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Nonlinear Observers for Sensorless Control of Permanent Magnet Synchronous Motors

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- **SINGULAR PERTURBATION FRAMEWORK**

Contents

Introduction

Due to their versatility, electrical machines are even more used in very different applications, e.g. classical automation, industrial and domestic robotics, energy conversion (from mechanical to electrical energy), up to the "new" automotive scenario. Versatility means high efficiency and high compactness, i.e. efficiency ($\sim 20\%$) and dimensions of an Internal Combustion Engine despite the efficiency ($\sim 80\%$) and dimensions of an Electrical Machine. Electrical machines are perfect bidirectional machines, in fact they work efficiently as a converter from mechanical to electrical energy, and vicerversa, e.g. used either as a engine to provide movement and as a Kinetic Energy Recovery System (KERS) in autotive applications.
Electrical machines can be divided into two main categories: DC machines, i.e. with the reactive magnetic field on the stator and as a consequence with the needs of brushes to carry the armature current to the rotor, and AC machines, i.e. with the reactive magnetic field on the rotor and with no needs of brushes. In the category of the AC machines other two subcategories can be identified, based on how the reactive magnetic field is created there are Induction Machines (IM) and Permanent Magnet Synchronous Machines (PMSM). For IMs the reactive field is created by a portion of stator current so it is created every time and only if a stator current is flowing, while for PMSM the reactive magnetic field is created once and during the building of the electrical machines, so it is always present also with no flowing current.
Until few years ago, the trend describing the types of electrical motors used for standard applications has been driven by the control capability of each motor. The DC machines are the easiest to control, due to the fact that they must be commanded by continuous currents, while the AC machines are difficult to control, and only with an appropriate mathematical manipulation they can be controlled as simple as a DC machine. Therefore, DC machines were initially preferred and then have been substituted with the AC machines thanks to the introduction of the appropriate theory behind this mathematical manipulation and also thanks to the coming of high performances microcontroller that made this theory applicable without a high time consume.
With the coming of the "age" of the AC machines, the IM was initially pre-

ferred still for the (speed) control capability and for the stability properties shown with respect to PMSM, i.e. the reactive magnetic field generated by the stator current introduces an intrinsic stabilizing property, useful especially for the first form of IM speed control algorithms called V/f, also erroneously and commonly called Inverter as a synonym of variable speed control.
In the recent years, with the consolidation of control algorithms, among which we can identify Scalar control methods and Vector control methods, or in other words with no differences in terms of control capability and reached performances between IM and PMSM, other criteria are driving the choice of PMSM instead of IM in a given scenario, these criteria are the efficiency and the scalability. In terms of efficiency PMSM are more efficient due to the fact that no “extra” current is needed for the generation of the reactive magnetic field in the rotor. In term of scalability PMSMs are better than IMs in fact is simpler to replicate the realization scheme of a PMSM despite the productive scheme of an IM to realize even low size and large size AC machines.
Scalar control methods are based on V/f algorithms, initially designed for IM machines, which for PMSM machines are also called I/f methods. Vector control methods, based on a mathematical manipulation of the equations describing the electrical machine, are usually adopted to ensure a high performance regulation, nevertheless these methods require the perfect knowledge of the position of magnetic field in the rotor that reacting with the stator magnetic field (generated by the stator currents) give rise to a torque, of course when a speed control loop is implemented also the rotor speed measure for the feedback is needed, but knowing the position the speed is straightforwardly known.
Absolute encoders are able to cope with the task of measuring the rotor magnetic flux position, but the desire for reducing costs and the number of components, to improve at the same time the system reliability, has stimulated the research towards the so called *sensorless* control algorithms. A natural choice to implement this control alghoritms is to enrich the system with an observer and feed the controller with the estimated variables instead of the sensor measured variables.
An intense research activity has been carried out to cope with this problem, it has also been discussed in some monographs ([1], [2], [3]) concerning nonlinear and adaptive control solutions applied to electrical drives regulation. Despite the topic is mature, and many practical applications have been successfully implemented, there is still room for improving the estimation scheme performance, particularly under some well known critical conditions.
Literature on estimation of PMSM parameters is divided into two main categories. The first, usually referred as *signal-based*, includes all the approaches based on high frequency voltage signals injection, used to get complete position information exploiting the *magnetic saliency* (see [4], for instance). Differen-

tly, the second category, usually referred as *model-based*, covers methods where nominal models of PMSM are exploited in different ways to reconstruct the rotor magnet position and speed through the back-emf induced on stator windings.
Classical model-based solutions contain extended Kalman filters (see [5] and [7] among the others), or some other interesting solutions, for example in [8], [9], [10] significant approaches, concerning analysis and improvement of robustness with respect to parameters uncertainties, are presented along with some discussions on the stability properties of the adopted nonlinear schemes. Among the others, the two solutions recently presented in [5] and [11] are particularly attractive since, exploiting modern nonlinear observer design techniques ([12], [13]), rigorous stability analysis has been carried out. It is well known and quite consolidated that at low speed values, the performance of model-based methods abruptly decreases due to a lack of observability of the system hence these control methods are commonly adopted especially in medium or high speed range of operation, and this is the case of energy conversion applications for which zero speed is by definition avoided (see [5], [6] for more details on observability analysis).
Another common problem of these approaches is the sensitivity to parameters uncertainty, again particularly relevant at low speed. These two drawbacks are even more significant when a linear approximation of the machine model is taken to design the estimation system, hence the research effort has been devoted to develop nonlinear observers for these applications. The stability problems and performance decrease of the adaptive schemes at low speed are worsen from the use of solid state converter, also called (Voltage Source) Inverter ([22]-[27]), for this reason an appendix to explore the nature of the nonlinearities introduce by Inverters can be found.
Finally, the observer schemes presented in this thesis belongs to the family of model based observer for the advantages presented above and because they give the possibility to face with advanced stability theories, like Lyapunov stability theory or Adaptive theory.

Chapter 1

Electromagnetic models for a Permanent Magnet Synchronous Machine

Introduction

In this chapter some electromagnetic models for a 3-phase PMSM will be derived. First of all, some hypothesis will be done to derive simpler models for a PMSM. These are constitutive hypothesis because are based on how the motor is built, and how the motor is used.
The process to derive the models describing a PMSM starts with the description of the motor in a standard 3-phase stationary reference frame A, B, C, then exploiting a matrix transformation, called Park transformation, the model will be reported in the standard 2-phase stationary reference frame usually called α, β and then after the last coordinates transformation, called Clarke transformation, the motor will be described in the standard 2-phase rotating reference frame usually called d, q.

1.1 PMSM Models

The following hypothesis for the derivation of the model for a PMSM are supposed:

- **Linear** magnetic paths, i.e. no flux saturation in the magnetic paths
- Negligible **leakages**, i.e. hysteresis and Eddy current effects can be neglected
- No differences on windings **mutual inductances**

The hypothesis above are often satisfied but for particular PMSM realization these can be not true, especially the first of the three, for example in all those motors where high Torque (current) and small dimensions must be fulfilled together.

The electrical equations describing the three stator voltages u_{sA}, u_{sB}, u_{sC} on a 3-phase stationary reference frame, with the axes A, B, C aligned with the three stator windings, are the following:

$$\begin{bmatrix} u_{sA} \\ u_{sB} \\ u_{sC} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \begin{bmatrix} i_{sA} \\ i_{sB} \\ i_{sC} \end{bmatrix} + p\frac{d}{dt} \begin{bmatrix} \psi_A \\ \psi_B \\ \psi_C \end{bmatrix} \tag{1.1}$$

Where:

- p is the number of pole pairs,
- R_s is the stator winding resistance,
- i_{sA}, i_{sB}, i_{sC} are the three stator winding currents,
- ψ_A, ψ_B, ψ_C are the three stator linkage fluxes,

As for the stator, supposing for the rotor to have a 3-phase (rotating) reference frame a, b, c aligned with the 3-phase rotor windings, similar equations can be written for the rotor voltages u_{ra}, u_{rb}, u_{rc}

$$\begin{bmatrix} u_{ra} \\ u_{rb} \\ u_{rc} \end{bmatrix} = \begin{bmatrix} R_r & 0 & 0 \\ 0 & R_r & 0 \\ 0 & 0 & R_r \end{bmatrix} \begin{bmatrix} i_{ra} \\ i_{rb} \\ i_{rc} \end{bmatrix} + p\frac{d}{dt} \begin{bmatrix} \phi_a \\ \phi_b \\ \phi_c \end{bmatrix} \tag{1.2}$$

Where:

- R_r is the rotor winding resistance,

- i_{ra}, i_{rb}, i_{rc} are the three rotor winding currents,
- ϕ_a, ϕ_b, ϕ_c are the three rotor linkage fluxes,

The equations above are the starting equations for the description of a generic electrical motor. Therefore, if the described motor is an Induction Machine (IM) the following property can be used:

- $u_{ra} = u_{rb} = u_{rc} = 0$, in fact the rotor windings are shorted to have zero voltage

If the described motor is a Permanent Magnet Synchronous Machine (PMSM) the following property can be exploited

- $i_{ra} = i_{rb} = i_{rc} = 0$, in fact on the rotor a PMSM there are no currents

The following equations describe the stator linkage fluxes in the 3-phase stationary reference frame with the axes aligned with the stator windings

$$\begin{aligned} \begin{bmatrix} \psi_A \\ \psi_B \\ \psi_C \end{bmatrix} &= \begin{bmatrix} L_{s3} & M_{s3} & M_{s3} \\ M_{s3} & L_{s3} & M_{s3} \\ M_{s3} & M_{s3} & L_{s3} \end{bmatrix} \begin{bmatrix} i_{sA} \\ i_{sB} \\ i_{sC} \end{bmatrix} + \phi_M \begin{bmatrix} \cos(p\theta) \\ \cos(p\theta - 2\pi/3) \\ \cos(p\theta + 2\pi/3) \end{bmatrix} + ... \\ &...+ M_{sr} \begin{bmatrix} \cos(p\theta) & \cos(p\theta + 2\pi/3) & \cos(p\theta - 2\pi/3) \\ \cos(p\theta - 2\pi/3) & \cos(p\theta) & \cos(p\theta + 2\pi/3) \\ \cos(p\theta + 2\pi/3) & \cos(p\theta - 2\pi/3) & \cos(p\theta) \end{bmatrix} \begin{bmatrix} i_{ra} \\ i_{rb} \\ i_{rc} \end{bmatrix} \end{aligned} \tag{1.3}$$

Where

- L_{s3}, M_{s3}, are respectively, the auto-inductance of a stator winding, and the mutual-inductance between windings on the stator
- ϕ_M, is the value (amplitude) of the rotor flux
- θ, is the "mechanical" angle between the rotor winding a and the stator winding A, i.e. for a standard 3-phase electrical machine, the stator windings A, B, C are displaced of the mechanical angle $2\pi/(3p)$, the same happens for the rotor windings a, b, c. The angle $p\theta$ is the so called "electrical" angle, and the electrical angle between stator or rotor windings is $2\pi/3$
- M_{sr} is the mutual-inductance between rotor corrents and stator flux

The following equations describe the rotor linkage fluxes in the 3-phase rotating reference frame a, b, c with the axes a aligned with the rotor magnet

$$\begin{aligned} \begin{bmatrix} \phi_a \\ \phi_b \\ \phi_c \end{bmatrix} &= \begin{bmatrix} L_{r3} & M_{r3} & M_{r3} \\ M_{r3} & L_{r3} & M_{r3} \\ M_{r3} & M_{r3} & L_{r3} \end{bmatrix} \begin{bmatrix} i_{ra} \\ i_{rb} \\ i_{rc} \end{bmatrix} + \phi_M \begin{bmatrix} 1 \\ -1/2 \\ -1/2 \end{bmatrix} + ... \\ &...+ M_{sr} \begin{bmatrix} \cos(p\theta) & \cos(p\theta - 2\pi/3) & \cos(p\theta + 2\pi/3) \\ \cos(p\theta + 2\pi/3) & \cos(p\theta) & \cos(p\theta - 2\pi/3) \\ \cos(p\theta - 2\pi/3) & \cos(p\theta + 2\pi/3) & \cos(p\theta) \end{bmatrix} \begin{bmatrix} i_{sA} \\ i_{sB} \\ i_{sC} \end{bmatrix} \end{aligned} \tag{1.4}$$

Where

- L_{r3}, M_{r3}, are respectively, the auto-inductance of a rotor winding, and the mutual-inductance between windings on the rotor

The terms L_{s3}, L_{r3} are the auto-inductance of the stator and rotor windings, each of these terms is composed by two terms, a magnetizing term L_{sm3}, L_{rm3} which represent the part of flux that going out from the winding is able to generate magnetic cross coupling effects, and a leakage term L_{sd3}, L_{rd3} which despite to the first term represents the part of flux that does not go outside the winding and hence is not able to generate cross coupling effects.
The terms M_{s3}, M_{r3} are the mutual-inductance between the stator windings and between the rotor windings respectively. These terms are due to the fact that the windings in a 3-phase machine are displaced with an electrical angle of $2\pi/3$ and are coupled, in fact two windings are magnetically decoupled if the angle between the windings is $\pi/2$. By definition these terms are related only with L_{sm3}, L_{rm3}, in fact the terms L_{sd3}, L_{rd3} cannot give rise to coupling effects, because this part of flux does not go outside of the winding.
We have already said that for a PMSM the rotor currents are zero, $i_{ra} = i_{rb} = i_{rc} = 0$, so in the equations (1.3) (1.4) the term dependent on the rotor currents can be neglected.
Moreover, for a built-in property of a PMSM the linkage rotor flux is mainly due to permanent magnet flux, and the part of flux due to the stator currents coupled to the rotor can be neglected, hence the following simplification can be done

$$\phi_M \begin{bmatrix} 1 \\ -1/2 \\ -1/2 \end{bmatrix} \gg M_{sr} \begin{bmatrix} \cos(p\theta) & \cos(p\theta - 2\pi/3) & \cos(p\theta + 2\pi/3) \\ \cos(p\theta + 2\pi/3) & \cos(p\theta) & \cos(p\theta - 2\pi/3) \\ \cos(p\theta - 2\pi/3) & \cos(p\theta + 2\pi/3) & \cos(p\theta) \end{bmatrix} \begin{bmatrix} i_{sA} \\ i_{sB} \\ i_{sC} \end{bmatrix} \tag{1.5}$$

Therefore, rewriting the equations above for a PMSM, instead of a generic motor, and in more compact form, we obtain

$$\begin{bmatrix} u_{sA} \\ u_{sB} \\ u_{sC} \end{bmatrix} = \mathbf{R_s} \begin{bmatrix} i_{sA} \\ i_{sB} \\ i_{sC} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \psi_A \\ \psi_B \\ \psi_C \end{bmatrix} \tag{1.6}$$

$$\begin{bmatrix} \psi_A \\ \psi_B \\ \psi_C \end{bmatrix} = \mathbf{L_{s3}} \begin{bmatrix} i_{sA} \\ i_{sB} \\ i_{sC} \end{bmatrix} + \begin{bmatrix} \phi_A \\ \phi_B \\ \phi_C \end{bmatrix} = \mathbf{L_{s3}} \begin{bmatrix} i_{sA} \\ i_{sB} \\ i_{sC} \end{bmatrix} + \phi_M \begin{bmatrix} \cos(p\theta) \\ \cos(p\theta - 2\pi/3) \\ \cos(p\theta + 2\pi/3) \end{bmatrix} \tag{1.7}$$

$$\begin{bmatrix} \phi_a \\ \phi_b \\ \phi_c \end{bmatrix} = \phi_M \begin{bmatrix} 1 \\ -1/2 \\ -1/2 \end{bmatrix} \tag{1.8}$$

Where

- ϕ_A, ϕ_B, ϕ_C are the rotor fluxes ϕ_a, ϕ_b, ϕ_c (expressed in the 3-phase rotating reference frame a, b, c) reported in the 3-phase stationary reference frame A, B, C

- For the resistance and inductance matrices we supposed

$$\mathbf{L_{s3}} = \begin{bmatrix} L_{s3} & M_{s3} & M_{s3} \\ M_{s3} & L_{s3} & M_{s3} \\ M_{s3} & M_{s3} & L_{s3} \end{bmatrix} \quad , \quad \mathbf{R_s} = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & R_s \end{bmatrix} \tag{1.9}$$

Now, once obtained the model of PMSM in 3-phase stationary reference frame A, B, C it is possible to derive the same model in a 2-phase stationary reference frame α, β exploiting the so called Park transformation, that is identified by the following matrices

$$\begin{aligned} T_{2\leftarrow 3} &= k \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \\ T_{3\leftarrow 2} &= \frac{2}{3k} \begin{bmatrix} 1 & 0 \\ -1/2 & \sqrt{3}/2 \\ -1/2 & -\sqrt{3}/2 \end{bmatrix} \end{aligned} \tag{1.10}$$

Where the matrix $T_{2\leftarrow 3}$ transform a 3-phase vector in a 2-phase vector, and $T_{3\leftarrow 2}$ transform a 2-phase vector back to a 3-phase vector. There are infinite Clark transformations parameterized with k, in literature two particular choices of k are known

- $k = 2/3$, gives rise to the so called iso-amplitude transformation, with this choice the amplitude of a 3-phase vector transformed in its 2-phase version has the same amplitude.

- $k = \sqrt{2/3}$, gives rise to the so called iso-power transformation, with this choice the amplitude of a 3-phase vector transformed in a its 2-phase version does not has the same amplitude, but the inner products are maintained, hence the power does not change.

In the following will be supposed the iso-amplitude Clarke transformation, i.e. $k = 2/3$.
Applying the Clarke transformation to the 3-phase model of a PMSM reported in equations (1.6)(1.8)(1.7), after some simple analytical computations the PMSM model in the standard 2-phase stationary reference frame α, β can be obtained

$$\begin{bmatrix} u_{s\alpha} \\ u_{s\beta} \end{bmatrix} = \begin{bmatrix} R_{s_{\alpha\beta}} & 0 \\ 0 & R_{s_{\alpha\beta}} \end{bmatrix} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \psi_\alpha \\ \psi_\beta \end{bmatrix} \tag{1.11}$$

$$\begin{bmatrix} \psi_\alpha \\ \psi_\beta \end{bmatrix} = \begin{bmatrix} L_{s_{\alpha\beta}} & 0 \\ 0 & L_{s_{\alpha\beta}} \end{bmatrix} \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} + \begin{bmatrix} \phi_\alpha \\ \phi_\beta \end{bmatrix} \tag{1.12}$$

$$\begin{bmatrix}\phi_\alpha\\ \phi_\beta\end{bmatrix} = \phi_M \begin{bmatrix}\cos(p\theta)\\ \sin(p\theta)\end{bmatrix} \tag{1.13}$$

Where

- $R_{s_{\alpha\beta}} = R_s$, is the winding resistance for the model in the 2-phase stationary reference frame
- $L_{s_{\alpha\beta}} = L_{s3} - M_{s3}$, is the winding inductance for the model in the 2-phase stationary reference frame

From the equations (1.11)(1.12)(1.13), and for the sake of brevity defining $L_{s_{\alpha\beta}} = L$, $R_s = R$, and dropping out the subscript "s" for the variables where is redundant, the following state equations can be derived

$$\begin{aligned} L\dot{i}_\alpha &= -Ri_\alpha + p\omega\phi_\beta + u_\alpha\\ L\dot{i}_\beta &= -Ri_\beta - p\omega\phi_\alpha + u_\beta \end{aligned} \tag{1.14}$$

$$\begin{aligned} \dot{\psi}_\alpha &= u_\alpha - Ri_\alpha\\ \dot{\psi}_\beta &= u_\beta - Ri_\beta \end{aligned} \tag{1.15}$$

$$\begin{aligned} \dot{\phi}_\alpha &= p\omega\phi_\beta\\ \dot{\phi}_\beta &= -p\omega\phi_\alpha \end{aligned} \tag{1.16}$$

Now it is possible to derive the motor model in a generic 2-phase rotating reference frame called d, q using the so called Park transformation, and defined by the following matrix transformation

$$\begin{aligned} \begin{bmatrix}x_d\\ x_q\end{bmatrix} &= \begin{bmatrix}\cos(\epsilon_o) & \sin(\epsilon_o)\\ -\sin(\epsilon_o) & \cos(\epsilon_o)\end{bmatrix} \begin{bmatrix}x_\alpha\\ x_\beta\end{bmatrix}\\ \begin{bmatrix}x_\alpha\\ x_\beta\end{bmatrix} &= \begin{bmatrix}\cos(\epsilon_o) & -\sin(\epsilon_o)\\ \sin(\epsilon_o) & \cos(\epsilon_o)\end{bmatrix} \begin{bmatrix}x_d\\ x_q\end{bmatrix} \end{aligned} \tag{1.17}$$

Where the variable x is a generic variable, i.e. the stator voltages u_α, u_β the stator currents i_α, i_β, etc..., and the angle ϵ_0 is the angle between the axis d and the axis α, moreover, the speed of the rotating reference frame d, q is $\dot{\epsilon}_0 = \omega_0$. Hence, applying the Park transformation to the motor model equations (1.11) (1.12) (1.13) the motor model in the generic 2-phase rotating reference frame d, q is the following

$$\begin{bmatrix}u_{sd}\\ u_{sq}\end{bmatrix} = \begin{bmatrix}R_{s_{dq}} & 0\\ 0 & R_{s_{dq}}\end{bmatrix} \begin{bmatrix}i_{sd}\\ i_{sq}\end{bmatrix} + \omega_0 \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix} \begin{bmatrix}\psi_d\\ \psi_q\end{bmatrix} + \frac{d}{dt}\begin{bmatrix}\psi_d\\ \psi_q\end{bmatrix} \tag{1.18}$$

$$\begin{bmatrix}\psi_d\\ \psi_q\end{bmatrix} = \begin{bmatrix}L_{s_{dq}} & 0\\ 0 & L_{s_{dq}}\end{bmatrix} \begin{bmatrix}i_{sd}\\ i_{sq}\end{bmatrix} + \begin{bmatrix}\phi_d\\ \phi_q\end{bmatrix} \tag{1.19}$$

$$\begin{bmatrix}\phi_d \\ \phi_q\end{bmatrix} = \phi_M \begin{bmatrix}\cos(\epsilon_0 - p\theta) \\ -\sin(\epsilon_0 - p\theta)\end{bmatrix} \tag{1.20}$$

Where

- $R_{s_{dq}} = R_s$, is the winding resistance for the model in the 2-phase rotating reference frame
- $L_{s_{dq}} = L_{s_{\alpha\beta}}$, is the winding inductance for the model in the 2-phase rotating reference frame

From the equations (1.18)(1.19)(1.20), and for the sake of brevity defining $L_{s_{dq}} = L$, $R_{s_{dq}} = R$, and dropping out the subscript "s" for the variables where is redundant, the following state equations can be derived

$$\begin{aligned} L\dot{i}_d &= -Ri_d + \omega_0 L i_q + p\omega\phi_q + u_d \\ L\dot{i}_q &= -Ri_q - \omega_0 L i_d - p\omega\phi_d + u_q \end{aligned} \tag{1.21}$$

$$\begin{aligned} \dot{\psi}_d &= -Ri_d + \omega_0\psi_q + u_d \\ \dot{\psi}_q &= -Ri_q - \omega_0\psi_d + u_q \end{aligned} \tag{1.22}$$

$$\begin{aligned} \dot{\phi}_d &= (\omega_0 - p\omega)\phi_q \\ \dot{\phi}_q &= -(\omega_0 - p\omega)\phi_d \end{aligned} \tag{1.23}$$

To derive the expression of the motor torque at the shaft, we have to start from the electrical power equation at the stator in the 3-phase stationary reference frame and in the 2-phase rotating reference frame as follows

$$\begin{aligned} P_{s_{ABC}} &= u_{sA}i_{sA} + u_{sB}i_{sB} + u_{sC}i_{sC} \\ P_{s_{dq}} &= u_{sd}i_{sd} + u_{sq}i_{sq} \end{aligned} \tag{1.24}$$

The power expressions in the different reference frames are related as follows

$$\begin{aligned} &P \equiv P_{s_{ABC}} = \frac{2}{3k^2}P_{s_{dq}} \equiv \frac{3}{2}P_{s_{dq}} \\ &(k = 2/3 : \text{iso-amplitude}) \end{aligned} \tag{1.25}$$

Furthermore, using the equations (1.21) to derive the expression of the stator voltages and after some analytical computations, the following expression of the power can be derived

$$\begin{aligned} P &= \frac{3}{2}\left(L(i_d\dot{i}_d + i_q\dot{i}_q) + R(i_d^2 + i_q^2) + p\omega(\psi_d i_q - \psi_q i_d)\right) \\ &= \frac{dE_{\text{Magn}}}{dt} + P_d + T_m\omega \end{aligned} \tag{1.26}$$

Where, E_{Magn} is the electrical energy stored in the magnetic fields due to the inductances, P_d are the electrical loss due to the winding resistances and T_m is the motor torque at the shaft and its expression is the following

$$\begin{aligned} T_m &= \frac{3}{2}p(\psi_d i_q - \psi_q i_d) \\ &= \frac{3}{2}p(\phi_d i_q - \phi_q i_d) \end{aligned} \tag{1.27}$$

It is important to say that when the motor is built with some **saliency** property the magnetic circuits are not isotropic. The natural reference frame to describe these PMSM motors is the 2-phase rotating reference frame d, q, in fact for these motors the saliency is translated in a difference of inductances on the two magnetic paths, i.e. d and q paths (or axis). Therefore instead of a unique inductance we have two inductances, L_d and L_q, and the motor model, even based on the equations (1.21)(1.22)(1.23), becomes as follows

$$\begin{aligned} L_d \dot{i}_d &= -Ri_d + \omega_0 L_q i_q + p\omega\phi_q + u_d \\ L_q \dot{i}_d &= -Ri_d - \omega_0 L_d i_d - p\omega\phi_d + u_q \end{aligned} \tag{1.28}$$

$$\begin{aligned} \dot{\psi}_d &= -Ri_d + \omega_0 \psi_q + u_d \\ \dot{\psi}_q &= -Ri_q - \omega_0 \psi_d + u_q \end{aligned} \tag{1.29}$$

$$\begin{aligned} \dot{\phi}_d &= (\omega_0 - p\omega)\phi_q \\ \dot{\phi}_q &= -(\omega_0 - p\omega)\phi_d \end{aligned} \tag{1.30}$$

And the algebraic constitutive equations become

$$\begin{matrix} \psi_d = Li_d + \psi_d \\ \psi_q = Li_q + \psi_d \end{matrix} \quad \Rightarrow \quad \begin{matrix} \psi_d = L_d i_d + \psi_d \\ \psi_q = L_q i_q + \psi_d \end{matrix} \tag{1.31}$$

Also the torque equation shows some differences, and it becomes as follows

$$\begin{aligned} T_m &= \frac{3}{2}p(\psi_d i_q - \psi_q i_d) \\ &= \frac{3}{2}p(\phi_d i_q - \phi_q i_d + (L_d - L_q) i_d i_q) \end{aligned} \tag{1.32}$$

Where the term "$(L_d - L_q) i_d i_q$" is the part of the toque produced by the saliency, i.e. due to the difference of reluctance of the magnetic paths on d-axis and on q-axis.

Chapter 2

PMSM Speed and Stator Flux Adaptive Observer

Introduction

In this chapter a speed/stator flux observer for a PMSM is derived using adaptive and Lyapunov techniques. Adaptive technique is used for the estimation of the rotor speed (considered as a constant parameter to be estimated), and Lyapunov technique is used as a guide for the estimation of the stator fluxes. The stability proof of the observer is initially done during the derivation of the observer exploiting Lyapunov like theory for non linear systems, and then using the Adaptive Framework as reported in [17]. The observer is derived in a generic d, q rotating reference frame because of the rotor position is not known, this generic reference frame used for the observer is not aligned with the rotor magnet. The alignment condition is often needed to implement the so called Field-Oriented control (FOC), so if this condition is needed an external reference frame controller must be designed.
The chapter is organized as follows, section 2.1 is devoted to the step-by-step design of the observer with ongoing stability considerations driving the designing process, in subsection 2.1.1 the stability is carried out in a more rigorous way, putting the model and the observer in a standard Adaptive framework. In subsection 2.1.2 a modified version of the observer is given, to cope with practical stability issues arising from considerations contained in the previous section. Subsection 2.1.3 is devoted to design the reference frame controller, also called *Alignment Controller*. In subsection 2.1.4 the tuning procedure for the observer and the alignment controller is reported, with simulation results reported in the ending section 2.2.

I would like to thank you Doct. Eng. *Marcello Montanari* for the fundamental idea of the observer proposed in this chapter, and last but not least I would like to thank you Doct. Eng. *Andrea Tilli* for the help gave me for the study of this observer, especially for the idea of the stability proof carried out using the Adaptive framework and reported in section 2.1.1.

2.1 Observer Design

The speed/flux observer is derived in a generic rotating reference frame with angle ϵ_0 and speed ω_0 called (d, q).
In order to design the observer, the electro-magnetic dynamics of the PMSM are rewritten using as state variables the stator currents and stator fluxes instead of rotor fluxes. Stator and rotor fluxes are algebraically defined as:

$$\begin{aligned} \psi_d &= Li_d + \phi_d \\ \psi_q &= Li_q + \phi_q \end{aligned} \tag{2.1}$$

Using stator fluxes as state variable the PMSM electro-magnetic model can be rewritten as:

$$\begin{aligned} L\dot{i}_d &= -Ri_d + (\omega_0 - p\omega)Li_q + p\omega\psi_q + u_d \\ L\dot{i}_q &= -Ri_q - (\omega_0 - p\omega)Li_d - p\omega\psi_d + u_q \\ \dot{\psi}_d &= -Ri_d + \omega_0\psi_q + u_d \\ \dot{\psi}_q &= -Ri_q - \omega_0\psi_d + u_q \end{aligned} \tag{2.2}$$

To have a complete PMSM model the following equations must be added to the previous ones:

$$\begin{aligned} \dot{\theta} &= \omega \\ \dot{\epsilon}_0 &= \omega_0 \\ J\dot{\omega} &= \frac{3}{2}p(\psi_d i_q - \psi_q i_d) - T_{load} \end{aligned} \tag{2.3}$$

Expressing the electro-magnetic PMSM model as a function of stator fluxes instead of rotor fluxes allows for the design of a speed/flux observer according to Lyapunov method in a simpler way. Based on the electro-magnetic dynamics of the PMSM in the (d, q) reference frame, the observer is designed according to Lyapunov and adaptive control theory.
The observer is designed according to the following assumptions:

- Stator currents are known (measures), i.e. i_d, i_q are known through the knowledge of i_a, i_b and ϵ_0.
- Rotating reference frame angle ϵ_0 and speed ω_0 are known. The generic reference frame speed is a control variable and can be properly designed by a reference frame controller to achieve a particular configuration.
- PMSM parameters are supposed to be known, i.e. stator inductance L and resistance R, and in particular the permanent magnet flux amplitude Φ (i.e. bemf constant) are required to be known.
- Rotor angle θ and speed ω are not known, i.e. the speed will be estimated.

- Rotor fluxes ϕ_d, ϕ_q are not known, i.e. the stator fluxes are not measurable.

- Winding stator voltages are equal to the command stator voltages derived from the current controller u_d, u_q, the latter being the system input are a priori known. To avoid performance degradation it is necessary to compensate for dead-times effects and other inverter non-idealities that make command voltages different from the winding stator voltages.

Two main choices for the (d, q) generic reference frame are possible:

- *Stationary reference frame.* In this case $\epsilon_0 = \omega_0 = 0$, so the reference frame controller is not needed, and signals in the stationary reference frame are exploited. At steady-state conditions in this reference frame, currents, fluxes and voltages are sinusoidal signals with frequency $p\omega$, i.e. the "electrical" frequency.

- *Rotating and rotor aligned reference frame.* In this case the (d, q) rotating reference frame is defined in such a way that the d-axis tends asymptotically to be aligned to the permanent magnet axis, so a reference frame alignment controller is needed to cope with this task. At steady-state conditions it follows that $\omega_0 = p\omega$ and $\epsilon_0 = p\theta$, i.e. the initial misalignment of the rotor is compensated in order to be aligned with the permanent magnet, i.e. $\phi_d = \Phi, \phi_q = 0$. In this reference frame at steady state all signals are constant.

We can start to derive the speed/flux observer designing current and flux dynamics, as follows:

$$
\begin{aligned}
L\dot{\hat{i}}_d &= -R\hat{i}_d + (\omega_0 - p\hat{\omega})Li_q + p\hat{\omega}\hat{\psi}_q + u_d + \eta_d \\
L\dot{\hat{i}}_q &= -R\hat{i}_q - (\omega_0 - p\hat{\omega})Li_d - p\hat{\omega}\hat{\psi}_d + u_q + \eta_q \\
\dot{\hat{\psi}}_d &= -Ri_d + \omega_0\hat{\psi}_q + u_d + \xi_d \\
\dot{\hat{\psi}}_q &= -Ri_q - \omega_0\hat{\psi}_d + u_q + \xi_q
\end{aligned}
\tag{2.4}
$$

in which $\hat{i}_d, \hat{i}_q, \hat{\psi}_d, \hat{\psi}_q, \hat{\omega}$, are respectively stator current, stator flux and rotor speed estimates, while $\eta_d, \eta_q, \xi_d, \xi_q$ are auxiliary variables to be defined.
Defining the estimation error variables as

$$
\begin{gathered}
\tilde{i}_d = i_d - \hat{i}_d \ , \ \tilde{i}_q = i_q - \hat{i}_q \\
\tilde{\psi}_d = \psi_d - \hat{\psi}_d \ , \ \tilde{\psi}_q = \psi_q - \hat{\psi}_q \\
\tilde{\omega} = \omega - \hat{\omega}
\end{gathered}
\tag{2.5}
$$

the error system is as follows

$$\begin{aligned} L\dot{\tilde{i}}_d &= -R\tilde{i}_d - p\tilde{\omega}Li_q + p\omega\tilde{\psi}_q + p\tilde{\omega}\hat{\psi}_q - \eta_d \\ L\dot{\tilde{i}}_q &= -R\tilde{i}_q + p\tilde{\omega}Li_d - p\omega\tilde{\psi}_d + p\tilde{\omega}\hat{\psi}_d - \eta_q \\ \dot{\tilde{\psi}}_d &= \omega_0\tilde{\psi}_q - \xi_d \\ \dot{\tilde{\psi}}_q &= -\omega_0\tilde{\psi}_d - \xi_q \end{aligned} \tag{2.6}$$

We can start to analyze the stability properties of the system choosing the following Lyapunov function candidate

$$V = \frac{1}{2}\left(L\tilde{i}_d^2 + L\tilde{i}_q^2 + \frac{\tilde{\omega}^2}{\gamma}\right) \tag{2.7}$$

In order to cancel out from the time derivative Lyapunov function $\dot{V}$ the unknown terms $p\omega\tilde{\psi}_q$, $-p\omega\tilde{\psi}_d$ the following change of variables is exploited

$$\begin{aligned} \chi_d &= \omega\tilde{\psi}_d \\ \chi_q &= \omega\tilde{\psi}_q \end{aligned} \tag{2.8}$$

The error system can be rewritten as follows

$$\begin{aligned} L\dot{\tilde{i}}_d &= -R\tilde{i}_d + p\chi_q + p\tilde{\omega}(\hat{\psi}_q - Li_q) - \eta_d \\ L\dot{\tilde{i}}_q &= -R\tilde{i}_q - p\chi_d - p\tilde{\omega}(\hat{\psi}_d - Li_d) - \eta_q \\ \dot{\chi}_d &= \omega_0\chi_q - \omega\xi_d + \frac{\dot{\omega}}{\omega}\chi_d \\ \dot{\chi}_q &= -\omega_0\chi_d - \omega\xi_q + \frac{\dot{\omega}}{\omega}\chi_q \end{aligned} \tag{2.9}$$

Some considerations on the PMSM Model:

- In the stationary reference frame, the rotor flux dynamics is an autonomous pure oscillator at frequency $p\omega$ with non-null and unknown initial conditions, with known amplitude $\|(\phi_d, \phi_q)\| = \Phi$.
- In the generic rotating reference frame, the rotor flux dynamics is a pure oscillator at frequency $\omega_0 - p\omega$, hence in the rotor reference frame ($\omega_0 = p\omega$) rotor flux is constant.
- In the generic rotating reference frame, the stator flux dynamics are described by a pure non autonomous oscillator at frequency ω_0, fed by the difference of stator voltage and resistance voltage drop. It is a neutrally stable dynamics with unknown initial conditions (because of the rotor fluxes initial conditions are unknown).

- In the stationary reference frame, pure oscillator dynamics of the stator fluxes are substituted by two pure integrators.

Some considerations on the up to here designed Observer:

- From the Eq.2.6, the stator flux estimation error dynamic $\tilde{\psi}_d, \tilde{\psi}_q$ is a pure oscillator at frequency ω_0 fed by auxiliary terms which will be defined to stabilize the overall system (or equivalently, the neutrally stable stator flux dynamics is stabilized through feedback of the asymptotically stable stator current dynamics). In the stationary reference frame, the stator flux estimation error dynamic is described by two pure integrators, which take into account the unknown initial conditions.
- By Lyapunov method, it is not possible to cancel out terms $p\omega\tilde{\psi}_d, p\omega\tilde{\psi}_q$ if speed is not known. This is the reason for the introduction of variables χ_d, χ_q, and since they are unknown as will be shown later they will be estimated by $\hat{\chi}_d, \hat{\chi}_q$.
- In the stationary reference frame the stator flux estimation errors $\tilde{\psi}_a, \tilde{\psi}_b$, with $\xi_d = 0, \xi_q = 0$, are constant; hence, assuming constant speed, also χ_a, χ_b in the stationary reference frame are unknown constant to be estimated. Equivalently, in the rotating reference frame, these terms are sinusoidal signals to be estimated. Since they are expected to tend to zero, they will be sinusoidal signals whose amplitude will tend to zero.

Recalling the candidate Lyapunov function reported by Eq.2.7, its time derivative along the solution of the error system is as follows

$$\begin{aligned}\dot{V} = & - R\tilde{i}_d^2 - R\tilde{i}_q^2 + p\tilde{\omega}\left[(\hat{\psi}_q - L\hat{i}_q)\tilde{i}_d - (\hat{\psi}_d - L\hat{i}_d)\tilde{i}_q + \frac{\dot{\tilde{\omega}}}{\gamma}\right] + \dots \\ & - \eta_d\tilde{i}_d - \eta_q\tilde{i}_q + p\chi_q\tilde{i}_d - p\chi_d\tilde{i}_q\end{aligned} \tag{2.10}$$

Now we can define the following observation laws

$$\begin{aligned}\eta_d &= k_p\tilde{i}_d + p\hat{\chi}_q \\ \eta_q &= k_p\tilde{i}_q - p\hat{\chi}_q \\ \dot{\tilde{\omega}} &= \dot{\omega} - \dot{\hat{\omega}} = -\gamma\left[(\hat{\psi}_q - L\hat{i}_q)\tilde{i}_d - (\hat{\psi}_d - L\hat{i}_d)\tilde{i}_q\right]\end{aligned} \tag{2.11}$$

Terms η_d, η_q are composed by proportional terms (and integral terms, as can be seen in the following), while speed estimation is based on classical adaptive control theory.
It is worth noting that $(\hat{\psi}_q - L\hat{i}_q, -\hat{\psi}_d - L\hat{i}_d)$ are the estimated rotor fluxes. If rotor speed is assumed to be constant, as usual in adaptive theory for the

estimation of a constant parameter, in this case the rotor speed, the speed estimation law can be defined as

$$\dot{\hat{\omega}} = \gamma \left[(\hat{\psi}_q - L\hat{i}_q)\tilde{i}_d - (\hat{\psi}_d - L\hat{i}_d)\tilde{i}_q \right] \tag{2.12}$$

So we obtain the following time derivative of the Lyapunov function

$$\dot{V} = -(R + k_p)(\tilde{i}_d^2 + \tilde{i}_q^2) + p\tilde{\chi}_q\tilde{i}_d - p\tilde{\chi}\tilde{i}_q \tag{2.13}$$

As already said, terms χ_d, χ_q are unknown, and we try to estimate them by $\hat{\chi}_d, \hat{\chi}_q$. To take into account the errors associated to these new estimates, we add to the Lyapunov function the following terms $(\frac{p}{k_i}\tilde{\chi}_d^2, \frac{p}{k_i}\tilde{\chi}_q^2)$, and supposing constant speed, we obtain

$$\dot{V} = -(R + k_p)(\tilde{i}_d^2 + \tilde{i}_q^2) + \frac{p}{k_i}\tilde{\chi}_q(\dot{\tilde{\chi}}_q + k_i\tilde{i}_d) + \frac{p}{k_i}\tilde{\chi}_d(\dot{\tilde{\chi}}_d + k_i\tilde{i}_q) \tag{2.14}$$

So, to cancel out the last two terms we should have the following adaptation laws:

$$\begin{aligned} \dot{\tilde{\chi}}_d &= \omega_0\chi_q - \omega\xi_d - k_i\tilde{i}_q + \frac{\dot{\omega}}{\omega}\chi_d \\ \dot{\tilde{\chi}}_q &= -\omega_0\chi_d - \omega\xi_q + k_i\tilde{i}_d + \frac{\dot{\omega}}{\omega}\chi_q \end{aligned} \tag{2.15}$$

Obviously we can not use the equations above because ω is unknown, but supposing constant speed we can try to use the following

$$\begin{aligned} \dot{\hat{\chi}}_d &= \omega_0\hat{\chi}_q - k_i\tilde{i}_q \\ \dot{\hat{\chi}}_q &= -\omega_0\hat{\chi}_d + k_i\tilde{i}_d \end{aligned} \tag{2.16}$$

As it will be shown in the following, terms dependent on current estimation straightforward derives from Lyapunov analysis to cancel out coupling terms in $\dot{V}$. Defining the estimation errors $\tilde{\chi}_d = \chi_d - \hat{\chi}_d, \tilde{\chi}_q = \chi_q - \hat{\chi}_q$, for them we have the following dynamics

$$\begin{aligned} \dot{\tilde{\chi}}_d &= \omega_0\tilde{\chi}_q + k_i\tilde{i}_q + \frac{\dot{\omega}}{\omega}\chi_d - \omega\xi_d \\ \dot{\tilde{\chi}}_q &= -\omega_0\tilde{\chi}_d - k_i\tilde{i}_d + \frac{\dot{\omega}}{\omega}\chi_q - \omega\xi_q \end{aligned} \tag{2.17}$$

It is worth noting that dynamics $\tilde{\chi}_d, \tilde{\chi}_q$ describe a pure oscillator at frequency ω_0 fed by terms depending on current estimation and additional terms dependent on rotor speed and acceleration.

The overall 7^{th} order error system is here recalled (note that also χ_d, χ_q dynamics must be considered in the overall error dynamics because they are implicitly equivalent to the flux estimation error dynamics)

$$
\begin{aligned}
L\dot{\tilde{i}}_d &= -(R+k_p)\tilde{i}_d + p\tilde{\chi}_q + p\tilde{\omega}(\hat{\psi}_q - Li_q) \\
L\dot{\tilde{i}}_q &= -(R+k_p)\tilde{i}_q - p\tilde{\chi}_d - p\tilde{\omega}(\hat{\psi}_d - Li_d) \\
\dot{\tilde{\chi}}_d &= \omega_0\tilde{\chi}_q + k_i\tilde{i}_q + \frac{\dot{\omega}}{\omega}\chi_d - \omega\xi_d \\
\dot{\tilde{\chi}}_q &= -\omega_0\tilde{\chi}_d - k_i\tilde{i}_d + \frac{\dot{\omega}}{\omega}\chi_q - \omega\xi_q \\
\dot{\chi}_d &= \omega_0\chi_q - \omega\xi_d + \frac{\dot{\omega}}{\omega}\chi_d \\
\dot{\chi}_q &= -\omega_0\chi_d - \omega\xi_q + \frac{\dot{\omega}}{\omega}\chi_q \\
\dot{\tilde{\omega}} &= -\gamma\left[(\hat{\psi}_q - L\hat{i}_q)\tilde{i}_d - (\hat{\psi}_d - L\hat{i}_d)\tilde{i}_q\right]
\end{aligned}
\tag{2.18}
$$

Choosing now the following Lyapunov function

$$
V = \frac{1}{2}\left(L\tilde{i}_d^2 + L\tilde{i}_q^2 + \frac{\tilde{\omega}^2}{\gamma} + \frac{p}{k_i}\tilde{\chi}_d^2 + \frac{p}{k_i}\tilde{\chi}_q^2 + \chi_d^2 + \chi_q^2\right) \tag{2.19}
$$

The time derivative of V along the error system trajectories is

$$
\begin{aligned}
\dot{V} =& -(R+k_p)(\tilde{i}_d^2 + \tilde{i}_q^2) + \ldots \\
&+ \frac{p}{k_i}\tilde{\chi}_d\left(\frac{\dot{\omega}}{\omega}\chi_d - \omega\xi_d\right) + \frac{p}{k_i}\tilde{\chi}_q\left(\frac{\dot{\omega}}{\omega}\chi_q - \omega\xi_q\right) + \ldots \\
&+ \frac{\dot{\omega}}{\omega}\left(\chi_d^2 + \chi_q^2\right) - \omega\left(\xi_d\chi_d + \xi_q\chi_q\right)
\end{aligned}
\tag{2.20}
$$

Choosing null auxiliary terms $\xi_d = \xi_q = 0$ and supposing constant speed, it follows that

$$
\dot{V} = -(R+k_p)(\tilde{i}_d^2 + \tilde{i}_q^2) \leq 0 \tag{2.21}
$$

The time derivative of the Lyapunov function is negative semidefinite, hence the boundedness of $\tilde{i}_d, \tilde{i}_q, \tilde{\chi}_d, \tilde{\chi}_q, \chi_d, \chi_q$ and $\tilde{\omega}$ is straightforward assured. Applying Barbalat's Lemma to V can be proved that $\tilde{i}_d, \tilde{i}_q$ tend to zero asymptotically[1]. Supposing bounded ω, boundedness of $\chi_d, \chi_q \Rightarrow$ boundedness of $\tilde{\psi}_d, \tilde{\psi}_q$, and supposing boundedness of $\psi_d, \psi_q \Rightarrow$ boundedness of $\hat{\psi}_d, \hat{\psi}_q$. Looking at the first two equations of Eq.(2.18), to state that also $\dot{\tilde{i}}_d, \dot{\tilde{i}}_q$ tend asymptotically to zero, must be proved that $\dot{\hat{\psi}}_d, \dot{\hat{\psi}}_q$ and $\dot{i}_d, \dot{i}_q$ are bounded, and this is basically related just to external variables $u_d, u_q, i_d, i_q, \omega_0$, and this must be supposed.

[1]Lyapunov function V is *differentiable* and has a *finite limit*, because of it is lower bounder ($V >= 0$) and $\dot{V}$ is *uniformly continuous* (bounded $\ddot{V}$ is a suff. cond.), therefore $\dot{V} \to 0$.

Equivalently can be stated that in case of persistence of excitation (which always hold on practical operating conditions), it can be also proved that $\tilde{\chi}_d, \tilde{\chi}_q, \tilde{\omega}$ tend to zero.
It is important to remark that the oscillatory dynamics χ_d, χ_q is neutrally stable and autonomous, in other words, it cannot be proved that the flux estimation errors $\tilde{\psi}_d, \tilde{\psi}_q$ tend to zero, but we have an estimate of them looking at $\hat{\chi}_d, \hat{\chi}_q$.
During speed transient with bounded acceleration, from exponential stability of the origin of the overall error system, ultimate boundedness of estimation errors follows.
Since according to this analysis it follows that χ_d, χ_q are bounded and recalling that $\hat{\chi}_d, \hat{\chi}_q$ are an estimation of the flux estimation error ($\chi_d = \omega\tilde{\psi}_d, \chi_d = \omega\tilde{\psi}_q$), can be proved that the right estimation of the stator fluxes are the following

$$\begin{aligned} \bar{\hat{\psi}}_d &= \hat{\psi}_d + \frac{\hat{\chi}_d}{\hat{\omega}} \\ \bar{\hat{\psi}}_q &= \hat{\psi}_q + \frac{\hat{\chi}_q}{\hat{\omega}} \end{aligned} \tag{2.22}$$

The variables $\hat{\chi}_d, \hat{\chi}_q$ are related to the estimation of the initial values of the stator fluxes. In fact, looking at the stator flux estimates in Eq.(2.4), the designed observer is based on pure integration of the stator equations for the reconstruction of the stator fluxes. The initial values of the stator fluxes are constant value in the stationary reference frame a, b, and of course these are a pure oscillator at ω_0 in the generic rotating reference frame d, q. The idea behind this estimation scheme is based on the fact that apart from the initial values, the stator flux is known because its derivative is known, so the only parameters to be estimated are the initial values that are constant in a, b and rotating in d, q.
From Eq.(2.22), rotor fluxes can be algebraically derived as follows

$$\begin{aligned} \hat{\phi}_d &= \bar{\hat{\psi}}_d - L\hat{i}_d \\ \hat{\phi}_q &= \bar{\hat{\psi}}_q - L\hat{i}_q \end{aligned} \tag{2.23}$$

2.1.1 Adaptive Framework for the Stability Proof

First of all I want to thank you Doct. Eng. *Andrea Tilli* for the help gave me for the study of the observer proposed in the previous section, and especially for the idea of the stability proof carried out using the Adaptive framework and reported in the following.
For the observer design reported in the previous section, Lyapunov theory has been used as a guide to identify terms constituting the final observer, but a rigorous stability proof of the designed observer can be done in the Adaptive

Framework exploiting the stability results reported in [17].
The proof begins with some considerations on the motor model, that is knowing the initial conditions of the stator flux dynamics $(\psi_d(0), \psi_q(0))$ we can "measure" them by a pure integration of the stator voltages minus the resistance voltage drop, as indicated in the third and fourth equation in Eq.(2.2), therefore we can suppose to perfectly know the stator flux dynamics. Nevertheless these initial conditions are unknown, but they are constant by definition, an as all constant parameters we can try to estimate them using the adaptive theory. We can start dividing the stator fluxes into two parts, the first part is a known part that is related to the pure integration of the stator voltages and with null initial condition by definition, and an unknown part related to the initial conditions and that will be estimated. So the following can be stated

$$\begin{aligned} \psi_d(t) &= \psi_{d0}(t) + \bar{\psi}_d(t) \\ \psi_q(t) &= \psi_{q0}(t) + \bar{\psi}_q(t) \end{aligned} \tag{2.24}$$

Where the known part results as follows

$$\begin{aligned} \dot{\bar{\psi}}_d &= -Ri_d + \omega_0\bar{\psi}_q + u_d \\ \dot{\bar{\psi}}_q &= -Ri_q - \omega_0\bar{\psi}_d + u_q \\ \bar{\psi}_d(0) &= 0, \bar{\psi}_q(0) = 0 \end{aligned} \tag{2.25}$$

The unknown part is the following

$$\begin{aligned} \dot{\psi}_{d0} &= \omega_0\psi_{q0} \\ \dot{\psi}_{q0} &= -\omega_0\psi_{d0} \\ \psi_{d0}(0)&, \psi_{q0}(0) \quad \textit{unknown} \end{aligned} \tag{2.26}$$

It is important to underline that initial conditions ψ_{d0}, ψ_{q0} are time dependent because they are expressed in the generic rotating reference frame d, q, in fact the stator flux initial conditions are constant in the stationary reference frame but in the rotating reference frame they are rotating with a speed $-\omega_0$ and with constant amplitude.
Substituting Eq.(2.24) into the stator current dynamics in Eq.(2.2) we obtain

$$\begin{aligned} L\dot{i}_d &= -Ri_d + (\omega_0 - p\omega)Li_q + p\omega\psi_{q0} + p\omega\bar{\psi}_q + u_d \\ L\dot{i}_q &= -Ri_q - (\omega_0 - p\omega)Li_d - p\omega\psi_{d0} - p\omega\bar{\psi}_d + u_q \end{aligned} \tag{2.27}$$

From the previous equations can be seen that the initial conditions are multiplied by the term $p\omega$ so these products will be estimated instead of ψ_{d0}, ψ_{q0} beacuse also the speed is supposed to be constant. This estimation will be done exploiting the following change of coordinates

$$\chi_d = p\omega\psi_{d0} \quad , \quad \chi_q = p\omega\psi_{q0} \tag{2.28}$$

Therefore, the following dynamics for the new variables χ_d, χ_q can be stated

$$\begin{aligned} \dot{\chi}_d &= p\dot{\omega}\psi_{d0} + \omega_0\chi_q \\ \dot{\chi}_q &= p\dot{\omega}\psi_{q0} - \omega_0\chi_d \end{aligned} \tag{2.29}$$

The equations above show that the initial conditions, constant in a stationary reference frame, are rotating with the inverse of the speed of the reference frame in a generic rotating reference frame.
Supposing now constant speed ($\dot{\omega} = 0$) the following system, for which the observer must be designed, can be obtained

$$\begin{bmatrix} \dot{i}_d \\ \dot{i}_q \end{bmatrix} = \begin{bmatrix} -R/L & (\omega_0 - p\omega) \\ -(\omega_0 - p\omega) & -R/L \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 0 & 1/L & p\bar{\psi}_q/L \\ -1/L & 0 & -p\bar{\psi}_d/L \end{bmatrix} \begin{bmatrix} \chi_d \\ \chi_q \\ \omega \end{bmatrix} + \begin{bmatrix} u_d/L \\ u_q/L \end{bmatrix}$$
$$\begin{bmatrix} \dot{\chi}_d \\ \dot{\chi}_q \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & \omega_0 & 0 \\ -\omega_0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \chi_d \\ \chi_q \\ \omega \end{bmatrix} \tag{2.30}$$

The first two equations are the current dynamics that are also the measured output, the last three equations are the variables to be estimated, the stator flux initial condition and the rotor speed. The system can be expressed also as follows

$$\begin{aligned} \dot{x} &= Ax + \Gamma z + u \\ \dot{z} &= \Omega_0 z \end{aligned} \tag{2.31}$$

Where the following change of coordinates has been used

$$x = [i_d, i_q]^T \quad , \quad z = [\chi_d, \chi_q, \omega]^T \tag{2.32}$$

For this system the following full order observer can be designed

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + \Gamma\hat{z} + u + \eta \\ \dot{\hat{z}} &= +\Gamma^T\tilde{x} + \Omega_0\hat{z} \end{aligned} \tag{2.33}$$

Where $\hat{x}, \hat{z}$ are the estimates of x, z. Defining the estimation errors as

$$\tilde{x} = x - \hat{x} \quad , \quad \tilde{z} = z - \hat{z}$$

the following error system can be derived

$$\begin{aligned} \dot{\tilde{x}} &= A\tilde{x} + \Gamma\tilde{z} - \eta \\ \dot{\tilde{z}} &= -\Gamma^T\tilde{x} + \Omega_0\tilde{z} \end{aligned} \tag{2.34}$$

It is important to note that the term η should be used to place the poles of the subsystem error related to $\tilde{x}$, i.e. $\eta = F\tilde{x}$, otherwise the dynamic will be driven only by the system matrix A.
Now the Lyapunov analysis can be done with the following candidate

$$V = \frac{1}{2}\left(\tilde{x}^T\tilde{x} + \tilde{z}^T\tilde{z}\right)^2 \tag{2.35}$$

The following time derivative of V results

$$\dot{V} = \tilde{x}^T(A+F)\tilde{x} + \tilde{x}^T\Gamma\tilde{z} - \tilde{z}^T\Gamma^T\tilde{x} + \tilde{z}^T\Omega_0\tilde{z} \tag{2.36}$$

It is straightforward to show the function $\dot{V}$ results as follows

$$\dot{V} = \tilde{x}^T(A+F)\tilde{x} \tag{2.37}$$

and choosing opportunely the matrix F the Lyapunov function time derivative is negative semidefinite, hence the error system is bounded. Moreover, it can be shown that $\ddot{V}$ is bounded, in fact it is a combination of bounded terms, hence from direct application of Barbalat's lemma to V can be shown that

$$\lim_{t\to\infty}\dot{V} = 0 \quad \Rightarrow \quad i_d, i_q \to 0 \tag{2.38}$$

Other considerations on the stability of the error system are related to the properties of the function Γ, and these are related principally to the persistence of excitation of the two signals $\bar{\psi}_d, \bar{\psi}_q$, that will bring the system to the classical asymptotic stability of the origin.

2.1.2 Modified Stator Flux Dynamics

The stator flux adaptation laws (Eq.2.4) derived for the observer in the previous section are here recalled

$$\begin{aligned}\dot{\hat{\psi}}_d &= -Ri_d + \omega_0\hat{\psi}_q + u_d + \xi_d \\ \dot{\hat{\psi}}_q &= -Ri_q - \omega_0\hat{\psi}_d + u_q + \xi_q\end{aligned}$$

Supposing $\xi_d = \xi_q = 0$, these two equations represent the dynamic of a pure oscillator at frequency ω_0. These observation laws are model-based and they differ from the real one (see Eq.2.2) just for initial conditions, that for a PMSM are not known[2].

[2]For IM the rotor flux initial conditions are often supposed to be known because of the rotor flux is generated from measured stator currents.

It is well known that sensitivity to model uncertainties such as unknown resistance, measurement errors on stator currents, actuation errors such as inverter non-idealities (mismatching between command stator voltage and real stator voltage due to distortion introduced by dead-time and switching device non-idealities), is higher at low speed, i.e. at low electrical frequency.
In order to compensate for unknown initial conditions and to be robust to such uncertainties, a modified version of the stator flux estimation laws is defined. Defining the variable ξ_d, ξ_q as follows

$$\begin{aligned}\xi_d &= -k_\psi(\hat{\psi}_d - \psi_d^*)\\ \xi_q &= -k_\psi(\hat{\psi}_q - \psi_q^*)\end{aligned}$$

These two terms introduce a negative feed-back in a stable dynamic, which is the stator flux pure oscillator dynamic, and the modified flux adaptation laws is as follows

$$\begin{aligned}\dot{\hat{\psi}}_d &= -Ri_d + \omega_0\hat{\psi}_q + u_d - k_\psi(\hat{\psi}_d - \psi_d^*)\\ \dot{\hat{\psi}}_q &= -Ri_q - \omega_0\hat{\psi}_d + u_q - k_\psi(\hat{\psi}_q - \psi_q^*)\end{aligned} \tag{2.39}$$

The variables (ψ_d^*, ψ_q^*) can be seen as reference stator fluxes and can be defined as follows

$$\begin{aligned}\psi_d^* &= Li_d^* + \phi_d^*\\ \psi_q^* &= Li_q^* + \phi_q^*\end{aligned}$$

where $i_d^*, i_q^*, \phi_d^*, \phi_q^*$ are respectively current and rotor flux references to be defined.
In order to define these reference variables we have to look at the current controller. Supposing the current controller is designed in a priori known reference frame with angle ϵ_c and speed w_c, therefore in this reference frame we have current controller references $\bar{i}_d^*, \bar{i}_q^*$, and we also know the nominal rotor fluxes $\bar{\phi}_d^*, \bar{\phi}_q^*$. With this assumption we can algebraically derive the desired reference variables on the observer reference frame with the following equations

$$\begin{aligned}\begin{bmatrix} i_d^* \\ i_q^* \end{bmatrix} &= \begin{bmatrix} \cos(\epsilon_0 - \epsilon_c) & \sin(\epsilon_0 - \epsilon_c) \\ -\sin(\epsilon_0 - \epsilon_c) & \cos(\epsilon_0 - \epsilon_c) \end{bmatrix} \begin{bmatrix} \bar{i}_d^* \\ \bar{i}_q^* \end{bmatrix}\\ \begin{bmatrix} \phi_d^* \\ \phi_q^* \end{bmatrix} &= \begin{bmatrix} \cos(\epsilon_0 - \epsilon_c) & \sin(\epsilon_0 - \epsilon_c) \\ -\sin(\epsilon_0 - \epsilon_c) & \cos(\epsilon_0 - \epsilon_c) \end{bmatrix} \begin{bmatrix} \bar{\phi}_d^* \\ \bar{\phi}_q^* \end{bmatrix}\end{aligned}$$

Moreover, if the current controller is designed according to Field-Oriented Control (FOC), the current controller reference frame is aligned with the real rotor flux, i.e. the d-axis is aligned with the rotor magnet, therefore for the rotor flux reference variable the following result is known

$$\begin{aligned}\bar{\phi}_d^* &= \Phi\\ \bar{\phi}_q^* &= 0\end{aligned}$$

It is worth noting that if the observer is also designed in the same reference frame used for the current controller we have $\epsilon_0 = \epsilon_c$.
Looking at the modified stator flux observer (Eq.2.39), the pure integrator is substituted by a low-pass filter fed by the original input and an additional term dependent on the reference stator flux, and the stator flux estimation error dynamics become

$$\begin{aligned}\dot{\tilde{\psi}}_d &= \omega_0\tilde{\psi}_q - k_\psi\tilde{\psi}_d + k_\psi(\psi_d - \psi_d^*)\\ \dot{\tilde{\psi}}_q &= -\omega_0\tilde{\psi}_d - k_\psi\tilde{\psi}_q + k_\psi(\psi_q - \psi_q^*)\end{aligned}$$

The autonomous part of the stator flux error system is now exponentially stable, therefore the estimation error is bounded if the flux tracking error input is bounded, i.e. $(\psi_d - \psi_d^*, \psi_q - \psi_q^*)$ is bounded, moreover the estimation error tends to zero if the flux tracking error input also tends to zero.
It is not simple to obtain the proof of the stability of the modified observer exploiting Lyapunov theory because of the $\dot{V}$ function is not negative definite neither semidefinite negative. However, an explanation of the modified observer will be given by physical/practical insight.
In order to understand the behavior of the modified stator flux observer, assume for simplicity that $\omega_0 = 0$, i.e. the observer is designed in the stationary reference frame. Expressing the stator flux dynamics with complex variables and computing the Laplace transform we obtain

$$\begin{aligned}s\hat{\psi}(s) &= u(s) - Ri(s)\\ s\hat{\psi}_m(s) &= u(s) - Ri(s) - k_\psi\left[\hat{\psi}_m(s) - \hat{\psi}^*(s)\right]\end{aligned}$$

in which $\hat{\psi}, \hat{\psi}_m$ stand respectively for the unmodified stator flux estimation ($\xi_d = \xi_q = 0$) and modified stator flux estimation. Hence, it follows that

$$\hat{\psi}_m(s) = \frac{s}{s+k_\psi}\hat{\psi}(s) + \frac{k_\psi}{s+k_\psi}\hat{\psi}^*(s)$$

Some consideration on the result obtained

- At high frequency (higher than k_ψ) the modified stator flux is similar to the pure integrator dynamics, in fact we have $\hat{\psi}_m(s) \approx \hat{\psi}(s)$.
- At low frequency or frequency near k_ψ (when pure integration results critical) the integrator is substituted by a low-pass filter, hence the term dependent on $\hat{\psi}(s)$, which has a low signal to noise ratio, is reduced in order to avoid drift and estimation errors. The term dependent on $\hat{\psi}^*(s)$ allows for compensation of phase-shift and amplitude attenuation introduced by the substitution of the pure integrator with a low-pass filter and adds a forcing term at low frequency.

- Substitution of the pure integrator with the (asymptotically stable) low-pass filter allows for convergence to zero of the dynamics dependent on unknown initial conditions.

- At low frequency, the stator flux estimate tracks a low-pass-filtered version of the reference stator flux, i.e. the estimation is open-loop because the observer introduce a modification in the model to follow, and is based on the assumption that the flux is correctly tracked (in phase). As a consequence, operations at frequency lower than k_ψ are allowed only for a limited amount of time.

We can finally summarize the overall speed/flux observer in the generic reference frame with angle ϵ_0 and speed ω_0 with the following equations

$$\begin{aligned}
L\dot{\hat{i}}_d &= -R\hat{i}_d + (\omega_0 - p\hat{\omega})Li_q + p\hat{\omega}\hat{\psi}_q + u_d + \eta_d \\
L\dot{\hat{i}}_q &= -R\hat{i}_q - (\omega_0 - p\hat{\omega})Li_d - p\hat{\omega}\hat{\psi}_d + u_q + \eta_q \\
\dot{\hat{\psi}}_d &= -Ri_d + \omega_0\hat{\psi}_q + u_d - k_\psi(\hat{\psi}_d - \hat{\psi}_d^*) \\
\dot{\hat{\psi}}_q &= -Ri_q - \omega_0\hat{\psi}_d + u_q - k_\psi(\hat{\psi}_q - \hat{\psi}_q^*) \\
\dot{\hat{\chi}}_d &= \omega_0\hat{\chi}_q - k_i\tilde{i}_q \\
\dot{\hat{\chi}}_q &= -\omega_0\hat{\chi}_d + k_i\tilde{i}_d \\
\dot{\hat{\omega}} &= \gamma\left[(\hat{\psi}_q - L\hat{i}_q)\tilde{i}_d - (\hat{\psi}_d - L\hat{i}_d)\tilde{i}_q\right]
\end{aligned}
\qquad
\begin{aligned}
\tilde{i}_d &= i_d - \hat{i}_d \\
\tilde{i}_q &= i_q - \hat{i}_q \\
\eta_d &= k_p\tilde{i}_d + p\hat{\chi}_q \\
\eta_q &= k_p\tilde{i}_q - p\hat{\chi}_d \\
\psi_d^* &= Li_d^* + \phi_d^* \\
\psi_q^* &= Li_q^* + \phi_q^*
\end{aligned}
\tag{2.40}$$

2.1.3 Alignment Controller

The observer reference frame speed ω_0 is designed to guarantee exponential convergence of the d-axis to the permanent magnet axis, i.e. the reference frame controller acts as an *Alignment Controller*. We can define the reference frame speed as follows

$$\begin{aligned}
\dot{\epsilon}_0 &= \omega_0 \\
\omega_0 &= p\hat{\omega} + \eta_0
\end{aligned}
\tag{2.41}$$

The first of the two equations above recall the fact that the controlled variable is ϵ_0 and as a consequence the plant to control is a pure integrator, and η_0 is an auxiliary term control containing the control action. The rotor flux dynamics of a PMSM is as follows

$$\begin{aligned}
\dot{\phi}_d &= (\omega_0 - p\omega)\phi_q \\
\dot{\phi}_q &= -(\omega_0 - p\omega)\phi_d
\end{aligned}
\quad\Rightarrow\quad
\begin{aligned}
\dot{\phi}_d &= -p\tilde{\omega}\phi_q + \eta_0\phi_q \\
\dot{\phi}_q &= p\tilde{\omega}\phi_d + \eta_0\phi_d
\end{aligned}
\tag{2.42}$$

The objective of η_0 is to achieve asymptotic flux alignment, i.e. to guarantee that $\delta_\theta = \arctan(\phi_q/\phi_d) \to 0$, and assuming that the rotor flux is known, we try to define the control term as follows

$$\eta_0 = k_A \arctan\left(\frac{\phi_q}{\phi_d}\right) = k_A \delta_\theta \tag{2.43}$$

Differentiating δ_θ and using the rotor flux equations (Eq.2.42), it follows that

$$\dot{\delta}_\theta = \frac{\phi_d \dot{\phi}_q - \phi_q \dot{\phi}_d}{\phi_d^2 + \phi_q^2} = -\eta_0 + p\tilde{\omega} = -k_A \delta_\theta + p\tilde{\omega} \tag{2.44}$$

Therefore, the designed control law guarantee exponential stability (with time constant k_A) and boundedness of alignment error in presence of speed estimation error. In order to guarantee the alignment of the observer reference frame with the rotor flux also with speed estimation error, an integral term must be added. Hence, taking into account that the flux is not known but is estimated by the observer designed in the previous sections the alignment controller is as follows

$$\begin{aligned} \omega_0 &= p\hat{\omega} + \eta_0 \\ \eta_0 &= k_A \hat{\delta}_\theta + \chi_{\omega_0} \\ \dot{\chi}_{\omega_0} &= k_{Ai} \hat{\delta}_\theta \\ \hat{\delta}_\theta &= \arctan\left(\frac{\hat{\phi}_q}{\hat{\phi}_d}\right) \end{aligned} \tag{2.45}$$

Further, we can study what happens using estimated rotor fluxes (Eq.2.23) instead of real rotor fluxes. Defining $\tilde{\delta}_\theta = \delta_\theta - \hat{\delta}_\theta$, recalling the dynamic of the real angle error (Eq. 2.44) and the expression of η_0 (Eq.2.45) we have

$$\begin{aligned} \dot{\delta}_\theta &= -k_A \delta_\theta - \chi_{\omega_0} + k_A \tilde{\delta}_\theta + p\tilde{\omega} \\ \dot{\chi}_{\omega_0} &= k_{Ai}(\delta_\theta - \tilde{\delta}_\theta) \end{aligned} \tag{2.46}$$

Some considerations on the results

- Studying the equilibrium point of the system above we ca say that, $\hat{\delta}_\theta = \delta_\theta - \tilde{\delta}_\theta \to 0$ and $\chi_{\omega_0} \to p\tilde{\omega}$. Hence the controller with integral action allows for the asymptotic alignment of the observer reference frame to the estimated rotor flux and also an information about the speed estimation error.

- During speed transient, where of course we have a speed estimation error due to the adaptive nature of the observer, we have an observer reference frame misalignment, recovered with the dynamic imposed with k_A and k_{Ai}.

2.1.4 Parameter Tuning Rules

We start recalling that for the observer designed in sections 2.1 and 2.1.2 and for the alignment controller designed in section 2.1.3, the tuning parameters are

- $\mathbf{k_p}, \mathbf{k_i}$, proportional and integral gain involved in the current estimation
- γ, gain involved in the adaptive speed estimation
- $\mathbf{k}_\psi$, pole of the filter introduced by the modified observer
- $\mathbf{k_A}, \mathbf{k_{Ai}}$ proportional and integral gain involved in the alignment control

First parameter that can be tuned is k_ψ, because of this parameter does not influence the remaining estimation dynamics of the observer.
As already said, the aim of k_ψ is both to robustify the simple stable dynamic in the stator flux estimates and to compensate for unknown initial conditions in the stator flux estimates.
In terms of robustness of the observer, this parameter allows to filter out from pure integration of the input voltages in the stator flux estimates the non-ideality effects of the system, i.e. the dead-times. In terms of compensating for unknown initial condition, setting the value of k_ψ to 0 means that $\hat{\psi}_d, \hat{\psi}_q$ and $\hat{\chi}_d, \hat{\chi}_q$ are perfect sinusoidal signals not converging to the real values.
From the analysis reported in section 2.1.2, this parameter is the pole of the filter approximating the pure oscillator at frequency ω_0 in the stator flux estimation dynamics, so this parameter is related to the minimum electrical frequency for which the observer works properly, i.e. the approximation in the observer model is good enough, and can be set approximately one decade below this minimum electrical frequency ($k_\psi = p\omega_{min}/10$).
The minor is the value of k_ψ, and the minor will be the non-ideality filtering effect, and the longer will be the attenuation of the sinusoidal signals $\hat{\psi}_d, \hat{\psi}_q$ and $\hat{\chi}_d, \hat{\chi}_q$. It is worth noting that, as already said at the end of section 2.1, the right stator flux estimates are

$$\bar{\hat{\psi}}_d = \hat{\psi}_d + \frac{\hat{\chi}_d}{\hat{\omega}}$$
$$\bar{\hat{\psi}}_q = \hat{\psi}_q + \frac{\hat{\chi}_q}{\hat{\omega}}$$

Simulations show that, even if variables $\hat{\psi}_d, \hat{\psi}_q$ and $\hat{\chi}_d, \hat{\chi}_q$ are sinusoidal signals (with constant amplitude if $k_\psi = 0$, or with decreasing amplitude if $k_\psi \neq 0$), the sum of these variables generating $\bar{\hat{\psi}}_d, \bar{\hat{\psi}}_q$ are not sinusoidal, and of course these last stator fluxes estimates are better than $\hat{\psi}_d, \hat{\psi}_q$.
It is important to recall that the alignment controller discussed in section 2.1.3

is fed by the alignment estimation $\hat{\delta}_\theta$ that is related to rotor flux estimates, and of course if rotor flux estimates are derived from $\hat{\psi}_d, \hat{\psi}_q$ the alignment controller (a PI regulator) will be fed by a sinusoidal input, of course if $k_\psi \neq 0$ the amplitude of these sinusoidal variables will decrease, and after the transient imposed by k_ψ the alignment controller could be turn on. If instead of $\hat{\psi}_d, \hat{\psi}_q$ are used $\hat{\bar{\psi}}_d, \hat{\bar{\psi}}_q$ the alignment controller can be turn on with the observer at the same time, decreasing a lot the initial transient time.

Parameter k_p, k_i can be tuned with the following approximated procedure. Supposing $\omega_0 = \hat{\omega} = 0$ in the current estimation dynamics, we obtain two decoupled systems as follows:

$$\begin{bmatrix} \dot{\hat{i}}_d \\ \dot{\hat{\chi}}_q \end{bmatrix} = \begin{bmatrix} -\frac{(R+k_p)}{L} & \frac{1}{L} \\ -k_i & 0 \end{bmatrix} \begin{bmatrix} \hat{i}_d \\ \hat{\chi}_q \end{bmatrix} + \begin{bmatrix} (u_d + k_p i_d)/L \\ k_i i_d \end{bmatrix} \tag{2.47}$$

$$\begin{bmatrix} \dot{\hat{i}}_q \\ \dot{\hat{\chi}}_d \end{bmatrix} = \begin{bmatrix} -\frac{(R+k_p)}{L} & \frac{1}{L} \\ -k_i & 0 \end{bmatrix} \begin{bmatrix} \hat{i}_q \\ \hat{\chi}_d \end{bmatrix} + \begin{bmatrix} (u_q + k_p i_q)/L \\ -k_i i_q \end{bmatrix}$$

The eigenvalues of the matrix defining the homogeneous part of the system are the following

$$s_{1,2} = -\frac{R+k_p}{2L} \pm \sqrt{\frac{(R+k_p)^2}{4L^2} - \frac{k_i}{L}} \tag{2.48}$$

The equation above defines the exact equation of the "feed-back" system poles. In fact using a feed-back control scheme to express the current estimation dynamic, as shown on Figure 2.1 for the $\hat{i}_d$ current, can be seen that the "plant" to control is a pure integrator stabilized by a PI controller and with an additive feed-forward term depending on the resistance R.
From linear control theory is quite known that this kind of feed-back system gives raise to a zero with no possibility of cancellation, and of course the zero is introduced by the PI controller as usual.
The frequency of this zero depends not only by the PI coefficients, but also by the plant parameter due to the feed-forward term, in fact we have:

$$\omega_z = \frac{k_i}{R + k_p} \tag{2.49}$$

No cancellation is possible because the only pole of the plant is at zero frequency, i.e. a pure integrator, and to place at zero frequency we should have $k_p \to \infty$, that is of course not possible.
The PI controller can be tuned supposing to desire a small integral control action with respect to the proportional one. The reason of this choice is to

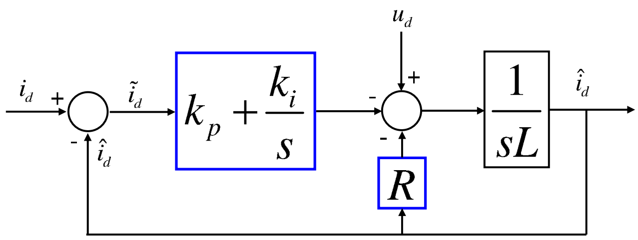

Figure 2.1: Feed-back scheme for the $\hat{i}_d$ current estimation dynamic. Blue bounded blocks refer to the PI stabilizing term for observer stability and a model-based term depending on the resistance R.

have a low bandwidth for the $\hat{\chi}_d, \hat{\chi}_q$ estimation dynamics, in fact recalling the adaptation law of these variables (Eq.2.40) coefficient k_i defines their cut-off frequency.
Variables $\hat{\chi}_d, \hat{\chi}_q$ and $\hat{\omega}$ act with their dynamics on the current estimation dynamic, and are driven both by the current estimation errors $\tilde{i}_d, \tilde{i}_q$.
If the dynamic of the variables $\hat{\chi}_d, \hat{\chi}_q$ is faster than the dynamic of the variable $\hat{\omega}$, the current estimation errors will feed $\hat{\chi}_d, \hat{\chi}_q$ despite to $\hat{\omega}$.
Because of $\hat{\chi}_d, \hat{\chi}_q$ are related with flux estimation, and $\hat{\omega}$ is by definition related with speed estimation, choosing a slower dynamic for $\hat{\chi}_d, \hat{\chi}_q$ than the dynamic of $\hat{\omega}$ is equivalent to prefer the estimation of the speed instead of the estimation of the stator fluxes and vice versa.
It is important to note that the stator flux estimates are used by the alignment controller, described in section 2.1.3, to achieve the correct reference frame alignment, so if the correct alignment is more important than speed estimation, one should think to choose a tuning procedure that makes $\hat{\chi}_d, \hat{\chi}_q$ dynamics faster the $\hat{\omega}$ one.

Choosing a small integral control action for the PI controller is equivalent to have one low frequency pole ($\omega_{p_{LF}}$) and one high frequency pole ($\omega_{p_{HF}}$). The low frequency pole is attracted by the zero, which will be at low frequency too, so to design the low frequency pole is possible to design a low frequency zero. From Eq.(2.48) and supposing a small integral action, we can neglect from the square root the term dependent on k_i, so we can derive the expression of the high-frequency pole and consequently define the value for k_p:

$$\omega_{p_{HF}} = \frac{R + k_p}{L} \Rightarrow k_p = \omega_{p_{HF}} L - R \tag{2.50}$$

The value of the high frequency pole $\omega_{p_{HF}}$ should take into account for the bandwidth of the current controller, for the noise on the current measures, and if the current estimates $\hat{i}_d, \hat{i}_q$ are used as a measure by the current controller

instead of the real current measure i_d, i_q. Current estimates are the filtered version of real current measure, and if current estimates are used by the current controller, a high filtering action cannot be imposed, to not give raise to a instability of the feed-back current control.
If current estimates are not used by the current controller, which in this case uses the real measure, one could think to use the current estimation dynamic to filter out the noise in the observer variables, of course introducing a phase lag in all estimated variables.
Once defined k_p, it is possible to define the value for k_i recalling the Eq.(2.49) as follows:

$$\omega_{p_{LF}} \approx \omega_z = \frac{k_i}{R + k_p} \Rightarrow k_i = (R + k_p)\omega_{p_{LF}} = (R + k_p)\frac{\omega_{p_{HF}}}{k} \tag{2.51}$$

Where the relation $\omega_{p_{LF}} = \omega_{p_{HF}}/k$ has been used, and a typical value for k could be $50 \div 100$.
Of course the value of each pole/zero must be realizable, especially for high frequency pole/zero in a discrete time realization, and for this purpose the effective expression of the poles/zeros of the system must be used, i.e using Eq.(2.48).
It is important to remark that the value of k_i defines the band-width of the estimation of $\hat{\chi}_d, \hat{\chi}_q$ and must be compared with the band-with of the speed estimation dynamic $\hat{\omega}$ that will be reported in the following part of the tuning procedure.
Another consideration about the value of parameter k_p, k_i in case of presence of dead-times non-ideality must be done.
Dead-times act as a disturbance voltages (d_d, d_q) on the input voltages u_d, u_q applied to the motor windings.
These disturbances are quite complex in frequency, in fact the frequency spectrum of d_d, d_q contains components from zero to high frequencies, of course with an amplitude decreasing with frequency. It is important to remark that dead-times disturbance voltages described in a stationary a, b reference frame are periodic signals with period $\approx 2\pi/p\omega$, so the Fourier analysis of dead-times give rise to a frequency discretized spectrum.
Dead-times act on input voltages but their effect can be seen on $\tilde{i}_d, \tilde{i}_q$, and of course it propagates to all the observer estimates. From simulations can be seen that the major effect of dead-times is on variables $\hat{\chi}_d, \hat{\chi}_q$, and as a consequence on variables $\bar{\hat{\psi}}_d, \bar{\hat{\psi}}_q$, so be careful when these variables are used to derive the rotor flux estimates used by the alignment controller. To attenuate the effect of dead-times on $\hat{\chi}_d, \hat{\chi}_q$ the parameter k_p must be increased, maintaining k_i unchanged.

For the definition of the adaptation gain γ, involved in the estimation of the speed, the following approximated procedure can be used. Supposing, $\omega_0 = 0$, $\hat{\phi}_d = \hat{\psi}_d - L\hat{i}_d \approx \phi$, $\hat{\phi}_q = \hat{\psi}_q - L\hat{i}_q \approx 0$, and considering $\hat{\chi}_d$ as an input (due to its slow dynamic imposed) the following system can be exploited:

$$\begin{bmatrix} \dot{\hat{\omega}} \\ \dot{\hat{i}}_q \end{bmatrix} = \begin{bmatrix} 0 & \gamma\phi \\ -\frac{p\phi}{L} & -\frac{(R+k_p)}{L} \end{bmatrix} \begin{bmatrix} \hat{\omega} \\ \hat{i}_q \end{bmatrix} + \begin{bmatrix} -\gamma\phi i_q \\ (u_q + k_p i_q - p\hat{\chi}_d)/L \end{bmatrix} \tag{2.52}$$

The eigenvalues of the homogeneous part of the system above are the solutions of the following II order equation:

$$s^2 + \frac{R+k_p}{L}s + p\gamma\frac{\phi^2}{L} \tag{2.53}$$

It is important to stress that the value of the coefficient k_p, designed from the estimation current tuning procedure, is involved also in the definition of the adaptive gain for the speed estimation, as shown in the equation above.
Recalling the expression of a II order equation using the damping coefficient δ and the natural pulsation ω_n as follows

$$s^2 + 2\delta\omega_n s + \omega_n^2 \tag{2.54}$$

the value of γ can be derived choosing a value for the natural pulsation ω_n as follows:

$$\gamma = \frac{L}{p}\left(\frac{\omega_n}{\phi}\right)^2 = \frac{L}{p}\left(\frac{h\omega_{p_{LF}}}{\phi}\right)^2 \tag{2.55}$$

A typical value for h can be $10 \div 50$. It important to note that the value of the damping parameter δ must be checked because with this tuning procedure it is just a result, and a typical value for δ could be 0.9.
Finally, it is important to remark that the dynamic of $p\hat{\chi}_q$ and $p\hat{\phi}_d\hat{\omega}$, in the equation of $\dot{\hat{i}}_q$, are pure integrators with bandwidth pk_i and $p\gamma\phi^2$ respectively, therefore these values must be chosen to prefere the dynamic of stator flux estimates instead of the dynamic of the speed estimate, or vice versa.

Once tuned the speed/stator flux observer, parameters k_A, k_{Ai} are still left to tune. The tuning procedure for these parameters is based on Eq.(2.41) and on the fact that the plant to control is a pure integrator, i.e. we act on ω_0 to control ϵ_0, and $\dot{\epsilon}_0 = \omega_0$. From theory of linear systems design, controlling a pure integrator with a PI controller to impose a damping δ and a natural frequency ω_n the following equations can be straightforward derived

$$\begin{aligned} k_A &= 2\delta\omega_n \\ k_{Ai} &= \omega_n^2 \end{aligned} \tag{2.56}$$

Typical values for these two parameters could be $\delta = 0.7 \div 1.0$ and $\omega_n = 1 \div 20$.

Motor inertia J $[Kgm^2]$	$n.a.$
Nominal angular speed ω_{nom} [rad/s]	1.60
Rotor flux Φ [Wb]	5.50
Nominal torque T_{nom} [Nm]	680×10^3
Stator resistance R [Ω]	9.0×10^{-3}
Stator inductance L [H]	3.0×10^{-3}
Number of pole pairs p	50

Table 2.1: Motor parameters for a realistic scenario simulation.

2.2 Simulation Results

In this section the performances of the proposed observer are simulated in an ideal set-up: without noise measure (i.e. current measures and DC-Bus voltage measure), without inverter non-idealities, and adopting a discrete time version of the observer ($f_s = 2.5$[kHz]). The observer is used as the core of a sensorless torque controller designed on the estimated $d-q$ reference frame for which the alignment with the rotor flux is achieved by the Alignment Controller.
Simulations are performed using machine parameters reported in Tab.2.1. With these machine parameters and following the parameter tuning procedure reported in section 2.1.4 the following standard parameterization has been imposed: $k_\psi = 3$, $k_p = 0.506$, $k_i = 1.138$, $\gamma = 0.4$, $k_A = 20.7$, $k_{Ai} = 132.25$. Results obtained with the standard parameterization are shown in Figure 2.2, in which the sensorless algorithm is turn on when the rotor is rotating at a speed of $0.42[rad/s]$ (nearly 1/3 of the nominal speed) and from this figure it is interesting to note that, i) the rotor flux alignment is reached (subplot (b)), ii) the presence of k_ψ reduces the amplitude of the sinusoidal part present in $\hat{\psi}_d$,$\hat{\psi}_q$ and $\hat{\chi}_d/\omega$,$\hat{\chi}_q/\omega$ (subplots (e)(f)). The parameter k_ψ is crucial when noise measure and inverter non-idealities are present, and in this case cannot be null.
In Figure 2.3 is shown the same simulation with different parameterization, i.e. with k_phi null, and with the alignment controller turn off. In this figure it is important to note that: i) the rotor flux alignment is not reached, ii) the sinusoidal part present in $\hat{\psi}_d$,$\hat{\psi}_q$ and $\hat{\chi}_d/\omega$,$\hat{\chi}_q/\omega$ is persistent, but can be easily seen that the sum: $\hat{\psi}_d + \hat{\chi}_d/\omega$, $\hat{\psi}_q + \hat{\chi}_q/\omega$ are constant at steady state due to the delay of π between the two components.

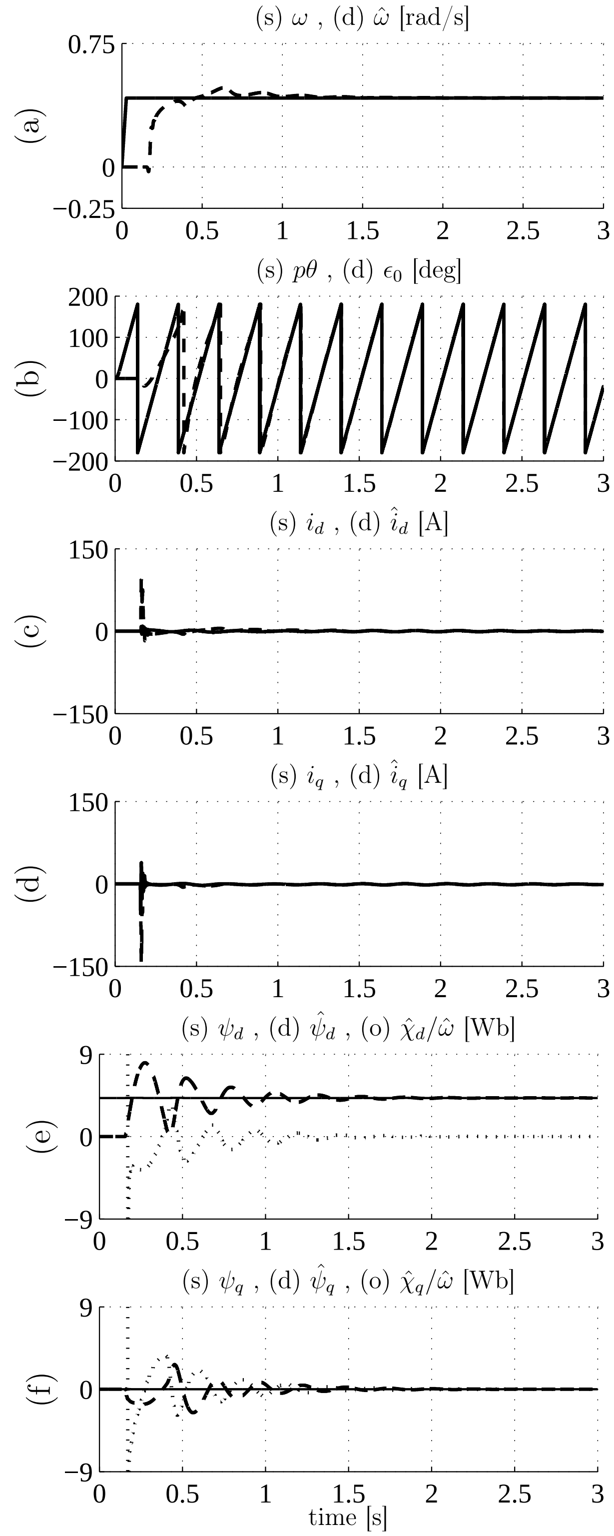

Figure 2.2: (s) solid, (d) dashed , (o) dotted. Convergence of the Observer variables supposing standard parameterization.

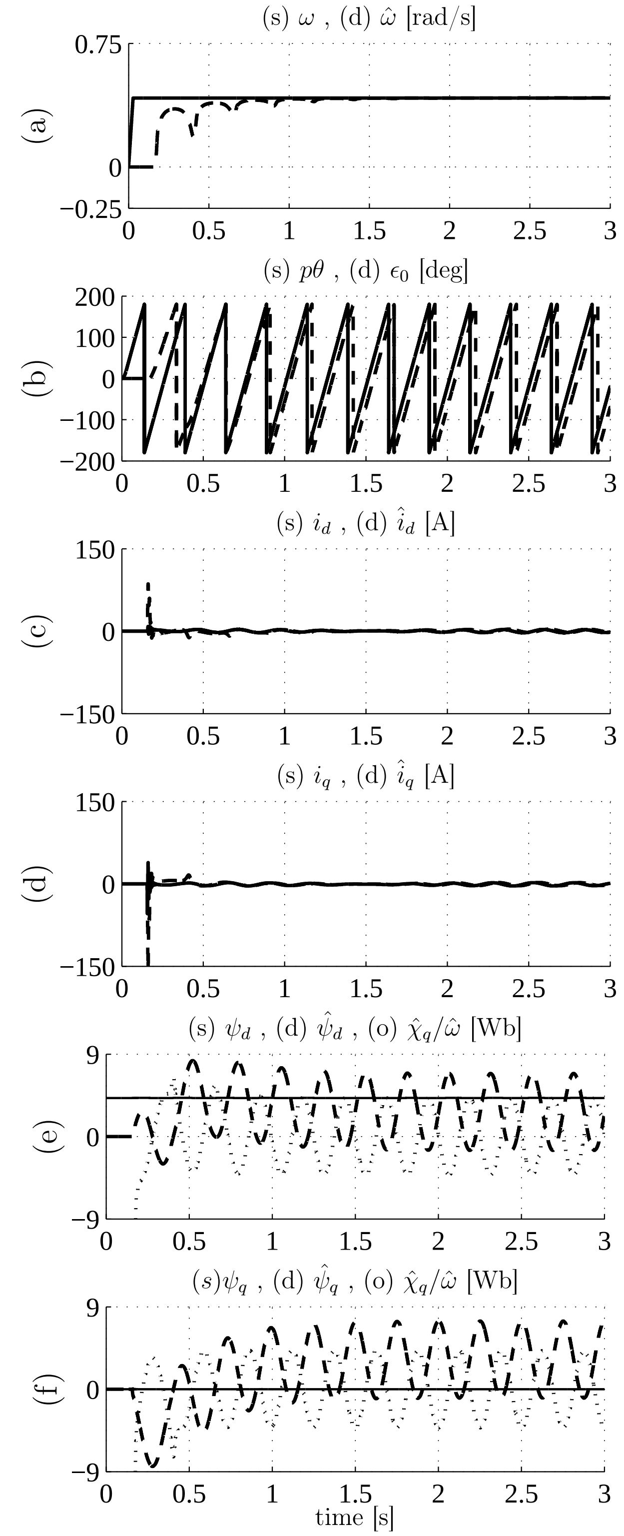

Figure 2.3: (s) solid, (d) dashed , (o) dotted. Convergence of the Observer variables supposing $k_\psi = 0$ and Alignment Controller turn off ($k_A = k_{Ai} = 0$).

Chapter 3

A Synchronous Coordinates Approach Observer for a PMSM

Introduction

This chapter focus on a new observer for a PMSM based on rotor flux dynamics[1].
In the observer proposed, the reference frame used for the observer design is a generic $\hat{d}, \hat{q}$ reference frame with angle $\hat{\theta}$. This reference frame is pushed toward the synchronous one by forcing it to be intrinsically aligned with the estimated *back-emf* vector dependent on rotor flux vector, in other words the generic reference frame $\hat{d}, \hat{q}$ is forced to be aligned with the Field-Oriented d, q reference frame.
The design of suitable adaptation laws allows for the estimation of speed, angle and *back-emf* amplitude. The intrinsic alignment of the reference frame with the rotor flux allows us to drop any form of alignment controller as mentioned in chapter 2.
Stator flux dynamics are not used in this approach, therefore the model based part of the observer will not contain the stator flux (simply) stable dynamics, improving the robustness of the solution adopted with respect to voltage and current measurement uncertainties, an allowing us to drop any form of observer modification to cope with this fact. The stability analysis of the observer is carried out using a singular perturbation framework and using Linear Control and Lyapunov theory.
The chapter is organized as follows, in section 3.1 first the definition of the

[1]The results achieved has been presented at MED 2012, 20^{th} *Mediterranean Conference on Control and Automation, Barcellona (ES).*

model of a PMSM in a generic u, v reference frame is reported, then the same model is described in the so called "field-oriented" d, q reference frame. After that, the Observer is proposed in the estimated field-oriented $\hat{d}, \hat{q}$ reference frame and the development of the Singular Pertubation Framework for the stability proof is reported. Section 3.2 is devoted for the tuning procedure of the Observer, with simulations results reported in the ending section 3.3.

3.1 Observer Design

This novel and simple observer for PMSM has been designed taking the cue from the approach proposed in [14], in which the main idea is to build an observer in a generic reference frame $\hat{d}, \hat{q}$, and imposing for the beck-emf vector a representation equivalent to the one it would have in the so-called Field-Oriented reference frame d, q.
In this way, an implicit alignment of the observer reference frame is forced between the adopted reference frame and the estimated *back-emf* vector, while its amplitude along with the rotor speed and position can be suitably adapted to obtain asymptotic reconstruction of the stator currents.
No pure integration of the stator flux dynamics are exploited, since the stator current dynamics are directly exploited as an indirect measurement of the *back-emf* vector. This leads to an intrinsic robustness to many kinds of voltage and current measurement uncertainties. Time scale separation between the stator current dynamics and the remainder of the observer dynamics is exploited to provide *practical semiglobal asymptotic stability* as will be shown later.
According to standard planar representation of three-phase electric motors, in the following the PMSM electro-magnetic model is reported in a generic 2-phase u, v reference frame rotated by an angle ϵ_0 with respect to a stator winding aligned stationary reference frame

$$\begin{aligned} \dot{i}_u &= -\frac{R}{L} i_u + \omega_0 i_v + \frac{p\omega\phi_v}{L} + \frac{u_u + d_u}{L} \\ \dot{i}_v &= -\frac{R}{L} i_v - \omega_0 i_u - \frac{p\omega\phi_u}{L} + \frac{u_v + d_v}{L} \\ \dot{\phi}_u &= -(p\omega - \omega_0)\phi_v \\ \dot{\phi}_v &= (p\omega - \omega_0)\phi_u \end{aligned} \tag{3.1}$$

Where, $\omega_0 = \dot{\epsilon}_0$ is the angular speed of the arbitrary selected reference frame u, v; p are the pole pairs; R, L are stator winding resistance and inductance; ω is the actual rotor mechanical speed; i_u, i_v are the stator currents; ϕ_u, ϕ_v are the components given by the projection in the considered reference frame of the rotor magnet flux vector, whose amplitude will be indicated as Φ.
In this framework, the angle θ and θ_e, such that $\dot{\theta} = \omega$ and $\dot{\theta}_e = p\omega$, can be used to represent respectively the mechanical and the so-called electrical angle of the rotor magnet flux vector with respect to the stator-aligned stationary reference frame. Finally, $u_u + d_u$ and $u_v + d_v$ give the voltages applied to the stator windings. It is worth noting that these voltages have been represented by the sum of the expected voltages u_u, u_v and unknown signals d_u, d_v. In fact, stator voltages are usually actuated by means of switching power converters (commonly referred as *inverters*) and direct measurements are not available or not accurate as they should be. Therefore, d_u, d_v account for measurement

errors and/or inverter non-idealities (such as Dead-Time effect, IGBT/MOS voltage drop, etc...).
From now on, and with no loss of generality, $p = 1$ will be taken, therefore ω will be directly the so-called electrical rotor speed, and the mechanical angle θ and the electric angle θ_e will be the same.
As it is well known (see for example [1]) defining a reference frame d, q with the d-axis aligned with the rotor flux vector, the *back-emf* component on the d-axis disappears, and the model (3.1) reads as follows

$$\begin{aligned}
\dot{i}_d &= -\frac{R}{L} i_d + \omega i_q + \frac{u_d + d_d}{L} \\
\dot{i}_q &= -\frac{R}{L} i_q - \omega i_d - \frac{\omega \Phi}{L} + \frac{u_q + d_q}{L} \\
\dot{\phi}_d &= 0 \\
\dot{\phi}_q &= 0
\end{aligned} \tag{3.2}$$

With this choice for the reference frame, the speed ω_0 and angle ϵ_0 become exactly the electrical rotor speed ω and the rotor flux vector angle θ. Therefore, this reference frame is usually referred as Field-Oriented reference frame, and d and q stand for *direct* and *quadrature* axis.

In sensorless control of PMSM a fundamental issue is to achieve an estimation of the rotor flux vector angle θ and speed ω, since no direct measurements are available. This goal is crucial to build standard and also some kind of advanced speed-torque controllers based on field-orientation concepts (see for example [1]).
Usually, in model-based observer approaches, speed and position estimation task is performed by defining a suitable observer exploiting the electromagnetic model of the PMSM, while no relevant information is assumed available on the mechanical model, due to the low accuracy of the mechanical model parameters, like inertia and viscous coefficient, and due to the problem arising from a good measurement of the load torque.
On the other hand, the speed dynamics is assumed much slower than the electromagnetic one, therefore the speed is assumed constant (or slowly varying) for the formulation of the above-mentioned estimation problem.
Beside this basic problem, also the estimation of the amplitude of the rotor magnet flux vector is often considered to enable very accurate torque control (the flux amplitude can be time varying because depending on the working temperature [1]).
Bearing in mind these considerations, the following general objectives can be defined for an observer based on the electromagnetic model of PMSM:

1. Guaranteeing estimation of rotor magnet vector position, θ, and speed ω along with its amplitude Φ, under constant speed condition ($\dot{\omega} = 0$),

assuming stator currents and expected stator voltages available from measures and actuations, respectively, and considering null voltage uncertainties (these conditions will be referred as *nominal conditions*).

2. Achieving as large as possible bandwidth in the estimation of the speed ω in order to compensate for the lack of knowledge of the mechanical model and cope with variable speed conditions.

3. Obtaining large voltage disturbances rejection, i.e. attenuation of the d_d, d_q disturbances.

The basic idea comes by imposing in a generic reference frame called $\hat{d}, \hat{q}$, with angle $\hat{\theta}$ and speed $\hat{\omega}$, the model (3.2) which is valid only in the Field-Oriented reference frame, for this purpose feedback estimation laws are designed in order to push the angle and the speed of the observer reference frame toward θ and ω of the Field-Oriented reference frame, therefore, the proposed observer reference frame can be seen as an estimation of the Field-Oriented reference frame.
An additional important step in the line defined above is the coordinate changing defining the *back-emf* components as state variables, therefore defining

$$\begin{aligned} \chi_{\hat{d}} &= \omega\phi_{\hat{d}} \\ \chi_{\hat{q}} &= \omega\phi_{\hat{q}} \end{aligned} \tag{3.3}$$

the synchronous model (3.2) can be revised leading to the following observer model in the $\hat{d}$-$\hat{q}$ reference frame

$$\begin{aligned} \dot{\hat{i}}_{\hat{d}} &= -\frac{R}{L} i_{\hat{d}} + \hat{\omega} i_{\hat{q}} + \frac{u_{\hat{d}}}{L} + \eta_d \\ \dot{\hat{i}}_{\hat{q}} &= -\frac{R}{L} i_{\hat{q}} - \hat{\omega} i_{\hat{d}} - \frac{\hat{A}}{L} + \frac{u_{\hat{q}}}{L} + \eta_q \\ \dot{\hat{A}} &= \nu_a \\ \dot{\hat{\theta}} &= \nu_\omega + \hat{\bar{\omega}} \qquad (\hat{\omega} \equiv \nu_\omega + \hat{\bar{\omega}}) \\ \dot{\hat{\bar{\omega}}} &= \eta_\omega \end{aligned} \tag{3.4}$$

where $i_{\hat{d}}$, $i_{\hat{q}}$ and $u_{\hat{d}}$, $u_{\hat{q}}$ are the stator currents and expected voltages, available from measurements and actuator commands and reported in $\hat{d}$-$\hat{q}$ frame by a trigonometric transformation, the variable $\hat{A}$ is the estimation of the *back-emf* term $\omega\Phi$ in (3.2), while the meaning of $\hat{i}_{\hat{d}}$, $\hat{i}_{\hat{q}}$, $\hat{\theta}$ and $\hat{\omega}$ is straightforward from the considerations already done.
Differently, η_d, η_q, ν_a, η_ω and ν_ω are feedback terms for observer convergence

and defined as follows

$$\begin{aligned}\eta_d &= k_p \tilde{i}_{\hat{d}}\\ \eta_q &= k_p \tilde{i}_{\hat{q}}\\ \nu_a &= -Lk_1 k_p \tilde{i}_{\hat{q}}\\ \eta_\omega &= \gamma \frac{\hat{A}}{Lk_p}\tilde{i}_{\hat{d}}\\ \nu_\omega &= k_2 \frac{\hat{A}}{Lk_p}\tilde{i}_{\hat{d}}\end{aligned} \tag{3.5}$$

where $\tilde{i}_{\hat{d}} = i_{\hat{d}} - \hat{i}_{\hat{d}}$, $\tilde{i}_{\hat{q}} = i_{\hat{q}} - \hat{i}_{\hat{q}}$. It is worth noting that a sort of PI structure has been adopted for the $\hat{\theta}$ estimation, but just $\hat{\omega}$ will be considered as output speed estimation of the proposed observer.
To better understand the observer equations in (3.4) can be useful to specify that the part of the observer devoted to the rotor flux estimation is not described using standard Cartesian coordinates, but it is described in the stator reference frame using Polar coordinates, where A is the amplitude and ω the speed of the real unknown *back-emf* rotating vector, and can be intuitively inferred that ν_a is devoted to the estimation of the amplitude of the rotor flux, while $\nu_\omega + \hat{\omega}$ to the estimation of the rotor flux angle (position). In fact, to describe a pure oscillator, i.e. the rotor flux on the a, b stationary reference frame, we can equivalently use standard Cartesian coordinates obtaining the following model

$$\begin{aligned}\dot{\phi}_a &= -p\omega\phi_b\\ \dot{\phi}_b &= p\omega\phi_a\end{aligned} \tag{3.6}$$

or using Polar coordinates obtaining the following model

$$\begin{aligned}\dot{\Phi} &= 0 \quad , \quad (\Phi = \sqrt{\phi_a^2 + \phi_b^2})\\ \dot{\theta} &= p\omega \quad , \quad (\dot{\omega} = 0)\end{aligned} \tag{3.7}$$

with appropriate initial conditions to have consistent systems.
Finally, recalling the motor model (3.1), substituting u, v with $\hat{d}, \hat{q}$ in it, and reformulating the equations using $\chi_{\hat{d}}, \chi_{\hat{q}}$ defined in (3.3), the PMSM model can be expressed in the $\hat{d}, \hat{q}$ observer reference frame as follows

$$\begin{aligned}\dot{i}_{\hat{d}} &= -\frac{R}{L} i_{\hat{d}} + \hat{\omega} i_{\hat{q}} + \frac{\chi_{\hat{q}}}{L} + \frac{u_{\hat{d}} + d_{\hat{d}}}{L}\\ \dot{i}_{\hat{q}} &= -\frac{R}{L} i_{\hat{q}} - \hat{\omega} i_{\hat{d}} - \frac{\chi_{\hat{d}}}{L} + \frac{u_{\hat{q}} + d_{\hat{q}}}{L}\\ \dot{\chi}_{\hat{d}} &= -(\omega - \hat{\omega})\chi_{\hat{q}} + \frac{\dot{\omega}}{\omega}\chi_{\hat{d}}\\ \dot{\chi}_{\hat{q}} &= \quad (\omega - \hat{\omega})\chi_{\hat{d}} + \frac{\dot{\omega}}{\omega}\chi_{\hat{q}}\end{aligned} \tag{3.8}$$

where $\chi_{\hat{d}}$ $\chi_{\hat{q}}$ enlighten the *back-emf* projections in the considered frame as already said. Variables $d_{\hat{d}}$, $d_{\hat{q}}$ and $\dot{\omega}$ has been reported because in section (3.2), with focus on a parameter tuning procedure, will be evaluated the effect on the observer performances of the voltage uncertainties and non-constant speed conditions.

In the remaining part of this section a stability analysis of the proposed solution is reported. As already mentioned, the stability analysis is carried out assuming nominal conditions defined in Objective 1 at the end of section 3.1, hence the perturbation introduced by the disturbances on the actuated voltages and by non constant rotor speed, appearing in (3.8), will be neglected. These additional input signals will be considered in next section, dedicated to derive the observer gains tuning rules according to Objectives 2-3 defined at the end of section 3.1.
A model to suitably represent the behavior of the observation error can be defined by considering the current errors $\tilde{i}_{\hat{d}}$ and $\tilde{i}_{\hat{q}}$, previously introduced, and adding the following errors variables related to the estimation of the *back-emf* components and speed

$$\tilde{\chi}_{\hat{d}} = \chi_{\hat{d}} - \hat{A} \ , \ \ \tilde{\chi}_{\hat{q}} = \chi_{\hat{q}} \ , \ \ \tilde{\omega} = \omega - \hat{\omega} \tag{3.9}$$

It is worth to note that, looking at the expression of $\tilde{\chi}_{\hat{q}}$ is clear the fact that the estimated value of the *back-emf* on axis q is forced to be 0, introducing the effect of the asymptotic alignment the observer reference frame to the Field-Oriented one.
By subtracting (3.4) from (3.8), neglecting $d_{\hat{d}}, d_{\hat{q}}, \dot{\omega}$, the dynamics of the above defined estimation errors is the following

$$\begin{aligned}
\dot{\tilde{i}}_{\hat{d}} &= -\eta_d + \frac{\tilde{\chi}_{\hat{q}}}{L} \\
\dot{\tilde{i}}_{\hat{q}} &= -\eta_q - \frac{\tilde{\chi}_{\hat{d}}}{L} \\
\dot{\tilde{\chi}}_{\hat{d}} &= -(\tilde{\omega} - \nu_\omega)\tilde{\chi}_{\hat{q}} - \nu_a \\
\dot{\tilde{\chi}}_{\hat{q}} &= (\tilde{\omega} - \nu_\omega)(\tilde{\chi}_{\hat{d}} + \hat{A}) \\
\dot{\tilde{\omega}} &= -\eta_\omega
\end{aligned} \tag{3.10}$$

Exploiting the adaptation laws defined in (3.5), defining $\epsilon = \frac{1}{k_p}$, and using the following change of coordinates

$$\begin{aligned}
\tilde{\chi}_{\hat{d}_1} &= \tilde{\chi}_{\hat{d}}/Lk_p \\
\tilde{\chi}_{\hat{q}_1} &= \tilde{\chi}_{\hat{q}}/Lk_p \\
\nu_{a_1} &= \nu_a/Lk_p \\
\hat{A}_1 &= \hat{A}/Lk_p
\end{aligned} \tag{3.11}$$

the system (3.10) can be expressed as

$$\begin{aligned}
\epsilon\dot{\tilde{i}}_{\hat{d}} &= -\tilde{i}_{\hat{d}} + \tilde{\chi}_{\hat{q}_1} \\
\epsilon\dot{\tilde{i}}_{\hat{q}} &= -\tilde{i}_{\hat{q}} - \tilde{\chi}_{\hat{d}_1} \\
\dot{\tilde{\chi}}_{\hat{d}_1} &= -(\tilde{\omega} - k_2\hat{A}_1\tilde{i}_{\hat{d}})\tilde{\chi}_{\hat{q}_1} + k_1\tilde{i}_{\hat{q}} \\
\dot{\tilde{\chi}}_{\hat{q}_1} &= (\tilde{\omega} - k_2\hat{A}_1\tilde{i}_{\hat{d}})(\tilde{\chi}_{\hat{d}_1} + \hat{A}_1) \\
\dot{\tilde{\omega}} &= -\gamma\hat{A}_1\tilde{i}_{\hat{d}}
\end{aligned} \tag{3.12}$$

This can be easily seen as a standard singular perturbation model [15], where the time scale separation between the current error dynamics and the *back-emf* and speed error dynamics is parametrized by the gain k_p. Therefore, assuming a sufficiently high value of k_p has been chosen (more details will be given in the next section), the problem of the estimates convergence can be approached by considering the overall system as the interconnection of a *fast subsystem*, represented by the current error variables $(\tilde{i}_{\hat{d}}, \tilde{i}_{\hat{q}})$, and a *slow subsystem* given by the other dynamics $(\tilde{\chi}_{\hat{d}_1}, \tilde{\chi}_{\hat{q}_1}, \tilde{\omega})$.
According to [15] and [16], we start by studying the so-called *boundary layer* system related to the fast dynamics. First, define the *quasi steady-state* value for the current errors imposing $\epsilon = 0$ in the fast subsystem as follows

$$\begin{aligned}
\tilde{i}_{\hat{d}} &= \tilde{\chi}_{\hat{q}_1}(t) \\
\tilde{i}_{\hat{q}} &= -\tilde{\chi}_{\hat{d}_1}(t)
\end{aligned} \tag{3.13}$$

Then, defining the following change of coordinates

$$\begin{aligned}
y_d &= \tilde{i}_{\hat{d}} - \tilde{\chi}_{\hat{q}_1} \\
y_q &= \tilde{i}_{\hat{q}} + \tilde{\chi}_{\hat{d}_1} \\
t &= \epsilon\tau
\end{aligned} \tag{3.14}$$

after some computation, consisting in *freezing* the slow varying variables by setting $\epsilon = 0$, the following boundary layer system is obtained

$$\frac{dy_d}{d\tau} = -y_d \ , \ \frac{dy_q}{d\tau} = -y_q \tag{3.15}$$

It's trivial to verify that the origin of (3.15) is globally exponentially stable, uniformly in both the slow variables and the time, since it is a LTI system with Hurwitz state matrix. Note that the quasi steady-state definition enlightens how the current errors can be used as indirect measure of the *back-emf* estimation errors, thanks to time scale separation imposed by k_p.
Again according to [15] and [16], we now put the focus on the reduced dynamics obtained by substituting the fast variables $i_{\hat{d}}$, $i_{\hat{q}}$ with their quasi steady-state

approximation, $\tilde{i}_{\hat{d}} = \tilde{\chi}_{\hat{q}1}(t)$, $\tilde{i}_{\hat{q}} = -\tilde{\chi}_{\hat{d}_1}(t)$, in the slow dynamics given by the last three equations in (3.12). After some computation the following reduced system results

$$\begin{aligned} \dot{\tilde{\chi}}_{\hat{d}_1} &= -(\tilde{\omega} - k_2\hat{A}_1\tilde{\chi}_{\hat{q}1})\tilde{\chi}_{\hat{q}1} - k_1\tilde{\chi}_{\hat{d}_1} \\ \dot{\tilde{\chi}}_{\hat{q}1} &= (\tilde{\omega} - k_2\hat{A}_1\tilde{\chi}_{\hat{q}1})(\tilde{\chi}_{\hat{d}_1} + \hat{A}_1) \\ \dot{\tilde{\omega}} &= -\gamma\hat{A}_1\tilde{\chi}_{\hat{q}1} \end{aligned} \tag{3.16}$$

It's worth noting that, the system is non-autonomous for the presence of the estimate of the *back-emf* $\hat{A}_1$, and this is a time dependent parameter independently from the constant speed hypothesis, hence uniformity for the following stability statement is not straightforward. To investigate the stability of (3.16) consider the following Lyapunov candidate function

$$V = \frac{1}{2}(\tilde{\chi}_{\hat{d}_1}^2 + \tilde{\chi}_{\hat{q}1}^2 + \frac{\tilde{\omega}^2}{\gamma}) \tag{3.17}$$

The following derivative along the system trajectories can be obtained

$$\dot{V} = -k_1\tilde{\chi}_{\hat{d}_1}^2 - k_2\hat{A}_1^2\tilde{\chi}_{\hat{q}1}^2 \leq 0 \quad (\forall k_1 > 0, \forall k_2 > 0) \tag{3.18}$$

From direct application of Barbalat's lemma ([15]) to V, it can be stated that

$$\lim_{t\to\infty} \dot{V} = 0 \quad \Rightarrow \quad \lim_{t\to\infty} \tilde{\chi}_{\hat{d}_1} = 0 \quad , \quad \lim_{t\to\infty} \tilde{\chi}_{\hat{q}1} = 0 \tag{3.19}$$

Applying the same Lemma to $\tilde{\chi}_{\hat{d}_1}$ and $\tilde{\chi}_{\hat{q}1}$, can be stated also that

$$\lim_{t\to\infty} \dot{\tilde{\chi}}_{\hat{d}_1} = 0 \quad , \quad \lim_{t\to\infty} \dot{\tilde{\chi}}_{\hat{q}1} = 0 \tag{3.20}$$

For the application of Barbalat's lemma to $\tilde{\chi}_{\hat{d}_1}$ and $\tilde{\chi}_{\hat{q}1}$, the following conditions are necessary:

1. $\tilde{\chi}_{\hat{d}_1}, \tilde{\chi}_{\hat{q}1}$ must have a finite limit;
2. $\dot{\tilde{\chi}}_{\hat{d}_1}, \dot{\tilde{\chi}}_{\hat{q}1}$ must exist and must be uniformly continuous;

First condition comes directly from (3.19), i.e. the limits exist and are 0 for both variables. Condition 2) can be fulfilled exploiting the following property

$$\ddot{\tilde{\chi}}_{\hat{d}_1}, \ddot{\tilde{\chi}}_{\hat{q}1} \text{bounded} \Rightarrow 2)$$

To have $\ddot{\tilde{\chi}}_{\hat{d}_1}, \ddot{\tilde{\chi}}_{\hat{q}1}$ bounded the following are needed:

- The Lyapunov candidate function V is lower bounded ($V \geq 0$) and not increasing ($\dot{V} \leq 0$) and these (sufficient) conditions ensure that $\tilde{\chi}_{\hat{d}_1}, \tilde{\chi}_{\hat{q}1}, \tilde{\omega}$ are bounded, and as a consequence also $\dot{\tilde{\omega}}, \dot{\hat{A}}, \dot{\hat{A}}_1$ are bounded;

- Supposing $Lk_p \neq 0$, and by the fact that if ω is bounded also $\chi_{\hat{d}}, \chi_{\hat{q}}$ are bounded, then also $\hat{A}, \hat{A}_1$ are bounded;

- $\dot{\tilde{\chi}}_{\hat{d}_1}, \dot{\tilde{\chi}}_{\hat{q}_1}$ are bounded because sum of bounded terms;

- $\ddot{\tilde{\chi}}_{\hat{d}_1}, \ddot{\tilde{\chi}}_{\hat{q}_1}$ are bounded because sum of bounded terms;

Therefore, the origin of the reduced dynamics is globally asymptotically stable. From the previous considerations and using the singular perturbation results as formulated in [16] (which actually covers also averaging and ISS analysis), the following proposition defining the properties for the overall error dynamics (3.12) can be drawn.

Proposition 1 *For the system (3.12), replacing for simplicity current coordinates, $\tilde{i}_{\hat{d}}$, $\tilde{i}_{\hat{q}}$, with the above defined $y_d = \tilde{i}_{\hat{d}} - \tilde{\chi}_{\hat{q}_1}$, $y_q = \tilde{i}_{\hat{q}} + \tilde{\chi}_{\hat{d}_1}$, there exist two class $\mathcal{KL}$ functions β_f and β_s such that, for each $\delta > 0$ and for every compact sets $\Omega_f \subset \mathbb{R}^2$ and $\Omega_s \subset \mathbb{R}^3$, there exists ϵ^* such that $\forall \epsilon = k_p^{-1} \in (0, \epsilon^*]$, the following inequalities hold*

$$\begin{aligned} \|[y_d(t), y_q(t)]^T\| \leq & \beta_f \left(\|[y_d(0), y_q(0)]^T\|, t/\epsilon\right) + \delta \\ & \forall [y_d(0), y_q(0)]^T \in \Omega_f \end{aligned} \tag{3.21}$$

$$\begin{aligned} \|[\tilde{\chi}_{\hat{d}_1}(t), \tilde{\chi}_{\hat{q}_1}(t), \tilde{\omega}(t)]^T\| \leq & \beta_s \left(\|[\tilde{\chi}_{\hat{d}_1}(0), \tilde{\chi}_{\hat{q}_1}(0), \tilde{\omega}(0)]^T\|, t\right) + \delta \\ & \forall [\tilde{\chi}_{\hat{d}_1}(0), \tilde{\chi}_{\hat{q}_1}(0), \tilde{\omega}(0)]^T \in \Omega_s \end{aligned} \tag{3.22}$$

Hence, *semiglobal practical stability* can be stated for the overall error dynamics (3.12), provided that a sufficiently large k_p has been selected.

3.2 Parameter Tuning Rules

In this section the tuning rules are defined according to time scale separation requirements derived during the observer stability analysis and the general objectives defined in section 3.1. A preliminary step toward this goal is to rewrite the error dynamics (3.10) taking into account the voltage disturbances and the perturbation given by non-constant speed as follows

$$\begin{aligned}
\dot{\tilde{i}}_{\hat{d}} &= k_p(-\tilde{i}_{\hat{d}} + \tilde{\chi}_{\hat{q}_1} + d_{\hat{d}_1}) \\
\dot{\tilde{i}}_{\hat{q}} &= -k_p(\tilde{i}_{\hat{q}} + \tilde{\chi}_{\hat{d}_1} + d_{\hat{q}_1}) \\
\dot{\tilde{\chi}}_{\hat{d}_1} &= -(\tilde{\omega} - \nu_\omega)\tilde{\chi}_{\hat{q}_1} - \nu_{a_1} + \frac{\dot{\omega}}{\omega}(\tilde{\chi}_{\hat{d}_1} + \hat{A}_1) \\
\dot{\tilde{\chi}}_{\hat{q}_1} &= (\tilde{\omega} - \nu_\omega)(\tilde{\chi}_{\hat{d}_1} + \hat{A}_1) + \frac{\dot{\omega}}{\omega}\tilde{\chi}_{\hat{q}_1} \\
\dot{\tilde{\omega}} &= -\eta_\omega + \dot{\omega}
\end{aligned} \tag{3.23}$$

Where the following definition has been used $d_{\hat{d}_1} = d_{\hat{d}}/Lk_p, d_{\hat{q}_1} = d_{\hat{q}}/Lk_p$.
The origin of the system, $[\tilde{\chi}_{\hat{d}_1}\ \tilde{\chi}_{\hat{q}_1}\ \tilde{\omega}] = 0$, is an equilibrium point, so linearizing the system near the origin and defining $\Phi_1 = \Phi/Lk_p$, we obtain the following LTI system

$$\begin{aligned}
\dot{\tilde{i}}_{\hat{d}} &= k_p(-\tilde{i}_{\hat{d}} + \tilde{\chi}_{\hat{q}_1} + d_{\hat{d}_1}) \\
\dot{\tilde{i}}_{\hat{q}} &= k_p(-\tilde{i}_{\hat{q}} - \tilde{\chi}_{\hat{d}_1} + d_{\hat{q}_1}) \\
\dot{\tilde{\chi}}_{\hat{d}_1} &= -\nu_{a_1} + \dot{\omega}\Phi_1 \\
\dot{\tilde{\chi}}_{\hat{q}_1} &= (\tilde{\omega} - \nu_\omega)\Phi_1\omega \\
\dot{\tilde{\omega}} &= -\eta_\omega + \dot{\omega}
\end{aligned} \tag{3.24}$$

The input $\dot{\omega}$ acts on $\dot{\tilde{\omega}}$ and $\dot{\tilde{\chi}}_{\hat{d}_1}$ and its presence is useful for the evaluation of the sensitivity of the error variables with respect to variable speed, i.e. to evaluate the observer bandwidth. Other inputs in the error dynamics (3.24) are the voltage disturbances $d_{\hat{d}}, d_{\hat{q}}$, and these variables allow to evaluate the disturbance rejection.
Applying to the error system (3.24) the results deriving from singular perturbation properties enlightened in subsection 3.1, the following quasi-steady state equations can be considered

$$-\tilde{i}_{\hat{d}} + \tilde{\chi}_{\hat{q}_1} + d_{\hat{d}_1} \approx 0 \qquad , \qquad -\tilde{i}_q - \tilde{\chi}_{d_1} + d_{\hat{q}_1} \approx 0 \tag{3.25}$$

From the equation (3.25) can be clearly seen that disturbance voltages propagates now up to the current errors, corrupting the measure of the *back-emf*. Thus, the reduced error dynamics can be linearized as follows

$$\begin{aligned}
\dot{\tilde{\chi}}_{\hat{d}_1} &= k_1(-\tilde{\chi}_{\hat{d}_1} + d_{\hat{q}_1}) + \dot{\omega}\Phi_1 \\
\dot{\tilde{\chi}}_{\hat{q}_1} &= \omega\Phi_1\tilde{\omega} - k_2(\omega\Phi_1)^2\left(\tilde{\chi}_{\hat{q}_1} + d_{\hat{d}_1}\right) \\
\dot{\tilde{\omega}} &= -\gamma\omega\Phi_1\left(\tilde{\chi}_{\hat{q}_1} + d_{\hat{d}_1}\right) + \dot{\omega}
\end{aligned} \tag{3.26}$$

The linear system derived has the following state matrix A_R and input matrix B_R with respect to the input vector $[d_{\hat{d}}\ d_{\hat{q}}\ \dot{\omega}]^T$

$$A_R = \begin{bmatrix} -k_1 & 0 & 0 \\ 0 & -k_2(\Phi_1\omega)^2 & \Phi_1\omega \\ 0 & -\gamma\hat{A}_1 & 0 \end{bmatrix},\ B_R = \frac{1}{Lk_p}\begin{bmatrix} 0 & k_1 & \Phi \\ -k_2(\omega\Phi_1)^2 & 0 & 0 \\ -\gamma\omega\Phi_1 & 0 & Lk_p \end{bmatrix} \tag{3.27}$$

State matrix A_R in (3.27) has the following eigenvalues

$$\lambda_1 = -k_1\ ,\ \lambda_{2,3} = \frac{k_2(\omega\Phi_1)^2}{2}\left[-1 \pm \sqrt{1 - \frac{4\gamma}{k_2^2(\omega\Phi_1)^2}}\right] \tag{3.28}$$

It is possible to find the value of k_2, γ to impose the damping coefficient δ and the angular natural frequency ω_n for the eigenvalues $\lambda_{2,3}$ using the following equations

$$k_2 = \frac{2\omega_n\delta}{(\omega\Phi_1)^2}\quad ,\quad \gamma = \frac{1}{(\omega\Phi_1)^2}\left[\omega_n^2(1-\delta^2) + \frac{k_2^2(\omega\Phi_1)^4}{4}\right] \tag{3.29}$$

With these formulas at hand, and bearing in mind the introduction of this section, the tuning parameter must be chosen to cope with:

1. **Frequency separation**, between fast dynamics of $\tilde{i}_{\hat{d}_1}$, $\tilde{i}_{\hat{q}_1}$ and slow dynamics of $\tilde{\chi}_{\hat{d}_1}$, $\tilde{\chi}_{\hat{q}_1}$, $\tilde{\omega}$.

2. **High Bandwidth Observer**, for good estimation during speed variations, i.e. low sensitivity to $\dot{\omega}$.

3. **Disturbance rejection** for high robustness to common disturbances due to Inverter non-idealities, i.e. low sensitivity to $d_{\hat{d}}$, $d_{\hat{q}}$.

Obviously, frequency separation can be obtained choosing large k_p. The upper bound for this parameter is usually related to the common discrete time realization of the observer. In fact k_p represents the bandwidth for the current $\hat{i}_{\hat{d}}$, $\hat{i}_{\hat{q}}$ reconstruction.
High observer bandwidth can be obtained acting on k_1, k_2 and γ but, actually, this is in contrast with disturbance rejection requirement.
First of all, a good practice is to chose k_1, k_2 and γ such that they identify three distinct eigenvalues for the matrix A_R, to avoid ill conditioned problem. Parameter k_1 is related only on the bandwidth of the $\tilde{\chi}_{\hat{d}_1}$ dynamic. Its value must be chosen to be lower than the faster dynamic imposed by k_p (e.g. $k_1 = k_p/50$), recalling that a low value for this parameter produces a low sensitivity to $\dot{\omega}$.
Parameter k_2 and γ can be chosen to impose damping coefficient δ and natural frequency ω_n of eigenvalues $\lambda_{2,3}$ of the reduced order system, as reported in

(3.29). For the value of ω_n, the same considerations as for k_1 hold, i.e. ω_n must be lower than the fast dynamic imposed by k_p (e.g. $w_n = k_p/80$), but not too low to not compromise the observer bandwidth.
The damping of the eigenvalues $\lambda_{2,3}$ ($\delta < 1$) can be chosen to lightly augment the frequency of the eigenvalue related to it, but its major effect is to introduce a resonant frequency behavior, giving low disturbance rejection for a particular disturbance band frequency.
For what concerns the disturbance rejection, a preliminary task is the identification of the disturbance band frequency. Inverter non-idealities introduce voltage disturbances with frequencies n-times the actual electrical frequency, and from practical experiments for the main disturbance component $n = 6$. The worst case is when the rotor rotates at low speed, when also the electrical speed and disturbances are at low frequency, and because the observer has as usual a low-pass filter behavior, so it has a relatively high gain at low frequency.

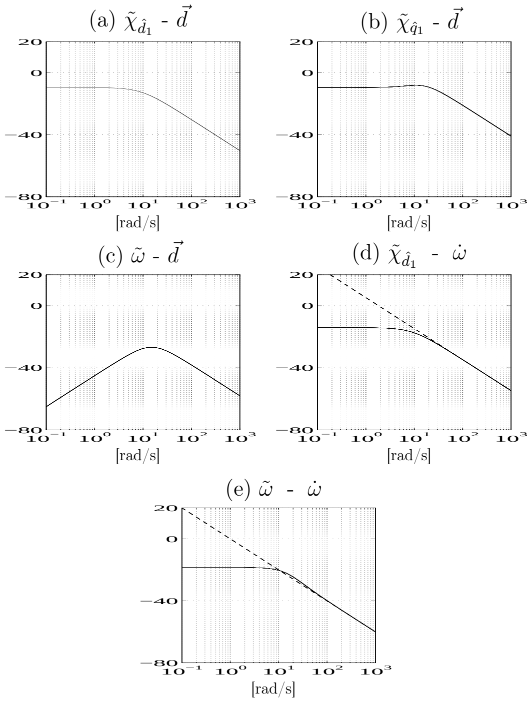

Figure 3.1: $\vec{d}$-Sensitivity Bode diagrams (a,b,c) ($\vec{d} = [d_{\hat{d}}\ d_{\hat{q}}]^T$) and $\dot{\omega}$-Sensitivity Bode diagrams (d,e).

To give an example of a realistic parameter tuning process following the rules mentioned above, the data of a realistic set-up reported in the second column of Tab.2.1 has been taken (these data will be exploited later for simulations). Supposing for the motor a speed range of $[33 - 70]$[rad/s], the main distur-

bances frequency is about $[200 - 420]$[rad/s]. Recalling [14] for the analysis of sensitivity to disturbances for the linearized system (3.27), it is possible to verify that the sinusoidal voltage disturbances can be taken counter-rotating or rotating equivalently, thanks to the structure of the couple A_R, B_R. Three transfer functions must be drawn, each for one of the error variables $\tilde{\chi}_{\hat{d}_1}$,$\tilde{\chi}_{\hat{q}_1}$, $\tilde{\omega}$ of the reduced system (3.27).
To have a good static disturbance rejection k_p can be set as 900, higher values for this parameter are not allowed due to its discrete time realization ($f_s = 2.5$[kHz]). The parameter k_1 has been chosen as 10, i.e. $k_1 \approx k_p/100$, and with $k_2 = 0.0075$ and $\gamma = 0.0627$ we obtain $\delta = 0.9$ and $\omega_n = 15$ supposing a rotor speed of 33[rad/s]. With these parameters, the three Bode diagrams of the sensitivity transfer functions related to voltage distrurbances and the two diagrams for evaluating the observer bandwidth are reported in Fig.3.2.
From plots (a), (b) and (c), it can be noted that, in the disturbances frequency band, we have a minimum attenuation of -36dB for $\tilde{\chi}_{\hat{d}_1}$, -27dB for $\tilde{\chi}_{\hat{q}_1}$ and -44dB for $\tilde{\omega}$.
To better evaluate the observer bandwidth, in plots (d) and (e) of Fig.3.2 the transfer function obtained with no feedback actions in the observer has been added. That is Φ_1/s for the Bode diagram of $\tilde{\chi}_{\hat{d}_1}$ - $\dot{\omega}$, and $1/s$ for the $\tilde{\omega}$ - $\dot{\omega}$. From these plots a bandwidth of 10[rad/s] can be inferred.

3.3 Simulation Results

In this section the performances of the proposed observer are simulated in a realistic set-up: with noise on current measures, inverter non-idealities, and adopting a discrete time version of the observer ($f_s = 2.5$[kHz]). The observer is used as the core of a sensorless torque controller designed on the $\hat{d}, \hat{q}$ reference frame. For this realistic set-up, a more accurate model of the electrical converter has been simulated, introducing PWM technique ($f_{\mathrm{PWM}} = 2.5$[kHz]) effects and inverter non-idealities (dead-times, solid state devices voltage drop and turn ON/OFF time delays), i.e. to introduce the disturbance voltages d_d, d_q, in order to validate the robustness of the estimator with respect to the most common disturbances in practical applications. Simulations of the scenario are performed using machine parameters reported in Tab.2.1 reported in section 2.2. The observer parameters, tuned according to linear analysis discussed in 3.2, are: $k_p = 928.8$, $k_1 = 9.29$, $k_2 = 0.0058$, $\gamma = 0.0375$. For this simulation scenario an estimate for the initial rotor speed is known, hence a good estimate for the initial value of the rotor speed itself ($\hat{\omega}$) and *bemf* ($\hat{A}$) are known.
Fig.3.2 allows for the performance evaluation of the proposed observer in a

torque tracking control scheme during a typical torque-speed profile for wind-turbine applications. The observer is turn on with a non null rotor speed, after a brief transient time the speed and the torque request augment reaching their respective nominal values. To better evaluate the observer performance some error variables are reported, speed error (plot b), torque error (plot d) and angle error (plot e).
It is worth noting that on plot (d), the torque error is due to the use of the rotor flux estimate ($\hat{\phi} = \hat{A}/\hat{\omega}$) for the transformation of the torque set-point to the current set-point, and when inverter non-idealities are present the rotor flux estimate is corrupted by the dc-component of the disturbance voltages.

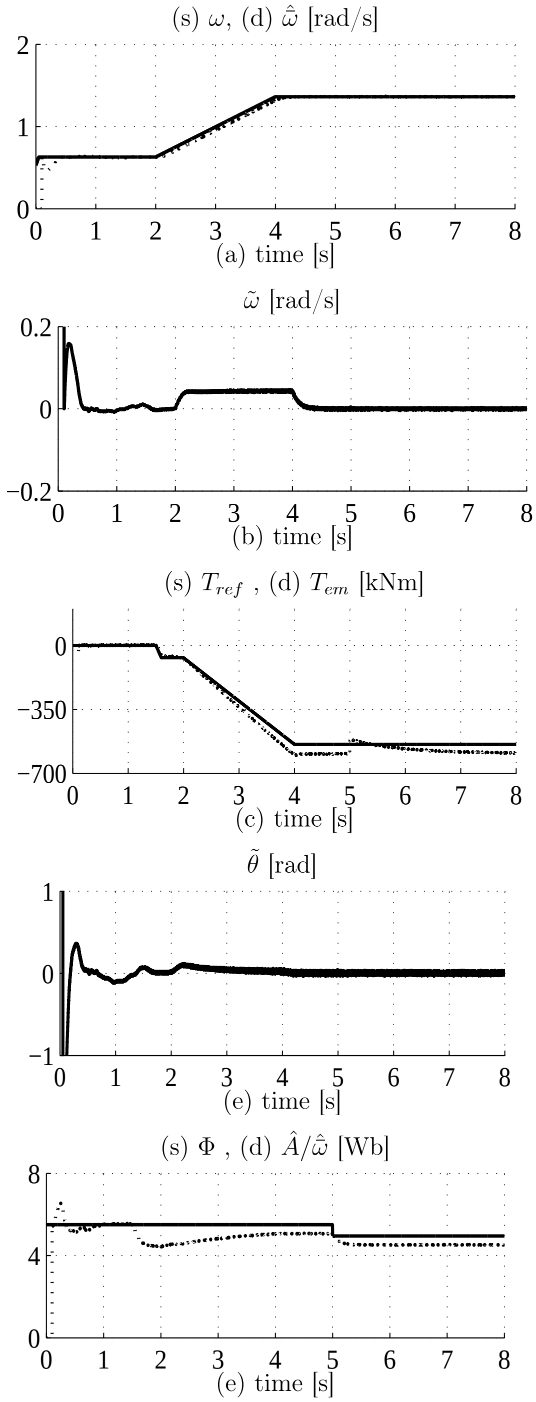

Figure 3.2: (s) solid, (d) dashed. Torque tracking performance in a carachteristic profile for a wind turbine application (with simulation of Dead-Time).

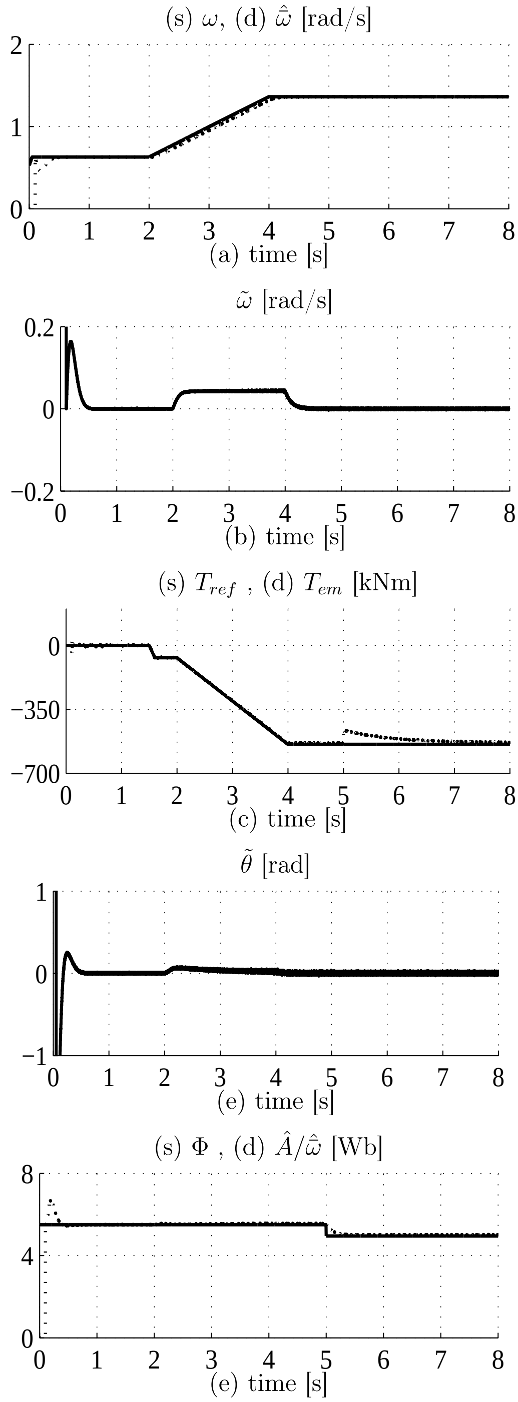

Figure 3.3: (s) solid, (d) dashed. Torque tracking performance in a carachteristic profile for a wind turbine application (with no Dead-Time simulation).

Conclusions

In this thesis, the industrial application of control a Permanent Magnet Synchronous Motor in a sensorless configuration (without encoder measure) has been faced, and in particular the task of estimating the unknown "parameters" necessary for the application of standard motor control algorithms. In literature several techniques have been proposed to cope with this task, but in this thesis only the techniques based on model-based nonlinear observer has been followed.

Model-based observers give the possibility to work with the physical nature of the motor, working on the estimation of its electromagnetic variables and parameters.

The hypothesis of neglecting the mechanical dynamics from the motor model has been applied due to practical and physical considerations deriving from the industrial field, in fact in these applications parameters and/or variables describing the mechanical motor model are unknown, or known with a very small precision, therefore only the electromagnetic dynamics has been used for the design of the observers, that are described by the motor voltage equations, supposing to know the stator currents (stator current sensors), and to know the effective stator voltages (no inverter nonlinearities).

First observer proposed (Chapter 2) is based on stator currents and *Stator Flux* dynamics described in a generic rotating reference frame. Stator current dynamics are dynamics that can be estimated and measured, so their estimation errors can be fed-back in all the observer dynamics, and this is a common feature also for the second observer proposed. Stator flux dynamics are known apart their initial conditions which are estimated, with the speed that is also unknown, through the use of the standard *Adaptive Theory*. Stability results are global and asymptotic, without exponential properties which can be just local results, as expected in the framework of the adaptive theory, for which the asymptotic behavior is guaranteed thanks to the persistence of excitation of the system, related to how the motor is used.

The stator flux dynamics is a pure oscillatory stable dynamic, with the known problems of estimated parameter drifting due to offset measures and parameter uncertainties, therefore practical and effective modifications has been proposed

to face with this problems.
The second observer proposed (Chapter 3) is based on stator currents and *Rotor Flux* dynamics described in a generic reference frame with the property of auto-alignment. These last dynamics are described in the stationary reference frame exploiting polar coordinates instead of classical Cartesian coordinates, by means the estimation of amplitude and speed of the rotor flux. The stability proof has been designed in a *Singular Perturbation* Framework, which allows for the use the current estimation errors as a measure of rotor flux estimation errors, rotor flux that is otherwise unmeasurable. Singular perturbation framework is a sort of frequency separation principle, on which certain variables (estimated current variables) are faster than other variables (estimated rotor fluxes and speed), and the overall system can be divided in two (or more) subsystems. The stability properties has been derived using a specific theory for systems with time scale separation, which guarantees for the overall system a semi-global practical stability.
It is important to remark that differently from the first observer, for which the observer reference frame alignment is obtained by means of an additive active controller called *alignment controller*, for the second observer the reference frame is aligned to the rotor permanent magnet automatically, without utilizing an additive active device.
For the two observer ideal simulations and real simulations have been performed to prove the effectiveness of the observers proposed, real simulations on which the effects of the Inverter nonlinearities have been introduced, showing the already known problems of the model-based observers for low speed applications, on which the signal to noise ratio is particularly small.

Appendix A

Modeling of Inverter Nonlinearities

Introduction

In this appendix a description of the inverter nonlinearities will be given, to allow for the simulation of these effects on inverter based schemes, that are generally all those schemes with DC or AC electric power control realized by means of power electronic devices, e.g. photo voltaic DC energy conversion, wind energy conversion trough control of electrical motor control, etc...
The inverter non linearities effects are important because they introduce in the system a corrupting noise on the voltages applied to the load, this is equivalent to say that the effective voltages applied to the load are not the expected command voltages coming out from the control algorithm, e.g. in a PMSM control scheme the command voltages are the stator voltages, hence with the inverter non linearities the stator voltages are corrupted and using the command values instead of the real ones introduce a simplification that comes untrue as far as the applied voltages are small. Therefore, when the applied voltages are small, the effect of inverter non linearities cannot be neglected and the signal-noise ratio increase, this is the reason why adaptive observers schemes proposed in chapters 2 and 3 fail at low speed.
Beside the simulation of the inverter nonlinearities modeling these nonlinearities gives the opportunities to understand their effects on the control system and gives the opportunity to realize algorithms for their compensation, with open loop or closed loop schemes.

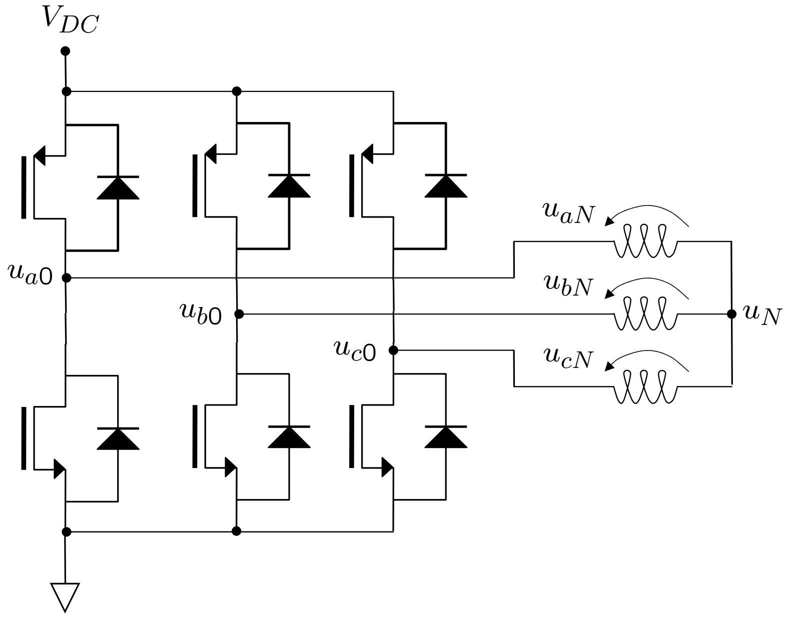

Figure A.1: 3-phase inverter with a pure inductive load.

A.1 Inverter Nonlinearities

A typical application based on Inverter is schematically reported in Figure A.1. In this scheme the power electronic core of the inverter is realized by means of MOS and free-wheeling Diodes, while the load is depicted as a pure inductive load, of course instead of MOS as active device other power electronic devices can be used, i.e. IGBT, PNP/NPN transistor, etc..., each one used for a specific application, but for our modeling purpose the choice of the active device is not important because only the main common effects of these devices will be considered, so for simplicity from now on it will be referred to the MOS as a general active device.
The inverter nonlinearities modeled are:

- Voltage drop on the MOS and on the free-wheeling Diode, from now on for brevity called Diode.
- Switch ON/OFF time delay
- Dead-Times effects

Referring to the voltage drop on the MOS/Diode, the overall effect can be modeled with a constant term, V_S and V_D respectively, plus a linear term proportional with the flowing current ($\prec RI$), but for our modeling purposes the linear term can be neglected due to the fact that these electronic devices

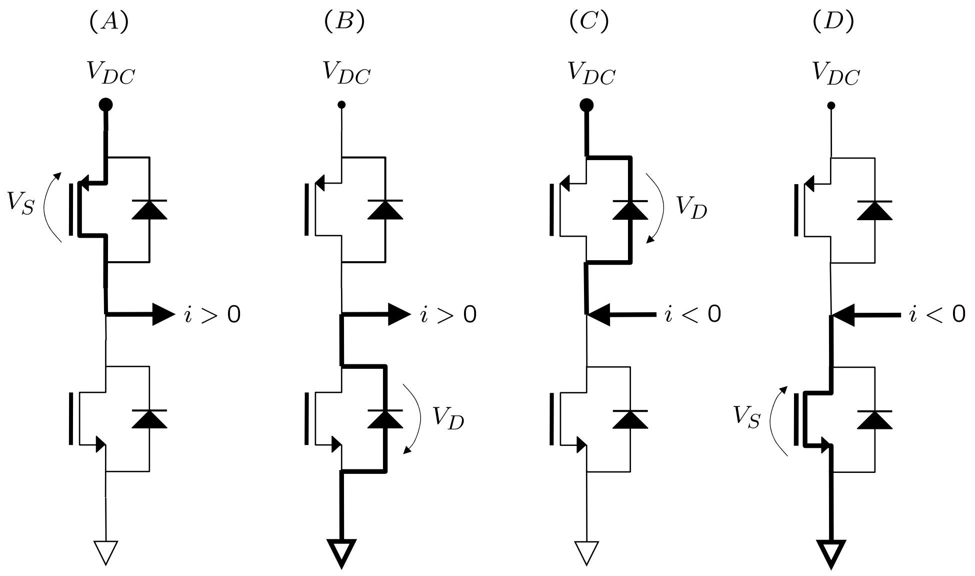

Figure A.2: Switch ON/OFF phases of an Inverter.

have a very low resistance.

As depicted in Figure A.2, for each leg of the Inverter, and depending on the current sign ($i > 0$: current flowing to the load, $i < 0$: current flowing to the Inverter), the following four 4 switch ON/OFF configurations arise:

A) When the high-side command of the inverter is ON and a positive current ($i > 0$) is present, the high-side MOS is ON and the current is flowing on it. Therefore, the voltage applied to the load is $V_{DC} - V_S$ instead of V_{DC}.

B) When the low-side command of the inverter is ON and a positive current ($i > 0$) is present, the low-side Diode is ON and the current is flowing on it. This configuration is active also during Dead-Time with the same sign of the current. Therefore, the voltage applied to the load is $-V_D$ instead of 0.

C) When the high-side command of the inverter is ON and a negative current ($i < 0$) is present, the high-side Diode is ON and the current is flowing on it. This configuration is active also during Dead-Time with the same sign of the current. Therefore, the voltage applied to the load is $V_{DC}+V_D$ instead of V_{DC}.

D) When the low-side command of the inverter is ON and a negative current ($i < 0$) is present, the low-side MOS is ON and the current is flowing on it. Therefore, the voltage applied to the load is $+V_S$ instead of 0.

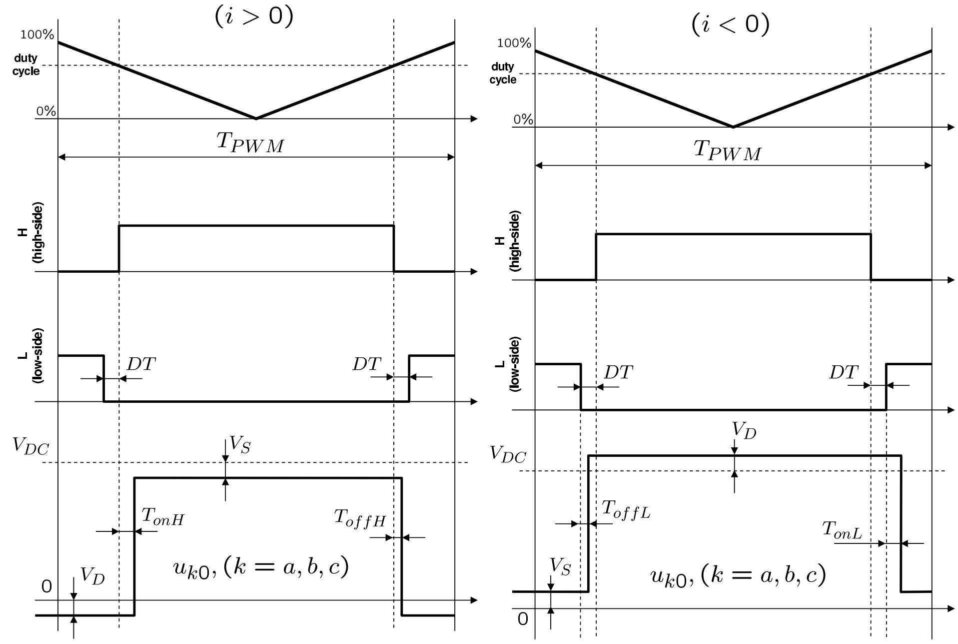

Figure A.3: High-side and low-side commands during a PWM period along with current sign.

Aside the voltage applied to the load as explained before, the following two effects are modeled:

- Dead-Time (DT), that is the time on which both the high-side and the low-side command are OFF to give to the MOS the opportunity of effectively turn OFF before to switch on the other MOS and to prevent the so called *leg short-circuit*. During this phase, as explained above, due to the fact that the current is a state variable, and therefore it cannot be discontinuous, it is flowing in the free-wheeling Diodes, which Diode is determined by the current sign.

- Turn OFF/ON time delays ($T_{onH}, T_{offH}, T_{onL}, T_{offL}$), that are the amount of time which the MOS (H: high-side, L: low-side) take to effectively turn OFF/ON, e.g. supposing the high-side MOS is ON and its switch OFF is commanded, the MOS does not immediately turn OFF, but it takes an amount of time to stop the current flowing on it, i.e. T_{offH}.

It is important to remark that Dead-time is introduced to take account for the four turn ON/OFF switching times, and of course Dead-time must be greater than the maximum between the turn ON and the turn OFF time. The Figure A.3 is reported to show how DT and $T_{onH}, T_{onL}, T_{offH}, T_{offL}$ interact during a period of a PWM (Pulse-Width Modulation), supposing to have the DT

contribute inserted just on the low-side commands.
Furthermore, given a desired duty-cycle ($\rho = u^{sp}/V_{DC}$) which correspond to a desired average voltage (u^{sp}) over a single PWM period, the effective average voltages applied to the load, depending on the current sign, are the following:

$$\textbf{(i > 0)}$$

$$\begin{gathered}
\tau_{on} = \left(\rho - \frac{T_{onH} - T_{offH}}{T_{PWM}}\right) T_{PWM} \quad , \quad \tau_{off} = T_{PWM} - \tau_{on} \\
u = \frac{\tau_{on}}{T_{PWM}} (V_{DC} - V_S) + \frac{\tau_{off}}{T_{PWM}} (-V_D) = u^{sp} - \delta u^+ \\
u^{sp} = \rho V_{DC} \\
\delta u^+ = \frac{T_{onH} - T_{offH}}{T_{PWM}} V_{DC} + \frac{\tau_{on} V_S + \tau_{off} V_D}{T_{PWM}}
\end{gathered} \tag{A.1}$$

$$\textbf{(i < 0)}$$

$$\begin{gathered}
\tau_{on} = \left(\rho + \frac{2DT - T_{offL} + T_{onL}}{T_{PWM}}\right) T_{PWM} \quad , \quad \tau_{off} = T_{PWM} - \tau_{on} \\
u = \frac{\tau_{on}}{T_{PWM}} (V_{DC} + V_D) + \frac{\tau_{off}}{T_{PWM}} (+V_S) = u^{sp} + \delta u^- \\
u^{sp} = \rho V_{DC} \\
\delta u^- = +\frac{2DT + T_{onL} - T_{offL}}{T_{PWM}} V_{DC} + \frac{\tau_{on} V_S + \tau_{off} V_D}{T_{PWM}}
\end{gathered} \tag{A.2}$$

The equations (A.1)(A.2) are the command voltage for each phase of the inverter and are referred to the negative of the DC-link, therefore for each phase k of the inverter the following voltage equation can be written as follows

$$u_{k0} = u^*_{k0} - \delta u_{k0} = u^*_{k0} - \begin{cases} +\delta u^+ & (i_k > 0) \\ -\delta u^- & (i_k < 0) \end{cases} \tag{A.3}$$

Furthermore, supposing to have a 3-phase system, it is possible to describe the effect of the voltage noise due to the inverter nonlinearities on the star center instead of on the negative of the DC-Link, and to do this it is necessary to define the star center voltage of a 3-phase system, that is as follow

$$u_N = \frac{1}{3} \sum_{k=a,b,c} u_{k0} = \frac{1}{3} \sum_{k=a,b,c} u^*_{k0} - \frac{1}{3} \sum_{k=a,b,c} \delta u_{k0} \tag{A.4}$$

Due to the fact that, if the three currents are not null at the same time, is not possible that all the three currents (i_a, i_b, i_c) have the same sign, the noise on the star center is quantized with the following two values

$$\delta u_N = \frac{1}{3} \sum_{k=a,b,c} \delta u_{k0} = \left\{ \frac{1}{3}(2\delta u^+ - \delta u^-), \frac{1}{3}(\delta u^+ - 2\delta u^-) \right\} \tag{A.5}$$

Therefore, the noisy winding voltages can be expressed as follows

$$
\begin{aligned}
u_{kN} &= u_{k0} - u_N = ... = u^*_{kN} - \delta_{wk} \quad , (k = a, b, c) \\
u^*_{kN} &= u^*_{k0} - \frac{1}{3} \sum_{k=a,b,c} u^*_{k0} \\
\delta_{wk} &= \delta u_{k0} - \frac{1}{3} \sum_{k=a,b,c} \delta u_{k0}
\end{aligned} \tag{A.6}
$$

Again, due to the fact that, if the three currents are not null at the same time, is not possible that the three currents (i_a, i_b, i_c) have the same sign, the noise on the voltage winding is quantized with the following four values

$$
\delta_{wk} = \left\{ -\frac{2}{3}(\delta u^+ + \delta u^-), -\frac{1}{3}(\delta u^+ + \delta u^-), +\frac{1}{3}(\delta u^+ + \delta u^-), \frac{2}{3}(\delta u^+ + \delta u^-) \right\} \tag{A.7}
$$

The quantization values reported in (A.7) are changed during the zero-crossing of the current for each phase, as shown on plot (d) in the figure A.4.
Furthermore, neglecting from the inverter nonlinearities analysis above the effects of the voltage drop on the electronic devices ($V_S = V_D = 0$), and the effects of the turn ON/OFF time delays ($T_{onH} = T_{offH} = T_{onL} = T_{offL} = 0$), it is possible to have an estimate of the major contributes of the voltage noise due to the inverter nonlinearities, in fact with this hypothesis is trivial to verify that

$$
\begin{aligned}
\delta u^+ &= 0 \\
\delta u^- &= \frac{2DT}{T_{PWM}} > 0
\end{aligned} \tag{A.8}
$$

The variables $\delta u_{a0}, \delta u_{b0}, \delta u_{c0}$ along with the currents i_a, i_b, i_c and the variables $\delta_{wa}, \delta_{wb}, \delta_{wc}$ are reported in the figure A.4, in a simulation scenario where the inverter is trying to impose a sinusoidal current on the stator windings of a PMSM to cope with a torque control task, and with a DC-Link voltage of $V_{DC} = 1070[V]$, with a PWM period of $T_{PWM} = 400[\mu s]$, and with a Dead-Time of $DT = 3[\mu s]$, therefore the maximum noise on the voltage windings is $\delta^{max}_{wk} = (2/3)\delta u^- = 10.7[V]$.
Furthermore, looking at the quantization levels reported in (A.7) and looking at the plot (d) of the figure A.4 it is possible to give a description of one of the three signals through a Fourier analysis, as follows

$$
\delta_{wk} = \frac{2}{3}(\delta u^+ + \delta u^-) \sum_{k=1}^{\infty} \left(\frac{\cos(k\pi) - 1 + \cos(\frac{2k\pi}{3}) - \cos(\frac{k\pi}{3})}{\pi k} \right) \sin\left(\frac{2\pi k t}{T_I}\right) \tag{A.9}
$$

Where T_I is the period of the signal δ_{wa} that is the period of the current as can be shown in the figure. It is straightforward to verify the for $k = 1$

$$\delta_{wka} = \frac{2}{3}(\delta u^+ + \delta u^-)\left(\frac{3}{\pi}\right)\sin\left(\frac{2\pi t}{T_I}\right) \sim \frac{2}{3}(\delta u^+ + \delta u^-)\sin\left(\frac{2\pi t}{T_I}\right) \quad \text{(A.10)}$$

The equation above shows that one of the most important contribute of the inverter nonlinearities has the same frequency of the current flowing to the load ($\omega = 2\pi f$), i.e. in a PMSM, where the current is synchronous with the rotor flux, in the rotating reference frame d, q we have a constant corrupting contribute on the stator voltages. While high order harmonics ($k > 1$) due to inverter nonlinearities can be seen through the current measure or through the stator voltage applied to have a sinusoidal current, in the d, q reference frame the 0 frequency term ($k = 1$) cannot be measured, so the high order noise harmonics can be canceled out enriching the state of the system, i.e. introducing a number of oscillators equal to the number of harmonics that will be compensated.
Another idea to exploit the analysis of the inverter nonlinearities, and especially this last Fourier analysis is that using the relation linking the amplitude of the harmonics each other (A.9), through the estimation of one of the high order harmonic is possible to estimate also that harmonics synchronous with the current, otherwise unmeasurable.

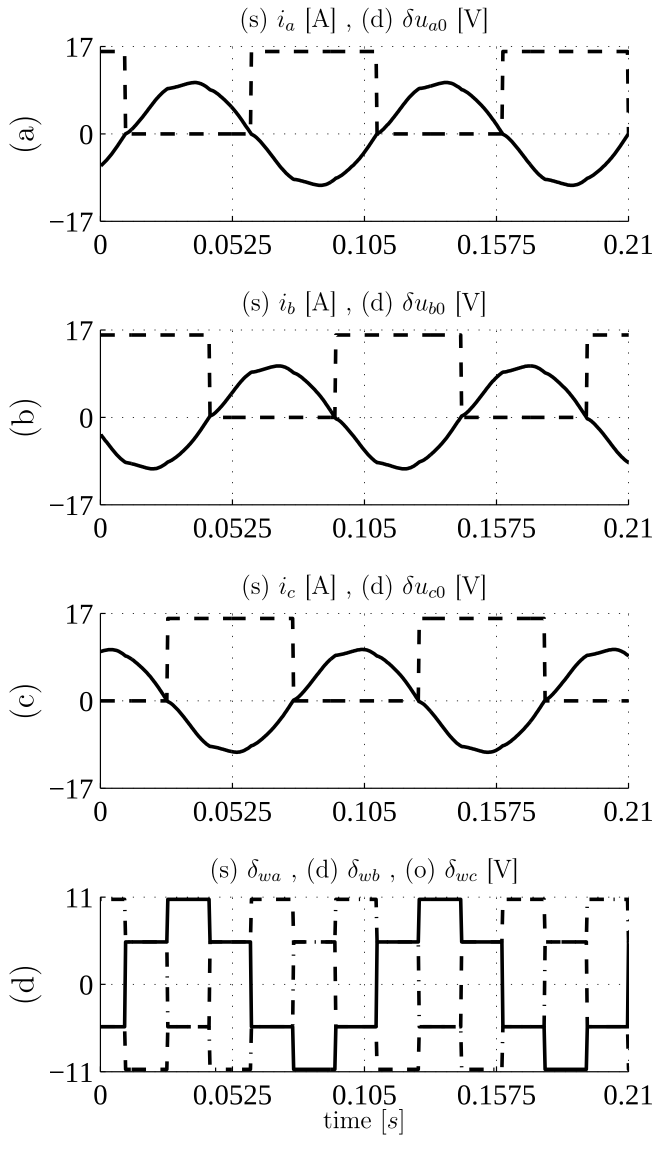

Figure A.4: (s) solid, (d) dashed , (o) dot-dashed. Inverter nonlinearities from a real simulation scenario with $V_{DC} = 1070[V]$, $T_{PWM} = 400[\mu s]$, $DT = 3[\mu s]$.

Appendix B

PMSM Characteristics of Functioning

Introduction

In this appendix the description of functioning characteristics of a PMSM are reported. Theses characteristics are extensively treated in literature ([29]-[31]) and are based on the iso-Torque, iso-Voltage and iso-Current loci. These characteristics give rise to the so called MTPA (Maximum Torque per Ampere), FW (Field-Weakening) and MTPV (Maximum Torque per Voltage) loci, and from these loci a polytopes, describing the area of correct functioning of the motor in the Cartesian plane i_d, i_q can be drawn.

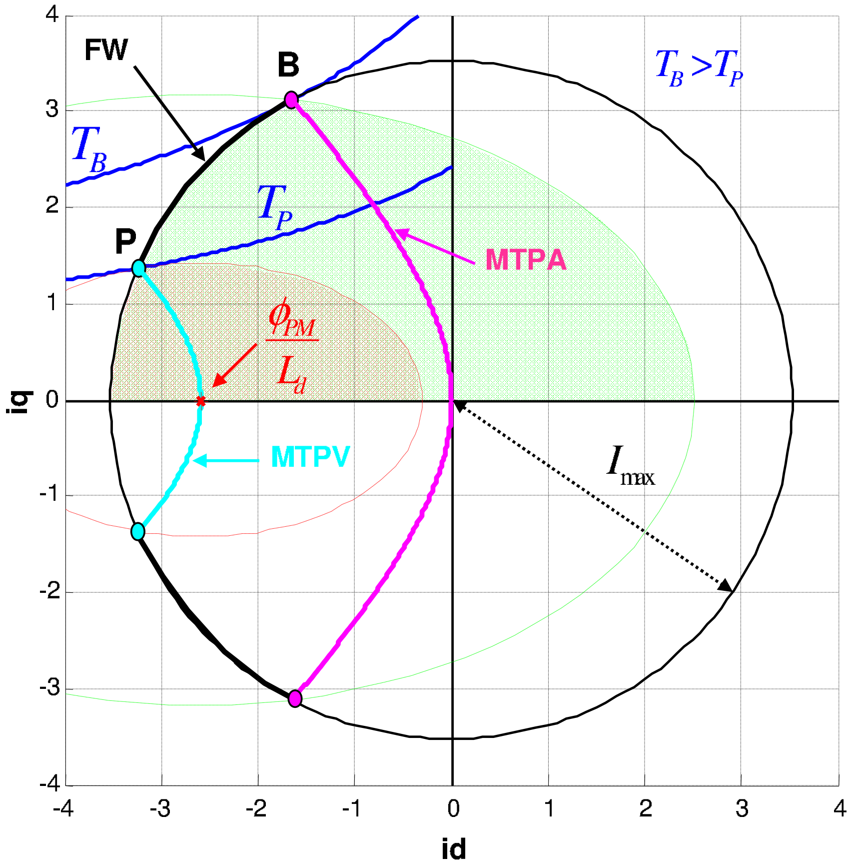

Figure B.1: Cartesian plane i_d, i_q with MTPA, FW and MTPV characteristics. Torque (hyperbolic) loci are in blue ($T_B > T_P$), Voltage (elliptic) loci are green (low speed, point B) and red (high speed, point P), Maximum Current (circle) locus is in black.

B.1 MTPA, FW, MTPV Loci

The standard MTPA, FW and MTPV loci (see Figure B.1) can be derived through the analytic manipulation of the stator current equations (1.21) and the Torque equation (1.32), described in the permanent magnet aligned reference frame ($\phi_d = \Phi_M, \phi_q = 0$), and supposing steady-state conditions ($\dot{i}_d = \dot{i}_q = 0$) as follows:

$$\begin{aligned} \frac{3}{2}p\left(\Phi_M i_q + (L_d - L_q)i_d i_q\right) &= T \leq T_{max} \\ \left(-\frac{Ri_d}{\omega} + L_q i_q\right)^2 + \left(\frac{Ri_q}{\omega} + L_d i_d + \Phi_M\right)^2 &= \frac{u_d^2 + u_q^2}{\omega^2} \leq \frac{U_{max}^2}{\omega^2} \\ i_d^2 + i_q^2 &= I_{ph}^2 \leq I_{max}^2 \end{aligned} \tag{B.1}$$

The three equations above describe respectively the Torque (hyperbolic) loci (blue lines in Figure B.1), the Voltage (elliptic) loci (green (low speed) and red (high speed) lines in Figure B.1) and the Current (circle) loci (black line in Figure B.1) in the i_d, i_q plane.
The MTPA locus is obtained from the intersection of the Torque equation and the Current equation, the FW locus is derived from the intersection of the Voltage equation and the Current equation, the MTPV locus can be analytically derived from the Torque equation and the Voltage equation.
It is important to remark that the presence of the MTPV locus depends on the value of the rotor flux, in fact if $\Phi_M/L_d \geq I_{max}$ the "end" point (high speed) of the MTPV is outside the maximum current circle, and the MTPV locus does not exist. Whenever $\Phi_M/L_d < I_{max}$ the motor can magnetically work for all speed, also for "infinite" speed, this is of course an abstraction.
When the motor work on the FW or the MTPV loci, the maximum motor torque is not available (the maximum torque is reached in the point B in Figure B.1), despite to the FW characteristic, in the MTPV the i_d value increase, therefore is erroneous to call Field-Weakening the MTPV characteristic.

Neglecting the resistive voltage drop ($R = 0$) the following three equations, for the expression of i_d as a function of the i_q, can be derived for the MTPA, the FW and the MTPV loci respectively:

$$i_{d,MTPA} = -\frac{\Phi_M}{2(L_d - L_q)} - \sqrt{\left(\frac{\Phi_M}{2(L_d - L_q)}\right)^2 + i_q^2} \tag{B.2}$$

$$i_{d,FW} = -\frac{\Phi_M}{L_d} - \sqrt{\frac{\Phi_M}{L_d}^2 - \frac{(p\omega L_q i_q)^2 + (p\omega\Phi_M)^2 - \|u\|^2}{(p\omega L_d)^2}} \tag{B.3}$$

$$\begin{aligned} i_{d,MTPV} = &-\left(\frac{\Phi_M}{2}\frac{2L_d - L_q}{L_q(L_d - L_q)}\right) + ... \\ &... + \sqrt{\left(\frac{\Phi_M}{2}\frac{2L_d - L_q}{L_q(L_d - L_q)}\right)^2 - \frac{\Phi_M^2 - (L_q i_q)^2}{L_d^2} - \frac{L_q\Phi_M^2}{L_d^2(L_d - L_q)}} \end{aligned} \tag{B.4}$$

B.2 From Torque set-point to i_d, i_q Current set-points

Usually, the motor is driven by a torque reference (T^*), also when a speed control loop is present, i.e. the mechanical dynamic is driven by a torque reference, therefore it can be useful to transform the torque reference into the two i_d, i_q current references, especially when the inner current control loops are present.

The algorithm to transform the Torque reference to the i_d, i_q current references, fulfilling the MTPA, FW and MTPV loci, is a minimum problem, and can be stated as follows:

$$\begin{array}{ll} P) & \min(\eta\,|s|) + \min((1-\eta)\Phi_M\,\|i\|) \\ c_1) & s \equiv T(\Phi_M, L_d, L_q, i_d, i_q) - T^* \leq 0 \\ c_2) & I_{Dmin} \leq i_d \quad , \quad (I_{Dmin} < 0) \\ c_3) & i_d \leq 0 \\ c_4) & \|i\|^2 \leq I_{max}^2 \\ c_5) & \|u\|^2 \leq U_{max}^2 \end{array} \tag{B.5}$$

Where, $P)$ is the minimum problem and $c_1)...c_5)$ are the constraints.
The function to minimize is composed by two terms, the first is related to the minimization of the Torque error (called s), and the second is related to the minimization of the norm of the current vector ($\|i\|$).
The constraint c_2 is a further constraint to impose to the i_d current to be greater than a minimum value (I_{Dmin}), to overcome the problem of the rotor demagnetization with high negative i_d current.
The constraint c_4 is the Current loci in Figure (B.1), while the constraint c_5 is the Voltage loci. The parameter η is a weight that allow to move the minimum search from the search of Torque minimum ($\eta \rightarrow 1$) to the search of Current minimum ($\eta \rightarrow 0$).

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