



ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

DOTTORATO DI RICERCA IN

FISICA

Ciclo XXXIIX

Settore Concorsuale: 02/A1 – FISICA SPERIMENTALE DELLE INTERAZIONI FONDAMENTALI

Settore Scientifico Disciplinare: FIS/01 - FISICA SPERIMENTALE

Proton source measurements in Pb—Pb collisions
at the LHC with femtoscopy using an EFT model
for the strong potential

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Acknowledgements

This work would not have been possible without my supervisors Francesca Bellini and Pietro Antonioli, to whom I owe a tremendous debt of gratitude for their professional guidance and personal support since the very first day of my arrival in Bologna. I hope that someday I will be worthy of the trust and time they invested in me.

I also express my sincere gratitude to my senior colleagues Nicolo Jacazio and Francesco Noferini. I cannot even count how many times their expertise helped me move forward with my research. You are my role models of what it means to be a researcher.

During these years in Bologna I met many amazing people who have come to occupy a large part of my heart and I am happy to consider them as friends. Chiara, David, Francesca E., Giovanni, Hanna, Marco, Nicola, Nicolo (yes, again), Riccardo, Sofia T., thank you for being present in my life. It never ceases to surprise me how lucky I am to have such beautiful, smart, kind, and generous people.

And of course, thanks to my colleagues Bianca, Marta, Sandro, Sofia S., Neelima and Gilda Scioli for all the chats, lunches, coffee and smoking breaks together. Because of you, it has always been a pleasure to come to the office.

Благодарю своих родителей и сестру Полину за то, что всегда меня поддерживали и даже несмотря на расстояние ваша теплота и любовь всегда продолжала ощущаться близко. Я очень люблю вас.

Ну и конечно не могу не вспомнить своих потрясающих друзей. Даня, Коля, Лена, Лизочка, Маша, Миша и Нелли, спасибо вам за то, что были и есть в моей жизни, за вашу поддержку, за то, что не дали нашей дружбе угаснуть за эти годы. Без вас я бы не смог пройти тот путь что прошел. Спасибо вам, люблю вас.

Abstract

In this thesis, the proton-emitting sources created in ultrarelativistic heavy-ion collisions are explored using methods of femtoscopy. The first part of this study addresses the importance of accounting for the strong interaction between protons from a theoretical point of view, and it develops an original analytical quantum mechanical calculation. The second part involves the experimental analysis of the data collected by ALICE during Run 3, presenting one- and three-dimensional femtoscopic measurements of the proton source in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.36$ TeV. The experimental analysis utilizes the theoretical formalism developed in this study.

The one-dimensional results, obtained as a function of the pair transverse momentum k_T and centrality, indicate that the radii scale with k_T due to the presence of radial flow in the system. The proton-emitting sources are found to be smaller in more peripheral collisions. The extracted radii are compared to the previous ALICE proton source measurements in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. The "scaling hypothesis" is tested by comparing multiplicity-dependent one-dimensional proton radii across different colliding systems. Signs of the common multiplicity scaling are found.

New to the heavy-ion collision field, the experimental measurement of the proton source is extended to three dimensions proposing a novel approach to the problem. Three-dimensional proton radii are extracted in three centrality and three k_T ranges, providing proof-of-principle that a three-dimensional experimental characterization of the proton-emitting source is viable. The behavior of the measured radii is examined and future perspectives for these measurements are discussed.

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Introduction

The Rutherford experiment on α particles scattering on a thin gold foil showed that atoms consist of heavy and positively charged nuclei [1] in 1910s. A subsequent scattering experiment bombarding nitrogen nuclei with α particles resulted in the first artificial nuclear reaction ${}^{14}_7\text{N} + {}^4_2\text{He} \rightarrow {}^{17}_8\text{O} + {}^1_1\text{H}$ and lead to the discovery of the proton [2]. This proved that atomic nuclei are not only positively charged but also complex objects.

The possibility of altering nuclei artificially inspired physicists to accelerate particles to conduct nuclear reactions. In the beginning of the 1930s another milestone was reached, the discovery of the neutron [3], a proton-like (how Rutherford predicted it) particle without an electric charge and the last constituent of atomic nuclei to be unveiled. This completed the picture of the nucleus structure, yet the mechanism that bound protons and neutrons together was unclear. There must have been a short-range nuclear force that were "strong" enough to counteract the Coulomb repulsion between protons. The choice of the name was pretty straightforward, *strong force*.

Empirical studies on nucleon-nucleon elastic scattering indicated that the nuclear force must be short-range, repulsive at sub-fm distances ($1\text{ fm} = 10^{-15}\text{m}$) and attractive at distances greater than 1 fm [4]. Such non-monotonic behavior of the strong potential (Fig. 1), together with the interference of other interactions in experiments, makes these studies particularly challenging. Nevertheless, experimental data on nucleon–nucleon scattering have long served – and continue to serve – as fundamental input for tuning and validating models of the strong potential.

In parallel to experiments, theorists proposed theoretical models aimed at providing an analytical description of the strong potential. One of such models proposed in 1935 by Yukawa suggested the existence of mediators of the strong interaction (like the photon is a mediator for electromagnetic interaction), a "massive boson" (or just *meson*) [6]. Twelve years later the existence of the π -meson was confirmed experimentally [7].

Even though pions were later proven to not to be the mediators, their discovery showed that nucleons were not the only particles interacting strongly. In the following years many more strongly-interacting particles, called *hadrons*, were discovered (K , η , ρ , ω ,...). By the

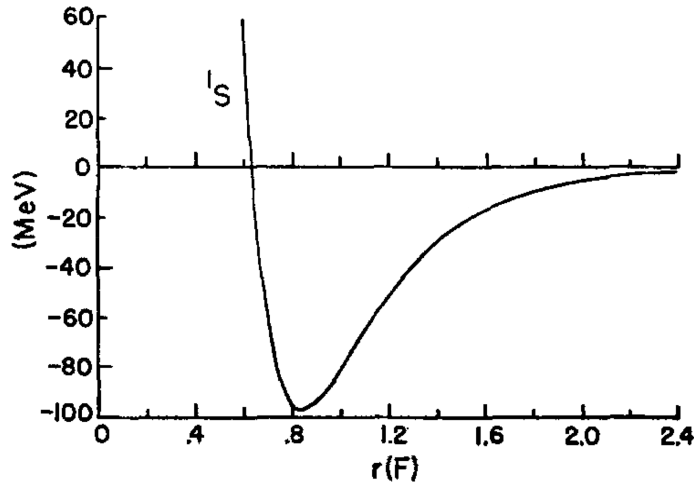


Figure 1. Nucleon-nucleon phenomenological potential shape for a singlet state with $l = 0$ [5].

beginning of 1960s it was clear that this "zoo" of strongly interacting particles needed to be structured.

The idea of hadrons consisting of more elementary particles was independently proposed by Gell-Mann [8] and Zweig [9] in 1964. The building blocks, now known as *quarks*, can be of six flavors (in the first works of Gell-Mann and Zweig there were only three types): up, down, charm, strange, top and bottom. The quark model was experimentally confirmed in 1969 at SLAC [10].

The evolution of the quark model led to the development of quantum chromodynamics (QCD) adding the concepts of color charge and gluons, mediators of the strong interaction. Various predictions of QCD were confirmed by particle accelerators: the gluon discovery at the Positron–Electron Tandem Ring Accelerator (PETRA) [11], the confirmation of energy dependence of the strong coupling constant α_s at the Large Electron–Positron Collider (LEP) [12], the discovery of J/ψ at the Brookhaven National Laboratory (BNL) [13] and Stanford Linear Accelerator Center (SLAC) [14] etc.

In 1973, Gross, Wilczek [15] and Politzer [16] introduced the concept of asymptotic freedom, paving the way for the idea that it is possible to deconfine hadronic matter by increasing the energy transferred to it. Nowadays, the Quark-Gluon Plasma (QGP), a deconfined state of QCD matter, is one of the main directions of QCD studies. The interest in studying the QGP prompted the development of the heavy-ion experimental programmes at two experimental high-energy facilities over the last decades, the Relativistic Heavy Ion Collider (RHIC) at the Brookhaven National Laboratory (USA) and the Large Hadron Collider (LHC) at CERN, where A Large Ion Collider Experi-

ment (ALICE) was specifically conceived to study and characterize the properties of the QGP.

The *femtoscopy* technique described in Chapter 2 has proven to be a powerful tool for investigating and characterizing quantitatively the fireball produced in heavy-ion collisions, of which an introduction is given in Chapter 1. The technique is based on momentum correlations between particles emitted by the source created in collisions and measured at the final state. This work focuses on correlations in proton pairs addressing the matter of importance of accounting for the strong interaction in femtoscopic studies with protons. The discussion is carried out employing an *original* theoretical work of embedding the strong potential in calculation of the proton pair wave function (WF). This is described in Chapter 3. The calculations follow a non-relativistic effective field theory (EFT) approach by incorporating the strong potential in a square-well shape allowing to obtain a *fully analytical solution* for the wave function.

The theoretical formalism developed as a first part of this study finds an immediate application in the experimental femtoscopic analysis of proton pairs measured in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.36$ TeV by ALICE, subject of the second part of this work and discussed in Chapter 5. The EFT model is employed for calculating the theoretical femtoscopic correlation function used for extraction of the geometrical properties of the proton-emitting source created in Pb–Pb collision. The source is explored in both one- and three-dimensions. The parameterization of the three-dimensional correlation function exploits a novel technique, developed originally in this work, enabling the first three-dimensional femtoscopic study with proton pairs conducted in heavy-ion collision experiments.

Chapter 1

Phenomenology of Heavy-Ion Collisions

After the quark theory proposed by Gell-Mann and Zweig in 1964 [8; 9] received its first experimental confirmation in 1969 at SLAC [10] the question of why don't we see quarks in an unbound (deconfined) state arose naturally. In just a few years Gross, Wilczek [15] and Politzer [16] proposed that the hadronic matter can be deconfined if large enough energy is deposited, introducing the concept of asymptotic freedom. In simple words, the strong coupling constant α_s that effectively characterizes the strength of the interaction decreases with increasing energy Q^2 deposited to the system (Fig. 1.1). Thus, a deconfined state could be achievable by increasing the energy transferred to the hadronic matter [17]. Experimentally it is achieved by colliding ultrarelativistic atomic nuclei.

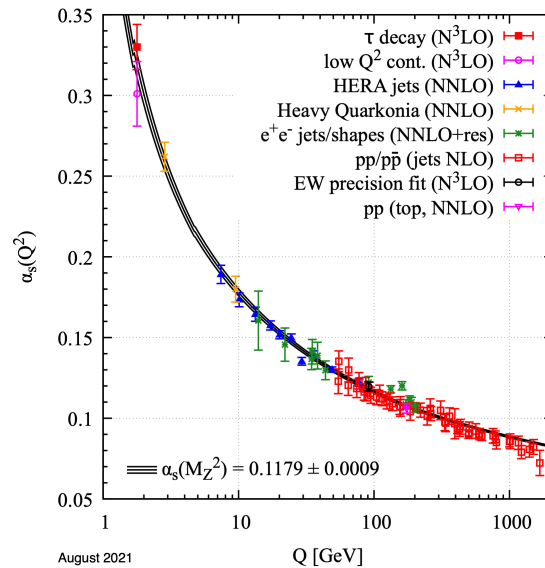


Figure 1.1. Strong coupling constant α_s in dependence of energy scale Q (modulus of four-momentum) calculated with perturbative QCD utilising experimental data [18].

As for the past couple of decades, the deconfined state – the Quark-Gluon Plasma (QGP) – is not a hypothesis but an established phenomenon. At present, the highest-energy operating "QGP factory" is the Large Hadron Collider (LHC). It is also believed that in the first microseconds after the Big Bang the matter in our Universe existed in a QGP phase which means that studying its evolution means learning about the evolution of our Universe itself at its earliest stage.

In this Chapter, the basic concepts and phenomenology of ultrarelativistic Heavy-Ion Collisions (HIC) in which the QGP is created, its signatures and evolution are introduced.

1.1 Space-time evolution of Heavy-Ion Collisions

At the LHC the nuclei of lead (^{208}Pb) are accelerated and collided at the energy of $\sim 5 \text{ TeV}^1$ per nucleon. In this regime the nuclei are not spherical but have a shape of "pancakes" due to the Lorentz contraction. At the energy per nucleon of $\sqrt{s_{\text{NN}}} \approx 5 \text{ TeV}$ in the center of mass (each colliding beam has half) the contraction factor (Lorentz factor) would be $\gamma \approx 2500$.

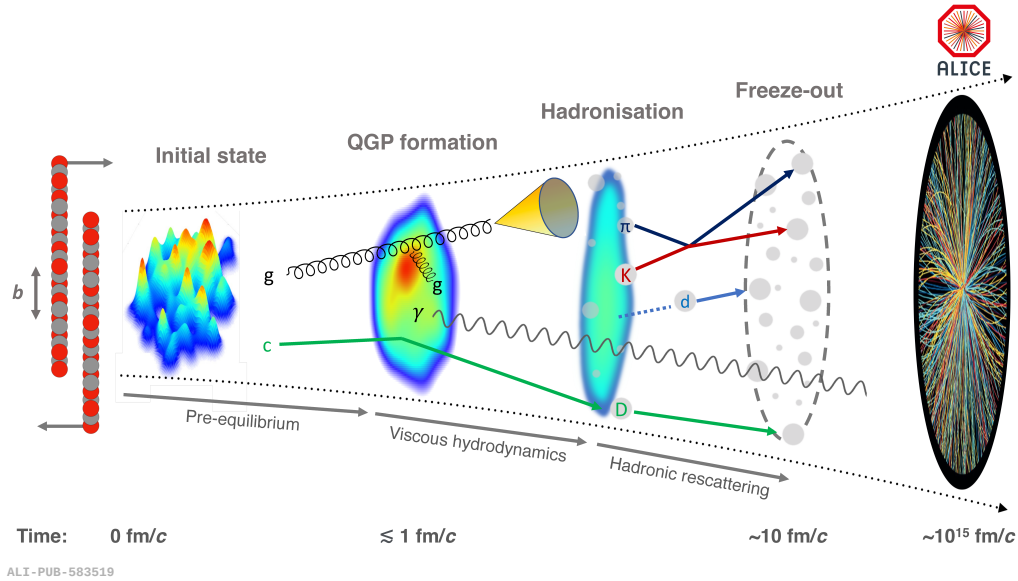


Figure 1.2. Schematic HIC evolution at the LHC energies [19].

Due to the fact that the larger the energy (momenta) of the colliding particles the smaller the scales of their interaction, at the *initial stage* ($t = 0 \text{ fm/c}$) of such high-energy collisions the matter first interacts at the partonic level. And since not only the lead nuclei but the

¹LHC started operating in 2009. During the Run 1 (2010-2013) data taking campaign the center-of-mass energy per nucleon pair in Pb–Pb collisions was $\sqrt{s_{\text{NN}}} = 2.76 \text{ TeV}$, in Run 2 (2015-2018) it was $\sqrt{s_{\text{NN}}} = 5.02 \text{ TeV}$ and in Run 3 (2022-ongoing) it reached $\sqrt{s_{\text{NN}}} = 5.36 \text{ TeV}$.

nucleons themselves have an internal structure, the energy transferred to the hadronic matter is not even across the collision's cross section. Interactions with large Q^2 produce highly energetic partons traveling away through the collision region and consequently resulting in *hard processes*. If the transferred energy Q^2 is smaller and mostly spent on deconfinement, droplets of QGP are created.

In the next moments ($t \lesssim 1 \text{ fm}/c$) the uneven profile of the initial stage levels out, the bits of QGP formed in the small- Q^2 processes couple producing the QGP fireball. It behaves as an almost perfect fluid [20] at extremely high temperature (the critical temperature at the LHC energies – with baryochemical potential $\mu_b \sim 0$ – is $T_{cr} \approx 156.5 \text{ MeV}$ at the energy density $\epsilon_{cr} = 0.42(6) \text{ GeV}/\text{fm}^3$ [21]) following the laws of thermodynamics and demonstrating collective behavior (see Sec. 1.3).

Later on, the QGP fireball cools down resulting in re-confinement of the partons forming hadrons again. This process is not immediate as partons can still interact with each other undergoing through scattering and recombination processes. Once the "chemical" composition of hadrons is established – so-called "chemical freeze-out" – the expansion of the matter, now in the form of a hadronic gas, continues. During this expansion the hadrons can still interact elastically due to the fact that right after the hadronisation the density and temperature of the hadronic gas is still high enough. When the elastic rescattering processes cease due to the further cooling and expansion, the hadrons stop interacting at the so-called "kinetic freeze-out" ($t \sim 10 \text{ fm}/c$) and keep traveling away from the interaction point until they are detected.

In the following sections we will discuss some of the main observables used to characterize the properties of HICs, and the QGP probes.

1.2 Characterizing the initial stage: multiplicity and centrality

In order to describe the initial stage of the matter right after the collision (Fig. 1.2) one needs to know the energy deposited to it. The latter depends not only on the energy of a collision but also on its geometry – i.e. the overlapping region of the colliding nuclei. Due to this interplay, it's more convenient to use energy density ϵ in order to parameterize the initial stage (Fig. 1.2). If the distance between the centers of the colliding nuclei – impact parameter b – is large, then the parts of the nuclei that don't overlap will keep traveling carrying a large fraction of initial energy/momenta (Fig. 1.3). Thus, the energy density ϵ in the overlapping region would be smaller than in the case of "central" collisions (small b).

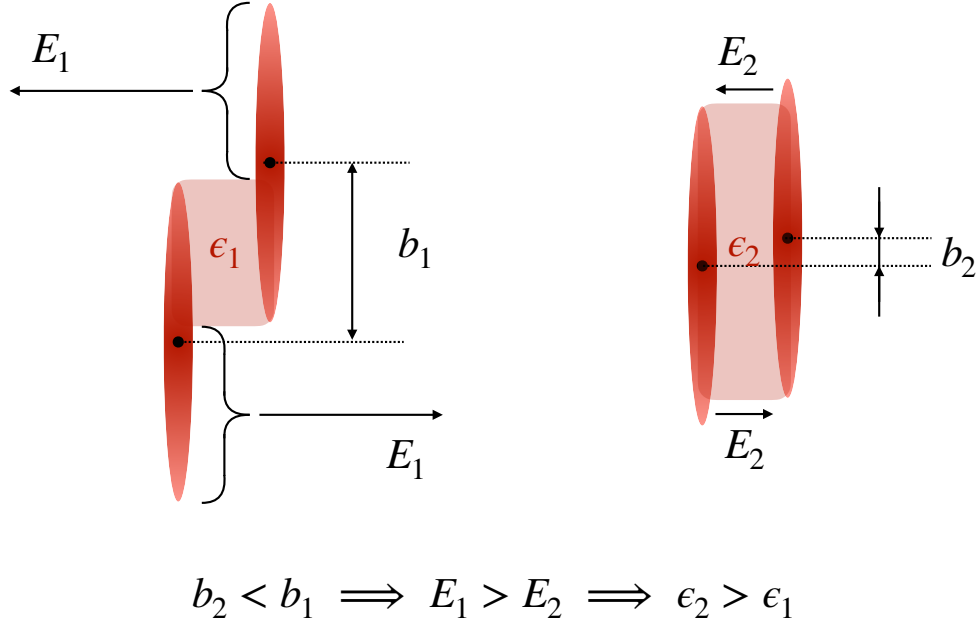


Figure 1.3. Schematic representation of how the impact parameter affects the final energy density deposited in the system in HICs.

Another variable to parameterize the initial geometry of a collision that is commonly used in HIC physics is *centrality*. The centrality value c is introduced as a fraction of the total A–A cross section. For a given impact parameter range $[0, b]$ one can write [22]:

$$c(b) = \frac{\int_0^b \frac{d\sigma}{db'} db'}{\int_0^\infty \frac{d\sigma}{db'} db'} = \frac{1}{\sigma_{AA}} \int_0^b \frac{d\sigma}{db'} db' \quad (1.1)$$

This can be interpreted as a probability of having a collision with an impact parameter in the $[0, b]$ range under the condition that the collision did occur. In other words, if we could plot the distribution of the number of recorded events in dependence on the impact parameter b , the value of $c(b)$ would give us the percentile of this distribution. The events with small impact parameter b (central) have small values of centrality c percentile (e.g. 0–5%) whereas events with large b (peripheral) have large values of centrality (e.g. 80–90%).

When the ultrarelativistic identical heavy nuclei with the same energy collide, it's natural to assume that the matter created in result of such collision is at rest – i.e. the Laboratory

frame coincides with the Center-of-Mass (CM) frame. In other words, the system possesses boost invariance along the beam direction (later referred as longitudinal boost invariance) right after the collision. In this case we can parameterize the system by a longitudinally Lorentz-invariant modulus of four-coordinate (including only the z component):

$$\tau_B = \sqrt{t^2 - z^2} = \text{const} \quad (1.2)$$

where τ_B is the Bjorken's *proper time* [23]. This defines a hyperbolic hypersurface where each value of τ_B defines an isohypersurface for the state of matter during its later evolution (Fig. 1.4).

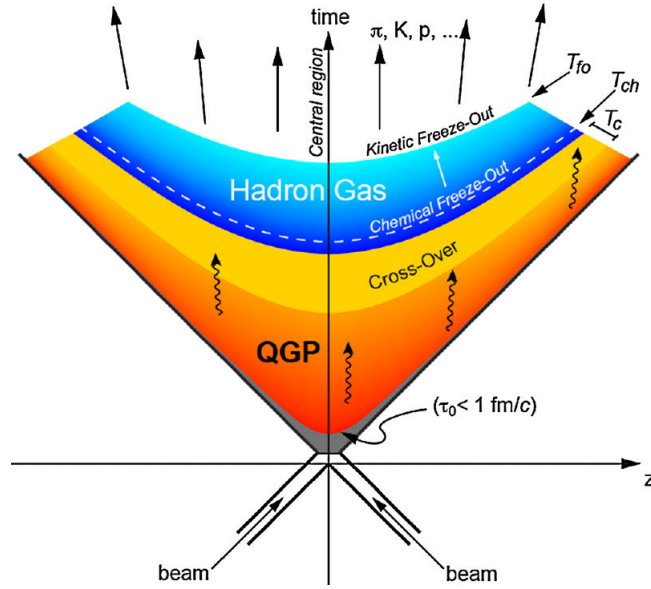


Figure 1.4. Bjorken's hypersurface profiles at different stages of HICs (Fig. 1.2). Credits for the picture: Ref. [24].

The corresponding kinematic variable with the longitudinal invariance used to characterize the particles emitted during the HIC's evolution is *rapidity*:

$$y = \frac{1}{2} \ln \frac{t+z}{t-z} = \frac{1}{2} \ln \frac{E+p_z}{E-p_z}, \quad (1.3)$$

where E is the energy of a particle, p_z is the momentum component along z -axis.

Another longitudinal invariant variable commonly used in HIC physics is *pseudorapidity* η :

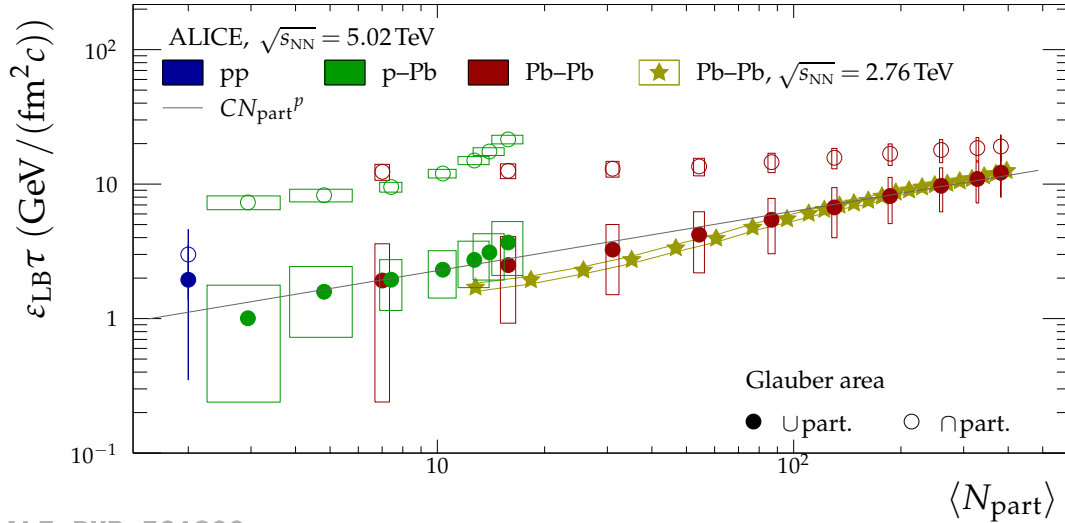
$$\eta = \frac{1}{2} \ln \frac{|\vec{p}| + p_z}{|\vec{p}| - p_z} = -\ln \left[\tan \left(\frac{\theta}{2} \right) \right]. \quad (1.4)$$

Comparing Eq. (1.3) and (1.4) one can see that for ultrarelativistic particles their values converge – i.e. $|\vec{p}| \gg m \Rightarrow E \approx |\vec{p}| \Rightarrow \eta \approx y$. The convenience of using pseudorapidity comes from the possibility of defining it via the polar angle θ (angle w.r.t. z axis) that is especially useful when dealing with the geometry of a detector (see Chapter 4).

If the energy deposited into the system is conserved, it translates into the amount of particles N – particle *multiplicity* – created later during the hadronisation and their mass/energy distributions. Thus, the number of detected particles per unit of rapidity can be used as the proxy to estimate the initial energy density ε [23]:

$$\varepsilon \sim \frac{dN_{ch}}{dy}. \quad (1.5)$$

The charged particle multiplicity is used because the HIC detectors are specifically designed to reconstruct (mainly) charged particles (see Chapter 4). The lower bound values of the energy density ε_{LB} times the Bjorken's proper time τ_B estimated by ALICE [25] employing the relation with dN_{ch}/dy distributions (1.5) are shown in Fig. 1.5. Assuming that the QGP phase transition occurs at $\tau_B \approx 1$ fm/c (Fig. 1.4 and Ref. [23]), the energy densities achieved in HICs at the LHC are well above the critical value $\varepsilon_{cr} = 0.42(6)$ GeV/fm³ [21].



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Figure 1.5. Lower bound of the Bjorken's energy density ε_{LB} times $\tau := \tau_B$ values estimated in pp, p-Pb, and Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV [25] in dependence of the average number of nucleons participating in a collision $\langle N_{part} \rangle$ (see next Section).

According to the Bjorken's model [23], the nuclei don't stop during the collision but rather pass through each other creating the region of highly condensed and hot partonic matter (Fig. 1.6) more or less evenly distributed at mid-rapidity. Therefore, the system

possesses a cylindrical symmetry (at mid-rapidity) and evenly distributed energy density. In such a case, the measured charged particle multiplicity density would be flat at mid-rapidity ($|y| \lesssim 1$), which is confirmed by HIC experiments such as ALICE at the LHC (e.g. Fig. 1.7).

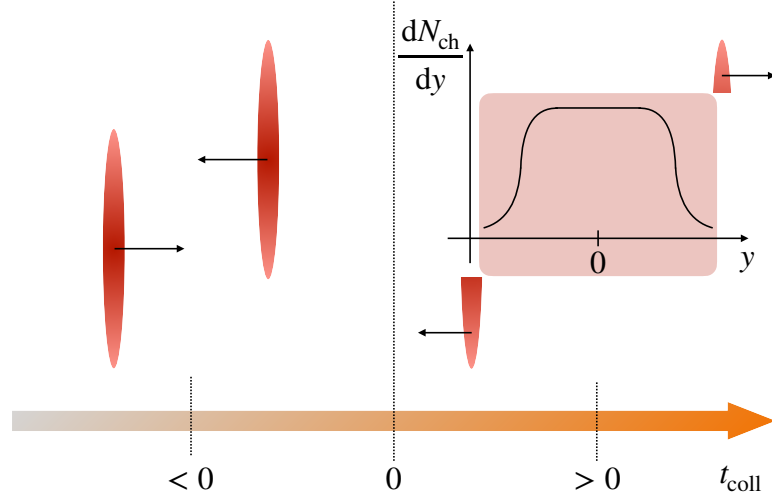


Figure 1.6. Particle production in HICs according to the Bjorken's model [26].

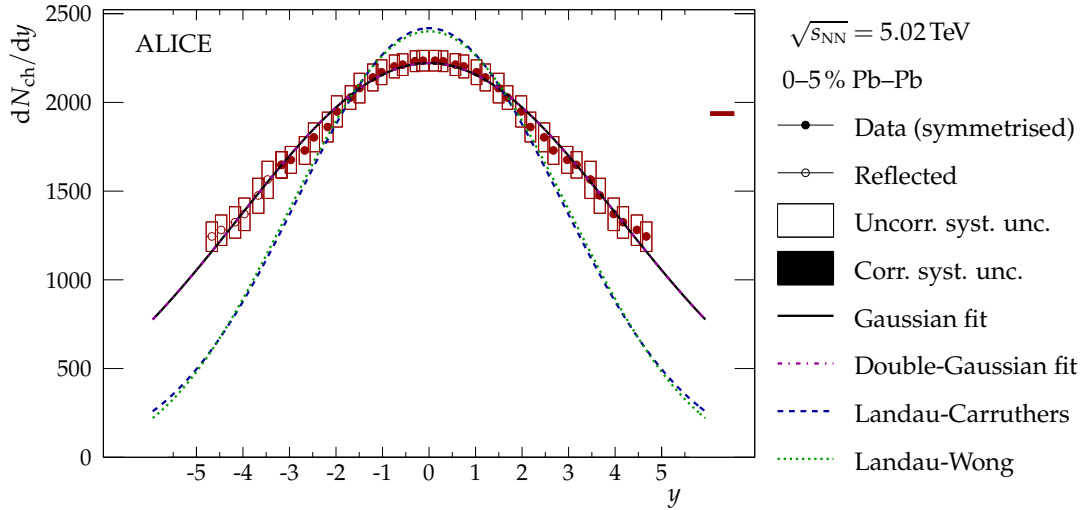


Figure 1.7. Charged particle multiplicity rapidity density measured by ALICE in Pb–Pb at $\sqrt{s_{\text{NN}}} = 2.76$ TeV collisions [27].

As the number of the charged particles N_{ch} produced in a collision is defined by the initial energy density ε which is in turn related to the initial geometry, the centrality value (1.1) can also be calculated via N_{ch} [22]:

$$c(N_{\text{ch}}) = \frac{\int_{N_{\text{ch}}}^{\infty} \frac{dn}{dN'_{\text{ch}}} dN'_{\text{ch}}}{\int_0^{\infty} \frac{dn}{dN'_{\text{ch}}} dN'_{\text{ch}}} = \frac{1}{N_{\text{ev}}} \int_{N_{\text{ch}}}^{\infty} \frac{dn}{dN'_{\text{ch}}} dN'_{\text{ch}}, \quad (1.6)$$

where n is the number of observed events as a function of the charged particle multiplicity ($n := n(N_{\text{ch}})$) and N_{ev} is the total number of events. The logic here is the same as before (1.1) and the $c(N_{\text{ch}})$ value provides a percentile for the $n(N_{\text{ch}})$ distribution.

The discussion about how the centrality is calculated in experiment can be found in the Sec. 4.2.

1.3 Quark-Gluon Plasma probes in Heavy-Ion Collisions

Since the formation of the QGP implies the formation of a unique state of matter, in order to establish its creation in HIC one needs to have a reference colliding system where the QGP is not expected to be formed. The most intuitive choice for such a system is proton-proton (pp) collisions at the same colliding energy per nucleon. If the the result of A–A collisions at the energies corresponding to the perturbative QCD (pQCD) regime cannot be explained as a superposition of incoherent nucleon-nucleon collisions then it can be considered as a manifestation of different physics. With this idea, we would like to start with one of the main QGP signatures that was also among the very first ones historically [28] – so-called *nuclear modification factor* R_{AA} .

The idea behind this observable is to compare the momentum distributions of particles (spectra) measured in events with (expected) QGP stage and without it. The nuclear modification factor is defined as

$$R_{\text{AA}} = \frac{1}{\langle N_{\text{coll}} \rangle} \frac{dN_{\text{AA}}/dp_T}{dN_{\text{pp}}/dp_T}, \quad (1.7)$$

where p_T is the momentum in the transverse plane (perpendicular to the beam axis); $\langle N_{\text{coll}} \rangle$ is the average number of binary nucleon-nucleon collisions in A–A event; dN_{AA}/dp_T is the number of particles per unit of p_T measured in A–A collision and dN_{AA}/dp_T – number of particles per unit of p_T measured in pp collision.

If there is no QGP stage and the particle spectra in a A–A collision is just a scaled up (by $\langle N_{\text{coll}} \rangle$) variant of the ones measured in pp collisions, then $R_{\text{AA}} \approx 1$. The measurements show that R_{AA} deviates from unity [29] in Pb–Pb collisions (Fig. 1.8).

The non-monotonic shape of R_{AA} (Fig. 1.8) comes from the difference between particle production mechanisms in different regions of momenta [30]. The high-momenta (or "hard"; $p_T \gtrsim 8$ GeV/c) particles are more likely to be a result of hadronization of the high-energy

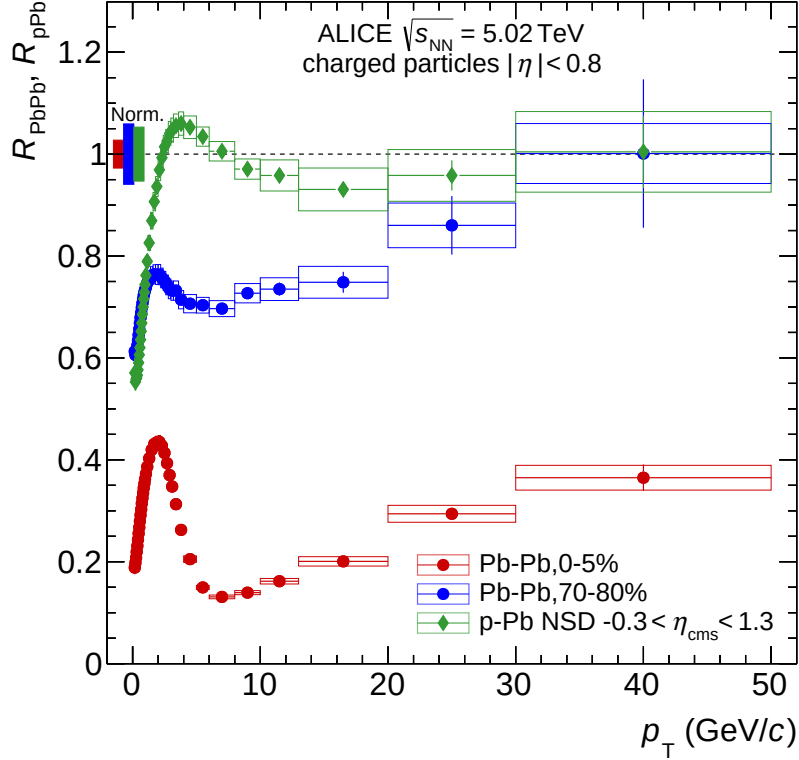


Figure 1.8. Nuclear modification factors measured in central (0–5% cent., red) and peripheral (70–80%, blue) Pb–Pb collisions and in p–Pb collisions (green) at $\sqrt{s_{\text{NN}}} = 5.02$ TeV [29].

partons created in binary inelastic nucleon-nucleon collisions, where large amounts of energy are suddenly released. Such partons "dress" themselves up with virtual quark-antiquark pairs and gluons that can fragment out creating a collimated shower of partons that later hadronise resulting in a spray of high-momenta particles, a *jet*.

The low-momenta (or "soft"; $p_T \lesssim 2$ GeV/c) particles are created as a result of inelastic collision of nuclear matter. While the hard particle production would scale with the number of binary collisions N_{coll} , the production of the soft particles would rather depend on the amount of nucleons N_{part} participating in a A–A collision [31]. The particle production mechanism in the intermediate p_T ($2 \lesssim p_T \lesssim 8$ GeV/c) region is an interplay between the two.

In case of Pb–Pb collisions at $\sqrt{s_{\text{NN}}} = 5.02$ TeV, the N_{part} is estimated to be ≈ 400 and $N_{\text{coll}} \approx 1900$ in 0 – 2.5% most central events [31]. Since the R_{AA} is normalized by N_{coll} (see Eq. 1.7) we can roughly estimate that the R_{AA} value in low- p_T region must be $N_{\text{part}}/N_{\text{coll}} \approx 0.21$ and indeed if we look at the Fig. 1.8 we find that the R_{AA} starts at around

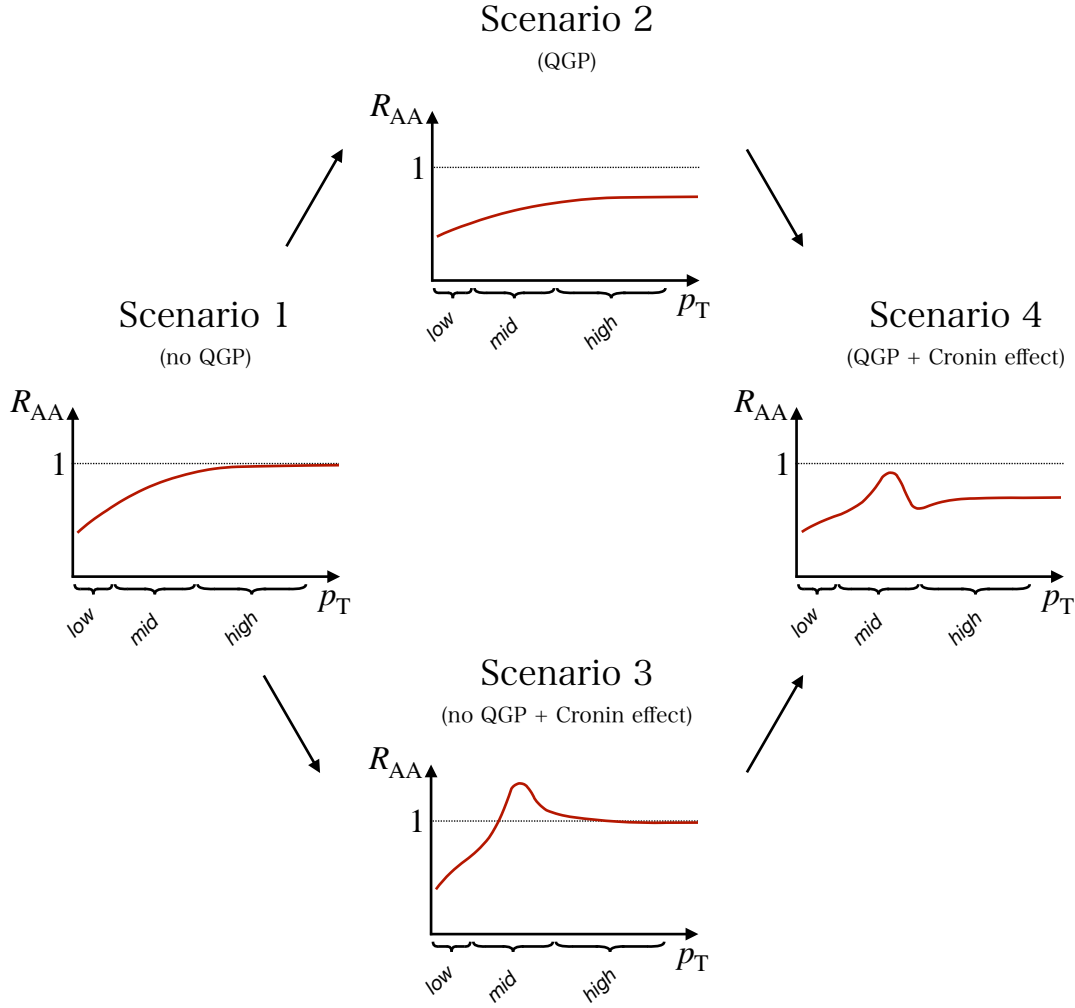


Figure 1.9. Different scenarios of nuclear modification factor p_T -dependence in A–A collisions.

0.2 for the case of 0 – 5% of most central events. In case of the absence of the QGP phase it would monotonically rise to unity saturating at high- p_T (Scenario 1 in Fig. 1.9) – i.e. at high- p_T the dN_{AA}/dp_T would match dN_{pp}/dp_T scaled up by $\langle N_{coll} \rangle$ manifesting the identity between the particle production mechanisms.

Assuming the presence of the QGP phase, "hard" particles are expected to be suppressed (Scenario 2 in Fig. 1.9) due to fact that the high-energy partons passing through the QGP medium would loose their energy/momentum. The loss might be caused by multiple scattering processes on the partons in the QGP and/or broadening of the jet cone as the production of quark-antiquark pairs is "cheaper" in a deconfined medium. Therefore, the resulting high- p_T hadrons get suppressed with respect to non-QGP events like pp collisions, leading to $R_{AA} < 1$ in Pb–Pb collisions in the high- p_T region (Fig. 1.8).

The rise in the intermediate p_T region is known as "Cronin effect" [32; 33] and to explain it we turn to the case of p–A collisions. Before the inelastic collision that produces "hard" partons happens, the partons in the projectile nucleon (proton) undergo a chain of subsequent elastic collisions with partons inside the target nucleus (A) therefore losing longitudinal momentum but gaining the transverse one (here we need to remember that at ultrarelativistic energies nuclei are highly contracted in the longitudinal direction). At the same time, it transfers additional momentum to the nuclear matter to the target nucleus [34]. As a result of this, the production of "soft" particles (coming from the interaction with the nuclear matter) gets shifted towards higher p_T , while "hard" particles gain additional transverse momenta. Both of these effects result in an enhancement of particles in mid- p_T range which is indeed the case for p–Pb measurements on Fig. 1.8. These effects can be present also in A–A collisions, as a projectile nucleus A is composed of many nucleons (i.e. an extended object).

And finally, if we join the Scenarios 2 and 3 in the Fig. 1.9, we would get the Scenario 4 that we indeed observe in A–A collisions (Fig. 1.8).

The suppression of hadron production in the high- p_T region is commonly defined as a "hard probe" of QGP and was historically among the very first of the sought-after QGP signatures to be experimentally confirmed [28]. Since its nature is explained via energy loss of hadrons in the strongly interacting color charged medium, it is particularly interesting to look at the R_{AA} spectra for the "color blind" particles – i.e. photons, $W^\pm$ and Z^0 vector gauge bosons – for which the medium is transparent. Indeed, Fig. 1.10 shows that the R_{AA} for such particles is unity in contrast to charged hadrons. This very elegant example provides an additional proof that the hadron suppression at high- p_T is a genuine signature of QGP.

While the suppression of high- p_T particles is an evidence of the presence of the QGP phase, there are probes related to the intrinsic QGP properties. In this section we will focus on the so-called *anisotropic flow* while another kind of collective behavior – radial flow – will be discussed later in the Sec. 2.2.

Considering a scenario in which identical nuclei collide with a non-zero impact parameter $b \neq 0$, the system created in such collision inherits the asymmetry of the nuclei overlapping region (Fig. 1.11).

This asymmetry is translated to the QGP fluid resulting in pressure gradients that depend on the direction in the transverse plane. In such case, particle p_T -differential spectra would depend on their azimuthal angle φ ². To characterize this effect, it is convenient to use the Fourier expansion and measure the corresponding coefficients:

²Due to the fact that even for the events with the same impact parameter its direction varies event-by-event, we need to define the angle φ with respect to the reaction plane – i. e. plane defined by the beam axis and the impact parameter vector $\vec{b}$ in each event.

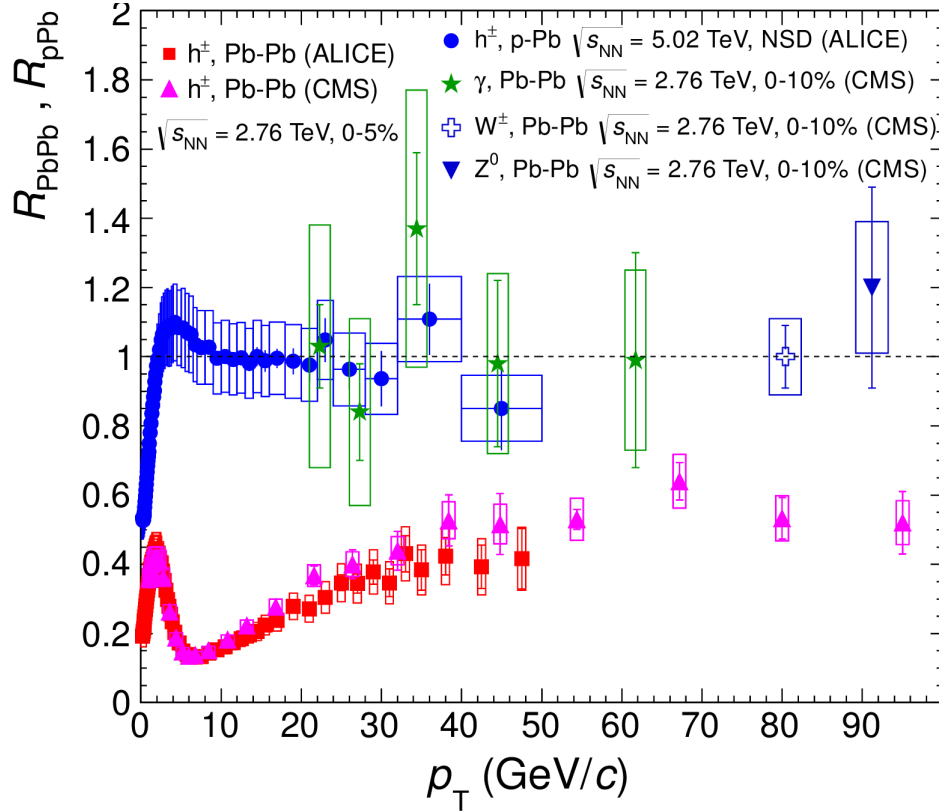


Figure 1.10. Nuclear modification factors R_{AA} measured by ALICE and CMS in Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV for charged hadrons $h^\pm$, direct photons γ , $W^\pm$ and Z^0 vector bosons. Additionally, Nuclear modification factors R_{pA} for charged hadrons measured by ALICE in p–Pb collisions at $\sqrt{s_{NN}} = 5.02$ [35].

$$\frac{dN_i}{dy p_T dp_T d\phi} = \frac{1}{2\pi} \frac{dN_i}{dy p_T dp_T} \left(1 + \sum_{n=1}^{\infty} 2 \left(v_n \cos(n\phi) + w_n \sin(n\phi) \right) \right), \quad (1.8)$$

where N_i is the number of particles of interest i .

Since we're considering the colliding system of identical nuclei, we can exploit the system's symmetries (Fig. 1.11): the symmetry with respect to x -axis would result in $\frac{dN_i}{dy p_T dp_T d\phi} = f(-\phi) = f(\phi)$ and therefore lead to the cancellation of $\sin(n\phi)$ terms; the symmetry w.r.t. y -axis would result in $\frac{dN_i}{dy p_T dp_T d\phi} = f(\phi) = f(\pi - \phi)$ and therefore the cancellation of odd terms with $\cos(n\phi)$. Using this, Eq. (1.8) can be rewritten as

$$\frac{dN_i}{dy p_T dp_T d\phi} = \frac{1}{2\pi} \frac{dN_i}{dy p_T dp_T} \left(1 + 2 \sum_{n=1}^{\infty} v_{2n} \cos(2n\phi) \right). \quad (1.9)$$

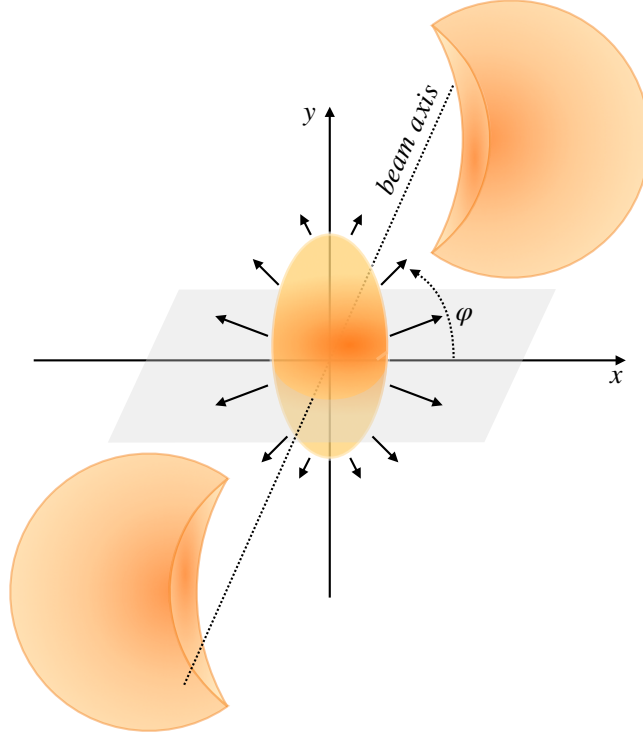


Figure 1.11. Schematic illustration of how the overlapping shape geometry translates to the geometry of the QGP fireball.

Thus, the non-zero values of the v_n coefficients would manifest the presence of the anisotropic flow in the system which was proposed as a signature of the QGP [36],

Since the first non-vanishing Fourier is v_2 we will focus on this. This coefficient is called the *elliptic flow* coefficient due to the shape the parametrization (1.9) produces (Fig. 1.12) and like the suppression of high- p_T particles, its measurement in A–A collisions was historically one of the first reported signatures of QGP [37].

As one can see from the Fig. 1.13 the elliptic flow v_2 measured in Pb–Pb collisions starts at its lowest in the most central collisions, characterized by a small impact parameter (the highest charged particle multiplicity), and grows with decreasing the centrality of the collision³, which can be intuitively understood by the fact that the overlapping region of nuclei (Fig. 1.11) is less symmetric in less central events. v_2 seems to reach its maximal values at $N_{\text{ch}} \approx 600$ and starts to descend towards lower multiplicities. This happens due to the fact that the QGP effects start to have a smaller contribution to the system's evolution process, almost vanishing in the most peripheral events. The behavior of v_2 measured in Xe–Xe collisions at $\sqrt{s_{NN}} = 5.44$ TeV is similar to the one measured in Pb–Pb but displaced

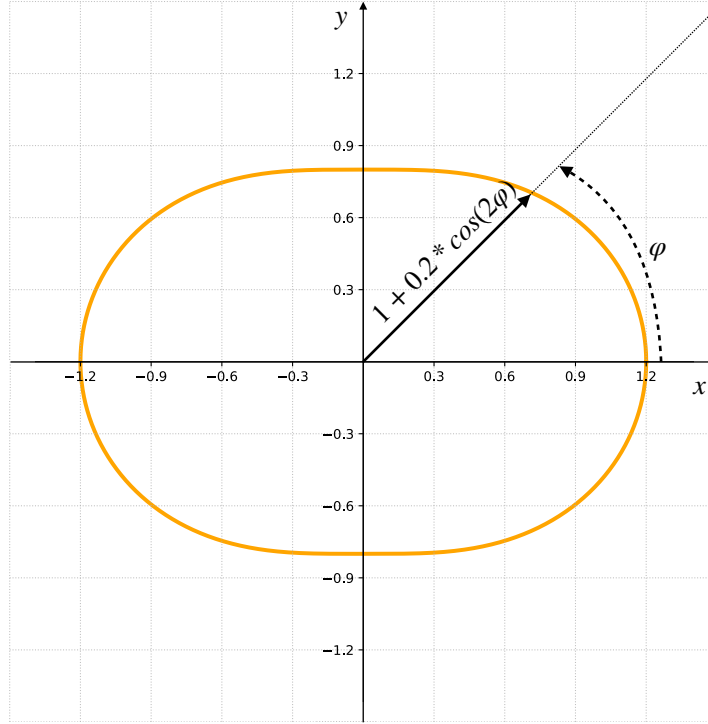


Figure 1.12. Schematic demonstration of the azimuthal distribution in the transverse plane for the expression in the brackets of Eq. (1.9) for the case of $v_2 = 0.1$ ($v_n = 0$ for $n > 2$).

towards lower N_{ch} . This can be intuitively understood by the fact that the ^{129}Xe nuclei have $\approx 38\%$ less nucleons (and therefore $\sim 15\%$ smaller radius) so even at the similar energy per nucleon the energy deposited to the system would be smaller resulting in smaller values of N_{ch} (plus there would be different asymmetry level due to the smaller radius of the nucleus).

The results from hybrid model simulations [39] including IP-Glasma to simulate impact-parameter-dependent initial conditions of the system, MUSIC for viscous relativistic hydrodynamics and UrQMD for hadronic transport also shown in Fig. 1.13 demonstrate a good agreement with the Pb–Pb and Xe–Xe experimental results. The calculations were performed assuming the constant shear viscosity over entropy density value of $\eta/s = 0.12$ during the hydrodynamical stage, i.e. QGP stage. This value is very close to the "most perfect fluid" lower bound of $\eta/s = 1/4\pi$ given by the Kovtun-Son-Starinets conjecture [40]. Therefore, the good agreement between the simulations and the experimental data suggests that QGP behaves as an almost perfect fluid.

Notably, v_2 at the lowest Pb–Pb multiplicities $N_{ch} \approx 30$ is compatible with the values measured in pp and p–Pb collisions where the QGP is not expected to be produced.

³Here we mean the events with small impact parameter and not the events with small value of the centrality percentile.

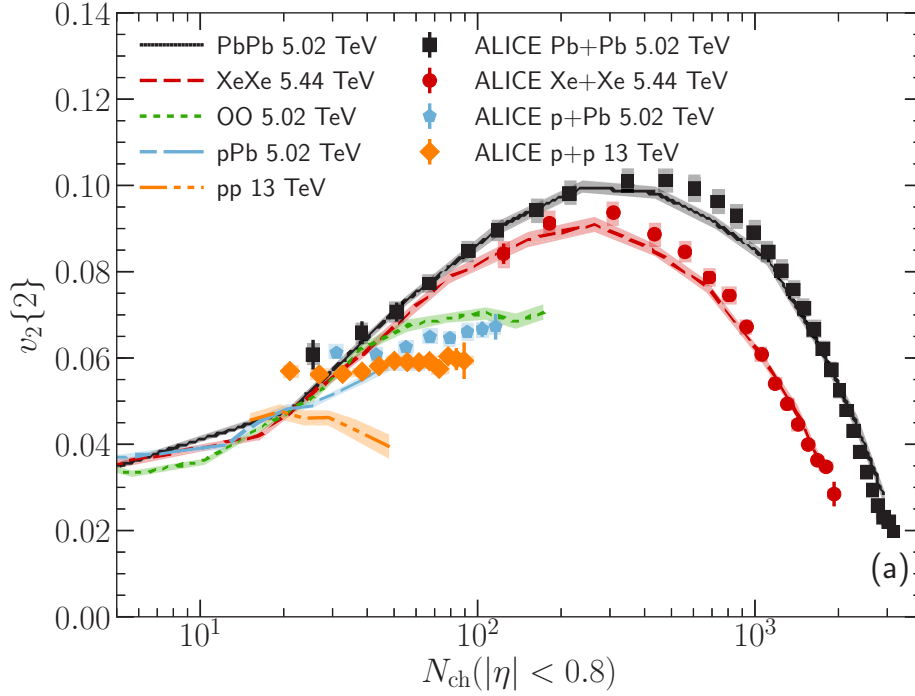


Figure 1.13. Elliptic flow coefficients v_2 in dependence on charged particle multiplicity measured in pp, p-Pb, Xe-Xe and Pb-Pb collisions measured by ALICE [38] and corresponding results from IP-Glasma + MUSIC + UrQMD model simulations [39].

The matter of a non-zero anisotropic flow in "small" systems suggesting the system's hydrodynamical nature and therefore presence of the QGP stage is one of open questions in HIC physics. Indeed, as one can see in Fig. 1.13 the model simulations noticeably underestimate the experimental results for pp and p-Pb systems while providing decent agreement for Pb-Pb and Xe-Xe at similar N_{ch} . This might be due to the fact that the calculations do not include the possible non-flow contributions [39]. Nevertheless, it could also be an evidence of inapplicability of the formalism used for "large" systems to the "small" ones.

With this we would like to conclude the current chapter in which we briefly discussed the HIC evolution (Fig. 1.2), gave the phenomenological explanation for the main characteristics of system's initial state continuing with "hard" and "soft" probes of QGP. In this chapter we didn't address the hadronization stage and the system's properties accessible at it. We will explore the possibility of accessing the space-time properties of the system created in HICs at its hadronization stage with femtoscopy in the following Chapter of this manuscript.

Chapter 2

Femtoscopy

Femtoscopy is a technique used in HICs for measuring the space-time properties of a particle-emitting source created in a collision based on momentum correlations caused by the interaction between hadrons in the final state (freeze-out stage in Fig. 1.2). It is usually referred that the idea of femtoscopy takes its roots back to Hanbury Brown–Twiss (HBT) interferometry [41] – a method of measuring the angular size of discrete sources of radiation through correlations of their intensities. This method was developed to measure angular size of stars and was successfully applied to measure the size of Sirius in 1956 [42].

In particle physics, the idea of measuring pair correlations was successfully applied in 1960 to explain the angular distribution of pions produced in $p\bar{p}$ annihilation [43] showing the importance of taking into account final-state interaction (FSI) (in this case only the effects of Bose-Einstein statistics were considered).

In this Chapter, the principles and concepts of femtoscopy that are essential pillars of this work are introduced with focus on the modern formalism.

2.1 Basic concepts

In order to explain the basic concept of how the distance between two emission points can be measured, we first consider a simple example of two point-like pion π^0 sources S_1 and S_2 with their spatial coordinates $\vec{r}_1$ and $\vec{r}_2$ in a given frame as shown in Fig. (2.1). The convenience of using neutral pions is due to the absence of the Coulomb interaction which simplifies the calculations dramatically. The matter of taking into account the Coulomb and the strong interactions will be discussed in the next Chapter.

Now, supposing that we measured two pions of momenta $\vec{p}_1$ and $\vec{p}_2$ with detectors D_1 and D_2 positioned at $\vec{d}_1$ and $\vec{d}_2$. If the sources are so close to each other that we cannot distinguish them, there are two possible scenarios:

1. D_1 measured a pion from S_1 and D_2 measured a pion from S_2 (blue lines in 2.1);
2. D_1 measured a pion from S_2 and D_2 measured a pion from S_1 (red lines in 2.1);

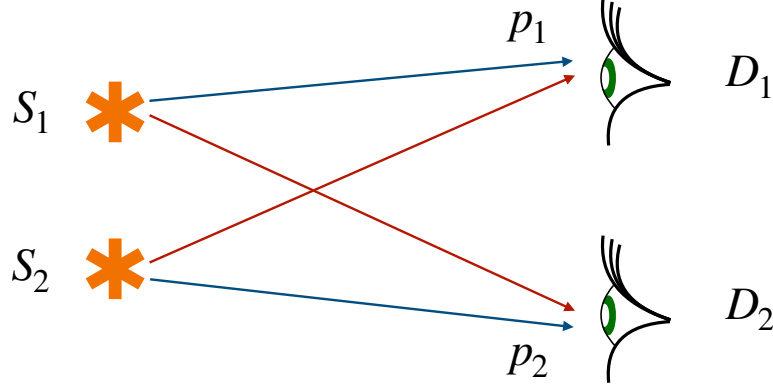


Figure 2.1. Schematic representation of interferometric experiment on π^0 pion pairs coming from different sources S_1 and S_2 , being measured by detectors D_1 and D_2 .

Considering pions as plane waves we can write the wave function (WF) of the system of two pions measured at $\vec{d}_1$ and $\vec{d}_2$ with momenta $\vec{p}_1$ and $\vec{p}_2$:

$$\Psi \propto e^{i(\vec{d}_1 - \vec{r}_1)\vec{p}_1} e^{i(\vec{d}_2 - \vec{r}_2)\vec{p}_2} + e^{i(\vec{d}_1 - \vec{r}_2)\vec{p}_1} e^{i(\vec{d}_2 - \vec{r}_1)\vec{p}_2}. \quad (2.1)$$

The first term in the Eq. (2.1) corresponds to the Scenario 1 and the second – to the Scenario 2. The probability of measuring two pions with momenta $\vec{p}_1$ and $\vec{p}_2$:

$$P = |\Psi|^2 = \Psi^* \Psi \propto 2 + e^{i(\vec{p}_2 - \vec{p}_1)(\vec{r}_2 - \vec{r}_1)} + e^{-i(\vec{p}_2 - \vec{p}_1)(\vec{r}_2 - \vec{r}_1)} \propto 1 + \cos(\Delta\vec{p}\Delta\vec{r}). \quad (2.2)$$

The final probability depends only on the measured momentum difference $\Delta\vec{p} = \vec{p}_2 - \vec{p}_1$ and the distance between the emission points $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$ but not on the positions of the detectors D_1 and D_2 . It means that if we could measure not only the momenta difference $\Delta\vec{p}$ but also the probability P of detecting a pion pair corresponding to that difference, we could extract the distance between the sources.

This very simple example explains the motivation of femtoscopy. Assuming that the sources S_1 and S_2 are not point-like but have some shapes and the probabilities of a particle to be emitted at a certain space point $\vec{r}$ are given by some distributions $S_1(\vec{r})$ and $S_2(\vec{r})$, called *source functions*. In this case the spectrum of pair relative momentum $C(\Delta\vec{p})$ measured from such sources can be expressed in the following way:

$$C(\Delta\vec{p}) = \frac{\int d^3r_1 d^3r_2 S_1(\vec{r}_1) S_2(\vec{r}_2) |\Psi(\Delta\vec{p}, \Delta\vec{r})|^2}{\int d^3r_1 d^3r_2 S_1(\vec{r}_1) S_2(\vec{r}_2)}. \quad (2.3)$$

The integral in Eq. (2.3) can be rewritten in the following form:

$$C(\Delta\vec{p}) = \int d^3r S'(\vec{r}^*) |\Psi(\Delta\vec{p}, \vec{r}^*)|^2, \quad (2.4)$$

where $\vec{r}^* := \Delta\vec{r}$ and

$$S'(\vec{r}^*) = \frac{\int d^3r_1 d^3r_2 S_1(\vec{r}_1) S_2(\vec{r}_2) \delta(\vec{r}^* - \vec{r}_2 + \vec{r}_1)}{\int d^3r_1 d^3r_2 S_1(\vec{r}_1) S_2(\vec{r}_2)}. \quad (2.5)$$

In the Eq. (2.4) the WF part $|\Psi(\Delta\vec{p}, \vec{r}^*)|^2$ can be interpreted as a kernel of an integral transform from a coordinate-space basis to a momentum one. Another important information one can derive from Eq. (2.4-2.5) is that any direct information about the structure of each source is lost and only the distribution of the relative positions of the emitted particle pairs is left. This may happen if one measures for example non-identical pairs (even if they are emitted in the result of the same collision). As for the work described in this manuscript, the focus will be on pairs of identical particles, we will proceed with the corresponding formalism.

If two identical particles come from the same source (Fig. 2.2), then $S_1(\vec{r}) \equiv S_2(\vec{r}) \equiv S(\vec{r})$. We will also assume that the probability of a particle to be emitted at point $\vec{r}$ is given by a spherically symmetric (i.e. $\sigma = \sigma_x = \sigma_y = \sigma_z$) Gaussian distribution:

$$S(\vec{r}) = e^{-\frac{(r_x - \mu_x)^2}{2\sigma_x^2} - \frac{(r_y - \mu_y)^2}{2\sigma_y^2} - \frac{(r_z - \mu_z)^2}{2\sigma_z^2}} = e^{-\frac{(\vec{r} - \vec{\mu})^2}{2\sigma^2}}. \quad (2.6)$$

This means that the probability of particle emission depends not on the three components of a radius vector $\vec{r}$ but on a single value of $|\vec{r} - \vec{\mu}|$. Therefore, the particle source can be characterized by only *one* parameter σ .

To further simplify the calculations it is convenient to consider the pairs in their Center-of-Mass (CM), or Pair Reference Frame (PRF)¹. Generally, the CM system of a pair of particles with masses m_1 and m_2 is defined such that the sum of their momenta $\vec{p}_1^*$ and $\vec{p}_2^*$ in that frame is zero and the CM position is

$$\begin{aligned} \vec{p}_1^* + \vec{p}_2^* &= 0, \\ \vec{r}_{CM} &= \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}. \end{aligned} \quad (2.7)$$

¹PRF is the common alias for the CM frame used in femtoscopic studies.

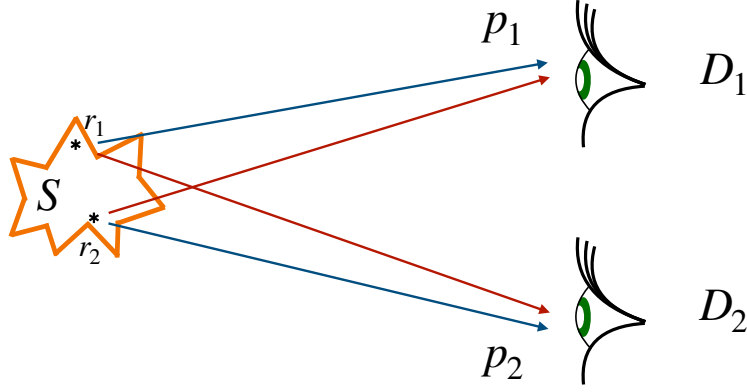


Figure 2.2. Schematic representation of interferometric experiment on π^0 pion pairs coming from the same source S , being measured by detectors D_1 and D_2 .

For the identical pairs, i.e. $m_1 = m_2$, the $\vec{r}_{CM} = \frac{\vec{r}_1 + \vec{r}_2}{2}$. Here comes the first profit from moving to the CM system: $\vec{\mu} = \vec{r}_{CM}$ in Eq. (2.6). Another one comes from the fact that (as we discuss later) the calculation of the WF is also more convenient to perform in the CM frame. In light of these considerations Eq. (2.5) can be simplified:

$$\begin{aligned}
 S'(\vec{r}^*) &= \frac{\int d^3r_1 d^3r_2 S(\vec{r}_1) S(\vec{r}_2) \delta(\vec{r}^* - \vec{r}_2 + \vec{r}_1)}{\int d^3r_1 d^3r_2 S(\vec{r}_1) S(\vec{r}_2)} = \frac{\int d^3r_2 S(\vec{r}_2 - \vec{r}^*) S(\vec{r}_2)}{\int d^3r_1 d^3r_2 S(\vec{r}_1) S(\vec{r}_2)} = \\
 &= \frac{S(\vec{r}_1) \int d^3r_2 S(\vec{r}_2)}{\int d^3r_1 d^3r_2 S(\vec{r}_1) S(\vec{r}_2)} = \frac{S(\vec{r}_1)}{\int d^3\vec{r}_1 S(\vec{r}_1)} = \frac{e^{-\frac{(\vec{r}_1 - \vec{r}_{CM})^2}{2\sigma_1^2}}}{\int d^3r_1 e^{-\frac{(\vec{r}_1 - \vec{r}_{CM})^2}{2\sigma_1^2}}} = \frac{e^{-\frac{\vec{r}^{*2}}{2\sigma_1^2}}}{\int d^3r_1 e^{-\frac{(\vec{r}_1 - \vec{r}_2)^2}{8\sigma_1^2}}} = \quad (2.8) \\
 &= \frac{1}{16\pi\sqrt{2\pi}\sigma_1^3} e^{-\frac{\vec{r}^{*2}}{8\sigma_1^2}}.
 \end{aligned}$$

Under the assumption of an uncorrelated (covariance is 0) particle emission, the dispersion σ of the relative distance $\vec{r}^*$ between the particles in a pair can be expressed in terms of their separate dispersions:

$$\sigma_{12}^2 = \sigma_1^2 + \sigma_2^2. \quad (2.9)$$

Since the particles in a pair come from the same source with the same dispersion (i.e. $\sigma_1 = \sigma_2$), we can rewrite Eq. (2.8):

$$S'(\vec{r}^*) = \frac{1}{8\sqrt{\pi^3}\sigma_{12}^3} e^{-\frac{\vec{r}^{*2}}{4\sigma_{12}^2}}. \quad (2.10)$$

Similarly to the Eq. (2.6), the probability of a pair of particles to be emitted with a given relative distance $\vec{r}^*$ depends on its absolute value – $S'(\vec{r}^*) = S'(r^*)$, where $r^* := |\vec{r}^*|$ – and can be described by a single value of σ_{12} . This form of the pair *source function* parametrization is called *one-dimensional* and commonly used in femtoscopy to model a spherically symmetric Gaussian source with the following alias: $R_{inv} := \sigma_{12}$, where R_{inv} is the one-dimensional *effective pair source size*² or *radius* (from now on we will use this notation).

Another commonly used notation in one-dimensional femtoscopy is $k^* = \frac{1}{2}|\vec{p}_2^* - \vec{p}_1^*|$ (the motivation of using this form will be discussed later in Sec. 3.1). Taking into account beforementioned, we can rewrite Eq. (2.4) as

$$C(k^*, R_{inv}) = \int d^3r^* S(r^*, R_{inv}) |\Psi(k^*, r^*)|^2, \quad (2.11)$$

$$S(r^*, R_{inv}) = \frac{1}{8\sqrt{\pi^3} R_{inv}^3} e^{-\frac{r^{*2}}{4R_{inv}^2}}.$$

The expression for $C(k^*, R_{inv})$ is commonly referred as the Koonin-Pratt formula [44; 45].

In the case of pairs of identical particles one can see from Eq. (2.11) that measuring the relative momentum difference k^* spectrum it becomes possible to extract the effective size R_{inv} by fitting the spectrum with this formula. This is the main motivation/goal of femtoscopy.

Now, we want to demonstrate a couple of illustrative examples of how would such spectra look like if we consider only the effects of quantum statistics (QS). A pair of free particles in their center of mass is described by the Schrodinger's equation³ in terms of their relative momentum k^* and relative distance r^* :

$$\frac{d^2\psi}{dr^{*2}} + k^{*2}\psi = 0. \quad (2.12)$$

The general solution to the Eq. (2.12) is a linear combination of sine and cosine:

$$\psi_{k^*}(r^*) = A \cos(k^* r^*) + B \sin(k^* r^*). \quad (2.13)$$

In order to define the coefficients A and B we will consider two illustrative cases:

²The term "effective" is used as the R_{inv} relates to the dispersion of the pair relative distance in the correlation region and doesn't define the whole particle-emitting source.

³Here we consider only the s-wave state for simplicity. See more about the case of higher partial waves in the Section 3.1

1. the particles in a pair are bosons $\implies$ the WF must be symmetrized, i.e. $\psi_{k^*}(r^*) = \psi_{k^*}(-r^*) \implies B = 0, A = 1$
2. the particles in a pair are fermions $\implies$ the WF must be antisymmetrized, i.e. $\psi_{k^*}(-r^*) = -\psi_{k^*}(r^*) \implies A = 0, B = 1$

Taking into account the laws of QS, we can express the expected spectrum of boson pairs as

$$C_b(k^*, R_{inv}) = \int dr^* e^{-\frac{r^{*2}}{4R_{inv}^2}} \cos^2(k^* r^*) = \left(1 + e^{-4R_{inv}^2 k^{*2}}\right). \quad (2.14)$$

Similarly, for a pair of fermions:

$$C_f(k^*, R_{inv}) = \int dr^* e^{-\frac{r^{*2}}{4R_{inv}^2}} \sin^2(k^* r^*) = \left(1 - e^{-4R_{inv}^2 k^{*2}}\right). \quad (2.15)$$

The practical effects of QS on measured pair relative momentum spectra are shown in Fig. 2.3. In the case of boson pairs an interferometric peak in the low k^* region is observed, whereas in case of fermion pairs it would be a depletion. The interpretation of this in terms of QS is that for fermions it is prohibited to occupy the same quantum state ($k^* \rightarrow 0$ means that they have same momenta) by the Pauli's law. Instead for bosons the situation is the opposite. The fact that the femtoscopic signal is present in the region with small pair relative momentum k^* means that the spatial region accessible by femtoscopy doesn't cover the whole fireball but only a part of it that emits particles with similar momenta. This region is commonly referred to as the region of *homogeneity*.

Generalizing, when there is a positive correlation, i.e. the particles in a pair are attracted to each other, $C(k^*) > 1$, when the correlation is negative, i.e. the particles in a pair are repelled by each other, $C(k^*) < 1$, and when the correlation is absent, $C(k^*) = 1$. Therefore the shape of the spectrum contains not only the information about the source but also about the interaction in a pair, which is not unexpected since the correlation function contains also the pair WF (2.11).

Of course, in reality we don't deal only with the effects of the QS but other Final State Interactions (FSI) as well, such as the Coulomb and the strong ones. The matter of taking into account them will be discussed in the Section 3.1.

Another assumption we made is related to the source structure. In our example we considered it to be Gaussian and spherically symmetric (2.11) which illustrates an idealized case. The case of a non-Gaussian particle source has been of a particular interest in the past years (e.g. Ref. [46; 47]) but since it's mostly relevant to "small" systems (such as proton-proton ultrarelativistic collisions) and not to the "large" ones (such as Pb-Pb collisions

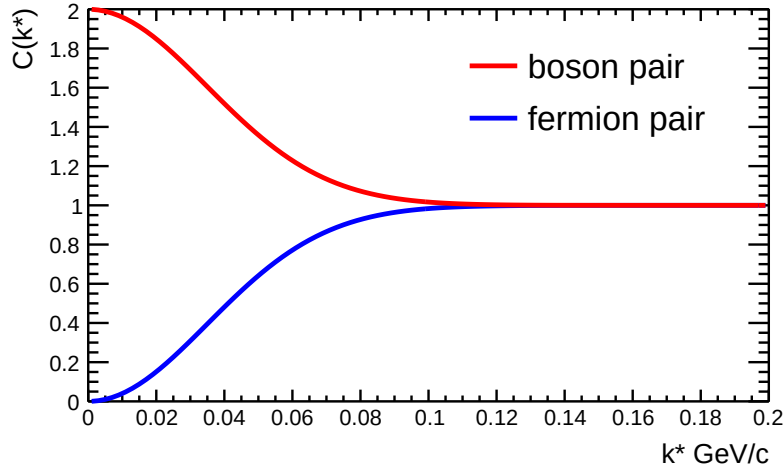


Figure 2.3. Interferometric effects of QS for a source size of $R_{inv} = 2$ fm.

that we will consider) [48], we will not discuss this in our study. Instead, the non-sphericity of the source is more relevant to the "large" systems. Thus, let us introduce the corresponding formalism.

2.2 Three-dimensional parametrization of the source

Although, the spatial information about the particle-emitting source can be accessed via one-dimensional femtoscopy, the latter is based on the assumption of a spherical source which might not be the case in HICs, especially the non-central ones (Sec. 1.3). To account for the asymmetric shape, the *three-dimensional* source parametrization is used. As we will see in this section, such parametrization allows not only to access the spatial information in three dimensions but also the temporal information characterizing the source evolution.

The commonly used coordinate system for three-dimensional femtoscopy is the Longitudinally Co-Moving System (LCMS) [49; 50]. It is defined as a system in which the total longitudinal momentum of a pair is 0 ($p_{z,1} + p_{z,2} = 0$) with the z axis (or *long*) aligned towards the beam, the x axis (or *out*) is directed along the pair total transverse momentum $\vec{P}_T = \vec{p}_{T,1} + \vec{p}_{T,2}$ and the y axis (or *side*) is perpendicular to the other two (Fig. 2.4).

The Gaussian source function in the LCMS is parameterized as

$$S(\vec{r}_{LCMS}, \vec{R}_{LCMS}) = \frac{1}{8\sqrt{\pi^3} R_{out} R_{side} R_{long}} e^{-\frac{r_{out}^2}{4R_{out}^2} - \frac{r_{side}^2}{4R_{side}^2} - \frac{r_{long}^2}{4R_{long}^2}}, \quad (2.16)$$

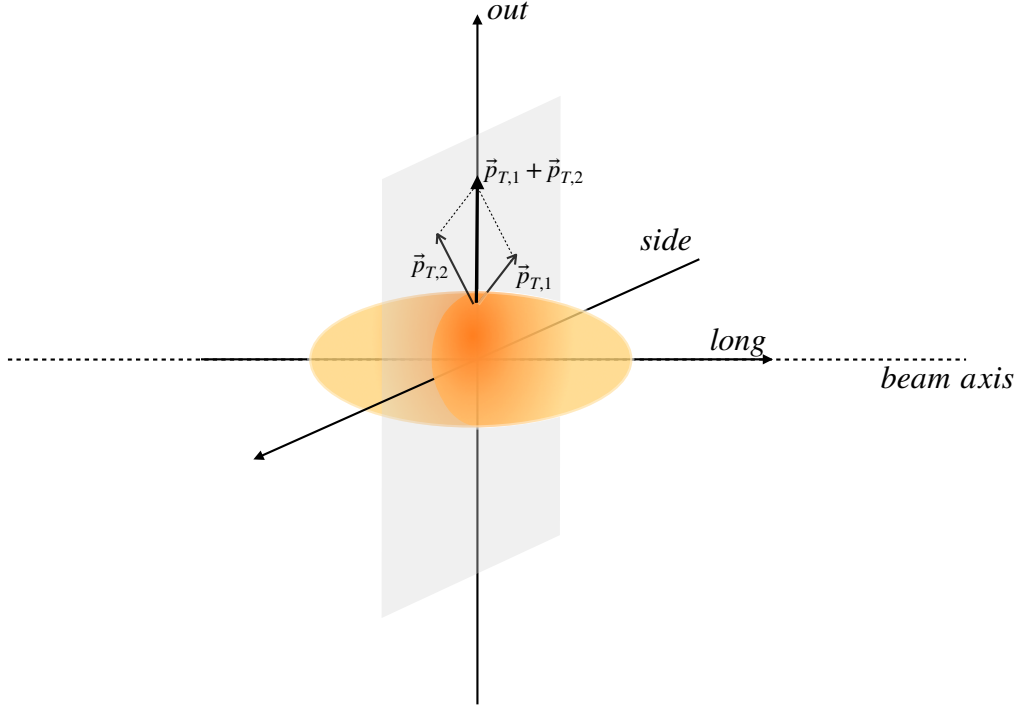


Figure 2.4. Longitudinally Co-Moving System (LCMS) frame.

where R_{out} , R_{side} and R_{long} are effective source sizes in the LCMS frame also commonly referred as *three-dimensional remtoscopic radii*.

Comparing to the PRF system where pairs are measured at the moment of their emission in the center of mass, in the LCMS frame (Fig. 2.4) there is a boost in the *out* direction. Thus, the pair relative distance in the *out* direction in the LCMS r_{out} would relate to the corresponding relative distance of emission points in the PRF r_{out}^* through the Lorentz transformation:

$$r_{out} = x_2 - x_1 = \gamma_T(x_1^* + v_T t_1^*) - \gamma_T(x_2^* + v_T t_2^*) = \gamma_T(r_{out}^* + v_T \Delta t^*), \quad (2.17)$$

where v_T is the average pair transverse velocity and γ_T is the corresponding Lorentz-factor. Then the corresponding relation between the corresponding dispersions of this values (i.e. source sizes) would be:

$$\begin{aligned}
R_{out}^2 &= \mathbb{E} \left[(r_{out} - \mathbb{E}(r_{out}))^2 \right] = \mathbb{E} \left[(\gamma_T(r_{out}^* + v_T \Delta t^*) - \mathbb{E}(\gamma_T(r_{out}^* + v_T \Delta t^*)))^2 \right] = \\
&= \gamma_T^2 \mathbb{E} \left[(r_{out}^* - \mathbb{E}(r_{out}^*) + v_T(\Delta t^* - \mathbb{E}(\Delta t^*)))^2 \right] = \gamma_T^2 \left\{ \mathbb{E} \left[(r_{out}^* - \mathbb{E}(r_{out}^*))^2 \right] + \right. \\
&\quad \left. + v_T^2 \mathbb{E} \left[(\Delta t^* - \mathbb{E}(\Delta t^*))^2 \right] + 2v_T \mathbb{E} \left[(r_{out}^* - \mathbb{E}(r_{out}^*)) (\Delta t^* - \mathbb{E}(\Delta t^*)) \right] \right\}. \tag{2.18}
\end{aligned}$$

The **first** and the **second** terms are just the dispersions of r_{out}^* and Δt^* which we denote as R_{out}^* and σ_τ ; whereas the **third** term cancels out, assuming that the coordinates and the times of emission are uncorrelated. Then we can rewrite

$$R_{out}^2 = \gamma_T^2 (R_{out}^{*2} + v_T^2 \sigma_\tau^2). \tag{2.19}$$

From Eq. (2.19) we can see that the source size component R_{out} in the LCMS contains both information about the transverse size of the emission region as well as the duration of particle emission. To disentangle these contributions and access the temporal information one could look at the ratio R_{out}/R_{side} since the *side* component is also laying in the transverse plane but due to the absence of boost (i.e. $R_{side} = R_{side}^*$) contains only the information about the source size.

Since this thesis is dedicated to the systems created in ultrarelativistic HICs, the QGP phase must be considered. Thus, the particle emission occurs at the kinetic freeze-out stage (Fig. 1.2), therefore $R_{out,side,long}^*$ are the effective sizes of the source at the freeze-out and σ_τ is the effective particle emission duration. Now, we would like to discuss how one can explore the dynamics of the QGP evolution studying the behavior of these scales.

In the simplest case of a pure thermal emission of particles, described by the laws of thermodynamics (Fig. 2.5, left), the source would expand with no acceleration and particles would have similar thermal velocities defined by the temperature T and the mass of a given particle species m_p : $V_{th} \sim \sqrt{\frac{T}{m_p}}$. Since particles (of the same species) have similar velocities and therefore momenta, the homogeneity region can approach the whole source.

In reality the QGP fireball evolves "explosively". The blast-wave would lead to a collective expansion of the source with a radial gradient of the collective velocity V_{coll} resulting in its larger values towards the surface (Fig. 2.5, right). The collective velocity contributes to the total velocity of particles $V = V_{th} + V_{coll}(r)$ and the closer the emission point to the surface the larger this contribution is. Thus, the particles with high transverse momenta p_T are more likely to come from the outer parts of the source, resulting in a smaller homogeneity region for such particles. Also, since heavier particles have smaller

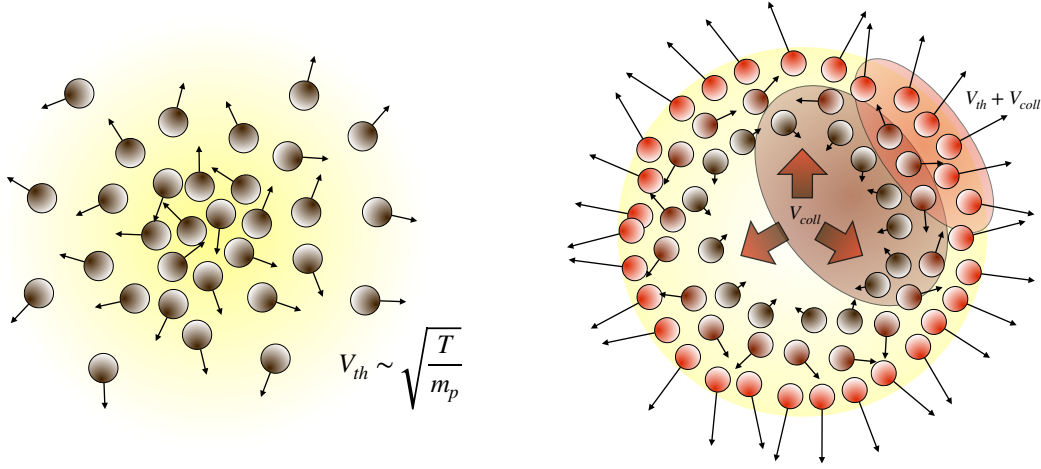


Figure 2.5. Left: schematic picture of particle emission by a thermal source. Right: schematic picture of particle emission by an "explosive" source incorporating radially dependent collective velocity.

thermal velocities they would be dominated by the radial flow and pushed towards the surface resulting in earlier emission and reduction of the homogeneity region.

As we discussed before (see Sec. 1.2) the evolution of the system is convenient to consider in the frame with cylindrical symmetry and boost-invariance in the longitudinal direction (Fig. 2.6), which is the reason why the LCMS is defined as a system with zero total pair momentum along the beam. Assuming that the emission happens at the Bjorken's proper time $\tau := \tau_B$ (1.2) of the freeze-out and that the radial flow is present only in the transverse plane, i.e. the expansion in the longitudinal direction is just thermal, the longitudinal size of the system at mid-rapidity could be simply expressed as

$$R_{long} = R_{long}^* \sim \tau V_{th} \sim \tau \sqrt{\frac{T}{m_T}}, \quad (2.20)$$

where $m_T = \sqrt{E^2 - p_z^2} = \sqrt{m^2 + p_T^2}$ is particle's transverse mass, Lorentz-invariant along the beam axis z .

Thus, measuring the size in the longitudinal direction R_{long} one can access the time of particle emission. In reality the R_{long} dynamics is more complex and Eq. (2.20) reflects only the asymptotic behavior at $m_T \gg T$ [51] in the absence of the transverse flow [52].

Overall, the homogeneity region would scale with m_T in all the three dimensions (Fig. 2.6). And since $m_T = \sqrt{m^2 + p_T^2}$ it would mean that the region for particles of the same mass would scale with p_T and vice versa – for different particles with the same p_T it would scale with mass. Both of this scalings are observed in the experimental results, however

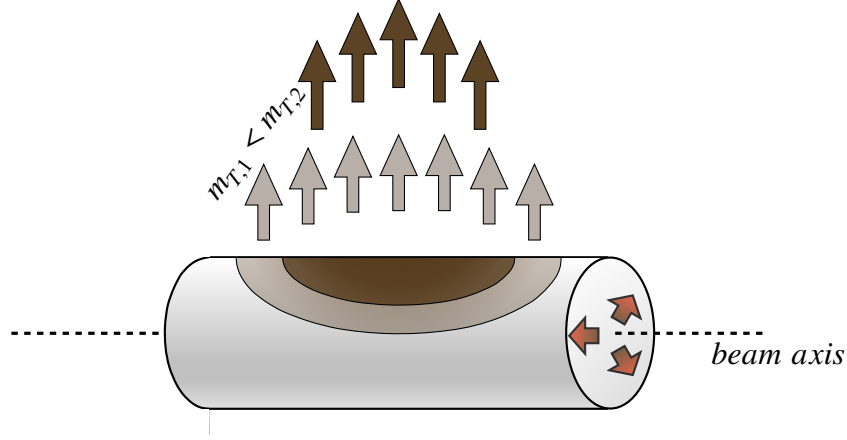


Figure 2.6. Schematic picture of a particle emission source in LCMS and scaling of the homogeneity region with the particles transverse mass m_T .

the common m_T -scaling was found to be broken. The discussion about the experimental femtoscopic results in HICs is given in the next Section, but before we would like to highlight the main concluding points of this one:

The main highlights of this section are:

- R_{out} – related to the transverse geometrical size of the source and the duration of particle emission σ_τ ; R_{side} – just to the geometrical size; R_{long} – to the time of particle emission τ .
- The femtoscopic radii are expected to scale with m_T .

2.3 Femtoscopy in heavy-ion collisions

In this section we omit the discussion on how the experimental femtoscopic function is obtained and the parameters are extracted. It will be discussed later in the Sec. 5.4 and 5.5 of the Chapter 5 dedicated to the experimental part of this thesis. Instead, here we will give a brief overview of the femtoscopic studies carried out in HIC experiments.

As we learnt in the previous section, femtoscopic observables can provide important spatio-temporal information about the particle-emitting sources created in HICs. Moreover, these observables were proposed as a benchmark for the creation of the QGP in HIC [53; 54],

e. g. m_T -dependence⁴ of the radii, reflecting presence of the radial flow and the "explosive behavior" of the source, and especially the R_{out}/R_{side} ratio as due to the particle emission phase it was expected to be greater than 1.

Relativistic Heavy Ion Collider (RHIC) was expected to be the first experimental facility to confirm the existence of the QGP state. The first femtoscopic study with identical charged pion pairs was performed by the STAR Collaboration at the RHIC in Au-Au collisions at $\sqrt{s_{NN}} = 130$ GeV [55]. The main femtoscopic QGP signature R_{out}/R_{side} was found to be ~ 1 while in case of the QGP presence it was expected to be $\gtrsim 1.5$ at RHIC energies [56] (Fig. 2.7). Thus, the experimental results did not confirm nor disprove the creation of QGP as some model studies suggested that the results may still be explained assuming the creation of the QGP [57]. It was clear that more data were needed.

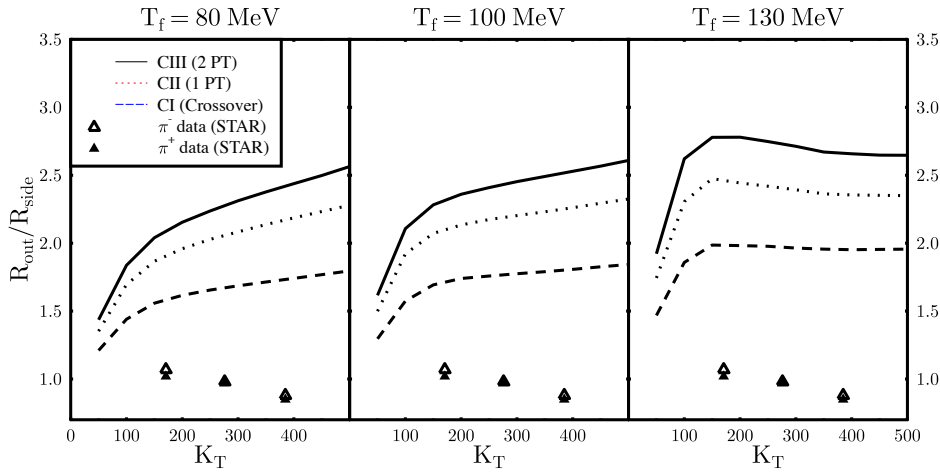


Figure 2.7. Chiral $SU(3) \times SU(3)$ model calculations of R_{out}/R_{side} ratio for central Au–Au collisions at $\sqrt{s_{NN}} = 130$ GeV for different freeze-out temperatures T_f and different phase transition scenarios compared to the experimental data from STAR [55].

Just a few months later the same year another STAR paper [28] showed its new measurements of the nuclear modification factor R_{NN} that favored QGP creation scenario in Au-Au collisions at $\sqrt{s_{AA}} = 130$ GeV [28]. But even when the creation of the QGP was confirmed at the RHIC [28; 37], one of the main QGP signatures – the R_{out}/R_{side} ratio – was still at or slightly below 1 even at the top RHIC energy [58] (Fig.2.8). This phenomenon became known as the "HBT puzzle".

⁴As we discuss pair femtoscopy, the transverse momentum and mass parameters that we introduced in the section before must be redefined for pairs: $m_T = \sqrt{m^2 + k_T^2}$, where $m = \frac{m_1 + m_2}{2}$ – average mass of a pair of the particles 1 and 2; $k_T = \frac{|\vec{p}_{T,1} + \vec{p}_{T,2}|}{2}$ – pair average transverse momentum.

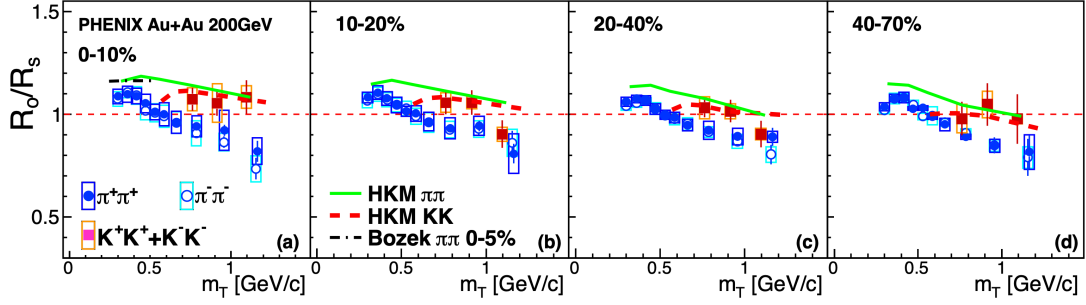


Figure 2.8. RHIC results in Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV [58]. R_{out}/R_{side} ratio for charged pions and kaons as a function of m_T .

Later on when the LHC started operating, the first ALICE's femtoscopic measurement with identical charged pion pairs for the 5% most central Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV (the QGP creation was not doubted at this energy and confirmed by other observables) again showed that the ratio $R_{out}/R_{side} \sim 1$ [59] (Fig. 2.9). Moreover, it was even smaller then for the RHIC [60] although one would expect larger emission duration for a larger colliding system at a higher colliding energy. However, the HydroKinetic Model (HKM) calculations [61] did provide a rather good description of the experimental results through a careful treatment of the collective transverse flow.

The m_T -dependence of R_{long} obtained in this work [59] allowed ALICE to extract the pion decoupling time τ (Fig. 2.10) which was (as expected) noticeably higher then those obtained for lower energies by other HIC experiments. However, the pion decoupling time $\tau_\pi \approx 10.4$ fm/c extracted in this work was obtained without taking into account the transverse expansion.

Later ALICE publishes a paper on kaon 3D femtoscopy [62] showing the the kaon radii are higher then the pion ones at similar m_T suggesting that the m_T -scaling is broken. The decoupling time for kaons τ_K was extracted considering also the transverse expansion [52] and resulted in $\tau_K = 11.6$ fm/c. The pion $R_{long}(m_T)$ dependence (Fig. 2.9) was also refitted taking account the transverse expansion which resulted in $\tau_\pi = 9.5$ fm/c.

The higher radii and maximal emission time for kaons was interpreted through the influence of the rescattering processes [63] involving the recombination of kaons with pions resulting to the regeneration of the $K^*(892)$ (with the life-time of $\sim 4 - 5$ fm/c) which decays into pions and kaons again. Since the kaons are more sensitive to this process, it would result in a delay of the average kaon emission time increasing the corresponding effective source sizes as well (Fig. 1.4). Taking into account the rescattering phase the HKM calculations [63] provide a good description of the experimental data.

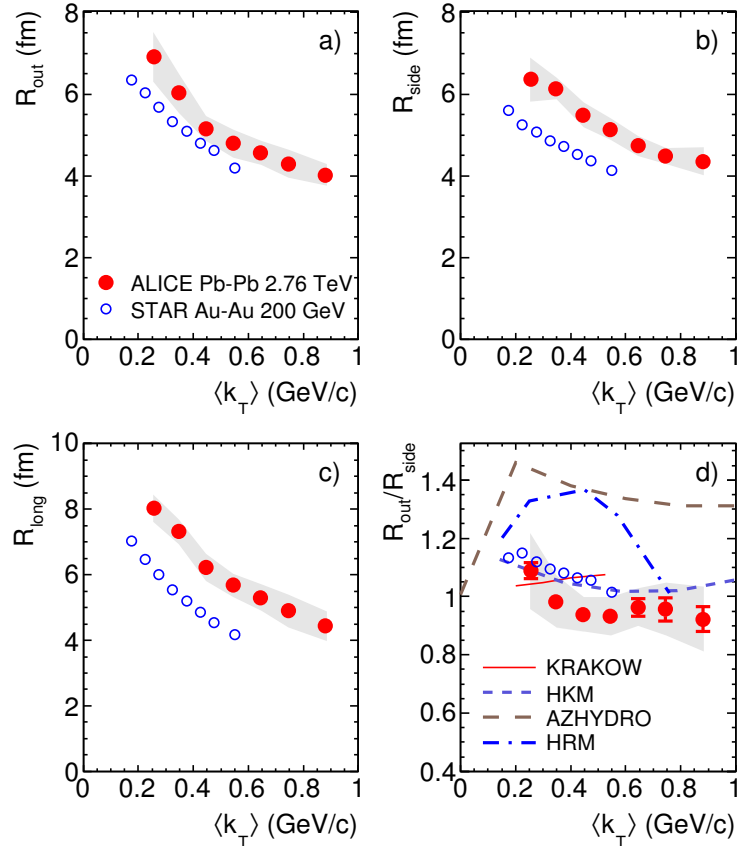


Figure 2.9. ALICE results on pion femtoscopy for the 5% most central Pb—Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, as function of k_T [59] in comparison to RHIC results [60] and model calculations.

Intuitively, in order to add more information to the common hadron m_T -dependence and clarify the hypothesis of the common m_T -scaling, one would want to add other particle species. After pions and kaons the most abundant identified hadrons are protons, and indeed such a comparison had been performed [64] (Fig. 1.6). The comparison of the 1D radii (Fig. 1.6) demonstrates that the source sizes of kaons and protons are consistent within the uncertainties and follow the same m_T -scaling trend. The discrepancy with pions was interpreted as a consequence of the Lorentz-boost [64].

This comparison though was performed using only the 1D radii because the extraction of 3D radii for protons is way more trickier (see Sec. 5.9) than for pions or kaons and, in fact, has never been done with the experimental data. Attempting to obtain the first ever experimental results of 3D femtoscopy with proton pairs is one of the main parts of this manuscript.

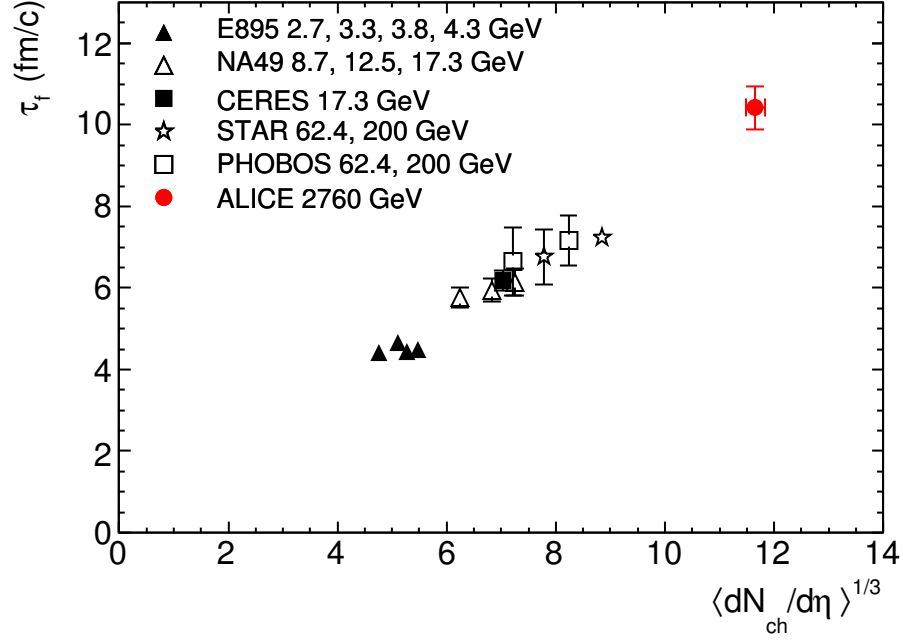


Figure 2.10. The decoupling time extracted from $R_{long}(m_T)$ extracted of pions in the 5% most central Pb—Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV measured by ALICE in comparison to the results from other HIC experiments [59].

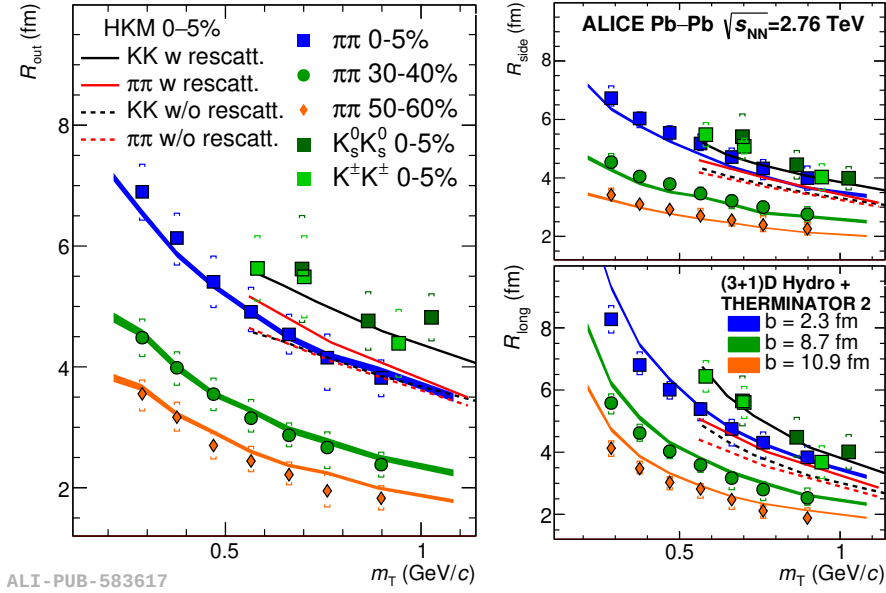


Figure 2.11. Pion and kaon pair m_T -dependencies of the femtoscopy 3D radii for different event centralities in Pb—Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV measured by ALICE and compared to different theoretical models with/without rescattering phase [19].

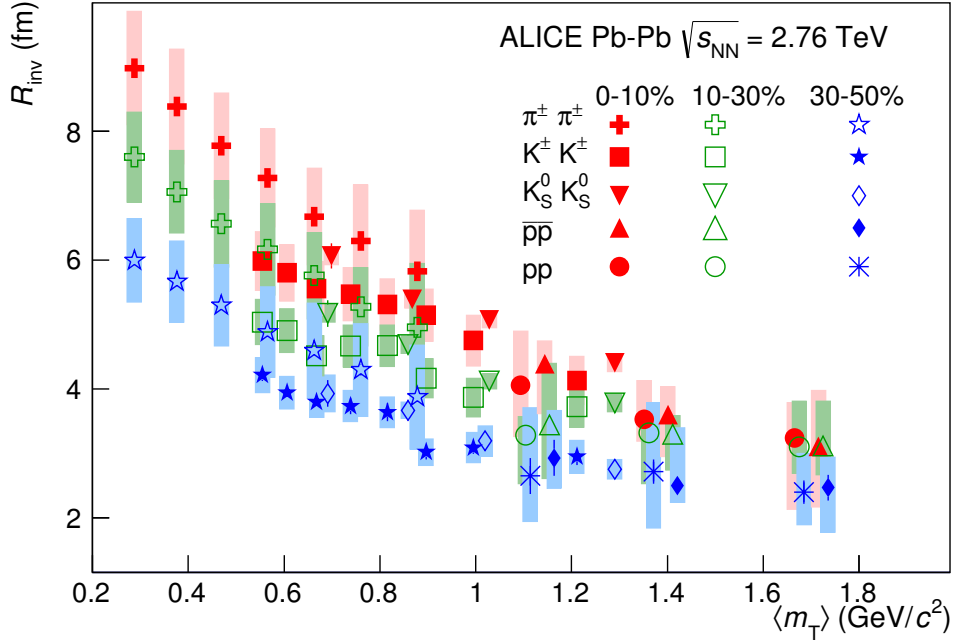


Figure 2.12. One-dimensional source sizes R_{inv} in dependence of pair transverse mass m_T obtained for pions, kaons and protons in Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV in three centrality classes by ALICE [64].

Another feature of proton (and baryon in general) femtoscopy emerges from the fact that unlike for pions or kaons, for which the strong interaction is insignificant compared to the effects of Quantum Statistics (QS) and the Coulomb interaction [65], proton femtoscopy is an exemplary case in which a proper account of the strong interaction becomes relevant [50]. And while the Coulomb interaction and laws of quantum statistics are well known, the strong interaction still remains an open question in physics up to these days. The difficulty in studying it comes from the fact that it is exerted over a very short range which makes any precise direct study challenging. For example, indirect methods of studying the strong potential utilizing the experimental information from nucleon scattering and/or data about light nuclei structure shed some light on the problem and led to the development of state-of-art models as Nijm93, Reid93 [66] and Argonne v18 [67]. Nevertheless, the final understanding is not yet reached.

Thus, any possible tool that could provide an access to measurements at small distances might give new insights about the strong interaction and it was suggested that femtoscopy in application to ultrarelativistic ion collisions could be such a tool [68]. Because of the extremely small distances between particles emitted by the source created in high-energy

collisions (e.g. in ALICE at the LHC the nucleon emitting source size is about 1 fm in pp and up to 9 fm in Pb–Pb collisions [19]), it is believed that one can probe fundamental interactions at such scales by using the methods of femtoscopy [69; 70] employing the fact that different strong potential shapes would translate in different shapes of the femtosopic CFs which would be especially evident in "small" colliding systems [68] (Fig. 2.13).

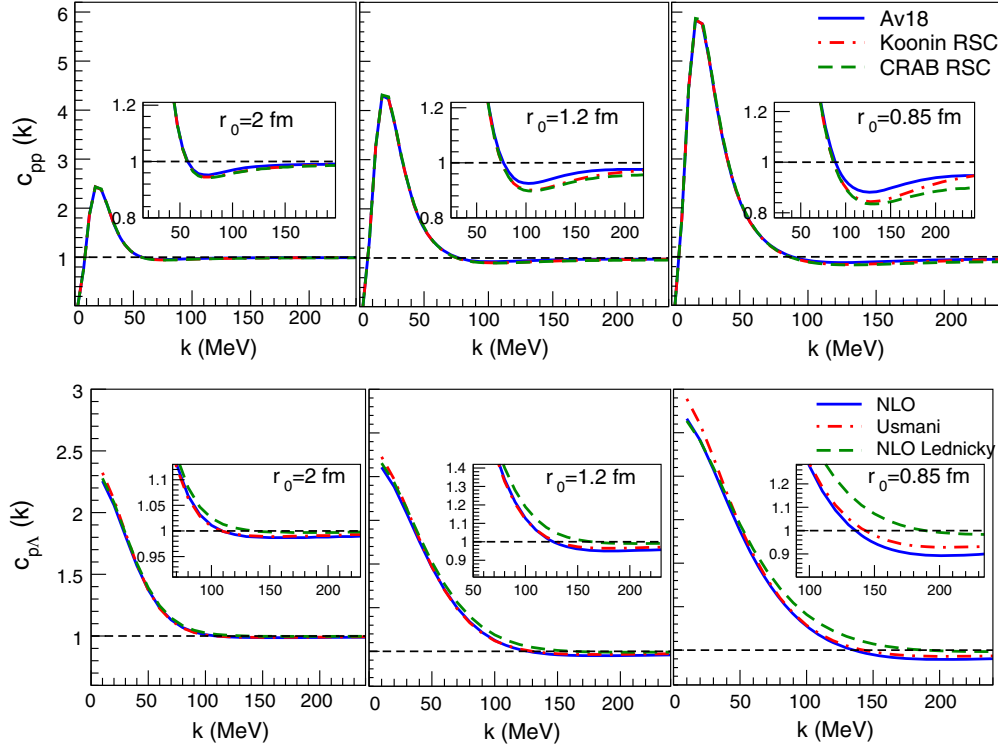


Figure 2.13. Theoretical CFs calculated for proton-proton (p-p) and proton-lambda (p- Λ) pairs for different strong interaction potentials and effective source sizes $r_0 := R_{inv}$ [68].

In the next Chapter we will discuss the importance of properly taking into account the strong potential for proton pairs. We will introduce an original way of an analytical effective field theory (EFT) treatment of the strong potential that was developed as a part of this study. The EFT approach is compared to other methods of accounting for the strong potential commonly used in the field. The idea of sensitivity of the femtosopic observables to the shape of the strong potential is also discussed.

Chapter 3

Effective field theory model

This Chapter is dedicated to the theoretical work on the effective field theory (EFT) treatment of the strong potential for the case of proton pair femtoscopy, developed *originally* for the study presented in this thesis.

The theoretical formalism of an analytical non-relativistic EFT treatment of the proton-proton pair wave function (WF) and its application to the femtoscopic CF are presented. More importantly, the possibility of probing the shape of the strong interaction potential with femtoscopy [71] is also discussed. First, a general solution of the Schrodinger's equation is obtained for a pair of fermions interacting through Coulomb and an arbitrary short-range potential in the shape of a simple box. The obtained formalism is then used to model the femtoscopic CF for the case of proton pairs accounting for both the Coulomb and the strong potentials. Finally, the obtained theoretical CF is compared with other models that have been used in the field [71; 72].

It is worth mentioning that the theoretical calculations discussed in this Chapter have been published on the the arXiv with Ref. [73].

3.1 Proton pair wave function

We have already mentioned in Sec. 2.1 why it's more convenient to consider particle pairs in their (pair's) Center-of-Mass (CM) coordinate system from the perspective of the parametrization of the emitting source. In this section we will show how we can profit from it even more for calculating the WF.

If we assume that the pair measured in the final state is isolated once emitted, namely that two particles interact only with each other, we can consider the pair in its CM frame (Fig. 3.1). To solve the problem we will use the following kinematic variables:

$$\begin{aligned}
\vec{r} &= \vec{r}_2 - \vec{r}_1, \\
\vec{k} &= \frac{m_2 \vec{p}_1 - m_1 \vec{p}_2}{m_1 + m_2}, \\
\mu &= \frac{m_1 m_2}{m_1 + m_2}.
\end{aligned} \tag{3.1}$$

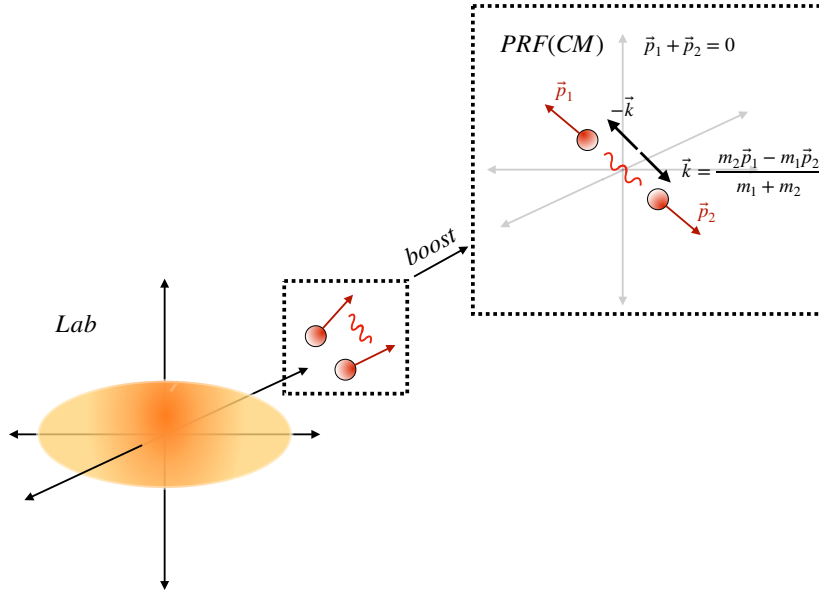


Figure 3.1. Boosting an isolated pair of particles emitted from a source created in HIC to the PRF ($\equiv$ CM) frame for solving the quantum mechanical problem of pair's interaction.

The convenience of using these variables comes from the fact that solving the problem for a pair of particles with masses m_1, m_2 and momenta $\vec{p}_1, \vec{p}_2$ in their CM is equivalent to solving it for a pair with effective masses $m_1 = m_2 = \mu$ and momenta $\vec{k}$ and $-\vec{k}$. Then, if their interaction potential is spherically symmetric – depends only on the distance $r = |\vec{r}|$ – each particle can be considered as being scattered on an effective center, which would be the Center-of-Mass. By finding the solution for just one particle in a pair we immediately find it for the other by mirroring $\vec{k} \rightarrow -\vec{k}$ (Fig. 3.2).

Thus, the number of dimensions for our problem gets reduced from six (three per each particle) to just three and the wave function $\psi(\vec{r})$ can be obtained as a solution to the Schrodinger's equation for a single particle in a potential $\hat{V}(\vec{r})$:

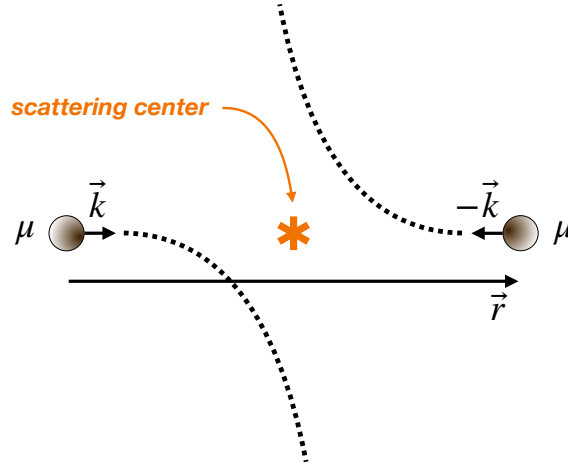


Figure 3.2. In the CM particles with the same mass μ and magnitude of momenta $|\vec{k}|$ are scattered off an effective center, which coincides with the CM.

$$\left[-\frac{\hbar^2}{2\mu} \nabla^2 + \hat{V}(\vec{r}) \right] \psi(\vec{r}) = E \psi(\vec{r}). \quad (3.2)$$

For the spherical potentials (such as the Coulomb and the strong ones), the orbital momentum is conserved and the solution (i.e. wave function) can be split into angular and radial parts, according to a well-known approach [74; 75; 76]. Thus, here we just anticipate the final form of the wave function:

$$\psi(\vec{r}) = \frac{1}{kr} \sum_{l=0}^{\infty} i^l (2l+1) e^{i\sigma_l} \varphi_l(r) P_l(\cos \theta), \quad (3.3)$$

where σ_l is a phase shift for an orbital momentum of value l ; $P_l(\cos \theta)$ are the Legendre polynomials; θ is the angle between $\vec{k}$ and $\vec{r}$, such that $\cos \theta = \frac{\vec{k} \cdot \vec{r}}{kr}$, and $\varphi_l(r)$ is the solution to the radial Schrodinger's equation:

$$\frac{d^2 \varphi_l(r)}{dr^2} + \left[k^2 - \frac{l(l+1)}{r^2} - \frac{2\mu}{\hbar^2} V(r) \right] \varphi_l(r) = 0, \quad (3.4)$$

with $r := |\vec{r}|$ and $k := |\vec{k}|$.

To build the wave function for the case where both the Coulomb and the short-range strong potential are considered, it is important first to understand how the wave for the Coulomb-only potential is built.

3.1.1 Solution for the Coulomb potential

The potential term in Eq. (3.4) for the case of Coulomb is

$$\frac{2\mu}{\hbar^2} V(r) = \frac{2\mu}{\hbar^2} \frac{e^2 Z_1 Z_2}{r} = \frac{2}{a_B r}, \quad (3.5)$$

where $a_B = \frac{\hbar^2}{\mu Z_1 Z_2 e^2}$ is the Bohr radius of the pair.

We can rewrite Eq. (3.4) taking into account Eq. (3.5) and the following substitution $r \rightarrow \rho = kr$:

$$\frac{d^2 \phi_l}{d\rho^2} + \left[1 - \frac{l(l+1)}{\rho^2} - \frac{2\eta}{\rho} \right] \phi_l = 0, \quad (3.6)$$

where $\eta = \frac{1}{ka_B}$.

We will look for a solution of Eq. (3.6) with the potential (3.5) in this form:

$$\phi_l(\rho) = \rho^{l+1} e^{i\rho} v_l(\rho). \quad (3.7)$$

After substituting Eq. (3.7) in Eq. (3.6), Eq. (3.4) can be brought to a Kimmer-like form:

$$z \frac{d^2 v_l}{dz^2} + \left(2(l+1) - z \right) v_l - (l+1 + i\eta) v_l = 0, \quad (3.8)$$

where $z = -2i\rho$.

In Mathematics the Kimmer's equation has *regular* and *singular* solutions [77]:

1. Kummer's (confluent hypergeometric) function ${}_1F_1(a, b, z)$,
2. Tricomi's (confluent hypergeometric) function $U(a, b, z)$.

Since the final wave function must be regular at $r = 0$, we choose the first option and applying it to Eq. (3.7) we can write the *regular* solution to the Eq. (3.6):

$$F_l(\eta, \rho) := \phi_l^{reg} = C_l \rho^{l+1} e^{i\rho} {}_1F_1(l+1 + i\eta, 2l+2, -2i\rho). \quad (3.9)$$

The function $F_l(\eta, \rho)$ is called the *regular Coulomb wave function* and the coefficient C_l given as

$$C_l = \frac{2^l e^{-\frac{\pi\eta}{2}} |\Gamma(l+1 + i\eta)|}{(2l+1)!} \quad (3.10)$$

is the *Coulomb normalization constant* needed to satisfy the required behavior of the solution at large distances $r \rightarrow \infty$ where the influence of the potential is neglected meaning that the WF at such distances must correspond to a plane wave. Therefore, the C_l is chosen such that $F_l \underset{r \rightarrow \infty}{\sim} \sin(\rho - \eta \ln(2\rho) - l\frac{\pi}{2} + \sigma_l)$. The $\sigma_l = \arg \Gamma(l+1+i\eta)$ is the Coulomb phase shift.

Thus, combining Eq. (3.9) and Eq. (3.3) one can write the general regular total WF ψ_c in terms of partial waves for the Coulomb potential as

$$\psi_c = \frac{1}{\rho} \sum_{l=0}^{\infty} i^l (2l+1) e^{i\sigma_l} F_l(\eta, \rho) P_l(\cos \theta). \quad (3.11)$$

Since we mentioned that the Eq. (3.6) has also irregular solutions, we would like to discuss them as well, especially considering their further importance. These solutions are called the *irregular Coulomb wave functions* and have the following asymptotic behavior:

$$\begin{aligned} u_l^{(\pm)}(\eta, \rho) &\underset{r \rightarrow \infty}{\sim} e^{\pm i(\rho - \eta \ln 2\rho - \frac{l\pi}{2})}, \\ G_l(\eta, \rho) &\underset{r \rightarrow \infty}{\sim} \sin(\rho - \eta \ln(2\rho) - l\frac{\pi}{2} + \sigma_l). \end{aligned} \quad (3.12)$$

The functions $u_l^{(\pm)}$ can be expressed via the Tricomi's (confluent hypergeometric) function $U(a, b, z)$, which is the irregular solution to Eq. (3.8).

The relations between the regular and irregular Coulomb wave functions are the following (here Messiah's [74] notation is used):

$$\begin{aligned} u_l^{(\pm)} &= e^{\mp i\sigma_l} (G_l \pm i F_l), \\ u_l^{(-)} &= u_l^{(+)*}, \\ F_l &= \frac{1}{2i} (u_l^{(+)} e^{i\sigma_l} - u_l^{(-)} e^{-i\sigma_l}), \\ G_l &= \frac{1}{2} (u_l^{(+)} e^{i\sigma_l} + u_l^{(-)} e^{-i\sigma_l}). \end{aligned} \quad (3.13)$$

Using these relations we can rewrite Eq. (3.11) in terms of the irregular Coulomb WFs:

$$\psi_c(\eta, \rho) = \frac{1}{2\rho} \sum_{l=0}^{\infty} (2l+1) i^{l+1} \left(u_l^{-}(\eta, \rho) - e^{2i\sigma_l} u_l^{+}(\eta, \rho) \right) P_l(\cos \theta). \quad (3.14)$$

The terms that contain $u_l^{(\pm)}$ can be interpreted as incoming and outgoing, or scattered, spherical waves.

3.1.2 Adding strong potential asymptotically

In case of the presence of an additional short-range potential that fades not slower than r^{-2} in the asymptotic region its influence in that region can be effectively taken into account by adding an additional phase shift [74]. Since the strong potential satisfies the condition, we will utilize this feature in our calculations.

Let δ_l be the phase shift related to the short-range potential. Thus, one obtains the wave function ψ_{c+s} , accounting for both the Coulomb and the strong potentials in the asymptotic region, by modifying the solution given by Eq. (3.14) the following way:

$$\psi_{c+s}(\eta, \rho) \underset{r \rightarrow \infty}{=} \frac{1}{2\rho} \sum_{l=0}^{\infty} (2l+1) i^{l+1} \left(u_l^-(\eta, \rho) - e^{2i\sigma_l} e^{2i\delta_l} u_l^+(\eta, \rho) \right) P_l(\cos \theta). \quad (3.15)$$

It is to be noted that here the notation $r \rightarrow \infty$ refers to the asymptotical region, where the short-range potential can be neglected, and not to the actual infinity.

The solution (3.15) can be also rewritten via the F_l and G_l functions (3.13):

$$\psi_{c+s}(\eta, \rho) \underset{r \rightarrow \infty}{=} \frac{1}{r} \sum_{l=0}^{\infty} (2l+1) i^l e^{i\sigma_l} \left(\frac{F_l(\eta, \rho)}{k} + f_l(k) (G_l(\eta, \rho) + i F_l(\eta, \rho)) \right) P_l(\cos \theta), \quad (3.16)$$

where $f_l(k)$ is the scattering amplitude [74; 75]:

$$f_l(k) = \frac{e^{2i\delta_l} - 1}{2ik} = \frac{1}{k \cot \delta_l - ik}. \quad (3.17)$$

3.1.3 Introducing short-range potential in a square-well form

To approach the problem in the non-asymptotic region we propose to consider the strong potential effectively in a shape of a simple square-well as represented in Fig. 3.3:

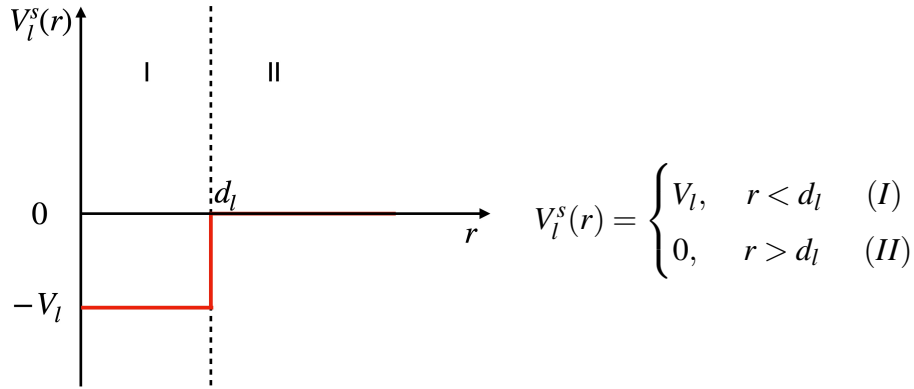


Figure 3.3. Square-well potential. Different widths d_l and depths V_l^s for different values of the orbital momentum l are used because it is suggested by different phase shift spectra measured in proton-proton scattering experiments (see Ref. [78] and Fig. 3.4).

To get the total solution for this type of short range potential, in addition to the Coulomb, we need to find the solutions separately for the region of its influence (I) and the asymptotic region (II) and then match them.

Since we already know the solution in the region (II), i.e. the asymptotic solution given by Eq. (3.15 -3.16), we have to find only the solution for the region (I). To take into account the potential V_l in that region (Fig. 3.3) one should add a corresponding term $\frac{2\mu}{\hbar^2} V_l$ to the radial Eq. (3.4). This term can be then substituted by the energy term k^2 since the potential is constant in the region (I):

$$\frac{d^2 \varphi_l^{(I)}}{dr^2} + \left[\tilde{k}_l^2 - \frac{l(l+1)}{r^2} - \frac{2}{a_B r} \right] \varphi_l^{(I)} = 0, \quad (3.18)$$

$$\tilde{k}_l = \sqrt{k^2 - \frac{2\mu}{\hbar^2} V_l}.$$

Noticing that the Eq. (3.18) has the same form as the Eq. (3.4) we can immediately write the full WF partial-wave expansion in the region (I) similarly to (3.11):

$$\psi_{c+s}^{(I)}(\tilde{\eta}_l, \tilde{\rho}_l) = \sum_{l=0}^{\infty} \frac{1}{\tilde{\rho}_l} (2l+1) i^l e^{i\tilde{\sigma}_l} F_l(\tilde{\eta}_l, \tilde{\rho}_l) P_l(\cos \theta), \quad (3.19)$$

where $\tilde{\eta}_l = \frac{1}{\tilde{k}_l a_B}$, $\tilde{\rho}_l = \tilde{k}_l r$ and $\tilde{\sigma}_l = \arg \Gamma(l+1 + i\tilde{\eta}_l)$.

We will look for a partial-wave expansion of the total WF in the whole coordinate space in the following form:

$$\psi_{c+s}(k, r) = \frac{1}{r} \sum_{l=0}^{\infty} (2l+1) i^l \varphi_l(k, r) P_l(\cos \theta). \quad (3.20)$$

Employing both solutions (3.19) and (3.16) for the regions (I) and (II), respectively, one obtains:

$$\varphi_l(k, r) = \begin{cases} A_l \varphi_l^{(I)} := A_l \frac{1}{\tilde{k}_l} e^{i\tilde{\sigma}_l} F_l(\tilde{\eta}_l, \tilde{\rho}_l) & r < d_l, \\ \varphi_l^{(II)} := e^{i\sigma_l} \left(\frac{F_l(\eta, \rho)}{k} + f_l(k) (G_l(\eta, \rho) + i F_l(\eta, \rho)) \right) & r \geq d_l. \end{cases} \quad (3.21)$$

Here A_l is a normalization constant that must be obtained from the matching condition at $r = d_l$ and ensure that the WF is continuous at $r = d_l$, namely

$$\varphi(k, d - \varepsilon) \underset{\varepsilon \rightarrow 0}{=} \varphi(k, d + \varepsilon).$$

The WF must be also smooth (differentiable) at $r = d_l$, namely

$$\left. \frac{d\varphi}{dr} \right|_{r=d_l-\varepsilon} \underset{\varepsilon \rightarrow 0}{=} \left. \frac{d\varphi}{dr} \right|_{r=d_l+\varepsilon}.$$

Therefore we can write the matching conditions in the following form:

$$\begin{aligned} A_l \varphi_l^{(I)}(k, r = d) &= \varphi_l^{(II)}(k, r = d), \\ \left. \frac{d}{dr} \left(\ln(A_l \varphi_l^{(I)}(k, r)) \right) \right|_{r=d} &= \left. \frac{d}{dr} \left(\ln(\varphi_l^{(II)}(k, r)) \right) \right|_{r=d}. \end{aligned} \quad (3.22)$$

From the first condition one finds A_l :

$$A_l = \frac{\tilde{k}_l}{F_l(\tilde{\eta}_l, \tilde{k}_l d)} e^{i(\sigma_l - \tilde{\sigma}_l)} \left[\frac{F_l(\eta, kd)}{k} + f_l(k) (G_l(\eta, kd) + i F_l(\eta, kd)) \right], \quad (3.23)$$

and from the second one:

$$\cot \delta_l = \frac{G_l(\eta, kd)}{F_l(\eta, kd)} \frac{\tilde{k} f_l(\tilde{\eta}, \tilde{k}d) - k g_l(\eta, kd)}{k f_l(\eta, kd) - \tilde{k} f_l(\tilde{\eta}, \tilde{k}d)}, \quad (3.24)$$

where the f_l and g_l are the logarithmic derivatives of F_l and G_l , respectively, given by

$$\begin{aligned} f_l(\eta, \rho) &= \frac{d}{dr} \left(\ln (F_l(\eta, \rho)) \right), \\ g_l(\eta, \rho) &= \frac{d}{dr} \left(\ln (G_l(\eta, \rho)) \right). \end{aligned} \quad (3.25)$$

From Eq. (3.20, 3.21, 3.23) we can finally assemble the total solution in terms of partial-wave expansion:

$$\begin{aligned} \psi_{c+s}(k, r) &= \frac{1}{r} \sum_{l=0}^{\infty} (2l+1) i^l e^{i\sigma_l} \varphi_l(k, r) P_l(\cos \theta), \\ \varphi_l(k, r) &= \begin{cases} \frac{F_l(\tilde{\eta}_l, \tilde{k}_l r)}{F_l(\tilde{\eta}_l, \tilde{k}_l d)} \left[\frac{F_l(\eta, kd)}{k} + f_l(k) (G_l(\eta, kd) + i F_l(\eta, kd)) \right], & r < d_l, \\ \frac{F_l(\eta, \rho)}{k} + f_l(k) (G_l(\eta, \rho) + i F_l(\eta, \rho)), & r \geq d_l. \end{cases} \end{aligned} \quad (3.26)$$

In practice, contributions from the terms with large orbital momentum l can be neglected with respect to the ones with low l . Now, if we assume that the short-range potential has influence only for a finite arbitrary number of angular momentum states $l = \{0, \dots, n\}$ (i.e. if $l > n$ then $\tilde{k} \equiv k \implies \tilde{\rho} \equiv \rho$ and $\tilde{\eta} \equiv \eta$ as well as $f_l(k) = 0$) the solution can be rewritten as

$$\begin{aligned} \psi_{c+s}(k, r) &= \sqrt{A_c(\eta)} e^{i\sigma_0} e^{i\vec{k}\vec{r}} {}_1F_1 \left(-i\eta, 1, i(kr - \vec{k}\vec{r}) \right) + \sum_{l=0}^n (2l+1) i^l e^{i\sigma_l} a_l(k, r) P_l(\cos \theta), \\ a_l(k, r) &= \begin{cases} \frac{F_l(\tilde{\eta}_l, \tilde{k}_l r)}{F_l(\tilde{\eta}_l, \tilde{k}_l d)} \left[\frac{F_l(\eta, kd)}{kr} + f_l(k) \frac{G_l(\eta, kd) + i F_l(\eta, kd)}{r} \right] - \frac{F_l(\eta, \rho)}{kr}, & r < d_l, \\ f_l(k) \frac{G_l(\eta, \rho) + i F_l(\eta, \rho)}{r}, & r \geq d_l. \end{cases} \end{aligned} \quad (3.27)$$

In the latter passage the following relation for the Coulomb WF partial wave expansion [74] was used:

$$\frac{1}{kr} \sum_{l=0}^{\infty} (2l+1) i^l e^{i\sigma_l} F_l(\eta, \rho) P_l(\cos \theta) = \sqrt{A_c(\eta)} e^{i\sigma_0} e^{i\vec{k}\vec{r}} {}_1F_1 \left(-i\eta, 1, i(kr - \vec{k}\vec{r}) \right), \quad (3.28)$$

where $A_c(\eta) = \frac{2\pi\eta}{e^{2\pi\eta}-1}$ is the Gamow factor (Coulomb penetration factor).

3.1.4 Pair wave function and possible pair states

As anticipated before, the solutions (3.26, 3.27) to the Schrodinger's equation (3.2) for the Coulomb and the strong potential with a square-well shape are obtained by considering a proton being scattered on an "effective center" (Fig. 3.2). Now we need to complete our calculations by building up a pair's wave function. Again, since the system possesses spherical symmetry the solution for the other constituent of the pair is obtained by just mirroring the frame, i.e. $\vec{k} \rightarrow -\vec{k}$. Thus, the pair's WF would be a superposition of $\psi_{\vec{k}}$ and $\psi_{-\vec{k}}$. Due to the fact that we want to consider fermions, the parity symmetry must be respected. For any pair of fermions the pair's WF is

$$\Psi_{ff}^S = \frac{\psi(\vec{r}, -\vec{k}) + (-1)^S \psi(\vec{r}, \vec{k})}{\sqrt{2}}, \quad (3.29)$$

where $S = s_1 + s_2$ is the total pair's spin of particles with spins s_1 and s_2 .

When using the partial-wave expansion (3.3), we have to consider which pair states with different orbital momentum are allowed.

In the general case of a nucleon pair, protons p and neutrons n can be regarded as two isospin states of the same isospin doublet, with $i_p = +\frac{1}{2}$ (or $|\uparrow\rangle$) and $i_n = -\frac{1}{2}$ (or $|\downarrow\rangle$). The total nucleon pair's isospin I can have only two values $I \in [0, 1]$. But since they are fermions they must obey the Pauli's principle: their isospin projections cannot be the same (as for the spin), therefore, not any nucleon-nucleon state is allowed. Considering the fact that the pair's isospin I_z can have only three projections, $[-1, 0, 1]$, we see that

$I_z = -1$: only possible for $|\downarrow\downarrow\rangle \equiv |nn\rangle$ and therefore the $|nn\rangle$ state can correspond only to the $I = 1$ case,

$I_z = 1$: only possible for $|\uparrow\uparrow\rangle \equiv |pp\rangle$, then $|pp\rangle$ state can correspond only to the $I = 1$ case,

$I_z = 0$: only possible for $|\uparrow\downarrow\rangle \equiv |np\rangle$, then $|np\rangle$ state can correspond to the $I = 0$ and $I = 1$ cases.

Using the partial wave expansion (3.3) we also need to consider states with different pair's orbital momentum L . As the pair's WF must be globally antisymmetric, it needs to obey the parity condition [79]:

$$(-1)^{L+S+I} = -1$$

The following two cases can be considered:

- $L - \text{odd}: \implies S + I - \text{even} \implies S + I = [0, 2]$ (as both S and I separately can be only 0 or 1) and therefore there can be only two cases:

$$[S = 0, I = 0] \implies |np\rangle \implies \text{possible sates } {}^1P, {}^1F, \dots$$

$$[S = 1, I = 1] \implies |np\rangle, |nn\rangle, |pp\rangle \implies \text{possible sates } {}^3P, {}^3F, \dots$$

- $L - \text{even}: \implies S + I - \text{odd} \implies S + I = 1$ and therefore there can be only two cases:

$$[S = 0, I = 1] \implies |np\rangle, |nn\rangle, |pp\rangle \implies \text{possible sates } {}^1S, {}^1D, \dots$$

$$[S = 1, I = 0] \implies |np\rangle \implies \text{possible sates } {}^3S, {}^3D, \dots$$

Above we used the common representation of spin-orbital states as ${}^{2S+1}L$, where $L = 1, 2, 3, 4, \dots$ correspond to $L = S, P, D, F, \dots$

To consider that the total pair's spin S and its orbital momentum L can have have different orientations with the corresponding degeneracy, it's convenient to use the total angular momentum variable $J = L + S$. For example, for the state 3S , the orbital momentum $L = 0$ but the spin $S = 1$ can have three different projections ($-1, 0$ and 1). Thus, using the notation ${}^{2S+1}L_J$, we can write the possible nucleon-nucleon states:

$$|np\rangle: {}^1S_0, {}^3S_1, {}^1P_1, {}^3P_0, {}^3P_1, {}^3P_2, {}^1D_2, {}^3D_1, {}^3D_2, {}^3D_3, \dots$$

$$|nn\rangle, |pp\rangle: {}^1S_0, {}^3P_0, {}^3P_1, {}^3P_2, {}^1D_2, \dots$$

Although in this study we are focusing on the proton-proton pairs, we think that looking at this general nucleon-nucleon system is important for understanding why we will be considering only the particular pair's states in our further calculations.

3.1.5 Defining potential parameters

In the previous sections we discussed the solutions (3.26, 3.27) to the Schrodinger's equation (3.2) for the case of Coulomb and an arbitrary short-range potential with a square-well shape. Our initial goal was to use the obtained WF for the application to proton pair femtoscopy taking into account the strong potential effectively. To do so, the effective parameters of the well must be set.

In this respect our EFT model is not different than such state-of-art strong potential models as Nijm93, Reid93 [66] and Argonne v18 [67]. We need to choose them so they describe the phase shifts spectra measured in proton-proton scattering experiments. For this

we will utilize the values provided by the Nijmegen database (<https://nn-online.org>) and based on proton-proton scattering data [78]. The original phase shift δ_l spectra that are given in dependence on the kinetic energy of the projectile proton T_{kin} in the Laboratory frame were converted¹ to the k^* -dependencies via $k^* = \sqrt{\frac{T_{kin}m_p}{2}}$, where m_p is the proton mass. The pair relative momentum k^* -dependent proton proton phase shift δ_l spectra are shown in Fig. 3.4. In this work we focus only on s- and p-waves and therefore will consider 1S_0 , 3P_0 , 3P_1 and 3P_2 states.

For the extraction of the effective square-well parameters for the strong potential one has to fit the dependence of the phase shifts δ_l on the pair relative momentum in the CM (k^*) with the matching condition relation we obtained before, Eq. (3.24).

The fits and the extracted effective strong potential parameters are reported in Fig. 3.4 and Tab. 3.1.

Table 3.1. Effective parameters of the strong potential in a shape of a square-well extracted from the fits of experimental phase shifts (Fig. 3.4) with Eq. 3.24.

L	Pair's state $^{2S+1}L_J$	Depth $\frac{2\mu}{\hbar^2} V_{2S+1L_J}$ [GeV]	Width d_{2S+1L_J} [fm]
0	1S_0	-0.01620	2.30394
1	3P_0	-0.00283	4.03726
	3P_1	0.00374	3.43222
	3P_2	-0.01023	2.18680

These square-well parameters characterize the strong potential in the well region (I) (see Fig. 3.3) and enter into the WF at $r < d$.

To take into account the influence of the strong potential in the asymptotic region (II) we used the scattering amplitude (3.17). Starting from the simplest case without the presence of the Coulomb interaction, the s-wave phase shifts (and therefore $f_0(k)$) can be parameterized with the following relation [80]:

$$k \cot \delta_l = -\frac{1}{f_0} + \frac{1}{2}d_0k^2 + (-Pd_0^3k^4) + O(k^6). \quad (3.30)$$

¹Here we deal with a non-relativistic case.

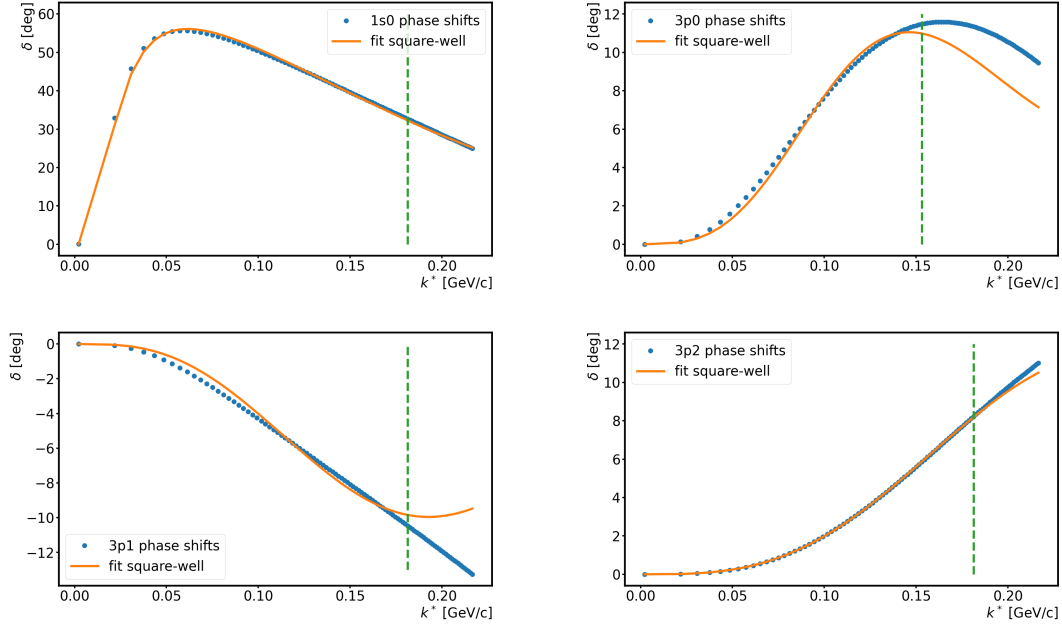


Figure 3.4. p-p phase shifts fitted with Eq. (3.24) to extract the effective strong potential parameters (Tab. 3.1) in a shape of a square-well for different pair states: 1S_0 (top left), 3P_0 (top right), 3P_1 (bottom left), 3P_2 (bottom right). Green line corresponds to the upper limit of the fit range.

The parametrization gets way more complex if one has to include the Coulomb interaction and higher order partial waves. To this end, in this work we exploited the solution given in Ref. [81]:

$$k^{2l+1} \left[A_c(\eta) (\cot \delta_l - i) + 2\eta H(\eta) \right] \prod_{n=1}^l \left(1 + \frac{\eta^2}{n^2} \right) = -\frac{1}{f_l} + \frac{1}{2} d_l k^2 + (-P d_l^3 k^4) + O(k^6), \quad (3.31)$$

where

$$H(\eta) \equiv \psi(i\eta) + \frac{1}{2i\eta} - \ln \left(-i\eta \operatorname{sgn} \left(-\frac{1}{a_B} \right) \right), \quad (3.32)$$

and $\psi(x)$ here is the digamma function.

The fits of the phase shifts performed with Eq. (3.31) are shown in Fig. 3.5 and the extracted parameters are listed below. The third parameter P for the cases of 3P_1 and 3P_2 pair states was fixed to 0 as the inclusion of the corresponding term in the Eq. (3.31) doesn't improve the performance of the fit in the considered k^* region.

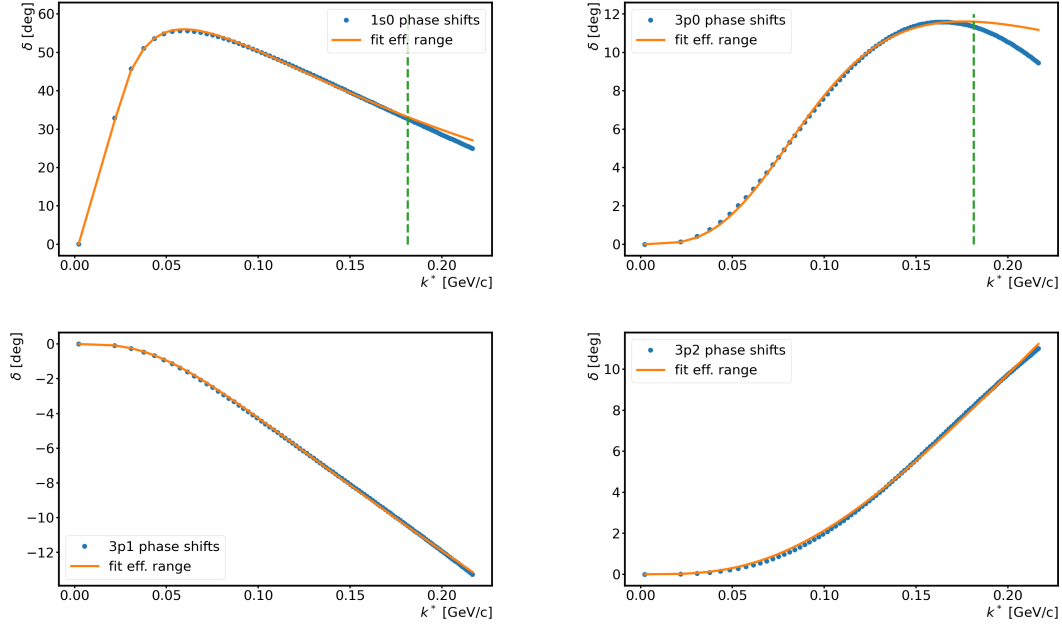


Figure 3.5. p-p phase shifts fitted with Eq. (3.31) to extract the effective range parameters (Tab. 3.2) for different pair states: 1S_0 (top left), 3P_0 (top right), 3P_1 (bottom left), 3P_2 (bottom right). Green line corresponds to the upper limit of the fit range.

Table 3.2. Effective range parameters f_l , d_l and P_l used in the parametrization (3.31) of the scattering amplitude to account for the strong potential in the asymptotic region.

L	Pair's state $^{2S+1}L_J$	f_{2S+1L_J} [fm]	d_{2S+1L_J} [fm]	P_{2S+1L_J}
0	1S_0	-7.52584	2.46092	-0.02547
1	3P_0	-2.48927	2.00572	-0.41956
	3P_1	1.86182	-7.81453	0 (fixed)
	3P_2	-0.44475	7.01762	0 (fixed)

3.2 Application to femtoscopy

In the previous sections we obtained a general solution (3.27) to the Schrodinger's equation (3.2) for a proton scattered of another proton in their center of mass system. Using this solution to build a proton pair's WF (3.29) we can plug it in the Koonin-Pratt formula (2.11) and write the theoretical femtoscopic CF. Starting with only s-wave terms and averaging over possible pair spin states one obtains

$$\begin{aligned}
C_{pp}(k, R_{inv}) &= \sum_{S=0}^1 \omega_S \int d^3\vec{r} S(\vec{r}, R_{inv}) \left| \Psi_{pp}^S \right|^2 = \\
&= \sum_{S=0}^1 \omega_S \int d^3\vec{r} S(\vec{r}, R_{inv}) \left| \frac{\psi(-\vec{k}, \vec{r}) + (-1)^S \psi(\vec{k}, \vec{r})}{\sqrt{2}} \right|^2,
\end{aligned} \tag{3.33}$$

where

$$\omega_S = \frac{2S+1}{(2s_1+1)(2s_2+1)} \tag{3.34}$$

are weights associated to a given pair's spin configuration $S = s_1 + s_2$ (in the proton-proton case $s_1 = s_2 = \frac{1}{2}$).

The proton WF ² $\psi(\vec{k}, \vec{r})$ for $l = 0$:

$$\begin{aligned}
\psi_{c+s}^{l=0}(k, r) &= \sqrt{A_c(\eta)} e^{i\sigma_0} e^{i\vec{k}\vec{r}} {}_1F_1\left(-i\eta, 1, i(kr - \vec{k}\vec{r})\right) + e^{i\sigma_0} \times \\
&\times \begin{cases} \frac{F_0(\tilde{\eta}_0, \tilde{k}_0 r)}{F_0(\tilde{\eta}_0, \tilde{k}_0 d)} \left[\frac{F_0(\eta, kd)}{kr} + f_0(k) \frac{G_0(\eta, kd) + iF_0(\eta, kd)}{r} \right] - \frac{F_0(\eta, \rho)}{kr}, & r < d_0, \\ f_0(k) \frac{G_0(\eta, \rho) + iF_0(\eta, \rho)}{r}, & r \geq d_0. \end{cases}
\end{aligned} \tag{3.35}$$

The WF part corresponding to the region outside the strong potential, $r \geq d_0$, is commonly known as the Lednicky-Lyuboshits formula [72; 82] (later referred as LL) and has been commonly used in femtoscopy in heavy-ion collision experiments [64; 83]. We argue that using the asymptotic part of the WF alone to calculate the theoretical CF and use it later to extract the source size from experimental CFs could give incorrect results. To demonstrate this one has to consider the behavior of the constituents of the WF at small distances ($r \rightarrow 0$), having in mind that the wave function must be regular at zero:

$$\begin{aligned}
{}_1F_1\left(-i\eta, 1, i(kr - \vec{k}\vec{r})\right) &\xrightarrow{r \rightarrow 0} 0, \\
F_l(\eta, \rho) &\xrightarrow{r \rightarrow 0} r^{l+1}, \\
G_l(\eta, \rho) &\xrightarrow{r \rightarrow 0} r^{-l},
\end{aligned} \tag{3.36}$$

²It is worth mentioning that a similar form of the WF (3.35) was discussed in [84] but it wasn't used in experimental data analyses (at least we couldn't find any experimental work in literature using this formalism).

Therefore

$$\psi_{c+s}^{l=0}(k, r \geq d_0) \xrightarrow{r \rightarrow 0} \frac{1}{r}$$

is singular at zero and cannot be used at small distances (not even mentioning that it's non-normalizable). A problem rises since one needs to perform the integration over the whole coordinate space to obtain the CF in Eq. (3.33). This problem did not emerge dramatically when the LL model was used to calculate the CF, probably do to the fact that the integration in Eq. (3.33) was carried out in the spherical coordinates with $d\vec{r} = r^2 \sin \theta dr d\phi d\theta$ such that the r^2 term in the Jacobian would cancel out the $|\Psi_{pp}|^2 \xrightarrow{r \rightarrow 0} \frac{1}{r^2}$ singular behavior. But the usage of the asymptotic part of the proton pair WF should still lead to an overestimation of the CF since the small- r region would contribute the most to the final CF value due to the nature of the Gaussian distribution used to parametrize the source (see Eq. 2.11 in Sec. 2.1), i.e. concentration of the distribution around $r \sim 0$.

Instead, if we consider our general solution (3.27) for an arbitrary l , we see that it behaves regularly at zero:

$$\psi_{c+s}^l(k, r) \xrightarrow{r \rightarrow 0} r^l.$$

By comparing the theoretical CFs (3.33) calculated with our EFT solution (3.35) for different sizes R_{inv} of a Gaussian source (2.11) to the CFs calculated only with the asymptotical part (i.e. LL), we will see that the LL model leads to a noticeable overestimation of the resulting proton-proton CF (Fig. 3.6), especially if the source size is small. The inability of the LL model to be applied to "small" systems was observed also in [69] where the exact solution to the Schrodinger's equation was calculated numerically employing the Argonne v18 strong potential model with the CATS package [71].

Another feature of the EFT model is that it can be extended to higher partial waves. This would require a modification of the Eq. (3.33) to average not only over pair spin states but also over L , and J . The resulting form of the CF is

$$C_{pp}(k, R_{inv}) = \sum_{S=0}^1 \omega_S \sum_{L=0}^{L_{max}} \sum_{J=|L-S|}^{L+S} \omega_{LJ} \int d^3\vec{r} S(\vec{r}, R_{inv}) \left| \frac{\psi(-\vec{k}, \vec{r}) + (-1)^S \psi(\vec{k}, \vec{r})}{\sqrt{2}} \right|^2, \quad (3.37)$$

where ω_S are still the weights for a given pair's spin configuration (see Eq. 3.34), the proton WF $\psi(\vec{k}, \vec{r})$ is given by Eq. (3.27) and

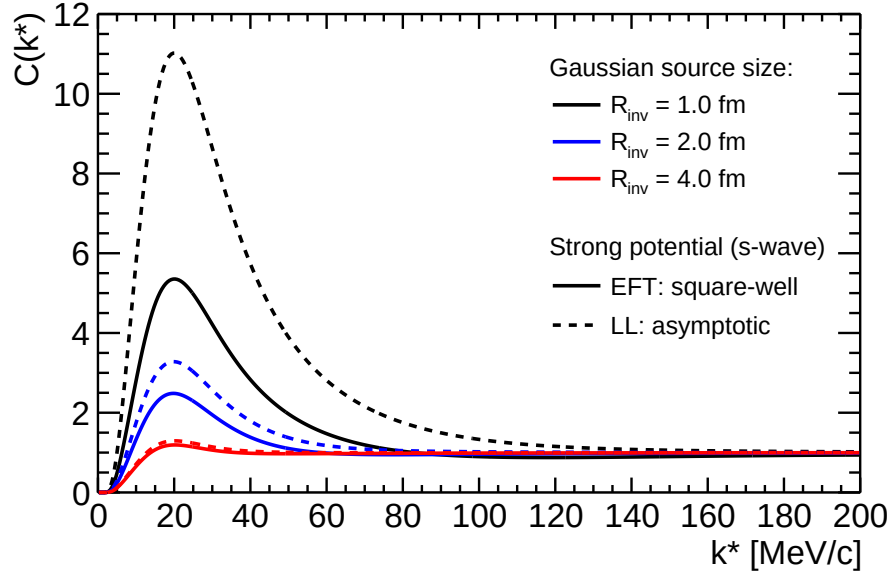


Figure 3.6. Comparison between the EFT model (s-wave only) and LL [72; 82] for different source sizes corresponding to the Gaussian source (3.35).

$$\omega_{LJ} = \frac{2J+1}{(2S+1)(2L+1)} \quad (3.38)$$

are weights corresponding to a given pair's spin-orbital configuration.

The next step is to include also the p-wave ($L_{max} = 1$) in addition to the s- one and compare it against the s-wave case we considered above. The comparison (Fig. 3.7) shows that the discrepancy between the s-wave and s+p-waves calculations is very mild (up to $\approx 6\%$ for the $R_{inv} = 1$ fm source size calculation) and the final result is strongly dominated by the s-wave contribution. This is why many effective calculations do not usually include higher partial waves (e.g. [74; 75]).

To understand the viability of the EFT approach discussed in this work we compared the theoretical CFs calculated with it against the calculations provided by the CATS framework [71]. CATS also uses the partial wave expansion (3.3) providing a *numerical solution* $\varphi_l(r)$ to the differential radial Schrodinger's equation (3.4) for a given partial wave term with a given shape of the interaction potential $V(r)$, being a combination of the short-range strong potential and long-range Coulomb one. The total WF $\psi(\vec{r})$ is built by running the sum over the partial wave terms until the required convergence condition is reached. The numerical approach to the problem allows the CATS framework to obtain solutions for more advanced

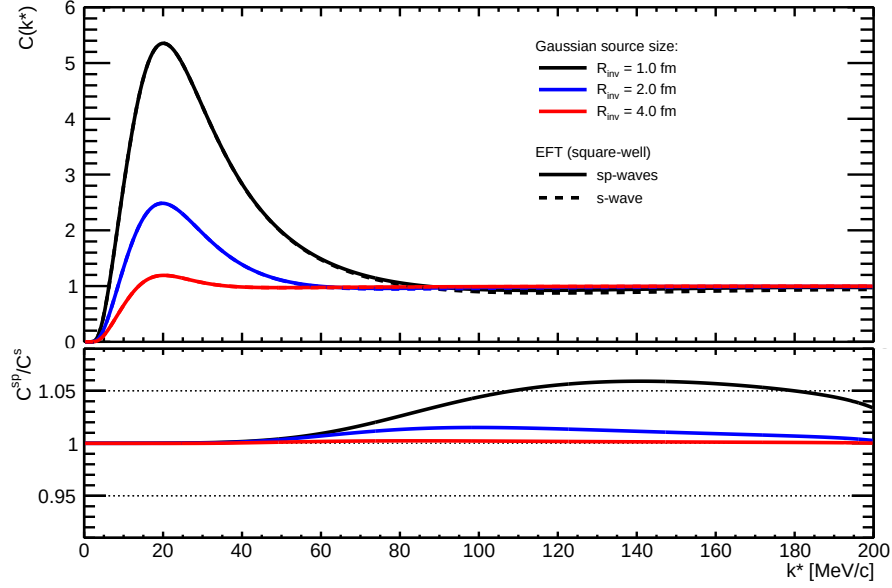


Figure 3.7. Comparison between s-wave only and s- and p-waves calculations with the EFT model for different source sizes corresponding to the Gaussian source (3.35).

and complex strong potential shapes that would be impossible with an analytical approach. Thus, the CATS provides a good reference to compare the EFT calculation against.

For the comparison we choose the CATS calculations for the p-p CF for different source sizes R_{inv} using the already mentioned Argonne v18 [67], since it was the one to be chosen for experimental application in Ref. [69] and disproved the LL approach. The results of the comparison for the cases of accounting the strong potential in only s- and both s- and p-waves are shown in Figs. 3.8 and 3.9 respectively. One can see that the CFs are quite close and exhibit a small discrepancy, localized in the interval $20 < k^* < 100$ MeV/c, which is more pronounced for smaller source sizes. Apart from the different potential shapes used in the calculation, the possible causes of discrepancy between the two approaches could be ascribed to *i*) a difference in the parametrization of the WF in the asymptotic region, and/or *ii*) a difference in the summation of the partial wave terms³, where CATS uses a finite number of terms needed to satisfy the convergence condition while the in the EFT model the infinite sum is used thankfully to Eq. (3.28).

However, such a small difference might be too insignificant (up to $\approx 4\%$ for the $R_{inv} = 1$ fm) in the application to experimental data. For example, in that interval the total uncertainties (quadratic sum of statistical and systematic uncertainties) of the proton pair CF reported in Ref. [69] range from $\approx 6.7\%$ at $k^* = 22$ MeV/c to $\approx 3.1\%$ at $k^* = 98$

³Here we need to remember that only the first two terms include the strong potential and the remaining ones include only the Coulomb one.

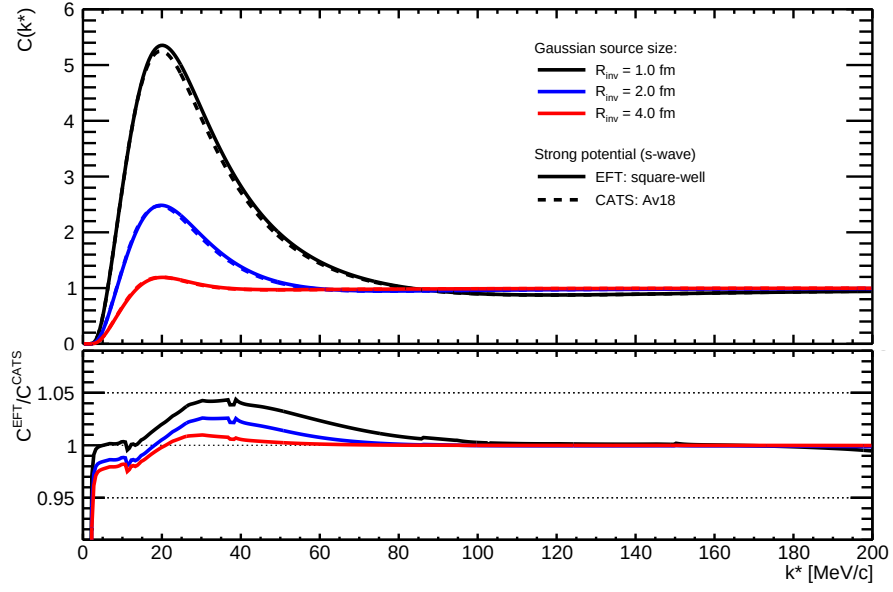


Figure 3.8. Comparison between the EFT and CATS calculation considering the strong interaction only in s-wave only for different source sizes corresponding to the Gaussian source (3.35).

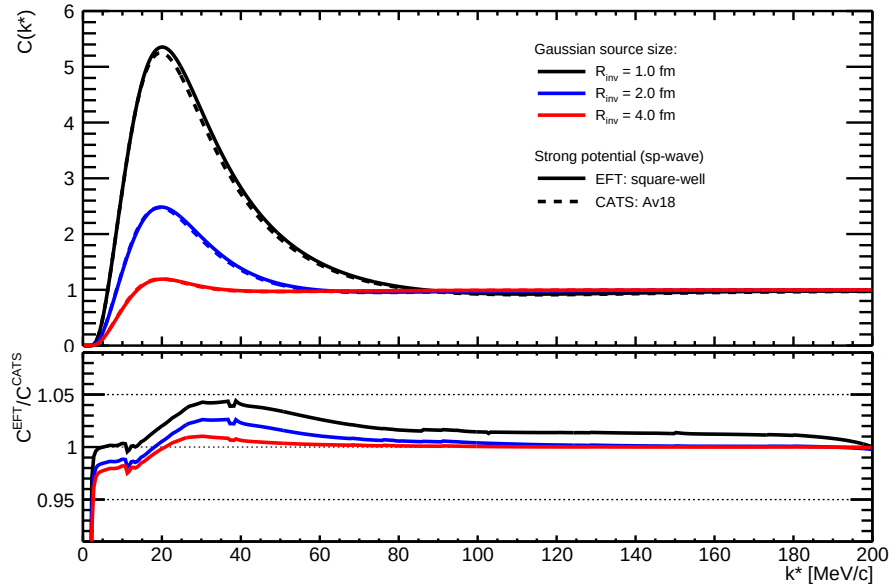


Figure 3.9. Comparison between the EFT and CATS calculation considering the strong interaction in s- and p-waves for different source sizes corresponding to the Gaussian source (3.35).

MeV/c, which exceeds the discrepancy between the CATS and the EFT calculations (see Figs. 3.8 and 3.9). Moreover, the experimental distributions are always given in a discrete form, meaning that each experimental value of CF corresponds to some range of k^* . This

additional built-in uncertainty of the argument of the CF results in a higher tolerance to the discrepancy.

Despite using the proton pair case as an example, the employed EFT formalism was obtained for a generic case of two particles interacting via the Coulomb and the strong potential. This idea suggests one of the main physical outcomes of this study: the femtoscopic observables might not be sensitive enough to the shape of the strong potential⁴ to use them to test different strong potential models. Remembering that the proton pair WF is obtained as a solution to the scattering problem one can recall that it has been well known for many years from the phase shift studies that there is no possibility to access the information about the shape of the strong potential (generally, of any short-range one) at low momenta [80]. Indeed, as it was shown in this work (see Fig. 3.4) even a simple square-well shape provides a satisfactory description of the phase shifts at low relative momenta k^* . We hypothesize that due to the fact that the femtoscopic signal (i.e. peak of the CF) is present exactly in the small- k^* region, the same argumentation applies. Thus, in the experimental analysis part of this thesis (see Chapter 5) we will utilize the EFT model as it provides a more than sufficient estimation of the proton pair CF being at the same time fully analytical and less complex in terms of the strong potential shape and calculation procedure.

Nevertheless, even if the EFT approach discussed in this work might be enough for proton pairs, one could go beyond pairs and study three-body correlations (e.g. [85]) or explore correlations between rarer baryons (like Λ and Ξ in [86]) that can only be observed in collider experiments. Thus, we believe that femtoscopy in high-energy collisions can still be a unique tool to study the strong interaction by employing configurations and/or involving hadronic states inaccessible by other kinds of experiments.

In this regard, another unique advantage of the EFT model arises. Originally, we used the effective range parametrization (3.31) for phase shifts in the asymptotic part of the WF (3.26). However, in principle, the definition of the whole WF can be reduced to only two parameters, width d_l and depth V_l by using Eq. (3.24) to calculate the phase shifts in the asymptotic part. Thus, one could use experimentally measured femtoscopic CFs of pairs of (rare) hadrons to solve the "reverse" problem of defining the effective strong potential parameters from the fits of experimental CFs.

These parameters can provide an effective estimate of the phase shifts at small energy scale since – as we mentioned before – even a simple square-well can provide a decent description ([80] at low momenta (Fig. 3.4). This opens a possibility of indirect phase shift measurements with rare hadronic states inaccessible by classical scattering experiments. The

⁴In this study we consider only the case of two particles focusing on proton pairs. The conclusion drawn for this case may not apply to other species and especially to the case of more than two particles.

CATS framework is unable to do it due to the intrinsic limitation of the numerical approach, namely it needs the explicit potential shape to calculate the WF. The same "reverse" approach was applied to extract the effective range parameters for K^-d pairs in [87] using the LL model that, as we showed before, is insufficient for the proper treatment of the strong potential. The EFT formalism on the other hand might provide a suitable way to approach the "reverse" problem and access the strong potential in non-trivial pairs measurable only in high-energy collisions.

Chapter 4

A Large Ion Collider Experiment

A Large Ion Collider Experiment (ALICE) is one of the four main experiments at the LHC, along with ATLAS, CMS and LHCb. The LHC started operating in 2009 and since then there have been three periods of data taking: Run 1 took place from 2010 to 2013, Run 2 from 2015 to 2018, and Run 3 is still ongoing since 2022, planned to end in 2026. Each operation period was followed with few years of the so-called Long Shutdowns (LS), which were used for upgrades aimed at improving the collider and the detector capabilities of collecting data at higher and higher interaction rates. To demonstrate the progress, we recall the concept of *luminosity* $\mathcal{L}$ ¹ that quantifies the number of events in a given colliding system with the cross section σ per unit of time $\frac{dN_{coll}}{dt}$,

$$\frac{dN_{coll}}{dt} = \mathcal{L} \cdot \sigma. \quad (4.1)$$

To quantify the number of collisions over a given period of time one has to use the *integrated luminosity* L :

$$L = \int \mathcal{L} dt. \quad (4.2)$$

The integrated luminosity has the inverse units of the cross section – i.e. barn and its derivatives. For example if we consider inelastic proton-proton (pp) collisions at $\sqrt{s} = 13$ TeV with $\sigma_{pp}^{inel} = 79.5$ mb [89], then an integrated luminosity of 1 fb^{-1} would correspond to 79.5×10^{12} events.

An example of the evolution of the LHC capacity of collecting data in terms of integrated luminosity in pp collisions is shown in Fig. 4.1, where one can see that it noticeably increases for each operational period, following in turn an increase in the interaction rate.

¹For a more detailed discussion of the concept of luminosity see Ref. [88].

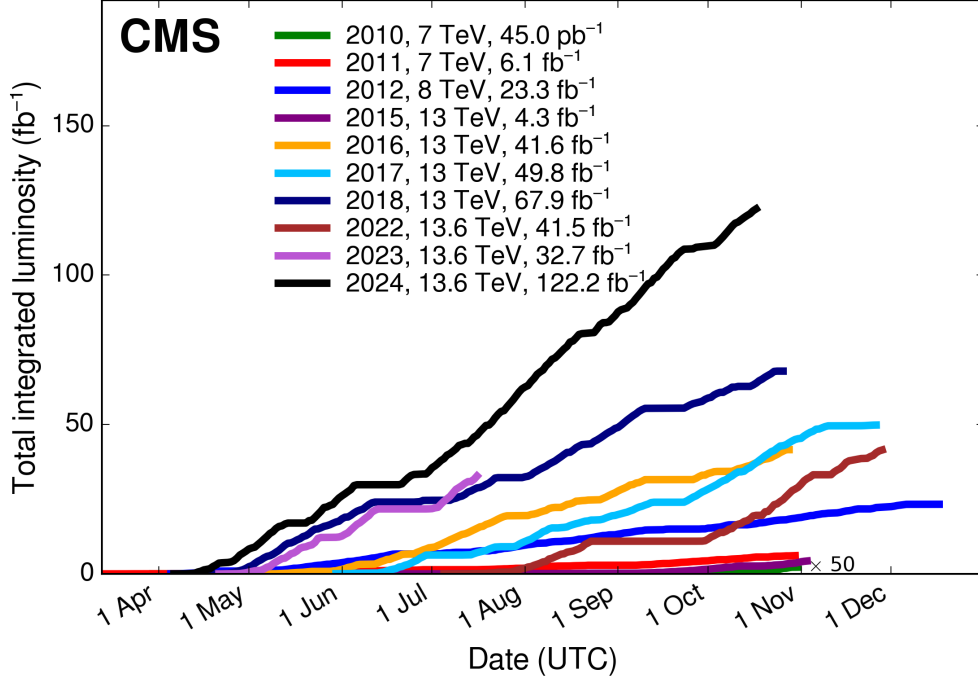


Figure 4.1. LHC's data taking performance in terms of integrated luminosity in pp collisions. The plot is provided by the CMS Collaboration [90].

Since for this study we used the data collected by ALICE during the Run 3 period, in this work we will focus on the ALICE detector system. The main goals of the ALICE upgrades during the LS2 [91] were to improve the detector capabilities in measuring the heavy-flavour particles and allowing for completely new measurements of dilepton thermal production in Pb–Pb collisions. Additionally, the upgrades enabled increased precision measurements of the production of light (anti)nuclei, femtoscopic correlations of hadrons and the effects of collectivity such as the anisotropic flow in high multiplicity pp collisions.

The capability to operate in a continuous readout mode [91] also allowed ALICE to sustain interaction rates in Pb–Pb collisions up to 50 times higher with respect to Run 2. Table 4.1 shows the progressively increasing integrated luminosity collected by ALICE in Pb–Pb collisions.

In the next Sections of this Chapter, the ALICE experimental setup is described, with a focus on the subsystems that are relevant for the experimental study carried out in this thesis.

Table 4.1. Amount of Pb–Pb collisions recorded by ALICE in term of integrated luminosity L over different operational years.

Year(s)	$\sqrt{s_{NN}}$ [TeV]	L [nb^{-1}]
2010, 2011	2.76	≈ 0.01
2015	5.02	≈ 0.02
2018	5.02	≈ 0.02
2023	5.36	≈ 1.54
2024	5.46	≈ 1.54

4.1 Experimental setup

Given that the main goal of the ALICE experiment is to study the QGP and its properties, the detector was specifically designed [92] to operate with high multiplicity events where the charged particle multiplicity density can reach up to $\frac{dN_{ch}}{d\eta} \approx 6000 - 8000$ (in most central Pb–Pb collisions²) at the same time maintaining excellent tracking and Particle Identification (PID) capabilities.

²When the LHC started operating it was found that the charged particle multiplicity density was significantly lower than embedded – e.g. $\frac{dN_{ch}}{d\eta} \approx 2000$ in central Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV.

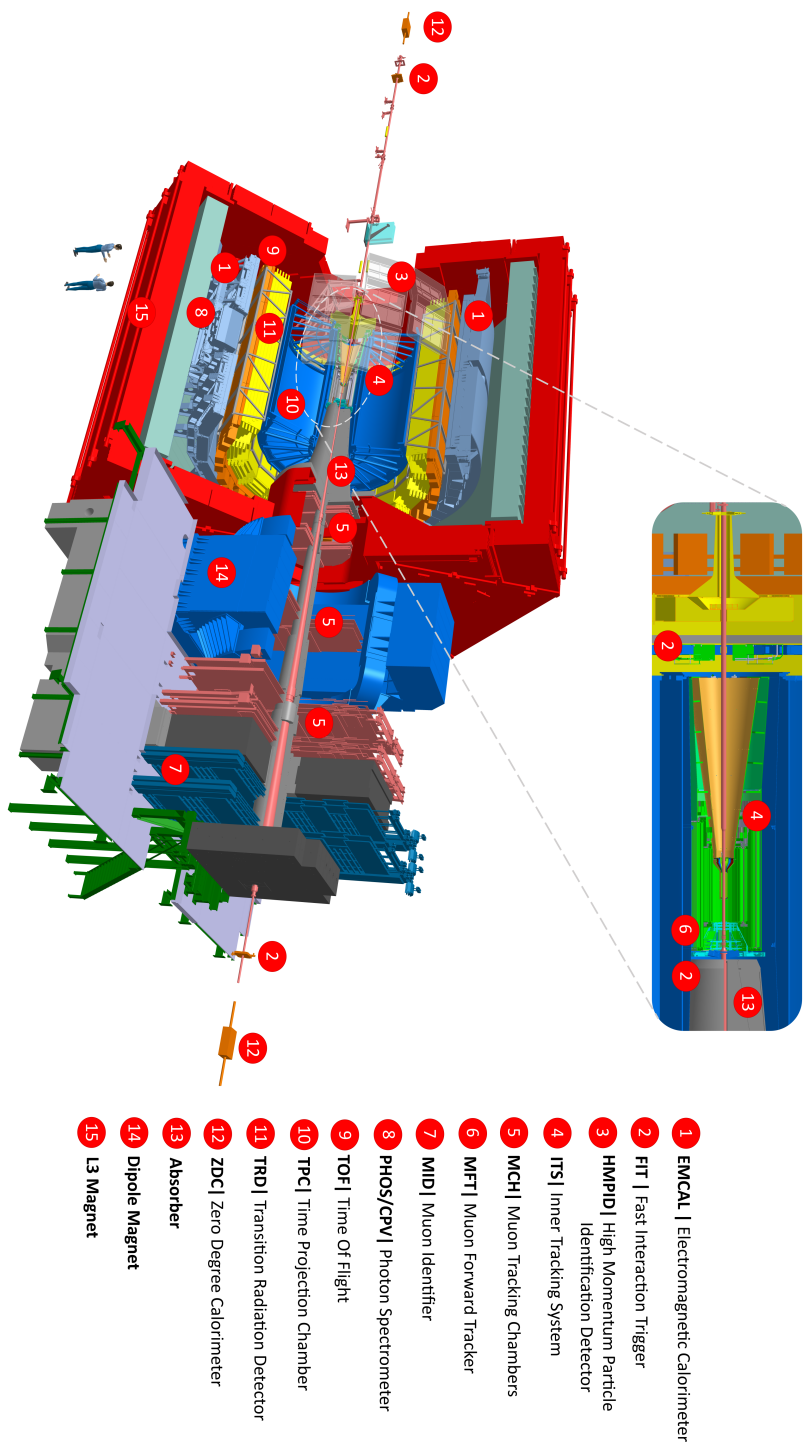


Figure 4.2. ALICE's detector scheme during the Run 3 datataking period [91]

Figure 4.2 is a schematic representation of the ALICE detector that we can divide into three groups of subsystems:

1. **Central barrel detectors:** detectors inside the L3 solenoidal magnet that provides a homogeneous magnetic field of $B = 0.5$ T for the inside detectors which are (from the closest to the beam to the furthest):
 - (a) Inner Tracking System (ITS) – a silicon based detector responsible for the primary collision vertex reconstruction and charged particle tracking (see more in the Sec. 4.3)
 - (b) Time Projection Chamber (TPC) – a gaseous detector responsible for charged particle tracking and PID (see more in the Sec. 4.4, 4.6)
 - (c) Transition Radiation Detector (TRD) – a gaseous detector used for charged particle tracking and electron identification
 - (d) Time of Flight (TOF) – a glass-gas based detector used for PID at intermediate momenta (see more in the Sec. 4.5, 4.6)
 - (e) Electromagnetic Calorimeter (EMCal) – a scintillator based calorimeter responsible for jet and electron reconstruction
 - (f) Photon Spectrometer (PHOS) – a scintillator based detector used for photon and electron reconstruction
 - (g) High Momentum Particle Identification Detector (HMPID) – RICH (ring-imaging Cherenkov) detector used for charged-hadron identification at high momenta.
2. **Muon spectrometer:** detectors placed at forward pseudorapidity following the Dipole Magnet and used to track and identify muons – Muon Tracking Chambers (MCH), Muon Identifier (MID) and the Muon Forward Tracker (MFT). Although, the latter is technically placed in the central barrel we would still put it into this group since all the detectors in it serve the same very specific purpose (plus covers only the forward rapidity region).
3. **Forward detectors:** detectors placed in forward pseudorapidity regions and used for measuring event characteristics:
 - (a) Fast Interaction Trigger (FIT) – a group of detectors providing the measurements of the time of a collision, an estimation of its vertex and the multiplicity (see more in the Sec. 4.2)

- (b) Zero Degree Calorimeter (ZDC) – detectors symmetrically positioned at ± 1.13 m from the geometrical center of ALICE and used to measure spectators to determine centrality of a collision.

In preparation for Run 3 several ALICE detectors – namely ITS, FIT and MFT – were replaced with the new ones, the TPC was upgraded with the new signal readout pads (see Sec. 4.4), and other detectors – TOF, TRD, etc. – underwent to the upgrades of the readout electronics. Overall, ALICE is a very versatile machine that covers a various aspects of QGP studies but since our work is focused only in a particular one, only the relevant detectors are discussed in this Chapter, for more details regarding the other ones see e.g. Ref. [22; 91].

4.2 Fast Interaction Trigger

The Fast Interaction Trigger (FIT) is a new detector family installed during the LS2 as a part of the ALICE's upgrade program in preparation for Run 3 [91; 93]. It consists of five sub-detectors installed at different positions along the beam (Fig. 4.3) with the overall resolution of 26 ps for the collision time and 0.8 cm for the vertex

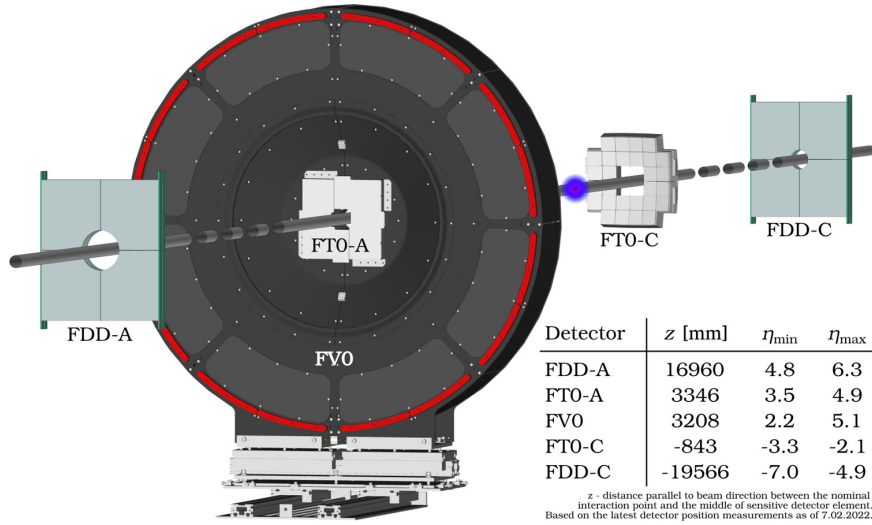


Figure 4.3. FIT detectors configuration with their pseudorapidity coverage and positions along the beam w.r.t. geometrical center of the central barrel [93].

The FDD (Forward Diffractive Detector) are scintillator-based detectors designed to detect charged particles at large pseudorapidity (see the table in Fig. 4.3) for the purpose of selecting ultra-peripheral events when the colliding nuclei don't overlap yet still interact via electromagnetic forces when passing by. The FT0 system consists of two Cherenkov arrays positioned on both sides of the interaction point serving the purpose of measuring the

collision time essential for the TOF PID (see more in the Sec. 4.5, 4.6). It is the successor of the T0 detector used before the Run 3 [22] and inherits its name where "T" stands for time. The last one is the scintillator based FV0 used for collision vertex determination and being the successor of the V0 detector used prior to the upgrade [22]. The "V" in the name stands for vertex.

For the work of this thesis the FIT detector was important for centrality determination. Previously in the Sec. 1.2 we introduced this essential event characteristic and discussed how it can be defined in terms of the impact parameter (1.1) or the number of charged particles produced in a collision (1.6). In practice, the centrality in Pb–Pb collisions is calculated from the FT0-C signal that measures the particles produced as a result of a collision at backward pseudorapidity $-3.3 < \eta < -2.1$. Since the FT0 doesn't reconstruct the particles, the raw signal amplitude is used to build the corresponding distribution and calculate the centrality percentiles (Fig. 4.4).

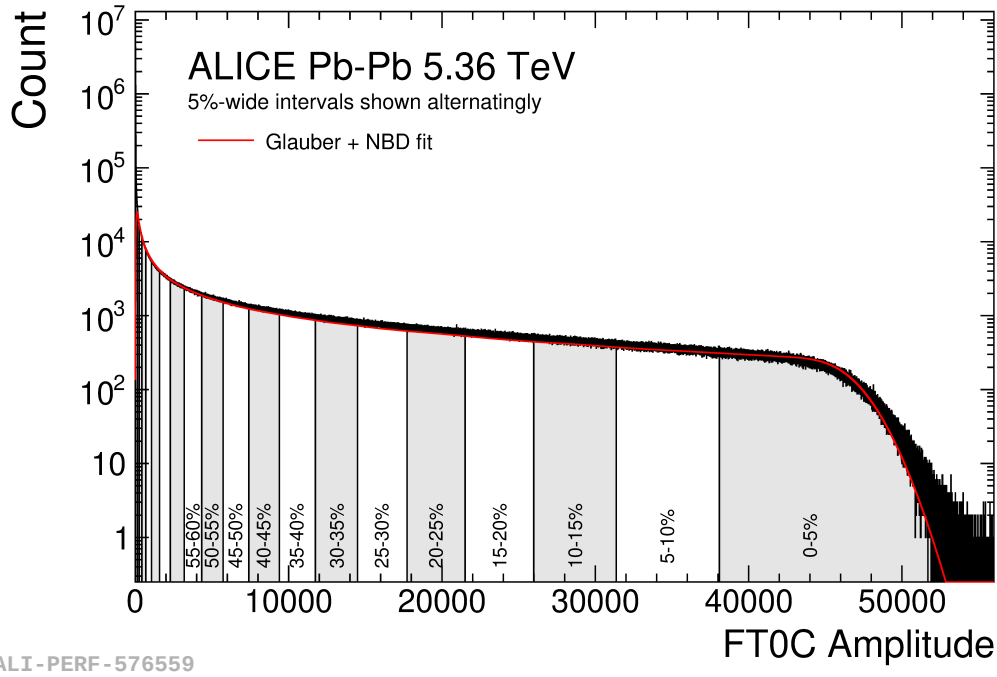


Figure 4.4. Centrality percentiles determination via FT0-C amplitude (arbitrary units) distribution measured in Pb–Pb at $\sqrt{s_{NN}} = 5.36$ TeV.

4.3 Inner Tracking System.

The Inner Tracking System (ITS2) is a 7 layer silicon-based pixel detector placed inside the central barrel of ALICE (Fig. 4.2) with the full azimuthal angle ϕ and $|\eta| < 1.3$ coverage.

It is the closest to the beam detector that serves as one of the main tracking detector and for the collision vertex reconstruction. During the LS2 the ITS used in Runs 1 and 2 was replaced with a new ITS2 detector (Fig. 4.5) in preparation for Run 3 with the motivation of improving the spatial resolution and to cope with the interaction rates up to 50 kHz in Pb–Pb and 400 kHz in pp collisions [91]. In this section we will briefly highlight the characteristics of the ITS2 and improvements with respect to the ITS that we find more relevant for this study (for more details see Ref. [91]).

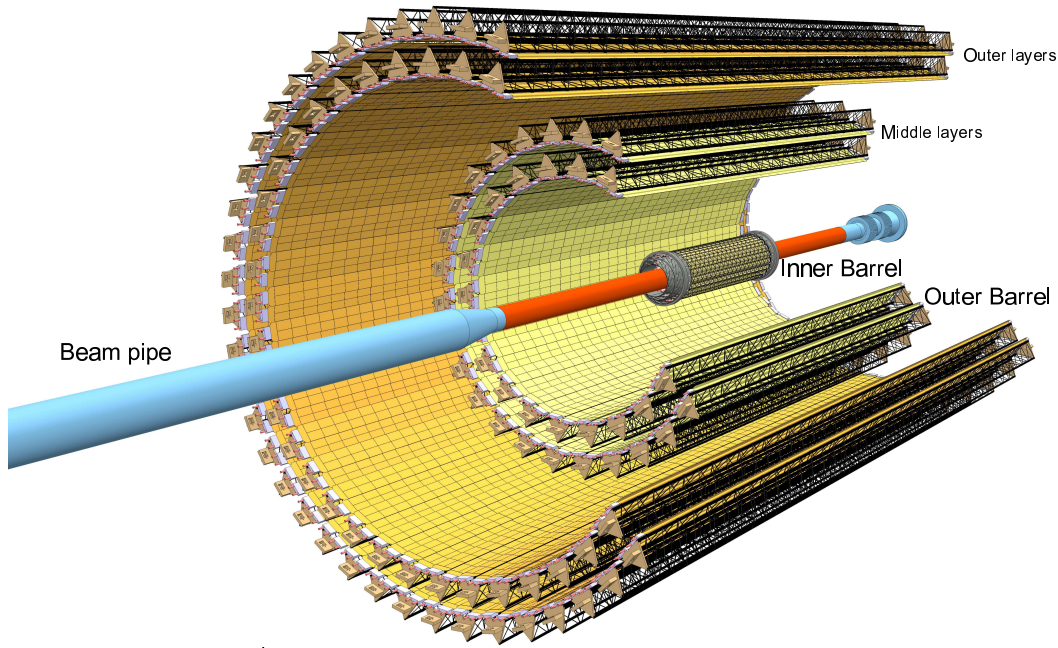


Figure 4.5. ITS2 layout [91]. The inner barrel consists of the 3 innermost layers, the outer barrel has 2 middle layers and 2 outer.

The ITS used in Runs 1 and 2 was a hybrid 6 layers detector system with two (innermost) Silicon Pixel Detectors, two (middle) Silicon Drift Detectors, and two (outermost) Silicon Strip Detectors [22] with a hybrid readout: digital for the pixel layers and analogue for the others. The layers of the ITS2 on the other hand are all based on Monolithic Active Pixel Sensors (MAPS) [94] with the digital readout that allows its usage at higher interaction rates.

The Inner Barrel (Fig. 4.5) of the ITS2 consists now of 3 layers instead of only 2 in ITS with the innermost one being placed closer to the beam – 22 mm instead 39 mm for the ITS. Also, the granularity of the layers has been increased by reducing the size of the pixels from $425 \times 50 \mu\text{m}^2$ to $27 \times 29 \mu\text{m}^2$. Overall, these upgrades resulted in the improvement of the spatial resolution and track reconstruction capabilities.

Fig. 4.6 demonstrates the improvement of the ITS2 performance compared to the ITS in terms of the resolution of the track impact parameter reconstructed in the transverse plane³. The impact parameter of a track (also known as the Distance of the Closest Approach (DCA) and later referred as such) is the minimal distance from the track's propagated trajectory to the Primary Vertex (PV) of a collision (Fig. 4.7). The p_T -dependence of the impact parameter resolution of the ITS/ITS2 is explained by the fact that for low-momenta particles it's more likely to undergo through multiple scattering processes in the material which worsens the reconstruction performance. As the interaction cross section decreases with increasing energy/momentum of a particle, high-momenta particles are less affected by the scattering and therefore can be reconstructed with a higher precision.

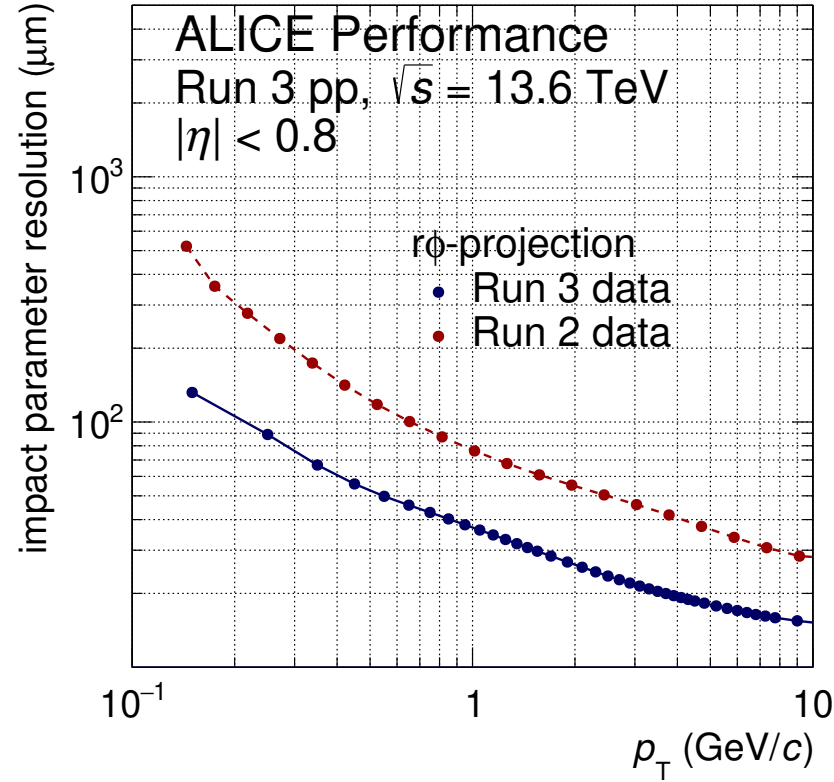


Figure 4.6. p_T -dependence of the impact parameter resolution reconstructed in the transverse plane $r\phi$ (or XY) measured by the ITS2 in pp collisions at $\sqrt{s_{NN}} = 13.6$ TeV during Run 3 in comparison to the ITS performance during Run 2.

³Due to the fact that the pixels in the ITS2 are almost squared (their sides differ by only $\approx 7\%$), the difference between the resolution performance in the transverse plane and along the beam is very mild and in this work will be neglected when we will be applying the corresponding DCA selection cuts.

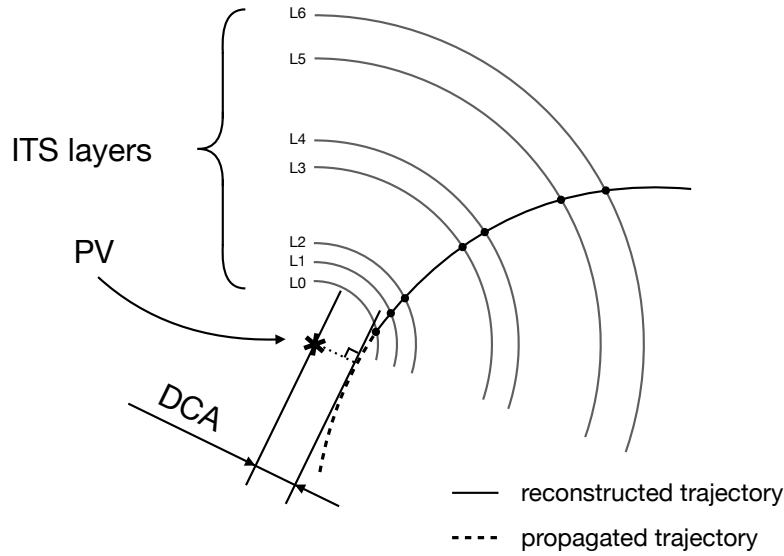


Figure 4.7. Schematic representation of the impact parameter (i.e. DCA) determination in the ITS2.

4.4 Time Projection Chamber

The Time Projection Chamber (TPC) is the main tracking and one of the main PID detectors placed in the central barrel (Fig. 4.2) covering the full azimuthal angle ϕ and the $|\eta| < 0.9$ region. Structurally, it is a cylindrical field cage (Fig. 4.8) filled with gas and a uniform electrostatic field formed by the central high voltage electrode placed in the middle of the chamber dividing it in two parts.

Being a drift detector the principle of TPC's operation is based on the fact that when charged particles travel through the chamber they ionize the gas leaving a trace of electrons and ions. The field of 400 V/cm created by the central electrode forces the electrons (ions) to drift towards the endplates. The gas mixture of $Ne - CO_2 - N_2$ (90-10-5) is chosen to optimize low diffusion for drifting electrons and a large mobility for ions. The field cage with the CO_2 layer is needed to isolate the HV field of the TPC from the other detectors.

The endplates (one on each side) consist of 18 azimuthal sectors each with 2 readout chambers – inner and outer. Each Inner Readout Chamber (IROC) has 63 tangential rows of readout pads measuring the charge signal from the drifting electrons. The total number of pads in IROC is 5280 and their size differs for each row, growing with the radius since the track occupancy per a unit of volume decreases with the distance from the interaction point. Similarly, each Outer Readout Chamber (OROC) consists of 89 pad rows⁴ with the

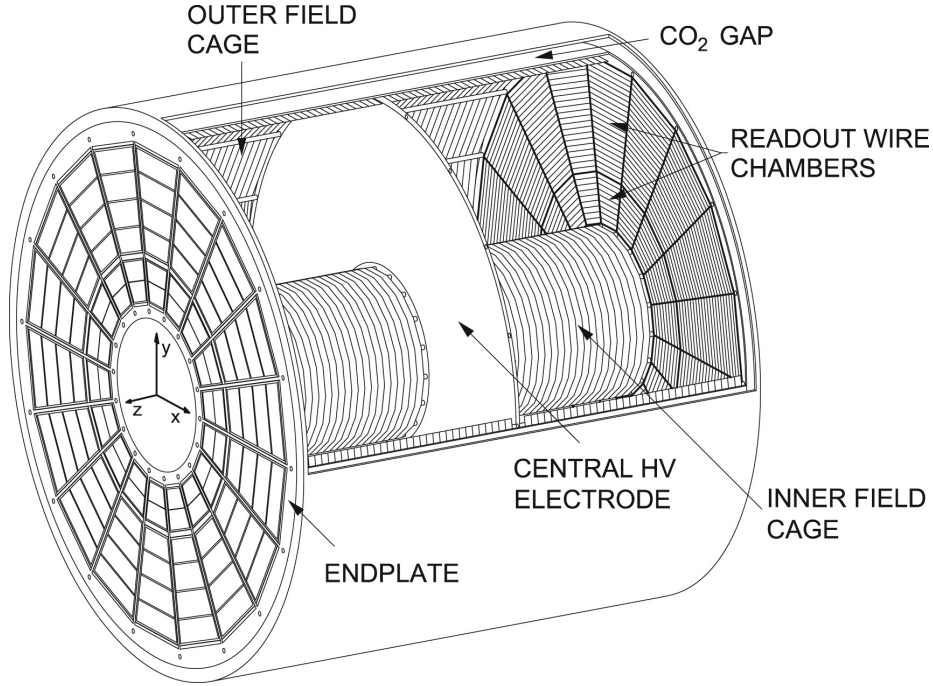


Figure 4.8. TPC layout [95].

total number of pads (in the chamber) being 9280. Such large granularity of readout channels provides precise tracking in a high multiplicity environment.

As we just mentioned, the pads receive the charge signal from the drifting electrons left as a trace of a charged particle traveling through the TPC. The "fired up" pads then represent the particle's trajectory in projection to the endplate plane – i.e. XY -plane (Fig. 4.9). As the electrostatic HV field is constant the electron drift velocity is constant as well being $2.58 \text{ cm}/\mu\text{s}$ for the TPC setup. Thus, measuring the electron drift time⁵ to the endplate one finds the drift length (Fig. 4.9) and can calculate the Z component completing the three-dimensional trajectory reconstruction.

Because all the detectors in the central barrel are placed inside the L3 solenoidal magnet (Fig. 4.2) creating a homogeneous magnetic field $B = 0.5 \text{ T}$, knowing the particle's trajectory one can then reconstruct its charge and momentum as they define the bending orientation and the curvature of the track. Another important feature of the TPC is its ability to measure the energy loss per length unit, dE/dx , of a particle. It is estimated as a truncated mean of the charge signals collected by pads associated to a given track. Together with the momentum

⁴The numbers given in this section (e.g. number of pads, pad rows, drift time, etc.) correspond to the upgraded TPC [96] design and slightly differ from the ones of the TPC used in Run 1-2 [95].

⁵The reference time must be independent of the TPC reference, i.e. collision time from the LHC.

information, the measurement of dE/dx allows to identify the particle type (more about the TPC PID in Sec. 4.6).

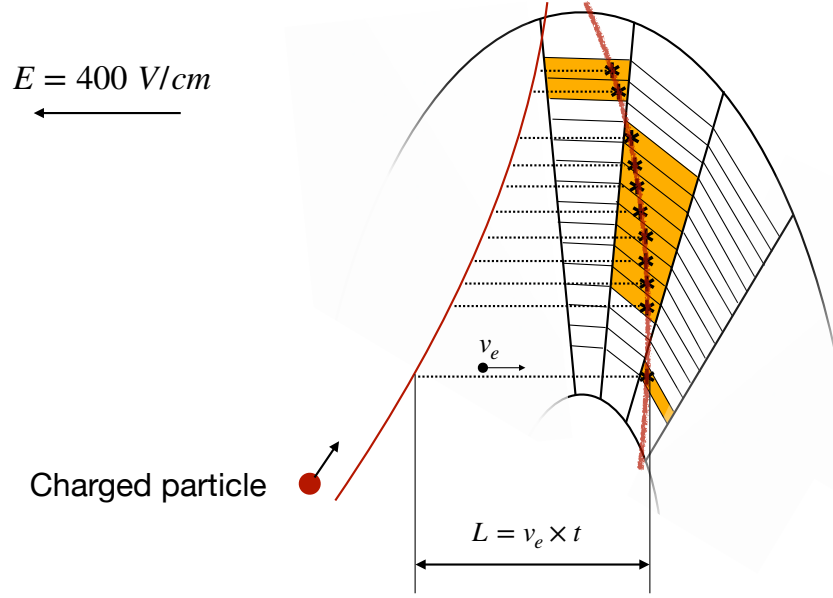


Figure 4.9. Schematic depiction of charged particle trajectory reconstruction in the TPC.

During the Runs 1 and 2 the readout chambers of the TPC were based on the Multi-Wire Proportional Chamber (MWPC) technology that operates with an acting ion grid collecting the ions coming from the signal [95]. The fact that this technology operates with "open" (collecting the signal) and "closed" (preventing the charge backward propagation) states implies limitations for the interaction rate, up to 1 kHz in Pb–Pb collisions. For the Run 3 the readout chambers had to be upgraded to allow operation in a continuous readout mode needed to sustain 50 times higher interaction rates. The new readout chambers of the TPC are based on Gas Electron Multipliers (GEMs) [91] arranged in stacks forming several amplification layers. Usage of this technology reduces the intrinsic dead time and avoid ion backward propagation allowing to sustain the readout rates of Run 3.

4.5 Time-Of-Flight

The Time-Of-Flight (TOF) [97; 98] is one of the main PID detectors of ALICE positioned in the central barrel (Fig. 4.2) with the full azimuthal angle φ and $|\eta| < 0.9$ coverage. Structurally the TOF is a cylindrical layer of glass based Multi-gap Resistive Plate Chamber

(MRPC) detectors arranged in 18 sectors (TOF supermodules) of the ALICE spaceframe (Fig. 4.10). Each supermodule covers the whole range of η acceptance and consists of 5 modules with 91 strips in total. Due to the necessity of reducing the material budget before the PHOS detector (Fig. 4.2) 3 central modules (45 strips) are missing at $\eta \sim 0$. Thus, the total number of strips is 1593.

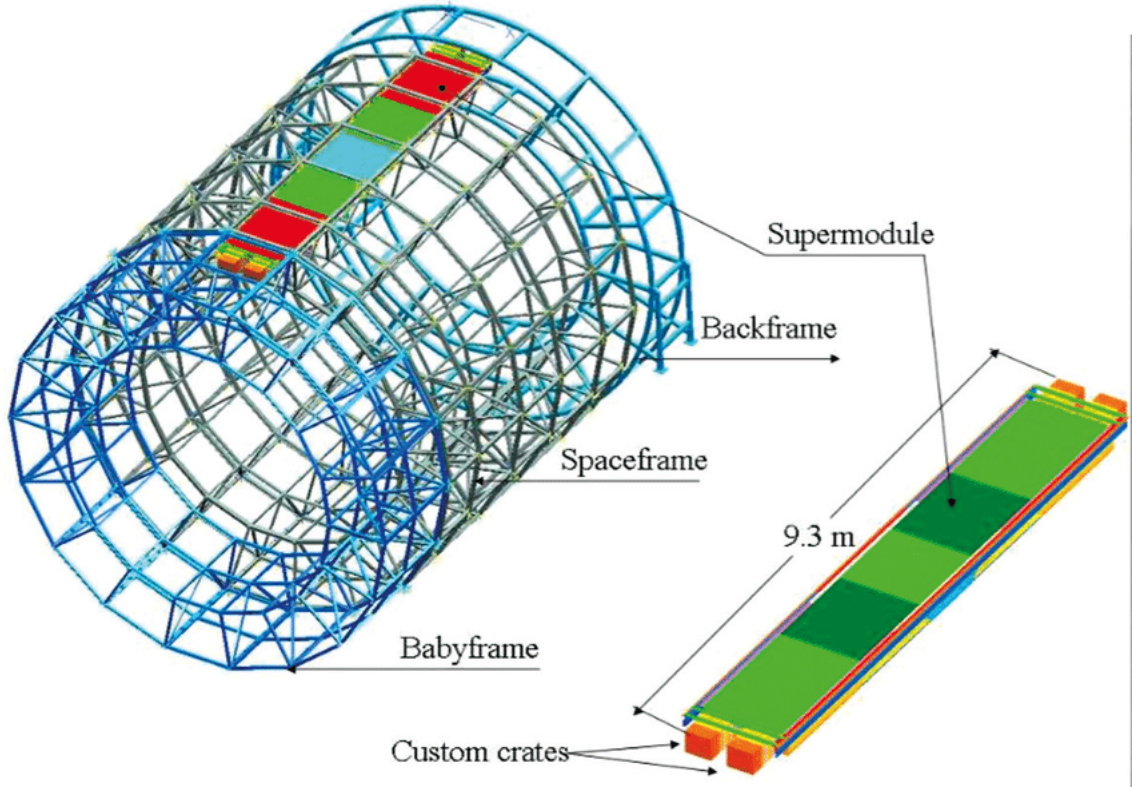


Figure 4.10. TOF layout [98].

The double stack MRPCs of the TOF are 2×5 glass layers separated by $250 \mu\text{m}$ gaps filled with a $\text{C}_2\text{H}_2\text{F}_2$ (93%) + SF_6 (7%) gas mixture. A strong electric field of 13 V is applied inside the plates between the cathodes and an anode (Fig. 4.11) such that when a charged particle travels through the layers ionizing the gas, it triggers a shower of electrons (ions). The anode and cathodes both read the signal allowing for a differential readout. The usage of the multilayer design with thin gaps allows to constrain the size of the shower resulting in a faster signal readout and reducing its fluctuations.

The only measurement the TOF performs is the time-of-flight of particles, hence imposing high standards for time resolution (TOF MRPC's time resolution is ~ 50 ps) as one must be able to separate them. The time-of-flight of a particle is calculated as the time it takes it to reach the TOF detector from the IP:

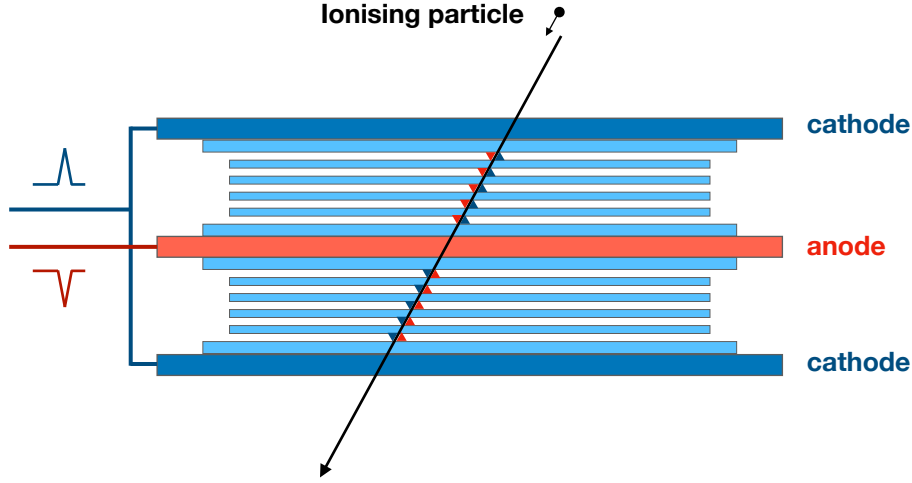


Figure 4.11. Schematic depiction of operation principle of the double stack MRPC technology used in the ALICE TOF detector.

$$TOF = t_{TOF} - t_{ev}, \quad (4.3)$$

where t_{TOF} is the time measured by the TOF; t_{ev} is the time of the collision estimated by the TOF itself [98] or given by the FT0 detector.

The information provided by the TOF is essential for the particle identification that we will discuss in the next section.

4.6 Particle Identification

Particle Identification (PID) in the ALICE TPC and TOF is based on their ability to distinguish the signal measured from different species of particles and effectively separate them. In case of the TPC the energy loss dE/dx is used. For charged particles (apart from electrons) the mean value of dE/dx caused by ionization or excitation of material atoms is given by the Bethe-Bloch formula:

$$-\left\langle \frac{dE}{dx} \right\rangle = 4\pi N_A r_e^2 m_e^2 c^2 z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta}{2} \right], \quad (4.4)$$

where N_A is the Avogadro's number; r_e is the classical electron radius; m_e is the electron mass; z is the charge of the incident particle; Z and A are the charge and atomic numbers

of the material; β and γ indicate the relativistic velocity and Lorentz factor of the ionizing particle; T_{max} is the maximal kinetic energy a free electron can get in a single collision; I is the mean excitation energy; δ represents the density effect correction to ionization energy loss.

For a given material the mean energy loss of an ionizing particle would depend only on its velocity and charge. Therefore, particles of the same momenta but with different masses would loose different amounts of energy in the material. As the TPC measures both momenta and energy loss, one can separate different particle species employing the dependence of the energy loss on the particle's rigidity $dE/dx(p/z)$ (Fig. 4.12). Here we use the dependence on the so-called "rigidity" p/z instead of p to take into account also the dependence on the charge of ionizing particles.

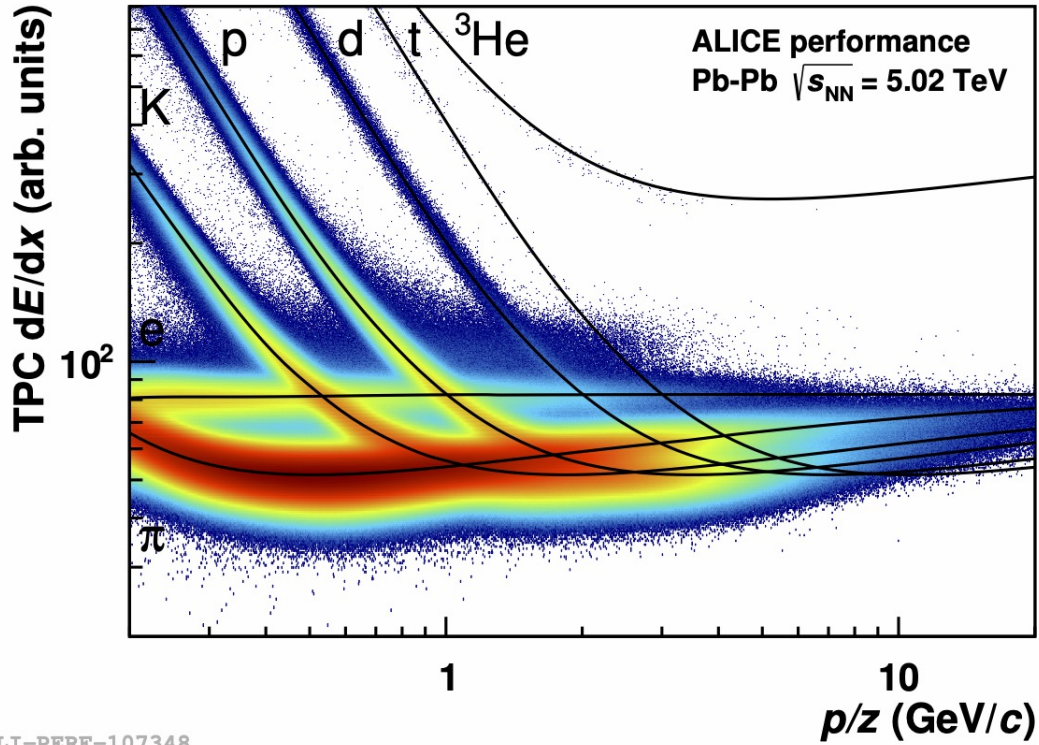


Figure 4.12. Energy loss in TPC dE/d in dependence on rigidity (p/z) measured in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV during the Run 2. The solid lines correspond to the fits of $dE/dx(p/z)$ with the ALEPH formula (4.5) for different PID hypotheses.

The measured energy loss dependence $dE/dx(p/z)$ is then parameterized with the formula proposed by the ALEPH Collaboration for the LEP experiment [99]:

$$f(\beta\gamma) = \frac{P_1}{\beta^{P_4}} \left[P_2 - \beta^{P_4} - \ln \left(P_3 + \frac{1}{(\beta\gamma)^{P_5}} \right) \right], \quad (4.5)$$

where P_{1-5} are fit parameters.

In practice the selection of a given particle species i is performed by "slicing up" over the p/z (or just p) the $dE/dx(p/z)$ distribution and selecting an area of a certain width around the fit line. The width is given by the numbers of dispersion σ_i^{TPC} values of dE/dx distributions for a slice at a given p/z (or just p):

$$N\sigma_i^{TPC} = \frac{\left. \frac{dE}{dx} \right|_{meas} - \left. \frac{dE}{dx} \right|_{i,exp}}{\sigma_i^{TPC}}. \quad (4.6)$$

An example of the $N\sigma^{TPC}$ distribution for the case of protons can be found in Sec. 5.2.

The PID with the TPC is usually used for low momenta particles ($p \lesssim 1$ GeV/c) because as one can see from the Fig. 4.12 the energy losses of different particles start to saturate around the same region and stop depending on the momenta (Fermi's plateau). To identify particles in the $1 \lesssim p \lesssim 3$ GeV/c the TOF is used. The TOF PID is based on the fact that particles of the same momenta but different masses have different speeds and therefore can be separated. In the Fig. 4.13 where the distribution of measured particle velocities β in dependence of their momenta p is reported. The velocity β is estimated as a ratio of the reconstructed trajectory length L corresponding to a given particle to its time-of-flight TOF (4.3):

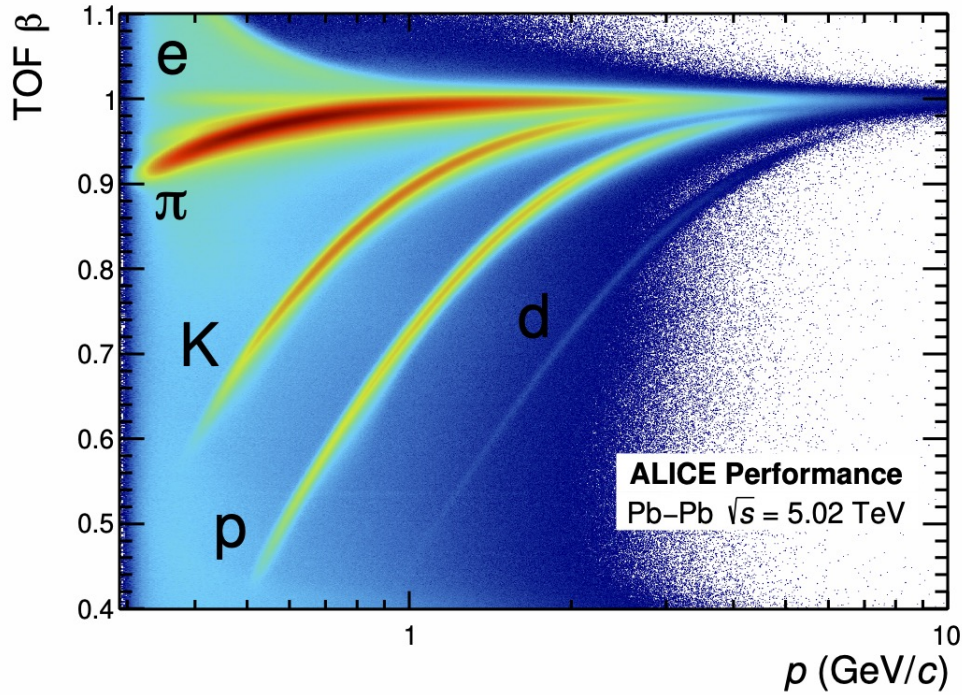
$$\beta c = \frac{L}{TOF}. \quad (4.7)$$

For the particle selection with the TOF one must choose a region of the TOF time (4.3) distribution around the expected time t_{exp} estimated from the mass hypothesis and momenta information from the TPC. Using the same $N\sigma$ approach one can write

$$N\sigma_{TOF} = \frac{TOF - t_{exp}}{\sigma_{TOF}} = \frac{t_{TOF} - t_{ev} - t_{exp}}{\sigma_{TOF}}. \quad (4.8)$$

The time-of-flight resolution σ_{TOF} consists of the time resolution of the TOF detector $\sigma_{t_{TOF}}$ (related to the MRPCs, readout electronics, calibration), the event time resolution $\sigma_{t_{ev}}$ and resolution of the expected time $\sigma_{t_{exp}}$ (related to the track reconstruction in the TPC):

$$\sigma_{TOF} = \sqrt{\sigma_{t_{TOF}}^2 + \sigma_{t_{ev}}^2 + \sigma_{t_{exp}}^2}. \quad (4.9)$$



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Figure 4.13. TOF β in dependence on momentum p measured in Pb–Pb at $\sqrt{s_{NN}} = 5.02$ TeV collisions during the Run 2.

An example of the TOF (4.3) distribution for the pion hypothesis at $p_T \approx 1.5$ GeV/c in the data collected in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.36$ TeV is shown in Fig. 4.14. One can also see that during the Run 3 data taking TOF demonstrated an excellent time resolution of ~ 70 ps (for pion case). This estimate is based on the Gaussian profile superimposed to the experimental data and doesn't include the exponential tails.

An example of the $N\sigma^{TOF}$ distribution for the case of protons can be found in the Sec. 5.2.

Overall, ALICE offers an excellent PID capabilities in a broad range of momentum (Fig. 4.15) allowing to perform various kinds of experimental studies of HICs. In the next Chapter we will present one of such studies dedicated to a femtoscopic analysis of proton pairs measured in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.36$ TeV.

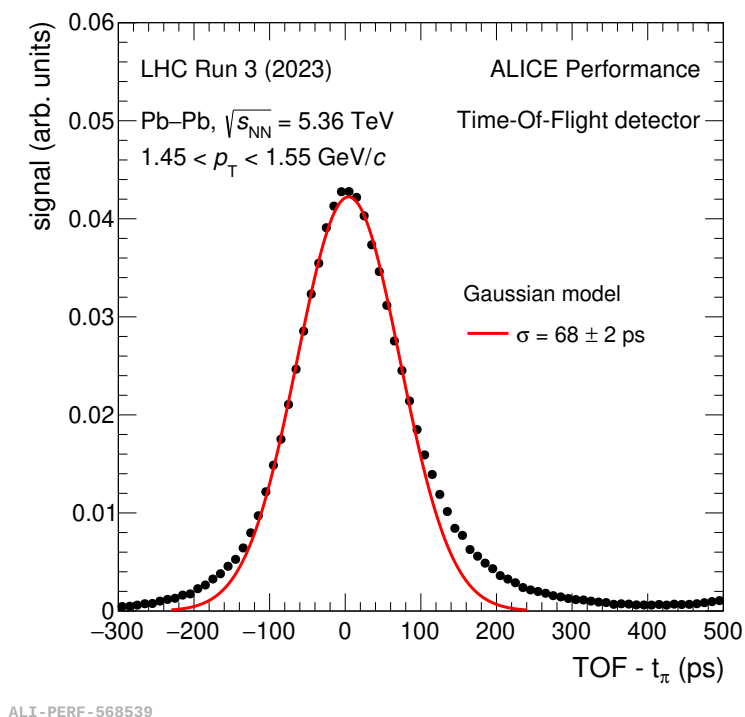


Figure 4.14. TOF timing performance for pions at 1.5 GeV. A Gaussian model with $\sigma = 68$ ps is superimposed to the data.

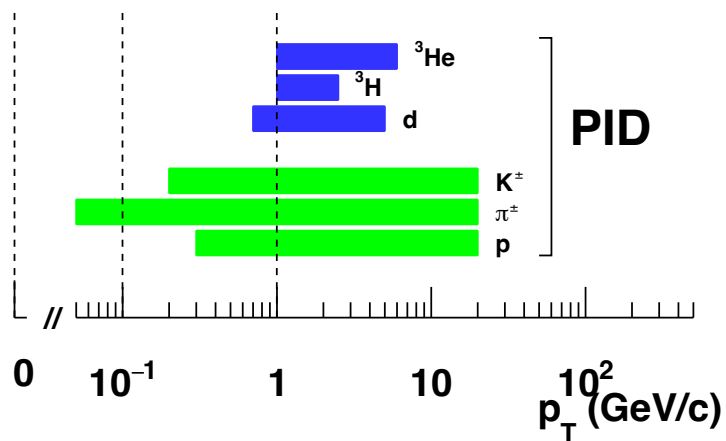


Figure 4.15. ALICE PID capabilities in dependence of particle transverse momenta p_T . The estimate is based on the published measurements in pp or Pb-Pb collisions.

Chapter 5

Proton femtoscopy in Pb–Pb collisions

This Chapter is dedicated to the experimental part of this study and focuses on the femtoscopic analysis of proton pairs measured in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.36$ TeV using the data collected by ALICE during 2023.

The first Sections discuss the event and the track selection, the identification of proton candidates, and the strategy for building the experimental femtoscopic CF, including pair and event mixing procedures. The extracted CFs are used for extracting the proton-emitting source sizes in different centrality classes via a fitting procedure that employs the EFT model discussed in Chapter 3. The resulting source sizes and their dynamics are discussed and compared to the previous ALICE’s measurements.

5.1 Data analysis, event and track selection

For the purposes of this study a software package *Femto3D*¹ was originally developed within the official ALICE’s analysis framework O2. The package implements a full analysis chain need to extract experimental CFs, namely, event and track selection, pair and event mixing, construction of the signal and background distributions to carry out both one- and three-dimensional femtoscopic analyses.

The analysis is performed on a sample of data of Pb–Pb collisions at $\sqrt{s_{NN}} = 5.36$ TeV collected during the 2023 data taking period. The events are selected if the reconstructed position of the collision vertex along the beam direction V_z lays within 10 cm (Fig. 5.1) from the geometrical center of the ALICE’s central barrel to maximize the number of tracks in the ITS and TPC acceptance.

¹<https://github.com/AliceO2Group/O2Physics/tree/master/PWGCF/Femto3D>

Only events in the $[0, 90\%]$ centrality range are adopted which are later split into four centrality classes (Fig. 5.2) for the purpose of exploring the dynamics of the femtoscopic observables in dependence on centrality and multiplicity. The centrality classes used in the analysis are reported in the Tab. 5.1 together with the corresponding values of the charged particle multiplicity density. The centrality values used in this study are calculated from the FT0-C signal distribution (see Sec. 4.2).

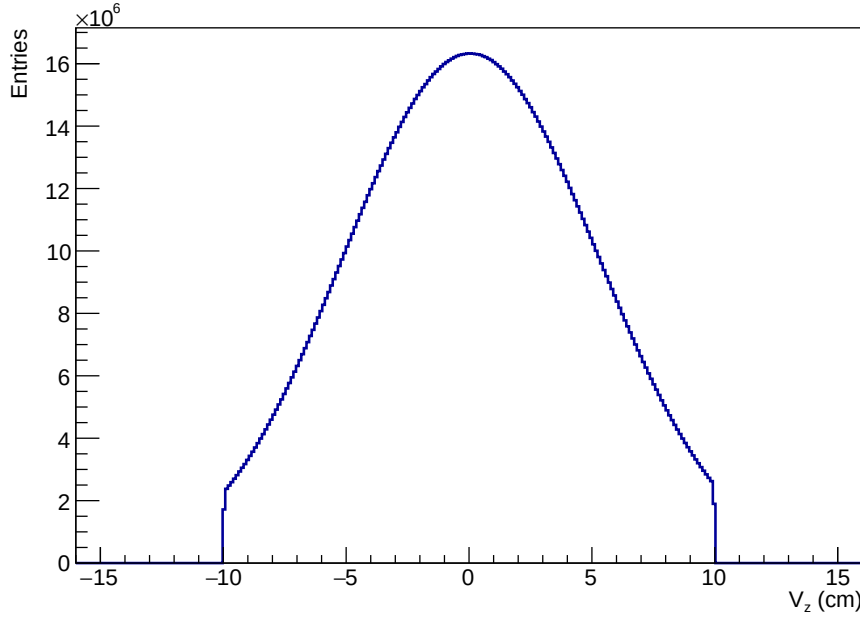


Figure 5.1. Distribution of the reconstructed collision vertex position along the beam direction V_z in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.36$ TeV after applying the $|V_z| < 10$ cm selection criteria.

Table 5.1. Average charged-particle multiplicity pseudorapidity density values at midrapidity, $\langle dN_{ch}/d\eta \rangle_{|\eta| < 0.5}$, corresponding to the centrality classes used in this analysis. The values of $\langle dN_{ch}/d\eta \rangle_{|\eta| < 0.5}$ are calculated as weighted average from the ones reported in Ref. [100]

Centrality [%]	$\langle dN_{ch}/d\eta \rangle_{ \eta < 0.5}$
0-10	1857.5
10-30	1050.5
30-50	455
50-90	97.8

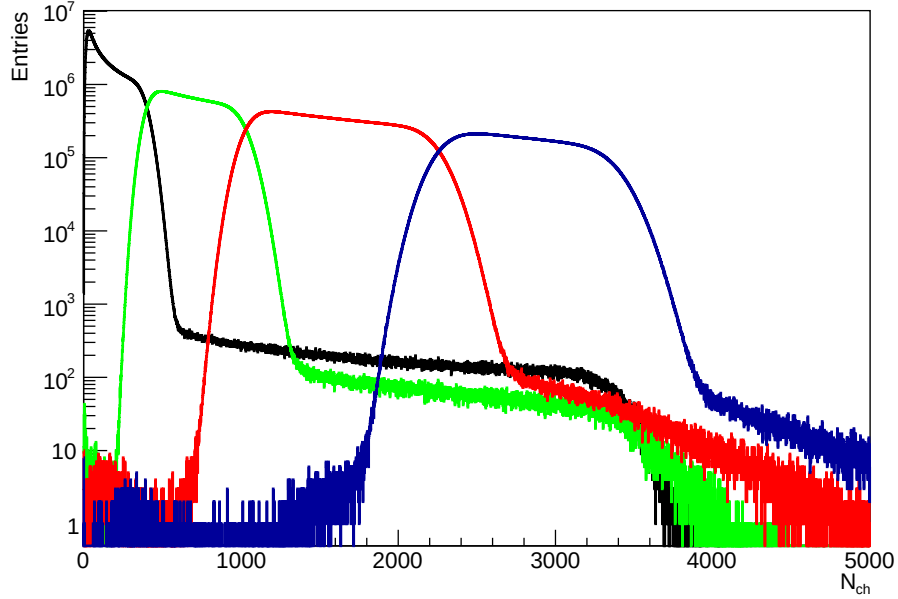


Figure 5.2. Distribution of the number of charged particles N_{ch} in the central barrel (defined as tracks reconstructed in the ITS and the TPC and matched to the primary vertex of the collision) in four centrality classes determined according to the FT0-C estimator (see text): blue line correspond to the 0 – 10% class, red to 10 – 30%, green to 30 – 50% and black to 50 – 90%.

Since the ^{208}Pb ions circulate in bunches, it is possible that more than one collision might occur within the data acquisition window of the detector and the particles produced in one collision could be wrongly associated to another one. In order to avoid this, the events for which the V_z reconstructed separately with the ITS and FT0 differ for more than 1 cm are rejected.

After applying the event selection cuts $1.87 \cdot 10^9$ events were accepted.

When selecting tracks one has to find the balance between keeping as many candidates as possible while maintaining a sufficient quality of the selected tracks. The track selection criteria used in this analysis were optimized to have the best compromise. They are listed in Tab. 5.2 and the same for all the centrality classes.

The selection on pseudorapidity η is conditioned by the fiducial region of the central barrel detectors acceptance (see Chapter 4).

The requirements on the number of the reconstructed ITS clusters N_{cls}^{ITS} and the quality of the fit over the clusters fired up by a particle passing through the ITS ($\chi^2/NDF|_{ITS}$) are optimized to have a sufficient estimation of the track DCA. The latter is important to

Table 5.2. Track selection criteria.

$ \eta $	< 0.8
N_{cls}^{ITS}	7
$\chi^2/NDF _{ITS}$	< 4
$ DCA_{XY} $ (cm)	$< 0.004 + 0.013/p_T$
$ DCA_Z $ (cm)	$< 0.004 + 0.013/p_T$
N_{cls}^{TPC}	≥ 120
$\chi^2/NDF _{TPC}$	< 2
$N_{cls}^{TPC}/N_{findable}^{TPC}$	> 0.9
$N_{shared}^{TPC}/N_{cls}^{TPC}$	< 0.3

constrain the track proximity to the collision vertex, thus avoiding contamination of non-primary particles, i.e. particles that don't originate from the source created in a collision. For example, protons might be a decay product of heavier baryons (e.g. Λ^0 , Σ^+ , etc.) that are also produced. If we consider a $\Lambda^0(uds)$ baryon (mean lifetime of $\tau_\Lambda = 2.631 \cdot 10^{-10}s$) with momentum $p = 1$ GeV/c created in a collision, then it would decay on average at ≈ 5.3 cm from the collision vertex, for the $\Sigma^+(uus)$ (mean lifetime of $\tau_\Sigma = 8.018 \cdot 10^{-10}s$) it would be ≈ 15.5 mm. Another source of non-primary protons contaminating the signal are the ones kicked out from the detector material. The p_T -dependent selection criteria on DCA used in this analysis (Tab. 5.2 and Fig. 5.3²) correspond to a $5\sigma_{ITS}$ range of the ITS impact parameter resolution σ_{ITS} (see Fig. 4.6).

Reconstructed tracks are required to have at least 120 out of 152 possible TPC space points (clusters, see Sec. 4.4) associated to its trajectory and as the latter is obtained via a fit over the clusters, the requirement for the fit quality $\chi^2/NDF|_{TPC} < 2$ is applied. For each particle crossing the TPC the possible (findable) number of clusters that might be associated to its trajectory $N_{findable}^{TPC}$ is estimated. The request on the $N_{cls}^{TPC}/N_{findable}^{TPC}$ ratio ensures that at least 90% of the findable clusters were found. Due to the high number of particles produced in a collision (Fig. 5.2), TPC clusters might be shared between different tracks. The selection on the $N_{shared}^{TPC}/N_{cls}^{TPC}$ limits the amount of the shared clusters for each selected track to 30%.

²The study on the single track quality and proton identification (discussed later) selection criteria and the corresponding figure presented in this work was carried out on a $\approx 10\%$ sub-sample of the whole Pb–Pb data used in the present analysis.

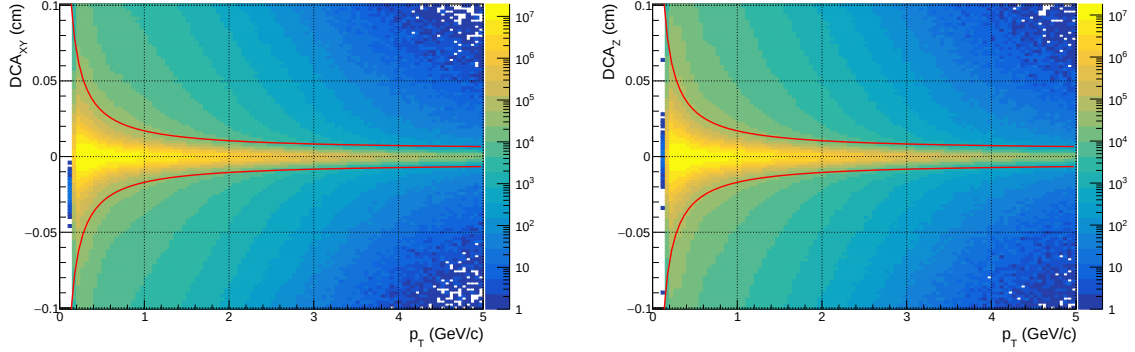


Figure 5.3. DCA distributions in the transverse plane XY (left) and along the beam direction Z (right) versus p_T of the tracks. The red curves correspond to the p_T -dependent DCA selection criteria listed in Tab. 5.2 and used in this analysis.

5.2 Proton identification

The criteria to identify proton candidates used in this analysis are based on the $N\sigma$ approach discussed in the Sec. 4.6.

In order to choose the PID criteria one has to examine the $N\sigma$ distributions prior to any selection except for track reconstruction quality, for a given particle species – in our case (anti)protons – starting with the TPC ones. As one can see in Fig. 5.4, the (anti)proton signal in terms of $N\sigma_{TPC}$ is well separated from the kaon one up to $p \approx 1$ GeV/c within the $-2 < N\sigma_{TPC,p} < 4$ range. However, if we look at the $p/\bar{p}$ momentum-dependent ratio of proton to antiproton number in this region (Fig. 5.5), we will see that it's non-flat and far above unity. As the production of matter and antimatter is the same in collisions at LHC energies [101; 102], deviations from unity in the uncorrected distributions are due to the fact that a fraction of anti-protons annihilates in the detector material and some of the proton candidates are either "fake" or knock out protons from the detector's material.

In order to avoid the contamination of the "fake" protons we applied the TOF PID selection (Tab. 5.3) across the whole momentum range. Due to the limitations of TOF momentum acceptance (Fig. 5.6), the candidates are selected with $p > 0.5$ GeV/c.

Similarly to the TPC PID, the $N\sigma_{TOF}$ selections (Tab. 5.3) are chosen by inspecting the corresponding distributions shown in Fig. 5.6. In addition to the $N\sigma_{TOF,p}$ selection for (anti)protons, kaon and pion rejection is applied: particles with $|N\sigma_{TOF,\pi,K}| < 5$ candidates are rejected. After applying the TOF PID cuts in addition to the TPC ones the momentum distributions of protons and antiprotons (Fig. 5.7) look consistent.

Thus, for the present analysis we used the combined TPC and TOD PID selection criteria listed in Tab. 5.3. It's worth mentioning that a similar approach of using the combined

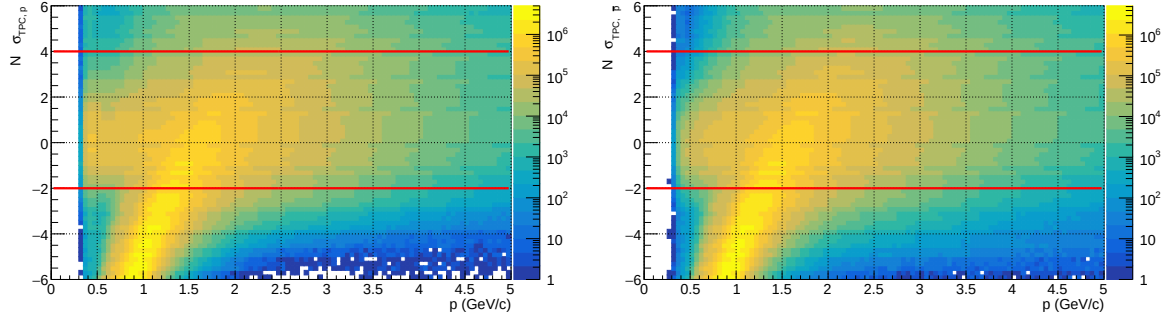


Figure 5.4. $N\sigma_{TPC,p}$ distributions of the selected tracks (Tab. 5.2) corresponding to the proton (left) and anti-proton (right) candidates as a function of the momentum p . The red lines correspond to the adopted TPC PID criteria (Tab. 5.3).

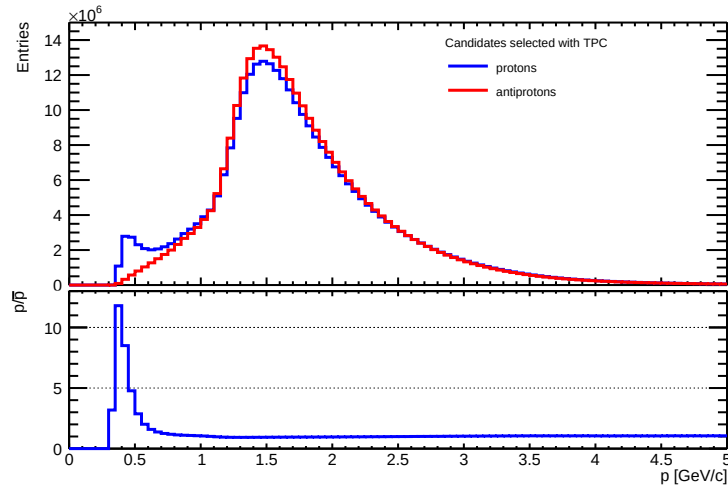


Figure 5.5. Upper panel: momentum p distributions of proton (blue line) and anti-proton (red line) candidates selected within the $-2 < N\sigma_{TPC,p} < 4$ range. Bottom panel: ratio of the proton over anti-proton distribution.

Table 5.3. Proton candidate selection criteria adopted for this analysis.

p (GeV/c)	PID cuts
$0.5 < p < 4.0$	$-2 < N\sigma_{TPC,p} < 4.0$
	$-2 < N\sigma_{TOF,p} < 3.0$
	$ N\sigma_{TPC,\pi,K} > 5.0$

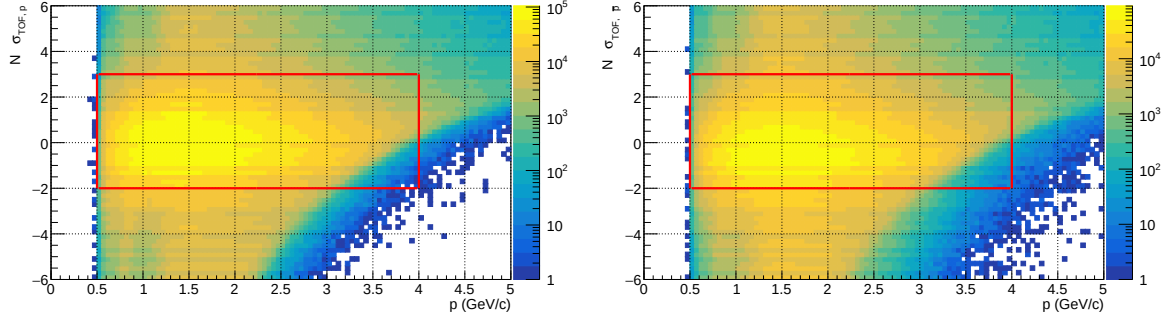


Figure 5.6. $N\sigma_{TOF,p}$ distributions of the selected according to Tab. 5.2 corresponding to the proton (left) and anti-proton (right) candidates versus p . The red lines correspond to the adopted TOF PID selection criteria (Tab. 5.3). The displayed distributions include the application of the rejection criteria for pion and kaon candidates $|N\sigma_{TOF,\pi,K}| < 5$.

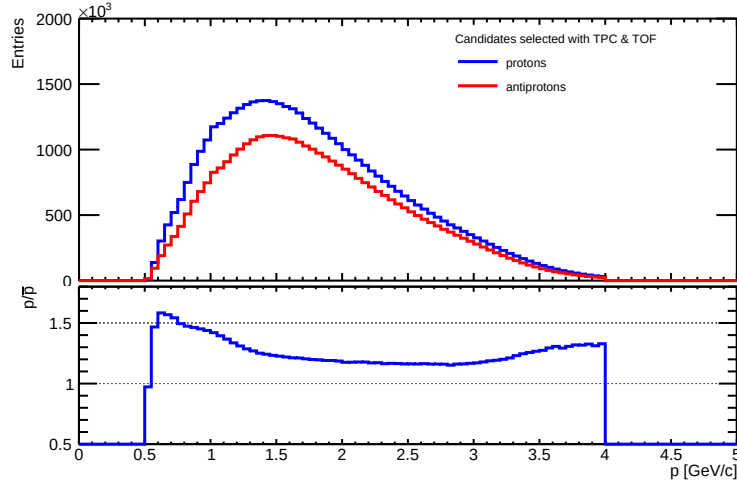


Figure 5.7. Upper panel: momentum p distributions of proton (blue line) and anti-proton (red line) candidates selected according to the PID selection reported in Tab. 5.3. Bottom panel: ratio of the proton distribution relative to the anti-proton one.

PID selections was used in a previous ALICE study on proton pair femtoscopy in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV data collected during Run 2 [48].

5.3 Event mixing and pair selection

The experimental femtosopic CF is built from pair relative momentum k^* distributions. The signal distribution $S(k^*)$ is obtained by combining pairs of particles produced in the same event (SE). In order to remove the combinatorial background it is divided by the background distribution $B(k^*)$ measured as a k^* -spectrum of pairs combined from different

(later referred as "mixed", ME) events where any physical correlations are excluded by construction. Therefore the experimental femtosopic CF is constructed as [50]

$$C_{exp}(k^*) = N \frac{S(k^*)}{B(k^*)} \equiv N \frac{SE(k^*)}{ME(k^*)}, \quad (5.1)$$

where N is a normalization factor.

The candidates selected according to the track (Tab. 5.2) and proton PID (Tab. 5.3) criteria are used to construct the pair distributions needed to obtain the experimental CFs (5.1). The mixed event (ME) pair distribution (denominator in Eq. 5.1) construction is performed in two steps:

1. Events in a given centrality class (Tab. 5.1) are divided in eight equidistant centrality bins simultaneously being split into 10 equidistant ranges in V_z . Each double differential sub-bin constitutes to a mixing pool of events with similar collision characteristics that are paired with each other;
2. Each proton coming from one event in the pool is paired with all the protons belonging to another event.

Before looking at the SE and ME distributions we have to address the matter of pair selection. As the femtosopic signal is present in a low pair relative momentum region (see Sec. 3.2), the kinematic characteristics of the particles in a pair are similar. This requires to take account for *double track effects*, arising from the reconstruction procedure (and performance).

There are two types of such effects – "merging" and "splitting". The former occurs when two different tracks are reconstructed as one as these tracks were too close to each other to be distinguished during the reconstruction process. The latter is the opposite – one track is reconstructed as two. These effects affect only the SE distribution since the track pairs constructed from different events are uncorrelated by definition. Thus, they affect the form of the experimental CF by introducing an imbalance to the SE/ME ratio unrelated to physics: merging lowers the experimental CF in the low k^* region, whereas splitting enhances it (Fig. 5.8).

In order to avoid any significant influence of the double-track effects, the pairs are selected by constrainig the difference of pseudorapidity values $\Delta\eta$, related to the polar angle (1.4), and the pair azimuthal angular distance.

Due to the presence of the magnetic field $B = 0.5$ T created by the L3 solenoid (Fig. 4.2), the trajectories of the charged particles are bent in the transverse plane. To take the

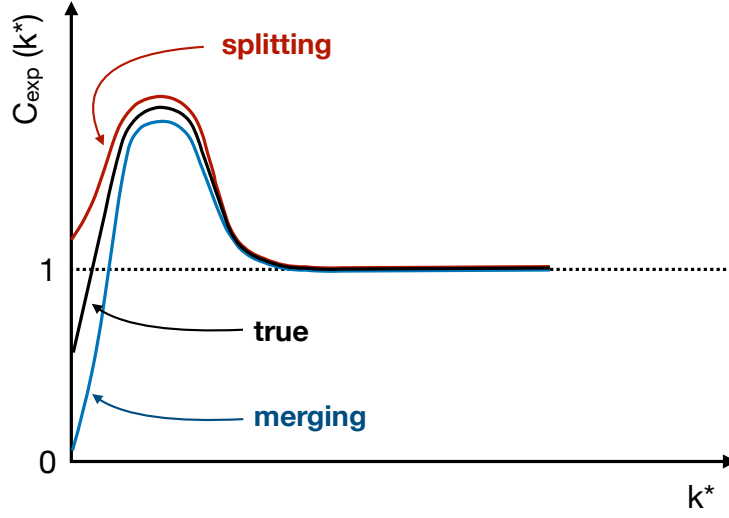


Figure 5.8. A schematic example of "merging" (blue line) and "splitting" (red line) double track effects on the "true" experimental CF (black line).

bending into account the particle azimuthal angle at the interaction point φ must be corrected to obtain the azimuthal angle φ^* at a distance R from the interaction point in the transverse plane as

$$\varphi^* = \varphi - \arcsin\left(\frac{-0.3 B z R}{2 p_T}\right), \quad (5.2)$$

where z is the particle's charge, R is in meters.

The selections on the pair relative angular distances $\Delta\varphi^*$ and $\Delta\eta$ are chosen by utilizing Monte Carlo (MC) simulations. The Angantyr model in PYTHIA8 [103] is used for the event generation and Geant4 [104] for particle propagation through the detector. These MC simulations incorporate only the detector effects whereas the experimental distributions are influenced also by the physical interaction between the particles in a pair. The events and tracks analysed in the simulated samples are treated "data like", i.e. the same event, track quality and PID selection criteria are applied to the corresponding reconstructed characteristics.

Similarly to the CF (5.1), in order to factor out the combinatorial background one has to divide the SE distribution by the ME one. The $\Delta\varphi^*(R=1)$ vs. $\Delta\eta$ distributions obtained with the MC are shown in Fig. 5.9. A distinct depletion in $\Delta\varphi^* \sim 0$ and $\Delta\eta \sim 0$ region

manifests the presence of the effects of track merging. To avoid it we use an elliptic exclusion criteria (Fig. 5.9):

$$\left(\frac{\Delta\eta}{\Delta\eta_{cut}}\right)^2 + \left(\frac{\Delta\varphi^*}{\Delta\varphi_{cut}^*}\right)^2 < 1, \quad (5.3)$$

with $\Delta\eta_{cut} = 0.2$ and $\Delta\varphi_{cut}^*(R=1) = 0.015$.

While the double track effects are present only in the SE distribution, the cuts must be applied to both the SE and ME experimental distributions to not to induce imbalance artificially.

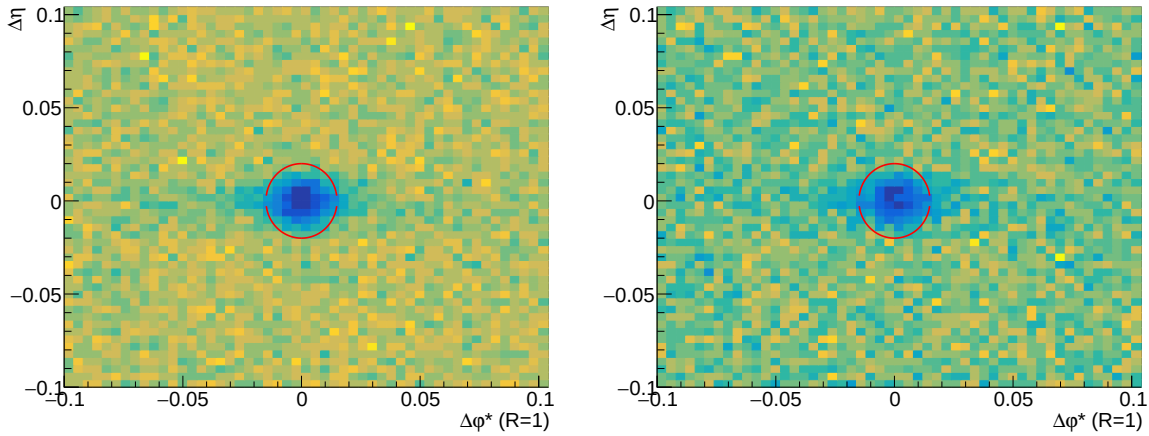


Figure 5.9. The SE/ME ratio for the $\Delta\varphi^*(R=1)$ vs. $\Delta\eta$ distributions obtained with MC simulations for proton (left) and antiproton (left) pairs. The red curves correspond to the elliptical exclusion cut (5.3) with $\Delta\eta_{cut} = 0.2$ and $\Delta\varphi_{cut}^*(R=1) = 0.015$.

Finally, the selected pairs are divided into four ranges (Fig. 5.10) in pair transverse momentum k_T (in GeV/c): $[0.4, 0.8]$, $[0.8, 1.0]$, $[1.0, 1.2]$ and $[1.2, 1.6]$. Together with the centrality one, the k_T binning makes this analysis double-differential allowing for a more detailed study of the proton-emitting source.

An example of the experimental SE and ME distributions of proton pairs ($p-p$) obtained in the first centrality class and the first k_T range is shown in Fig. 5.11. The difference in population between SE and ME distributions is explained by the fact the the number of pairs in the first scales as $\propto \langle N_p \rangle^2 - \langle N_p \rangle$ where $\langle N_p \rangle$ is the average number of particles per event, whereas the second scales as $\propto [\langle N_{ev} \rangle^2 - \langle N_{ev} \rangle] \times \langle N_p \rangle^2$ where $\langle N_{ev} \rangle$ is the average number of events in the mixing pool.

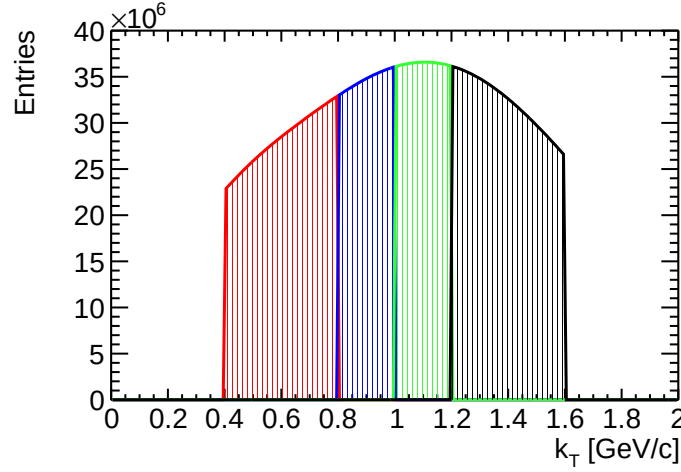


Figure 5.10. Experimental pair transverse momentum k_T distributions of $p-p$ pairs in the 0 – 10% centrality class.

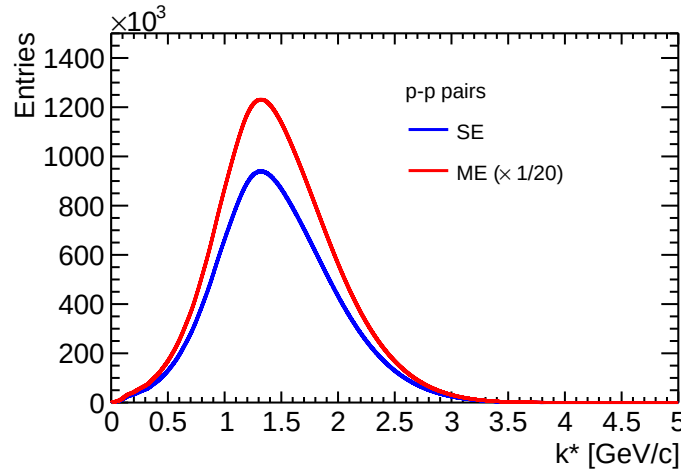


Figure 5.11. Experimental SE (blue line) and ME (red line) distributions of proton pairs in $0.3 < k_T < 0.8$ range for the 0 – 10% centrality class. The ME distribution is scaled by the factor of 1/20 for a better visualization.

5.4 Experimental proton pair correlation function

To compensate the imbalance in the number of entries between SE and ME distributions (Fig. 5.11) the experimental CF $C(k^*)$ built as their ratio must be normalized (5.1). The normalization region has to be chosen outside of the range where the femtoscopic signal is expected. The experimental $p-p$ and $\bar{p}-\bar{p}$ CFs for 4 centrality classes and 4 k_T ranges shown in Fig. 5.12 are normalized to unity³ in the $k^* = [0.4, 0.6]$ GeV/c range.

³The CFs must be normalized to 1 since the $C(k^*) = 1$ in the absence of interaction (see Sec. 2.1).

As it can be seen from Fig. 5.12, the $p - p$ and $\bar{p} - \bar{p}$ CFs displayed in the signal region fully are compatible (within uncertainties), which is expected since the interaction in proton and antiproton pairs is the same [105] due to the CPT theorem. Therefore, from now on we will be considering the common $p - p \oplus \bar{p} - \bar{p}$ CFs.

An example of the $p - p \oplus \bar{p} - \bar{p}$ CF obtained for the 0 – 10% centrality class in the $k_T = [0.3, 0.8]$ GeV/c range is shown in Fig. 5.13. The positive correlation ($C(k^*) > 1$) at large k^* might be explained by the jet contribution or a violation of the momentum-energy conservation laws in the ME [50]. To factor out the influence of these non-femtoscopic correlations, the experimental CFs are fitted with a second-order polynomial function (Fig. 5.13) outside the signal region ($k^* = [0.15, 3.0]$ GeV/c) and then divided by its profile in the whole k^* range. This profile is commonly referred to as *baseline* and the corresponding correction as the *baseline correction*.

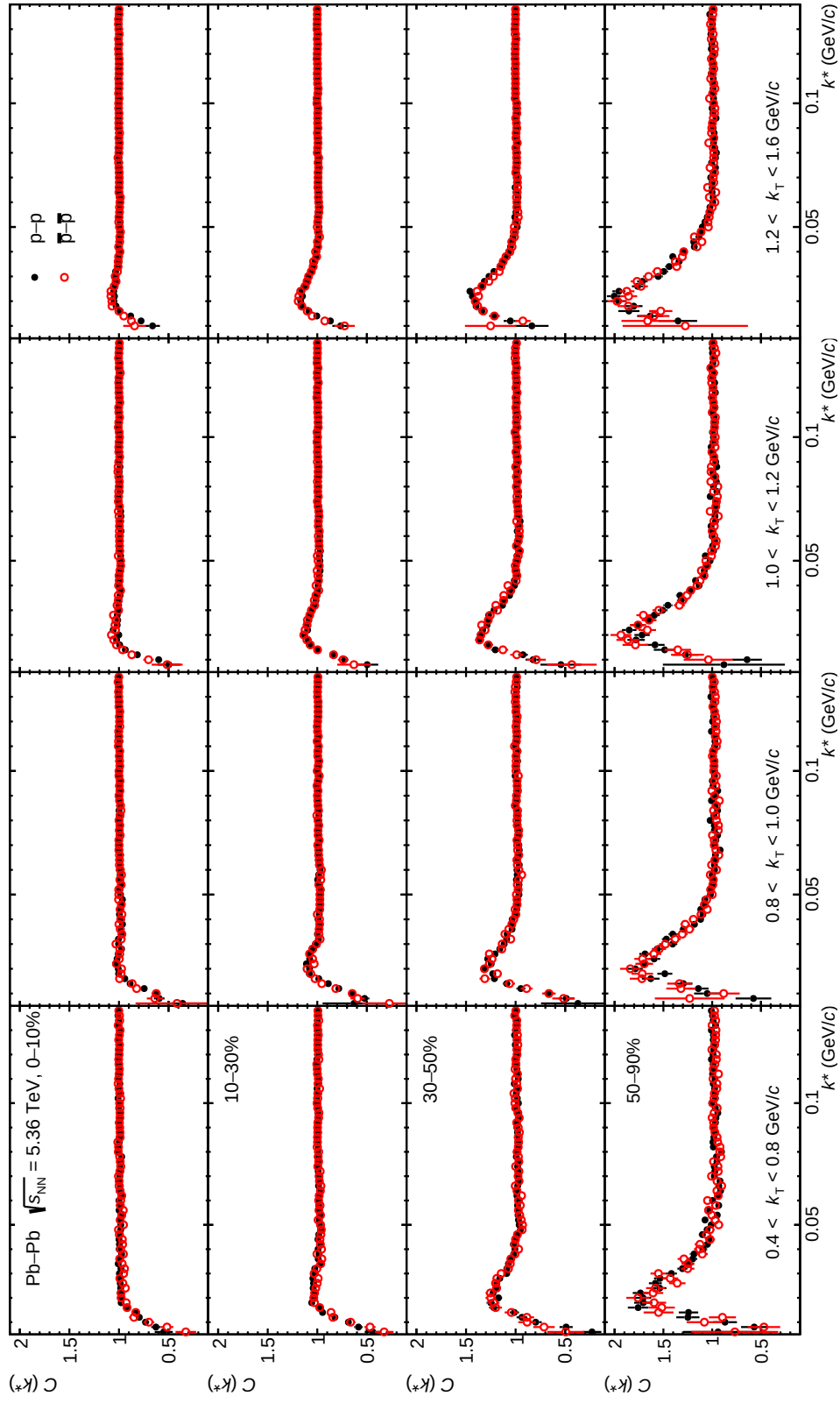


Figure 5.12. $p - p$ and $p - \bar{p}$ CFs for four centrality classes (presented vertically) and four k_T ranges (presented horizontally).

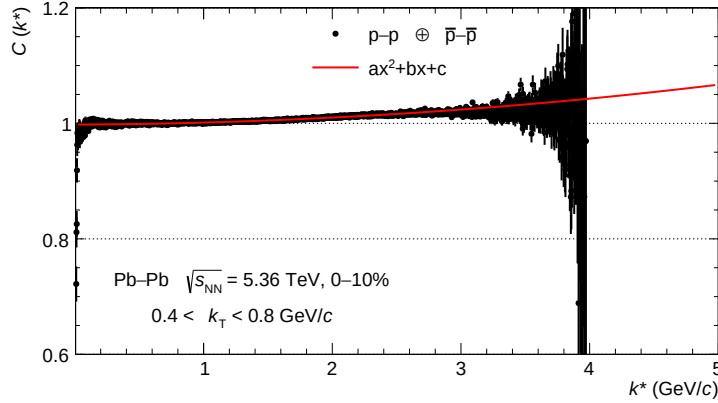


Figure 5.13. Experimental $p - p \oplus \bar{p} - \bar{p}$ CF in $0.3 < k_T < 0.8$ GeV/c range for the 0 – 10% centrality class. The baseline (red line) used for the corresponding correction is estimated from the fit of the CF in $k^* = [0.15, 3.0]$ GeV/c range.

5.5 Fitting procedure

Once we have the experimental proton CFs normalized and corrected for the baseline we can use them for the extraction of the proton source size parameters by fitting them with the following formula:

$$C_{exp}(k^*, R_{inv}) = 1 + \lambda [C_{th}(k^*, R_{inv}) - 1], \quad (5.4)$$

where the $C_{th}(k^*, R_{inv})$ is the theoretical femtoscopic CF calculated with Eq. (3.37) utilizing the proton pair WF (3.27) originally developed as a part of this study (see Sec. 3.1); R_{inv} is the one-dimensional source size parameter extracted from the fit, and λ is another fit parameter used to accommodate for the possible divergence of the pure theoretical proton pair CF and the one obtained from the experiment.

The deviation of the experimental CF from the theoretical one can be caused by a various reasons such as the non-Gaussianity of the particle-emitting source [46; 47], its asymmetry, as well as contributions of the non-primary protons and/or misidentified protons in the experimental sample, etc. A more detailed discussion on the λ parameter will be carried out in the next Section.

Another source of discrepancy one has to account for, is the effect caused by the finite momentum resolution. As a particle momentum cannot be reconstructed precisely by a detector but with some uncertainty, this consequently propagates to the experimental k^* pair distributions. This is commonly referred as *momentum smearing*. Since it's a detector effect,

we once again used the MC simulations to estimate it as they contain both the information of the particle momenta before and after particles are propagated through the detector simulating the reconstruction procedure. Figure 5.14 (left) reports the *resolution matrix* $M(k_{true}^*, k_{rec}^*)$ representing the correspondence between the "true" k_{true}^* values calculated before particle propagation through the detector and the "reconstructed" k_{rec}^* ones.

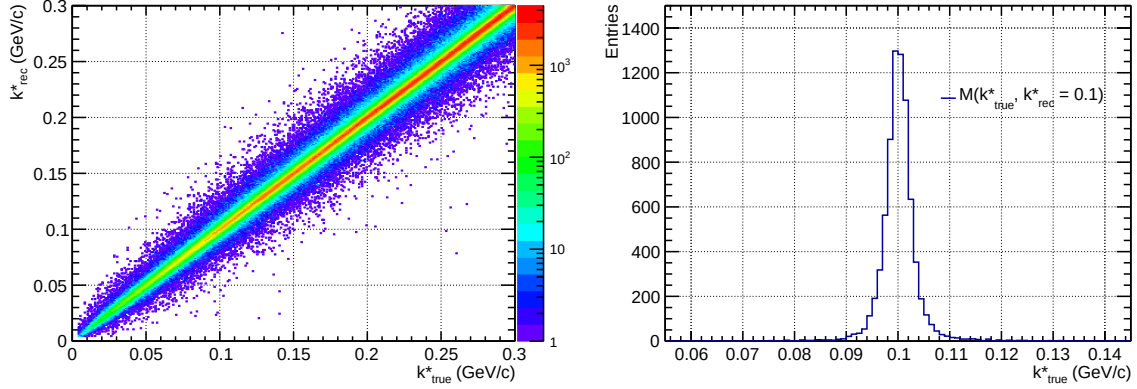


Figure 5.14. Left: resolution matrix $M(k_{true}^*, k_{rec}^*)$ calculated with MC simulations for $p - p \oplus \bar{p} - \bar{p}$ pairs in $0.3 < k_T < 0.8$ GeV/c range for the 0 – 10% centrality class. Right: k_{true}^* profile at $k_{rec}^* = 0.1$ GeV/c.

As one can see from the figure, each reconstructed value of k_{rec}^* may correspond to different values of k_{true}^* (e.g. Fig. 5.14, right). Using $M(k_{true}^*, k_{rec}^*)$ we can define the probability of k_{true}^* being reconstructed with a given k_{rec}^* by introducing the corresponding probability density function (PDF):

$$\rho_{k_{rec}^*}^*(k_{true}^*) = \frac{M(k_{true}^*, k_{rec}^*)}{\int M(k_{true}^*, k_{rec}^*) dk_{true}^*}, \quad (5.5)$$

which we use to simulate the momentum smearing effect on the fit function (5.4) via the following convolution:

$$\begin{aligned} C_{exp}(k_{rec}^*, R_{inv}) &= \int \left\{ 1 + \lambda [C_{th}(k_{true}^*, R_{inv}) - 1] \right\} \cdot \rho_{k_{rec}^*}^*(k_{true}^*) dk_{true}^* = \\ &= 1 + \lambda \left[\frac{\int C_{th}(k_{true}^*, R_{inv}) \cdot M(k_{true}^*, k_{rec}^*) dk_{true}^*}{\int M(k_{true}^*, k_{rec}^*) dk_{true}^*} - 1 \right]. \end{aligned} \quad (5.6)$$

The $C_{exp}(k_{rec}^*, R_{inv})$ can be interpreted as a mathematical expectation value $\mathbb{E}[C_{exp}(k_{true}^*, R_{inv})]$ given that k_{true}^* is a random variable with a PDF defined for each k_{rec}^* (see Eq. 5.5) from the resolution matrix $M(k_{true}^*, k_{rec}^*)$. An example of the smearing

effect on the theoretical proton CF calculated for $R_{inv} = 2$ fm is presented in Fig. 5.15.

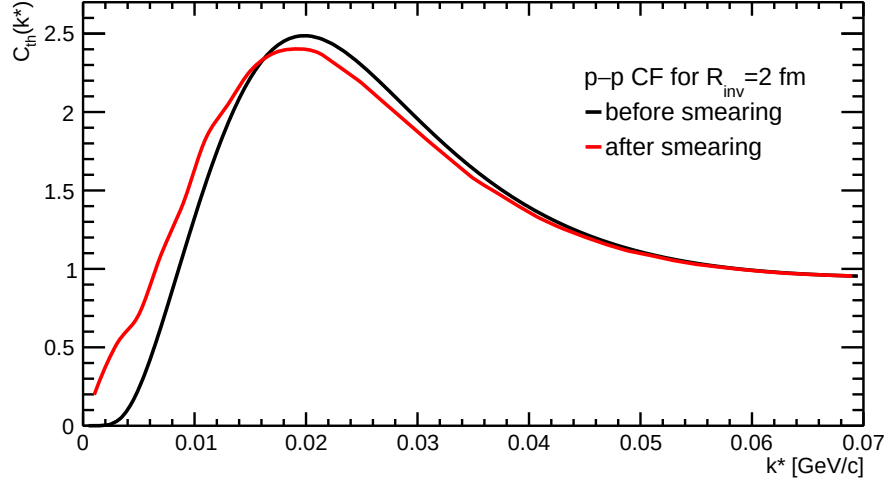


Figure 5.15. Theoretical proton CF (i.e. $\lambda = 1$) calculated for $R_{inv} = 2$ fm before (black curve) and after (red curve) the convolution according to Eq. (5.6) with the resolution matrix $M(k_{true}^*, k_{rec}^*)$ (Fig. 5.14).

The Eq. (5.6) is used to fit the experimental CFs (Fig. 5.16) and extract the one-dimensional source size R_{inv} and λ parameters that we discuss in the following sections.

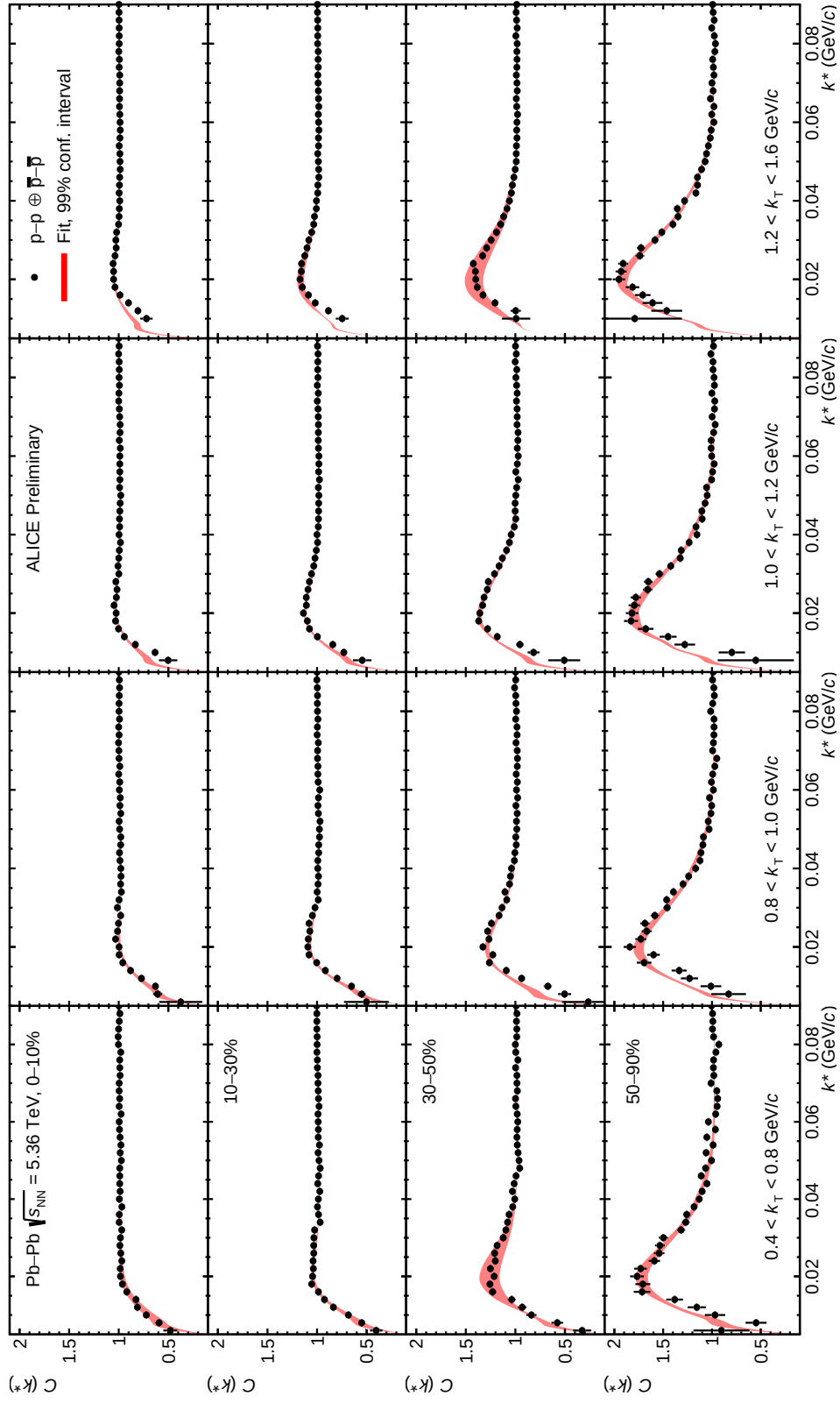


Figure 5.16. $p - p \oplus \bar{p} - \bar{p}$ CFs for four centrality classes (presented vertically) and four k_T ranges (presented horizontally) fitted with the Eq. 5.6. The red bands represent the fit within a 99% confidence interval.

5.6 Systematic uncertainties

In order to estimate the robustness of the extracted from the fit parameters (Fig. 5.17 and Tab. 5.5, 5.5), a study on the systematic uncertainties was performed by varying track selection, PID and pair selection criteria as well as the k^* range within which the fit is done. The variations we used are listed in Tab. 5.4 and for each of them the corresponding experimental CFs were obtained and one-dimensional radii R_{inv} and λ parameters were extracted.

The choice of the variations (Tab. 5.4) for the single track selection criteria is motivated by the sensitivity of the femtoscopic CF to the quality of the momentum reconstruction in the TPC. Therefore, the requested number of clusters N_{cls}^{TPC} and the shared fraction $N_{shared}^{TPC}/N_{cls}^{TPC}$ are varied. The variation of the DCA selection is needed to assess the influence of non-primary protons on the experimental CFs.

Since we deal with proton pairs we need to estimate the uncertainty related to the PID criteria, which is done by varying separately the TPC and TOF selection (Tab. 5.4).

The variation on the double-track selection is important due to their direct impact on the shape of the experimental CFs as they are used to rule out the track-merging effect observed in this work (see Sec. 5.3). In this work the requirements (Tab. 5.4) on $\Delta\eta$ and $\Delta\phi^*$ are varied simultaneously by reducing/enlarging the corresponding elliptic area (5.3).

To account for the uncertainty associated with the fitting procedure, the upper limit of the k^* range over which the fit is performed was varied by $\pm 10\%$.

Variations of the normalization region (by $\pm 15\%$) and of the fit range used to determine the baseline were performed, but the resulting uncertainties were found negligible and not reported here.

For each considered source i of systematic uncertainty (Tab. 5.4), the variation that results to the largest deviation from the default value of the extracted parameter is kept. All the contributions are then summed in quadrature to obtain the total systematic uncertainty:

$$\sigma_{tot}^{sys} = \sqrt{\sum_i (\sigma_i^{sys})^2}. \quad (5.7)$$

Tables 5.5 and 5.6 report the total uncertainty values and its contributors i calculated for radii R_{inv} and λ parameters. Fig. 5.17 shows the k_T dependencies of the extracted R_{inv} and λ parameters with their systematic uncertainties for different classes of centrality.

The larger contributions to the systematic uncertainties of R_{inv} and λ come from the variation of the double-track selection which is due to the fact that they directly affect the shape of the experimental CF in the signal region (see Sec. 5.3). The next largest contributions are associated to the PID criteria variation that define the fraction of the genuine proton

Table 5.4. Considered sources of systematic uncertainties and their variations.

	Default	Variation
Single track	$ DCA_{XY,Z} < 0.004 + 0.013/p_T$	$ DCA_{XY,Z} < 0.04$
	$N_{cls}^{TPC} \geq 120$	$N_{cls}^{TPC} \geq 110$ $N_{cls}^{TPC} \geq 130$
	$N_{shared}^{TPC}/N_{cls}^{TPC} \leq 0.3$	$N_{cls}^{TPC} \leq 0.25$ $N_{cls}^{TPC} \leq 0.35$
PID	$-2 < N\sigma_{TPC,p} < 4$ $-2 < N\sigma_{TOF,p} < 3$	$-2 < N\sigma_{TPC,p} < 3$ - // -
		- // - $-3 < N\sigma_{TOF,p} < 4$
		- // - $-2 < N\sigma_{TOF,p} < 2$
Double track (exclusion)	$\left(\frac{\Delta\eta}{0.02}\right)^2 + \left(\frac{\Delta\phi^*}{0.015}\right)^2 < 1$	$\left(\frac{\Delta\eta}{0.022}\right)^2 + \left(\frac{\Delta\phi^*}{0.017}\right)^2 < 1$ $\left(\frac{\Delta\eta}{0.018}\right)^2 + \left(\frac{\Delta\phi^*}{0.013}\right)^2 < 1$
Fit Range	$0.005 < k^* < 0.15$	$\pm 10\%$ (upper limit)

Table 5.5. Extracted 1D proton radii R_{inv} with the relative (%) statistical uncert. and relative systematical uncertainties calculated for each source as well as the total ones.

uncertainties [%]											
cent. (%)	k_T GeV/c	R_{inv}	stat.	sys.							tot. (sys.)
				DCA	N_{cls}^{TPC}	$N_{shared}^{TPC}/N_{cls}^{TPC}$	$N\sigma_{TPC,p}$	$N\sigma_{TOF,p}$	2trk	fitrange	
0-10	[0.4-0.8]	6.23	0.34	1.60	2.90	3.50	2.80	1.00	1.70	2.80	6.60
	[0.8-1.0]	5.43	0.11	0.90	1.30	0.50	0.80	1.80	4.70	1.00	5.50
	[1.0-1.2]	4.74	0.06	0.20	0.80	0.40	2.30	5.20	5.60	0.20	8.10
	[1.2-1.6]	4.32	0.05	1.30	1.60	0.30	4.40	7.00	12.00	0.50	14.80
10-30	[0.4-0.8]	4.72	0.06	0.80	0.20	1.00	0.00	0.80	1.30	0.40	2.10
	[0.8-1.0]	4.49	0.04	1.70	0.80	1.10	0.70	1.40	3.50	0.00	4.50
	[1.0-1.2]	3.99	0.04	0.40	1.10	0.10	0.70	2.90	5.80	0.70	7.60
	[1.2-1.6]	3.48	0.04	0.70	2.70	1.30	1.40	3.60	10.20	1.10	11.50
30-50	[0.4-0.8]	3.46	0.07	0.00	0.50	1.10	0.40	1.20	3.00	0.50	3.70
	[0.8-1.0]	3.22	0.06	0.70	0.40	0.50	1.00	0.60	7.30	1.30	9.30
	[1.0-1.2]	2.82	0.06	0.10	0.80	1.20	0.40	1.10	4.80	0.70	6.90
	[1.2-1.6]	2.59	0.04	0.90	2.70	1.40	0.80	2.10	4.10	0.50	6.10
50-90	[0.4-0.8]	2.35	0.08	1.30	3.20	3.50	0.20	2.10	1.30	0.70	5.60
	[0.8-1.0]	1.99	0.06	2.00	0.70	0.69	0.00	0.90	2.30	0.20	4.00
	[1.0-1.2]	2.05	0.06	1.50	0.60	1.10	0.70	2.40	1.80	0.60	4.70
	[1.2-1.6]	1.87	0.04	0.10	3.20	1.70	1.40	1.60	1.10	0.10	4.40

Table 5.6. Extracted λ parameters with the relative (%) statistical uncert. and relative systematical uncertainties calculated for each source as well as the total ones.

uncertainties [%]											
cent. (%)	k_T GeV/c	λ	stat.	sys.							
				DCA	N_{cls}^{TPC}	$N_{shared}^{TPC}/N_{cls}^{TPC}$	$N\sigma_{TPC,p}$	$N\sigma_{TOF,p}$	2trck	fitrange	tot. (sys.)
0-10	[0.4-0.8]	0.77	0.05	1.50	6.00	3.80	0.50	5.90	9.70	3.20	13.90
	[0.8-1.0]	0.78	0.04	1.50	4.60	5.70	0.50	7.30	11.40	3.10	16.80
	[1.0-1.2]	0.64	0.04	1.50	7.20	4.80	2.60	12.90	26.00	7.30	32.60
	[1.2-1.6]	0.49	0.03	12.70	15.60	1.70	17.10	23.60	46.30	6.10	59.30
10-30	[0.4-0.8]	0.87	0.04	1.40	4.50	4.30	0.70	6.00	4.00	5.50	11.40
	[0.8-1.0]	0.89	0.04	0.80	4.50	4.10	0.20	2.20	14.80	7.00	19.10
	[1.0-1.2]	0.73	0.03	7.20	3.30	1.20	1.20	7.00	23.70	5.20	30.50
	[1.2-1.6]	0.56	0.02	4.10	12.00	6.30	1.50	13.00	33.30	6.00	39.40
30-50	[0.4-0.8]	0.82	0.07	5.60	2.50	3.70	0.90	3.80	10.20	2.70	14.00
	[0.8-1.0]	0.69	0.05	8.90	1.10	1.90	2.60	1.30	20.00	5.60	28.50
	[1.0-1.2]	0.59	0.04	5.20	3.20	4.30	2.40	3.40	11.10	2.60	19.00
	[1.2-1.6]	0.54	0.03	8.10	7.30	4.00	3.20	4.60	5.40	1.80	15.10
50-90	[0.4-0.8]	0.72	0.07	0.60	8.60	8.90	0.80	5.90	6.30	2.30	15.30
	[0.8-1.0]	0.52	0.04	8.10	3.20	2.80	0.40	1.50	3.40	0.50	10.70
	[1.0-1.2]	0.59	0.04	7.90	2.30	4.10	2.10	6.30	4.90	1.70	13.70
	[1.2-1.6]	0.57	0.03	5.20	6.80	3.70	4.20	3.90	6.60	0.40	12.90

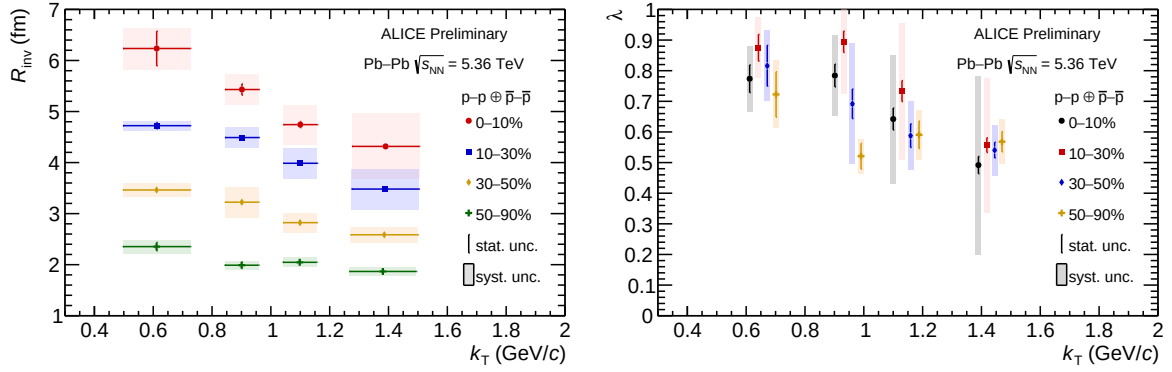


Figure 5.17. Left: One-dimensional proton radii R_{inv} in dependence on the pair transverse momentum k_T in different centrality classes in Pb–Pb collisions. Right: corresponding λ parameters in dependence on the pair transverse momentum k_T in different centrality classes. The k_T values and the corresponding uncertainties – estimated as a standard deviation from the pair k_T distributions (see Fig. 5.10) – for the λ parameters are the same as for the radii (left), they were modified for a better visualization.

pairs in the analyzed sample (see next Section). The λ parameters have greater systematic uncertainties with respect to R_{inv} due to the fact that the experimental CF C_{exp} (5.4) depends linearly on λ while its dependence on the R_{inv} is not that straightforward.

5.7 The λ parameter

As we anticipated in the previous section, the λ parameter value might be affected by various factors. In this work we don't discuss those related to possible non-Gaussianity of the proton source function as it's more relevant to "small" systems (~ 1 fm) such as pp collisions [47] and can be neglected⁴ for "large" ones (up to ~ 7 fm) as Pb–Pb collisions [48]. Instead, we will explore the effects related to the contribution of non-primary and misidentified protons therefore assuming that the λ value represents a fraction of the genuine $p - p \oplus \bar{p} - \bar{p}$ correlations between primary (anti-)protons in the experimental pair sample.

In order to do this MC simulations are used as they provides both the information about the "true"⁵ species and origin of a particle as well its reconstructed characteristics. Thus, the λ_{MC} parameter is factorized into two terms:

$$\lambda_{MC} = \lambda_{purity}^{p-p \oplus \bar{p}-\bar{p}} \times \lambda_{primary}^{p-p \oplus \bar{p}-\bar{p}}, \quad (5.8)$$

where $\lambda_{purity}^{p-p \oplus \bar{p}-\bar{p}}$ is called as *pair purity*, being the fraction of "true" (anti)proton pairs in the whole sample of pairs in the signal distribution; similarly, $\lambda_{primary}^{p-p \oplus \bar{p}-\bar{p}}$ – *pair primary fraction* is the the fraction of primary (anti)proton pairs.

To estimate the first, one has to know the *single-particle purity* of (anti)protons that is obtained as a fraction of the "true" (anti)protons in the pool of the "reconstructed" (anti)protons, i.e. proton candidates selected with the proton PID cuts, utilizing the MC data. The momentum dependence of the (anti)proton single purity values is shown in Fig. 5.18 (left). It can be seen that up till $p = 1$ GeV/c the purity is ≈ 1 which means 100% effectiveness of the PID cuts in this region. From $p = 1$ to 2.5 GeV/c the purity starts to drop due to the kaon contamination, reaching ≈ 0.92 and continuing to decrease for larger p .

This distribution is then used as an input to calculate the (anti)proton pair purity with the following procedure ("toy" MC):

1. pairs with momenta p_1 and p_2 are randomly sampled employing the experimental momentum distribution (Fig. 5.7);
2. the pair purity value is calculated as a product of the single purity values for given p_1 and p_2 ;

⁴For example the adequacy of using the Gaussian distribution for large sources might be motivated by the central limit theorem of statistics.

⁵We use the term "true" for particles of a given species that were correctly identified.

3. the k_T -dependent pair purity histogram is filled (e.g. Fig. 5.18, right) using the pair purity value as a weight for each entry;
4. the procedure is repeated until the desired statistical significance is reached⁶,

The obtained k_T -dependent pair purity distributions are then averaged with the weights calculated from the experimental k_T spectra to obtain the average (anti)proton pair purity values reported in Tab. 5.7.

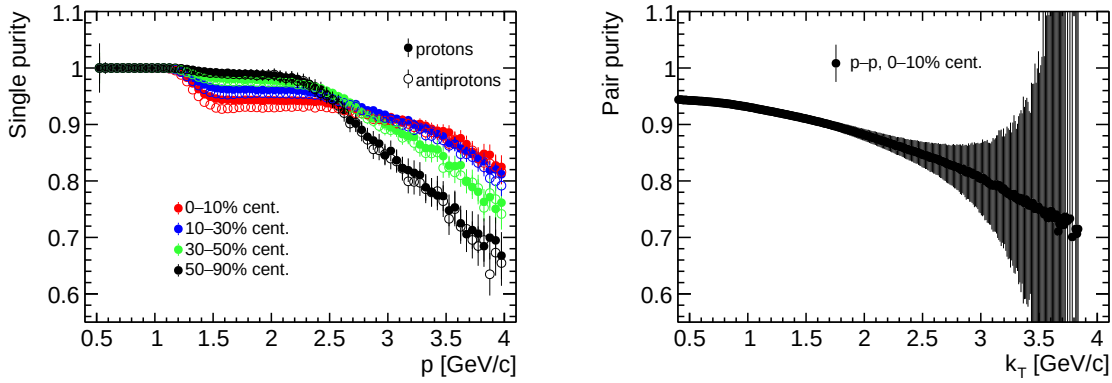


Figure 5.18. Left: proton single purity values in dependence of momentum p calculated with MC simulations as a ratio between "true" protons and particles selected with the proton PID cuts (Tab. 5.3). Right: an example of proton pair purity values in dependence of pair transverse momentum k_T calculated as a product of the single purity values.

Likewise, to estimate the proton pair primary fraction, one needs the single proton primary fraction. There are three possible origins of protons: primary protons come from the source created in a collision; "weak" ones are the products of weak decays of heavier baryons created in the collision; "material" are protons knocked out from the detector's material by particles created in the collision. The last contribution is neglected for antiprotons as the probability (in terms of the cross-section) of them being created via interaction with the material is negligible.

The proton primary fraction was extracted by fitting the experimental DCA_{XY} ⁷ distributions with the ones obtained from the MC simulations for the considered three components. An example of such fit is shown in Fig. 5.19 for a single p_T bin. The resulting p_T -dependent proton primary fractions are presented in Fig. 5.20 (left). By using a similar procedure as for calculating the pair purity we obtain the k_T -dependent proton pair primary fraction (e.g. Fig. 5.20, right) and its weighted average values for the four k_T ranges considered in this work

⁶Being a "toy" Monte Carlo the number of iterations is arbitrary.

⁷As the analysis is done at mid-rapidity.

(Tab. 5.4). Combining these with the average pair purity one obtains the λ_{MC} defined in Eq. (5.8).

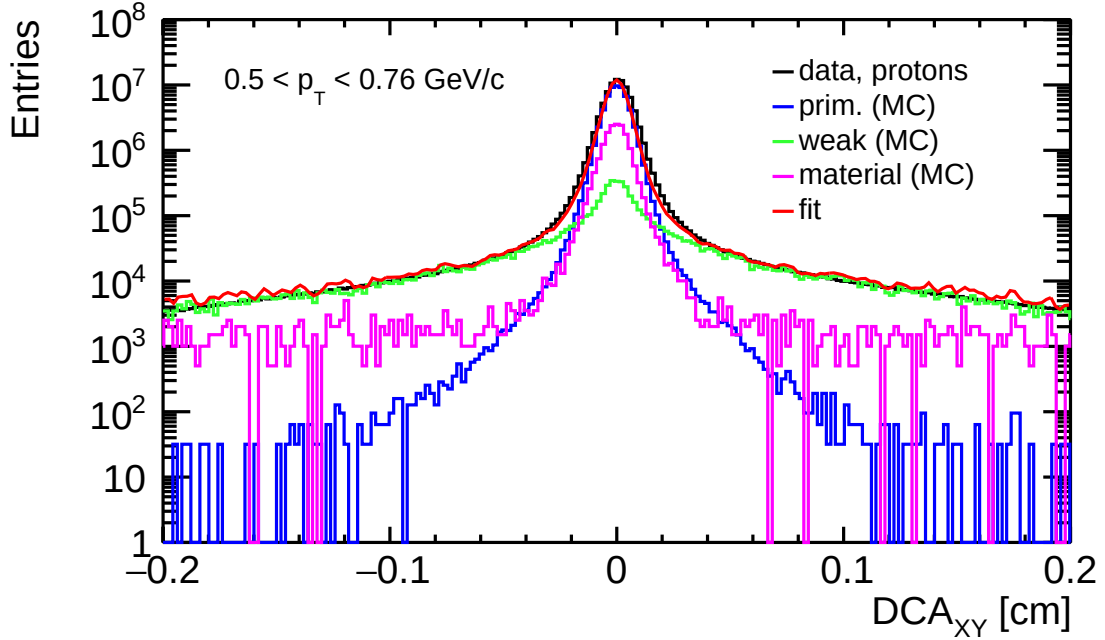


Figure 5.19. Example of a fit of the experimental DCA_{XY} distribution of proton candidates in the $p_T = [0.5, 0.76]$ GeV/c range selected with the criteria listed in Tab. 5.3. The fit is performed with a function built as a linear combination of primary, "material" and "weak" DCA_{XY} distributions obtained from MC simulations.

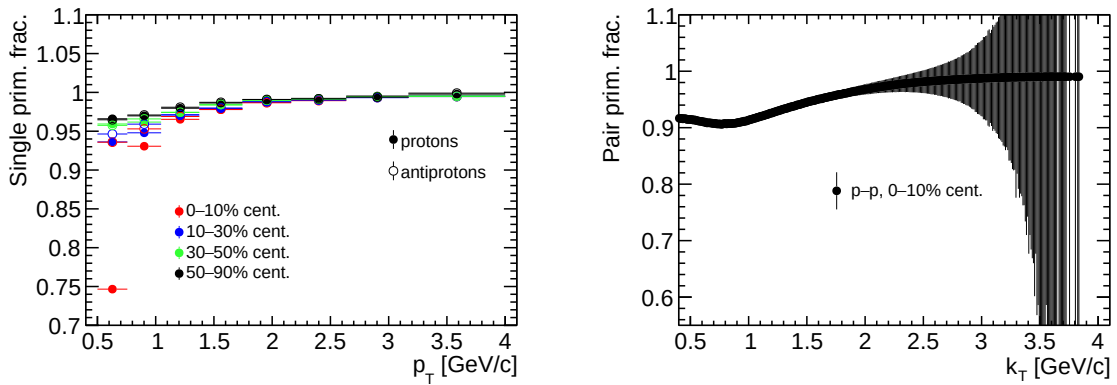


Figure 5.20. Left: single-proton primary fraction values as a function of the transverse momentum p_T estimated with MC via DCA_{XY} fits (e.g. Fig. 5.19). Right: an example of a proton pair primary fraction as a function of the pair transverse momentum k_T calculated as a product of the single primary fraction values.

In some works on femtoscopy (e.g. Ref. [47; 69]) the λ_{MC} values obtained with MC simulations are used to fix the λ in the fit function (5.4, 5.6). In our study we choose to keep it as a free fit parameter for the following reasons:

- we consider only one genuine proton-proton correlation contribution to the fit function;
- we think that reducing the number of the sources of discrepancy between the theory and the experiment to only two – misidentification and contribution of non-primary protons – might be insufficient. Since, the shape of the fit function (5.4, 5.6) depends on both R_{inv} and λ , we don't want to bias the extraction of the radii.

Although we extract the λ from the fit, comparing it to the λ_{MC} is not only an important cross-check of the overall analysis consistency but it also may help us to understand its nature. Such a comparison is presented in Fig. 5.21. The λ_{MC} values differ slightly with centrality and barely depend on k_T unlike the λ values extracted from the experimental data.

In the 0-10% and 10-39% centrality classes λ_{MC} and λ are in a good agreement in the first two k_T bins. However, the λ extracted from the data tend to lay lower for the next two k_T bins with a discrepancy growing towards higher k_T . This might be caused by the increase of the influence of radial flow (see Sec. 2.2, in particular Figs. 2.5 and 2.6) on the homogeneity region pushing it towards the surface of the source and making less symmetric, i.e. less spherical.

In the last two centrality classes – 30-50% and 50-90% – the disagreement between λ_{MC} and λ is present already at low k_T (also growing with k_T). A possible explanation to this is that the particle emitting source created in more peripheral collisions has a more ellipsoidal shape (see Sec. 1.2) and cannot be parameterized with a spherical source function as given by Eq. (2.11) with the same effectiveness as for the central collisions.

If our assumptions are correct, then the usage of the 3D source parametrization (2.16) might help in resolving the disagreement. The possibility of measuring femtoscopic radii in three dimensions with proton pairs will be discussed later in Sec. 5.9.

Table 5.7. Pair purity and primary fraction values for (anti)proton pairs in four k_T ranges for four centrality classes obtained with MC simulations. The $\lambda_{p-p\oplus\bar{p}-\bar{p}}^{p-p\oplus\bar{p}-\bar{p}}$ are calculated as a weighted average of the corresponding purity and primary fraction values.

cent. (%)	k_T (GeV/c)	pair purity			pair prim. frac.		
		$p-p$	$\bar{p}-\bar{p}$	$\lambda_{purity}^{p-p\oplus\bar{p}-\bar{p}}$	$p-p$	$\bar{p}-\bar{p}$	$\lambda_{primary}^{p-p\oplus\bar{p}-\bar{p}}$
0-10	[0.4-0.8]	0.92	0.91	0.92	0.91	0.95	0.93
	[0.8-1.0]	0.92	0.90	0.91	0.91	0.94	0.93
	[1.0-1.2]	0.91	0.89	0.90	0.92	0.95	0.93
	[1.2-1.6]	0.90	0.88	0.89	0.94	0.96	0.95
10-30	[0.4-0.8]	0.94	0.94	0.94	0.94	0.95	0.95
	[0.8-1.0]	0.94	0.93	0.93	0.94	0.95	0.95
	[1.0-1.2]	0.93	0.92	0.92	0.95	0.95	0.95
	[1.2-1.6]	0.92	0.91	0.91	0.95	0.96	0.96
30-50	[0.4-0.8]	0.96	0.95	0.96	0.95	0.96	0.95
	[0.8-1.0]	0.95	0.94	0.95	0.95	0.96	0.95
	[1.0-1.2]	0.94	0.94	0.94	0.96	0.6	0.96
	[1.2-1.6]	0.93	0.92	0.92	0.96	0.96	0.96
50-90	[0.4-0.8]	0.96	0.96	0.96	0.96	0.96	0.96
	[0.8-1.0]	0.95	0.95	0.95	0.96	0.96	0.96
	[1.0-1.2]	0.94	0.93	0.94	0.96	0.96	0.96
	[1.2-1.6]	0.92	0.91	0.91	0.97	0.97	0.97
							λ_{MC} (Eq. 5.8)
							0.85
							0.84
							0.84
							0.84
							0.89
							0.88
							0.88
							0.87
							0.91
							0.90
							0.90
							0.89
							0.92
							0.91
							0.90
							0.88

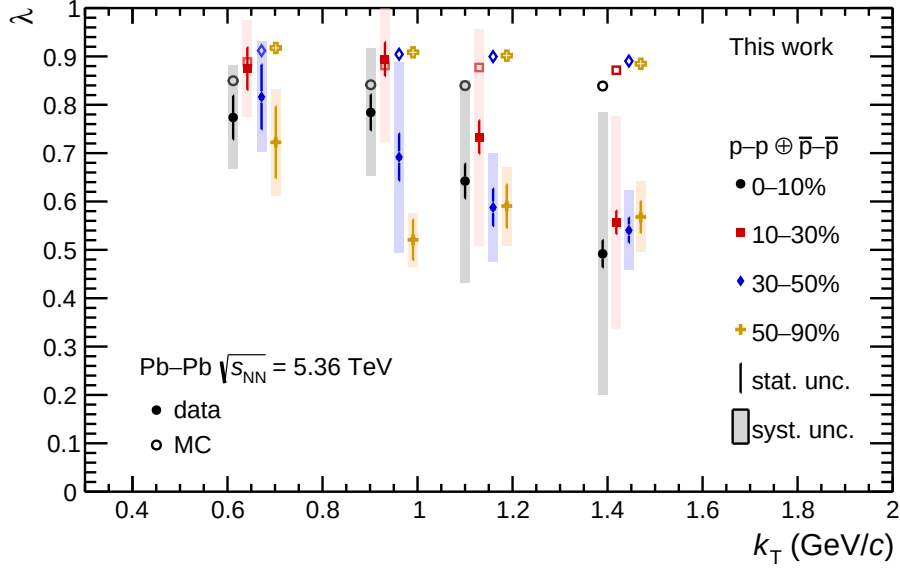


Figure 5.21. λ parameters obtained from the fits of the experimental CFs in comparison to λ_{MC} ones calculated with MC simulations.

5.8 1D radii: discussion and comparison

The extracted one-dimensional radii of the proton source are presented in Fig. 5.22 in dependence on the pair transverse mass m_T . The effective source sizes are smaller for more peripheral events which relates to a smaller size of the initial overlapping volume of the colliding nuclei. The radii scale with m_T reflecting the presence of the radial flow in the system (see Sec. 2.2). The m_T -dependence flattens out towards more peripheral events indicating that the flow is weaker and the system expands less explosively in such events. Overall, the behavior of the proton radii is similar to what is observed for pion and kaon ones (see Sec. 2.3).

The results of this study, obtained in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.36$ TeV collected by ALICE during Run 3 are compared with similar measurements based on the Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV data collected during Run 2 [48]. Although, the colliding system is the same, the collision energy is different. Thus, the radii are compared in dependence of the average charged-particle multiplicity density $\langle dN_{ch}/d\eta \rangle_{|\eta| < 0.5}$. The $\langle dN_{ch}/d\eta \rangle_{|\eta| < 0.5}$ values corresponding to this work are listed in Tab. 5.1. As the radii depend not only on centrality/multiplicity but on the m_T as well, the comparison must be carried out for similar values of m_T to be meaningful. The $\langle dN_{ch}/d\eta \rangle_{|\eta| < 0.5}^{1/3}$ -dependent radii with the comparison between the Run 2 and Run 3 results are shown in Fig. 5.23-5.24.

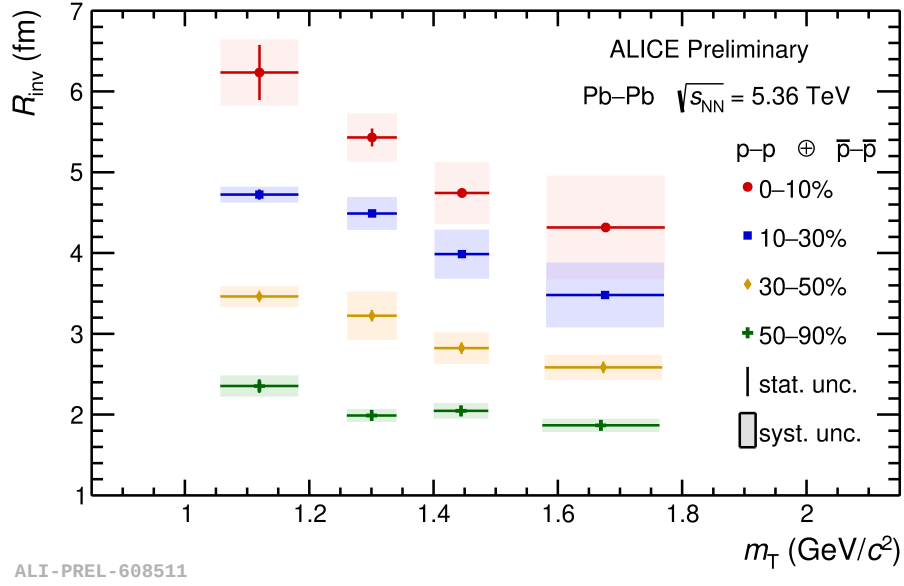


Figure 5.22. One-dimensional proton radii R_{inv} measured in Pb–Pb $\sqrt{s_{NN}} = 5.36$ TeV as a function of the pair transverse mass m_T in different centrality classes.

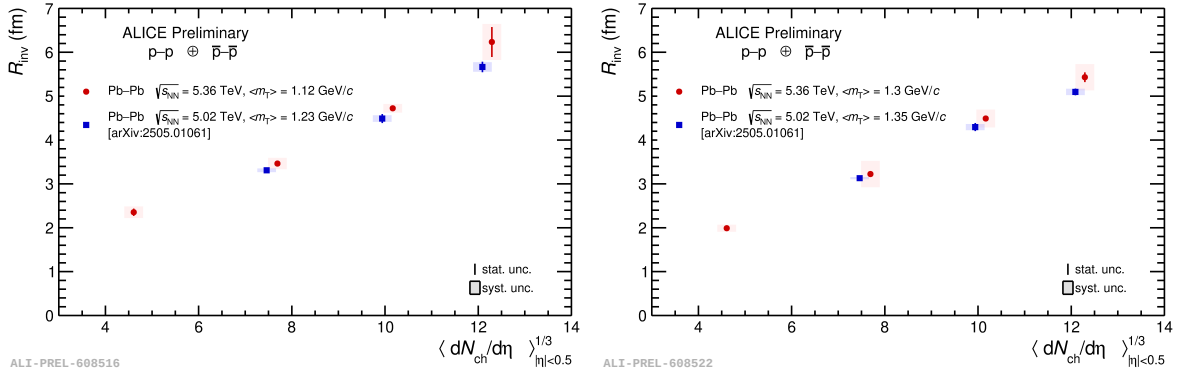


Figure 5.23. One-dimensional proton radii R_{inv} radii as a function of the cube root of the charged-particle multiplicity density $\langle dN_{ch}/d\eta \rangle_{|\eta|<0.5}^{1/3}$ in the *first* (left) and *second* (right) m_T ranges measured in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.36$ TeV compared with results obtained in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV [48].

The Run 3 results presented in this work are preliminary. They have larger uncertainties with respect to the Run 2 ones due to the fact that the present analysis was performed only with the Pb–Pb data collected during 2023 and doesn't include the 2024 data as they were not fully reconstructed at the moment. Also, the reconstruction of the 2023 data used in the study is not final. Thus, the precision of the measurements is expected to improve significantly in the future in terms of both statistical (by adding the 2024 data) and systematical (due to a more robust reconstruction) uncertainties.

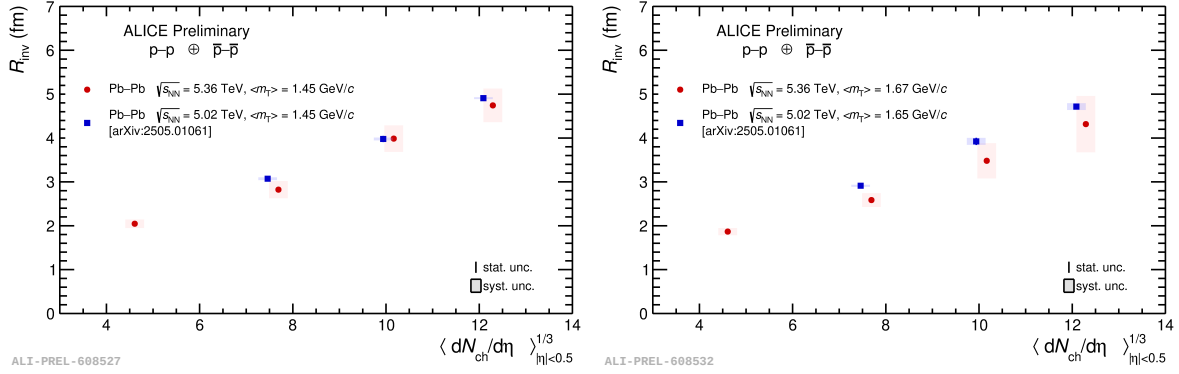


Figure 5.24. One-dimensional proton radii R_{inv} radii as a function of the cube root of the charged-particle multiplicity density $\langle dN_{ch}/d\eta \rangle_{|\eta|<0.5}$ in the *third* (left) and *fourth* (right) m_T ranges measured in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.36$ TeV compared with results obtained in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV [48].

Nevertheless, the new Run 3 results obtained in this study are in a good agreement, within uncertainties, with the ones of Run 2. The novelty of these results is the extension of the proton source size measurement to more peripheral events with respect to the Run 2 ones. Considering the recent ALICE's campaign on collecting data in O–O and Ne–Ne collisions (very first physics results are already out, see Ref. [106]), the measurement of the proton radii in peripheral events is particularly important to compare the multiplicity dependent radii in Pb–Pb collisions with the smaller colliding systems.

Another feature emerging in Figs. 5.23 and 5.24 is the approximate linear scaling of the proton source with $\langle dN_{ch}/d\eta \rangle_{|\eta|<0.5}^{1/3}$ suggesting that the cube root of the charged-particle multiplicity density could be used as a proxy for the source size. The same behavior was observed by ALICE for pion and kaon sources [107; 108]. Moreover, a common scaling with the multiplicity across different colliding systems is suggested. This is referred in the literature as the "scaling hypothesis" [50]. The comparison of the multiplicity-dependent pion [107] and kaon [108] radii across different colliding systems performed by ALICE demonstrated that for the pion case the scaling is broken, while for kaons it was indecisive. Proton femtoscopy measurements can provide an important insight to this problem. Such comparison is shown in Fig. 5.25 and includes proton radii measured in pp collisions at 0.9 TeV, 7 TeV [69], 13 TeV [109] and p–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV [109] in addition to the Pb–Pb radii presented in Fig. 5.23 (left).

By providing insights on the phenomenology of the hadronic phase, the multiplicity-dependent measurements of the femtoscopic radii find interesting applications in other HIC studies, such as on short-lived resonance suppression and rescattering [110] or on production mechanisms of light (anti)(hyper)nuclei [111]. Specifically, coalescence-based models used

to predict the production of light (anti)(hyper)nuclei adopt the results of the femtoscopic measurements straight away to parametrize the nucleon-emitting source [112].

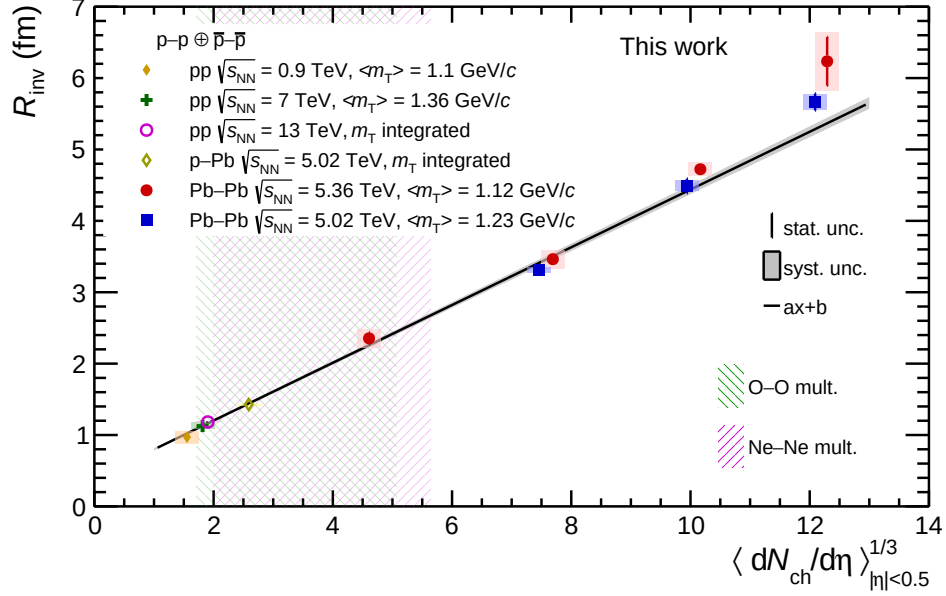


Figure 5.25. One-dimensional proton radii R_{inv} as a function of the cube root of the charged-particle multiplicity density $\langle dN_{ch}/d\eta \rangle_{|\eta|<0.5}$ measured in pp collisions at 0.9 TeV, 7 TeV [69], 13 TeV [109], p-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV [109], Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV [48] and $\sqrt{s_{NN}} = 5.36$ TeV. Shaded areas correspond to the expected multiplicity regions covered by the O-O and Ne-Ne collisions collected by ALICE in 2025.

Returning to the discussion of the results of this study, the radii across all the considered colliding systems were fitted with a linear function $a \cdot x + b$ (Fig. 5.25) to quantify the common scaling as a function of $\langle dN_{ch}/d\eta \rangle_{|\eta|<0.5}$. The resulting fit quality is characterized by a value of $\chi^2/NDF = 2.76$, indicating the signatures of a common scaling of the proton radii with the cube root of the charged-particle multiplicity density. The coefficients extracted from the fit are $a = 0.405 \pm 0.006$ and $b = 0.394 \pm 0.016$. However, in order to draw a conclusion, more experimental measurements are needed in the $27 \lesssim \langle dN_{ch}/d\eta \rangle_{|\eta|<0.5} \lesssim 64$ range as it corresponds to the intermediate multiplicity region between small and large colliding systems. In this regard the new O-O and Ne-Ne collision data collected by ALICE in 2025 (few months before the release of this thesis) are of particular interest as they will be the "bridge" connecting small and large systems that is indicated in Fig. 5.25 by the shaded areas. This is true not only for femtoscopic studies but for any other measurements in HICs as theoretical models that are tuned to describe large system rarely can describe the small ones and vice versa (e.g. see Fig. 1.13 in Sec. 1.3). So, having the experimental measurements

in the intermediate-size systems would enrich our understanding of the physical processes occurring in ultrarelativistic ion collisions.

5.9 3D femtoscopy with proton pairs

This section is dedicated to the three-dimensional femtoscopic analysis of the proton-emitting source created in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.36$ TeV. Such analysis is completely unique for the experimental HIC physics as the proton-emitting source has never been measured before in three dimensions. Below we will discuss the original methodology developed for this study and the results obtained with it.

The three-dimensional (3D) experimental CF is build the same way as the one-dimensional (1D) one (5.1) but instead of using 1D same event $S(k^*)$ and mixed event $B(k^*)$ distributions one has to use 3D pair distributions of pair relative momentum in the LCMS, $\vec{q}_{LCMS} = (q_{out}, q_{side}, q_{long})$ (see Sec. 2.2). Thus, the three-dimensional femtoscopic CF in the LCMS is defined as

$$C_{exp}(\vec{q}_{LCMS}) = N \frac{S(\vec{q}_{LCMS})}{B(\vec{q}_{LCMS})}. \quad (5.9)$$

The increase in the number of dimensions in the experimental distributions leads to the need of increasing the number of entries in these distributions required to obtain a distinguishable correlation signal shape. Due to this, we had to reduce both the number of the centrality classes and k_T bins from four to three: $[0 - 20] \%$, $[20 - 50] \%$ and $[50 - 90] \%$ for centrality; $[0.3 - 0.6] \text{ GeV}/c$, $[0.6 - 1.0] \text{ GeV}/c$ and $[1.0 - 2.0] \text{ GeV}/c$ for k_T . The k_T ranges used for the 3D analysis also have a different overall span due to the different kinematics of the LCMS with respect to the PRF coordinate system used in the 1D analysis. Examples of the experimental three-dimensional proton pair CFs obtained in the $[20 - 50] \%$ centrality class are shown in Fig. 5.26 in three projections onto *out*, *side* and *long* axes.

For the cases of charged pion and kaon pairs, the Bowler–Sinyukov formula [113; 114] is used to calculate the theoretical 3D CF [59; 62; 107]. The formula is based on the assumption that in case of accounting only for Coulomb interaction and Bose-Einstein quantum statistical (QS) effects, the CF $C_{Coulomb+QS}(\vec{q}_{LCMS}, \vec{R}_{LCMS})$ can be effectively factorized into two terms:

$$C_{Coulomb+QS}(\vec{q}_{LCMS}, \vec{R}_{LCMS}) = C_{Coulomb}(k^*, R_{inv}) \times C_{QS}(\vec{q}_{LCMS}, \vec{R}_{LCMS}), \quad (5.10)$$

where $\vec{R}_{LCMS} = (R_{out}, R_{side}, R_{long})$ are the effective 3D source size parameters in the LCMS; $C_{Coulomb}(k^*, R_{inv})$ is usually called the *Coulomb correction factor* and represents the correlation function for a pair of particles interacting only via Coulomb, i.e. non-identical, calculated in the PRF ($\equiv$ CM); and the $C_{QS}(\vec{q}_{LCMS}, \vec{R}_{LCMS})$ term is the CF for a pair of free bosons that includes only the QS effects. The latter is calculated in the LCMS system as it's calculation is fairly simple and can be done in an arbitrary frame. The $C_{Coulomb}(k^*, R_{inv})$ term is obtained in the CM because only in that system the WF can be calculated (see Sec. 3.1). Therefore, the k^* variable has to be used. Practically it is done by mapping each set of the q_{out} , q_{side} and q_{long} components to the corresponding value of k^* and filling an additional 3D histogram. As the values of the correction factor $C_{Coulomb}(k^*, R_{inv})$ depend on a given effective size of the system, the one-dimensional radius R_{inv} obtained in a complementing one-dimensional analysis is usually used.

The factorization solves the major problem, the usage of different frames for the source parametrization and the WF calculation. It was shown that such approximation works well for pion and kaon cases where only the Coulomb+QS is considered (e.g. see Ref. [115]). Instead, for the proton case where taking the strong interaction into account is crucial, such factorization is not possible and one has to perform the full calculation of the 3D CF.

In Sec. 2.2 we wrote the relation between the pair relative distance in the *out* direction in the LCMS and the corresponding relative distance in the PRF (see Eq. 2.17). To build the theoretical three-dimensional CF we will use the reverse Lorentz boost from the LCMS to the PRF under the assumption of the same emission time of protons, i.e. $r_{out}^* = \gamma_T r_{out}$. Similarly, for the pair relative momentum *out* component $q_{out}^* = \frac{q_{out}}{\gamma_T}$. Thus, the theoretical three-dimensional CF is calculated as⁸

$$C_{th}(\vec{q}_{LCMS}, \vec{R}_{LCMS}) = \sum_{S=0}^1 \omega_S \int dr_{out} dr_{side} dr_{long} S(\vec{r}_{LCMS}, \vec{R}_{LCMS}) \left| \Psi_{pp}^S(\vec{k}^*, \vec{r}^*) \right|^2, \quad (5.11)$$

$$\vec{k}^* = \frac{1}{2} \left(\frac{q_{out}}{\gamma_T}, q_{side}, q_{long} \right),$$

$$\vec{r}^* = (\gamma_T r_{out}, r_{side}, r_{long}),$$

where the source function $S(\vec{r}_{LCMS}, \vec{R}_{LCMS})$ is given by Eq. 2.16; $\Psi_{pp}^S(\vec{k}^*, \vec{r}^*) = \frac{\psi(-\vec{k}^*, \vec{r}^*) + (-1)^S \psi(\vec{k}^*, \vec{r}^*)}{\sqrt{2}}$ with $\psi(\vec{k}^*, \vec{r}^*) := \psi(\vec{k}, \vec{r})$ given by Eq. 3.35; γ_T is the Lorentz factor in the LCMS $\rightarrow$ PRF boost.

⁸Here we use only the s-wave terms as the contribution of the p-wave is small but it's inclusion to the 3D CF calculation requires more computing power.

The γ_T values are stored in an additional 3D histogram that is filled in parallel to the ME distribution during the event mixing procedure. For each set of the q_{out} , q_{side} and q_{long} components the corresponding value of γ_T is calculated and used as a weight to the histogram entry. The final distribution is divided by the ME one to factor out the combinatorial background. The normalization in this case is not needed as both the γ_T and the ME distributions are filled in parallel, and therefore the entries are distributed identically over the q_{out} , q_{side} and q_{long} components.

The experimental three-dimensional CF is build similarly to the one used in the one-dimensional analysis (5.4):

$$C_{exp}(\vec{q}_{LCMS}, \vec{R}_{LCMS}) = 1 + \lambda(C_{th}(\vec{q}_{LCMS}, \vec{R}_{LCMS}) - 1). \quad (5.12)$$

This function is used to fit the experimental 3D CFs to extract the three-dimensional proton radii $\vec{R}_{LCMS}$ and λ that are free fit parameters. Examples of such fits for the [20 – 50] % centrality class are shown in Fig. 5.26 in three projections. The fit catches the shape of the experimental distributions quite well.

The 3D radii and λ parameters extracted from the fit for three centrality classes and three k_T ranges are shown in Fig. 5.27. The 3D radii exhibit a dynamics similar to the 1D ones, i.e. they are smaller for more peripheral collisions and scale with k_T (m_T). However, the slopes of the k_T -dependencies are different for different effective source size projections. The k_T -dependencies of R_{long} seem to have the steepest slope among the three, while the R_{out} ones have the flatter. A similar behavior is also observed for pion [107] and kaon pairs [62] and is typical for HICs. The radii are also about the same magnitude suggesting that the homogeneity region of protons is rather spherical in the LCMS.

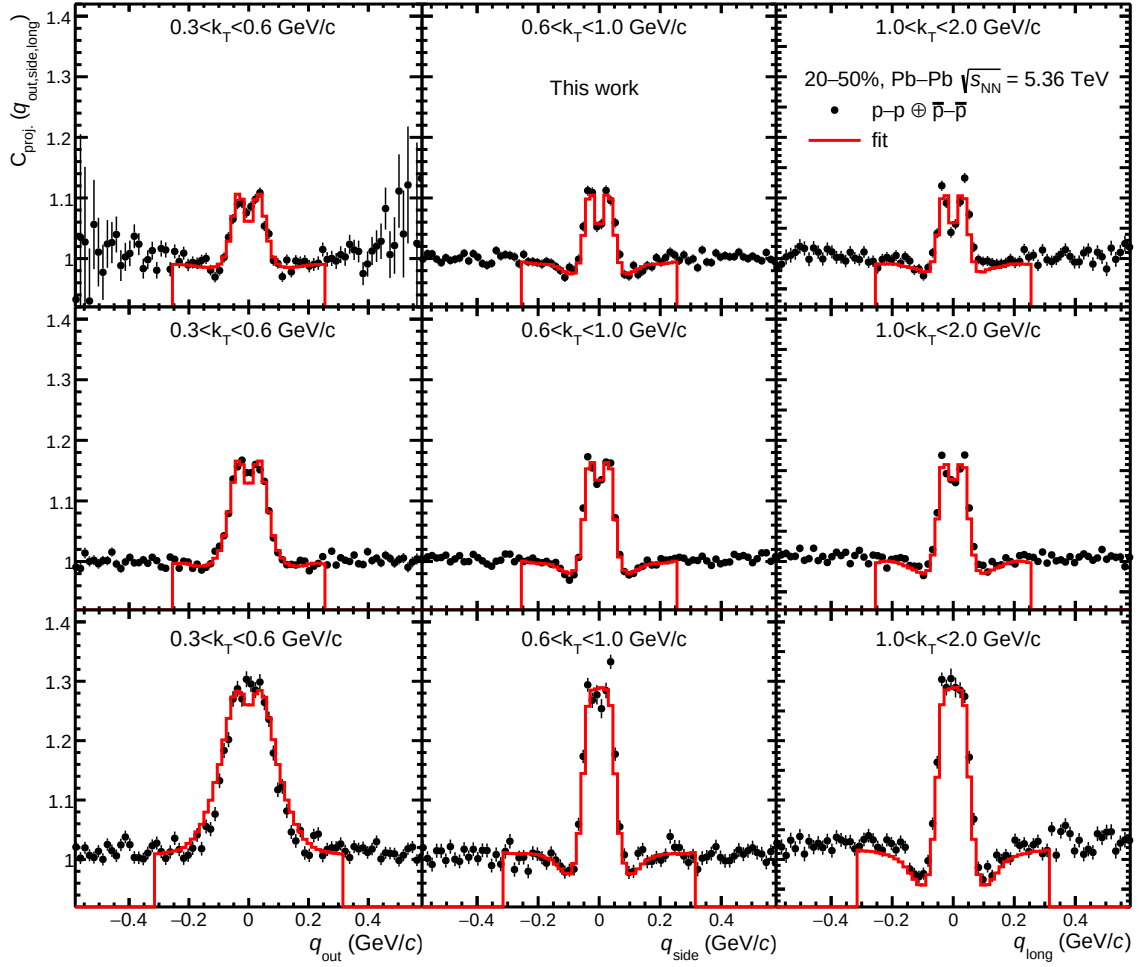


Figure 5.26. 3D proton correlation functions (black markers) obtained in the 20 – 50% centrality class. The CFs are presented in projections onto the q_{out} (left column), q_{side} (middle column), and q_{long} (right column) axes for three k_T ranges fitted with Eq. 5.12 (red lines). To project onto q_i ($i = out, side, long$) component, the others are integrated over the range $q_i < 0.045$ GeV/c to illustrate the femtoscopic effect for the corresponding axis.

This work constitutes the first effort to carry out an experimental 3D femtoscopy analysis with proton pairs. However, there are examples of analyses carried out on simulated data. In Ref. [116] Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV are simulated employing a hybrid LHyquid+Therminator2 model where the first is used to simulate the hydrodynamical stage and the second is used for the hadronization one. Even though the author uses a different approach for the 3D radii extraction that is based on the Spherical Harmonics formalism (for more details see Ref. [116]), the dynamics of the radii is rather similar⁹, the qualitative

⁹Due to the significant difference in the collision energy, centrality and k_T we cannot really compare directly their results with ours.

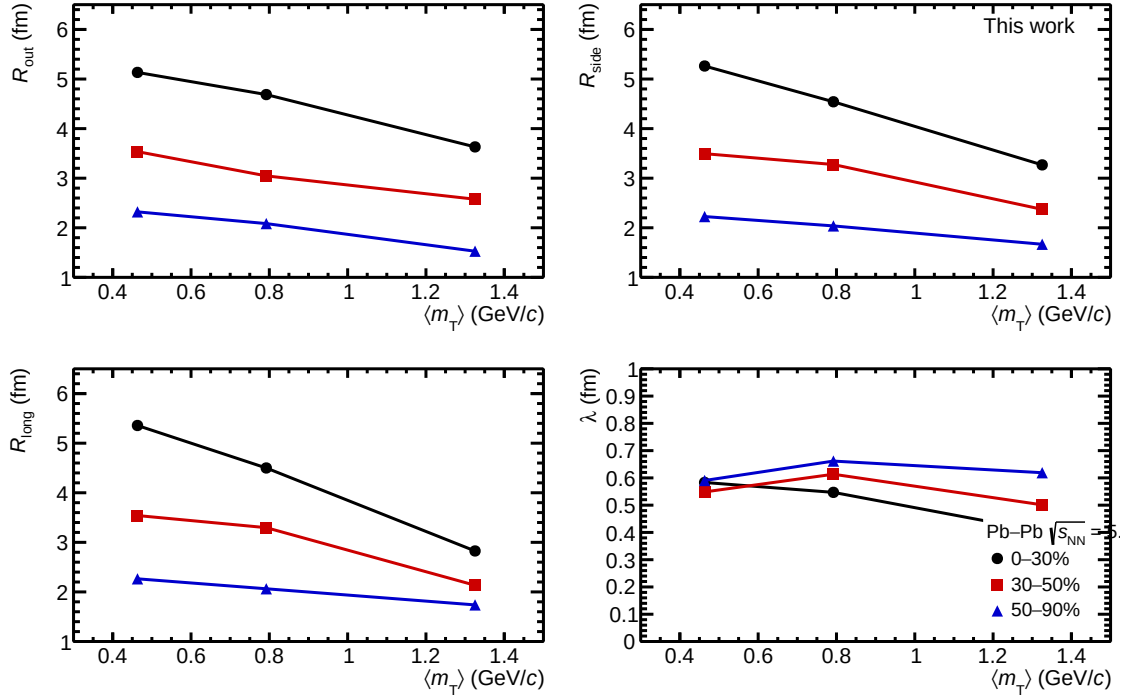


Figure 5.27. Three-dimensional proton radii R_{out} , R_{side} , R_{long} and λ parameters measured in Pb–Pb $\sqrt{s_{NN}} = 5.36$ TeV in dependence on the average pair transverse momentum k_T in different centrality classes.

consistency with the model study can be regarded as a first indirect validation of methodology used in this work.

The next steps of the analysis include applying the momentum resolution correction and performing the estimation of the systematic uncertainties. Nevertheless, these preliminary results constitute proof of principle that the experimental 3D femtoscopic analysis with proton pairs is attainable, making the work presented in this thesis utterly novel and original.

Conclusions

The research work reported in this thesis focuses on exploring the properties of the strong interaction between protons via the methods of femtoscopy, and on extracting the size parameters of the proton-emitting source created in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.36$ TeV.

An original theoretical model that accounts for the strong interaction in proton pairs was developed and discussed in Chapter 3. The model exploits a fully analytical non-relativistic EFT treatment of the strong potential in a shape of a square-well to calculate the wave function. The performance of the EFT model in terms of its application to femtoscopy was evaluated by comparing it against calculations provided by the Lednický-Lyuboshits model and the CATS package, as both of them are commonly employed in the field. The calculations were performed for the case of a spherically-symmetric Gaussian proton-emitting source.

By comparing the theoretical CFs calculated with the EFT model against the ones calculated with the LL one, it was found that latter leads to an overestimation of the resulting proton-proton CF. The discrepancy between the CFs increases towards smaller source sizes. This is caused by the fact that the LL model accounts for the strong potential in the asymptotic region, i.e. outside of the region of its influence, and therefore its practical application to femtoscopy is questioned. The inability of the LL model to provide a sufficient description of the theoretical proton-proton femtoscopic CF was observed also in [69] where CATS was used to calculate the exact solution to the Schrodinger's equation employing the Argonne v18 strong potential model. Another distinct feature of the EFT model with respect to the LL is that it allows to extend the wave function calculation to higher partial waves whereas the LL model includes only the s-wave term.

The comparison against the CATS calculations showed that both the EFT model and CATS provide very similar CFs with up to $\approx 4\%$ discrepancy for $R_{inv} = 1$ fm source size. Since the application to experimental data might be insensitive to such a small difference, the EFT model is considered to be justified for its direct use in the proton source measurement. A unique advantage of the EFT model with respect to CATS is being fully analytical. As such, it could provide the possibility to extract the strong potential effective parameters from the fit to the experimental CFs by solving the "reverse" problem. Although it is

irrelevant for the pairs of particles for which phase shift measurements exist, in perspective, femtoscopic analyses of pairs of (rare) hadrons measurable only in high-energy collisions could provide new insights on the strong interaction between non-trivial hadronic states.

The experimental part of this work, discussed in Chapter 5, presents a measurement of the proton-emitting source created in Pb–Pb collisions with ALICE data. The one- (three-) dimensional proton radii were extracted utilizing the EFT model in four (three) centrality classes and four (three) ranges of pair transverse momentum (mass) k_T (m_T). The radii scale with centrality/multiplicity, decreasing towards more peripheral events where the correlation volume is smaller due to smaller sizes of the nuclei overlapping region. The radii exhibit a distinct m_T -dependence, i.e. scaling, due to the presence of radial flow, which is observed to get weaker from central to peripheral events.

The one-dimensional proton radii obtained in this study were compared to the results of similar measurements based on the data collected by ALICE during Run 2 in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV [48]. For similar m_T values, the comparison between the radii as a function of the cube root of the charged-particle multiplicity density $\langle dN_{ch}/d\eta \rangle_{|\eta| < 0.5}$ show consistency with the results previously released by ALICE. Additionally, a comparison of the multiplicity-dependent one-dimensional proton radii extracted in various colliding systems was carried out to examine possible signatures of a common scaling. The results suggest that such a scaling might be present, thus emphasizing the importance of the recent ALICE campaign on collecting data in O–O and Ne–Ne collisions that bridge pp and p–Pb collisions with Pb–Pb collisions in terms of multiplicity.

Moreover, this thesis proposes a novel study of the three-dimensional radii in the LCMS, resulting into the first measurement of this kind for proton pairs in heavy-ion collisions to date. Indeed, although the three-dimensional femtoscopic analyses with pion and kaon pairs are common in the HIC field, it has never been conducted with protons before. The reason of that was the inability of using the Bowler–Sinyukov formula based on the assumption that the CF can be effectively factorized into the Coulomb and quantum statistical terms. As for the proton case such approach is inapplicable due to the need to account for the strong interaction, the calculation of the three-dimensional CF has posed a major challenge. To resolve it, an original method of bridging the two different frames, namely the LCMS, used for the source parametrization, and the PRF, needed to calculate the WF, was developed in this thesis. The methodology was successfully applied to extract the three-dimensional proton radii using ALICE Pb–Pb data. The radii exhibit a behavior similar to the one observed with pion and kaon pairs. The applicability of the methodology developed in this work was finally substantiated by the comparison with a model study of the 3D proton source

performed on simulated Pb–Pb collision data.

In summary, this thesis offers an example of a joint study consisting of a theoretical and an experimental analyses complementary to each other. The results of the study provide interesting insights on the phenomenology of ultrarelativistic heavy-ion collisions and open new perspectives for experimental measurements.

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