



ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

DOTTORATO DI RICERCA IN
SCIENZE E TECNOLOGIE AEROSPAZIALI

Ciclo 38

Settore Concorsuale: 09/A1 - INGEGNERIA AERONAUTICA, AEROSPAZIALE E NAVALE

Settore Scientifico Disciplinare: ING-IND/03 - MECCANICA DEL VOLO

ADVANCED DYNAMICS MODELING AND INNOVATIVE ESTIMATION AND
CONTROL STRATEGIES FOR AIR DELIVERY APPLICATIONS

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Esame finale anno 2026

Abstract

This dissertation investigates estimation and control strategies for cooperative aerial transportation with multirotor vehicles carrying cable-suspended payloads. The central challenge is the dynamic coupling introduced by the suspended mass, which renders the system underactuated, highly nonlinear, and prone to oscillations that degrade safety and performance. A high-fidelity simulator based on first-principles models of single and multi-vehicle slung-load systems is developed, serving as a design and verification platform under model uncertainty, exogenous disturbances, and sensor noise. Three main technical advances are contributed. (i) A compact nonlinear formation controller derived from an Eulerian model exhibits two-time-scale behavior: fast modes stabilize formation geometry and damp payload swing, while slow modes achieve trajectory tracking. Stability and gain selection are guided by singular perturbation arguments and Lyapunov criteria. (ii) A baseline controller from a Lagrangian formalization introduces a physically insightful parameterization of payload swing, enabling simultaneous formation keeping, trajectory tracking, and oscillation suppression in a single feedback law. (iii) A robustness layer based on $\mathcal{L}_1$ adaptive control augments the baseline while preserving its nominal properties, compensating matched and unmatched uncertainties, external disturbances, and actuator faults without compromising closed-loop stability. Complementing control, a discrete-time estimator requir-

ing only standard onboard inertial measurements is proposed. Using a Fading Gaussian Deterministic Filter, the method estimates payload swing angles and rates without extra sensors. The framework is further extended to a single multirotor transporting two serially-connected payloads, enabling multi-parcel delivery within a single flight. All methods are validated through high-fidelity simulations and experimental campaigns, demonstrating reliable swing attenuation, accurate trajectory tracking, and robustness to disturbances and parameter uncertainty on resource-constrained avionics. These contributions yield a unified framework for aerial mobility and delivery scenarios, applicable to logistics, search-and-rescue, and disaster-relief operations.

Acronyms

AC Adaptive Controller

BET Blade Element Theory

BL Buttline

CAC Collision Avoidance Controller

CG Center of Gravity

CICLoPE Centre for International Cooperation in Long Pipe Experiments

CP Center of Pressure

CTC Cooperative Transportation Controller

DOB Disturbance Observer

EKF Extended Kalman Filter

ENU East–North–Up

ESC Electronic Speed Controller

FC Formation Controller

FCC Flight Companion Computer

FCU Flight Control Unit

FGDF Fading Gaussian Deterministic Filter

FLOP Floating Point Operation per Second

GCS Ground Control Station

GEUB Globally Exponentially Ultimately Bounded

GNSS Global Navigation Satellite System

H Hook point

IMU Inertial Measurement Unit

ISA International Standard Atmosphere

LQR Linear Quadratic Regulator

MHE Moving Horizon Estimation

MoCap Motion Capture

MPC Model Predictive Control

MT Momentum Theory

NED North–East–Down

NMPC Nonlinear Model Predictive Control

PC Payload Controller

PCMPC Perception–Constrained Model Predictive Control

PD Proportional Derivative

PID Proportional–Integral–Derivative

PWM Pulse–Width–Modulated

RISE Robust Integral of the Sign of the Error

RMSE Root Mean Square Error

ROS Robot Operating System

ST Stationline

UAMD Urban Air Mobility and Delivery

UAV Unmanned Aerial Vehicle

UKF Unscented Kalman Filter

VC Velocity Controller

VRS Vortex Ring State

WGS84 World Geodetic System 1984

WL Waterline

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CHAPTER 1

Introduction

1.1 Background and Motivation

Urban Air Mobility and Delivery (UAMD) envisions on-demand air transport of passengers and goods using highly automated Unmanned Aerial Vehicles (UAVs) in urban airspace. Recent reviews synthesize evidence that UAMD concepts can reduce point-to-point travel time by roughly 30–40% on average (and up to 40–50% in large metropolitan areas) relative to ground transport, while also flagging persistent hurdles of integration, safety/airworthiness, and low-altitude traffic management [1]. Beyond travel time and sustainability signals are increasingly favorable: systematic reviews on last-mile logistics report significant cost reductions for short distances compared with traditional express delivery, contingent on airspace access and scale effects [2, 3].

These aerial platforms emerge as a disruptive solution to reduce ground traffic congestion and its environmental impact by exploiting the vertical dimension [4]. Societally critical use cases include rapid transport of medical supplies and blood products, diagnostics, and vaccines, as well as disaster-relief tasks, owing to the ability to reach restricted areas and provide critical

aid in emergencies [5, 6, 7]. The floods in Emilia Romagna (Italy) in May 2023 underscore the need for resilient, rapidly deployable aerial support: evacuation challenges and transportation delays exposed bottlenecks in ground-based response [8]. While post-event mapping with UAVs proved invaluable [9], these emergencies also outline the critical need for resilient, all-weather suspended load capabilities to deliver vital cargo during peak crises, a gap addressed by the control strategies developed herein. These dramatic events highlighted how UAVs enabled continuous monitoring when traditional infrastructure failed, with the Copernicus Emergency Management Service integrating drone-collected data for real-time disaster response coordination [10]. Additional high-impact domains include firefighting, hazardous-environment operations, and search-and-rescue, where human intervention may be risky. In wild land fire response, UAVs are used for early detection, hot-spot mapping, perimeter tracking, and real-time situational awareness for incident commanders. Recent surveys report persistent monitoring with multi-UAV fleets and onboard perception tuned to smoke and flame signatures [11, 12, 13]. In hazardous settings, unmanned vehicles enable remote inspection and sensing (e.g., gas leaks, radiation mapping) and reduce worker exposure during assessments in chemical plants, refineries, offshore assets, and nuclear facilities [14]. For search-and-rescue, comprehensive reviews indicate that UAVs accelerate area coverage, improve detection of missing people in low-visibility conditions, and enhance mission safety [15].

1.2 State of the Art

In aerial delivery operations, two principal methods are commonly employed [16]: the payload may be either rigidly fixed to the vehicle or suspended by a cable. The rigid attachment approach is prevalent, typically realized through the use of clamps or by leveraging a sufficiently large fuselage. However, this solution often results in increased take-off weight and may be unsuitable for transporting large payloads or for missions requiring aerial delivery where landing is not feasible [17, 18]. By contrast, suspended-load configurations

enable aerial delivery without the need to land, thereby enhancing mission efficiency while preserving rotorcraft performance and system simplicity. The introduction of a suspended mass, however, generates disturbance forces and moments that significantly influence vehicle dynamics. The resulting slung-load system is under-actuated, namely it has fewer independent control inputs than degrees of freedom, and challenging to control. Moreover, the resulting oscillations can pose risks to both the payload and surrounding environment due to potential collisions [19].

The problem of a single unmanned vehicle carrying a load has been addressed with the goal of minimizing payload swing to enable safe transport of delicate or sensitive items, thereby broadening the range of goods deliverable by air. A variety of control approaches have been proposed, often inspired by crane control. Palunko et al. used dynamic programming to generate swing-free trajectories, but open-loop performance limitations motivated closed-loop alternatives [20]. de Angelis et al. derived a highly nonlinear controller that enforces a two-time-scale behavior, with fast oscillation dynamics and slow trajectory tracking, invoking singular perturbation theory and providing a proof of almost-global asymptotic stability [21]. Nicotra et al. presented a controller that steers the aerial robot to a desired reference while limiting payload swing, proving equilibrium stability via nested saturation for disturbance rejection [22]. Guerrero et al. adopted a Hamiltonian approach to suppress oscillations independently of the swing angle, under a stiff, massless cable and point-mass payload model [23]. Zhou et al. compared PD and sliding-mode control for a quadrotor with a slung load, showing that the second control method offers superior robustness to cable-induced disturbances [24]. Other approaches include active model-based control supported by Kalman filtering and adaptive controllers to estimate unknown payload parameters [25, 26]. Liang et al. proposed a minimum-time trajectory planner that accounts for nonlinear dynamics and system constraints [27]. Input shaping has also been explored to reduce payload swing: Huo et al. combined input shaping with PID position control to improve trajectory tracking and load stabilization [28], while enhanced integrations with sliding-mode and LQR methods were analyzed in Refs. [29, 30].

While a single aerial robot may suffice for certain applications, many transportation tasks, such as lifting payloads beyond a single rotorcraft’s capacity or maintaining functionality after a vehicle failure, require the use of multi-agent systems. Cooperative payload transportation has therefore received considerable attention in the literature. Cable-suspended cooperative methods are commonly categorized by control architecture: UAV-based (decentralized) and payload-based (centralized). In both approaches, stabilizing payload swing is paramount, and many solutions borrow from overhead-crane control [31].

In decentralized frameworks, the emphasis is on maintaining UAV formation while treating payload swing as a perturbation that complicates position control [32, 33, 34, 35, 36]. For example, Meissen et al. proposed passivity-based formation control, showing a continuum of equilibria with stability when cables remained taut [32]. Klausen et al. accounted for unknown payload mass and wind disturbances, with a Lyapunov-based design validated in simulations and field experiments [33]. Tognon et al. presented a Lyapunov-based controller for aerial co-manipulation with two UAVs [34], while Pizetta et al. employed nonlinear feedback linearization to enable trajectory tracking, swing mitigation, and obstacle avoidance in a two-UAVs formation [35].

In centralized architectures, the objective is precise trajectory tracking of the payload while accounting for full dynamic coupling among all components [37, 38, 39, 40, 41]. This coupling poses substantial challenges for control design. Masone et al. adopted a modeling approach rooted in reconfigurable cable-driven parallel robots and addressed full-system dynamics [37]. Giménez et al. proposed a multi-objective controller for trajectory tracking, formation keeping, and collision avoidance using two helicopters to transport a point-mass payload [38]. Shirani et al. introduced a sub-optimal LQR–PID controller to minimize swing by aligning payload and UAV velocities within a designated formation [39]. Geng et al. took a hierarchical approach in which the payload (leader) computes the net wrench required to follow a trajectory, and a tension-distribution problem is solved via sequential least-squares programming [40]. Azzam et al. explored reinforcement learning with centralized training and decentralized execution, employing a reward that combines positive and

negative components [41].

While existing centralized and decentralized control approaches show promise, literature gaps remain in developing a unified framework that simultaneously addresses formation geometry keeping, trajectory tracking, payload swing damping, and adaptability across various rotary-wing platforms. More recent studies began to fill these gaps considering more sophisticated methodologies for enhanced performance and robustness. Tasooji et al. proposed an obstacle-aware, event-triggered distributed Nonlinear Model Predictive Control (NMPC) scheme that dynamically balances global planning and real-time load control during cooperative transportation. A trajectory-based full-pose motion planner for agile, whole-body multi-robot maneuvers through narrow passages is developed in Ref. [42], where the aerial agents can achieve accelerations up to eight times greater than state-of-the-art methods. Wang et al. presented Auto-Multilift, a learning-augmented distributed model predictive control (MPC) that auto-tunes hyperparameters through deep neural networks for scalable multi-agent load tasks [43]. Villa et al. elaborated a robust integral of the sign of the error (RISE) controller is integrated with model-reference adaptation in virtual-structure formations. Numerical simulations and experimental results demonstrate the effectiveness of the method, leading to improved trajectory tracking and showing strong disturbance rejection capabilities [44].

Accurate load-state measurement is essential for anti-swing capability, and many methods are inspired by bridge-crane applications [45]. Motion-capture solutions are highly accurate but costly and impractical for many operations [46]. Onboard vision is a viable alternative, yet increases weight, power draw, computational load, and sensitivity to environmental variability [47, 48]. Non-optical solutions include inclinometers attached to the payload [49] and potentiometers on the vehicle to measure appendage orientations in aerial manipulation [50]. High-precision GNSS-based methods are feasible provided that both the payload and vehicles carry dedicated receivers [51].

With respect to the estimation algorithms, methods differ in computational complexity, disturbance robustness, and sensor requirements. The Extended Kalman Filter (EKF) is widely adopted for its simplicity and effec-

tiveness on nonlinear systems. In this regard, de Angelis et al. [52] developed a nonlinear EKF that relies solely on onboard IMU measurements to estimate swing angles and cable forces, while compensating for external aerodynamic disturbances such as wind. This approach is validated in both simulation and experimental flight tests, demonstrating improved robustness and energy efficiency. In parallel, the Unscented Kalman Filter (UKF) has been employed to better capture nonlinearities and sensor noise in dynamic transport scenarios, sometimes combined with adaptive schemes that tune covariances online [53]. More recently, estimation and control have been integrated within perception-aware frameworks. Li et al. proposed a Perception-Constrained Model Predictive Control (PCMPC) that fuses monocular vision and IMU data to estimate cable dynamics at high frequency, while enforcing visibility constraints. This approach achieves precise trajectory tracking in visually constrained environments and reduces onboard computational burden, while preserving estimation accuracy in cluttered scenes [54]. To enhance robustness under bounded disturbances and model uncertainties, researchers started investigating zonotopic state estimators. Rego et al. estimated the position and orientation of suspended loads and integrated the filter within a discrete-time state-feedback mixed H_2/H_∞ control scheme, demonstrating strong performance with limited a priori disturbance knowledge at the cost of higher computation [55]. Moving Horizon Estimation (MHE) is a powerful tool for handling nonlinearities and constraints, but its slower adoption in suspended-load transportation is mainly due to significant computational cost and implementation complexity. However, recent advances in embedded solvers and parallelized MHE are making real-time application increasingly feasible for short to medium horizons [56]. Complementary to model-based approaches, learning-based estimators are rapidly emerging. Jin et al. introduced a hybrid model that combines first-principle dynamics with a neural predictor to capture unmodeled residual forces/torques, stemming from payload dynamics for multirotor UAVs. Embedded within a MPC framework, the approach reduces both force and torque estimation errors in simulation and real-world tests relative to purely learning-based models [57]. Although these methods have demonstrated potential in capturing unmodeled dynamics

and compensating for sensor delays, challenges related to their generalization and safety assurance remain unresolved, limiting their current applicability in safety-critical aerial systems.

Despite these significant advancements, a unified and scalable framework for cooperative slung-load transportation remains an open challenge. Specifically, a control strategy is needed that simultaneously ensures formation geometry and trajectory tracking, actively damps payload swing, and provides high robustness to uncertainties (e.g., unknown load mass and external disturbances) across a generic rotary-wing platform configuration. Furthermore, existing state estimation methods often rely on specialized, costly hardware or struggle with real-time complexity under significant aerodynamic disturbances. Addressing these gaps, particularly through computationally efficient control strategies and sensor-minimal state estimation, is the focus of this thesis.

Beyond the algorithmic landscape, the practical viability of these control and estimation strategies is intrinsically tied to the state-of-the-art in onboard computational hardware. Modern aerial platforms typically rely on a dual-tier architecture: low-level Flight Control Units (FCUs) and high-level Flight Companion Computers (FCCs). The FCUs, such as the Pixhawk series [58, 59] based on STM32 microcontrollers, provide the real-time determinism and high-frequency I/O (often exceeding 1 kHz) essential for inner-loop attitude stabilization. However, these units possess limited memory and floating point performance, often precluding the execution of complex optimization-based methods. To handle sophisticated estimation and control logic, FCCs such as the Raspberry Pi and NVIDIA Jetson are employed [60, 61]. While these platforms offer significant Floating Point Operation per Second (FLOPs), they introduce challenges related to power-to-weight ratios, thermal management, and non-deterministic latencies inherent in Linux-based environments and middleware like the Robot Operating System (ROS). This hardware-software ecosystem is often integrated via the PX4 autopilot stack, which serves as the industry-standard firmware for research-grade UAVs. Consequently, there is a growing demand for computationally lean control architectures that bridge this gap, aiming to maintain the robustness of higher-order nonlinear methods. This thesis specifically investigates estimation and control methodologies

designed to operate within these embedded computation envelopes, ensuring seamless transition from high-fidelity simulation to physical hardware.

1.3 Research Contributions

This research provides several original contributions.

- A nonlinear control strategy, based on the Eulerian mathematical modeling of a system of two multirotors carrying a suspended-load, that recasts cooperative transportation as a formation-stabilization problem, treating payload swing as an external disturbance to mitigate. In particular, a formation control scheme is proposed for two agents with the aim to cooperatively perform trajectory-tracking and formation geometry-keeping with the additional stabilization of the oscillatory dynamics. In this framework, controller gains are tuned to make the loaded formation exhibit a two-time-scale behavior: the fast dynamics develops with simultaneous formation geometry stabilization and payload swing damping, while the slow dynamics is related to the trajectory-tracking task. Under these conditions, stability and gain tuning are based on singular perturbation and conservative Lyapunov criteria [62], allowing practical relaxation of time-scale separation.
- A baseline formation control strategy obtained from a Lagrangian-based mathematical modeling of a formation of two multirotors cooperatively transporting a suspended-load. Thanks to the novel parameterization introduced to describe the payload position, the resulting system representation is simple and physically insightful. This simplicity allows the resulting baseline controller to provide a generalized approach for simultaneous (i) formation-keeping, (ii) trajectory-tracking, and (iii) payload swing damping, valid for (iv) a generic rotary wing configuration, thereby filling a significant literature gap.
- A $\mathcal{L}_1$ Adaptive Control augmentation for the state-feedback baseline controller of the previous point. Despite the recent promising advance-

ments, $\mathcal{L}_1$ AC remains largely unexplored in cooperative rotorcraft transportation scenarios, where the coordination of multiple multirotors gives the possibility to carry heavier payloads and accomplish tasks even in case of vehicle failure. The proposed $\mathcal{L}_1$ controller maintains the baseline properties described above, while also ensuring high robustness to matched and unmatched uncertainties, external disturbances, and actuator failures. In this respect, by enforcing a clear separation between the estimation and control loops, the augmented layer preserves the baseline controller's stability while rapidly compensating for parameter uncertainties and disturbances. The cooperative aspect of this adaptive architecture is that the $\mathcal{L}_1$ AC layer can be implemented and deployed onboard one vehicle only. Here, it computes the necessary control inputs for the entire formation, acting as a centralized adaptive supervisor that leverages the formation structure to maintain stability, performance, and robustness.

- A discrete-time estimator is proposed to sense the suspended load's position in cooperative transportation scenarios while fulfilling a strict practical constraint: no dedicated hardware is required beyond standard onboard Inertial Measurement Units (IMUs). The adopted Fading Gaussian Deterministic Filter (FGDF) methodology, after a dedicated tuning procedure, provides the best compromise between modeling errors and measurement noise, while simultaneously ensuring robustness against external disturbances and unmodeled dynamics. The estimated swing angle and rate are then integrated in a dedicated controller to stabilize payload oscillations, implemented as an auxiliary contribution to available control loops developed for formation guidance and navigation. Within this framework, two critical features are integrated. First, a procedure is introduced to estimate the three-dimensional aerodynamic disturbance force acting on each vehicle, which significantly enhances estimation reliability and system performance in adverse wind conditions [63]. Second, in light of practical air delivery and search-and-rescue operations, where the cargo may be either unknown or variable, an

algorithm for estimating payload mass from onboard sensors data is also proposed.

- Swing state estimation and control for the novel scenario involving a multirotor aerial vehicle transporting two serially-connected suspended payloads. This addresses the practical aerial logistics challenge of delivering multiple parcels in a single mission to reduce costs and increase sustainability. The estimation problem is solved recursively according to the FGDF methodology, as described in the previous point. The estimated swing variables are used in a proportional-derivative feedback control logic that stabilizes oscillations for both loads, which is designed as an auxiliary contribution to available control loops specifically developed for rotorcraft guidance and navigation task. The proposed approach brings several novelties. Concerning air delivery missions, multirotor transportation is swift and effective, avoiding the limitations of traditional ground-based traffic. Being able to stabilize multiple serially-connected payloads allows the simultaneous delivery of more parcels during one mission, thus optimizing delivery routes and cutting operational costs. Experimental flight tests are conducted to demonstrate that the proposed estimation and control architecture remains easily implementable in commercial-off-the-shelf solution of wide usage [59]. Finally, this approach is widely applicable and can be either easily extended to various rotorcraft configurations or to scenarios involving a larger number of payloads, outlining ground-breaking potential for future research directions.

1.4 Outline of the Thesis

This dissertation is organized into three main parts. Chapter 2 introduces the mathematical modeling of a multirotor aerial vehicle equipped with a cable-suspended payload, laying the theoretical foundations for the subsequent developments. Chapter 3, which constitutes the core of the thesis, focuses on cooperative transportation of suspended loads by multiple multirotors. Several

control strategies are formulated to achieve simultaneous formation-keeping, trajectory-tracking and suppression of payload oscillations. In this context, a Fading Gaussian Deterministic Filter is proposed to sense the load's position in cooperative scenarios, enabling the design of a ad-hoc controller for oscillation minimization. Building upon these results, Chapter 4 extends the proposed estimation and control methodologies to air delivery missions involving a multirotor carrying serially-connected suspended payloads, a typical scenario in practical aerial logistics applications. The performance of the developed approaches is validated through high-fidelity numerical simulations conducted in MATLAB[®]-Simulink environment, followed by dedicated experimental campaigns (Chapters 5 and 6). The thesis ends with a chapter of concluding remarks that summarizes the key findings and outlines promising future research directions.

CHAPTER 2

Mathematical Models

This chapter develops the mathematical model for a multirotor slung-load system, establishing a rigorous foundation for the estimation and control strategies explored later in this dissertation. Since the focus is on cooperative transportation, this chapter details the dynamics of a single, generic agent i and a suspended payload. Each lifting vehicle is modeled as a six-degree-of-freedom (6-DOF) rigid body, while suspended loads are considered as point-masses. This modeling approach is specifically chosen to capture the critical dynamic couplings, such as cable-induced wrenches and aerodynamic effects, while ensuring the resulting equations remain computationally efficient for real-time execution on embedded hardware. In this respect, some simplifying assumptions are adopted: a flat, non-rotating Earth, quasi-steady aerodynamics, and mass-less cables. In what follows, after introducing the relevant reference frames, Newton-Euler equations are formulated for a single vehicle, enriched with the mathematical description of propellers and power-plant. The resulting equations underpin control and estimation methods presented in subsequent chapters. For the remainder of this work, the individual multirotors in a cooperative formation are denoted as UAV i

$i \in \{1, \dots, n\}$.

2.1 Reference Frames

An Earth-fixed North-East-Down (NED) frame, $\mathcal{F}_E = \{O_E; \mathbf{x}_E, \mathbf{y}_E, \mathbf{z}_E\}$, is first considered, according to standard practice in flight mechanics. This system is assumed to be inertial under the assumption of flat and non-rotating Earth. Provided P_i is the center of gravity (CG) of the i -th agent, two sets of right-handed orthogonal reference frames are introduced:

1. Two Body-fixed frames, $\mathcal{F}_{B_i} = \{P_i; \mathbf{x}_{B_i}, \mathbf{y}_{B_i}, \mathbf{z}_{B_i}\}$, $i = 1, 2$. The longitudinal axis $\mathbf{x}_{B_i}$ is positive out of the nose of the i -th platform in its selected plane of symmetry, $\mathbf{z}_{B_i}$ aims in the direction of the fuselage/frame bottom, and $\mathbf{y}_{B_i}$ completes a right-handed triad.
2. A vehicle structural reference frame, $\mathcal{F}_S = \{O_S; \mathbf{x}_S, \mathbf{y}_S, \mathbf{z}_S\}$, used to locate the center of gravity and all platform components: the axes are parallel to the Body-fixed frame axes such that $\mathbf{x}_S = -\mathbf{x}_B$, $\mathbf{y}_S = \mathbf{y}_B$, and $\mathbf{z}_S = -\mathbf{z}_B$. Stationlines (ST) are measured positively aft along the longitudinal axis. Buttlines (BL) are lateral distances, positive to the right, and waterlines (WL) are measured vertically, positive upward. In what follows, without loss of generality, a sample multirotor configuration is considered. In such a case, O_S lies on the top surface of the frame and is aligned vertically with the vehicle's geometric center over the $\mathbf{x}_S - \mathbf{y}_S$ plane.

A sketch of the i -th agent, including the selected reference frames, is shown in Figure 2.1. Defining $c = \cos(\cdot)$ and $s = \sin(\cdot)$, the vector transformation between $\mathcal{F}_E$ and $\mathcal{F}_{B_i}$ is provided by the rotation matrix

$$\mathbf{T}_{BE}(\boldsymbol{\eta}) = \begin{bmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\phi s\theta c\psi - c\phi s\psi & s\phi s\theta s\psi - c\phi c\psi & s\phi c\theta \\ c\phi s\theta c\psi - s\phi s\psi & c\phi s\theta s\psi - s\phi c\psi & c\phi c\theta \end{bmatrix} \quad (2.1)$$

where $\boldsymbol{\eta} = [\phi, \theta, \psi]^T$ defines the attitude of the i -th vehicle through the classical ‘roll’, ‘pitch’, and ‘yaw’ angles that characterize the ZYX Tait-Bryan

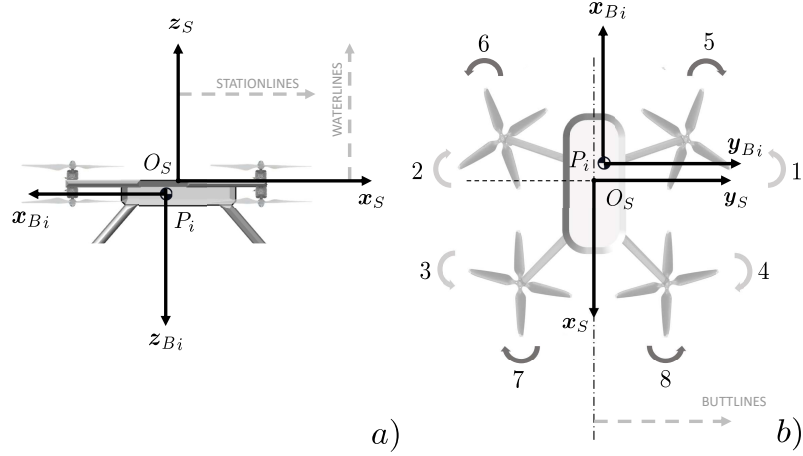


Figure 2.1: Multirotor body-fixed reference frames.

extrinsic rotation sequence [64, 65]. In what follows, all vector quantities are expressed in terms of body-frame components, unless otherwise stated. The subscript ‘ B ’ will be omitted, for the sake of notation conciseness.

2.2 Single Vehicle Dynamics

2.2.1 Equations of Motion

Without loss of generality, since the formation is composed by identical multirotors, only the i -th agent dynamics is detailed. For the sake of brevity, the subscript i will be omitted in what follows to not weight down the notation. In particular, Newton-Euler equations of motion projected in $\mathcal{F}_B$ yield

$$\dot{\mathbf{v}} = -\dot{\boldsymbol{\omega}} \times \mathbf{v} + \mathbf{F}/m \quad (2.2)$$

$$\dot{\boldsymbol{\omega}} = \mathbf{J}^{-1} [-\boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} + \mathbf{M}] \quad (2.3)$$

where $\mathbf{v} = [v_x, v_y, v_z]^T$ is the linear velocity, $\boldsymbol{\omega} = [\omega_x, \omega_y, \omega_z]^T$ is the angular velocity, $\mathbf{J} = \text{diag}(J_x, J_y, J_z)$ is the inertia tensor about the center of gravity

with respect to $\mathcal{F}_B$ and m is the total mass of the rotorcraft. Moreover, $\mathbf{F} = [F_x, F_y, F_z]^T$ and $\mathbf{M} = [M_x, M_y, M_z]^T$ are the external force and moment vectors, respectively.

Rotorcraft attitude kinematics, which relates the generalized velocity $\dot{\boldsymbol{\eta}}$ and the angular velocity $\boldsymbol{\omega}$ is given by

$$\dot{\boldsymbol{\eta}} = \begin{bmatrix} 1 & \sin\phi \tan\theta & \cos\phi \tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix} \boldsymbol{\omega} \quad (2.4)$$

Meanwhile, the position of the lifting vehicle $\mathbf{r} = [r_x, r_y, r_z]^T$, with components expressed in the inertial frame $\mathcal{F}_E$, is obtained from the equation

$$\dot{\mathbf{r}} = \mathbf{T}_{BE}(\boldsymbol{\eta})^T \mathbf{v} \quad (2.5)$$

The external force and moment vectors are made of gravity, aerodynamic, and cable contributions, namely

$$\mathbf{F} = \mathbf{F}^{(g)} + \mathbf{F}^{(a)} + \mathbf{T}_{BE} \mathbf{F}_E^{(c)} \quad (2.6)$$

$$\mathbf{M} = \mathbf{M}^{(a)} + \mathbf{M}^{(c)} \quad (2.7)$$

Considering Equation (2.1), the gravity force vector expressed in the body frame is

$$\mathbf{F}^{(g)} = \mathbf{T}_{BE}(\boldsymbol{\eta}) \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} = mg \begin{bmatrix} -\sin\theta \\ \sin\phi \cos\theta \\ \cos\phi \cos\theta \end{bmatrix} \quad (2.8)$$

where g is the gravitational acceleration, described according to WGS84 Taylor series model [66].

2.2.2 Frame and Rotor Aerodynamics

The total aerodynamic load acting on each vehicle is written as the sum of the airframe contribution and the N individual rotors contribution

$$\mathbf{F}^{(a)} = \mathbf{F}^{(f)} + \mathbf{F}^{(r)} \quad (2.9)$$

$$\mathbf{M}^{(a)} = \mathbf{M}^{(f)} + \mathbf{M}^{(r)} \quad (2.10)$$

For airframe aerodynamics, the equivalent flat plate drag area model is exploited, from which the force vector expressed in the body frame results

$$\mathbf{F}^{(f)} = -\frac{1}{2} \rho [A_x |v_x| v_x, \quad A_y |v_y| v_y, \quad A_z |v_z| v_z]^T \quad (2.11)$$

where A_x , A_y , and A_z are the equivalent flat-plate areas facing each of the body-frame axes and ρ is the air density. This force $\mathbf{F}^{(f)}$ will generate a certain moment $\mathbf{M}^{(f)} = \mathbf{d}_{CP} \times \mathbf{F}^{(f)}$, where $\mathbf{d}_{CP} = [ST_{CG} - ST_{CP}, BL_{CP} - BL_{CG}, WL_{CG} - WL_{CP}]^T$ is the vector directed from CG to the center of pressure (CP) of the airframe, which is assumed to be a constant. Following Ref. [67], the influence of rotor downwash on Equation (2.11) is neglected.

Without loss of generality, a planar vehicle configuration with N rotors is considered for each agent of the formation. The thrust T_j generated by the j -th rotor, $j = 1, \dots, N$, is directed along $\mathbf{z}_B$ and points upward. The net rotor forces and moments are

$$\mathbf{F}^{(r)} = \sum_{j=1}^N \mathbf{F}_j^{(r)} \quad (2.12)$$

$$\mathbf{M}^{(r)} = \sum_{j=1}^N \mathbf{M}_j^{(r)} \quad (2.13)$$

assuming that the j -th rotor has radius R , area $A = \pi R^2$, and position with respect to the body frame $\mathcal{F}_B$ given by $\mathbf{d}_{R_j} = [ST_{CG} - ST_{R_j}, BL_{R_j} - BL_{CG}, WL_{CG} - WL_{R_j}]^T$. Under the assumption of incompressible flow, the j -th rotor aerodynamics is based on Momentum Theory (MT) and Blade Element Theory (BET) [67, 68], specialized for small multirotors. For clarity, the underlying assumptions are collected here.

Assumption 1. *The following assumptions hold:*

- *Rigid, hingeless blades ($\epsilon = K_\beta = \tan \delta_3 = 0$).*
- *No dynamic inflow; induced velocity v_i is solved quasi-statically.*
- *Blade stall is neglected.*
- *Pre-cone angle a_0 is constant; flapping dynamics are omitted.*

According to Ref. [68], two set of N right-handed local frames are introduced for detailing the rotor aerodynamic forces and moments.

- A set of Rotor–Hub frames, $\mathcal{F}_{Hj} = \{H_j; \mathbf{x}_{Hj}, \mathbf{y}_{Hj}, \mathbf{z}_{Hj}\}$, $j = 1, \dots, N$. Each frame has its origin, H_j , at the j -th rotor hub center. The $\mathbf{z}_{Hj}$ axis is collinear with the rotor shaft and points upward, $\mathbf{x}_{Hj}$ is chosen forward in the $\mathbf{x}_B - \mathbf{z}_B$ plane, and $\mathbf{y}_{Hj}$ completes the triad.
- A set of Rotor–Wind frames, $\mathcal{F}_{Wj} = \{H_j; \mathbf{x}_{Wj}, \mathbf{y}_{Wj}, \mathbf{z}_{Wj}\}$, $j = 1, \dots, N$. Each frame is obtained by rotating $\mathcal{F}_{Hj}$ about $\mathbf{z}_{Hj}$ such that $\mathbf{x}_{Wj}$ aligns with the instantaneous in-plane velocity with components u_H and v_H , respectively. The misalignment between $\mathcal{F}_{Hj}$ and $\mathcal{F}_{Wj}$ is the rotor-disk azimuth, namely the sideslip angle $\beta_w = \text{atan2}(v_H, u_H)$.

The Rotor–Wind reference system is exploited in the calculation of both out-of-plane and in-plane rotor aerodynamic contributions. Concerning the former, the vertical thrust, T , and the torque, Q , are obtained as

$$\begin{aligned}
 T = & \frac{1}{2} n_b \rho a c R^3 \left\{ \frac{\lambda}{2} \Omega^2 \right. \\
 & + \theta_0 \left[\frac{1}{3} + \frac{v_H^2 + u_H^2}{2 R^2} \right] + \frac{\theta_t}{4} \left[1 + \frac{v_H^2 + u_H^2}{R^2} \right] \\
 & \left. + \frac{\sqrt{v_H^2 + u_H^2}}{4 R} (p_H \cos \beta_w + q_H \sin \beta_w) \right\} \quad (2.14)
 \end{aligned}$$

and

$$\begin{aligned}
 Q = & \frac{1}{2} n_b \rho a c R^4 \left\{ \frac{\delta}{4 a} \left[\Omega^2 + \frac{v_H^2 + u_H^2}{R^2} \right] \right. \\
 & - \frac{\theta_0}{3} \left[\lambda \Omega^2 + \frac{\sqrt{v_H^2 + u_H^2}}{2 R} (p_H \cos \beta_w + q_H \sin \beta_w) \right] \\
 & - \left[\frac{\theta_t}{4} + \frac{\lambda}{2} \right] \lambda \Omega^2 - \frac{1}{8} (-p_H \sin \beta_w + q_H \cos \beta_w)^2 \\
 & \left. - \frac{1}{8} (p_H \cos \beta_w + q_H \sin \beta_w)^2 \right\} \quad (2.15)
 \end{aligned}$$

The horizontal drag, H_w , and the lateral force, Y_w , are given by

$$\begin{aligned}
 H_W = & \frac{1}{2} n_b \rho a c R^3 \left\{ \frac{\delta \Omega}{2 a} \frac{\sqrt{v_H^2 + u_H^2}}{R} \right. \\
 & - \frac{\theta_0}{6} \left[3 \lambda \frac{\sqrt{v_H^2 + u_H^2}}{R} + p_H \cos \beta_w + q_H \sin \beta_w \right] \Omega \\
 & - \frac{\theta_t}{8} \left[2 \lambda \frac{\sqrt{v_H^2 + u_H^2}}{R} + p_H \cos \beta_w + q_H \sin \beta_w \right] \Omega \\
 & \left. - \frac{\lambda}{2} (p_H \cos \beta_w + q_H \sin \beta_w) \Omega \right\}
 \end{aligned} \tag{2.16}$$

and

$$\begin{aligned}
 Y_W = & \frac{1}{2} n_b \rho a c R^2 (\Omega R) \left\{ -\frac{\theta_0}{4} (p_H \sin \beta_w - q_H \cos \beta_w) \right. \\
 & - \frac{\theta_t}{4} (p_H \sin \beta_w - q_H \cos \beta_w) \\
 & \left. - \frac{\lambda}{2} (p_H \sin \beta_w + q_H \cos \beta_w) \right\},
 \end{aligned} \tag{2.17}$$

where all the symbols are defined in Table 2.1. In the present framework the in-plane moments L_w and M_w are identically zero under Assumption 1.

Consider Ω_j as the j -th rotor angular rate, such that $V_{tip_j} = \Omega_j R$ is the corresponding blade tip speed. In order to obtain slightly more accurate results, some of the assumptions in Ref. [68] have been refined. In particular, rotor blades are characterized by airfoils with 2D lift-curve slope $a < 2\pi$ 1/rad, a more suitable choice for the small-scale propellers treated in this work [69]. According to Talbot et al. [68], the profile drag coefficient for the j -th rotor averaged along the blade, δ_j , is estimated as

$$\delta_j = C_{d_0} + 0.3 \left(\frac{6 C_{T_j}}{\sigma a} \right)^2 \tag{2.18}$$

where the two terms on the right-hand side account for blade parasitic drag and blade induced drag, respectively. Concerning the former, it is important to stress that in large-scale rotors is reasonable to directly assume $\delta_j = C_{d_0}$ as a constant independent from Reynolds number, being the flow

Table 2.1: Nomenclature for Equations (2.14)–(2.17)

Symbol	Description	Unit
T	Rotor thrust	N
Q	Rotor torque	N m
H_W	Hub horizontal force (drag)	N
Y_W	Hub lateral force	N
n_b	Number of blades	–
ρ	Air density	kg/m ³
a	Lift–curve slope	1/rad
c	Blade chord	m
R	Rotor radius	m
λ	Inflow ratio ($v_i/\Omega R$)	–
Ω	Rotor angular speed	rad/s
θ_0	Collective pitch at root	rad
θ_t	Linear twist rate	rad
u_H	Longitudinal hub velocity component	m/s
v_H	Lateral hub velocity component	m/s
p_H	Hub roll rate about $\mathbf{x}_B$	rad/s
q_H	Hub pitch rate about $\mathbf{y}_B$	rad/s
β_w	Rotor-disk azimuth angle	rad
δ	Profile drag coefficient	–

fully turbulent. Instead, in small-scale rotors, the Reynolds number typically ranges between $10^4 - 10^5$, which implies the flow can be assumed to be fully laminar [70], leading to $C_{d_0} = C_{d_0}(Re)$. Considering the j -th rotor, define $Re_j = \rho c_{75} V_{75_j} / \mu_{air}$ as the Reynolds number evaluated at 75% of the blade radius [70], where μ_{air} is air dynamic viscosity, c_{75} is the local airfoil chord, and $V_{75_j} = \sqrt{v_{i_j}^2 + (0.75 V_{tip_j})^2}$ is the local relative airspeed, being v_{i_j} the induced speed at the j -th rotor disc. The second term accounts for blade induced drag and depends on both the thrust coefficient $C_{T_j} = T_j / (\rho A V_{tip_j}^2)$ and the solidity ratio $\sigma = n_b c / (\pi R)$. In typical multirotor applications, σ has a limited variability, with average values in the order of 0.1 for classical two-bladed configurations [17].

Concerning rotor inflow, several non-ideal effects, such as non-uniform

induced velocity, wake swirl, vortex ring state (VRS) regime, are lumped into an induced power factor k_{ind} . For the present framework, this value is assumed constant, with values typically ranging between $1.1 \leq k_{ind} \leq 1.35$ for lightly loaded multirotor applications [69]. The proximity of the rotor to the ground is modeled with the Padfield in-ground-effect coefficient

$$K_{IGE} = \frac{1 + \left(\frac{\|V_H\|}{v_i} \right)^2}{1 - \frac{1}{16} \left(\frac{R}{z_g} \right)^2}, \quad (2.19)$$

where $V_H = [u_H, v_H, w_H]^T$ is the total relative airflow on the rotor, v_i is the induced velocity, and z_g is propeller height with respect to the ground [71]. The computation of thrust in Equation (2.14) requires solving the following inflow implicit equation

$$\lambda = \frac{w_H}{\Omega R} - \frac{C_T}{2\sqrt{\lambda^2 + \mu^2}} \quad (2.20)$$

where w_H is the vertical hub velocity component and $\mu = \sqrt{u_H^2 + v_H^2} / (\Omega R)$ is the rotor advance ratio. By defining

$$f \triangleq \lambda - \frac{w_H}{\Omega R} - \frac{C_T}{2\sqrt{\lambda^2 + \mu^2}}, \quad (2.21)$$

the iterative procedure is solved through Halley's method according to the estimate update

$$\lambda_{n+1} = \lambda_n - \frac{2 f(\lambda_n) f'(\lambda_n)}{2 [f'(\lambda_n)]^2 - f(\lambda_n) f''(\lambda_n)} \quad (2.22)$$

where $f'(\lambda_n)$ and $f''(\lambda_n)$ are the first and second derivative of f , respectively. The initial guess for the iterative solver, $\lambda_0 = v_{i0} / (\Omega R)$, is obtained by simple algebraical calculation in hover. Finally, air parameters are calculated from the International Standard Atmosphere (ISA) model as a function of rotorcraft altitude [72].

2.2.3 Propulsive System

A model-based approach is exploited to design the multirotor power-plant, which includes an electronic speed controller (ESC) and a three-phase electrical

motor. The idea is to perform an equivalent modeling of the propulsive system, treated as a transfer function that maps the throttle command to the rotor rotational speed. The j -th motor is driven by an electronic speed controller that reads a pulse-width-modulated (PWM) command between roughly $1000 \mu\text{s}$ (idle) and $2000 \mu\text{s}$ (full throttle). The ESC behaves like a three-phase inverter, converting the DC current from the battery into a set of three sinusoidal currents whose electrical frequency, f_{em} , is proportional to the demanded duty cycle. For a generic brushless AC synchronous motor it is $\Omega = 60 f_{em} / p_{em}$, where p_{em} is the number of pair poles. Let PWM_j be the signal sent to j -th motor and PWM_{idle} the arming value at which the rotor is still stationary ($\Omega_j = 0$), then $\delta_j = PWM_j - PWM_{idle}$ is the command increment with respect to the idle condition $PWM_j = PWM_{idle}$. Bench tests show that the rotational speed of the j -th rotor obeys to the power law

$$\Omega_j = k_\Omega \delta_j^n \quad (2.23)$$

with coefficients $k_\Omega > 0$ and $0 < n < 1$.

The high-level controllers generate a vector of control outputs $\mathbf{u} = [\delta_{roll}, \delta_{pitch}, \delta_{yaw}, \delta_v]^T$. These signals represent the stabilization commands for the roll (δ_{roll}), pitch (δ_{pitch}), and yaw dynamics (δ_{yaw}), and the vertical velocity regulation channel (δ_v). These outputs are mapped into the per-motor command vector $\boldsymbol{\delta} = [\delta_1, \dots, \delta_N]^T$ through a mixing matrix. To ensure the vehicle operates around a stable equilibrium, a constant trim offset δ_h (hover throttle) is added to each motor command. This offset is tuned such that a 50% throttle command corresponds to a near-hover condition, preserving maneuverability margins along the vertical axis.

In this thesis, two configurations were investigated: a light-lift X-quadrotor ($N = 4$) and a heavy-lift coaxial octarotor ($N = 8$). For the quadrotor, the motor-mixing scheme is:

$$\begin{aligned} \delta_1 &= -\delta_{roll} + \delta_{pitch} + \delta_{yaw} + \delta_v + \delta_h \\ \delta_2 &= -\delta_{roll} - \delta_{pitch} - \delta_{yaw} + \delta_v + \delta_h \\ \delta_3 &= +\delta_{roll} - \delta_{pitch} + \delta_{yaw} + \delta_v + \delta_h \\ \delta_4 &= +\delta_{roll} + \delta_{pitch} - \delta_{yaw} + \delta_v + \delta_h \end{aligned} \quad (2.24)$$

For the octarotor the motor-mixing scheme is:

$$\begin{aligned}
 \delta_1 &= -\delta_{roll} + \delta_{pitch} + \delta_{yaw} + \delta_v + \delta_h \\
 \delta_2 &= +\delta_{roll} + \delta_{pitch} - \delta_{yaw} + \delta_v + \delta_h \\
 \delta_3 &= +\delta_{roll} - \delta_{pitch} + \delta_{yaw} + \delta_v + \delta_h \\
 \delta_4 &= -\delta_{roll} - \delta_{pitch} - \delta_{yaw} + \delta_v + \delta_h \\
 \delta_5 &= -\delta_{roll} + \delta_{pitch} - \delta_{yaw} + \delta_v + \delta_h \\
 \delta_6 &= +\delta_{roll} + \delta_{pitch} + \delta_{yaw} + \delta_v + \delta_h \\
 \delta_7 &= +\delta_{roll} - \delta_{pitch} - \delta_{yaw} + \delta_v + \delta_h \\
 \delta_8 &= -\delta_{roll} - \delta_{pitch} + \delta_{yaw} + \delta_v + \delta_h
 \end{aligned} \tag{2.25}$$

The sign of each entry reflects how a control channel modifies the thrust of a specific rotor to produce the desired body-axis moment or force.

2.3 Suspended-Payload and Cable Dynamics

The suspended load is assumed to be a point with mass m_l , connected by a cable to the multirotor at the hook point H with coordinates $\mathbf{h} = [STACG - STA_H, BL_H - BL_{CG}, WL_{CG} - WL_H]^T = [h_x, h_y, h_z]^T$ in $\mathcal{F}_B$. Payload dynamics is described by Newton-Euler equations projected in $\mathcal{F}_E$, namely

$$m_l \dot{\mathbf{v}}_l = \mathbf{F}_{l_E} = \begin{bmatrix} 0 \\ 0 \\ m_l g \end{bmatrix} + \mathbf{F}_{l_E}^{(d)} + \mathbf{F}_{l_E}^{(c)} \tag{2.26}$$

where $\mathbf{F}_{l_E}^{(c)} = -\mathbf{F}_E^{(c)}$ is the force exerted by the multirotor on the load through an elastic cable with Hooke's constant $K > 0$ and nominal length L . $\mathbf{F}_{l_E}^{(d)}$ is the aerodynamic force acting on the payload, calculated as

$$\mathbf{F}_{l_E}^{(d)} = -\frac{1}{2} \rho A_l C_{d_l} \|\mathbf{v}_l\| \mathbf{v}_l \tag{2.27}$$

where C_{d_l} is the payload drag coefficient and A_l is a reference area for drag computation. The cable is assumed to be mass-less and its aerodynamic drag

is disregarded. Assuming $\mathbf{r}_l$ is the position of the load obtained from the equation

$$\dot{\mathbf{r}}_l = \mathbf{v}_l \quad (2.28)$$

the vector describing the actual orientation and length of the cable in $\mathcal{F}_E$ may be expressed as $\mathbf{c}_E = \mathbf{r}_l - \mathbf{r} - \mathbf{T}_{BE}^T \mathbf{h}$.

On the basis of Hooke's model, the force $\mathbf{F}_E^{(c)}$ is obtained from the unilateral linear-elastic equation:

$$\mathbf{F}_E^{(c)} = \max(0, K\Delta L)\hat{\mathbf{c}}_E \quad (2.29)$$

where $\Delta L = \|\mathbf{c}_E\| - L$ is the elastic deformation and $\hat{\mathbf{c}}_E = \mathbf{c}_E/\|\mathbf{c}_E\|$ is the unit vector directed along the cable. This formulation explicitly accounts for the physical nature of the cable, which can only exert tensile forces. If the distance between the hook point and the payload is less than the nominal length L , the cable becomes slack and the force drops to zero, preventing unfeasible compression loads. Finally, the moment generated on the multirotor by the force $\mathbf{F}_E^{(c)}$, referred to $\mathcal{F}_B$, is given by

$$\mathbf{M}^{(c)} = \mathbf{h} \times [\mathbf{T}_{BE} \mathbf{F}_E^{(c)}] \quad (2.30)$$

2.4 Discussion on Payload Modeling

The point-mass representation of the suspended payload adopted in this work is a simplification that warrants careful discussion, as it neglects some physical phenomena that become increasingly important in cooperative transportation scenarios.

2.4.1 Distributed Attachment Points

In the current formulation, all cables attach to a single point coincident with the payload center of gravity. This geometric constraint forces the system to maintain equilibrium through lateral cable tension components. For a cooperative planar formation, these lateral components increase the required thrust at equilibrium beyond the simple weight-sharing scenario.

Extending the payload modeling to a rigid body with spatially distributed hook points, allows the suspended cables to remain nearly vertical at equilibrium, significantly reducing tension magnitude and thus energy consumption.

However, for the aerial delivery applications targeted in this work, typical payload dimensions are significantly smaller than cable length ($D_{\text{load}} \ll L$). Under these conditions, modeling distributed attachment points as a single central point introduces negligible geometric error. Furthermore, the lateral force components, while present, remain manageable for the thrust capabilities of modern multirotors and do not compromise the feasibility of the proposed control strategies. The numerical and experimental campaigns presented in Chapters 5 and 6, respectively, demonstrate successful trajectory tracking and swing damping, validating the practical sufficiency of the single-point assumption for the proposed framework.

2.4.2 Payload Attitude Control

The point mass assumption precludes any consideration of payload attitude control. In reality, when $n \geq 3$ agents carry a rigid payload with distinct attachment points, the formation possess full authority over payload orientation. With $n = 2$ vehicles, partial attitude control is possible, though rotational motion can still occur about the axis connecting the two hook points.

However, for typical delivery operations, attitude control is not a primary requirement. Standard packages are designed to tolerate arbitrary orientations during transport, and delivery protocols generally do not impose strict orientation constraints upon release. In cases where orientation matters, passive stabilization through appropriate packaging and center-of-mass positioning is the industry standard, rather than active aerial control. Moreover, the proposed control framework focuses on the more fundamental challenges of formation stabilization, trajectory tracking, and swing damping capabilities that are prerequisites for any future extension to attitude-aware transportation. The modular architecture presented in Chapter 3 could accommodate attitude control loops if extended to rigid-body payload models, making the current work a natural stepping stone toward such developments.

2.4.3 Fast Rotational Dynamics

The point-mass model neglects the payload's rotational inertia and the resulting attitude dynamics. When the cable hook point does not coincide with the payload center of mass, a moment arm exists that couples translational and rotational motion. The resulting fast attitude dynamics could interact with cable oscillations and multicopter control loops, potentially degrading stability and performance. This coupling is particularly pronounced during aggressive maneuvers or when transporting elongated objects.

However, such scenarios fall outside the intended scope of the present work, which focuses on aerial transportation in light of future UAMD applications. On the one hand, this implies that rather than aggressive maneuvers smooth and conservative ones are considered to minimize the dynamic effort on the load. On the other hand, the focus is on concentrated payloads, which exhibit minimal rotational inertia effects with respect to beam-like ones. When the characteristic payload dimension D_{load} satisfies $D_{\text{load}} \ll L$, the rotational time scale becomes much faster than the pendular oscillation time scale $\tau_{\text{swing}} \sim \sqrt{L/g}$. This causes attitude perturbations to damp rapidly through aerodynamic drag without significantly coupling to the slower swing dynamics that dominate system behavior.

CHAPTER 3

Cooperative Payload Transportation

This chapter develops and evaluates three cooperative control architectures of increasing complexity and robustness. The first section presents a rigorous two-time-scale nonlinear controller that establishes strong theoretical stability guarantees. To enhance modularity and practical implementation, a simpler baseline formation controller is derived using an insightful Lagrangian parameterization. Recognizing the limitations of this baseline in dealing with real-world uncertainties and non-nominal conditions, a $\mathcal{L}_1$ Adaptive Control layer augmentation is implemented to achieve enhanced performance and robustness. The chapter ends with the development of the Fading Gaussian Deterministic Filter for estimating payload swing state in cooperative transportation scenarios, providing the essential feedback signals required for the practical implementation of the proposed control architectures.

3.1 Two-Time-Scale Control

Cooperative aerial load transport poses three concurrent objectives: (i) maintain a prescribed inter-vehicle geometry, (ii) track a reference trajectory for

the overall formation, and (iii) actively damp payload swing. To address these requirements, this section develops a nonlinear control strategy for two aerial vehicles cooperatively transporting a suspended payload. The proposed method leverages a two-time-scale structure: a fast outer loop regulates formation errors and shapes the slung-load dynamics to behave as a well-damped virtual mass, while a slow guidance loop drives the formation's center of mass along the mission trajectory. The section proceeds from an Eulerian modeling of the three objects to an interpretable control law with stability guarantees based on time-scale separation, and finally concludes with practical implementation guidance on a PX4-style stack [59].

3.1.1 Equations of Motions

This subsection introduces the kinematics and Newton-Euler dynamics for the i -th vehicle ($i = 1, 2$) and the payload, defines the cable geometry, and collects the forces that couple the three objects. The goal is to obtain compact error dynamics expressions for formation keeping and trajectory tracking of the system's center of mass. Let $\mathbf{r}_i = [r_{xi}, r_{yi}, r_{zi}]^T$ be the position of the i -th multirotor and $\mathbf{v}_i = [v_{xi}, v_{yi}, v_{zi}]^T$ its velocity with respect to the inertial frame. A suspended load with mass m_l is connected in P_l by a cable to the i -th multirotor, as depicted in Figure 3.1. The dynamics of each agent as expressed in $\mathcal{F}_E$ is described by the following equations:

$$\dot{\mathbf{r}}_i = \mathbf{v}_i \quad (3.1)$$

$$\dot{\mathbf{v}}_i = \frac{1}{m} (m \mathbf{g} + \mathbf{f}_{di} + \mathbf{f}_{li} + \mathbf{f}_{ci}) \quad (3.2)$$

where m is the aerial vehicle's mass, $\mathbf{g} = [0, 0, g]^T$ is the local gravitational acceleration vector, $\mathbf{f}_{di}$ is the vehicle aerodynamic drag, and $\mathbf{f}_{li}$ is the force induced by the load on the i -th multirotor through the cable. The total thrust vector $\mathbf{f}_{ci} = [\mathbf{f}_{cxi}, \mathbf{f}_{c yi}, \mathbf{f}_{c zi}]^T$, directed along $\mathbf{z}_{Bi}$ and pointing upwards, represents the only control input to Equation (3.2). Provided ρ is the air density, $\mathbf{f}_{di}$ principally accounts for the contribution of the vehicle frame, for which a simple flat-plate area model is proposed,

$$\mathbf{f}_{di} = -\frac{1}{2} \rho A_{eq} C_{deq} \|\mathbf{v}_i\| \mathbf{v}_i \quad (3.3)$$

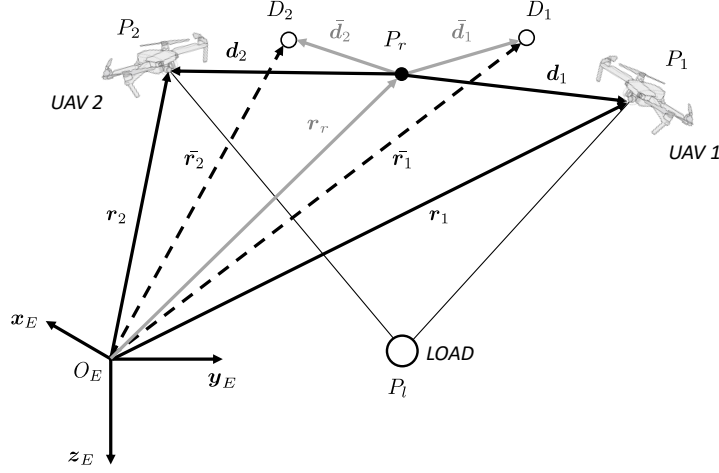


Figure 3.1: Definition of current (P_i) and desired (D_i) position of the i -th UAV.

where A_{eq} and C_{deq} represent an equivalent area and drag coefficient for the multirotor. While a more detailed, frame-fixed model is provided in Equation (2.11) for high-fidelity simulation, this isotropic approximation is adopted here to maintain the mathematical tractability of the cooperative control law. It is assumed that the i -th cable is directly connected to the center of gravity of the i -th multirotor, such that $\mathbf{f}_{di}$ has no effect on attitude motion. Payload dynamics is described by Newton-Euler equations projected in $\mathcal{F}_E$, namely:

$$\dot{\mathbf{v}}_l = \frac{1}{m_l} (m_l \mathbf{g} + \mathbf{f}_{dl} - \mathbf{f}_{l1} - \mathbf{f}_{l2}) \quad (3.4)$$

where $\mathbf{v}_l = [v_{xl}, v_{yl}, v_{zl}]^T$ is the load velocity and $\mathbf{f}_{dl}$ is the payload aerodynamic force, obtained as

$$\mathbf{f}_{dl} = -\frac{1}{2} \rho A_l C_{dl} \|\mathbf{v}_l\| \mathbf{v}_l \quad (3.5)$$

provided C_{dl} is the payload drag coefficient and A_l is a reference area for drag computation. Each cable is mass-less and its aerodynamic drag is disregarded. Let $\mathbf{r}_l = [r_{xl}, r_{yl}, r_{zl}]^T$ be the position of the load, obtained from

$$\dot{\mathbf{r}}_l = \mathbf{v}_l, \quad (3.6)$$

the vector describing the orientation and length of the i -th cable is $\mathbf{c}_i = \mathbf{r}_l - \mathbf{r}_i$. According to Hooke's law, the force $\mathbf{f}_{l_i}$ is calculated from the linear-elastic equation

$$\mathbf{f}_{l_i} = K \Delta L_i \hat{\mathbf{c}}_i \quad (3.7)$$

where K is the elastic constant, L is the nominal cable length, $\Delta L_i = \|\mathbf{c}_i\| - L$ is the elastic deformation, and $\hat{\mathbf{c}}_i = \mathbf{c}_i / \|\mathbf{c}_i\|$.

3.1.2 Control System Design

The main goal of this subsection consists in designing a nonlinear Cooperative Transportation Controller (CTC) to simultaneously perform (i) formation keeping, (ii) trajectory tracking, and (iii) load swing stabilization. Assume the i -th vehicle is required to keep a specified distance from a reference point P_r , identified by the vector $\mathbf{r}_r$ and moving with velocity $\mathbf{v}_r$. Provided D_i is the i -th agent required position, identified by position vector $\bar{\mathbf{r}}_i$ (see Figure 3.1), it is

$$\mathbf{r}_r + \mathbf{d}_i = \mathbf{r}_i \quad (3.8)$$

and

$$\mathbf{r}_r + \bar{\mathbf{d}}_i = \bar{\mathbf{r}}_i, \quad (3.9)$$

where $\mathbf{d}_i = [d_{xi}, d_{yi}, d_{zi}]^T$ and $\bar{\mathbf{d}}_i = [\bar{d}_{xi}, \bar{d}_{yi}, \bar{d}_{zi}]^T$ respectively indicate the current and the required relative distances between the i -th agent and the reference point. The following equation is derived:

$$\bar{\mathbf{d}}_i - \mathbf{d}_i = \bar{\mathbf{r}}_i - \mathbf{r}_i \quad (3.10)$$

Let $\dot{\bar{\mathbf{r}}}_i = \bar{\mathbf{v}}_i$ and consider Equation (3.1). The time derivative of Equation (3.10) yields

$$\dot{\bar{\mathbf{d}}}_i - \dot{\mathbf{d}}_i = \bar{\mathbf{v}}_i - \mathbf{v}_i, \quad (3.11)$$

where $\bar{\mathbf{v}}_i$ represents the desired velocity of the i -th vehicle. Assuming the desired velocity $\bar{\mathbf{v}}_i$ equals the velocity of the reference point $\mathbf{v}_r$, the position dynamics of the i -th vehicle with respect to the reference point becomes:

$$\dot{\mathbf{d}}_i = \dot{\bar{\mathbf{d}}}_i - \mathbf{v}_r + \mathbf{v}_i \quad (3.12)$$

Define $\boldsymbol{\delta}_i = \bar{\mathbf{d}}_i - \mathbf{d}_i$ and $\boldsymbol{\epsilon}_i = \mathbf{v}_r - \mathbf{v}_i$ as the relative position and trajectory error variables, respectively. By taking into account Equations (3.2) and (3.12), formation error dynamics results in

$$\dot{\boldsymbol{\delta}}_i = \boldsymbol{\epsilon}_i \quad (3.13)$$

$$\dot{\boldsymbol{\epsilon}}_i = -\mathbf{g} - \frac{1}{m} (\mathbf{f}_{di} + \mathbf{f}_{li} + \mathbf{f}_{ci}) + \dot{\mathbf{v}}_r \quad (3.14)$$

The formation error dynamics is extended to account for the residual degree of freedom related to payload oscillation. Let G be the center of gravity of the complete system with total mass $M = 2m + m_l$ made of multirotors and payload, with position given by

$$\mathbf{r} = \frac{1}{M} (m \mathbf{r}_1 + m \mathbf{r}_2 + m_l \mathbf{r}_l) \quad (3.15)$$

Assume the center of gravity G is also required to track the reference point P_r , such that $\boldsymbol{\rho} = \mathbf{r}_r - \mathbf{r}$ is a relative position error vector. Let $\boldsymbol{\sigma} = \dot{\mathbf{r}}_r - \dot{\mathbf{r}} = \mathbf{v}_r - \dot{\mathbf{r}}$, such that

$$\dot{\boldsymbol{\rho}} = \boldsymbol{\sigma} \quad (3.16)$$

Based on the definition of G in Equation (3.15), the dynamics of $\boldsymbol{\sigma}$ is derived combining Equations (3.2) and (3.4) as

$$\dot{\boldsymbol{\sigma}} = \dot{\mathbf{v}}_r - \ddot{\mathbf{r}} = \dot{\mathbf{v}}_r - \mathbf{g} - \frac{1}{M} (\mathbf{f}_{c1} + \mathbf{f}_{c2}) \quad (3.17)$$

With this in mind, the i -th elastic disturbance term defined in Equation (3.7) and affecting Equation (3.14) is conveniently expressed as a function of errors δ_i and ρ . To this end, payload position is first expressed as a function of $\mathbf{r}$ from Equation (3.15):

$$\mathbf{r}_l = \frac{1}{m_l} (-m \mathbf{r}_1 - m \mathbf{r}_2 + M \mathbf{r}) \quad (3.18)$$

Taking into account Equation (3.8) and the definitions of formation error vectors, it follows $\mathbf{r}_i = \mathbf{r}_r + \mathbf{d}_i = \mathbf{r}_r + \bar{\mathbf{d}}_i - \boldsymbol{\delta}_i$ and $\mathbf{r} = \mathbf{r}_r - \boldsymbol{\rho}$ for $i = 1, 2$. By substitution into Equation (3.18) one has

$$\begin{aligned} \mathbf{r}_l &= \frac{1}{m_l} [-m (\mathbf{r}_r + \bar{\mathbf{d}}_1 - \boldsymbol{\delta}_1) - m (\mathbf{r}_r + \bar{\mathbf{d}}_2 - \boldsymbol{\delta}_2) + M (\mathbf{r}_r - \boldsymbol{\rho})] \\ &= \mathbf{r}_r - \frac{1}{m_l} [m (\bar{\mathbf{d}}_1 - \boldsymbol{\delta}_1) + m (\bar{\mathbf{d}}_2 - \boldsymbol{\delta}_2) + M \boldsymbol{\rho}] \end{aligned} \quad (3.19)$$

and the vector directed from UAV 1 to the payload becomes

$$\begin{aligned}
\mathbf{c}_1 &= \mathbf{r}_l - \mathbf{r}_1 \\
&= \mathbf{r}_l - (\mathbf{r}_r + \bar{\mathbf{d}}_1 - \boldsymbol{\delta}_1) \\
&= -\frac{1}{m_l} [m (\bar{\mathbf{d}}_1 - \boldsymbol{\delta}_1) + m (\bar{\mathbf{d}}_2 - \boldsymbol{\delta}_2) + M \boldsymbol{\rho}] - \bar{\mathbf{d}}_1 + \boldsymbol{\delta}_1 \\
&= -(\bar{\mathbf{d}}_1 - \boldsymbol{\delta}_1) \left(1 + \frac{m}{m_l}\right) - (\bar{\mathbf{d}}_2 - \boldsymbol{\delta}_2) \frac{m}{m_l} - \boldsymbol{\rho} \frac{M}{m_l}
\end{aligned} \tag{3.20}$$

Similarly, for UAV 2 is:

$$\mathbf{c}_2 = -(\bar{\mathbf{d}}_2 - \boldsymbol{\delta}_2) \left(1 + \frac{m}{m_l}\right) - (\bar{\mathbf{d}}_1 - \boldsymbol{\delta}_1) \frac{m}{m_l} - \boldsymbol{\rho} \frac{M}{m_l} \tag{3.21}$$

The application framework is defined by the following operating assumption.

Assumption 2. *It is supposed that:*

- *each vehicle has knowledge of both position and velocity of other objects in the formation, including the payload;*
- *inner-loop control systems allow the i -th vehicle to track the desired control force $\mathbf{f}_{ci}$ with sufficient accuracy;*
- *control design is performed on a simplified model, where aerodynamic drag acting on vehicles and payload is neglected.*

Remark 1. *The estimation of the oscillatory state of the payload in the cooperative scenario is a challenging issue that was recently solved by the author's research group [73, 36]. Stemming from the approach in Ref. [74], a method was derived to estimate the swing angle and rate in cooperative multirotor suspended-load systems relying only on the onboard IMU, without any extra sensors. In this novel approach, accelerometer data and dynamic model information are fused through a Fading Gaussian Deterministic Filter (FGDF), whose theoretical framework ensures the best compromise between model uncertainties and measurement noise. The cooperative FGDF is presented in detail in Section 3.4. However, other methods based on current*

technology can be envisaged to perform payload state determination. They include motion capture ground systems [75, 46], onboard visual detection devices [47, 76], or sensors that are fixed with respect to the load [77, 50].

Remark 2. *This thesis does not delve into the inner-loop problem of forcing each rotorcraft to follow a commanded thrust vector. A wide spectrum of well-established attitude controllers can accomplish that task, ranging from conventional PID laws [78] and LQR designs [79] to active-disturbance-rejection schemes [80], back-stepping or feedback-linearized regulators [81], and various sliding-mode formulations [82, 83]. Whichever strategy is adopted, a common requirement emerges: the bandwidth of the attitude-and-thrust loop must exceed, by a comfortable margin, the bandwidth associated with cooperative load transport. Proper gain tuning, together with hard limits on commanded attitudes, angular rates and accelerations, ensures that separation. Under this premise the overall system naturally exhibits three distinct time-scales. Attitude dynamics form the fastest layer, the intermediate layer comprises formation keeping and swing damping, and the slowest layer is the trajectory-tracking mission itself. The controller proposed here is expressly structured around that hierarchy.*

Based on Assumption 2, a preliminary result is provided to characterize the formation geometry adopted in the present framework.

Lemma 3.1.1. *Let $\mathbf{a} = [a_x, a_y, 0]^T$ be the prescribed mutual distance vector between the two agents of the formation, with components expressed in $\mathcal{F}_E$. Provided $a_x, a_y \in \mathbb{R}$ are selected such that $a = \sqrt{a_x^2 + a_y^2} < 2L$, it follows that desired formation geometry is defined by*

$$\bar{d}_{x_2} = -\bar{d}_{x_1} = a_x/2, \quad \bar{d}_{y_2} = -\bar{d}_{y_1} = a_y/2, \quad (3.22)$$

and

$$\bar{d}_{z_2} = \bar{d}_{z_1} = -\frac{m_l}{2M} \sqrt{4\bar{L}^2 - a^2} \quad (3.23)$$

where $\bar{L}$ is actual cable length of loaded formation, obtained as the only real positive solution of equation:

$$p_0 + p_1 \bar{L} + p_2 \bar{L}^2 + p_3 \bar{L}^3 + p_4 \bar{L}^4 = 0 \quad (3.24)$$

that satisfies $\bar{L} > L$, where

$$\begin{aligned} p_0 &= -K^2 L^2 a^2 \\ p_1 &= 2 K^2 L a^2 \\ p_2 &= 4 K^2 (L^2 - a^2/4) - m_l^2 g^2 \\ p_3 &= -8 K^2 L \\ p_4 &= 4 K^2 \end{aligned} \tag{3.25}$$

Proof of Lemma 1. See Appendix A. $\square$

Consider the formation geometry described in Lemma 3.1.1. Let $\mathbf{z}_i = [\boldsymbol{\delta}_i^T, \boldsymbol{\epsilon}_i^T]^T \in \mathbb{R}^6$ be the i -th extended error vector and define $\mathbf{x} = [\boldsymbol{\rho}^T, \boldsymbol{\sigma}^T]^T \in \mathbb{R}^6$. Provided $\mathbf{z} = [\mathbf{z}_1^T, \mathbf{z}_2^T]^T \in \mathbb{R}^{12}$, the dynamics of center of gravity tracking error $\mathbf{x}$ and formation error $\mathbf{z}$ are respectively expressed as

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, \mathbf{z}) + \mathbf{L}(t) \tag{3.26}$$

and

$$\dot{\mathbf{z}} = \mathbf{G}(\mathbf{x}, \mathbf{z}) + \mathbf{M}(t) \tag{3.27}$$

where $\mathbf{L}(t) = [\mathbf{0}_{1 \times 3} \ \dot{\mathbf{v}}_r^T]^T \in \mathbb{R}^{6 \times 1}$, $\mathbf{M}(t) = [\mathbf{L}^T \ \mathbf{L}^T]^T \in \mathbb{R}^{12 \times 1}$,

$$\mathbf{F}(\mathbf{x}, \mathbf{z}) = \begin{bmatrix} \boldsymbol{\sigma} \\ -\mathbf{g} - \frac{1}{M} (\mathbf{f}_{c1} + \mathbf{f}_{c2}) \end{bmatrix} \tag{3.28}$$

and

$$\mathbf{G}(\mathbf{x}, \mathbf{z}) = \begin{bmatrix} \boldsymbol{\epsilon}_1 \\ -\mathbf{g} - \frac{1}{m} \left[K \left(1 - \frac{L}{\|\mathbf{c}_1\|} \right) \mathbf{c}_1 + \mathbf{f}_{c1} \right] \\ \boldsymbol{\epsilon}_2 \\ -\mathbf{g} - \frac{1}{m} \left[K \left(1 - \frac{L}{\|\mathbf{c}_2\|} \right) \mathbf{c}_2 + \mathbf{f}_{c2} \right] \end{bmatrix} \tag{3.29}$$

are determined from Equations (3.13)–(3.21) under Assumption 2.

Let $\mathbf{K}_\delta = \text{diag}(k_{\delta_x}, k_{\delta_y}, k_{\delta_z})$, $\mathbf{K}_\epsilon = \text{diag}(k_{\epsilon_x}, k_{\epsilon_y}, k_{\epsilon_z})$, $\mathbf{K}_\rho = \text{diag}(k_{\rho_x}, k_{\rho_y}, k_{\rho_z})$, and $\mathbf{K}_\sigma = \text{diag}(k_{\sigma_x}, k_{\sigma_y}, k_{\sigma_z})$ be positive gain matrices. Define

$\mathbf{K}_x = [\mathbf{K}_\rho \ \mathbf{K}_\sigma] \in \mathbb{R}^{3 \times 6}$ and $\mathbf{K}_z = [\mathbf{K}_\delta \ \mathbf{K}_\epsilon] \in \mathbb{R}^{3 \times 6}$. The following controller is proposed for $i = 1, 2$:

$$\mathbf{f}_{ci} = \bar{\mathbf{f}}_{ci} - \lambda_i \boldsymbol{\nu}_i - \mu_i \boldsymbol{\Sigma} \bar{\mathbf{d}}_i + m \mathbf{K}_z (\mathbf{z}_i - \mathbf{R} \boldsymbol{\alpha}) \quad (3.30)$$

where $\bar{\mathbf{f}}_{ci}$ is the trim contribution derived according to Lemma 3.1.1, $\boldsymbol{\Sigma} = \text{diag}(m_l, m_l, M)$, and $\mathbf{R} = [\mathbf{I}_3 // \mathbf{0}_{3 \times 3}] \in \mathbb{R}^{6 \times 3}$ is a selection matrix obtained by vertical concatenation of $\mathbf{I}_3$ and $\mathbf{0}_{3 \times 3}$. Functions $\lambda_1, \lambda_2, \mu_1, \mu_2 \in \mathbb{R}$ and $\boldsymbol{\nu}_1, \boldsymbol{\nu}_2 \in \mathbb{R}^3$ are defined as

$$\lambda_1 = K L \left(\frac{1}{m_l L} - \frac{1}{\|\boldsymbol{\nu}_1 + \boldsymbol{\Sigma} \bar{\mathbf{d}}_1\|} \right) \quad (3.31)$$

$$\lambda_2 = K L \left(\frac{1}{m_l L} - \frac{1}{\|\boldsymbol{\nu}_2 - \boldsymbol{\Sigma} \bar{\mathbf{d}}_2\|} \right) \quad (3.32)$$

$$\mu_1 = K \left(\frac{1}{\|\boldsymbol{\Sigma} \bar{\mathbf{d}}_1\|} - \frac{1}{\|\boldsymbol{\nu}_1 + \boldsymbol{\Sigma} \bar{\mathbf{d}}_1\|} \right) \quad (3.33)$$

$$\mu_2 = K \left(\frac{1}{\|\boldsymbol{\Sigma} \bar{\mathbf{d}}_2\|} - \frac{1}{\|\boldsymbol{\nu}_2 - \boldsymbol{\Sigma} \bar{\mathbf{d}}_2\|} \right) \quad (3.34)$$

$$\boldsymbol{\nu}_1 = M \boldsymbol{\rho} - (m + m_l) \boldsymbol{\delta}_1 - m \boldsymbol{\delta}_2 \quad (3.35)$$

$$\boldsymbol{\nu}_2 = -M \boldsymbol{\rho} + (m + m_l) \boldsymbol{\delta}_2 + m \boldsymbol{\delta}_1 \quad (3.36)$$

while $\boldsymbol{\alpha} \in \mathbb{R}^3$ is obtained as a solution of the the implicit equation

$$\boldsymbol{\alpha} = -\frac{m_l}{M} \mathbf{g} + \mathbf{K}_x \mathbf{x} + K L \left(\frac{\boldsymbol{\beta}_1}{\|\boldsymbol{\beta}_1\|} + \frac{\boldsymbol{\beta}_2}{\|\boldsymbol{\beta}_2\|} - \frac{\boldsymbol{\beta}_1 + \boldsymbol{\beta}_2}{m_l L} \right) \quad (3.37)$$

where $\boldsymbol{\beta}_1 = \boldsymbol{\alpha} + \boldsymbol{\rho} + \boldsymbol{\Sigma} \bar{\mathbf{d}}_1$ and $\boldsymbol{\beta}_2 = \boldsymbol{\alpha} + \boldsymbol{\rho} + \boldsymbol{\Sigma} \bar{\mathbf{d}}_2$. The structure of this controller can be interpreted physically. The term $\bar{\mathbf{f}}_{ci}$ establishes the static equilibrium force for the desired formation geometry. The terms involving λ_i and μ_i act as a nonlinear cancellation, effectively neutralizing the complex, state-dependent elastic forces from the cables. The term $m \mathbf{K}_z (\mathbf{z}_i - \mathbf{R} \boldsymbol{\alpha})$ represents the core feedback action: a fast-acting PD-like control on the formation error $\mathbf{z}_i$, biased by a slow-acting guidance acceleration $\boldsymbol{\alpha}$. This $\boldsymbol{\alpha}$ term, computed implicitly, smoothly translates the high-level trajectory tracking objective from the center of mass into a coherent command for the fast formation-keeping loop.

3.1.3 Stability and Control Analysis

Asymptotic stability for the origin $[\mathbf{x}^T \mathbf{z}^T]^T = \mathbf{0}_{18 \times 1}$ of the closed-loop system in Equations (3.26)–(3.29) is addressed by the following theorem.

Theorem 3.1.2. *Consider the system in Equations (3.26)–(3.29) with the control law in Equations (3.30)–(3.37). Provided $\dot{\mathbf{v}}_r \in L_2[0, \infty) \cap L_\infty[0, \infty)$, there exist $\mathbf{K}_x$ and $\mathbf{K}_z$ such that $\|\mathbf{x}\| \rightarrow 0$ and $\|\mathbf{z}\| \rightarrow 0$ as $t \rightarrow +\infty$.*

Proof of Theorem 1. See Appendix B. $\square$

Guidance, navigation, and control tasks are performed, without loss of generality, through a cascaded controller based on PX4 architecture and notation [59]. The control solution stabilizes the i -th multirotor according to different flight modes such as the tracking of an Earth-fixed set-point position $\mathbf{p}_{sp_i}$, velocity $\mathbf{v}_{sp_i}$, or acceleration $\mathbf{a}_{sp_i}$ (see Figure 3.2, where $\mathbf{a}_{sp_i}^{(VC)}$ is the acceleration set-point generated by Velocity Controller). In this framework, desired acceleration for the i -th multirotor is obtained as $\mathbf{a}_{sp_i} = \mathbf{a}_{sp_i}^{(CTC)} = \mathbf{f}_{ci}/m$, where the control input $\mathbf{f}_{ci}$ is calculated according to the Cooperative Transportation Controller (CTC) in Equations (3.30)–(3.37).

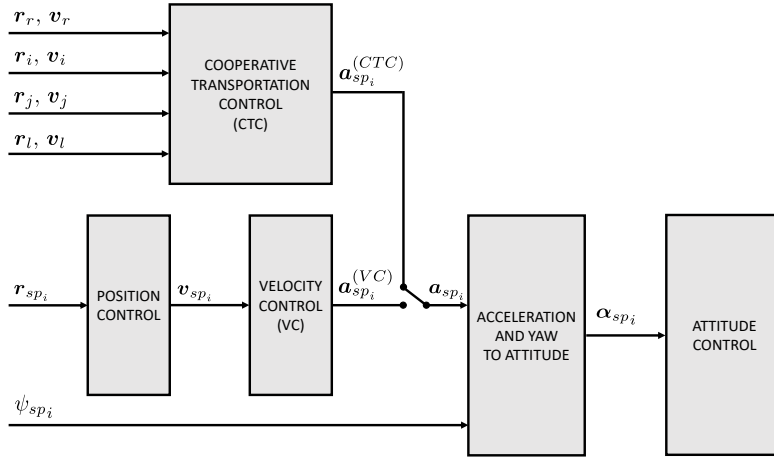


Figure 3.2: The i -th multirotor control scheme based on PX4 autopilot .

3.2 Baseline Formation Control

This section develops a baseline cooperative controller for two aerial vehicles transporting a suspended payload using a Lagrangian formulation. The three objects are modeled as point masses linked by massless, inextensible cables of equal length. The objective is twofold: stabilize a desired planar formation while optionally damping payload swing. To keep the design modular, the per-vehicle input is split into a Formation Controller (FC) that regulates inter-vehicle geometry and trajectory-tracking, and a Payload Controller (PC) that acts along the swing direction to dissipate oscillations. The subsequent sections introduce the modeling choices, the parameterization used to describe payload motion, the linearized dynamics around hover, and a simple yet effective control law with tuning guidance and implementation notes for PX4-like stacks [59].

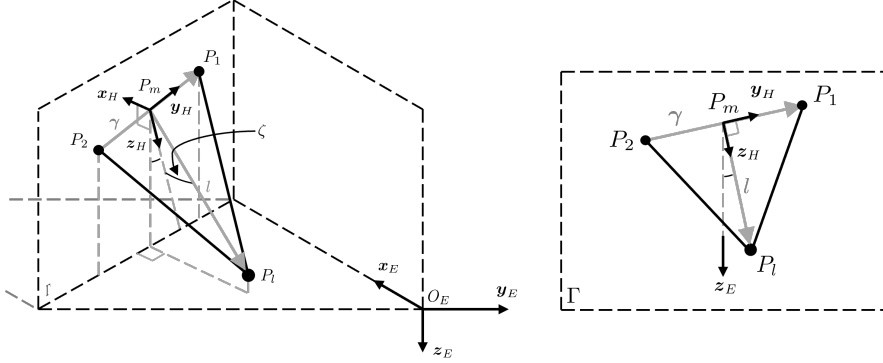
3.2.1 Equations of Motion

This subsection specifies the kinematics and forces for the vehicles and payload and introduces an ad-hoc $\mathcal{F}_H$ frame attached to the formation. The reference system $\mathcal{F}_H$ isolates the plane that contains the vehicles and parametrizes payload motion by a single swing angle ζ . This parameterization leads to compact expressions for load's position and velocity in the inertial frame that depend only on few formation geometry parameters and swing variables. Let $\mathbf{r}_i = [r_{xi}, r_{yi}, r_{zi}]^T$ be the position of the i -th agent ($i = 1, 2$) and $\mathbf{v}_i = [v_{xi}, v_{yi}, v_{zi}]^T$ its velocity with respect to the inertial frame. A suspended load with mass m_l is connected at P_l by a cable to the i -th agent. The vehicles and the suspension cables are assumed to be identical. The dynamics of each agent, as expressed in $\mathcal{F}_E$, is described by the following equations:

$$\dot{\mathbf{r}}_i = \mathbf{v}_i \quad (3.38)$$

$$\dot{\mathbf{v}}_i = \frac{1}{m} (m \mathbf{g} + \mathbf{f}_{li} + \mathbf{f}_{di} + \mathbf{f}_{ci}) \quad (3.39)$$

where m is the mass of the multirotor, $\mathbf{g} = [0, 0, g]^T$ is the gravitational acceleration vector, $\mathbf{f}_{li}$ is the force exerted on the i -th agent through the


 Figure 3.3: Definition of oscillation angle ζ .

cable, and $\mathbf{f}_{di}$ is the vehicle aerodynamic disturbance. The net thrust $\mathbf{f}_{ci}$, directed along $\mathbf{z}_{Bi}$ and pointing upwards, represents the only control input to Equation (3.39). It is assumed that the i -th cable is directly connected to the center of gravity of the i -th agent such that $\mathbf{f}_{li}$ has no effect on attitude motion. Each cable is modeled as massless and rigid with a constant nominal length L .

With respect to the payload position $\mathbf{r}_l = [r_{x_l}, r_{y_l}, r_{z_l}]^T$, an ad hoc parameterization is proposed. Let P_m be the midpoint between P_1 and P_2 , with the position vector obtained by the average of $\mathbf{r}_1$ and $\mathbf{r}_2$, namely $\mathbf{r}_m = (\mathbf{r}_1 + \mathbf{r}_2)/2$ (see Figure 3.3). Define $\boldsymbol{\gamma} = \mathbf{r}_1 - \mathbf{r}_2$ and $\boldsymbol{\beta} = (\boldsymbol{\gamma}/\gamma) \times \mathbf{z}_E$, where $\gamma = \|\boldsymbol{\gamma}\|$. Consider the reference frame $\mathcal{F}_H = \{P_m; \mathbf{x}_H, \mathbf{y}_H, \mathbf{z}_H\}$, characterized by unit vectors $\mathbf{x}_H = \boldsymbol{\beta}/\beta$, $\mathbf{y}_H = \boldsymbol{\gamma}/\gamma$, and $\mathbf{z}_H = \mathbf{x}_H \times \mathbf{y}_H = (\boldsymbol{\beta}/\beta) \times (\boldsymbol{\gamma}/\gamma)$, with $\beta = \|\boldsymbol{\beta}\|$. Note that $\mathbf{y}_H$ and $\mathbf{z}_H$ lie on the local vertical plane Γ that contains the vehicles, while $\mathbf{x}_H$ identifies the plane's normal direction, as shown in Figure 3.3. The rotation matrix $\mathbf{T}_{EH} = [\mathbf{x}_H \ \mathbf{y}_H \ \mathbf{z}_H]$ is defined by unit vector components expressed in $\mathcal{F}_E$, namely

$$\mathbf{x}_H = [r_{y_1} - r_{y_2}, r_{x_2} - r_{x_1}, 0]^T / \beta \quad (3.40)$$

$$\mathbf{y}_H = [r_{x_1} - r_{x_2}, r_{y_1} - r_{y_2}, r_{z_1} - r_{z_2}]^T / \gamma \quad (3.41)$$

$$\mathbf{z}_H = \left[-(r_{x_1} - r_{x_2})(r_{z_1} - r_{z_2}), -(r_{y_1} - r_{y_2})(r_{z_1} - r_{z_2}), \right. \\ \left. (r_{x_1} - r_{x_2})^2 + (r_{y_1} - r_{y_2})^2 \right]^T / (\beta \gamma) \quad (3.42)$$

where

$$\beta = \sqrt{(r_{x_1} - r_{x_2})^2 + (r_{y_1} - r_{y_2})^2} \quad (3.43)$$

$$\gamma = \sqrt{(r_{x_1} - r_{x_2})^2 + (r_{y_1} - r_{y_2})^2 + (r_{z_1} - r_{z_2})^2} \quad (3.44)$$

Let $\hat{\mathbf{l}} = \mathbf{l} / \|\mathbf{l}\|$ be the unit vector directed from P_m to P_l , where $\mathbf{l} = \mathbf{r}_l - \mathbf{r}_m$ and $\|\mathbf{r}_l - \mathbf{r}_m\| = \sqrt{L^2 - \gamma^2/4}$. The payload position is defined by the angle $\zeta \in]-\pi/2, \pi/2[$, which characterizes rotations about $\mathbf{y}_H$. Given $\hat{\mathbf{l}} = \mathbf{T}_{EH} \hat{\mathbf{l}}_H$, being $\hat{\mathbf{l}}_H = [\sin \zeta, 0, \cos \zeta]^T$, the payload position is calculated as a function of ζ , $\mathbf{r}_1$, and $\mathbf{r}_2$. It follows that

$$\begin{aligned} \mathbf{r}_l &= \mathbf{r}_m + \|\mathbf{l}\| \mathbf{T}_{EH} [\sin \zeta, 0, \cos \zeta]^T \\ &= (\mathbf{r}_1 + \mathbf{r}_2)/2 + (\sqrt{L^2 - \gamma^2/4}) \mathbf{T}_{EH} [\sin \zeta, 0, \cos \zeta]^T \end{aligned} \quad (3.45)$$

Let $\boldsymbol{\lambda} = [\mathbf{r}_1^T, \mathbf{r}_2^T, \zeta]^T \in \mathbb{R}^7$ be a vector of generalized coordinates. The Lagrangian equation with respect to $\boldsymbol{\lambda}$ is

$$\frac{d}{dt} \left(\frac{\partial E}{\partial \dot{\boldsymbol{\lambda}}} \right) - \frac{\partial E}{\partial \boldsymbol{\lambda}} = \boldsymbol{\tau} \quad (3.46)$$

where $E = K - U$ is the Lagrangian function and $\boldsymbol{\tau} \in \mathbb{R}^7$ is the vector of generalized forces. In the present framework, K is the system kinetic energy given by

$$K = \frac{1}{2} m (\dot{\mathbf{r}}_1^T \dot{\mathbf{r}}_1 + \dot{\mathbf{r}}_2^T \dot{\mathbf{r}}_2) + \frac{1}{2} m_l \dot{\mathbf{r}}_l^T \dot{\mathbf{r}}_l \quad (3.47)$$

where $\dot{\mathbf{r}}_l$ denotes the load velocity obtained from Equation (3.45), namely

$$\begin{aligned} \dot{\mathbf{r}}_l &= \frac{\dot{\mathbf{r}}_1 + \dot{\mathbf{r}}_2}{2} - \frac{\gamma \dot{\gamma}}{2 \sqrt{4L^2 - \gamma^2}} \mathbf{T}_{EH} [\sin \zeta, 0, \cos \zeta]^T + \frac{\sqrt{L^2 - \gamma^2}}{4} \\ &\quad \left\{ \dot{\mathbf{T}}_{EH} [\sin \zeta, 0, \cos \zeta]^T + \mathbf{T}_{EH} [\cos \zeta, 0, -\sin \zeta]^T \dot{\zeta} \right\} \end{aligned} \quad (3.48)$$

With respect to the time derivatives $\dot{\mathbf{T}}_{EH}$ and $\dot{\gamma}$ in Equation (3.48), one has

$$\dot{\mathbf{T}}_{EH} = [\dot{\mathbf{x}}_H \quad \dot{\mathbf{y}}_H \quad \dot{\mathbf{z}}_H] \quad (3.49)$$

while

$$\dot{\mathbf{x}}_H = (\dot{\boldsymbol{\beta}} \boldsymbol{\beta} - \boldsymbol{\beta} \dot{\boldsymbol{\beta}}) / \beta^2 \quad (3.50)$$

$$\dot{\mathbf{y}}_H = (\dot{\gamma} \gamma - \gamma \dot{\gamma}) / \gamma^2 \quad (3.51)$$

$$\dot{\mathbf{z}}_H = [(\dot{\boldsymbol{\beta}} \boldsymbol{\beta} - \boldsymbol{\beta} \dot{\boldsymbol{\beta}}) / \beta^2] \times (\gamma / \gamma) + [(\dot{\gamma} \gamma - \gamma \dot{\gamma}) / \gamma^2] \times (\boldsymbol{\beta} / \beta) \quad (3.52)$$

with

$$\dot{\boldsymbol{\beta}} = [\dot{r}_{y_1} - \dot{r}_{y_2}, \dot{r}_{x_2} - \dot{r}_{x_1}, 0]^T \quad (3.53)$$

$$\dot{\beta} = [(\dot{r}_{x_1} - \dot{r}_{x_2})(r_{x_1} - r_{x_2}) + (\dot{r}_{y_1} - \dot{r}_{y_2})(r_{y_1} - r_{y_2})]/\beta \quad (3.54)$$

$$\dot{\boldsymbol{\gamma}} = [\dot{r}_{x_1} - \dot{r}_{x_2}, \dot{r}_{y_1} - \dot{r}_{y_2}, \dot{r}_{z_1} - \dot{r}_{z_2}]^T \quad (3.55)$$

$$\begin{aligned} \dot{\gamma} = & [(\dot{r}_{x_1} - \dot{r}_{x_2})(r_{x_1} - r_{x_2}) + (\dot{r}_{y_1} - \dot{r}_{y_2})(r_{y_1} - r_{y_2}) \\ & + (\dot{r}_{z_1} - \dot{r}_{z_2})(r_{z_1} - r_{z_2})]/\gamma \end{aligned} \quad (3.56)$$

The potential energy U is related to the vertical displacement of the agents and the suspended load and is given by

$$U = -m g (r_{z_1} + r_{z_2}) - m_l g r_{z_l} \quad (3.57)$$

Given the energy contributions in Equations (3.47) and (3.57), and taking into account the state of the load as described by Equations (3.45) and (3.48), the Lagrangian equation is manipulated into the system:

$$\dot{\boldsymbol{\lambda}} = \boldsymbol{\mu} \quad (3.58)$$

$$\mathbf{M}(\boldsymbol{\lambda}) \dot{\boldsymbol{\mu}} + \mathbf{C}(\boldsymbol{\lambda}, \boldsymbol{\mu}) \boldsymbol{\mu} + \mathbf{G}(\boldsymbol{\lambda}) = \boldsymbol{\tau} \quad (3.59)$$

where $\boldsymbol{\mu} = [\mathbf{v}_1^T, \mathbf{v}_2^T, \dot{\zeta}]^T \in \mathbb{R}^7$. For brevity, the highly nonlinear and lengthy expressions of terms $\mathbf{M}(\boldsymbol{\lambda})$, $\mathbf{C}(\boldsymbol{\lambda}, \boldsymbol{\mu})$, and $\mathbf{G}(\boldsymbol{\lambda})$ are not detailed in the present thesis, but can be provided to the reader upon request. The extended force vector $\boldsymbol{\tau} = \boldsymbol{\tau}_c + \boldsymbol{\tau}_d$ consists of a control contribution, $\boldsymbol{\tau}_c = [\mathbf{f}_{c1}^T, \mathbf{f}_{c2}^T, 0]^T$, and a disturbance term, $\boldsymbol{\tau}_d = [\mathbf{f}_{d1}^T, \mathbf{f}_{d2}^T, 0]^T - \mathbf{D} \boldsymbol{\mu}$. This last term accounts for the effects of payload drag at low oscillation speeds, where $\mathbf{D} = \text{diag}(0, 0, 0, 0, 0, 0, d) \in \mathbb{R}^{7 \times 7}$ and $d > 0$ is a drag coefficient.

Remark 3. *It should be noted that the Lagrangian approach developed in this section implicitly assumes a rigid distance constraint, effectively modeling the cables as inextensible rods. This formulation does not naturally account for slack-cable conditions (unilateral constraints). To ensure the physical validity of the results, Equations (3.38) and (3.39) serve as a post-processing tool to reconstruct the internal cable tension $\mathbf{f}_{li}$. This reconstruction is a necessary safety check to guarantee that, as long as the calculated tension remains strictly*

positive, the rigid-link model remains a valid representation of the slung-load system. In the numerical simulations presented in Chapter 5.2, this condition was monitored to verify that no nonphysical compression occurred during the proposed maneuvers.

3.2.2 System Linearization

The highly nonlinear model derived in Equations (3.58) and (3.59) is linearized about a hovering condition with the payload at rest, allowing for stability, observability, and controllability investigations within an analytical framework. Let $\bar{\gamma} = [\bar{\gamma}_x, \bar{\gamma}_y, 0]^T$ be the prescribed mutual distance vector between the two agents of the formation, with components expressed in $\mathcal{F}_E$, selected such that $\bar{\gamma} = \sqrt{\bar{\gamma}_x^2 + \bar{\gamma}_y^2} < 2L$. The system in Equation (3.59) is linearized about the equilibrium $(\bar{\mathbf{y}}, \bar{\mathbf{u}})$, namely

$$\bar{\mathbf{y}} = [\bar{\boldsymbol{\lambda}}^T, \bar{\boldsymbol{\mu}}^T]^T = [\mathbf{r}_1^T, (\mathbf{r}_1 - \bar{\boldsymbol{\gamma}})^T, \mathbf{0}_{1 \times 8}]^T \in \mathbb{R}^{14} \quad (3.60)$$

which is representative of a hovering flight where the three point-masses lie on the local vertical plane Γ (see Figure 3.3), provided the aerodynamic disturbances, $\bar{\mathbf{f}}_{d_1}$ and $\bar{\mathbf{f}}_{d_2}$, are constant. The term $\bar{\mathbf{u}} = [\bar{\mathbf{u}}_1^T, \bar{\mathbf{u}}_2^T]^T \in \mathbb{R}^6$ represents the proper equilibrium force, where $\bar{\mathbf{u}}_1 = \bar{\mathbf{f}}_{c_1} + \bar{\mathbf{f}}_{d_1}$ and $\bar{\mathbf{u}}_2 = \bar{\mathbf{f}}_{c_2} + \bar{\mathbf{f}}_{d_2}$. In this condition $\bar{\zeta} = 0$, $\dot{\bar{\zeta}} = 0$, and UAV 2 is required to maintain the prescribed position $\bar{\mathbf{r}}_2 = \mathbf{r}_1 - \bar{\boldsymbol{\gamma}}$. The generalized equilibrium force is obtained as $\bar{\boldsymbol{\tau}} = \bar{\boldsymbol{\tau}}_c + \bar{\boldsymbol{\tau}}_d = \mathbf{G}(\bar{\boldsymbol{\lambda}})$. Let $M = 2m + m_l$ be the total system mass and define $\xi_i = 2(1 - i) + 1$. Taking into account the first six components in $\bar{\boldsymbol{\tau}}$, the proper equilibrium force $\bar{\mathbf{u}}_i$ is obtained as

$$\bar{\mathbf{u}}_i = \left[\frac{g m_l \xi_i \bar{\gamma}_x}{2\sqrt{4L^2 - \bar{\gamma}^2}}, \frac{g m_l \xi_i \bar{\gamma}_y}{2\sqrt{4L^2 - \bar{\gamma}^2}}, -\frac{g M}{2} \right]^T \quad (3.61)$$

while the control force generated by the i -th agent is

$$\bar{\mathbf{f}}_{c_i} = [\bar{f}_{x_{c_i}}, \bar{f}_{y_{c_i}}, \bar{f}_{z_{c_i}}]^T = \bar{\mathbf{u}}_i - \bar{\mathbf{f}}_{d_i} \quad (3.62)$$

Theorem 3.2.1. *Consider the slung-load system described by Equation (3.59). The equilibrium point $(\bar{\mathbf{y}}, \bar{\mathbf{u}})$, characterized by Equation (3.61) and obtained*

by the control force in Equation (3.62), satisfies the second-order equation:

$$a_i m_l^2 + b_i m_l + c_i = 0 \quad (3.63)$$

where

$$a_i = \frac{g^2}{4} \left(\frac{\bar{\gamma}^2}{4L^2 - \bar{\gamma}^2} - \tan^2 \bar{\alpha}_i \right) \quad (3.64)$$

$$b_i = -g \left[\xi_i \frac{\bar{f}_{x_{d_i}} \bar{\gamma}_x + \bar{f}_{y_{d_i}} \bar{\gamma}_y}{\sqrt{4L^2 - \bar{\gamma}^2}} + (\bar{f}_{z_{d_i}} + m g) \tan^2 \bar{\alpha}_i \right] \quad (3.65)$$

$$c_i = \bar{f}_{x_{d_i}}^2 + \bar{f}_{y_{d_i}}^2 - (\bar{f}_{z_{d_i}} + m g)^2 \tan^2 \bar{\alpha}_i \quad (3.66)$$

for $i = 1, 2$, provided $\bar{\alpha}_i > 0$ is the angle between $-\mathbf{z}_E$ and the i -th equilibrium control force $\bar{\mathbf{f}}_{c_i}$. In the case when the agents of the formation are multirotors, it follows $\bar{\alpha}_i = \arccos(\cos \bar{\phi}_i \cos \bar{\theta}_i)$, provided $\bar{\boldsymbol{\eta}}_i = [\bar{\phi}_i, \bar{\theta}_i, \bar{\psi}_i]^T$ is the equilibrium attitude of the i -th agent.

Proof. See Appendix C. □

Remark 4. Assume the disturbance force components $\bar{f}_{x_{d_i}}$, $\bar{f}_{y_{d_i}}$, and $\bar{f}_{z_{d_i}}$ can be estimated and the attitude variables $\bar{\phi}_i$ and $\bar{\theta}_i$ are measured in a hovering condition. Equation (3.63) can be solved to estimate m_l in all those cases where payload mass is uncertain or not known a priori. In this respect, an initial hovering maneuver can be planned where the formation is stabilized in the desired equilibrium condition and the estimated payload mass $\tilde{m}_l$ is computed as a steady-state solution to Equation (3.63). For values of practical application, two real-distinct solutions are obtained, between which the negative one is excluded. The analysis can be performed for either vehicle, nominally leading to the same solution even though different disturbances and attitude variables characterize the multirotors. As an alternative, without loss of generality, UAV 1 can be assumed as the formation leader where all necessary information converge to allow all estimation procedures. With respect to the estimation of disturbance forces, different methods are available in the literature, where disturbance observers (DOBs) are typically designed to support the control action in rejecting both external disturbances and effects of model uncertainties [84].

Remark 5. To prevent singularities in the proposed mathematical formulation, a constraint is imposed on the design of the prescribed mutual distance vector between the two agents, namely $\bar{\gamma} = \sqrt{\bar{\gamma}_x^2 + \bar{\gamma}_y^2} < 2L$. Assuming UAV 1 is stabilized at $\mathbf{r}_1$ and UAV 2 lies at $\mathbf{r}_1 - \bar{\gamma}$, the determinant of $\mathbf{M}(\boldsymbol{\lambda})$ becomes:

$$\det(\mathbf{M}) = \frac{m_l m^4}{\bar{\gamma}^2} \left[m_l (2m_l + m) L^4 + m^2 \bar{\gamma}^2 \left(L^2 - \frac{\bar{\gamma}^2}{4} \right) \right] \quad (3.67)$$

Given m , m_l , and L , the configuration becomes ill-posed when $\det(\mathbf{M}) = 0$, which occurs for values of $\bar{\gamma}$ that satisfy the equation

$$m^2 \bar{\gamma}^4 - 4m^2 L^2 \bar{\gamma}^2 - 4m_l (2m_l + m) L^4 = 0, \quad (3.68)$$

that does not depend on the particular swing state of the payload. The 4-th order polynomial in Equation (3.68) has only one real positive root, given by $\bar{\gamma}^* = 2L \sqrt{1 + m_l/(2m)}$. Since $\bar{\gamma}^* > 2L$, the suggested constraint allows for a well-posed and safe configuration of the formation.

Let $\mathbf{y} = [\boldsymbol{\lambda}^T, \boldsymbol{\mu}^T]^T \in \mathbb{R}^{14}$ and $\mathbf{u} = [\mathbf{u}_1^T, \mathbf{u}_2^T]^T \in \mathbb{R}^6$ be the state and input vectors respectively describing the dynamics in Equations (3.58) and (3.59), linearized about the considered equilibrium condition. It is possible to obtain the standard state-space representation

$$\dot{\mathbf{y}} = \mathbf{A} \mathbf{y} + \mathbf{B} \mathbf{u} \quad (3.69)$$

where $\mathbf{A} \in \mathbb{R}^{14 \times 14}$ is the state matrix,

$$\mathbf{A} = \left[\begin{array}{cccc|c} & & & & \mathbf{I}_7 \\ \hline & \mathbf{0}_{7 \times 7} & & & \\ A_{8,1} & A_{8,2} & \dots & A_{8,7} & \\ A_{9,1} & A_{9,2} & \dots & A_{9,7} & \\ \vdots & \vdots & \ddots & \vdots & \\ A_{14,1} & A_{14,2} & \dots & A_{14,7} & \\ \hline & & & & \mathbf{0}_{7 \times 7} \end{array} \right], \quad (3.70)$$

and $\mathbf{B} \in \mathbb{R}^{14 \times 6}$ is the input matrix,

$$\mathbf{B} = \left[\begin{array}{cccc} \hline & \mathbf{0}_{7 \times 6} & & \\ B_{8,1} & B_{8,2} & \dots & B_{8,6} \\ B_{9,1} & B_{9,2} & \dots & B_{9,6} \\ \vdots & \vdots & \ddots & \vdots \\ B_{14,1} & B_{14,2} & \dots & B_{14,6} \end{array} \right] \quad (3.71)$$

In the present framework, $\mathbf{I}_p$ is the unit matrix with dimension p , while $\mathbf{0}_{q \times r}$ is the null matrix with dimension $q \times r$. The components of matrices in Equations (3.70) and (3.71) are detailed in Appendix D.

It is important to note that the linearization detailed in this section is performed around a static hover condition. The resulting controller is therefore expected to perform well for near-hover operations and slow maneuvers, but its performance may degrade during highly aggressive trajectories or very large amplitude payload oscillations where nonlinear effects become dominant. However, these cases are beyond the intended scope of the current investigation. The envisaged application context, in fact, comprises logistics and aerial delivery missions, where flight paths are typically conservative to reduce risks to the payload and the surrounding environment.

3.2.3 Control System Design

A controller is designed and empirically validated to simultaneously fulfill (i) formation-keeping, (ii) trajectory-tracking, and (iii) active payload stabilization. Control tasks are performed, without loss of generality, through a cascaded algorithm based on PX4 architecture and notation [59]. Such control solution is known to stabilize the i -th octarotor according to different flight modes that include the tracking of an Earth-fixed set-point position, velocity, or acceleration (see Figure 3.4).

Provided the relative position and trajectory error variables $\boldsymbol{\delta}_i$ and $\boldsymbol{\epsilon}_i$ defined in Equations (3.13) and (3.14), the following FC is proposed

$$\mathbf{u}_i^{(FC)} = \bar{\mathbf{u}}_i + m (\mathbf{K}_\delta \boldsymbol{\delta}_i + \mathbf{K}_\epsilon \boldsymbol{\epsilon}_i) \quad (3.72)$$

where $\bar{\mathbf{u}}_i$, $i = 1, 2$, is a trim contribution, while $\mathbf{K}_\delta = \text{diag}(K_{\delta_x}, K_{\delta_y}, K_{\delta_z})$ and $\mathbf{K}_\epsilon = \text{diag}(K_{\epsilon_x}, K_{\epsilon_y}, K_{\epsilon_z})$ are positive gain matrices. Without loss of generality, it is assumed that when the i -th agent reaches the desired position D_i (namely $\boldsymbol{\delta}_i = \mathbf{0}_{3 \times 1}$, $\boldsymbol{\epsilon}_i = \mathbf{0}_{3 \times 1}$, $i = 1, 2$), the formation midpoint P_m exactly tracks P_r . The trim contribution $\bar{\mathbf{u}}_i$, $i = 1, 2$, derived in Equation (3.61), allows the system described in Equations (3.58) and (3.59) to maintain the equilibrium $\bar{\mathbf{y}} = [\bar{\boldsymbol{\lambda}}^T, \bar{\boldsymbol{\mu}}^T]^T = [\mathbf{r}_1^T, (\mathbf{r}_1 - \bar{\boldsymbol{\gamma}})^T, \mathbf{0}_{8 \times 1}]^T \in \mathbb{R}^{14}$, which represents

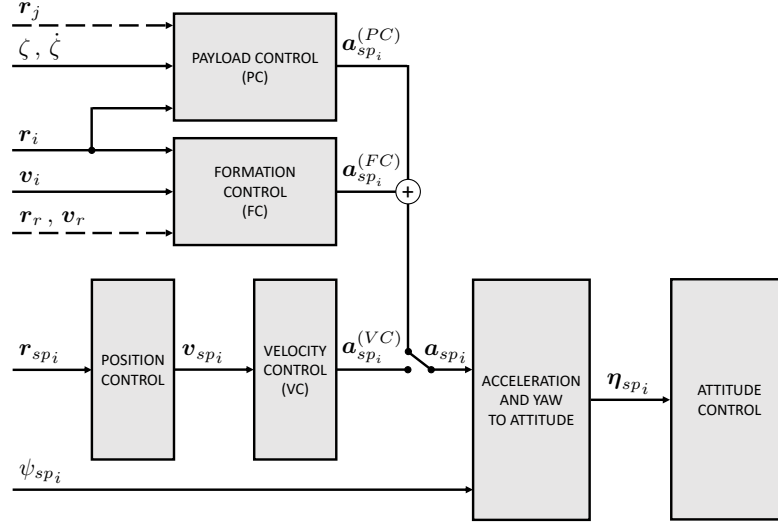


Figure 3.4: The i -th multirotor control scheme based on PX4 autopilot architecture.

a hovering flight where the three point masses lie on the local vertical plane Γ (see Figure 3.3). The proposed PC is structured as

$$\mathbf{u}_1^{(PC)} = \mathbf{u}_2^{(PC)} = m (k_p \zeta + k_d \dot{\zeta}) \mathbf{x}_H, \quad (3.73)$$

and is directed over the local horizontal plane in the direction of $\mathbf{x}_H$, where k_p and k_d are positive gains. The proportional contribution in Equation (3.73) allows the formation to track the payload position over the local horizontal plane along the preferred direction $\mathbf{x}_H$. At the same time, the derivative contribution damps the motion of the relative position vector $\mathbf{l}$ with respect to $\mathcal{F}_H$. This approach follows strategies used for stabilizing inverted pendulums on moving carts, where various control strategies are employed, ranging from standard PID [85] to MPC algorithms [86]. It should be noted that the payload control action $\mathbf{u}_i^{(PC)}$ is directed along $\mathbf{x}_H$, meaning it specifically targets oscillations occurring out of the local vertical plane Γ . In maneuvers where the formation moves in-line (namely, the direction of motion is aligned with $\mathbf{y}_H$), the resulting swing occurs within Γ . In such cases, damping is provided solely by the formation controller $\mathbf{u}_i^{(FC)}$, which stabilizes the relative positions of the vehicles.

3.2.4 Stability and Control Analysis

In this subsection the stability of the coupled slung-load system described in Equations (3.58) and (3.59), with the control law $\mathbf{u}_i = \mathbf{u}_i^{(FC)} + \mathbf{u}_i^{(PC)}$ detailed in Equations (3.72) and (3.73), is addressed within a linear framework. The stability of the linearized system with the control action in Equations (3.72) and (3.73) is first addressed in the case when $\mathbf{v}_r = [0, 0, 0]^T$ such that $\dot{\mathbf{v}}_r = [0, 0, 0]^T$ and $\mathbf{r}_r$ is a constant. Let $\mathbf{e} = \mathbf{y}_d - \mathbf{y}$ be the error with respect to the desired state $\mathbf{y}_d = [\bar{\mathbf{r}}_1^T, \bar{\mathbf{r}}_2^T, \mathbf{0}_{1 \times 8}]^T$. The error dynamics are obtained from Equation (3.69) as

$$\dot{\mathbf{e}} = \mathbf{A} \mathbf{e} - \mathbf{B} \mathbf{u} \quad (3.74)$$

Based on a simple manipulation of Equations (3.72) and (3.73) in the linear framework, we have $\mathbf{u} = \mathbf{K} \mathbf{e}$, where $\mathbf{K} \in \mathbb{R}^{6 \times 14}$. The feedback-controlled system evolves according to $\dot{\mathbf{e}} = \hat{\mathbf{A}} \mathbf{e}$, where $\hat{\mathbf{A}} = \mathbf{A} - \mathbf{B} \mathbf{K}$ and

$$\mathbf{K} = m \begin{bmatrix} \mathbf{K}_\delta & \mathbf{0}_{3 \times 3} & k_p \bar{\gamma} & \mathbf{K}_\epsilon & \mathbf{0}_{3 \times 3} & k_d \bar{\gamma} \\ \mathbf{0}_{3 \times 3} & \mathbf{K}_\delta & k_p \bar{\gamma} & \mathbf{0}_{3 \times 3} & \mathbf{K}_\epsilon & k_d \bar{\gamma} \end{bmatrix} \quad (3.75)$$

To outline the dynamic modes in $\hat{\mathbf{A}}$, the error vector is conveniently rearranged according to the order

$$\mathbf{y}_{flip} = [r_{x1}, v_{x1}, r_{x2}, v_{x2}, \zeta, \dot{\zeta}, r_{y1}, v_{y1}, r_{y2}, v_{y2}, r_{z1}, v_{z1}, r_{z2}, v_{z2}]^T \quad (3.76)$$

Without loss of generality, consider the situation where $\bar{\gamma}_x = 0$ and $\bar{\gamma}_y = \bar{\gamma}$ for a prescribed $\bar{\gamma}$, which allows the dynamic analysis to be performed about the $\mathcal{F}_H$ -frame axes. Based on the results in Appendix D, it is straightforward to verify that the closed-loop dynamic matrix assumes the block-diagonal form:

$$\hat{\mathbf{A}}_{flip} = \left[\begin{array}{c|c} \hat{\mathbf{A}}_{fs} & \mathbf{0}_{6 \times 8} \\ \hline \mathbf{0}_{8 \times 6} & \hat{\mathbf{A}}_{lv} \end{array} \right] \quad (3.77)$$

where

$$\hat{\mathbf{A}}_{fs}(\nu_{fs1}, \nu_{fs2}, \nu_{fs3}, \nu_{fs4}, \nu_{fs5}, \nu_{fs6}) \quad (3.78)$$

with eigenvalues ν_{fsj} , $j = 1, \dots, 6$, is the subsystem matrix that describes the coupling between the vehicles' forward motion (along $\mathbf{x}_H$) and the payload swing state, while

$$\hat{\mathbf{A}}_{lv}(\nu_{lv1}, \nu_{lv2}, \nu_{lv3}, \nu_{lv4}, \nu_{lv5}, \nu_{lv6}, \nu_{lv7}, \nu_{lv8}) \quad (3.79)$$

with eigenvalues ν_{lvk} , $k = 1, \dots, 8$, is related to the lateral and vertical behavior of the formation (along $\mathbf{y}_H$ and $\mathbf{z}_H$, respectively). In the most general case, the closed-form expressions of the eigenvalues in Equations (3.78) and (3.79) cannot be determined. In this context, numerical methods can be employed for a dedicated analysis of the system's eigenstructure [87]. Also, there exist $\mathbf{K}$ with the structure in Equation (3.75) such that both Equations (3.78) and (3.79) are Hurwitz. Consider now the general case where $\mathbf{v}_r \neq \mathbf{0}_{3 \times 1}$ and $\dot{\mathbf{v}}_r \neq \mathbf{0}_{3 \times 1}$. Provided the origin of the feedback dynamic system $\dot{\mathbf{e}} = \hat{\mathbf{A}} \mathbf{e}$ is stable and $\mathbf{v}_r, \dot{\mathbf{v}}_r \in \mathcal{L}_2[0, \infty) \cap \mathcal{L}_\infty[0, \infty)$, one still obtains $\|\mathbf{e}\| \rightarrow 0$ as $t \rightarrow +\infty$ as a consequence of Lemma 1 in Ref. [88].

In the case when $\mathbf{K} = \mathbf{0}_{6 \times 14}$, formation open-loop dynamics are analyzed. In particular, $\hat{\mathbf{A}}_{fs}$ is characterized by null eigenvalues with multiplicity 2 and a set of purely complex poles given by

$$\nu_{fs1,2} = \pm j \sqrt{\frac{g m_l}{2 m \sqrt{4 L^2 - \bar{\gamma}^2}}} \quad (3.80)$$

and

$$\nu_{fs3,4} = \pm j \sqrt{\frac{g M}{m \sqrt{4 L^2 - \bar{\gamma}^2}}} \quad (3.81)$$

Similar considerations hold for $\hat{\mathbf{A}}_{lv}$, which is defined by four null eigenvalues and two pairs of complex poles:

$$\nu_{lv1,2} = \pm j \sqrt{\frac{g m_l M L^2}{m (2 M L^2 - m \bar{\gamma}^2) \sqrt{4 L^2 - \bar{\gamma}^2}}} \quad (3.82)$$

$$\nu_{lv3,4} = \pm j \sqrt{\frac{2 g m_l M L^2}{m (2 m_l L^2 + m \bar{\gamma}^2) \sqrt{4 L^2 - \bar{\gamma}^2}}} \quad (3.83)$$

Given the absence of dissipative effects related to the aerodynamic drag (which is disregarded), all dynamic modes related to Equations (3.80)–(3.83) are representative of a purely oscillating motion about the equilibrium with no damping. Since open-loop modes are undamped, aggressive trajectory commands or wind gusts can trigger long-lived oscillations if the payload controller is absent or under-tuned. The eigenvalue expressions provide a

useful sanity check: as $\bar{\gamma}$ approaches $2L$, the effective pendulum length shrinks and the swing frequency increases, telling that extreme spans must be avoided in practice (see Remark 5).

3.3 $\mathcal{L}_1$ Adaptive Augmentation

A $\mathcal{L}_1$ Piecewise-Constant Adaptive Law (which stems from the idea proposed by Hovakimyan et al. [89]) is designed on top of the baseline controller presented in Section 3.2. This augmented methodology allows to simultaneously (i) maintain formation geometry, (ii) track a desired trajectory, (iii) optionally damp payload swing, while (iv) preserving robustness against external disturbances and both matched and unmatched uncertainties. Exploiting the decoupled nature of the problem, first introduced in Equation (3.77), the control system is independently developed for the forward-swing and lateral-vertical channels. In what follows, without loss of generality, only the control architecture implemented for the former dynamics is presented in detail.

3.3.1 Control System Design

Consider the forward-swing system dynamics

$$\begin{aligned} \dot{\mathbf{x}}_{fs} &= \hat{\mathbf{A}}_{fs} \mathbf{x}_{fs} + \mathbf{B}_{m_{fs}} \left(\boldsymbol{\omega}_{fs} \mathbf{u}_{fs} + \mathbf{f}_{1_{fs}} \right) \\ &\quad + \mathbf{B}_{um_{fs}} \mathbf{f}_{2_{fs}}, \quad \mathbf{x}_{fs}(0) = \mathbf{x}_{fs_0} \\ \mathbf{y}_{fs} &= \mathbf{C}_{m_{fs}} \mathbf{x}_{fs} \end{aligned} \tag{3.84}$$

where $\mathbf{x}_{fs} = [r_{x_1}, v_{x_1}, r_{x_2}, v_{x_2}, \zeta, \dot{\zeta}]^T \in \mathbb{R}^6$ is the state vector, $\mathbf{u}_{fs} = [u_{x_1}, u_{x_2}]^T \in \mathbb{R}^2$ is the control input, $\mathbf{y}_{fs} = [r_{x_1}, r_{x_2}]^T \in \mathbb{R}^2$ is the regulated output, and $\boldsymbol{\omega}_{fs} \in \mathbb{R}^{2 \times 2}$ is the uncertain system input gain matrix. $\hat{\mathbf{A}}_{fs} \in \mathbb{R}^{6 \times 6}$ is Hurwitz and defines the desired closed-loop dynamics, $\mathbf{B}_{m_{fs}} \in \mathbb{R}^{6 \times 2}$ is the input matrix, $\mathbf{C}_{m_{fs}} \in \mathbb{R}^{2 \times 6}$ is a known constant matrix. Note that the pair $(\hat{\mathbf{A}}_{fs}, \mathbf{B}_{m_{fs}})$ must be completely controllable, while $(\hat{\mathbf{A}}_{fs}, \mathbf{C}_{m_{fs}})$ must be completely observable. $\mathbf{B}_{um_{fs}} \in \mathbb{R}^{6 \times 4}$ is a constant matrix such that $\mathbf{B}_{m_{fs}}^T \mathbf{B}_{um_{fs}} = \mathbf{0}$ and $\text{rank}([\mathbf{B}_{m_{fs}}, \mathbf{B}_{um_{fs}}]) = 6$ (full). Consider $\mathbf{B}_{fs} \triangleq [\mathbf{B}_{m_{fs}}, \mathbf{B}_{um_{fs}}]$ and let $\mathbf{f}_{1_{fs}}$ and $\mathbf{f}_{2_{fs}}$ be two unknown, bounded,

nonlinear functions satisfying

$$\begin{bmatrix} \mathbf{f}_{1_{fs}}(\cdot) \\ \mathbf{f}_{2_{fs}}(\cdot) \end{bmatrix} = \mathbf{B}_{fs}^{-1} \mathbf{f}_{fs}(\cdot) \quad (3.85)$$

where $\mathbf{f}_{1_{fs}}(\cdot)$ and $\mathbf{B}_{um_{fs}} \mathbf{f}_{2_{fs}}(\cdot)$ respectively account for matched and unmatched uncertainties [89].

Remark 6. *The desired closed-loop dynamics, encapsulated by the Hurwitz matrix $\hat{\mathbf{A}}_{fs}$, are precisely the one achieved by applying the baseline formation controller from Section 3.2. The gains $\mathbf{K}_\delta$, $\mathbf{K}_\epsilon$, k_p , and k_d are chosen via pole placement to make the nominal linearized system stable with a desired response. The $\mathcal{L}_1$ adaptive layer is then designed to compensate for any deviations from this ideal behavior caused by uncertainties ($\boldsymbol{\omega}_{fs}$), external disturbances, and unmodeled nonlinearities ($\mathbf{f}_{1_{fs}}, \mathbf{f}_{2_{fs}}$).*

The $\mathcal{L}_1$ AC implemented in this work exploits a fast estimation scheme based on a piecewise constant law, whose adaptation rate can be directly associated with the sampling rate of the available CPU. The state predictor assumes the form

$$\begin{aligned} \dot{\hat{\mathbf{x}}}_{fs} &= \hat{\mathbf{A}}_{fs} \hat{\mathbf{x}}_{fs} + \mathbf{B}_{m_{fs}} (\boldsymbol{\omega}_{0_{fs}} \mathbf{u}_{fs} + \hat{\boldsymbol{\sigma}}_{1_{fs}}) \\ &\quad + \mathbf{B}_{um_{fs}} \hat{\boldsymbol{\sigma}}_{2_{fs}}, \quad \hat{\mathbf{x}}_{fs}(0) = \mathbf{x}_{fs_0} \\ \hat{\mathbf{y}}_{fs} &= \mathbf{C}_{m_{fs}} \hat{\mathbf{x}}_{fs} \end{aligned} \quad (3.86)$$

where $\boldsymbol{\omega}_{0_{fs}}$ is a known, nominal system input gain contained in a compact convex set $\boldsymbol{\Omega} \subset \mathbb{R}^{2 \times 2}$, while $\hat{\boldsymbol{\sigma}}_{1_{fs}} \in \mathbb{R}^2$ and $\hat{\boldsymbol{\sigma}}_{2_{fs}} \in \mathbb{R}^4$ are the adaptive estimates. The adaptation laws for $\hat{\boldsymbol{\sigma}}_{1_{fs}}$ and $\hat{\boldsymbol{\sigma}}_{2_{fs}}$ are defined as

$$\begin{bmatrix} \hat{\boldsymbol{\sigma}}_{1_{fs}} \\ \hat{\boldsymbol{\sigma}}_{2_{fs}} \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{\sigma}}_{1_{fs}}(k T_s) \\ \hat{\boldsymbol{\sigma}}_{2_{fs}}(k T_s) \end{bmatrix}, \quad t \in [k T_s, (k+1) T_s] \quad (3.87)$$

with

$$\begin{bmatrix} \hat{\boldsymbol{\sigma}}_{1_{fs}}(k T_s) \\ \hat{\boldsymbol{\sigma}}_{2_{fs}}(k T_s) \end{bmatrix} = - \begin{bmatrix} \mathbb{I}_2 & \mathbf{0}_{2 \times 4} \\ \mathbf{0}_{4 \times 2} & \mathbb{I}_4 \end{bmatrix} \mathbf{B}_{fs}^{-1} \boldsymbol{\Phi}_{fs}^{-1}(T_s) \boldsymbol{\mu}_{fs}(k T_s) \quad (3.88)$$

for $k = 0, 1, 2, \dots$ and given $T_s > 0$ is the adaptation sampling time, directly related to the available CPU characteristics. In this regard, the matrix

$\Phi_{fs} \in \mathbb{R}^{6 \times 6}$ is

$$\Phi_{fs}(T_s) \triangleq \hat{\mathbf{A}}_{fs}^{-1} \left(e^{\hat{\mathbf{A}}_{fs} T_s} - \mathbb{I}_6 \right), \quad (3.89)$$

while the vector $\boldsymbol{\mu}_{fs} \in \mathbb{R}^6$ is

$$\boldsymbol{\mu}_{fs}(k T_s) = e^{\hat{\mathbf{A}}_{fs} T_s} \tilde{\mathbf{x}}_{fs}(k T_s) \quad (3.90)$$

provided $\tilde{\mathbf{x}}_{fs} = \hat{\mathbf{x}}_{fs} - \mathbf{x}_{fs}$ is the forward-swing estimation error. Finally, the $\mathcal{L}_1$ forward-swing control is modulated by a feedback gain matrix $\mathbf{K}_{fs} \in \mathbb{R}^{2 \times 2}$ and by a strictly proper transfer matrix $\mathbf{D}_{fs}(s) \in \mathbb{R}^{2 \times 2}$ that, for any choice of $\boldsymbol{\omega}_{fs}$, gives a strictly proper stable

$$\mathbf{C}(s) \triangleq \boldsymbol{\omega}_{fs} \mathbf{K}_{fs} \mathbf{D}_{fs}(s) (\mathbb{I}_2 + \boldsymbol{\omega}_{fs} \mathbf{K}_{fs} \mathbf{D}_{fs}(s))^{-1} \quad (3.91)$$

where $\mathbb{I}_2 \in \mathbb{R}^{2 \times 2}$ is the identity matrix related to the forward-swing state dimension. The $\mathcal{L}_1$ control input is defined as

$$\mathbf{u}_{fs}(s) = -\mathbf{K}_{fs} \mathbf{D}_{fs}(s) \hat{\boldsymbol{\eta}}_{fs}(s) \quad (3.92)$$

where $\hat{\boldsymbol{\eta}}_{fs}(s)$ is the Laplace transform of

$$\hat{\boldsymbol{\eta}}_{fs}(t) \triangleq \boldsymbol{\omega}_{0_{fs}} \mathbf{u}_{fs}(t) + \hat{\boldsymbol{\eta}}_1(t) + \hat{\boldsymbol{\eta}}_{2_m}(t) - \mathbf{r}_g(t) \quad (3.93)$$

with all quantities defined exactly as in Chapter 3.3 of Ref. [89].

Remark 7. *The $\mathcal{L}_1$ AC layer for the forward-swing dynamics relies on the reduced-state $\mathbf{x}_{fs} = \begin{bmatrix} r_{x1}, v_{x1}, r_{x2}, v_{x2}, \zeta, \dot{\zeta} \end{bmatrix}^T$, which includes both the oscillation angle ζ and rate $\dot{\zeta}$. The estimation of the oscillatory state of the payload in the cooperative scenario is a challenging issue that was recently solved by the author, as already pointed out in Remark 1 [73, 36].*

Definition 3.3.1 ($\mathcal{L}_1$ induced norm). *For a stable, strictly proper MIMO transfer function $\mathbf{G}(s)$ with impulse response $\mathbf{g}(\mathbf{t})$,*

$$\|\mathbf{G}\|_{\mathcal{L}_1} \triangleq \int_0^\infty \|\mathbf{g}(\mathbf{t})\|_1 \, d\mathbf{t},$$

where $\|\cdot\|_1$ is the matrix 1-norm. In particular, $g_e := \|\mathbf{C}\|_{\mathcal{L}_1}$ with $\mathbf{C}(s)$ defined in Equation (3.91).

Lemma 3.3.1 (Discrete contraction). *Let $\hat{\mathbf{A}}_{fs}$ be Hurwitz. Then there exist $P = P^T > 0$, $q > 0$, and $\bar{T} > 0$ such that for all $T_s \in (0, \bar{T}]$,*

$$\Phi_d^T P \Phi_d - P \leq -q P, \quad \Phi_d \triangleq e^{\hat{\mathbf{A}}_{fs} T_s}.$$

Equivalently, there exists $\alpha \in (0, 1)$ with $\|\Phi_d\| \leq 1 - \alpha$.

Without loss of generality, a proof of global exponential ultimate boundedness for the forward-swing subsystem is provided in what follows. The proposed methodology, however, can be straightforwardly extended to the lateral-vertical channel.

Theorem 3.3.2. *Let $\hat{\mathbf{A}}_{fs}$ be Hurwitz and let $\mathbf{P} = \mathbf{P}^T > 0$ be a symmetric, positive-definite matrix that solves the Lyapunov equation $\hat{\mathbf{A}}_{fs}^T \mathbf{P} + \mathbf{P} \hat{\mathbf{A}}_{fs} = -\mathbf{Q}$ with $\mathbf{Q} = \mathbf{Q}^T > 0$. Let $\mathbf{B}_{fs} := [\mathbf{B}_{m_{fs}}, \mathbf{B}_{um_{fs}}] \in \mathbb{R}^{6 \times 6}$ be non-singular and $\mathbf{B}_{m_{fs}}^T \mathbf{B}_{um_{fs}} = \mathbf{0}$. Let the nonlinear functions $\mathbf{f}_{1_{fs}}(t)$ and $\mathbf{f}_{2_{fs}}(t)$ be bounded by some known quantities Δ_1 and Δ_2 , namely $\|\mathbf{f}_{1_{fs}}(t)\| \leq \Delta_1$, $\|\mathbf{f}_{2_{fs}}(t)\| \leq \Delta_2$ for all $t > 0$. Let $\mathbf{D}_{fs}$ be strictly proper, stable with DC gain satisfying $\|\boldsymbol{\omega}_{fs} \mathbf{K}_{fs} \mathbf{D}_{fs}(0) (\mathbb{I}_2 + \boldsymbol{\omega}_{fs} \mathbf{K}_{fs} \mathbf{D}_{fs}(0))^{-1}\| < 1$ according to the small-gain inequality theorem [89]. Assume $T_s > 0$ is chosen such that $\|e^{\hat{\mathbf{A}}_{fs} T_s}\| \leq 1 - \alpha$ with $0 < \alpha < 1$. Consider $\tilde{\mathbf{x}}_{fs} = \hat{\mathbf{x}}_{fs} - \mathbf{x}_{fs}$ and $\tilde{\boldsymbol{\sigma}}_{fs} = \hat{\boldsymbol{\sigma}}_{i_{fs}} - \mathbf{f}_{i_{fs}}$ to be the state prediction and parameter estimation errors, respectively. Then, by properly picking an adaptation-gain matrix $\boldsymbol{\Lambda}$ so that $\lambda_{\max}(\boldsymbol{\Lambda})$ satisfies Equation (E.13), the closed-loop forward-swing subsystem is Globally Exponentially Ultimately Bounded (GEUB). In particular, there exist constants $\rho \in (0, 1)$ and $V_\infty > 0$ for which the Lyapunov candidate function*

$$V(t) = \tilde{\mathbf{x}}_{fs}^T \mathbf{P} \tilde{\mathbf{x}}_{fs} + \tilde{\boldsymbol{\sigma}}_{fs}^T \boldsymbol{\Lambda}^{-1} \tilde{\boldsymbol{\sigma}}_{fs} \quad (3.94)$$

satisfies, for all $t \geq 0$, the geometric inequality

$$V(t) \leq \rho^{\lfloor t/T_s \rfloor} V(0) + V_\infty. \quad (3.95)$$

Moreover, the tracking error is bounded

$$\|\mathbf{e}_{fs}(t)\|_\infty \leq g_e \sqrt{\lambda_{\max}(\boldsymbol{\Lambda}) V_\infty} \quad (3.96)$$

where $g_e := \|C(s)\|_{\mathcal{L}_1}$ is the $\mathcal{L}_1$ induced norm of Equation (3.91).

Proof. See Appendix E. □

3.4 Swing State Estimation

This section derives the discrete-time estimator required to provide the payload swing state feedback $(\zeta, \dot{\zeta})$ for the controllers previously developed. A reduced second-order model is obtained from the full Lagrangian dynamics, and a scalar, virtual measurement of the oscillation angle ζ is reconstructed from agent accelerations and control inputs projected onto the horizontal frame $\mathcal{F}_H$. Estimation is performed with a Fading Gaussian Deterministic Filter, which can be cast as a Kalman-like recursive scheme with a fading factor $\beta \in (0, 1)$. An implementation architecture and a global tuning strategy using a filter bank over the fading factor β conclude the present chapter.

3.4.1 FGDF Recursive Equations

Let $\mathbf{x} = [\zeta, \dot{\zeta}]^T \in \mathbb{R}^{n_x}$ with $n_x = 2$ be the state vector representing payload oscillation dynamics only. The linear time-invariant state-space model

$$\dot{\mathbf{x}} = \mathbf{A}_\zeta \mathbf{x} + \mathbf{B}_\zeta \mathbf{u} \quad (3.97)$$

is extracted from Equations (3.70) and (3.71), where

$$\mathbf{A}_\zeta = \begin{bmatrix} 0 & 1 \\ A_{14,7} & 0 \end{bmatrix}, \quad (3.98)$$

and

$$\mathbf{B}_\zeta = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ B_{14,1} & B_{14,2} & B_{14,3} & B_{14,4} & B_{14,5} & B_{14,6} \end{bmatrix} \quad (3.99)$$

The considered dynamic model is reformulated into difference equations where the true state vector $\mathbf{x}^{(k)}$ at time k propagates according to $\mathbf{x}^{(k+1)} = \Phi^{(k)} \mathbf{x}^{(k)} + \Psi^{(k)} \mathbf{u}^{(k)}$. The state transition matrix for the estimation algorithm is obtained as $\Phi^{(k)} = \Phi = \exp(\mathbf{A}_\zeta \Delta t)$, where $\Delta t = t_{k+1} - t_k$ is the sampling interval. The control-input model is calculated as $\Psi^{(k)} = \Psi = \mathbf{A}_\zeta^{-1}(\Phi - \mathbf{I}_2) \mathbf{B}_\zeta$. The linearization of system dynamics in Equation (3.59) provides additional information related to vehicles acceleration on the local horizontal plane. As an example, the following equation is obtained for UAV 1

$$\mathbf{a}_1 = \boldsymbol{\alpha}_1 [\mathbf{r}_1^T \mathbf{r}_2^T]^T + \beta_1 \mathbf{u} + \boldsymbol{\lambda}_1 \zeta, \quad (3.100)$$

where $\mathbf{a}_1 = [a_{x_1}, a_{y_1}, a_{z_1}]^T = [\dot{v}_{x_1}, \dot{v}_{y_1}, \dot{v}_{z_1}]^T$ is the acceleration of UAV 1 as expressed in $\mathcal{F}_E$, provided

$$\boldsymbol{\alpha}_1 = \begin{bmatrix} A_{8,1} & A_{8,2} & A_{8,3} & A_{8,4} & A_{8,5} & A_{8,6} \\ A_{9,1} & A_{9,2} & A_{9,3} & A_{9,4} & A_{9,5} & A_{9,6} \\ A_{10,1} & A_{10,2} & A_{10,3} & A_{10,4} & A_{10,5} & A_{10,6} \end{bmatrix}, \quad (3.101)$$

$$\boldsymbol{\beta}_1 = \begin{bmatrix} B_{8,1} & B_{8,2} & B_{8,3} & B_{8,4} & B_{8,5} & B_{8,6} \\ B_{9,1} & B_{9,2} & B_{9,3} & B_{9,4} & B_{9,5} & B_{9,6} \\ B_{10,1} & B_{10,2} & B_{10,3} & B_{10,4} & B_{10,5} & B_{10,6} \end{bmatrix}, \quad (3.102)$$

and $\boldsymbol{\lambda}_1 = [A_{8,7}, A_{9,7}, 0]^T$ are matrix subsets of $\mathbf{A}$ and $\mathbf{B}$, with $\boldsymbol{\alpha}_1, \boldsymbol{\beta}_1 \in \mathbb{R}^{3 \times 6}$, and $\boldsymbol{\lambda}_1 \in \mathbb{R}^{3 \times 1}$. It is noted from Equations (3.40)–(3.42) that the reference frame $\mathcal{F}_H$ in the equilibrium condition is defined by unit vectors $\bar{\mathbf{x}}_H = [\bar{\gamma}_y, -\bar{\gamma}_x, 0]^T / \bar{\gamma}$, $\bar{\mathbf{y}}_H = [\bar{\gamma}_x, \bar{\gamma}_y, 0]^T / \bar{\gamma}$, and $\bar{\mathbf{z}}_H = [0, 0, 1]^T$. By applying the transformation $\bar{\mathbf{T}}_{HE} = \bar{\mathbf{T}}_{EH}^T = [\bar{\mathbf{x}}_H \quad \bar{\mathbf{y}}_H \quad \bar{\mathbf{z}}_H]^T$ to both sides of Equation (3.100), one obtains

$$\bar{\mathbf{T}}_{HE} \left(\mathbf{a}_1 - \boldsymbol{\alpha}_1 [\mathbf{r}_1^T \mathbf{r}_2^T]^T - \boldsymbol{\beta}_1 \mathbf{u} \right) = \left[\frac{g m_l}{2m} \zeta, 0, 0 \right]^T \quad (3.103)$$

where the definition of $\boldsymbol{\lambda}_1$ is accounted with the values of $A_{8,7}$ and $A_{9,7}$ reported in Appendix D. Assume that, at time k the input $\mathbf{u}^{(k)}$ is known, while acceleration $\mathbf{a}_1^{(k)}$ and position information $\mathbf{r}_1^{(k)}$ and $\mathbf{r}_2^{(k)}$ are made available from sensors. A key insight of this estimation approach is that the swing angle ζ , while not directly measured, is dynamically coupled to the vehicle's horizontal acceleration. By algebraically inverting Equation (3.103), the oscillation angle ζ is retrieved as a function of known or measured quantities: one agent's acceleration $\mathbf{a}_1$, its position relative to the other vehicle, and the applied control forces $\mathbf{u}$. This yields a virtual measurement of the swing angle ζ_m , namely

$$\zeta_m^{(k)} = \frac{2m}{g m_l} \bar{\mathbf{x}}_H^T \left(\mathbf{a}_1^{(k)} - \boldsymbol{\alpha}_1 [\mathbf{r}_1^{(k)T} \mathbf{r}_2^{(k)T}]^T - \boldsymbol{\beta}_1 \mathbf{u}^{(k)} \right) \quad (3.104)$$

The observation $z^{(k)} = \zeta_m^{(k)}$ tracks the considered dynamic process as in $z^{(k)} = \mathbf{H} \mathbf{x}^{(k)}$, where $\mathbf{H} = [1, 0]$ is the time-invariant observation matrix.

Measurements are assumed to be corrupted by noise $\nu^{(k)}$ with variance R , that is $z^{(k)} = \mathbf{H} \mathbf{x}^{(k)} + \nu^{(k)}$. Given the time series of data $z_0, z_1, z_2, \dots, z_k$, the aim is to estimate in a time series the state vector $\tilde{\mathbf{x}}_0, \tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2, \dots, \tilde{\mathbf{x}}_k$, by taking into account the model in Equations (3.97)–(3.99). In what follows, such a problem is solved by a set of Kalman-like recursive equations, where the gain matrix is computed through the formal minimization of a cost function, with no other assumption [90]. Let R^* be an assigned weighting scalar and $\beta \in (0, 1)$ be the fading factor. Consider the cost function:

$$\varepsilon_k = \sum_{h=0}^k (z_{k-h} - \mathbf{H} \tilde{\mathbf{x}}_{k-h/k})^T R^{*-1} (z_{k-h} - \mathbf{H} \tilde{\mathbf{x}}_{k-h/k}) \beta^h \quad (3.105)$$

where

$$\tilde{\mathbf{x}}_{k-h/k} = \begin{cases} \tilde{\mathbf{x}}_k & \text{if } h = 0 \\ \Phi^{-h} \tilde{\mathbf{x}}_k - \gamma_{k-h/k} & \text{if } h > 0 \end{cases} \quad (3.106)$$

and

$$\gamma_{k-h/k} = \sum_{j=0}^{h-1} \Phi^{-(j+1)} \Psi \mathbf{u}_{k-h+j} \quad (3.107)$$

is defined for $h > 0$. Note that the estimated state vector appears in any term of the sum in Equation (3.105) as $\tilde{\mathbf{x}}_{k-h/k}$, which is the back-projection of the current estimate, given all data and input history up to time k . By minimizing the cost function, the best fit between the current estimate $\tilde{\mathbf{x}}_k$ and all previous available data is thus sought. Turning the estimation problem into a curve fitting exercise also allows, thanks to elementary statistical analysis, the true error covariance to be directly estimated from the data without prior knowledge of R (which is also estimated, as part of the process). In this regard, it is necessary to stress the difference between the “filter-propagated” error covariance matrix and the true error covariance matrix, which are, in general, two different objects. This is the reason why the proposed formulation uses two different symbols: $\mathbf{E}_k$ for the filter-computed matrix and $\mathbf{P}_k$ for the true error covariance. The least squares solution to $\tilde{\mathbf{x}}_k$ is found through differentiation of Equation (3.105) with respect to $\tilde{\mathbf{x}}_k$, according to the procedure described in detail by de Angelis et al. adapted

to a time-invariant scenario [91]. Given $\mathbf{H}$, Φ , and β , the filter recursive equations are derived below.

Filter Recursive Equations

1. Input: $z_k, \tilde{\mathbf{x}}_k^-, \mathbf{E}_k^-$

2. Compute:

$$\tilde{\mathbf{x}}_k = \tilde{\mathbf{x}}_k^- + \mathbf{M}_k (z_k - \mathbf{H} \tilde{\mathbf{x}}_k^-) \quad (3.108)$$

where

• *standard formulation*

$$\mathbf{M}_k = \mathbf{E}_k^- \mathbf{H}^T (\mathbf{H} \mathbf{E}_k^- \mathbf{H}^T + R^*)^{-1} \quad (3.109)$$

$$\mathbf{E}_k = (\mathbf{I}_{n_x} - \mathbf{M}_k \mathbf{H}) \mathbf{E}_k^- \quad (3.110)$$

• *alternative formulation*

$$\mathbf{E}_k^{-1} = \mathbf{H}^T R^{*-1} \mathbf{H} + (\mathbf{E}_k^-)^{-1} \quad (3.111)$$

$$\mathbf{M}_k = \mathbf{E}_k \mathbf{H}^T R^{*-1} \quad (3.112)$$

3. Project ahead:

$$\tilde{\mathbf{x}}_{k+1}^- = \Phi \tilde{\mathbf{x}}_k + \Psi \mathbf{u}_k \quad (3.113)$$

$$\mathbf{E}_{k+1}^- = \frac{\Phi \mathbf{E}_k \Phi^T}{\beta} \quad (3.114)$$

In the present framework, the terms characterized by the minus sign superscript represent the predicted (a priori) filter estimates, obtained from the model-based forward projection described in Equations (3.113) and (3.114). The updated (a posteriori) filter estimates, corrected on the basis of current observations and gain matrix $\mathbf{M}$ computation, are provided by Equations (3.108) and (3.110) (or Equation (3.111) for the alternative formulation).

Remark 8. Input $\mathbf{u}^{(k)}$ in Equation (3.104) is assumed to be known, accounting for both disturbance and control forces. With respect to the knowledge of control

force, an approximate solution is provided for a multirotor platform, based on the approach in Ref. [52]. Starting from the definition in Section 3.2.1, the i -th vehicle control force is expressed as $\mathbf{f}_{ci} = -T_i \mathbf{z}_{Bi} = \mathbf{T}_{BE}^T(\boldsymbol{\eta}_i) [0, 0, -T_i]^T$, where T_i is net thrust generated along $\mathbf{z}_{Bi}$. Taking into account Equation (3.39) and including the estimated external disturbance $\tilde{\mathbf{f}}_{d_i}$, the reconstructed thrust magnitude $\tilde{T}_i$ is calculated as

$$\tilde{T}_i = \left\| m \mathbf{a}_i - m \tilde{\mathbf{g}} - \tilde{\mathbf{f}}_{l_i} - \tilde{\mathbf{f}}_{d_i} \right\| \quad (3.115)$$

where $\tilde{\mathbf{g}}$ is measured gravity acceleration and $\tilde{\mathbf{f}}_{l_i}$ is the estimated force induced by the load on the i -th multirotor. Under the simplifying assumption that maneuvers are performed near the equilibrium $(\bar{\mathbf{y}}, \bar{\mathbf{u}})$, it is possible to assign $m \tilde{\mathbf{g}} + \tilde{\mathbf{f}}_{l_i} \approx -\bar{\mathbf{u}}_i$, where $\bar{\mathbf{u}}_i$ is retrieved from Equation (3.61). Reconstructed control force for filtering purposes is thus $\tilde{\mathbf{u}}_i = \mathbf{T}_{BE}^T(\tilde{\boldsymbol{\eta}}_i) [0, 0, -\tilde{T}_i]^T$, provided $\tilde{\boldsymbol{\eta}}_i$ is obtained from measured attitude and $\tilde{T}_i = \left\| m \mathbf{a}_i + \bar{\mathbf{u}}_i - \tilde{\mathbf{f}}_{d_i} \right\|$. Extension of the proposed method to conventional helicopter configurations can be conceived with minimum effort by accounting for both tail rotor and main rotor contributions in the estimation of net control force.

Figure 3.5 suggests a suitable architecture for the implementation of the estimator in a real application scenario. Without loss of generality, UAV 1 is assumed to be the leader, where both payload mass and swing state estimation routines are computed. On condition that each vehicle is equipped with a DOB, the amount of transmission data is reduced to a minimum, that includes position $\mathbf{r}_2$ and reconstructed input force $\mathbf{u}_2$ (from UAV 2 to UAV 1) and estimated swing state $\tilde{\mathbf{x}}$ (from UAV 1 to UAV 2), the latter being available to generate a common payload control action. With respect to the disturbance observation and the control force estimation respectively discussed in Remarks 5 and 8, an alternative method of practical application will be proposed in Chapter 5, based on the particular control strategy of PX4 standard [59].

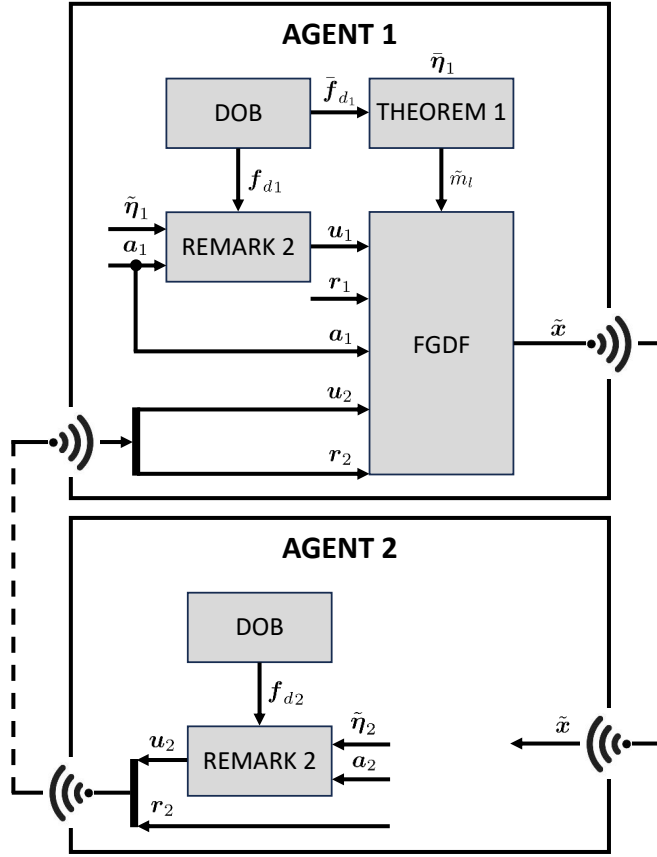


Figure 3.5: Suitable architecture for FGDF implementation.

3.4.2 FGDF Global Optimization

In this subsection the global optimization of the Fading Gaussian Deterministic Filter is discussed, based on the particular nature of the estimation approach. Two subsequent steps are considered: the tuning of the fading parameter and the scaling of the noise covariance matrix. To this aim, an ensemble of filters is implemented, all sharing the same data input z_k , dynamical model Φ , measurement model H , and assigned R^* , but different values of the fading factor β . A tuning statistic $\chi_k^2(\beta)$ and a scaling statistic $\xi_k^2(\beta)$ are evaluated as a function of β , whose definitions can be found in Ref. [91]. For each run of the filter bank (that is, for each value of β), time averaging of the tuning statistic over a certain time window is performed, providing the

averaged value $\overline{\chi^2}(\beta)$. The filter is then considered to be tuned for a value $\beta = \beta_T$ for which the minimum averaged tuning statistic is obtained, namely $\overline{\chi^2}(\beta_T) = \min_{\beta \in (0,1)} [\overline{\chi^2}(\beta)]$. It is thus postulated that the optimal β (in the sense of minimum estimation error) is the one for which the rate of fading of the past residuals yields a globally best fit (over β) of the measurements to the state vector estimates, namely at the best compromise configuration that makes use of both the dynamic model and the measurement information. In the tuned condition $\beta = \beta_T$ the average scaling statistic $\overline{\xi^2}(\beta_T)$ is used as the best estimate of a gain factor (see Appendix B in Ref. [91]) that allows the assigned R^* to be scaled to an estimate $\tilde{R}$ of the true noise covariance matrix R . In other words:

$$\tilde{R} = \overline{\xi^2}(\beta_T) R^* \quad (3.116)$$

As a by-product, the true error covariance $\mathbf{P}_k$ can be estimated from the filter-computed error covariance $\mathbf{E}_k$ as

$$\tilde{\mathbf{P}}_k(\beta_T) = \overline{\xi^2}(\beta_T) \mathbf{E}_k(\beta_T) \quad (3.117)$$

In the tuned and scaled condition, where the nominal covariance matches the true error covariance, a minimum to (the trace of) $\mathbf{P}_k$ is located, proving that the considered approach is able to identify the globally optimum filter.

3.4.3 Control System Design

A controller is designed and empirically validated to fulfill (i) formation-keeping and trajectory-tracking, (ii) inter-agent collision avoidance, and (iii) active payload stabilization. Control tasks are performed, without loss of generality, through a cascaded algorithm based on PX4 architecture and notation [59]. Such control solution stabilizes the i -th octarotor according to different flight modes that include the tracking of an Earth-fixed set-point position $\mathbf{r}_{sp_i}$, velocity $\mathbf{v}_{sp_i}$, or acceleration $\mathbf{a}_{sp_i}$ (see Figure 3.6), where $\mathbf{a}_{sp_i}^{(VC)}$ is the acceleration set-point generated by the available Velocity Controller).

Desired acceleration $\mathbf{a}_{sp_i} = [a_{sp_{xi}}, a_{sp_{yi}}, a_{sp_{zi}}]^T$ for the i -th lifting vehicle is designed as

$$\mathbf{a}_{sp_i} = \mathbf{a}_{sp_i}^{(VC)} + \mathbf{a}_{sp_i}^{(CAC)} + \mathbf{a}_{sp_i}^{(PC)} \quad (3.118)$$

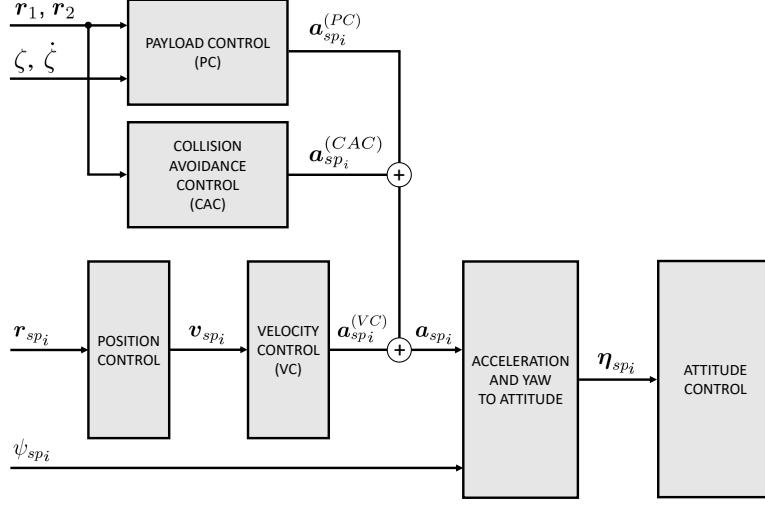


Figure 3.6: The i -th multirotor control scheme based on PX4 autopilot architecture.

where $\mathbf{a}_{sp_i}^{(VC)}$ accounts for formation stabilization, $\mathbf{a}_{sp_i}^{(CAC)}$ represents Collision Avoidance Controller (CAC), and $\mathbf{a}_{sp_i}^{(PC)}$ implements payload control strategies.

With respect to $\mathbf{a}_{sp_i}^{(CAC)}$, the method proposed in Ref. [92] is adopted, based on the definition of potential function

$$V_p(\gamma) = \frac{1}{4} \left[\frac{(\bar{\gamma} - \gamma)^T (\bar{\gamma} - \gamma)}{\gamma^T \gamma - R_{sph}^2} \right]^2 \quad (3.119)$$

where $R_{sph} < \bar{\gamma}/2$ is the radius of a sphere, centered at P_i , representing a safety zone for the i -th agent. The acceleration contribution allowing for inter-agent collision avoidance is thus:

$$\mathbf{a}_{sp_i}^{(CAC)} = \xi_i K_a \nabla V_p|_{\gamma} \quad (3.120)$$

where $K_a > 0$ is a control gain and

$$\nabla V_p|_{\gamma} = -\frac{(\bar{\gamma} - \gamma) \|\bar{\gamma} - \gamma\|^2}{(\gamma^T \gamma - R_{sph}^2)^2} - \frac{\gamma \|\bar{\gamma} - \gamma\|^4}{(\gamma^T \gamma - R_{sph}^2)^3} \quad (3.121)$$

is the gradient of potential function in Equation (3.119) with respect to γ .

The proposed PC is structured as

$$\mathbf{a}_{sp_i}^{(PC)} = (k_p \zeta + k_d \dot{\zeta}) \mathbf{x}_H, \quad (3.122)$$

being directed over the local horizontal plane in the direction of $\mathbf{x}_H$, where k_p and k_d are positive gains and ζ and $\dot{\zeta}$ are estimated through the proposed FGDF.

CHAPTER 4

Double-Payload Transportation

Modern Urban Air Mobility and Delivery operations increasingly demand efficient multi-stop logistics, where a single drone mission must serve multiple delivery points to maximize operational potential and minimize energy consumption. Transporting two serially-connected payloads enables such multi-parcel delivery within a single flight: after releasing the first payload at an intermediate waypoint, the drone continues to the final destination with the remaining load. This operational paradigm reduces the number of required flights and total mission time compared to sequential single-payload trips, offering significant advantages in terms of cost efficiency and environmental impact. However, the dual-payload configuration introduces coupled oscillatory dynamics that complicate flight control: both loads exhibit independent pendular motion while simultaneously interacting through cable tension, creating a strongly nonlinear, under-actuated system prone to instabilities during maneuvering and payload release. Ensuring safe and precise operation requires robust estimation of both swing states and coordinated control strategies that can damp oscillations while maintaining trajectory tracking performance.

This chapter develops a dynamics, estimation, and control framework for a multirotor transporting two serially-connected suspended payloads. The main goal is to (i) derive a tractable yet physically meaningful model suitable for control design, (ii) extract a reduced swing subsystem capturing the dominant lateral and frontal oscillations, (iii) design a discrete-time estimator that is implementable onboard and robust to modeling mismatch, and (iv) synthesize a pragmatic and intuitive controller that stabilizes the vehicle while damping payloads oscillations. This controller is deliberately designed around the center of gravity of the payload system to ensure simplicity, ease of tuning, and inherent robustness to variations in payload mass, which are key requirements for real-world aerial delivery missions. The analysis proceeds from a Lagrangian formulation, a small-angle linearization about hover, and a separation between swing dynamics on the horizontal plane and vertical motion.

4.1 Equations of Motion

Let $\mathbf{r} = [r_1, r_2, r_3]^T$ be the position of multirotor and $\mathbf{v} = [v_1, v_2, v_3]^T$ its velocity with respect to $\mathcal{F}_E$. Payload A , located in P_A , is assumed to be a point with mass m_A and is connected to the multirotor through a cable with constant length L_A . Similar considerations hold for payload B with mass m_B , located in P_B and connected to the upper payload by a cable with length L_B . Each cable is modeled as massless, under the assumption that its mass is negligible compared to the total mass of the system. It is further treated as a perfectly rigid element, implying no deformation under load and the ability to transmit both tensile and compressive forces, akin to an ideal rigid link, although, in practice, the transmission of compressive forces through cables is highly improbable. The dynamics of the vehicle as expressed in $\mathcal{F}_E$ (the subscript E will be dropped for simplicity) is described by:

$$\dot{\mathbf{r}} = \mathbf{v} \tag{4.1}$$

$$m \dot{\mathbf{v}} = \mathbf{f}_g + \mathbf{f}_A + \mathbf{f}_d + \mathbf{f}_c \tag{4.2}$$

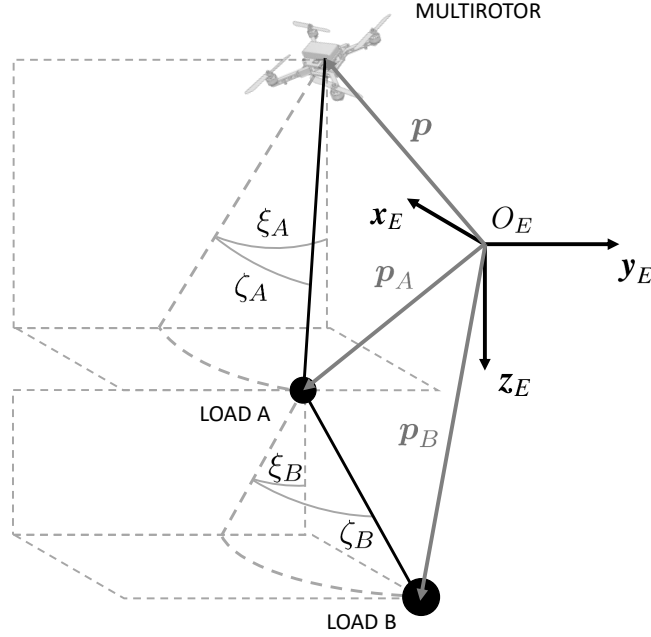


Figure 4.1: Definition of lateral and frontal upper (ξ_A, ζ_A) and lower (ξ_B, ζ_B) swing angles.

where m is mass of the rotorcraft, $\mathbf{f}_g = [0, 0, mg]^T$ is gravity force vector, $\mathbf{f}_A$ is the force induced by the upper load on the rotorcraft through its cable, and $\mathbf{f}_d = [f_{d1}, f_{d2}, f_{d3}]^T$ is vehicle aerodynamic force, that accounts for the contributions of rotorcraft frame drag and rotor in-plane forces. The net thrust vector $\mathbf{f}_c = [f_{c1}, f_{c2}, f_{c3}]^T$, directed along $\mathbf{z}_B$, represents the only control input to Equation (4.2). The position of payload A with respect to the rotorcraft is parametrized by angles $-\pi/2 < \xi_A < \pi/2$ and $-\pi/2 < \zeta_A < \pi/2$, respectively obtained from load rotations about $\mathbf{x}_E$ and $\mathbf{y}_E$. The absolute position of payload A , identified by $\mathbf{r}_A = [r_{A1}, r_{A2}, r_{A3}]^T$, is thus expressed as a function of ξ_A, ζ_A , and $\mathbf{r}$ as

$$\mathbf{r}_A = \mathbf{r} + L_A [\sin \zeta_A, -\sin \xi_A \cos \zeta_A, \cos \xi_A \cos \zeta_A]^T \quad (4.3)$$

Correspondingly, the position of payload B , given by $\mathbf{r}_B = [r_{B1}, r_{B2}, r_{B3}]^T$, is described as a function of $-\pi/2 < \xi_B < \pi/2$, $-\pi/2 < \zeta_B < \pi/2$, and $\mathbf{r}_A$ as

$$\mathbf{r}_B = \mathbf{r}_A + L_B [\sin \zeta_B, -\sin \xi_B \cos \zeta_B, \cos \xi_B \cos \zeta_B]^T \quad (4.4)$$

Let $\boldsymbol{\lambda} = [\boldsymbol{r}^T, \xi_B, \zeta_B, \xi_A, \zeta_A]^T \in \mathbb{R}^7$ be a vector of generalized coordinates. The Lagrangian equation with respect to the generalized coordinate vector $\boldsymbol{\lambda}$ is

$$\frac{d}{dt} \left(\frac{\partial E}{\partial \dot{\boldsymbol{\lambda}}} \right) - \frac{\partial E}{\partial \boldsymbol{\lambda}} = \boldsymbol{\tau} \quad (4.5)$$

where $E = K - U$ is the Lagrangian function and $\boldsymbol{\tau} \in \mathbb{R}^7$ is the vector of generalized forces. In the present framework, K is system kinetic energy, given by

$$K = \frac{1}{2} m \dot{\boldsymbol{r}}^T \dot{\boldsymbol{r}} + \frac{1}{2} m_A \dot{\boldsymbol{r}}_A^T \dot{\boldsymbol{r}}_A + \frac{1}{2} m_B \dot{\boldsymbol{r}}_B^T \dot{\boldsymbol{r}}_B \quad (4.6)$$

where $\dot{\boldsymbol{r}}_A$ and $\dot{\boldsymbol{r}}_B$ denote load velocities as obtained from Equations (4.3) and (4.4), respectively. It is:

$$\begin{aligned} \dot{\boldsymbol{r}}_A = \dot{\boldsymbol{r}} + L_A \left[\dot{\zeta}_A c\zeta_A, \quad -\dot{\xi}_A c\xi_A c\zeta_A + \dot{\zeta}_A s\xi_A s\zeta_A, \right. \\ \left. -\dot{\xi}_A s\xi_A c\zeta_A - \dot{\zeta}_A c\xi_A s\zeta_A \right]^T \end{aligned} \quad (4.7)$$

and

$$\begin{aligned} \dot{\boldsymbol{r}}_B = \dot{\boldsymbol{r}}_A + L_B \left[\dot{\zeta}_B c\zeta_B, \quad -\dot{\xi}_B c\xi_B c\zeta_B + \dot{\zeta}_B s\xi_B s\zeta_B, \right. \\ \left. -\dot{\xi}_B s\xi_B c\zeta_B - \dot{\zeta}_B c\xi_B s\zeta_B \right]^T \end{aligned} \quad (4.8)$$

where $s(\cdot) = \sin(\cdot)$ and $c(\cdot) = \cos(\cdot)$. Potential energy U is related to the vertical displacement of the multirotor and the suspended loads, namely:

$$U = -m g r_3 - m_A g r_{A3} - m_B g r_{B3} \quad (4.9)$$

Given the energy contributions in Equations (4.6) and (4.9) and taking into account the state of the loads as described by Equations (4.3), (4.4), (4.7), and (4.8), the Lagrangian equation is manipulated into the system:

$$\dot{\boldsymbol{\lambda}} = \boldsymbol{\mu} \quad (4.10)$$

$$\boldsymbol{M}(\boldsymbol{\lambda}) \dot{\boldsymbol{\mu}} + \boldsymbol{C}(\boldsymbol{\lambda}, \boldsymbol{\mu}) \boldsymbol{\mu} + \boldsymbol{G}(\boldsymbol{\lambda}) = \boldsymbol{\tau} \quad (4.11)$$

where $\boldsymbol{\mu} = [\boldsymbol{v}^T, \dot{\xi}_B, \dot{\zeta}_B, \dot{\xi}_A, \dot{\zeta}_A]^T \in \mathbb{R}^7$. The generalized force vector $\boldsymbol{\tau}$ is made of an active control contribution, $\boldsymbol{\tau}_c = [\boldsymbol{f}_c^T, 0, 0, 0, 0]^T$, and a disturbance term, $\boldsymbol{\tau}_d = [\boldsymbol{f}_d^T, 0, 0, 0, 0]^T - \boldsymbol{D} \boldsymbol{\mu}$, made of two contributions. The former accounts for aerodynamic drag effects on the multirotor, while the latter term represents a simplified damping model for the payload oscillations. In general it is $\boldsymbol{D} = \text{diag}(0, 0, 0, d_{\dot{\xi}_B}, d_{\dot{\zeta}_B}, d_{\dot{\xi}_A}, d_{\dot{\zeta}_A})$, but in this thesis the isotropic case $\boldsymbol{D} = \text{diag}(0, 0, 0, d, d, d, d)$ is considered.

Remark 9. *The linear damping model in $\mathbf{D}$ is a significant simplification of the actual physics. Aerodynamic drag on suspended payloads is proportional to the square of the velocity magnitude and depends on the full payload velocity vector (combining both angular swing motion and vehicle translation). However, such nonlinear terms would preclude analytical eigenvalue analysis and complicate the linearization framework adopted here. The linear damping coefficient d is therefore introduced as a modeling convenience that can represent:*

- *Small-amplitude energy dissipation in numerical simulations*
- *Effective damping from structural friction in cables and connections*
- *A conservative approximation valid in the low-velocity, small-angle regime (those of interest)*

More importantly, the controller design and stability analysis in Sections 4.3 and 4.4 are performed for the undamped case ($d = 0$), ensuring that the control system does not rely on any dissipative effects for stabilization. Any physical damping present in the real system (aerodynamic or otherwise) can only improve performance relative to this conservative baseline.

The elements of $\mathbf{M} \in \mathbb{R}^{7 \times 7}$ and $\mathbf{C} \in \mathbb{R}^{7 \times 7}$ are reported in Appendix F, while $\mathbf{G} \in \mathbb{R}^7$ is defined as

$$\mathbf{G}(\boldsymbol{\lambda}) = g \begin{bmatrix} 0 \\ 0 \\ (m + m_A + m_B) \\ L_B m_B \sin \xi_B \cos \zeta_B \\ L_B m_B \cos \xi_B \sin \zeta_B \\ L_A m_A \sin \xi_A \cos \zeta_A \\ L_A m_A \cos \xi_A \sin \zeta_A \end{bmatrix} \quad (4.12)$$

4.2 Swing State Estimation

The aim of this section is to estimate the payloads swing state from readily available onboard signals. Starting from the linearized model, the oscillatory

subsystem is isolated and a discrete-time representation suitable for real-time computation is derived. Synthetic measurements of the upper load oscillation angles are retrieved from inertial accelerations and known control inputs, and a Fading Gaussian Deterministic Filter is implemented to fuse model predictions with these measurements. Particular attention is paid to tuning the fading factor and measurement noise scaling so as to balance model mismatch against sensor noise.

4.2.1 System Linearization

The system in Equations (4.10) and (4.11) is linearized about the equilibrium $(\bar{\mathbf{y}}, \bar{\mathbf{u}})$, where $\bar{\mathbf{y}} = [\bar{\boldsymbol{\lambda}}^T, \bar{\boldsymbol{\mu}}^T]^T = [\bar{\mathbf{r}}^T, \mathbf{0}_{1 \times 11}]^T \in \mathbb{R}^{14}$ is representative of a hovering flight in $\mathbf{r} = \bar{\mathbf{r}}$ with the payloads at rest ($\bar{\xi}_A = \bar{\zeta}_A = \bar{\xi}_B = \bar{\zeta}_B = 0$, $\dot{\bar{\xi}}_A = \dot{\bar{\zeta}}_A = \dot{\bar{\xi}}_B = \dot{\bar{\zeta}}_B = 0$), provided the aerodynamic disturbance is a constant, $\bar{\mathbf{f}}_d$. The term $\bar{\mathbf{u}} \in \mathbb{R}^3$ represents the proper equilibrium force, with $\bar{\mathbf{u}} = \bar{\mathbf{f}}_c + \bar{\mathbf{f}}_d$. The generalized equilibrium force is thus obtained as $\bar{\boldsymbol{\tau}} = \bar{\boldsymbol{\tau}}_c + \bar{\boldsymbol{\tau}}_d = \mathbf{G}(\bar{\boldsymbol{\lambda}})$. Let $M = m + m_A + m_B$ be the total system mass. Taking into account the first 3 components in $\bar{\boldsymbol{\tau}}$, the proper equilibrium force is obtained as $\bar{\mathbf{u}} = [0, 0, -gM]^T$ while the control force generated by the rotorcraft is

$$\bar{\mathbf{f}}_c = [\bar{f}_{c1}, \bar{f}_{c2}, \bar{f}_{c3}]^T = \bar{\mathbf{u}} - \bar{\mathbf{f}}_d \quad (4.13)$$

Let $\mathbf{y} = [\boldsymbol{\lambda}^T, \boldsymbol{\mu}^T]^T \in \mathbb{R}^{14}$ and $\mathbf{u} \in \mathbb{R}^3$ be the state and input vectors respectively describing the dynamics in Equations (4.10) and (4.11), linearized about $(\bar{\mathbf{y}}, \bar{\mathbf{u}})$. One can derive the standard state-space representation

$$\dot{\mathbf{y}} = \mathbf{A} \mathbf{y} + \mathbf{B} \mathbf{u} \quad (4.14)$$

where $\mathbf{A} \in \mathbb{R}^{14 \times 14}$ is the state matrix

$$\mathbf{A} = \left[\begin{array}{c|cc|c|cc} \mathbf{0}_{7 \times 3} & & \mathbf{0}_{7 \times 4} & & \mathbf{I}_7 & & \\ \hline & 0 & 0 & 0 & A_{8,7} & & 0 & 0 & 0 & A_{8,14} \\ & 0 & 0 & A_{9,6} & 0 & & 0 & 0 & A_{9,13} & 0 \\ & 0 & 0 & 0 & 0 & & 0 & 0 & 0 & 0 \\ \mathbf{0}_{7 \times 3} & A_{11,4} & 0 & A_{11,6} & 0 & \mathbf{0}_{7 \times 3} & A_{11,11} & 0 & A_{11,13} & 0 \\ & 0 & A_{12,5} & 0 & A_{12,7} & & 0 & A_{12,12} & 0 & A_{12,14} \\ & A_{13,4} & 0 & A_{13,6} & 0 & & A_{13,11} & 0 & A_{13,13} & 0 \\ & 0 & A_{14,5} & 0 & A_{14,7} & & 0 & A_{14,12} & 0 & A_{14,14} \end{array} \right] \quad (4.15)$$

and $\mathbf{B} \in \mathbb{R}^{14 \times 3}$ is the input matrix,

$$\mathbf{B} = \left[\begin{array}{c|cc} & \mathbf{0}_{7 \times 3} & \\ \hline B_{8,1} & 0 & 0 \\ 0 & B_{9,2} & 0 \\ 0 & 0 & B_{10,3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & B_{13,2} & 0 \\ B_{14,1} & 0 & 0 \end{array} \right] \quad (4.16)$$

In the present framework, $\mathbf{I}_p$ represents the unit matrix with dimension p , while $\mathbf{0}_{q \times r}$ is the null matrix with dimension $q \times r$. Non-null and non-unity

terms in matrices $\mathbf{A}$ and $\mathbf{B}$ are, respectively:

$$A_{8,7} = -A_{9,6} = \frac{g(m_A + m_B)}{m} \quad (4.17)$$

$$A_{8,14} = -A_{9,13} = \frac{d}{L_A m} \quad (4.18)$$

$$A_{11,4} = A_{12,5} = -A_{11,6} = -A_{12,7} = -\frac{g(m_A + m_B)}{m_A L_B} \quad (4.19)$$

$$A_{11,11} = A_{12,12} = \frac{-d(m_A + m_B)}{L_B^2 m_A m_B} \quad (4.20)$$

$$A_{11,13} = A_{12,14} = A_{13,11} = A_{14,12} = \frac{d}{L_A L_B m_A} \quad (4.21)$$

$$A_{13,4} = A_{14,5} = \frac{g m_B}{m_A L_A} \quad (4.22)$$

$$A_{13,6} = A_{14,7} = -\frac{g(m_A + m_B)(m + m_A)}{m m_A L_A} \quad (4.23)$$

$$A_{13,13} = A_{14,14} = \frac{-d(m + m_A)}{L_A^2 m m_A} \quad (4.24)$$

$$B_{8,1} = B_{9,2} = \frac{1}{m} \quad (4.25)$$

$$B_{10,3} = \frac{1}{M} \quad (4.26)$$

$$B_{13,2} = -B_{14,1} = \frac{1}{m L_A} \quad (4.27)$$

4.2.2 FGDF Recursive Equations

Let $\mathbf{x} = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8]^T = [\xi_B, \zeta_B, \xi_A, \zeta_A, \dot{\xi}_B, \dot{\zeta}_B, \dot{\xi}_A, \dot{\zeta}_A]^T \in \mathbb{R}^{n_x}$ with $n_x = 8$ be the state vector representing payload oscillation dynamics only. The linear time-invariant state-space model

$$\dot{\mathbf{x}} = \mathbf{A}_{pl} \mathbf{x} + \mathbf{B}_{pl} \mathbf{u} \quad (4.28)$$

is extracted from Equations (4.15) and (4.16), where

$$\mathbf{A}_{pl} = \left[\begin{array}{cc|cc} \mathbf{0}_{4 \times 4} & & & \\ \hline A_{11,4} & 0 & A_{11,6} & 0 \\ 0 & A_{12,5} & 0 & A_{12,7} \\ A_{13,4} & 0 & A_{13,6} & 0 \\ 0 & A_{14,5} & 0 & A_{14,7} \end{array} \middle| \begin{array}{cc} \mathbf{I}_4 & \\ \hline A_{11,11} & 0 & A_{11,13} & 0 \\ 0 & A_{12,12} & 0 & A_{12,14} \\ A_{13,11} & 0 & A_{13,13} & 0 \\ 0 & A_{14,12} & 0 & A_{14,14} \end{array} \right] \quad (4.29)$$

and

$$\mathbf{B}_{pl} = \begin{bmatrix} \mathbf{0}_{6 \times 3} \\ 0 & B_{13,2} & 0 \\ B_{14,1} & 0 & 0 \end{bmatrix} \quad (4.30)$$

The reduced-order dynamic model defined by Equations (4.28)–(4.30) is reformulated using difference equations where the true state vector $\mathbf{x}^{(k)}$ at time $t^{(k)}$ evolves according to $\mathbf{x}^{(k+1)} = \Phi^{(k)} \mathbf{x}^{(k)} + \Psi^{(k)} \mathbf{u}^{(k)}$. The state transition matrix used in the estimation algorithm is $\Phi^{(k)} = \Phi = \exp(\mathbf{A}_{pl} \Delta t)$, where $\Delta t = t^{(k+1)} - t^{(k)}$ is the sampling interval. The control-input model is determined as $\Psi^{(k)} = \Psi = \mathbf{A}_{pl}^{-1}(\Phi - \mathbf{I}_8) \mathbf{B}_{pl}$. Equations (4.28)–(4.30) also provide:

$$a_1 = A_{8,7} \zeta_A + B_{8,1} u_1 \quad (4.31)$$

$$a_2 = A_{9,6} \xi_A + B_{9,2} u_2 \quad (4.32)$$

where $a_1 = \dot{v}_1$ and $a_2 = \dot{v}_2$ are vehicle acceleration components along $\mathbf{x}_E$ and $\mathbf{y}_E$, respectively. A key insight for the estimator design is that although the lateral and frontal swing angles are not directly measured, they are dynamically coupled to the vehicle's horizontal acceleration. By algebraically inverting Equations (4.31) and (4.32), a virtual measurement of the upper payload's swing angles, ξ_A and ζ_A , is formulated using only the vehicle's measured acceleration components (a_1, a_2) and known control inputs (u_1, u_2). The measurement model for the upper cable swing angles is thus derived as follows

$$\xi_A^{(k)} = \frac{a_2^{(k)} - B_{9,2} u_2^{(k)}}{A_{9,6}} \quad (4.33)$$

$$\zeta_A^{(k)} = \frac{a_1^{(k)} - B_{8,1} u_1^{(k)}}{A_{8,7}} \quad (4.34)$$

The observation $\mathbf{z}^{(k)} = [\xi_A^{(k)}, \zeta_A^{(k)}]^T$ with dimension $n_z = 2$ approximately tracks the considered dynamic process as in $\mathbf{z}^{(k)} = \mathbf{H} \mathbf{x}^{(k)}$, where

$$\mathbf{H} = \left[\mathbf{0}_{2 \times 2} \mid \mathbf{I}_2 \mid \mathbf{0}_{2 \times 4} \right] \quad (4.35)$$

is a time-invariant observation matrix. It is finally assumed that measurements are corrupted by noise $\boldsymbol{\nu}^{(k)}$ with variance $\mathbf{R}$, that is $\mathbf{z}^{(k)} = \mathbf{H} \mathbf{x}^{(k)} + \boldsymbol{\nu}^{(k)}$.

Given the time series of data $\mathbf{z}^{(0)}, \mathbf{z}^{(1)}, \mathbf{z}^{(2)}, \dots, \mathbf{z}^{(k)}$, the aim is to compute estimates to the state vector, namely $\tilde{\mathbf{x}}^{(0)}, \tilde{\mathbf{x}}^{(1)}, \tilde{\mathbf{x}}^{(2)}, \dots, \tilde{\mathbf{x}}^{(k)}$, by taking into account the model in Equations (4.28)–(4.30). Such a problem is solved by the FGDF method as shown in Section 3.4, where the gain matrix is computed through the formal minimization of a cost function, with no other assumption [91]. The estimated swing states $\tilde{\mathbf{x}}^{(k)}$ from the estimator serve as critical inputs to the control system designed in the next section.

Remark 10. *The global optimization of the FGDF follows the methodology detailed in Section 3.4. The procedure involves tuning the fading factor β by running a filter bank to find the value β_T that minimizes an averaged tuning statistic, $\overline{\chi^2}(\beta)$. Subsequently, an averaged scaling statistic, $\overline{\xi^2}(\beta_T)$, is used to estimate the true measurement noise covariance $\tilde{\mathbf{R}}$ and the true error covariance $\tilde{\mathbf{P}}^{(k)}$, ensuring the filter achieves globally optimal performance.*

4.3 Control System Design

Without loss of generality, a cascaded controller based on PX4 architecture and notation [59] is adopted to stabilize the octarotor across various flight modes, ranging from high-level position control to low-level attitude regulation (see Figure 4.2). While a comprehensive analysis of the PX4 control framework lies beyond the scope of this thesis, it is assumed that the vehicle is stabilized in accordance with specified flying qualities through appropriate tuning of the control parameters. Desired acceleration $\mathbf{a}_{sp} = [a_{sp_1}, a_{sp_2}, a_{sp_3}]^T$ is designed as

$$\mathbf{a}_{sp} = \mathbf{a}_{sp}^{(VC)} + \mathbf{a}_{sp}^{(PC)} \quad (4.36)$$

where $\mathbf{a}_{sp}^{(VC)}$ allows trajectory-following through the tracking of setpoint speed $\mathbf{v}_{sp}$ (Velocity Controller) while $\mathbf{a}_{sp}^{(PC)}$ implements Payload Control strategies. Let $k_p > 0$ and $k_v > 0$ be control gains. The following heuristic controller is proposed:

$$\mathbf{a}_{sp}^{(PC)} = \text{diag}(1, 1, 0) [k_p (\mathbf{r}_C - \mathbf{r}_H) + k_v (\mathbf{v}_C - \mathbf{v}_H)] \quad (4.37)$$

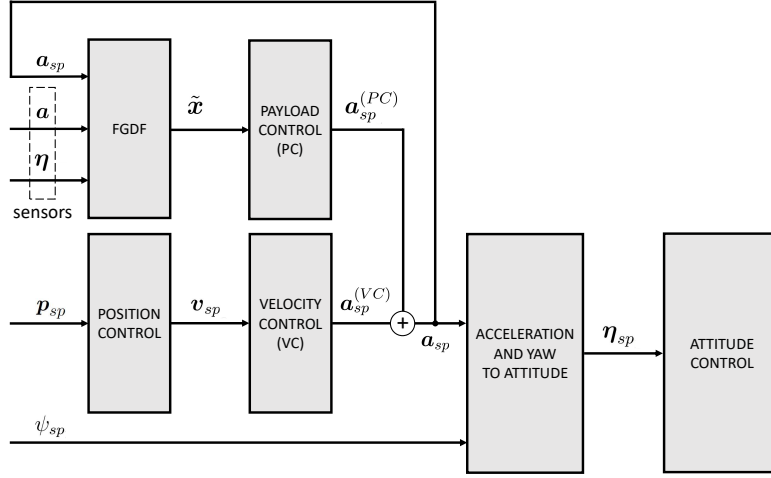


Figure 4.2: Multirotor control scheme based on PX4 autopilot architecture.

where $\mathbf{r}_C$ is the absolute position of the center of gravity C of the payload system, obtained as

$$\mathbf{r}_C = \frac{m_A \mathbf{r}_A + m_B \mathbf{r}_B}{m_A + m_B} \quad (4.38)$$

while $\mathbf{v}_C$ is the corresponding speed, namely

$$\mathbf{v}_C = \frac{m_A \mathbf{v}_A + m_B \mathbf{v}_B}{m_A + m_B} \quad (4.39)$$

Assuming $\mathbf{r}_H \approx \mathbf{r}$ and considering Equation (4.38), one has:

$$\mathbf{r}_C - \mathbf{r}_H \approx \mathbf{r}_C - \mathbf{r} \approx \frac{m_A (\mathbf{r}_A - \mathbf{r}) + m_B (\mathbf{r}_B - \mathbf{r})}{m_A + m_B} \quad (4.40)$$

Taking into account Equations (4.3) and (4.4), one derives

$$\begin{aligned} \mathbf{r}_C - \mathbf{r}_H \approx & L_A [\sin \zeta_A, \quad -\sin \xi_A \cos \zeta_A, \quad \cos \xi_A \cos \zeta_A]^T \\ & + \frac{m_B L_B}{m_A + m_B} [\sin \zeta_B, \quad -\sin \xi_B \cos \zeta_B, \quad \cos \xi_B \cos \zeta_B]^T \end{aligned} \quad (4.41)$$

In the same way, by considering Equations (4.7) and (4.8), it follows:

$$\begin{aligned} \mathbf{v}_C - \mathbf{v}_H \approx & L_A [\dot{\zeta}_A \cos \zeta_A, \quad -\dot{\xi}_A \cos \xi_A \cos \zeta_A + \dot{\zeta}_A \sin \xi_A \sin \zeta_A, \\ & -\dot{\xi}_A \sin \xi_A \cos \zeta_A - \dot{\zeta}_A \cos \xi_A \sin \zeta_A]^T \\ & + \frac{m_B L_B}{m_A + m_B} [\dot{\zeta}_B \cos \zeta_B, \quad -\dot{\xi}_B \cos \xi_B \cos \zeta_B + \dot{\zeta}_B \sin \xi_B \sin \zeta_B, \\ & -\dot{\xi}_B \sin \xi_B \cos \zeta_B - \dot{\zeta}_B \cos \xi_B \sin \zeta_B]^T \end{aligned} \quad (4.42)$$

Intuitively, $\mathbf{a}_{sp}^{(PC)}$ adds proportional-derivative feedback on the projected center of gravity displacement and velocity in the horizontal plane, providing a swing-damping contribution for the overall payloads system. In this framework swing angles and rates are obtained from the estimation algorithm proposed in Section 4.2 and then used in the control logic implementation according to Equations (4.41) and (4.42).

Remark 11. *The controller described in Equation (4.37) relies on the heuristic feedback of the relative position of payload system's center of gravity with respect to the hook point. This approach follows the one adopted in Ref. [63], where a single payload is considered, being representative of a more general case. However, in situations where $m_B = 0$ kg, such as delivery tasks where the lower payload and cable may be dropped, the closed-loop system reverts to the single load scenario and $\mathbf{r}_C \equiv \mathbf{r}_A$. In this scenario, the relative position and velocity of the remaining load is used as a feedback term proportional to the swing angles and rates, thus inferring flexibility and continuity in usage of the same control law over the entire mission.*

The choice of this heuristic, center-of-gravity-based controller is deliberate. While a model-based approach such as LQR could be designed using the linearized dynamics, the proposed PD controller offers significant practical advantages. First, it is computationally simple and easy to tune with intuitive gains (k_p, k_v) . Second, by acting on the combined center of gravity, it is inherently more robust to uncertainties in the individual payload masses (m_A, m_B) , as the feedback signal naturally adapts to the mass distribution. Finally, as detailed in Remark 11, this structure provides a seamless transition to the single-payload scenario, a crucial feature for missions involving multiple parcels delivery. These benefits of simplicity and robustness are paramount for deployment on resource-constrained flight controllers in real-world operational environments.

Remark 12. *In accordance with the PX4 autopilot control architecture adopted in Section 5.5, a systematic and effective approach is employed to reconstruct the input force, defined as $\mathbf{u} = \mathbf{f}_c + \mathbf{f}_d$. Under the as-*

sumption of ideal electric motor behavior, the thrust magnitude is approximated as $\tilde{T} = (m + m_A + m_B) \|\mathbf{a}_{sp}\|$. Taking into account the vehicle's attitude dynamics, the control force expressed in the inertial frame $\mathcal{F}_E$ is estimated as $\tilde{\mathbf{f}}_c = \mathbf{T}_{BE}^T(\tilde{\boldsymbol{\eta}}) [0, 0, -\tilde{T}]^T$. With regard to the characterization of external disturbances, it is noted that the velocity control module shown in Figure 4.2 is implemented as a conventional PID controller [59]. The latter converts the velocity-tracking error into a desired acceleration, expressed as $\mathbf{a}_{sp}^{(VC)} = \mathbf{a}_{sp}^{(VC)}\big|_P + \mathbf{a}_{sp}^{(VC)}\big|_I + \mathbf{a}_{sp}^{(VC)}\big|_D$, where the subscripts P , I , and D respectively denote the proportional, integral, and derivative contributions. By considering quasi-steady external disturbances and taking advantage of the integral component's well-known ability to reject such disturbances [93], the disturbance force is conservatively modeled as $\tilde{\mathbf{f}}_d = -(m + m_A + m_B) \mathbf{a}_{sp}^{(VC)}\big|_I$. Accordingly, the estimated total input force is given by $\tilde{\mathbf{u}} = \mathbf{T}_{BE}^T(\tilde{\boldsymbol{\eta}}) [0, 0, -\tilde{T}]^T - (m + m_A + m_B) \mathbf{a}_{sp}^{(VC)}\big|_I$, where the latter contribution accounts for the aerodynamic drag force exerted on the whole system, payloads included.

4.4 Stability and Control Analysis

In what follows, the stability of the coupled slung-load system described in Equations (4.10) and (4.11), with the control law $\mathbf{u}_{sp} = \mathbf{u}^{(VC)} + \mathbf{u}^{(PC)}$ detailed in Equations (4.36) and (4.37), is addressed within a linear framework. Let $\mathbf{e} = \mathbf{y}_d - \mathbf{y}$ be the error with respect to the desired state $\mathbf{y}_d = [\bar{\mathbf{p}}^T, \mathbf{0}_{1 \times 11}]^T$. The error dynamics are obtained from Equation (4.14) as

$$\dot{\mathbf{e}} = \mathbf{A} \mathbf{e} - \mathbf{B} \mathbf{u} \quad (4.43)$$

Based on simple manipulations of Equations (4.36) and (4.37) in the linear framework, it is $\mathbf{u} = \mathbf{K} \mathbf{e}$, where $\mathbf{K} \in \mathbb{R}^{3 \times 14}$. The feedback-controlled system evolves according to $\dot{\mathbf{e}} = \hat{\mathbf{A}} \mathbf{e}$, where $\hat{\mathbf{A}} = \mathbf{A} - \mathbf{B} \mathbf{K}$ and

$$\mathbf{K} = m \begin{bmatrix} K_{\delta_z} & 0 & 0 & 0 & \frac{k_p L_B \mu_B}{\mu_B + 1} & 0 & k_p L_A & K_{\epsilon_z} & 0 & 0 & 0 & \frac{k_v L_B \mu_B}{\mu_B + 1} & 0 & k_v L_A \\ 0 & K_{\delta_y} & 0 & -\frac{k_p L_B \mu_B}{\mu_B + 1} & 0 & -k_p L_A & 0 & 0 & K_{\epsilon_y} & 0 & -\frac{k_v L_B \mu_B}{\mu_B + 1} & 0 & -k_v L_A & 0 \\ 0 & 0 & K_{\delta_z} & 0 & 0 & 0 & 0 & 0 & 0 & K_{\epsilon_z} & 0 & 0 & 0 & 0 \end{bmatrix} \quad (4.44)$$

with $\mathbf{K}_\delta = \text{diag}(K_{\delta_x}, K_{\delta_y}, K_{\delta_z})$ and $\mathbf{K}_\epsilon = \text{diag}(K_{\epsilon_x}, K_{\epsilon_y}, K_{\epsilon_z})$ positive gain matrices, and $\mu_B = m_B/m_A$.

To outline the dynamic modes in $\hat{\mathbf{A}}$, the error vector is conveniently rearranged according to the order

$$\mathbf{y}_{flip} = [r_1, \zeta_B, \zeta_A, v_1, \dot{\zeta}_B, \dot{\zeta}_A, r_2, \xi_B, \xi_A, v_2, \dot{\xi}_B, \dot{\xi}_A, r_3, v_3]^T \quad (4.45)$$

It is straightforward to verify that the closed-loop dynamic matrix assumes the block-diagonal form:

$$\hat{\mathbf{A}}_{flip} = \left[\begin{array}{c|c|c} \hat{\mathbf{A}}_{fs} & \mathbf{0}_{6 \times 6} & \mathbf{0}_{6 \times 2} \\ \hline \mathbf{0}_{6 \times 6} & \hat{\mathbf{A}}_{ls} & \mathbf{0}_{6 \times 2} \\ \hline \mathbf{0}_{2 \times 6} & \mathbf{0}_{2 \times 6} & \hat{\mathbf{A}}_v \end{array} \right] \quad (4.46)$$

where

$$\hat{\mathbf{A}}_{fs}(\nu_{fs1}, \nu_{fs2}, \nu_{fs3}, \nu_{fs4}, \nu_{fs5}, \nu_{fs6}) \quad (4.47)$$

with eigenvalues ν_{fsj} , $j = 1, \dots, 6$, is the subsystem matrix that describes the coupling between the vehicle's forward motion and the payload frontal swing motion,

$$\hat{\mathbf{A}}_{ls}(\nu_{ls1}, \nu_{ls2}, \nu_{ls3}, \nu_{ls4}, \nu_{ls5}, \nu_{ls6}) \quad (4.48)$$

with eigenvalues ν_{lsk} , $k = 1, \dots, 6$, is the subsystem matrix related to the coupling between the vehicle's lateral motion and the payloads lateral swing state, and

$$\hat{\mathbf{A}}_v(\nu_{v1}, \nu_{v2}) \quad (4.49)$$

with eigenvalues ν_{vj} , $j = 1, \dots, 2$, is the subsystem matrix accounting for the vehicle's vertical dynamics. In the most general case, the closed-form expressions of the eigenvalues in Equations (4.47)–(4.49) cannot be determined. As pointed out in Section 3.2.4, numerical methods can be employed for a dedicated analysis of the system's eigenstructure [87]. Also, there exist $\mathbf{K}$ with the structure in Equation (4.44) such that Equations (4.47)–(4.49) are Hurwitz.

For compactness, the useful ratios $r = L_A/L_B$, $\mu_B = m_B/m_A$, and $\mu_0 = m/m_A$ are defined. The open-loop stability analysis is performed for the case when $\mathbf{K} = \mathbf{0}_{3 \times 14}$ and $d = 0$ in $\mathbf{D} = \text{diag}(0, 0, 0, d, d, d, d)$, corresponding

to the undamped dynamics. This represents the most conservative scenario and reveals the intrinsic oscillatory modes that the controller must stabilize. In this case, $\hat{\mathbf{A}}_{fs}$ is characterized by null eigenvalues with multiplicity 2 and a set of purely complex poles given by

$$\nu_{fs1,2} = \pm j \sqrt{\frac{g}{L_A}} \sqrt{\frac{\theta - \sqrt{\theta^2 - 4\Delta}}{2}} \quad (4.50)$$

and

$$\nu_{fs3,4} = \pm j \sqrt{\frac{g}{L_A}} \sqrt{\frac{\theta + \sqrt{\theta^2 - 4\Delta}}{2}} \quad (4.51)$$

provided that

$$\theta = (1 + \mu_B) \left(r + 1 + \frac{1}{\mu_0} \right), \quad (4.52)$$

$$\Delta = r(1 + \mu_B) \left[(1 + \mu_B) \left(1 + \frac{1}{\mu_0} \right) - \mu_B \right] \quad (4.53)$$

Owing to the symmetry of the problem, similar considerations hold for $\hat{\mathbf{A}}_{ls}$, which is defined by null eigenvalues with multiplicity 2 and a set of purely complex poles given by:

$$\nu_{ls1,2} = \pm j \sqrt{\frac{g}{L_A}} \sqrt{\frac{\theta - \sqrt{\theta^2 - 4\Delta}}{2}} \quad (4.54)$$

and

$$\nu_{ls3,4} = \pm j \sqrt{\frac{g}{L_A}} \sqrt{\frac{\theta + \sqrt{\theta^2 - 4\Delta}}{2}} \quad (4.55)$$

Finally, $\hat{\mathbf{A}}_v$ is defined by two null eigenvalues. Given the absence of dissipative effects related to the aerodynamic drag (which is set to zero), all dynamic modes related to Equations (4.50)–(4.55) are representative of a purely oscillating motion about the equilibrium with no damping. The two purely imaginary pairs in the forward and lateral subsystems correspond to the classic double-pendulum modes: an in-phase (lower frequency) and an out-of-phase (higher frequency) swing of the two loads. The expressions in Equations (4.50) and (4.51) (and their lateral counterparts) reduce to

well-known limits: as $m_B \rightarrow 0$ or $L_B \rightarrow 0$ the lower mode collapses and the higher mode approaches $\sqrt{g/L_A}$, namely the single-pendulum result.

These analytical expressions for the open-loop modes are not merely a theoretical curiosity. They confirm that the controller is tasked with stabilizing two distinct, undamped oscillatory modes in each plane and provide crucial physical insight into how the system's natural frequencies depend on cable lengths and mass distribution. The control design does not assume or require any damping for stability, making it robust to variations in actual aerodynamic conditions.

CHAPTER 5

Numerical Validation

5.1 Two-Time-Scale Control Validation

This section validates the two-time-scale cooperative transportation controller (CTC) on a formation of two octarotors carrying a cable-suspended payload. First, the simulation platform is detailed, then performance indicators focused on safety, efficiency, and swing attenuation are introduced. Two representative scenarios are studied: (1) a hover-to-hover repositioning with gain perturbations to assess robustness and (2) a waypoint-tracking mission emulating an air delivery operation. All results are compared against a conventional velocity controller (VC) baseline under identical actuator limits, delays, and sensor noise.

5.1.1 Simulation Setup

The proposed control method is validated through high-fidelity numerical simulations in MATLAB®-Simulink environment with a 4-th order Runge-Kutta solver at a frequency of 200 Hz. The agents of the formation are two identical octarotors modeled as rigid bodies. The payload

is assumed to be a point mass attached through viscoelastic cables to a hook point H_i , coincident with rotorcraft center of gravity P_i . The suspended mass is affected by aerodynamic drag according to Equation (3.5), provided $A_l = \pi R_l^2$ is the frontal area of a sphere with radius $R_l = 0.5$ m and drag coefficient $C_{dl} = 0.5$. The octarotor lifting vehicles are characterized by a set of 4 counter-rotating propulsion units (see Figure 2.1). The cross-shaped planar configuration is described by $STA_{R_1} = STA_{R_2} = STA_{R_5} = STA_{R_6} = -STA_{R_3} = -STA_{R_4} = -STA_{R_7} = -STA_{R_8} = -0.690$ m, $BL_{R_1} = BL_{R_4} = BL_{R_5} = BL_{R_8} = -BL_{R_2} = -BL_{R_3} = -BL_{R_6} = -BL_{R_7} = 0.690$ m, $WL_{R_1} = WL_{R_2} = WL_{R_3} = WL_{R_4} = -0.024$ m, and $WL_{R_5} = WL_{R_6} = WL_{R_7} = WL_{R_8} = -0.238$ m. The most relevant system parameters are reported in Table 5.1.

Rotor aerodynamics is described by the classical Blade Element Momentum Theory. For control purposes only the quadratic contributions to thrust and torque are retained, namely $T_j = k_T \Omega_j^2$ and $Q_j = k_Q \Omega_j^2$, where the dimensional coefficients scale with air density according to $k_T = \gamma \bar{k}_T$ and $k_Q = \gamma \bar{k}_Q$ with $\gamma = \rho/\bar{\rho}$. Because of the wake interference, the upper rotors (1–4) are more efficient than the lower ones (5–8), namely $\bar{k}_T|_{up} = 2.90 \cdot 10^{-3}$ N/(rad/s)² and $\bar{k}_T|_{down} = 2.20 \cdot 10^{-3}$ N/(rad/s)² [94]. Experimental calibration at the reference density $\bar{\rho} = 1.1229$ kg/m³ gives $\bar{k}_Q = 1.25 \cdot 10^{-4}$ N m/(rad/s)². The induced-velocity equation is solved at each time step with MATLAB[®] implementation of Dekker’s hybrid algorithm, using an initial guess of -10 m/s [95]. Atmospheric properties vary with altitude according to the International Standard Atmosphere (ISA) model [96]. The elastic force $\mathbf{f}_{li}$ in Equation (3.7) is calculated from $\mathbf{f}_{li} = (K\Delta L_i + C\dot{\Delta L}_i) \hat{\mathbf{c}}_i$, that with respect to the classical Hooke’s model also accounts for cable internal damping by the coefficient C .

Guidance, navigation, and control tasks are performed following the methodology introduced in Section 3.1.2, where the i -th multirotor is required to track an Earth-fixed acceleration set-point $\mathbf{a}_{sp_i}$ (see Figure 3.2). In this framework, desired acceleration for the i -th multirotor is obtained as $\mathbf{a}_{sp_i} = \mathbf{a}_{sp_i}^{(CTC)} = \mathbf{u}_i/m$, where $\mathbf{u}_i$ is calculated according to the CTC in Equations (3.30)–(3.37). To get a better insight into the performance of the cooperative controller, a classical stabilization method is discussed as an

Table 5.1: Relevant system parameters.

Parameter	Symbol	Value	Units
Multirotor			
Mass	m	70	kg
Center of gravity position	$STA_P = BL_P$	0	m
	WL_P	-0.15	m
Moments of inertia	J_{11}	10.61	kg m ²
	J_{22}	10.31	kg m ²
	J_{33}	19.74	kg m ²
	J_{12}	0.037	kg m ²
	J_{13}	-0.043	kg m ²
	J_{23}	-0.003	kg m ²
Center of pressure position	$STA_{CP} = BL_{CP}$	0	m
	WL_{CP}	-0.125	m
Frame drag areas	$A_1 = A_2$	0.22	m ²
	A_3	1.03	m ²
Propeller			
Number of blades	n_b	2	
Radius	R	0.5	m
Mean aerodynamic chord	$\bar{c}$	0.086	m
Chord @ 75% R	c_{75}	0.103	m
Lift curve slope	a	5.9	1/rad
Pre-cone angle	a_0	0	rad
Root pitch angle	θ_0	0.7854	rad
Total twist	θ_t	-0.6981	rad
Load			
Mass	m_l	100	kg
Reference area	A_l	0.785	m ²
Drag coefficient (sphere)	C_{dl}	0.5	
Cable			
Nominal length	L	15	m
Hooke's constant	K	90 950	N/m
Damping coefficient	C	215	N s/m
Hook point position	$STA_H = BL_H$	0	m
	WL_H	-0.2	m

alternative to the proposed solution. Taking into account Figure 3.2, it is assumed that set-point acceleration of the i -th vehicle is generated by Velocity Controller (VC) rather than by cooperative transportation algorithm, namely $\mathbf{a}_{sp_i} = \mathbf{a}_{sp_i}^{(VC)}$. Provided a detailed analysis of PX4 control functions is out of the scope of this thesis, it is supposed that inner-loop stabilization of attitude parameters is performed with prescribed requirements in terms of accuracy and rapidity, according to Remark 2. Non-ideal sensor behavior is considered in terms of noise and sensor positioning errors. The on-board computer is located at $STA_{PIX} = BL_{PIX} = 0$ m and $WL_{PIX} = -0.15$ m, with axes aligned with $\mathcal{F}_B$ -axes. In order to simulate the effects of sensor noise on attitude control, zero-mean Gaussian white noise is added to each Euler angle and angular velocity measurement, with standard deviations $\sigma_\alpha = 0.5$ deg and $\sigma_\omega = 0.1$ deg/s, respectively. The on-board computer generates control signals at a sampling frequency of 100 Hz (sample time $\Delta_t = 0.01$ s). Finally, a fixed time delay $t_d = 0.04$ s is added to account for local processing and communication issues between the agents of the formation. For safety, every motor command is clipped to the valid SBUS throttle range, namely 202 to 1802 counts, such that 202 corresponds to zero-throttle (no thrust) and 1802 corresponds to full throttle (maximum thrust). Each electric propulsion unit, consisting of an Electronic Speed Controller (ESC) and a brushless motor, is modeled by a first-order transfer function with time constant $\tau_{em} = 0.05$ s. This transfer function captures the combined electrical and mechanical lags of the ESC-motor pair, mapping the saturated rotor-speed command to the actual shaft speed delivered by the motor.

5.1.2 Performance Indicators

Indicators are defined to characterize the performance of the cooperative transportation controller with respect to the standard velocity controller.

- Maneuver time: t_m . It is the total time required to conclude a prescribed maneuver, according to a stop criterion. Based on the expected asymptotic nature of convergence, the maneuver is considered terminated at time t_m when the total error $[\mathbf{x}^T, \mathbf{z}^T]^T$ falls below a certain threshold,

namely $\|[\mathbf{x}^T, \mathbf{z}^T]^T\| < 0.25$.

- Average swing angle: ζ_{av} . The oscillation angle ζ is defined by the intersection of (i) the local-vertical plane that contains the multirotors (plane 1) and (ii) the plane that contains all the objects of the formation, including the payload (plane 2). Given $\hat{\mathbf{c}}_1$ and $\hat{\mathbf{c}}_2$, the unit vector orthogonal to plane 2 is obtained as $\mathbf{e}_\perp = (\hat{\mathbf{c}}_2 \times \hat{\mathbf{c}}_1) / \|\hat{\mathbf{c}}_2 \times \hat{\mathbf{c}}_1\|$. The oscillation angle is retrieved as $\zeta = -\arcsin(\mathbf{e}_{\perp z})$, provided $\mathbf{e}_{\perp z}$ is the third component of $\mathbf{e}_\perp$ as expressed in $\mathcal{F}_E$. According to the selected convention, a positive oscillation angle is obtained from positive rotation about vector $\mathbf{r}_1 - \mathbf{r}_2$. Finally, the average swing angle is calculated as $\zeta_{av} = I_\zeta(t_m)/t_m$, where the integral

$$I_\zeta(t_m) = \int_0^{t_m} |\zeta(s)| ds \quad (5.1)$$

is evaluated over maneuver time and provides an indication of how much and how long plane 2 remains inclined with respect to plane 1.

- Average swing rate: $\dot{\zeta}_{av}$. It is calculated as $\dot{\zeta}_{av} = I_{\dot{\zeta}}(t_m)/t_m$, where the integral

$$I_{\dot{\zeta}}(t_m) = \int_0^{t_m} |\dot{\zeta}(s)| ds \quad (5.2)$$

is adopted to characterize the swing rate during the entire maneuver.

- Load kinetic energy, E_{kinl} . It is given by

$$E_{kinl} = \frac{1}{2} m_l \mathbf{v}_l^T \mathbf{v}_l \quad (5.3)$$

and provides a direct measure of residual swing and undesired motion.

- Total propulsive energy, E_{prop} . It is the mechanical energy delivered by electrical motors to propellers, calculated by the integral

$$E_{prop} = \int_0^{t_m} \sum_{j=1}^8 P_{shj}(s) ds \quad (5.4)$$

where $P_{shj} = Q_j \Omega_j$ is the output shaft power generated by the j -th motor, $j = 1, \dots, 8$, obtained from required torque Q_j and angular

rate Ω_j under the assumption of null friction torque. Without loss of generality, E_{prop} is calculated for UAV 1 only.

5.1.3 Test Cases

In the following simulations, consider the formation identified by desired geometry $\mathbf{a} = [0, -10, 0]^T$ m. With this in mind, Lemma 3.1.1 gives $a = 10$ m, $\bar{d}_{x_2} = -\bar{d}_{x_1} = 0$ m, and $\bar{d}_{y_2} = -\bar{d}_{y_1} = -5$ m. Coefficients in Equation (3.25) result to be $p_0 = -1.69 \cdot 10^{14}$ N²m², $p_1 = 3.38 \cdot 10^{13}$ N²m, $p_2 = 5.07 \cdot 10^{12}$ N², $p_3 = -1.352 \cdot 10^{12}$ N²/m, and $p_4 = 6.76 \cdot 10^{10}$ N²/m². As predicted by theory, 4 real distinct solutions are provided (3 positive, 1 negative). The only solution that satisfies $\bar{L} > L$ is $\bar{L} = 10.0044$ m, related to $\overline{\Delta L} = \bar{L} - L = 0.0044$ m cable elongation. Given $\bar{L}$, one derives $\bar{d}_{z_2} = \bar{d}_{z_1} = -3.6105$ m from Equation (3.23), where $M = 2m + m_l = 240$ kg. From Equations (A.2) and (A.3) it is $\bar{\mathbf{c}}_1 = [0, -5, 8.6653]^T$ m and $\bar{\mathbf{c}}_2 = [0, 5, 8.6653]^T$ m, such that the unit vectors aligned with the cables are, respectively, $\hat{\mathbf{c}}_1 = [0, -0.4998, 0.8662]^T$ and $\hat{\mathbf{c}}_2 = [0, 0.4998, 0.8662]^T$. Trim forces are finally derived according to Equation (A.5), where $\bar{\mathbf{u}}_1 = [0, 282.93, -1176.80]^T$ N and $\bar{\mathbf{u}}_2 = [0, -282.93, -1176.80]^T$ N. Based on simple geometric considerations, desired attitude of vehicles results to be $\boldsymbol{\eta}_{SP_2} = -\boldsymbol{\eta}_{SP_1} = -[\arctan(-\bar{u}_{y_1}/\bar{u}_{z_1}), 0, 0]^T = [-0.2359, 0, 0]^T$ rad, provided $\psi_{sp_2} = \psi_{sp_1} = 0$ rad. Control gain matrices are $\mathbf{K}_\delta = 0.25 \cdot \mathbf{I}_3$ 1/s², $\mathbf{K}_\epsilon = 2.25 \cdot \mathbf{I}_3$ 1/s, $\mathbf{K}_\rho = 0.1 \cdot \mathbf{I}_3$ 1/s², and $\mathbf{K}_\sigma = 0.1 \cdot \mathbf{I}_3$ 1/s. For the given selection of control gain matrices, the eigenvalues of $\mathbf{H}$ in Equation (B.11) (degenerate slow subsystem) are $-0.051 \pm j0.31$ with multiplicity 3. On the other hand, the eigenvalues of $\bar{\mathbf{H}}$ in Equation (B.13) (boundary-layer fast subsystem) are given by -0.12 and -2.13 , both with multiplicity 6. From the former, one obtains $f_{slow} \approx 0.008$ Hz, while considering the slowest eigenvalues of the fast dynamics one has $f_{fast} \approx 0.02$ Hz. For the attitude loop, which relies on the PX4 architecture, it is safe to assume $f_{att} \approx 3$ Hz [59]. These values support the validity of the three-time-scale separation claimed in Remark 2. With respect to Equation (3.37), a nonlinear system solver based on Levenberg-Marquardt method is implemented through the

MATLAB[®] routine *fsolve*, initialized with null first-guess solution [97].

Test Case 1

In the first simulation test case, the multirotors start from the initial state defined by $\mathbf{r}_1(0) = [0, 5, -15]^T$ m, $\mathbf{r}_2(0) = [0, -5, -15]^T$ m, and $\mathbf{v}_1(0) = \mathbf{v}_2(0) = [0, 0, 0]^T$ m/s, with the payload at rest. In such equilibrium condition, payload position is given by $\mathbf{p}_l(0) = [0, 0, -6.3347]^T$ m and formation center of gravity is identified by $\mathbf{r}(0) = [0, 0, -11.3895]^T$ m. For $t > 5$ s, the reference point is a constant equal to $\mathbf{r}_r(t) = \mathbf{r}(0) + [15, 0, 0]^T$ m, thus simulating a sample hover-to-hover maneuver. To probe robustness, the fast subsystem gain $\mathbf{K}_z$, responsible for formation control and swing attenuation in the two-time-scale architecture, is perturbed by $\pm 20\%$. The resulting trajectories are reported in Figures 5.1–5.3, where the CTC is compared to the baseline VC.

Figure 5.1 characterizes the payload state in terms of oscillation angle ζ and rate $\dot{\zeta}$, as defined in Section 5.1.2. The CTC reduces both variables relative to the VC, demonstrating faster decay and more damping.

In Figure 5.2 both load kinetic energy and UAV 1 propulsive energy are reported as a function of the maneuver time. The cooperative controller significantly reduces the former throughout the trajectory, indicating a substantial suppression of pendular motion. Interestingly, this improved oscillation behavior is obtained with a minimal additional energy consumption. The propulsive energy is only slightly increased compared to the velocity controller, despite the enhanced stabilization performance, due to the longer maneuver time.

Figures 5.1 and 5.2 also include $\pm 20\%$ perturbations of the fast-subsystem gain $\mathbf{K}_z$. The perturbed curves closely bracket the nominal CTC response, confirming robustness to moderate gain variations. Analogously, energy histories show minimal deviation from nominal. Together with Table 5.2, these results demonstrate that the proposed CTC outperforms the VC even under non-nominal operating conditions.

To further assess the validity of the cooperative control architecture, the load factor components experienced by the suspended payload are analyzed.

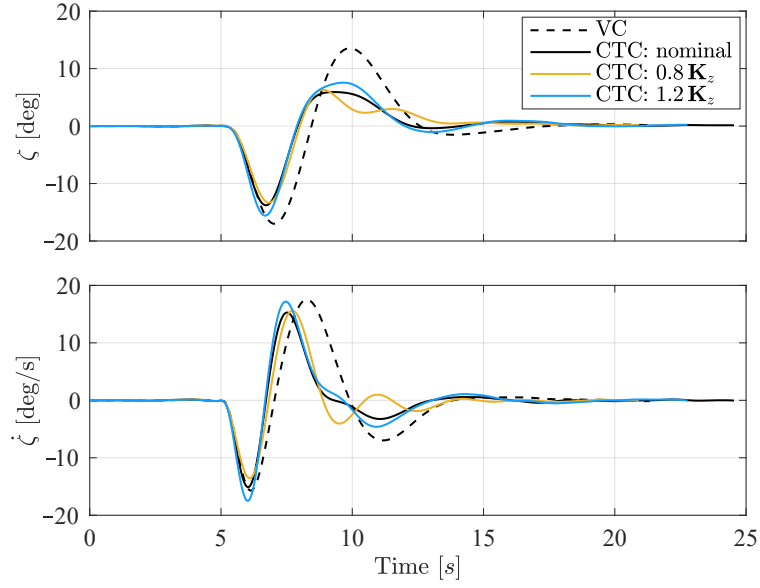


Figure 5.1: Test Case 1: Oscillation angle and rate in the presence of commanded acceleration as generated by cooperative transportation controller (CTC) and velocity controller (VC).

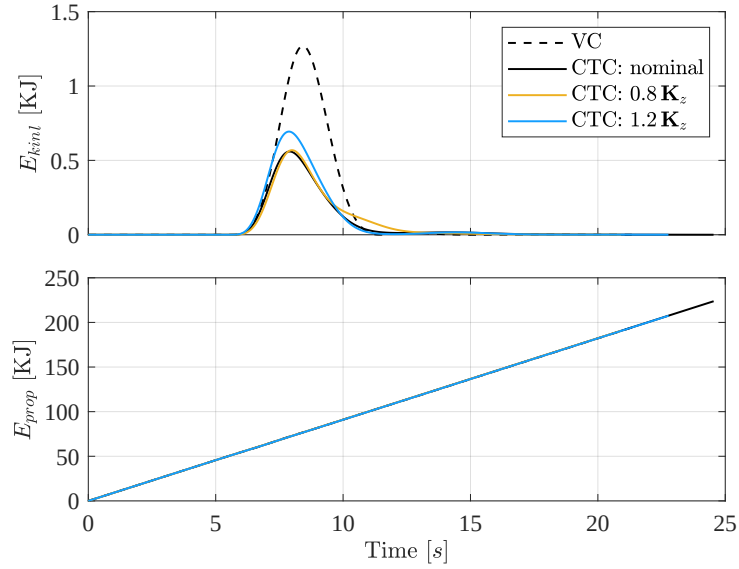


Figure 5.2: Test Case 1: System energy contributions in the presence of commanded acceleration as generated by cooperative transportation controller (CTC) and velocity controller (VC).

5.1. TWO-TIME-SCALE CONTROL VALIDATION

Table 5.2: Test Case 1: CTC robustness to $\pm 20\%$ perturbations in $\mathbf{K}_z$.

Indicator	Cooperative Transportation Controller (CTC)		
	Nominal	$0.8 \mathbf{K}_z$ (-20%)	$1.2 \mathbf{K}_z$ ($+20\%$)
t_m [s]	24.54	20.91 (-14.8%)	22.77 (-7.21%)
$I_\zeta(t_m)$ [deg s]	39.67	38.90 (-1.94%)	46.44 ($+17.07\%$)
ζ_{av} [deg]	1.61	1.86 ($+15.08\%$)	2.04 ($+26.17\%$)
$I_{\dot{\zeta}}(t_m)$ [deg]	0.138	0.140 (-1.03%)	0.231 ($+63.64\%$)
$\dot{\zeta}_{av}$ [deg s]	0.0058	0.0067 ($+16.15\%$)	0.0102 ($+76.37\%$)
E_{prop} [Wh]	223.74	190.65 (-14.8%)	207.53 (-7.24%)

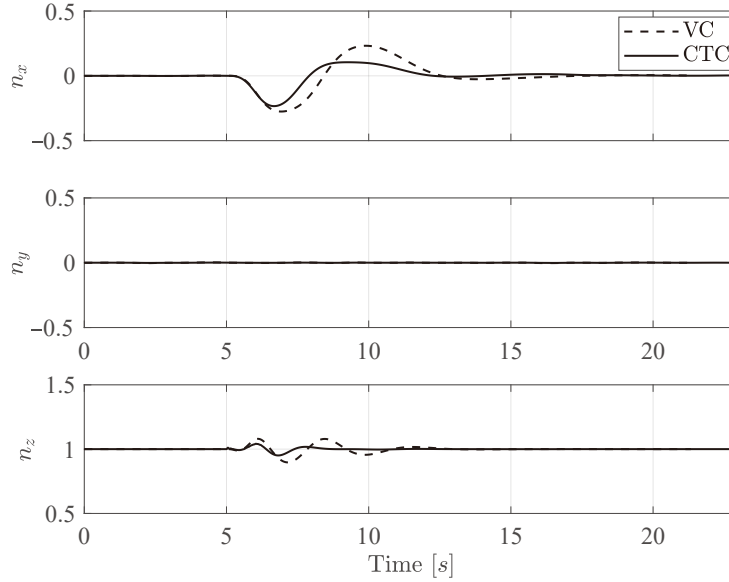


Figure 5.3: Test Case 1: Load factor components along the primary translational axis in $\mathcal{F}_E$ in the presence of commanded acceleration as generated by cooperative transportation controller (CTC) and velocity controller (VC).

The load factor, defined as the ratio of the total inertial force to the gravitational force alone, is given by $\mathbf{n} = (\mathbf{g} - \mathbf{a}_t)/\|\mathbf{g}\|$, providing an indication of the dynamic effort imposed on the cables and payload system. Figure 5.3 illustrates the time evolution of the load factor components along the primary translational axis in $\mathcal{F}_E$. These results confirm the effectiveness of the CTC

in attenuating fluctuations compared to the conventional velocity controller. This reduction in load factor excursions directly correlates with a decrease in payload oscillation and tension variability along the suspension cables. The outcome demonstrates the controller's ability to stabilize the suspended mass while preserving formation and tracking performance, thereby enhancing both safety and reliability during transport operations. Finally, Table 5.3 summarizes the most relevant performance indicators for the first simulation test case.

Table 5.3: Test Case 1: Relevant performance indicators.

Index	VC	CTC
t_m [s]	21.33	24.54 (+15.05%)
$I_\zeta(t_m)$ [deg s]	67.39	39.67 (-41.14%)
ζ_{av} [deg]	3.16	1.61 (-48.84%)
$I_\dot{\zeta}(t_m)$ [deg]	0.15	0.14 (-8.26%)
$\dot{\zeta}_{av}$ [deg/s]	0.0072	0.0058 (-20.26%)
E_{prop} [Wh]	194.29	223.74 (+15.05%)

Test Case 2

In the second test case, the formation is required to track a sequence of six waypoints $\mathbf{W}\mathbf{P}_i = [x_i, y_i, z_i]^T$, $i = 1, \dots, 6$. Following from the formulation of Section 3.1.2, the i -th vehicle's desired position is based on the reference point trajectory and is given by $\bar{\mathbf{r}}_i = \mathbf{r}_r + \bar{\mathbf{d}}_i$. Without loss of generality, a waypoint is considered to be reached when the positioning error of UAV 1, defined as $\varepsilon_P = \|\mathbf{W}\mathbf{P}_i - \mathbf{r}_1\|$, remains below 0.5 m for at least 5 s. This simulation scenario envisages a sample urban parcel-delivery mission as detailed in Table 5.4.

Figure 5.4 depicts the reference and the multirotors trajectories over the horizontal plane $\mathbf{x}_E\text{--}\mathbf{y}_E$ for the two control strategies, both of which achieve accurate waypoint tracking. Despite receiving an acceleration setpoint, rather than a direct position setpoint as reference input, the CTC allows for

Table 5.4: Test Case 2: Waypoints of the delivery mission.

i -th Waypoint	x_i [m]	y_i [m]	z_i [m]
1	0	0	−11.39
2	10	0	−11.39
3	20	−10	−11.39
4	20	10	−11.39
5	30	0	−11.39
6	30	10	−11.39

slightly smoother transitions, especially near abrupt path changes, suggesting enhanced formation coordination.

In Figure 5.5 the temporal evolution of the swing state is reported. The cooperative methodology reduces both the peak amplitude and the duration of oscillations during and after each maneuver, particularly evident around 0 s, 20 s, and 50 s. The results demonstrate that the proposed controller attenuates payload swing far more effectively than the baseline scheme, yet preserves path-following accuracy. Such improvements are critical in practical delivery tasks, where minimizing payload motion enhances safety, reliability, and energy efficiency. Test Case 2 is specifically designed to emulate a typical aerial delivery mission: the formation is required to follow a sequence of waypoints while concurrently damping the pendular motion of the suspended mass. The observed performance therefore satisfies the principal requirements of current unmanned logistics practice. Quantitative metrics are presented in detail in Table 5.5.

Table 5.5: Test Case 2: Relevant performance indicators.

Index	VC	CTC
t_m [s]	80.72	79.43 (−1.60%)
$I_\zeta(t_m)$ [deg s]	139.67	82.84 (−40.69%)
ζ_{av} [deg]	1.73	1.04 (−39.72%)
$I_{\dot{\zeta}}(t_m)$ [deg]	0.0953	0.0071 (−92.57%)
$\dot{\zeta}_{av}$ [deg/s]	0.0012	0.00009 (−92.45%)
E_{prop} [Wh]	736.73	724.35 (−1.68%)

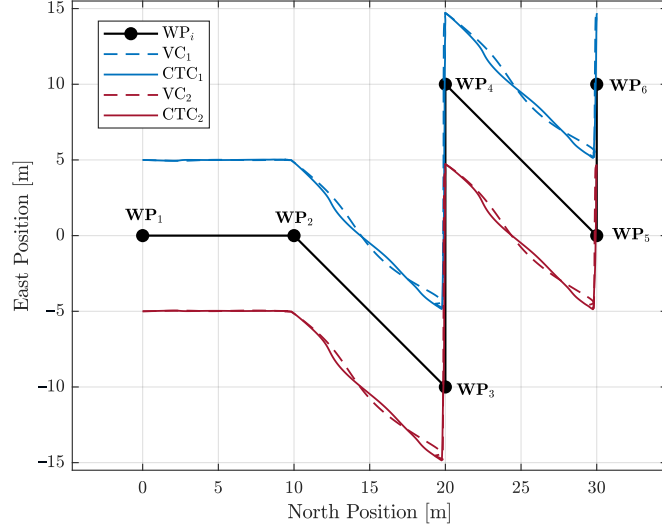


Figure 5.4: Test Case 2: Delivery mission over the horizontal plane x_E - y_E in the presence of commanded acceleration as generated by velocity controller (VC) and cooperative transportation controller (CTC).

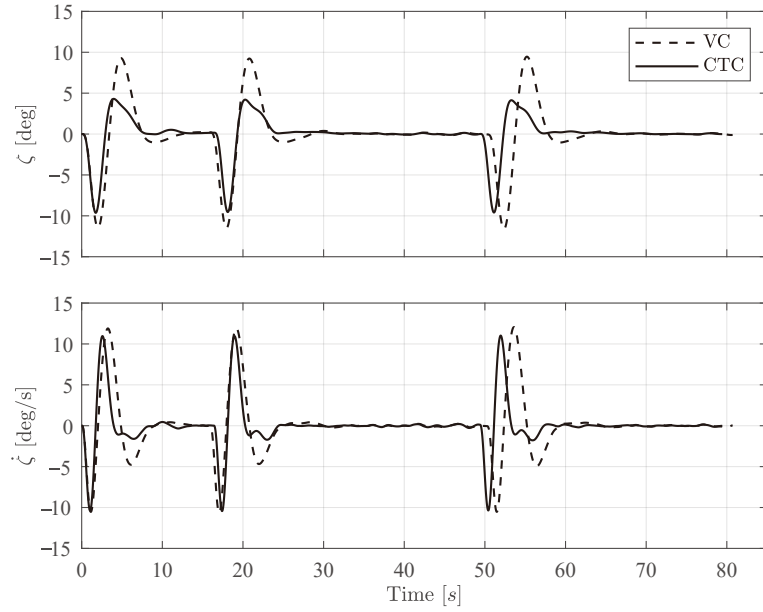


Figure 5.5: Test Case 2: Oscillation angle and rate in the presence of commanded acceleration as generated by cooperative transportation controller (CTC) and velocity controller (VC).

5.2 Baseline Formation Control Validation

This section presents a comprehensive numerical validation of the baseline cooperative formation controller (FC) introduced in Section 3.2, both with and without the auxiliary Payload Controller (PC). In what follows the simulation platform is described and the main control design is briefly recalled. A stability analysis at hover quantifies the beneficial effect of the PC contribution in stabilizing the oscillatory modes of the system. Finally, two representative test cases demonstrate formation-keeping and trajectory-tracking capabilities under realistic conditions. The primary objective is to assess payload swing attenuation without degrading formation performance.

5.2.1 Simulation Setup

The proposed control method is validated through high-fidelity numerical simulations in MATLAB[®]-Simulink environment with a 4-th order Runge-Kutta solver at a frequency of 500 Hz. The lifting vehicles are two identical octarotors, each modeled as a rigid body, and the suspended load is a point mass attached through viscoelastic cables to a hook point H_i , different from the rotorcraft center of gravity P_i . The simulation platform and methodology are identical to the ones described in Section 5.1.1, while the multirotor and payload most relevant parameters are detailed in Table 5.1.

Guidance, navigation, and control tasks are performed following the approach described in detail in Section 3.2.3, where the i -th agent tracks an Earth-fixed acceleration set-point $\mathbf{a}_{sp_i}$ (see Figure 3.4). Within this framework, the i -th multirotor control input is the sum of Equations (3.72) and (3.73), providing formation control (FC) and load swing stabilization (PC), respectively.

Non-ideal sensing behavior is considered where noise, bias, and sensor positioning errors are introduced and described from a numerical standpoint. Onboard computer is located in $STA_{PIX} = BL_{PIX} = 0$ m and $WL_{PIX} = -0.1$ m, with axes aligned with $\mathcal{F}_B$ -axes. To simulate the effects of sensor noise, zero-mean Gaussian white noise is added to each Euler angle and angular velocity

measurement, with standard deviations $\sigma_\alpha = 0.018$ deg and $\sigma_\omega = 0.095$ deg/s, respectively. The body-fixed accelerometer readings are obtained by adding white Gaussian noise with $\sigma_{acc} = 0.015$ m/s² and constant bias $\mathbf{b}_{acc_1} = [0.015, -0.01, 0.002]^T$ m/s² and $\mathbf{b}_{acc_2} = [-0.004, 0.007, -0.02]^T$ m/s² to the simulated values of Vehicle 1 and 2, respectively. Position and velocity measurement errors (expressed in $\mathcal{F}_E$) with standard deviations $\sigma_p|_{xy} = 0.074$ m and $\sigma_v|_{xy} = 0.009$ m/s affect the motion over the local-horizontal plane, while $\sigma_p|_z = 0.034$ m and $\sigma_v|_z = 0.005$ m/s characterize the vertical channel. The onboard computer operates at a frequency of 100 Hz (sample time $\Delta t = 0.01$ s) and a fixed time delay, $t_d = 0.04$ s, is accounted for local processing and communication issues between the agents of the formation. Note that, among all broadcast signals, the reference trajectory information (provided, for example, by a ground control station) must be included. Same considerations hold for the mutual transmission of agents position data to reconstruct the $\mathbf{x}_H = \mathbf{x}_H(\mathbf{r}_1, \mathbf{r}_2)$ -term in Equation (3.73).

5.2.2 Performance Indicators

The following performance indicators are defined to quantitatively characterize the performance of the closed-loop system with and without the auxiliary support of the proposed PC.

- Maneuver time, t_m . The maneuver is considered complete at time t_m when both of the following conditions are satisfied simultaneously: (1) the formation control residual error $e_r = \|[\boldsymbol{\delta}_1^T \boldsymbol{\epsilon}_1^T \boldsymbol{\delta}_2^T \boldsymbol{\epsilon}_2^T]\|$ falls below 0.2, where $\boldsymbol{\delta}_i$ and $\boldsymbol{\epsilon}_i$ are expressed in m and m/s, respectively, and (2) the absolute value of ζ remains below 0.5 degrees for a predetermined time of 5 s. These thresholds represent practical delivery accuracy requirements, where positioning errors below 20 cm and mitigated swing angles below 0.5 deg indicate successful payload placement.
- Average swing angle, ζ_{av} . This is calculated as $\zeta_{av} = I_\zeta(t_m)/t_m$, where the integral

$$I_\zeta(t_m) = \int_0^{t_m} |\zeta(s)| ds \quad (5.5)$$

is evaluated over the maneuver time domain and provides an indication of how much and for how long the payload deviates from the local vertical plane Γ .

- Propulsive energy, E_{prop} . This measures the total energy delivered by the motors to the propellers and is calculated through the integral

$$E_{prop} = \int_0^{t_m} \sum_{k=1}^8 P_{shk}(s) ds \quad (5.6)$$

where $P_{shk} = Q_k \Omega_k$ is the output shaft power generated by the k -th motor, obtained from required torque Q_k and angular rate Ω_k under the assumption of null friction torque. In the present case, E_{prop} is evaluated for UAV 1 only.

5.2.3 Stability Analysis

In this subsection a numerical stability analysis is performed to investigate the stability of both the open- and closed-loop systems, with and without the auxiliary payload controller contribution. In what follows, the poles are computed using the data in Tables 5.1, namely $m = 70$ kg, $m_l = 100$ kg, $L = 15$ m, and $g = 9.80665$ m/s².

Figure 5.6 plots the open-loop eigenvalues analytically described in Section 3.2.4. The non-null forward-swing poles in Equations (3.80) and (3.81) are $\nu_{fs1,2} = \pm j 0.7038$ 1/s and $\nu_{fs3,4} = \pm j 1.0903$ 1/s, while for the lateral-vertical ones, Equations (3.82) and (3.83) give $\nu_{lv1,2} = \pm j 0.7278$ 1/s and $\nu_{lv3,4} = \pm j 1.0143$ 1/s.

For the closed-loop dynamics, formation control gains are selected as $\mathbf{K}_\delta = 0.1 \cdot \mathbf{I}_3$ N/m and $\mathbf{K}_\epsilon = 0.6 \cdot \mathbf{I}_3$ Ns/m, while the payload controller is characterized by $k_p = 1$ m/(rad s²) and $k_d = 5$ m/(rad s). Table 5.6 lists the eigenvalues obtained for the closed-loop dynamic matrices $\hat{\mathbf{A}}_{fs}$ and $\hat{\mathbf{A}}_{lv}$ in Equations (3.78) and (3.79), with and without the auxiliary contribution of the PC. In both cases, the hovering equilibrium is stable, as both $\hat{\mathbf{A}}_{fs}$ and $\hat{\mathbf{A}}_{lv}$ are Hurwitz matrices. While the effect of the PC does not

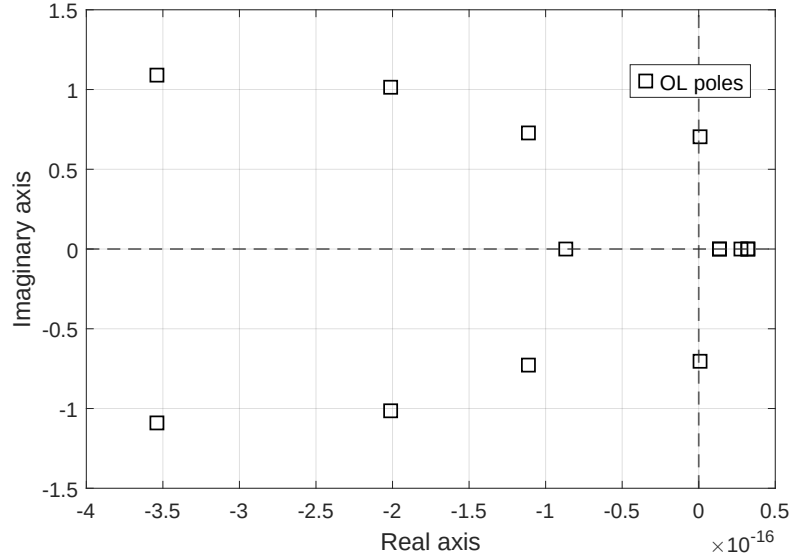


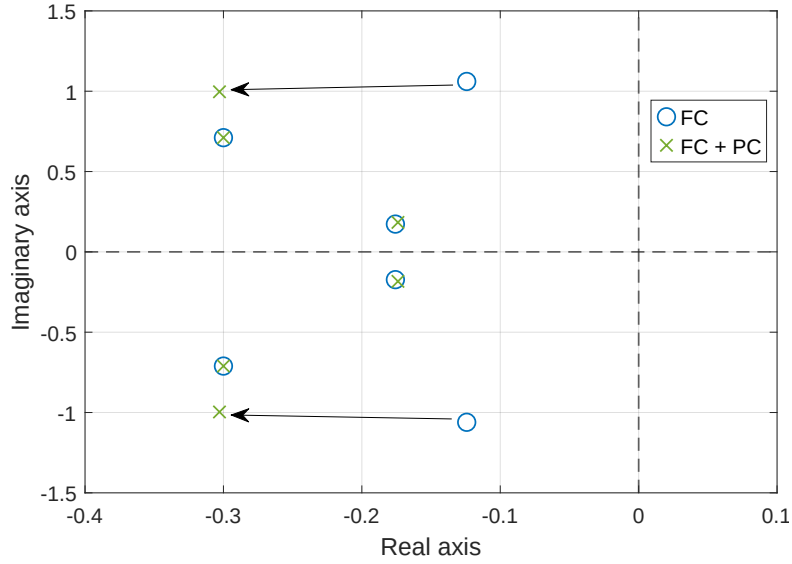
Figure 5.6: Open-loop dynamics eigenvalues.

influence the linearized lateral-vertical behavior, it does affect the forward-swing motion, particularly the complex-conjugate poles $-0.1242 \pm j 1.0607$ 1/s, which shift to $-0.3029 \pm j 0.9966$ 1/s, resulting in increased damping of the oscillatory behavior. Specifically, the damping ratio increases from 0.116 to 0.291 (+151%), while the natural frequency slightly decreases from 1.068 rad/s to 1.042 rad/s (−2.43%). In Figures 5.7 and 5.8 the closed-loop eigenvalues for the two dynamics subsystems are reported. In particular, the beneficial effect of the payload controller in stabilizing the oscillatory modes is made evident by the black arrows in Figure 5.7, accounting for the poles shifting provided by PC activation.

Note that lateral-vertical dynamics remains unaffected by the PC, since this auxiliary contribution acts primarily in the forward direction through the swing angle ζ , which couples predominantly with the longitudinal motion. This implies the lateral-vertical behavior remains governed by the formation control contributions under the small-angle linearization assumptions.

Table 5.6: Eigenvalues of the feedback-controlled subsystems at hover $[\text{s}^{-1}]$.

Subsystem	FC	FC + PC
$\hat{\mathbf{A}}_{fs}$	$-0.3000 \pm j 0.7109$	$-0.3000 \pm j 0.7109$
	$-0.1758 \pm j 0.1729$	$-0.1739 \pm j 0.1835$
	$-0.1242 \pm j 1.0607$	$-0.3029 \pm j 0.9966$
$\hat{\mathbf{A}}_{lv}$	$-0.2913 \pm j 0.7330$	$-0.2913 \pm j 0.7330$
	$-0.1750 \pm j 0.1671$	$-0.1750 \pm j 0.1671$
	$-0.1756 \pm j 0.1722$	$-0.1756 \pm j 0.1722$
	$-0.1648 \pm j 0.9936$	$-0.1648 \pm j 0.9936$


Figure 5.7: Closed-loop forward-swing dynamics eigenvalues.

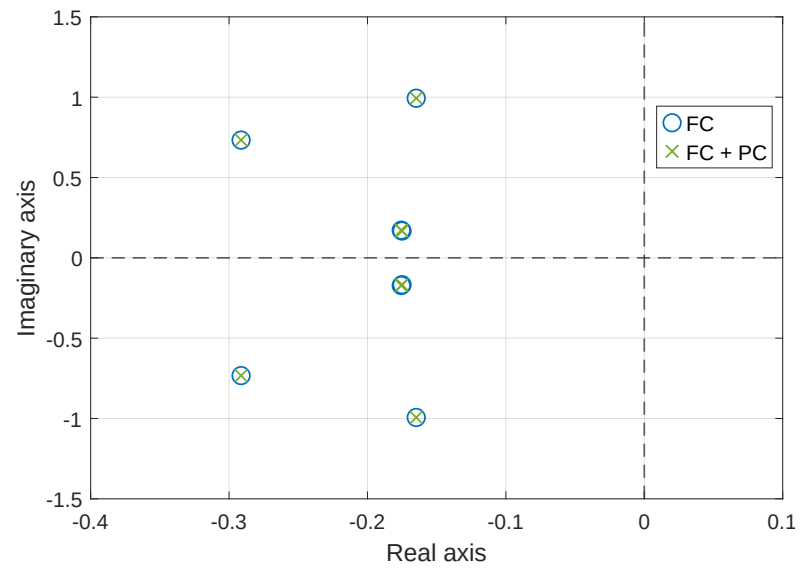


Figure 5.8: Closed-loop lateral-vertical dynamics eigenvalues.

5.2.4 Test Cases

Test Case 1

Consider the formation identified by $\bar{\gamma} = [0, 10, 0]^T$ m, with components expressed in $\mathcal{F}_E$. It follows that $\bar{\mathbf{d}}_1 = -\bar{\mathbf{d}}_2 = [0, 5, 0]^T$ m, given $\bar{\gamma}_x = 0$ m and $\bar{\gamma}_y = 10$ m. In the first test case, the lifting vehicles are required to leave the initial state $\mathbf{r}_1(0) = [0, 5, -20]^T$ m, $\mathbf{r}_2(0) = [0, -5, -20]^T$ m, and $\mathbf{v}_1(0) = \mathbf{v}_2(0) = [0, 0, 0]^T$ m/s at $t_0 = 0$ s, with the payload at rest. Under nominal conditions (no external disturbances), the trim contributions from Equation (3.61) are $\bar{\mathbf{u}}_1 = [0, 173.36, -1176.80]^T$ N and $\bar{\mathbf{u}}_2 = [0, -173.36, -1176.80]^T$ N, respectively. The initial attitude is represented by $\boldsymbol{\eta}_1(0) = -\boldsymbol{\eta}_2(0) = [0.1462, 0, 0]$ rad. The reference point is constant at $\mathbf{r}_r(t) = [10, 0, 0]^T$ m, and the desired yaw angle is $\psi_{sp1} = \psi_{sp2} = 0$ rad. A comparison is made between the sub-case with simple formation stabilization (FC) and the one where active payload stabilization is added to the control action (FC + PC). Figure 5.9a shows the time history of the formation-keeping error e_r , characterized by an asymptotic behavior that is essentially unaltered by the PC contribution. Figure 5.9b shows the effectiveness of the PC, where the maneuver time is reduced from 23.5 s to 17.9 s and the swing behavior rapidly fades. In Figure 5.10, the control mechanism is exemplified. To this end, the set-point acceleration components generated for UAV 1 over the local horizontal plane are reported. The provided PC is operative, and $\mathbf{a}_{sp1} = \mathbf{a}_{sp1}^{(FC)} + \mathbf{a}_{sp1}^{(PC)}$, with the two input contributions depicted separately to show partial vector opposition. For example, in the very first part of the maneuver ($t \leq 3$ s), the induction of a negative oscillation angle determines $a_{sp_{x1}}^{(PC)} < 0$ in opposition to $a_{sp_{x1}}^{(FC)} > 0$, thus mitigating the aggressiveness of the maneuver for reduced payload swing. Table 5.7 lists the main performance parameters and percentage variations. The activation of the PC, which improves disturbance rejection capabilities, produces a noticeable reduction in propulsive energy to conclude the delivery task. Moreover, considering the quantity I_ζ , namely the integral of the oscillation angle over the maneuver time t_m , a reduction of -30.5% is obtained. The same considerations apply to the average value ζ_{av} , for which a reduction of -9.2% is calculated.

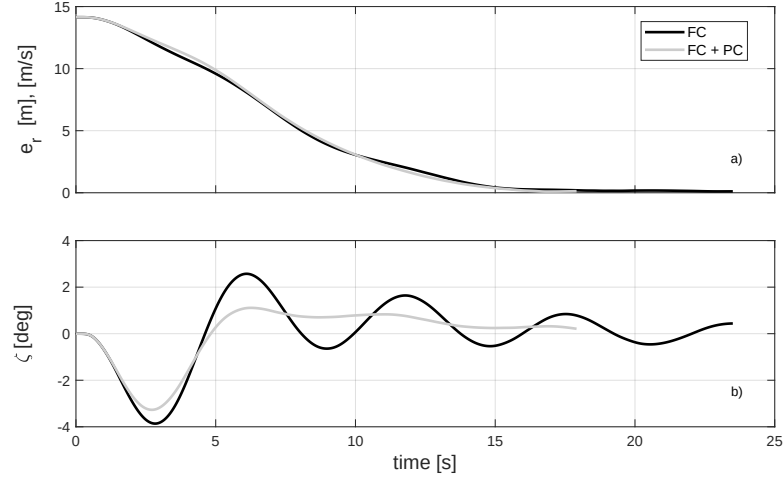


Figure 5.9: Test Case 1: Stabilization of (a) formation-keeping residual error and (b) payload swing angle.

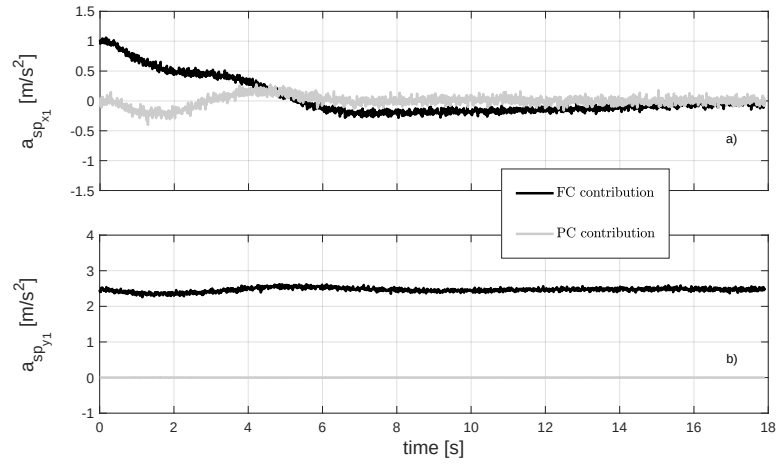


Figure 5.10: Test Case 1: Set-point acceleration contributions for UAV 1 with operative PC.

Table 5.7: Test Case 1: Relevant performance indicators.

Index	FC	FC + PC	Variation [%]
t_m [s]	23.5	17.9	-23.8
I_z [deg s]	22.96	15.95	-30.5
ζ_{av} [deg]	0.98	0.89	-9.2
E_{prop} [kJ]	327.05	249.31	-23.8

Test Case 2

The multirotors start from the same initial condition as Test Case 1 and are required to track a rectilinear trajectory described by the reference point position $\mathbf{r}_r(t) = [\lambda t, 0, -20]^T$ m and velocity $\mathbf{v}_r = [\lambda, 0, 0]^T$, given $\lambda = 2$ m/s and $\mathbf{v}_r \notin \mathcal{L}_2[0, \infty)$. The desired formation geometry is here reconfigured to $\bar{\boldsymbol{\gamma}} = [10, 0, 0]^T$ m such that $\bar{\mathbf{u}}_1 = [173.36, 0, -1176.80]^T$ N and $\bar{\mathbf{u}}_2 = [-173.36, 0, -1176.80]^T$ N. A constant wind $\mathbf{w} = [0, -5, 0]^T$ m/s perturbs the system during time interval $13 \leq t \leq 15$ s, thus simulating a discrete gust. In this case, the proportional formulation in Equation (3.72) for a system affected by external (drag) disturbances is expected to determine the steady-state residual errors in terms of $\boldsymbol{\delta}_1$ and $\boldsymbol{\delta}_2$. Conversely, the tracking of the desired speed $\mathbf{v}_r$ theoretically occurs with zero residuals, that is, $\|\boldsymbol{\epsilon}_1\|, \|\boldsymbol{\epsilon}_2\| \rightarrow 0$ m/s for $t \rightarrow +\infty$. With this in mind, the condition defining the maneuver time in Section 5.2.2, is here adapted to the reduced-order speed error $e_{rs} = \|[\boldsymbol{\epsilon}_1^T, \boldsymbol{\epsilon}_2^T]\|$, required to fall below 0.02 m/s. Figure 5.11a depicts the time history of e_{rs} , which indicates the correct stabilization of both agents to the reference velocity. Figure 5.11b shows the swing angle ζ , where the effect of wind gusts is evident in the induction of undesired oscillations in addition to the deviation of the formation from the prescribed trajectory (see gray areas).

The time behavior of the positioning error is depicted in Figure 5.12, where the formation residuals remain bounded with $\|\boldsymbol{\delta}_1\|, \|\boldsymbol{\delta}_2\| \rightarrow 1.99$ m for $t \rightarrow +\infty$. In steady-state conditions, the components of $\boldsymbol{\epsilon}_1$ and $\boldsymbol{\epsilon}_2$ oscillate about zero with standard deviations of 0.011 m/s over the horizontal plane and 0.004 m/s along the local vertical. Finally, the results of the performance analysis are reported in Table 5.8, highlighting the favorable effect of the PC. The proposed control strategy represents a starting point for implementing more complex algorithms with improved disturbance-rejection capabilities. For example, the PD formulation in Equation (3.72) could benefit from an additive integral term applied to the formation error $\boldsymbol{\delta}_i$. In such case, the accuracy of both formation-keeping and trajectory-tracking tasks would improve at the cost of extending the system state in Equation (3.74).

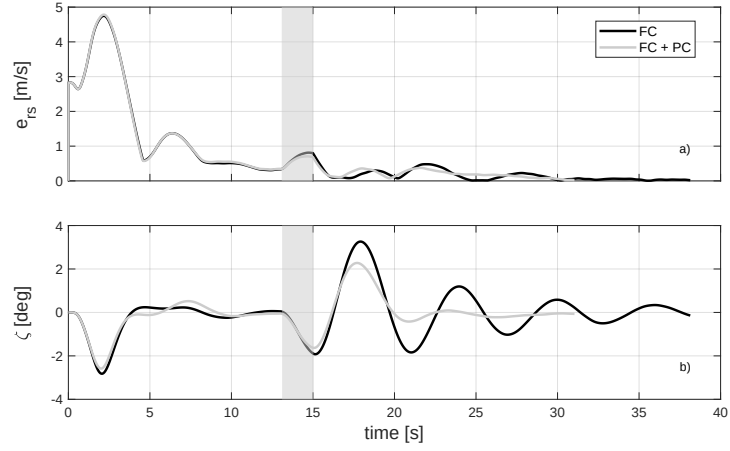


Figure 5.11: Test Case 2: Stabilization of (a) formation-keeping residual error and (b) payload swing angle.

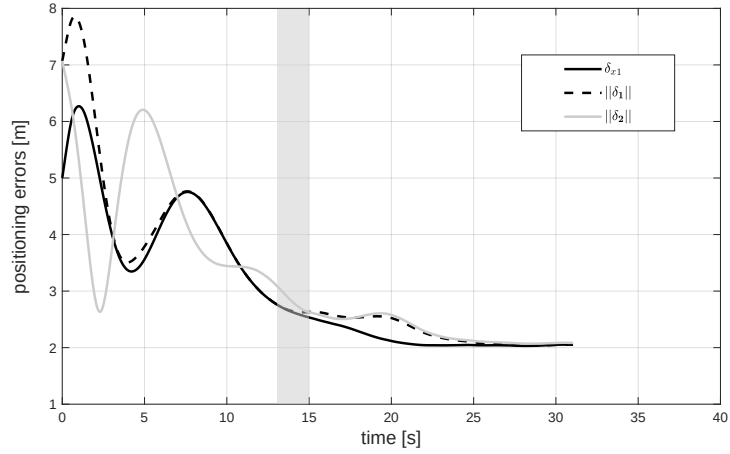


Figure 5.12: Test Case 2: Convergence of positioning errors to a steady-state value (FC + PC).

Table 5.8: Test Case 2: Relevant performance indicators.

Index	FC	FC + PC	Variation [%]
t_m [s]	38.1	31.0	-18.6
I_ζ [deg s]	26.90	15.62	-41.9
ζ_{av} [deg]	0.71	0.50	-28.7
E_{prop} [kJ]	530.53	431.77	-18.61

This test case evaluates the in-line configuration, where the formation axis $\bar{\gamma}$ is aligned with the trajectory. As predicted by the stability analysis in Section 3.2.4, the PC contribution is null in this direction. However, the system remains stable, even under the effect of external perturbations, owing to the action of the FC. This demonstrates the efficacy of the proposed method in non-nominal conditions, providing, for example, the opportunity to navigate narrow passages where abreast flight is not possible. While there is no active swing damping in this specific direction, the results confirm that the formation controller is robust enough to ensure bounded stability during transition maneuvers.

5.3 $\mathcal{L}_1$ Adaptive Augmentation Validation

This section evaluates the $\mathcal{L}_1$ adaptive augmentation layer's ability to enhance robustness against model uncertainties, external disturbances, and actuator faults. Building upon the baseline architecture validated in Section 5.2, the adaptive controller is tested in two demanding scenarios: (1) a hover-to-hover maneuver affected by wind gusts, atmospheric turbulence, and propeller failure, and (2) a multi-waypoint urban air delivery mission with significant payload mass variations. Performance improvements are quantified through comparison with the non-augmented baseline controller.

5.3.1 Simulation Setup

The simulation platform defined in Section 5.2.1 is maintained for consistency, enabling direct and fair comparison between baseline and augmented controllers.

5.3.2 Performance Indicators

The following performance indicators are introduced to better assess the augmented controller performance compared to the baseline one.

- Maneuver time, t_m . It is the total time required to conclude the considered maneuver. According to the different test cases, it can be defined a priori or based on a stopping criterion. In the latter scenario, the simulation starts at $t = 0$ s and is assumed to be concluded when three conditions are simultaneously satisfied: (i) the position error $e_p = \|[\delta_1^T \delta_2^T]\|$ falls below 0.2 m, (ii) the velocity error $e_v = \|[\epsilon_1^T \epsilon_2^T]\|$ falls below 0.2 m/s, and (iii) the absolute value of ζ remains bounded below 0.5 deg for 5 s.
- Average position and velocity errors, $e_{p_{av}}$ and $e_{v_{av}}$. They are calculated as $e_{p_{av}} = Ie_{p_{av}}(t_m)/t_m$ and $e_{v_{av}} = Ie_{v_{av}}(t_m)/t_m$ respectively, given the

integrals

$$I_{\mathbf{e}_{p_{av}}}(t_m) = \int_0^{t_m} |\mathbf{e}_p(s)| ds \quad (5.7)$$

$$I_{\mathbf{e}_{v_{av}}}(t_m) = \int_0^{t_m} |\mathbf{e}_v(s)| ds. \quad (5.8)$$

- Maximum position and velocity error, $\mathbf{e}_{p_{max}}$ and $\mathbf{e}_{v_{max}}$.
- Total propulsive energy, E_{prop} . It is the mechanical energy delivered by electrical motors to propellers, calculated as

$$E_{prop} = \int_0^{t_m} \sum_{j=1}^8 P_{sh_j}(s) ds \quad (5.9)$$

where $P_{sh_j} = Q_j \Omega_j$ is the output shaft power generated by the j -th motor, $j = 1, \dots, 8$, obtained from required torque Q_j and angular rate Ω_j under the assumption of null friction torque. Without loss of generality, E_{prop} is calculated for UAV 1 only.

5.3.3 Test Cases

In this subsection consider the formation identified by $\bar{\boldsymbol{\gamma}} = [0, 10, 0]^T$ m, with components expressed in $\mathcal{F}_E$. It is $\bar{\gamma}_x = 0$ m and $\bar{\gamma}_y = 10$ m, such that $\bar{\mathbf{d}}_1 = -\bar{\mathbf{d}}_2 = [0, 5, 0]^T$ m. In the absence of external disturbances, the nominal trim contributions are identical to the one reported in Section 5.2.4. With respect to the baseline controller, adopted gains are $\mathbf{K}_\delta = \text{diag}(0.25, 0.25, 0.25)$ $1/s^2$, $\mathbf{K}_\epsilon = \text{diag}(1.5, 1.5, 1.5)$ $1/s$, $k_p = 0$ $1/s^2$, and $k_d = 0$ $1/s$. Regarding the most relevant $\mathcal{L}_1$ control parameters, a trial-and-error tuning is performed separately for the forward-swing and lateral-vertical dynamics. For the former, it is $\boldsymbol{\omega}_{0_{fs}} = \mathbb{I}_2$, $\mathbf{K}_{fs} = \text{diag}(1.5, 1.5)$, and $\mathbf{D}_{fs} = \text{diag}(1/(15s + 1), 1/(15s + 1))$. For the latter, it is $\boldsymbol{\omega}_{0_{lv}} = \mathbb{I}_4$, $\mathbf{K}_{lv} = \text{diag}(5, 5, 7.5, 7.5)$, and $\mathbf{D}_{lv} = \text{diag}(1/(10s + 1), 1/(10s + 1), 1/(15s + 1), 1/(15s + 1))$.

Test Case 1

The multirotors are characterized by the initial state $\mathbf{r}_1(0) = [0, 5, -20]^T$ m, $\mathbf{r}_2(0) = [0, -5, -20]^T$ m, and $\mathbf{v}_1(0) = \mathbf{v}_2(0) = [0, 0, 0]^T$ m/s at $t_0 = 0$ s, with

the payload at rest. The initial attitude is represented by $\boldsymbol{\eta}_1(0) = -\boldsymbol{\eta}_2(0) = [0.1462, 0, 0]$ rad, while the desired yaw angle is $\psi_{sp1} = \psi_{sp2} = 0$ rad. The first simulation case consists in performing a hover-to-hover delivery maneuver, with an intermediate ten seconds stop, under disturbances and actuator fault, with an imposed overall maneuver time $t_m = 50$ s. In particular, a constant wind $w = [0, -10, 0]^T$ m/s perturbs the system during the time interval $5.5 \leq t \leq 8.5$ s, thus simulating a discrete gust. In the second part of the maneuver, atmospheric turbulence is introduced to disturb the state of both the octarotor and the payload. The turbulence is modeled using a dedicated Simulink Aerospace Blockset component, which implements the Von Kàrmàn spectral representation. This model follows the mathematical formulations provided by the Military Specification MIL-F-8785C and the Military Handbook MIL-HDBK-1797B [98, 99]. The following parameters are set:

- specification: MIL-F-8785C;
- model type: continuous Von Kàrmàn (+q +r);
- wind speed at 6 m (used to define the low-altitude wind intensity): 5 m/s;
- wind direction at 6 m (degrees clockwise from North): 90 deg;
- scale length at medium/high altitudes: 533.4 m (default);
- band-limited noise sample time: 0.002 s;
- MR wingspan: 2.38 m (maximum octarotor frontal size);
- load wingspan: 1 m (maximum load frontal size);

Moreover, for $t \geq 28$ s, UAV 1 experiences a malfunctioning of propeller 2 caused by an abrupt and persistent 15% rotational speed loss. The reference trajectory $\mathbf{r}_r(t)$ is a C^1 cubic polynomial of the following form

$$\mathbf{r}_r(t) = \begin{cases} [-0.089t^3 + 0.2t^2, 0, -20]^T & 0 < t \leq 15 \text{ s} \\ [15, 0, -20]^T & 15 < t \leq 25 \text{ s} \\ [-0.0119\tau^3 + 0.267\tau^2 + 15, 0, -20]^T & 25 < t \leq 40 \text{ s} \end{cases} \quad (5.10)$$

where $\tau = t - 25$.

A comparison is made between the sub-case with baseline formation control only and the sub-case where $\mathcal{L}_1$ AC is added to the control action. In Figure 5.13, the East positions of the three objects are reported with respect to time. The additive $\mathcal{L}_1$ contribution provides high robustness against both the wind gust in the first part of the maneuver and the actuator fault in the second one. The agents, after being perturbed, show a very fast reaction in re-gaining the desired formation geometry.

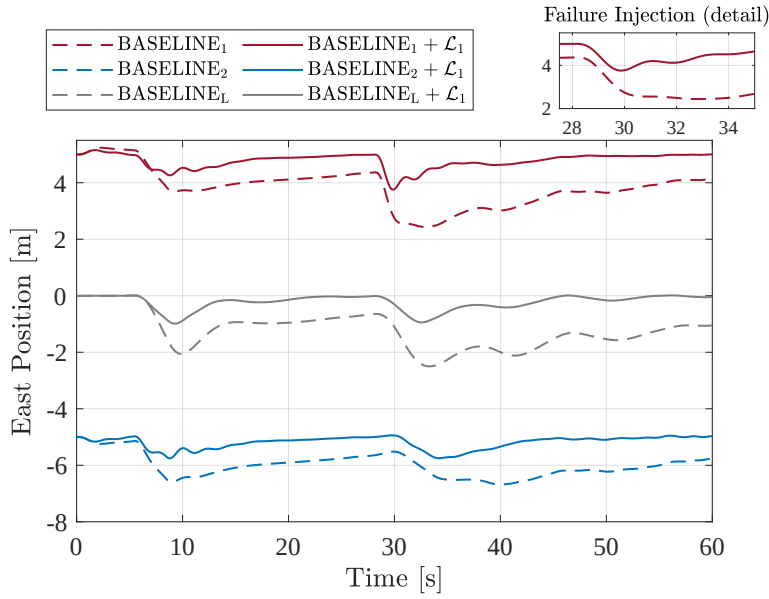


Figure 5.13: Test Case 1: Multirotors and load East positions with and without $\mathcal{L}_1$ AC.

Similar considerations hold for Figure 5.14, where the vertical positions of the three objects are depicted. Here the main concern is the altitude drop caused by the partial loss of a propeller of UAV 1. The augmented layer immediately accounts for the failure and stabilizes the system without significantly losing height, while the baseline controller lacks of robustness leading to height loss even once the desired position is reached.

Finally, Figure 5.15 reports the position and velocity errors, respectively. While the velocity error $\mathbf{e}_v$ remains almost unaltered, the position error $\mathbf{e}_p$ is significantly smaller, thanks to the beneficial contribution provided by the $\mathcal{L}_1$

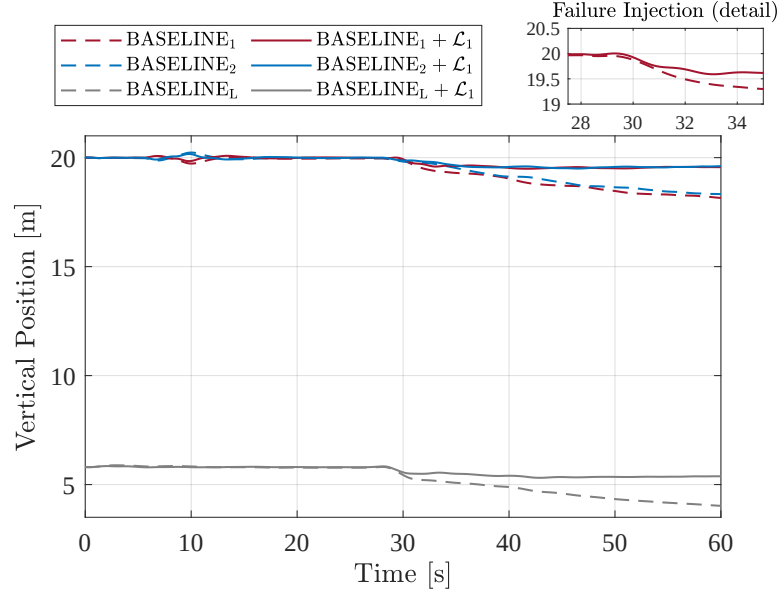


Figure 5.14: Test Case 1: Multirotors and load vertical positions with and without $\mathcal{L}_1$ AC.

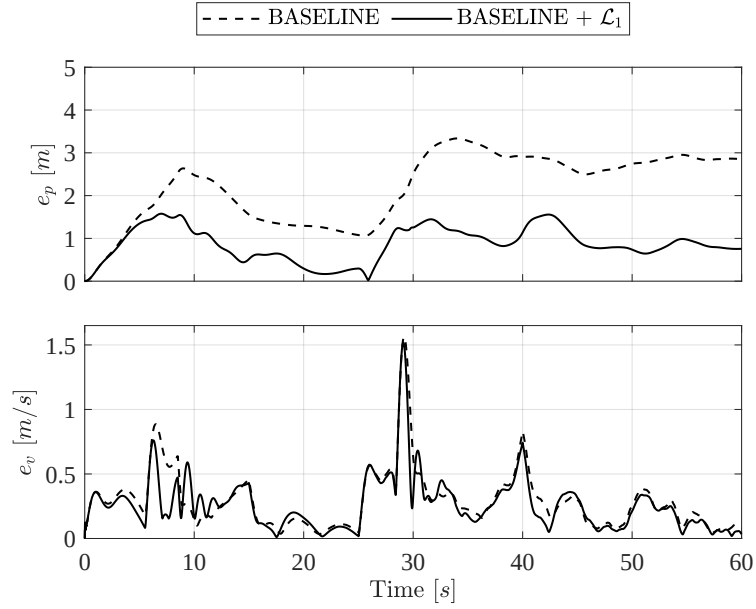


Figure 5.15: Test Case 1: Position and velocity errors with and without $\mathcal{L}_1$ AC.

augmentation. In this regard, performance indicators obtained through the two different control layers are detailed in Table 5.9. For the same maneuver time, there is a slight improvement in the propulsive energy reduction, which can be considered negligible. However, the other indicators are notably improved, demonstrating the favorable contribution of the proposed method.

Table 5.9: Test Case 1: Relevant performance indicators.

Index	BASELINE	BASELINE + $\mathcal{L}_1$
$e_{p_{av}}$ [m]	2.20	0.87 (-50.45%)
$e_{v_{av}}$ [m/s]	0.29	0.26 (-10.01%)
$e_{p_{max}}$ [m]	3.34	1.58 (-52.76%)
$e_{p_{max}}$ [m/s]	1.56	1.52 (-2.91%)
E_{prop} [Wh]	564.90	557.97 (-1.29%)

Test Case 2

In this case, the formation is required to track a sequence of seven waypoints $\mathbf{WP}_i = [x_i, y_i, z_i]^T$, $i = 1, \dots, 7$. Following the formulation in Section 3.1.2, the i -th vehicle's desired position is based on the reference point trajectory and is given by $\bar{\mathbf{r}}_i = \mathbf{r}_r + \bar{\mathbf{d}}_i$. Without loss of generality, a waypoint is considered to be reached when the positioning error of UAV 1, defined as $\varepsilon_P = \|\mathbf{WP}_i - \mathbf{r}_1\|$, remains below 0.5 m for at least 5 s. This multi-waypoint scenario is representative of urban last-mile air delivery operations where a single UAV formation services multiple drop-off locations from a central depot, with varying payload masses as parcels are delivered sequentially. Starting from a nominal hover condition, the formation immediately moves to a first pick-up location ($\mathbf{WP}_1$) and then visits five drop-off sites before ending the operation when returning to the initial hovering position ($\mathbf{WP}_7$). Abrupt payload-mass changes at selected waypoints (see Table 5.10) model loading and unloading events. The 50% mass increase at $\mathbf{WP}_1$ (from 100 kg to 150 kg) represents a worst-case scenario for testing $\mathcal{L}_1$ adaptive augmentation robustness. While such substantial changes may be less common in practice, they serve to stress-test the adaptation mechanism. Controller performance

with and without $\mathcal{L}_1$ AC are evaluated under the same load mass variation disturbances.

Table 5.10: Test Case 2: Waypoints of the delivery mission.

i -th waypoint	x_i [m]	y_i [m]	z_i [m]	m_i [kg]
1	10	0	-50	150
2	10	-10	-50	125
3	-10	10	-50	115
4	-10	0	-50	105
5	-10	-10	-50	105
6	0	-10	-50	95
7	0	0	-50	95

In Figure 5.16, the trajectory followed by UAV 1 is reported. It is evident that, when load mass is increased by 50% ($\mathbf{WP}_1$), the baseline controller lacks sufficient robustness, resulting in a vertical deviation of several meters and reducing the accuracy of trajectory tracking. The augmented control system shows improved robustness and fast adaptation against non-nominal conditions, allowing to fulfill the delivery tasks more smoothly. In this respect, maneuver time with the $\mathcal{L}_1$ adaptive layer active is reduced from 143.89 s to 126.96 s, allowing for a concurrent decrease of power consumption (see Table 5.11). Position error $\mathbf{e}_p$ is depicted in Figure 5.17, while Table 5.11 details significant reduction in both the average and the maximum position errors (more than 30 % and 7 %, respectively).

Table 5.11: Test Case 2: Relevant performance indicators.

Index	BASELINE	BASELINE + $\mathcal{L}_1$
t_m [s]	143.89	126.96 (-11.77%)
$\mathbf{e}_{p_{av}}$ [m]	7.88	5.42 (-31.20%)
$\mathbf{e}_{p_{max}}$ [m]	31.11	28.80 (-7.44%)
E_{prop} [Wh]	1245.10	1116.60 (-10.32%)

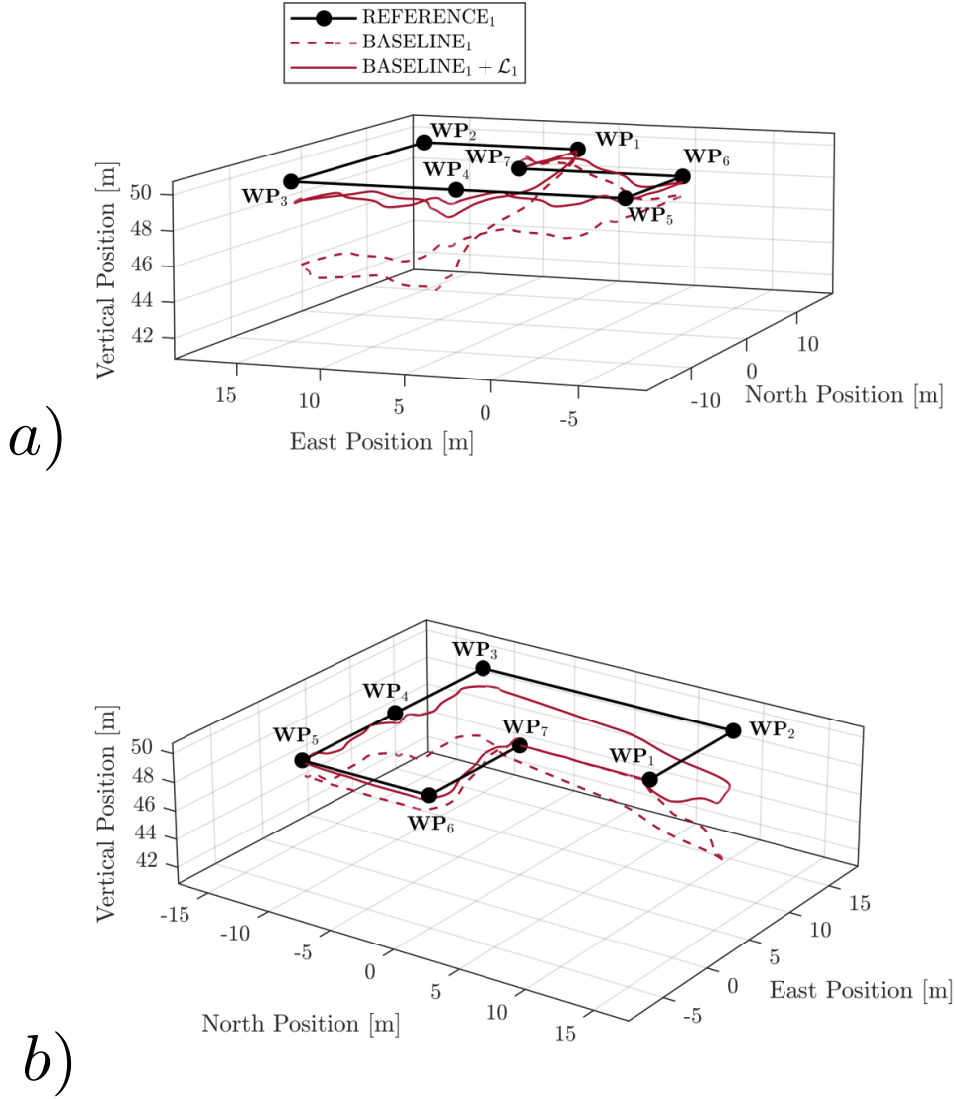


Figure 5.16: Test Case 2: Waypoint-based 3D trajectory followed by UAV 1 with and without $\mathcal{L}_1$ AC from the different camera views (a) and (b).

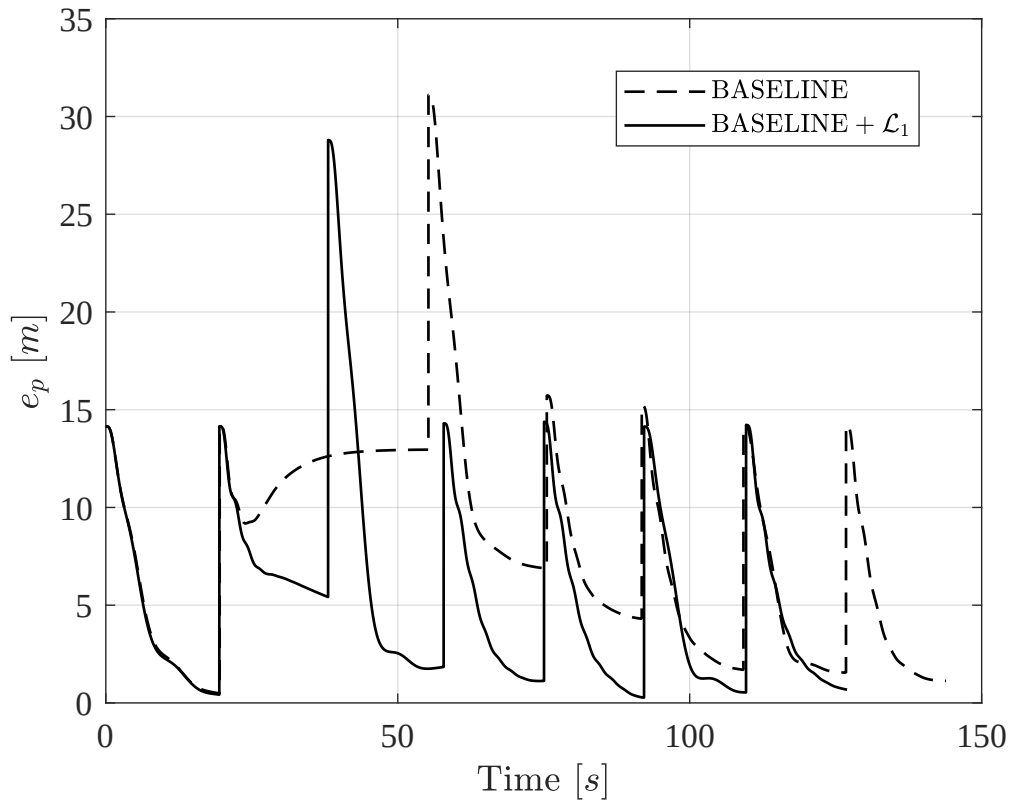


Figure 5.17: Test Case 2: Position error with and without $\mathcal{L}_1$ AC.

5.4 Swing State Estimation Validation

This section presents the numerical validation of the proposed swing state estimation and control methodology for cooperative payload transportation. Simulation setup and performance indicators are identical to the ones adopted in Section 5.2, while the most relevant system parameters are detailed in Table 5.1. The validation encompasses three distinct test cases designed to demonstrate different aspects of the system's performance: payload mass estimation capabilities, filter tuning and configuration, and trajectory tracking with disturbance rejection. Each test case progressively increases in complexity, moving from basic formation acquisition to dynamic maneuvers affected by external disturbances. The results validate the effectiveness of the Fading Gaussian Deterministic Filter for estimating the payload swing state variables, which are subsequently used in a closed-loop feedback architecture to stabilize load oscillations. Significant performance improvements are achieved through the proposed payload control strategy in terms of maneuver time, energy consumption, and swing suppression.

5.4.1 Simulation Setup

The simulation platform is identical to the one described in Section 5.2.1. Guidance, navigation, and control tasks are performed following the approach described in detail in Section 3.4.3, where the i -th agent tracks an Earth-fixed acceleration set-point $\mathbf{a}_{sp_i}$ (see Figure 3.6). Within this framework, the i -th multirotor control input is given by Equation (3.118) and is made of three contributions. In particular, $\mathbf{a}_{sp_i}^{(VC)}$ provides formation-keeping and trajectory-tracking, $\mathbf{a}_{sp_i}^{(CAC)}$ deals with collision avoidance tasks, and $\mathbf{a}_{sp_i}^{(PC)}$ concerns payload oscillations stabilization.

5.4.2 Performance Indicators

The same performance indicators defined in Section 5.2.2 are adopted.

5.4.3 Test Cases

In what follows consider the desired formation geometry identified by $\bar{\gamma} = [0, 10, 0]^T$ m. It follows $\bar{\mathbf{d}}_1 = -\bar{\mathbf{d}}_2 = [0, 5, 0]^T$ m, provided $\bar{\gamma}_x = 0$ m and $\bar{\gamma}_y = 10$ m. Since $\bar{\gamma} = \|\bar{\gamma}\| < 2L = 15$ m, the designed formation is well-posed according to Remark 5 (in this case $\bar{\gamma}^* = 2L\sqrt{1 + m_l/(2m)} \approx 39.28$ m). Concerning collision avoidance, a virtual radius $R_{sph} = 2$ m is selected, while CAC gain is $K_a = 0.05$ m²/s². With respect to PC, it is $k_p = 9$ m/(rad s²) and $k_d = 2$ m/(rad s).

Test Case 1

The first simulation case validates the formation acquisition capabilities and the payload mass estimation method discussed in Theorem 3.2.1 and Remark 4 while $\mathbf{a}_{sp_i}^{(PC)} = \mathbf{0}$ m/s². The octarotors are required to leave the initial state defined at $t = 0$ s by $\mathbf{r}_1(0) = [2, 5.2, -18]^T$ m, $\mathbf{r}_2(0) = [-4, -5.2, -18]^T$ m, and $\mathbf{v}_1(0) = \mathbf{v}_2(0) = [0, 0, 0]^T$ m/s, with $\zeta(0) = 3.6$ deg and $\dot{\zeta}(0) = 0$ deg/s. The initial attitude is represented by $\boldsymbol{\eta}_1(0) = -\boldsymbol{\eta}_2(0) = [8.89, -5.19, 0]$ deg. The reference point is a constant, $\mathbf{r}_r(t) = [0, 0, -20]^T$ m, and the desired yaw angle is $\psi_{sp1} = \psi_{sp2} = 0$ deg. The maneuver is performed in the presence of a constant wind $\mathbf{w} = [-2, 1.5, 0]^T$ m/s, with components expressed in $\mathcal{F}_E$. During the maneuver, performed with a prescribed 60 s duration, Equation (3.63) is solved on board Vehicle 1 and the solution $\hat{m}_l = (-b_1 + \sqrt{b_1^2 - 4a_1c_1})/(2a_1)$ is considered.

In Figure 5.18a, the residual error e_r is plotted as a function of time, showing correct stabilization of formation geometry. In Figure 5.18b, the estimated payload mass is reported, provided a low-pass filter is applied to the computed value, as obtained by a first-order transfer function with time constant $\tau_{LP} = 0.5$ s. After a rapid initial transient ($t \leq 2$ s), the algorithm reaches plausible values and stabilizes to a steady-state solution. By averaging the final 10 seconds of data, the estimated payload mass converges to $\hat{m}_l = 99.76$ kg, which infers satisfactory accuracy. It is noted that mass estimation error (-0.24%) accounts for the presence of a hook point that is distinct from vehicle's center of gravity, which goes against the hypotheses at

5.4. SWING STATE ESTIMATION VALIDATION

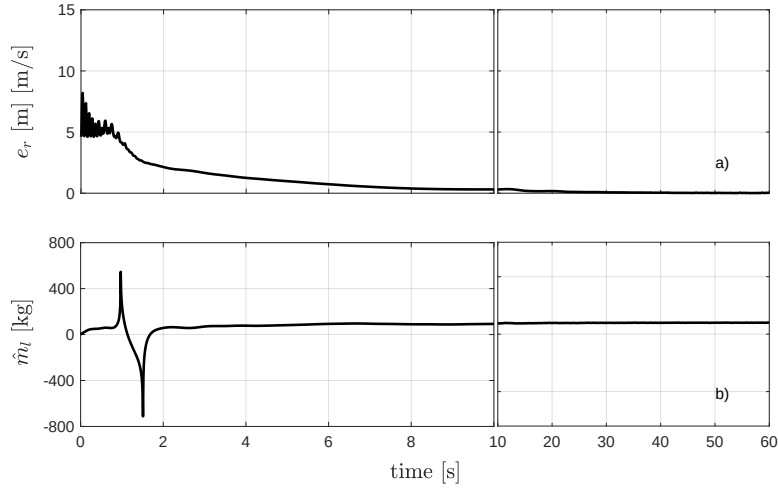


Figure 5.18: Test Case 1: (a) Stabilization of formation error. (b) Static estimation of payload mass.

the basis of proposed algorithms (and is responsible for the high-frequency oscillations in Figures 5.18–5.20).

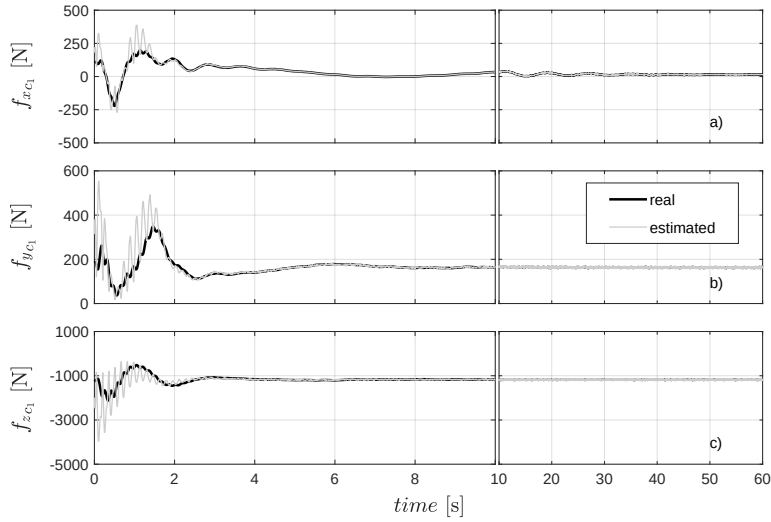


Figure 5.19: Test Case 1: Comparison between real and estimated control forces.

The estimation of control and disturbance forces plays a fundamental role, as it was pointed out in Remarks 4 and 8. In this respect, Figures 5.19 and 5.20 report a comparison between real and estimated values for Vehicle 1,

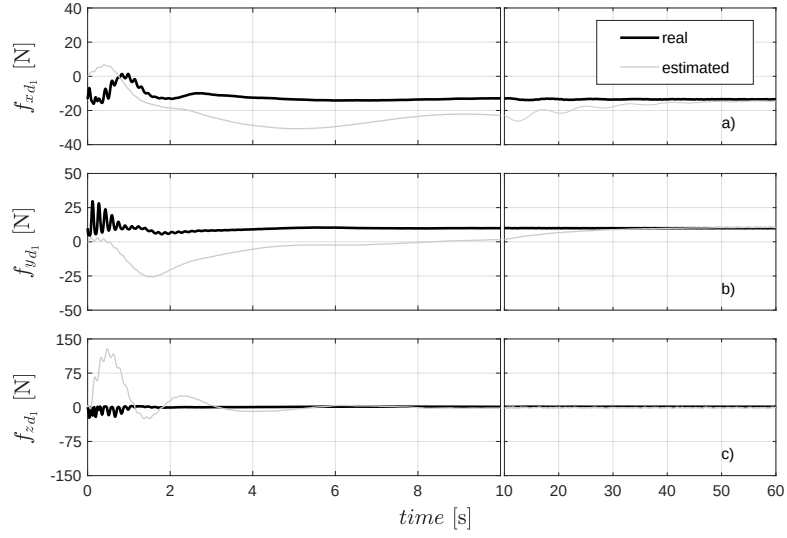


Figure 5.20: Test Case 1: Comparison between real and estimated disturbance forces.

which reaches the steady-state measured attitude identified by $\bar{\phi}_1 = 7.88$ deg, $\bar{\theta}_1 = -0.71$ deg, and $\bar{\alpha}_1 = \arccos(\cos \bar{\phi}_1 \cos \bar{\theta}_1) = 7.91$ deg. During the maneuver, payload swing is excited according to Figure 5.21, until the rest condition is practically acquired. The system results to be evidently under-damped, provided dissipative effects are (directly) generated by payload aerodynamic drag and (indirectly) by the stabilization of formation speed errors, ϵ_1 and ϵ_2 .

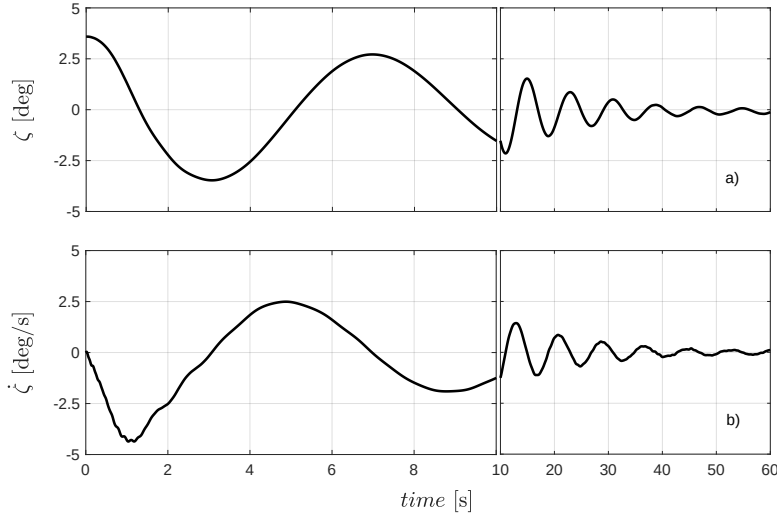


Figure 5.21: Test Case 1: Stabilization of (a) swing angle ζ and (b) rate $\dot{\zeta}$.

Test Case 2

The second maneuver is adopted as a test case to perform filter tuning, based on payload mass value $\hat{m}_l$ estimated above. The octarotors leave the initial state defined by $\mathbf{r}_1(0) = [0, 5, -20]^T$ m, $\mathbf{r}_2(0) = [0, -5, -20]^T$ m, and $\mathbf{v}_1(0) = \mathbf{v}_2(0) = [0, 0, 0]^T$ m/s, with the payload at rest in the absence of wind. The initial attitude is $\boldsymbol{\eta}_1(0) = -\boldsymbol{\eta}_2(0) = [8.38, 0, 0]$ deg. The reference point is a constant, $\mathbf{r}_r(t) = [20, 0, -20]^T$ m, and the desired yaw angle is $\psi_{sp1} = \psi_{sp2} = 0$ deg. Dynamic system matrices in Equations (3.98) and (3.99) are, respectively:

$$\mathbf{A}_\zeta = \begin{bmatrix} 0 & 1 \\ -1.1876 & 0 \end{bmatrix}, \quad (5.11)$$

$$\mathbf{B}_\zeta = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -0.5051 & 0 & 0 & -0.5051 & 0 & 0 \end{bmatrix} \cdot 10^{-3} \quad (5.12)$$

The state-transition matrix is computed by MATLAB[®] matrix exponential function $\text{expm}(\cdot)$, according to the method in Ref. [100],

$$\boldsymbol{\Phi} = \begin{bmatrix} 0.9999 & 0.010 \\ -0.0119 & 0.9999 \end{bmatrix} \quad (5.13)$$

while the control-input model is:

$$\Psi = \begin{bmatrix} -0.0025 & 0 & 0 & -0.0025 & 0 & 0 \\ -0.5051 & 0 & 0 & -0.5051 & 0 & 0 \end{bmatrix} \cdot 10^{-5} \quad (5.14)$$

For the sake of brevity, the tuning and scaling procedures to find the globally-optimum Fading Gaussian Deterministic filter configuration are not described in detail, but they stem from the methodology proposed by de Angelis [74]. Final results are directly provided, where $\beta = 0.998$ is the best fading factor at which the estimated measurement noise covariance matrix $\tilde{R}$ satisfies $\tilde{R} = R^* \approx R$, where R^* and R are, respectively, the assigned and the true noise covariance values. In the present case $R = 1.98 \cdot 10^{-5} \text{ rad}^2$ and the assigned value is $R^* = 1.95 \cdot 10^{-5} \text{ rad}^2$. The algorithm is initialized with null state vector and nominal error covariance $(\mathbf{E}^-)^{(0)} = \text{diag}(8, 9) \cdot 10^{-8}$. The time history of formation residual e_r in the case when $\mathbf{a}_{sp_i}^{(PC)} = \mathbf{0} \text{ m/s}^2$ is reported in Figure 5.22a, provided that maneuver time is $t_m = 71.2 \text{ s}$. Filter stabilization occurs as in Figure 5.22b, where the trace of the nominal covariance matrix $\mathbf{E}$ (and, therefore, the gain of the estimator) sets on a constant value. With respect to filter performance, a comparison is reported in Figure 5.23 between the true, the filter-estimated, and the measured oscillation angle according to Equation (3.104). In Figure 5.24 satisfactory performance of the recursive algorithm is assessed also for the estimation of oscillation rate.

Table 5.12: Test Case 2: Relevant performance indicators.

Index	without PC	with PC	Variation [%]
t_m [s]	71.2	42.1	-40.9
$I_\zeta(t_m)$ [deg s]	299.71	172.03	-42.6
ζ_{av} [deg]	4.21	4.08	-3.1
E_{prop} [kJ]	995.7	589.6	-40.8

In the case when PC is activated according to Equation (3.73), based on the feedback of estimated variables, the same maneuver is concluded with $t_m = 42.1 \text{ s}$. The beneficial effect of $\mathbf{a}_{sp_i}^{(PC)}$ is also evident in Figure 5.25, where oscillation variables are shown to be rapidly damped. With respect to other performance indicators, Table 5.12 summarizes main results and computes

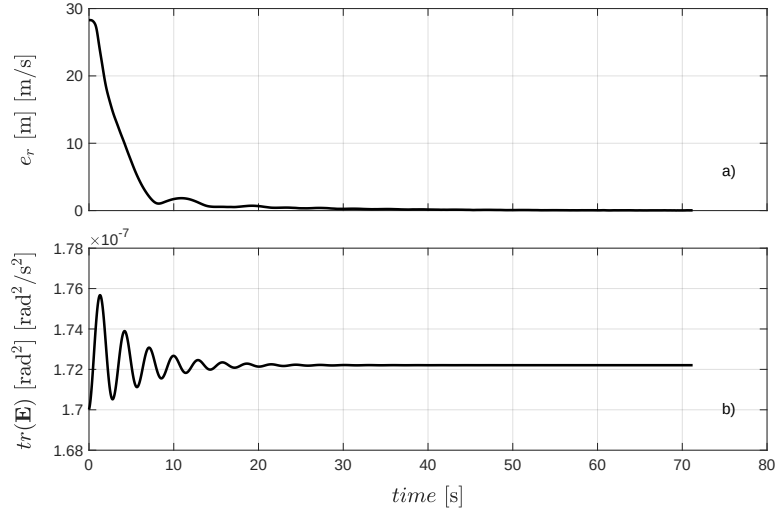


Figure 5.22: Test Case 2: (a) Formation error stabilization. (b) Estimation convergence for the nominal covariance matrix.

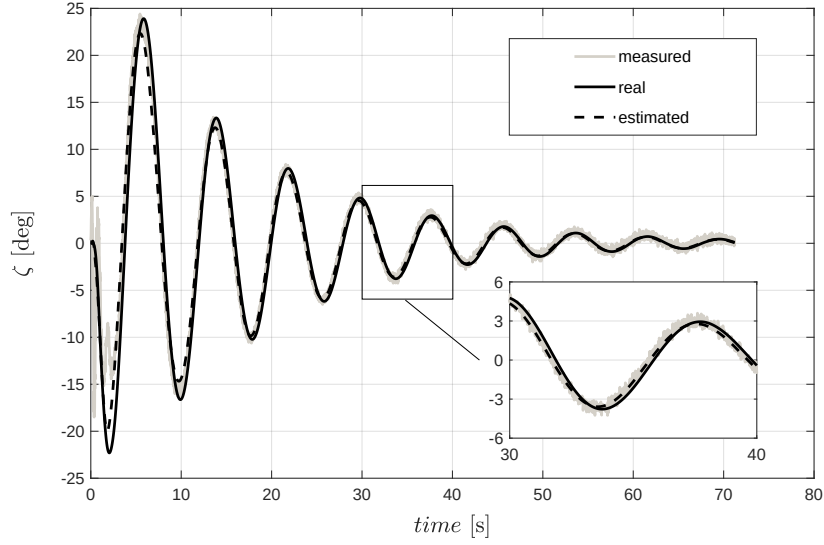


Figure 5.23: Test Case 2: Comparison between the real, the measured, and the estimated oscillation angle.

percentage errors. It is noteworthy that a 40.8% reduction of required energy is obtained thanks to PC strategy.

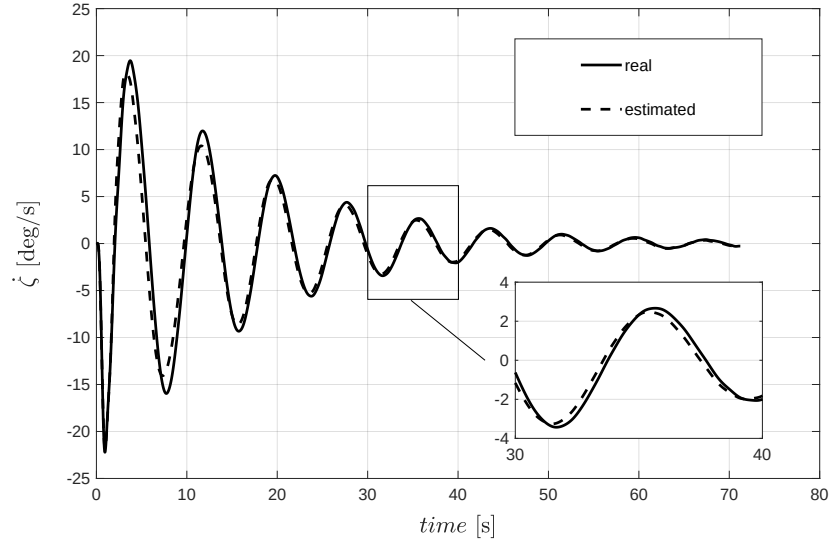


Figure 5.24: Test Case 2: Comparison between the real and the estimated oscillation rate.

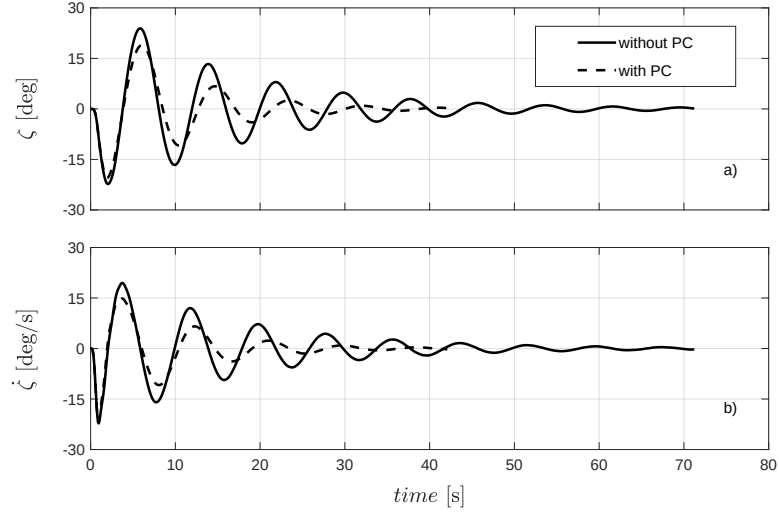


Figure 5.25: Test Case 2: Stabilization of oscillation (a) angle ζ and (b) rate $\dot{\zeta}$ with and without PC.

Test Case 3

In the last simulation case the octarotors start from a hovering condition characterized by $\mathbf{r}_1(0) = [2, 0, -20]^T$ m and $\mathbf{r}_2(0) = [-2, 0, -20]^T$ m with

the payload at rest in the presence of steady wind $\mathbf{w} = [-0.5, 2, -1]^T$ m/s. Desired formation geometry and FGDF equations are reconfigured with $\bar{\boldsymbol{\gamma}} = [4, 0, 0]^T$ m and null set-point yaw angles. The tuning, scaling, and initialization parameters given above for the estimation algorithm are left unchanged to validate filter configuration robustness with respect to (at least bounded) formation geometry variations. Reference trajectory is defined by $\mathbf{r}_r(t) = [t, 2, -20]^T$ m ($\mathbf{v}_r(t) = [1, 0, 0]^T$ m/s) for $t < 20$ s and $\mathbf{r}_r(t) = [2t - 20, 0, -20]^T$ m ($\mathbf{v}_r(t) = [2, 0, 0]^T$ m/s) for $t \geq 20$ s. An additive disturbance signal $\mathbf{a}_{sp_1}^{(D)} = [-3, 0, 0]^T$ m/s² is assumed to perturb Equation (3.118) for $i = 1$ during time interval $30 \leq t \leq 31$ s, thus simulating a short-term malfunction of control system in UAV 1. Provided the focus is posed on speed tracking capabilities, the condition that defines maneuver time in Section 5.2.2, is here adapted to the reduced-order speed error $e_{rs} = \|\boldsymbol{\epsilon}_1^T \boldsymbol{\epsilon}_2^T\|$, required to fall below 0.02 m/s.

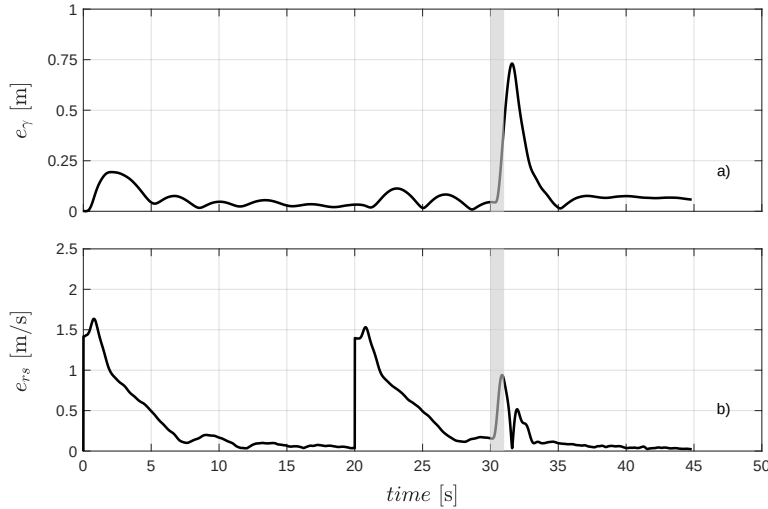


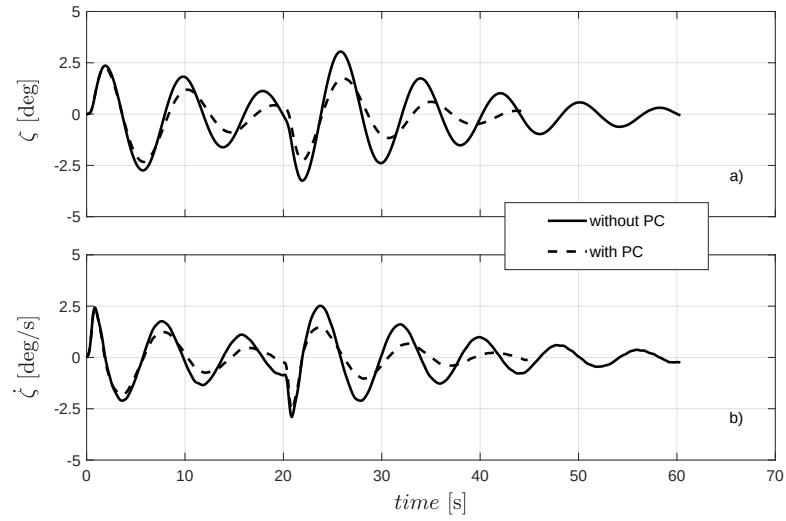
Figure 5.26: Test Case 3: Stabilization of (a) formation geometry and (b) residual speed error with PC.

Figure 5.26a reports the time history of error $e_\gamma = \|\bar{\boldsymbol{\gamma}} - \boldsymbol{\gamma}\|$, showing stabilization of close formation for $R_{sph} = 1$ m. The beneficial effect of CAC is evident during time interval $30 \leq t \leq 31$ s (see gray areas), where safety margin with respect to eventual collision is guaranteed. In Figure 5.26b, the

Table 5.13: Test Case 3: Relevant performance indicators.

Index	without PC	with PC	Variation [%]
t_m [s]	60.4	44.8	-25.7
$I_\zeta(t_m)$ [deg s]	62.60	34.76	-44.5
ζ_{av} [deg]	1.04	0.78	-25.2
E_{prop} [kJ]	845.6	630.5	-25.4

speed error e_{rs} is plotted with satisfactory accuracy of the speed-tracking capabilities. Table 5.13 resumes main indicator values with evident advantages of the proposed estimation and control strategy, especially in terms of required energy and maneuver time. Finally, Figure 5.27 depicts the true oscillation angle and rate as obtained with and without PC, showing reduced swing in the former case.

**Figure 5.27:** Test Case 3: Stabilization of oscillation (a) angle ζ and (b) rate $\dot{\zeta}$ with and without PC.

5.5 Double-Payload Transportation Validation

This section validates the estimation and control framework for an octarotor transporting two serially-connected payloads in a double-slung configuration. After presenting the simulation setup and defining quantitative performance metrics, two representative test cases are investigated: (1) a tuning maneuver used to select the optimal fading factor, and (2) a realistic waypoint parcel-delivery mission where payloads are picked and released. Results include estimator convergence, tuning statistics for the filter bank, and mission-level performance indices that quantify reduction in payload swing and energy consumption.

5.5.1 Simulation Setup

Table 5.1 lists the main multirotor data, while Table 5.14 reports the payload system parameters used in the simulations.

Guidance, navigation, and control tasks are performed following the approach described in detail in Section 4.3, where the lifting vehicle tracks an Earth-fixed acceleration set-point $\mathbf{a}_{sp_i}$ (see Figure 4.2). Within this framework, the multirotor control input is given by Equation (4.36) and is made of two contributions. In particular, $\mathbf{a}_{sp_i}^{(VC)}$ allows trajectory-following through the tracking of setpoint speed $\mathbf{v}_{sp}$ and $\mathbf{a}_{sp_i}^{(PC)}$ implements payload control strategies.

5.5.2 Performance Indicators

Performance indicators are defined to quantitatively characterize the performance of the closed-loop system with and without the payloads oscillations stabilization system. For the aim of the present analysis, let t_m be the total time required to conclude the desired maneuver. Cable swing angles ε_A and ε_B are respectively defined as the angles between the local vertical axis and each cable's direction, namely

$$\varepsilon_A = \arccos(\mathbf{z}_E \cdot \hat{\mathbf{c}}_A), \quad \varepsilon_B = \arccos(\mathbf{z}_E \cdot \hat{\mathbf{c}}_B) \quad (5.15)$$

Table 5.14: Double slung-payload system parameters.

Parameter	Symbol	Value	Units
Load A			
Mass	m_A	60	kg
Reference area	A_A	0.785	m ²
Drag coefficient (sphere)	C_{dA}	0.5	-
Cable A			
Nominal length	L_A	10	m
Hooke's constant	K_A	136 424	N/m
Damping coefficient	C_A	250	N s/m
Load B			
Mass	m_B	40	kg
Reference area	A_B	0.385	m ²
Drag coefficient (sphere)	C_{dB}	0.5	-
Cable B			
Nominal length	L_B	5	m
Hooke's constant	K_B	272 847	N/m
Damping coefficient	C_B	250	N s/m

Average swing angles are defined as $\bar{\varepsilon}_A = I_{\varepsilon A}(t_m)/t_m$ and $\bar{\varepsilon}_B = I_{\varepsilon B}(t_m)/t_m$, where the integrals

$$I_{\varepsilon A}(t_m) = \int_0^{t_m} |\varepsilon_A(s)| ds \quad (5.16)$$

and

$$I_{\varepsilon B}(t_m) = \int_0^{t_m} |\varepsilon_B(s)| ds \quad (5.17)$$

are evaluated over t_m . Similarly, average swing rates $\bar{\nu}_A$ and $\bar{\nu}_B$ are introduced. Let $\nu_A = \dot{\varepsilon}_A$ and $\nu_B = \dot{\varepsilon}_B$ be the time derivatives of $\varepsilon_A(t)$ and $\varepsilon_B(t)$, respectively. Then, $\bar{\nu}_A$ and $\bar{\nu}_B$ are calculated through the integrals

$$\bar{\nu}_A = \sqrt{\frac{1}{t_m} \int_0^{t_m} \nu_A^2(s) ds} \quad (5.18)$$

and

$$\bar{\nu}_B = \sqrt{\frac{1}{t_m} \int_0^{t_m} \nu_B^2(s) ds} \quad (5.19)$$

E_{prop} is the mechanical energy delivered by electrical motors to propellers, calculated by the integral

$$E_{prop} = \int_0^{t_m} \sum_{j=1}^8 P_{shj}(s) ds \quad (5.20)$$

and expressed in Wh, where $P_{shj} = Q_j \Omega_j$ is the output shaft power generated by the j -th motor, $j = 1, \dots, 8$, obtained from required torque Q_j and angular rate Ω_j , under the assumption of null friction torque. Finally, the amplitudes of the accelerations of the two payloads are given, provided $\mathbf{a}_A = \dot{\mathbf{v}}_A$ and $\mathbf{a}_B = \dot{\mathbf{v}}_B$. Average values are respectively obtained as

$$\bar{a}_A = \frac{1}{t_m} \int_0^{t_m} \|\mathbf{a}_A(s)\| ds \quad (5.21)$$

and

$$\bar{a}_B = \frac{1}{t_m} \int_0^{t_m} \|\mathbf{a}_B(s)\| ds \quad (5.22)$$

5.5.3 Stability Analysis

In this subsection a numerical stability analysis is performed to investigate the stability of both the open- and closed-loop systems. First, the open-loop poles are reported with and without the rotational viscous damping coefficient d . Then, the closed-loop modes are addressed with and without the auxiliary payload controller contribution. In what follows, the poles are computed using the data in Tables 5.1 and 5.14, namely $m = 70$ kg, $m_A = 60$ kg, $m_B = 40$ kg, $L_A = 10$ m, $L_B = 5$ m, and $g = 9.80665$ m/s².

In Figure 5.28 the open-loop eigenvalues without aerodynamic damping ($d = 0$) analytically described in Section 4.4 are reported, together with the ones obtained considering $d = 0.25$ N m s. Regarding the former, the nonzero forward-swing poles in Equations (4.50) and (4.51) are purely oscillatory, namely $\nu_{fs1,2} = \pm j 2.1071$ 1/s and $\nu_{fs3,4} = \pm j 1.2017$ 1/s. Thanks to symmetry, the lateral-swing modes are identical to the forward-swing ones,

as reported in Equations (4.54) and (4.55). For the latter case, the nonzero forward-swing modes become $\nu_{fs1,2} = -0.0002 \pm j 2.1071$ and $\nu_{fs3,4} = -4.7185 \cdot 10^{-5} \pm j 1.2017$. With $d = 0.25$ N m s, the swing pairs acquire small negative real parts, indicating weak, but non-negligible aerodynamic dissipation.

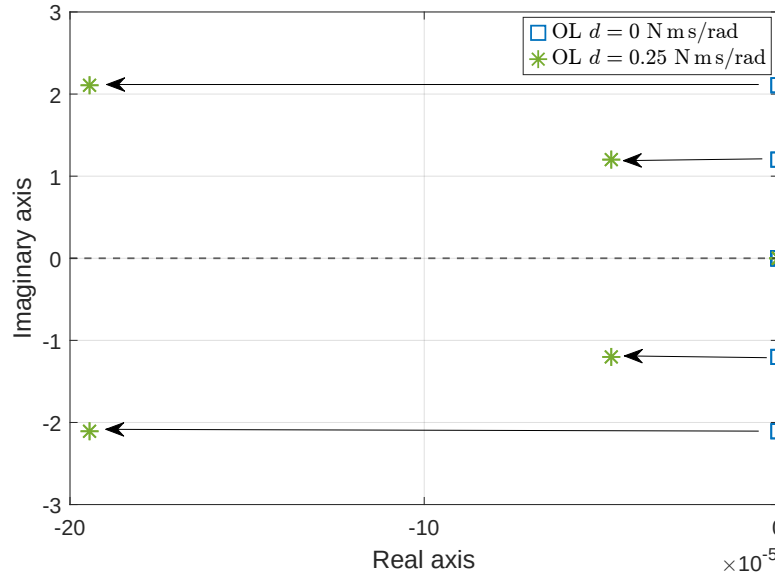


Figure 5.28: Open-loop dynamics eigenvalues with and without the aerodynamic damping contribution.

For the closed-loop dynamics, position and velocity control gains are selected as $\mathbf{K}_p = \text{diag}(0.1, 0.1, 0.6)$ N/m and $\mathbf{K}_v = \text{diag}(2.244, 2.244, 4)$ N s/m, while the payload controller is characterized by $k_p = -1.2$ m/(rad s²) and $k_v = -0.4$ m/(rad s). Table 5.15 lists the eigenvalues obtained for the closed-loop dynamic matrices $\hat{\mathbf{A}}_{fs}$, $\hat{\mathbf{A}}_{ls}$, and $\hat{\mathbf{A}}_v$ in Equations (4.47)–(4.49), with and without the auxiliary contribution of the PC. In all the cases, the hovering equilibrium is stable, as $\hat{\mathbf{A}}_{fs}$, $\hat{\mathbf{A}}_{ls}$, and $\hat{\mathbf{A}}_v$ are Hurwitz matrices. While the effect of the PC does not influence the vertical behavior, it does affect the forward and lateral swing motion. On one hand, the low-frequency complex-conjugate poles $-0.1604 \pm j 0.9371$ 1/s are shifted to $-0.5165 \pm j 0.9164$ 1/s, resulting in increased damping of the oscillatory behavior. Specifically, the damping ratio increases from 0.169 to 0.491 (+190.53%), while the nat-

ural frequency from 0.951 rad/s to 1.052 rad/s (+10.64%). On the other hand, the high-frequency pair is moved from $-0.0385 \pm j 2.0583$ 1/s to $-0.0592 \pm j 2.0406$ 1/s. This implies the damping ratio is slightly increased from 0.019 to 0.029 (+52.63%), while the natural frequency remains almost unchanged, namely from 2.059 rad/s to 2.041 rad/s (−0.87%). While the PC stabilizing contribution is very effective on the low-frequency pair, its action is mitigated for the high-frequency poles. This behavior reflects the center-of-gravity-based controller’s feedback, which couples strongly to the in-phase swing, but only weakly to the out-of-phase motion between the two loads.

Figure 5.29 reports the closed-loop eigenvalues for the forward-swing subsystem. The beneficial effect of the payload controller in stabilizing the oscillatory modes is made evident by the black arrows, indicating the PC-induced pole shifts. The same considerations hold for the lateral-swing poles, which, by symmetry, are omitted for brevity.

Table 5.15: Eigenvalues of the feedback-controlled subsystems at hover [s^{−1}].

Subsystem	PC OFF	PC ON
$\hat{\mathbf{A}}_{fs}$	−1.8002	−1.4450
	−0.0465	−0.0481
	$-0.0385 \pm j 2.0583$	$-0.0592 \pm j 2.0406$
	$-0.1604 \pm j 0.9371$	$-0.5165 \pm j 0.9164$
$\hat{\mathbf{A}}_{ls}$	−1.8002	−1.4450
	−0.0465	−0.0481
	$-0.0385 \pm j 2.0583$	$-0.0592 \pm j 2.0406$
	$-0.1604 \pm j 0.9371$	$-0.5165 \pm j 0.9164$
$\hat{\mathbf{A}}_v$	−0.1633	−0.1633
	−1.8367	−1.8367

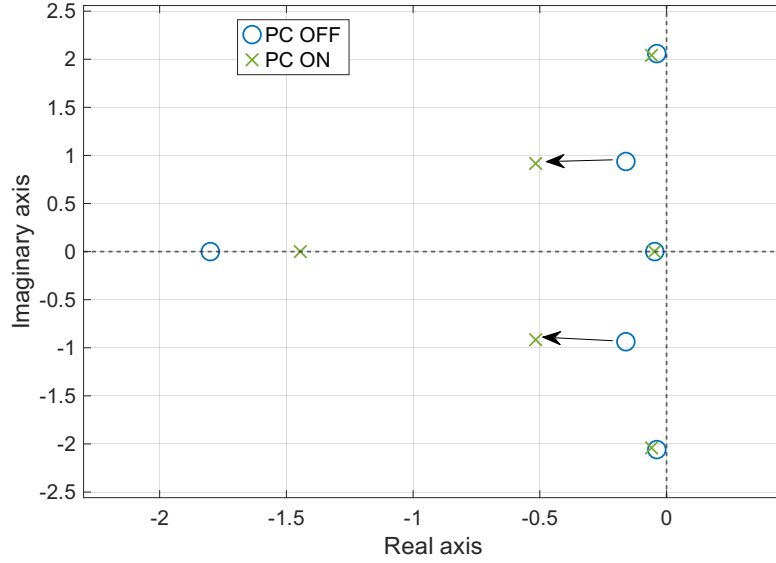


Figure 5.29: Closed-loop forward-swing dynamics eigenvalues.

5.5.4 Test Cases

This subsection introduces two test cases to numerically validate the swing state estimation and control algorithms in the double-payload transportation scenario. First, a simple hovering maneuver is taken into account to describe in detail the FGDF tuning procedure from a practical standpoint, such that the optimal filter configuration is retrieved. Then, the proposed strategies are validated in a realistic delivery mission, in which the aerial vehicle either drops or picks up suspended masses. In what follows, consider the system parameters reported in Tables 5.1 and 5.14. Equations (4.29) and (4.30) respectively provide:

$$A_{pl} = \left[\begin{array}{c|c} \mathbf{0}_{4 \times 4} & \mathbf{I}_4 \\ \hline \begin{matrix} -3.2689 & 0 & 3.2689 & 0 \\ 0 & -3.2689 & 0 & 3.2689 \\ 0.6538 & 0 & -3.0354 & 0 \\ 0 & 0.6538 & 0 & -3.0354 \end{matrix} & \begin{matrix} -2.5 \cdot 10^{-4} & 0 & 5 \cdot 10^{-5} & 0 \\ 0 & -2.5 \cdot 10^{-4} & 0 & 5 \cdot 10^{-5} \\ 5 \cdot 10^{-5} & 0 & -4.6429 \cdot 10^{-5} & 0 \\ 0 & 5 \cdot 10^{-5} & 0 & -4.6429 \cdot 10^{-5} \end{matrix} \end{array} \right] \quad (5.23)$$

and

$$B_{pl} = \left[\begin{array}{c|c} \mathbf{0}_{6 \times 3} & \\ \hline \begin{matrix} 0 & 0.0014 & 0 \\ -0.0014 & 0 & 0 \end{matrix} & \end{array} \right] \quad (5.24)$$

The state-transition matrix is computed by MATLAB[®] matrix exponential function $\text{expm}(\cdot)$ and yields

$$\Phi = \begin{bmatrix} 0.9998 & 0 & 0.0002 & 0 & 0.0100 & 0 & 5.473 \cdot 10^{-7} & 0 \\ 0 & 0.9998 & 0 & 0.0002 & 0 & 0.0100 & 0 & 5.473 \cdot 10^{-7} \\ 3.2687 \cdot 10^{-5} & 0 & 0.9998 & 0 & 1.1146 \cdot 10^{-7} & 0 & 0.0100 & 0 \\ 0 & 3.2687 \cdot 10^{-5} & 0 & 0.9998 & 0 & 1.1146 \cdot 10^{-7} & 0 & 0.0100 \\ -0.0327 & 0 & 0.0327 & 0 & 0.9998 & 0 & 0.0002 & 0 \\ 0 & -0.0327 & 0 & 0.0327 & 0 & 0.9998 & 0 & 0.0002 \\ 0.0065 & 0 & -0.0304 & 0 & 3.3187 \cdot 10^{-5} & 0 & 0.9998 & 0 \\ 0 & 0.0065 & 0 & -0.0304 & 0 & 0 & 3.3187 \cdot 10^{-5} & 0.9998 \end{bmatrix} \quad (5.25)$$

The control-input function is

$$\Psi = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0.0007 & 0 \\ -0.0007 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0.1428 & 0 \\ -0.1428 & 0 & 0 \end{bmatrix} \cdot 10^{-4} \quad (5.26)$$

according to the formulation reported in Section 4.2.

Test Case 1

The first simulation case consists in a representative maneuver designed to facilitate filter optimization by tuning the fading factor β . The filter's initial state estimate is set to $\tilde{\mathbf{x}}^{(0)-} = \mathbf{0}_{8 \times 1}$ and the nominal covariance matrix is initialized as $\mathbf{E}^{(0)-} = \text{diag}(10^{-6}, 10^{-6}, 10^{-8}, 10^{-8}, 10^{-6}, 10^{-6}, 10^{-7}, 10^{-7})$. Acceleration measurements are converted to measured oscillation angles through Equations (4.33) and (4.34). In this case, the acceleration measurement variance σ_{acc}^2 is scaled by the Jacobian factor $1/A_{8,7}$ (equal to $= |1/A_{9,6}|$), which reduces to $m/(g(m_A + m_B))$ under the assumed linearization. This yields the exact covariance matrix $\mathbf{R}_{syn} = 1.44 \cdot 10^{-6} \cdot \mathbf{I}_2 \text{ rad}^2$, related to the synthetic measurement vector $\mathbf{z}$. Within this setup, the assigned measurement noise covariance $\mathbf{R}^*$ is intentionally chosen to be three times the value of the synthesized covariance $\mathbf{R}_{syn}$, resulting in $\mathbf{R}^* = 4.32 \cdot 10^{-6} \cdot \mathbf{I}_2 \text{ rad}^2$. The octarotor is required to maintain the initial hover condition at $t_0 = 0$ s with $\mathbf{r}(0) = [0, 0, -30]^T \text{ m}$, $\mathbf{v}(0) = [0, 0, 0]^T \text{ m/s}$, and $\boldsymbol{\eta}(0) = [0, 0, 0]$

rad, provided that $\xi_A(0) = \zeta_A(0)$ deg, $\dot{\xi}_A(0) = \dot{\xi}_B(0) = \dot{\zeta}_A(0) = \dot{\zeta}_B(0)$ deg/s, and $\xi_B(0) = \zeta_B(0) = 5$ deg. The set point position is thus a constant equal to $\mathbf{r}_{sp} = \mathbf{r}(0)$, the desired yaw angle is $\psi_{sp} = 0$ rad, and payload control is initially deactivated, $\mathbf{a}_{sp}^{(PC)} = \mathbf{0}_{3 \times 1}$ m/s².

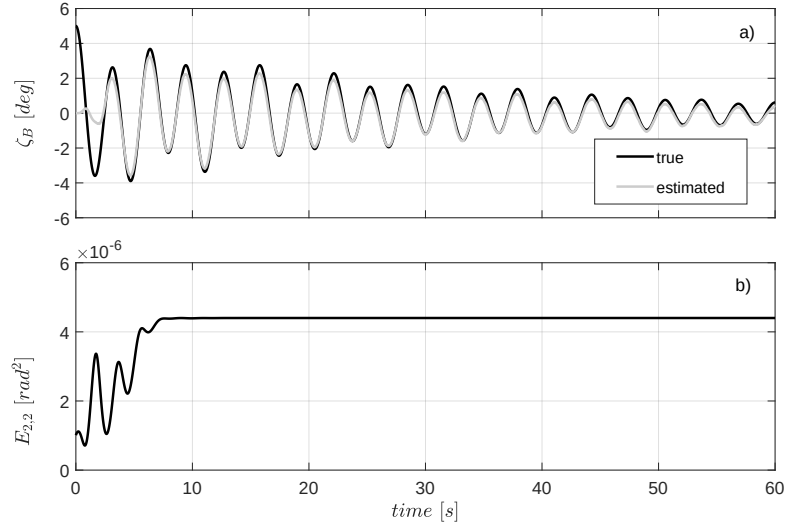


Figure 5.30: Test Case 1: (a) Comparison between the true and the estimated ζ_B angle. (b) Estimation convergence for the nominal covariance component $E_{2,2}$.

Figure 5.30 presents the results of a sample run with $\beta = 0.99$ in calm atmospheric conditions, illustrating the estimator's rapid convergence to a steady state. Specifically, the plot compares the true value, $x_2 = \zeta_B$, with its estimate, $\tilde{x}_2$, over a 60 s interval. During this time, the nominal covariance component $E_{2,2}$, and thus the estimator gain, settles at a constant value. A similar behavior is observed in the estimation of the angular rate $x_6 = \dot{\zeta}_B$, as shown in Figure 5.31. With respect to payload A , results are detailed in Figure 5.32, where satisfactory accuracy characterizes the estimation task in the presence of oscillations with a higher harmonic content. Similar results are obtained for the lateral oscillatory dynamics.

Figure 5.33 shows time-averaged results obtained in the estimator's steady state condition for simulations performed over 60 s. A range of β from 0.96 to 0.99, distributed over 31 filters, shows the effects predicted by theory. The tuning statistic $\overline{\chi^2}$ goes through a minimum on about the 22-nd filter,

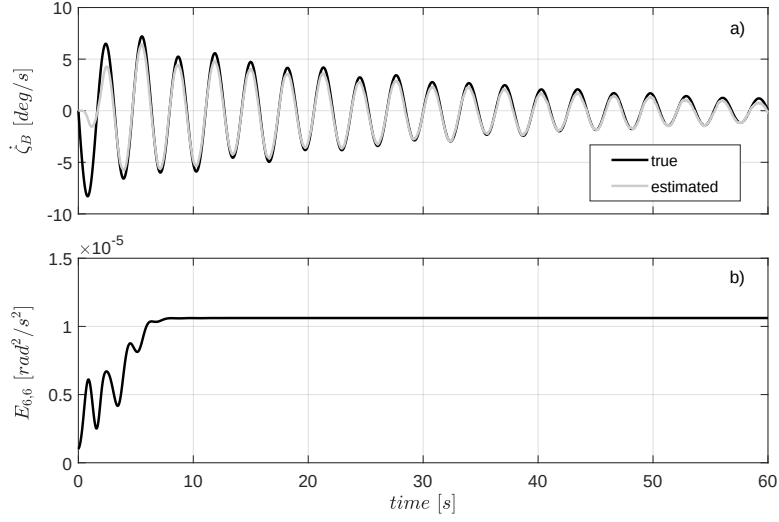


Figure 5.31: Test Case 1: (a) Comparison between the true and the estimated $\dot{\zeta}_B$ angular rate. (b) Estimation convergence for the nominal covariance component $E_{6,6}$.

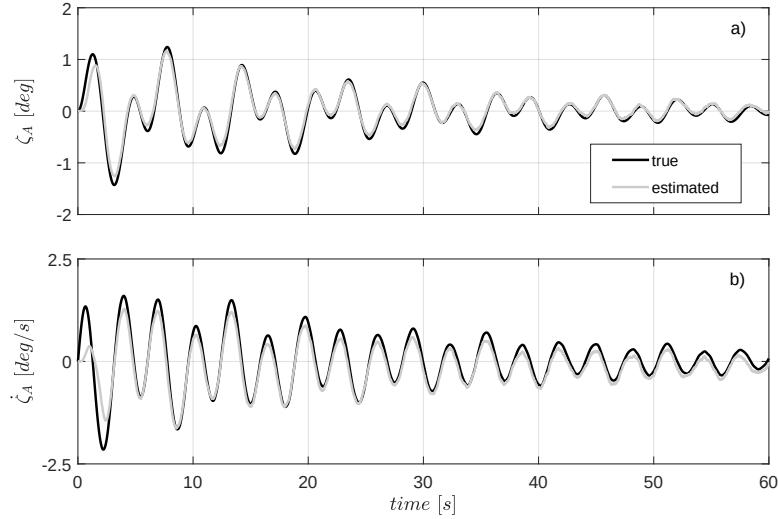


Figure 5.32: Test Case 1: Comparison between the true and the estimated values of (a) ζ_A angle and (b) $\dot{\zeta}_A$ angular rate.

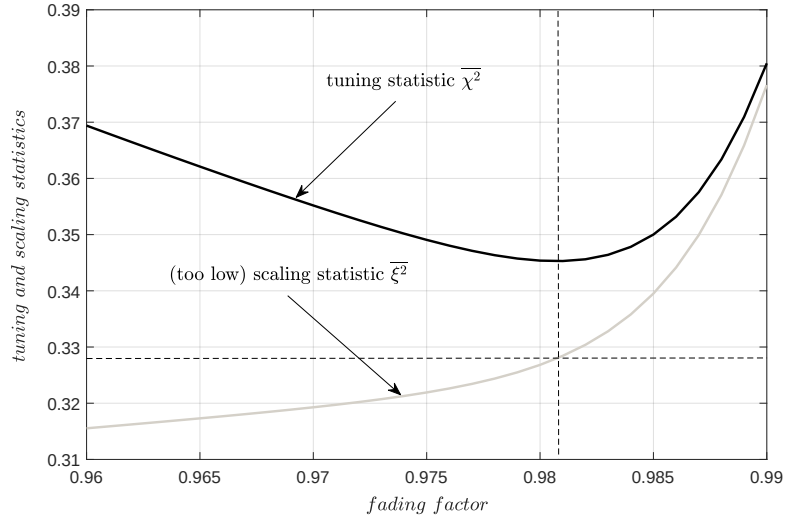


Figure 5.33: Test Case 1: Analysis of the averaged tuning and scaling statistics over the filter bank (assigned $\mathbf{R}^*$).

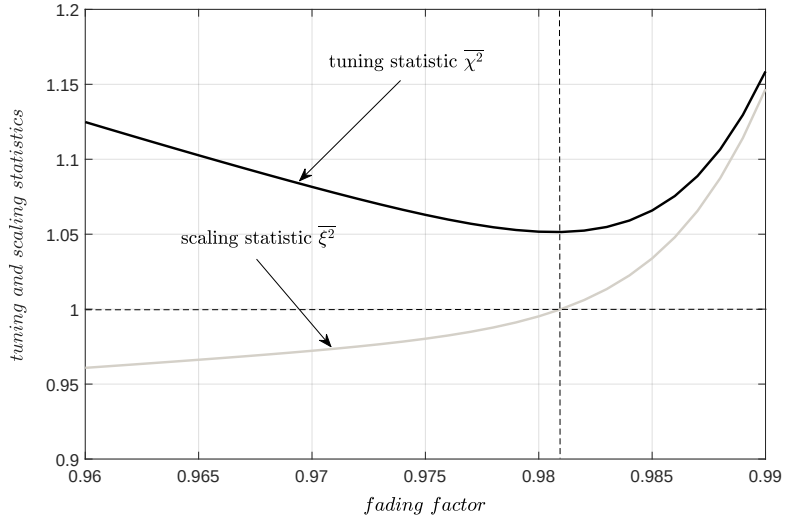


Figure 5.34: Test Case 1: Analysis of the averaged tuning and scaling statistics over the filter bank (scaled $\mathbf{R}^*$).

where $\beta = \beta_T = 0.981$. As expected, the value of the scaling statistic results to be too low, with $\bar{\xi}^2(\beta_T) = 0.3284 < 1$, meaning that the assigned $\mathbf{R}^*$ is too high. At the best value $\beta = \beta_T$, the average scaling statistic $\bar{\xi}^2(\beta_T) = 0.3284$ is thus used to properly scale $\mathbf{R}^*$ to an improved estimate of

$\mathbf{R}_{syn}$, namely $\tilde{\mathbf{R}} = \overline{\xi^2}(\beta_T) \mathbf{R}^* = 1.42 \cdot 10^{-6} \cdot \mathbf{I}_2 \text{ rad}^2 \approx \mathbf{R}_{syn}$, according to the procedure described in detail by de Angelis et al. [91]. The current estimate $\tilde{\mathbf{R}}$, representing the expected value of the noise covariance matrix, is employed to recompute the filter bank across the same range of β (see Figure 5.34). The tuning statistic continues to identify the same optimal filter, although the corresponding minimum value differs. Concurrently, the scaling statistic now crosses unity at the tuned condition, indicating that the newly assigned $\mathbf{R}^*$ constitutes the best available estimate of $\mathbf{R}_{syn}$. As shown in Figure 5.35, the trace of the true covariance $\mathbf{P}$, evaluated over the updated filter bank using $\mathbf{R}^*$, attains its minimum near β_T , further validating the effectiveness of the tuning statistic in identifying the globally optimal filter. Additionally, the intersection between the true and the nominal error covariance curve as obtained through the trace operator, occurs close to this tuned condition at about $\beta_{P_{min}} = 0.978$ (-0.36% error with respect to β_T), showing that the nominal error covariance approximates the true covariance under the tuned and scaled configuration.

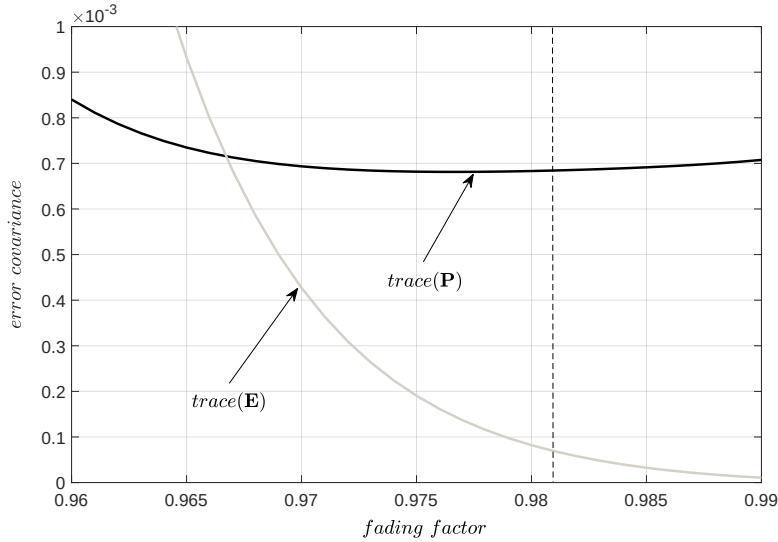


Figure 5.35: Test Case 1: Analysis of the trace of nominal and true error covariance matrices over the filter bank (scaled $\mathbf{R}^*$).

Test Case 2

The second test case simulates a real-world air delivery maneuver in which the aerial vehicle either picks or deliver parcels upon reaching dedicated locations. The octarotor is initialized under the same conditions adopted for the tuning procedure, with the payloads at rest, and the filter is setup to deliver optimal performance ($\beta = \beta_T = 0.981$ and $\mathbf{R}^* = 1.42 \cdot 10^{-6} \cdot \mathbf{I}_2 \text{ rad}^2$) under the loading configuration discussed in Section 5.5.4. The vehicle is required to follow a sequence of prescribed waypoints $\mathbf{WP}_i = [x_i, y_i, z_i]^T$, $i = 1, \dots, 5$, as listed in Table 5.16. Each waypoint is deemed to be reached when two conditions are simultaneously satisfied: (1) the positioning error, defined as $\varepsilon_P = \|\mathbf{WP}_i - \mathbf{r}\|$, falls below 0.3 m and (2) both cable oscillation angles $\varepsilon_A \geq 0$ and $\varepsilon_B \geq 0$ remain bounded under 3 deg for a continuous duration of 10 s. Delivery tasks are performed according to the program detailed in the 5-th and 7-th column of Table 5.16, which define the expected payload conditions, denoted as m_{A_i} and m_{B_i} , when $\mathbf{WP}_i$ is designated as the setpoint. For instance, at $i = 1$, the octarotor departs from its initial position and moves toward $\mathbf{WP}_1$ carrying $m_{A1} = 60 \text{ kg}$ and $m_{B1} = 40 \text{ kg}$. Upon arrival at $\mathbf{WP}_1$, load B is released, resulting in $m_{A2} = 60 \text{ kg}$ and $m_{B2} = 0 \text{ kg}$, and the vehicle continues toward the next setpoint, $\mathbf{WP}_2$.

Table 5.16: Test Case 2: Waypoints of the delivery mission.

WP	$[x_i, y_i, z_i] \text{ [m]}$	$m_A \text{ [kg]}$	$A_A \text{ [m}^2\text{]}$	$m_B \text{ [kg]}$	$A_B \text{ [m}^2\text{]}$
1	[10, 0, -50]	60	0.785	40	0.385
2	[20, -10, -50]	60	0.785	0	0
3	[20, 10, -50]	60	0.785	20	0.193
4	[30, 0, -50]	60	0.785	48	0.462
5	[30, 10, -50]	60	0.785	0	0

Certain waypoints, such as $\mathbf{WP}_2$ and $\mathbf{WP}_3$, serve as loading points where the value of m_B increases. In contrast, waypoints like $\mathbf{WP}_1$, $\mathbf{WP}_4$, and $\mathbf{WP}_5$, correspond to delivery locations where part of the payload is dropped. In the latter case, the closed-loop system reverts to the single payload scenario, such that $\mathbf{r}_C \equiv \mathbf{r}_A$, and continuity of operation characterizes the control strategy

in Equation (4.37), according to Remark 11. In such a case, one has:

$$\mathbf{r}_C - \mathbf{r}_H \approx L_A [\sin \zeta_A, -\sin \xi_A \cos \zeta_A, \cos \xi_A \cos \zeta_A]^T \quad (5.27)$$

and

$$\mathbf{v}_C - \mathbf{v}_H \approx L_A [\dot{\zeta}_A c\zeta_A, -\dot{\xi}_A c\xi_A c\zeta_A + \dot{\zeta}_A s\xi_A s\zeta_A, -\dot{\xi}_A s\xi_A c\zeta_A - \dot{\zeta}_A c\xi_A s\zeta_A]^T \quad (5.28)$$

Accordingly, the estimation procedure switches to the method presented in Ref. [63], where the EKF is used to estimate the lateral ($\xi = \xi_A$) and the frontal ($\zeta = \zeta_A$) oscillation angle of the remaining payload through the same acceleration and attitude measurements of the FGDF. The two estimators are thus executed in parallel, provided the EKF is initialized with $\tilde{\mathbf{x}}^{(0)-}|_{EKF} = [\tilde{\xi}_A(0), \tilde{\zeta}_A(0), \tilde{\xi}_A(0), \tilde{\zeta}_A(0), \tilde{f}_{d1}, \tilde{f}_{d2}, \tilde{f}_{d3}]^T = \mathbf{0}_{7 \times 1}$ and $\mathbf{E}^{(0)-}|_{EKF} = \text{diag}(10^{-6}, 10^{-6}, 10^{-6}, 10^{-6}, 1, 1, 10^{-5})$, the latter being the estimated covariance matrix. In this respect, the assigned noise covariance matrix is $\mathbf{R} = 2.25 \cdot 10^{-4} \cdot \mathbf{I}_3$ m²/s⁴ and process noise is accounted by $\mathbf{Q} = \text{diag}(10^{-5}, 10^{-5}, 10^{-5}, 10^{-5}, 1, 1, 10^{-5})$, both matrices assumed to be constant. For the sake of brevity, the EKF tuning procedure to find the optimal configuration of $\mathbf{R}$ and $\mathbf{Q}$ is not detailed. It is noted that, while the EKF always operates in the tuned condition during single-payload mission phases, the FGDF results to be suboptimal in loading configurations where $m_B \neq 40$ kg. The deliberate, albeit conservative, omission of the fading parameter adjustment is intended to assess filter performance in the presence of model uncertainties and errors.

During the mission, gusts are introduced to disturb the states of both the octarotor and the payloads. The turbulence is modeled using a dedicated Simulink Aerospace Blockset component, which implements the Von Kármán spectral representation. The setting is identical to the one introduced in Section 5.3.3, thus it will be here omitted for the sake of brevity. However, in addition to the considered turbulence contribution, a horizontal East wind with constant 10 m/s intensity is considered.

In Figure 5.36 the trajectory followed by the octarotor is reported for the case when no active payload stabilization is performed, namely $\mathbf{a}_{sp}^{(PC)} = \mathbf{0}_{3 \times 1}$ m/s².

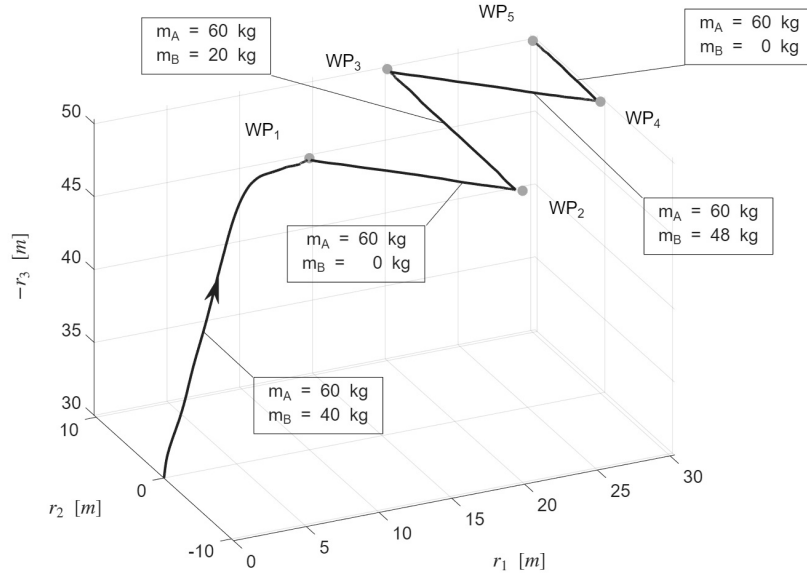


Figure 5.36: Test Case 2: Trajectory followed by the octarotor based on a set of prescribed waypoints ($\mathbf{a}_{sp}^{(PC)} = \mathbf{0}_{3 \times 1} \text{ m/s}^2$).

Figure 5.37 presents a comparison between the true frontal oscillation variables of load A and their respective estimates obtained using the FGDF (solid gray line) and the EKF (dotted gray line). Correspondingly, Figure 5.38 illustrates the same comparison for load B, restricted to the time intervals during which $m_B \neq 0 \text{ kg}$. As expected, both estimation tasks exhibit satisfactory accuracy under nominal and near-nominal payload conditions. Reduced accuracy is observed during the mission phase between **WP**₂ and **WP**₃, where the knowledge of m_B is affected by a -50% uncertainty. Nonetheless, the FGDF continues to capture the macroscopic behavior, albeit with increased estimation noise.

Figures 5.39 and 5.40 compare the true and estimated lateral oscillation states for loads A and B, respectively. A bias appears in the estimation of the oscillation angles ξ_A and ξ_B (leading to a maximum positioning error of approximately 0.7 m for the lower load), which is attributed to the influence of the constant East wind contribution. This wind induces a persistent lateral oscillation angle due to aerodynamic drag on the payloads, an effect that is not

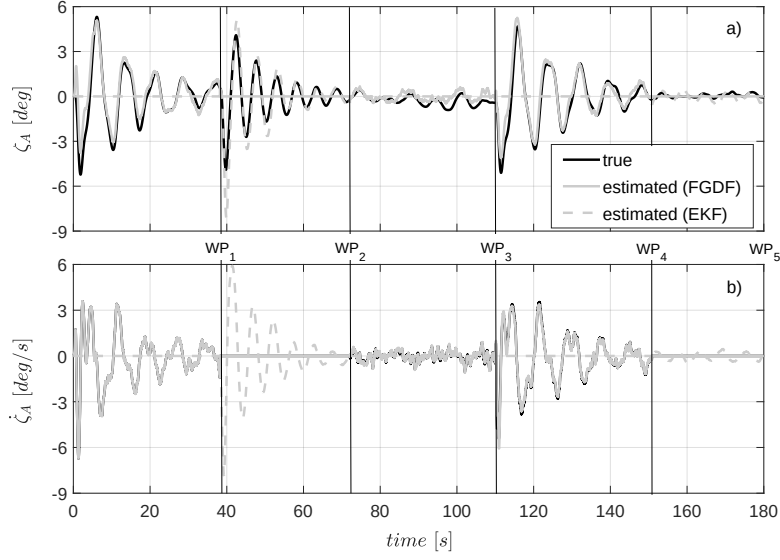


Figure 5.37: Test Case 2: Comparison between the true and the estimated frontal oscillation states of load A ($\mathbf{a}_{sp}^{(PC)} = \mathbf{0}_{3 \times 1} \text{ m/s}^2$).

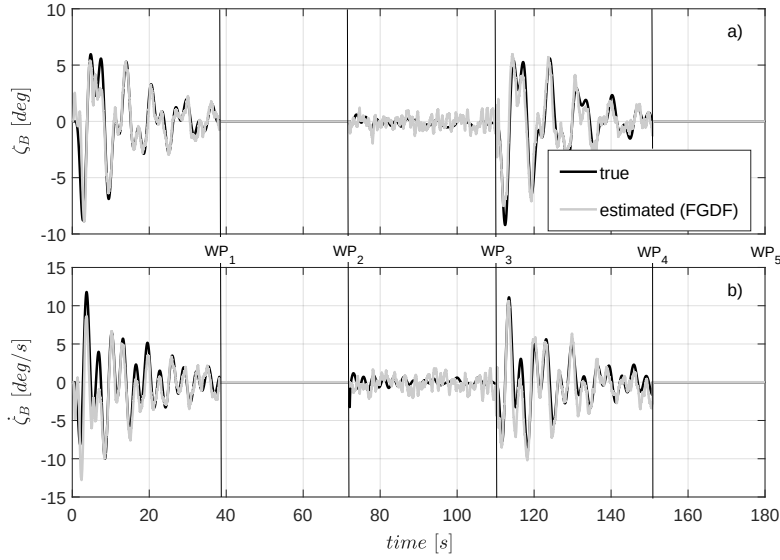


Figure 5.38: Test Case 2: Comparison between the true and the estimated frontal oscillation states of load B ($\mathbf{a}_{sp}^{(PC)} = \mathbf{0}_{3 \times 1} \text{ m/s}^2$).

captured by the observation model described in Equations (4.31) and (4.32). Specifically, under steady-state conditions where $a_2 = 0 \text{ m/s}^2$ and $u_2 = 0 \text{ N}$, the lateral angle measured by Equation (4.33) is indeed zero. In addition, the nominal scenario in which both cables align with the local vertical no longer corresponds to a true equilibrium point. Hence, the eventual application of the auxiliary controller in Equation (4.37) apparently becomes inconsistent with the system's actual dynamics, since the term $\text{diag}(1, 1, 0) [k_p (\mathbf{r}_C - \mathbf{r}_H)]$ would provide a residual, steady-state contribution. In contrast, the term $\text{diag}(1, 1, 0) [k_v (\mathbf{v}_C - \mathbf{v}_H)]$ would contribute with a significant damping effect, regardless of the filter's limitations, based on the accurate estimation of $\mathbf{v}_C - \mathbf{v}_H$. In what follows, the implementation of PC is discussed, while clarifying the effect of $\text{diag}(1, 1, 0) [k_p (\mathbf{r}_C - \mathbf{r}_H)]$ from a practical standpoint.

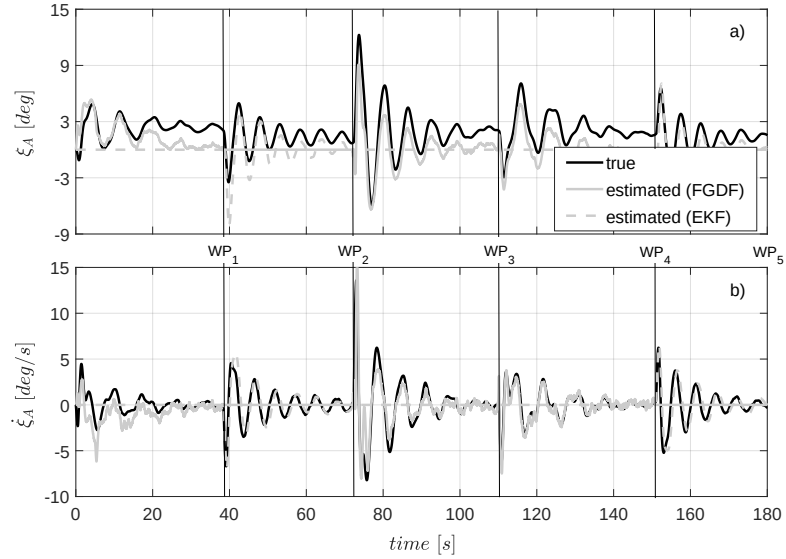


Figure 5.39: Test Case 2: Comparison between the true and the estimated lateral oscillation states of load A ($\mathbf{a}_{sp}^{(PC)} = \mathbf{0}_{3 \times 1} \text{ m/s}^2$).

In the case when PC is activated based on the FGDF information, the mission is executed with performance indicators listed in the third column of Table 5.17, introduced to quantify the beneficial effects of the auxiliary controller. A comparison, summarized in the second column, is also provided with the case when $\mathbf{a}_{sp}^{(PC)} = \mathbf{0}_{3 \times 1} \text{ m/s}^2$.

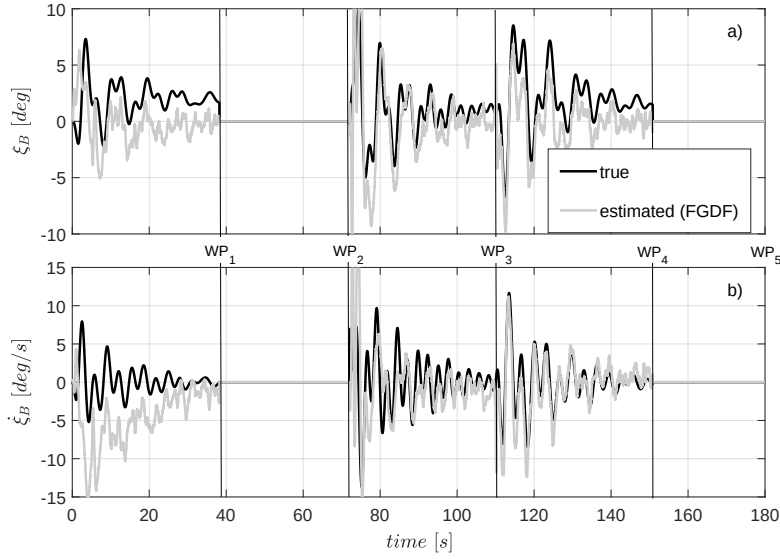


Figure 5.40: Test Case 2: Comparison between the true and the estimated lateral oscillation states of load B ($\mathbf{a}_{sp}^{(PC)} = \mathbf{0}_{3 \times 1} \text{ m/s}^2$).

Table 5.17: Test Case 2: Relevant performance indicators.

Index	without PC	with PC (FGDF+EKF)
t_m [s]	180.7	177.4 (-1.8 %)
E_{prop} [Wh]	1 161.0	1 140.2 (-1.8 %)
$\bar{\varepsilon}_A$ [deg]	2.52	2.15 (-14.7 %)
$\bar{\varepsilon}_B$ [deg]	2.03	1.69 (-16.7 %)
$\bar{\nu}_A$ [deg/s]	2.05	1.68 (-18.0 %)
$\bar{\nu}_B$ [deg/s]	3.17	2.23 (-29.7 %)
$\bar{a}_A$ [m/s ²]	0.39	0.27 (-30.8 %)
$\bar{a}_B$ [m/s ²]	2.22	1.71 (-23.0 %)

Table 5.17 shows that the estimation strategy, combined with the proposed payload control method, enables the system to complete the same mission in approximately the same maneuver time and with comparable battery consumption. Notably, the swing motion of both payloads is significantly reduced. This is particularly evident in the average swing angles, which decrease by approximately 14.7 % for the upper load and 16.7 % for the lower load. Similar considerations hold for the average swing rates, with diminutions

amounting respectively to 18 % and 29.7 %. Even more promising are the acceleration results, with amplitude reductions of about 30.8 % and 23 %, respectively, which represents a critical objective when ensuring the payload is protected from undesired mechanical stress is of paramount importance. Although initial concerns were raised, it is observed that the proportional term $\text{diag}(1, 1, 0) [k_p (\mathbf{r}_C - \mathbf{r}_H)]$ proves effective in attenuating oscillation angles during phases of accelerated motion. Nonetheless, it provides no contribution under steady-state conditions, where residual angular deviations inherently persist in the physical system but remain unperceived by the filter and, consequently, by the controller. In this context, the inherent limitations of the filter actually translate into a beneficial reduction of control activity.

CHAPTER 6

Experimental Validation

This chapter presents experimental validation of the estimation and control methodologies developed throughout this dissertation. The experimental work is organized into two campaigns addressing distinct system configurations. Section 6.1 validates the cooperative swing state estimator of Section 3.4 through flight tests conducted during the candidate’s research visit to the Robotics and Mechatronics department of the University of Twente, the Netherlands. Section 6.2 validates the complete estimation and control architecture for dual serially-connected payloads introduced in Chapter 4, demonstrating real-time implementability and closed-loop performance on commercial hardware.

6.1 Swing State Estimation Validation

This section validates the Fading Gaussian Deterministic Filter for swing-angle estimation in cooperative transportation scenarios introduced in Section 3.4.

6.1.1 Experimental Setup

The experimental platform is designed to be a physical instantiation of the systems modeled in the previous chapters, comprising two identical multirotor aerial vehicles, a shared suspended payload, a ground-based control station running the core estimation and control algorithms, and a Motion Capture (MoCap) computer.

Hardware Platform

The aerial vehicles employed in this research are two Holybro X500 V2 quadcopters with take-off mass $m = 1.824$ kg and a wheelbase of 0.5 m. The avionics consists of a Pixhawk[®] 6C FCU and a Raspberry Pi 4B as (FCC). The propulsive system comprises 4 Holybro 2216 KV920 brushless motors, each managed by a BLHeli S 20 A ESC. The system is powered by a 4S Li-Po battery and equipped with 1045-size propellers [101]. Each vehicle is equipped with a thin, high-tensile strength cable with nominal length $L = 1.35$ m, suspended by a hook positioned 0.20 m below the quadrotor top surface (i.e. $STA_H = BL_H = 0$ and $WL_H = -0.22$ m). The payload is a small cylinder positioned over a 3D printed board (for properly placing the motion capture markers) with mass $m_l = 0.537$ kg. Figure 6.1 shows the overall experimental platform used for the campaign.

Environment and Ground Truth

The experimental campaign is conducted within the indoor flight arena of the University of Twente equipped with an OptiTrack motion capture system [102]. This setup, comprising multiple high-speed infrared cameras, provides high-frequency (100 Hz), sub-millimeter accuracy position and orientation data for both the quadrotors and the suspended payload. Within this framework, the OptiTrack system serves as the definitive ground truth for these experiments, providing baseline information against which the performance of the FGDF is evaluated. OptiTrack information is streamed over the local network via wired Ethernet connection to a dedicated MoCap computer running the software

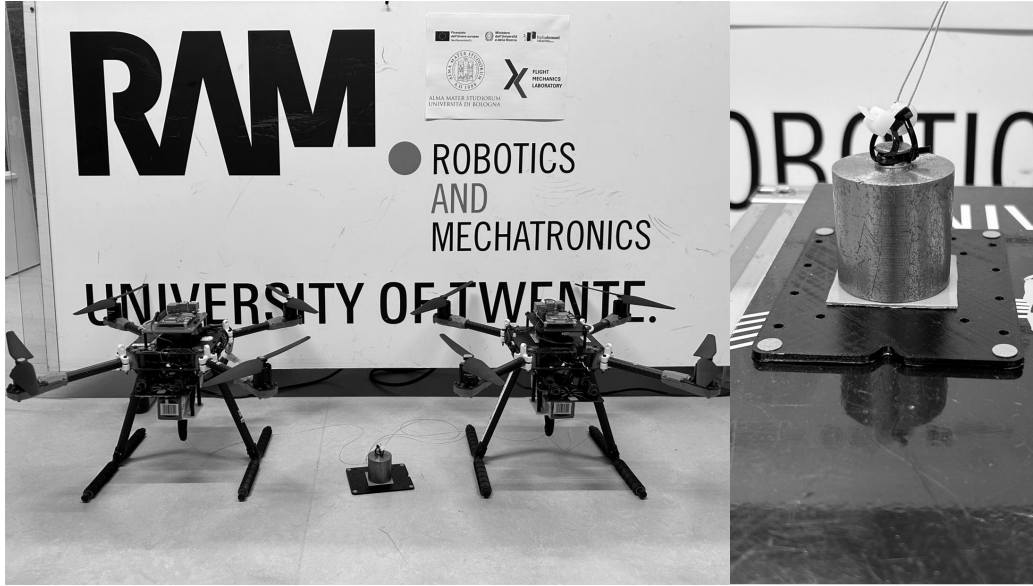


Figure 6.1: Experimental platform used for the experimental campaign: Holybro X500 V2 multirotors and cable suspended-payload.

Motive.

Ground Control Station

The Ground Control Station (GCS) computer is a Dell Precision 7780 with processor Intel Core i7-13850 HX vPro and operating system Ubuntu 20.04.6 LTS (Focal Fossa). This laptop serves two primary functions. On the one hand, it launches QGroundControl, an open source software providing ground control station capabilities to any computer, useful for streaming telemetry, changing firmware's parameters, and downloading log files [103]. On the other hand, it runs MATLAB®-Simulink, where the high-level control and estimation algorithms, specifically the FGDF, were executed OFFBOARD.

Software and Communication Architecture

Communication between the Pixhawk 6C and Raspberry Pi 4 is established via UART serial connection using the MAVLink protocol, which provides a lightweight message marshaling system for mutually exchanging vehicle

state data, commands, and telemetry. A high-bandwidth physical link is established using the TELEM2 port on the FCU configured at a 921.6 kbaud rate to ensure reliable bidirectional communication.

On the software side, the system employs Robot Operating System (ROS) Noetic, a widely used robotics middleware framework that provides publish-subscribe messaging, hardware abstraction, and distributed computation capabilities [104]. In particular, the MAVROS package serves as bridge between MAVLink messages from the FCU and ROS topics and services running on the FCC. MAVROS handles coordinate frame transformations (converting from aerospace NED coordinates to ROS ENU conventions), publishes vehicle state estimates and sensor data as ROS topics, and enables OFFBOARD control through service calls and setpoint messages [105].

The workflow is structured as follows.

Communication Workflow

1. **State Feedback.** The OptiTrack system publishes the six degrees-of-freedom poses of each vehicle and the payload's position to the ROS network. Simultaneously, the Pixhawk autopilot on each UAV streams its onboard sensor data via MAVROS. All these data are subscribed to by the Simulink environment through the dedicated ROS Toolbox.
2. **OFFBOARD Flight Mode.** The core estimation and control logic is executed within the Simulink model. The ground truth vehicles and load's positions are used for performance analysis only, while the UAVs IMU data and commanded control inputs are fed into the FGDF to generate the real-time swing state estimate $(\zeta, \dot{\zeta})$.
3. **Commands Generation.** The high-level controller implemented in Simulink generates position, velocity, or acceleration setpoints that are published back to the ROS network.
4. **Vehicles Control.** MAVROS, running onboard each Raspberry Pi, relays the setpoints to the respective FCU. To ensure a seamless transition of authority, it is critical that this command stream

is already active before the vehicles are switched into OFFBOARD flight mode. The underlying low-level PX4 controllers, responsible for attitude stabilization, are utilized without modification.

Wireless communication between the UAVs and the GCS relies on a 5 GHz (802.11ac WiFi) local network provided by a dedicated router. The 5 GHz band is preferred over the 2.4 GHz for aerial robotics applications due to its higher data throughput and reduced interference in environments with multiple wireless devices. Although this band offers a shorter range, its superior bandwidth is crucial for transmitting ROS topics containing sensor data, control commands, and real-time telemetry. This is particularly critical for multi-agent systems, where multiple vehicles must communicate simultaneously with a ground station or exchange information peer-to-peer. Of course, the dual-band capability of the router ensures fallback to 2.4 GHz when signal strength at 5 GHz becomes insufficient, guaranteeing continuous connectivity.

Figure 6.2 describes the overall communication architecture.

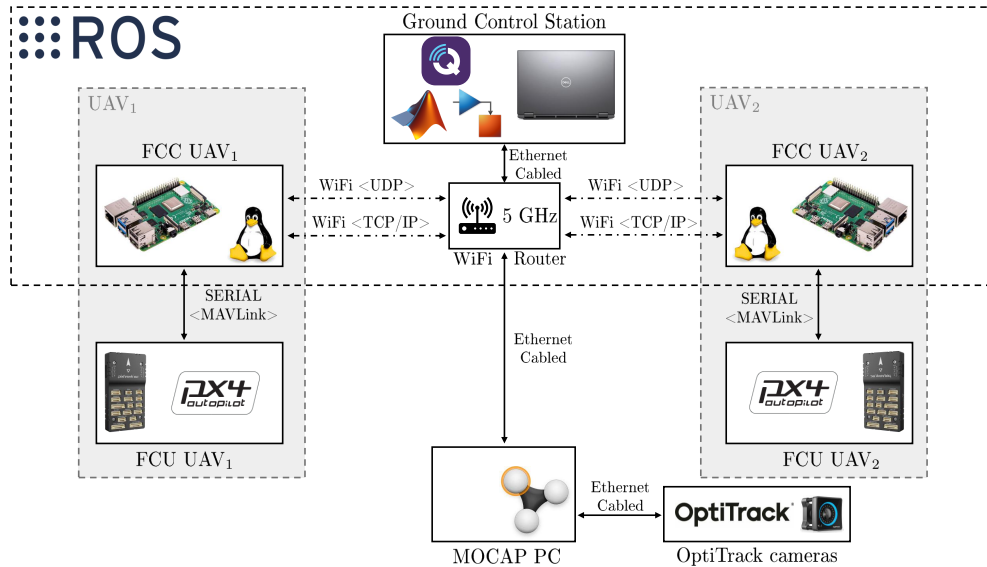


Figure 6.2: Communication architecture: Ground Control Station, Motion Capture System, UAV 1 and UAV 2.

6.1.2 Experimental Methodology

Mission Profile

Due to the spatial constraints of the indoor flight arena together with the UAVs dimensions, the validation is based on simple yet effective hover-to-hover maneuvers. The procedure is detailed as follows:

1. **Manual Take-Off.** For safety reasons, the two quadrotors are armed and taken off manually. Each pilot guides a vehicle to a desired hover position at a safe altitude, ensuring the payload is lifted clear of the ground.
2. **OFFBOARD Mode.** The pilots, through a dedicated stick on the radio controller, switch both vehicles to OFFBOARD Flight Mode, at which point control authority is transferred to the Simulink-based GCS.
3. **Formation Acquisition.** Upon engaging OFFBOARD flight mode, the GCS commands the vehicles to autonomously navigate to their predefined initial formation positions. The system holds this configuration until any pendular motion induced by the transition has dampened and the payload is at rest.
4. **Maneuvers Execution.** Two kinds of maneuvers are performed:
 - A simultaneous step input is applied to the position reference of each vehicle, commanding a back-and-forth longitudinal translation. This aggressive command profile is intentionally designed to induce significant payload swing, providing a clear signal for the estimator to track.
 - The formation's virtual center point is manually piloted through a smooth trajectory using a dedicated radio controller interfaced with MATLAB®-Simulink running on the GCS. This allows the formation to be maneuvered as a single, unified entity according to the baseline FC introduced in Section 3.2. The resulting smoother, lower-frequency motion profile is used to evaluate the FGDF's performance in tracking less aggressive, slowly varying oscillations.
5. **Final Hover.** The formation holds the final position, allowing the

payload’s residual swing to decay naturally while the estimator continues to track its state.

6. **Manual Landing.** For safety reasons, once the desired maneuver is completed, the pilots regain manual control to land both vehicles safely.

Data Collection and Processing

During each experimental run, all relevant ROS topics are logged to a `rosvbag` file for post-mission analysis. Among all, this involves the vehicles and payload ground truth poses from MoCap, local positions and velocities provided by the FCU, IMU data from both vehicles, and commanded setpoints sent from Simulink.

A critical step in the processing is the computation of the virtual swing angle measurement in Equation 3.104 using real sensor data. This requires one vehicle’s inertial acceleration and the applied control forces. The former is directly obtained from the `linear_acceleration` field of the `/uav1/mavros/imu/data` message after compensating for gravity. This signal is inherently noisy and contains biases, presenting a realistic challenge for the filter. Instead, the control force is not directly measurable. This input is therefore reconstructed from the estimated attitude and thrust magnitude, as detailed in Remark 8.

The ground truth swing angle, ζ_T , is calculated geometrically given the motion capture positions of the two vehicles and the payload. All data streams are time synchronized during post-processing operations to guarantee a valid comparison.

6.1.3 Results and Analysis

Experimental Case 1

In the first experimental test case, the vehicles take off by manual pilot command and are then switched to OFFBOARD flight mode, reaching the desired initial positions $\bar{\mathbf{r}}_{1_{sp}}^i = [0.15, -1, -1.70]^T$ m and $\bar{\mathbf{r}}_{2_{sp}}^i = [0.15, 0.10, -1.70]^T$ m, respectively. Once formation stabilizes, the two drones are commanded to per-

form a forward hover-to-hover maneuver, along the $-\mathbf{x}_E$ direction, to arrive at the final locations $\bar{\mathbf{r}}_{1sp}^f = [-0.8, -1, -1.70]^T$ m and $\bar{\mathbf{r}}_{2sp}^f = [-0.8, 0.10, -1.70]^T$ m, respectively. When payload oscillations are attenuated, the vehicles are brought back to their initial hovering positions through another step-input OFFBOARD command.

The FGDF algorithm runs in Simulink on the GCS computer with a frequency of 100 Hz and no active payload stabilization is performed at this stage. Concerning dynamic system matrices in Equations (3.98) and (3.99), they are

$$\mathbf{A}_\zeta = \begin{bmatrix} 0 & 1 \\ -9.1254 & 0 \end{bmatrix}, \quad (6.1)$$

$$\mathbf{B}_\zeta = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0.2223 & 0 & 0 & 0.2223 & 0 & 0 \end{bmatrix} \quad (6.2)$$

The state-transition matrix is computed by MATLAB[®] matrix exponential function $\text{expm}(\cdot)$

$$\Phi = \begin{bmatrix} 0.9995 & 0.0100 \\ -0.0912 & 0.9995 \end{bmatrix} \quad (6.3)$$

while the control-input model is

$$\Psi = \begin{bmatrix} 0.0111 & 0 & 0 & 0.0111 & 0 & 0 \\ 2.2231 & 0 & 0 & 2.2231 & 0 & 0 \end{bmatrix} \cdot 10^{-3} \quad (6.4)$$

The algorithm is initialized with zero state and nominal error covariance $(\mathbf{E}^-)^{(0)} = \text{diag}(0.65, 6.97) \cdot 10^{-4}$, while the assigned noise covariance is $R^* = 0.0041$. Finally, after several experimental trials, the tuned fading factor is $\beta = 0.995$.

In Figure 6.3a, the proposed FGDF tracks the motion capture ground truth ζ_T with good fidelity while rejecting the high-frequency noise visible in the measured angle ζ_m . Across the 30 s run the estimator attains an angle Root Mean Square Error (RMSE) of 2.19 deg, a 95th-percentile absolute error of 4.87 deg, and a maximum absolute error of 8.35 deg.

Remark 13. *Metrics are computed on synchronized ROS timestamps from the same `rosbag` used during the OFFBOARD experiments, thus reflecting*

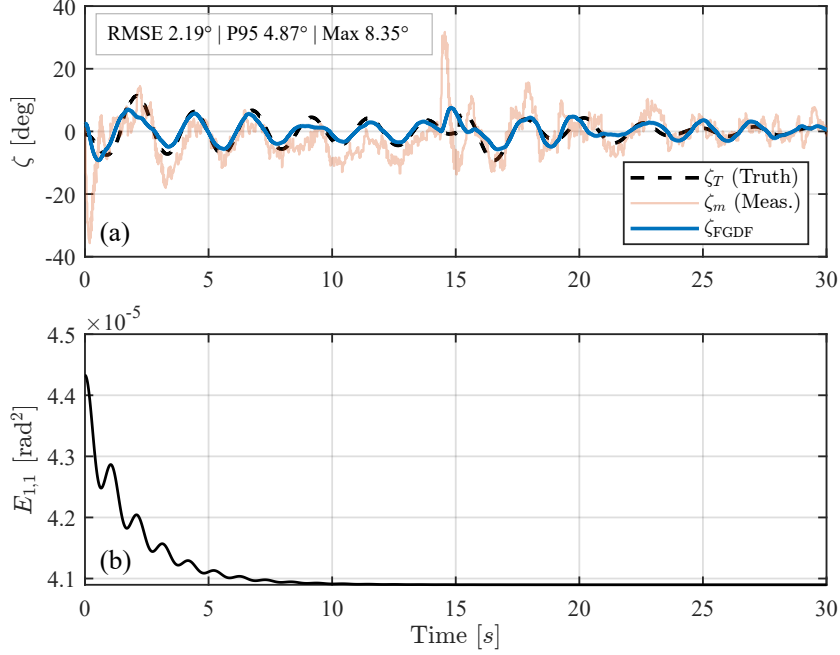


Figure 6.3: Experimental Case 1: (a) Comparison between the real, the measured, and the estimated oscillation angle. (b) Estimation convergence for the nominal covariance component $E_{1,1}$.

the estimator performance under the actual communication and computation stack. In this section, all RMSE, 95th-percentile, and max errors are computed over the entire maneuver time, with a single for each experimental case due to the complexity of the system.

Figure 6.3b reports the first-state covariance, $E_{1,1}$, which converges to approximately $4.1 \times 10^{-5} \text{ rad}^2$. This corresponds to a steady-state standard deviation $\sigma_\zeta \approx \sqrt{4.1 \times 10^{-5}} = 0.0064 \text{ rad} \approx 0.37 \text{ deg}$. Compared with the empirical errors in Figure 6.3a, the covariance is mildly optimistic, which is expected for several reasons. First of all, residual modeling mismatch, especially because the FGDF, in particular the measurement model, relies on the reconstructed input control forces. Even though the campaign was carried out indoor, the flight arena was quite small for flying two Holybro X500 simultaneously, creating a lot of aerodynamic interference and disturbances. Secondly, the Inertial Measurement Units mounted on both the drones were

degraded (likely due to previous crashes), producing unusable acceleration signals that were overly noisy. In this respect, derived local velocity signals coming from the FCU are exploited to overcome this limitation. Finally, a fixed MoCap transport latency relative to the estimator clock that, even if irrelevant for offboard control, may inflate point-wise discrepancies. All this implies the covariance components should be interpreted as an internal confidence measure rather than a strict error bound. The author is confident that, within a larger flight room and with properly working IMUs, the measured angle would perfectly track the ground truth and estimator performance would be further enhanced as well.

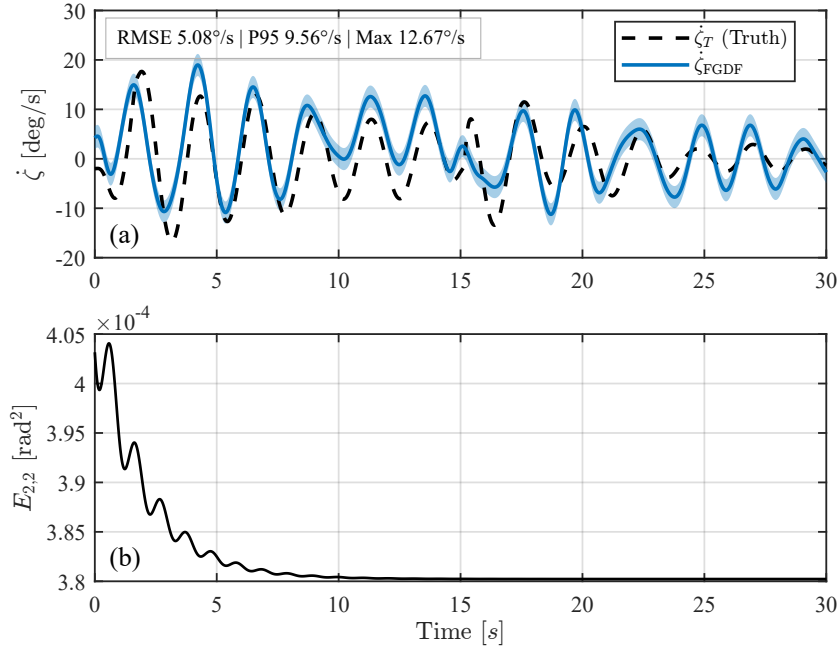


Figure 6.4: Experimental Case 1: (a) Comparison between the real and the estimated oscillation rate. (b) Estimation convergence for the nominal covariance component $E_{2,2}$.

Figure 6.4a demonstrates that the estimator captures the dominant oscillatory content of the payload angular rate while effectively suppressing the high-frequency noise. Over the same 30 s interval, the rate RMSE is 5.08 deg/s, with 95th-percentile and maximum error of 9.56 deg/s and 12.67 deg/s, respectively. The corresponding covariance dynamics in Figure 6.4b

exhibits a short transient and convergence of $E_{2,2}$ to approximately $4.0 \cdot 10^{-4} \text{ rad}^2/\text{s}^2$, namely a steady-state standard deviation $\sigma_{\dot{\zeta}} \approx 1.15 \text{ deg/s}$. As with the angle channel, the empirical errors exceed the nominal $\pm 2\sigma_{\dot{\zeta}}$ envelope, indicating optimistic covariance calibration. This can be attributed to the numerical differentiation used to obtain the “truth” rate from MoCap angles (which amplifies noise and accentuates small mismatches), together with the modeling and input-reconstruction uncertainties discussed above.

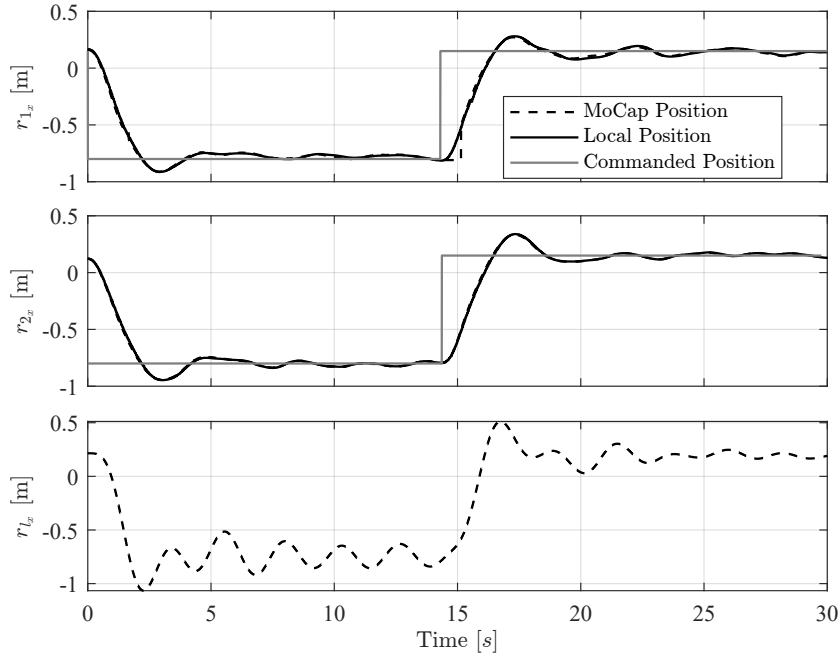


Figure 6.5: Experimental Case 1: Longitudinal position tracking along the x_E -axis. (Top) Commanded, MoCap (ground truth), and local position estimates for UAV 1. (Middle) Commanded, MoCap, and local position estimates for UAV 2. (Bottom) Resulting payload longitudinal motion (MoCap).

To provide context for the payload dynamics and the estimation challenges, the vehicle-level tracking performance is shown in Figures 6.5 and 6.6. Figure 6.5 details the motion along the longitudinal x_E axis. The top two subplots (r_{1_x} and r_{2_x}) show the commanded setpoint (gray solid line), the ground-truth MoCap position (black dashed line), and the onboard local position (black solid line). The plot clearly captures the aggressive nature of the two hover-to-hover maneuvers:

1. The initial forward maneuver (from $t = 0$ to $t \approx 5$ s) executing the step command from $x = 0.15$ m to $x = -0.8$ m.
2. The return maneuver (starting at $t \approx 15$ s) executing the step command back to $x = 0.15$ m.

The resulting longitudinal motion of the payload itself (r_{l_x}) is shown in the bottom subplot, confirming that these maneuvers are the primary source of the large-scale oscillations seen in Figure 6.3a.

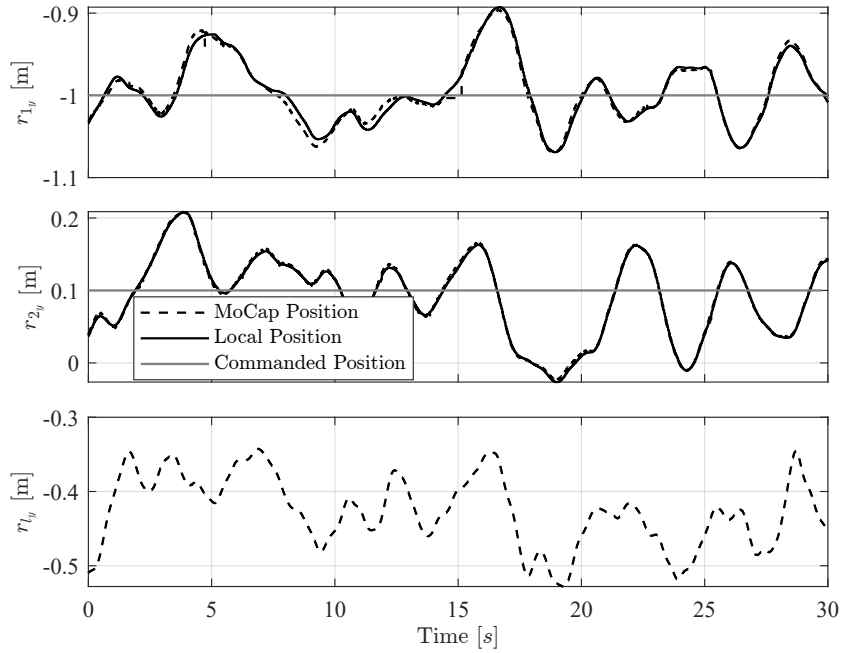


Figure 6.6: Experimental Case 1: Lateral position tracking along the $\mathbf{y}_E$ -axis. (Top) Commanded, MoCap (ground truth), and local position estimates for UAV 1. (Middle) Commanded, MoCap, and local position estimates for UAV 2. (Bottom) Resulting payload lateral motion (MoCap).

Figure 6.6 presents the corresponding motion along the lateral $\mathbf{y}_E$ axis. While the vehicles maintained their constant lateral setpoints (two upper subplots), the payload's lateral position r_{l_y} (bottom subplot) exhibits some uncontrolled oscillatory drift. This behavior is characteristic of an under-actuated pendular dynamic in the lateral plane, where the cable tension forces provide a restoring effect, but any perturbation (such as aerodynamic

disturbances or coupling from the longitudinal maneuver) excites this natural oscillation. This unmodeled lateral dynamic visually confirms the challenging conditions and disturbances discussed previously, which contribute to the estimation errors and the optimistic covariance.

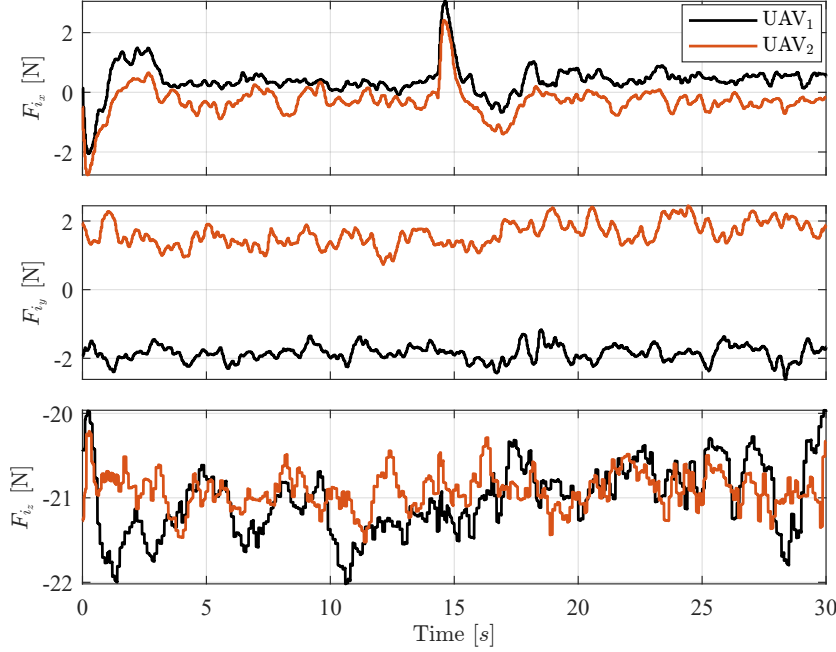


Figure 6.7: Experimental Case 1: Reconstructed control forces for UAV 1 (black) and UAV 2 (orange). (Top) Longitudinal forces (F_x). (Middle) Lateral forces (F_y). (Bottom) Vertical thrust forces (F_z).

Finally, Figure 6.7 displays the reconstructed control forces for both vehicles, which are used as the input for the FGDF measurement model. The top subplot (F_x) clearly shows the large longitudinal forces applied to execute the forward and return maneuvers, correlating with the accelerations seen in Figure 6.5. The middle subplot (F_y) provides direct evidence for the lateral dynamics. The forces are non-zero and opposed, confirming that the vehicles must maintain a constant lateral trim force to counteract the payload's pull and maintain the formation's lateral separation. This trim creates the restoring force for the lateral pendulum seen in Figure 6.6. The high-frequency noise visible across all three axes underscores the challenging flight environment that the FGDF must robustly handle.

Experimental Case 2

The second experimental case is designed to validate the functional objectives of the baseline formation controller (FC) introduced in Section 3.2, namely, simultaneous formation keeping and trajectory tracking. To achieve this, the formation’s virtual center point is manually piloted using a dedicated radio controller, providing a smooth, time-varying trajectory reference, as described in Section 6.1.2.

Concurrently, the baseline FC logic running on the GCS commands the vehicles to track this moving reference while maintaining the desired lateral separation $\bar{\mathbf{d}}_1 = [0, -0.6, 0]^T$ and $\bar{\mathbf{d}}_2 = [0, 0.6, 0]^T$. This implies the desired mutual distance vector is $\bar{\boldsymbol{\gamma}} = [0, 1.2, 0]^T$ for the present scenario. While the theoretical FC framework proposed a direct acceleration-level control law according to Equation (3.72), a more practical, high-level strategy is employed here to avoid the risks of sending direct acceleration commands in a constrained environment. Position setpoints ($\bar{\mathbf{r}}_{1sp}$ and $\bar{\mathbf{r}}_{2sp}$) are computed based on the virtual center’s trajectory and the required formation geometry. These setpoints are streamed to the FCUs, which rely on their robust onboard low-level controllers for stabilization. This implementation, therefore, realizes the same core objectives of the theoretical controller by layering the formation-level logic (formation keeping and trajectory tracking) on top of the standard OFFBOARD position control.

This maneuver profile, characterized by smoother, lower-frequency motion than the step inputs in Experimental Case 1, is used to evaluate the FGDF’s performance in tracking less aggressive, slowly varying oscillations. The FGDF algorithm runs with the same parameters, tuning, and frequency as in the first case, namely $\mathbf{A}_\zeta$, $\mathbf{B}_\zeta$, $\boldsymbol{\Phi}$, $\boldsymbol{\Psi}$, $(\mathbf{E}^-)^{(0)}$, $R^\star$, and β are all unchanged.

The angle estimation results for this smoother maneuver are shown in Figure 6.8a. The FGDF estimate again tracks the ground truth ζ_T with high fidelity, successfully filtering the noise from the virtual measurement ζ_m . As expected, the overall magnitude of the swing is less extreme compared to Experimental Case 1. For this 30-seconds run, the estimator achieves an angle RMSE of 1.61 deg, a 95th-percentile error of 3.09 deg, and a maximum

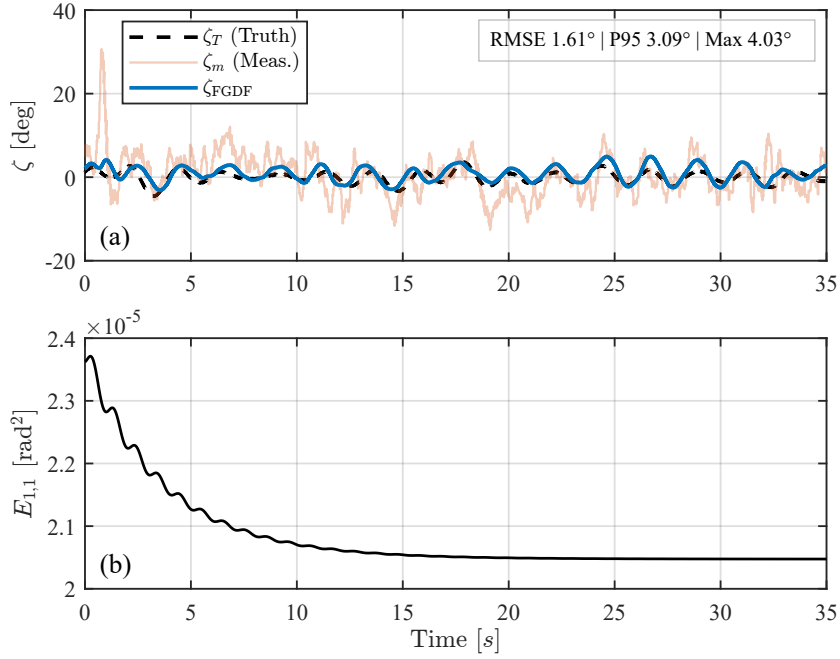


Figure 6.8: Experimental Case 2: (a) Comparison between the real, measured, and estimated oscillation angle. (b) Estimation convergence for the nominal covariance component $E_{1,1}$.

error of 4.03 deg. These metrics underscore comparable performance with respect to the previous case, demonstrating the filter’s robustness in different dynamic regimes. Figure 6.8b shows the convergence of the $E_{1,1}$ covariance component. It converges to a similar steady-state value of approximately $2.05 \cdot 10^{-5} \text{ rad}^2$, corresponding to $\sigma_\zeta \approx 0.26 \text{ deg}$. The empirical errors are once again mildly optimistic, as the underlying systemic challenges (modeling mismatch, sensor noise, aerodynamic disturbances) remain present in this experiment.

Figure 6.9a shows the angular rate estimation. The estimator successfully captures the dominant, lower-frequency content of the true angular rate. The corresponding RMSE is 5.28 deg/s, with 95th-percentile and maximum errors of 9.53 deg/s and 12.51 deg/s, respectively. The $E_{2,2}$ covariance component in Figure 6.9b converges to approximately $1.92 \cdot 10^{-4} \text{ rad}^2/\text{s}^2$, with the same optimistic calibration as noted before.

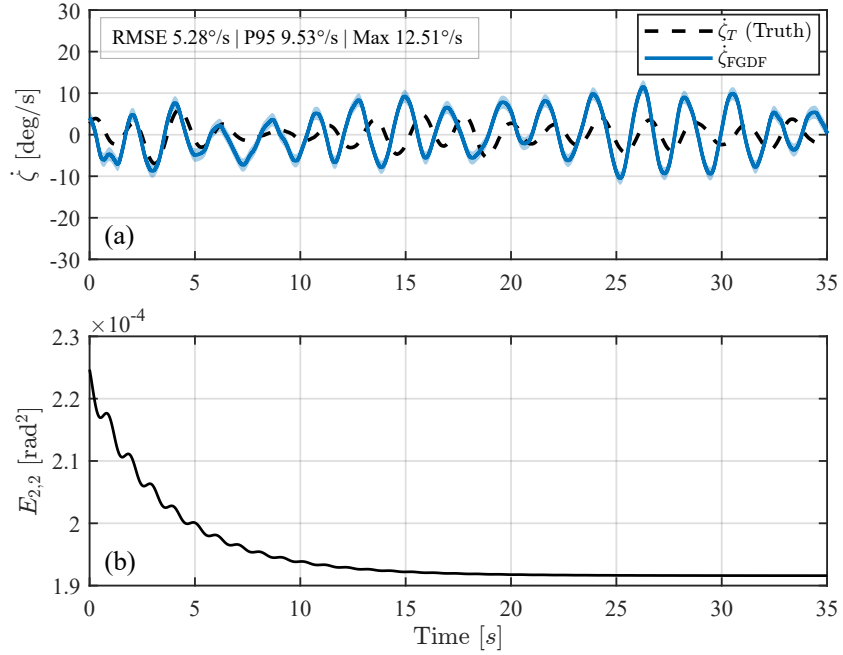


Figure 6.9: Experimental Case 2: (a) Comparison between the real and the estimated oscillation rate. (b) Estimation convergence for the nominal covariance component $E_{2,2}$.

To provide context for this maneuver, the vehicle-level performance is shown in Figures 6.10 and 6.11. Figure 6.10 shows the longitudinal motion of the formation. As expected from the manual piloting, the trajectory is much smoother and lacks the sharp, aggressive step commands of the previous case. Figure 6.11 provides the key validation for the proposed FC strategy. By simultaneously plotting all lateral positions, the figure clearly demonstrates that UAV 1 (blue line) and UAV 2 (orange line) successfully track their respective lateral setpoints (dotted lines). This active tracking maintains the desired lateral separation throughout the maneuver, even as the formation's center point is moving. The payload's true lateral position (r_{ly}) is also shown, presenting limited unmodeled lateral drifting compared to Experimental Case 1.

Taken together, these two experimental cases validate the FGDF's performance across two distinct and complementary regimes: tracking large, aggressive, step-induced oscillations (Case 1) and filtering smoother, lower-

6.1. SWING STATE ESTIMATION VALIDATION

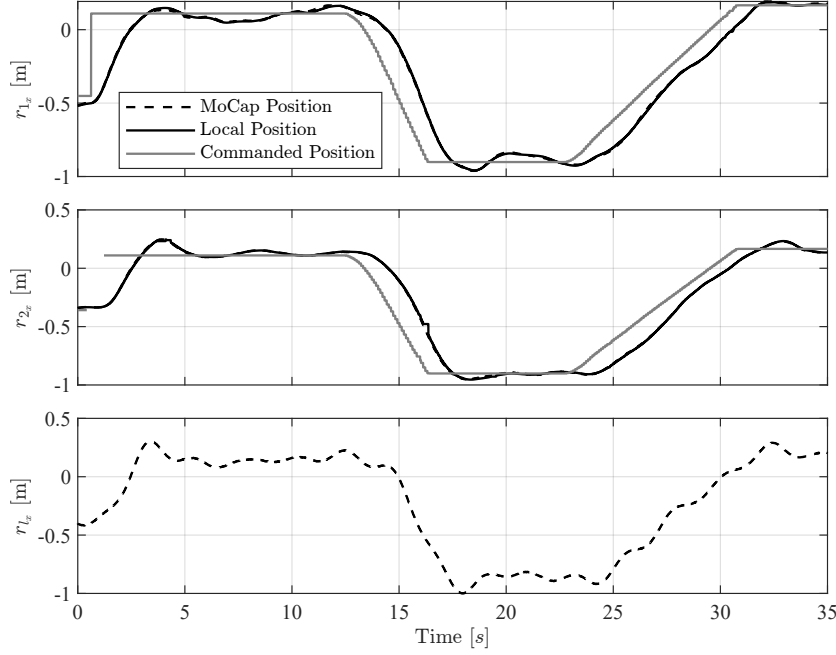


Figure 6.10: Experimental Case 2: Longitudinal position tracking along the x_E -axis. (Top) Commanded, MoCap (ground truth), and local position estimates for UAV 1. (Middle) Commanded, MoCap, and local position estimates for UAV 2. (Bottom) Resulting payload longitudinal motion (MoCap).

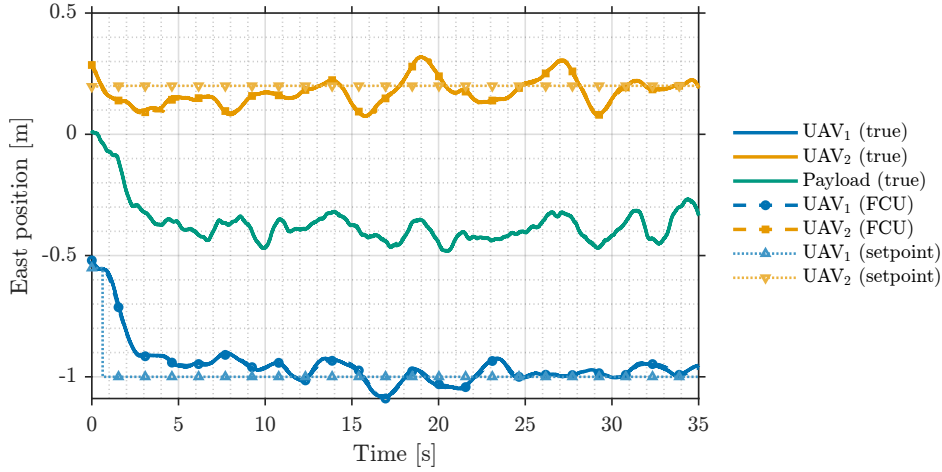


Figure 6.11: Experimental Case 2: Lateral formation keeping performance along the y_E -axis. The plot shows UAV 1 and UAV 2 tracking their respective setpoints, thereby maintaining a constant lateral separation. The payload's true position oscillates between them.

frequency, manually-induced swings during active formation control (Case 2).

6.2 Double-Payload Transportation Validation

This section validates the double-payload estimation and control framework introduced in Chapter 4. The focus is twofold: (i) real-time estimation of frontal and lateral swing angles for two serially connected payloads and (ii) swing motion damping thanks to the auxiliary payload controller based on filter-estimated oscillation angles. Owing to the external MoCap infrastructure, the true oscillation angles are reconstructed from vision pose data to provide a ground-truth baseline.

6.2.1 Experimental Setup

The experimental activity is conducted using the ANT-X *DroneLab* platform, a research and educational facility developed by ANT-X s.r.l. for safe and repeatable indoor flight testing. The system consists of one or more **antx_v2** multirotors, a Ground Control Station, a Motion Capture setup, and the proprietary *DroneLab* software that manages the integration between hardware components and the flight control architecture [106].

Hardware Platform

The aerial vehicle employed in this research is the **antx_v2** micro quadcopter with take-off mass $m = 0.350$ kg and a wheelbase of 0.2 m. It features three-bladed propellers and is powered by a 3S LiPo battery with nominal capacity of 1.3 Ah. The main avionics components are a Pixracer Pro (FCU) and a NanoPi (FCC). The quadcopter is outfitted with a double-pendulum suspension system constructed from two thin woven ropes of known lengths (see Figs. 6.12.a–6.12.c), denoted as $L_A = 0.51$ m and $L_B = 0.37$ m, respectively. The suspended payloads respectively consist of one (load A) and two (load B) steel hardware nuts, each positioned on a small flat plate

marked with a distinctive camera-tracking pattern (see Fig. 6.12.c). The combined mass of load A (one nut plus one plate) is $m_A = 0.026$ kg, while that of load B (two nuts plus one plate) is $m_B = 0.037$ kg.

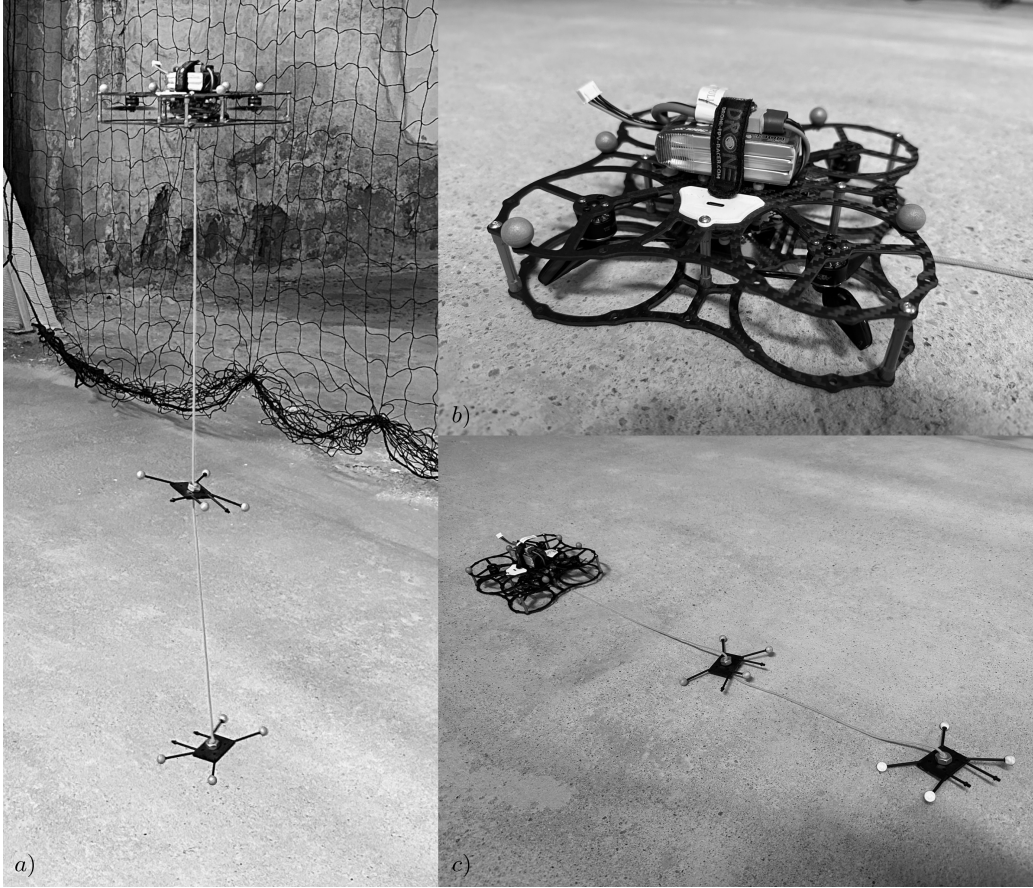


Figure 6.12: a) The slung-load system in flight. b) The drone `antx_v2 DL00021`. c) The slung-load system in the flight arena.

Environment

Flights take place in the University of Bologna Flight Mechanics Laboratory indoor arena at CICLoPE, with dimensions $3.5 \times 5 \times 3$ m, enclosed by a protective net. It is equipped with an OptiTrack MoCap system, comprising 4 Prime^X 13 and 4 Prime^X 13W cameras, providing high-precision pose tracking of aerial vehicles within the flightpace. Custom estimation and control blocks

are auto-generated from MATLAB®-Simulink and directly deployed onboard the autopilot via the ANT-X *SLXtoPX4* software tool.

Ground Control Station

The GCS laptop is the one already described in Section 6.1.1 and serves the same functions. However, in this campaign, the estimation and control algorithms are not executed in Simulink (OFFBOARD), instead they are directly embedded into the FCU.

Firmware Deployment

Custom control laws and estimation modules are implemented and deployed using the ANT-X proprietary software *SLXtoPX4*. This tool enables automatic code generation and firmware flashing starting from MATLAB®-Simulink models, providing a direct and intuitive interface with the PX4 flight stack. In particular, Simulink diagrams describing the proposed control and estimation architecture are converted into C++ source code through *SLXtoPX4*, compiled within the PX4 build environment, and finally deployed on the FCU onboard the `antx_v2` DL00021.

Payload-Controller Actuation

With respect to the control methodology described in Section 4.3, the following experimental payload controller is implemented:

$$\boldsymbol{\omega}_{sp}^{(PC)} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} [k_p (\mathbf{r}_C - \mathbf{r}_H) + k_v (\dot{\mathbf{r}}_C - \dot{\mathbf{r}}_H)] \quad (6.5)$$

where $\boldsymbol{\omega}_{sp}^{(PC)}$, computed from the onboard swing state estimates, is added to the setpoint body-fixed angular rate vector generated by the PX4 attitude controller. For the aim of the present analysis, it is $k_p = 0$ rad/s/m and $k_v = 1$ rad/m.

6.2.2 Experimental Methodology

Two experimental cases are presented to validate the proposed estimation and control strategies. In Case 1, the micro drone is operated via manually generated position commands (POSITION flight mode), issued by the pilot from a remote controller. Here, the estimation filter is active to assess tracking accuracy, but payload control is not engaged. In Case 2, the full control system architecture is validated in a dynamic environment, provided it is activated during the flight. Here, the vehicle executes a sample delivery mission, following a sequence of predefined waypoints in OFFBOARD flight mode. In both scenarios, the mission starts from the initial take-off position at $t_0 = 0$ s with approximately $\mathbf{r}(0) = [0, 0, 0]^T$ m and $\mathbf{v}(0) = [0, 0, 0]^T$ m/s and ends when the quadcopter lands back at the same location after completing the assigned maneuvers.

Data Collection and Processing

Thanks to the onboard deployment of the estimation and control stack, all flight data are recorded by the Pixhawk autopilot in the `.ulg` format and retrieved via QGroundControl at the end of each experiment. The logs include high-rate IMU measurements, local position and velocity estimates, controller set-points and outputs, and standard PX4 topics. In addition, quantities specific to this study are logged: oscillation angles and rates estimated by the FGDF, measured upper load swing angles as defined in Equations (4.33) and (4.34), and the auxiliary contributions injected by the payload controller.

External vision pose data for the vehicle and the two suspended loads, not available in the `.ulg` file, are streamed in real-time by the OptiTrack system at 100 Hz and recorded in a dedicated `rosvbag` for ground truth validation purposes. In particular, the true lateral and frontal oscillation angles of both loads are computed from the recorded MoCap data to enable comparison with the filter estimates.

6.2.3 Results and Analysis

Experimental Case 1

During this test, the pilot commands manual translations along the frontal and lateral axes (see Fig. 6.13) to excite the swing dynamics of both payloads, with the micro drone in POSITION flight mode. This allows for the assessment of the FGDF's capability to accurately estimate the oscillation states in real-time.

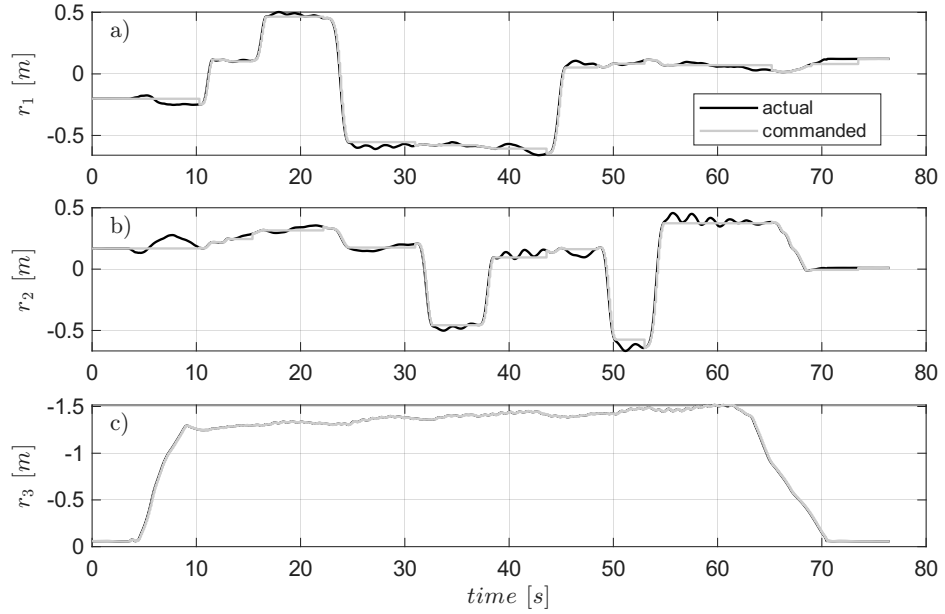


Figure 6.13: Experimental Case 1: Commanded and actual multirotor trajectory during manual flight maneuvers along a) frontal, b) lateral, and c) vertical directions.

Figures 6.14 and 6.15 present the results of the estimation process for the lateral and frontal oscillation angles of load B and A, respectively. The gray lines represent the estimates obtained using the FGDF, while the black lines correspond to the computed ground truth data. The comparison indicates that the proposed method effectively tracks the oscillation dynamics of both payloads, maintaining high-fidelity even during rapid maneuvers. To provide a quantitative assessment of the estimation performance, the error covariance matrix is analyzed over the active maneuvering window. The analysis reveals

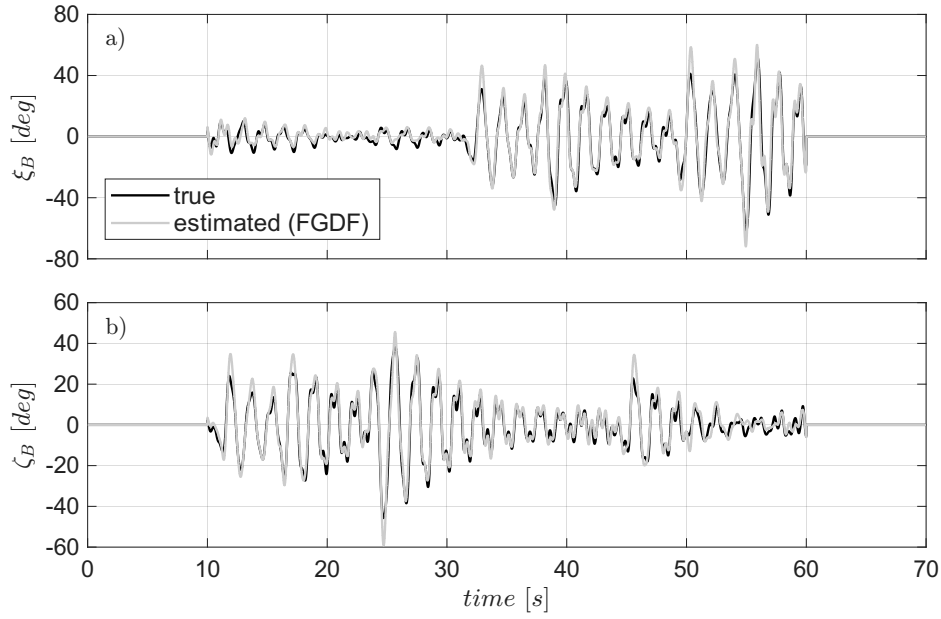


Figure 6.14: Experimental Case 1: Comparison between the true and filter-estimated lateral and frontal oscillation states of load B.

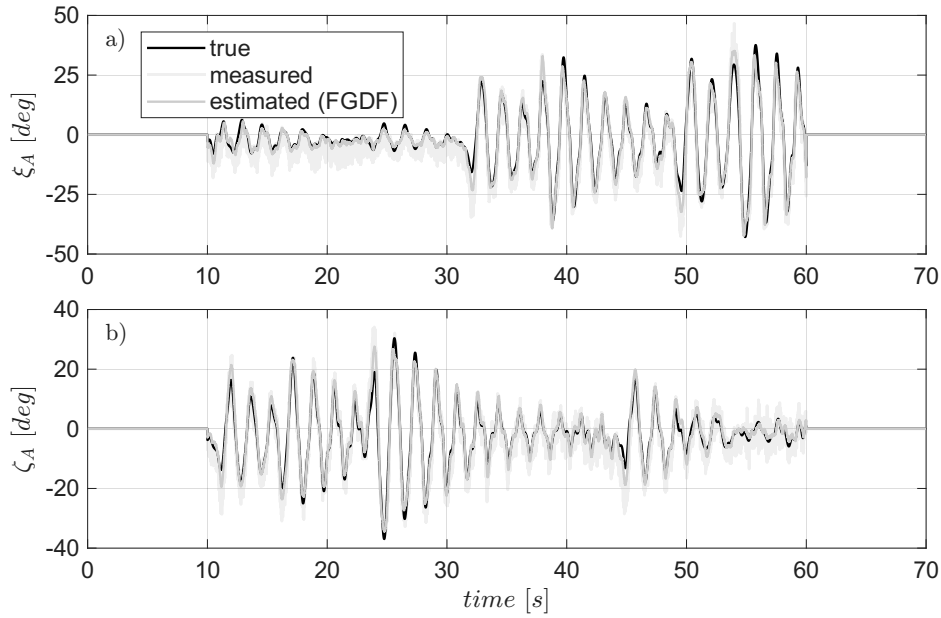


Figure 6.15: Experimental Case 1: Comparison between the true, filter-measured, and filter-estimated lateral and frontal oscillation states of load A.

that the average estimation error variance across the frontal and lateral axes is $1.72 \cdot 10^{-3} \text{ rad}^2$ for the upper cable (load A) and $4.44 \cdot 10^{-3} \text{ rad}^2$ for the lower one (load B). These values correspond to a mean standard deviation of approximately 2.37 deg and 3.82 deg, respectively. The slightly higher uncertainty observed in the lower payload is consistent with the physical propagation of disturbances along the double-pendulum chain. The computed cross-covariance terms are negligible, confirming that the filter effectively decouples the frontal and lateral dynamics. Finally, the mean error remains below 0.8 deg for all axes, indicating that the proposed estimator is unbiased and free from significant systematic drift.

Experimental Case 2

In this scenario, the multicopter follows a predefined sequence of waypoints in OFFBOARD flight mode, simulating an autonomous small-scale transportation mission. The objective is to evaluate the effectiveness of the payload control system in a operational environment, where maneuvers are dictated by pre-programmed mission-level navigation rather than manual inputs. The delivery mission, including take-off and landing phases, is illustrated in Fig. 6.16 with active payload control, where both the commanded and actual trajectories of the multirotor are depicted.

To quantitatively evaluate the contribution of the auxiliary payload controller, the same validation metrics introduced in Section 5.5.2 for numerical simulations are adopted. The impact of the active payload control system on the oscillation dynamics of both loads is assessed by comparing the controlled flight (with PC) against a baseline mission where the auxiliary control effort is null (namely, $\omega_{sp}^{(PC)} = \mathbf{0}_{3 \times 1} \text{ rad/s}$). Figures 6.17 and 6.18 illustrate the lateral and frontal true oscillation angles of loads B and A, respectively, under both conditions. The visual comparison highlights the effectiveness of the control strategy in mitigating undesired payload swing during the autonomous delivery task. In particular, in the scenario without payload controller, each waypoint transition induces high-amplitude oscillations in both the lateral (ξ_A, ξ_B) and frontal (ζ_A, ζ_B) planes, with peaks reaching

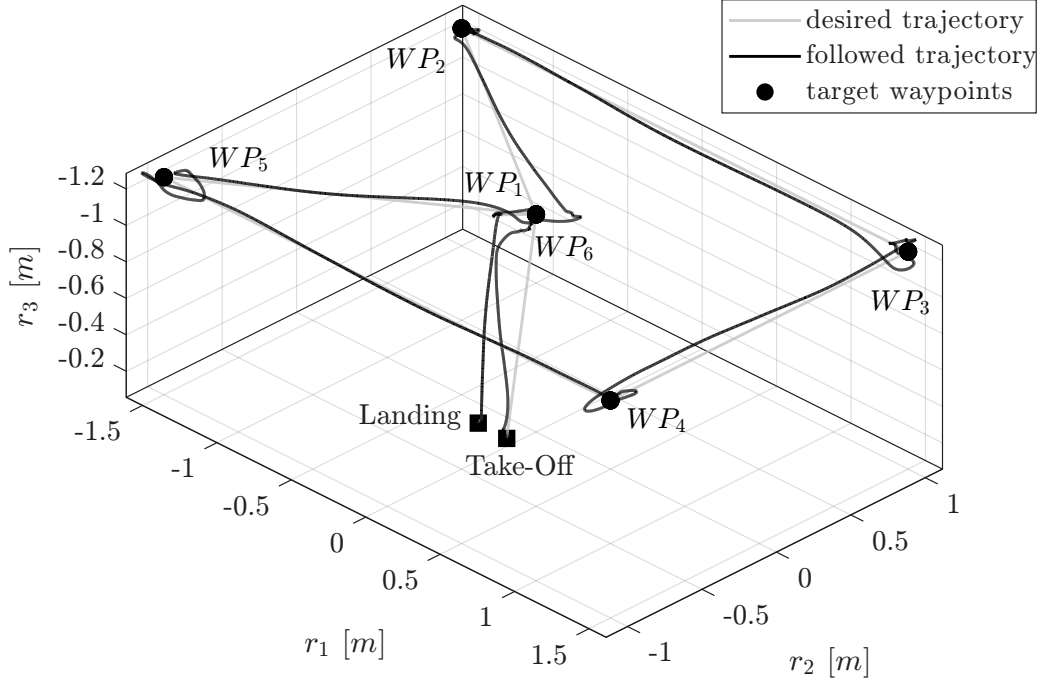


Figure 6.16: Experimental Case 2: Commanded and actual multirotor trajectory during the autonomous delivery mission with active payload control system.

nearly ± 30 deg. When the control system is engaged, based on FGDF estimates, a significant swing motion reduction is observed, particularly for the lower payload (load B). In addition, the settling time after each maneuver is notably decreased, indicating an enhanced dynamic response of the overall slung-load system.

The performance indices $\bar{\varepsilon}_A$ and $\bar{\varepsilon}_B$ provide a definitive validation of the proposed FGDF-based control architecture. When the payload controller is active, $\bar{\varepsilon}_A$ is reduced by 20.40 % (from 9.20 deg to 7.32 deg), while $\bar{\varepsilon}_B$ experiences a substantial 47.86 % improvement (from 10.16 deg to 5.30 deg). This demonstrates the controller's capability to effectively dampen oscillations, particularly for the lower payload, which is more susceptible to dynamic disturbances due to its position in the double-pendulum configuration. The RMS values of the average oscillation angles, which penalize larger amplitude transients, further corroborate these findings. Specifically, $RMS_{\bar{\varepsilon}_A}$ and $RMS_{\bar{\varepsilon}_B}$ decrease by 15.21 % and 43.66 % for load A and load B, respectively.

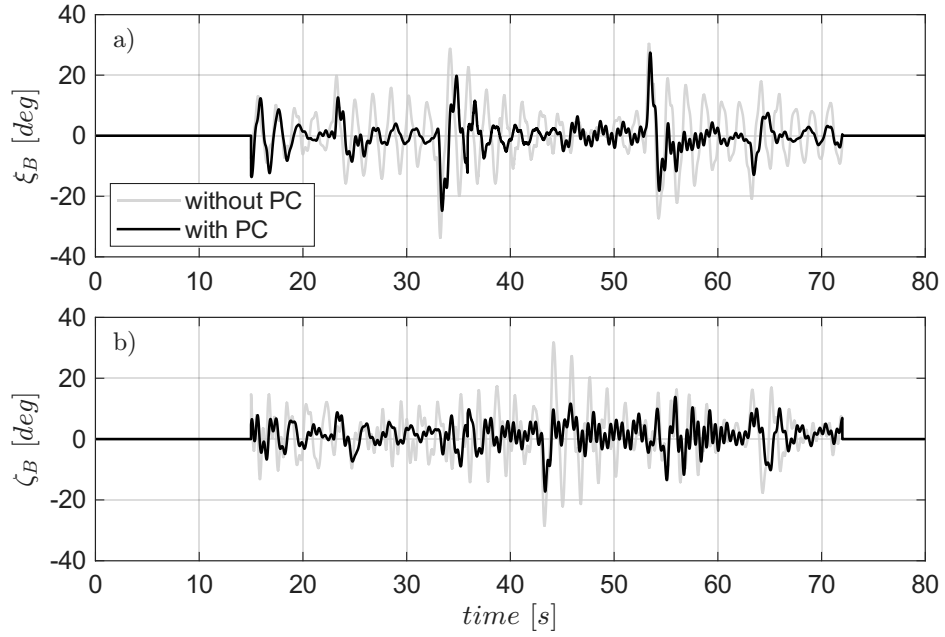


Figure 6.17: Experimental Case 2: Lateral and frontal true oscillation angles with and without payload control system for load B.

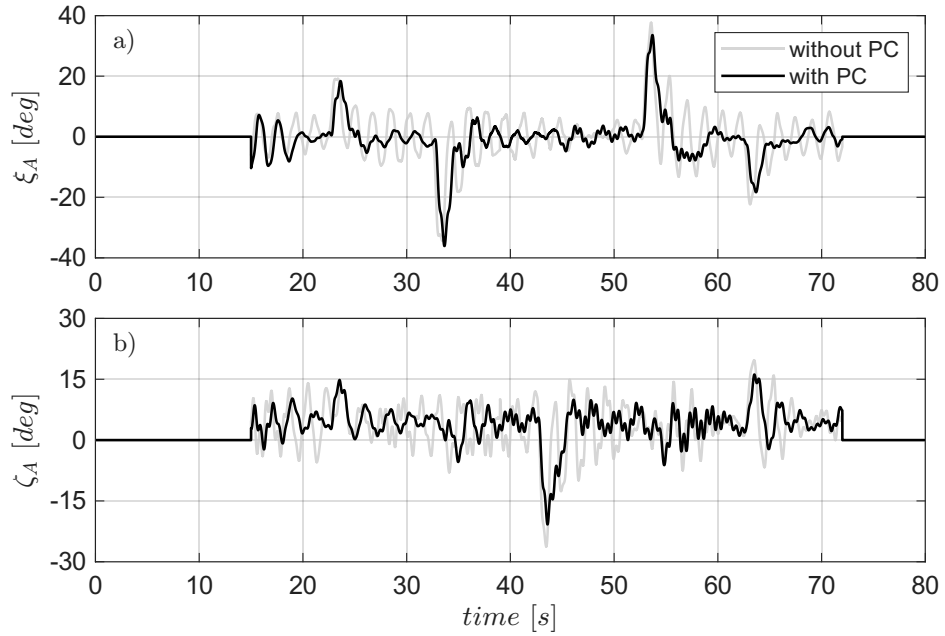


Figure 6.18: Experimental Case 2: Lateral and frontal true oscillation angles with and without payload control system for load A.

This indicates that the control system not only reduces average oscillations but also effectively mitigates peak deviations, enhancing overall payload stability during the mission. Overall, this experimental campaign demonstrates the effectiveness of the proposed estimation and control methods, underscoring practical deployability and real-time implementation on embedded hardware and ease of integration with commercial off-the-shelf autopilots.

CHAPTER 7

Conclusion

This dissertation has addressed the challenging problem of cooperative aerial transportation of cable-suspended payloads, a critical enabling technology for future Urban Air Mobility and Delivery applications. The core difficulty lies in the inherent dynamic complexity of these systems: they are underactuated, highly nonlinear, and susceptible to persistent oscillations that jeopardize safety and mission effectiveness. The research presented herein developed a comprehensive framework, progressing from first-principles modeling to robust adaptive control and practical state estimation, to provide a suite of solutions for safe and efficient slung-load operations.

The contributions of this thesis are organized around a systematic progression from theoretical rigor to practical applicability. First, a rigorous mathematical foundation was established for both single- and dual-vehicle configurations. A detailed Eulerian formulation was developed for a two-agent system, enabling the design of a nonlinear two-time-scale controller with strong stability guarantees based on singular perturbation theory. This approach decouples fast formation-keeping and swing damping from slower trajectory tracking, providing a principled tool for verifiable system design. To

enhance modularity and physical insight, a complementary Lagrangian-based model was then derived. While both Eulerian and Lagrangian approaches yield equivalent dynamics for the same physical system, the energy-based Lagrangian framework naturally exposes the system’s inertial structure and facilitates systematic control design for multi-body configurations. A key innovation of this model is an ad hoc parameterization that describes out-of-plane payload motion with a single swing angle, simplifying the representation and revealing a useful decoupling between forward-swing and lateral-vertical dynamics. Building on this baseline, a modular formation controller with auxiliary payload-oscillation damping and collision-avoidance contributions was synthesized. While this control method, designed and analyzed in a linear framework, proved effective for nominal operations, it also exposed the performance degradation inherent in non-adaptive schemes under significant uncertainty. To overcome these limitations, by exploiting the control-affine representation inherent to Lagrangian mechanics, an $\mathcal{L}_1$ augmentation was implemented. This piecewise-constant adaptive law was layered on top the baseline architecture, providing guaranteed transient performance and high robustness to both matched and unmatched uncertainties. The effectiveness of this augmentation was demonstrated in the presence of severe off-nominal conditions, including large, unmodeled payload mass variations, atmospheric disturbances, and actuator failures. This thesis intentionally develops parallel modeling pathways, Eulerian force-balance and Lagrangian energy-based, to serve complementary purposes. The Eulerian formulation in the two-agent cooperative scenario provides transparent physical interpretation of inter-vehicle coupling and constraint forces, making it well suited for the two-time-scale control decomposition. The Lagrangian methodology, adopted for both single-vehicle dual-payload and formation control scenarios, offers systematic scalability to multi-body systems and naturally reveals the control-affine structure to implement the $\mathcal{L}_1$ adaptation law. Both approaches support equivalent baseline controllers. The choice to augment the Lagrangian-based design reflects the clearer separation between nominal model dynamics and parametric uncertainties, rather than any inherent limitation of the Eulerian method. The dual-formalism presentation demonstrates that the core control

insights are independent of the modeling formalism, reinforcing the generality of the proposed framework.

A critical enabler for these strategies was the development of a practical, sensor-minimal swing-state estimation framework. By algebraically inverting the linearized dynamics, a virtual measurement of the swing state was constructed from standard onboard IMUs data and known control inputs, and fused with a model-based prediction using a Fading Gaussian Deterministic Filter with systematically addressed tuning. This approach obviates the need for external Motion Capture systems or complex vision-based hardware, representing a meaningful step toward operational viability and onboard deployment. Finally, the versatility of the framework was demonstrated by extending both estimation and control strategies to the more complex case of a single vehicle transporting two serially-connected payloads. This double-pendulum scenario, representative of multi-parcel delivery missions, validated the scalability of the underlying principles to multi-body slung-load systems.

Beyond the methodological arc, the thesis established clear evidence of effectiveness. All components were first examined in high-fidelity numerical simulations in MATLAB®-Simulink environment that included actuator limits, aerodynamic effects, parametric uncertainty, measurement noise, and exogenous disturbances. These studies confirmed that the two-time-scale design preserves the intended fast-slow separation and that the $\mathcal{L}_1$ augmentation restores near-nominal performance under substantial modeling error. Complementing the simulations, the experimental campaigns successfully validated the cooperative filter and demonstrated both swing-state estimation and control for the double-payload scenario. Notably, these results were obtained without relying on external motion capture or vision in the control loop, underscoring the practicality of the sensor-minimal approach for onboard deployment.

The scope of the guarantees is naturally tied to the modeling envelope. The most conservative statements apply to moderate swing amplitudes, bounded cable elasticity and damping, and actuation that remains within saturation limits. Extremely aggressive maneuvers, pronounced cable nonlinearities,

and tight inter-vehicle communication constraints may require explicit limitations handling, delay compensation, and additional robustness margins in the adaptation layer. However, such scenarios fall beyond the intended scope of the thesis, which focuses on smooth maneuvers typical of UAMD operations. Looking ahead, the framework invites several direct extensions. Integrating constraint-aware planning, such as robust model predictive control, on top of the two-time-scale core would enable native handling of actuation limits and safety envelopes during more aggressive tasks. Generalizing the cooperative layer to teams larger than two and establishing string-stability properties would broaden applicability to logistics swarms. In this respect, the central concern of this work has been to confront the inherent complexity of underactuated slung-load systems with verifiable control and estimation mechanisms, that remain physically intuitive and require minimal sensors.

In conclusion, the dissertation investigated the state-of-the-art by delivering a theoretically grounded, numerically rigorous, and experimentally verified framework for slung-load transportation scenarios. The proposed estimation and control strategies provide concrete steps toward enabling safer and more efficient Urban Air Mobility and Delivery systems of the future.

APPENDIX A

Proof of Lemma 3.1.1

The hypotheses in Lemma 3.1.1 and the definition of error $\boldsymbol{\rho}$ uniquely determine desired formation geometry, which is depicted in Figure A.1. When the i -th agent reaches the desired position D_i ($\boldsymbol{\delta}_i = \mathbf{0}_{3 \times 1}$, $\boldsymbol{\epsilon}_i = \mathbf{0}_{3 \times 1}$, $i = 1, 2$) and the formation center of gravity G exactly tracks P_r ($\boldsymbol{\rho} = \mathbf{0}_{3 \times 1}$, $\boldsymbol{\sigma} = \mathbf{0}_{3 \times 1}$), all elements P_1 , P_2 , P_l , and G lie on a local vertical plane and the relative distance vectors $\mathbf{d}_1 = \bar{\mathbf{d}}_1$ and $\mathbf{d}_2 = \bar{\mathbf{d}}_2$ are organized according to a symmetric configuration that makes the derivation of results in Equation (3.22) straightforward. In order to evaluate $\bar{d}_{z_1}$ and $\bar{d}_{z_2}$, the equilibrium of forces acting on the payload along the local vertical is analyzed. To this end, let $\|\bar{\mathbf{f}}_{l_i}\| = K \bar{\Delta L}$ be the i -th elastic force intensity in the equilibrium condition (see Equation (3.7)), where $\bar{\Delta L} = \bar{L} - L$ and $\bar{L}$ are, respectively, desired cable elongation and length. Taking into account Equation (3.4), it is

$$2 K \bar{\Delta L} \cos \bar{\xi} = m_l g \quad (\text{A.1})$$

where $\bar{\xi} = \cos^{-1}(c/\bar{L})$ is the desired cable inclination with respect to the local vertical, with $c = \sqrt{\bar{L}^2 - (a/2)^2}$. Then Equation (A.1) provides, after manipulation, Equations (3.24) and (3.25). Provided $p_2 > 0$, which occurs for values of practical application, the sequence of signs for the polynomial in

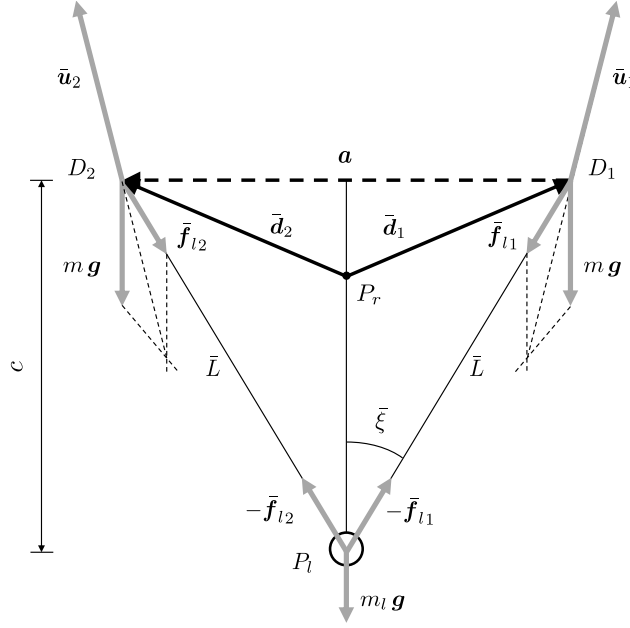


Figure A.1: Desired formation geometry (relative distance a is assigned).

Equation (3.24) is $- + + - +$. According to Descartes' rule three real positive solutions and one real negative solution are derived [107]. The only positive solution which is representative of the case where $\overline{\Delta L} > 0$ and the payload lies below the UAV is the largest one, which can be determined analytically or by means of a root-finding numerical algorithm [108, 109]. Let $\bar{\mathbf{c}}_i$ be cable orientation vector in the equilibrium condition. From Equation (3.20) it is

$$\bar{\mathbf{c}}_1 = -\bar{\mathbf{d}}_1 \left(1 + \frac{m}{m_l} \right) - \bar{\mathbf{d}}_2 \frac{m}{m_l} \quad (\text{A.2})$$

and

$$\bar{\mathbf{c}}_2 = -\bar{\mathbf{d}}_2 \left(1 + \frac{m}{m_l} \right) - \bar{\mathbf{d}}_1 \frac{m}{m_l} \quad (\text{A.3})$$

It is noted that the parameter $c > 0$ in Figure A.1 represents the third component of both $\bar{\mathbf{c}}_1$ and $\bar{\mathbf{c}}_2$. With this in mind, Equation (A.2) provides

$$c = -\bar{d}_{z1} \left(1 + \frac{m}{m_l} \right) - \bar{d}_{z2} \frac{m}{m_l} = -\bar{d}_{z1} \frac{M}{m_l} \quad (\text{A.4})$$

with the knowledge that $\bar{d}_{z2} = \bar{d}_{z1}$. By equating the right-hand side of Equation (A.4) to the expression of c obtained from the analysis of right triangles in Figure A.1, one finally derives the result in Equation (3.23).

As a by-product of Lemma 3.1.1, the trim control forces $\bar{\mathbf{f}}_{c1}$ and $\bar{\mathbf{f}}_{c2}$ required for the UAV to maintain the considered formation geometry are also derived. Static equilibrium from Equation (3.2) gives:

$$\bar{\mathbf{f}}_{ci} = -m\mathbf{g} - \bar{\mathbf{f}}_{li} = -m\mathbf{g} - K\overline{\Delta L}\hat{\mathbf{c}}_i \quad (\text{A.5})$$

where $\hat{\mathbf{c}}_i = \bar{\mathbf{c}}_i / \|\bar{\mathbf{c}}_i\|$ is calculated for $i = 1, 2$ by taking into account Equations (A.2) and (A.3).

APPENDIX B

Proof of Theorem 3.1.2

First the exponential stability of $[\mathbf{x}^T \mathbf{z}^T]^T = \mathbf{0}_{18 \times 1}$ is addressed in the case when $\dot{\mathbf{v}}_r = \mathbf{0}_{3 \times 1}$. Equations (3.26)–(3.29) with the control law in Equations (3.30)–(3.37) provide:

$$\mathbf{F}(\mathbf{x}, \mathbf{z}) = \begin{bmatrix} \boldsymbol{\sigma} \\ -\mathbf{g} - \frac{1}{M} \left[\bar{\mathbf{f}}_{c1} + \bar{\mathbf{f}}_{c2} + \lambda_1 \boldsymbol{\nu}_1 + \lambda_2 \boldsymbol{\nu}_2 + \mu_1 \boldsymbol{\Sigma} \bar{\mathbf{d}}_1 \right. \\ \left. + \mu_2 \boldsymbol{\Sigma} \bar{\mathbf{d}}_2 + 2m \mathbf{K}_z \mathbf{R} \boldsymbol{\alpha} + m \mathbf{K}_z (\mathbf{z}_1 + \mathbf{z}_2) \right] \end{bmatrix} \quad (\text{B.1})$$

and

$$\mathbf{G}(\mathbf{x}, \mathbf{z}) = \begin{bmatrix} \boldsymbol{\epsilon}_1 \\ -\mathbf{g} - \frac{1}{m} \left[\left(1 - \frac{L}{\|\mathbf{c}_1\|} \right) \mathbf{c}_1 + \bar{\mathbf{f}}_{c1} + \lambda_1 \boldsymbol{\nu}_1 \right. \\ \left. + \mu_1 \boldsymbol{\Sigma} \bar{\mathbf{d}}_1 + m \mathbf{K}_z (\mathbf{z}_1 + \mathbf{R} \boldsymbol{\alpha}) \right] \\ \boldsymbol{\epsilon}_2 \\ -\mathbf{g} - \frac{1}{m} \left[\left(1 - \frac{L}{\|\mathbf{c}_2\|} \right) \mathbf{c}_2 + \bar{\mathbf{f}}_{c2} + \lambda_2 \boldsymbol{\nu}_2 \right. \\ \left. + \mu_2 \boldsymbol{\Sigma} \bar{\mathbf{d}}_2 + m \mathbf{K}_z (\mathbf{z}_2 + \mathbf{R} \boldsymbol{\alpha}) \right] \end{bmatrix} \quad (\text{B.2})$$

Taking into account Equations (3.20) and (3.21), and the results obtained in Assumption 2, it can be shown that the latter equation simply becomes

$$\mathbf{G}(\mathbf{x}, \mathbf{z}) = \bar{\mathbf{H}} \mathbf{z} - \mathbf{\Gamma} \boldsymbol{\gamma} \quad (\text{B.3})$$

where $\bar{\mathbf{H}}, \mathbf{\Gamma} \in \mathbb{R}^{12}$ are respectively given by

$$\bar{\mathbf{H}} = \left[\begin{array}{c|c} \mathbf{R}^T & \mathbf{0}_{6 \times 6} \\ \hline -\mathbf{K}_z & \mathbf{R}^T \\ \hline \mathbf{0}_{6 \times 6} & -\mathbf{K}_z \end{array} \right] \quad (\text{B.4})$$

and

$$\mathbf{\Gamma} = \left[\begin{array}{c|c} \mathbf{0}_{3 \times 6} & \mathbf{0}_{6 \times 6} \\ \hline \mathbf{K}_z \mathbf{S} & \mathbf{0}_{3 \times 6} \\ \hline \mathbf{0}_{6 \times 6} & \mathbf{K}_z \mathbf{S} \end{array} \right], \quad (\text{B.5})$$

where $\boldsymbol{\gamma} = [\mathbf{0}_{1 \times 3} \ \boldsymbol{\alpha}^T \ \mathbf{0}_{1 \times 3} \ \boldsymbol{\alpha}^T]^T \in \mathbb{R}^{12}$ and

$$\mathbf{S} = \begin{bmatrix} \mathbf{0}_{3 \times 3} & \mathbf{I}_3 \\ \mathbf{I}_3 & \mathbf{0}_{3 \times 3} \end{bmatrix} \quad (\text{B.6})$$

Assume that a suitable selection of control gain matrices $\mathbf{K}_x$ and $\mathbf{K}_z$ forces the closed-loop system to exhibit a two-time-scale behavior. In the present framework, the scenario where formation-keeping and payload swing damping corresponds to the fast dynamics is considered, while trajectory tracking is related to the slow dynamics. In this way $\mathbf{x}$ and $\mathbf{z}$ are the vectors containing the slow and the fast variables, respectively. Also, it is noted that control signal in Equation (3.37) is a function of the slow variable only, namely $\boldsymbol{\alpha} = \boldsymbol{\alpha}(\mathbf{x})$ and $\boldsymbol{\gamma} = \boldsymbol{\gamma}(\mathbf{x})$. With this in mind, the system in Equations (3.26), (3.27), (B.1), and (B.3) can be rewritten in the standard form:

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, \mathbf{z}, \varepsilon) \quad (\text{B.7})$$

$$\varepsilon \dot{\mathbf{z}} = \mathbf{G}(\mathbf{x}, \mathbf{z}, \varepsilon) \quad (\text{B.8})$$

where the artificial parameter ε (whose nominal value is 1) is inserted to represent $\mathbf{z}$ as the fast variable, according to the forced singular perturbation method described in [110, 111].

Given the system in Equations (B.7) and (B.8), the proof follows from the stability results for singularly perturbed systems (see Theorem 11.4 in Ref. [62]). In particular, it is $\mathbf{F}(\mathbf{0}_{6 \times 1}, \mathbf{0}_{12 \times 1}, \varepsilon) = \mathbf{0}_{6 \times 1}$ and $\mathbf{G}(\mathbf{0}_{6 \times 1}, \mathbf{0}_{12 \times 1}, \varepsilon) = \mathbf{0}_{12 \times 1}$. The equation $\mathbf{0}_{12 \times 1} = \mathbf{G}(\mathbf{x}, \mathbf{z}, 0)$ has a single isolated root

$$\mathbf{z}_0 = \mathbf{h}(\mathbf{x}) = \bar{\mathbf{H}}^{-1} \Gamma \gamma(\mathbf{x}) = -[\alpha^T(\mathbf{x}) \mathbf{0}_{1 \times 3} \alpha^T(\mathbf{x}) \mathbf{0}_{1 \times 3}]^T \quad (\text{B.9})$$

that, given the definition of α in Equation (3.37), satisfies $\mathbf{h}(\mathbf{0}_{6 \times 1}) = \mathbf{0}_{12 \times 1}$. Also, functions $\mathbf{F}$, $\mathbf{G}$, $\mathbf{h}$, and their partial derivatives are bounded up to the second order and $\mathbf{z}_0(t)$ represents the degenerate solution for $\mathbf{z}(t, \varepsilon)$, which is used in turn to evaluate the unperturbed problem:

$$\dot{\mathbf{x}}_0 = \mathbf{F}(\mathbf{x}_0, \mathbf{h}(\mathbf{x}_0), 0) \quad (\text{B.10})$$

and compute the degenerate solution $\mathbf{x}_0(t)$ for $\mathbf{x}(t, \varepsilon)$. In particular, Equation (B.10) becomes:

$$\dot{\mathbf{x}}_0 = \mathbf{H} \mathbf{x}_0 \quad (\text{B.11})$$

where $\mathbf{H} = [\mathbf{R}^T // -\mathbf{K}_x]$. Since $\mathbf{H}$ is a Hurwitz matrix, the origin of the degenerate system in Equation (B.11) is exponentially stable. Consider now the change of variable $\mathbf{y} = \mathbf{z} - \mathbf{h}(\mathbf{x})$ and define $\tau = t/\varepsilon$. The boundary-layer system

$$\frac{d\mathbf{y}}{d\tau} = \mathbf{G}(\mathbf{x}, \mathbf{y} + \mathbf{h}(\mathbf{x}), 0) \quad (\text{B.12})$$

assumes the form:

$$\frac{d\mathbf{y}}{d\tau} = \bar{\mathbf{H}} \mathbf{y} \quad (\text{B.13})$$

which is independent of $\mathbf{x}$ and whose origin is exponentially stable since $\bar{\mathbf{H}}$ is a Hurwitz matrix. Then, according to Theorem 11.4 in Ref. [62], there exists $\varepsilon^* > 0$ such that for all $\varepsilon < \varepsilon^*$, the origin of the nominal system in Equations (B.7) and (B.8) is exponentially stable.

Returning now to the complete system in Equations (3.26)–(3.29), note that it involves the extra term $\dot{\mathbf{v}}_r$. Since $\dot{\mathbf{v}}_r \in L_2[0, \infty) \cap L_\infty[0, \infty)$, one still obtains $\|\mathbf{x}\| \rightarrow 0$ and $\|\mathbf{z}\| \rightarrow 0$ as $t \rightarrow +\infty$ as a consequence of Lemma 1 in Ref. [88].

Remark 14. *The proof of Theorem 3.1.2 shows that the closed loop remains exponentially stable once the fast subsystem is reinstated, provided the scaling parameter ε is chosen small enough to preserve a clear separation of time-scales. In singular-perturbation language, setting $\varepsilon = 0$ gives the reduced (or degenerate) model, whereas the original, singularly perturbed formulation represents the full plant. When the origin of the reduced model is exponentially stable, Khalil's converse-Lyapunov result (Theorem 11.4 in Ref. [62]) ensures that the same property extends to the full system, as long as the neglected fast dynamics evolves sufficiently quickly. While that theorem guarantees the existence of an upper bound on ε , it does not provide a recipe for computing it. Here, ε is introduced only as an analytical device; in practice the designer controls the degree of separation by selecting appropriate feedback gains. The intermediate steps of the proof make this link explicit: by comparing the slow subsystem in Equation (B.11) with the fast boundary-layer subsystem in Equation (B.13), one can inspect their eigen-spectra and gauge directly how a given gain set stretches the gap between the two dynamic modes.*

APPENDIX C

Proof of Theorem 3.2.1

The angle $\bar{\alpha}_i > 0$ between $-\mathbf{z}_E$ and the i -th equilibrium control force $\bar{\mathbf{f}}_{c_i}$ is given by:

$$\tan \bar{\alpha}_i = -\frac{\sqrt{\bar{f}_{x_{c_i}}^2 + \bar{f}_{y_{c_i}}^2}}{\bar{f}_{z_{c_i}}} \quad (\text{C.1})$$

After squaring both members of Equation (C.1) and taking into account Equations (3.61) and (3.62), one has:

$$\begin{aligned} & \left(\frac{g m_l \xi_i \bar{\gamma}_x}{2\sqrt{4L^2 - \bar{\gamma}^2}} - \bar{f}_{x_{d_i}} \right)^2 + \left(\frac{g m_l \xi_i \bar{\gamma}_y}{2\sqrt{4L^2 - \bar{\gamma}^2}} - \bar{f}_{y_{d_i}} \right)^2 \\ & - \left(-\frac{g M}{2} - \bar{f}_{z_{d_i}} \right)^2 \tan^2 \bar{\alpha}_i = 0 \end{aligned} \quad (\text{C.2})$$

Since $\xi_i^2 = 1$ and $\bar{\gamma}^2 = \bar{\gamma}_x^2 + \bar{\gamma}_y^2$, Equation (C.2) is finally manipulated to obtain the polynomial expression in Equation (3.59), where payload mass m_l is collected. In the case when the agents of the formation are multirotors, $\bar{\mathbf{f}}_{c_i}$ is directed along $\mathbf{z}_{B_i}$, such that $\cos \bar{\alpha}_i$ represents the element with position (3, 3) of the direction cosine matrix $\mathbf{T}_{BE}(\bar{\boldsymbol{\eta}}_i)$ defined in Section 2.1. For the selected attitude representation, irrespective of set-point yaw angle $\bar{\psi}_i$, it

is $\cos \bar{\alpha}_i = \cos \bar{\phi}_i \cos \bar{\theta}_i$, where $\bar{\phi}_i$ and $\bar{\theta}_i$ are the equilibrium roll and pitch angles, respectively [63].

APPENDIX D

Elements of Matrices **A** and **B**

In what follows, the elements of matrices **A** and **B** respectively defined in Equations (3.70) and (3.71) are listed.

$$\begin{aligned} A_{8,1} &= -\frac{g m_l (2 M L^2 - m \bar{\gamma}_y^2)}{2 m (2 M L^2 - m \bar{\gamma}^2) \sqrt{4 L^2 - \bar{\gamma}^2}} \\ &= -A_{8,4} = -A_{11,1} = A_{11,4} \end{aligned} \quad (\text{D.1})$$

$$\begin{aligned} A_{8,2} &= -\frac{g m_l \bar{\gamma}_x \bar{\gamma}_y}{2 (2 M L^2 - m \bar{\gamma}^2) \sqrt{4 L^2 - \bar{\gamma}^2}} \\ &= -A_{8,5} = A_{9,1} = -A_{11,2} = A_{11,5} = -A_{12,1} = A_{12,4} \end{aligned} \quad (\text{D.2})$$

$$\begin{aligned} A_{8,3} &= -\frac{g m_l^2 \bar{\gamma}_x L^2}{m \bar{\gamma}^2 (2 m_l L^2 + m \bar{\gamma}^2)} \\ &= -A_{8,6} = A_{11,3} = -A_{11,6} \end{aligned} \quad (\text{D.3})$$

$$A_{8,7} = \frac{g m_l \bar{\gamma}_y}{2 m \bar{\gamma}} = A_{11,7} \quad (\text{D.4})$$

$$\begin{aligned}
 A_{9,2} &= -\frac{g m_l (2 M L^2 - m \bar{\gamma}_x^2)}{2 m (2 M L^2 - m \bar{\gamma}^2) \sqrt{4 L^2 - \bar{\gamma}^2}} \\
 &= -A_{9,5} = -A_{12,2} = A_{12,5}
 \end{aligned} \tag{D.5}$$

$$\begin{aligned}
 A_{9,3} &= -\frac{g m_l^2 \bar{\gamma}_y L^2}{m \bar{\gamma}^2 (2 m_l L^2 + m \bar{\gamma}^2)} \\
 &= -A_{9,6} = A_{12,3} = -A_{12,6}
 \end{aligned} \tag{D.6}$$

$$A_{9,7} = -\frac{g m_l \bar{\gamma}_x}{2 m \bar{\gamma}} = A_{12,7} \tag{D.7}$$

$$\begin{aligned}
 A_{10,1} &= -\frac{g m_l^2 \bar{\gamma}_x L^2}{m (4 L^2 - \bar{\gamma}^2) (2 M L^2 - m \bar{\gamma}^2)} \\
 &= -A_{10,4} = A_{13,1} = -A_{13,4}
 \end{aligned} \tag{D.8}$$

$$\begin{aligned}
 A_{10,2} &= -\frac{g m_l^2 \bar{\gamma}_y L^2}{m (4 L^2 - \bar{\gamma}^2) (2 M L^2 - m \bar{\gamma}^2)} \\
 &= -A_{10,5} = A_{13,2} = -A_{13,5}
 \end{aligned} \tag{D.9}$$

$$\begin{aligned}
 A_{10,3} &= -\frac{g m_l M L^2}{m (2 m_l L^2 + m \bar{\gamma}^2) \sqrt{4 L^2 - \bar{\gamma}^2}} \\
 &= -A_{10,6} = -A_{13,3} = A_{13,6}
 \end{aligned} \tag{D.10}$$

$$A_{10,7} = 0 = A_{13,7} = A_{14,1} = A_{14,2} = A_{14,3} = A_{14,4} = A_{14,5} = A_{14,6} \tag{D.11}$$

$$A_{14,7} = -\frac{g M}{m \sqrt{4 L^2 - \bar{\gamma}^2}} \tag{D.12}$$

$$B_{8,1} = \frac{1}{m} - \frac{m_l (m + m_l) L^2 \bar{\gamma}_x^2}{m [4 m_l M L^4 + m^2 \bar{\gamma}^2 (4 L^2 - \bar{\gamma}^2)]} = B_{11,4} \tag{D.13}$$

$$B_{8,2} = -\frac{m_l (m + m_l) L^2 \bar{\gamma}_x \bar{\gamma}_y}{m [4 m_l M L^4 + m^2 \bar{\gamma}^2 (4 L^2 - \bar{\gamma}^2)]} \quad (D.14)$$

$$= B_{9,1} = B_{11,5} = B_{12,4}$$

$$B_{8,3} = \frac{m_l (m + m_l) L^2 \bar{\gamma}_x \sqrt{4 L^2 - \bar{\gamma}^2}}{m [4 m_l M L^4 + m^2 \bar{\gamma}^2 (4 L^2 - \bar{\gamma}^2)]} \quad (D.15)$$

$$= B_{10,1} = -B_{11,6} = -B_{13,4}$$

$$B_{8,4} = -\frac{m_l \bar{\gamma}_x^2 (2 L^2 - \bar{\gamma}^2)}{2 [4 m_l M L^4 + m^2 \bar{\gamma}^2 (4 L^2 - \bar{\gamma}^2)]} = B_{11,1} \quad (D.16)$$

$$B_{8,5} = -\frac{m_l \bar{\gamma}_x \bar{\gamma}_y (2 L^2 - \bar{\gamma}^2)}{2 [4 m_l M L^4 + m^2 \bar{\gamma}^2 (4 L^2 - \bar{\gamma}^2)]} \quad (D.17)$$

$$= B_{9,4} = B_{11,2} = B_{12,1}$$

$$B_{8,6} = -\frac{m_l \bar{\gamma}_x (2 L^2 - \bar{\gamma}^2) \sqrt{4 L^2 - \bar{\gamma}^2}}{2 [4 m_l M L^4 + m^2 \bar{\gamma}^2 (4 L^2 - \bar{\gamma}^2)]} \quad (D.18)$$

$$= -B_{10,4} = -B_{11,3} = B_{13,1}$$

$$B_{9,2} = \frac{1}{m} - \frac{m_l (m + m_l) L^2 \bar{\gamma}_y^2}{m [4 m_l M L^4 + m^2 \bar{\gamma}^2 (4 L^2 - \bar{\gamma}^2)]} = B_{12,5} \quad (D.19)$$

$$B_{9,3} = \frac{m_l (m + m_l) L^2 \bar{\gamma}_y \sqrt{4 L^2 - \bar{\gamma}^2}}{m [4 m_l M L^4 + m^2 \bar{\gamma}^2 (4 L^2 - \bar{\gamma}^2)]} \quad (D.20)$$

$$= B_{10,2} = -B_{12,6} = -B_{13,5}$$

$$B_{9,5} = -\frac{m_l \bar{\gamma}_y^2 (2 L^2 - \bar{\gamma}^2)}{2 [4 m_l M L^4 + m^2 \bar{\gamma}^2 (4 L^2 - \bar{\gamma}^2)]} = B_{12,2} \quad (D.21)$$

$$B_{9,6} = -\frac{m_l \bar{\gamma}_y (2 L^2 - \bar{\gamma}^2) \sqrt{4 L^2 - \bar{\gamma}^2}}{2 [4 m_l M L^4 + m^2 \bar{\gamma}^2 (4 L^2 - \bar{\gamma}^2)]} \quad (D.22)$$

$$= -B_{10,5} = -B_{12,3} = B_{13,2}$$

$$B_{10,3} = \frac{1}{m} - \frac{m_l (m + m_l) L^2 (4 L^2 - \bar{\gamma}^2)}{m [4 m_l M L^4 + m^2 \bar{\gamma}^2 (4 L^2 - \bar{\gamma}^2)]} = B_{13,6} \quad (\text{D.23})$$

$$B_{10,6} = \frac{m_l (2 L^2 - \bar{\gamma}^2) (4 L^2 - \bar{\gamma}^2)}{2 [4 m_l M L^4 + m^2 \bar{\gamma}^2 (4 L^2 - \bar{\gamma}^2)]} = B_{13,3} \quad (\text{D.24})$$

$$B_{14,1} = -\frac{\bar{\gamma}_y}{m \bar{\gamma} \sqrt{4 L^2 - \bar{\gamma}^2}} = B_{14,4} \quad (\text{D.25})$$

$$B_{14,2} = \frac{\bar{\gamma}_x}{m \bar{\gamma} \sqrt{4 L^2 - \bar{\gamma}^2}} = B_{14,5} \quad (\text{D.26})$$

$$B_{14,3} = B_{14,6} = 0 \quad (\text{D.27})$$

APPENDIX E

Proof of Theorem 3.3.2

Consider the forward-swing state prediction error

$$\tilde{\mathbf{x}} = \hat{\mathbf{x}}_{fs} - \mathbf{x}_{fs} \in \mathbb{R}^6 \quad (\text{E.1})$$

and the parameter estimation error

$$\tilde{\boldsymbol{\sigma}} = \begin{bmatrix} \tilde{\boldsymbol{\sigma}}_1 \\ \tilde{\boldsymbol{\sigma}}_2 \end{bmatrix} = \begin{bmatrix} \hat{\boldsymbol{\sigma}}_{1fs} - \mathbf{f}_{1fs} \\ \hat{\boldsymbol{\sigma}}_{2fs} - \mathbf{f}_{2fs} \end{bmatrix} \in \mathbb{R}^6 \quad (\text{E.2})$$

Note that there exist constants $\boldsymbol{\Delta}_1, \boldsymbol{\Delta}_2$ such that the parameter estimation error is bounded. In the open interval $t \in [kT_s, (k+1)T_s)$, between two sampling instants, the error dynamics is

$$\dot{\tilde{\mathbf{x}}} = \hat{\mathbf{A}}_m \tilde{\mathbf{x}} + \mathbf{B}_m (\tilde{\boldsymbol{\omega}} \mathbf{u} + \tilde{\boldsymbol{\sigma}}_1) + \mathbf{B}_{um} \tilde{\boldsymbol{\sigma}}_2 \quad (\text{E.3})$$

where $\hat{\mathbf{A}}_m := \hat{\mathbf{A}}_{fs}$, $\mathbf{B}_m := \mathbf{B}_{mfs}$, $\tilde{\boldsymbol{\omega}} := \boldsymbol{\omega}_{fs} - \boldsymbol{\omega}_0$, $\mathbf{u} := \mathbf{u}_{fs}$, and $\mathbf{B}_{um} := \mathbf{B}_{umfs}$. Let the gain-mismatch term be $\mathbf{d} := \tilde{\boldsymbol{\omega}} \mathbf{u}$ and bounded, namely $\|\mathbf{d}\| \leq \boldsymbol{\Delta}_\omega$. Let one pick the following Lyapunov candidate function

$$V(t) = \tilde{\mathbf{x}}^T \mathbf{P} \tilde{\mathbf{x}} + \tilde{\boldsymbol{\sigma}}^T \boldsymbol{\Lambda}^{-1} \tilde{\boldsymbol{\sigma}} \quad (\text{E.4})$$

where $\mathbf{P}$ is the solution of the Lyapunov matrix equation $\hat{\mathbf{A}}_{fs}^T \mathbf{P} + \mathbf{P} \hat{\mathbf{A}}_{fs} = -\mathbf{Q}$ and $\mathbf{\Lambda}$ is a symmetric, positive-definite adaptation gain matrix. By taking the time derivative of Equation (E.4) on an open interval and plugging it in Equation (E.3), one obtains

$$\dot{V}(t) = -\tilde{\mathbf{x}}^T \mathbf{Q} \tilde{\mathbf{x}} + 2\tilde{\mathbf{x}}^T \mathbf{P} \mathbf{B}_m \tilde{\boldsymbol{\eta}}_1 + 2\tilde{\mathbf{x}}^T \mathbf{P} \mathbf{B}_{um} \tilde{\boldsymbol{\eta}}_2 \quad (\text{E.5})$$

where $\tilde{\boldsymbol{\eta}}_1 := \mathbf{d} + \tilde{\boldsymbol{\sigma}}_1$ and $\tilde{\boldsymbol{\eta}}_2 := \tilde{\boldsymbol{\sigma}}_2$. By taking into account the Young inequality

$$2\mathbf{a}^T \mathbf{b} \leq \epsilon \|\mathbf{a}\|^2 + \frac{1}{\epsilon} \|\mathbf{b}\|^2 \quad (\text{E.6})$$

it is possible to derive eigen-bounds for both the matched and unmatched parts. In particular, it is

$$\|\tilde{\mathbf{x}}\|^2 \leq \frac{1}{\lambda_{\min}(\mathbf{P})} V \quad (\text{E.7})$$

$$\begin{aligned} \|\tilde{\boldsymbol{\eta}}_1\|^2 &\leq \lambda_{\max}(\mathbf{\Lambda}) V + (\mathbf{\Delta}_1 + \mathbf{\Delta}_\omega)^2 \\ &\leq \lambda_{\max}(\mathbf{\Lambda}) V + \bar{\mathbf{\Delta}}_1^2 \end{aligned} \quad (\text{E.8})$$

$$\|\tilde{\boldsymbol{\eta}}_2\|^2 \leq \lambda_{\max}(\mathbf{\Lambda}) V + \mathbf{\Delta}_2^2 \quad (\text{E.9})$$

where $\bar{\mathbf{\Delta}}_1 = \mathbf{\Delta}_1 + \mathbf{\Delta}_\omega$. By applying the Young inequality with $\epsilon = \lambda_{\min}(\mathbf{Q})/4$ and introducing the above eigen-bounds, one obtains the following ordinary differential inequality

$$\dot{V}(t) \leq \xi V(t) + d_0 \quad (\text{E.10})$$

where

$$\xi = -\frac{\lambda_{\min}(\mathbf{Q})}{2\lambda_{\min}(\mathbf{P})} + \frac{4\lambda_{\max}(\mathbf{\Lambda})}{\lambda_{\min}(\mathbf{Q})} (\|\mathbf{P} \mathbf{B}_m\|^2 + \|\mathbf{P} \mathbf{B}_{um}\|^2) \quad (\text{E.11})$$

and

$$d_0 = \frac{4}{\lambda_{\min}(\mathbf{Q})} (\|\mathbf{P} \mathbf{B}_m\|^2 \|\bar{\mathbf{\Delta}}_1\|^2 + \|\mathbf{P} \mathbf{B}_{um}\|^2 \|\mathbf{\Delta}_2\|^2) \quad (\text{E.12})$$

Remark 15. Note that the exponential decay of the homogeneous part of Equation (E.10) is guaranteed by picking $\mathbf{\Lambda}$ such that $\xi < 0$. A sufficient condition, obtained directly from Equation (E.11), can be

$$\lambda_{\max}(\mathbf{\Lambda}) < \frac{\lambda_{\min}^2(\mathbf{Q})}{8 \lambda_{\min}(\mathbf{P}) (\|\mathbf{P} \mathbf{B}_m\|^2 + \|\mathbf{P} \mathbf{B}_{um}\|^2)}. \quad (\text{E.13})$$

Since $\mathbf{\Lambda}$ is a design parameter, the requirement can be always fulfilled with a proper choice of this adaptation gain matrix.

The solution of Equation (E.10) is obtained from the corresponding linear, first-order ordinary differential equation with initial condition $V(t_0) = V_0 \geq 0$ as

$$V(t) \leq (1 - \beta) V_0 + \frac{d_0 \beta}{|\xi|} \quad (\text{E.14})$$

where $\beta := 1 - e^{\xi T_s} \in (0, 1)$. By choosing $T_s > 0$ such that $\|e^{\mathbf{A}_{m_{fs}} T_s}\| \leq 1 - \alpha$ and applying it to Equation (E.14) at the sampling instant $t_{k+1} = (k + 1) T_s$, one has

$$V(t_{k+1}) \leq (1 - \alpha) V(t_{k+1}^-), \quad (\text{E.15})$$

where t_{k+1}^- is the value right before the update and α introduces a contraction factor between two consecutive samples. The combination of Equations (E.10) and (E.15) yields an expression for the generic $k+1$ time step as $V_{k+1} \leq \rho V_k + q$ where $V_{k+1} := V((k + 1) T_s)$, $V_k := V(k T_s)$, $\rho := (1 - \alpha)(1 - \beta)$, and $q := (1 - \alpha)\beta \frac{d_0}{|\xi|}$. By definition $0 < \rho < 1$, thus the classical geometric-series argument gives

$$\begin{aligned} V_k &\leq \rho^k V_0 + \frac{q}{1 - \rho} = \rho^k V_0 + \frac{(1 - \alpha)\beta d_0}{1 - \rho |\xi|} \\ &= \rho^k V_0 + V_\infty \end{aligned} \quad (\text{E.16})$$

where

$$V_\infty = \frac{(1 - \alpha)\beta d_0}{1 - \rho |\xi|}. \quad (\text{E.17})$$

Since the closed-loop map from $\tilde{\sigma}$ to the output error $\mathbf{e}_{fs}$ is exactly $\mathbf{e}_{fs} = C(s)\tilde{\sigma}$, and $C(s)$ is strictly proper and stable, its $\mathcal{L}_1$ induced gain is finite. Then, for every $t \geq 0$, the convolution inequality yields

$$\|\mathbf{e}_{fs}\|_\infty \leq g_e \sup_{0 \leq \tau \leq t} \|\tilde{\sigma}(\tau)\|. \quad (\text{E.18})$$

From the Lyapunov analysis it is possible to derive the uniform-ultimate bound

$$\sup_{t \geq 0} \|\tilde{\sigma}(\tau)\| \leq \sqrt{\lambda_{\max}(\mathbf{\Lambda}) V_\infty}, \quad (\text{E.19})$$

with V_∞ already defined in Equation (E.16). By combining Equations (E.18) and (E.19), one finally obtains

$$\|\mathbf{e}_{fs}(t)\|_\infty \leq g_e \sqrt{\lambda_{\max}(\mathbf{\Lambda}) V_\infty} \quad \forall t \geq 0, \quad (\text{E.20})$$

which coincides exactly with the performance bound stated in Theorem 3.3.2.

APPENDIX F

Elements of matrices $\mathbf{M}$ and $\mathbf{C}$

In what follows, nonzero elements of $\mathbf{M} \in \mathbb{R}^{7 \times 7}$ and $\mathbf{C} \in \mathbb{R}^{7 \times 7}$ in Equation (4.11) are reported.

$$M_{1,1} = m + m_A + m_B = M_{2,2} = M_{3,3} \quad (\text{F.1})$$

$$M_{1,5} = L_B m_B \cos \zeta_B = M_{5,1} \quad (\text{F.2})$$

$$M_{1,7} = L_A (m_A + m_B) \cos \zeta_A = M_{7,1} \quad (\text{F.3})$$

$$M_{2,4} = -L_B m_B \cos \xi_B \cos \zeta_B = M_{4,2} \quad (\text{F.4})$$

$$M_{2,5} = L_B m_B \sin \xi_B \sin \zeta_B = M_{5,2} \quad (\text{F.5})$$

$$M_{2,6} = -L_A (m_A + m_B) \cos \xi_A \cos \zeta_A = M_{6,2} \quad (\text{F.6})$$

$$M_{2,7} = L_A (m_A + m_B) \sin \xi_A \sin \zeta_A = M_{7,2} \quad (\text{F.7})$$

$$M_{3,4} = -L_B m_B \sin \xi_B \cos \zeta_B = M_{4,3} \quad (\text{F.8})$$

$$M_{3,5} = -L_B m_B \cos \xi_B \sin \zeta_B = M_{5,3} \quad (\text{F.9})$$

$$M_{3,6} = -L_A (m_A + m_B) \sin \xi_A \cos \zeta_A = M_{6,3} \quad (\text{F.10})$$

$$M_{3,7} = -L_A (m_A + m_B) \cos \xi_A \sin \zeta_A = M_{7,3} \quad (\text{F.11})$$

$$M_{4,4} = L_B^2 m_B \cos^2 \zeta_B \quad (\text{F.12})$$

$$M_{4,6} = L_A L_B m_B (\cos \xi_A \cos \zeta_A \cos \xi_B \cos \zeta_B + \sin \xi_A \cos \zeta_A \sin \xi_B \cos \zeta_B) = M_{6,4} \quad (\text{F.13})$$

$$M_{4,7} = L_A L_B m_B (-\sin \xi_A \sin \zeta_A \cos \xi_B \cos \zeta_B + \cos \xi_A \sin \zeta_A \sin \xi_B \cos \zeta_B) = M_{7,4} \quad (\text{F.14})$$

$$M_{5,5} = L_B^2 m_B \quad (\text{F.15})$$

$$M_{5,6} = L_A L_B m_B (-\cos \xi_A \cos \zeta_A \sin \xi_B \sin \zeta_B + \sin \xi_A \cos \zeta_A \cos \xi_B \sin \zeta_B) = M_{6,5} \quad (\text{F.16})$$

$$M_{5,7} = L_A L_B m_B (\sin \xi_A \sin \zeta_A \sin \xi_B \sin \zeta_B + \cos \xi_A \sin \zeta_A \cos \xi_B \sin \zeta_B + \cos \xi_A \cos \zeta_A) = M_{7,5} \quad (\text{F.17})$$

$$M_{6,6} = L_A^2 (m_A + m_B) \cos^2 \zeta_A \quad (\text{F.18})$$

$$M_{7,7} = L_A^2 (m_A + m_B) \quad (\text{F.19})$$

$$C_{1,5} = -L_B m_B \dot{\zeta}_B \sin \zeta_B \quad (\text{F.20})$$

$$C_{1,7} = -L_A (m_A + m_B) \dot{\zeta}_A \sin \zeta_A \quad (\text{F.21})$$

$$C_{2,4} = L_B m_B (\dot{\xi}_B \sin \xi_B \cos \zeta_B + \dot{\zeta}_B \cos \xi_B \sin \zeta_B) \quad (\text{F.22})$$

$$C_{2,5} = L_B m_B (\dot{\xi}_B \cos \xi_B \sin \zeta_B + \dot{\zeta}_B \sin \xi_B \cos \zeta_B) \quad (\text{F.23})$$

$$C_{2,6} = L_A (m_A + m_B) (\dot{\xi}_A \sin \xi_A \cos \zeta_A + \dot{\zeta}_A \cos \xi_A \sin \zeta_A) \quad (\text{F.24})$$

$$C_{2,7} = L_A (m_A + m_B) (\dot{\xi}_A \cos \xi_A \sin \zeta_A + \dot{\zeta}_A \sin \xi_A \cos \zeta_A) \quad (\text{F.25})$$

$$C_{3,4} = -L_B m_B (\dot{\xi}_B \cos \xi_B \cos \zeta_B - \dot{\zeta}_B \sin \xi_B \sin \zeta_B) \quad (\text{F.26})$$

$$C_{3,5} = L_B m_B (\dot{\xi}_B \sin \xi_B \sin \zeta_B - \dot{\zeta}_B \cos \xi_B \cos \zeta_B) \quad (\text{F.27})$$

$$C_{3,6} = L_A (m_A + m_B) (-\dot{\xi}_A \cos \xi_A \cos \zeta_A + \dot{\zeta}_A \sin \xi_A \sin \zeta_A) \quad (\text{F.28})$$

$$C_{3,7} = L_A (m_A + m_B) (\dot{\xi}_A \sin \xi_A \sin \zeta_A - \dot{\zeta}_A \cos \xi_A \cos \zeta_A) \quad (\text{F.29})$$

$$C_{4,4} = -L_B^2 m_B \dot{\zeta}_B \sin \zeta_B \cos \zeta_B \quad (\text{F.30})$$

$$C_{4,5} = -L_B^2 m_B \dot{\xi}_B \sin \zeta_B \cos \zeta_B \quad (\text{F.31})$$

$$\begin{aligned}
 C_{4,6} = & L_A L_B m_B \dot{\xi}_A (-\sin \xi_A \cos \zeta_A \cos \xi_B \cos \zeta_B \\
 & + \cos \xi_A \cos \zeta_A \sin \xi_B \cos \zeta_B) \\
 & - L_A L_B m_B \dot{\zeta}_A (\cos \xi_A \sin \zeta_A \cos \xi_B \cos \zeta_B \\
 & + \sin \xi_A \sin \zeta_A \sin \xi_B \cos \zeta_B)
 \end{aligned} \tag{F.32}$$

$$\begin{aligned}
 C_{4,7} = & -L_A L_B m_B \dot{\xi}_A (\cos \xi_A \sin \zeta_A \cos \xi_B \cos \zeta_B \\
 & + \sin \xi_A \sin \zeta_A \sin \xi_B \cos \zeta_B) \\
 & + L_A L_B m_B \dot{\zeta}_A (\cos \xi_A \cos \zeta_A \sin \xi_B \cos \zeta_B \\
 & - \sin \xi_A \cos \zeta_A \cos \xi_B \cos \zeta_B)
 \end{aligned} \tag{F.33}$$

$$C_{5,4} = L_B^2 m_B \dot{\xi}_B \sin \zeta_B \cos \zeta_B \tag{F.34}$$

$$\begin{aligned}
 C_{5,6} = & L_A L_B m_B \dot{\xi}_A (\sin \xi_A \cos \zeta_A \sin \xi_B \sin \zeta_B \\
 & + \cos \xi_A \cos \zeta_A \cos \xi_B \sin \zeta_B) \\
 & + L_A L_B m_B \dot{\zeta}_A (\cos \xi_A \sin \zeta_A \sin \xi_B \sin \zeta_B \\
 & - \sin \xi_A \sin \zeta_A \cos \xi_B \sin \zeta_B)
 \end{aligned} \tag{F.35}$$

$$\begin{aligned}
 C_{5,7} = & L_A L_B m_B \dot{\xi}_A (\cos \xi_A \sin \zeta_A \sin \xi_B \sin \zeta_B \\
 & - \sin \xi_A \sin \zeta_A \cos \xi_B \sin \zeta_B) \\
 & + L_A L_B m_B \dot{\zeta}_A (\sin \xi_A \cos \zeta_A \sin \xi_B \sin \zeta_B \\
 & + \cos \xi_A \cos \zeta_A \cos \xi_B \sin \zeta_B - \sin \zeta_A \cos \zeta_B)
 \end{aligned} \tag{F.36}$$

$$\begin{aligned}
 C_{6,4} = & L_A L_B m_B \dot{\xi}_B (-\cos \xi_A \cos \zeta_A \sin \xi_B \cos \zeta_B \\
 & + \sin \xi_A \cos \zeta_A \cos \xi_B \cos \zeta_B) \\
 & - L_A L_B m_B \dot{\zeta}_B (\cos \xi_A \cos \zeta_A \cos \xi_B \sin \zeta_B \\
 & + \sin \xi_A \cos \zeta_A \sin \xi_B \sin \zeta_B)
 \end{aligned} \tag{F.37}$$

$$\begin{aligned}
 C_{6,5} = & -L_A L_B m_B \dot{\xi}_B (\cos \xi_A \cos \zeta_A \cos \xi_B \sin \zeta_B \\
 & + \sin \xi_A \cos \zeta_A \sin \xi_B \sin \zeta_B) \\
 & + L_A L_B m_B \dot{\zeta}_B (\sin \xi_A \cos \zeta_A \cos \xi_B \cos \zeta_B \\
 & - \cos \xi_A \cos \zeta_A \sin \xi_B \cos \zeta_B)
 \end{aligned} \tag{F.38}$$

$$\begin{aligned}
C_{7,4} = & L_A L_B m_B \dot{\xi}_B (\sin \xi_A \sin \zeta_A \sin \xi_B \cos \zeta_B \\
& + \cos \xi_A \sin \zeta_A \cos \xi_B \cos \zeta_B) \\
& + L_A L_B m_B \dot{\zeta}_B (\sin \xi_A \sin \zeta_A \cos \xi_B \sin \zeta_B \\
& - \cos \xi_A \sin \zeta_A \sin \xi_B \sin \zeta_B)
\end{aligned} \tag{F.39}$$

$$\begin{aligned}
C_{7,5} = & L_A L_B m_B \dot{\xi}_B (\sin \xi_A \sin \zeta_A \cos \xi_B \sin \zeta_B \\
& - \cos \xi_A \sin \zeta_A \sin \xi_B \sin \zeta_B) \\
& + L_A L_B m_B \dot{\zeta}_B (\sin \xi_A \sin \zeta_A \sin \xi_B \cos \zeta_B \\
& + \cos \xi_A \sin \zeta_A \cos \xi_B \cos \zeta_B - \cos \zeta_A \sin \zeta_B)
\end{aligned} \tag{F.40}$$

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Fundings

Finanziato dall'Unione europea- Next Generation EU, Missione 4, Componente 2, Investimento 3.3 (D.M. 117/2023) CUPJ33C22001920002.

