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Introduction

This dissertation comprises four essays on the topic of environmental economics and industrial organization. In the first essay, we develop a two-country world differential game model with a polluting firm in each country to investigate the equilibrium of the game between firms when they decide to trade or not and to see under which conditions social welfare coincides with the market equilibrium. In the second essay, we built a model where firms strategically choose whether to participate in an auction/lottery to attain pollution permits, or instead invest in green R&D, to show that, somewhat counterintuitively, a desirable side effect of the auction is in fact that of fostering environmental R&D in an admissible range of the model parameters. The third essay investigates a second-best trade agreement between two countries when pollution spillovers are asymmetric to examine the strategic behavior of governments in using pollution taxes and tariffs under trade liberalization. The fouth essay studies the profitability of exogenous output constraint in a differential game model with price dynamics under the feedback strategies.

In the first chapter, we have theoretically addressed the effects of trade liberalization in a two country world differential polluting oligopoly game where there is transportation cost to investigate the equilibrium of the game between firms when they decide to trade or not and to see under which conditions social welfare coincides with the market equilibrium. We find out while in the static game bilateral trade is always the equilibrium for any acceptable transportation cost, in the dynamic game social planner can prevent the inefficient outcome by imposing and determining the proper amount of corrective taxation. We figure out the market equilibrium under the open-loop and feedback strategies and determine which one of the three cases of bilateral trade, unilateral trade or autarchy is the equilibrium of the game between two firms according to the amount of transportation cost and corrective tax. Then, we determine the extent of tax amount for various quantities of negative externality to which social welfare coincides with market equilibrium.

In the second chapter, we examine the welfare implications of trade liberalization when governments behave strategically using environmental policy with asymmetric pollution spillovers. We investigated a second-best trade agreement between two countries to examine the strategic behavior of governments in using pollution taxes and tariffs under trade liberalization.

We found that when the marginal cost of pollution of the domestic firm increases, the pollution-shifting motive is enhanced and government wants to raise production taxes and surprisingly the rent-seeking behavior is observed and government raises import tariffs. On the other hand, when the marginal cost of pollution of the foreign firm increases, government want to reduce the level of tax and interestingly the level of tariff as well.

The third chapter is on non-tradable pollution permit and incentives for investments in green technologies. Acquired wisdom has it that the allocation of pollution rights to firms hinders their willingness to undertake uncertain R&D projects for environmental-friendly technologies. We revisit this issue in a model where firms strategically choose whether to participate in an auction/lottery to attain pollution permits, or instead invest in green R&D, to show that, somewhat counterintuitively, a desirable side effect of the auction is in fact that of fostering environmental R&D in an admissible range of the model parameters.

Finally, in the last chapter, we investigate the profitability of exogenous output constraints. In a series of papers Gaudet and Salant (1991a,b) show that, in the case of Cournot competition among producers of perfect substitutes, a marginal contraction is strictly beneficial if and only if the number of firms in the designated subset exceeds the "adjusted" number of firms outside it by strictly more than one. In the special case of linear cost and demand functions, the firms in the subset will gain from an exogenously marginal contraction of their output if and only if they outnumber the firms outside the subset by more than one.

In this paper we generalize this result to the case of dynamic competition instead of looking at the one-shot game. While in the standard Cournot model any output constraint is not to the benefit of constrained firms, in this paper, we show that when firms play a dynamic Cournot game with Markov-perfect strategies, exogenous output constraint by a subset of firms results in: (i) increase in the value of unconstraint firms irrelevant of the amount of constraint because of having less intensive competition, (ii) increase in the market price for any output constraint below the optimal level and slightly above that because of lowering the total output caused by less competition and (iii) increase in the value of constrained firms for a viable range of parameters and initial conditions because of increasing the price during the price path. Our analysis has some applications to voluntary export restraints (VER), Mergers, Economics Sanctions, etc.

Part I

Essays in Environmental Economics

Chapter 1

A Dynamic Approach to the Environmental Effects of Trade Liberalization

1.1 Introduction

Controlling the emission of environment-damaging pollution caused by increased economic activity has received a considerable attention in the field of environmental economics. Given that the pollution function is increasing in the output of the industry, we have the usual trade-off between the price effect and the negative externality. If we restrict the output the environment is cleaner but the price is higher.

International trade is playing an important role in expanding global economic activities and there is an increasing amount of literature regarding trade and the environment in trade theory[1]. However, there are not too many contributions regarding the effects of trade liberalization in a dynamic context. What creates negative externality is the stock of pollution not just the current emission of pollution. Thus, we need a dynamic model to study the environmental effect of trade liberalization due to the fact that pollution is accumulated over time. Fujiwara (2009) investigates the effects of free trade on global stock of pollution using a two country differential game model. We develop a two-country world

[1] See Copeland and Taylor (2003), Antweiler *et al.* (2001), *inter alia.*

differential game model, where there is a polluting firm in each country, to derive the open-loop and feedback equilibria of the game between firms in case of autarky, unilateral and bilateral trade when there is transportation cost and also a Pigouvian tax is introduced to reduce damaging emissions.

Most of the existing contributions in the field of environmental economics examine the existence of Pigouvian taxation aimed at inducing firms to reduce damaging emissions directly[2] or indirectly[3]. Accordingly, the common approach to deal with this problem in all of these studies is to derive the first best, where a social planner chooses a welfare maximizing production plan, and introduce corrective taxes to induce profit-seeking firms to produce at socially optimum level. In our study, the game between social planners is not technically solvable. As a result, it is not possible to outline the social optimum. However, we figure out the market equilibrium and determine which one of the three cases of bilateral trade, unilateral trade or autarky is the equilibrium of the game between two firms according to the transportation cost and Pigouvian tax quantity. Then, we determine the extent of tax amount for various quantities of negative externality to which social welfare coincides with market equilibrium.

The remainder of the paper is structured as follows. Section 2 constructs a basic model. Section 3 briefly outlines the static version of the game. In section 4, the differential game is illustrated and the open-loop and feedback equilibria under autarky, unilateral and bilateral trade are characterized. Profits and social welfares are assessed in section 5. Section 6 concludes the paper.

1.2 The Setup

There are two similar countries, indexed by $i = 1, 2$. In each country there is a firm which produces a single output. Firms supply a homogenous good and

[2] See Bergstrom *et al.* (1981), Karp and Livernois (1992, 1994), Benchekroun and Long (1998, 2002) and Tsur and Zemel (2008).

[3] To this regard, see Downing and White (1986), Milliman and Prince (1989), Damania (1996), Chiou and Hu (2001) and Tsur and Zemel (2002), Dragone *et al.* (2009).

their productions, q_i, have two parts:

$$q_i = q_{ii} + q_{ij}, \qquad i, j = 1, 2 \textit{ and } j \neq i,$$

where q_{ii} and q_{ij} denote the amounts of output produced by firm i and consumes in domestic market and is exported to the other country, respectively. It is obvious that the second part becomes zero if there isn't any export.

Exporting firm must pay an iceberg transportation cost which depends on the amount of export. In our setting, $m \in (0, 1]$ captures the effect of transportation cost. If there is no transportation cost, m is equal to one. Therefore, the inverse demand function in each country is

$$p_i = a - (q_{ii} + mq_{ji}), \qquad i, j = 1, 2 \textit{ and } j \neq i,$$

where q_{ji} is the amount of goods which is exported by the firm j into country i.

Technology is the same for both firms and production takes place at constant returns to scale (CRS), with a constant marginal cost c. It is summarized by the cost function $C_i = cq_i(t)$. Hence, firm i's instantaneous profits are

$$\pi_i (t) = p_i (t) q_{ii} (t) + p_j (t) mq_{ij} (t) - cq_i (t) .$$

The production of the final output creates a negative externality in the form of polluting emissions' flow which we assume $E(t) = Q(t)$, and it increases the stock of pollution, Z. Pollution is accumulated over time and is transboundary. The accumulation process of the world pollutant follows:

$$\dot{Z} (t) = \sum_{i=1}^{2} q_i (t) - kZ (t) , \qquad k > 0, \tag{1.1}$$

where k is the natural purification rate of the pollutant.

The stock of pollution lowers the consumer surplus by the following rule:

$$CS_i (t) = \frac{(q_{ii} (t) + mq_{ji} (t))^2}{2} - h\frac{Z (t)^2}{2}, \qquad h > 0,$$

where h measures the effect of negative externality on consumers. However, the instantaneous social welfare in each country is the aggregate amount of firm's profits and consumer surplus:

$$SW_i (t) = \pi_i (t) + CS_i (t) . \tag{1.2}$$

By knowing this setting, we are deriving firms' profit equilibria in autarky, unilateral and bilateral trade. We will compare these profits as well as social welfares to obtain the trade strategy from the viewpoints of the both, the social planner and the firms.

1.3 The Static Problem

As a preliminary step, in this section, we consider the static Cournot game in order to examine the case where firms maximize their profit functions without taking into account the negative externality because of the lack of corrective tax. We consider the game in figure 1 in which firms make their trade strategy decision, where π_i^A, π_i^T (π_i^{NT}) and π_i^{BT} denote the optimal profit of firm i in the case of autarky, trade (not trade) in unilateral and bilateral trade, respectively.

		firm 2	
		NT	T
firm 1	NT	$(\pi_1{}^A, \pi_2{}^A)$	$(\pi_1{}^{NT}, \pi_2{}^T)$
	T	$(\pi_1{}^T, \pi_2{}^{NT})$	$(\pi_1{}^{BT}, \pi_2{}^{BT})$

Figure 1.1: The game between two firms when they decide to trade (T) or not (NT).

In autarky case, there is no trade between the two countries and each firm is monopolist in its own country with the optimal quantity level of $(a-c)/2$. In the unilateral and bilateral trade where firms play *à la Cournot*, the equilibrium amount of outputs is summarized in lemma 1 and 2.

Lemma 1 *The equilibrium amounts of firms' output in unilateral trade in the static Cournot competition is*

$$q_{ii}^T = \frac{a-c}{2}, \qquad q_{ij}^T = \frac{(a+c)\,m - 2c}{3m^2},$$

$$q_{jj}^{NT} = \frac{(a-2c)\,m + c}{3m}.$$

Proof. The maximization problem of trading and not trading firms are

$$\pi_i^T = \max_{q_{ii},q_{ij}} (a - q_{ii})q_{ii} + (a - q_{jj} - mq_{ij})(mq_{ij}) - c(q_{ii} + q_{ij}), \tag{1.3}$$

$$\pi_j^{NT} = \max_{q_{jj}} (a - q_{jj} - mq_{ij})q_{jj} - cq_{jj}, \tag{1.4}$$

with the following first-order conditions (FOCs):

$$\frac{\partial \pi_i^T}{\partial q_{ii}} = a - 2q_{ii} - c = 0, \tag{1.5}$$

$$\frac{\partial \pi_i^T}{\partial q_{ij}} = am - mq_{jj} - 2m^2 q_{ij} - c = 0, \tag{1.6}$$

$$\frac{\partial \pi_i^{NT}}{\partial q_{jj}} = a - 2q_{jj} - mq_{ij} - c = 0. \tag{1.7}$$

Consequently, the resulting levels of individual output are

$$q_{ii}^T = \frac{a-c}{2}, \quad q_{ij}^T = \frac{(a+c)\,m - 2c}{3m^2}, \quad q_{jj}^{NT} = \frac{(a-2c)\,m + c}{3m}.$$

■

Lemma 2 *The equilibrium in bilateral trade under static Cournot competition is*

$$q_{ii}^{BT} = \frac{(a-2c)\,m + c}{3m}, \qquad q_{ij}^{BT} = \frac{(a+c)\,m - 2c}{3m^2}.$$

Proof. The maximization problem of firms in case of bilateral trade, which is the same for both because of symmetry, would be

$$\pi_i^{BT} = \max_{q_{ii},q_{ij}} (a - q_{ii} - mq_{ji})q_{ii} + (a - q_{jj} - mq_{ij})(mq_{ij}) - c(q_{ii} + q_{ij}). \tag{1.8}$$

The first order conditions of this problem w.r.t. controls are

$$\frac{\partial \pi_i^{BT}}{\partial q_{ii}} = a - 2q_{ii} - mq_{ji} - c = 0, \tag{1.9}$$

$$\frac{\partial \pi_i^{BT}}{\partial q_{ij}} = am - mq_{jj} - 2m^2 q_{ij} - c = 0, \tag{1.10}$$

which leads to this solution:

$$q_{ii}^{BT} = \frac{(a-2c)\,m + c}{3m}, \quad q_{ij}^{BT} = \frac{(a+c)\,m - 2c}{3m^2}.$$

■

Comparing the corresponding profits on autarky, unilateral and bilateral trade, it is clear that $\pi_i^T > \pi_i^A$, $\pi_i^{BT} > \pi_i^{NT}$ and $\pi_i^A > \pi_i^{BT}$. Therefore:

Proposition 3 *Under the static framework trade is dominant strategy for both firms and (T, T) is the Nash equilibrium of the game where firms decide to trade or not. This is a prisoner's dilemma game.*

Proof. This follows from equilibrium in autarky and lemmas 1 and 2. ■

Now, we are interested in welfare comparison across the four cases which is summarized in:

Corollary 4 *Under the static framework bilateral trade is welfare improving if and only if*

$$h < \frac{k^2 m^2 (5am + c(17m - 22))}{4(m-2)(c(4 + m(-4 + 7m)) - am(2 + 5m))}, \tag{1.11}$$

which coincides the equilibrium of the firms' game. Otherwise, social welfare has higher value in the case of autarky.

Proof. By plugging q_{ii}, q_{ij}, q_{jj} and q_{ji} into the stationary condition $\dot{Z} = 0$, the steady state stock of pollution is obtained which in turn can be plugged into (1.2) in order to get social welfare amounts in autarky, unilateral and bilateral trade. Comparing the acquired welfares, we obtain the inequality. ■

Corollary 5 *The less transportation cost is, the more bilateral trade is socially preferable.*

Proof. The right hand side of the inequality (1.11) is increasing in m which means in order for bilateral trade becomes socially acceptable, h can have a larger value when transportation cost decreases. This concludes the proof. ■

However, trade liberalization would increase firms' output which has two contradictory effects on consumer surplus. Output increase, on the one hand, would directly raise consumer surplus, on the other hand, increases the stock of pollution which in turn reduces consumer surplus. Now, if inequality (1.11) holds or in the other words h is small enough, pollution increase does not reduce the consumer surplus too much and consumers will benefit from output enlargement.

1.4 The Dynamic Game

As it is said before, the production of final output creates a cross-boundary negative externality which is accumulated over time and follows the dynamic (4.31). Now, by introducing a corrective (Pigouvian) tax, in quadratic form, the firms are forced to internalize the negative externality of pollution in a dynamic framework. Therefore, the firm i's optimization problem is formulated as:

$$\max_{q_i} \Pi_i \equiv \int_0^\infty e^{-rt} \left[p_i \; q_{ii} + p_j \; m q_{ij} - c \; (q_{ii} + q_{ij}) - \frac{s}{2} Z^2 \right] dt, \tag{1.12}$$

subject to (4.31) and $Z(0) = Z_0$. Parameter $r > 0$ is a constant rate of discount common to all firms and parameter s is a policy instrument that policy maker by manipulating it modifies taxation. This taxation is not the same if firms play open-loop or feedback.

In the remainder of this section, the problem is solved for the open-loop equilibrium and feedback equilibrium as well.

1.4.1 Open-Loop Solution

Here we characterize the open-loop equilibria of the three cases, starting with the autarky which is the simplest one because there is only one supplier in each country.

Proposition 6 *At the open-loop Nash equilibrium under autarky, the steady state levels of the price and the individual output are*

$$p_i^{OLA} = a - q_{ii}^{OLA}, \qquad q_{ii}^{OLA} = \frac{k(a-c)(k+r)}{2(k(k+r)+s)},$$

where OLA denotes the open-loop equilibrium at autarky. Such a steady state is saddle point stable.

Proof. The Hamiltonian equation of firm i is:

$$\begin{aligned} H_i^A(t) \quad = \quad & e^{-\rho t} \Big\{ q_{ii}(t) \left(a - q_{ii}\left(t\right) - c \right) - \frac{s}{2} Z^2(t) \\ & + \lambda_i(t) \left[q_{ii}(t) + q_{jj}(t) - kZ(t) \right] \Big\}, \end{aligned} \tag{1.13}$$

where $\lambda_i(t) = \mu_i(t)\, e^{\rho t}$ and $\mu_i(t)$ is the co-state variable associated with $Z(t)$. Consider the first-order condition w.r.t. $q_{ii}(t)$:

$$\frac{\partial H_i^A(t)}{\partial q_{ii}(t)} = a - 2q_{ii}(t) - c + \lambda_i(t) = 0. \tag{1.14}$$

This yields the optimal open-loop output for the firm i as follows[4]:

$$q_{ii}(t) = \begin{cases} \frac{1}{2}(a - c + \lambda_i(t)) & \text{if} \quad a > c - \lambda_i(t), \\ 0 & \text{otherwise.} \end{cases} \tag{1.15}$$

The adjoint equation for the optimum is

$$\frac{\partial \lambda_i(t)}{\partial t} = r\lambda_i(t) - \frac{\partial H_i^A(t)}{\partial Z(t)} = (k + r)\,\lambda_i(t) + sZ(t)\,, \tag{1.16}$$

and the associated transversality condition is

$$\lim_{t\to\infty} \mu_i(t)\,.Z(t) = 0.$$

Differentiating (1.15), using (1.16) and symmetry assumption, we obtain

$$\frac{dq(t)}{dt} \equiv \dot{q}(t) = \frac{1}{2}\left[(k + r)\,\lambda(t) + sZ(t)\right]. \tag{1.17}$$

From (1.14), we know

$$\lambda(t) = -a + 2q(t) + c.$$

By substituting this into (1.17), we have

$$\dot{q}(t) = -\frac{1}{2}\left[(k + r)(a - 2q(t) - c) - sZ(t)\right]. \tag{1.18}$$

Therefore, the dynamic system can be rewritten in matrix form as follows:

$$\begin{bmatrix} \dot{q} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} k + r & \frac{s}{2} \\ 2 & -k \end{bmatrix} \begin{bmatrix} q \\ Z \end{bmatrix} + \begin{bmatrix} -\frac{1}{2}(k + r)(a - c) \\ 0 \end{bmatrix}. \tag{1.19}$$

Since the determinant of the above two-by-two matrix is negative, the equilibrium point is a saddle, with

$$p_i^{OLA} = a - q_{ii}^{OLA}, \qquad q_{ii}^{OLA} = \frac{k(a - c)(k + r)}{2(k(k + r) + s)}.$$

■

[4] In the remainder, we consider the positive solution.

Proposition 7 *At the open-loop Nash equilibrium under unilateral trade, the steady state levels of the price and the individual outputs are*

$$
\begin{aligned}
p_i^{OLT} &= a - q_{ii}^{OLT}, \qquad p_j^{OLNT} = a - q_{jj}^{OLNT} - mq_{ij}^{OLT}, \\
q_{ii}^{OLT} &= \frac{3km^2(a-c)(k+r) + a(m-2)(m-1)s}{6km^2(k+r) + (4+m(7m-4))s}, \\
q_{ij}^{OLT} &= \frac{2k(c(m-2)+am)(k+r) + 4a(m-1)s}{6km^2(k+r) + (4+m(7m-4))s}, \\
q_{jj}^{OLNT} &= \frac{2km(am+c-2cm)(k+r) - a(m^2+m-2)s}{6km^2(k+r) + (4+m(-4+7m))s},
\end{aligned}
\tag{1.20}
$$

where OLT and $OLNT$ denote the open-loop equilibrium in unilateral trade for trading and not trading firms, respectively. Such a steady state is saddle point stable.

Proof. In unilateral trade, only one firm exports. The Hamiltonian for the trading and not trading firms are

$$
\begin{aligned}
H_i^T(t) = e^{-\rho t} \Big\{ & \Big[p_i(t) q_{ii}(t) + p_j(t) m q_{ij} - c q_i - \frac{s}{2} Z^2(t) \Big] \\
& + \lambda_i(t) \left[q_{ii}(t) + q_{ij}(t) + q_{jj}(t) - kZ(t) \right] \Big\},
\end{aligned}
\tag{1.21}
$$

$$
\begin{aligned}
H_j^{NT}(t) = e^{-\rho t} \Big\{ & \Big[p_j(t) q_{jj} - c q_j - \frac{s}{2} Z^2(t) \Big] \\
& + \lambda_j(t) \left[q_{ii}(t) + q_{ij}(t) + q_{jj}(t) - kZ(t) \right] \Big\}.
\end{aligned}
\tag{1.22}
$$

The first-order necessary conditions w.r.t. control variables, adjoint equations and associated transversality conditions for the optimum are

$$\frac{\partial H_i^T(t)}{\partial q_{ii}(t)} = a - 2q_{ii}(t) - c + \lambda_i(t) = 0, \tag{1.23}$$

$$\frac{\partial H_i^T(t)}{\partial q_{ij}(t)} = am - 2m^2 q_{ij}(t) - mq_{jj}(t) - c + \lambda_i(t) = 0, \tag{1.24}$$

$$\frac{\partial H_i^{NT}(t)}{\partial q_{jj}(t)} = a - 2q_{jj}(t) - mq_{ij}(t) - c + \lambda_j(t) = 0, \tag{1.25}$$

$$\frac{\partial \lambda_i(t)}{\partial t} = r\lambda_i(t) - \frac{\partial H_i^T(t)}{\partial Z(t)} = (k+r)\lambda_i(t) + sZ(t), \tag{1.26}$$

$$\frac{\partial \lambda_j(t)}{\partial t} = r\lambda_j(t) - \frac{\partial H_j^{NT}(t)}{\partial Z(t)} = (k+r)\lambda_j(t) + sZ(t), \tag{1.27}$$

$$\lim_{t\to\infty}\mu_i(t).Z(t)=0, \quad \lim_{t\to\infty}\mu_j(t).Z(t)=0.$$

Differentiating FOCs w.r.t. time and using adjoint equations we obtain the following control dynamical system:

$$\begin{cases} \dot{q}_{ii}(t) = -\frac{1}{2}\left[(k+r)(a-2q_{ii}(t)-c)-sZ(t)\right], \\ \dot{q}_{ij}(t) = -\frac{1}{3m^2}\left[(k+r)(m(a-3mq_{ij}(t)+c)-2c)-s(2-m)Z(t)\right], \\ \dot{q}_{jj}(t) = -\frac{1}{3m}\left[(k+r)(am-3mq_{jj}(t)-c(2m-1))-s(2m-1)Z(t)\right]. \end{cases} \tag{1.28}$$

Solving (1.28) together with (4.31), yields the stable steady state equilibrium point in (1.20). ■

Proposition 8 *At the open-loop Nash equilibrium under bilateral trade, the steady state levels of the price and the individual outputs are*

$$p_i^{OLBT} = a - q_{ii}^{OLBT} - mq_{ji}^{OLBT},$$

$$q_{ii}^{OLBT} = \frac{km(am+c-2cm)(k+r)+2a(1-m)s}{3km^2(k+r)+4(m(m-1)+1)s}, \tag{1.29}$$

$$q_{ij}^{OLBT} = \frac{k(c(m-2)+am)(k+r)+2a(m-1)s}{3km^2(k+r)+4(m(m-1)+1)s}.$$

where $OLBT$ denotes the open-loop equilibrium at bilateral trade. Such a steady state is saddle point stable.

Proof. As mentioned before, the two firms and two countries are symmetric. Then, the Hamiltonian function of each firm in bilateral trade is

$$\begin{aligned} H_i^{BT}(t) &= e^{-\rho t}\left\{\left[p_i(t)q_{ii}(t)+p_j(t)mq_{ij}-cq_i-\frac{s}{2}Z^2(t)\right]\right. \\ &\quad \left. +\lambda_i(t)\left[q_{ii}(t)+q_{ij}(t)+q_{jj}(t)-q_{ji}(t)-kZ(t)\right]\right\}. \end{aligned} \tag{1.30}$$

Considering the first-order conditions, adjoint equations and associated transversality conditions:

$$\frac{\partial H_i^{BT}(t)}{\partial q_{ii}(t)} = a-2q_{ii}(t)-mq_{ji}(t)-c+\lambda_i(t)=0,$$

$$\frac{\partial H_i^{BT}(t)}{\partial q_{ij}(t)} = m(a-2mq_{ij}(t)-q_{jj}(t))-c+\lambda_i(t)=0,$$

$$\frac{\partial \lambda_i(t)}{\partial t} = r\lambda_i(t)-\frac{\partial H_i^{BT}(t)}{\partial Z(t)} = (k+r)\lambda_i(t)+sZ(t),$$

$$\lim_{t\to\infty} \mu_i(t).Z(t) = 0,$$

yields the dynamics of firm i's controls:

$$\begin{cases} \dot{q}_{ii}(t) = -\frac{1}{3m}\left[(k+r)\left(am - 3mq_{ii}(t) - c(2m-1)\right) - s(2m-1)Z(t)\right], \\ \dot{q}_{ij}(t) = -\frac{1}{3m^2}\left[(k+r)\left(m\left(a - 3mq_{ij}(t) + c\right) - 2c\right) - s\left(2-m\right)Z(t)\right]. \end{cases} \tag{1.31}$$

Solving (1.31) accompanied by the dynamics of firm j's control variables and (4.31), fully characterizes the stable steady state equilibrium point in (1.29). ■

1.4.2 Feedback Solution

Here, we characterize a subgame perfect Cournot equilibrium in Markov strategies where firms employ pollution dependent decision rules when maximizing their discounted profit. Therefore, changes in the stock of pollution stimulate responses, through Pigouvian tax, by all players that are reflected in their quantity choices.

Proposition 9 *At the feedback Nash equilibrium under autarky, the steady state levels of the price and the individual output are*

$$p_i^{FA} = a - q_{ii}^{FA}, \qquad q_{ii}^{FA} = \frac{1}{2}\left(a - c + e^A Z + f^A\right),$$

where FA denotes the feedback equilibrium at autarky and

$$\begin{aligned} e^A &= \frac{1}{3}\left(2k + r - \sqrt{(2k+r)^2 + 6s}\right), \\ f^A &= \frac{2(a-c)e^A}{2(k+r) - 3e^A}. \end{aligned}$$

Proof. The Bellman equation of firm i in autarky is

$$\begin{aligned} rV_i\left(Z(t)\right) &= \max_{q_{ii}(t)} \left\{ q_{ii}(t)\left[p_i(t) - c\right] - \frac{s}{2}Z^2(t) \right. \\ &\quad \left. + \frac{\partial V_i\left(Z(t)\right)}{\partial Z(t)}\left[q_{ii}(t) + q_{jj}(t) - kZ(t)\right] \right\}, \end{aligned} \tag{1.32}$$

where $V_i\left(Z(t)\right)$ is the value function of firm i. Given the linear quadratic form of the maximand, we assume the quadratic value function:

$$V_i(Z) = \frac{e_i}{2}Z^2 + f_i Z + g_i, \tag{1.33}$$

so that

$$\frac{\partial V_i\left(Z\right)}{\partial Z} = e_i Z + f_i. \tag{1.34}$$

where e_i, f_i and g_i are unknown coefficients and the indication of time is omitted to ease the exposition. Taking the FOC w.r.t. q_{ii} and using (1.34), we obtain:

$$q_{ii}^{FA} = \frac{1}{2}\left(a - c + e^A Z + f^A\right), \quad p_i^{FA} = a - q_{ii}^{FA}, \tag{1.35}$$

where e^A and f^A can be calculated by using (1.35) and rewriting (1.32) as follows:

$$\beta_1 Z^2 + \beta_2 Z + \beta_3 = 0, \tag{1.36}$$

where

$$\beta_1 = \frac{1}{4}\left[e(3e^A - 4k - 2r) - 2s\right], \tag{1.37}$$

$$\beta_2 = \frac{1}{4}\left[4e^A(a-c) + 2f^A(3e^A - 2\left(k+r\right))\right], \tag{1.38}$$

$$\beta_3 = \frac{1}{4}\left[(a-c)^2 + f^A(4\left(a-c\right) + 3f^A) - 4g^A r\right]. \tag{1.39}$$

Equation (1.36) is satisfied if expressions (1.37)-(1.39) are simultaneously zero. This results to the following solution:

$$\begin{aligned} e^A &= \frac{1}{3}\left(2k + r - \sqrt{(2k+r)^2 + 6s}\right), \\ f^A &= \frac{2(a-c)e^A}{2(k+r) - 3e^A}. \end{aligned}$$

■

Proposition 10 *At the feedback Nash equilibrium under unilateral trade, the steady state levels of the prices and the individual outputs are*

$$p_i^{FT} = a - q_{ii}^{FT}, \qquad p_j^{FNT} = a - q_{jj}^{FNT} - m q_{ij}^{FT},$$

$$q_{ii}^{FT} = \frac{1}{2}\left(a - c + e^T Z^{FT} + f^T\right),$$

$$q_{ij}^{FT} = \frac{(2-m)(e^T Z^{FT} + f^T - c) + am}{3m^2},$$

$$q_{jj}^{FNT} = \frac{c + m(a-2c) + (2m-1)(e^T Z^{FT} + f^T)}{3m},$$

where FT and FNT denote the feedback equilibrium in unilateral trade for trading and not trading firm and

$$\begin{aligned} e^T &= \frac{9m^2(2k+r) - 3\sqrt{9m^4(2k+r)^2 + 2m^2(16+m(37m-28))s}}{16+m(37m-28)}, \\ f^T &= \frac{e^T(c(16+m(25m-22)) - am(11m+8))}{e^T(16+m(37m-28)) - 18m^2(k+r)}. \end{aligned}$$

Proof. The Bellman equations of trading and not trading firms in unilateral trade are[5]:

$$\begin{aligned} rV_i(Z(t)) &= \max_{q_i(t)} \left\{ \left[p_i(t)q_{ii}(t) + p_j(t)mq_{ij} - cq_i - \frac{s}{2}Z^2(t) \right] \right. \\ &\quad \left. + \frac{\partial V_i(Z(t))}{\partial Z(t)} \left[q_{ii}(t) + q_{ij}(t) + q_{jj}(t) - kZ(t) \right] \right\}, \end{aligned} \tag{1.40}$$

$$\begin{aligned} rV_j(Z(t)) &= \max_{q_j(t)} \left\{ \left[p_j(t)q_{jj} - cq_j - \frac{s}{2}Z^2(t) \right] \right. \\ &\quad \left. + \frac{\partial V_j(Z(t))}{\partial Z(t)} \left[q_{ii}(t) + q_{ij}(t) + q_{jj}(t) - kZ(t) \right] \right\}, \end{aligned} \tag{1.41}$$

with the same value function form that was introduced before. Taking the FOCs w.r.t. controls and using (1.34), we obtain:

$$q_{ii}^{FT} = \frac{1}{2}\left(a - c + e^T Z^{FT} + f^T\right), \tag{1.42}$$

$$q_{ij}^{FT} = \frac{1}{2m^2}\left(am - c + e^T Z^{FT} + f^T - mq_{jj}^{FNT}\right), \tag{1.43}$$

$$q_{jj}^{FNT} = \frac{1}{2}\left(a - c + e^T Z^{FT} + f^T - mq_{ij}^{FT}\right). \tag{1.44}$$

By solving (1.43) and (1.44) simultaneously, the amounts of q_{ij}^{FT} and q_{jj}^{FNT} is taken. Using these and rewriting (1.40) or (1.41) as (1.36) and as the same procedure as the previous proof, we can calculate e^T and f^T. ■

Proposition 11 *At the feedback Nash equilibrium under bilateral trade, the steady state levels of the price and the individual outputs are*

$$p_i^{FBT} = a - q_{ii}^{FBT} - mq_{ji}^{FBT},$$

$$q_{ii}^{FBT} = \frac{c + m(a-2c) + (f^{BT} + e^{BT}z)(2m-1)}{3m},$$

[5] We omit the full calculations but they are available upon request.

$$q_{ij}^{FBT} = \frac{(2-m)(e^{BT}Z^{FBT} + f^{BT} - c) + am}{3m^2},$$
$$Z^{FBT} = \frac{2(am(m+1) + 2(f^{BT} - c)(m(m-1)+1))}{3km^2 - 4e^{BT}(m(m-1)+1)},$$

where FBT denotes the feedback equilibrium in bilateral trade and

$$e^{BT} = \frac{9m^2(2k+r) - \sqrt{81m^4(2k+r)^2 + 36m^2(22 - 28m + 22m^2)s}}{2(22 - 28m + 22m^2)},$$
$$f^{BT} = \frac{e^{BT}(m^2(5a - 16c) + m(5a + 22c) - 16c)}{9m^2(k+r) - e^{BT}(22 - 28m + 22m^2)}.$$

Proof. When there is trade between two countries, the Bellman equation of firm i is

$$rV_i(Z(t)) = \max_{q_i(t)} \left\{ \left[p_i(t)q_{ii}(t) + p_j(t)mq_{ij} - cq_i - \frac{s}{2}Z^2(t) \right] + \frac{\partial V_i(Z(t))}{\partial Z(t)} \left[q_{ii}(t) + q_{ij}(t) + q_{jj}(t) - q_{ji}(t) - kZ(t) \right] \right\}. \tag{1.45}$$

Taking the first order necessary conditions and using the similar procedure with the previous proofs leads to find the Nash equilibrium of the game in bilateral trade[6]. ■

Remark 12 *The parameter m must be belong to $(\underline{m}, 1]$ in which $\underline{m}$ is*

- $\frac{2c}{a+c}$ *in the static game,*
- $\frac{2ck(k+r) + 2as}{k(a+c)(k+r) + 2as}$ *in the open-loop equilibrium,*
- *the positive root of $k(f-c)(2-m) + a(km + 2e(1-m)) = 0$ in the feedback equilibrium where (e, f) is equal to (e^T, f^T) and (e^{BT}, f^{BT}) for unilateral and bilateral trade,respectively,*

otherwise firms do not have an incentive to trade due to high transportation cost. This results from the condition $q_{ij} > 0$.

Corollary 13 *In the dynamic equilibria, the maximum acceptable transportation cost decreases when s increases and in the limit when s goes to infinity, it must be zero.*

Proof. Differentiating $\underline{m}$, illustrated in remark 8, in the open-loop and the feedback equilibria w.r.t. s we found that $\frac{\partial \underline{m}}{\partial s} > 0$. Thus, increasing s leads to increasing the minimum acceptable value of m or in the other word lowering the maximum rate of tranpostation cost by which trade is doable. ■

[6] The full calculations are available upon request.

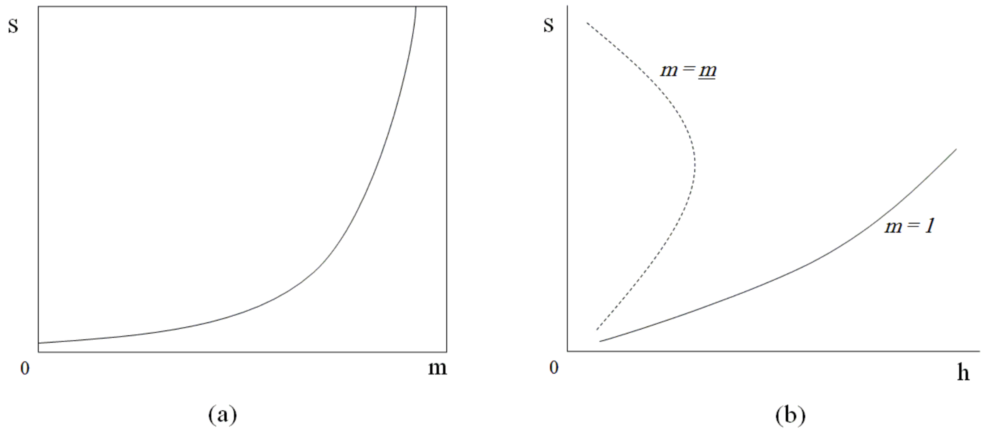

Figure 1.2: (a) Profit comparison according to the level of transportation cost and tax rate under the feedback information; (b) welfare comparison according to the level of negative externality and tax rate under the open-loop information

1.5 Profit and Welfare Assessment

In this section, by using equilibrium values, we compare firms' profits in autarky, unilateral and bilateral trade to determine the equilibrium of the game between firms where they decide to trade or not, in the open-loop and in the feedback solutions. In addition, we will look into the case which leads to the efficient level of social welfare.

Because of having too many parameters, comparing the results is difficult. Therefore, we use a numerical analysis to assess the profits and welfares in the three cases for the open-loop and the feedback equilibria, respectively. In our setting, the parameters a, c, k and r are given and in the remainder we assume that they have definite and plausible values of $10, 0, 0.5$ and 0.05, respectively.

In figure 2a, the profits of firms in different cases, under open-loop equilibria, is compared according to the amounts of transportation cost and Pigouvian tax. As it can be seen in this figure, the equilibrium of the game illustrated in figure 1 depends on the amounts of m and s. In the region below the curve, trade is dominant strategy for both firms which leads to the equilibrium $\left(\pi_1^{OLBT}, \pi_2^{OLBT}\right)$ and due to the fact that in this region the profit of firms in autarky is greater

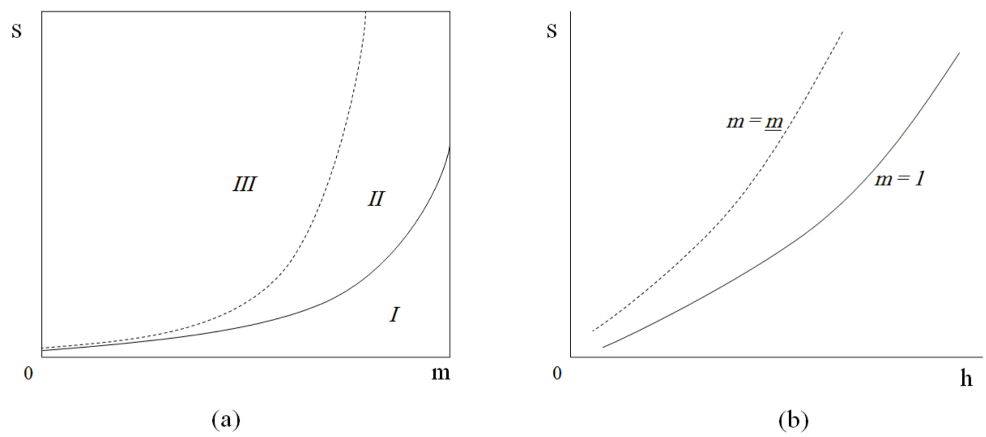

Figure 1.3: (a) Profit comparison according to the level of transportation cost and tax rate under the feedback information; (b) welfare comparison according to the level of negative externality and tax rate under the feedback information

than bilateral trade, this game is a prisoner's dilemma. In the region above the curve, the condition of remark 8 is not satisfied. Therefore, non of them choose trade strategy and $\left(\pi_1^{OLA}, \pi_2^{OLA}\right)$ is the equilibrium of the game.

Figure 2b depicts the regions that conditioned on the value of parameters h, s and m bilateral trade (autarky) becomes the preferable case from the social welfare point of view. In this figure, for $m = 1$, the solid line divides the region of parameters h and s into two areas where in the upper region bilateral trade is socially preferable and in the lower area autarky. The dashed line does the same but for $m =\underline{m}$. For any other amount of m we have an analogous boderline between the solid and the dash lines. As it can be seen in the figure, when m decreases the area where bilateral trade is socially efficient shrinks. However, depending on the amount of existing h, policy maker can determine tax rate in such a way that either bilateral trade or autarky become socially efficient.

Figure 3a compares the profits of firms in different cases according to the amounts of transportation cost and Pigouvian tax for feedback information. As it can be seen in this figure, in region I, where m is close to one and s is not too large, trade is dominant strategy for both firms which leads to the equilibrium $\left(\pi_1^{FBT}, \pi_2^{FBT}\right)$. In this region the profit of firms in autarky is greater than bilateral trade, therefore, this equilibrium is not pareto efficient. In region *II*, there is not any unique equilibrium and firms play a chicken game. If firms play simultaneously, they make a systematic mistake to reach the equilibrium, and if they play sequentially, the problem is who plays first and gains the enormous benefits of the trade. In the last region, *III*, because of high transportation cost, trade is not possible and autarky is the equilibrium.

In figure 3b, it is shown that which one of the three cases (autarky, unilateral and bilateral trade) is socially efficient according to the amounts of negative externality and corrective tax rate. Similar to figure 2b the solid line is for $m = 1$ and the dashed line is for $m =\underline{m}$. In the region above the curves, bilateral trade is efficient from the social planner point of view. In the other region autarky is socially efficient. Note that in some situations unilateral trade can be the efficient case socially. It means that if what one country gains is more than what the other looses, over all, they gain. But this makes a huge coordination problem. The problem is that which country accepts not to sell to the other country. In this case, there should be a side payment. Hence, unilateral trade is very hard to sustain.

However, if the social planner makes a deal about taxation, he makes a deal about s as well and this deal is different if he knows firms are playing open-loop equilibrium or feedback equilibrium.

Consequently, if firms play under the open-loop strategies, in order for the socially efficient equilibrium coincides with the market equilibrium, according to the amounts of h and m,social planner must determine s in a way that it characterizes a point in the lower (upper) region of figure 2a and the uper (lower) region in figure 2b. The most efficient point for the welfare (if it is applicable) takes place on the dividing curves (depended on m) in figure 2b.

If firms play under the feedback rule, bilateral trade can be the most preferable case if social planner can determine the tax rate, according to the amounts of h and m, in a way that it characterizes a point in region I of figure 3a and the upper region in figure 3b. Otherwise, he must choose s such that the equilibrium characterizes a point in region *III* of figure 3a and the lower region in figure 3b where autarky is the preferable case.

However, it is not clear to social planners whether firms are playing open-loop or feedback. Considering the figure 2, if social planners assume that firms are playing under open-loop equilibrium and they determine s in order to induce bilateral trade, they may face an unexpected outcome. Because if firms are playing feedback instead of open-loop, autarky may be welfare improving provided that the point places in the region above the curve in figure 2b and below the curve in figure 3b. Therefore, to avoid this problem policy makers must determine s for any given exogenous pair of (h, s) to satisfy the stricter constraint.

1.6 Concluding Remarks

In this paper, we have theoretically addressed effects of trade liberalization in a two country world differential polluting oligopoly game. We found out when firms decide to trade or not, if the transportation cost is not too large, under the open-loop information they play a prisoner's dilemma in which trade is the dominant strategy for both otherwise they play autarky. In order for trade to be dominant strategy in feedback information, the Pigouvian tax and transportation cost must have relatively lower values. For larger amounts of transportation cost and corrective tax, the equilibrium can be unilateral trade or autarky.

By comparing social welfares in autarky, unilateral and bilateral trade, we showed that, depending on the effects of negative externality on consumer and the transportation cost, policy maker can determine the amount of Pigouvian tax so that market equilibrium coincides with socially efficient equilibrium. This taxation is different if firms are playing open-loop equilibrium as compare to the feedback equilibrium.

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Chapter 2

Strategic environmental policies under international competition with asymmetric pollution spillovers

2.1 Introduction

The environmental consequences of trade liberalization have received a considerable attention in trade theory and environmental economics. International trade is playing an important role in expanding global economic activities and, therefore, many individuals have argued that trade liberalization will lead to an increase in world pollution.

Although globalization brings about many benefits and opportunities, some environmentalists have resisted freer trade, because governments which are unable to use trade policy may lower their environmental standards to give competitive advantage to existing domestic industries and protect their economy. This has led some economists to investigate the relationship between trade and the environment.[1]

The established literature on trade and environment suggests that, while

[1] Copeland and Taylor (2003) provides a comprehensive review of the link between trade and the environment.

each country can gain from trade, it expands global pollution. Fujiwara (2009) investigates the effects of free trade on global stock of pollution and he finds that under trade liberalization the stock of pollution is larger as compared to the autarky.

Another part of the literature deals with the links between strategic environmental policies and the patterns of trade and pollution levels. Stem from the Brander and Spencer (1985) model, Rauscher (1994), Kennedy (1994), Barrett (1994), Walz and Wellisch (1997) and Tanguay (2001) all show that governments can have incentives to use environmental policies to subsidize their exports. It is beneficial for rent-shifting governments to set an environmental tax below the Pigouvian level in an international oligopoly. Such a weak environmental regulation to support domestic firms has been called ecological dumping.

The aim of this study is to examine the welfare implications of trade liberalization when governments behave strategically using environmental policy in the presence of transboundary pollution. However, we model the transboundary pollution in such a way that it allows drawing the results also in pure local pollution and global environmental problem.

In this paper, we consider two symmetric countries with a single firm in each producing a homogenous good. The two firms may export a part of their production to the other country. In our model trade of the same product occurs between countries.[2] Thereby, we have a two-stage game where in the first stage governments decide about the environmental and trade policies, and the two firms compete 'a la Cournot in the second stage. The most important difference of this study with the aforementioned literature is that we allow for assymetric environmental damages between the two countries in our model.

We find that when the marginal cost of pollution of the domestic firm increases, the pollution-shifting motive is enhanced and government wants to raise production taxes and the rent-seeking behavior is observed and government

[2] Brander (1981) and Brander and Krugman (1983) showed that intraindustry trade occurs because each firm perceives each country as a separate market and makes distinct output decisions for each.

raises import tariffs. On the other hand, when the marginal cost of pollution of the foreign firm increases, government want to reduce the level of tax and the level of tariff as well.

In addition, contrary to existing literature, it is shown that the global pollution decreases in bilateral trade compared to autarky provided that the difference between the emission rates of the two firms is sufficiently large. This result holds even for the case of pure local pollution. Furthermore, it is shown that how the asymmetric pollution emissions affects the firms' profit and countries' welfare.

The rest of the paper is organized as follows. Section 2 constructs the general framework of the model and describes autarkic equilibria. Section 3 devoted to the firms' equilibrium. In Section 4 we turn to the games between the two governments. Comparing the trade equilibria with the autarkic equilibria takes place in section 5. Section 6 concludes the paper.

2.2 The fundamentals

2.2.1 The setup

There are two countries, indexed by $i = 1, 2$. In each country there is a firm which produces a single output. Their productions, q_i, have two parts:

$$q_i = q_{hi} + q_{ei}, \qquad i, j = 1, 2,$$

where q_{hi} and q_{ei} denote the amounts of output produced by firm i and consumes in the domestic market and is exported to the other country, respectively.

The inverse demand function in each country is

$$p_i = a - (q_{hi} + q_{ej}), \qquad i, j = 1, 2 \; and \; j \neq i,$$

where q_{ej} is the amount of good which is exported by the firm j into country i.

Production takes place at constant returns to scale (CRS), with a constant marginal cost c which is summarized by the cost function $C_i = cq_i(t)$.

The production of firm i, q_i, creates a constant per unit emission level, η_i. While firms are homogenous in their cost functions, it is assumed that they are hetrogeneous in their environmental damage functions, E_i

$$E_i(q_i) = \eta_i q_i = \eta_i \left(q_{hi} + q_{ei}\right), \qquad i, j = 1, 2,$$

which is not confined to the country where the production takes place and gives rise to a transboundary pollution problem.

The foreign production results in a negative externality in home country at the fixed level ψ, per unit of its environmental damage. Hence, the negative externality caused by home and foreign production in country i is

$$ex_i(q_i, q_j) = E_i(q_i) + \psi E_j(q_j).$$

where $\psi \in [0, 1]$, and $\psi = 0$ denotes the case of pure local pollution and $\psi = 1$ denotes the case of pure global environmental problem.

In order to protect the environment, country i levies an environmental tax, τ_i, on its polluting production and imposes a tariff , θ_i,on imported items. Hence, firm i's instantaneous profits are

$$\pi_i = p_i q_{hi} + p_j q_{ei} - c q_i - \tau_i q_i - \theta_j q_{ei}, \qquad i, j = 1, 2 \; and \; j \neq i,$$

Tax revenues are distributed in the form of a lump sum to the consumers. Thus, the social welfare in each country is the aggregate amount of firm's profits, consumer surplus, tax and tariff revenues minus negative environmental externality caused by home and foreign firms productions:

$$W_i = \pi_i + CS_i + \tau_i q_i + \theta_i q_{ej} - ex_i, \tag{2.1}$$

where $CS_i = \left(q_{hi} + q_{ej}\right)^2 / 2$.

2.2.2 The autarkic equilibrium

Now, we consider a closed economy where there is no trade between countries and each firm is monopolist in its own country. Therefore, given the government environmental policy τ_i, the firm i maximizes her monopolistic profit $\pi_i = p_i q_i - c q_i - \tau_i q_i$, $i = 1, 2$. By first-order condition (FOC), we obtain $q_i^A =$

$\frac{1}{2}(a - c - \tau_i)$ where the superscript A denotes the autarky. At the equilibrium, firm i's reaction to the tax policy is $\partial q_i^A / \partial \tau_i = -1/2$.

In the autarky, the government's first-best environmental policy is introduced by the Pigovian tax $\tau_i^A = \eta_i$. Note that because of transboundary pollution, the foreign firm's production creates negative externality in the home country but it is not affected by the home government policy.

Therefore, the Cournot-Nash equilibrium in the autarky is

$$q_i^A = \frac{1}{2}(a - c - \eta_i), \tag{2.2}$$

$$\pi_i^A = \frac{1}{4}(a - c - \eta_i)^2, \tag{2.3}$$

$$W_i^A = \frac{3}{8}(a - c - \eta_i)^2 - \frac{1}{2}\psi\eta_j\left(a - c - \eta_j\right), \tag{2.4}$$

$$E_i^A = \frac{1}{2}\eta_i(a - c - \eta_i), \tag{2.5}$$

$$ex_i^A = E_i^A + \psi E_j^A = \frac{1}{2}\left[(a - c)\left(\eta_i + \psi\eta_j\right) - \eta_i^2 - \psi\eta_j^2\right], \tag{2.6}$$

$$ex_G^A = ex_i^A + ex_j^A = \frac{1}{2}(1 + \psi)\left[(a - c)\left(\eta_i + \eta_j\right) - \eta_i^2 - \eta_j^2\right], \qquad j \neq i, \tag{2.7}$$

where G denotes the global negative environmental externality.

2.3 Trade liberalization

In this section, we want to investigate the firms behavior and governvent policies after trade liberalization. In what follows, we construct a two-stage game. In the first stage, governments determine the level of tax and tariff and in the second stage, the two firms simultaneously choose their outputs.

2.3.1 The firms' equilibrium

By backward induction, we first solve the two international Cournot competitors problem when choosing their export and home production levels, q_{ej} and q_{hi} respectively. The problem facing firm i is

$$\max_{q_{hi}, q_{ei}} \pi_i = (a - q_{hi} - q_{ej})\, q_{hi} + (a - q_{hj} - q_{ei} - \theta_j)\, q_{ei} - (c + \tau_i)(q_{hi} + q_{ei}),$$

Taking the FOCs, we obtain the following reaction functions

$$\frac{\partial \pi_i}{\partial q_{hi}} = a - 2q_{hi} - q_{ej} - c - \tau_i = 0,$$

$$\frac{\partial \pi_i}{\partial q_{ei}} = a - q_{hj} - 2q_{ei} - c - \tau_i - \theta_j = 0.$$

Solving the FOCs of both firms simultaneously, we find

$$q_{hi}^{CN} = \frac{1}{3}\left(a - c - 2\tau_i + \tau_j + \theta_i\right), \tag{2.8}$$

$$q_{ei}^{CN} = \frac{1}{3}\left(a - c - 2\tau_i + \tau_j - 2\theta_j\right), \tag{2.9}$$

where CN denotes the Cournot-Nash equilibrium. From equations (2.8)-(2.9) it is found that $\partial q_{hi}^{CN}/\partial \tau_i = \partial q_{ei}^{CN}/\partial \tau_i = -2/3 < \partial q_i^A/\partial \tau_i = -1/2$, which implies that, first, the firm i reacts to the tax levied by the home government by reducing her output and, second, this reaction is stronger compared to the autarky. However, the firm reaction to the foreign tax is opposite. As the foreign government increases the tax rate, the domestic firm enhances her output, i.e. $\partial q_{hi}^{CN}/\partial \tau_j = \partial q_{ei}^{CN}/\partial \tau_j > 0$.

Furthermore, the level of import decreases as the government of the home country increases the level of tariff ($\partial q_{ej}^{CN}/\partial \theta_i < 0$), and, consequently, the home firm's production increases ($\partial q_{hi}^{CN}/\partial \theta_i > 0$).

2.3.2 The noncooperative government policies

Now, knowing the firms' behavior in the second stage, we move to the first stage where the environmental taxes and tariffs are determined by governments as Stackelberg strategic leaders. The total welfare of each country is defined as the summation of consumer surplus, the firm's profits, tax and tariff revenues minus the negative environmental externality caused by both home and foreign firms. Thus, the government's problem in country i is defined

$$\max_{\tau_i, \theta_i} W_i^{CN} = \pi_i^{CN} + CS_i^{CN} + \tau_i q_i^{CN} + \theta_i q_{ej}^{CN} - ex_i^{CN}. \tag{2.10}$$

We consider a non-cooperative game where each country unilaterally make decision about the environmental tax and tariff to maximize its own national welfare, and ignores its impact on the other. This problem is done with governments choosing their pollution tax τ_i and tariff θ_i and knowing the reactions of both firms in the second stage. Thus, the FOCs are

$$\frac{\partial W_i^{CN}}{\partial \tau_i} = \frac{1}{9}\left(12\eta_i - 6\psi\eta_j - 7\tau_i - \tau_j + 3\theta_i + 2\theta_j - 4(a-c)\right) = 0,$$

$$\frac{\partial W_i^{CN}}{\partial \theta_i} = \frac{1}{3}\left(a - c - \eta_i + 2\psi\eta_j + \tau_i - \tau_j - 3\theta_i\right) = 0.$$

Solving the FOCs of the problems of both governments simultaneously, we find the equilibrium amount of pollution tax and tariff

$$\tau_i^* = \frac{1}{96}\left[167\eta_i - 43\eta_j + 32\psi\left(\eta_i - 2\eta_j\right) - 28(a-c)\right], \tag{2.11}$$

$$\theta_i^* = \frac{1}{48}\left[16(a-c) + 19\eta_i - 35\eta_j + 16\psi\left(\eta_i + \eta_j\right)\right]. \tag{2.12}$$

As it can be seen from (2.11), $\partial\tau_i^*/\partial\eta_i = (167 + 32\psi)/96 > \partial\tau_i^A/\partial\eta_i = 1$, therefore, as the firms' emission rates increase the home government increases the tax level. And, surprisingly, this taxation is stronger as the rate of spill-over rises. Futhermore, in the international competition, firms faces a stronger environmental taxation compared to autarky. However, the government's reaction to the incease in the foreign firm's pollution is reduction in levied tax on his home firm, i.e. $\partial\tau_i^*/\partial\eta_j < 0$. Also, interestingly, we can see that the equilibrium level of tariff on imports increases with the domestic rate of pollution production, i.e. $\partial\theta_i^*/\partial\eta_i > 0$. In addition, this tariff decreases when the foreign pollution increases, $\partial\theta_i^*/\partial\eta_j < 0$. Furthermore, we can see that $\partial\tau_i^*/\partial\psi > 0$ when $\eta_j < \eta_i/2$, and $\partial\theta_i^*/\partial\psi > 0$.

Finally, the market equilibrium becomes

$$\begin{aligned} q_{hi}^* &= \frac{1}{96}\left[52(a-c) - 113\eta_i + 61\eta_j - 32\psi\left(\eta_i - 2\eta_j\right)\right], \\ q_{ei}^* &= \frac{1}{96}\left[20(a-c) - 79\eta_i + 59\eta_j - 32\psi\left(2\eta_i - \eta_j\right)\right]. \end{aligned}$$

Then the total output of firm i is

$$q_i^* = \frac{1}{4}\left[3(a-c) - 8\eta_i + 5\eta_j - 4\psi\left(\eta_i - \eta_j\right)\right]. \tag{2.13}$$

Consequently, the negative externality produced by the home and foreign firms in country i is

$$ex_i^T = \eta_i q_i^* + \psi \eta_j q_j^*, \tag{2.14}$$

where T denotes the case of trade liberalization. Therefore, the global pollution and the total negative externality caused by firms' production are

$$E_G^T = \eta_i q_i^* + \eta_j q_j^*, \tag{2.15}$$

$$ex_G^T = (1+\psi) E_G^T, \tag{2.16}$$

where G stands for the global.

2.4 Trade *vs* autarky

In this section, we compare the autarkic equilibrium with the noncooperative restricted trade equilibria. We want to know how the asymmetric pollution spill-over makes some differences.

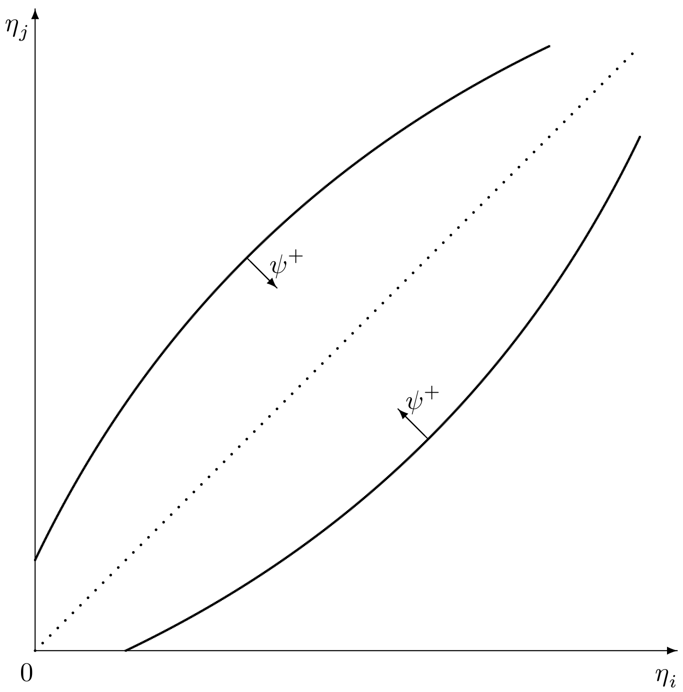
2.4.1 Global pollution

Comparing (2.7) and (2.16), yields

$$\begin{aligned}\Delta ex_G &= ex_G^T - ex_G^A \\ &= \frac{1}{4}(1+\psi)\left[(a-c)(\eta_i+\eta_j) - \left(6\eta_i^2 + 6\eta_j^2 - 10\eta_i\eta_j + 4\psi(\eta_i-\eta_j)^2\right)\right]\end{aligned} \tag{2.17}$$

In the symmetric case where $\eta_i = \eta_j = \eta$, we have $\Delta ex_G|_{i=j} = \frac{\eta}{2}(1+\psi)(a-c-\eta)$. Since from (2.2) we know that $a-c-\eta \geq 0$, the total externality and globa pollution in bilateral trade is larger than autarky. However, in the asymmetric case $\eta_i \neq \eta_j$, there exists a range of parameter in which global pollution and consequently total negative externality in the autarkic equilibrium are larger than the restricted trade. In figure 1, for a given value of $a-c$, Δex_G is depicted in the space of (η_i, η_j). In the region between the two curves Δex_G is positive, therefore, trade liberalization is detrimental for the environment if the two firms' rates of emissions are almost equal. Note that the dotted line in this and the following figures represent the points where $\eta_i = \eta_j$. On this points we have $E_G^T > E_G^A$, which is consistent with the existing literature with symmetric emission rate.

Figure 1: Global pollution comparison.



The region beyond the curves represents the points where $\Delta ex < 0$. Therefore, for a wide range of asymmetric emission rates, trade liberalization not only is not a bad news for the environment but also it could even make reduction in the environmental damages. This result is contrary to the almost all of the previous studies where they argue that trade liberalization leads to increase in environmental pollution.

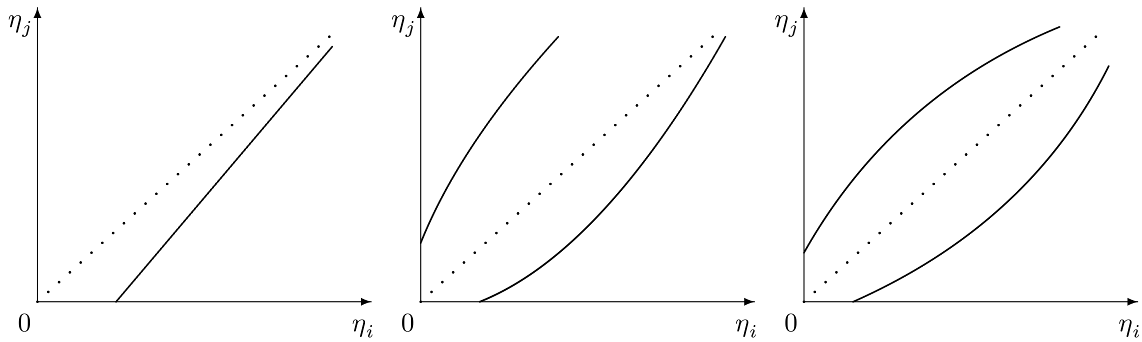
As it is shown in figure 1, as the rate at which pollution crosses borders, ψ, increases the region where the restricted trade is environmentally detrimental shrinks. Thus, for a pure global environmental problem (i.e. $\psi = 1$) the region where international trade compared to autarky is environmental friendly becomes even larger.

2.4.2 Externality in home country

A part of the negative externality caused by polluting production is created by the domestic firm and another part by the foreign firm because of having transboundary pollution ($\psi > 0$). Therefore, in the case of autarky we still have the negative environmental effect of foreign firm activity. In order to compare the negative externality in country i in the international trade framework with the autarky, we should compare (2.6) with (2.14) which yields

$$\Delta ex_i = ex_i^T - ex_i^A = \frac{1}{4}\eta_j \left[(a-c) + 5\eta_i - 6\eta_j + 4\psi\left(\eta_i - \eta_j\right)\right]. \qquad (2.18)$$

Figures 2: Negative externality comparison in country i where a) $\psi = 0$, b) $\psi = 1/2$, c) $\psi = 1$.



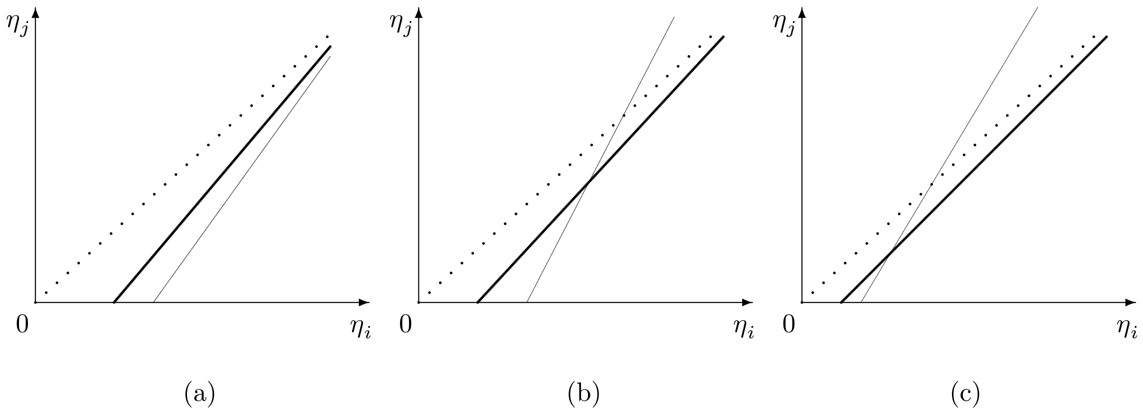
(a) (b) (c)

It can be easily shown that in the case of international trade competition environmental damages is larger than the autarky provided that $\eta_i = \eta_j$. For the general values of emission rates, in figures 2, we have plotted Δex_i in the space of (η_i, η_j), for the case of: a) pure local pollution, $\psi = 0$, b) an example of transboundary pollution, $\psi = 1/2$, and c) pure global pollution, $\psi = 1$. In these figures, only in the regions between the two curves (in figure 2a, between the curve and vertical axis) trade will increase the negative environmental externality in country i.

2.4.3 Output

Considering (2.2) and (2.13), in figures 3, we have shown the region on the right side of the ticker curves where firm i produces more in international competition. Thus, provided that the firm i's pollution spill-over is sufficiently larger than her rival's; her total production in the presence of international trade is lower than the case of autarky. This is a very good news for environmentalist which even noncooperative environmental and trade policies make the more environmentally inefficient firm to reduce her production.

Figures 3: Total output comparison for firm i where a) $\psi = 0$, b) $\psi = 1/2$, c) $\psi = 1$.

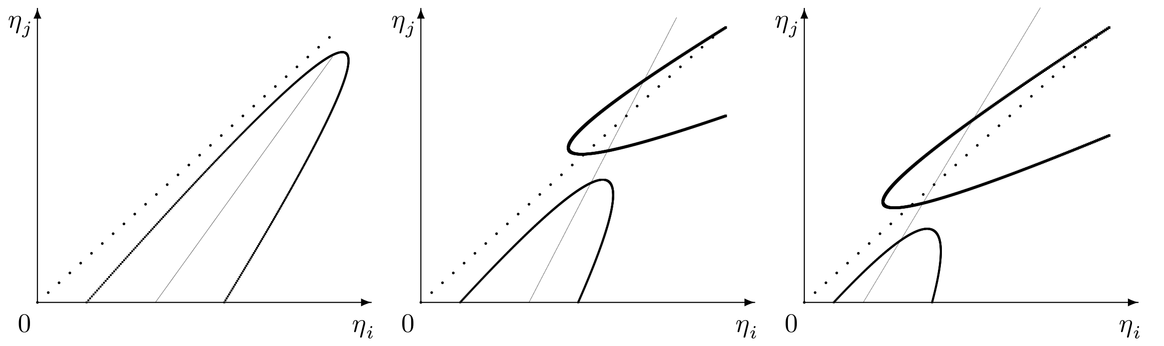


In the figures 3 and 4, the thinner lines represent the points where below them $q^*_{ei} < 0$, and therefore, there is not any export by the firm i to the country j.

2.4.4 Profits and welfare

Finally we want to examine the profitability and welfare consequences of trade liberalization.

Figures 4: Profit and welfare comparison in country i where a) $\psi = 0$, b) $\psi = 1/2$, c) $\psi = 1$.



(a) (b) (c)

Figures 4, in the space of (η_i, η_j), shows the regions inside the curves where firm's profit and social welfare of country i under the autarky are larger than the ones in international competition. The lower curves represent the points where firm's profits in autarky and international trade are equal, and the upper curves characterizes the points where the social welfare in autarky and trade are the same. Consitent with the other studies, firms' profits in the case of symmetric pollution spill-overs decreases in trade liberalization. However, in the case of asymmetric pollution spill-over, trade liberalization decreases the firm's profits provided that her pollution spill-over is sufficiently larger than her rival in international competition.

In the case of pure local pollution, trade liberalization always increases total welfare. As ψ increases the regions where firms profits in autarky is larger than international trade shrink and the regions where social welfare in autarky is larger than international trade expand. However, although in the presence of transboundary pollution governments prefer autarky rather than international competition when firms production functions are the same, they prefer restricted trade where they use environmental and trade policies rather than autarky provided that the home firm's pollution spill-over is lower than her rival.

2.5 Concluding Remarks

In this paper, we considered a two-country world model with a single polluting firm in each country to examine the welfare implications of trade liberalization when governments behave strategically using environmental policy with asymmetric pollution spillovers. We investigated a second-best trade agreement between two countries to examine the strategic behavior of governments in using pollution taxes and tariffs. We found that when the marginal cost of pollution of the domestic firm increases, the pollution-shifting motive is enhanced and government wants to raise production taxes and surprisingly the rent-seeking behavior is observed and government raises import tariffs. On the other hand, when the marginal cost of pollution of the foreign firm increases, government want to reduce the level of tax and interestingly the level of tariff as well. Furthermore, it is shown that how the level of taxes may increase or decrease when/as the rate at which pollution crosses borders rises. We also show that, because of asymmetric pollution spill-over, the global pollution may decrease after trade liberalization.

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Chapter 3

Non-Tradeable Pollution Permits as Green R&D Incentives

3.1 Introduction

The regulation of industries producing negative environmental effects is a hot issue in the current literature. Most of the existing contributions in the field of environmental economics examine the existence of Pigouvian taxation aimed at inducing firms to reduce damaging emissions directly[1] or indirectly[2] (for an exhaustive overview, see Bovenberg and Goulder, 2002; and Requate, 2005). Another possibility consists in assigning firms pollution rights, which in turn can be tradable.[3] The latter, in general, is indeed a short run remedy that in principle does not modify the nature of the production technology used by firms, while clearly in the long run it would be best to attain new environmental-friendly technologies.

A comparatively limited number of contributions investigate the link between some forms of environmental regulation and the incentives to generate

[1] See Bergstrom *et al.* (1981), Karp and Livernois (1992, 1994), Benchekroun and Long (1998, 2002), Poyago-Theotoky (2007) and Tsur and Zemel (2008).

[2] To this regard, see Downing and White (1986), Milliman and Prince (1989), Damania (1996), Chiou and Hu (2001), Tsur and Zemel (2002) and Dragone *et al.* (2010).

[3] See von der Fehr (1993), Sartzetakis (1997), Tietenberg (2003) and MacKenzie (2011), *inter alia*. For a modelization of the auction design for the allocation of pollution rights and the resulting R&D incentives to abate pollution in a Cournot duopoly, see Sunnevåg (2003).

and adopt green technologies or pollution-abatement measures.[4] In particular, Laffont and Tirole (1996) argue that pollution permits diminish or eliminate altogether firms' incentives towards green R&D because once a firm has acquired the right to pollute then she finds it more convenient to leave aside any uncertain and costly project eventually yielding a green technology. A qualitatively similar conclusion is reached by Damania (1996) in a supergame where quantity-setting firms aims at stabilising collusion while considering the feasibility of green R&D project, being subject to Pigouvian taxation.

Our aim is to nest into this debate by modelling the interplay between the costly acquisition of pollution rights on one side and green innovation incentives on the other, so as to single out the possibility for a public agency aiming at preserving the environment to design the distribution of pollution rights as an instrument to foster environmental R&D. The mechanism yielding this result can be intuitively explained as follows. Instead of modelling an auction for pollution rights, we envisage the possibility that, in order to acquire them, firms must participate to a lottery controlled by the government. If the outcome of such lottery is the assignment of pollution rights to a limited number of firms (say, one), the losers face two alternatives: the first is to stay out of the market, the other is to enter with a clean technology. In view of this, the regulator may set up the lottery with this in mind, expecting to get two eggs in one basket. That is, awarding, say, monopolistic pollution rights to a single firm may not necessarily force the regulator to accept a suboptimal trade-off between market power (and the associated negative price effect) and pollution abatement, provided that - with some positive probability - losers are going to innovate and enter the market with new clean technologies. To illustrate this perspective, we adopt a simple model involving two firms, that choose whether to participate in the lottery or try their luck in an uncertain R&D project aimed at the attainment of a green technology. We characterise (i) the equilibria of the

[4] See Jung *et al.* (1996); Denicolò (1999); and Scotchmer (2011). Montero (2002) compares the R&D incentives across a number of possible policy instruments, including emission standards and either auctioned or tradeable permits.

game between firms, based on expected profit incentives, (ii) the consequences on consumer surplus, and (iii) the social preferences over alternative scenarios. The outcome of our analysis is that there exists a non-empty region of parameters where social and private incentives are indeed aligned, in such a way that at least one firm prefers to invest in R&D, so that it appears that assigning pollution rights via the lottery can be taken - at least indirectly - as a means to drive profit-seeking firms to invest their resources in green technologies even in absence of taxation or subsidization.

The remainder of the paper is structured as follows. Section 2 illustrates the setup. Section 3 briefly outlines the problem from the consumers' viewpoint. In section 4, the firms' equilibrium behaviour is illustrated. Section 5 assesses the social welfare consequences of market equilibria. An example based on the Cournot model is contained in section 6. Section 7 concludes the paper.

3.2 The setup

Consider a one-shot non cooperative game played by two single-product firms, indexed by $i = 1, 2$, supplying a homogeneous good with the same marginal cost. Initially, they share the same brown technology, whereby the production of the final output creates a negative externality in the form of polluting emissions. We suppose that, to mitigate the environmental implications of this technology, the government introduces a regulation according to which if a firm wants to produce she must not pollute the environment or she has to buy the pollution right which is sold by the government only to one firm. Therefore, at the outset, each firm faces the following perspective:

- she can take part in a lottery for emission rights. The exogenous individual probability of winning the lottery is $p = 1/2$, and the winner must pay a fixed fee F to the government in order to acquire the emission permit. Since we may suppose that F is redistributed among consumers as windfall money, the total effect of these costs on welfare is nought. The loser incurs a fixed cost Γ to shut down and quit the market. Alternatively,

- the firm may invest a given amount, K, to attain a green technology which comes out of the R&D division with probability $\alpha \in [0, 1]$. If so, she has the right to produce as her technology is now clean. If, instead, the R&D project yields no results, the firm, besides K, incurs a fixed cost Γ to quit the market. The innovation is patentable; in case both firms innovate, the authority allows both of them to patent the new technology and a symmetric green duopoly obtains.

In line with this setting, we have three cases: (i) both firms participate in the lottery for pollution rights; (ii) both invest in R&D looking for a green technology; (iii) one buys the pollution rights while the other invests in search of the green technology. In all cases, the marginal cost of production remains the same, the only difference between the two technologies being that one is clean and the other is not.

Therefore, denoting the participation in the lottery as L and the search for a green technology as G, we have the 2×2 game shown in matrix 1.

		2: L	2: G
1	L	$E\pi^{LL}, E\pi^{LL}$	$E\pi^{LG}, E\pi^{GL}$
1	G	$E\pi^{GL}, E\pi^{LG}$	$E\pi^{GG}, E\pi^{GG}$

Matrix 1

Here, $E\pi^{GG}$, $E\pi^{GL}$, $E\pi^{LG}$ and $E\pi^{LL}$ are firms' expected profits when both invest in green technology, one of them invests in green technology and the other buys the pollution permit, or both take part in the lottery, respectively.

Consider first the scenario where both firms are participating in the lottery. In this case, the winner becomes a monopolist and makes monopoly profits, while the loser gets no revenues and also incurs a fixed cost Γ. Therefore, the individual expected profits in this case are

$$E\pi^{LL} = \frac{\pi_M - F - \Gamma}{2}, \tag{3.1}$$

where π_M is gross monopoly profit. The non-negativity of $E\pi^{LL}$ requires $\pi_M > F + \Gamma$.

Alternatively, when both firms invest in R&D for a green technology, the expected profits for each firm are

$$E\pi^{GG} = -K + \alpha\left[(1-\alpha)\pi_M + \alpha\pi_D\right] - (1-\alpha)\,\Gamma. \tag{3.2}$$

Expression (3.2) consists of the R&D cost and the sum of (i) monopoly profits if the firm succeeds in innovating before the other and get the exclusive patent, and (ii) gross duopoly profits, $\pi_D < \pi_M$, if both firms show up simultaneously at the patent office with the green technology to get it patented on parallel, as it is rational for a smart government to have a totally green duopoly combining environmental friendly production with the equally desirable of output expansion on market price and therefore on consumer surplus.

In the third case, in which one invests to attain a clean technology and one participates in the lottery, since the firm which takes part in the lottery is the only potential buyer, she will obtain the pollution permit for sure and her expected payoff is

$$E\pi^{LG} = -F + (1-\alpha)\pi_M + \alpha\pi_D, \tag{3.3}$$

which depends on whether the rival succeeds in innovating or not. Accordingly, the maximum willingness to pay for the emission right cannot exceed $(1-\alpha)\pi_M + \alpha\pi_D$. In this scenario, the expected payoff for the firm activating the R&D project is

$$E\pi^{GL} = -K + \alpha\pi_D - (1-\alpha)\,\Gamma. \tag{3.4}$$

Hence, the game has a two-stage structure, where the first stage describes the firms' choice between taking part in the lottery or investing in green R&D, and the second models market behaviour. Moves are simultaneous in both stages, with complete, symmetric and imperfect information in each, while strategies taken at the first stage are observable to firms prior to playing the second stage. The solution concept is subgame perfection by backward induction.

3.3 The consumers' view point

We shall now have a look at the level of consumer surplus generated by consumption (and therefore gross of the redistribution of F) in the three different perspectives:

(G,G) In this case, the expected surplus for consumers is the aggregate amount of the monopolistic consumer surplus if one firm innovates and the other one does not, and the duopolistic consumer surplus if both firms attain the innovation:

$$ECS^{GG} = 2\alpha(1-\alpha)CS_M + \alpha^2 CS_D, \tag{3.5}$$

where CS_M and CS_D are the levels of consumer surplus in monopoly and duopoly, respectively.

(L,G) or (G,L) If one firm buys the pollution permit and the other invests in R&D, the expected consumer surplus becomes

$$ECS^{LG} = ECS^{GL} = (1-\alpha)CS_M + \alpha CS_D - (1-\alpha)E_M - \alpha E_D, \tag{3.6}$$

illustrating the fact that depending on the probability of innovation, consumers incur some amount of negative externality either in monopoly, E_M, or in the asymmetric duopoly where only one of the two firms creates a negative externality, $E_D (< E_M)$.

(L,L) If both firms participate in the lottery, one of them wins it and becomes a monopolist with the existing brown technology, which obviously entails a negative externality for consumers:

$$ECS^{LL} = CS_M - E_M. \tag{3.7}$$

By comparing these functions, we have

$$\Delta_{cs}^{gl,ll} \equiv ECS^{GL} - ECS^{LL} = \alpha \left[CS_D - CS_M + E_M - E_D \right], \tag{3.8}$$

$$\Delta_{cs}^{gg,gl} \equiv ECS^{GG} - ECS^{GL} = (1-\alpha)\left[(2\alpha-1)\,CS_M - \alpha CS_D + E_M - E_D\right] + E_D, \tag{3.9}$$

$$\Delta_{cs}^{gg,ll} \equiv ECS^{GG} - ECS^{LL} = (2\alpha(1-\alpha)-1)\,CS_M + \alpha^2 CS_D - E_M. \tag{3.10}$$

Since we know that $CS_D > CS_M$ and $E_M > E_D$, it is easily shown that $\Delta_{cs}^{gl,ll} > 0$ which means that consumers prefer the perspective in which one firm goes for the green technology rather than that in which both take part in the lottery. For the other two expressions, (3.9) and (3.10), we find that as α increases $\Delta_{cs}^{gg,gl}$ and $\Delta_{cs}^{gg,ll}$ increase monotonically and, for sufficiently large values of α, they become positive. Therefore, while from the consumers' standpoint having one firm investing in green technologies and the other buying pollution rights is more desirable than having both involved in the lottery for the pollution rights, consumers dislike the idea that both firms may disregard pollution permits and to invest symmetrically in clean technologies unless the probability of successful innovation be sufficiently high.

Having characterised consumer preferences concerning the strategi behaviour of firms, there remains to assess the pivotal role of the R&D cost K in determining whether there exists a parameter range wherein social and private incentives are indeed reciprocally aligned.

3.4 Equilibrium analysis

Here, we characterise the subgame perfect equilibrium solution of the non cooperative game between the two firms, based on the examination of matrix 1. The shape of firms' strategic behaviour is essentially determined by probability α as well as the relative size of costs F and K.

Considering (3.1), (3.2), (3.3) and (3.4), we have

$$\Delta_{\pi}^{gl,ll} \equiv E\pi^{GL} - E\pi^{LL} = \frac{1}{2}(F - \pi_M) + \alpha\pi_D - \left(\frac{1}{2} - \alpha\right)\Gamma - K, \tag{3.11}$$

$$\Delta_{\pi}^{gg,lg} \equiv E\pi^{GG} - E\pi^{LG} = F - (1-\alpha)^2 \pi_M - \alpha(1-\alpha)\pi_D - (1-\alpha)\,\Gamma - K, \quad (3.12)$$

$$\Delta_{\pi}^{gg,ll} \equiv E\pi^{GG} - E\pi^{LL} = \frac{1}{2}F - \left(\frac{1}{2} - \alpha\,(1-\alpha)\right)\pi_M + \alpha^2\pi_D - \left(\frac{1}{2} - \alpha\right)\Gamma - K. \quad (3.13)$$

If the right hand sides of (3.11)-(3.13) are simultaneously positive, investing in search of the green technology is a dominant strategy for both firms and (G, G) emerges as the unique and Pareto-efficient equilibrium of the game. The game is instead a prisoners' dilemma with (G, G) as the unique but Pareto-inefficient equilibrium if the RHS of (3.11-3.12) is positive while (3.13) is negative. Independently of the nature of the resulting equilibriium, we may compare $\Delta_{\pi}^{gl,ll}$, $\Delta_{\pi}^{gg,lg}$ and $\Delta_{\pi}^{gg,ll}$, which results in:

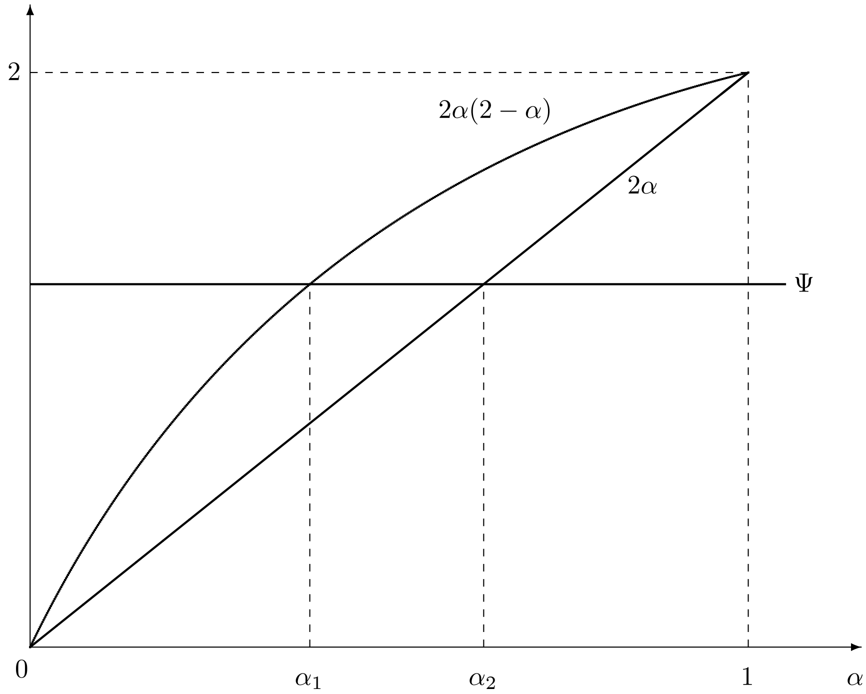
$$\Delta_{\pi}^{gg,lg} \gtrless \Delta_{\pi}^{gl,ll} \,\forall\, 2\alpha\,(2-\alpha) \gtrless \frac{\pi_M - F + \Gamma}{\pi_M - \pi_D} \equiv \Psi > 1, \quad (3.14)$$

$$\Delta_{\pi}^{gg,lg} \gtrless \Delta_{\pi}^{gg,ll} \,\forall\, 2\alpha \gtrless \frac{\pi_M - F + \Gamma}{\pi_M - \pi_D} \equiv \Psi > 1, \quad (3.15)$$

$$\Delta_{\pi}^{gg,ll} > \Delta_{\pi}^{gl,ll} \text{ always.} \quad (3.16)$$

The fact that inequality (3.16) is met over the entire admissible parameter space means that if a firm finds that the case where she invests in R&D and her rival buys the pollution right is more profitable than the case in which both take part in the lottery for pollution rights, certainly she prefers the symmetric green R&D outcome rather than the symmetric lottery; therefore, $E\pi^{GG} > E\pi^{GL}$. In order to assess the sign of the other two inequalities, in figure 1 we plot the two curves $2\alpha\,(2-\alpha)$ and 2α and the straight line Ψ. Depending on the value of α, we have three domains where the sign of the inequalities changes: $(0, \alpha_1)$, (α_1, α_2) and $(\alpha_2, 1)$.

Figure 1 :



For $\alpha \in (0, \alpha_1)$,[5] we have $\Delta_{\pi}^{gg,lg} < \Delta_{\pi}^{gl,ll}$ and $\Delta_{\pi}^{gg,lg} < \Delta_{\pi}^{gg,ll}$. Thus, if $F - K$ is high enough (i.e. the cost of R&D is sufficiently lower than the cost of pollution rights) such that $\Delta_{\pi}^{gg,lg} > 0$, both firms find it profitable to invest in green technologies and (G, G) is the Pareto efficient equilibrium of matrix 1. If instead $F - K$ is sufficiently low such that $\Delta_{\pi}^{gl,ll} < 0$, the equilibrium of the game is (L, L). The remaining situation is where $\Delta_{\pi}^{gg,lg} < 0$ and $\Delta_{\pi}^{gl,ll} > 0$. In this case, as it is discussed in Proposition 1, we have asymmetric equilibria along the secondary diagonal in chicken game where one firm buys the pollution right while the other invests in green technology.

In the region $(\alpha_1, 1)$, we have $\Delta_{\pi}^{gg,lg} > \Delta_{\pi}^{gl,ll}$. Therefore, if $\Delta_{\pi}^{gl,ll} > 0$, the unique and Pareto efficient equilibrium is (G, G). Where instead $\Delta_{\pi}^{gg,lg} < 0$, the equilibrium is (L, L) and in (α_1, α_2) this is a Pareto inefficient equilibrium of a

[5] It is worth mentioning that since $\Psi > 1$, this region always exists. In the cases $\Gamma > \pi_M - 2\pi_D + F$ or $\pi_D \gg F$, Ψ could become higher than 2 and, therefore, $\alpha_1 = 1$.

prisoner's dilemma game. In the other cases (i.e. $\Delta_{\pi}^{gl,ll} < 0$ and $\Delta_{\pi}^{gg,lg} > 0$), the matrix becomes a coordination game with the two equilibria on the main diagonal of matrix 1, i.e., either both firms participate in the lottery or both invest in search of the green technology. In such a coordination game and in the region $(\alpha_2, 1)$, we have $E\pi^{GG} > E\pi^{LL}$.

The above discussion yields the following result:

Proposition 14 *A chicken game, with $E\pi^{GL} > E\pi^{LL}$ and $E\pi^{LG} > E\pi^{GG}$ and therefore (G, L) and (L, G) are Nash equilibria, may arise provided that Γ and α are sufficiently large. Otherwise, only symmetric equilibria are observed.*

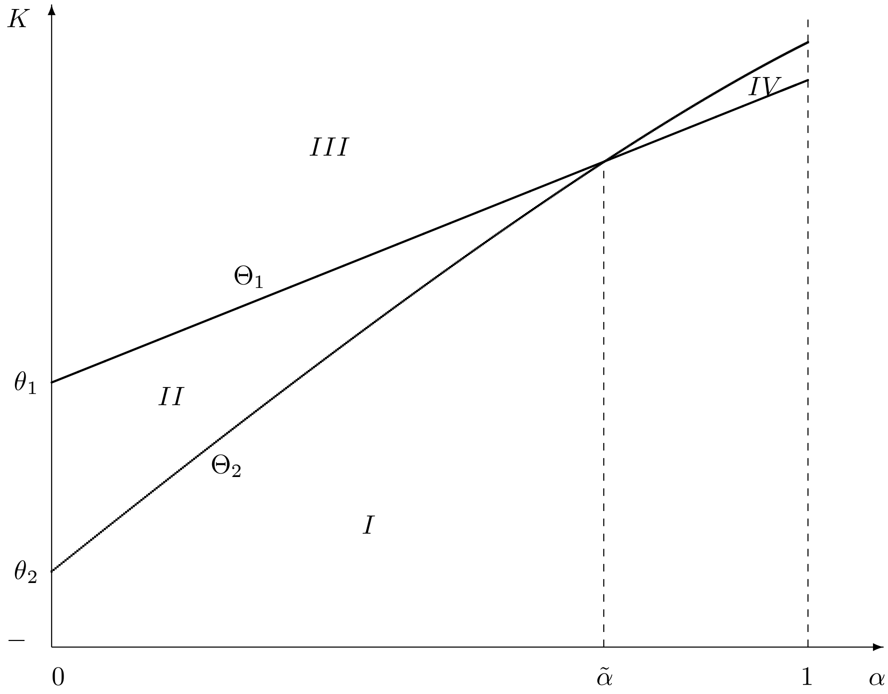
Proof. Considering (3.11) and (3.12), we have

$$\Delta_{\pi}^{gl,ll} > 0 ... \forall .. K < -\frac{1}{2}\pi_M + \alpha\pi_D - \left(\frac{1}{2} - \alpha\right)\Gamma + \frac{1}{2}F \equiv \Theta_1, \tag{3.17}$$

$$\Delta_{\pi}^{gg,lg} < 0 ... \forall .. K > -(1-\alpha)^2\pi_M - \alpha(1-\alpha)\pi_D - (1-\alpha)\Gamma + F \equiv \Theta_2. \tag{3.18}$$

It is obvious that both Θ_1 and Θ_2 are upward sloping with respect to α. In figure 2, we plot the two curves Θ_1 and Θ_2 against α.

Figure 2 :



In this figure, as it can be checked from (3.17) and (3.18), at $\alpha = 0$, we have $\theta_2 \left(= F - \pi_M - \Gamma\right) < \theta_1 \left(= \frac{1}{2}\left(F - \pi_M - \Gamma\right)\right) < 0$. There could be an intersection, $\tilde{\alpha}$, between the curves provided that $\Gamma < \pi_M - 2\pi_D + F$, otherwise, $\tilde{\alpha} = 1$. This yields $\Theta_1 > \Theta_2$ in the region $(0, \tilde{\alpha})$. Therefore, there exists a viable range of parameters between the two curves, region II, where matrix 1 becomes a chicken game with asymmetric equilibria along the secondary diagonal of the 2×2 game where one firm buys the pollution right and the other goes for the green technology. However, in the vicinity of $\alpha \rightsquigarrow 0$, in order to have at least one firm investing in green technologies K should be negative, which is economically inadmissible. Thus the probability of successful innovation must be high enough so as for the firms to have an incentive to invest in R&D. Finally, in regions I, III and IV we observe a pure coordination game along the main diagonal, as both (G, G) and (L, L) are Nash equilibria. ■

3.5 Social optimum

We are now in a position to put together consumers' likings and firms' incentives so as to evaluate social preferences according to the expected welfare levels arising in the three possible cases.

Since the pollution permit fees are going to be redistributed across consumers, the social planner decides only on the basis of K (and not F). Then, the expected amount of social welfare in each case is as follows:

$$ESW^{GG} = -2K + 2\alpha(1-\alpha)SW_M + \alpha^2 SW_D - 2(1-\alpha)\Gamma, \tag{3.19}$$

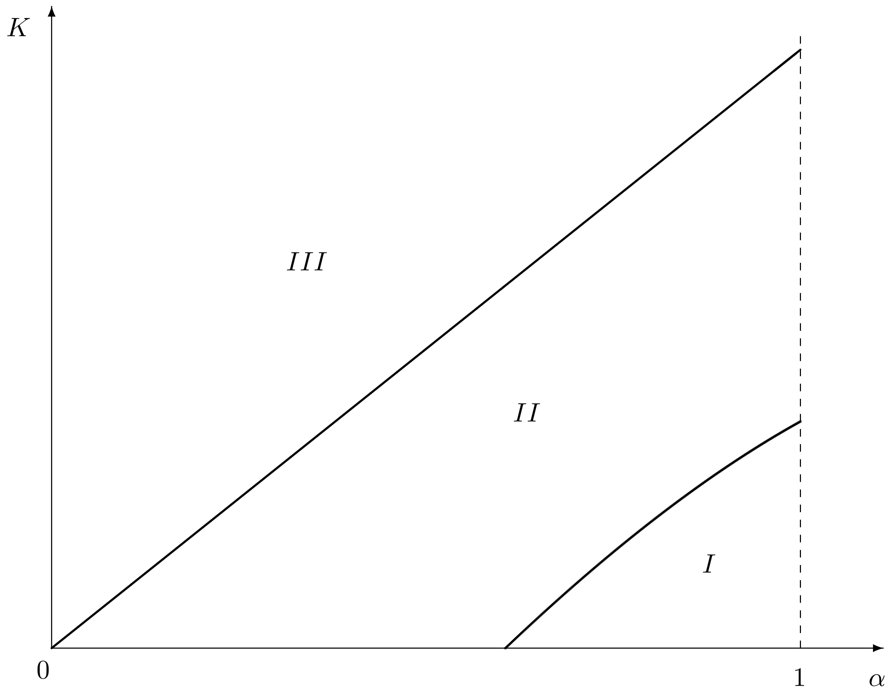
$$ESW^{LG} = ESW^{GL} = -K + (1-\alpha)SW_M + \alpha SW_D - (1-\alpha)E_M - \alpha E_D - (1-\alpha)\Gamma, \tag{3.20}$$

$$ESW^{LL} = SW_M - E_M - \Gamma. \tag{3.21}$$

in which SW_M and SW_D are the social welfare levels (gross of external effects) in monopoly and duopoly, respectively.

By comparing (3.19), (3.20), (3.21) and knowing that $SW_D > SW_M$ and $E_D < E_M$, we can characterise social preferences in the space (α, K) as in figure 3.

Figure 3: The upper and lower solid curves, respectively, characterise the points where expected soc



This figure shows that social welfare is highest in GG, LG (or GL) and LL if (α, K) are such that the industry allocation falls in region I, II and III, respectively.

Now, we have to ascertain whether the regions in figure 3 overlap, at least to some extent, with the corresponding regions in figure 2. More precisely, we are looking for conditions ensuring that the two regions labelled as II in figures 2 and 3 do overlap.

Proposition 15 *Provided firms incur a sufficiently large cost to quit the market, there exists a range of parameters wherein profit incentives yield asymmetric equilibria generated by a chicken game where* $E\pi^{GL} > E\pi^{LL}$ *and* $E\pi^{LG} > E\pi^{GG}$ *and such equilibria are also socially efficient.*

Proof. In order to prove the validity of this claim, it suffices to observe that there are infinitely many admissible values of F such that the curves delimiting region II in figure 2 intersect the horizontal axis in figure 3 between the origin and the curve dividing regions I and II in figure 3. ■

For illustrative purposes, in the next section we lay out an example based on the linear Cournot model.

3.6 Example

Consider a market where 2 symmetric firms producing the same homogeneous good with zero marginal cost $c = 0$. The inverse demand function is defined as $p = A - Q$, where $Q = \sum q_i$, $i = 1, 2$ and $q_i \geq 0$ is the individual output of firm i. If firm i wins the lottery or succeeds in her innovation, q_i is strictly positive; otherwise she is not allowed to operate in the market. Accordingly, the industry can be either a monopoly or a duopoly. Therefore, the optimal level of output as well as the corresponding profits are either $q_M = A/2$, $\pi_M = A^2/4$ or $q_D = A/3$ and $\pi_D = A^2/9$, and the resulting social welfare levels (gross of negative externalities) are $SW_M = 3A^2/8$ and $SW_D = 4A^2/9$. To model pollution, we assume that the negative externality is a quadratic function of output, $E = bQ^2/2$. Hence, externalities in the two cases are $E_M = bA^2/8$ and $E_D = bA^2/18$.

Then, plugging profits, social welfare levels and externalities in inequalities (3.12), (3.11) and (3.13), we get

$$E\pi^{GG} > E\pi^{LG} \quad \text{if} \quad k < f - \frac{(1-\alpha)(9-5\alpha)}{36} - (1-\alpha)\gamma, \tag{3.22}$$

$$E\pi^{GL} > E\pi^{LL} \quad \text{if} \quad k < \frac{f}{2} - \frac{(9-8\alpha)}{72} - \left(\frac{1}{2} - \alpha\right)\gamma, \tag{3.23}$$

$$E\pi^{GG} > E\pi^{LL} \quad \text{if} \quad k < \frac{f}{2} - \frac{\left(10\alpha^2 - 18\alpha + 9\right)}{72} - \left(\frac{1}{2} - \alpha\right)\gamma, \tag{3.24}$$

where $k = K/A^2$, $f = F/A^2$ and $\gamma = \Gamma/A^2$.

Note that, in order for firms to have an incentive to take part in the lottery, F must not be greater than π_D, i.e. $f \leq 1/9$, as the firm participating in the lottery may expect the other firm to come up with new technology.

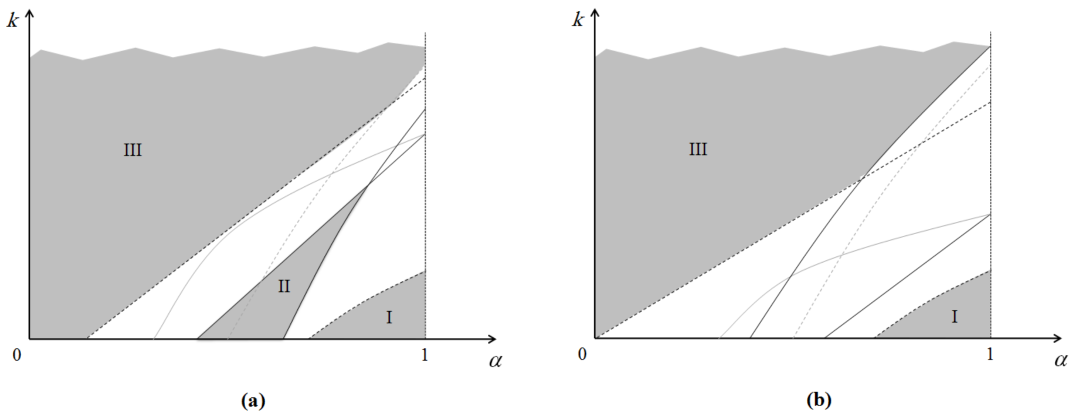


Figure 3.1: The gray aresa represent the regions where firms' strategic incentives and social incentives are reciprovally aligned. In (a) Γ is strictly positive, in (b) $\Gamma \sim 0$.

By comparing (3.19), (3.20) and (3.21) we find

$$ESW^{GG} > ESW^{LG} \text{ if } k < \frac{b(9-5\alpha)-(1-\alpha)(27-22\alpha)}{72} - (1-\alpha)^2\gamma, \quad (3.25)$$

$$ESW^{GL} > ESW^{LL} \text{ if } k < \frac{5\alpha\,(1+b)}{72} - (\frac{1}{2}-\alpha)\gamma, \quad (3.26)$$

$$ESW^{GG} > ESW^{LL} \text{ if } k < \frac{9b+54\alpha-22\alpha^2-27}{72} - \left((1-\alpha)^2+(\frac{1}{2}-\alpha)\right)\gamma. \quad (3.27)$$

Now, we can perform a numerical simulations by normalizing all but two parameters. For instance, taking plausible values $b = 1/5$, $f = 1/10$ and $\gamma = 1/10$, we can plot k against α to assess inequalities (3.22), (3.23), (3.24), (3.25), (3.26) and (3.27). The outcome is illustrated in figure 4.

In regions I, II and III, firms and the social planner alike prefer GG, LG (GL) and LL, respectively. Therefore, it can be seen that, when Γ is not close to zero, there indeed exists a viable range of parameters (area II) where we have a chicken game whose equilibria are also welcome from the planner's viewpoint. This confirms our main point that using the instrument of assigning pollution right through the simple lottery we have modelled here may indeed serve the purpose of creating a side incentive for firms losing the lottery in the first place or deciding not to participate in it to take the alternative root which is to finance

R&D efforts for green technologies.

3.7 Concluding Remarks

There are two main lines of research in modelling the abatement of polluting emissions: the optimal assignment of pollution rights and the introduction of corrective taxes or subsidies to internalize the externality and provide firms with R&D incentives that otherwise would not arise spontaneously.

We have taken an alternative route to highlight the possibility that a mechanism for the costly acquisition of pollution rights might actually turn the losers into green innovators. According to our analysis, it seems indeed that controlling pollution rights may exert - somewhat unexpectedly - some positive long-run impacts on the environmental performance of industries by virtue of indirect innovation incentives that can be considered as the side-effect of the allocation of pollution rights.

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Part II

Dynamic Cournot Oligopoly

Chapter 4

Exogenous Output Constraint in a Dynamic Oligopoly

4.1 Introduction

Consider an industry consisting of N symmetric firms each producing a homogenous output. Then, the output levels of a subset of M $(< N)$ firms are constrained into a constant level. If the remaining firms simultaneously make the best reply to this exogenous constraint, we want to investigate under what circumstances is this to the benefit of constrained subset.

In a series of papers Gaudet and Salant (1991a,b) show that, in the case of Cournot competition among producers of perfect substitutes, a marginal contraction is strictly beneficial if and only if the number of firms in the designated subset exceeds the "adjusted" number of firms outside it by strictly more than one. In the special case of linear cost and demand functions, the firms in the subset will gain from an exogenously marginal contraction of their output if and only if they outnumber the firms outside the subset by more than one.

In this paper we generalize this result to the case of dynamic competition instead of looking at the one-shot game. While in the *standard Cournot model* any output constraint is not to the benefit of constrained firms, in this paper, we show that when firms play a *dynamic Cournot game with Markov-perfect* *strategies* exogenous output constraint by a subset of firms results in: (i) increase in the value of unconstrained firms irrelevant of the amount of constraint because of having less intensive competition, (ii) increase in the market price for any output constraint below the optimal level and slightly above that because of reduction in the total output caused by less competition and (iii) increase in the value of constrained firms for a viable range of parameters and initial conditions because of increasing the price during the price path.

Our analysis has some applications to voluntary export restraints (VER), Mergers, Economics Sanctions, etc. Mai and Hwang (1988) examine VERs in a static duopoly model by using a conjectural variations approach. They find that if the free trade equilibrium is Cournot, a VER set at the free trade level of imports will have no impact on profits. We show that if the free trade equilibrium is Cournot played with Markovian (subgame-perfect) strategies, then the imposition of a VER at the free trade level of imports increases the market price and the profits of the foreign and domestic firm. Hence, the VER is voluntary in the dynamic Cournot model.

Suppose that the subset of firms represents firms that are part of a cartel. Our study explains how it is to their benefit when they agree on producing a constant level of quantity, for example thier optimal steady state level of output before the merger.

Consider the international market in which a group of countries are exporting a specific good. Now, assume that one of these exporting countries is sanctioned by part (not all) of the importing countries. This imposed economic sanction force that country to be constrained to a lower output level which, however, can be to her advantage. Our analysis characterizes circumstances under which economic sanctions are not effective.

The rest of the paper is organized as follows. Section 2, present the model. In section 3, the dynamic equilibria are derived before and after the exogenous output constraint. Circumstances under which the exogenous output constraint is profitable is examined in section 4. In section 5, robustness of the result is checked by conjectural variations equilibrium. Some applications are presented in section 6. Finally, section 7 concludes the paper.

4.2 A dynamic oligopoly model

Consider a dynamic oligopoly market consisting of N symmetric firms each producing a homogenous output. Firms are assumed to produce with strictly concave technologies described by the cost functions

$$C(q_i(t)) = \frac{1}{2}q_i^2(t), \quad i = 1, ..., N, \tag{4.1}$$

where $q_i(t) \geq 0$ is the output of firm i produced at time t. The equilibrium price, $\bar{p}(t)$, in period t is related to industry output by means of an inverse demand function which in its linear version is given by

$$\bar{p}(t) = a - \sum_{i=1}^{N} q_i(t) \tag{4.2}$$

where the units of measurement are chosen such that the slope of the demand curve is -1. Thus, the single period profit function of firm i is given by

$$\pi_i(t) = [a - \sum_{i=1}^{N} q_i(t)]q_i(t) - \frac{1}{2}q_i^2(t). \tag{4.3}$$

Equation (4.3) represents a classical one-shot Cournot game. However, in this paper we want to look at the continuous time dynamic competition where firms are assumed to maximize the discounted stream of profits over an infinite planning horizon with $r > 0$ as the constant discount rate. We are interested in deriving Markov-perfect equilibria for this game. In order to solve for those equilibria, we make use of the "sticky price" model introduced by Fershtman and Kamien (1987). It is given by

$$\max \Pi_i = \int_0^{\infty} e^{-rt}\{p\,(t)\,q_i\,(t) - \frac{1}{2}q_i^2\,(t)\}dt, \tag{4.4}$$

subject to

$$\dot{p}\,(t) = s[a - \sum_{i=1}^{N} q_i\,(t) - p\,(t)];\ p\,(0) = p_0. \tag{4.5}$$

In (4.4) and (4.5) it is assumed that the actual market price deviates from its level given by the demand function but moves towards it with a constant speed of adjustment denoted by s ($0 < s \leq \infty$). Thus, we have sticky prices.

The strategy spaces available to the firms should be specified in order to clearly define the dynamic Cournot game (4.4). The assumption that the industry equilibrium is identified as a subgame-perfect Cournot equilibrium in Markov strategies means that firms design their optimal policies as decision rules dependent on the state variables of the game (in our case price). This means that firms take into account the rivals reactions to their own actions as expressed by the state variables of the game. This is exactly the characteristic present in the case of conjectural variations equilibrium.

4.3 Dynamic equilibria

As motivated in the introduction, we are interested to see whether firms benefit from being forced to act non-strategically or not. To this end, in this section we want to derive the dynamic equilibrium of game (4.4) under two different scenarios. First, we solve for the equilibrium when all the N firms in the industry are strategic players. Next, we consider the scenario where M strategic players are eliminated by being forced to be constrained to a constant level of output and we derive the equilibrium in this scenario.

If value of non-strategic firm increases compared to the unconstrained case, the answer to the question is yes. This is what we focus on in next section.

4.3.1 Unconstrained oligopoly equilibrium

We derive the equilibrium of the model in which firms employ price dependent decision rules when maximizing their discounted profits. Thus, changes in the market price stimulate responses by all players that are reflected in their quantity choices. This corresponds to the recognized interdependence present in oligopolistic markets.

Theorem 16 *There exists a Markov perfect equilibrium of the "sticky price"* *model in an N firm dynamic Cournot oligopoly given by*

$$q^* (p) = p(1 - sK) + sE, \tag{4.6}$$

$$V (p) = \tfrac{1}{2}Kp^2 - Ep + g, \tag{4.7}$$

and

$$p(t) = p^* + (p_0 - p^*)e^{Dt}, \tag{4.8}$$

where p_0 is the initial price and p^* is the steady state price

$$p^* = \frac{a - NsE}{1 + N(1 - sK)}, \tag{4.9}$$

K, E, g and D are defined as

$$K = \frac{2s(N+1) + r - \sqrt{[2(N+1)s + r]^2 - 4s^2(2N-1)}}{2s^2(2N-1)}, \tag{4.10}$$

$$E = \frac{-sKa}{s(N+1) + r - s^2K(2N-1)}, \tag{4.11}$$

$$g = \frac{s^2E^2(2N-1) - 2sEa}{2r}, \tag{4.12}$$

$$D = s[N(sK - 1) - 1]. \tag{4.13}$$

Proof. See Appendix A. ■

The results of Theorem 1 have the following implications. Firstly, the equilibrium quantities of the infinite horizon game do not coincide with that of the one shot game if firms employ Markov strategies. Secondly, firms produce more (and hence market price is lower) in the dynamic game compared to the classical Cournot model. The interpretation of this result arises from the price dependent decision rules (4.6). In particular, with an increase in price firms react by producing more. To see why this causes equilibrium quantities to be closer to the competitive equilibrium consider the following scenario. Assume that a firm i finds it profitable to reduce its equilibrium quantity. This causes the market price to increase. Given the feedback decision rules of the competitors their optimal response to the increasing price is to increase their equilibrium quantities thus offsetting firm i's action. This behavior causes in equilibrium all firms to produce beyond the level of simple Cournot quantities.

4.3.2 Exogenous output constraint

After having characterized the unconstrained equilibrium we assume that a subset of M ($M < N$) strategic players are eliminated by being constrained to a constant level of output, $\bar{q}$ ($0 < \bar{q} < a$). Moreover, we assume that these firms cannot deviate as they are constrained to these output levels. Thus, the game played by the $N - M$ strategic players in the model of sticky prices becomes

$$\max \Pi_i^C = \int_0^\infty e^{-rt}\{p(t)\, q_i(t) - \frac{1}{2} q_i^2(t)\}dt, \quad i = M+1, ..., N, \tag{4.14}$$

subject to

$$\dot{p}(t) = s[a - M\bar{q} - \sum_{i=M+1}^{N} q_i(t) - p(t)];\ p(0) = p_0. \tag{4.15}$$

This provides us with the following result.

Theorem 17 *If a subset of M firms in an N firm dynamic Cournot oligopoly is forced to act non-strategically through being exogenously constrained to the output choices $\bar{q}$, there exists a Markov perfect equilibrium of the "sticky price" model, where the remaining $N-M$ firms play strategically the dynamic Cournot game, given by*

$$\tilde{q}(p) = \hat{p}(1 - s\hat{K}) + s\hat{E}. \tag{4.16}$$

$$\hat{V}(p) = \tfrac{1}{2}\hat{K}p^2 - \hat{E}p + \hat{g}, \tag{4.17}$$

and

$$\hat{p}(t) = \tilde{p} + (p_0 - \tilde{p})e^{\hat{D}t}, \tag{4.18}$$

where p_0 is the initial price and $\tilde{p}$ is the steady state price

$$\tilde{p} = \frac{a - (N-M)s\hat{E} - M\bar{q}}{1 + (N-M)(1 - s\hat{K})}, \tag{4.19}$$

$\hat{K}$, $\hat{E}$, $\hat{g}$ and $\hat{D}$ are defined as

$$\hat{K} = \frac{2s(N-M+1) + r - \sqrt{[2(N-M+1)s + r]^2 - 4s^2(2(N-M)-1)}}{2s^2(2(N-M)-1)}, \tag{4.20}$$

$$\hat{E} = \frac{-s\hat{K}a - s\hat{K}M\bar{q}}{s(N-M+1) + r - s^2\hat{K}(2(N-M)-1)}, \tag{4.21}$$

$$\hat{g} = \frac{s^2\hat{E}^2(2(N-M)-1) - 2s\hat{E}a + 2s\hat{E}M\bar{q}}{2r}. \tag{4.22}$$

$$\hat{D} = s[(N-M)(s\hat{K}-1)-1]. \tag{4.23}$$

And the present value of a non-strategic firm becomes

$$\hat{V}^C = Ap + \hat{g}^C, \tag{4.24}$$

where A and $\hat{g}^C$ are

$$A = \frac{\bar{q}}{r-\hat{D}},$$

$$\hat{g}^C = \frac{\bar{q}(\hat{D}(2\tilde{p}-\bar{q}) + r\bar{q})}{2r(\hat{D}-r)}.$$

Proof. See Appendix B. ■

It is important to note, however, that the behavior of the firms in the subset after being non-strategic does not correspond to an equilibrium. The strategically-playing firms, however, are in dynamic Cournot equilibrium.

4.4 Profitable output constraint

Theorem 18 *Assume that the subset of M strategic players are eliminated by being exogenously constrained to the output choices $\bar{q}$, whereas the remaining $N-M$ firms react strategically to this exogenous change in a dynamic Cournot game with Markov-perfect strategies. This results in an*

(a) increase in the market price for any $\bar{q} \leq q^*$;

(b) increase in the present value of strategic firms irrespective of the amount of $\bar{q}$;

(c) increase in the present value of non-strategic firms for a viable range of parameters and initial conditions.

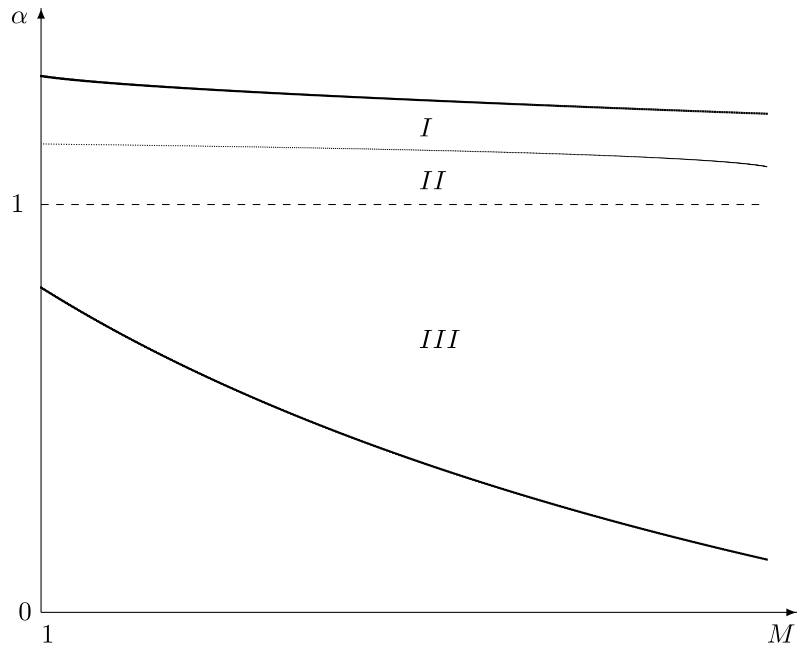
Proof. See Appendix C. ■

However, the steady state price $\tilde{p}$ could be larger than p^* even for some values of $\bar{q}$ above the q^*. Consider $\bar{q} = \alpha q^*$ where $\alpha > 0$, therefore, we have

$$\tilde{p} > p^* \iff 0 < \alpha < -\frac{N\hat{D}}{Ms} - \frac{(N-M)D(a(1-s\hat{K})+s\hat{E})}{Ms(a(1-sK)+sE)},$$

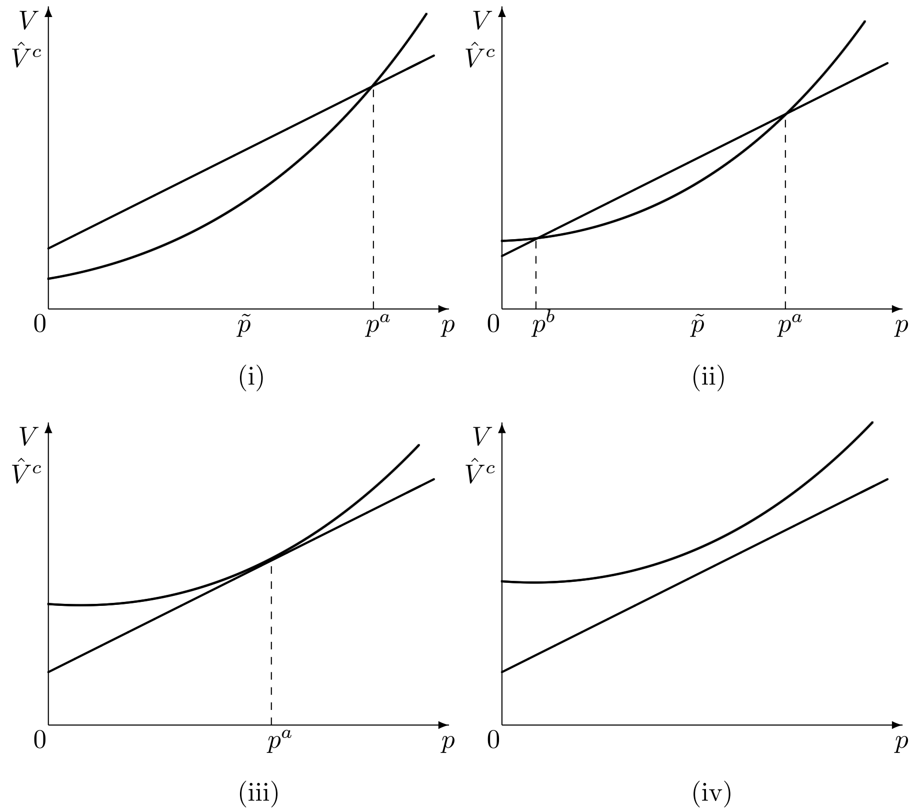
The expression in the right hand side of the inequality is always greater than 1, for all plausible amounts of parameters. The thinner curve in figure 1 represents the ranges of parameters in the space of (M, α), for a given values of other parameters, where the two steady state prices, $\tilde{p}$ and p^*, are equal. In the region below the curve $\tilde{p}$ is larger than p^*. Therefore, the market price in constrained equilibrium at every instant is higher compared to unconstrained equilibrium.

Figure 1: Profitability of acting non-strategically for the firms in the subset. On the dashed line $\alpha = 1$ ($\bar{q} = q^*$). On the thin solid curve $\tilde{p} = p^*$ and below (above) it, is larger (smaller) than p^*. Beyond the two thick solid curves exogenous output constraint is never profitable for non-strategic firms.



In figure 1, in the regions between the thick curves, non-strategic firms can benefit from an exogenous output constraint. In the region II constrained firms benefit by producing more and selling them at a higher price at every instant. In the region I, while constrained firms are constrained to a large output level, they can still benefit since $D < \hat{D} < 0$ and, therefore, price in constrained equilibrium moves to its steady state level more slowly. Region III, represents the points where excluding some of strategic players from the game make the competition in the industry less aggressive and pushes the price up in such a way that constrained firms benefit even with a substantial decrease in their quantity.

Figures 2: Comparing the value functions of a non-strategic firm before (V, the curve) and after ($\hat{V}^c$, the straight line) the exogenous output constraint.



However, provided that the parameters characterize a point in region I, II or III, the profitability of output constraint depends on the initial condition and how far the initial price is different from the steady state level. This is shown in figures 2. In figures 2 (i) and 2 (ii), if the initial price belongs to $(0, p^a)$ (or (p^b, p^a)), we can argue that $\hat{V}^C(p)$ always has a larger value than $V(p)$. Whereas, in the case where p_0 is outside the $(0, p^a)$ (or (p^b, p^a)) , output constraining is not to the benefit of the non-strategic firms in so far as $\hat{p}(t)$ arrives to the interval and it becomes profitable afterwards. Figures 2 (iii) and 2 (iv) corresponds to the points beyond the thick curves in figure 1 and illustrate the case where constrained firms do not benefit for any initial condition.

Note that, in our analysis, we consider general output constraint $\bar{q} > 0$ and examine the profitability of it in a dynamic context. However, at the steady state and for output constraint $\bar{q} = q^*$, we have the following corollary.

Corollary 19 *The steady state profits of non-strategic firms increase when they are constrained to their equilibrium output level q^*.*

Proof. As it was indicated before, after eliminating some strategic players, the steady state price increases ($\tilde{p} > p^*$). Therefore, since the output level does not change, the firm's revenue will increase while the cost remains the same as before. Hence, the non-strategic firms make higher profits in steady state. ■

4.5 Robustness of results

We have shown that, in a dynamic Cournot oligopoly when firms employ Markovian strategies, eliminating a subset of strategic players from the competition can be to the benefit of all the strategic and non-strategic players. Although we make use of the sticky price model, results do not correspond to the price stickiness. In this section, the robustness of results is evaluated through a conjectural variations analysis.[1] As it is shown in Dockner (1992), a static conjectural variations analysis approximates long-run dynamic interactions. Hence, we are interested in conjectural variations equilibrium in both unconstrained and constrained cases, and, then, examining the profitability of being a non-strategic player.

In the unconstrained equilibrium, all firms are strategic players. Firms have symmetric profit functions given by

$$\pi_i = p(Q)q_i - C(q_i), \tag{4.25}$$

where Q is the industry output, $p(Q)$ is a general inverse demand curve and $C(q_i)$ is a general cost function. First order conditions in the case of conjectural

[1] The conjectural variation is the firm's conjectures about her rivals' behavior.

variations equilibrium are given by

$$\frac{\partial \pi_i}{\partial q_i} = p(Q) + p'(Q)q_i - C'(q_i) + p'(Q)q_i[\sum_{j=1, j\neq i}^{N} \frac{\partial q_j}{\partial q_i}] = 0, \tag{4.26}$$

where $\frac{\partial q_j}{\partial q_i}$ is the conjecture of firm i about firm j's behavior. The industry output, price and cost functions are assumed to be $Q = \sum_{i=1}^{N} q_i$, $p(Q) = a - Q$ and $C(q_i) = \frac{1}{2}q_i^2$, respectively. Thus, the equilibrium corresponding to the F.O.C. of (4.26) is

$$q_{cv}^* = \frac{a}{2 + N + \lambda (N-1)},$$

where the subscript *cv* denotes the conjectural variations equilibrium, and firms are presumed to have identical conjectures $\lambda = \lambda_{ij} = \frac{\partial q_j}{\partial q_i}$. This conjecture belongs to the interval $[\lambda_0, 0]$ where $\lambda_0 \in (-1, 0)$ is the minimum viable conjecture which solves $\pi^* = p_{cv}^* q_{cv}^* - \frac{1}{2}q_{cv}^{*2} = 0$, and $\lambda = 0$ replicates the standard Cournot oligopoly.

However, in a *consistent conjecture equilibrium (CCE)*[2], the conjectural variation must be equal to the reaction function. The firm's reaction function is the firm's actual behavior and is defined by $q_i = \rho_i(q_j)$ which solves (4.26). The implicit differentiation of (4.26) yields

$$[1 + (\lambda + 1)(N-1)] \frac{\partial \rho_i}{\partial q_j} p'(Q) + p'(Q) - \frac{\partial \rho_i}{\partial q_j} = 0.$$

Considering symmetric reaction functions, $\frac{\partial \rho_i}{\partial q_j} = \frac{\partial \rho}{\partial q}$ and equating conjectural variation and reaction function, i.e. $\frac{\partial \rho}{\partial q} = \lambda$, the consistent conjecture is obtained[3]

$$\lambda^* = -\frac{N + 1 - \sqrt{5 + N(N-2)}}{2N - 2}.$$

It can be easily shown that $\lambda^* \in (\lambda_0, 0)$. Therefore, the slope of consistent conjecture lies between -1 and 0 which refer to Bertrand and Cournot competitions. Hence, in a CCE, the competition among firms in an oligopoly is more aggressive compared to the Cournot.

[2] Consistent conjectures equilibrium is discussed comprehensively in Bresnahan (1981).
[3] There exists a second root which is lower than -1 and is not acceptable.

Now, we want to know the consequences of excluding a subset of strategic players from the competition. Let us force a subset of M firms to be non-strategic players by constraining them to a constant output levels $\bar{q}$ where this constraint is binding. Thus, the remaining $N-M$ strategic firms solve the first order conditions

$$\frac{\partial \pi_i}{\partial q_i} = p(Q) + p'(Q)q_i - C'(q_i) + p'(Q)q_i[\sum_{j=M+1, j\neq i}^{N} \frac{\partial q_j}{\partial q_i}] = 0. \tag{4.27}$$

Evaluating this first order condition along the equilibrium of (4.26) yields

$$\left.\frac{\partial \pi_i}{\partial q_i}\right|_{q=q^*_{cv}} = -p'(Q)q^*_{cv}[\sum_{j=1, j\neq i}^{M} \frac{\partial q_j}{\partial q_i}] < 0. \tag{4.28}$$

This, however, implies (given the second order conditions) that industry output shrinks when a subset of firms is constrained to their equilibrium in the unconstrained case. Hence, market price increases and both strategic and non-strategic players benefit, irrespective of the size of the M.

Now, consider a general output constraint $\bar{q} = \alpha q^*_{cv}$, $\alpha \in (0, 2)$. Assuming symmetry between the $N-M$ strategic players, the equilibrium output level of (4.27) becomes

$$\tilde{q}_{cv} = \frac{a\left(2+N+(N-1)\lambda - M\alpha\right)}{\left(2+N+\lambda(N-1)\right)\left(N-M+2+\lambda(N-M-1)\right)},$$

and the resulting market price is

$$\tilde{p}_{cv} = a - M\alpha q^*_{cv} - (N-M)\,\tilde{q}_{cv}.$$

Therefore, the unconstrained and constrained firms' profits are

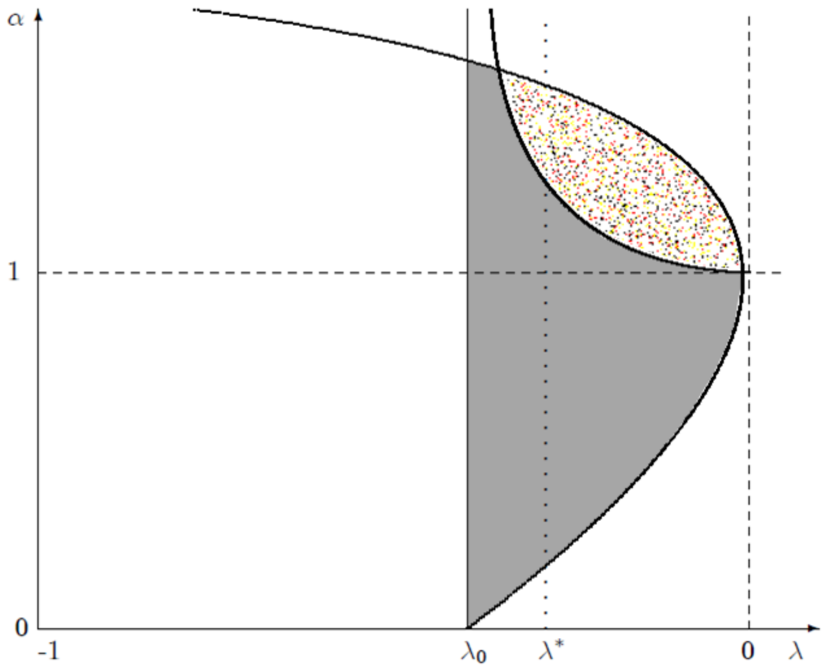
$$\tilde{\pi} = \tilde{p}_{cv}\tilde{q}_{cv} - \frac{1}{2}\tilde{q}^2_{cv}, \tag{4.29}$$

$$\tilde{\pi}_c = \tilde{p}_{cv}\bar{q} - \frac{1}{2}\bar{q}^2, \tag{4.30}$$

where $\tilde{\pi}$ denotes the profits in constrained case, and subscript c stands for the constrained firms.

Figure 3 shows the range of parameters in the space of (λ, α) where unconstrained and constrained firms can benefit from output constraint. The two

Figure 4.1: Profitability of exogenous output constraint in conjectural variations equilibrium in the space of (λ, α). The lower curve and the upper one represent the points where firms' profits in the two cases are equal for the constrained and unconstrained firms, respectively. On the right hand side of the curves both type of fims benefit.



curves in this figure are the locus of the points where firms have the same profits in unconstrained and constrained equilibria. However, the gray area represents the points where both type of firms benefit while in the dotted area only constrained firms benefit.

As it can be seen, firms' profits can increase even if they are constrained to an output level higher than the unconstrained equilibrium. The profitability of output constraint decreases as firms' conjectures goes to zero. These results are consistent with the results of the dynamic competition when firms employ Markovian strategies.

In the figure, $\lambda = 0$ corresponds to the standard static Cournot competition in which any output constraint in not beneficial neither for the constrained firms nor for the unconstrained ones.

4.6 Applications

4.6.1 Voluntary export restraints

The study we have conducted has many applications among which voluntary export restraints (VERs) is the most obvious one. It is of importance to international trade policy to answer the question whether domestic and/or foreign firms benefit from the imposition of so-called 'voluntary' export restraints by the foreign producer. If the foreign producer's profit increases by restraining export to the domestic market, VERs are indeed 'voluntary'.

Dockner and Haug (1991) analyses VERs in a differential game model with a domestic and foreign producer of a homogenous good sold in the domestic market. There are several differences between this contribution and present study. First, Dockner and Haug (1991) analysis is restricted in a speed of price adjustment that goes to infinity. However, in our model it is possible to investigate price behavior in determining the profitability of VERs. Second, with the model presented here we can consider more than one foreign and domestic firm which provides us the chance to examine the incentive for VER in relation to the number of constrained firms and the level of output they are constrained to.[4]

In addition to Dockner and Haug (1991) that shows the imposition of a VER at the free trade level of imports increases the market price and the profits of all firms in the industry, our analysis implies that: market price increases for any output constraint which in below the free trade level of imports; it is always to the benefit of domestic firms for any level of exports that foreign firms are restricted to and finally in part (c) of theorem 5 it is comprehensively explained under which conditions and for what level of output export restraint is profitable for foreign firms.

4.6.2 Horizontal Mergers and Cartels

When in an N-firm industry a subset of M firms is constrained to a constant level of output, since there is strategic interaction among $N-M+1$ firms rather than N firms, the level of competition in the industry will decrease which is always to the benefit of unconstrained firms as it is proved in theorem 5. We show when the subset of firms are constrained to q* which is their steady state equilibrium level before the exogenous output constrained, the anticompetitive forces due to an exogenous output constraint can be strong enough to benefit the subset of firms as well. Theorem 5 also discusses about conditions and other output levels that being constrained to it can be advantageous for the subset of firms.

The same story holds when we consider the profitability of mergers and cartels. Our model does not precisely fit the horizontal merger problem in which firms solve their strategic problem to determine the equilibrium output level. However, in general, output contraction creates the same results that horizontal mergers and cartels can create that are reduction in aggregate output, increase in the market price and therefore increase in the profit of $N-M$ outside firms. Now, assume that the subset represents firms that are part of a cartel. Here, we

[4] For another contribution on VERs in a differential game you can see Calzolari and Lambertini (2007) who study the impact of VERs in a duopoly game with a Ramsey capital accumulation dynamics

can define cartel as an agreement in which firms in the subset agree on being constrained to a constant level of output (for example q^*) and as it is shown in theorem 5, it can be profitable for them. It is difficult for antitrust authority to recognize such a cartel in which a subset of firms is constrained to their steady state equilibrium level before the exogenous output constraint. Dockner and Gaunersdorfer (2001), Benchekroun (2003), Esfahani (2012) and Esfahani and Lambertini (2012)[5] using a dynamic model with sticky prices, investigate the profitability of horizontal mergers in the specific case of instantaneous price adjustment.

4.6.3 Economic sanctions

Economic sanctions are punishments imposed on a country by one or a group of countries due to various reasons. Economic sanctions may take a number of forms including: embargo on exports, embargo on imports, financial controls, transportation and communication controls, sequestration of property, preemptive purchasing and other measures. For extensive discussion, see Bornstein (1968).

We are considering import restrictions from the target country into the participants which attempts to reduce the target country's foreign exchange earnings. There is a debate over the effectiveness of economic sanctions in their ability to achieve its intention even if any import restrictions enacted by sanctioners ensures income reduction in target country. However, our analysis can address the question of whether sanctions can reduce the target country's income.

Suppose that M firms in the subset represent firms in the target country. The rest of the $N - M$ Firms are outside the target country. Sanctioning countries by enforcing import restrictions are the cause of exogenous output constraint in the target country. Part (c) of theorem 5 explains how sanctions can be designed by imposing countries in order to decrease the present value of firms

[5] They considered non-linear demand function and the open-loop equilibrium.

in the imposed country through the level of output that they force the target country to be constrained to.

4.7 Conclusion

In the case of static Cournot competition among producers of perfect substitutes, output constraint is never to the benefit of constrained firms. When firms use feedback strategies, eliminating a subset of strategic players by exogenously constrained them to a constant level of output results in: (i) increase in the value of strategic firms irrelevant of the amount of constraint because of having less intensive competition, (ii) increase in the market price for any output constraint below the optimal level and slightly above that because of total output reduction caused by less competition and (iii) increase in the value of non-strategic firms for a viable range of parameters and initial conditions because of increase in the price during the price path.

APPENDIX

Appendix A:

Proof of Theorem 1: The proof is carried out for symmetric interior solutions. We use dynamic programming. The Bellman equation is given by

$$rV^i(p) = \max_{q_i}\{pq_i - \frac{1}{2}q_i^2 + sV_p^i(p)[a - \sum_{i=1}^{N} q_i - p]\}, \tag{4.31}$$

where $V^i(p)$ is the optimal value function of firm i. Since the game is symmetric and linear quadratic we conjecture symmetric, quadratic value functions

$$V^i(p) = \tfrac{1}{2}Kp^2 - Ep + g, \tag{4.32}$$

which implies that

$$V_p^i(p) = Kp - E, \tag{4.33}$$

where K, E and g are constants that need to be determined. Maximizing the right hand side of equation (4.31) gives

$$q_i = p - sV_p^i(p). \tag{4.34}$$

Thus, the feedback rules are given by

$$q_i\,(p) = p(1 - sK) + sE. \tag{4.35}$$

Substituting this last expression and using the quadratic value function (4.32) into the Bellman equation yields

$$\begin{aligned}\tfrac{1}{2}p^2 \left(1 - rK - 2sK(N+1) + s^2K^2(2N-1)\right) \\ + p(asK + E(r+s) + sNE - s^2EK(2N-1)) \\ + s^2E^2(2N-1) - sEa - rg = 0.\end{aligned} \tag{4.36}$$

The requirement that this equation be satisfied for all values of p implies that K, E and g have to satisfy

$$1 - rK - 2sK(N+1) + s^2K^2(2N-1) = 0, \tag{4.37}$$

$$asK + E(r+s) + sNE - s^2EK(2N-1) = 0, \tag{4.38}$$

$$s^2E^2(2N-1) - sEa - rg = 0. \tag{4.39}$$

The solutions to equations (4.37)-(4.39) are given by

$$K = \frac{2s(N+1) + r \pm \sqrt{[2(N+1)s + r]^2 - 4s^2(2N-1)}}{2s^2(2N-1)}, \tag{4.40}$$

$$E = \frac{c - sKa}{s(N+1) + r - s^2K(2N-1)}, \tag{4.41}$$

$$g = \frac{s^2E^2(2N-1) - 2sEa}{2r}. \tag{4.42}$$

With the decision rules (4.35) the price equation (4.5) becomes

$$\dot{p} + ps[N(1 - sK) + 1] = s(a - NsE), \tag{4.43}$$

which is a linear first order differential equation. A solution to this equation is given by

$$p(t) = p^* + (p_0 - p^*)e^{Dt}, \tag{4.44}$$

where p^* is the steady state price

$$p^* = \frac{a - NsE}{1 + N(1 - sK)}, \tag{4.45}$$

p_0 is the initial price and D is the constant

$$D = s[N(sK-1)-1].$$

This constant is only negative, and hence the Markov-perfect equilibrium is globally stable if we choose the negative root of (4.40). Equations (4.35) and (4.40) to (4.45) give us the Markov-perfect equilibrium in linear strategies for the differential game (4.4) and (4.5) for any finite s. This completes the proof.

Appendix B:

Proof of Theorem 2: The Bellman equation of the problem (4.14)-(4.15) is given by

$$r\hat{V}^i(p) = \max_{q_i}\{pq_i - \frac{1}{2}q_i^2 + s\hat{V}_p^i(p)\,[a - \sum_{j=1}^{M}\bar{q}_j - \sum_{i=M+1}^{N} q_i - p]\}, \tag{4.46}$$

where $\hat{V}^i(p)$ is the optimal value function of firm i, which is an unconstrained firm in the constrained case. Maximization of the right hand side of the Bellman equation gives

$$\hat{q}_i(p) = p - s\hat{V}_p^i(p), \tag{4.47}$$

Substituting (4.47) into (4.46) and inducing symmetry yields

$$\begin{aligned} r\hat{V}(p) &= p(p - s\hat{V}_p(p)) - \frac{1}{2}(p - s\hat{V}_p(p))^2 \\ &\quad + s\hat{V}_p(p)\,[a - p - M\bar{q} - (N-M)(p - s\hat{V}_p(p))]. \end{aligned} \tag{4.48}$$

As with the unconstrained case, we propose the following quadratic value function

$$\hat{V}(p) = \tfrac{1}{2}\hat{K}p^2 - \hat{E}p + \hat{g},$$

which implies that

$$\hat{V}_p(p) = \hat{K}p - \hat{E},$$

where $\hat{K}$, $\hat{E}$ and $\hat{g}$ are constants that need to be determined. Thus, the feedback rules are given by

$$\hat{q}_i(p) = p(1 - s\hat{K}) + s\hat{E}. \tag{4.49}$$

Substituting $\hat{V}(p)$ and $\hat{V}_p(p)$ in (4.48) and collecting with respect to p, we obtain

$$\beta_1 p^2 + \beta_2 p + \beta_3 = 0, \tag{4.50}$$

where

$$\beta_1 = \tfrac{1}{2}\left(1 - r\hat{K} - 2s\hat{K}(N-M+1) + s^2\hat{K}^2(2(N-M)-1)\right), \tag{4.51}$$

$$\beta_2 = as\hat{K} + \hat{E}(r+2s) + s\hat{E}(N-M-1) - s\hat{K}M\bar{q} - s^2\hat{E}\hat{K}(2(N-M)-1), \tag{4.52}$$

$$\beta_3 = s^2\hat{E}^2(N-M-\tfrac{1}{2}) + s\hat{E}M\bar{q} - s\hat{E}a - r\hat{g}. \tag{4.53}$$

The equation (4.50) is satisfied if expressions (4.51)-(4.53) are simultaneously zero. This results to the following solution

$$\begin{aligned} \hat{K} &= \frac{2s(N-M+1) + r \pm \sqrt{[2(N-M+1)s + r]^2 - 4s^2(2(N-M)-1)}}{2s^2(2(N-M)-1)}, \\ \hat{E} &= \frac{-s\hat{K}a - s\hat{K}M\bar{q}}{s(N-M+1) + r - s^2\hat{K}(2(N-M)-1)}, \\ \hat{g} &= \frac{s^2\hat{E}^2(2(N-M)-1) - 2s\hat{E}a + 2s\hat{E}M\bar{q}}{2r}. \end{aligned} \tag{4.54}$$

Using (4.49), a solution to equation (4.15) is given by

$$\hat{p}(t) = \tilde{p} + (p_0 - \tilde{p})e^{\hat{D}t},$$

where $\tilde{p}$ is the steady state price

$$\tilde{p} = \frac{a - (N-M)s\hat{E} - M\bar{q}}{1 + (N-M)(1 - s\hat{K})},$$

p_0 is the initial price and $\hat{D}$ is the constant

$$\hat{D} = s[(N-M)(s\hat{K} - 1) - 1].$$

This constant is only negative and if we choose the negative root of (4.54) the Markov-perfect equilibrium is globally stable.

The discounted present value of the constrained firm is derived from

$$\hat{V}^C(p(t)) = \int_t^\infty e^{-r(\tau - t)}[p(\tau) - \frac{1}{2}\bar{q}]\bar{q}d\tau, \tag{4.55}$$

where $\hat{V}^C$ is the value function of the constrained firms and $p(.)$ is the price given by (4.18). Substituting (4.18) in (4.55), we have

$$\hat{V}^C(p(t)) = e^{rt}\bar{q}\left[\int_t^\infty e^{-r\tau}(\tilde{p} - \frac{1}{2}\bar{q})d\tau + \int_t^\infty e^{-(r-D)\tau}(p_0 - \tilde{p})d\tau\right], \quad (4.56)$$

which results to

$$\hat{V}^C = \frac{1}{r}(\tilde{p}\bar{q} - \frac{1}{2}\bar{q}^2) + \frac{1}{r-\hat{D}}(p_0 - \tilde{p})\bar{q}e^{\hat{D}t}, \quad (4.57)$$

Thus, we obtain

$$\hat{V}^C = Ap + \hat{g}^C,$$

where A and $\hat{g}^C$ are

$$A = \frac{\bar{q}}{r-\hat{D}},$$

$$\hat{g}^C = \frac{\bar{q}(\hat{D}(2\tilde{p} - \bar{q}) + r\bar{q})}{2r(\hat{D} - r)}.$$

This proves the theorem.

Appendix C:

Proof of Theorem 3: Looking at (4.10) and (4.20), it is obvious that K and $\hat{K}$ have the same functional form with this difference that instead of N we have $N - M$ in $\hat{K}$. Thus, since it can be easily shown that $\partial K/\partial N < 0$, we find that $\hat{K} > K > 0$. We have the similar story to compare E with $\hat{E}$ and g with $\hat{g}$. Substituting (4.10) in (4.11), we can show that $\partial E/\partial N > 0$. Thus, as the number of firms decreases the coefficient E will decrease. This together with having the negative term $-s\hat{K}M\bar{q}$ in equation (4.21) we can argue that, as long as $\bar{q}$ is positive, $\hat{E} < E < 0$. With the same procedure we can show that always $\hat{g} > g > 0$. Therefore, comparing (4.7) and (4.17), for all values of p we obtain $\hat{V}(p) > V(p)$. This concludes (b).

Considering (4.10) and (4.20), it can be easily shown that for all values of parameters $sK < s\hat{K} < 1$. So, comparing (4.13) and (4.23), we find that $D < \hat{D} < 0$. Furthermore, looking at steady state prices it can be shown that for $\bar{q} \leq q^*$, the steady state price (4.19) is larger than (4.9). Therefore, it can be simply proven that the price path (4.18) is greater than (4.8) which concludes (a).

Now, in order to examine (c) we have to compare (4.7) and (4.24). Since $V(p)$ is a convex function and $\hat{V}^C(p)$ is a linear function of p, by equating these two equations, we obtain

$$p^l = \frac{A + E \pm \sqrt{(A+E)^2 - 2K(g - \hat{g}^C)}}{K}, \quad l = a, b, \tag{4.58}$$

where $p^a > p^b$ (if there exist any p^b). Therefore, in principal, for positive values of p, the two value functions may have (i) one intersection if the radicand is larger than zero and $\hat{g}^C > g$, (ii) two intersections provided that the radicand has a positive value and $g > \hat{g}^C$, (iii) one tangency point when the radicand is zero, and (iv) no intersection when the radicand is negative. Therefore, having a viable range of parameters for which acting non-strategically is to the benefit of firms mainly depends on the amount of $(A+E)^2 - 2K(g - \hat{g}^C)$. Here, we are interested in its positive values. Using a numerical analysis, a range of parameters in the space of (M, α) is depicted in figure 1by means of two dividing curves (the thicker ones) where between them the amount of radicand is positive and beyond them it is negative. Situation (iv) corresponds to the regions beyond the curves. However, in the region between the two curves, we do have a viable range of parameters for output constraint to be profitable for the non-strategic firms which corresponds to (i) or (ii). Figures 2 show (i), (ii), (iii) and (iv) graphically.

The condition (iii) occurs in the limit where s goes to zero, because we have

$$\lim_{s \to 0} (A+E)^2 - 2K(g - \hat{g}^C) = 0,$$

and, therefore, there is a single common point in $V(p)$ and $\hat{V}^C(p)$. Furthermore, the slope of $V(p)$ at $p^a = \frac{A+E}{K}$ is$V'(p) = Kp - E = A$, which is the same as the slope of $\hat{V}^C(p)$. Thus, the contacting point is a tangency point. Hence, when the price does not adjust at all (i.e. $s = 0$), acting non-strategically is never to the benefit of the non-strategic firms. However, for $0 < s < \infty$ there is a range of parameters where the non-strategic firms also benefit from the output constraining. Comparing the steady state prices (4.9) and (4.19) and prices driven from (**??**), we found that

$$p^* < \tilde{p} \in \begin{cases} (0, p^a) & \text{in (i)}, \\ (p^b, p^a) & \text{in (ii)}. \end{cases}$$

Now, looking at (4.18), we know that $\hat{p}$ starts at the initial price p_0 and moves towards the steady state price $\tilde{p}$. Therefore, provided that the initial price belongs to $(0, p^a)$ (or (p^b, p^a)), we can argue that $\hat{V}^C(p)$ always has a larger value than $V(p)$. Whereas, in the case where p_0 is outside the aforementioned interval, output constraining is not to the benefit of the non-strategic firms in so far as $\hat{p}(t)$ arrives to the interval and it becomes profitable afterwards.

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