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Cross Section Measurements of a 200 MeV/u ^{12}C beam on C target with the FOOT Detector

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Abstract

Accurate knowledge of how charged particles interact with matter is fundamental in applied nuclear physics research. In particular, the scarce understanding of nuclear fragmentation processes in the 200–800 MeV/u energy range represents an important limitation for the applications in Particle Therapy and Space Radioprotection, where an accurate modeling of fragment production is essential.

Particle Therapy is a medical treatment where the localized energy deposition of charged particles enables precise irradiation of deep-seated tumors while minimizing damage to healthy tissue. However, nuclear fragmentation of both projectile and target nuclei inside the patient generates secondary fragments that contribute to the dose distribution, producing a tail beyond the Bragg peak and altering the entrance dose. Accurate measurements of fragmentation cross sections are essential to refine physical models, thereby increasing the reliability of Treatment Planning Systems and thus the quality of the patient's clinical treatment.

Space Radioprotection is a field of growing importance with the prospect of long-duration manned missions outside the Earth's orbit, where astronauts are exposed to high-energy heavy ions from Galactic Cosmic Rays or Solar Particle Events. Experimental data on nuclear fragmentation are essential to improve radiation risk assessment in terms of the biological impact of cosmic radiation and to optimize nuclear process evaluations for spacecraft shielding design.

The FOOT (FragmentatiOn Of Target) experiment was conceived to fill the current gaps in fragmentation data by measuring double differential cross sections in angle emission and in kinetic energy for light ions ($Z \leq 8$) in the 200–800 MeV/u energy range. To achieve this, FOOT employs two complementary setups: an Electronic Setup, optimized for heavier fragments ($3 \leq Z \leq 8$) in an angular acceptance of 10° and an Emulsion Spectrometer dedicated to light fragments ($Z \leq 3$) emitted at large angles (up to 45°).

This Ph.D. thesis presents results from the 2024 campaign at CNAO, where the Electronic Setup was used with a 200 MeV/u ^{12}C beam on a C target. The work includes the characterization of the MIMOSA-28 vertex detector, focusing on threshold calibration, noise studies and efficiency evaluation. Furthermore, the validated reconstruction and analysis framework is applied to experimental data, leading to the first measurements of total and angular differential cross sections within the complete setup. For selected fragments, even isotopic cross sections are obtained. Complementary research within the SIRMIO project on Timepix3 calibration for small-animal proton Computed Tomography contributes to reducing range uncertainties in ion beam applications.

Sommario

L'interazione di particelle cariche con la materia è un importante argomento di studio nella fisica nucleare applicata. In particolare, la scarsa comprensione dei processi di frammentazione nucleare nel range energetico di 200–800 MeV/u rappresenta un limite importante in Adroterapia e Radioprotezione Spaziale, ambiti in cui una modellizzazione di questi processi è essenziale.

L'Adroterapia è un trattamento medico in cui le particelle cariche rilasciano energia in maniera localizzata e ciò consente di irradiare con precisione tumori profondi riducendo al minimo il danno ai tessuti sani. Tuttavia, la frammentazione nucleare del proiettile e del bersaglio all'interno del paziente genera frammenti secondari che modificano la distribuzione di dose, aumentandola all'ingresso e producendo una coda oltre il picco di Bragg. Misure accurate delle sezioni d'urto di frammentazione aumenterebbero l'affidabilità del sistema per i piani di trattamento e, di conseguenza, la sua qualità a livello clinico.

La Radioprotezione Spaziale è un campo di crescente importanza in vista di missioni umane di lunga durata al di fuori dell'orbita terrestre, durante le quali gli astronauti sono esposti a raggi cosmici fortemente energetici. Dati sperimentali accurati sulla frammentazione nucleare sono cruciali per migliorare la valutazione del rischio radiobiologico legato all'impatto della radiazione cosmica sul corpo umano e per ottimizzare la progettazione della schermatura dei veicoli spaziali.

L'esperimento FOOT (FragmentatiOn Of Target) è stato concepito per colmare le attuali lacune sperimentali misurando sezioni d'urto doppiamente differenziali in angolo di emissione ed energia cinetica per ioni leggeri ($Z \leq 8$) nel range 200–800 MeV/u. A tale scopo, FOOT è composta da due apparati complementari: un apparato elettronico, ottimizzato per i frammenti più pesanti ($3 \leq Z \leq 8$) entro un'accettanza angolare di 10° , e un setup a emulsioni, specifico per frammenti leggeri ($Z \leq 3$) emessi a grandi angoli (fino a 45°).

Questa tesi di dottorato presenta i risultati della campagna sperimentale di CNAO svolta nel 2024, utilizzando l'Electronic Setup per misure di un fascio di ^{12}C a 200 MeV/u su un bersaglio di carbonio. Il lavoro comprende la caratterizzazione del rivelatore di vertice MIMOSA-28, con particolare attenzione alla calibrazione delle soglie, allo studio del rumore e alla valutazione dell'efficienza. Inoltre, il framework di ricostruzione e analisi è validato e applicato ai dati sperimentali, portando alle prime misure delle sezioni d'urto totali e differenziali angolari ottenute con l'intero setup elettronico. Per alcuni frammenti selezionati, sono state ottenute anche sezioni d'urto isotopiche. Un'attività complementare è stata svolta nell'ambito del progetto SIRMIO, dedicata alla calibrazione dei rivelatori Timepix3 per la tomografia a protoni su piccoli animali, al fine di ridurre le incertezze di range nell'ambito dell'Adroterapia.

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Introduction

The way charged particles interact with matter and the biological effects of the resulting ionizing radiation form the basis of many applied studies where physics, biology and medicine are deeply intertwined. When an energetic charged particle, like a proton or a heavier ion, passes through a material, it mainly loses energy through electromagnetic interactions with atomic electrons. However, the particle can also undergo nuclear interactions with atomic nuclei, leading to the creation of secondary fragments. While electromagnetic energy loss is well understood, current particle transport codes face challenges to accurately model nuclear fragmentation processes due to a lack of sufficiently precise experimental data on the relevant nuclear cross sections. This limitation, in the particular focus of particles with energies between 200 MeV/u and 800 MeV/u, directly affects key application fields such as Particle Therapy and Space Radioprotection, where an accurate description of nuclear fragmentation is essential for both performance optimization and safety assessment.

In Particle Therapy, charged ion beams (such as protons or carbon ions) are employed to treat deep-seated tumors, since they offer a more favorable depth-dose distribution than conventional radiotherapy. While photon dose decreases exponentially as they enter a material after an initial turn-on, charged particles release a lower dose in the entrance channel and exhibit a sharp maximum of energy deposition near the end of the particle range (Bragg Peak), which can be precisely tuned to coincide with the tumor position. However, when ions heavier than protons are used, projectile fragmentation inside the human body generates secondary fragments with a longer range than the primary ion, causing a dose deposition tail beyond the Bragg Peak, potentially damaging adjacent Organs at Risk. Conversely, target fragmentation produces low-energy and short-range fragments with high ionization density that modify the dose deposited in healthy tissues, particularly in the entrance channel of the beam (dose build-up). Since the current estimation of these effects relies on physical models, the lack of precise cross section measurements prevents an accurate use of Monte Carlo and deterministic codes for Treatment Planning Systems, where conservative margins are therefore applied. Refining nuclear fragmentation measurements would reduce dose discrep-

ancies in the models below the clinical tolerance of 3-5%, allowing to better exploit the full potential of Particle Therapy.

Similarly, in Space Radioprotection, accurate cross section data are fundamental to ensuring the safety of long-term, deep-space manned missions, with the goal of reaching Mars by 2030. Under such conditions, astronauts are exposed to high doses of heavy ions originating from Galactic Cosmic Rays and Solar Particle Events, which are distributed over an energy range spanning more than 10 orders of magnitude, with a peak between 400 and 1000 MeV/u. Effective shielding design requires optimizing material composition of spacecraft hulls to maximize both electromagnetic stopping power and beneficial nuclear fragmentation (which breaks down high-LET primaries into less biologically harmful fragments). However, current risk models and particle transport codes, used to predict radiation hazards and optimize shielding, strongly depend on fragmentation cross sections. Existing data gaps hinder accurate code benchmarking, leading to large uncertainties in cancer risk estimates and nuclear process evaluations within shielding materials. Therefore, experimental measurements, particularly of double differential cross sections (which provide the most complete information on fragment kinetic energy and emission angle), are required to validate the models and ensure acceptable safety margins for astronauts.

The FOOT (FragmentatiOn Of Target) experiment was proposed explicitly to bridge the aforementioned gaps in nuclear data to measure double differential cross sections for light nuclei ($Z \leq 8$) between 200 and 800 MeV/u, targeting an accuracy better than 5% for projectile fragmentation and 10% for target fragmentation measurements. The experimental campaign uses light ion beams (^4He , ^{12}C , ^{16}O) on targets simulating tissue (H, C, O) and shielding materials (PMMA, C_2H_4). To reach such goals, FOOT is based on two complementary, portable setups. An Electronic Setup is optimized for heavier fragments ($3 \leq Z \leq 8$) within a $\pm 10^\circ$ acceptance, incorporating a magnetic spectrometer, a Time-of-Flight system and a calorimeter. An Emulsion Spectrometer is suitable for lighter fragments ($Z \leq 3$) up to large polar angles (45°).

This Ph.D. thesis focuses on experimental data acquired during the campaign conducted at CNAO in November 2024, exploiting the Electronic Setup.

In particular, *Chapter 1* provides an overview of the fundamental physical principles governing charged particle interactions, followed by the core motivation of this research in Particle Therapy and Space Radioprotection in *Chapter 2*. *Chapter 3* describes the experimental configuration of both setups, justifying the design to reach FOOT achievements.

Chapter 4 details the process of characterization of the high resolution pixel detector MIMOSA-28 used for vertexing, including threshold scanning and efficiency analysis critical for precise tracking. Such results are obtained using proton beams

of 70-230 MeV. The implementation of the reconstruction software is described in *Chapter 5*, focusing on GENFIT global tracking and the rigorous methodology for defining selection cuts necessary for cross section extraction. The implementation of the analysis chain is discussed in *Chapter 6*, where it is validated via a closure test on simulated and reconstructed data. It is then applied to data acquired during the CNAO 2024 campaign, where a ^{12}C 200 MeV/u beam impinging on a C target. The main goal in this part is to measure, for the first time, elemental and angular differential cross sections with the full electronic setup. Finally, among preliminary fragment mass reconstructions, first exploratory measurements of integral and differential isotopic cross sections have been performed, in the direction of a complete isotopic characterization of the measured fragmentation processes. In parallel, *Appendix A* summarizes research performed within the SIRMIO project framework, dedicated to optimizing small animal Proton Computed Tomography through the calibration and equalization of Timepix3 detectors, in synergy with the FOOT experiment's objective of reducing range uncertainties in Particle Therapy through accurate nuclear studies.

Chapter 1

Passage of Particles through Matter

The physical principles of charged particle interaction with matter and the main biological aspects of ionizing radiation are the basis for the most important studies of *Particle Therapy (PT)* and *Space Radioprotection (SR)*.

When a charged particle like a proton or a heavier ion enters an absorbing medium, it starts to slow down due to a large number of inelastic interactions with the atomic electrons (Coulomb interactions). Moreover, the particle can also deflect its path since elastic interactions with the atomic nuclei occur (Multiple Scattering). In addition to the above mentioned electromagnetic interactions, nuclear reactions play an important role, causing in particular the production of secondary particles (nuclear fragmentation).

Supposing that the medium is a biological material, these physical processes have a non negligible biological effect on its cellular structures and a deep knowledge of all these aspects is fundamental for a valid understanding of the capabilities and criticalities of PT and SR.

In this chapter, an overview of the interaction mechanisms of charged particles with matter is given, highlighting what happens in the energy range of interest. Finally, the main biological processes that occur after ionizing radiation are outlined, focusing on the effects related to PT and SR.

1.1 Electromagnetic energy deposition

For ions in the energy regime suitable for PT and SP, the dominant process of energy loss is via inelastic electromagnetic interactions with the atomic electrons of the medium. Indeed, every electron acquires impulse from the crossing particle due to the attractive Coulomb force, sufficient to be excited to an external orbital level (*excitation*) or enough to be ejected away (ionization). Moreover, the latter electrons (usually called *δ -rays*) may induce further ionization in the medium.

Assuming that a particle moves rapidly compared with an electron and with enough energy to ionize it, from classical kinematics it is possible to calculate the maximum energy that a charged particle can lose in colliding with an electron initially free and at rest [1]:

$$W_{max} = \frac{4EMm_e}{(M + m_e)^2} \quad (1.1)$$

where E and M are respectively the kinetic energy and the mass of the incoming particle, m_e is the mass of the electron.

For a particle heavier than the electron ($M \gg m_e$), the maximum transferred energy is $W_{max} = 4Em_e/M$, which represents only a small fraction of the overall particle energy. This means that many interactions of this kind are experienced before the particle totally stops. Thus, the energy loss by the incoming particle can be considered as a continuous process and, intuitively, it is expected to increase gradually while slowing down because more time is spent near each atomic electron.

1.1.1 The Bethe-Bloch formula

The assumptions described so far were introduced by Bohr in agreement with the classical calculation of momentum impulse transfer [2] and then formalized by Bethe, Bloch and others considering quantum mechanical effects [3][4] in the so called *linear stopping power* S formula:

$$S = \left\langle -\frac{dE}{dx} \right\rangle = -K \frac{\rho Z_t}{A_t} \frac{Z^2}{\beta^2} \left[\frac{1}{2} \ln \left(\frac{2m_e c^2 \beta^2 \gamma^2 W_{max}}{I_t^2} \right) - \beta^2 - \frac{\delta}{2} - \frac{C}{Z} \right] \quad (1.2)$$

It represents the mean energy loss dE per unit path length dx by the incoming particle of charge Z inside a homogeneous material of density ρ , where

- β is the ratio of the relative velocity v to the speed of light c and γ is the Lorentz factor $1/\sqrt{1 - \beta^2}$ for the incident particle.
- K is a constant defined as $K = 4\pi r_e^2 N_A m_e c^2 = 0.307075 \text{ MeV cm}^2/\text{g}$, where N_A , m_e and r_e are respectively Avogadro's number, the electronic mass (0.511 MeV/c²) and the electron classical radius (2.8 fm).
- Z_t , A_t and I_t are the atomic number, the mass number and the mean excitation potential of the absorber medium, where t stands for target. The latter has to be derived experimentally from stopping power data and recommended values are given by the ICRU (*International Commission on Radiation Units and Measurements*) Reports 37, 49 and 90 [5]. In particular, the standard value of water ($\rho = 0.998 \text{ g cm}^{-3}$), also present in simulations, is $I_t = 78 \text{ eV}$.

However, an empirical estimate obtained from the quantum-mechanical derivation of Eq. 1.2 is the following:

$$I_t \cong \begin{cases} 19.0 \text{ eV}, & Z = 1 \text{ (hydrogen)} \\ (11.2 + 11.7Z) \text{ eV}, & 2 \leq Z \leq 13 \\ (52.8 + 8.71Z) \text{ eV}, & Z > 13. \end{cases} \quad (1.3)$$

- $W_{\max}$ is the maximum energy transfer to an electron in a single collision (relativistic adaptation of 1.1), given by

$$W_{\max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2} \quad (1.4)$$

where M is the mass of the incoming particle. At low energy regime ($2\gamma m_e \ll M$) the expression approximates to $W_{\max} = 2m_e c^2 \beta^2 \gamma^2$, which is often implicit in the Bethe-Bloch formula. This approximation holds up to $\sim 10^6$ MeV for protons, so it is also suitable for the purposes of PT and of SP. For example, a proton of 200 MeV in water has an energy release of ~ 500 keV per electron, losing all energy after roughly 400 collisions.

On the contrary, at high energies ($2\gamma m_e \gg M$) the value turns into $W_{\max} = Mc^2 \beta^2 \gamma$, where the electron rest mass effects became negligible [6].

- δ represents the density correction, which implies energy loss reduction due to polarization effects in the medium, significant only at high energies.
- C represents the shell correction, which is not negligible when the energy of the incident particle is low enough that its velocity is comparable with that of orbital electrons. For example, at 10 MeV/u the shell correction modifies the stopping power of protons by about 6% [7].
Moreover, at this low energy regime, the projectile charge Z can be replaced by the effective charge Z_{eff} due to recombination effects, that can be obtained by the Barkas formula [8]:

$$Z_{\text{eff}} = Z \left(1 - e^{-125\beta Z^{-2/3}} \right) \quad (1.5)$$

At the range of interest, the energy loss due to electromagnetic interaction with nuclei is negligible since, substituting the mass of a nucleus to the one of an electron in Eq. 1.1, it is possible to infer that the energy transfer is suppressed by the mass ratio. Moreover, it may be noted that, from a macroscopic perspective, the stopping power S can be interpreted as an effective frictional force whose dependence on the ion velocity is intrinsically nonlinear.

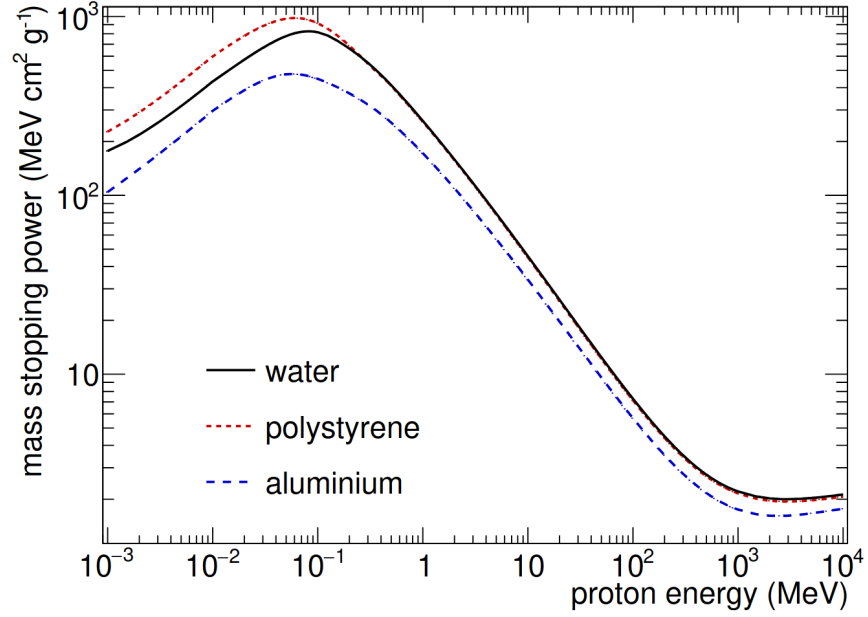


Figure 1.1: Stopping power of protons in water, polystyrene and aluminium vs proton kinetic energy. Adapted from [9].

In order to limit the dependency on the density of the medium, the mass stopping power is obtained as:

$$\frac{dE}{d\chi} = \frac{1}{\rho} \frac{dE}{dx} \quad (1.6)$$

which is measured in $\text{MeV cm}^2 \text{g}^{-1}$ and χ represents the mass thickness in units of g/cm^2 .

The behavior of the mass stopping power of a proton crossing several materials is shown in Fig. 1.1. Regardless of the crossing material, it is possible to inspect a common pattern for each energy loss: at higher energies than 10 GeV there is a logarithmic rise proportional to $\ln(\beta^2)$ up to a density effect plateau; the minimum value is reached around 1000 MeV ($\beta\gamma \sim 3$) and for lower energies there is a behavior proportional to $1/\beta^2$ until tens of eV when the shell correction becomes relevant and the curve falls off due to electron pickup with the particle. Moreover, the mass stopping power does not differ greatly for materials with similar atomic composition, as can be seen for the curves of water and polystyrene, all materials made of light elements.

1.1.2 Energy straggling

The value obtained by Eq. 1.2 is a mean energy loss because the interaction process suffers from statistical fluctuations (*energy straggling*). Given a particle crossing a specific material thickness, three different theories are validated to describe the energy loss distribution, according to the following selection parameter [10]:

$$k = \frac{\langle \Delta E \rangle}{W_{\max}}, \quad (1.7)$$

where $\langle \Delta E \rangle$ is the approximate mean energy loss and $W_{\max}$ is the maximum energy loss possible in a single event described by Eq. 1.4. When the material is thick enough ($k \geq 0.1$), Bohr's theory [3] assumes that the central limit theorem holds due to many individual collisions and the energy straggling can be parametrized by a Gaussian distribution:

$$f(\Delta E) = \frac{1}{\sigma_{\Delta E} \sqrt{2\pi}} \exp \left(-\frac{(\Delta E - \overline{\Delta E})^2}{2\sigma_{\Delta E}^2} \right). \quad (1.8)$$

where $\sigma_{\Delta E}$ is the distribution standard deviation.

In an intermediate thicker absorber ($k \sim 0.1$), fewer individual collisions where rare high energy transfers to electrons occur, with the rise of a long asymmetric tail. This is described by the Landau Theory [11] with an expression of the energy distribution given by:

$$f(\Delta E, \rho x) d\Delta E = \frac{1}{\langle \Delta E \rangle} \varphi_L(\lambda_L) d\lambda_L \quad (1.9)$$

where ρ is the mass density, x is the thickness of the absorber and the parameter $\varphi_L(\lambda_L)$ roughly corresponds to the deviation from the mean energy loss and was defined by Landau as

$$\varphi_L(\lambda_L) = \frac{1}{\pi} \int_0^\infty e^{-y (\ln y - \lambda_L y)} \sin(\pi y) dy \quad (1.10)$$

given λ_L as the so called *Landau parameter*, which is related to the energy loss $d\Delta E$ and the thickness of the material x in the following way [12] :

$$\lambda_L = \frac{(\Delta E - \overline{\Delta E})}{W_{\max} x} - \ln(x) \quad (1.11)$$

A generalization for all the values of k (so even for the thinner material of $k \leq 0.01$) was introduced by Vavilov's theory [15], where the expression for ΔE is given by

$$f(\Delta E, \rho x) d\Delta E = \frac{1}{\xi} \varphi_\nu(\lambda_\nu, k, \beta^2) d\lambda_\nu, \quad (1.12)$$

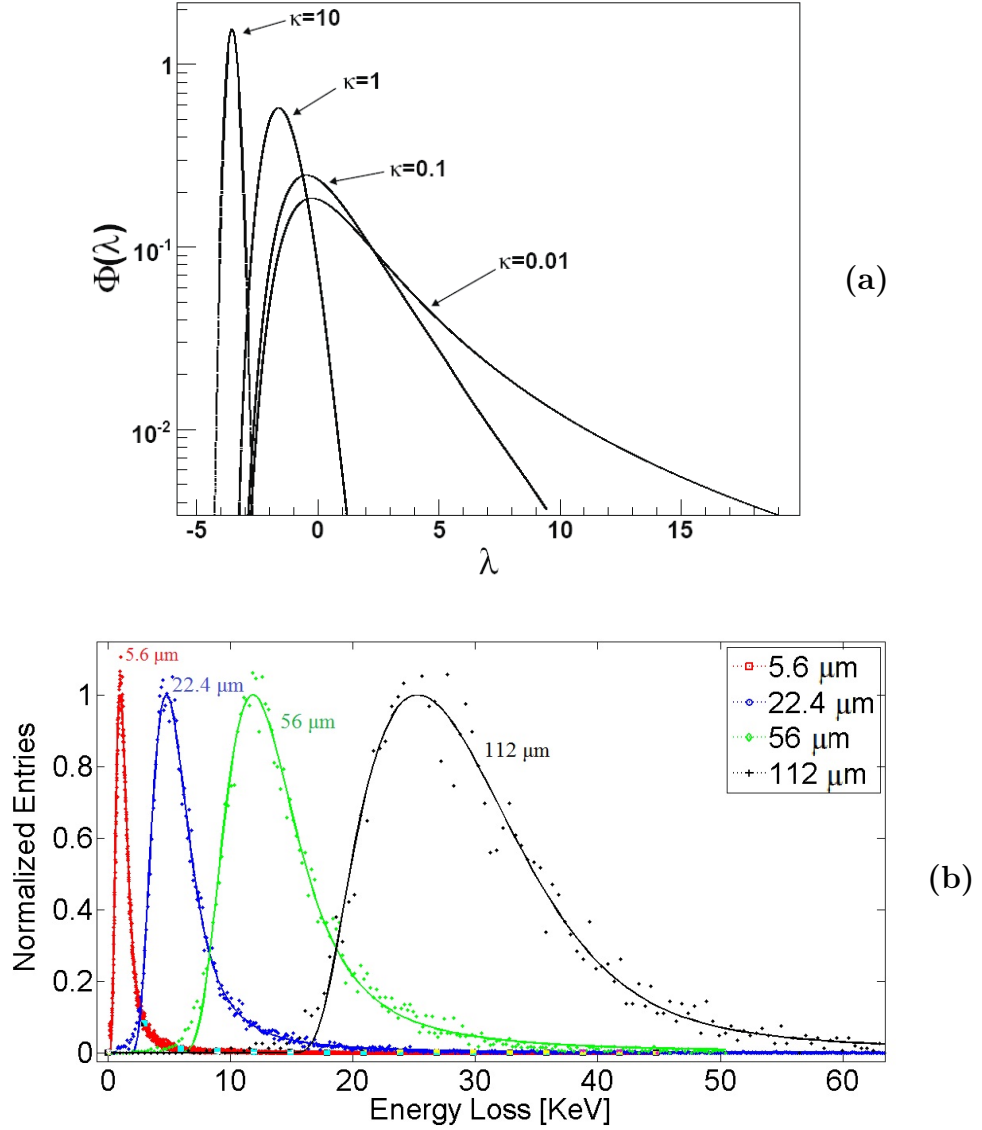


Figure 1.2: (a) The Vavilov distribution $\Phi(\lambda)$ for various parameters k , analytically computing Eq. 1.12. From [13]. (b) Energy loss distributions with fits for 12 GeV protons passing through several silicon thicknesses. From [14].

where

$$\varphi_\nu(\lambda_\nu, k, \beta^2) = \frac{1}{\pi} e^{k(1+\beta^2\gamma)} \int_0^\infty e^{kf_1} \cos(\lambda_\nu y + kf_2) dy. \quad (1.13)$$

$$\lambda_\nu = \frac{(\Delta E - \overline{\Delta E})}{W_{max}x} - k(1 + \beta^2 - \gamma) \quad (1.14)$$

$$f_1(y) = \beta^2 [\ln y + \text{Ci}(y)] - \cos y - y \text{Si}(y), \quad (1.15)$$

$$f_2(y) = y [\ln y + \text{Ci}(y)] + \sin y + y\beta^2 \text{Si}(y), \quad (1.16)$$

$$\text{Si}(y) = \int_0^y \frac{\sin u}{u} du \quad (\text{sine integral}), \quad (1.17)$$

$$\text{Ci}(y) = \int_{-\infty}^y \frac{\cos u}{u} du \quad (\text{cosine integral}). \quad (1.18)$$

and γ is Euler's constant.

Several parameterizations of the Vavilov distribution are reported in Fig. 1.2-a, while an example of energy loss of protons in different silicon media is reported in Fig. 1.2-b. Increasing the material thickness, more and more collisions occur, shifting the energy distribution from a Landau to a Gaussian.

Generally, this function is computationally expensive, so other approximations can be used in practical scenarios, like Moyal [16] distribution for $k \leq 0.1$ and Skew Gaussian [17] distribution for $0.1 \leq k \leq 1$.

1.1.3 Range

According to the stochastic nature of energy loss already discussed, all the particles with the same charge and initial kinetic energy do not cross the same path before stopping in a medium. However, in the approximation that the fluctuations in energy loss are negligible and that the particle continuously loses energy along its path, the so called *Continuous Slowing Down Approximation* (CSDA), it is possible to introduce the range of a particle as:

$$R = \int_0^R dx = \int_{E_k}^0 \left(\frac{dE}{dx} \right)^{-1} dE \quad (1.19)$$

where E_k is the initial kinetic energy.

The structure of Eq. 1.19 leads to useful scaling laws to extrapolate the relation between the range of two heavily charged particles at the same initial speed β in the medium:

$$\frac{R_1(\beta)}{R_2(\beta)} = \frac{Z_2^2 M_1}{Z_1^2 M_2}, \quad (1.20)$$

where Z_1 and Z_2 , M_1 and M_2 are respectively the particle charges and rest masses. In Fig. 1.3, the range of several particles is compared. For example, it is possible to appreciate how the range of ^{12}C is $\sim 12/6^2 \sim 1/3$ that of protons.

Another important consideration, which is also inferred by Fig. 1.3, is the relation

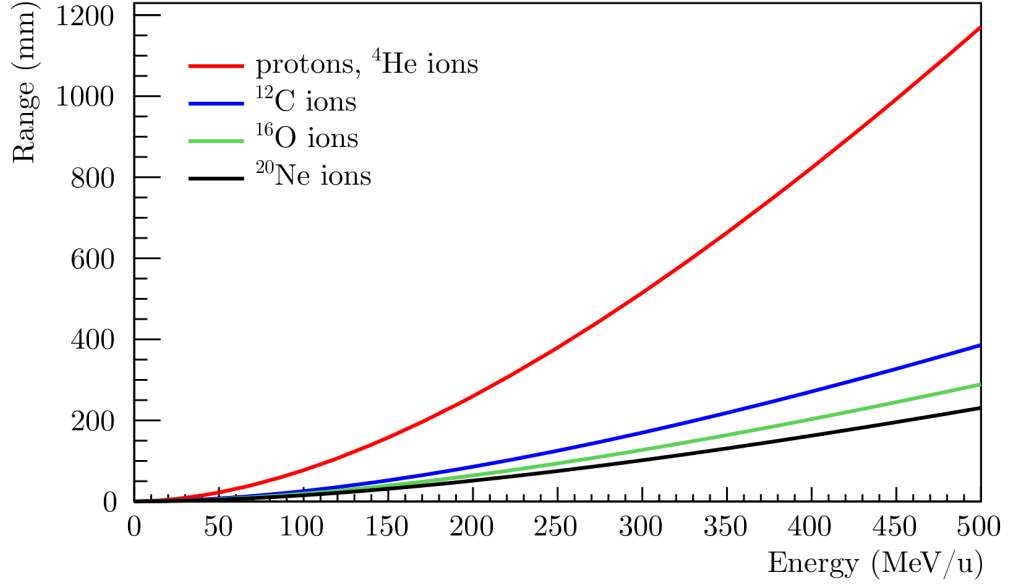


Figure 1.3: Range in water for different particles as a function of the energy. From [18].

between the range and the kinetic energy of the particle. In fact, Eq. 1.2 implies that, in a non relativistic approximation:

$$\frac{dE}{dx} \propto \frac{1}{\beta^2} \rightarrow \frac{dE}{dx} \propto E_{kin}^{-1}$$

and thus, putting this dependency into Eq. 1.19, it results $R \propto E_{kin}^2$. In general, the range - energy relation has been parametrized by the Bragg–Kleeman rule [19]:

$$R = A E_0^p \quad (1.21)$$

where R is the range in CSDA approximation, E_0 is the kinetic energy of the particle, p is a dimensionless power and A is a constant. According to that, by tuning the particle's kinetic energy, it is possible to control its stopping depth. This principle forms the basis of PT, where a beam of charged particles is tuned in order to be stopped where a tumor is located (see Sec. 2.1). In the range of therapeutic protons, in particular, (E_0 between 50 and 250 MeV), $p = 1.77$ and $A = 0.0022 \text{ cm/MeV}^p$, with a standard error of the order of 1.5% [20].

In the Gaussian approximation of energy loss straggling, it is possible to retrieve the range straggling variance as [21]:

$$\sigma_R^2 = \int \left(\frac{dE}{dx} \right)^{-3} \frac{d\sigma_{\Delta E}}{dx} dE \quad (1.22)$$

where σ_{Δ_E} can be obtained from Eq.1.8. For example, a 200 MeV proton has a range of approximately 25.8 cm, with a straggling of about 2.5 mm (roughly 1% of the range).

The relative range straggling of a particle of energy E and mass M is nearly constant:

$$\frac{\sigma_R}{R} = \frac{1}{\sqrt{M}} \phi \left(\frac{E}{Mc^2} \right) \quad (1.23)$$

where ϕ is a slowly varying function, depending on the target material. In relation to that, a useful scaling law can be provided to compute the relation of range straggling of two particles at the same range:

$$\frac{\sigma_{R1}}{\sigma_{R2}} = \sqrt{\frac{M_2}{M_1}}. \quad (1.24)$$

For example, the straggling of a beam of ^{12}C is $1/\sqrt{12} \sim 1/3.5$ times lower than the one of ^1H .

Thus, range straggling depends on the beam type and represents an important source of uncertainty to be handled in PT treatments (see Sec. 2.1.3).

1.1.4 Multiple Coulomb scattering

If the longitudinal spread of a particle crossing a material is described

by the range straggling, the lateral spread is due to elastic Coulomb scattering with the atomic nuclei, usually expressed as Multiple Coulomb Scattering (MSC).

The mechanism of MSC in a thick material is described by Moliere's theory which introduces the distribution function of the scattering angle θ [22]:

$$f(\theta) = \frac{1}{4\pi\theta_M^2} \sum_k \frac{f^k(\theta')}{B^k}, \quad (1.25)$$

where θ_M is the characteristic MCS angle (the scale of angular deflection), $\theta' = \frac{\theta}{\sqrt{2}\theta_M}$ is the reduced scattering angle, B is the logarithm of the effective number of collisions on the target and f^k is related to the Bessel function. In the first-order approximation of $k = 0$, the distribution becomes Gaussian and the standard deviation of the characteristic MCS angle can be obtained as [23]:

$$\sigma_\theta = \frac{13.6 \text{ MeV}}{pv} Z \sqrt{\frac{x}{X_0}} \left[1 + 0.0038 \ln \left(\frac{x}{X_0} \right) \right]. \quad (1.26)$$

where x is the material thickness, p , v , and Z are the momentum, velocity, and atomic number of the particle, respectively, while X_0 is the radiation length of the material.

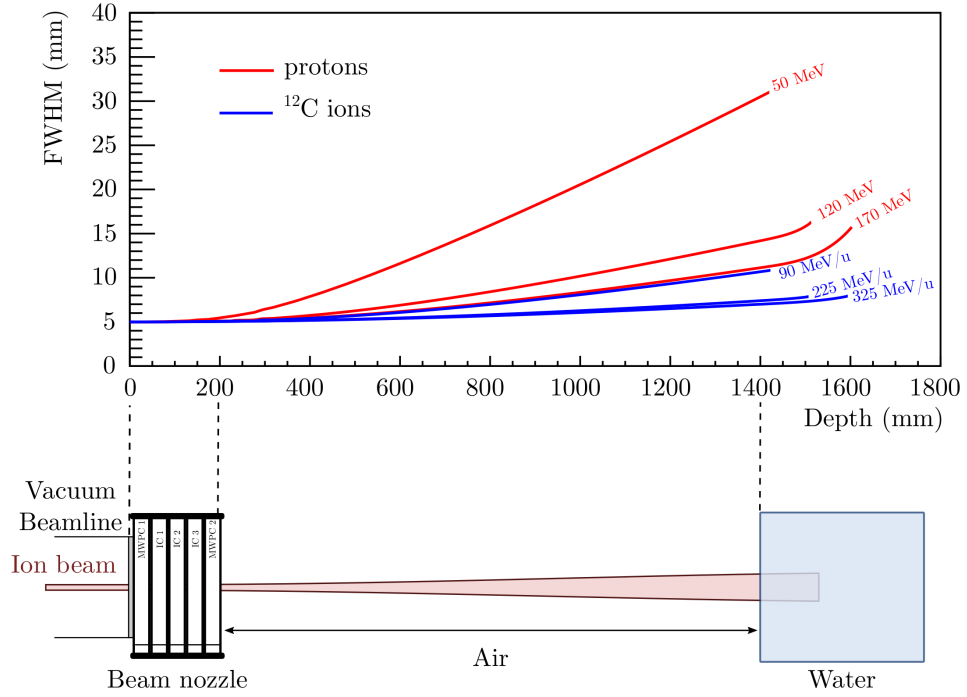


Figure 1.4: The figure (bottom) represents the simulation of an experimental setup to measure the range of a given beam of charged particles. The particles cross a set of detectors (beam nozzle) to reconstruct the feature of the beam (like its spatial distribution) and then they reach a water tank where they are fully stopped. In the plot (top) the FWHM of the different particle types (protons and ^{12}C) at different energies is shown. From [18].

Fig. 1.4 shows the lateral diffusion of several beams of protons and ^{12}C ions in terms of FWHM and their relation with the range of the particle. It increases for lighter particles, up to tens of mm in the region where the particle stops. As well as longitudinal straggling, even the MCS lateral spread represents a critical aspect for PT (see Sec. 2.1.3). For instance, a 200 MeV proton in water has a range of approximately 25.6 cm and a lateral displacement of about 5 mm, which is significant enough to impact a PT treatment plan.

1.2 Nuclear interactions

Particles crossing the medium can also incur into nuclear interactions that, despite a lower probability than electromagnetic ones, have a crucial role in the field of PT and SR.

At the energy range of interest, a charged particle could interact with the medium via an elastic process which is characterized by a deflection of the particle with no variation in the kinetic energy nor in the composition of the nuclei. On the other hand, for inelastic nuclear interactions, projectile and target react with variations in the energy due to excitation processes or even cause a breakup of both into secondary particles. The latter are projectile fragments originating from the primary ions or target ones coming from the medium.

Nuclear-nuclear interactions can be described through the so-called *abrasion-ablation model* [24] which is split into two-step processes. During abrasion, the geometrical overlap of the projectile and the target causes the removal of nucleons from the original status (*fireball*), while the rest is not affected by the reaction (*spectators*). During the ablation phase, both fireball and spectators are de-excited emitting nucleons, light clusters and gamma rays. A sketch of the processes is shown in Fig.1.5, where the role of overlap is highlighted. In particular, at the typical kinetic energy of PT, it is possible to deduce that peripheral collisions are more likely than complete disintegration.

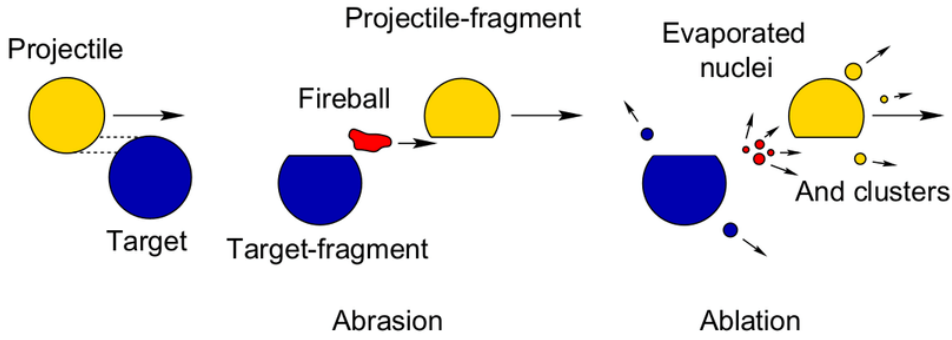


Figure 1.5: Sketch of the Abrasion - Ablation model

1.2.1 Nuclear Reaction Cross Section

A physical quantity that can be used to describe fragmentation processes is the *total reaction cross section*:

$$\sigma_R = \sigma_T - \sigma_{el} \quad (1.27)$$

defined as the difference between total cross section σ_T and elastic one σ_{el} .

In a geometric approximation, nuclei are assumed to be opaque spheres and the reaction cross section is proportional to the projectile and target overlapping area [25]:

$$\sigma_R = \pi (a_p + a_T)^2 \quad (1.28)$$

where $a = r_0 A^{1/3}$ is the nuclear radius, with $r_0 = 1.2671$ fm and A is the mass number of the nucleus. An empirical model was introduced by Bradt and Peters [26]:

$$\sigma_R = \pi r_0^2 \left(A_p^{1/3} + A_T^{1/3} - b \right)^2 \quad (1.29)$$

where A_p and A_t are respectively the projectile and target mass number, r_0 is the nucleon radius (1.2 fm) and b is the overlap transparency parameter. The model is suited for high energy reactions (≥ 1.5 GeV/n) and heavy nuclei, while it is less accurate at the lowest energies because the cross section is more energy dependent. In PT where nuclear reactions at low energies are of critical importance, the parametrization of the b value is introduced as a function of energy [21] and this semi-empirical parametrization of Eq.1.29 can adequately reproduce experimental data, as shown in Fig. 1.6. In the field of SR, a general expression of cross section used in cosmic radiation transport calculation is the following [28]:

$$\sigma_R(E) = \pi r_0^2 c_1(E) \left(A_p^{1/3} + A_T^{1/3} - c_2(E) \right)^2 \quad (1.30)$$

where r_0 , $c_1(E)$, $c_2(E)$ are parameters depending on the kinetic energy.

A usual approach to experimentally retrieve the cross section σ_R is based on the attenuation of the primary ion beam along the depth x of the target:

$$\Phi(x) = \Phi_0 e^{-\mathcal{N}\sigma x}, \quad (1.31)$$

where $\mathcal{N}$ is the atomic density of target nuclei in the material per unit volume, σ is the total reaction cross section and Φ_0 and $\Phi(x)$ are, respectively, the fluence at the beginning of the path and after a given depth x . A comparison of fluence attenuations of different charged particles is reported in Fig.1.7, where it is possible to infer that the heaviest particles have the highest attenuation, up to 30 % for O, a criticality that has to be faced in PT (see Sec. 2.1.3).

In a more complete approach, the quantity σ_R has to be considered as a *total* reaction cross section, obtained by an integration of a double differential cross section both in kinetic energy E_k and in angle emission Ω :

$$\sigma_R = \int_0^\Omega \int_0^\infty \frac{d^2\sigma}{dE_K d\Omega} dE_K d\Omega. \quad (1.32)$$

The importance of nuclear cross sections in PT and SR is discussed in Sec. 2.1.5 and Sec. 2.2.5 respectively.

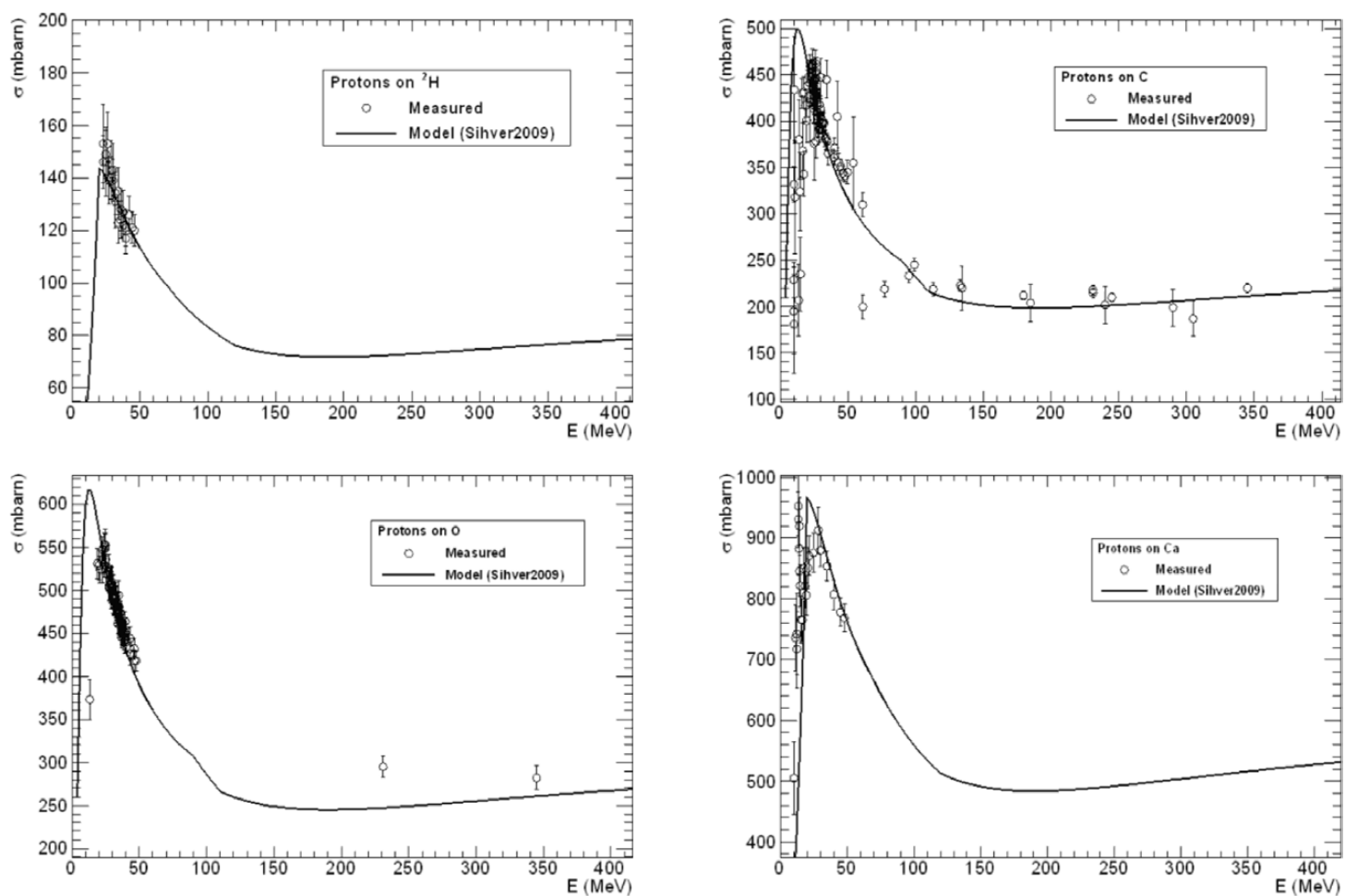


Figure 1.6: Fragmentation cross section of a beam of protons with different nuclei present in human tissues. The model is compared with experimental data of [27].

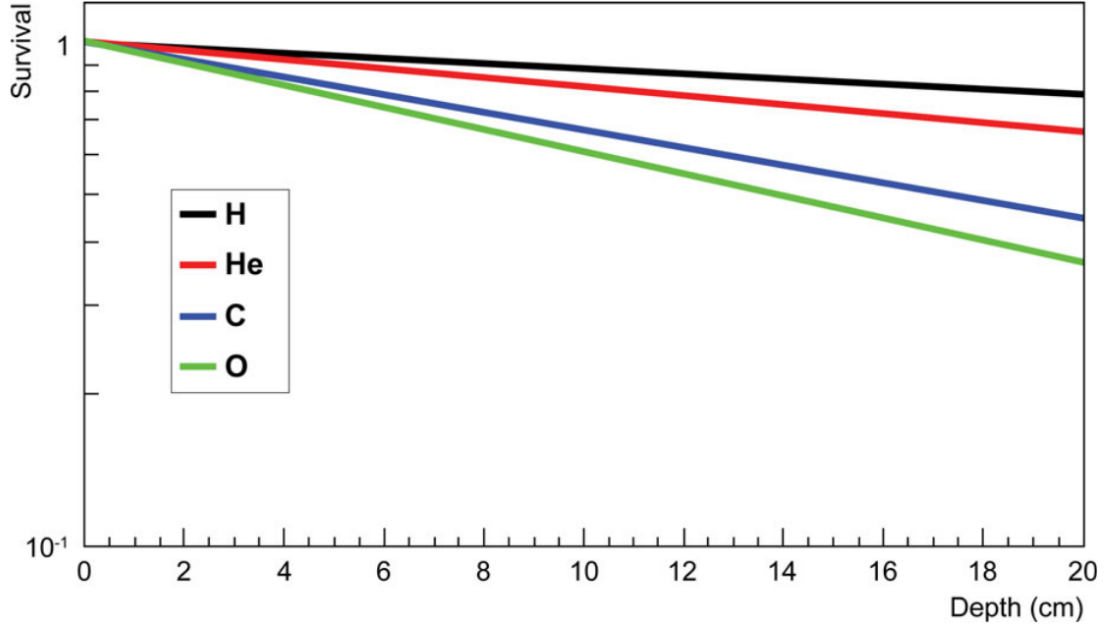


Figure 1.7: Attenuation of different charged particles as percentage (defined as *Survival*) with the same range of ~ 20 cm in water. From [21]

1.3 Depth - Dose profile

1.3.1 Absorbed Dose and the Bragg Peak

An important physical quantity that introduces the effects of radiation exposure on the biological medium is the *absorbed dose* D , defined as the mean energy dE absorbed by ionizing radiation in a mass element dm :

$$D = \frac{dE}{dm} \quad (1.33)$$

and it is measured in Gray, where $1 \text{ Gy} = 1 \text{ J/kg}$.

This quantity is also correlated to the the stopping power S :

$$D [\text{Gy}] = 1.6 \times 10^{-9} \times S \left[\frac{\text{keV}}{\mu\text{m}} \right] \times \Phi \left[\frac{1}{\text{cm}^2} \right] \times \frac{1}{\rho} \left[\frac{\text{cm}^3}{\text{g}} \right]. \quad (1.34)$$

where Φ is the fluence of a mono energetic ion beam and ρ is the density of the target material.

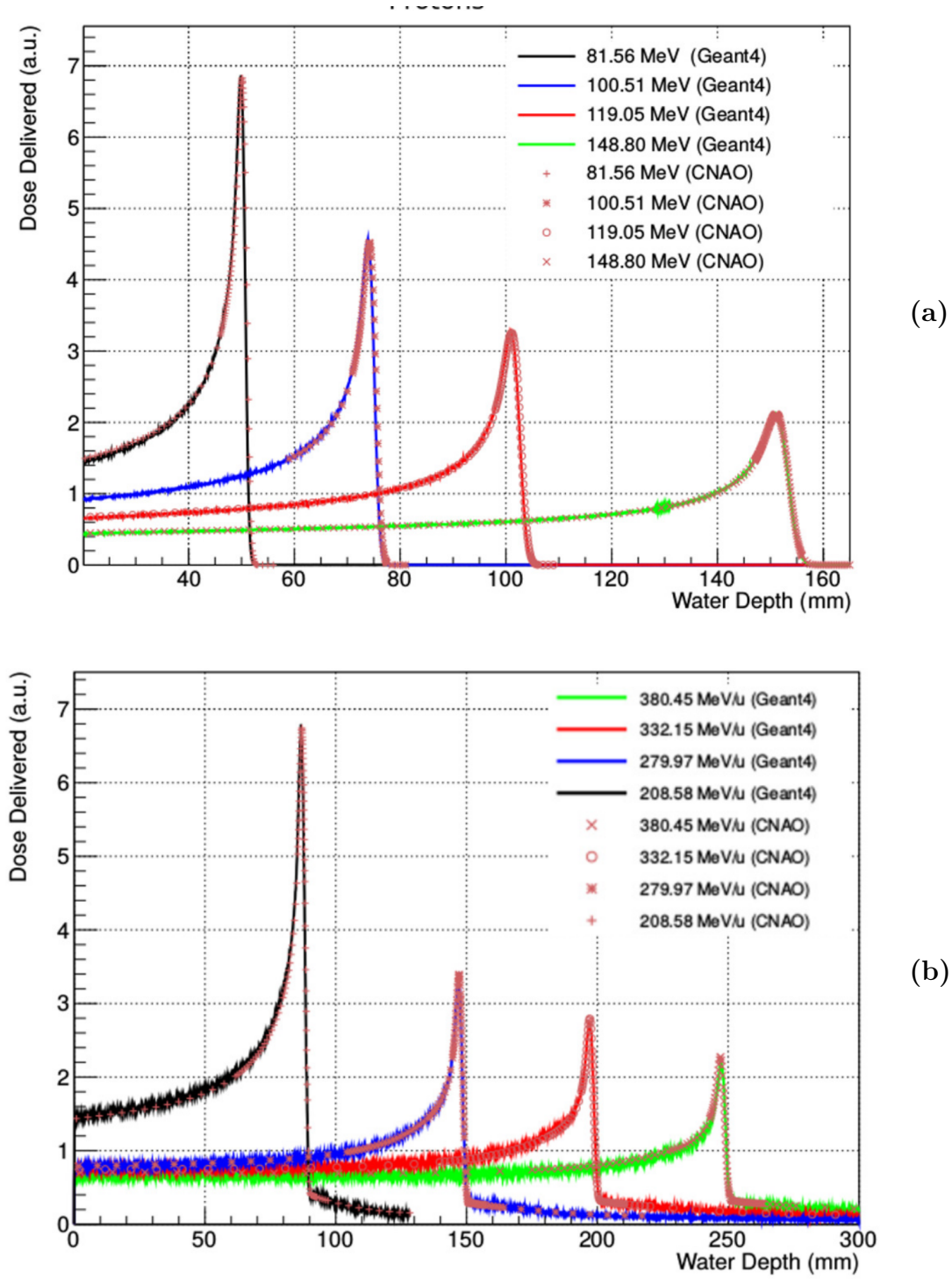


Figure 1.8: Depth-dose deposited in a water phantom by protons (a) and ^{12}C (b), normalized in area at the dose deposited by the beam with lower energy. Data taken from CNAO are compared with GEANT4 simulation. From [29].

The depth-dose profile is a representation of the absorbed dose deposited by a particle beam or other radiation source as it penetrates a medium, plotted as a function of depth. An example for ^1H and ^{12}C ions at different energies crossing a water tank is shown in Fig. 1.8. Both profiles are characterized by a step increase of dose toward the end of the ion range, which is called *Bragg Peak*. According to Eq.1.34, the energy loss increases as the ion slows down ($D \propto S \propto 1/\beta^2$) and this feature plays a fundamental role in the applications of PT (see Sec. 2.1.3). The dose tail after the Bragg Peak for the ^{12}C ion is due to dose released by secondary fragments generated during nuclear interactions, and this contribution became more and more relevant for heavier ions, as highlighted in the following section.

1.3.2 The role of fragmentation

The fragmentation of primary ions through matter is a process that critically affects the Depth Dose profile.

In Fig.1.9, the dose profiles of ^{28}Si and ^{48}Ti heavy ions are reported, separating the electromagnetic and nuclear dose contributions. The attenuation of the primary ion beam is mainly due to *projectile fragmentation* into nuclei with similar kinetic energy per nucleon but lower atomic number. Thus, since $S \propto Z^2$ (see Eq.1.2) and $R \propto S^{-1}$ (see Eq. 1.19), they have a lower energy loss and thus a larger range, which contributes to the tail after the Bragg peak. This effect is more pronounced for ^{48}Ti , which traverses a greater amount of material. As a result, it undergoes more fragmentation compared to ^{28}Si , leading to a lower number of primary particles reaching the Bragg peak depth. Consequently, the dose in the entrance channel remains higher.

In Fig.1.10-a, the profile of ^1H and ^4He light ions is reported. Compared to He, the Bragg peak for H is higher due to the absence of nuclear fragmentation of the primary beam. Protons cannot undergo projectile fragmentation, as they are elementary particles without substructures to break apart in this energy range. Although hadron production can occur through proton-proton interactions, causing beam attenuation, the kinetic energy threshold for pion production is approximately 280 MeV, with a cross section of about 50 mb that increases with energy [30]. At the energies considered in this study, this contribution is negligible. As a result, the characteristic dose tail after the Bragg peak for H is absent.

However, zooming the entrance channel for both the particles (Fig.1.10-b), a small dose enhancement before energy loss attenuation can be appreciated, in particular for protons. This phenomenon (called as *dose build-up*) is due to nuclear interactions that cause *target fragments*, which are emitted mainly in forward direction [32]. The effects stop when a dynamic equilibrium is reached, namely the

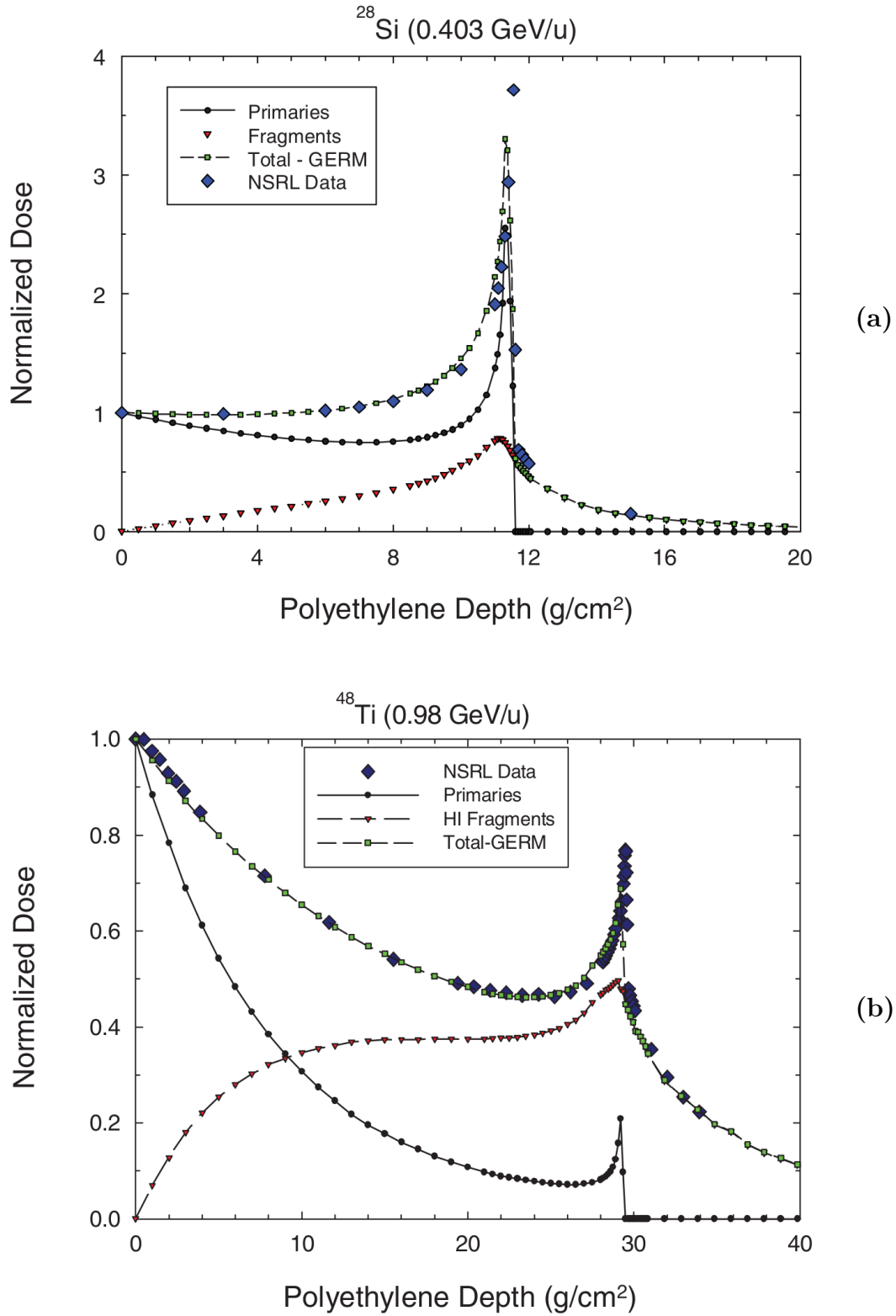


Figure 1.9: Total normalized dose and relative contributions of primary ions and fragments for: (a) ^{28}Si (b) ^{48}Ti . From [28].

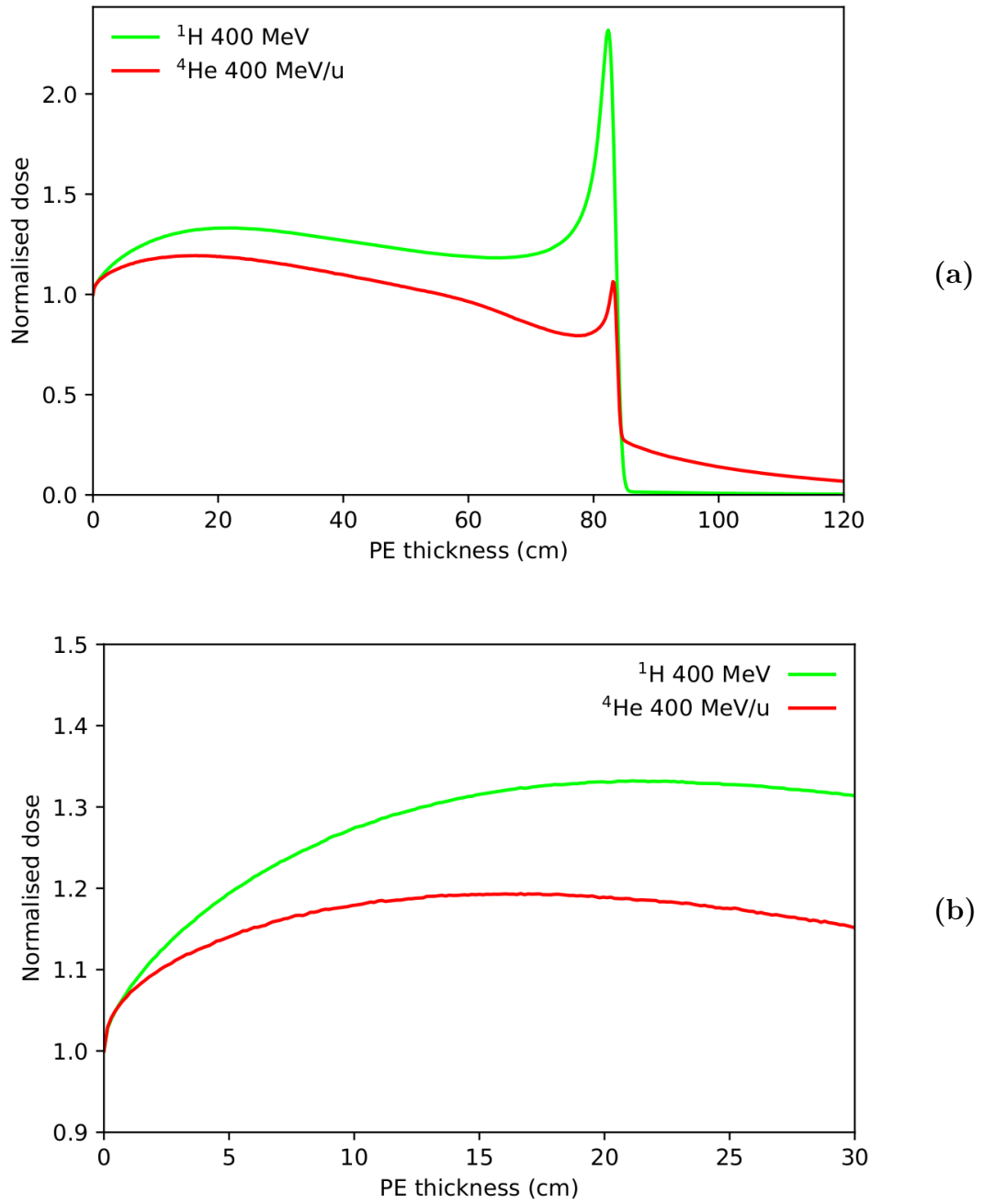


Figure 1.10: (a) Simulation of normalized dose curves at entrance point for ^1H and ^4He ions at 400 MeV/u in a Polyethylene material. (b) Zoom-in of the first 30 cm where the dose build-up effect is visible. From [31].

number of target fragments that stop is the same as the ones that are subsequently created. For protons, this equilibrium is maintained up to the Bragg peak. In contrast, for heavier ions, dose attenuation is enhanced by projectile fragmentation, which reduces the dose build-up effect.

Despite the electromagnetic interaction being the most probable, these processes are not negligible because they have a critical impact on the applications of PT (Sec. 2.1.3) and SR (Sec. 2.2.4).

1.4 Biological Aspects of Ionizing Radiation

1.4.1 LET

While crossing a material, charged particles lose enough energy to ionize orbital electrons (δ rays) which can deposit their energy to other bound electrons and beyond. A radial dose is then released around the track of the charged particle and the local concentration of energy deposition can be measured by introducing the concept of *Linear Energy Transfer LET*:

$$LET = \frac{dE_{\Delta}}{dx}. \quad (1.35)$$

It represents the ionization energy lost by the charged particles dE , crossing a distance dx in a material. Δ can define the spatial radius around the track of the incoming particle, inside which the energy release has to be considered. Otherwise, it delimits the maximum value of energy emission for a secondary particle. It has to be underlined, moreover, that in the limit of $\Delta \rightarrow \infty$, all the energy emissions by particles are considered and then *LET* and the stopping power S correspond (called as *unrestricted LET*).

1.4.2 DNA damages

According to how particles interact with matter, *LET* is also correlated to the ionization density. Low *LET* particles like photons or electrons are considered as *sparsely-ionizing* radiations (see Fig. 1.11-a, upper part) because the interaction processes happen with low probability along their track and the number of indirect ionizations per unit path is small [33], with a mean distance of 100 nm. Heavy charged particles are considered *densely ionizing* radiations because a high quantity of δ rays is emitted along the track, with a mean range up to the order of 1 nm (see Fig. 1.11-a, lower part).

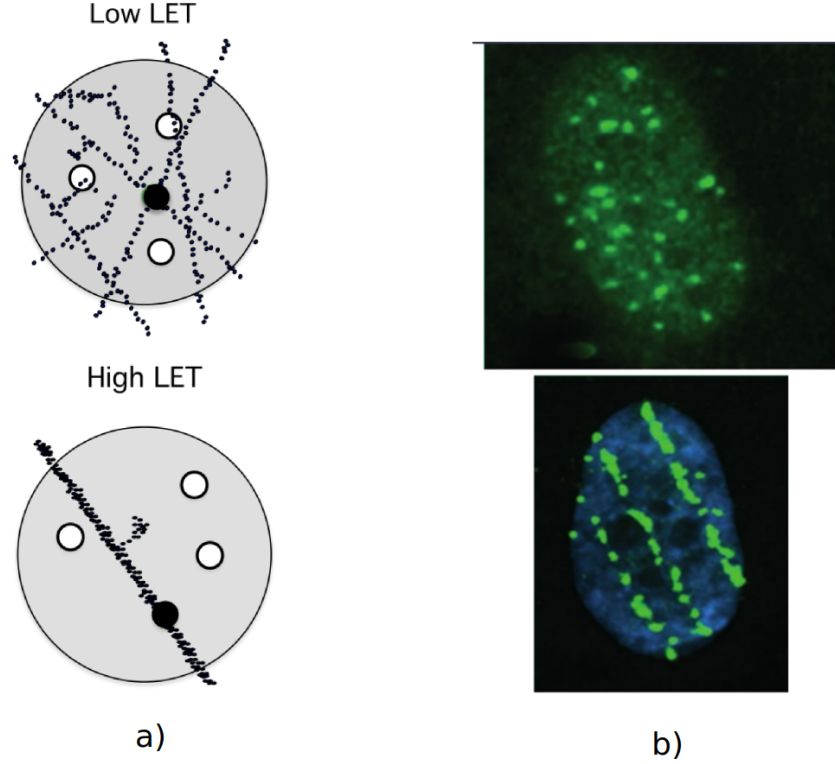


Figure 1.11: Left: representation of ionization density of low *LET* (upper part) and high *LET* (lower part) particles from [34]. Right: visualization of damages on cells after an irradiation with X-ray (upper part) and protons (lower part). From [35].

Since energy deposition causes the break of chemical bonds, the main consequence of biological matter irradiation is cellular damage due to DNA lesions. Ionization can cause a direct break of the DNA strands (*direct damage*) or originate reactions with the overall environment that indirectly affect the DNA (*indirect damage*), like the production of free radicals which are toxic for the cells [36].

The cellular response to DNA damage is based on repair mechanisms developed in years of evolution, aiming at restoring the normal nucleotide sequence and DNA structure. The main damages are single strand breaks (SSB) which can be removed by replacing the damage with the right information in the symmetrical strand. The most critical, on the contrary, is the double strand breaks (DSB) which affect both the helix of the DNA structure and thus there is no satisfactory template for repair, causing mutations or apoptosis of the cells [38].

In Fig. 1.11-b, DNA DSB on nuclei of human fibroblasts are visualized thanks to a technique which connects fluorescence proteins to the chromatin surrounding

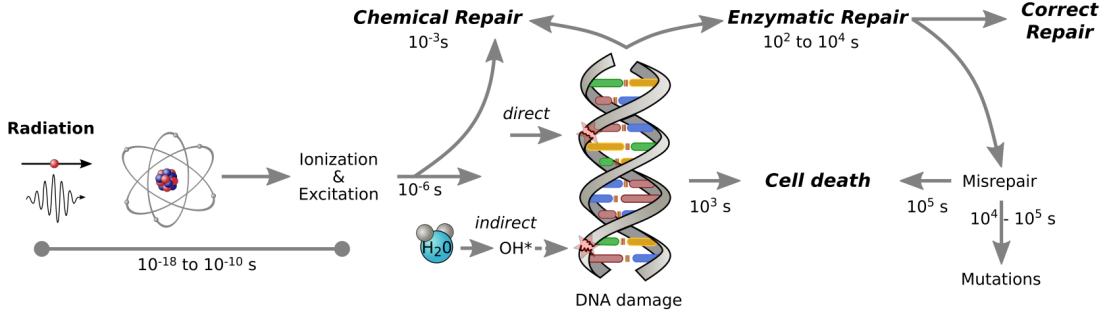


Figure 1.12: Steps from cellular damage to repair or death. From [37].

the break [39], emphasizing the correlation between DSB and ionization density of low (upper part) and high (lower part) *LET* particles. Moreover, for low *LET* radiation, the SSB to DSB ratio is notably high, around 20:1, while for high *LET* radiation it drops to less than 3:1, indicating that high *LET* radiation (namely heavy ions) tends to cause fewer but more complex DNA damage events per unit dose [40]. All the process of DNA damage is schematized in Fig. 1.12.

1.4.3 Relative Biological Effectiveness

It is important to note that, due to the energy dependence of *LET* (see Eq.1.2), particles with the same *LET* can exhibit different ionization densities [41]. Moreover, a given particle has a higher *LET* near the end of its range compared to the entrance channel (see Chapter1.3.1).

To study the biological effects of different ion species the *Relative Biological Effectiveness* (*RBE*) has been introduced, defined as follows:

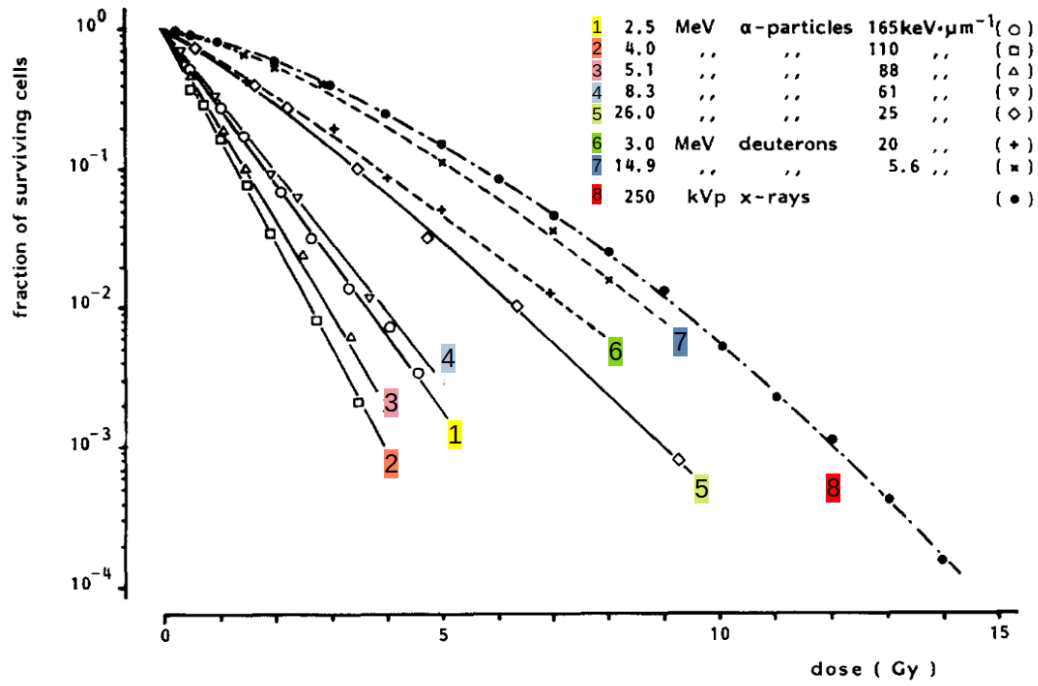
$$RBE = \frac{D_{ref}}{D_{ion}} \Big|_{\text{same effect}} \quad (1.36)$$

where D_{ion} is the ion dose (Eq.1.33) released to obtain the same biological effects of D_{ref} , which is generally evaluated using photon beams (γ -rays from ^{60}Co decay) as reference, due to its well-characterized properties and consistent energy output (1.17 MeV).

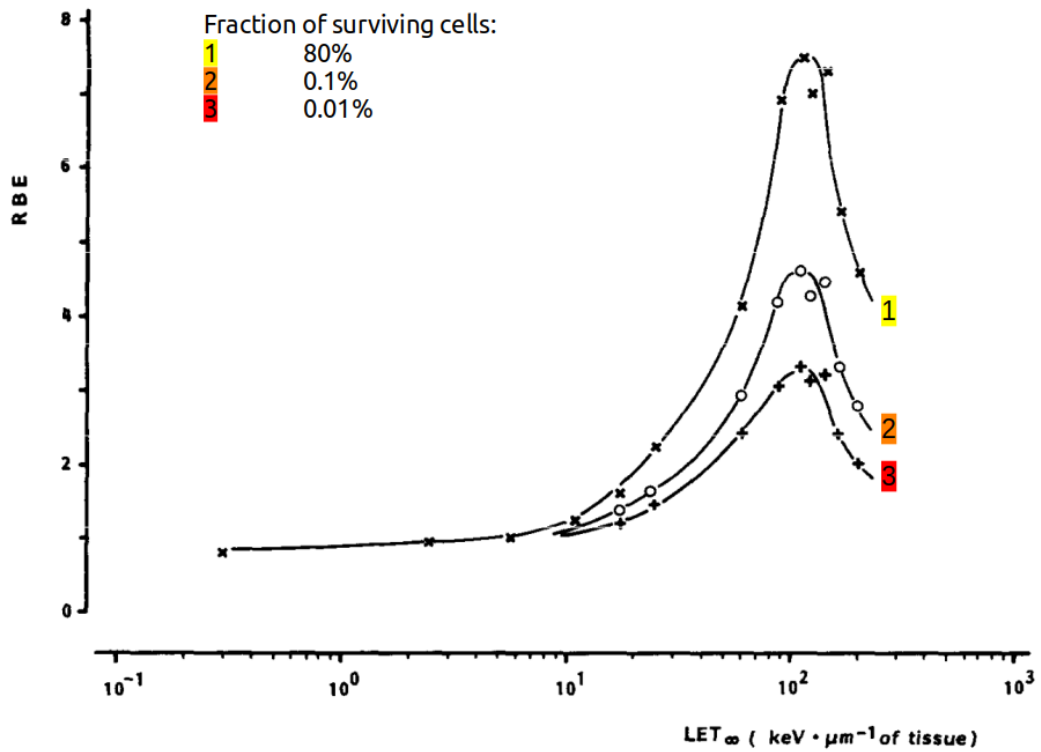
The effects are measured on living cells cultured in a controlled environment, as a biological response to exposure. The most common method involves generating cell survival curves ξ , using the *linear-quadratic model*:

$$\xi = e^{-\alpha D - \beta D^2} \quad (1.37)$$

where ξ is the fraction of survived cells, D is the absorbed dose and α and β are the model parameters experimentally obtained .



(a)



(b)

Figure 1.13: Left: Survival curves for cells exposed to radiation of different LET . The slope becomes steep as the LET of the radiation increases. Right: Variation of RBE with LET for survival of mammalian cells. Different survival levels correspond to different absorbed doses. From [42].

In particular, the α/β ratio defines the experimental curves and can be used to compare the biological effects of different types of radiation, constraining all the other variables. Inspecting Fig.1.13-a, it is possible to infer that for high *LET* particles (α particles) the linear part of the model is sufficient for their characterization. Comparing these curves with the ones of X-rays, it is possible to obtain *RBE* at different fractions of survival as a function of *LET* (see Fig.1.13-b). *RBE* increases as *LET* rises until a peak around 100-200 keV/ μ m. After that, the value decreases due to overkilling effects, which means that the dose is high enough to obtain the death of the cells, regardless of the type of radiation. Additionally, the maximum values of *RBE* for high *LET* particles correspond to the *LET* at the end of their range, namely the Bragg Peak [43].

1.4.4 OER

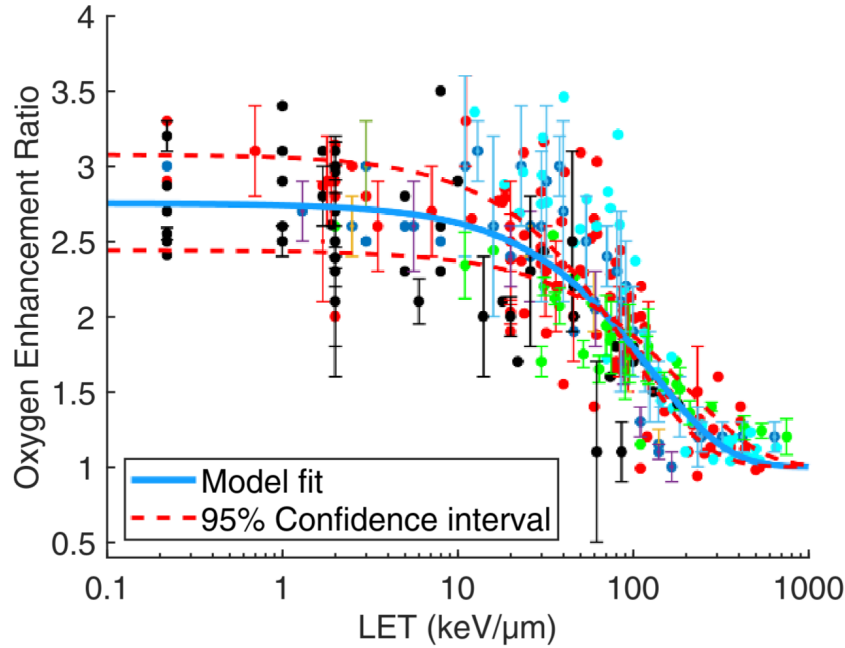


Figure 1.14: *OER* measurements of different mammalian and human cell lines and fit. From [44].

In conventional radiotherapy, the presence of oxygen around the tumor has been demonstrated to boost cell kill, by releasing free radicals as indirect damage (Sec. 1.4.2), which brings important benefits [45]. Thus, it represents an important parameter to investigate about treatment response in radiotherapy and PT and it

is generally expressed in terms of *oxygen enhancement ratio* (*OER*), defined as:

$$OER = \frac{D_{hypoxia}}{D_{air}} \Bigg|_{\text{same effect}} \quad (1.38)$$

which represents the ratio of doses in the absence of oxygen (hypoxia) and under highly oxygenated conditions (e.g. in air) to obtain the same biological effects.

Fig. 1.14 shows the variation of *OER* at different *LET* of the radiation. For low *LET* radiation, such as in X-ray radiotherapy or for low energy PT, the enhancement is boosted to a factor of approximately three, decreasing to unity at very high *LET*s. It depends on the type of damages that are generated by the radiation: at low *LET*, indirect damages dominate and it has been demonstrated that oxygen molecules prevent easy chemical repair due to free radicals, resulting in improved kill [46]. For high *LET*, on the contrary, direct damages to DNA increases and the presence of oxygen becomes no longer predominant in the generation of biological effects.

This property has important biological implications: while many tumors have hypoxic areas that are resistant to traditional photon therapy, charged particle therapy can overcome this resistance [47].

1.4.5 Equivalent Dose

In the field of PT, the *RBE* of different radiation types is incorporated in the concept of *Equivalent Dose* D_{eq} , in order to assess the biological effectiveness of the radiation delivered to patients. It is defined as follows:

$$D_{eq} = \sum_R w_R D_T, \quad (1.39)$$

where R are the types of radiation, D is the absorbed dose (see Eq.1.33) received by an organ or tissue T and w_R is the radiation weighting factor. The latest accepted values of w_R by the *International Commission on Radiological Protection* (ICRP) uses RBE values to derive standard w_R factors and are reported in Tab.1.1 [48].

A value of w_R higher than 1 means that the radiation type R is more effective than the reference one (X and γ rays).

Since every organ or tissue has a different biological response to irradiation, the *Effective Dose* D_{eff} is introduced to account for uniform or non-uniform body expositions to radiation:

$$D_{eff} = \sum_T w_T D_{eq}, \quad (1.40)$$

where w_T are weighting factors of specific tissues or organs T and D_{eq} is the Equivalent Dose given by a radiation type. The w_T factors are based on epidemiological

Radiation type	w_R
X and γ rays	1
Electrons and muons	1
Protons and charged pions	2
α particles and heavy ions	20
Neutrons	2 to 20

Table 1.1: Radiation weighting factors for different radiation types.

studies on tumor induction in exposed populations and are fractionated in such a way that the total sum in a full-body exposition is equal to 1 (see Tab.1.2 [48]).

Tissue	w_T	$\sum w_T$
Red bone marrow, colon, lungs, stomach	0.12	0.72
Breast, remaining tissues *	0.12	
Gonads	0.08	0.08
Bladder, esophagus, liver, thyroid	0.04	0.16
Bone surface, brain, salivary glands, skin	0.01	0.04
Total		1.00

Table 1.2: Tissue weighting factors for different tissue types.

*Remaining tissues: adrenal glands, extrathoracic region, gallbladder, heart, kidneys, lymph nodes, oral mucosa, skeletal muscle, pancreas, prostate (σ), small intestine, spleen, thymus, uterus/cervix (φ).

Both D_{eq} and D_{eff} are expressed in sievert (Sv), to highlight their biological nature in contrast to the physical one used for D , measured in gray (Gy). These are relative quantities that relate biological effectiveness to physical dose. For instance, an exposure of 1 Sv from a given radiation type produces the same biological effect as 1 Gy of the reference radiation (usually X-rays or γ rays).

In the field of SR, the choice of a uniform w_R value of 20 for all types and energy levels of heavy charged particles is considered a too conservative estimate. A more accurate and realistic approach is adopted replacing w_R by the *Quality Factor* $Q_{ICRP}(L)$, introduced by ICRP [49] to reflect the *RBE* of different *LET* radiation, where L is the unrestricted *LET* in water, defined as:

$$Q_{ICRP}(L) = \begin{cases} 1, & L < 10 \text{ keV } \mu\text{m}^{-1} \\ 0.32L - 2.210, & 10 \text{ keV } \mu\text{m}^{-1} \leq L \leq 100 \text{ keV } \mu\text{m}^{-1} \\ \frac{300}{\sqrt{L}}, & L > 100 \text{ keV } \mu\text{m}^{-1} \end{cases}. \quad (1.41)$$

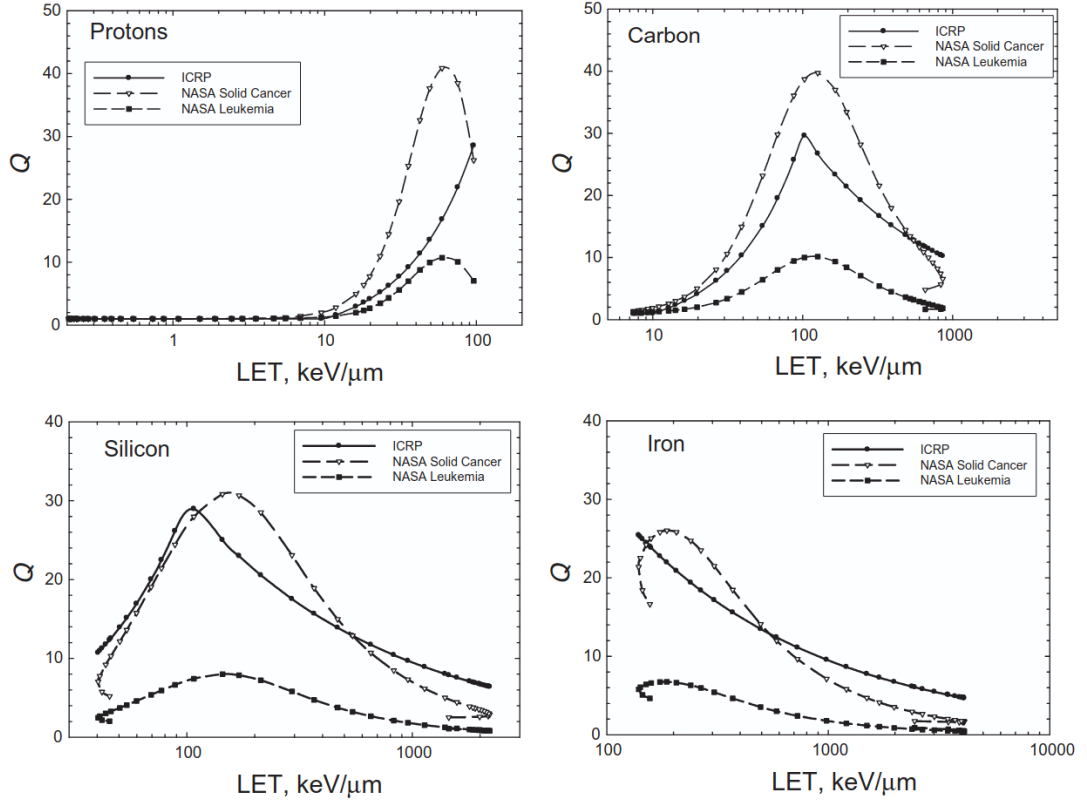


Figure 1.15: Quality factor as a function of LET for H, C, Si and Fe charge particles as defined by ICRP ($Q_{ICRP}(L)$) and by NASA ($Q_{NASA}(L)$) for leukemia and solid tumors. From [51].

NASA has introduced new models in which the quality factor $Q_{NASA}(L)$ is also redefined in terms of the energy E and the atomic number Z of the particle [50], distinguishing between leukemia and solid cancers. The different definitions of $Q(L)$ are reported for H, C, Si and Fe in Fig. 1.15.

For each organ or tissue T, a tissue-mean radiation quality factor Q_T can be calculated considering:

$$Q_T = \frac{1}{m_T D_T} \int_{m_T} \int_L Q(L) D_L dL dm \quad (1.42)$$

where m_T , D_T are respectively the total mass and total dose absorbed by the organ or tissue T, $Q(L)$ is the quality factor introduced before and $D_L = dD/dL$ is the distribution of D in L (for charged particles in water) at the point of interest in the tissue or organ.

1.5 Final remarks

In summary, both PT and SR rely on a detailed understanding of how charged particles interact with matter. While the Bethe-Bloch formula (Eq. 1.2) describes their energy loss, factors like energy straggling (Sec. 1.1.2) and multiple Coulomb scattering (Sec. 1.1.4) refine the description. Furthermore, nuclear interactions and subsequent fragmentation of particles (Sec. 1.3.2) significantly impact the dose profile.

Biologically, LET dictates the density of ionization (Eq. 1.35) and influences the RBE (Eq. 1.36) of different particles. Concepts like Equivalent Dose D_{eq} and the Quality Factor $Q(L)$ (Sec. 1.4.5) are crucial for assessing the biological impact and risks in both PT and SR scenarios.

These intertwined physical and biological aspects are essential for the development and refinement of PT and SR, which are introduced in the following chapter.

Chapter 2

Applications

2.1 Charged Particle Therapy

2.1.1 General aspects

Charged Particle Therapy (PT) is an advanced form of radiotherapy that employs charged nuclear particles (from protons to light ions up to ^{16}O) to treat deep-seated tumors.

The main advantage over conventional X-Ray radiotherapy (RT) is given by the favorable Depth-Dose profile (see Sec.1.3). As Fig. 2.1-a refers, photons deliver a relatively low dose in the entrance channel of the medium, increasing rapidly until a maximum (build-up region), whose depth depends on the energy of the beam. That's mainly due to excitation - and relative δ rays - generated by photon interactions. After the peak, the dose decreases exponentially until reaching the exit point [52]. On the other hand, charged particles are characterized by a small dose in the entrance channel until they release a maximum at the end of the range, the Bragg Peak (BP) (see Sec. 1.3.1).

In terms of treatment, this assures that the same radiation dose to the tumor (supposedly at depth reachable by the BP for charged particles) can be achieved by a lower integral dose to the healthy tissues surrounding it.

Despite the large volume of the tumor with irregular shapes, the effectiveness of PT guarantees a uniform dose distribution over it through the Spread Out Bragg Peak (SOBP). In fact, as shown in Fig. 2.1-b, varying the energy of the beam, it is possible to broaden the BP into a plateau, overlapping precisely the entire depth of the cancer region. This is particularly important in complex cases where tumors are adjacent to critical structures, called *Organs at Risks* (OAR); thanks to these features, beams from only a few directions are needed for PT to treat a tumor, while cross-fires from many different angles are required for conventional RT to avoid them, as shown in Fig. 2.2.

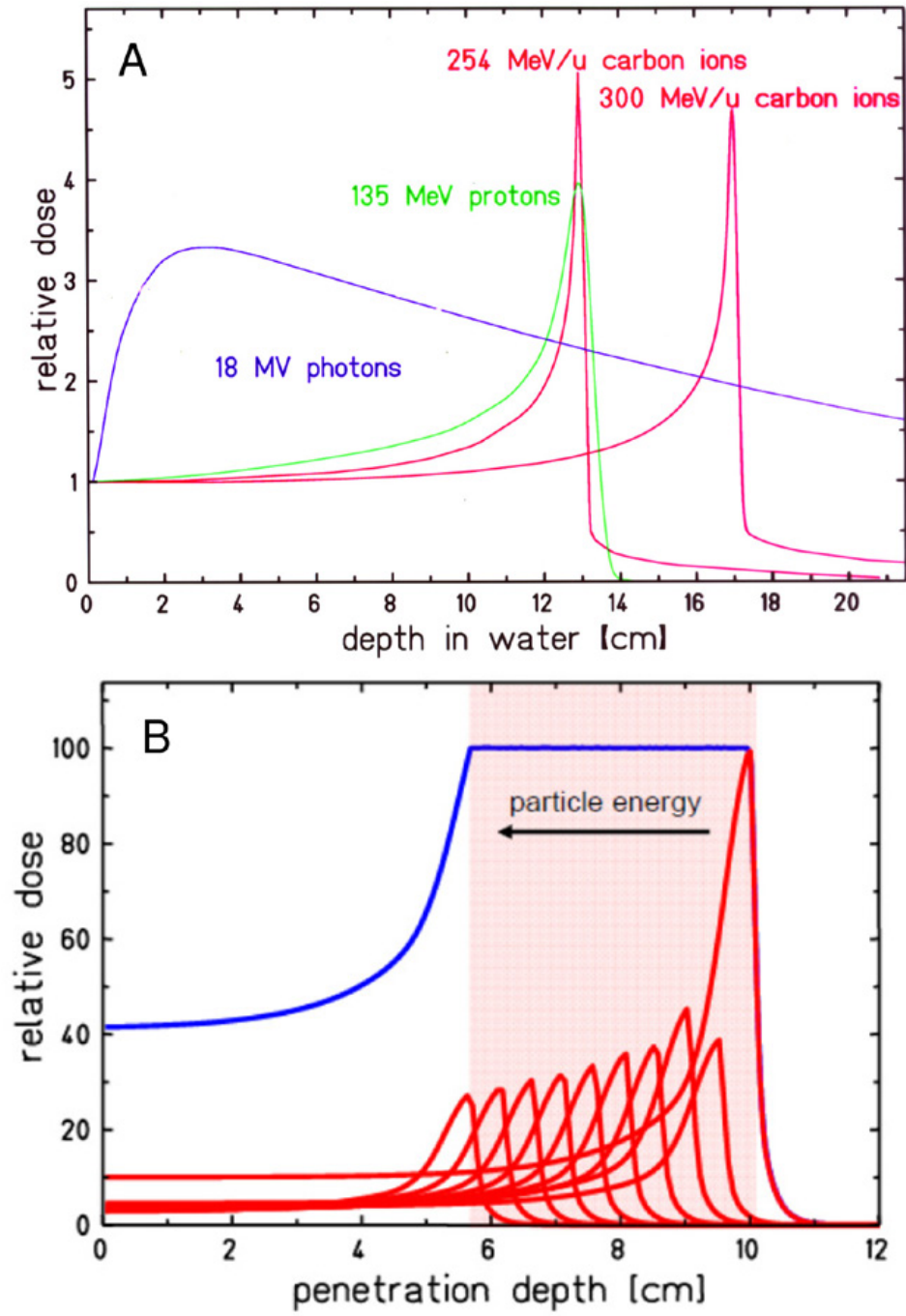


Figure 2.1: a) Depth-Dose profile of photons for RT, protons and carbon ions for PT. b) SOBP of a proton beam to cover a target between 6 and 10 cm of depth. Figures from GSI Helmholtz Center library, Darmstadt, Germany

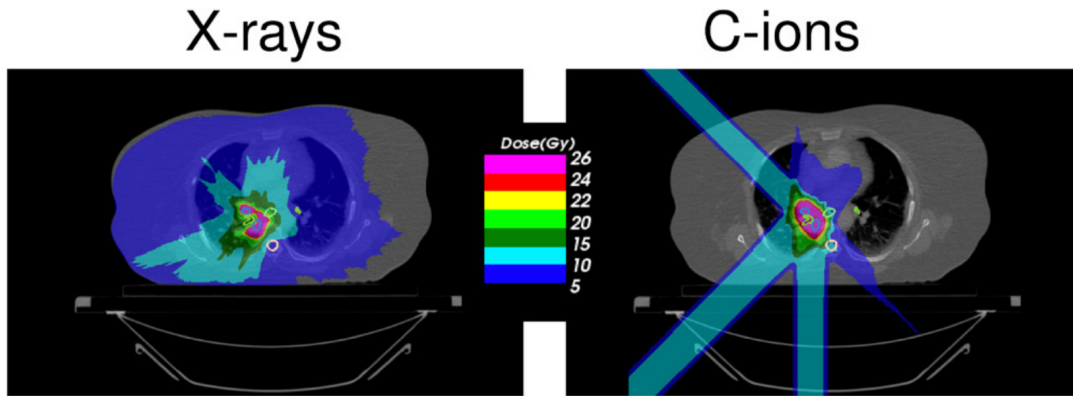


Figure 2.2: Treatment of a lung cancer by stereotactic body radiation therapy (left) or carbon ions (right). The plans are designed for a treatment with high-dose single-fraction. Only 3 beam directions can be used with particles with a higher sparing of healthy tissues. From [21].

2.1.2 State of the art

Particle therapy has a rich history dating back to the mid-20th century. The concept of using charged particles for cancer treatment was first proposed by physicist Robert R. Wilson in 1946, while the first proton therapy treatments were conducted in the 1950s at institutions like Lawrence Berkeley National Laboratory (LBNL) in the United States and the Svedberg Laboratory (TSL) in Sweden [54] using particle physics accelerators, while dedicated proton centers started to spread out from 1990s. Carbon ion therapy began in 1977 at LBNL and gained prominence with the initiation of clinical trials at the National Institute of Radiological Sciences (NIRS) in Japan in 1994 [55].

Globally, particle therapy has expanded to include 17 carbon ion centers and over 100 proton facilities, with Europe alone projected to reach 50 centers by 2026 [53]. The operating PT centers and the ones under construction are reported in Fig. 2.3. The CATANA proton therapy center (Catania, Italy) was the first Italian facility to utilize proton beams for treating ocular melanomas and played a pivotal role in initiating proton therapy studies in the country. The Trento Proton Therapy Centre (Trento, Italy) began its clinical activities in October 2014 and is equipped with two gantry rooms and a research area. National Center for Oncological Hadrontherapy (CNAO) in Pavia, Italy offers both proton and carbon ion therapy using a synchrotron. An expansion of the infrastructure will include a new proton accelerator with a rotating gantry and a compact neutron accelerator to start Boron Neutron Capture Therapy (BNCT) [56] in the next few years. Recently, a proton center was opened at the European Institute of Oncology (IEO) in

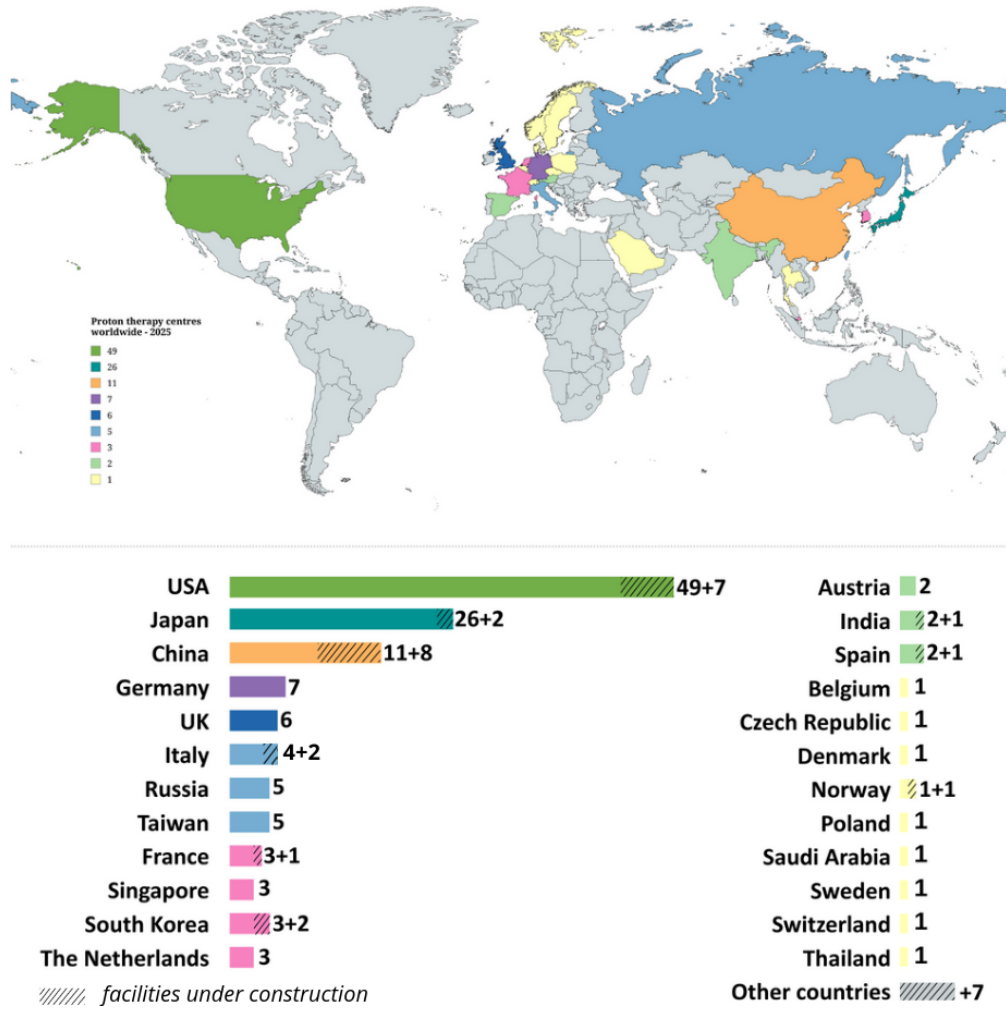


Figure 2.3: Operating worldwide particle therapy centers, based on PTCOG data [53].

Milan, Italy while two new ones are under construction in Rozzano (Milan, Italy) and Aviano (Udine, Italy), making Italy a key player in the global landscape of particle therapy.

As well as CNAO, other important centers that combine physics, biology, and medicine to improve the efficacy and safety of particle therapy are the German Heidelberg Ion Beam Therapy Center (HIT) in Heidelberg and the Helmholtz Centre for Heavy Ion Research (GSI) in Darmstadt. In CNAO, GSI and HIT several data taking sessions were carried out for the FOOT experiment.

2.1.3 Ion sources

Ion sources of p, ^4He , ^{12}C and ^{16}O are designed for current or future PT treatments and what to use in a specific case is chosen according to physical and biological reasons.

In [57], the depth dose distributions of beams of p, He, C and O were studied, changing their range from 2.5 cm to 32.5 cm. Fig. 2.4-a, in particular, shows the distributions for protons. Analogously, the ones of the other particles were obtained tuning the energy: since the stopping power $S \propto Z^2$ (see Eq. 1.2), heavier ions need higher energy to reach the same deep positions.

The longitudinal spread is a consequence of the energy straggling (Sec. 1.1.2) and it was shown that its variance inversely depends on the mass of the ion $\sigma_r \propto 1/\sqrt{m}$ (Eq. 1.23). As a consequence of that, for example, at the energy of 100 MeV/u the straggling goes from 15 mm for protons down to 5 mm for ^{16}O . In particular, the peak width at different ranges, which is also related to the longitudinal straggling, decreases for heavier ions (Fig. 2.4-b) with the consequent higher dose release (so biological effectiveness) around the BP.

The lateral spread is instead due to MCS, where the variance proportionality $\theta_0 \propto Z/pv$ (Eq. 1.26) holds. In Fig. 2.4-c, the relative dose is presented as a function of the lateral depth: supposing the same range, it is larger for lighter particles. For larger distances, the Gaussian approximation (see Sec. 1.1.4) is no longer valid because beam contamination and nuclear scattering increase their impact [58].

From these considerations, ^{16}O seems the most promising species that should cover all the PT treatments since the longitudinal and lateral spreads are the lowest, but another critical aspect has not been introduced yet: the nuclear fragmentation. As already mentioned in Sec. 1.3.2, projectile fragments are generated via nuclear interactions for ions heavier than protons, causing the release of energy in the tail after the BP. Further to this point, Fig. 2.4-d shows what is the impact of the tail after BP for the different species compared at the same range, going from 2% for ^4He to more than 10% for ^{16}O of the peak dose. Along with complexity in beam control and operation costs, fragmentation is also the main reason why ions heavier than oxygen are not commonly used in particle therapy [59].

The overall discussion leads to the conclusion that there is not an "optimal particle" for treatments but that the decision has to be inspected case by case. In general, protons are preferred for solid tumors close to OAR, due to their precision in energy delivery and lack of projectile fragments. They are also widely used for pediatric tumors, which helps to reduce the risk of long-term side effects. Carbon ions have an increased biological effectiveness due to the high *LET* around the BP,

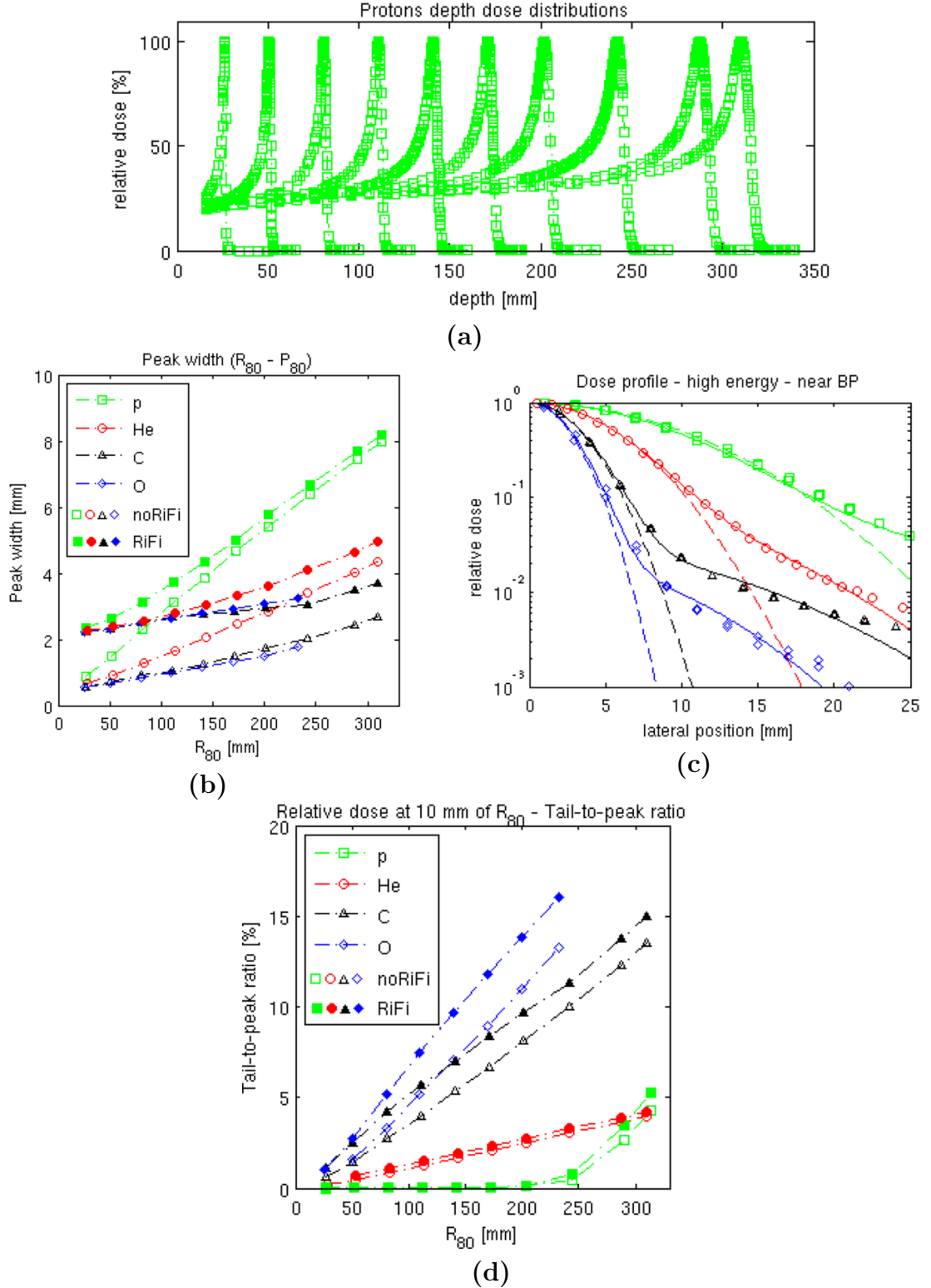


Figure 2.4: a) Depth dose distributions results for proton. Analogously, He, C, O beams were used to obtain BP with the same ranges, with a modulating ripple filter (RiFi) and without (noRiFi). b) Peak-widths of the depth dose distributions as a function of R_{80} . R_{80} (P_{80}) is the depth on the distal (proximal) side of the Bragg peak where the dose falls to 80% of the peak dose. c) Lateral dose distribution. d) Tail-to-peak ratio of different ion beams as a function of R_{80} . From [57].

making them particularly effective against radio-resistant tumors. Helium ions have a smaller lateral scattering than protons, with improved dose distribution and a lower fragmentation than carbon ions, being a good compromise between the two. Finally, Oxygen ions, despite the non negligible fragmentation, seem to be particularly beneficial for treating hypoxic tumors (see Sec. 1.4.4).

Protons and carbon ions are currently the most commonly used particles in cancer treatment. Helium is primarily being explored and used as a beam at the forefront of research and treatment at HIT, while oxygen ions remain under investigation for potential future therapeutic applications [47].

2.1.4 Range uncertainties

PT demonstrates comparable or superior effectiveness to conventional photon radiotherapy in specific clinical scenarios, particularly in reducing severe side effects and improving outcomes for certain cancer types [60][61]. However, the full potential of PT is constrained by its susceptibility to range uncertainties and the consequent impact. In Fig. 2.5, two schematic treatment plans for a brain tumor are shown, where conventional RT and PT are used, respectively. Despite the important PT advantages of dose conformity already introduced, range uncertainties are very harmful to PT since, considering the steep dose gradient at the end of the BP, they can cause an acute release of dose in OAR (e.g. brainstem). For radiotherapy, on the other hand, the exponentially decreasing dose gradient is negligibly affected by them.

Thus, range uncertainties are incorporated as systematics in PT treatment plans, putting additional margins in the defined SOBP of 3.5% of the beam range plus a constant offset in mm, which is relatively conservative and prevents us from exploiting the full potential of PT [64].

The main sources of range uncertainties in clinical treatments are reported in Tab. 2.1, ordered according to their dependence on dose calculation. Among them, patient setup depends on clinical workflow, and any misalignment in the treatment room can cause range discrepancy due to tissue density variations [65]. Even with perfect positioning, range uncertainties arise from internal changes such as weight fluctuations, sinus filling, tumor shrinkage between treatments, or organ motion during a fraction [66].

Range degradation is strictly connected to the longitudinal (Sec. 1.1.3) and lateral (Sec. 1.1.4) spread of the charged particle crossing the material and represents the main physical aspect that has to be handled. A prerequisite for treatment planning is the identification and delineation of the target volume and OAR nearby, which is generally supported by Computed Tomography (CT) imaging. However, CT provides images in Hounsfield Units (HU), which reflect tissue density, but they have to be converted into terms of the stopping power (SP) of tissues to tune

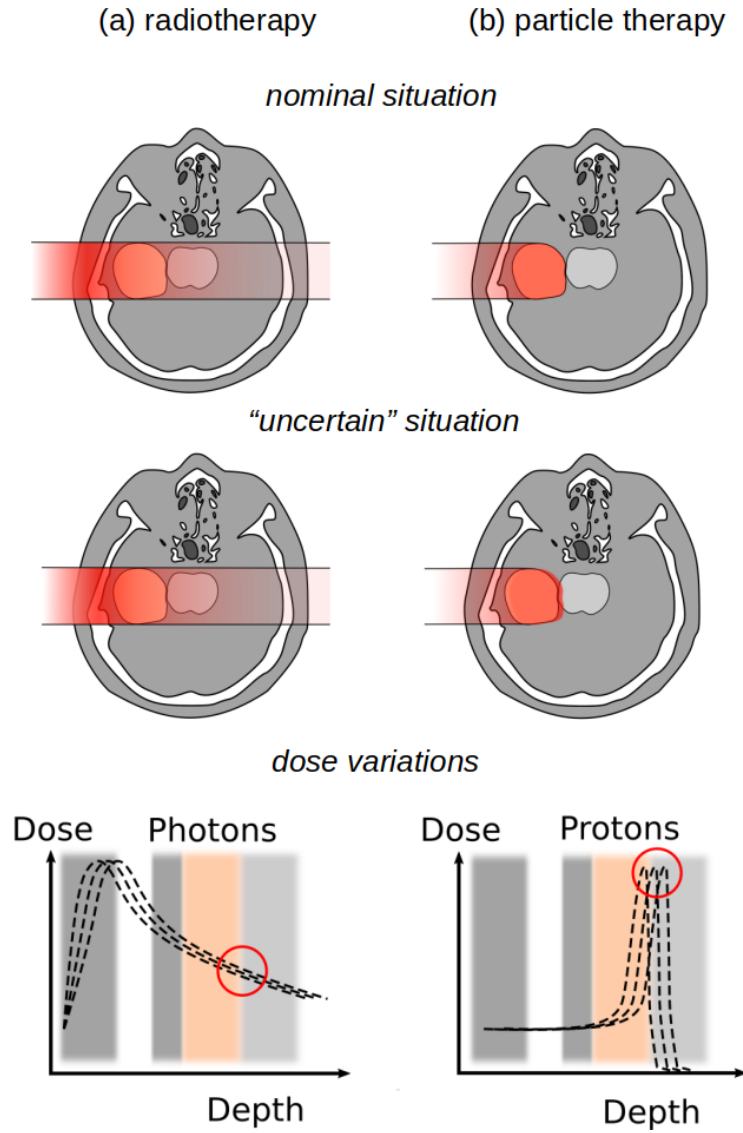


Figure 2.5: The impact of range uncertainties in conventional radiotherapy (a) and in PT (b) and the consequent differences in dose distribution. Illustration from [62], adaptation of [63].

the beam in the right way. Such a process involves uncertainties inherent to the CT imaging, like calibration and resolution [67] and the conversion itself [68]. To mitigate them, researchers are exploring alternative imaging modalities like ion CT, which works directly in terms of SP. The latter attempt was developed, in particular, in the frame of the SIRMIO project [69], where I actively contributed to the development of a proton CT scanner using silicon pixel detectors. The de-

Source of range uncertainty in the patient	Range uncertainty (% or mm)
Independent of dose calculation:	
Measurement uncertainty in water for commissioning	± 0.3 mm
Compensator design	± 0.2 mm
Beam reproducibility	± 0.2 mm
Patient setup	± 0.7 mm
Dose calculation:	
Biology (always positive)	$+ \sim 0.8\%$
Mean excitation energy (I -values) in tissues	$\pm 1.5\%$
Range degradation; complex inhomogeneities (negative)	-0.7%
Range degradation; local lateral inhomogeneities	$\pm 2.5\%$
CT imaging and calibration	$\pm 0.5\%$
CT conversion to tissue (excluding I -values)	$\pm 0.5\%$
CT grid size	$\pm 0.3\%$
Total (excluding biology and lateral inhomogeneities)	$2.7\% + 1.2$ mm
Total (excluding biology)	$4.6\% + 1.2$ mm

Table 2.1: Sources of range uncertainty and their estimation

scription of the setup and the main performance results are reported in Appendix A.

To reduce the source of uncertainties, imaging devices could also be used to monitor the treatment while it is delivered, exploiting the secondary radiation (positrons, prompt gamma, nuclear fragments) generated by the beam mainly during the interaction with the tumoral region, in a promising field of research called *in vivo range verification* [70].

2.1.5 The contribution of nuclear physics to PT

A Treatment Planning System (TPS) is an analytical software used in radiation therapy to optimize the delivery of ion beam radiation for cancer patients, to maximize tumor control and minimize damage to surrounding healthy tissues. After the target volume definition using imaging techniques, dose optimization can be performed.

The dose $D(x)$ provided by a pencil beam (PB) of a given fluence Φ is generally calculated as follows:

$$D(x) = \frac{1}{\rho(r)} \sum_{\text{PB}} \Phi(\text{PB}) \cdot g(r, E, z) \cdot d(E, z) \quad (2.1)$$

where the depth d and the lateral components g depend on the actual algorithm provided by the commercial TPS [71]. All the physical processes of interaction

explained in Chapter 1, which are fundamental for dose optimization, are introduced using deterministic methods in clinical settings due to their ease of integration and established clinical validation. Monte Carlo (MC) simulation codes are thus fundamental to verify and correct the prediction from deterministic TPS but, while electromagnetic interactions can be simulated accurately, the nuclear fragmentation processes present large uncertainties, forcing the TPS to operate in a conservative approach [72].

Fig. 2.6 shows the prediction of dose distributions for carbon ions with a deep narrow SOBP (a) and a wide superficial SOBP (b) of 2 Gy (RBE) [73]. They are obtained using TRiP TPS software [74] which is based on SHIELD-HIT [75], an MC code which simulates the transport of particles through various media, such as biological tissues. The variations in the modeling of nuclear interactions (from A to F of Fig. 2.6) in the TPS software lead to substantial differences in the dose distribution, with variations up to 10%. The clinical tolerance commonly used by certified particle therapy software of TPS is generally about 3% dose difference for plan validation and quality assurance [76]. Thus, refining nuclear cross section measurements will limit the discrepancies between simulations and what is required for TPS accuracy.

Dosimetry in radiotherapy is generally measured in terms of water-equivalent dose, while tissue conversion is only provided by MC calculations. These are not yet tuned to consider also the nuclear contribution of nuclear-stopping power and for ions heavier than protons this approximation is no longer accurate [77].

For PT plans, a constant value of $RBE = 1.1$ is generally used as a standardized approach to decouple the process from biological effects. Instead, in Carbon ion therapy the TPS are optimized considering also the RBE of the particle, introducing also the biological complexity. Alongside biological uncertainties due to in vitro and in vivo experimental data, RBE model robustness can be increased only with a good knowledge of the produced nuclear fragments. In these specific cases, total charge-changing cross sections are no longer sufficient, but double-differential ones (Sec. 1.2.1) are necessary [78] to improve also the radiobiological models.

The previous considerations are of evident importance even to reduce range uncertainties that affect the full capabilities of PT.

Nuclear interactions of hadron beams at therapeutic energies with nuclei composing human tissues (made mainly of H, O, C) are then fundamental to improve TPS, as a benchmark for MC simulation transport codes coupled with radiobiological models [79]. Differential fragmentation cross section data are poor in the range of interests and new measurements are thus required.

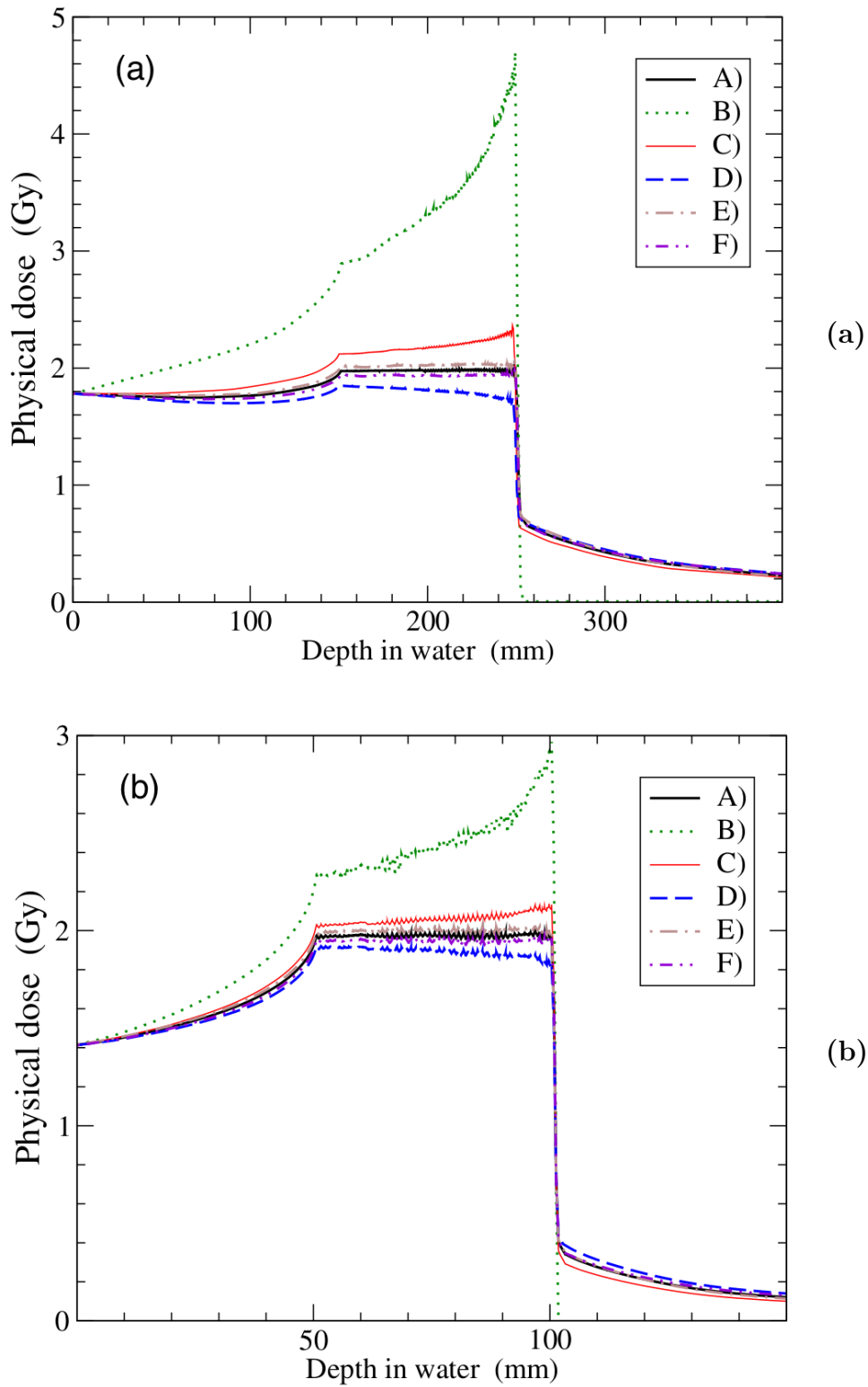


Figure 2.6: Dose distributions for carbon ions in water using the six different cases (A)–(F) of modeling nuclear interactions as specified in [73], optimized to 2 Gy for two different SOBP scenarios.

2.2 Space radiation protection

2.2.1 Space exploration

The current century is characterized by a renewed interest in space exploration since several national space agencies and private companies are aiming at suitable space destinations for humans. For example, NASA is planning a sustainable human presence on the Moon in the 2030s [80] and a first manned mission to Mars in the 2040s for a duration of at least one year, as a combination of scientific, technological, economic, and geopolitical motivations [81].

The humankind evolved on Earth in conditions which are strongly different from the space environment and thus several health problems can be faced by astronauts in long term missions [82]. A microgravity environment can cause physiological issues to the musculoskeletal system as well as immune system dysfunctions [83], while isolation for a dozen weeks can induce serious neurobehavioural difficulties in adaptation and medical problems for what concerns autonomous medical care. Other risks are related to the exposure to space radiation (SR) which, according to its nature, can cause acute or late biological effects resulting in carcinogenesis or brain and cardiovascular damage. According to the Human Research Roadmap of NASA, while countermeasures are already considered for psychological and physiological issues, radiation exposure is one of the main health concerns due to high uncertainty on risk estimates and the lack of effective countermeasures [28]. Moreover, it also represents a critical challenge for the functionality of electronic systems during space missions [84]. Many solutions are being examined, starting from a deep knowledge of the sources of SR.

2.2.2 Space environment

The radiation environment in the Solar System is a compound of charged particles in a wide energy range originating from solar, galactic and extragalactic sources. What represents most an issue for health risks and electronic hazard in long term missions is radiation of Solar Particle Events (SPEs) and Galactic Cosmic Rays (GCRs).

Among electromagnetic radiation, charged particles are continuously emitted by the Sun. Low energy protons and electrons (between 100 eV and 3.5 keV) propagate constantly through the so-called *solar wind* with fluxes between 10^{10} to 10^{12} particles $\text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$ but, since they can be stopped within few μm in skin or shields, have negligible concern for astronauts.

However, for what concerns SP, the most dangerous activity of the sun is the sudden appearance of SPEs, where large currents and moving magnetic fields ac-

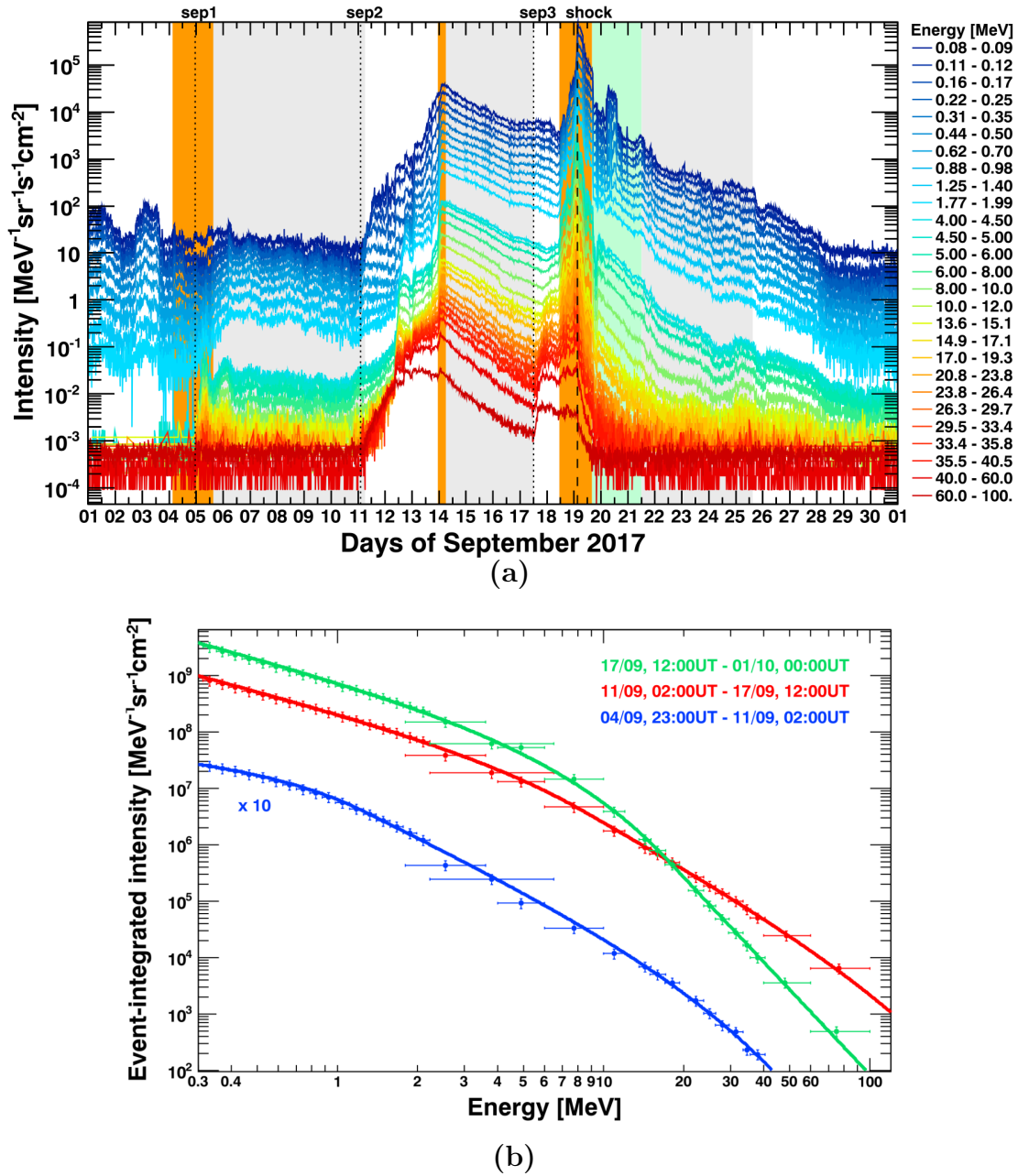


Figure 2.7: a) Temporal profiles of proton intensities measured by the Solar Electron and Proton Telescope (SEPT), Low Energy Telescope (LET), and High Energy Telescope (HET) instruments onboard the Solar Terrestrial Relations Observatory-Ahead (STEREO-A) during September 2017. The vertical dotted lines mark the onset of the SPE and the time of the shocks, respectively. The green, orange, and gray areas indicate the periods of the interplanetary coronal mass ejections, magnetic cloud, and high speed streams, respectively. b) Time-integrated energy spectra of the 4, 11, and 17 September 2017 solar energetic particle events (blue, red, and green points, respectively) measured by STEREO-A. From [85].

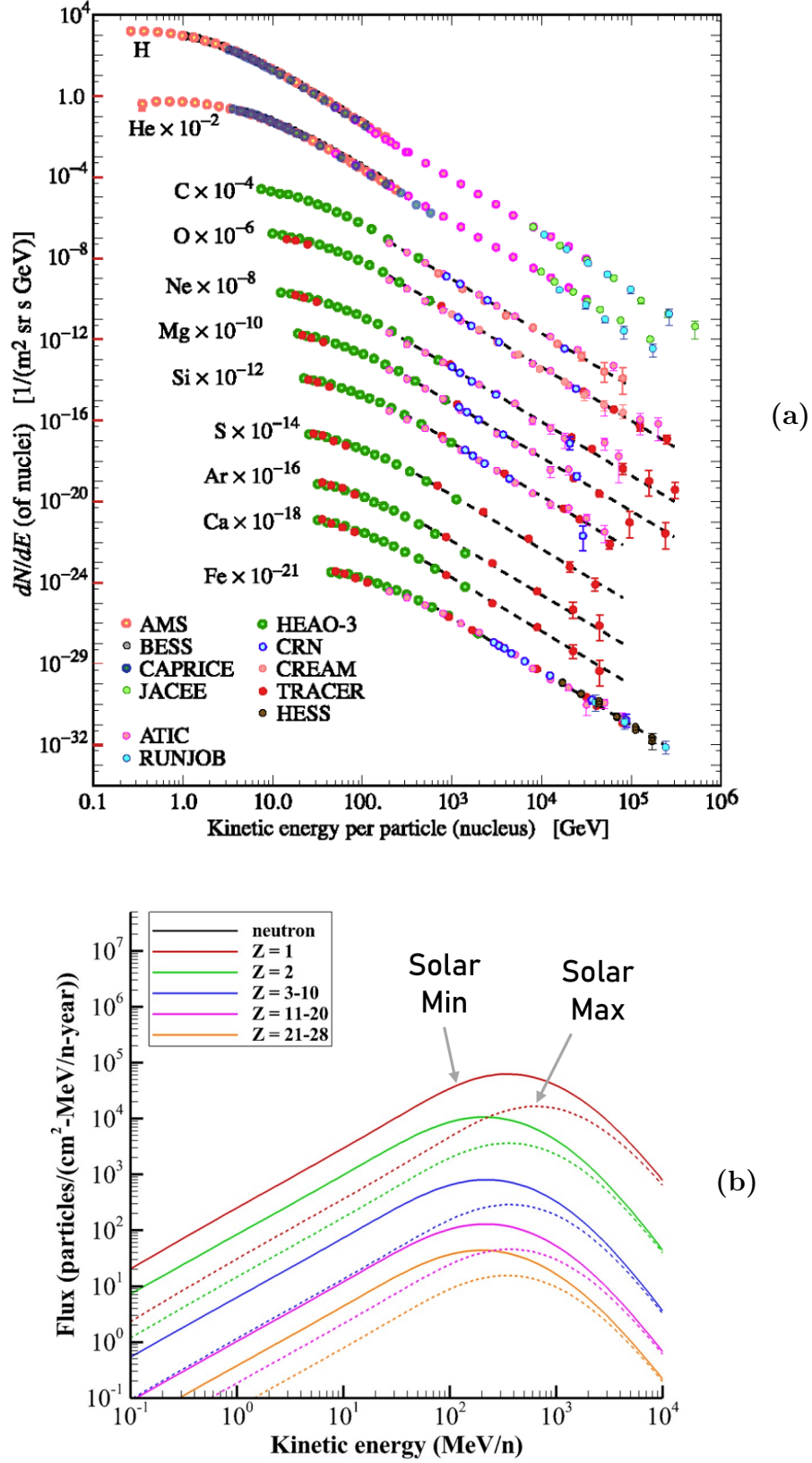


Figure 2.8: a) Fluxes of nuclei of the primary cosmic radiation in particles per energy-per-nucleus are plotted vs energy-per-nucleus using data from [86]. b) Differential energy spectra of GCR during solar minimum and maximum phases. From [87].

celerate coronal particles [88] during the so-called *solar flares*. In this kind of event, mainly protons (98%) and heavier ions are accelerated up to several GeV and emitted in the planetary space. They can last from several hours to days, with an observed frequency of one per month which increases during the maximum phase of the Sun cycle. One of the latest extreme solar activities occurred in September 2017 during the late decay phase of the solar cycle (Fig. 2.7-a), with three SPEs registered at Earth (Fig. 2.7-b). In particular, the event of 11 September was so intense that fluxes of protons could be observed as Ground Level Events (GLE), a special subset of SPE with enough energy to generate effects measurable even at Earth's surface [85].

GCRs are charged particles accelerated outside the Solar System due to high energy phenomena such as supernova explosions, neutron stars, pulsars or other astrophysical sources [89]. They are distributed isotropically on Earth, with abundances made of 2% of electrons and 98% of baryons. Among them 85% is composed of protons, 14 % of helium and the other high charge Z and high energy E (HZE) heavier nuclei represent the remaining 1%.

Their fluxes and energies span several orders of magnitude (see Fig. 2.8-a). In particular, up to energies of $10\text{-}10^2$ GeV, their spectrum is modulated by the solar cycle (see Fig. 2.8-b) with a fluctuation roughly around 15%. The GCR flux is lower during solar maximum due to increased solar magnetic activity, which more effectively shields the inner heliosphere from low energy charged particles.

Another radiation which is modulated by the solar cycle consists of protons up to 600 MeV and electrons up to 7 MeV trapped in the Earth's magnetic sphere (the so-called *Van Allen belts*). They are relevant for Low Energy Orbit (LEO) missions but of low relevance for risks and radiation protection purposes.

2.2.3 Doses and Risks of exposure

The equivalent dose introduced in Sec. 1.4.5 is used in space to assess the biological effects of radiation on astronauts. Taking into account the different types of radiation introduced in Sec. 2.2.2, Table 2.2 schematizes the main dose exposures in different scenarios. Earth has a strong magnetic field and a dense atmosphere (1.2 kg/m^3) that effectively shields its surface from most cosmic radiation. Leaving the planet at the ground level, astronauts on the International Space Station (ISS) are still quite protected by the magnetosphere and by the spacecraft structure (with an areal density of 20 g/cm^2 for shielding quantification). On the Moon, the weak magnetic field and no atmosphere aren't able to absorb or scatter radiation, leaving its surface totally exposed, while on Mars a thin atmosphere (2% of Earth's value) lets a shielding similar to that on ISS.

Exposure scenario	D_{eq} rate
Earth [90]	3.3 mSv year ⁻¹
ISS [91]	0.2 – 0.5 mSv day ⁻¹
Moon [92]	1.4 mSv day ⁻¹
Mars [93]	0.6 mSv day ⁻¹

Table 2.2: Equivalent dose rates for different exposure scenarios.

NASA has developed a risk-based strategy for setting radiation exposure which also evaluates the uncertainties associated with risk assessments. A simplified form of the risk equation for solid cancer given by a specific condition ξ is the following [94]:

$$\text{Risk}_\xi = R_0(\text{age, gender}) \frac{F \cdot L \cdot Q}{DDREF} \left\{ \frac{x_{R_0} x_{\text{phys}} x_Q}{x_{D_R}} \right\}_\xi \quad (2.2)$$

where R_0 is a risk coefficient generally derived from epidemiology data of the atomic-bomb survivors [95], the effective dose is written as the product of the particle fluence (F), LET (L), and quality factor (Q). The Dose and Dose-Rate Effectiveness Factor (DDREF) is introduced generally to distinguish the risk among acute and late exposures, while x_R , x_{phys} , x_{D_R} , and x_Q represents corrections factors in the biophysical model. Fig. 2.9-a provides a qualitative estimate of the uncertainties associated with this model. The dominant source of uncertainty arises from the method used to derive radiation quality from radioepidemiological data: specifically, the extrapolation of effects observed on Earth from X-rays, γ -rays, β -particles, and α -particles to those induced by protons and HZE particles in space.

Moreover, Fig. 2.9-b shows the relative contributions of different nuclei from GCRs in flux, dose and equivalent dose according to a simulation behind a thin 5 g cm⁻² aluminium shield in space. Even if the relative abundance of HZE represents only 1% of the total radiation fluence, the equivalent dose is higher than the one for protons because of high Z and Q .

Risk acceptance levels are set at 3% *Risk of Exposure-Induced Death* (REID) for radiation carcinogenesis which, due to the high uncertainties, exceed this value for extended missions to the Moon and Mars up to now [96]. While research experimental programs are ongoing to reduce risk uncertainty on the biological and physical side [97], operational countermeasurements play a crucial role in mitigating the impact of radiation on both human health and electronics reliability. Among them, shielding enables safer and more sustainable space exploration.

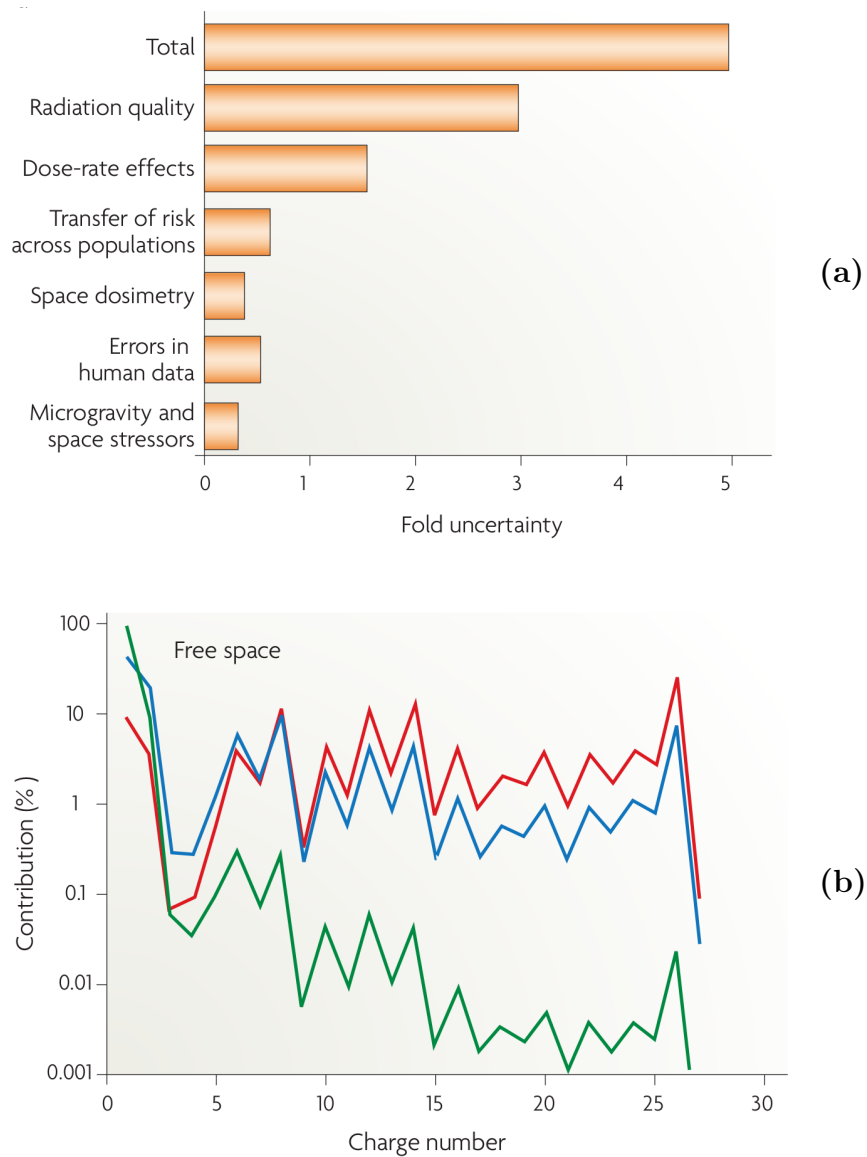


Figure 2.9: a) Estimates of uncertainties in projecting cancer risks for space from terrestrial exposures. Several factors, such as radiation-quality effects, space physics and microgravity, do not contribute on Earth and lead to large increases in risk projections. b) The contribution in fluence (green), dose (blue), and dose equivalent (red) of different nuclei in galactic cosmic radiation. From [96].

2.2.4 Shielding

The general strategy of radiation protection on Earth is based on three main principles:

1. increasing the distance from the radiation source
2. minimizing the time of exposure
3. using radiation absorbers as shielding.

In space, the first aspect is not feasible because GCRs and SPEs are isotropically distributed while the second is problematic because it could contradict the goals of a deep space mission like colonization of the Moon or Mars. However, there are some attempts to find innovative propulsion technologies to limit the transit time of spacecraft in deep space [98].

Thus, a shielding approach is the last suitable in a space environment, with the aim to reduce exposure as much as possible according to the nature of the radiation and the specific spacecraft design.

The best criteria for shielding material selection is based on HZE interaction process with matter and consists of finding the best compromise between electromagnetic energy loss and nuclear fragmentation (see Sec.1).

According to the Bethe-Bloch formula (Eq. 1.2):

$$\frac{dE}{d\chi} [\text{MeVcm}^2\text{g}^{-1}] \propto \frac{Z_t}{A_t} \quad (2.3)$$

where $\frac{dE}{d\chi}$ is the mass stopping power introduced in Eq. 1.6 and represents the stopping power of a particle S per density unit and Z_t and A_t are the atomic and atomic mass numbers of the shielding material, respectively. Since the number of neutrons per unit proton increases with Z , low- Z materials have the highest $\frac{Z_t}{A_t}$ fraction and thus are the best choice for slowing down HZE and stopping them before reaching the hull of the craft.

On the other hand, according to nuclear fragmentation cross section models (Eq. 1.29):

$$\sigma_{mass} [\text{cm}^2\text{g}^{-1}] \propto \frac{N_a \sigma_R}{A_t} \propto A_t^{-1/3} \quad (2.4)$$

where σ_{mass} represents the total reaction cross section per unit of mass, N_a/A_t is the number of targets per unit mass and A_t is the atomic mass number of the shielding material. Therefore, low- Z materials, due to the low A_t , are good candidates even for maximizing nuclear fragmentation processes. In fact, when a HZE particle is not completely absorbed, fragmentation is desirable because there is a reduction of high- LET radiation and biological effectiveness and thus, according

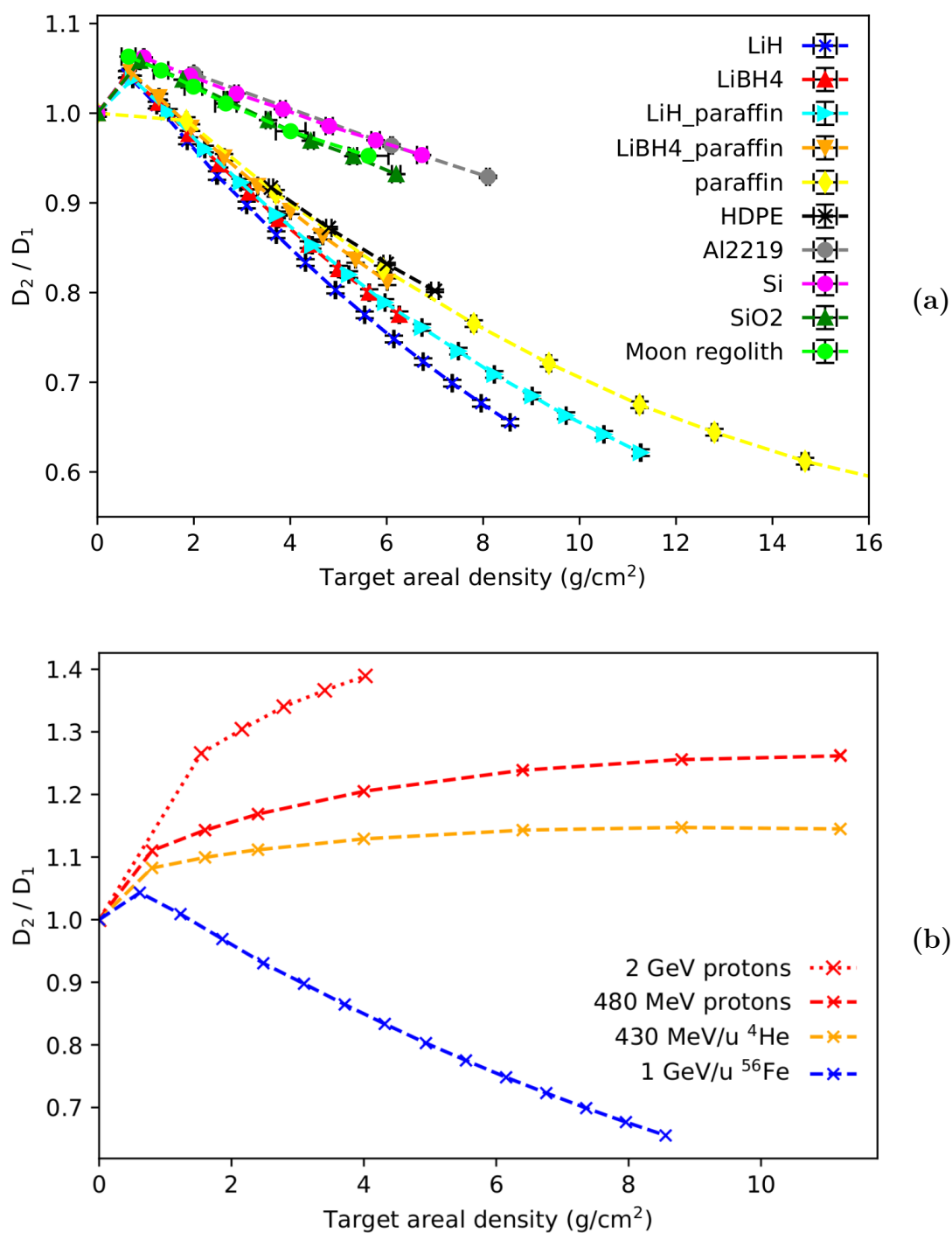


Figure 2.10: a) Dose attenuation of 1 GeV/n ^{56}Fe beam with different materials at different thicknesses. b) Dose attenuation of different beams against a LiH target of different thicknesses. From [99].

to Eq. 1.40, the effective dose of secondary fragments is lower than that of the incoming particle, nevertheless a higher range [59].

According to that, the best candidates are hydrogen-rich materials such as polyethylene which is considered the gold standard for space exploration [100]. In Fig. 2.10-a, the dose ratio of a 1 GeV/n ^{56}Fe beam is reported, crossing several materials of different mass thickness, with measurements done at GSI [99]. In particular, Aluminium, which is the main material of spacecraft, presents the worst shielding capabilities, followed by SiO_2 , the main component of Moon and Mars regolith. Lithium hydrates (LiH compounds) show better performances even than polyethylene (HDPE), confirming that the lowest Z -materials have the highest attenuation power. As well as the composition, engineering considerations about the material strength, temperature resistance, material degradation and flammability need to be done. For this reason, a compound of Lithium hydrates and paraffin with more chemical and mechanical stability was also tested. As shown in Fig. 2.10-b, the shielding power of LiH compounds is however different according to the beam. The absorbed dose decreases exponentially for 1 GeV/n ^{56}Fe beam due to projectile fragmentation processes it undergoes, while a strong dose build-up due to target fragmentation (see Sec. 1.3.2) is present for light particles. This latter condition could be attenuated by raising the mass thickness of the shielding, with the inevitable consequence of weight gain and thus mission costs increase.

The ISS shielding has a nominal thickness on the order of tens g/cm^2 , enable to stop protons up to 100 MeV and thus adequate for trapped radiation and most SPEs shielding. Small area with heavy shield (so called *storm shelter*) higher than 20 g cm^{-2} can also be introduced in crafts against acute SPEs [101]. On the other hand, even according to the previous considerations, shielding against GCRr remains a significant challenge, requiring further investigations into materials and tailored solutions for specific missions.

2.2.5 The contribution of nuclear physics on SR

In order to understand the effects of space radiation on both humans and electronic systems in space missions, transport codes are used, which are predictions of the interaction of space radiation with spacecraft and biological tissues.

A deterministic method called HZETRN [102] is used by NASA for space radio-protection and consists of solving analytically all the processes that a particle can undergo moving in a specific direction. An example of a differential transport equation is the following:

$$\Omega \cdot \nabla \phi_j(\mathbf{x}, \Omega, E) = \sum \int \sigma_{jk}(\Omega, \Omega', E, E') \phi_k(\mathbf{x}, \Omega', E') d\Omega' dE' - \sigma_j(E) \phi_j(\mathbf{x}, \Omega, E) \quad (2.5)$$

where $\phi_j(\mathbf{x}, \Omega, E)$ is the flux density of particles j of direction $\mathbf{x}$ and energy E , and $\phi_k(\mathbf{x}, \Omega', E')$ the flux of particles k of direction Ω' and energy E' . The first term is linked to the cross section $\sigma_{jk}(\Omega, \Omega', E, E')$ which represents the probability of interaction between the projectile particle j and the target one k while the second term is connected to the total cross section $\sigma_j(E)$ for the particle j interaction with the medium. The cross-sections used in the HZETRN code are derived from various sources, including experimental data and theoretical models.

The solution of Eq. 2.5 involves hundreds of multidimensional equations which are usually approximated. To solve complex exposure scenarios, MC transport methods are then preferred, where single particle tracks are simulated step by step between consecutive physics interactions. The step length z is sampled by the probability function:

$$p(z) = \mu(z)e^{-\int_0^z \mu(z')dz'}, \quad (2.6)$$

where μ is a property of the medium proportional to the interaction cross section. The main MC transport methods are based on specific tools such as FLUKA, Geant4 and PHITS, where different algorithms and physical interaction models are used.

In Fig. 2.11, the dose released by a source of GCR after a shielding of Aluminum (top) and Polyethylene (bottom) of different mass thicknesses is calculated using both the deterministic and the MC transport methods introduced before [103]. In both cases, a dose decrease is given due to the absorption of more and more low energy GCR components. Polyethylene shows a higher shielding power thanks to its low-Z composition (see 2.2.4), which limits fragmentation and takes the dose release at almost constant values for higher thicknesses. For Aluminum, after the minimum dose release, there is a rise while the thickness increases. In fact, fragmentation occurs and hadronic showers are produced, generally with enough energy to make particles escape the shielding material and reach astronauts or electronics. In both cases, a variability in the results can be appreciated, in particular for Aluminum where differences go beyond 15%. This emphasizes the need for improved modeling accuracy and experimental validation of nuclear cross sections, the core component of radiation protection shielding.

Double differential nuclear cross section measurements are thus essential in SR as a benchmark of transport code accuracy that could be used for shielding design optimization; however, there are significant gaps in the energy range of interest. On the other hand, they are also fundamental to support stochastic risk models improving the dose based quantities that are used (as DDREF in Eq. 2.2).

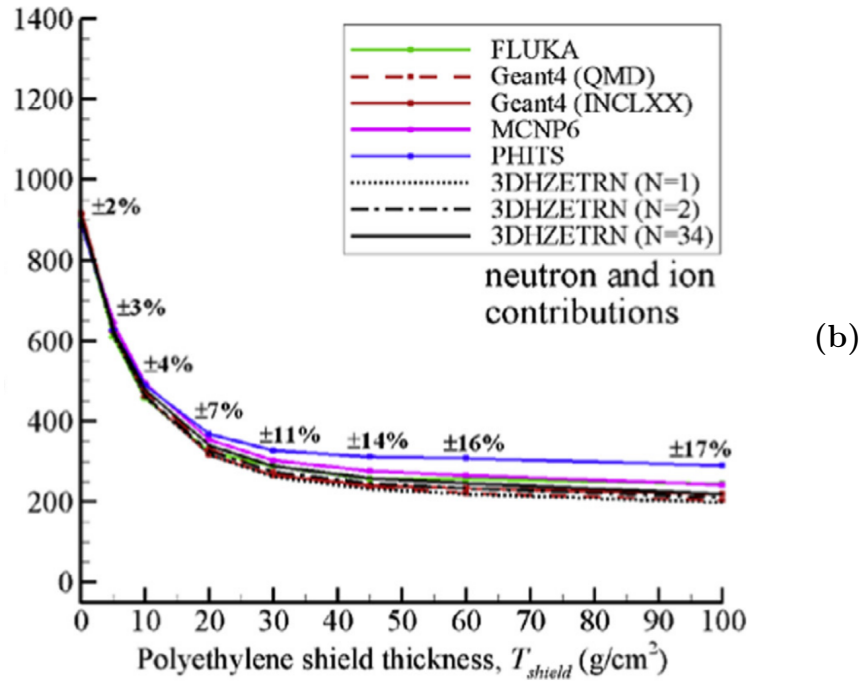
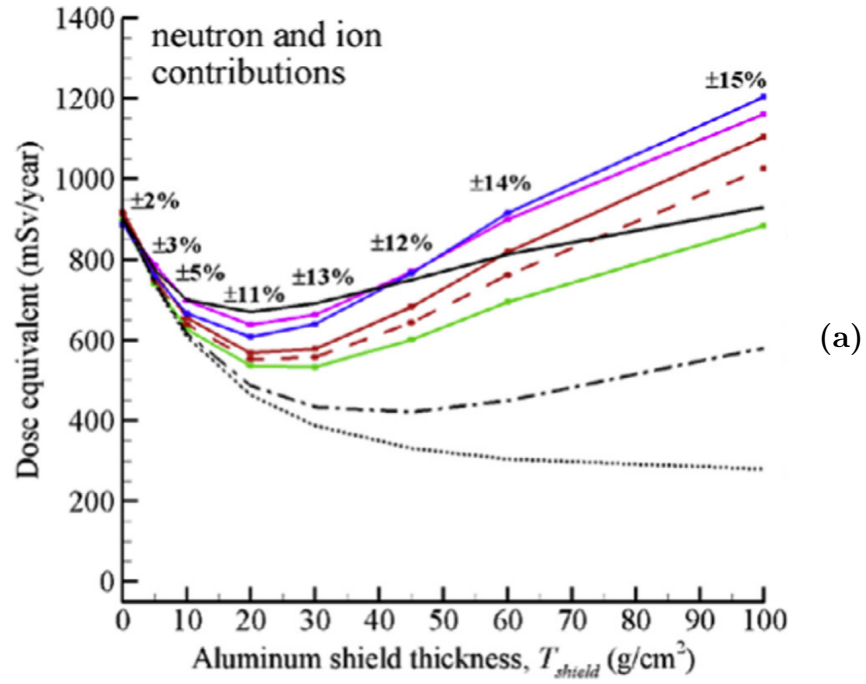


Figure 2.11: Dose equivalent from neutrons and ions as a function of aluminum (a) and polyethylene (b) shield thickness for the full GCR boundary condition. From [103].

2.2.6 Final remarks

Both PT and SR are significantly impacted by nuclear fragmentation. In PT, fragmentation of ions heavier than protons can affect dose distribution and healthy tissues. In SR, fragmentation of HZE interacting with shielding materials influences the type and energy of radiation reaching astronauts or electronics. Accurate modeling of nuclear fragmentation is crucial to improve transport codes in both fields, to optimize TPS in PT or design effective shielding and predict radiation risks in SR.

A significant challenge is the lack of sufficient and precise experimental data on nuclear cross sections and, therefore, new experimental data are needed to improve the accuracy of models and advance both fields.

Chapter 3

The FOOT experiment

The FOOT (FragmentatiOn Of Target) experiment has been conceived to perform double differential cross section measurements of nuclear fragmentation of light particles ($Z \leq 8$) in the energy range between 200 and 800 MeV/u, which is of interest in the fields of PT and SR. The final goal, in particular, is to measure differential cross sections with respect to the kinetic energy ($d\sigma/dE_{\text{kin}}$) for the target fragmentation process with an accuracy better than 10% and double differential cross sections with respect to the kinetic energy and angle emission ($d^2\sigma/d\Omega dE_{\text{kin}}$) for the projectile fragmentation process with an accuracy better than 5% [104].

As described in Sec. 2.1.5, a factor that limits the full exploitation of PT is fragmentation of the beam with the atomic nuclei of patient tissues. Both target and projectile fragments can significantly alter the dose distribution, impacting treatment precision. The relative target fragmentation cross sections of the proton–nucleus (p–N) reaction can be measured by FOOT studying the interaction of ^{12}C and ^{16}O beams at 50–200 MeV/u against C and C_2H_4 targets, exploiting an inverse kinematic approach (see Sec. 3.1). FOOT also performs direct measurements of double differential cross sections of projectile fragments from the interaction of 250 MeV/u ^4He , 400 MeV/u ^{12}C and ^{16}O beams with C, C_2H_4 and PMMA targets. The measurement precision aimed by the experiment is needed to include nuclear fragmentation contributions in TPS within a 3–5% dose accuracy required for radiotherapy applications [105].

In Sec. 2.2.5, it was stated that shielding design optimization as a countermeasure of SR can be achieved by improving the knowledge of fragmentation cross sections of cosmic rays with the spacecraft material. To this end, FOOT will measure direct differential fragmentation cross sections of 700–800 MeV/u ^4He , ^{12}C and ^{16}O beams, mimicking the cosmic rays spectrum for light elements around their

maximum energy, against C, C₂H₄ and PMMA targets. These are low Z materials that are favorable for shielding purposes (see Sec. 2.2.4).

The aforementioned measurements conducted by FOOT also present an opportunity to investigate fundamental nuclear physics research. The experiment, in particular, is well-suited to study alpha clustering, a phenomenon where nuclei are tightly bound in nucleons in substructures resembling alpha particles (⁴He). The theoretical model will be constrained by studying nuclear fragmentations of ¹²C and ¹⁶O beams at the energy range of 150-400 MeV/u, which are intermediate energies beyond the Fermi level characterized by lack of experimental data [106].

3.1 Inverse and Direct kinematics

Detection of target fragments is problematic, as their range doesn't exceed the order of 10-100 μm , causing them to remain confined within the target material. However, if a proton-target interaction produces only target fragments which are not measurable, it is possible instead to study the inverse fragmentation process, using ⁴He, ¹²C, or ¹⁶O beams impinging on hydrogen targets.

What is actually used is a solid hydrogen-rich target such as C₂H₄. This expedient compensates for the inability to use a gaseous or liquid hydrogen target directly due to low reaction rates, safety concerns and logistical complexities, making solid composite targets a more practical and effective choice. The differential cross section measurements of H can then be obtained through a subtraction method between results using C₂H₄ and C targets [107]:

$$\frac{d\sigma}{d\Omega}(\text{H}) = \frac{1}{4} \left(\frac{d\sigma}{d\Omega}(\text{C}_2\text{H}_4) - 2 \frac{d\sigma}{d\Omega}(\text{C}) \right). \quad (3.1)$$

At this point, applying a suitable Lorentz transformation, there will be a reference frame in which the roles of projectile and target are effectively reversed: the X-p process will be inverted in a p-X scenario, where target fragments correspond to the original projectile ones, enabling cross section measurements inaccessible otherwise.

The overall approach (*Inverse Kinematic Approach*) introduces some additional uncertainties which have to be considered but allows for measurements impossible otherwise. As well as precise knowledge of the beam before the interaction, mass reconstruction is needed for the Lorentz transformation. Thus, isotopic identification, along with charge reconstruction, is an important aspect to apply this approach within the FOOT setup.

As the requirements are fulfilled for inverse kinematics, direct cross section data are implicitly guaranteed.

3.1.1 General design

The FOOT setup has the aim to detect, track and identify all the charged fragments produced in beam-target collisions in order to reach the final cross section goal. The experimental designs need to face several constraints, primarily driven by its physics goals and the practicalities of conducting such measurements.

Fig. 3.1 shows the angular and the kinetic energy distribution of all the parti-

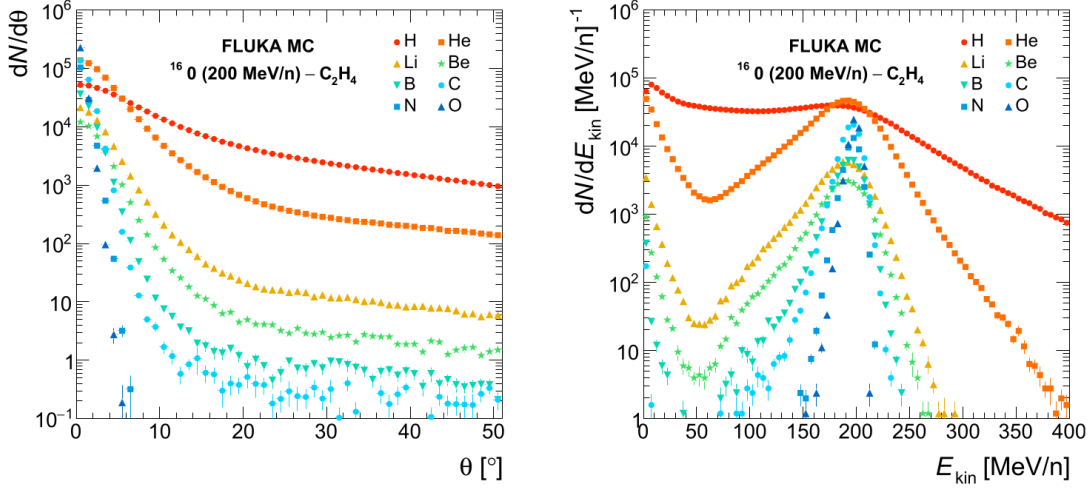


Figure 3.1: MC calculation of the angular (Left) and kinetic energy (Right) distributions of different fragments produced by a 200 MeV/u ^{16}O beam impinging on a C_2H_4 target.

cles ($Z \leq 8$) emitted after a 200 MeV/u ^{16}O beam interaction with a C_2H_4 target. Heavier fragments ($Z \geq 2$) have a forward angular peak contained in a -10° , $+10^\circ$ range and have a kinetic energy around the corresponding beam value, while light fragments follow wider distributions. According to these features, the detector design must have sufficient geometrical acceptance to capture the produced fragments.

The most suitable centers for the experiment are CNAO (Pavia, Italy), GSI (Darmstadt, Germany) and HIT (Heidelberg, Germany), as they are equipped with experimental rooms where ^4He , ^{12}C and ^{16}O beams can be provided at the energy of interest. Portability and size are thus two crucial aspects to consider for an experiment that needs to be "movable" to fit different facility space constraints. Finally, to provide high acceptance, efficiency and particle identification capability in a finite size, it is essential to proceed with a strategic selection of the detectors, also considering the cross section resolutions that must be reached. To face all these requirements and criticalities, two complementary setups have been developed: a magnetic spectrometer and a nuclear emulsion spectrometer.

3.2 Electronic setup

The FOOT electronic setup is a multi-detector system tailored for univocal particle identification of fragments with $Z \geq 3$ and an angular acceptance of 10° with respect to the beam axis. Due to the high resolution requirements and the compactness and portability of the design, a redundant system approach was defined. In particular, a $\Delta E - E$ telescope made of plastic scintillators and inorganic crystals provides the Time of Flight (ToF), the energy loss in a thin layer of scintillator and the kinetic energy E_k of the fragments. Moreover, a magnetic spectrometer made of permanent magnets and Silicon detector stations retrieves the rigidity p/Z of the fragments from the deflection of their tracks, where p and Z are respectively their momentum and charge. The charge Z can be obtained from the ToF and information taken from the scintillators, exploiting the Bethe-Bloch formula (see Eq. 1.2). An improved identification can also be obtained by the

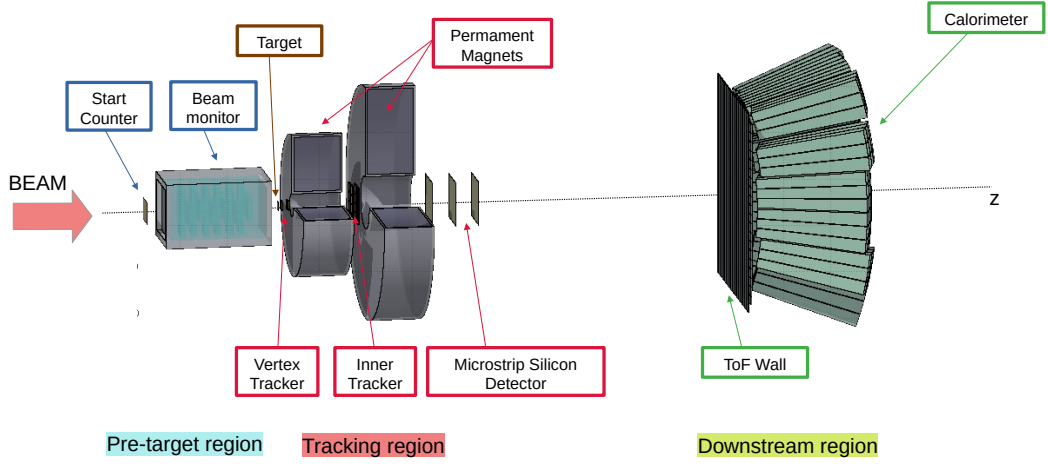


Figure 3.2: View of the electronic setup

energy loss information in the silicon detectors.

Finally, the mass number A can be computed from momentum p , velocity β (a function of ToF) and kinetic energy E_k , exploiting the following relations:

$$A_1 = \frac{E_k}{Uc^2(\gamma - 1)} \quad A_2 = \frac{p^2c^2 - E_k^2}{2 \cdot U \cdot c^2 E_k} \quad A_3 = \frac{p}{U \cdot c\beta\gamma} \quad (3.2)$$

where c is the speed of light, γ is the Lorentz factor and U is the atomic mass unit. Based on all the previous considerations, the electronic setup for the FOOT ex-

periment was designed as shown in Fig. 3.2 which, according to MC studies, is able to achieve the following experimental resolutions:

- $\sigma(p)/p \simeq 4\text{--}5\%$;
- $\sigma(\text{ToF}) \simeq 70\text{--}250\text{ ps}$;
- $\sigma(E_{\text{kin}})/E_{\text{kin}} \simeq 1.5\text{--}2.5\%$;
- $\sigma(\Delta E)/\Delta E \simeq 3\text{--}10\%$

These resolutions are crucial for achieving a charge identification capability at the 2–3% level and an isotopic identification precision of about 5%, enabling the discrimination of both elemental charge and isotopic mass required for the fragmentation cross section measurements of the FOOT experiment.

The final setup is divided into three regions which are described in the following sections.

3.2.1 Upstream region

The upstream region is composed of pre-target detectors whose aim is to monitor the beam and define its direction until its interaction with the target. As mentioned before, a good knowledge of the direction of the primary is important for the Inverse Kinematic approach (see Sec. 3.1). The configuration consists of a plastic scintillator and a drift chamber.

Start Counter

The Start Counter (SC) is made of a EJ-200 plastic scintillator foil of 250 μm thickness and $5 \times 5\text{ cm}^2$ active surface. It is closed with a thin layer of aluminized mylar via two square windows surrounded by 48 $3 \times 3\text{ mm}^2$ SiPM, 12 per side, connected to 8 electronic channels, as shown in Fig. 3.3-a. It is held by a 3D-printed box, as reported in Fig. 3.3-b. The read-out and the power supply of the SiPMs are provided by the WaveDAQ system [108], which is suitable for sampling signals up to 5 Gsamples/s and provides a gain between 0.5 and 100 of the scintillating signal before the digitization to face different beam types and energies. According to the latest results from CNAO 2024 campaign, the SC time resolution is of the order of 50 ps with 400 MeV/u ^{12}C ions, while an efficiency for primary detection close to 1 was observed for a rate up to 10 kHz [109]. The SC provides the minimum bias trigger (see Sec. 3.2.4), measures the incoming ion flux, provides the reference time t_0 for other detectors and is used as the start time for *ToF* measurements.

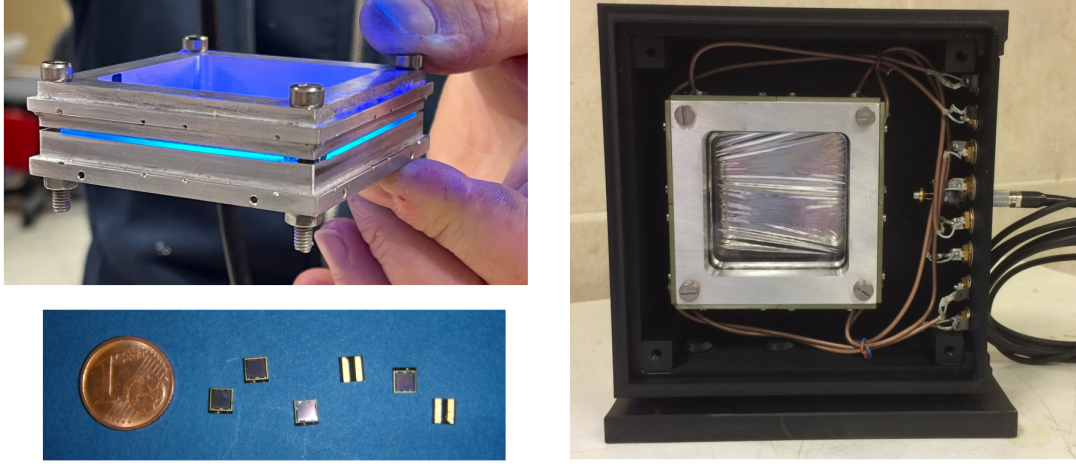


Figure 3.3: a) SC plastic scintillator among the metal windows (upper part) is surrounded on every side by SiPM at the edges of the window (lower part), b) everything held inside the black printed box.

Beam Monitor

The Beam Monitor (BM) is a drift chamber consisting of twelve wire layers, with three drift cells per layer. The direction of the wires is transverse to the beam direction (z axis) and alternated in x and y axes for the beam profile reconstruction in both views. Layers in the same directions are staggered by half a cell to solve left-right ambiguities in track reconstruction [109]. A schematic view is shown in Fig. 3.4. Every cell has the shape of $16 \times 10 \text{ mm}^2$, for a total length of 21 cm and a transversal region of 13 cm. BM operates at a pressure of approximately 0.9 bar with an 80/20% gas mixture of Ar/CO₂ with a high voltage power supply. A primary detection efficiency of around 90% has been achieved, reaching a lower limit on the track spatial resolution of $100 \text{ } \mu\text{m}$ in the central part of the BM cell, with an angular resolution lower than 2 mrad [110]. A picture of the detectors and its technical draw are shown in Fig. 3.5.

The BM is positioned between the SC and the target and plays a significant role in characterizing the incoming beam. A primary function is to measure the direction of the beam, to be able to reconstruct the further direction of the fragments after the target interaction, fundamental for the Inverse Kinematic Approach (see Sec. 3.1). It also plays a crucial role in addressing the pile-up ambiguity, namely more than one primary particle recorded in the same event, and discarding events where secondary fragmentation occurred in the SC. Finally, it provides essential

information about the beam spot size and it is very efficient for monitoring very low intensity beams (needed by FOOT data acquisition) that standard facility devices could not perform otherwise. The readout process is based on TDC channels connected to sense wires, which measure drift time for position reconstruction via lookup tables.

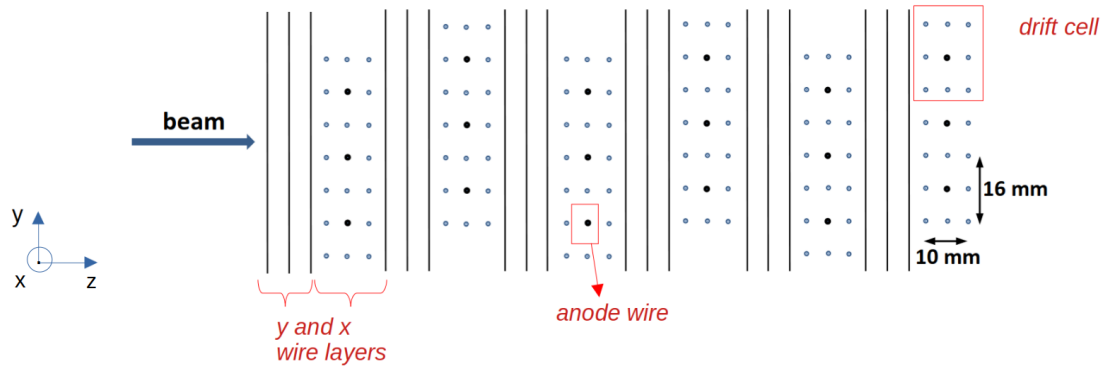


Figure 3.4: Schematic view of the BM detector with a description of its details. From [110].

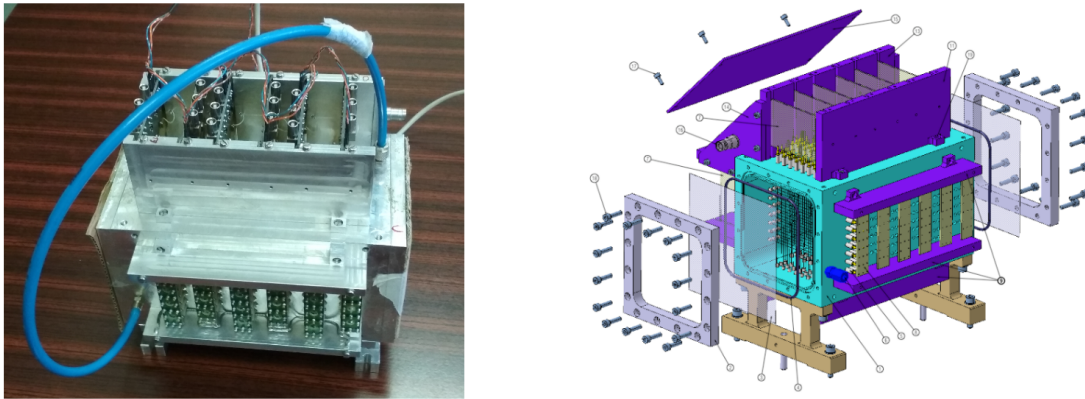


Figure 3.5: A picture (left) and a technical draw (right) of the BM detector. From [111].

3.2.2 Tracking region

The tracking region is a crucial component of the FOOT experimental setup whose primary role is to determine the momentum of the charged fragments produced in the target through tracking in a magnetic field. This region is situated

after the target and comprises three stations of silicon detectors strategically placed upstream, between, and downstream of two permanent magnets, as shown in Fig. 3.6 (and in Fig. 3.2). The design has been developed in order to satisfy the re-

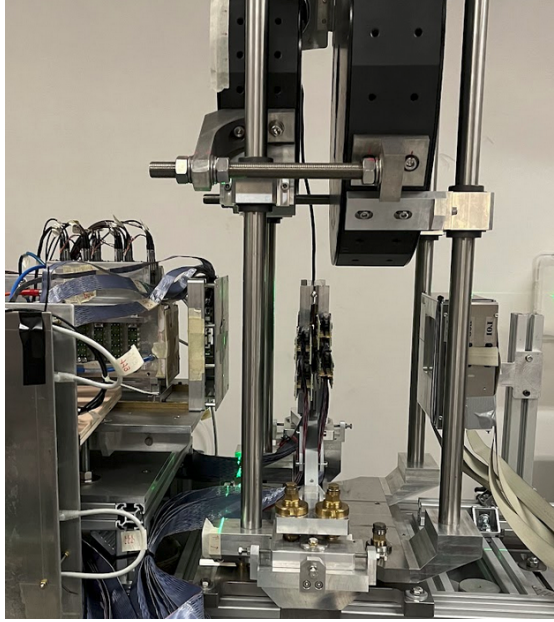


Figure 3.6: Picture of the tracking region during CNAO 2023 data taking. In particular, from left to right after the BM: the Vertex detector, the Inner Tracker and the Micro Strip detectors, respectively upstream, between, and downstream of two permanent magnets, which are out of the beam line in this specific configuration.

quired momentum resolution and the global angular acceptance, limiting as much as possible the secondary fragmentation and the multiple scattering in sensors and mechanical structure.

Magnets

Sectors of Neodymium-Iron-Boron material are organized to form two permanent magnets in a cylindrical shape for a total of 140 kg. The depth of each magnet is of 10 cm and the internal radius of, respectively, 2.5 cm and 5.5 cm (see Fig. 3.7-a), with a distance of 5 cm, enough to host a detector between them. They are arranged in a Halbach configuration, in which an almost constant magnetic field is produced inside the cylinders (of 1.4 T and 0.9 T respectively), while outside it is negligible as shown in Fig. 3.7-b. They are installed on a structure that allows vertical displacement, giving the opportunity to take data even in configurations without the magnetic field contribution, for example, for inter-alignment purposes.

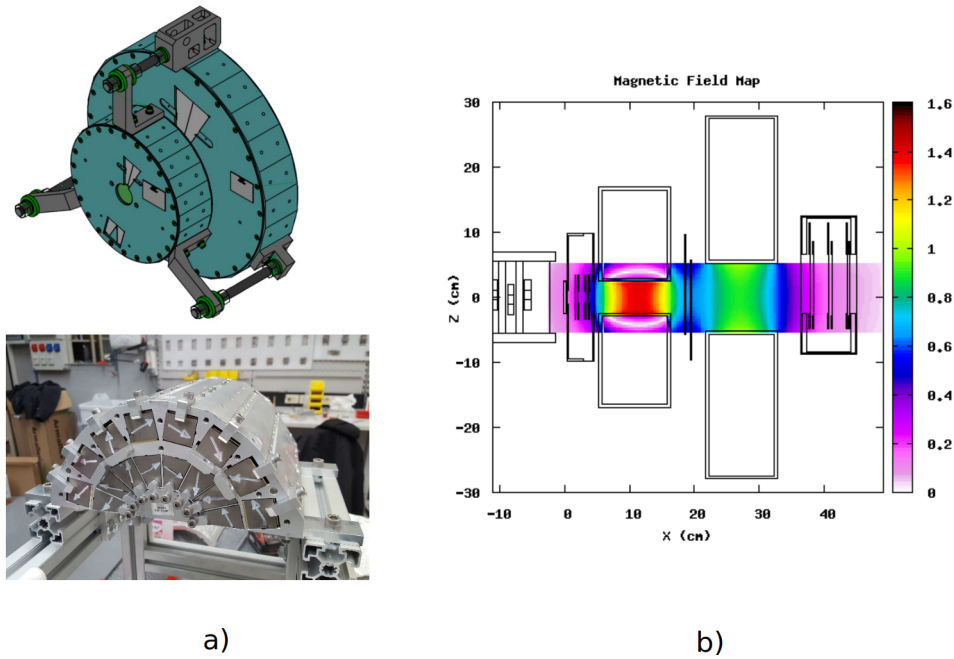


Figure 3.7: a) Schematic view of the FOOT permanent magnets (upper part) and the internal structures of a section taken during construction (lower part). b) Map of the resulting magnetic field in the spectrometer.

Vertex and Inner Tracker

The Vertex (VTX) and Inner Tracker (IT) detectors utilize monolithic pixel sensors for high precision tracking. The VTX reconstructs the interaction points of primaries inside the target (vertex) and initial trajectory information, while the IT, located within the magnetic field, provides further information on the direction and x, y position of particle tracks.

The MIMOSA-28 (M28) Monolithic Active Pixel Sensors (MAPS) technology has been adopted, where each M28 sensor consists of a matrix with 928 rows and 960 columns of pixels, with a pixel pitch of $20.7 \mu\text{m}$. The total active area covers a size of $2 \times 2 \text{ cm}^2$, with a material budget of $50 \mu\text{m}$ of thickness (see Fig. 3.8-a). Developed by the Strasbourg CNRS PICSEL group, it was designed for tracking in the STAR-PXL detector at RHIC at Brookhaven National Laboratory [112]. A rolling shutter readout technique is employed with a total of $186 \mu\text{s}$ of integration time, setting the threshold of the experiment trigger rate at the order of kHz. More detailed information on the M28 chip will be presented in Chap. 4.

The VTX is made of 4 layers of M28 chips, for a total material budget of $200 \mu\text{m}$. The layers are positioned respectively at 0.6, 0.9, 2.1, and 2.4 cm from the

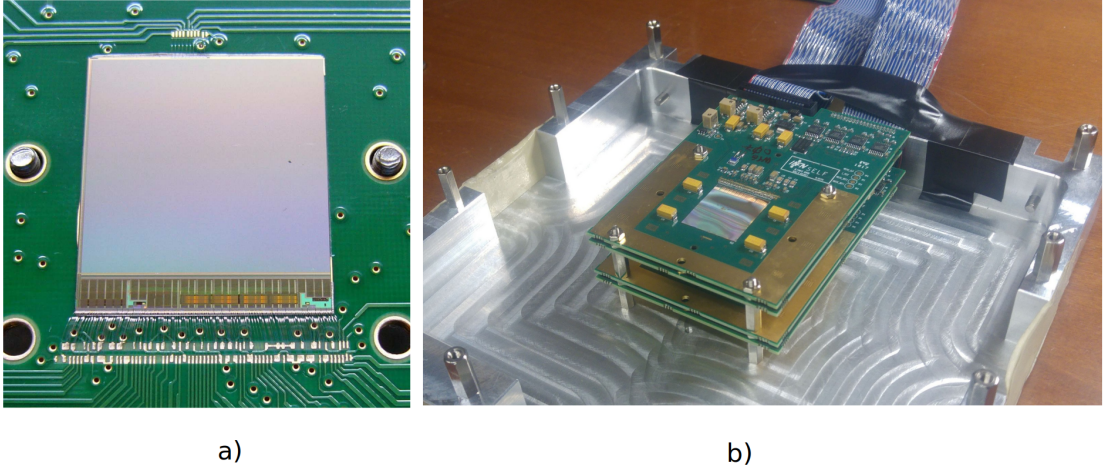


Figure 3.8: a) Picture of MIMOSA-28 chip. b) View of the for layers of M28 chips of the VTX detector, positioned inside an aluminum holder.

target center, along the beam axis (see Fig. 3.8-b), providing a geometrical acceptance of about 40° for the emitted fragments, while the target is attached to the first one. A high cluster spatial resolution of $5 \mu\text{m}$ is achieved [113] and the VTX reconstructed track, matched with the BM, provides a precise identification of the interaction point of the fragments and their emission angle.

The correlation between the charge of a fragment and the number of fired pixels after the sensor crossing can be exploited to improve the Z identification already performed by other detectors [114].

The IT is made of 2 planes of pixel sensors, each covering a sensitive area of $8 \times 8 \text{ cm}^2$, for a total of 16 M28 sensors per layer. In particular, these chips are not expected to be influenced by the residual magnetic field present between the two magnets. They are glued in modules composed of 4 M28 each (see Fig. 3.9-a), implemented in a double-sided layout of ladders in a similar way to the ones used in the PLUME project [115]. The two staggered planes are made of two ladders each, installed on a SiC support structure.

The full detector has been completed and tested in dedicated acquisitions at BTF in LNF (Italy) and introduced in the experiment acquisitions at CNAO in 2023 and 2024 (see Fig. 3.9-b). The advantage of using the same technology for VTX and IT simplifies the architecture of the Data Acquisition system.



Figure 3.9: a) A module of 4 M28 chips with electronics, which forms one of the two extensions of a ladder. b) View of the two planes of the IT in the CNAO 2023 configuration, followed by VT.

Micro Strip Detector

The Micro Strip Detector (MSD) is located downstream of the second magnet in the experimental setup. It consists of three x - y strip detectors, each with an active area of $9.6 \times 9.3 \text{ cm}^2$, separated by a 2 cm gap along the beam direction, ensuring the right angular acceptance for measuring ions with $Z \geq 2$. To limit the material budget, every plane is made of two perpendicular Single-Sided Silicon Detector (SSSD) sensors with a thickness of $150 \text{ }\mu\text{m}$ (see Fig. 3.10-a,b), for a total thickness of $900 \text{ }\mu\text{m}$. In particular, each SSSD has a strip size of $50 \text{ }\mu\text{m}$ and a strip pitch of $150 \text{ }\mu\text{m}$, resulting in a total of 640 channels per sensor.

In contrast to VTX and IT, the MSD uses an analog readout, which achieves a spatial resolution estimated to be around $13.3 \text{ }\mu\text{m}$ for fragments heavier than protons [116]. This analogic approach, enhancing the tracking capability of the setup, also has the advantage of providing a valuable measurement of energy loss dE/dx , completing the charge identification capabilities of the downstream detector.

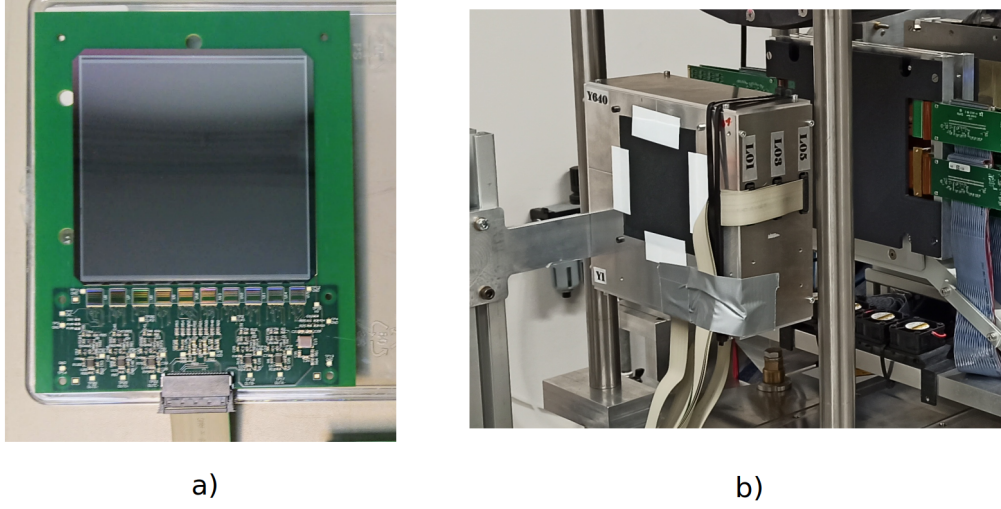


Figure 3.10: a) Picture of one SSSD of the MSD, placed together with another rotated sensor to obtain a MSD plane. b) Picture of the MSD box installed in the tracking region during CNAO 2023 data taking, followed (on the right) by IT.

3.2.3 Downstream region

Placed at least 1 m away from the target, the downstream region has the aim to measure the energy loss, the ToF and the kinetic energy E_k to evaluate the charge and the mass of the fragments produced in the nuclear interactions of the beam with the target. It is composed by two layers of plastic scintillators (ToF Wall) for $\Delta E/\Delta z$ and ToF measurements and by BGO crystals (Calorimeter) for the kinetic energy information.

ToF Wall

The ToF Wall Detector (TW) is composed of two orthogonal layers, each containing 20 plastic scintillator bars EJ-200 of dimensions 0.3 cm in thickness, 2 cm in width, and 44 cm in length. The total active area of this arrangement is 40×40 cm² (see Fig. 3.11-a,b) with a granularity of 2 cm, carefully chosen to match the expected angular distribution of the heavier fragments and to keep the pile-up of multiple fragments in the same bar below 1%. The ends of each bar are optically matched to 4 Silicon SiPMs with a 3×3 mm² active area and 25 μ m microcell pitch. Two channels per bar carry the signal, digitized by the WaveDAQ system with a trigger scheme already adopted by the SC. The detector is designed to achieve a ToF resolution better than 100 ps and an energy loss resolution $\sigma_{\Delta E}/\Delta E$ of approximately 5% for heavier fragments [117]. Moreover, from the time measure-

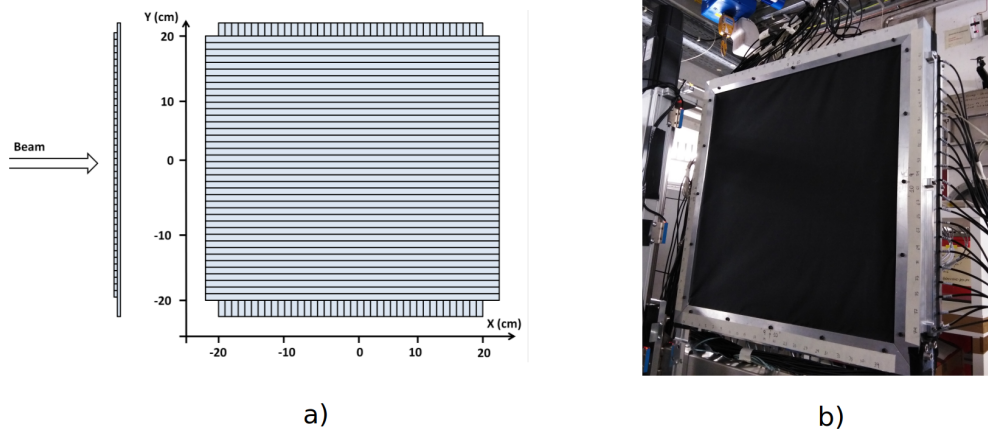


Figure 3.11: a) Schematic view of the TW detector b) Picture of the detector on the beam line with its shielding from background light.

ment provided by SiPM, it is possible to reconstruct the actual position of the hit along the bar, reaching a spatial precision better than 8 mm.

Coupled with the starting time t_0 from the SC, TW allows for the calculation of the ToF of the fragment. Considering also the energy loss ΔE in the layers of the detector, atomic number Z can be measured according to the Bethe Bloch formula (see Sec.1.1.1). The Z identification is also fundamental as a value to retrieve the momentum p measurement from rigidity p/Z , a quantity that can be obtained from global tracking, where the TW point, moreover, has to be used as a seed for track extrapolation and fitting (see Sec. 5.3).

Calorimeter

The calorimeter (CAL) is constructed from 320 $\text{Bi}_4\text{Ge}_3\text{O}_{12}$ (BGO) crystals arranged in an approximately disk-like geometry with a radius of about 20 cm and mechanically divided into modules of 3×3 crystals for ease of handling. Each crystal has a length of 24 cm in a truncated pyramid shape, with front and back faces of approximately $2 \times 2 \text{ cm}^2$ and $3 \times 3 \text{ cm}^2$ respectively (see Fig. 3.12-a), a size chosen to align with the granularity of the previous TW. Each BGO crystal is optically coupled in the rear face to a 25 SiPMs matrix with an active surface of $2 \times 2 \text{ cm}^2$, employing microcells with a $15 \mu\text{m}$ pitch to ensure a linear response up to approximately 10 GeV (see Fig. 3.12-b). The front-end board interfaces with the WaveDAQ system, the same system utilized for the SC and TW, enabling signal sampling at 1 Gsample/s. It also includes a SiPM temperature sensor that can be used to equalize the calorimeter response in case of non negligible variations.



Figure 3.12: a) Module of two BGO crystals, coupled at one end to SiPM and its readout. b) Picture of the SiPM used for energy measurements.

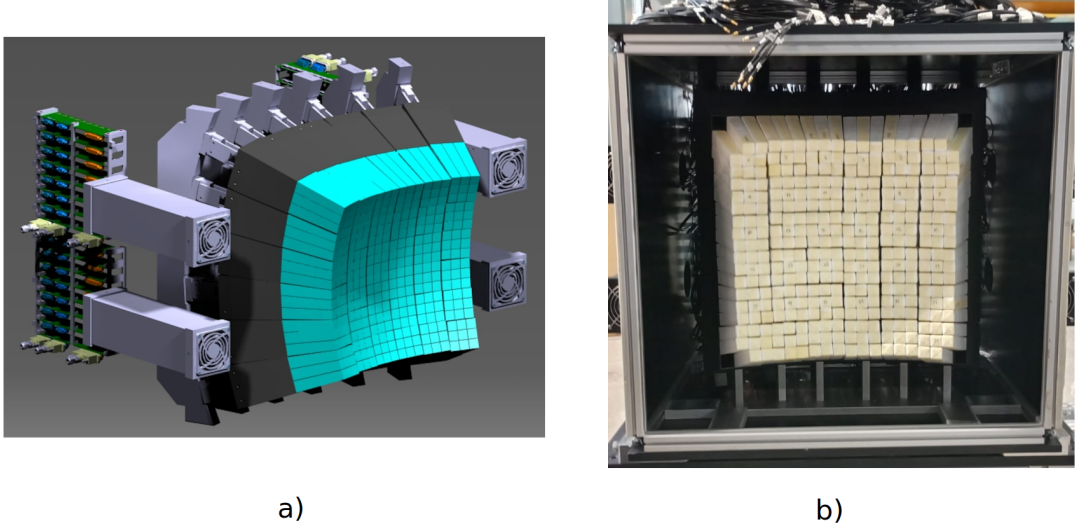


Figure 3.13: a) Sketch of the final design of the CAL detector. b) Picture of the FOOT calorimeter assembly during CNAO 2024 data taking.

CAL aims to measure the kinetic energy of the fragments as a fundamental step in mass reconstruction. According to the aim of the experiment, the chosen design is good for a proper containment of fragments up to ^{12}C and ^{16}O in the energy range of 200-400 MeV/u. In this circumstance, linearity and an energy resolution $\sigma_{E_{kin}}/E_{kin}$ below 2% for heavier fragments has been reached [118]. In general, despite the high BGO density ($\rho = 7.13 \text{ g/cm}^3$) which guarantees a high stopping power, there is still a small fraction of the energy that, through nuclear interactions, generates neutrons that can escape the detector. Moreover, above the threshold of 280 MeV/u, $\pi^\pm$ production is also possible (see Sec. 1.3.2), causing an

energy leak from the calorimeter and a corresponding performance degradation. The final configuration of the CAL, including the total number of crystals, was established during the data-taking campaign at CNAO in 2024, shown in Fig. 3.13-a,b. In this phase, the energy response was calibrated using protons and ^{12}C ions at various energies ranging from 70 to 400 MeV/u.

An overall description of the Electronic Setup experiment detectors and their resolution is reported in Tab. 3.1.

Detector	Resolution	Technology
Start Counter	Time resolution: ~ 50 ps	Plastic scintillator EJ-200, silicon photomultipliers
Beam Monitor	Spatial resolution: ≤ 150 μm	Drift chamber
Vertex Detector	Spatial resolution: ≤ 5 μm	Silicon pixel sensors (MIMOSA-28)
Inner Tracker	Spatial resolution: ≤ 5 μm	Silicon pixel sensors (MIMOSA-28)
Microstrip Silicon Detector	Spatial resolution: ~ 15 μm	Silicon microstrip sensors
ToF-Wall Detector	Energy resolution: $\sim 5\%$, Time resolution: ≤ 100 ps	Plastic scintillator EJ-200, silicon photomultipliers
Calorimeter	Energy resolution: $\sim 2\%$	Bismuth germanium oxide (BGO) crystals, silicon photomultipliers

Table 3.1: Summary of detector resolutions and technologies.

3.2.4 Trigger and Data Acquisition System

The Trigger and Data Acquisition (TDAQ) system is a flexible, hierarchical and distributed system whose main aim is the selection of events of interest during the data taking (trigger) and the management of the data flow from the detectors to the storage (data acquisition). It comprises Linux PCs, VME crates and boards, detector-integrated readout systems and standard communication links like Ethernet, USB and optical fibers [119], as schematized in Fig. 3.14.

The data acquisition system is made of different elements that communicate via a self-standing network. In particular, a switch is used to collect all the data from the detectors, namely 20 DE10nano or DE10 Terasic board for VTX, IT and MSD and the WaveDAQ for SC, TW and CAL via 1 GbE Ethernet connections. The Storage PC manages the data acquisition on a SSD (Solid State Drive) with a writing speed of ~ 400 MB/s and it is connected via a 10 GbE fiber to the switch, in order to avoid any bottleneck. An offline HHD-based storage composed of a 30 TB NAS (Network Attached Storage) is also connected via a 10 GbE link to the local network switch and filled in idle times.

The Storage PC is also connected via a dedicated fiber SFP to a VME crate housing a CAEN V2495 board, which serves as the central trigger module. Alongside a custom board, it manages the coordination of trigger and busy signals across

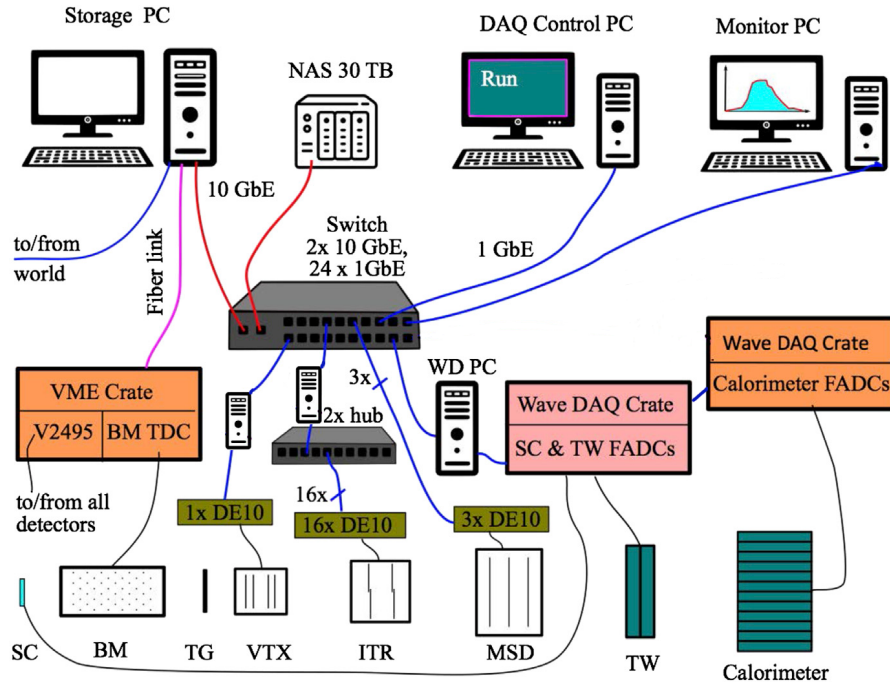


Figure 3.14: FOOT TDAQ infrastructure

all subdetectors of the experiment, to which all associated electronics are connected. The VME crate hosts, moreover, a CAEN V2718 board which operates as a bridge for reading/writing to/from VME crate and CAEN V1190B board which represents the overall electronics (TDC and discriminators) for triggering and data acquisition of the BM.

All these components are installed in the experimental room, while a Control PC is remotely managed in the control room, which hosts the DAQ GUI interface for starting/stopping runs and controlling/configuring other system nodes. Finally, a Monitor PC is equipped for online monitor tools, such as on-line histograms of detector occupancy, particle arrival times, particle energies and collected charges. Furthermore, fast online event reconstruction is performed on a fraction of the events to monitor fragment momentum spectra, ToF , reconstructed charges and masses [120].

The two main trigger logics used by the experiment are *minimum bias* (MB), intended to start the acquisition when a beam particle enters the SC, and the *fragmentation trigger*, generated when MB holds and the bars of the TW register a pulse height compatible with the passage of a fragment particle through the setup. Both of these triggers are generated by the WaveDAQ [121] and issued by the V2495 board to all the detectors. While the system has been tested at higher rates, the typical sustained acquisition rate for physics data taking, limited by factors like the beam conditions and the slowest detectors read-out (M28 chips), is around 1 kHz, which corresponds approximately to a collection of 26 million events per day with MB trigger.

3.3 Emulsion setup

A nuclear emulsion spectrometer has been adopted for identifying low Z ($Z \leq 3$) fragments emitted up to large polar angles (approximately 70°), designed as a complementary detector to the Electronic Setup to cover the wide angular emission of light fragments (see Sec. 3.1.1). They achieve the highest spatial resolution among tracking devices used in nuclear physics for vertex detection and tracking, as well as a good charge identification capability.

The one used by the FOOT experiment is an Emulsion Cloud Chamber (ECC) structure, where the emulsion film consists of two $70 \mu\text{m}$ thick sensitive layers deposited on both sides of a $210 \mu\text{m}$ plastic base, resulting in a total thickness of $350 \mu\text{m}$.

The sensitive regions are made of AgBr crystals of $0.2 \mu\text{m}$ diameter scattered in a gelatin binder which, when crossed by charged particles, develop a sequence of AgBr crystals along their trajectory. A chemical development process triggers the

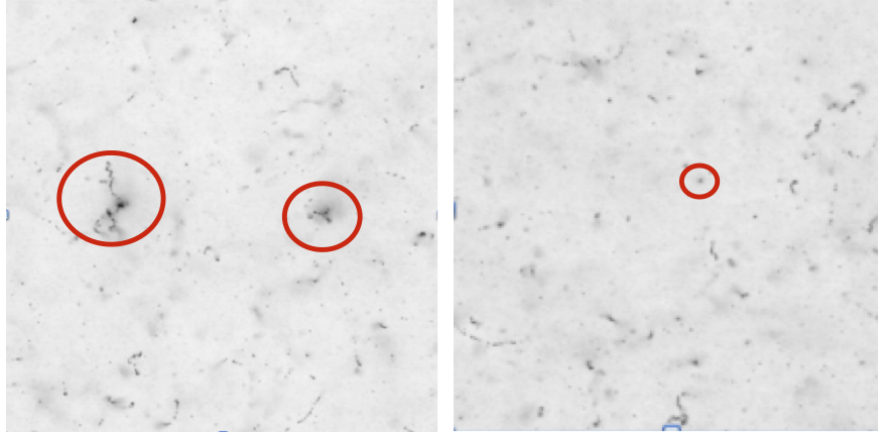


Figure 3.15: Tracks in emulsion generated by passing through carbons (left) and protons (right) impinging perpendicularly to the emulsion surface. In the left image, delta rays emitted from the carbon interaction are also visible. View size is $300 \times 300 \mu\text{m}^2$. From [122]

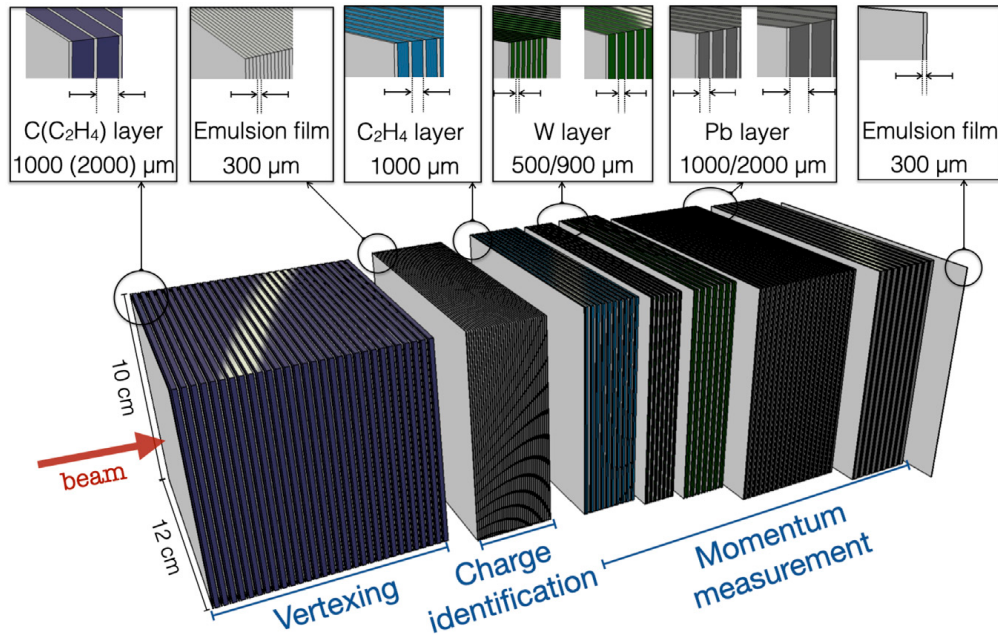


Figure 3.16: Schematic view of the FOOT Emulsion Cloud Chamber setup.

growth of silver grains making them visible under optical microscopes, which now employ ultra-fast scanning techniques [123]. After an automated scanning system, a software takes aligned dark "pixels" of silver grains to form tracks and from their density along the particle's path, the fragment charge can be retrieved. An example of the detection of fragments generated from ^{12}C and proton beams on a single emulsion layer is shown in Fig. 3.15.

The nuclear emulsion spectrometer advantageously integrates target and high resolution detector without requiring readout electronics during exposure, but on the contrary, it can only be used once for a given beam and energy due to the permanent nature of particle track recording. The ECC is shown in Fig. 3.16 and it is composed of three regions described in the following sections.

3.3.1 Interaction and Vertexing region

The Vertexing region is made of several cells, each composed of targeting elements like Carbon or C_2H_4 interleaved with emulsion films. The length of this section is tailored for each experimental run according to the beam species and energy and target configuration, in order to guarantee enough fragmentation processes. In particular, this section is chosen in such a way as to have the partial (almost 20%) absorption of primary particles, letting only the nuclear fragments reach the following sections. The produced secondary fragments are subsequently detected in the downstream sections of the ECC and, by tracking, the interaction vertex position can be retrieved from this region.

3.3.2 Charge identification region

The grain density generated by particles crossing the emulsion material saturates for those that overcome the minimum of their ionizing power (MIPs), suppressing the charge identification. To increase the dynamic range of the emulsions, different thermal treatments are applied to the films after exposure but before chemical development: in this way, a controlled fading is achieved and, correlating grain density in films with different thermal treatments, charge identification of light fragments ($Z \leq 3$) is obtained [122]. The charge identification region consists of nine cells, each containing four emulsion films. These films are subjected to four different thermal treatments, denoted as R0 (no thermal enhancement), R1 (28°C), R2 (34°C), and R3 (36°C), respectively. Each track is thus characterized by four track volume variables (VR0, VR1, VR2, VR3) which are proportional to the grain density after each thermal treatment. In Fig. 3.17, the correlation among these variables is reported, for beams of 200 MeV/u ^{16}O against carbon and 400 MeV/u ^{16}O against C_2H_4 respectively, during a data taking of the ECC FOOT experiment at GSI [124].

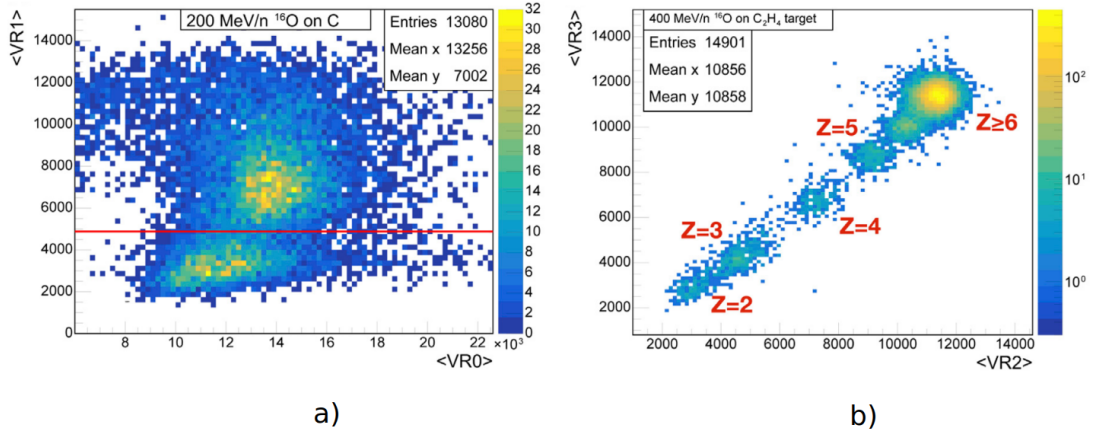


Figure 3.17: a) $\langle VR1 \rangle$ vs. $\langle VR0 \rangle$ distribution for all tracks, classified as $Z = 1$ and as $Z = 2$, below and above the red cut respectively. b) $\langle VR3 \rangle$ vs. $\langle VR2 \rangle$ distribution for all tracks identified as signal. From [124].

3.3.3 Momentum Measurement region

The Momentum Measurement region is made of layers of emulsion films interleaved with layers of passive materials such as Lexan, Tungsten and Lead. It is specifically designed to determine the momentum of charged fragments traversing the detector. A first technique consists in estimating momentum from the measured total track length of a stopped particle, according to the correlation between

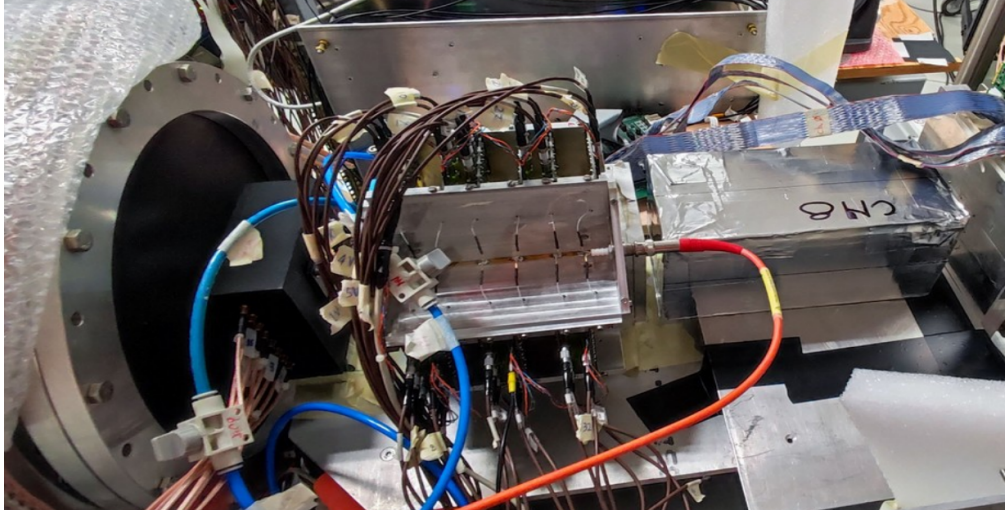


Figure 3.18: ECC setup during CNAO 2023 data taking. From left to right, after the beam pipe: the SC and BM for beam monitoring followed by the ECC.

range and kinetic energy. Another method, instead, provides an independent momentum estimation by analyzing the small deflections in the particle's trajectory as it traverses the emulsion films and passive layers due to MCS (see Sec. 1.1.4). A mass identification of the fragments is thus performed.

Since 2019, the ECC has been used in several data takings at GSI and CNAO. The setup in the experimental room also includes the SC and BM for beam monitoring purposes, already used in the upstream region of the Electronic Setup, but completely decoupled by ECC (see Fig. 3.18).

Chapter 4

Vertex MIMOSA-28 characterization

As already mentioned in Chap. 3, the MIMOSA-28 pixel sensors (M28) are high spatial resolution detectors suitable for particle trajectory reconstruction. In the FOOT experiment, 4 layers of M28 chips are used as VTX to identify the interaction of the primary particles with the target.

In the following paragraphs, after a description of the M28 architecture and its working principles, the characterization of the VTX detector during CNAO 2024 data collection is shown, focusing on its threshold optimization and efficiency analysis.

4.1 M28 overview

4.1.1 MAPS detector

The M28 chip belongs to the family of Monolithic Active Pixel Sensor (MAPS) in a CMOS technology. As schematized in Fig. 4.1, every pixel consists of a thin p-doped Epitaxial layer (P-) interleaved between a high p-doped Substrate (P++) and a P-well (P++). The external layers represent a mechanical and electrical foundation for the device, providing good isolation. N-well (N++) highly n-doped regions are implanted among the P-well, creating a pn-junction with the epitaxial layer. In particular, a depletion region free of charges is generated near the interface of the two layers, making the pn-junction the building block of diodes.

Unlike conventional fully depleted CMOS sensors, the standard MAPS pn-junction is only partially depleted. As a consequence of that, when a charged particle crosses the sensitive area of the detector, generated electron-hole pairs move mainly due to diffusion for concentration gradients rather than drifting in the electric field.

The latter became predominant only at the boundary of the epitaxial layer where built-in voltages are induced among the two different doped regions. Here, ions move toward the collection diode by drifting and a signal is stored in a diode-parasitic capacitance before amplification. The analog read-out electronics are also embedded in every pixel.

As a consequence of diffusion, the electrons produced by a single particle crossing the epitaxial layer of a pixel can be collected in adjacent ones, causing the formation of *clusters* of pixels as detection.

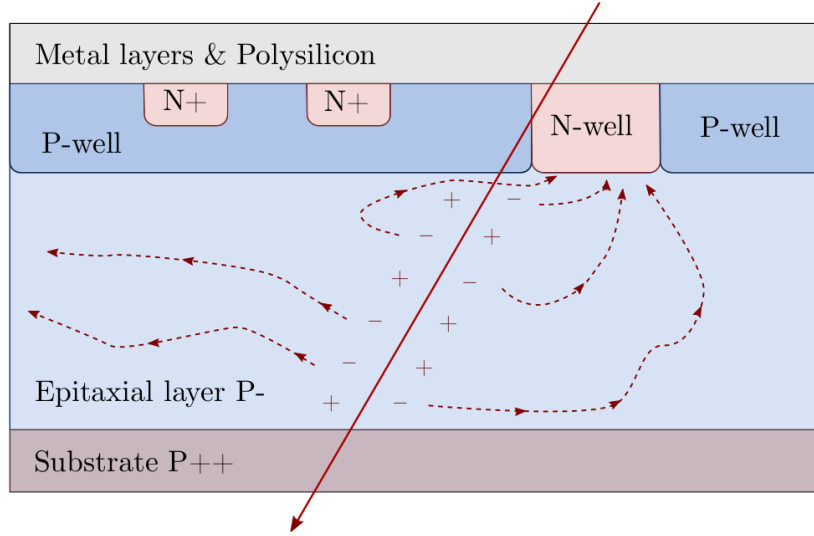


Figure 4.1: Schematization of a monolithic active pixel sensor for charged particle detection. Ionization processes occur after the passage of a charged particle inside the epitaxial layer. The produced electrons diffuse thermally (red dotted lines) and are collected by the N-well diode. From [18].

4.1.2 M28 General architecture

Fig. 4.2 represents the functional block diagram of a M28 chip [125]. It includes a pixel array of 928 rows and 960 columns of $20.7 \mu\text{m}$ pitch with the technology described before, for an overall sensitive area of $\sim 3.8 \text{ cm}^2$. The sensor has a thickness of $50 \mu\text{m}$ with a high resistivity epitaxial layer ($400 \Omega\cdot\text{cm}$) of $14 \mu\text{m}$. Every pixel includes an amplification and a Correlated Double Sampling (CDS). It is a signal processing technique that measures the signal twice, after the sensor reset and after the collected charge, to remove offset voltages and noise and thus to increase the Signal to Noise Ratio (SNR). In order to ensure that the pixel signal is stabilized during the CDS sampling, each column is organized in 58 groups of

16 pixels each to reduce the output node capacitance.

The overall readout works line by line in a rolling shutter mode, meaning that every row is sequentially read, from top to bottom, at slightly different times. The read-out time of each line is 200 ns in which the pixels are not active. Considering the number of lines, the total integration time of the sensor is of 186.5 μ s.

A total of 960 discriminators are used in the M28, each for every column. A column level CDS, similar to the one already explained, is implemented to remove pixel-to-pixel offsets in input signals, in order to have the same response [126]. A reference voltage is also applied to all the switches for discrimination but, to limit as much as possible the charge injection of nearby devices, the 960 discriminators are organized into 4 groups of 240 columns each (called A,B,C,D) where bias voltage is provided by separate bias Digital to Analog Converters (DAC). As shown in Sec. 4.2, the noise level of every group can be evaluated in order to provide an adequate adjustable threshold for noise suppression purposes.

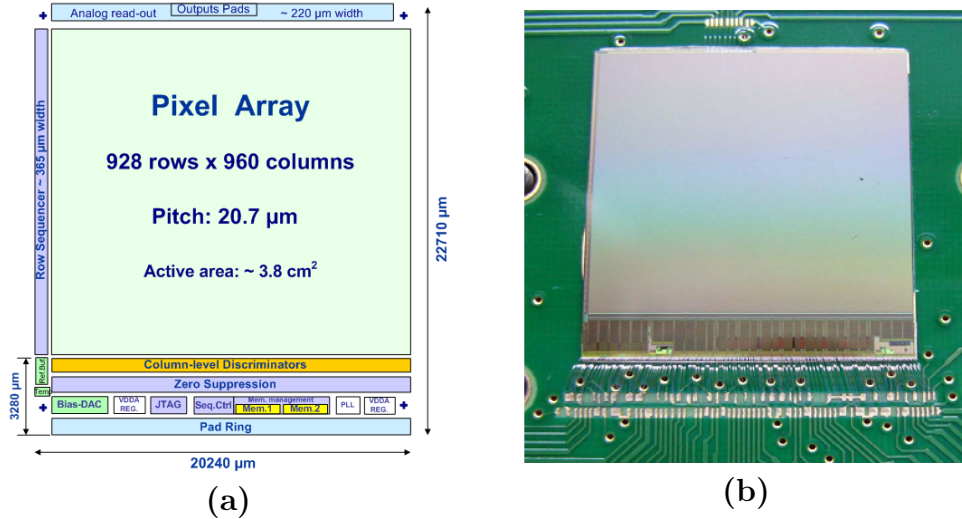
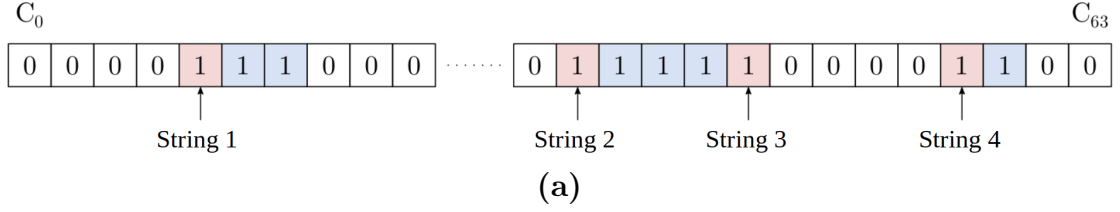


Figure 4.2: a) Block diagram of M28 pixel sensor. b) Picture of M28 pixel sensor, embedded in the readout electronics. From [18].

The digital output signal of every line is then processed via zero suppression, which transmits only the relevant signals of fired pixels, excluding empty data [127]. This increases the readout speed and bandwidth efficiency. The process consists of three steps.

- At first, every row is organized in 15 banks of 64 pixels each. According to a sparse data scan algorithm (SDS), up to 6 *strings* (4 contiguous pixels) are collected for every bank, as shown in Fig. 4.3-a. In particular, every string



String															
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of hits		Address of the column												Not used	

(b)

Status															
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of states				Address of the line											Overflow

(c)

Figure 4.3: Sparsification process implemented in the M28 architecture. a) Every row of 960 pixels is organized in 15 banks of 64 pixels each. In every bank, a maximum of 6 *strings* is implemented, each recording up to 4 fired pixels (so a total of 24 pixels for each bank). b) A *string*, in particular, is made of 16 bits, including the number of fired pixels and their address. c) Two multiplexers reduce the signal of 15 banks in a total of 9 *strings* by row, in a *status* format of 16 bits, which includes the number of *strings*, their address and the overflow bit. Thus, for every line, 1 status plus 9 strings are recorded for a maximum of 10×16 bits, while the maximum number of fired pixels is 36 out of 960. From [18].

is made of 16 bits, including the number of hits (up to 4) and the address of the column (see Fig. 4.3-b).

- After that, two multiplexers reduce the signal of 15 banks up to 9 strings by row, in a *status* format which includes the row address, the number of strings in the entire row, the overflow bit if the latter exceeds 9 and the line addresses (see Fig. 4.3-c).
- Finally, the status format outcomes are written in memory, composed of 2 buffers to perform read and write operations simultaneously.

The collection of sparsified data in a frame (a status plus 9 strings per row, equivalent to 10×16 bit words) is serialized and sent out via two low-voltage differential signal (LVDS) outputs running at 160 MHz, where the writing process operates on words of 2×16 bits for each cycle. Then, considering 10×16 bits per row, produced by the M28 architecture every 200 ns, the required data flow is of 800 Mbit/s. The buffers can theoretically support up to $160 \text{ MHz} \times 2 \times 16 \text{ bits} = 5120 \text{ Mbit/s}$, a capability much higher than the required value by M28 chip.

Another point to highlight is that, according to the implemented sparsification, a maximum of 9 strings can be collected for every line, which represents a maximum of 36 "fired" pixels out of 960. However, this feature does not represent an issue for the FOOT setup, since at the flux of primaries used in the data taking and due to the TDAQ architecture, each frame typically contains only the signal from the primary particle or its fragments. After a good characterization of the VTX detector, this ensures that the sparsification threshold is never exceeded.

4.2 M28 Characterization

The VTX detector is of fundamental importance for tracking reconstruction because it gives the first information about the origin and direction of the fragments after the interaction of the beam with the target. Therefore, a characterization of the detector is needed, in particular of its four layers of M28 chip to run the detector under the best working conditions. This characterization focuses on two crucial aspects: noise measurements with threshold scanning and beam efficiency computation, as it will be shown in the next sections.

4.2.1 Threshold scanning

In architectures such as M28 sensors, where pixel detection is coupled to a discrimination system, the discriminator is activated when the input signal is above a certain level, called *threshold*. According to its value, an output signal can be

obtained when the pixel is fired by a physical signal, but it could also react to random noise. Hence, scanning on the value of the threshold, it is possible to measure the characteristics of the noise. The so-called *pedestal value* p represents the baseline value of a detector signal at which no specific particle interaction is expected but for which true signals are detected above it. Pedestal can arise from electronic noise and background signals and its width can be parametrized in terms of a σ of the equivalent Gaussian (i.e. standard deviation of the noise level). To remove the noise in practical situations, it is usual to set the actual threshold at a given number n of σ , as:

$$T_{\text{thres}} = p + n \cdot \sigma \quad (4.1)$$

where the value n can be chosen to suppress a given amount of noise or to retain a given amount of physical signal, thus fixing the detection efficiency. To experimentally evaluate the pedestal characteristics, it is possible to follow this procedure:

- A range of thresholds $[T_{\min}, T_{\max}]$ and a threshold step ΔT are defined.
- A scan on the possible threshold values, $T_k = T_{\min} + k \cdot \Delta T$, with k a natural number in the range $[0, (T_{\max} - T_{\min})/\Delta T + 1]$.
- For each value of T_k , N measurements are performed, resulting in m_k counts above the T_k threshold.
- For each T_k value, a noise probability n_k and its error δn_k is evaluated as follows:

$$n_k = \frac{m_k + 1}{N + 2}, \quad \delta n_k = \sqrt{\frac{n_k(1 - n_k)}{N + 2}}$$

- Then the distribution of n_k values is fitted with a logistic function to evaluate the pedestal position and the width of the pedestal noise.

The chosen estimation of noise probability and its error comes from a binomial distribution with a Bayesian uniform prior. This choice is due to the fact that very far from the pedestals m_k is approximated to N or m_k to 0, so that a standard max-likelihood estimation of n_k will provide a biased n_k probability (respectively towards 1 or 0).

Logistic functions

To fit the scanning data, several logistic functions can be used, where the main feature is that $f(x) \rightarrow 1$ when $x \rightarrow -\infty$, $f(x) \rightarrow 0$ when $x \rightarrow +\infty$ and $f(0) = 0.5$. In particular, the used function should be of the type of $f_i((x - p)/a_i)$, where p

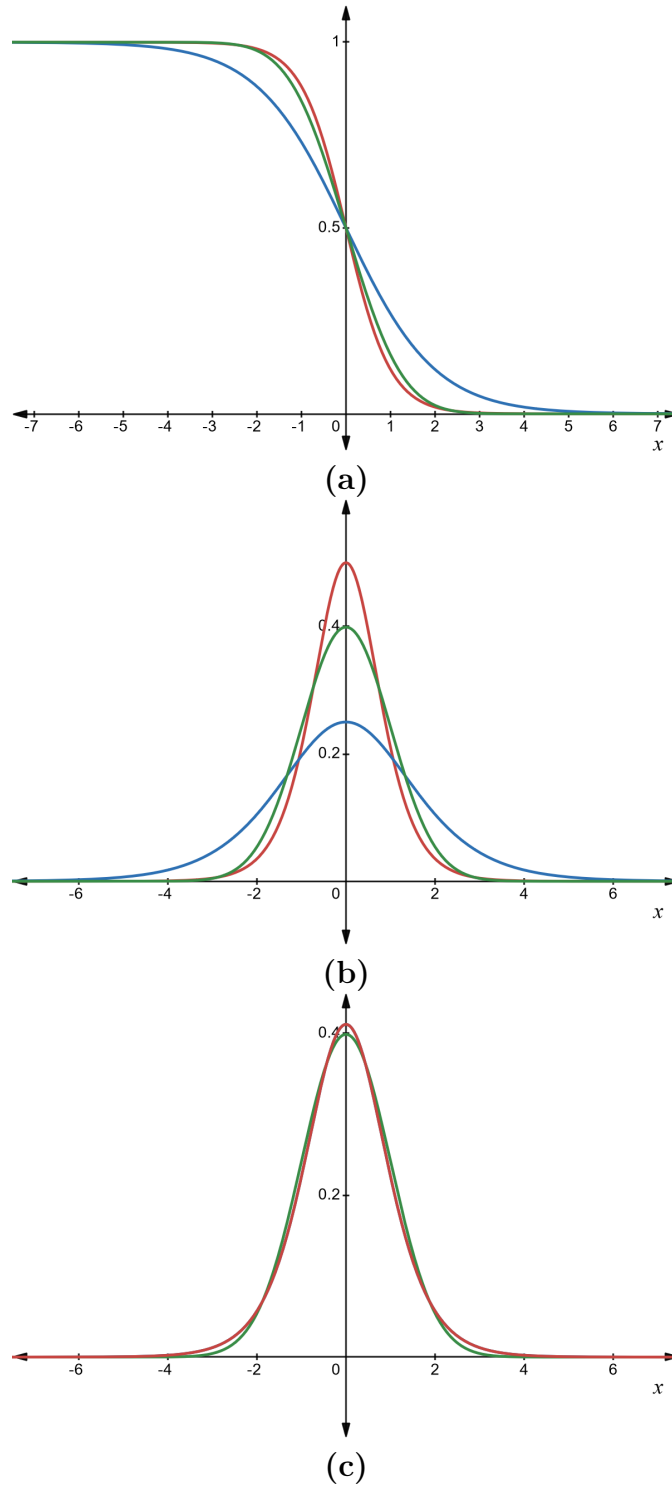


Figure 4.4: Representation of the function of Tab. 4.1. In particular, α is shown in green, β in red and γ in blue. a) Logistic functions. b) Pedestal shapes, all with $p = 0$, $a_i = 1$. c) Pedestal shapes, all with $p = 0$ and the same standard deviation $\sigma_i = 1$. The blue curve is completely covered by the red one.

	$f_i(x)$	$p_i(x)$	σ_{p_i}	FWHM_{p_i}
α	$1 - \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-x'^2/2} dx'$	$\frac{1}{\sqrt{2\pi}a_1}$	a_1	$2\sqrt{2\ln 2} a_1$
β	$\frac{1}{2} [1 - \tanh(x)]$	$\frac{1}{2a_2}$	$0.8225 a_2$	$2 \ln(\sqrt{2} + 1) a_2$
γ	$\frac{e^{-x}}{1 + e^{-x}}$	$\frac{1}{4a_3}$	$1.645 a_3$	$4 \ln(\sqrt{2} + 1) a_3$

Table 4.1: Examples of logistic functions f_i , its pedestal p_i (derivative of f_i in $x = 0$), standard deviation σ_{p_i} and Full Width Half Maximum FWHM_{p_i} .

is the pedestal position and a_i is related to the slope at $x = 0$ and hence to the width σ of the pedestal noise.

The shape of the pedestal distribution can thus be easily obtained with the derivative of the functions:

$$p_i(x, p, a_i) = -\frac{df_i((x - p)/a_i)}{dx} \quad (4.2)$$

while a_i interpretation is more complex because it is related to the characteristics of the $p_i(x, p, a_i)$ specific function. In Tab. 4.1, three examples of logistic functions f_i are introduced: f_α is related to a cumulative of a Gaussian with mean 0 and $\sigma = 1$, f_β and f_γ are related in such a way that $f_\beta(x) = f_\gamma(2x)$. They share the same general properties but differ in the region close to $x = 0$, as seen in Fig. 4.4-a. The pedestals p_i are obtained according to Eq. 4.2 at $x = 0$ and are reported in Tab. 4.1 as well. Fig. 4.4-b highlights the differences in the width of the three functions, since their σ_{p_i} or the full width at half maximum FWHM_{p_i} are related in different ways with the parameter value a_i (in this example $a_i = 1$). On the contrary, the three pedestal distributions p_i are shown in the case where $\sigma_i = 1$ for all in Fig. 4.4-c. In particular, p_β and p_γ have wider tails, so for a fixed σ , they are narrower than a Gaussian distribution.

Threshold scanning for CNAO 2024 data taking

Threshold scanning for each M28 chip of the VTX detector was done before and after every shift during the CNAO 2024 data acquisition. The main aim, of course, was to optimize the detector thresholds in a balance between signal detection and noise suppression. As already stated in Sec. 4.1.2, the pixel array of every M28 chip can be organized into 4 groups of 240 columns each (A,B,C,D), for which a threshold scanning can be evaluated.

A script has been implemented in order to measure, for every group of every sensor, the fraction of active columns in relation to the thresholds, set in a range from 0

to 255 ADC in steps of 1 ADC, following the procedure explained in Sec. 4.2. The chosen sigmoid function to fit the obtained data is the cumulative Gaussian $f_\gamma(x)$ of Tab. 4.1 and its derivative for $x = 0$ ($p_\gamma(x)$) has been considered to extract the pedestal p and the width of the noise function a_γ .

In Figs. 4.5, 4.6, 4.7, 4.8, the threshold scans for the four regions A,B,C,D of the 4 M28 chips layers are reported. These have been measured in the electronic noise environment of CNAO, shortly before the beam was delivered to the FOOT setup in the 2024 data taking. The pedestal value varies from region to region, while the σ width remains consistent among all: this means that, despite having different offsets for the working point, the noise sensitivity of every detector shares the same features. Only σ of Sensor 0 appears slightly lower than those of the other sensors. This may reflect a reduced sensor sensitivity, as will be further confirmed by analyzing other aspects.

Threshold scans were done at different moments in order to verify the stability condition throughout the whole data taking. In Fig. 4.9, the variation of p and σ for each sector and each sensor is reported as box plots, where the mean value, first and third quartile and outliers (if present) are shown. The numerical value of the mean and of the standard error of every distribution is also reported in Tab. 4.2 for p and in Tab. 4.3 for σ . As reported in Fig. 4.9-a, the pedestal values of all sensors remain stable over time despite different offsets, with very peaked distributions around the mean value. For what concerns the σ distribution in Fig. 4.9-b, three sensors (1,2,3) show stability in the distribution with mean values around 4 ADC and a temporal width variation of 1% - 4%. On the other hand, Sensor 0 turned out to be problematic as already pre-announced, showing a variation higher than 35 % in its temporal width for the first three sectors, with an outlier value shifted from 2.7 ADC as mean to 4.5 ADC. This behavior indicates potential instability in its response.

In general, the obtained values of the pedestal and noise width proved to be reliable enough to be used as a reference to set threshold levels during the actual data taking and for subsequent efficiency studies, as described in the following paragraph.

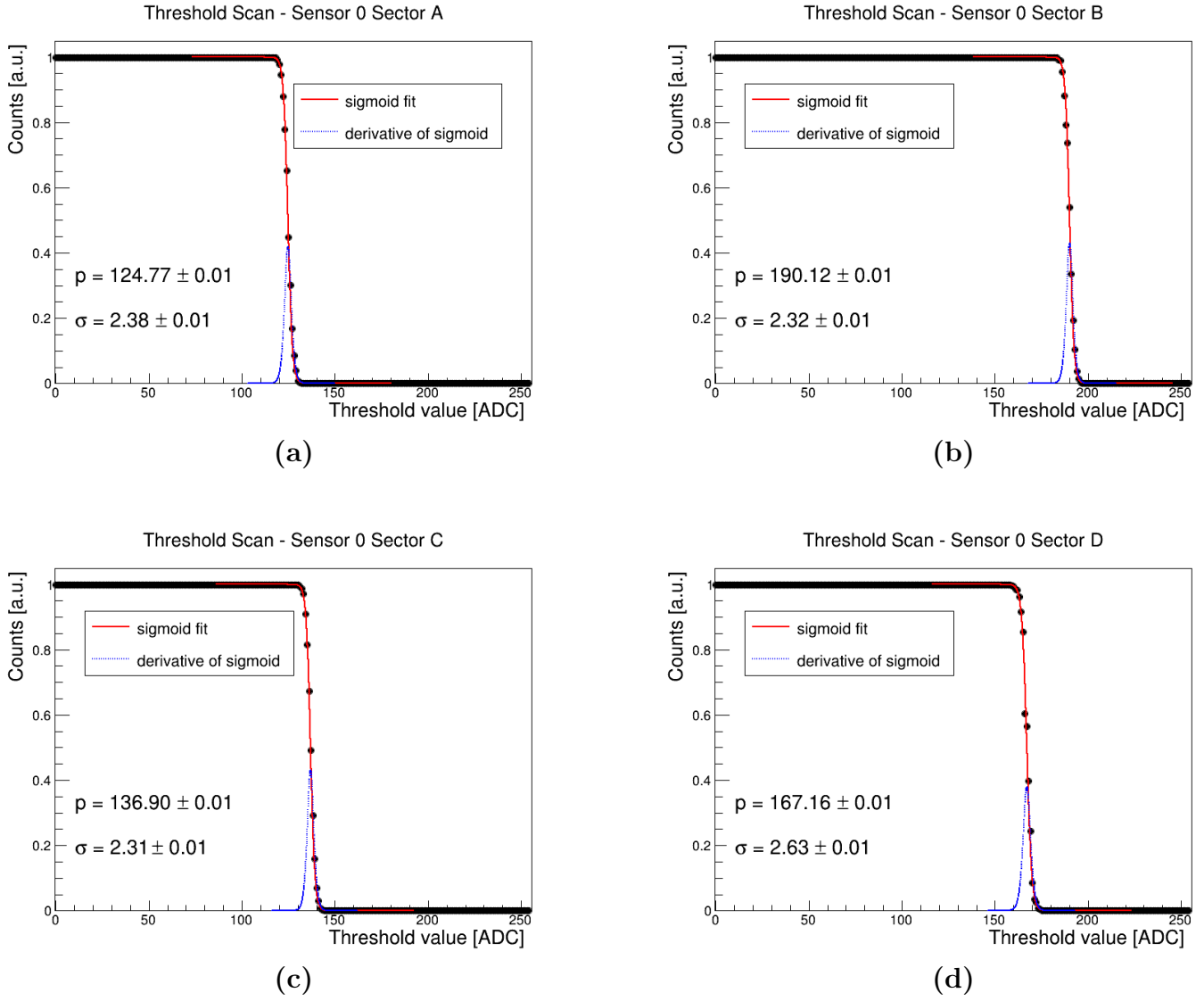


Figure 4.5: Threshold scans of the four groups of the first layer of M28 chip. The scans illustrate the fraction of active columns as a function of the applied threshold (in ADC) for each region. The values of pedestal and σ correspond to the derivative of a sigmoid function.

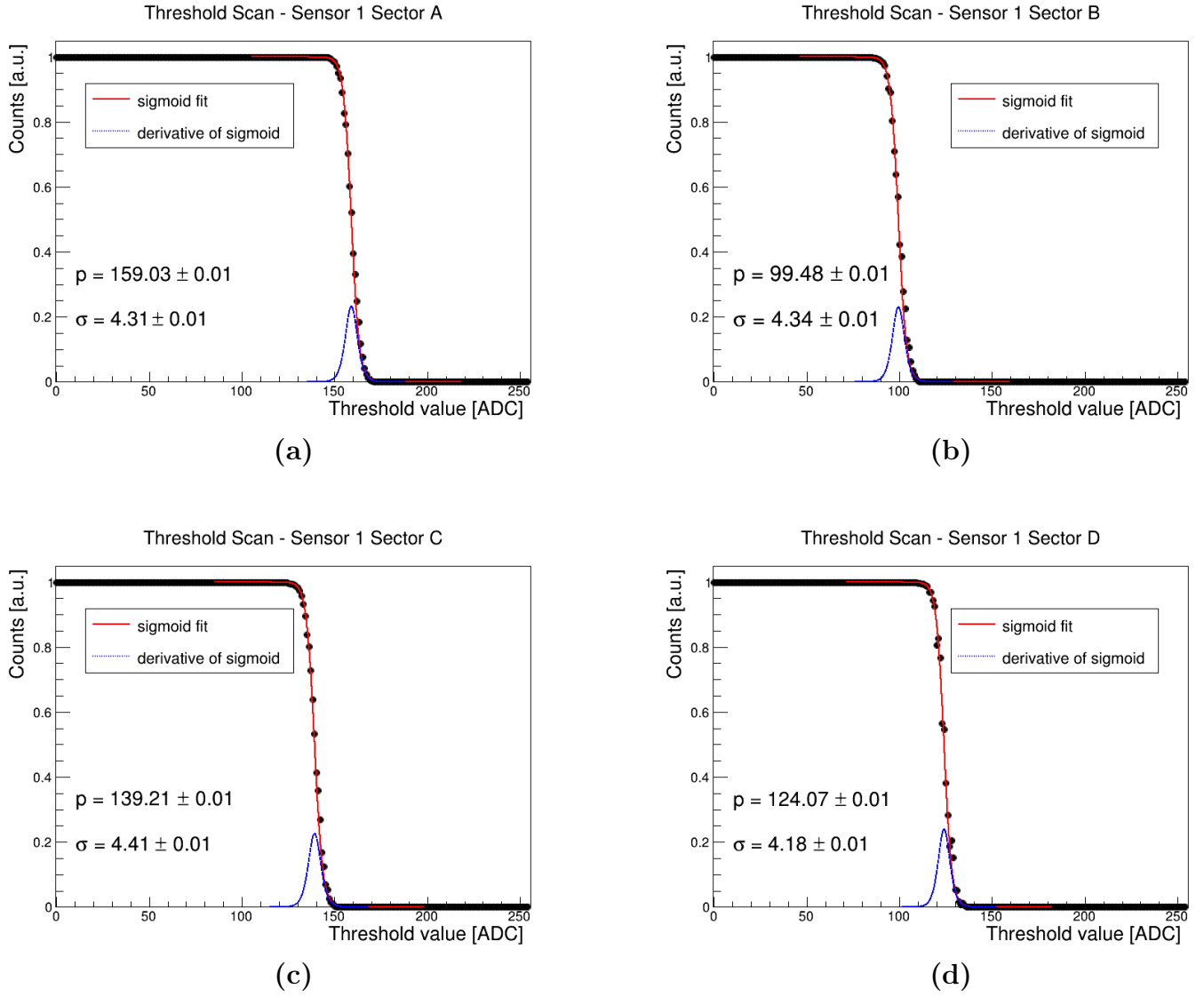


Figure 4.6: Threshold scans of the four groups of the second layer of M28 chip. The scans illustrate the fraction of active columns as a function of the applied threshold (in ADC) for each region. The values of pedestal and σ correspond to the derivative of a sigmoid function.

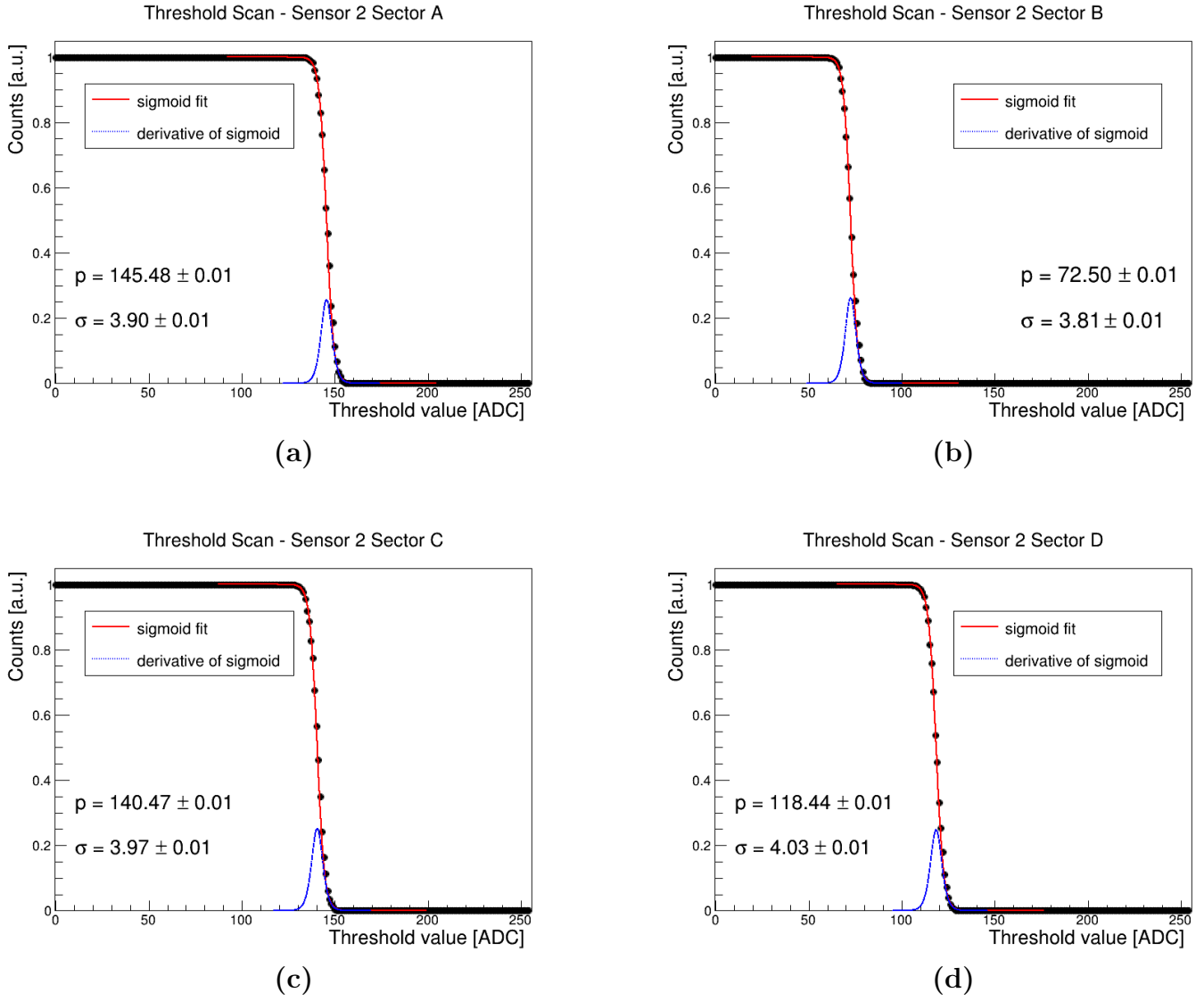


Figure 4.7: Threshold scans of the four groups of the third layer of M28 chip. The scans illustrate the fraction of active columns as a function of the applied threshold (in ADC) for each region. The values of pedestal and σ correspond to the derivative of a sigmoid function.

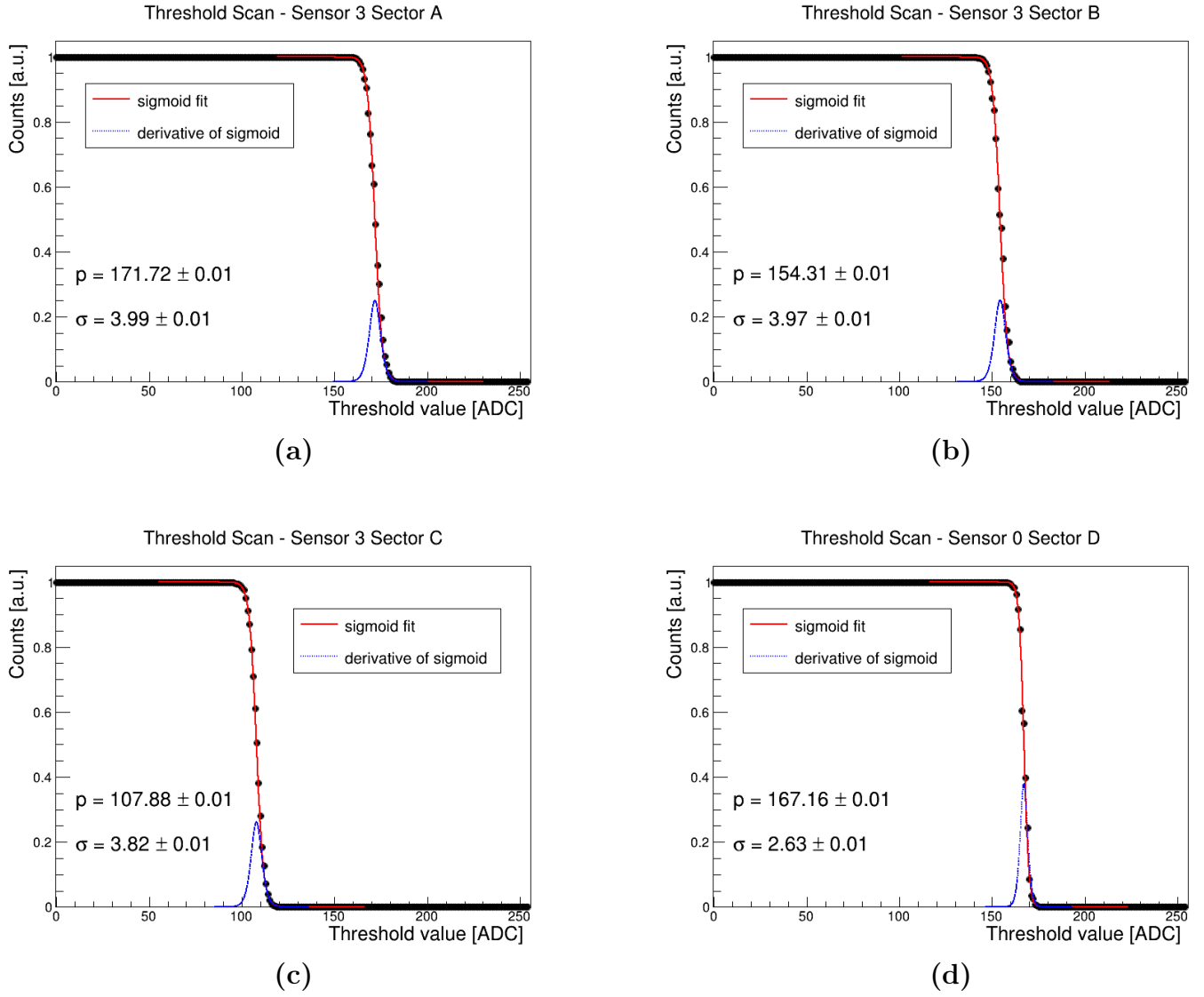


Figure 4.8: Threshold scans of the four groups of the fourth layer of M28 chip. The scans illustrate the fraction of active columns as a function of the applied threshold (in ADC) for each region. The values of pedestal and σ correspond to the derivative of a sigmoid function.

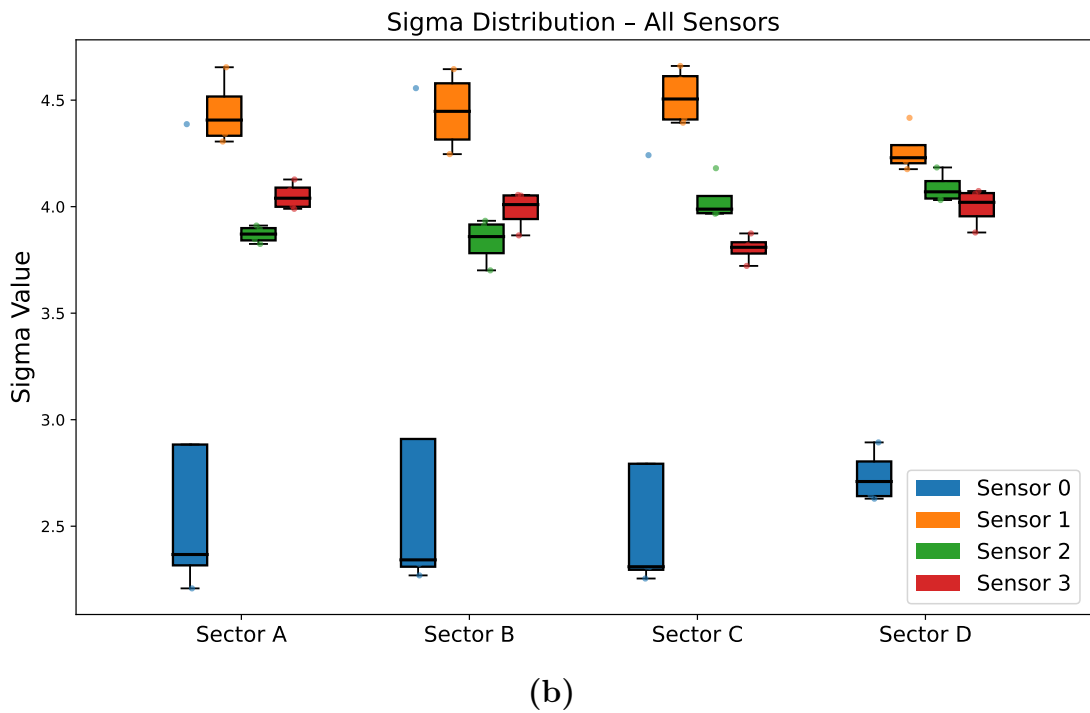
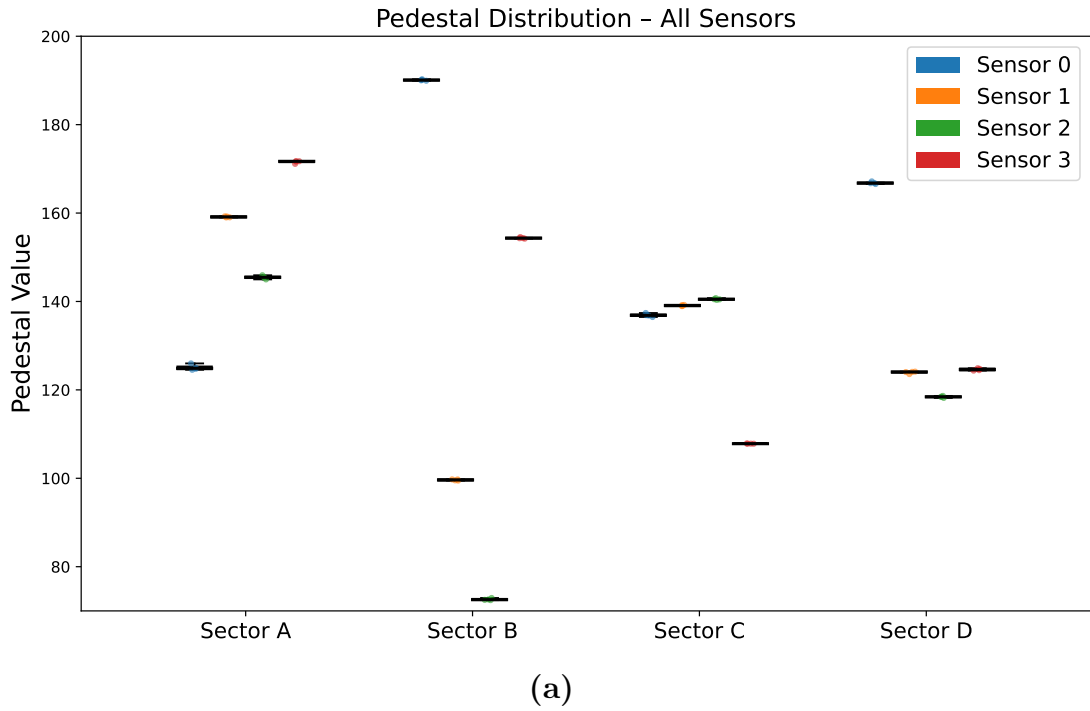


Figure 4.9: Box plots of pedestal (a) and sigma (b) distribution of every sensor and sector for different measurements throughout the data taking. Pedestal values are very stable, while temporal noise distribution shows instabilities for sectors A,B,C of sensor 0.

Sensor	Sector A	Sector B
0	125.07 ± 0.63 [0.5%]	190.09 ± 0.14 [0.1%]
1	159.13 ± 0.12 [0.1%]	99.63 ± 0.14 [0.1%]
2	145.46 ± 0.38 [0.3%]	72.61 ± 0.22 [0.3%]
3	171.57 ± 0.29 [0.2%]	154.36 ± 0.15 [0.1%]
Sensor	Sector C	Sector D
0	136.90 ± 0.37 [0.3%]	166.82 ± 0.25 [0.1%]
1	139.08 ± 0.13 [0.1%]	123.95 ± 0.25 [0.2%]
2	140.53 ± 0.17 [0.1%]	118.42 ± 0.18 [0.2%]
3	107.84 ± 0.03 [0.0%]	124.62 ± 0.25 [0.2%]

Table 4.2: Temporal variation of pedestal values for every sector of every sensor, in terms of mean $\pm$ std [%]. They remained stable throughout all the data taking of CNAO 2024.

Sensor	Sector A	Sector B
0	2.83 ± 1.04 [36.7%]	2.88 ± 1.12 [38.9%]
1	4.44 ± 0.16 [3.5%]	4.45 ± 0.19 [4.2%]
2	3.87 ± 0.04 [1.0%]	3.84 ± 0.11 [2.8%]
3	4.05 ± 0.06 [1.6%]	3.98 ± 0.09 [2.2%]
Sensor	Sector C	Sector D
0	2.78 ± 0.98 [35.1%]	2.74 ± 0.12 [4.5%]
1	4.52 ± 0.13 [2.9%]	4.26 ± 0.11 [2.5%]
2	4.03 ± 0.10 [2.5%]	4.09 ± 0.07 [1.7%]
3	3.80 ± 0.06 [1.7%]	4.00 ± 0.09 [2.2%]

Table 4.3: Temporal variation of σ values of noise for every sector of every sensor, in terms of mean $\pm$ std [%]. The highest instability concerns sectors A,B,C of Sensor 0, with a variation of more than 35% throughout all the data taking.

4.2.2 M28 Efficiency

In order to optimize the VTX detector for physics data taking, it is possible to study its performance when irradiated by a beam. In the following section, several types of efficiency are evaluated, varying the threshold set to each sensor, the energy and the type of incoming charge particles.

Threshold efficiency

According to Eq. 4.1, noise thresholds of each M28 chip were determined using the values of p and σ evaluated in Sec. 4.2.1. With protons at 230 MeV, data acquisition occurred by changing the threshold in units of σ , respectively, at values of $n = 4, 4.5, 5, 5.5, 6, 7$. In this way, detector efficiency is performed at different discrimination values.

The involved experimental setup used for efficiency measurements includes the SC, BM and VTX, as sketched in Fig. 4.10. Once the SC is triggered by the primary, an event is recorded. The reconstructed track inside the BM will represent a reference to the four layers of the VTX detector. To make the study as clear as possible, only events where one BM reconstructed track is considered, whose projection is inside the 4 layers of M28 geometrical acceptance. Moreover, since the interests focus on the capability of the primary beam detection, only events with only one VTX reconstructed track are taken into consideration. In particular:

- the *sensor efficiency* is obtained by projecting the BM track to each layer of the VTX. The closest cluster to the track intersection is identified and, if its distance is closer than 2 mm, the sensor efficiency is considered successful

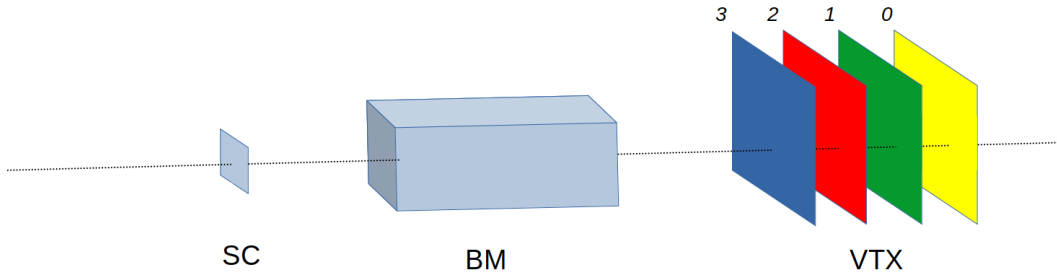


Figure 4.10: Set up used for efficiency studies. Events with a BM track reconstructed within the VTX geometrical acceptance are considered. VTX clusters and tracks are studied with respect to BM tracks distances. The index of M28 sensors is shown according to their spatial position with colors that will be used in the following plots.

for the specific event:

$$\epsilon_{sensor} = \frac{\text{events with VTX cluster matching the BM track}}{\text{events with BM tracks inside VTX acceptance}}; \quad (4.3)$$

- the *track efficiency* is computed as a comparison between the BM track and the VTX track. The latter is obtained by a reconstruction algorithm in which, starting from the last layer backward, at least 3 clusters of different layers are identified as a track, if validated through a χ^2 selection (see Sec. 5.2.2). If the BM track projection at the target matches the analogous VTX track projection (at a distance lower than 2 mm), the track efficiency holds, meaning they belong to the same crossing particle:

$$\epsilon_{track} = \frac{\text{events with VTX track matching the BM track}}{\text{events with BM tracks inside VTX acceptance}}. \quad (4.4)$$

Every sensor efficiency as a function of the discrimination value n of the threshold is reported in Fig. 4.11-a. Sensors 1,2,3 show a similar pattern with an efficiency over 90% for thresholds higher than 5σ , while a drop emerges for lower values. An explanation of that could be linked to the higher noise at lower thresholds. As explained in Sec. 4.1.2, a maximum of 36 pixels can be read out for every row of the M28 pixel array: if the acquisition is very noisy, the good signal pixel can be hidden by them. Sensor 0, instead, presents quite a stable efficiency for all the threshold values, which are still lower than 90%. Being aware that this layer features instabilities (as marked in Sec. 4.2.1), the lower efficiency could be associated with the fact that noisy pixels invalidate signal detection in several columns. To limit acquisition dead time, they were masked via hardware before physics data acquisition.

The tracking efficiency is reported in Fig. 4.11-b. Since each VTX track is formed by at least three clusters, a good reconstruction is achieved even with the lower efficiency of sensor 4. In fact, tracking performance is comparable to that of sensors 1, 2, and 3.

Fig. 4.12 reports the mean cluster size in terms of number of pixels, for protons at 230 MeV crossing the sensors at several thresholds. Following a behavior in agreement with the previous evaluations, Sensors 1,2,3 reach a maximum cluster size of ~ 5 pixels in the range between 4.5 and 5σ , decreasing for higher thresholds. The pattern is also similar for Sensor 0, but with lower cluster size values as a mean. This fact could explain the lower cluster efficiency of this sensor with respect to the others, since low-deposition-energy particles could not be reconstructed in the proper way.

Another fundamental aspect to consider is the cluster noise, which represents the number of reconstructed clusters not belonging to a track. The chosen events

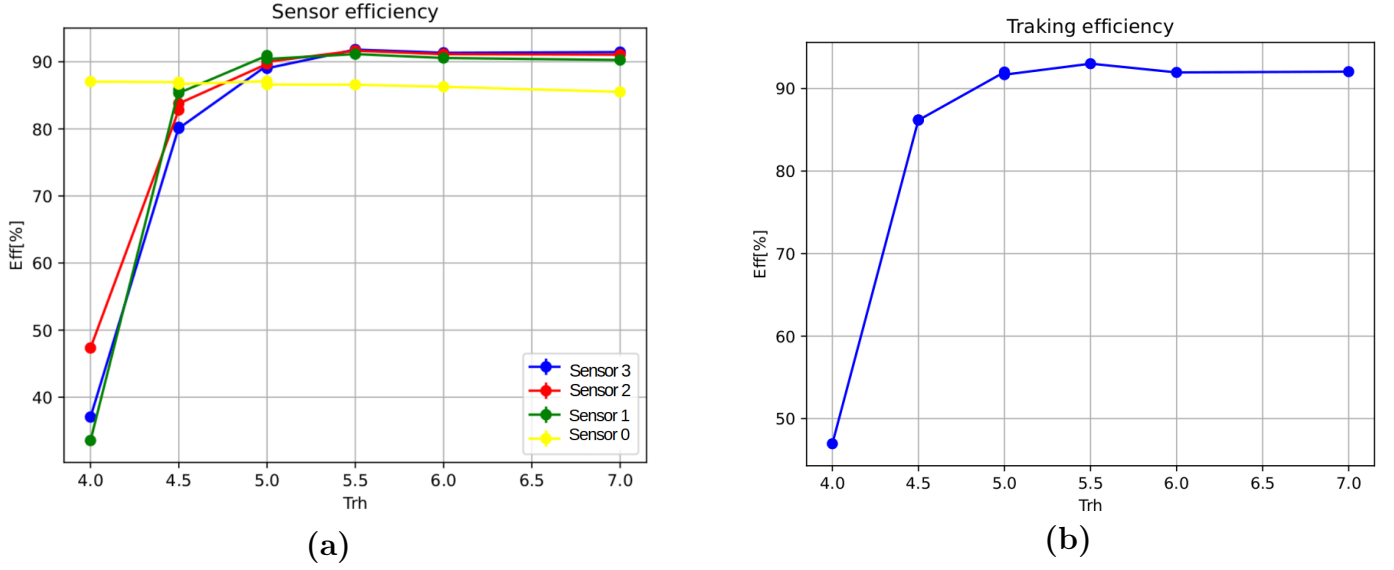


Figure 4.11: a) Sensor efficiency as reported in Eq. 4.3 as a function of the threshold. b) Track efficiency as reported in Eq. 4.4 as a function of the threshold. A proton beam at 230 MeV is used.

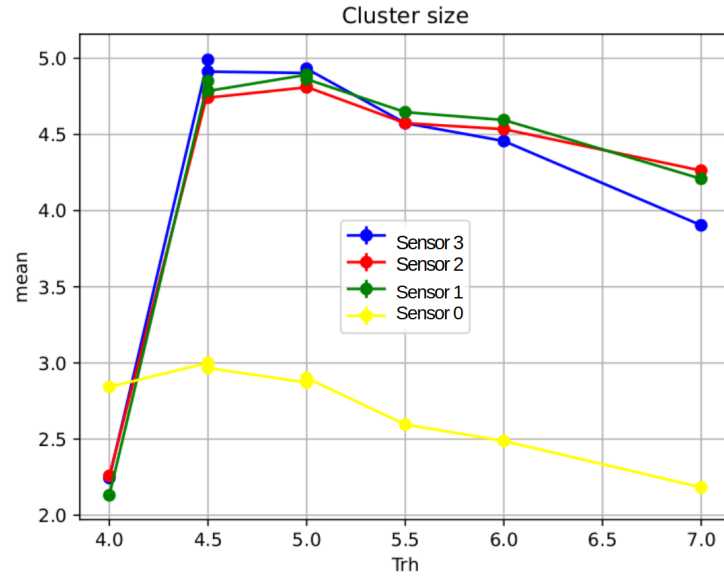


Figure 4.12: Average cluster size for every Sensor as a function of the threshold for a proton beam at 230 MeV.

are required as before, and the counts of noisy clusters per event are reported in Fig. 4.13. Up to a threshold value of 5σ , the noisy level is significantly high,

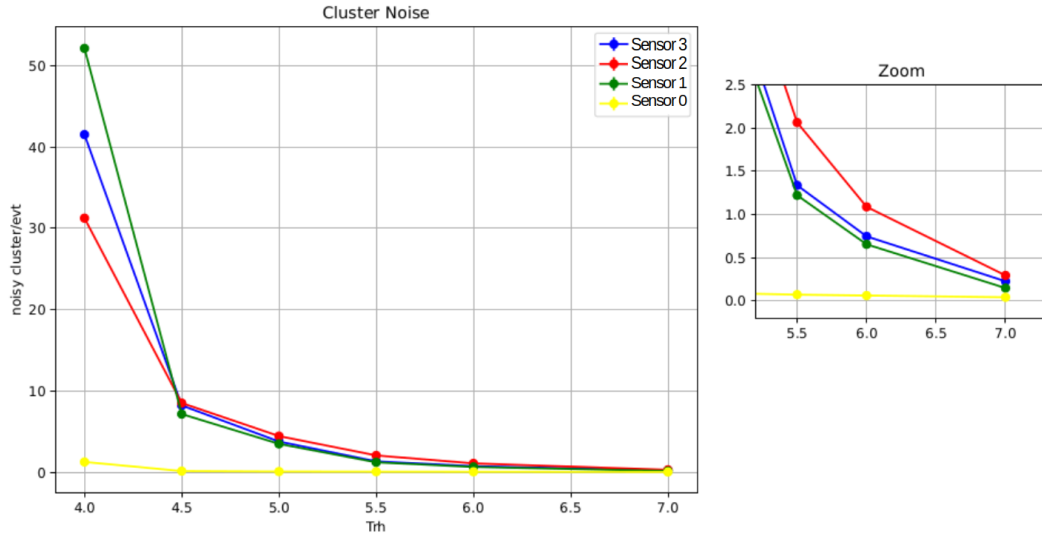


Figure 4.13: Cluster noise per event as a function of the threshold, namely the number of clusters not belonging to a beam track per event. A proton beam at 230 MeV is used.

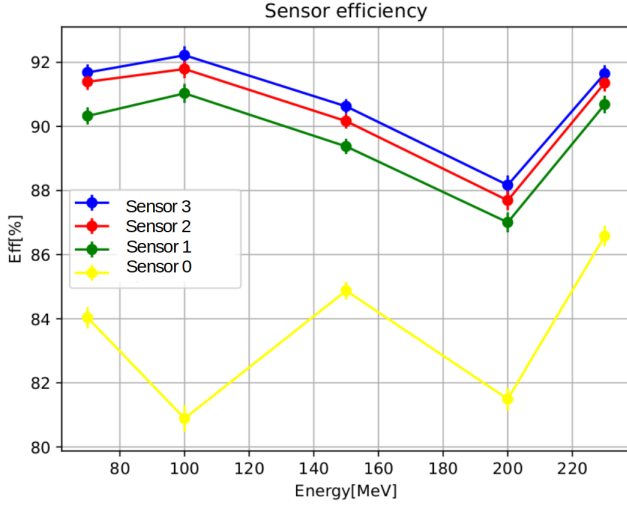
becoming reasonable (≤ 2 clusters) for values higher than 5.5σ - 6σ . This trend is followed by Sensors 1,2,3 and, again, it is lower for Sensor 0 because of its overall lower efficiency in detection.

It is also measured that the cluster size due to noise is generally 1 pixel per event, for every chip.

At this stage, considering sensor and tracking efficiency, average cluster size and the contribution from noisy clusters, it is possible to define an optimal working point for both data acquisition and analysis. A reasonable compromise is a threshold set at 6σ , which ensures a tracking efficiency above 90% while reducing the noise level to approximately one cluster per event. Although the average cluster size decreases to fewer than 5 pixels, it remains sufficiently large to allow reliable discrimination from noise.

Cluster efficiency as a function of the deposited energy

Once the threshold for all M28 sensors is set at 6σ , it is possible to study the behavior of VTX with protons at different energies, ranging from 70 to 230 MeV. Sensor and track efficiencies, with an analogous definition as in the previous paragraph, are reported in Fig. 4.14 as a function of energy.



(a)



(b)

Figure 4.14: a) Sensor efficiency as reported in Eq. 4.3 as a function of the beam energy. b) Track efficiency as reported in Eq. 4.4 as a function of the beam energy.

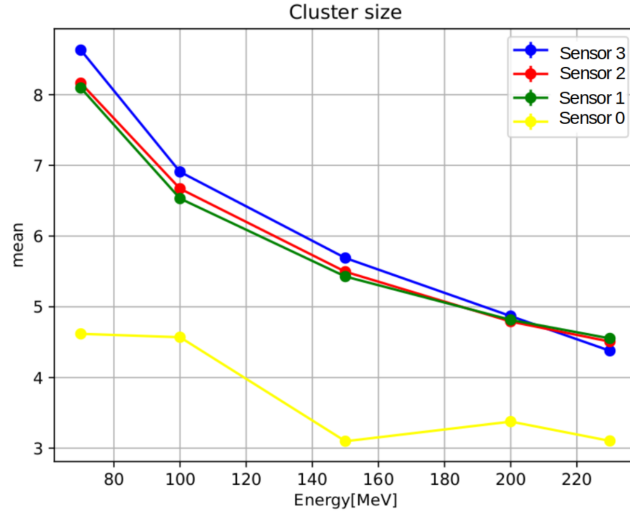


Figure 4.15: Average cluster size as a function of the proton beam.

Sensor 0 shows a lower efficiency than the others, around 82%, probably due to its softer response. Sensors 1,2,3 show a value around 90%, which seems energy-dependent at first glance. However, systematic uncertainty around 2% has been estimated in the BM-VTX selection process, putting the efficiency variations as a consequence of systematic fluctuations. As previously noted for threshold studies, tracking efficiency closely follows the behavior of the most efficient layers. Overall,

the results indicate that setting the threshold at 6σ yields an efficiency of approximately 90% for proton detection, largely independent of energy.

To better interpret the dependence of the detection efficiency on the proton energy, it is useful to relate the nominal beam energy to the one actually deposited in the silicon sensors. Table 4.4 summarizes the estimated deposited energy in 50 μm of detector and 15 μm of active material, as well as the corresponding number of generated electron–hole pairs, assuming a mean ionization energy of 3.6 eV per pair [128].

Proton Energy [MeV]	E_{dep} (50 μm) [keV]	E_{dep} (15 μm) [keV]	e–h pairs [$\times 10^3$]
70	180	54	15
100	140	42	12
150	110	33	9
200	93	28	8
230	85	25	7

Table 4.4: Estimated energy deposition and corresponding number of electron–hole pairs produced by protons of different kinetic energies in silicon. Values are derived from NIST PSTAR stopping power data, assuming a silicon density of 2.33 g/cm³ and an average ionization energy of 3.6 eV per pair.

The deposited energy decreases with increasing proton kinetic energy, reflecting the typical $1/\beta^2$ behavior of the stopping power (Eq. 1.2). Consequently, the number of generated electron–hole pairs is slightly larger for low-energy protons. The efficiency dependence on the beam energy shown in Fig. 4.14 can therefore be reinterpreted as an effect of the varying deposited energy and thus charge collection within the epitaxial layer. Nevertheless, the resulting change in the number of generated electron–hole pairs is limited, leading to the nearly constant efficiency observed across the investigated proton energies.

Noise considerations

Once the efficiency has been evaluated as constant at different proton energies, the cluster noise is compared in three different configurations: in runs with carbon ions at 200 MeV/u, with protons at 230 MeV and without beam. For beam runs, the cluster noise is evaluated as previously stated. In particular, with the C beam, a further cut on clusters due to fragmentation in the VTX has been implemented, in order to limit their misrecognition as noisy clusters as much as possible.

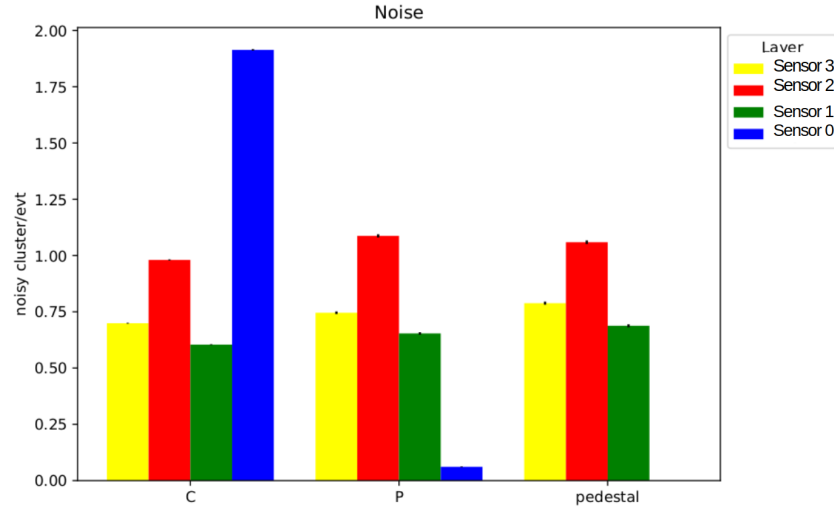
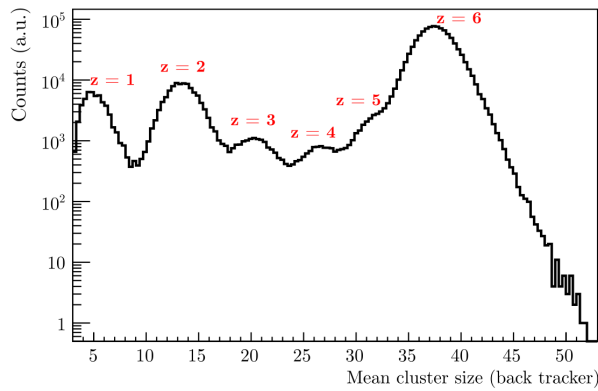
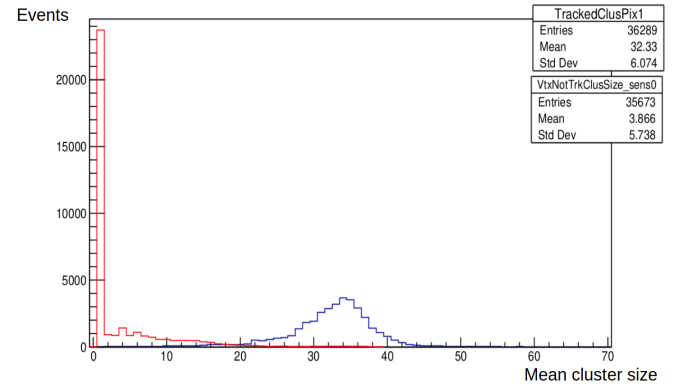


Figure 4.16: Noisy Cluster size for runs with carbon ions at 200 MeV/u, with protons at 230 MeV and without beam.



(a)



(b)

Figure 4.17: a) Mean cluster size distribution measured for 287.50 MeV/u carbon ions, from [114]. b) Cluster size distribution for noise clusters (red), and tracked ones (blue), with a beam of ^{12}C at 200 MeV/u.

For runs without beam, on the other hand, where tracks are not reconstructed, all the clusters are considered as noisy and then averaged by the number of events. Noisy cluster results are reported in Fig. 4.16. For Sensors 1,2,3, the result of ~ 1 noisy cluster per event, regardless of the type of run, is consistent with what was already found. The variation of noisy pixels in Sensor 0, on the other hand, is a consequence of its temporal instability, as already studied with Fig. 4.9, where the performance of the sensor under the same conditions changed with time.

Fig. 4.17-a shows the cluster size of elements generated after the fragmentation of ^{12}C at 287.50 MeV/u using M28 sensors [114]. Since $dE/dx \propto z^2$ by Eq. 1.2, the cluster size increases for heavier fragments, when more and more pixels are fired due to diffusion after the particle crossing. In particular, the cluster size increases from protons by a factor of 2 for He, up to 8 for C, discriminating signal and noise more effectively. This can clearly be seen, for example, in the difference of cluster size of noisy clusters and ^{12}C in Fig. 4.17-b, where a beam at 200 MeV/u was deployed.

Following the threshold and energy studies with different beam particles, the presented results provide strong evidence that the chosen working point for the four M28 layers of the VTX detector is well-optimized for effective data taking. In particular, the detector demonstrates reliable performance even with protons, particles with the lowest energy release, ensuring robust detection of all the fragments and enabling accurate analysis of elemental cross sections.

Chapter 5

Data processing and Analysis Software

SHOE (Software for Hadrontherapy Optimization Experiment) is a multipurpose software developed by the FOOT Collaboration, designed to cover all the event reconstruction steps, from data acquisition or simulation to final analysis results, handling every sub-detector processing. The flexible framework follows a C++ object-oriented programming logic and is based on ROOT libraries [129], including also external ones for specific purposes. The workflow is schematically reported in Fig. 5.1 and will be discussed in the next sessions, focusing on the steps developed in the scope of this thesis.

5.1 Data Input

Raw experimental data is a binary data stream written by the TDAQ system (see Sec. 3.2.4) as a hierarchical structure, encoded in a digital format of words of 4 bytes each. It is composed of a series of *full events* (see Fig. 5.2-a) which consist of a header that describes the entire event (fragment size, time stamp, run number etc.) and a payload, a collection of data *fragments*, namely data from all the triggered relevant subdetectors. In turn, every fragment is composed of a header which contains metadata, among which a subsystem univocal ID, and detector raw data (see Fig. 5.2-b).

On the other hand, simulations are obtained using FLUKA [130] or GEANT4 [131], Monte Carlo packages widely used in particle and nuclear physics for modeling the interaction and transport of particles through matter. In these packages, the magnetic field and the geometry and material of the detectors are implemented to correctly simulate the interactions and fragmentation throughout the overall ex-

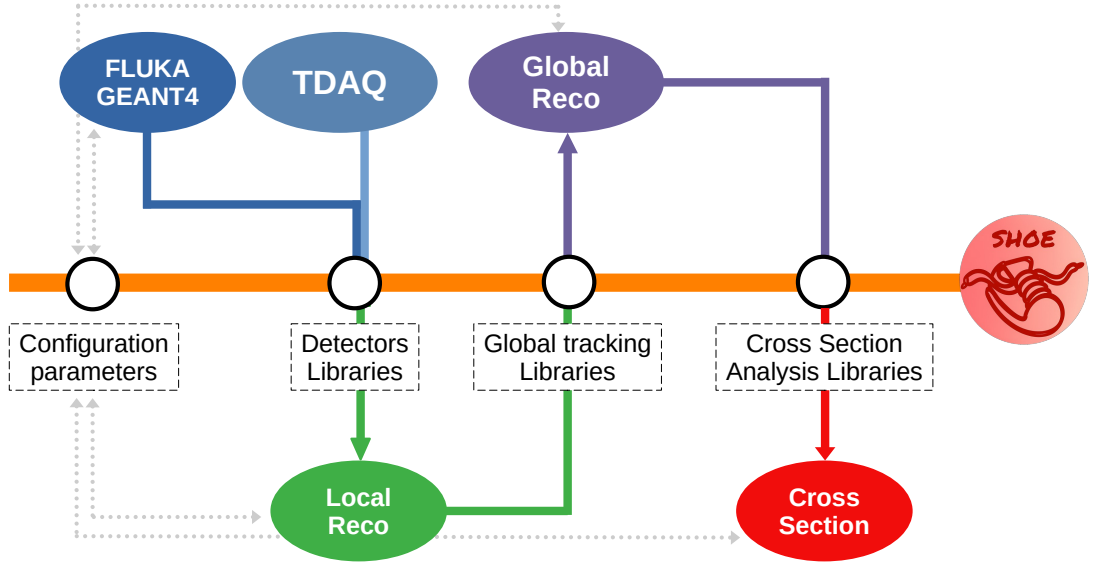


Figure 5.1: SHOE workflow. Data (from simulations or experimental acquisitions) are processed by libraries specific to every detector (*local reconstruction*), and put together in the reconstruction of global tracks (*global reconstruction*). The main physical quantities from global tracks (charge, momentum, kinetic energy, etc.) are retrieved for cross section measurements.

perimental setup, with detector positions set to be as consistent as possible with those used during the experimental campaigns. The FOOT implementation of the FLUKA code is specifically configured to produce a customized event-by-event output, adapted to store information that enables a complete understanding and reconstruction of events, including space-time coordinates, kinematics and energy depositions within the active detectors [132]. The results of the simulation of each event are then converted in a ROOT format and stored on file.

5.2 Local Reconstruction

5.2.1 Level 0

SHOE contains a reader process whose aim is to decode the raw data stream of the TDAQ system. The metadata contained inside the headers of both fragments and full events guides the parsing of detector data and lets the reader jump directly to the specific fragments' payload. At this point, 4-byte words are translated back into meaningful physics quantities, organized by their source and channel in

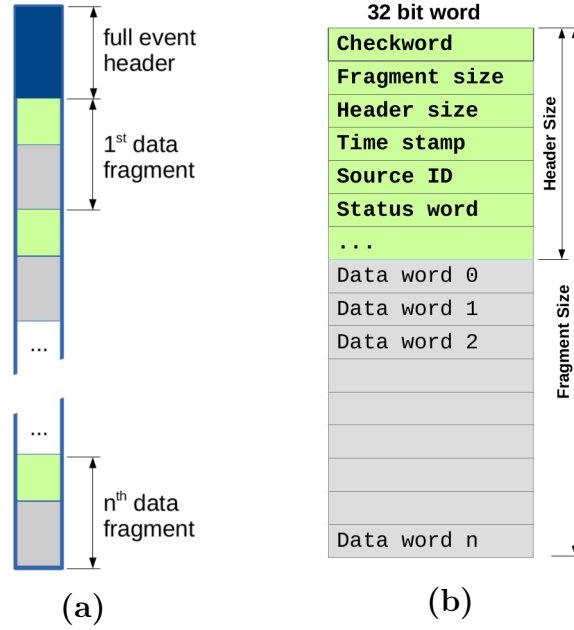


Figure 5.2: a) Composition of a *full event*, made of a header and a payload. b) The payload is a collection of *fragments*, each of which contains a subdetector header and its raw data. Every word is made of 4 bytes.

a SHOE format.

On the other hand, simulated data undergo a digitization process, where the information of the Monte Carlo particle crossing a material is converted into digital signals that mimic the real detector response. For example, by retrieving the crossing position of a particle into a layer of the VTX and its energy loss, it is possible to reconstruct the size and shape of fired pixels by a reasonable model of charge sharing. Results are obtained in a format comparable to the experimental one. This allows the same analysis procedure to be applied to both simulated and experimental data within the same SHOE software.

5.2.2 Detector Reconstruction

Coming from the level 0 reconstruction, the raw information of every detector is processed to obtain suitable objects for the analysis. Based on the specific data acquisition, a *campaign manager* has been developed in order to address the appropriate configuration, geometry and calibration files during reconstruction. Moreover, it is possible to choose which detector to process and which information to save, in an output which is still in a ROOT format, containing histograms and/or trees according to the needs.

Among all the processes, vertex tracking and charge identification are two fundamental algorithms for an accurate fragment reconstruction.

Vertex Tracking

A VTX track contains the point of interaction of the primary with the target and its projection in the four layers of the VTX. At first, beginning with the fired

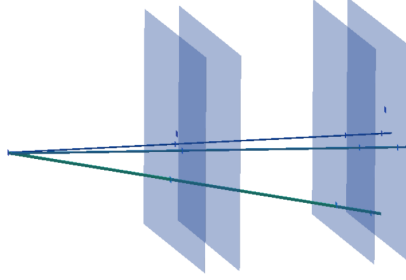


Figure 5.3: GEANT4 Event display showing a reconstructed VTX point belonging to three tracks, with two additional clusters not associated.

pixels of the vertex, a clustering algorithm is used to reconstruct the crossing point of the ionizing particle through the sensor, giving spatial information. The vertex tracking algorithm, then, exploits the information of the cluster of each layer of the VTX detector (4 layers of M28 chips as stated in Sec. 3.2.2 and Chap. 4) to reconstruct the particle track.

The track reconstruction algorithm involved in SHOE was inherited from the FIRST experiment and is described in detail in [133]. For each event, the cluster from the farthest layer from the target is selected. Then, all possible combinations of this cluster with those of the previous layers are taken, creating a preliminary track as a twice 2D (x-z and y-z) fitted line through the cluster coordinates, which is updated for every cluster added. Once the closest layer is reached, if the track includes at least three clusters and passes a χ^2 quality criterion, it is accepted and stored and the associated clusters are excluded from further reconstruction attempts. This procedure is repeated for all clusters in the fourth and third farther layers to account for the minimum requirement of three clusters.

Once all the tracks of an event are reconstructed, it is crucial to accurately identify the common point from which they originate, the so-called *vertex point*. Each track is extrapolated to a given plane, which is chosen to be orthogonal to the beam axis and within the target volume. The centroid of the transversal track positions on that plane is calculated, which defines the transversal vertex point

position. Then, the longitudinal (Z) position is determined by minimizing the average of track distances from the centroid as a function of the Z value, utilizing a binary search algorithm to improve convergence [133]. An example of VTX point and tracks reconstruction is shown in Fig. 5.3.

Charge reconstruction

A TW point represents the spatial position of interaction of a charged particle with the TW (see Sec. 3.2.3). It is obtained by matching hits from the TW's two orthogonal layers, assuring unique values of energy deposited (ΔE), Time of Flight (ToF) and position for every point. The raw energy loss on the TW, in particular, is calibrated position-by-position using Monte Carlo simulations to obtain the final calibrated ΔE [117].

The Charge Z identification algorithm exploits the Bethe-Bloch formula (see Eq. 1.2), which describes the relationship between ΔE and ToF for particles of a specific charge Z. In particular, Bethe-Bloch curves of every charge hypothesis are derived from Monte Carlo and fitted as a function of ToF , as reported in Fig. 5.4. Every TW point is then assigned to the closest Bethe-Bloch curve among various charge hypotheses using a dichotomic algorithm. A purity correction is also applied to address instances of mis-identification, in particular for the regions where there is no clear separation between different fragment charges. More details are reported in [134].

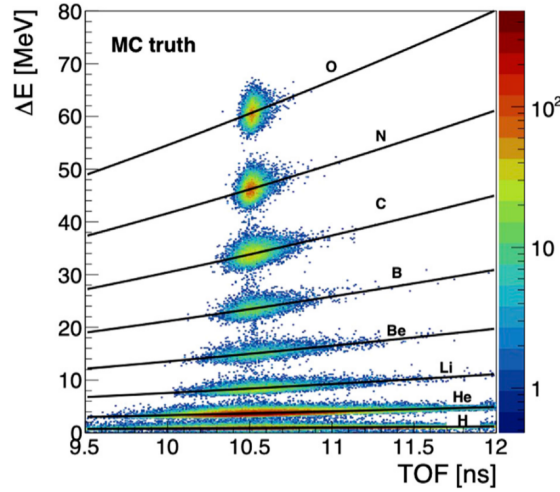


Figure 5.4: ΔE vs. TOF for fragments detected by TW detector (front layer) using the Monte Carlo truth information from a simulation of a 400 MeV/u ^{16}O beam interacting with a C target.

5.3 Global Reconstruction

From the local reconstruction, the main observables from each detector are obtained, including the physical position where the charged particle interacts with the sensors. The next goal is thus the reconstruction of the particle's trajectory across the entire experimental setup. This process is referred to as *global tracking*, as it combines the individual local hits to form a coherent global track.

The algorithm used in the analysis of this thesis is developed by the Bologna group and its operation is based on the GENFIT library (Sec. 5.3.1) which is tuned properly for the FOOT purposes.

5.3.1 GENFIT toolkit

GENFIT [135] is an open-source, experiment-independent C++ toolkit that performs particle tracking reconstruction in high energy physics and nuclear physics fields. Fully integrated with ROOT and GEANT4, it offers a flexible structure capable of handling different detector types and measurements, even in environments with significant noise and background. GENFIT is built upon three main pillars: *measurements*, *track representation* and *track fitting*.

The measurements encapsulate the observables obtained from every detector, including their uncertainties. In particular, GENFIT contains predefined classes for different detector types (pixels, strips, wires, etc.) and their deformation (bending, misalignment, etc.), which will be taken into consideration for the tracking reconstruction.

A track representation consists of defining a set of parameters that univocally represent the state of a particle track in a local position and of an extrapolation model that solves the equation of motion of the charged particle in a magnetic field in any position. The GENFIT track representation involved in SHOE is called **RKTrackRep** and considers a 5 coordinate vector to parameterize the state $\mathbf{x}$ of a particle track with position $\vec{r}$ and direction $\vec{a}$:

$$\begin{aligned}
 \mathbf{x} &= (q/p, u', v', u, v) \\
 u' &= \frac{\vec{a} \cdot \hat{u}}{\vec{a} \cdot \hat{n}} \\
 v' &= \frac{\vec{a} \cdot \hat{v}}{\vec{a} \cdot \hat{n}} \\
 u &= (\vec{r} - \vec{o}) \cdot \hat{u} \\
 v &= (\vec{r} - \vec{o}) \cdot \hat{v}
 \end{aligned} \tag{5.1}$$

where the state is parametrized in a local plane with origin $\vec{o}$ and normal versor $\hat{n}$, q and p are the charge and momentum of the particle and $\hat{u}$ and $\hat{v}$ are the

vectors of the local coordinate reference frame of the plane. u and v represent the coordinates of the track in the local reference frame of the plane, while u' and v' are the track direction cosines with respect to the $\hat{u}$ and $\hat{v}$ vectors defined on the plane. (see Fig. 5.5). The solution of the equation of motion is numerically obtained using a fourth order Runge-Kutta-Nyström (RKN) method [136], which takes into account magnetic field inhomogeneities that make analytical solutions difficult otherwise.

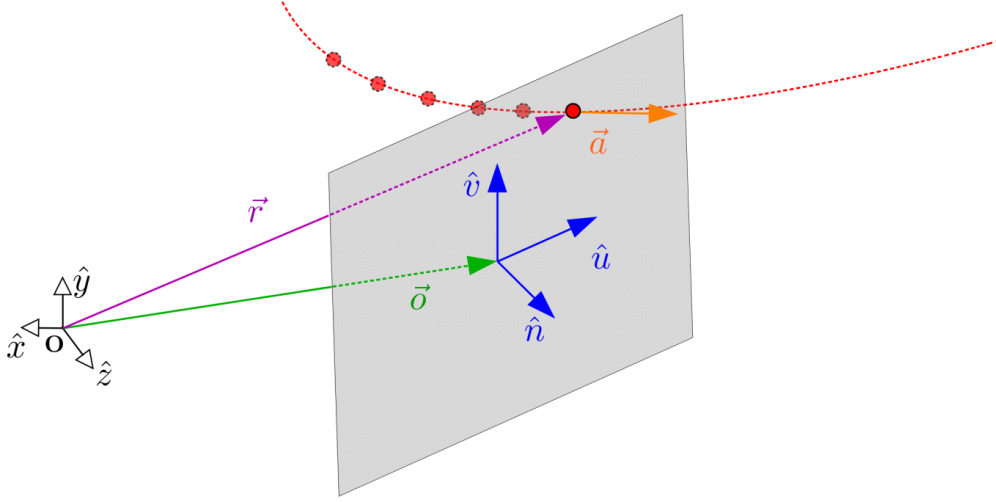


Figure 5.5: Visualization of a track parameterization in GENFIT. The origin $\vec{o}$ and local directional vectors $\hat{u}$, $\hat{v}$ and $\hat{n}$ of the plane are shown. From [120].

Measurements and track representations are then used for track fitting. Among several algorithms provided by GENFIT, the one implemented in SHOE is based on Kalman Filter [139], a recursive algorithm that aims to extract the best estimate of a system state vector (Eq. 5.1) and its parameters' covariant matrix from noisy measurements. In the first stage, the current state vector and covariance matrix are extrapolated (using the RKN model of the track propagator) to the next measurement point, considering all the material effects like energy loss and multiple scattering in the so-called "Noisy matrix" (*prediction state*). Then, the state vector and the covariance matrix are refined incorporating the new measurement through a "Kalman Gain" matrix which states the weight influence of every measurement (*update state*). A sketch of Kalman Filter procedure is shown in Fig. 5.6.

In comparison with a generic least squares fit, Kalman Filter provides a significant advantage in maintaining reconstruction accuracy, in particular for the low energy (and thus low momentum) fragments studied by FOOT, for which the effects of

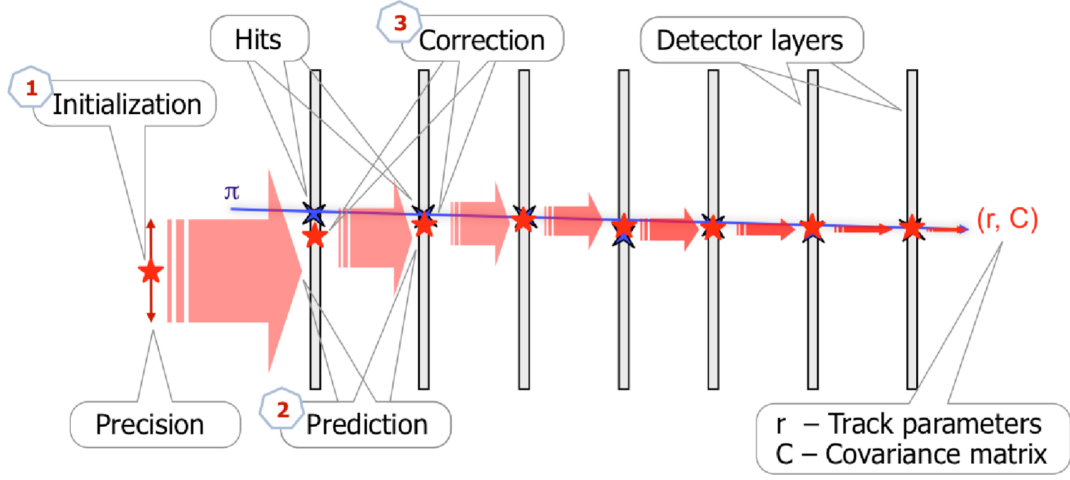


Figure 5.6: Sketch of a Kalman filter based track fitting algorithm. 1) The representation of a track starts from a seed (red stars). 2) Track is extrapolated to the next measurement point (red arrows) in the prediction state. 3) Fit is corrected taking into account the position of measurements and their resolution (blue stars) in the update state. This process is repeated for every measurement until the final fit, in which the representation of the track (parameters and covariance matrix) is finally obtained. From [137].

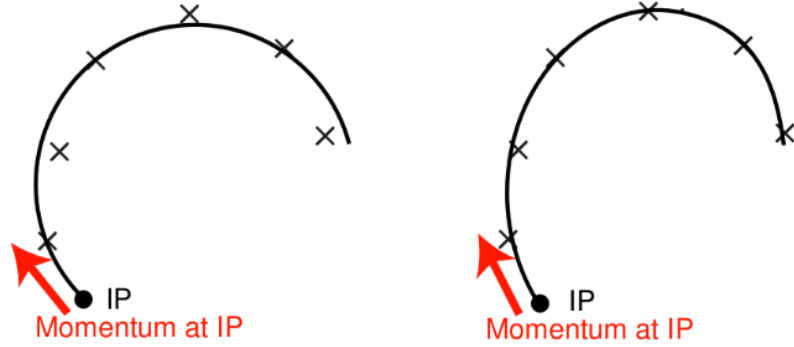


Figure 5.7: Comparison between normal least squares fitting (left) and Kalman filter based fitting (right) applied to the same trajectory. The track parameters are evaluated only once in the first case, while they are updated at each added point in the Kalman Filter approach and all the interactions with the material are taken into account in the propagation. From [138].

multiple Coulomb scattering and energy loss in the materials between tracking layers are far from negligible. A comparison between the two methods is reported in Fig. 5.7.

Another feature of the Kalman Filter algorithm introduced in GENFIT is that the fitting is progressively applied in forward and backward direction of the track, in order not to be biased by the initial seed state vector. This smoothing process gives final parameters as a weighted average of the two, increasing the robustness of the procedure.

5.3.2 Global Tracking

The first step in developing the SHOE global tracking system is to import the experimental setup geometry and magnetic field map into a GENFIT-compatible format, as defined in the configuration files managed by the campaign manager. Then, event by event, measurements from all the selected FOOT detectors are considered and uploaded to GENFIT.

At this point, a track finding algorithm is required to generate suitable track candidates, where all the processes of construction, parameterization and extrapolation are featured by GENFIT `RKTrackRep` track representation described in Sec. 5.3.1. Considering an event where the FOOT setup contains clusters from VTX, IT, MSD and TW in a magnetic field (Fig. 5.8-a), the initial candidate is the VTX track (see Sec. 5.2.2), which is used as a seed of parametrization for the global track. A linear extrapolation is then applied to all the geometry of the IT, including the closest clusters within a tolerance distance (Fig. 5.8-b).

From this stage, the RKN model is used for further extrapolations. However, since it has to solve the equation of motion of a charged particle in a magnetic field (as stated in Sec. 5.3.1), a seed state of its rigidity $R = q/p$ is needed, but, at the level of VTX and IT, no information about the possible charge and momentum of the particle is still known. Thus, different track representations are propagated in parallel, considering the charge Z reconstructed at every TW point (see Sec. 5.2.2) as a hypothesis. The momentum p , instead, is taken assuming the mass number A of the most abundant isotope of the corresponding Z , with a β close to that of the primary, in agreement with the behavior of a projectile fragment (see Sec. 1.3.2). The track is then extrapolated using RKN model to all the planes of MSD ((Fig. 5.8-c), adding a new cluster candidate if the distance to the track projection is below a certain threshold. For every plane, the track is fitted using Kalman Filter for all the representations, discarding the ones that do not satisfy the convergence criteria. Finally, the track is RKN extrapolated to the TW ((Fig. 5.8-d), looking for a match with a TW point. If added, the Z chosen in the previous representation is checked and corrected if needed, with the momentum p calculated for the Z , ToF and length of the track candidate.

It is possible to appreciate how, at the stage of track finding, the three pillars of *measurements*, *track representation* and *track fitting* of GENFIT are exploited together.

In the final step, all the track candidates attempt to be fitted using a Kalman Filter, according to specific requirements of p -value and $\chi^2/n_{d.o.f}$ of the fit during the iterations. The tracks which satisfy the fitting requirements are saved in a ROOT format as output of the all global reconstruction, in order to be used for further analysis.

Track efficiency

Track efficiency evaluates the performance of the implemented global tracking algorithm discussed in Sec. 5.3.2, disentangled from the behaviors of the involved detectors. An MC simulation of the setup is computed in order to compare the reconstructed quantities with the MC ones. Hence, the tracking efficiency is defined as:

$$\varepsilon_{track} = \frac{N_{reco}}{N_{ref}}. \quad (5.2)$$

N_{ref} represents the reference set of MC particles provided at the generation level, generated in the target and emitted in the geometrical acceptance of the setup, in particular crossing the TW. These are the particles that should be reconstructed within the FOOT experiment as global tracks. N_{reco} is the number of global tracks that are successfully fitted. This efficiency can be evaluated separately as a function of charge Z , angular emission θ or both.

The errors on ε were calculated from the variance of a binomial distribution:

$$\delta\varepsilon = \sqrt{\frac{\varepsilon(1 - \varepsilon)}{N_{ref}}} \quad (5.3)$$

In particular, only the reconstructed tracks associated to an MC particle that belongs to the reference set are considered. In this way, all sources of misreconstruction not related to the algorithm itself are omitted, such as fragmentation out of the target or other background sources. An evaluation of the capability of the algorithm to reconstruct reliable events is obtained with the purity:

$$p = \frac{N_{good\ reco}}{N_{all\ reco}} \quad (5.4)$$

where $N_{all\ reco}$ is the count of all reconstructed global tracks, while $N_{good\ reco}$ represents the counts of the global tracks associated to a reference MC particle. In other words, it represents the fraction of selected tracks that are actually true signal, as opposed to spurious or background events. Uncertainties are calculated using Eq. 5.3 as well.

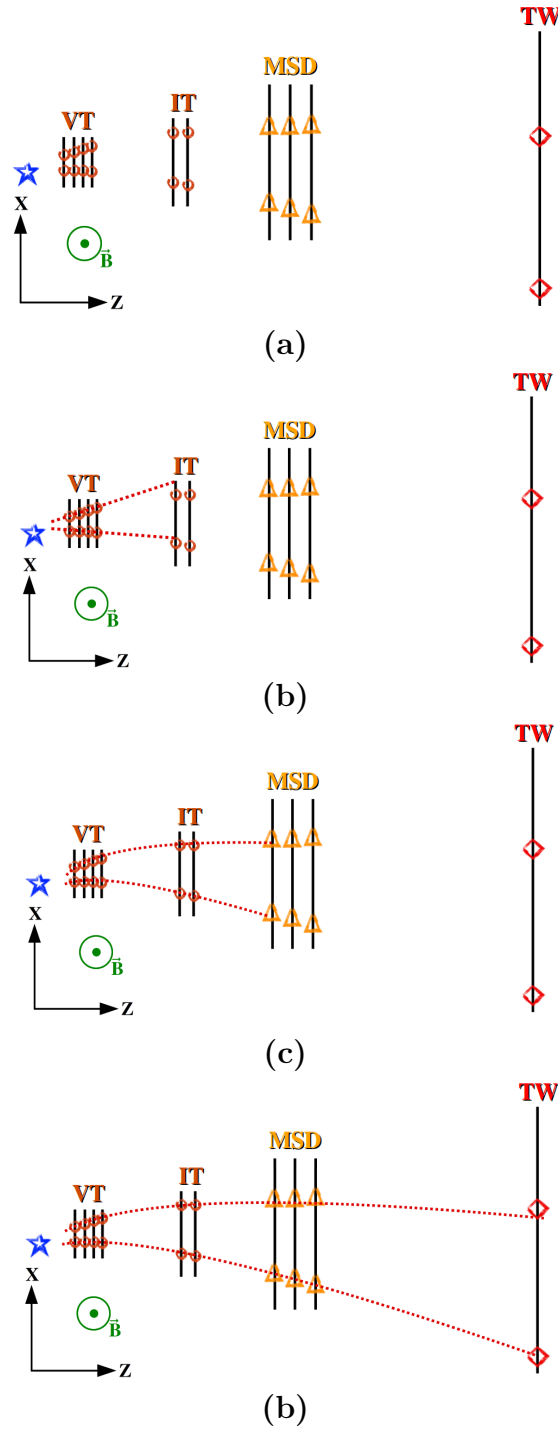


Figure 5.8: Tracking selection implemented in SHOE for a) an event of FOOT setup involving VTX, IT, MSD, TW in a magnetic field. b) The VTX tracks are considered as seeds and linearly extrapolated to the IT. c) The new candidate is RKN extrapolated to the MSD and d) then to the TW using the RKN model as well. From [120].

5.4 Cross Section Analysis

The SHOE analysis part is designed to exploit all the reconstructed physical quantities associated to global tracks, such as charge Z , angle of emission θ , momentum p and velocity β to extract cross section measurements (see Sec. 1.2.1). Its development represents an important core of this thesis and of my work during the PhD. In particular, starting from local procedures developed for specific tasks by the FOOT collaboration, I was eager to build a complete multi-purpose library that follows all the steps for cross section measurements, from event selections to data representation. Thanks to configuration parameters, the code is flexible to handle all FOOT experimental campaigns in which global tracking is involved but is still adaptive to satisfy specific requirements of every data taking, such as the absence of a detector or a different setup.

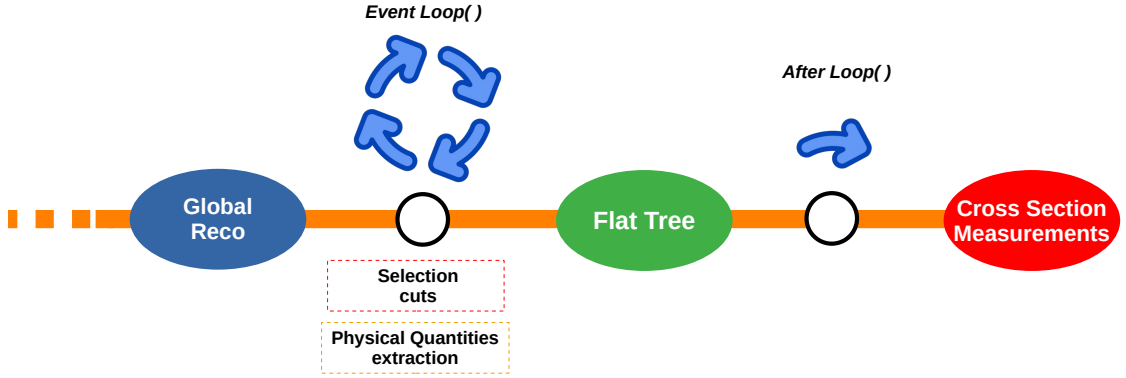


Figure 5.9: Cross Section analysis workflow. For every event (from experiment or simulation), selection cuts are computed and physical quantities are retrieved from global reconstruction of SHOE, filling a flat tree. Quantities in the tree (physical quantities and selection cuts) are used to extract cross section measurements.

The main structure of the cross section library is illustrated in Fig. 5.9. Starting from the global reconstructed SHOE dataset, every event is subjected to a series of selection criteria. Alongside these selections, relevant physical quantities for cross section measurements are stored in dedicated containers. They are organized in events or in tracks and further separated into reconstructed and Monte Carlo variables. At the end of the event processing, the overall information is stored in a flat TTree, which serves as a comprehensive dataset for performing cross section measurements and all the relevant studies, such as closure tests.

In the next sessions, a description of the main modules in which the analysis has been divided will be presented.

5.4.1 Selection cuts

In order to carry out an analysis on data with minimal background contamination, it is necessary to apply selection cuts that classify and filter both events and tracks. For this purpose, a dedicated selection cut class is implemented to evaluate several conditions on data and results are stored in containers that will serve as the basis for further analysis.

In particular, parallel cuts are performed on both MC (if present) and reconstructed information. The formers rely on generated truth values, used to investigate the exact origin of the particle and kinematic values in specific regions. The latters, instead, are based on detector measurements and investigates quality factors, cluster positions and thresholds.

Event cuts		
1	SCcut	There is NOT pileup in the SC.
2	BMcut	The BM detector reconstructs only one BM track.
3	TWnum	The number of GENFIT reconstructed global tracks is the same as the number of reconstructed TW points.
4	NTracksCut	The number of GENFIT reconstructed global tracks is higher than 1.
Track cuts		
5	VTXposCut	The reconstructed VTX point is positioned within the target dimensions.
6	TWclone	The specific global track has the same TW point as <i>at least</i> another track.
7	TrackQuality	The specific global track has <i>the worst</i> residual < 0.04 cm and a p-value > 0.01 .
<i>MC based Track cuts (valid only on simulations)</i>		
8	MC_isGoodReco	The most frequent MC ID of the global track is used to retrieve the corresponding MC truth particle. The particle is a primary or a fragment generated in the TG that crosses both TW planes and goes beyond (no secondary fragmentation).

Table 5.1: Numbered list of event and track selection cuts for reconstructed quantities, including simulation only cuts.

The main event and global track cuts concerning reconstructed quantities are listed in Tab. 5.1. Cuts 1 - 7 are introduced to filter out all sorts of misrecon-

Event cuts		
9	MCgoodEvent	The MC particle crosses the TG, meaning no fragmentation occurred before reaching it.
Track cuts		
10	GoodParticle	The particle is a primary or a fragment generated in the TG that crosses both planes of the TW and goes beyond, indicating no secondary fragmentation.

Table 5.2: Event and track selection cuts based on MC truth information (simulation only).

struction, thereby improving the purity of yields in track measurements. Cuts 1 - 4 focus on the topology of the event. **SCcut** and **BMcut**, in particular, are used to classify events where no pile-up of primary particles occurs at level of SC or BM, which could affect measurements of fragments due to their non-univocal origin. **TWnum** relies on global track efficiency for tracks to match with all the TW points present in the event, while **NTracksCut** is introduced to disentangle all the events with only the primary. Cuts 5 - 7 rely on features of specific tracks. **VTXposCut** checks if the VTX point is reconstructed in the right target position and matches with the BM track, while **TWclone** identifies all global tracks reconstructed with the same TW point. Finally, valid only in simulations, Cut 8 **MC_isGoodReco** establishes the association between the track (reconstruction level) and a MC particle (truth level). In particular, if the MC particle originates in the target crossing the TW, it represents exactly the fragments to be measured by the FOOT setup, classifying the track as a *good* global track. In other words, this cut distinguishes tracks of those particles which represent the true signal for the FOOT experiment (*good* tracks) from background processes or misreconstruction.

In simulations, it is crucial to classify all events in which fragmentation occurs correctly, even at the truth MC level. The relevant selection criteria are listed in Tab. 5.2. Specifically, Cut 9 **MCgoodEvent** defines a *good* event as one where the beam properly traverses the target, which at the reconstruction level corresponds to the presence of a single BM track (as in **BMcut**). Cut 10 **GoodParticle**, on the other hand, identifies the *good* particles that are the actual focus of the FOOT experiment: fragments originating in the TG that pass through all detectors up to the TW. MC particles satisfying Cut 10 (*good* MC particles) are compared to global tracks satisfying Cut 8 (*good* global tracks) for performance evaluations.

The values chosen for every cut and their implementation could differ according to the data taking. Their application on data of CNAO 2024 will be shown in Chap. 6, justifying them with a performance evaluation of every cut exploiting an MC simulation.

5.4.2 Cross Section approach

The differential cross section $\frac{d\sigma}{d\Omega}$ of nuclear fragmentation production as a function of the fragment charge Z and emission angle θ can be estimated experimentally following the empirical formula:

$$\frac{d\sigma}{d\Omega}(Z, \theta) = \frac{Y(Z, \theta)}{N_{\text{beam}} N_{\text{target}} \Delta\Omega(\theta) \epsilon(Z, \theta)}. \quad (5.5)$$

$Y(Z, \theta)$ is the number of yields, namely the number of global tracks reconstructed with charge Z and emission angle θ . N_{beam} represents the total number of particles of the incident beam and it has been computed as the number of *fragmentation triggers* (see Sec. 3.2.4) collected in data. Both Y and N_{beam} undergo selection cuts of Sec. 5.4.1.

N_{target} represents the surface nuclear density of the target, calculated as:

$$N_{\text{target}} = \frac{\rho \Delta x N_A}{A} \quad (5.6)$$

where ρ is the target density, Δx is the target thickness, N_A the Avogadro number and A the mass number. In particular, for the target of graphite (C) used in the analysis of this thesis, $\rho = 1.83 \text{ g/cm}^3$, $\Delta x = 0.5 \text{ cm}$ and $A = 12.0107$.

$\Delta\Omega$ represents the phase space, i.e. the portion of solid angle inside which the counting measurement is taken. It is possible to assume full azimuthal symmetry ($\phi \in [0, 2\pi]$) since there is no physical evidence of an asymmetric emission of fragments in the direction of the transverse plane. Then, in the conversion from the polar angle θ to the solid angle Ω in spherical coordinates, integrating over ϕ gives:

$$\Delta\Omega(\theta) = \int_{\theta_1}^{\theta_2} \int_0^{2\pi} \sin \theta d\phi d\theta = 2\pi [\cos \theta_1 - \cos \theta_2] \quad (5.7)$$

where the finite polar angle bin $[\theta_1, \theta_2]$ generally corresponds to the angular resolution of the setup. In most of the analyzed data, the bin width is chosen to match the angular resolution of the TW, corresponding for most of the experimental campaigns to 0.6° .

$\epsilon(Z, \theta)$ represents the convolution of the efficiencies that have to be handled in the process of cross section measurements. They can be related to the detection capability or to the tracking algorithm performances, as VTX track efficiency (Eq.

4.4) and global track efficiency (Eq. 5.2).

The total cross section σ_{tot} for the production of a fragment with charge Z can be obtained by integrating the differential cross section $\frac{d\sigma}{d\Omega}(Z, \theta)$ over the angular range covered by the experimental setup. It can thus be expressed as:

$$\sigma_{tot}(Z) = \int_{\theta_{min}}^{\theta_{max}} \frac{d\sigma}{d\theta}(Z, \theta) d\theta. \quad (5.8)$$

As a result, the obtained quantity corresponds to a fiducial total cross section, since it represents the fraction of the production cross section accessible to the FOOT experiment. For the lighter elements, whose angular distribution goes beyond the experiment acceptance, it does not necessarily coincide with the full production cross section.

After processing all events, the calculation of Eq. 5.5 is carried out within the analysis software using the ROOT `RDataFrame` class [140]. This modern, high-level interface provides a robust and well-tested framework for data analysis in ROOT. Event and track level selections are implemented through its filtering functionalities, while yields are computed according to the defined variables. The results are then aggregated into histograms, following the specified binning. Besides keeping the structure very manageable, the implementation of `RDataFrame`, thanks to its internal multithreading processing, makes the procedure very fast, even when analyzing millions of events.

Chapter 6

CNAO 2024 Data Analysis

During November of 2024, the FOOT Collaboration performed a data taking at CNAO (Sec 2.1.2) with both the electronic and emulsion setups. The installed electronic setup is reported in Fig. 6.1-a, which, for the first time, comprehends the full complete apparatus (Sec. 3.2): the upstream region (SC and BM), the tracking region (VTX, IT, MSD and magnets) and the downstream region (TW and CAL).

In particular, the data acquisition section was organized according to three main purposes:

- *CAL calibration*, to study the energy response of all 320 BGO crystals using a beam of protons of 70 - 230 MeV (around 15 million events)
- *VTX threshold scan*, to study the performance of VTX detector at different noise conditions and proton energies (70 - 230 MeV) as reported in Sec. 4 (around 1.5 million events)
- *physics*, using a beam of ^{12}C ions at 200 MeV/u impinging on a 5 mm C target, with B field. Exploiting the full setup, charge, momentum and mass of the fragments are reconstructed, in the direction of cross section measurements (around 1.9 million events).

In the following sections, all the reconstruction performance evaluations and results for cross section measurements at CNAO 2024 data taking are described. For such studies, a FLUKA MC simulation of 10 million events of ^{12}C ions at 200 MeV/u impinging on a 5 mm C target was generated exploiting the full setup apparatus (see Fig. 6.1-b).

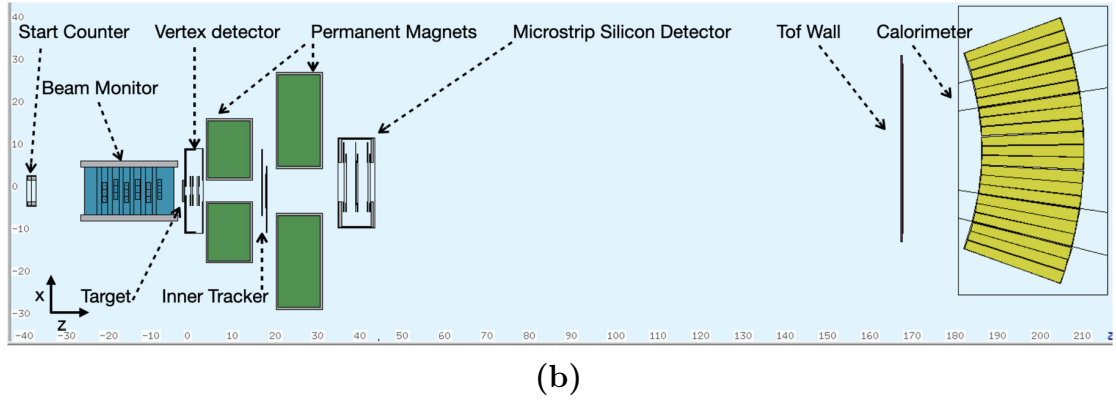
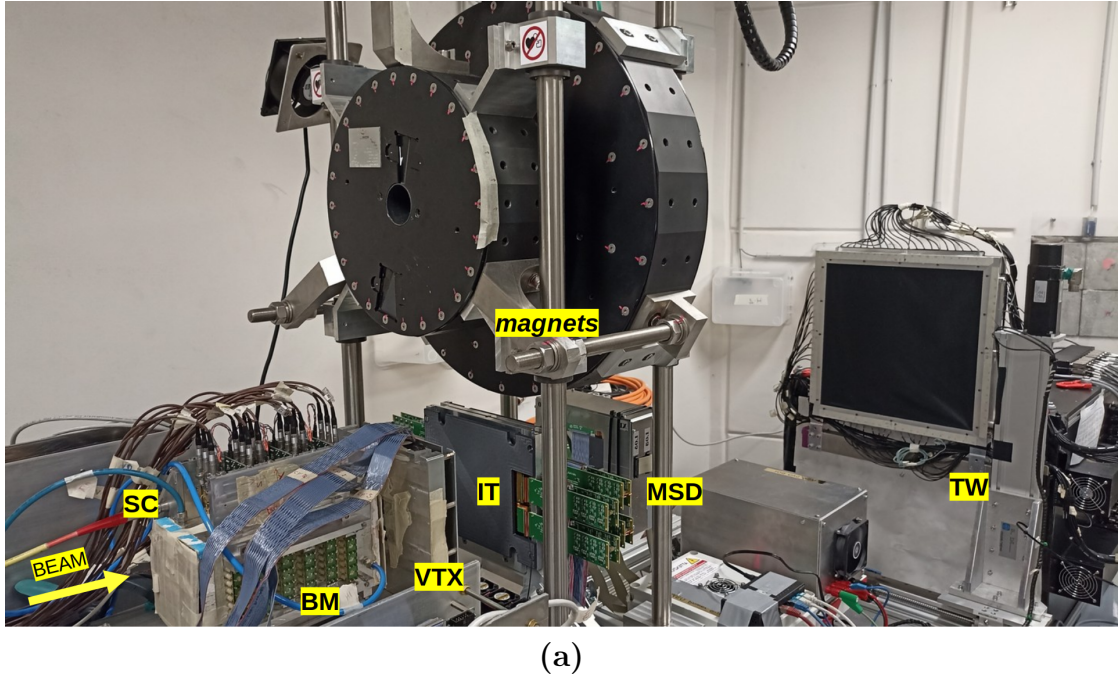


Figure 6.1: a) FOOT electronic setup during CNAO 2024 data taking, from left to right: SC and BM (upstream region), VTX, IT, MSD and magnets (tracking region) and TW and CAL (downstream region). Magnets can be positioned above or among tracking detectors according to the measurement purposes. CAL is located behind the TW and is being moved to its pedestal to align with the beam direction line. b) Simulation of the FOOT electronic setup geometry in FLUKA, including all the detectors present at CNAO 2024. From [132].

6.1 Event Selection

To achieve the goal of the FOOT experiment, a precise identification and characterization of nuclear fragments is necessary, beginning with the most accurate selection of events. Such tasks are obtained with the help of the MC FLUKA simulation of the CNAO 2024 campaign, in order to evaluate the performance in reconstruction.

As stated in Sec. 5.3.2, global tracks are reconstructed using the GENFIT library, applying a Kalman filter approach for fitting. An example of the reconstruction of a primary ^{12}C ion beam global track is reported in Fig. 6.2.

One of the main sources of background for the FOOT experiment is the generation of out-of-target fragments, which could spoil the global reconstruction and thus the required measurements. Although the setup has been designed with a low material budget where the fragment travels, there is a non negligible probability of interaction of the beam ion (or of its fragments) with air or active and passive layers throughout the detectors, making the background fragmentation reach up to 25% of that happening inside the target. Thus, event selection has the main purpose of retaining interaction events and tracks produced in the target, removing background sources or algorithm artifacts.

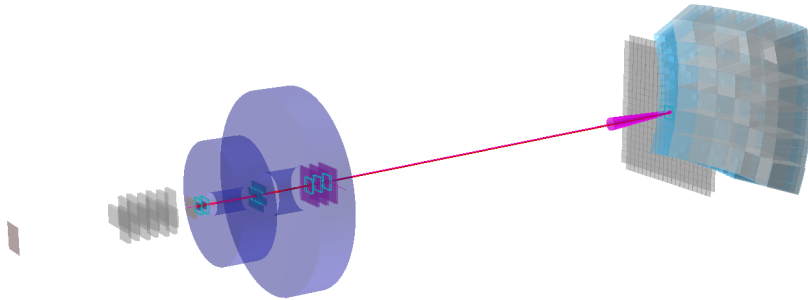


Figure 6.2: GENFIT event display of an event of CNAO 2024 simulation, in which the reconstruction of a global track of ^{12}C has been achieved. According to the GENFIT-based algorithm, the track is reconstructed from the VTX to the TW.

The first selection consists of the definition of "clean" events, namely events where the fundamental conditions for track reconstruction hold. At first, only 1 track of the BM is required, to discard all the events in which more than 1 beam ion is recorded in the same temporal interval (beam pile-up). The VTX tracklet (see Sec. 5.2.2) has to match with the BM track to ensure that the fragment was generated by the beam ion, while the VTX point needs to be reconstructed in

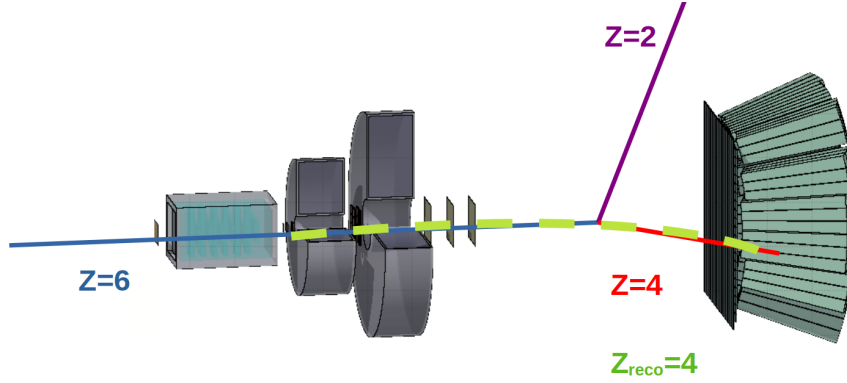


Figure 6.3: Example of an event in which out-of-target fragmentation occurs. A $Z=6$ primary interacts after the tracking region, generating two fragments of $Z=2$ and $Z=4$, where the former goes outside the geometrical acceptance of the setup. The reconstructed global track (dashed green line) has the feature of the $Z=6$ primary particle, but with a different reconstructed charge. These kinds of events are not distinguishable from a genuine fragment generated in the target.

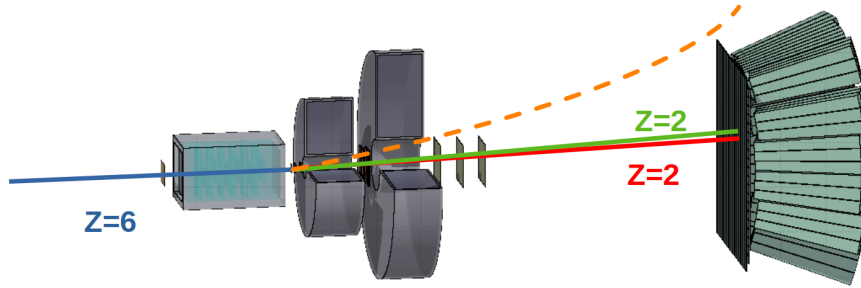


Figure 6.4: Example of an event in which track reconstruction is limited by TW granularity. Two light fragments of $Z=2$ have an angular emission such that they cross the same TW bar, creating a misreconstructed TW point, shared by the two tracks. Moreover, the number of reconstructed tracks (two $Z=2$ lines and a dashed line) is different from the number of TW points in the event. These kinds of events don't allow a suitable reconstruction of the fragmentation process.

a reasonable position within the target geometry. The global tracks are reconstructed with at least 7 clusters (which represent around 70 % of total), including points of VTX, IT, MSD and TW. This requirement contains Selections 1,2,5 of Sec. 5.4.1 and will be referred to as **Cut_0**.

The next selection requires the quality of the global tracks reconstructed on the basis of the Kalman Filter fit. Defining the residual as the distance between the cluster and fit position, all the tracks with the worst residual higher than 0.04 cm are discarded (Fig. 6.6-c). An important metric that assesses how well the fit describes the observed measurements of the particle's track is the p -value, strictly correlated to a χ^2 test (Fig. 6.6-b). Since p -value represents the probability of obtaining such results if the fitting model is wrong, a discard of all tracks whose p -value ≤ 0.01 is introduced. This requirement is defined as **Cut_1** and contains Selections 1,2,5,7 of Sec. 5.4.1.

A ^{12}C ion that does not interact within the target can undergo fragmentation outside it, which can occur anywhere along its trajectory. As a result, fragments may be produced in air or in the detector layers and emitted beyond the geometrical acceptance of the setup as shown in Fig. 6.3, where, e.g., two fragments of $Z=4$ and $Z=2$ are generated after the tracking region, with $Z=2$ not interacting with the TW. In this topology of events, a single global track is generally reconstructed, typically with features of a primary but with a different reconstructed charge. Since this situation cannot be distinguished from a genuine target fragment in experimental data, only events with more than one reconstructed global track are considered, applying Selections 1,2,4,5,7 of Sec. 5.4.1, (**Cut_2**).

One of the intrinsic limits of TW charge identification (see Sec. 5.2.2) is given by its granularity. Two light fragments, e.g. ^4He ions, can be generated with a very low emission angle and may reach the TW crossing the same bar (Fig. 6.4). Only one TW point would be created and, since $\Delta E \propto Z^2$ (see Eq. 1.2), the energy released by two ^4He ions can be mis-reconstructed by the charge algorithm as $Z = 3$. The global tracking algorithm has enough granularity to disentangle the two fragments, but they would be reconstructed with the same (wrong) TW point. Thus, all the tracks embedded with the same TW point are discharged. Moreover, in order to remove all the events in which in-target fragmentation cannot be properly detected, e.g. the one of Fig. 6.4 where a fragment goes outside the acceptance of the TW, the same number of reconstructed global tracks and TW points is required. For **Cut_3**, selections 1,2,3,4,5,6,7 of Sec. 5.4.1 hold. A short recap of cuts is reported in Tab. 6.1.

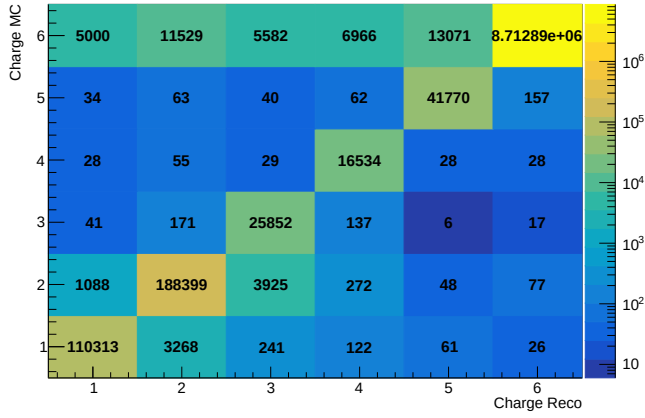
A justification of the applied cuts can be given only by studying the reconstruction performances, varying the selections. Fig. 6.5 shows the *charge changing matrix* (CCM). For every global track, its reconstructed charge Z_{reco} and the MC charge of the most frequent particle Z_{MC} are reported, respectively, on the x and

Cut	Selections	Description
Cut_0	1,2,5	Global tracks with BM matching and no pile-up
Cut_1	1,2,5,7	χ^2 and residual track quality cuts
Cut_2	1,2,4,5,7	Only multitracks
Cut_3	1,2,3,4,5,6,7	TW points pile-up cuts

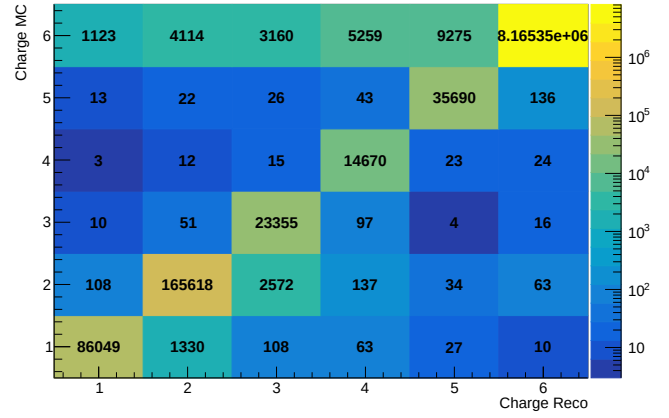
Table 6.1: Short recap of the cuts used in CNAO 2024 analysis

y axis of the plot. The ideal reconstruction should present entries as close as possible on the diagonal of the CCM, indicating that almost all of the tracks were reconstructed with the true charge. Fig. 6.5-a shows the CCM after **Cut_0**, where there is evidence of elements out of the diagonal. The application of χ^2 and residuals cuts of **Cut_1**, to remove bad reconstructed tracks, improves the CCM of Fig. 6.5-b, where, alongside a reduction of 3% of the good tracks, there is a rejection of more than 30% of events out of diagonal. The populated row of $Z_{MC}=6$ is still present and can be associated with all the events in which the fragmentation of the primary happens out of target: applying **Cut_2**, it is completely removed, as well as all the events affected by secondary fragmentation. The drawback of this selection is the fact that also good tracks in the diagonal of the CCM are removed. Applying a cut for all the events in which only one track is reconstructed, in fact, can also distress those in which, in the generation of two or more fragments, only one reaches the TW within the setup geometrical acceptance. Exploiting the efficiency of the method for such events, however, their counts will be recovered. Inspecting tracks with $Z_{MC}=2$, it is possible to notice a non-negligible quantity of $Z_{reco}=3$ bad reconstructions. For this kind of event, a TW point is associated with the wrong charge due to the pile up of different particles impinging on the same region of the detector. Applying **Cut_3**, an improvement is appreciable, limiting e.g. the bad reconstruction of $Z_{MC}=3$ from 15% to 1.5%.

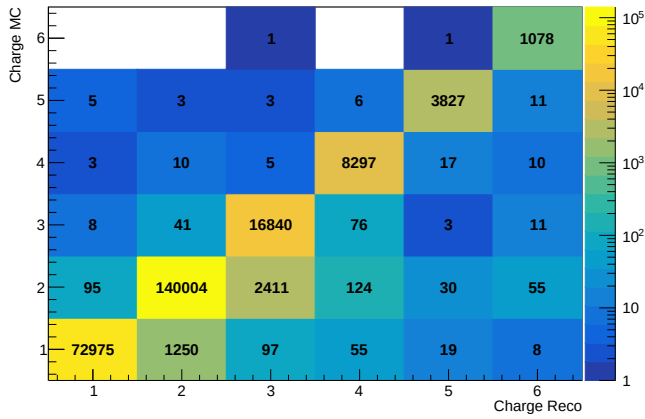
Fig. 6.6-a shows the origin, on the z axis of the setup, of the particles that cross the TW for every track. After the first peak at the origin position of the beam in the simulation, the others lie near the location of the detectors or of the target, associated with fragmentation in them. For **Cut_0**, all the peaks are present, together with continuous lines in the upstream, tracking and downstream region, showing how the fragmentation out of the target affects the measurements. After applying **Cut_1**, an improvement is appreciable in particular in the tracking region, since tracks with bad χ^2 and residuals could be, in part, generated by particles fragmented between the two magnets, whose trajectory does not fit the one expected if generated in the target. The effects of **Cut_2** are clearly visible even in this plot, removing completely the reconstruction of fragments originating



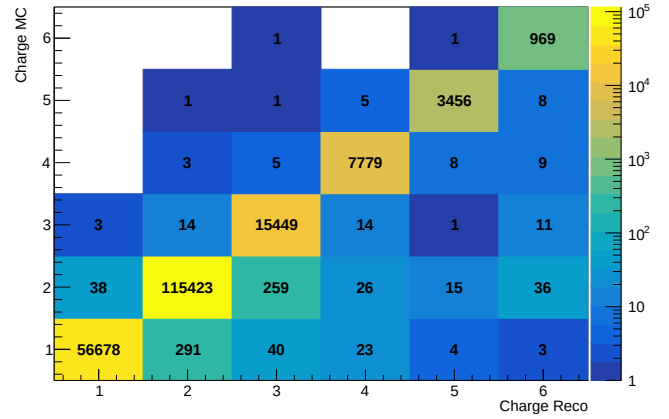
(a) Cut_0



(b) Cut_1



(c) Cut_2



(d) Cut_3

Figure 6.5: CCM of global track reconstructed in the simulated data for a) Cut_0, b) Cut_1, c) Cut_2 and d) Cut_3.

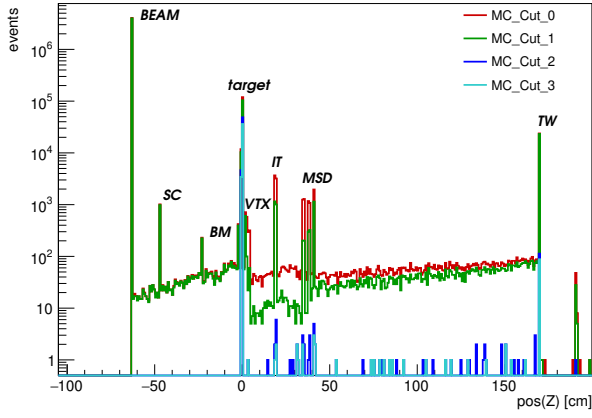
in the upstream region and drastically reducing the ones in the others. Only the origin peak around the target remains present, as well as for **Cut_3**, demonstrating the great rejection capability of the cuts.

Fig. 6.6-b reports reduced χ^2 track distributions for different cuts. The application of **Cut_1** highlights the presence of a tail in the **Cut_0** distribution, which is associated with badly reconstructed tracks that are thus removed. Moreover, after that **Cut_2** and **Cut_3** hold, the data become closer and closer to the theoretical probability density function of a reduced χ^2 with mean closer to 1, a sign of improved track quality that enhances the validity of the model, namely the trajectory of charged particles crossing the FOOT setup while surrounded in a magnetic field.

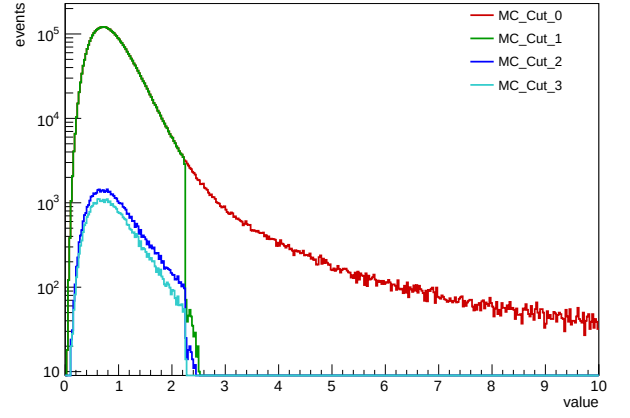
Residuals, defined as the difference between the actual measurement at a point and the estimate predicted by the Kalman filter's global track state, are reported in Fig. 6.6-c for every selection. A value of 0.04 cm has been chosen for **Cut_1** to remove the tail, easily visible in **Cut_0** residual distribution. Cuts application shows smaller residuals that suggest improved fit quality, better error estimates and less influence of measurement noise or systematic errors on the global track.

Given the points of a track, it is possible to inspect their MC nature and infer the track quality as the percentage of points belonging to the generated MC particle (see Fig. 6.6-d). After applying cuts, it is possible to see how the mean quality tends to increase, more than 60% after **Cut_3**. The observed reduction in tracks with 100% quality is not due to any deterioration after applying the cuts, but rather to the effect of **Cut_1**, which removes the $Z = 6$ particles, representing over 90% of the statistics in that bin and are not of interest for cross section measurements.

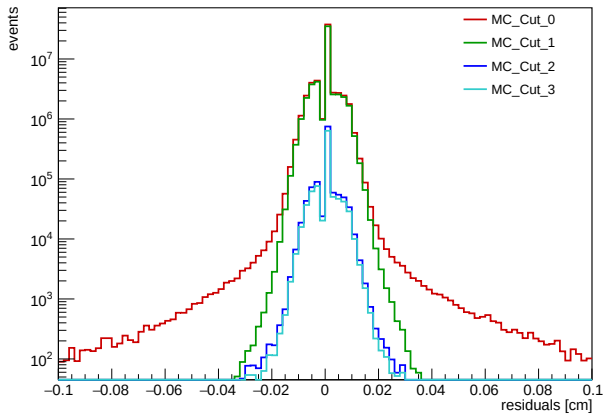
The performance of the implemented selection can also be assessed by examining the efficiency (Eq. 4.4) and purity (Eq. 5.4) of the global track reconstruction. Figure 6.7 presents the efficiency as a function of Z (Fig. 6.7-a) and, for each Z , as a function of the polar angle θ (Fig. 6.7-b,f), defined as the emission angle of the fragment with respect to that of the primary (as measured by BM) before interaction. In every case, the efficiency decreases when further selections are introduced. The highest drop, in particular, is seen for the **Cut_2** application, which rejects all events with only one reconstructed track. The worst case is for events where a single $Z=5$ track has been reconstructed because, in such situations, it is very likely that p fragments are generated outside the geometrical acceptance and thus not reconstructed. As already stated, this efficiency could be exploited to take into account this kind of event in the final cross section evaluation. Analog distributions in Z and polar angle θ are reported even for purity in Fig. 6.8. Here, the application of **Cut_1** is fully justified, as the purity reaches values close to unity for every Z . The effect, in particular, is evident at lower emission angles,



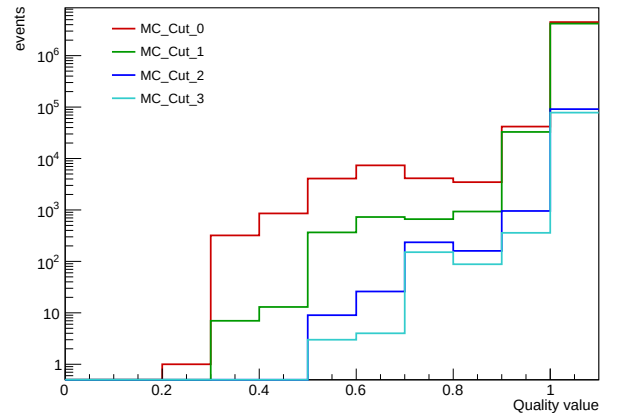
(a)



(b)

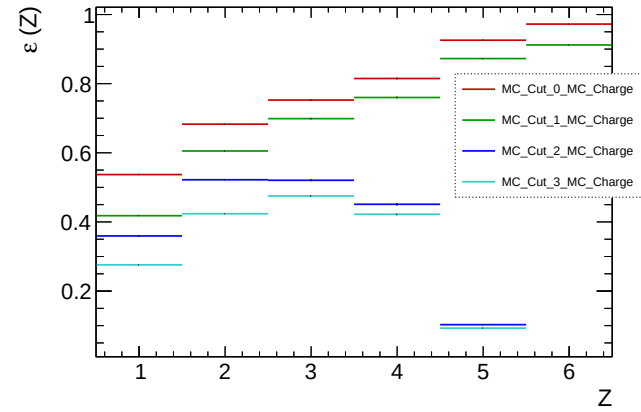


(c)

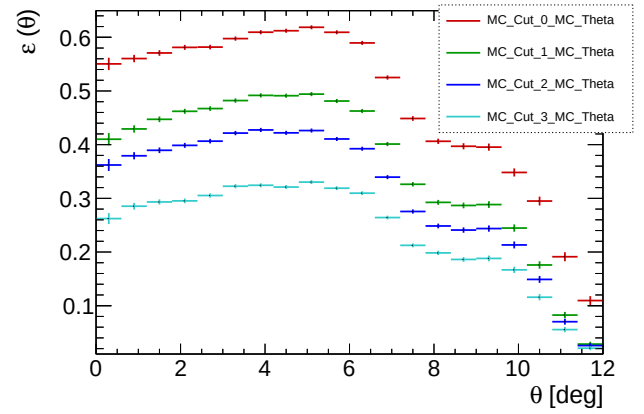


(d)

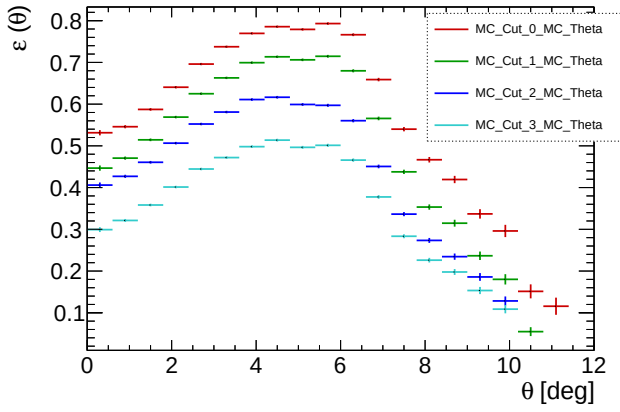
Figure 6.6: Performance plots studying a) the origin of track particles crossing the TW, b) reduced χ^2 track distribution, c) residual distribution and d) track quality plots for Cut_0 in red, Cut_1 in green, Cut_2 in blue and Cut_3 in light blue. The plots are based on a $5 \cdot 10^6$ MC events.



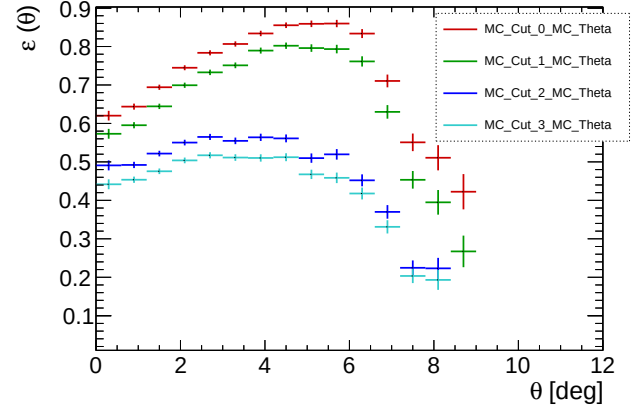
(a) Total



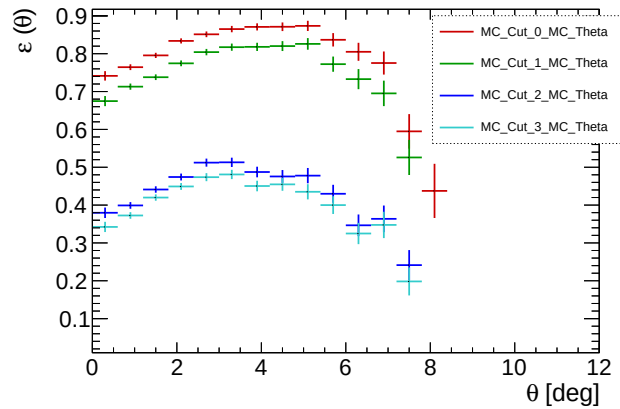
(b) H



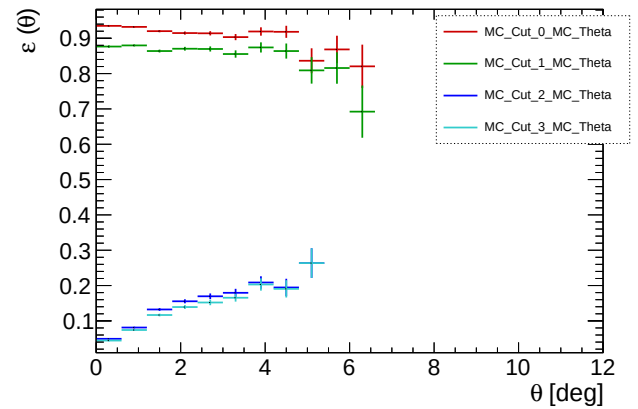
(c) He



(d) Li

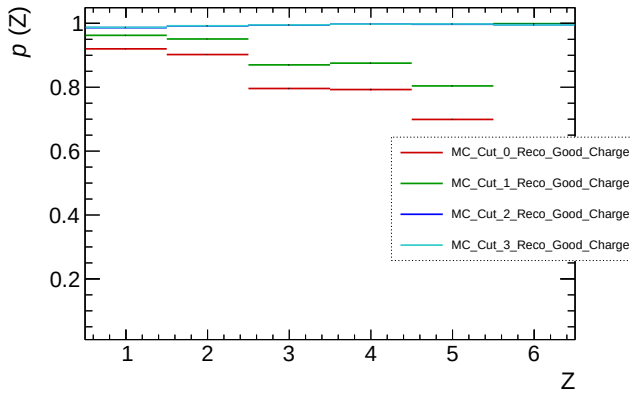


(e) Be

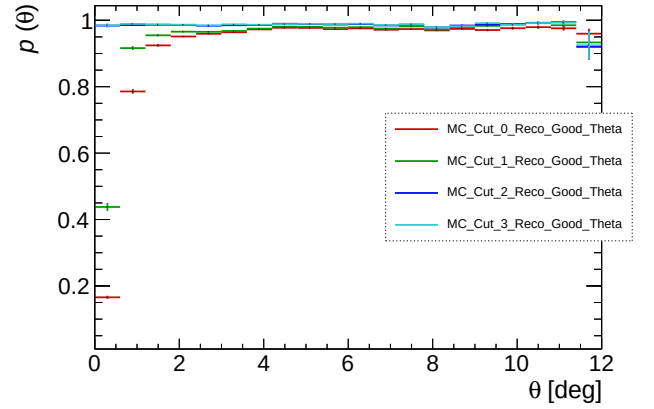


(f) B

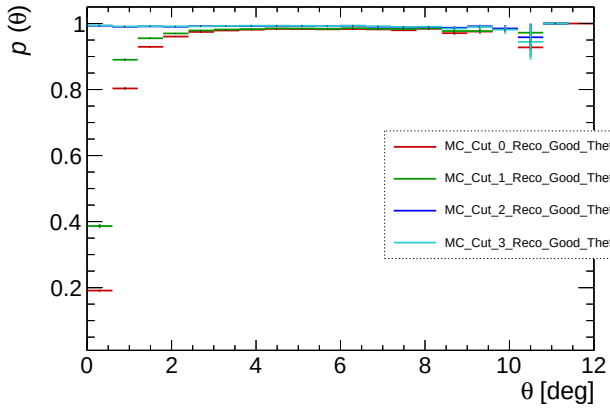
Figure 6.7: Total (a) and angular (b-f) global track reconstruction charge efficiency for CNAO 2024 simulation



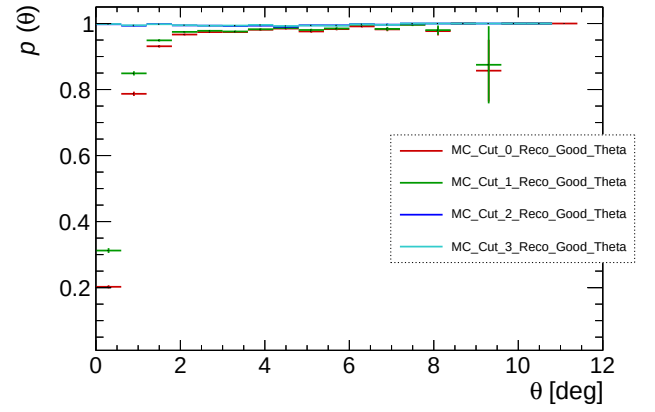
(a) Total



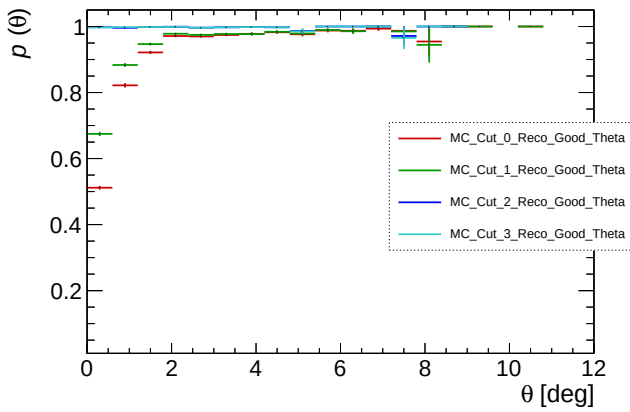
(b) H



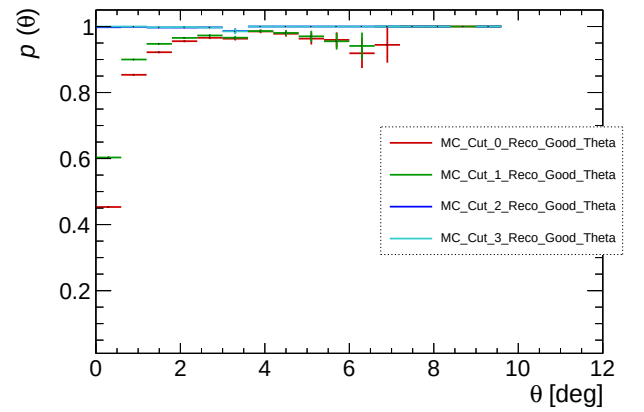
(c) He



(d) Li



(e) Be



(f) B

Figure 6.8: Total (a) and angular (b-f) global track reconstruction charge purity for CNAO 2024 simulation

where most of the particles are concentrated and where the separation from pile-up and out of target fragmentation is intrinsically more challenging. It is important to state that, in the trade-off between efficiency and purity, prioritizing purity is preferable: a cleaner reconstruction of the selected sample ensures more reliable results during experimental data taking, while a possible loss in efficiency can be mitigated by increasing the available statistics.

6.2 Closure test

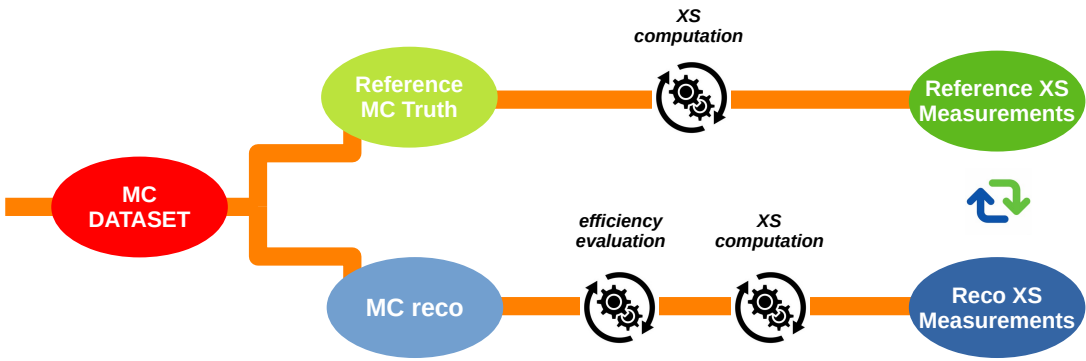
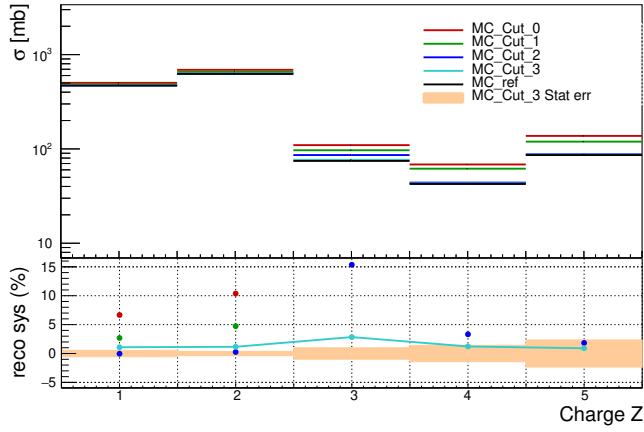


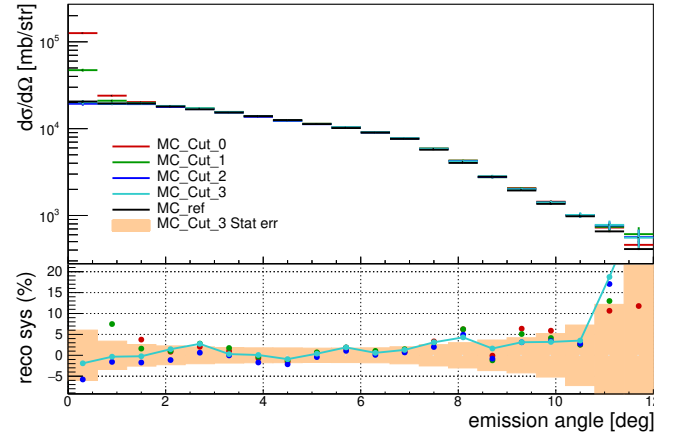
Figure 6.9: Closure test workflow.

To evaluate the reliability and statistical robustness of the analysis method, a closure test can be performed on simulated data. This validation tool leverages the FLUKA MC dataset, which is designed to reproduce experimental geometry and nuclear processes, employing two complementary approaches (see Fig. 6.9). In the first case, the true MC values are used (applying Selections 9 and 10 of Sec. 5.4.1) to obtain reference cross section measurements of Eq. 5.5, representing the ideal results in the absence of systematics, inefficiencies or noise. In the second case, the quantities are reconstructed as if they were experimental data applying `Cut_0`, `Cut_1`, `Cut_2`, `Cut_3` and, after efficiency evaluations, Eq. 5.5 for reconstruction cross section measurements is computed. Comparing the two types of results, it is possible to assess the consistency of the method and to quantify the systematic uncertainties introduced throughout the analysis chain.

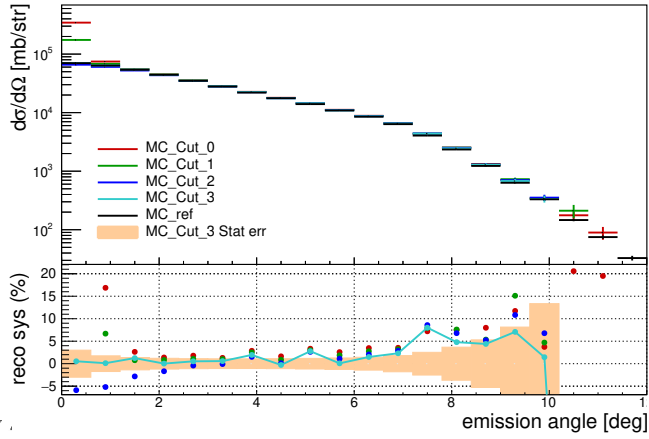
Fig. 6.10 reports the closure test of total and differential angular fragmentation cross sections of ^{12}C 200 MeV/u projectile impinging on C, as simulated for the CNAO 2024 campaign. The upper plot reports the cross section measurements obtained for each cut together with the reference values, while the lower panel displays their relative variations, including the statistical uncertainty of the last



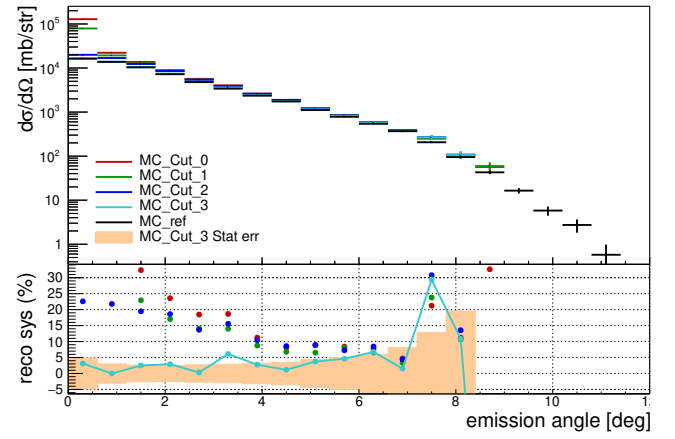
(a) Total



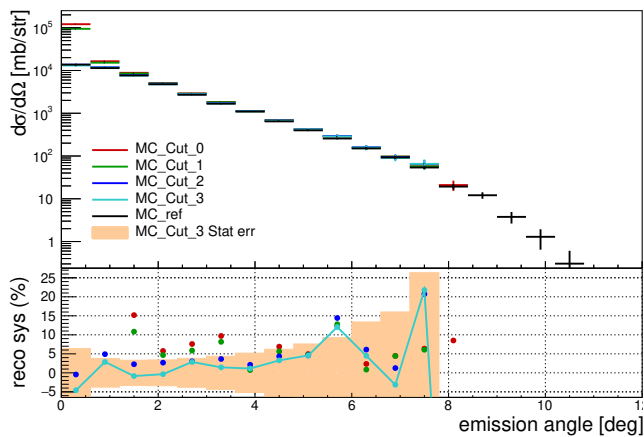
(b) H



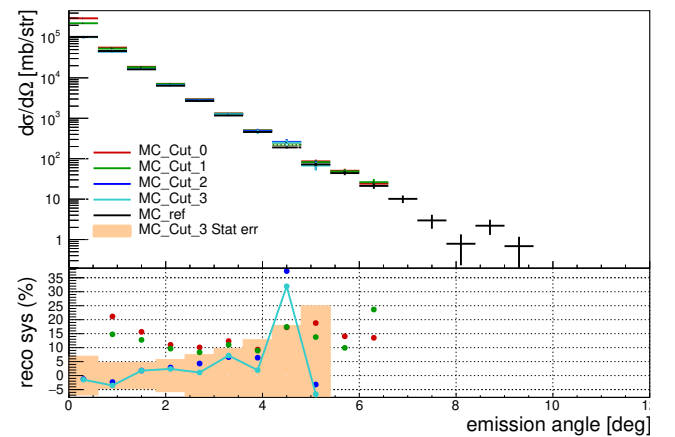
(c) He



(d) Li



(e) Be



(f) B

Figure 6.10: Total (a) and differential (b-f) fragmentation cross section closure test for CNAO 2024 simulation for ^{12}C 200 MeV/u ions impinging on a C target. In the upper part, cross section measurements of Cut_0 (red), Cut_1 (green), Cut_2 (blue), Cut_3 (light blue) and MC_ref (black) are reported. In the lower part, the relative variation of cuts with MC reference is shown (in dots), together with the statistical uncertainties of the last cut.

cut considered. From the analysis of all cuts discussed in Sec. 5.4.1, the highest consistency is achieved with `Cut_3`, where the systematic discrepancy is reduced to about 2–5% across all results. The most significant improvement is observed for low emission angles and for $Z = 3$, in agreement with the discussion in the previous section. A residual systematic difference below 5% in the cross-section evaluation suggests the presence of hidden processes not yet considered in the reconstruction; nevertheless, it remains comparable to the overall statistical fluctuations.

Overall, the closure test demonstrates the reliability of the analysis method, implemented for total and angular differential cross section measurements. In particular, a compatibility better than 5% with the MC reference confirms the robustness of the workflow and supports its application to experimental data.

6.3 Momentum and Mass reconstruction

The global track representation also contains the information of momentum, which, according to the GENFIT library (see Sec. 5.7), can be extrapolated to the point of interaction of the particle in the target. The momentum reconstruction can be combined with the ToF of the particle to retrieve the atomic mass A as follows:

$$A = \frac{p}{\beta\gamma m_u} \quad (6.1)$$

where $\beta = \frac{l}{ToF \cdot c}$ is the relativistic velocity of the particle, l is the length of the global track, ToF is given by TW point, c is the speed of light and $m_u = 931.5$ MeV/ c^2 is the unit of atomic mass.

Fig. 6.11 and Fig. 6.12 show, respectively, the momentum and mass distributions for all fragments (from $Z = 1$ to $Z = 5$) and primary ^{12}C ions from the simulated CNAO 2024 campaign, compared to the MC reference one. Generally, reconstruction results are in good agreement with the expected values, in particular after the application of cuts. The most appreciable improvement is obtained for Be ions, where, after applying `Cut_2`, the yields around $p \sim 5$ GeV/ c and for $A \simeq 8$ are completely suppressed. This is expected since ^8Be is an unstable isotope with a half-life $t_{1/2} \sim 10^{-17}$ s [141], far too short to be detected across the FOOT setup, where the mean ToF is about 11 ns. The observed artifact is instead associated with out-of-target fragmentation, as illustrated in Fig. 6.3. When a secondary fragment is produced after the tracking region of the FOOT setup, the reconstructed global track inherits the kinematic features of the primary ^{12}C , in particular its momentum, but the charge of the fragment is assigned by TW point. Since the GENFIT reconstruction algorithm measures the rigidity p/Z , the momentum is then evaluated using the fragment charge from TW point, while the bending still corresponds to the primary ^{12}C . As a consequence, the final

reconstructed momentum is rescaled by the factor

$$p_{\text{rec}} = p_{\text{prim}} \cdot \frac{Z_{\text{frag}}}{Z_{\text{primary}}}, \quad (6.2)$$

which explains the appearance of tracks wrongly reconstructed as ^8Be ions.

Inspecting C plots, besides the peak of the primary ^{12}C at 200 MeV/u of kinetic energy, corresponding to $p \sim 7.8$ GeV/c and $A=12$, other distributions are present, slightly reconstructed after the application of `Cut_2`. These correspond to the isotopes ^9C , ^{10}C and ^{11}C , which are produced by rare processes, such as out-of-target fragmentation and interactions with fragments undergoing backscattering from the CAL.

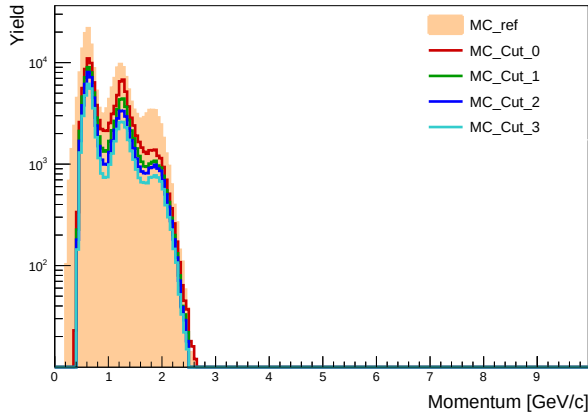
An important result of Fig. 6.12 is that the p and ToF performances allow clear mass reconstruction, with the main isotopes well discriminated. However, separation decreases with increasing mass number despite similar p and ToF resolution, because the relative mass separation becomes lower, going from 100% for H isotopes to less than 9% for Be fragments. It is important to note that the experiment also enables mass reconstruction through two additional methods (see Sec. 3.2), and the combination of all three will provide more precise results.

The mass spectra exhibit a systematic overestimation, directly inherited from momentum reconstruction, which reflects the same behavior as well. To further inspect this aspect, the momentum computed after applying `Cut_3` p_{cut_3} has been compared to p_{MC} according to the following formula:

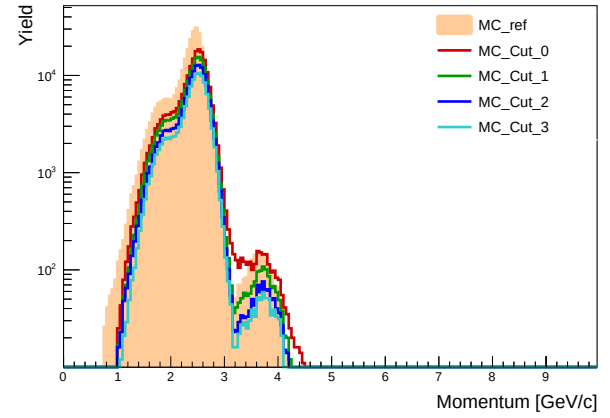
$$\frac{\Delta p}{p} = \frac{p_{\text{cut}_3} - p_{MC}}{p_{MC}}. \quad (6.3)$$

The mean values of this distribution for every fragment are shown in Fig. 6.13-a. Ideally close to 0%, they instead vary by 2–4% across isotopes, confirming the systematic overestimation of momentum reconstruction previously noted. This effect arises from the GENFIT track fitting algorithm or from its extrapolation of track parameters, which systematically reconstructs higher rigidity p/Z . As a result, tracks appear less bent, leading to a shorter path length l and a lower velocity β . Although a systematic offset is present and requires further investigation, the momentum resolution remains below 3% for all reconstructed fragments (Fig. 6.13-b), outperforming the 4–5% design requirement (Sec. 3.2). This demonstrates that the GENFIT algorithm still provides very good performance.

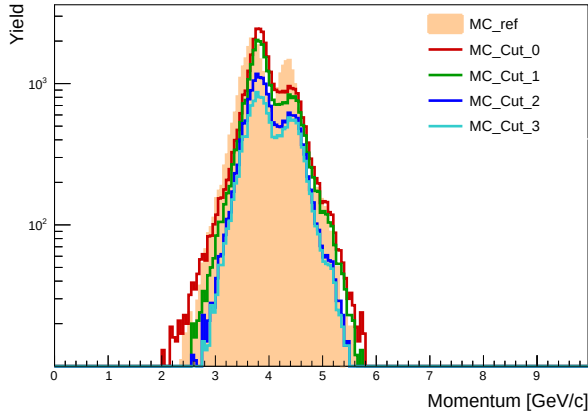
In these sections, the analysis algorithm has been shown to perform well when applied to FLUKA MC campaigns, both in the measurement of cross sections through the closure test and in the reconstruction of mass and momentum through dedicated performance studies. Therefore, this procedure is now applied to the experimental data in the following section.



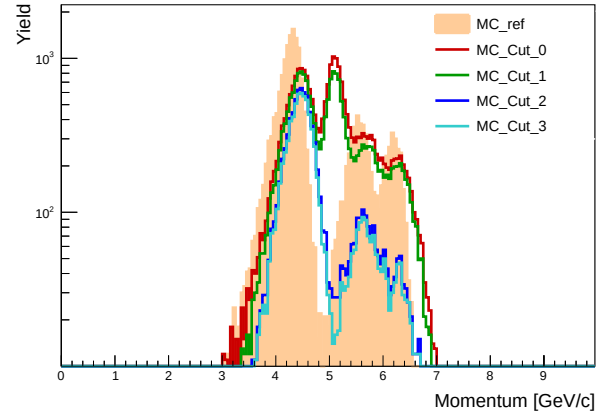
(a) H



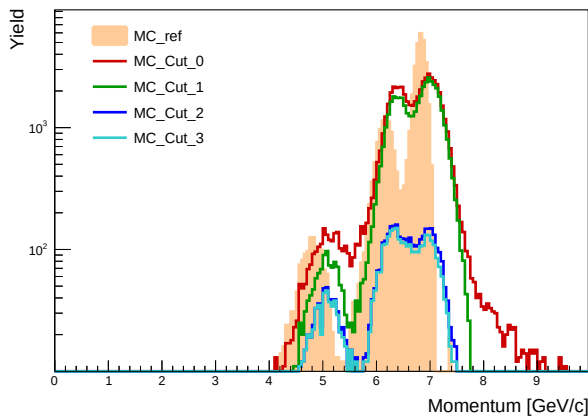
(b) He



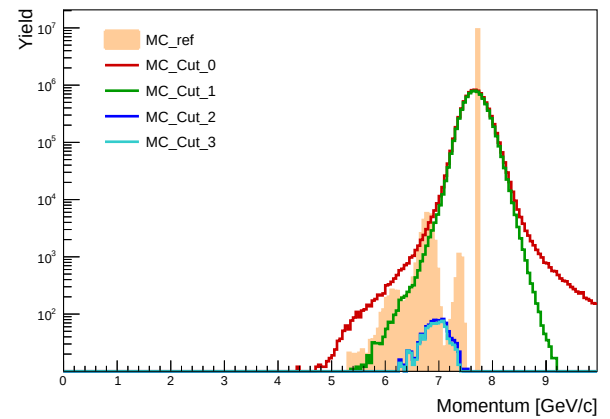
(c) Li



(d) Be

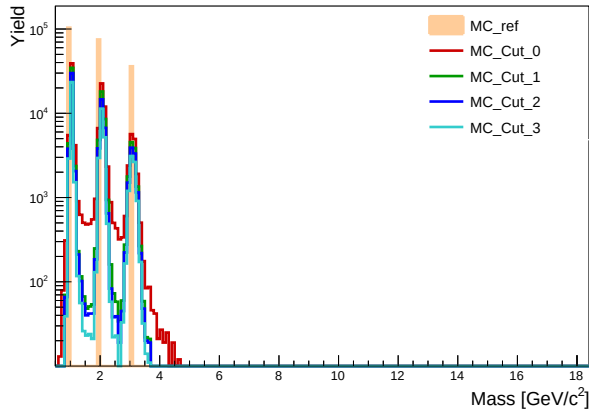


(e) B

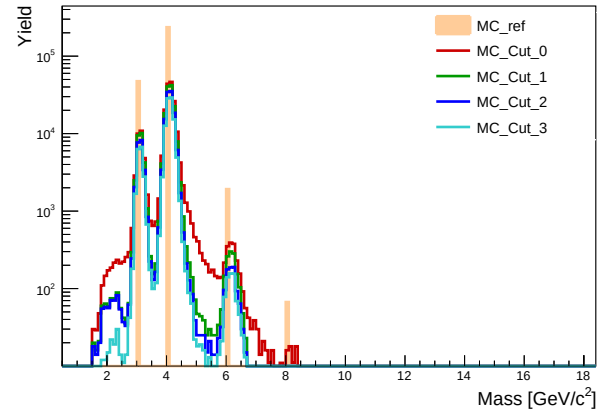


(f) C

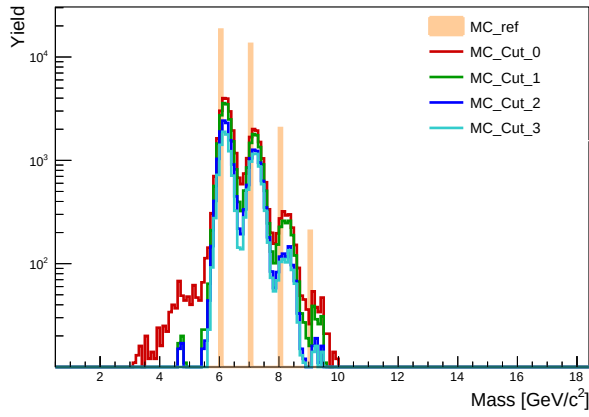
Figure 6.11: Momentum reconstruction for fragments (a-e) and primary particles (f), applying Cut_0 (red), Cut_1 (green), Cut_2 (blue) and Cut_3 (light blue), in comparison with MC_ref (light orange), from MC FLUKA simulation.



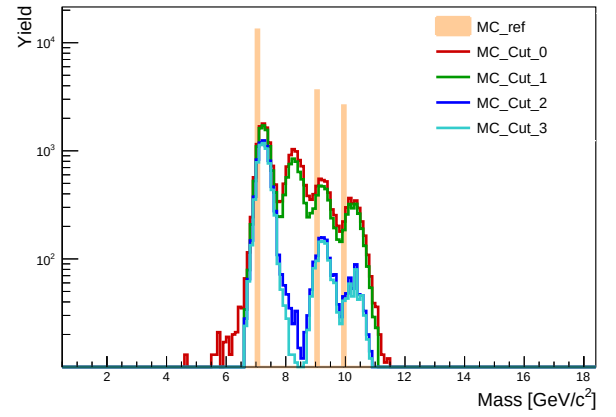
(a) H



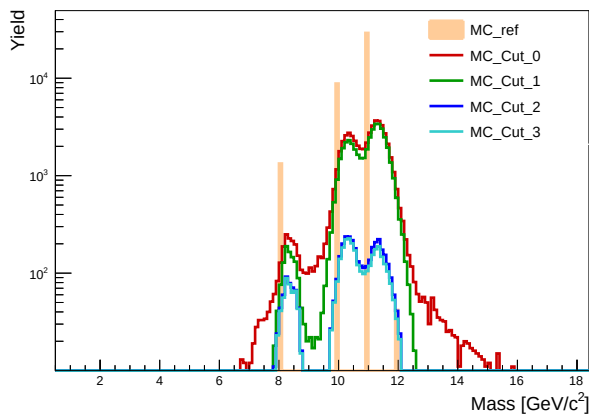
(b) He



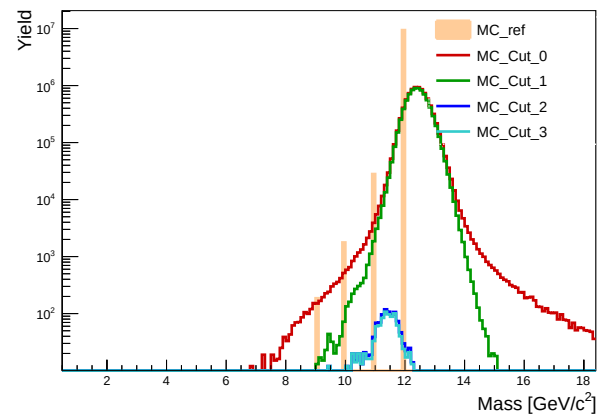
(c) Li



(d) Be



(e) B



(f) C

Figure 6.12: Mass reconstruction for fragments (a-e) and primary particles (f), applying Cut_0 (red), Cut_1 (green), Cut_2 (blue) and Cut_3 (light blue), in comparison with MC_ref (light orange), from MC FLUKA simulation.

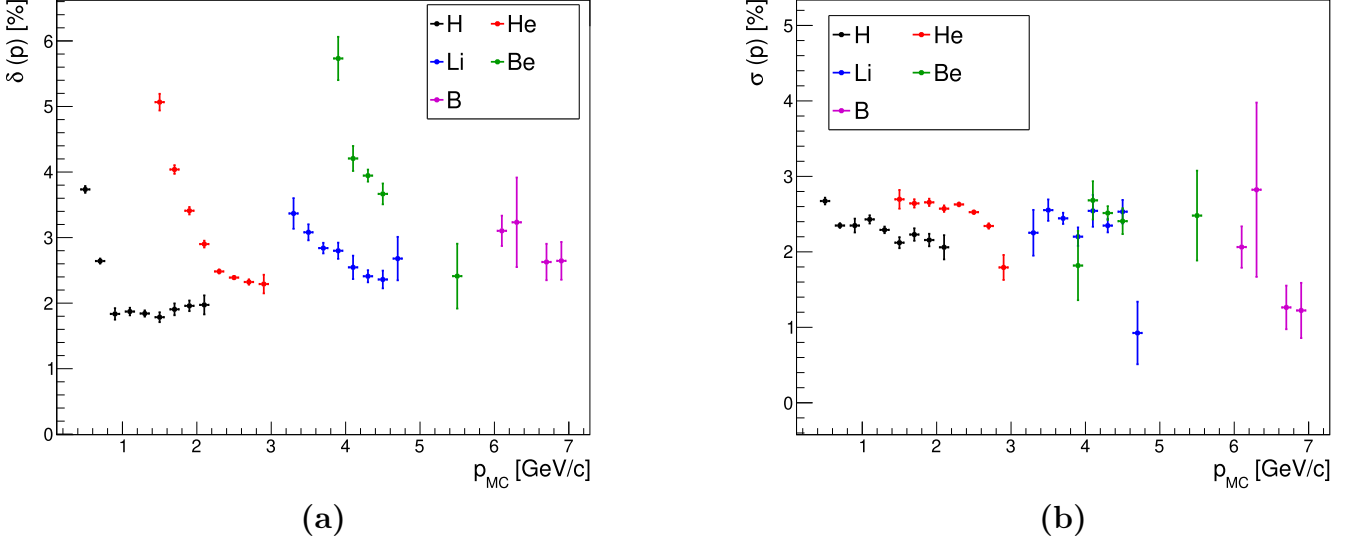


Figure 6.13: $\Delta p/p$ Momentum Resolution after Cut_3. a) Mean variation (offset δ) and b) standard deviation σ of its distribution.

6.4 Experimental Results

The following results are based on 1.9 million data taken during the CNAO campaign of November 2024 with the full FOOT setup, using a ^{12}C 200 MeV/u kinetic energy ion beam impinging on a 5 mm C target, in a *minimum bias* trigger configuration.

In the global track reconstruction, VTX, MSD and TW clusters are considered, while IT requires further performance studies before it can be reliably integrated into the reconstruction. In such conditions, the processes for cross section measurements and momentum and mass reconstruction are presented in the following sections.

6.4.1 Yields

GENFIT algorithm succeeds in tracking reconstruction, even in the magnetic field environment which has been introduced in experimental data analysis for the first time. Fig. 6.14, e.g., shows an event of the experimental data, in which up to three fragments have been successfully reconstructed from detector clusters.

Figure 6.15 shows the reconstructed tracks per event. Out of 1.6 million events, about 1 million tracks were reconstructed, with $\sim 1.1\%$ of events containing mul-

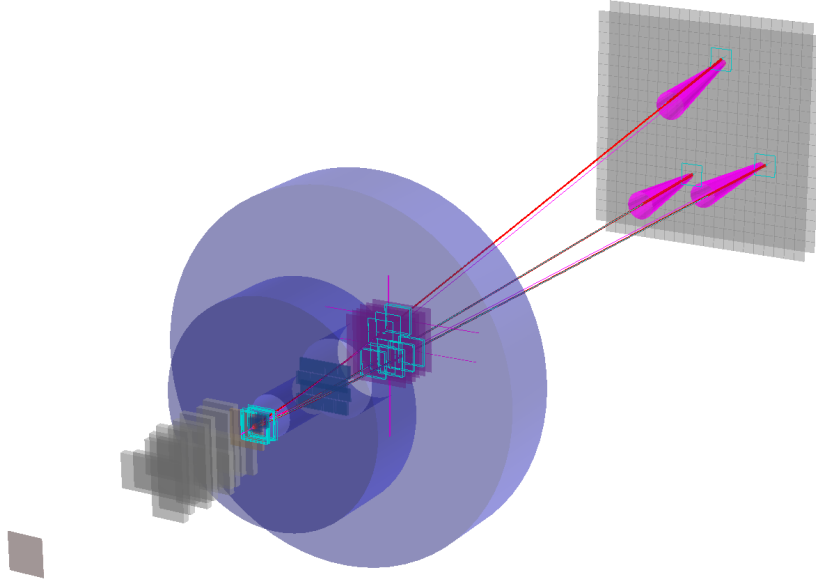


Figure 6.14: GENFIT event display of an event of CNAO 2024 experimental data, in which the reconstruction of three fragments has been achieved, considering clusters of VTX, MSD and TW and the presence of the magnetic field in the tracking region.

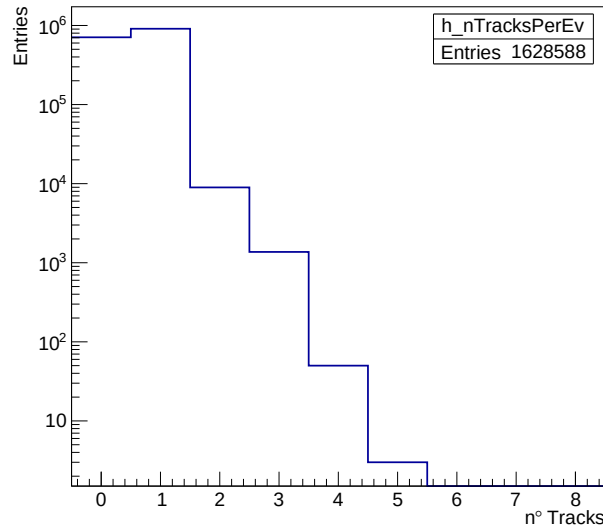
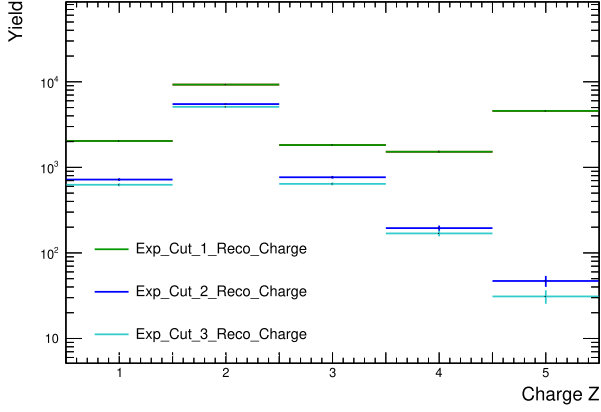
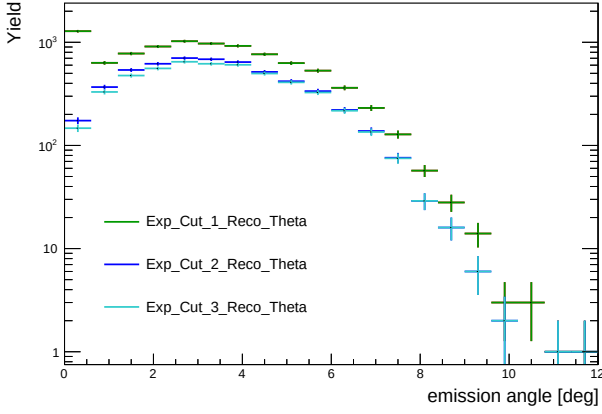
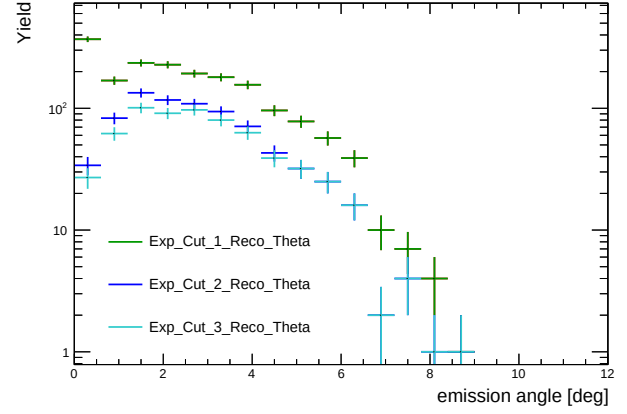


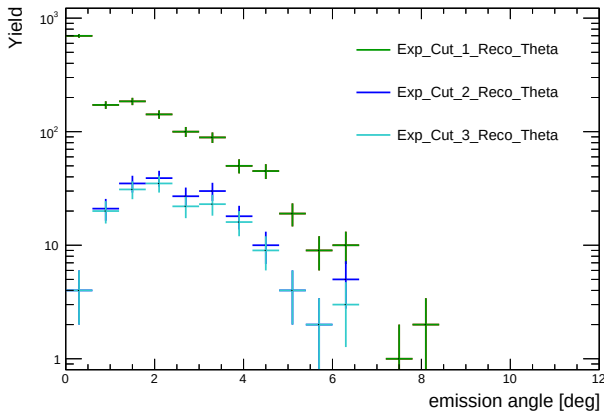
Figure 6.15: Distribution of the number of tracks reconstructed in each event. Up to 5 fragments are fitted by GENFIT algorithm.

(a) *Total*

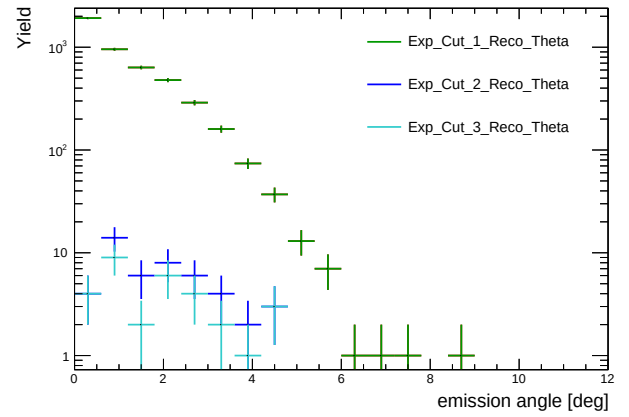
(b) He



(c) Li



(d) Be



(e) B

Figure 6.16: Yields reconstruction as a function of charge (a) and fragments angular distribution (b-d), applying Cut_1 (green), Cut_2 (blue) and Cut_3 (light blue) on experimental data.

multiple tracks, consistent with target fragmentation probability in the FOOT setup. Up to five tracks per event are obtained, confirming the robustness of the GENFIT algorithm. On the other hand, the $\sim 600,000$ events with no tracks are under investigation and expected to be resolved with improved alignment and detector calibration.

All selections of **Cut_0**, **Cut_1**, **Cut_2** and **Cut_3** of Sec. 5.4.1 are applied, with two only main exceptions: at first, the p -value cut of **TrackQuality** selection of Sec. 5.4.1 has been made softer to maintain an appropriate statistic, still considering the removal of bad tracks. Moreover, the Selection **TWnum**, for which the number of reconstructed tracks is the same of the number of TW points, is turned off because a higher inefficiency of reconstruction has been faced on experimental data, which affects the statistics of events to be considered. Yields are shown in Fig. 6.16 as a function of charge and fragment angular distribution. In particular, **Cut_0** visualization is omitted due to its almost complete overlap with **Cut_1**. As visible in Fig. 6.16-a, the largest drop in tracks occurs after **Cut_2**, namely the requirement of multitrack reconstruction per event, as already stated in the description of MC efficiencies (Sec. 5.2). This effect is particularly evident for $Z \geq 4$, where the primary generally fragments into two or more tracks, mainly made of protons. Together with particles which go outside the detector acceptance, as already discussed in Chap. 4, a possible VTX inefficiency for such particle detection makes this cut especially relevant for yield suppression. Regarding the angular distributions of Fig. 6.16-b,e, the reconstruction positively reaches the setup geometric acceptance (around 10°), with values consistently decreasing while fragments get heavier. These counts are ready to be used for cross section measurements.

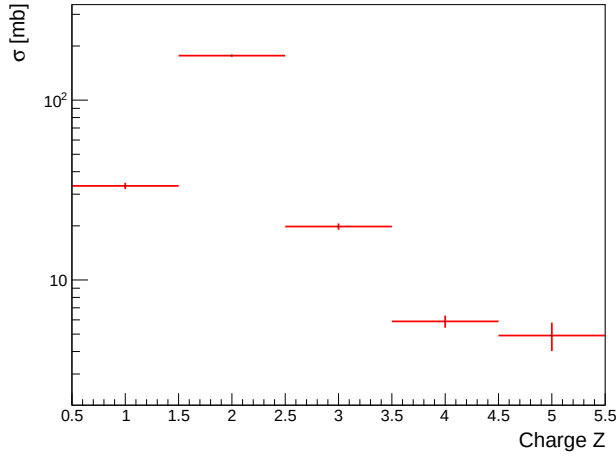
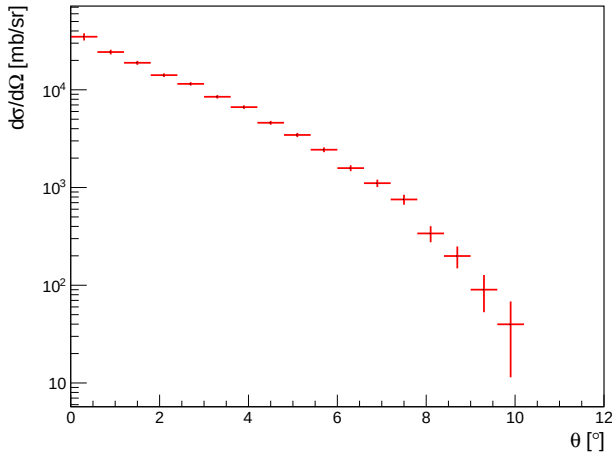
6.4.2 Cross Sections measurements on data

Cross section measurements are obtained following the approach of Sec. 5.4.2 and, in particular, Eq. 5.5 is reported for the sake of clarity:

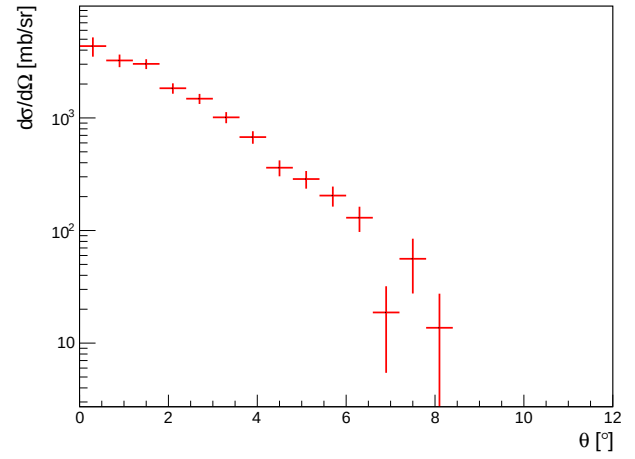
$$\frac{d\sigma}{d\Omega}(Z, \theta) = \frac{Y(Z, \theta)}{N_{\text{beam}} N_{\text{target}} \Delta\Omega(\theta) \varepsilon(Z, \theta)}.$$

In this analysis, yields $Y(Z, \theta)$ are the one obtained in Sec. 6.4.1, in a phase space with $\Delta\theta$ of 0.6° . $\varepsilon(Z, \theta)$ is represented by the global tracking efficiency obtained by the FLUKA MC simulation reported in Sec. 5.2. N_{beam} finally, is the counts of primary particles impinging on the target, where Selection Cuts 1,2 are applied to consider only the eligible events, namely the ones with not SC pileup and with BM track reconstructed.

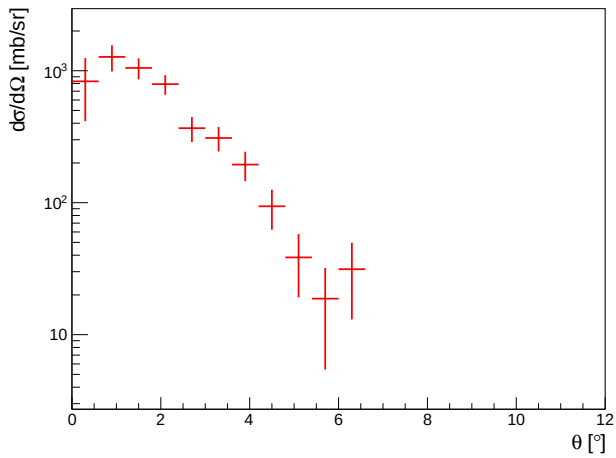
After application of all selections up to **Cut_3**, experimental cross section measurements are reported in Fig. 6.17. At present, only the statistical error is shown,

(a) *Total*

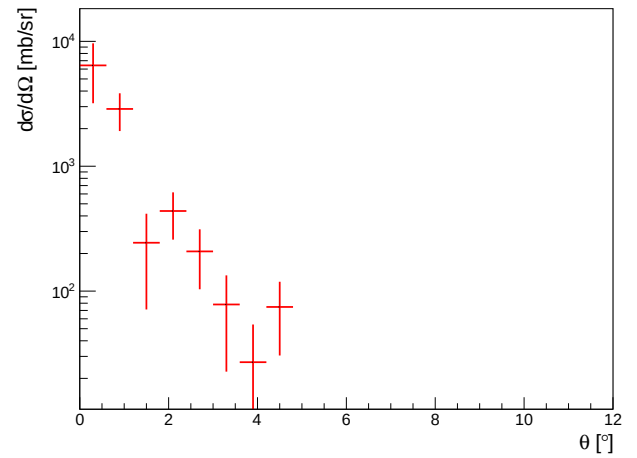
(b) He



(c) Li

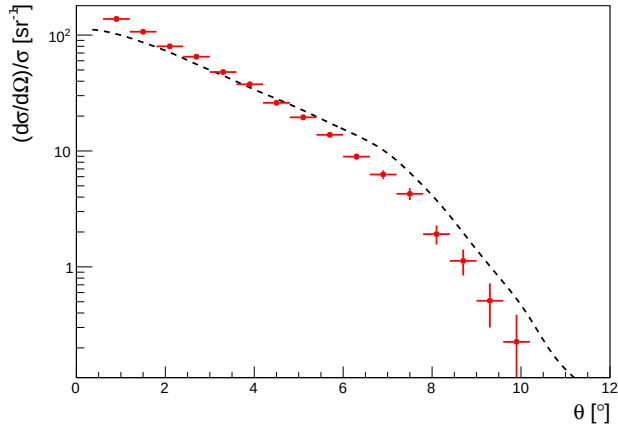


(d) Be

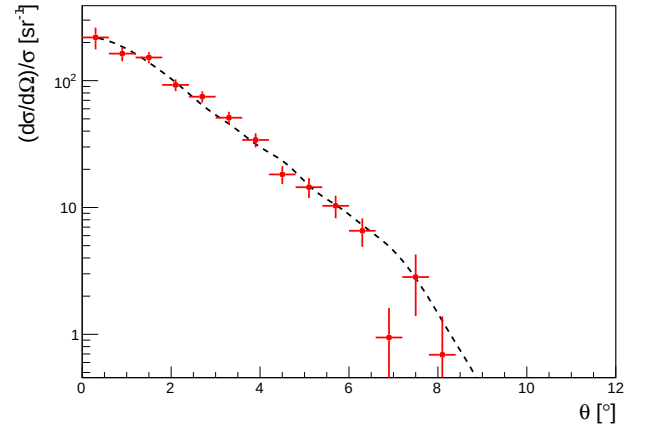


(e) B

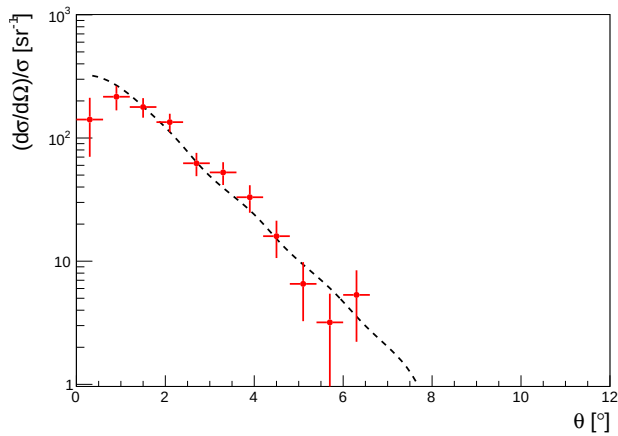
Figure 6.17: Elemental cross section (a) and differential nuclear fragmentation cross sections in angle (b-d) of ^{12}C 200 MeV/u ion beam impinging on a 5 mm C target on experimental data.



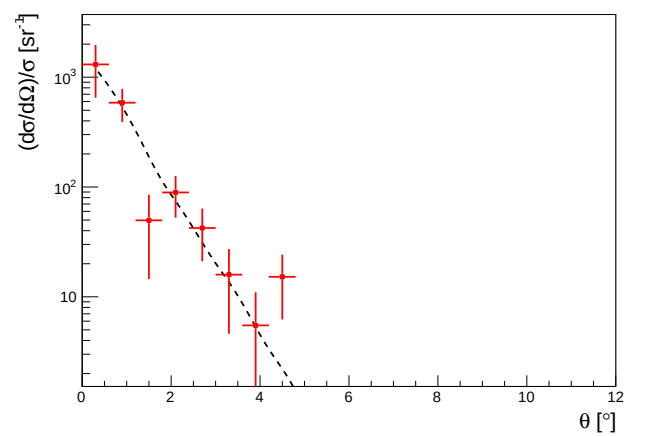
(a) He



(b) Li



(c) Be



(d) B

Figure 6.18: Comparison of differential nuclear fragmentation cross sections normalized on total cross section, for experimental (red dots) and MC reference (black dashed line) ones.

while the systematics will be evaluated through comparison with MC reference results. This will provide insight into the robustness of the reconstruction and selection algorithm, without affecting the main goal of obtaining MC independent cross section measurements.

In such comparison, measured cross-section values show a systematic underestimation with respect to the MC ones from the closure test (Sec. 6.2), whose causes can be attributed to VTX efficiency, not yet finalized MSD clustering, magnets mapping and detector alignment. A meaningful way to assess the reliability of the cross section measurements is to compare their normalized distributions, a process that allows bypassing the effect of overall inefficiencies without altering the physical content of the measured values.

Fig. 6.18, in particular, shows the experimental (MC) differential cross sections of Fig. 6.17-b,e (Fig. 6.10-c,f), normalized for the fiducial total elemental one (Eq. 5.8) of Fig. 6.17-a (Fig. 6.10-a). After their comparison, a good agreement is observed, indicating that the MC model used in FLUKA is performing well for this type of interaction, as it has already been demonstrated in a similar situation [142]. Incidentally, this indicates that the whole analysis chain is performing as expected on the differential cross section shapes, notwithstanding the problems encountered on the data. Values are, within their statistical uncertainties, generally consistent with the MC ones. The statistical uncertainty, in particular, becomes larger for $Z \geq 4$, where the limited statistics represent the main constraint; however, this does not prevent the reconstruction of such fragments even at large emission angles.

Overall, these results suggest that, with the planned improvements, even more complete and reliable cross-section measurements will be achievable.

6.4.3 Mass measurements on data

Eq.6.1 is used for mass reconstruction, where p is obtained by GENFIT track fitting and ToF from TW points. The distributions of the main fragments ($Z \leq 5$) and of the primary C are shown in Fig. 6.19, after the application of all cuts. A comparison with MC reconstruction from Fig. 6.12 is also reported, normalized to the same number of primaries for qualitative purposes. After the selection cuts, aside from the expected decrease in statistics, which is particularly pronounced for some elements as already observed for the charge yields (Sec. 6.4.1), a clearer separation between the main peaks is observed.

This motivates the use, for the cleanest selection (`Cut_2`), of a multi-Gaussian fit in order to quantitatively discriminate the individual peaks. `Cut_2`, is reported in Fig. 6.20, with normalized MC for qualitative reference. In general, the main isotopes are well reconstructed, with an overall resolution of the order of $\sim 10\%$

(see Tab. 6.2). Isotopes $Z \leq 4$ are discriminated, while the resolution is not enough to distinguish the isotopes of B. In the reconstruction of C, some fragments of the primary survive the selection cuts, while the available statistics are not sufficient to distinguish other Carbon isotopes.

With the current setup, mass reconstruction has been successfully achieved by combining the momentum measured by the tracking system with the *ToF* (Eq. 6.1). These results represent the first isotope identification in FOOT obtained using the tracking setup. Building on this achievement, the observed separation between the mass peaks enabled the definition of yields as a function of mass and charge $Y(Z, A)$. This, in turn, motivated an additional step: the evaluation of isotopic cross sections $d\sigma/d\Omega(Z, A)$ for the main isotopes of He, Li, and Be.

As a preliminary study, a relatively smooth discrimination has been adopted, with the main goal of verifying whether isotopic cross sections of this kind can be extracted and to obtain first exploratory results. Based on the fit parameters reported in Tab. 6.2, fiducial mass ranges were defined for each isotope, as summarized in Tab. 6.3. These intervals have been chosen to ensure a good balance between purity and statistics in the selected sample.

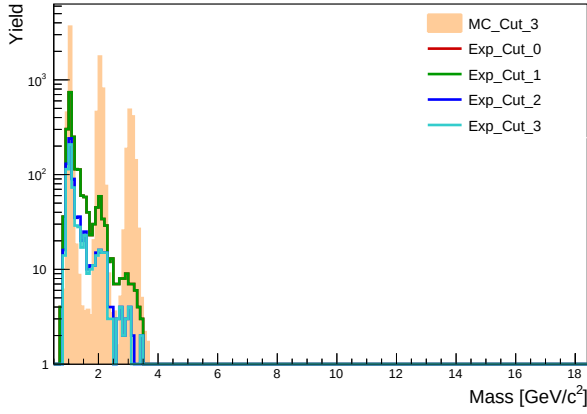
The corresponding calculations are presented in the following section.

Isotopes	$\mu \pm \sigma$ [%]	Isotopes	$\mu \pm \sigma$ [%]
^1H	1.1 ± 0.2 [18%]	^7Li	7.2 ± 0.4 [5.5%]
^2H	2.1 ± 0.2 [9.5%]	^7Be	7.2 ± 0.2 [2.7%]
^3He	3.1 ± 0.2 [6.5%]	^9Be	9.1 ± 1.0 [10.9%]
^4He	4.1 ± 0.2 [4.8%]	^{10}B	11.0 ± 0.8 [7.2%]
^6Li	6.1 ± 0.2 [3.2%]	^{12}C	12.4 ± 0.6 [4.8%]

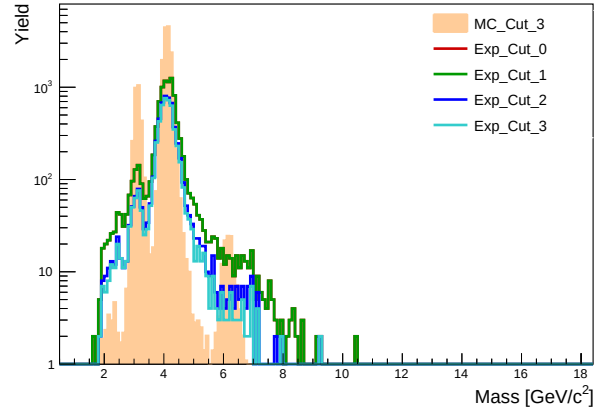
Table 6.2: Fit values of main isotopes for mass reconstruction, after the application of all the cuts on experimental events.

Isotope	Fiducial range on A	Isotope	Fiducial range on A
^3He	$2.5 < A < 3.5$	^7Li	$6.6 < A < 7.6$
^4He	$3.5 < A < 5.0$	^7Be	$6.5 < A < 7.5$
^6Li	$5.4 < A < 6.6$	^9Be	$8.5 < A < 9.5$

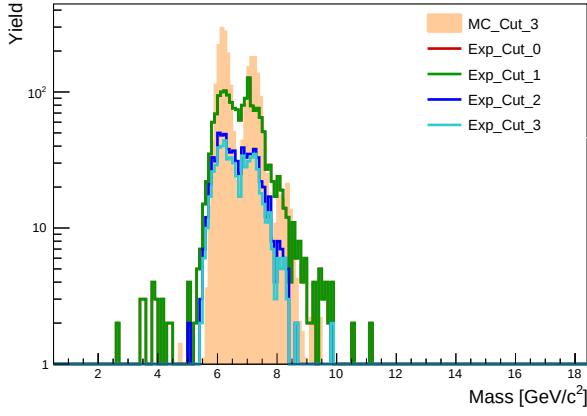
Table 6.3: Fiducial mass intervals that will be adopted for the preliminary extraction of isotopic cross sections $d\sigma/d\Omega(Z, A)$, defined from the reconstructed mass distributions and the fit results in Tab. 6.2.



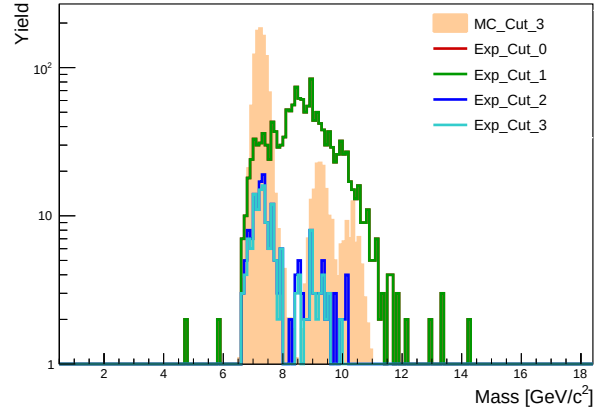
(a) H



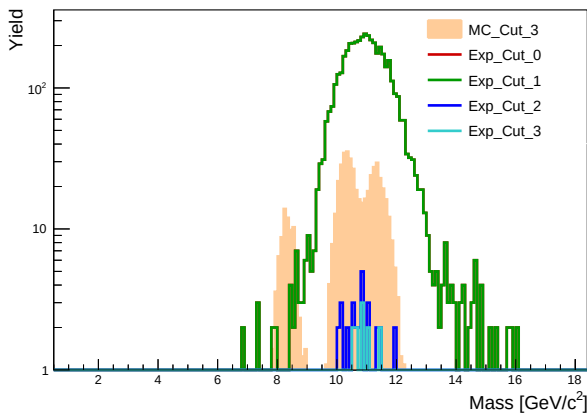
(b) He



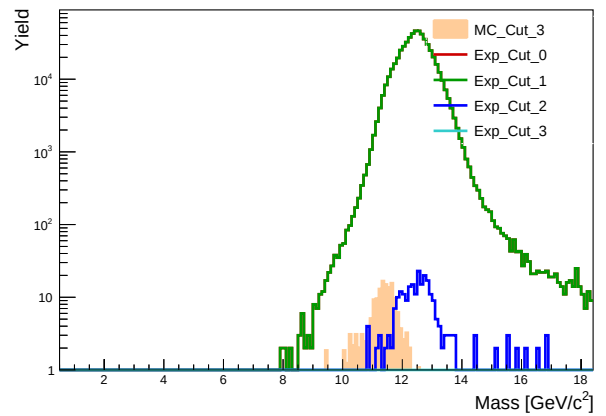
(c) Li



(d) Be

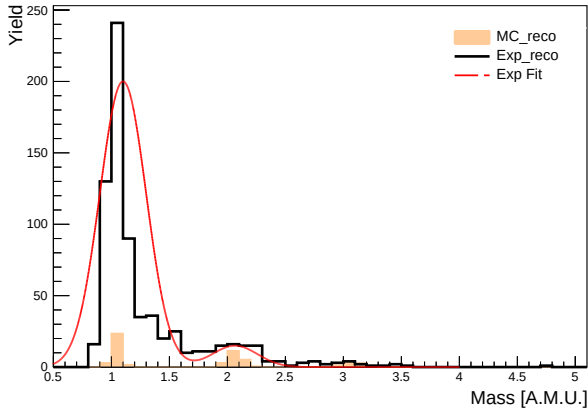


(e) B

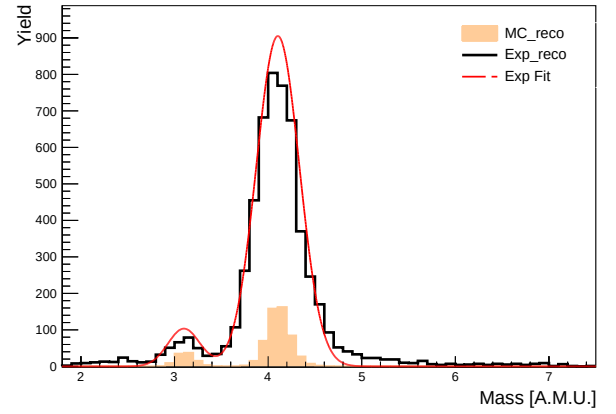


(f) C

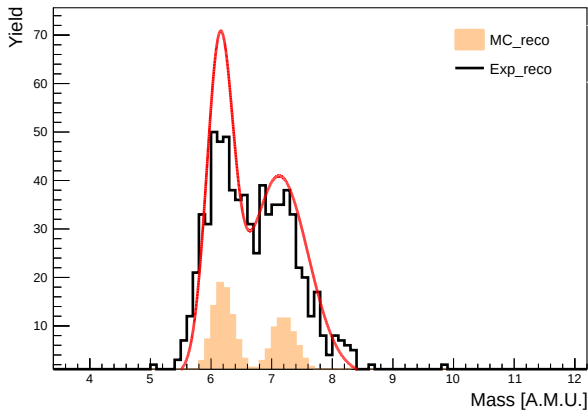
Figure 6.19: Mass reconstruction for fragments (a-e) and primary particles (f) obtained on experimental data, applying **Cut_0** (red), **Cut_1** (green), **Cut_2** (blue) and **Cut_3** (light blue), in comparison with MC (light orange) from MC FLUKA simulation, normalized to experimental N_{beam} .



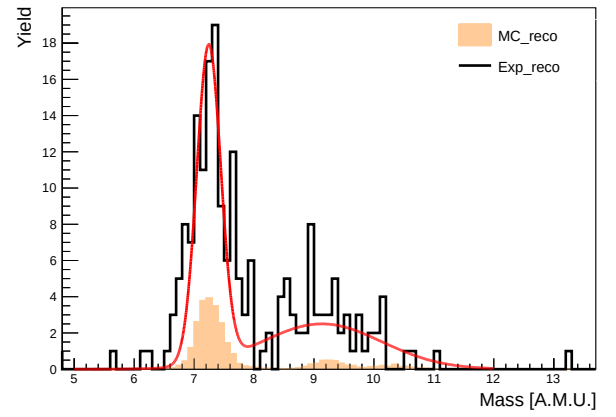
(a) H



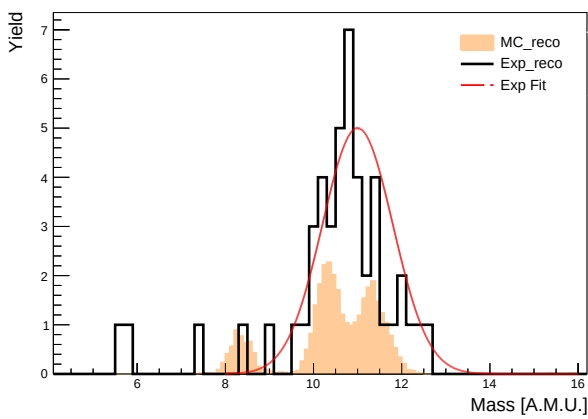
(b) He



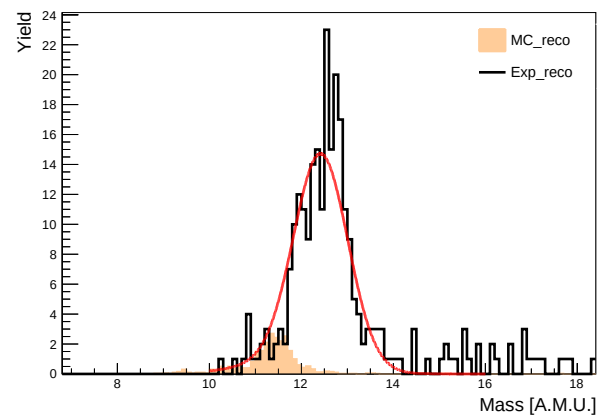
(c) Li



(d) Be



(e) B



(f) C

Figure 6.20: Mass reconstruction for fragments (a-e) and primary particles (f), applying Cut_2 (blue) with multifitting. It is also in comparison with MC (light orange) from MC FLUKA simulation, normalized only for qualitative purposes.

6.4.4 Isotopic Cross Sections measurements on data

The adopted approach for differential cross section in Z (see Sec. 1.2.1) is followed, with the addition of the atomic mass number A identification. This allows the extension of the previous analysis, providing a more detailed characterization of the fragmentation process. Accordingly, the expression for the differential cross section is modified to include the isotopic dependence, becoming:

$$\frac{d\sigma}{d\Omega}(Z, A, \theta) = \frac{Y(Z, A, \theta)}{N_{\text{beam}} N_{\text{target}} \Delta\Omega(\theta) \varepsilon(Z, A, \theta)}. \quad (6.4)$$

where $Y(Z, A, \theta)$ is the number of detected fragments with atomic number Z and mass number A , N_{beam} is the number of incident primary ions, N_{target} is the target areal density, $\Delta\Omega$ is the solid angle subtended by the detection system for every bin and $\varepsilon(Z, A, \theta)$ represents the tracking efficiency of fragments (Eq. 4.4) according to their atomic number Z and mass number A . This definition naturally reduces to the elemental (charge-resolved) case when summing over all the isotopes A of a given element Z :

$$\frac{d\sigma}{d\Omega}(Z, \theta) = \sum_A \frac{d\sigma}{d\Omega}(Z, A, \theta).$$

The isotopic efficiencies of the main He, Li, and Be isotopes are shown in Fig. 6.21 for the angular distributions. Their trend closely follows the characteristic behavior observed for the corresponding charge efficiencies, indicating the overall consistency of the reconstruction and selection chain also at the isotopic level. These efficiencies have been incorporated into the isotopic differential cross section closure test in Fig. 6.22, as outlined in Sec. 6.2. Within the statistical uncertainty of the last cut `Cut_3`, a systematic discrepancy below 5% is observed for the He and Li isotopes up to $\sim 8^\circ$ angular emission, in agreement with the corresponding elemental results shown in Fig. 6.10. Conversely, a systematic underestimation of about 15% is observed for the Be isotopes. This effect could be related to an excessively loose discrimination among the Be isotopes in the definition adopted in Tab. 6.3, suggesting that further refinements may be required either in the yield extraction or in the efficiency evaluation. Nevertheless, the obtained values are satisfactory for proceeding towards the assessment of experimental isotopic cross sections.

Similar to Fig. 6.16, isotopic experimental yields are reported in Fig. 6.23 as a function of angular emission, after the application of all the experimental cuts. Despite the statistical uncertainty increasing while isotope abundance decreases, a clear spectral shape can be identified for all particles, up to an angular emission of $\sim 10^\circ$, in agreement with the detector geometrical acceptance. Following Eq. 6.4, using yields of Fig. 6.23 and efficiencies of Fig. 6.21, the experimental isotopic

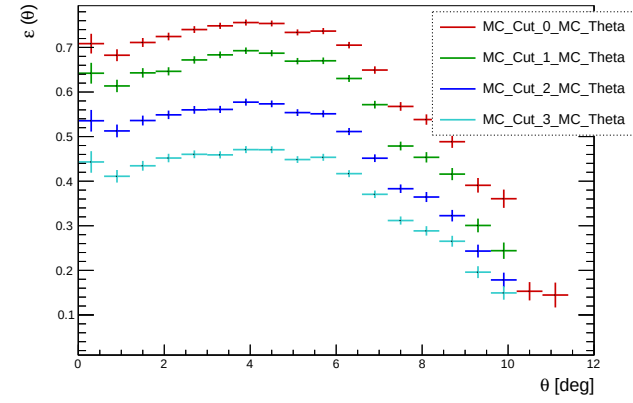
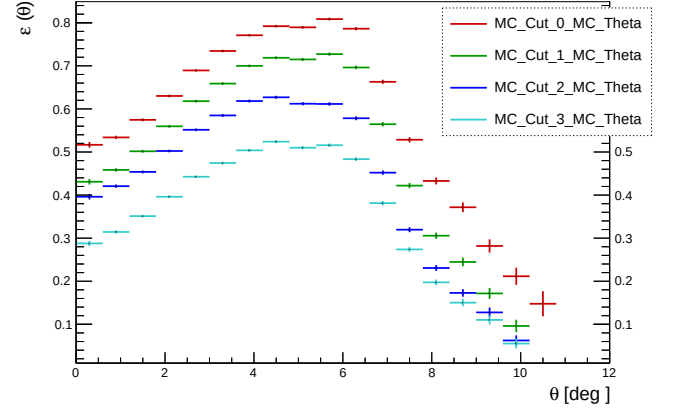
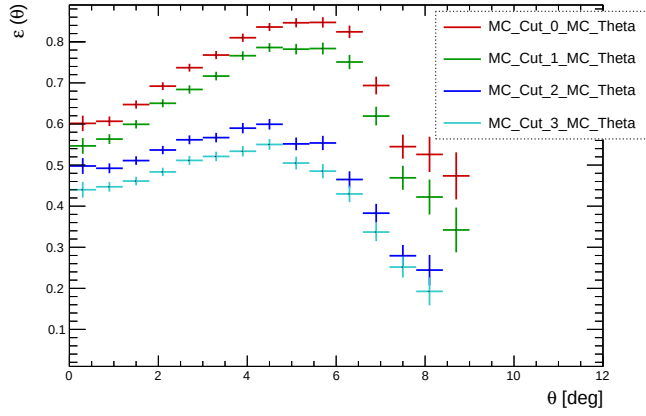
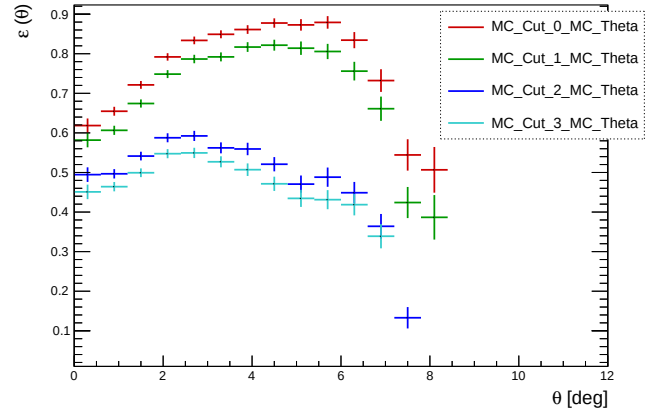
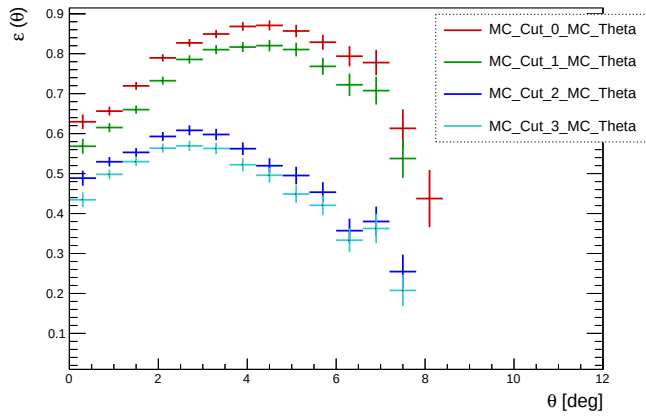
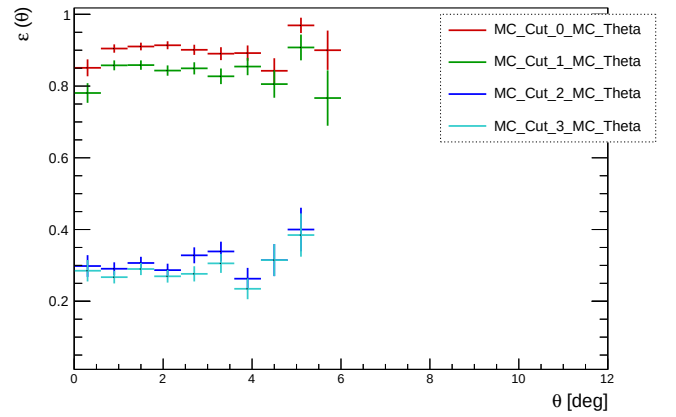
(a) ^3He (b) ^4He (c) ^6Li (d) ^7Li (e) ^7Be (f) ^9Be

Figure 6.21: Global track reconstruction isotopic efficiency (a-f) for CNAO 2024 simulation.

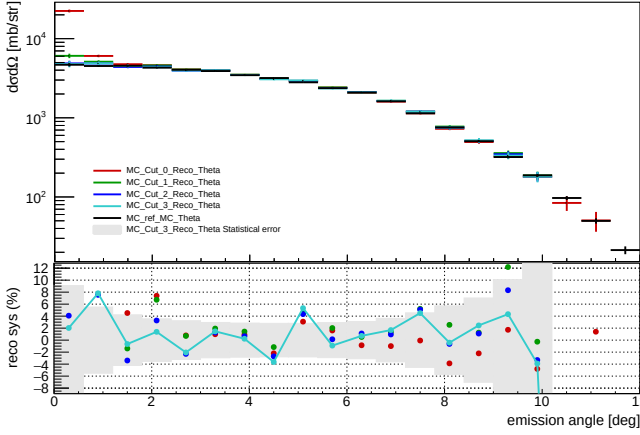
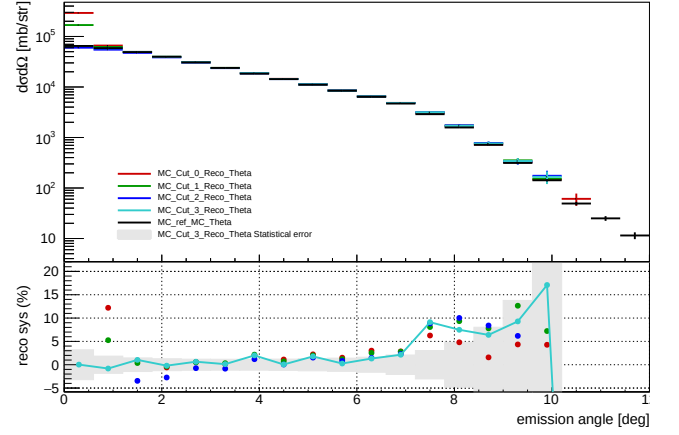
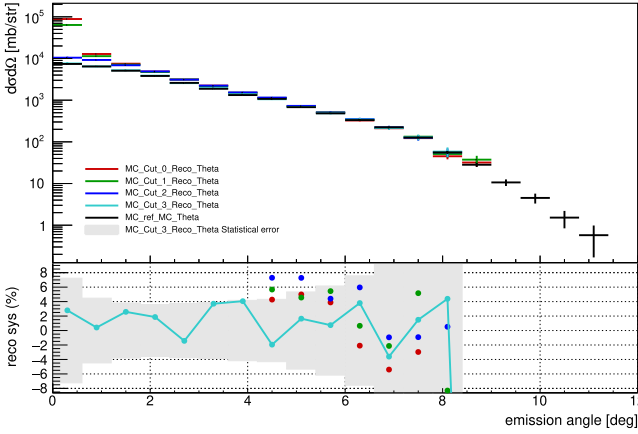
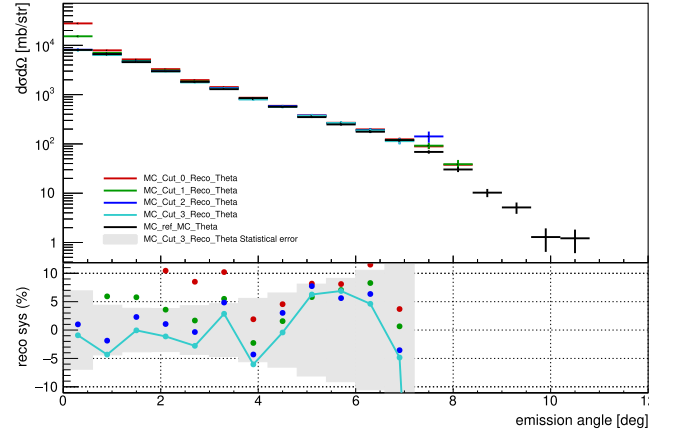
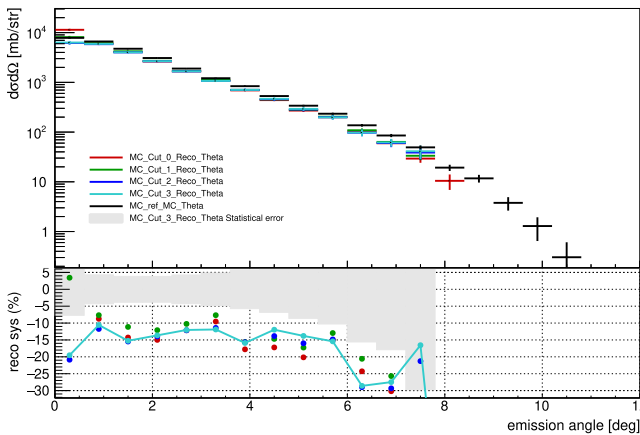
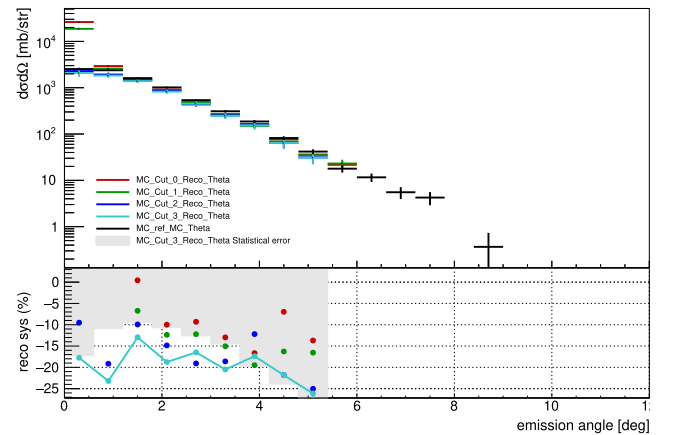
(a) ^3He (b) ^4He (c) ^6Li (d) ^7Li (e) ^7Be (f) ^9Be

Figure 6.22: Differential (a-f) fragmentation isotopic cross section closure test for CNAO 2024 simulation for ^{12}C 200 MeV/u ions impinging on a C target. In the upper part, cross section measurements of Cut_0 (red), Cut_1 (green), Cut_2 (blue), Cut_3 (light blue) and MC_ref (black) are reported. In the lower part, the relative variation of cuts with MC reference is shown (in dots), together with the statistical uncertainties of the last cut.

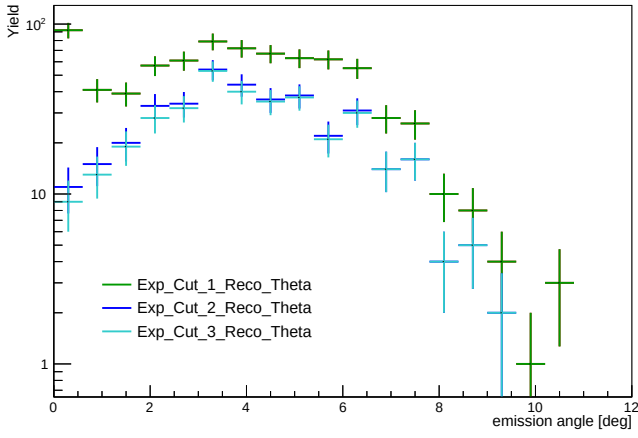
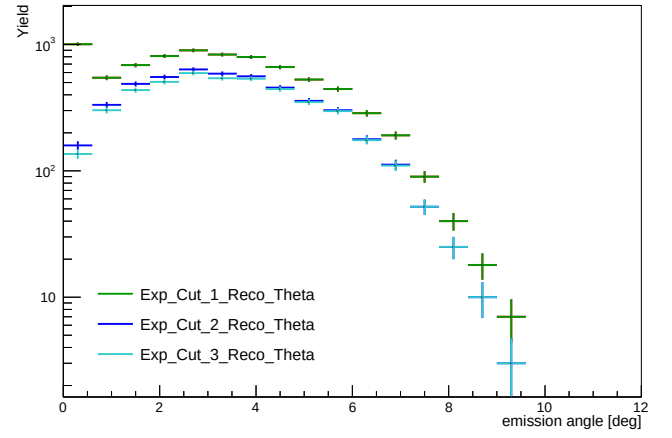
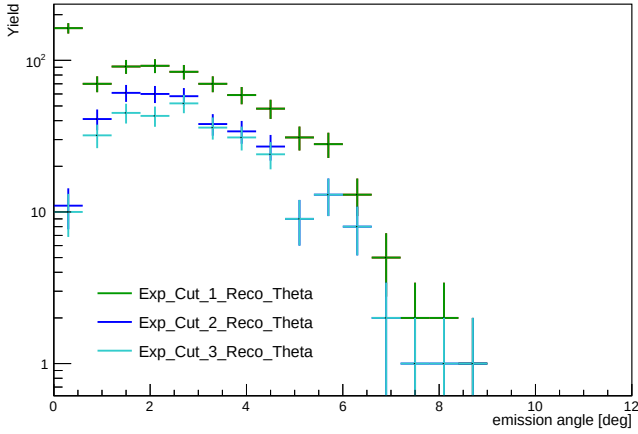
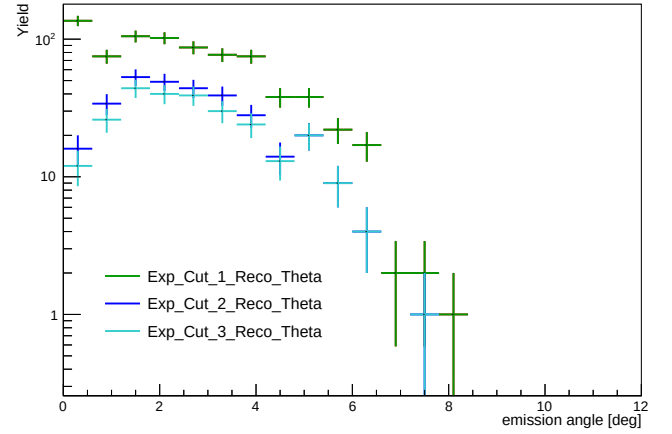
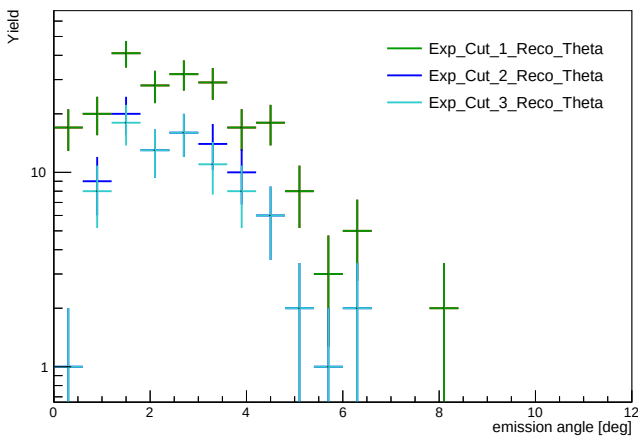
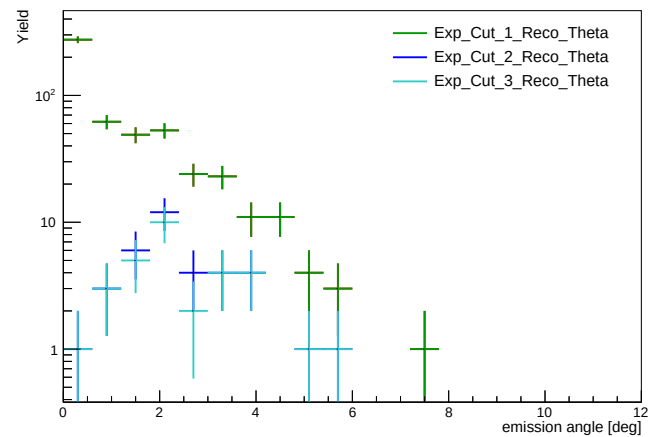
(a) ^3He (b) ^4He (c) ^6Li (d) ^7Li (e) ^7Be (f) ^9Be

Figure 6.23: Yields reconstruction as a function of isotopes angular distribution (a-f), applying Cut_1 (green), Cut_2 (blue) and Cut_3 (light blue) on experimental data.

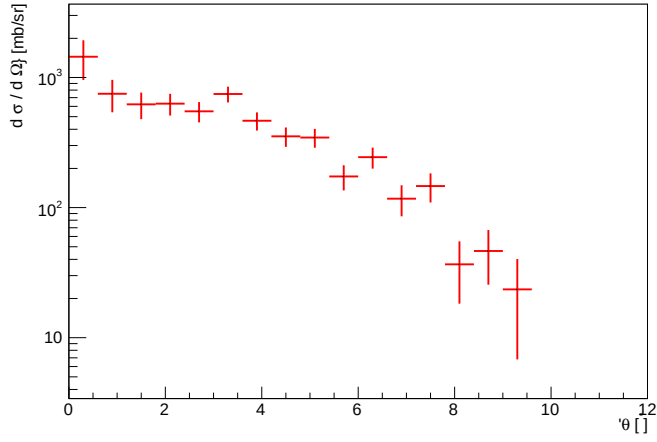
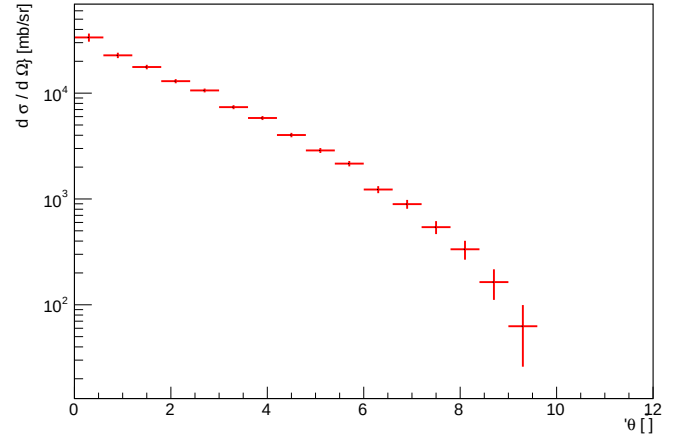
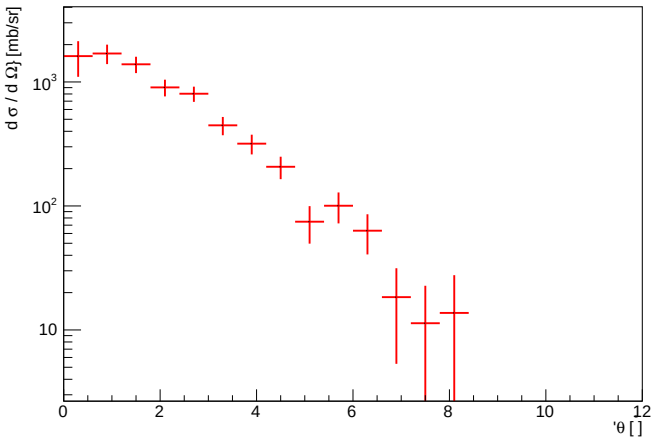
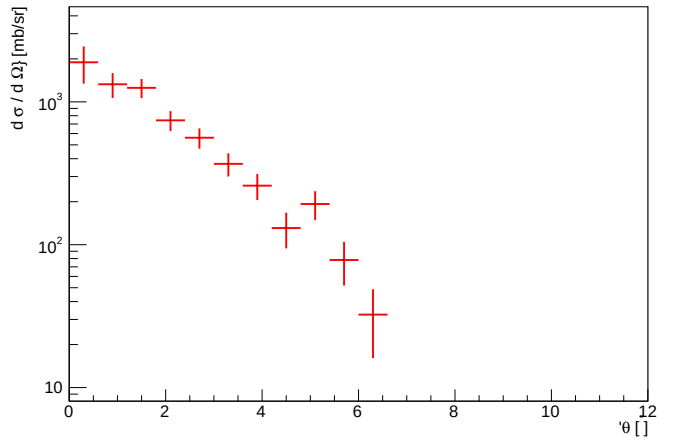
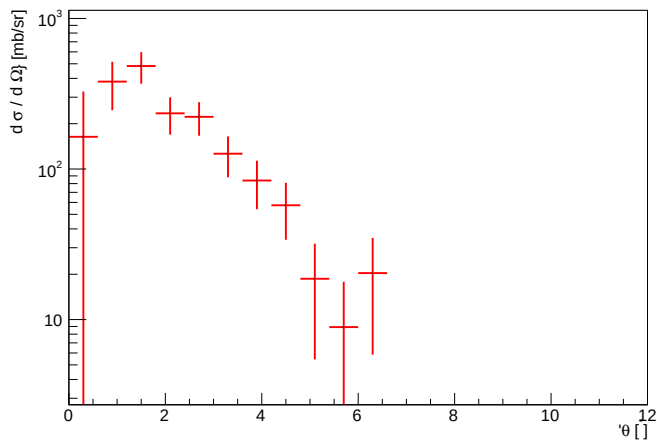
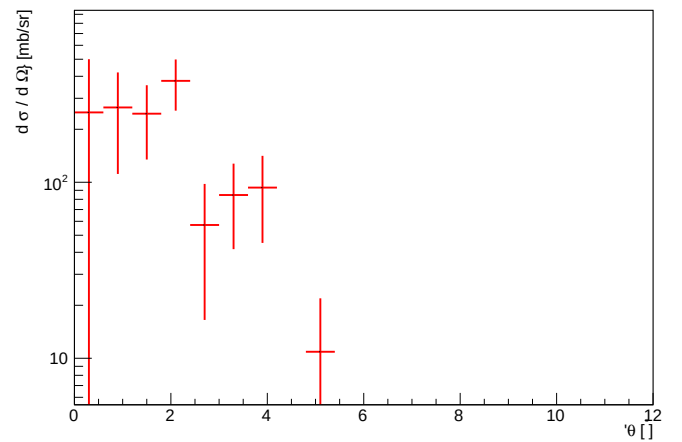
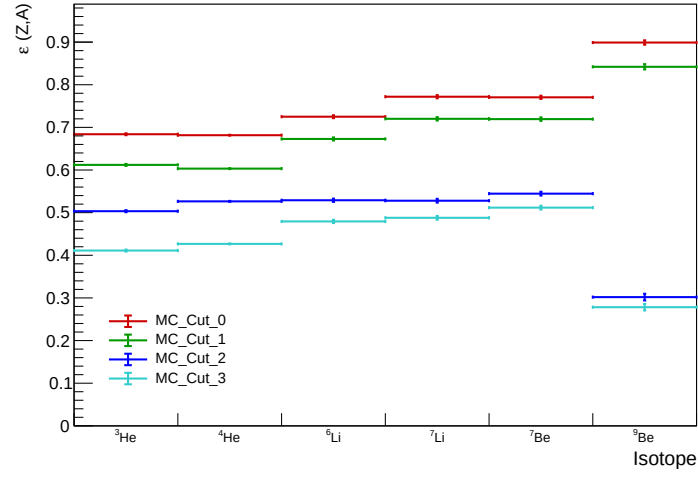
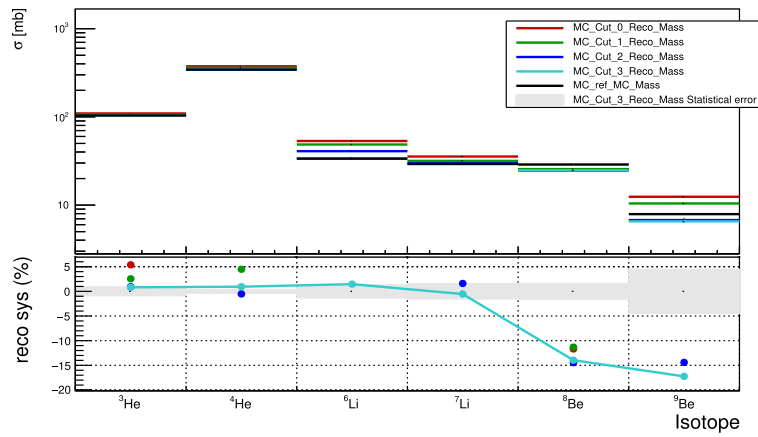
(a) ^3He (b) ^4He (c) ^6Li (d) ^7Li (e) ^7Be (f) ^9Be

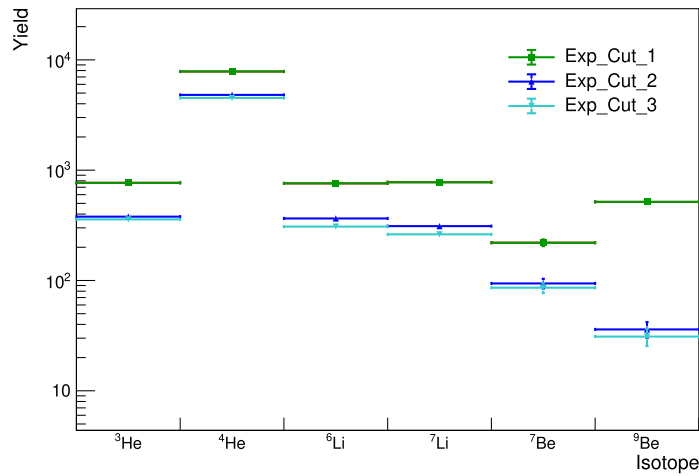
Figure 6.24: Differential isotopic fragmentation cross sections in angle (a-d) of ^{12}C 200 MeV/u ion beam impinging on a 5 mm C target on experimental data.



(a)



(b)



(c)

Figure 6.25: For the main isotopes: (a) global track efficiencies (simulation), (b) isotopic cross-section closure test (simulation), (c) isotopic yields of CNAO 2024 experimental data.

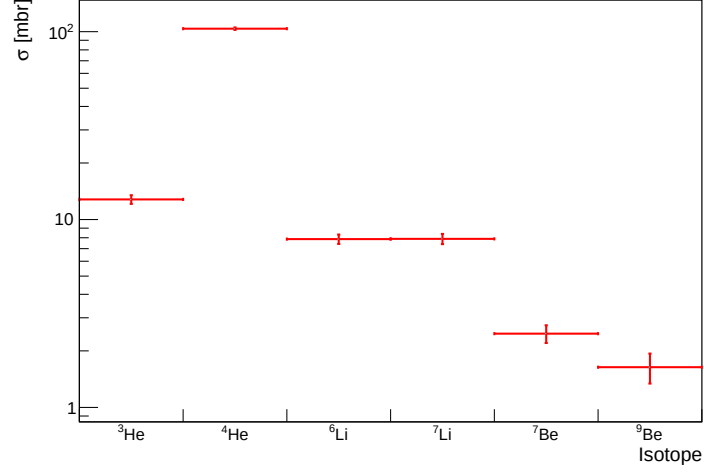


Figure 6.26: Experimental isotopic cross section of CNAO 2024 campaign for the main isotopes.

differential cross sections $d\sigma/d\Omega (Z, A, \theta)$ are reported in Fig. 6.24, where only statistical uncertainties are shown.

Furthermore, the isotopic differential cross sections can be integrated over the angular aperture to obtain the total isotopic cross sections, following the same approach adopted in Eq. 5.8 for the elemental case:

$$\sigma_{\text{tot}}(Z, A) = \int_{\theta_{\min}}^{\theta_{\max}} \frac{d\sigma}{d\theta}(Z, A, \theta), d\theta. \quad (6.5)$$

Figure 6.25-a shows the global track efficiencies for each isotope, which are also used in the isotopic cross-section closure test displayed in Fig. 6.25-b. The comparison between reconstructed and Monte Carlo results reveals a level of agreement consistent with that observed for the differential case. The isotopic yields of the main fragments, after the full selection chain, are reported in Fig. 6.25-c. By combining these yields with the efficiencies of Fig. 6.25-a, the resulting experimental total isotopic cross sections, obtained through Eq. 6.5, are presented in Fig. 6.26. An accurate comparison with MC results is essential for a proper assessment of the systematic uncertainty, therefore a first attempt in this direction consists of comparing the normalized differential cross sections (each differential cross section of Fig. 6.24 (Fig. 6.22) divided by its total value of Fig. 6.26 (Fig. 6.25-b) for experimental (MC) measurements), taking into account all the considerations discussed in Sec. 6.4.2 regarding the interpretation of this comparison.

Normalized isotopic cross sections are reported in Fig. 6.27, where a consistent trend is observed between the two quantities. With the present experimental ap-

paratus, operated in a complete setup within a magnetic field, it has been possible for the first time to perform this type of measurement and extend the analysis to isotopic differential cross sections in the research field of 200 - 800 MeV/u nuclei. The results represent a pioneering step in this field, with clear potential for further improvement through continued optimization of the analysis.

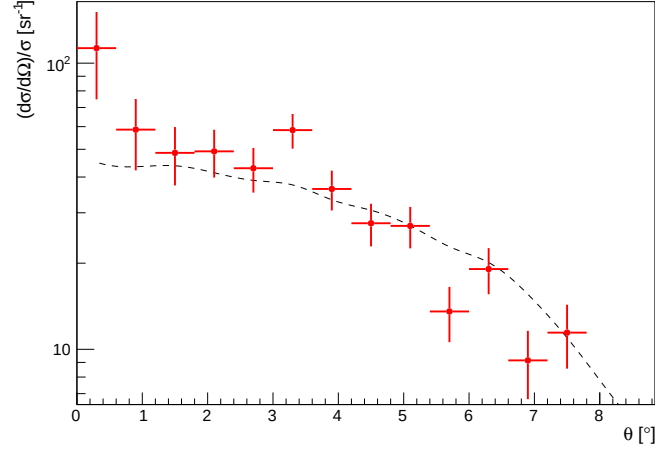
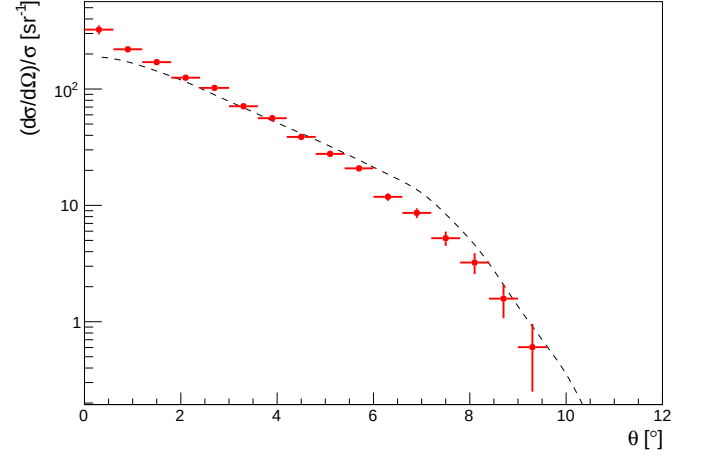
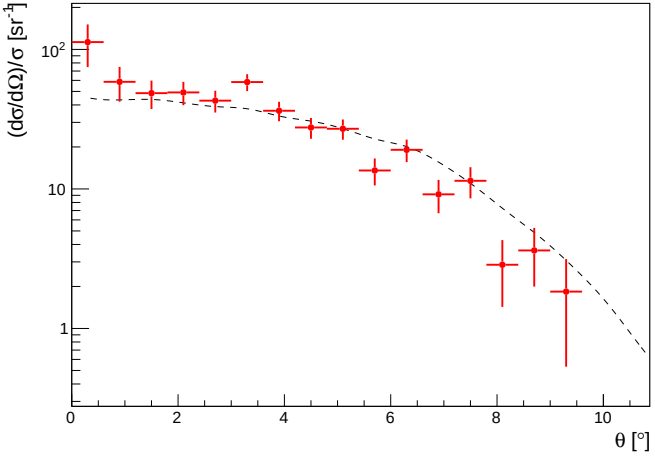
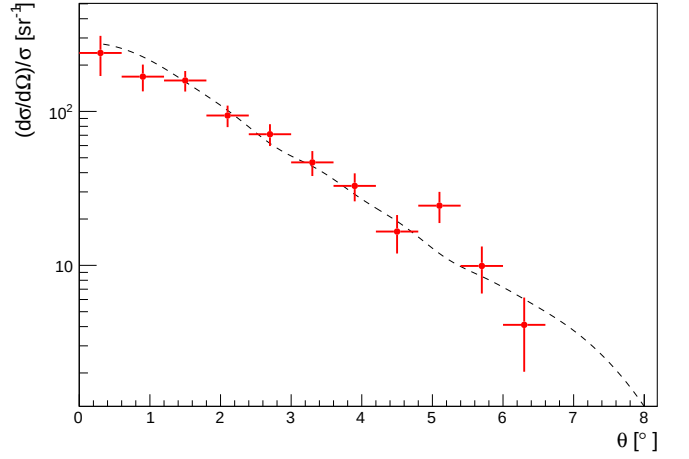
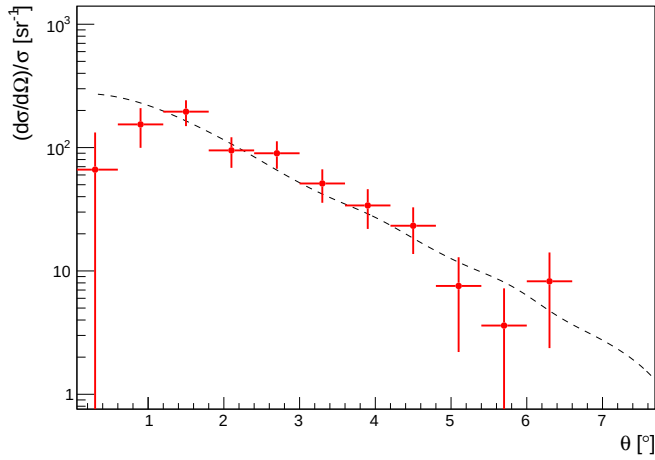
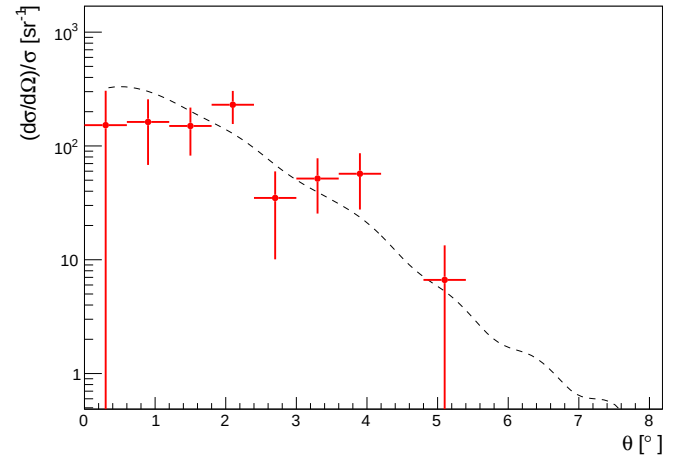
(a) ${}^3\text{He}$ (b) ${}^4\text{He}$ (c) ${}^6\text{Li}$ (d) ${}^7\text{Li}$ (e) ${}^7\text{Be}$ (f) ${}^9\text{Be}$

Figure 6.27: Comparison of differential nuclear fragmentation cross sections normalized on total cross section, for experimental (red dots) and MC reference (black dashed line) ones.

Conclusions

The FOOT experiment was conceived with the aim of measuring double differential cross sections in kinetic energy and emission angle for nuclear fragmentation reactions of light ions ($Z \leq 8$) in the 200–800 MeV/u energy range, providing data essential for refining Treatment Planning Systems in Particle Therapy and improving radiation risk assessments for long-term manned Space missions. To achieve those goals, FOOT utilizes two complementary, portable setups: the Electronic Setup, optimized for heavier fragments ($3 \leq Z \leq 8$) within a 10° angular acceptance, which includes a magnetic spectrometer, a Time of flight system and a calorimeter; and the Emulsion Spectrometer, dedicated to lighter fragments ($Z \leq 3$) emitted at large angles (up to 45°).

This Ph.D. work focuses on the experimental campaign conducted at CNAO in November 2024, employing the full Electronic Setup with a 200 MeV/u ^{12}C beam impinging on a carbon target. Two main and complementary tasks have been addressed: the detailed characterization of the MIMOSA-28 vertex detector and the measurement of the integral and differential cross sections of the fragmentation process between the beam and the target.

The performance evaluation of the MIMOSA-28 vertex detector was focused on threshold calibration, noise studies and efficiency evaluation. Analysis revealed that operating at the optimized threshold of 6σ on the sigmoid response curve of the threshold scan ensured that noise was successfully limited to approximately one cluster per event. At the same time, the detection efficiency for the stable sensors (1, 2, 3) remained consistently above 90%. The overall vertex tracking efficiency also exceeded 90%, a performance found to be largely independent of the incident proton energy, in the range of 70 to 230 MeV. Cluster analysis also showed robust signal discrimination from noise.

The analysis framework flows from event selections exploiting a Monte Carlo FLUKA simulation. In particular, several signal improvements are obtained, including the rejection of more than 30% of misidentified events (**Cut_1**), the complete removal of background associated with out-of-target fragmentation (**Cut_2**) and limiting the bad reconstruction of $Z = 3$ (Li) fragments due to TW pile-up from 15% down to 1.5% (**Cut_3**).

A closure test confirmed the reliability of the method, demonstrating that the systematic discrepancy between reconstructed and simulated cross sections was reduced to approximately 2–5% after applying the final cuts (**Cut_3**), achieving consistency with the required accuracy goals.

Furthermore, the magnetic spectrometer achieved high kinematic resolution: after cuts application, while momentum reconstruction showed a systematic offset of 2–4%, the momentum resolution ($\sigma(p)/p$) remained below 3% for all fragments, surpassing the 4–5% design requirement.

The core analysis presented in this thesis is based on experimental data collected during the CNAO 2024 campaign, conducted using the complete Electronic Setup within a magnetic field. The global tracking reconstruction successfully yielded more than 1 million global tracks, demonstrating the robustness of the GENFIT algorithm, with roughly 1.1% of events containing multiple tracks (up to 4), consistent with fragmentation probability. After applying the selection cuts and retrieving the tracking efficiencies from Monte Carlo (MC) simulations, both integral and differential elemental cross sections were measured. The comparison of the normalized differential cross sections showed good agreement in spectral shape with the MC reference, confirming the reliability of the analysis procedure. At present, the comparison with the MC simulation indicates a systematic underestimation of the cross section, likely due to reconstruction inefficiencies, which are currently under investigation.

Combining momentum (p) from tracking and Time of Flight, experimental mass reconstruction was evaluated, successfully discriminating the main isotopes for fragments with $Z \leq 4$. The overall mass resolution was approximately $\sim 10\%$, with specific resolutions measured at 4.8% for ${}^4\text{He}$ (4.1 ± 0.2 a.m.u.) and 5.5% for ${}^7\text{Li}$ (7.2 ± 0.4 a.m.u.). Building on the defined isotopic yields $Y(Z, A)$, the analysis was extended to extract the first angular isotopic differential cross sections $d\sigma/d\Omega(Z, A, \theta)$ using the complete magnetic spectrometer. This extension further confirmed a consistent trend between the normalized experimental isotopic cross sections and the MC prediction, establishing these results as a milestone in high-precision fragmentation identification for the FOOT experiment.

Overall, the research demonstrates the capabilities of the FOOT experiment to achieve its fragmentation cross-section measurements goals, supported by a set of highly efficient and well-characterized detectors and by a robust and reliable analysis framework.

Appendix A

Proton Imaging for the SIRMIO project

Thanks to the Marco Polo mobility scholarship funded by the University of Bologna, I conducted a 6-month internship in the Department of Experimental Physics – Medical Physics at Ludwig-Maximilians-Universität München, under the supervision of Prof. Dr. Katia Parodi, Head of the Department of Medical Physics.

The research focused on investigating the performance of a setup used in small animal radiographic proton imaging, improving the calibration and equalization approach of two TimePix3 pixel detectors, within the framework of the ERC funded project SIRMIO (Small Animal Proton Irradiator for Research in Molecular Image-guided) [69]. The goal is to enhance the robustness of imaging technique to improve animal positioning and to minimize range uncertainties in pre-clinical Particle Therapy. These results are complementary to those of the FOOT experiment, whose precise nuclear measurements provide an independent contribution to the reduction of such uncertainties.

After an overview on ion imaging principles, the SIRMIO project is introduced, focusing on the aim and the application of its TimePix3 detector setup. The process of calibration and equalization is explained, with results highlighting the quality and quantitative accuracy of the acquired proton images during CNAO 2022 data taking.

A.1 Medical imaging with ion beams

The fundamental principle of imaging is to infer the internal structure of an object based on its interaction with radiation. In this regard, X-ray Computed Tomography (xCT) and Ion Computed Tomography (iCT) share the same purpose

but differ in their mechanisms.

As discussed in Sec. 2.1, photons are either fully absorbed or scattered by a crossed medium or they pass without interaction, making photon attenuation the key quantity used for image reconstruction in xCT. In contrast, charged particles lose energy continuously as they traverse matter (see Sec. 1.1) but, if provided with sufficiently high initial energy, they can fully traverse the object. In iCT, the typically measured quantity for image reconstruction is then the residual energy or range of the particles after crossing the material.

Imaging plays a crucial role in medicine and in particular in Particle Therapy (PT), where an accurate knowledge of the tissues along the beam is required to infer where the beam stops. In comparison with xCT, iCT shows an important improvement from the clinical point of view: the dose exposure is reduced [62] along with range uncertainties [143] (see Sec. 2.1.4). Moreover, on-site iCT exploits the same delivery system of PT, opening new opportunities to innovate the treatments with daily adaptation and personalization [144].

In the next sections, the principles of iCT are deepened, followed by a description of the main iCT systems.

A.1.1 Principles of iCT

As stated in Sec. 2.1, the principle of PT is to modulate the energy of the ion beam so that its range ends precisely at the tumor site, exploiting the proportionality $R \propto E^2$ (see Eq. 1.21). However, since the stopping power depends on the properties of the material (in particular $dE/dx \propto \rho$, see Eq. 1.2), the beam energy must be scaled according to the density of the traversed tissues, which can vary significantly within the patient.

This variation can be measured in terms of Relative Stopping Power (RSP), a quantity defined as the ratio between the ion stopping power in a material (S_m) and that in water (S_w):

$$RSP = \frac{S_m(I_m(\mathbf{r}), E)}{S_w(I_w, E)} = -\frac{dE}{dx} \cdot \frac{1}{S_w}. \quad (\text{A.1})$$

Inspecting Eq. 1.2, it is possible to infer that S_m is a function of the mean excitation energy of the traversed material $I_m(\mathbf{r})$, varying with the position $\mathbf{r}$ but that it also depends on the energy E of the impinging ion. However, in the energy range from 80 MeV to 300 MeV typical of PT, the RSP could be considered energy-independent since the variations are less than 0.7% [145].

As a consequence, the energy and position dependent variables from Eq. A.1 can be separated and this allows to perform a linear integration of RSP along the

particle trajectory Γ of length L :

$$\int_L RSP(\Gamma(l)) d\Gamma(l) = - \int_{E_{in}}^{E_{out}} \frac{1}{S_w} dE \equiv WEPL \quad (A.2)$$

where E_{in} and E_{out} represent the energy of the ion before and after crossing the object. The right-hand side of the equation yields the total path length of the ion through various materials expressed as an equivalent thickness of water, also called Water-Equivalent Path Length $WEPL$).

Systems of iCT are able to compute $WEPL$ values by measuring the energy loss (knowing I_w), or calibrating the detector response to material of known RSP and thickness. Hence, every $WEPL$ measurement can be related to the required RSP value, obtaining a RSP map of interest for PT. Moreover, Eq A.2 represents a Radon Transform, at the mathematical basis for tomographic images [146].

In the next section, the main instrumentation designs for iCT are described.

A.1.2 Detector system for iCT

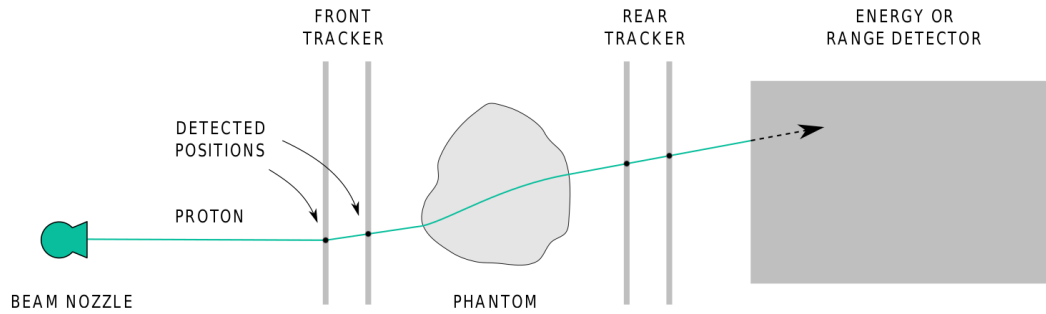


Figure A.1: Sketch of a typical iCT system for single-particle tracking. Individual ions are detected by a tracking system surrounding the object to reconstruct their trajectories. A calorimeter or range detector is placed after the rear tracker inside which the particle usually stops, for energy or range measurements. From [147].

To retrieve the RSP map of an imaged object, an iCT setup requires detectors for ion tracking and for measuring ion range or residual energy. A typical iCT system is reported in Fig. A.1 (*single-particle tracking system*): a low intensity ion beam is provided with enough energy to cross the object of interest (or a patient); 2D tracking detectors are placed before and after it, in order to reconstruct the ion direction and path $\Gamma(l)$, needed for integration of Eq. A.2. In particular, a high resolution tracking system should also account for Multiple Coulomb Scattering (see Sec. 1.1.4) that can occur in the interaction of the beam inside the object.

Additionally, a calorimeter or a range telescope is added after the rear tracker to measure the residual energy or range of ions which usually stop inside it, quantities that can be correlated to the *WEPL* information.

Another approach exploits a signal from an integrated number of particles behind the imaged object (*integration-mode system*). In this setup, single particle trackers are not required, but only a segmented absorption detector is placed downstream of the object (or patient). In this way, the measured energy is strictly related to *WEPL*, while the path information is retrieved by statistical procedures. The setup presents a simplified and cost-effective configuration, but the currently available technologies demand a higher dose exposure.

In the next section, the development of an iCT detector in the context of small animal imaging is presented.

A.2 Small Animal Proton Imaging

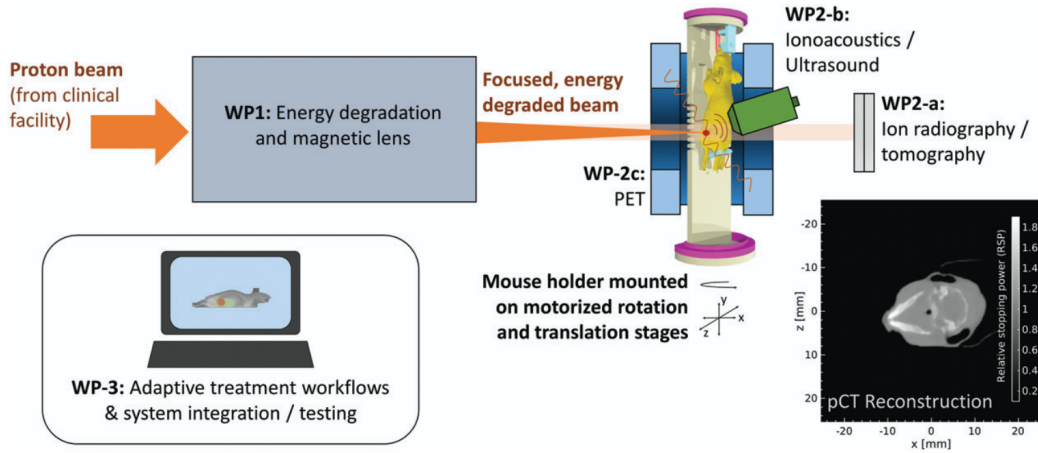


Figure A.2: Schematics of the SIRMIO project. In particular, it involves five work packages, concerning beam line design (WP1), pre-treatment imaging with pCT (WP2-a), in vivo range verification with an ultrasound (WP2-b), positron emission tomography (PET) (WP2-c) system and treatment planning (WP3), respectively. It also includes the design of a dedicated mouse holder, as schematically reported. From [69].

Currently, iCT is not yet used at a clinical stage. However, a lot of effort is put into pre-clinical studies on small animals to develop and improve innovative approaches before human trials. Protons are typically employed in these studies, with energies ranging from 20 to 50 MeV for therapeutic purposes and from 70 to

90 MeV for imaging, using a technique commonly referred to as Proton Computed Tomography (pCT).

To bridge the gap between ion therapy and pre-clinical research, the project SIRMIO developed a novel small animal irradiation platform for integration in clinical facilities [69]. The portable prototype system aims to follow all the steps for a treatment, from planning imaging and treatment planning to intensity modulated irradiation, exploiting the beamline with the biological specimen already set up in treatment position (see Fig. A.2).

For what concerns the work package of pre-treatment imaging with pCT, three different small animal pCT detectors are explored in SIRMIO. The first one is a *single-particle tracker* based on Micromegas tracking detectors around the object and a Time Projection Chamber with vertical Mylar absorbers behind it, working as a range telescope [148]. Another one is an *integrated mode* setup based on a single planar CMOS detector placed after the object, working also with intense fluxes of a synchrocyclotron accelerator [149]. The last one is a "hybrid" setup employing Timepix3 (TPX) pixel detectors placed after the object, with the aim to measure both the position and the energy of single protons, merging properties of both the previous setup configurations [150]. The work done during my internship was focused on the study of the performance of this latter detector setup, which is described in detail in the next paragraphs.

A.2.1 Timepix3 detector for pCT

The detector used for pCT is an AdvaPIX TPX [151] that contains CERN Timepix detectors for particle tracking and imaging. In particular, the sensor features a 50 μm thick silicon layer, organized into a 256×256 pixel matrix, where every pixel is bump-bonded to readout electronics. Each pixel is of size $55 \times 55 \mu\text{m}^2$, for a total sensitive area of $14 \times 14 \text{ mm}^2$ (see Fig. A.3-a). According to the aim, the TPX can be used in different modes:

- Particle Counting: it measures the number of hits in each pixel during the exposure time.
- Time-over-Threshold (ToT): it measures the number of periods of 40 MHz clock signal (25 ns step) when the amplifier amplitude output signal stays over the energy threshold for every pixel (see Fig. A.3-b).
- Time-of-Arrival (ToA): it measures the periods of 40 MHz clock signal (25 ns step) from start of exposure until the event is registered for every pixel
- Fast-Time-of-Arrival (FTtoA): periods of 640 MHz clock signal (1.56 ns step) from start of exposure till the event is registered for every pixel.

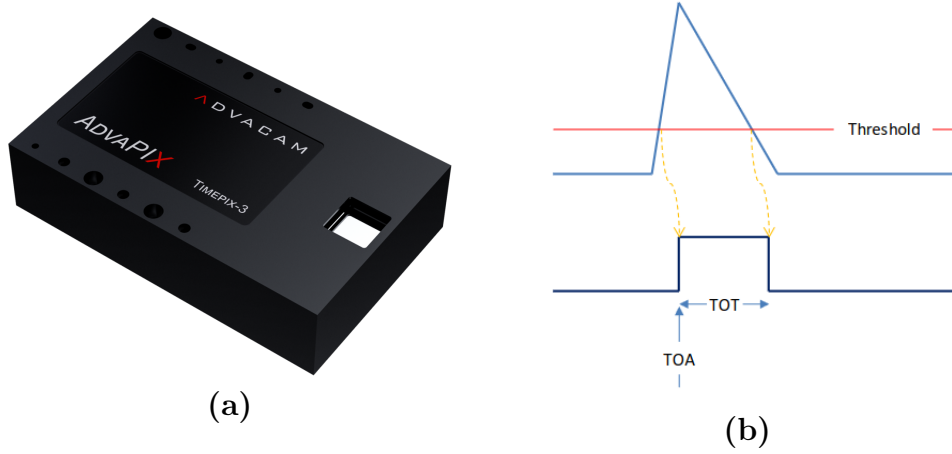


Figure A.3: a) AdvaPiX TPX. The sensor is embedded inside a protection cover, containing a +5V Power supply, USB 3.0 and I/O Digital connectors. b) Sketch of ToT and ToA operational modes: when a signal is detected, ToT measures time it stays over a threshold, while ToA measures clocks of signals of exposures from the arrival to the end of an event.

ToA and FTtoA can be combined via software giving an accurate assessment of time of arrival with a resolution of 1.56 ns, increasing its capability of single particle detection for its low deadtime. Finally, ToT can be converted into deposited energy using a per-pixel-calibration function, obtaining energy loss measurements with a resolution of order 10 keV [152].

The detectors used in the pCT setup are connected via USB 3.0 with 5 Gbit/s transfer rate to an acquisition computer where the provided PIXet Pro (ADVACAM, Czech Republic) software is used for acquisition. Data acquisition, separately for each detector, is used in pixel-based measurement, where a pixel hit is registered as an event and read out immediately after it. The operating mode is FTtoA + ToT, with an acquisition time of 6s per measurement. The DAQ settings like the bias voltage or the bias current are arranged according to the configuration of the manufacturer.

A.2.2 pCT setup in SIRMIO

Fig. A.4 reports a schematic of the pCT setup used during data taking. On the left, protons cross a Beam Line Box (BLB), containing a triplet of permanent magnet quadrupoles to focus beams to the treatment isocenter, as well as degraders and graphite collimators to adjust the emittance in front of the magnets. The BLB is filled with low-density gas in order to reduce scattering due to interaction with air and, moreover, two multi-strip Ionization Chambers are set at the borders as

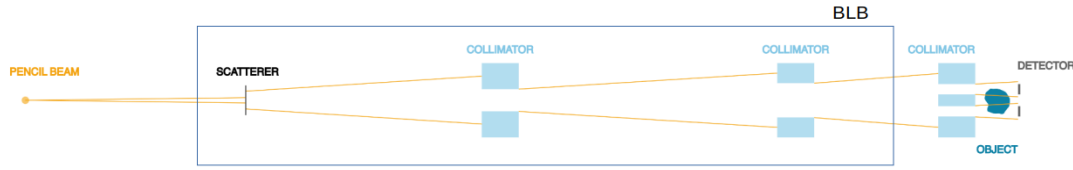


Figure A.4: Schematics of the pCT setup. Protons cross a Beam Line Box (BLB) containing a scattering foil and collimators to shape the beam before reaching the target object. After that, 2 TPX detectors are placed for imaging. Courtesy of Guyue Hu.

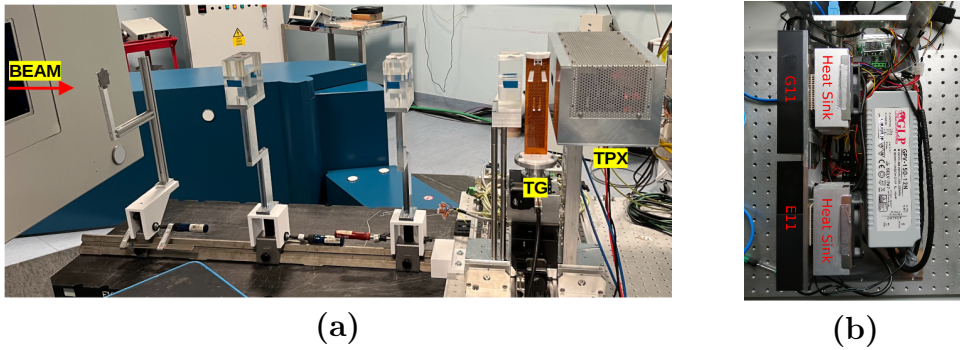


Figure A.5: a) Test setup of pCT for the SIRMIO project. Collimators are followed by the target and by the aluminum box in which two TPX detectors are held. Courtesy of Guyue Hu. b) View of the aluminum box: two TPX detectors with temperature sensors and a cooling setup, from [152].

beam monitors. In imaging mode, these components are moved to the side and are thus replaced by a scattering foil and two imaging collimators. Fig. A.5-a, in particular, shows the imaging setup, where the collimators are present without the BLB. The last collimator, in particular, is placed before the target (TG) to filter protons that do not travel in parallel to the direction of the TPX sensors, an important constraint for the operation of the system (as explained in Sec. A.3.1). After the target (TG), two AdvaPIX TPX detectors E11-W0048 and G11-W0048 (abbreviated by E11 and G11) are held in an aluminum plate (see Fig. A.5-b), with the sensors oriented toward each other, at a distance of 16 mm. Temperature sensors and a cooling setup made of fans are also provided.

In the position of the TG, a 3-D printed mouse holder is set to accommodate all requirements for the foreseen biological studies, but the supports can be arranged to place different thicknesses PMMA targets for calibration and equalization, as well as phantoms (as used in Sec. A.3). The same setup was used for several data takings, among which the one at CNAO in 2022 and at DCPT in 2023 and 2024.

A.3 Calibration and Equalization

The proton imaging process involves a multi-step workflow before the actual image reconstruction, including pixel clustering, equalization, energy calibration, position calibration and finally radiography. The following sections refer to data acquired at CNAO in 2022 using a proton beam of 85 MeV. In this section, equalization and energy calibration processes are discussed in detail, as a predominant task covered during my internship.

A.3.1 WET measurement

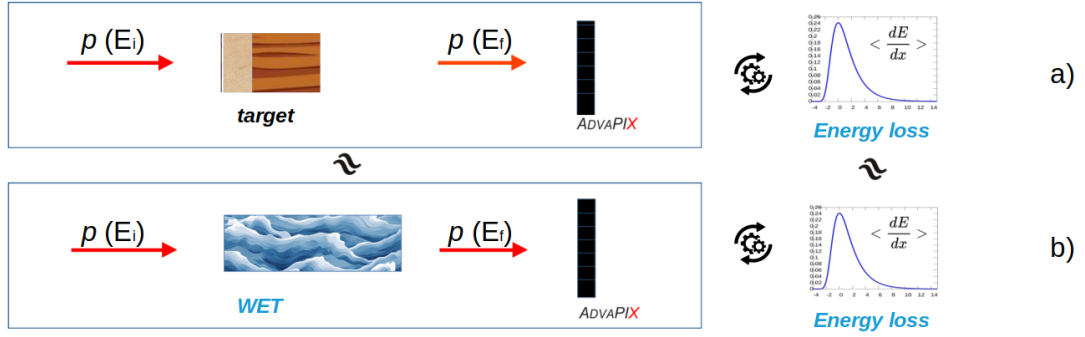


Figure A.6: Conceptual operation of the pCT system. a) Protons with initial energy E_i cross a material and are measured downstream with energy E_f . b) The process is analogous to protons with the same initial energy E_i crossing a target of water with a specific WET , if the energy release E_f in the detector is the same. After the calibration of the sensor, it is possible to retrieve WET from residual energy measurements.

As said in Sec A.1.1, the main quantity to be measured for pCT is the spatially resolved $WEPL$ of the proton through the target, however this information is not trivial to be obtained in the aforementioned TPX setup.

In fact, it is not possible to define a precise path $\Gamma(l)$ of the particle crossing the imaged object, as required in Eq. A.2, since there are no tracking detectors at the farthest edges of the target that would be able to identify protons, which may have also scattered within the BLB region. Thus, the trajectory of every particle is assumed to be parallel to the beam direction, a condition enforced by the presence of collimators (as said in Sec. A.2.2), which filter out all the particles outside the geometrical acceptance of the setup. In a good approximation, the particle trajectory is hence reduced to the physical thickness of the crossed blocks

of material t and Eq. A.2 becomes:

$$\int_L \text{RSP}(l) d(l) = \text{WEPL} \sim \sum_i \text{RSP}_i \cdot t_i \equiv \text{WET} \quad (\text{A.3})$$

where i represents the i -th crossed material with specific RSP and thickness t and the sum of it takes the name of Water Equivalent Thickness (WET). It is important to note that, since WEPL is a path integral, an average of WEPL values for many particles gives an estimate of WET [153].

The procedure of WET extraction from the pCT system is illustrated in Fig. A.6. Let's consider a proton of energy E_i that crosses a target, it loses part of its energy inside it and reaches the TPX detector with a final energy E_f (see Fig. A.6-a). According to Eq. A.3, the same results would be obtained if the target were replaced by an equivalent thickness of water, namely WET , as shown in Fig. A.6-b. In this equivalent scenario, a proton with the same E_i would reach the detector with the same final energy E_f , resulting in an identical energy loss measurement inside the TPX. Thus, WET can be extracted from energy loss measured by the detector and then RSP can be retrieved, which represents the main goal of pCT systems. In the next paragraphs, the procedure of Energy loss - WET conversion is described, considering all aspects of calibrations.

A.3.2 Energy - WET curve

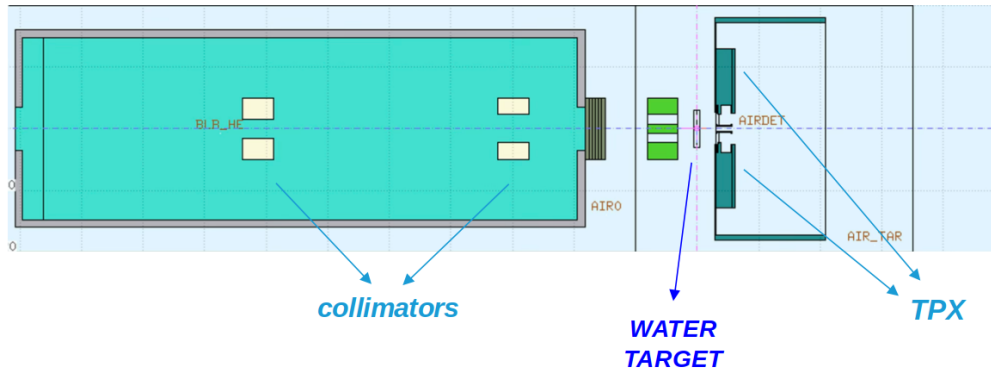


Figure A.7: View of the FLUKA simulation of the pCT setup. In every simulation, the size of the water target is changed, from 0 mm to 32 mm in steps of 0.1 mm.

To apply Eq. A.3, quantitative information of the energy loss by protons in the imaged object is needed. What is measured by the detector is ultimately the residual particle energy downstream the target, without precise knowledge of the energy loss that occurred throughout the setup upstream the target.

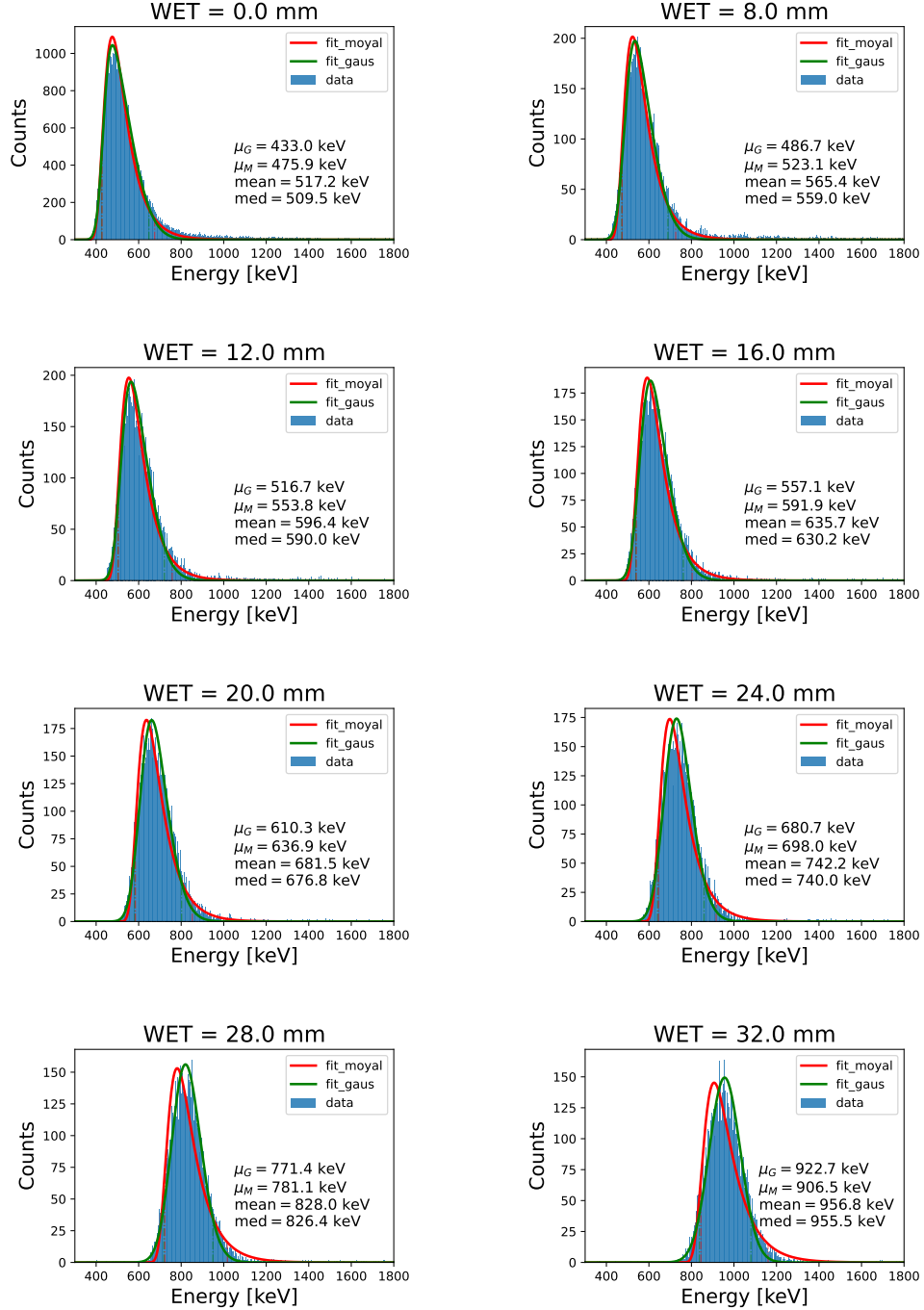
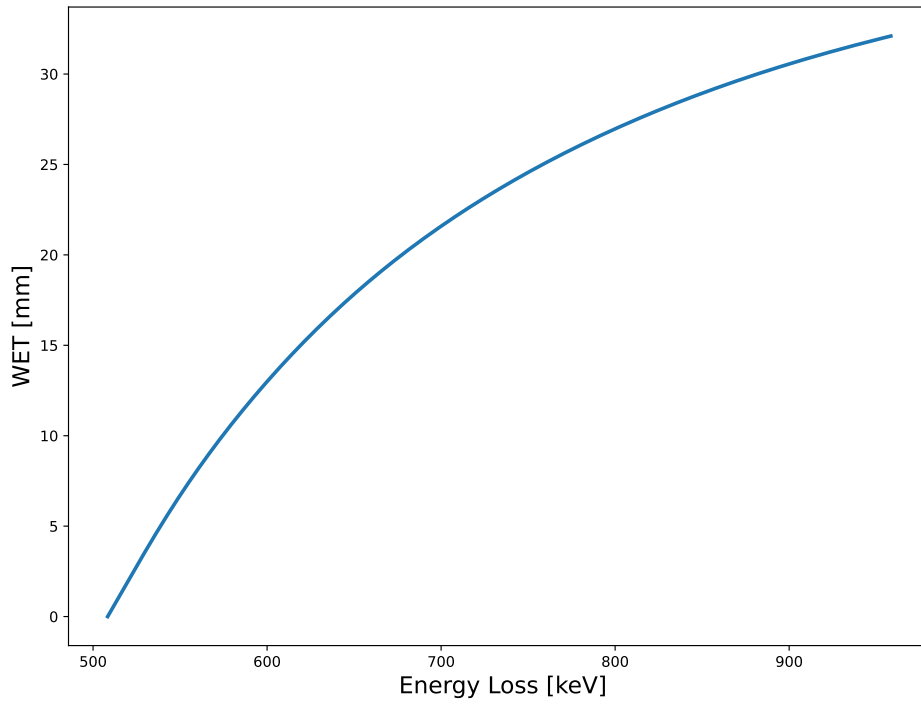
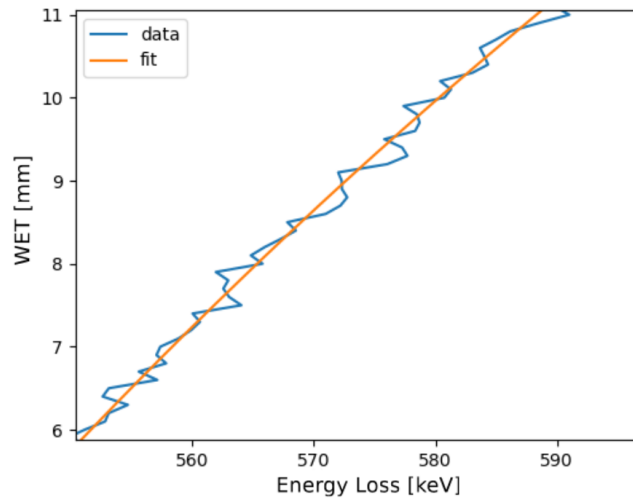


Figure A.8: Energy loss distribution in simulations of the pCT setup with target of WET: 0 mm, 8 mm, 12 mm, 16 mm, 20 mm, 24 mm, 28 mm, 32mm. Every distribution is fitted with a Moyal (green) and Skew Gaussian (red) distribution. Increasing the WET, the energy release broadens due to energy loss straggling, making the distribution more Gaussian.



(a)



(b)

Figure A.9: a) Energy Loss - WET curve, obtained from simulation of WET from 0 mm to 32 mm. The energy value is the median of every distribution, taken within 5^{th} and 95^{th} percentiles of a Skew Gaussian fit. b) A polynomial fit smooths the data and ensures an injective Energy Loss– WET conversion, avoiding ambiguities.

To address this constraint, the energy loss measured by the detector can be converted in terms of *WET* by performing a set of FLUKA Monte Carlo (MC) simulations of the experimental setup. The beam properties are defined according to available models and the target is modeled as a water column of variable thickness placed upstream of the sensors, as shown in Fig. A.7. Every simulation is computed with 1 million primaries, changing the water thickness from 0 mm to 32 mm in steps of 0.1 mm. For each primary, the energy released in the silicon sensor is scored, allowing a direct correlation between energy loss distribution and *WET* that, after the proper calibrations, can be applied to experimental data.

The simulated energy loss distributions recorded by each sensor for various *WET* values are shown in Fig. A.8. As discussed in Sec. 1.1.2, energy loss straggling can be theoretically described by a Vavilov distribution. However, for practical reasons, Moyal and Skew Normal functions are used here as computationally efficient approximations of the Vavilov distribution. Despite the constant beam energy and identical detector active area (i.e., the Δx of $\Delta E/\Delta x$), the shape of the energy loss distribution becomes more symmetric as *WET* increases. This is because protons undergo more collisions in thicker water layers, leading to a broader spread in their energy losses. The beam becomes less monoenergetic and, by the Central Limit Theorem, the energy loss distribution tends to a Gaussian form.

Consequently, at lower *WET* values, both the Moyal and Skew Normal functions provide similarly good fits. However, as *WET* increases, the distribution becomes increasingly symmetric, and the Gaussian-like approximation becomes more accurate, with the skewness progressively decreasing.

In addition, the mean value of the energy loss increases with *WET*. This occurs because, as the protons progressively lose energy, their velocity decreases and, according to Eq. 1.2 where $dE/dx \propto 1/\beta^2$, the energy loss increases. At this point, a filtering on the fitted data is applied, taking only the energy values in the interval between 5th and 95th percentiles of the Skew Gaussian distribution. Then, the median value of the distribution is extracted for every *WET*, obtaining the Energy Loss - *WET* curve of Fig. A.9-a. Since every measurement is affected by statistical uncertainty, a polynomial regression is applied to smooth the calibration curve. This fitting procedure ensures that the resulting curve is an injective function, eliminating any ambiguity in the Energy Loss - *WET* conversion when applied to experimental data. The effectiveness of this approach is illustrated in Fig A.9-b, which shows a zoomed view of the curve within 5 mm of *WET*.

A.3.3 PMMA calibration

Despite all the components of the experimental setup are being considered in the simulation, some slight variations could be expected in comparison with experimental data. Thus, the Energy Loss - *WET* curve needs to be calibrated using a

specific procedure. Several homogenous polymethyl methacrylate (PMMA) slabs



Figure A.10: a) PMMA slabs for *WET* calibration. b) Combination of the slabs to be put after the BLB and before the TPX sensors for an open field imaging. From [154].

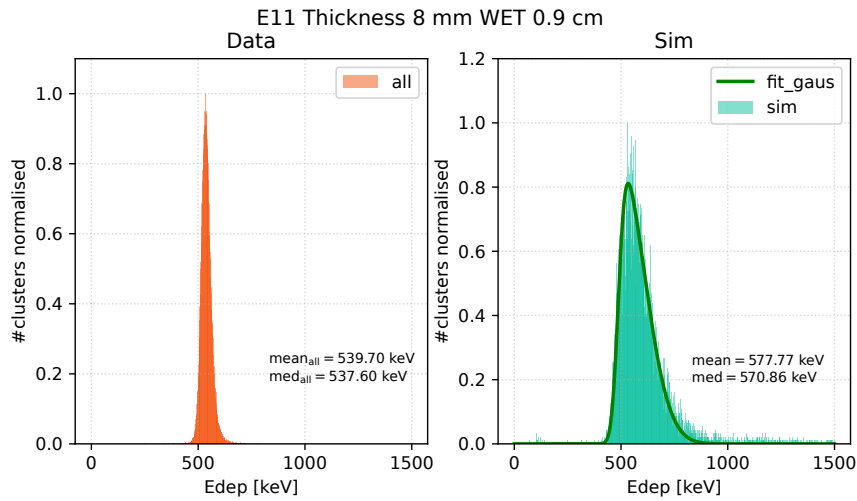


Figure A.11: Energy deposition measured (left) and from simulation (right) for a 8 mm *WET* PMMA slab for *WET* calibration. Results are for E11 (a) sensor.

with known *WET* (see Fig. A.10-a) were combined to obtain blocks of different thicknesses of 0, 4, 8, 12, 16, 20, 24, 28 mm respectively. For every combination, an irradiation with open field was done, obtaining experimental data of energy loss distribution measured separately by the two sensors (see Fig. A.10-b).

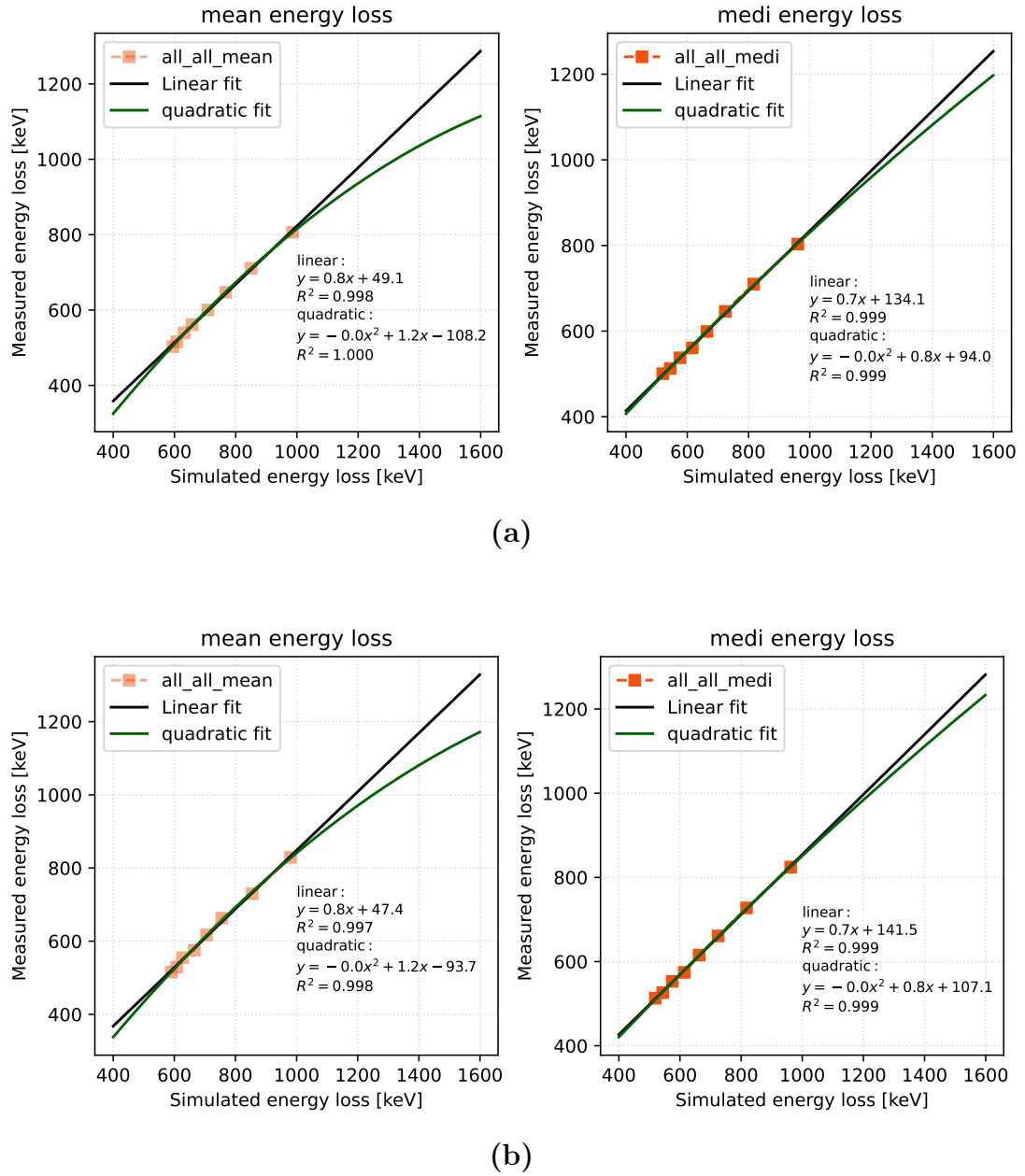


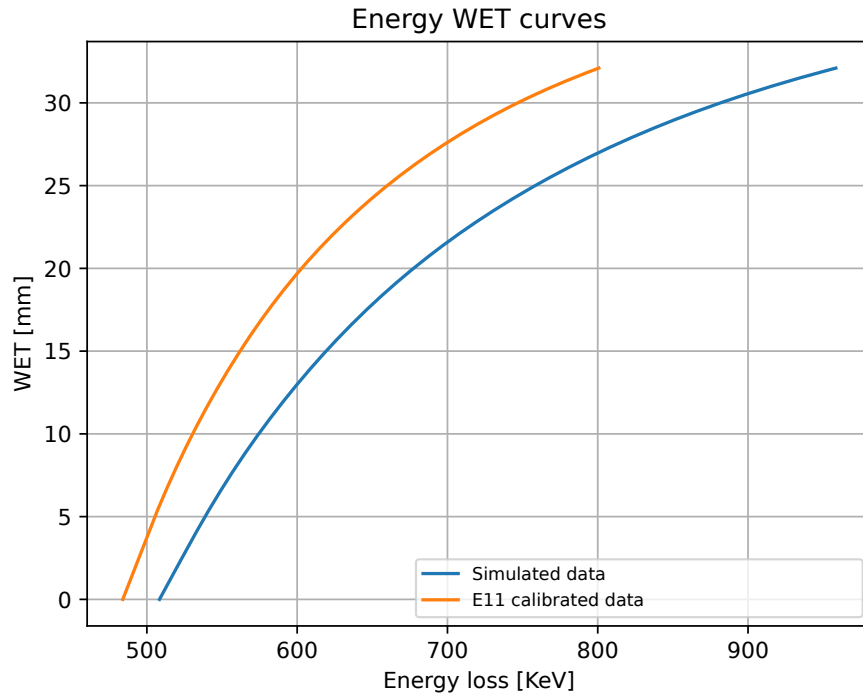
Figure A.12: Relationship between experimental and simulated energy, defined considering mean (left) and median (right) as best estimators. Linear and quadratic fits are calculated, showing a good overimposition in the energy range of the pCT system using median. Results are obtained for E11 (a) and G11 (b) sensors.

Subsequently, new FLUKA MC simulations using a water target with the specific *WET* of the PMMA slabs are computed, in order to compare the experimental and simulated results.

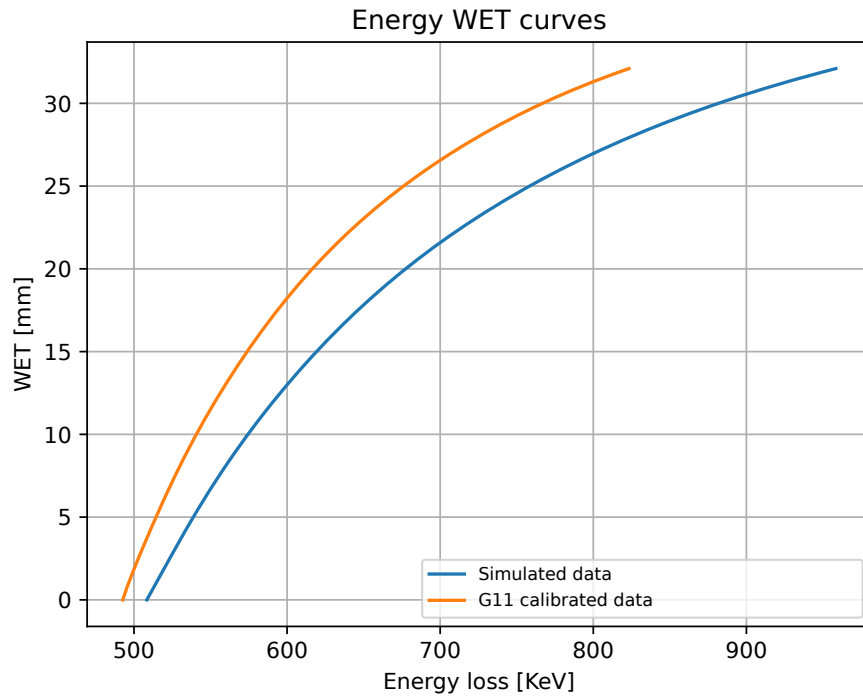
An example of comparison of the energy distribution of experimental and simulated data for PMMA slabs of 8 mm is shown in Fig. A.11. The experimental distribution, like the simulated one (see Sec. A.3.2), exhibits a Gaussian shape but is more symmetrically peaked around its mean value, which is systematically lower. This discrepancy could be attributed to several factors. One possible cause is an inconsistency between the experimental and simulated setups, which may lead to differences in the detector response or beam conditions. Another contributing factor could be the absence of certain physical processes in the simulation, particularly those related to the detector read-out. For instance, charge sharing [155]—the spreading of charge among multiple adjacent pixels—can modify the effective signal distribution and, consequently, influence the behavior and accuracy of the clustering algorithm. As previously applied, the distribution is filtered within 5th and 95th percentiles of the fit, taking the mean and the median of it as the best estimators of the residual energy.

In Fig. A.12, the relation between experimental and simulated energy losses at various *WET* values is shown for both sensors, with linear and quadratic fits applied. Both the median and the mean are considered as estimators. For both sensors, the median appears to be the most suitable choice, as in this case the two fits are nearly superimposed over the energy range covered by the simulation and start to diverge only for values beyond the pCT system's measurement necessities. Moreover, the median's robustness to outliers, potentially introduced by background events or mis-reconstruction, further supports its use as the preferred estimator.

Finally, calibration coefficients obtained by this linear correlation are applied to the simulated Energy Loss - *WET* curve of Fig. A.9 and results are reported in Fig. A.13. The obtained calibrated Energy loss - *WET* curve, each for every detector, will be used for a more reliable image reconstruction of the pCT system (see Sec. A.4).



(a)



(b)

Figure A.13: Energy loss - WET curves for pCT measurements before (blue) and after (orange) PMMA calibrations for E11 (a) and G11 (b) sensors.

A.3.4 Energy Equalization

To study the effects of homogeneity in pixel response of the two TPX, the following metric is introduced:

$$res = \frac{E_{all} - E_{pixel}}{E_{all}} \cdot 100 \quad (A.4)$$

which represents the percentage residual of the energy loss distribution, where E_{all} is the median estimation of the overall distribution used for calibrations (see Par. A.3.3) and E_{pixel} is the mean estimation of the energy loss of all the hits in a specific pixel coordinate. The residual distribution for both detectors in open field at 0 mm WET is shown in Fig. A.14. Fitted by a Gaussian distribution, the mean value is centered around 0% with a standard deviation lower than 4%, in agreement with the energy resolution of the TPX detector [156].

However, for the final pCT system purposes, a residual 2D distribution plot

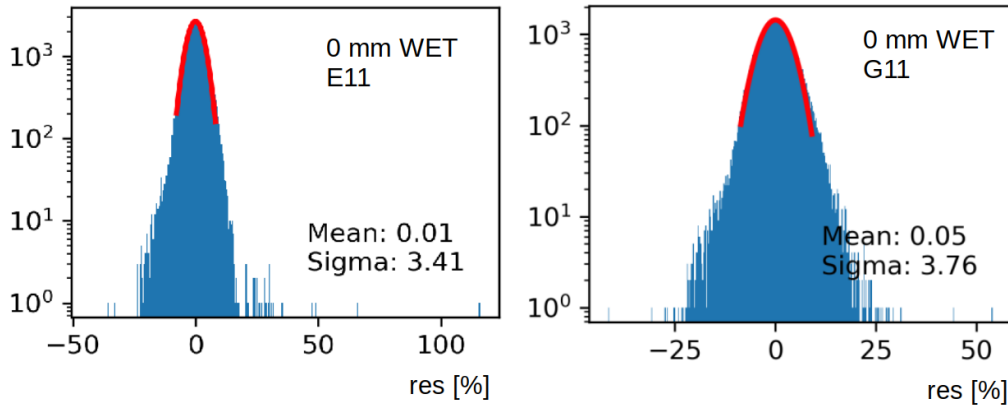


Figure A.14: Energy loss residual distribution for E11 and G11 TPX sensors. The resolution is in agreement with the one of the detector.

is more interesting. Fig. A.15 shows a residual map of energy for both detectors in pixel coordinates at 0 mm WET . The variation is in agreement with Fig. A.14, but with a systematic increase in their external vertical parts. The consequences are not negligible since, after the energy loss to WET conversion, the spatial dishomogeneity could introduce artifacts in the final imaging, giving different WET values for the same crossed material. This effect may arise from several factors. First, the per-pixel calibration from FToA + ToT to Energy provided by the manufacturer (see Sec. A.2.1), based on radioactive sources, can be influenced by sensor temperature and aging effects, which vary between data takings. In addition, the relative positioning of detectors and the collimator with respect to the

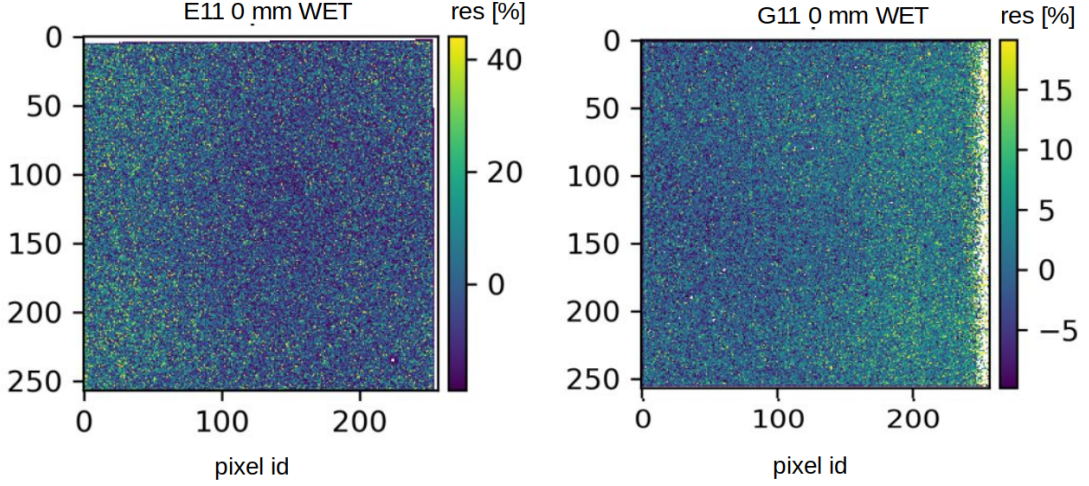


Figure A.15: 2D energy loss residual map of E11 and G11 sensors after irradiation with 0 mm *WET*.

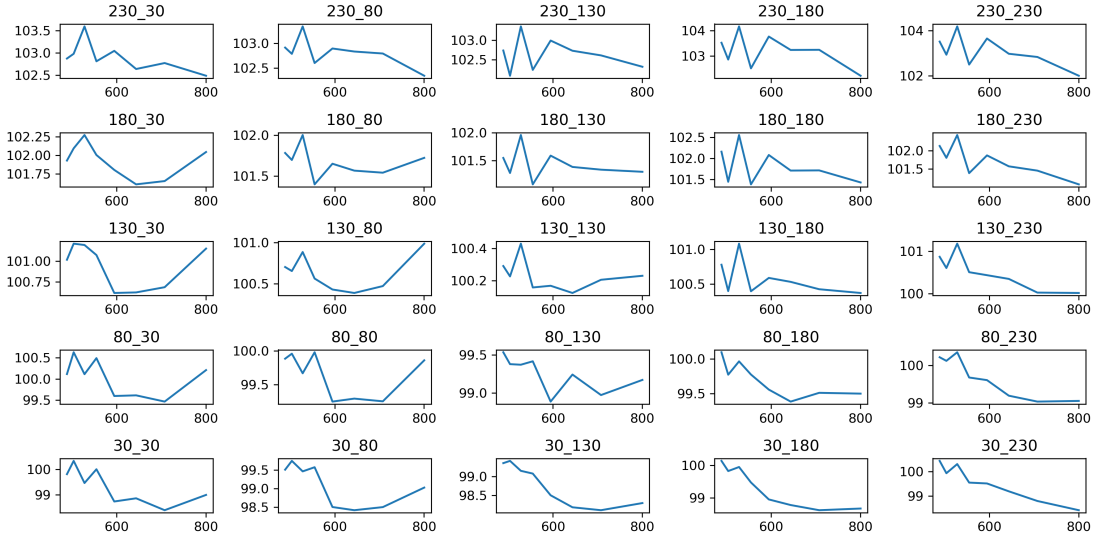


Figure A.16: Residual behavior of 25 different pixels for E11 detector as a function of the energy loss. A systematic variation of the relative energy loss is seen, from center to external edge for both sensors.

beam can affect the spatial response of the sensors. Similar features are observed with all the different PMMA slab thicknesses. So, while these aspects are under investigation, open field imaging at different *WET* can be exploited for a position and energy dependent equalization procedure.

A 50×50 pixel central region, less affected by spatial inhomogeneities, is used as the reference for each PMMA thickness measurement. For each pixel and *WET* thickness, the energy loss variation relative to this reference is then calculated. Figure A.16 shows the residual of Eq. A.4 as a function of energy loss (and thus of *WET* thickness of the slab) for 25 representative pixels of the E11 sensor. The oscillation amounts to approximately 2% for most pixels, with greater stability observed in the central region. Toward the detector edges, the residual values exhibit systematic variations depending on position. The mean residual per pixel shows localized regions with values exceeding 100%, while lower values are found in the opposite areas. Finally, a linear fit is applied to the deviation of the residuals for each pixel, and the resulting coefficients define the equalization map used to correct the energy data.

Figure A.17 shows the energy loss maps from the E11 and G11 sensors for a *WET* of 0 mm, before and after equalization. Alongside the 2D plots, 1D projections along both axes are shown, where each point corresponds to the mean value of the respective row or column. In the *y*-axis projection before equalization, an instability is visible, with energy loss increasing toward the upper edge, a behavior already noted in Fig. A.15 (plotted in a different reference system). After the equalization, a high improvement is obtained in terms of homogeneity, evident from inspecting the projection plots. In particular, the systematic highest variation in the inner part is corrected.

A side effect of this process is to scale even pixel-by-pixel fluctuations, mainly due to statistical limitations from the low number of events per pixel, but also due to nuclear reactions and large-angle Multiple Coulomb Scattering (MCS). To minimize these effects, a statistical 3σ -filter can be applied to energy loss values [157]. All the results of calibration reported in Sec. A.3.3 contain equalization for the PMMA energy loss measurements. Moreover, the equalization calibration is performed at the cluster level in the detector reference frame, before applying any physical transformations.

These calibration and equalization results were subsequently employed to apply pCT on a phantom, to study the performance of *WET* measurements, as described in the following section.

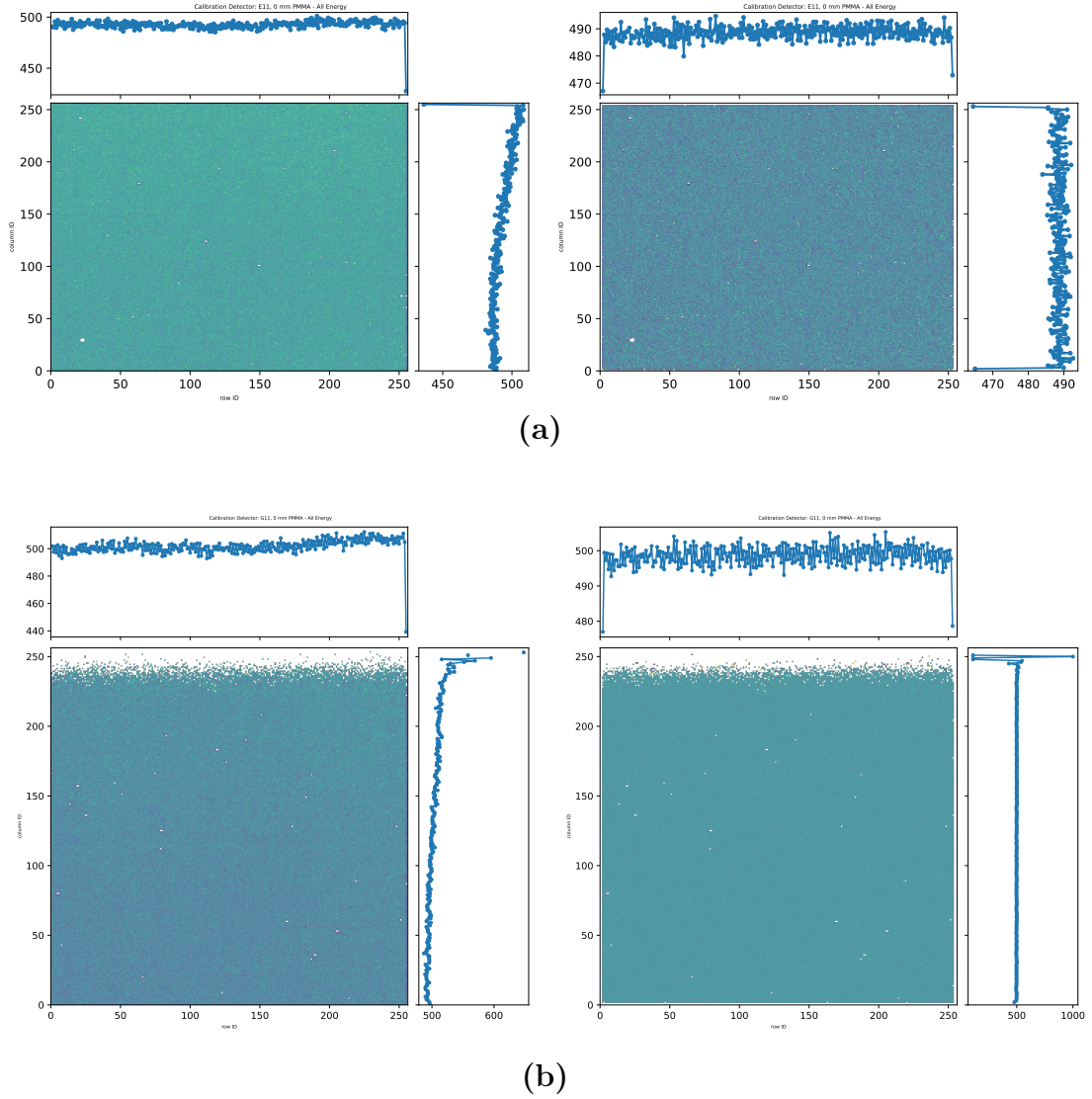


Figure A.17: Energy loss map of E11 (a) and G11 (b) sensors before (left) and after (right) energy loss equalization. Inspecting the 1D axis projections, it is possible to appreciate the improvement of homogeneity after the process.

A.4 Phantom radiography

The analysis presented in this chapter aims to evaluate the capabilities of the described pCT system for *WET* and *RSP* measurements. To this end, imaging of a well-characterized phantom was performed by placing the object in front of the TPX detector within the SIRMIO setup at CNAO in 2022. The results are compared with the nominal values of the phantom inserts, with particular attention to the potential impact of the equalization process discussed earlier.

A.4.1 SmART Phantom

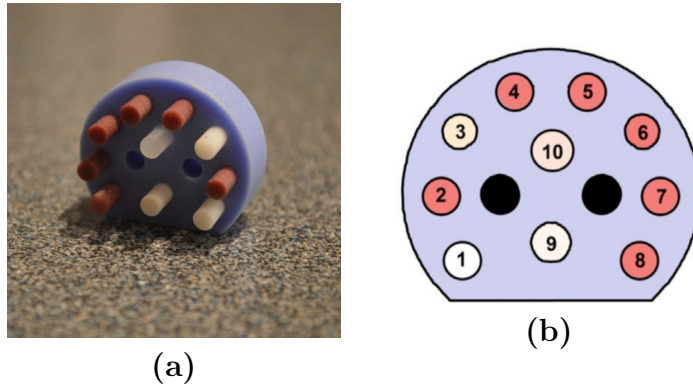


Figure A.18: a) Picture of the micro-CT phantom used for *WET* performance evaluations. It is composed of a base of 30 mm in diameter holding 10 tissue-equivalent inserts with different *RSP*. b) Schematic of the phantom with insert associated to numbers, for a better clarification of Tab. A.1.

A micro-CT calibration phantom (SmART Scientific Solutions, Netherlands) is used for radiography and quantitative *WET* evaluations (Fig. A.18-a). It is made from a 10mm-thick water-equivalent base plate with a diameter of 30 mm that holds 10 tissue-equivalent inserts (Gammex materials[®]; 3.5mm diameter, 16mm length). A sketch of the phantom is shown in Fig. A.18-b, while the properties of every insert are reported in Table A.1, considering the effective atomic number Z_{eff} , relative-to-water electron density ρ_e^w , mass density ρ_{ref} , the relative weight of every element, *RSP* and *WET* (in cm). The *RSP* was measured using carbon ions with a resolution of 0.2% [158] and, knowing the thickness, it is possible to retrieve *WET* by Eq. A.3.

Tissue insert	No.	Z_{eff}	ρ_e^w	ρ_{ref}	H	C	N	O	P	Cl	Ca	RSP	WET
CB2-50%	1	12.54	1.47	1.56	4.77	41.61	1.52	32.00	0.00	0.08	20.02	1.43	2.29
B200 Bone	2	10.42	1.10	1.16	6.65	55.51	1.98	23.64	3.24	0.11	8.87	1.12	1.76
CB2-30%	3	10.90	1.28	1.33	6.68	53.47	2.12	25.61	0.00	0.11	12.01	1.28	2.04
Solid Water	4	7.74	0.99	1.02	8.00	67.29	2.39	19.87	0.00	0.14	2.31	1.00	1.59
Adipose	5	6.21	0.93	0.95	9.06	72.29	2.25	16.27	0.00	0.13	0.00	0.94	1.51
Inner Bone	6	10.42	1.09	1.13	6.67	55.65	1.96	23.52	3.23	0.11	8.86	1.09	1.74
Breast	7	6.93	0.96	0.98	8.59	70.10	2.33	17.90	0.00	0.13	0.00	0.97	1.55
Liver	8	7.74	1.06	1.10	8.06	67.01	2.47	20.01	0.00	0.14	0.23	1.08	1.72
SB3 Cort. Bone	9	13.64	1.69	1.82	3.41	31.41	1.84	36.50	0.00	0.04	26.80	1.62	2.60
Brain	10	6.09	1.04	1.05	10.83	72.54	1.69	14.86	0.00	0.08	0.00	1.03	1.70

Table A.1: Tissue-equivalent inserts: effective Z , relative electron density ρ_e^w , reference density ρ_{ref} , element composition (%), RSP and WET (cm).

A.4.2 WET measurements

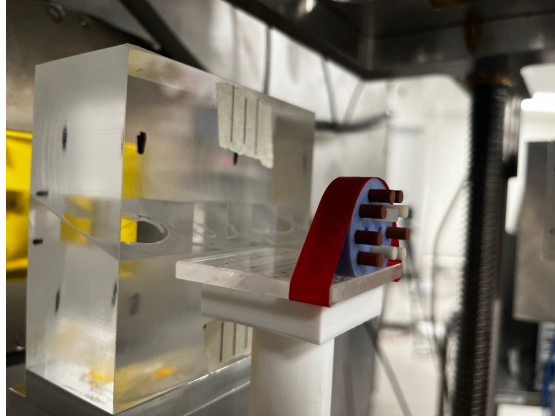
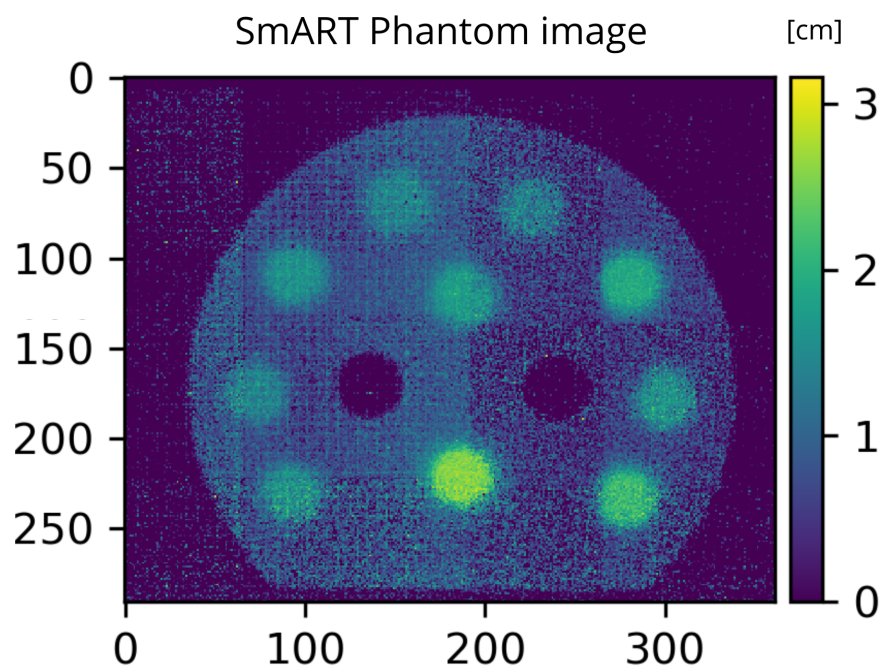
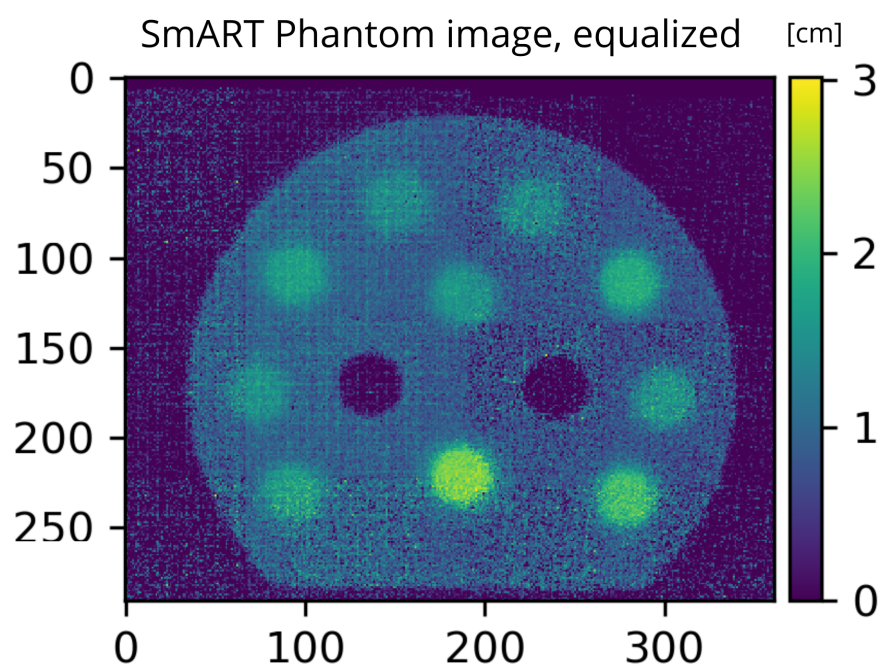


Figure A.19: Insert of the SmART Phantom as target in the SIRMIO setup in DCPT 2024.

The SmART Phantom is held in the target region thanks to a support, between the sensors and the collimator as for PMMA measurements (Fig. A.19). Some stages allow the target to shift in X and Y positions while the sensor is at rest. In this way, with several acquisitions, it is possible to obtain an image of the overall object, since its dimensions are higher than the field of view of the sensors. According to what is stated in Sec. A.3.4, the energy loss distribution of every acquisition is first Gaussian-filtered to reduce image noise. At this point, the



(a)



(b)

Figure A.20: Image of SmART Phantom taken at CNAO in 2022, before (a) and after (b) equalization. Qualitatively, it is possible to appreciate an higher homogeneity in the phantom reconstruction. Vertical and horizontal axes represents the physical scale, the color scale represents the measured WET

equalization coefficients already measured using the PMMA slabs are applied to overcome the spatial dishomogeneity of measured energy loss. The images of the Smart Phantom in terms of WET , before and after the application of filters and equalization, are shown in Fig. A.20. The granularity is a consequence of low intensity beam and thus low statistics in some pixels of the sensor. What can be appreciated is the qualitatively highest homogeneity of values within the region of every insert after the application of equalization, improving the edges and separation in regions.

In the next paragraph, a quantitative analysis of the obtained image is carried out.

A.4.3 Results

A region of interest (ROI) has been considered for every insert of Fig. A.20, consisting of a circular area of 1.5 mm diameter centered at each insert. Considering the WET data inside every ROI, a Gaussian 2σ -filter is applied for smoothing and noise reduction, in order to minimize the effects of nuclear reactions and large-angle MCS events [157] (see Sec. A.3.4). Margins of 11 pixels are also chosen to define a subregion of data disentangled from spatial blur at the edges. The median WET is measured and then compared to the ground-truth (GT) value, according to the following formula:

$$\text{Relative } WET = \frac{WET_{meas} - WET_{GT}}{WET_{GT}} \cdot 100 . \quad (\text{A.5})$$

Relative WET for every insert is shown in Fig. A.21, in a comparison of values before and after applying the equalization. Numerical results are also reported in Tab. A.2.

Inspecting values for inserts from 1 to 8, the effect of the equalization procedure is clearly evident, showing a systematic increase in WET values after equalization application. This behavior can be understood by considering the effect of data homogenization within each insert. When the distribution of measured values becomes more uniform, local variations that previously contributed to systematic trends are significantly reduced. In particular, the trend of progressively decreasing values observed along a specific direction in Fig. A.15 is largely suppressed, indicating that the equalization process effectively mitigates position-dependent fluctuations in the detector response. As a result, the overall mean WET values increase after equalization. Due to this, the percentage deviation from the GT value improves by several percentage points overall, reaching up to 7% improvement for the Inner Bone insert. Slight degradations are, however, observed for the Solid Water and B200 Bone inserts. As visible in Fig. A.18-b, these regions correspond to the portion of the reconstruction predominantly measured by the TPX E11 sensor. This behavior could be attributed to residual variations between

the calibration setup and the radiography setup, which may require further investigation.

Insert	No.	GT WET [cm]	WET [cm]	Rel. diff
Cortical bone 50%	1	2.289	2.231	-2.54%
Bone B200	2	1.758	1.800	2.38%
Cortical bone 30%	3	2.039	2.020	-0.91%
Solid water	4	1.593	1.640	2.95%
Adipose	5	1.507	1.501	-0.39%
Inner bone	6	1.743	1.620	-7.07%
Breast	7	1.555	1.460	-6.12%
Liver	8	1.723	1.660	-3.64%
Cortical bone SB3	9	2.595	2.580	-0.58%
Brain	10	1.698	1.800	6.01%
Average of absolute WET diff.			3.26%	

(a)

Insert	No.	GT WET [cm]	WET [cm]	Rel. diff
Cortical bone 50%	1	2.289	2.280	-0.39%
Bone B200	2	1.758	1.832	4.15%
Cortical bone 30%	3	2.039	2.042	0.14%
Solid water	4	1.593	1.740	9.21%
Adipose	5	1.507	1.511	0.23%
Inner bone	6	1.743	1.741	-0.15%
Breast	7	1.555	1.590	2.29%
Liver	8	1.723	1.759	2.14%
Cortical bone SB3	9	2.595	2.500	-3.64%
Brain	10	1.698	1.669	-1.68%
Average of absolute WET diff.			2.30%	

(b)

Table A.2: Comparison of GT *WET* and measured *WET* before (a) and after (b) equalization correction, for all inserts.

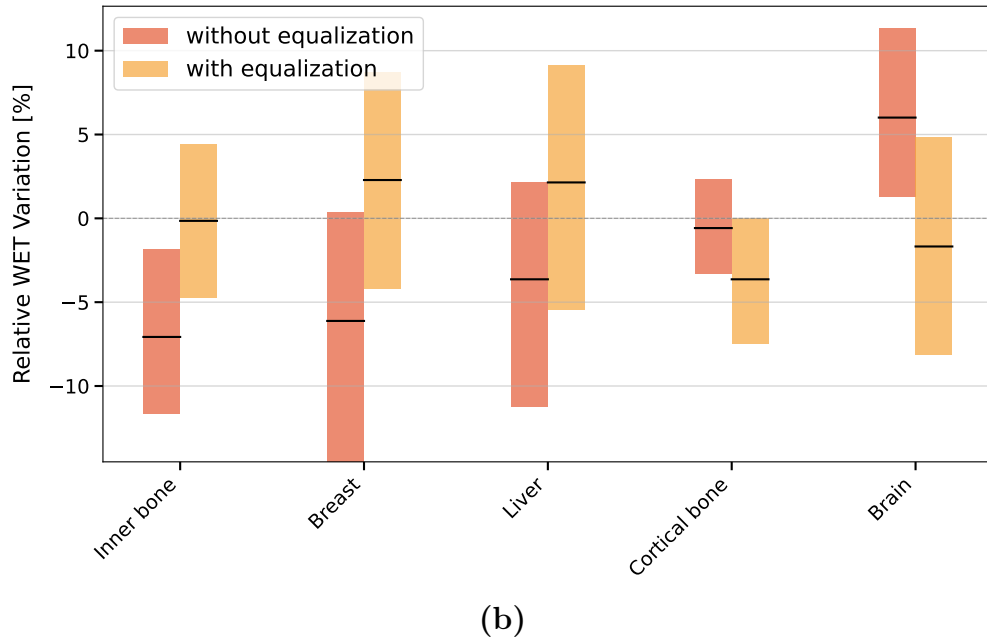
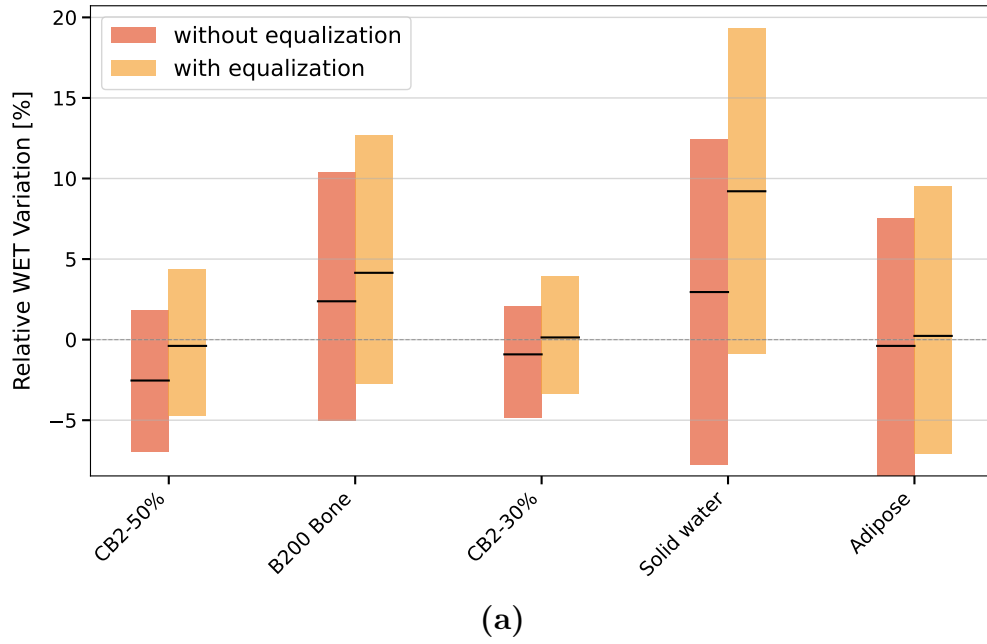


Figure A.21: Relative *WET* measurements (mean and standard deviation) for every insert, before (red) and after equalization (orange) for SmART Phantom, for inserts 1-5 in (a) and 6-10 in (b).

A somewhat different situation is observed for inserts 9 and 10, where the equalization, instead, leads to a decrease in WET . Considering their spatial position (Fig. A.18-b), these inserts fall within the region where the radiography is acquired by the two distinct sensors with limited overlap. In this area, an inaccurate WET estimation could arise from residual misalignment or calibration inconsistencies in the transition region between the sensors, affecting the reconstructed values. Overall, the system highlights a solid robustness in WET reconstruction, even in regions affected by sensor transitions. The application of the equalization procedure is therefore well justified, as it induces a clear and consistent modification of the reconstructed values, leading to an overall improvement in WET resolution, from approximately 3.3% to 2.3% on average.

A.5 Conclusions

SIRMIO established a novel small-animal irradiation platform integrated within clinical facilities, with a particular focus, relevant to the present work, on pre-treatment imaging through pCT. The SIRMIO pCT setup adopted a “hybrid” approach employing TPX pixel detectors capable of measuring both the position and residual energy of individual protons.

The conversion of these measurements into WET values required a dedicated calibration workflow centered on energy loss to WET calibration and equalization. The initial Energy Loss – WET correlation was derived from FLUKA Monte Carlo (MC) simulations of a water column of variable thickness, and subsequently refined through experimental data obtained with PMMA slabs of known WET (PMMA calibration). The comparison between simulated and experimental data showed that the median of the energy loss distribution provided the most stable estimator for correlating measurements with simulation results.

To further improve data quality, an energy equalization procedure was implemented to correct spatial inhomogeneities in the pixel response across the TPX sensors. Systematic variations, especially towards the detector edges, were found to introduce artifacts in the reconstructed images. The equalization process, based on open-field acquisitions at different WET s, employed a linear fit to the variance of residuals for each pixel, significantly enhancing the uniformity of the resulting energy maps.

The effectiveness of the full calibration and equalization workflow was validated through radiography of a SmART Phantom containing ten tissue-equivalent inserts. Qualitatively, the images acquired at CNAO in 2022 exhibited improved homogeneity within the regions of interest after equalization. Quantitatively, the average absolute WET deviation from ground-truth values decreased from approximately 3.3% to 2.3%, while for specific materials—such as the Inner Bone

insert—the deviation improved by up to 7%.

These results confirm the robustness of *WET* reconstruction within the SIRMIO pCT system and represent a promising step forward in the development of proton imaging. They highlight the potential of this approach to accelerate the clinical translation of pCT, ultimately contributing to more precise and effective Particle Therapy treatments.

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