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Model and Control of Tendon Actuated Robots

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This thesis was written in LATEX 2ε on a Debian GNU/Linux system with GNU Emacs.

To my wife Sonia

Abstract

The use of tendons for the transmission of the forces and the movements in robotic devices has been investigated from several researchers all over the world. The interest in this kind of actuation modality is based on the possibility of optimizing the position of the actuators with respect to the moving part of the robot, in the reduced weight, high reliability, simplicity in the mechanic design and, finally, in the reduced cost of the resulting kinematic chain.

After a brief discussion about the benefits that the use of tendons can introduce in the motion control of a robotic device, the design and control aspects of the UB Hand 3 anthropomorphic robotic hand are presented. In particular, the tendon-sheaths transmission system adopted in the UB Hand 3 is analyzed and the problem of force control and friction compensation is taken into account.

The implementation of a tendon based antagonistic actuated robotic arm is then investigated. With this kind of actuation modality, and by using transmission elements with nonlinear force/compression characteristic, it is possible to achieve simultaneous stiffness and position control, improving in this way the safety of the device during the operation in unknown environments and in the case of interaction with other robots or with humans. The problem of modeling and control of this type of robotic devices is then considered and the stability analysis of proposed controller is reported.

At the end, some tools for the realtime simulation of dynamic systems are presented. This realtime simulation environment has been developed with the aim of improving the reliability of the realtime control applications both for rapid prototyping of controllers and as teaching tools for the automatic control courses.

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Introduction

While tendons, or more in general cables, are widely used in many mechanical devices since the 19th century, the use of tendons in robotic applications has been studied since the early 80's, and several tendon actuated robots have been developed all over the world, both in research laboratories and in industries. Often, tendons are used in robotic hands [1, 2, 3] and in parallel robots [4, 5, 6]. The main reasons of the interest in robotic tendon applications are their efficiency in the transmission of the forces from remotely located actuators to the moving parts of the robot, the reliability and the simplicity of implementation of this kind of transmission system, and because they allow to reduce the weight and the cost of the overall device. The main drawbacks of this transmission modality are, first of all, the limitation to both the static and the dynamic performance due to the non-negligible tendon elasticity and, depending also on the routing systems that guide the tendons from the actuator to the joint, the distributed friction along the tendon path and the necessity of maintaining a suitable tendon pretension to avoid the cable slack.

In the human body, or, more in general, in the biologic organisms, the transmission of the movements is realized by means of the muscles, that in many cases act as linear actuators, connected to the articulations, the joints, through tendons. Moreover, in almost all the articulations of the human body, more than one muscle-tendon couple works in antagonistic configuration to realize the movement. This actuation structure gives to humans an optimal behavior both in the free space movements and during the interaction with the external environment. On the base of these considerations, an increasing interest has been posed, in the last years, in the study and in the development of antagonistic actuated robots, as a way to realize variable stiffness devices. Since at least two cooperating actuators must be used to adjust simultaneously both the position and the stiffness of a single joint, the use of tendon transmission systems in antagonistic actuated robots allows to optimize the mass distribution by placing the actuators remotely with respect to the joints, while in other implementations both the actuators are placed near to the joint, resulting in a considerable increment of the links inertia.

The starting point of my research activity has been the development of the UB Hand 3[1], with particular attention for the design of both the sensory apparatus and the actuation

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[1]University of Bologna Hand, version 3.

system. As a consequence of the complexity of this kind of devices, due the number of actuators and sensors involved, a great effort has been devoted also to the definition and the development of the realtime control system of the UB Hand 3. After the evaluation of the advantages and of the drawbacks of the particular implementation adopted, an intensive study of the tendon-sheath transmission system of the UB Hand 3 has been made to overcome the limitations to both the static and the dynamic performance of the finger position and force control.

As a natural extension of this research activity, the application of tendons in antagonistic configuration together with transmission elements with nonlinear force/compression characteristic has been studied. This actuation modality allows the simultaneous control of both the position and the stiffness of robotic devices. The research on this topic has been carried out during my stay at the German Aerospace Center (DLR) in Oberpfaffen-hofen.

A great attention has been posed also in the development of tools for the programming and the testing of realtime control applications in the RTAI-Linux environment. A software library for the realtime simulation of dynamic systems has been build and a live Linux distribution has been realized to collect all the more useful tools for the RTAI control applications developer. This environment has been successfully used as teaching tool in automatic control courses.

This thesis is organized as follow:

- In **Chapter 2** the UB Hand 3 project is presented, and the more important features of this device are discussed. The activities for the development of the purposely designed sensors and actuators are reported and the structure of the control system of the UB Hand 3 is illustrated together with the preliminary activities for the evaluation of the manipulation capabilities of this device.

- In **Chapter 3** the tendon-sheath transmission system adopted in the UB Hand 3 is investigated in deep with the aim of improving the performance of the system in the control of the force that the tendons apply to the hinges. Suitable solutions for the compensation of the friction and for the control of the output tension of the tendon are proposed.

- The analysis on the dynamics of antagonistic actuated robots is presented in **Chapter 4**. The conditions that allow to achieve simultaneous stiffness and position control of anthropomorphic robotic arms are discussed, taking into account the characteristics of the transmission system adopted to drive the robot. The feedback linearization of the system is presented and the simulation results of a 2-link manipulators are reported.

- The **Chapter 5** presents the analysis of the antagonistic actuated joint implemented at the DLR and the experimental activity carried out to evaluate the effectiveness of the design approach for the realization of a variable stiffness device.

- Due to the fact that very often linear filters are used instead of state observers for the estimation of the velocity from the joint position information, the stability analysis of feedback linearization control of robotic manipulators based on filtered position information is reported in **Chapter 6**.

- The development of realtime algorithms for the simulation of dynamic systems is presented in **Chapter 7**. This approach allows to design and test the control application using the simulated system, improving the safety of the tuning phase of the controller and improving the reliability of the final control application.

- In **Chapter 8** some conclusion about the overall work are reported together with considerations about the open issues and plans for the future research activities.

2

The UB Hand 3 Project

2.1 Introduction

In dexterous robotic hands the reproduction of human hand compliance during the interaction with the objects to be manipulated (as demonstrated in [7], [8], [9]) is fundamental for grasp adaptability and stability. Furthermore, a continuous soft cover increases the level of protection against external agents and unexpected impacts, thus increasing the reliability of the robotic hand. Even if these advantages are widely recognized, few humanoid robotic hands, so far, have been designed specifically to optimize this aspect. In most cases they are covered with thin layers of elastomeric material, capable to provide high surface friction but not enough thick to actually work as real compliant pads. One of the main reasons why it is so difficult to host pads with adequate thickness and therefore deformability is the usual adoption of mechanical finger structures based prevalently on the exoskeletal pattern, which means rigid hollow frames with transmissions or actuation inside [10], [11].

(a)   (b)

Figure 2.1: Detail of the UB Hand 3 with the soft cover (a) and the UB Hand 3 in comparison to the human hand (b).

The UB Hand 3 project addresses alternative design solutions in order to substitute the exoskeletal structure with an endoskeletal articulated frame with a tendon-based actuation, aiming to reach the desired external compliance and to simplify the overall mechanical complexity of the hand. The endoskeletal structures may be implemented successfully considering different morphologies of the joints: among the potential solutions, biologically-inspired joints with rolling or sliding conjugated profiles or non biologically-inspired solutions like compliant hinges to substitute articulations. The goals are to reduce the complexity of the articulated structure by reducing the number of components, to reduce the effects of drawbacks like backlash or friction and to increase the reliability of the mechanical structure. Moreover, the reconsideration of a tendon transmission for remote actuation of joints is coherent with the perspective of hand-arm integration and with a more generalized design of the finger structure, not conditioned by the type of the actuators.

The general aim of the project is to test non-conventional design solutions, understanding what may be their advantages and their limits by means of theoretical investigation, practical implementation and testing as well. A strong issue is to test not only the validity of such solutions in terms of theoretical behavior, but also to point out and to evaluate technological aspects related to their application and their compatibility with general specifications, like the adoption of proper sensory equipment or the application of specific control strategies. One of the key choices of the UB Hand 3 project is to investigate advantages and limits of articulated structures obtained with serial compliant mechanisms actuated by means of tendons: a series of different finger architectures have been built and evaluated based on this concept and described in previous works [12], [13], [14], [15].

This project has been, for my research activity, a starting point for the evaluation of various problematics related to the modeling of compliant structures and the control of tendon actuated systems, with particular attention for sheath-based tendon routing systems and to antagonistic actuation. Another important point of interest for me has been the development of realtime control application for such a complex mechatronic device like the UB Hand 3. The overall project has been developed thanks to the joined work of the Department of Mechanical Engineering (DIEM) and Department of Electronics, Computer Science and Systems (DEIS) of the University of Bologna.

This chapter illustrates the hand architecture that is the result of the evolution performed so far: it will probably be improved in the future, but it seems now a valid base to start on-field evaluation of the proposed concepts. Some preliminary results of the hand operating capability are presented in the final part of the chapter. They have been obtained with a prototype that only partially implements all the prospected solutions, but already confirm that the proposed approach exhibits very high potential.

External coating (skin) Soft pad Internal endoskeleton

Figure 2.2: Structure of the finger module of the UB Hand 3.

2.2 Architecture and Kinematics of the Hand

Due to the particular actuation modality adopted in this robotic device, the kinematics of each finger can be seen as the result of the combination both of its mechanical structure and of the connection of the tendons to the actuators and perhaps to the coupling between the movements of the different tendons that driver the finger. All these aspects, together with the mechanical structure of the UB Hand 3, will be illustrated in this section.

2.2.1 Mechanical Structure of the Hand

The present prototype of the UB Hand 3 is characterized by a modular structure in which four identical fingers and one opposable thumb are assembled on a carpal frame, that will be connected to a wrist. The tendons are routed from the forearm, where the actuators are placed, to the fingers passing trough the wrist and the carpal frame, miming in this way the human hand configuration. A compliant layer, reproducing the role of human hand soft tissues, covers the endoskeletal structure, as shown in Fig. 2.1(a) and sketched in Fig. 2.2 for a single finger. The overall dimensions of the hand are very similar to the human one and in Fig. 2.1(b) a direct comparison is proposed.

The internal articulated structure is designed according the "compliant mechanism" concept so that the mobility of the phalanges is obtained by means of elastic joints (the

(a)   (b)

Figure 2.3: Adoption of coiled spring in elastic hinges.

Figure 2.4: The tendons path inside the finger.

hinges) connected to the rigid parts (the phalanges, see Fig. 2.3(a)). The compliant elements are made with close-wound helical springs and the bending movement of the joints is then obtained by means of the action of pulling tendons. A suitable choice of the material of the springs allows to have large joint displacements with a limited number of coils while avoiding permanent deformations and buckling phenomena. The structure of the fingers is then obtained by plastic moulding with inclusion of continuous steel springs. The parts that are not covered by the plastic material preserve the capacity of relative movement due to the flexibility of the springs, while the other parts become rigid forming the phalanges. The overall finger structure obtained in this way show a good reliability: in the stress experiments that has been done under different load conditions, no failures occurred after thousands of working cycles

As sketched in Fig. 2.3(b), multiple springs can be placed in parallel in order to obtain high torsional stiffness in the orthogonal rotating directions with respect to the one which the hinge is designed for. The actuation tendons are routed across the coiled springs which form at the same time the hinges and the routing paths (see Fig. 2.4). The springs are then used both as structural elements and as sheaths for the routing of the tendons. This solution allows a simplified design with appreciable kinematics properties. In this way, it is possible to consider the movement of each joint independent for the movements of the others.

In the UB Hand 3 prototype (see Fig. 2.5) each finger can have up to 4 degrees of mobility, obtaining a total number of 20 degrees of mobility. In order to find o good trade-off between the complexity of the actuation system and the dexterity of the hand, 16 degrees of mobility are actively actuated whereas the others are locked or coupled. The

Figure 2.5: CAD design of the UB Hand 3 internal structure.

thumb and the index fingers have 4 d.o.f. each one, the middle and the little finger have 3 d.o.f. while the ring finger has 2 d.o.f. (see Tab. 2.1). This configuration, similar to that implemented in the Robonaut hand project [16], is adopted in order to have a five fingered hand suitable to perform power and enveloping grasps, in which only three fingers (the thumb, the index and the middle) have fully mobility to execute dexterous manipulation tasks.

2.2.2 Finger Kinematics

Significant efforts were performed in developing the proximal joints of the fingers. Different design solutions are adopted for the upper fingers and the opposable thumb. In the upper finger, the yaw joint and the flexural bending of the proximal phalanges are obtained through two orthogonal single axis hinges (see Fig. 2.6(a)), while the articulation at the base of the thumb is obtained by a single two d.o.f. helicoidal hinge as shown in Fig. 2.6(b). This last joint is actuated by means of three cooperating tendons that allow

(a)	(b)

Figure 2.6: Two degrees of freedom articulation of the upper fingers (a) and of the thumb (b).

Figure 2.7: The experimental setup used for the identification of the kinematic properties of the finger.

the thumb to bend on a plane having variable direction.

An experimental setup (see Fig. 2.7) is used to verify this kinematic properties of the finger hinges. The experimental result show that the rotational center of such compliant flexures may be considered fixed in the whole angular range of the joint (0° - 90°). Therefore, the kinematic behavior can be modeled with good approximation as an ideal revolute joint with low-level of torsional stiffness. Accordingly it is possible to exploit the usual kinematic relations between joint configuration and cartesian position given by the Denavit-Hartemberg parameters, and to treat the fingers like standard robots with revolute pairs. The position of the rotational center of medial and distal joints are depicted in Fig. 2.8.

Figure 2.8: Rotational center of medial hinge.

2.2.3 Configuration of the Tendons

It is worth to notice that, due to the inelastic tendons used, the estimation of the hand configuration from the motor position provides excellent results. In Fig. 2.9 the relation between tendon elongation and joint angle is shown. Such a relation, experimental ob-

Figure 2.9: Static relation between tendon elongation and joint angle.

tained by means of a setup including a video-camera and tendon position/tension sensors (see Fig. 2.7), shows that the motion of the hinge, driven by the tendon, is quite repeatable, as can be seen in the plot 2.9, where the circles corresponding to different measures are perfectly overlying. Due to this useful property, the relation between i-th tendon elongation h_i and the i-th joint angle displacement θ_i can be approximated, with little errors, with the simple expression:

$$h_i = h_{i0} - \sqrt{r_i^2 + d_i^2 - 2r_i d_i \cos(\theta_{i0} - \theta_i)} \qquad (2.1)$$

where h_{i0} is the tendon length corresponding to the zero joint position reference θ_{i0} while r_i and d_i are the geometric parameters of the hinge, as indicated in Fig. 2.10. The joint angle can be computed from the tendon position measurement by inverting the eq. (2.1):

$$\theta_i = -\theta_{i0} + \arccos\left[\frac{(h_{i0} - h_i)^2 - r_i^2 - d_i^2}{2r_i d_i}\right] \qquad (2.2)$$

Alternatively, providing a desired trajectory of joints, from eq. (2.1) it is possible to derive the corresponding trajectory of the motors [1].

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[1]Obviously, due to the presence of the rotative motor, the further relation $h = R \cdot \theta_m$ is introduced, where θ_m is the motor position and $R = 6mm$ is the radius of pulley.

Figure 2.10: Internal articulated finger structure.

The finger design allows operating with different tendon configurations. In the simplest case (single-acting system), each tendon bends the related joint, while the return is obtained by means of the flexures elasticity. In this configuration the finger stiffness is depending on the hinge stiffness and it can't be fully controlled. The other possible solution is to implement a double-acting system by adding two antagonistic tendons able to cooperate with the hinge in the return phase: one related to the yaw joint and the other, in the dorsal part of the finger, that works against the finger bending. The antagonistic solution for the motion control of the finger is under investigation, and it will be illustrated in chapter 4, and its implementation in the next prototype of the UBHand 3 is planed.

Table 2.1: Degrees of mobility for each fingers

Finger	Degree of mobility for each joint			
	Yaw	Proximal	Medial	Distal
Index	A	A	A	A
Middle	L	A	A	A
Ring	L	A	A	C
Little	A	A	A	C
Thumb	A^3		A	C

A - Actively actuated
L - Locked
C - Coupled with the medial joint
A^3 - 2-d.o.f. joint actuated by 3 tendons

The adopted kinematic configuration allows to change the actual number of d.o.f. without changing the prototype structure. This strategy, for example, may be applied to couple the distal and medial bending motion to mimic the human finger behavior. Furthermore, by adopting elastic coupling devices between the linked joints it is possible to obtain self-adapting grasping procedures.

2.3   Finger Control

The kinematic considerations presented in sec. 2.2.2 allow to implement a control "at the motor level" with an acceptable error in the operating space. A simple impedance control on each motor, with classical elastic and damping terms, has been implemented.

Figure 2.11: Transformations from the cartesian space to the joints space and from the joints space to the tendons space

Taking into consideration the cartesian space, an impedance controller with stiffness K_d and damping D_d can be written as:

$$K_d(p - p_d) + D_d\ \dot{p} + F_e = 0 \tag{2.3}$$

where p and p_d indicate the measured fingertip position and the desired one respectively, and F_e is the force applied to the environment. The mass of the finger is neglected for simplicity because of its very small value. It is important to note that, on the basis of the considerations about the kinematics of the tendons, the fingertip position p can be estimated by means of a measure of the tendon displacements using the eq. (2.2) and the Denavit-Hartemberg parameters of the finger. In the joint space, the eq. (2.3) can be written as:

$$J_j^T(q)\left[K_d(p - p_d) + D_d\ \dot{p} + F_e\right] = 0 \tag{2.4}$$

in which $J_j(q)$ is the Jacobian matrix of the finger and q is the vector of the joint angular positions. The relation that describes the static equilibrium of the finger is:

$$K_e q = \tau + J_j^T(q)\ F_e \tag{2.5}$$

where K_e is the stiffness of the hinges (given by the bending of the steel springs) and τ is the vector of the torques applied by the tendons to the joints. Then, by applying the control law:

$$\tau = J_j^T(q)\left[K_d(p - p_0) + D_d\ \dot{p}\right] + K_e q \tag{2.6}$$

the desired behavior of the finger eq. (2.4) is achieved but, since the finger is driven by means of tendons, the control law in eq. (2.6) must be rewritten in the state space of the tendons:

$$F_h = J_t^T(h)\ \tau = J_t^T(h)\left\{J_j^T(q)\left[K_d(p - p_0) + D_d\ \dot{p}\right] + K_e q\right\} \tag{2.7}$$

where F_h is the vector of the tendon forces, h is the vector of the tendon displacements and $J_t(h)$ is the Jacobian matrix transform between the tendons space and the joints space. The connection between the cartesian space, the joints and the tendons space is depicted in Fig. 2.11.

From eq. (2.7) it is possible to conclude that the control of the tendon tension is fundamental to implement the cartesian stiffness control of the finger. The use of the steel springs as sheaths for the routing of the tendons introduces nonlinear effects in the transmission system due to the presence of elasticity and friction. These considerations justify the analysis of the tendon-sheath transmission system reported in chapter 3.

Another important consideration is related to the unidirectionality of the static equilibrium of the finger eq. 2.5 due the tendon configuration. Without the use of an antagonistic actuation, it is then possible to control the impedance of the finger only during the closing movement. The analysis on the antagonistic actuated kinematic chains reported in chapter 4 can be useful, from this point of view, to improve the performance of the control system of the finger.

The control law eq. (2.7) is suitable for dexterous manipulation [17, 18], in particular when hands with such a structure (with a back-drivable actuation chain and with a soft cover) are used. Despite the "almost ideal" static behavior of the hand (concerning the relation tendon length /joint angle and the center of rotation) the use of tendons routed inside flexible sheaths, the compliant hinges and the soft pads introduce dynamic terms, which can easily make the system unstable, especially during physical interactions with the environment, if suitable and accurate measurements of the joint positions and torques are not available. On the contrary, by exploiting this kind of actuator level controller, the manipulation of objects of different size and weight has been performed in a safe manner, and the performance of the system can be improved by a suitable knowledge both of the model of the transmission system and of the finger kinematics.

It is worth to highlight the role of visco-elastic cover in performing such operations. On one side the dissipative terms introduced by this material contribute to the dynamic stability of the hand/object system. On the other side, it has been noticed that a thick layer of compliant material can compensate the positioning error of a "not very rigid" and "not very precise" endoskeletal structure, allowing tasks otherwise quite prohibitive.

2.4 Sensory Apparatus

The non-conventional structure of the hand imposes the design of *ad hoc* force and position sensors. A systematic analysis of devices able to detect the bending angle of the joints has been performed [14]. Different sensing technologies have been compared: optical sensors, piezo-resistive sensors, hall-effect sensors and strain-gauge based sensors. From the analysis the last solution seems preferable. The sensor, depicted in Fig. 2.12, exploits the bending torque exerted by one of the springs composing the hinge on a miniaturized load cell, which is integrated into each phalanx. This sensor has been chosen because it offers a number of advantages with respect to other solutions. From its more

<center>(a)   (b)</center>

<center>Figure 2.12: Position sensor: FEM model (a) and actual implementation (b).</center>

remarkable properties there are a working range that includes both positive and negative angle displacements, a fairly linear characteristic (as can be seen in Fig. 2.13) and the insensitivity with respect to lateral bending and compression of the hinge. The sensing principle, based on strain gauge, allows to simplify the structure of the electronics for the signal conditioning and acquisition. In fact, the same principle will be exploited for tendon tension sensors and a single acquisition chain, with a suitable multiplexing stage for all sensors, can be used for the sampling of all the sensors of the finger. Since the signals produced by a (half-)bridge of strain gauges are very small and easily influenced by electrical noise (in particular that produced by motors), the amplification chain will be located directly on the back of each finger.

<center>Figure 2.13: Output characteristic of the position sensor.</center>

The miniaturized load cell that monitors the force applied by the tendons to the joints, is properly constrained in the lower side of each phalanx and, at the same time, it sets

Figure 2.14: Tendon force sensor: FEM model (a), working principle (b).

up the mechanical link between the tendon and the phalanx. In Fig. 2.14 the structure and the working mechanism of the sensor are reported. Two strain gauges, disposed in an half-bridge circuit, are used to measure the deformation of the connector. In Fig. 2.15 the output characteristic (with respect to the force applied to the sensor and the joint configuration) of a prototype is reported. Note that the sensor exhibit an excellent linearity ($e_L \approx 1\%$) and, in particular, it is insensitive to the joint angle and, therefore, to the tendon orientation.

Figure 2.15: Output characteristic of a tendon tension sensor prototype with respect to applied force and joint angle.

2.5   Actuation Module

The actuation of the hand is provided by a system that include 16 modular multi-sensorized motors. Each actuator is based on a very low-cost DC-brushed motor (Hitec HS 475 HB),

Figure 2.16: Instrumented actuation module.

that is equipped with a position sensor, a tendon force sensor and custom power electronics (see Fig. 2.16). The tendon is wrapped around a sprocket, which is fixed to the output shaft of the reduced DC motor, and then it wraps around a pulley to get the proper direction at the beginning of the sheath. This pulley is fixed on a instrumented support, where two strain gauges measure the strength applied to the tendon. The modules are hosted in a purposely designed structure (see Fig.2.17(b)), where each motor is placed radially respect to the forearm axis with the tendons directed along the forearm axis. The overall structure of the UB Hand 3 is shown in Fig. 2.17(a).

The electronics originally integrated into the motors (which provides a position control) has been modified in order to implement an hardware current/torque control loop, more suitable for robotic applications, integrated directly into the power electronic of the actuation module. The potentiometer and the gearbox integrated into the motor module introduce non negligible backslash and friction effects in the motor characteristic. Then, besides the hardware torque controller, an outer software control loop is necessary to control the tension of the tendon at the actuator side and to compensate for these nonlinear effects, In the adopted configuration, the actuation system is able to apply to the tendon a force of $70N$ and to close a finger joints in $0.36sec$.

2.6 The UB Hand 3 Realtime Control System

In order to provide the motor electronics with the desired torque set-point and to acquire the (position/force) sensor signals, an I/O board has been exploited and a realtime control is performed on a standard personal computer. The I/O board is a Sensoray 626 Model, a low cost PCI analog and digital I/O Card with four 14-bit D/A outputs, sixteen 16-bit differential A/D input. The PC, used for realtime control, is a Pentium 4 at 1.8 GHz that is equipped with RTAI-Linux operating system [19]. This OS provides realtime func-

(a)   (b)

Figure 2.17: The UB Hand 3 (a) and the a detail of the forearm (b).

tionality both in the kernel and in the user space and it supports IPC mechanisms like semaphores, FIFOs and shared memory. These functionalities are used to build a kernel module for realtime control algorithm and to implement monitoring tasks and user interfaces in standard user applications. This system (PC with RTAI-Linux OS equipped with the I/O board) represents a valid tool for rapid prototyping of robotic systems and allows to reduce the developing time. Since the Sensoray 626 does not own a sufficient number of input and output channels, 2 interfacing boards for the multiplexing/demultiplexing of the signals have been built (see Fig. 2.18). These 2 interfacing boards will be redesigned with SMD components in order to reduce the overall size of the electronics and to improve the reliability of the cabling system.

The structure of the realtime controller of the UB Hand 3 is modular and hierarchical, as can be seen from Fig. 2.6. The kinematic of the overall hand is computed, on the basis of the setpoint information coming from the supervisor, by an independent task that manage the hand data structure and the connected finger (or thumb) data structures. In this way, the system kinematic task can handle simultaneously more than one hand without any modifications to its internal structure, and each hand can have fingers with different dimension or kinematic configuration. An handler is devoted to the description of the

Figure 2.18: The connection between UB Hand 3 and I/O card.

thumb due to its particular kinematic configuration.

The actuation of the computed control and the acquisition and the filtering of the input channels are managed by a control computation task. This task uses a controller for each joint of the controlled finger, giving in this way to the control designer to possibility to choose between different control strategies for the various joints (PID position control is used at this moment). Each joint controller is provided with a descriptor of the joint actuator and sensors, and independent digital filter functionality is given to the controller to acquire and filter all the necessary data form its sensors.

The control system supervisor communicates with all these tasks. The communication process can be split in two independent parts. The former is driven by the user requests and by the scheduling of the data acquisition and actuation processes and it is indicated by the green arrows in Fig. 2.6. These channels are used to update setpoint information in the hand kinematic task and from the actuator and sensor modules to sample input and output data. The latter, indicated by the red arrows, is driven by the supervisor and is used to monitor the execution of the kinematic and control tasks and to stop the controller in case of miss-function.

On the other side, the supervisor manages the communications with the low-level data acquisition driver (the COMEDI library [20]) and with the user interface, see Fig. 2.6. From the user point of view, The UB Hand 3 can operate in different modalities thanks to the possibility of choosing between different user interfaces. The xrtailab [21] graphic interface can be used to monitor the behavior of the overall system, to record data or to adjust the parameters of the controller, while the movements of the hand can be controlled via programmed trajectories or by means of a virtual reality data glove.

By means of 4 reading/writing cycles, it is possible to provide the 16 value of torque to

Figure 2.19: The structure of the UB Hand 3 realtime control system.

the motors and to acquire all the sensor signals. Due to the timing constraints of the Sensoray 626 data acquisition board, the minimum achievable sampling time of the control loop is quite large ($\approx 4ms$), which is acceptable in the first phase of rapid prototyping. For the future development, we planed to change the data acquisition subsystem to achieve to a faster and more complex control algorithm, both for improving the overall performances and to implement multipoint manipulation applications. Thanks to the used of the above described structure of the realtime controller, the modifications on the control software are reduced to the minimum. In particular, only the low-level data acquisition hardware interface must be redesigned. If the new acquisition subsystem is COMEDI-complaint, only the adjustments related to the number of the available I/O channels are needed.

2.7   Experimental Activities

While a number of experiments have been carried out on single parts of the UB Hand 3, i.e. elastic hinges, compliant cover, position and force sensors, the complete structure has not been tested yet. Aim of this stage of the project is to demonstrate the capability of the robotic hand to perform both grasping operations and manipulation tasks, and to highlight the drawbacks of the proposed structure in order to further improve the technology of compliant mechanisms and soft pads, by means of structural modifications and/or suitable control strategies.

Figure 2.20: The communication of the realtime controller with the DAQ hardware and the user.

Therefore, grasping and manipulation sequences have been planned for objects with different shapes and dimensions:

- enveloping grasp of a glass bottle, Fig. 2.21(a);

- grasping and fingertip manipulation of middle-sized/small objects, like a can or a cylindrical box, Fig. 2.21(b);

- fingertip manipulation of a pen, Fig. 2.22.

(a)   (b)

Figure 2.21: The UB Hand 3 grasping a bottle (a) and a cylindrical box (b).

Figure 2.22: Manipulation sequence of a pen.

In this phase, all the operations are based on the feedback of the signals coming from the motors, while a direct measure of the joint positions and tendon forces is not available yet.

2.8   Conclusions

The practical implementation of the five fingered hand prototype is not concluded yet, but some motions, grasp and manipulation experiments have been performed with success. As to the mechanical design, the structure with spring-based compliant hinges proves to be quite satisfactory in terms of structural simplification, ease of manufacturing and assembly. Moreover, it shows good properties in terms of kinematic behavior, it is very reliable and fully compatible with the requirements related to hosting the distributed sensory equipment and the compliant external layers. The drawbacks due to the limited stiffness of elastic-hinged fingers with respect to transverse loads do not seem an insurmountable obstacle for application to light manipulation tasks, where, on the contrary, a limited passive compliance of the finger can play a positive role. A systematic approach to hinge stiffness evaluation is under development and should provide a valuable tool, not

yet available in the literature, for optimizing the hinge design. Severe limitations have been found in the present implementation of the sheath-guided tendon actuation system, that seems the weakest point of the proposed architecture. Again, it seems that a wide potential of technological improvement can exist and the present drawbacks can be at least reduced, if not eliminated with a suitable knowledge of the behavior of the tendon-sheath transmission system. Changing the pattern of tendon routing can be another way to further improve the system performance: to this purpose the present finger design is ready to host an additional extensor tendon in order to improve finger dynamics. The rough manipulation experiments performed so far can not be assumed as representative of the functional capability of the hand; when performed with a previous device with very reduced compliance (the UB Hand 2, 1995) these experiments proved to be very critical and many times were failing. Now they succeed, even in presence of incomplete sensory feedback and simplified control algorithms. It is an early, partial result that does not allow to say that the followed approach is in absolute the best for designing hands, but it simply confirms that it worths going-on.

3

Model and Control of Tendon-Sheath Transmission Systems

3.1 Introduction

Tendon-sheath transmission systems are used in several robotic applications. In particular, they are adopted in robotic hands [12, 14, 15, 22, 23], because very often it is not possible to place actuators inside the fingers. On the other hand, the use of pulleys to route the tendons increases the mechanical complexity and the dimension of the structure, and implies that the tendons must be properly preloaded to prevent that they route off from the guides. With this respect, the use of sheaths reduces the size, the complexity, the cost and, at the same time, increases the reliability of the overall system.

This transmission modality introduces some side effects (e.g. deadzones, hysteresis, direction-dependent behaviors in the force control) due to the tendon compliance and to the friction between tendon and sheath [24, 25, 26, 27, 28]. In particular, the non linear behavior arising in force control is of great interest for tendon driven robotic fingers, since the control of the output tendon tension plays a key role in the definition of impedance controllers that can improve the manipulation capabilities of the robotic hand [17].

A simple tendon-sheath static model that highlight nonlinearities and direction-dependent behavior is presented in [25]. Starting from this model, some additional considerations have been made about environment disturbance and tension distribution along the tendon. The environment is modeled as a mass linked to a mobile constrain by a spring. The disturbance is represented by the movement of the constrain and by an external force applied to the environment mass. This model is adequately modified to consider the effect of non-constant preload and environment disturbance.

In [26], a lumped parameters dynamical model is used to validate the input-output relation derived from the experiments. We used a similar model in simulation to study the tension distribution inside the tendon and the effect of the environment load. In the analysis of the quasi-static behavior of the tendon-sheath transmission system, this model shown a strange behavior in stationary condition due to the considered static friction model (Coulomb friction). A dynamical friction model (Dahl model [29]) has been introduced to corrects this drawback.

The new three-mass model presented in this chapter, very simple from a computational point of view but incorporating the main nonlinear phenomena present in tendon-sheath transmission systems, may help in the simulation and design of suitable tension control strategies.

In order to compensate the nonlinear behavior of the driving system, a suitable controller with a friction compensator is designed. The friction compensation law presents discontinuities when the sign of the tendon velocity changes, so the bandwidth of the actuators must be very high to obtain good performance. This switching control action introduces dynamics behaviors in the system, such as vibrations in the tendon, that are not considered in the static tendon model. Simulation results show that this controller is able to compensate both friction effects and also the environment disturbances.

3.2 Tendon-Sheath Transmission Characteristic

The static model of the tendon-sheath driving system presented in [25] is (see Fig. 3.1):

$$
\begin{aligned}
F_f &= \mu N \,\text{sign}(\dot{\varepsilon}) \\
\Delta T &= -F_f \\
\delta &= \frac{1}{EA}T = \rho T
\end{aligned}
$$

where E, A, and δ are respectively the Young modulus, the cross sectional area and the elongation of the tendon, $\dot{\varepsilon}$ and T are the tendon velocity and tension, F_f and N are the friction force and the normal load acting between the tendon and the sheath respectively, and

$$
\mu = \begin{cases} \mu_s & if \; \dot{\varepsilon}=0 \\ \mu_d & if \; \dot{\varepsilon} \neq 0 \end{cases}
$$

is the Coulomb friction coefficient. In Fig. 3.1, the equilibrium condition of a small tendon element is shown to highlight the fact that the friction force F_f between tendon and sheath is mainly caused by the sheath curvature rather than the tendon weight, usually negligible.

Assuming that $\dot{\varepsilon}$ has the same sign along the whole length of the tendon, and neglecting the effect of stiction, we can model an infinitesimal tendon element as:

$$
N = T d\gamma = T\frac{dx}{R} \tag{3.1}
$$

$$
dT = -F_f = -\mu T \frac{dx}{R}\,\text{sign}(\dot{\varepsilon}) \tag{3.2}
$$

where $d\gamma$ is the angle subtended by the arc of length dx and R is the radius of the sheath curvature. From these equations we can write:

$$
\begin{bmatrix} \frac{dT}{dx} \\ \frac{d\delta}{dx} \end{bmatrix} = \begin{bmatrix} -\frac{\mu}{R}sign(\dot{\varepsilon}) & 0 \\ \frac{1}{EA} & 0 \end{bmatrix} \begin{bmatrix} T \\ \delta \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{EA} \end{bmatrix} T_0
$$

Figure 3.1: Equilibrium of a tendon element.

or, in more compact form

$$\frac{d\theta}{dx} = D\theta + \Lambda T_0 \tag{3.3}$$

where T_0 is the tendon preload. The solution of this system is:

$$T(x) = \begin{cases} T_{in} \exp\left[-\frac{\mu}{R} x \, sign(\dot{\epsilon})\right] & if \ x < L_1 \\ T_0 & if \ x \geq L_1 \end{cases} \tag{3.4}$$

$$\delta(x) = \begin{cases} \left[H(x) - T_0 x + \frac{R}{\mu} T_{in} sign(\dot{\epsilon})\right] \rho & if \ x < L_1 \\ \left[H(L_1) - T_0 L_1 + \frac{R}{\mu} T_{in} sign(\dot{\epsilon})\right] \rho & if \ x \geq L_1 \end{cases} \tag{3.5}$$

where

$$H(x) = -\frac{R}{\mu} T_{in} sign(\dot{\epsilon}) \, \exp\left[-\frac{\mu}{R} x \, sign(\dot{\epsilon})\right] \tag{3.6}$$

and

$$L_1 = \min\{x \in T(x) = T_0\}$$

When L_1 becomes equal to L, to variation of the input tension is immediately transmitted to the output side, with a ratio of:

$$\frac{dT_{out}}{dT_{in}} = \begin{cases} e^{-v} & \text{if } \dot{\epsilon} \geq 0 \\ e^{v} & \text{if } \dot{\epsilon} < 0 \end{cases} \tag{3.7}$$

where

$$v = \frac{\mu}{R} L$$

This implies that $\dot{\epsilon}$ has the same sign along the tendon, positive during the pulling phase and negative during the loosening one.

Figure 3.2: The Dahl friction model.

This model is obtained by considering only the static Coulomb friction. With a dynamical friction model, in our case the Dahl model (see Fig. 3.2), it is possible to see that the slope of the exponential term of the (3.4) can assume all the intermediate values between v and $-v$. To explain this phenomenon, we can note that in (3.2), the variation of the tendon tension is due only to the Coulomb friction. In the Dahl model, the value of friction can vary between F_c and $-F_c$, as can be seen in Fig. 3.2, where F_c is the absolute value of the Coulomb friction.

Introducing the Dahl friction model into eq. (3.1) and (3.2), the normal load N that the tendon element exerts on the sheath arc of length dx can be expressed as:

$$N = T d\gamma = T\frac{dx}{R} \tag{3.8}$$

$$\dot{F}_f = \sigma\left[\dot{x} - \frac{F_f|\dot{x}|}{\mu N}\right] \tag{3.9}$$

$$\eta A \ddot{x} dx + b \dot{x} = T - (T + dT) - F_f \tag{3.10}$$

where σ and b are respectively the stiffness of the contact bristles and the viscous friction coefficient that characterize the contact between tendon and sheath, η and A are the tendon density and cross sectional area respectively ($\eta A dx = m$), $\ddot{x}$ and $\dot{x}$ are the acceleration and the velocity of the infinitesimal tendon element. Note that in eq.(3.10) also the dynamics of the infinitesimal tendon element has been included: this will help in the definition of the tendon-sheath lumped parameters model in the next section. Considering quasi-static conditions, the same analysis that has been done to found the solution of the model eq. (3.1) and (3.2) can be made also for the model (3.8)-(3.10) if the hypothesis of uniform velocity direction along the tendon is satisfied.

The Dahl dynamical friction model, Eq. (3.9), is here adopted, [30]. Note that a more complete friction model that takes into account more friction phenomena like stickbeck

Figure 3.3: Tendon tension distribution using the Coulomb friction model (a) and using the Dahl friction model (b).

and elastoplastic effects i.e. the LuGre model or the Bliman and Sorine model [30, 31] could be considered as well. In this model the sheath is considered infinitely stiff and non movable in space.

3.3 Tendon Dynamic Model

A lumped parameters tendon model similar to the one presented in [26] is used for simulation.

Figure 3.4: Lumped parameters tendon model.

With reference to the tendon-sheath lumped parameters model sketched in Fig. 3.4, the equilibrium of each tendon element is given by:

$$m_i\ddot{\varepsilon}_i - c_i(\dot{\varepsilon}_{i+1} - 2\dot{\varepsilon}_i + \dot{\varepsilon}_{i-1}) = T_i^+ + T_i^- - F_{fi} \tag{3.11}$$

$$T_i^+ + T_i^- = k_i(\varepsilon_{i+1} - 2\varepsilon_i + \varepsilon_{i-1}) \tag{3.12}$$

$$F_{fi} = \frac{\mu}{2R}(T_i^+ - T_i^-)\tanh(k_v\dot{\varepsilon}_i) =$$
$$= \frac{\mu}{2R}k_i(\varepsilon_{i+1} - \varepsilon_{i-1})\tanh(k_v\dot{\varepsilon}_i) \tag{3.13}$$

Figure 3.5: Simulation results: tendon tension input-output characteristic.

where m_i, ε_i, c_i, F_{fi}, T_i^+, T_i^-, k_v and k_i are respectively the mass, the position, the damping, the friction, the input side tension, the output side tension, the friction parameter in the Karnopp model and the stiffness of each tendon element. Note that the $sign(\cdot)$ function in the Coulomb friction model is replaced in the Karnopp model by the $tanh(\cdot)$ function, eq. (3.13), to avoid numerical problems during simulation [29].

The simulation results highlight that in stationary conditions the tendon tension distribution shows a behavior without physical meaning, see Fig. 3.3(a). In particular, it is possible to note that the output tension increases without an input tension variation, as if the effect of friction is vanishing. This fact is due to the friction model used, that neglects the stiction effect. The use of the Dahl friction model [29] allows to take into account the effects of the friction also when the velocity goes to zero and for very small movements. Considering the Dahl friction model instead of the Coulomb model, the equilibrium condition of each tendon element can be written as:

$$\dot{x}_i = v_i \tag{3.14}$$

$$\dot{v}_i = \frac{1}{m_i}\big[c_i(v_{i+1} - 2v_i + v_{i-1}) + $$

$$+ k_i\,(x_{i+1} - 2x_i + x_{i-1}) - F_{fi}\big] \tag{3.15}$$

$$\dot{F}_{fi} = \sigma\left[v_i - \frac{2nR\,F_{fi}\,|v_i|}{\mu L\,k_i(x_{i+1} - x_{i-1})}\right] \tag{3.16}$$

where m_i, x_i, c_i, F_{fi}, T_i^+, T_i^-, and k_i are respectively the mass, the position, the damping, the friction, the input side tension, the output side tension and the stiffness of each tendon element. The internal tension distribution along the tendon for a constant input of the model (3.14)-(3.16) is reported in Fig. 3.3(b). In this model, also the friction force F_{fi}

Figure 3.6: Simulation results: tendon tension distribution in the lumped parameters model with sinusoidal input.

is a state variable. Eq. (3.16) is the Dahl friction derivative for the considered system. To validate this model, simulations have been performed, studying a tendon with a total length $L = 0.1 \ m$, subdivided in $n = 40$ elements modeled by Eq. (3.14)-(3.16). The parameter γ is chosen equal to $\pi/2$ so the radius of curvature is $R = 2L/\pi$.

The transmission characteristic obtained from simulations of this model is shown in Fig. 3.5, where four main phases can be identified: pulling phase (A), relaxation dead zone (B), loosening phase (C) and stretching dead zone (D). Fig. 3.6 shows the distribution of tension along the tendon, supposed here with length $L = 0.4 \ m$ and with $R = 2L/\pi$, when a sinusoidal input force is considered.

3.4 Experimental Results

In order to validate the simulation results and also to characterize the friction parameters of different materials, for both tendon and sheath (alloy-alloy, Kevlar-alloy, Teflon-alloy, Teflon-Teflon, spectra-alloy, spectra-Teflon), some experiments have been conducted. In the laboratory setup (see Fig. 3.7 for a scheme of the acquisition system), the tendon is connected at each end to an actuator, composed by a DC motor with encoder, gearheads and ballskrew. The input and output tensions are measured by two load cells, placed at each tendon end. The sheath curvature radius R can be changed, replacing the black plastic cylinder visible in Fig. 3.8, and the possible bending angles γ are $\pi/2$, π and $3\pi/2$.

The experimental results in Fig. 3.9(a) and Fig. 3.9(b) confirm that the transmission characteristic (alloy-alloy) does not depend on the tendon length, but only on the bending angle γ. The parameters of friction can be determined from the transmission characteristic

Figure 3.7: Acquisition system.

Figure 3.8: Experimental setup for the testing of the different materials.

(see Fig. 3.10(a)). The comparison between simulation and experimental results (see Fig. 3.10(b)) validates the proposed model.

From Fig. 3.10(b) it is also possible to note that for high tendon tension, the compliance of the sheath is not negligible. A model that takes into account also the elasticity and the dynamics of the sheath is under development.

3.5   The 'Three-Mass' Model

Since the lumped parameters model presented in the Section 3.3 is computationally complex, and therefore not very convenient if several simulations have to be performed, a simplified version, called here the "three-mass model", is now presented. The model of the tendon-sheath driving system is now obtained considering the tendon constituted by a single macro element having the same properties of the infinitesimal tendon element of

Figure 3.9: Experiment results: transmission characteristic from constant bending angle (a) and constant tendon length (b).

Fig. 3.1. Therefore in this model the tendon is described by eq. (3.8), (3.9) and (3.10).

Moreover, also the inertia of both the actuator and the load are now considered, thus obtaining the three mass model shown in Fig. 3.11 where m_1 represents the mass of the actuator, m_2 is the load and b_1, b_2 are the viscous friction coefficients of the actuator and of the load respectively. F_{act} is the force applied from the actuator to the tendon and F_{env} represents any other external force applied to the load. In the UB Hand 3 [14, 15] the tendon is attached to the phalanx that is affected by the elastic force of the hinge and by the external forces due to the contact with a grasped object. By considering a suitable projection of the elastic and grasping forces on the space of the tendons, it possible to include the dynamic model of the hinge in the of the environment eq. (3.18).

The overall system is described by:

$$F_{act} = m_1\ddot{x}_1 + b_1\dot{x}_1 + k(x_1 - x_3) \tag{3.17}$$

$$F_{env} = m_2\ddot{x}_2 + b_2\dot{x}_2 + k(x_2 - x_3) \tag{3.18}$$

$$-F_f = m_3\ddot{x}_3 + b_3\dot{x}_3 + k(-x_1 - x_2 + 2x_3) \tag{3.19}$$

$$\dot{F}_f = \sigma\left[\dot{x}_3 - \frac{F_f|\dot{x}_3|}{\mu\,\gamma\,k(x_2 - x_1)}\right] \tag{3.20}$$

where x_1, x_2 and x_3 are the position of m_1, m_2 and m_3 respectively, γ is the sheath curvature angle and b_3 is the viscous friction coefficient between tendon and sheath, introduced since the Dahl friction model does not take into account this friction phenomenon. Note that the tendon *internal* viscous friction is neglected because it is very small whit respect to other friction effects, therefore the tendon is modeled only with its mass m_3 and its stiffness k.

The three-mass model of the tendon seems to be suitable if the shape of the sheath is constituted by an arc of constant radius. However, since the tendon path can be decomposed in several arcs of the same radius such that the total bending angle γ is conserved without

Figure 3.10: Experimental results: identification of friction parameters (a) and comparison between simulation and experimental results (b).

alteration of the behavior of the transmission system [25, 26, 28], this model can be used also for different tendon paths. Analogous considerations can be made also if the shape of the sheath is constituted by several arcs with different radius and/or different bending angles.

3.5.1   Validation of the Three-Mass Model

The simulation results of the three-mass model are now compared with those of the lumped parameters model and with experimental results, see Fig. 3.12(a) and 3.12(b). In the laboratory setup (see Fig. 3.13), the tendon is connected at each end to an actuator, composed by a DC motor (of the same type used in the UB Hand 3) with a potentiometer for absolute position measure, connected to the output shaft, and a multi-stage gearheads with total reduction ratio of 300:1. The input and output tensions are measured by means of two load cells, placed at each tendon end. The sheath curvature radius R can be changed, and the possible bending angles γ are $\pi/2$, π and $3\pi/2$. The simulation results of the lumped parameters model and of the three-mass model reported in Fig. 3.12(a) and 3.12(b) are obtained using the physical parameters (length, angles, ...) of the experimental setup.

Figure 3.11: Scheme of the three-mass tendon-sheath transmission model.

Figure 3.12: Tendon tension input-output characteristic: comparison between the three-mass and of the lumped parameters models (a) and comparison of simulation and experimental results for $\gamma = \pi/2$ (b).

3.5.2 Geometric Properties of the Model

In this section, the properties of the system eq. (3.17)-(3.19) will be analyzed and the conditions for the solution of the problems of disturbance decoupling will be checked. These considerations are useful for the design of the a suitable controller for the compensation of the friction in the tendon-sheath transmission system.

If F_{act} is considered as the control input of the system, and F_{env}, F_f as disturbance inputs, the system described by Eq. (3.17)-(3.19) is a linear system that can be written as:

$$\begin{aligned} \dot{x} &= Ax + Bu + Hd \\ y &= Cx \\ w &= Ex \end{aligned}$$

Figure 3.13: Laboratory setup for the testing of the tendon tension controller.

where $x = [x_1 \ \dot{x}_1 \ x_2 \ \dot{x}_2 \ x_3 \ \dot{x}_3]^T$ is the state of the system, $u = F_{act}$ is the control input, $d = [F_{env} \ F_f]^T$ is the disturbance input, $y = [x_1 \ x_2 \ T_{in} \ T_{out}]^T$ is the output vector, and $w = T_{out}$ is the controlled output.

The matrices of the system are defined as:

$$
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
\frac{-k}{m_1} & \frac{-b_1}{m_1} & 0 & 0 & \frac{k}{m_1} & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & \frac{-k}{m_2} & \frac{-b_2}{m_2} & \frac{k}{m_2} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\frac{k}{m_3} & 0 & \frac{k}{m_3} & 0 & \frac{-2k}{m_3} & \frac{-b_3}{m_3}
\end{bmatrix}
\quad , \quad
B = \begin{bmatrix}
0 \\
\frac{1}{m_1} \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\quad , \quad
H = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
\frac{1}{m_2} & 0 \\
0 & 0 \\
0 & \frac{-1}{m_3}
\end{bmatrix}
\tag{3.21}
$$

$$
C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
-k & 0 & 0 & 0 & k & 0 \\
0 & 0 & k & 0 & -k & 0
\end{bmatrix}
\quad , \quad
E = \begin{bmatrix} 0 & 0 & k & 0 & -k & 0 \end{bmatrix}
\tag{3.22}
$$

Matrix C and the output vector y are chosen as described by Eq. (3.22) because in the laboratory setup the available information are the position of the actuator (x_1) and of the environment load (x_2), and the input (actuator side) and output (environment side) tension of the tendon (T_{in} and T_{out} respectively). It can be easily shown that the system described by matrices (3.21)-(3.22) has an eigenvalue in the origin and it is completely controllable (hence stabilizable) from the input u, and completely observable (hence detectable) from the output y. With respect to the control output w, this system is not completely observable and it is not detectable. The minimum set of output information that makes this system completely observable is $y_m = [x_2 \ F_{out}]^T$. With this choice it is possible to reduce the number of sensors required to control the tendon. The system under analysis is completely observable also from $[x_1 \ F_{in}]^T$ but this makes the control of the output tension an open loop control, based only on the model of the system and this can increase the effects of parameters uncertainties.

It can be easily shown that for this system:

$$
\mathcal{H} \nsubseteq \mathcal{V}^*_{\mathcal{B},\mathcal{E}}
\tag{3.23}
$$

and also:

$$
\mathcal{H} \nsubseteq \left(\mathcal{V}^*_{\mathcal{B},\mathcal{E}} + \mathcal{B} \right)
\tag{3.24}
$$

where $\mathcal{H}$ is the image of the matrix H, $\mathcal{B}$ is the image of the matrix B and $\mathcal{V}^*_{\mathcal{B},\mathcal{E}}$ is the maximum output nulling controlled invariant [32].

Equations (3.23) and (3.24) show that both the disturbance decoupling and measurable signal decoupling problems are not solvable for the system described by the matrices (3.21) and (3.22) because the structural conditions for the solution of these problems are not satisfied [32].

While the problem of perfect decoupling between d and w cannot be solved, it is possible to minimize the norm of the transfer functions between d and w while considering

Figure 3.14: Setpoint, control action, tendon output tension (a) and tracking error (b).

Figure 3.15: Setpoint, control action, output tension (a) and tracking error (b) with the boundary layer.

a suitable limitation on the control action u [32]. Since this can be done with a suitable algebraic state feedback, the knowledge of the entire state x of the system is required. Since only two components of the state x are directly deducible from the output y (x_1 and x_2), the problem of the design of a suitable state observer for the three-mass model of the tendon must be addressed, as discussed in the Section 3.6.2.

3.6   Tendon Transmission Control

In this Section, two possible control strategies are analyzed. First, a control law based on static friction compensation is presented. This controller is very simple to be implemented but requires very fast actuators and can generate high frequency oscillations in the system. Then, an optimal controller is presented. This latter controller is more complex, but allows to control the tendon driving system considering explicitly also the effects of friction, of environment forces, and the limitations of the actuators. At the moment, both controllers have been verified only by means of simulations.

3.6.1 Friction Compensation

The control of the output tension of the tendon, with a compensation of friction and elasticity effects, is among the main targets of this research. For this purpose, a friction compensator is designed to add a proper action to the input tension in order to reach the desired value for the output tension.

The friction compensation law is not based on the tendon velocity, but on the tension tracking error because if the environment constraint is considered fix, a positive tracking error means that the tendon must be pulled and then the tendon velocity must become positive and vice versa. So the control action $F_c = F_{act}$ is:

$$F_c = T_s \exp \left[\mu \frac{L}{R} \operatorname{sign}(e) \right] \tag{3.25}$$

where T_s is the tension setpoint and $e = T_s - T_{out}$ is the tension tracking error.

In Fig. 3.14 it is possible to see that the tracking error is bounded but presents fast variations due to the switching nature of the friction compensation action (3.25). As a matter of fact, the controller is very similar to a sliding mode controller, because the control action switches from a maximum to a minimum value related to the magnitude of the disturbance (friction). In order to reduce the chattering of this control system, a boundary layer of proper width is introduced. The new friction compensation action is:

$$F_c = \begin{cases} T_s \exp \left[\mu \frac{L}{R} \operatorname{sign}(e) \right] & \text{if } |e| \geq e_{th} \\ T_s \left(1 + \frac{|e|}{e_{th}} \exp \left[\mu \frac{L}{R} \operatorname{sign}(e) \right] \right) & \text{if } |e| < e_{th} \end{cases} \tag{3.26}$$

where e_{th} is the amplitude of the boundary layer. The response and the tracking error of this controller are shown in Fig. 3.15.

With this control law, it is also possible to compensate for the effects of a non fixed environment constraint. This is possible by overestimating the friction effects, and therefore increasing the magnitude of the action computed from the friction compensator. Fig. 3.16.a,b show the response of the system to a fast input variation when the control law (3.26) is applied and the friction parameters are overestimated. In Fig. 3.16.c,d the response of this system to a sinusoidal input is reported while Fig. 3.16.e,f show the response to a sinusoidal load disturbance applied at $t = 2\ s$.

It is important to note that the control action (3.26) is very easy to implement because the exponential term can be computed only once for positive tracking error and once for negative tracking error if the other parameter are not variables in time. It is also important to note that measurement noise in the output tension is very critical for this controller, since it is based on the sign of the output tracking error.

Figure 3.16: Response of the controller (3.26) with disturbance overestimation.

3.6.2 Optimal Controller Design

In order to take into consideration the effects of control input and disturbances on the system output, the system described by matrices (3.21)-(3.22) can be rewritten as:

$$\dot{x} = Ax + Bu + Hd \tag{3.27}$$
$$y = Cx + \quad + D_1 d \tag{3.28}$$
$$w = Ex + D_2 u \tag{3.29}$$

In this case, recalling that only two components of the state vector x are known (hence the necessity of a state observer), and by considering the dual system, the state observer can be designed to minimize the index:

$$J_{obs} = \int_0^\infty \left(d^T H^T H d + d^T D_1^T D_1 d \right) dt \tag{3.30}$$

where D_1 is chosen very small to reduce the effects of disturbances on the state estimation, but this increase the gain of the output feedback of the observer and, consequently, the effects of the measurement noise in the state observation. The order of this observer can

Figure 3.17: Scheme of the tendon tension optimal controller.

be reduced to estimate only the state components that are not directly deducible from the outputs. To improve the performance in terms of estimation error, it is possible to design a nonlinear state observer in order to estimate also the friction force, but this increase the complexity of the observer. To ensure the convergence to zero of the state estimation error, also the measure (or a suitable estimation) of the environment force F_{env} is required.

The state feedback is designed to minimize:

$$
\begin{aligned}
J_{fb} &= \int_0^\infty \left(w_r^T w_r + u^T D_2^T D_2 u \right) dt = \\
&= \int_0^\infty \left(x_r E^T E x_r + u^T D_2^T D_2 u \right) dt
\end{aligned}
\tag{3.31}
$$

$$
w_r = w_s - w_d
\tag{3.32}
$$

$$
x_r = x_s - E^\dagger w_d
\tag{3.33}
$$

where x_s and w_s are respectively the state estimated by the observer and the output of the observer, w_d is the desired output tension (i.e. the set point), $E^\dagger$ is the pseudo-inverse of matrix E. The parameter D_2 is chosen taken into consideration that is possible to reduce the tracking error reducing D_2 but this increase the required control action and vice versa.

In Fig. 3.18.a,b the response of the optimal controller to a fast input variation is reported while Fig. 3.18.c,d and Fig. 3.18.e,f show the response of the same system to a sinusoidal input and to a sinusoidal load disturbance applied at $t = 2 \ s$ respectively. Form these plots it is possible to note that the performances, in terms of tracking error, of the optimal controller are better than the ones of (3.26) in all the cases here presented, even if the solicitations of the control action due to a fast input variation are greater than in Fig. 3.16(a).

The complete control scheme for the tendon is shown in Fig. 3.17 where K is the algebraic feedback matrix computed from the solution of the continuous algebraic Riccati

Figure 3.18: Response of the optimal controller.

equation (CARE) that minimize the (3.31), L is the observer feedback matrix computed from the solution of the CARE that minimize the (3.30) and $Q = E^{\dagger}$ is the pseudo-inverse of the matrix E.

3.7 Conclusions

A dynamic model of an actuation system based on a tendon-sheath mechanism has been presented. The model, because of its computational simplicity, may be used for the simulation and the design of suitable control laws for devices using this type of transmission system. The results obtained so far, basically simulations and comparisons with experimental data, are quite satisfactory and show the validity of the model.

With respect to future activities, implementation of the optimal control laws on the laboratory setup and some improvements of the dynamic model considering also additional effects are planned. In particular, it is foreseen to define also models of the actuators and of the tendon (with a proper parameters estimation), as well as a more complex dynamic friction model (e.g. the LuGre model).

With respect to control performance, the simulations show that a possible improve-

ment could derive from considering saturation in the control action, that are physically present, but that have not been considered in the design phase. These could be easily taken into account with a suitable frequency shaped design of the optimal controller.

Moreover, in this type of driving system, a fundamental rule is played also by the sheath, whose dynamics has not been considered yet. A simple model of the sheath, taking into account its finite stiffness, is currently under development. Note that the dynamics of the non constrained parts of the sheath is very important for modeling the force transmission dynamics into the tendon since, in practice, the mass of the sheath (subject to motion) is much larger than the mass of the tendon.

Finally, also the dynamic model of the finger will be included to evaluate the performance of the overall finger control and, in order to implement an impedance control for the robotic finger, an output tendon stiffness controller will be designed.

4

Antagonistic Actuated Robots

4.1 Introduction

Standard industrial robots are usually designed to have very rigid links, that implies a considerable increment of the link masses, and to minimize the effects of the elastic coupling between the actuators and the joints due to the deformation of the transmission elements like long shafts, belts or harmonic drives. These design goals are usually maintained also for the design of the control law of these robots. This approach is justified because industrial tasks usually require accuracy, repeatability and simplicity in the implementation of the control law. On the other hand, it is well know that neglecting the elastic coupling between the actuators and the robot joints can lead to vibrations in the kinematic chain and reduce both the dynamic and static performance of the overall system [33, 34, 35, 36].

In the last years, a large variety of robots have been developed to accomplish a completely different class of tasks, like space and submarine activities, cooperative manipulation and, in particular, to interact with humans for entertainment, domestic activities and assistance to elder or handicapped people. The main requirements for the introduction of robots in the human environment are safety and dependability of the robotic system [37, 38, 39]. These requirements exclude the use of standard industrial robots for the interaction with humans. Also incrementing the sensorization and improving the performances of the controller, there are intrinsic limitations on the safety of industrial robots due to the inertia of the links and to the magnitude of the torque that the actuators can apply [39].

The development of lightweight robotic arms [40, 41] is carried out to improve the dynamic performance and reduce the weight-to-payload ratio. While this approach is suitable in case of devices for special applications, in particular for space activities, to improve the safety of robotic arms different projects have been developed, besides maintaining a low level of inertia, introducing also an high compliance at the mechanical level both in the joints of the robot and in the interface between the robot and the environment. Concerning the joint compliance in order to obtain an adequate level of accuracy preserving safety, several variable stiffness devices, and in particular antagonistic actuated joints, have been developed [39, 42, 43]. Continuous high compliant structures with antagonistic actuation are also applied to reduce the mechanical complexity, the weight and the cost of robotic hands [12, 14, 15, 44].

In this chapter, the dynamic model of a robot with antagonistic actuated joints similar to the one reported in [39] but without direct coupling between the antagonistic actuators is presented, and the problem of full linearization via static state feedback of both the position and the stiffness of the joint simultaneously is analyzed. The modulation of the mechanical stiffness of the joint is achieved by using transmission elements with nonlinear relation between the displacement and the actuated force [39, 42, 43]. In particular, the cases of transmission elements with quadratic [42] and exponential [43] force-length characteristic are analyzed as examples of application of this methodology. The time dependence of the functions is omitted for brevity in the sequel of the chapter.

4.2   Dynamic Model of Robots with Antagonistic Actuated Joints

In this section, the dynamic model of a robotic arm with N antagonistic actuated joints is reported. Some assumptions have been made to obtain this dynamic model. In particular, we assume that the actuators have uniform mass distribution and center of mass on the rotation axis [45] and that their rotor kinetic energy is due only to their spinning angular velocity [35]. Another assumption is that each joint is independently actuated by 2 motors in an antagonistic configuration. From now, we refer to this kind of mechanical structure as fully antagonistic actuated kinematic chain. Applications of this methodology to mechanical structures with coupled antagonistic actuation (or non-fully antagonistic, to distinguish from the former case) [12, 44] are object of future research.

The considered model is then composed by $3N$ rigid bodies (N links and $2N$ actuators) with nonlinear elastic coupling between their positions. The state dimension of the model of a robot with N spatial DOFs is then equal to $6N$ (position and velocity of each rigid body) while the input dimension is $2N$ (the torques commanded to the actuators). In this case it is necessary to distinguish between the spatial and the stiffness DOFs. The former is the possibility of modifying the position of the system while the latter means the possibility of adjusting the mechanical stiffness of the device. In this terms, a robot with N antagonistic actuated joints[1] has a total of $2N$ DOFs (N spatial + N stiffness).

It is important to stress the fact that, for mechanical stiffness, we mean the compliance of the mechanical coupling between the link and the actuation. Usually this characteristic is imposed by the mechanical design, and in particular by the elasticity of transmission elements, while in this case it can be modulated with antagonistic actuation and nonlinear transmission elements. This allows to increase the safety of the robot arm in the case of unexpected collision with i.e. a human operator. In [38] a meaningful analysis of how the mechanical coupling between the link and the actuation affects the safety of a robotic arm operating in the human environment is reported.

---

[1]For mechanical structures with coupled antagonistic actuation, the total number of DOFs N_{sp+st} (spatial + stiffness) is $N < N_{sp+st} < 2N$.

Figure 4.1: A robotic arm with 3 antagonistic actuated joints.

As output of the system, the positions of the joints and of the actuators are considered, obtaining a output of dimension equal to $3N$.

In Fig. 4.1 a sketch of a robotic arm with antagonistic actuated joints is reported. With reference to this figure, α_i and θ_i ($i = 1, \dots, N$) are positive in the counterclockwise direction while β_i is positive in clockwise direction. $\phi_i = \phi_i(\alpha_i, \theta_i)$ and $\psi_i = \psi_i(\beta_i, \theta_i)$ are the nonlinear coupling functions between the position of the joint and of the two actuators (see also Fig. 4.2).

The dynamic model of the robot with antagonistic actuated joints can be obtained form the standard Lagrangian formulation and can be written in a convenient form grouping the dynamics of the joints and considering separately the dynamics of the two groups of actuators:

$$M(\theta)\ddot{\theta} + C_j(\theta, \dot{\theta})\dot{\theta} + D_j\dot{\theta} + g_f(\theta) + \varphi(\alpha, \beta, \theta) = 0 \tag{4.1}$$

$$J\ddot{\alpha} + D_m\dot{\alpha} + \phi(\alpha, \theta) = \tau_\alpha \tag{4.2}$$

$$J\ddot{\beta} + D_m\dot{\beta} + \psi(\beta, \theta) = \tau_\beta \tag{4.3}$$

where θ is the vector of joint positions, α and β are the vectors of actuator positions, $M(\theta)$ is the inertia matrix of the robot, J is the diagonal matrix of the inertia moments of the actuators, $C_j(\theta, \dot{\theta})$ is the matrix of the centrifugal and Coriolis terms of the robot, D_j and D_m are the diagonal matrices of the viscous friction coefficients of the robot and of the actuators respectively, $g_f(\theta)$ is the vector of gravity effects, $\varphi(\alpha, \beta, \theta)$ is the combined effect of the positions of the actuators on the joints, τ_α and τ_β are the vectors of the torques commanded to the actuators.

The input of the system and the vector of output information are:

$$u = \begin{bmatrix} \tau_\alpha^T & \tau_\beta^T \end{bmatrix}^T, \qquad y = \begin{bmatrix} \theta^T & \alpha^T & \beta^T \end{bmatrix}^T \tag{4.4}$$

Since we are interested to control both the position and the stiffness of the joints of the robot, it is useful to define a new output vector that contains explicitly these information's:

$$
y_c = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_N \\ \frac{\partial \varphi_1(\alpha_1, \beta_1, \theta_1)}{\partial \theta_1} \\ \vdots \\ \frac{\partial \varphi_N(\alpha_N, \beta_N, \theta_N)}{\partial \theta_N} \end{bmatrix} = \begin{bmatrix} \theta \\ S \end{bmatrix} \tag{4.5}
$$

where S is the vector of the mechanical stiffness of the joints that can be expressed as in eq.(4.5) because we suppose all the joints and all the actuators to be independent (no coupling between the movements of the joints or the actuators). From eq. (4.5) it is possible to note that, in general, while it is difficult to measure the stiffness of the joints, a suitable knowledge of the coupling functions $\varphi(\alpha, \beta, \theta)$ enables us to derive the joints stiffness S from the measured output y.

Considering the new output vector y_c, the input u and the state vector

$$
x = \begin{bmatrix} \theta^T & \dot{\theta}^T & \alpha^T & \dot{\alpha}^T & \beta^T & \dot{\beta}^T \end{bmatrix}^T \tag{4.6}
$$

the model of the robot with antagonistic actuated joints described by eq. (4.1)-(4.3) can be rewritten in the input-affine state space form:

$$
\dot{x} = f(x) + g(x)u \tag{4.7}
$$
$$
y_c = h(x) \tag{4.8}
$$

where $x \in \mathbb{R}^{6N}$ and $u, y_c \in \mathbb{R}^{2N}$. In particular, for the model of the robot with antagonistic actuated joints:

$$
f(x) = \begin{bmatrix} \dot{\theta} \\ M(\theta)^{-1}(-C_j(\theta, \dot{\theta})\dot{\theta} - D_j\dot{\theta} - g_f(\theta) - \varphi(\alpha, \beta, \theta)) \\ \dot{\alpha} \\ J^{-1}(-D_m\dot{\alpha} - \phi(\alpha, \theta)) \\ \dot{\beta} \\ J^{-1}(-D_m\dot{\beta} - \psi(\beta, \theta)) \end{bmatrix} \tag{4.9}
$$

$$
g(x) = \begin{bmatrix} 0_{N \times N} & 0_{N \times N} & 0_{N \times N} & J^{-1} & 0_{N \times N} & 0_{N \times N} \\ 0_{N \times N} & 0_{N \times N} & 0_{N \times N} & 0_{N \times N} & 0_{N \times N} & J^{-1} \end{bmatrix}^T \; , \; h(x) = \begin{bmatrix} \theta \\ S \end{bmatrix} \tag{4.10}
$$

The eq. (4.7)-(4.10) define a square nonlinear system that can be linearized via static feedback if suitable conditions are satisfied [46, 47].

4.3   Static Feedback Linearization

In this section, necessary and sufficient conditions for the solution of the problem of full linearization via static state feedback of the system described by the eq. (4.1)-(4.5) are analyzed [46]. In particular, we have to check if the decoupling matrix of this system is nonsingular and if the sum of the relative degrees of the outputs are equal to the state dimension of the system.

With the aim of simplifying the explanation, the notation used in the previous section is extended. Since all the components of both the input, the output and the state vectors belong to $\mathbb{R}^N$, where N is the number of robot joints, we can redefine the dimensions of the input u, the output y_c and the state x of the system to 2, 2 and 6, respectively. Also the notation of Lie derivative is extended to work with vectors in $\mathbb{R}^N$.

The problem of static feedback linearization consists in transforming a nonlinear system of the type (4.7)-(4.8), via static state feedback and coordinate transformations into a fully controllable and observable linear system that can be represented in the form:

$$\dot{z} = Az + B\left(\begin{bmatrix} L_f^{r_1} h_1(\Phi^{-1}(z)) \\ \vdots \\ L_f^{r_m} h_m(\Phi^{-1}(z)) \end{bmatrix} + Q(\Phi^{-1}(z))u\right) \tag{4.11}$$

$$y_c = Cz \tag{4.12}$$

where A, B and C are matrices of proper dimensions given by the Brunowsky canonical form, m is the number outputs of the system, $r_1, \ldots, r_m$ are the relative degrees of each output, $L_f h(x)$ denotes the Lie derivative of $h(x)$ along the vector function $f(x)$, $z = \Phi(x)$ is the coordinate transformation from the original to the new state space and $Q(x)$ is the so-called decoupling matrix.

First of all, we have to define the (vector) relative degree of the outputs of the system. For this purpose, it is useful to firstly define the state transformation from the original state space to the state space of the linearized system:

$$\Phi(x) = \begin{bmatrix} h_1(x) \\ \vdots \\ L_f^{(r_1-1)} h_1(x) \\ \vdots \\ h_m(x) \\ \vdots \\ L_f^{(r_m-1)} h_m(x) \end{bmatrix} \tag{4.13}$$

The relative vector degree of the outputs can now be easily found by looking when the inputs appearing explicitly in eq. (4.13). In particular, for the output θ it possible to find

that:

$$L_{g_{(\alpha,\beta)}} L_f^k h_\theta(x) = 0_{N\times N} \quad , \quad i = 0, \cdots, 2 \tag{4.14}$$

$$L_{g_\alpha} L_f^3 h_\theta(x) = -J^{-1} M(\theta)^{-1} \frac{\partial \varphi(\alpha,\beta,\theta)}{\partial \alpha} \tag{4.15}$$

$$L_{g_\beta} L_f^3 h_\theta(x) = -J^{-1} M(\theta)^{-1} \frac{\partial \varphi(\alpha,\beta,\theta)}{\partial \beta} \tag{4.16}$$

where $h_\theta(x)$ denotes the restriction of the output vector to θ only while for the output S:

$$L_{g_{(\alpha,\beta)}} h_S(x) = 0_{N\times N} \tag{4.17}$$

$$L_{g_\alpha} L_f h_S(x) = J^{-1} \frac{\partial S}{\partial \alpha} \tag{4.18}$$

$$L_{g_\beta} L_f h_S(x) = J^{-1} \frac{\partial S}{\partial \beta} \tag{4.19}$$

where $h_S(x)$ denotes the restriction of the output vector to its component S. L_{g_α} and L_{g_β} denote the restriction of the Lie derivative to the τ_α and τ_β component of the input vector respectively while $L_{g_{(\alpha,\beta)}}$ denotes both these cases.

From this result we can state that, if the derivatives of $\varphi(\alpha,\beta,\theta)$ in eq. (4.15), (4.16), (4.18), (4.19) are not null, the vector relative degree of θ is 4 while the one of S is 2. The sum of the vector relative degrees of the output is then equal to the dimension of the state of the system (i.e. 6), so that the condition for the existence of noninteracting control via static state feedback is satisfied. In particular, if $\varphi(\alpha,\beta,\theta)$ depends linearly on their arguments, as in [33, 35, 36, 48], the terms in eq.(4.18), (4.19), and also the successive derivatives, are always zero. Hence the vector relative degree of S is not defined and therefore the mechanical stiffness of the joints is not controllable.

Now, to verify that the system has no zero dynamics, we have to check if the decoupling matrix is nonsingular. The decoupling matrix of this system is:

$$Q(x) = B_d(\theta) \begin{bmatrix} \frac{\partial \varphi(\alpha,\beta,\theta)}{\partial \alpha} & \frac{\partial \varphi(\alpha,\beta,\theta)}{\partial \beta} \\ \frac{\partial S}{\partial \alpha} & \frac{\partial S}{\partial \beta} \end{bmatrix} \tag{4.20}$$

where

$$B_d(\theta) = \begin{bmatrix} -J^{-1} M(\theta)^{-1} & 0_{N\times N} \\ 0_{N\times N} & J^{-1} \end{bmatrix} \tag{4.21}$$

From this equation and from eq.(4.5), it is possible to see that the rank of $Q(x)$ depends on the nature of $\varphi(\alpha,\beta,\theta)$, hence this property has to be checked for mechanical implementation. If $\varphi(\alpha,\beta,\theta)$ depends linearly on their arguments, S is constant and therefore Q is singular.

Now, supposing that $Q(x)$ is not singular, by defining the new input

$$u = Q^{-1}(x) \left(\begin{bmatrix} -L_f^4 h_\theta(x) \\ -L_f^2 h_S(x) \end{bmatrix} + \begin{bmatrix} v_\theta \\ v_S \end{bmatrix} \right) \tag{4.22}$$

we obtain the linear model of the robot with elastic joints

$$\dot{z} \;=\; Az + Bv \;\;,\;\; v = \begin{bmatrix} v_\theta \\ v_S \end{bmatrix} \tag{4.23}$$

$$y_c \;=\; Cz \tag{4.24}$$

where

$$A = \begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \;,\; B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ I & 0 \\ 0 & 0 \\ 0 & I \end{bmatrix} \;,\; C = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \end{bmatrix} \tag{4.25}$$

$$z = \begin{bmatrix} \theta^T & \dot{\theta}^T & \ddot{\theta}^T & \theta^{[3]T} & S^T & \dot{S}^T \end{bmatrix}^T \tag{4.26}$$

in which I and 0 are the identity and the zero matrix of dimension N. From eq. (4.23)-(4.26) it is then possible to note that:

$$\begin{bmatrix} \theta^{[4]} \\ \ddot{S} \end{bmatrix} = \begin{bmatrix} v_\theta \\ v_S \end{bmatrix} \tag{4.27}$$

4.4 Control Strategy

Since the system (4.23),(4.24) is completely controllable and observable, the state of the system can be reconstructed by means of an asymptotic observer and of the change of coordinates (4.13) or, since the position of each rigid body is directly measurable, the velocities can be estimated in many ways e.g. by means of state variable filters or adaptive windowing algorithms [49].

From eq.(4.27), applying the control laws:

$$v_\theta \;=\; \theta_d^{[4]} + K_{3_\theta}\left[\theta_d^{[3]} - L_f^3 h_\theta(x)\right] + K_{2_\theta}\left[\ddot{\theta}_d - L_f^2 h_\theta(x)\right] +$$
$$+ K_{1_\theta}\left[\dot{\theta}_d - L_f h_\theta(x)\right] + K_{0_\theta}\left[\theta_d - h_\theta(x)\right] \tag{4.28}$$

$$v_S \;=\; \ddot{S}_d + K_{1_S}\left[\dot{S}_d - L_f h_S(x)\right] + K_{0_S}\left[S_d - h_S(x)\right] \tag{4.29}$$

with diagonal gain matrices $K_{3_\theta}, \ldots, K_{0_S}$ such that

$$\lambda^4 + \lambda^3 K_{3_{\theta_i}} + \lambda^2 K_{2_{\theta_i}} + \lambda K_{1_{\theta_i}} + K_{0_{\theta_i}} \;=\; 0 \;,\; i = 1, \ldots, N \tag{4.30}$$

$$\lambda^2 + \lambda K_{1_{S_i}} + K_{0_{S_i}} \;=\; 0 \;,\; i = 1, \ldots, N \tag{4.31}$$

are Hurwitz polynomials, the convergence to zero of the tracking error is ensured. If the desired trajectory θ_d is continuous together with its derivatives up to the 4th order and

S_d is continuous together with its derivatives up to the 2nd, also the asymptotic trajectory tracking is achieved.

It is important to note that all the Lie derivative appearing in eq. (4.22),(4.28) and (4.29) can be written as function of the measurable quantities x, so the computation of the time derivatives of θ (up to 4th order) and S (up to 2nd order) are not necessary.

The control law in eq. (4.28),(4.29) is equivalent to a static state feedback plus feedforward in the state space of the linearized system:

$$v = v_d + K(z_d - \Phi(x)) \tag{4.32}$$

$$v_d = \left[\begin{array}{cc} \theta_d^{[4]T} & \ddot{S}_d^T \end{array}\right]^T \quad , \quad z_d = \left[\begin{array}{cccccc} \theta_d^T & \dot{\theta}_d^T & \ddot{\theta}_d^T & \theta_d^{[3]T} & S_d^T & \dot{S}_d^T \end{array}\right]^T \tag{4.33}$$

$$K = \mathrm{diag}[K_{0_\theta}\ K_{1_\theta}\ K_{2_\theta}\ K_{3_\theta}\ K_{0_S}\ K_{1_S}] \tag{4.34}$$

The matrix K can be also obtained via direct eigenvalues assignment or through the solution of the CARE equation with a suitable choice of the weight matrices.

This approach ensures that, in case of an undesired event e. g. a collision with an obstacle, the coupling between the joint and the actuators has the desired stiffness without the intervention of the controller, avoiding in this way any problem related to limited control bandwidth, sensorization, delay in the control loop and so on. If the stiffness S depends itself on θ, this behavior is a linearization of the system stiffness around the desired position θ_d.

4.5   Properties of the Transmission Elements

To show some properties of different types of nonlinear coupling between the joint and the actuators, transmission elements with quadratic and exponential relation between the displacement and the force will be investigated. This analysis is performed considering a two-links planar arm with antagonistic actuated joints. In Fig. 4.2 a detail of the antagonistic joint is given. From this picture, the displacement between the i-th joint and its actuators can be defined as:

$$\varepsilon_{\alpha_i} = r_m\alpha_i + r_j\theta_i \tag{4.35}$$

$$\varepsilon_{\beta_i} = r_m\beta_i - r_j\theta_i \tag{4.36}$$

where $\varepsilon_{\alpha_i}, \varepsilon_{\beta_i}$ are the displacements (compression or extension) of the transmission elements and r_m, r_j are the radii of the pulley of the actuator and of the joint respectively. In order to simplify the following calculations and without loss of generality, r_m and r_j are supposed to be equals for all the actuators and all the joints. In [42, 43] the transmission elements are connected to the actuators and to the joint by means of metallic cables. To avoid cable slack, a suitable operating region of α, β, θ must be defined considering the geometry of the device. However, the condition $\varepsilon_{\alpha_i,\beta_i} < 0$ can be achieved by properly

setting the zero position of the actuators. From Fig. 4.2 it is also possible to establish the relationship between the coupling functions of the joint and of the actuators:

$$\varphi_i(\alpha_i, \beta_i, \theta_i) = \frac{r_j}{r_m} [\phi_i(\alpha_i, \theta_i) - \psi_i(\beta_i, \theta_i)] \tag{4.37}$$

4.5.1 Quadratic Force-Displacement Transmission Elements

In order to provide a stiffness that is a linear function of the displacement and independent from the position of the joint, transmission elements with quadratic force-compression characteristic must be used [42]:

$$F = k_2 \varepsilon^2 + k_1 \varepsilon + k_0 \tag{4.38}$$

where F is the force, ε is the displacement and $k_{2,1,0}$ are proper constants. The torques applied to the actuators result:

$$\phi_i(\alpha_i, \theta_i) = r_m(k_2 \varepsilon_{\alpha_i}^2 + k_1 \varepsilon_{\alpha_i} + k_0) \tag{4.39}$$

$$\psi_i(\beta_i, \theta_i) = r_m(k_2 \varepsilon_{\beta_i}^2 + k_1 \varepsilon_{\beta_i} + k_0) \tag{4.40}$$

and the torque applied to the i-th joint is:

$$\varphi_i(\alpha_i, \beta_i, \theta_i) = r_j[k_2(\varepsilon_{\alpha_i}^2 - \varepsilon_{\beta_i}^2) + k_1(\varepsilon_{\alpha_i} - \varepsilon_{\beta_i})] =$$
$$= r_j[k_2 r_m(\alpha_i + \beta_i) + k_1][r_m(\alpha_i - \beta_i) + 2r_j \theta_i] \tag{4.41}$$

Deriving this expression with respect to θ_i the value of the joint stiffness is obtained:

$$\frac{\partial \varphi_i(\alpha_i, \beta_i, \theta_i)}{\partial \theta_i} = S_i = 2r_j^2[k_2 r_m(\alpha_i + \beta_i) + k_1] \tag{4.42}$$

In this case, the joint stiffness does not depend on the position of the joint: this result can be achieved only with quadratic force-displacement relation as eq. (4.38). The decoupling matrix for the two-links planar arm can be be written as:

$$Q(x) = r_j r_m B_d(\theta) \begin{bmatrix} -(2k_2 \varepsilon_{\alpha_1} + k_1) & 0 & (2k_2 \varepsilon_{\beta_1} + k_1) & 0 \\ 0 & -(2k_2 \varepsilon_{\alpha_2} + k_1) & 0 & (2k_2 \varepsilon_{\beta_2} + k_1) \\ 2k_2 r_j & 0 & 2k_2 r_j & 0 \\ 0 & 2k_2 r_j & 0 & 2k_2 r_j \end{bmatrix} \tag{4.43}$$

In this case, $Q(x)$ is singular if $\varepsilon_{\alpha_i} = 0$ or $\varepsilon_{\beta_i} = 0$ and $k_1 = 0$ (purely quadratic force-displacement characteristic, zero stiffness for zero displacement) and is always singular if $k_2 = 0$, confirming that the stiffness of the joints is not adjustable using transmission elements with linear compliance. Another condition of singularity of $Q(x)$:

$$\alpha_i + \beta_i = -\frac{k_1}{k_2 r_m} \tag{4.44}$$

that corresponds, from eq. (4.42), to the case in which $S_i = 0$, that means no coupling between the actuators and the joint, a nonsense. This condition can be avoided imposing a suitable minimum value of the joint stiffness.

Figure 4.2: Detail of the antagonistic actuated joint.

4.5.2   Exponential Force-Displacement Transmission Elements

In this example, the transmission elements are composed by plastic elements (rubber balls) compressed inside a cylinder [43]. The geometry and the characteristic of the material gives the nonlinear force-compression relation [50, 51, 52]:

$$F = k(e^{\gamma \varepsilon} - 1) \tag{4.45}$$

where F is the force, ε is the displacement and $k > 0$, $\gamma > 0$ are suitable constants that depend on the shape and on the material used in the transmission elements. The torques applied to the actuators of the i-th joint are then:

$$\phi_i(\alpha_i, \theta_i) = r_m k[e^{\gamma \varepsilon \alpha_i} - 1] \tag{4.46}$$

$$\psi_i(\beta_i, \theta_i) = r_m k[e^{\gamma \varepsilon \beta_i} - 1] \tag{4.47}$$

The resulting torque applied to the joint using these elements in antagonistic configuration is given by:

$$\varphi_i(\alpha_i, \beta_i, \theta_i) = r_j k[e^{\gamma \varepsilon \alpha_i} - e^{\gamma \varepsilon \beta_i}] \tag{4.48}$$

The resulting stiffness of the joint can be then expressed in the form:

$$\frac{\partial \varphi_i(\alpha_i, \beta_i, \theta_i)}{\partial \theta_i} = S_i = k\gamma r_j^2[e^{\gamma \varepsilon \alpha_i} + e^{\gamma \varepsilon \beta_i}] \tag{4.49}$$

The decoupling matrix for the two-links planar arm can be be written as:

$$Q(x) = k\,\gamma\,r_j r_m B_d(\theta) \begin{bmatrix} -e^{\gamma \varepsilon \alpha_1} & 0 & e^{\gamma \varepsilon \beta_1} & 0 \\ 0 & -e^{\gamma \varepsilon \alpha_2} & 0 & e^{\gamma \varepsilon \beta_2} \\ \gamma r_j e^{\gamma \varepsilon \alpha_1} & 0 & \gamma r_j e^{\gamma \varepsilon \beta_1} & 0 \\ 0 & \gamma r_j e^{\gamma \varepsilon \alpha_2} & 0 & \gamma r_j e^{\gamma \varepsilon \beta_2} \end{bmatrix} \tag{4.50}$$

It is important to note that in this case $Q(x)$ is always nonsingular thanks to the properties of the exponential function that characterize the transmission elements.

Another important point is that, due to the exponential relation between the compressions and the forces of transmission elements (see eq. (4.49)), the joint stiffnesses are always strictly positive, hence a suitable translation of the origin of the linearized system is necessary. The value of the joint stiffnesses S_0 that corresponds to the condition $\varepsilon_{\alpha,\beta} = 0$ is given by:

$$S_{0i} = 2k\gamma r_j^2, \quad i = 1, \dots, N \tag{4.51}$$

The state of the linearized system is then:

$$z = \begin{bmatrix} \theta^T & \dot{\theta}^T & \ddot{\theta}^T & \theta^{[3]T} & (S - S_0)^T & \dot{S}^T \end{bmatrix}^T \tag{4.52}$$

Note that all the system state values x_{S_0} that satisfy the condition $\varepsilon_{\alpha,\beta} = 0$ are equilibrium points for the system (4.1)-(4.3). Also the desired stiffness trajectory S_d must be chosen taking into account the value of the joint stiffness S_0 in the equilibrium points of the original system.

4.6 Simulation of the Two-Link Antagonistic Actuated Arm

In the following the validity of the proposed approach is reported by presenting the simulation results of a planar two-link antagonistic actuated arm. Due to space limitations, the well-known model of the planar two-link arm together with the solution of the previous equations for this system are omitted. For the same reason, only the simulations about the exponential force-displacement transmission elements case are reported.

In the simulation scheme, only the joint and actuator positions are known and the corresponding velocities are estimated by means of state variable filters. The trajectories are generated through filters of appropriate order to estimate also their derivatives up to the necessary order. The control strategy has been chosen as in eq. (4.33) and the matrix K is obtained from the solution of the CARE equation with a diagonal state weights matrix.

In Fig.4.3 the positions of the joints of the antagonistic actuated arm are reported. Both step and sinusoidal joint trajectories are used together with coordinated movements to show the stabilizing properties of the controller. The joint stiffnesses and the stiffness errors are reported in Fig.4.4. During the test, the stiffness of the joint 2 is constant while the one of joint 1 is sinusoidal. It is important to note that the joint stiffness trajectories are not affected by the changes of the joint positions and vice versa.

4.7 Conclusions

In this chapter, the non-interacting static feedback linearization of a antagonistic actuated arm has been presented. This problem can be solved if the transmission elements have nonlinear force-displacement characteristic and if the transmission element stiffness with zero deformation $S_0 \neq 0$.

Figure 4.3: (a) Joint positions and (b) trajectory tracking errors

Figure 4.4: (c) Joint stiffnesses and (d) stiffness tracking errors

The simultaneous stiffness-position control of a two-link robot arm has been validated through simulation. Even if only the joint and the actuator positions are known, the velocities estimation through state variable filters does not compromise the stability of the closed loop system.

Future activities consist in the definition a suitable nonlinear state and external load observer for the antagonistic actuated arm. A different analysis of the overall system, based on singular perturbations, can lead to the definition of a simpler control law, especially for robots with a high number of DOF. Moreover, the introduction of a robust controller can improve the reliability of the system in the case of model uncertainties or undesired effects.

The analysis of the behavior of robots with antagonistic actuated joints during the interaction with an external environment and the problem of external load compensation for this system are under analysis while the extension of this approach to non-fully antagonistic actuated robots is object of future research.

5

The DLR's Antagonistic Actuated Joint

5.1 Introduction

With the aim of verifying the properties and of evaluating the behavior of antagonistic variable stiffness devices, a testbed composed by a single rotative joint has been realized at DLR[1] (see Fig 5.1). In this implementation, the modulation of the mechanical stiffness of the joint is achieved by means of transmission elements with exponential relation between the displacement and the actuated force in antagonistic configuration [39, 42, 43]. These transmission elements are connected to the actuators (two brushless motors) and to the joint by means of alloy cables (tendons). The only information available on the system are the joint and the actuators positions, measured by means of optical encoder, while no force sensors are used to simplify the overall system. The forces applied by the transmission elements will be estimated from the position information by means of a suitable model of the their force/deformation relation. The estimation of both the joint and the actuators velocities are obtained by means of state variable filters.

The control system is composed by a personal computer equipped with a 3GHz Pentium IV processor, 1GB of RAM, the QNX realtime OS and a Sensoray 626 data acquisition board. The development platform used to design the controller and to monitor the control system is a standard PC with Windows OS and the MatLab/Simulink/ Realtime-Workshop development environment.

In the previous chapter, the analysis and the feedback linearization of a *N*-DOF robotic arm with antagonistic actuated joint has been presented. In this chapter, the same principles are applied for the analysis of the DLR's antagonistic actuated joint to design a suitable joint position and stiffness controller. In the case of 1-DOF system, such as in the considered implementation, or in the case of very light structure, in which the coupling between the link dynamics can be neglected, a more simple controller can be defined considering the static relation between the actuator positions and the joint position and stiffness, as show in the sequel.

The chapter is organized as follows. In section 5.2 the nonlinear characteristic of the transmission elements is analyzed and a suitable model is proposed. In section 5.3 the DLR's antagonistic actuated joint is presented and analyzed and the way how to achieve

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Figure 5.1: The DLR's antagonistic actuated joint.

the joint variable stiffness is explained. Neglecting the joint dynamics, two suitable approaches for the simultaneous control of the joint position and stiffness in the case of backdrivable and non-backdrivable actuators are presented in section 5.4. An alternative approach for the simultaneous stiffness and position control of the joint, based on the analysis reported in the previous chapter, is reported in section 5.5. In section 5.6 the problem of the identification of the parameters of the transmission elements is taken into account. Some conclusions about this work and some plans for future research activities on this field are reported in section 5.7.

This research activity has been carried out during my stay at the DLR, in the period February-September 2006.

5.2   Characterization of the Transmission Elements

It is well known that, to achieve simultaneous position and stiffness control of a robotic device via antagonistic actuation, transmission elements with nonlinear relation between the elements compression and the applied forces must be used [42]. In this section, the transmission elements used in the implementation of the DLR's antagonistic actuated joint are described.

With the aim of reducing the weight, the flexibility of application and the dimensions of the transmission elements, the properties of elastic materials, such as plastic polymers,

Figure 5.2: Detail (a) and working principle (b) of the transmission element.

forged in a suitable shape, in this case sphere, are exploited to achieve a nonlinear relation between the element compression and the applied force. In Fig. 5.2 it is possible to see a detail of a transmission element and in Fig. 5.2(b) a scheme of the working principle of the transmission element is reported. With reference to Fig. 5.2(b), the transmission elements are composed by an aluminum shell (A), connected at one end to the actuator (a pulley installed to the output shaft of a brushless rotative motor) by means of an alloy cable (B), that contains from 2 to 6 rubber balls (C). These rubber balls can be compressed by means of a plate (D), connected to the joint by means of another alloy cable (E). The geometry and the material of the transmission elements, and in particular of the rubber balls, gives a nonlinear relation between compression ε and applied force F (see eq. (5.1) and (5.2)). Adjustable stiffness is achieved by means of two of these elements connected to the joint in antagonistic configuration and driven by two independent actuators.

The relation between the compression of the transmission elements ε and the applied force F can be modeled using the following nonlinear elastic relation:

$$F(t) = \begin{cases} k[e^{\gamma \varepsilon(t)} - 1], & \varepsilon(t) \geq 0 \\ 0, & \varepsilon(t) < 0 \end{cases} \tag{5.1}$$

where k and γ are constant parameters depending on the material, on the dimensions and on the shapes of the elastic elements (in this case the rubber balls) and on the implementation of the transmission elements. A more complete model that takes into account also the damping introduced by the transmission elements is the nonlinear visco-elastic model, also known as the Hunt-Crossley model [50]:

$$F(t) = \begin{cases} k[e^{\gamma \varepsilon(t)} - 1] + b\,\dot{\varepsilon}(t)e^{\gamma \varepsilon(t)}, & \varepsilon(t) \geq 0 \\ 0, & \varepsilon(t) < 0 \end{cases} \tag{5.2}$$

where b is the nonlinear damping coefficient. Other models can be choose for the description of the characteristic of the viscoelastic materials [51, 52], but experimental results have shown that the proposed model is more reliable for this implementation.

It is important to note that the compression to force relation is unilateral. This is due to the particular implementation of the transmission elements and to the use of tendons. It implies that, both from the model and from the control point of view, the necessary conditions to maintain the transmission elements in the proper working region ($\varepsilon(t) \geq 0$) must be ensured.

For simulation purpose, the model reported in eq. (5.2) is used to ensure a more reliable behavior of the system while for control purpose, the elastic nonlinear model reported in eq. (5.1) can be used to obtain a more simple control law. Note that the additional damping, introduced by the transmission elements and neglected during the design of the controller, may increase the stability of the overall system.

5.3   System Analysis

With the aim of testing the behavior and of evaluating the benefits and the drawbacks of this actuation modality, an experimental setup has been built. The system is driven by means of two brushless motors and each of these motors is commanded by an embedded current control unit that includes also the power electronic. The output information are the joint and the motors positions, measured by means of optical encoders connected to the joint and to the motor shafts respectively.

Description	Symbol	Value	
Joint inertia	J_j	1.15e-2	$kg \cdot m^2$
Joint viscous friction coefficient	b_j	0.001	$N \cdot s \cdot m^{-1}$
Joint pulley radius	r_j	0.0355	m
Joint mass	m_j	0.541	kg
Link center of mass	l_c	0.085	m
Link length	l	0.3	m
Motors inertia	J_m	6.6e-5	$kg \cdot m^2$
Motors viscous friction coefficient	b_m	0.00462	$N \cdot s \cdot m^{-1}$
Motors pulley radius	r_m	0.00755	m
Motors torque constant	K_m	0.106	$N \cdot m \cdot A^{-1}$

Table 5.1: Parameters of the DLR's antagonistic actuated joint.

In Fig. 5.3 a scheme of the antagonistic actuated joint implemented at DLR is presented while in Tab. 5.1 the parameters of both the joint and the actuators are reported. The identification of the parameters of the transmission elements will be illustrated in the section 5.6.

Due to the particular implementation, the relation between the actuators and the joint positions and the transmission elements compression can be written as:

$$\varepsilon_\alpha = r_m\alpha + r_j\theta \tag{5.3}$$

$$\varepsilon_\beta = r_m\beta - r_j\theta \tag{5.4}$$

where α, β and θ are the position of the two actuators and of the joint respectively and r_m and r_j are, respectively, the radii of the actuator pulleys and of the joint pulley.

5.3.1 Static Response

In this section, the behavior of the system in steady state conditions is analyzed, so the nonlinear elastic model reported in eq. (5.1) is considered to evaluate the relations between the actuator positions and the joint position and stiffness, taking into account also external load torques applied to the joint.

The torques applied to the actuators by the transmission elements are:

$$\phi(\alpha,\theta) = r_m k[e^{\gamma\varepsilon_\alpha} - 1] \ , \ \varepsilon_\alpha \geq 0 \tag{5.5}$$

$$\psi(\beta,\theta) = r_m k[e^{\gamma\varepsilon_\beta} - 1] \ , \ \varepsilon_\beta \geq 0 \tag{5.6}$$

The resulting torque T applied to the joint by the transmission elements in antagonistic configuration is:

$$\varphi(\alpha,\beta,\theta) = T = \frac{r_j}{r_m}[\phi(\alpha,\theta) - \psi(\beta,\theta)] = r_j k[e^{\gamma\varepsilon_\alpha} - e^{\gamma\varepsilon_\beta}] \tag{5.7}$$

In the derivation of this expression, both the weight and the inertia terms due to the mass of the transmission elements are neglected because they are very small with respect to the involved forces and because we are here interested to study the static behavior of the system, obtaining in this way an algebraic expression of the torques applied both to the actuators and to the joint with respect to their positions. Deriving the eq. (5.7) with respect to θ, the joint stiffness equation is obtained:

$$\frac{\partial\varphi(\alpha,\beta,\theta)}{\partial\theta} = S = k\gamma r_j^2[e^{\gamma\varepsilon_\alpha} + e^{\gamma\varepsilon_\beta}] \tag{5.8}$$

From eq. (5.8) it is possible to note that the joint stiffness depends, besides on the actuator positions, also on the joint position. In [42] the use of transmission elements with quadratic compression-force relation allows to obtain an expression of the joint stiffness that is independent from the joint position.

Solving the eq. (5.8) and (5.7) with respect to α and β, it is possible to find the inverse model that gives the value of the actuator positions given the actual joint torque and the actual joint stiffness:

$$\alpha, \beta = \mp\frac{r_j}{r_m}\theta + \frac{1}{\gamma r_m}\ln\frac{S \pm \gamma r_j T_l}{2\gamma r_j^2 k} = \mp\frac{r_j}{r_m}\theta + \frac{1}{\gamma r_m}\ln\frac{S \pm \gamma r_j T_l}{S_0} \tag{5.9}$$

Figure 5.3: Scheme of the antagonistic actuated joint.

where T_l is and external disturbance torque (the load torque) applied to the joint and $S_0 = 2\gamma r_j^2 k$ is an important design parameter that represent the minimum value of the joint stiffness that it is possible to achieve, corresponding to the condition (see eq. (5.8)):

$$S|_{\varepsilon_\alpha=0,\varepsilon_\beta=0} = 2k\gamma r_j^2 = S_0 \tag{5.10}$$

From the eq. (5.3), (5.4) and (5.9) and from the conditions in the eq. (5.5) and (5.6), a condition on the minimum value of the joint stiffness value can be found:

$$r_m\{\alpha,\beta\} \pm r_j\theta = \varepsilon_{\alpha,\beta} = \frac{1}{\gamma}\ln\frac{S \pm \gamma r_j T_l}{S_0} \geq 0 \;\Rightarrow\; S \pm \gamma r_j T_l \geq S_0 \;\Rightarrow\; S \geq S_0 + \gamma r_j |T_l| \tag{5.11}$$

Other conditions on the actuator positions that can be found starting from the same equations are:

$$\varepsilon_{\alpha,\beta} \geq 0 \;\Rightarrow\; \alpha,\beta \geq \mp\frac{r_j}{r_m}\theta \tag{5.12}$$

From the eq. (5.5), (5.6) and (5.7), considering the minimum values of the transmission elements compression ε_α and ε_β needed to compensate for an external disturbance torque T_l one can find:

$$T_l|_{\min\varepsilon} = \begin{cases} r_j k[e^{\gamma\varepsilon_\alpha} - 1], & T_l > 0, \;\; \varepsilon_\beta = 0 \\ -r_j k[e^{\gamma\varepsilon_\beta} - 1], & T_l < 0, \;\; \varepsilon_\alpha = 0 \end{cases} \tag{5.13}$$

or, in other words:

$$T_l|_{\min\varepsilon} = \begin{cases} \phi(\alpha,\theta), & T_l > 0 \\ -\psi(\beta,\theta), & T_l < 0 \end{cases} \tag{5.14}$$

From the eq. (5.13) and (5.14) it is possible to note that the system behaves as if just one transmission element is working and the other is completely detached from the system. In this conditions, the joint stiffness assumes the minimum achievable value:

$$\min S = \left.\frac{d|T_l|}{d\theta}\right|_{\min \varepsilon} = \gamma k r_j^2 e^{\gamma \varepsilon} = \gamma k r_j^2 [+1 - 1 + e^{\gamma \varepsilon}] = \frac{1}{2} S_0 + |T_l|\big|_{\min \varepsilon} \qquad (5.15)$$

Now, evaluating the expression of the joint stiffness (5.8) for small but positive displacements one can obtain:

$$\min S = k\gamma r_j^2 [e^{\gamma \varepsilon_\alpha} + e^{\gamma \varepsilon_\beta}] \simeq 2k\gamma r_j^2 \quad (\,=\, S_0\,) \qquad (5.16)$$

Comparing the eq. (5.15) with the eq. (5.16), it is possible to note that the system presents a discontinuity in the joint stiffness when the compression of one of the two transmission elements goes to zero ($\varepsilon_{\alpha,\beta} \to 0$). This is caused by the unilateral characteristic of the transmission elements, see eq. (5.5) and (5.6). To avoid this conditions, a suitable preload of the transmission elements must be guaranteed adding a threshold value S_T to the desired joint stiffness. Moreover, the value of the desired joint stiffness must be chosen according to the condition in the eq. (5.11) to avoid cable slack. If the value of the external load torque is known, this information can be included in the design of the joint stiffness setpoint, otherwise a suitable evaluation of the load torque is necessary.

5.3.2 Dynamic Response

To verify how the joint dynamics is influenced by the programmed joint stiffness S, the visco-elastic transmission elements model eq. (5.2) is adopted. Considering an external load torque T_l, the dynamic model of the joint can be written as:

$$J_j \ddot{\theta} + b_j \dot{\theta} + \varphi(\alpha, \beta, \theta) + r_j b \left(\dot{\varepsilon}_\alpha e^{\gamma \varepsilon_\alpha} - \dot{\varepsilon}_\beta e^{\gamma \varepsilon_\beta}\right) = T_l \qquad (5.17)$$

Fixing the value of α and β, considering in this way as variable only the parameter θ, one can write:

$$\dot{\varepsilon}_{\alpha,\beta} = \pm r_j \dot{\theta} \qquad (5.18)$$

and then:

$$J_j \ddot{\theta} + \left[b_j + r_j^2 b \left(e^{\gamma \varepsilon_\alpha} + e^{\gamma \varepsilon_\beta}\right)\right] \dot{\theta} + \varphi(\alpha, \beta, \theta) = T_l \qquad (5.19)$$

or, introducing the stiffness expression:

$$J_j \ddot{\theta} + \left[b_j + \frac{b}{k\gamma} S\right] \dot{\theta} + \varphi(\alpha, \beta, \theta) = T_l \qquad (5.20)$$

The dynamic model of the joint is then linearized with respect to the joint state variables $x = [\theta \ \dot{\theta}]^T$ obtaining the state space model:

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{J_j}\left[S + b\gamma r_j^3 \left(e^{\gamma \varepsilon_\alpha} - e^{\gamma \varepsilon_\beta}\right)\dot{\theta}\right] & -\frac{1}{J_j}\left[b_j + \frac{b}{k\gamma} S\right] \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J_j} \end{bmatrix} T_l \quad (5.21)$$

Figure 5.4: Response of the antagonistic actuated joint to a step joint position variation for different values of the commanded stiffness.

It is important to note that the term $b\gamma r_j^3 (e^{\gamma\epsilon_\alpha} - e^{\gamma\epsilon_\beta})$ is always zero in the equilibrium condition for null input. Considering the neighborhood of the equilibrium point $x_{eq} = [\theta_{eq} \ 0]^T$, the linearized dynamic system becomes:

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{S}{J_j} & -\frac{b_{eq}}{J_j} \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J_j} \end{bmatrix} T_l \qquad (5.22)$$

where $b_{eq} = b_j + \frac{b}{k\gamma}S$. The transfer function from the load torque T_l to the output position θ is:

$$G(s) = \frac{\theta(s)}{T_l(s)} = \frac{\frac{1}{J_j}}{s^2 + \frac{b_{eq}}{J_j}s + \frac{S}{J_j}} \qquad (5.23)$$

from which it is possible to define the natural frequency ω_n and the damping factor δ:

$$\omega_n = \sqrt{\frac{S}{J_j}}, \ \ \delta = \frac{b_{eq}}{2\sqrt{J_j S}} = \frac{b_j + \frac{b}{k\gamma}S}{2\sqrt{J_j S}}$$

It is important to note that, increasing the joint stiffness, both the natural frequency and the damping factor increase. The maximum percent over-elongation:

$$E_\% = 100 \, e^{-\frac{\delta\pi}{\sqrt{1-\delta^2}}} = 100 \, e^{-\frac{\pi b_{eq}}{\sqrt{4J_j S - b_{eq}^2}}}$$

goes to zero for $\delta \to 1$. Neglecting the joint damping b_j, the minimum damping factor of the transmission system $\delta_{min} = \frac{b}{2k\gamma}\sqrt{\frac{S_0}{J_j}}$ can be then suitably chosen designing the

minimum stiffness parameter S_0. Another important thing to note is that, if the damping term b introduced by the transmission elements is neglected, considering in this way the static nonlinear elastic model eq. (5.1), the damping factor $\delta \rightarrow 0$ if $S \rightarrow \infty$, while if the visco-elastic model eq. (5.2) is considered introducing b, the damping factor $\delta \rightarrow \infty$ if $S \rightarrow \infty$. This fact shows as the nonlinear damping contribution given by the transmission elements increase the stability of the system with respect to the case of pure elastic force/compression relation. In Fig. 5.4 the experimental response of the system to a step joint position variation of amplitude 0.1 *rad* for different value of the commanded mechanical stiffness S is reported to show the effect of the damping introduced by the transmission elements. These results confirm the effectiveness of the previous analysis. The controller adopted during these experiments is described in sec. 5.4.2.

5.4 Actuator-Level Stiffness/Position Control

In this section, the simultaneous stiffness/position control of the antagonistic actuated joint is taken into account. With the aim of design a very simple controller, the static relation between the actuator positions and the joint position and stiffness eq. (5.9) is exploited, and the the control objective is fulfilled controlling the position of the actuators. This actuator-level controller of the joint position and stiffness is suitable simple and light mechanical structures, like robotic fingers. In these cases, this control strategy allows to obtain a good dynamic behavior with a very simple controller.

Typically, to reduce the weight, the size and the power consumption of robotic devices, small and light electrical (DC) motors with very high reduction ratio are used, resulting in a non backdrivable actuation. For this reason, a very simple stiffness/position controller for non-backdrivable actuators is presented.

Since in the DLR's antagonistic actuated joint direct drive synchronous motors are used, also the problem of the joint stiffness and position control with backdrivable actuation has been analyzed and some experimental results are reported.

5.4.1 Non-Backdrivable Actuators

Non-backdrivable actuators allow to neglect the coupling between the actuator position and the torque applied to the actuator by the transmission element expressed by the eq. (5.5) and (5.6). Hence, the dynamic equation of the actuators are:

$$J_m\ddot{\alpha} + b_m\dot{\alpha} = \tau_{c_\alpha} - f_{f_\alpha} \tag{5.24}$$
$$J_m\ddot{\beta} + b_m\dot{\beta} = \tau_{c_\beta} - f_{f_\beta} \tag{5.25}$$

where τ_{c_α}, τ_{c_β} are the control torque applied to the actuators and f_{f_α}, f_{f_β} are the actuator friction forces.

A possible, simple, approach to the control of the joint position and stiffness can be

implemented with the static model inversion + actuators PID position control:

$$\tau_{c_{\alpha,\beta}}(s) = \left(K_p + \frac{K_i}{s} + \frac{K_d\,s}{1+N\,s}\right)\Delta_{\alpha,\beta}(s) \tag{5.26}$$

$$\Delta_\alpha(s) = (\alpha_d - \alpha(s)) \tag{5.27}$$

$$\Delta_\beta(s) = (\beta_d - \beta(s)) \tag{5.28}$$

$$\alpha_d = -\frac{r_j}{r_m}\theta_d + \frac{1}{\gamma r_m}\ln\frac{S_{d2} + \gamma r_j T_l}{S_0} \tag{5.29}$$

$$\beta_d = \frac{r_j}{r_m}\theta_d + \frac{1}{\gamma r_m}\ln\frac{S_{d2} - \gamma r_j T_l}{S_0} \tag{5.30}$$

$$S_{d2} = \begin{cases} S_d, & S_d \geq S_0 + S_T + \gamma r_j|T_l| \\ S_0 + S_T + \gamma r_j|T_l|, & S_d < S_0 + S_T + \gamma r_j|T_l| \end{cases} \tag{5.31}$$

where θ_d and S_d are the joint position and the joint stiffness setpoint respectively. The term S_T is a threshold stiffness value introduced to avoid the condition $S = S_0$ in which the system shows a discontinuity in the stiffness as shown in the previous section, see eq. (5.15) and (5.16). K_p, K_i and K_d are the proportional, the integral and the derivative gain of the PID controller respectively while N is introduced to reduce the effect of the derivative term for high frequency signals, such as noise. If the friction forces can be neglected, the control law (5.26) ensure that the position of the actuators approach asymptotically to the desired value ($\alpha, \beta \rightarrow \alpha_d, \beta_d$ as $t \rightarrow \infty$). Otherwise, the integral action can compensate for the static effects of friction but this approach can introduce also limit cycles due to the nonlinear behavior of the friction. In this latter case, a suitable friction compensation law can be considered.

If the external load is known, just the actuator position measures are necessary to control the system. In the cause of unknown external disturbance torque, it is important to note that, in steady-state conditions:

$$T_l = T = r_j k[e^{\gamma\epsilon_\alpha} - e^{\gamma\epsilon_\beta}] \tag{5.32}$$

This implies that the external load applied to the joint can be estimated by means of the joint and the actuator position information.

5.4.2 Backdrivable Actuators

If backdrivable actuation is used, also the torque applied by the transmission elements to the actuators must be included in the dynamic equations of the actuators:

$$J_m\ddot{\alpha} + b_m\dot{\alpha} + \phi(\alpha,\theta) = \tau_{c_\alpha} - f_{f_\alpha} \tag{5.33}$$

$$J_m\ddot{\beta} + b_m\dot{\beta} + \psi(\beta,\theta) = \tau_{c_\beta} - f_{f_\beta} \tag{5.34}$$

In this case, considering the equilibrium of the torques at the actuator side, if the integral action is set to zero ($K_i = 0$) to avoid high values of the control action due to the

Figure 5.5: Response of the actuator level controller.

integral term and neglecting friction for simplicity, the steady-state equilibrium point of the system is given by the solution of the equation:

$$r_m k[e^{\gamma \varepsilon_{\alpha,\beta}} - 1] \quad = \quad K_p \, \Delta_{\alpha,\beta} \tag{5.35}$$

In Fig. 5.5 the response of the DLR's antagonistic actuated joint is reported. In this experiment, the controller (5.26)-(5.31) is adopted with $K_i = 0$. From this plot, it is possible to note that the joint stiffness is significantly influenced by the oscillations of the joint during the transitory. An high value of the stiffness setpoint is used to highlight the steady state stiffness error due to the non null position error of the actuators.

To restore the equilibrium point to the desired actuator positions, a forward action can be added to compensate for the torque applied by the transmission elements:

$$\tau_\alpha \quad = \quad \tau_{c_\alpha} + r_m k[e^{\gamma \varepsilon_\alpha} - 1] \tag{5.36}$$
$$\tau_\beta \quad = \quad \tau_{c_\beta} + r_m k[e^{\gamma \varepsilon_\beta} - 1] \tag{5.37}$$

This approach decouples, in steady state conditions, the actuators form the transmission elements but it can introduce undesired effects due to unmodeled dynamics, as can be seen in Fig. 5.6. From this figure it is possible to note the steady state stiffness error tends to zero but the system shows a more oscillating behavior.

Another possible approach is to add to the actuator setpoint a suitable term such that the equilibrium point given from the eq. (5.35) has the desired value. Form the eq. (5.3),

Figure 5.6: Response of actuator level controller with feedforward action.

(5.4), (5.35) and (5.9) one can obtain:

$$\Delta_{\alpha,\beta} = \frac{r_m k}{K_p} \left[\frac{S_{d2} \pm \gamma r_j T_l}{S_0} - 1 \right] \tag{5.38}$$

From this equation the new values of the actuator position setpoint is obtained:

$$(\alpha,\beta)_{d2} = (\alpha,\beta)_d + \frac{r_m k}{K_p} \left[\frac{S_{d2} \pm \gamma r_j T_l}{S_0} - 1 \right] =$$

$$= \mp \frac{r_j}{r_m}\theta_d + \frac{1}{\gamma r_m} \ln \frac{S_{d2} \pm \gamma r_j T_l}{S_0} + \frac{r_m k}{K_p} \left[\frac{S_{d2} \pm \gamma r_j T_l}{S_0} - 1 \right] \tag{5.39}$$

Fig. 5.7 shows the response of the system with the setpoint compensation introduced in eq. (5.39). From this plot it is possible to note that, in this case, the steady state stiffness error tends to zero and the oscillations of the system are similar to the case of simple PD actuator position control (see Fig. 5.5).

It is important to note that, also in this case, the joint position information is required if the external load torque is not know.

Figure 5.7: Response of the actuator level controller with setpoint compensation.

5.5 Feedback Linearization

In the chapter 4 the problem of system linearization via static state feedback of an N d.o.f. robotic arm with antagonistic actuated joints is analyzed considering transmission elements with static nonlinear force/compression relation, such as eq. (5.1). If the visco-elastic model eq. (5.2) is considered, the problem of full static feedback linearization of the system cannot be achieved, as shown in section 5.5.1. In this case, a dynamic state feedback controller must be adopted to achieve the full system linearization, see section 5.5.2.

From the general dynamic model of the antagonistic actuated robot eq. (4.1)-(4.3), the dynamics the overall system, considering the visco-elastic behavior of the transmission elements described by the eq. (5.2) can be written as:

$$J_j\ddot{\theta} + b_j\dot{\theta} + \varphi(\alpha,\beta,\theta) + r_j b\left[\dot{\varepsilon}_\alpha e^{\gamma\varepsilon_\alpha} - \dot{\varepsilon}_\beta e^{\gamma\varepsilon_\beta}\right] = T_l \tag{5.40}$$

$$J_m\ddot{\alpha} + b_m\dot{\alpha} + \phi(\alpha,\theta) + r_m b\dot{\varepsilon}_\alpha e^{\gamma\varepsilon_\alpha} = \tau_\alpha \tag{5.41}$$

$$J_m\ddot{\beta} + b_m\dot{\beta} + \psi(\beta,\theta) + r_m b\dot{\varepsilon}_\beta e^{\gamma\varepsilon_\beta} = \tau_\beta \tag{5.42}$$

where θ is the joint positions, α and β are the actuator positions, J_j is the inertia of the joint, J_m is the inertia of the actuators, b_j and b_m are the viscous friction coefficients of the joint and of the actuators respectively, $\varphi(\alpha,\beta,\theta)$ is the combined effect of the positions of the actuators on the joint, τ_α and τ_β are the torques commanded to the actuators. In

the dynamical model eq. (5.40)-(5.42), both the friction torques acting on the joint and on the actuators are neglected together with the inertia of the transmission elements for simplicity. The actuators friction can be compensated with a low level controller while the joint friction can be included in the external load torque. The transmission elements inertia is neglected because its value is many times smaller than the involved transmission forces.

The control input of the system, the disturbance input and the vector of output information are:

$$u = \begin{bmatrix} \tau_\alpha & \tau_\beta \end{bmatrix}^T, \qquad w = T_l \ , \qquad y = \begin{bmatrix} \theta & \alpha & \beta \end{bmatrix}^T \tag{5.43}$$

$$x \ = \ \begin{bmatrix} \theta & \dot\theta & \alpha & \dot\alpha & \beta & \dot\beta \end{bmatrix}^T \tag{5.44}$$

The whole state information can be reconstructed from the output y by means of several velocity estimation techniques, i.e. filtered position derivation, as we have done during the experiments, so we suppose in the sequel that the state of the system is known.

Since we are interested to control the both the position and the stiffness of the joint, it is useful to define a new output vector that contains directly these information:

$$y_c = \begin{bmatrix} \theta & S - S_0 \end{bmatrix}^T \tag{5.45}$$

where S is the mechanical stiffness of the joints that can be expressed considering the derivative of the left side term of eq.(5.40) with respect to θ:

$$S \ = \ \gamma r_j^2 \left[k(e^{\gamma\varepsilon_\alpha} + e^{\gamma\varepsilon_\beta}) + b(\dot\varepsilon_\alpha e^{\gamma\varepsilon_\alpha} + \dot\varepsilon_\beta e^{\gamma\varepsilon_\beta}) \right] \tag{5.46}$$

$$S_0 \ = \ 2\gamma r_j^2 k \tag{5.47}$$

$$\dot\varepsilon_{\alpha,\beta} \ = \ r_m\{\dot\alpha, \dot\beta\} \pm r_j\dot\theta \tag{5.48}$$

S_0 is the stiffness value computed in the equilibrium condition of the system. The set of the equilibrium points of the system (5.40)-(5.42) is given by:

$$x_{eq} = \left\{ x \in \mathbb{R}^6 \ : \ \varepsilon_{\alpha,\beta} = 0 \right\} \tag{5.49}$$

Also in this case, as already discussed in the sections 5.3.1 and 5.4.1, a suitable threshold value must be added to the equilibrium stiffness S_0 to ensure an adequate cable pretension and to avoid the discontinuity of the joint stiffness. From eq. (5.5), (5.6) and (5.8), and evaluating the equilibrium conditions of the system (5.40)-(5.42), it is possible to find the constant value of the actuator torques that gives to the joint the desired stiffness S_T:

$$\tau_{S_T} = \frac{r_m}{2\gamma r_j^2} (S_T - S_0) \tag{5.50}$$

The torque τ_{S_T} is then added to both the actuators to modify the equilibrium condition of the system. Note that this implies a change of the equilibrium stiffness and actuator positions but has no influence on the equilibrium position of the joint. The new equilibrium

stiffness of the system can be then considered S_T, and the equilibrium actuator positions can be found using eq. 5.9. In the sequel, the analysis of the system is then carried out considering the equilibrium conditions for null input without loss of generality.

We suppose that the two actuators are independent (no coupling between the movements of the actuators). From the eq. (5.46) it is possible to note that, in general, while the stiffness of the joint are difficult to measure, a suitable knowledge of the relation between the compression of the transmission elements and the torque applied to the joint makes it possible to derive the joint stiffness S from the state of the system.

Considering the new output vector y_c, the input u and the state vector x, the dynamic model of the antagonistic actuated joint described by eq. (5.40)-(5.42) can be rewritten in the state space form:

$$\dot{x} = f(x) + g(x)u + d(x)w \tag{5.51}$$
$$y_c = h(x) \tag{5.52}$$

where $x \in \mathbb{R}^6$, $u, y_c \in \mathbb{R}^2$ and $w \in \mathbb{R}$. In particular, for the model of the antagonistic actuated joint:

$$f(x) = \begin{bmatrix} \dot{\theta} \\ J_j^{-1}\left\{-b_{j\theta}\dot{\theta} - b_{j\alpha}\dot{\alpha} - b_{j\beta}\dot{\beta} - \varphi(\alpha,\beta,\theta)\right\} \\ \dot{\alpha} \\ J_m^{-1}\left\{-b_{j\alpha}\dot{\theta} - b_{m\alpha}\dot{\alpha} - \phi(\alpha,\theta)\right\} \\ \dot{\beta} \\ J_m^{-1}\left\{-b_{j\beta}\dot{\theta} - b_{m\beta}\dot{\beta} - \psi(\beta,\theta)\right\} \end{bmatrix} \tag{5.53}$$

$$g(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ J_m^{-1} & 0 \\ 0 & 0 \\ 0 & J_m^{-1} \end{bmatrix}, \quad d(x) = \begin{bmatrix} 0 \\ J_j^{-1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad h(x) = \begin{bmatrix} \theta \\ S - S_0 \end{bmatrix} \tag{5.54}$$

where:

$$b_{j\theta} = b_j + r_j^2 b\left(e^{\gamma\varepsilon\alpha} + e^{\gamma\varepsilon\beta}\right) \tag{5.55}$$
$$b_{j\alpha} = r_j r_m b e^{\gamma\varepsilon\alpha} \tag{5.56}$$
$$b_{j\beta} = -r_j r_m b e^{\gamma\varepsilon\beta} \tag{5.57}$$
$$b_{m\alpha} = b_m + r_m^2 b e^{\gamma\varepsilon\alpha} \tag{5.58}$$
$$b_{m\beta} = b_m + r_m^2 b e^{\gamma\varepsilon\beta} \tag{5.59}$$

Considering the control input u and the output information y_c, the eq. (5.51)-(5.54) define a square nonlinear system that can be linearized via output feedback if suitable conditions are satisfied [53, 46, 47].

Starting from the analysis on the feedback linearization of antagonistic actuated robots carried out in the previous chapter, we have to check, first of all, some conditions on the relative degrees of the output information. In this preliminary analysis only the controllable input u of the system is taken into account while the external torque T_l applied to the joint is neglected. In this case, for the output θ it is possible to write:

$$L_{g_{(\alpha,\beta)}} L_f^i h_\theta(x) \;=\; 0 \quad, \quad i = 0,1 \tag{5.60}$$

$$L_{g_\alpha} L_f^2 h_\theta(x) \;=\; -\frac{b_{j\alpha}}{J_m J_j} \tag{5.61}$$

$$L_{g_\beta} L_f^2 h_\theta(x) \;=\; \frac{b_{j\beta}}{J_m J_j} \tag{5.62}$$

while for the output S:

$$L_{g_\alpha} h_S(x) \;=\; \frac{\gamma r_j^2 r_m b e^{\gamma \varepsilon_\alpha}}{J_m} \tag{5.63}$$

$$L_{g_\beta} h_S(x) \;=\; \frac{\gamma r_j^2 r_m b e^{\gamma \varepsilon_\beta}}{J_m} \tag{5.64}$$

where L_{g_α} and L_{g_β} denote the restriction of the Lie derivative respectively to the τ_α and τ_β component of the input vector while $L_{g_{(\alpha,\beta)}}$ denotes both the cases above mentioned.

If the disturbance input, the external torque applied to the joint, is considered, the relative degrees of the output θ can be defined as:

$$L_d h_\theta(x) \;=\; 0 \tag{5.65}$$

$$L_d L_f h_\theta(x) \;=\; \frac{1}{J_j} \tag{5.66}$$

while for the output S:

$$L_d h_S(x) \;=\; \frac{\gamma r_j^3 b (e^{\gamma \varepsilon_\alpha} - e^{\gamma \varepsilon_\beta})}{J_j} \tag{5.67}$$

From this considerations it is possible to state that the joint position cannot be decoupled from the external torque while it is possible to annihilate the effects of the external torque on the joint stiffness if a measure (or a suitable estimation) of the external torque is available, since the relative degree of the stiffness is the same both for the control input and for the disturbance. The external torque T_l will be neglected in the sequel to simplify the analysis. The problem of feedback linearization under the effects of an external torque will be object of future research.

5.5.1   Static Feedback Linearization

From the eq. (5.60)-(5.64) we can state that, considering the controllable input u, the vector relative degree of θ is 3 while the one of S is 1. The sum of the vector relative

degrees of the output is then not equal to the dimension of the state of the system, so the conditions for the solution of the full feedback linearization problem are not satisfied [46]. Anyway, the system can be partially linearized via static feedback if it is possible to define a nonsingular coordinates transformation from the space of the original state variables to the state space of the partially linearized one [54]. To solve this problem, since the additional entries of the coordinates transformation (the ones that must be added to make the coordinates transformation nonsingular) can be arbitrary chosen, it is convenient, if possible, to make the nonlinear part of the new system independent from the inputs.

The state of the partially linearized system z can be split into the linear part:

$$z_l = \begin{bmatrix} z_1 & z_2 & z_3 & z_4 \end{bmatrix}^T = \begin{bmatrix} \theta & \dot{\theta} & \ddot{\theta} & S - S_0 \end{bmatrix}^T \tag{5.68}$$

and its nonlinear part:

$$z_n = \begin{bmatrix} z_5 & z_6 \end{bmatrix}^T \tag{5.69}$$

where z_5 and z_6 will be chosen in suitable way as shown in the sequel. Hence, the overall system state in the new coordinate system is:

$$z = \begin{bmatrix} z_l \\ z_n \end{bmatrix} \tag{5.70}$$

The dynamics of the partially linearized system can be then written as:

$$\dot{z}_l = Az_l + Bu_l \tag{5.71}$$
$$\dot{z}_n = \eta(z_l, z_n, u_l) \tag{5.72}$$
$$y_c = Cz_l \tag{5.73}$$

$$u_l = \begin{bmatrix} L_f^3 h_\theta(\Phi^{-1}(z)) \\ L_f h_S(\Phi^{-1}(z)) \end{bmatrix} + Q(\Phi^{-1}(z))u \tag{5.74}$$

in which $Q(\Phi^{-1}(z))$ is the so-called decoupling matrix and the matrices A, B and C are in the Brunowsky canonical form

$$A = \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array} \right], \quad B = \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ \hline 0 & 1 \end{array} \right], \quad C = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \tag{5.75}$$

where eq. (5.72) express the nonlinear part of the system and $z = \Phi(x)$ represents the coordinates transformation between the original and the partially linearized system:

$$\Phi(x) = \begin{bmatrix} h_\theta(x) & L_f h_\theta(x) & L_f^2 h_\theta(x) & h_S(x) & \Phi_5(x) & \Phi_6(x) \end{bmatrix}^T \tag{5.76}$$

where obviously $L_f h_\theta(x) = \dot{\theta}$, $L_f^2 h_\theta(x) = \ddot{\theta}$ can be expressed in terms of the state vector x by using the second element of the eq. (5.53), while $\Phi_5(x)$ and $\Phi_6(x)$ must be chosen in such a way to make the coordinate transformation $\Phi(x)$ nonsingular.

It is important to note that the nonlinear residual part of the system $\eta(z_l, z_n, u_l)$ is unobservable from the outputs, as can be seen looking at the eq. (5.73). In particular the stability of the zero dynamics

$$\dot{z}_n = \eta(0, z_n, 0) \tag{5.77}$$

is a necessary condition for the controllability with bounded internal state of the overall system [46].

Looking at the first four rows of $\frac{d\Phi(x)}{dx}$:

$$\frac{d\Phi}{dx} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \star & \star & k_1(\dot{\varepsilon}_\alpha)e^{\gamma\varepsilon_\alpha} & k_2 e^{\gamma\varepsilon_\alpha} & -k_1(\dot{\varepsilon}_\beta)e^{\gamma\varepsilon_\beta} & -k_2 e^{\gamma\varepsilon_\beta} \\ \star & \star & k_3(\dot{\varepsilon}_\alpha)e^{\gamma\varepsilon_\alpha} & k_4 e^{\gamma\varepsilon_\alpha} & k_3(\dot{\varepsilon}_\beta)e^{\gamma\varepsilon_\beta} & k_4 e^{\gamma\varepsilon_\beta} \end{bmatrix}$$

where

$$\begin{aligned} k_1(\xi) &= \frac{\gamma r_m r_j(k + b\xi)}{J_j} \\ k_2 &= \frac{r_m r_j b}{J_j} \\ k_3(\xi) &= \gamma^2 r_m r_j^2(k + b\xi) \\ k_4 &= \gamma r_m r_j^2 b \end{aligned}$$

Choosing $z_5 = \dot{\alpha}$ and $z_6 = \dot{\beta}$, the coordinate transformation $\Phi(x)$ becomes nonsingular:

$$\frac{d\Phi}{dx} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \star & \star & k_1(\dot{\varepsilon}_\alpha)e^{\gamma\varepsilon_\alpha} & k_2 e^{\gamma\varepsilon_\alpha} & -k_1(\dot{\varepsilon}_\beta)e^{\gamma\varepsilon_\beta} & -k_2 e^{\gamma\varepsilon_\beta} \\ \star & \star & k_3(\dot{\varepsilon}_\alpha)e^{\gamma\varepsilon_\alpha} & k_4 e^{\gamma\varepsilon_\alpha} & k_3(\dot{\varepsilon}_\beta)e^{\gamma\varepsilon_\beta} & k_4 e^{\gamma\varepsilon_\beta} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{5.78}$$

From this last expression, it is important to note that the state space transformation $\Phi(x)$ is nonsingular in the whole state space, and in particular in the origin. The state space transformation $\Phi(x)$ is then:

$$\begin{aligned} z_1 &= \theta \\ z_2 &= \dot{\theta} \\ z_3 &= J_j^{-1}\left[-b_{j\theta}\dot{\theta} - b_{j\alpha}\dot{\alpha} - b_{j\beta}\dot{\beta} - \varphi(\alpha, \beta, \theta)\right] \\ z_4 &= \gamma r_j^2\left[k(e^{\gamma\varepsilon_\alpha} + e^{\gamma\varepsilon_\beta} - 2) + b(\dot{\varepsilon}_\alpha e^{\gamma\varepsilon_\alpha} + \dot{\varepsilon}_\beta e^{\gamma\varepsilon_\beta})\right] \\ z_5 &= \dot{\alpha} \\ z_6 &= \dot{\beta} \end{aligned}$$

The state space transformation form the original to the linearized system can be then expressed only by means of the state space information of the original system. The dynamics equations that describe the nonlinear part of the system are then:

$$
\dot{z}_5 = J_m^{-1} \left[-b_{j\alpha}\dot{\theta} - b_{m\alpha}\dot{\alpha} - \phi(\alpha,\theta) + \tau_\alpha \right]
$$
$$
\dot{z}_6 = J_m^{-1} \left[-b_{j\beta}\dot{\theta} - b_{m\beta}\dot{\beta} - \psi(\beta,\theta) + \tau_\beta \right]
$$

Even if the nonlinear residual part of the system depends on the input, it is important to note that the zero dynamics

$$
\begin{bmatrix} \dot{z}_5 \\ \dot{z}_6 \end{bmatrix} = \begin{bmatrix} -\dfrac{b_{m\alpha}}{J_m} & 0 \\ 0 & -\dfrac{b_{m\beta}}{J_m} \end{bmatrix} \begin{bmatrix} z_5 \\ z_6 \end{bmatrix} \tag{5.79}
$$

is asymptotically stable in the whole state space, so the system can be controlled with bounded internal state.

To define the input of the linearized system, see eq. (5.74), we have to define first the nonlinear terms $L_f^3 h_\theta(x)$ and $L_f h_S(x)$ and the decoupling matrix $Q(x)$.

$$
L_f^3 h_\theta(x) = J_j^{-1} \Big\{ -b_j\dddot{\theta} - b_m\ddot{\alpha} - b_m\ddot{\beta} - r_j b(\ddot{\varepsilon}_\alpha e^{\gamma\varepsilon_\alpha} - \ddot{\varepsilon}_\beta e^{\gamma\varepsilon_\beta}) +
$$
$$
-\gamma r_j k(\dot{\varepsilon}_\alpha e^{\gamma\varepsilon_\alpha} - \dot{\varepsilon}_\beta e^{\gamma\varepsilon_\beta}) - \gamma r_j b(\dot{\varepsilon}_\alpha^2 e^{\gamma\varepsilon_\alpha} - \dot{\varepsilon}_\beta^2 e^{\gamma\varepsilon_\beta}) \Big\} \tag{5.80}
$$
$$
L_f h_S(x) = \gamma r_j^2 \Big\{ k\gamma(\dot{\varepsilon}_\alpha e^{\gamma\varepsilon_\alpha} + \dot{\varepsilon}_\beta e^{\gamma\varepsilon_\beta}) + b\left[(\gamma\dot{\varepsilon}_\alpha^2 + \ddot{\varepsilon}_\alpha)e^{\gamma\varepsilon_\alpha} + (\gamma\dot{\varepsilon}_\beta^2 + \ddot{\varepsilon}_\beta)e^{\gamma\varepsilon_\beta} \right] \Big\} \tag{5.81}
$$

where

$$
\ddot{\varepsilon}_{\alpha,\beta} = r_m\{\ddot{\alpha},\ddot{\beta}\} \pm r_j\ddot{\theta} \tag{5.82}
$$

It is important to note that the joint and the actuator accelerations $\ddot{\theta}$, $\ddot{\alpha}$ and $\ddot{\beta}$ in eq. (5.80)-(5.82) can be expressed in terms of the state of the system x by using eq. (5.53). Concerned to the decoupling matrix $Q(x)$, considering eq. (5.61)-(5.64):

$$
Q(x) = \frac{r_m r_j b}{J_m} \begin{bmatrix} -\dfrac{e^{\gamma\varepsilon_\alpha}}{J_j} & \dfrac{e^{\gamma\varepsilon_\beta}}{J_j} \\ \gamma r_j e^{\gamma\varepsilon_\alpha} & \gamma r_j e^{\gamma\varepsilon_\beta} \end{bmatrix} \tag{5.83}
$$

or, in equivalent form:

$$
Q(x) = \frac{1}{J_m} \begin{bmatrix} -k_2 e^{\gamma\varepsilon_\alpha} & k_2 e^{\gamma\varepsilon_\beta} \\ k_4 e^{\gamma\varepsilon_\alpha} & k_4 e^{\gamma\varepsilon_\beta} \end{bmatrix} \tag{5.84}
$$

The resulting decoupling matrix is then nonsingular in the overall state space of the system. The control law that allows to obtain the linearized system is then:

$$
u = Q^{-1}(x) \left(\begin{bmatrix} -L_f^3 h_\theta(x) \\ -L_f h_S(x) \end{bmatrix} + \begin{bmatrix} v_\theta \\ v_S \end{bmatrix} \right) \tag{5.85}
$$

where $v = [v_\theta \ v_S]^T$ is the new input of the system. Note that this control law is static, since it can be be computed directly from the system state information x and from the new reference input v. The dynamics of the system eq. (5.40)-(5.42) with the control law eq. (5.85) is, in the z coordinates:

$$\dot{z}_l = Az_l + Bv \tag{5.86}$$
$$\dot{z}_n = \eta(z_l, z_n, v) \tag{5.87}$$
$$y_c = Cz_l \tag{5.88}$$

By recalling the definition of the matrices A, B and C in eq. (5.75), it is possible to write:

$$\begin{bmatrix} \theta^{[3]} \\ \dot{S} \end{bmatrix} = \begin{bmatrix} v_\theta \\ v_S \end{bmatrix} \tag{5.89}$$

An important point to note is that if the damping coefficient of the transmission elements b tends to zero (considering also the case in which b can be neglected), the control law eq. (5.85) becomes ill conditioned and the zero dynamics eq. (5.79) is governed by the damping coefficient of the actuators b_m.

Due to the structure of the matrices A, B and C, the system described by eq. (5.86) and (5.88) is completely controllable and observable, while eq. (5.87) represents its unobservable part. This implies that the state z_l can be estimated by means of an asymptotic state observer and of the change of coordinates $\Phi(x)$ or, since the position of each rigid body is directly measurable, the velocities can be estimated in many ways e.g. by means of state variable filters or adaptive windowing algorithms [49].

For the control of the partially linearized system, an outer control loop can be used to solve the problem of simultaneous decoupled stiffness and position trajectory tracking. From eq.(5.89), applying the control laws:

$$v_\theta = \theta_d^{[3]} + K_{2_\theta}[\ddot{\theta}_d - L_f^2 h_\theta(x)] + K_{1_\theta}[\dot{\theta}_d - L_f h_\theta(x)] + K_{0_\theta}[\theta_d - h_\theta(x)] \tag{5.90}$$
$$v_S = \dot{S}_d + K_{0_S}[S_d - h_S(x)] \tag{5.91}$$

where K_{2_θ}, K_{1_θ}, K_{0_θ} and K_{0_S} are such that

$$\lambda^3 + \lambda^2 K_{2_\theta} + \lambda K_{1_\theta} + K_{0_\theta} = 0 \tag{5.92}$$
$$\lambda + K_{0_S} = 0 \tag{5.93}$$

are Hurwitz polynomials, the convergence to zero of the tracking error is ensured. If the desired trajectories θ_d is continuous together with its derivatives up to 3th and S_d is continuous together with its fist derivative, also the asymptotic trajectory tracking is achieved. The control law in eq. (5.90),(5.91) is equivalent to a static state feedback plus feedforward in the state space of the linearized system:

$$v = v_d + K(z_d - \Phi_{(\theta,S)}(x)) \tag{5.94}$$

$$v_d = \begin{bmatrix} \theta_d^{[3]} & \dot{S}_d \end{bmatrix}^T \quad , \quad z_d = \begin{bmatrix} \theta_d & \dot{\theta}_d & \ddot{\theta}_d & S_d \end{bmatrix}^T \tag{5.95}$$

$$K = \mathrm{diag}[K_{0_\theta}\ K_{1_\theta}\ K_{2_\theta}\ K_{0_S}] \tag{5.96}$$

and $\Phi_{(\theta,S)}(x)$ stay for the restriction of the coordinate transformation to the state component z_l:

$$\Phi_{(\theta,S)}(x) = z_l = \begin{bmatrix} \theta \\ \dot{\theta} \\ J_j^{-1}\left\{-b_{j_\theta}\dot{\theta} - b_{j_\alpha}\dot{\alpha} - b_{j_\beta}\dot{\beta} - \varphi(\alpha,\beta,\theta)\right\} \\ \gamma r_j^2 \left\{k(e^{\gamma\varepsilon_\alpha} + e^{\gamma\varepsilon_\beta} - 2) + b(\dot{\varepsilon}_\alpha e^{\gamma\varepsilon_\alpha} + \dot{\varepsilon}_\beta e^{\gamma\varepsilon_\beta})\right\} \end{bmatrix} \tag{5.97}$$

The matrix K can be also obtained via direct eigenvalues assignment or through the solution of the CARE equation with an opportune choice of the weight matrices.

5.5.2 Dynamic Feedback Linearization

Since the sum of the relative degrees of the inputs is not equal to the dimension of the state space of the system, to achieve full state linearization and to solve the problem of a vanishing damping of the transmission elements b, a dynamic feedback compensator must be designed.

By choosing, for the linearized system, the new state vector:

$$z = \begin{bmatrix} \theta & \dot{\theta} & \ddot{\theta} & \theta^{[3]} & S - S_0 & \dot{S} \end{bmatrix}^T \tag{5.98}$$

the dynamics of the linearized system can be then written as:

$$\dot{z} = Az + Bu_l \tag{5.99}$$
$$y_c = Cz \tag{5.100}$$

$$u_l = \begin{bmatrix} L_f^4 h_\theta(\Phi^{-1}(z)) \\ L_f^2 h_S(\Phi^{-1}(z)) \end{bmatrix} + Q(\Phi^{-1}(z))\dot{u} + R(\Phi^{-1}(z))u \tag{5.101}$$

where

$$L_f^4 h_\theta(x) = J_j^{-1}\left\{-b_j\theta^{[3]} - b_m\alpha^{[3]} - b_m\beta^{[3]} - r_j b(\varepsilon_\alpha^{[3]}e^{\gamma\varepsilon_\alpha} - \varepsilon_\beta^{[3]}e^{\gamma\varepsilon_\beta}) + \right.$$
$$- 3\gamma r_j b(\ddot{\varepsilon}_\alpha\dot{\varepsilon}_\alpha e^{\gamma\varepsilon_\alpha} - \ddot{\varepsilon}_\beta\dot{\varepsilon}_\beta e^{\gamma\varepsilon_\beta}) - \gamma r_j k(\ddot{\varepsilon}_\alpha e^{\gamma\varepsilon_\alpha} - \ddot{\varepsilon}_\beta e^{\gamma\varepsilon_\beta}) +$$
$$\left. -\gamma^2 r_j k(\dot{\varepsilon}_\alpha^2 e^{\gamma\varepsilon_\alpha} - \dot{\varepsilon}_\beta^2 e^{\gamma\varepsilon_\beta}) - \gamma^2 r_j b(\dot{\varepsilon}_\alpha^3 e^{\gamma\varepsilon_\alpha} - \dot{\varepsilon}_\beta^3 e^{\gamma\varepsilon_\beta})\right\} \tag{5.102}$$

$$L_f^2 h_S(x) = \gamma r_j^2 \left\{k\gamma^2(\dot{\varepsilon}_\alpha^2 e^{\gamma\varepsilon_\alpha} + \dot{\varepsilon}_\beta^2 e^{\gamma\varepsilon_\beta}) + k\gamma(\ddot{\varepsilon}_\alpha e^{\gamma\varepsilon_\alpha} + \ddot{\varepsilon}_\beta e^{\gamma\varepsilon_\beta}) + \right.$$
$$\left. +3b\gamma(\ddot{\varepsilon}_\alpha\dot{\varepsilon}_\alpha e^{\gamma\varepsilon_\alpha} + \ddot{\varepsilon}_\beta\dot{\varepsilon}_\beta e^{\gamma\varepsilon_\beta}) + b\gamma^2(\dot{\varepsilon}_\alpha^3 e^{\gamma\varepsilon_\alpha} + \dot{\varepsilon}_\beta^3 e^{\gamma\varepsilon_\beta}) + b(\varepsilon_\alpha^{[3]}e^{\gamma\varepsilon_\alpha} + \varepsilon_\beta^{[3]}e^{\gamma\varepsilon_\beta})\right\} \tag{5.103}$$

$$R(x) = \frac{1}{J_m} \begin{bmatrix} -k_1(\dot{\varepsilon}_\alpha)e^{\gamma\varepsilon_\alpha} & k_1(\dot{\varepsilon}_\beta)e^{\gamma\varepsilon_\beta} \\ k_3(\dot{\varepsilon}_\alpha)e^{\gamma\varepsilon_\alpha} & k_3(\dot{\varepsilon}_\beta)e^{\gamma\varepsilon_\beta} \end{bmatrix} \tag{5.104}$$

while $Q(\Phi^{-1}(z))$ is the same as in eq. (5.83) and the matrices A, B and C change in:

$$A = \left[\begin{array}{cccc|cc} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right], \quad B = \left[\begin{array}{cc} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{array}\right], \quad C = \left[\begin{array}{cccc|cc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \tag{5.105}$$

It is important to note that, also in this case, all the terms appearing in eq. (5.102) and (5.103) can be expressed as algebraic functions of the system state x.

Note that the matrix $R(x)$ is well defined even if the coefficient b vanishes. The coordinates transformation between the original and the linearized system is then:

$$\Phi(x) = \begin{bmatrix} h_\theta(x) & L_f h_\theta(x) & L_f^2 h_\theta(x) & L_f^3 h_\theta(x) & h_S(x) & L_f h_S(x) \end{bmatrix}^T \tag{5.106}$$

Now, by defining the dynamic compensation law:

$$\dot{u} = Q^{-1}(x)\left(-\begin{bmatrix} L_f^4 h_\theta(x) \\ L_f^2 h_S(x) \end{bmatrix} - R(x)u + \begin{bmatrix} v_\theta \\ v_S \end{bmatrix}\right) \tag{5.107}$$

we obtain the fully linearized system

$$\dot{z} = Az + Bv \tag{5.108}$$
$$y_c = Cz \tag{5.109}$$

that, together with eq. (5.107), give the whole dynamics of the antagonistic actuated joint plus the dynamic controller. By recalling the definition of the matrices A, B and C in eq. (5.105), it is possible to write:

$$\begin{bmatrix} \theta^{[4]} \\ \ddot{S} \end{bmatrix} = \begin{bmatrix} v_\theta \\ v_S \end{bmatrix} \tag{5.110}$$

The zero dynamics of the system is now inside the controller:

$$\dot{u} = -Q^{-1}(0)R(0)u = -\frac{r_m r_j(\gamma k + b)}{2} \begin{bmatrix} \gamma r_j + \frac{1}{J_j} & \gamma r_j - \frac{1}{J_j} \\ \gamma r_j - \frac{1}{J_j} & \gamma r_j + \frac{1}{J_j} \end{bmatrix} u \tag{5.111}$$

Hence, the zero dynamics of the system is clearly asymptotically stable. It is important to note that now the dynamic compensation law eq. (5.107) is well defined also for vanishing b, because if the matrix $Q(x)$ becomes zero, the dynamic system in eq. (5.107) can be seen

Figure 5.8: Response of the feedback linearization control.

as a singular perturbed system. Then, assuming that the system in eq. (5.107) has already reached its steady state by posing $\dot{u} = 0$, the control law becomes:

$$u = R^{-1}(x) \left(- \begin{bmatrix} L_f^4 h_\theta(x) \\ L_f^2 h_S(x) \end{bmatrix} + \begin{bmatrix} v_\theta \\ v_S \end{bmatrix} \right) \tag{5.112}$$

that is the same control law reported in the previous chapter for transmission elements without the damping term b [53].

Also in this case, an outer control loop can be used to achieve trajectory tracking. By posing $v = [v_\theta \ v_S]^T$, the control law:

$$v = v_d + K(z_d - \Phi(x)) \tag{5.113}$$

$$v_d = \begin{bmatrix} \theta_d^{[4]} & \ddot{S}_d \end{bmatrix}^T \quad , \quad z_d = \begin{bmatrix} \theta_d & \dot{\theta}_d & \ddot{\theta}_d & \theta_d^{[3]} & S_d & \dot{S}_d \end{bmatrix}^T \tag{5.114}$$

$$K = \mathrm{diag}[K_{0_\theta} \ K_{1_\theta} \ K_{2_\theta} \ K_{3_\theta} \ K_{0_S} \ K_{1_S}] \tag{5.115}$$

ensures the asymptotic tracking of the desired trajectory, where $\Phi(x)$ is the coordinates transformation eq. (5.106).

In Fig. 5.8 the experimental response of the static feedback linearization control (5.112)-(5.115) is reported. The joint stiffness and position trajectories have been chosen smooth, with continuous time derivative up to the 2nd and to the 4th order respectively, and slow enough to avoid the excitation of the unmodeled system dynamics. From Fig. 5.8(a) it is possible to note the position tracking error is very small and its not influenced by the stiffness variation, while from Fig. 5.8(b) one can see that there is a small coupling between the joint movements and the stiffness error. This effects can be caused by the error on the velocity estimation obtained by means of variable state filters.

5.6 Identification of the Transmission Element Parameters

In this section, we deal with the the identification of the transmission element parameters. Former, an offline estimation technique is presented and latter the problem of the online parameters estimation for the visco-elastic model of the transmission elements reported in eq. (5.2) is analyzed.

5.6.1 Offline Identification Procedure

Since no direct information about the force applied to the joint is available, experiments in quasi static conditions has been made to avoid problems caused by unmodeled dynamics and the gravitational forces are used as known external load. In these experiments, the joint is pulled up using just one actuator to avoid problems related to the coupling between the two transmission elements. The position of the actuator is controlled over a sinusoidal trajectory by means of a PD controller and both the positions of the joint and of the actuator are registered.

A crucial point for the application of the offline identification technique is the initialization of the system. The lack of information about the torque applied to the joint and the absence of absolute position information make not possible to obtain an unbiased measurement of the transmission element compression since, from eq. (5.3) and (5.4), without these information is not possible to find which values of the positions of the joint and of the actuator give $\varepsilon_{\alpha,\beta} = 0$. To consider this problem explicitly, the model in eq. (5.2) is modified in this convenient way:

$$\varepsilon_{id}(t) = \gamma\varepsilon(t) - \varepsilon_0 \tag{5.116}$$

$$F(t) = \begin{cases} k\left(e^{\varepsilon_{id}(t)} - 1\right) + b\,\dot{\varepsilon}(t)e^{\varepsilon_{id}(t)}, & \varepsilon_{id}(t) \geq 0 \\ 0, & \varepsilon_{id}(t) < 0 \end{cases} \tag{5.117}$$

An offline estimation technique based on symmetric adaptive windowing [49] is then used to estimate the compression velocity of the transmission element $\dot{\varepsilon}(t)$. The Matlab

Figure 5.9: Scheme of the offline identification experiment.

Optimization Toolbox is then used for the estimation of the parameters γ, k, b and ε_0 in the eq. (5.116)-(5.117). In particular, the parameter ε_0 depends on the system initialization and it is the more responsible of the variance of the other parameters. In the Tab. 5.2 the mean value of the parameters γ, k and b estimated on the base of several experiments are reported together with their percent errors. In Fig 5.10 the experimental data relative to one single identification experiment are shown as example. Fig. 5.10(a) reports the comparison between the gravitational force acting on the joint and the force applied by the transmission element computed from the eq. (5.116)-(5.117) using the estimated parameters while in Fig. 5.10(b) the estimation error is reported.

This offline estimation technique is quite simple and it gives useful information for the implementation of the joint controller but, since the parameter ε_0 must be evaluated at any new initialization of the system, additional consideration are necessary.

Description	Symbol	Value	Unit	$\Delta\%$
Stiffness coeff.	k	14.229	N	$\simeq 10\%$
Damping coeff.	b	98.476	$N \cdot s \cdot m^{-1}$	$\simeq 10\%$
Exponential factor	γ	1209.5	m^{-1}	$\simeq 1\%$

Table 5.2: Mean value of the transmission elements estimated parameters and their percent errors.

Figure 5.10: Offline estimation results.

5.6.2   Online Identification Algorithm

An online identification algorithm can be used to obtain an adaptive estimation of the transmission element parameters. This latter approach, besides to be more complex in the implementation that the former, has the advantage to give also an online estimation of the parameter ε_0, simplifying in this way the initialization of the system.

For the analysis of the online identification algorithm, the eq. (5.116)-(5.117) must be moved to the discrete time domain simply substituting the continuous time variable t with the discrete time one n.

Since the transmission element model in eq.(5.116)-(5.117) is not-linear into their parameters, the online estimation algorithm must be split in two parts. The first estimator identifies the linear parameters k and b minimizing the cost function:

$$P_1(k,b) = \frac{1}{N} \sum_{n=0}^{N} f_1^2(n) \tag{5.118}$$

where the error function $f_1(n)$ is:

$$f_1(n) = T_l(n) - \hat{k} \left(e^{\varepsilon_{id}(n)} - 1 \right) + \hat{b}\, \dot{\varepsilon}(n) e^{\varepsilon_{id}(n)} \tag{5.119}$$

$T_l(n)$ is the gravitational load applied to the joint and $\hat{k}$ and $\hat{b}$ are the estimated values of the parameters k and b respectively.

Defining the estimated parameter vector, the input and the output term as:

$$\hat{\Gamma}_1(n) = \left[\begin{array}{c} \hat{k}(n) \\ \hat{b}(n) \end{array} \right], \ \ y_1(n) = T_l(n), \ \ u_1(n) = \left[\begin{array}{c} e^{\varepsilon_{id}(n)} - 1 \\ \dot{\varepsilon}(n) e^{\varepsilon_{id}(n)} \end{array} \right] \tag{5.120}$$

the estimation error can be rewritten in the linear form:

$$f_1(n) = y_1(n) - u_1^T(n)\, \hat{\Gamma}_1(n) \tag{5.121}$$

Figure 5.11: Online identification algorithm.

The second estimator identifies the parameters of eq. (5.116) minimizing the cost function

$$P_2(\gamma, \varepsilon_0) = \frac{1}{N} \sum_{n=0}^{N} \ln^2 f_2(n) \tag{5.122}$$

where the error function $f_2(n)$ is:

$$f_2(n) = \frac{T_l(n) + k}{(k + b\,\dot{\varepsilon}(n))\,\exp(\hat{\gamma}\varepsilon(n) - \hat{\varepsilon}_0)} = 1 + \frac{f_1(n)}{(k + b\,\dot{\varepsilon}(n))\,\exp(\hat{\gamma}\varepsilon(n) - \hat{\varepsilon}_0)} \tag{5.123}$$

$$\ln f_2(n) = \ln\left(\frac{T_l(n) + k}{k + b\,\dot{\varepsilon}(n)}\right) - (\hat{\gamma}\varepsilon(n) - \hat{\varepsilon}_0) \tag{5.124}$$

Defining the estimated parameter vector, the input and the output term as:

$$\hat{\Gamma}_2(n) = \left[\begin{array}{c} \hat{\gamma}(n) \\ \hat{\varepsilon}_0(n) \end{array}\right] \quad,\quad u_2(n) = \left[\begin{array}{c} \varepsilon(n) \\ -1 \end{array}\right] \tag{5.125}$$

$$y_2(n) = \ln\left(\frac{T_l(n) + k}{k + b\,\dot{\varepsilon}(n)}\right) \tag{5.126}$$

the estimation error can be rewritten in the linear form:

$$\ln f_2(n) = y_2(n) - u_2^T(n)\,\hat{\Gamma}_2(n) \tag{5.127}$$

Both the two observers can be implemented in the standard recursive form

$$\hat{\Gamma}(n+1) = \hat{\Gamma}(n) + Q(n+1)\left[y(n+1) - u^T(n+1)\,\hat{\Gamma}(n)\right] \tag{5.128}$$

$$Q(n+1) = R(n)\,u(n+1)\left[\rho + u^T(n+1)\,R(n)\,u(n+1)\right]^{-1} \tag{5.129}$$

$$R(n+1) = \frac{1}{\rho}\left[I - Q(n+1)\,u^T(n+1)\right]R(n) \tag{5.130}$$

where ρ is the forgetting factor

In Fig. 5.11 a scheme of the online identification algorithm is reported. The convergence of the overall estimation algorithm can be analyzed looking at how the error of one

Transmission elements parameters

Figure 5.12: Online estimation results.

estimator is influenced by the error of the other one [52, 55]. The result of the online estimation algorithm for the same experiment in Fig. 5.10 are reported in Fig. 5.12. Only the first part of the experiment is shown to highlight the variation of the parameters estimation. After this period there are no appreciable variation in the estimated parameters. The parameter ε_0 is not reported because it depends on the initialization and it is different for any experiment. The estimation procedure is enabled at $t = 1\,s$ to avoid the estimation drift due to the initialization phase of the system, necessary to establish a suitable tendon pretension before the start of the controller. The final value of the parameter estimations using the online estimation algorithm is reported in Tab. 5.3.

The online estimation of the parameter ε_0 leads to a set of estimated parameters k, b and γ with a very little variation between the various experiments also for different initial conditions both for the parameters and for the system state. For this reason the parameter

Description	Symbol	Value	Unit
Stiffness coeff.	k	14.732	N
Damping coeff.	b	99.710	$N \cdot s \cdot m^{-1}$
Exponential factor	γ	1211.9	m^{-1}

Table 5.3: Mean value of the transmission elements parameters estimated with the online algorithm.

Figure 5.13: Online estimation results for a generic joint movement.

variations between the various experiments is not reported in Tab. 5.3.

In Fig. 5.13 the results of the online estimation algorithm applied to another experiment with different initial conditions and different joint movement. Also in this case the estimation procedure is enabled at $t = 1$ s and the parameter estimations converge to values close to the ones reported in Tab. 5.3.

5.7 Conclusions

In this chapter, the analysis of the DLR's antagonistic actuated joint is carried out and some different control strategies are presented together with some preliminary experimental results.

The actuator-level controller, other than to be very simple to implement, shows a good behavior also for fast setpoint variations. The introduction of a suitable setpoint compensation allows to recover to zero the steady state stiffness error, even if backdrivable actuators are used.

The feedback linearization control shows a stable response if smooth setpoint functions are considered, and the tracking of the joint position ad stiffness trajectory is achieved with very limited error.

All the experiments that has been carried out have shown that there are some limitations to the dynamic performance of the system due to the unmodeled dynamics of the

transmission elements, and in particular to the orthogonal movements of these elements with respect to the direction of the transmission forces caused by the use of long tendons to simplify the assembling procedure of the device. These undesired effects can be reduced by choosing tendons of opportune length and with the adoption of a suitable mechanism to ensure a minimum level of tendon pretension. Also the nonlinear transmission elements (see. Fig. 5.2) can be redesigned to reduce both their mass and their dimensions.

An interesting point for future activities will be the evaluation of the system behavior during the interaction with the external environment. The response of the system in case of unexpected collision will be investigated together with the benefit introduced by the use of a variable stiffness device in the force/impedance control of robotic systems.

6

Robots Feedback Linearization Control Based on Joint Position Measurements

6.1 Introduction

In this chapter, the stability problem for robotic control schemes based on feedback linearization and velocity estimation is addressed. As a matter of fact, in several control schemes for robotic systems the knowledge of the whole state of the system, i.e. of the position and the velocity of each joint, is required while, on the other hand, the problem of state reconstruction is often neglected. In many practical application, in fact, optical encoders or potentiometers are the only sensors in the robotic manipulator, so only position information are directly measurable, while the joint velocities must be estimated by a suitable filtering of the position data [56, 49].

The feedback linearization of a robotic manipulator, besides requiring the knowledge of the dynamic model of the robot, requires also the joint velocities to compute and compensate the centripetal and Coriolis terms of the robot dynamics [57, 58], while only the joint position information is necessary for the compensation of the gravitational terms. If both the exact model and the true state of the system are known, the feedback linearization control allows to obtain a linear model of the robot, and it is trivial to verify its stability and to design a suitable high level joint position controller. Also the problem of asymptotic trajectory tracking can be solved with a "feedback linearization+PD" controller [57]. In practice, the velocity estimation and the uncertainties on the model parameters introduce a limitation in the achievable performances and, in some cases, destabilizing effects.

From a theoretical point of view, the velocity estimation problem could be solved by means of proper nonlinear observers of the state variables [59, 60, 61, 62]. On the other hand, quite often in practical applications a simple numerical filter is used to obtain velocity from position information.

With the aim of showing the effects of the estimation of the joint velocities with simple filters, and then of the imperfect cancellation of the centripetal and Coriolis terms, the Lyapunov direct method is used in this chapter to find suitable conditions on the parameters of both the "feedback linearization+PD" controller and of the velocity estimation process that ensure the stability of the overall system. In this chapter, the continuous-time filtered position derivation is used as velocity reconstruction technique.

6.2   Dynamics of Robotic Manipulators

We consider the dynamic model of a *n* DOF robotic manipulator having only revolute joints, rigid links, ideal actuators and no friction at the joints, given by the classic Lagrangian formulation [63]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{6.1}$$

where q is the vector of the joint positions, $M(q)$, $C(q,\dot{q})$ are the matrices of the inertia and of the centripetal/Coriolis terms of the robot respectively, $g(q)$ is the vector of the gravitational effects and τ is the vector of the torques applied to the robot joints. The time dependence of the variables is omitted throughout the chapter for notation ease.

Supposing, as usual, that the complete state of the system $[q, \dot{q}]$ is known, a controller based on the feedback linearization of the robot dynamics [57, 58], consists in the static nonlinear control law:

$$\tau = M(q)u + C(q,\dot{q})\dot{q} + g(q) \tag{6.2}$$

so that the robot dynamics results in a linear system constituted by a cascade of two integrators

$$\ddot{q} = u \tag{6.3}$$

By defining:

$$\tilde{q} = q_d - q \tag{6.4}$$

where q_d is the vector of the desired joint positions, it is straightforward to show that the control law

$$u = \ddot{q}_d + K_p \tilde{q} + K_d \dot{\tilde{q}} \tag{6.5}$$

with K_p and K_d positive definite (usually diagonal) matrices, allows the exact tracking of the desired joint trajectories $[q_d, \dot{q}_d, \ddot{q}_d]$. As a matter of fact, by assigning $K_p = diag\{\omega_{n,i}^2\}$, $K_d = diag\{2\delta_i\omega_{n,i}\}$, where δ_i, $\omega_{n,i}$, $i = 1,\dots,n$ are proper design parameters, preassigned dynamic behaviors can be achieved. A complete review on the stability and the tracking properties of the system defined by eq. (6.1)-(6.5) is reported e.g. in [57].

6.3   Feedback Linearization via Filtered Velocity

In this section, we want to verify the stability of the robotic manipulator in the case that both the feedback linearization, eq. (6.2), and the control law, eq. (6.5), are computed considering an estimation of the joint velocities, given by a numerical derivation of the joint position data.

In this chapter, the estimation of the joint velocities is made by means of filtered derivation of the joint position information q. A possible state space representation of this process is:

$$\begin{aligned} \dot{\xi} &= -D\xi - D^2 q \tag{6.6} \\ \zeta &= \xi + Dq \tag{6.7} \end{aligned}$$

where $D = \text{diag}\{d_i\}$ is a positive definite diagonal matrix, ζ is the estimated velocity, and ξ is the state of the velocity estimation filter. In the Laplace transform domain, the estimation process described by eq. (6.6) and (6.7) can be written as:

$$\begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \vdots \\ \zeta_n \end{bmatrix}(s) = \begin{bmatrix} \frac{d_1 s}{s+d_1} & 0 & \cdots & 0 \\ 0 & \frac{d_2 s}{s+d_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{d_n s}{s+d_n} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix}(s) \tag{6.8}$$

or, in a more compact form and with some abuse of notation:

$$G(s) = \frac{\zeta(s)}{q(s)} = \text{diag}\left\{\frac{d_i s}{s+d_i}\right\} \tag{6.9}$$

¿From the two last equations, it is possible to note that we assume to estimate the joint velocities with a battery of n first order independent filters, one for each joint, whose bandwidth is defined by the parameter d_i. For the sake of simplicity and without lost of generality, we can assume also $d_i = d$, $i = 1, \ldots, n$, as usual in practice.

Since the joint velocities are not available, the velocity tracking error $\dot{\tilde{q}}$ used in eq. (6.5) is no longer the time derivative of eq. (6.4), and must be replaced with:

$$\hat{q} = \dot{q}_d - \zeta \tag{6.10}$$

where ζ is the output of the velocity estimation process and $\hat{q}$ is the velocity tracking error calculated with respect to the estimated velocity ζ, eq. (6.7). We use this notation to highlight that $\hat{q}$ is not the derivation of a variable but it is the difference between two known quantities. Therefore, eq. (6.5) becomes:

$$u = \ddot{q}_d + K_p \tilde{q} + K_d \hat{q} \tag{6.11}$$

Also eq. (6.2) must be modified in accordance with the velocity estimation process:

$$\tau = M(q)u + C(q, \zeta)\zeta + g(q) \tag{6.12}$$

The resulting system dynamics is then:

$$\ddot{\tilde{q}} = -K_p \tilde{q} - K_v \hat{q} - \alpha(\zeta, q, \dot{q}) \tag{6.13}$$

where

$$\alpha(\zeta, q, \dot{q}) = M^{-1}(q)[C(q, \zeta)\zeta - C(q, \dot{q})\dot{q}] \tag{6.14}$$

6.4 Stability of Feedback Linearization Based on Velocity Estimation

The stability of the system described by eq. (6.6), (6.7), and (6.13) is now discussed. We assume a constant position reference q_d, so that $\dot{q}_d = \ddot{q}_d = 0$. The dynamics of the system results:

$$\ddot{q} = K_p\tilde{q} - K_v\zeta + \alpha(\zeta, q, \dot{q}) \tag{6.15}$$

By choosing the state vector $[\zeta^T \ \tilde{q}^T \ \dot{q}^T]^T$, and considering that the time derivative of eq. (6.7) is:

$$\dot{\zeta} = \dot{\xi} + D\dot{q} = -D\xi - D^2 q + D\dot{q} = -D\zeta + D\dot{q} \tag{6.16}$$

the dynamics of the overall system can be written as:

$$\frac{d}{dt}\begin{bmatrix} \zeta \\ \tilde{q} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -D\zeta + D\dot{q} \\ -\dot{q} \\ K_p\tilde{q} - K_v\zeta + \alpha(\zeta, q, \dot{q}) \end{bmatrix} \tag{6.17}$$

It can be easily shown that the origin of the state space is an equilibrium point for this system. To show the unicity of the equilibrium point, it is convenient to rewrite eq. (6.17) as:

$$\frac{d}{dt}\begin{bmatrix} \zeta \\ \tilde{q} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -D & 0 & D \\ 0 & 0 & -I \\ K_\zeta(q, \zeta) & K_p & K_{\dot{q}}(q, \dot{q}) \end{bmatrix}\begin{bmatrix} \zeta \\ \tilde{q} \\ \dot{q} \end{bmatrix} \tag{6.18}$$

where

$$\begin{aligned} K_\zeta(q, \zeta) &= -K_v + M^{-1}(q)C(q, \zeta) \\ K_{\dot{q}}(q, \dot{q}) &= -M^{-1}(q)C(q, \dot{q}) \end{aligned}$$

or, in an equivalent form:

$$\frac{d}{dt}\begin{bmatrix} \zeta \\ \tilde{q} \\ \dot{q} \end{bmatrix} = A(\zeta, q, \dot{q})\begin{bmatrix} \zeta \\ \tilde{q} \\ \dot{q} \end{bmatrix} \tag{6.19}$$

Since K_p and D are positive definite, and therefore nonsingular, the matrix $A(\zeta, q, \dot{q})$ is always nonsingular. From this fact, it is possible to conclude that the origin is the unique equilibrium point of the system (6.17).

6.4.1 Lyapunov Function Candidate

To analyze the stability of the origin of the feedback linearized robot manipulator, expressed by eq. (6.15), we define the following Lyapunov function candidate:

$$V(\zeta, \tilde{q}, \dot{q}) = \frac{1}{2}\begin{bmatrix} \zeta \\ \tilde{q} \\ \dot{q} \end{bmatrix}^T P \begin{bmatrix} \zeta \\ \tilde{q} \\ \dot{q} \end{bmatrix} \tag{6.20}$$

where

$$
P = \begin{bmatrix} K_v D^{-1} & 0 & -K_p^{-1} \\ 0 & K_p & -K_v^{-1} \\ -K_p^{-1} & -K_v^{-1} & I \end{bmatrix}
\tag{6.21}
$$

The Lyapunov candidate function can also be written as:

$$
V(\zeta, \tilde{q}, \dot{q}) = \frac{1}{2} \zeta^T K_v D^{-1} \zeta + \frac{1}{2} \dot{q}^T \dot{q} + \frac{1}{2} \tilde{q}^T K_p \tilde{q} - \dot{q}^T K_v^{-1} \tilde{q} - \dot{q}^T K_p^{-1} \zeta
\tag{6.22}
$$

This function is clearly zero in the origin. If matrix P is positive definite, the candidate Lyapunov function is a quadratic form in the state variables, and is also globally defined and radially unbounded. In order to verify if matrix P is positive definite, the following bounds can be defined for the terms of eq. (6.22):

$$
\zeta^T K_v D^{-1} \zeta \geq \frac{\lambda_m\{K_v\}}{\lambda_M\{D\}} \|\zeta\|^2
\tag{6.23}
$$

$$
\dot{q}^T \dot{q} = \|\dot{q}\|^2
\tag{6.24}
$$

$$
\tilde{q}^T K_p \tilde{q} \geq \lambda_m\{K_p\} \|\tilde{q}\|^2
\tag{6.25}
$$

$$
-\dot{q}^T K_v^{-1} \tilde{q} \geq -\frac{1}{\lambda_m\{K_v\}} \|\dot{q}\| \|\tilde{q}\|
\tag{6.26}
$$

$$
-\dot{q}^T K_p^{-1} \zeta \geq -\frac{1}{\lambda_m\{K_p\}} \|\dot{q}\| \|\zeta\|
\tag{6.27}
$$

where $\lambda_m\{A\}$ and $\lambda_M\{A\}$ are the minimum and the maximum eigenvalues of the matrix A respectively. Then, a lower bound for the candidate Lyapunov function can be written as:

$$
V(\zeta, \tilde{q}, \dot{q}) \geq \frac{1}{2} \begin{bmatrix} \|\zeta\| \\ \|\tilde{q}\| \\ \|\dot{q}\| \end{bmatrix}^T P_l \begin{bmatrix} \|\zeta\| \\ \|\tilde{q}\| \\ \|\dot{q}\| \end{bmatrix}
\tag{6.28}
$$

where:

$$
P_l = \begin{bmatrix} \frac{\lambda_m\{K_v\}}{\lambda_M\{D\}} & 0 & -\frac{1}{\lambda_m\{K_p\}} \\ 0 & \lambda_m\{K_p\} & -\frac{1}{\lambda_m\{K_v\}} \\ -\frac{1}{\lambda_m\{K_p\}} & -\frac{1}{\lambda_m\{K_v\}} & 1 \end{bmatrix}
\tag{6.29}
$$

We have now to verify if the conditions that make the matrix P_l positive define are satisfied. Recalling that K_p, K_v and D are symmetric (diagonal) and positive definite and using the Gershgoring theorem [58], we obtain that P_l is positive definite if and only if:

$$
\frac{\lambda_m\{K_v\}}{\lambda_M\{D\}} > \frac{1}{\lambda_m\{K_p\}}
\tag{6.30}
$$

$$
\lambda_m\{K_p\} > \frac{1}{\lambda_m\{K_v\}}
\tag{6.31}
$$

$$
1 > \frac{1}{\lambda_m\{K_p\}} + \frac{1}{\lambda_m\{K_v\}}
\tag{6.32}
$$

These conditions can be satisfied with a suitable choice of the matrices K_p, K_v and D. In particular, matrix P_l is positive definite if K_p and K_v are "large" enough.

6.4.2 Time Derivative of the Lyapunov Function Candidate

The time derivative of the Lyapunov function candidate (6.22) can be written as:

$$\dot{V}(\zeta,\tilde{q},\dot{q}) = -\zeta^T K_v \zeta + \zeta^T K_v \dot{q} + \dot{q}^T K_p \tilde{q} - \dot{q}^T K_v \zeta + \dot{q}^T \alpha(\zeta,q,\dot{q}) + $$
$$- \tilde{q}^T K_p \dot{q} + \dot{q}^T K_p^{-1} D\zeta - \dot{q}^T K_p^{-1} D\dot{q} - \zeta^T \tilde{q} - \zeta^T K_p^{-1} \alpha(\zeta,q,\dot{q}) + $$
$$- \tilde{q}^T K_p K_v^{-1} \tilde{q} + \tilde{q}^T \zeta + \dot{q}^T K_v^{-1} \dot{q} - \tilde{q}^T K_v^{-1} \alpha(\zeta,q,\dot{q}) \tag{6.33}$$

Removing all the terms with opposed sign we have:

$$\dot{V}(\zeta,\tilde{q},\dot{q}) = -\zeta^T K_v \zeta + \dot{q}^T \alpha(\zeta,q,\dot{q}) + \dot{q}^T K_p^{-1} D\zeta - \dot{q}^T K_p^{-1} D\dot{q} + $$
$$- \zeta^T K_p^{-1} \alpha(\zeta,q,\dot{q}) - \tilde{q}^T K_p K_v^{-1} \tilde{q} + \dot{q}^T K_v^{-1} \dot{q} - \tilde{q}^T K_v^{-1} \alpha(\zeta,q,\dot{q}) \tag{6.34}$$

We have now to show that $\dot{V}(\zeta,\tilde{q},\dot{q}) \leq 0$. For this purpose, it is convenient to select a suitable set of bounds for each term in eq. (6.34).

The first term can be easily bounded by:

$$-\zeta^T K_v \zeta \leq -\lambda_m \{K_v\} \|\zeta\|^2 \tag{6.35}$$

Then

$$\dot{q}^T K_p^{-1} D\zeta \leq \lambda_M \{K_p^{-1}\} \lambda_M \{D\} \|\zeta\| \|\dot{q}\| \tag{6.36}$$

or, in an equivalent form:

$$\dot{q}^T K_p^{-1} D\zeta \leq \frac{\lambda_M \{D\}}{\lambda_m \{K_p\}} \|\zeta\| \|\dot{q}\| \tag{6.37}$$

For the other terms:

$$-\dot{q}^T K_p^{-1} D\dot{q} \leq -\frac{\lambda_m \{D\}}{\lambda_M \{K_p\}} \|\dot{q}\|^2 \tag{6.38}$$

$$-\tilde{q}^T K_p K_v^{-1} \tilde{q} \leq -\frac{\lambda_m \{K_p\}}{\lambda_M \{K_v\}} \|\tilde{q}\|^2 \tag{6.39}$$

$$\dot{q}^T K_v^{-1} \dot{q} \leq \frac{1}{\lambda_m \{K_v\}} \|\dot{q}\|^2 \tag{6.40}$$

The bounds for the terms involving $\alpha(\zeta,\tilde{q},\dot{q})$ must be carefully defined. For this purpose, let us recall the following property of matrix $C(q,x)y$ [57].

Property 1: *For robots having exclusively revolute joints, there exists a number $k_{C1} > 0$ such that:*

$$\|C(q,x)y\| \leq k_{C1} \|x\| \|y\| \tag{6.41}$$

for all q, x, $y \in \mathbb{R}^n$.

By using Property 1, the term $\alpha(\zeta, q, \dot{q})$ can be bounded by:

$$\alpha(\zeta, q, \dot{q}) \leq k_m \left(\|\zeta\|^2 + \|\dot{q}\|^2 \right) \tag{6.42}$$

where

$$k_m = \frac{k_{C1}}{\lambda_m \{M(q)\}}$$

The following bounds can be then defined:

$$\dot{q}^T \alpha(\zeta, q, \dot{q}) \leq k_m \|\dot{q}\| \left(\|\zeta\|^2 + \|\dot{q}\|^2 \right) \tag{6.43}$$

$$-\zeta^T K_p^{-1} \alpha(\zeta, q, \dot{q}) \leq k_m \frac{\|\zeta\|}{\lambda_m \{K_p\}} \left(\|\zeta\|^2 + \|\dot{q}\|^2 \right) \tag{6.44}$$

$$-\tilde{q}^T K_v^{-1} \alpha(\zeta, q, \dot{q}) \leq k_m \frac{\|\tilde{q}\|}{\lambda_m \{K_v\}} \left(\|\zeta\|^2 + \|\dot{q}\|^2 \right) \tag{6.45}$$

By defining:

$$\eta = k_m \left[\|\dot{q}\| + \frac{\|\zeta\|}{\lambda_m \{K_p\}} + \frac{\|\tilde{q}\|}{\lambda_m \{K_v\}} \right] \tag{6.46}$$

it is possible to note that η defines a ball, in the neighborhood of the origin of the state space, in which we are interested to study if $\dot{V}(\zeta, \tilde{q}, \dot{q}) \leq 0$. It is also important to note that the dimension given by the term $\|\zeta\|$ and $\|\tilde{q}\|$ of the ball η depend also on $\lambda_m \{K_p\}$ and $\lambda_m \{K_v\}$ respectively.

The time derivative of the candidate Lyapunov function can be then upper bounded by:

$$\dot{V}(\zeta, \tilde{q}, \dot{q}) \leq - \begin{bmatrix} \|\zeta\| \\ \|\tilde{q}\| \\ \|\dot{q}\| \end{bmatrix}^T Q(\|\zeta\|, \|\tilde{q}\|, \|\dot{q}\|) \begin{bmatrix} \|\zeta\| \\ \|\tilde{q}\| \\ \|\dot{q}\| \end{bmatrix} \tag{6.47}$$

where

$$Q(\|\zeta\|, \|\tilde{q}\|, \|\dot{q}\|) = \begin{bmatrix} \lambda_m \{K_v\} - \eta & 0 & -\frac{1}{2} \frac{\lambda_M \{D\}}{\lambda_m \{K_p\}} \\ 0 & \frac{\lambda_m \{K_p\}}{\lambda_M \{K_v\}} & 0 \\ -\frac{1}{2} \frac{\lambda_M \{D\}}{\lambda_m \{K_p\}} & 0 & \frac{\lambda_m \{D\}}{\lambda_M \{K_p\}} - \eta - \frac{1}{\lambda_m \{K_v\}} \end{bmatrix} \tag{6.48}$$

To verify if matrix $Q(\|\zeta\|, \|\tilde{q}\|, \|\dot{q}\|)$ is positive definite, keeping in mind the positiveness of K_p, K_v, D and recalling again the Gershgoring theorem, we have the following conditions:

$$\lambda_m \{K_v\} - \eta \; > \; \frac{1}{2} \frac{\lambda_M \{D\}}{\lambda_m \{K_p\}} \tag{6.49}$$

$$\frac{\lambda_m \{D\}}{\lambda_M \{K_p\}} - \eta - \frac{1}{\lambda_m \{K_v\}} \; > \; \frac{1}{2} \frac{\lambda_M \{D\}}{\lambda_m \{K_p\}} \tag{6.50}$$

The first condition, eq. (6.49), can be easily satisfied for sufficiently high value of $\lambda_m \{K_v\}$. It is possible to verify that the second condition, eq. (6.50), can be satisfied with a suitable choice of K_p and D. In particular, with the simplifying assumption $\lambda_M \{K_v\} = \lambda_m \{K_v\}$ and $\lambda_M \{D\} = \lambda_m \{D\}$ the condition given by eq. (6.50) is always satisfied for a sufficiently high value of $\lambda_m \{D\}$. These conditions are not in contrast with the ones given by eq. (6.30)-(6.32), so it is possible to conclude that the system is asymptotically stable in the ball defined by η centered in the origin of the state space.

In conclusion, the following Theorem can be formulated.

Theorem 1: *Given the robot dynamics (6.1) and the velocity estimation filter defined by (6.7), the feedback linearization control (6.11), (6.12) is stable if conditions (6.30)-(6.32) and (6.49), (6.50) hold.*

6.4.3   Comments

Considering the conditions (6.30)-(6.32) and (6.49), (6.50) it is possible to find a set of parameters of the controller and of the estimation process such that the overall system results asymptotically stable in a neighborhood η of the origin of the state space. It is important to note that η can be made as large as desired by a suitable choice of the gain matrices K_p, K_v and D.

It is also important to note that the condition (6.30) implies the condition (6.31) if $\lambda_M \{D\} \geq 1$, while condition (6.31) implies (6.30) if $\lambda_M \{D\} < 1$.

Moreover, the condition (6.32) is certainly satisfied if both $\lambda_m \{K_p\}$ and $\lambda_m \{K_p\}$ are greater than 2.

Finally, condition (6.30) implies (6.49) if $\eta < \frac{1}{2} \frac{\lambda_M \{D\}}{\lambda_m \{K_p\}}$, while (6.49) implies (6.30) otherwise. In any case, the strongest condition is the (6.50), because it relates the minimum and the maximum eigenvalue of both the matrices K_p and D. To highlight this fact, we can rewrite the condition (6.50) as:

$$\frac{\lambda_m \{D\}}{\lambda_M \{K_p\}} - \frac{1}{2} \frac{\lambda_M \{D\}}{\lambda_m \{K_p\}} > \eta + \frac{1}{\lambda_m \{K_v\}} \tag{6.51}$$

Therefore, the left hand side of eq. (6.51) must be not only strictly positive, but also "large" enough.

To conclude, it is possible to state that if $\lambda_M \{D\} \geq 1$ and $\lambda_m \{K_p\}, \lambda_m \{K_v\} > 2$, only the conditions (6.30) and (6.50) must be checked to find a region η in which the system (6.17) is asymptotically stable.

6.5   Case Study

In order to show the validity and the features of the main result of the previous Section, i.e. Theorem 1, a simple two DOF manipulator is now considered. The main dynamic parameters of the manipulator are reported in Table I.

Parameter	Value	
b_j	0.001	$N \cdot s \cdot m^{-1}$
a_{c1}	0.085	m
a_{c2}	0.085	m
a_1	0.3	m
I_1	1.15e-4	$kg \cdot m^2$
I_2	1.15e-4	$kg \cdot m^2$
m_1	0.541	kg
m_2	0.541	kg

Table 6.1: Parameters of the two DOF manipulator considered in the simulations.

The control parameters are $K_p = \mathrm{diag}\{3,3\}$ and $K_v = \mathrm{diag}\{3,3\}$. The set point is a step of amplitude 0.2 at $t = 0$ *sec* for joint 2, and a step of amplitude -2 at $t = 10$ *sec* for joint 1.

We analyze then two different situations. In the former, the velocity estimation parameters are set to $D = \mathrm{diag}\{8,8\}$, so that all the conditions for the existence of a stability region hold. Fig. 6.1 and Fig. 6.2 show that the system has a stable behavior.

In the latter, we set the velocity estimation parameters to $D = \mathrm{diag}\{1,1\}$, so that the condition (6.50) is not satisfied, and then it is not possible to define a stability region. In Fig. 6.3 and Fig. 6.4 it is possible to see that, while the (small) step on the second joint does not destabilize the system, the (large) step on the first joint make the system unstable.

6.6 Conclusions

In this chapter, the stability problem of control laws based on feedback linearization for n-DOF robotic manipulators is taken into account. The stability of this kind of controllers is analyzed considering that, in many practical application, the joint velocities must be estimated from position information by means of suitable filtering techniques.

After a brief discussion on the unicity of the equilibrium point in the case of constant setpoints, sufficient conditions for the local asymptotic stability of the system are given, and it is shown how the stability region can be made arbitrarily large by means of a suitable choice of the parameters of both the controller and the velocity estimation process.

Future activities include the application of the proposed analysis to the problem of trajectory tracking, to the case of adaptive control laws, and external load estimation.

Figure 6.1: Positions and position errors of the two DOF robot.

Figure 6.2: Velocity estimation errors.

Figure 6.3: Positions and position errors of the two DOF robot with $D = \text{diag}\{1,1\}$.

Figure 6.4: Velocity estimation errors with $D = \text{diag}\{1,1\}$.

7

Realtime Simulation

7.1 Introduction

Several CACSD[1] software packages are available for the simulation of dynamic systems and the design of digital controllers, e.g. Scilab/Scicos, Matlab/Simulink, Mathematica, and so on, [64, 65, 66, 67]. These CACSD tools allow the user to study the behavior of a dynamical system and to verify the effectiveness of a control law during the designing phase of the controller. It is also a standard practice to adopt an iterative procedure based on simulation of the closed loop system for the tuning of the parameters of the controller.

These tools are normally used by the designer to obtain the 'best' controller that has to be then translated in the real hardware/software control system. This latter phase implies to re-write the control law in a standard programming language (e.g. C) taking care of the hardware interface between the controller and the physical world. This design step may introduce some side effects in the final controller, because time delays and quantizations introduced by digital electronic devices can modify the behavior or, in the worst case, destabilize the closed-loop system.

It is a standard feature of modern CACSD tools the possibility of automatically converting the control scheme developed into a ready-to-run application for several realtime platform and also to interface directly the simulation packages with real time I/O devices. All these features are very useful for rapid-prototyping of control systems because they simplify the development and reduce the time required for the synthesis of the controller.

In the last years, the issue of rapid prototyping of digital control systems has been addressed by several researchers in order to reduce the time and the effort needed to obtain the control system for the given plant, [68].

However, this approach requires that the final controller must be tested in the real plant to verify its effectiveness. It is well known that this phase is very critical because during this test it is possible to cause hardware damages and to reach dangerous conditions.

As a matter of fact, the interface between the control algorithm and the plant is based on devices such as DAC, ADC, digital I/O, encoders, and so on. Many of these devices are usually grouped in a single board, called Data Acquisition Board (or DAQ board). There are many commercially available general purpose DAQ boards, each of them with specific

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[1]Computer Aided Control System Design

Figure 7.1: Left: typical software structure; Right: software structure with the COMEDI library.

software interfaces, limited compatibility with other boards even of the same producer and, in general, no compatibility with boards of other producers. As a consequence, a control application designed for a specific DAQ board is not portable to a different hardware without, at least, the redefinition of the I/O interface.

To avoid this problem, the COMEDI[2] library [20] provides an unified API[3] for control applications. This library allows to obtain applications that are independent from the specific DAQ board used to interface the control system to the plant. The major part of commercial DAQ boards are already supported by COMEDI, and it is possible to extend the support to other boards by writing a specific driver. COMEDI also supports the realtime extension of Linux (RTLinux and RTAI [69, 70]) natively.

With the aim of simplifying the development and testing phase of digital control systems, in this chapter we extend the traditional use of the COMEDI drivers to the realtime simulation of the overall plant. With this architecture, it is possible to monitor the behavior of both the controller and the plant (or, more precisely, the dynamic model of the plant) without really connecting the control system to the process. Fig. 7.1 reports a simple scheme of the adopted approach. Besides being useful in the design phase, this may improve the reliability of the control system, because it is possible to test critical conditions without risk, reducing also the possibility of miss-functions of the controller when it is applied to the real process. This architecture has been also successfully used as a teaching tool in Mechatronics/Digital control courses to allows the students to test their digital controllers.

In [71] a similar architecture has been presented, and an RTAI application for the

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[2]COntrol and MEasurement Device Interface.
[3]Application Programming Interface.

simulation of dynamic system has been derived directly from the Modelica [66] model of the plant. Both the Scilab/Scicos CACSD platform [72, 73] and the Matlab/Simulink/ RealTimeWorkshop environment can be used for the design of the digital controller because they allow to obtain ready-to-run realtime control applications with support for both RTAI-Linux and the COMEDI library.

In any case, a crucial point is to ensure the realtime execution of the basic integration algorithm used to solve the differential equations that describe the plant in the simulation task. For this purpose, the GNU Scientific Library [74] has been used in this chapter because, besides being opensource, it provides all the necessary functions for the implementation of the simulation algorithms.

This chapter is organized as follows. In Section 7.2 the realtime simulation algorithm for linear and non-linear systems is briefly discussed. In Section 7.3 the implementation the COMEDI realtime simulation driver is described, while in Sections 7.4 and 7.5 two examples are reported and discussed. Section 7.6 concludes with final remarks and plans for future activities.

7.2 Realtime Simulation of Dynamic Systems

In this environment, the realtime simulation of the plant is implemented by means of a fixed step periodic RTAI task that executes all the necessary operations to compute the time evolution of the system. The plant is modeled with the classical state space representation for dynamic systems:

$$
\begin{align}
\dot{x}(t) &= f(x(t), u(t), t), \quad x(t_0) = x_0 \tag{7.1}\\
y(t) &= g(x(t), u(t), t) \tag{7.2}
\end{align}
$$

where x is the state vector, u and y are the input and output vectors, t is the time, $f(\cdot)$ is the state evolution function, $g(\cdot)$ is the output function and x_0 is the initial state.

When the plant is described by linear stationary differential equations, it is possible to define its discrete time state space evolution in the classical linear form using standard discretization techniques:

$$
\begin{align}
x(k+1) &= A_d x(k) + B_b u(k), \quad x(0) = x_0 \tag{7.3}\\
y(k) &= C_d x(k) + D_d u(k) \tag{7.4}
\end{align}
$$

where A_d, B_d, C_d, D_d are matrices of proper dimensions and k is the discrete-time variable. Eq. (7.3)-(7.4) are very simple to be implemented with a periodic realtime task, and the required computational effort is very low.

When the plant is described by means of generic non-linear differential equations, it is possible to write its dynamics in the form of eq. (7.1)-(7.2).

In order to compute the time evolution of the plant, the ordinary differential equation system (ODE) defined by eq. (7.1) must be solved by means of a numerical integration

algorithm over the period of the realtime task. This may require a very high computational effort, especially if a variable step integration algorithm is used.

In this chapter, the GNU Scientific Library (GSL) is used for the integration of generic differential equations, such as eq. (7.1), but proper modifications to its source code have been made since GSL are user space libraries, while COMEDI board drivers are Linux kernel modules. For this reason, the code of the GSL library must be modified to make it running in kernel space, and therefore to ensure that the integration algorithm is executed taking into account the constraints of the realtime environment. This approach allows to use all the integration algorithms available in GSL (such as Runge-Kutta, Prince-Dormand, Bulirsch-Stoer algorithm and so on), with fixed or variable integration step, for the implementation of the COMEDI driver of the plant. It is important to note that RTAI supports also kernel space floating point operations, therefore no modification on the data type used by GSL for its computations is required.

When a *fixed step* algorithm is selected, the step size must be chosen considering the computational capabilities of the platform in which the simulation algorithm must be implemented. In general, the error on the solution of the differential equations that describe the system increases with the step size. In this case, the period of the simulation task corresponds to the integration step. This approach is suitable if the differential equations are smooth enough, even though there is no control on the integration error. Experiments show that the execution time of the integration algorithm in the fixed step case depends mainly on the order of the system and on the computational capabilities of the computer, and that it is practically constant during the simulation.

The use of a *variable step* size integration algorithm may improve the reliability of the solution. In this case, the integration step is set to the same period of the simulation task at the beginning of each period in order to estimate the first possible solution. Then, the step doubling error checking method [75] is applied in order to evaluate the relative integration error and, in case, to subdivide the integration step until the requested error is achieved. In this case, the computational effort can be very high, especially if large bandwidth/non-smooth functions are considered.

Controls on the integration error, the step size and the execution time are introduced to avoid system starvation and to satisfy the realtime constraints of the hardware platform. Therefore, it is possible to force the simulator to solve the dynamic equations with a given precision, in terms of relative or absolute error: if the time required by the integration algorithm during the iterations necessary to solve the ODE system with the prescribed error is too large, the algorithm is stopped and the last computed solution is returned. In Fig. 7.2 the flowchart of the realtime integration algorithm is reported. In general, the period of the simulation task must be grater or, at least, equal than the controller period. In the case of variable step integration algorithm, the simulation task period can be chosen equal to the period of the controller without problems: the error checking procedure can refine the integration step if necessary to satisfy the specifications on the integration error. The use of fixed step integration algorithms can introduce a large error drift in the solution of the system, since there is no control on the integration error.

Obviously, the period of the simulation task must be chosen large enough to allow

Figure 7.2: Flowchart of the realtime integration algorithm.

the solver to compute at least a possible solution of the ODE system. In case, the time scale of the simulation and of the controller can be expanded to fit the computational capabilities of the hardware platform. On the other hand, if the time required by the integration algorithm is small enough with respect to the simulation step, it is possible to compress the time scale to evaluate the behavior of the controller in a shorter time.

7.3 The COMEDI Realtime Simulation Driver

The COMEDI library provides to the control applications a set functions, called COMEDI API, to exchange data with the various devices on the DAQ board, to configure the devices and to convert and to manage the acquired data. This library creates a unified interface for the communications with many different data acquisition boards. With COMEDI it is possible to transfer data from the processing unit to the interface devices and vice versa using both synchronous and asynchronous communication via buffered data acquisition or direct data acquisition instructions respectively.

COMEDI is composed by two different parts: a userspace library that implements the

COMEDI API for user space applications, some kernel modules for the kernel space API support and a set of drivers for the various DAQ boards. These drivers are implemented as kernel modules and each of these modules implements all the specifics functions for a particular board (or board family). Since COMEDI is opensource, it is also possible to develop new drivers for acquisition boards that are not supported yet, and eventually to extend the functionality of existing drivers.

In COMEDI, the communications with the acquisition hardware is implemented in the standard UNIX way using characters device files. Since the association between the board and the file is specified dynamically via command line during the COMEDI configuration, it is possible to support simultaneously many (different) boards on the same system by associating each of them to a different file.

In this chapter, this mechanism is used to create a COMEDI driver for a virtual board that implements, besides the necessary functions to emulate the acquisition hardware, the realtime task for the simulation of the plant. Each data sent to the DAQ board changes the value of the input variables (or some parameters) of the plant simulation task and the data read functions return the value of the plant outputs.

There are some properties of the DAQ boards like setting time, conversion time, and so on, that are not taken explicitly into account by COMEDI. These parameters can be considered during the development of the COMEDI driver for the plant and introduced in the communication functions in order to simulate also the delays caused by electronic devices.

In order to implement the COMEDI driver for the realtime simulation of a given dynamic system, it is only necessary to write in C eq. (7.1)-(7.2) and to insert the code into the generic realtime simulation COMEDI driver. The generation of the C code for eq. (7.1)-(7.2) can be automated using, for example, Mathematica [67] or Modelica.

7.4   The Inverted Pendulum

The software architecture described in the previous Sections has been used for the simulation and control design of a number of mechatronic devices. Here, as a case study, the swing-up and balance control of the Quanser [76] rotary inverted pendulum is discussed as a test bed to show the features and compare the responses of the realtime simulator and of the real plant.

7.4.1   The Control System

The control system is based on a standard PC, with a Pentium III processor at 800 MHz, 256 MB of RAM and a Sensoray 626 DAQ board. The operating system is Debian-GNU Linux SID with kernel 2.6.9, RTAI 3.2 and GCC 3.3.4. The COMEDI library has been obtained from the CVS[4] archive. Note that this software platform has been chosen

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[4]Concurrent Versions System, an opensource tool to manage the distributed development of software projects [77].

Figure 7.3: Rotary inverted pendulum.

because all its components are opensource.

With the aim of testing this software platform on different hardware, a Knoppix-based [78] Linux Live-CD distribution called RTAI-Knoppix[5] has been developed. This live distribution contains, besides the software components described above, all the necessary tools like IDE[6], editors, libraries, examples and source code for the development of re-altime software and others applications for office, internet ecc. RTAI-Knoppix provides also an auto-configurable GUI[7] with xrtailab [72, 73], the graphic interface for RTAI applications.

7.4.2 The COMEDI Driver of the Rotary Inverted Pendulum

A sketch of the rotary inverted pendulum is reported in Fig. 7.3, while its parameters are reported in Tab. 7.1. By using the standard Lagrangian formulation, the dynamic model of the inverted pendulum can be written as:

$$M(q)\ddot{q} + C(\dot{q}, q)\dot{q} + D\dot{q} + g(q) = \tau - \tau_f \tag{7.5}$$

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[5]RTAI-Knoppix is available for download at: http://www-lar.deis.unibo.it/people/gpalli/files/rtai-knoppix.iso

[6]Integrated Development Environment.

[7]Graphic User Interface.

Description	Name	Value	Unit
Motor torque constant	K_m	0.00767	$N\,m\,A^{-1}$
Armature resistance	R_m	2.6	Ω
Total gear ratio	G_r	60.5	
Arm radius	l_1	0.2	m
Arm mass	m_1	0.128	Kg
Total arm inertia	I_1	0.0044	$Kg\,m^2$
Arm viscous friction	d_α	0.0001	$N\,m\,s\,rad^{-1}$
Pendulum half length	l_2	0.43	m
Pendulum mass	m_2	0.14	Kg
Total pendulum inertia	I_2	0.019	$Kg\,m^2$
Pendulum viscous friction	d_θ	0.0152	$N\,m\,s\,rad^{-1}$
Potentiometer sensitivity	P_s	1.637	$rad\,V^{-1}$
Encoder resolution	E_r	651.9	$rad\,step^{-1}$

Table 7.1: The parameters of the Quanser rotary inverted pendulum.

where $q = [\alpha\ \theta]^T$ is the vector of generalized coordinates (respectively the angle between the arm and a reference position and the angle of the pendulum with respect to the vertical position), τ and τ_f are the vectors of the torque and of static friction acting on the joints, $M(q)$ is the inertia matrix, $C(\dot{q}, q)$ is the matrix of the Coriolis and centripetal effects, D is the viscous friction matrix and $g(q)$ is the vector of gravity effects. In particular, for this system:

$$M(q) = \begin{bmatrix} I_1 + m_2(l_1^2 + l_2^2 \sin^2(\theta)) & -m_2 l_1 l_2 \cos(\theta) \\ -m_2 l_1 l_2 \cos(\theta) & I_2 \end{bmatrix} \tag{7.6}$$

$$C(\dot{q}, q) = -m_2 l_2 \sin(\theta) \begin{bmatrix} l_2 \cos(\theta)\dot{\theta} & l_2 \cos(\theta)\dot{\alpha} + l_1 \dot{\theta} \\ -l_2 \cos(\theta)\dot{\alpha} & 0 \end{bmatrix} \tag{7.7}$$

$$D = \begin{bmatrix} d_\alpha & 0 \\ 0 & d_\theta \end{bmatrix}, \quad g(q) = \begin{bmatrix} 0 \\ -m_2 l_2 g \sin(\theta) \end{bmatrix}, \quad \tau = \begin{bmatrix} \tau_m \\ 0 \end{bmatrix}, \quad \tau_f = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{7.8}$$

where τ_m is the torque applied by the DC motor used as actuator to the arm. Note that the static friction is neglected (eq. (7.8)).

A simplified model of the DC motor is considered, being its electrical time constant negligible with respect to the mechanical one:

$$\tau_m = \frac{K_m G_r}{R_m} v - \frac{K_m^2 G_R^2}{R_m} \dot{\alpha} = K_v v - K_\alpha \dot{\alpha} \tag{7.9}$$

where v is the DC motor driving voltage provided by the Sensoray 626 DAQ board analog output.

From eq. (7.6)-(7.9), assuming as state vector $x = [\alpha\ \theta\ \dot{\alpha}\ \dot{\theta}]^T$ and as input vector

$u = [v]$, eq. (7.5) can be rewritten in the form of eq. (7.1):

$$\dot{x}_1 = x_3 \tag{7.10}$$

$$\dot{x}_2 = x_4 \tag{7.11}$$

$$\dot{x}_3 = \frac{M_{12}C_{21} - M_{22}(C_{11} + D_{11} + K_\alpha)}{M_{11}M_{22} - M_{12}M_{21}}x_3 +$$

$$+ \frac{[M_{12}(C_{22} + D_{22}) - M_{22}C_{12}]x_4 + M_{12}g_2}{M_{11}M_{22} - M_{12}M_{21}} +$$

$$+ \frac{M_{22}K_v}{M_{11}M_{22} - M_{12}M_{21}}v \tag{7.12}$$

$$\dot{x}_4 = \frac{-M_{21}\dot{x}_3 - C_{21}x_3 - (C_{22} + D_{22})x_4 - g_2}{M_{22}} \tag{7.13}$$

Two sensors are available for the measurement of the positions α, θ of the arm (a potentiometer) and of the pendulum (an encoder), see Tab. 7.1. Then, eq. (7.2) can be written as:

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1/P_s & 0 & 0 & 0 \\ 0 & 1/E_r & 0 & 0 \end{bmatrix} x \tag{7.14}$$

Eq. (7.10)-(7.14) are implemented in C in the COMEDI driver to test the realtime simulation of the system. The realtime integration of these equations is performed with an adaptive variable step 4th-order Runge-Kutta algorithm, with relative error set to 10^{-6} and sampling time of 1 *ms*.

The input u and the output y of this system are then converted into the driver to the standard unsigned integer representation used by COMEDI for the data acquired from hardware devices. This mechanism allows also to consider the data quantization introduced by ADC and DAC devices using the same word length during the conversion of these data in the unsigned integer form.

In particular, in the COMEDI driver of the inverted pendulum, the channel 0 of the DAC subdevice is the DC motor driving voltage, that corresponds to the input u, the channel 0 of the ADC subdevice is the value of the output voltage of the potentiometer y_1 an the channel 0 of the counter subdevice is the output of the encoder y_2. The same assignment of channels and subdevices to input and output signals is maintained in the Quanser pendulum to switch from the simulated to real plant without any modification on the controller.

7.4.3 Experimental Results

A swing-up and balance controller has been developed. The swing-up control algorithm is based on an unstable pendulum position control with limitation on the arm angle to avoid the collision between the pendulum and the table during its oscillations, while the balance controller is a LQR state feedback control based on the pendulum model, linearized at the upward vertical position. The controller starts with the unstable position control and the

Figure 7.4: The rotary inverted pendulum is balanced.

pendulum in the downward vertical position. When the pendulum is sufficiently near to the vertical upward position, the controller switches to the balance control algorithm.

Estimation of angular velocities of both the pendulum and of the arm are essential to realize a robust balance control. For this purpose, two state variable filters are used.

The control algorithms described above have been implemented in two different ways, using Matlab/Simulink/RealTimeWorkshop and in C language. These two controllers did not show any appreciable difference. The sampling time of the controller has been set to 1 *ms*.

After having verified that the controllers have the requested performance on the simulated plant, the control applications are switched -without any modification- to the real plant and the response of the system has been recorded. The switch between the simulated and the real system is done simply by moving the association to the device file `/dev/comedi0` (used by the control application to acquire and send data from the COMEDI realtime simulation driver) to the driver of the Sensoray 626 DAQ board using the instruction `comedi_config`[8]. In Fig. 7.4 it is possible to view the Quanser pendulum maintained in the vertical upward position by the balance controller.

The comparison between the response of the simulated and the real plant are reported in Fig. 7.5 and 7.6. In Fig. 7.5, it is possible to observe some differences between the responses of the two systems, caused by some parameters uncertainties and some neglected

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[8]see COMEDI documentation for more details.

Figure 7.5: Comparison between the positions of the real and the realtime simulated plant.

effects like static friction (see the first plot in Fig. 7.6).

During the test of the controller on the COMEDI realtime simulation driver, the execution time of the simulation task has been monitored. During several minutes of simulation, the execution time has never exceeded the value of 16 μs, with an average value of 8.5 μs and a minimum value of 8.4 μs even under heavy system load caused by standard Linux applications, like massive hard-disk access, multimedia streaming or network navigation, and the simultaneous execution of other RTAI tasks like latency test or xrtailab graphic interface.

7.5 The Tendon-Sheath Lumped Parameter Model

In this section, the implementation of a realtime simulation driver for a lumped parameter model is described and the response of the system is compared with the results of the simulation obtained using Matlab/Simulink. The lumped parameter model of a tendon-sheath transmission system presented in [28] has been chosen because, while the dimension of the state space is larger that in the case of a standard mechatronic device like the one reported in Sec. 7.4, the system dynamics is described by non-smooth nonlinear differential equations.

This experiment has been conducted with the aim of testing the performances of the realtime simulation environment in the case of systems with large state space dimension and non-smooth differential equations.

Figure 7.6: Difference of positions and comparison between the velocities of the real and the realtime simulated plant.

7.5.1 The Control System

In this case the simulations have been executed on a laptop computer, with Intel Centrino processor at 1.6 GHz and 512 MB of RAM. The operating system is Debian-GNU Linux SID, as in the previous case, but with kernel 2.6.14, RTAI 3.3 and GCC 3.3.6. The COMEDI library version is the same used in the previous case.

7.5.2 The COMEDI Driver of the Tendon-Sheath System

A scheme representing the lumped parameter model of the tendon-sheath transmission system is reported in Fig.7.7. The tendon-sheath lumped parameter model is described by

Figure 7.7: Representation of the tendon-sheath lumped parameter model.

the equations:

$$\dot{x}_0 = v_0 \tag{7.15}$$

$$\dot{v}_0 = \frac{1}{m_0}\left[-c_0(v_0 - v_1) - k_0(x_0 - x_1) + u\right] \tag{7.16}$$

$$\dot{x}_i = v_i \tag{7.17}$$

$$\dot{v}_i = \frac{1}{m_i}\left[c_i(v_{i+1} - 2v_i + v_{i-1}) + k_i(x_{i+1} - 2x_i + x_{i-1}) - F_{fi}\right] \tag{7.18}$$

$$\dot{F}_{fi} = K_b\left[v_i - \frac{2n\,F_{fi}\,|v_i|}{\mu\gamma\,k_i(x_{i+1} - x_{i-1})}\right] \tag{7.19}$$

$$i = 1,\dots,n$$

$$\dot{x}_{n+1} = v_{n+1} \tag{7.20}$$

$$\dot{v}_{n+1} = \frac{1}{m_{n+1}}\left[-c_{n+1}(v_n - v_{n+1}) - k_{n+1}(x_n - x_{n+1}) - k_{env}x_{n+1} + d\right] \tag{7.21}$$

where x_i, v_i and F_{fi} are respectively the position, the velocity and the friction force acting on the i-th tendon element, x_0 and v_0 are the position and the velocity of the actuator and x_{n+1} and v_{n+1} are the position and the velocity of the load, u is the input force applied by the actuator (e.g. the motor torque), d is the disturbance force applied to the load (e.g. the gravity). The description of the other system parameters is reported in Tab. 7.2.

Eq. (7.19) describes the dynamics of the friction force of the i-th tendon element according to the Dahl friction model [29]. It is important to note that this differential equation, besides nonlinear, is non-smooth in $v_i = 0$ because of the presence of the absolute value function and the typical high value of the contact bristle stiffness K_b.

The output vector of the system is:

$$y = \begin{bmatrix} x_0 & x_{n+1} & k_0(x_0 - x_1) & k_{n+1}(x_n - x_{n+1}) \end{bmatrix}^T \tag{7.22}$$

This set of output variables are chosen because in the experimental setup two potentiometers are used to measure the position of output shaft of the actuator and the position of the load and two load cells are used to measure the input and output tension of the tendon.

Description	Name	Value	Unit
Tendon internal viscous friction	$c_{0,\dots,n+1}$	0.004	$N\,s\,m^{-1}$
Tendon element stiffness	$k_{0,\dots,n+1}$	$6 \cdot 10^4$	$N\,m^{-1}$
Tendon element mass	$m_{1,\dots,n}$	10^{-5}	Kg
Coulomb friction coefficient	μ	0.1186	
Contact bristle stiffness	K_b	10^3	$N\,m^{-1}$
Tendon curvature angle	γ	$\pi/4$	rad
Tendon elements	n	15	
Environment stiffness	k_{env}	10^2	$N\,m^{-1}$

Table 7.2: The parameters of the tendon-sheath lumped parameter model.

Figure 7.8: Comparison between Matlab/Simulink and the realtime environment.

Eq. (7.15)-(7.22) are implemented in C in the COMEDI driver to test the realtime simulation of the system. The realtime integration of these equations is performed with an adaptive variable step 4th-order Runge-Kutta algorithm, with both relative and absolute error set to 10^{-3} and sampling time of 1 ms.

The input u and the output y of this system are then converted into the driver to the standard unsigned integer representation used by COMEDI for the data acquired from hardware devices. This mechanism allows also to consider the data quantization introduced by ADC and DAC devices using the same word length during the conversion of these data in the unsigned integer form.

In particular, in the COMEDI driver, the channel 0 of the DAC subdevice is the input u, the channels from 0 to 3 of the ADC subdevice provide the value of the output vector y.

7.5.3 Experimental Results

In this case only the openloop response of the system is reported and compared with the results of the simulation on the Matlab/Simulink environment, see Fig. 7.8. It is important to note that, while both input and output data in the COMEDI driver of the tendon are quantized because of the presence of ADC and DAC, in Matlab/Simulink the simulation data are not affected by quantization. This factor must be considered in the comparison of the simulations. Also the duration of the simulation with Matlab/Simulink is many times longer than the effectives 10 s on the same calculator.

The parameter n (the number of tendon elements) has been set to 15 to reduce the simulation time and the results granularity. The considered tendon length is 0.1 m. It is important to note that, with $n = 15$, the dimension of the state space is 49. The tendon-sheath lumped parameters model can be simulated in realtime with a sample time of 1 ms and an average integration time of $\sim$400 μs. If the value of n is increased, the execution time of the realtime integration algorithm increases very quickly as well. With $n = 20$ (the state space dimension is 64) the integration algorithm is not able to provide a solution that satisfy the desired relative error. The integration algorithm is stopped before the desired precision is achieved to prevent the operating system starvation. In this case, to simulate the system in realtime mode, a bigger integration error or a more powerful calculator is necessary.

7.6 Conclusions

In this chapter, the implementation of a realtime environment for rapid prototyping of digital control systems has been presented. The environment is based on RTAI-Linux and on the COMEDI library, and its main feature is that it offers the possibility of simulating in real time both the plant and the control law. Once the desired controller is obtained, the control algorithm can be simply switched from the simulated to the real process without any change in the software. Although being somehow limited to the RTAI-Linux environment, the system would facilitate the implementation and rapid prototyping of digital control systems.

This tool, besides for design purposes, has also been used as a teaching support in courses on digital control as an alternative to standard laboratory setups. In this case, the student may be asked to face both standard control design problems and aspects related to implementation of multi-task realtime software.

Note that, if a proper model of the plant is available, with this system it is possible to run simultaneously two identical controllers, one connected to the plant and one to the simulator. This could help in the real time evaluation of the response of the plant to the given commands before their real application, and also for supervision/fault detection purposes.

An environment to obtain directly the COMEDI realtime simulation driver for a plant described with the Modelica language, and the integration of this system with a 3D graphic representation tools are object of future activity.

8

Conclusions

In this thesis, some problems related to the control of tendon actuated robots are considered, and suitable control solutions are proposed. The most important lesson learned from the research activity carried out on this topic is that, while the use of tendons simplifies the mechanical design and allows to reduced the dimension, the weight and the cost of the robotic devices, it introduces some challenging problems from the control point of view due, first of all, to the elasticity of the tendons and to the friction distributed along the transmission chain. In antagonistic actuated robot, the use of tendons allows to partially reduce the increment of mechanical complexity intrinsic in this actuation modality, but it rises up the problem of tendon pretension due to their unilateral transmission characteristic. These problems can be solved by a suitable choice of both the control law and the mechanical implementation, even if, especially in the case of tendon-sheaths transmission, also a deeper comprehension of the dynamic model of the system can help in the definition of controllers with better performance.

From my point of view, the use of tendon transmission systems are not so widespread due to the above mentioned limitations. In this thesis, my aim is to show that solutions exist to overcome the limits of this actuation modality. Both simulation results and experimental activities confirm the effectiveness of tendon actuated robots in different fields, like in object manipulation with the UB Hand 3 or in simultaneous stiffness/position control with the DLR's antagonistic actuated joint.

The research activity presented in this thesis introduces some novel contributions in the field of model and control of tendon actuated robotic devices:

- The development of a set of sensors and actuators for the control of tendon actuated lightweight mechanical structures, like the fingers of the UB Hand 3;

- The definition of a new dynamic model of the tendon-sheath transmission systems and of suitable solutions for the implementation of tendon tension control;

- The solution to the problem of the feedback linearization and decoupled stiffness/position control of antagonistic actuated robotic arms;

- The analysis of both the static and the dynamic characteristics of the DLR's antagonistic actuated joint and the implementation of different stiffness/position controllers for this device;

- The stability analysis of the feedback linearization (computed torque) control of a robotic manipulator based on velocity reconstruction from position informations by means of linear filters.

Also in the field of realtime control systems, some new contributions are presented:

- A modular and device independent software architecture has been developed for the realtime control of a complex mechatronic device, like the UB Hand 3. This software architecture can be easily used for the control of several devices and with different data acquisition systems;

- A live RTAI-Linux distribution has been built to give to the control system designer a reliable software environment and a collection of useful tools for the development of realtime control applications, avoiding all the problems related to the installation and the tweak of the realtime operating system;

- A realtime algorithm for the simulation of dynamic systems. This realtime simulation environment has been successfully used as a teaching tool in automatic control courses.

Future research activities can be addressed to improve the model of the tendon-sheath transmission system, to the definition of new control laws for this actuation modality, to the experimental evaluation of low-level tendon transmission controller integrated in a more complex device like the UB Hand 3, to the evaluation of the response of robots with antagonistic actuated joints, or more in general variable stiffness devices, during the interaction with an external environment.

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