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Essays in Structural Heuristics

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To Elena, without whom nothing of this would have been possible

En toute circonstance, la nature réalise la morphologie locale la moins complex compatible avec les données initiales locales

(Renè Thom, *Modèles mathématiques de la morphogenèse*, 1980, p. 144)

In a sense, an appreciation of symmetry allows a blending of relativist and determinist theorizing

(Philip Mirowski, *More heat than light*, 1989, p. 95)

Just as a scissors cannot cut paper without two blades, a theory of thinking and problem solving cannot predict behavior unless it encompasses both an analysis of the structure of task environments and an analysis of the limits of rational adaptation to task requirements

(Allen Newell and Herbert A. Simon, *Human problem solving*, 1972, p. 55)

Contents

Chapter 1 **Introduction: Structural Heuristics and Economic Knowledge**

1.1 Structural Heuristics and Economic Knowledge……………………… 7

1.2 References……………………………………………………… 14

Chapter 2 **Plurality in Economic Axiomatics: a ‘Structural Heuristic’ Reconstruction**

2.1. Economic axiomatics and the ‘structural heuristic’ reconstruction…… 19

2.2. Axioms………………………………………………………… 24

2.2.1 From atoms to molecules……………………………………… 24

2.2.2 The justification of axioms: the route between ‘analytic’ and ‘synthetic’ statements……………………………………... 26

2.2.3 Degrees of empirical commitment…………………………... 33

2.3. Rules of inference: ‘mathematics’ style vs. ‘logics’ style of axiomatics...35

2.4. Theorems……………………………………………………… 44

2.5. The vertical specification of axiomatic structure: Velupillai’s proposal for a ‘Computable’ and ‘Constructive’ Economics………………………51

2.6 Concluding Remarks…………………………………………….57

2.7 References……………………………………………………….58

Chapter 3 **Structural realism in econophysics: the case of Giuseppe Palomba**

3.1 Introduction……………………………………………………...70

3.2 What is it like to be a 'structural realist'?..71

3.3 Giuseppe Palomba's view of economic dynamics………………………77

3.3.1 Multilayered axiomatization, 'natural philosophy,' and physics…79

3.3.2 Empirical progress as historical progress………………………...87

3.4 Palomba's structural realism……………………………………………91

3.5 Concluding remarks……………………………………………………100

3.6 References…………………………………………………………...101

Chapter 4 **Atoms, structures and hierarchies in the analysis of economic production**

4.1 Introduction……………………………………………………………110

4.2 The 'atoms' of economic production: Fundamental Units of Analysis (FUAs)…………………………………………………………………… 113

4.3 Production as chemistry……………………………………………...118

4.4 From atoms to structures………………………………………………123

4.4.1 Combinatorics as structuralism……………………………………124

4.4.2 Time and structure…………………………………………………126

4.4.3 Symmetry-breaking………………………………………………… 127

4.4.4 Hierarchic systems……………………………………………128
4.5 On structuralist approaches in the theory of production
4.5.1 Piero Sraffa: against the 'generalized' marginalistic principle in production………………………………………………………130
4.5.2 Structuralist requirements for the theories of production: the analysis of sectoral interdependencies……………………………………134
4.6 Concluding remarks……………………………………………………139
4.7 Appendix A. The 'language of production': some atomistic and structuralist remarks……………………………………………………141
4.8 References…………………………………………………………...146

Chapter 5 **Numerals as triggers of System 1 and System 2 in the 'bat and ball' problem** (co-authored with Antonio Mastrogiorgio)

5.1 Introduction………………………………………………………… 158
5.2 State of the art
5.2.1 The Cognitive Reflection Test (CRT)………………………….. 159
5.2.2. The 'bat and ball' problem…………………………………….. 160
5.3 The hypothesis…………………………………………………………162
5.4 Methodology…………………………………………………………...165
5.4.1 The experiment………………………………………………………166
5.5 Results……………………………………………………………………169
5.6 Discussion…………………………………………………………….....172
5.7 Concluding remarks……………………………………………….....176

5.8 References……………………………………………………………...177

Chapter 6 **Concluding remarks and Lines of Further Research………..184**

List of Figures and Tables

Figure 1: The conceptual space of 'axiomatics' and its internal structure…. 22

Figure 2: Structure of 'terms' and structure of 'sentences' in an axiomatic system…………………………………………………………. 25

Figure 3: Time lapse in Palomba's framework……………………………… 90

Figure 4: Characterizations of a FUA as 'metric' and 'entity'……………115

Figure 5: Alternative 'time-structures' of FUA(A) and FUA(B)………… 127

Figure 6: The input-output matrix………………………………………... 135

Table 1: A synthesis of Palomba's 'axiomatic stratification'………………86

Table 2: Relative frequencies………………………………………….. 169

Table 3: Contingency table……………………………………………..170

List of Abstracts

Dissertation's abstract……………………………………………………….. 6

Abstract for Chapter 2………………………………………………………… 18

Abstract for Chapter 3………………………………………………………… 69

Abstract for Chapter 4……………………………………………………….. 109

Abstract for Chapter 5……………………………………………………….. 157

Dissertation's abstract

This dissertation introduces and develops a new method of rational reconstruction called *structural heuristics*. Structural heuristics takes assignment of structure to any given object of investigation as the starting point for its rational reconstruction. This means to look at any given object as a system of relations and of transformation laws for those relations. The operational content of this heuristics can be summarized as follows: *when facing any given system the best way to approach it is to explicitly look for a possible structure of it*. The utilization of structural heuristics allows *structural awareness*, which is considered a fundamental epistemic disposition, as well as a fundamental condition for the rational reconstruction of systems of knowledge.

In this dissertation, structural heuristics is applied to reconstructing the domain of economic knowledge. This is done by exploring four distinct areas of economic research: (i) economic axiomatics; (ii) realism in economics; (iii) production theory; (iv) economic psychology. The application of structural heuristics to these fields of economic inquiry shows the flexibility and potential of structural heuristics as epistemic tool for theoretical exploration and reconstruction.

Chapter 1

Introduction:

Structural Heuristics and Economic Knowledge

1.1 Structural heuristics and economic knowledge

'Structuralism' is a controversial notion in intellectual history. The histories of disciplines such as linguistics, anthropology and literary criticism are there to show such a state of affairs. The transition from structuralism to post-structuralism (and even beyond) has marked a watershed, so that the legitimacy of a structuralist perspective is nowadays subject, particularly in those disciplines, to strict qualifications[1].

Associated with the decline of the notion of structuralism in many social sciences is a serious collateral risk, the hard-to-avoid 'throwing out the baby with the bathwater': the ostracism of the notion of 'structure' along with that of 'structuralism'. Once more, this is truer

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[1] For the history of 'structuralism' as intellectual movement, see Dosse (1997a and 1997b). Essays that provide conceptual reconstructions of the notion of 'structuralism' tend typically to focus on the specific disciplinary domains of linguistics, anthropology and literary criticism. A still unsurpassed exception is Jean Piaget (1971). It is also worth to mention the interdisciplinary work of René Thom (1980).

for some disciplines than for others. This risk, however, can turn out to be an opportunity for structuralist analyses: the continuous pressure to specify, update and refine the notion of structure, and the frameworks in which it is employed, is an invaluable opportunity for structural analysis. A chance this that could more properly be exploited, in a climate of more freedom, in disciplines in which the structuralist/non-structuralist divide has been harsh but not disruptive.

The domain of economics provides an interesting field of investigation for structural analysis. The origins of economics as a scientific discipline are firmly structuralist: the *Classical Political Economy* of Smith, Ricardo and Marx is a 'science of structures' at full extent. Only with the *Neoclassical revolution* of the 1870s, the structuralist gist of economics has been overshadowed and neglected. However, with respect to the intellectual history of other social sciences, the case of economics looks rather peculiar, since the Neoclassical school has merely marked the advent of a '*non*-structuralist' economics, and not of an '*anti*-structuralist' one: that is to say, the intellectual role of structural analysis has never been criticized or put into question within the Neoclassical transition. This is true to such an extent that structuralist-inspired theorists have continued to mark the history of 20th century economics, even well after the 'Marginalist' revolution: among them, notably, economists such as Wassily Leontief, Piero Sraffa, Adolph Lowe, John R. Hicks, and Luigi L. Pasinetti have to be mentioned (see Hagemann and Scazzieri, 2010; Scazzieri, 2012)[2]. In light of these considerations, the

[2] Among other structuralist approaches one can mention structuralist macroeconomics (e.g. Taylor, 1983; for the historical reconstruction of structural macroeconomics see Gibson, 2003) and the structuralist approach in econometrics (see e.g. Reiss and Wolak, 2007). The approaches listed here and in the text vary enormously in the way they conceive and use the notion of structure. However, as Jean Piaget points out "there is really an

development of structural analysis in economics is in line with a persistent foundational thread of analysis that has characterized economics since its birth, and which has never been conceptually questioned. I may say that in economics the structuralist paradigm has never been conceptually superseded, since even mainstream (Neoclassical) economics has *pre-structuralist* foundations. This state of affairs can be important for the other social sciences as well. Economics could become a *structuralist laboratory*, pointing to a possible change of status of structural analysis in the social sciences.

This dissertation addresses two principal objectives:

i. to contribute to the theoretical development of the notion of structure;
ii. to frame and apply this theoretical development to the domain of economics.

This dissertation will not consider specific structures, as they have been developed in economic analyses, but will consider the overall *status* of structure as *analytical category* in economics. In particular, this dissertation proposes to reconceptualize structural analysis by reconsidering the *epistemological* as well as *ontological* status of structures in the discipline, by shifting the emphasis from the latter to the former. In a nutshell, *I propose the reconceptualization of the notion of structure shifting the emphasis from its substantive notion to its heuristic notion*. The substantive notion of structure is mainly related to the *ontological* characterization of structures: these are assumed to exist here and now and to constraint representations and actions within their scope and extent. The heuristic notion of structure

ideal of intelligibility held in common, or at least aspired after, by all structuralist, even though their critical objectives vary enormously" (Piaget, 1971, p. 4). This is true also for economics.

rejects such a picture, by *epistemologically* accepting the existence of structures but not the strict subordination of epistemology to ontology.

A couple of examples from the intellectual histories of cognitive psychology and of rationality studies in economics could help to clarify the approach followed in this dissertation. In cognitive psychology, we see the shift from William Wundt's *structural psychology* to *Gestalt* psychology (see e.g Koffka, 1935). Wundt thought that cognition resulted from the composition of elementary elements – called 'sensations' – that mirror the external structure of the environment. The *Gestalt* school points out that structural psychology is unable to tackle cases in which, given a fixed structure of the environment, subjects give different accounts of it (think of the well-known Wittgenstein's duck-rabbit illusion). The conceptual shift proposed by *Gestalt* psychology is that structures are better conceptualized as heuristics of subjects. Given whatever material configuration of the environment, it is human cognition that heuristically gives structure to it.

In the history of economic thought, a shift closely related to the one described above was the shift in the notion of 'rationality' due to Herbert A. Simon (1976, 1978). Simon proposed to shift from a *substantive* notion of rationality to a *procedural* one. This means that rationality should not anymore be assessed on the basis of a pre-specified normative content. Alternatively, according to Simon, rationality has to be assessed: i) on the basis of the situations in which agents behave and 2) on the basis of the 'adequacy' of the pattern of interaction chosen by agents relative to the situation in which they behave. This turn is at the same time an epistemological and a heuristic one. It is epistemological because rationality is not seen anymore as a concept aspiring to gain an independent, and all the less an existence status. Rationality is an emerging notion, which takes from time to time different meanings and expressions, and thus escapes reification. The heuristic turn associated with procedural rationality concerns the new definition of rationality in terms of 'adequacy' to the environment in which the action takes place. If the environment determines the adequacy of behavior, the same environment exerts a selective pressure towards the epistemic procedures more apt to interact with it. Heuristics emerge from selective pressure, either ontogenetic or phylogenetic (Todd, 2000; Gigerenzer and Selten, 2002). A huge amount of research has been provided studying the adequacy of heuristics to environments by emphasizing the structural properties of the environment as the fundamental tracts that trigger the selective pressure (e.g. Gigerenzer et al. 1999; Gigerenzer and Gassmaier, 2011). In this perspective, heuristics are evolutionary response to the structural properties of environments, so that heuristics are highly responsive to those structures[3].

From the above considerations, structural heuristics, as conceived in this dissertation, is meant to be an *explorative cognitive device* to be employed in different domains of inquiry. If we want to make such heuristics operational, we could say that *when facing an environment the best way to approach at it is explicitly looking for a possible structure of it*. Looking for structures means, basically, to *rationally reconstruct* the environment, i.e to try to characterize the environment as constituted by a fundamental system of relationships. The benefits coming from the employment of structural heuristics as rational reconstruction method are in the

[3] This does not mean that each heuristics that is responsive to the structure of a particular environment is effective in that environment. Although specific structures of environments trigger specific heuristics, the adaptive function played by those heuristics in those environment may not be actual anymore. The conflict between *evolutionary* and *current adequateness* of heuristics to environments is at the root of what are commonly labeled 'errors' in the 'heuristics and biases' approach to rationality (see e.g. Tversky and Kahneman, 1974).

process of discovery of structures, not in the output (structure) discovered *per se*. It might be said, actually, that what the application of the structural heuristics does is to ease awareness of environments.

This dissertation applies the method of structural heuristics to the domain of economics. Its four essays make use of structural heuristics to address four specific topics: i) economic axiomatics, ii) realism in economics; iii) economic production; iv) economic psychology. The unifying thread is provided by structural heuristics in its use as method for rational reconstruction. In what follows, I summarize the content of the four essays, emphasizing why employing structural heuristics has been useful to shed light on the specific topics they address. By considering the specific content of each essay, I want also to deal with the details concerning the implementation of structural heuristics in different domains. The objective of this dissertation is in fact twofold: i) to show the fruitfulness of the application of structural heuristics as method of rational reconstruction and ii) to show the way to implement it in different domains (environments) of research.

Chapter 2 considers 'Pluralism in economic axiomatics: a 'structural heuristic' reconstruction'. In this essay, I specify the notion of structural heuristics at a deeper extent (in particular, I specify it against the alternative notion of 'historical heuristics'). This is the main reason why this essay constitutes the first one in the sequence of the four essays of the dissertation. Then, I apply the notion of structural heuristics in a case of conceptual reconstruction, i.e. the reconstruction of the concept of economic axiomatics. The employment of structural heuristics to the concept of economic axiomatics allow me to reconstruct systematically the plurality that lays behind such a concept.

Chapter 3 deals with 'Structural realism in econophysics: the case of Giuseppe Palomba'. In this essay, I apply structural heuristics to the history of economic thought. In particular, through the application of structural heuristics, I identify the figure of Giuseppe Palomba, second-generation member of the School of post-Paretian economics, as a proto-structural realist thinker in economics and econophysics. This acknowledgment allows me also to contribute to the growing body of research on structural realism in economics. In this respect, I identify econophysics as a disciplinary domain in which the suitability of the notion of structural realism (originally devised in physics) to economics could best be assessed.

Chapters 4 and 5 are methodologically connected, in the sense that Chapter 4 is an application of *positive* structural heuristics, whereas Chapter 5 is meant to be an application of *negative* structural heuristics, along the lines that will be specified in what follows.

Chapter 4 considers 'Atoms, structures and hierarchies in the analysis of economic production'. In this essay, I acknowledge that economics and the neighboring disciplines dealing with production phenomena start their analysis identifying atomistic units of analysis, which I label *Fundamental Units of Analysis* (FUAs). In this essay, I show that a simple atomistic framework is not suitable to deal with production phenomena: in this sense, a framework for production has to be complemented with structuralist addenda. I identify, in this vein, some structuralist *basic requirements* that a framework for economic production have to satisfy in order to address properly the production phenomena. These basic requirements are identified in the structuralist notions of combinatorics, symmetry-breakings and hierarchic systems. The essay shows that structural heuristics is employed in a *positive* sense: in particular, by employing it in the domain of production it is able to let emerge the limits of atomism and the need to complement them with structural considerations.

Chapter 5 deals with 'Numeral as triggers of System 1 and System 2 in the bat and ball problem'. This essay (co-authored with Antonio Mastrogiorgio) contributes to the strand of economic psychology literature that inquires into the distinction between the automatic and deliberative ways of thinking. The 'bat and ball' problem is employed as testbed in an experiment on the two-systems dynamics of activation. We administer a classical 'bat and ball' tasks to the control group and a 'bat and ball modified' task to the experimental group. In the latter, only the *numerals* of the task were changed. In this experimental setting, we identified a different dynamics of activation of the two systems between the experimental and the control group. By conceptualizing the 'bat and ball' as a structure, and the numerals in which the task is arranged as elements of that structure, we are able to say that what determines performance in 'bat and ball'-like problems is their numerical arrangement, and not only the mere (logical) structure of the problem. The design of the experiment can so be seen as an application of *negative* structural heuristics, i.e. heuristics able to highlight how (logical) structures of problems are not the end of the story for what concerns the dual-system dynamics of activation.

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Chapter 2

Plurality in economic axiomatics: a 'structural heuristic' reconstruction

Abstract

The practice of the reconstruction of concepts in economics has mostly followed a historicist approach, i.e the reconstruction of economic concepts has been equated to their historical reconstruction. In this essay, it is shown that a critical reconstruction of concepts in economics has not to commit to historicization too early. An orientation, even provisional, within the target concept of reconstruction is needed before historicization. Such a criterion of orientation can be identified with the *structure* of the concept, which can be employed as a *heuristics* in the process of critical reconstruction. Such a reconstructive strategy will be labeled 'structural heuristics'. In this essay, this reconstructive strategy is proved in the case of the concept of economic axiomatics. Through the disentanglement of the concept of axiomatics into its structural constituents, i.e. 1) axioms, 2) rules of inference and 3) theorems in a horizontal specification, and 4) philosophy of proof in a vertical specification, a reconstruction for each level of the structure will be provided. In this respect, the structural heuristic reconstruction is meant to let emerge the intrinsic plurality behind the notion of axiomatics: the structuralist view of economic axiomatics can be employed to better address the controversies and misunderstandings arisen over such a concept.

Keywords: structural heuristics, conceptual reconstruction, conceptual plurality, axiomatics in economics, controversies on axiomatics

2.1. Economic axiomatics and the 'structural heuristic' reconstruction

As Roy E. Weintraub (2002, p.72) points out, concepts such as *axiomatization*, *mathematization*, *rigor* and *formalism* are often employed somehow informally in economics. Although this statement may sound paradoxical, this points to a real condition and suggests the need to provide a critical reconstruction of these concepts in order to gain a reliable framework for economic discourse. In this respect, Weintraub further states that

> "[m]odern controversies over formalism [axiomatization, mathematization and rigor] in economics rest on misunderstandings about the history of mathematics, the history of economics, and the history of the relationship between mathematics and economics" (ibid.)

calling attention to the privileged role that historical reconstruction plays in the reconstruction of these concepts in economics. The perspective of the primacy and self-sufficiency of the historical perspective in reconstructive attempts expresses the idea that an historical projection of concepts is always *decisive* for the clarification of their meaning[4]. Many successful attempts have been pursued in economics in this direction, employing the historical perspective as the privileged way to demonstrate the spatiotemporal embeddedness of concepts. For the historicization of concepts such as axiomatization, rigor, formalism and mathematization see, for instance, Ingrao and Israel (1990); Debreu (1991); Punzo (1991); Golland (1996); Weintraub (2002)[5]; Giocoli (2003); Blaug (2003), Mirowski (2012).

Contrary to the above points of view, I shall maintain that historical reconstruction alone might *not always* be enough in order to gain an adequate critical reconstruction of concepts.

[4] For the methodology and philosophy of historicism see the classics Mach (1883) and Enriques (1906).

[5] Weintraub (2002) indicates as a remarkable antecedent in this tradition Israel (1980), who provides the historical reconstruction of the concept of "rigor" in mathematics.

Histories are always specific inquiries, so they unavoidably privilege some reconstructive paths rather than others. Relying on historical reconstruction means following, selectively, a subset of reconstructive paths. In order not to neglect potentially eligible paths – this is a cognitive suggestion at least – a preliminary analysis is required. The methodological core of this essay is based on the idea that one should not rely too early on the historicization of concepts in the process of "conceptual exploration" (Hausman, 1989, 115).

What is needed is, first of all, to get an analytical glimpse into the complexity of the concept to be reconstructed. In order to get such a glimpse, we might benefit from considering a concept as representable by means of a semantic space, which would be provided of its own *structure* (Gärdenfors, 2000). This structure, once identified, can be employed as the basis of critical reconstruction, and would thus precede historical reconstruction, both conceptually and operationally. Imre Lakatos (1970) set out a method, labeled 'rational reconstruction', in which the historical reconstruction was preceded by an explicit process of orientation, which he conceived as a heuristic process[6]. As he states, "any historical study must be preceded by a *heuristic* study: history of science without philosophy of science is blind" (Lakatos, 1970, p. 138, emphasis added). My perspective, on this line of argument, is that of conceiving *structures as heuristics* in the process of conceptual reconstruction. In this sense, a heuristics labeled 'structural heuristics' may be proposed, in which the structural configuration of concepts should be identified before undertaking the historical reconstruction.

This essay examines the heuristic rule of structures (conceived as semantic spaces) in the reconstruction of concepts in economics. This will be done by addressing the concept of

[6] For the notion of rational reconstruction in economics see Scazzieri (1989)

‘axiomatics’ in economics. Economic axiomatization concerns the systematization of economic theories according to specific criteria (Vilks, 1998; see also Thomson, 2001). Axiomatization is characterized by a stable structure across different disciplinary domains (although domain-specific considerations inevitably inform each axiomatic process[7]). A ‘minimalist’ account sees any given axiomatic system as consisting of a *horizontal* and a *vertical* structure. From a horizontal point of view, any given axiomatic system consists of three conceptually separable sub-systems:

1) the sub-system of axioms;

2) the sub-system of rules of inference;

3) the sub-system of derived theorems[8].

If a vertical perspective is adopted instead, various alternative 4) ‘philosophies of proof’ may be considered. These ‘philosophies of proof’ express, ultimately, controversies on the logical foundations of mathematics, and provide different answers to the question ‘what is to have a *good proof*’, and thus ‘what is a good axiomatization’. These philosophies, although disagreeing on many points, substantially accept the tripartite structure of axiomatization.

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[7] An example of domain-specific issue in economic axiomatization may concern, for instance, the allowance or not – following the debate triggered by Milton Friedman’s methodological position – of ‘unrealistic’ axioms in the axiomatic system.

[8] David Hilbert, considered the father of the modern axiomatic method, identifies (Hilbert, 1917) as necessary and sufficient conditions for an axiomatic system the provision of 1) the sub-system of axioms and 2) the sub-systems of rules of inference. I consider the sub-system of theorems as a strict consequence of that provision. This is, as just noticed, and as we will briefly see in the next paragraph, a very minimalist account of axiomatic structure. For a description of the axiomatic structure from the point of view of formal systems see, for instance, Smullyan (1961, ch.1).

These distinctions are rarely taken into explicit consideration by historians of economics – although they are considered fundamental by practitioners of the axiomatic method. My claim is that those distinctions represent the 'conceptual space' for axiomatics, i.e. its structure (see Figure 1).

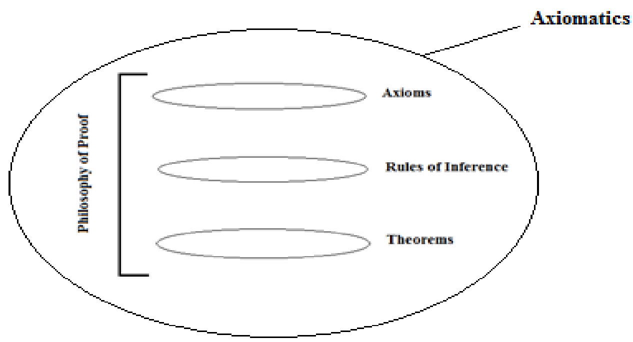

Figure 1: The conceptual space of 'axiomatics' and its internal structure

Each layer of the conceptual space shown in Figure 1 should be considered as an autonomous object of reconstruction. 'Structural heuristics' can so be thought as focusing device within the conceptual space. The concept of economic axiomatics is subject to variations along each structural layer. The different configuration of each structural layer can lead to altogether different versions of axiomatics. For instance, different views of the empirical status of axioms may lead to different versions of axiomatics. More generally, this approach to structural disentanglement along the structural layers highlights the *plurality* of economic axiomatics. Recollecting all the varying conceptions related to the structural layers yields the (always-provisional and mobile) boundaries of the plurality of economics axiomatics.

The heuristic nature of the structural investigation proposed here is worth of some further remarks. A reconstruction employing the structural heuristic method can be useful either i) when one already has in mind a precise question involving the concept to be reconstructed (e.g. when one wishes to reconstruct ‘in which way Von Neumann conceived axiomatics’) or ii) when one is just looking for orientation within the concept in question (e.g. ‘what is axiomatization?’ and ‘from where may I start reconstructing it?’). In both cases, we may appreciate the heuristic nature of structures. In fact, structural specification drives the reconstruction but, at the same time, the purpose of inquiry itself may be important in selecting which structural specification, in the case there are many, is more suitable for the inquiry at hand (see the next section). The nature of structures employed for reconstructive purposes is, in the end, instrumental and dialectical with one’s purposes[9].

This essay is organized as follows. Each section examines a specific layer of the axiomatic structure: axioms, rules of inference and theorems as horizontal layers, the philosophy of proof as vertical layer.

[9] Notwithstanding the approach employed in this essay can be called structuralist – for it considers structure(s) a central issue in the process of rational reconstruction – it is not ‘structuralist’ in the reconstructive perspective introduced by J. Sneed (1971) and W. Stegmüller (1979). First of all and decisively, our approach is primarily meant to reconstruct concepts, while Sneed and Stegmüller’s objects are theories. Further, the structure conceived by Sneed and Stegmüller is somehow fixed, while ours is heuristic and flexible. For an overview and discussion of the “structuralist” view as applied to economic theories see Hands (1985).

2.2. Axioms

We may start the analysis of axioms as autonomous object of reconstruction in economic axiomatics by asking the following question: 'to which extent may various conceptualizations of the notion of axiom be conceived as source of plurality for economic axiomatics?'. To put it another way: 'how many axiom modalities have been conceived – or may be virtually conceived – in the practice of economic axiomatization?'. This pattern of inquiry will be replicated in each of the subsequent sections, when dealing with the other layers of the axiomatic structure.

2.2.1. From atoms to molecules

In a 'minimalist' structure of axiomatics we may conceive of axioms as the fundamental units of analysis. However, if we employ a more refined specification of the structure of axiomatics, we can see this is not the end of the story. From a structural point of view, axioms are not 'atoms', i.e. indivisible units of analysis. Alfred Tarski's structural model of axiomatization (1956, ch. 10) provides, in fact, the further distinction between *terms* and *sentences* of an axiomatic system. While axioms belong to the domain of sentences, a more elementary level of analysis involves the terms composing such sentences. Among the latter, Tarski identifies a further sub-structure. 'Primitive' (or 'undefined') terms are the new atoms of an axiomatic system: combinations of 'primitive' terms yield 'defined' terms by means of the 'rules of definition' (see also Suppes, 1991). In an axiomatic system, the structuration of terms is analogue to the structuration of sentences, but pertain to a different process. The symmetry of these two levels of structuration (Tarski, 1956, p. 297) is shown in Figure 2.

Figure 2: Structure of *terms* and structure of *sentences* in an axiomatic system

Figure 2 shows the relationship between the 'minimalist' and Tarski's models of axiomatic system (Figure 2 shows, in particular, the hierarchic embedding of terms structure into sentence structure, in the specific layer of axioms). In many cases, this structural embedding may be justified according to the perspective that "a set of axioms is sometimes said to give an implicit definition of its primitive terms" (Haak, 1978, p. 245). According to this point of view, terms are not assumed as sequentially pre-determined and self-sufficient: terms and sentences may undergo a process of reciprocal and simultaneous assignment of meaning. Tarski's structural model of axiomatization shows that the specification of an axiomatic system is flexible within some structural constraints. Structural specification is subject to degrees of freedom, and these degrees of freedom play an important role in the reciprocal influence between heuristic tools and problems at hand.

The plural conceptualization of axioms may be addressed, at a first level of approximation, by assessing the gap between the common sense and the technical notion of axiom. When the layman refers to axioms, she is used to establishing an informal identity between axioms and some 'first principles' (see the entry 'axiom' in the *Oxford English Dictionary*). This approach holds true for disciplinary domains, in which axioms are thought to be the 'first principles' of a given discipline. However, this account is unacceptably vague. What is a 'first principle'? This is just the kind of question practitioners of the axiomatic method tackle, implicitly or explicitly, in their everyday job, and they give, still implicitly or explicitly, various and different answers to such a question. Robert Clower, pointing out the oft-misconceived role played by axiomatics in economics, stated that "unless axiomatics can be made to play a *critical* as contrasted to a *constructive* role, it is likely to be as little use to an empirical scientist as a broken saw to a carpenter" (1995, p. 310, emphasis in the original). If we were ready to look at axiomatics through the proper lens suggested by Clower, we would more easily recognize that there is no general agreement on the way in which to conceptualize axioms in axiomatics. On the contrary, the notion of axiom is one of the most controversial and 'plural' ones in economic theory.

2.2.2 The justification of axioms: the route between 'analytic' and 'synthetic' statements

Penelope Maddy (1988), emphasizing the often-overlooked point that axioms are in the actual practice a matter of discretional choice, claims that the correct question to ask regarding axioms is why "believing the axioms". Axioms, from this point of view, must undergo some process of justification. In economics, this point has been raised long ago by John R. Hicks, who asked the following question:

"Pure economics has a remarkable way of producing rabbits out of a hat – apparently *a priori* propositions which apparently refer to reality. It is fascinating to try to discover how the rabbits got in; for those of us who do not believe in magic must be convinced that they got in somehow." (Hicks, 1946[1939], p. 23)

With respect to the axioms of set theory, which were the objects of Maddy's analysis, the justification of axioms of empirical disciplines, such as physics or economics, is subject to further requirements of adequateness. In this respect, Herbert Simon claims:

"an axiom system may be constructed for a theory of empirical phenomena with any of a number of goals in mind. Some of these goals are identical with those that motivate the axiomatization of mathematical theories, hence relate only to the formal structure of the theory – its syntax. Other goals for axiomatizing scientific theories relate to the problems of verifying the theories empirically, hence incorporate semantic considerations" (Simon, 1959, p.443)

What differs between the axiomatization of "mathematical theories" and the axiomatization of "theor[ies] of empirical phenomena", according to Simon, is a sort of conceptual factorization of the latter into: (1) a sub-system (within the sub-system of axioms) that is equivalent to some axiom system for a part of logic and mathematics (2) and a remainder in which axioms correspond to *observation sentences* (p.444, emphasis added). 'Theories of empirical phenomena' would be characterized, following this point of view, by undefined terms such as 'commodity' and 'prices', if we look at the case of economics. The empirical dimension and the mathematical dimensions should be both explicit and detectable, i.e. factorizable.

Whether this factorization is attainable, or even desirable, is a matter I will consider closely. But before that, some remarks on economics as an empirical discipline are required. For also the characterization of economics as empirical science is not unambiguous, and exerts influence on the way we see plurality in economic axiomatics. Simon's characterization of the empirical component of axiomatic systems as corresponding to 'observation sentences' is a specific way of conceiving empirics, which we borrowed from the empirical methodological status of physics[10]. If observation is foundational for the empirical status of economics[11], 'observables' would correspondingly be the objects of economics. However, this is not uncontroversial. Uskali Mäki claims that economics' objects do not consist in 'observables' but, more extensively, in the so-called 'commonsensibles' (2000). While the observational status of 'prices' and 'commodities' is more or less acceptable, that of 'utilities' or 'preferences' is not unambiguous. Samuelson's notion of 'revealed preferences' has not proved to be conclusive in this respect (Wong, 1978). Even physics, however, suffers from the so-called 'T-theoretical' terms (Stegmüller, 1979; see Hands, 1985, pp. 308-309), such as 'force' and 'mass', whose empirical status cannot be detached from a specific theory that defines the way of their 'observability'. Observation is not however the only reliable source of economic principles. The tradition that Klant (1984) named 'empirical apriorism' according to which Classics such as Cairnes, Mill and Senior are associated with Austrians such as von Mises and Rothbard, claimed that first principles in economics are not established through observation. 'Introspection', in this perspective, is another foundational epistemic act for the empirical status of economics[12]. The epistemological justification of the empirical status of economics deeply influences economics, and all the more the conception of its axiomatics.

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[10] Simon (1959) discusses primarily axiomatics in physics. The observational treatment of axiomatics in physics goes back to Hans Reichenbach (1924).

[11] It would be too complex dealing with the nature and varieties of observations in empirical disciplines. Nonetheless, also at this level we cannot freely assume the homogeneity of the notion of observation.

[12] According to William Nassau Senior,"[premises] rest on a very few general propositions, which are the result of observation, *or consciousness*, and which almost every man, as soon as he hears them, admits, as familiar to his thoughts, or at least, as included in his previous knowledge" (Senior, 1827, p. 7, italics added). Still, according to Slutsky, diminishing marginal utility "can be founded upon some sort of *internal evidence*, not on the facts of

As claimed by a hopeful Georgescu-Roegen, "the fact that many economic controversies […] were caused only by the confusion between synthetic and analytic truths has paved the way for the increasing use of axiomatization" (1954, p.506). Axiomatization as foundational practice was welcome as the 'peacekeeper', as the appeaser of long-lasting controversies and dichotomies in the foundations of economics and the other empirical disciplines: the *apriori/aposteriori* one and the *analytic/synthetic* one (for a reconstruction of these dichotomies in the context of economics see Mongin 2006, 2007). Consequently, the issue of the empirical status of economics would have found resolution too[13]. What the supporters of axiomatization had not foreshadowed was that different answers could be proposed as resolution of these ancient dychotomies, raising new issues and controversies.

The recognized benchmark of economic axiomatics is that established by Gérard Debreu (1959) in his work on competitive general equilibrium. Debreu's work expresses what has been labeled the *formalist* approach to axiomatics[14], an approach that in principle aims at solving the analytic/synthetic ambiguity. Before treating in detail the technical features of Debreu's axiomatics, it is worth to emphasize what are the 'external' reasons of Debreu's popularity. These reasons, which concern the context in which Debreu proposed his

economic conduct. The generality of this conviction authorizes us to call it *faith in the consciousness of economic conduct*" (in Mirowski, 1989, p. 362).

[13] Stigum (1991) undertakes his monumental work of axiomatization of economics in view of its empirical foundations, i.e. econometrics.

[14] Debreu's axiomatic attempt is historically preceded by Von Neumann and Morgenstern (1944) for the sub-domain of game theory and by Arrow (1951) for the sub-domain of social choice. Both von Neumann and Morgenstern's and Arrow's works lack the programmatic radicalness associated to the work of Debreu. For the ambiguous reception of von Neumann and Morgenstern work as an example of 'formalist' work see Giocoli (2003, p. 30).

methodological standard, play a great deal in the *justification* process Maddy was concerned about. Two ingredients, in particular, contributed to Debreu's success: 1) the programmatic attempt to follow a higher standard of 'rigor' (a suggestive word, indeed) in axiomatics; 2) the amount of effort spent by Debreu in popularizing his axiomatic methodology. For what concerns the first point, many scholars (e.g. Mirowski and Weintraub, 1994, Düppe, 2010) have interpreted Debreu's program – that "the theory […] is treated here with the standards of rigor of the contemporary formalist school of mathematics" (Debreu, 1959, p. viii) – as if Debreu referred exclusively to the Bourbaki foundationalist methodology for mathematics (Bourbaki, 1950) that at that time exerted a decisive influence. Velupillai (2012) questions this reconstructive account, either on the programmatic ground or on the implementation ground. He claims, in fact, that the Debreuan work and the tradition of mathematical economics inspired by him is consistent with the approach to mathematics held by the *Polish School of Mathematics* in the interwar period, breaking in this way a decisive channel of Debreu's legitimation. For what concerns the second point, Düppe (2010) provided a careful reconstruction of how, after 1983 – when he was awarded the Nobel Prize in Economics –, Debreu pursued a massive operation of popularization of his way to look at mathematical economics. An operation that, according to Weintraub (2002), had a decisive influence upon the 'image' acquired by mathematical economics.

Debreu puts rather simply the distinctive feature of his methodology as follows: "an axiomatized theory has *a mathematical form that is completely separated from its economic content. If one removes the economic interpretations* […] *its bare mathematical structure must still stand*" (Debreu, 1986, p. 1265, emphasis added). The formalist standard convincingly endorse Simon's factorization: on the one hand, the syntax of logic and mathematics while, on the other, the semantics of economics. The programmatic acceptance of the factorization is however radicalized here, to the extent that mathematical form and economic content seem independent the one from the other (Mirowski and Weintraub, 1994). The only phase in which the mathematical form meets the economic content, breaking their orthogonality, is in the elaboration phase of the theory. In this phase Debreu lessen the extent of the disconnection by claiming that: “although an axiomatic theory may flaunt the separation of its mathematical form and its economic content in print, their interaction is sometimes close in the *discovery* and elaboration phase” (1986, p.1266, emphasis added). Debreu seems here to explicitly endorsing the fundamental distinction between discovery and justification contexts in axiomatics. While discovery allows communication, it is the justification that implies and requires disconnection.

Some economists addressed severe criticism to Debreu’s framework, assessing Debreu’s work through the lens of Debreu’s self-stated methodological prescriptions. Robert Clower (1995) convincingly shows that Debreu’s work suffers the continuous interference between syntactic and semantic domains. What should be syntax in Debreu’s intentions, Clower demonstrates[15], turns out to be in fact semantics. In line with Clower’s position, Philippe Mongin claims: “axiomatizations [à la Debreu] deeply alter the distinction between syntax and semantics” (2003, p.117, my translation). Ken Dennis (1996) on this point states that the mathematics itself used by Debreu prevents the perfect factorization of syntax and semantics. To achieve the objective of factorization, a complete first-order formalization would be required

[15] Basically, Clower demonstrates that the “language of the theory” is decisively molded around the economic content it bears. In this way, the language of the theory is not open to *any* new interpretation, but to the subset of interpretations allowed by the original structure.

(Dennis, 1996[16]). Debreu's methodological prescriptions look, in the light of these remarks, quite *paradoxical* in nature. In fact, rather than being a practical impediment, blurring the distinction between syntax and semantics (blurring that was doctrinally refused) proves to be an effective device in exploiting the deductive richness of the mathematical system (Mongin, ibid.).

Endorsing the factorization between mathematical form and economic content does not, however, say anything about the primacy of the former or the latter, which is, in the end, the point of major debate among different points of view on economic axiomatics. This is mostly visible in the debate concerning the notion of *truth* in axiomatics. Boylan and O'Gorman (2007) show that this notion was at the center of strong debate at the birth of the axiomatic method, when the 'formalist' view on axiomatics conflated with an 'intuitionist' view (Resnik, 1972). While David Hilbert claimed that the truth of an axiomatic system depends on the consistency of its axioms, Gottlob Frege claimed that truth would require a source of sense coming from outside the axiomatic system. Frege calls this source as 'propaedeutic' to the axiomatic system. Axioms, in Frege's view, should be definite semantic entities for the axiomatic system to be 'true'. The formalist/intuitionist debate has, as we have seen and we shall see further, its counterpart in economics too, according to the legitimation source of axiomatics: the issue is whether the requirement of consistency of the axiomatic base is *one* of the requirement or the *only* requirement of the axiomatic system. The notion of truth, quite out of practice in the economic discourse, is explicitly re-proposed by Clower, who connects it, first and foremost, to the empirical status of axioms. He claims, in fact, that "for disciplines rooted in plausible

[16] Dennis (2002) provides a framework in which to accomplish such an axiomatization, which is called method of 'numerics'.

reasoning [such as economics], *truth is of paramount importance*; the aim is not to state and prove theorems but to persuade others that proffered empirical conjectures are sufficiently plausible to deserve closer scrutiny" (Clower, 1995, p. 308, italics added).

2.2.3 Degrees of empirical commitment

The most tangible implication of the adherence to formalism concerns the empirical commitment of axiomatics. An axiom system devised following the formalist standard is not judged anymore according to its representativeness of an underlying empirical phenomenon. The requirement of consistency in formalist axiomatics has led, according to Giocoli (2003, p.3), to the rise of "non-descriptive" axiomatic systems. The empirical and semantic content of formalist systems becomes programmatically 'minimalist', since that empirical content is not anymore a decisive criterion of successful axiomatics. On the contrary, the detailed specification of the empirical and semantic content of theories is inversely proportional to the possibility to gain new interpretations of axiomatic systems.

However, after the methodological benchmark was established, economists devoted more effort in the treatment of axioms as working tools rather than as objects of discussion concerning the foundations of mathematical economics. The issue of the empirical status of axioms has re-gained attention even in an allegedly formalist context. In this connection, Frank Hahn (1985) specifically clarified the distinction between *axioms* and *assumptions* in economic axiomatics. According to Hahn:

> "[…] we need to distinguish between axioms and assumptions. That people have preferences and try to satisfy them we treat as an axiom, while universal perfect competition, for instance, must concern as an assumption. By this I mean that neither introspection neither observation makes itself evident up to an acceptable

margin of error that agents are price-takers in all markets. […] They serve to simplify – that is to make an argument possible – and they also often try to encapsulate a sort of causal empiricism" (1985, p.18)

The role of assumptions was not new in economic axiomatics (Debreu used the axioms/assumptions distinction in 1959[17]), but Hahn clearly envisaged the existence of degrees of empirical commitment: axiomatization starts by stating very general propositions (axioms) and tries to assign to assumptions the role of empirical specification[18]. Note that Hahn refers to both observation and introspection (the 'intuition' of empirical apriorists), and judge them unsuitable foundations for the empirical status of assumptions. It is as if assumptions were judged at the same time more 'empirical' than axioms, but not so much as to gain a complete empirical status. Hahn addresses the question of the empirical status of assumptions in terms of the relationship between axioms and assumptions: this approach of co-determination seems similar to the approach of co-determination between undefined terms and axioms considered above. Axioms and assumptions are, according to Hahn, 'semanticized' entities.

Lastly, it is worth to mention the view on the empirical status of axioms outlined by Janòs Kornai (1971). Kornai endorses a 'representationalist' perspective on economic axiomatics, more explicitly devoted to express what economists mean with the notion of

[17] About the distinction axioms-assumptions for Debreu, Düppe remarks: "Assumptions are not specified on primitive concepts, let alone their referential meaning, but on the mathematical objects that substitute for these primitive concepts. To speak about 'assumptions' is misleading in the sense that Debreu does not assume something to be the case which, as with hypotheses, suppositions, or basic beliefs, could be at stake when theorizing, What Debreu calls 'assumption' are, rather than epistemic priors, the *result* of the axiomatization, - the axioms" (2010, p. 12. italics in the original)

[18] This distinction may be considered a natural reaction to the problem of 'falsification' of axiomatic systems: using a Lakatosian terminology, while assumptions constitute the 'protective belt', axioms constitute the 'core' of an axiom system. Establishing an empirical asymmetry between axioms and assumptions in an axiom system decreases the degree of falsifiability of the system.

‘model’ (Gibbard and Varian, 1978). In this way, he wants to preserve the rigor in the constitution of an axiom system (e.g. the consistency of axioms, their independence, etc.) while explicitly assigning a fixed semantics to the axiom system under consideration. Axioms immediately express a ‘vision’ of the economic environment – that is, in the case of Kornai, they outline “systemic vision” – articulated in levels (*institutions*, *organization*, and *units* within individual organization) that interact as processes (*real processes* and *control processes*). Kornai claims that this reform (the direct semanticization) in the conception of axiomatics in economics would lead to the provision of “axioms of *more general force*” (1971, p. 374) , emphasis added). The general forces expresses in the multiple economic phenomena, such as disequilibrium states other than equilibrium states, which can be encompassed by the same set of axioms. Kornai’s perspective on axiomatization differs from Debreu’s, because in his case semantics is not “disconnected” from syntax (not even programmatically), so that interpretation is somehow stabilized; but it also differs from Hahn’s because it is does not entail a difference of empirical status among economic propositions.

The reconstruction of the multiple conceptions of axioms into an axiomatic system has showed that at this structural layer the crucial (aspirational) question of the distinction between *analytic* and *synthetic* statements emerges decisively.

2.3. Rules of Inference: ‘mathematics’ style vs. ‘logics’ style of axiomatics

In economics – as in other disciplines relying heavily on axiomatics – we see typically much debate concerning the issue of ‘accuracy’ and ‘carefulness’ of actual axiomatizations.

Such a debate, however, does not seem to be driven by methodological awareness that axiomatics is a proper *method* of inquiry, and not only a *technical practice*. The fluctuating conceptualization of axiomatics as a method of inquiry or as a technical practice has important consequences for the appropriate categorization of economic axiomatics.

The different accounts of axiomatics provided by Wilder (1967) and Dalla Chiara and Toraldo di Francia (1981) reflect such ambiguities. On the one hand, Wilder distinguishes among three "types" of "axiomatic *methods*". He identifies 1) an "Euclidean type", in which "primitive terms are not treated as undefined and only one model is described" (p. 122); 2) a "type used by the so called 'working mathematician[s]' [among whom the author places himself] […] careful to list our primitive terms, but we list neither the logical nor the set-theoretic rules by which we shall abide; hence the term 'naïve axiomatics'" (p. 124); 3) a "third type of axiomatics, in which the logical apparatus not only enters the discussion, but is explicitly formalized" (p.126). On the other hand, Dalla Chiara and Toraldo di Francia propose their tripartite distinction among

"*Semi-axiomatization*: a theory is semi-axiomatized when the distinction between its initial hypotheses and the demonstrated propositions is not completely clear;

Non-formal axiomatization: a theory is axiomatized in a non-formal way when all its initial hypotheses are rigorously made clear, although its language is not formal (completely controllable) but a vague one, characterized by the ambiguities of common language;

Formal axiomatization: a theory is axiomatized in a formal way when it is rigorous either linguistically or deductively" (Dalla Chiara and Toraldo di Francia, 1981, p.10, my translation).

Although these two accounts look apparently the same one, they differ insofar as Wilder proposes tripartition as a sequence of distinct types of *axiomatic methods*, while Dalla Chiara and Toraldo di Francia conceive the tripartition as a sequence of level of *axiomatic perfection*. This is an important difference. For Wilder's account, while emphasizes the methodological character of different axiomatics, leads to a *pluralistic* account of axiomatics; while Dalla Chiara and Toraldo di Francia's, which emphasizes the technical nature of axiomatization, leads to a *monistic* account. From the latter point of view, all actual axiomatizations can be subject to evaluation in terms of accomplishment relative to a uniform standard. Rules of inference are the structural elements of axiomatics usually called into question when assessing 'accuracy'. They are the structural elements responsible of the inferential process in axiomatization. The trigger of my inquiry is that in actual economic axiomatization they are usually not well explicit. This is, I claim, an issue that is worth of attention. Such an acknowledgment – i.e. the lack of explicit rules of inference in axiomatization – would lead to a negative judgment by supporters of the monist position: they would argue that axiomatics needs explicit rules of inference. But what about the supporters of pluralist axiomatics? Is it conceivable that axiomatizations could be executed well while not making explicit and accountable its deductive apparatus? This is the issue dealt with in this section. In particular, in this section I will consider the plausibility of the distinction, in economic axiomatics, between a 'mathematics style' of axiomatization and a 'logic style' of axiomatization. Many axiomatizations in economics have a similarity of *gestalt* with axiomatizations in mathematics, in particular for what concerns a relative lack of attention for the rules of inference. Others axiomatizations are closer to formal logic, for they make explicit and are attentive towards the rules of inference they use. This section examines whether such empirical categorizations rest solely on an intuitive basis, or if they have a deep theoretical underpinning. To such an objective, a digression in the philosophies of logic and mathematics is inescapable.

To inquire the conceptual and disciplinary relationship between mathematics and logic the issue of 'logicism' has to be considered in the first place. 'Logicism', as Jakko Hintikka states, "can be characterized as the doctrine according to which mathematics is, or can be understood as being, a branch of logic […] Another formulation says that according to logicism mathematics can be reduced to logics" (2009, p. 271). A preliminary appraisal of the feasibility of the logicist program(s) – in particular its 'reductionist' version – would consider the extent of commensurability of the respective objects of these two disciplines. For what concerns mathematics, it is common wisdom that contemporary mathematics is primarily a 'science of structure' (Corry, 1996). In this respect, as Hittinka claims, "[m]athematical theorems deal with what is true in a certain structure […]. In contrast, logical principles deal with logical truths. These are not subclass of truth simpliciter, that is truths in some one structure. They are truth in every possible structure". In this way, "mathematical and logical system are not commensurate in a natural and widely accepted perspective" (Hintikka, ibid.). This and other major technical and conceptual problems lead to the acknowledgment that the reductionist program was unattainable: logic and mathematics should properly be built in parallel instead of the one reduced to the other. Starting from David Hilbert, the reductionist program have been re-considered as an issue of 'metatheory' or of 'metamathematics', as it is currently known, i.e. a study of formal systems.

Logicism has however survived in its more interesting version, i.e. considering mathematics as a branch of logic. The logicist program today consists in inquiring the '*shift of emphasis from axiomatic reductions of mathematics to logic to comparisons of mathematical and logical modes of reasoning*" (ibid. p. 284, emphasis added). A distinction between mathematics and logic can be properly supported, in this perspective, on the ground of inferential processes. Set theory plays here a fundamental role, as "axiomatic set theory came to be considered widely as the natural medium of mathematical reasoning and theorizing" (ibid., p. 279): in this way, set theory is nowadays widely accepted, in its axiomatic form, "as a theory of mathematical modes of inference" (ibid., p. 284)[19]. In this way, we could categorize, at a first level of approximation, axiomatizations that rely on set theory as employing a 'mathematical' and not strictly a 'logical' mode of inference.

This is not the end of the story however, since we can go even further, characterizing the provisional distinction between mathematics style of axiomatization and logic style of axiomatization (a distinction that will be useful when coming back to axiomatizations in economics) on the ground of different interpretations of the notion of "logical consequence". Logical consequence is defined as a relation between claims – for instance A and B – in which B is 'logical consequence' of A whereas B follows *logically* from A. But what does 'follows logically' mean? In a widely accepted perspective, the notion of logical consequence is assimilated to that of *deduction*; in particular, for the manipulation of symbols by means of rules of inference guarantees the correct accomplishment of deductions, logical consequence in this perspective is associated with the notion of *deducibility*. Nonetheless, that of deduction is not the only way the notion of logical consequence may be conceived, neither the most up-to-date. The notion of *model-theoretic consequence*, complementary to that of *deductive consequence*, emphasizes the semantic (truth-value) side of the relation of consequence. A *model* of a formal language is a function that assigns a truth-value to each closed formula of

[19] That axiomatic set theory constitutes a theory of mathematical modes of inference is however not uncontroversial. As Hittinka perceptively notices, there is somehow an ambiguity of categories whereas the Axiom of Choice, which codifies the "mathematical inference pattern *par excellence*" is treated in axiomatic set theory as an axiom and not as a theorem.

the language or, as Tarski states, "a possible realization in which all valid sentences of a theory T are satisfied is called a model of T" (as quoted in Suppes, 1960, p. 287)[20]. In formal systems, a deductive apparatus have to be thought as separated from a logical consequence apparatus. Surely, "a central question that arises for a formal system with a model-theoretic apparatus is that of the coincidence between the relation of model-theoretic consequence and the relation of deducibility" (Blanchette, 2001). Formal systems should be devised in principle so as to satisfy both apparatuses: these are the requirements of *soundness* (of the model-theoretic apparatus) and *completeness* (of the deductive-apparatus) of a formal system. In terms of Toraldo di Francia and Dalla Chiara, the satisfaction of both soundness and completeness would be a requirement of 'perfection' of the axiomatic system. However, in practice, it is not sure whether the model-theoretic apparatus and the deductive apparatus are orthogonal, in the sense that the satisfaction of the deductive requirements is not even harmful to the model-theoretical apparatus. As Suppes remarks, "does being a meaningful [model-theoretic] logical consequence imply derivability by the rules of inference? This problem has not been solved […]" (Suppes, 1965, p. 368). It is still currently a problem indeed. As Hintikka states, "from the vantage point of the services that the axiomatic method can provide, any restriction to mechanizable *consequence relations is not only unnecessary but positively harmful*". In fact "proof-theoretical consequence relation cannot capture all the actual model-theoretical consequences of the axioms that the system is calculated to capture as its theorems" (Hintikka, 2011, p.76, emphasis added). At a certain point, it seems, a choice between model-theoretic way of thinking

[20] In this way, "where ∑ is a set of formulas of a formal system S and ϕ is a formula of S, we say that ϕ is a *model-theoretic consequence* in S of ∑ if every one of S's models that assigns *true* to each member of ∑ also assigns *true* to ϕ" (see Blanchette, 2001).

and proof-theoretic way of thinking is necessary. This choice seems to involve the emphasis to put on the rules of inference conceived as part of the deductive apparatus too.

In economics, Debreu's axiomatization of general equilibrium follows consciously a model-theoretic way of thinking, founded on a set-theoretic language. It was an explicit filiation of mathematical foundationalism *à la* Bourbaki. The model-theoretic interest is patently expressed when Debreu states that "the divorce of form and content", that is the supposed gist of Debreu's axiomatics – "immediately yields a new theory whenever a novel interpretation of a primitive concept is discovered" (Debreu, 1986, p. 1265). Theoretical progress, according to Debreu, is related to the shift from the *standard* model of the theory to *non-standard* models of it. At this point one may ask, legitimately, whether the lack of explicit rules of inference is for Debreu a sort of theoretical choice – being an expression of the mathematical style of axiomatization – or a simple inaccuracy in axiomatization[21].

Nonetheless, in economics we find instances of axiomatization that explicitly employ the tools of propositional calculus and predicate calculus. These contributions make their deductive apparatus (rules of inference) explicit. Axiomatizations of this type are particularly accomplished in the domain of game theory (Bacharach 1982, Aumann, 1999; Lismont and Mongin, 1994; Kaneko, 2002), which make use of epistemic logic – a branch of modal logic, on its turn an extension of classical logic – specifically designed to deal with reasoning about knowledge. Axiomatizations using this logic, to make an interesting example, relies on the so-called 'rule of epistemization', which is a specific rule of inference of epistemic logic. Here,

[21] Debreu has never dropped the emphasis on correct deductions in theorizing: on the contrary, he emphasized it programmatically (see Debreu, 1986). Suppes (1960) attests the 'consistency' of Debreu's axiomatization. In Debreu, however, we find the usual non-formal proofs characteristic of 'working mathematicians'.

the explicitness of the rules of inference plays a full-fledged role, not serving only the purpose of deducibility, but also the semantic content of the system (the ‘rule of epistemization’ is a rule of inference that expresses at the same time a relation of deducibility in the system and the semantic content that when people ‘state’ something they ‘know’ that thing)

Mongin (2003) explicitly supports the distinction between a mathematical way to axiomatize (labeled ‘set-theoretical axiomatization’) and a logic way to axiomatize in economics. Mongin’s benchmark – against which he compares both the mathematical way and the logic way – is that of ‘formal systems’ (see Smullyan, 1961). He lists the essential requirements of formal system (e.g., ibid, ch.1) and checks whether the two kinds of axiomatizations satisfy these requirement. For what concerns the rules of inference, Mongin claims that in ‘set-theoretical’ axiomatization there are rules of inference, “at least analogically” (p. 109). “Even not specifying a notion of rule of inference, at least implicitly they [theorists who follows a set-theoretical way] delimit allowed operations on signs” (p. 110), such as vector spaces manipulations and set-theoretical operations in general. Relying on the benchmark notion of formal system, Mongin seems to re-conceptualize the distinction mathematics/logic in a form of monism, neglecting – in a loose interpretation of the notion of rule of inference – the *fundamental difference between rules of mathematical structure and general rules of logic reasoning*. It may in fact be argued that adopting formal systems as benchmark for all axiomatizations could be detrimental to assessing the plurality of axiomatizations. As Armatte (2004) argues, in response to Mongin, formal systems are not any more the benchmark for mathematics. This is explicitly expressed by Backhouse (1998, p. 1848), who claims: “if mathematics is informal, then perhaps we should accept that economics must be informal too”.

The search for 'analogical similarities' between formal systems and actual pieces of axiomatization is, according to this perspective, somehow redundant.

A last remark for economic axiomatics is in order. From what I have discussed regarding logical consequence, it seems as if employing set theory and model theory would be naturally incompatible with taking care of the deductive apparatus. Put it that way would be a nonsense: the fact that some interference could emerge in theory and practice is not a sufficient reason not to try to accomplish both as best as possible. Bernt Stigum (1990) developed his axiomatic attempt with this goal in mind. He tries to develop a set-theoretic, first-order axiomatization explicitly considering logical rules of inference, like 'modus ponens'. The limits of such an approach however, are well described by Wolfgang Stegmüller, who claims that constructing scientific theories through first-order logic is simply "not humanly possible" (1979, p. 5).

This section has argued that the issue of accuracy in axiomatization is often a 'false problem'; it actually hides deeper conceptual issues, which lead to different categorizations of axiomatics. This point has been discussed by examining the structural level of rules of inference, while acknowledging that they are not always made explicit in actual axiomatizations. It has been shown that the explicit character of rules of inference cannot be reduced to that of accuracy. Accuracy involves instead the distinction between 'truth in mathematical structures' and 'logical truth' as well as the distinction between the notion of 'deductive logical consequence' and that of 'model-theoretic logical consequence'. Insofar as these two pairs of notions are not implementable hand-by-hand in a definitive way, categorical distinction in economic axiomatics is introduced: that between a 'mathematical' way to axiomatize and a 'logic' way to axiomatize. This result has an important consequence for economic axiomatics, as it calls attention to the true fact that hanging on the benchmark of formal system may be a will o' the wisp. In short, a common normative schema for the axiomatics assessment in economics, hinging on the benchmark of formal systems, could not be the best way to aspire to[22].

2.4. Theorems

Theorems are the output of the process of axiomatization. They are propositions to which the derivational process awards a prominent status. In this section, I provide some distinctions that may be useful to address the methodological and epistemological status of theorems in axiomatics. The following passage by Alfred Tarski, concerning what he labels 'equipollent' (deductive) systems, can be useful in this respect:

"It is important to realize that we have a large amount of freedom in selecting the primitive terms and the axioms; it would be quite erroneous to believe that, in a given theory, certain expressions cannot possibly be defined, or that certain statements cannot in any way be derived, and that ,therefore, they have to be regarded as primitive terms or as axioms, respectively. Let us call two systems of sentences of a given theory EQUIPOLLENT, if each sentence of the first system can be derived from the sentences of the second […] and conversely if every sentence of the second system can be derived from the sentences of the first […] If a situation of this kind arises, then, from the theoretical point of view, *one could reconstruct the entire theory in such a manner that the*

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[22] In this section, I focused on the distinction between 'mathematic' style and 'logic' style of axiomatization, triggered by the phenomenal lack of interest in rules of inference in many actual axiomatizations. Given the prominence of this topic into the section, I leave to this note a further way in which the structural elements of rules of inference could make the notion of axiomatics plural. Hintikka (1998, pp. 513-514) distinguishes between 'definitory' rules of inferences, employed in the normative construction of axiomatization and 'strategic' rules of inference, conceived as rules of search within a given information space (in this case, the axiomatic base). Hintikka's distinction could be employed to characterize, in a structural way, Suppes' categorical distinction in axiomatics between 'heuristic' and 'non-heuristic' axiomatizations (1983).

statements of the new system are taken as axioms, while the former axioms are proved as theorems” (p.121, capital letters in the original, italics added)

As a matter of fact, we can conceive two axiomatic systems in which the axioms of the first can be proven as theorems of the second and, conversely, the axioms of the second can be proven as theorems of the first. This implies that, in axiomatization, we cannot associate to specific propositions a specific *epistemic status* – i.e the status of precondition to knowledge (as axiom) or of output of knowledge (as theorem) - *once and for all*. This epistemological issue clearly collapses into a methodological one. It is up to the designer of the axiomatic system – the theorist – to choose which propositions to treat as axioms and which to treat as theorems in the process of axiomatization. This choice is made in order to make the axiomatic process a knowledge process that satisfies the knowledge requirements of the discipline in which the axiomatization is attempted. This issue is well expressed by Frank Hahn, when he states

> “that there is no axiom to the effect that an economy is in equilibrium at all times. If there were, then Arrow and Debreu certainly wasted their time. For it would be exceedingly odd to accept this axiom and prove at the same time that equilibrium exists” (Hahn, 1985, p. 21)

The paradoxical case envisaged by Hahn is that of a ‘tautological system’, in which the axioms coincide with the theorems to be proved, i.e. the extreme case in which two equipollent systems collapse into the same one. A paradoxical case indeed, because this would be the case of knowledge process yielding no knowledge at all, neither a wrong one. This entails that theorists who rely on axiomatics have to make a definite choice, of which propositions to treat as axioms and which one as theorem, to avoid axiomatic triviality and, possibly, to gain economic salience.

This state of affairs has important consequences for the categorization of axiomatics. Two alternative approaches to economic axiomatics are consistent with such a picture. On the one hand, we can conceive economic axiomatic as a method whose scope is always to prove some *specific* theorems, and not others. Consider the case of the notion of economic 'equilibrium', the example most historically important. From Adam Smith to Arrow-Debreu and beyond, economic equilibrium has been something to 'prove' – i.e. to justify in some way – rather than to 'assume'[23]. Following this perspective, the axiomatic process consists in a process in which the axiomatic base is selected, in order to prove desired theorems. The manipulation of axiomatic base becomes central here. Arnis Vilks (1992) shows that the core notion of equilibrium (*à la* Arrow-Debreu) is compatible only with the notion of equilibrium as 'simultaneous optimization' while, if a stronger notion of equilibrium is desired, like for instance "stationary state", further assumptions are required. That of selecting a minimal axiomatic base to reach a desired theorem seems the most important skill involved in axiomatization; to the extent that axiomatization itself is often identified with such a kind of procedure.

Another perspective on axiomatization is identifiable. This consists in assigning to axioms a role well beyond that of a preliminary condition for knowledge: in this approach, axioms become part of the non-negotiable content of knowledge itself. Accordingly, axioms acquire a pivotal role, and then the process of axiomatization starts not only formally, but also substantially, from them. The axiomatization of normative economics, i.e. that branch of

[23] Henry Woo (1984, p. 74) labels the notion of economic 'equilibrium' – aside others such as 'symmetry' and 'evolution' – as "epistemic archetypes" in economic theorizing. I claim, however, that the notion of 'epistemic archetype' might be misleading in this context: archetypes are commonly conceived as *a priori* entities that do not need any justification, while 'equilibrium' has always underwent some process of justification in economics.

economic theory founded on value judgments, fits this axiomatic perspective. Mongin (2003) considers Arrow's (1951) as the first example of axiomatics of normative theories. Mongin identifies, in this way, the category of *normative axiomatics*, according to which axioms (as expression of value judgments) are treated as non-negotiable priors, whose extent is inquired by means of rules of inference. That is, what is inquired are all the potential implications (the 'space of implications') of such priors. The justification of this epistemic procedure can be traced back to Samuelson, who had written that "[i]t is a legitimate exercise of economic analysis to examine the consequences of various value judgments, whether or not they are shared by the theorist." (Samuelson 1947, p. 220).

The category of normative axiomatics can be generalized, so as to encompass cases other than that of value judgments alone. In this broader sense, axiomatization is the process of inquiring (by means of rules of inference) the extent of any premise taken as non-negotiable content. Think, for instance, of the already-mentioned tradition of 'empirical apriorism' or, more broadly, of the apriorist traditions in general, whose content does not need axiomatic justification, and so does not require the formulation of theorems. The label of *constructive axiomatics*[24] can be used to encompass under a unique label all such cases – such as that of normative axiomatics – in which axiomatization starts from whatever apriorist position. On the other hand, *descriptive axiomatics* is oriented to prove specific theorems by means of the manipulation of the axiomatic base, as I have already discussed. The distinction between these two kinds of axiomatics (Krygowska, 1971; De Villiers, 1986) is meaningful only insofar as

[24] Reichenbach originally introduced the label 'constructive axiomatics' to characterize that notion of axiomatization founded on 'observables' (see also note 10). I employ this notion here with a different meaning, following the current standard.

the epistemic distinction between axioms and theorems stands. In other words, if one treats a given content as a theorem, one is implicitly endorsing that this content needs justification within the axiomatic process. On the other hand, the treatment of the same content as axiom would be legitimate only if this content gains legitimacy somewhere else. Krygowska, (1971) and De Villiers (1986), who explicitly introduce the dichotomy between *constructive* and *descriptive* axiomatics, propose this distinction in the context of the didactic use of axiomatization, an employment that has been often neglected in economics.

A slightly different distinction in axiomatics is that introduced by Mongin (2003) between *definitional* axiomatics and *theorematic* axiomatics. They are characterized such that "the former establishes a general framework, within which axioms do not contribute to results directly; the latter uses axioms to prepare a mathematical results directly" (Agliardi, 2004, p. 123). In other words, pieces of definitional axiomatics could in principle neglect the requirement of *completeness* of an axiomatic system, insofar as the axiomatic base exceeds the minimalist configuration necessary to prove the theorems of the system. Definitional axiomatics and theorematic axiomatics are *ex-post* categorizations, for they reconstruct actual pieces of axiomatization, while nothing, from the theoretical point of view, prevents axiomatizations to be at the same time 'definitional' or 'theorematic' at the same time (see, for instance, Bacharach, 1987).

The status of theorems as 'derived' propositions from axioms raises the issue of the semantic relationship between axioms and theorems. I have touched on this issue while distinguishing a model-theoretic logical consequence, essentially aimed at preserving truth (semantics) in derivations, and a proof-theoretic logical consequence, aimed at preserving deductive (syntax) rigor. *Completeness* is that property of formal systems according to which all propositions that are true are provable and, conversely, *soundness* is that property of systems for which all provable sentences are true. This conceptual scheme helps us to tackle Gottlob Frege's question of 'what means of a proposition to be *contained* in the premises'. Famously, Frege wondered what 'contained in the premises' means. According to Frege, the term contained can be assigned two different meanings: contained "as plants are contained in their seeds, [or] as beams are contained in a house" (Frege, 1953, p. 101). To put it in a relevant way for economics, from the premises that people have such and such kind of preferences (together with other assumptions), in which sense can we say that market is in equilibrium? In which sense can this relation be 'semantically' addressed? Which is the semantic relation between 'preferences' and 'equilibrium'? Do the derivations come from the premises like in the seeds-plant relation or like in the beams-house one? In other words, in which way the derivations are semantically traceable back to the premises? The issue of 'containment' between axioms and theorems can be re-interpreted in terms of the *information* that flows from axioms to the theorem. This new point of view on this issue, emphasizing the role of information flowing through axiomatization, expresses decisively the operational role of axiomatics. In Hintikka's words, the problem is expressed in a very simple way: "here we seem to have a paradox, maybe even a contradiction, in our hands. The very purpose of the axiomatic method was said to be the study of the models of the theory by deriving theorems purely logically from the axioms. But it is now suggested that purely logical inferences cannot yield new information. Something appears to be quite wrong here" (Hintikka, 2011, p. 77). Hintikka (2007) tries to overcome this apparent 'paradox' of information, developing the notion of 'surface information' and that of 'depth information'. According to Hintikka, what the logical derivation of a theorem conveys is a set of abstract disjuncts about the world, i.e. 'surface information' or 'explicit information'. 'Depth information' or 'implicit information' is the residual information about the world that cannot be assessed by means of purely logical operations, and that can be only assessed probabilistically. This distinction allows us to assess the relationship between axioms and theorems on the basis of the relationship between surface and depth information they convey.

A last taxonomical point, for what concerns theorems in axiomatics, is related to *robustness* of axiomatizations. The relationship between economic modelling and robustness has been at the center of much inquiry in the last decades (Gibbard and Varian, 1978; more recently, see Kuorikoski, Lehtinen and Marchionni, 2010). The notion of robustness concerns the status of economic theories, whose derivations are stable under some range of different assumptions. Robustness, in other words, falsifies Deidre McCloskey's provocative 'A-prime/C-prime' theorem on the development of economic theories, stating that:

> "For each and every sets of assumptions A implying a conclusion C, there exists a set of alternative assumptions, A', arbitrarily close to A, such that A' imply an alternative conclusion, C', arbitrarily far from C" (McCloskey, 1993, p. 235).

Robust derivations gain a higher degree of reliance than fragile results, to the extent that fragile results might be deemed as 'idiosyncrasies'[25]. In addition to this notion of robustness, labeled 'inferential robustness', Woodward (2006) identifies other three types of robustness, among whom that of 'derivational robustness' has a particular importance in this context. Derivational robustness applies to the cases in which a theory is parametrized, so that the outcome of a theorem is not sensitive to different parameters, otherwise we are in the case of 'derivational fragility'. In the context of economic axiomatics, these two notions, inferential and derivational robustness, could be source of different categorizations. To the general

[25] Cartwright (1991) and Woodward (2006) however denies that robust results should be awarded a higher status of truthfulness than non-robust ones.

distinction between *robust axiomatics* and *ad hoc axiomatics*, that of *inferentially robust* axiomatizations and *derivationally robust* axiomatizations could be further added. In particular, the notion of derivational robustness can be employed to assess the model-theoretic apparatus of a theory.

In this section, I have tried to consider the different ways in which the notion of theorem may be considered source of plurality in economic axiomatization. A series of methodological remark have been proposed and, accordingly, a series of taxonomic distinction have been provided, such as that between *constructive* and *descriptive* axiomatics (as a generalization of the distinction between *normative* and *descriptive* axiomatics), that between *definitional* and *theorematic* axiomatics, that between *surface information* and *depth-information* in axiomatics, that between *robust* and *ad-hoc* axiomatics, and lastly that between *inferentially robust* and *derivationally robust* axiomatics.

2.5. The vertical specification of axiomatic structure: Velupillai's proposal for a *Computable* and *Constructive* Economics

This section examines the vertical specification of axiomatic structure by considering in particular the foundational framework for mathematical economics proposed by the economist K. Vela Velupillai. The reason for this choice is twofold. First, Velupillai proposes a new conceptualization of the foundations of mathematical economics that involves – as far as the implications for axiomatics are concerned –all the three layers of the axiomatic structure at the same time (in this way, it is a vertical perspective on axiomatics). Second, I claim that Velupillai's proposal stems from a genuine 'structural heuristic' point of view on the history of economic thought. Reconstructing the ‘structural heuristic’ gist of Velupillai’s theorizing will allow us to inquire into the ‘generative’ role of ‘structural heuristics’, showing that awareness of structures can be the starting point of new theorizing in economics.

The background of Velupillai’s theorizing is provided by the controversies arisen over the nature and role of the notion of *infinity* in mathematics in the early decades of 20th century. ‘Finitism’ is, broadly speaking, a methodological perspective according to which either the objects of mathematics or the mathematical derivation procedures have to be in some way finite. Hilbert (1926) developed a specific notion of ‘finitism’ – strictly linked to his notion of axiomatics – in which the requirement of finiteness is satisfied through a finite number of variables and derivations in the axiomatic systems, while the mathematical objects are allowed to be *not finite in nature*. The set-theoretical foundations of mathematics express fully this perspective. The notion of infinity underlying this perspective is that of the ‘actual infinity’, as developed by the mathematician Georg Cantor, according to which infinity is just intuitively ‘assumed’, independently by its effective attainability. Poincarè puts in contrast to this notion that of ‘potential infinity’, according to which mathematical objects have to be thought as ‘recursive’ or ‘recursively enumerable’, that is reachable in principle starting from a finite procedure of count. The employment of the notion of ‘actual infinite’ in proofs would lead, according to Poincaré, to major epistemological problems. As Poincaré claims:

> “a theorem must be capable of proof but since we ourselves are finite, we can only deal with finite objects. Thus, even though the notion of infinity plays a role in the statement of a theorem, *there must be no reference to it in the proof*: otherwise the proof is impossible” (Poincarè, in Boylan and O’Gorman, 2007, p.436).

Boylan and O'Gorman label Poincaré's argument "anthropological-epistemological principle" (2007, p.440), identifying the violation of this principle as a source of illegitimate epistemological status of mathematical proofs.

Velupillai's primary focus concerns the satisfaction of the 'antropological-epistemological principle' in mathematical economics. In modern terms, this means inquiring into the *computability* and *constructivity* status of proofs in mathematical economics. Classical foundations of mathematical economics in the Bourbaki, set-theoretic tradition entail that proofs do not in general satisfy principles of *computability* and *constructivity* as conceived by those branches of logic and mathematics such as proof theory, computability theory and the various forms of constructive mathematics (Mathias, 1992). Computability and constructivity of mathematical objects concern, in general, the property of these objects to be reached by means of finite instructions or calculations, i.e algorithms[26]. Through the label *Computable and Constructive Economics* Velupillai makes a plea for an 'algorithmic revolution' (Velupillai, 2011) in mathematical economics, in order economics to be inspired by computable and constructive principles and methodology[27]. As he points out:

"The key word [of *Computable Economics*] is 'effective' referring to a procedure whose execution is specified in a finite series of instructions, each of which is finite in length and where all the detail of the execution

[26] In this respect, emblematic is Bourbaki's stance that "anything that was purely the result of calculation was not considered by us a good proof" (as quoted in Hintikka, 2011).

[27] Mirowski (2002) depicts the story of 19th century economics as progressively characterized by computational methods and metaphors. Nonetheless, according to Mirowski, Velupillai's argument is a step more radical: "[Velupillai's] argument seems to be that since the neoclassical orthodoxy persists in treating the agent in a sloppy and unrigorous way as a utility computer, why not go the whole 9 yards and make the agent a formal abstract computer?" (Mirowski, 2004, p. 126), shifting the story to that of 'computability'. In this way, the label *Computable economics* has not to be confounded with that of *Computational economics*.

are specified exactly, whereby leaving no room for magic, miracles, or such metaphysical entities […]"(Velupillai, 2000, p.1)

Effectivity is, from the computable and constructive point of view, matter of a 'Terrestrial Paradise', ruling out too heavy metaphysical assumption (e.g. the existence of the 'actual infinity' as a *fiat* entity) of the 'Cantorian Paradise' in which Hilbert placed axiomatization. What might be considered at a first glance as an intellectualistic shift – for economists use mathematics in an apparently unproblematic way, one would say – is in fact a perspective full of pragmatic implications. In fact, "an economic theory that relies on a mathematics that cannot be computed cannot, by definition, be quantitative in numerical modes. How can economists maintain the fiction that their subject is numerically meaningful and use their formal propositions, derived by using a non-numerical mathematics, to claim applicable policy prescriptions of significance to the daily lives of people, societies and nations?" (Velupillai, 2005).

The adoption of the computable and constructive point of view, and so the adoption of 'effectivity' as primary criterion in mathematical economics, leads to critically re-consider the usual practices of mathematical economics. For instance, the quest for mere *existence* proofs in axiomatized theories (e.g. market equilibrium, Arrow and Debreu, 1954; games' solutions, Von Neumann and Morgenstern, 1953) is considered, from the computable and constructive points of view, conceptually misleading. In the same vein, conceptually flawed is considered the employment of 'indirect proof methods' (IPM). A proof based on IPM is "a demonstration which works by first supposing that the object whose existence we want to assert does not exist and then showing that this assumption leads to a contradiction" (Giocoli, 2003, p. 21). The validity of this proof method relies on the implicit assumption that given the object A, the relation $\neg(\neg A) \Leftrightarrow A$ holds, i.e. that proof of the contradiction of the existence of A *implies the existence* of A (Velupilla, 2005). An assumption that is refused in the constructive mathematics tradition. The *pars destruens* of Velupillai's research program leads him, more generally, to review all the economic theories (most of them axiomatized) finding sources of uncomputability and unconstructivity. Among the various sources of uncomputability and unconstructivity, it is still worth to mention, for instance, the use of (undecidable) *infinitary disjunctions*, which are present in the widely-used Bolzano-Weierstrass theorem. By checking all computable and constructive requirements, Velupillai (2006) claims in the end that Arrow-Debreu's version (1954), Scarf's version (1973) and the 'recursive' version (see e.g. Lucas, Stockey and Prescott, 1989) of CE are uncomputable and uncostructable[28].

It is important to emphasize how, although Velupillai's starting point is the unsatisfying status of *proof* in economics, the consequences of adopting a computable and constructive perspective in axiomatics spread to all other levels of the axiomatic structure. This presupposes a distinctively 'vertical' perspective on axiomatization. When Velupillai says that keeping the point of view of recursion-theoretic formalism "we will have to view economic entities, economic actions and economic institutions as computable objects or algorithms" (Velupillai, 2000, p. 3), we can see how the changing foundations informs also the other elements of the axiomatic structure, i.e. axioms and theorems other than 'mechanical' rules of inference. Emphasizing this transversal impact on the whole structure of axiomatics, Rosser, on the wake of Greenleaf, points out that following the recursion-theoretic perspective "mathematics is

[28] Velupillai's works are plenty of examples of 'uncomputable', 'undecidabile' and 'unconstructive' pieces of mathematical economics, most of them pieces of economic axiomatized theories (such as, e.g., the existence of Nash equilibria in finite games). For a complete survey, see Velupillai (2000).

reduced to being essentially a programming language, so that the old trinity of *assumption, proof, conclusion* becomes *input data, algorithm, output data*" (2012, p.5). The 'trinity' structure has remained, but has changed its nature.

It is also worth emphasizing how Velupillai comes to assume his 'computable' perspective from the point of view of the 'structural heuristic' approach outlined in this essay. In fact, I argue, Velupillai's process towards the computable alternative in economics follows a genuine 'structural heuristic' process. He conceives, in fact, 'proof' as a structural element in axiomatics whose status needs new foundations. Velupillai then projects this need, in search of alternative foundational options, onto the history of economic thought. Velupillai here implements the methodological intuition of Axel Leijonhufvud (2006), who represents the history of economic thought as the outcome of a decision-tree, composed by sequential decision branches. This tree shows at the same time the routes taken and those not-taken due to past, locking, theoretical choices characterizing economics. He states that

> " 'backtracking' along the decision tree may help in the efforts to get out of the blind alley or the swamp if one can gain some sense of the 'alternative futures' that once were possible- although they were never realized. The forks in the decision-tree that is the history of the subject contains a collection of 'alternative futures'- the predecessors we create for ourselves" (Valupillai, 1996, p.255)

The problem of the current unsatisfying epistemological status of proofs in economics – an acknowledgment driven by structural awareness – is addressed by going to the historical fork in which set-theoretic formalism was imported into mathematical formalism in economics. From there a 'counterfactual history' is written, developing the before neglected 'computable' alternative. Historically, the set-theoretic, non-computable mathematics developed by Bourbaki was imported into economics by Debreu because of its readier 'availability'. Velupillai depicts the historical competition between the set-theoretic formalism and the computable alternative in the 1950s as a competition between "theoretical technolog[ies]" (ibid., p.295) at different stages of development. In terms of 'theoretical returns', the Hilbertian 'Cantorian paradise' was at that time decisively superior. It was only shortly after that computability theory, under the impulses of Alan Turing and Alonzo Church, became a theoretically competitive option. Velupillai was able to pick up that broken thread by means of his 'structural heuristic' awareness projected onto the history of economic thought: a piece of superb structural heuristics indeed

2.6. Concluding remarks

The objective of this essay has been twofold. First, to introduce in economic analysis a reconstructive method labeled 'structural heuristics', which complements and procedurally precedes historical reconstruction. The basic tenet of such a methodology – principally devoted to conceptual reconstruction – is to consider the internal structural articulation of concepts as a privileged schema to orientate conceptual reconstruction. An even provisional structural schema representing a concept can constitute, I claim, the orientation map from which its reconstruction can start. Each element or layer of this structure can be subject, according to this methodology, to its own reconstruction, in a sort of *ceteris paribus* clause. The primary and most natural scope of this methodology is to ease the reconstruction of one concept's plurality. Encompassing all the reconstructed variations at each structural layer through a virtual boundary will yield the always-provisional boundary of any given concept's plurality.

Second, I have employed such a methodology in the reconstruction of the intrinsic plurality of the notion of 'economic axiomatics' as a case study. I have devised a 'minimalist' structure for this concept, identifying 1) axioms; 2) rules of inference and 3) theorems as structural layers in a horizontal perspective; 4) philosophy of proof has been identified as a further structural layer, in a vertical perspective. Each layer of this structure was reconstructed as an autonomous object of reconstruction, with the aim of reconstructing the plurality of the notion of economic axiomatics.

As we have seen, the major challenge in conceptual reconstruction is that of categorization. Categorizing is a difficult process but, I claim, a highly rewarding one. Correct, fine-grained and rich categorizations are central also for conceptual progress: categorization triggers focused research. The main objective of 'structural heuristics' is, in the end, to foster structural awareness as epistemic condition for focalization.

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Chapter 3

Structural realism in econophysics: the case of Giuseppe Palomba

Abstract

Don Ross (2008) has argued that economics, due to its developed mathematical formalism, is a discipline in which the notion of 'structural realism' (originally devised in philosophy of physics) could suitably be applied. Econophysics, the discipline that explicitly employs physical theories in economics, seems the natural place the evaluation of the 'structural realist' hypothesis in economics could start from. This essay links the theoretical hypothesis of structural realism in econophysics with the historical analysis of one of the fathers of econophysics, the Italian economist of the late Paretian School Giuseppe Palomba, and interprets his framework as a proto-structural realist framework in economics. The essay shows that Palomba attempted the re-semanticization of physical theories in the economic domain having in mind a clear and original structural framework. Still, this essay emphasizes that mathematical structures as borrowed from physics are employed by Palomba in the explanation and exploration of 'new facts' in the economic domain. Econophysics, in light of these considerations, is an interesting and stimulating case for cross-domain structural realism.

Keywords: structural realism in economics; constructive and re-constructive structural realisms; econophysics; history of economic thought; philosophy of economics.

3.1 Introduction

The hypothesis of the 'unity of science', ambitious as it is, requires the effort of the special sciences to be properly assessed. Economics, on its part, cannot escape this task. The philosophy and historiography of economics have to join the effort to assess the categories employed by philosophers of science to inquire the unity hypothesis. 'Structural realism' is one of these categories, maybe currently the most qualified. This essay is specifically meant to contribute to the implementation of the research on structural realism into economics (Ross, 2008). Since structural realism is a category of scientific realism originally devised for the domain of physics, it seems natural, as a start, to investigate it in the disciplinary domain that is at the exact intersection between economics and physics, that is econophysics[29]. In this respect, my argument is mainly historiographical, since I go back at the origins of econophysics, reconstructing the thought of one of the founding fathers of econophysics, the post-Paretian economist Giuseppe Palomba. In my reconstruction, I highlight how in Palomba there is a clear and accurately developed structural realist framework. Palomba's thought can so be seen as an example of proto-structural realist thought in economics. His way of conceiving the relationship between economics and physics is still relevant, as I am going to show, in the assessment of cross-disciplinary structural realism.

The essay is organized as follows. Section 2 identifies, through the review of the literature on structural realism, a conceptual core of what means to be a structural realist thinker. Section 3 introduces the framework set out by Palomba to study economic dynamics and

[29] For the disciplinary relationships between economics, physics and econophysics see, for instance, Schinckus (2010).

change. Section 4, by identifying the structural realist features of Palomba's thought, lays some conceptual basis for the assessment of structural realism in econophysics.

3.2. What is it like to be a 'structural realist'?

This section introduces the notion of 'structural realism' as it has been conceived in the philosophical literature. Although there is more than one variety of structural realism, later in this section I will set out a 'minimalist' account of what being a 'structural realist' means.

A couple of preliminary clarifications are required. First, it is helpful to note that the notion of 'realism' in the philosophical literature has a meaning distinct from the way the term is used in economics; there "is a major terminological discontinuity between the two disciplines" (Mäki, 2008), which must not be overlooked. Usually, economics does not distinguish between 'realism' and 'realisticness'[30], and uses the former in the sense of the latter (Mäki, 1989). But since we will need to consider several varieties of realism drawn from the philosophical literature, it will be easier to elucidate the meaning of these notions if it is recognized that the term 'realism', throughout this essay, does not specifically mean 'realisticness'. Second, the philosophical notion of 'structural realism' can most easily be understood as a particular form of the broad current in philosophy named 'scientific realism'[31].

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[30] Mäki (1989, p. 194) distinguishes between "realism as designating a collection of ontological and semantic doctrines" from "realisticness as designating a collection of attributes predicable of representations".

[31] Although scientific realism can itself be seen as a particular form of 'realism' in philosophy, I will not need to be concerned with clarifying what 'realism' in general means in philosophy. The debate on realism in philosophy is, in any case, so multifaceted that a unified, simple account of realism risks becoming misleading. For purposes

'Scientific realism', roughly speaking, is centrally concerned with the relationship which a systematic enterprise of knowledge, that is, science – and in particular scientific theories – bears to the 'real' world, considered mainly as a reference domain; scientific realism accepts that there is some such relationship, while investigating what the relationship is. In this respect, the ultimate goal of 'scientific realism' is to assess the notion of 'truth' in scientific activity in general, and in scientific theories in particular[32]. On the other hand, 'scientific anti-realism' interprets science and scientific theories as having mainly instrumental value, not as being true or false. Scientific anti-realists do not assume that science is committed to saying something about the 'real' world (that is, science needs not to have any ontological commitment).

Within the field of scientific realism, one important position is 'structural realism', which has been recently revitalized by the philosopher John Worrall[33] (1989). Worrall describes 'structural realism' as a position reconciling scientific realism and scientific anti-realism, under the attractive perspective that theoretical disputes would be synthesized by taking "the best of both worlds". 'Structural realism' would reconcile and do justice to the strongest arguments of realists and anti-realists: respectively, the 'no miracles' argument (e.g see Musgrave, 1988) and the 'pessimistic meta-induction' argument (see Laudan, 1981). Realists claim that it would be a 'miracle' if scientific theories were at the same time effective[34] (as they are) and not true – in terms of telling something about substantive reality. On the other hand, anti-realists claim that

of this paper, it is enough to focus on scientific realism and, especially, on structural realism, without considering realism in general.

[32] For an overview on the notion of 'scientific realism' see Chakravartty (2011) and references therein.

[33] Worrall indicates as precursors and reference points the mathematician Henri Poincaré and the philosopher Glover Maxwell. See references in Worrall.

[34] Worrall conceives effectiveness as predictive power, i.e. "correct empirical predictions" (p.101).

talking of an 'approximation' of theories to a substantive reality would presuppose a continuity among theories in time, in the sense that changes across theories would be in some way cumulative. Apparently, this is not always the case: think of the notion of 'incommensurability' of theories introduced by Kuhn (1962). The aim of 'structural realism' is to overcome this incompatibility by considering the *structural* content of theories. In particular, 'structural realism' emphasizes a continuity of *mathematical structures* among theories. By means of this characterization, a) the 'no miracles' argument would be met simply by attributing truth value primarily to structures and b) the 'pessimistic induction' argument would be met by considering structures as invariants with respect to the (incommensurable) specific content of theories. The often-cited historical example brought up by structural realists in support of their perspective is that of the classical wave theory of light (based on the assumption of existence of elastic solid ether) of Augustin-Jean Fresnel, superseded by the electromagnetic theory of light of James Clerk Maxwell. The latter denies the existence of ether at all. Nonetheless:

> "[t]here was continuity or accumulation in the shift, but the continuity is one of *form* or structure, not of content […] if we restrict ourselves to the level of mathematical equations – *not* notice the phenomenal level – there is in fact complete continuity between Fresnel's and Maxwell's theories" (Worrall, 1989, pp.117; 119, italics in the original).

This example goes hand in hand with that of mathematical continuity between Newtonian and Einsteinian physics; they constitute some of the most strikingly successful historical examples of structural continuity conceived as examples of structural realism. To sum up, entities whose existence is asserted are mathematical structures[35].

Going a step deeper into 'structural realism', James Ladyman (1998) distinguishes, as a refinement of Worrall's position, between an Epistemic Structural Realism (ESR) and an Ontic Structural Realism (OSR). This distinction is also represented by Ladyman, in slightly different terms, by using respectively the labels of 'syntactic' and 'semantic' structural realism. Statis Psillos conceives the distinction in these terms[36]:

> "SR has two options available. Either there is something other than the structure – call it X – in the world, which however cannot be known, or there is nothing else in the world to be known. On the first disjunct, the restriction imposed by SR is epistemic. […] On the second disjunct, the restriction is ontic: there is nothing other than the structure to be known, because there is nothing other than structure" (2001, pp. S18-S19)

The gist of the distinction is in the way to conceive the relationship between *structures* and their *objects of predication*. For ESR, objects of predication are somehow unknowable, in a Kantian fashion (Ladyman, 2009), leaving space and prominence to the knowledge of structures alone. For OSR instead, "there are no unknowable *objects* lurking in the shadow" (French, 1998, p. 203, italics in the original). While OSR denies "that there are objects that are not structures" it nonetheless claims that "there will always be a place for objects, understood as objects of predication; but [there is] no reason why objects in this sense should precisely line up with the constituents of reality" (Saunders, 2003, p. 130). On a skeptic side, Psillos maintains

[35] Beyond a simplistic notion of mathematical structure, Anjan Chakravartty claims that mathematical structures relevant to structural realism are structures that express "relations between first order, causal properties" in the languages of logic or mathematics (Chakravartty , 2004).

[36] For further refinements of this distinction, see Ladyman & Ross et al. (2007) and Ladyman (2009).

the impossibility of separating, either epistemically or ontologically, the nature and the structure of entities which, as he claims, "form a continuum" (Psillos, 1995, p.15). The disentanglement between objects and structure is however the way in which structures can be provided of cross-domain referential specification, and then of truth value, following the tradition of the 'semantic' or 'model-theoretic' approach to theories (Chakravartty, 2001).

For what we are concerned with in this essay, a disciplinary remark is key: 'structural realism' was originally conceived as a realist position for physics. It is a recent turn of the debate on 'structural realism' to consider its extension to other scientific domains (Ladyman, 2008), among which are social sciences (Kinkaid, 2008). Ross (2008), supporting an OSR perspective on economics, claims that "one […] should not expect to motivate OSR by appeal to disciplines such as sociology that lack distinctive canonical formal theory for interpreting quantitatively parameterized models", and that instead "among social sciences, economics has the requisite mathematical structure, in which it expresses a suite of standard theories" (ibid., p. 733). According to Ross, the OSR framework, as it has been conceived for physics, can be projected onto economics. The application of OSR to both physics and economics is guaranteed by a substantial homogeneous "metaphysical significance" (p. 741) of objects within the two domains of inquiry. There are "structures" and their relative objects of predications ("relata") in the domains of both physics and economics: just in the same way rocks and tables are the objects of the structure of the force of gravity, people are the objects of the structure of economic games. "In this respect, economic theory exactly resembles physical theories, just as Jevons and Walras hoped it would" (ibid., p. 742), realizing a long-lasting dream in economic theory (see Mirowski, 1989).

A huge debate developed in philosophy of economics, precisely about the possibility of borrowing 'realist' frameworks originally devised for physics. The outcome of that debate consisted in recognizing either the "irrelevancy" of the notion of realism in economics if borrowed from physics (Hausman, 1998) or the necessity to adjust the realist framework of physics to the specificities of economics (Mäki, 2000). The point to be ascertained is whether 'structural realism' has to be taken either as a *monist* or a *pluralistic* philosophical position, that is whether there is a notion of structural realism that is good for all disciplines or not (for monist vs. pluralistic realism see Mäki, 2011).

A last point, of a rather technical nature – which has much to do with the scope of this essay – is related to the importance of the mathematics of group theory for the individuation and characterization of structures. A huge number of people, who would define themselves as 'structural realists', believe that the identification of structures is above all a matter of identifying groups of transformations. Once more, the privileged field of application of group theory has been physics and, in particular, quantum physics (French, 1998) and relativity theory. It is not by chance that people like Hermann Weyl and Arthur Eddington, who championed the use of group theory in physics, were among the most influential precursors of structural realism (Ladyman, 2009). Re-actualizing the Galilean motto that the 'book of nature is written in the language of mathematics', Weyl and Eddington claim that the uniformity of physics and mathematics could be pursued by means of group theory, by identifying structural invariances among physical theories and then ('realistically' considering them as substantive structures) into the physical world. To a certain extent, it is difficult to operationalize the structural realist perspective, or – more radically – to be a structural realist at all, if this fundamental technical dimension is neglected.

I can sum up by isolating, from the above considerations, a set of elements that could characterize a 'minimalist structural realist' position[37]:

- what is knowable (ESR) and/or what constitute the ultimate essence of reality (OSR) are structures;
- these structures are mathematical structures;
- identity of mathematical structures is identifiable mainly by means of groups of transformations;
- these groups of transformations are meant to identify structures also across radically different theories, i.e. across paradigm changes, and across radically different phenomena.

3.3. Giuseppe Palomba's view of economic dynamics

The figure of the economist Giuseppe Palomba is almost completely forgotten[38]. Typically, dismissive accounts of Palomba's work emphasize how Palomba's ultimate framework is distinctively *sui generis*[39]. This is mentioned to such an extent that one is led to wonder whether the thin line between heterodoxy and idiosyncrasy is for Palomba irremediably crossed, compromising the possibility of reaching a meaningful and effective reconstruction of

[37] Similar 'minimalist' attempts have been made not infrequently, *e.g*. Kincaid (2008).

[38] Mirowski (1986, p.202) points out the neglect of Palomba, and places his name beside other eminent 'forgotten' thinkers, like Ladislaus von Bortkiewicz and Benoît Mandelbrot.

[39] Palomba's early economic writings, despite being highly original, could be given a more orthodox interpretation (see Gandolfo, 2008). For some biographical ls, see Fusco (1986).

his thought. It is instead precisely the aim of what follows to provide a suitable reconstructive thread.

Although the developed form of Palomba's thought is considered *sui generis*, its roots and scope are easily identifiable within the Italian Paretian School, whose main objective was to render dynamic the 'Paretian-type' competitive framework (Pomini and Tusset, 2010). Palomba, a pupil of Luigi Amoroso, is generally considered as a second-generation member of the school (Tusset, 2004). At the roots of post-Paretianism

> "the progressive abandonment of the evolutionary perspective, followed by strengthening of the static neoclassical method widened the interest in dynamic features. The logical outcome should have been dynamic equilibrium, but economists needed an instrument that would also enable them to explain the deep and rapid transformations affecting economic reality and productive systems from the nineteenth century to the twentieth century. The result was economic dynamics as an analysis of economic changes" (Tusset, 2009, p. 269)

The need to address the issue of *change* in economic dynamics lies in the background of Palomba's theorizing. Palomba answers the need by providing an altogether new framework in which to account for economic change. The usual time-variable framework, i.e. an approach to modeling in which dynamics is dependent upon a time variable, is superseded by an axiomatic framework, in which dynamics is represented by an 'axiomatic stratification'[40] of distinct but interconnected theories (see Tusset, 1998). The lapse of time is rendered through the switch from one axiomatic layer to the next, so that Palomba claims that "one can say that

[40] The expression is drawn from Palomba (1967a). We will see that the expression 'axiomatic stratification' is well-suited to describe the overall point of view held by Palomba on economic science, either as a description of his constructive theorizing or as a description of his reconstructive point of view on economics, as a historian of economic thought (see section 4).

the time variable is absorbed into the spatial variable" [si può dire che la variabile temporale è assorbita in quella spaziale] (1977, p. 1208).

3.3.1 Multilayer axiomatization, *Natural Philosophy* and Physics

Palomba's axiomatic program is pursued in the light of the conception of economic science as a *Natural Philosophy*[41] ("Anglo Saxon-style", as he explicitly states). The evocative point of this label is easy to guess. For one thing, economics has to be driven by an 'experienced' reality: this is the "Anglo Saxon-style" character of his framework. But this is not all. Palomba further specifies that the meaning of his own *Natural Philosophy* has to be understood in a Galilean sense, and not in an Aristotelian one (1977, p. 1187). This specification is meant basically to express the principle of *modern science* for which empiricism cannot be detached from a formal element which gives structure to it.

Palomba, following in the footsteps of Pareto and Amoroso, is fascinated by treating economics on the same footing as physics. This may be more than an analogy for him: Palomba's work shows the full acceptance of the Galilean point of view for which the 'book of nature is written in the language of mathematics'. The link with physics is construed, as we are going to see, at the foundational level. That is, the interest is not primarily in borrowing physical 'images' like motions, forces, equilibria and perturbations *per se,* but rather in replicating the mathematical foundations of physics, which are geometrical in nature:

[41] In English in the text.

“According to [Jean-Marie] Souriau[42] <<the physical universe U is a geometrical universe>>, so that given a *set* E provided with a spatial structure such that any movement is recordable and referable to it, the set has a *universe-structure*. Further there must be an *operator* and a *field* in U which characterize its evolution in such a way that physical laws determine a family of *invariants* the field belongs to, so as to say that the latter, which constitutes ‘sensible reality’, can belong to a certain family of ‘purely abstract possibilities’. To build such a physics, the properties of U, of the operator and of the family of invariants must be specified or defined. In such a way no one can prevent us from speaking of an ‘economic physics’ when we can set up such a framework” (1977, pp. 1187-1188, italics in the original)

[Dice il Souriau che ‘l’universo fisico U è un universo geometrico’, nel senso che dato un *insieme* E munito di struttura spaziale, e tale che qualsiasi suo spostamento sia registrabile e ad esso riferibile, l’insieme possiede appunto una *struttura d’universo*. Inoltre devono esistere in U un *operatore* e un *campo* che ne caratterizzano l’evoluzione de guisa che le leggi fisiche determinano una famiglia d’*invarianti* F a cui il campo appartiene, vale a dire che quest’ultimo, costituente la <<realtà sensibile>>, deve poter rientrare in una certa famiglia di <<possibilità puramente astratte>>. Per l’elaborazione di una fisica, bisogna precisare o definire le proprietà di U, dell’operatore e della famiglia d’invarianti F. In tal senso nessuno può impedirci di parlare di una <<fisica economica>> allorché siamo in grado si giungere alle conclusioni di cui sopra]

From this dense quote, a series of fundamental features of Palomba’s framework may be extracted:

- analogous to the assertion that “the physical universe U is a geometrical universe”, Palomba’s ‘economic physics’ poses that the “*economic universe* U is a geometrical universe”. Abstractness of the geometric structure provides the logical possibility to build up different configurations within the abstract economic universe: Palomba claims that “pure economics, methodologically, is a genuine geometry” and that “from the logical point of view, there are as

[42] Jean-Marie Soriau, French mathematician, was a pioneer and worked in the fields of *differential geometry* and *differential topology*.

many pure economies as there are economic systems which may be thought of or tried out" (1950, p.253) [l'economia pura, nel suo metodo, è una vera e propria geometria"- Palomba claims – "dal punto di vista logico, esistono tante economie pure quanti sono gli ordinamenti economici che si possono ipotizzare o sperimentare];

- configurations of the abstract structure are modeled relying on different *metric spaces*, which define the geometrical characteristics of that universe (see also Palomba, 1969b, pp. 593-594);

- each configuration of the economic universe is meant to represent an economic *phenomenal field* (such as competitive states, monopoly, oligopoly, planned economy);

- if different economic phenomena are rendered by means of changes of metric among economic universes, suitable transformations – associated with *groups of transformations* and accomplished by related *operators* – allow the reduction of all different kinds of phenomena to fundamental economic invariant principles (see section 4). Operators – aside from playing this unifying role – allow tackling distortionary effects (induced by changes of Lemetric) among economic instantiations within each phenomenal field (distortions among actual competitive states, actual monopolies, etc.), and in this way allow performance comparisons[43].

[43] To illustrate the notion of operator in the domain of economics Palomba proposes a very simple example, drawn from Paretian theory of general equilibrium. The theory of Paretian equilibrium can be conceived, in a stylized way, as a transformation that *maps* point-to-point any given initial quantity q into a final quantity q'. Think of an economic agent who possesses, in an initial state, n goods in the quantities $q_1, q_2, \ldots, q_n$ and that, if the n prices of the goods are known, sells and buys these goods according to a personal criterion of preferences

Through this analytical schema, characterized by the quadruple *economic universe – metric space – phenomenal field – transformation group*, Palomba proposes four[44] layers of successive axiomatizations[45], meant to encompass all the kinds of phenomena in the economic domain. Although the content of each of Palomba's axiomatic layers may sound rather 'technical' and 'specific', it will be useful here to provide just a sketchy account of these four (for a more exhaustive description see Tusset, 1998) to emphasize their methodological relevance.

The first axiomatic layer (Palomba, 1968) represents a *probabilistic* (economic) *universe*. The associated phenomenal *field* consists in the generalization of the Pareto-Amoroso framework of competition, so as to encompass, within a common axiomatization, either a 'perfect and unperturbed competitive state', a 'perfect but perturbed competitive state' or a 'quasi-perfect and perturbable competitive state' ['stato di concorrenza imperturbato e

a_{rs} (which express the willingness to exchange r for s). Assuming that the sum of exchange preferences is normalized to 1 and that $\sum_r a_{rs} = \sum_s a_{rs} = 1$, the transformation can be expressed as:

$$[q][\, a_{rs}] = [q']$$

that defines a group of transformations, according to the linear operator [a_{rs}], which is a matrix that expresses the complete set of preferences of an agent, or to put it differently his exchange behavior (Palomba, 1968, pp. 770-771).

[44] In addition, the Paretian- and the Amoroso-type general equilibrium models represent a 0th axiomatic layer (see Table 1, below).

[45] Palomba proposes a technical illustration of the four axiomatizations in three articles that appeared in the *Giornale degli economisti e Annali di economia* between 1969 and 1970 (Palomba 1968, 1969a, 1969b). Subsequently, under pressure "by more than one reader" Palomba provides a non-technical illustration of previous articles, in the same journal (Palomba, 1970a). The framework remains substantially untouched in subsequent presentations.

perfetto', a 'stato di concorrenza perfetto ma perturbato' or a 'stato di concorrenza perturbabile e quasi-perfetto'] (pp. 792-793). The shift from the Pareto to the Amoroso characterization of equilibrium carries with it a theoretical problem to be addressed, since – unlike Paretian static equilibrium – Amoroso type dynamic equilibrium no longer guarantees that firm profits are zero at any point in time (Amoroso, 1942). This acknowledgment gives rise – according to Palomba – to the need to tackle the problem of profit determination at the intra-firm level. This issue looks particularly troublesome since profit has to be determined according to the profit and loss account and the balance sheet, two reporting systems that are intrinsically different (all the more since they are both vulnerable to the subjective choices of the bookkeeper). To deal with profit determination, the *hermitian operator* Ψ is introduced (the same operator used in the Schrödinger equation to deal with the quantum-mechanics problem of determining position and momentum of a particle). The role of the operator Ψ is to minimize the discrepancies in the probabilistic distributions of profit that emerge from the profit and loss account and the balance sheet (see also, Palomba, 1967).

The second axiomatic layer (Palomba, 1969a) represents an *inertial universe*. The associated phenomenal *field* consists in the market forms of monopolies and oligopolies. The emergence of systematic and persistent market disequilibria generates the theoretical problem of commensurability of economic performances for different economic units in the market. To solve the problem of commensurability Palomba relies on *Lorentz transformations*. He explicitly translates findings of Einsteinian special relativity – according to which accounts of space and time of two observers are specific to their reference frame – stating, analogously, that 'economic time' is not the same for different economic units (which are assumed to act in different reference frames). According to Palomba, the flow of 'economic time' in each reference frame would be driven by a specific economic variable, which is the amortization speed of fixed capital. This variable would determine a sort of deformation of the space-time of the firm: the time shrinks (i.e. amortization speed is faster) for a more prosperous firm, but expands (i.e. amortization speed decelerates) for a worse-performing one. In order to provide a comparison among economic units in different ‘economic times’, appropriate corrections are needed, and can be obtained by Lorentz transformations. Analogous to the speed of light in special relativity theory, Palomba identifies an upper bound for the speed of amortization of firms. Importantly – still in accordance with special relativity – relativistic corrections become necessary for economic units when the speed of amortization approximates to its upper bound level; otherwise, usual (Galilean) Lagrangian equations apply as special cases[46].

[46] It can be useful to provide a mathematical tasting of Palomba’s framework. In this axiomatic layer, for instance, Palomba relaxes the assumption that time is homogeneous among economic units: in productive environments in which the amount of fixed capital in production is great, time flows according to the variable of the amortization speed of capital v. In a situation in which v of a firm is close to the <<modal>> v of the industry, the determination of the net profit y can expressed by the relation

$$y = Y - vt$$

where Y is the ‘gross’ profit. At this point, Palomba introduces the constant w, i.e. the limit value of the amortization speed of capital. When v is high enough, the ratio v/w is not negligible and a problem of commensurability among firms emerges. Relativistic correction are necessary to reach the correct value of $Y$and t. These corrections are provided by the *Lorentz* transformations (see Palomba, 1969a, p.185 for the ‘group’ considerations concerning Lorentz transformations):

$$Y' = \frac{Y - vt}{\sqrt{1 - \frac{v^2}{w^2}}}$$

and

$$t' = \frac{t - \frac{v}{w^2}Y}{\sqrt{1 - \frac{v^2}{w^2}}}$$

The third axiomatic layer (Palomba, 1969b) represents a *gravitational universe*. The related phenomenal *field* coincides with that of the preceding layer – i.e. monopolies and oligopolies – but it is meant to generalize it. It is devised, in fact, so as to encompass firms' varying (accelerating or decelerating) amortization speeds. A four-dimensional space is considered for each economic unit (each coordinate is meant to represent one of the four factors of production: labor, capital and land, plus time). The analysis chiefly aims at identifying the 'proper time' of development[47] of economic units (Palomba refers here mainly to macroeconomic units such as regions, nations, and continents), and at providing relativistic corrections in order to make comparison possible among these units. The operator associated with the gravitational space is a *metric tensor*, like that used in Einstein's general relativity.

The fourth axiomatic level (Palomba, 1969b) represents a *planning universe*. The *phenomenal* field consists of the phenomenon of economic planning. The intuition informing this axiomatic layer is that an economic plan is built within the constraints of an autonomously expanding universe. Plan execution, to be effective, should be provided with a reconceptualization either of calendar time or of the 'proper time' of development. In other words, to reconcile the calendar time with the time of the plan, a proper 'planning time' has to be considered. Accordingly, a suitable curvature has to be impressed on the universe in order to reconcile calendar and planning time. A 'maximum time' of plan execution has to be set in

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As we can see, when v is negligible the ratio v/w is close to zero, and the profit determination collapses in the special case of the <<modal>> v among the firm of the industry. In better performing firms, $Y'>Y$ in a time $t'<t$. In the next layer, the assumption that v is uniform will be further relaxed.

[47] Palomba (1970, p. 62) refers to the works of François Perroux, Ferenc Jánossy and Giovanni Demaria as forerunners of the notion of 'proper time' in economics. More recently, Mandelbrot re-presents a similar conceptualization of time, specific to each economic unit (see Mandelbrot and Hudson, 2004).

a range whose minimum is the 'proper time' of development itself and whose maximum is the limit of the amortization speed of capital. These corrections are made according to the relativity model proposed by Fantappiè, by means of *Fantappiè group*, or *final relativity group*. Table 1 provides a conceptual organization of Palomba's axiomatics.

	Economic Universe	**Metric Spaces**	**Phenomenal Field / Theoretical problems emerged and solved through transformations at each level**	**Operator/ Transformation Group**
Zeroth axiomatic layer	Stationary/ Inertial	Euclidean	Competitive states as conceived by Stationary Paretian Economic General Equilibrium and Dynamic Amoroso-type Economic General Equilibrium	Linear
First axiomatic layer	Probabilistic	Pseudo-Euclidean	Competitive states (generalized)/ profit determination at the intra-firm level	Hermitian Operator Ψ
Second axiomatic layer	Inertial	Pseudo-Euclidean	Monopolies and Oligopolies/ problem of commensurability among economic units (at a fixed speed of amortization)	Lorentz Transformations
Third axiomatic layer	Gravitational	Riemannian	Monopolies and Oligopolies/ generalized problem of commensurability among economic units (variable speed of amortization)	Metric Tensor
Fourth axiomatic layer	Planning	Pseudo-Euclidean	Planning Phenomena / reconcile the calendar time with the 'plan time'	Fantappié Group/ "Final" Group

Table 1: A synthesis of Palomba's 'axiomatic stratification'

By construction, as a fundamental characteristic of Palomba's architecture, each subsequent axiomatic layer has been devised as to encompass – mathematically and phenomenally – the previous one as a special case. As Palomba claims, using the words of the philosopher of mathematics Albert Lautman, his axiomatics aspires to represent an "ascension towards the absolute" (Palomba, 1970a, p. 48) [ascesa verso l'assoluto]. In this way, Palomba explicitly poses (1968, p. 772) his framework of economic progressive generalization and abstractness in direct connection with the 'Erlangen Program' of geometry.

3.3.2 Empirical progress *as* historical progress

While each axiomatic layer depicts a specific phenomenal reality (i.e. different forms of competition, monopolies, oligopolies and economic planning), it encompasses phenomena of previous layers as special cases (i.e. economic planning is a generalization both of competition and asymmetric competition depicted at previous layers, while asymmetric competition is a generalization of perfect competition, and so on). In this sense, one can easily say that Palomba's axiomatics is meant to be 'empirically progressive', the 'retention' of previous phenomena being a fundamental ingredient of progressiveness.

However, Palomba is aware of how the 'empirical progressiveness' of economic theories cannot be that of physical theories. In particular, two substantial differences of the economic domain from the physical domain have consequences for the conception of economic science. First, while physics "covers all the reality to be explained, economic physics covers only part of it. Part is left to the study of past as history, part to the creation of the future" (Palomba, 1981, p.165) [copre tutta la realtà da spiegare, [l]a fisica economica ne copre soltanto una parte. Parte è lasciata allo studio del passato come storia, parte alla creazione dell'avvenire]. Second, humans interacting in time, intentionally or unintentionally, produce "new facts" (*e.g.*

Palomba, 1977, p.1188) which are not explicable by already existing theories. The driver of economic theorizing is precisely the attempt to give an account of continuously emerging 'new facts'.

The issue of 'new facts' is indissolubly linked with the interpretation of economic dynamics as 'change'. This is a characterization that the Italian Paretian School internalized due to the direction traced by Maffeo Pantaleoni (see Giocoli, 2003). Not by chance, Pantaleoni (1907) was also the author of the attracting metaphor of 'cinematograph', introduced to represent the evolution of economics as science. It is under the lens of this metaphor – I argue – that Palomba's axiomatics can be aptly read: the succession of axiomatic layers can be considered as the succession of frames of the 'movie' of economic evolution. More properly – if we wish to reconcile the economic perspective with the physical one – we can say that Palomba conceives the notion of change more a succession of different *kinematic* states (which Palomba labels 'axiomatizations') than as a dynamics expressed through a function depending on the time variable.

In this framework, the movie projected by Palomba is precisely that of economic history. Following 'new facts' and accounting for them means expressing precise historical judgments. Palomba states that a 'Smith-Pareto' world, that is a world characterized by the phenomenon of competition, is not actual anymore. Accordingly, the zeroth axiomatic layer is insufficient and has to be supplemented. While the inclusion of probabilistic considerations led Palomba to propose a general axiomatics for competitive states, in which different parameter values are able to account for more complex forms of competition (first axiomatic layer), this is not sufficient anymore after the historical onset of monopolies and oligopolies (1977, p. 1188) which "breaks up the 'ergodicity' of variables which compose the set under observation"

(1969, p. 795) [rompe l’<<ergodicità>> delle variabili che compongono l’insieme in osservazione]. New axiomatic layers are needed (second and third axiomatic layers). The same is true for the historical appearance of planned economies (fourth axiomatic layer). Palomba answers these historical challenges by providing more general axiomatic layers able to encompass them. This empirical progressivity is conceived in this framework as historical progressivity. Economics, according to Palomba, is the discipline able to dissolve the constitutive tension between ‘history’ and ‘nature’, so that Palomba conceives “economics as *a synthesis* of history and nature” (Palomba, 1975, p.16, emphasis added).

To get a step deeper into Palomba’s perspective, it would be useful to read it in the light of Joan Robinson’s (1980) categories of *logical* as opposed to *historical* time. ‘Logical time’ is for Palomba not *longitudinal*[48] to the ‘historical time’, as in most of economic theories, but *transversal* to (split by) it; that is, ‘logical time’ is an analytic category which assumes meaning only within each axiomatic layer. Each axiomatic layer has its characterizing ‘logical time’: think of axiomatic layers 2 and 3, in which time is respectively constant and variable (Palomba, 1974). Historical time – that is the succession of historical changes – is instead represented by means of the kinematic sequence of axiomatic layers (see Figure 3 below).

[48] ‘Longitudinality’ has to be intended as the property of following the time as it lapses (e.g. the notion of ‘longitudinal data’).

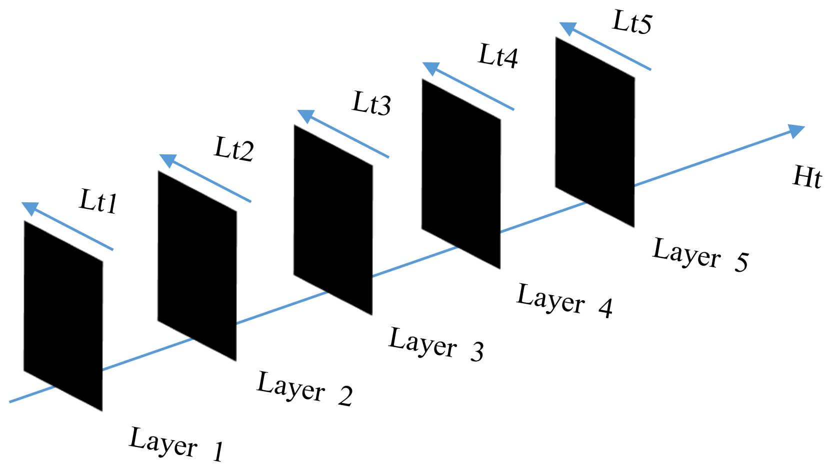

Figure 3: Time lapse in Palomba's framework (the historical time lapse (Ht) is represented through the succession of axiomatic layers. Each axiomatic layer has its own logical time (Lt))

To sum up, Palomba's conception of axiomatics is aimed at two simultaneous objectives: the aim of *empirical adequacy* (in a historical sense) and that of *retention* and *generalization*[49] (the inclusion of previous axiomatic layers and of their phenomena as special cases). The first objective is pursued by means of qualitative techniques of historical sensitivity;

[49] I will only address in this note (though it would deserve a more careful inquiry), the extent to which Palomba's framework is *technically* successful – according to the standards provided by himself. A preliminary look at Table 1 suffices to attest that Palomba does not succeed in ensuring that each axiomatic layer is a generalization of the preceding layer. For instance, if we consider the criterion of metric spaces, we can see that from a Riemann space, in the third layer, we go back to a more restricted Euclidean space at the fourth layer. Further, Palomba recognizes that a satisfying economic interpretation of technical devices borrowed from physics is not always attained (see Palomba, 1969b).

the second objective is pursued by means of the formal instrument provided by abstract geometry. In Palomba's words:

> "an economic science exists if it can explain hard facts in front of it; an economic science exists if it can encompass such a reality in a geometry or transformation group from which to extract invariants and constants" (1981, p. 164)[50]

> [esiste una scienza economica se essa riesce a spiegare i fatti che le si presentano come realtà inoppugnabili; esiste una scienza economica se riesce ad immettere questa realtà in una geometria o gruppo di trasformazione da cui estrarre degli invarianti e delle costanti]

3.4. Palomba's structural realism

The main objective of this essay is to inquire whether – and to what extent – Palomba's theoretical architecture is susceptible of a 'structural realist' interpretation. A series of methodological problems – both of general and specific kinds – are connected to such an attempt. As Mäki rightly points out, for instance, the ontological commitment of theories may not coincide with the ontological beliefs of theorists: the impossibility of freely assuming that

[50] I argue that far from isolated and idiosyncratic, Palomba's conception of economic science and of the best tools for pursuing it was shared by other Italian economists of the time like Giovanni Demaria. Palomba, in reference to "some early writings which, for about forty years, I have thought over and worked on, reaching partial formulations which I now summarize", credits Demaria for inspiring the "reduction of economic phenomena to abstract algebra" (1969, pp. 758-759) [in reference to "alcuni scritti giovanili sui quali ho meditato e lavorato per circa un quarantennio a formulazioni parziali che oggi condenso" Palomba says that Demaria inspired to him the "riduzione del fenomeno economico all'algebra astratta […]"]. Giovanni Demaria in some late writings (e.g. 1981) explicitly poses the fundamental question of the relationship between "symmetries and political economy". For his part, however, Demaria was clear in stating that the notion of symmetry should not be identified with that of structures, basically for the simple reason that not all structures are symmetric (p. 534).

these ontologies coincide leads generally to considering theories as 'instrumentalist' products[51] (Mäki, 2001, p.10). Furthermore, it is difficult to find, in economic texts, expressions that unambiguously express an ontological commitment; and it is still more difficult to find the use of terms like 'realism' (in its proper ontological sense). In our case, Palomba's intellectual *milieu* was not acquainted with the English-speaking world's philosophy of science categories, so that it is hasty even to guess whether Palomba could have been aware of the possibility of being realist in the current sense[52]. Nonetheless, it will be my task to provide a 'structural realist' interpretation of Palomba's thought using contemporary categories – as far as it seems useful. Such an exercise would be fruitful not only in order to shed light on Palomba's thought, but also to test analytic categories in context different from those in which were originally devised.

In section 1, I have provided an account of what is broadly meant by 'structural realism'. In section 2, I have provided a sketch of Palomba's axiomatics. My thesis is that – even before considering the possibility of structural realism in economics – Palomba endorses structural realism in physics. This is reasonably implicit in his endorsement of the conception of physics as founded on an abstract 'geometrical universe'. Within the logical structure of the 'geometrical universe' Palomba looks at the succession of physical theories as following an

[51] The champion of the instrumentalist interpretation of economic theories is Friedman (1953).

[52] Hands (1990, p. 74) claims that there is consensus in economic methodology that the founders of neoclassical economics were 'realist' in the current sense of the term. This means that 'realism' is a sort of default interpretative heuristics for the assessment of early economists' theories.

approach either of *retention of structures* or of *generalization*[53]. He wants to import this framework into economics.

The issue of groups of transformations appears to be the main way to get into the 'structural realist' interpretation of Palomba's economics. The label of 'group structural realism' has been recently introduced (Roberts, 2011) in philosophy of science literature. It is meant to express that the mathematics of groups of transformations constitutes more than a mere technical device (such as calculus) for theory development. As far as I am concerned, I reject the idea that any use whatsoever of groups of transformations in economics would suffice to show that we are dealing with an example of 'structural realism'. That would be too simplistic, and misleading. In a genuinely 'structural realist' perspective, groups of transformations have to be used as bridges to establish continuity among theories. In Palomba's axiomatics, groups of transformations play a double role. The primary role is that each group of transformations is associated with an axiomatic layer (which includes a phenomenal field), and in this way – as we have seen – is able to compare *all* varieties of phenomena at that layer. For instance, for the second axiomatic layer, Lorentz transformations are able to make comparable each pair of economic units with different (but constant, otherwise the third layer

[53] Reporting the words of the German astronomer August Kopff, Palomba expresses his point of view on physics in this way: "Physics [hat to be] intended in the widest sense of the word; so not inquire of specific phenomena of nature or of experience and search for their explanations, but comprehension of ALL THE PHYSICAL PHENOMENA FROM A UNITARY POINT OF VIEW" (Palomba, 1952, p. 57, italics in the original, capital letters added; reported in Tusset, 2004, p. 149).

["*Fisica* intesa nel *più ampio significato della parola*; non cioè esame dei singoli fenomeni della natura o dell'esperienza e ricerca delle loro spiegazioni, ma comprensione di tutti i fenomeni fisici da un punto di vista unitario"] applies) speeds of amortization. But in this axiomatic architecture, the groups of transformations at different axiomatic layers are not mutually independent. A link connects the axiomatic layers – this link being the second role played by groups of transformations. This link consists in the 'higher degree of generality' of the group at the next axiomatic layer with respect to the group at the previous one. Palomba establishes a hierarchy among axiomatic layers, by means of the sequential 'generality' of transformation groups. When economic phenomenal world changes, with the arrival or prevalence of monopolistic, oligopolistic or planning phenomena:

> "the central point is what follows: *when the current group does not encompass all phenomena to be studied or to be theorized, it is necessary to rely on a wider group which includes the latter as a special case*" (1977, p. 1207, italics in the original)
>
> [il punto fondamentale è allora il seguente: *qualora il gruppo a cui si è fatto ricorso non ingloba tutta la fenomenica da studiare o da teorizzare, bisognerà far ricorso ad un gruppo più ampio di quello di cui si tratta che ne costituisce così un caso particolare*]

As we have seen, the objective of Palomba is to provide an axiomatic structure that is 'empirically progressive' or, better, 'historically progressive'. Groups of transformations are constitutive elements in this project. Summarizing, I can claim that Palomba conceives groups of transformations in a twofold way: not only *within* each axiomatic layer to exhaust empirical variety at each level, but also *between* axiomatic layers to account for 'new facts', and to establish empirical progress conceived as 'historical progress', by means of retention of structures and generalization.

A question closely related to the issue of retention of structures and generalization concerns the *content* of structures. They are, as I have already noticed, mathematical structures.

They are conceived so as to encompass previous axiomatizations as special cases. But there is more. From Palomba's point of view, they are an expression – in fact progressively generalized expressions – of an underlying invariant that is semantically retained. According to Palomba's never suppressed Paretian orthodoxy, this economic invariant consists in the notion of 'equilibrium', and in the related 'marginalist principle' that informs equilibrium. As Palomba clearly states:

"our aim is just to reach, by means of suitable axiomatizations, more and more general views of Paretian economics to which correspond more universal extensions of the concept of equilibrium and of the related fundamental invariant, constituted by the marginalist principle and by the ophelimitarian maximum deduced from it"" (1970a, p 47)

[Il nostro scopo è proprio quello di giungere, coll'ausilio di assiomatizzazioni adeguate, a visioni sempre più generali dell'economia paretiana onde riemergano concezioni corrispondentemente più universali del concetto di equilibrio e dell'invariante fondamentale che vi si connette costituito dal principio marginalistico e dal massimo ofelimitario che se ne deduce]

Further, Palomba explicitly states that the invariant that is retained in different axiomatizations is not only a 'theoretical' entity:

"the marginalist principle […] is an invariant property […] not only of any equilibrium at any level of (reasonable) axiomatization; but also of any (reasonable) economic system, be it individualistic or socialistic in nature" (ibid., p. 49, n.10)

[il principio marginalistico […] costituisce una proprietà invariantiva […] non solo d'un qualsiasi equilibrio ad un qualsiasi livello di assiomatizzazzione (naturalmente ragionevole); ma anche di ogni ordinamento economico, sia a base individualistica, sia a base socialistica (naturalmente anch'esso plausibile)]

The invariant that is postulated at the theoretical level by means of successive axiomatizations is found in real-world economic systems, and thus is thought as an element of reality itself.

The issue of the retention either of the equilibrium or of the marginalistic principle in successive axiomatizations involve directly the core questions of structural realism, i.e. those of the role of interpretation of mathematics and of the place of objects of predication in theories (see section 2). The issue of semantics is clear to Palomba from the outset of his research. In an early passage that may be read as skeptical, Palomba claims that "the invariant properties being discussed are what they are only if conceived from a purely formal point of view, while if one inquires into their content even a little, a series of related contradictions begin to appear" (1950, p. 337) [le dette proprietà invariantive sono tali soltanto se considerate da un punto di vista puramente formale, mentre se per poco si approfondisce il loro significato sostanziale le contraddizioni che esse implicano cominciano a manifestarsi]. Twenty years later, the conclusion of his inquiry is that "these generalizations do not at all imply that equilibrium and the marginalist principle continue to mean the same things which they meant and still mean in the Paretian universe" ["tali generalizzazioni non significano affatto che equilibrio e principio marginalistico continuino a significare le stesse cose che significavano e continuano a significare nell'universo paretiano"] (1970a, p. 48). Interpretation plays a primary role, as we can see, in Palomba's framework. Although the equilibrium and the marginalistic principle have been thought of as constituents of reality itself, they are theoretically legitimated only after the mathematics is interpreted. This primacy of the uninterpreted notion of structure is consistent with the Ontic or semantic notion of structural realism, as I introduced it in section 2[54].

Although substantial parts of Palomba's axiomatics are consistent with a 'structural realist' interpretation, it is worth considering some traits that do not fit so well with the literature on structural realism. It would be interesting to see whether these traits would be detrimental to my interpretation or whether, conversely, they would give useful insights for the implementation of 'structural realism' in a cross-domain or cross-disciplinary case. First, as I have noticed, Palomba places a decisive emphasis on the issue of sequential *generalization* of axiomatic layers. The literature on 'structural realism' has often considered cases in which retention of structure in theory change has proceeded by means of assimilation and generalization of previous theories (think of the case of Galilean and Einsteinian theories). However, one may argue that generalization is not a necessary requirement of structural realism. One may argue that retention of structure – which is the core of 'structural realism' – may be accomplished in different ways that do not necessarily involve sequential generalization.

This consideration is linked to a second fundamental point to consider. Think, for instance, of what for Psillos is the only tenable core of 'structural realism': a 'structural realism' which consists merely in recognizing that there "has been a lot of structural continuity in theory-

[54] Palomba seems to agree with the 'formalist' methodological standard for economic axiomatization set out in the late 1950s by Gerard Debreu, according to which "an axiomatized theory has a mathematical form that is *completely separated* from its economic content. If one removes the economic interpretations […] its bare mathematical structure must still stand" (Debreu, 1986, p. 1265, emphasis added). Clower (1995), following the same line of Psillos's criticism of structural realism, denies that the detachment between mathematical and economic content can be attained, at least not in the Debreu's framework.

change" (2001, p. S23). In this sense, 'structural realism' has generally been conceived as a *re-constructive* enterprise, that is a perspective which emphasizes structural permanence as a common characteristic identified *ex post* among theories. Palomba instead conceives structural realism in a *constructive* sense, structural permanence being taken as an *ex ante* constraint of theorizing.

The *constructive*[55] side of Palomba's theorizing is surely the aspect that is more prominent at a first glance. Nonetheless, a couple of remarks are in order to underline how Palomba's thought is not devoid of re-constructive stimuli. First, we have seen that at the roots of his axiomatic attempts was the economic historian's acknowledgement that the phenomenal domain has changed. Using Palomba's label, the "Smith-Pareto" world was over. Palomba infers from this that the theories, too, of Smith and Pareto were over. The practice of linking each historical period with *its own* representative theory may be read as a proper and fine reconstructive activity, and a prerequisite to constructive theorizing. In a second sense, too, the work of Palomba is extremely attentive towards reconstructive considerations. Palomba the 'historian of economic thought' reconstructed the history of economics trying to identify lineages among theories on the basis of structural considerations (see Palomba, 1967a), and further tried to reconcile perspectives in the history of economic thought that seemed incommensurable (*e.g.* linking Pareto and Marx) – again, proposing structural arguments (see Palomba, 1973). However, even prior to be a historian of economic thought Palomba shows himself as a structuralist historian of physics: the sequence of mathematical theories that he takes as underlying structure of his economic axiomatic layers is nothing more than a sequence

[55] The label 'constructive' has not to be intended in connection with the philosophical anti-realist position of Van Fraassen (1980) labeled 'constructive empiricism'.

of the most important physical theories of the 20th century, selected by virtue of their structural continuity. In all the three senses sketched above, reconstructive structuralism looks like a precondition of constructive structuralism.

As a last remark, let consider what Ladyman and Ross (2007, p. 67) consider one of the major concern with structural realism, i.e. that it could be exposed to the charge of being an *ad hoc* position. *Adhocness* means that structural permanence would not be a substantial characteristic of theories, but rather an epistemic selection and manipulation of theories in order to let emerge some structural permanence. Under the latter perspective, many claim, structuralism would lose the realist value. To avoid such a risk, Worrall claimed that structuralism can gain the realist status only when structures can be employed to attain "correct empirical *predictions*" (Worrall, 1989, p. 101, emphasis added, see note 34 supra). Under a certain point of view, it is true that Palomba's perspective could be charged of adhocness: in fact, axiomatics is evidently 'constructively' postulated. I claim, however, that adhocness conceived as cross-disciplinary 'constructivity' is in Palomba's framework not a *vulnus* but, on the contrary, the source and legitimation of Palomba's realist claims. Physics structures in fact are not 'forced' to accord to well-established economic facts but, on the contrary, they are employed mainly to explore and assess 'new' economic facts. According to Palomba, in fact

> "the problem is to see whether and to what extent [operators employed in physics] are able to explain [economic] facts that are still left or *unexplored* or unexplainable (Palomba, 1977, p. 1208, emphasis added) [tutto il problema è di vedere se e fino a qual punto essi [gli operatori impiegati in fisica] riescono a spiegare fatti finora rimasti inesplorati o inesplicabili].

3.5. Concluding remarks

The reconstruction of the work of Giuseppe Palomba allows us to draw some concluding remarks. The first one concerns the meaning of 'heterodoxy' as a way to describe the character and extent of Palomba's thought. As is easily arguable – but hardly really acknowledged – heterodox thought is always meant to fill some gap in its orthodox rival. In our case, I claim that Giuseppe Palomba took more seriously than orthodox economists the parallel between economics and physics. As pointed out by Mirowski

> "whereas in physics there was a continuous research program whose purpose was to come to understand the nature and limits of the new reified invariant [i.e. energy], it seems not one of the neoclassical economists even understood what was at stake in the problem of invariance" (Mirowski, 1989, p. 271).

Palomba instead put right at the center of his analytical focus the issue of invariance in economics[56], mainly conceived of as structural invariance, inquired for the first time through the advanced tools of group theory[57].

The second major remark concerns the interpretation of this attention to structural invariance as an example of 'structural realism'. My reconstructive effort towards Palomba's structural line of inquiry has been characterized by a constant tension between interpretative boundaries imposed by the literature on structural realism and the need to give a salient and accurate account of Palomba's economic thought. We have seen how the 'econophysicist'

[56] Palomba (1968) introduces the substitution of the Lagrangian equations with the Hamiltonian equations in the formalization economic dynamics, in this way demonstrating a deep comprehension of the issue of invariance as conceived in mechanics (i.e. the issue of conservation of energy).

[57] Tusset (1998) does justice to Palomba, acknowledging his primacy in the introduction of the mathematics of transformation groups in economics. Previously Samuelson had wrongly attributed such an honor to Sato (1981, in the preface).

Palomba endorses basically the structural realism in physics, conceiving axiomatization as an attempt to give an economic interpretation to the sequence of the most important physics theories of the XXth century. This is not, however, the end of the story. In Palomba's perspective, structures as borrowed from physics are not only re-semanticized in the domain of economics but, more decisively, they are continuously employed to inquire and explore unexplained or even 'new' economic facts. In this essay, I have characterized as *constructive* structural realism the position, typical of a discipline like econophysics, in which structures are willfully employed to explore, assess or even predict facts in a cross-disciplinary perspective.

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Chapter 4

Atoms, structures and hierarchies in the analysis of economic production

Abstract

Production is currently a neglected topic in the economics literature. This state of affairs is connected, I argue, with the lack of an adequate methodological assessment of the notion of 'economic production'. Critical methodological problems undermine the validity of widely used conceptual frameworks that are currently employed in the analysis of production in economics. The objective of this essay so is mostly theoretical, as the essay is devoted to inquire into and assess the *fundamental properties* that a conceptual framework of economic production *must satisfy* in order to be adequate to deal with the nature of production phenomena. In order to do so, my starting point is that the different theories of production have the identification of *Fundamental Units of Analysis* (FUAs) as the basis of their conceptual framework. These FUAs are then thought to be subject to some operations of 'aggregation'. I claim that such an 'atomistic' and 'aggregative' framework is insufficient to properly conceptualizing economic production. In particular, the atomistic framework has to be complemented with an explicitly structural point of view, i.e. with a framework in which the 'atoms' express a productive value only with respect to the relative positioning of the other atoms. In this way, the proper way to think of the 'aggregation' of FUAs is in a proper 'combinatorial' framework. Further, I extend the structural analysis of production by assessing a more complex structuralist view, through the consideration of the properties of symmetry-breakings and complex-systems modeling. The analogy between production and chemistry, which is more and more used in the literature on economic production, is constantly employed as a heuristics. The final part of the essay is devoted to the assessment of explicitly structural approaches to the theory of production in economics. From the early methodological 'anti-marginistic' considerations of Piero Sraffa – which demonstrate a remarkable point of contact with the later Georgescu-Roegen – I pass to

assess the structural fundamental requirements identified in the essay in the case of the input-output framework of economic production

Keywords: theories of production, atomism, structuralism, symmetry-breaking, hierarchic systems

4.1 Introduction

The reason why economics has relinquished the inquiry into the production phenomena as part of its core mission[58] is a question that still needs a proper answer. While the historical trajectory of such an eclipse has been reconstructed in detail[59], we have not yet a satisfactory explanation. This essay argues that the reason for such a state of affairs may be either the under-conceptualization or, to a certain extent, the mis-conceptualization of the notion of 'economic production'. This essay looks for the origin of this under/mis-conceptualization and outlines a framework in which such a notion can be properly addressed. The main scope of the essay, to state it more explicitly, is that of identifying the *fundamental properties* a framework of

[58] For example, the *New Palgrave Dictionary of Economics* (Blume & Durlauf, 2008) has not any entry for 'economics of production', 'production economics' and the like, while there are entries for 'economics of religion' and 'economics of marriage'. Further, in the *IDEAS/RePEC* archive the only journal that shows 'production' as its core topic (*International Journal of Production Economics*) is at 129th in the aggregate ranking of economics journals.

[59] Two historiographical traditions converge on the same account. On the one hand, those who see in the history of economic thought the breaking between 'political economy' and 'economics' (or 'catallactics') (e.g. Bharadwaj, 1978; Hicks, 1974) and on the other hand those who see the history of economic thought as structuring around the main ideas of 'production' and 'exchange' (e.g. Pasinetti, 1981; Quadrio-Curzio and Scazzieri, 1986; Pasinetti, 2007).

economic production *must satisfy* in order to be suitable to deal with the nature of production phenomena.

In order to identify the fundamental properties of a production framework, the notion of economic production will be addressed by integrating different points of view, such as the ontological, the epistemological and the linguistic ones. According to Nicholas Georgescu-Roegen, production in economics has never been the subject of "epistemological analysis" (e.g. 1986, p. 254). He identifies this situation as a primary cause of the under/mis-conceptualization of production. Nonetheless – as the work of Georgescu-Roegen itself shows – epistemological analysis alone is inadequate, unless i) the nature of the production phenomenon is clarified, ii) the clarification is complemented by an analysis of the language used to represent production itself. This essay integrates the three points of view (epistemological, ontological and linguistic). It will also make use of insights from disciplines other than economics.

The specific issue addressed in this essay is the *insufficiency* of the 'atomistic' framework in the analysis of economic production and the possible way to complement it. Since the sharp distinction provided by John Stuart Mill between 'laws of production' and 'laws of distribution' (Mill, 1848, p.21), many theorists of production have adopted 'units of analysis' whose status is similar to that of atoms in the physical world. This essay maintains that this 'atomistic' aspiration is either incomplete or misleading. It can be worth to remark in this context that, as we shall see, even disciplines that in the *vulgata* relies on atomistic frameworks, such as physics and chemistry, are putting into question their atomistic ontology[60]. If the

[60] It is interesting to note that ontologists who look at physics as source of ontological inspiration for other disciplines claim that physics does not anymore support an 'atomistic' view of the world (see Ladyman and Ross et al., 2007)

naturalistic analogy holds, this brings to the fore the need to identify the limits of the atomistic framework in economic production too, along the identification of the proper way to complement such framework. To put it another way the analytical core of this essay is to provide adequate foundation to the statement that *the whole is not simply the sum of its parts* in economic production.

The essay is organized as follows. Section 2 reconstructs the different ways in which the ‘atoms’ of production have been conceived in the theories of economic production. Section 3 assesses the ‘chemical analogy’ that is more and more employed as a heuristics in the theories of economic production. Section 4 identifies ‘structuralism’, in its different forms, as a possible alternative to a naïve atomism. In particular, while Sub-section 4.1 discusses the notion of combinatorics as a form of structuralism, section 4.3 and 4.4 discuss respectively the importance of two structuralist notions, symmetry-breakings and hierarchic systems, for the analysis of production. Section 5.1 examines the complementary roles of Piero Sraffa and Nicholas Georgescu-Roegen as opponents of the atomistic analysis of production and identifies them as opponents of what I will call the ‘generalized marginalistic principle’ in production. Section 6.2 examines the approaches to production that explicitly express a structuralist point of view on production: it in particular focus on the input-output models. Appendix A discusses, in light of the atomist/structuralist dichotomy, the languages employed in the analysis of economic production (in particular, infinitesimal calculus), and emphasizes the inadequacy of those languages for the representation of the phenomena under consideration.

4.2 The ‘atoms’ of economic production: *Fundamental Units of Analysis* (FUAs)

Most of the theories dealing with economic production, in economics as well as in the neighboring disciplines, starts from the identification of some units of analysis that become the focal point of the theories. These units are conceptually conceived as ‘a-toms’, i.e. etymologically indivisible units of analysis, so that in what follows they will be labeled *Fundamental Units of Analysis* (FUAs) of production theory. The objective of this section is to survey the literature on economic production, identifying the most important types of FUAs that have been conceived there.

Analysts of economic production have reached a high level of conceptual refinement in the identification of FUAs. Georgescu-Roegen, for instance, takes seriously Plato’s suggestion to “curve nature at its joints” (Phaedrus 265d–266a) but, at the same time, seems to reject it. As he says, “because actuality is a seamless whole we can slice it wherever we may please […] And actuality has no joints to guide a carver” (1970, p. 3). Then, the only criterion that can guide our identification attempt has to be a criterion of convenience. It is in these very same lines that Georgescu-Roegen introduces some cracks in this for other ways perfect conventionalist and pragmatist picture, claiming that “as economists we know only too well the unsettled issue of where the natural boundary of the economic process lies [for slicing reality is] an operation that cannot be performed without some intimate knowledge of the corresponding phenomenal domain” (ibid. ; see also 1971, p. 213). The question raised by Georgescu-Roegen can be posed in more specific terms nowadays, claiming that there is a continuous interplay between ‘transitive’ and ‘intransitive’ domains of reality (Lawson, 1997), i.e. between what is subject to changes according to different conceptualizations and what is not. The choice of FUAs pertains to the former, but is somehow bounded by the latter. That is the same to say that nature has joints indeed, although they are multiple, and then cutting is always a matter of choice among them.

The notion of ‘unit’ of analysis may be ambiguous however. It reflects, I claim, an inherent dichotomy lying in the background of all the conceptualizations of production. On the one hand, a unit can be conceived as a *metric*. A metric is supposed to measure an underlying object. In this context, unit is an elementary part of a *quantitative* measurement system. For a metric to be feasible, some requirements have to be met. The relationship between a metric and its reference object should be that the latter is either 1) homogeneous or 2) properly homogenizable[61]. The economists trained in the Cambridge-UK tradition, for instance, know very well that in the (Neo-Classical) ‘production function approach’ the metric employed to measure the aggregate capital (K) does not satisfy these requirements most of the times (see Harcourt, 1972). The most radical conception of metric in production analysis attempts to identify a single measure able to give account of the whole process of economic production (such measure has been often identified with the unit of a monetary system[62]).

On the other hand, FUA can be conceived as *entity*. Trivially, one could say that while a metric is supposed to measure an underlying object, an entity is that object itself. Nonetheless, for what this conception may be right, I are not going to emphasize this aspect. The FUA as entity, as it is conceived here, is a conceptual boundary designed to capture an elementary and fundamental part of production activity. A ‘production unit’, for instance, is a FUA conceived

[61] This is a necessary but not sufficient condition of course for measurability.

[62] In the case of heterogeneous capital, money can be seen as a metric for it, but it is not an autonomous (and then ‘fundamental’) one because it depends on income distribution (see Kurz and Salvadori, 1995).

as entity. Although it is possible to give a quantitative account of the internal components of a production unit, the production unit in itself is a *qualitative* entity (see Katzner, 1983). Figure 4 represents the difference between FUA as metric and FUA as entity by juxtaposing them: what characterizes a FUA as metric is its 'thinness', while for FUA as entity is its "thickness"[63].

Figure 4: Characterizations of a FUA as *metric* (left side) and *entity* (right side)

Other features contribute to characterize a FUA, as they have emerged in the literature. Among them, I could mention, for instance, the distinction between *economic unit* and *technical unit*, which is an implication of the distinction between *economic* and *technical indivisibility* (see Morroni, 1992, pp.26-27, pp.144-5). In Georgescu-Roegen's words, this distinction can be rendered by saying that "[a]n engineer, for example, may draw a boundary between the furnace with melted glass and the rolling machines of a plate glass factory, but not so an economist [...]. For melted glass is not [...] a commodity." (Georgescu-Roegen, 1965, pp. 79-80). That is the same to say that it is not marketable, where the market is identified as the source of economic

[63] 'Thickness', in this case, simply refers to the properties of a referent to be characterized in a plurality of ways.

value. On the line of this argument, Georgescu-Roegen identifies the FUA for production as the 'elementary production process', i.e. "the process defined by a boundary such that only one unit or only one normal batch is produced. The most instructive illustration is the sequence of operations by which an automobile is produced on an assembly line" (Georgescu-Roegen, 1984). This is an economic unit. Accordingly, Morroni defines an 'elementary technical unit' as the minimum set of production elements that can be activated separately to produce a unit of output (Morroni, 1992, pp. 25-8). This is, by definition, a technical unit. The notion of elementary process and that of elementary technical unit are strictly related, in the sense that each economic unit has a counterpart in terms of technical units[64].

The dichotomy human/material is also another source of FUAs' categorical distinctions. The distinction between the human and social dimensions of production on the one side and its material dimension on the other has been recently proposed by Landesmann (1986). This distinction has also disciplinary references. Engineering emphasizes the material/technical dimension of production (e.g. Pahl et al., 2007), while organization science emphasizes the human and relational dimension of production[65] (e.g. Roberts, 2004). The units of the engineering approach could be 'operations' that once properly measured and taken as inputs give an engineering nuance to production functions (Chenery, 1949). The units of the organization approach, on the other hand, could be 'organizational units', and these could be identified as inputs too in some sort of 'organizational' production function (Fioretti, 2007).

[64] On the contrary, a technical unit has not necessarily a given economic unit as its counterpart; in this way, they are not 'dual' notions.

[65] For the natural but sometimes overlooked link between organization and production activity see Langlois and Foss (1999).

Another interesting dimension characterizing a FUA is the extent of its domain. According to this criterion, there can be a *transversal* or a *longitudinal* unit of production: while the former attempts to encapsulate into a single notion a network of economic and technical relationships belonging to different spheres of production, the latter focus on a specific sphere of production. Landesmann and Scazzieri (1996) identify the content of a production process as being constituted by three types of elements: i) tasks, i.e. "completed operation usually performed without interruption on some particular object. […] [for instance, a task could be described by the instruction] 'cut a wood of type A'" (Scazzieri 1993, pp. 84-85); ii) agents participating in the process, that can be physical objects (tools or machines) or human beings; iii) materials subject to transformation in the course of the production process. In this respect, they identify a 'production unit' as a network of elements belonging to each of the previous layer of elements (1996, p. 219). This FUA can be thought as encapsulating in theory all the different elements involved in a production process; in this respect, it has a 'transversal' character. Landesmann and Scazzieri further state that economists have generally concentrated on one of the subsets, avoiding a complete description and analysis of the production process in terms of interaction of these heterogeneous elements (ibid., p. 194). Each subset of productive elements can become itself a FUA of an analysis of the productive process longitudinally oriented to deepen a specific layer of the production process. For instance, Scazzieri (1993) focalizes on the benefits that analysis in terms of tasks may bring to the conceptualization of the production process, whereas Andreoni (2013) emphasized the role of 'productive capabilities' of agents in production.

Alternative criteria to identify FUAs can further be listed. However, there is a characteristic which is common to all FUAs, no matter the way in which they are defined: this is their *microeconomic* character[66]. In other words, FUAs represent always a conceptual partition of the production phenomenon under scrutiny[67], so that the production phenomenon under consideration is the output of the composition of FUAs. This may appear a trivial remark, at a first glance, even tautological. But it is a fundamental remark to introduce the central point of this essay: *FUAs are, by definition, units subjected to composition*. The proper compositional framework of these FUAs will be at the center of the analysis that follows.

4.3 Production as Chemistry

The common linguistic characterization of the FUAs of production as the 'a-toms' of production has inevitably recalled the analogy between production and chemistry. The core operation of both chemistry and production would be, from this perspective, the act of *combining* their atoms, their FUAs, thus grounding the analogy between the two disciplines on their common manipulative roots. In fact, the analogy between production and chemistry also reflects the goal of reducing production phenomena to those of the 'hard sciences', even if

[66] The microeconomic character of the units of analysis in economic production is emphasized by Tani (1986) and Morroni (1992). Still, it should be emphasized that not all microeconomic units are FUAs in the strict sense the term is used here (Georgescu-Roegen's criterion that a FUA cannot be further decomposed once chosen for the analysis in scope): in fact nothing prevents a microeconomic unit to be further subject to analytical decomposition within the same analytical attempt. What I wish to emphasize, however, is that by converse all FUAs are microeconomic units instead.

[67] If the object of analysis is production at the plant level, a FUA for that phenomenon has to be conceptually littler than a plant, while if the object of analysis is production at the firm level, a plant can now be properly a FUA.

chemistry instead of physics and mechanics is taken as the benchmark discipline[68][69]. Mirowski (1989, Ch.6) showed in detail that the analysis of production can hardly be carried out in terms of the mechanics-borrowed formalism of Neoclassical Economics, so raising the general issue that any attempt to establish the analogy between production and other scientific disciplines have to be carefully scrutinized in its consistency. The analogy between production and chemistry shifts the disciplinary background of Mirowski's issue: if chemistry, paraphrasing Marshall, is the new 'Mecca' of economic production, both the *coherence* and *consistency* (Lakoff and Johnson, 1980) [70] of the chemical metaphor for production have to be carefully assessed. This is the objective of this section.

The combinatorial origins of production and technology have been laid down by Schumpeter, who famously claimed that technological innovation consists in "a *new combination* of means of production, that is, as a change in the factors of production (inputs) to produce products (outputs)' (Schumpeter, 1939, p. 87, emphasis added)[71]. This account of 'combination', however, carries a certain ambiguity, which must be clarified. In particular, we

[68] It is often claimed that chemistry can be reduced lastly to physics. This is a very debated point in fact, whose arguments against reductionism can be found, for instance, in Scerri and McIntyre (1997). However, the salience of the choice of chemistry as metaphoric field in place of physics remains untouched by this point.

[69] The analogy between production and chemistry can be well due to their intuitive combinatorial foundations; nonetheless, let me also propose Vela Velupillai's perceptive judgment that "it *is a sign of the times* that our metaphors have 'evolved' from the mechanical to the chemical" (2000, p. 153, n. 10, emphasis added). Chemistry would express better than mechanics the intuitive idea of production in a de-materialized world.

[70] 'Metaphorical coherence' represents the feature of a metaphor not to lead to contradictions in domain shift, while 'metaphorical consistency' represents the feature of the metaphor to be robust over the different instantiations of the domains, i.e. to hold in all the phenomena of the domains connected by the metaphor.

[71] A precursor might still be found in Jean Baptiste Say (1821, pp. 4-5), when he says that the production of values comes from the spatial reorganization of existing matter (see Mirowski, 1989, p. 289).

may ask what does ‘new combination’ mean[72]? Does it mean that old inputs are re-combined or that new inputs are introduced, or both? A similar kind of ambiguity emerges if we distinguish between combination of old inputs and introduction of a new technique. As Stefano Zamagni points out, the production function approach is not able to “distinguish between variations in the coefficients [of production] owing to the introduction of a new technique and variations arising from a reorganization of processes based on the old technique” (Zamagni, in Morroni, 1992, p. 13). In which way a ‘new technique’ is different from a reorganization of processes based on the ‘old technique’? The Schumpeterian, combinatorial approach, seems to be quite ‘inclusive’ in treating these phenomena, reducing them to the plural phenomenology of ‘combinatorics’. Recently, Brian W. Arthur (2009), on the footsteps of Schumpeter, re-proposes the notion of *combinatorial evolution* as the distinctive, ontological trait of production and technology[73]. Under this point of view, the gist of production is pure combinatorics, and the distinctions between old/new inputs or old/new techniques become different ‘epi-phenomena’ of the fundamental category of combination.

Strands of current literature on economic production emphasize the combinatorial dimension of production in a way that leads to an explicit analogy between production and chemistry (e.g. Padgett et. al. 2003, Grandori & Furnari, 2008; Padgett et. al. 2012). Grandori and Furnari, in particular, postulate the analogy between ‘classes of organizational elements’

[72] Leontief emphatically says: “In itself, [the term ‘new combinations’] is not wrong, but it is so vague that it would be better to say nothing at all” (Leontief, 1991 [1928], p. 183).

[73] Arthur quotes an important passage from Schumpeter in which it is expressed, even more clearly, the combinatorial point of view on production and technology: “To produce means to combine material and forces within our reach […] To produce the same things, or other things by a different method means to combine these materials and forces differently” (in Arthur, 2009, p. 19).

(their focus is on organizational units) and the 'table of elements' in natural chemistry (Grandori and Furnari, 2009, p. 68)[74]. In this framework, they look for some *'technical' laws of combination*:

> "these substantive basic rules of combination are arguably a component of design that can be conceived as a science: as much as an architect, while devising a house, should respect the basic laws of the static of construction and of physics; or as much a chemist, devising a new drug should respect the basic laws of chemistry; so the organization designer, while devising organizational arrangements, should respect some basic laws of composition of organizational elements" (2008, p.460).

The identification of these technical laws of combinations would play, according to Grandori and Furnari, a double role in the *science* of production. i) They would allow the identification of the sub-space of technically feasible combinations of organizational elements; ii) they would be a starting point of the second step, the provision of a "theory of predicting combinations" (p. 260), i.e. a theory which is able to predict the properties of the combined entities.

This account of the analogy between production and chemistry, however seems to neglect the often debated *status* of chemistry as a science, since chemistry itself does not seem to be provided any necessary implication from the conditions to be satisfied by any technical combination of elements and the prediction of the properties of the combined outcomes. It was Georgescu-Roegen himself who first raised this point clearly:

> "chemistry is not interested only in how the finite number of chemical elements combine themselves into numberless other chemical substances. As noted above, chemistry is also interested (and even more so) in the various qualities of substances IN THE BULK. And the brute fact is that most of these qualities cannot be deduced

[74] For the distinction between a *naturalistic* and a *constructivist* approach to organizational design, see Avenier (2010).

from the simple properties of the elements involved in the chemical formula. The rules that are now used for predicting the qualities of a substance from its chemical formula are spotty" (ibid. p. 115, capital letters in the original).

The connection between the process by which elements are combined and the prediction of the properties of compounds breaks down continuously:

> "from the viewpoint of extant knowledge, therefore, almost every new compound is a NOVELTY in some respect or other. That is why the more chemical compounds chemistry has synthetized the more baffling has become the irregularity of the relation between chemical structure and qualitative properties". (ibid., p. 115, capital letters in the original)

Georgescu-Roegen, who had also argued that "each special science should build its analytical framework on those elements which represent atomic units within its particular domain" (1976, p. 205), is well aware of the impossibility to draw an analogy between production and the 'chemical method':

> "[g]iven that the 'chemical' doctrine fails to work in the chemical domain, it would be foolhardy to count on its success in social sciences, where the number of compounds is almost limitless and quality dominates the scene to an incomparably greater degree than in the domain of elementary matter" (Georgescu-Roegen, 1971, p. 327)

The point raised by Georgescu-Roegen is clear, and is all meant to prevent the *naïve* analogy between production and chemistry based on the atomistic and combinatorial framework. At this point we face the following alternatives either i) the analogy between economic production and chemistry could be ruled out as simply misleading (after all the so-called 'chemical method' does not work even in chemistry!) or ii) I may try to understand the differences and similarities between production and chemistry at a deeper level of analysis[75]. This would lead us to inquire into the extent and limits of atomism in a cross-disciplinary perspective.

4.4 From atoms to structures

Acknowledging the inadequacy of the analytic notion of 'atom' and of the analytic framework of 'atomism' leads suggests reverting to the opposite analytic notion of 'structure', and to the framework of 'structuralism'. The distinction between aggregates of atoms and structures has been expressed by Jean Piaget as follows:

> "fundamental [is] the contrast between *structures* and *aggregates*, the former being wholes, the latter composites formed of elements that are independent of the complexes into which they enter. To insist on this distinction is not to deny that structures have elements, but the elements of a structure are subordinated to laws, and it is in terms of these laws that the structure *qua* whole or system is defined. Moreover, the laws governing a structure's composition are not reducible to cumulative one-by-one association of its elements: they confer on the whole as such over-all properties distinct from the properties of its elements" (Piaget, 1971, p. 7)

The strong version of the structuralist argument claims that structures are irreducible to atoms, and denies the relevance of the notion of atom itself. However, there is a softer version of structuralism that claims that the properties of wholes depend crucially by the *relative position* that atoms assume within a structure. This soft version of structuralism, it has to be remarked, is a full-fledged position in the debate on ontology, which has been proposed as

[75] Ladyman, Ross et al. (2007, p. 24) emphasize that the analogies with chemistry are often pursued at the level of A-level chemistry, i.e. naïve chemistry. They claim that the analogies with chemistry should consider a more sophisticated view of chemistry.

intermediate ontological position between the strong versions of atomism and structuralism (e.g. Lewis, 1986). In what follows, I shall inquire in which way the framework of structuralism conceived as *relative positioning of atomic entities* may be a suitable framework for the analysis of economic domain.

4.4.1 Combinatorics as structuralism

The term 'combination' is often employed in common usage in an extensive way, and covers a wide set of heterogeneous practices. In section 2 above, we have seen that re-combining a fixed set of atoms and combining new atoms among themselves – *re*-combination and *new*-combination – are two different operations. In this way, we can better understand the difficulty arising from the analogy between production and chemistry. Chemistry, strictly speaking, *is basically the science of the new-combination and not that of re-combination*. In fact, it meaningless to say that two atoms of O are re-combined with an atom of H, since the output is always a molecule of water, H_2O, under any combination of the two elements. Another way to say this is that chemistry is a science that is mostly inattentive to the relative positions of the FUAs, once a fixed set of FUAs are given[76]. While the overall properties of chemical compounds are relatively invariant under changes in the relative positions of a fixed set of elements, the overall properties of productive and technological compounds do vary

[76] Such a statement may sound really indigestible to those who are aware of stereochemistry as one of the most important branches of chemistry. Stereochemistry is the branch that inquires the different properties of compounds that are composed by the same elements that have different spatial dislocations. Nonetheless, I claim that my statement can be accepted at a very abstract level.

systematically under the re-combination of FUAs. This provide a further argument against the conceptual reducibility of production compounds and chemical compounds.

Re-combination crucially depends on *relative positioning*, according to which the characteristics of the whole depend neither exclusively on the atoms involved nor strictly on the rules of combinations. To understand this point, let me introduce and discuss the notion of *modularity* of a given structure. Modularity concerns the property of a system to be decomposable in modules, i.e conceptually and operationally divisible sub-system units. Modularity is the pre-requisite of re-combination conceived as source of change, so that this concept acquires a special status in the analysis of production and technology (e.g. Baldwin and Clark, 1997; Brusoni and Prencipe, 2001; Langlois, 2002). Ulrich (1995) developed the approach to design known as *modular product design*, while Buensdorf (2005), on the same vein, suggested the notion of *modular production process*. The fundamental point to elucidate, in order to understand properly the notion of modularity, is the distinction between *ex-ante* and *ex-post* framework for modularity. If modularity is conceived in an *ex-post* framework, each module is assigned a specific function, so that the aggregate of modules would have a specific teleological objective. In this sense, modularity allows innovation in parts of a system without changes in the other components of the same system. In this vein, Buensdorf maintains that in a perfectly modular (*ex post*) architecture “both architectural and radical innovations are incompatible. Innovation is thus restricted to modifications of the individual components” (p. 232). The opposite situation is depicted in an *ex-ante* modular framework. In this framework, modules are not assigned any functional role to the modules, and the property of decomposition is the source of architectural and radical innovations. The case of Meccano-production (Swan, 1956) is the most striking expression of ex-ante modularity: three identical pieces of Meccano can be composed – thus varying their relative positions – in radically different ways. The same, coming back to the chemical analogy, cannot be said of chemistry: three ‘pieces’ of O cannot be combined in any way that is not O_3 (ozone).

4.4.2. Time and structure

The work of Georgescu-Roegen (1971) has highlighted the central importance of time in the analytical representation of production[77]. Time is often said to be another differential characteristic able to break the analogy between production and chemistry[78]. In particular, the possibility of introducing different time arrangements of FUAs increases conceptually the irreducibility of production to its constitutive elements. Not only the relative position of FUAs in *space* is important, but also their relative positions with respect to *time* are of central importance. The time arrangement of FUAs in a time-explicit framework identifies the *time-structure* of production, and production itself can only be conceived as a *production process*. Both the spatial and time notion of structure have to be taken into consideration, if we wish to identify the sources of variability of productive systems. Figure 5 shows a ‘minimalist’ example

[77] The origins of a theory of economic production inspired by structural considerations with respect to time might be traced back to the work of Carl Menger (1871) who clearly distinguished different ‘stages’ within the ‘structure of production’. For the Neo-Austrian approach to production see Hicks (1973) and Amendola and Gaffard (1998). For the heritage of Georgescu-Roegen’s contribution on time in the economic theory of production, see Vittucci-Marzetti (2013). An attempt to reconcile the time dimension with the neoclassical production framework is undertaken by Winston (1981).

[78] Once more, this is a stylized characteristic of chemistry. Time is a crucial variable, for instance, in chemical reactions.

of the time arrangement of FUA(A) and FUA(B), conceived as operation A and operation B respectively.

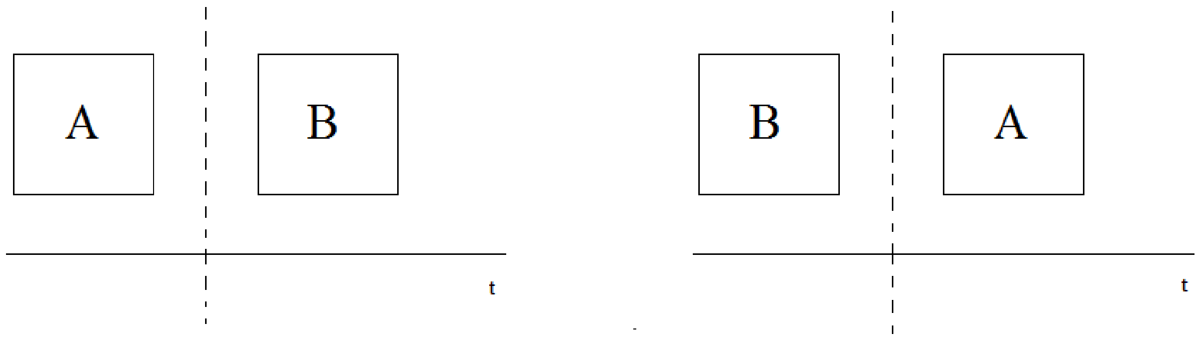

Figure 5: Alternative 'time-structures' of FUA(A) and FUA(B)

4.4.3. Symmetry-breaking

The notion of structure associated with the consideration of relative positioning in space and time entails that the phenomenon of production is subject to some principles of *order*. In fact, in the analysis of structures, *symmetry* identification is one of the most powerful heuristics, and symmetry itself one of the most basic principles of order (Weyl, 1952). The identification of symmetry with order is a natural epistemic principle of symmetric beings like humans. Nonetheless, symmetry by itself does not guarantee order neither in the natural nor in the artificial domain. The notion of 'symmetry breaking'[79], which is the creation of "asymmetric

[79] The notion of symmetry-breaking has been introduced in the domain of chemistry and physics by Anderson (1972). See also Thom (1980).

outcomes in the symmetric environment" (Matsuyama, 2008), generalizes the notion of symmetry and order, and captures the set of phenomena in which *the relative positions of atoms remains unchanged while the aggregate properties of the system change*. An example of symmetry-breaking is given in Figure 5 (above): the outcome of the 'minimalist' production process is not invariant with respect to the symmetric time recombination of the two operations, although the two FUAs remain spatially symmetric the one relative to the other. The framework of symmetry breaking conceptually underpins the combinatorial analysis discussed above. In fact, the relationship between order and symmetry breaks down so frequently in nature that, as it has been claimed, "order is broken symmetry" (Salam, in Mirowski, 1986).

4.4.4. Hierarchic systems

The previous analysis has considered a concept of structure according to which the fundamental characteristics of any given system depend on the relative (spatial and time) positioning of the elementary components within the system. The objective of this section is to enrich this account of structure on the basis of Herbert Simon's notion of 'hierarchic system'. According to Simon, a hierarchic system is "a system that is composed of interrelated subsystems, each of the latter being, in turn, hierarchic in structure until we reach some lowest level of elementary subsystem" (Simon, 1962, p. 468). This account of the notion of system allows us to conceptualize structure as a 'partitioned' system, in which each partition is composed, in turn, by some 'atomic' elements. In this new conceptualization, the mutual influence between the 'atomic' elements of the system is not necessarily 'direct' (so that each element can be assessed on the basis of its relative positioning with respect with *everyone else* element of the system), but is mediated by the influence between the different partitions that have been identified in the system[80]. This means that the mutual dependence of 'atoms' is mediated by the mutual dependence of the partitions in which they are. The concept of a 'hierarchic system' complements the notion of structure along two different directions.

First, the notion of structure as the relative positioning of FUAs takes a dual meaning: i) structure can be seen as the relative positioning of atoms *within* a given partition of the system, or ii) structure as the relative positioning of atoms *among* the partitions of the systems. In the latter case, the degree of *decomposability* of the system determines whether the relationship between FUAs belonging to two different partitions is a relevant structural feature or not. Whether two partitions of a system are not interdependent (i.e. the two partitions are fully decomposable), the relationship between two atoms, each one placed in its partition, is negligible and their relative position does not explain anything.

Second, the notion of hierarchic system creates a link between different hierarchies of 'atoms'. Whether, for instance, workers' capabilities and organizational units are different 'atoms' according to alternative frameworks, the conceptualization of a hierarchic system is able to create a link between them, by showing how organizational units may 'emerge' from the simple interaction of capabilities. The 'emergentist' framework recalls, once more, and in a more sophisticated way, the analogy between production and chemistry. Following the emergentist line of inquiring, John Padgett and his colleagues (Padgett, Lee and Collier, 2003; Padgett and Powell, 2012) have developed a framework in which the interaction of capabilities

[80] Simon (1962, pp. 468-469, n. 6) is clear saying that "by hierarchy I mean the partitioning in conjunction with the relations among the subset". So, hierarchy is partitioning with variable degrees of independence among the parts.

leads to the emergence of a higher production order. Twpo further points are worth emphasizing in this context:

i) "Intermediate stable forms" are an essential step in the emergence of higher-level structures. In the process of combination of lower level atoms, some combinations have certain properties that are more stable than others, and can be considered as an intermediate basis towards the emergence of the system as a whole.

ii) The notion of emergence calls into question the possibility to conceive the arrangement of production as 'design'. Emergence is an evolutionary property of the system, thus it is not completely reducible to a 'constructive' framework. This leads to another important considerations, as in the evolutionary framework the object of evolution are structures and not units (see Dosi and Nelson, 1994, p. 156).

4.5. On the structuralist approaches in the theory of production

4.5.1 Piero Sraffa: against the 'generalized' marginalistic principle in production

This essay has argued so far that a comprehensive conceptual framework for the study of production has still to be devised. What is most clear is, in particular, the *insufficiency* of the atomistic framework in production. To state the rejection of the atomistic framework in economics is the same as to reject the 'marginalistic' framework of neoclassical economics, which is still the framework mostly employed to analyze the phenomenon of production in the discipline.

Piero Sraffa is one of the fiercest opponents of the marginalistic method in economic analysis and, maybe more decisively, he is the one who identified in the domain of economic production the privileged domain in which the employment of such a method could be better criticized. His "arguments against marginism" (Marcuzzo and Rosselli, 2011) demonstrate an advanced and sophisticated view of the issue atoms/structures. Sraffa did not refuse the employment of units conceived as metrics, nonetheless such units had to be employed "not for measurement, but for conception"[81] (ibid. p. 220). The opposition to 'marginism' lies all in the refusal of the Marshallian *continuum hypothesis* borrowed from the Newtonian mechanics, that 'Natura non facit saltum' (D3/12/42/8, ibid. p. 224). As Marcuzzo and Rosselli argue,

> "Sraffa held that change in economic realities hardly ever manifested itself in the form of infinitesimal variations in magnitudes that leave the overall structure unchanged. […] [He] does not seem to have been satisfied with criticizing the continuity hypothesis for its lack of realism, but went on to demonstrate the logical impossibility of continuity in the production function with infinitesimal variations in one factor alone if there were more than two factors" (Marcuzzo and Rosselli, 2011, p. 224).

The argument against the *continuity hypothesis* follows from John A. Hobson's criticism (1909) of marginalism, according to which when a marginal dose of one factor is introduced in production the variation in the total product cannot be attributed just to the last dose, since what changes is the interaction between the factors held fixed and the variable factor. This argument is very important, and it may be helpful if we want to read Piero Sraffa's

[81] This stands at the opposite of Robert Solow's conception of unit, which has significance only in an empirical context of measurement.

work not only as a generic opposition to marginalism, but as a 'structuralist' criticism of marginalism. An example of Sraffa's 'structuralism' can be found in his unpublished papers. There Sraffa claims that "[w]here marginism goes astray is in (falsely) assuming […] that it has general applicability, whereas in fact it only applies exceptionally (*in cases where partial change is feasible, there is independence, the whole is not affected*)' (D3/12/42/9, emphasis added)[82]. The italicized expression shows in full clarity the principles of functioning of a hierarchic system, and in particular shows deep awareness of both independences and interdependences among the components of the system[83].

Another important point of Sraffa's criticism of the marginalist analysis has been identified by Sen (2003), who claims that Sraffa considered as a constituent part of the marginalistic framework the possibility to rely on 'counterfactuals'. Counterfactuals are surely problematic in a context in which small variations in a variable can have significant effects on the system as a whole, since this conceptual framework should assume that the systemic effects are predictable. As a result, a purely marginalistic analysis should be conceived only as an *ex-post* analysis. Marginalism appears to be an inadequate analytic framework in the context of production design, which is a typical *ex ante* setting.

[82] This is also the conceptual underpinning of Sraffa's opposition to the 'coeteris paribus' clause.

[83] Piero Sraffa is a fundamental point in that lineage of scholars, starting from Keynes, who assume a *structuralist* view, conceived as a point of view emphasizing the in/inter-dependencies and relative positioning of the components of systems, one of whom is Luigi L. Pasinetti. It is not by chance that Pasinetti's structuralist strand of thought in economics conceives production as its privileged domain of inquiry (see Pasinetti, 2007). Pasinetti (2007, p. 267) emphasizes the so-called 'fallacy of composition', i.e. fallacy of inferring properties of the singular to the properties of the whole, as the specific trait of the Keynesian, anti-Marginalist tradition.

Marcuzzo and Rosselli acknowledge that "the core of the [Sraffian] critique [to political economy] might have been the marginal method itself". However, as they also claim, "we are left to wonder what would have come after the prelude [of this critique]" (p. 219). A tentative answer to this question can be provided going back to the early thought of Sraffa, when Sraffa himself, however fascinated by the hard sciences, in particular physics and chemistry, refused to carry out production analysis in terms of "atomic analysis" (D3/12/7: 161 (3) and D3/12/13: 16 (9), 18, in Kurz and Salvadori, 2005, section 4). It is interesting to quote at length the motivation of such decision:

> "if we take [the] natural science point of view [to which he ascribed the 'atomic analysis'], we must start by assuming that for every effect there must be a sufficient cause*, that the causes are identical with their effects, and that there can be nothing in the effect which was not in the causes*" (Sraffa D3/12/7: 161, in Kurz and Salvadori, 2005, p. 432, emphasis added).

This is something that Sraffa could not accept for the domain of production, in which the phenomenon of symmetry-breakings is continuously challenging the continuity hypothesis.

Sraffa's refusal to apply the principle of 'sufficient reason' in the domain of production is an example of subtle epistemological analysis of the domain of production. Georgescu-Roegen's claim that production had not underwent any epistemological analysis (see Section 1) is wrong, if we consider the Sraffian contribution to political economy. On the contrary, Georgescu-Roegen's epistemolgical contribution to production may be read as a prosecution of Sraffa's one. While Kurz and Salvadori (2003) rightly emphasize the differences between Sraffa and Georgescu-Roegen, I see them as sustaining a homogeneous epistemological position, in the substantive critique of the marginalistic analysis. A common epistemological position that may be labeled as a criticism of the 'generalized' marginalistic principle. A criticism of marginalism in the domain of quantities and metrics leads *a fortiori* to a criticism of the marginalistic principle as employed in relation to any FUA in the production domain.

4.5.2 Structuralist requirements for the theories of production: the analysis of sectoral interdependencies

The features I identified as fundamental requirements for a structuralist framework of economic production (*relative positioning*, *symmetry-breakings* and *hierarchy*) can in principle be used to assess all theories of production proposed in the economic literature, in order to identify what may be called their 'degree of structuralism'. Different theories of production may in fact present different emphasis on structuralist requirements (or subsets of them), and may thus vary in structuralist attitude and intensity. While the 'marginalist' or Neo-classical approach to production is the prototype of atomistic and non-structuralist production analysis, other theories of production embody, although to different degrees, structuralist considerations. The gist of the often-heard expression 'opening the black box of economic production' coincides, to a large extent, with the introduction of structuralist ingredients in production analysis. The analysis of 'relations' is the distinctive feature of the structuralist turn. This structuralist gist is clearly visible in approaches to production such as the input-output models (Quesnay, 1752; Leontief, 1941; Sraffa, 1960; for the historical reconstruction of the long tradition of input-output models see Kurz and Salvadori, 2000), the vertical integration approach (Pasinetti, 1975b, see also Scazzieri, 1990), and the Georgescu-Roegen's flow-fund model (e.g. Georgescu-Roegen, 1969; Georgescu-Roegen, 1971; for the development of this approach see Vittucci-Marzetti, 2013). In this section, my objective is to assess to what extent the structuralist requirements are met in those theories that focus upon *sectoral interdependencies*, i.e. upon the relationships between sectors within the economic system.

The analysis of sectoral interdependencies can be pursued by following two approaches, in which sectoral interdependencies are represented according to two alternative but complementary points of view: the *horizontal* point of view and the *vertical* point of view. From a horizontal perspective, the productive system is represented as a ‘circular flow’ (Leontief, 1991 [1928)], in which means of production are used in producing final consumption goods as well as in their own material re-production. The vertical perspective, on the other hand, represents the productive system as an ‘allocative’ and ‘transformational’ system, according to which production is conceived either as the allocation of productive inputs to different production processed or as a process of transformation of these inputs in final, consumption goods. For a detailed reconstruction of the notions of horizontal and vertical specification of the productive structure in economic theory, see Baranzini and Scazzieri, 1990). Figure 6 gives the standard analytical representation of the productive system as a matrix of technical input-output coefficients[84] (see Pasinetti, 1975a, ch. 4).

$$\begin{pmatrix} \boldsymbol{a_{11}} & \boldsymbol{a_{12}} & . & \cdots & \boldsymbol{a_{1n}} \\ \boldsymbol{a_{21}} & \boldsymbol{a_{22}} & . & . & \boldsymbol{a_{2n}} \\ \vdots & . & \ddots & . & . \\ \vdots & . & . & \ddots & . \\ \boldsymbol{a_{n1}} & \boldsymbol{a_{n2}} & . & . & \boldsymbol{a_{nn}} \end{pmatrix}$$

Figure 6. The input-output matrix

[84] The economy is composed by *n* sectors and *n* products in this case, i.e. the input-output matrix is a square matrix. A generic technical coefficient a_{ij} (i=1,…,n; j=1,…,n) is defined as the ratio of input *i* per unit of output *j*.

In each row of the input-output matrix we find the technical coefficients relative to the utilization of the commodity produced in the sector *i* (*i*=1,..,*n*) in all the other productive sectors of the economy. On the other hand, in each column we find the technical coefficients relative to the utilization of the different commodities produced in the system as inputs of sector *i* (*i*=1,..,*n*). In this way, the horizontal specification of the system is expressed following the rows, so that we can identify not only the contribution that a sector makes to the production of other sectors, but also the re-integrative contribution of any given sector to itself (diagonal coefficients). It has to be remarked that the horizontal and vertical specification of the system are different ways of representing the *same* system: in this way, they are *dual* notions.

In the input-output framework, *Fundamental Units of Analysis* (FUAs) are identified both at the 'metric' and at the 'entity' levels (see Section 2). The coefficients of production express metrically the purely technical relations among the 'entities' of the framework, i.e. the productive sectors. Inter-sectoral relationships, conceived as "directly observable basic structural relationships" (Leontief, 1987, p. 870), are expressed through the technical coefficients, and structural change is conceived as a change in input-output relationships. A major drawback of the basic input-output framework is that while input-output coefficients can express synthetically any sectorial relationship among sectors in the substantive productive system, they are not able to describe these relationships at a more fundamental level. As already remarked in Section 3, the input-output framework is not able to distinguish between changes of technical coefficients stemming from a reorganization of sectors from changes of technical coefficients stemming from the introduction of a new technique (see also Scazzieri, 1983): more in general, any given technical coefficients are compatible with a series of different production arrangements in the substantive production sector. This drawback could be fixed if we provide the input-output matrix with additional conceptual features, such as the introduction of some principles of ‘order’. The fact that two sectors are ‘proximate’ or ‘distant’ in the input-output matrix is not explicative of anything in the basic input-output framework: what matters are just technical coefficients, no matter their ‘order’ in the matrix. The organization of sectors in the matrix according to some ‘category’ criteria (be it geographical, merceological, etc.) would allow the identification of salient features of the system that *determine*, as unambiguously as possible, technical coefficients. In this way, different *combinations* (according to the selected principle of order) of sectors in the matrix are able to express different salient features of the substantive productive system important for the variability of production coefficients[85]. Technically, the incorporation of ‘proximity’ and ‘combinatorial’ principles into the input-output framework is possible by associating, for instance, to the input-output matrix an *adjacency matrix* able to represent the ‘network configuration’ of the sectoral productive structure, where the network description embodies salient properties (e.g. the geographical distribution of sectors) of the substantive productive system[86]. The analytical description of the substantive productive system is accomplished in terms of *input-output productive networks* (McNerney et al, 2013; Blöchl et al, 2011)[87]. In this way, the network configuration may incorporate explicitly, i.e. describing it, some classical sources of variability of the input-output

[85] If we introduce in the input-output matrix, for instance, a ‘geographical’ principle of order we might render, by varying the relative position of sectors in the matrix, the fact that geographical distance might affect the suitability of a sector as input to other sectors (in this case, the input-output matrix, does not express purely technical relations).

[86] Forerunners of this approach have been Chiou-Shuang and Ames (1965), who associated an ‘order matrix’ to the usual input-output matrix.

[87] For recent developments in input-output analysis see Carvalho (2012)

coefficients (see e.g. Sonis et al., 1996 and references therein). In this way, description of inter-industry configurations is informative of the *structural causes* of production coefficients. The possibility of introducing proximity and combinatorial principles, and of identifying them as sources of variability of sectoral coefficients, would allow assessment of phenomena of symmetry-breakings within the production infrastructure. Even small variations in the proximity and combinatorial configurations of the production structure could determine large variations in the production coefficients.

A last structural requirement that has to be assessed for the input-output framework is that of hierarchy. In this respect, two distinct notions of hierarchy can be considered: the notion of hierarchy as 'relation of dependence' among parts of a system, and the more specific notion of hierarchy as the 'multilayered structure' of complex systems as devised by Simon (see section 4.4). In the former case, the notion of interdependence can be directly rendered in terms of hierarchy. Not only the input-output matrix can assign an explicit status of priority among sectors[88], but also the technical coefficients can do it indirectly. The notion of 'basic' and 'non-basic' sectors, introduced by Sraffa to identify respectively those sectors that participate in the production process of other sectors or not, is a form of hierarchy (the cross-technical coefficients of non-basic sectors with other sectors are, in fact, null). Still, the in the Leontief input-output framework, supply and demand considerations (which are not present in the Sraffian framework, which is a purely 'technical' framework) are able to introduce the common notion of hierarchy among sectors, identifying more 'strategic' (in terms of demand and supply) sectors than others. As for Simon's notion of hierarchy, the input-output framework allows

[88] Network specification of input-output models also put right at the conceptual center the analysis of hierarchy conceived as relation of importance in the network (see e.g. Blöchl et al, 2011).

identification of hierarchy of this type if we consider Pasinetti's reformulation of the input-output framework in terms of *vertically integrated* sectors, *vertically integrated* labor coefficients and *vertically integrated* technical coefficients. Vertically integrated sector is an 'intermediate entity' between the sector conceived as 'atomistic entity' and the system as a whole. It is also important to note that the matrix representation of input-output relationships allows the description of the economic system to be granular at will. Input-output tables can be given of nations, but they can also be given of *regions* at a more granular level, or of international flows at a more aggregate level.

4.6. Concluding remarks

The scope of this essay has been mainly methodological, devoted to identify and assess the *fundamental properties* a framework of economic production has to satisfy in order to be congruent with the nature of the production phenomena. The main result of this methodological inquiry is that the atomistic framework founded on FUAs is not self-sufficient. To address properly the issue of 'aggregation' of FUAs, the atomistic framework has to be complemented with structural considerations. First of all, a framework for production has to consider the relative positioning of FUAs, and so the fact that the outcomes of production are inevitably sensitive to variations of positions of the productive components. This means that production is sensible to some principles of 'order'; still, order is a 'multi-dimensional' phenomenon, so that while order can be preserved according to some dimensions, it may be lost according to other dimensions. Beyond the simple positional analysis, a framework for production can benefit from internalizing the hierarchic systems perspective, as set up for instance by Simon. In this way, the interdependencies within the elements of the system become central.

The approach of the essay has been to identify a framework in which the shortcomings of one specific approach to the analysis of production is complemented by relying on a subsequent, more complete approach. In this way, a sequential schema has been provided to analyze the phenomenon of economic production: *atoms, structures, symmetry-breakings, complex systems.*

In the course of the analysis, the analogy between production and chemistry – that I have showed being more and more used in the literature on economic production – has been employed as a heuristics. The manipulative roots of both chemistry and production, associated with the linguistic habit of calling 'atoms' the FUAs of production, induced many to draw an analogy between chemistry and production. I have showed that the chemistry-production analogy suffers from the different ways the notion of structure is conceptualized in the two domains. In particular, structure as relative positioning of the same set of FUAs is particularly important in the domain of production while this is not the case in the domain of chemistry. However, the notions of symmetry-breaking and complex system hierarchy allow to go beyond a simplistic chemical analogy (which comes from a simplistic 'image' of chemistry), casting new possibilities for the fruitful employment of the chemical analogy in production.

Lastly, the reconstruction of the theories of production that I provided allowed to be aware of those, in the history of economic thought, who clearly revealed the limits of atomism and the necessity of structuralism to deal analytically with production phenomena. In this way, the epistemological analyses of Piero Sraffa and Nicholas Georgesu-Roegen have been considered as particularly foundational.

4.7. Appendix A - The ‘language of production’: some atomistic and structuralist remarks

In the course of the essay, I have inquired the fundamental structuralist requirement of a theory of production from both the epistemological as well as the ontological points of view. I leave to this appendix the completion of the methodological analysis, discussing the linguistic dimension of the frameworks of production in light of the atomistic/structuralist heuristics. The language used in the representation and analysis of production phenomenon embodies the ontological and epistemic assumptions of the framework and, at the same time, allows practitioners to carry out their everyday activity. This critical role is well-expressed by Michael Polanyi, when he says that “the inventor sketchbook is his laboratory” (1962, p. 88); exactly the same can be said of the economist or the production engineer, whose ideas take shape first and foremost through the language they employ when dealing with production.

The relationship between language and economic production can be conceived in many different ways. For instance, language can be considered as an ‘input’ in the productive process. In this sense, Morroni (2013) extends and complement Sraffa’s framework, claiming that commodities are created by means of commodities that, in turn, are created by means of processes that, lastly, are created by means of ‘creation of knowledge’. The creation of language for economic production is itself a productive – maybe the primitive productive – act. The philosopher Ferruccio Rossi-Landi (1968) emphasizes another dimension of the relationship between language and production. He identify a 'homology'[89] between the process of language production and that of material production. The homological scheme of language and material production is a sequential schema, which starts from the production of the respective constituent elements of these domains, language and goods, to get to more and more complex forms within these domains. For example, the schema starts from the basics of the 'phonic material' in the domain of language and of the 'physical material' in the domain of goods, to reach a stage consisting in 'all the linguistic production of humanity' and in 'all the material production of humanity' respectively (see Rossi-Landi, 1968; see also Rossi-Landi, 1975).

The homological framework of production proposed by Rossi-Landi is important for the purpose of my structuralist inquire. If the homology between language production and goods production holds, we have however to take seriously the fact that the linguistic model proposed by Rossi-Landi – a Sassurian model of structural linguistic, according to which the language is composed by progressively more elementary parts – has been decisively challenged in linguistics itself. The Chomskyan revolution in linguistic has shown for instance the shortcomings of the Sassurian models of sentence production, where the sentences producible from elementary components are numberless, so the impossibility to assess them from a simple additive procedure (for the challenges of the Chomskyian paradigm in linguistics see Searle, 1972). The 'atomistic' model of sentence production described in Chomsky, in which a new linguistic particle is progressively added to the sentence, resembles the economic marginalist model of production. What constitutes the basic ontological model of languages is, according

[89] The term 'homology' is explicitly preferred to the alternatives of 'identity', 'analogy' and 'isomorphism' (p.151): it is meant to emphasize the common root of the general act of 'producing' in different domains.

to Chomsky, that of 'syntactic structures'. This implies the consequent delimitation of linguistics to the domain of synthax, i.e. of combinatorial rules. If the atomistic model of language production does not hold anymore, how the homology with economic production should now be assessed?

The revolutionary collapsing of semantics with synthax – the specific trait of the Chomskyian revolution – has been internalized also in the design of many technical languages for specific domains, so that the problem of semantics has been practically ruled out. Very explicit in this sense is also Brian Arthur, in his account of the relationship between production and language:

> "technology as a whole [is] a collection of several languages […]. [I]t means that the key activity in technology – engineering design – is *a form of composition*. It is *expression within a language* (or several). […] [J]ust as utterances in a language must be put together according to the rules of that language, so must designs be architected according to the rules of allowable combinations in that domain. I will call such rules a *grammar*" (Arthur, 2009, p. 76, emphasis added)

Grandori and Furnari (2008) (see Section 3 above) followed this point of view when looking for the 'rules of combination' of the domain of production organization (it is not by chance that Arthur just here says "[t]hink of a grammar as the 'chemistry', so to speak, of a domain, ibid.). However, 'rules of combination' for production, i.e. a production synthax (see Leoncini, Lombardi & Montresor, 2009), cannot exhaust the semantic analysis of the domain of production. While syntax looks at the rules of composition between the elementary parts of the linguistic composition, semantics looks at the meaning and truth-value of the resulting outcome. While syntax defines a set of feasible combinations, semantics identifies the 'effective' and 'successful' ones, according to the objective of the language under consideration. The set of 'effective' combinations is a (weak) subset of feasible combinations. The two subsets cannot be identified in principle. This is most clear in the domain of production in which some productive combinations that are considered to work 'in principle' do not 'actually' work[90].

Economics has no specific language to cope with production phenomena. The usual language of the discipline is that of infinitesimal calculus, employed in the production functions in which the variables are the quantities of inputs and the quantities of output. In the language of calculus there is nothing specific to the representation of production. The language of mathematics may be labeled as a 'general-purpose' language, employed *also* in the context of production. This is why Mirowski claimed that in the production function approach "the problem of production has been reduced to a matter of semantics"[91] (1989, p. 316). Still, the language of mathematics is not able to express some fundamental epistemological requirements of an analysis of production phenomena. For instance, Wassily Leontief (1991[1928), p.181) claimed that the purpose of a theory of economic production is to identify "causal relationship" within the economic sphere of production, while Judea Pearl (2000, section 5.1), a leading causality scholar, claims that the mathematical language used in economics does not provide an unambiguous way to assess causal relationships. Another problem associated with the employment of calculus in the analysis of production phenomena is embodied in Mirowski's remark that "there is no reason to believe that the algebras of economic quantities are

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[90] In the domain of economic production, there is a particular link between the semantic and pragmatic dimensions of the language, dimensions that seem to be indiscernible (for the introduction of the distinction between syntax, semantics and pragmatics see Morris, 1938).

[91] A similar position was held by Marshall and Schumpeter who claimed that production is a 'semantic tempest in a teapot' (see Mirowski, 1989),

isomorphic to the algebras used to characterize their physical manifestations" (Mirowski, 1986). The problem of isomorphism between quantitative and qualitative representations of production is above all evident if we consider as inputs into the production function 'atoms' different from standard quantities. Think, for instance, of the 'tasks' as inputs of the production function. Velupillai (2005, p. 855-856) notes that at the origins of mathematical calculus, the independent variables of functions were, in fact, 'tasks' (such as 'cut a piece of wood longitudinally') and the dependent variable was the output of a series of accomplished tarks. Velupillai, through an enlightening example, remarks clearly that the algebra of mathematics is not suitable to treat with these alternative 'units'. If we consider the production functions:

(1) $f(x, y) = (x + y)^2$

(2) $g(x, y) = x^2 + 2xy + y^2$

they are in principle mathematically equivalent expressions. But if x and y are interpreted as 'tasks' to be performed in the transformative process of the object y, the two expressions are not equivalent. This example clearly shows that not only the manipulation of functions to represent economic production is a matter of semantics, but also the syntax of mathematical calculus is not able to deal with production phenomena. The language of calculus is not able to cope with the plurality of 'atoms' that may be thought as the 'inputs' of production.

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Chapter 5

Numerals as triggers of System 1 and System 2 in the 'bat and ball' problem[92]

Abstract

The 'bat and ball' is one of the problems most frequently employed as a testbed for research on the dual-system hypothesis of reasoning. Frederick (2005) is the first to envisage the possibility that different numerical arrangements of the 'bat and ball' problem could lead to different dynamics of activation of the dual-system, and so to different performances of subjects in task accomplishment. This possibility has triggered a strand of research oriented to accomplish 'sensitivity analyses' of the 'bat and ball' problem. The scope of this paper is to test experimentally the specific hypothesis that *numerals* are responsible for the selective activation of the two systems of reasoning in this task. In particular, we argue that their role goes beyond and cannot be reduced to that of numbers conceived as magnitudes. To test our hypothesis, we devise an experimental setting in which the role of numbers (as magnitudes) is rendered irrelevant. We find experimental results consistent with our hypothesis. We further provide a link between the literature on mathematical problem solving and that on mathematical cognition research, in particular that branch labeled *embodied mathematical cognition*.

Keywords: cognitive reflection, 'bat and ball' problem, dual-system theory, magnitudes, numerals, embodied mathematical cognition

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5.1 Introduction

The dual-system theory of reasoning is a major current field of research into human rationality. This theory distinguishes between automatic/intuitive/fast and reflective/deliberate/slow processes of reasoning (e.g. Pacini and Epstein 1999; for overviews see Osman 2004, Evans 2008; for the debate on this topic see Macchi et al. 2012), often labeled respectively ‘System 1’ and ‘System 2’ (Kahneman 2003, 2011). Individuals sometimes express uncontrolled, impulsive judgments – which may be the source of errors – while other times they adopt a more deliberate and controlled process of reasoning, which is supposed to prevent from committing such errors.

The Cognitive Reflection Test (CRT), introduced by Shane Frederick (2005), has been employed to assess individuals’ ability to suppress impulsive (and then potentially wrong) judgments in favor of reflective and deliberate ones. Lots of research has flourished on this topic, whose common denominator lies in the emphasis on the individual side of the phenomenon: it aspires to identify relevant individual variables (IQ, numeracy, risk preferences, etc.) – and then relative differences among individuals – capable of discriminating performances in the CRT. This approach is related to the so-called *individual differences* approach to cognition and rationality (see Stanovich and West 2000).

The emphasis on individual differences, however, may risk to overshadow a fundamental and complementary part of the story. The well-known metaphor of scissors, introduced by Newell and Simon (1972, p. 55), claims that a theory of reasoning must focus not only on the cognition of individuals but also, as a necessary condition, on the task environment (see also Callebaut 2007). As Gigerenzer and Gaissmaier claim, “if one looks only at one blade, cognition, one cannot understand why and when it succeeds or fails” (2011, p.

457). On the basis of such an argument, research on System 1 and System 2 could be re-addressed accordingly, so as to identify also environmental variables that are responsible for a selective dual-system dynamics.

This paper is precisely devoted to such an inquiry. Our aim is to study the way in which *numerals*, part of the arithmetical arrangement of mathematical tasks, are related to the dual-system dynamics of activation. In what follows, we will inquire such an argument in the context of the 'bat and ball' problem, part of the the CRT. To such a purpose we will run an experiment, whose literature connections, hypothesis, methodology and results will be respectively made explicit and discussed in the next sessions. In the last section, we will make some broader remarks on the theoretical framework suitable to support our results.

5.2 State of the art

5.2.1 The Cognitive Reflection Test (CRT)

The vast literature on the CRT can be summarized into a few main strands. A first strand of research is devoted to characterize the CRT, so as to identify what the CRT measures, and how and if it is the right way to measure what it is supposed to measure. The debate is oriented to assess whether the CRT is "another intelligence test" and what 'cognitive reflection' really means (see Frederick 2005 himself; Campitelli and Labollita 2010).

Connected with this issue, another stream of research is devoted to understanding the determinants of performance in the CRT. This is accomplished mainly by investigating which individual variables are able to predict performance in the CRT. The individual variable that gained more attention in this literature is the numeracy of individuals: experimental studies show a strong correlation between performance in numeracy tests and performance in the CRT. Liberali et al. (2012) investigate this relation by comparing objective and subjective numeracy scales (the broader relationship between numeracy and decision-making has been the object of inquiry in Peters et al. 2006). Stupple, et al. (2013) investigate the role of other individual variables (e.g. working memory capacity) as predictors of performance in the CRT. This kind of approach, in which individual variables are hypothesized as determinants of performance, is particularly meaningful in the 'heuristics-and-biases' research program (see Toplak et al. 2011), according to which heuristic decision-making is decisively linked to minor cognitive abilities. Oechssler et al. (2009) and Hoppe and Kusterer (2011) investigate the relationship between performance in the CRT and specific cognitive biases.

Another strand of literature, though of far less impact, is related to the issue of 'debiasing' the CRT (see also the next section). Whereas the CRT has been conceived as a test specifically designed to induce the activation of System 1, this kind of research focuses on the ways in which the CRT may be rearranged in order that the System 1 is suppressed. This strand of research on the CRT can be considered as part of a broader research program, in the context of the dual-system hypothesis of reasoning, which studies the environmental determinants of the dual-system dynamics of activation (see Alter et al. 2007; Söllner et al. 2013).

5.2.2 The 'bat and ball' problem

Although the CRT is composed of three different tasks, the 'bat and ball' problem ("a bat and a ball cost $1.10 in total. The bat costs $1 more than the ball. How much does the ball cost?") is the most widely considered, also for having been studied in isolation before the CRT was devised. The dynamics of respondents' interaction with the 'bat and ball' problem is well-represented by the *attribute substitution* heuristics (Kahneman and Frederick 2002), according to which individuals substitute, in task accomplishment, an unknown target attribute with an available, but potentially wrong, heuristic attribute. In the 'bat and ball' problem the solving process collapses, in fact, into the available, but wrong, subtraction 1.10 – 1 = 0.10, and this subtraction substitutes the correct, but less intuitive, structure of the problem (notice that the correct answer is that the ball costs $ 0.05). More generally, Morewedge and Kahneman (2010) show that, along with attribute substitution, other two features of associative memory – i.e. associative coherence and processing fluency – give rise to errors of this kind.

The 'bat and ball' problem has gained more attention than the other tasks in the CRT also due to Frederick's explicit acknowledgment that people "do much better on analogous problems that *invite more computation*" (p. 28, emphasis added), suggesting a slightly different version as the 'banana and bagel' problem ("A banana and a bagel cost 37 cents. The banana costs 13 cents more than the bagel. How much does the bagel cost?"). Such acknowledgement has triggered research on the 'sensitivity analysis' of the 'bat and ball' problem, inquiring whether and what specifics of the 'bat and ball' problem could be changed in order to get a better performance in it. De Neys et al. (2013) have highlighted that attribute substitution is "graded": exposed to different arithmetical specifications of the 'bat and ball' problem, subjects are partially aware of their substitution process so as to exhibit various degrees of confidence in their response. Macchi and Bagassi (2012) have shown that performance in the 'bat and ball' can be related to the rhetorical arrangement of the problem: performance improves whether the question is formulated in a way more adequate to the aim of the task. Still, Bourgeois-Gironde and Van der Henst (2009) have shown that 'debiasing' can be pursued by modulating different dimensions of the 'bat and ball' problem: in particular, they show how variations pertaining to arithmetical processing, priming dynamics, intuitive dimension and accessibility systematically affect the dual-system dynamics.

5.3 The Hypothesis

Our contribution, along the line of research investigating the dual-system response to task environment modulation, aims at formulating and testing a new hypothesis, concerning the conceptual distinction between numbers (as expression of magnitudes) and numerals, and the implications of such distinction for the dual-system dynamics. As a testbed we employ the 'bat and ball' problem, already exploited in the literature to inquire dual-system's sensitivity to different arithmetical specifications. In order to disclose the content of our hypothesis step by step, it is important to notice that, in the reviewed literature discussed above, the 'bat and ball' problem has been administered to subjects as a problem characterized by two degrees of absolute arithmetical freedom:

"a bat and a ball cost $ X in total. The bat costs $ Y more than the ball. How much does the ball cost?"

Both the difference in price between the two items (Y) and the sum of the prices (X) are subject to unconstrained experimental variations. When the 'bat and ball' problem and its varying version (i.e. the 'banana and bagel' problem) were experimentally administered so as to inquire the dual-system differential responses to them, no attention has been paid to the issue of magnitudes involved in it. The 'bat and ball' and the 'banana and bagel', whose data are respectively \$1.10 - \$1 and \$0.37 - \$0.13, do not strictly share any arithmetical homogeneity. Bourgeois-Gironde and Van der Henst (2009), and De Neys et al. (2013) administered the two problems in juxtaposition having as only explicit constraint their algebraic "isomorphism" (De Neys et al. 2013, p. 270). The strong hypothesis underlying the literature on the 'sensitivity analysis' of the 'bat and ball' is that *numbers* involved in the 'bat and ball' are responsible for dual-system dynamics. There are numbers, according to Frederick's authentic interpretation (2005, p. 28), which "invite more computation" than others, in this way activating cognitive reflection.

In this essay, we want to go a step further with respect to this formulation of the story; we think that additional considerations concerning the arithmetical specifications of the varying 'bat and ball' problems are a central issue. If we go so as to devise two different versions of the 'bat and ball' problem that are not only algebraically isomorphic but that also possess some form of arithmetical similarity, we could be able to say something more than the usual hypothesis. In particular, if the 'debiasing' of subjects holds over with arithmetical similarity of magnitudes, we could state that determinants of the dual-system dynamics are *numerals* and not *numbers*, i.e. we could rule out the role of magnitudes in the dual-system dynamics, identifying a further determinant.

The distinction between numerals and numbers is fundamental in mathematical cognition research and is discriminative in our hypothesis. While numbers are ideal entities which express magnitudes, numerals are their ostensive (real-world) representations (see e.g. Cohen Kadosh and Walsh 2009, p. 313). For instance, 19 is a numeral which expresses a particular magnitude within the base-10 numeral system, but the same magnitude is represented by another numeral in the base-2 numeral system (i.e. 10011). In general, holding constant the number-magnitude we can have different numeral representations (think also of different numeral systems that share the same base-10 but that use different numerals to express the same number: 19 is XIX in the Roman numeral system).

Numerals are relevant not only because they are different ways of representing the same number, but also because they are what individuals manipulate in order to perform arithmetical operations, that is to say, computation requires specific arrangements of numerals to be performed and it is sensitive to such arrangements. Referring to such a point, and misunderstandings, Lakoff and Núñez state: "when we learn procedures for adding, subtracting, multiplying, and dividing, we are learning algorithms for manipulating *symbols-numerals*, not *numbers*. What is taught in grade school as arithmetic is, for the most part, not ideas about numbers but automatic procedures for performing operations on numeral-procedures that give consistent and stable results" (2000, p. 86, emphasis added).

On the line of such arguments, mathematical tasks that are isomorphic in algebraic structure and share arithmetical similarity (in terms of magnitudes), are close to be the 'same task' – so they are expected to be solved in the same manner – than tasks which do not share any. If a systematically different dual-system dynamics is involved in tasks that are equivalent along both the arithmetical and algebraic dimensions, then the residual determinant that explains such dynamics is related to how such tasks are embodied in a numeral system, that is to say, in our hypothesis, *numerals are able to account for dual system dynamics, holding constant the issue of magnitudes*.

5.4 Methodology

In what follows we are going to identify a method apt to test the hypothesis that *numerals, holding magnitudes similar, account for the dual-system dynamics in 'bat and ball'-like problems.* To such an objective, we have devised an identification strategy based on 4 steps, so as to rule out all factors that work as confounders of our working hypothesis.

1) We require our experiment to allow us to rule out individuals' differences (IQ, numeracy, risk-attitude, etc.) that could account for the differential of performances in the experiment. In other words, our experiment should be able to go directly to the other blade of Simon's scissors, from individuals' cognition to the task environment. In order to do so, we have devised a randomized control trial (RCT) in the administration of the 'bat and ball' problem in different versions.

2) We require our experiment to be based on 'bat and ball'-like tasks, i.e. tasks that share the identical logical (algebraic) structure with the original 'bat and ball'. The tasks assigned to the control and to the experimental groups have to be structurally identical to the 'bat and ball' problem.

3) The tasks administered to the control and to the experimental groups have to be identical with respect to any other characteristic that is not the object of experimental study and that could change the 'frame' of the task. For instance, we will not assign a task labeled 'bat and ball' to one group and another labeled 'banana and bagel' to the other, because this could result in a further unwarranted degree of freedom in the experimental design.

4) As a specific characteristic of our experimental setting, we will administer two tasks, one to the experimental and the other to the control group, which share the same overall magnitude (arithmetical similarity), in order the two tasks to be not only algebraically identical

but also arithmetically homogeneous. Therefore, we will be able to identify possibly different dual-system responses because of the numerals involved in the task.

5.4.1 The experiment

The methodological principles previously set out are implemented through an experiment. We selected 60 subjects from a virtual social network. Our sample is characterized by variability according to four main features: Sex: 35 females (58%) and 25 males (42%); Age: 47 subjects of 20-40 years old (78%) and 13 subjects of >40 years old (22%); Education: 22 subjects with high school education (37%) and 38 subjects with university education (63%); Country: 55 subjects of the same country (Italy) (92%), 5 subjects of other countries (8%).

We randomly assigned such subjects to two groups: 30 subjects to the *control group* and 30 subjects to the *experimental group*. By randomizing, we rule out that the correlation between individual differences and the assignment may lead to biased results.

We use as a bedtest the “bat and ball” problem. In particular, we use the “bat and ball” problem in its original formulation:

“A bat and a ball cost $1.10 in total. The bat costs $1 more than the ball. How much does the ball cost?”

and a “bat and ball modified” problem that we created:

“A bat and a ball cost $1.10 in total. The bat costs $ 1.01 more than the ball. How much does the ball cost?”.

The 'bat and ball' problem and 'bat and ball modified' problem are obviously characterized by the same algebraic structure to the extent they require the same equations to be solved (the correct answer to the modified problem is that the ball costs \$ 0.045). Furthermore, both problems are characterized by substantially the same magnitudes of numbers: 1.10 and 1 in 'bat and ball' problem; 1.10 and 1.01 in 'bat and ball modified' problem. The difference of just 0.01 between the two versions of the problem can be considered negligible, from the point of view of magnitudes[93] and this fact can be considered as the central identifying assumption of our experimental design. Our experiment aims to articulate Kahneman's idea that the physiognomy of the number – the 'natural separation' of \$1 and 10 cents (Kahneman 2003, p. 1450) – is explicative of the dual-system dynamics in the 'bat and ball' problem. On the line of this argument, we maintain that the physiognomic hypothesis is fundamental in the context of the 'bat and ball' problem: as we are going to demonstrate, a rather negligible difference in the magnitudes of the problems entails two different numerical physiognomies of the tasks and, hence, a different pattern of activation of the dual-system dynamics. As we shall discuss in detail in section 6, the relevance of the physiognomy of numerals for the dual-system dynamics is specific to each numeral system, i.e each numeral system (such as the decimal system) provides physiognomies that determine different patterns of activation of mathematical reasoning.

We administered the original "bat and ball problem" to the control group and the 'bat and ball modified' problem to the experimental group. We provided a half A4 paper and a pen

[93] We avoid, in our setting, to provide versions of the task in which magnitudes were identical (e.g. relying on fractions Vs. decimals) because the experimental outcome could be imputed to the effect of 'frame' (see Druckman 2001) and not, strictly, to 'numerals', even if it was actually the case.

to the respondents. We limited the amount of time to come to a solution to 10 minutes[94].Our experimental design is consistent with the four arguments discussed in the previous section, insofar as it neutralizes:

1) the effect of individual differences, by means of groups randomization;

2) the effect of logical structure of the problem, by administering problems equivalent in logical structure,

3) the effect of non-structural and 'framing' characteristics, such as different names and words involved in the task[95];

4) the effect of magnitudes (numbers), by using problems that are very similar in their arithmetical magnitudes.

Consistent with our hypothesis, we expect a different dual-system dynamics of activation depending solely on the specific numerals 1.10, 1, 1.01 involved in the tasks, holding constant 1) individual differences, 2) the algebraic structure of the task, 3) the 'frame' of the task; 4) the magnitudes/numbers involved in the task.

As in the CRT, we analyze not only the answers to the test but also the introspective verbal reports (see Ericsson and Simon 1980); in particular, we asked subjects to reconstruct and narrate in detail, immediately after they accomplished the task, all the solving process they had used to come to the solution.

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[94] We do not find in Frederick (2005) specific indications on how much time in CRT accomplishment is allocated to answering the tasks. Since he gives 45 minutes to perform the three tasks and a questionnaire, it seems reasonable to give 10 minutes for the 'bat and ball' alone.

[95] De Neys et al. (2013) and Bourgeois-Gironde and Van der Henst (2009) are not attentive to such a factor.

The results of the experiment have been codified in terms of two types of answers provided by respondents: i) incorrect answer (that we assume based on the use of System 1), ii) correct answers (that we assume based on the use of System 2[96]).

5. 5 Results

We present the experimental results in Table 2 below.

	Incorrect answers (System 1)	**Correct answers (System 2)**	
'bat and ball' problem: control group (n=30)	43.3%	56.7%	**100%**
'bat and ball modified' problem: experimental group (n=30)	20%	80%	**100%**

Table 2: Relative frequencies (N = 60)

The evidence shows neatly that there is a difference in the percentages of correct answers between the experimental and the control groups. The control group's results ('bat and ball' problem) are in line with the ones of Frederick (2005): 43.3% of subjects make the wrong answer ("10 cents") and 56.6% of subjects make the right one using System 2. In the

[96] The assumptions 'incorrect answer=use of System 1' and 'correct answer= use of System 2' can be partially relaxed (see e.g. Oechssler et al., 2009, p. 148) considering also the cases in which the subjects are not able to technically implement the correct resolution process, although not being biased by the problem. Here, however, we prefer to avoid such a further degree of freedom, performing the experiment in order to be comparable with the other experiments in this literature.

experimental group, 20% of subjects provide the wrong answer, and up to 80% of the subjects provide the correct answer (relying on System 2).

In order to test the statistical significance of our results we perform a Fisher's exact test for the contingency table (see Table 3 below). The results of the statistical test is p = 0.0473: it allows us to say that there is a statistically significant relation between treatment and outcome[97].

Contingency table		**Treatment**		**Row totals**
		"bat and ball"	*"bat and ball modified"*	
Outcome	Incorrect answer	13	6	**19**
	Correct answer	17	24	**41**
	Column totals	**30**	**30**	60

Table 3: Contingency table

p=0.0473 (Fisher's exact test)

The evidence suggests that the specific *numerals* involved in the task are responsible for the dual-system dynamics in our experimental setting. In particular, when subjects handle numerals like 1.10 and 1, the solution collapses in the use of an automatic/intuitive judgment. In the case of numerals like 1.10 and 1.01 used in the experimental group, the reflective system

[97] We use the Fisher's exact test because the chi-square test was not suitable to our experimental data, since one cell of the contingency table has a numerosity less than 10 and we have just 1 degree of freedom. The Fisher's test we implement is one-tailed since, consistently with our hypothesis and the CRT, we are not interested in testing the hypothesis that harder computations entail an increasing activation of System 1. To check the result, we also performed a one-tailed Barnard test for the contingency table; Barnard's test is considered more powerful than Fisher's one because it maximizes with respect to the nuisance parameter when calculating p. The result is p=0.0351.

is triggered to a higher degree. Verbal reports of respondents provide some insight concerning the cognitive processes involved in the solving processes. The reports are represented by unstructured notes of what subjects declared immediately after they solved the task; the arguments have been organized into 4 categories related to the experimental design: 2 Treatments ('bat and ball', 'bat and ball modified') x 2 Outcomes (Incorrect, Correct):

a) Treatment: 'bat and ball' problem, Outcome: Incorrect.

Subjects reported that it was "natural" to "subtract 10 cents from 1"; when they realized that the problem was a "matter of subtraction" they automatically performed 1.10 – 1 = 0.10 and reported the wrong solution.

b) Treatment: 'bat and ball modified' problem, Outcome: Incorrect.

A limited number of subjects reported a wrong answer based on subtraction 1.10 – 1.01.

c) Treatment: 'bat and ball' problem, Outcome: Correct.

Some subjects reported that they directly modeled the problem in terms of a "system of equations". Others adopted a systematic process once they realized, after checking, that 10 cents was not the right answer; in other words, alerted by the simplicity of the solution, they were tempted to check it.

d) Treatment: 'bat and ball modified' problem, Outcome: Correct.

Subjects reported that they started to "try a subtraction", but in the meantime they realized that they needed a more systematic approach to model the problem: so they built a system of equations. A few subjects admitted that they were tempted to subtract 1.10 – 1.01, and it seems that deliberate process aroused as a consequence of the impossibility of performing a fast subtraction.

The experimental evidence suggests the necessity to discuss our most important findings, in particular the role of numerals (as distinguished by numbers as magnitudes) in the dual-system dynamics of the 'bat and ball' problem. This is what we shall do in the next session and in the conclusion.

5.6 Discussion

As we expected, our experimental design succeeded in creating a 'debiasing' effect, i.e. individuals in the experimental group were not induced to give an impulsive answer to the problem, resulting in a better performance. This result, in light of our experimental design, allows us to draw some conclusions.

- As we discussed in some length in section 4, our experimental design implements an identification strategy that allows us to say something more about the practice of varying parameters in the 'bat and ball' problem: neutralizing the effects of 1) the individual differences, 2) the algebraic structure of the task, 3) the 'frame' of the task, 4) the magnitudes/numbers involved in the task, we are able to say that in our setting *the dual-system dynamics is due to varying the numerals and not the numbers of the 'bat and ball'*. Notice that we are not stating that numbers/magnitudes do not matter; we are just specifying that they are not the end of the story, and that numerals play a decisive role when magnitudes are very similar.

- The way in which the magnitudes involved in the task are related to the degree of computational difficulty of the task is itself an issue that deserves further investigation. Frederick explicitly states that people "do much better [in the 'bat and ball'] on analogous problems [like the ‘banana and bagel’] that *invite more computation*” (p. 28, emphasis added). But, we ask, what does ‘more computation’ mean? Which mathematical task can be properly said to “invite more computation” than another one? Why do the numbers 0.37 and 0.13 entail more computation than 1.10 and 1? A tentative answer could be proposed, for instance, by considering what electronic calculators would consider as ‘more computation’. As far as two mathematical operations differ in magnitude – although they share the same algebraic structure – the computational load involved in processing is, strictly speaking, not the same; so one operation entails, by definition, more computation than the other. But this is not the case when two problems involve equivalent magnitudes. In our case, holding constant both the logical dimension (type of problem) and the magnitudes (its instances) the computational load of the two problems is the same to the extent that they are fungible for a computer (see Goldreich 2008). In fact, from a purely computational point of view, our experimental setting neutralizes the argument that the difference in performances between the ‘bat and ball’ and the ‘bat and ball modified’ is due to the computational load: the two versions are, by the experimental conditions, characterized by the same computational load (same logical structure and magnitudes).

- The picture changes if we look at the problem of ‘computation’ through the lens of human cognition. This change of perspective requires the reformulation of the notion of arithmetical manipulation and of the very nature of computation too. In fact, from the point of view of human cognition, two problems, which share the same logical structure and magnitudes, could be perceived as requiring a different computational effort, depending on the specific instances involved in the task. Wulf Albers’ *prominent theory* (Albers and Albers 1983; Albers 2002) “models the construction of numerical responses and the perception of numerical stimuli in the *decimal system*" by identifying the so-called *prominent numbers* or *full-step numbers* "..., 1, 2, 5, 10, 20, 50, 100,… , i.e., the powers of ten, their halves, and their doubles" (Albers 2002, p. 297, emphasis added); prominent numbers can be seen as pivots and facilitators in arithmetical computations in the decimal system. Arithmetical operations characterized by more prominent numbers require less computational effort with respect to equivalent operations in which no prominent numbers are at stake, triggering reliance on arithmetical automatism. Number 1 used in the original 'bat and ball' problem is more prominent with respect to the number 1.01 used in the 'bat and ball modified' problem. In spite of the negligible difference in their magnitude, their different prominence fosters a different dual-system dynamics (notice that the numbers employed in Frederick's 'banana and bagel' problem are also less prominent with respect to those of the 'bat and ball' problem).

- Current research in mathematical cognition develops a distinction, analogue to that of dual-process theory in decision-making and problem solving, discriminating between automatic and intentional number processing (Tzelgov et al. 1992). This research inquires the interaction between human cognition and arithmetical manipulations emphasizing the ostensive dimension of numbers. The shift from numbers to numerals means that number processing is not 'modality independent' (see for a review Cohen Kadosh and Walsh 2009). A fundamental result in such a strand of research is that numerical symbols do not automatically activate substantive quantity representations (Cohen 2009, p. 336). Numerical surface form (i.e. the number's 'physiognomy'), for instance, is discovered to be a primary determinant of numerical processing (Campbell and Fugelsang 2001; Jackson and Coney 2007; for the debate on the interpretation of these results see Zhang et al. 2010). In decision-making research, the fact that numerals (which are independent by quantity interpretations) affect decisions, has been object of inquiry for instance in the context of cooperative behavior (Furlong and Opfer 2009). Peters et al. (2008) show that people who are able to infer better substantive magnitudes from numerals are better in decision making. In the context of such a broader and much debated field of research in mathematical cognition, we want to emphasize the promising turn that such a debate has been assuming in the last decade, i.e. its increasing relationship with the *embodied cognition* research. Such a strand of research attempts at identifying *body correlates* at the roots of mathematics and mathematical processing (e.g. Lakoff and Núñez 2000, see Núñez 2008 and references therein).

In particular, *counting systems* are the link between embodied cognition and the dual-system dynamics in mathematical cognition. This point may be emphasized in particular in a cross-cultural perspective (e.g. Núñez 2011; Göbel et al. 2011). Different cultures develop different counting systems, each based on different parts of the body as references (Gibbs 2006, p. 105). For instance, the Mamuo counting system in Africa has both fingers and toes as references, so that the resulting numeral system is vigesimal (the sum of fingers and toes), i.e. the numeral system is based on 20. More commonly, the Arab numeral system – coming from a counting system based only on fingers – is a 10-base numeral system. The way in which the body-counting system shapes the numeral system is the same in which it triggers mathematical automatism for special numerals in each numeral system. To put it more bluntly, each numeral system has its own numerals that foster arithmetical automatism, depending on the body correlates of the underlying counting system. What Albers calls 'prominent numbers' are actually 'prominent numerals' in the 10-base numeral system.

- Another point that deserves specific mention is the idea of 'task environment' that emerges from our research. The task environment, from the pioneering article of Simon (1956), has been thought in its structural or, more precisely, syntactical dimension: this is still the case in the research program on *ecological rationality* that today assigns, systematically, a decisive emphasis on the conceptualization of the task environment (Bullock and Todd 1999). Our research highlights the importance of what is generally beyond the attention of a naïve structural analysis: by focusing the attention on the specific modal content embedded in structures, our research claims in favor of an enriched re-conceptualization of the notion of structure in current rationality paradigms.

5.7. Concluding remarks

This last section is oriented to frame our attempt to identify the crucial role of numerals in mathematical problem-solving and dual-system research as part of a broader intellectual framework. The current research in mathematical cognition emphasizes the role of numerals in a twofold way:

1) numerals are the ways in which a mathematical task is arranged in real-world situations, so that they are the artifacts in which abstract mathematical tasks are e*mbodied*: the *interaction* between the problem-solver and these artifacts becomes central in the assessment of the performance in the task;

2) numerals are expression of specific numeral systems. Differences of base among numeral systems depend crucially on the underlying counting systems. For instance, the decimal system stems from a counting system based on the 10 fingers of both hands. These

body correlates explain lastly the special status of prominent numerals in the decimal system (Albers' 'prominent numbers') and, accordingly, the special status of different prominent numerals in different numeral systems. This special status plays a decisive role in the dynamics of activation of the dual-system in mathematical cognition.

This turn in mathematical cognition research (see e.g. Cohen Kados and Walsh 2009 and the related Open Commentary) follows the advent of the *situated* and *embodied cognition* paradigm in cognitive psychology (for an overview see Calvo and Gomila 2008), which sees respectively interaction and body correlates as determinants of cognition. The internalization of this new paradigm in the research on the dual-system could be devoted, more generally, to identify the situational and body determinants of the dual-system dynamics of activation. In our experimental setting numerals can be conceived as embodied entities in the ways expressed above. The situated and embodied approach could be seen as a promising perspective in the light of that attempt to identify "common principles" (Kruglanski and Gigerenzer 2011) at the roots of the dual-system dynamics of activation.

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Chapter 6

Concluding Remarks and Lines of Future Research

The aim of these concluding remarks is to overview the line of argument and contribution of each chapter in this dissertation, and to identify lines of further research. This exercise will take advantage of the tools of structural heuristics developed through the previous chapters.

Chapter 1 on 'Plurality in economic axiomatics: a 'structural heuristic' reconstruction' has applied structural heuristics to a specific case of conceptual reconstruction. In particular, the essay has argued for the need to move beyond the approach of the last three decades of research in economic methodology, which has seen historical heuristics as the privileged standpoint from which to explore and critically reconstruct economic axiomatics. The essay has proposed structural heuristics as an epistemic device to be employed *before* historical reconstruction in order to trigger awareness of the concept to be reconstructed and to guide conceptual exploration. The essay has shown that the application of structural heuristics to the study of economic axiomatics is useful in two different ways. First, it has allowed the *systematic* reconstruction of the plurality of criteria behind the notion of economic axiomatics. In this respect, the essay provides the basis for both new taxonomical distinctions and the systematization of old distinctions in a comprehensive conceptual organization. Second, the essay suggests a way to overcome long-lasting controversies in economic axiomatics by means of a conceptual scheme allowing better understanding of the different points of view.

Chapter 2 on ‘Structural realism in econophysics: the case of Giuseppe Palomba’ aims to contributing to both the history of economic analysis and the philosophy of economics. Structural heuristics applied to the history of economic analysis has allowed the interpretation of the work of Giuseppe Palomba, one of the founding fathers of econophysics, as an original contribution to structural thinking in economics. In this respect, the contribution of Chapter 2 is twofold. First, the chapter emphasizes the structuralist gist of Palomba’s work. Second, and maybe more interestingly, the chapter connects Palomba’s structuralist framework with contemporary philosophy of science research on ‘structural realism’. The essay has argued that Palomba’s contribution can be interpreted as an example of the proto-structural realist approach in economics. A distinctive aspect of the essay concerns identification of the disciplinary domain of econophysics. This type of investigation is seen as of critical importance in assessing how suitable the notion of structural realism is in its application to the domain of economics. In fact, physics is the disciplinary domain to which the notion of structural realism was first applied. For this reason, econophysics would be where to start when discussing structural realism in economics.

Chapter 3 on ‘Atoms, structures and hierarchies in the analysis of economic production’ has provided an assessment of the ontological and epistemological status of the theories of production in economics, and a framework aimed at solving the methodological problems that have emerged in the assessment phase. In particular, structural heuristics bring to the fore the shortcomings of naïve atomistic frameworks in theories of production. In this respect, the chapter is meant to be an example of *positive* structural heuristics. Structural heuristics as applied to the reconstruction of theories of production has allowed the identification of structures of relations as a framework for production analysis more suitable than the atomistic one. The chapter has identified the features of structural heuristics more apt to the study of production phenomena. In particular, and starting from the notion of structure as 'relative positioning', the chapter has examined the structuralist notion of 'symmetry-breaking' and the notion of complex system architecture as conceived by Herbert Simon. This analysis has led to a reassessment of the analogy between production and chemistry, and has highlighted similarities and differences between these two domains in the way they conceptualize the notion of structure. This case study brings to the foreground the effectiveness of structural heuristics as epistemic tool for cross-disciplinary and cross-domain methodological assessment.

Chapter 4 on 'Numerals as trigger of System 1 and System 2 in the "bat and ball' problem" ' is a contribution to experimental economic psychology. It proposes and tests a new hypothesis in the context of the dual-system theory of reasoning and of the Cognitive Reflection Test (CRT). Structural heuristics have played a decisive role in this investigation, since it has triggered acknowledgment that the logical structure of mathematical tasks cannot by itself explain the variability of the dual-system dynamics of problem-solvers involved in the task. The arrangement of the mathematical task is in fact the most important determinant of the dual-system dynamics of activation in problems that share the same logical structure. In this sense, this experiment is an exploration in *negative* structural heuristics, since it emphasizes that (logical) structures are not the end of the story. In particular, the experimental setting has allowed testing the new hypothesis that the *numerals* involved in the problem, and not the *numbers*, are responsible for different patterns of activation of the dual-system in a specific mathematical task (the 'bat and ball' problem). This means that the sign-tokens in which the task is arranged, and not the magnitudes described by such tokens, are responsible for adoption of specific activation patterns. This is a novel conceptual distinction in the literature in decision-making and mathematical problem solving. Further support for this conceptual distinction (numerals/numbers) is found by linking research on mathematical problem solving with the psychological literature on *embodied mathematical cognition*.

Taken in their unity the four essays point to the central importance of structural heuristics as a tool for rational reconstruction in the domain of economic knowledge. In this respect, a full-fledged program of research in structural heuristics could be further pursued along two main directions. First, attention could be devoted to the origins and status of structural heuristics in the context of what is called the human reason's 'tool-box' (Gigerenzer). This type of inquiry should be devoted to specifying the phylogenetic and ontogenetic evolutionary features determining under which environmental conditions such heuristics are effective. This investigation would be inherently interdisciplinary, and would need collaboration among disciplines such as evolutionary psychology, cognitive psychology, philosophy of science, and decision-making and problem solving research. Second, the implementation of the research programme on structural heuristics would need the *systematic* application of such heuristics to a specific knowledge domain. This dissertation has applied structural heuristics to the domain of economics by considering a number of specific fields of economic investigation. A systematic and extensive application of this heuristics would be required in order to assess its full scope in the rational reconstruction of economic knowledge.