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NON-LINEAR PRICING IN IMPERFECTLY COMPETITIVE MARKETS

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Part I

Preliminary Material

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Abstract

This thesis is dedicated to the analysis of non-linear pricing in oligopoly. Non-linear pricing is a fairly predominant practice in most real markets, mostly characterized by some amount of competition. The sophistication of pricing practices has increased in the latest decades due to the technological advances that have allowed companies to gather more and more data on consumers preferences.

The first essay of the thesis highlights the main characteristics of oligopolistic non-linear pricing. Non-linear pricing is a special case of price discrimination. The theory of price discrimination has to be modified in presence of oligopoly: in particular, a crucial role is played by the competitive externality that implies that product differentiation is closely related to the possibility of discriminating. The essay reviews the theory of competitive non-linear pricing by starting from its foundations, mechanism design under common agency. The different approaches to model non-linear pricing are then reviewed. In particular, the difference between price and quantity competition is highlighted. Finally, the close link between non-linear pricing and the recent developments in the theory of vertical differentiation is explored.

The second essay shows how the effects of non-linear pricing are determined by the relationship between the demand and the technological structure of the market. The chapter focuses on a model in which firms supply a homogeneous product in two different sizes. Information about consumers' reservation prices is incomplete and the production technology is charac-

terized by size economies. The model provides insights on the size of the products that one finds in the market.
Four equilibrium regions are identified depending on the relative intensity of size economies with respect to consumers' evaluation of the good. Regions for which the product is supplied in a single unit or in several different sizes or in only a very large one. Both the private and social desirability of non-linear pricing varies across different equilibrium regions.

The third essay considers the broadband internet market. Non discriminatory issues seem the core of the recent debate on the opportunity or not of regulating the internet. One of the main questions posed is whether the telecom companies, owning the networks constituting the internet, should be allowed to offer quality-contingent contracts to content providers. The aim of this essay is to analyze the issue through a stylized two-sided market model of the web that highlights the effects of such a discrimination over quality, prices and participation to the internet of providers and final users. An overall welfare comparison is proposed, concluding that the final effects of regulation crucially depend on both the technology and preferences of agents.

Chapter 1

Introduction

This thesis is dedicated to the theory of non-linear pricing in imperfect market forms. A common theme characterizing the essays that make up the thesis are price discrimination and product differentiation, both standing at the foundation of the modern theory of non-linear pricing. These topics also characterize a good part of the recent literature on industrial organization, developed since the 60s and 70s on oligopoly and strategic interaction between firms. In this already classical stream of economics literature, a primary role is played by the pricing strategies of firms, the heterogeneity of consumers preferences and the product lines firms decide to propose on the market.

The thesis focuses on two particular issues in the outlined framework. First, the theory of non-linear pricing is evaluated. It is well known that a monopolist, even in presence of imperfect information, has the possibility to induce consumers to self-select by supplying different price-quantity combinations. The mechanism allows the firm to increase its profits, as it extracts the surplus from consumers more efficiently. Non-linear pricing, however, is a very diffused practice also in more competitive market structures. In those settings the effects of this strategy are far less clear cut. The first essay of this thesis attempts to review and present consistently some of the most recent approaches and the the main results in the literature of non-linear pricing.

One of the most striking features of the literature on the topic is the focus on the features of the consumers' demand, while the features of production have often standard properties. The contribution of the second essay is to take a closer look at the interaction between the demand and supply side in determining the effects of non-linear pricing both for producers and consumers. The results are particularly interesting as they can be interpreted in term of the size of the products that firms commercialize: price decisions and features of both demand and supply determine the feature of the product lines chosen by firms in equilibrium.

The theory of non-linear pricing has been developed with reference to situations of both quantity discounts and situations in which different price-quality combinations are available on the market. The qualitative interpretation is dual to the quantitative one and the theoretical principles do not change substantially. However, there are a number of further possible applications of the theory when considering quality supply. A reference framework similar to the one adopted in the second essay is also widely used in the literature on product differentiation. This literature mainly deals with issues like the supply of more or less heterogenous product lines, quality supply and the incentives to enter the market or even the incentives to bring to the market a good which is of a lower quality and costs more to be produced than a high quality one. This thesis explores a different application in the field of regulation. Can the theoretical principles analyzed shed any light on a policy debate that is involving many important actors both in the United States and in Europe? Is it possible to model theoretically the most important characteristics of the broadband internet market? This is actually the goal of the third essay dedicated to the "network neutrality" debate and that is trying to shed light on the "pros" and "cons" of regulating the web.

The remaining of this chapter is dedicated to the motivation and a short description of the structure of the thesis.

1.1 Motivation

The latest decades witnessed a relentless development of applications of game theory to many fields in economics; between these the analysis of imperfectly competitive market forms is one of the most prominent. Strategic aspects have been acknowledged as the distinguishing feature of the interaction between firms. This very simple observation allowed to rewrite almost completely or reinterpret most of the existing theories in industrial organization. The new approach has driven to the formulation of important hypotheses on firms' behaviour that can be empirically tested.

Oligopolistic analysis emphasizes the behaviour and the decisions of firms in more or less direct competition between each other. It is not surprising, then, that the analysis of price decision of firms is still at the centre of the attention of the modern theory of industrial organization. It is well known since Dupuit[22], who first described what is now known as consumers' "screening", that firms have an incentive to adopt every possible strategy to exploit their market power to extract as much consumers' surplus as possible. It is a familiar, almost daily, experience to most of us to select a product from a menu of reporting many possible combinations of prices, quantities, qualities and other optional services: one can think, for example, to the size of soft drinks or chips when ordering a meal at a fast food; when shopping in the supermarket one's cart is just a bundle of goods chosen from an almost infinite possible combinations of products, each sold in packets of several different sizes, not to mention the several types of discounts to be obtained through coupons, membership cards or other type of analogous fidelization practices; when taking the car for a periodic revision one can opt for a minimal testing obeying the regulations or for a more in-depth full service of the car; when booking a flight ticket, the price quoted is varying through time and is also linked to another series of determinants that make almost impossible to make any credible forecast on which the price will at a distance of just a few days; when subscribing to cable or pay-tv one is faced with several possible packets featuring different types of thematic channels. These are only a few examples from an almost potentially infinite list of real world cases.

The preponderance of those price practices does not seem to be correlated to business cycles or slowed down by the several crises that hit the economy: the adoption of discriminatory pricing strategies follows a rapidly increasing trend that does not seem to find an end in the near future. The dramatic technological developments of the latest decades deeply changed not only the way of doing business but society as a whole. These technological advances have allowed companies to gather and store an unprecedented amount of data on consumers and their behaviour. The information is highly relevant and can be used profitably by firms in a number of creative fashions that allow to extract from consumers the highest possible surplus.

A question arising in the outlined framework regards the effect of competition and strategic interaction in determining the pricing decisions of firms. As observed by Stole[75], "Economic reality (. . .) largely lies somewhere between the textbook extremes, and most economists agree that price discrimination arises in oligopoly settings". It is not surprising then, as observed by Armstrong that: "much of the recent literature has focused on price discrimination in competitive settings". Given the wide range of different practices adopted by firms in pricing no unique and clear-cut answer has emerged. In general and hardly surprising, however, the effect of competition is to restrict the ability of firms to extract surplus and discriminate, as rival firms may challenge them to capture the marginal consumers. In presence of competition, then, firms may face a prisoners' dilemma situation in which they would be better off if they could commit not to use discriminatory practices. Although this is not the only possible strategic situation that firms may face, the theme is recurrent in the literature and will characterize also part of this thesis.

The effects of competition are not limited to firms: what is the effect of sophisticated pricing practices on consumers? The effects of complicated pricing strategies and competition on consumers and aggregate social welfare are far from obvious. The literature has proposed a number of answers. In a monopolistic market, consumers may enjoy positive effects if firms price in a more sophisticated way in case these allow to extend the market served; this is not necessarily true in presence of competition. For the same reason for which firms may want to commit not to discriminate, it may well happen that consumers end up with a larger share of the surplus in case competition drives firms to an equilibrium suboptimal for them.

A further important characteristic of price discrimination in oligopoly is its close link with product differentiation. In many cases, although not always, competition implies that a minimal amount of differentiation is necessary to avoid repeated undercutting of prices and the erosion of all profits. An immediate option when analyzing firms'decisions is to assume prices as the choice variable of firms. In this case, product differentiation is strictly linked to the possibility of discriminating. As underlined by Feuerstein[28] assertion that "(...) all firms that have some scope to set prices have an incentive to price discriminate if they act on sub-markets with different demand structures". The literature has dealt in a number of ways with product differentiation and analyzing its effects: this theme will also characterize this thesis, as issues related to both horizontal and vertical differentiation will be addressed.

Finally, another concern when analyzing pricing practices and in presence of product differentiation is the effect of those on other important strategic choices of firms. As witnessed by McAfee[54], "(. . .) pricing ought to be at the core of business strategy (. . .), pricing strategies are important determinants of the profitability of R&D, service contracts, warranties, market segmentation, and other strategic choices". Pricing as well as the different type of interaction between firms in the market may actually determine a wide array of important decisions for a firm. R&D incentives, service contracts and warranties, endogenous market segmentation and the decision of firms to merge with competitors are examples. More closely related to the goals of this thesis the interaction between price decisions, the technological structure available to firms' production and the demand are interpreted as one possible determinants of why different products are commercialized in different ways: many are supplied in a single unit, some in packets of several different sizes while some others only in packets of a larger size. Another application regards the role of pricing decisions and the freedom to price discriminate in the decision of whether or not (and how) to regulate the internet broadband sector. In this market, the telecom companies which own the network are in principle able to discriminate between different content providers according to their demand of bandwidth and the priority needed in the delivery of their content. This type of price discrimination, not allowed at the moment in name of the neutrality of the network, would have a clear impact on both content providers and final users of the internet.

This brief introduction of the main themes faced when studying price discrimination, non-linear pricing and their applications allowed to highlight many of the issues that will be analyzed in greater depth in the rest of the thesis. The next paragraph outlines the structure of the thesis.

1.2 Structure of the Thesis

The thesis is structured as follows. Chapter 2 outlines the theoretical framework of analysis of price discrimination in oligopoly and, in particular, reviews the recent literature on the theme of non-linear pricing. It is highlighted, in particular, the different approach in case it is assumed that the choice variable of firms are prices as opposed to quantities or consumers' types. The relationship between quality and quantity interpretations of non-linear pricing is then explored.

Chapter 3 analyzes quantity discounts and the size of products. It is a familiar experience for most people nowadays to buy products on offer at a discount when the quantity bought is large. This chapter highlights one possible interpretation of why some products are sold with a quantity discount, some others are sold only in large packets while others only in packets of a single unit and so on. The explanation put forward is based on two fundamental ingredients: the first is the price regime adopted by firms; the second is the interaction between the technological structure available to firms and the features of the demand side.

Chapter 4 is dedicated to a different application of the theory of non-linear pricing to the supply of quality. The problem tackled is the one of "network neutrality". This is the headline usually adopted to describe the important debate on the future of broadband internet. Both in Europe and especially in the United States, many remarkable actors took part in the discussion in the last few years. Trying to simplify, on one side stands who believes that internet should be regulated to make sure the neutrality of the network, that allows everyone to access it and provide content, is preserved; on the other side, instead, everyone believing that firms owning the networks should be allowed to charge content providers to supply them with internet highways able to prioritize the content and deliver it efficiently. The chapter proposes a stylized economic analysis of the issue and the result indicate features which may be relevant for the decision of whether to regulate or not the internet.

Chapter 5 complete the thesis by providing the concluding remarks and possible future extensions of the research presented.

Part II

Non-Linear Pricing in Oligopoly

Chapter 2

Competitive Non-Linear Pricing: An Overview

Non-linear pricing is a specific type of price discrimination taking place when the information regarding relevant consumers' features is not complete. The theory of oligopolistic price discrimination has a number of distinguishing features with respect to monopolistic price discrimination, as presented by Varian[78] and Phlips[63]. Two features are particularly relevant: first, there is a strong link between the ability of firms to price discriminate and the differentiation of the products supplied in the market; second, the results provided by theoretical models in terms of profits of firms, consumers' surplus and aggregate social welfare are overall much less clear cut than the case of monopoly.

As far as product differentiation is considered, the classical example of Bertrand competition well illustrates the point. Two firms supplying an homogeneous product have far less opportunities to induce consumers to self-select in a way that they can enforce different prices. Companies' market power, and hence their opportunity to enforce sophisticated price schemes, is hindered by the competitive pressure exerted by the presence of rival firms. This simple example is sufficient to illustrate the main effect of competition on the ability of firms to price discriminate.

As competition is taken into account, the theoretical model used to address competitive non-linear pricing needs to be modified accordingly. In particular, firms have to differentiate along one or more relevant characteristic in order to dilute the effect of competition. Product differentiation is an obvious option to achieve the goal and this is why it plays such a relevant role when dealing with competitive non-linear pricing.

A further effect of competition is to modify the impact of non-linear pricing on the share of the market served, profits and consumers' welfare. A monopolist is able to use more sophisticated pricing to extract surplus from consumers, increasing its profits. In presence of competition instead, non-linear pricing maybe detrimental for firms' profits as compared to a uniform price. The strategic situation faced by firms may then have an undesired outcome: despite non-linear pricing being the privately optimal choice, it drives to a suboptimal allocation when all firms choose it.

These important issues are illustrated while reviewing the major theoretical approaches to tackle non-linear pricing in oligopoly, with occasional reference also to other types of price discrimination. In particular, the chapter is structured as follows. First, the theoretical background of imperfectly competitive non-linear pricing is traced. As the monopolistic theory can be seen as a particular case of the principal-agent model, the competitive extension involves considering games of common agency. Second, the main theoretical approaches to model non-linear pricing in oligopoly are outlined. The features and the main results are illustrated for both the case of price schedule and market share competition. Finally, a parallel between non-linear pricing and vertical quality competition is drawn. The models adopted to address product line choice by competing firms are closely related to the models of non-linear pricing in which firms choose the market share to be served. The review is completed by exploring how the model can be extended to address situations in which the market structure features platforms that are intermediaries between two sides of the market.

2.1 The Theoretical Background

From a purely theoretical point of view, non-linear pricing can be seen as an application of the theory of mechanism design in a context of incomplete information. The monopolistic non-linear pricing model, as reviewed for example by Varian[78], can be seen as a principal-agent type of model in which the principal-monopolist does not have complete information about the agent-consumers. In more formal terms, both the principal and the agent are characterized by a Von-Neumann-Morgenstern utility function $u_i(y, \theta)$ where θ represents the type of the agent while $y = \{x, t\}$ is called an allocation and is composed by a decision x and a transfer t from the principal to the agent, and $i = \{A, P\}$ identify the players. The utility of the principal is decreasing in the amount of the transfer t as opposed to the one of the agent which is strictly increasing with the transfer. Both functions are twice continuously differentiable.

In this game, a mechanism is a contract m defines a message space and a game form to announce the messages sent by all agents. As information on agent's type is incomplete, the allocation can be conditioned only on messages sent by agents. The timing of the game is the following: first, the principal designs a mechanism; second, the agent accepts or not the mechanism and third the game is played according to the designed mechanism otherwise both players get their reservation utilities.

In such a context a role of primary importance is played by the *revelation principle*: this result, first presented by Gibbard[33], states that the principal can focus on direct mechanisms or, in other words, mechanisms for which the message space is the type space, all agents accept the mechanism and truthfully announce their type as the game is played. The revelation principle allows to restrict the set of implementable allocations: in particular, the allocations implementable need to be incentive compatible or, in other words, the agent should have no incentive to mimic a different type. Between the implementable mechanisms, the principal select the ones which maximize his expected utility. The problem of the principal can be then summarized as:

$$\begin{aligned} &\max_{x(\theta),t(\theta)} E_\theta u_P(x(\theta), t(\theta), \theta) \\ \text{s.t. } &u_A(x(\theta), t(\theta), \theta) \geq u_A(x(\theta'), t(\theta'), \theta) \quad \forall \theta, \theta' \\ &u_A(x(\theta), t(\theta), \theta) \geq \underline{u}_A \qquad \forall \theta \end{aligned}$$

Under a number of more restrictive assumptions, the problem can be simplified so that it is possible to show that an optimal decision exists and can also be characterized. Between these assumptions[1], it is worth recalling the Spence-Mirlees single-crossing condition on the agent's utility function, a monotone hazard rate of the distribution function and, more interestingly for the goals of this thesis, that the agent's reservation utility is type independent.

This methodology of analysis has been applied to a variety of contexts, including auctions, regulatory problems, optimal taxation, public good games and bargaining. In order to address a number of issues, the theory of mechanism design has evolved in several directions. Multi-dimensional type spaces have been considered. In that case θ is a vector characterizing preferences of agents. It turns out that this case is hardly tractable and some specific results have been obtained by Rochet[65], Rochet- Choné[66], Armstrong[3] and Armstrong-Rochet[4]. A second direction of research in mechanism design has considered the possibility that the principal can contract with multiple agents, as Myerson[57], Demski-Sappington[19] and Ma-Moore-Turnbull[46]. In this context, it assumes relevance the possibility of agents to report about other agents against the possibility of "cheating" of agents. Further, the case

[1] A complete treatment of the principal-agent problem and the assumptions needed to guarantee existence of the optimal contract can be found in Fudenberg-Tirole[29].

of informed principals, in which principals have a trade-off between revealing their private information and the advantage of conditioning the contract proposed to the information itself, has been tackled by Myerson[58] and Maskin-Tirole ([52]-[53]). Epstein-Peters[27] and Yamashita[81] focus instead on the case of multiple principals and multiple agents.

As far as non-linear pricing is concerned the case of so called "common agency" is particularly interesting. Competitive non-linear pricing, in fact, can be seen as a particular case in the general theory of common agency games under incomplete information. This type of games were pioneered by the papers of Martimort[47] and Stole[73]. A distinction can be made between *intrinsic* common agency, in which the agent decides whether to contract with both agents or neither, as compared with the *delegated* common agency case in which the agent can contract with only one of the principals. In presence of multiple principals competing for a single agent, the revelation principle may not be valid. Calzolari-Pavan[10] identify three fundamental problems with the revelation principle: first, agents may serve as a coordination device for the principals and restricting the space of messages may affect the generality of the outcome; second, equilibria involving randomization over indirect mechanisms may exist; third, out of equilibrium allocations may be part of the equilibrium mechanism. The first two problems have to do mainly with mixed strategy equilibria; as in the thesis only equilibria in pure strategies are considered, the third problem is more interesting. In a common agency setting, principals can not contract with agents independently, as they need to take into account the strategic effect on the other principals. This implies that the usual characterization of the problem through participation constraint and incentive compatibility constraint may not be satisfactory. The use of the revelation principle in searching for a Nash equilibrium between principals is harmed by the possibility of existence of out-of-equilibrium allocations in the equilibrium menu offered by principals. The point is more clearly illustrated by the following example proposed by Martimort-Stole[52].

In order to capture the difficulties with the revelation principle it sufficient to focus on a complete information setting. There exist two principals $P1$ and $P2$ and agent A. The strategies available to the principals are $S_i = \{A, B, C\}$ while the payoffs are given by $U = \{u_{P1}, u_{P2}, u_A\}$. The game is summarized by the payoff matrix in Table 2.1.

Table 2.1 Payoffs for a Simple Common Agency Game.

	$s_2 = A$	$s_2 = B$	$s_2 = C$
$s_1 = A$	$1, 1, 1$	$2, 0, 2$	$-1, 5, 10$
$s_1 = B$	$0, 2, 2$	$1, 1, 1$	$0, 0, 0$
$s_1 = C$	$5, -1, 10$	$0, 0, 0$	$0, 0, 0$

A Perfect Bayesian-Nash equilibrium of the game is given by the following menu of contracts: each principal offers the menu $s_i^* = \{B, C\}$ and the agent chooses B. C is offered by the principal as an off-equilibrium choice by principal 1 to discourage principal 2 by offering A. If principal 1 were to offer only B then principal 2 best response would be to offer A. By simply inserting C in the contract menu, principal 1 discourages principal 2 from offering A which would leave him with a payoff of -1, as the agent would then choose C from principal 1. Epstein-Peters[27] suggest that enlarging the message space to a universal set, comprehending market information, would restore the revelation principle: this universal set, however, may be of little help in applications as it may not be easy to characterize. The solution proposed by the Martimort-Stole[52] and Peters ([60]-[62]) comes from the use of the generalization of the taxation principle of Guesnerie-Laffont[35], called the delegation principle: instead of focusing on direct mechanisms, it is possible to focus on a restricted class of indirect mechanisms in which principals design menus of alternatives while the choice of the option is delegated to the agent.

Given the challenging theoretical issues and the interesting economic applications, the topic has received a great deal of attention in the last two decades. The following contributions are worth to be mentioned. Mezzetti[55] considers a case of intrinsic common agency in which principals are horizontally differentiated: in this case, the differentiation of principals implies that the agent faces countervailing incentives in reporting his type, limiting the distortions highlighted by Stole[73] and Martimort[47]. Peters[61] identifies a set of restrictions on players' preferences, called the "no-externalities condition" under which there is no loss of generality in restricting the principal's strategies to be take-it or leave-it offers. Calzolari-Pavan[11] discuss the case of sequential contracting in common agency: they conclude that the use of menus may not be equivalent to the use of general mechanisms. Martimort-Stole[53] analyze the problem of the multiplicity of equilibria in a delegated common agency game proposing a refinement of "local truthfulness of equilibria". Laussel-Lebreton[45] and Chiesa-Denicolò[15] consider trading under complete information: the first contribution rationalizes the use of truthful revelation in presence of ex-ante uncertainty over agent's characteristic while the second focuses on a special class of common agency games for which not always the truthful revelation equilibrium is the Pareto dominant for the principals.

2.2 Competitive Non-Linear Pricing: Price Competition

After outlining the theoretical background, the theory of competitive non-linear pricing can be presented. The two major approaches to modelling competitive non-linear pricing are reviewed: the first assumes that prices are the firms' choice variables while the second focuses on the share of consumers served or, in other words, quantity supplied. The crucial role of product differentiation in allowing oligopolistic firms to choose more sophisticated pricing strategies is underlined: in particular, it will be shown that product differentiation is strictly linked to price discrimination not only when firms have prices as choice variables but also when the product line is chosen.

Following Armstrong[2], non-linear pricing as compared to uniform pricing is a case of firms having more *information* to price discriminate. The effect of more information is going to be discussed and the implications in terms of profits and welfare are presented.

2.2.1 Price Competition

The first approach to model oligopolistic interaction between firms practicing non-linear pricing is to assume that their choice variables are price schedules. A first tentative analysis of such a setting have been proposed by Borenstein[9]: he assumes consumers are heterogeneous both in terms of vertical (v_i) and horizontal preferences (t_i) for the qualitatively differentiated good. Given the free entry structure of the model, the demand faced by firms can be decomposed into two segments: local monopoly and competitive. Firms can price discriminate by choosing one price for each segment. The model does not have a closed form solution but the indications of numerical simulations highlight that if sorting is based on horizontal characteristics, higher discounts take place when the fraction of competitive demand is relatively large. If firms are sorting with respect to consumers' vertical preference parameter, then the opposite is true. Moreover, price discrimination has a positive effect on both firms' entry and total output, independently of the type of sorting. However, consumers are damaged when discrimination is based on the horizontal parameter.

Analytical results, however, are provided only by Spulber[72] in a monopolistic competition framework. He shows that when consumers have an elastic demand and their tastes are horizontally differentiated, non-linear pricing and quantity discounts arise in a Bertrand-Nash equilibrium of the free-entry game. In particular, given the competitive pressure of the rival, the discount received by a consumer increases with the distance from firm's location. Two further interesting results are: (1) as the fixed entry costs tend to zero, the equilibrium tends to the perfectly competitive marginal cost pricing equilibrium; (2) non-linear pricing imply greater variety and total output supplied in equilibrium.

The paper of Stole[74], however, has set up perhaps the reference framework for non-linear pricing in oligopoly. The main characteristics of the approach are: first, Stole[74] considers both a duopolistic and a free-entry monopolistic competition setting; second and more important, he considers products that are qualitatively differentiated both horizontally and vertically[2].

Despite the generality of the framework, there are considerable technical difficulties in dealing with multi-dimensional non-linear pricing. The difficulties arising are of the same kind of the mechanism design literature under multi-varied type spaces, as recalled in Section 2.1. Then, although consumers' tastes are vertically and horizontally heterogeneous, heterogeneity in only one dimension at a time is considered to be private information of agents. Stole, then, analyzes the two cases independently: the simplifying assumption is that firm can perfectly price discriminate with respect to the other dimension of preference. As discussed above, non-linear pricing under horizontal differentiation in monopolistic competition was analyzed by Spulber[72]; however, the case of vertical differentiation is original.

Assume that firms know and supply the preferred quality to all customers, discriminating perfectly with respect to a vertical attribute v. Brand preference (or location, x) instead is private information. Each firm $i = 1, 2$ aims to maximize its profit by choosing a tariff such that customers buy and truthfully report their type (location). The problem differs from the monopoly benchmark because the outside option in the individual rationality constraint

[2] As underlined by Stole[75], one should be careful in distinguishing between pure vertical product differentiation, as briefly reviewed in Section 2.3.1, and vertical heterogeneity in which the preference of a consumer for quality increase with his type, no matter what firm supplies them.

is now defined as:

$$\max\{0,\, u_{-i}(q(\hat{x}), x, v) - p_{-i}(\hat{x})\}$$

in which $\hat{x}$ is the reported location and q is the quality supplied and the index $-i$ refers to the rival firm. The interpretation of the situation is easy: not only the consumer have the alternative of not buying, she can also buy from the rival firm. The individual rationality constraint then is *type dependent*. The incentive compatibility constraint, requiring that types reveal truthfully their location, is not affected by the presence of the outside option.

Having assumed firm 1 is located at the left extreme of a linear horizontal quality space and firm 2 at the right end, it is possible to solve the firms' problems. Despite the participation constraint being type dependent, it is possible to prove that the relevant outside option for firm 1 is increasing in θ while it is decreasing for firm 2. This implies that an indifferent type $\hat{\theta}$ exists so that the determination of the market share and the decision on the quality schedule are independent problems. Stole joins the conclusion that the market is segmented in a local monopoly, where the usual rationality constraint binds, and a competitive area, where the outside option is represented by the rival firm offer. Quality distortion is registered and is related to the distribution function of customers.

The real contribution of Stole[74] is, however, on vertical differentiation. In this case, given that all types of consumers are ranked equally by firms, competition would bring down prices to cost, jeopardizing the possibility of any type of price discrimination. The assumption of perfect discrimination (or delivery) in the horizontal space allows to overcome both the problem of equal rankings and of multiple dimensions of differentiation at once. In this context type dependency implies that it is no more as clear-cut which participation constraint binds. It turns out that not necessarily the firm chooses a schedule for which the constraint binds for the lowest type: a further effect of competition is to increase the quality received from the lowest type.

Champsaur-Rochet[13] analyze a slightly different situation: first firms commit to a range of qualites/product lines and then compete in prices choosing the optimal non-linear schedule. The timing of the game is related to the observation that prices decisions are more easily reversible than product lines choices. This assumption is another solution to the problem of the same ranking of types that, as recalled above, does not allow to price discriminate. In deciding their product lines firms take into account two contrasting effects: first, discrimination would require a wider range of qualities to better meet the tastes of consumers; second, however, the competitive effect between neighbouring qualities calls for a larger amount of differentiation. In the quality subgame, firms strategy are restricted to be the intervals $Q_1 = [q_1^-, q_1^+]$ and $Q_2 = [q_2^-, q_2^+]$ and the firms serve a continuous of consumers indexed in their preferences for quality by the $\left[\underline{\theta}, \overline{\theta}\right]$. The authors focus on two polar cases. First, if the qualities are not overlapping, i.e. if $q_1^+ < q_2^-$, then the only form of interaction between firms is in the determination of the indifferent consumer $\hat{\theta}$. From that, the market shares and the extreme of the quality ranges q_1^+ and q_2^- and their prices can be derived and are respectively:

$$p_1(q_1^+) = C(q_1^+) + [u_1(\hat{\theta}, q_2^-) - u_1(\hat{\theta}, q_1^+)]\frac{F(\hat{\theta})}{f(\hat{\theta})}$$

$$p_2(q_2^-) = C(q_2^-) + [u_2(\hat{\theta}, q_2^-) - u_2(\hat{\theta}, q_1^+)]\frac{1 - F(\hat{\theta})}{f(\hat{\theta})}$$

The solution implies "bunching" or in other words that a set of consumers of positive measure demands the extreme qualities and it is compatible with the case in which one of the two firms monopolize the market. Second, it is interesting the case in which qualities can overlap, i.e. when $q_2^- \leq q_1^+$. Quite intuitively, the non-linear price schedule involves marginal cost pricing for all the range of qualities supplied by both firms. The last result they obtain is to characterize the more general game in which firms do not have constraints on the choice of the product line. Under the assumption of linear utility and quadratic costs, they join the conclusion that firms make positive profits by supplying a single quality: in other terms, the differentiation incentive dominates the segmentation one. The result is robust to less strict specifications, implying that in equilibrium firms supply disjoint intervals of qualities.

In the last few years several papers adopted the Stole[74] or a strongly related framework: Valletti[77] considered discrimination over vertical preferences when there are two different types of customers, instead of a continuous of them. The model proves to be useful in the analysis of the optimal location/product design of firms, which is actually non monotonic in the heterogeneity parameter. Jensen[40] uses the same model to analyze the optimal tariff to be implemented, with a special focus on the mobile telecommunications industry. Jorge-Pires[43] compare non-linear pricing with respect to the vertical quality dimension both in case the product is delivered to consumers and when the price is determined at the mill. In the long run, delivered non-linear pricing are to be preferred for low entry costs or when customers types are not too similar.
A similar but different perspective is taken by Gal-Or[32]: she attempts to analyze the social welfare impact of non-linear pricing in oligopoly. The model is then different from the one of Stole for the way of modelling product differentiation and tastes' heterogeneity. However, the characterization of the (type dependent) individual rationality and the incentive compatibility constraint is analogous. There are n firms choosing the amount $T(q)$ to charge for a given quantity q of the brand produced. Each brand y is differentiated and customers have n indirect utility functions, so that the favourite one is defined as:

$$w(y) = \max_q \left\{\phi(q, y_i) - T_i(q)\right\} \geqq \max\left\{0, \max_q \left\{\phi(q, y_{-i}^{\max}) - T_{-i}(q)\right\}\right\} = u(y_{-i}^{\max})$$

where $\phi(q, y_i)$ is the direct utility derived by consuming variety y_i. Firms know consumers' preferences only in terms of the n-variate distribution function $F(Y)$ where Y is the vector representing all the varieties available. This representation of consumers preferences is quite general, so compatible with

different types of product differentiation analyzed before. The price to pay for generality, however, it is the impossibility of characterizing the equilibrium; however, as far as the welfare analysis is concerned, the following comparative statics results can be stated:
1. an increase in the number of firms/brands supplied enhances price competition and then causes a higher number of types of customers being served;
2. an increase in the number of firms/brands supplied lowers marginal prices for all types, provided that the distribution function is never decreasing in type.

A different modelling approach was introduced by Rochet-Stole[67]. They consider consumers whose utility for each brand is given by a deterministic common value plus a random signal, so that each consumer has a multidimensional type given by the vector $(\theta, \varepsilon_1, \ldots, \varepsilon_n)$. It turns out that for a monopolist the newly formulated problem can be solved endogenizing the stochastic participation constraint and using control theory techniques. The result is that the distortions are reduced with respect to the case of a monopolist facing deterministic participation. The intuition for the result can be traced to the trade off between lowering the surplus extracted with increased expected market participation. The relevance of the model for competitive non-linear pricing relies on the fact that it encompasses as special cases a class of models that allow to address multi-dimensional heterogeneity: between these the model used in Armstrong-Vickers ([5])-([6]), characterized by horizontal differentiation *à la Hotelling*. The interest of the model relies on the fact that it allows to provide a comparison of the effect of competition on non-linear pricing as compared to linear pricing.

Suppose that firms $i = 1, 2$ are located at the extremes of an Hotelling line and consumers have both a preference for the horizontal characteristic and elastic demand for vertical quality of each of the $j = 1...n$ products supplied. This is captured by the following utility function:

$$V(\theta, q_i) = u(\theta, q_i) - td_i(x) - T^i$$

where T^i is the amount charged by the firm, t is a measure of "choosiness" of consumers and $d_i(x)$ measures the distance of a consumer located at x from the firm, so that $d_i(x) = x$ for firm 1 and $d_i(x) = 1 - x$ for firm 2. Armstrong-Vickers show that if the market is covered, the two-part tariff:

$$T_i = t + \sum_{j=1}^{n} c_i q_i$$

is an equilibrium and the aggregate market profits are $\pi_{NLP} = t$. Less straightforward is the case of linear pricing and unfortunately it is not possible to identify the shape of the equilibrium schedule in that case; the important result, however, is that it is possible to show that the aggregate profits decrease in presence of linear pricing, or $\pi_{LP} < \pi_{NLP}$.

This, however, is not the only possibility. Analogously to the analysis of Stole[74], suppose that perfect discrimination takes place with respect to the vertical quality characteristic; firms also discriminate on the basis of location[3]. A consumer chooses firm 1 as long as:

$$v(\theta, q) + tx - p_1 \geq v(\theta, q) + t(1 - x) - p_2$$

implying that the symmetric equilibrium price schedules are:

$$p_i = \begin{cases} (1 - 2x)t & \text{if} \quad x \leq \frac{1}{2} \\ (2x - 1)t & \text{if} \quad x > \frac{1}{2} \end{cases}$$

Consumers obtain the product from the closest firm, in a socially efficient-transportation cost minimizing way. On the other hand, if firms are constrained to a uniform price, the equilibrium would imply $p_i = t$. The consequence is that equilibrium prices are lower in presence of price discrimination and so are firms' profits. The relevance of the analysis proposed is to

[3] A similar analysis is provided by Thisse-Vives[76] and Armstrong[2].

show that not always the adoption of ornate pricing practices is beneficial to firms. As in the latter example, they may face a sort of prisoners' dilemma situation: they would be better off in case commitment to non-discriminate was possible. However, the higher pricing freedom is privately profitable but damaging firms collectively: in this case competition has a beneficial effect for consumers. Corts[16] has given an interpretation of what is determining whether more price flexibility is advantaging or damaging firms. The key concept is best response symmetry or asymmetry. Suppose a submarket is "strong" for a firm if the elasticity of demand is relatively low and it "weak" if the elasticity is high. *Best response symmetry* takes place if firms evaluate the same sub-markets as being the "strong" one and the "weak" one respectively; when the ranking is different, then *best response asymmetry* takes place. The first example provided by Armstrong-Vickers ([5])-([6]) involves best response symmetry, the one of Thisse-Vives[76] best response asymmetry.

A final word deserves the case in which consumers can buy differentiated goods from different providers at the same time. In that case, Martimort-Stole[50] adapt the theory of common agency, surveyed in Section 2.1, to non-linear pricing. The firms' problem changes in a similar way as in Stole[74]: the participation constraints are now type dependent, due to the strategic effect of the competing firm. The results depend on whether common agency is intrinsic or delegated. In case consumers can either buy from both buyers or stay out of the market, the equilibrium outcome involves more distortions than in the monopoly benchmark. If they can opt to buy the product from one firm only, then the result depends on whether the goods contracted are complements or substitutes. Market participation increases when the goods are substitutes and vice versa if they are complements. Ivaldi-Martimort[39] go a step further: they consider the case of consumers in common between firms and characterized by private information in two dimensions (θ_1, θ_2). The trick to deal with this complicated case is to adopt a change of variable such that consumers' heterogeneity is captured by a single dimensional summary statistic. The model is also parametrized in a way that can be fitted to empirical data from the French energy market.

2.3 Quantity Competition

A second stream of literature considers competition in non-linear pricing models when firms compete in quantities supplied. The choice variable in these models is either quantity or the share of consumers served or the product line: the interpretation may be more convenient in one case rather than others but the principles and the results are not affected.

The first analysis of Cournot competition and firms are allowed to practice second degree price discrimination can be traced back to Oren-Smith-Wilson[59]. Consider n symmetric firms supplying an homogeneous good to a continuous of consumers' types θ. Consumers of type θ will buy the quantity $q(\theta)$ if and only if the price-quantity combination is maximizing their consumers surplus (i.e. the incentive compatibility constraint holds) and yields a non-negative consumer surplus (i.e. the individual rationality constraint holds). In equilibrium there can be two situations: the optimal policy being either serving all θs or restricting the market to the first θ_1 highest types. The main difference with the price competition approach is that the i-th firm takes into account these constraints to maximize its profits defined on the residual demand function. In other words, given the $(n-1)$ rival firms decisions, i acts as a monopolist on her own share of demand.

In the outlined framework Oren-Smith-Wilson[59] show how the problem can be expressed in several alternative ways as the choice variable, the total supply as function of type-disaggregated parameters, is differently interpreted. Firms can either choose directly the quantity supplied to each type or select the value of the last consumer's type served, as it is the case in Ireland[38].

The elasticity interpretation of pricing in the original Cournot framework can be adapted to the case of non-linear pricing case: for each level of quantity demanded q, in fact, the equilibrium mark-up ratio is equal to the inverse of the elasticity of demand evaluated at that specific point of the demand schedule times the number of oligopolistic suppliers:

$$\frac{p(q) - c(q)}{p(q)} = \frac{1}{n\varepsilon_N(q)}$$

where $p(q)$ is the price schedule while $c(q)$ is the marginal cost of supplying a given quantity while n is the number of firms and $\varepsilon_N(q)$ is the elasticity relative to a specific cumulative demand $N(p(q), q)$ up to quantity q.

The model works as well in the quality interpretation and several authors have used similar approaches to analyze the optimal product line choice by oligopolistic firms. Next then, this parallel is exploited further, after briefly outlining the theory of vertical quality differentiation.

2.3.1 Vertical Quality Differentiation

Product Differentiation is a classical topic in economics: a long time passed since Hotelling[36] and Chamberlin[12] formalized the idea that firms have incentives to strategically differentiate to soften competition and supply products with heterogeneous characteristics to prevent a perfectly competitive market outcome. A relevant distinction can be made between horizontal and vertical differentiation. Broadly speaking, *horizontal product differentiation* can be defined as when there exist no ranking between the varieties supplied: consumers do not agree in their preferences. A typical example of horizontal differentiation is the preference for objects that are identical in all respects but their colour, as for example t-shirts, pens or cars: different individuals will have a different ranking of preference between the different colours of the same object. *Vertical differentiation*, instead, applies to situations in which all consumers rank the qualities supplied in the same way. *Ceteris paribus*, very few people would opt for a Fiat 500 instead of a Ferrari F430. However, even this example can be tricky, as there might be Fiat 500 fans club members who would never exchange their car, not even for a way fancier Ferrari. A further definition might link vertical differentiation to the cost of production: according to Shaked-Sutton[70] "pure" vertical differentiation takes place when the highest quality is the only demanded even if the goods are supplied competitively at marginal cost. This definition is not immune from shortcomings either: it is however sufficient for the goals of this thesis.

Consider a qualitatively differentiated product about which, *ceteris paribus*, all consumers have the same ranking in judging the quality of varieties. Quality preferences are indexed through a parameter θ continuously distributed over an interval. All consumers prefer an high θ to a lower one. The preferences for the product of one of the two firms i can then be expressed as:

$$U(v_i, \theta) = v_i\theta - p_i$$

Firms face a cost $c_i = c \; \forall i$ to produce one unit of the good. This assumption can be easily relaxed to take into account that higher quality is more costly to supply. Assuming further that consumers are heterogeneous enough in their tastes for quality and that the market is covered, a price equilibrium exists in which the higher valuation consumers demand the high quality variety of the good while the lower valuation consumers the low quality variety. Without loss of generality, if $v_2 > v_1$ then firm 2 charges the highest price, supplies the highest possible quality in the range and gains higher profits. Firm 1 on the other hand chooses the lowest possible quality and the market outcome is maximum differentiation.

This extremely simple framework is characterized by each firm supplying only one variety of the product. However, it is often observed that firms supply an entire product line constituted of varieties of several different qualities. A large body of literature, building on the seminal contributions of Mussa-Rosen[56] and Maskin-Riley[51], has focused on the supply of products of different quality by a monopolist. The problem of quality supply can be seen in that case as one of informational differentiation. The monopolist does not have information on the valuation of consumers for quality and so tries to have them self-selected in the contract designed for them. Defining the situation in these terms, the link with non-linear pricing should be immediately clear. The product line then must be designed in a way that induces consumers not to arbitrage between the contract designed for his type and the other ones. For example, supposing that the product line is constituted of two qualities $v_2 > v_1$, and consumers can have either a high $(\overline{\theta})$ or a low valuation $(\underline{\theta})$ of quality. The high type should demand quality level 2, and not find convenient to buy quality 1; in other terms:

$$v_2\overline{\theta} - p_2 \geq v_1\overline{\theta} - p_1$$

The relation stated is just the incentive compatibility constraint. The firm will then have to maximize her profits taking into account both the incentive compatibility constraint and not only that consumers take part in the market. The usual individual rationality constraint should in fact hold for the low types: the price-quality combination they demand leaves them with a non-negative utility; in other terms:

$$v_1\underline{\theta} - p_1 \geq 0$$

Self-selection requires a strategic manipulation of quality below optimum; using the words of Mussa-Rosen: «The optimal policy "smokes out" consumer preferences, separates markets, and assigns different customer types to different varieties of goods, thereby permitting partial discrimination among consumers of varying intensities of demand». A result which is robust in both specifications is that the monopolist supplies a low quality that is under the socially desirable level. An interesting extension is proposed by Champsaur-Rochet[13]. Although the case they address is not compatible with pure vertical differentiation, they address the interesting case in which the outside option of the consumers is of a higher quality. Suppose there is a continuous of types of consumers distributed between $\left[\underline{\theta}, \overline{\theta}\right]$ and the monopolist can supply intervals of quality. The participation constraints is affected by the presence of a high quality outside option $\overline{v}$; for the generic type θ this becomes:

$$v(\theta)\theta - p(\theta) \geq \overline{v}\theta - p_{\overline{v}}$$

The implication is that in equilibrium the firm's quality range is now distorted towards the higher range of qualities. Clearly, the examples quoted involve non-linear pricing, so the further developments in oligopoly will be discussed in what follows.

Both Gal-Or[31] and De Fraja[17] develop models in which both price and quality are endogenously determined. Gal-Or[31] considers the following model: a continuous of consumers is indexed by θ representing preferences for quality. Firms decide their supply which is linked to the product line selected $q(v(\theta))$. The solution for a symmetric equilibrium allows to identify two types of consumers, receiving the following utilities:

$$u(\theta) = \begin{cases} U(0,\theta) - U(0,\hat{\theta}) & \text{if } \hat{\theta} \leq \theta < \theta^* \\ \int\limits_{\theta^*}^{\theta} U_\theta(v(t),t)dt + U(0,\theta^*) - U(0,\hat{\theta}) & \text{if } \theta^* \leq \theta < \overline{\theta} \end{cases}$$

the first being consumers "bunched" to receive the lowest quality level normalized to zero while the second group demands the higher levels of quality. If the distribution of consumers is uniform, the existence of a unique Nash equilibrium is guaranteed. The main result is that further entry encourages firms to lower the quality supplied. In other words:

$$\begin{array}{c} \frac{d\hat{\theta}}{dn} \leq 0 \\ \frac{d\theta^*}{dn} > 0 \end{array}$$

while $\frac{dv(\theta)}{dn}$ for $\theta > \theta^*$, which is the increased competition does not have an effect on the supply of higher qualities. The ability to segment the market declines with increased competition: similarly to what found by Champsaur-Rochet[13], firms need to increase the width of the product line. However, in this context this is achieved by increasing the share of "bunched" consumers served with the lowest quality. The effect is to decrease the average quality.

De Fraja[17] remarks that the previous case does not involve "pure" vertical differentiation as defined by Shaked-Sutton[70]. Instead, it is assumed that every firm can supply a finite number of specifications $i = 1..n$ whose cost is linear in the quantity demanded; not necessarily, however, the cost of a higher quality has to be superior. Having defined $g_i(\theta)$ the poorest consumer who demands quality i, it is possible to show that the price schedule is non-decreasing in quality:

$$p_i(\theta) - p_{i-1}(\theta) = \frac{v_i - v_{i-1}}{v_i v_{i-1}} \sum_{j=1}^{i+1} v_{j-1} \left[g_j(\theta) - g_{j-1}(\theta)\right]$$

As opposed to Gal-Or[31], first it is proven that the only possible equilibrium has to be symmetric; second, the equilibrium involves "pure" differentiation and non-linear price schedules; third and more importantly, firms may leave gaps in their product lines even in case the marginal cost is an increasing function of quality. The latter result is to be ascribed to the relationship between preferences and technology. In particular, this happens if the quality not supplied would not be the consumers' favourite, even in case all the superior ones were not supplied. In other words, "pure" vertical differentiation has to take place for all varieties in order to find a complete product line in equilibrium.

The latest and more general approach to vertical differentiation is introduced by Johnson-Myatt, in a series of closely related papers ([41]-[42]) based on the so called 'upgrades approach'. This new technique allows to deal both with asymmetric multi-product Cournot duopoly[41] and with symmetric Cournot n-firms competition[42]. As the framework is quite similar to De Fraja[17], non-linear pricing arise in equilibrium. The new approach allows to reformulate the problem: an '*upgrade*' is defined as the supply to the mass of customers who demands a certain quality or more. In other words, the

i-th upgrade is defined as:

$$Z_{ir} = z_{ir} + Z_{i+1r}$$

which is the demand for the i-th quality *plus* the demand for all varieties of superior quality supplied by the firm r. Despite the fact that there is not an intuitive explanation for the concept of 'upgrade' as used by the authors, reasoning in this dimension allows one to highlight many features of multi-product quality Cournot competition. In particular, in the symmetric case the Oren-Smith-Wilson[59] interpretation of non-linear pricing can be re-expressed in terms of upgrades for the case of finite product quality supply:

$$\frac{P_i(Z_i) - C_i}{P_i(Z_i)} = \frac{\xi_i(Z_i)}{n}$$

where $\xi_i(Z_i)$ is the reciprocal of the price elasticity for the upgrade i. The usual intuitive properties of Cournot equilibria apply to upgrades also in the asymmetric cases: the increase of the cost of upgrade i supplied by firm r, for example, results in a lower supply of the upgrade, in a general reduction of the output in the industry and of the profits of firm r. As every upgrade is independent of each other, reasoning in terms of upgrades allows to find existence and uniqueness results and interesting comparative statics properties by simply applying the usual Cournot logic to every single upgrade.

Returns to quality are defined by the authors as the ratio between the cost, for a given firm, of producing a certain upgrade, and the price of that upgrade: in other words, firms enjoy positive returns when $\frac{C_i}{Q_i}$ is strictly decreasing. A further feature of interest of the Johnson-Myatt approach is that it traces back the product line choices of firms to returns to quality and price sensitivity. In a symmetric equilibrium in which the marginal cost of each upgrade is positive, the lowest quality one is supplied if either there are negative returns in supply or there is a decreasing price sensitivity. The close relationship between technological and demand parameters is rather uncommon in the literature but it will play a crucial role in this thesis. In terms of equilibrium quality line, Johnson-Myatt[41] find that De Fraja[17] results hold as a particular case of their model: however, the symmetric outcome with holes is not the only possibility and asymmetric equilibrium configurations arise as a response to entry and increased competition in the market.

2.3.2 Vertical Differentiation in Two-Sided Markets

The theory of vertical differentiation and product line choice can be extended to analyze market structures in which platforms are intermediaries between two different sides of the economy. Competitive models of markets with intermediaries and network externalities usually assume that platforms are horizontally differentiated, as in Armstrong[1] and Rochet-Tirole[68]. This convenient simplifying assumption may not be suitable when considering markets characterized by vertical quality differentiation. Hermalin-Katz[44] extend the two-sided market model to analyze product line restrictions imposed on platforms supplying different qualities. Although their analysis is mainly performed in a monopolistic market, their results are robust to an extension considering an oligopolistic market structure. Suppose the platform faces demand for quality q from a group of heterogeneous agents on side 1 indexed by the parameter θ. The link between the two sides of the platform is given by agents in group 1 serving the other group of agents on side 2 whose utility depends on the quality received and quantity consumed $u(\frac{x}{\theta q(\theta)})$; their total utility is quasi-linear can be then written as:

$$U = \int_{\Theta} \int_{0}^{x(\theta)} u(\frac{z}{\theta q(\theta)}) f(\theta) dz d\theta + y$$

where $q(\theta)$ is the quality, $x(\theta)$ is the quantity demanded and y is a numeraire. However, it should be noticed that consumers of group 2 are homogeneous in their evaluation of the benefit of participating in the market. The game has three stages. In the first, the platform sets its price $p(q)$ to the group 1 of agents and an access fee h to the second group. In the second stage, group 1 selects the quality desired $q(\theta)$ and the unit price t to charge to group 2. Finally group 2 selects whether to connect to the platform and the levels of quality and consumption desired. The technical assumptions needed in order to solve the model are not very different from the one used in the standard principal-agent model and they rely on the revelation principle presented in Section 2.1. The authors focus then on three relevant situations: first, a social welfare maximizing benchmark; second, the unrestricted monopoly solution and, third, a situation in which some restriction is imposed on the range of qualities the platform can supply to group 1. The extension of the model to the case of duopoly relies on horizontal differentiation *à la Hotelling*: given th assumption on vertical differentiation, the model can be solved and the properties are similar to the monopoly benchmark. In particular, the results obtained extend to a two sided market a few well known facts: first, agents in group 1 that would buy low quality under unrestricted monopoly are now excluded from the market, the medium levels of quality supplied maybe too high as compared with the first best while the top quality maybe too low. Further, overall welfare is likely to be harmed by product line restriction.

Chapter 3

Size (*of the product*) matters...

3.1 Introduction

Walking through the shelves of local shops and supermarkets everyone notices that not all products are commercialized the same way: some of them are sold in a single size, some in different sizes, some others in packets of several units. Products sold in different sizes or packets are also often sold at a discount linked to quantity. The aim of this chapter is to propose one interpretation of why different products are commercialized differently. The answer provided relies on the pricing policy of firms and on the interaction between the demand and the supply side of the economy.

The following examples can clarify the phenomenon this chapter wants to address. The following figure is taken from a website comparing the price of goods in the four main british supermarket chains. It is clear from Figure 3.1 that all firms supply Cola in bottles of different sizes (1.25 L and 2 L bottles) and packed differently (a single 2 L bottle as opposed to a pack of 4). Moreover, as illustrated by Table 3.1 quantity discounts linked to the overall amount of Cola bought. The proportion of the discount varies slightly across different supermarkets but all chains propose it.The approach adopted in the chapter is not limited to case of products supplied in different sizes


PRICE CHECK Comparison results

Cola Bottle 1&2ltr

Product description	Tesco price	Sainsbury's price	Morrisons price	ASDA price
COCA COLA 2 LTR BOTTLE ANY	£1.49	£1.49	£1.49	£1.49
COCA COLA 4 X 2 LTR BOTTLE ANY	£4.71	£4.95	£4.95	£4.92
COCA COLA/DIET COKE BOTTLE 1.25L	£1.25	£1.25	N/F	£1.25

* All prices listed were collected between 28 April 2008 and 30 April 2008.

Figure 3.1: Products and packets of different sizes: an example. Source: Tesco.com.

but can also fit the situation of quality supply. This is the case illustrated by Figure 3.2. Broadband internet connection is supplied by several providers in many western economies. Each provider offers users a menu of different contracts; for each contract the monthly charge is linked to the quality of the connection and other ancillary services. As in the example, it is very common that the monthly charge is proportionally increasing less than the quality of the contract.

Table 3.1 Products and Packets of Different Sizes: Unit Prices

Price/Ltr	Tesco	Sainsbury's	Morrisons	ASDA
CC 1.25Ltr	1.000	1.000	N/F	1.000
CC 2 Ltr	0.745	0.745	0.745	0.745
CC 4 x 2Ltr	0.588	0.619	0.619	0.615

The examples highlight the main features of the approach adopted in this chapter. The analysis will focus on an oligopolistic model in which n firms compete by supplying an homogeneous product. The comparison between non-linear pricing and linear pricing strategies will allow to join conclusions on the size of the product that can be found on the market[1]. In

[1] Although the quality interpretation fits rather well the model described in this chapter,

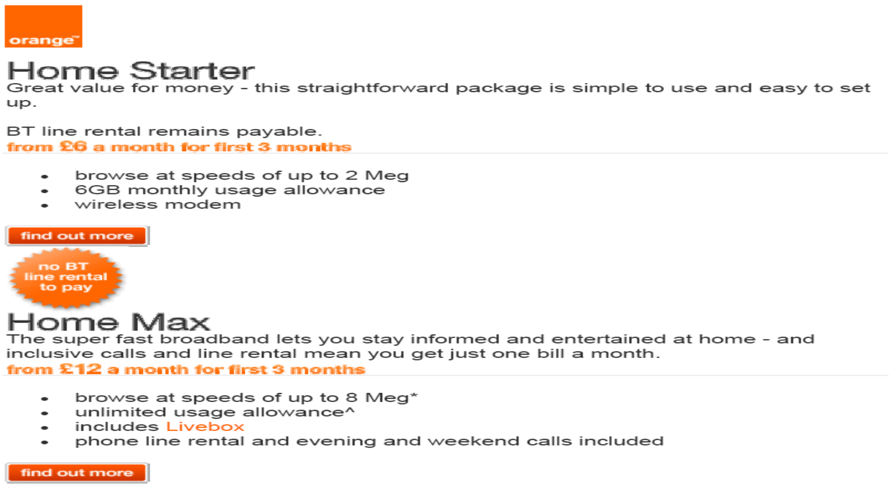

Figure 3.2: Packets of different quality: an example. Source: Orange.co.uk.

particular, the results suggest that there is an important relationship between the pricing strategy adopted by firms and the technological structure of the market. Under non-linear pricing, in fact, firms may find optimal to supply the packets in two different sizes or, in case cost savings related to supplying a larger product size are relevant enough, they may opt to supply the product only in a large size. If the strategy involves linear pricing, instead, firms may either supply the product in packets of a single unit or packets of two units or, if production costs are large enough, they may focus on bringing the product to the market only in the single unit size. The equilibria found, then, suggest that the size of products found on the market are related to the pricing strategy of firms and to the parameters of the demand and supply side. These results maybe relevant for policy purposes in cases where the regulator cares about the size of the products supplied by firms; for example, public health concerns arose recently due to the tendency of selling food and drinks in supermarkets in always larger and larger sizes.

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the analysis will stress the quantity interpretation. The next chapter, however, extends the model to focus on its implications in terms of the quality supplied.

The results of the chapter are not limited to the positive indication on which type of product is supplied. The analysis is completed by the comparison between the equilibria registered under the two pricing regimes (non-linear pricing vs. linear pricing). The results obtained are not trivial and can be linked to the literature on the topic. The extensive literature on non-linear pricing has focused on the screening role of the practice. A monopolist uses his market power and the possibility to offer more sophisticated pricing schemes to screen the tastes of consumers and, by doing that, to increase profits. It is no surprise, then, that most authors focused on the role of demand side, assuming away complications linked to different possible technological structures. The same observation applies to the more recent advances on non-linear pricing, considering oligopolistic market settings. The monopolistic results maybe confirmed or controverted when interaction between firms is taken into account. The result is related to the symmetry or asymmetry of best response schedule faced by firms: if the firms face best response asymmetry then competition might not affect the profitability of price discrimination; on the other hand in case of best response symmetry, non-linear pricing leads to lower profits as compared to linear pricing[2]. However, even in a competitive context, the relationship between the demand and the supply side in determining the shape of the pricing schedules is not a central concern. Remarkable exceptions are given by De Meza[18] and Ireland[38]. De Meza[18] shows that in duopoly economies of scale are not the only way to justify non-linear pricing: in a model where firms first choose on pricing and then they decide on output, consumers maybe favored by non-linear pricing as compared to linear. Ireland[38], on the other hand, takes another stand. In a competitive model of non-linear pricing he shows that scale or size economies are necessary to justify non-linear pricing on welfare grounds. The profit increase guaranteed to firms is more than offset by the losses

[2] On best response symmetry/asymmetry, see Corts[16], Feuerstein[28], Stole[75] and Armstrong[2].

made by consumers. The robustness of the results of Ireland[38], however, is questioned by Cheung-Wang[14]: they show that allowing for more general distribution functions, the results might be controverted. Moreover, linear pricing are not necessarily benefiting consumers because they might lead to a restriction of the packets supplied. Our approach is closely linked to Ireland and Cheung-Wang, making the results directly comparable. The main caveat is that focusing on the packets size and the technology to produce them leads to a wealth of situation, including ones in which consumers are benefiting and firms are losing under non-linear pricing.

The structure of the chapter is as follows. Section 3.2 presents the model adopted for the analysis. Section 3.3 solves the model in the two polar cases of non-linear and linear pricing. Section 3.4 enquires on the properties of the equilibria devised while Section 3.5 provides conclusive remarks.

3.2 The Model

The framework considered is based on Ireland[38] and it is adapted to take into account technological issues on the supply side. Consider n symmetric firms producing an homogeneous good that can be sold in packets of different size: for simplicity and without loss of generality, it is assumed that only one-unit packets and two-unit packets are supplied. Defining q the size and p_q the price of a packet of size q then prices are linear if $p_2 = 2p_1$; otherwise prices are non-linear. Assuming perfect rationality of consumers, under the assumptions adopted the two-unit packets are demanded if and only if $p_2 \leq 2p_1$. No arbitrage is possible.The supply side is characterized by a production technology which allows cost savings related to the size of products. An intuitive way of justifying the assumption relates to the packaging and selling costs to bring the product to the market. Suppose that the total cost is a function of three inputs:

$$TC = wl + rk + P_q$$

in which l represents labor and w is its given price, k is capital whose price is r and the size-dependent cost of packaging devices is denoted by P_q. It is clear that the packaging cost component depends on size q and it is assumed that P_q/q is weakly decreasing in q, i.e. $P_1 \geq P_2/2$. This implies that the marginal and average costs of production are constant with respect to demanded quantity, but they decline with size q. The previous discussion allows us to state Assumption 1[3]:

Assumption 1 *The production of each firm takes place according to the following cost function:*

$$C(q, x_1, x_2, Q_q) = \begin{cases} c\, D_1(x_1, x_2, Q_1) & \text{if} \quad q = 1 \\ 2\theta c\, D_2(x_1, x_2, Q_2) & \text{if} \quad q = 2 \end{cases}$$

in which $\theta \in [1/2, 1]$.

The parameter θ has an intuitive interpretation: it can be thought as a measure of the savings in packaging costs related to size, i.e.:

$$\theta = \frac{P_2/2}{P_1}$$

Two limiting cases are encompassed in this description: if P_2 is exactly the double of P_1 the model is the same one as in Ireland while if the cost P is fixed and does not depend on the size of the packet, i.e. when $P_1 = P_2$, the value of θ is equal to $1/2$.

The demand side is constituted by a continuum of consumers characterized by a type parameter x that expresses their willingness to pay for one unit of the good. Consumers are distributed according to a distribution function $f(x)$ that is assumed to be continuous, twice differentiable and with domain

[3]In the light of the previous discussion, it is worth to underline further the difference between size and scale economies. The present chapter focuses on the former and not on the latter.

$x \in [0,1]$. Each consumer can demand either nothing or the one-unit packet or the two-unit packet. According to the following utility function:

$$U(q,x) = \begin{cases} 0 & \text{if} \quad q = 0 \\ x - E(q) & \text{if} \quad q = 1 \\ bx - E(q) & \text{if} \quad q = 2 \end{cases}$$

in which $x \in [0,1]$, $q = 0,1,2$ are the units of product bought, $E(q)$ is the expenditure necessary to buy the desired packet, the marginal utility of consumption is non-increasing. This is captured by the parameter b, about which the following holds:

Assumption 2 *The marginal decrease of utility in consuming a second unit of the good is the same across all consumers so that the willingness to pay for the second unit in the package is $(b-1)x$, $b \in [1,2]$.*

All consumers aim to maximize their utility, i.e. choose q such that:

$$max\{0,\, x - p_1,\, bx - p_2\}$$

The demand faced by each firm is derived by identifying the marginal consumers, given a set of prices (p_1, p_2). Agents of type x_1 are indifferent between buying nothing or one unit if:

$$x_1 - p_1 = 0 \Leftrightarrow x_1 = p_1$$

Customers of type x_2 are indifferent between one unit or two units if:

$$(b-1)x_2 = p_2 - p_1$$

from which:

$$x_2 = \frac{p_2 - p_1}{b-1}$$

To summarize consumer's choices, then:

$$q = \begin{cases} 0 & \text{if} \quad 0 < x < x_1 \\ 1 & \text{if} \quad x_1 < x < x_2 \\ 2 & \text{if} \quad x_2 < x < 1 \end{cases}$$

from which the total demand is computed as follows:

$$D_q(x_1, x_2, Q_q) = \begin{cases} 0 & \text{if } \ 0 < x < x_1 \\ F(x_2) - F(x_1) - Q_1 & \text{if } \ x_1 < x < x_2 \\ 1 - F(x_2) - Q_2 & \text{if } \ x_2 < x < 1 \end{cases}$$

3.3 Product Evaluation, Cost Savings and the Size of a Good

Firms' problem can now be stated under both price regimes: non-linear and linear pricing. The main result of the chapter on the size of products brought to the market is then stated.

3.3.1 Non-Linear Pricing

The problem of firm i can be stated as follows:

$$\begin{aligned} \max_{p_1, p_2} \pi_i &= \sum_{j=1}^{2} [p_j D_{qj}(x_1, x_2, Q_q) - C(q, x_1, x_2, Q_q)] \\ \text{s.t.} \quad p_1 &= x_1 \\ p_2 &= (b-1)x_2 - p_1 \end{aligned}$$

Given the symmetry of firms, the first order conditions for a maximum can be written as:

$$\frac{\partial \pi_i}{\partial x_1} = \frac{1 - F(x_1)}{n} - f(x_1)(x_1 - c) = 0 \tag{3.1}$$

$$\frac{\partial \pi_i}{\partial x_2} = (b-1)\frac{1 - F(x_2)}{n} - f(x_2)[(b-1)x_2 - (2\theta - 1)c] = 0 \tag{3.2}$$

This two equations define implicitly and independently x_1^* and x_2^*, i.e. the firms' optimal choices when price discrimination is allowed.

The first order conditions can be interpreted in terms of a well known trade-off: the left part represents the additional profits on infra-marginal consumers while the right part represents the losses due to serving a marginal consumer with a higher reservation value. As opposed to standard models, in this case such a trade-off holds for all the size of packets that firms bring to the market.

3.3.2 Linear Pricing

Suppose that, for some reasons, firms can not price discriminate and

The problem of firm i can be stated as follows:

$$\begin{aligned}
\max_{p_1,p_2} \pi_i &= \sum_{j=1}^{2} [p_j D_{qj}(x_1, x_2, Q_q) - C(q, x_1, x_2, Q_q)] \\
\text{s.t.} \quad p_1 &= x_1 \\
p_2 &= (b-1)x_2 - p_1 \\
p_2 &= 2p_1
\end{aligned}$$

Given the symmetry of firms, maximizing with respect to x_2 yields the following first order condition:

$$\begin{aligned}
&-(b-1)f[(b-1)x_2][(b-1)x_2 - c] - f(x_2)[(b-1)x_2 - (2\theta - 1)c] + \\
&+(b-1)\frac{2 - F(x_2) - F[(b-1)x_2]}{n} = 0 \qquad (3.3)
\end{aligned}$$

Firms face the inframarginal gains-marginal loss trade-off but are further constrained by the linearity of their price schedule. The implication is that they have one less degree of freedom which is reflected in the first order condition. The latter along with the condition $x_1 = (b-1)x_2$ are denoted by x_1' and x_2' and represent the firms' optimal choices under linear pricing.

Finally, two aspects should be underlined. First, the two problems just presented are clearly related. Firms' choices under linear pricing can be seen as a special case of their choice under non-linear pricing once taken into account the further constraint. This is reflected in the following relationship between the first order conditions. Express (3.1), (3.2) and (3.3) as functions of x as follows:

$$FOC_{NLP1}(x) = 0$$
$$FOC_{NLP2}(x) = 0$$
$$FOC_{LP}(x) = 0$$

The first order conditions, then, are linked by the following relation:

$$(b-1)FOC_{NLP1}[(b-1)x] + FOC_{NLP2}(x) = FOC_{LP}(x) \tag{3.4}$$

Second, sufficient conditions for a maximum should be checked under both price regimes. This often overlooked aspect is important in the light of Cheung-Wang[14]: the authors show how a deeper analysis of distribution functions that satisfy second order conditions allows to obtain more general results on non-linear pricing. A few remarks on sufficiency conditions are reported in the Appendix.

3.3.3 The Size of a Product

If a unique non-linear pricing equilibrium exists, it is described by (3.1)-(3.2). A well-behaved equilibrium in this context has the following characteristics:

$$c < x_1^* < x_2^* < 1$$

or, in other words, the market is segmented between consumers who choose the one unit packets and others who choose the two unit packets. Proposition 1 establishes when this is the case. Before that, however, it is necessary to introduce a further piece of notation. Define the following expression as:

$$\psi(b) = 1 - \frac{2-b}{2c}$$

Moreover, define also:

$$\vartheta(b) = \frac{b}{2} + \frac{1 - F\left(\frac{2\theta-1}{b-1}c\right)}{f\left(\frac{2\theta-1}{b-1}c\right)} \frac{(2-b)(b-1)}{2}$$

The following result on the size of products supplied under non-linear pricing can be now stated.

Proposition 1 *(i) Suppose $\theta \geq \vartheta(b)$, then a non-linear pricing equilibrium in which $c < x_1^* < x_2^* < 1$ is the outcome of firms' profit maximization problem. (ii) If $\theta \leq \psi(b)$, then the non-linear pricing equilibrium is characterized by $c < x_1^{**} \equiv x_2^{**} < 1$.*

The proposition provides sufficient conditions on the parameters to register a well-behaved equilibrium. In that case both one and two unit packets are supplied, which is exactly when the inequality $\theta \geq \vartheta(b)$ holds. However, the proposition is also providing a sufficient condition under which a corner solution is found: if $\theta \leq \psi(b)$, then all firms will supply only two-unit packets and not the one-unit ones.

The intuition for the result in Proposition 1 is that firms in equilibrium choose to supply both sizes packets (one and two units) when the effect of size economies (θ) is not too intense relative to the consumers' valuation of a second unit of product (b). In other words, when the cost savings related to size economies are relatively important, firms find it optimal to supply only two-units packets.

The parameter space ($bO\theta$) results then split into several areas by the relations derived: non-linear pricing equilibria with different properties will take place for different combinations of parameters. It is not possible however to completely identify equilibria. For example, it is *a priori* not possible to predict what characteristics the equilibria will have when the parameters satisfy:

$$\psi(b) < \theta < \vartheta(b)$$

The linear pricing equilibrium also deserves an accurate analysis. A linear pricing equilibrium is described by (3.3) and $x'_1 = (b-1)x'_2$. A well-behaved linear pricing equilibrium requires:

$$c < x'_1 < x'_2 < 1$$

As pointed out by Cheung-Wang[14] this needs not always to be the case. The same applies to the model presented here. Define the following expression as:

$$\phi(b) = \frac{nc(b-1)f(b-1) + (b-1)^2 f(b-1)[1-F(b-1)]+}{2f(1)\{nc+(b-1)[1-F(b-1)]\}}$$

$$\frac{-n[(b-1)^2 f(b-1) - F(1)]}{2f(1)\{nc+(b-1)[1-F(b-1)]\}} + \frac{1}{2}$$

The function $\phi(b)$ determines the size of products supplied under linear pricing:

Proposition 2 *Suppose $\theta > \phi(b)$, then the linear pricing equilibrium is characterized by $c < x'_1 < x'_2 < 1$. If $\theta \leq \phi(b)$, then the linear pricing equilibrium is characterized by $c < x''_1 < x''_2 = 1$.*

Proposition 2 states that a well behaved equilibrium in which firms supply both one and two unit packets is found if $\theta > \phi(b)$. A different scenario, however, takes place if $\theta \leq \phi(b)$: only one-unit packets, then, can be found on the market. The function $\phi(b)$ can be reformulated in terms of the marginal cost of production c to allow a more intuitive interpretation of the result of Proposition 2[4]:

$$c < c^* = \frac{(b-1)^2 f(b-1) - f(1)}{(b-1)f(b-1) - (2\theta-1)f(1)} - \frac{(b-1)[1-F(b-1)]}{n}$$

[4]An analogous result is obtained by Cheung-Wang[14] in the context of the original model.

In other words, when firms choose linear pricing there exist a threshold value c^* of the marginal cost over which the firms do not find profitable to produce and sell two units packets.

The results obtained identify two regions in which the parameters space is split:

$$\begin{cases} \text{if} \quad \theta \leq \phi(b): \quad c \geq c^* \quad \Rightarrow \quad q = 1 \text{ only} \\ \text{if} \quad \theta > \phi(b): \quad c < c^* \quad \Rightarrow \quad q = 1,\, 2 \end{cases}$$

The intuition for the results presented is clear using the latest interpretation of $\phi(b)$: under linear pricing a threshold value for the unit cost exists below which firms find it optimal to supply both one and two unit packets and above which only one unit packets are supplied.

3.3.4 Size and Pricing Regime

The results in the previous section have shown how the size of products supplied by firms may vary depending on the relevant characteristics of the demand and supply parameters and the pricing regime chosen by firms. It is then possible to state the conditions under which firms provide products under different price regimes. This point is well illustrated by taking a linear demand function. In this case, the condition $\theta \gtreqless \phi(b)$ in Section 3.3.3 can be expressed as:

$$\theta \gtreqless \frac{nc(b-1) + (b-1)^2(2-b) + nb(2-b)}{2[nc + (b-1)(2-b)]} + \frac{1}{2}$$

As Figure 3.3 witnesses for plausible combinations of c and n, the purple function $\theta = \phi(b)$ is downward sloping.

Even simpler is the case of non-linear pricing. If the distribution is uniform, the relation $\theta \geq \vartheta(b)$ becomes:

$$\theta \geq \frac{b^2 - 4b + bc - 2c + 2}{2(cb - 2c - 1)}$$

The relation $\theta \leq \psi(b)$ does not change and it is clearly an increasing function of b. The functions are depicted in Figure 3.3 in yellow and green respectively.

Proposition 1 and Proposition 2 provide results about the size of products supplied under non-linear and linear pricing. In this context, they give rise to four types of equilibrium regions characterized by different combinations of products sizes and prices depending on the price regime.

Table 3.2 Parameters and Types of Equilibria

	$\theta \leq \phi(b)$	$\theta > \phi(b)$
$\theta \geq \vartheta(b)$	Type 1	Type 2
$\theta \leq \psi(b)$	Type 3	Type 4

Type1 equilibria is the parameter subspace for which firms supply the product in different sizes under non-linear pricing while under linear they commercialize the product in only one unit size. The characteristics of this equilibria were also found by Cheung-Wang[14] of which this region of parameter constitutes a generalization. It is located in the north-western part of Figure 3.3. Type 2 equilibria are characterized by the product being supplied in packets of different sizes, no matter the pricing regime. This is the generalization of the well behaved type of setting considered by Ireland[38] and can be graphically identified in the small triangle in the north-east of Figure 3.3. Type 3 equilibria are rather particular: under both price regimes a corner solution takes place. Under non-linear pricing firms focus on supplying a large size product while under linear pricing commercialize the product in a single unit. This happens in the south-eastern part of Figure 3.3. In Type 4 equilibria firms still supply only product in large size packets but they supply two different sizes of product under linear pricing. In graphical terms, it can be identified in the eastern part of Figure 3.3.

Figure 3.3 illustrates the space of the parameters of the model $bO\theta$ for $n = 3$ and $c = 0.25$ and the partition deriving from results of Proposition 1 and Proposition 2. The existence of a 'cone' of '*a priori*' non identifiable equilibria can be noticed between the yellow and green schedules.

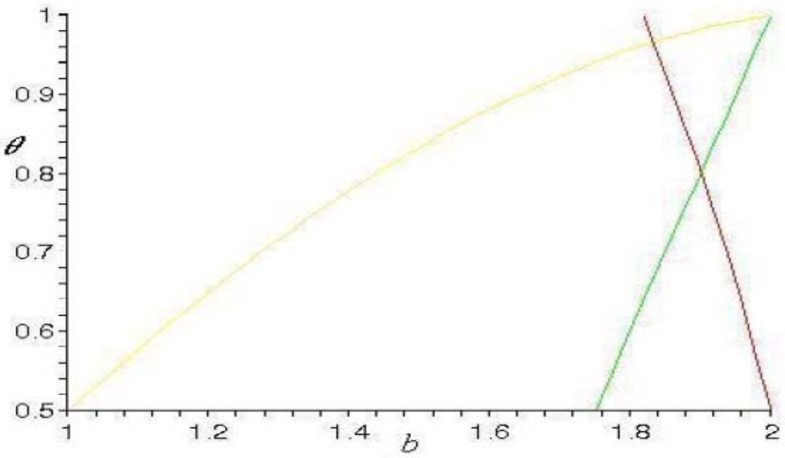

Figure 3.3: Equilibrium Regions for $n = 3$ and $c = 0.25$.

3.4 Output, Profit and Welfare Analysis

In this section, the analysis of the model's results is presented. Not all types of equilibrium regions are analyzed completely and independently; the most important results, instead, are outlined in each subsection. The complete characterization of all equilibrium regions can be found in Appendix 3.6.

3.4.1 Generalizing Cheung-Wang

Cheung-Wang[14] pointed out that firms may not always want to supply both packets under linear pricing. When the parameters of demand and cost function are such that under linear pricing it is optimal to supply only one-unit packets, then firms have profit advantages to practice non-linear prices. Furthermore this also drives to a Pareto-superior equilibrium: both firms and consumers are better off in the latest situation. These features also characterize Type 1 equilibrium.

Proposition 3 states the relations between non-linear and linear pricing equilibrium:

Proposition 3 *Equilibria of Type 1, when $\theta > \psi(b)$ and $\theta \leq \phi(b)$, are characterized by:*

1.
$$x_1^* \equiv x_1'' \ \forall x \in [0,1] \tag{3.5}$$

2.
$$Q^{NLP} \geq Q^{LP} \ \forall x \in [0,1]$$

3.
$$\pi^{NLP} \geq \pi^{LP} \ \forall x \in [0,1]$$

4.
$$CS^{NLP} \geq CS^{LP} \ \forall x \in [0,1]$$

This is a general and strong result: according to (3.5) the marginal customer choosing to consume one unit is the same under non-linear and linear pricing. This obviously implies that the total output is always larger under non-linear pricing, under which both one and two unit packets are supplied. Moreover and more importantly, not only firms' profits but also consumers' surplus are always higher under non-linear pricing in this case.
These conclusions are the exact generalization of Proposition 6 in Cheung-Wang[14], that proves to be robust to the extension presented, featuring size economies. If price-discrimination allows to sell products in different sizes, then it has overall positive welfare effects. The result stated parallels a very well known result concerning third degree price discrimination: this practice can be welfare enhancing in situations in which it allows to serve a share of demand that would stay out of the market if a uniform price were practiced.

3.4.2 Are Ireland's Results Robust?

Equilibria of Type 2 are a generalization of Ireland's benchmark: his analysis is encompassed as a special case $\theta = 1$. Under both pricing regimes all firms supply both the one and the two unit packets (as both $\theta \geq \vartheta(b)$ and $\theta > \phi(b)$ hold). This allows an important comparison with the results of Ireland[38] and a deeper analysis of the role played by size economies in this particular model and, more generally, with respect to the practice of non-linear pricing. The equilibrium expressions for the non-linear and the linear pricing case are summarized in Appendix 3.6 while the main results regarding this case are stated in the following:

Proposition 4 *In Type 2 equilibrium with linear demand: (i) a larger share of customers is served with one-unit packets under linear pricing while under non-linear pricing a larger share of two-unit packets is supplied; (ii) the total output is the same under both pricing regimes; (iii) firms' profits are always larger under non-linear pricing.*

Two observations about the above results are in order. First, since the total output is constant in both equilibria, the output of each firm should be as well, i.e. $\Delta Q_i = 0$. Now, decomposing the output change in its components:

$$\Delta Q_i = 2\Delta(\frac{1 - x_2}{n}) + \Delta(\frac{x_2 - x_1}{n}) = 0$$

it can be noticed that not only the two components must have opposite signs [which was known from Point (i)] but the increase in the share of consumers served with one-unit packets under linear prices has to be twice as big as the decrease in the share of consumers buying two-units packets once firms switch from non-linear to linear pricing.

Second, the results of Ireland original model about economy's total output and firms' profits are robust and hold in the case production displays size economies: output is constant and profits are always higher under non-linear pricing. This property is quite relevant since it is driving Ireland[38] results in the benchmark model: non-linear pricing is welfare dominated since, in the linear demand case, it does not provide an output expansion effect.

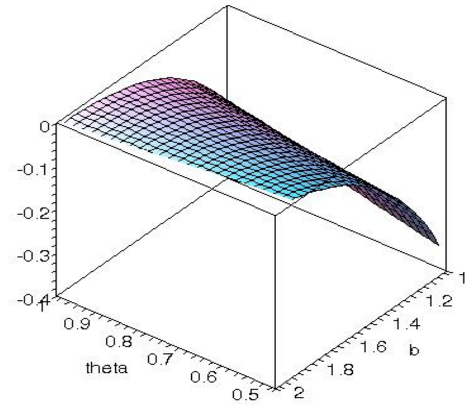

Figure 3.4: Consumers' Surplus Differential in Type 2 Equilibrium.

There are no immediate analytical conclusions that can be derived for what regards the comparison between consumers' surplus and social welfare under non-linear and linear pricing. It is possible, however, to get insights from simulation evidence. Consider first the differential between consumers' surplus in the two situations(non-linear vs. linear):

$$\Delta CS = CS^{*} - CS' = \frac{[b - 2 + 2cn(\theta - 1)][(2 - b)(1 + 2n) + 2cn(\theta - 1)]}{2b(n + 1)^2}$$

A priori the expression can not be signed. Fixing the values of n and c at plausible levels, though, the relation ΔCS can be interpreted as a function of θ and b. Figure 3.4 illustrates the point: the chosen parameters are $n = 3$ and $c = 0.25$.

Only negative values of the function are represented in the graph while positive combinations are left blank. Inspection of the relevant region, the eastern, confirms that the function is negative for all combinations of parameters. This leads to the following remark:

Remark 1 *Ireland's conclusion that consumers are better-off if non-linear prices are prohibited is robust to the extension of the model allowing for size economies.*

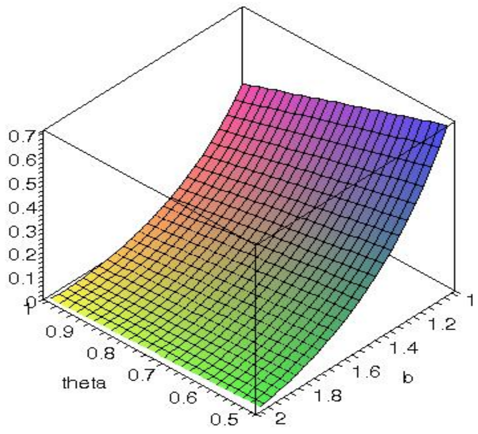

Figure 3.5: Social Welfare Differential in Type 2 Equilibria.

The same, though, does not hold for social welfare: as Figure 3.5 makes clear, the gains in profits under non-linear pricing more than compensate the losses suffered by consumers and ΔW results to be positive in the relevant region defining Type 2 equilibria.

The intuition for this result, that is in contrast with Ireland's conclusions, is that non-linear pricing imposes no restrictions on firms, allowing them to be more flexible and effective in taking advantage of the cost savings deriving from size economies.

Remark 2 *In presence of size economies on the supply side, the higher flexibility allowed by non-linear pricing implies a gain in efficiency which more than offset the losses imposed on consumers.*

In conclusion, when firms find convenient to supply the product in different sizes no matter the price regime, consumers always result damaged while overall welfare may be enhanced as non-linear pricing allow more flexibility in taking advantage of economies related to the size of the product.

3.4.3 Output Effects

As underlined, in Type 1 equilibrium, output expansion under non-linear pricing implies a welfare gain. A similar result is obtained in Type 3 equilibrium, however the intuition is quite different. If $\theta \leq \psi(b)$ then only two-unit packets of the good are sold under non-linear pricing while as $\theta \leq \phi(b)$ only one-unit packets are sold under linear pricing. This type of equilibrium is obviously quite peculiar as the pricing regimes drive to corner solution which imply the presence on the market of only one packet of goods: either large or small. The following result is obtained:

Proposition 5 *Assuming the demand function is linear, in Type 3 equilibrium: (i) the share of consumers served with two units under non-linear pricing is larger than the share supplied with one unit packets under linear pricing; (ii) total output is larger under non-linear pricing.*

The result can be contrasted with the received literature: while output expansion under non-linear pricing is often achieved by expanding the share of customers served, as illustrated for example by Type 1 equilibrium, in this case output increases despite firms are serving an identical share of demand under both regimes. To summarize:

Remark 3 *In Type 3 equilibria the share of consumers served remains constant while the output is expanded under non-linear pricing.*

The discussion of the profit, consumers' surplus and welfare implications of this result is delayed to the next section.

Turning to Type 4 equilibrium, one more interesting result is available regarding the output and the share of consumers' served. If $\theta \leq \psi(b)$ then only two-unit packets of the good are supplied under non-linear pricing while as $\theta > \phi(b)$ then both one and two-unit packets are sold under linear pricing. The following results is obtained:

Proposition 6 *In Type 4 equilibrium with linear demand: (i) a larger share of customers is served with two-unit packets under non-linear pricing than under linear; (ii) total output is larger under non-linear pricing.*

The striking feature of this result is that, despite the fact that firms supply both the one and two unit packets under linear-pricing, total output is larger under non-linear pricing. This is due to the fact that a larger share of two-unit packets is supplied under non-linear pricing. This effect is not compensated by the one-unit packets that are offered only under linear pricing. The price flexibility allowed by non-linear pricing and the more extreme demand and supply conditions encourage firms to specialize on the two unit packets and this has a positive effect on overall output.

3.4.4 A Prisoner's Dilemma?

Type 3 equilibria are interesting for the peculiar configuration of packets supplied, for the output results but also for the profit effects. In the previous section it was shown that if firms supply two unit packets under non-linear pricing while only one unit under linear pricing then overall output is expanded. However, this does not imply that the profits of firms are necessarily larger under non-linear pricing. To illustrate the point, as in Section 3.4.2, the strategy of fixing plausible values of the parameters c and n is adopted. Figure 3.5 represents $\Delta\pi$, the differential between profits under non-linear and linear pricing. The relevant region for this case is the southern part: for $n = 3$ the function is mainly positive, apart for the half moon shaped region in the south-eastern part of the figure, representing rather extreme combinations of parameters. The negative area expands as n increases[5]: a general conclusion, then, on the profit effects can not be reached.

The uncertainty over the profit effects of non-linear pricing, however,

[5] The results of simulations for different values of n and c are available upon request from the author.

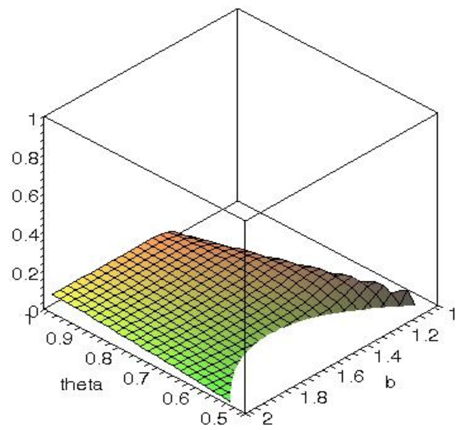

Figure 3.6: Profit Differential in Type 3 Equilibria.

highlights a very interesting mechanism related to the oligopolistic nature of the model. The model encompasses a monopolistic market structure. The only profit maximizing firm should always be able to do at least as well using non-linear pricing: that strategy, in fact, encompasses linear pricing in the limit. This needs not to be the case as the number of firms increases. The reason why non-linear pricing may be dominated by linear pricing is related to very last nature of firms' maximizand function: according to Bergstrom-Varian[8] this is a combination of aggregate consumer surplus and aggregate profits. Following that approach, Ireland[38] shows that the function firms maximize, $G(Y)$ can be written as:

$$G(Y) = \frac{(n-1)CS + n\pi}{n} \tag{3.6}$$

This specification, *mutatis mutandis*[6], is robust to the extended version of the model considered here. Ireland[38] insight is that, as $G(Y)$ is maximized freely in a non-linear pricing equilibrium and $G(Y)$ is maximized under

[6] The expressions of aggregate consumers surplus and profit are different in this case: this does not affect the validity of the analysis.

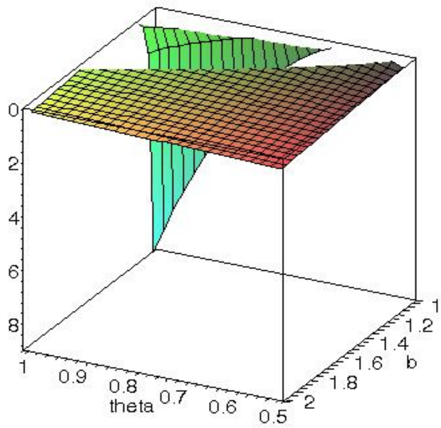

Figure 3.7: Consumers' Surplus Differential in Equilibria of Type 3.

constraints in the linear pricing equilibrium, then necessarily the following relation holds:

$$G(Y^{NLP}) > G(Y^{LP})$$

assuming the constraint is strictly binding. As the consumer surplus is always higher in a linear pricing equilibrium, equation (3.6) implies that profit must be higher in the non-linear pricing one. As shown by Figure 3.7, in the case analyzed here consumers surplus is always *higher* in a non-linear pricing equilibrium, so the relation between profits π^* and π' can go either way, without this fact compromising the logical consistency of the model.

The intuition behind this result is that strategic interaction may determine all firms ending up in a sub-optimal outcome: this is due to the externality they exercise on each other when maximizing their profits. The results obtained imply that in case production is characterized by size economies and firms supply only one type of packet under each price regime, firms may face a prisoners' dilemma being better off in case they can commit not to use non-linear pricing.

3.4.5 Welfare Effects

A few comments deserve the overall welfare results of the analysis conducted. Suppose the regulator assigns the same weight to firms' profits than to the consumers' surplus. In all the equilibrium types identified is then possible to join conclusions on the overall welfare effect of non-linear pricing as opposed to linear pricing. Although not always the analytical results are conclusive, one feature of allowing production to display economies related to the size of the product is that these might help non-linear pricing to be welfare enhancing. Under non-linear pricing, in fact, firms are more efficient in taking advantage of the characteristics of the supply side and this has positive effects that are reflected in the social welfare as a whole. As far as Type 1 and Type 4 equilibria are concerned, welfare enhancement is linked to a positive effect of non-linear pricing on both profits and consumers' surplus, allowing to identify a Pareto superior equilibrium. Things are not quite as straightforward in case of Type 2 and Type 3 equilibria. In Type 2 equilibria, however, the effect of size economies allows firms' gains in profits to more than compensate consumers' losses under non-linear pricing. The opposite happens in Type 3 equilibria when possible firms' losses are compensated by a positive effects on the consumers' surplus. The conclusion of this analysis is then that non-linear pricing is very likely to improve overall welfare unless products would be supplied in several sizes also under linear pricing. In that case, non-linear pricing can be still welfare enhancing but only in case economies of size are important enough.

3.5 Concluding Remarks

This chapter provided a theoretical analysis of the size of the products that firms propose to consumers on the market. The interpretation put forward relies on two main elements: the first is the pricing regime and the second is the role of the economies related to the size of the product.

If firms can price non-linearly bundles of different size, then either both the small and the large size packets or only a very large size can be found on the market. Whether the firm focuses only on the production of the large packets or not is linked to the consumers' evaluation of the good and to the extent of cost savings linked to the production of the larger size.

On the other hand, if firms stick to linear pricing, they are either selling the product in a single unit or in packets of both sizes. Firms are focusing on production of only one unit of the good when the unit production costs are relatively high.

These results allow to identify four possible type of market outcomes, depending on whether firms are pricing linearly or non-linearly. In Type 1 equilibria firms supply only packets of one unit under linear pricing while both packets when non-linear pricing is the strategy adopted. In Type 2 equilibria both sizes of product are available on the market, no matter the pricing regime. In Type 3 equilibria, firms only supply the large size under non-linear pricing while they limit themselves to produce the small size under linear pricing. In Type 4 equilibria, finally, firms supply the large size only under non-linear pricing while both sizes under linear pricing. The analysis of the property of the four equilibrium types provides interesting results on pricing, output, profits, consumers' surplus and social welfare. The main conclusions can be summarized in the following table:

Table 3.3 Comparison of Non-Linear Pricing to Linear Pricing in different types of equilibria

	ΔQ	$\Delta\Pi$	ΔCS	ΔW
Type 1	+	+	+	+
Type 2	=	+	-	+
Type 3	+	+/-	+	+
Type 4	+	+	+	+

It is clear from Table 3.3 that when firm do not supply the large unit packets under linear pricing, consumers are always worse off. This is linked to the overall output restrictions taking place under linear pricing in this case. A noticeable result is that in Type 3 equilibria output increases despite being constant the share of consumers served and participating in the market. An interesting feature displayed by the same region is that for some parameters' combinations profits may be lower under non-linear pricing, despite linear pricing being a special case of the former. The result is linked to the existence of the oligopoly externality: the results highlights how an oligopoly externality is at work also in a Cournot-like model of non-linear pricing. Finally, considering the possibility that firms enjoy economies related to the size of the products produced highlights that non-linear pricing is usually welfare enhancing. Moreover, in most types of equilibria discussed non-linear pricing is likely to be preferred by all agents involved in the economy.

The chapter proposes one of the first analysis of the interaction between the demand and technology sides of the market under non-linear pricing and derives the recalled results. However, this contribution might be seen as a first step in the study of what determines the size of the products that arrive on the market. Firstly, the analysis proposed is confined to the case of a small and a large size packets: the model, however, is easily extended to encompass the case of a generic number of packets of different sizes being sold. Secondly, the analysis has very strong empirical predictions which maybe worth to be addressed by gathering suitable data on the phenomenon. Thirdly, an interesting result is that the oligopolistic externality is present in the model and this may see firms ending up with a lower profits when they have more pricing flexibility. It would be desirable, in a later stage to characterize the complete payoff matrix of the game faced by firms. This would allow to verify whether and under which conditions firms face a prisoners' dilemma type of situation: both would like to commit to linear pricing but they end up in a dominated non-linear pricing equilibrium. Finally, it is often observed that the same product is sold both in packets of different sizes and in a larger packet of one size too. One example is provided by beer or crisps: both a pack with several cans or small packets and a large bottle or packet are often sold at the same time. This type of situation is not encompassed in our model but it would be interesting to be tackled in future research.

3.6 Appendix: All Equilibrium Regions

This appendix presents the equilibrium expressions identifying all equilibrium regions discussed in the chapter.

The first equilibrium configuration is characterized by both packets supplied under non-linear pricing (as $\theta \geq \vartheta(b)$) and only the one-unit packets under linear pricing (as $\theta \leq \phi(b)$): it constitutes the generalization of Type 1 equilibrium of Cheung-Wang[14]. The equilibrium is described by the following tables.
Table 4 reports the equilibrium values when firms practice non-linear pricing:

Table 3.4 Non-Linear Pricing Equilibrium of Type 1

x_1^*	$\frac{1+cn}{1+n}$
x_2^*	$\frac{(b-1)+nc(2\theta-1)}{(b-1)(n+1)}$
Q^*	$\frac{n\{2[(b-1)-c(\theta-1)]-cb\}}{(b-1)(n+1)}$
π_i^*	$\frac{b^2-b+bc^2-4\theta c[(b-1)+(1-\theta)c]}{(n+1)^2(b-1)}$
CS^*	$\frac{n^2\{b^2-b+bc^2-4\theta c[(b-1)+(1-\theta)c]\}}{2(n+1)^2(b-1)}$

Table 5 reports the equilibrium variables in case linear pricing are chosen by all firms:

Table3. 5 Linear Pricing Equilibrium of Type 1

x_1''	$\frac{1+cn}{n+1}$
x_2''	$\nexists$
Q''	$\frac{n(1-c)}{(n+1)}$
π_i''	$\frac{(1-c)^2}{(n+1)^2}$
CS''	$\frac{n^2(1-c)^2}{2(n+1)^2}$

The second type of equilibrium is a generalization of Ireland's benchmark case: his analysis is encompassed as a special case $\theta = 1$. Under both pricing regimes all firms supply both the one and the two unit packets (as both $\theta \geq \vartheta(b)$ and $\theta > \phi(b)$ hold).

The equilibrium expressions for the non-linear and the linear pricing case are summarized in the following tables:

Table 3.6 Non-Linear Pricing Equilibrium of Type 2

x_1^*	$\frac{1+cn}{1+n}$
x_2^*	$\frac{(b-1)+nc(2\theta-1)}{(b-1)(n+1)}$
Q^*	$\frac{n\{2[(b-1)-c(\theta-1)]-cb\}}{(b-1)(n+1)}$
π_i^*	$\frac{b^2-b+bc^2-4\theta c[(b-1)+(1-\theta)c]}{(n+1)^2(b-1)}$
CS^*	$\frac{n^2\{b^2-b+bc^2-4\theta c[(b-1)+(1-\theta)c]\}}{2(n+1)^2(b-1)}$

Table 3.7 Linear Pricing Equilibrium of Type 2

x_1'	$\frac{2(b-1)+cn(b+2\theta-2)}{b(n+1)}$
x_2'	$\frac{2(b-1)+cn(b+2\theta-2)}{b(b-1)(n+1)}$
Q'	$\frac{n\{2[(b-1)-c(\theta-1)]-cb\}}{(b-1)(n+1)}$
π_i'	$\frac{4c+4cn+4n-8bn-6cb+2cb^2+4b^2n-6cbn+2cb^2n+2bc^2\theta^2n}{b(b-1)n(n+1)^2}$ $\frac{-2b^2c\theta-4nc\theta-4c\theta+10nbc\theta+6bc\theta-6nb^2c\theta+2n^2bc^2\theta^2}{b(b-1)n(n+1)^2}$ $\frac{-n^2b^2c^2\theta^2-2c^2bn^2\theta+c^2n^2b^2\theta-2c^2bn\theta+c^2b^2n\theta}{b(b-1)n(n+1)^2}$
CS'	$\frac{b^3+2b^3n+b^3n^2-10b^2n-b^2n^2-5b^2+b^2c^2n^2\theta^2+}{2b(b-1)(n+1)^2}$ $\frac{-4b^2n^2c\theta+16bn+8b+4bcn^2\theta-8n-4}{2b(b-1)(n+1)^2}$

Type 3 equilibrium is original and peculiar at the same time: in both linear and non-linear pricing, under the given combinations of the relevant parameters, firms do not find it profitable to supply both packets. In a sense, no price discrimination exists in equilibrium under either one price regime or the other; the shape of the equilibrium is nevertheless determined by the degree of price freedom firms enjoys.

Only two-units packets are on the market under non-linear pricing as $\theta \leq \psi(b)$ and as the following table describing this equilibrium makes clear:

Table 3.8 Non-Linear Pricing Equilibrium of Type 3

x_1^{**}	$\nexists$
x_2^{**}	$\frac{(b-1)+nc(2\theta-1)}{(b-1)(n+1)}$
Q^{**}	$\frac{2n[(b-1)-(2\theta-1)c]}{(b-1)(n+1)}$
π_i^{**}	$\frac{[(b-1)-(2\theta-1)c](b^2-b-bcn-2\theta cb+2\theta c+2\theta cn)}{(n+1)^2(b-1)^2}$
CS^{**}	$\frac{bn^2[(b-1)-(2\theta-1)c]^2}{2(n+1)^2(b-1)^2}$

whereas, only one-unit packets are offered under linear pricing as and as it can be seen in the table below.

Table 3.9 Linear Pricing Equilibrium of Type 3

x_1''	$\frac{1+cn}{n+1}$
x_2''	$\nexists$
Q''	$\frac{n(1-c)}{(n+1)}$
π_i''	$\frac{(1-c)^2}{(n+1)^2}$
CS''	$\frac{n^2(1-c)^2}{2(n+1)^2}$

In Type 4 equilibrium firms supply only two-unit packets under non-linear pricing since the condition $\theta \leq \psi(b)$ holds. Under linear pricing, nevertheless both one and two-unit packets are supplied as the relation $\theta > \phi(b)$ is verified.

The expressions characterizing the non-linear pricing equilibrium are summarized in the following table:

Table 3.10 Non-Linear Pricing Equilibrium of Type 4

x_1^{**}	$\nexists$
x_2^{**}	$\frac{(b-1)+nc(2\theta-1)}{(b-1)(n+1)}$
Q^{**}	$\frac{2n[(b-1)-(2\theta-1)c]}{(b-1)(n+1)}$
π_i^{**}	$\frac{[(b-1)-(2\theta-1)c](b^2-b-bcn-2\theta cb+2\theta c+2\theta cn)}{(n+1)^2(b-1)^2}$
CS^{**}	$\frac{bn^2[(b-1)-(2\theta-1)c]^2}{2(n+1)^2(b-1)^2}$

The equilibrium when linear prices are practiced is characterized by the expressions reported in Table 3.11:

Table 3.11 Linear Pricing Equilibrium of Type 4

x_1'	$\frac{2(b-1)+cn(b+2\theta-2)}{b(n+1)}$
x_2'	$\frac{2(b-1)+cn(b+2\theta-2)}{b(b-1)(n+1)}$
Q'	$\frac{n\{2[(b-1)-c(\theta-1)]-cb\}}{(b-1)(n+1)}$
π_i'	$\frac{4c+4cn+4n-8bn-6cb+2cb^2+4b^2n-6cbn+2cb^2n+2bc^2\theta^2n}{b(b-1)n(n+1)^2}$ $\frac{-2b^2c\theta-4nc\theta-4c\theta+10nbc\theta+6bc\theta-6nb^2c\theta+2n^2bc^2\theta^2}{b(b-1)n(n+1)^2}$ $\frac{-n^2b^2c^2\theta^2-2c^2bn^2\theta+c^2n^2b^2\theta-2c^2bn\theta+c^2b^2n\theta}{b(b-1)n(n+1)^2}$
CS'	$\frac{b^3+2b^3n+b^3n^2-10b^2n-b^2n^2-5b^2+b^2c^2n^2\theta^2+}{2b(b-1)(n+1)^2}$ $\frac{-4b^2n^2c\theta+16bn+8b+4bcn^2\theta-8n-4}{2b(b-1)(n+1)^2}$

3.7 Appendix: Proofs

This appendix contains the proofs of Proposition 1 to Proposition 6.

Proof of Proposition 1 It is firstly established that firms do not price below marginal cost. To prove that $x_1^* > c$ and $x_2^* > \frac{(2\theta-1)c}{(b-1)}$ notice that x_1^* and x_2^* are defined respectively by $FOC_{NLP1}(x) = 0$ and $FOC_{NLP2}(x) = 0$. If second order conditions are met, (3.1)-(3.2) are also monotonically decreasing in x. Now, since $FOC_{NLP1}(c) = \frac{1-F(c)}{n} > 0$ and $FOC_{NLP2}(\frac{(2\theta-1)c}{b-1}) = \frac{1-F[\frac{(2\theta-1)c}{(b-1)}]}{n} > 0$, the claim is verified.
Part (i) has then to be proven: if $\theta \geq \vartheta(b)$ then $x_1^* < x_2^*$. Consider the function:

$$\Omega(x) = FOC_{NLP2}(x) - FOC_{NLP1}(x)$$

for a generic, given x. Notice first that by assumptions, both $FOC_{NLP1}(x)$ and $FOC_{NLP2}(x)$ are continuous and weakly decreasing in x. Moreover, it can be shown that $FOC_{NLP1}(x)$ is decreasing at a higher rate than $FOC_{NLP2}(x)$; having defined $\Xi(x) = SOC_{NLP2}(x) - SOC_{NLP1}(x)$ it is verified that: $\Xi(x) > 0 \ \forall x \in [0, 1]$.
By algebraic manipulations, it is found that $\Omega(x)|_{x=\frac{2\theta-1}{b-1}c} \geq 0 \Leftrightarrow \theta \geq \vartheta(b)$. As $FOC_{NLP1}(x)$ is decreasing at a faster rate, this is sufficient to ensure that x s.t. $\{FOC_{NLP_2}(x) = 0\} > x$ s.t.$\{FOC_{NLP_1} = 0\}$ which is equivalent to say $x_2^* > x_1^*$.
As to part (ii), it is be derived that: $\Omega(x)|_{x=1} \leq 0 \Leftrightarrow \theta \leq \psi(b)$. This a sufficient condition to ensure that the equilibrium collapses to $x_1^{**} \equiv x_2^{**}$ *Q.E.D.*

Proof of Proposition 2 The first inequality ($c < x_1'$) is showed to hold by checking that $FOC_{LP}[c/(b-1)] > 0$ and noticing that also $FOC_{LP}(x)$ is monotonically decreasing in x. The second inequality ($x_1' < x_2'$)is verified by definition. To see that the last inequality holds notice that $x_2' < 1$ if and only if $FOC_{LP}(1) < 0$ which requires:

$$FOC_{LP}(1) = -(b-1)[(b-1)-c]f(b-1) - f(1)[(b-1)-(2\theta-1)c] +$$

$$+\frac{(b-1)}{n}[1-F(b-1)] < 0$$

This inequality can be expressed as a relation between θ and b:

$$\theta > \frac{nc(b-1)f(b-1) + (b-1)^2 f(b-1)[1-F(b-1)]+}{2f(1)\{nc+(b-1)[1-F(b-1)]\}}$$

$$\frac{-n[(b-1)^2 f(b-1) - F(1)]}{2f(1)\{nc+(b-1)[1-F(b-1)]\}} + \frac{1}{2} = \phi(b)$$

$x_2'' \equiv 1$ in case $\theta \leq \phi(b)$ *Q.E.D.*

Proof of Proposition 3 Point 1. derives by simply observing that x_1^* and x_1' are identified by the same first order condition:

$$x_1 \text{ s.t. } \frac{1-F(x_1)}{n} - f(x_1)(x_1 - c) = 0$$

this, in turn, implies they coincide. Point 2. and 3. are direct implications of the result in 1. while 4. is the result of direct comparison between the equilibrium expressions of the consumers' surplus under non-linear and linear pricing. *Q.E.D.*

Proof of Proposition 4 Point (i) descends from direct comparisons of the equilibrium expressions for the choice variables. First of all, $\Delta x_2 = x_2^* - x_2' \leq 0$ and is equal zero only in the extreme case $b = 1$. Furthermore, once again by direct comparison, it is obtained $\Delta x_1 = x_1^* - x_1' > 0$ under the assumptions made on the parameters. These results imply that the share of customers served with two-units packets is (weakly) larger under non-linear pricing:

$$1 - x_2^* \geq 1 - x_2'$$

while, under linear pricing, is larger the share of consumers buying the one-unit packet:

$$x_2^* - x_1^* < x_2' - x_1'$$

Point (ii) is immediate by looking at Q^* and Q'. Point (iii) comes from observing that $\Delta\pi_i = \pi_i^* - \pi_i' > 0$ for all the combinations of the relevant parameters. *Q.E.D.*

Proof of Proposition 5 Since, by direct comparison, $x_1'' \geq x_2^{**}$ it is immediate that both Point (i) and (ii) are verified. *Q.E.D.*

Proof of Proposition 6 Point (i) follows by direct comparison: $\Delta x_2 = x_2^{**} - x_2' \leq 0$ and it is equal to zero only in the special case $b = 2$ and $\theta = 1$. From this, it is immediate to see that $1 - x_2^{**}$ is larger than $1 - x_2'$.
As to point (ii), direct comparison and the restriction $\theta \leq \psi(b)$ allow to show that $\Delta Q = Q^{**} - Q' \geq 0$ for all the feasible combinations of the parameters, so that the output under non-linear pricing results larger or at least equal to the one under linear prices. *Q.E.D.*

Chapter 4

Network Neutrality and the Non-Discrimination Issue

4.1 Introduction

Network neutrality is the word used to refer to the debate over the management and regulation of the internet in the future[1].

The relevance of this industry has grown dramatically in the recent past and it is clear the importance of managing the development of the sector effectively. Nevertheless, many economics issue arise when considering the broadband internet industry and this feature complicates attempts to outline a widely supported regulatory policy.

The ultimate goal of a network neutrality regulation is to guarantee competition and innovation on the internet. The key to achieve this goal seems to have been identified by the proponents of state intervention in the protection principles of *inter-operability* and *non-discrimination.*

Inter-operability has to do with the degree of homogeneity of the networks owned by different companies or institutions. On one side, the homogeneity of networks guarantees identical opportunities to all customers and the

[1] An overview of the topic is provided by Economides [23]-[24].

compatibility of communications between all networks. On the other end, some authors suggest that the differentiation of networks could benefit customers by favouring an efficient specialization on different internet services. A typical example is provided by e-mail as opposed to VOIP services: those applications have very different technical requirements in their supply and also very different priority for the success of the service.

Non discrimination deals with the possibility of networks' owners of versioning the access to the services from both content providers and final users. According to a recent quote, this issue strongly characterize the whole debate: 'Network Neutrality is the term commonly used to describe the battle between the telecom industry and a varied coalition of groups arguing over whether content providers should be able to pay in order for surfers to get faster access to their sites'(ABC News, 23/06/06).

The two recalled issues are, clearly, strongly interrelated. This chapter, however, focuses on non-discrimination and offers a possible interpretation of the economic implications of the evolution of the network neutrality debate. In particular, an assessment of the implications of different pricing and quality supply policies from the network owners is provided.

According to Sidak[71] «the analysis of optional tariffs sheds light on the network neutrality debate. A network operator could offer content providers one tariff schedule for priority delivery of data packets and another tariff schedule for unprioritized delivery». The same author put forward the argument of Ramsey pricing to conclude that: «differential pricing for content providers for the priority delivery of packets is a Pareto improvement over a 'neutrality' regime that required that a single price be charged». One of the aims of this chapter is to show that whereas most of the usual intuition from non-linear pricing theory carries on when analyzing a stylized but specific model on the non-discrimination issue, there are circumstances over which results can be controverted. These circumstances depend in fact on the very specific features of the demand side of the market but also the supply side and the essential two-sidedness nature of the internet.

The approach adopted in this chapter differs from most of the literature quoted, whose approach characterizes much of the debate on network neutrality. More rare are economics contribution on the issue. The two more closely related papers are Economides-Tag[25] and Hermalin-Katz[44]. Both contribution recognize the two-sided nature of the internet and focus on the economics effects of an eventual network neutrality regulation. However, Economides-Tag[25] does not focus on issues related to quality of network access. Their approach can be seen as a short run analysis of the impact of network neutrality regulation, as it is very unlikely that in the short run the network owners can introduce improvements to the internet, allowing to offer a differentiated product to content providers. Hermalin-Katz[44], instead, focus exactly on supply of quality as the screening device adopted by owners to discriminate providers with different willingness to pay. The spirit and the approach of their paper is quite close to this one, as it is recognized in the long run networks can be improved and network neutrality regulation may have an effect on quality supply. However, the two papers differ in many respect. First, our specification naturally deals with the oligopolistic market structure which seems to characterize the broadband internet industry. Second, the richer framework adopted in this chapter allows to focus on several aspects that the contribution of Hermalin-Katz[44] does not address, as for example network externalities. Finally, while Hermalin-Katz[44] focus on whether neutrality is superior or not to discrimination, this chapter highlights which market fundamentals play in favour of network neutrality and which ones against regulation.

The rest of the chapter is structured as follows: Section 4.2 provides an extremely synthetic and non-exhaustive overview of the network neutrality debate. Section 4.3 introduces the model and its relation with the economics literature. Section 4.4 provides an overview and an illustration of the two situations analyzed: a network neutral benchmark case and an equilibrium in which network owners are allowed to offer different qualities of access to the internet to different content providers. Section 4.5 analyzes the model by discussing the role of demand and supply characteristics on both sides of the market in determining the effect of network neutrality regulation. Section 4.6 provides a few concluding remarks.

4.2 The Network Neutrality Debate

In extreme synthesis the non-discrimination problem can be summarized in the two following questions. Should network owners be able to discriminate content providers? Or should the openness and democracy of the web be preserved?
Before turning to the analysis of these specific questions, a synthetic summary of the network neutrality debate is provided.
Wu[80] categorizes the actors of the whole network neutrality debate as "Openists vs. Deregulationists". Broadly speaking,'openists' highlight that the success of the internet built on the 'innovation commons' principle, which is the possibility of everyone to access everyone else's advances. This can only be achieved by a public network/infrastructure on which interconnections are guaranteed by creative users and developers. Telecom companies should not be allowed to appropriate the great value of the positive externalities existing throughout the web. This principle suggests favour for a regulation guaranteeing network neutrality. Ingram[37] cites among network neutrality supporters companies like Google, Yahoo!, Ebay, Amazon and Microsoft. On the other end, 'deregulationists' seem to refer to the idea of 'media-convergence', according to which there is a «natural technological progression towards a single network for communications services[2]». This convergence can be efficiently achieved by guaranteeing property over different network for different services. Coupled with deregulation, this seems the only way to guarantee

[2] Wu[80], pp.71-79.

an efficient structure to the internet in the future. Verizon, AT&T, Qualcomm, Comcast, Cisco, according to Ingram[37], stand against the purpose of regulating the internet to achieve network neutrality.
The players and the importance of the sector witness how hot the debate is at the current moment. Ingram[37] points out, nevertheless, the distinguishing traits of the American and European telecom/information technology business environments. Overall the broadband sector appears more healthily competitive in Europe and, at the current moment, there seems to be no real worry justifying the need of anti-discriminatory or net neutrality policies to guarantee the rights of content providers and final web users. This feature can explain the different impact the policy debate is having in these two major economic areas.
Addressing the debate from an economic theory perspective seems crucial in order to assess the relative merits and shortcomings of each position. According to Reynolds[64], the conclusions on the need of a network neutrality regulation crucially depend on whether the markets are efficient enough. The focus should be on checking that the conditions for an efficient market allocation are respected by the network structure of the internet. If this is not the case, there maybe scope for regulation or *ex-post* competition authorities' intervention.
A closer look to the wide array of economics issues arising in the broadband internet debate is in order at this stage. A variety of those topics are discussed in greater detail in papers by Wu[80], Yoo[82] and more recently Sidak[71], but a non-exhaustive list is as follows:

- Effects of scale economies on networks' size and concentration of the industry. According to Baumol-Swanson[7] in this kind of industry banning differential pricing maybe socially detrimental not only because it may determine some customers not participating in the market but also because forces firms out of the market as they may not be able to cover their own fixed costs without price discriminating;

- Demand size economies/externalities and the size of the networks: the claim from network neutrality proposers is that having different networks from different services may reduce the positive externalities linked to joining the network;

- Congestion and social cost of network usage: 'deregulationists' claim that price freedom can help firms in dealing with different levels of bandwidth usage and possible congestion. The existence of different networks for services differing in bandwidth usage can determine a more rational allocation of capacity;

- Vertical integration at the different level of supply of broadband internet services: competitive concern arise in case of integration between network owners and content or service providers;

- End to end principles: engineering principles should shed light on costs and benefits of rendering the network more sophisticated at intermediate levels. On the one hand, regulation would secure more freedom in network development; on the other hand few players on the market could ensure more coordination in the choice of the networks' structure;

- Product differentiation, the variety of supplied services and the problem of connectivity between networks;

- Competition and innovation in the industry. The basic questions here seem to be: which regime would foster more competition in the internet sector? and which one, on the other end, would guarantee the highest rate of innovation? These seems crucial questions to look at the future as innovation is recognized to be one of the main ingredients in determining the rapid expansion of the industry as a whole.

The debate looks extremely complex. The scope of this chapter, nevertheless, it is not to outline a model that can address all those issues. A stylized

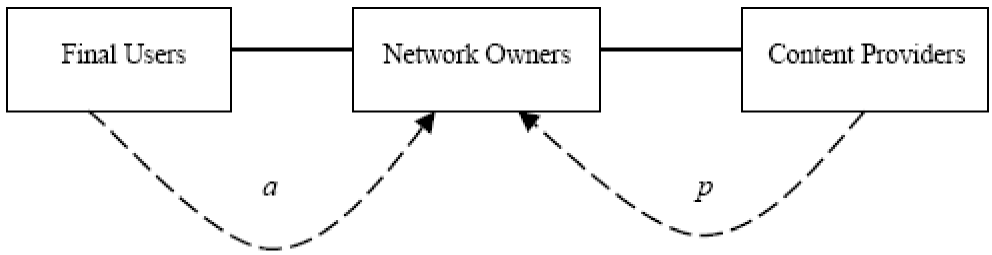

Figure 4.1: The two-sided structure of the internet.

two-sided market model of the internet is purposed to focus on one specific aspect of the network neutrality debate: the effects of the pricing strategy by network owners/telecom companies to internet content providers appear to be one of the key ingredients of all the discussion. This is the reason why it is worth to analyze the topic more in depth.

4.3 The Model

The framework used to analyze the pricing issues related to network neutrality explicitly recognizes the two-sided market structure of the broadband internet industry. As Economides-Tag[25] and Katz-Hermalin[44], it is assumed network owners act as intermediaries between heterogeneous content providers and final users.

The model adopted is related to the work of Mussa-Rosen[56] who were the first to address the issue of quality supply under incomplete information on consumers' tastes in one-sided markets. Maskin-Riley[51] formalization of non-linear pricing closely parallel the one selected here despite assuming that a continuous of price-quantity combinations can be supplied. Both of those models assume the goods are supplied by a monopolist who can provide different level of qualities of the product. Monopolistic is also the market studied by Deneckere-McAfee[20] in their analysis of 'damaged goods', i.e.

goods for which the supply of the lower quality is more expensive then the high quality one. The theoretical structure builds upon the oligopoly model introduced by Ireland[38] and used in Chapter 3, extended to allow for two sided markets and network externalities. The latter papers focus on the effects of non-linear pricing as compared to linear pricing when firms supply packets of different size; this chapter explicitly addresses the issue of quality supply.

4.3.1 Network Owners

Access to the network is supplied by n telecommunications companies. These companies are intermediaries between final internet users who demand access to several services and contents on the web and companies supplying these contents and services. The quality of the access that internet content providers receive is a variable q_L, which for convenience it is normalized to 1. The chapter will focus on two polar cases.
First, the case in which companies are allowed to supply access to only one quality at an identical price for all providers. The contract in this case takes the form $(q, p(q))$. Broadly speaking, this case can be considered as the benchmark/current situation or the result of enforcing a network neutrality regulation.
Second, the case in which companies are allowed to introduce a new superior quality of service in order to screen different content providers. The contracts offered in this case will be: $\{(q_L, p_L(q_L)), (q_H, p_H(q_H))\}$ For analytical convenience it is assumed that q_H is exactly t times better than q_L and this proportion is fixed. This implies: $q_H = tq_L$ with $t > 1$.
The technology used by firms displays constant returns to scale with respect to quantity but not necessarily with respect to quality. The technology available to firms in fact can be described by the following cost function:

$$C_i(q_i, x_i, Q_i) = \begin{cases} q_L c\, D_L(x_L, x_H, n) & \text{per unit of } \quad q_L \text{ supplied} \\ \theta q_H c\, D_H(x_L, x_H, n) & \text{per unit of } \quad q_H \text{ if supplied} \end{cases}$$

where D_i represents the demand function as defined in the next subsection. These cost functions imply that the marginal cost of supplying quality q_i is constant. However this cost depends on the quality herself and on the parameters θ. This parameter captures the relative cost savings/dissavings in supply of high quality. The feature allows to deal with a wide range of possible technologies, ranging from cost savings on quality, diminishing returns in supply of quality and also the extreme but possible case in technological markets of damaged goods. Despite Giovannetti-D'Ignazio[34] highlight the role of asymmetry as a key relevant determinant for the scope of network neutrality regulation[3], the assumption that firms are symmetric is used throughout the chapter and it can be justified by the fact that big players in the telecom sector, owning the internet network, can be considered of comparably big dimension. The focus here is, in fact, on pricing and not on inter-connectivity issues.

4.3.2 Content Providers

Internet content providers have heterogeneous preferences for the quality of access provided by telecom companies to their site. This heterogeneity is captured by the type parameter x that expresses their willingness to pay for quality of access whose distribution function is $f(x)$ with $x \in [0,1]$. Despite not being necessary in order to find an equilibrium of the model, it will be assumed that the providers' types are distributed according to a uniform distribution. This can be interpreted as companies facing a linear demand function for a given level of quality and considerably simplifies the rest of the analysis.

Each provider demands one quality of access for its web-site: either low or

[3]Giovanetti-D'Ignazio[34] use ISP level data in order to assess the importance of different incentives (business stealing effects as opposed to the network externality effect) in determining the decision of interconnection between networks. Heterogeneity between ISPs seems the key determinant of weather one or the other can prevail.

high.
Marginal utility is decreasing so that the higher quality access gives a proportionally lower utility to the provider. A key assumption is that the diminishing marginal returns to quality is the same across all providers.
A positive network externality exists so that the utility of content providers will depend on the total proportion of final users who access the web. This can be interpreted as follows: more internet users implies more probability of your content being used, your service being demanded or your site being visited; this probability increases the income the content providers can get from advertisers or other sponsors.
Consistently with the described features of providers' tastes, the utility function can be described as:

$$U(q_i, x) = \begin{cases} x + \gamma(1 - G(\hat{y})) - p_L(q_L) & \text{for} \quad q_L \\ bx + \gamma(1 - G(\hat{y})) - p_H(q_H) & \text{for} \quad q_H \end{cases}$$

in which $x \in [0, 1]$ describes the type of provider, $p(q)$ is the price of the contract signed with the telecom company for the quality demanded and the parameter b captures the marginal returns to quality perceived by content suppliers.
In case network owners supply both qualities, the utilities of content providers should respect the following constraints:

$$U(q_L, x) = x + \gamma(1 - G(\hat{y})) - p_L(q_L) \geq 0$$

$$U(q_H, x) = bx + \gamma(1 - G(\hat{y})) - p_H(q_H) \geq U(q_L, x)$$

4.3.3 Final Users

Final users are heterogeneous in their evaluation of the access to the web y, a continuous variable distributed according to $g(y)$, $y \in [0, 1]$. Users' utility however depends also on the quality of the service they receive. As users surf on a number of web-sites, it can be assumed that the quality they receive is proportional to the number of providers who demand a high quality access to the network and providers who opt for the lower quality option. The following quality index is then employed:

$$Q(\hat{x}_L, \hat{x}_H) = [1 - F(\hat{x}_H)]h + [F(\hat{x}_H) - F(\hat{x}_L)]$$

where h is the *perceived* quality increase if accessing a high quality web content. Moreover, final users face a double network externality. Their utility depends on the number of users on the web and increases as the content provided on the web increases. Under this assumptions the utility of web surfers can be described by the following function:

$$U_y = y + \alpha Q(\hat{x}_L, \hat{x}_H) + \mu_1(1 - G(\hat{y})) + \mu_2(1 - F(\hat{x}_L)) - a$$

Clearly, for final users to participate in the market $U_y \geq 0$ must hold.

4.4 Solution of the Model

This section characterizes the solution of the model under two possible situations of interest: the neutral network regulated equilibrium and the case in which the network owners supply more than one quality of access to the network both to final users and content providers. The equilibria are characterized in terms of market shares served and equilibrium prices. Numerical examples illustrate the profits of the network owners and the surplus of both the final users and the content providers.

4.4.1 The Neutral Network Equilibrium: One Quality-One Price of Access

This section analyzes the case of network neutrality: all network owning companies offer access to both providers and final users at a uniform price and quality.

The first step is to derive the demand functions faced by firms on both sides. Assuming that firms only offer one level of quality q_L and the distribution of both content providers and final users is uniform, content providers will choose to buy or not access to the network depending on:

$$\max\{0, x + \gamma(1 - \hat{y}) - p(q_L)\}$$

The previous expression implies that a content provider is indifferent between buying access or staying out of the web if:

$$\hat{x} = p(q_L) - \gamma(1 - \hat{y})$$

An analogous way of reasoning allow to identify the final users who are indifferent between accessing the internet or not. These are defined by:

$$\hat{y} = \frac{an - \mu_1}{n - \mu_1} - \frac{(\alpha + \mu_2)n(1 - \hat{x})}{n - \mu_1}$$

All providers with an evaluation of access superior to $\hat{x}$ and all final users characterized by a type y will subscribe to one of the telecom's network. The profit function faced by each network is then:

$$\pi_j = [p_L(q_L) - c]\, D_x(\hat{x}) + (a - k)D_y(\hat{y}) = [\hat{x} + \gamma(1 - y) - c]\left[\frac{1 - \hat{x}}{n}\right] +$$
$$+\left[\hat{y} + \alpha(1 - \hat{x}) + \frac{\mu_1(1 - \hat{y})}{n} + \mu_2(1 - \hat{x}) - k\right]\left[\frac{1 - \hat{y}}{n}\right] \qquad \forall j = 1..n$$

The first order conditions are, respectively:

$$\frac{\partial \pi_j}{\partial \hat{x}} = \frac{1 - \hat{x}}{n} - [\hat{x} + \gamma(1 - \hat{y}) - c] = 0$$

$$\frac{\partial \pi_j}{\partial \hat{y}} = \frac{1 - \hat{y}}{n} - \left[\hat{y} + \alpha(1 - \hat{x}) + \mu_1\frac{(1 - \hat{y})}{n} + \mu_2(1 - \hat{x}) - k\right] = 0$$

The indifferent final users and content providers can then be characterized as a function of the models parameters:

$$\hat{x} = \hat{x}(c, k, \alpha, \mu_1, \mu_2, \gamma, n)$$

$$\hat{y} = \hat{y}(c, k, \alpha, \mu_1, \mu_2, \gamma, n)$$

The equilibrium expressions of the benchmark case can then be characterized in terms of the indifferent consumers as:

Table 4.1 The Network Neutrality Equilibrium

MS_x	$\frac{1-\hat{x}}{n}$
MS_y	$\frac{1-\hat{y}}{n}$
$p(q_L)$	$\hat{x} + \gamma(1 - \hat{y})$
a	$\hat{y} + \alpha(1 - \hat{x}) + \mu_1 \frac{(1-\hat{y})}{n} + \mu_2(1 - \hat{x})$

A numerical example, useful to illustrate the network neutrality equilibrium, is proposed in section 4.4.3.

4.4.2 Quality Discrimination and Network Access

If companies supply two types of access contracts, then content providers decide whether to access the internet and select the contract $(q_i, p_i(q_i))$ according to the following criterium:

$$max\{0,\, x + \gamma(1 - y^*) - p_L(q_L),\, bx + \gamma(1 - y^*) - p_H(q_H)\}$$

To define the demand functions it is crucial to characterize the marginal providers, given the set of prices $(p_L(q_L), p_H(q_H))$. Content provider x_L^* is indifferent between buying nothing or one unit if:

$$x_L^* + \gamma(1 - y^*) - p_L(q_L) = 0 \Leftrightarrow x_L^* = p_L(q_L) - \gamma(1 - y^*)$$

The provider x_H^* is indifferent between the low or the high quality if:

$$(b - 1)x_H^* = p_H(q_L) - p_L(qL)$$

The preceding equation implies:

$$x_H^* = \frac{p_H(q_H) - p_L(q_L)}{b - 1}$$

The demand for each quality, faced by a generic network owner, is:

$$D_j(x_L^*, x_H^*, n) = \begin{cases} 0 & \text{if} \quad 0 < x < x_L^* \\ \frac{x_H^* - x_L^*}{n} & \text{if} \quad x_L^* < x < x_H^* \\ \frac{1 - x_H^*}{n} & \text{if} \quad x_H^* < x < 1 \end{cases}$$

Analogous reasoning allows to define the demand faced on the other side of the market from final users. These are willing to access the web if and only if:

$$U_y = y + \alpha Q(x_L^*, x_H^*) + \mu_1(1 - y^*) + \mu_2(1 - x_L^*) - a \geq 0$$

implying that the indifferent consumer is defined by:

$$y^* = \frac{\alpha Q(x_L^*, x_H^*) + \mu_1 + \mu_2(1 - x_L^*)}{1 - \mu_1} - \frac{a}{1 - \mu_1}$$

The profit function of each firm is:

$$\begin{aligned} \pi_j \quad = \quad & \left[\frac{x_H^* - x_L^*}{n}\right] [x_L^* + \gamma(1 - y^*) - c] + \left[\frac{1 - x_H^*}{n}\right] [(b-1)x_H^* + x_L^* + \gamma(1 - y^*) - t\theta c] + \\ & + \{y^* + \alpha Q(x_L^*, x_H^*) + \mu_1(1 - y^*) + \mu_2(1 - x_L^*) - k\} \left[\frac{1 - y^*}{n}\right] \end{aligned}$$

after having substituted for $p_H(q_H)$, $p_L(q_L)$ and a the relative expressions in terms of the indifferent web users and providers.
The first order conditions with respect to x_L^* , x_H^* and $y^*$are then given by:

$$\begin{aligned} \frac{\partial \pi_j}{\partial x_L^*} &= \frac{1 - x_L^*}{n} - [x_L^* + \gamma(1 - y^*) - c] - [\alpha + \mu_2] \left[\frac{1 - y^*}{n}\right] = 0 \\ \frac{\partial \pi_j}{\partial x_H^*} &= (b-1)\frac{1 - x_H^*}{n} - [(b-1)x_H^* - (t\theta - 1)c] - \alpha t = 0 \\ \frac{\partial \pi_j}{\partial y^*} &= (1 - \mu_1) \left[\frac{1 - y^*}{n}\right] - \{y^* + \alpha Q(x_L^*, x_H^*) + \mu_1(1 - y^*) + \mu_2(1 - x_L^*) - k\} + \\ -\gamma &\left[\frac{1 - x_L^*}{n}\right] = 0 \end{aligned}$$

Under the assumptions made, it is possible to solve the system of equations to find the equilibrium indifferent consumers as a function of the parameters of the models:

$$\begin{aligned} x_L^* &= x_L^*(b, c, k, h, \theta, t, \alpha, \gamma, \mu_1, \mu_2, n) \\ x_H^* &= x_H^*(b, c, k, h, \theta, t, \alpha, \gamma, \mu_1, \mu_2, n) \\ y^* &= y^*(b, c, k, h, \theta, t, \alpha, \gamma, \mu_1, \mu_2, n) \end{aligned}$$

This equilibrium can be characterized in term of market served and prices on the two sides of the market as follows:

Table 4.2 Non-Regulated Quality Competition Equilibrium

MS_{x1}	$1 - x_1^*$
MS_{x2}	$x_2^* - x_1^*$
MS_y	$1 - y^*$
p_L	$x_L^* + \gamma(1 - y^*)$
p_H	$(b-1)x_H^* + x_L^* + \gamma(1 - y^*)$
a	$y^* + \alpha Q(x_1^*, x_2^*) + \mu_1(1 - y^*) + \mu_2(1 - x_1^*)$

The following numerical example is useful to illustrate the unregulated equilibrium and the features of the model presented.

4.4.3 A Numerical Illustration

Suppose the following values are set for the relevant parameters of the model:

Parameter	c	k	α	γ	μ_1	μ_2	n	h	θ	t	b
Value	0.25	0.2	0.1	0.1	0.1	0.1	3	2	1	2	1.8

In the network neutrality benchmark, the model has a "well behaved" solution characterized by the following values of the relevant variables:

$\hat{x}$	0.553
$\hat{y}$	0.517
MS_x	0.447
MS_y	0.483
$p(q_L)$	0.601
a	0.622
π_j	0.120
SCP	0.100
SFU	0.113
SW	0.574

where SCP and SFU stand for content providers and final users surpluses respectively and SW represents aggregate social welfare.

The model can be solved under quality discrimination. A "well behaved" solution is then characterized by the following values of the relevant economics variables:

x_L^*	0.389
x_H^*	0.689
y^*	0.498
MS_x	0.611
MS_y	0.502
$p(q_L)$	0.439
$p(q_H)$	0.750
a	0.606
π_j	0.113
SCP	0.300
SFU	0.122
SW	0.761

The example provided is useful to illustrate the properties of the model adopted. The effect of quality discrimination as compared to a network neutrality benchmark can be summarized in three main effects: first, the share of both final users and content providers participating in the market largely increases when discrimination is allowed; second, both the price charged to final users and of the access to low quality decrease as compared with the benchmark while the price of high quality is expectedly higher; third, given the first two effects, the effect on surplus of final users and content providers is positive. This implies an overall positive effect on social welfare, as measured by the sum of welfare of all agents involved.

It should be clear, as already pointed out by Hermalin-Katz[44], that in the context of this model there seems to be little point to push for network neutrality regulation on the basis of non-discrimination concerns. However, the type of modelling approach adopted here and in Hermalin-Katz[44] gives clearly the best chances to non-linear pricing to have positive welfare effects. The point this chapter would like to stress, instead, is what elements of the internet industry structure seem to make network neutrality more or less socially desirable. That type of analysis is provided in the following section.

4.5 Analysis of the Results and Implications

This section focuses on the effect of the relevant features of the network owners' supply and of the users' and providers' demand for internet access in determining the effects of an eventual network neutrality regulation.

The approach of analysis adopted is to focus on the effects of the parameters which characterize the relevant features of the preferences and of the technology on all sides of the market. This is done by considering the network neutral equilibrium as compared with the non-regulated equilibrium. In this section, basically, it will be presented the effect of the parameters in determining the effect of network neutrality regulation with respect to the relevant variables. The pictures that will be presented in what follows represent the difference in the levels of a variable when passing from a situation of quality supply freedom and discrimination to a regulated one with network neutrality. So, for the variable X, a graph would represent the function:

$$\Delta X = \hat{X} - X^*$$

in which a positive value implies network neutrality is increasing the amount of the variable with respect to quality discrimination, while the opposite is true if the value is negative. A similar strategy of analysis is adopted by Armstrong-Vickers[6] in their study of the relevant features influencing the effects of non-linear pricing and bundling as compared with linear pricing.

4.5.1 The Effect of Quality

The first and most important question posed by the network neutrality debate is what happens if network owners are allowed to discriminate between different content providers. In other words, if some of the providers can benefit of the so called internet highways, how will this impact low end content providers, demanding standard quality, and final users?

The approach taken is that the effect of a quality discrimination will depend both on the effective quality increase, as captured by t, and by the perceived quality increase by both final users and content providers, as captured respectively by h and b.

Figure 4.2 represent the differential between the variables in the two situations considered as a function of the actual increase in quality t, assuming that $\theta = 1$ and $h = 2$, i.e. that there are constant returns to supplying quality and the final users perceive that high quality is twice as good as the low. The yellow line represents the profits differential. As it can be seen, if the actual quality is not very different with respect to the low quality, then network neutrality reduces owner's profits. However, as soon as the actual quality increases, approaching and exceeding $t = h = 2$ profits of the firms are enhanced by network neutrality. This can be interpreted as follows: supposing that actual quality is higher than the quality perceived by content providers, then it is better for owners to commit not to supply the high quality, as it is

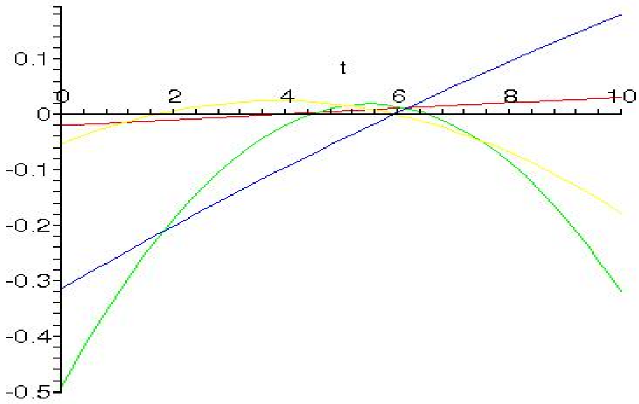

Figure 4.2: Differential of Profits (Yellow), Final Users' Surplus (Blue), Content Providers'Surplus (Purple) and Social Welfare (Green) plotted against actual quality t.

the case in presence of network neutrality regulation. Values of parameters larger than $t = 5$ or $t = 6$ do not look very realistic, as it is very unlikely that providers do not feel such a major increase in quality. In technological markets, however, it is not rare the case in which further sensible advancements in technology are not perceived as important by the beneficiaries of it: this is the case, for example, of high speed internet connections. It is really difficult for users to discern in many situations the speed of a connection. In such a case it is the right part of the graph to become relevant and, regarding profits, it is clear that quality discrimination is a much better deal for network companies. The purple line describes the content providers' surplus differential. Net neutrality seems to harm them by preventing them to access the high quality of the version. However, when the actual quality becomes too large without them realizing it, they are damaged by the increase in prices for the two qualities and the corresponding restriction of the market served[4].

[4]The evidence regarding the differential in prices and shares of market served is not

The blue line represents the differential in the surplus of final users: these are clearly damaged by network neutrality regulation for most of the likely values of actual quality. The green line, finally, presents the total surplus differential. In this case too, for the most likely values of the actual quality network neutrality seems have a strongly negative effect on the economy: this effect is explained by net neutrality implying on average larger prices and a lower share of the market served. Only for intermediate values of quality, when profits are not too damaged by regulation and a slight positive effect is registered for users and providers, the net effect on surplus is positive. A dynamic interpretation of the interaction between actual and perceived quality can be put forward. Suppose, in fact, as suggested by Economides-Tag[25], that in the beginning the supply of a high quality version of internet access to content providers is not implying an actual increase in quality. This would imply that $t = 1$ while providers and final users may think to receive higher quality than they actually do. *Mutatis mutandis*, the result obtained is the opposite of Economides-Tag[25]: network neutrality has an absolutely negative impact on welfare. It can be supposed that quality increases in the long run and becomes equal to the perceived one: the strong negative welfare effect of regulation is consistent with the findings of Hermalin-Katz[5]. In the long run technology can overshoot perception, with the consequences that were analyzed above.

Figure 4.3 is analogous to Figure 4.2 but the variables are plotted against the perceived increase in quality h, given that actual quality is $t = 2$ and final users evaluation is $b = 1.8$. The effect of perceived quality is monotone on the variables analyzed. As the blue function highlights, network owners' profits increase under network neutrality as perceived quality increases. Network neutrality, instead, damages final users (purple function) and to a larger extent content providers (yellow function). This is quite reasonable: as the

presented in the paper but it is available upon request from the author.

[5]Hermalin-Katz[44], Proposition 6.

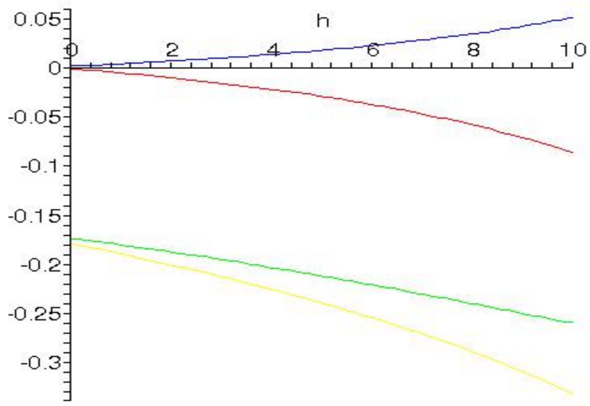

Figure 4.3: Differential of Profits (Blue), Final Users' Surplus (Purple), Content Providers'Surplus (Yellow) and Social Welfare (Green) plotted against quality perceived by final users h.

perceived quality increases, network neutrality denying companies to supply the higher quality hurts more and more final users, who can not transform their preferences in higher utility. As for content providers, the restriction to supply only the low quality implies a lower ability to satisfy users' demand and so to self-select them; this translates in a higher average price faced and lower share of them taking part into the market. The overall welfare effect (green function) is then clearly negative.

Figure 4.4 deals instead with the differential in the four variables considered plotted against the evaluation of high quality of content providers. It is supposed that the actual quality is $t = 2$, there are constant returns to quality $\theta = 1$ and perceived quality is $h = 2$. Network neutrality, as witnessed by the purple function, has a positive effect on network owners' profits. However, as decreasing marginal returns in the demand of quality are less intense, the profit differential after joining its maximum tends to become thinner until it vanishes. Final users surplus (blue line) and content

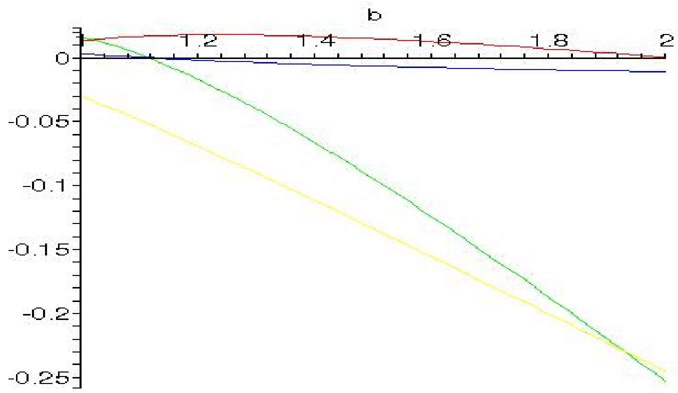

Figure 4.4: Differential of Profits (Purple), Final Users' Surplus (Blue), Content Providers'Surplus (Yellow) and Social Welfare (Green) plotted against content providers' returns to quality b.

providers' surplus (yellow line) suffer even sharper losses from the adoption of regulation. These losses become larger as the marginal evaluation of high quality increases. Intuitively, the effect is similar to what was found regarding the quality perceived by final users: as network neutrality impedes the supply of high quality, content providers suffer out of it more and more as their evaluation of the product increases. This is reflected also on final users who face higher prices and a lower share of them is served under net neutrality. The overall welfare effect (green curve) is also negative.

4.5.2 The Effect of Technology

The cost of providing access to users and content providers afforded by the network owners may play an important role in determining the effect of network neutrality regulation. As underlined by Cheung-Wang[14], in a one-sided market bundling model, a relatively high unit cost of production may suggest a detrimental effect of a regulation impeding discrimination.

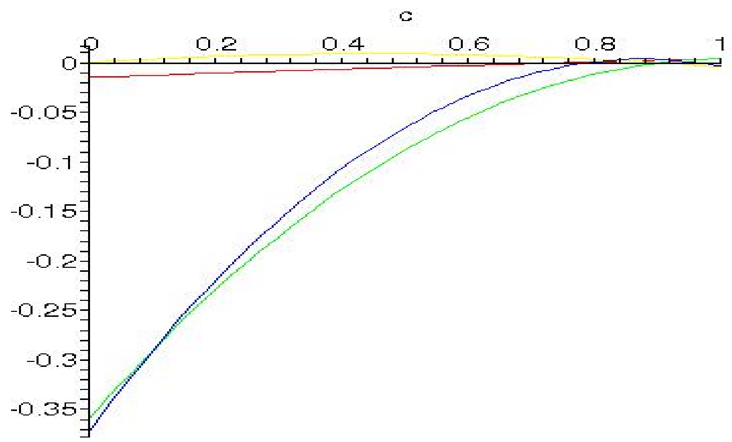

Figure 4.5: Differential of Profits (Yellow), Final Users' Surplus (Purple), Content Providers'Surplus (Green) and Social Welfare (Blue) plotted against the marginal cost of serving content providers c.

Figure 4.5 displays the differentials between network neutrality and quality discrimination plotted against the marginal cost of serving content providers c. Network neutrality tends to have a weak positive effect on profits for intermediate values of the marginal cost. The difference tends to be negligible for more extreme values, both on the lower and higher end of marginal costs. Negative, instead, is the effect of regulation on both final users and content providers. The more efficient network owners in supplying the content providers, the more damaging regulation would be for content providers themselves and social welfare as a whole. There are, however, extremely high values of the marginal cost for which regulation is increasing the welfare of all agents involved. The final interpretation is then the following: net neutrality regulation can have a positive effect and should be considered if the sector displays extremely high costs of providing access to content providers.

Figure 4.6 displays the effects of regulation as compared to quality differentiation as a function of the marginal cost of serving final users. As the

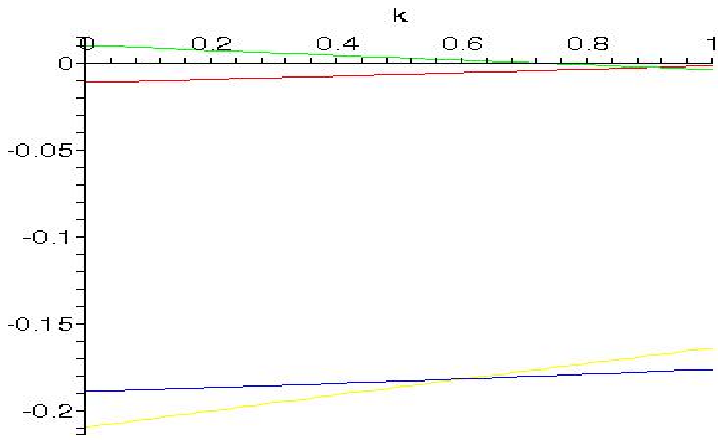

Figure 4.6: Differential of Profits (Green), Final Users' Surplus (Purple), Content Providers'Surplus (Yellow) and Social Welfare (Blue) plotted against the marginal cost of serving final users k.

cost is low, network owners, whose profits differential is in green, have an advantage from the network neutrality regulation as opposed to final users who are actually damaged and would prefer quality discrimination. As the cost of serving them increases, however, the two regimes do not differ substantially, as the purple function shows. Strongly negative, instead, is the effect of regulation for content providers: as witnessed by the blue function, as the marginal cost of serving the other side increases there is only a non-substantial improvement for their welfare.

Figures 4.7 displays the differential between the network neutral and the quality discrimination equilibria as a function of the parameter θ which is capturing the returns to quality in supply. The yellow function highlights that profits are always reduced by network neutrality when supplying the high quality implies non-negligible positive cost savings. In presence of non-negligible cost savings, network neutrality becomes less and less desirable also

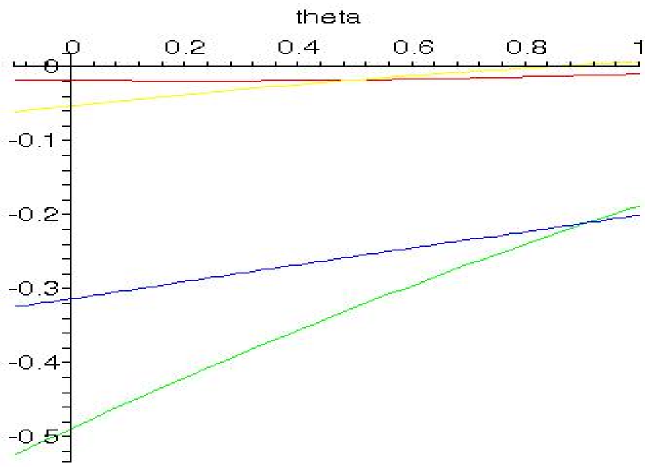

Figure 4.7: Differential of Profits (Purple), Final Users' Surplus (Yellow), Content Providers'Surplus (Blue) and Social Welfare (Green) plotted against the marginal returns in supplying quality θ.

for final users, content providers' and, obviously, the aggregate social welfare. The differential for these variables are depicted in the figure in purple, blue and green respectively.

The surplus of final users, however, is not decreasing monotonically with θ: for rather extremely high returns to quality, comprehending also the case of damaged goods ($\theta < 0$), the effect of network neutrality keeps being negative, but progressively less so. The cause of this effect is probably to be traced back to prices of access: as θ decreases, the access fee paid by final users becomes relatively lower under network neutrality and this is compensating the relative decrease in consumers served.

4.5.3 The Effects of Network Externalities

Network externalities play a very important role when dealing with two-sided networks: the utility of an agent depends on how many agents participate in

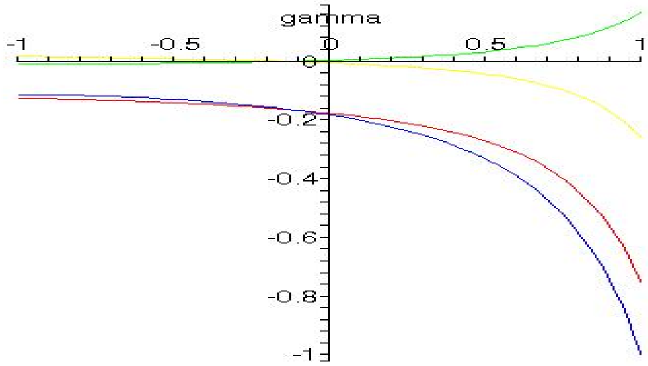

Figure 4.8: Differential of Profits (Green), Final Users' Surplus (Yellow), Content Providers'Surplus (Blue) and Social Welfare (Purple) plotted against the effect of network externalities of final users on content providers γ.

the market both on his side and on the other side. The model set up allows to evaluate both the effect of such externalities on final users and on content providers, both in case there is a positive participation externality and in case more participation may imply congestion.

Figure 4.8 displays the effects of network neutrality regulation on the variables of interest for a wide range of combinations of γ, the externality by final users on content providers. Results differ in case final users' participation implies a positive effect for content providers, as opposed to the case they imply, instead, a negative congestion effect. As expected, when more users imply congestion, network neutrality may have positive effects. The orthants on the left in the figure display that network neutrality has a positive effect for final users: when the content provided is of a uniform quality, then the share of consumers served is reduced, implying reduced congestion. As illustrated by the blue curve, also the negative effect of regulation on content providers is milder in presence of congestion. A weak but negative effect related to regulation is registered also with respect to profits (green curve). This is overturned and definitely more intense when the externality of final users on content providers is positive. In that case, however, network neutrality displays a strong negative effect on final users, content providers and social welfare (purple curve): the reason is once more to be traced to the share of market served, which increases on both sides as quality discrimination is allowed.

Figure 4.9 focuses on the effect of externalities of final users on themselves: more users accessing the web may have a positive effect for everyone or be a source of congestion. However, no matter the sign of the externality, the effect of regulation is positive for profits, as the green function witnesses. The amount of the advantages of net neutrality for owners is increasing in μ_1. A negative effect is registered for final users' surplus (yellow curve): as intuitively expected, the magnitude of the effect increases with μ_1, i.e. with the impact that final users who access the web have on themselves. This result is to be coupled with the negative impact of net neutrality on the share of users served. Participation decreases under net neutrality, and more severely, for content providers: this explains the strong negative effect on their surplus, as in the purple curve. This is only partially offset when there is congestion. The yellow line displays how net neutrality negatively impacts social welfare, no matter the sign of externality.

Finally, Figure 4.10 shows the effect of the externality exerted by content providers on final users. This is positive when users display preference for variety of the content when navigating. However, it may be negative when the amount of information provided is too large so that the users find difficult to surf the web efficiently for their needs. Once more, preference for variety determines a positive effect of net neutrality for profits, as witnessed by the yellow function in the positive orthant.

Network owners, however, would be better off to quality differentiate when final users are negative affected by the *plethora* of contents available online. These results are exactly reversed when dealing with final users:

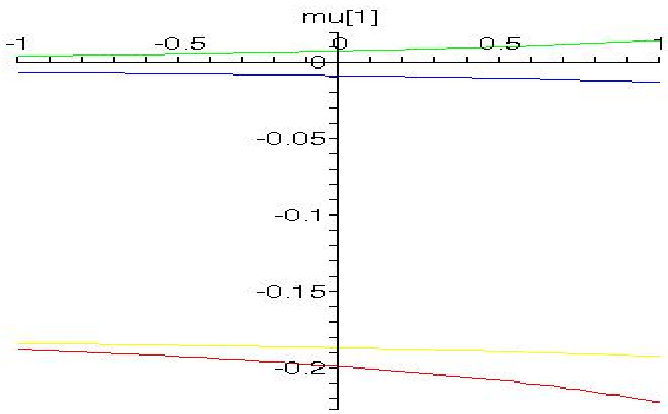

Figure 4.9: Differential of Profits (Green), Final Users' Surplus (Blue), Content Providers'Surplus (Purple) and Social Welfare (Yellow) plotted against the effect of externalities of final users on themselves μ_1.

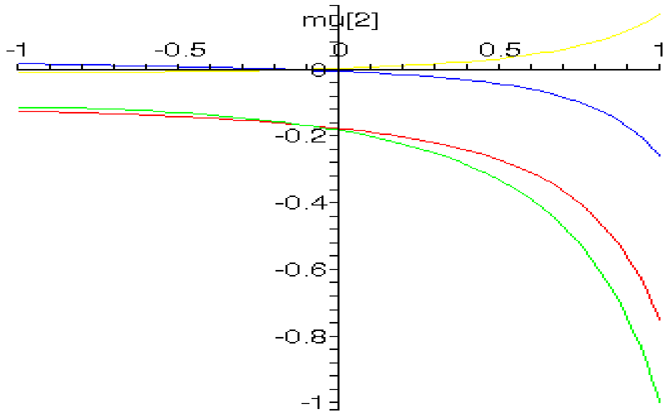

Figure 4.10: Differential of Profits (Yellow), Final Users' Surplus (Blue), Content Providers'Surplus (Green) and Social Welfare (Red) plotted against the effect of the externality of content providers on final users μ_2.

the blue line shows as they are better off under net neutrality only when they are negatively affected by the amount of information supplied. This effect is absolutely analogous to the one described when dealing with the externality of final users on content providers: as net neutrality implies a decrease in participation by content providers, then final users are better off. These negative effect on participation of content providers, however, harms the welfare of this last category of agents: as the green line displays, network neutrality damages them, the effect is only mitigated in presence of a negative externality while is magnified when it is positive. Network neutrality has an overall negative effect, more intense when the externality is positive.

4.6 Concluding Remarks

This chapter aimed to analyze the effects of a non-discrimination requirement on network owners, in case network neutrality regulation is adopted. This has been achieved by introducing a model of the broadband internet which recognizes explicitly its two-sided market structure. Network owners are intermediaries between final users of the web and content providers. This tentative analysis is based on a number of simplifying assumptions and on an overly stylized representation of reality: for this reason, its policy implications should not be overemphasized. It is however important to underline that this research allows to draw the attention on a few interesting aspects related to the impact of an eventual adoption of network neutrality regulation. In particular, the role of demand and supply side characteristics on all sides of the market is taken into account in determining the effects of regulation on all the involved agents.The results seem to suggest that regulation is likely to restrict the options available to content providers and as such reduce theirs and final users' welfare. Factors which mitigate this effect are high costs of supply of the two sides of the market, high actual quality with respect to the perception of users, relatively low evaluation of quality of content providers, the presence of congestion on content providers' side and excess information, implying for example higher searching costs, on the final users' side.

The analysis provided has been confronted with the results in the most closely related literature. There are, however, two further issues which should be addressed in the developments of this research to exhaust the analysis of net neutrality within the model adopted. First, nothing has been said about how network neutrality and quality discriminating outcomes compare with the social welfare maximizing outcome. Secondly a few authors, including Economides-Tag[25], adopt a rather extreme view regarding network neutrality and non-discrimination. According to them, in fact, the network is really neutral if and only if access is provided for free. Within our framework this would imply full coverage on the content providers side. The effect would be to neutralize all the gains related to a larger share of the market under quality discrimination. A comparison of our results with this more extreme regulatory regime maybe of some interest. Finally, despite the wealth of simplifying assumptions, the model is extremely rich and it is this wealth of details which allows to capture many effects not completely highlighted by the previous literature. Future developments, however may try to simplify further the setting in order to focus on the effects of quality supply in two sided markets. This topic seems quite interesting and rather unexplored in the theoretical literature.

There is a wealth of issues of the net neutrality debate which may be worth to address in a two-sided market perspective. Important dynamic aspects relate to a non-discrimination regulation: competition, the possibility of entry and the incentives to innovate are crucial for the destiny of the internet industry in the long run. The oligopolistic structure of a market characterized by large telecom companies owning the network seems to fit the presented model in which they compete for the market share *à la Cournot*. This, however, may suggest the possibility of tacit collusion between network owners. A repeated interaction structure of the model can help shed light on how different patterns of regulation impact on likelihood of collusion. The vertical structure of the internet can also be considered: the possibility of vertical mergers and the formations of coalitions between network owners and content providers may have an important effect on welfare.

Part III

Conclusions

Chapter 5

Final Considerations

Non-linear pricing is a widespread practice in most real world markets. These markets are characterized, although to different extents, by competition: the extremes cases of monopoly and perfect competition are in most cases just interesting textbook caricatures not very suitable for the interpretation of reality. The sophistication of pricing practices has increased in the latest decades due to the technological advances in information technology. Companies try to gather more and more data on consumers to reconstruct their preferences and demographic data. This information is precious and can be productively used to personalize offers and pricing. The thesis is focused on the effect of both increased information and competition on the effects of non-linear pricing on agents and their welfare. The analysis is conducted adopting the perspective of the new theory of industrial organization, mainly developed in the last three decades, that captures strategic interaction between firms as a non-cooperative game. This approach allows to model effectively the strategic issues involved in non-linear pricing under competition. Along these lines, Chapter 1 provides the motivation for writing a thesis on non-linear pricing and some of its applications.

Chapter 2 reviews the theory of oligopolistic non-linear pricing. Non-linear pricing is a special case of price discrimination taking place in presence of asymmetric information. The theory of price discrimination has to be modified in presence of oligopolistic competition. A crucial role is played by the competitive externality imposed by rival firms. This effect tends to erode firms profits. In order to avoid an outcome similar to the "Bertrand paradox" and explain the prevalence of non-linear pricing in real world markets, two approaches have been taken in the literature. The first assumes that firms compete in pricing schedules but their products are differentiated. Product differentiation is then closely related to the possibility of discriminating and plays a crucial role within this approach. The foundations of the approach rely on mechanism design under common agency: firms compete for a common mass of consumers. Exploiting this parallel it is shown that firms face a type dependent participation constraint. The other modelling option is quantity competition. In this case horizontal product differentiation is not crucial as firms behave as monopolists facing a residual demand for each type of agents. The usual Cournot intuition that the mark up is related to the demand elasticity and the number of firms carries on in this case, when considering the different types independently. It turns out that there is a close link between non-linear pricing and the recent developments in the theory of vertical differentiation. Suppose firms are selecting a product line characterized by different qualities of a good, then most economic principles used to find the optimal non-linear price schedule apply also in this case. This parallel has been exploited in the literature to study the strategies of firms when deciding whether to introduce products of higher or lower quality to the range they already offer. The model can be extened to show how the same economic intuition on quality supply applies to the case of two-sided markets.

One remarkable result, no matter the approach adopted, is that in a competitive setting not necessarily the possibility of discriminating favours firms. In fact, in presence of "best response asymmetry", firms face a prisoners' dilemma strategic situation: the private gain from non-linear pricing is transormed in a public loss when all firms adopt the same strategy with the results that firms would be better off if they could commit to price uniformly.

Chapter 3 focuses on technology and its relation with non-linear pricing. The motivating question is linked to everyday's shopping experience suggesting that many products can be purchased in several different sizes, while others are supplied in one size only. The explanation provided suggests a relationship between the demand and the technological structure of the market that determines the size of the products supplied by firms in equilibrium. The chapter focuses on a model in which firms supply a homogeneous product in packets of different sizes. Information about consumers' reservation prices is incomplete and the production technology is characterized by size economies. Four equilibrium regions are identified depending on the relative intensity of size economies with respect to consumers' evaluation of the good. Regions are characterized by the product being supplied either in a single unit or in several sizes or in only a large size, including different units bundled together. Both the private and social desirability of non-linear pricing varies across different equilibrium regions; the general conclusion, however, is that in presence of size economies the overall welfare effect is likely to be positive, although profits might decrease as compared with the linear pricing case.

Chapter 4 considers the broadband internet market, an extremely dynamic sector, which is attracting the attention of politicians and other actors, besides the companies operating in the sector. Broadly speaking, the market is characterized by oligopolistic telecom companies owning the networks. They provide modern fast-speed internet services being intermediaries between content providers and final users. Content providers are users who own a web site and provide contents or services over the net to final users that benefit of the contents and services available. The framework is then carachterized by huge cross platform externalities. In such a situation, non-discriminatory issues stand the core of the recent debate on the opportunity or not of regulating the internet. According to some, the success of the internet is due to its accessibility. In the past this was always guaranteed for free to whoever desired to contribute to its development. This accessibility could be jeopardized in the context of the new bradband internet structure. One of the main questions posed, in fact, is whether the telecom companies should be allowed to offer quality-contingent contracts to content providers. In other words, the problem is the effect of discrimination on users who require different type of access services for their business. The chapter provides an analysis of the issue through a stylized two-sided market model of the web that highlights the effects of such a discrimination over quality, prices and participation to the internet of both providers and final users. An overall welfare comparison is proposed, concluding that the effects of regulation crucially depend on both the technology and preferences of agents. Although network neutrality seems to disadvantage both content providers and final users, there are situations in which a case for regulation can be made.

The work presented in this thesis is not immune of limitations and aspects of not complete satisfaction. Many of the current limitations can be seen as important triggers for further research and are discussed in what follows.

The research on size economies is a first attempt to analyze the size of products that can be found on the market. One possible extension should take into account the possibility that both a single size and a bundle made of several units of the good are supplied. This is an often seen feature and several good are sold in different packets at the same time as, for example, beer or crisps. Further, the analysis should be generalized in order to weaken a few of the assumptions. First, the model can be extended to the case of price as the choice variable of firms. This robustness check would allow to compare the results with the alternative approach in the theory of non-linear pricing, reviewed in Chapter 2. Second, the shape of both preferences and of technologies considered is not general enough. The assumption that a further unit is evaluated proportionally the same by all consumers should be relaxed. Moreover, the technological structure allowing for negative returns to size or the even more extreme case of damaged goods has to be considered to complete the analysis. Finally, the main limitation of the model is that it does not allow to fully characterize the strategic interaction between firms. It would be desirable, instead, to analyze the payoff when one firm adopts non-linear pricing while the other prices linearly. This would complete the matrix of a game in which pricing strategies are the choice variables: the conjecture is that under some conditions firms might face a prisoners' dilemma, a new finding in presence of quantity competition. Technical difficulties, however, did not allow to prove that yet.

With reference to the research on network neutrality, as underlined in the text, there is a wide array of economic issues to be taken into account when considering the topic. One immediate extension of the research would involve to "calibrate" the model in the text with estimates of the parameters provided by the literature or by actual data regarding the extent of network externalities, costs or congestion effects. This exercise would make the example more relevant and authoritative; the problem, however, consists in the availability of the data needed for such a parametrization. One way to get around these difficulties is to consider estimation models of the type often applied in competition policy analysis and based on the available data on the UK case: this option is being explored at the current moment. The analysis can also be improved by providing comparative statics on more than one parameter at the same time: as the model is quite rich, the difficulty is represented by the dimension of the equilibrium expressions obtained. In the context of a parametrized example, however, it would be possible to show the comparative statics with respect to a subset of the parameters at each time. The model proposed can be simplified or enriched at need in order to accommodate further related relevant issues like incentives to innovation, compatibility of applications, vertical integration and the structure of the internet from backbone to the last mile or the effect of fixed costs of construction and improvement of the network.

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