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Designing Electricity Auctions in the Presence of Transmission Constraints

Presentata da: Mario Blazquez de Paz

Coordinatore Dottorato: Matteo Cervelati

Relatori: Giacomo Calzolari, Emanuele Tarantino

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Designing Electricity Auctions in Presence of Transmission Constraints.

Mario Blazquez de Paz*

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Abstract

This paper analyzes the effect that different designs in the access to financial transmission rights has on spot electricity auctions. In particular, I characterize the equilibrium in the spot electricity market when financial transmission rights are assigned to the grid operator and when financial transmission rights are assigned to the firm that submits the lowest bid in the spot electricity auction. When financial transmission rights are assigned to the grid operator, my model, in contrast with the models available in the literature, works out the equilibrium for any transmission capacity. Moreover, I have found that an increase in transmission capacity not only increases competition between markets but also within a single market. When financial transmission rights are assigned to the firm that submits the lowest bid in the spot electricity auction, firms compete not only for electricity demand, but also for transmission rights and the arbitrage profits derived from its hold. I have found that introduce competition for transmission rights reduces competition in spot electricity auctions.

KEYWORDS: electricity auctions, transmission constraint, market design.

1 Introduction.

Electricity transmission facilities have long been recognized as central elements in the efficient planning and operation of electricity systems. Traditionally, the role of large, interutility transmission paths has been to permit transactions between utilities that exploit regional differences in consumption seasonality and generation costs. As the electricity generation industry is deregulated, however, transmission facilities will also provide important competitive links between potentially isolated markets, thus mitigating the potential for exercise of market power.

Pioneering research on electricity markets in which transmission lines are congested was done by Schweppe et al. (1988). They concluded that the short-term price of transmission services between any two locations is the difference of spot prices between

*PhD Student at the University of Bologna. Mail: mario.blazquezdepaz2@unibo.it

those two points. Hogan (1992) introduces the concept of contract network which provides a mechanism for allocating long-term transmission capacity rights subject to maintaining short-run price efficiency. Chao and Peck (1996) use the physical rights approach to incorporate network externality impacts into the competitive trading mechanism. When competition in the spot electricity market is perfect, the mechanisms proposed by (Hogan, 1992) and (Chao and Peck, 1996) generates the efficient equilibrium predicted by Schweppe et al. (1988). However, as Joskow and Tirole (2000) have shown, when competition in the spot electricity market is imperfect, the way transmission rights are assigned modifies the equilibrium outcome on the spot electricity market.

Joskow and Tirole (2000) assume in their analysis that the equilibrium price in one of the markets is a parameter. Under this assumption, they work out the equilibrium in the other market using two types of transmission rights, financial and physical. By contrast, my model works out the equilibrium simultaneously in both markets. My aim is to characterize the equilibrium in the spot electricity market when competition is imperfect and financial transmission rights are assigned to the grid operator or to the firm that submits the lowest bid in the spot electricity auction. Borenstein, Bushnell and Stoft (2000) work out the equilibrium when the firms compete in quantities and financial transmission rights are assigned to the grid operator. In contrast with their model, my model characterizes the equilibrium for any transmission capacity. When the financial transmission rights are assigned to the firm that submits the lowest bid in the spot electricity auction, the firms compete not only for electricity demand, but also for transmission rights and the arbitrage profits derived from its hold. Therefore, under this set up, I analyze whether the introduction of competition for the transmission rights exacerbates competition in the spot electricity market, or on the contrary, the firms behave less aggressively.[1]

My analysis proceeds by first considering a simple duopoly model similar to the one in Fabra et al. (2006), which is then varied in several directions. In the basic set up, two suppliers with symmetric capacities and (marginal) costs, are allocated in two different markets (North and South) connected by a transmission line. The two firms face a demand in each market that is assumed to be perfectly inelastic and known with certainty when suppliers submit their offer prices. Each supplier must submit a single price offer for its entire capacity. The assumption of price-inelastic demand can be justified by the fact that the vast majority of consumers purchase electricity under regulated tariffs that are independent of the prices negotiated in the wholesale market, at least in the short run.

The assumption that suppliers have perfect information concerning market demand is reasonable when applied to markets in which offers are "short lived", such as in Spain, where there are 24 hourly day-ahead markets each day. In such markets suppliers can be

[1]There are other important assignment rules of financial transmission rights that I do not analyze in the paper. First, I do not characterize the equilibrium when transmission rights are assigned to one firm arbitrarily. Second, I do not characterize the equilibrium when there exist two markets that run sequentially: first, firms compete for transmission rights and later, firms compete in the spot electricity market taking into account the transmission rights they hold. Mahenc and Salanie's (Mahenc and Salanie, 2004) paper can provide interesting insights to work out the equilibrium when the markets run sequentially and firms compete in prices. Third, Küpper and Willems (2007) characterize the equilibrium when there exists a monopoly firm producing in both markets. They work out the equilibrium when the monopolist remains the only user of the transmission line and when the monopolist has to share the transmission line with arbitrageurs. However, my model differ from their model because I work out the equilibrium when there are two firms competing in the market.

assumed to know the total demand they face in any period with a high degree of certainty. In markets in which offer prices remain fixed for longer periods, e.g., a whole day, like in Australia and in the former markets in England and Wales, on the other hand, it is more accurate to assume that suppliers face some degree of demand uncertainty, or volatility, at the time they submit their offers.

When the transmission network is congested, market clearing prices will vary among locations on the network. Prices are higher at locations that are import constrained and lower at locations that are export constrained. Since demand and transmission capacity availability both vary over time, the incidence of network congestion, the differences in locational prices, and congestion charges can also vary widely over time. The associated variations in prices create a demand by risk-averse buyers and sellers for instruments to hedge price fluctuations. To satisfy this demand, several Independent System Operators have created and assigned "financial-transmission-rights" to market participants. These financial rights give the holders a claim on the congestion rents created when the network is constrained and allow them effectively to hedge variations in differences in nodal prices and associated congestion charges. As I have explained above, in the paper I characterize the equilibrium in the spot electricity market when the financial transmission rights are assigned to the grid operator and when the financial transmission rights are assigned to the firm that submits the lowest bid in the spot electricity auction.

Under this set of assumptions, when the financial transmission rights are assigned to the grid operator, if the realization of demand is low, the two producers have enough capacity to satisfy the demand in both markets. Therefore, they compete fiercely to be dispatched first in the auction. Hence, the equilibrium is the typical Bertrand equilibrium in which firms submit bids equal to their marginal costs. If the realization of demand is high, the equilibrium strategies pair is in mixed strategies. When the realization of demand is such that the transmission constraint does not affect the payoff function of the firms, the equilibrium is symmetric even when the realization of the demand is different in both markets, and the expected price is equal in both markets even when the realization of demand can be substantially different between markets. By contrast, when the realization of demand is such that the transmission constraint modifies the payoff function of the firms, the equilibrium is asymmetry. The firm located in the high demand market assigns higher probability to high bids; and so, the expected value of bids in that market is higher.

My model, in contrast with Borenstein, Bushnell and Stoft (2000) characterizes the equilibrium for any transmission capacity. Therefore, I can analyze the effect that any increase in transmission capacity has on equilibrium. In particular, I find that increases in transmission capacity not only increase competition between markets, but also induces changes in the payoff functions that could facilitate the entry of new firms in the long term and so increase competition within a market. Moreover, the results that I obtain complements the literature that characterizes the equilibrium in a Bertrand model when some type of asymmetry is introduced. In particular, increases of transmission capacity, or equivalently, reductions in the asymmetry in the access to demand, as in (Deneckere and Kovenock, 1986) and (Osborne and Pitchik, 1986), increases smoothly the competition between both markets. This result contrasts with Shitovitz (1973) that predicts a sharp change through perfect competition when the asymmetry decreases.

When the financial transmission rights are assigned to the firm that submits the lowest bid in the spot electricity auction, the firms compete not only for electricity demand, but also for the transmission rights and the arbitrage profits derived from its hold. When the realization of demand is low, the equilibrium is in pure strategies and coincide with the equilibrium when the financial transmission rights are assigned to the grid operator. When the demand is high, the unique symmetric mixed strategies equilibrium is when the realization of the demand belongs to the 45 degree line. The possibility to obtain arbitrage profits made vanish the symmetry mixed strategies equilibrium for any other realization of demand. If the realization of demand is uniformly distributed, i.e., the probability that the realization of the demand is above the 45 degree line is one-half, then assigning the financial transmission rights to the firm that submits the lowest bid in the energy auction reduces the welfare of consumers and increases the expected payoff of firms.

The article proceeds as follows, in section two, I describe the set up, the timing and equilibrium in a two node electricity market where the financial transmission rights are assigned to the grid operator. In section three, I describe the set up, the timing and equilibrium in a two node electricity market where the financial transmission rights are assigned to the firm that submits the lowest bid in the spot electricity auction. Section four concludes. Proofs are in the Appendix.

2 Transmission rights assigned to the transmission grid operator

The aim of this section is to characterize the equilibrium in a spot electricity market when the financial transmission rights are assigned to the grid operator. I also run different comparative statics analysis, focusing mainly on the effect of a reduction in transmission capacity on the equilibrium.

2.1 The model

Set up of the model. There exist two electricity markets, market North and market South, that are connected by a transmission line with capacity T.

There exist two duopolists with capacities k_n and k_s, where subscript n means that the firm is located in market North and subscript s means that the duopolist is located in market South. Suppliers' marginal costs[2] of production are c_n and c_s. The level of demand in any period, θ_n in market North and θ_s in market South, is a random variable that is independent of the market price, i.e., perfectly inelastic. In particular, $\theta_i \in [\underline{\theta}_i, \overline{\theta}_i] \subseteq [0, k+T]$ is distributed according to some known distribution function $G(\theta_i)$, $i = n, s, i \neq j$

The capacity of the transmission line is lower than the installed capacity in each market $T \leq k$, i.e. the transmission line could be congested for some realization of demands (θ_s, θ_n). The term "congested" is used throughout this article in the electrical engineering sense: a line is congested when the flow of power is equal to the line's capacity, as determined by engineering standards.

[2]My aim in this paper is analyze the effect that asymmetries in the access to demand has on equilibrium. Therefore, in the rest of the paper, I will assume that firms are symmetric in capital $k_n = k_s = k > 0$ and symmetric in costs $c_n = c_s = c = 0$.

Timing of the game. Having observed the realization of demands $\theta \equiv (\theta_s, \theta_n)$, each supplier simultaneously and independently submits a bid specifying the minimum price at which it is willing to supply up to its capacity, $b_i \leq P$, $i = n, s$, where P denotes the "market reserve price", possibly determined by regulation.[3] Let $b \equiv (b_s, b_n)$ denote a bid profile. On the basis of this profile the auctioneer calls suppliers into operation. If suppliers submit different bids, the lower-bidding supplier's capacity is dispatched first. Without loss of generality, assume that $b_n < b_s$. If the capacity of supplier n is not sufficient to satisfy the total demand $(\theta_s + \theta_n)$ in the case of the transmission line not congested, or $(\theta_n + T)$ in the case of the transmission line congested,[4] the higher-bidding supplier's capacity, firm s is then dispatched to serve residual demand, $(\theta_s + \theta_n - k)$ if $(\theta_s > k - \theta_n$ and $\theta_n \in [k - T, k])$, or $(\theta_s - T)$ if $(\theta_s > T$ and $\theta_n \in [0, k - T])$. If the two suppliers submit equal bids, then supplier i is ranked first with probability ρ_i, where $\rho_n + \rho_s = 1$, $\rho_i = 1$ if $\theta_i > \theta_j$, and $\rho_i = \frac{1}{2}$ if $\theta_i = \theta_j$, $i = n, s$, $i \neq j$. The tie breaking rule implemented is such that if the bids of both firms are equal and the demand in market i is greater than the demand in market j, the auctioneer dispatches first the supplier located in market i. Since electricity transmission through the grid is costly and both firms are equally efficient at generating electricity, if both firms submit equal bids, the auctioneer gives priority in the dispatch to the firm located in each own market.

The output allocated to supplier $i, i = n, s$, denoted by $q_i(\theta, b)$, is given by

$$q_i(b; \theta, T) = \begin{cases} L_i \equiv min\left\{\theta_i + \theta_j, \theta_i + T, k_i\right\} & \text{if } b_i < b_j \\ T_i \equiv \rho_i min\left\{\theta_i + \theta_j, \theta_i + T, k_i\right\} + & \\ \qquad [1 - \rho_i] max\left\{0, \theta_i - T, \theta_i + \theta_j - k_j\right\} & \text{if } b_i = b_j \\ H_i \equiv max\left\{0, \theta_i - T, \theta_i + \theta_j - k_j\right\} & \text{if } b_i > b_j \end{cases} \tag{1}$$

The output function has an important role determining the equilibrium, therefore I will explain it in greater detail. Below, I describe the construction of firm n's output function, the one for firm s is symmetric.

The total demand that can be satisfied by firm n when it submits the lower bid $(b_n < b_s)$ is defined by $min\left\{\theta_n + \theta_s, \theta_n + T, k\right\}$. The realization of (θ_s, θ_n) determines three different areas (left panel in figure 1).

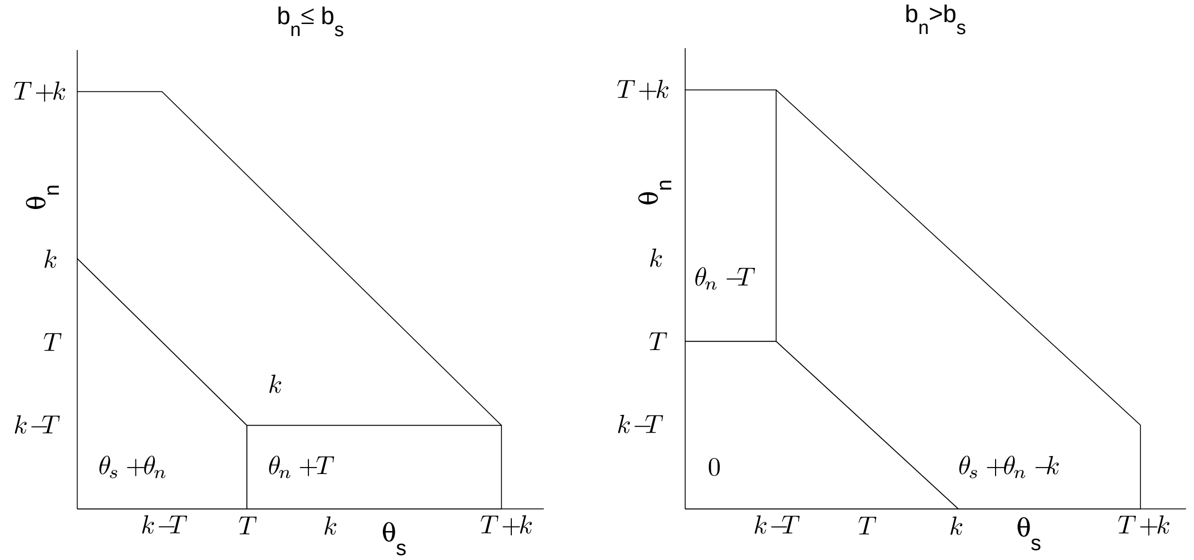
$$min\left\{\theta_n + \theta_s, \theta_n + T, k\right\} = \begin{cases} \theta_s + \theta_n & \text{if } \theta_n \leq k - \theta_s \text{ and } \theta_s < T \\ \theta_n + T & \text{if } \theta_n < k - T \text{ and } \theta_s > T \\ k & \text{if } \theta_n > k - \theta_s;\ \theta_s \in [0, T] \\ & \text{or if } \theta_n > k - T;\ \theta_s \in [T, k + T] \end{cases}$$

When demand in both markets is low, firm n can satisfy the total demand $(\theta_s + \theta_n)$. If the demand in market South is greater than the transmission capacity $\theta_s > T$, firm n cannot satisfy the demand in market South even when it has enough generation capacity

[3]P can be interpreted as the price at which all consumers are indifferent between consuming and not consuming, or a price cap imposed by the regulatory authorities. See von der Fehr and Harbord (1993, 1998).

[4]When the demand in market South is greater than the transmission line capacity $\theta_s > T$, firm n can only satisfy the demand in its own market and T units of demand in market South $(\theta_n + T)$. Below in this section, I will explain with detail the total demand that can be satisfied by each firm and the residual demand that can be satisfied by each firm.

Figure 1: Output function for firm n. ($k_n = k_s = 60, T = 40$)



to do so, therefore the total demand that firm n can satisfy is $(\theta_n + T)$. Finally, if the demand is big enough the total demand that firm n can satisfy is its own generation capacity.

The residual demand that firm n satisfies when it submits the higher bid $(b_n > b_s)$ is defined by $max\,\{0, \theta_n - T, \theta_s + \theta_n - k\}$. The realization of (θ_s, θ_n) determines three different cases (right panel in figure 1).

$$max\,\{0, \theta_n - T, \theta_s + \theta_n - k\} = \begin{cases} 0 & \text{if } \theta_n < T;\ \theta_s \in [0, k-T] \\ & \text{or } \theta_n < k - \theta_s;\ \theta_s \in [k-T, k] \\ \theta_n - T & \text{if } \theta_n > T \text{ and } \theta_s \in [0, k-T] \\ \theta_s + \theta_n - k & \text{if } \theta_n > k - \theta_s;\ \theta_s \in [k-T, T+k] \end{cases}$$

When demand in both markets is low, firm s satisfies the total demand, therefore the residual demand that remains to firm n is zero. When the total demand is large enough, firm s cannot satisfy the total demand and some residual demand $(\theta_s + \theta_n - k)$ remains to firm n. Due to the transmission constraint, the total demand that firm s can satisfy diminishes. As soon as demand in market North is greater than the transmission capacity $(\theta_n > T)$, firm s cannot satisfy it, therefore some residual demand $(\theta_n - T)$ remains to firm n.

Finally, the payments are worked out by the auctioneer. I will assume that the auctioneer runs a uniform price auction[5], the price received by a supplier for any positive quantity dispatched by the auctioneer is equal to the highest accepted bid in the auction

[5]Under the set up described in the model, the uniform and the discriminatory auction perform equally. Without loss of generality, I assume that the equilibrium price in market i is lower than the equilibrium price in market j. If the auction is run using a uniform price setting. The payoff for firm i is $p_i\theta_i + p_jT$, where $p_i\theta_i$ is the payoff obtained from the sales in its own market and p_jT is the payoff obtained from the sales in its rival's market. However, the firm located in market i will not collect the congestion rents, the grid operator will do. Therefore, the payoff for firm i will be $p_i(\theta_i + T)$ instead of $p_i\theta_i + p_jT$, but $p_i(\theta_i + T)$ is precisely the payoff for firm i when the auction is run using the discriminatory setting. Therefore, the uniform and the discriminatory auction performs equally under the set up of the model.

in its own market. As in Borenstein et al. (2000), I assume that electricity flows from the market with the lower price to the market with the higher price and that the grid operator collects the congestion rents. Hence, for a given realization of $\theta \equiv (\theta_s, \theta_n)$ and a bid profile $b \equiv (b_s, b_n)$, supplier n's profits, $i = n, s$, can be expressed as

$$\pi_i(b;\theta,T) = \begin{cases} [b_i - c_i]q_i(b;\theta,T) & \text{if } b_i < b_j \text{ and } (\theta_i + \theta_j \leq k_i) \text{ and } (\theta_j \leq T) \\ [b_i - c_i]q_i(b;\theta,T) & \text{if } b_i > b_j \text{ and } (\theta_i + \theta_j > k_j) \text{ or } (\theta_i > T) \\ [b_j - c_i]q_i(b;\theta,T) & \text{otherwise} \end{cases} \tag{2}$$

As in the case of the production function, the payoff function has an important role determining the equilibrium, therefore I will explain it in greater detail. Below, I describe the construction of firm n's payoff function, the one for firm s is symmetric.

If $b_n < b_s$ and $(\theta_n + \theta_s \leq k_n)$ and $(\theta_s \leq T)$. Firm n submits the lower bid, has enough capacity to satisfy the total demand and the transmission line is not congested, therefore it sets the price in the auction and its payoff is $\pi_n(b;\theta,T) = [b_n - c_n]q_n(b;\theta,T)$. Instead, if $b_n > b_s$ and $(\theta_n + \theta_s > k_s)$ or $(\theta_n > T)$. Firm n submits the higher bid, firm s has not enough capacity to satisfy the total demand or the transmission line is congested, therefore firm n sets the price in the auction and its payoff is $\pi_n(b;\theta,T) = [b_n - c_n]q_n(b;\theta,T)$. In the remaining cases, firm s instead of firm n sets the price in the auction, therefore $\pi_n(b;\theta,T) = [b_s - c_n]q_n(b;\theta,T)$.

2.2 Equilibrium analysis

In this section, I characterize the equilibrium in the spot electricity market when the financial transmission rights are assigned to the grid operator. I start proving, that in general, a pure strategy equilibrium does not exist.

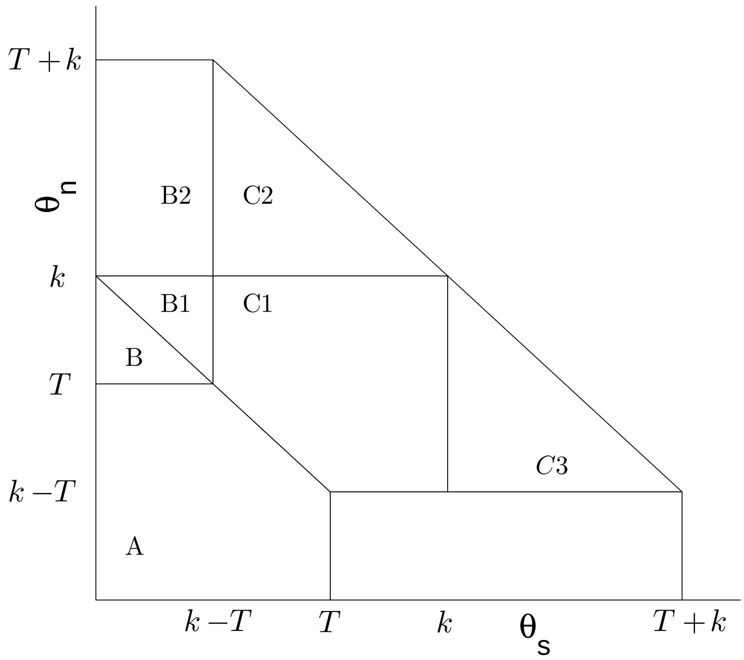
Proposition 1. When the realization of demands (θ_s, θ_n) belongs to area A, the equilibrium is in pure strategies. When the realization of demands (θ_s, θ_n) belongs to area $B, B1, B2, C1, C2$ or $C3$, there no exist neither a symmetric nor an asymmetric pure strategy equilibrium (figure 2).

Proof. When the realization of demands (θ_s, θ_n) belongs to area A the two producers have enough capacity to satisfy the demand in both markets. Therefore, they compete fiercely to be dispatched first in the auction. Hence, the equilibrium is the typical Bertrand equilibrium in which both firms submit bids equal to their marginal cost.

When the realization of demand belong to areas $B, B1, B2, C1, C2$ or $C3$ and two producers submit the same bid b such that $c < b < P$. According with the tie breaking rule, the firm allocated in the market in which the demand is higher is dispatched first. Therefore, the firm dispatched last has incentives to shading its bid an arbitrarily small amount ϵ to be dispatched first. Therefore, a symmetric pure strategy equilibrium does not exist.

When the realization of demand belong to areas $B, B1, B2, C1, C2$ or $C3$ and both firms submit a different bid b_i such that $c < b_i < P \ \forall i, j \mid i \neq j$, there no exist an asymmetric pure strategy equilibrium. Without loss of generality, I will assume that $b_i > b_j$.

Figure 2: Equilibrium areas ($k_n = k_s = 60, T = 40, c_n = c_s = 0, P = 7$)



If (b_i, b_j) satisfies $b_i max\{0, \theta_n - T, \theta_s + \theta_n - k\} = b_j min\{\theta_i + \theta_j, \theta_i + T, k\}$, then firm i has no incentives to deviate. By contrast, firm j has incentives to deviate shading b_i an arbitrarily small amount ϵ increases its payoff because $(b_j - \epsilon) min\{\theta_i + \theta_j, \theta_i + T, k\} \geq b_i min\{\theta_i + \theta_j, \theta_i + T, k\}$. Therefore, an asymmetric pure strategy equilibrium does not exist $\square$

As I have proved in proposition one, a pure strategy equilibrium does not exist. However, the payoff functions satisfy the requirements that guarantee the existence of a mixed strategy equilibrium. The discontinuities of $\pi_i, \forall i, j$ are restricted to the strategies such that $b_i = b_j$. Furthermore, it is simple to confirm that by lowering its price from a position where $b_i = b_j$, a firm discontinuously increases its profit. Therefore, $\pi_i(b_i, b_j)$ is everywhere left lower semi-continuous in b_i, and hence weakly lower semi-continuous. Obviously $\pi_i(b_i, b_j)$ is bounded. Finally, $\pi_i(b_i, b_j) + \pi_j(b_i, b_j)$ is continuous, because discontinuous shifts in clientele from one firm to another occur only where both firms derive the same profit per customer. Therefore, theorem five in Dasgupta and Maskin (1986) applies, hence a mixed strategy equilibrium exists.[6]

Lemma 1. In a mixed strategy equilibrium none firm submits a bid lower than bid[7] ($\underline{b}_i^I$) such that $\underline{b}_i^I min\{\theta_i + \theta_j, \theta_i + T, k\} = P max\{0, \theta_i - T, \theta_i + \theta_j - k\}$. Moreover, the sup-

[6] The model presented before satisfies the properties that guarantee that a mixed strategy equilibrium exists. However, Dasgupta and Maskin (1986) does not provide an algorithm to work out the equilibrium. However, using the approach proposed by (Karlin, 1959; Beckmann, 1965; Shapley, 1957; Shilony, 1977; Varian, 1980; Deneckere and Kovenock, 1986; Osborne and Pitchik, 1986; Fabra et al., 2006) the equilibrium characterization is guarantee by construction. In the annex, I have described the steps required to work out the mixed strategies equilibrium.

[7] The superscript I denote support, profits, bids and strategies referred to model I, the one that allocate the transmission rights to the grid operator.

port for the mixed strategies equilibrium for both firms is $S^I = \left[max\left\{\underline{b}_i^I, \underline{b}_j^I\right\}, P\right]$.

Proof. Each firm can guarantees to itself the payoff $Pmax\{0, \theta_i - T, \theta_i + \theta_j - k\}$, because each firm always can submit the highest bid and satisfies the residual demand. Therefore, in a mixed strategies equilibrium, none firm submits a bid that generate a payoff equilibrium lower than $Pmax\{0, \theta_i - T, \theta_i + \theta_j - k\}$. Hence, none firm submits a bid lower than $\underline{b}_i^I$, where $\underline{b}_i^I$ solves $\underline{b}_i^I min\{\theta_i + \theta_j, \theta_i + T, k\} = Pmax\{0, \theta_i - T, \theta_i + \theta_j - k\}$.

None firm can rationalize submit a bid lower than $\underline{b}_i^I, i = n, s$. In the case that $\underline{b}_i^I = \underline{b}_j^I$, the mixed strategy equilibrium and the support is symmetric. In the case that $\underline{b}_i^I < \underline{b}_j^I$, firm i knows that firm j never submits a bid lower than $\underline{b}_j^I$. Therefore, in a mixed strategy equilibrium, firm i never submits a bid b_i^I such that $b_i^I \in \left(\underline{b}_i^I, \underline{b}_j^I\right)$, because firm i can increases its expected payoff choosing a bid b_i^I such that $b_i^I \in \left[\underline{b}_j^I, P\right]$. Hence, the equilibrium strategies support for both firms is $S^I = \left[max\left\{\underline{b}_i^I, \underline{b}_j^I\right\}, P\right]$ □

Using this ancillary result, I can present the main result of this section.

Proposition 2. The characterization of the equilibrium strategies fall into one of the next two categories (figure 2).

i Area A (low demand). The equilibrium strategies pair is pure and equal to $b_n = b_s = c = 0$. The equilibrium payoff is zero for both firms. No electricity flows through the grid.

ii Area $B, B1, B2, C1, C2, C3$ (high demand). The equilibrium strategies pair is in mixed strategies.
The mixed strategy equilibrium support is defined by lemma 1

$$S^I = \left[max\left\{\underline{b}_i^I, \underline{b}_j^I\right\}, P\right] \tag{3}$$

The cumulative distribution function is defined by

$$F_i^I(b) = \frac{L_j^I(b) - L_j^I(\underline{b})}{L_j^I(b) - H_j^I(b)} \forall i, j \tag{4}$$

The payoff function is

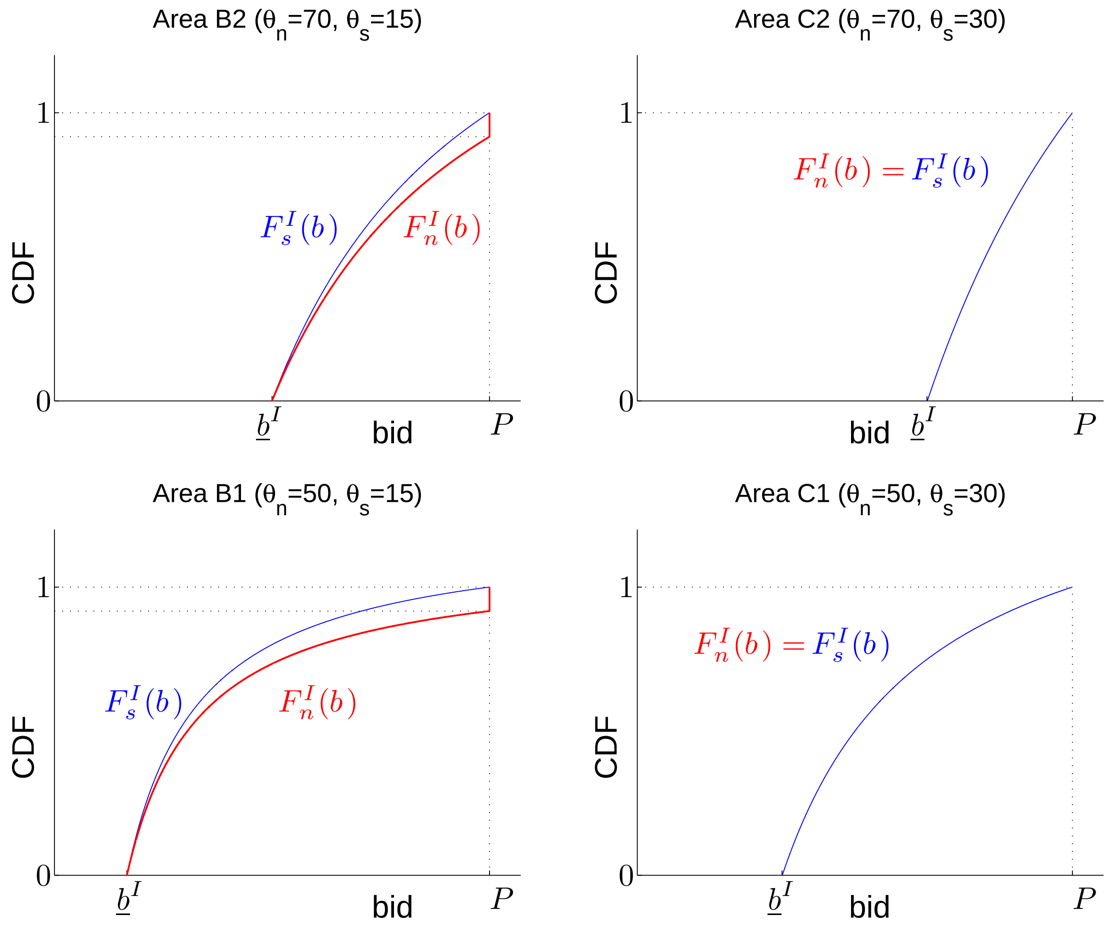
$$\overline{\pi}_i^I = \underline{b}^I min\{\theta_i + \theta_j, \theta_i + T, k\} \forall i, j \tag{5}$$

The expected bid is defined by

$$E(b_i^I) = \int_{\underline{b}^I}^{P} b\frac{\partial F_i^I(b)}{\partial b}\partial b \forall i, j \tag{6}$$

The mixed strategies equilibrium characterized by proposition two have been represented graphically in figure 3. In order to explain the main insights derived from proposition two, I discuss in detail the mixed strategies equilibrium in areas $B1$ and $C1$. In area $C1$ (the same logic applies in area $C2$), the transmission constraint does not affect the payoff functions of the firms, therefore the equilibrium will be symmetric even when the realization of the demand is different in both markets, therefore, the expected price is

Figure 3: Equilibrium Strategies ($k_n = k_s = 60, T = 40, c_n = c_s = 0, P = 7$)



equal in both markets, even when the realization of demand can be substantially different between markets. By contrast, in area $B1$ (the same logic applies in area $B2$), the transmission constraint modifies the payoff functions of the firms, therefore the equilibrium will be asymmetric. In particular, the cumulative distribution function of firm s is more concave, therefore it submits lower bids with higher probability. Moreover, the cumulative distribution function of firm s, $F_s^I(b)$, is continuous in the support, by the contrary, the cumulative distribution function of firm n, $F_n^I(b)$, is discontinuous at the upper bound of the support, P, therefore it assigns a positive probability to the maximum price allowed by the auctioneer. Moreover, $F_n^I(b) \leq F_s^I(b) \forall b \in S^I$. Hence, $F_s^I(b)$ stochastic dominate $F_n^I(b)$ and so the expected value of bids in market North, $E(b_n^I)$, is greater than the expected value of bids in market South, $E(b_s^I)$.

Proposition two characterizes the mixed strategies equilibrium. However, many interesting insights can be obtained from the comparative static analysis of the main variables of the model. In corollaries one, two and three, I analyze the effect that a reduction in demand in market South, a reduction in demand in market North and a reduction in transmission capacity have on the mixed strategies equilibrium.

Corollary 1. When the realization of the demand belongs to areas[8] $B1$ or $B2$. A reduction in the demand in market South has no effect on the lower bound of the strategies support; increases the probability that firm n assigns to high bids; increases the expected value of bids in market North, does not modify the expected value of bids in market South; has no effect on the expected payoff of firm n and reduces the expected payoff of firm s. Finally, when the realization of the demand belongs to areas $C1$ or $C2$. A reduction in the demand in the South reduces the lower bound of the strategies support; has no effect on the probability that the firm n assigns to high bids; reduces the expected value of bids in both markets and reduces the expected payoff of both firms.

In order to facilitate the comprehension of the main insights derived from Corollary one, I focus on the equilibrium in areas $B1$ and $C1$. In area $B1$ (the same logic applies in area $B2$), a reduction of demand in market South does not modify neither the lower bound of the support nor the cumulative distribution function of firm s, therefore the expected value of bids in market South does not change; however, due to the reduction in demand in market South, the expected payoff of firm s decreases. A reduction of demand in market South increases the probability that firm n assigns to the highest bid allowed by the auctioneer. The probability that firm n assigns to the maximum bid allowed by the auctioneer represents the opportunity cost of submit high bids for firm s. A reduction of demand in market South increases the opportunity cost of submit high bids for firm s because in case of be dispatched last, the residual demand that it faces is very low, then firm n must increase the probability that assigns to high bids to made firm s stay in equilibrium. Hence, the expected value of bids in market North increases. However, the expected payoff of firm n does not change because its residual demand does not change with a reduction of the demand in market South. In area $C1$ (the same logic applies in area $C2$), a reduction in demand in market South reduces the lower bound of the support. A reduction in demand in market South reduces the residual demand; then, by lemma one, the bid that made any firm be indifferent between submit a high bid and satisfy the residual demand and submit a low bid and satisfy the total demand decreases. Therefore, the expected bid and the expected payoff decrease for both firms.

Table 1 and figure 3 provides numerical and graphical examples that summarizes the results enumerated in corollary one.
Corollary 2. When the realization of the demand belongs to areas $B1$ or $B2$. A reduction in the demand in market North reduces the lower bound of the strategies support; has no effect on the probability that firm n assigns to high bids; reduces the expected value of bid in both markets and reduces the expected payoff of both firms. Finally, when the realization of the demand belongs to areas $C1$ or $C2$. A reduction in the demand in the North reduces the lower bound of the strategies support; has no effect on the probability that firm n assigns to high bids; reduces the expected value of bids in both markets and reduces the expected payoff of both firms.

To made the discussion simple, I focus in the equilibrium in areas $B1$ and $C1$. In area $B1$ (the same logic applies in area $B2$), a reduction of demand in market North reduces the lower bound of the support. The lower bound of the support represents the opportunity cost of submit high bids for firm n. A reduction of the demand in market North reduces

[8] I will focus on the equilibrium in areas $B1$, $B2$, $C1$ and $C2$. The equilibrium in area B presents minor and irrelevant differences with respect the equilibria in areas $B1$ and $B2$.

Table 1: $(\bigtriangledown\theta_n, \bigtriangledown\theta_s)$, $(k_n = k_s = 60, T = 40, c_n = c_s = 0, P = 7)$

Area	$\underline{b}^I$	$F_n^I(P)$	$E(b_n)$	$E(b_s)$	$\overline{\pi}_n$	$\overline{\pi}_s$
Area B1						
$(\theta_n = 50, \theta_s = 10)$	1.17	0.83	3.25	2.51	70	58.33
$(\theta_n = 50, \theta_s = 15)$	1.17	0.91	2.88	2.51	70	64.17
Area B2						
$(\theta_n = 70, \theta_s = 10)$	3.5	0.83	5.21	4.85	210	175
$(\theta_n = 70, \theta_s = 15)$	3.5	0.91	5.03	4.85	210	192.5
Area C1						
$(\theta_n = 50, \theta_s = 25)$	1.75	1	3.23	3.23	105	105
$(\theta_n = 50, \theta_s = 30)$	2.33	1	3.84	3.84	140	140
Area C2						
$(\theta_n = 70, \theta_s = 25)$	4.08	1	5.28	5.28	245	245
$(\theta_n = 70, \theta_s = 30)$	4.67	1	5.67	5.67	280	280

the residual demand that firm n faces in case of submit the highest bid in the auction. Therefore, submit high bids is less attractive to firm n. To be more precise, by lemma one, the bid that made firm n be indifferent between submit a low bid and satisfy the total demand and submit a high bid a satisfy the residual demand decreases. Moreover, the probability that firm n assign to the maximum bid allowed by the auctioneer does not change. Both effects induce a reduction of the expected value of bids in market North and South and a reduction of the expected payoff of firm n and firm s. In area $C1$ (the same logic applies in area $C2$), a reduction of demand in market North reduces the lower bound of the support and, as I have explained above, reduces the expected value of bids in both markets and reduces the expected payoff of both firms.

Table 1 and figure 3 provides numerical and graphical examples that summarizes the results enumerated in corollary two.

Corollary 3. When the realization of the demand belongs to areas $B1$ or $B2$. A reduction of transmission capacity increases the lower bound of the strategies support; increases the probability that firm n assigns to high bids; increases the expected value of bids in both markets; increases the expected payoff of firm n and increases the expected payoff of firm s when the transmission capacity is below the threshold $(\theta_n > 2T + \theta_s)$ and decreases the expected payoff when the transmission capacity is above the threshold. When the transmission capacity reduces to zero, the asymmetry in the access to the demand becomes extreme, two monopolies emerge, one in each market.

A reduction of transmission capacity increases the residual demand of firm n and, by lemma one, increases the lower bound of the support. Moreover, a reduction of transmission capacity made submit high bids more attractive to firm n, therefore the probability that firm n assigns to high bids increases. Both effects induce an increase of the expected value of bids in both markets and induce an increase on the expected payoff of firm n. However, the effect on the expected payoff of firm s is ambiguous. When the transmission capacity is high enough, $T \geq k$, both markets are integrated, therefore, the competition

Table 2: ($\bigtriangledown T$), ($\theta_n = 55, \theta_s = 5$), ($k_n = k_s = 60, c_n = c_s = 0, P = 7$)

	$\underline{b}^I$	$F_n^I(P)$	$E(b_n)$	$E(b_s)$	$\overline{\pi}_n$	$\overline{\pi}_s$
$T = 60$	0	1	0	0	0	0
$T = 50$	0.58	0.92	2.03	1.58	35	32.08
$T = 40$	1.75	0.75	4.17	3.23	105	78.75
$T = 30$	2.92	0.58	5.47	4.37	175	102.08
$T = 20$	4.08	0.42	6.28	5.28	245	102.08
$T = 10$	5.25	0.25	6.76	6.04	315	78.75
$T = 0$	7	0	7	7	385	35

between firms is fierce and the equilibrium price and payoff are zero for both firms. When the transmission capacity decreases the equilibrium prices increase in both markets and firm s can sell part of its capacity in market North, therefore the payoff of firm s increases. However, when the transmission capacity satisfies $T \geq \frac{\theta_n + \theta_s}{2}$, with the parameters of the example, $T \geq 25$, a reduction in transmission capacity induces an increase in prices, but, the total demand that firm s faces decreases and so its expected payoff. Therefore, a reduction of transmission capacity has an ambiguous effect on the expected payoff of firm s. Table 2 summarizes these results.

As in (Deneckere and Kovenock, 1986) and (Osborne and Pitchik, 1986) a reduction of transmission capacity, or equivalently, an increase in the asymmetry in the access to the demand, reduces smoothly the competition between both markets. This result contrasts with Shitovitz, (1973) that predicts a sharp change trough monopoly when the asymmetry increases. Moreover, as can be observed in table 2 increases in transmission capacity always generates an increase in competition between markets. My model, in contrast with the model developed by Borenstein, Bushnell and Stoft (2000), gives us the opportunity to evaluate the effect that an increase in transmission capacity has on equilibrium when the transmission line is congested.[9]

To conclude, I would like to emphasized that an increase in transmission capacity not only increases the competition between both markets, but also induces changes on the payoff functions that could facilitate the entry of new firms in the long term and so increase competition. To motivate the argument, I introduce the next example. As can be observed in the last raw in table 2, when the transmission capacity is zero, the expected payoff of firm s is the monopoly profit, 35. Imagine that in market South exists a potential entrant with the same capacity $k = 60$ and the same marginal cost $c = 0$ that firm s, but it faces an entry cost of 35. If the two markets are isolated, due to the fix cost, the potential competitor can not enter in market South. However, if the transmission capacity increases to $T = 20$, the expected payoff in market South increases to $102, 08$. Therefore, the potential competitor could enter in market South.[10] This

[9]Borenstein, Bushnell and Stoft (2000) are unable to characterize the equilibrium when the transmission line is congested. Therefore, they can not provide any results to evaluate increases in transmission capacity when the transmission line is congested. By contrast, my model fully characterizes the equilibrium for any transmission capacity.

[10]In the third chapter of my thesis, I would like to analyze the effect that an increase in transmission capacity could have not in the competition between markets but in the competition within a single

simple example[11] shows that even a small increase in transmission capacity could have a big impact on equilibrium prices not only increasing competition between both markets, but within a single market.[12]

3 Transmission rights assigned to the firm that submits the lowest bid in the spot electricity auction

When the financial transmission rights are assigned to the firm that submits the lowest bid in the spot electricity auction, the firms compete not only for the electricity demand, but also for the financial transmission rights and the arbitrage profits derived from its hold. The strategies of the firms will be affected not only by the transmission line constraint, but also for the possibility to obtain arbitrage profits. The aim of this section is characterize the equilibrium when the financial transmission rights are assigned to the firm that submits the lowest bid in the spot electricity auction and analyze if the introduction of competition for the transmission rights exacerbate competition in the spot electricity market, or by the contrary, the firms behave less aggressively.

3.1 The model

Set up of the model. The same set up that in section two.

Timing of the game. The same timing that in section two. However, the payoff function is different. I will explain it in detail.

Finally, the payments are worked out by the auctioneer. I am going to assume that the auctioneer runs a uniform price auction[13], the price received by a supplier for any positive quantity dispatched by the auctioneer is equal to the highest accepted bid in the market.

[11] The same logic applies to explain the impact of an increase in trade between countries. In the example described above, market North could be Europe, high demand market, and market South could be Africa, low demand market. As I have explained in the example, even a small increase in trade between both markets, increase of quotas allowances, could facilitate the entry of new firms in the market with the lowest demand and induce a reduction on the prices that consumers face on equilibrium. Moreover, the model that I have developed can be useful to complement trade models that analyze the effect that quotas has on equilibrium (Bhagwati, 1968; Itoh and Ono, 1982; Shibata, 1968; Yadav, 1968). My model, in contrast with the models enumerated, works out the equilibrium simultaneously in both markets.

[12] Borenstein, Bushnell and Stoft (2000) can not analyze the effect that an increase in transmission capacity has on the equilibrium when the transmission line is congested. Therefore, using their model, we can not address the effect that an increase in transmission capacity could have on the competition within a single market. By contrast, the model that I present in this paper gives us the opportunity to evaluate the effect that an increase in transmission capacity could have on generation investment decisions. Moreover, the model that I present in the paper gives us the opportunity to use the industrial organization tools to analyze competition in electricity markets.

[13] Under the set up described in the model described in this section, the uniform and the discriminatory auction performs differently. Without loss of generality, I will assume that the equilibrium price in market i is lower than the equilibrium price in market j. If the auction is run using a uniform price setting. The payoff for firm i will be $p_i\theta_i + p_jT$, where $p_i\theta_i$ is the payoff obtained from the sells in its own market and p_jT is the payoff obtained from the sells in its rival market. If the auction is run using a discriminatory price setting. The payoff for firm i will be $p_i(\theta_i + T)$ instead of $p_i\theta_i + p_jT$, . Therefore, when the firms compete simultaneously for the demand and for the transmission right, the uniform and the discriminatory auction does not performs equally.

auction in its own market. As in the model described in the previous section, I am going to assume that electricity flows from the market with the lower price to the market with the higher price. Hence, for a given realization of $\theta \equiv (\theta_s, \theta_n)$ and a bid profile $b \equiv (b_s, b_n)$, supplier n's profits, $i = n, s$, can be expressed as

$$\pi_i(b;\theta,T) = \begin{cases} [b_i - c_i](\theta_i + \theta_j) & \text{if } b_i < b_j \text{ and } (\theta_i + \theta_j \leq k_i) \text{ and } (\theta_j \leq T) \\ [b_i - c_i]min\,\{\theta_i, k_i\} + & \\ [b_j - c_i]max\,\{0, min\,\{T, k_i - \theta_i\}\} & \text{if } b_i < b_j \text{ and } (\theta_i + \theta_j > k_i) \text{ or } (\theta_j > T) \\ [b_i - c_i]max\,\{0, \theta_i - T, \theta_i + \theta_j - k_j\} & \text{if } b_i > b_j \text{ and } (\theta_i + \theta_j > k_j) \text{ or } (\theta_i > T) \end{cases} \tag{7}$$

The payoff function will have an important role determining the equilibrium, therefore I am going to describe it in greater detail. Equation 7 can be written in a less compact way in order to facilitate its interpretation.

$$\pi_i(b;\theta,T) = \begin{cases} [b_i - c_i](\theta_i + \theta_j) & \text{if } b_i < b_j \text{ and } (\theta_i + \theta_j \leq k_i) \text{ and } (\theta_j \leq T) \\ [b_i - c_i]\theta_i + & \\ [b_j - c_i]min\,\{T, k_i - \theta_i\} + & \text{if } b_i < b_j \text{ and } ((\theta_i + \theta_j > k_i) \text{ or } (\theta_j > T)) \text{ and } (\theta_i \leq k_i) \\ [b_i - c_i]k_i & \text{if } b_i < b_j \text{ and } ((\theta_i + \theta_j > k_i) \text{ or } (\theta_j > T)) \text{ and } (\theta_i > k_i) \\ [b_i - c_i] & \\ max\,\{0, \theta_i - T, \theta_i + \theta_j - k_j\} & \text{if } b_i > b_j \text{ and } (\theta_i + \theta_j > k_j) \text{ or } (\theta_i > T) \end{cases}$$

Below, I describe the construction of firm n's payoff function, the one for firm s is symmetric.

If $b_n < b_s$ and $(\theta_n + \theta_s \leq k_n)$ and $(\theta_s \leq T)$, firm n submits the lowest bid, it has enough capacity to satisfy the total demand and the transmission line is not congested, therefore it sets the price in the auction and its payoff is $\pi_n(b;\theta,T) = [b_n - c_n](\theta_n + \theta_s)$. If $b_n < b_s$ and $((\theta_n + \theta_s > k_n)$ or $(\theta_s > T))$ and $(\theta_n < k_n)$, firm n submits the lowest bid, but it has not enough capacity to satisfy the total demand or the demand in the South is higher than the transmission capacity, therefore, firm s sets the price in the auction, firm n satisfies the demand in the North, θ_n, at price b_n and sells the rest of its capacity, $k_n - \theta_n$, up to the transmission line capacity, T, into market South at price b_s, its payoff is $\pi_n(b;\theta,T) = [b_n - c_n](\theta_n) + [b_s - c_n]min\,\{T, k_n - \theta_n\}$. If $b_n < b_s$ and $((\theta_n + \theta_s > k_n)$ or $(\theta_s > T))$ and $(\theta_n > k_n)$, firm n submits the lowest bid, but it has not enough capacity to satisfy the total demand, therefore, firm s sets the price in the auction, firm n satisfies the demand in the North up to its capacity, therefore, its payoff is $\pi_n(b;\theta,T) = [b_n - c_n]k_n$. Finally, when firm n submits the highest bid in the auction, firm s has not enough capacity to satisfy the total demand or the transmission line is congested, firm n sets the price in the auction and satisfies the residual demand, its payoff is $\pi_n(b;\theta,T) = [b_n - c_n]max\,\{0, \theta_n - T, \theta_n + \theta_s - k_s\}$.

3.2 Equilibrium analysis

I this section, I will characterize the equilibrium when the financial transmission rights are assigned to the firm that submits the lowest bid in the spot electricity auction. As in the previous section, I will start proving, that in general, a pure strategy equilibrium does not exist.

Proposition 3. When the realization of demands (θ_s, θ_n) belongs to area A, the equilibrium will be in pure strategies. When the realization of demands (θ_s, θ_n) belongs to area $B, B1, B2, C1$ or $C2$, there no exist neither a symmetric nor an asymmetric pure strategy equilibrium (figure 4).

The proof use the same logic that the one used in proposition one $\square$

As I have explained in the previous section, the payoff functions satisfy the properties that guarantee the existence of a mixed strategy equilibrium.

Lemma 2. In a mixed strategy equilibrium none firm submits a bid lower than bid[14] $(\underline{b}_i^{II})$ such that

$$\underline{b}_i^{II} min\left\{\theta_i, k_i\right\} + E(b_j \mid b_j \geq \underline{b}^{II}) max\left\{0, min\left\{T, k_i - \theta_i\right\}\right\} = \\ Pmax\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\}.$$

Moreover, the support for the mixed strategy equilibrium for both firms is $S^{II} = \left[max\left\{\underline{b}_i^{II}, \underline{b}_j^{II}\right\}, P\right]$. Furthermore, $\underline{b}_i^{II} \leq \underline{b}_i^{I}, \forall i = n, s$.

The proof of the two first statements of the lemma use same logic that the one used in lemma one.

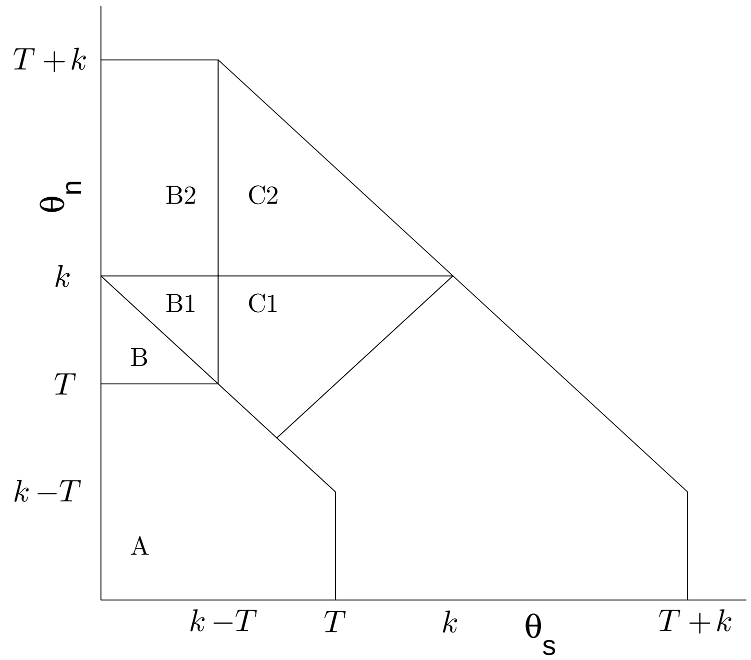
The proof for the latest statement is as follows. When $\theta_i > k$, the lower bound of the support is defined by $\underline{b}_i k = Pmax\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\}$, both for the model that assigns the transmission rights to the grid operator and for the model that assigns the transmission rights to the firm that submits the lowest bid in the spot electricity auction. Hence, if $\theta_i < k$, $\underline{b}_i^{II} = \underline{b}_i^{I}, \forall i = n, s$. Instead, when $\theta_i < k$, the lower bound of the support is defined by $\underline{b}_i^{I} k = Pmax\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\}$ for the model that assigns the transmission rights to the grid operator and by $\underline{b}_i^{II}\theta_i + E(b_j \mid b_j \geq \underline{b}^{II})(k_i - \theta_i) = Pmax\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\}$ for the model that assigns the transmission rights to the firm that submits the lowest bid in the spot electricity auction. The right hand side of the equations is equal in both models. However, in the latest model, the firm that submits the lowest bid in the auction sells part of its capacity in the other market at a higher price. Therefore, it can lower its bid, b_i^{II}, because it compensate a reduction in profit in its own market with an increase in profit in the other market. Hence, if $\theta_i < k$, $\underline{b}_i^{II} < \underline{b}_i^{I}, \forall i = n, s$ $\square$

Using this ancillary result, I can present the main result of this section.

Proposition 4. The characterization of the equilibrium strategies fall into one of the next two categories (figure 4).

[14] The superscript II denote support, profits, prices and strategies referred to model II, the one that assigns the transmission rights to the firm that submits the lowest bid in the spot electricity auction.

Figure 4: Equilibrium areas ($k_n = k_s = 60, T = 40, c_n = c_s = 0, P = 7$)



i Area A (low demand). The equilibrium strategies pair is pure and equal to $b_n = b_s = c = 0$. The equilibrium payoff is zero for both firms. No electricity flows through the grid.

ii Area $B, B1, B2, C1, C2$ (high demand). The equilibrium strategies pair is in mixed strategies[15].
The mixed strategy equilibrium support is defined by lemma 1

$$S^{II} = \left[max\left\{\underline{b}_i^{II}, \underline{b}_j^{II}\right\}, P\right]$$

The cumulative distribution function is defined by

$$F_i^{II}(b) = \frac{L_j^{II}(b) - L_j^{II}(\underline{b})}{L_j^{II}(b) - H_j^{II}(b)} \forall i, j$$

The payoff function is

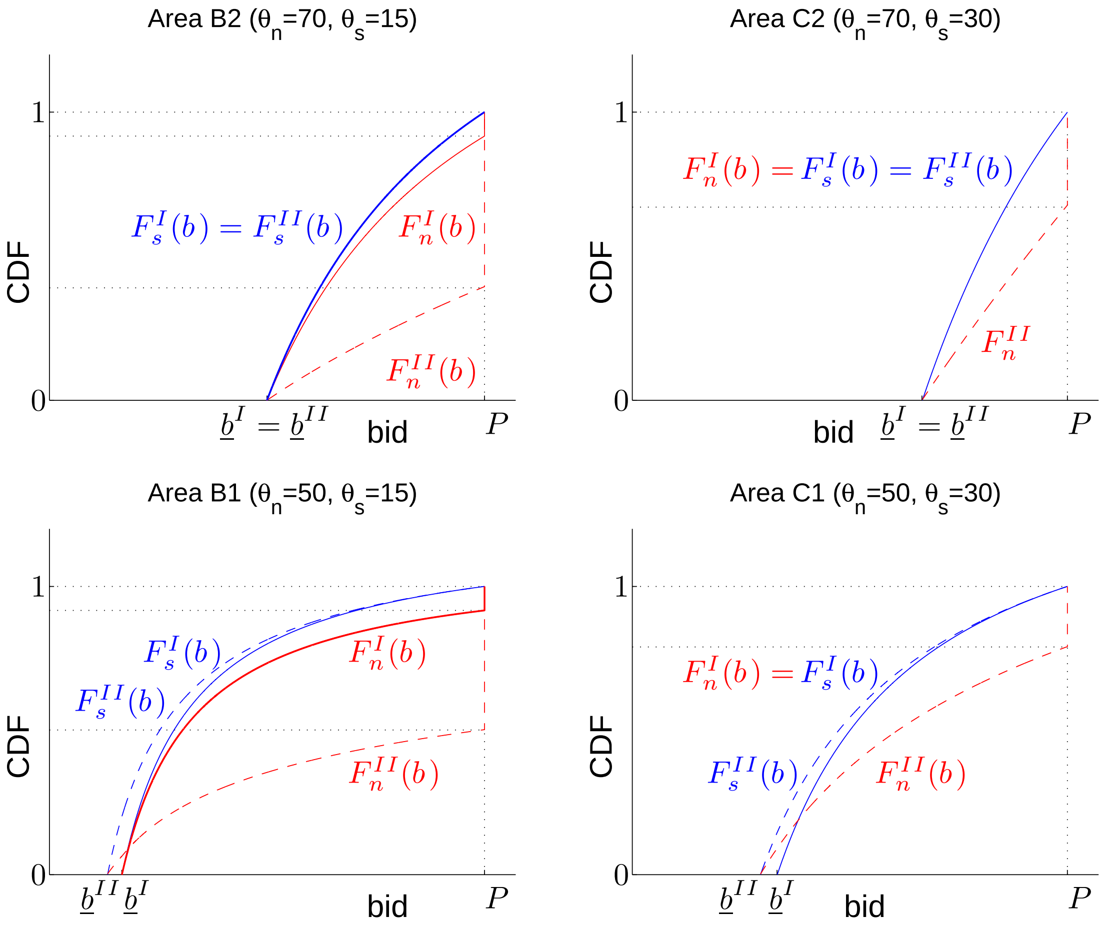
$$\overline{\pi}_i^{II} = \underline{b}^{II} min\left\{\theta_i + k\right\} + E(b_j \mid b_j \geq b) max\left\{0, min\left\{T, k - \theta_i\right\}\right\} \forall i, j$$

The expected bid, $E(b_i^{II})$, can only be calculated using an approximation.

The mixed strategies equilibrium characterized in proposition four have been represented graphically in figure 5. As can be observed, the cumulative distribution function of firm s is more concave for any realization of demand over the 45 degree line, therefore

[15]It is important to remark that, even when the equilibrium is guaranteed because the payoff function satisfies the properties that guarantee the existence of the equilibrium, the cumulative distribution function and so the support, the payoff function and the expected bid have not a close form solution. However, I have proposed and algorithm that gives me the opportunity to work out an approximation of the cumulative distribution function. In the annex, I present evidence of the algorithm convergence.

Figure 5: Equilibrium Strategies ($k_n = k_s = 60, T = 40, c_n = c_s = 0, P = 7$)



it submits lower bids with higher probability than firm n. Moreover, the cumulative distribution function of firm s, $F_s^{II}(b)$, is continuous at the upper bound of the support; by the contrary, the cumulative distribution function of firm n, $F_n^{II}(b)$, is discontinuous at the upper bound of the support, P, therefore it assigns a positive probability to the maximum price allowed by the auctioneer. Moreover, $F_n^{II}(b) \leq F_s^{II}(b) \forall b \in S^{II}$. Hence $F_s^{II}(b)$ stochastic dominate $F_n^{II}(b)$ and so the expected value of bids in market North, $E(\underline{b}_n^{II})$, is greater than the expected value of bids in market South, $E(\underline{b}_s^{II})$, for any realization of demand over the 45 degree line.

As can be observed in figure 5, when the transmission rights are assigned to the firm that submits the lowest bid in the spot electricity auction, the strategies change completely with respect to the model in which the transmission rights are assigned to the grid operator. Therefore, a deeper analysis is required to stablish comparisons between both models. Proposition five do it so.

Proposition 5. If the financial transmission rights are assigned to the firm that submits the lowest bid in the auction, set up II, instead of been assigned to the transmission grid operator, set up I.

i In Areas $B2$, $C2$ (figure 4). The lower bound of the strategies support does not

Table 3: $(k_n = k_s = 60, T = 40, c_n = c_s = 0, P = 7)$

Model	Area	$\underline{b}$	$F_n(P)$	$E(b_n)$	$E(b_s)$	$\overline{\pi}_n$	$\overline{\pi}_s$
	Area B1						
I	$(\theta_n = 50, \theta_s = 15)$	1.17	0.91	2.88	2.51	70	64.17
II	$(\theta_n = 50, \theta_s = 15)$	0.93	0.50	4.89	2.32	70	230.4
	Area B2						
I	$(\theta_n = 70, \theta_s = 10)$	3.5	0.83	5.03	4.85	210	175
II	$(\theta_n = 70, \theta_s = 15)$	3.5	0.91	5.21	4.85	210	192.5
	Area C1						
I	$(\theta_n = 50, \theta_s = 30)$	2.33	1	3.84	3.84	140	140
II	$(\theta_n = 50, \theta_s = 30)$	2.06	0.79	4.56	3.68	140	247.2
	Area C2						
I	$(\theta_n = 70, \theta_s = 30)$	4.67	1	5.67	5.67	280	280
II	$(\theta_n = 70, \theta_s = 30)$	4.67	0.67	6.17	5.67	280	355.2

The values of the variables for model II are worked out using the algorithm that I have described in the annex.

change (figure 5). The cumulative distribution function of firm s does not change and the cumulative distribution function of firm n is lower for all the bids in the support, i.e., $F_n^I(b)$ stochastic dominates $F_n^{II}(b)$. Moreover, the expected value of the bids in market North increases, the expected value of the bids in market South decreases, the expected payoff of firm n does not change and the expected payoff of firm s increases.

ii In Areas $B1$, $C1$ (figure 4). The lower bound of the strategies support decreases (figure 5). The cumulative distribution function of firm s is higher for all the bids in the support, i.e., $F_s^{II}(b)$ stochastic dominates $F_s^I(b)$. The cumulative distribution function of firm n $F_n^{II}(b)$ is higher than $F_n^I(b)$ for low bids, but no rank between $F_n^{II}(b)$ and $F_n^I(b)$ can be established for high bids, therefore none stochastic dominance rank of the cumulative distribution function can be made for firm n. Moreover, the expected value of the bids in market South decreases, the expected payoff of firm n does not change and no rank can be established neither on the expected value of the bids in market North, nor on the expected payoff of firm s.

I will start comparing the equilibrium when the transmission rights are assigned to the grid operator and when the transmission rights are assigned to the firm that submits the lowest bid in the spot electricity auction in area $C2$ (the same logic applies in area $B2$, but in area $B2$, the asymmetry in the access to the demand induced by the transmission constraint reinforces the results). When the realization of demand is such that $\theta_n > k$, as I have explained in lemma two, firm n can not sell any electricity in market South in case of submit the lowest bid in the auction, then its payoff function is identical in both models, and so the lower bound of the support, $\underline{b}^{II} = \underline{b}^I$, the expected payoff of firm n, $\overline{\pi}_n^{II} = \overline{\pi}_n^I$, and the expected value of bids in market South, $E(b_s^{II}) = E(b_s^I)$. The probability that firm n assigns to the highest bid allowed by the auctioneer increases when the transmission rights are assigned to the firm that submits the lowest bid in the auction, $F_n^{II}(P) \leq F_n^I(P)$. As I have explained in corollary one, the probability that firm n assigns to the maximum bid allowed by the auctioneer represents the opportunity cost of submit high bids for firm s. When the transmission rights are assigned to the firm that submits the lowest bid in the auction, the opportunity cost of submit high bids for firm s increases because in case of be dispatched last, the residual demand that it faces is very low, but also because it losses the transmission rights and the arbitrage profits derived from its hold; therefore firm n must increases the probability that assigns to high bids to made firm s stay in equilibrium. Hence, the expected value of bids in market North increases and so the expected payoff of firm s. As can be observed in table 3 when the transmission rights are assigned to the firm that submits the lowest bid in the auction, the change induced on the expected value of bids and on the expected payoff of the firms can be quite big, and so welfare. In particular, consumers in market North are worse, firm n and consumers in market South are equal and firm s is better.

I will continue comparing the equilibrium when the transmission rights are assigned to the grid operator and when the transmission rights are assigned to the firm that submits the lowest bid in the spot electricity auction in area $C1$ (the same logic applies in area $B1$, but in area $B1$, the asymmetry in the access to the demand induced by the transmission constraint reinforces the results). The residual demand that firm n faces does not change, and so its expected profit, $\overline{\pi}_n^{II} = \overline{\pi}_n^{I}$. However, as I have shown in lemma two, the lower bound of the support decreases. Moreover, $F_s^{II}(b) \geq F_s^{I}(b) \forall b \in S^{II}$, then $F_s^{II}(b)$ stochastic dominates $F_s^{I}(b)$, hence $E(b_s^{II}) \leq E(b_s^{I})$. By the contrary, it can not be established any rank between $F_n^{II}(b)$ and $F_n^{I}(b)$, and so between the expected bid value of firm n. However, using the algorithm that I have described in the annex, I can conclude that when the transmission rights are assigned to the firm that submits the lowest bid in the auction, the expected bid value in market North increases. As can be observed in table 3, when the transmission rights are assigned to the firm that submits the lowest bid in the auction, the change induced on the expected value of bids and on the expected payoff of the firms can be quite big, and so welfare. In particular, consumers in market North are worse, firm n is equal and firm s and consumers in market South are better.

When the transmission rights are assigned to the firm that submits the lowest bid in the auction, the unique symmetric equilibrium will be when the realization of the demand belongs to the 45 degree line. The possibility to obtain arbitrage profits made vanish the symmetry mixed strategies equilibrium for any other realization of the demand. In particular, in areas $C1$ and $C2$, the asymmetry in the equilibrium is due to the arbitrage profits and in areas $B1$ and $B2$, the asymmetry in equilibrium is due to the arbitrage profits and to the asymmetry in the access to the demand induced by the transmission constraint. Therefore, I can disentangle the effect that the arbitrage profits and the transmission constraint have on equilibrium for any realization of the demand. Moreover, the change on equilibrium induced by a change in the assignment of the transmission rights have a huge impact on welfare. In particular, when the realization of demand is over the 45 degree line, the payoff of firm n does not change, the payoff of firm s increases, the welfare of consumers in market South improves slightly and the welfare of consumers in market North worsens substantially. If the realization of demand is uniformly distributed, i.e., the probability that the realization of the demand is above the 45 degree line is one-half, then assign the transmission rights to the firm that submits the lowest bid in the auction reduces the welfare of consumers and increases the expected payoff of firms.

4 Conclusions

In this paper I have analyzed the effect that different designs in the access to financial transmission rights has on spot electricity auctions. In particular, I have characterized the equilibrium in the spot electricity market when transmission rights are assigned to the grid operator and when the transmission rights are assigned to the firm that submits the lowest bid in the spot electricity auction. The model that I have presented in this paper, in contrast with Borenstein, Bushnell and Stoft, (2000) characterizes the equilibrium for any transmission capacity. I have found that an increase of transmission capacity not only increases the competition between markets, but also induces changes on the payoff functions that could facilitate the entry of new firms in the long term and so increase competition within a market. Moreover, the results that I have obtained complements the literature that characterize the equilibrium in a Bertrand model when some type of asymmetry is introduced. When the transmission rights are assigned to the firm that submits the lowest bid in the spot electricity auction; if the realization of demand is uniformly distributed, i.e., the probability that the realization of the demand is above the 45 degree line is one-half, then assign the transmission rights to the firms that submits the lowest bid in the auction reduces the welfare of consumers and increases the expected payoff of firms.

My analysis, however, does not take into consideration other possible assignments of transmission rights. In particular, further analysis is necessary to characterize the equilibrium when the transmission and the spot electricity markets run sequentially; first the firms acquire transmission rights in the transmission rights market and later they compete in the spot electricity market.

Finally, the model that I have developed gives us the opportunity to use the Industrial Organization tools to analyze competition in electricity markets. In the third chapter of my thesis, I will analyze the effect that an increase in transmission capacity has not in the competition between markets, but within a single market.

Annex

Proposition 2.

Proof:

The model presented in section two satisfies the properties established by Dasgupta and Maskin (1986), that guarantee that a mixed strategy equilibrium exists. However, Dasgupta and Maskin (1986) did not provide an algorithm to work out the equilibrium. Nevertheless, using the approach proposed by (Karlin, 1959; Beckmann, 1965; Shapley, 1957; Shilony, 1977; Varian, 1980; Deneckere and Kovenock, 1986; Osborne and Pitchik, 1986; Fabra et al., 2006), the equilibrium characterization is guaranteed by construction. I will use the approach proposed by this branch of the literature to work out the mixed strategy equilibrium.

First step, the payoff function for any firm is:

$$\begin{aligned}\pi_i^I(b) &= b\left[F_j^I(b)max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}+(1-F_j^I(b))min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}\right]=\\ &= -bF_j^I(b)\left[min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right]+\\ &\quad bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\}\end{aligned} \tag{8}$$

Second step, $\pi_i^I(b)=\overline{\pi}_i^I\forall b\in S_i, i=n,s$, where S_i is the support of the mixed strategies. Then,

$$\begin{aligned}\overline{\pi}_i^I &= -bF_j^I(b)\left[min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right]+\\ &\quad bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\}\Rightarrow\\ F_j^I(b) &= \frac{bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-\overline{\pi}_i^I}{b\left[min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right]}\end{aligned} \tag{9}$$

Third step, at $\underline{b}^I$, $F_i^I(\underline{b}^I)=0\forall i=n,s$. Then,

$$\overline{\pi}_i^I = \underline{b}^I min\left\{\theta_i+\theta_j,\theta_i+T,k\right\} \tag{10}$$

Fourth step, Plug in 10 into 9, I obtain the mixed strategies for both firms.

$$\begin{aligned}F_j^I(b) &= \frac{bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-\underline{b}^I min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}}{b\left[min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right]}=\\ &= \frac{L_i^I(b)-L_i^I(\underline{b})}{L_i^I(b)-H_i^I(b)}\forall i=n,s\end{aligned} \tag{11}$$

It is easy to verify that equation $F_j^I(b)\forall i,j$ is indeed a cumulative distribution function. First, in the third step, I have established that $F_j^I(\underline{b})=0$. Second, $F_j^I(b)$ is an increasing function in b. At $\underline{b}$, $L_i^I(\underline{b})=H_i^I(b)$, for any $b>\underline{b}$, $L_i^I(\underline{b})<H_i^I(b)$; moreover, $\dfrac{\partial L_i^I(b)}{\partial b}>0$, $\dfrac{\partial L_i^I(\underline{b})}{\partial b}=0$ and $\dfrac{\partial H_i^I(b)}{\partial b}>0$, therefore, $\dfrac{\partial\left(L_i^I(b)-L_i^I(\underline{b})\right)}{\partial b}>\dfrac{\partial\left(L_i^I(b)-H_i^I(b)\right)}{\partial b}$. Third,

$F_j^I(b) \leq 1 \forall b \in S_i$. Finally, $F_j^I(b)$ is continuous in the support because $L_i^I(b) - L_i^I(\underline{b})$ and $L_i^I(b) - H_i^I(b)$ are continuous functions in the support.

To conclude the proof, I will work out the support, the cumulative distribution function, the expected bid and the expected payoff for any realization of the demand in figure 2.

First, I work out the support of the cumulative distribution function in each area.

In the border between areas $B1 - C1$ and $B2 - C2$, $\theta_s = k - T$. In these borders, $\underline{b}_n^I$ solves $\underline{b}_n^I min\left\{\theta_n + \theta_s, \theta_n + T, k\right\} = Pmax\left\{0, \theta_n - T, \theta_s + \theta_n - k\right\}$, therefore $\underline{b}_n^I = \dfrac{P(\theta_n - T)}{k}$ and $\underline{b}_s^I$ solves $\underline{b}_s^I min\left\{\theta_n + \theta_s, \theta_s + T, k\right\} = Pmax\left\{0, \theta_s - T, \theta_s + \theta_n - k\right\}$, therefore $\underline{b}_s^I = \dfrac{P(\theta_n + \theta_s - k)}{\theta_s + T}$. Plug in the value of θ_s in the border between these areas into $\underline{b}_s^I$ formula, I obtain $\underline{b}_s^I = \dfrac{P(\theta_n + k - T - k)}{k - T + T} = \dfrac{P(\theta_n - T)}{k} = \underline{b}_n^I$. Therefore, in the border between these areas, $\underline{b}_s^I = \underline{b}_n^I = \dfrac{P(\theta_n - T)}{k}$.

In areas B, $B1$ and $B2$, $\underline{b}_n^I > \underline{b}_s^I$. Taking partial derivatives $\dfrac{\partial \underline{b}_n^I}{\partial \theta_s} = 0$ and $\dfrac{\partial \underline{b}_s^I}{\partial \theta_s} = \dfrac{P(k + T - \theta_n)}{(\theta_s + T)^2} > 0$. Therefore, in areas B, $B1$ and $B2$, $\underline{b}_n^I > \underline{b}_s^I$. Hence, in areas B, $B1$ and $B2$, $S^I = \left[max\left\{\underline{b}_n^I, \underline{b}_s^I\right\}, P\right] = \left[\underline{b}_n^I, P\right] = \left[\dfrac{P(\theta_n - T)}{k}, P\right]$.

In areas $C1$, $C2$ and $C3$ it is straight forward to check that $\underline{b}_s^I = \underline{b}_n^I = \dfrac{P(\theta_s + \theta_n - k)}{k}$. Therefore, in areas $C1$ and $C2$, $S^I = \left[max\left\{\underline{b}_n^I, \underline{b}_s^I\right\}, P\right] = \left[\underline{b}_n^I, P\right] = \left[\dfrac{P(\theta_s + \theta_n - k)}{k}, P\right]$.

Second, I work out the mixed strategies for both firms.

Using equation 11. The mixed strategies in area B is defined by:

$$F_s^I(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{\theta_n + \theta_s}{\theta_s + T} \dfrac{b - \underline{b}^I}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$
$$F_n^I(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{\theta_s + T}{\theta_s + T} \dfrac{b - \underline{b}^I}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases} \tag{12}$$

Other useful results in area B.

$$
\begin{aligned}
F_s^I(P) &= \frac{\theta_n+\theta_s}{\theta_s+T}\frac{P-\dfrac{P(\theta_n-T)}{\theta_n+\theta_s}}{P}=1 \\
F_n^I(P) &= \frac{P-\dfrac{P(\theta_n-T)}{\theta_n+\theta_s}}{P}=\frac{\theta_s+T}{\theta_n+\theta_s}<1
\end{aligned}
$$

Using equation 11. The mixed strategies in areas $B1$ and $B2$ is defined by:

$$
F_s^I(b)=\begin{cases} 0 & \text{if } b<\underline{b} \\ \dfrac{k}{T+k-\theta_n}\dfrac{b-\underline{b}^I}{b} & \text{if } b\in(\underline{b},P) \\ 1 & \text{if } b=P \end{cases}
$$

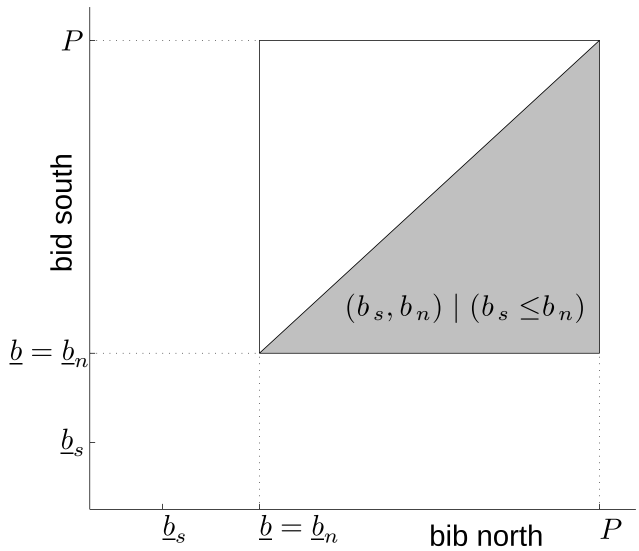
$$
F_n^I(b)=\begin{cases} 0 & \text{if } b<\underline{b} \\ \dfrac{\theta_s+T}{T+k-\theta_n}\dfrac{b-\underline{b}^I}{b} & \text{if } b\in(\underline{b},P) \\ 1 & \text{if } b=P \end{cases} \tag{13}
$$

Other useful results in areas $B1$ and $B2$.

$$
\begin{aligned}
F_s^I(P) &= \frac{k}{T+k-\theta_n}\frac{P-\dfrac{P(\theta_n-T)}{k}}{P}=1 \\
F_n^I(P) &= \frac{\theta_s+T}{T+k-\theta_n}\frac{P-\dfrac{P(\theta_n-T)}{k}}{P}=\frac{\theta_s+T}{k}<1
\end{aligned}
$$

$Prob(b_s<b_n)$ is determined by the integral of the joint distribution in the grey area in figure 6.

Figure 6: $(b_s, b_n) \mid b_s < b_n$



$$
\begin{aligned}
prob(b_s < b_n) &= \left(\int_{\underline{b}^I}^{P} f_n^I(b_n)\left(\int_0^{b_n} f_s^I(b_s)\partial b_s\right)\partial b_n\right) + F_s^I(P) - F_n^I(P) = \\
&= \int_{\underline{b}^I}^{P} f_n^I(b_n)F_s^I(b_n)\partial b_n + F_s^I(P) - F_n^I(P) = \\
&= \int_{\underline{b}^I}^{P} \frac{\theta_s + T}{k + T - \theta_n}\frac{\underline{b}^I}{b^2}\frac{k}{k + T - \theta_n}\frac{b - \underline{b}^I}{b}\partial b + 1 - \frac{\theta_s + T}{k} \\
&= \frac{(\theta_s + T)\underline{b}^I k}{(k + T - \theta_n)^2}\left[\int_{\underline{b}^I}^{P}\frac{\partial b}{b^2} - \int_{\underline{b}^I}^{P}\frac{\underline{b}^I}{b^3}\partial b\right] + 1 - \frac{\theta_s + T}{k} = \\
&= \frac{(\theta_s + T)\underline{b}^I k}{(k + T - \theta_n)^2}\left[\frac{\underline{b}^I}{4b^4} - \frac{1}{3b^3}\right]_{\underline{b}^I}^{P} + 1 - \frac{\theta_s + T}{k} = \\
&= \frac{(\theta_s + T)\underline{b}^I k}{(k + T - \theta_n)^2}\left[\frac{3\underline{b}^I - 4b}{12b^4}\right]_{\underline{b}^I}^{P} + 1 - \frac{\theta_s + T}{k} = \\
&= \frac{(\theta_s + T)\underline{b}^I k}{(k + T - \theta_n)^2}\left[\frac{3\underline{b}^I - 4P}{12P^4} + \frac{1}{12\underline{b}^3}\right]_{\underline{b}^I}^{P} + 1 - \frac{\theta_s + T}{k} = \\
&= \frac{(\theta_s + T)\underline{b}^I k}{(k + T - \theta_n)^2}\frac{(3\underline{b}^I - 4P)\underline{b}^3 + P^4}{12P^4\underline{b}^3} + 1 - \frac{\theta_s + T}{k}
\end{aligned}
$$

Using equation 11. The mixed strategies in areas $C1$, $C2$ and $C3$ is defined by:

$$
F_s^I(b) = F_n^I(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{k}{2k - \theta_n - \theta_s}\dfrac{b - \underline{b}^I}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases} \tag{14}
$$

Other useful results in areas $C1$, $C2$ and $C3$.

$$F_s^I(P) = \frac{k}{2k-\theta_n-\theta_s}\frac{P-\dfrac{P(\theta_n+\theta_s-k)}{k}}{P} = 1$$

$$Prob(b_s < b_n) = \frac{1}{2}.$$

Third, I work out the expected bid for both firms.

Using equation 12. The expected bid in Area B is:

$$\begin{aligned} f_s^I(b) &= \frac{\partial F_s^I(b)}{\partial b} = \frac{\theta_n+\theta_s}{\theta_s+T}\frac{\underline{b}^I}{b^2} \\ f_n^I(b) &= \frac{\partial F_n^I(b)}{\partial b} = \frac{\underline{b}^I}{b^2} \end{aligned}$$

$$\begin{aligned} E(b_s^I) &= \int_{\underline{b}^I}^P b f_s^I(b_s)\partial b = \int_{\underline{b}^I}^P \frac{\theta_n+\theta_s}{\theta_s+T}\frac{\underline{b}^I}{b}\partial b = \frac{\theta_n+\theta_s}{\theta_s+T}\underline{b}^I\left[ln(b)\right]_{\underline{b}^I}^P \\ E(b_n^I) &= \int_{\underline{b}^I}^P b f_n^I(b_n)\partial b = \int_{\underline{b}^I}^P \frac{\underline{b}^I}{b^2}\partial b = \underline{b}^I\left[ln(b)\right]_{\underline{b}^I}^P + \left(1-F_n^I(P)\right)P \end{aligned} \tag{15}$$

Using equation 13. The expected bid for both firms in areas $B1$ and $B2$ is:

$$\begin{aligned} f_s^I(b) &= \frac{\partial F_s^I(b)}{\partial b} = \frac{k}{T+k-\theta_n}\frac{\underline{b}^I}{b^2} \\ f_n^I(b) &= \frac{\partial F_n^I(b)}{\partial b} = \frac{\theta_s+T}{T+k-\theta_n}\frac{\underline{b}^I}{b^2} \end{aligned}$$

$$\begin{aligned} E(b_s^I) &= \int_{\underline{b}^I}^P b f_s^I(b_s)\partial b = \int_{\underline{b}^I}^P \frac{k}{T+k-\theta_n}\frac{\underline{b}^I}{b}\partial b = \frac{k}{T+k-\theta_n}\underline{b}^I\left[ln(b)\right]_{\underline{b}^I}^P \\ E(b_n^I) &= \int_{\underline{b}^I}^P b f_n^I(b_n)\partial b = \int_{\underline{b}^I}^P \frac{\theta_s+T}{T+k-\theta_n}\frac{\underline{b}^I}{b}\partial b = \\ & \frac{\theta_s+T}{T+k-\theta_n}\underline{b}^I\left[ln(b)\right]_{\underline{b}^I}^P + \left(1-F_n^I(P)\right)P \end{aligned} \tag{16}$$

Using equation 14. The expected bid for both firms in areas $C1$ and $C2$ is:

$$f_s^I(b) = f_s^I(b) = \frac{\partial F_s^I(b)}{\partial b} = \frac{k}{2k-\theta_n-\theta_s}\frac{\underline{b}^I}{b^2}$$

$$E(b_s^I) = E(b_n^I) = \int_{\underline{b}^I}^P b f_s^I(b_s)\partial b = \int_{\underline{b}^I}^P \frac{k}{2k-\theta_n-\theta_s}\frac{\underline{b}^I}{b}\partial b = \frac{k}{2k-\theta_n-\theta_s}\underline{b}^I\left[ln(b)\right]_{\underline{b}^I}^P \tag{17}$$

Fourth, I work out the expected payoff for both firms.

Using equation 10. The payoff function in areas B is:

$$\begin{aligned}\overline{\pi}^I_n &= \underline{b}^I(\theta_n+\theta_s)\\ \overline{\pi}^I_s &= \underline{b}^I(\theta_s+T)\end{aligned} \tag{18}$$

Using equation 10. The payoff function in areas $B1$ and $B2$ is:

$$\begin{aligned}\overline{\pi}^I_n &= \underline{b}^I k\\ \overline{\pi}^I_s &= \underline{b}^I(\theta_s+T)\end{aligned} \tag{19}$$

Using equation 10. The payoff function in areas $C1$, $C2$ and $C3$ is:

$$\overline{\pi}^I_n = \overline{\pi}^I_s = \underline{b}^I k \tag{20}$$

Corollary 1.

Proof:

Areas $B1$ and $B2$.

$$\frac{\partial \underline{b}^I}{\partial\theta_s} = 0$$

$$\frac{\partial F^I_n(P)}{\partial\theta_s} = \frac{1}{k} > 0$$

$$\begin{aligned}\frac{\partial E(b^I_n)}{\partial\theta_s} &= \frac{1}{k+T-\theta_n}\frac{P(\theta_n-T)}{k} ln\left(\frac{P}{\underline{b}^I}\right)\frac{P(k-\theta_s-T)}{k} + \\ &\quad \frac{\theta_s+T}{k+T-\theta_n}\frac{P(\theta_n-T)}{k} ln\left(\frac{P}{\underline{b}^I}\right)\frac{-P}{k} = \\ &= \frac{P^2(\theta_n-T)}{k^2(k+T-\theta_n)} ln\left(\frac{P}{\underline{b}^I}\right)(k-2(\theta_s-T)) < 0 \Leftrightarrow k < 2(\theta_s-T)\end{aligned}$$

$$\frac{\partial E(b^I_s)}{\partial\theta_s} = 0$$

$$\frac{\partial \overline{\pi}^I_n}{\partial\theta_s} = 0$$

$$\frac{\partial \overline{\pi}^I_s}{\partial\theta_s} = \underline{b}^I > 0$$

Areas $C1$ and $C2$.

$$\frac{\partial \underline{b}^I}{\partial \theta_s} = \frac{P}{k} > 0$$

$$\frac{\partial F_n^I(P)}{\partial \theta_s} = 0$$

$$\begin{aligned}\frac{\partial E(b_n^I)}{\partial \theta_s} = \frac{\partial E(b_s^I)}{\partial \theta_s} &= \frac{k}{(2k-\theta_n-\theta_s)^2}\frac{P(\theta_n+\theta_s-k)}{k} ln\left(\frac{P}{\underline{b}^I}\right) + \\ & \frac{k}{(2k-\theta_n-\theta_s)}\frac{P}{k} ln\left(\frac{P}{\underline{b}^I}\right) + \\ & \frac{k}{(2k-\theta_n-\theta_s)}\frac{P(\theta_n+\theta_s-k)}{k}\frac{\underline{b}^I}{P}\frac{P}{k} > 0\end{aligned}$$

$$\frac{\partial \overline{\pi}_n^I}{\partial \theta_s} = \frac{\partial \overline{\pi}_s^I}{\partial \theta_s} = P > 0$$

Corollary 2.

Proof:

Areas $B1$ and $B2$.

$$\frac{\partial \underline{b}^I}{\partial \theta_n} = \frac{P}{k} > 0$$

$$\frac{\partial F_n^I(P)}{\partial \theta_n} = 0$$

$$\begin{aligned}\frac{\partial E(b_n^I)}{\partial \theta_n} &= \frac{\theta_s+T}{(k+T-\theta_n)^2}\frac{P(\theta_n-T)}{k}\frac{P(k-\theta_s-T)}{k} ln\left(\frac{P}{\underline{b}^I}\right) + \\ & \frac{\theta_s+T}{k+T-\theta_n}\frac{P}{k}\frac{P(k-\theta_s-T)}{k} ln\left(\frac{P}{\underline{b}^I}\right) + \\ & \frac{\theta_s+T}{k+T-\theta_n}\frac{P(\theta_n-T)}{k}\frac{P(k-\theta_s-T)}{k}\frac{\underline{b}^I}{P}\frac{P}{k} > 0\end{aligned}$$

$$\begin{aligned}\frac{\partial E(b_s^I)}{\partial \theta_n} &= \frac{k}{(T+k-\theta_n)^2}\frac{P(\theta_n-T)}{k} ln\left(\frac{P}{\underline{b}^I}\right) + \\ & \frac{k}{T+k-\theta_n}\frac{P}{k} ln\left(\frac{P}{\underline{b}^I}\right) + \\ & \frac{k}{T+k-\theta_n}\frac{P(\theta_n-T)}{k}\frac{\underline{b}^I}{P}\frac{P}{k} > 0\end{aligned}$$

$$\frac{\partial \pi_n^I}{\partial \theta_n} = P > 0$$

$$\frac{\partial \overline{\pi}_s^I}{\partial \theta_n} = \frac{P}{k}(\theta_s + T) > 0$$

Areas $C1$ and $C2$.

$$\frac{\partial \underline{b}^I}{\partial \theta_n} = \frac{P}{k} > 0$$

$$\frac{\partial F_n^I(P)}{\partial \theta_n} = 0$$

$$\begin{aligned}\frac{\partial E(b_n^I)}{\partial \theta_n} = \frac{\partial E(b_s^I)}{\partial \theta_s} &= \frac{k}{(2k - \theta_n - \theta_s)^2} \frac{P(\theta_n + \theta_s - k)}{k} ln\left(\frac{P}{\underline{b}^I}\right) + \\ &\quad \frac{k}{2k - \theta_n - \theta_s} \frac{P}{k} ln\left(\frac{P}{\underline{b}^I}\right) + \\ &\quad \frac{k}{2k - \theta_n - \theta_s} \frac{P(\theta_n + \theta_s - k)}{k} \frac{\underline{b}^I}{P} \frac{P}{k} > 0\end{aligned}$$

$$\frac{\partial \overline{\pi}_n^I}{\partial \theta_s} = \frac{\partial \overline{\pi}_s^I}{\partial \theta_s} = P > 0$$

Corollary 3.

Proof:

Areas $B1$ and $B2$.

$$\frac{\partial \underline{b}^I}{\partial T} = \frac{-P}{k} < 0$$

$$\frac{\partial F_n^I(P)}{\partial T} = \frac{1}{k} > 0$$

$$\begin{aligned}\frac{\partial E(b_n^I)}{\partial T} &= \frac{(k + T - \theta_n) - (\theta_s + T)}{(k + T - \theta_n)^2} \frac{P(\theta_n - T)}{k} ln\left(\frac{P}{\underline{b}^I}\right) \frac{P(k - \theta_s - T)}{k} + \\ &\quad \frac{(\theta_s + T)}{k + T - \theta_n} \frac{-P}{k} ln\left(\frac{P}{\underline{b}^I}\right) \frac{P(k - \theta_s - T)}{k} + \\ &\quad \frac{(\theta_s + T)}{k + T - \theta_n} \frac{P(\theta_n - T)}{k} \frac{\underline{b}^I}{P} \frac{-P}{k} \frac{P(k - \theta_s - T)}{k} + \\ &\quad \frac{(\theta_s + T)}{k + T - \theta_n} \frac{P(\theta_n - T)}{k} ln\left(\frac{P}{\underline{b}^I}\right) \frac{-P}{k} < 0\end{aligned}$$

$$\begin{aligned}\frac{\partial E(\underline{b}_s^I)}{\partial T} &= \frac{-k}{(T+k-\theta_n)^2}\frac{P(\theta_n-T)}{k}ln\left(\frac{P}{\underline{b}^I}\right)+\\ &\frac{k}{T+k-\theta_n}\frac{-P}{k}ln\left(\frac{P}{\underline{b}^I}\right)+\\ &\frac{k}{T+k-\theta_n}\frac{P(\theta_n-T}{k}\frac{\underline{b}^I}{P}\frac{-P}{k}<0\end{aligned}$$

$$\frac{\partial \overline{\pi}_n^I}{\partial T} = -P<0$$

$$\frac{\partial \overline{\pi}_s^I}{\partial T} = \frac{-P}{k}(\theta_s+T)+\frac{P(\theta_n-T)}{k} = \frac{P(\theta_n-2T-\theta_s)}{k}>0 \Leftrightarrow \theta_n>2T+\theta_s$$

Finally, I will work out the equilibrium when the transmission capacity is very low (the asymmetry in the access to the demand is high).

As can be observed in figure 7, when the transmission capacity decreases a new area emerge (area N). In the limit (when the transmission capacity collapses to zero), the areas A, B, $B1$, $B2$, $C1$, $C2$ and $C3$ disappear and N will be the unique area. Below, I will work out the equilibrium in this area.

First, the support of the cumulative distribution function. $\underline{b}_n^I = \frac{P(\theta_n-T)}{\theta_n+T}$ and $\underline{b}_s^I = \frac{P(\theta_s-T)}{\theta_s+T}$. When the realization of the demand is over the diagonal $(\theta_n,\theta_s)=(\theta+\epsilon,\theta)$. Therefore, $\underline{b}_n^I = \frac{P(\theta+\epsilon-T)}{\theta+\epsilon+T}$ and $\underline{b}_s^I = \frac{P(\theta-T)}{\theta+T}$. Hence, $\underline{b}_n^I > \underline{b}_s^I \iff (\theta+\epsilon-T)(\theta+T) > (\theta+\epsilon+T)(\theta-T) \iff 2T\epsilon>0$. Therefore, when the realization of the demand is over the diagonal, $\underline{b}_n^I > \underline{b}_s^I$. Hence,$S^I = \left[max\left\{\underline{b}_n^I,\underline{b}_s^I\right\},P\right] = \left[\underline{b}_n^I,P\right] = \left[\frac{P(\theta_n-T)}{\theta_n+T},P\right]$.

Second, I work out the mixed strategies equilibrium. I proceed in four different steps.

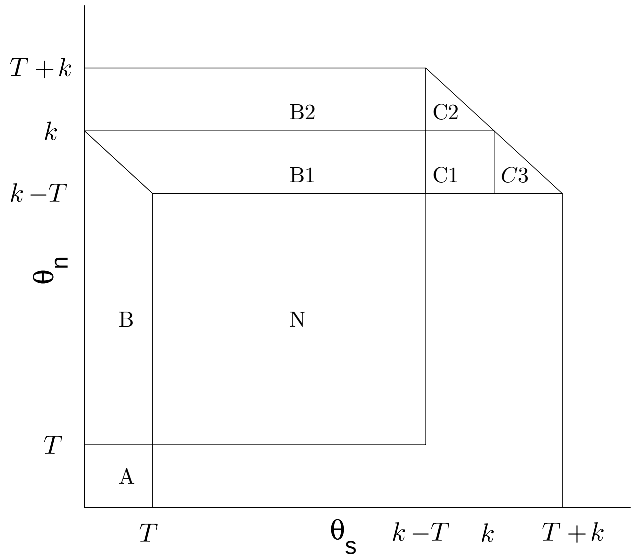
First step, the profits for both firms are:

$$\begin{aligned}\pi_n^I(b) &= b\left[F_s^I(b)max\left\{0,\theta_n-T,\theta_s+\theta_n-k\right\}+(1-F_s^I(b))min\left\{\theta_n+\theta_s,\theta_n+T,k\right\}\right]=\\ &= b\left[F_s^I(b)(\theta_n-T)+(1-F_s^I(b))(\theta_n+T)\right]\\ \pi_s^I(b) &= b\left[F_n^I(b)max\left\{0,\theta_s-T,\theta_s+\theta_n-k\right\}+(1-F_n^I(b))min\left\{\theta_n+\theta_s,\theta_s+T,k\right\}\right]=\\ &= b\left[F_n^I(b)(\theta_s-T)+(1-F_n^I(b))(\theta_s+T)\right]\end{aligned} \tag{21}$$

Second step, $\pi_i^I(b) = \overline{\pi}_i^I \forall b \in S_i, i=n,s$, where S_i is the support of the mixed strategies. Then,

$$\begin{aligned}\overline{\pi}_n^I &= -F_s^I(b)b(2T)+b(\theta_n+T) \Rightarrow F_s^I(b) = \frac{b(\theta_n+T)-\overline{\pi}_n^I}{b2T}\\ \overline{\pi}_s^I &= -F_n^I(b)b2T+b(\theta_s+T) \Rightarrow F_n^I(b) = \frac{b(\theta_s+T)-\overline{\pi}_n^I}{b2T}\end{aligned} \tag{22}$$

Figure 7: Equilibrium areas when transmission capacity is low ($k_n = k_s = 60, T = 10$)



Third step, at $\underline{b}^I$, $F_n^I(\underline{b}^I) = F_s^I(\underline{b}^I) = 0$. Then,

$$\begin{aligned} \overline{\pi}_n^I &= \underline{b}^I(\theta_n + T) \\ \overline{\pi}_s^I &= \underline{b}^I(\theta_s + T) \end{aligned} \tag{23}$$

Fourth step, plug in 23 into 22, we obtain the mixed strategies for both firms.

$$\begin{aligned} F_s^I(b) &= \frac{\theta_n + T}{2T}\frac{b - \underline{b}^I}{b} \\ F_n^I(b) &= \frac{\theta_s + T}{2T}\frac{b - \underline{b}^I}{b} \end{aligned} \tag{24}$$

Third, I work out the expected bid for both firms. Using 24. The expected bid in area N is:

$$\begin{aligned} f_s^I(b) &= \frac{\partial F_s^I(b)}{\partial b} = \frac{\theta_n + T}{2T}\frac{\underline{b}^I}{b^2} \\ f_n^I(b) &= \frac{\partial F_n^I(b)}{\partial b} = \frac{\theta_s + T}{2T}\frac{\underline{b}^I}{b^2} \end{aligned}$$

$$\begin{aligned} E(b_s^I) &= \int_{\underline{b}^I}^{P} b f_s^I(b_s)\partial b = \int_{\underline{b}^I}^{P} \frac{\theta_n + T}{2T}\frac{\underline{b}^I}{b}\partial b = \frac{\theta_n + T}{2T}\underline{b}^I \left[ln(b)\right]_{\underline{b}^I}^{P} \\ E(b_n^I) &= \int_{\underline{b}^I}^{P} b f_n^I(b_n)\partial b = \int_{\underline{b}^I}^{P} \frac{\theta_s + T}{2T}\frac{\underline{b}^I}{b}\partial b = \\ & \frac{\theta_s + T}{2T}\underline{b}^I \left[ln(b)\right]_{\underline{b}^I}^{P} + \left(1 - F_n^I(P)\right) P \end{aligned} \tag{25}$$

Fourth, the equilibrium pay off for both firms is determined by 23.

Proposition 4.

Proof:

Using the same steps that I have used in Proposition two, I will work out the equilibrium strategies.

First step, the payoff function for any firm is:

$$\begin{aligned}\pi_i^{II}(b) &= b\left[F_j^{II}(b)max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right] \\ &\quad +(1-F_j^{II}(b))\left[bmin\left\{\theta_i,k\right\}+E(b_j\mid b_j\geq b)max\left\{0,min\left\{T,k-\theta_i\right\}\right\}\right]= \\ &= -F_j^{II}(b)\left[bmin\left\{\theta_i,k\right\}+E(b_j\mid b_j\geq b)max\left\{0,min\left\{T,k-\theta_i\right\}\right\}\right] \\ &\quad -F_j^{II}(b)\left[-bmax\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right]+ \\ &\quad bmin\left\{\theta_i,k\right\}+E(b_j\mid b_j\geq b)max\left\{0,min\left\{T,k-\theta_i\right\}\right\}\end{aligned} \tag{26}$$

Second step, $\pi_i^{II}(b)=\overline{\pi}_i^{II}\forall b\in S_i, i=n,s$, where S_i is the support of the mixed strategies. Then,

$$\begin{aligned}\overline{\pi}_i^{II} &= -F_j^{II}(b)\left[bmin\left\{\theta_i,k\right\}+E(b_j\mid b_j\geq b)max\left\{0,min\left\{T,k-\theta_i\right\}\right\}\right] \\ &\quad -F_j^{II}(b)\left[-bmax\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right]+ \\ &\quad bmin\left\{\theta_i,k\right\}+E(b_j\mid b_j\geq b)max\left\{0,min\left\{T,k-\theta_i\right\}\right\}\Rightarrow \\ F_j^{II}(b) &= \frac{bmin\left\{\theta_i,k\right\}+E(b_j\mid b_j\geq b)max\left\{0,min\left\{T,k-\theta_i\right\}\right\}-\overline{\pi}_i^{II}}{L_i(b)-H_i(b)}\end{aligned} \tag{27}$$

Third step, at $\underline{b}^{II}$, $F_i^{II}(\underline{b}^{II})=0\forall i=n,s$. Then,

$$\overline{\pi}_i^{II} = \underline{b}^{II}min\left\{\theta_i,k\right\}+E(b_j\mid b_j\geq \underline{b}^{II})max\left\{0,min\left\{T,k-\theta_i\right\}\right\} \tag{28}$$

Fourth step, Plug in 28 into 27, we obtain the mixed strategies for both firms.

$$F_j^{II}(b) = \frac{L_i^{II}(b)-L_i^{II}(\underline{b}^{II})}{L_i^{II}(b)-H_i^{II}(b)}\forall i=n,s \tag{29}$$

It is easy to verify that equation $F_j^{II}(b)\forall i,j$ is indeed a cumulative distribution function. First, in the third step, I have established that $F_j^{II}(\underline{b}^{II})=0$. Second, $F_j^{II}(b)$ is an increasing function in b. At $\underline{b}^{II}$, $L_i^{II}(\underline{b}^{II})=H_i^{II}(b)$, for any $b>\underline{b}^{II}$, $L_i^{II}(\underline{b}^{II})<H_i^{II}(b)$; moreover, $\dfrac{\partial L_i^{II}(b)}{\partial b}>0$, $\dfrac{\partial L_i^{II}(\underline{b}^{II})}{\partial b}=0$ and $\dfrac{\partial H_i^{II}(b)}{\partial b}>0$, therefore, $\dfrac{\partial\left(L_i^{II}(b)-L_i^{II}(\underline{b}^{II})\right)}{\partial b}>\dfrac{\partial\left(L_i^{II}(b)-H_i^{II}(b)\right)}{\partial b}$. Third, $F_j^{II}(b)\leq 1\forall b\in S_i$. Finally, $F_j^{II}(b)$ is continuous in the support because $L_i^{II}(b)-L_i^{II}(\underline{b}^{II})$ and $L_i^{II}(b)-H_i^{II}(b)$ are continuous functions in the support.

To conclude the proof, I will work out the support, the cumulative distribution function, the expected bid and the expected payoff for any realization of the demand in figure 4.

First, I work out the support of the cumulative distribution function in each area.

In the diagonal, the payoff function for both firms is equal, therefore $\underline{b}_n^{II} = \underline{b}_s^{II}$.

In area B, $\underline{b}_n^{II} \geq \underline{b}_s^{II}$. The residual demand in Area B for the firm allocated in the South is zero, therefore $\underline{b}_s^{II} = 0$, moreover $\underline{b}_n^{II} = \dfrac{P(\theta_n - T)}{k}$. Therefore $S^{II} = \left[max\left\{\underline{b}_n^{II}, \underline{b}_s^{II}\right\}, P\right] = \left[\underline{b}_n^{II}, P\right] = \left[\dfrac{P(\theta_n - T)}{k}, P\right]$.

In areas $B2$ and $C2$, $\underline{b}_n^{II} = \underline{b}_s^{II}$. In areas $B2$ and $C2$, $\theta_n > k > \theta_s$. Therefore, using lemma two, $b_n^{II} = b_n^{I}$ and $b_s^{II} \leq b_s^{I}$. Using proposition two, I know that in areas $B2$ and $C2$ $b_n^{I} > b_s^{I}$. Hence $b_s^{II} < b_s^{I} < b_n^{I} < b_n^{II}$. Then, in area $B2$, $S^{II} = \left[max\left\{\underline{b}_n^{II}, \underline{b}_s^{II}\right\}, P\right] = \left[\underline{b}_n^{II}, P\right] = \left[\dfrac{P(\theta_n - T)}{k}, P\right]$ and in area $C2$, $S^{II} = \left[max\left\{\underline{b}_n^{II}, \underline{b}_s^{II}\right\}, P\right] = \left[\underline{b}_n^{II}, P\right] = \left[\dfrac{P(\theta_s + \theta_n - k)}{k}, P\right]$.

In area $C1$, $\underline{b}_n^{II} \geq \underline{b}_s^{II}$. According with lemma two, in area $C1$, $\underline{b}_n^{II}$ solves $\underline{b}_n^{II}\theta_n + E\left(b_s \mid b_s \geq \underline{b}_n^{II}\right)(k - \theta_n) = P(\theta_n + \theta_s - k)$ and $\underline{b}_s^{II}$ solves $\underline{b}_s^{II}\theta_s + E\left(b_n \mid b_n \geq \underline{b}_s^{II}\right)(k - \theta_s) = P(\theta_n + \theta_s - k)$. The right hand side of both expressions is equal. Therefore $\underline{b}_n^{II}\theta_n + E\left(b_s \mid b_s \geq \underline{b}_n^{II}\right)(k - \theta_n) = \underline{b}_s^{II}\theta_s + E\left(b_n \mid b_n \geq \underline{b}_s^{II}\right)(k - \theta_s)$. When the realization of the demand is an ϵ over the diagonal $(\theta_n, \theta_s) = (\theta_n, \theta_n - \epsilon)$. Therefore, $\underline{b}_n^{II}\theta_n + E\left(b_s \mid b_s \geq \underline{b}_n^{II}\right)(k - \theta_n) = \underline{b}_s^{II}(\theta_n - \epsilon) + E\left(b_n \mid b_n \geq \underline{b}_s^{II}\right)(k - \theta_n + \epsilon)$. My claim is that in area $C1$, $b_n^{II} > b_s^{II}$. In order to prove it, I will start assuming that when we move an ϵ to the left of the diagonal $E\left(b_s \mid b_s \geq \underline{b}_n^{II}\right) = E\left(b_n \mid b_n \geq \underline{b}_s^{II}\right)$.[16] Under this assumption, $\underline{b}_n^{II}\theta_n + E\left(b_s \mid b_s \geq \underline{b}_s^{II}\right)(k - \theta_n) = \underline{b}_s^{II}(\theta_n - \epsilon) + E\left(b_s \mid b_s \geq \underline{b}_s^{II}\right)(k - \theta_n + \epsilon)$. After simple algebra, I obtain $\underline{b}_n^{II}\theta_n = \underline{b}_s^{II}\theta_n + \epsilon\left[E\left(b_s \mid b_s \geq \underline{b}_s^{II}\right) - \underline{b}_s^{II}\right]$, where $\epsilon\left[E\left(b_s \mid b_s \geq \underline{b}_s^{II}\right) - \underline{b}_s^{II}\right] > 0$. Therefore $\underline{b}_n^{II} > \underline{b}_s^{II}$. Therefore, using lemma 2,

$$S^{II} = \left[max\left\{\underline{b}_n^{II}, \underline{b}_s^{II}\right\}, P\right] = \left[\frac{P(\theta_n + \theta_s - k) - E\left(b_s \mid b_s \geq \underline{b}_n^{II}\right)(k - \theta_n)}{\theta_n}, P\right]$$

Finally, in area $B1$, $b_n^{II} \geq b_s^{II}$. In areas B, $B1$ and $C1$ (all the areas that border $B1$), $b_n^{II} \geq b_s^{II}$ and $b_i^{II} \forall i = n, s$ is a continuous and monotone function. Therefore, $b_n^{II} \geq b_s^{II}$. Hence, using lemma 2,

[16]First, in the diagonal $\theta_n = \theta_s$, moreover the firms are symmetric in capacity and costs, therefore in the diagonal the mixed strategies and the support are symmetric. Hence, assume $E\left(b_s \mid b_s \geq \underline{b}_n^{II}\right) = E\left(b_n \mid b_n \geq \underline{b}_s^{II}\right)$ when the realization of demand is an ϵ over the diagonal is reasonable. Second, if under the assumption $E\left(b_s \mid b_s \geq \underline{b}_n^{II}\right) = E\left(b_n \mid b_n \geq \underline{b}_s^{II}\right)$, I obtain that $b_n^{II} > b_s^{II}$, then $F_s(b)^{II}$ stochastic dominate $F_n(b)^{II}$ and $E\left(b_s \mid b_s \geq \underline{b}_n^{II}\right) \leq E\left(b_n \mid b_n \geq \underline{b}_s^{II}\right)$, but if I assume $E\left(b_s \mid b_s \geq \underline{b}_n^{II}\right) \leq E\left(b_n \mid b_n \geq \underline{b}_s^{II}\right)$, the proof shows even stronger evidence in favour of $b_n^{II} > b_s^{II}$.

$$S^{II} = \left[max\left\{\underline{b}_n^{II}, \underline{b}_s^{II}\right\}, P\right] = \left[\frac{P(\theta_n - T) - E\left(b_s \mid b_s \geq \underline{b}_n^{II}\right)(k - \theta_n)}{\theta_n}, P\right]$$

Second, I work out the mixed strategies for both firms.

Using equation 29. The mixed strategies in area B is defined by:

$$\begin{aligned}
F_s^{II}(b) &= \begin{cases} 0 & \text{if } b < \underline{b}^{II} \\ \dfrac{\theta_n + \theta_s}{\theta_s + T}\dfrac{b - \underline{b}^{II}}{b} & \text{if } \underline{b}^{II} \leq b < P \\ 1 & \text{if } P \leq b \end{cases} \\
F_n^{II}(b) &= \begin{cases} 0 & \text{if } b < \underline{b}^{II} \\ \dfrac{\theta_s\left(b - \underline{b}^{II}\right) + T\left(E(b_n \mid b_n \geq b) - E\left(b_n \mid b_n \geq \underline{b}^{II}\right)\right)}{b\theta_s + E\left(b_n \mid b_n \geq b\right)T} & \text{if } \underline{b}^{II} \leq b < P \\ 1 & \text{if } P \leq b \end{cases}
\end{aligned} \tag{30}$$

Other useful results in area B.

$$\begin{aligned}
F_s^{II}(P) &= \frac{\theta_n + \theta_s}{\theta_s + T}\frac{P - \dfrac{P(\theta_n - T)}{\theta_n + \theta_s}}{P} = 1 \\
F_n^{II}(P) &= \frac{\theta_s\left(P - \underline{b}^{II}\right) + T\left(P - E\left(b_n \mid b_n \geq \underline{b}^{II}\right)\right)}{P(\theta_s + T)} < 1
\end{aligned}$$

Using equation 29. The mixed strategies in area $B1$ is defined by:

$$\begin{aligned}
F_s^{II}(b) &= \begin{cases} 0 & \text{if } b < \underline{b}^{II} \\ \dfrac{\theta_n\left(b - \underline{b}^{II}\right) + (k - \theta_n)\left(E(b_s \mid b_s \geq b) - E\left(b_s \mid b_s \geq \underline{b}^{II}\right)\right)}{bT + E\left(b_s \mid b_s \geq b\right)(k - \theta_n)} & \text{if } \underline{b}^{II} \leq b < P \\ 1 & \text{if } P \leq b \end{cases} \\
F_n^{II}(b) &= \begin{cases} 0 & \text{if } b < \underline{b}^{II} \\ \dfrac{\theta_s\left(b - \underline{b}^{II}\right) + T\left(E(b_n \mid b_n \geq b) - E\left(b_n \mid b_n \geq \underline{b}^{II}\right)\right)}{b(k - \theta_n) + E\left(b_n \mid b_n \geq b\right)T} & \text{if } \underline{b}^{II} \leq b < P \\ 1 & \text{if } P \leq b \end{cases}
\end{aligned} \tag{31}$$

Other useful results in area $B1$.

$$\begin{aligned}
F_s^{II}(P) &= 1 \\
F_n^{II}(P) &= \frac{\theta_s\left(P - \underline{b}^{II}\right) + T\left(P - E\left(b_n \mid b_n \geq \underline{b}^{II}\right)\right)}{P(T + k - \theta_n)} < 1
\end{aligned}$$

Using equation 29. The mixed strategies in area $B2$ is defined by:

$$F_s^{II}(b) = \begin{cases} 0 & \text{if } b < \underline{b}^{II} \\ \dfrac{k}{T+k-\theta_n}\dfrac{b-\underline{b}^{II}}{b} & \text{if } \underline{b}^{II} \leq b < P \\ 1 & \text{if } P \leq b \end{cases}$$

$$F_n^{II}(b) = \begin{cases} 0 & \text{if } b < \underline{b}^{II} \\ \dfrac{\theta_s\left(b-\underline{b}^{II}\right)+T\left(E(b_n \mid b_n \geq b)-E\left(b_n \mid b_n \geq \underline{b}^{II}\right)\right)}{b(k-\theta_n)+E\left(b_n \mid b_n \geq b\right)T} & \text{if } \underline{b}^{II} \leq b < P \\ 1 & \text{if } P \leq b \end{cases} \quad (32)$$

Other useful results in area $B2$.

$$\begin{aligned} F_s^{II}(P) &= 1 \\ F_n^{II}(P) &= \frac{\theta_s\left(P-\underline{b}^{II}\right)+T\left(P-E\left(b_n \mid b_n \geq \underline{b}^{II}\right)\right)}{P(T+k-\theta_n)} < 1 \end{aligned}$$

Using equation 29. The mixed strategies in area $C1$ is defined by:

$$F_s^{II}(b) = \begin{cases} 0 & \text{if } b < \underline{b}^{II} \\ \dfrac{\theta_n\left(b-\underline{b}^{II}\right)+(k-\theta_n)\left(E(b_s \mid b_s \geq b)-E\left(b_s \mid b_s \geq \underline{b}^{II}\right)\right)}{b(k-\theta_s)+E\left(b_s \mid b_s \geq b\right)(k-\theta_n)} & \text{if } \underline{b}^{II} \leq b < P \\ 1 & \text{if } P \leq b \end{cases}$$

$$F_n^{II}(b) = \begin{cases} 0 & \text{if } b < \underline{b}^{II} \\ \dfrac{\theta_s\left(b-\underline{b}^{II}\right)+(k-\theta_s)\left(E(b_n \mid b_n \geq b)-E\left(b_n \mid b_n \geq \underline{b}^{II}\right)\right)}{b(k-\theta_n)+E\left(b_n \mid b_n \geq b\right)(k-\theta_s)} & \text{if } \underline{b}^{II} \leq b < P \\ 1 & \text{if } P \leq b \end{cases}$$

Other useful results in area $C1$.

$$\begin{aligned} F_s^{II}(P) &= 1 \\ F_n^{II}(P) &= \frac{\theta_s\left(P-\underline{b}^{II}\right)+(k-\theta_s)\left(P-E\left(b_n \mid b_n \geq \underline{b}^{II}\right)\right)}{P(2k-\theta_n-\theta_s)} < 1 \end{aligned}$$

Using equation 29. The mixed strategies in area $C2$ is defined by:

$$F_s^{II}(b) = \begin{cases} 0 & \text{if } b < \underline{b}^{II} \\ \dfrac{k}{2k-\theta_n-\theta_s}\dfrac{b-\underline{b}^{II}}{b} & \text{if } \underline{b}^{II} \leq b < P \\ 1 & \text{if } P \leq b \end{cases}$$

$$F_n^{II}(b) = \begin{cases} 0 & \text{if } b < \underline{b}^{II} \\ \dfrac{\theta_s\left(b-\underline{b}^{II}\right)+(k-\theta_s)\left(E(b_n \mid b_n \geq b)-E\left(b_n \mid b_n \geq \underline{b}^{II}\right)\right)}{b(k-\theta_n)+E\left(b_n \mid b_n \geq b\right)(k-\theta_s)} & \text{if } \underline{b}^{II} \leq b < P \\ 1 & \text{if } P \leq b \end{cases}$$

Other useful results in area $C2$.

$$
\begin{aligned}
F_s^{II}(P) &= 1 \\
F_n^{II}(P) &= \frac{\theta_s\left(P-\underline{b}^{II}\right)+(k-\theta_s)\left(P-E\left(b_n \mid b_n \geq \underline{b}^{II}\right)\right)}{P(2k-\theta_n-\theta_s)} < 1
\end{aligned}
$$

Third, the expected bid can not be worked out using the derivative of the cumulative distribution function, because the cumulative distribution function has not a close form solution. However, the algorithm that I will introduce below in the annex gives me the opportunity to work out the expected bid for both firms.

Fourth, I work out the payoff function.

Using equation 28. The payoff function in area B is:

$$
\begin{aligned}
\overline{\pi}_n^{II} &= \underline{b}^{II}(\theta_n+\theta_s) \\
\overline{\pi}_s^{II} &= \underline{b}^{II}\theta_s + E\left(b_n \mid b_n \geq \underline{b}^{II}\right)T
\end{aligned} \tag{33}
$$

Using equation 28. The payoff function in area $B1$ is:

$$
\begin{aligned}
\overline{\pi}_n^{II} &= \underline{b}^{II}\theta_n + E\left(b_s \mid b_s \geq \underline{b}^{II}\right)(k-\theta_n) \\
\overline{\pi}_s^{II} &= \underline{b}^{II}\theta_s + E\left(b_n \mid b_n \geq \underline{b}^{II}\right)T
\end{aligned} \tag{34}
$$

Using equation 28. The payoff function in area $B2$ is:

$$
\begin{aligned}
\overline{\pi}_n^{II} &= \underline{b}^{II}k \\
\overline{\pi}_s^{II} &= \underline{b}^{II}\theta_s + E\left(b_n \mid b_n \geq \underline{b}^{II}\right)T
\end{aligned} \tag{35}
$$

Using equation 28. The payoff function in area $C1$ is:

$$
\begin{aligned}
\overline{\pi}_n^{II} &= \underline{b}^{II}\theta_n + E\left(b_s \mid b_s \geq \underline{b}^{II}\right)(k-\theta_n) \\
\overline{\pi}_s^{II} &= \underline{b}^{II}\theta_s + E\left(b_n \mid b_n \geq \underline{b}^{II}\right)(k-\theta_s)
\end{aligned} \tag{36}
$$

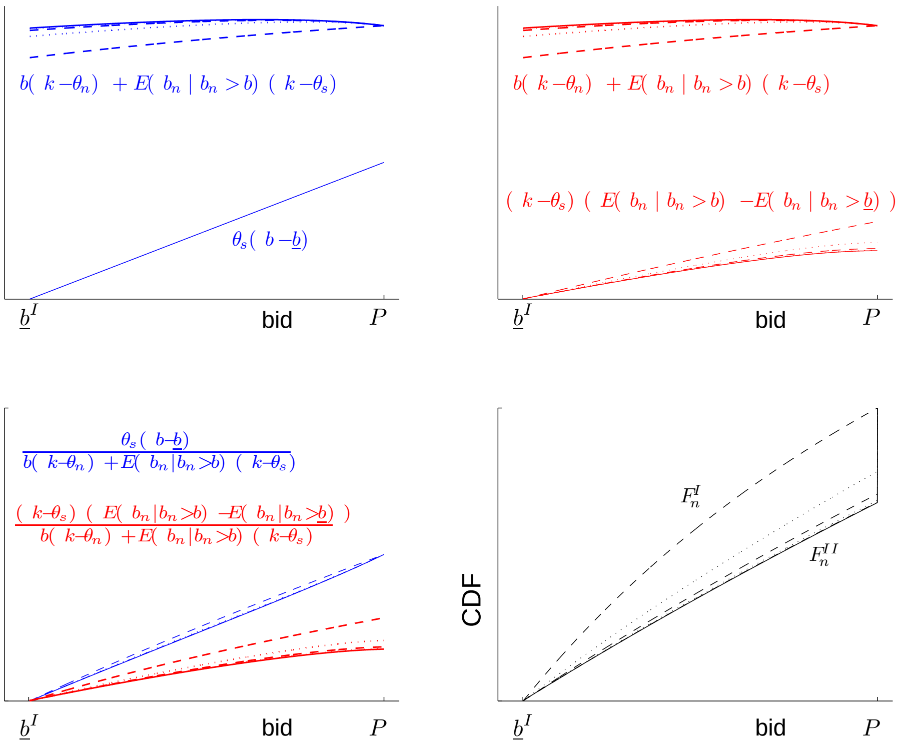
Using equation 28. The payoff function in area $C2$ is:

$$
\begin{aligned}
\overline{\pi}_n^{II} &= \underline{b}^{II}k \\
\overline{\pi}_s^{II} &= \underline{b}^{II}\theta_s + E\left(b_n \mid b_n \geq \underline{b}^{II}\right)(k-\theta_s)
\end{aligned} \tag{37}
$$

Algorithm to work out the cumulative distribution function.

The payoff function of the model in which the transmission rights are assigned to the firm that submits the lowest bid in the spot electricity auction satisfies the properties that guarantee that a mixed equilibrium exists, however the cumulative distribution function defined by equation 29 is a function of its own expected value, therefore it does not exist a close form solution for it. In the next lines, I present an algorithm that gives

Figure 8: Existence and Uniqueness of the CDF



me the opportunity to work out an approximation of the cumulative distribution function.

To made the exposition easier, I will focus only in the equilibrium in Area $C2$ in figure 4. The support and mixed strategies equilibrium in area $C2$ are defined by the next equations:

$$S^{II} = \left[max\left\{\underline{b}_n^{II}, \underline{b}_s^{II}\right\}, P\right] = \left[\underline{b}_n^{II}, P\right] = \left[\frac{P(\theta_s+\theta_n-k)}{k}, P\right] \tag{38}$$

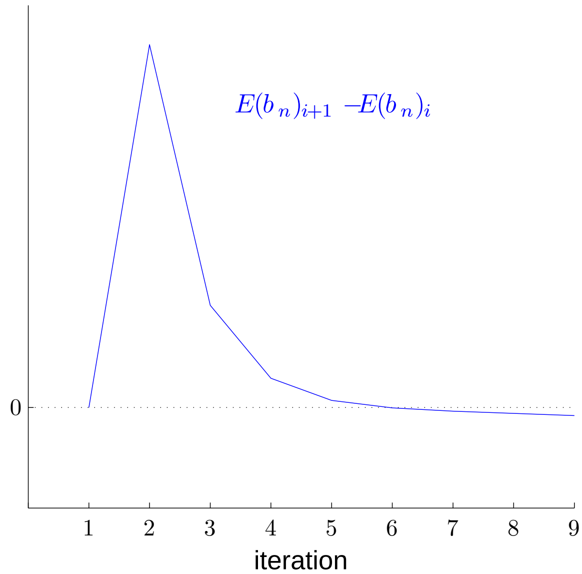
$$F_s^{II}(b) = \frac{k(b-\underline{b}^{II})}{(2k-\theta_n-\theta_s)b} \tag{39}$$

$$F_n^{II}(b) = \frac{\theta_s(b-\underline{b}^{II})+(k-\theta_s)\left[E\left(b_n \mid b_n \geq b\right)-E\left(b_n \mid b_n \geq \underline{b}^{II}\right)\right]}{b(k-\theta_n)+E\left(b_n \mid b_n \geq b\right)(k-\theta_s)} \tag{40}$$

Equations 38 and 39 do not depend of any expectation. Therefore, they can be easily computed. Nevertheless, equation 40, depends on its own expectation. Therefore, it does not exist a close form solution for it. To work out the cumulative distribution function defined by 40 I have developed the algorithm that I describe below. The key point in the algorithm is guarantee that the prior expected value that I use to work out 40 and the posterior expected value calculated using 40 coincide, i.e., a fix point exist.

The algorithm to work out the cumulative distribution function defined in 40 consists in the next sequence of iterations. In the first iteration, I have taken the cumulative distribution function for area $C2$ when the transmission rights are assigned to the grid

Figure 9: Existence of a fix point



operator defined by equation 14 . In the rest of iterations, I use the cumulative distribution function generated in the previous iteration to work out the cumulative distribution function defined in 40. In each iteration, I work out the difference in the expected value between two consecutive iterations, (figure 9 summarizes this information). The iteration process concludes when the difference in means between two consecutive iterations is zero.

In the next lines, I provide evidence that the algorithm that I have described above converge to a fix point.

Equation 40 can be split in two equations:

$$E_1(b) = \frac{\theta_s(b - \underline{b}^{II})}{b(k-\theta_n) + E\left(b_n \mid b_n \geq b\right)(k-\theta_s)} \tag{41}$$

$$E_2(b) = \frac{(k-\theta_s)\left[E\left(b_n \mid b_n \geq b\right) - E\left(b_n \mid b_n \geq \underline{b}^{II}\right)\right]}{b(k-\theta_n) + E\left(b_n \mid b_n \geq b\right)(k-\theta_s)} \tag{42}$$

In figure 8, I have plotted the first six iterations of the algorithm that I have described above. In the first panel (starting from the top left), I have plotted in blue the iterations for the numerator and the denominator of equation 41. As can be observed the numerator does not change and the denominator converges quickly. In the second panel, I have plotted in red the six first iterations for the numerator and denominator of equation 42. As can be observed, both converge quickly. In the third panel, I have plotted in blue the six first iterations for equation 41 and in red the six first iteration for equation 42. As can be observed both equations converges quickly. Finally, in the last panel, I have plotted in black the six first iterations for equation 40, as can be observed the iteration process converges quickly.

In figure 9 I have plotted the difference in means between two consecutive iterations.

Table 4: ($\bigtriangledown T$), ($\theta_n = 55, \theta_s = 5$), ($k_n = k_s = 60, c_n = c_s = 0, P = 7$)

	$\underline{b}^{II}$	$E(b_s)$	$\overline{\pi}_n = \underline{b}^{II}\theta_n + E(b_s)(k - \theta_n)$	$\overline{\pi}_n = P(\theta_n - T)$
$T = 60$	0	0	0	0
$T = 50$	0.50	1.49	$(0.5 \cdot 55) + (1.49 \cdot 5) = 34.95$	35
$T = 40$	1.62	3.14	$(1.62 \cdot 55) + (3.14 \cdot 5) = 104.8$	105
$T = 30$	2.79	4.31	$(2.79 \cdot 55) + (4.31 \cdot 5) = 175$	175
$T = 20$	3.97	5.23	$(3.97 \cdot 55) + (5.23 \cdot 5) = 244.5$	245
$T = 10$	5.18	6.02	$(5.18 \cdot 55) + (6.02 \cdot 5) = 315$	315
$T = 0$	7	7	$(7 \cdot 55) + (7 \cdot 5) = 385$	385

As can be observed, in the second iteration the difference in means is big and in latter iterations the difference decreases smoothly. The difference in mean becomes zero between iteration six and iteration seven. This means that there exists a cumulative distribution function between the one obtained in iteration six and the one obtained in iteration seven for which the prior mean used to work out the cumulative distribution function and the posterior mean derived from the cumulative distribution function coincide, i.e., a fix point exists.

Above, I have described the algorithm to work out the equilibrium and I have shown that the algorithm converges. In the next lines, I provide evidence that the algorithm do not have internal mistakes, i.e., generates the results predicted by the model. Lemma two stablish $\underline{b}^{II}\theta_i + E(b_j \mid b_j \geq \underline{b}^{II})(k_i - \theta_i) = P(\theta_i - T)$. Column four in table 4 presents the expected profit for firm n using the values generated by the algorithm $(\underline{b}^{II}\theta_i + E(b_j \mid b_j \geq \underline{b}^{II})(k_i - \theta_i))$. Column five presents the expected profit predicted by the theory $(P(\theta_i - T)$). As can be observed both values coincide.

Proposition 5.

Areas $B2$ and $C2$. Using proposition two and four it is straight forward to check that the lower bound of the support, the expected value of the bids of the firm located in the South and the expected payoff of the firm located in the North are equal in both models.

In area $B2$, $F_n^I(\underline{b}^I) = F_n^{II}(\underline{b}^{II}) = 0$. $F_n^I(P) = \dfrac{(\theta_s + T)(P - \underline{b}^I)}{P(T + k - \theta_n)}$ and $F_n^{II}(P) = \dfrac{\theta_s(P - \underline{b}^{II}) + T(P - E(b_n \mid b_n \geq \underline{b}^{II})}{P(T + k - \theta_n)}$. As I have shown before, $\underline{b}^I = \underline{b}^{II}$, moreover $E(b_n \mid b_n \geq \underline{b}^{II}) \geq \underline{b}^I)$, therefore, $F_n^{II}(P) \leq F_n^I(P)$. Furthermore, $F_n^{II}(b)$ and $F_n^I(b)$ are monotone increasing and continue in the support. Therefore, $F_n^I(b) \geq F_n^{II}(b) \forall b \in [\underline{b}, P]$. Hence, $F_n^I(b)$ stochastic dominate $F_n^{II}(b)$. Then, $E(b_n^{II}) \geq E(b_n^I)$. Moreover, the expected payoff of the firm located in the South $\overline{\pi}_s^{II} \geq \overline{\pi}_s^I$. The same logic applies in area $C2$.

Areas $B1$ and $C1$. Using proposition two and four it is straight forward to check that the lower bound of the support when the transmission rights are assigned to the firm that submits the lowest bid in the spot electricity auction is lower than the lower bound of the support when the transmission rights are assigned to the grid operator. Moreover, the expected payoff of the firm located in the North is equal in both models, $\overline{\pi}_n^I = \overline{\pi}_n^{II}$.

In area $B1$, $F_n^I(\underline{b}^I) = 0 \leq F_n^{II}(\underline{b}^I) = 0$. $F_n^I(P) = \dfrac{(\theta_s + T)(P - \underline{b}^I)}{P(T + k - \theta_n)}$ and $F_n^{II}(P) = \dfrac{\theta_s(P - \underline{b}^{II}) + T(P - E(b_n \mid b_n \geq \underline{b}^{II})}{P(T + k - \theta_n)}$. As I have shown before, $\underline{b}^{II} \leq \underline{b}^I$, $E(b_n \mid b_n \geq \underline{b}^{II}) \geq \underline{b}^{II})$. However, with this information, $F_n^{II}(P)$ and $F_n^I(P)$ can not be ranked. Therefore, it can not be determined the stochastic dominate relation between $F_n^{II}(b)$ and $F_n^I(b)$. And so, the expected value of bids for the firm located in the North. The same logic applies in area $C1$.

In area $B1$, $F_s^I(\underline{b}^I) = 0 \leq F_s^{II}(\underline{b}^I)$. $F_s^I(P) = F_s^{II}(P) = 1$. Moreover, $F_s^{II}(b)$ and $F_s^I(b)$ are monotone increasing and continue in the support. Therefore, $F_s^I \leq F_s^{II} \forall b \in [\underline{b}^{II}, P]$. Hence, $F_s^{II}(b)$ stochastic dominate $F_s^I(b)$. Then, $E(b_s^I) \geq E(b_s^{II})$. The same logic applies in area $C1$.

Finally, in areas $B1$ and $C1$, $E(b_s^I) \geq E(b_s^{II})$, but the relation between $E(b_n^I)$ and $E(b_s^{II})$ can not be determined. Hence, the relation between the expected payoff of the firm located in the South can not be ranked in both models.

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