Demographic Change,

Intergenerational Conflict and Economic Growth

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Contents

List of Figures

Overview

Overview

This doctoral thesis contributes to the political economy of institutions, and focuses in particular on political interactions between economic and demographic factors. We are especially interested in the social conflicts arising among intergenerational groups, specifically young, adults and old. While these groups can be differentiated on the basis of their economic characteristics, demographic dynamics are also at work when these interactions are considered. This research will address these questions from a theoretical perspective where policy prescriptions will also be outlined.

Focusing on intergenerational conflict and the interaction among socio-demographic variables, economic institutions, political institutions and macroeconomic outcomes, this work can be divided into two lines.

The first line of this research focuses on the impact of intergenerational conflict for technology adoption and, consequently, for economic growth, in the presence of endogenous evolution of life expectancy. The idea is to join together, into a unique theoretical framework, two important recent research strands within the field of political economy: the one that studies the interaction among demographic variables, with special focus on life expectancy evolution and economic growth, and the one that investigates the political mechanism that drives the economy toward different innovation policies.

Specifically, Chapter 1 presents "*A Politico-Economic Model of Aging, Technology Adoption and Growth*". This model provides a positive theory that explains how an economy evolves when the longevity of its citizens is jointly determined with the process of economic development. We propose a three periods OLG politico-economic model with human capital accumulation. Agents' decisions embrace two dimensions: the private choice about education and the public one upon innovation policy. Analyzing an economic model in which endogenous changes in life expectancy, education, technological improvements and economic growth interact with each other, we find that (a) poverty traps can arise in the accumulation process of human capital and have long-lasting effects on aggregate output; (b) at the individual level a higher life expectancy increases the incentive to innovate for both young and adults; (c) at the aggregate level different political configurations can arise depending on endogenous demographic structures; (d) depending on initial conditions and parameter values, in the long run both "Innovation" and "No Innovation" can be feasible steady states.

The second part of this research develops a theoretical investigation of the interactions between demographic conflict and the differential performance of political institutions. We focus on the role of intergenerational conflict for the determination of both the size of the State and the redistribution programs adopted, in the presence of endogenous human capital production and of an aging society. This work relies on the dynamic political economy literature that incorporates forward-looking decision makers in a multidimensional policy space without commitment. In particular this work relates to two main streams of literature. On the one hand, it supports and gives new theoretical fundamentals to the existing literature on social security sustainability, which recognizes the link between productive and redistributive public spending. On the other hand, this part also contributes to the growing literature on dynamic politico-economic models. Starting from the seminal work of Krusell et al. (1997)[1], the main interesting issue in the dynamic political economy literature concerns the modeling of economies where endogenous dynamic feedbacks between private and political choices are explicitly considered. Due to theoretical complexities, to extend standard static models to encompass fully dynamic policy-making has proved to be difficult, even in the case of one-dimensional policy environments both in the finite- and infinite-horizon environment.

Chapter 2 presents "*A Dynamic Politico-Economic Model of Intergenerational Contracts*" that investigates the conditions for the emergence of implicit intergenerational contracts without assuming reputation mechanisms, commitment technology and altruism. We present a tractable dynamic politico-economic model in an OLG environment where politicians play

[1] Krussell, P., Quadrini, V. and Rìoss-Rull, J. V., 1997, "Politico-Economic Equilibrium and Economic Growth", *Journal of Economic Dynamics and Control*, 21, $243-272$.

Markovian strategies in a probabilistic voting environment, setting multi-dimensional political agenda. Both backward and forward intergenerational transfers, respectively in the form of pension benefits and higher education investments, are simultaneously considered in an endogenous human capital setting with distortionary income taxation. On the one hand, social security sustains investment in public education; on the other hand investment in education creates a dynamic linkage across periods through both human and physical capital, driving the economy toward different welfare state regimes. Embedding a repeated-voting setup of electoral competition, we find that under the dynamic efficiency scenario both forward and backward intergenerational transfers simultaneously arise.

Chapter 3 presents "*Time Consistent Public Expenditure with Intergenerational Exchange*". In this chapter, we study the optimal choice of intergenerational public expenditures when there is no way of committing to future policy and "reputational" mechanisms are not operative. Restricting our attention to Markov equilibria, as in the previous chapter, we solve for a dynamic game between successive governments and the private sector, whose underlying state variables are the physical and the human capital stocks. We characterize equilibria in terms of an intertemporal first-order condition - *Generalized Euler Equation* - for the government using the methodology proposed by Krusell et al. (2008) and we base analytical computations on it. Comparing the resulting optimal public expenditure level with the politico-economic equilibrium in probabilistic voting environment, we find that: The equilibrium allocation is education efficient but, due to political overrepresentation of the elderly, the electoral competition process induces overtaxation compared with a *time-consistent* Central Planner solution with balanced welfare weights.

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Chapter 1

A Politico-Economic Model of Aging, Technology Adoption and Growth

"The political economy of technological change is only dimly understood. [...] the vigor of youth is followed by the caution of maturity and finally the feebleness of old age. [...] If we are to understand why the fires of innovation die down, we must propose a model in which technological progress creates the condition for its own demise." (*Mokyr 1990 : 261*)

1.1 Introduction

Over the last two centuries the Western world has experienced an extraordinary change in the economic environment and in all aspects of human life. During this period, OECD countries have been characterized by dramatic improvements in economic conditions, the longevity of their population and education attainments. Simultaneously, the traditional social structure has greatly changed: the share of both schooling age and retired people has increased significantly and, as a consequence, the proportion of the working population has shrunk.

Some specific facts provide a better description of this evolution. In the last one hundred and fifty years life expectancy has increased tremendously. Focusing on the US, it shifted from less than 60 years (Lee, 2001) in 1850, to almost 80 years today (Fogel, 1994). At the same time, both the portion of lifetime devoted to education and retirement have increased. In 1850 about 10% of the population was enrolled in primary school and, on average, the time devoted to education was negligible. Considering both formal and informal schooling (domestic education), people now study for around 20 years, about a quarter of their expected lifetime. The length of time spent in retirement shows a similar trend. In 1850, less than three years were devoted to retirement. Today, especially in Europe due to the introduction of social security systems after World War II, people enjoy retirement for almost 20 years: again, one quarter of their lifetime (Latulippe, 1996). Figure 1.1 shows how life expectancy and its composition, in terms of agents' economic roles, have evolved between 1850 and 1990 in the United States.[1] This trend is even more evident in the case of Europe: in particular, life expectancy has grown more rapidly (surpassing the United States). It was around 40 years in England in 1850, while today it has reached almost 80 years (Galor, 2005). The length of retirement has increased even more.[2]

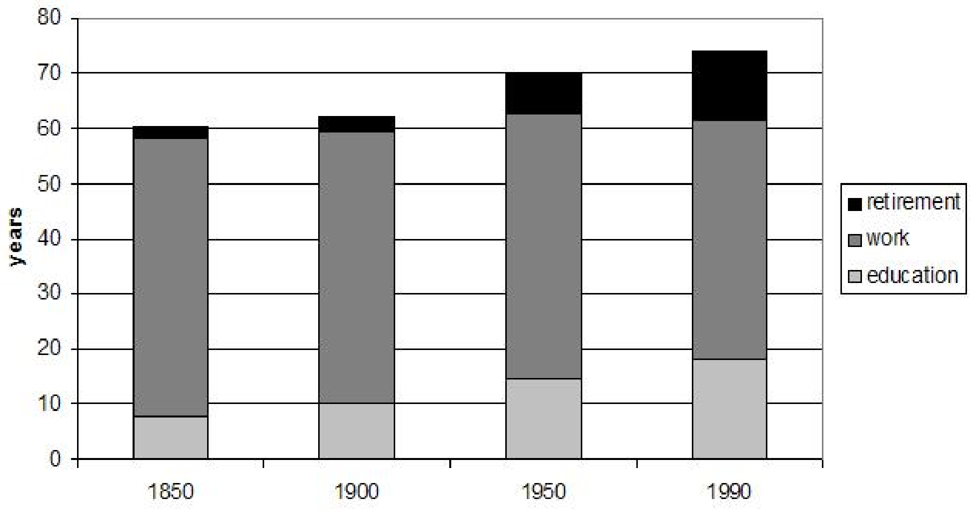

Figure 1.1. Life expectancy and economic roles in the US.

One of the main implications of this trend is that the socio-demographic

[1] Figure 1.1 plots the average length of life and the distribution of time spent in educating, working and retiring for a 20-years-old person for the United States. Source: Lee (2001) and www.bls.gov.

[2] For European data see Galasso and Profeta (2004).

structure of developed and, to some extent, developing countries are experiencing important changes. This movement creates a system in which the preferences of both young and old people are becoming more and more important in the political debate competing with the traditional interests of adult workers. We observe the transition from a sort of *"workers' dictatorship"* - defined as a situation where the mass of the workers represents a large majority in the population - to a more diluted political representation.

The purpose of this work is to provide an investigation of how an economy evolves when life expectancy affects both individual and aggregate preferences concerning the production side of the economy and, therefore, the growth process. We propose a three periods OLG model where agents, during their lifetime, cover different economic roles characterized by different time horizons and, consequently, incentive structures. Agents' decisions embrace two dimensions: the private choice about education and the public one related to innovation policy. The theory focuses on the crucial role played by heterogeneous interests in determining innovation policies.

Our model economy does not create new technologies; it simply adopts those that are already disposable. The adoption process is costly. We refer to a systemic innovation as to a type of innovation that, in order to be implemented, has to pass through the endorsement of a political mechanism, where, in general, the interests of different groups of agents do not coincide. In our framework the contrast evolves among different age groups. The public nature of systemic innovation, in contrast with the Schumpeterian view of innovations developed by firms running for the best cost-saving technology, comes from the historical point of view according to which the implementation of a new technology is rarely the outcome of pure profit-maximization by firms. Following Mokyr (1998*a*, 2002) and Olson (1982), in this study we focus our attention on systemic innovation as a growth-enhancing technology. Bauer (1995) points out that a decentralized market outcome seems to be a poor description of many technology breakthroughs. Economic convenience is certainly not irrelevant, but, as Mokyr (1998*a*) suggests, *"there usually is, at some level, a non-market institution that has to approve, license or provide some other imprimatur without which firms cannot change their production methods. The market test by itself is not always enough. In the past, it almost never was."* (p. 219) Thus, as reported by Olson (1982), the decision whether to adopt a new technology is likely to be resisted by those who lose by it through some kind of activism aimed at influencing the decision by the above-mentioned institutions.

Consequently, we construct a model in which technology adoption is delegated to a regulatory institution, the democratic vote. We formalize the idea that an innovation, before being introduced in large-scale production, has to be approved by some non-market institution.[3] Its adoption is ex-post disposable for all individuals in the economy, but ex-ante the choice to adopt it or not can be affected by the interests of different age groups.[4] To capture the evolving clash between resistive and innovative interests, we consider an economy that, at any point in time, is populated by three different overlapping groups of agents. In fact, besides the increasing human capital accumulation, productivity improvements come from the innovation process. A systemic innovation is implemented if and only if there is a political consensus for it: because its net benefits are not spread equally among the different age classes, in a heterogeneous setting there is always room for suboptimal provision of the innovation itself. According to Krusell and Ríos-Rull (1996) as well as Aghion and Howitt (1998), we assume that the public choice is carried out by means of a democratic majority voting where the interests of the absolute majority of the population prevail.

We find that a conflict of interests on which technology to adopt will arise between workers and students, on one side, and retired people, on the other. If the former will tend to support innovations, the latter are likely to resist technological change given that their income is not related to the current technology but rather to the previous innovation cycle. Another potential conflict opposes young people to adults. For the youngest cohort, an innovation has long lasting effects, since it affects both their future productivity in the labour market and their children's future capacity to acquire human capital. For the adults, however, a new technology will only have an

[3] We assume that there is no uncertainty in the outcome of a new technology: once the decision to shift to the new technology is undertaken, with probability one a productivity enhancement takes place. It follows that we are not dealing with the risky process of producing new ideas, but with the process of implementing existing ideas in new ways that are more efficient, although not for everybody in the same way.

[4] According to Bellettini and Ottaviano (2005), the central authority can be seen as a licensing system that has some agency to approve new technologies before they are brought to the market. Again in Mokyr (1998*a*)'s words: "*almost everywhere some kind of non-marketing control and licensing has been introduced*". A recent example is the creation of standard-setting agencies such as the International Organization of Standardization (ISO).

effect on the ability of future generations to finance their pension. These different incentive structures would hardly coincide.

This paper contributes to two important recent research strands within the field of economic growth: life expectancy and growth (e. g. Blackburn and Cipriani (2002), Chakraborty (2004), Cervellati and Sunde (2005))[5] and vested interest and growth (e. g. Krusell and Rios-Rull (1996), Canton et al. (2002), Bellettini and Ottaviano (2005)).[6] Building on the existing literature, this paper analyzes an economic model in which the interactions among endogenous changes in life expectancy, education, technological improvements and economic growth, suggest that (a) poverty traps can arise in the accumulation process of human capital and have long-lasting effects on aggregate output; (b) at the *individual* level a higher life expectancy increases the incentive to innovate for both young and adults; (c) at the *aggregate* level different configurations can arise depending on endogenous demographic structures; (d) depending on initial conditions and parameter values in the long run both *"Innovation"* and *"No Innovation"* can be feasible steady state. Due to interplay between demographic structures and the private incentives that endogenously change, the transition path to steady states can be characterized by three switches between *"No Innovation"* and *"Innovation"* regimes.

[5] The important role played by life expectancy in determining the optimal education decisions of individuals has already been pointed out by models that analyze the relationship between demographic variables and development. In a recent study, Blackburn and Cipriani (2002) endogenize life expectancy. As a result, their model generates multiple development regimes depending on initial conditions. Endogenizing life expectancy allows Blackburn and Cipriani (2002) to explain jointly the main changes that take place during the demographic transition of economies, such as greater life expectancy, higher levels of education, lower fertility and later timing of births. Cervellati and Sunde (2005) analyze a model in which human capital formation, technological progress and life expectancy are endogenously determined and reinforce each other. In a microfounded theory the authors show that the inclusion of endogenous life expectancy helps to explain the long-term development of economies and, in particular, the industrial revolution experienced by many countries as an endogenous result in the process of development. Chakraborty (2004) also endogenizes life expectancy and assumes that the survival probability depends on the public investment in health. In his model low life expectancy is detrimental for growth because on the one hand, low expectations of surviving make individuals less patient and willing to save and invest and, on the other hand, lower life expectancy also reduces the returns of investing in education. See Galor (2005) for an overview on the literature.

[6] To the best of our knowledge only Canton et al. (2002) have analyzed the relationship between vested interest and economic growth with the focus on the role played by an aging population in determining the optimal technology adoption. The authors argue that when older people face a higher cost of adopting new technologies, political pressure in a democratic system may slow down innovation adoption in an ageing society.

The paper is organized as follows. In section 1.2 we present the mechanics of the model, describing the economic environment and solving both the individual education problem and the aggregate innovation one. Section 1.3 concludes. All proofs are contained in the Appendix.

1.2 The Model

Time is discrete and indexed by $t \in \mathbb{N}$. The economy is populated by a finite number of overlapping generations of homogeneous agents. Each generation consists of a unit mass of individuals ($N_t = N = 1$) living up to three periods. Every agent born at time t survives with probability one from youth to adulthood and with probability p_{t+2} to old age. When people of generation t are young they split their unit time endowment between schooling (e_t) and working as unskilled ($1 - e_t$). Their income comes from their productivity multiplied by time spent working. An innovation cost has to be paid today to implement a new technology in the next period. This innovation cost is a fixed share of income and takes the values $i \in (0, 1)$ or zero in case the innovation is decided or not, respectively. We define the indicator function of i_t, denoted by $\Phi(i_t)$, as follows:

$$\Phi(i_t) = \begin{cases} 1 & if & i_t = i \\ 0 & if & i_t = 0 \end{cases} .$$

Each adult works as skilled and has a single child. Adults' human capital is a function of average human capital of the previous period and the effort they made when young. They produce combining their human capital with a TFP parameter that increases if a new technology is endorsed the period before. This income is divided between consumption, a constant share s that goes, in a PAYGO fashion, in paying their parents' pensions[7] and, in case, the innovation cost i_{t+1}. When old, they consume the pension that their children pass to them, net of the innovation cost i_{t+2}. The scheme of the timing for an agent born at time t is represented in Figure 1.2.

[7] We do not discuss the way in which the pension system is implemented and if it can be politically self-sustaining as, among others, Bellettini and Berti Ceroni (1999) do. We assume that a commitment between generations is in place and no one can default on it.

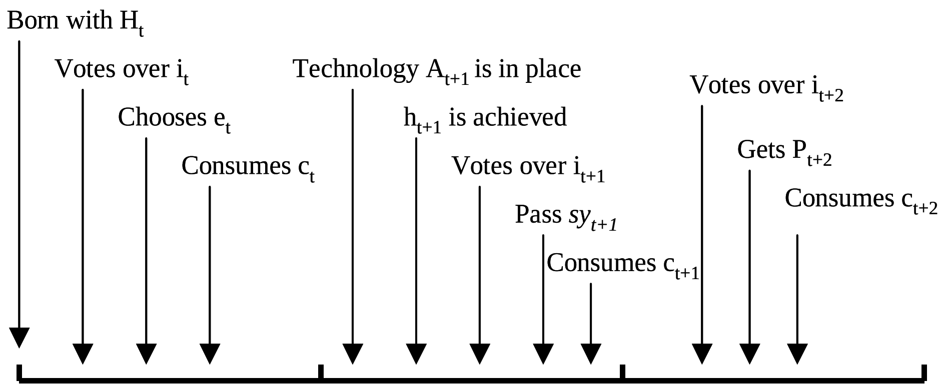

Figure 1.2. Timing for an agent born at time t.

In every period, adult agents individually produce a single homogenous good employing human capital as the sole input, using the publicly available technology A_t. Agents' political lever is characterized by their ability to vote, every period of their life, for a systemic innovation to be implemented in the next period. We replicate the stylized facts that young people show a lower turnout rate at elections – defined as the percentage of people who actually vote among those having the right to – with respect to adults and old.[8] Thus, young's weight in the political process is represented by an exogenous parameter $\eta \in (0, 1]$. All adults and old vote at each period t, so their measure is 1 and p_t, respectively, where p_t is the share of old alive.

1.2.1 Production by Skilled Adults

Each skilled adult produces a homogenous private good using a decreasing return function of human capital, combined with the available technology vintage. The production function at time t is:[9]

$$y_t = A_t h_t^{\gamma} \tag{1.1}$$

[8] As Galasso and Profeta (2004) report, not all potential electors actually vote. In some countries, elderly voters have a higher turnout rate at elections than the young, thus leading to an overrepresentation of the elderly. This voting pattern is strongest in the US, where turnout rates among those aged 60 – 69 years is twice as high as among the young (18 – 29 years). Significant differences appear also in other countries: in France, the turnout rate of the elderly (60 – 69 years) is almost 50% higher than that of the young (18 – 29 years).

[9] Since $N_t = N = 1$ the aggregate production function at time t, Y_t, is $Y_t = y_t$.

where $\gamma \in (0, 1)$. h_t is adult endowment of human capital and A_t is the technological coefficient. Changes in A_t reflect therefore TFP changes. The level of technology employed at time t in the production of output, A_t, depends on the political choice of the previous period $(t-1)$. The TFP parameter A_t is equal to A_{t-1} in case a new technology is not implemented (i.e. $\Phi(i_{t-1}) = 0$), while $A_t = (1+\theta) A_{t-1}$ in case a new technology is implemented (i.e. $\Phi(i_{t-1}) = 1$). At time $t = 0$, $A_0 = A > 0$. A compact formulation for the dynamic evolution of technology parameter, A_{t+1}, is

$$A_{t+1} = (1 + \theta\Phi(i_t)) A_t \tag{1.2}$$

where θ denotes the growth rate of the technology and is a strictly positive scalar.

1.2.2 Investment in Human Capital

In the first period of her life, a member of generation t invests in human capital. The acquisition of skills requires the individual's effort in schooling and a stock of existing human capital, whose average level is $\frac{H_t}{N_t} = H_t$ because $N_t = 1$. The elasticity of past human capital in the production of new human capital is ε. The human capital that an adult gets at time $t+1$, h_{t+1}, is:

$$h_{t+1} = \Upsilon(e_t, H_t, i_t) \equiv \lambda\left((1 - \delta\Phi(i_t)) e_t H_t^{\epsilon}\right) \tag{1.3}$$

where λ is a scale parameter. The properties of the production function of human capital are as follows:

1. *The individuals' level of human capital is an increasing function of the individual's effort in schooling* $\left(\text{i.e. } \frac{\partial\Upsilon}{\partial e_t}(e_t, H_t, i_t) > 0\right)$.

   The importance and the empirical significance of the individual's effort in schooling inputs is well documented in the literature. For a comprehensive survey of the related literature see Mincer (1974).

2. *The individuals' level of human capital is an increasing function of the parental level of human capital* $\left(\text{i.e. } \frac{\partial\Upsilon}{\partial H_t}(e_t, H_t, i_t) > 0\right)$.

   The importance of the parental education input in the formation of the human capital of the child has been explored theoretically as well as empirically. The empirical significance of the parental effects has been documented by Becker and Tomes (1986), as well as others.

3. *There exist diminishing returns to the parental human capital effect* i.e. $\left(\frac{\partial^2 \Upsilon}{\partial H_t^2}\left(e_t, H_t, i_t\right) < 0\right)$.

4. *The level of human capital depreciates by a factor* $(1-\delta)$ *in case an innovation is decided at time* t.

   The assumption is that when new technologies are implemented, human capital produced in schools based upon previous types of technology is less useful. The concept of vintage human capital has been explicitly used in the 90s to treat some specific issues related to technology diffusion, inequality and economic demography. In a world with a continuous pace of innovations, a representative individual faces the typical question of whether to stick to an established technology or to move to a new and better one. The trade-off is the following: switching to the new technique would allow him to employ a more advanced technology but he would lose the expertise, the specific human capital, accumulated on the old technique. For a comprehensive survey of *vintage human capital* literature see Boucekkine et al. (2006).

5. *Ranges for the parameters are* $\lambda > 0$, $0 \leq \delta < 1$ *and* $0 < \epsilon < 1$.

1.2.3 Utility Function and Budget Constraints

Agents born at time t evaluate consumption according to the following intertemporal, non altruistic, expected utility function defined over the vector $c^t \equiv \left(c_t^t, c_{t+1}^t, c_{t+2}^t\right) \in \mathbb{R}^3$:

$$u_t^t = u\left(c_t^t\right) + \alpha u\left(c_{t+1}^t\right) + p_{t+2}\beta u\left(c_{t+2}^t\right) \tag{1.4}$$

where $\alpha, \beta \in (0,1)$ are the impatient factors for the adult and old age consumption, respectively. p_{t+2} is the probability to survive until old age. The function $u(\cdot)$ is concave, twice continuously differentiable and satisfies the Inada condition, i.e. $\lim_{c_t \to 0} u_c\left(c_t\right) = \infty$. Assume that preferences exhibit logarithmic form, i.e. $u(\cdot) = \log(\cdot)$.

Individuals' budget constraints of agents in the three periods are as follows.

$$c_t^t \leq \omega\,(1-e_t)\,(1-i_t) \tag{1.5}$$

Consumption of a member of generation t at time t, c_t^t, is the income generated working as unskilled net of the innovation cost. When young each agent works as unskilled getting a constant wage, ω, that, for simplicity, we normalize to 1. The time devoted to work is $(1-e_t)$. Because of the assumptions $N_t = 1$ and setting $\omega = 1$, young's gross income is $(1-e_t)$.

$$c_{t+1}^t \leq y_{t+1}\,(1-s-i_{t+1}) \tag{1.6}$$

Consumption of a member of generation t at time $t+1$, c_{t+1}^t, is the income received in the skilled sector net of the innovation cost and the pension contribution, required to finance the pension of her parent. s must satisfy the condition: $s < 1-i$.

$$c_{t+2}^t \leq P_{t+2}\,(1-i_{t+2}) \tag{1.7}$$

Consumption of a member of generation t at time $t+2$, c_{t+2}^t, is the pension benefit net of the innovation cost. In the third period of her life, a member of generation t receives

$$P_{t+2}^t = \frac{s y_{t+2}^{t+1}}{p_{t+2}} = \frac{s A_{t+2} h_{t+2}^{\gamma}}{p_{t+2}} \tag{1.8}$$

The pension is the share s of income that an adult of generation $t+1$ disbursed in the PAYGO system, divided by p_{t+2} that takes into account the share of people surviving to old age.

Remark 1 *Ceteris paribus, the pension benefit for an old agent decreases with the lengthening of life expectancy, i.e. with* p_{t+2}.

1.2.4 Individual Optimization with Given Innovation Policy

Agents choose their optimal schooling time when young taking as given the innovation policy.[10] Maximization of Eq. (1.4) subject to the individual budget constraints (1.5), (1.6) and (1.7), in which we previously plugged the human capital production function, (1.3), and Eq. (1.8), yields the

[10] We will add the case of endogenous innovation policy in the paragraph 1.2.6.

optimal schooling time, e_t^*,

$$e_t^* = \frac{\gamma[\alpha + \beta\epsilon p_{t+2}]}{1 + \gamma[\alpha + \beta\epsilon p_{t+2}]} \tag{1.9}$$

Remark 2 *The longer is the life expectancy, the higher is the time investment needed to finance their prolonged consumption, consistently with existing literature.*[11]

The positive effect of p_{t+2} on e_t^* arises because agents know that the only way to get higher pension benefits from their children is to invest in their own education. This, in turns, positively affects their children's human capital and, ultimately, their children's income.

Substituting Eq. (1.9) in Eq. (1.3) and writing h_t instead of H_t (since in equilibrium $h_t = H_t/N_t$ and by assumption $N_t = 1$) we get the accumulation function of human capital as a function of the previous level of human capital, the innovation policy chosen the period before and the fraction of time young spend in education. We obtain:

$$h_{t+1} = \lambda\left(1 - \delta\Phi\left(i_t\right)\right)\frac{\gamma[\alpha + \beta\epsilon p_{t+2}]}{1 + \gamma[\alpha + \beta\epsilon p_{t+2}]}h_t^{\epsilon} \tag{1.10}$$

The human capital accumulation function shows a concave shape (given that $0 < \epsilon < 1$) and undergoes a reduction in case an innovation takes place (i.e. $\Phi\left(i_t\right) = 1$).

1.2.5 Endogenous life expectancy

In this subsection we allow for the level of life expectancy to increase with the aggregate human capital level.[12] For an agent born at time t the probability to reach old age is, therefore, $p_{t+2} = p\left(H_t\right)$. We impose some restrictions on $p\left(H\right)$, in order to get simple results. $p\left(0\right) = p^L > 0$ avoids the extreme case of a disappearing old age, while $\frac{\partial p(H)}{\partial H} \geq 0$ replicates the empirical evidence of a positive correlation between life expectancy and human capital.[13] Since

[11] The positive effect of longevity on education is emphasized by Blackburn and Cipriani (2002), Chakraborty (2004) and Cervellati and Sunde (2005). For further evidence on the effect of health and living conditions on education attainments, see De la Croix and Licandro (1999), Lagerlof (2003) and Galor (2005).

[12] As in, among others, Blackburn and Cipriani (2002), Boucekkine et al. (2002), Cervellati and Sunde (2005).

[13] Empirically, both private and aggregate endowment of human capital are conductive to a longer life, although we focus on the aggregate view: on the one hand, demographic

p is a probability, we assume that $\lim_{H\to+\infty} p(H) = p^H \leq 1$. For simplicity, we set $p^H = 1$. Simple algebra and the identity $h_t \equiv H_t$ allow us to rewrite the expression of human capital accumulation, (1.10), as follows:

$$h_{t+1} = \Gamma(h_t; i_t)\, h_t^{\epsilon}$$

The function Γ always takes positive values, is non decreasing in h ($\Gamma_1(h_t; i_t) \geq 0$) and, for the restrictions imposed on the function p, is limited from above by some finite number.

Proposition 1 *For a given i_t it is always possible to explicitly find a continuous increasing function $\Gamma(h_t; i_t)$ such that $h_{t+1} = \Gamma(h_t; i_t)\, h_t^{\epsilon}$ shows multiple steady states.*

Proof. (See Appendix). ■

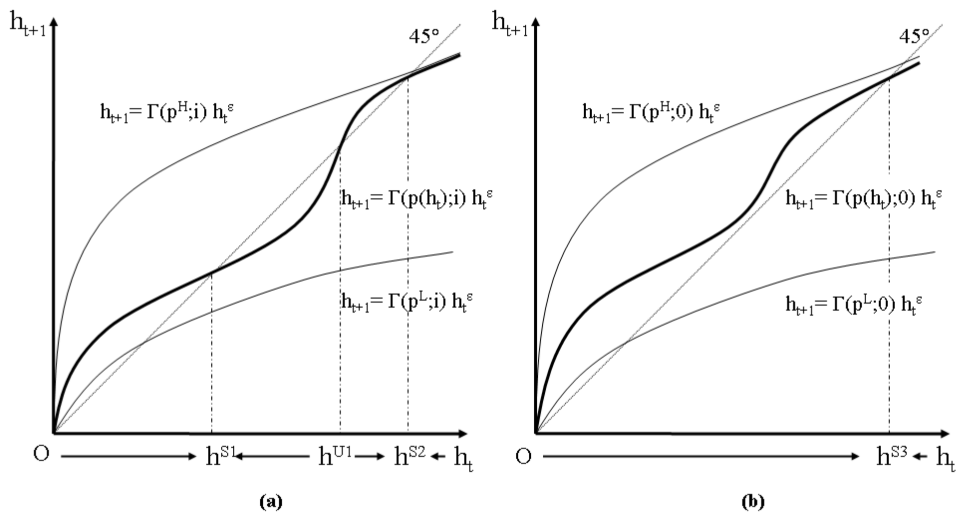

Figure.1.3. Human capital Equilibrium in the case of Innovation, Panel (*a*), and no Inn., Panel (*b*).

and historical evidence suggests that the level of human capital profoundly affect the longevity of people. For example, the evidence presented by Mirowsky and Ross (1998) supports strongly the notions that better educated people are more able to coalesce health-producing behaviour into a coherent lifestyle, are more motivated to adopt such behaviour by a greater sense of control over the outcomes in their own lives, and are more likely to inspire the same type of behaviour in their children. Schultz (1993, 1998) evidences that children's life expectancy increases with parent's human capital and education. On the other hand, there is evidence that the human capital intensive inventions of new drugs increases life expectancy (Lichtenberg, 1998, 2003) and societies endowed with an higher level of human capital are more likely to innovate, especially in research fields like medicine (Mokyr, 1998*b*).

In Figure 1.3 we show the peculiar case in which the introduction of an innovation leads the economy from a unique steady state case to a multiple steady state one. Panel (a) highlights the case of *innovation* (i.e. $i_t = i$). h^{S1} and h^{S2} are stable equilibria, while h^{U1} is the unstable, positive one. Relying on the concept of vintage human capital described above, the whole graph of h_{t+1} lies below the one of no innovation. It can be, therefore, the case that if innovation takes place there is room, due to the depreciation of human capital, for two stable steady states, while in the case of no innovation only one stable steady state occurs. Panel (b) shows the case of *no innovation* (i.e. $i_t = 0$). The graph of h_{t+1} is higher and only one stable steady state, h^{S3}, arises.

Apart from the innovation policy, increases in the weight of both adult (α) and old age (β) consumption, the constant of proportionality (λ), the productivity of human capital in final good production (γ) and the elasticity of past human capital in the production of new human capital (ϵ) shift h_{t+1} upward, leading both to higher level of human capital for any steady state and, in case, to the disappearance of the low steady state, h^{S1} in Figure 3.

Remark 3 *The fact that* (i) *the growth of human capital is bounded and* (ii) *human capital is the only factor of production and its accumulation function does not depend upon the level of the TFP parameter allows us to study, in an "additive" way, how human capital and production evolve.*

For example, once human capital reaches a steady state, using Eq. (1.1) we can keep track of the final production looking solely at the innovation policy undertaken. Therefore the steady state production is a constant level in the case of no innovation, $y^* = A_0 \left(h^{S*}\right)^{\gamma}$, while it will increase at the constant rate θ in the case of innovation, $y_t = A_0 \left(1+\theta\right)^t \left(h^{S*}\right)^{\gamma}$. The value h^{S*} represents one of the stable steady states reached by the human capital function.

1.2.6 Endogenous Innovation Policy: Aggregation Rule and Individual Choices

In this section we endogenize the process of technology adoption by means of a majority voting mechanism. At every point in time the agents belonging to the three age classes vote for a new technology to be implemented in the next period. The decision to adopt a new technology is endorsed if the majority of agents votes in favor of it.[14] At time t young of generation t, adults of generation $(t-1)$ and (survived) old born at time $(t-2)$ are alive. Their political weights, whose sum is normalized to one, are

$$\frac{\eta}{\eta+1+p_t}, \frac{1}{\eta+1+p_t} \text{ and } \frac{p_t}{\eta+1+p_t},$$

respectively.

Remark 4 *The longer the life expectancy is, the larger is the political weight of old and the smaller is that of both young and adults.*

Lemma 1 *For values of old's life expectancy p_t smaller than the threshold p^O:*

$$p_t < p^O = 1 - \eta$$

a "workers' dictatorship" arises at time t: no matter what young and old prefer, adults alone will set the agenda in terms of innovation. There are no values of p_t such that another age class alone can decide upon innovation.

Proof. (See Appendix). ■

In the early stages of development $\left(\text{i.e. } p_t < p^O\right)$ the political power is, therefore, in the hands of adult alone. Meanwhile the accumulation of human capital leads to longer life expectancy and ultimately to smaller shares of both young and adults. Once p_t reaches and exceeds p^O decisions about innovation cannot be supported by adults alone. In order to implement a new technology, the economy needs the consensus of at least two age classes. We call this subsequent stage of development *"diluted power"*. Note that the specific cost-benefit setup of the innovation implies that old people are always *against* innovation: they are supposed to pay today a fraction of their income for a new technology that will be available once they are dead. In the case of *"workers' dictatorship"* this feature is not influential, since adults have the absolute majority. On the contrary in the case of $p_t > p^O$ an innovation is implemented if and only if both young and adults vote in favor of a progressive policy.[15] Therefore, if either young, adults or both

[14] It is possible to restate the mechanism of deciding upon technology adoption in terms of median voter, but we find this approach very clear and intuitive.

[15] With *progressive policy* we indicate the adoption of a new technology. Conversely, *conservative policy* means no adoption.

these age classes vote against innovation, a conservative policy will be put in place.

Definition 1 v_t^j *is the individual preference over the innovation policy voted by an agent of age* j *at time* t *(with* $j \in \{Y; A; O\}$ *standing for young, adults and old, respectively).* v_t^j *can take the states* $\{\iota; \nu\}$*, indicating a vote in favor of innovation and a vote against innovation, respectively.*

Note that old's choice is always to vote against innovation, as will be shown below: $v_t^O = \nu, \forall t \in \mathbb{N}$. The function M_t aggregates the votes of the three generations alive at time t and its outcome is the majority choice:

$$M_t(v_t^Y; v_t^A; v_t^O) = \begin{cases} I & \text{if} \quad \begin{cases} v_t^Y = v_t^A = \iota \text{ and } p_t \geq p^O \\ v_t^A = \iota \text{ and } p_t < p^O \end{cases} \\ \\ N & \qquad\qquad \text{otherwise} \end{cases} \tag{1.11}$$

Whenever $M_t = I$ the innovation cost applies to every agent alive at time t and the new technology A_{t+1} is disposable at time $(t+1)$. Conversely, if $M_t = N$ agents do not pay any innovation cost and they produce, at time $(t+1)$, with technology A_t. The majority choice $M_t = \{I; N\}$ maps, through the biunivocal function $i_t = i\,(M_t)$, into the set $\{0; i\}$.

In order to have an intertemporal voting equilibrium it is required that, in every period, agents optimally choose the innovation policy, taking future outcomes as given. Since people live up to three periods, young face three-period sequences of policies, adults two-periods ones and old have just one policy choice to do.[16]

Now we turn to the analysis of how each age class votes taking into account the optimal future political and economic choices. An agent belonging to age class j at time t bases her choice on the difference between the utility she gets in the case she votes in favor or against innovation. The stream

[16] Being the two values of policy variable $M = \{I; N\}$ ("innovation" and "no innovation", respectively), young born at time t face eight possible streams of policies: $\{I_t; I_{t+1}; I_{t+2}\}$; $\{I_t; I_{t+1}; N_{t+2}\}$; $\{I_t; N_{t+1}; I_{t+2}\}$; $\{I_t; N_{t+1}; N_{t+2}\}$; $\{N_t; I_{t+1}; I_{t+2}\}$; $\{N_t; I_{t+1}; N_{t+2}\}$; $\{N_t; N_{t+1}; I_{t+2}\}$; $\{N_t; N_{t+1}; N_{t+2}\}$. Adults at time t face four possible streams: $\{I_t; I_{t+1}\}$; $\{I_t; N_{t+1}\}$; $\{N_t; I_{t+1}\}$; $\{N_t; N_{t+1}\}$. Old people just face the decision $\{I_t\}$ or $\{N_t\}$.

of future majority choices and outcomes over which the agent forms correct expectations is $V_{t+1} = \{M_{t+1}; e^*_{t+1}; M_{t+2}; e^*_{t+2}; ...\}$. For every age class $j \in \{Y; A; O\}$ we define the differential utility as

$$\Delta u_t^j (V_{t+1}) = u_t^j \left(v_t^j = \iota; V_{t+1} \right) - u_t^j \left(v_t^j = \nu; V_{t+1} \right)$$

that collapses to

$$\Delta u_t^j (V_{t+1}) = u_t^j \left(v_t^j = \iota \right) - u_t^j \left(v_t^j = \nu \right)$$

because of the specification of the utility function described above. In fact, the outcome of future innovation policies and educational choices do not influence agent's differential utility: income and substitution effects of the innovation cost cancel out. Since at the beginning of their life agents cannot commit themselves to a specific stream of votes, at each moment in time each of them votes to maximize her expected future lifetime utility. For a young agent born at time t the expected future lifetime utility is

$$u_t^Y = \log c_t^t + \alpha \log c_{t+1}^t + p_{t+2} \beta \log c_{t+2}^t \tag{1.12}$$

that coincides with Eq. (1.4). Expected future lifetime utility for an adult born at time $(t-1)$ is defined as

$$u_t^A = \alpha \log c_t^{t-1} + p_{t+1} \beta \log c_{t+1}^{t-1} \tag{1.13}$$

while the one of an old agent born at time $(t-2)$ is

$$u_t^O = \beta \log c_t^{t-2} \tag{1.14}$$

In the last expression the probability p_t does not appear because only survived old choose. The single age classes choose how to vote as follows.

Old Old people, in the case of a progressive policy, only incur in costs: once the new technology is in place, they will be dead. Their differential utility is therefore

$$\Delta u_t^O = u_t^O \left(v_t^O = \iota \right) - u_t^O \left(v_t^O = \nu \right) = \beta \log (1 - i) < 0, \forall i \in (0, 1)$$

where we plugged Eq. (1.7) and Eq. (1.8) into Eq. (1.14).

Remark 5 *Old's optimal choice is to always vote against innovation.*

Adults When adult, agents vote over the innovation that will be implemented the next period. As described above, their differential utility depends only on present innovation choices.

$$\Delta u_t^A = u_t^A \left(v_t^A = \iota \right) - u_t^A \left(v_t^A = \nu \right) \tag{1.15}$$

By substituting Eq. (1.6), (1.7), (1.10) and (1.13) into (1.15), we get:

$$\begin{aligned} \Delta u_t^A \left(p_{t+1} \right) = \alpha \log \left(1 - i - s \right) &+ p_{t+1} \beta \log \left(1 + \theta \right) \\ + p_{t+1} \beta \gamma \log \left(1 - \delta \right) &- \alpha \log \left(1 - s \right) \end{aligned} \tag{1.16}$$

The first and fourth terms jointly show the differential negative impact of the innovation cost on the net income when adult: in the case of innovation the share of income going to finance adult age consumption shrinks. The second term represents the gain in productivity attached to the pension income when old, weighted by the probability to survive. The third term is the negative impact of an innovation on the stock of human capital acquired by adult's child: this translates in smaller pensions benefits for the adult herself when old.

Let us assume from now on that

$$\left(1 + \theta \right) \left(1 - \delta \right)^{\gamma} > 1 \iff \theta > \left(1 - \delta \right)^{-\gamma} - 1 \tag{1.17}$$

This condition on the relative magnitude of TFP improving parameter and human capital depreciation parameter states that the productivity improvements in the production of final good (θ) exceeds an increasing function of both the rate of depreciation of the human capital in the case of innovation (δ) and its productivity in the production of the final good (γ). We rewrite (1.16) in a compact way, since it will be useful in the next subsection.

Remark 6 *Adults' differential utility can be represented by a linear positive relation linking Δu^A to p, dropping the time index for simplicity:*

$$\Delta u^A \left(p \right) = m^A \left(\theta, \delta, \gamma, \beta \right) p + q^A \left(s, i, \alpha \right) \tag{1.18}$$

where $m^A = \beta \log\left((1+\theta)(1-\delta)^\gamma\right)$ *and* $q^A = \alpha \log\left(\frac{1-s-i}{1-s}\right)$.

Lemma 2 *Adults vote for the adoption of a new technology if and only if they achieve a life expectancy* p_{t+1} *larger than the threshold* p^A*, defined as*

$$p^A = \frac{\alpha \log\left(\frac{1-s}{1-s-i}\right)}{\beta \log\left((1+\theta)(1-\delta)^\gamma\right)} \tag{1.19}$$

Conversely, if $p_{t+1} < p^A$*, they are against.*

Proof. (See Appendix). ■

Adults vote for an innovation if and only if they will get higher resources (net of innovation costs) when old, in the form of pensions paid by their adult children. The threshold p^A is a positive function of i: the more expensive is the adoption of a new innovation, the less the adult will be innovation-prone. The same consideration holds for δ: due to the adoption of a new technology, the more the human capital depreciates, the less the adult will be in favour of implementing the new technology itself. Conversely, higher growth rates of TFP make adults to prefer innovations. Note that the elasticity of past human capital in the production of the new human capital (ϵ) is not involved in adult's decisions: we will see below that only young take into account how the past level of human capital affects the next period's human capital accumulation. The higher the share of adult's income going to finance old's pensions is (s), the less the adult will be innovation-prone. The higher is the preference for adult age consumption (α), the more they will be against innovation. Conversely, preference for old age consumption (β) leads to preference for innovation. This is because of the structure of innovative process: it is a cost today and it gives benefits tomorrow. Lastly, an increase in the elasticity of human capital in the production of final good (γ) works against innovation: innovation makes part of the human capital achieved during youth to depreciate, and the higher its effectiveness in production is, the higher the loss is in terms of pensions paid by adults' adult children.

Young Young vote over innovation taking into account their expected future lifetime utility but, for the same arguments stated above, what will happen at time $(t+1)$ and $(t+2)$ does not influence young's vote today.

Young's differential utility is therefore:

$$\Delta u_t^Y = u_t^Y \left(v_t^Y = \iota\right) - u_t^Y \left(v_t^Y = \nu\right) \tag{1.20}$$

By substituting Eq. (1.5), (1.6), (1.7), (1.10) and (1.12) into (1.20), we get:

$$\begin{aligned}\Delta u_t^Y \left(p_{t+2}\right) =& \log\left(1-i\right) + \alpha\log\left(1+\theta\right) + \alpha\gamma\log\left(1-\delta\right) \\ &+ p_{t+2}\beta\log\left(1+\theta\right) + p_{t+2}\beta\epsilon\gamma\log\left(1-\delta\right)\end{aligned} \tag{1.21}$$

Young, in case of innovation, again directly benefit from the technologic parameter θ, but now it impacts both on their labour income when adults and on their pension benefits when retired. In this last case the benefit from innovation is proportional to p_{t+2}, so a longer life gives them more time to enjoy higher consumption. The cost structure is similar: a constant cost is due to the depreciation of human capital when young become adults, through a smaller marginal productivity in the production of final good. Another cost, proportional to p_{t+2}, takes into account the depreciation of human capital of young's children: two periods later, in fact, today's young will get a pension that will be, in terms of human capital, depreciated because of today's choice to innovate. Therefore the depreciation term is mitigated by two terms, ϵ and γ: the former takes into account the elasticity between the production of new human capital and the past stock of human capital, the latter the elasticity of human capital in the production of final good.

Consistently with the case of adults, we rewrite Eq. (1.21) in the same fashion.

Remark 7 *Young's differential utility can be represented by a linear positive relation linking Δu^Y to p, their life expectancy:*

$$\Delta u^Y\left(p\right) = m^Y\left(\theta,\delta,\gamma,\epsilon,\beta\right)p + q^Y\left(\theta,\delta,\gamma,i,\alpha\right) \tag{1.22}$$

where $m^Y = \beta\log\left(\left(1+\theta\right)\left(1-\delta\right)^{\gamma\epsilon}\right)$ *and* $q^Y = \log\left(\left(1-i\right)\left(\left(1+\theta\right)\left(1-\delta\right)^{\gamma}\right)^{\alpha}\right)$.

Lemma 3 *Young vote for the adoption of a new technology if and only if* *they achieve a life expectancy p_{t+2} larger than the threshold p^Y, defined as*

$$p^Y = \frac{-\left[\log\left((1-i)\left((1+\theta)(1-\delta)^{\gamma}\right)^{\alpha}\right)\right]}{\beta\log\left((1+\theta)(1-\delta)^{\gamma\epsilon}\right)} \tag{1.23}$$

Conversely, if $p_{t+2} < p^Y$, young are against innovation.

Proof. (See Appendix). ■

Young's choices over innovation show similar determinants as adult's ones. Again the threshold level is negatively correlated with the TFP growth rate (θ) induced by innovation. The depreciation of human capital in the case of innovation (δ) is a factor that discourages young, as long as adult, to vote for innovation. For young, increases in both adult and old age consumption preferences make them to be more prone to innovation.

The effect of the elasticity of past human capital in the production of human capital (ϵ) on p^Y is positive: $\frac{\partial p^Y}{\partial \epsilon} > 0$. A high inertia in the transmission of human capital from one generation to the other leads to less interest in innovation because, as in Boucekkine et al. (2002). *Ceteris paribus*, the more the accumulation of human capital relies on past human capital, the more it depreciates in case of innovation. Differently from the case of adults, for young preference for both adult (α) and old (β) age consumption is conducive to innovation.

	p^O	p^A	p^Y	p^*
political weight of young people (η)	-	0	0	0
preference for adult age consumption (α)	0	+	-	+
preference for old age consumption (β)	0	-	-	+
productivity gains from innovation (θ)	0	-	-	0
frictional costs of innovation (i)	0	+	+	-
depreciation of human capital due to innovation (δ)	0	+	+	-
productivity of human capital in final good production (γ)	0	+	+	+
elasticity of past h in the production of new h-capital (ϵ)	0	0	+	+
share of adults' income used to pay parents' pensions (s)	0	+	0	0
constant of proportionality in h-capital production (λ)	0	0	0	+

Table 1.1. Partial effects of parameters on thresholds.

In Table 1.1 we sum up the partial effects of the parameters on the thresholds p^O, p^A and p^Y. Moreover, we add the effects of the same parameters on all the steady state values (p^*) of the function $h_{t+1} = \Gamma_1 (h_t; i_t)\, h_t^\epsilon$, derived in the previous section. This will turn out to be useful in the section where we jointly study the economic and political mechanisms.

1.2.7 Political Outcome

We now define the *static* political outcome at each point in time, given the life expectancies of every age class. We derive some propositions, in terms of key parameters of the economy, that allow to classify the economy's dynamic features, analyzed in the next subsection. To resume how the political choice works, in Figure 1.4 we plot the graphs of $\Delta u^A(p)$ and $\Delta u^Y(p)$ (Eq. (1.18) and Eq. (1.22), respectively) and report, on the p axis, the value of p^Y, p^A, 1 and (an arbitrary) p^O.

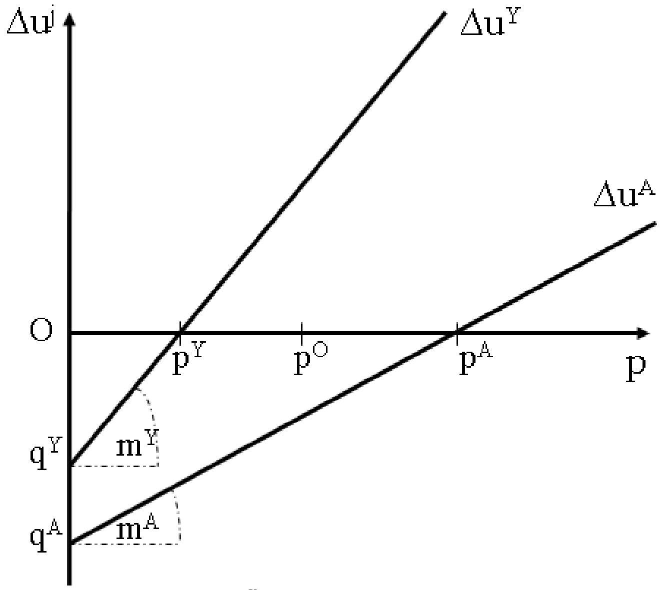

Fig. 1.4. Differential utilities and thresholds.

At time t the three generations alive are represented by their own life expectancies: p_t for old people born at time $(t-2)$, p_{t+1} for adults born at time $(t-1)$ and p_{t+2} for young people born at time t.[17] Life expectancy of each of the three age class is therefore compared with the corresponding threshold: p_t with p^O, p_{t+1} with p^A and p_{t+2} with p^Y. From (1.11) we know

[17] More precisely, the best interpretation of old people's p_t is not in term of life expectancy, but as their mass in the political choice at time t.

that for $p_t < p^O$ adults alone decide upon innovation. If $p_t > p^O$, innovation takes place only if both $p_{t+1} > p^A$ and $p_{t+2} > p^Y$.

Proposition 2 *With standard intertemporal discounting behavior, i.e. $\beta = \alpha^2$, it is never the case of $p^A < p^Y < 1$. Moreover, in the case of $\beta > \alpha^2$,it is possible to find a parametrization characterized by small values of α, large values of β and intermediate values of i such that $p^A < p^Y < 1$ is feasible.*

Proof. (See Appendix). ■

The Proposition above states that when the discount factor is independent from the time index but it only depends on the *distance* between two points in time (the case $\beta = \alpha^2$), there are not feasible values of life expectancy such that adults are in favor of innovation while young are not. That is because young get a double benefit from innovation, both during their adulthood and old age. Discounting them in the same way, it is intuitive that once they became adult they cannot "do better" than when they were young, in the sense that they discount old age consumption in the same way as before, but now the gains will only be from one side (i.e. higher pension contributions by their children) and will be only a *fraction*, depending on s, of their child's gain in productivity. The second part of the Proposition states that if people attach a large weight on old age consumption, for some values of life expectancy it can be the case that, when young, they are not in favor of innovation, while adults are. This is because the variable part of net gains young get with innovation (the last two terms in Eq. (1.23)) are only in part linked to life expectancy, and therefore they are less reactive to large values of β. Adults' variable part and constant part of net gains are instead directly linked by the parameters α and β (see Eq. (1.19)). By allowing $\frac{\beta}{\alpha}$ to increase, the differential expected future lifetime utility of adults increases at an higher rate than that of young, giving rise to the case of $p^A < p^Y < 1$.

Corollary 1 *Young are in favor of innovation for any given level of life expectancy if innovation costs are small enough, i.e. $i < 1-((1+\theta)(1-\delta)^{\gamma})^{-\alpha} \equiv i^*$.*

Proof. (See Appendix). ■

Intuitively, since young get benefits in adult age and adulthood is reached with probability one, for large enough productivity improvement from innovation (θ) they are favorable to innovation if it is cheap ($i < i^*$), no matter what is their life expectancy. An implication of this Corollary is that the decision to adopt a new technology is therefore in the hand of adults alone when frictional costs are small.

Noting that whenever $p^Y < p^A$ the political outcomes in the case of *"workers' dictatorship"* and *"diluted power"* are the same,[18] we derive the following key Proposition.

Proposition 3 *Whenever $\beta = \alpha^2$ the decision to adopt a new technology is made by adults alone: no matter the value of p^O, innovation is implemented iff $p_{t+1} > p^A$.*

Proof. (See Appendix). ■

The previous Proposition, that is only based on *utility* parameters, rules out the case of adult deciding alone to innovate against the will of both young and old. *Workers' dictatorship* does not arise whenever intertemporal discounting shows usual exponential behavior. However, the case of $p^A < p^Y < 1$ is only a necessary condition for an innovative *workers' dictatorship* to arise: when reaching life expectancy p^A adults need to be the absolute majority, so to implement their preferred policy. We resume this in the following Lemma.

Lemma 4 *Whenever $p^A < p^Y < 1$ and $\eta < 1 - p^A$ an innovative workers' dictatorship arises for adults' life expectancy $p_{t+1} \in \left(p^A; \min\{p^Y; p^O\}\right)$.*

Proof. (See Appendix). ■

This Lemma states that, although observationally equivalent, innovation episodes arise from very different sources: it can be a strong majority of adults alone, coupled with a "light" presence of young, that decides to implement an innovation or it can be a coalition of young and adults. In this last case the weight of young in the political decision is not important, since adults and young together hold always an absolute majority of votes.

We have now a number of Propositions and Lemmas that allow to taxonomize and describe the dynamic evolution of the economy. This is done in the next Subsection.

[18] It is clear inspecting Eq. (1.11).

1.2.8 Dynamics and Discussion

The transitional dynamic of the economy during the adjustment toward the steady state is the core analysis of this section. The artificial economy we describe is one in which initial life expectancy is small but increasing and people are not yet in favor of innovation, in order to give an example of some dynamic behaviors of the economy. We leave to the reader the analysis of other kinds of dynamics, easily derivable from the economy's properties described up to this point.[19] We therefore assume two restrictions to hold for initial life expectancy p_0:[20]

$$p_0 < \min\{p^O; p^A; p^Y\} \tag{1.24}$$

$$p\left(h^{U_0}\right) < p_0 < p\left(h^{S_0}\left(i_t\right)\right) \tag{1.25}$$

where (1.25) means that initial life expectancy p_0 lies between the values that $p(h)$ takes at the two successive unstable and stable steady states of $h_{t+1} = \Gamma_1(h_t; i_t) h_t^{\epsilon}$, respectively. These assumptions ensure that at time $t = 0$ life expectancy is monotonically increasing toward its steady state value $p^S = p\left(h^{S_0}(i_t)\right)$. The preferred policy is N because, due to Eq. (1.19) and (1.23), agents vote against innovation. We further assume that two more restrictions hold: $p^A < 1$ and $p^Y < 1$, so that both adults and young can, in principle, be in favor of innovation for large enough values of life expectancy.[21] We keep track of the evolution of p_t knowing that it converges monotonically toward its steady value p^S. The evolution of p_t allows us to describe the (possible) variations in the innovation policy adopted. The political outcome, defined by Eq. (1.11), depends on (i) the relative ordering of $\{p^O; p^A; p^Y\}$ and (ii) the one-to-one comparison between the triplets $\{p_t; p_{t+1}; p_{t+2}\}$ and $\{p^O; p^A; p^Y\}$. Moreover, where p^S is located with respect to p^O, p^A and p^Y affects the long run policy implemented.

Given assumptions (1.17), (1.24) and (1.25), up to four dynamic scenar-

[19] As an example, other kinds of dynamics include cases in which initial life expectancy is *decreasing* toward a lower steady state or cases in which the initial undertaken policy is I.

[20] Without loss of generality, we assume that p_0 is the life expectancy of young born at time $t = 0$. Note that $p_0 \neq \underline{p}$: the former is the value of life expectancy that the economy shows at time $t = 0$, the latter is the value of life expectancy that function $p(h)$ takes for $h = 0$.

[21] The inequalities $p^A < 1$ and $p^Y < 1$ resolve in $\frac{1-s}{1-s-i} < \left((1+\theta)(1-\delta)^{\gamma}\right)^{\frac{\beta}{\alpha}}$ and $\frac{1}{1-i} < (1+\theta)^{\alpha+\beta}(1-\delta)^{\gamma(\alpha+\beta\epsilon)}$, respectively.

ios are possible.

Proposition 4 *The evolution over time of an economy characterized by an increasing life expectancy, whose initial value is* $p_0 < \min\{p^O; p^A; p^Y; p^S\}$ *and* δ *is small, shows up to four different dynamics in term of innovation policy. Which of the four dynamics the economy experiences depends on the four thresholds' ordering. The ordering of the thresholds depends, in turns, on the underlying parameters. The four possible dynamics are the following:*

1. *The economy never engages in innovation whenever* (1.*a*) $\beta = \alpha^2$*, the share of income going to pension system* s *is large and the productivity parameter* γ *is small or* (1.*b*) β/α^2 *is sufficiently larger than* 1 *and human capital production is not very effective* (*small* γ, ε *and/or* λ) *or* (1.*c*) β/α^2 *is sufficiently larger than* 1*, young's political weight* η *is large and human capital production is not too effective.*

2. *Whenever* (2.*a*) $\beta = \alpha^2$ *and the scale parameter* λ *is large or* (2.*b*) β/α^2 *is substantially larger than* 1 *and* α *takes small absolute values, the economy at some point switches to a regime of steady innovation adoption, ending up in a steady state in which output grows over time at a rate* θ.

3. *The economy experiences innovation for a limited time span. Before and after this limited period of enhanced output growth the economy evolves without innovating, ending up in a steady state in which output is constant over time. This is the case for* β/α^2 *larger than* 1*, small absolute values of* α *and relatively small young's political weight* η.

4. *The economy experiences two waves of innovation, the second of which lasts forever. The economy behaves as in the previous point* (3)*, but before reaching the human capital steady state* h^{S_0} $\left(\text{and, therefore, } p = p^S\right)$ *it again incurs in preference for innovation. Its output's steady state growth rate is* θ*. This dynamic behavior is achieved for values of* α *larger than in the previous case.*

Proof. (See Appendix). ■

The four scenarios depicted in the previous proposition describe the four different regimes that an economy characterized by endogenous increase in life expectancy and centralized decisions upon innovation policy can show.

Note that regimes 3 and 4 and, in general, ordering of $\{p^O; p^A; p^Y; p^S\}$ in which $p^A < p^Y$ are not feasible in the case the intertemporal discounting behavior of agents is characterized by $\beta = \alpha^2$.

1.3 Conclusions

Over the past century, all OECD countries have been characterized by a dramatic increase in economic conditions, life expectancy and education attainments. This paper examines the unexplored interactions among aging, human capital formation, technology adoption and economic growth. Assuming that longevity is positively correlated with the level of human capital, it demonstrates that an increase of life expectancy is, in principle, a growth-enhancing factor. However, its effectiveness can be harmed by two phenomena, one related to human capital accumulation and the other to aggregation issues about technology adoption.

We reach Blackburn and Cipriani's (2002) same conclusions about the pure economic effects of an increase in longevity. Due to the positive causal effect of human capital on expected life expectancy, it can be the case that small levels of human capital lead to a short life, and this in turn disincentives people to invest in education, giving rise to a poverty trap. At this stage of development, life expectancy is short and human capital stock is small.

About the political features of our economy, we find that a variation in life expectancy affects both the individual incentives to innovate and the aggregate choices of the economy, since political representativeness of different age classes changes. At individual level a higher life expectancy increases the incentive to innovate for both young and adults. However, at the aggregate level different configurations can arise due to the endogenous changes in the demographic structure. Relatively to the predictions about the transition toward the steady state, we find that during the first stages of development, when (i) human capital is negligible, (ii) life expectancy is short and (iii) retired people are few, the political power is in the hand of adult workers alone. The decision to innovate or not coincides, therefore, with adults' choice. In the case their incentives to innovate are small (i.e. a large share of labour income going to finance the pension system, a large elasticity of the human capital used in production or a high concern in adult age consumption) they impose to the whole economy a no innovation regime. In developed economies, where (i) life expectancy is long, (ii) human capital endowment is large and (iii) retired people are several, a political majority that enforces an innovation policy can be achieved only by means of a coalition. Since elderly people are innovation averse, the only way for an innovation to be implemented is that both young and adult are in favour of innovation. Therefore, if on the one hand a longer life expectancy pushes people's incentives toward innovation, on the other hand it makes the political weight of old to increase, making the achievement of a consensus for innovation potentially more difficult. This is true, in particular, when young's incentives for innovation are lower than the ones of adult, in the case of a high inertia in the transmission of human capital from one generation to the next one and when the preference for old age consumption is large. However, if intertemporal discounting is standard, the case of adults in favor of innovation and, at the same time, young against is not feasible. With this paper we provide the basis for joining together two strands of the literature on economic growth that are gaining importance in the research and political debate: technologic innovation and aging population. We stress how different links run between these two phenomena, defining the possible conflict of interests among different generations and showing how the lengthening of life expectancy changes the way this conflict of interests is solved. Moreover, we stress how private and public choices combine (or not) in order to give birth to a human capital abundant, growing economy.

1.4 Technical Appendix

Proof of Proposition (1). For simplicity, we drop the time index and substitute H with h. Let $p(h) = \frac{p^L + p^H \left(\frac{h}{h^F}\right)^{\sigma}\left(\frac{\sigma-1}{\sigma+1}\right)}{1+\left(\frac{h}{h^F}\right)^{\sigma}\left(\frac{\sigma-1}{\sigma+1}\right)}$, with $\sigma > 1$ and $0 < h^F < \infty$. Straightforward calculations lead to $p'(h) \geq 0$, $p''(h) \gtreqless 0$ for $h \lesseqgtr h^F$, and $p'\left(h^F\right) = \frac{\left(p^H - p^L\right)}{4h^F}\left(\frac{\sigma^2-1}{\sigma}\right)$. h^F is therefore the value of h such that $p(h)$ shows an inflection point. Note that $p'(h)\,|_{h=h^F} > 0$ and $\frac{\partial p'(h)|_{h=h^F}}{\partial \sigma} > 0$. From (1.10) we build the function $\tilde{\Gamma}(p(h);h) = \Gamma(h_t;i_t)\,h_t^{\epsilon}$, where we (i) separate the dependency from human capital and life expectancy and (ii) drop the innovation variable i_t. Playing with λ we obtain that for $h = h^F$ the limiting functions $\tilde{\Gamma}\left(p^L;h\right)$ and $\tilde{\Gamma}\left(p^H;h\right)$ take values below and above h^F, respectively i.e. $\tilde{\Gamma}\left(p^L;h^F\right) < h^F$ and $\tilde{\Gamma}\left(p^H;h^F\right) > h^F$. Since $\lim_{\sigma\to+\infty} p'\left(h^F\right) \to +\infty$, the function $\tilde{\Gamma}(p(h);h)$ takes values $\tilde{\Gamma}\left(p\left(h^F - \triangle h\right);\left(h^F - \triangle h\right)\right) < h^F$ and $\tilde{\Gamma}\left(p\left(h^F + \triangle h\right);\left(h^F + \triangle h\right)\right) > h^F$ for any $\triangle h = o(h) > 0$ and $1 < \sigma^M(\triangle h) < \sigma < +\infty$, where $\sigma^M(\triangle h)$ is a threshold related to the (arbitrary small) magnitude of $\triangle h$. For continuity of $\tilde{\Gamma}(p(h);h)$ there is a steady state at h^F where function $\tilde{\Gamma}(p(h);h)$ crosses the 45 degrees line from below. This steady state is therefore unstable. Calculus inspection shows that $\frac{\partial\left(\tilde{\Gamma}(p(h);h)\right)}{\partial h} > 0$ in $[0;\infty)$, $\lim_{h\to0^+}\frac{\partial\left(\tilde{\Gamma}(p(h);h)\right)}{\partial h} \to +\infty$ and $\lim_{h\to+\infty}\frac{\partial\left(\tilde{\Gamma}(p(h);h)\right)}{\partial h} \to 0^+$. With $\tilde{\Gamma}(p(0);0) = 0$ we can prove that the function $\tilde{\Gamma}(p(h);h)$ shows four steady states, alternatively unstable and stable. These are $h^{U0} = 0$, $0 < h^{S1} < h^F$, $h^{U1} = h^F$ and $h^F < h^{S2} < +\infty$. ■

Proof of Lemma (1). Adults get the absolute majority if and only if their share is bigger than $\frac{1}{2}$: imposing $\frac{1}{\eta+1+p_t} > \frac{1}{2}$ we obtain, solving for p_t, $p_t < 1-\eta$. For similar considerations it is possible to show that both $\frac{\eta}{\eta+1+p_t}$ and $\frac{p_t}{\eta+1+p_t}$ can not exceed $\frac{1}{2}$. ■

Proof of Lemma (2). The expression of p^A is obtained from (1.16) solving $\Delta u_t^A(p_{t+1}) = 0$ for p_{t+1}. Given (1.17) and $i > 0$, the graph of $\Delta u_t^A(p_{t+1})$ has a negative intercept and crosses the $\Delta u_t^A = 0$ axis from below, proving the Lemma. ■

Proof of Lemma (3). The expression of p^Y is obtained from (1.21) solving $\Delta u_t^Y(p_{t+2}) = 0$ for p_{t+2}. ■

Proof of Proposition (2). The strategy we follow to prove the first part of the Proposition is to break the two inequalities and to show that both can not simultaneously hold for any parametrization of the model. Let us define

$$\begin{aligned}\Psi(\alpha)_1 &= \alpha^2\left(1-p^Y\right)\\ &= \alpha^2+\alpha\frac{\log\left((1+\theta)(1-\delta)^{\gamma}\right)}{\log\left((1+\theta)(1-\delta)^{\epsilon\gamma}\right)}+\frac{\log(1-i)}{\log\left((1+\theta)(1-\delta)^{\gamma\epsilon}\right)}\end{aligned}$$

and

$$\begin{aligned}\Psi(\alpha)_2 &= \alpha^2\left(p^A-p^Y\right)\\ &= \alpha\left(\frac{\log\left((1+\theta)(1-\delta)^{\gamma}\right)}{\log\left((1+\theta)(1-\delta)^{\epsilon\gamma}\right)}+\frac{\log\left(\frac{1-s}{1-s-i}\right)}{\log\left((1+\theta)(1-\delta)^{\gamma}\right)}\right)\\ &+\frac{\log(1-i)}{\log\left((1+\theta)(1-\delta)^{\epsilon\gamma}\right)}\end{aligned}$$

using (1.19), (1.23) and substituting $\beta=\alpha^2$. We only write the dependency of both Ψ_1 and Ψ_2 on α for brevity. It turns out that the two inequalities $p^A<p^Y$ and $p^Y<1$ are both satisfied if both $\Psi_2<0$ and $\Psi_1>0$ hold, respectively. In Figure 1.5 we show the shapes of these two functions in terms of α.

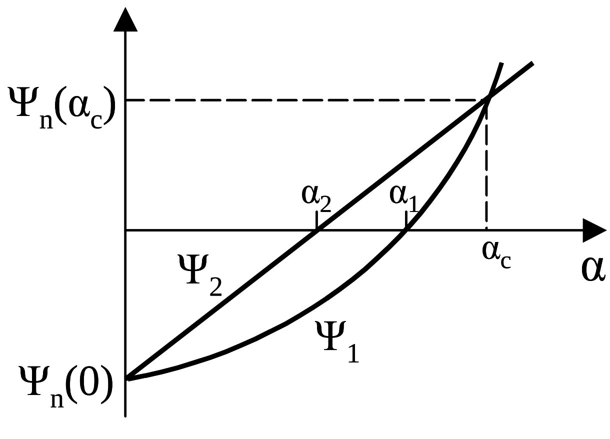

Figure. 1.5. Shapes of Ψ_1 and Ψ_2, their intersections with the axis and their crossing points.

The first order derivatives of Ψ_1 and Ψ_2 with respect to α are $\Psi_1' = \frac{\log((1+\theta)(1-\delta)^{\gamma})}{\log((1+\theta)(1-\delta)^{\epsilon\gamma})} + 2\alpha$ and $\Psi_2' = \frac{\log((1+\theta)(1-\delta)^{\gamma})}{\log((1+\theta)(1-\delta)^{\epsilon\gamma})} + \frac{\log\left(\frac{1-s}{1-s-i}\right)}{\log((1+\theta)(1-\delta)^{\gamma})}$. The second derivatives are $\Psi_1'' = 2$ and $\Psi_2'' = 0$, implying that Ψ_1 is a quadratic function of α and for $\alpha > 0$ it is increasing at an increasing rate. Ψ_1 is a positively sloped straight line. For $\alpha = 0$, both Ψ_1 and Ψ_2 take the same value $\Psi(0)_1 = \Psi(0)_2 = \Psi(0)_n = -\frac{\log\left(\frac{1}{1-i}\right)}{\log((1+\theta)(1-\delta)^{\epsilon\gamma})} < 0$, as shown in the graph. Moreover, Ψ_2 is steeper than Ψ_1, for any values of the parameters and for some small values of α $\left(\text{from an inspection of } \Psi_1' \text{ and } \Psi_2', \text{ until } \frac{\log\left(\frac{1-s}{1-s-i}\right)}{\log((1+\theta)(1-\delta)^{\gamma})} > 2\alpha\right)$. Therefore for $\alpha : 0 \leq \alpha < \alpha_2$, $\Psi_2 < 0$ holds, while it is not the case for $\Psi_1 > 0$. Because of their shapes and their crossing in $\alpha = 0$, they also have to cross again for some positive value of α. If the crossing point $(\Psi(\alpha_c)_n)$ of Ψ_1 and Ψ_2 lies below the α-axis and $\alpha_c < 1$, this means that there are values of α: $\alpha_1 < \alpha < \alpha_2$ such that both $\Psi_2 < 0$ and $\Psi_1 > 0$ hold at the same time. This cannot be the case because equating Ψ_1 to Ψ_2 gives $\alpha_c = \frac{\log\left(\frac{1-s}{1-s-i}\right)}{\log((1+\theta)(1-\delta)^{\gamma})}$, that plugged into Ψ_1 or Ψ_2 gives $\Psi(\alpha_c)_1 = \Psi(\alpha_c)_2 = \Psi(\alpha_c)_n = \frac{\log\left(\frac{1-s}{1-s-i}\right)^2}{(\log((1+\theta)(1-\delta)^{\gamma}))^2} + \frac{\log\left(\frac{1-s-i+is}{1-s-i}\right)}{\log((1+\theta)(1-\delta)^{\epsilon\gamma})} > 0$, for any values of $\theta, s, i, \delta, \gamma$ and ϵ in their supports. The first part of the Proposition is therefore proved. The second part relies again on splitting the inequality $p^A < p^Y < 1$ in two. $p^A < p^Y$ can be rewritten as $\frac{m^Y}{m^A} < -\frac{\log(1-i)+\alpha\log((1+\theta)(1-\delta)^{\epsilon\gamma})}{\alpha\log\left(\frac{1-s}{1-s-i}\right)}$ that is satisfied for $\alpha \to 0$ since the left hand side is constant and independent from α while the right hand side goes to $+\infty$. This result does not depend on the value of β while it depends on the value of i: although for $\alpha \to 0$ the inequality holds, for a range of α such that $i < 1 - ((1+\theta)(1-\delta)^{\gamma})^{-\alpha}$, the right hand side numerator is negative. That is why for some positive values of α the investment cost can not be too small. $p^Y < 1$, on the other hand, can be written as $(1-i)^{-1} < (1+\theta)^{\alpha+\beta}(1-\delta)^{\gamma(\alpha+\epsilon\beta)}$. With α and β approaching their lower and upper bounds (0 and 1,respectively) the inequality holds for small values of i and it also holds for some right interval of α that is compatible with the previous inequality $p^A < p^Y$. This proves the second part of the Proposition. ■

Proof of Corollary (1). We need that $p^Y < 0$ for some small values of i. Under assumption (1.17), it is enough to show that $q^Y > 0$, by graphical considerations based on Figure 1.4. This is true if and only if

$(1-i)\left((1+\theta)(1-\delta)^{\gamma}\right)^{\alpha} > 1$. By simple algebra the Corollary is proved. ■

Proof of Proposition (3). $\beta = \alpha^2$ ensures that $p^Y < p^A$. For small i, say $i < 1 - \left((1+\theta)(1-\delta)^{\gamma}\right)^{-\alpha}$, young are in favor of innovation $\forall p \geq 0$. This leads to a share of voters (young) in favor of innovation that is $\frac{\eta}{\eta+1+p_t}$ for $p_{t+1} < p^A$ and a share of voters (young + adults) in favor of innovation $\frac{\eta+1}{\eta+1+p_{t+1}}$ for $p_{t+1} > p^A$. Since $p^O = 1-\eta$ it is straightforward to show that $\frac{\eta}{\eta+1+p_t} < \frac{1}{2} \forall \eta \in (0;1]$ and $\frac{\eta+1}{\eta+1+p_t} > \frac{1}{2} \forall \eta \in (0;1]$. In the case of $i > 1 - \left((1+\theta)(1-\delta)^{\gamma}\right)^{-\alpha}$ for the positive values of life expectancy $0 < p_{t+2} < p^Y$ nobody is in favor of innovation, while for $p^Y < p_{t+2}$ and $p_{t+1} < p^A$ only young are in favor. Being them a minority the political outcome is unchanged with respect to the case of nobody backing innovation. Once $p_{t+1} > p^A$ is achieved, the previous analysis applies. ■

Proof of Lemma (4). When $p_{t+1} < p^A < p^Y < 1$ there is no way for the economy to support innovation, while with $p_{t+1} > p^A$ (at least) adults vote for innovation. If young's vote is not so "heavy" in the political arena, $\left(\text{i.e. } \eta < 1 - p^A\right)$ this implies, using the definition $p^O = 1-\eta$, that p^O is larger than p^A. In a right interval of p^A innovation takes place because adults alone want it: their voting share is $\frac{1}{\eta+1+p_{t+1}}$ and it is larger than 1/2 until $p_{t+1} < p^O$ In case $p^A < p^Y < p^O$ innovation takes again place with $p_{t+1} > p^A$ but in the interval $\{p^Y; p^O\}$ it would have taken place even if adults had not an absolute majority. above p^Y also young contribute in backing innovation. ■

Proof of Proposition (4). As the Proposition, this Proof is divided in four points. The assumption of small δ ensures that, whenever the economy switches between no innovation to innovation (or vice versa) the ordering of $p^O; p^A; p^Y$ and p^S does not change.

1. In case (1.a) $\beta = \alpha^2$ ensures, from Proposition (3), that $p^Y < p^A$. Sufficiently large values of s make p^A to be larger than p^S. In turns, small values of γ lower the steady state level of human capital, and therefore p^S. Independently from η, the orderings $p^Y < p^S < p^A$ and $p^S < p^Y < p^A$ are consistent with a no innovation policy outcome for any values of $p < p^S$. In case (1.b) β/α^2 substantially larger then 1 leads to $p^A < p^Y$: in this case the inequality $p^S < p^A$, ensured by small γ, ε and/or λ, is a sufficient condition for the case of constant no

innovation to be in place. In case (1.c) the strong political power of young impedes adults to decide for innovation alone, since $p^O < p^A < p^Y$. The limited effectiveness of human capital production ensures $p^S < p^Y$.

2. As in case (1.a), in (2.a) $\beta = \alpha^2$ ensures that $p^Y < p^A$. At the same time, large values of λ ensures that $p^A < p^S$ holds. For values of life expectancy above p^A output is increasing at at rate θ because both adults and young vote for innovation. Again, as in case (1.b), β/α^2 substantially larger then 1 leads to $p^A < p^Y$ also in case (2.b). Moreover, small values of α implies a small p^A, so that for value of life expectancy larger than p^A innovation is always chosen.

3. This case takes place if and only if $p^A < p^O < p^S < p^Y$ holds. This configuration requires β/α^2 larger then 1 as a necessary condition so to have $p^A < p^Y$. Moreover young's weight η must be small in order to have $p^O < p^S$. Small absolute values of α make the inequality $p^A < p^O$ to be satisfied.

4. This case takes place if and only if $p^A < p^O < p^Y < p^S$ holds. Conditions on the parameters are the same as in case (3) but α must be slightly larger in order to have young in favor of innovation for values of life expectancy in the interval (p^Y, p^S).

■

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Chapter 2

A Dynamic Politico-Economic Model of Intergenerational Contracts

"Why should I care about future generations? What have they done for me?"
(Groucho Marx)

2.1 Introduction

The implementation of intergenerational redistribution programs is a crucial issue in the current political debate. On the one hand, the public system can be manipulated for political purposes; on the other hand, it is not clear how a transfer scheme should be designed to be optimal and, thus, less responsive to political pressure. For these reasons, it becomes critical to explore the conditions under which intergenerational transfers, as outcome of a political voting game, can be implemented and why the welfare system developed so far has become a stable institution of modern society.

Since intergenerational redistribution is carried out by means of democratic voting, an important source of heterogeneity across individuals concerns their difference in age. Heterogeneous agents account for a big component of the variability in asset holdings as well as in sources of income. As a consequence, the conflict between different age-classes is likely to arise on a broader set of fiscal instruments than the size of social-security transfers.

Given the special focus of our analyses on the age-class heterogeneity and the inter-classes political conflicts, among all the redistributive programs, we point out the relevant role played by two critical age-target policies: public higher education spending and PAYG social security. These intergenerational redistributive programs, strongly interrelated with each other, have deep redistributive impact and have recently experienced even stronger political support in modern democracies. Following the terminology adopted by Rangel (2003), we refer to public higher education spending as forward (i.e. productive) intergenerational transfers and to unfunded pension as backward (i.e. pork barrel and log-rolling) intergenerational transfers. The former are transfers going forward in time generating a cost for the present generation and a benefit for the future one, being crucial for future productive capacity through human capital production. By contrast, the latter are transfers going backward generating a cost for the present generation and a benefit for the past one, giving adults a claim on the future productivity of their young.[1] This different timing of exchange generates different incentive problems.

Furthermore, the aging of population plays a relevant role in stressing even more the timing of the intergenerational bargaining from both a demographic and a political point of view. On the one hand, the quantitative component of population aging, i.e. the mass effects due to population growth dynamics (demographic aging), has a direct economic impact on the financial solvency of the public system, since the fraction of recipients – the retirees – tends to increase, while the share of contributors – the workers - tends to decreases. On the other hand, the qualitative component of population aging, i.e. the political ideology influence of the old age-class in the electoral competition process (political aging), has an indirect economic impact through the electoral vote. As population ages, so do the voters. In democratic society population aging leads to an increase in the political representation of the elderly, who gather a larger share of votes.[2] As politi-

[1] The welfare of a generation depends on the action taken by past generations and, in turn, affects the well-being of the future one. For example, the development of each generation of youth depends on the resources for education and sustenance that it receives from workers through taxation system. At the same time the well-being of the elderly depends on social programs that provide income support.

[2] The political influence of the old is magnified by their homogenous preferences in terms of economic policies. According to Mulligan and Xala-i-Martin (1999) old agents are *"single-minded"*.

cians seek re-election, they will try to address the needs of the crucial voting group – the old – with generous social security policies.

Empirical motivation relies on recognizing that since the Second World War developed countries have experienced dynamically efficient growth path, i.e. the economic growth rate has fallen below the interest rate, and have been characterized by underaccumulation on physical capital (Abel et al., 1989). Under such an economic scenario, exacerbated by the recent demographic transition, there would be no elements in the previous literature to justify the implementation of PAYG social security programs,[3] which would depress savings further and, consequently, intertemporal consumption and, in turn, economic growth. Furthermore, even in the case of a dynamic inefficiency scenario, a PAYG social security scheme is a dynamically inconsistent agreement between successive generations. Adult generations would be better off discontinuing the PAYG scheme and setting up a new one. However, quite surprisingly, the share of per-capita GDP used to finance social security following retributive schemes remains substantial.[4] For these reasons, the existence of unfunded pension schemes seems puzzling. Hence the question arises of why PAYG schemes survive.

Departing from previous literature, we support the existence of a pension system also in an economy experiencing a dynamically efficient path and characterized by underaccumulation on physical capital, conditionally on the existence of public investment in human capital in a *time consistent* scenario. The following idea is defended: selfish adults buy insurance for their future old age both by paying productive education transfers for their children and by taking care of their parents. Obviously the contract works only if the cost of providing a productive transfer is low with respect to the value of receiving a pork-barrel transfer when old. Therefore, if a PAYG pension scheme is introduced,[5] its future beneficiaries may become supportive of

[3] There are many explanations in the literature on why pay-as-you-go (PAYG) social security has been introduced and then expanded. The classical solution on the puzzle is that, if the economy is on dynamically inefficient path i.e. the interest rate falls below the economic growth rate, then the introduction of a PAYG social security system is Pareto improving since it reduces the capital deepening. Among others, see Azariadis and Galasso (2002).

[4] OECD data show how public tertiary education and social security transfers become increasingly important and strategic among the main components of public expenditure in modern welfare countries. Focusing on European Union members, in 2007 public expenditures on higher education took on average of 1.46 percent of GDP (OECD, 2008) and pension transfers were on average 7.8 percent of GDP (OECD, 2008).

[5] PAYG pension schemes in which pensions are financed by contributions from current

higher funding to public education via taxes. In other words, the existence of a retributive social security system gives incentives to invest optimally in human capital and, as a consequence, it becomes growth enhancing for the economic system. Thus, the two age-specific redistributive programs may self-sustain under a dynamic efficiency scenario and under certain economic and institutional conditions reach Pareto optimality.

Technically, this paper highlights two main features concerning fiscal policies. First, several political choices have to be set at the same time, so the political space cannot be reduced to a mere unidimensional problem. Second, since political decisions and private intertemporal choices are mutually affected over time, then selfish perfect forward-looking agents internalize how political current choices will influence the evolution of the economy and the implementation of future policies.

The aim of this chapter is to provide a tractable dynamic politico-economic theory to analyze how intergenerational conflicts affect, through the political mechanisms in the form of democratic vote, the size and composition of public expenditures in a context of population aging. Focusing on target-specific transfers, our main objective concerns the determination of the economic and institutional conditions which may induce the emergence of a decentralized implicit intergenerational contract based on *side payments* in the form of PAYG and public education transfers. The economy we study is characterized by overlapping generations living three periods: youth, adulthood and old age. Besides their private consumption, both the adults and the elderly value the public transfers; the presence of a political system is justified by the need to finance the provision of public spending. In our environment there are two types of selfish agents: the private players choose the optimal saving and vote their political representatives and the elected "public player" decides on public policies. The electoral competi-

workers have often been criticized as detrimental to growth. According to Feldstein (1974) such pension schemes have a negative effect on capital accumulation since they discourage private saving and, unlike in the case of a funded pension system, the payments into the PAYG scheme do not contribute to the national saving. Moreover, the implicit rate of return on contributions to a PAYG scheme typically falls short of the interest rate. Therefore according to such analysis, PAYG pension systems reduce per capita income. This standard argument is focused on physical capital accumulation and fails to take notice of the effect of PAYG pension systems have on the accumulation of human capital, particularly through public education. Primary and secondary education is now overwhelmingly publicly financed in all OECD countries, and universities also receive substantial funding from public sources.

tion takes place in a majoritarian probabilistic environment, where political representatives compete proposing multidimensional fiscal platforms, concerning both the income tax level and the provision of intergenerational transfers in the benefit formula, subject to intra-period budget balance. We assume away the provision of public goods - a key element in the political economy of fiscal policy[6] - in order to bring out more clearly the impact of political institutions on intergenerational transfers.

The focal point of this paper is the characterization of time-consistent public policies in a multidimensional dynamic political setting. Following Maskin and Tirole (2001), we embody the "*minor causes should have minor effect*" principle to implement differentiable Stationary Markov Subgame Perfect Equilibria (*SMPE*), where the size of the income tax rate and the amount of intergenerational transfers are conditioned on the two payoff-relevant asset variables: physical and human capital. We determine the political policy rules as equilibrium outcomes in a finite horizon environment when time goes to infinity. As a result we are able to overcome the main limit related to trigger strategies equilibria, which are not robust to such refinement.[7]

Ruling out commitment devices and reputation mechanisms, solving backward and making the time horizon go to infinity, we determine *time-consistent* policy rule reaching the following results: 1) the dynamic efficiency condition is necessary for the simultaneous existence of public education and PAYG programs; 2) the equilibrium political decisions are *no* education *strategic*, while due to distortionary taxation and to the politicians' opportunistic behavior, strategic *persistency* underlies the determination of the income tax rate; 3) three different welfare state regimes arise depending on institutional variables, i.e. the adults' and the elderly's relative bargaining power, and on economic variables, i.e. the endogenous level of physical capital; 4) demographic aging increases the equilibrium per-capita level in public education spending and depending on the welfare state regime has an ambiguous effect on the size of government.

The paper is organized as follows. Section 2.2 reviews the literature.

[6] This issue is yet well investigated by Tabellini (1991), Lizzeri and Persico (2001), Hassler, Storesletten and Zilibotti (2005).

[7] As natural benchmark in Chapter 3 we will compare the political equilibrium outcome with the *time-consistent* Central Planner solution in an infinite horizon environment in order to point out normative predictions.

In section 2.3 we present the model characterizing the economic environment and solving the individual economic problem. Section 2.4 presents the politico-economic equilibrium in the cases of both a myopic scenario and a perfect forward-looking one. We provide a complete characterization of both the transition dynamics and the long run of the economy. Section 2.5 concludes. All proofs are contained in the Appendix.

2.2 Literature Review

This chapter relies on the dynamic political economy literature that incorporates forward-looking decision makers in a multidimensional policy space without commitment. In particular our paper relates to two main streams of literature.

On the one hand, it supports and gives new theoretic foundations to the existing literature on social security sustainability, which recognizes the link between productive and redistributive public spending. From a pure economic point of view Boldrin and Montes (2005) formalize public education and PAYG systems as two parts of an intergenerational contract where public pensions are the return of the investment into the human capital of the next generation. The authors show how an interconnected pension and public education system can replicate the allocation achieved by complete markets. Allowing issue-by-issue voting, Rangel (2003) studies in a three-period OLG model the ability of non-market institutions to optimally invest in "forward intergenerational goods" and "backward intergenerational goods". Bellettini and Berti Ceroni (1999) incorporate politics in an OLG model to analyze how societies might sustain public investments (e.g. education) even if the interests of those benefitting from the investment are not represented in the political process. Restricting voting to a binary choice of the tax rate and education, the authors study whether a given system can be maintained but do not determine the level of investment in education or social security. As a main shortcoming the previous studies have assumed voters played trigger strategies. Although trigger strategies may be analytical convenient, they lead to multiplicity of equilibria. Furthermore, they require coordination among agents and costly enforcement of a punishment technology which may not work when agents are not patient enough. Finally, they are not robust to refinements such as backward induction in a finite horizon economy when time tends to infinity. As a main consequence time inconsistent equilibria may be attained in an environment characterized by no credible punishment devices, therefore the second best cannot be achieved (see Klein et al., 2008). Unlike the previous literature, rather than emphasizing complementarity between education and pension payments purely, which are purely sustained because of reputation mechanisms, our model adopts a different perspective. It focuses on the resolution of the intergenerational conflict over the determination of the amount of the two public spending components in a Markovian environment.

On the other hand, this paper also contributes to the growing literature on dynamic politico-economic models. Starting from the seminal work of Krusell et al. (1997), the most interesting issue in the dynamic political economy literature concerns the modelling of economies where endogenous dynamic feedbacks between private and political choices are explicitly considered. Due to theoretical complexities, to extend standard static models to understand fully dynamic policy-making has proved to be difficult, even in the case of one-dimensional policy environments. Krusell and Ríos-Rull (1999) embed a distortionary income tax system into the neoclassical growth model in a repeated voting setting and adopt a median voter framework. They solve the model numerically making predictions on the long run size of government. In a simpler underlying economic environment Hassler et al. (2003) develop an OLG model of the welfare state where tax revenues are used to finance public goods and in each period the level of benefits is determined by majority voting. Studying a linear-quadratic economy, they provide analytical solutions in one-dimensional policy space but the voting strategies equilibrium turns out to be either constant or independent of fundamentals. Hassler et al. (2005) extend this approach to a richer economic environment in which the welfare state provides an insurance system.[8] Departing from the past literature we find analytical solutions in a multi-dimensional political space where equilibrium voting strategies become non trivially dependent on fundamental asset variables in the political environment.

[8] More recent studies extend the dynamic politico-economic modelling to the infinite-horizon Central Planner environment as in Klein et al. (2008), Azzimonti et al. (2009) and Martin (2009).

2.3 The Model

Time is discrete and indexed by t. The economy is populated by an infinite number of overlapping generations of homogenous agents, living up to three-periods: youth, adulthood and old age. Every agent born at time t survives with probability one until old age. Population grows at a constant rate $n \in (-1, 1)$, thus the mass of a generation born at time j and living at time t is equal to $N_t^j = N_0 (1+n)^t$. When young, an agent spends all his/her time endowment in acquiring skills without having access to private credit markets. When adult, the individual works and contributes to the public spending through taxes, while when old, the individual retires. In every period, the economy produces a single homogeneous good combining human capital with physical capital.

At the beginning of each period public policy choices are taken through a repeated voting system according to a majoritarian rule where ideological bias is taken into account in the candidates' electoral competition.[9] Both adults and old have voting power.[10] In order to cover both efficiency and equity aspects concerning intergenerational conflicts, each candidate proposes a multidimensional platform where both the size of government and intergenerational income redistribution are simultaneously considered. The set of political variables, f_t, includes education (i.e. forward looking) transfers, e_t, social security (i.e. backward looking) transfers, p_t, and a proportional labor income tax rate, τ_t. The public financial system is assumed to be balanced in every period.

The sequential politico-economic game in the repeated voting setting can be viewed as a Stackelberg game and it is solved by backward induction to guarantee time-consistent solutions. First, the agents determine the optimal level of savings given the fiscal stance (*Economic Equilibrium*). Second, short-lived office-seeking politicians determine both the optimal level of taxation and the optimal amount of backward and for-

[9] Given the Markov structure and the evolution of the asset variables, allowing for each year election instead of each generation one would not change the political outcome of the model.

[10] We replicate the stylized facts that young people show a much lower turnout rate at elections with respect to adults and old. As Galasso and Profeta (2004) report in some countries elderly people have a higher rate at elections than the young. In the U.S. turnout rates among those aged 60-69 years in twice as high as among the young $(19 - 29$ years). Again in France it is almost 50% higher.

ward transfers in order to maximize the probability of winning the elections (*Politico-Economic Equilibrium*). We allow for fully rational and forward-looking voters, restricting the notion of politico-economic equilibrium to the differentiable political *SMPE* concept as equilibrium refinement of subgame perfect equilibria.[11]

2.3.1 Production

At each time t a homogenous private good, Y_t, is produced using a linear technology both in labor, L_t, and capital, K_t, which fully depreciates. The linearity of the production function can be derived as an equilibrium outcome in a context of perfect international capital mobility and factor price equalization in the presence of goods trade. The production function at time t is:

$$Y_t = w_t L_t + R K_t \tag{2.1}$$

where the wage rate, $w_t = \omega(1 + h_t)$, and the gross rental price to capital, R, are determined by the marginal productivity condition for factor price ($\partial Y_t / \partial L_t = w_t$ and $\partial Y_t / \partial K_t = R$).[12] At any time t, each adult supplies inelastically one unit of labor,[13] $L_t = N_t^{t-1}$, with productivity equal to ω augmented by the level of human capital acquired the period before, h_t. Without loss of generality we normalize $\omega = 1$.

The human capital per worker, h_t, is an increasing function in both parental human capital and public higher education spending.[14] The public education transfer is supplied in an egalitarian way, consequently, to each

[11] The Markov-perfect concept implies that outcomes are history-dependent only in the fundamental state variables. The stationary part is introduced to focus only on the current value of the payoff relevant state variable. Consequently the vector of equilibrium policy decision rules is not indexed by time, i.e. the structural relation among payoff-relevant state variables and political controls is not time variant. The differentiable part is a convenient requirement to avoid multiplicity of equilibrium outcomes and in order to give clear positive political predictions.

[12] Given the absence of capital income tax, in our economy the gross rental price to physical capital and the net one coincide, $R = r + \delta$ where $\delta = 1$ for the assumption of full depreciation of K_t.

[13] Since adults supply labor inelastically, income taxation does not distort individual labor supply decision at the margin.

[14] The importance and the empirical relevance of both the public spending in schooling inputs and the parental education input in the formation of the human capital of the young people has been explored theoretically as well as empirically. For a comprehensive survey of the related literature see Becker and Tomes (1986).

individual is given the same level of it. Thus, the acquisition of skills requires the public transfers and a stock of existing human capital. In aggregate terms the following Cobb-Douglas human capital technology is adopted:

$$H_{t+1} = \left(\alpha H_t + (1-\alpha)\bar{H}\right)^{\theta} E_t^{1-\theta} \tag{2.2}$$

where $H_t = h_t N_t^{t-1}$, $\bar{H} = \bar{h} N_t^{t-1}$ and $E_t = e_t N_t^t$. After simple algebra we obtain the following per-capita human capital technology:

$$h_{t+1} = H[e_t, h_t] \equiv \left(\frac{\alpha h_t + (1-\alpha)\bar{h}}{1+n}\right)^{\theta} e_t^{1-\theta} \tag{2.3}$$

where $\theta \in (0,1)$. $\bar{h}$ is the constant society endowment of human capital and h_t is the dynasty's human capital at time t.

Physical capital fully depreciates each period. Consequently, the level of savings, s_t, determines the dynamics of per-capita physical capital accumulation. The capital market clears when:

$$(1+n)\,k_{t+1} = s_t \tag{2.4}$$

2.3.2 Households

Agents born at time $t-1$ evaluate consumption according to the following intertemporal, non altruistic, expected utility function defined over the vector $c_t \equiv (c_{1,t}, c_{2,t+1}) \in \mathbb{R}^2_{++}$:

$$U_{t-1}[c_t] = u[c_{1,t}] + \beta u[c_{2,t+1}] \tag{2.5}$$

where $\beta \in (0,1)$ is the time discount factor. $c_{1,t}$ represents the consumption at time t when adult and $c_{2,t+1}$ is the consumption at time $t+1$ when old. In the first period of life (youth), the individual does not consume. The function $u[\cdot]$ is concave, twice continuously differentiable and satisfies the Inada condition, i.e. $\lim_{c_t \to 0} u_c[c_t] = \infty$. Assume that preferences exhibit the logarithmic form, i.e. $u[\cdot] = \log[\cdot]$.

The individual budget constraints of the agents are as follows:

$$c_{1,t} \leq C_{1,t}[\tau_t, h_t, k_{t+1}] \tag{2.6}$$

where $C_{1,t}[\tau_t, h_t, k_{t+1}] \equiv (1+h_t)(1-\tau_t) - (1+n)\,k_{t+1}$. When adult, agents consume their labor income net of the proportional labor tax and individual savings.

$$c_{2,t+1} \leq C_{2,t+1}\left[p_{t+1}, k_{t+1}\right] \tag{2.7}$$

where $C_{2,t+1}\left[p_{t+1}, k_{t+1}\right] \equiv R\left(1+n\right)k_{t+1} + p_{t+1}$. When old, agents consume their total income, equal to the sum of pension benefits that their children pass to them in a PAYG fashion, and the capitalized savings at a fixed gross rental price R.

The net present value at time t of the lifetime wealth of an agent born at time $t-1$ is then:

$$I_t = \left(1+h_t\right)\left(1-\tau_t\right) + \frac{p_{t+1}}{R} \tag{2.8}$$

2.3.3 Individual Optimal Decisions

Adults choose their optimal savings taking as given fiscal and redistributive policies.[15] Maximizing Eq. (2.5) subject to the individual budget constraints (2.6) and (2.7), the following first order condition for interior solutions must hold in equilibrium:

$$0 = \eta\left[\tau_t, p_{t+1}, h_t, k_{t+1}\right] \equiv u_{c_1,t}\left[C_{1,t}\left[\tau_t, h_t, k_{t+1}\right]\right] - R\beta u_{c_2,t+1}\left[C_{2,t+1}\left[p_{t+1}, k_{t+1}\right]\right] \tag{2.9}$$

Then, in equilibrium by the implicit function theorem there exists a unique saving function, k_{t+1}, which satisfies the condition (2.9). We can rewrite the optimal level of savings in terms of lifecycle after-tax endowment as:

$$k_{t+1} = K\left[\left(1+h_t\right)\left(1-\tau_t\right), p_{t+1}\right] \tag{2.10}$$

Given any separable additive intertemporal utility, Eq. (2.10) emphasizes the income and substitution effects due to a variation of the implemented policies on the individual saving choice.[16]

Definition 2 *Given the sequence of taxes and intergenerational transfers,*

[15] From now on, we adopt the following notation. Let $z\left[x, q\left[x\right]\right]$ be a function in the variable x, $z_x \equiv \frac{\partial z[x]}{\partial x}$ is the partial derivative and $\frac{dz}{dx}$ is the total derivative.

[16] Under logarithmic utility function the equilibrium saving is as follows:

$f_t = \{\tau_t, e_t, p_t\}_{t=0}^{\infty}$, *and the initial conditions* (h_0, k_0), *an Economic Equilibrium is defined as a set of functions* $\{c_t, h_{t+1}, k_{t+1}\}_{t=0}^{\infty}$ *such that individual choices are consistent with the law of motion of the economy described in Eq. (2.3) and Eq. (2.10). Markets clear at any point in time.*

At time t the indirect utility, $W_{1,t}[\cdot]$, of an adult born a time $t-1$, is then equal to:

$$W_{1,t}[\tau_t, p_{t+1}, h_t, k_{t+1}] \equiv \max_{k_{t+1}} \{U_{t-1} \mid I_t\} \tag{2.11}$$

where $W_{1,t}[\cdot] = u[C_{1,t}[\tau_t, h_t, k_{t+1}]] + \beta u[C_{2,t+1}[p_{t+1}, k_{t+1}]]$.

For an old individual born a time $t-2$ the indirect utility, $W_{2,t}[\cdot]$, at time t is as follows:

$$W_{2,t}[p_t, k_t] \equiv u[C_{2,t}[p_t, k_t]] \tag{2.12}$$

We define as *laissez-faire* indirect utility, $W_{1,t}^{LF}$, the lifetime utility of an adult born at time $t-1$, when no public taxation and spending are considered:

$$W_{1,t}^{LF}[k_{t+1}] \equiv \max_{k_{t+1}} \{U_{t-1} \mid I_t = 1\} \tag{2.13}$$

Suppose there is no government that has the authority to levy taxes. As a consequence, adults keep the *entirety* of their labor income to purchase the final good and to save. Capital earns a gross return of R, used by the old to buy the consumption good. Clearly, the economy converges to the unique steady state in at most one period, where $h^{LF} = 0, k^{LF} = \frac{\beta}{(1+\beta)(1+n)}, c_1^{LF} = \frac{1}{1+\beta}, c_2^{LF} = \frac{\beta}{1+\beta}R$ and $w^{LF} = 1$.

2.3.4 Government balanced budget constraints

The government's budget is balanced in every period. This implies that in each period the total benefits paid to old and young equalize total contributions collected from working generations:

$$(1+n)\,k_{t+1} = \frac{\beta}{(1+\beta)} w_t (1-\tau_t) - \frac{1+n}{R(1+\beta)} (w_{t+1}\tau_{t+1} - (1+n)\,e_{t+1})$$

$$(1+h_t)\,\tau_t = (1+n)e_t + \frac{p_t}{1+n} \tag{2.14}$$

Ceteris paribus, the more the population ages, the higher the aggregate pension benefits for old agents are and the lower the aggregate education transfers for young people are. The condition above allows us to reduce the multidimensionality of the political platform to $f_t \in \mathbb{R}^2$ where $f_t \equiv (e_t, \tau_t)$. Let $\hat{e}_t$ and $\hat{p}_t$ be the maximum feasible values of education and pension transfers at each time t.

Definition 3 *A feasible allocation is a sequence of education transfers, pensions, labor income tax rates and saving decisions* $\{e_t, p_t, \tau_t, k_{t+1}\}_{t=0}^{\infty}$ *that satisfies the implementability constraint, (2.10), the balanced budget constraints, (2.14),* $e_t \in (0, \hat{e}_t)$, $p_t \in (0, \hat{p}_t)$ *and* $\tau_t \in (0,1)$ $\forall t$.

2.4 Politico-Economic Equilibrium

In this section we consider a government of politically-motivated but short-lived representatives that have the authority to levy labor income taxes and to transfer income across generations.[17] Public policies are chosen through a repeated voting system without commitment where elections take place at the beginning of each period. Young have no political power. To characterize the behavior of politicians we consider a probabilistic voting setting.[18] In this environment there are two policy-maker candidates who compete in a majoritarian election proposing their own political multidimensional

[17] We assume that once the fiscal plan is implemented no one can default on it.

[18] Due to the multidimensionality of the political platform $\{\tau_t, e_t, p_t\}$ Condorcet winner generally fails to exist. Consequently the median voter theorem doesn't hold (Plot, 1967). In the literature there are three main influential approaches for making predictions when the policy space is multi-dimensional. The first is the implementation of structure-induced equilibria. By following Shepsle (1979), agents vote simultaneously, yet separately (i.e. issue by issue), on the issues at stake. Votes are then aggregated over each issue by the median voter. See Condez-Ruiz and Galsso (2005) for a more detailed discussion of this approach. The second is the legislative bargaining approach, which stems from the seminal work of Baron and Ferejohn (1989) and develops from Battaglini and Coate (2006). This approach applies when legislators' first loyalty is to their constituents and legislative coalitions are fluid across time and issues. The last approach, which will be exploited in this paper, concerns the adoption of probabilistic voting rule. While it dates back to the 1970s, its resurgence in popularity stems from Lindbeck and Weibull (1987) . It applies to political environments where party discipline is strong and the winning political party simply implements its platform. See Persson and Tabellini (2000) for a survey of this framework.

platform. Under a balanced budget constraint the political platform is represented by the pair (e_t, τ_t). Since politicians can extract rents from being in power, the objective of each candidate is to maximize support among the currently living voters to win elections and implement the proposed policy, with no ability to commit to future policies (*opportunistic framework*). In the Nash equilibrium of the electoral competition game both candidates propose the same policy platform, implementing the utilitarian optimum with respect to current voters.[19] It follows that the political objective function, W_t, which aggregates the political preferences of the two generations having the right to vote has the following structural form:

$$W_t \equiv (1+n) W_{1,t} [f_t, f_{t+1}, h_t, h_{t+1}, k_{t+1}] + \phi W_{2,t} [f_t, k_t] \tag{2.15}$$

where $W_{1,t} [\cdot]$ and $W_{2,t} [\cdot]$ are obtained after plugging condition (2.14) respectively into Eq. (2.11) and Eq. (2.12).[20] In the probabilistic voting setting a relevant factor is the presence of unobservable *ex-ante* ideological bias in voters' preferences toward political candidates. We consider the case in which the adults' ideological weight is normalized to unity and the old's political power distribution is denoted by ϕ.[21] If $0 < \phi < 1$ then, on average, the old cohort cares less about ideology and has more "swing-voters" than the adult one. For $\phi > 1$ the opposite holds, where the preferences of the old in the political debate represent the political majority. Finally, when $\phi = 1$, all voters are equally represented.

Summarizing, individual electorate choices depend on the proposed fiscal platform, on the impact of political programs on agents' private behavior, and on an "ideology" that is orthogonal to the fundamental policy dimensions. While elderly people care for current taxation and redistribution only, the political choices for adults are also affected by future expected policies.

Remark 8 *Given the probabilistic environment described above the resulting political power distribution of voters, i.e. current old and adults, is*

[19] Since candidates have no intrinsic preferences over taxation and redistribution, they are assumed to implement their promised platform.

[20] For an analytical derivation see Persson and Tabellini (2000).

[21] In other terms the parameter ϕ is a measure of how strongly the old generation pursues her own interest. In our framework, due to multidimensionality in the aggregation of political preferences, ϕ cannot be equal to zero, excluding possibility of dictatorship.

represented by the finite-dimensional vector:

$$\Im^p_{(3)} \equiv (\phi, (1+\beta)(1+n)) \in \mathbb{R}^2 \tag{2.16}$$

Thus the relative political bargaining weights of current adults and old, whose sum is one, are:

$$\Omega_A \equiv \frac{(1+n)(1+\beta)}{\phi + (1+n)(1+\beta)} \quad \text{and} \quad \Omega_O \equiv \frac{\phi}{\phi + (1+n)(1+\beta)} \tag{2.17}$$

respectively.

Remark 9 *The more population ages* (*i.e.* n *decreases and* ϕ *increases*)*, the smaller is the relative political weight of the adults* (Ω_A) *and the larger is the relative political weight of the old* (Ω_O)*.*

At each time voting over a political platform generates dynamic linkages of policies across periods. The standard logic of competitive models, where agents optimize taking future equilibrium outcomes as given breaks down when political choices are considered. Due to the non-negligible impact of current political actions on future equilibria, rational agents internalize these dynamic feedbacks. In our framework dynamic linkages generated by physical and human capital arise both directly, affecting asset accumulation decision (*direct dynamic feedbacks*), and indirectly affecting future political choices (*indirect dynamic feedbacks*).[22] While the implemented labor income tax rate has a one-period lagged impact on the physical capital stock due to full depreciation, intergenerational transfers in the form of education have a two-period lagged impact on physical capital and infinite persistency on human capital due to the complementarity between parental human capital and education. Because of the different *intensities* of the dynamic feedbacks, the internalization degree of future expectations drives the economy towards different equilibrium outcomes. In order to explore the implications concerning the dynamic persistence of the welfare state allowing for different equilibrium concepts, we gradually incorporate all the dynamic feedbacks generated by political choices. Focusing on Markov strategies, in which the players' actions depend on the level of the fundamental state variables only,

[22] The persistency of dynamic linkages depends on the "type" of general equilibrium effects including in the model (i.e. endogenous prices, asset accumulation).

physical capital and human capital, we analyze three different scenarios. First, we explore the equilibrium dynamic policies under the assumption of myopic voters. In this environment agents are supposed not to be able to internalize the future.[23] Second, we allow for dynamic rational expectations in a finite time horizon. In this scenario agents are able to fully internalize the overall direct and indirect impact of taxation and redistribution through the evolution of assets and politicians are now endowed with strategic behavior. The basic idea is that by affecting the evolution of the relevant state variables current representatives may act strategically choosing the level of both the labor income tax rate and intergenerational transfers which induce higher total expected direct and indirect gains, fully anticipating the total future impact of current choices.[24] Finally in the next chapter we will analyze an infinite time horizon economy with a benevolent Central Planner, which enables us to internalize the infinitely-persistent feedback effects of policies on fundamental assets without political competition.

2.4.1 Myopic Politico-Economic Equilibrium

In myopic voting setting, where agents play Markovian strategies, the implemented policy rules, $f_t^m \equiv (\tau_t^m, e_t^m)$, turns out to be serially uncorrelated at each time. The apex m stands for myopic. No dynamic feedback effects generated by political choices are internalized in equilibrium. This is equivalent to model static expectations, where future policies are taken as given, $\bar{f}^m \equiv (\bar{\tau}^m, \bar{e}^m)$, ensuring a dynamically consistent sequence of policies. We now provide the formal definition of myopic politico-economic equilibrium.

Definition 4 *A myopic political SMPE is defined as a vector of differentiable policy decision rules, $F^m = (\mathbf{T}^m, \mathbf{E}^m)$, where $T^m : \mathbb{R} \times \mathbb{R} \longrightarrow (0,1)$ and $E^m : \mathbb{R} \times \mathbb{R} \longrightarrow (0, \hat{e}_t^m)$ are the taxation policy rule and the public*

[23] The myopic approach studies the politico-economic equilibrium concept under the assumption that voter's ability to predict is restricted. In this setup, even though the future equilibrium paths need to be predicted for each current policy, the agents are not required to take into account the impact of current political decisions on future ones. Voters only take into account direct positive/negative impacts of current policies on their intertemporal utility. Consequently, the policies are not serially correlated and the maximization program is essentially static, excluding the possibility to adopt strategic behavior.

[24] In our environment governments don't try to manipulate voting behavior as Hassler et al. (2003) do. This is due to the fact that we consider productive asset variables as fundamental instead of demographic variables.

higher education policy rule, $\tau_t^m = \mathbf{T}^m [h_t, k_t]$ *and* $e_t^m = \mathbf{E}^m [h_t, k_t]$*, respectively. Given the political indirect utility, Eq. (2.15), the following condition must hold:*

$$\digamma^m [h_t, k_t] \equiv \arg\max_{f_t^m} W [f_t^m, h_t, k_t]$$

subject to the following set of constraints:

1. $V [h_t, k_t] \equiv W [\digamma^m [\cdot], h_t, k_t] \geq W^{LF} [k_t]$

2. $\Phi [h_t, k_t] = \begin{cases} k_{t+1} = K [f_t^m, h_t] \\ h_{t+1} = H [e_t^m, h_t] \end{cases}$

3. $f_t^m \in \Pi [h_t, k_t]$

where $W^{LF} [\cdot]$ *is the Eq. (2.15) in the laissez-faire case,* $H [\cdot]$ *and* $K [\cdot]$ *are defined in Eq. (2.3) and Eq. (2.10), and* $\Pi [\cdot]$ *is a continuous convex correspondence.*

The equilibrium condition requires that the policy control variables, f_t^m, have to be chosen by politicians in order to maximize the probability of winning election when future policy outcomes $\bar{f}^m$ are taken as given. The vector of the implemented policy platform is feasible if the individual rationality constraint and the transformation constraints hold. To solve for the equilibrium policy rules $\digamma^m$, we take the first order derivatives of Eq. (2.15) with respect to f_t^m applying the envelope theorem. The following first order conditions are achieved for τ_t^m and e_t^m, respectively:

$$0 = \underbrace{\phi (1 + n) (1 + h_t) u_{C_{2,t}}}_{\text{old's direct benefit}} - \underbrace{(1 + n) (1 + h_t) u_{C_{1,t}}}_{\text{adults' direct cost}} \tag{2.18}$$

$$0 = \underbrace{-\phi(1 + n)^2 u_{C_{2,t}}}_{\text{old's direct cost}} + \underbrace{\beta(1 + n)^2 \frac{dh_{t+1}}{de_t^m} \bar{\tau}^m u_{C_{2,t+1}}}_{\text{adults' expected direct benefit}} \tag{2.19}$$

The incentive scheme in the myopic case is characterized only by direct effects of political choices on voters' indirect utility in terms of costs and benefits. Let us first refer to Eq. (2.18). At each time an interior solution for the income tax rate is simply determined as the outcome of a weighted bargaining between current old and adults,[25] who get benefits and sustain costs by a variation in the tax level. The first term in Eq. (2.18) represents the old's marginal benefits in terms of PAYG social security due to the increase in the income tax rate. Since tax levying on labor income makes adults sustain the whole tax burden, the second term captures the adults' marginal cost caused by a positive variation on the fiscal dimension. Similarly, redistributive choices are taken as the outcome of a weighted bargaining between current old and future ones.[26] An increase in public higher education transfers is a "double-edge sword". On the one hand it makes the current old sustain direct costs due to a reduction in social security contributions, represented by the first part of Eq. (2.19). On the other hand the future old enjoy direct benefits from the expected return of productive investment in human capital, whose effects are captured by the second part of Eq. (2.19).[27]

Political *SMPE* with *Myopia*

Solving the above system of FOCs, we yield the *myopic* political *SMPE* for interior solutions.[28] Let $\Upsilon^m \equiv (K_t^{m,\tau}, H_t^{m,\tau}) \cap H_t^{m,e}$ be the state-space in which interior fiscal and redistributive policy rules are simultaneously obtained, where $(K_t^{m,\tau}, H_t^{m,\tau}) = \left\{(k_t, h_t) \,|\check{k}_t^m < k_t < \hat{k}_t^m\right\}$ and $H_t^{m,e} = \{h_t | h_t \in (0, \infty)\}$ if $\bar{h} < \frac{1}{(1-\alpha)\psi^m}$ otherwise $H_t^{m,e} = \left\{h_t | h_t \in \left(\tilde{h}^m, \infty\right)\right\}$.[29] Then the following Proposition applies:

[25] The bargaining absolute power weights for old and adults on the fiscal dimension in the myopic case are respectively ϕ and $1 + n$. Clearly, the stronger is the old's political power, the higher is the equilibrium level in the income tax rate.

[26] The bargaining absolute power weights of current and future old are respectively equal to ϕ and $\beta (1 + n)$.

[27] It should be noted that the expected direct benefits enjoyed by future old crucially depends on the value of $\bar{\tau}^m$. Thus, we need a criterion based on rational expectation for the policy makers to correctly make predictions on future taxation and redistribution policies as in the next paragraph.

[28] At any time t, ϕ is different from zero. Given the concavity in both the instantaneous utility function and human capital production, we attain an interior solution for both τ_t and e_t at each time t. Equating Eq. (2.18) and (2.19) to zero and solving the system, if interiority in both dimensions is yielded then human capital production has to be characterized by decreasing return in education transfers. Otherwise we get either corner solution in one of the two dimensions or indefiniteness in the structural determination of the two policy rules. We rule out such circumstances adopting a Cobb-Douglas technology.

[29] For the exact characterization of the threshold values $\check{k}_t^m, \hat{k}_t^m$ and $\tilde{h}^m$ see the proof of Proposition 5 reported in Appendix.

Proposition 5 *Let* $\psi^m \equiv \left(\frac{1-\theta}{R}\bar{\tau}^m\right)^{\frac{1}{\theta}}$. *For any* $(h_t, k_t) \in \Upsilon^m$, *the set of feasible rational policies,* $f_t^m \equiv (\tau_t^m, e_t^m)$, *which can be supported by a myopic political SMPE, has the following functional form:*

(i)

$$\mathbf{E}^m[h_t] = a_1^m h_t + a_0^m \tag{2.20}$$

where $a_1^m \equiv \frac{\alpha}{1+n}\psi^m$ *and* $a_0^m \equiv \frac{1-\alpha}{1+n}\bar{h}\psi^m$;

(ii)

$$\mathbf{T}^m[h_t, k_t] = -b_3^m \frac{k_t}{1+h_t} + b_2^m \frac{h_t}{1+h_t} + b_1^m \frac{1}{1+h_t} + b_0^m \tag{2.21}$$

where $b_3^m \equiv R\Omega_A$, $b_2^m \equiv \alpha\psi^m \left(\Omega_A + \frac{1}{(1-\theta)}\Omega_O\right)$, $b_1^m \equiv \bar{h}\left(\frac{1-\alpha}{\alpha}\right) b_2^m + \frac{\Omega_O}{R}\left((1+n)\bar{\tau}^m - (1+n)^2\bar{e}^m\right)$ *and* $b_0^m \equiv \Omega_O$.

For any $(h_t, k_t) \notin \Upsilon^m$, *corner solutions result in at least one of the two dimensions.*

Proof. (See Appendix). ■

This Proposition characterizes the behavior of politicians in a myopic environment when Markov strategies are implemented. From a structural point of view, while the policy rule associated to education transfers is linear in human capital production, the fiscal policy rule is a linear function in physical capital but not in the human capital level. The equilibrium conditions predict the simultaneous existence of both sides of the redistributive program for $(h_t, k_t) \in \Upsilon^m$.

Finally, the exact quantitative characterization of the fiscal policies crucially depends on the expected values on both future policy dimensions $(\bar{e}^m, \bar{\tau}^m)$. Given the equilibrium Euler condition[30] of e_t, i.e. $\left.\frac{dh_{t+1}}{de_t}\right|_{e_t=\mathbf{E}^m} = \frac{R}{\bar{\tau}^m}$, and the decreasing return in education investment, $\bar{\tau}^m$ positively affects the level of forward transfers through the parameter $\psi^m[\bar{\tau}^m]$. The higher the expectation of the future income tax level is, the greater is the political support in public education spending in order to increase the future taxable income and, in turn, compensate the lower private savings with pension benefits. Moreover, the parameter $b_1[\bar{e}^m, \bar{\tau}^m]$ of Eq. (2.21) captures the expected value of the *minimum pension benefits,* defined as a social program whose

[30] See Proof of Proposition 5 in Appendix for the analytical derivation.

contributors are just the unskilled workers, i.e. $\check{p}^m = (1+n)\,\bar{\tau}^m - (1+n)^2\bar{e}^m$. The higher the expected value on $\check{p}^m$ is, the greater is the size of government.

A more structured approach to correctly internalize the expectations on future policies is then necessary in order to fully catch the strategic component of political equilibrium decisions. To this aim, in the following paragraph we introduce and develop a perfect forward-looking approach.

2.4.2 Politico-Economic Perfect Forward-looking Equilibrium

The deletion of the myopic information constraints modifies dramatically the dynamic programming problem, generating serial correlation among present and future political choices. Agents are now able to strategically vote over the political space internalizing both the direct dynamic feedbacks and the partially indirect ones due to persistence of current policies on future political variables. As suggested by Krusell et al. (1997), in order to restrict the set of possible equilibrium outcomes, we employ the differentiable political *SMPE* with *perfect foresight* as equilibrium concept of our economy. In Markov equilibria, the current political decisions may affect the future state variables, i.e. the current level of physical capital and human capital, and thus the future labor income tax rate, education transfers and pension benefits. The definition of the equilibrium is given by:

Definition 5 *A perfect foresight political SMPE is defined as a vector of differentiable policy decision rules,* $F = (\mathbf{T}, \mathbf{E})$*, where* $\mathbf{T} : \mathbb{R} \times \mathbb{R} \longrightarrow (0,1)$ *and* $\mathbf{E} : \mathbb{R} \times \mathbb{R} \longrightarrow (0, \hat{e}_t)$ *are the taxation policy rule and the public higher education policy rule,* $\tau_t = \mathbf{T}[h_t, k_t]$ *and* $e_t = \mathbf{E}[h_t, k_t]$*, respectively. Given the political indirect utility, Eq. (2.15), the following conditions must hold:*

(*i*)

$$F[h_t, k_t] = \arg\max_{f_t} W[f_t, f_{t+1}, h_t, k_t]$$

subject to the following set of constraints:[31]

$$1. \quad V\left[h_t, k_t\right] \equiv W\left[\digamma\left[\cdot\right], h_t, k_t\right] \geq W^{LF}\left[k_t\right]$$

$$2. \quad \Phi\left[h_t, k_t\right] = \begin{cases} k_{t+1} = K\left[f_t, \digamma\left[h_{t+1}, k_{t+1}\right], h_t\right] \\ h_{t+1} = H\left[e_t, h_t\right] \end{cases}$$

$$3. \quad f_t \in \Pi\left[h_t, k_t\right]$$

where $W^{LF}[\cdot]$ *is the Eq. (2.15) in the laissez-faire case,* $H[\cdot]$ *and* $K[\cdot]$ *are defined in Eq. (2.3) and Eq. (2.10), and* $\Pi[\cdot]$ *is a continuous convex correspondence.*

(ii)

$$V\left[h_t, k_t\right] = M\left(V\right)\left[h_t, k_t\right]$$

where the functional $M : \mathbb{C}^{\infty}\left(R^2\right) \rightarrow \mathbb{C}^{\infty}\left(R^2\right)$ *is defined as follows:*

$$M\left(V\right)\left[h_t, k_t\right] := \max_{f^c \in \Pi^c[h,k]} W\left[f_t, f_{t+1}, h_t, k_t\right]$$

The first equilibrium condition requires the political control variables, f_t, have to be chosen in order to maximize the decisive voter's indirect utility function (2.15), taking into account that future redistribution and taxation depend on the current policy choices via both the equilibrium private decision and future equilibrium policy rules. The second condition requires that, if an equilibrium exists, it must satisfy the fixed point properties, i.e. $M(\cdot)$ is a contraction. At this point note that the sequential resolution of the optimization program leads to a *time-inconsistent* allocation. Since the policies at time $t+1$ influence individual saving choices in period t, the reoptimization in any period $s > t$ would yield a first order condition violating the one achieved at time t. To avoid the emergence of *time-inconsistent* equilibrium policies, we allow for the current government to set its political platform correctly foreseeing how the future government will set political instruments. From a technical point of view, we are looking for two differentiable policies which obey the recursive rules given by the vector of

[31] Differently from Definition 4, in the perfect forward looking case the politicians take care also of the impact current policies have on next-period one. Consequently the political internalization relaxes the constraints requirements.

functions $f_t = \mathcal{F}[k_t, h_t]$, where $\mathcal{F}$ is an infinite dimensional object and the key endogenous variable of the problem. The second fundamental element we are looking for is a function which describes the private sector response to a *one-shot deviation* of the government, when agents expect future policies to be set by politicians according to $\mathcal{F}$ as a function of the current state and of political control variables, $k_{t+1} = \tilde{K}[f_t, h_t]$.[32]

Before solving recursively for the equilibrium policy rule $\mathcal{F}$, we investigate the marginal impact of τ_t and e_t on the welfare of the two decisive voters' groups. Maximizing Eq. (2.15) with respect to the policy vector $f_t \in \Pi[h_t, k_t]$ and applying the envelope theorem that cancels out the effect of the two political control variables via $k_{t+1,}$ we obtain the following system of first order conditions:

$$0 = \underbrace{\phi(1+n)(1+h_t)u_{C_{2,t}}}_{\text{old's direct benefit}} - \underbrace{(1+n)(1+h_t)u_{C_{1,t}}}_{\text{adults' direct cost}} \tag{2.22}$$
$$\underbrace{+\beta(1+n)^2 u_{C_{2,t+1}} \left((1+h_{t+1}) \frac{d\tau_{t+1}}{d\tau_t} - (1+n) \frac{de_{t+1}}{d\tau_t} \right)}_{\text{adults' exp. cost/benefit}}$$

[32] The function $\tilde{K}$ is known only conditioning on the existence of $\mathcal{F}$.To derive $\tilde{K}$ start from Eq. (2.10):

$$k_{t+1} = K[f_t, f_{t+1}, h_t]$$

Function K describes the equilibrium behavior of private agents as a function of current state and both current and future policies. If there exists a differentiable function $\mathcal{F}$, which describes the policy behavior followed by politicians in equilibrium, this rule can be internalized by fully rational private agents. It follows that:

$$k_{t+1} = K[f_t, \mathcal{F}[k_{t+1}, h_{t+1}], h_t]$$

Plugging the Eq. $h_{t+1} = H[e_t, h_t]$ into the above equation and rearranging the terms we get:

$$k_{t+1} = \tilde{K}[f_t, h_t]$$

Due to the full depreciation of physical capital, $\tilde{K}$ is not a function of current level of physical capital, which strongly simplifies the analyses.

$$0 = \underbrace{-\phi(1+n)^2 u_{C_{2,t}}}_{\text{old's direct cost}} \tag{2.23}$$
$$\underbrace{+\beta(1+n)^2 u_{C_{2,t+1}} \left(\frac{dh_{t+1}}{de_t}\tau_{t+1} + (1+h_{t+1})\frac{d\tau_{t+1}}{de_t} - (1+n)\frac{de_{t+1}}{de_t} \right)}_{\text{adults' expected direct/indirect cost/benefit}}$$

Differently from the FOCs resulting in the myopic case, conditions (2.22) and (2.23) internalize the *strategic effects*, capturing how politicians can affect future policies through their current choices of f_t. If $\frac{d\tau_{t+1}}{d\tau_t} > 0$ (< 0) and $\frac{d\tau_{t+1}}{de_t} > 0$ (< 0) agents know that a higher income tax rate and larger education transfers lead to a higher (lower) tax rate in the future. Thus, differently from the case where the current political choices do not affect future policy outcome, representatives may strategically increase (reduce) τ_t and e_t in order to distort the tax rate outcome of tomorrow. The same idea holds for e_{t+1}.

Political *SMPE* with *Perfect Foresight*

Due to the non-linearity and bidimensionality in the political space, the system of partial differential equations (2.22) and (2.23) cannot be easily solved using integration methods.[33] We start by solving simultaneously for the maximization of the decisive voter with respect to the income tax rate and the level of public higher education transfers. As reported in Klein et al. (2008) the equilibrium is obtained as the limit of a finite-horizon equilibrium, whose characteristics do not significantly depend on the time horizon, as long as the time horizon is long enough. Consequently our resolution strategy consists in a constructive approach (*induction method*). We compute the FOCs defining the feasible equilibrium policy rules in a finite-horizon environment via backward induction. We start at a final round $t < \infty$ and we re-compute the equilibrium policy rules, $\digamma_t = (\mathbf{E}_t, \mathbf{T}_t)$, as long as all the *direct dynamic feedbacks*, induced by political choices on private one, have been internalized. In particular, due to two-periods lagged impact of e_t on

[33] See for example Grossman-Helpman (1996) and Azariadis-Galasso (2002) frameworks in which by applying the envelope theorem the differential equation becomes linear and solution results straightforward to determine.

private saving choice, we will perform recursive maximization until period $t-2$. At each time the political objective function, described in Eq. (2.15), has to be simultaneously maximized with respect to its arguments, i.e. the pair (e_t, τ_t), subject to the Euler condition of the economic optimization problem, the balanced budget constraint, the individual rationality condition and the equilibrium policy rules of the following periods, computed via backward procedure. Once a recursive structure is identifiable, by making the time horizon go to infinity for all the time-variant coefficients determined so far, we obtain the equilibrium policy rules as fixed point of the recursive problem in a multidimensional environment.

Fixing $\theta = \frac{1}{2}$, we analytically determine a *fundamental* equilibrium capturing the effects that are inherent in the dynamic game itself, which turns out to be unique. Let $\Upsilon^p \equiv (K_t^\tau, H_t^\tau) \cap H_t^e$, defined as in the previous paragraph, be the state-space in which interior policy rules are obtained. Furthermore, let $\bar{R} \equiv \frac{1+n}{R-(1+n)}$ be an index measuring the economy's dynamic efficiency. The following Proposition then applies:

Proposition 6 *Let* $\psi^* = \frac{1}{\alpha}\left(\frac{2R}{\alpha}\left(R - \sqrt{R^2-\alpha}\right) - 1\right)$. *Under the dynamic efficiency condition, for any* $(h_t, k_t) \in \Upsilon^p$ *the set of feasible rational policies,* $f_t \equiv (e_t, \tau_t)$, *which can be supported by a perfect foresight political SMPE, has the following functional form:*

(i)

$$\mathbf{E}[h_t] = a_1 h_t + a_0 \tag{2.24}$$

where $a_1 \equiv \frac{\alpha}{1+n}\psi^*$ *and* $a_0 \equiv \frac{1-\alpha}{1+n}\bar{h}\psi^*$;

(ii)

$$\mathbf{T}[k_t, h_t] = -b_3 \frac{k_t}{1+h_t} + b_2 \frac{h_t}{1+h_t} + b_1 \frac{1}{1+h_t} + b_0 \tag{2.25}$$

where $b_0 \equiv \Omega_O$, $b_1 \equiv \bar{h}\frac{1-\alpha}{\alpha}b_2 + \bar{R}\left(1+\bar{h}(1-\alpha)\psi^*\right)\Omega_O$, $b_2 \equiv \alpha\psi^*(\Omega_A + 2\Omega_O)$ *and* $b_3 \equiv R\Omega_A$.

Otherwise, for any $(h_t, k_t) \notin \Upsilon^p$ *corner solutions result in at least one of the two dimensions.*

Proof. (See Appendix). ■

The Proposition characterizes the equilibrium outcomes of public choices in a fully rational environment when Markov strategies are implemented.

The dynamic efficiency requirement, $R > 1 + n$, is a necessary condition for the simultaneous existence of PAYG and public education programs. In our economy, during the transition path, the *implicit net return* to pensions is determined by both the population growth rate and the marginal increase in taxable income due to human capital investment net of the future resources devoted to education. As long as the *implicit net return* is higher than the capital rental price, there will emerge incentives in investing simultaneously in both sides of the redistribution programs. By contradiction, suppose that the population growth rate exceeds the net rental price to physical capital, then it is straightforward to prove that b_1 tends to infinity[34] and consequently the asset variable $H[\cdot]$ has a negative marginal impact on the size of government. Thus, according to Eq. (2.3) and (2.25), an increase in education spending would determine a positive variation in the stock of human capital and in turn a decrease in tax rate. Consequently, physical capital increases inducing further reduction in the future tax level. This cannot be an equilibrium since, given $R < 1 + n$, agents always have an incentive to deviate by choosing a higher level of income tax rate in order to depress private saving and guarantee a higher future level in pension contributions even without investment in education. As long as the economy is dynamic inefficient the simultaneous existence of both forward and backward transfers is excluded. We depart from the traditional literature on redistributive policies, where no endogenous human capital formation is modelled, which states that social security survives just in an economy characterized by a population growth rate higher than the rental price.[35]

The two policy rules in the perfect foresight equilibrium are structurally equivalent to the policy functions in the myopic case. The political decision on the education transfers solely depends linearly on the stock of human capital, while the fiscal tax is a non-linear combination of k_t and h_t. Instead, the major difference between the two equilibrium concepts concerns their quantitative predictions through the two distinct channels illustrated in the previous section. First, all the elements of Eq. (2.24) and (2.25) affected by h_t are scaled by the difference between ψ^* and ψ^m, which incorporates

[34] See Proof of Proposition 6 in Appendix for the derivation of b_1.

[35] Contrary to the previous literature based on dynamic inefficiency condition, our framework has the advantage to be more realistic. Consistently with data from World War II onward, OECD countries faced higher rental price to capital with respect to population growth rate. For a complete discussion see Abel (1987).

how perfectly rational voters, differently from myopic ones, create their own expectation on the future income tax. Second, the coefficient b_1 internalizes the expectations on the future value of the pension benefits, mostly incorporated in the dynamically efficient index $\bar{R}$. the more the population growth rate tends to the return to physical capital, which induces an increase in $\bar{R}$, the more voters are willing to substitute private savings with public ones. This turns out in a higher political support for income tax.

As depicted in Figure 2.1, for any non-zero level of the income tax rate, the larger the human capital is, the more political support the education program receives, i.e. $\frac{de_t}{dh_t} = a_1 > 0$. Two different configurations may arise depending on the level of society's human capital endowment. As shown in Panel (a), as long as $\bar{h} < \frac{1}{(1-\alpha)\psi^*}$, $\mathbf{E}[h_t]$ lies within the feasibility boundaries, $(0, \hat{e}_t)$, for any level of human capital. Instead, as reported in Panel (b), if $\bar{h} \geqslant \frac{1}{(1-\alpha)\psi^*}$, there exists a threshold value of parental human capital, $\tilde{h} \equiv \frac{1-\bar{h}(1-\alpha)\psi^*}{\alpha\psi^*-1}$, such that for any level of h_t lower than $\tilde{h}$ boundary solution is attained, i.e. $\mathbf{E}[h_t] = \hat{e}_t$.

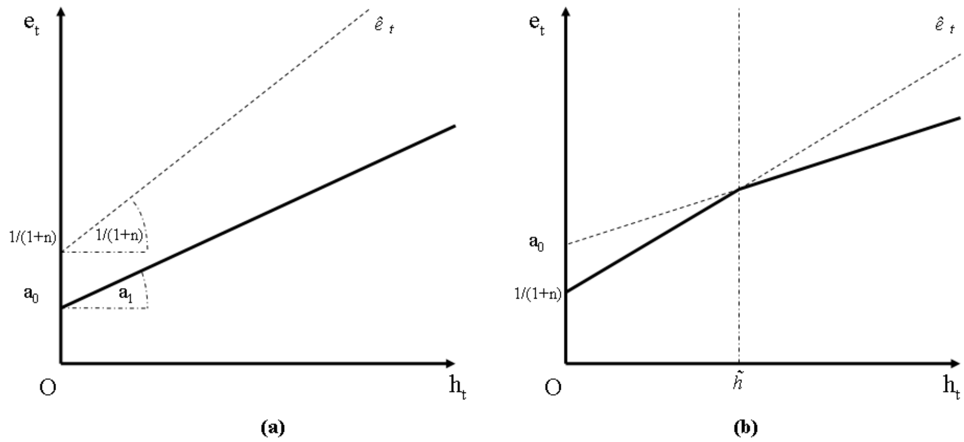

Figure 2.1: Education Transfers Policy Rule

In other terms, due to complementarity between the inputs employed in the skill technology, the whole tax revenue is devoted to investment in public education and no social security program is implemented. Otherwise, if h_t is higher than $\tilde{h}$, the larger the stock of human capital is, the lower is the variation in education transfers and consequently the flatter is the equilibrium policy function. Indeed, due to the decreasing returns in parental human capital, in equilibrium politicians set positive transfers both for education and social security.[36]

Remark 10 $\mathbf{E}[h_t]$ *does not depend on strategic political components embedded in the parameter* ϕ. *For the determination of the transfers' level, only the mass effect component,* n, *matters.*

As reported in Proposition 6, in equilibrium the amount of education transfers has to be equal to the highest feasible value of forward spending which maximizes the *net implicit rate* of future pensions. In other terms $\mathbf{E}[h_t]$ maximizes the intertemporal utility of current adults without considering the political distortions due to the old's bargaining power. This result sounds counterintuitive because, as shown in Eq. (2.23), the old actually have incentives in reducing the education amount at the minimal level. This in turn, under dynamic efficiency, would remove the adults' incentives in substituting private saving with public one. As a final result the laissez-faire economy would be established. It cannot be an equilibrium for the setting of an intergenerational contract and, as a consequence, the emergence of a public education program not distorted by the political bias is justified.

Figure 2.2 reports the equilibrium fiscal policy rule described in Eq. (2.25). For illustrative purposes, it is useful to analyze separately the effects of the two asset variables on $\mathbf{T}[h_t, k_t]$. Panel (a) describes the structural relation between the equilibrium tax rate and the level of h_t where the intercept, $\mathbf{T}[k_t, 0]$, is a decreasing function in physical capital. As long as $k_t < \tilde{k}$ where $\tilde{k} \equiv \frac{b_1 - b_2}{b_3}$, the larger the human capital is, the higher is the opportunity cost to tax levy, i.e. $\frac{d\tau_t}{dh_t} < 0$. If instead $k_t \geqslant \tilde{k}$, incentives to increase simultaneously the taxable income and the income tax rate arise, i.e. $\frac{d\tau_t}{dh_t} \geqslant 0$.

[36] Note that the scenario characterized by the whole tax revenue devoted to public higher education investments, i.e. no current pension benefits, is an equilibrium outcome only as long as one-period future pension transfers are allocated to current adults. In other terms, when $\bar{h} \geq \frac{1}{(1-\alpha)\psi^*}$ and $h_t < \tilde{h}$, there exists an initial condition $\tilde{h}_0$ such that for any $h_0 > \tilde{h}_0$, due to public investments in higher education, future human capital level exceeds the threshold level $\tilde{h}$, i.e. $h_{t+1} \geq \tilde{h}$. In this case adults have incentive in taxing their income because of the future expected benefits in terms of PAYG social security. Thus there emerges a one-period-equilibrium characterized by an intergenerational contract with current backward transfers equal to zero. Otherwise, if $h_0 < \tilde{h}_0$, then no future pensions will be set for current adults and no incentive to implement an intergenerational contracts may emerge.

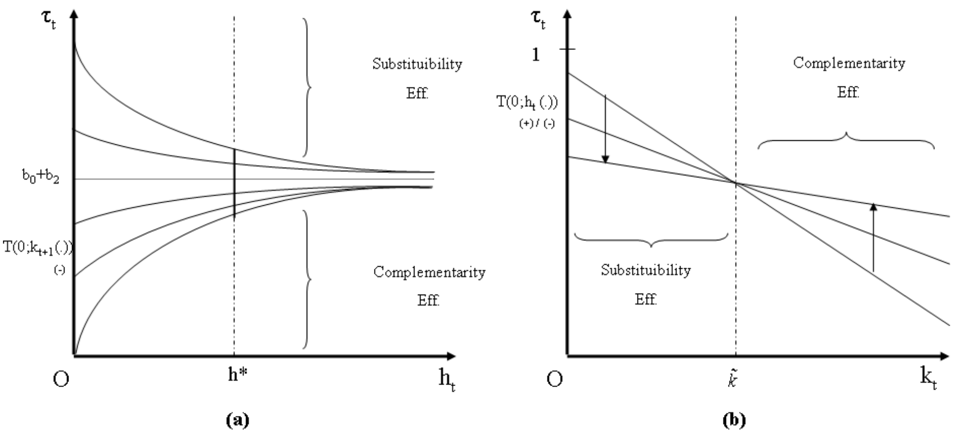

Figure 2.2: Income Tax Policy Rule

Panel (b) illustrates the structural relation between the equilibrium tax rate and the level of k_t. The equilibrium predicts for any value of k_t the higher the physical capital is, the lower is the income tax rate, consistently with previous literature.[37] The intuition for the fiscal policy function to be non-increasing in the capital stock is the following. By contradiction, if $\mathbf{T}[h_t, k_t]$ were increasing in k_t, current adults would have incentive to save in order to provide the next generation with a higher level of capital and therefore receive a higher pension. This cannot be an equilibrium, since the higher amount of backward transfer reduces the level of saving that workers are willing to make.

Remark 11 $\mathbf{T}[h_t, k_t]$ *crucially depends on both the strategic political component embedded in the parameter* ϕ *and the demographic component,* n*, for the determination of the size of government.*

Due to the distortions induced by taxation on saving choices and the politicians' opportunistic behavior, a strategic persistency criterion drives the setting of the income tax rate. In our environment human capital plays a crucial role in two different ways. On the one hand it *mitigates* the politicians' strategic behavior. Precisely, the higher the level of human capital is, the flatter is the equilibrium policy function and the lower is the elasticity of $\mathbf{T}[h_t, k_t]$ with respect to physical capital. The lower responsiveness of taxation policy decisions on the level of private savings weakens the strategic

[37]See among others Grossman and Helpman (1998), Forni (2005), Bassetto (2008).

channel through which politicians can extract rent to win elections. On the other hand human capital, through the choice in education transfers, *perturbs* the political choice concerning the size of government. Depending on the political bargaining intensity between adults and old embedded in the coefficients b_1 and b_2 of Eq. (2.25), the marginal impact of human capital on taxation decisions can be either positive or negative, as already pointed out in the above analyses of the equilibrium tax structure. Formally, let us define $\bar{\Omega}_O \equiv \frac{(\alpha-(1-\alpha)\bar{h})\psi^*}{(2(1-\alpha)\bar{h}+\alpha)\psi^*+\bar{R}}$, the following relation holds:

$$\begin{cases} b_2 < b_1 & \text{iff} \quad \Omega_O > \bar{\Omega}_O \\ b_2 \geqslant b_1 & \text{iff} \quad \Omega_O \leq \bar{\Omega}_O \end{cases} \tag{2.26}$$

The relation states that an economy where $\Omega_O \leq \bar{\Omega}_O$ experiences a political competition characterized by weak bargaining power of the old and $b_1 \leqslant b_2$. If $\Omega_O > \bar{\Omega}_O$, then the old exert a strong bargaining power and $b_1 > b_2$.

To summarize, a complete description of the recursive Markovian structure including both the economic environment and the political scenario is represented in Figure 2.3.

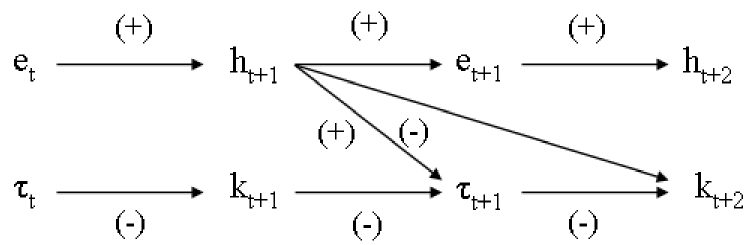

Figure 2.3: Markovian Structure

The picture points out the strategic relations which provide the necessary incentives to selfish agents to sustain simultaneously backward redistributive policies and forward ones, i.e. (e_t, p_t), as described above.

Welfare State Regimes

Figure 2.2 points out the strategic structural relation between the income tax rate and human capital in the Markovian environment, which drives the economy towards different welfare state regimes. If pure political factors matter in splitting the public spending, then a political welfare regime will emerge. If economic factors are also relevant, then a politico-economic welfare regime will arise. The following Corollary fully characterizes the conditions for the identification of the different regime configurations:

Corollary 2 *Given the stationary equilibrium policy rules* $\mathbf{T}[h_t, k_t]$ *and* $\mathbf{E}[h_t]$*:*

(a) *if* $b_1 \leqslant b_2$*, then the Politico Complementarity Welfare Regime, PCR, arises, i.e.* $\frac{d\tau_t}{dh_t} \geqslant 0$*;*

(b) *if* $b_1 > b_2$ *and* $k_t \geqslant \tilde{k}$*, then the Politico-Economic Complementarity Welfare Regime, PECR, arises, i.e.* $\frac{d\tau_t}{dh_t} \geqslant 0$*;*

(c) *if* $b_1 > b_2$ *and* $k_t < \tilde{k}$*, then the Politico-Economic Substitutability Welfare Regime, PESR, arises, i.e.* $\frac{d\tau_t}{dh_t} < 0$.

Proof. (See Appendix). ■

While economic factors driving the system into different welfare state regimes are endogenously determined by capital asset accumulation through the saving choices, i.e. $k_t \gtreqless \tilde{k}$, political factors depend on the relative bargaining power between the adults and the old, i.e. $b_2 \gtreqless b_1$. An economy characterized by a weak level of old bargaining power in the political process, i.e. $b_1 \leqslant b_2$, will experience a *PCR*, for any level of k_t. Contrarily, an economy with a strong level of old bargaining power in the political arena, i.e. $b_1 > b_2$, will experience a *PECR* if the system is high-capitalized, i.e. $k_t > \tilde{k}$, otherwise a *PESR* will emerge if the economy is low-capitalized, i.e. $k_t < \tilde{k}$.

Intuitively, as already pointed out, in equilibrium a higher level of the current income tax rate will determine a decrease of the future physical capital stock and, consequently, an increase of the future tax rate. In the *PCR* welfare state regime, adults anticipate that, if they invest in education today, an increase in future human capital will determine a further positive variation in the level of income tax rate tomorrow. Given the increase in both the future tax rate and taxable income, i.e. gross future pension benefits, which maximize the adults' intertemporal utility, *PCR* emerges as the only sustainable welfare state regime when adult bargaining power prevails.

To fully characterize the public spending process, based on the welfare state regime criterion, we move the analyses to the equilibrium characterization for pension benefits.

Corollary 3 *Under decreasing return in education, the impact of education spending on social security transfers is always positive, i.e.* $\frac{dp_{t+1}}{de_t} > 0$.

Proof. (See Appendix). ■

Remark 12 *The existence of a PAYG social security program supports public investment in higher education even in absence of altruism.*[38]

Independently from the welfare state regime characterizing the economy, an increase in public education transfers induces higher pension benefits in the future, creating the incentive for adults in supporting the education program. *Ceteris paribus*, by supporting a higher education cost today, the adults internalize that it will generate a higher taxable income tomorrow, guaranteeing a higher level of pension benefits when they will be old, for any level of $\mathbf{T}[\cdot]$.

The interaction between political and economic institutions determines the amount and the dynamic evolution of the pension system.

Corollary 4 *At each time t, for any given level of human capital, in PESR pension benefits are lower then the PCR and larger then the PECR, i.e.* $p_t^{PECR} < p_t^{PESR} < p_t^{PCR}$.

Proof. (See Appendix). ■

Under rational expectations, when the adults' bargaining power is sufficiently strong, i.e. $b_1 \leqslant b_2$ and PCR arises, the equilibrium pension benefits reach the highest feasible level. Otherwise, when the old prevail in the political debate, depending on the physical capital stock, the pension benefits are lower in a high-capitalized economy then in a low-capitalized one.

To resume graphically, in Figure 2.4 we plot on the state-space (h_t, k_t) as illustrative case the welfare state regime configurations which arises under certain parameters' conditions when $\bar{h} > \frac{1}{(1-\alpha)\psi^*}$ and $h_0 > \tilde{h}_0$.[39] Panel (a) shows the case in which a weak level of adult bargaining power characterizes

[38] When no public education transfers are provided, $e_t = 0$ the bidimensional political space degenerates to the unidimensional case in an economy characterized by no human capital accumulation and consequently general equilibrium effects. This type of economy was studied among others by Grossman and Helpman (1996) and Azariadis and Galasso (2002).

[39] It should be noted that if $\bar{h} \leq \frac{1}{(1-\alpha)\psi^*}$, then human capital does not play any role in splitting public spending between education and retirement transfers. In other terms, it avoids the interesting case with pension benefits set to zero.

the political scenario. Contrarily, Panel (b) allows for a strong bargaining power of the adults.

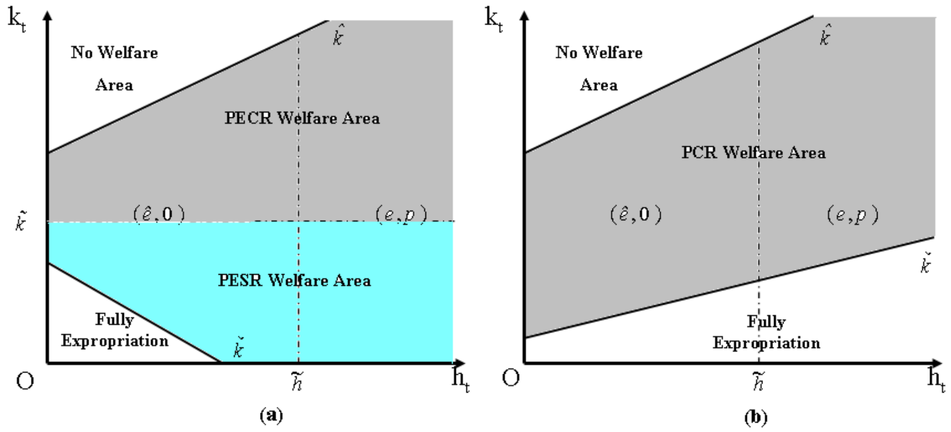

Figure 2.4: Panel (a) shows the case for $b_2<b_1$, Panel (b) shows the case for $b_2>b_1$.

As long as $k_t < \check{k}_t$ in both cases full expropriation occurs. The tax rate, equal to 100% of labor income, is assigned either to finance only the public education program if $h_t < \tilde{h}$ or to support both redistributive social programs if $h_t \geqslant \tilde{h}$. Differently, as long as $k_t \geqslant \hat{k}_t$ the *laissez-faire* equilibrium emerges. Panel (a) reports the politico-economic parameters' configurations which make *PECR* and *PESR* arise, i.e. $b_2 + b_0 < 1$ and $b_1 + b_0 > 1$, whereas panel (b) shows the emergence of *PCR* due the pure political factors, i.e. $b_2 > b_1$.

Aging

Quantitatively, one of the most severe challenges concerning the intergenerational transfer system in developed economies regards the impact of population aging both in demographic (n) and political (ϕ) terms. Demographic aging, which represents the quantitative component of the aging phenomenon, decreases partially the returns from a PAYG system in our economy characterized by endogenous human capital formation. Political aging, which represents the qualitative component of aging phenomenon, gives retirees stronger claim over pension benefits even on constant demographic terms. Based on the characterization of the political equilibrium, we now consider how aging affects the policy decisions of representatives who face electoral constraints in the form of both the size of the welfare state, represented by the tax rate $\mathbf{T}$, and the amount of intergenerational transfers, $\mathbf{E}$ and $\mathbf{P}$. Focusing on political aging the following Corollary holds:

Corollary 5 *Political aging, i.e. the increase in ϕ, has no quantitative impact on the education transfers, $\frac{d\mathbf{E}}{d\phi} = 0$, and induces an increase in the income tax rate, $\frac{d\mathbf{T}}{d\phi} > 0$. It follows that, for any level of $\bar{h}$, $\frac{d\mathbf{P}}{d\phi} > 0$.*

Proof. (See Appendix). ■

The political effect is captured by a decrease in the political weight of the adults, that is, an increase in the political weight of the old. A stronger ideological pressure of the old in the political debate implies a higher income tax rate. This in turn determines a larger social security system supported by voting. Given the efficiency criterion driving the implementation of public education policy, the overall effect of political aging does not distort **E.**

Corollary 6 *The demographic aging, i.e. the decrease in n, induces an increase in education transfers, $\frac{d\mathbf{E}}{dn} < 0$, and has an ambiguous impact on the income tax rate, $\frac{d\mathbf{T}}{dn} \gtreqless 0$. It follows $\frac{d\mathbf{P}}{dn} \gtreqless 0$.*

Proof. (See Appendix). ■

Departing from previous literature suggesting the size of social security to be increasing in population growth, our model predicts under which parametric condition the inverse relation also appears. Specifically, demographic aging has an ambiguous impact on the amount of pension transfers in per-capita terms. A first interesting case arises when the margin $R - (1 + n)$ is sufficiently small, which in turns implies, even without considering the human capital return, the implicit return to pensions to be close to the gross return to private saving. It gives incentives in a younger society to opt for higher pension benefits due to their larger demographic return, i.e. $\frac{d\mathbf{P}}{dn} > 0$. A second illustrative case emerges when the relative political weight of the adults is larger than $\bar{R}$ and $\bar{h}$ is sufficiently high. In this scenario, even if population ages and, in turn, the demographic pension returns decrease, adults have incentives to depress the current level of savings in order to compensate the smaller number of future tax payers with a higher tax rate level tomorrow, i.e. $\frac{d\mathbf{P}}{dn} < 0$.

Dynamics and Steady States

We now discuss the transition dynamics of the economy during the adjustment towards the steady state.

Definition 6 *The laws of motion of the collection* $\{e_t, \tau_t, h_t, k_t\}_t$ *are definite as the mappings:*

$$h_{t+1} = H\left[\mathbf{E}\left[h_t\right], h_t\right], \qquad k_{t+1} = \tilde{K}\left[\mathbf{E}\left[h_t\right], \mathbf{T}\left[h_t, k_t\right], h_t\right],$$
$$e_{t+1} = \mathbf{E}\left[H\left[\mathbf{E}\left[h_t\right], h_t\right]\right], \qquad \tau_{t+1} = \mathbf{T}\left[\tilde{K}\left[\mathbf{E}\left[h_t\right], \tau_t, h_t\right], H\left[\mathbf{E}\left[h_t\right], h_t\right]\right].$$

The economy's dynamics are basically driven by human capital evolution which affects both the education transfers' law of motion and the transition dynamics of taxation policy. While the former is directly influenced only by human capital, the latter is affected by human capital both directly and indirectly through physical capital. This implies that convergence conditions in the state-space are also sufficient for the stable convergence of the policy rules evolution. The following Lemma states the conditions for the economy's convergence.

Lemma 5 *Let* $\underline{\phi} \equiv \beta\left(R - (1+n)\right)$ *and* $\underline{n} \equiv \sqrt{2R\left(R - \sqrt{R^2 - \alpha}\right) - \alpha} - 1$. *Given any feasible initial condition* (h_0, k_0)*, if* $\phi > \underline{\phi}$ *and* $n > \underline{n}$*, then the collection* $\{e_t, \tau_t, h_t, k_t\}_t$ *is characterized by stable monotonic convergence. The speed of convergence for* τ_t *crucially depends on the initial condition, the exogenous society's human capital endowment and the welfare state regime characterization.*

Proof. (See Appendix). ■

Given the differentiability of the policy functions, the interior solution conditions and Lemma 5, the following proposition holds:

Proposition 7 *A feasible steady state* $\{e^*, \tau^*, h^*, k^*\}$ *exists and is unique.*

Proof. (See Appendix). ■

Thus, depending on the initial condition, (h_0, k_0), and the level of the exogenous human capital endowment, $\bar{h}$, the control and the state variables converge monotonically to the unique feasible steady state. According to the specific emerging welfare state regime different speeds of convergence and amounts of intergenerational transfers characterize the economy.

2.5 Conclusions

In this paper we investigate the conditions for the emergence of implicit intergenerational contracts without assuming reputation mechanisms, commitment technology and altruism. We present a tractable dynamic politico-economic model in an OLG environment where political representatives compete by proposing multidimensional fiscal platforms. Both backward and forward intergenerational transfers, respectively in the form of pension benefits and higher education investments, are simultaneously considered in an endogenous human capital setting with distortionary income taxation when agents play Markovian strategies.

The dynamic mechanisms driving our results are intuitive: The social security system sustains investment in public education that, in turn, creates a dynamic linkage across periods through both human and physical capital driving the economy towards different welfare state regimes.

We show that intergenerational contracts can be politically sustained uniquely as long as the economy is dynamically efficiency, i.e. the rental gross price of capital is larger than the economic growth rate, with underaccumulation of physical capital. Departing from the previous literature, our economic environment is in line with empirical findings on the dynamic efficiency status of most developed countries, especially after the demographic transition. By endogenizing human capital formation through public education investments, backward and forward redistributive programs may optimally self-sustain each other even in the absence of a benevolent Central Planner. In equilibrium political decisions are *no* education *strategic*, while due to distortionary taxation and the politicians' opportunistic behavior, strategic *persistency* underlies the determination of the income tax rate.

Relatively to the predictions about the transition towards the steady state, we find that three different welfare state regimes may emerge depending on both the relative political bargaining power between adults and old and the endogenous capital asset accumulation. The emergence of different regimes leads the economy towards different dynamic paths and *persistence* degrees of distortionary redistribution. Under rational expectations, in the regime supported by the adults, the equilibrium pension benefits reach the highest feasible level.

Demographic aging increases the equilibrium per-capita level of forward transfers, i.e. public education spending. Due to the decreasing return in human capital accumulation aging does not always exacerbate the generous behavior of the politicians towards the elderly. Political aging has instead a positive impact on taxation but no effects on the level of public education investments.

Our analysis leaves some natural direction open for future research. We have assumed that only adults and old compete in the political debate. Using the developed methodology, relaxing the voting rule by considering youth enfranchisement would generate even further distortions on the determination of education transfers and the government size. Another direction for future research concerns the introduction of a dynamic electoral stage by endogenizing the probability of re-election, which would introduce another source of distortion.

2.6 Technical Appendix

Proof of Proposition (5). In order to prove Proposition (5), we look for a feasible interior solution of $f_t^m \equiv (e_t^m, \tau_t^m) \in \Pi[h_t, k_t]$, when the rationality constraint and the transformation constraints hold. The proof will be performed in two steps. First, we compute the first order condition and we check the feasibility conditions. Second, we will show that the specific solution found satisfies the first order necessary and the second order sufficient conditions of the problem and, therefore, it is a proper solution.

First step

Given $\bar{f}^m \equiv (\bar{e}_t^m, \bar{\tau}_t^m)$, maximizing equation (2.15) with respect to the vector f_t^m, and applying the envelope theorem, we get the first order conditions given by Eq. (2.18) and (2.19). After some algebra the following system of Euler equations is attained:

$$\begin{cases} \frac{u_{C_{2,t}}}{u_{C_{1,t}}} = \frac{1}{\phi} \\ \frac{u_{C_{2,t}}}{u_{C_{1,t}}} = \frac{\bar{\tau}^m}{\phi R}\frac{dh_{t+1}}{de_t^m} \end{cases} \tag{1A}$$

Equating the two conditions above, for $\frac{dh_{t+1}}{de_t^m} = \frac{1-\theta}{(1+n)^\theta}\left(\frac{\alpha h_t + (1-\alpha)\bar{h}}{e_t^m}\right)^\theta$ we determine the equilibrium value of public higher education policy rule, $e^m[h_t]$, as follows:

$$\mathbf{E}^m[h_t] = a_1^m h_t + a_0^m \tag{2A}$$

where $\psi^m \equiv \left(\frac{1-\theta}{R}\bar{\tau}^m\right)^{\frac{1}{\theta}}$, $a_0^m \equiv \frac{\bar{h}(1-\alpha)}{1+n}\psi^m$ and $a_1^m \equiv \frac{\alpha}{1+n}\psi^m$. Plugging Eq. (2A) into the first order condition and after some rearrangements, the equilibrium fiscal policy rule, $\mathbf{T}^m[h_t, k_t]$, has the following functional form:

$$\mathbf{T}^m[h_t, k_t] = -b_3^m \frac{k_t}{1+h_t} + b_2^m \frac{h_t}{1+h_t} + b_1^m \frac{1}{1+h_t} + b_0^m \tag{3A}$$

where $b_0^m \equiv \Omega_O$, $b_1^m \equiv (1-\alpha)\bar{h}\psi^m(\Omega_A + \frac{1}{1-\theta}\Omega_O) + \frac{\Omega_O}{R}((1+n)\bar{\tau}^m - (1+n)^2\bar{e}^m)$, $b_2^m \equiv \alpha\psi^m(\Omega_A + \frac{1}{1-\theta}\Omega_O)$ and $b_3^m \equiv R\Omega_A$.

To determine the interiority conditions of the equilibrium fiscal policy rule we check on the double inequality condition $0 < \mathbf{T}^m[h_t, k_t] < 1$ where $\hat{k}_t^m \equiv \frac{b_2^m + b_0^m}{b_3^m}h_t + \frac{b_1^m + b_0^m}{b_3^m}$ and $\check{k}_t^m \equiv \frac{b_2^m + b_0^m - 1}{b_3^m}h_t + \frac{b_1^m + b_0^m - 1}{b_3^m}$ are the feasible upper and lower capital threshold value, respectively. We denote the bidimensional state-space, which delimits the interior solutions for the income tax rate, as follows:

$$\left(K_t^{m,\tau}, H_t^{m,\tau}\right) = \left\{(h_t, k_t) \,|\check{k}_t^m < k_t < \hat{k}_t^m\right\}$$

If $k_t < \check{k}_t^m$, then the full expropriability regime is theoretically reachable. Otherwise, if $k_t > \hat{k}_t^m$, then the *laissez-faire* economy characterized by zero tax rate is the equilibrium solution. To determine the state-space conditions for interior solution on redistributive dimension, we check on the double inequality condition $0 < \mathbf{E}^m[h_t] \leq \hat{e}_t^m$. The feasible upper threshold value of education transfer, $\hat{e}_t^m = \frac{1}{1+n} + \frac{1}{1+n}h_t$, is obtained from balanced budget constraint, Eq. (2.14), when 1) the equality condition holds, $\mathbf{E}^m[h_t] = \hat{e}_t^m$, 2) tax revenue is maximum, $(1+n)(1+h_t)$, and 3) pension transfers are equal to zero. Plugging $\hat{e}_t^m$ into the above inequality, we require the conditions $0 < \frac{\bar{h}(1-\alpha)}{1+n}\psi^m + \frac{\alpha}{1+n}\psi^m h_t < \frac{1}{1+n} + \frac{1}{1+n}h_t$ to hold. The state-space, which delimits interior solutions for the education transfers policy, is then equal to:

$$H_t^{m,e} = \begin{cases} \{h_t | h_t \in (0,\infty)\} & \text{if } \bar{h} < \frac{1}{(1-\alpha)\psi^m} \\ \left\{h_t | h_t \in (\tilde{h}^m, \infty)\right\} & \text{if } \bar{h} \geq \frac{1}{(1-\alpha)\psi^m} \end{cases}$$

where $\tilde{h}^m \equiv \frac{1-\bar{h}(1-\alpha)}{\alpha\psi^m - 1}\psi^m$. Otherwise, if $h_t < \tilde{h}^m$ the whole tax revenue is devoted to public higher education transfers and no positive pension system is feasible. Considering jointly the interiority feasibility conditions for τ_t^m and e_t^m, we obtain the state-space, Υ^m, which delimits interior solutions of the myopic maximization problem:

$$\Upsilon^m \equiv \left(K_t^{m,\tau}, H_t^{m,\tau}\right) \cap H_t^{m,e}$$

Second step

We now check for the second order sufficient condition of the problem. Let $z[x, q[x]]$ be a function in the variable x. From now on, we adopt the following notation $z_{x_t y_t} \equiv \frac{d^2 z}{dx_t dy_t}$, where x_t and y_t are equal to e_t or τ_t, to indicate the second total differential. The second order conditions are:

$$W_{\tau\tau} = (1+n)(1+h_t)\left((1+h_t)\left(\phi(1+n)u_{C_{22,t}} + u_{C_{11,t}}\right) + (1+n)\frac{dk_{t+1}}{d\tau_t^m}u_{C_{11,t}}\right)$$

$$W_{\tau e} = (1+n)^2(1+h_t)\left(-\phi(1+n)u_{C_{22,t}} + \frac{dk_{t+1}}{de_{m,t}}u_{C_{11,t}}\right)$$

$$W_{ee}=(1+n)^2\left(\phi(1+n)^2 u_{C_{22,t}}+\frac{\bar{\tau}^m}{R}\frac{d^2h_{t+1}}{de_t^m de_t^m}u_{C_{1,t}}-\frac{1+n}{R}\bar{\tau}^m\frac{dh_{t+1}}{de_t^m}\frac{dk_{t+1}}{de_t^m}u_{C_{11,t}}\right)$$
$$W_{e\tau}=-\phi(1+n)^3(1+h_t)u_{C_{22,t}}-\frac{(1+n)^2}{R}\bar{\tau}^m\frac{dh_{t+1}}{de_t^m}u_{C_{11,t}}\left((1+n)+(1+h_t)\frac{dk_{t+1}}{d\tau_t^m}\right)$$

where under logarithmic structure, from equation (2.10), $\frac{dk_{t+1}}{d\tau_t^m}=-\frac{\beta}{(1+\beta)(1+n)}(1+h_t)$ and $\frac{dk_{t+1}}{de_t^m}=-\frac{1}{R(1+\beta)}\bar{\tau}^m\frac{dh_{t+1}}{de_t^m}$.

Note that $W_{\tau\tau}<0$, $W_{\tau e}>0$, $W_{ee}<0$ and $W_{e\tau}>0$. The determinant of the Hessian matrix is then larger than zero only if $W_{\tau\tau}W_{ee}>W_{\tau e}W_{e\tau}$. Given the equilibrium condition described in Eq. (1A), $\frac{\bar{\tau}^m}{R}\frac{dh_{t+1}}{de_t^m}=1$, after rearranging the terms, the above inequality condition is equivalent to:

$$\frac{(1+n)^3}{R}\bar{\tau}^m(1+h_t)\frac{d^2h_{t+1}}{de_t^m de_t^m}u_{C_{1,t}}\left(\phi(1+n)(1+h_t)u_{C_{22,t}}+(1+h_t+(1+n)\frac{dk_{t+1}}{d\tau_t^m})u_{C_{11,t}}\right)>0 \tag{4A}$$

Due to concavity of utility function and human capital production, LHS of Eq. (4A) is proved to be greater or equal to zero in all the parameter space. Thus the objective function is locally concave in the bidimensional space, Υ^m, and the equilibrium policy rules (2A) and (3A) are the feasible solution of the problem. ■

Proof of Proposition (6). Following Klein et al. (2008), our resolution strategy consists in two stages. In the *first step* we will compute the first order conditions subject to: *1*) the Euler condition of the economic optimization problem, Eq. (2.9), *2*) the balanced budget constraint, Eq. (2.14), and *3*) the equilibrium policy rules of the following periods, computed via backward procedure. After having determined the conditions for the existence of fixed points, in the *second step* we will show that the specific solution found, satisfying the first order necessary and second order sufficient conditions of the problem, is a proper solution.

First step

Suppose the economy ends at time $t<\infty$ and that adults at that time have one period temporal-horizon. Thus, the political objective function is as follows:

$$W_t \equiv (1+n)u[C_{1,t}[\tau_t,h_t]]+\phi u[C_{2,t}[p_t,k_t]] \tag{5A}$$

where $C_{1,t}\equiv(1+h_t)(1-\tau_t)$ and $C_{2,t}\equiv(1+n)Rk_t+p_t$. At time t there are no incentives in investing in education, $e_t^*=0$. Assuming interior solution, the fiscal dimension, τ_t, is determined according to the Euler condition, as follows:

$$\frac{u_{C_{2,t}}}{u_{C_{1,t}}} = \frac{1}{\phi} \tag{6A}$$

Under logarithmic utility, the functional form of the equilibrium fiscal policy rule at time t is $\tau_t = -R\Omega_{A,t}\frac{k_t}{1+h_t} + \Omega_{O,t}$ where $\Omega_{A,t} \equiv \frac{1+n}{1+n+\phi}$ and $\Omega_{O,t} \equiv \frac{\phi}{1+n+\phi}$. Consequently, the equilibrium policy rules, $\digamma_t = (\mathbf{E}_t, \mathbf{T}_t)$, are equal to:

$$\digamma_t : \begin{cases} \mathbf{T}_t = -b_{1(0)}\frac{k_t}{1+h_t} + b_{0(0)} \\ \mathbf{E}_t = 0 \end{cases} \tag{7A}$$

where $b_{1(0)} \equiv R\Omega_{A,t}$ and $b_{0(0)} \equiv \Omega_{O,t}$. The number in the brackets represents the number of iterations.

Next we consider period $t-1$, in which adults born at time $t-2$ live up three periods. Due to three-periods effects of the political variable e_t not all the intergenerational *direct dynamic feedbacks* are internalized at time $t-1$ and further recursion is necessary. The political objective function is now as follows:

$$W_{t-1} \equiv (1+n)\, W_{1,t-1}\left[f_{t-1}, f_t, h_{t-1}, h_t, k_t\right] + \phi W_{2,t-1}\left[f_{t-1}, k_{t-1}\right] \tag{8A}$$

where $W_{1,t-1}[\cdot] \equiv u\left[C_{1,t-1}\left[\tau_{t-1}, h_{t-1}, k_t\right]\right] + \beta u\left[C_{2,t}\left[k_t, p_t\right]\right]$ and $W_{2,t}[\cdot] \equiv u[C_{2,t-1}[k_{t-1}, p_{t-1}]]$. After plugging the equilibrium policy rules (7A) of the previous period into Eq. (8A), we maximize with respect to $f_{t-1} \equiv (e_{t-1}, \tau_{t-1})$. Applying envelope theorem, after some algebra, we get the following system of Euler equations:

$$\begin{cases} \frac{u_{C_{2,t-1}}}{u_{C_{1,t-1}}} = \frac{1+\beta}{\phi+\beta(\phi+1+n)} \\ \frac{u_{C_{2,t-1}}}{u_{C_{1,t-1}}} = \frac{1}{R}\left(\frac{1+\beta}{\phi+\beta(\phi+1+n)}\right)\frac{dh_t}{de_{t-1}} \end{cases} \tag{9A}$$

Equating the two conditions in (9A), we get the necessary condition for the determination of the equilibrium level of e_{t-1}, i.e. $\frac{dh_t}{de_{t-1}} = R$. Recalling that at time t, $h_t = \left(\frac{\alpha h_{t-1}+(1-\alpha)\bar{h}}{1+n}\right)^{\theta} e_{t-1}^{1-\theta}$, then plugging $\frac{dh_t}{de_{t-1}}$ into the equilibrium condition, we derive the equilibrium public education transfers at time $t-1$. Let us denote $\psi_{(1)} \equiv \left(\frac{1-\theta}{R}\right)^{\frac{1}{\theta}}$ and $\gamma_{(1)} = \frac{1+n}{R}$. Solving the system (9A), the equilibrium policy rules are then equal to:

$$\digamma_{t-1}:\begin{cases}\mathbf{T}_{t-1}=-b_{4(1)}\frac{k_{t-1}}{1+h_{t-1}}+b_{3(1)}\frac{h_{t-1}}{1+h_{t-1}}+b_{2(1)}\frac{\bar{h}}{1+h_{t-1}}+b_{1(1)}\frac{1}{1+h_{t-1}}+b_{0(1)}\\ \mathbf{E}_{t-1}=a_{1(1)}h_{t-1}+a_{0(1)}\end{cases}\tag{10A}$$

where $a_{0(1)} \equiv \frac{(1-\alpha)\bar{h}}{1+n}\psi_{(1)}$, $a_{1(1)} \equiv \frac{\alpha}{1+n}\psi_{(1)}$ and $b_{0(1)} \equiv \Omega_{O,t-1}$, $b_{1(1)} \equiv \gamma_{(1)}\Omega_{O,t-1}$, $b_{2(1)} \equiv (1-\alpha)\,\Omega_{A,t-1}\psi_{(1)}$, $b_{3(1)} \equiv \alpha\left(\Omega_{A,t-1}+\frac{1}{(1-\theta)}\Omega_{O,t-1}\right)\psi_{(1)}$ and $b_{4(1)} \equiv R\Omega_{A,t-1}$. Now $\Omega_{O,t-1} \equiv \Omega_O \equiv \frac{\phi}{\phi+(1+n)(1+\beta)}$ and $\Omega_{A,t-1} \equiv \Omega_A \equiv \frac{(1+n)(1+\beta)}{\phi+(1+n)(1+\beta)}$ are, respectively, the indexes of the relative old's and adults' political power in an economy that lasts more than one period.

Finally let us consider time $t-2$. At that all the *direct dynamic feedbacks* are internalized. The political objective function is equivalent to equation (8A), then it is not reported. The recursive problem is now subject to the equilibrium policy rules (7A) and (10A) of the previous two periods. Maximizing the political objective function with respect to $f_{t-2} \equiv (e_{t-2}, \tau_{t-2})$ the system of Euler conditions are:

$$\begin{cases}\frac{u_{C_{2,t-2}}}{u_{C_{1,t-2}}}=\frac{1}{\phi+(1+n)\beta}\\ \frac{u_{C_{2,t-2}}}{u_{C_{1,t-2}}}=\frac{1}{R(\phi+(1+n)\beta)}\left(1+\frac{\alpha\theta}{1-\theta}\left(\frac{1-\theta}{R}\right)^{\frac{1}{\theta}}\right)\frac{dh_{t-1}}{de_{t-2}}\end{cases}\tag{11A}$$

Let us now denote with $\psi_{(2)} \equiv \left(\frac{\alpha\theta}{R}\left(\frac{1-\theta}{R}\right)^{\frac{1}{\theta}}+\frac{1-\theta}{R}\right)^{\frac{1}{\theta}}$ and $\gamma_{(2)} \equiv \frac{1+n}{R}+\left(\frac{1+n}{R}\right)^2$. Furthermore, let us introduce the following notation $g_{(2)} \equiv \frac{1+n}{R}\psi_{(1)}+\psi_{(2)}$. As before, solving the system (11A) we yield the following pair of equilibrium policy rules at time $t-2$:

$$\digamma_{t-2}:\begin{cases}\mathbf{T}_{t-2}=-b_{4(2)}\frac{k_{t-1}}{1+h_{t-1}}+b_{3(2)}\frac{h_{t-1}}{1+h_{t-1}}+b_{2(2)}\frac{\bar{h}}{1+h_{t-1}}+b_{1(2)}\frac{1}{1+h_{t-1}}+b_{0(2)}\\ \mathbf{E}_{t-2}=a_{1(2)}h_{t-1}+a_{0(2)}\end{cases}\tag{12A}$$

where $b_{0(2)} \equiv b_{0(1)}$, $b_{1(2)} \equiv \gamma_{(2)}\Omega_O$, $b_{2(2)} \equiv (1-\alpha)\left(\left(\Omega_A+\frac{1}{1-\theta}\Omega_O\right)\psi_{(2)}+\frac{\theta}{1-\theta}\Omega_O g_{(2)}\right)$, $b_{3(2)} \equiv \alpha\psi_{(2)}(\Omega_A+\frac{1}{1-\theta}\Omega_O)$, $b_{4(2)} \equiv b_{4(1)}$ and $a_{0(2)} \equiv \frac{(1-\alpha)\bar{h}}{1+n}\psi_{(2)}$, $a_{1(2)} \equiv \frac{\alpha}{1+n}\psi_{(2)}$.

It is straightforward to show that $\psi_{(2)}$ can be derived as a differentiable monotonic transformation of $\psi_{(1)}$, $m\,[.]$, characterized by $m\,[0] > 0$, $m_\psi > 0$ and $m_{\psi\psi} > 0$. In particular $m\left[\psi_{(1)}\right] = \left(\frac{\alpha\theta}{R}\psi_{(1)}+\frac{1-\theta}{R}\right)^{\frac{1}{\theta}}$. The argument can be repeated for each time $j > 0$ such that:

$$\psi_{(j+1)} = m\left[\psi_{(j)}\right] \tag{13A}$$

Furthermore for each j the following series can be derived:

$$\gamma_{(j)} \equiv \sum_{l=1}^{j} \left(\frac{1+n}{R}\right)^{l}$$
$$g_{(j)} \equiv \left(\frac{1+n}{R}\right)^{j-1} \psi_{(1)} + \left(\frac{1+n}{R}\right)^{j-2} \psi_{(2)} + ... + \psi_{(j)}$$

Using the above notation, starting from $t-3$ we can finally derive the recursive structure which characterizes the political problem:

$$F_{t-j} : \begin{cases} \mathbf{T}_{t-j} = -b_{4(j)}\frac{k_{t-j}}{1+h_{t-j}} + b_{3(j)}\frac{h_{t-j}}{1+h_{t-j}} + b_{2(j)}\frac{\bar{h}}{1+h_{t-j}} + b_{1(j)}\frac{1}{1+h_{t-j}} + b_{0(j)} \\ \mathbf{E}_{t-j} = a_{1(j)}h_{t-j} + a_{0(j)} \end{cases} \tag{14A}$$

where $a_{0(j)} \equiv \frac{(1-\alpha)\bar{h}}{1+n}\psi_{(j)}$, $a_{1(j)} \equiv \frac{\alpha}{1+n}\psi_{(j)}$ and $b_{0(j)} \equiv b_{0(1)}$, $b_{1(j)} \equiv \gamma_{(j)}\Omega_O$, $b_{2(j)} \equiv (1-\alpha)\left((\Omega_A + \frac{1}{1-\theta}\Omega_O)\psi_{(j)} + \frac{\theta}{1-\theta}\Omega_O g_{(j)}\right)$, $b_{3(j)} \equiv \alpha\psi_{(j)}\left(\Omega_A + \frac{1}{(1-\theta)}\Omega_O\right)$, $b_{4(j)} \equiv b_{4(1)}$.

If a political *SMPE* exists, then the limits for $j \to \infty$ of the set of time-variant parameters $\left\{a_{0(j)}, a_{1(j)}, b_{0(j)}, b_{1(j)}, b_{2(j)}, b_{3(j)}, b_{4(j)}\right\}$ exist and are finite. Note that the fixed points determination for the two stationary policy rules crucially depends on the existence of the fixed point of the policy e and, in final instance, on the determination of the limit for $\psi_{(j)}$. Thus we start with the redistributive policy dimension. The computation consists in solving the non-linear difference equation (13A). The $\lim_{j\to\infty}\psi_{(j)}$ is equivalent to the solution(s), if any, of such difference equation given ψ_0 as initial condition. Let us denote with $\hat{\psi}_j$ the value of ψ_j such that $\left(\frac{dm[\psi_j]}{d\psi_j}\right)_{\psi_j=\hat{\psi}_j} = 1$. We yield respectively zero, one or two fixed points as solution of the difference equation iff $m\left[\hat{\psi}_j\right] \gtreqless \hat{\psi}_j$. $\hat{\psi}_j$ is then equal to:

$$\hat{\psi}_j = \frac{1}{\theta}\left(\frac{R}{\alpha}\right)^{\frac{1}{1-\theta}} - \frac{1-\theta}{\alpha\theta} \tag{15A}$$

Note that $R > \alpha^{\theta}$ in all the parameters' space. Such condition guarantees the existence of at least one stable fixed point. For analytical tractability we determine the solutions for quadratic form case. For $\theta = \frac{1}{2}$ under the above condition the two fixed points are:

$$\psi^*_{1,2} = \frac{1}{\alpha}\left(\frac{2R}{\alpha}\left(R \pm \sqrt{R^2-\alpha}\right) - 1\right)$$

We focus on the stable equilibrium, denoted by $\psi^* = \frac{1}{\alpha}\left(\frac{2R}{\alpha}\left(R - \sqrt{R^2-\alpha}\right) - 1\right)$ and we take $\psi_0 = \psi^*$ as initial condition. The solution of the difference equation (13A) is represented in Figure 2.5.

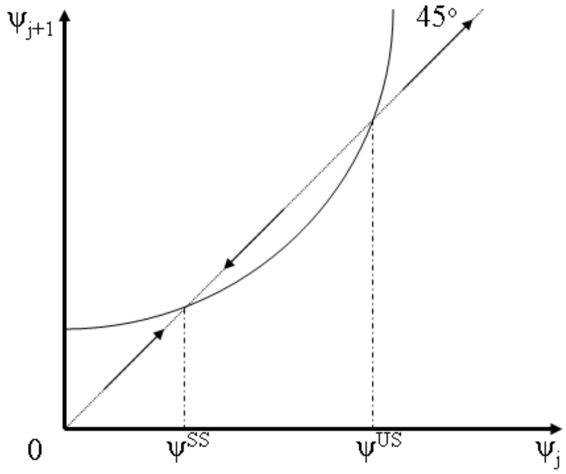

Figure 2.5: $\psi_{(j+1)}{=}m\left[\psi_{(j)}\right]$

Under the condition $R > (1+n)$ the $\lim_{j\to\infty} \gamma_{(j)} < \infty$ is equal to $\frac{1+n}{R-(1+n)} \equiv \bar{R}$. Consequently the $\lim_{j\to\infty} g_{(j)} = \lim_{j\to\infty} \psi^* \sum_{l=1}^{j} \left(\frac{1+n}{R}\right)^l < \infty$ is equal to $\frac{\bar{R}}{\alpha}\left(\frac{2R}{\alpha}\left(R-\sqrt{R^2-\alpha}\right) - 1\right)$. Under such convergence conditions the fixed points are finally attained. Rearranging the terms we can reformulate the individual rational fiscal and redistribution policies as follows:

$$\mathbf{T}\left[h_t, k_t\right] = -b_3 \frac{k_t}{1+h_t} + b_2 \frac{h_t}{1+h_t} + b_1 \frac{1}{1+h_t} + b_0 \tag{16A}$$

where $b_0 \equiv \Omega_O$, $b_1 \equiv \bar{h}\left(1-\alpha\right)\psi^*(\Omega_A + (2+\bar{R})\Omega_O) + \bar{R}\Omega_O$, $b_2 \equiv \alpha\psi^*(\Omega_A + 2\Omega_O)$ and $b_3 \equiv R\Omega_A$;

$$\mathbf{E}\left[h_t\right] = a_1 h_t + a_0 \tag{17A}$$

where $a_0 \equiv \frac{1-\alpha}{1+n}\bar{h}\psi^*$ and $a_1 \equiv \frac{\alpha}{1+n}\psi^*$.

For what concerns feasibility conditions of fiscal and redistributive policies, the arguments reported in the proof of Proposition (1) continue to hold. We denote with $(K_t^{\tau}, H_t^{\tau}) = \{(k_t, h_t) \,|\check{k}_t^m < k_t < \hat{k}_t^m\}$ where $\hat{k}_t^m \equiv \frac{b_2+b_0}{b_3}h_t + \frac{b_1+b_0}{b_3}$ and $\check{k}_t^m(h_t) \equiv \frac{b_2+b_0-1}{b_3}h_t + \frac{b_1+b_0-1}{b_3}$. While

$$H_t^e = \begin{cases} \{h_t | h_t \in (0, \infty)\} & \text{if } \bar{h} < \frac{1}{(1-\alpha)\psi^*} \\ \{h_t | h_t \in (\tilde{h}, \infty)\} & \text{if } \bar{h} \geq \frac{1}{(1-\alpha)\psi^*} \end{cases}$$

where $\tilde{h} \equiv \frac{1-\bar{h}(1-\alpha)\psi^*}{\alpha\psi^*-1}$. Jointly considering the above feasibility conditions for both fiscal and redistributive dimensions, non-degenerate policies, i.e. $\tau_t \in (0,1)$ and $e_t \in \left(0, \hat{k}_t\right)$, are achieved at each time for any $(k_t, h_t) \in (K_t^{\tau}, H_t^{\tau}) \cap H_t^e$.

Second step

We now check for the second order conditions in order to get proper solutions. At each time the FOC can be rewritten as follows:

$$W_{\tau} : (1+n)(\phi(1+h_t)v_{C_{2,t}} + (\frac{1+n}{R}(1+h_{t+1})\frac{d\tau_{t+1}}{d\tau_t} - (1+h_t))v_{C_{1,t}})$$
$$W_e : (1+n)^2(-\phi v_{C_{2,t}} + \frac{1}{R}(\frac{dh_{t+1}}{de_t}\tau_{t+1} + \frac{d\tau_{t+1}}{de_t}(1+h_{t+1}) - (1+n)\frac{de_{t+1}}{de_t})v_{C_{1,t}})$$

where $\frac{d\tau_{t+1}}{d\tau_t} = \frac{d\tau_{t+1}}{dk_{t+1}}\frac{dk_{t+1}}{d\tau_t}$, $\frac{de_{t+1}}{de_t} = \frac{de_{t+1}}{dh_{t+1}}\frac{dh_{t+1}}{de_t}$ and $\frac{d\tau_{t+1}}{de_t} = \frac{d\tau_{t+1}}{dk_{t+1}}\frac{dk_{t+1}}{de_t} + \frac{d\tau_{t+1}}{dh_{t+1}}\frac{dh_{t+1}}{de_t}$. ■

Proof of Corollary (2). The proof is straightforward. The derivative of Eq. (2.25) with respect to h_t is equal to:

$$\frac{d\tau_t}{dh_t} = \frac{b_3k_t + b_2 - b_1}{(1+h_t)^2} \tag{18A}$$

For any level of k_t, if $b_1 \leqslant b_2$, then $\frac{d\tau_t}{dh_t} \geqslant 0$. Otherwise, if $b_1 > b_2$, then the sign of Eq. (18A) depends on the value reached by k_t. When $k_t < \tilde{k}$ where $\tilde{k} \equiv \frac{b_1-b_2}{b_3}$ the income tax rate is a decreasing function of h_t, i.e. $\frac{d\tau_t}{dh_t} < 0$. The opposite holds for $k_t \geqslant \tilde{k}$. ■

Proof of Corollary (3). Given the balanced budget constraint (2.14), let us denote with $\mathbf{P}[h_t, k_t] \equiv (1+n)(1+h_t)\,t\,[h_t, k_t] - (1+n)^2 e\,[h_t]$ the equilibrium pension policy rule. Under the decreasing return in education and the equilibrium level of policy rules, Eq. (2.24) and Eq. (2.25), the total amount of pension contributions can be rewritten as follows:

$$p_{t+1} = \mathbf{P}\left[h_{t+1}, k_{t+1}\right] \equiv \tag{19A}$$
$$(1+n)\left[-b_3 k_{t+1} + (b_2 + b_0 - (1+n)\, a_1)\, h_{t+1} + (b_1 + b_0 - (1+n)\, a_0)\right]$$

The derivative of (19A) with respect to e_t is equal to:

$$\frac{dp_{t+1}}{de_t} = (1+n)\left(-b_3 \frac{dk_{t+1}}{de_t} + (b_2 + b_0 - (1+n)\, a_1)\frac{dh_{t+1}}{de_t}\right) \tag{20A}$$

where under log utility $\frac{dk_{t+1}}{de_t} = -\frac{(b_2+b_0-a_1(1+n))}{R(1+\beta)}\frac{dh_{t+1}}{de_t}$. After some algebra, the derivative (20A) is as follows:

$$\frac{dp_{t+1}}{de_t} = \frac{R(1+\beta)(1+n)(b_2 + b_0 - a_1(1+n))}{R(1+\beta) - b_3}\frac{dh_{t+1}}{de_t} \tag{21A}$$

Noting that $(b_2 + b_0 - a_1(1+n)) > 0$ and $R(1+\beta) - b_3 > 0$, Eq. (21A) takes always positive values for any welfare state regime and in the whole state space. ■

Proof of Corollary (4). Let us denote with $\rho = \frac{b_2+b_0}{b_1+b_0}$ a measure of the welfare state regimes' *intensity*. According to Eq. (2.26), the higher the adults' relative power is, the larger is the value of ρ. Normalizing the Eq. (19A) by the factor $(b_1 + b_0)$, we obtain:

$$\mathring{p}_t = (1+n)\left[-\mathring{b}_3 k_t + (\rho - (1+n)\,\mathring{a}_1)\, h_t + (1 - (1+n)\,\mathring{a}_0)\right] \tag{22A}$$

where $\mathring{p}_t \equiv \frac{p_t}{b_1+b_0}$, $\mathring{b}_3 \equiv \frac{b_3}{b_1+b_0}$, $\mathring{a}_0 \equiv \frac{a_0}{b_1+b_0}$ and $\mathring{a}_1 \equiv \frac{a_1}{b_1+b_0}$. Taking the derivatives of Eq. (22A) with respect to ρ and k_t, the marginal impacts $\frac{d\mathring{p}_t}{d\rho} = (1+n)\, h_t > 0$ and $\frac{d\mathring{p}_t}{dk_t} = -(1+n)\,\mathring{b}_3 < 0$ are attained. In other words, the higher the level of ρ and the lower the level of physical capital are, the larger is the amount of pension benefits. ■

Proof of Corollary (5). The equilibrium education transfer chosen by politicians is the linear policy rule $\mathbf{E}[h_t] = a_1 h_t + a_0$, with a_1 and a_0 defined in Proposition 6. Political population aging, an increase in ϕ, does not affect at all the amount of equilibrium forward transfers, then $\frac{d\mathbf{E}}{d\phi} = 0$. The equilibrium level of income tax rate is instead a linear function of k_t and non linear in h_t, $\mathbf{T}[k_t, h_t] = -b_3 \frac{k_t}{1+h_t} + b_2 \frac{h_t}{1+h_t} + b_1 \frac{1}{1+h_t} + b_0$, where the coefficients are fully described in Proposition 2. A variation in the exogenous political ideological bias ϕ determines the following marginal changes in the structural parameters: $\frac{db_3}{d\phi} = -\frac{R(1+n)(1+\beta)}{(\phi+(1+n)(1+\beta))^2} < 0$, $\frac{db_2}{d\phi} = \frac{\alpha\psi^*(1+n)(1+\beta)}{(\phi+(1+n)(1+\beta))^2} > 0$, $\frac{db_1}{d\phi} = \frac{(1+n)(1+\beta)\left(1+n+(1-\alpha)\psi^*\bar{h}R\right)}{(R-(1+n))(\phi+(1+n)(1+\beta))^2} > 0$ and $\frac{db_0}{d\phi} = \frac{(1+n)(1+\beta)}{(\phi+(1+n)(1+\beta))^2} > 0$. Then, for any level of $\bar{h}$ $\frac{d\mathbf{T}}{d\phi} > 0$, which implies positive correlation between the pension benefits and the ideological bias in favor of old agents. Finally, using the above results, the derivative of pensions transfers obtained by balanced budget constraint, $\mathbf{P}[h_t, k_t] = (1+n)\left((1+h_t)\mathbf{T}[h_t, k_t] - (1+n)\mathbf{E}[h_t]\right)$, with respect to the political aging parameter is $\frac{d\mathbf{P}}{d\phi} = (1+n)\left((1+h_t)\frac{d\mathbf{T}}{d\phi}\right) > 0$. ■

Proof of Corollary (6). To determine the effect of demographic population aging on the level of education transfers chosen by politicians, i.e. a decrease in n, note that $\frac{da_1}{dn} = -\frac{\alpha}{(1+n)^2}\psi^* < 0$ and $\frac{da_0}{dn} = -\frac{1-\alpha}{(1+n)^2}\bar{h}\psi^* < 0$. Then it follows $\frac{d\mathbf{E}}{dn} < 0$. Concerning the impact of n on the political equilibrium level of income tax rate the following marginal changes in the structural parameters hold: $\frac{db_3}{dn} = \frac{\phi+R(1+\beta)}{(\phi+(1+n)(1+\beta))^2} > 0$, $\frac{db_2}{dn} = -\frac{\alpha\psi^*\phi(1+\beta)}{(\phi+(1+n)(1+\beta))^2} < 0$, $\frac{db_1}{dn} = D_0 + D_1 D_2 \gtreqless 0$, where $D_0 \equiv \frac{\phi}{(R-(1+n))(\phi+(1+\beta)(1+n))} > 0$, $D_1 \equiv \frac{\phi\left(1+n+\bar{h}\psi^*(1-\alpha)R\right)}{(R-(1+n))(\phi+(1+\beta)(1+n))} > 0$ and $D_2 \equiv \left(\frac{1}{R-(1+n)} - \frac{1+\beta}{\phi+(1+\beta)(1+n)}\right) = \frac{1}{1+n}(\bar{R} - \Omega_A) \gtreqless 0$ if $\bar{R} \gtreqless \Omega_A$, finally $\frac{db_0}{dn} = -\frac{\phi(1+\beta)}{(\phi+(1+n)(1+\beta))^2} < 0$. Then it follows that $\frac{d\mathbf{T}}{dn} \gtreqless 0$ depending on the difference $\left(\bar{R} - \Omega_A\right)$ and on the level of $\bar{h}$. In particular a sufficient condition to yield $\frac{d\mathbf{T}}{dn} < 0$ is $\bar{R} < \Omega_A$ and $\bar{h}$ high enough. Finally the marginal variation of pension benefits due to population growth is equal to $\frac{d\mathbf{P}}{dn} = (1+n)\left((1+h_t)\frac{d\mathbf{T}}{dn} - (1+n)\frac{d\mathbf{E}}{dn}\right) \gtreqless 0$. ■

Proof of Lemma (5). Let us first consider the transition dynamics of h_t and e_t. Plugging the equilibrium education transfers, Eq. (2.24), into the human capital production, Eq. (2.3), we obtain the law of motion $h_{t+1} = H^d[h_t]$, which is equal to:

$$h_{t+1} = \lambda_1 h_t + \lambda_0 \tag{23A}$$

where $\lambda_0 \equiv \frac{(1-\alpha)\bar{h}}{1+n}\sqrt{\psi^*}$ and $\lambda_1 \equiv \frac{\alpha}{1+n}\sqrt{\psi^*}$. It should be noted the serial correlation between current and future level of human capital is always positive, i.e. $\lambda_1 \geqslant 0$, To determine the law of motion of the redistributive policy we plug Eq. (2.3) into the equilibrium education policy rule at time $t+1$.

The law of motion $e_{t+1} = \mathbf{E}^d [h_t]$ is then as follows:

$$e_{t+1} = \xi_1 h_t + \xi_0 \tag{24A}$$

where $\xi_0 \equiv a_0 \left(\frac{a_1}{\sqrt{\psi^*}} + 1 \right)$ and $\xi_1 \equiv \frac{a_1^2}{\sqrt{\psi^*}}$. Note that, if the dynamics of h_t is characterized by stable convergence, i.e. $\lambda_1 < 1$, then also the dynamics of e_t is convergent toward the steady state. Thus, using the expression of λ_1, the sufficient condition for the convergence stability of both h_t and e_t requires:

$$n > \underline{n} \tag{25A}$$

where $\underline{n} \equiv \sqrt{2R\left(R - \sqrt{R^2 - \alpha}\right) - \alpha} - 1$. Due to linearity, both h_t and e_t converge monotonically toward the steady states.

Let us now analyze the transition dynamics of k_t and τ_t. First, consider the following recursive formulation for the equilibrium saving under log-utility, $k_{t+1} = \tilde{K}[e_t, \tau_t, h_t]$, which is obtained plugging the human capital production, Eq. (2.3), and the expected equilibrium policies e_{t+1} and τ_{t+1} according to Eq. (2.24) and (2.25). The saving function can then be rewritten as follows:

$$k_{t+1} = \frac{\beta R (1 + h_t)(1 - \tau_t)}{(R(1+\beta) - b_3)(1+n)} - \frac{(b_0 + b_2 - (1+n) a_1) H[e_t, h_t]}{R(1+\beta) - b_3} + \frac{(b_0 + b_1 - (1+n) a_0)}{R(1+\beta) - b_3} \tag{26A}$$

Plugging the equilibrium policy rules, Eq. (2.24) and Eq. (2.25), into Eq. (26A), we obtain the law of motion $k_{t+1} = K^d [h_t, k_t]$:

$$k_{t+1} = \pi_2 k_t + \pi_1 h_t + \pi_0 \tag{27A}$$

where:

$$\pi_2 \equiv \frac{R\beta b_3}{(1+n)(R(1+\beta) - b_3)}$$

$$\pi_1 \equiv -\left(\frac{(b_0 + b_2 - a_1 (1+n)) \lambda_1}{(R(1+\beta) - b_3)} + \frac{R\beta (b_0 + b_2 - 1)}{(1+n)(R(1+\beta) - b_3)} \right)$$

$$\pi_0 \equiv -\left(\frac{b_0 + b_1 - a_0 (1+n)}{(R(1+\beta) - b_3)} + \frac{(b_0 + b_2 - a_1 (1+n)) \lambda_0}{(R(1+\beta) - b_3)} + \frac{R\beta (b_0 + b_1 - 1)}{(1+n)(R(1+\beta) - b_3)} \right)$$

It should be noted that current and future level of physical capital are positively interrelated each other, $\pi_2 \geqslant 0$, on the contrary the way h_t perturbs

k_{t+1} depends on the welfare state regimes' *intensity* embedded in the parameter π_1.

Under condition (25A), the dynamics of physical capital is characterized by stable convergence if $\pi_2 < 1$, which requires:

$$\phi > \underline{\phi} \tag{28A}$$

where $\underline{\phi} \equiv \beta(R-(1+n))$. Let us denote by $Q_t^h \equiv \frac{1+h_t}{1+h_{t+1}}$. Plugging Eq. (23A) and (26A) into the equilibrium income tax policy at time $t+1$, after some manipulations, we attain the law of motion $\tau_{t+1} = \mathbf{T}^d[\tau_t, h_t]$, as follows:

$$\tau_{t+1} = \sigma[h_t]\,\tau_t + \zeta[h_t] \tag{29A}$$

where:

$$\sigma[h_t] \equiv \frac{R\beta b_3}{(1+n)(R(1+\beta)-b_3)} Q_t^h$$

$$\zeta[h_t] \equiv \frac{R(1+\beta)(1+n)(b_1-b_2)+(1+n)^2(a_1-a_0)\,b_3}{(R(1+\beta)-b_3)(1+n)} \frac{1}{1+\lambda_1 h_t+\lambda_0} - \frac{\beta R b_3}{(R(1+\beta)-b_3)(1+n)} \frac{1+h_t}{1+\lambda_1 h_t+\lambda_0} + \frac{R(1+\beta)(b_0+b_2)-(1+n)\,b_3 a_1}{R(1+\beta)-b_3}$$

Note that, under Eq. (25A), the convergence condition for k_t, Eq. (28A), is also sufficient for the convergence of τ_t, i.e. $\sigma[h^*] < 1$. Furthermore the speed of convergence for τ_t basically depends on the welfare state regime characterizing the economy jointly with the exogenous human capital society endowment. To show how such elements may affect the type of convergence let us take the derivative of $\Gamma[h_t]$ with respect to the human capital asset. We obtain:

$$\frac{d\zeta[h_t]}{dh_t} = \frac{-b_3\left(\beta R(1+\lambda_0-\lambda_1)+(1+n)^2\lambda(a_1-a_0)\right)}{(1+n)(R(1+\beta)-b_3)(1+\lambda_1 h_t+\lambda_0)^2} - \frac{R(1+\beta)(1+n)(b_1-b_2)\lambda_1}{(1+n)(R(1+\beta)-b_3)(1+\lambda_1 h_t+\lambda_0)^2}$$

It is straightforward to show how the sign of $\frac{d\zeta[h_t]}{dh_t}$ crucially depends on the differences (a_1-a_0) and (b_1-b_2) and in final instance on the level of social culture, $\bar{h}$, and on the relative political power weights of adults and old embedded in the coefficients b_1 and b_2. When $\frac{d\zeta[h_t]}{dh_t} \gtrless \left(\lesseqgtr\right) 0$ and $\tau_0 \lesseqgtr \left(\gtrless\right) \tau^*$ then the speed of convergence toward the steady state is lower (higher) than in the opposite case.

From a qualitative point of view the dynamics of e_t and τ_t are mirror image respectively to the dynamics of h_t and k_t. They mainly differ from an autoregressive component of infinite order in the past level of public education, which arises because of the infinite persistence of education spending on the future level of human capital through the parental transmission.

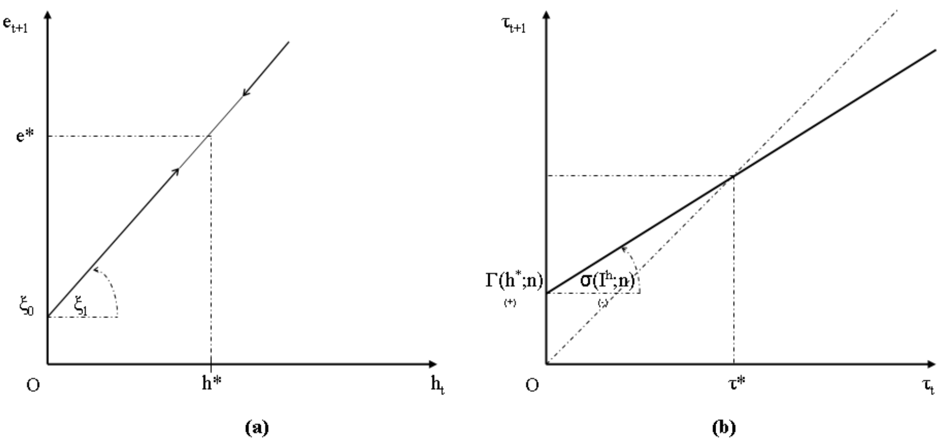

Figure 2.6: Panel (a) shows the law of motion of e_t, Panel (b) shows the law of motion of τ_t.

The Figure 2.6 emphasizes the dynamics of the political variables. The Panel (a) shows that, once the human capital converges to the steady state also the education policy reaches its balanced growth path. Differently, the Panel (b) highlights how the convergence condition of h_t is necessary but not sufficient for the stable convergence of the fiscal policy rule, which also requires the dynamic stability of k_t. ∎

Proof of Proposition (7). Under Lemma 3, due to linearity of the laws of motion, Eq. (23A), (24A), (27A) and (29A), there exists a unique steady state $\{e^*, \tau^*, h^*, k^*\}$. Equating $h_{t+1} = h_t = h^*$ in Eq. (23A) and $k_{t+1} = k_t = k^*$ in Eq. (27A), the following steady state levels for the state variables are obtained:

$$h^* = \frac{(1-\alpha)\,\bar{h}\sqrt{\psi^*}}{(1+n) - \alpha\sqrt{\psi^*}} \tag{30A}$$

$$k^* = \frac{\beta R\,(b_0+b_2-1) + (1+n)\,(b_0+b_2-(1+n)\,a_1)\,\lambda_1}{b_3\,((1+n)+\beta R) - R\,(1+\beta)\,(1+n)}h^* \tag{31A}$$
$$+\frac{((1+n)\,(1+\lambda_0)+\beta R)\,b_0 + ((1+n)+\beta R)\,b_1 + (1+n)\,b_2\lambda_0 - (1+n)^2\,(a_1\lambda_0+a_0) - \beta R}{b_3\,((1+n)+\beta R) - R\,(1+\beta)\,(1+n)}$$

Plugging Eq. (30A) and (31A) into the equilibrium policy rules described in Proposition 6, we obtain the following the steady states levels for the political control variables:

$$e^{*}=\frac{(1-\alpha)\,\bar{h}\psi^{*}}{(1+n)-\alpha\sqrt{\psi^{*}}} \tag{32A}$$

$$\begin{aligned}\tau^{*}=&-\frac{(1+n)\left(R\,(1+\beta)\,(b_1-b_2)+(1+n)\,(a_1-a_0)\,b_3\right)}{b_3\left((1+n)+\beta R\right)-R\,(1+\beta)\,(1+n)}\frac{1}{1+h^{*}} \\ &+\frac{\beta R b_3}{b_3\left((1+n)+\beta R\right)-R\,(1+\beta)\,(1+n)}-\frac{(1+n)\left(R\,(1+\beta)\,(b_0+b_2)-(1+n)\,b_3 a_1\right)}{b_3\left((1+n)+\beta R\right)-R\,(1+\beta)\,(1+n)}\end{aligned} \tag{33A}$$

By balanced budget constraint the pension steady state level is:

$$p^{*}=(1+n)\,(1+h^{*})\,\tau^{*}-(1+n)^{2}\,e^{*}$$

■

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Chapter 3

Time Consistent Optimal Public Expenditures with Intergenerational Exchange

3.1 Introduction

Long-lasting governments cannot always implement public policies under full commitment. Starting from the seminal work by Kydland and Prescott (1977) the issue has become a subject of increasing interest for economists in general and policymakers in particular. To this point, the literature on *time consistent* fiscal policies has confined itself to simple environments where taxes are used to finance a flow of public goods or services that are rapidly exhausted. In contrast, the benefits of government spending have been mainly documented for durable public goods that can be accumulated over time. This fact is ignored in recent studies because introducing public goods which are not rapidly depletable means introducing in the analyses an additional state variable, which significantly complicates the characterization of the optimal discretionary policy.

This paper, therefore, focuses on that part of public expenditures concerning intergenerational exchange, which is typically characterized by the provision of public goof with both long- and short-lasting impact. In this context the problem of understanding how the absence of government commitment affects the provision of public expenditure becomes crucial, as well as the implied welfare effects over an economy's transition to its long-run equilibrium.

Among previous works on optimal public investment, Glomm and Ravikumar (1994, 1997), characterize the optimal policy under full commitment only. More recent papers analyze optimal fiscal policy in the absence of commitment, but in environments where public goods cannot be accumulated. Klein et al. (2008) analyze the trade off between providing a consumable public good and its financing, Hassler et al. (2005) study time-consistent redistribution under repeated voting, and Azzimonti et al. (2006) explore the distortionary effects of income taxes on the evolution of wealth inequality. In contrast to these papers, our analysis focuses on the provision of a durable public good (i.e. education) that expands the production frontier. More closely to our theoretic environment Azzimonti et al. (2009) characterize Markov-perfect equilibria in a setting where the absence of government commitment affects public investment in physical capital in terms of infrastructure.

Related to these papers, solving for differentiable Subgame Markov Perfect Equilibrium ($SMPE$), the contribution on public expenditure of our work is threefold: (i) to illustrate how intergenerational exchange can affect macroeconomic outcomes through endogenous policy making in a normative perspective; (ii) to characterize recursively, using functional-equation methods, *time-consistent* equilibria in the presence of non-depletable public goods and human capital accumulation; (iii) to determine closed form solutions for a specific economic environment and perform comparative statics with the positive predictions attained under political competition (see Ch. 2).

In our environment heterogeneity is explicitly taken into account and it concerns age as well as production factors' ownership. In particular there are two classes: adults/workers who receive the return from human capital and old/capitalists who receive physical capital returns. In order to emphasize the intergenerational conflicts due to economic interests and the intergenerational exchange which will result with the implementation of public expenditures, the setup is characterized by a linear technology which uses human capital and physical capital as perfect substitutes. Proportional income taxation is used to simultaneously fund both transfers going lump-sum to capitalists as pork-barrel and transfers going in a productive way after one-period human capital accumulation to workers as public education investments. Saving decisions, made when adult, are affected both by productive transfers (positively) and by expected pork-barrel transfers (negatively). Long-run persistency of the productive impact coming from public good provision is guaranteed by the parental transmission of knowledge across dynasties. Furthermore because of fixed prices income/substitution effects are not perfectly compensated. Consequently different welfare state regimes emerge depending on the relative weights assigned to each cohort by the Central Planner and time-consistency is not generally guaranteed in the absence of commitment devises.

The latter point becomes a relevant part of our analysis, which concerns the discussion about the sequential nature of the Central Planner decision making: the current government only sets current policies, without direct influence on the decisions of future governments. This lack of commitment is a binding restriction in general and can be viewed as follows: each future government takes its initial capital stock as inelastically supplied, whereas the current government sees it as elastically supplied. Differently from previous analyses we do not only obtain a first order characterization ($GEEs$) in the case in which the commitment constraint turns out to be binding, stressing the divergence with respect to the first best allocation, but we also completely characterize the closed form solution for a specific case of interest in which intergenerational exchanges have long-lasting impact. Comparing the optimal predictions with the positive one obtained under probabilistic voting competition, presented in Ch. 2 -, we find that the equilibrium allocation is education efficient but, due to political overrepresentation of elderly agents, the electoral competition process induces overtaxation compared with a time-consistent Central Planner solution with balanced welfare weights.

The paper is organized as follows. In section 3.2 we present the underlying economic environment. Section 3 characterizes the Pareto optimal allocation with full commitment and no distortionary taxation. In section 3.4 the first-order characterization of a time-consistent Central Planner is provided and the distortions with respect to the first best allocation are discussed. In section 3.5 we use the Generalized Euler Conditions derived in the previous section as a base for analytical computation in the case of a simple economy. Section 3.6 compares the normative predictions of the model with the positive results attained under probabilistic voting competition. Section

3.7 concludes. All proofs are contained in the Appendix.

3.2 The Model

In this section we describe the specific setup. We then define a benchmark solution to an optimal-policy problem where the government can commit to future policies and taxation is not distortionary (first best allocation). After that, we proceed toward a definition of a time consistent equilibrium where the government does not have the ability to commit.

3.2.1 Economic Environment

Consider a discrete-time OLG economy populated by an infinite number of homogenous agents, living up to three-periods: youth, adulthood and old age. Agents in the same class are identical. The population growth rate is exogenous and equal to $n \in (-1, 1)$, thus the mass of a generation born at time j and living at time t is equal to $N_t^j = N_0 (1+n)^t$. The instantaneous preferences of a representative agent born at time $t-1$ are then defined as follows:

$$U_{t-1}[c_t] = u[c_{1,t}] + \beta u[c_{2,t+1}] \tag{3.1}$$

where $\beta \in (0, 1)$ is the individual discount rate and $u[\cdot]$ is twice continuously differentiable, with $u_c > 0$, $u_{cc} < 0$, and the usual Inada conditions hold. When young, the agent spends all his time endowment in acquiring skills if education is publicly provided without having access to private credit markets and does not consume. When adult, the individual works, makes saving choices, $s_{1,t}$, and contributes to public spending through proportional labor income taxes, τ_t, while when old only consumes his entire income which is composed by both capitalized private savings and pork-barrel transfers, p_t. The individual budget constraints are as follows:

$$c_{1,t} \leq w_t (1 - \tau_t) - s_t \tag{3.2}$$

$$c_{2,t+1} \leq R_{t+1} s_t + p_{t+1} \tag{3.3}$$

At each time a single consumption good is produced using a linear technology, in which human and physical capital are perfect substitute, $Y_t = w_t H_t + RK_t$. Here the interest rate is treated as constant as in a small open economy. Physical capital fully depreciates, i.e. $(1+n)k_{t+1} = s_t$. At any time t, each adult supplies inelatically one unit of labor with productivity equal to ω augmented by the level of human capital acquired the period before, h_t. Without loss of generality, ω is normalized to unity. Human capital is produced according to a CRS technology which uses parental education and public investment in education, e_t, as complement factors, $h_{t+1} = H(h_t, e_t)$, such that $H(h_t, 0) = N_t^{t-1}$, $H_{h_t} > 0$, $H_{e_t} > 0$, $H_{h_t e_t} > 0$ and $H_{e_t e_t} \leq 0$.

3.3 The Pareto Optimal Allocation

Before describing the optimal outcome in the Central Planner case without commitment,[1] it is useful to characterize the efficient allocation chosen by a benevolent planner with a commitment technology and in the absence of distortionary taxation. Among other public expenditures we focus on intergenerational transfers by distinguishing between productive transfers that go forward in time in terms of education investment, e_t, and pork-barrel transfers that go backward to sustain old agents consumption, p_t. We can think about the latter in terms of PAYG transfers. The planner takes the initial level of human and physical capital as given, and chooses the sequence of policies $\{e_t, p_t, c_{1,t}, c_{2,t}, h_{t+1}, k_{t+1}\}_{t=0}^{\infty}$ that maximizes the weighted sum of utilities, where the Welfare weight of each representative dynasty is given by δ. Lump sum taxation is used to finance public education and pension contributions. The corresponding maximization problem becomes as follows:

$$\max_{\{e_t, p_t, c_{1,t}, c_{2,t}, h_{t+1}, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (1+n)^t \, \delta^t \, (u[c_{1,t}] + \beta u[c_{2,t+1}]) \tag{3.4}$$

under linear production and CRS human capital technology and the following aggregate resource constraint:

[1] In this scenario the government cannot credible promise to abide by a sequence of future tax rates to finance public transfers. Hence, setting taxes once and for all at time zero results in policy announcements that are not credible, since, in each subsequent period, policymakers take the states they inherit as given and do not account for the impact of distortionary taxes on previous investment decisions.

$$c_{1,t}N_t^{t-1} + c_{2,t}N_t^{t-2} + (1+n)\,k_{t+1}N_t^{t-1} + e_tN_t^t = Y_t \tag{3.5}$$

where both the forward transfers, e_t, and the backward transfers, p_t, are sustained by adults, as an opportunity cost in terms of private consumption.[2] Since the policies at time $t+1$ influence individual saving choices in period t, the reoptimization in any period $s > t$ would yield a first order condition violating the one achieved at time t. As long as the planner gives positive weights to both the adult and old cohorts, the following Euler conditions for the optimal allocation of education and pension transfers must be satisfied:[3]

$$u_{c_{1,t}} = \delta H_{e_t} u_{c_{1,t+1}} \tag{3.6}$$

$$\delta u_{c_{1,t}} = \beta u_{c_{2,t}} \tag{3.7}$$

The first condition reflects the direct effect of e_t on the utility of the adults in terms of current cost and expected benefits. The second condition captures the current wedge between adults and old about the pension system. The optimal investment choice satisfies:

$$u_{c_{1,t}} = \beta R u_{c_{2,t+1}} \tag{3.8}$$

Hence, the planner chooses k_{t+1} to equate the marginal cost in terms of foregone consumption to the discounted marginal benefits of savings.

Note that in the absence of lump sum taxation a distortion emerges that creates a wedge, or gap, in the above conditions. Furthermore, to the degree that policies other than lump sum taxes are used, such policies will generally be time inconsistent. In general, we define *first best gaps* or *wedges* in the public expenditure as:

$$\Delta_e^{fb} \equiv u_{c_{1,t}} - \delta H_{e_t} u_{c_{1,t+1}} \tag{3.9}$$

$$\Delta_p^{fb} \equiv \delta u_{c_{1,t}} - \beta u_{c_{2,t}} \tag{3.10}$$

Similarly, we define a *first best wedge* in the efficient private investment

[2] We will characterized steady-state Pareto allocations in the Technical Appendix.

[3] We use $u_{c_{1t}}$ to indicate $u_c(c_t)$, H_{e_t} to indicate $H_e(e_t, H_t)$, etc. to simplify notation when the context is clear.

decision as:

$$\Delta_{k'}^{fb} \equiv u_{c_{1,t}} - \beta R u_{c_{2,t+1}} \tag{3.11}$$

where $\Delta_{k'}^{fb} = 0$ under first best allocations.

3.4 Central Planner Equilibrium without Commitment

We now move toward the definition of Markov Equilibrium for Central Planner (CP) without commitment. A benevolent government can implement distortionary taxation and intergenerational exchange through a system of public transfers financed by proportional income taxation, subject to the following balanced budget condition:

$$N_t^{t-1}(1+h_t)\,\tau_t = N_t^t e_t + N_t^{t-2} p_t \tag{3.12}$$

In the previous chapter we proved the existence of a time consistent bidimensional fiscal plan in the case of electoral competition and repeated voting. The SMPE was also characterized in closed form as a finite-horizon equilibrium, whose limit when time goes to infinity is well-defined. We now implement the CP optimal allocation under a zero-cost enforceability constraint and we use it as a normative benchmark to make policy predictions. The induction procedure adopted for the resolution of the Markov perfect political problem suffers of "*end of horizon*" effects for the determination of education transfers. Due to the one-period lagged impact of education investments in human capital production, the equilibrium education transfer appears degenerative in the last period of a finite horizon economy. As a consequence the limit of the finite horizon game does not coincide with the differentiable *SMPE* of the corresponding infinite-horizon economy.[4] Thus the CP solution requires an infinite dimensional strategies space and turns out to be quantitatively different from the political equilibrium.

As in the political game, we exclude private agents' default on the implemented fiscal plan within the period. Furthermore, under a balanced

[4] See Fundeberg and Levine (1986) for the characterization of the necessary and sufficient condition for equilibria of a game to arise as limits of ε-equilibria of games with smaller strategy spaces (for example finite horizon).

budget constraint the government platform is characterized by the vector $f_t^c \equiv (e_t^c, \tau_t^c)$, where the apex c stands for CP. Given the initial conditions (h_0, k_0), we first define the CP optimization program in a sequential version. Let us denote the equilibrium ex-ante instantaneous consumption behavior respectively for adults and old living at time t:

$$C_{1,t}\left[\tau_t^c, h_t, k_{t+1}\right] \equiv (1+h_t)\left(1-\tau_t^c\right) + (1+n)\, k_{t+1} \tag{3.13}$$

$$C_{2,t}\left[f_t^c, h_t, k_t\right] \equiv (1+n)\, Rk_t + p_t^c \tag{3.14}$$

Thus the sequential version of the CP maximization program turns out to be equal to:

$$\max_{\{f_t^c\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (1+n)^t \, \delta^t B\left[f_t^c, h_t, k_t, k_{t+1}\right] \tag{3.15}$$

subject to the following constraints:

$$\Phi^c\left[h_t, k_t\right] = \begin{cases} k_{t+1} = K\left[f_t^c, f_{t+1}^c, h_t\right] \\ h_{t+1} = H\left[e_t, h_t\right] \end{cases} \tag{3.16}$$

where $B[\cdot]$ is a concave function defined as:

$$B\left[f_t^c, h_t, k_t, k_{t+1}\right] \equiv \beta u\left[C_{2,t}\left[f_t^c, h_t, k_t\right]\right] + (1+n)\, \delta u\left[C_{1,t}\left[\tau_t^c, h_t, k_{t+1}\right]\right]$$

$K[\cdot]$ is the function which fully describes private saving behavior, which under concave separable additive preferences depends both on current and future expected policies. $H[\cdot]$ is the adopted human capital technology, which depends on both parental human capital and public investments in education, exploiting complementarity effects.

The CP assigns a Welfare weight δ to each dynasty. Let us consider the restriction $\delta < \bar{\delta} \equiv \frac{1}{1+n}$, which induces weak deterrence power, justifying the implementation of an optimal fiscal plan without imposing commitment devices.

Remark 13 *In the infinite-horizon CP environment the agents' Welfare weight distribution is represented by the following infinite-dimensional vec-*

tor:

$$\Im^c_{(\infty)} \equiv \left(\beta, \delta\left(1+n\right), \beta\delta\left(1+n\right), \delta^2\left(1+n\right)^2, ..., \beta\delta^j\left(1+n\right)^j, \delta^{j+1}\left(1+n\right)^{j+1}, ...\right) \in \mathbb{R}^\infty \tag{3.17}$$

Then the resulting relative Welfare weights are:

$$\Omega^c_R \equiv \frac{\delta\left(1+n\right)\left(1+\beta\right)}{\beta+\delta(1+n)} \quad \text{and} \quad \Omega^c_O \equiv \frac{\beta\left(1-\delta\left(1+n\right)\right)}{\beta+\delta\left(1+n\right)} \tag{3.18}$$

In the infinite-horizon game the CP takes into account both the relative Welfare weight of the representative agent, Ω^c_R, and the old's bargaining power gap between current and future pensioners, Ω^c_O.

Remark 14 *The more population ages* (*i.e.* n *decreases*), *the smaller is the relative Welfare weight of the representative agent* (Ω^c_R), *the larger is the old's bargaining power gap* (Ω^c_O) .

Clearly, the sequential optimization dynamic program (3.15) features similar dynamic inconsistency problems as in the political game. Without having access to a commitment mechanism, the government cannot choose future taxes and transfers directly, but it still wants to maximize her objective function, and it still needs to select an allocation among decentralized equilibria. In order to get time-consistent policy equilibrium, the current government should set the current policy perfectly foreseeing how the future governments will set intergenerational transfers. The key insight here is that the policies set in any period will depend on the relevant stocks the economy is endowed with at the beginning of the period, i.e. physical and human capital. In other terms we are looking for a multidimensional government policy that obeys a recursive rule given by the function:

$$f^c_t = \digamma^c\left[k_t, h_t\right] \tag{3.19}$$

where $\digamma^c\left[\cdot\right]$ is the key endogenous variable, which we restrict to be differentiable.

As in Klein at al. (2008),[5] let us rewrite in a recursive way the sequential CP program in order to derive the government *Generalized Euler Equations*

[5] Unlike Klein et al. (2008), however, and more generally models with a single state variable, the derivation of the *GEEs*, which characterizes the solution, is substantially more involved with two states. We show that such a deviation remains analytically tractable in our economy.

($GEEs$), which capture the CP optimal trade-offs between taxation and redistribution wedges over time.[6] Due to stationarity, we will omit the time subscript, denoting by the prime symbol next-period values.[7] In order to specify the CP's problem, we need its key inputs: a view of how the private sector responds to its current fiscal choices. The specification of this response must include what will happen in the future in response to the current fiscal choices. Let us denote with $\eta\left[f^c, f^{c\prime} h, h', k'\right]$ the economic first order condition coming from utility maximization and the equilibrium saving choices by private agents, such that $\eta\left[f^c, f^{c\prime} h, h', k'\right] = 0$. In equilibrium, by the implicit function theorem, there exists a unique $k' = K\left[f^c, f^{c\prime} h, h'\right]$ satisfying $\eta\left[f^c, f^{c\prime} h, h', K\left[\cdot\right]\right] = 0$. If there exists a policy rule $\digamma^c\left[h, k\right]$ which solves the CP optimization program, then under the transformation function of human capital, $h' = H(e^c, h)$, we derive the recursive formulation of $K\left[\cdot\right]$, whose functional form is then equal to $k' = \tilde{K}\left[f^c, h\right]$. The recursive economic first order condition becomes $\eta\left[f^c, h, \tilde{K}\left[f^c, h\right]\right] = 0$. Derivating the function $\eta\left[\cdot\right]$ with respect to its arguments we obtain $\tilde{K}_{f^c} = -\frac{\eta_{f^c}}{\eta_{k'}}$ and $\tilde{K}_h = -\frac{\eta_h}{\eta_{k'}}$.

After some manipulations, Eq. (3.15) can be reformulated in terms of a Bellman equation, as follows:[8]

$$V^c\left[h, k\right] = \max_{\{f^c, h', k'\}} B\left[f^c, h, k, k'\right] + (1+n)\,\delta V^c\left[h', k'\right] \tag{3.20}$$

We now provide the formal definition of the *time-consistent* CP equilibrium policy decision rules as solution of a dynamic programming equation:

Definition 7 *An infinite-horizon SMPE of the CP problem is defined as a vector of differentiable policy decision rules, $\digamma^c = (\mathbf{T}^c, \mathbf{E}^c)$, where $\mathbf{T}^c : \mathbb{R} \times \mathbb{R} \longrightarrow (0,1)$ and $\mathbf{E}^c : \mathbb{R} \times \mathbb{R} \longrightarrow (0, \hat{e}_t)$ are the taxation policy rule and the public higher education policy rule, $\tau^c = \mathbf{T}^c\left[h, k\right]$ and $e^c = \mathbf{E}^c\left[h, k\right]$, respectively. Given the Bellman Eq. (3.20), the following conditions must hold:*

[6] The GEE is the FOC of the government maximization program. It is obtained deriving the Bellman equation with respect to the political control variables, f^c. GEE can be equivalently derived by using Bellman's principle to identify Markov equilibrium with the solution of the sequential version of the central planner program. The Euler equation of this sequential problem is exactly the GEE.

[7] The stationarity requirement allows us to focus only on the current level of asset stocks, ruling out explicit dependence on any history beyond.

[8] See Appendix B for the derivation of both Bellman equation and $GEEs$.

(*i*)

$$F^c[h,k] = \arg\max_{f^c} B[f^c,h,k,k'] + (1+n)\,\delta V^c[h',k'] \tag{3.21}$$

subject to the following set of constraints:

$$1)\ V^c[F^c[\cdot],h,k] \geq V^{c,LF}[h,k]$$

$$2)\ \Phi^c[h,k] = \begin{cases} k' = \tilde{K}[f^c,h] \\ h' = H[e^c,h] \end{cases} \tag{3.22}$$

$$3)\ f^c \in \Pi^c[h,k]$$

(*ii*)

$$V^c[h,k] = M(V^c)[h,k] \tag{3.23}$$

where the functional form $M : \mathbb{C}^\infty(R^2) \to \mathbb{C}^\infty(R^2)$ *is defined as follows:*

$$M(V^c)[h,k] := \max_{f^c \in \Pi^c[h,k]} B\left[f^c,h,k,\tilde{K}[\cdot]\right] + (1+n)\,\delta V^c\left[H[\cdot],\tilde{K}[\cdot]\right] \tag{3.24}$$

The first condition requires that the political variables, f^c, have to be chosen by CP in order to maximize the utilitaristic social welfare, internalizing the equilibrium private saving decision and all the *direct and indirect feedback* effects. The second requirement is the fixed point condition, given the mapping $M(V^c)$.

3.4.1 First order characterization: *GEEs*

In section 3.3 we saw how in the presence of lump sum taxes, the government would set all distortions to zero, attaining first-best allocations. In contrast, in our setting, distortionary taxes and long-lasting intergenerational transfers induce wedges in the intertemporal conditions describing the efficient provision of public transfers and private capital, Eq. (3.9), (3.10) and (3.11). The following proposition states that the optimal discretionary policy is such that it sets a linear weighted sum of these distortions to zero.

Proposition 8 *Let* $\chi_1 \equiv (1+h')\left(\frac{H'_{h'}}{H'_{e'}}\frac{\eta'_{e'}}{\eta'_{\tau'}} - \frac{\eta'_{h'}}{\eta'_{\tau'}}\right)$ *and* $\chi_2 \equiv \chi_1 + (1+n)\frac{H'_{h'}}{H'_{e'}}$. *Then, in terms of wedges, the GEEs of the sequential CP program with* *respect of e and τ are as follows:*

$$0 = \Delta_e + \delta \frac{\eta_e}{\eta_{k'}} \Delta_{k'} + \delta H_e \tilde{\Delta}'_{\tau e} \tag{3.25}$$

$$0 = (1+h)\,\Delta_\tau + \delta(1+n)\frac{\eta_\tau}{\eta_{k'}}\Delta_{k'} \tag{3.26}$$

where we denote by Δ the following intra-/inter-temporal wedges:[9]

$\Delta_\tau \equiv \beta u_{C_2} - \delta u_{C_1}$	*taxation wedge*
$\tilde{\Delta}_{\tau e} \equiv -\chi_1 \delta u_{C_1} + \chi_2 \beta u_{C_2}$	*"persistency" fiscal wedge*
$\Delta_h \equiv \beta \tau u_{C_2} + \delta\,(1-\tau)\,u_{C_1}$	*human capital endowment wedge*
$\Delta_e \equiv -\beta u_{C_2} + \delta H_e \Delta'_{h'}$	*forward redistribution wedge*
$\Delta_{k'} \equiv u_{C_1} - \beta R u_{C'_2}$	*savings/consumption wedge*

Proof. (See Appendix) ■

The GEEs in terms of a weighted sum of deviations from efficient intertemporal decisions even if somewhat cumbersome in terms of computation, give meaningful economic insights. Under a differentiability condition of policy rules we are able to provide a non-trivial formulation of the government first order condition in the case of no commitment. The above inter- and intra-temporal wedges can be interpreted as deviations from the efficient intertemporal decisions and they acquire straightforward economic meaning in the recursive dynamic environment. First, note that only the current and the subsequent period matter directly. Even though both the current tax rate and the public education investment choices have repercussions into the infinite future, the marginal costs and benefits in equilibrium can be summarized by terms involving only two consecutive periods. As a consequence, the *GEE* can also be viewed as resulting from a *variational* (two-periods) problem (Klein et al., 2008).[10] Recalling that the *SMPE* in

[9] The strategic wedges Δ_τ, Δ_e, Δ_h and $\Delta_{k'}$ are derived as the marginal direct impact on the intertemporal agents' utility respectively of a variation in taxation, education investments, human capital endowment and individual savings. For example, a marginal variation in the income tax rate determines a direct cost for current adults equal to $\delta\,(1+n)\,(1+h)\,u_{c_1}$ and a direct benefits for current old equal to $\beta\,(1+n)\,(1+h)\,u_{c_2}$. The intergenerational taxation wedge becomes then $\Delta_\tau \equiv \beta\,(1+n)\,(1+h)\,u_{c_2} - \delta\,(1+n)\,(1+h)\,u_{c_1}$ which normalized by $(1+n)\,(1+h)$ is equal to $\Delta_\tau \equiv \beta u_{c_2} - \delta u_{c_1}$. The same characterization hold for Δ_e, Δ_h and $\Delta_{k'}$.

[10] Think of our *variational* problem as follows: given the state variables (h, k) and (h'', k'') fixed, let us vary (h', k') through the controls (τ, τ') and (e, e'), in order to obtain the highest possible utility.

the political case has been obtained as the limit of a finite horizon economy, whose convergence has been attained after two periods, we may easily conjecture no structural differences between the two equilibrium policy rules. For this reason in the following paragraph we will use the guess of the political equilibrium to verify the *GEE* and obtain the CP solution without commitment.

Let us discuss in more details the main economic and technical implications coming from the first order necessary conditions of the Central Planner maximization problem.

3.4.2 How far from the Pareto optimal frontier?

Our analysis indicates that by implementing Markov-perfect strategies can lead to considerably different allocations in the long run with respect to the Pareto optimal allocation, moving from an economy with government commitment to one with discretion. This outcome basically arises because of the greater emphasis that Markov governments place on short-run gains relative to a Ramsey planner. In particular, although the economy with commitment achieves higher long-run consumption relative to the regime with discretion, the tax policy chosen under discretion implies higher consumption in the early stages of the transition relative to the Ramsey equilibrium. This effect, therefore, partially offsets welfare losses incurred in the long run.

Before solving quantitatively the CP problem, let us interpret the *GEE* rewritten in terms of a linear weighted combination of wedges. First consider Eq. (3.26). Due to a marginal increase in distortionary taxation, τ, the inefficiency of private savings emerges. Such inefficiency is captured by the intertemporal savings distortion, $\Delta_{k'}$, which is scaled by the reduction in household savings, $\tilde{K}_\tau = -\frac{\eta_\tau}{\eta_{k'}} < 0$. Furthermore an increase in the income tax rate determines an increase in the gap between u_{C_2} and u_{C_1}which is captured by the intratemporal utility distortion, Δ_τ. Note that, due to full depreciation of physical capital k'' is equal to $\tilde{K}\left(f', h'\right)$ and it is not a function of k'. Then a variation in the current tax rate does not affect next period's wedges through its effect on future levels of physical capital. More cumbersome distortions emerge instead from the equilibrium determination of public education transfers, Eq. (3.25). As before an increase in e makes private savings inefficiency emerge, which is now scaled by the variation in household savings, $\tilde{K}_e = -\frac{\eta_e}{\eta_{k'}} < 0$, which is negative due to the substitution effects with public savings that are increased via the retributive pension scheme. The second component, Δ_e, represents the intertemporal utility distortion due to an increase in education transfers today, which determines both a decrease in the utility of current old and simultaneously an increase of the sum of the next-period adults' and old's weighted utility, $\beta\tau' u_{C_2} + \delta (1-\tau') u_{C_1}$, since they benefit from the augmented human capital h'. Finally, differently from τ, a variation in the current level of education transfers also affects next period's wedges through its effect on h', which induces a variation of both k'' and h''. More intuitively the last term of Eq. (3.25) can be rewritten in the following terms:

$$\delta H_e \tilde{\Delta}'_{\tau e} = \delta H_e \left(B'_{\tau'} \left(\frac{\tilde{K}'_{e'}}{\tilde{K}'_{\tau'}} \frac{H'_{h'}}{H'_{e'}} - \frac{\tilde{K}'_{h'}}{\tilde{K}'_{\tau'}} \right) - B'_{e'} \frac{H'_{h'}}{H'_{e'}} \right) \tag{3.27}$$

where the term $-H_e \frac{H'_{h'}}{H'_{e'}}$ is equal to the variation of e' which prevents h'' from a variation, while the term $H_e \left(\frac{\tilde{K}'_{e'}}{\tilde{K}'_{\tau'}} \frac{H'_{h'}}{H'_{e'}} - \frac{\tilde{K}'_{h'}}{\tilde{K}'_{\tau'}} \right)$ is equal to the variation of τ' which prevents k'' from a variation induced by current investment in education. In terms of wedges this variation in e determines an increase in the gap between $u_{C'_2}$ and $u_{C'_1}$, i.e. $\tilde{\Delta}'_{\tau e}$, which is affected by the described distortions.

3.5 A Closed Form Economy

We now use a parametric example to illustrate some of the results in our model. Under the assumption of log-linear utility, $u[c] = \log[c]$, we solve the CP optimization problem by guessing a time consistent bidimensional policy structurally equivalent to Eq. (2.24) and (2.25) in Ch. 3, which verifies the conditions (3.25) and (3.26). Fixing $\theta = \frac{1}{2}$, let $\Upsilon^c \equiv \left(K^{\tau^c}, H^{\tau^c}\right) \cap H^{e^c}$ be the state-space in which interior policy rules are obtained. Then the next Proposition characterizes the optimal feasible time-consistent policy rules:

Proposition 9 *Under the dynamic efficiency condition, for any* $(h, k) \in \Upsilon^c$ *the set of feasible rational policies,* $f^c \equiv (e^c, \tau^c)$, *which can be supported by a CP SMPE with perfect foresights, has the following functional form:*

(i)

$$\mathbf{E}^c[h] = a_1^c h + a_0^c \tag{3.28}$$

where $a_1^c = a_1$ *and* $a_0^c = a_0$;

(*ii*)

$$\mathbf{T}^c[h,k] = -b_3^c \frac{k}{1+h} + b_2^c \frac{h}{1+h} + b_1^c \frac{1}{1+h} + b_0^c \tag{3.29}$$

where $b_0^c \equiv \Omega_O^c$, $b_1^c \equiv \bar{h}\frac{1-\alpha}{\alpha}\frac{R}{R-(1+n)}b_2^c + \bar{R}(\Omega_O^c - \bar{h}(1-\alpha)\psi^*)$, $b_2^c \equiv \frac{\alpha\sqrt{\psi^*}}{R-\alpha\sqrt{\psi^*}}(\Omega_O^c + R\sqrt{\psi^*}\Omega_R^c - \alpha\psi^*)$ *and* $b_3^c \equiv R\Omega_R^c$.

For any $(h,k) \notin \Upsilon^c$ *corner solutions result in at least one of the two dimensions.*

Proof. (See Appendix). ■

Specifically, in equilibrium both the CP and the office-seeking politicians in a probabilistic voting environment set the same amount of forward transfers, inducing *education-efficient* political fiscal plans, as already discussed in Paragraph 2.4.1, i.e. $\mathbf{E}^c[h] = \mathbf{E}[h]$ for any level of human capital. The main difference concerns their quantitative predictions on the taxation policy dimension, which are fully captured by the policy parameters. As already noted such divergence comes from the fact that the finite horizon equilibrium is not epsilon-perfect according to Fudenberg and Levine (1986). In the following paragraph we discuss in details the divergence of the political equilibrium from the CP optimal allocation.

3.6 Are the political choices on pensions and education optimal?

Both the politicians and the social planner have incentives to provide intergenerational transfers in an environment with a linear technology with human capital accumulation and log-linear preferences. Moreover, their time-consistent equilibrium policies share similar structural properties.[11] However the quantitative differences detected so far imply distinct predictions in terms of regimes' identification and political behavior. For this reason we now examine how politicians act relatively to the CP in terms of taxation design. In other words, we determine to what extent the interior political *SMPE* chosen by the politicians diverges from the equilibrium policy rules

[11] To make comparison let us refer to the politico-economic equilibrium under perfect foresight of Ch. 2.

achieved by the CP without commitment. In order to obtain clear predictions, we normalize the vector of Welfare weights by δ assigning $\phi \equiv \frac{\beta}{\delta}$. Consequently we are able to write the relative Welfare weights, Eq. (3.18), in terms of political weights, making the two solutions comparable.[12] Let us introduce the following definitions $\Lambda \equiv \left\{(\phi, n) \in \left(\underline{\phi}, \infty\right) \times (\underline{n}, \overline{n}) \mid b_2 \geqslant b_1\right\}$ and $\Lambda^c \equiv \left\{(\phi, n) \in \left(\underline{\phi}, \infty\right) \times (\underline{n}, \overline{n}) \mid b_2^c \geqslant b_1^c\right\}$. In other terms (Λ, Λ^c) delimits the parametric space in which *PCR* emerges respectively for the political and the CP cases. The following Corollary resumes the conditions for the Welfare regimes' comparison between the political and CP cases in the parametric space (ϕ, n).

Corollary 7 *For any level of* $\bar{h}$ *and* $n \in (\underline{n}, \overline{n})$ *the following* $\Lambda \subset \Lambda^c$ *holds*

Proof. (See Appendix). ■

The parametric space in which *PCR* emerges is always larger in the CP environment than in the political one. Furthermore let $\bar{\phi}$ be a sufficiently large value of the ideological bias,[13] such that for any $\phi < \bar{\phi}$, the following Proposition is stated.

Proposition 10 *Under dynamic efficiency, for any* $\delta < \bar{\delta}$ *and for any* $\phi < \bar{\phi}$*, the political SMPE induces overtaxation with respect to the time-consistent Central Planner SMPE, i.e.* $\mathbf{T}[h_t, k_t] > \mathbf{T}^c[h_t, k_t]$ *for any* $(h_t, k_t) \in \Upsilon^p \cap \Upsilon^c$.

Proof. (See Appendix). ■

According to the above proposition, if the CP adopts a politically equivalent system of welfare weights, for any level of human and physical capital the level of the income tax rate is always lower than in the political case, i.e. $\mathbf{T}[h_t, k_t] > \mathbf{T}^c[h_t, k_t]$. Then, the politicians involved in a Markov game among successive generations of players deliver the time consistent social optimum if they reduce the political weight they assign to the old agents. Given the invariant level of education transfers achieved by both the politicians and the social planner, a high tax rate implies too generous pension

[12] In particular the relative Welfare weights rewritten in terms of political weights are equal to:

$$\Omega_R^c \equiv \frac{(1+n)(1+\beta)}{\phi + (1+n)} \quad \text{and} \quad \Omega_O^c \equiv \frac{\phi - \beta(1+n)}{\phi + (1+n)}$$

[13] See proof of Proposition 5 for the exact determination of $\bar{\phi}$.

benefits. These distortions come from the politicians' strategic behavior. In determining taxation rules, short-lived politicians take into account that future politicians will compensate the fiscal cost of current adults by paying pensions in their old age. This stems from the fact that higher taxes today lead to lower private wealth in old age, i.e. to a lower state variable in the following period, thereby triggering more transfers from the future politicians. The policy response of the future politicians thus reduces the current (electoral) cost of transferring resources to the elderly and leads to overspending, unless the adults enjoy an unusually large political power. Consequently, by transferring too much resources to old age due to both the overrepresentation of the current elderly agents and the policy response of future politicians, the politicians fail to provide the optimal income tax rate policy.

3.7 Conclusions

We characterize Markov-perfect equilibria in a model in which the absence of government commitment affects public expenditures in intergenerational transfers. Through the *GEEs* of the Central Planner's maximization problem we show that in choosing the tax rate and the type of redistribution, the government trades off intertemporal distortions in the provision of public expenditures over two consecutive periods only.

In particular we find closed form solution in a simple economy subject to a binding time-consistency constraint and characterized by a linear technology, productive public expenditures and log-additive preferences. In our environment intergenerational conflicts especially arise because of production factors ownership. The equilibrium turns out to be characterized by a bidimensional time-varying policy rule non trivially related to the relevant state variables of our economy.

Finally, due to the distortions generated by the repeated political competition process and by the political overrepresentation of elderly agents, political equilibrium under probabilistic voting is characterized by overtaxation compared with a *time-consistent* Central Planner solution.

Our analysis leaves some natural directions open for future research, especially from a technical point of view. Closed form solutions in the differentiable Markov strategies' game will enable us to implement numerical methods, such as the Projection Method, to test the robustness of the algorithm and the sensitivity of the model to structural variations.

3.8 Technical Appendix

3.8.1 Derivation of recursive formulation and Generalized Euler Equation

We derive the recursive formulation of the CP program starting from its sequential version:

$$V_0^c\left[h_0,k_0\right]=\max_{\{f_t^c,h_{t+1},k_{t+1}\}_{t=0}^{\infty}}\sum_{t=0}^{\infty}\left(1+n\right)^t\delta^t B\left[f_t^c,h_t,k_t,k_{t+1}\right] \tag{1B}$$

where (h_0,k_0) are the initial conditions of the payoff-relevant state variables of the dynamic optimization program and $B\left[f_t^c,h_t,k_t,k_{t+1}\right]\equiv\beta u\left[C_{2,t}\left[f_t^c,h_t,k_t\right]\right]+\left(1+n\right)\delta u\left[C_{1,t}\left[\tau_t^c,h_t,k_{t+1}\right]\right]$. Equivalently we rewrite the above value function in the following terms:

$$V_0^c\left[h_0,k_0\right]=\max_{\{f_0^c,k_1\}}B\left[f_0^c,h_0,k_0,k_1\right]+\max_{\{f_t^c,h_{t+1},k_{t+1}\}_{t=1}^{\infty}}\sum_{t=1}^{\infty}\left(1+n\right)^t\delta^t B\left[f_t^c,h_t,k_t,k_{t+1}\right] \tag{2B}$$

$$=\max_{\{f_0^c,k_1\}}B\left[f_0^c,h_0,k_0,k_1\right]+\left(1+n\right)\delta\max_{\{f_t^c,h_{t+1},k_{t+1}\}_{t=1}^{\infty}}\sum_{t=0}^{\infty}\left(1+n\right)^t\delta^t B\left[f_t^c,h_t,k_t,k_{t+1}\right]$$

By definition, we have:

$$V_1^c\left[h_1,k_1\right]=\max_{\{f_t^c,h_{t+1},k_{t+1}\}_{t=1}^{\infty}}\sum_{t=0}^{\infty}\left(1+n\right)^t\delta^t B\left[f_t^c,h_t,k_t,k_{t+1}\right] \tag{3B}$$

Due to stationarity condition the indirect utility function satisfies $V_0^c\left[\cdot\right]\equiv V_1^c\left[\cdot\right]\equiv\ldots\equiv V_t^c\left[\cdot\right]\ldots$ We omit time indexes and denote by prime symbol next period variables. Plugging Eq. (3B) into Eq. (2B) we yield the following Bellman equation:

$$V^c\left[h,k\right]=\max_{f^c}B\left[f^c,h,k,k'\right]+\left(1+n\right)\delta V^c\left[h',k'\right]$$

subject to the following set of constraints:

$$1)\ \Phi^c[h,k] = \begin{cases} k' = \tilde{K}[f^c,h] \\ h' = H[e^c,h] \end{cases}$$

$$2)\ f^c \in \Pi^c[h,k]$$

which can be rewritten as follows:

$$V^c[h,k] = \max_{f^c \in \Pi^c[h,k]} B\left[f^c, h, k, \tilde{K}[f^c,h]\right] + (1+n)\,\delta V^c\left[H[e^c,h], \tilde{K}[f^c,h]\right] \tag{4B}$$

The *GEE* are obtained as the first order condition of the CP optimization plan. The derivation below follows the method proposed by Klein et al. (2008) extending to the OLG case with two political controls in bidimensional state-space. In the following let us denote with $Y_x \equiv \frac{\partial Y}{\partial x}$ the partial derivative of Y with respect to x, while $\frac{dY}{dx}$ denotes total derivative. Furthermore, for simplicity of notation we will omit the apex c. The political first order conditions of Eq. (4B) with respect to $f \equiv (e,\tau)$ are equal to:

$$0 = B_e + B_{k'}\tilde{K}_e + (1+n)\,\delta\left(V_{h'}H_e + V_{k'}\tilde{K}_e\right) \tag{5B}$$

$$0 = B_\tau + B_{k'}\tilde{K}_\tau + (1+n)\,\delta V_{k'}\tilde{K}_\tau \tag{6B}$$

Using *Benveniste-Scheinkman* formula we obtain the following expression for V_h and V_k:

$$V_h = B_h + B_{k'}\tilde{K}_h + (1+n)\,\delta\left(V_{h'}H_h + V_{k'}\tilde{K}_h\right) \tag{7B}$$

$$V_k = B_k \tag{8B}$$

From Eq. (5B) and (6B) we obtain the expression for $V_{h'}$ and $V_{k'}$:

$$V_{h'} = \frac{1}{(1+n)\,\delta H_e}\left(\frac{B_\tau \tilde{K}_e - B_e \tilde{K}_\tau}{\tilde{K}_\tau}\right) \tag{9B}$$

$$V_{k'} = -\frac{B_\tau + B_{k'}\tilde{K}_\tau}{(1+n)\,\delta \tilde{K}_\tau} \tag{10B}$$

Plugging Eq. (9B) and (10B) into (7B) we get the final expression for V_h :

$$V_h = B_h + \frac{B_\tau \tilde{K}_e - B_e \tilde{K}_\tau}{\tilde{K}_\tau}\frac{H_h}{H_e} - B_\tau \frac{\tilde{K}_h}{\tilde{K}_\tau} \tag{11B}$$

Using stationarity condition and plugging Eq. (8B) and (11B) into (5B) and (6B), we obtain the GEEs of the CP problem respectively for e and τ:

$$0 = B_e + B_{k'}\tilde{K}_e + (1+n)\,\delta\left(\left(B'_{h'} + \frac{B'_{\tau'}\tilde{K}'_{e'} - B'_{e'}\tilde{K}'_{\tau'}}{\tilde{K}'_{\tau'}}\frac{H'_{h'}}{H'_{e'}} - B'_{\tau'}\frac{\tilde{K}'_{h'}}{\tilde{K}'_{\tau'}}\right) H_e + B'_{k'}\tilde{K}_e\right) \tag{12B}$$

$$0 = B_\tau + \left(B_{k'} + (1+n)\,\delta B'_{k'}\right)\tilde{K}_\tau \tag{13B}$$

From definition of $B[\cdot]$, we have:

$$\begin{aligned}
B_e &= \beta u_{C_2} C_{2,e} = -\beta(1+n)^2 u_{C_2} \\
B_\tau &= \beta u_{C_2} C_{2,\tau} + \delta\,(1+n)\,u_{C_1} C_{1,\tau} = (1+n)\,(1+h)\,(\beta u_{C_2} - \delta u_{C_1}) \\
B_h &= \beta u_{C_2} C_{2,h} + \delta\,(1+n)\,u_{C_1} C_{1,h} = (1+n)\,(\beta\tau u_{C_2} + \delta\,(1-\tau)\,u_{C_1}) \\
B_k &= \beta u_{C_2} C_{2,k} = \beta R\,(1+n)\,u_{C_2} \\
B_{k'} &= \delta\,(1+n)\,u_{C_1} C_{1,k'} = -\delta\,(1+n)^2\,u_{C_1}
\end{aligned}$$

Using the above partial derivatives and rewriting $\tilde{K}_j$ where $j \in (f^c, h)$ in terms of $\eta\,[\cdot]$, we get the GEEs as a weighted combination of intergenerational wedges:

$$0 = \Delta_e + \delta\frac{\eta_e}{\eta_{k'}}\Delta_{k'} + \delta H_e \tilde{\Delta}'_{\tau'} \tag{14B}$$

$$0 = (1+h)\,\Delta_\tau + \delta(1+n)\frac{\eta_\tau}{\eta_{k'}}\Delta_{k'} \tag{15B}$$

where Δ are defined as:

$\Delta_\tau \equiv \beta u_{C_2} - \delta u_{C_1}$	taxation wedge
$\tilde{\Delta}_\tau \equiv -\chi_1 \delta u_{C_1} + \chi_2 \beta u_{C_2}$	"modified" taxation wedge
$\Delta_h \equiv \beta \tau u_{C_2} + \delta (1-\tau) u_{C_1}$	human capital endowment wedge
$\Delta_e \equiv -\beta u_{C_2} + \delta H_e \Delta'_{h'}$	forward redistribution wedge
$\Delta_{k'} \equiv u_{C_1} - \beta R u_{C'_2}$	savings/consumption wedge

where $\chi_1 \equiv (1+h')\left(\frac{H'_{h'}\eta'_{e'} - H'_{e'}\eta'_{h'}}{H'_{e'}\eta'_{\tau'}}\right)$ and $\chi_2 \equiv \chi_1 + (1+n)\frac{H'_{h'}}{H'_{e'}}$.

Proof of Proposition (9). Let us guess as equilibrium policy functions for the time-consistent Central Planner solution the following functional form respectively for e and τ:

$$e^g = a_1^c h + a_0^c \bar{h} \tag{16B}$$

$$\tau^g = b_3^c \frac{k}{1+h} + b_2^c \frac{h}{1+h} + b_1^c \frac{1}{1+h} + b_0^c \tag{17B}$$

which are structurally equivalent to the equilibrium policy rules in the political case. If Eq. (16B) and (17B) are the equilibrium of the Central Planner problem, then they must satisfy simultaneously the *GEEs* given by conditions (12B) and (13B). Let us manipulate the *GEEs*, plugging the expressions for each partial derivative. We obtain for τ and e, respectively:

$$0 = -\beta u_{C_2} + \delta \left(\begin{array}{c} \left(\beta \tau' u_{C'_2} + \delta (1-\tau') u_{C'_1}\right) \\ + (1+h')\left(\beta u_{C'_2} - \delta u_{C'_1}\right)\left(\frac{\tilde{K}'_{e'}}{\tilde{K}'_{\tau'}}\frac{H'_{h'}}{H'_{e'}} - \frac{\tilde{K}'_{h'}}{\tilde{K}'_{\tau'}}\right) \\ +\beta(1+n) u_{C'_2} \frac{H'_{h'}}{H'_{e'}} \end{array} \right) H_e \tag{18B}$$

$$0 = \beta u_{C_2} - \delta u_{C_1} \tag{19B}$$

Using the equation of $H[\cdot]$, the following expressions result:

$$H_e = \frac{\alpha h + (1-\alpha)\bar{h}}{2(1+n)h'} \tag{20B}$$

$$\frac{H'_{h'}}{H'_{e'}} = \frac{\alpha e'}{\alpha h' + (1-\alpha)\bar{h}} \tag{21B}$$

Under logarithmic utility and linear production function, we characterize the following saving choice:Plugging the guess given by Eq. (16B) and (17B) into the saving function, we obtain the following recursive function for saving choice:

$$k' = \tilde{K}\left[e,\tau,h\right] = \frac{\beta R}{\left(1+n\right)\left(b_3^c + R\left(1+\beta\right)\right)}\left(1+h\right)\left(1-\tau\right) \tag{23B}$$
$$- \frac{b_2^c + b_0^c - \left(1+n\right)a_1^c}{b_3^c + R\left(1+\beta\right)}\sqrt{\frac{\left(\alpha h + \left(1-\alpha\right)\bar{h}\right)e}{1+n}}$$
$$- \frac{b_1^c + b_0^c - \left(1+n\right)a_0^c \bar{h}}{b_3^c + R\left(1+\beta\right)}$$

Using Eq. (21B) and (23B) and simplifying, we get:

$$\frac{\tilde{K}'_{e'}}{\tilde{K}'_{\tau'}}\frac{H'_{h'}}{H'_{e'}} - \frac{\tilde{K}'_{h'}}{\tilde{K}'_{\tau'}} = \frac{1-\tau'}{1+h'}$$

Finally rearranging all the terms, Eq. (18B) becomes as follows:

$$0 = -u_{C_2} + \delta\left(1 + \left(1+n\right)\frac{\alpha e'}{\alpha h' + (1-\alpha)\bar{h}}\right)u_{C'_2}H_e \tag{25B}$$

Using the political Euler condition $\beta u_{C_2} - \delta u_{C_1} = 0$ and the economic one $u_{C_1} - R\beta u_{C'_2} = 0$, Eq. (25B) simplifies to:

$$1 = \left(1 + \left(1+n\right)\frac{\alpha e'}{\alpha h' + (1-\alpha)\bar{h}}\right)\frac{1}{R}H_e \tag{26B}$$

which is also equivalent to:

$$e = \left(\left(1 + \left(1+n\right)\frac{\alpha e'}{\alpha h' + (1-\alpha)\bar{h}}\right)\frac{1}{2R}\right)^2 \frac{\alpha h + (1-\alpha)\bar{h}}{1+n} \tag{27B}$$

Let us now make a further assumption on the guess on e, considering the following variant of Eq. (16B) :

$$e^g = a_1^c\left(\psi^g\right)h + a_0^c\left(\psi^g\right)\bar{h} \tag{28B}$$

such that $a_1^c\left(\psi^g\right) = \frac{\alpha}{1+n}\psi^g$ and $a_0^c\left(\psi^g\right) = \frac{1-\alpha}{1+n}\psi^g$, i.e. we guess the policy e as a linear convex combination between parental human capital h and human capital society endowment $\bar{h}$ scaled by a constant which has to be determined, ψ^g. Then Eq. (27B) can be rewritten as follows:

$$e = \frac{\alpha}{1+n}\tilde{\psi}^g h + \frac{1-\alpha}{1+n}\tilde{\psi}^g \bar{h} \tag{29B}$$

where $\tilde{\psi}^g \equiv \left(\left(1+(1+n)\frac{\alpha e'}{\alpha h'+(1-\alpha)\bar{h}}\right)\frac{1}{2R}\right)^2$. Plugging the guess of e given by Eq. (28B) into the expression of $\tilde{\psi}^g$ and simplifying we get:

$$\tilde{\psi}^g = (1+\alpha\psi^g)^2\left(\frac{1}{2R}\right)^2 \tag{30B}$$

By fixed-point condition $\tilde{\psi}^g = \psi^g$ which yield the following solutions:

$$\psi^g_{1,2} = \frac{1}{\alpha}\left(\frac{2R}{\alpha}\left(R \pm \sqrt{R^2-\alpha}\right) - 1\right)$$

Similar arguments as in Proof of Proposition 2 can be made. Then let us consider the stable root $\psi^* = \frac{1}{\alpha}\left(\frac{2R}{\alpha}\left(R - \sqrt{R^2-\alpha}\right) - 1\right)$ as feasible solution. It immediately follows that:

$$a^c_1 = \frac{\alpha}{1+n}\psi^* \quad \text{and} \quad a^c_0 = \frac{1-\alpha}{1+n}\psi^* \tag{31B}$$

are the solutions for the guess on e which turns out to be equivalent to the political outcome After plugging the guesses, Eq. (16B) and Eq. (17B), and the recursive saving function, Eq. (23B), into Eq. (19B), the GEE for the policy τ is as follows:

$$\beta\frac{1}{(1+n)Rk+(1+n)(1+h)\tau-(1+n)^2e^g} = \delta\frac{1}{(1+h)(1-\tau)-(1+n)\tilde{K}\left[e^g,\tau,h\right]} \tag{32B}$$

After some algebraic manipulations we obtain the following well-defined system:

$$\begin{cases} b_0^c = \frac{\beta\left(R+b_3^c\right)}{R(\beta+\delta(1+n)(1+\beta))+(\beta+\delta(1+n))b_3^c} \\ \\ b_1^c = \frac{(1+n)\beta\left(b_0^c+b_1^c-(1-\alpha)\bar{h}\psi\right)-(1-\alpha)\bar{h}\sqrt{\psi}\left(-\beta\left(b_0^c+b_2^c\right)-\delta(1+n)\left(R(1+\beta)+b_3^c\right)\sqrt{\psi}+\alpha\beta\psi\right)}{\left(1+\bar{h}\right)\left(R(\beta+\delta(1+n)(1+\beta))+(\beta+\delta(1+n))b_3^c\right)} \\ \\ b_2^c = \frac{\bar{h}\alpha\sqrt{\psi}\left(\beta\left(b_0^c+b_2^c\right)+\delta(1+n)\left(R(1+\beta)+b_3^c\right)\sqrt{\psi}-\alpha\beta\psi\right)}{R(\beta+\delta(1+n)(1+\beta))+(\beta+\delta(1+n))b_3^c} \\ \\ b_3^c = -\frac{R(1+n)\delta\left(R(1+\beta)+b_3^c\right)}{R(\beta+\delta(1+n)(1+\beta))+(\beta+\delta(1+n))b_3^c} \end{cases}$$

Solving the system we obtain the following two solutions for τ :

$$\tau^{c1} = b_3^{c1}\frac{k}{1+h} + b_2^{c1}\frac{h}{1+h} + b_1^{c1}\frac{1}{1+h} + b_0^{c1} \tag{33B}$$

where, under $\Omega_R^c \equiv \frac{\delta(1+n)(1+\beta)}{\beta+\delta(1+n)}$ and $\Omega_O^c \equiv \frac{\beta(1-\delta(1+n))}{\beta+\delta(1+n)}$:

$$\begin{array}{ll} b_2^{c1} = \frac{\alpha\sqrt{\psi^*}}{R-\alpha\sqrt{\psi^*}}(\Omega_O^c + R\sqrt{\psi^*}\Omega_R^c - \alpha\sqrt{\psi^*}); & b_3^{c1} = -R\Omega_R^c; \\ b_1^{c1} = \bar{h}\frac{1-\alpha}{\alpha}\frac{R}{R-(1+n)}\mu_2 + \bar{R}\left(\Omega_O^c - \bar{h}\left(1-\alpha\right)\psi^*\right); & b_0^{c1} = \Omega_O^c; \end{array}$$

and

$$\tau^{c2} = b_3^{c2}\frac{k}{1+h} + b_2^{c2}\frac{h}{1+h} + b_1^{c2}\frac{1}{1+h} + b_0^{c2} \tag{34B}$$

where:

$$\begin{array}{ll} b_3^{c2} = -R; & b_1^{c1} = (1-\alpha)\,\psi^*\bar{h}; \\ b_2^{c1} = \alpha\psi^*; & b_0^{c1} = 0 \end{array}$$

Note that the Eq. (33B) is equivalent to Eq. (34B) under the condition $\Omega_R^c = 1$ and $\Omega_O^c = 0$, which implies $\delta = \frac{1}{1+n}$. Recall that, for the existence of the fix point, the condition $\delta < \frac{1}{1+n}$, which induces Ω_O^c to be strictly greater than zero, is required. Consequently the Eq. (34B) is not feasible. ■

Proof of Corollary (7). ■

Proof of Proposition (10). Let us first consider the following normalization of the relative Welfare weights, Eq. (3.18), after assigning $\phi \equiv \frac{\beta}{\delta}$:

$$\Omega_R^c \equiv \frac{(1+n)(1+\beta)}{\phi+(1+n)} \quad \text{and} \quad \Omega_O^c \equiv \frac{\phi-\beta(1+n)}{\phi+(1+n)} \tag{3.30}$$

Using the weights (3.30) and comparing the parameters of the policy rules of Eq. (2.25) and Eq. (3.29) we obtain for any $\phi < \bar{\phi}$ where:

$$\bar{\phi} \equiv \frac{1}{2}\left(\frac{-1-n-}{\frac{\sqrt{(1+n)^2\left(-1+2R\sqrt{\psi}-\alpha\psi\right)\left(-(1+2\beta)^2+2R(1+2\beta(1+\beta))\sqrt{\psi}-\alpha\psi\right)}}{-1+2R\sqrt{\psi}-\alpha\psi}}\right)$$

the following inequalities must hold:

$$\begin{array}{ll} b_0^c < b_0 & b_2^c < b_2 \\ b_1^c < b_1 & b_3^c > b_3 \end{array}$$

Then we conclude $\mathbf{T}[h_t, k_t] > \mathbf{T}^c[h_t, k_t]$ for any $(h_t, k_t) \in \Upsilon^p \cap \Upsilon^c$. $\blacksquare$

3.9 Technical Appendix

3.9.1 Derivation of Pareto optimal allocation in steady states

To obtain the equilibrium value for e_t, using the economic Euler condition and given the equivalence $\delta u_{c_{1,t}} = \beta u_{c_{2,t}}$ we can rewrite the FOC with respect the public education transfer as follows:

$$-Ru_{c_{1,t+1}} + H_{e_t} u_{c_{1,t+1}} = 0$$

After simplifying, we obtain:

$$H_{e_t} = R \Rightarrow e_t = \frac{1}{1+n}\left(\frac{1-\theta}{R}\right)^{\frac{1}{\theta}} \left(\alpha h_t + (1-\alpha)\bar{h}\right)$$

In equilibrium by implicit function theorem there exists a unique saving function, k_{t+1}, which satisfies the condition (3.6):

$$k_{t+1} = \frac{\beta}{(1+n)(1+\beta)}\left[(1+h_t) - \left(\frac{p_t}{1+n} + e_t(1+n)\right)\right] - \frac{1}{(1+n)R(1+\beta)} p_{t+1}$$

Plugging the equilibrium saving choice into the FOC with respect the pension transfer, we obtain:

$$-\delta \frac{1}{(1+h_t) - \left(\frac{p_t}{1+n} + e_t(1+n)\right) - (1+n)k_{t+1}} + \beta \frac{1}{(1+n)Rk_t + \frac{p_t}{1+n}} = 0$$

Solving for p_t :

$$p_t = -\frac{\beta(1+n)^2}{\beta+\delta} k_{t+1} - \frac{R\delta(1+n)^2}{\beta+\delta} k_t + \frac{\beta(1+n)}{\beta+\delta} h_t - \frac{\beta(1+n)^2}{\beta+\delta} e_t + \frac{\beta(1+n)}{\beta+\delta}$$

Fixing $\theta = \frac{1}{2}$ and equating $h_{t+1} = h_t = h^*$, the following steady state levels for the state variable H and education transfer are obtained:

$$h^* = \frac{(1-\alpha)}{2(1+n)R-\alpha}\bar{h}$$
$$e^* = \frac{(1-\alpha)}{4(1+n)R^2-2R\alpha}\bar{h}$$

Equating $k_{t+1} = k_t = k^*$ and solving simultaneously for k^* and p^*, we get :

$$k^* = \frac{\left(4(1+n)R^2 - \bar{h}(1+n-2R)(1-\alpha) - 2R\alpha\right)\beta(1+n-R\delta)}{2(1+n)R(2(1+n)R-\alpha)(nR\delta+\beta(n+(R-1)(R\delta-1)))}$$

$$p^* = \frac{(1+n)\left(4(1+n)R^2 - \bar{h}(1+n-2R)(1-\alpha) - 2R\alpha\right)\beta(R\delta-1)}{2(2(1+n)R-\alpha)(nR\delta+\beta(n+(R-1)(R\delta-1)))}$$

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