_{P}^{L}}{F_{P}} - 1 \text{ if } V_{H} < 2V_{L}.\\ \text{Consider } \Delta W &= (\delta_{I}(\beta(V_{H} - V_{L})^{2} - V_{L}^{2}) + \delta_{U}V_{L}^{2})E + (\delta_{I} - \delta_{U})V_{L}^{2}B^{L}.\\ \text{If } (\delta_{I}(\beta(V_{H} - V_{L})^{2} - V_{L}^{2}) + \delta_{U}V_{L}^{2}) \geq 0, B^{L} \text{ can be as small as } 0. \ (\delta_{I}(\beta(V_{H} - V_{L})^{2} - V_{L}^{2}) + \delta_{U}V_{L}^{2}) \geq 0 \text{ if } \delta_{I} < \delta_{U} \frac{V_{L}^{2}}{V_{L}^{2} - \beta(V_{H} - V_{L})^{2}}. \text{ For a separating equilibrium to exist, } \delta_{I} > \delta_{U}(1 + \tau) \text{ if } V_{H} < 2V_{L}. \text{ For } 1 > \delta_{I} > 0, \delta_{U} \frac{V_{L}^{2}}{V_{L}^{2} - \beta(V_{H} - V_{L})^{2}} > \delta_{I} > \delta_{U}(1 + \tau). \text{ The condition on } \delta_{I} \text{ is satisfied if } \tau < \frac{V_{L}^{2}}{V_{L}^{2} - \beta(V_{H} - V_{L})^{2}} - 1. \text{ From the existence condition of the separating equilibrium, } \tau > \frac{\delta_{I}}{\delta_{U}} \frac{F_{P}^{L}}{F_{P}} - 1. \text{ In order} \end{split}$$

for a positive  $\tau$  to exist,  $\frac{V_L^2}{V_L^2 - \beta(V_H - V_L)^2} - 1 > \frac{\delta_I}{\delta_U} \frac{F_P^L}{F_P} - 1$ . This is satisfied for  $\delta_I < \delta_U \frac{F_P}{F_P^L} \frac{V_L^2}{V_L^2 - \beta(V_H - V_L)^2}$ . The latter condition does not violate the coexistance condition  $\delta_I > \delta_U \frac{F_P}{F_P^L}$ . Therefore, there exist a Pareto improving tax  $min\{\frac{V_L^2}{V_L^2 - \beta(V_H - V_L)^2} - 1, \frac{\delta_I}{\delta_U} - 1\} > \tau > \frac{\delta_I}{\delta_U} \frac{F_P^L}{F_P} - 1$  if

$$\min\{\delta_U \frac{F_P}{F_P^L} \frac{V_L^2}{V_L^2 - \beta (V_H - V_L)^2}, \delta_U \frac{F_H - F_L}{F_H - F'_H}\} > \delta_I > \delta_U \frac{F_P}{F_P^L}.$$
 (B.1)

Otherwise,  $B^L$  has to be strictly larger than 0. In the following, a such  $B^L$  is derived. Therefore, I characterize a  $B^L(V_H)$ , that yields a non-negative difference  $W_S - W_P$  for any  $V_H$ , given  $V_L$ . Conjecture, the larger  $V_H$ , the smaller can be  $B^L$ . A change in  $V_H$  affects the welfare difference

$$\frac{\partial \Delta W}{\partial V_H} = \frac{(1-\beta)(2\beta\delta_I E(V_H - V_L)^2 + (\delta_I - \delta_U)\frac{\partial B^L(V_H)}{\partial V_H}V_L^2)}{2c}$$

In order to derive the conditions for which  $\Delta W > 0$ , I study the minimum of  $\Delta W$ . A candidate minimum of  $\Delta W$  will have to satisfy  $\frac{\partial \Delta W}{\partial V_H} = 0$  and  $\frac{\partial^2 \Delta W}{\partial V_H^2} \ge 0$ . From the necessary condition, an ordinary differential equation is obtained

$$\frac{\partial B^L(V_H)}{\partial V_H} = -\frac{2\beta\delta_I(V_H - V_L)}{(\delta_I - \delta_U)V_L^2}E$$

The function for  $B^L(V_H)$  satisfying the necessary condition is  $B^L(V_H) = \frac{2\beta\delta_I(V_H V_L - \frac{V_H^2}{2})}{(\delta_I - \delta_U)V_L^2}E$ . In order for  $B^L(V_H) > 0$ ,  $V_H < 2V_L$ . From the existence condition of the separating equilibrium,  $\frac{(\delta_I - (1+\tau)\delta_U)F_H}{\delta_I F'_H - (1+\tau)\delta_U F_L}E > B^L$ . In order to show that there exists a  $B^L > 0$  satisfying both the existence condition of the separating equilibrium and the minimum condition of the welfare difference,

 $\frac{(\delta_I - (1+\tau)\delta_U)F_H}{\delta_I F'_H - (1+\tau)\delta_U F_L} > \frac{2\beta\delta_I (V_H V_L - \frac{V_H^2}{2})}{(\delta_I - \delta_U)V_L^2}.$  The latter inequality is satisfied if

$$V_H < 2V_L, \tag{B.2}$$

$$\delta_I > \delta_U(1+\tau) \quad \leftrightarrow \quad \frac{\delta_I}{\delta_U} - 1 > \tau \text{ and}$$
 (B.3)

$$\frac{\delta_I}{\delta_U} \frac{V_H}{V_L} - 1 > \tau \tag{B.4}$$

 $\frac{\delta_I}{\delta_U} - 1 > \tau$  conicides with the existence condition and is hence always satisfied. Since  $\frac{\delta_I}{\delta_U} \frac{V_H}{V_L} > \frac{\delta_I}{\delta_U}$ , condition B.4 is satisfied as well. With  $B^L(V_H)$ ,

$$\Delta W = \frac{(1-\beta)(\delta_U - (1-\beta)\delta_I)V_L^2}{2c}E\tag{B.5}$$

$$\begin{split} \Delta W > 0 \text{ if } (\delta_U - (1 - \beta)\delta_I) > 0 \text{, i.e. } \frac{\delta_U}{1 - \beta} > \delta_I \text{. Remains to be shown that} \\ \text{there exists a positive } \delta_I \text{ which satisfied both the non-negativity condition} \\ \text{of } \Delta W \text{ and the existence condition of the separating equilibrium: } \frac{\delta_U}{1 - \beta} > \delta_I. \\ \text{This condition poses restrictions on (i) } \frac{\delta_U}{1 - \beta} > \delta_I > \delta_U (1 + \tau) \text{ if } \tau < \frac{\beta}{1 - \beta} \text{ and} \\ \text{(ii) } \frac{\delta_U}{1 - \beta} > \delta_I > \delta_U \frac{F_P}{F_P^L}. \ \frac{1}{1 - \beta} > \frac{F_P}{F_P^L} \text{ if } \beta > \frac{2V_H - 3V_L}{V_H - V_L}. \end{split}$$

There exists a Pareto improving tax  $min\{\frac{\delta_I}{\delta_U}-1,\frac{\beta}{1-\beta}\} > \tau > 0$ , if

$$B^{L} = \frac{2\beta \delta_{I} (V_{H} V_{L} - \frac{V_{H}^{2}}{2})}{(\delta_{I} - \delta_{U}) V_{L}^{2}} E,$$
 (B.6)

$$2V_L > V_H > V_L, \tag{B.7}$$

$$\frac{\delta_U}{1-\beta} > \delta_I > \delta_U \frac{F_P}{F_P^L} \text{ and } \tag{B.8}$$

$$\beta > \frac{2V_H - 3V_L}{V_H - V_L}.$$
 (B.9)

## Appendix C

## Optimal Timing of Asset Purchases

### C.1 Incentive Compatibility in the separating equilibrium in T=1

This is to show incentive compatibility in the separating equilibrium in period T = 1. I have to demonstrate now, that  $B_1^{\omega}$  is the optimal choice. First, I show that each type  $\omega$  of informed trader does not want to mimic the other type  $-\omega \neq \omega$ . Write I's ICs as:

$$-P_1^{\omega}B_1^{\omega} + \delta_{I1}(-P_2^{\omega}B_2^{\omega} + \delta_{I2}(B_1^{\omega} + B_2^{\omega})V_{\omega}) \ge -P_1^{-\omega}B_1^{-\omega} + \delta_{I1}(-P_2^{-\omega}B_2^{-\omega} + \delta_{I2}(B_1^{-\omega} + B_2^{-\omega})V_{\omega})$$

With  $P_1^{\omega} = \delta_{U1}\delta_{U2}V_{\omega}$ ,  $P_2^{\omega} = \delta_{U2}V_{\omega}$ ,  $P_1^{-\omega} = \delta_{U1}\delta_{U2}V_{-\omega}$  and  $P_2^{-\omega} = \delta_{U2}V_{-\omega}$ . Rewrite the ICs as follows:

$$-P_{1}^{\omega}B_{1}^{\omega} + \delta_{I1}\delta_{I2}B_{1}^{\omega}V_{\omega} \ge -P_{1}^{-\omega}B_{1}^{-\omega} + \delta_{I1}\delta_{I2}B_{1}^{-\omega}V_{\omega} + \delta_{I1}(-P_{2}^{-\omega}B_{2}^{-\omega} + \delta_{I2}B_{2}^{-\omega}V_{\omega} - (-P_{2}^{\omega}B_{2}^{\omega} + \delta_{I2}B_{2}^{\omega}V_{\omega}))$$

The second line is the negative IC in T = 2 and hence non-positive by

construction. The first line is satisfied if

$$\frac{(-\delta_{U1}\delta_{U2} + \delta_{I1}\delta_{I2})V_H}{(-\delta_{U1}\delta_{U2}V_L + \delta_{I1}\delta_{I2}V_H)}B_1^H \ge B_1^L \ge max\{0, \frac{(-\delta_{U1}\delta_{U2}V_H + \delta_{I1}\delta_{I2}V_L)}{(-\delta_{U1}\delta_{U2} + \delta_{I1}\delta_{I2})V_L}B_1^H\}$$

This is a sufficient condition for the ICs to be satisfied in T = 1

# C.2 Optimality in the separating equilibrium in T=1

In order to show that either informed type does not want to choose any other quantity  $B_1' \neq B_1^{\omega}$ .

$$-P_1^{\omega}B_1^{\omega} + \delta_{I1}(-P_2^{\omega}B_2^{\omega} + \delta_{I2}(B_1^{\omega} + B_2^{\omega})V_{\omega}) \ge -P_1'B_1' + \delta_{I1}(-P_2'B_2' + \delta_{I2}(B_1' + B_2')V_{\omega})$$

With  $P_1^{\omega} = \delta_{U1}\delta_{U2}V_{\omega}$ ,  $P_2^{\omega} = \delta_{U2}V_{\omega}$ ,  $P_1' = \delta_{U1}\delta_{U2}V_H$  and  $P_2' = \delta_{U2}V_H$ . Rewrite the ICs as

$$-P_{1}^{\omega}B_{1}^{\omega} + \delta_{I1}\delta_{I2}B_{1}^{\omega}V_{\omega} \ge -P_{1}^{\prime}B_{1}^{\prime} + \delta_{I1}\delta_{I2}B_{1}^{\prime}V_{\omega} + \delta_{I1}(-P_{2}^{\prime}B_{2}^{\prime} + \delta_{I2}B_{2}^{\prime}V_{\omega} - (-P_{2}^{\prime}B_{2}^{\prime} + \delta_{I2}B_{2}^{\prime}V_{\omega}))$$

The second line is satisfied the IC in T = 2 and at most zero. Therefore it suffices that

$$B_1^H \ge B_1' \quad B_1^H = E$$
  
$$B_1^L \ge max\{0, \frac{\delta_{I1}\delta_{I2}V_L - \delta_{U1}\delta_{U2}V_H}{(\delta_{I1}\delta_{I2} - \delta_{U1}\delta_{U2})V_L}B_1'\} \text{ with } B_1' = E$$

$$\frac{(-\delta_{U1}\delta_{U2} + \delta_{I1}\delta_{I2})V_H}{(-\delta_{U1}\delta_{U2}V_L + \delta_{I1}\delta_{I2}V_H)}E \ge B_1^L \ge max\{0, \frac{(-\delta_{U1}\delta_{U2}V_H + \delta_{I1}\delta_{I2}V_L)}{(-\delta_{U1}\delta_{U2} + \delta_{I1}\delta_{I2})V_L}E\}$$

### C.3 Incentive Compatibility in the pooling equilibrium in T=1

This can be rewritten as

$$-P_{1}B_{1}^{P} + \delta_{I1}\delta_{I2}B_{1}^{P}V_{\omega} \ge -P_{1}'B_{1}' + \delta_{I1}\delta_{I2}B_{1}'V_{\omega} + \delta_{I1}(-P_{2}'B_{2}' + \delta_{I2}B_{2}'V_{\omega} - (-P_{2}B_{2}^{P} + \delta_{I2}B_{2}^{P}V_{\omega}))$$

From T = 2, we know that the second line is more binding for the H-type and negative by construction. The first line is again more binding for the H-type. There is no profitable deviation for the informed trader if  $B_1^P = E$ . This implies that the informed trader wants to buy everything in the first period and nothing in the second period.

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