

UNIVERSITY OF BOLOGNA

DIEM - Department of Mechanical Engineering

SINGULARITY-FREE FULLY-ISOTROPIC
TRANSLATIONAL PARALLEL
MANIPULATORS

A dissertation presented

by

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in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

in

Mechanics of Machines

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2001

Keywords:

- *Parallel Mechanisms*
- *Translational Mechanisms*
- *Constraint Singularities*
- *Direct Kinematics Singularities*
- *Isotropy*

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ABSTRACT

Parallel mechanisms show desirable characteristics such as a large payload to robot weight ratio, considerable stiffness, low inertia and high dynamic performances. In particular, parallel manipulators with fewer than six degrees of freedom have recently attracted researchers' attention, as their employ may prove valuable in those applications in which a higher mobility is uncalled-for.

The attention of this dissertation is focused on translational parallel manipulators (TPMs), that is on parallel manipulators whose output link (platform) is provided with a pure translational motion with respect to the frame.

The first part deals with the general problem of the topological synthesis and classification of TPMs, that is it identifies the architectures that TPM legs must possess for the platform to be able to freely translate in space without altering its orientation.

The second part studies both constraint and direct singularities of TPMs. In particular, special families of fully-isotropic mechanisms are identified. Such manipulators exhibit outstanding properties, as they are free from singularities and show a constant orthogonal Jacobian matrix throughout their workspace. As a consequence, both the direct and the inverse position problems are linear and the kinematic analysis proves straightforward.

SOMMARIO

I meccanismi paralleli possiedono caratteristiche vantaggiose, quali un elevato carico pagante rispetto al peso del robot, una considerevole rigidezza, bassa inerzia e alte prestazioni dinamiche. In particolare, i manipolatori paralleli con meno di sei gradi di libertà hanno recentemente attratto l'attenzione dei ricercatori, potendo il loro impiego risultare utile in applicazioni nelle quali non sia necessaria una più alta mobilità.

L'oggetto della presente tesi è costituito dai manipolatori paralleli traslanti (TPM), così detti in quanto il proprio membro terminale (piattaforma) esibisce un moto di pura traslazione rispetto al telaio.

La prima parte affronta, in termini generali, il problema della sintesi topologica, nonché della classificazione, dei TPM. In particolare, s'individuano le architetture che le gambe di un TPM devono possedere affinché la piattaforma del meccanismo possa traslare liberamente nello spazio senza alterare il proprio orientamento.

La seconda parte studia le singolarità dei TPM, tanto quelle di vincolo quanto quelle dirette, e individua speciali famiglie di meccanismi completamente isotropi. Tali manipolatori mostrano proprietà notevoli, in quanto sono privi di singolarità e possiedono una matrice jacobiana ortogonale e costante in tutto lo spazio di lavoro. I problemi di posizione sia diretto che inverso risultano lineari e l'analisi cinematica immediata.

INTRODUCTION

A typical *parallel mechanism* (PM) consists of a moving platform connected to a fixed base by means of several kinematic chains, called *legs*. Only some kinematic pairs are actuated, in number generally equal to the number n of degrees of freedom (dofs) that the platform possesses with respect to the base; the other joints are passive. Usually, the number of legs is also equal to n . This makes it possible to actuate only one pair per leg, allowing all motors to be mounted close to the base. Such mechanisms show desirable characteristics like a large payload to robot weight ratio, considerable stiffness, low inertia and high dynamic performances. With respect to serial manipulators, disadvantages are a lower dexterity, a smaller workspace and more serious consequences caused by kinematic singularities (which are configurations in which the functioning of the mechanism is disrupted).

Since their first designs [1, 2], 6-dof PMs have been extensively studied, whilst only in relatively recent times have manipulators with fewer dofs attracted researchers' attention. As they are potentially architecturally simpler and cheaper than their 6-dof counterparts (they require less parts and actuators), their use may be advantageous in all those applications in which less than six dofs are required.

Different architectures of 3-dof mechanisms have been presented in the literature. Depending on the kind of motion exhibited by the platform, PMs can be divided into:

- mixed (MPMs);
- spherical (SPMs);
- translational (TPMs).

The first allow the platform to both translate and rotate and may be employed as motion simulators, wrists of hybrid serial-parallel robots and mixed orienting/positioning systems [3-7].

The second enable the travelling plate to rotate about a fixed point and may be used in those applications that require orienting a body in space, be it a solar panel, an antenna, a telescope, a gun, the end-effector of a robot, a human or humanoid artificial limb, etc. [8-18].

The last provide the output link with a pure translational motion and may be particularly valuable in the fields of automated assembly and machine tools as

alternatives to traditional serial positioning systems. The attention of this dissertation is focused on TPMs.

Throughout the text, the kind of kinematic pair will be addressed using the following symbols (in parentheses, the number of dofs of the joint is also specified):

- P for prismatic pair (1 dof);
- R for revolute pair (1 dof);
- H for screw pair (1 dof);
- PA for planar parallelogram¹ (1 dof);
- C for cylindrical pair (2 dof);
- U for universal pair (2 dof);
- S for spherical pair (3 dof).

In many architectures presented in the literature, all legs exhibit the same topological structure. In this case, the mechanism's type can be addressed by specifying the number of legs and the sequence of joints distributed along any of them, from the base to the platform.

The first TPM, called Delta Robot, was presented in the late Eighties [19, 20]. It comprises three legs, each including three R-pairs and a spatial parallelogram containing ball-and-socket joints. Due to its outstanding dynamic performances, it was attributed a widespread success.

In 1991, Hervé and Sparacino [21] introduced a whole class of new TPMs. The concept at the basis of the novel architectures is quite simple. If the axes of the revolute joints contained in a 4-dof leg² are all parallel to a unit vector \mathbf{w}_i , the terminal link, other than translating, can only be allowed to turn about \mathbf{w}_i . Therefore, a platform connected to the base by at least two legs of this kind (whose R-pairs are parallel to the lines \mathbf{w}_h and \mathbf{w}_k respectively) is deprived of any rotation mobility, provided that $\mathbf{w}_h \neq \mathbf{w}_k$.

In the literature, several TPMs have been proposed that belong to the family discovered by Hervé and Sparacino. Some have been presented by Hervé himself and his co-workers, like the Y-STAR manipulator [22], the H- and the Prism- Robots

¹ The planar parallelogram can be seen as a single kinematic pair providing two links with a relative motion of circular translation in the plane perpendicular to the parallelogram axis (which is defined as the direction of the axes of the mechanism's revolute joints).

² In the context of this dissertation, the number of dofs of a leg are meant to be those with which the leg provides its output or terminal link, i.e. the platform, with respect to the base.

[23], the 3-RPAPAR micro finger [24]. Some others have been studied in detail by other researchers, like the University of Maryland manipulator, whose 3-RRPAR architecture can be obtained from the Delta Robot by replacing the spatial parallelograms with planar ones [25-27], and the 2- and 3- PRRR mechanisms, whose several different arrangements (in some of which the P-pair is not mounted on the base and/or is replaced by a PA-pair) have been investigated in [21, 28, 29].

In 1996, Tsai [30] introduced for the first time a TPM that does not fall into the Hervé and Sparacino family: the 3-UPU mechanism. Each leg of this manipulator has five dofs and deprives the platform of a single dof of rotation, its U-pairs always remaining parallel to each other. The 3-UPU architecture was later recognized as a particular case of a wider class of mechanisms that can be addressed as the Tsai family [31-33].

The entire class of TPMs whose legs possess five dofs was exhaustively studied by Frisoli et al. [34] and, independently, by Carricato and Parenti-Castelli [35-36]. This has led to the definition of novel architectures, different from the ones derived from the 3-UPU manipulator and groupable under the name of the Frisoli and Carricato family.

The present dissertation approaches the problem of the topological synthesis of TPMs in a general way and focuses on the identification of families of singularity-free mechanisms. The work is essentially divided into two parts.

The first, grouping the first two chapters, deals, in a systematic and exhaustive form, with the general problem of the topological synthesis and classification of a TPM, or rather of its distinguishing elements, the legs. Whilst there are no particular bonds on the topological design of the legs of a 6-dof PM, as any 6-dof kinematic chain is consistent with a constraintless motion of the platform, when topologically designing the leg of a TPM, a fundamental question must be given a response: which architecture must the leg possess for the platform to be able to freely translate in space without altering its orientation? In this perspective, a TPM leg must satisfy the following requirements. It must:

- deprive the platform of one or more dofs of rotation, so that, by virtue of the contribution of all legs, platform turning is completely prevented;
- be consistent with a 3-dof translational motion of the platform.

These considerations lead to the classification of the TPM legs on the basis of the number of dofs of rotation eliminated from the platform, or, alternatively, the number of dofs that remain:

- type T_3 legs, having three dofs and depriving the platform of all its dofs of rotation;
- type T_4 legs, having four dofs and depriving the platform of two dofs of rotation;
- type T_5 legs, having five dofs and depriving the platform of one dof of rotation.

The first chapter, in particular, studies the geometric conditions that such legs must fulfill in order to satisfy the previously mentioned requirements; the second describes how they must be assembled in order to generate a TPM. Apart from type T_3 legs, resulting into trivial architectures containing prismatic pairs only (T_3 TPMs), it is proved that TPMs comprising type T_4 legs are only those belonging to the Hervé and Sparacino family (here called T_4 TPMs) and TPMs exhibiting type T_5 legs are only those belonging to the families of Frisoli and Carricato and of Tsai (here named T_5' and T_5'' TPMs respectively). The presented discussion provides the supra cited published material with a general and organic frame.

The second part of the dissertation, grouping the last two chapters, deals with the problem of singularities and in particular with the identification of classes of singularity-free TPMs.

Both *direct* and *constraint* singularities are addressed. The former are common to any PM and represent configurations in which one or more of the existing dofs of the platform become uncontrollable and a nonzero output motion exists even when the actuator velocities are zero. The latter are peculiar to manipulators having less than six dofs and lead the output link to acquire additional previously constrained dofs [37]. For TPMs, in particular, constraint singularities are configurations in which the platform loses its capability of purely translating and gains an instantaneous mobility of rotation. It is clear that both kinds of singularities deeply affect the kinematic behavior of the manipulator and their study is especially important.

The third chapter proves that only the mechanisms of the Tsai family exhibit constraint singularities [38-40], whilst T_3 , T_4 and T_5' TPMs are constraint-singularity-free.

The fourth chapter focuses on the identification of special families of mechanisms exhibiting outstanding properties: the *fully-isotropic* TPMs. In isotropic configurations the Jacobian matrix has the condition number, as well as the determinant, equal to one and the manipulator performs very well with regard to its force and motion transmission capabilities [41,42]. A manipulator is defined isotropic if it possesses at least one isotropic configuration. In this work, it is defined *fully-isotropic* if it is isotropic in its entire workspace. A fully-isotropic manipulator

does not show direct singularities, as the determinant of its Jacobian matrix is always equal to one. This work presents families of fully-isotropic mechanisms that exhibit a constant orthogonal Jacobian matrix throughout their workspace. As a consequence, both the direct and the inverse position problems³, besides the velocity and acceleration ones, are linear and the kinematic analysis proves straightforward. Most of the shown manipulators are utterly *singularity-free*, since they do not even present constraint singularities.

³ In the direct kinematics, the actuated joint variables and their derivatives are given and the position of the platform and its derivatives are calculated. In the inverse kinematics, the position of the platform and its derivatives are assumed assigned and the joint variables and their derivatives are computed.

1 TOPOLOGICAL CLASSIFICATION OF THE LEGS OF A TPM

This chapter provides an exhaustive classification of all the topological conformations that the legs of a PM may assume for the mechanism to be translational, that is for its platform to exhibit a purely translational motion.

In order to reduce the number of kinds of kinematic pairs to be taken into consideration, in the following discussion the C-pair will be considered as the ensemble of an R-pair and a P-pair mutually coaxial and the U-pair as the ensemble of two intersecting and nonparallel R-pairs. Moreover, since the functional difference between the R- and the H- pair is unessential for the aim that has been set (both have one dof and allow the relative rotation about a single axis), only the R-pair will be considered, it being understood that what is said for one applies to the other as well.

Now, let a PM be considered, comprising n_g legs, each possessing n_i linearly independent dofs. If all the dofs removed by the legs from their output link (in number of $6-n_i$ each) were linearly independent, the total number of dofs left to the platform would be

$$(1) \quad n = 6 - \sum_{i=1}^{n_g} (6 - n_i)$$

that is

$$(2) \quad n = \sum_{i=1}^{n_g} n_i - 6(n_g - 1)$$

Actually, since different legs may impose similar constraints, it is

$$(3) \quad \sum_{i=1}^{n_g} n_i - 6(n_g - 1) \leq n \leq \min \{n_i\}$$

As explained in the Introduction, a TPM leg must be designed so as to:

- deprive the platform of one or more dofs of rotation (so that, by virtue of the contribution of all legs, platform turning is completely prevented);
- be consistent with a 3-dof translational motion of the platform.

In this perspective, in order to define the topological conformation that a leg of a TPM may assume, what follows must be taken into consideration:

- since any dof taken away by a leg cannot be restored by another, all legs must provide the platform with at least three dofs of translation;
- no one of the kinematic pairs of a leg may allow a dof that has to be removed from the platform. For instance, let a leg that must prevent the platform rotation about the line \mathbf{w} be considered. If such a rotation were allowed by one of the leg joints, this would be sufficient, maintaining all the other ones blocked, to make the platform rotate about \mathbf{w} and hence violate the imposed constraint.

As a consequence of the last consideration, within any leg that must deprive the platform of one or more dofs of rotation:

- S-pairs cannot exist;
- there cannot be R-pairs having their axes parallel to the lines about which the rotation must be prevented;
- there are no particular constraints on the location and arrangement of P- and PA-pairs, which are always consistent with any rotation constraint imposed on the output member.

Furthermore, in order to guarantee the linear independence of the leg dofs, the following cannot exist:

- two parallel prismatic pairs;
- three prismatic pairs parallel to the same plane;
- four revolute pairs parallel to each other.

Bearing in mind what is stated above, the kinds of legs that can be comprised in a TPM will now be examined in detail. A leg will be called type T_{n_i} if it does not constrain the platform translation, has n_i dofs and consequently removes $6-n_i$ dofs of rotation from the platform.

1.1 Type T_3 legs

A leg of such a kind prevents any turn of the platform. Therefore it cannot comprise any joint allowing relative rotations, but it can only include P- or PA- pairs. A couple of examples are shown in Fig. 1.

1.2 Type T_4 legs

Such legs, investigated for the first time in [21], allow the platform to rotate about a unique axis, identified by a unit vector \mathbf{w} . It follows that all R-pairs must have their axes parallel to \mathbf{w} and their number must be equal to or greater than one (for the platform to be able to rotate about \mathbf{w}) and less than four (for the dofs to be linearly independent).

Actually, it can be proved that, in a TPM, a type T_4 leg must possess at least two R-pairs. In fact, if there were only one, the angular velocity $\boldsymbol{\omega}$ of the output link would be

$$(4) \quad \boldsymbol{\omega} = \dot{\theta} \mathbf{w}$$

where θ is the angular variable that defines the relative rotation between the links connected by the examined joint. However, if the leg is employed within a TPM, the platform must have zero angular velocity and so $\dot{\theta}$ should be identically nought: in conclusion, the joint would be idle and the leg would be type T_3 .

If θ_j is the angular variable relative to the j -th R-pair, when there are two of them it must be

$$(5) \quad \boldsymbol{\omega} = (\dot{\theta}_1 + \dot{\theta}_2) \mathbf{w} = \mathbf{0}$$

and so

$$(6) \quad \dot{\theta}_1 + \dot{\theta}_2 = 0$$

If there are three, it is analogously

$$(7) \quad \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 = 0$$

If there are only two R-pairs, they provide the platform with a rotational and a translational dof, the latter parallel to the plane Π perpendicular to \mathbf{w} . Hence, two P- or PA- pairs are needed to provide the platform with the other two translational dofs, one on and the other out of Π . The former may be realized by either a P-pair not having its axis parallel to \mathbf{w} or an anyhow oriented PA-pair (in fact, whatever the orientation of the PA-pair axis is, at least one component of the translational movement that it provides would lie on Π). The latter may be realized by means of a P-pair not having its axis perpendicular to \mathbf{w} or a PA-pair not having its axis parallel to \mathbf{w} .

If there are three R-pairs, a single translational joint is needed to permit the movement out of Π . If a P-pair is used, its axis must not lie on such a plane; if a PA-pair is adopted, its axis must not be parallel to \mathbf{w} .

Figure 2 shows some examples.

1.3 Type T_5 legs

A type T_5 leg prevents the platform turning about a single axis identified by a unit vector \mathbf{n} and permits its rotation about any other line perpendicular to \mathbf{n} . It follows that such a leg must comprise at least two R-pairs nonparallel to each other and parallel to the plane normal to \mathbf{n} . Any other R-pair must be parallel to this plane in its turn. Figure 3 provides some examples.

Unlike what happens for type T_3 and T_4 legs, parallelism conditions valid in a given instant do not necessarily hold throughout the motion, since the orientation of the joint axes may vary during the movement. In the examples depicted in Figs. 3a and 3b, all R-pairs are always parallel to a plane perpendicular to a certain \mathbf{n} , but this one does not maintain its posture constant during the motion. In the examples sketched in Figs. 3c and 3d, all R-pairs are drawn parallel to the same plane, but it is easy to realize that other configurations are possible in which this does not happen.

Therefore, it is clear that the precise definition of the topological conformation of a type T_5 leg requires a deeper analysis, aimed at identifying the geometric conditions that need to be satisfied for the leg to actually prevent the rotation of the platform about a line at any instant.

First, it must be observed that a type T_5 leg belonging to a TPM must comprise at least four R-pairs. If there were only two, the angular velocity $\boldsymbol{\omega}$ of the output link being equal to zero, it should be

$$(8) \quad \boldsymbol{\omega} = \dot{\theta}_1 \mathbf{w}_1 + \dot{\theta}_2 \mathbf{w}_2 = \mathbf{0}$$

where θ_j is the angular variable relative to the j -th R-pair and \mathbf{w}_j a unit vector along its axis. Since by hypothesis \mathbf{w}_1 and \mathbf{w}_2 are not parallel, Eq. (8) could not be satisfied for nonzero values of $\dot{\theta}_1$ and $\dot{\theta}_2$.

If the leg comprised three R-pairs, the constraint on the platform mobility would become

$$(9) \quad \boldsymbol{\omega} = \dot{\theta}_1 \mathbf{w}_1 + \dot{\theta}_2 \mathbf{w}_2 + \dot{\theta}_3 \mathbf{w}_3 = \mathbf{0}$$

If the axes of two joints were parallel, Eq. (9) could be satisfied only if the axis of the third one were so as well, violating the hypotheses. It follows that \mathbf{w}_1 , \mathbf{w}_2 and \mathbf{w}_3 should be linearly dependent, but not parallel to each other. As a consequence of a generic instantaneous translational motion of the platform, these vectors would become

$$(10) \quad \mathbf{w}'_1 = \mathbf{w}_1$$

$$(11) \quad \mathbf{w}'_2 = \mathbf{w}_2 + \dot{\theta}_1 \mathbf{w}_1 \times \mathbf{w}_2 dt$$

$$(12) \quad \mathbf{w}'_3 = \mathbf{w}_3$$

where \mathbf{w}_1 and \mathbf{w}_3 would not change their orientation because they are axes of pairs either fixed to the frame and the platform respectively or connected to them by means of series of prismatic joints. It follows that, for \mathbf{w}'_1 , \mathbf{w}'_2 and \mathbf{w}'_3 to be still linearly dependent, \mathbf{w}'_2 should be expressed as a linear combination of \mathbf{w}_1 and \mathbf{w}_3 and hence of \mathbf{w}_1 and \mathbf{w}_2 . Equation (11) proves that this is impossible, since \mathbf{w}_1 and \mathbf{w}_2 are not parallel.

The cases in which the leg possesses four or five revolute joints must be studied in detail.

1.3.1 Type T_5 legs with four R-pairs

The kinematic model of a type T_5 leg including four R-pairs (R_j , $j=1,2,3,4$) is shown in Fig. 4. The P-pair cannot alter the orientation of the platform and thus can be placed anywhere along the kinematic chain. \mathbf{w}_j is a unit vector along the axis of the j -th revolute joint, θ_j the relative rotation variable and γ_{j+1} the angle comprised between \mathbf{w}_j and \mathbf{w}_{j+1} ($j=1,2,3$). The directions of the vectors \mathbf{w}_j are chosen so that γ_{12} , γ_{23} and γ_{34} are acute angles. The quantities

$$(13) \quad \begin{aligned} \mathbf{w}_1 \cdot \mathbf{w}_2 &= \cos \gamma_{12} \\ \mathbf{w}_2 \cdot \mathbf{w}_3 &= \cos \gamma_{23} \\ \mathbf{w}_3 \cdot \mathbf{w}_4 &= \cos \gamma_{34} \end{aligned}$$

are construction parameters.

By hypothesis, the axes of the R-pairs cannot all be parallel to each other. It will be now proved that not even the axes of three revolute joints may be mutually

parallel. Since the angular velocity of the platform must be zero, the following relation must hold

$$(14) \quad \boldsymbol{\omega} = \dot{\theta}_1 \mathbf{w}_1 + \dot{\theta}_2 \mathbf{w}_2 + \dot{\theta}_3 \mathbf{w}_3 + \dot{\theta}_4 \mathbf{w}_4 = \mathbf{0}$$

If the axes of the revolute joints were all parallel except one (say \mathbf{w}_k), Eq. (14) could be satisfied only if R_k were idle ($\dot{\theta}_k=0$) and hence the leg was type T_4 . In conclusion, the R-pair axes may be parallel only two by two.

In order to prevent the platform rotating about the line identified by the unit vector \mathbf{n} , all \mathbf{w}_j must be perpendicular to \mathbf{n} , that is

$$(15) \quad \mathbf{n}^T \left(\sum_{j=1}^4 a_j \mathbf{w}_j \right) = 0$$

for any value of the scalar quantities a_j . Equation (15) requires two of the vectors \mathbf{w}_j to be linear combinations of the other two.

Now, sufficient conditions must be found that ensure Eq. (15) to be verified at any instant if it holds at a certain moment, namely the initial one (when the leg is assembled).

As a result of a generic translational instantaneous motion of the platform, the vectors \mathbf{w}_j (which originally satisfy Eq. (15)) turn into the following

$$(16) \quad \mathbf{w}'_1 = \mathbf{w}_1$$

$$(17) \quad \mathbf{w}'_2 = \mathbf{w}_2 + \dot{\theta}_1 \mathbf{w}_1 \times \mathbf{w}_2 dt$$

$$(18) \quad \mathbf{w}'_3 = \mathbf{w}_3 + (\dot{\theta}_1 \mathbf{w}_1 + \dot{\theta}_2 \mathbf{w}_2) \times \mathbf{w}_3 dt$$

$$(19) \quad \mathbf{w}'_4 = \mathbf{w}_4$$

where \mathbf{w}_1 and \mathbf{w}_4 do not change their orientation because they are axes of pairs either attached to the frame and the platform respectively or connected to them by means of a prismatic joint.

In the new configuration, the vectors \mathbf{w}'_j are still perpendicular to a common line only if special geometric conditions are met. If this happens, it can be inferred that the leg remains type T_5 during the entire motion, since the reasoning may be repeated at any instant.

Because the axes of the R-pairs may be parallel only two by two, the following cases have to be examined:

- i. $\gamma_{12} = 0, \gamma_{23} \neq 0, \gamma_{34} = 0.$
- ii. $\gamma_{12} \neq 0, \gamma_{34} \neq 0;$
- iii. $\gamma_{12} = 0, \gamma_{23} \neq 0, \gamma_{34} \neq 0;$
- iv. $\gamma_{12} \neq 0, \gamma_{23} \neq 0, \gamma_{34} = 0;$

1.3.1.1 Case $\gamma_{12} = 0, \gamma_{23} \neq 0, \gamma_{34} = 0$

In this case it is

$$(20) \quad \mathbf{w}_1 = \mathbf{w}_2$$

$$(21) \quad \mathbf{w}_3 = \mathbf{w}_4$$

and Eq. (15) is always satisfied, since the axes of all the revolute joints are always perpendicular to the unit vector

$$(22) \quad \mathbf{n}_2 = \frac{\mathbf{w}_2 \times \mathbf{w}_3}{\sin \gamma_{23}}$$

Equation (14) becomes

$$(23) \quad (\dot{\theta}_1 + \dot{\theta}_2) \mathbf{w}_1 + (\dot{\theta}_3 + \dot{\theta}_4) \mathbf{w}_3 = \mathbf{0}$$

which infers

$$(24) \quad \dot{\theta}_1 + \dot{\theta}_2 = 0$$

and

$$(25) \quad \dot{\theta}_3 + \dot{\theta}_4 = 0$$

A leg meeting the requirements (20) and (21) is said to be type T_5' . An example is sketched in Fig. 3a.

Such an architecture was presented for the first time by Frisoli et al. [34] and, independently, by Carricato and Parenti-Castelli [35].

1.3.1.2 Case $\gamma_{12} \neq 0, \gamma_{34} \neq 0$

The unit vectors perpendicular to the pairs $(\mathbf{w}_1, \mathbf{w}_2)$ and $(\mathbf{w}_4, \mathbf{w}_3)$ are respectively

$$(26) \quad \mathbf{n}_1 = \frac{\mathbf{w}_1 \times \mathbf{w}_2}{\sin \gamma_{12}}$$

$$(27) \quad \mathbf{n}_3 = \frac{\mathbf{w}_4 \times \mathbf{w}_3}{\sin \gamma_{34}}$$

Equation (15) infers

$$(28) \quad \mathbf{n}_1 = \mathbf{n}_3$$

As a result of a generic translational instantaneous motion of the platform, the lines perpendicular to the axes of R_1 and R_2 and of R_3 and R_4 become respectively

$$(29) \quad \mathbf{n}'_1 = \frac{\mathbf{w}'_1 \times \mathbf{w}'_2}{\sin \gamma_{12}}$$

$$(30) \quad \mathbf{n}'_3 = \frac{\mathbf{w}'_4 \times \mathbf{w}'_3}{\sin \gamma_{34}}$$

and hence, by virtue of Eqs. (16)-(19) and (26)-(27)

$$(31) \quad \mathbf{n}'_1 = \mathbf{n}_1 + \frac{\dot{\theta}_1}{\sin \gamma_{12}} \mathbf{w}_1 \times (\mathbf{w}_1 \times \mathbf{w}_2) dt$$

$$(32) \quad \mathbf{n}'_3 = \mathbf{n}_3 + \frac{\dot{\theta}_1}{\sin \gamma_{34}} \mathbf{w}_4 \times (\mathbf{w}_1 \times \mathbf{w}_3) dt + \frac{\dot{\theta}_2}{\sin \gamma_{34}} \mathbf{w}_4 \times (\mathbf{w}_2 \times \mathbf{w}_3) dt$$

By imposing

$$(33) \quad \mathbf{n}'_1 = \mathbf{n}'_3$$

and considering Eq. (28), the following equation is obtained

$$(34) \quad \dot{\theta}_1 [\sin \gamma_{34} \mathbf{w}_1 \times (\mathbf{w}_1 \times \mathbf{w}_2) - \sin \gamma_{12} \mathbf{w}_4 \times (\mathbf{w}_1 \times \mathbf{w}_3)] = \dot{\theta}_2 \sin \gamma_{12} \mathbf{w}_4 \times (\mathbf{w}_2 \times \mathbf{w}_3)$$

Since

$$(35) \quad \mathbf{w}_1 \times (\mathbf{w}_1 \times \mathbf{w}_2) = \cos \gamma_{12} \mathbf{w}_1 - \mathbf{w}_2$$

$$(36) \quad \mathbf{w}_4 \times (\mathbf{w}_1 \times \mathbf{w}_3) = \cos \gamma_{34} \mathbf{w}_1 - (\mathbf{w}_1 \cdot \mathbf{w}_4) \mathbf{w}_3$$

$$(37) \quad \mathbf{w}_4 \times (\mathbf{w}_2 \times \mathbf{w}_3) = \cos \gamma_{34} \mathbf{w}_2 - (\mathbf{w}_2 \cdot \mathbf{w}_4) \mathbf{w}_3$$

Equation (34) becomes

$$(38) \quad \dot{\theta}_1 \left[\sin(\gamma_{34} - \gamma_{12}) \mathbf{w}_1 - \sin \gamma_{34} \mathbf{w}_2 + \sin \gamma_{12} (\mathbf{w}_1 \cdot \mathbf{w}_4) \mathbf{w}_3 \right] = \dot{\theta}_2 \sin \gamma_{12} \left[\cos \gamma_{34} \mathbf{w}_2 - (\mathbf{w}_2 \cdot \mathbf{w}_4) \mathbf{w}_3 \right]$$

Equation (38) is satisfied for any $\dot{\theta}_1$ and $\dot{\theta}_2$ if and only if

$$(39) \quad \sin(\gamma_{34} - \gamma_{12}) \mathbf{w}_1 - \sin \gamma_{34} \mathbf{w}_2 + \sin \gamma_{12} (\mathbf{w}_1 \cdot \mathbf{w}_4) \mathbf{w}_3 = 0$$

and

$$(40) \quad \cos \gamma_{34} \mathbf{w}_2 = (\mathbf{w}_2 \cdot \mathbf{w}_4) \mathbf{w}_3$$

Equation (40) can be fulfilled only if \mathbf{w}_2 and \mathbf{w}_3 are parallel, that is

$$(41) \quad \mathbf{w}_2 = \mathbf{w}_3$$

which means

$$(42) \quad \gamma_{23} = 0$$

Equation (41) involves

$$(43) \quad \mathbf{w}_3 \cdot \mathbf{w}_4 = \mathbf{w}_2 \cdot \mathbf{w}_4$$

and hence, remembering the definitions (13), the fulfillment of Eq. (40).

By virtue of Eq. (41), Eq. (39) becomes:

$$(44) \quad \sin(\gamma_{34} - \gamma_{12}) \mathbf{w}_1 = \left[\sin \gamma_{34} - \sin \gamma_{12} (\mathbf{w}_1 \cdot \mathbf{w}_4) \right] \mathbf{w}_2$$

Since \mathbf{w}_1 and \mathbf{w}_2 are not parallel ($\gamma_{12} \neq 0$), Eq. (44) holds only if

$$(45) \quad \sin(\gamma_{34} - \gamma_{12}) = 0$$

and

$$(46) \quad \sin \gamma_{34} = \sin \gamma_{12} (\mathbf{w}_1 \cdot \mathbf{w}_4)$$

Equation (45) yields

$$(47) \quad \gamma_{34} = \gamma_{12}$$

whilst Eq. (46), together with Eq. (47), gives

$$(48) \quad \mathbf{w}_1 = \mathbf{w}_4$$

In conclusion, the conditions (42) and (47) must be fulfilled at a design level and the condition (48) during the assembly. A leg meeting the above requirements is said to be type T_5 and is the kind studied by Tsai [30] and Di Gregorio and Parenti-Castelli [31]. An example is sketched in Fig. 3c.

For such a leg, Eq. (14) becomes

$$(49) \quad (\dot{\theta}_1 + \dot{\theta}_4) \mathbf{w}_1 + (\dot{\theta}_2 + \dot{\theta}_3) \mathbf{w}_2 = \mathbf{0}$$

which infers

$$(50) \quad \dot{\theta}_1 + \dot{\theta}_4 = 0$$

and

$$(51) \quad \dot{\theta}_2 + \dot{\theta}_3 = 0$$

1.3.1.3 Case $\gamma_{12} = 0$, $\gamma_{23} \neq 0$, $\gamma_{34} \neq 0$

Since in this case

$$(52) \quad \mathbf{w}_1 = \mathbf{w}_2$$

Equations (15), (17) and (18) may be respectively written in the form

$$(53) \quad \mathbf{w}_2 \cdot (\mathbf{w}_4 \times \mathbf{w}_3) = 0$$

$$(54) \quad \mathbf{w}'_2 = \mathbf{w}_2$$

$$(55) \quad \mathbf{w}'_3 = \mathbf{w}_3 + (\dot{\theta}_1 + \dot{\theta}_2) \mathbf{w}_2 \times \mathbf{w}_3 dt$$

By imposing

$$(56) \quad \mathbf{w}'_2 \cdot (\mathbf{w}'_4 \times \mathbf{w}'_3) = 0$$

and considering Eqs. (19), (54) and (55), the following equation can be derived

$$(57) \quad \mathbf{w}_2 \cdot (\mathbf{w}_4 \times \mathbf{w}_3) + (\dot{\theta}_1 + \dot{\theta}_2) \mathbf{w}_2 \cdot [\mathbf{w}_4 \times (\mathbf{w}_2 \times \mathbf{w}_3)] dt = 0$$

By virtue of Eq. (53) and the properties of the vector triple product, Eq. (57) becomes

$$(58) \quad \mathbf{w}_4 \cdot [\mathbf{w}_2 \times (\mathbf{w}_2 \times \mathbf{w}_3)] = 0$$

The vector $\mathbf{w}_2 \times (\mathbf{w}_2 \times \mathbf{w}_3)$ lies on the same plane as \mathbf{w}_2 , \mathbf{w}_3 and \mathbf{w}_4 and is perpendicular to \mathbf{w}_2 . Then, Eq. (58) may hold if and only if

$$(59) \quad \mathbf{w}_2 = \mathbf{w}_4$$

but this condition is not acceptable since it causes the axes of three revolute joints to be parallel to each other. It can be concluded that there cannot be type T_5 legs so that $\gamma_{2,i} = 0$, $\gamma_{23,i} \neq 0$, $\gamma_{34,i} \neq 0$.

1.3.1.4 Case $\gamma_{12} \neq 0$, $\gamma_{23} \neq 0$, $\gamma_{34} = 0$

In this case

$$(60) \quad \mathbf{w}_3 = \mathbf{w}_4$$

and Eq. (15) leads to

$$(61) \quad \mathbf{w}_3 \cdot (\mathbf{w}_1 \times \mathbf{w}_2) = 0$$

In a way analogous to that adopted in Section 1.3.1.3, it can be proved that the leg is type T_5 only if

$$(62) \quad \mathbf{w}_1 = \mathbf{w}_3$$

but again this condition is not acceptable since it causes the axes of three revolute joints to be parallel to each other.

1.3.2 Type T_5 legs with five R-pairs

Type T_5 legs with five R-pairs may be simply obtained by adding a fifth revolute joint (R_5) to the architectures found in Sections 1.3.1.1 and 1.3.1.2, adjacent and parallel to any of the R-pair already present. In fact, as far as the mobility is concerned, two contiguous parallel revolute joints are equivalent to the ensemble of an R-pair and a P-pair whose axes are perpendicular to each other.

Adding R_5 to the architecture found in Section 1.3.1.1 yields the topologies illustrated in Figs. 5a and 5b, which will still be considered type T_5 's. For the former case, Eq. (24) becomes

$$(63) \quad \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_5 = 0$$

where θ_5 is the angular variable relative to R_5 . For the latter, Eq. (25) becomes

$$(64) \quad \dot{\theta}_3 + \dot{\theta}_4 + \dot{\theta}_5 = 0$$

Adding R_5 to the architecture found in Section 1.3.1.2 yields the topologies illustrated in Figs. 5c, 5d and 5e, which will still be considered type T_5'' . For the first and the third case, Eq. (50) becomes

$$(65) \quad \dot{\theta}_1 + \dot{\theta}_4 + \dot{\theta}_5 = 0$$

For the second case, Eq. (51) turns into

$$(66) \quad \dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_5 = 0$$

The legs depicted in Figs. 3b and 3d are type T_5' and T_5'' respectively: the former is analogous to that sketched in Fig. 5a, the latter to that shown in Fig. 5c.

1.4 Type T_6 legs

A leg having $n_i=6$ does not constrain the movement of the platform at all. Therefore it is not useful in the context of the definition of the motion characteristic of a TPM.

2 GENERATION OF A TPM

The previous chapter provided an exhaustive classification of all possible topologies that a leg of a PM may have in order to:

- provide some rotational constraint to the platform movement;
- be consistent with a purely translational motion of the platform.

In the present chapter, how to assemble such legs in order to realize a TPM will be discussed.

Basically, any combination of type T_3 , T_4 and T_5 legs that, as a whole, would prevent the platform rotating about three linearly independent axes would generate a TPM. However, a minimum number of constraints is imposed only if each one of them is not a repeated one. This can only be accomplished by using:

1. a single type T_3 leg, which prevents all platform rotations;
2. one type T_4 leg and one type T_5 leg, the former eliminating two dofs of rotation, the latter one dof;
3. three type T_5 legs, each one of which eliminates one rotational dof.

However, for the manipulator to be able to take full advantage of its potentialities as a parallel mechanism (for instance, in terms of high stiffness and good dynamic performance), it should possess at least three legs, so that there would be at least two closed-loop chains and all three motors could be mounted on the base. The first two of the presented solutions clearly do not meet such a requirement. Actually, the former even represents a serial manipulator. For the TPM to have three legs, overconstrained architectures must also be taken into account, i.e., architectures in which two or more legs eliminate the same dofs. An additional desirable feature, that would give symmetry to the design and could reduce the manufacturing cost, is making use of legs exhibiting the same topology.

According to the above specifications, the following families of TPMs are identified:

- T_3 TPMs, exhibiting three identical type T_3 legs;
- T_4 TPMs, exhibiting three identical type T_4 legs;
- T_5 TPMs, exhibiting three identical type T_5 legs.

Clearly, the mechanisms belonging to the first two families are overconstrained.

The above families will be studied in detail in the following chapters and, in particular, new classes of TPMs exhibiting outstanding properties will be presented.

3 CONSTRAINT SINGULARITIES

Constraint singularities, characteristic of manipulators having less than six dofs, cause the platform to be no longer constrained to the intended motion [37]. For TPMs, in particular, they are configurations in which the platform acquires an instantaneous mobility of rotation. The following sections show that only T_5 ' TPMs exhibit constraint singularities [38-40], whilst T_3 , T_4 and T_5 ' TPMs are constraint-singularity-free.

3.1 T_3 TPMs

Such manipulators possess three type T_3 legs, each one of which comprises three linearly independent P- or PA- pairs. Since each leg contains only translational joints, the rotation of the platform is impossible, whatever the leg configurations are. Hence, T_3 TPMs do not exhibit constraint singularities.

3.2 T_4 TPMs

The manipulators belonging to such a family possess three identical type T_4 legs. Each leg allows the platform to rotate only about an axis \mathbf{w}_i ($i=1,2,3$) and the orientation of this axis remains constant throughout the motion. If two legs are assembled so that $\mathbf{w}_h \neq \mathbf{w}_k$, then each one prevents the rotation allowed by the other, rendering any rotation constructively impossible, regardless of the leg configurations. It follows that T_4 TPMs do not exhibit constraint singularities.

3.3 T_5 ' TPMs

The manipulators belonging to this family exhibit three type T_5 ' legs. Like in the examples illustrated in Figs. 3a, 3b, 5a and 5b, each leg ($i=1,2,3$) comprises:

- two contiguous rotational pairs whose axes are parallel to a unit vector \mathbf{w}_{1i} ;
- two contiguous rotational pairs whose axes are parallel to a unit vector \mathbf{w}_{3i} nonparallel to \mathbf{w}_{1i} ;
- a prismatic pair, which can be replaced by a fifth rotational joint that can be located adjacent and parallel to any one of the others.

Such a leg prevents the platform rotation about the line \mathbf{n}_{2i} perpendicular to the plane determined by \mathbf{w}_{1i} and \mathbf{w}_{3i} . If the legs are assembled so that \mathbf{n}_{21} , \mathbf{n}_{22} and \mathbf{n}_{23} are linearly independent, the platform rotation is prevented about any line and its instantaneous motion can only be a translation. In this case, Eqs. (24) and (25) (respectively substituted by Eqs. (63) and (64) if a fifth R- or H- pair is present) hold and neither \mathbf{w}_{1i} and \mathbf{w}_{3i} nor \mathbf{n}_{2i} change their orientation because of the movement. This means that, if \mathbf{n}_{21} , \mathbf{n}_{22} and \mathbf{n}_{23} are linearly independent at the outset, they continue to be so during the entire motion and the platform translates throughout the workspace. In other words, T_5' TPMs do not exhibit constraint singularities.

3.4 T_5'' TPMs

The manipulators of this family exhibit three type T_5'' legs. Like in the examples illustrated in Figs. 3c, 3d, 5c, 5d and 5e, each leg ($i=1,2,3$) comprises:

- two rotational pairs whose axes are parallel to a unit vector \mathbf{w}_{1i} ;
- two contiguous rotational pairs, interposed between the previous ones, whose axes are parallel to a unit vector \mathbf{w}_{2i} nonparallel to \mathbf{w}_{1i} ;
- a prismatic pair, which can be replaced by a fifth rotational joint that can be located adjacent and parallel to any one of the others.

Such a leg prevents the platform rotation about the line \mathbf{n}_{1i} perpendicular to the plane determined by \mathbf{w}_{1i} and \mathbf{w}_{2i} . If the legs are assembled so that \mathbf{n}_{11} , \mathbf{n}_{12} and \mathbf{n}_{13} are linear independent, the platform turning is prevented about any line and its instantaneous motion can only be a translation. In this case, Eqs. (50) and (51) (respectively substituted by Eqs. (65) and (66) if a fifth R- or H- pair is present) hold.

The orientation of \mathbf{w}_{2i} , and therefore of \mathbf{n}_{1i} , does not remain constant during the motion, since it depends on θ_{1i} (and also θ_{5i} , if there is a fifth rotational joint). This means that there can be configurations in which \mathbf{n}_{11} , \mathbf{n}_{12} and \mathbf{n}_{13} are no longer linear independent, i.e.

$$(67) \quad \mathbf{n}_{11} \cdot (\mathbf{n}_{12} \times \mathbf{n}_{13}) = 0$$

Anytime Eq. (67) is satisfied, the rotation of the platform is no longer prevented and its motion ceases being purely translational. Such configurations are constraint singularities. This result, valid for all the mechanisms that belong to the examined family, was obtained for the first time by Di Gregorio and Parenti-Castelli [38] for the 3-UPU manipulator, a particular architecture presented by Tsai [30]. The

constraint singularity locus for such a mechanism has been shown in [40] and a geometric interpretation has been given in [39].

4 SINGULARITY-FREE FULLY-ISOTROPIC MANIPULATORS

Let \mathbf{x} be the position vector, in a given fixed reference frame, of a generic point P embedded in the platform. \mathbf{x} is a function of the three displacement variables q_i ($i=1,2,3$) of the actuated joints, assumed to be distributed one per leg.

Let, for each leg, a constant direction \mathbf{u}_i ($i=1,2,3$) exist so that the displacement of P along it, with respect to a reference configuration \mathbf{x}_0 , is determined by no other variable than q_i

$$(68) \quad \mathbf{u}_i^T (\mathbf{x} - \mathbf{x}_0) = f_i(q_i) \quad i = 1, 2, 3$$

Equation (68) can be expressed in the following form

$$(69) \quad \mathbf{J}(\mathbf{x} - \mathbf{x}_0) = \mathbf{f}(\mathbf{q})$$

where

$$(70) \quad \mathbf{q} = [q_1 \quad q_2 \quad q_3]^T$$

$$(71) \quad \mathbf{f}(\mathbf{q}) = [f_1(q_1) \quad f_2(q_2) \quad f_3(q_3)]^T$$

$$(72) \quad \mathbf{J} = \begin{bmatrix} \mathbf{u}_1^T \\ \mathbf{u}_2^T \\ \mathbf{u}_3^T \end{bmatrix}$$

\mathbf{J} being constant (\mathbf{u}_i is constant by hypothesis), differentiating Eq. (69) with respect to time yields

$$(73) \quad \mathbf{J}\dot{\mathbf{x}} = \frac{d\mathbf{f}}{d\mathbf{q}}\dot{\mathbf{q}}$$

where

$$(74) \quad \frac{d\mathbf{f}}{d\mathbf{q}} = \begin{bmatrix} df_1/dq_1 & 0 & 0 \\ 0 & df_2/dq_2 & 0 \\ 0 & 0 & df_3/dq_3 \end{bmatrix}$$

Equation (73) shows that \mathbf{J} is the Jacobian matrix. The systems (69) and (73) are invertible only if \mathbf{J} is not singular, that is if \mathbf{u}_1 , \mathbf{u}_2 and \mathbf{u}_3 are linearly independent. This has a clear physical meaning. Because of Eq. (68), each motorized joint basically represents the direct actuation of one of the translational dofs of the platform, which is provided with a 3-dof motion only if the actuated axes of translation are linearly independent. In short, if a TPM leg architecture can be devised so that Eq. (68) holds and the three legs are mounted so that their corresponding axes \mathbf{u}_i are linearly independent, the resulting manipulator has a constant nonsingular Jacobian matrix and does not exhibit direct singularities.

Moreover, if the axes \mathbf{u}_i are chosen mutually perpendicular, \mathbf{J} becomes orthogonal and so

$$(75) \quad \mathbf{J}^T \mathbf{J} = \mathbf{I}$$

where \mathbf{I} is the identity matrix.

Since Eq. (75) is necessary and sufficient for isotropy [42], it follows that the examined manipulators are fully-isotropic in their whole workspace.

By virtue of Eq. (75) the inversion of the systems (69) and (73) becomes very simple

$$(76) \quad \mathbf{x} = \mathbf{J}^T \mathbf{f}(\mathbf{q}) + \mathbf{x}_0$$

$$(77) \quad \dot{\mathbf{x}} = \mathbf{J}^T \frac{d\mathbf{f}}{d\mathbf{q}} \dot{\mathbf{q}}$$

Both the direct and the inverse kinematic analyses are hence straightforward.

In addition, since \mathbf{u}_1 , \mathbf{u}_2 and \mathbf{u}_3 are mutually perpendicular, the coordinate axes of the fixed reference frame may be assumed parallel to them without loss of generality. In this case, it is

$$(78) \quad \mathbf{J} = \mathbf{I}$$

In the following sections some families of singularity-free fully-isotropic TPMs will be presented. Such manipulators exhibit neither constraint nor direct singularities and have a constant orthogonal Jacobian matrix throughout their workspace.

4.1 Orthogonal T_3 TPMs

The manipulators belonging to the T_3 TPM family possess three identical type T_3 legs. Each leg comprises three linearly independent P- or PA- pairs and deprives the output link of all its rotational dofs. Clearly, such an architecture is overconstrained, since any constraint is repeated three times. As shown in Section 3.1, it does not exhibit constraint singularities.

A T_3 TPM is defined *orthogonal* if each leg comprises three orthogonal P-pairs and the actuated axes are mutually perpendicular. An example is sketched in Fig. 6. In this case, if \mathbf{v}_{ji} is a unit vector along the axis of the j -th P-pair of the i -th limb ($j=1,2,3; i=1,2,3$) and v_{ji} is the displacement along it with respect to the reference configuration, the following relation holds for each leg

$$(79) \quad \mathbf{x} - \mathbf{x}_0 = v_{1i} \mathbf{v}_{1i} + v_{2i} \mathbf{v}_{2i} + v_{3i} \mathbf{v}_{3i} \quad i = 1, 2, 3$$

Assuming that the actuated joints are mounted on the frame, it is

$$(80) \quad v_{1i} = q_i \quad i = 1, 2, 3$$

and therefore from Eq. (79)

$$(81) \quad \mathbf{v}_{1i}^T (\mathbf{x} - \mathbf{x}_0) = q_i \quad i = 1, 2, 3$$

In matrix form, Eq. (81) becomes

$$(82) \quad \mathbf{J}(\mathbf{x} - \mathbf{x}_0) = \mathbf{q}$$

where

$$(83) \quad \mathbf{J} = \begin{bmatrix} \mathbf{v}_{11}^T \\ \mathbf{v}_{12}^T \\ \mathbf{v}_{13}^T \end{bmatrix}$$

is the Jacobian matrix, constant and orthogonal.

4.2 Fully-isotropic T_4 TPMs

The manipulators of this family possess three identical type T_4 legs, each one of which allows the platform to rotate about a constant axis \mathbf{w}_i only. Such an architecture is overconstrained, since each leg eliminates two dofs, but, on the whole,

only three dofs are eliminated. As shown in Section 3.2, it does not exhibit constraint singularities.

As the following will show, a fully-isotropic T_4 TPM may be designed by conceiving leg architectures that allow the “direct” actuation of either the translational dof out of the plane Π_i perpendicular to \mathbf{w}_i (type I) or one of the translational dofs lying on such a plane (type II).

4.2.1 Type I fully-isotropic T_4 TPMs

The generic i -th leg ($i=1,2,3$) of a T_4 TPM must comprise, other than two or three rotational pairs whose axes are all parallel to the unit vector \mathbf{w}_i , at least one translational joint, called V_i , permitting the movement out of the plane Π_i perpendicular to \mathbf{w}_i . This can be a P-pair whose axis does not lie on such a plane, an H-pair whose axis is parallel to \mathbf{w}_i or a PA-pair whose axis is not parallel to \mathbf{w}_i .

Let \mathbf{v}_i be a unit vector along the axis of V_i and α_i the angle that it forms with respect to \mathbf{w}_i . Such an angle remains constant throughout the motion, since the other pairs produce either rotations about \mathbf{w}_i or translations. Let v_i also be the displacement variable relative to V_i . If V_i is a P- or an H- pair, v_i is the displacement along the joint axis with respect to the reference configuration. If V_i is a PA-pair, v_i is the rotation of the parallelogram cranks about \mathbf{v}_i with respect to the line $\mathbf{w}_i \times \mathbf{v}_i$ (the motion of one crank in the plane perpendicular to \mathbf{v}_i and the projection on \mathbf{w}_i of its end displacement are depicted in Fig. 7, where l_i is the crank length).

Let V_i be the only joint responsible for the platform movement out of Π_i . This implies that, if another translational joint is present in the leg, it only allows movements parallel to Π_i (and that there are no H-pairs other than the one that, if present, provides the translation along \mathbf{w}_i). Therefore, if V_i is a P- or an H- pair, the following relation must hold

$$(84) \quad \mathbf{w}_i^T (\mathbf{x} - \mathbf{x}_0) = v_i \cos \alpha_i \quad i = 1, 2, 3$$

or, if V_i is a PA-pair,

$$(85) \quad \mathbf{w}_i^T (\mathbf{x} - \mathbf{x}_0) = l_i \sin v_i \sin \alpha_i \quad i = 1, 2, 3$$

If V_i is assumed actuated, then

$$(86) \quad v_i = q_i \quad i = 1, 2, 3$$

and Eqs. (84) and (85) can be written in the form

$$(87) \quad \mathbf{w}_i^T (\mathbf{x} - \mathbf{x}_0) = f_i(q_i) \quad i = 1, 2, 3$$

where

$$(88) \quad f_i(q_i) = \begin{cases} (\cos \alpha_i) q_i, & \text{if } V_i \text{ is a P- or an H-pair} \\ (l_i \sin \alpha_i) \sin q_i, & \text{if } V_i \text{ is a PA-pair} \end{cases}$$

In matrix form, Eq. (87) becomes

$$(89) \quad \mathbf{J}(\mathbf{x} - \mathbf{x}_0) = \mathbf{f}(\mathbf{q})$$

where

$$(90) \quad \mathbf{J} = \begin{bmatrix} \mathbf{w}_1^T \\ \mathbf{w}_2^T \\ \mathbf{w}_3^T \end{bmatrix}$$

is the constant Jacobian matrix. If \mathbf{w}_1 , \mathbf{w}_2 and \mathbf{w}_3 are chosen linearly independent \mathbf{J} is always nonsingular; if they are chosen mutually perpendicular \mathbf{J} is also orthogonal. Figure 8 shows some leg architectures that make the manipulator fully-isotropic, provided that the legs are assembled so that \mathbf{w}_1 , \mathbf{w}_2 and \mathbf{w}_3 are mutually perpendicular. Figure 9 shows the entire manipulator in the case in which the linear actuation is provided by means of H-pairs mounted on the frame with α_i equal to zero.

4.2.2 Type II fully-isotropic T_4 TPMs

A type II fully-isotropic T_4 TPM may be designed by conceiving a leg architecture that allows the “direct” actuation of one of the two dofs that permit translation on the plane Π_i perpendicular to \mathbf{w}_i . As shown in Section 1.2, such translation may be realized by means of either

- two rotational pairs, or
- one rotational pair and a P-pair not having its axis parallel to \mathbf{w}_i (or a however oriented PA-pair).

Because each rotational pair (be it R- or H-) necessarily affects both translational dofs on Π_i , the former realization is not applicable: both dofs would always depend on the displacement variables of both pairs in a coupled way.

Therefore, let the latter solution be considered. On the whole, the leg comprises two rotational pairs (whose relative rotation variables θ_{1i} and θ_{2i} satisfy Eq. (6)) and two translational ones. Let \mathbf{v}_i be a constant unit vector perpendicular to \mathbf{w}_i , Γ_i the vector plane containing \mathbf{w}_i and \mathbf{v}_i , and $\mathbf{u}_i = \mathbf{w}_i \times \mathbf{v}_i$. Also, let the translational joints be arranged so as to span Γ_i . For instance, they can be:

- two nonparallel P-pairs whose axes lie on Γ_i (Fig. 10a);
- a P-pair whose axis lies on Γ_i and a PA-pair whose axis is perpendicular to it (Fig. 10b);
- two PA-pairs whose axes are perpendicular to Γ_i (Fig. 10c).

Since \mathbf{v}_i and \mathbf{u}_i must be constant vectors, the translational joints must be attached to either the frame or the platform. Only a P-pair whose axis is parallel to \mathbf{w}_i can be interposed between the rotational pairs, for it would not change its orientation.

Because of such an arrangement, any movement along \mathbf{u}_i can only depend on the rotational pairs. Therefore, referring to Fig. 11, it is

$$(91) \quad \mathbf{u}_i^T (\mathbf{x} - \mathbf{x}_0) = l_i \sin \theta_{1i} \quad i = 1, 2, 3$$

where l_i is the distance between the axes of the rotational pairs (clearly, with a simple sign change, θ_{2i} could be used in the place of θ_{1i}).

If θ_{1i} (or θ_{2i}) is assumed actuated, then

$$(92) \quad \theta_{1i} = q_i \quad i = 1, 2, 3$$

and Eq. (91) becomes

$$(93) \quad \mathbf{u}_i^T (\mathbf{x} - \mathbf{x}_0) = f_i(q_i) \quad i = 1, 2, 3$$

where

$$(94) \quad f_i(q_i) = l_i \sin q_i$$

In matrix form, Eq. (93) becomes

$$(95) \quad \mathbf{J}(\mathbf{x} - \mathbf{x}_0) = \mathbf{f}(\mathbf{q})$$

where

$$(96) \quad \mathbf{J} = \begin{bmatrix} \mathbf{u}_1^T \\ \mathbf{u}_2^T \\ \mathbf{u}_3^T \end{bmatrix}$$

is the constant Jacobian matrix. As in the previous sections, if \mathbf{u}_1 , \mathbf{u}_2 and \mathbf{u}_3 are chosen linearly independent \mathbf{J} is always nonsingular. If they are chosen mutually perpendicular \mathbf{J} is also orthogonal, like in the example shown in Fig. 12.

4.3 Fully-isotropic T_5 TPMs

The manipulators belonging to such a family possess three identical type T_5' or T_5'' legs, each one of which prevents the platform rotating about a single axis, so that, on the whole, three rotational dofs are eliminated. Such mechanisms are not overconstrained.

4.3.1 Fully-isotropic T_5' TPMs

The generic i -th leg ($i=1,2,3$) of a T_5' TPM comprises:

- two contiguous R- or H- pairs whose axes are parallel to a unit vector \mathbf{w}_{1i} and whose relative rotation variables θ_{1i} and θ_{2i} satisfy Eq. (24);
- two contiguous R- or H- pairs whose axes are parallel to a unit vector \mathbf{w}_{3i} nonparallel to \mathbf{w}_{1i} and whose relative rotation variables θ_{3i} and θ_{4i} satisfy Eq. (25);
- a translational pair, which may be replaced by a fifth rotational pair parallel and adjacent to any one of the others.

An example is sketched in Fig. 13. \mathbf{n}_{2i} is the vector defined as $\mathbf{w}_{1i} \times \mathbf{w}_{3i}$; γ_{23i} is the angle, different from zero, comprised between \mathbf{w}_{1i} and \mathbf{w}_{3i} ; l_{ji} is the distance between the j -th and the $(j+1)$ -th rotational pair (l_{1i} and l_{3i} are different from zero); Π_{1i} and Π_{3i} are the vector planes perpendicular to \mathbf{w}_{1i} and \mathbf{w}_{3i} respectively.

Manipulators of this kind do not exhibit constraint singularities and the orientations of \mathbf{w}_{1i} and \mathbf{w}_{3i} , as well as of \mathbf{n}_{2i} , remain constant throughout the motion.

A fully-isotropic T_5' TPM may be designed by conceiving leg architectures that allow the “direct” actuation of the movement along either \mathbf{w}_{3i} (type I) or \mathbf{w}_{1i} (type II).

Let the former case be considered. If the translational joint is chosen so that it does not produce motion along \mathbf{w}_{3i} , any displacement along this axis can only depend on the rotational pairs whose axes are parallel to \mathbf{w}_{1i} . As illustrated in Fig. 14 (in which θ_{1i} is negative), such a displacement is equal to

$$(97) \quad \mathbf{w}_{3i}^T (\mathbf{x} - \mathbf{x}_0) = -l_{1i} \sin \gamma_{23i} \sin \theta_{1i} \quad i = 1, 2, 3$$

Assuming l_{1i} constant and θ_{1i} actuated ($q_i = \theta_{1i}$), and defining the function

$$(98) \quad f_i(q_i) = -l_i \sin \gamma_{23i} \sin q_i \quad i = 1, 2, 3$$

Equation (97) yields

$$(99) \quad \mathbf{J}(\mathbf{x} - \mathbf{x}_0) = \mathbf{f}(\mathbf{q})$$

where

$$(100) \quad \mathbf{J} = \begin{bmatrix} \mathbf{w}_{31}^T \\ \mathbf{w}_{32}^T \\ \mathbf{w}_{33}^T \end{bmatrix}$$

Since the orientation of \mathbf{w}_{3i} does not change during the motion, \mathbf{J} is a constant matrix. Figure 15 shows three examples of leg architectures consistent with Eq. (97), whilst Fig. 16 provides the sketch of a type I fully-isotropic T_5 ' TPM, in which the legs have been arranged so that \mathbf{w}_{31} , \mathbf{w}_{32} and \mathbf{w}_{33} are mutually perpendicular.

In type II fully-isotropic T_5 ' TPMs, the translational joint is chosen so as not to produce motion along \mathbf{w}_{1i} , so that any displacement along this axis only depends on the rotational pairs whose axes are parallel to \mathbf{w}_{3i} . As illustrated in Fig. 17, such a displacement is equal to

$$(101) \quad \mathbf{w}_{1i}^T(\mathbf{x} - \mathbf{x}_0) = l_{3i} \sin \gamma_{23i} \sin \theta_{3i} \quad i = 1, 2, 3$$

Assuming l_{3i} constant and θ_{3i} actuated ($q_i = \theta_{3i}$), and defining the function

$$(102) \quad f_i(q_i) = l_{3i} \sin \gamma_{23i} \sin q_i \quad i = 1, 2, 3$$

Equation (101) yields

$$(103) \quad \mathbf{J}(\mathbf{x} - \mathbf{x}_0) = \mathbf{f}(\mathbf{q})$$

where

$$(104) \quad \mathbf{J} = \begin{bmatrix} \mathbf{w}_{11}^T \\ \mathbf{w}_{12}^T \\ \mathbf{w}_{13}^T \end{bmatrix}$$

As in the former case, \mathbf{J} is a constant matrix and can be easily made orthogonal. The disadvantage of such a solution is that the motors cannot be mounted on the frame.

4.3.2 Fully-isotropic T_5 TPMs

The generic i -th leg ($i=1,2,3$) of a T_5 TPM comprises:

- two R- or H- pairs whose axes are parallel to a unit vector \mathbf{w}_{1i} and whose relative rotation variables θ_{1i} and θ_{4i} satisfy Eq. (50);
- two contiguous R- or H- pairs, interposed between the previous ones, whose axes are parallel to a unit vector \mathbf{w}_{2i} nonparallel to \mathbf{w}_{1i} and whose relative rotation variables θ_{2i} and θ_{3i} satisfy Eq. (51);
- a translational pair, which may be replaced by a fifth rotational joint that can be located parallel and adjacent to any one of the others.

An example is provided in Fig. 18. \mathbf{n}_{1i} is the vector defined as $\mathbf{w}_{1i} \times \mathbf{w}_{2i}$; γ_{12i} is the angle, different from zero, comprised between \mathbf{w}_{1i} and \mathbf{w}_{2i} ; l_{ji} is the distance between the j -th and the $(j+1)$ -th rotational pair (l_{2i} is different from zero); Π_{2i} is the plane perpendicular to \mathbf{w}_{2i} .

As proved in Section 3.4, \mathbf{w}_{2i} and \mathbf{n}_{1i} change orientation during the motion and constraint singularities exist.

The projection of the platform displacement on any line perpendicular to \mathbf{w}_{1i} always depends, in general, on both θ_{1i} and θ_{2i} .

Therefore, let the projection on \mathbf{w}_{1i} be considered. Since any rotation about the joints having their axes parallel to \mathbf{w}_{1i} cannot provide a contribution, such a projection is equal to (Fig. 19)

$$(105) \quad \mathbf{w}_{1i}^T (\mathbf{x} - \mathbf{x}_0) = l_{2i} \sin \gamma_{12i} \sin \theta_{2i} \quad i = 1, 2, 3$$

where it has been assumed that the translational joint is chosen so as not to produce motion along \mathbf{w}_{1i} (as in the example provided in Fig. 18). Assuming l_{2i} constant, θ_{2i} actuated ($q_i = \theta_{2i}$) and defining the function

$$(106) \quad f_i(q_i) = l_{2i} \sin \gamma_{12i} \sin q_i \quad i = 1, 2, 3$$

Equation (105) yields

$$(107) \quad \mathbf{J}(\mathbf{x} - \mathbf{x}_0) = \mathbf{f}(\mathbf{q})$$

where

$$(108) \quad \mathbf{J} = \begin{bmatrix} \mathbf{w}_{11}^T \\ \mathbf{w}_{12}^T \\ \mathbf{w}_{13}^T \end{bmatrix}$$

Even though \mathbf{J} is constant and may be easily made orthogonal, the manipulators belonging to this class are not singularity-free, because of the existence of constraint-singularities. Moreover, the motors cannot be mounted on the frame. Such disadvantages make the presented architecture less significant than the ones proposed in the previous sections.

CONCLUSIONS

This dissertation focused on translational parallel mechanisms (TPMs), that is on parallel manipulators whose platform is provided with a pure translational motion with respect to the frame.

In the first part, the problem of the topological synthesis and classification of TPMs was dealt with in a systematic and exhaustive form, providing previously published material with a general and organic frame.

It was seen how a TPM leg must be designed in order to allow the platform to be able to freely translate in space without altering its orientation. More precisely, which topological and geometric conditions such a leg must satisfy in order to, on the one hand, deprive the platform of one or more dofs of rotation (so that, by virtue of the contribution of all legs, platform turning is completely prevented) and, on the other, be consistent with a 3-dof translational motion of the platform.

It was then shown how such legs must be assembled in order to generate a TPM.

The second part dealt with the problem of singularities and in particular with the identification of classes of singularity-free TPMs.

Constraint singularities, which are configurations in which the platform loses its capability of purely translating and acquires an instantaneous mobility of rotation, were investigated for all families of TPMs. It was shown that, whilst some families exhibit this kind of singularity, others are constraint-singularity-free.

Finally, whole families of singularity-free fully-isotropic mechanisms were presented. These manipulators show outstanding properties. In particular:

- they exhibit a constant orthogonal Jacobian matrix throughout their workspace;
- they do not present either constraint or direct singularities;
- both the direct and the inverse kinematic analyses are straightforward.

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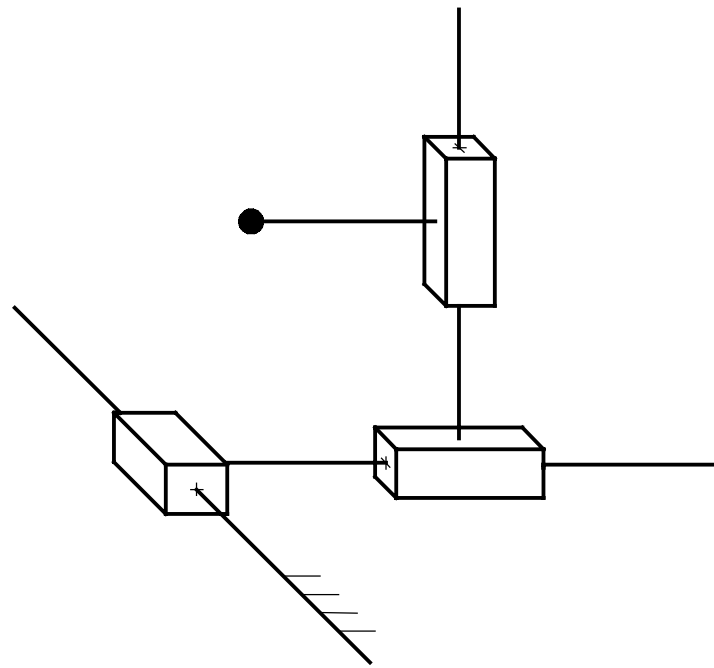
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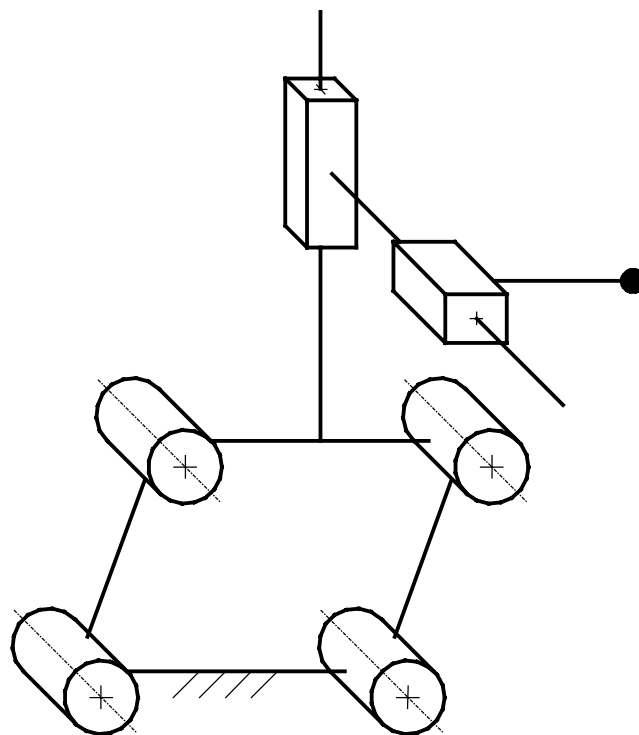
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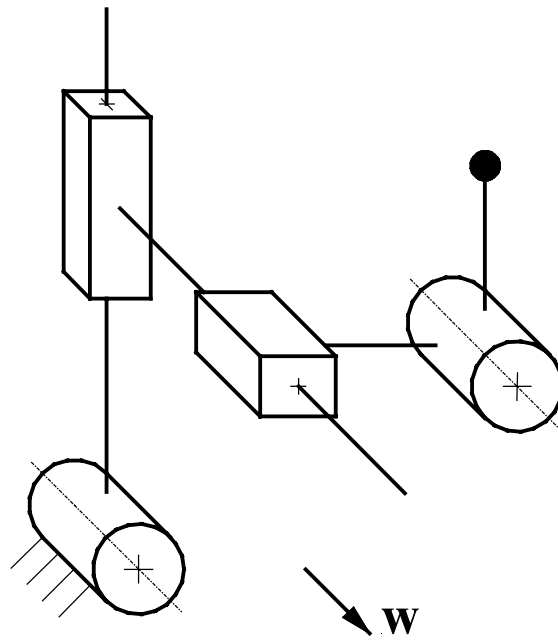


(a)

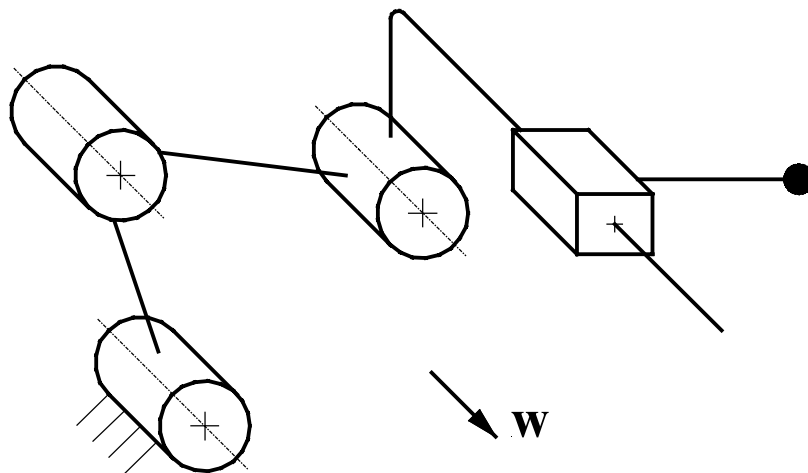


(b)

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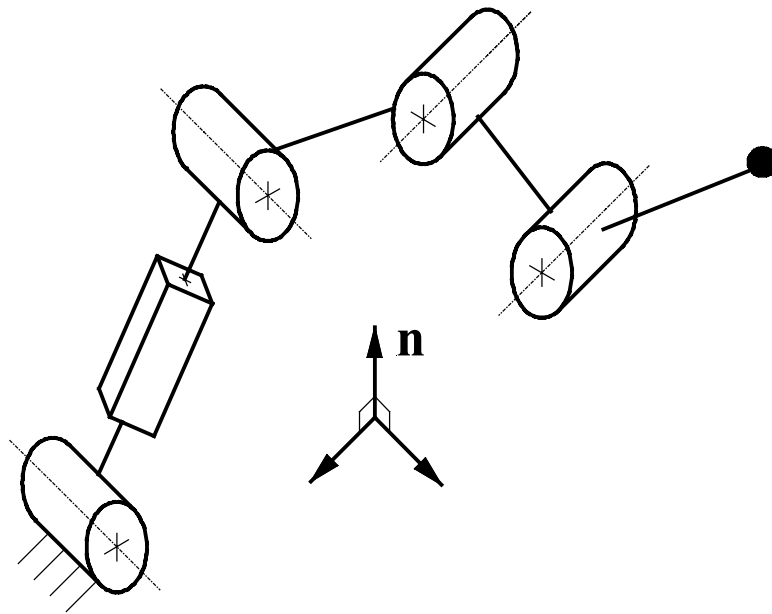


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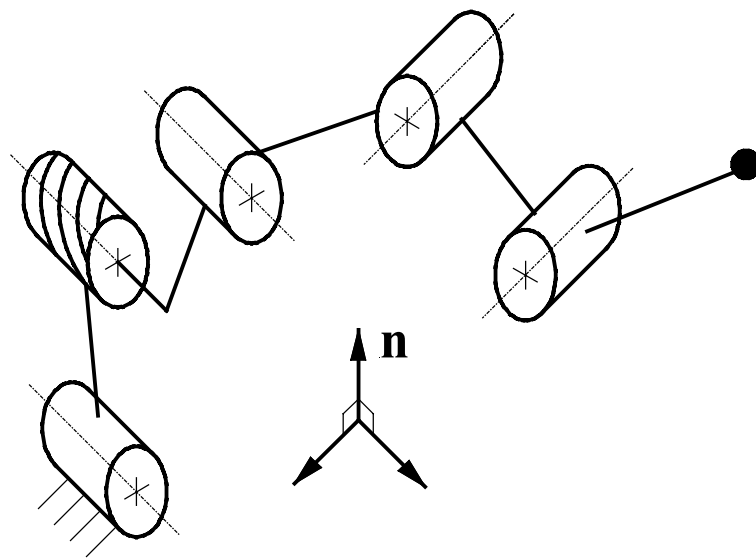


(b)

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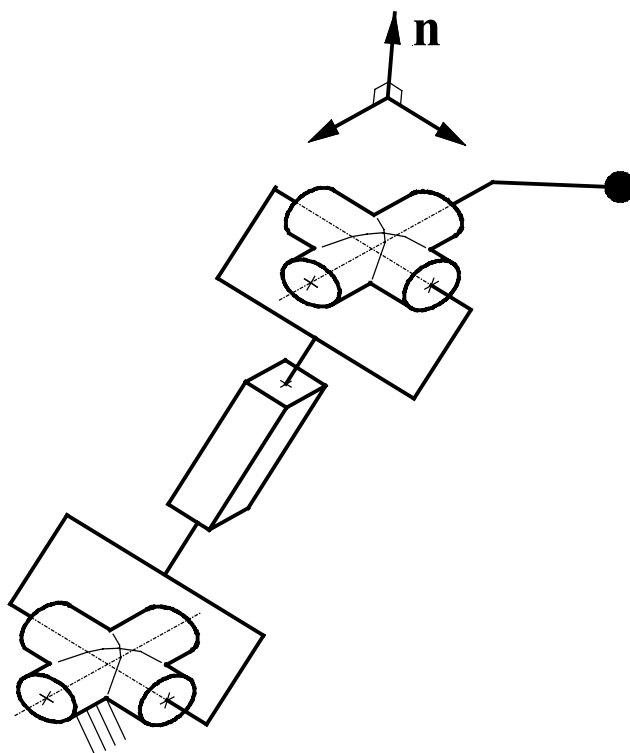


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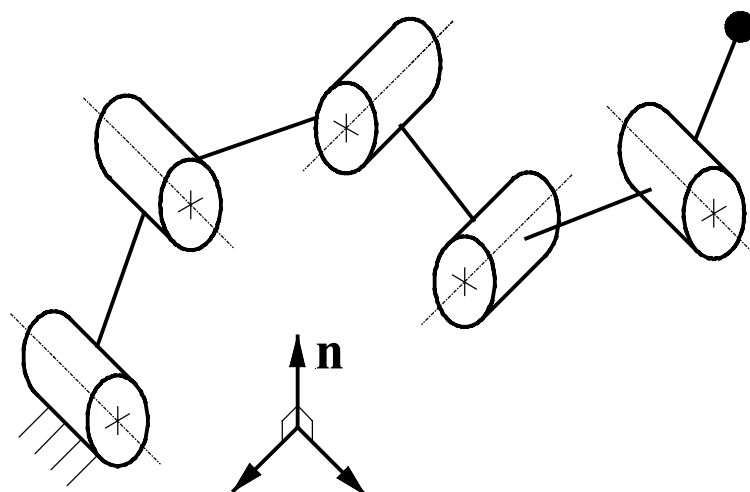


(b)

Fig. 3: Type T_5 legs.



(c)



(d)

Fig. 3: Type T_5 legs.

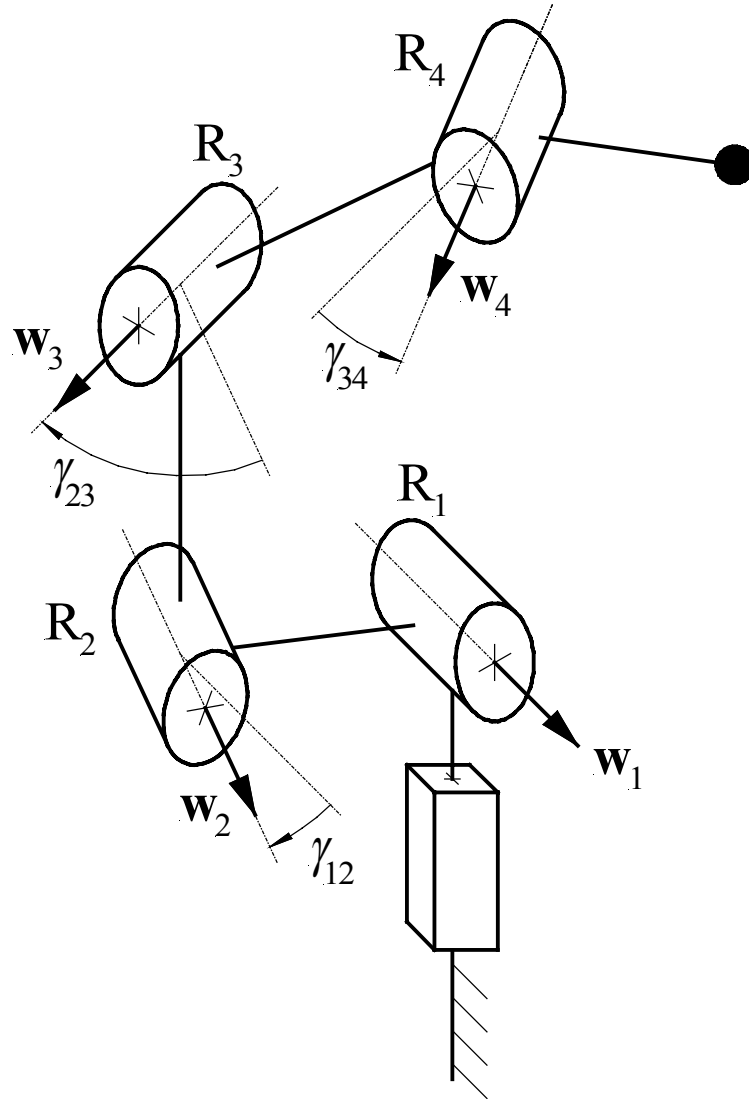
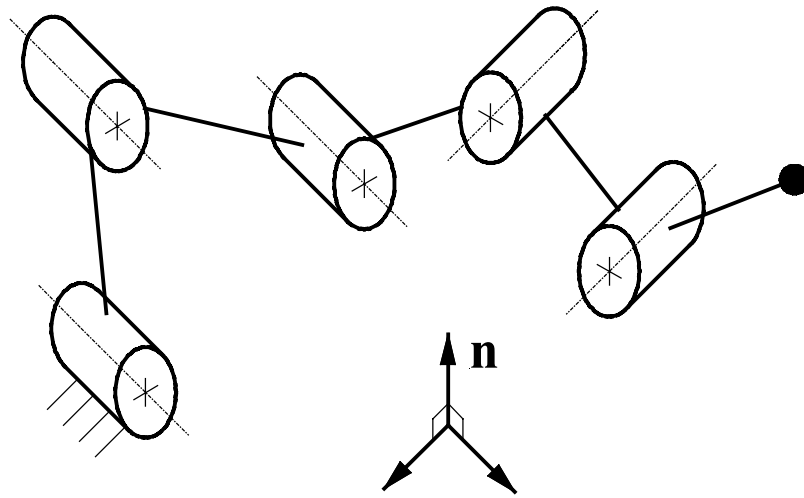
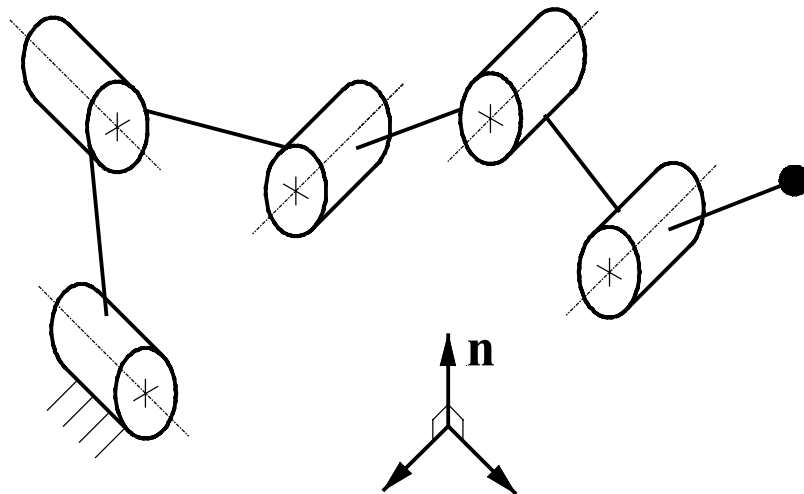


Fig. 4: Kinematic model of a type T_5 leg including four R-pairs.



(a)



(b)

Fig. 5: Type T_5 legs including five R-pairs.

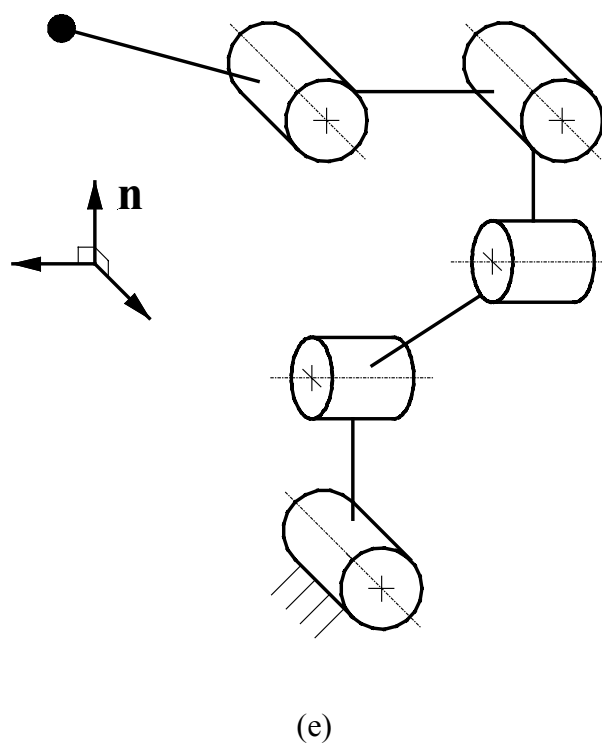
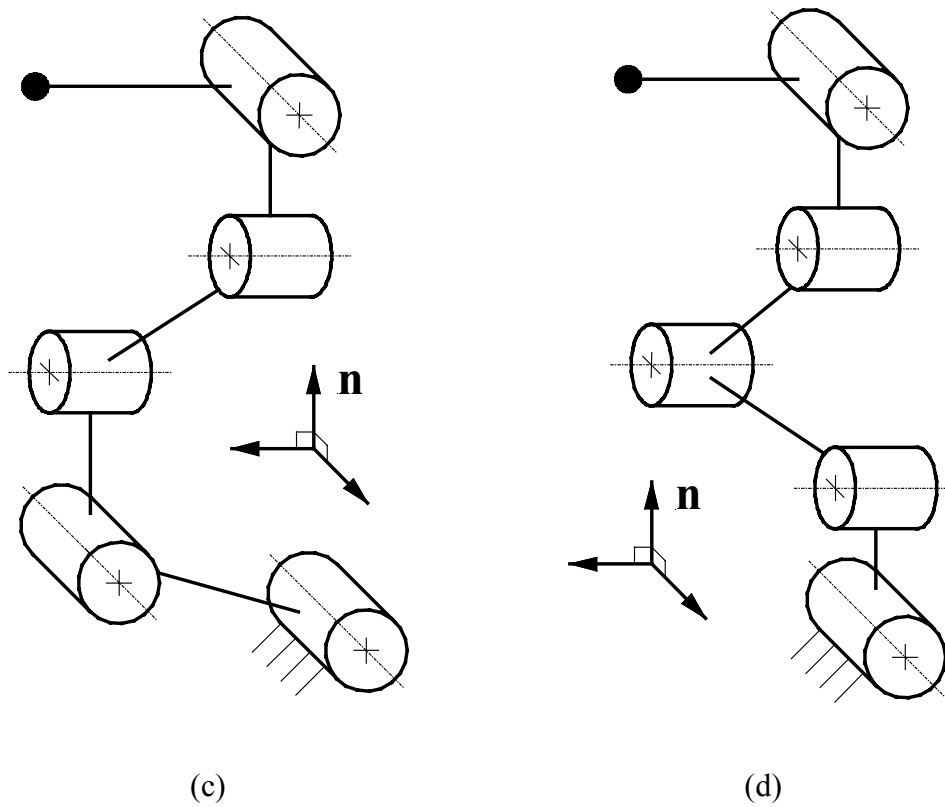


Fig. 5: Type T_5 legs including five R-pairs.

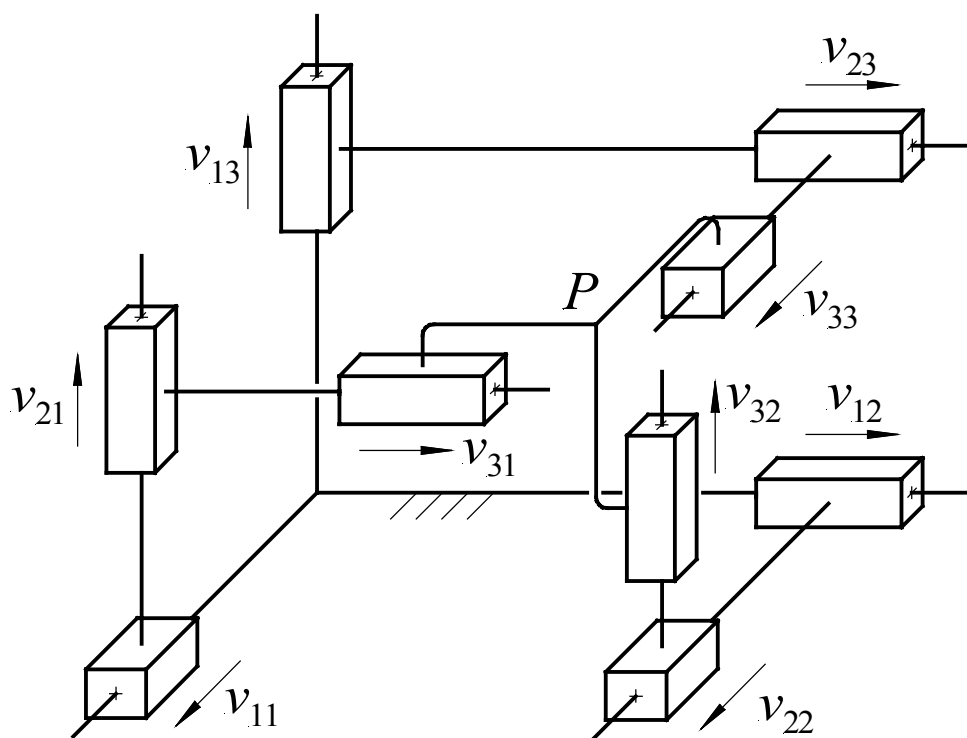


Fig. 6: An orthogonal T_3 TPM.

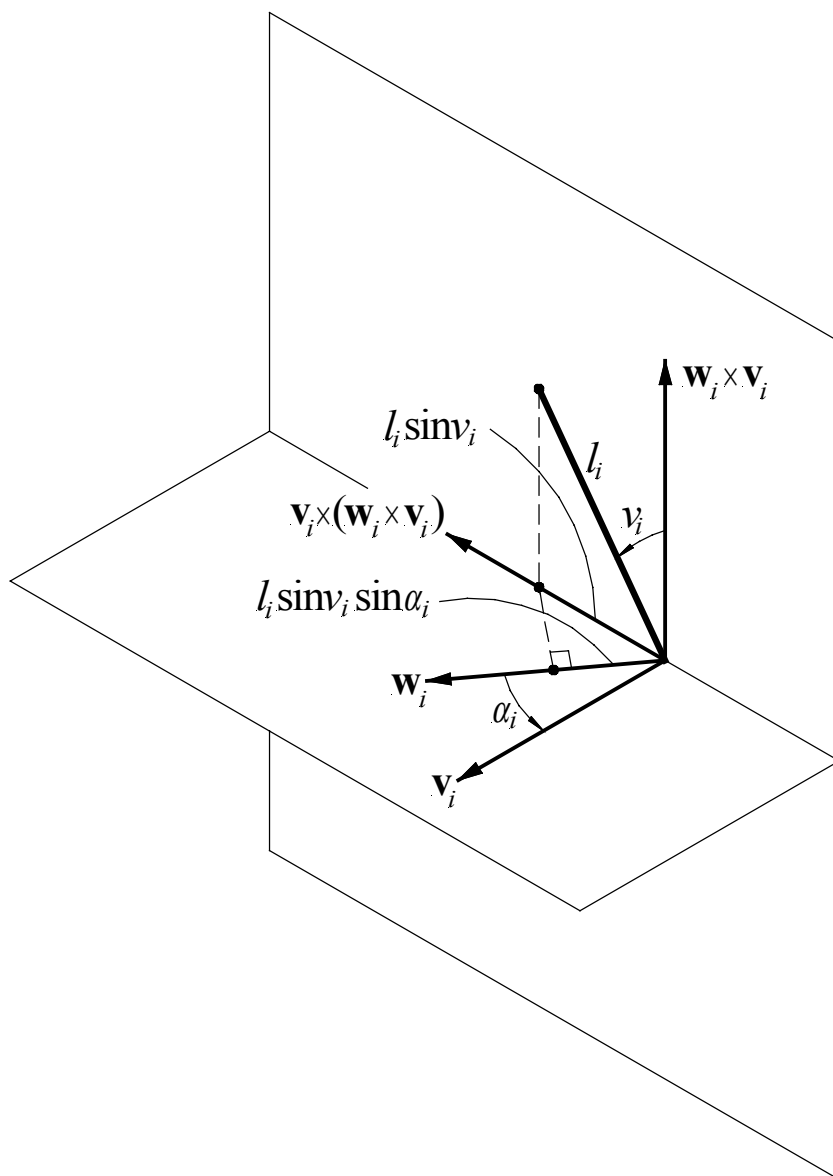
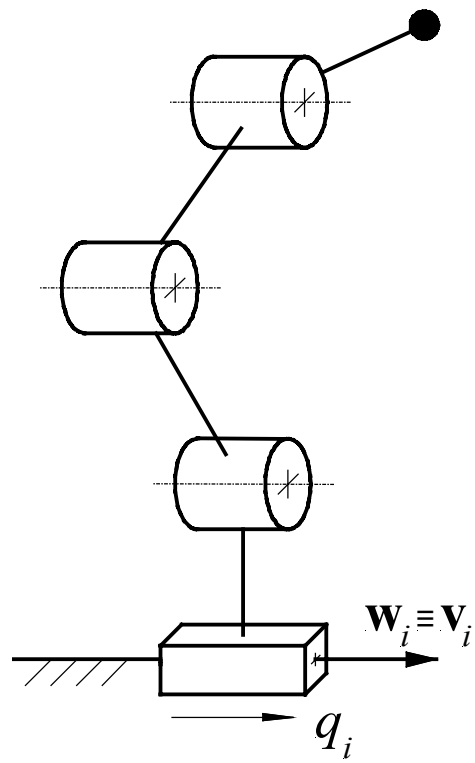
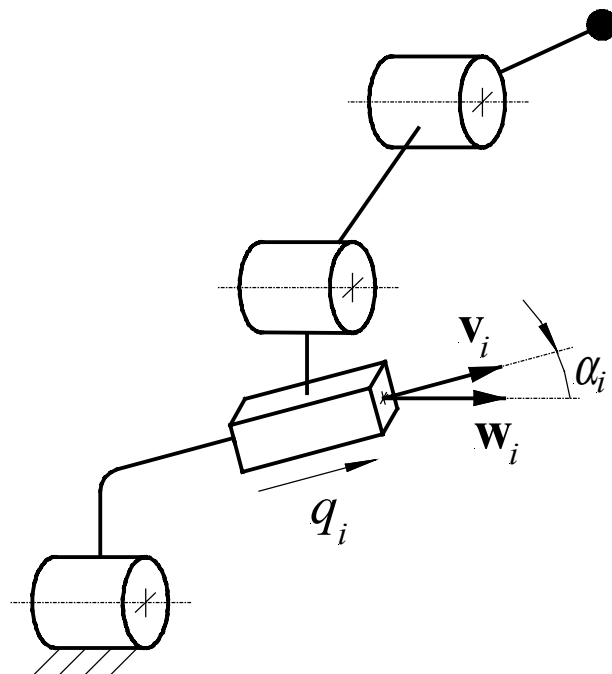


Fig. 7: Motion of one of the cranks of a parallelogram that permits, in a type T_4 leg, the platform's movement out of the plane perpendicular to w_i .

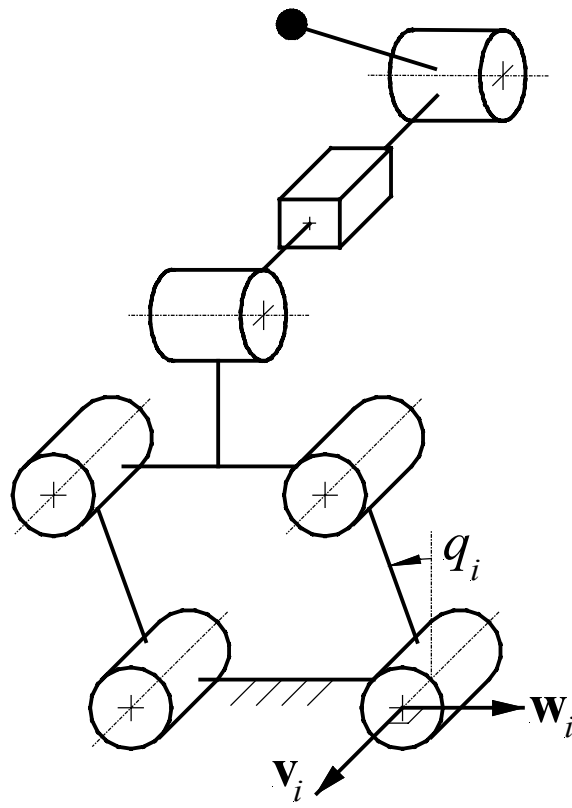


(a)

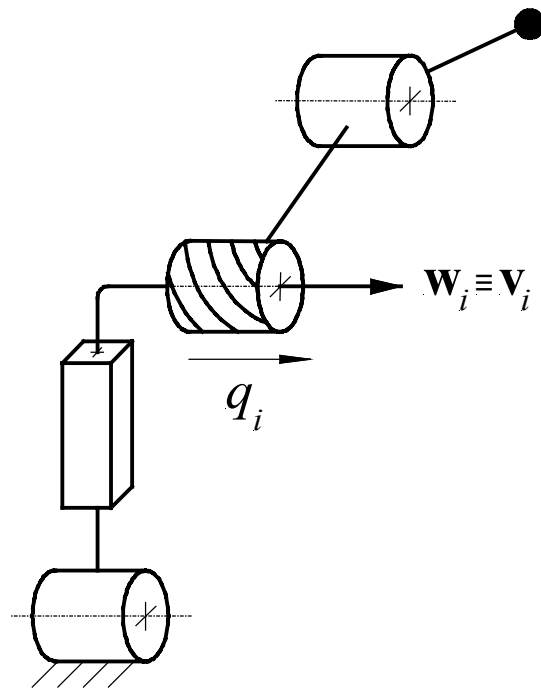


(b)

Fig. 8: Examples of architectures of type I fully-isotropic T_4 TPM legs.



(c)



(d)

Fig. 8: Examples of architectures of type I fully-isotropic T_4 TPM legs.

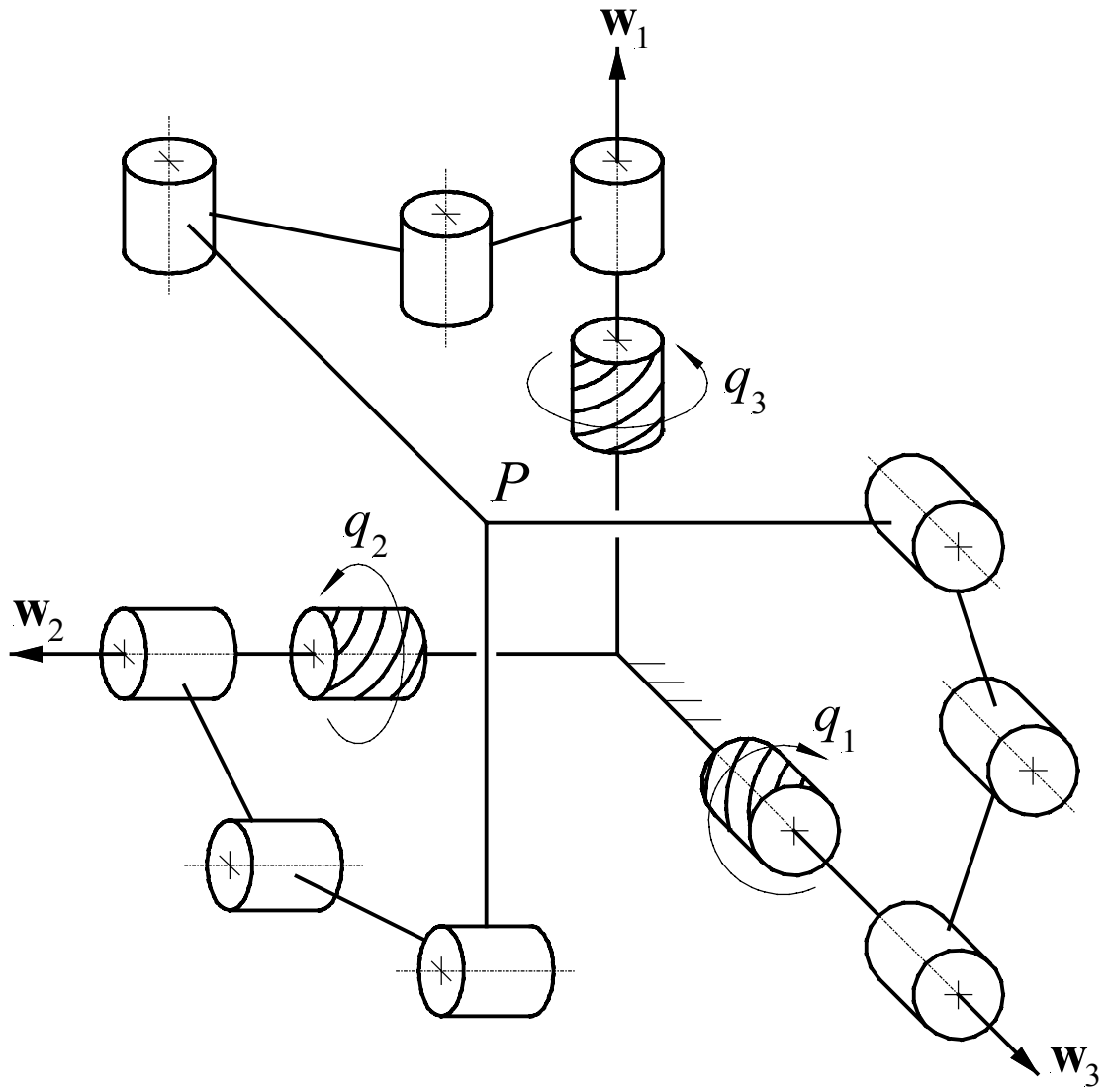
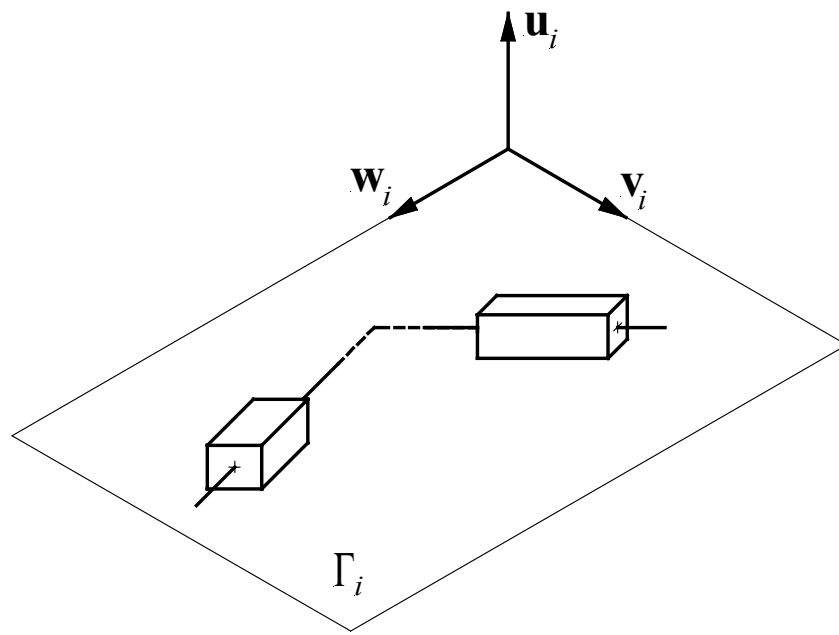
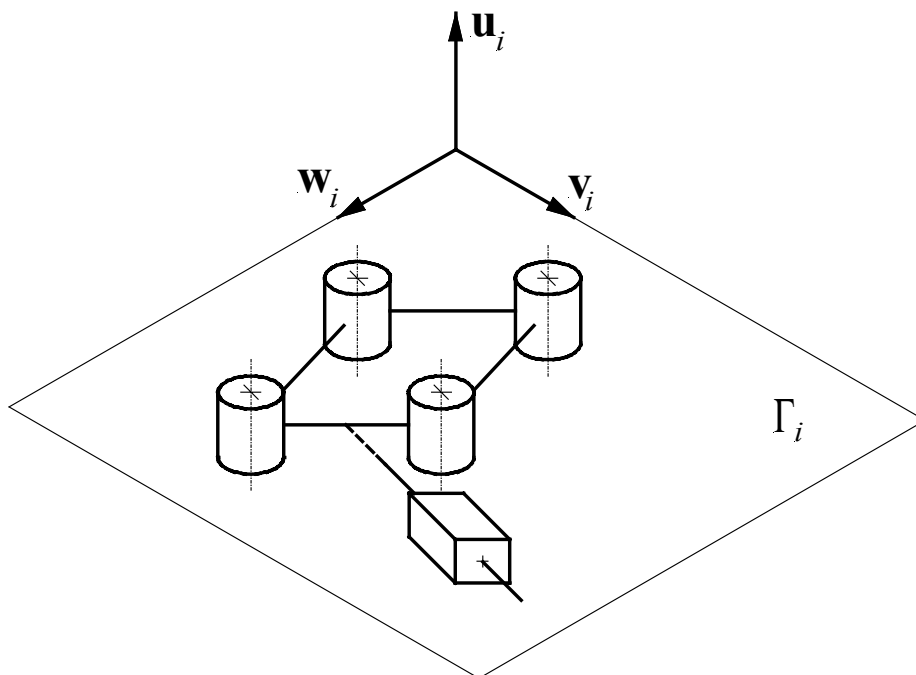


Fig. 9: A type I fully-isotropic T_4 TPM.



(a)



(b)

Fig. 10: Arrangement of the translational joints in a type II fully-isotropic T_4 TPM leg.

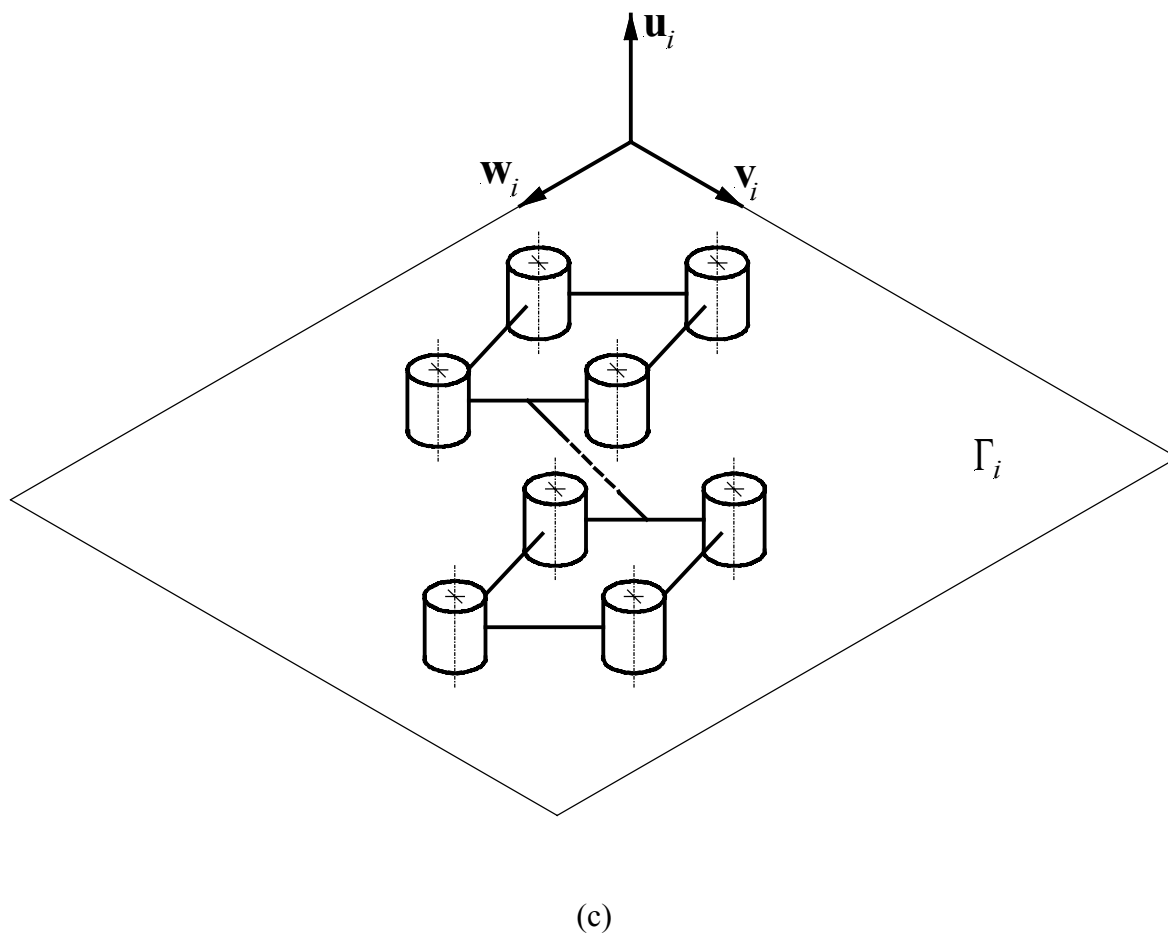


Fig. 10: Arrangement of the translational joints in a type II fully-isotropic T_4 TPM leg.

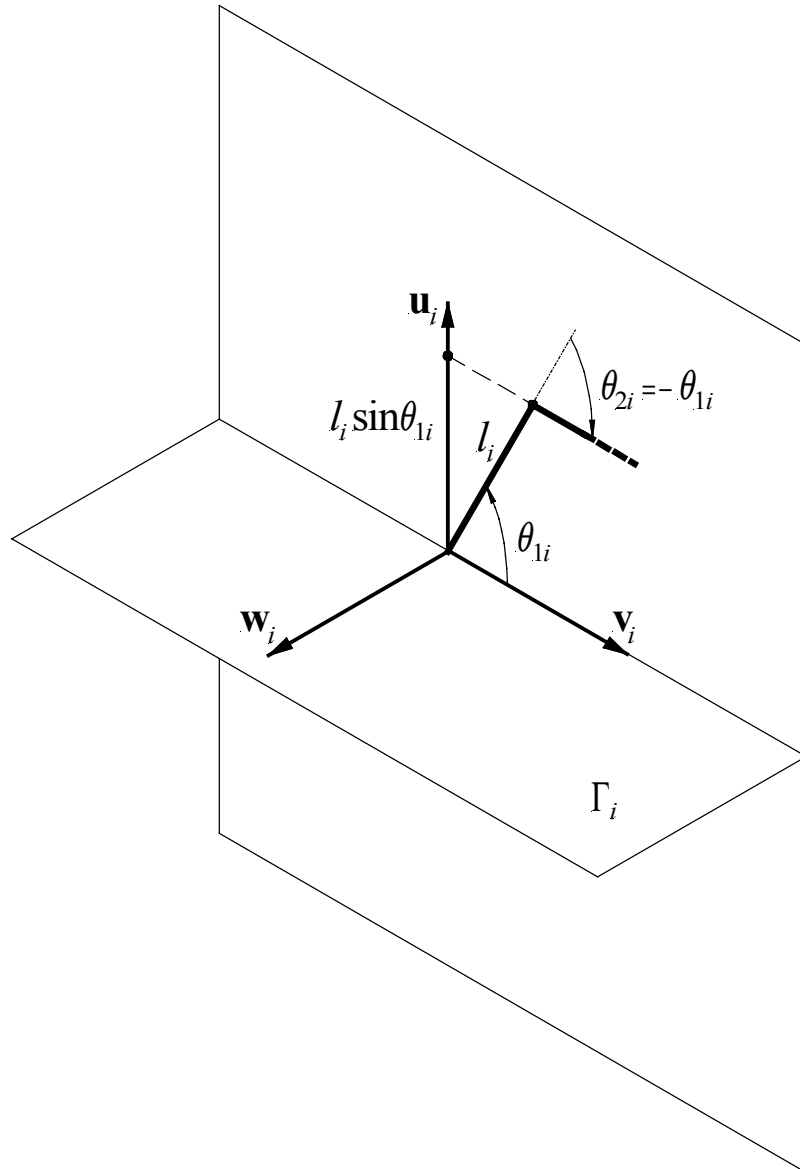


Fig. 11: Platform displacement along the constant direction \mathbf{u}_i in a type II fully-isotropic T_4 TPM.

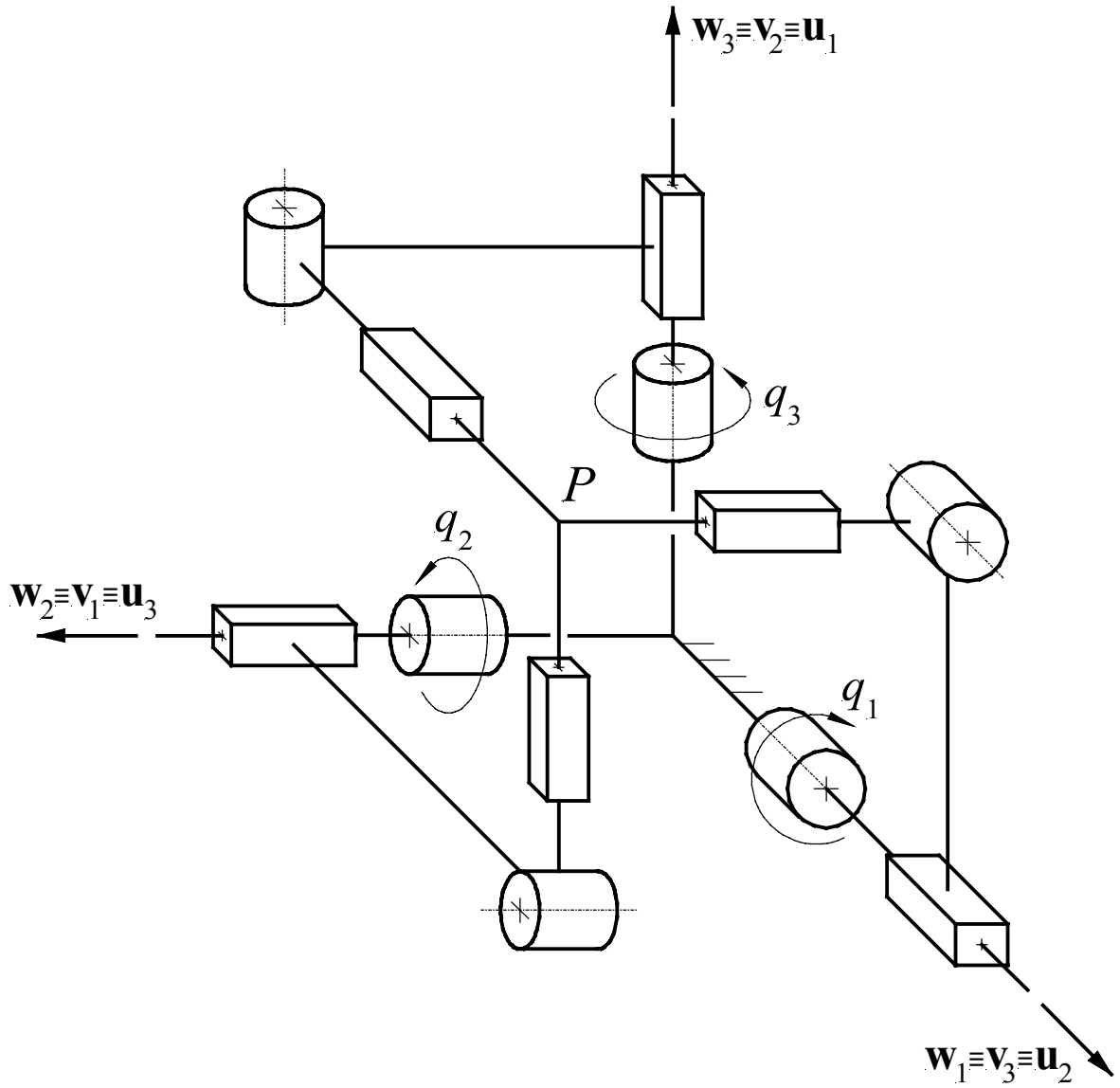


Fig. 12: A type II fully-isotropic T_4 TPM.

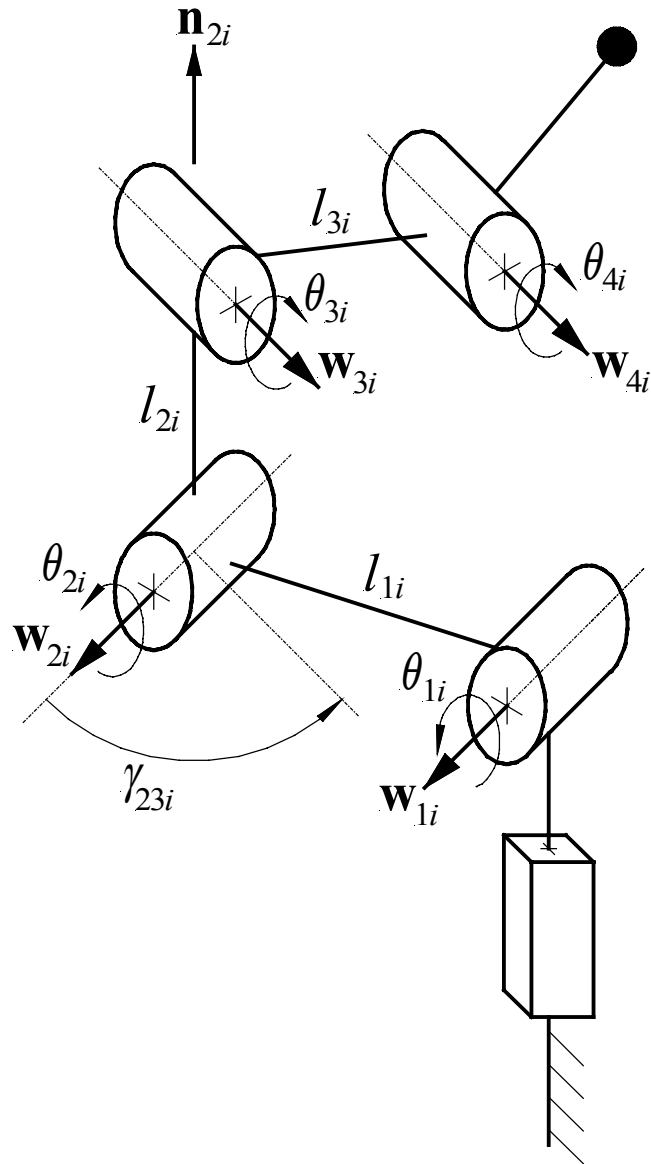


Fig. 13: Generic leg of a T_5' TPM.

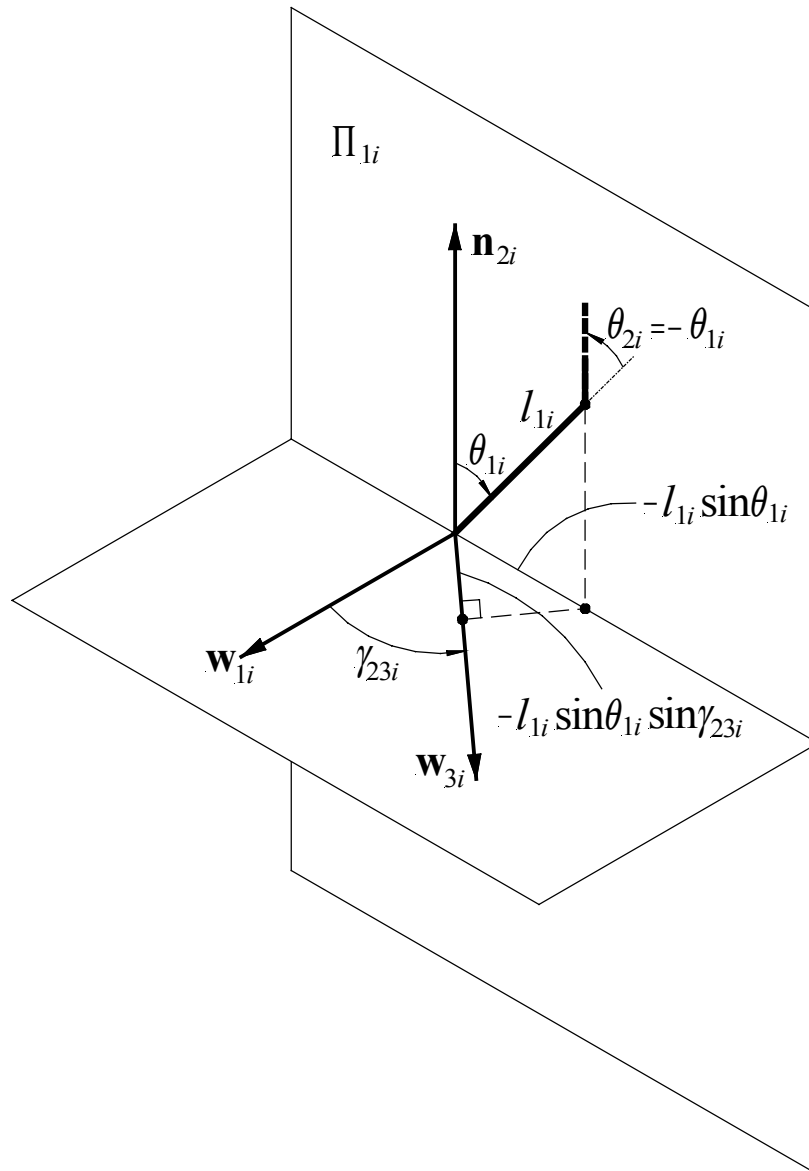


Fig. 14: Platform displacement along the constant direction \mathbf{w}_{3i} in a type I fully-isotropic T_5' TPM.

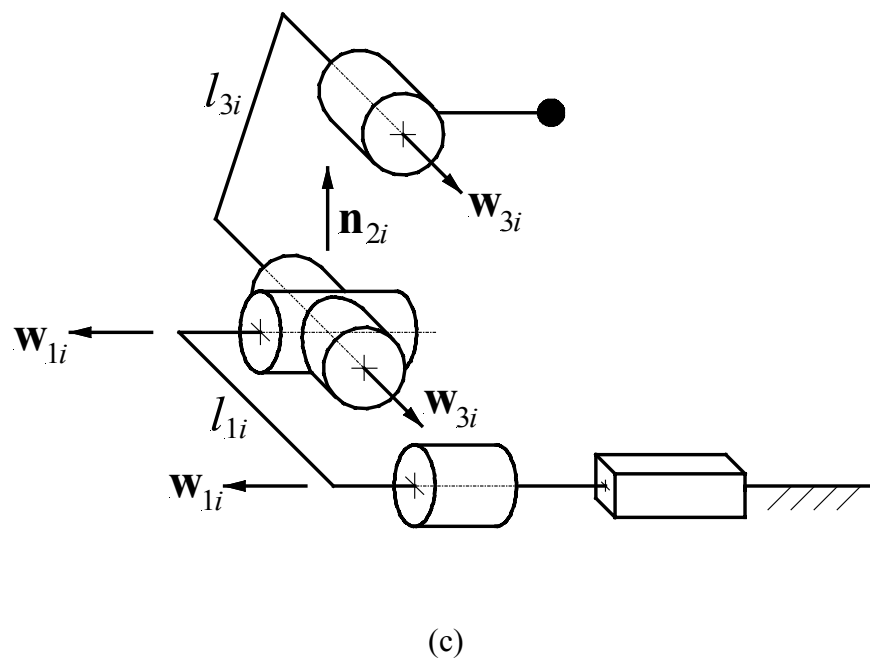
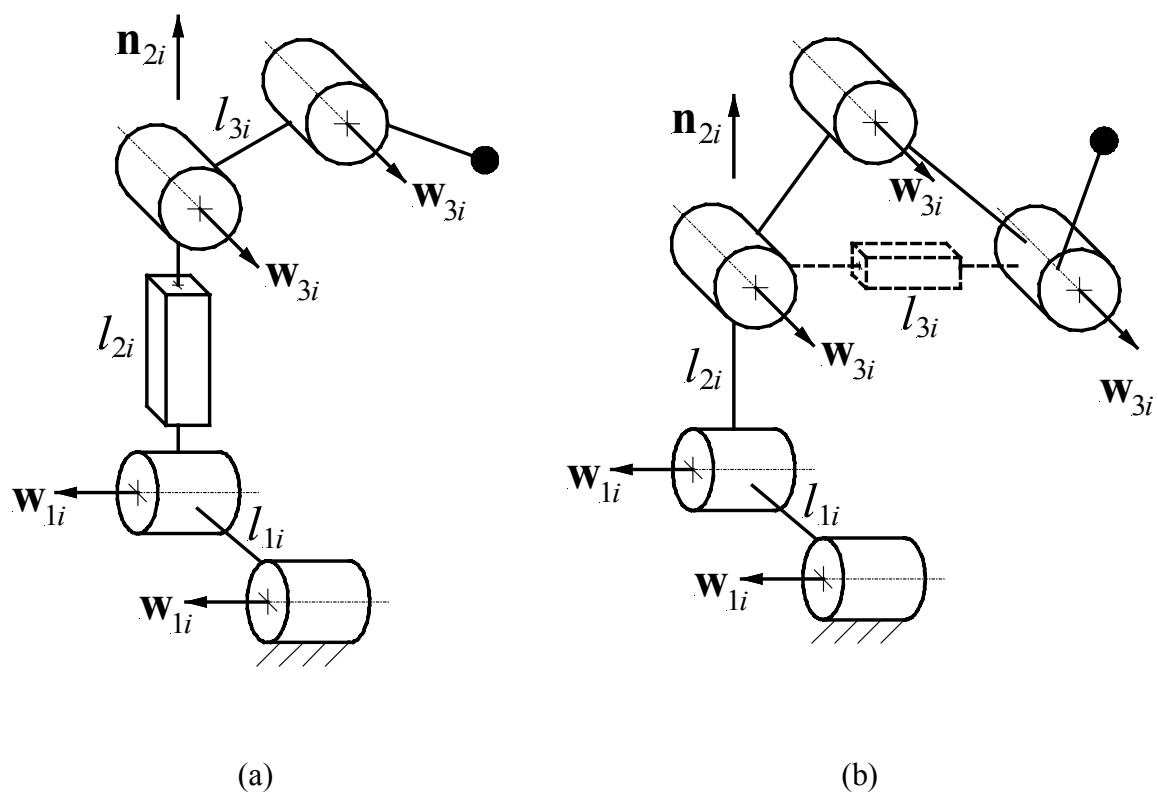


Fig. 15: Examples of leg architectures of type I fully-isotropic T_5' TPMs.

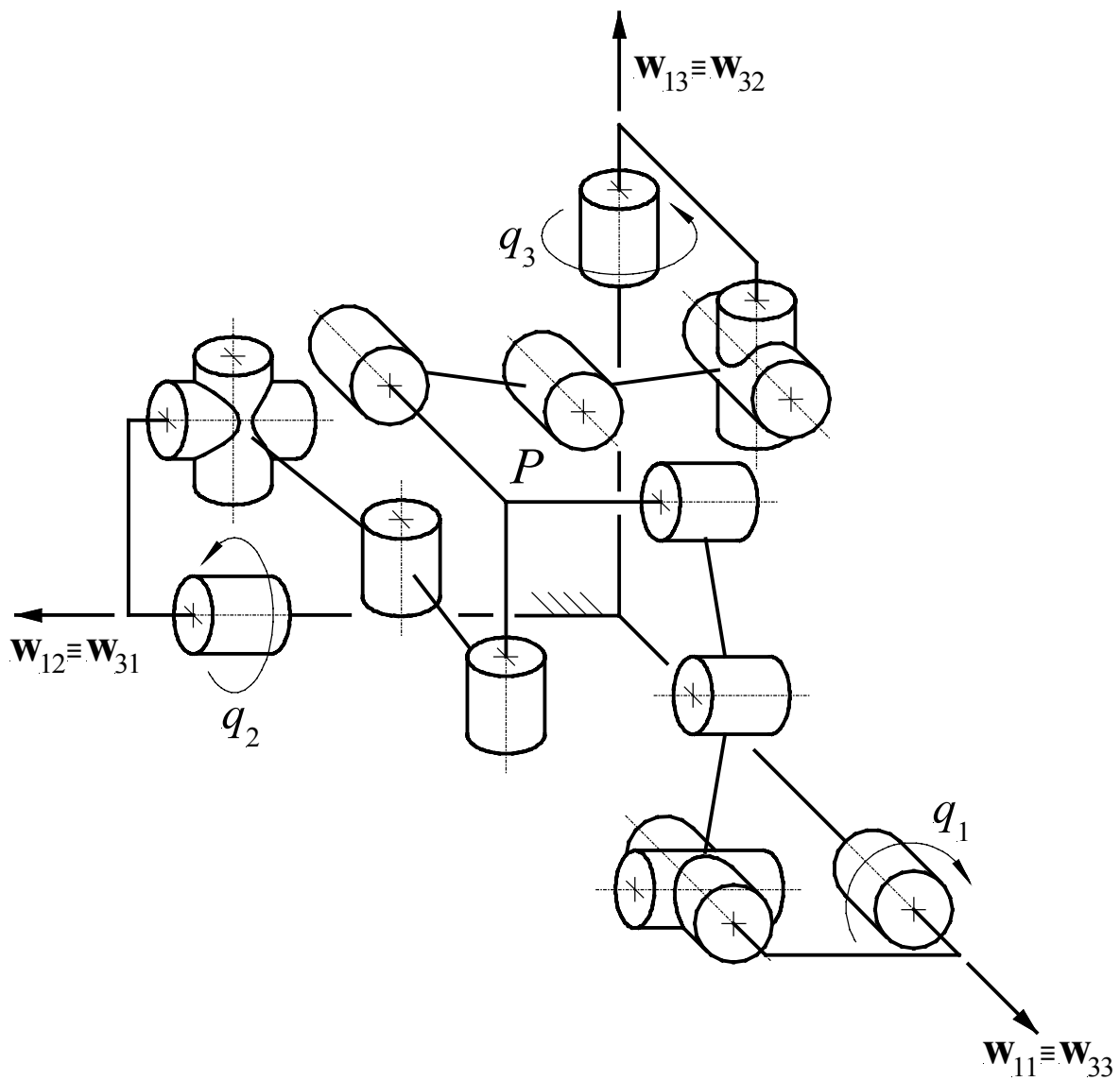


Fig. 16: A type I fully-isotropic T_5' TPM.

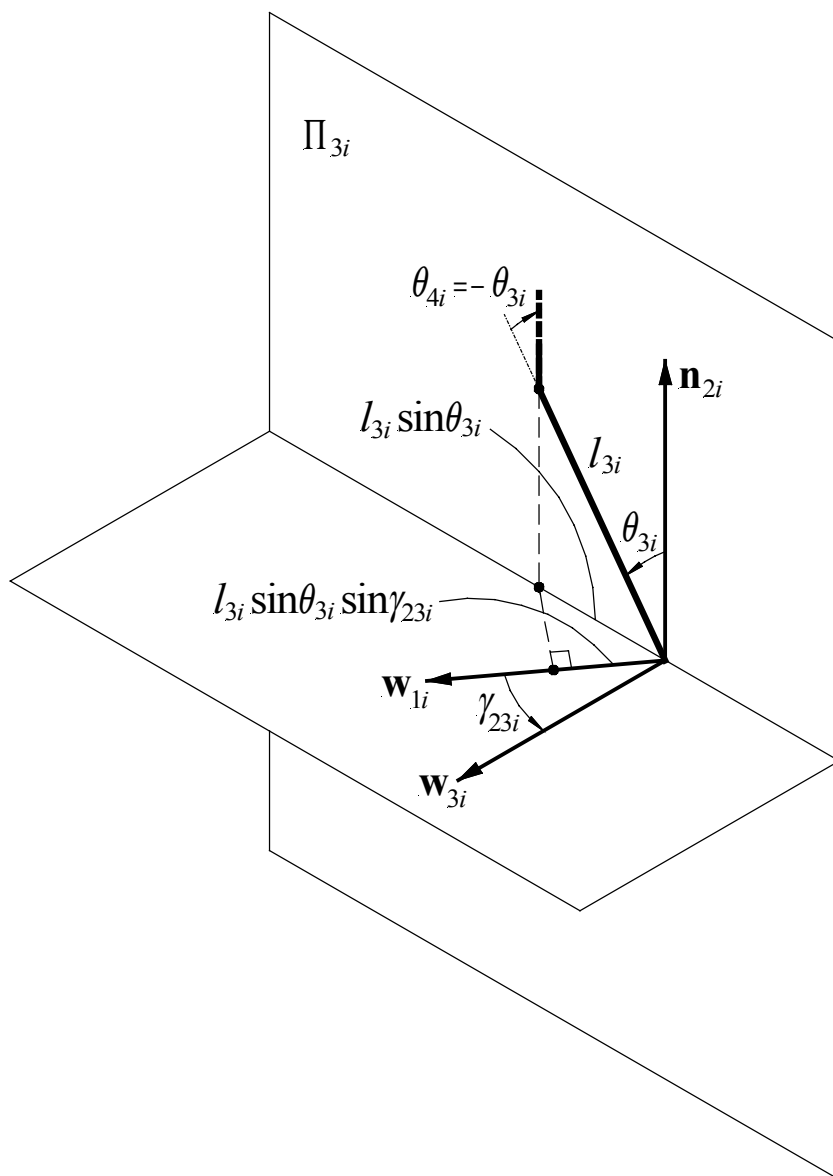


Fig. 17: Platform displacement along the constant direction \mathbf{w}_{1i} in a type II fully-isotropic T_5' TPM.

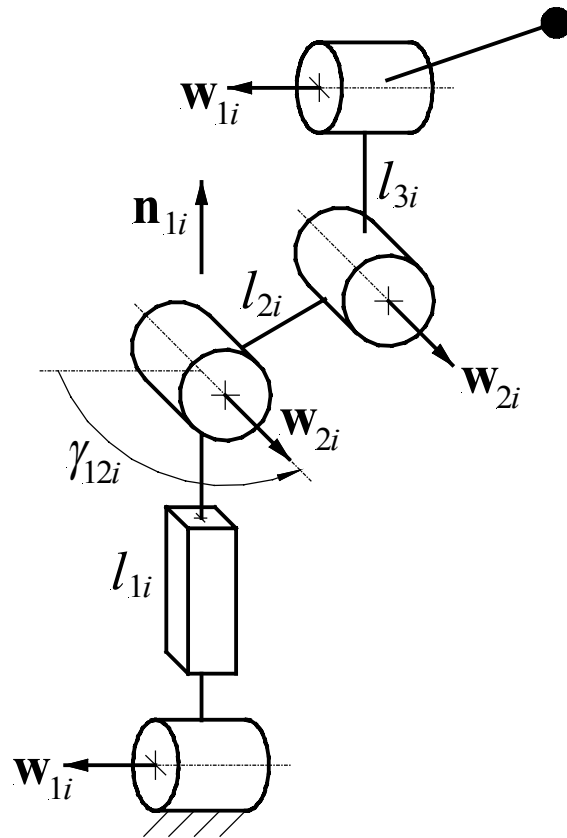


Fig. 18: Generic leg of a T_5 TPM.

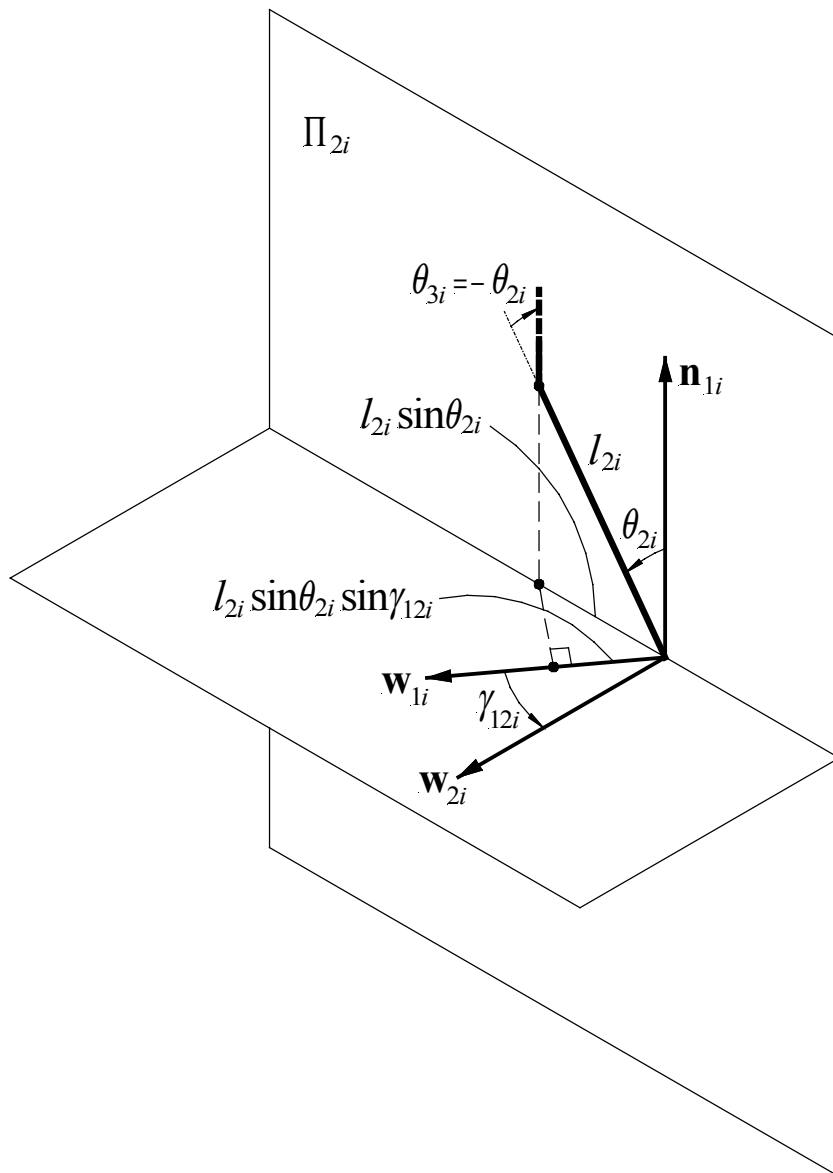


Fig. 19: Platform displacement along the constant direction \mathbf{w}_{1i} in a fully-isotropic T_5'' TPM.