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MEASURING, HEDGING, AND MITIGATING CLIMATE RISK IN FINANCIAL MARKETS AND ENVIRONMENTAL RESOURCES

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Abstract

This thesis investigates the interaction between climate risk and financial markets, focusing on transition risk, physical risk, and their implications for asset pricing and hedging. Transition risk, stemming from the economic adjustments required to address climate change, is inherently challenging to quantify due to its reliance on regulatory and market dynamics. The study examines potential proxies, including European carbon allowance returns and a transition risk index, to measure transition risk in stock and bond markets. However, both proxies were found statistically insignificant, indicating limited sensitivity of financial markets to these variables or their inadequacy as measures of transition risk. Physical risk, caused by climate-related extreme events, demonstrated a more substantial influence on bond market pricing. A novel pricing model incorporating climate variables into the stochastic hazard rate framework was proposed, allowing for the assessment of physical risk exposure. This approach provided actionable insights for ranking corporate issuers based on their sensitivity to physical risk factors. The thesis also explores weather derivatives as hedging instruments for climate risk. For temperature-based derivatives, a market-aligned pricing model was introduced by defining a tradable "forward temperature" asset, addressing inefficiencies in existing methods. Despite these advances, the market for temperature derivatives remains underdeveloped, with significant underpricing. Additionally, innovative derivative contracts, such as Rainfall Quanto Options and Basin Level Cash-or-Nothing Options, were proposed to hedge water scarcity risks. These tools demonstrated effectiveness in addressing geographic and market limitations, offering flexibility beyond traditional insurance mechanisms. By analyzing the integration of climate risks into financial markets and proposing novel hedging instruments, this work provides valuable insights for policymakers, asset managers, and financial institutions to better assess, manage, and mitigate the financial impacts of climate change.

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Introduction

The main focus of this work is to analyze climate risk and its interactions with financial markets. In this context, climate risk refers to risks associated with climate change, typically divided into these two types: transition risk and physical risk.

Transition risk arises from potential losses linked to the technological and operational changes required to address climate change. Although it is closely related to regulatory risk, it extends beyond this domain, as regulatory decisions are usually based on scientific data and research, independent of political influence. Ultimately, companies may need to adopt more sustainable production methods, which could result in losing market position to competitors better aligned with environmental regulations. Such risks directly impact the market values of stocks and bonds, as a company's profitability affects its ability to meet financial obligations. If a company fails to adapt, its non-compliance may decrease the value of its stocks and bonds, negatively affecting the portfolios of financial institutions such as banks and insurance companies. While the concept of transition risk is clear, quantifying it remains challenging, unlike other financial risks measured using tools like Value at Risk (VaR) and Expected Shortfall.

Physical risk, by contrast, involves the economic damage caused by extreme climate events, which are occurring with increasing frequency. This risk differs from transition risk, as it directly concerns the financial impact of climate-related disasters.

The first part of this thesis focuses on transition risk, as this is the most challenging to quantify with a single variable. This difficulty arises from its intrinsic link to regulatory risk, driven by policymakers' decisions, which in turn depend on voter preferences. These preferences may lead to more or less stringent environmental regulations, making adaptation more difficult—a factor challenging to measure numerically. Nevertheless, this thesis will explore whether it is possible to identify quantitative variables that could serve as indirect proxies for transition risk, suitable for statistical analysis. Drawing on the existing literature, we will test two potential proxies. The first is the returns on carbon allowances, which represent the market cost of CO2 in a cap-and-trade system and, in equilibrium, approximate the marginal cost of emissions reduction. Therefore, market prices can be interpreted as the average marginal cost of green investments. The second variable is derived from the difference in credit default swap (CDS) spreads between green and non-green companies; some researchers interpret this spread as the additional premium investors pay to protect themselves against default by non-green companies. Both variables will be tested as predictors to explain the returns of European companies primarily operating in "hard-to-abate" sectors, aiming to identify any dependencies useful for policy analysis as well as for forecasting and asset allocation. Unfortunately, both variables proved to be statistically insignificant, suggesting either that stock market traders are not overly concerned with transition risk or that these variables may not be the best proxies, despite being reasonable candidates given their definitions. Similar results were observed in bond market analysis, both for green and traditional bonds, aligning with the stock market findings.

The second part of this thesis examines how to incorporate physical climate risk into the pricing of risky bonds. Existing research indicates that climate variables are statistically significant predictors for explaining the yields of both green and non-green bonds. However, the literature lacks guidance on incorporating these factors into continuous-time pricing models. Therefore, the second part of this work will focus on defining a pricing approach that incorporates these factors into risky bond pricing models, using stochastic hazard rate models and indirect inference for estimation. Even though our approach is theoretically sound and able to catch markets effects current market data reflect a degree of incompleteness that is beyond modeling since market prices seems do not reflect the current market information meaning that we found that the weather derivatives are quite underpriced.

The third and final part of the thesis focuses on weather derivatives as hedging instruments against meteorological events. The literature on weather derivatives has primarily focused on temperature derivatives, pricing them based solely on temperature characteristics while largely ignoring market effects. While this approach provides a way to price derivatives, it has not explained the discrepancy between quoted and estimated prices, effectively ignoring market influences. In this work, we propose a model that incorporates market effects by defining a primitive asset, which we call "forward temperature." This definition is based on Cooling Degree Days (CDD), Heating Degree Days

(HDD), and Cumulative Degree Days (CAT) contracts, allowing us to redefine weather derivative pricing through a market-based model.

Finally, we address a new area in weather derivatives literature: designing hedging instruments to protect against water scarcity due to either insufficient rainfall or low reservoir levels. This topic is almost entirely absent from weather derivatives literature. We will show that the instruments we propose provide effective economic coverage for these phenomena, even in cases of geographic misalignment. The thesis will be organized as follow:

Chapter 1 explores how transition risk can be measured in stock returns to determine whether this risk is factored into pricing and to estimate its impact. Given the challenges of quantifying transition risk, two candidate variables are tested. The first is the log-returns of European emissions allowances, chosen because, in a cap-and-trade system, the price of carbon allowances reflects the marginal cost of reducing emissions. Higher carbon prices and returns indicate increased capital needs and risks for emissions-reducing investments, which should negatively affect firms with high transition risk exposure while positively impacting green firms. The second variable is the transition risk index from Blasberg et al. 2021, with similar expected relationships: positive for green firms and negative for non-green ones.

Chapter 2 analyzes how climate-related factors affect bond returns, with a focus on both physical climate risk and climate transition risk. Previous studies indicate that physical climate risk variables significantly influence excess returns in both green and non-green bond markets. However, attempts to link carbon allowance returns to bond returns as a measure of transition risk have shown no statistical significance, suggesting that carbon allowances may not effectively capture transition risk. The same result applies to the transition risk index by Blasberg et al. 2021. Nonetheless, climate variables continue to have a substantial impact on bond returns, showing that the market prices in physical climate risk. The chapter then explores integrating these climate risk factors into bond pricing using stochastic intensity rate models, following the approach of Duffie et al. 1999.

Chapter 3 explores the use of weather derivatives to hedge climate-related risks and address insurance gaps, focusing first on temperature-based derivatives and then on new contracts to hedge water scarcity. The market for temperature derivatives is underdeveloped, with inefficiencies such as pricing mismatches that fail to reflect actual temperature trends. To improve pricing accuracy, we introduce a tradable asset concept—forward temperature to better align prices with temperature movements and address market incompleteness.
In the second part, new weather derivatives are designed to hedge water scarcity risks, including options for low rainfall and low basin levels. A Rainfall Quanto Option links payouts to average rainfall and water prices, while a Basin Level Cash-or-Nothing Option activates based on basin levels and water prices at maturity. The chapter concludes with simulations demonstrating the effectiveness of these hedging tools.

1 Transition risk

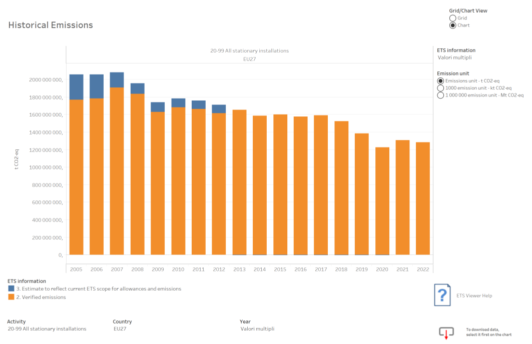
1.1 Introduction

This chapter will be devoted to the analysis of the component of the climate change risk known as transition risk. The transition risk is defined as the risk associated with the change of the structure of the economy, namely its transition from a high carbon-intensive economy to a carbon-neutral economy. Such transformation requires a deep change in the business structure at any level for all firms. The main issue in evaluating such risk (which is due to a technological change) stays in its long-run horizon (Bolton, Depres, et al. 2020). Even so, with the great difficulty of evaluating it, international institutions like the ECB are asking banks and financial corporations to estimate the impact of such risk on their portfolio and also allocate suitable reserves to absorb losses due to the transition risk. One of the issues with the transition risk is the fact that its definition does not refer to an economic variable that can be measured as in the case of default risk or portfolio market risk. Default risk refers to the inability of debtors to repay their obligations which means that an analyst has to model the default probability by analyzing debts, income assets' value, and so on, but such figures can be obtained from market data (stock price) and balance sheet. Market risk refers to the downside risk of a portfolio, which leads to techniques like value at risk and expected shortfall, which rely again on market data. So, to deal with transition risk we need a definition that relies on available data on which it is possible to construct models and extract the transition risk information. Therefore, we will try two different variables: the first candidate variable for measuring the transition risk in stock markets will be the price of the European carbon allowances thanks to its correlation structure with the other energy commodities such as natural gas and coal. The choice for studying if the dependence between energy commodities and stocks can be an indicator of transition risk or not is because energy production is the most important source of emissions in the atmosphere; moreover, it has been documented the existence of a link between energy commodities (like natural gas and coal) with the carbon allowances, which is the cost for emitting one tonne of CO_2. The second candidate variable that will be tested, is the difference in the credit default swap spread, with 20 years of maturity, between green corporations and brown corporations; such transition risk measure has been proposed by Blasques et al. 2014 and it was intended as the differential credit risk exposure of brown versus green firms.

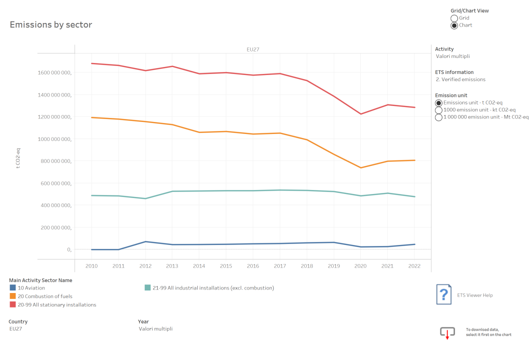
1.2 The emission trading system

To be compliant with the Kyoto Protocol of 1997 many nations has set up an emission cap and trade scheme as part of the efforts to reduce greenhouse emissions. A cap and trade system is a market where emission allowances are exchanged between firms. Emission allowances are certificates that allow the owner to emit 1 tonne of CO2, they are auctioned each year on what can be thought of as a primary market, and then the corporations can exchange them in the market determining in this way the fair market price for the emissions. Such a system has transformed an externality like pollution into a commodity, that increases the cost of production of goods and services and so encourages firms to abate their emissions and reduce the cost associated with them. The key point behind a cap and trade system is the equality (in equilibrium) between the marginal abatement cost (m.a.c.) and the market price of the certificates, in this situation when the m.a.c. is lower than the market price, corporations will invest in technologies for abating the emissions until the m.a.c. is greater or equal the market price for certificate; on the contrary, when the market price of allowances is lower than the m.a.c. the increase in the demand for certificates will raise the price until is greater or equal to the marginal abatement cost (see Aïd et al. 2023). At the end of each year, the authority checks if all the corporations of the cap and trade have in their portfolio a number of certificates enough to cover their total emissions in the solar year otherwise they have to pay a penalty (in the case of the EU-ETS the penalty is of 100 Euros per tonne). During the last 20 years there are several nations that have implemented or tried to implement a cap and trade system to reduce emissions: Australia initiated a carbon market for New South Wales in 2003, but this was canceled in July 2014 as part of the national shift away from such schemes, which also included the repeal of the carbon tax. Meanwhile, China took its first steps toward carbon trading in November 2011 by launching pilot programs in various provinces and cities, including Beijing, Shanghai, and Guangdong, each with distinct pricing. In 2021, China extended the program nationwide, though initial implementation was limited to the power sector due to challenges in gathering emissions data, with plans to gradually expand to other sectors. South Korea introduced its national emissions trading scheme in 2015, covering 525 entities across 23 sectors, and established a three-year emissions cap of 1.8687 billion tCO2e, making it the second-

largest carbon market globally after the EU ETS. In contrast, the United States has no national emissions trading scheme, prompting several states on the east and west coasts to create their own cap-and-trade programs in response to federal inaction. India, in 2014, launched a mandatory energy efficiency trading scheme targeting sectors that account for 54% of the country's industrial energy consumption, with the goal of reducing emission intensity by 20-25% from 2005 levels.
In 2005 the European Commission created the European emissions trading scheme[1] (EU-ETS), which has produced a reduction in the level of emissions[2] as shown in Fig. 1a and 1b



(a) EU emissions



(b) EU emissions by sector

Figure 1: EU-ETS emissions over years

The EU-ETS has been divided into four "trading periods": The first ETS trading period lasted three years, from January 2005 to December 2007, the second trading period ran from January 2008 until December 2012, the third trading period lasted from January 2013 to December 2020, and the last from January 2021 until December 2030. To increase the efficiency of the system in 2015, it was agreed to set up a Market Stability Reserve (MSR) as a long-term solution to the surplus of allowances on the EU carbon market. Aiming to rebalance supply and demand and make the carbon market more resilient to major future shocks, the MSR was established in 2018 and began operating in 2019.

1.3 Literature review

There is a unanimous consensus that climate change is real and humanity is responsible for it, as further highlighted by the IPCC 2021 report. While this has multiple implications for the whole economy, a keen interest has developed with respect to the implications of climate change for financial markets and financial risks. It is easy to understand why: as a significant issue for business continuity, a central topic for public discourse and regulation, and potentially an issue for financial stability, climate change poses several questions to financial academics and practitioners. The issue has received growing attention in recent literature. Starting from Carney 2015, more and more academic and industry studies have been proposed to investigate the topic. In this regard, when looking at climate change and its impacts, an important distinction needs to be made, i.e. the one between physical and transition risk. While the former denotes the risks associated with chronic (i.e. increased frequency and intensity of extreme events) and acute (i.e. permanent shifts in weather patterns) impacts of climate change, the latter looks at the risks imposed by a sustainable transition on companies and markets, stemming from new policies or technologies, for example, via the generation of stranded assets, see Monasterolo et al. 2017. Transition risk has been the most studied in the literature, from different perspectives; a stream of literature moving from S. Battiston et al. 2017, Battiston et al. 2019 and, Roncoroni et al. 2021 has investigated its systemic implications, a topic that has also been investigated by multiple central banks (for example, see Clerc et al. 2021, Vermuelen et al. 2021) also in conjunction with physical risk. Studies have also focused on disclosure and sentiment around climate risks as in Kolbel et al. 2020, Bingler et al. 2021, Engle et al. 2020, and Faccini et al. 2021.

The dependence structure between carbon allowances (EUAs), natural gas, coal, and oil (Brent) have been studied with different techniques: in Chevallier et al. 2019 the authors model the yearly future EUAs, ECF, (exchanged on ICE), the monthly TTF natural gas (exchanged on ICE), the

[1]https://climate.ec.europa.eu/eu-action/eu-emissions-trading-system-eu-ets_en
[2]https://www.eea.europa.eu/data-and-maps/dashboards/emissions-trading-viewer-1

monthly future contract on Brent (exchanged on ICE), the monthly coal ICE API2 with ARMA processes with TGARCH volatilities; the dependence structure between the four contracts with a vine copula, finding a positive link between the ECF and the TTF natural gas, COAL and Brent, and TTF natural gas and Brent, but nothing statistically significant between Coal and ECF. On the contrary in Kanwal et al. 2021, the authors found a statistically significant relationship between ECF and Brent and between ECF and Coal, by representing the correlation structure via a t-copula function and modeling the energy commodities with ARMA processes with GARCH volatilities with marginal Student-t distribution. In Meier et al. 2020 and Lovcha et al. 2022 with a VAR model for the yearly future ECF, the monthly TTF natural gas, the monthly future contract on Brent, the monthly coal ICE API2 have findings in line with Kanwal et al. 2021. In Balcılar et al. 2016, Chen et al. 2019 and Chuliá et al. 2019 it is shown that the carbon emission markets are linked to changes in the electricity, natural gas, and coal futures markets, and more significantly so in the case of the EUA market. The link is formed through the effects of the forces that drive volatility in the energy market as well as time-varying risk transmissions from these energy markets to the carbon market, both in terms of the cross-market correlations and volatility spillovers. There are several studies about spillover effects between carbon prices and stock markets: in Garcia-Jorcano et al. 2022 the CO_2 emission allowance returns are assumed to be a market proxy for changes in climate risk, and that financial markets price carbon risks asymmetrically. The authors found asymmetric tail dependence, indicating that the risk exposure of industry returns depends on the climate risk scenario and on whether downside risk or upside risk prevails in the market. Moreover, they found that downside risk is exacerbated when changes in CO2 emission allowance prices indicate a favorable (green) climate scenario, whereas the opposite is true when they indicate an adverse (brown) one. In Hanif et al. 2021 the authors studied the dependence structure, via copula functions, between six renewable energy indices and EUAs to daily prices spanning from May 18, 2011, to March 05, 2020. The copula results show that the European emission allowance prices are predominantly symmetrically related, i.e., in the center and in the tails, with the clean energy indices that they considered. In Dutta et al. 2018 it is found that the link between the carbon emission market and the market of clean energy stocks is usually statistically insignificant by employing the bivariate VAR-GARCH approach. More importantly, this finding holds for both the US and European markets. Hsu et al. 2023 found the presence of a pollution premium that couldn't be explained by existing systematic risks, investor preferences, market sentiment, political connections, or corporate governance; they found that firms with more toxic emissions are associated with higher current profitability and more environmental litigation; high-emission firms' future profitability is lower after governments impose stricter environmental regulations; moreover, high-emission firms observe a favorable shock in response to Donald Trump's 2016 U.S. presidential election win, which suggests a connection between emission-related return predictability and changes in environmental policies. In Azar et al. 2021 it is found that higher ownership by the Big Three (BlackRock, Vanguard, and State Street Global Advisors) is followed by lower carbon emissions. In Blasberg et al. 2021 is proposed a transition risk measure based on CDS spread by grouping the corporations by carbon intensity per unit of revenue; then they defined firms below the first quintile as "green" and gathered their CDS spreads in the set $\mathcal{G}_t^m$. Analogously, they defined firms above the last quintile as "brown" and gathered their CDS spreads in the set $\mathcal{B}_t^m$. Then, by taking the median cost of default protection of green and brown firms by calculating the median m-year CDS spread level for each tenor $m \in \{1, 3, 5, 10, 30\}$ at every time t is denoted as $G_t^m = \mathrm{Med}(\mathcal{G}_t^m)$, $B_t^m = \mathrm{Med}(\mathcal{B}_t^m)$. In the end, they calculated the difference between the median CDS spreads of brown and green firms. This difference, or wedge, represents the differential credit risk exposure of brown versus green firms. They called this the carbon risk (CR) factor:

$$CR_t^m = B_t^m - G_t^m \tag{1}$$

Essentially, CR mimics the dynamics of a portfolio in which default protection is bought for a representative (median) brown company and sold for a representative (median) green firm. When policy events trigger a rise in carbon risk (e.g. expectation of a tighter future regulatory framework), the demand for protection of more (less) exposed firms increases (decreases), resulting in a widening of the wedge. Conversely, if the market expects a loosening of the regulatory framework, there is a narrowing of the wedge (or possibly even a negative wedge). These changes in perceived exposure to carbon risk are aptly represented by the behavior of CR. As such, they considered CR to be an observable proxy for lenders' perception of carbon risk exposure. The previous indicator was tested by Livieri et al. 2023 to asses the jump nature of the transition risk in the bond price by implementing the Merton model for bond pricing.

1.4 Data

The data were collected from Refinitiv. The commodities used are quoted on ICE (Intercontinental Commodity Exchange), Coal ICE API2 CIF ARA Nr Mth \$/MT[3] (COAL hereafter), RFV Natural Gas TTF NL 1st Fut. Mth[4] (TTF hereafter), Crude Oil Brent ICE M1 UK 1200 hrs[5] (Brent hereafter) and ICE EUA Yearly Energy Future[6] (ECF hereafter). The MSCI price index[7], the VIX index[8]. The stock dataset (Table 19) covers the following sectors: Utilities (69), Technology (10), Industrials (136), Energy (50), Financials (9), Basic Materials (69), Consumer Cyclicals (7), Healthcare (6), Consumer Non-Cyclicals (7), and Real Estate (2). For the CDS used to construct the index proposed by Blasberg et al. 2021, we downloaded the CDS from Refinitiv and used only those for which the Emission Score[9] was available; a high emissions score means that the company is green, i.e. it is a small polluter, while small values of the index mean that the company is a big polluter and so it is brown. In the end, the companies included in the sample were 93 (Appendix A.2.1). Then, on the basis of the value of the emission score the sample was divided into quintiles, and for the first and last quintile we took the median for each day for tenors, 5, 10, and 20 years (Fig. 43). We found, that there was a systematic positive difference between the CDS of the brown companies and the ones from the green companies.

1.5 Score driven models

Before going into the analysis of the data, let's make a resume of the score-driven models that will be used in this chapter. Score-driven models were proposed by Creal et al. 2013 (a well-done summary of practical examples can be found in Artemova et al. 2022). Such models were proposed to better represent the stochastic volatility of financial time series than classic GARCH models introduced by Engle 1982, this is because they are based on the score of the log-likelihood function instead of just on the noise of the process allowing the model to be less sensitive to extreme values, especially when dealing with non-Gaussian distributions, as in the case of financial time series of log-returns. In fact, if you consider the GARCH(p,q) model the financial returns r_t are modeled as:

$$\begin{aligned} r_t &= \sigma_t \epsilon_t \\ \sigma_t^2 &= \omega + \sum_{k=1}^{p} \alpha_k \epsilon_{t-k}^2 + \sum_{k=1}^{q} \phi_k \sigma_{t-k}^2, \end{aligned} \tag{2}$$

where σ_t is the stochastic volatility at time t and ϵ_t is a martingale difference sequence. The model above is the well-known GARCH(p,q), such a model has the weakness of directly depending on ϵ_{t-k}^2 which can be problematic to handle in the estimation process when dealing with non-Gaussian data with high probability of having extreme observations in the data, which require to use fat tail distribution, as the Student-t distribution, for the noise ϵ_t. The score drive models instead model the financial returns r_t as

$$\begin{aligned} r_t &= \sigma_t \epsilon_t \\ \sigma_t^2 &= \omega + \sum_{k=1}^{p} \alpha_k s_{t-k} + \sum_{k=1}^{q} \phi_k \sigma_{t-k}^2, \end{aligned} \tag{3}$$

where $s_t = S_k \cdot \nabla_t$ is the score of the log-likelihood function and S_t is a scaling matrix and $\nabla_t = \frac{\partial \log(p(r_t|\sigma_t^2,\theta))}{\partial \sigma_t^2}$. There are no restrictions on the choice for matrix S_k, anyway, it is usually chosen to be an identity matrix or the Fisher information matrix with respect to the variable of interest, σ_t^2 in this case. If $\epsilon_t \sim \mathcal{N}(0,1)$ the score-driven model is exactly the GARCH model, but

[3]https://www.ice.com/products/243/API2-Rotterdam-Coal-Futures
[4]https://www.ice.com/products/27996665/Dutch-TTF-Natural-Gas-Futures
[5]https://www.ice.com/futures-europe/brent
[6]https://www.ice.com/products/197/EUA-Futures
[7]https://www.msci.com/documents/10199/178e6643-6ae6-47b9-82be-e1fc565ededb
[8]https://www.cboe.com/tradable_products/vix/
[9]https://www.lseg.com/en/data-analytics/sustainable-finance/esg-scores

in the case of the Student-t distribution with ν degrees of freedom

$$p(r_t|\sigma_t^2) = \frac{1}{Beta(\frac{1}{2}, \frac{\nu}{2})\sqrt{\nu\sigma_t^2}}\left(1 + \frac{r_t^2}{\nu\sigma_t^2}\right)^{-\frac{\nu+1}{2}} \tag{4}$$

$$s_t = \frac{1}{2}\left(\frac{(\nu+1)\nu^{-1}r_t^2\sigma_t^{-4}}{1+\nu^{-1}r_t^2/\sigma_t^2} - \frac{1}{\sigma_t^2}\right) \tag{5}$$

$$S_t = \mathcal{I}^{-1} = \frac{2(3+\nu)\sigma_t^4}{\nu}, \tag{6}$$

where $Beta(1/2, \nu/2)$ is the beta function evaluated at $1/2$ and $\nu/2$ and $\mathcal{I}$ is the Fisher information matrix. The model can be easily extended to account for lags and exogenous variables. Then, we have $r_{t+1} = \mathrm{E}[r_{t+1}|\mathcal{F}_t] + \sigma_t\epsilon_t$, where the most classic form for $\mathrm{E}[r_{t+1}|\mathcal{F}_t]$ is the linear model. So, substitute $y_t = r_t - \mathrm{E}[r_{t+1}|\mathcal{F}_t]$ instead of r_t in the Eq. 4, 5 and 6. Model 3 with the specification given by Eq. 4, 5 and 6 requires bounds and linear restrictions to ensure for the process σ_t^2 being positive, i.e. $\alpha_k \geq 0$ and $\phi_k \geq 0$, $\phi_k - \frac{(3+\nu)}{\nu}\alpha_k > 0$. Instead of model 3 we will use the score driven beta-t-EGARCH model

$$r_t = e^{\frac{1}{2}\lambda_t}\epsilon_t \tag{7}$$

$$\lambda_t = \omega + \sum_{k=1}^{p}\alpha_k s_{t-k} + \sum_{k=1}^{q}\phi_k\lambda_{t-k} \tag{8}$$

$$p(r_t|\sigma_t^2) = \frac{\exp(-\frac{1}{2}\lambda_t)}{Beta(\frac{1}{2}, \frac{\nu}{2})\sqrt{\nu}}\left(1 + \frac{r_t^2}{\nu\exp(\lambda_t)}\right)^{-\frac{\nu+1}{2}} \tag{9}$$

$$s_t = \frac{1}{2}\left(\frac{(\nu+1)\nu^{-1}r_t^2\exp(-\lambda_t)}{1+\nu^{-1}r_t^2/\exp(-\lambda_t)} - 1\right) \tag{10}$$

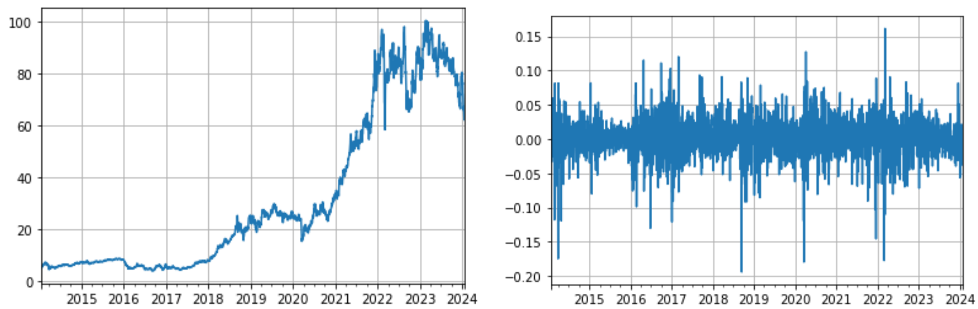
$$S_t = \mathcal{I}^{-1} = \frac{2(3+\nu)}{\nu}, \tag{11}$$

where $\lambda_t = \log(\sigma_t^2)$, $Beta(1/2, \nu/2)$ is the beta function evaluated at $1/2$ and $\nu/2$ and $\mathcal{I}$ is the Fisher information matrix. The model expressed by Eq. 7, 8, 9 10, and 11 has been shown to outperform the regular GARCH and GARCH-t models (Harvey et al. 2014, Blazsek et al. 2016, Catania et al. 2020, Artemova et al. 2022).

1.6 Energy commodity market

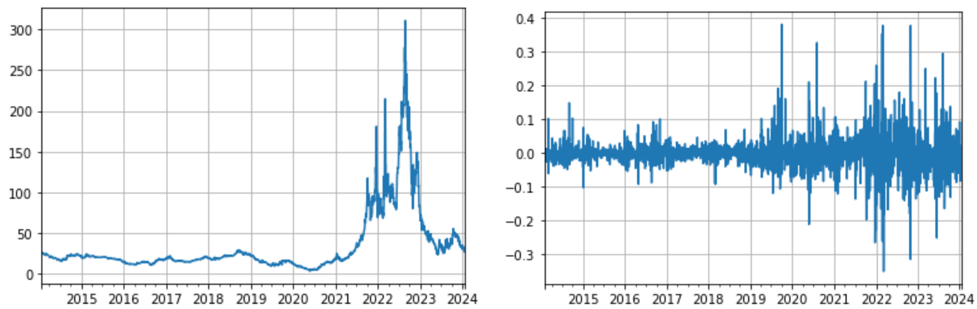
The objective of analyzing the correlation among these commodities is to pinpoint a potential indicator for transition risk. Leading the list of contenders is the Carbon Allowances Futures (ECF), which reflects the cost linked to CO_2 emissions. A positive correlation between ECF and other energy commodities would imply a substantial connection between emissions and the fundamental economic activities of developed nations. As a result, companies tied to these costs would encounter increased exposure to transition risk. Next, we will delve into the relationship between futures of energy commodities:

1. Coal ICE API2 CIF ARA Nr Mth $/MT (COAL hereafter), Figure 4
2. RFV Natural Gas TTF NL 1st Fut. Mth (TTF hereafter), Figure 3
3. Crude Oil Brent ICE M1 UK 1200 hrs (Brent hereafter), Figure 5
4. ICE EUA Yearly Energy Future (ECF hereafter), Figure 2



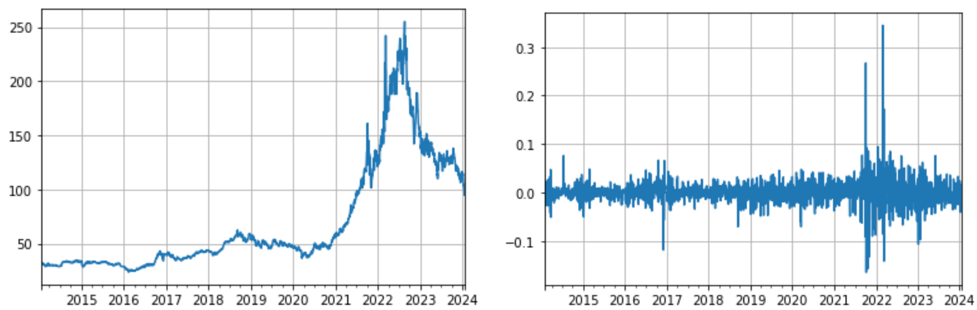
(a) ECF prices (b) ECF log-returns

Figure 2: ICE EUA Yearly Energy Future



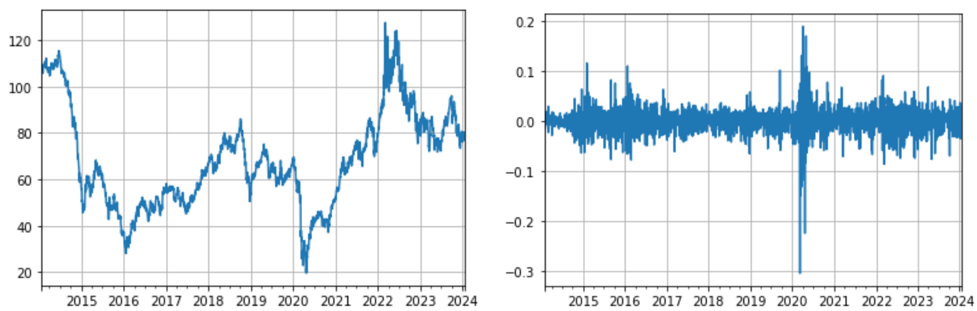
(a) TTF prices (b) TTF log-returns

Figure 3: RFV Natural Gas TTF NL 1st Fut. Mth



(a) Coal prices (b) Coal log-returns

Figure 4: Coal ICE API2 CIF ARA Nr Mth $/MT



(a) Brent prices (b) Brent log-returns

Figure 5: Crude Oil Brent ICE M1 UK 1200 hrs

Commodity	Statistic	P-value
ECF	-21.7019	0
TTF	-9.1323	0
Coal	-12.7371	0
Brent	-9.8674	0

(a) Augmented Dickey-Fuller test for energy commodities

Commodity	Statistic	P-value
ECF	369.6412	0
TTF	790.4517	0
Coal	1505.9755	0
Brent	808.5559	0

(b) D'Agostino and Pearson's test for energy commodities

Table 1

Table 1a reports the Augmented Dickey-Fuller test performed on log returns of energy commodities; the test rejects the null hypothesis of non-stationarity. Table 1b reports the D'Agostino and Pearson's test performed on log returns of energy commodities; the test rejects the null hypothesis of normally distributed returns. We start by modeling the time series of log-returns with AR(0) and AR(1) models with score-driven stochastic volatility as in Eq. 8. So, the model will be:

$$r_t = \mu + \beta r_{t-1} + e^{\frac{1}{2}\lambda_t}\epsilon_t \tag{12}$$

$$\lambda_t = \omega + \sum_{k=1}^{p} \alpha_k s_{t-k} + \sum_{k=1}^{q} \phi_k \lambda_{t-k} \tag{13}$$

$$p(y_t|\sigma_t^2) = \frac{\exp(-\frac{1}{2}\lambda_t)}{Beta(\frac{1}{2}, \frac{\nu}{2})\sqrt{\nu}} \left(1 + \frac{y_t^2}{\nu \exp(\lambda_t)}\right)^{-\frac{\nu+1}{2}} \tag{14}$$

$$s_t = \frac{1}{2}\left(\frac{(\nu+1)\nu^{-1} y_t^2 \exp(-\lambda_t)}{1 + \nu^{-1} y_t^2 / \exp(-\lambda_t)} - 1\right) \tag{15}$$

$$S_t = \mathcal{I}^{-1} = \frac{2(3+\nu)}{\nu}, \tag{16}$$

where $y_t = r_t - \mu - \beta r_{t-1}$ and ϵ_t is a martingale difference sequence with standard Student-t distribution with ν degrees if freedom. The estimation of the parameters for the score-driven models has been done via maximum-likelihood technique (Blasques et al. 2014)

$$l(y_t|\theta) = -\frac{1}{2}\lambda_t - \log\left(Beta\left(\frac{1}{2}, \frac{\nu}{2}\right)\right) - \frac{1}{2}\log(\nu) - \frac{-\nu+1}{2}\log\left(1 + \frac{y_t^2}{\nu\exp(-\lambda_t)}\right) \tag{17}$$

$$\arg\max_{\theta\in\Theta} l(y_t|\theta), \tag{18}$$

where θ is the vector of parameters. The solution of the problem 18 are reported in Table 3

Parameter	ECF	TTF	Coal	Brent
μ	0.000904	-0.000698	0.000456	-0.000151
ω	-3.590503	-0.289547	-4.062835	-5.060092
α_1	0.106137	0.157852	0.110176	0.096789
ϕ_1	0.536742	0.961114	0.559774	0.300335
ν	4.972422	4.421094	2.398765	4.002215
loglikelihood	5824.4814	5353.1128	7022.2451	6061.1813

Table 2: Maximum likelihood estimates AR(0) score tEGARCH(1,1)

Parameter	ECF	FFT	Coal	Brent
μ	0.000592	0.001707	0.000107	0.000179
ω	-4.646901	-0.584501	-2.613796	-5.059639
α_1	0.146892	0.207759	0.201986	0.109379
ϕ_1	0.400889	0.921003	0.718884	0.298071
β_1	-0.046909	0.013861	-0.021439	-0.000047
ν	4.361597	4.871364	1.868990	3.003859
loglikelihood	5820.7983	5323.1596	7037.757	6016.0204

Table 3: Maximum likelihood estimates AR(1) score tEGARCH(1,1)

Table 2 reports the estimates of the model described by equations 7, 8, 9, 10, and 11, while Table 3 reports the estimates of the model described by equations 12, 13, 14, 15, and 16. To choose between the models we perform the likelihood ratio test. Let l_0 be the loglikelihood function for the model AR(0) score tEGARCH(1,1), and let l_1 be loglikelihood function for the model AR(1) score tEGARCH(1,1), then the likelihood ratio statistic is $LR(l_0, l_1) = -2(l_0 - l_1)$ which is asymptotically distributed with a $\chi^2(1)$.

1. $H_0 : LR(l_0, l_1) = 0$
2. $H_1 : LR(l_0, l_1) > 0$

	TTF	ECF	Coal	Brent
$LR(l_0, l_1)$	-7.366325	-59.906370	31.023761	-90.321840
P-value	1	1	0	1

Table 4: Likelihood ratio test energy commodities

The results of the test reported in Table 4, show that for the TTF, the ECF and the Brent the AR(0) score t-EGARCH(1,1) proved to be the better model, while for the Coal the AR(1) score t-EGARCH(1,1) proved to better at 1% level of significant. For the ECF, TTF, and Brent model described by Eq. 7, 8, 9 10, and 11 was enough to remove all the autocorrelation from the residuals (Fig. 40a, 40b, 40d). However, in the case of Coal, both the AR(0) and AR(1) models with score-driven t-EGARCH(1,1) volatility weren't able to remove the autocorrelation from the residuals, making statistical tests unreliable. Given the structure of the autocorrelation function for both AR(0) and AR(1) (Fig. 40c, 41c), the returns of the coal will be modeled with a MA(1) with score-driven t-EGARCH(1,1) model:

$$r_t = \mu + \eta_1 \epsilon_{t-1} + e^{\frac{1}{2}\lambda_t}\epsilon_t \tag{19}$$

$$\lambda_t = \omega + \sum_{k=1}^{p} \alpha_k s_{t-k} + \sum_{k=1}^{q} \phi_k \lambda_{t-k} \tag{20}$$

$$p(y_t|\sigma_t^2) = \frac{\exp(-\frac{1}{2}\lambda_t)}{Beta(\frac{1}{2}, \frac{\nu}{2})\sqrt{\nu}} \left(1 + \frac{y_t^2}{\nu \exp(\lambda_t)}\right)^{-\frac{\nu+1}{2}} \tag{21}$$

$$s_t = \frac{1}{2}\left(\frac{(\nu+1)\nu^{-1}y_t^2 \exp(-\lambda_t)}{1 + \nu^{-1}y_t^2/\exp(-\lambda_t)} - 1\right) \tag{22}$$

$$S_t = \mathcal{I}^{-1} = \frac{2(3+\nu)}{\nu}. \tag{23}$$

Such a model removes the serial autocorrelation from the coal log-returns (Fig. 42) making the estimates (5) reliable.

	μ	η_1	ω	α_1	ϕ_1	ν
Coal	-0.005016	-0.000038	-1.165828	0.572274	0.848452	5.006709
Log-likelihood				6608.99353		

Table 5: Estimates MA(1) score t-EGARCH(1,1) Coal

The results in Table 18 in Appendix A.1 provide compelling insights into the dynamics of European Carbon Futures (ECF) in relation to various commodities. It becomes evident that, since 2015 (Paris Agreement), the ECF exhibits a positive correlation with the price of natural gas (TTF), a trend consistently observed across the examined period. Moreover, beginning in 2016, this correlation extends to coal, echoing findings documented in prior literature. In contrast, the relationship between ECF and Brent crude oil appears without any sign of a statistically significant correlation, distinguishing it from the patterns observed with natural gas and coal. This discrepancy prompts further exploration into the underlying factors shaping these dynamics. The observed positive correlation between ECF, TTF, and coal can be elucidated by examining the interplay of energy demand dynamics. Notably, the surge in energy demand precipitates a corresponding increase in the consumption of natural gas and coal, thereby driving up emissions levels. Consequently, there arises a heightened demand for carbon allowances to offset these emissions, thus perpetuating a cyclical relationship between energy demand, emissions, and carbon markets. This mechanism underscores the importance of considering the price of carbon allowances as a pivotal factor in assessing transition risks. Corporations less susceptible to fluctuations in emission costs are poised to exhibit greater resilience in navigating the challenges posed by the ongoing green transition. Hence, understanding and mitigating such risks becomes imperative for organizational sustainability and long-term viability in a rapidly evolving environmental landscape.

1.7 Stock market

To test the impact of the climate variable on the stock market we will follow a multi-step approach. In the first step, we check those stocks with statistically significant autocorrelation (via Ljung-Box test statistics) in the residuals of a regression with no mean component (so an AR(0) structure for the mean). Then we discarded those stocks with no autocorrelation since in that case there is nothing in the past that explains the log-returns. In the second step, the whole sample of stocks was reduced from 292 to 233 stocks.

In the third step, we compare via likelihood ratio test of the following two models:

$$r_t = \mu + \beta_1 r_{t-1} + e^{-\lambda_t}\epsilon_t \tag{24}$$

$$r_t = \mu + \beta_1 r_{t-1} + \sum_{i=1}^{2} \eta_i X_{i,t-1} + e^{-\lambda_t}\epsilon_t \tag{25}$$

$$\lambda_t = \omega + \alpha_1 s_{t-1} + \phi_1 \lambda_{t-1}, \tag{26}$$

A pure AR(1) model, Eq. 24, against an AR(1) with exogenous variables X_1 (the MSCI index) and X_2 (the VIX index), Eq. 25. The estimates for the AR(1) t-EGARCH(1,1) model, Eq. 24 are reported in Table 24, while the estimates for the AR(1) t-EGARCH(1,1) with exogenous variables Eq. 25 are reported in Table 25. We compare the estimates only for those stocks whose T-test rejects the null hypothesis (i.e. that the regressor was statistically insignificant in explaining the stock log-returns) at 5% of significant level. The results of the likelihood ratio test are reported in Table 22. For 92 stocks the null hypothesis was rejected, meaning that the parametrization with two exogenous variables was better than the one with just the autoregressive component. In Appendix 30 reports the Ljung-Box test for serial autocorrelation of the square residuals, showing that accepting the null hypothesis of absence of correlation.

In the last step, we tested via likelihood-ratio test, at 5% significant level, on the 92 stocks found in the previous step, the model described by Eq. 25 against the model:

$$r_t = \mu + \beta_1 r_{t-1} + \sum_{i=1}^{3} \eta_i X_{i,t-1} + e^{-\lambda_t}\epsilon_t \tag{27}$$

$$\lambda_t = \omega + \alpha_1 s_{t-1} + \phi_1 \lambda_{t-1}, \tag{28}$$

with exogenous variables X_1 (the MSCI index) and X_2 (the VIX index), and X_3 are the log-returns of the ECF. The results of the test are reported in Table 23. Among the remaining 92 stocks only 22 of them, the log-returns of the ECF were a statistically significant regressor, meaning that just only for the $7,5\%$ of a sample of 292 stocks, moreover for those stocks with statistically relevant coefficients, such coefficients were smaller than 10^{-4}, meaning that considering the information in the ECF market as a transition risk driver is not supported by data.

The cases when $X_3 = CR_t^m$ is variable proposed by Blasberg et al. 2021 are reported in tables 27 and 28 were in the first case the $X_3 = CR_t^{10y}$ and in the second $X_3 = CR_t^{20y}$. Anyway, in both cases the CR_t^m index wasn't statistically significant at any level (T-tets). So, the previous model has shown that both the ECF log-returns and the CR_t^i index proposed by Blasberg et al. 2021 aren't good regressors for stock returns. Then, the last check would be to see if there is any type of dependence between the noise of the ECF model (Eq. 7) and the correct specification of the stock data based on the previous estimates of the models, i.e.

$$\begin{aligned} r_t &= e^{\frac{1}{2}\lambda_t}\epsilon_t^{ecf} \\ r_t &= \mu_t + e^{\frac{1}{2}\lambda_t}\epsilon_t^{stock} \\ &KT(\epsilon_t^{ecf}, \epsilon_t^{stock}), \end{aligned}$$

where KT is the Kendall-Tau correlation index. The choice for the Kendall-Tau is due to the fact that financial returns aren't Gaussian and the Kendall-Tau is a non-parametric index. Contrary to the previous results, in this case the noise of the ECF log-returns has proven to be statistically related to the noise stock log-returns, in fact, it is found that 159 over 292 have a significant statistical Kenadll-Tau index, so around 54% of them show to be related to ECF.

From Figure 6 we can see that the Kendall-Tau is usually positive meaning that stocks and ECF are concordant but the values are not very high (most of them are in the order of 10^{-2}). Unfortunately, this result cannot be evidence for the ECF being a transition risk measure, the observed effect is mainly due to economic activity, meaning that if the economy is growing, corporations produce more and make higher profits, resulting in a higher demand for Carbon Allowances which lead to an increase of the price concordant to a price increase of the stocks. On the contrary, if the economy is depressed corporations produce less, and make fewer profits resulting in a decrease in stock prices, and the lower levels of production mean reduced demand for Carbon Allowances which reduce the Carbon price.

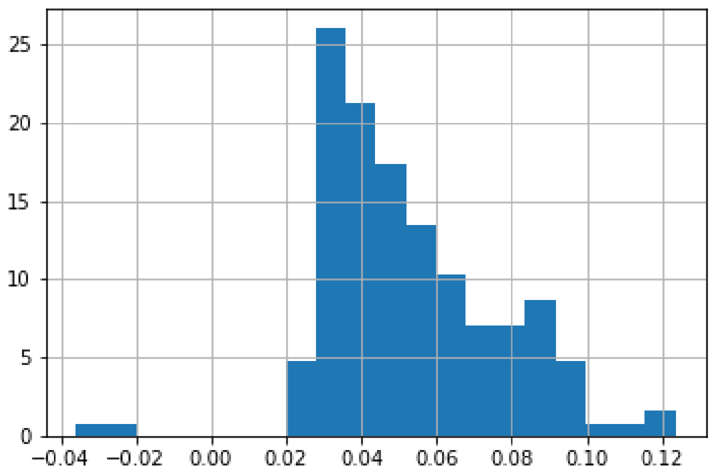


Figure 6: Kendall-Tau distribution

So, in this section, we have seen that both the ECF log-returns aren't a significant regressor for stock returns, and so removing such variables from the candidates as a transition risk factor; only for 22 stocks over a sample of 292 (around 7.5%) belonging to sectors that have to cover their emissions with Carbon Allowances, we found a statistically significant relationship between stock returns and ECF returns. With so few numbers of stocks, it is reasonable to think that the ECF is not a transition risk driver even for phases 3 and 4 at the moment. The index proposed by Blasberg et al. 2021 has had an even worse performance than ECF, since it was never statistically significant. Moreover, in my opinion, beyond the statistical significance, such an index has two main problems: the first one is that it doesn't have an economic theory behind it i.e. the fact that such a positive spread is the transition risk is a sort of assumption, that can be meaningfully, but still an assumption. So, the absence of a theory behind it, that cannot be tested if it is true or not, wouldn't allow us to discern if it is transition risk or a different kind of economic relationship. The second problem is related to the fact that such a measure deeply depends on the choice of the CDS used in the construction of the index, namely that analysts with different data could get different results but still have a correct transition risk index. So, in our case, we used every CDS to which we had access in our database but still, we cannot be sure if the index is correct.

1.7.1 Dependence in the extreme

From the results reported in the previous section, it is clear that the ECF log-returns, the CR_t^{10y}, and the CR_t^{20y} aren't good variables for explaining the impact of the transition risk in the mean. However, the analysis presented so far cannot say much about the impact of such variables in the extreme. To test for the impact of such variables on the stock log-returns we perform quantile regression analysis[10] (Koenker et al. 1978)

$$\arg\min_{\mu,\beta} \left\{ q \sum_{r_t \geq \mu + Z_t\beta} |r_t - \mu + Z_t\beta| + (1-q) \sum_{r_t < \mu + Z_t\beta} |r_t - \mu + Z_t\beta| \right\}, \tag{29}$$

where $q = 0.05, 0.025, 0.01$ is the quantile of interest, r_t are the stock log-returns, μ is the intercept, Z_t is a vector of regressors, which are, r_{t-1}, the MSCI, the VIX, and ECF/CR^{10y}/CR^{20y}, and β is the vector of coefficients for the regressors Z_t. In Appendix A.2.3, are reported only the estimates for those stocks whose parameters were at least significant at 5%. Table 31 reports the 5% quantile regression with the ECF as transition risk variable; we can see that only for 22 stocks, over a sample of 292, the 5% quantile was statistically dependent of the ECF, while, for both, the 2.5% quantile and 1% quantile the ECF was never significant. Similar results hold also when replacing the ECF with the CR^{10y} and CR^{20y} index proposed by Blasberg et al. 2021. In the case of the CR^{20y}, again only for 22 stocks the 5% quantile showed a statistically significant relationship with the transition risk variable (Table 35), while, for both, the 2.5% quantile and 1% quantile the CR^{20y} was never significant. In the case of the CR^{10y} index we got that for the 5% quantile 22 stocks showed a statistically significant relationship with the CR^{10y} index (Table 32), 18 stocks showed a statistically significant relationship with the CR^{10y} index (Table 33), and 14 stocks showed a statistically significant relationship with the CR^{10y} index (Table 34). It is worth noticing that the group of stocks for each regression is varying, i.e. the stocks whose 5% quantile depends on the CR^{10y} is not the same for 2.5% still considering the CR^{10y}. This fact, combined with the very small number of stocks with a statistically significant relationship, suggests that the three variables, ECF, CR^{10y}, and CR^{20y} aren't a good transition risk proxy.

[10] For the quantile regressions we relied on the Python package Statsmodels `https://github.com/statsmodels/statsmodels/`

2 Climate risk and bond pricing

2.1 Introduction

In 2020, the Authority began a pilot project with 29 volunteer banks to assess their exposure to climate risk and evaluate the overall stability of the financial system. The results, released in 2021, noted that the methods used were just a starting point and would need improvement. The report highlighted the need for better ways to measure climate risk and called for more attention to the amount of green financial instruments in bank portfolios. It also stressed the importance of understanding how climate risk impacts banks' balance sheets. These points were echoed in the 2022 EBA climate risk stress test report. Therefore, in this chapter, we analyze green and non-green bonds to understand better if climate variables are statistically significant for explaining the excess of returns in bond markets due to physical climate risk, while for the climate transition risk, we have tested if the returns of the carbon allowances are statistically significant for explaining the excess of returns of bonds, green and non-green. In Bartolini et al. 2024 the econometric analysis was performed with an ARMA model with exogenous variables with GARCH-type volatility and assuming a Student-t distribution since returns rejected the hypothesis of being normally distributed. In line with the previous literature on credit default swap (Livieri et al. 2023), it was found that the carbon allowance returns weren't statistically significant drivers for the returns of bonds meaning that even though they represent the cost for emissions, that ends up in balance sheet. This means that carbon allowances aren't the variable responsible for measuring the impact of climate transition risk. On the contrary climate variables seem to have a statistically significant impact on many bond returns meaning that the market is aware and is pricing the climate risk physical risk. For the statistical results, we refer to Bartolini et al. 2024. Despite the abundance of econometric analyses using different approaches, there are few methods that include climate risk in bond pricing. To our knowledge, only Agliardi et al. 2021 and Livieri et al. 2023 have seriously addressed this issue. Our goal is to fill this gap with a methodology that differs from those of Agliardi et al. 2021 and Livieri et al. 2023. Developing bond pricing techniques is important because these models allow for a better assessment of model sensitivity to parameters and different components, such as external factors. We aim to propose a pricing technique that evaluates the impact of external climate risk factors on bond pricing and assesses a corporation's resilience to climate risks. This will enable us to rank corporations based on their exposure to climate risk. Moreover our approach will directly link the probability of default of a corporation with the climate factors making them easy to interpret.

Then, the chapter will be devoted to how to include in pricing models the climate risk factors. To do so we will employ the methodology based on stochastic intensity rates models introduced by Duffie et al. 1999. The procedure for including climate variables in pricing models is divided into two parts. First, we perform an econometric analysis to identify the variables that explain internal rates of return. Then, we calibrate two pricing models: one without explanatory variables and one with them, and test whether the second model performs better than the first. This is necessary because econometric models typically explore linear relationships between a variable and its regressors, while pricing models are usually highly non-linear. As a result, the outcomes can sometimes differ. We chose reduced-form models (see Duffie et al. 1999 and Section 2.3.1) to price risky zero-coupon bonds for two main reasons. First, this approach is flexible and makes no assumptions about a corporation's debt structure, unlike structural models (see Merton 1974 and Section 2.3.2). Second, it encapsulates all relevant market information about a corporation (e.g., profitability, assets, liabilities) into a stochastic process, the hazard rate, which drives the probability of default. This framework also allows the direct inclusion of external factors, such as how they impact default probabilities. Additionally, the structure introduced in Duffie et al. 1999 supports a wide range of stochastic processes to model the hazard rate, while still producing semi-analytic formulas. These formulas are typically smooth and easy to handle numerically, with minimal error and computational time. In Section 2.3.3, we will propose a model where the stochastic hazard rate is a linear combination of firm-specific and climate-related external factors. Using the Lévy-Khintchine formula, we can obtain a semi-analytic solution for pricing risky zero-coupon bonds. Furthermore, in Section 2.3.3, we will show that our approach allows for the calculation of the long-term impact of climate variables. While the estimates themselves do not directly indicate a firm's sensitivity to climate change, if sufficient sectoral data is available, analysts can calculate betas and long-term impacts for any corporation in the sector, ranking firms based on their relative performance. Unfortunately, we lacked enough sectoral data to perform this analysis, but this is how the model is intended to be used. In the final part of this chapter, we will also demonstrate that our model outperforms one that does not incorporate external factors. Moreover, we will assess the time-dependent nature of the beta factor by estimating it monthly. However, we chose not to give it a parametric structure to avoid complicating the estimation process. A parametric structure would have made the pricing formulas more complex, leading to a significantly more difficult calibration. This version maintains the meaning of the original text while improving clarity, reducing repetition, and simplifying the language.

2.2 Literature review

Previous work on climate risk includes Allman 2021, Agliardi et al. 2021, Bats et al. 2023, Bolton and Kacperczyk 2019, and Po-Hsuan et al. 2023. Allman 2021 focuses on one indicator of physical risk, available in the United States: the Sea Level Rise (SLR) index. The risk is found to indeed be priced by the market: corporate bonds of companies with greater exposure to SLR risk bear a climate risk premium upon issuance. Furthermore, the premium is larger for geographically concentrated firms. Our work considers different physical risk indicators, with the selection being based on the previous academic literature, on the EBA Climate Stress Tests and Exercises, and on the Climate Risk Landscape reports by the United Nations Environment Programme Finance Initiative. For each variable, the granularity of the available data is also a factor in its selection, with preference given to those available at the higher frequency.

Agliardi et al. 2021 propose a structural credit-risk model incorporating both uncertainty about earnings and uncertainty due to climate risks. The theoretical framework derives explicit expressions for bond prices from balance sheet values impacted by sudden climate policy shocks. They also study the interplay among the various risk drivers. Our proposed framework also leads to the adaptation of a credit risk model, in our case intensity-based, but results from the formalization of a time-series analysis on corporate bond spreads, used to identify the relevant risk factors. The choice of the intensity-based model is motivated by the frequency of the data in our study, which is well-suited for a credit risk model calibrated on daily bond prices, such as the intensity-based one, instead of balance sheet items, such as the structural one.

Bats et al. 2023 study climate risk premia in Euro area corporate bond markets. As gauges of climate risk, they use text-based indices based on news content. They find that physical risk is significantly priced in corporate bonds with longer-term maturities. It is also found in shorter-term maturities, but for the latter the premium is smaller and less significant. Our work directly uses weather variables and risk indicators, provided by European data services (such as the European Drought Observatory and the Copernicus Data Service), and translates the results into an updated credit risk model.

Transition risk has also been found to be a potential explanatory variable of financial returns, with Bolton and Kacperczyk 2019 and Po-Hsuan et al. 2023 focusing on the equity market. Both works aim to gauge whether stock prices reflect investors' demand for compensation for exposure to carbon emission risk. The variables under study are carbon dioxide emissions, in Bolton and Kacperczyk 2019, and a wider measure of toxic emissions on the part of firms, in Po-Hsuan et al. 2023. Both papers find that the stocks of companies with higher emissions earn higher returns: a carbon premium that cannot be explained by traditional risk factors. For this reason, a potential proxy of transition risk is also included in our work. This expands the analysis beyond the equity market, and evaluates the potential impact on fixed-income instruments. Therefore, we aim to contribute to the existing literature by investigating the relationship between transition risk and corporate bond spreads, and whether it is consistent with the results relating to the stock market.

Blasberg et al. 2021 used Credit Default Swap spreads, for constructing a forward-looking, market-implied carbon risk factor and show that carbon risk affects firms' credit spread. The effect is larger for European than North American firms and varies substantially across industries, suggesting the market recognizes where and which sectors are better positioned for a transition to a low-carbon economy. They studied how carbon risk affects firms' creditworthiness, and found a positive relationship between lenders' perceived exposure to carbon risk and firms' cost of default protection. The relevance of the observed relationship is significantly stronger in Europe than in North America. In addition, using quantile regression, they found that the magnitude of the exposure to carbon risk differs considerably along the entire distribution of CDS spread returns.

Livieri et al. 2023 derived formulas for the pricing of defaultable coupon bonds and Credit Default

Swaps to empirically demonstrate that a jump-diffusion credit risk model in which the downward jumps in the firm value are due to tighter green laws can capture, at least partially, the transition risk.

2.3 Methodology

2.3.1 The intensity-based model

In this section, we report the main theoretical results, developed by Duffie et al. 1999 for the intensity-based models, that allow us to recover the pricing formula in Eq. (49) as a particular case. An extended exposition of this methodology can be found in Rutkowski et al. 2004 and McNeil et al. 2015. The strength of the methodology relies on the fact that it does not require any assumption about the nature of the firm's liabilities, as opposed to the structural model of Merton 1974. The approach in Duffie et al. 1999 is based on the modeling of the stochastic hazard rate that drives the survival probability of a firm, and the only assumption required is that it must be a positive process.

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space and let $\mathcal{F}_t = \sigma(\{\psi_t : s \leq t\})$ be the filtration generated by some observed background process. Define the random time τ on $\mathcal{F}$, with $\tau > 0$ a.s., and denote by $Y_t = \mathbf{1}_{\{\tau \leq t\}}$ the associated jump indicator and by $\mathcal{H}_t = \sigma\{\mathbf{1}_{\{\tau \leq s\}} : s \leq t\}$ the filtration generated by Y_t. Then, for our purposes, define the general filtration as

$$\mathcal{G}_t = \mathcal{F}_t \vee \mathcal{H}_t,$$

where τ is a stopping time with respect to $\mathcal{G}_t$ and $\mathcal{H}_t$, but not necessarily with respect to $\mathcal{F}_t$.

Definition 2.1. A random time τ is said to be doubly stochastic if there exists a positive $\mathcal{F}_t$-adapted process γ_t, such that $\Gamma_t = \int_0^t \gamma_s ds$ is strictly increasing and finite for every $t > 0$ and such that, for all $t \geq 0$,

$$\mathbb{P}(\tau > t | \mathcal{F}_\infty) = e^{-\int_0^t \gamma_s ds}. \tag{30}$$

In such a case, γ_t is referred to as the $\mathcal{F}_t$-conditional hazard process of τ.

Lemma 2.1. For every $t \leq 0$, the following statement holds:

$$\mathcal{G}_t^* = \{A \in \mathcal{G}_t : \exists B \in \mathcal{F}_t, A \cap \{\tau > t\} = B \cap \{\tau > t\}\}.$$

This lemma states that, before the default time, the only known events are those related to the background filtration $\mathcal{F}_t$, from which derives the following lemma.

Lemma 2.2. Let τ be a random time (not necessarily doubly stochastic) such that $P(\tau > t|\mathcal{F}_t) > 0$ for all $t \leq 0$, then, for every integrable random variable:

$$\mathbb{E}^{\mathbb{Q}}[\mathbf{1}_{\{\tau > t\}} X | \mathcal{G}_t] = \mathbf{1}_{\{\tau > t\}} \frac{\mathbb{E}^{\mathbb{Q}}[\mathbf{1}_{\{\tau > t\}} X | \mathcal{F}_t]}{\mathbb{Q}(\tau > t | \mathcal{F}_t)}. \tag{31}$$

Corollary 2.2.1. Let $T > t$ and assume τ is doubly stochastic with hazard process γ_t if then $\tilde{X}$ is integrable and $\mathcal{F}_T$ measurable, then:

$$\mathbb{E}^{\mathbb{Q}}\{\tilde{X}\mathbf{1}_{\{\tau > T\}} | \mathcal{G}_t\} = \mathbf{1}_{\{\tau > t\}} \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_t^T \gamma_s ds} \tilde{X} \middle| \mathcal{F}_t\right] \tag{32}$$

Theorem 2.3. Suppose that, under $\mathbb{Q}$, τ is doubly stochastic with background filtration $\mathcal{F}_t$ and hazard process γ_t. Define $R_s = r_s + \gamma_s$ and assume that the following random variables are integrable with respect to $\mathbb{Q}$:

1. $e^{-\int_t^T r_s ds}|X|$,
2. $\int_t^T |v_s| \exp\{\int_t^s r_u du\} ds$,
3. $\int_t^T |Z_s \gamma_s| \exp\{\int_t^s R_u du\} ds$,

where v_s is a continuous dividend, Z_τ is the value of the claim at the time of the default and γ_t is the stochastic hazard rate. Then, the following hold:

$$\mathbb{E}^{\mathbb{Q}}\left[\mathbf{1}_{\{\tau > T\}} e^{-\int_t^T r_s ds} X \middle| \mathcal{G}_t\right] = \mathbf{1}_{\{\tau > t\}} \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_t^T R_s ds} X \middle| \mathcal{F}_t\right], \tag{33}$$

$$\mathbb{E}^{\mathbb{Q}}\left[\int_t^T \mathbf{1}_{\{\tau > s\}} v_s e^{-\int_t^s r_u du} ds \middle| \mathcal{G}_t\right] = \mathbf{1}_{\{\tau > t\}} \mathbb{E}^{\mathbb{Q}}\left[\int_t^T v_s e^{-\int_t^s R_u du} ds \middle| \mathcal{F}_t\right], \tag{34}$$

$$\mathbb{E}^{\mathbb{Q}}\left[\mathbf{1}_{\{t < \tau \leq T\}} e^{-\int_t^\tau r_s ds} Z_\tau \middle| \mathcal{G}_t\right] = \mathbf{1}_{\{\tau > t\}} \mathbb{E}^{\mathbb{Q}}\left[\int_t^T Z_\tau \gamma_s e^{-\int_t^s R_u du} ds \middle| \mathcal{F}_t\right]. \tag{35}$$

Theorem 2.3[11] allows us to recover the explicit formula for the zero coupon bond with the further assumption of conditional independence between each hazard rate component and the short rate and with the assumption that the payment in case of default will happen at maturity.

2.3.2 The structural model

Let's consider the Merton model (Merton 1974), such model is the prototype of all firm-value models. Consider a firm whose asset value follows some stochastic process (S_t). The firm finances itself by equity (i.e. by issuing shares) and by debt. In Merton's model, debt consists of zero-coupon bonds with common maturity T; the nominal value of debt at maturity is given by the constant B. Moreover, it is assumed that the firm cannot pay out dividends or issue new debt. The values at time t of equity and debt are denoted by S_t and B_t. Default occurs if the firm misses a payment to its debtholders, which in the Merton model can occur only at the maturity T of the bonds. To sum up, the main assumptions of the model are:

1. The debt is only financial type debt so it implies that debts towards suppliers are zero, i.e. instantaneous payment for goods and services, or are assumed to be equivalent to financial debts.
2. The risk-free interest rate is deterministic and equal to $r \geq 0$.
3. The firm's asset-value process (V_t) is independent of the way the firm is financed, and in particular it is independent of the debt level B.
4. The asset value (V_t) can be traded on a frictionless market, and the asset value dynamics are given by the geometric Brownian motion $dV_t = \mu_V V_t dt + \sigma_V V_t dW_t$

Such assumptions allow us to express the stock price S_T and the debt level B_T at time T as:

1. $S_T = (V_T - B, 0)^+$
2. $B_T = B - (B - V_t)^+$

i.e. we have expressed the stock price and the debt value as the payoff of a European call option and a European put option, with which the assumption of Geometric Brownian Motion for V_t allows us to get:

$$S_t = C^{BS}(t, V_t; r, \sigma_V, B, T) = Vt\Phi(d_{t,1}) - Be^{-r(T-t)}\Phi(d_{t,2}), \tag{36}$$

$$B_t = Be^{-r(T-t)} - (Be^{-r(T-t)}\Phi(-d_{t,2}) - V_t\Phi(-d_{t,1})) \tag{37}$$

where

$$\begin{aligned} d_{t,1} &= \frac{\ln \frac{Vt}{B} + (r + \frac{1}{2}\sigma_V^2)(T-t)}{\sigma_V\sqrt{T-t}}, \\ d_{t,2} &= d_{t,1} - \sigma_V\sqrt{T-t}. \end{aligned} \tag{38}$$

Then, the default probability is:

$$P(V_T \leq B) = P(\ln V_T \leq \ln B) = \Phi\left(\frac{\log(B/V_t) - (\mu_V - \frac{1}{2}\sigma_V^2)(T-t)}{\sigma_V\sqrt{T-t}}\right) \tag{39}$$

Even though these assumptions allow us to recover nice formulas for the stock price, the debt value, and the default probability, they present a lot of critical issues, and usually, everyone focuses on the independence between the firm value V_t and the debt level B, which is questionable, because a very high debt level, and hence a high default probability, may adversely affect the ability of a firm to generate business, hence affecting the value of its assets. The second common critique is the fact that such a model is based on the availability of traded stocks for calibrating the model and then computing the default probability. But the most important issue with the Merton methodology is the structure of the firm's debt, i.e. the assumption that the debt is a zero-coupon bond and in this way treating debt towards suppliers and employees as a financial debt. When evaluating the firm's value the difference between financial and operational debt is crucial since usually they have different maturities and different balance sheet items in the asset part as a guarantee. The fact that in the Merton model there is this simplification is too much restrictive and unrealistic, so we have moved to intensity-based models that do not require anything like this.

[11]Details about the proofs can be found in McNeil et al. 2015 and Rutkowski et al. 2004

2.3.3 An intensity-based climate risk model

To further study the dependence between the default probabilities implied by risky bonds, on the one hand, and climate variables, on the other, we exploit the methodology based on stochastic hazard rates proposed by Duffie et al. 1999. As shown in Driessen 2005, this approach performs well when dealing with bond pricing that includes external factors, which corresponds to our setting. We thus propose an extension of it, to include physical risk factors. We assume that the payment to happen at a deterministic known time in case of default, which we assume to coincide with the maturity for simplicity.

Then, given a probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$, and letting γ_t be an $\mathcal{F}_t$-adapted process and τ the time of default as in definition 2.1 allow us to express the price of a risky zero coupon bond $P_R(0,T)$, with maturity T, as

$$\begin{aligned}
P_R(0,T) &= P_{RF}(0,T)\mathbb{E}^{\mathbb{Q}}[P(\tau > T|\mathcal{F}_\infty)|\mathcal{F}_0] + \delta P_{RF}(0,T)\mathbb{E}^{\mathbb{Q}}[P(\tau \leq T|\mathcal{F}_\infty)|\mathcal{F}_0] \\
&= P_{RF}(0,T)\mathbb{E}^{\mathbb{Q}}[P(\tau > T|\mathcal{F}_\infty)|\mathcal{F}_0] + \delta P_{RF}(0,T)(1 - \mathbb{E}^{\mathbb{Q}}[P(\tau > T|\mathcal{F}_\infty)|\mathcal{F}_0]) \\
&= P_{RF}(0,T)\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T \gamma_u du}\middle|\mathcal{F}_0\right] + \delta P_{RF}(0,T)\mathbb{E}^{\mathbb{Q}}\left[1 - e^{-\int_0^T \gamma_u du}\middle|\mathcal{F}_0\right],
\end{aligned} \tag{40}$$

where γ_t, $t \geq 0$, is the hazard rate, δ is the recovery rate, which we assume to be a constant, $P_{RF}(0,T)$ is the risk-free discount factor with maturity T, and $\mathbb{Q}$ is the risk-neutral measure equivalent to the physical risk measure $\mathbb{P}$. The change of measure is defined by the Radon-Nikodym derivative

$$\frac{d\mathbb{Q}}{d\mathbb{P}}\bigg|_T = \exp\Big\{-\int_0^T \pi_u dB_{0,u} - \frac{1}{2}\int_0^T \pi_u^2 du\Big\}, \tag{41}$$

where π_t is the market risk premium and $B_{0,t}$ is the Brownian Motion driving the firm-specific factor of the hazard rate process.

The stochastic hazard rate γ_t is a linear combination of the firm-specific factor, $\gamma_{0,t}$, modeled with Cox–Ingersoll–Ross (C.I.R) process (Duffie et al. 1999 and Driessen 2005) and which incorporates everything that is not explicitly modeled, and the physical risk proxies included in our study. These are the Fire Weather Index $\gamma_{1,t}$, the de-seasonalized average daily Eastward wind speed, $\gamma_{2,t}$, the de-seasonalized average daily Northward wind speed, $\gamma_{3,t}$, the flood index, $\gamma_{4,t}$ and the drought index, $\gamma_{5,t}$:

$$\gamma_t = \gamma_{0,t} + \beta_1\gamma_{1,t} + \beta_2\gamma_{2,t} + \beta_3\gamma_{3,t} + \beta_4\gamma_{4,t} + \beta_5\gamma_{5,t}, \tag{42}$$

where the parameters $\beta_i \in \mathbb{R}^+$, $i = 1, ..., 5$, measure the impact of the i-th factor on the survival probability.

The Fire Weather Index and the de-seasonalized average daily Eastward and Northward wind speed are modeled as Ornstein-Uhlenbeck processes driven by pure jump Lévy processes, as in Benth, Persio, et al. 2018 and Benth, Christensen, et al. 2021. The flood and drought indices, on the other hand, are modeled as C.I.R. processes. The choice of this type of process is due to the non-normality of the corresponding time series, while the reason for using pure jump Lévy processes for the fire and wind indices is the need to allow for a non-zero probability of the event when they have a value or an increment of zero.

Therefore, the dynamics of the Fire Weather Index and the de-seasonalized average daily Eastward and Northward wind speed are

$$\begin{aligned}
d\gamma_{i,t} &= -k_i\gamma_{i,t}dt + dL^{\mathbb{Q}}_{i,t}, \\
\gamma_{i,t} &= \gamma_{i,t_0}e^{-k_i(t-t_0)} + \int_{t_0}^t e^{-k_i(t-u)}dL^{\mathbb{Q}}_{i,u},
\end{aligned} \tag{43}$$

where $L^{\mathbb{Q}}_{i,t}$, with $i = 1,2,3$, are independent compound Poisson processes with intensity λ_i and exponential jump size of expected value η_i. The dynamics of the firm-specific factor, flood, and drought indices are represented by

$$d\gamma_{i,t} = k_i(\theta_i - \gamma_{i,t})dt + \sigma_i\sqrt{\gamma_{i,t}}dB^{\mathbb{Q}}_{i,t}, \tag{44}$$

where $B^{\mathbb{Q}}_{i,t}$, with $i = 0, 4, 5$, are independent Brownian Motions. We assume that the physical risk variables have the same dynamics under $\mathbb{P}$ and $\mathbb{Q}$, i.e. $L^{\mathbb{Q}}_{i,t} = L^{\mathbb{P}}_{i,t}$, $i = 1, 2, 3$, and $B^{\mathbb{Q}}_{i,t} = B^{\mathbb{P}}_{i,t}$, $i = 4, 5$.

For C.I.R. processes[12], the explicit solution of

$$\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_{t_0}^{t}\gamma_u du}\middle|\mathcal{F}_0\right],$$

which appears inside Eq. (40), is given by

$$\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^t \gamma_{i,u} du}\middle|\mathcal{F}_0\right] = \exp\{A_i(0,t) - C_i(0,t)\gamma_{i,t}\}, \tag{45}$$

where,

$$\begin{aligned}
C_i(0,t) &= \frac{2(\exp\{td_i\} - 1)}{2d_i + (k_i + d_i)(\exp\{td_i\} - 1)}, \\
A_i(0,t) &= \frac{2k_i\theta_i}{\sigma_i^2}\log\left\{\frac{2d_i\exp\{(k_i + d_i)t/2\}}{2d_i + (k_i + d_i)(\exp\{td_i\} - 1)}\right\}, \\
d_i &= \sqrt{k_i^2 + 2\sigma_i^2}.
\end{aligned} \tag{46}$$

On the other hand, for the pure-jump Lévy-driven Ornstein-Uhlenbeck process, the solution is given by

$$\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^t \gamma_{i,u} du}\middle|\mathcal{F}_0\right] = \exp\{H_i(0,t) + M_i(0,t)\gamma_{i,t}\}, \tag{47}$$

where,

$$\begin{aligned}
M_i(0,t) &= \frac{1}{k_i}\left(1 - e^{-k_i t}\right), \\
H_i(0,t) &= \int_0^t \lambda_i\left(\frac{\eta_i}{\eta_i + \frac{1}{k_i}(1 - e^{-k_i(t-s)})} - 1\right)ds.
\end{aligned} \tag{48}$$

The solutions for the C.I.R. dynamic and for the Lévy OU process, in Eq. (45) and (47) respectively, are then used in conjunction with Eq. (42) inside Eq. (40), to recover explicit formulas for model-implied zero coupon bond prices. Conditional independence is then assumed between each hazard rate component and the risk-free short rate. The resulting pricing equation for the risky zero coupon bond is:

$$\begin{aligned}
P_R(0,T) &= P_{RF}(0,T)\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T \gamma_u du}\middle|\mathcal{F}_0\right] + \delta P_{RF}(0,T)\mathbb{E}^{\mathbb{Q}}\left[1 - e^{-\int_0^T \gamma_u du}\middle|\mathcal{F}_0\right] \\
&= P_{RF}(0,T)\prod_{i=0,4,5}\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T \beta_i\gamma_{i,u} du}\middle|\mathcal{F}_0\right]\prod_{i=1,2,3}\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T \beta_i\gamma_{i,u} du}\middle|\mathcal{F}_0\right] \\
&+ \delta P_{RF}\left(1 - \prod_{i=0,4,5}\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T \beta_i\gamma_{i,u} du}\middle|\mathcal{F}_0\right]\prod_{i=1,2,3}\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T \beta_i\gamma_{i,u} du}\middle|\mathcal{F}_0\right]\right) \\
&= P_{RF}(0,T)\prod_{i=0,4,5} e^{A_i(0,T) - C_i(0,T)\gamma_{i,0}}\prod_{i=1,2,3} e^{H_i(0,T) - M_i(0,T)\gamma_{i,0}} \\
&+ \delta P_{RF}\left(1 - \prod_{i=0,4,5} e^{A_i(0,T) - C_i(0,T)\gamma_{i,0}}\prod_{i=1,2,3} e^{H_i(0,T) - M_i(0,T)\gamma_{i,0}}\right),
\end{aligned} \tag{49}$$

where A_i, C_i, H_i and M_i are defined in Eq. (46) and (48).

Proof. Under the complete filtration $\mathcal{G}_t$ and fixing $t = 0$, the price of the zero coupon bond is expressed as:

$$P_R(0,T) = \mathbb{E}^{\mathbb{Q}}\left[1_{\{\tau > T\}}e^{-\int_0^T r_u du}\middle|\mathcal{G}_0\right] + \delta\mathbb{E}^{\mathbb{Q}}\left[1_{\{0 < \tau \leq T\}}e^{-\int_0^T r_u du}\middle|\mathcal{G}_0\right].$$

[12] For details see Björk 2009

By Theorem 2.3, we have that

$$\begin{aligned}\mathbb{E}^{\mathbb{Q}}\left[1_{\{\tau>T\}}e^{-\int_0^T r_u du}\middle|\mathcal{G}_0\right] &= \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T (r_u+\gamma_u)du}\middle|\mathcal{F}_0\right] = \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T r_u du}\middle|\mathcal{F}_0\right]\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T \gamma_u du}\middle|\mathcal{F}_0\right]\\ &= P_{RF}(0,T)\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T \gamma_u du}\middle|\mathcal{F}_0\right],\end{aligned}$$

where in the last step we have used the assumption of conditional independence between the hazard rate components and the risk-free short rate.
By Lemma 2.2, we have that

$$\begin{aligned}\mathbb{E}^{\mathbb{Q}}\left[1_{\{0<\tau\leq T\}}e^{-\int_0^T r_u du}\middle|\mathcal{G}_0\right] &= \frac{\mathbb{E}^{\mathbb{Q}}\left[1_{\{\tau<T\}}e^{-\int_0^T r_u du}\middle|\mathcal{F}_0\right]}{\mathbb{Q}(\tau>0|\mathcal{F}_0)}\\ &= \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T r_u du}\mathbb{E}^{\mathbb{Q}}\left[(1-1_{\{\tau>T\}})\middle|\mathcal{F}_T\right]\middle|\mathcal{F}_0\right]\\ &= \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T r_u du}\middle|\mathcal{F}_0\right] - \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T r_u du}\mathbb{E}^{\mathbb{Q}}\left[(1_{\{\tau>T\}})\middle|\mathcal{F}_T\right]\middle|\mathcal{F}_0\right]\\ &= \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T r_u du}\middle|\mathcal{F}_0\right] - \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T r_u du}\mathbb{Q}(\tau>T|\mathcal{F}_T)\middle|\mathcal{F}_0\right]\\ &= \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T r_u du}\middle|\mathcal{F}_0\right] - \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T r_u du}e^{-\int_0^T \gamma_u du}\middle|\mathcal{F}_0\right]\\ &= \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T r_u du}\middle|\mathcal{F}_0\right] - \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T r_u du}\middle|\mathcal{F}_0\right]\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T \gamma_u du}\middle|\mathcal{F}_0\right]\\ &= \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T r_u du}\middle|\mathcal{F}_0\right]\left(1-\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T \gamma_u du}\middle|\mathcal{F}_0\right]\right)\\ &= P_{RF}(0,T)\mathbb{E}^{\mathbb{Q}}\left[1-e^{-\int_0^T \gamma_u du}\middle|\mathcal{F}_0\right],\end{aligned}$$

where in the last step we have again used the assumption of conditional independence between the risk-free short rate and the hazard rate components. Then, we find that

$$\begin{aligned}P_R(0,T) &= P_{RF}(0,T)\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T \gamma_u du}\middle|\mathcal{F}_0\right] + \delta P_{RF}(0,T)\mathbb{E}^{\mathbb{Q}}\left[1-e^{-\int_0^T \gamma_u du}\middle|\mathcal{F}_0\right]\\ &= P_{RF}(0,T)\prod_{i=0,4,5}\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T \gamma_{i,u} du}\middle|\mathcal{F}_0\right]\prod_{i=1,2,3}\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T \gamma_{i,u} du}\middle|\mathcal{F}_0\right]\\ &+ \delta P_{RF}\left(1-\prod_{i=0,4,5}\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T \gamma_{i,u} du}\middle|\mathcal{F}_0\right]\prod_{i=1,2,3}\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T \gamma_{i,u} du}\middle|\mathcal{F}_0\right]\right)\\ &= P_{RF}(0,T)\prod_{i=0,4,5}e^{A_i(0,T)-C_i(0,T)\gamma_{i,t}}\prod_{i=1,2,3}e^{H_i(0,T)-M_i(0,T)\gamma_{i,T}}\\ &+ \delta P_{RF}\left(1-\prod_{i=0,4,5}e^{A_i(0,T)-C_i(0,T)\gamma_{i,T}}\prod_{i=1,2,3}e^{H_i(0,T)-M_i(0,T)\gamma_{i,T}}\right),\end{aligned}$$

where the functions $A(0,T)$ and $C(0,T)$, as defined in Section **??**, are the well-known results for the C.I.R. model. Their proof can be found in the original paper, Cox et al. 2005.

$$\begin{aligned}C_i(0,T) &= \frac{2(\exp\{Td_i\}-1)}{2d_i+(\beta_i k_i+d_i)(\exp\{Td_i\}-1)},\\ A_i(0,T) &= \frac{2\beta_i k_i\theta_i}{(\beta_i\sigma_i)^2}\log\left\{\frac{2d_i\exp\{(\beta_i k_i+d_i)T/2\}}{2d_i+(\beta_i k_i+d_i)(\exp\{Td_i\}-1)}\right\}.\end{aligned}$$

For what concerns the functions $M_i(0,T)$ and $H_i(0,T)$, we follow the proof in Rocha-Arteaga et al. 2001. Let us consider $\gamma_{i,t} = \gamma_{i,t_0}e^{-k_i(t-t_0)} + \int_{t_0}^t e^{-k_i(t-u)}dL^{\mathbb{Q}}_{i,u}$. For the sake of legibility, from here onward the index i will be dropped. Therefore, we have $\gamma_t = \gamma_{t_0} e^{-k(t-t_0)} + \int_{t_0}^t e^{-k(t-u)} dL_u$, with

$$\begin{aligned}
\int_{t_0}^t \gamma_u du &= \int_{t_0}^t \left(\gamma_{t_0} e^{-k(u-t_0)} + \int_{t_0}^u e^{-k(u-s)} dL_s \right) du \\
&= \gamma_{t_0} \frac{1}{k} \left(1 - e^{-k(t-t_0)} \right) + \int_{t_0}^t \int_{t_0}^t 1_{s<u} e^{-k(u-s)} dL_s du \\
&= \gamma_{t_0} \frac{1}{k} \left(1 - e^{-k(t-t_0)} \right) + \int_{t_0}^t \left(\int_s^t e^{-k(u-s)} du \right) dL_s \\
&= \gamma_{t_0} \frac{1}{k} \left(1 - e^{-k(t-t_0)} \right) + \int_{t_0}^t \left[\frac{1}{k} \left(1 - e^{-k(t-s)} \right) \right] dL_s.
\end{aligned}$$

Now, let us define $g(s) = \frac{1}{k} \left(1 - e^{-k(t-s)} \right)$.

Then, we have $\int_{t_0}^t \gamma_u du = \gamma_{t_0} \frac{1}{k} \left(1 - e^{-k(t-t_0)} \right) + \int_{t_0}^t g(s) dL_s$. Let us compute the characteristic function of the process $\int_{t_0}^t \gamma_u du$ given the filtration $\mathcal{F}_{t_0}$:

$$\mathbb{E}\left[e^{i\xi \int_{t_0}^t \gamma_u du} \middle| \mathcal{F}_{t_0} \right] = e^{\frac{i\xi\gamma_{t_0}}{k} \left(1 - e^{-k(t-t_0)} \right)} \mathbb{E}\left[e^{i\xi \int_{t_0}^t g(s) dL_s} \right],$$

$$\begin{aligned}
\mathbb{E}\left[\exp\left\{ i\xi \int_{t_0}^t g(s) dL_s \right\} \right] &= \mathbb{E}\left[\lim_{n\to+\infty} \exp\left\{ i\xi \sum_{j=1}^n g(s_{j-1})(L_{s_j} - L_{s_{j-1}}) \right\} \right] \\
&= \lim_{n\to+\infty} \prod_{j=1}^n \mathbb{E}\left[\exp\left\{ i\xi g(s_{j-1})(L_{s_j} - L_{s_{j-1}}) \right\} \right] \\
&= \lim_{n\to+\infty} \prod_{j=1}^n \exp\left\{ \psi(\xi g(s_{j-1}))(s_j - s_{j-1}) \right\} \\
&= \exp\left\{ \lim_{n\to+\infty} \sum_{j=1}^n \psi(\xi g(s_j))(s_j - s_{j-1}) \right\} \\
&= \exp\left\{ \int_{t_0}^t \psi(\xi g(s)) ds \right\},
\end{aligned}$$

where ψ is the characteristic exponent of the Lévy process L_s. The switch between the integral end limit operators is possible since the function $g(s)$ is bounded and continuously differentiable, then the Lebesgue Dominated Convergence Theorem can be applied. Therefore, the pricing formula can be obtained just by setting $\xi = -i$. Recalling that $L_s = \sum_{n=1}^{N_s} J_n$ is a Compound Poisson process with i.i.d. exponential jumps J_n, $J_n \sim \exp(\eta)$, having characteristic function

$$\varphi(\xi) = \exp\left\{ s\lambda \left(\frac{\eta}{\eta - i\xi} - 1 \right) \right\},$$

and by setting $\xi = -ig(s)$, we obtain the expression for $H(t_0, t)$. Finally, we set $t_0 = 0$ and $t = T$ and get the solutions for A_i, C_i, H_i and M_i defined in Eq. (45) and (47):

$$\begin{aligned}
M_i(0,T) &= \frac{\beta_i}{k_i} \left(1 - e^{-k_i T} \right) \\
H_i(0,T) &= \int_0^T \lambda_i \left(\frac{\eta_i}{\eta_i + \frac{\beta_i}{k_i}(1 - e^{-k_i(T-s)})} - 1 \right) ds.
\end{aligned}$$

□

Finally, we seek to rank the impact of the different risk factors on the hazard rate of each issuer. Each climate variable has a different scale, so a simple comparison of the β_i coefficients is not sufficient. The mean reverting structure of the dynamics allows us to define the expected long-level impact of each variable for a given issuer as $\lim_{T\to+\infty} \beta_i \mathbb{E}^{\mathbb{Q}}[\gamma_T]$. For the C.I.R. process, this limit equates to $\beta_i\theta_i$, while for the OU Lévy processes it is $\beta_i\lambda_i/(\eta_i k_i)$. These quantities represent the expected long-range level of the climate variable on the hazard rate and can be used as a way to rank the climate resilience of a corporation for each variable.

Proof. By the definition of the stochastic hazard rate in Eq. (42) we have that:

$$\gamma_t = \gamma_{0,t} + \sum_{i=4,5} \beta_i \gamma_{i,t} + \sum_{i=1,2,3} \beta_i \gamma_{i,t}.$$

Then, by taking the conditional expectation we have

$$\begin{aligned}
\mathbb{E}_0[\gamma_T] &= \mathbb{E}_0[\gamma_{0,T}] + \sum_{i=4,5} \beta_i \mathbb{E}_0[\gamma_{i,T}] + \sum_{i=1,2,3} \beta_i \mathbb{E}_0[\gamma_{i,T}] \\
&= \gamma_{0,0} e^{-k_0 T} + \theta_0 (1 - e^{-k_0 T}) + \sum_{i=4,5} \beta_i \Big(\gamma_{i,0} e^{-k_i T} + \theta_i (1 - e^{-k_i T}) \Big) \\
&+ \sum_{i=1,2,3} \beta_i \Big(\gamma_{i,0} e^{-k_i T} + \lambda_i \frac{1}{\eta_i} \int_0^T e^{-k_i (T-s)} ds \Big) \\
&= \gamma_{0,0} e^{-k_0 T} + \theta_0 (1 - e^{-k_0 T}) + \sum_{i=4,5} \beta_i \Big(\gamma_{i,0} e^{-k_i T} + \theta_i (1 - e^{-k_i T}) \Big) \\
&+ \sum_{i=1,2,3} \beta_i \Big[\gamma_{i,0} e^{-k_i T} + \lambda_i \frac{1}{k_i \eta_i} \Big(1 - e^{-k_i T} \Big) \Big]
\end{aligned}$$

and by letting $T \to +\infty$ we get

$$\lim_{T \to +\infty} \mathbb{E}_0[\gamma_T] = \theta_0 + \sum_{i=4,5} \beta_i \theta_i + \sum_{i=1,2,3} \beta_i \frac{\lambda_i}{k_i \eta_i},$$

where each component of the summation gives the long-run mean impact of the i-th factor on the hazard rate. □

2.4 The Data

Our initial sample is comprised of corporate bonds from Eurozone firms that have issued at least one green bond. Due to the information intensity of some of the required regressors (variables such as leverage and Return On Equity require the manual collection of data from balance sheets over an almost 10-year period), we have restricted the analysis to Italian and German issuers. Germany was selected because of its size, being the largest Eurozone economy by GDP, while Italy (the third-largest Eurozone economy by GDP, after France), was selected because of its geographic location, representative of the Mediterranean area, and its exposure to physical risk factors. We can therefore consider the data as representative of the largest European areas, in economic and spatial terms: continental and Mediterranean Europe. The sample includes 43 German and 19 Italian firms, which are all green-bond issuers of the respective countries. The time frame under consideration goes from 01/01/2014 to 27/03/2023. To facilitate estimator convergence, we restrict our sample to bonds with at least 100 observations. To remove the impact of exchange rate risk, which is not of interest to this study, we only consider euro-denominated bonds. Data on bond bid, ask, and mid prices, bond YTMs, IRS rates, government yield curves, the VIX index, Eurozone corporate bond indices, national stock market indices, and EU carbon allowance prices are taken from Refinitiv Datastream. Balance sheet data is taken from the AIDA database, for some Italian firms. For others, and for German issuers, it is retrieved from the publicly available individual balance sheets of each firm. As for physical risk variables, the SMA is provided by the Copernicus European Drought Observatory, while the FWI, the wind speed, and the temperature indicators are taken from the Copernicus *ERA5 hourly data on single levels from 1940 to present* database. They are averaged daily and on the latitude and longitude coordinates, converted to the EPSG:4326 Geodetic coordinate system, falling within the boundaries of each country.

2.5 Results

2.5.1 Fitting and evaluating the intensity-based model

The initial step of the procedure requires fitting the stochastic process to the physical risk climate variables of the two countries, Italy ("IT") and Germany ("DE"). The estimation for the C.I.R. and Lévy-driven Ornstein-Uhlenbeck processes is carried out via indirect inference, as in Gourieroux et al. 1993. The auxiliary process, for all the cases, is an AR(1), while the structural process is

either the C.I.R. or the Lévy Ornstein-Uhlenbeck, depending on the variable at hand. The continuous time process in the indirect inference algorithm has been simulated via Euler discretization of the SDE governing the corresponding process. In the case of the C.I.R., the Matlab built-in function is used.
The results of the fits are in Table 6, for the Ornstein-Uhlenbeck Lévy-driven processes, and in Table 7 for the C.I.R. processes.

	k_i	λ_i	η_i
IT Eastw. Wind	0.6381	1.3748	0.0866
IT Northw. Wind	0.6022	1.2114	0.0929
IT FWI	0.0180	0.0147	0.2280
DE Eastw. Wind	0.4859	1.2501	0.1020
DE Northw. Wind	0.6590	1.6652	0.0846
DE FWI	0.1207	0.0676	0.0817

Table 6: Calibrated parameters of the Lévy-driven Ornstein-Uhlenbeck processes for weather variables

	k_i	θ_i	σ_i
IT Drought	0.0017	0.3256	0.0399
IT Flood	0.0017	0.3623	0.0542
DE Drought	0.0004	0.9938	0.0145
DE Flood	0.0005	0.8482	0.0335

Table 7: Calibrated parameters of the C.I.R. processes for weather variables

The second step of the procedure requires fitting the appropriate intensity-based models, each day, to the bond prices of each issuer. In doing so, we recover the daily parameters $k_0, \theta_0, \sigma_0, \gamma_{0,t_0}$ driving the firm-specific component of the hazard rate, $\gamma_{0,t}$, as well as the subset β of the parameters β_i, $i = 1, ..., 5$, which represent the impact of the relevant physical risk factors on each issuer's hazard rate. Given that the model is constructed to incorporate those factors that increase the riskiness of the firm, as it assumes a positive process for the hazard rate, for each issuer we only include the climate variables that display a positive (and statistically significant) coefficient. The calibration is performed via least squares, minimizing the sum of daily squared distances between the prices implied by the model in Subsection 2.3.3, denoted by $P_R^{Model}(t, t_N)$, and the observed mid quotes of daily closing market prices, denoted by $P_R^{Market}(t, t_N)$.

$$\underset{\beta, k_0, \theta_0, \sigma_0, \gamma_{0,t_0}}{\arg\min} \sum_{t=1}^{T} \left(P_R^{Market}(t, t_N) - P_R^{Model}(t, t_N) \right)^2, \tag{50}$$

where T is the last day in our observation window, and t_N is the maturity of each bond. For issuers where green bonds were not negatively affected by physical risk drivers, we only fit the model on the time series of prices of non-green bonds. For firms belonging to the second setting, in which both green and non-green bonds have a comparable relationship with the climate risk proxy of interest, we fit the model on the time series of prices of both types of bonds. Lastly, for issuers belonging to the third setting, we calibrate the model only on green bonds, and only if they have a risky relationship with at least one climate variable.

	AEROPORTI DI ROMA SPA	ALPERIA SPA	ASSICURAZIONI GENERALI SPA	BANCO BPM SPA
k_0	4.36413	0.77504	2.00981	7.27899
θ_0	0.23149	0.13273	0.81920	0.03213
σ_0	0.28353	1.57245	0.05291	8.20626
γ_{0,t_0}	1.80475	0.17044	0.00891	0.06405
$\beta_{Drought}$	-	-	-	0.35635
β_{Flood}	4.80387	0.00216	0.13268	0.37458
β_{FWI}	0.73911	-	-	-
$\beta_{Northw.Wind}$	0.01201	-	-	-

Table 8: Calibrated Parameters of Relevant Italian Issuers

	COMMERZ BANK AG	EUROGRID GMBH	EUSOLAG EUROPEAN SOLAR AG	RWE AG	LANDESBANK BADEN WUERTTEM-BERG	MUENCH. HYPO THEKEN BANK EG
k_0	1.42740	2.79104	4.75668	0.02306	0.84043	0.29921
θ_0	0.75358	0.40001	0.13413	3.12325	0.05207	0.08771
σ_0	1.22375	1.59228	0.09267	3.17051	0.04064	0.02103
γ_{0,t_0}	0.09814	0.86957	0.00162	0.45080	0.01461	0.00001
$\beta_{Eastw.wind}$	-	0.00072	-	0.00153	-	-
$\beta_{Drought}$	-	-	-	-	0.08895	0.06028
β_{Flood}	2.56516	-	-	-	-	-
$\beta_{Northw.wind}$	·	-	0.00731	-	-	-

Table 9: Calibrated Parameters of Relevant German Issuers

The results are shown in Tables 8 and 9. For this final set of issuers in both countries, the deseasonalized daily mean temperature was not a risk factor of interest. It was thus excluded from the fit of the models. Interestingly, all relevant Italian issuers were exposed to the flood risk indicator, with the airport being the most negatively affected. This seems reasonable, considering the dependence of the index on rainfall and negative weather conditions. As for German issuers, the resulting risk factors were divided by industry: the most frequent was the deseasonalized average daily wind speed, which exclusively affected all renewable energy producers in the relevant sample. On the other hand, all of the banks were exposed to either drought or flood risk. The relative dimension of the different beta coefficients allows for a comparison between the contribution of the different risk factors to the hazard rate of the issuer, and thus to its default probability. However, the size of the beta is not the only element to consider: the total impact of the factor will also depend on the parameters of its process and, in general, on its expected value. Tables 10 and 11 display the long-term impact estimates of each climate variable, computed with the procedure described in Section 2.3.3.

	AEROPORTI DI ROMA SPA	ALPERIA SPA	ASSICURAZIONI GENERALI SPA	BANCO BPM SPA
Drought	-	-	-	0.35414
Flood	4.07464	0.00183	0.11254	0.31772
FWI	5.06911	-	-	-
Northw. Wind	0.35882	-	-	-

Table 10: Long-Range Climate Impact on Relevant Italian Issuers

	COMMERZ BANK AG	EUROGRID GMBH	EUSOLAG EUROPEAN SOLAR AG	RWE AG	LANDESBANK BADEN WUERTTEM-BERG	MUENCH. HYPO THEKEN BANK EG
Eastw. Wind	-	0.01817	-	0.03871	-	-
Drought	-	-	-	-	0.08839	0.05990
Flood	2.17577	-	-	-	-	-
Northw. Wind	-	-	0.21842	-	-	-

Table 11: Long-Range Climate Impact on Relevant German Issuers

2.5.2 Robustness check

We finally perform a robustness check of the climate risk extension of the intensity-based model, from hereon referred to as the "proposed model". On the same data, for each issuer, we fit another model, henceforth referred to as the "alternative model", which excludes all weather variables and only includes the firm-specific component of the hazard rate in Eq. (42), i.e. $\gamma_{0,t}$. According to the theory, this component incorporates all other non-explicitly specified risk drivers that affect a company's probability of default. We test the goodness of the model by comparing the means μ_P and μ_A of the objective functions, as defined in Eq. (50), minimized daily by the proposed model and by the alternative model, respectively. This is done through the one-tailed Welch's t-test for comparing means with unequal variances. The null hypothesis is $H_0 : \mu_P = \mu_A$, while the alternative hypothesis is $H_1 : \mu_P > \mu_A$, meaning that the proposed model performs worse and has, on average, a higher objective function than the alternative one. The resulting statistics are reported in Table 36, disaggregated over multiple months. Welch's t-test mostly fails to reject the null hypothesis at the 5% and 1% significance levels, but the quality of the fit changes across the months, suggesting a time-changing nature in the link with the risk factors. For some issuers, the opposite alternative hypothesis $H_1 : \mu_P < \mu_A$, representing the superiority of the proposed model, is accepted at the 5% or 1% significance levels.

3 Hedging and Mitigating Climate Risks: Strategies for Financial Markets and Natural Resources

3.1 Introduction

In this chapter, we analyze the role of weather derivatives can have, in managing the physical risk (intended as financial losses, induced by weather events). The chapter will be divided into two parts: in the first one we focus on the current weather derivatives written on temperature, while in the second one, we propose new contracts for managing the financial losses induced by water scarcity.

Weather derivatives are a tool for hedging some of the risks arising from climate change and helping fill the corresponding insurance protection gap. Such contracts emerged in the late 1990s as financial instruments to hedge risks related to weather conditions. The market began when Enron and Koch Industries executed the first weather-related contracts in 1997. These instruments gained popularity as businesses, particularly in energy and agriculture, sought ways to manage risks from unpredictable weather. The Chicago Mercantile Exchange (CME) introduced weather futures and options in 1999. However, their market is however still in its infancy, and the lack of sufficient contracts and trading activity highlights a discrepancy between the behavior of quoted prices and that of the underlying physical variables. Additionally, a matter of concern is the incompleteness of the market, caused by the fact that the asset underlying the derivatives instrument - a climate variable - is not itself traded. This incompleteness affects not only the weather derivatives market but also all markets in which climate variables are a risk factor of interest. HDD/CDD and CAT contracts are employed by a wide variety of enterprises, ranging from energy and agriculture, to breweries, and amusement parks. CME Weather derivatives offer a useful tool for hedging volumetric risks related to adverse temperature and climatic conditions. Energy companies, for example, have been known to sell HDD or CDD contracts to manage the risk of diminished revenues under mild weather conditions, noting that the quantity of energy sold is heavily contingent upon consumer demand driven by temperatures; Retailers whose sales are sensitive to weather conditions might control inventory costs more effectively through the use of HDD or CDD contracts; Agricultural production has a well-known sensitivity to weather, with adverse conditions impacting both the quality and quantity of crops yields. In Stulec et al. 2016, the authors show that effectiveness of weather derivatives in crops grapes, corn, wheat, barley, soybeans, and cotton productions; utility companies, may utilize HDD or CDD contracts to guard against the volumetric risks due to the quantity of energy that might be expected to be marketed throughout the course of a heating or cooling season, for example if the daily average temperatures during a winter season were abnormally high, utility firms might face depressed demand for heating.

In this chapter then, it is analyzed the current weather derivatives market and the prices of the quoted instruments to ensure a fair valuation of temperature-based derivatives. Most research in this field has concentrated solely on temperature trends, largely disregarding the impact of market dynamics, distortions, and mispricing. However, the market is currently quite inefficient, quoted prices do not accurately represent the underlying temperature trends, and the observed pricing anomalies suggest that buyers of the contracts are paying considerably less than what would be expected. This underpricing creates a significant market imbalance, leading to concerns about the long-term viability and reliability of weather derivatives as financial instruments. We examine data of contracts on various American cities and show that pricing issues are consistent across both Cooling Degree Days (CDD) and Heating Degree Days (HDD) futures contracts. By analyzing historical price data and comparing it to realized payoffs, we demonstrate a clear disconnect between market quotes and expected outcomes. This mispricing, in turn, undermines the value of traditional pricing models, calling for an approach able to account for market anomalies. We propose a market model inspired by established interest rate models, such as those developed by Brace et al. 1997, Musiela et al. 1997 and Hagan et al. 2008. This model addresses the gaps in the current literature through its ability to accommodate market effects and speculator behaviors. These factors are expected to become increasingly important, as climate-related risks and the demand for weather derivatives grow. Furthermore, the framework addresses the problem of incompleteness by redefining contracts in terms of a tradable primitive asset: the forward temperature. This approach provides a robust and adaptable model, maintaining a link between prices and temperatures while ensuring the absence of arbitrage opportunities in an incomplete market. It is better suited for capturing the complexities and uncertainties inherent in the weather derivative markets.

The temperature rising associated to climate change is leading to increased water scarcity, unpredictability, and pollution, which disrupts the water cycle and threatens sustainable development, biodiversity, and access to clean water and sanitation. Flooding and rising sea levels can contaminate land and water supplies with saltwater. The rapid melting of glaciers, ice caps, and snow fields—key sources of freshwater for major rivers—disrupts freshwater regulation, impacting millions in downstream areas. Droughts and wildfires are destabilizing communities, sparking civil unrest, and forcing migration. Loss of vegetation and tree cover worsens soil erosion and reduces groundwater replenishment, worsening water shortages and food insecurity. Water demand, is rising due to population growth, energy production, and production/extraction of critical materials (rare lands) necessary for the digital and green transition (Figure 7). Moreover, unsustainable

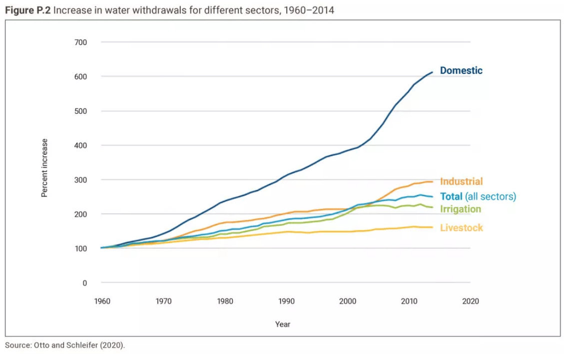


Figure 7: Demand of freshwater

farming practices have strained water resources, making access to freshwater increasingly difficult. With only 3% of the world's water being freshwater and much of it locked in glaciers or otherwise inaccessible, more than 1.1 billion people currently lack access to safe drinking water. Climate change has also amplified the frequency and intensity of droughts, flooding, and irregular rainfall patterns, which further stress global water supplies. Projections suggest that nearly two-thirds of the world's population may face water shortages by 2025. In response to this escalating crisis, governments and organizations have implemented various strategies, including desalination, rainwater harvesting, and wastewater reuse. These methods aim to improve water distribution and efficiency. However, as the water crisis deepens, financial instruments like water weather derivatives could represent tools for managing water-related risks. These derivatives provide a financial buffer against the volumetric risks associated with water shortages, such as low water reservoir levels or insufficient rainfall, both of which have severe implications for industries and economies that rely heavily on water. Water markets has emerged as tools to manage water-related risks, emphasizing the importance of water rights (prior appropriation and riparian rights) in ensuring efficient and sustainable water use. Such markets exist in countries like Australia, Chile, and parts of the U.S., allowing the trading of water rights as financial assets. However, the market lacks the presence of hedging instruments, such as options and weather derivatives, to hedge against volumetric risk associated to water scarcity.

The role of water weather derivatives in water management systems could be particularly significant due to their dual nature, offering both financial and physical hedging. By linking contracts to natural variables like rainfall, these derivatives allow businesses, farmers, and even governments to hedge against the risks posed by fluctuating water availability. This financial protection can stabilize industries that are vulnerable to droughts and water shortages, offering a way to safeguard agricultural productivity and economic stability in the face of increasingly erratic water supply patterns.
Pricing these derivatives requires an approach that accounts for both physical variables—such as rainfall or water indices—and financial variables like market conditions and asset values. Similar to catastrophic bonds (Cat Bonds) used in earthquake insurance, water weather derivatives are structured so that the likelihood of a trigger event, like drought, does not change when moving from the physical probability to a market-based measure. While market operators cannot influence physical occurrences like rainfall, they can influence the pricing of financial contracts, making accurate pricing models crucial for these instruments to function effectively.
By following a pricing model that captures this complexity, water weather derivatives can offer

reliable hedging strategies, such as buy-and-hold, which protect against long-term financial damage caused by water scarcity. These instruments can reduce the economic risks posed by natural variability in water availability, ultimately contributing to more resilient water management systems. In conclusion, water weather derivatives could play a pivotal role in addressing the growing challenge of water scarcity. By providing a financial safety net against natural water risks, these instruments enable industries and governments to manage water more effectively, ensuring economic resilience and stability in the face of climate change. As part of a broader, multi-faceted approach to water management, they offer a vital tool for navigating the uncertainties of a future with increasing water stress.

3.2 Literature review

The field of weather derivatives, particularly those focused on temperature-based contracts, offers a variety of mathematical models designed to accurately capture temperature dynamics for pricing purposes. One of the foundational approaches in this area is the use of the Ornstein-Uhlenbeck (OU) process. This model is used frequently, due to its mean-reverting properties, which make it a suitable candidate for modeling temperature fluctuations. However, researchers have identified and addressed several of its limitations, leading to a variety of enhancements and alternative methodologies. A number of studies, such as F. E. Benth and Šaltytė-Benth 2005, F. E. Benth and J. š. Benth 2007 F. E. Benth and Šaltytė Benth 2011, F. E. Benth, J. Š. Benth, et al. 2007, Šaltytė Benth et al. 2012, extend the traditional OU process to account for more complex temperature dynamics. They achieve this by employing the Lévy-based Ornstein-Uhlenbeck process, allowing for jumps and other non-continuous behaviors within the temperature data. This advancement helps capture the more erratic aspects of temperature shifts, contributing to a more robust model for weather derivatives. Hess 2018 take the OU model a step further by incorporating partial information from future weather forecasts. This innovative approach acknowledges that weather derivatives pricing could benefit from leveraging actual meteorological predictions. By integrating forecast data into the model, Hess opens the door to a more accurate representation of temperature trends, potentially leading to better pricing of weather derivatives. Brody et al. 2002 approach the pricing of weather derivatives by employing a fractional Ornstein-Uhlenbeck process. This variation of the OU model introduces a key characteristic: long memory. The concept of long memory recognizes that temperature data can exhibit dependencies over extended periods, which traditional OU processes might not adequately capture. The fractional OU approach thus allows for a more nuanced representation of temperature trends over time. Groll et al. 2016 propose a two-factor model for temperature, combining two distinct OU processes. This model acknowledges that temperature could be influenced by multiple underlying factors, each with its own mean-reverting characteristics. By combining these two processes, the model provides a more comprehensive framework for capturing the complexity of temperature dynamics, potentially leading to more accurate weather derivatives pricing. Gyamerah et al. 2018 take a different approach by introducing a regime-switching model. In this model, temperature can follow either a classic Ornstein-Uhlenbeck process or a Lévy OU process, depending on the value of a latent variable. This regime-switching mechanism allows for greater flexibility in modeling temperature, acknowledging that temperature dynamics might exhibit different behaviors under varying condition. Alfonsi et al. 2023 propose a more complex model, incorporating an Ornstein-Uhlenbeck process with stochastic volatility driven by a Cox-Ingersoll-Ross (C.I.R.) process. The presence of stochastic volatility adds another layer of complexity, reflecting the varying levels of uncertainty inherent in temperature data. Furthermore, they provide closed-formulas for estimating model parameters, contributing to a more practical and applicable framework for pricing weather derivatives. Despite these advancements, a significant gap remains in replicating market prices for weather derivatives. As Geman and Leonardi 2005 highlights, the existing models often fail to account for the market price of risk, leading to discrepancies between theoretical pricing and market behavior. This points to the need for further research to address the correct market measure reflecting the true price dynamics in weather derivatives markets. Härdle et al. 2012 address the issue directly and estimate the implied market price of risk from the quoted price of Cumulative Average Temperature (CAT) futures. They assume that it corresponds to the kernel of the Radon-Nikodym derivative in a change of measure going from the physical probability to the martingale measure under which the contracts are priced. The study leaves open the matter of market incompleteness, on which this thesis instead focuses. This challenge underscores the need for further research to address the correct market measure that reflects the true risk dynamics in weather derivatives markets. In summary, the scientific community has made substantial progress in modeling temperature for weather derivatives, utilizing and enhancing the Ornstein-Uhlenbeck process in various ways. Yet,

key issues persist, particularly in aligning theoretical models with actual market prices. Future research must continue to explore innovative methods to bridge this gap, ensuring that weather derivatives pricing reflects real-world market conditions and investor behaviors. The present thesis builds on the aforementioned literature, using an OU process with stochastic volatility to model temperatures, and aims to fill the highlighted gaps.
Additionally, we propose a framework aimed at addressing the problem of market incompleteness, to enable delta hedging of temperature derivatives. It is based on a primary asset, of the type defined in Arrow et al. 1954, where the necessary and sufficient conditions for markets to be complete and in equilibrium are introduced. Furthermore, our primary asset is a forward contract on a non-traded underlying, following the model presented in Black 1976. Black's work demonstrates that forward contracts (or futures, in the case of deterministic rates) in commodity markets can be used to price options by exploiting the non-arbitrage relationship between spot and future markets. In our framework, such primary assets become the underlying securities used for delta hedging. By considering a setting in which the non-traded underlying is the sole source of risk (thus confined to cases of non-stochastic volatility), this approach allows us to complete the market and construct perfect hedging strategies. We also draw from the branch of literature that identifies - and attempts to model - the presence of stochastic volatility in the log-returns of the S&P500 index. The first work of interest on this topic is Heston 1993, which documents the phenomenon and proposes a modeling approach. Importantly, the volatility is identified as an additional risk factor, which causes market incompleteness whenever it is not possible to trade it, directly, but only the underlying asset is present on the market. However, a possible opportunity for completing the market is identified in the literature: the VIX index[13], which tracks the implied volatility on options on the S&P500, and allows the construction of hedging strategies against movements in the volatility of the underlying index. This solves the problem of market incompleteness, ensuring that the additional risk factor represented by the S&P 500 volatility is also quoted. Derivatives on the VIX are also traded, thus enriching the hedging opportunities at the disposal of investors, and they include options and futures.
In the first part of this chapter, then, we propose a market model for weather derivatives, which is able to incorporate the market effects, and addresses the problem of incompleteness. This is done in line with the literature, i.e. by introducing a tradable primary asset able to complete the market.

The combined effects of climate change and population growth present a significant threat to water systems, even in the most developed nations. As highlighted by Asif et al. 2023, regions in North America are expected to experience increased water stress due to climate change. Understanding the regional impact of climate change on water resources is challenging because of the spatial and temporal variability involved. Climate change has a pronounced effect on the annual streamflow and runoff of rivers across North America. To protect water resources and meet future demand, it is crucial to implement regional management strategies. Employing cost-effective and decentralized methods and infrastructures is vital for reducing storm flow runoff and improving rainfall infiltration. The exacerbation of water crises due to climate change is already evident in North America and is likely to continue. Therefore, an integrated approach to water resource management is urgently needed, incorporating innovative technologies like artificial intelligence (AI) and smart sensors. These approaches can help address regional water stress issues comprehensively, considering environmental, social, and economic factors.

Hung et al. 2022 examine human adaptation and water scarcity uncertainties within the Colorado River Basin (CRB). Their study highlights the complexities of modeling human behavior, especially in agricultural water use. They emphasize the increasing concerns about water scarcity and the urgent need for reforms in water management practices within the basin. Molden 2020 found that poor water management exacerbates the mismatch between water supply and demand, leading to greater water scarcity. Tzanakakis et al. 2020 stress the need to re-evaluate water management strategies, especially in areas undergoing demographic changes and facing increased climate-related risks. Their research advocates for the adoption of advanced technologies and methodologies to improve water use efficiency among consumers. This should be a central goal of water management efforts to minimize water losses and enhance the resilience of water resources. Hartman et al. 2017 found out that governmental acts such as Oklahoma's Water for 2060 Act had an impact on at least the decision-making of water managers, and boosting innovation in the water management system.

[13] https://www.cboe.com/tradable_products/vix/

Dhakal et al. 2022 focus on desalination methods to address water supply challenges, particularly in India, China, and South Africa. Rosa et al. 2020 and Dolan et al. 2021 analyze water scarcity indicators and variables used to evaluate water scarcity phenomena. Hristov et al. 2021 investigate the use of reclaimed or treated water from urban wastewater treatment plants for irrigation in Europe. Their analysis found that using treated water is still too costly, requiring financial support for farmers. Ungureanu et al. 2020 emphasize the effectiveness of irrigation systems in reducing wastewater and alleviating water demand pressures.

Another area of research focuses on the legal and economic frameworks for optimal water allocation to reduce wastewater and economic losses. Water's nature as a public good complicates its efficient allocation. Geman and Kanyinda 2007 argue that water cannot be considered a commodity like natural gas or coal, as there is no close substitute. However, with increasing scarcity, water markets are emerging to improve allocation efficiency and reduce waste. Water trading involves the rights to use water, not ownership of the resource. Garrick et al. 2009 illustrated that water markets and a transactional approach to reallocating water rights can be crucial strategies for implementing environmental flows. They also highlighted that markets are simply extensions of existing institutional arrangements and governance systems.

Seidl et al. 2020 studied the Murray-Darling Basin and identified two types of market operators: those mitigating shortages and securing supply, and those trading for financial gain or hedging. Water is used as a financial asset for hedging due to the lack of storage costs. Brewer et al. 2007 show that water prices reflect the growing demand pressures from agricultural and urban sectors. Browne et al. 2023 found that the water market had reduced transaction costs and uncertainty in water delivery leading to a more efficient resource allocation, with surface water users standing more to gain. Wight et al. 2024 documented an increasing trend in water transactions, finding also that temperature, groundwater levels, and commodity prices for rice and cotton are correlated with water transactions.

The water market in California has grown significantly, with voluntary reallocation preferred over government-imposed allocation. Schwabe et al. 2020 report that 89% of water transactions in California involve environmental and agricultural uses. Brozovic et al. 2002 note benefits for farms trading water, such as better responses to seasonal conditions and economic gains. The water market in Pakistan has also improved the equitable distribution of water resources, benefiting both small and large farmers. Razzaq et al. 2019 show that the market mechanism allows small farmers to buy water at market prices, while large farmers are incentivized to sell excess water. Similar findings are reported by Howitt et al. 2005, Av et al. 2016, and Bajaj et al. 2022 for India and California.

Previous studies have qualitatively analyzed water markets, demonstrating their importance in promoting efficient water use by treating water as a commodity. However, hedging water is different from traditional financial assets due to external factors like rainfall. Additionally, when pricing water derivatives, it is essential to consider the appropriate pricing measure, as the situation differs from the Black-Scholes model. This is similar to catastrophe bonds, where both financial variables and catastrophic events influence the bond. Literature such as Vaugirard 2003, Burnecki et al. 2011, and Nowak et al. 2013 show that while physical variables retain their probability distribution, financial variables change under different measures, allowing the pricing of derivatives as martingales.

3.3 Data

Quoted prices of CDD and HDD futures are recovered from Refinitiv Datastream. Risk-free rates are proxied by SOFR rates and obtained from the Federal Reserve Bank of New York. Finally, temperature data is downloaded from the NASA Prediction Of Worldwide Energy Resources (POWER) Data Access Viewer Enhanced (DAVe) interface. For each contract, the time series of temperatures is downloaded at the precise Weather Bureau Army Navy (WBAN) latitude and longitude coordinates of the stated measuring station. For each city, the location of the reference station is an airport.

3.4 Weather Derivatives: Managing Climate Risks in Financial Markets

3.4.1 Inefficiency of the weather market

In order to study the presence of arbitrage opportunities in the temperature derivatives market, we analyze the prices of a few quoted CDD (Cooling Degree Days) and HDD (Heating Degree Days) futures. The payoff of these instruments depends on the realized temperatures of all days T_i, $i = 1, \ldots, n$, within a reference period $[T_1, T_n]$, usually equal to one calendar month. In the case of CDD contracts, we have that the payoff at maturity T_n is given by

$$CDD_{T_n} = \eta \sum_{i=1}^{n} (X_{T_i} - c)^+,$$

where X_{T_i} is the average temperature of day T_i, c is a threshold temperature level (or strike) for the contract, and η is the monetary value of one degree of temperature. The average daily temperature is defined as $X_{T_i} = \frac{X_{max}+X_{min}}{2}$, where X_{max} and X_{min} are the maximum and minimum temperatures of day T_i. For HDD contracts, we have

$$HDD_{T_n} = \eta \sum_{i=1}^{n} (c - X_{T_i})^+.$$

We take the prices of CDD and HDD contracts traded on the CME, for which $\eta = 20$ USD per index point, temperatures are expressed in degrees Fahrenheit, and $c = 65$°F, for U.S. cities. For each location, we recover the time series of temperatures registered at the locations stated inside the contract[14], with coordinates taken from the Weather Bureau Army Navy (WBAN) Station Locations[15]. The minimum price fluctuation on the respective CME Degree Days Index futures is 1 index point. Then, for every day within the reference period of the contract, we compute the corresponding realized portion of the CDD or HDD payoff. We then find the cumulated realized payoff of the instrument at each date, and compare it with its quoted price. As in the case of Asian options, the cumulated realizations of the underlying should be reflected in the market prices as shown in Geman and Yor 1993. In the case of weather derivatives, as the end of the reference period approaches, the quoted price should reflect the realized portion of the reference HDD or CDD index, and the difference between them should go to zero. We however observe that for most instruments in our sample, this is not true. Figures 8-15 hold the plots of cumulated realized payoffs and of quoted prices for CDD and HDD futures of the U.S. cities in our sample: Atlanta, Chicago, Cincinnati, Dallas, Las Vegas, Minneapolis, New York, and Sacramento.

The absence of such basic property signals a strong inefficiency of the regulated market, which fails to reflect available information about the underlying. The reason behind the discrepancy is market incompleteness, which prevents agents from exploiting clear arbitrage opportunities, as the underlying asset is not itself traded. In such a setting, any attempt to fit a traditional weather derivatives model, which is constrained to the temperature dynamic, would not be able to replicate the market quotes. However, any reasonable alternative pricing method would still have to reflect the actual information about the realizations of the underlying over time. We, therefore, propose a market model based on a tradable primitive security, which can satisfy this basic property, maintain a link with the physical variable, and is capable of addressing market incompleteness.

[14] https://www.cmegroup.com/content/dam/cmegroup/rulebook/CME/IV/400/403/403.pdf

[15] https://19january2017snapshot.epa.gov/sites/production/files/documents/STATION_LOCATIONS.PDF

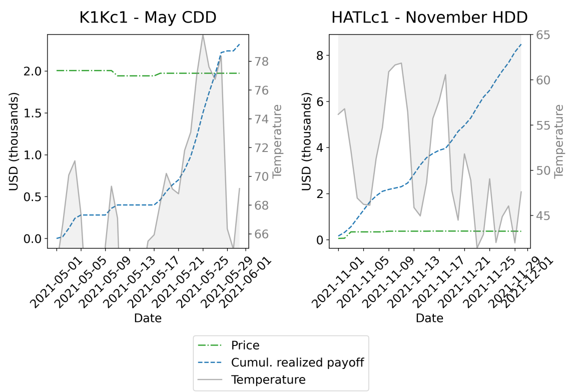


Figure 8: Atlanta CDD and HDD contracts

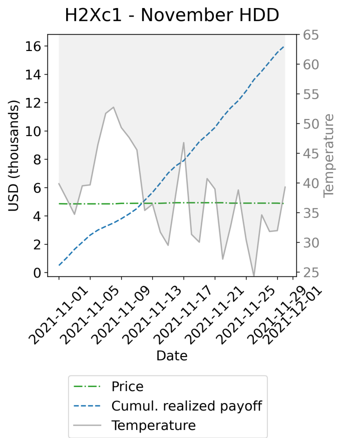


Figure 9: Chicago CDD and HDD contracts

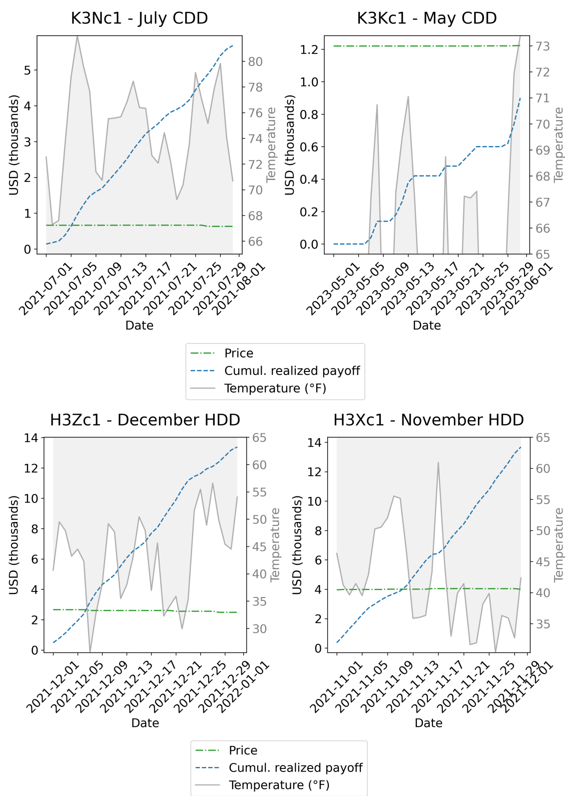


Figure 10: Cincinnati CDD and HDD contracts

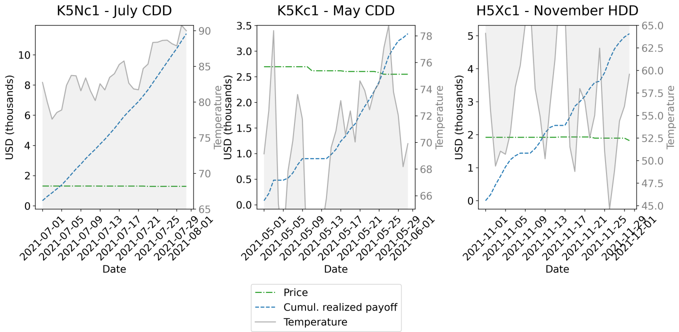


Figure 11: Dallas CDD and HDD contracts

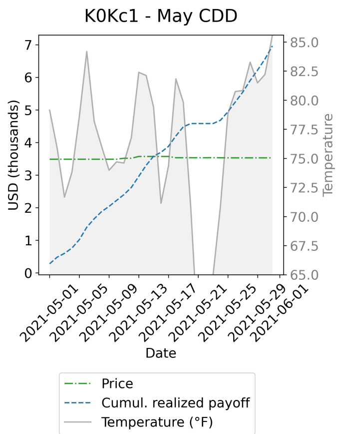


Figure 12: Las Vegas CDD and HDD contracts

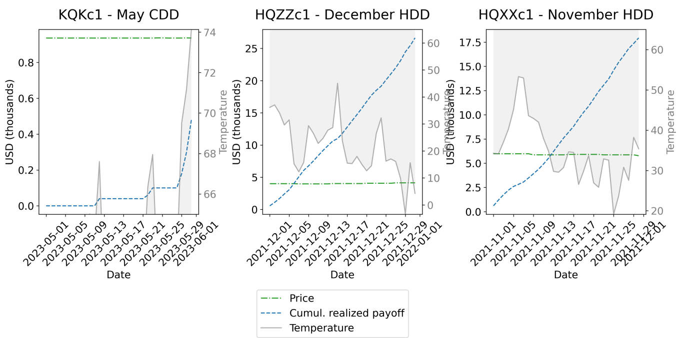


Figure 13: Minneapolis CDD and HDD contracts

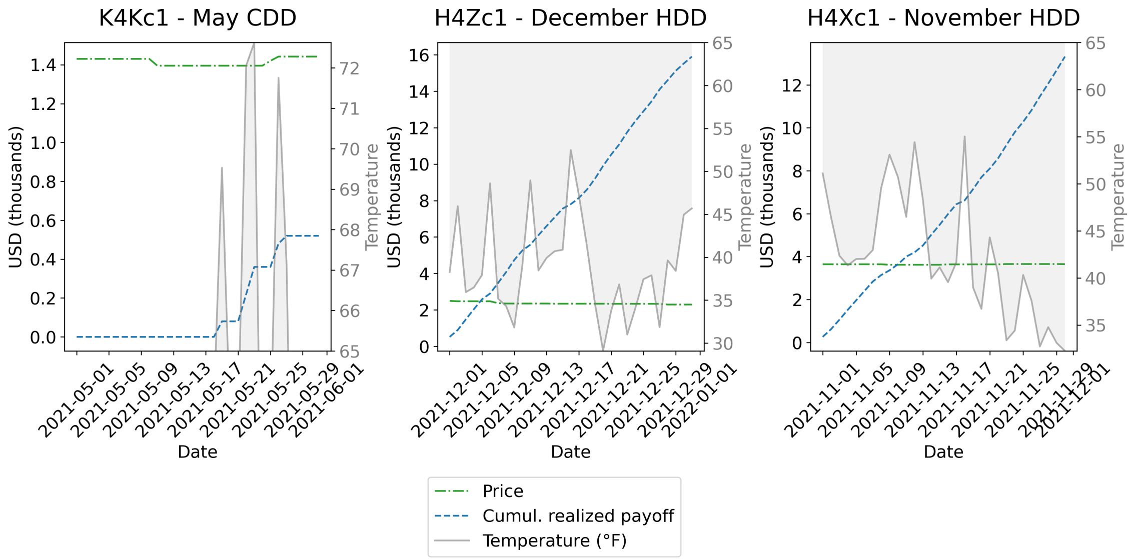


Figure 14: New York CDD and HDD contracts

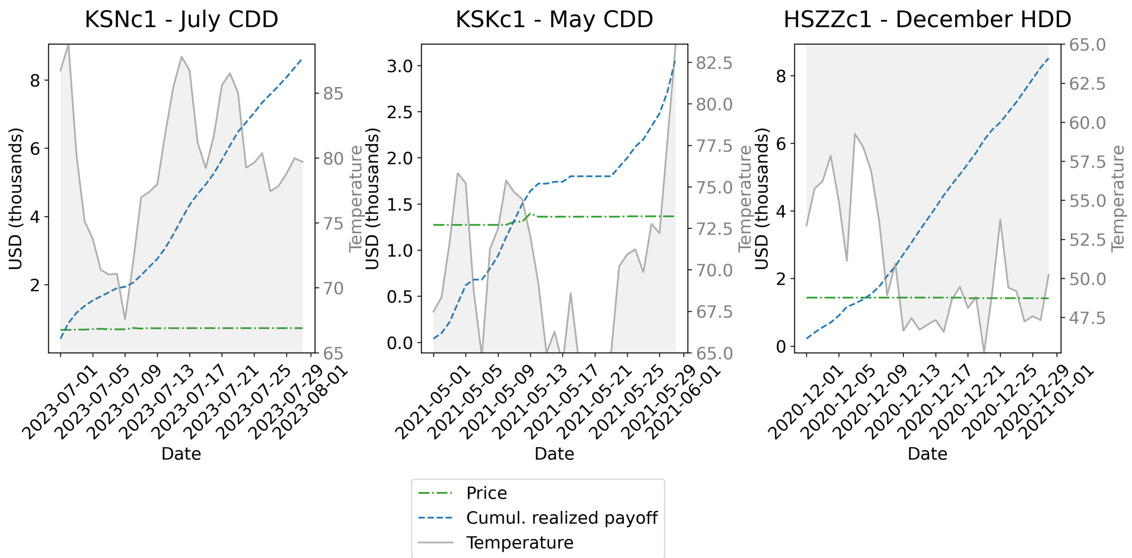


Figure 15: Sacramento CDD and HDD contracts

3.4.2 The primitive asset

Let us begin by assuming to be in an arbitrage-free market setting with deterministic interest rates. We then define the process $Y_t = \eta X_t$, $t \geq 0$, as the monetary value of temperature at time t. Here, η denotes a constant *tick* representing the monetary value of one degree of temperature. Temperatures are not, in themselves, tradable, so we consider a different primitive asset. We define it as a contract between two counterparties, in which the amount $Y_T - F(t,T)$ is exchanged at a maturity date T, with $0 \leq t < T$. We call $F(t,T)$ the *forward price* of the temperature at time t and maturity T. Its amount is agreed upon by the two parties at t, the time of inception of the contract, to ensure fairness under the forward temperature measure. We therefore have that $F(t,T)$ satisfies

$$\mathbb{E}_t^{\mathbb{Q}}[Y_T - F(t,T)] = 0,$$

which implies that

$$\mathbb{E}_t^{\mathbb{Q}}[Y_T] = F(t,T), \tag{51}$$

where $\mathbb{Q}$ is the *risk-neutral* measure. We observe that, at maturity T, $F(T,T) = \mathbb{E}_T^{\mathbb{Q}}[Y_T] = Y_T$. We note that, at any time t, multiple forward contracts can exist, each with a different maturity T. The market will be complete as long as the number of non-redundant and liquid primitive assets of different maturities is at least equal to the number of sources of risk arising from the temperature

process. In Section ?? we introduce the temperature model that will be used in our work, which incorporates stochastic volatility. Therefore, in the present setting, the number of sources of risk tied to the underlying is two. As a consequence, at any point in time t, two primitive securities are required to ensure market completeness: two liquid forward contracts of different maturities and which are non-redundant in terms of correlation.
Finally, we highlight that, since the temperature process X_t changes based on its geographic placement, the primitive assets are also location-specific. Here, the model is introduced for one location: for ease of legibility, we adopt a simplified notation and avoid subscripts referring to the spatial dimension of the processes.

3.4.3 Modeling temperature

Let $(\Omega, \mathbb{F}, \{\mathcal{F}_t)\}_{t\geq 0}, \mathbb{P})$ be a filtered probability space. Let $\mathbb{P}$ be the physical probability measure under which we observe $\{X_t\}_{t\geq 0}$, an $\mathcal{F}_t$-adapted process representing the average daily temperature at any generic location. The average daily temperature is defined as $X = \frac{X_{max}+X_{min}}{2}$, where X_{max} and X_{min} are the maximum and minimum temperatures of each day. We adopt the following stochastic volatility model for the temperature process, drawing from Alfonsi et al. 2023

$$
\begin{aligned}
X_t &= s(t) + \tilde{X}_t, \\
d\tilde{X}_t &= -a\tilde{X}_t + \sqrt{\nu_t}(\rho dW^{\mathbb{P}}_{1,t} + \sqrt{1-\rho^2} dW^{\mathbb{P}}_{2,t}), && (52) \\
d\nu_t &= k(\theta - \nu_t)dt + \sigma\sqrt{\nu_t} dW^{\mathbb{P}}_{1,t}, && (53)
\end{aligned}
$$

where $W^{\mathbb{P}}_{1,t}$ and $W^{\mathbb{P}}_{2,t}$ are independent Brownian Motions under $\mathbb{P}$, $a, \theta, \sigma, k > 0$, $\rho \in [-1, 1]$, and $\tilde{X}$ is the non-seasonal component of daily temperature. Finally, $s(t)$ is the temperature seasonality function, defined as the Fourier decomposition

$$
s(t) = \alpha_0 + \beta_0 t + \sum_{i=1}^{N_s} \alpha_i \sin\left(i\frac{2\pi}{365}t\right) + \sum_{i=1}^{N_s} \beta_i \cos\left(i\frac{2\pi}{365}t\right).
$$

As in F. E. Benth, J. Š. Benth, et al. 2007 and Šaltytė Benth et al. 2012, $N_s = 1$ is taken. The parameters are then estimated via the procedure in Alfonsi et al. 2023, detailed in Appendix C.

3.4.4 Modeling the dynamic of the primary asset

We allow the dynamic of the primary asset to not be directly dictated by that of the underlying, following instead a market-driven approach. This choice, entailing greater flexibility, serves to accommodate the role of market forces on pricing. In a more liquid market, it would be directly calibrated on primary assets bootstrapped from quoted CAT futures of different maturities.
The dynamic of $F(t, T)$ is driftless under $\mathbb{Q}_T$, due to its martingality under such measure, and maintains a link with the underlying through the volatility. The current state of the market is such that it is not possible to observe the volatility of this primary asset. We propose what we believe to be a reasonable historical metric: the volatility of the temperature process. Since this quantity is stochastic, as demonstrated by Alfonsi et al. 2023, the model is of the Heston 1993 type. The dynamic of the primary asset is described by

$$
\begin{aligned}
dF(t,T) &= F(t,T)\sqrt{\nu_t}(\rho dW^{\mathbb{Q}_T}_{1,t} + \sqrt{1-\rho^2} dW^{\mathbb{Q}_T}_{3,t}), && (54) \\
d\nu_t &= k(\tilde{\theta} - \nu_t)dt + \sigma\sqrt{\nu_t} dW^{\mathbb{Q}_T}_{1,t}, && (55) \\
\mathbb{E}^{\mathbb{Q}_T}&[dW^{\mathbb{Q}_T}_{1,t} dW^{\mathbb{Q}_T}_{3,t}] = \rho dt,
\end{aligned}
$$

where k and σ are the same parameters as Eq. 53, $\tilde{\theta} > 0$ is the long-run level, $W^{\mathbb{Q}_T}_{3,t}$ is a Brownian Motion under $\mathbb{Q}_T$, independent of $W^{\mathbb{Q}_T}_{1,t}$, and ρ is the correlation coefficient in Eq. 52. The change from the probability measure $\mathbb{P}$ to $\mathbb{Q}_T$ is described by the following Radon-Nikodym derivative

$$
\left.\frac{d\mathbb{Q}_T}{d\mathbb{P}}\right|_T = \exp\left\{\int_t^T \lambda_{1,s} dW^{\mathbb{P}}_{1,s} + \int_t^T \lambda_{3,s} dW^{\mathbb{P}}_{3,s} - \frac{1}{2}\int_t^T (\lambda^2_{1,s} + \lambda^2_{3,s}) ds\right\}, \tag{56}
$$

where $\lambda_{1,t} = \frac{\mu}{\rho\sqrt{\nu_t}} - \frac{\sqrt{1-\rho^2}}{\rho\sqrt{\nu_t}}$, and $\lambda_{3,t} = \frac{1}{\sqrt{\nu_t}}$. Then, by Girsanov's theorem, $dW^{\mathbb{Q}_T}_{1,t} = dW^{\mathbb{P}}_{1,t} - \lambda_{1,t} dt$, and $dW^{\mathbb{Q}_T}_{3,t} = dW^{\mathbb{P}}_{3,t} - \lambda_{3,t} dt$. Additionally, the following relationship holds $\tilde{\theta} = \theta - \frac{\sigma\mu}{\rho k} + \frac{\sqrt{1-\rho^2}}{\rho k}$

and can be recovered by performing the change of measure from $\mathbb{P}$ to $\mathbb{Q}_T$ on Eq. 53.
Although this model is inherently incomplete due to the stochastic nature of volatility, it doesn't allow for arbitrage opportunities, unlike a pricing model solely based on temperature dynamics. The definition of the above primary asset allows for the pricing of Cumulative Average Temperature (CAT), Cooling Degree Days (CDD), and Heating Degree Days (HDD) futures through separate building blocks. Each building block can then be expressed in terms of the primary asset.

3.5 Pricing derivatives

The definition of the above primitive assets allows for the pricing of Cumulative Average Temperature (CAT), Cooling Degree Days (CDD), and Heating Degree Days (HDD) futures through separate building blocks. Each building block can then be expressed in terms of a primitive asset.

3.5.1 Pricing the CAT Futures

The futures contract on the Cumulative Average Temperature (CAT) index, where the index refers to times $T_i, i = 1, ..., n$, can again be written as the sum of primitive assets, each with maturity T_i

$$\begin{aligned} F_{CAT}(t, T_1, T_n) &= \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left\{-\int_t^{T_n} r_s ds\right\}\sum_{i=1}^n Y_{T_i}\right] \\ &= \sum_{i=1}^n \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left\{-\int_t^{T_n} r_s ds\right\} Y_{T_i}\right] \\ &= \sum_{i=1}^n F(t, T_i)P(t, T_n), \end{aligned} \tag{57}$$

where $P(t, T_n)$ is the price in t of a zero-coupon-bond with maturity T_n which, under deterministic rates, satisfies $P(t, T_n) = \exp\left\{-\int_t^{T_n} r_s ds\right\}$. We also exploit the fact that, by definition of the contract, $F(T_i, T_i) = \mathbb{E}_{T_i}^{\mathbb{Q}}[Y_{T_i}] = Y_{T_i}$.

3.5.2 Pricing the CDD Futures

In this framework, a futures contract on the Cooling Degree Days (CDD) index, where the index refers to times $T_i, i = 1, ..., n$, can be priced as

$$\begin{aligned} CDD_t &= \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left\{-\int_t^{T_n} r_s ds\right\}\sum_{i=1}^n (Y_{T_i} - \eta c)^+\right] \\ &= \sum_{i=1}^n \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left\{-\int_t^{T_n} r_s ds\right\}(Y_{T_i} - \eta c)^+\right] \\ &= \sum_{i=1}^n \mathbb{E}_t^{\mathbb{Q}}\left[(F(T_i, T_i) - \eta c)^+\right]P(t, T_n), \end{aligned} \tag{58}$$

where c is the threshold temperature.

3.5.3 Pricing the HDD Futures

In this framework, a futures contract on the Heating Degree Days (HDD) index, with reference times for the index $T_i, i = 1, ..., n$, can be priced as

$$\begin{aligned} HDD_t &= \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left\{-\int_t^{T_n} r_s ds\right\}\sum_{i=1}^n (\eta c - Y_{T_i})^+\right] \\ &= \sum_{i=1}^n \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left\{-\int_t^{T_n} r_s ds\right\}(\eta c - Y_{T_i})^+\right] \\ &= \sum_{i=1}^n \mathbb{E}_t^{\mathbb{Q}}\left[(\eta c - F(T_i, T_i))^+\right]P(t, T_n). \end{aligned} \tag{59}$$

3.5.4 The fundamental absence-of-arbitrage relationship

We conclude the section by showing how the fundamental absence-of-arbitrage relationship between CAT, CDD, and HDD futures is maintained in this framework. Let us recall the put-call parity relationship between options

$$C_t - P_t = S_t - KP(t, T_n). \tag{60}$$

Then, in the case of weather derivatives, we have that

$$\begin{aligned}
CDD_t - HDD_t &= \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}}\Big[(F(T_i, T_i) - \eta c)^+\Big]P(t, T_n) - \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}}\Big[(\eta c - F(T_i, T_i))^+\Big]P(t, T_n) \\
&= \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}}\Big[(F(T_i, T_i) - \eta c)^+ - (\eta c - F(T_i, T_i))^+\Big]P(t, T_n) \\
&= \sum_{i=1}^{n} (F(t, T_i) - \eta c)P(t, T_n) \\
&= CAT_t - \eta cnP(t, T_n),
\end{aligned}$$

which is the analogue of the classical put-call parity relationship for option pricing (Eq. 60) which follows from the no-arbitrage assumption.

3.5.5 Simulation study

We now use the above model to price the weather derivatives presented in Section 3.4.1. For each asset, we evaluate prices from the beginning of the accumulation month to its end. CDD and HDD contracts are treated as the sum of multiple call and put options, respectively, as in Eq. 58 and 59, having maturities T_i, $i = 1, ..., n$, which correspond to each day of the month. As time goes on, the realized payoffs of the days for which average temperatures have already been observed make up an increasingly larger portion of the price. In parallel, the stochastic component of the price, corresponding to the sum of the expected payoffs of the remaining days, shrinks overtime. At the end of the last accumulation day, the entirety of the cumulative payoff is known. Given that all weather derivatives in the sample concern U.S. cities, the discount factor takes as risk-free rate the Secured Overnight Financing Rate (SOFR) quoted at the time of each instrument's evaluation.
The primary asset of each maturity is assumed to follow the dynamic in Eq. 54, with volatility in Eq. 55. Pricing is performed via Monte Carlo, using as input, for each city, the estimated temperature process parameters in Table 37, the estimated realized volatility at the day of pricing (T_0), and, for the realized payoff, the historical daily average temperatures. For the purpose of simulation, it is assumed that the futures temperature price for maturity T_0 coincides with ηT_0, i.e. the monetary value of the temperature on the same day. Furthermore, it is worth noting that, under $\mathbb{P}$, we have that $\tilde{\theta} = \theta$, the long-term mean of the temperature process. However, crucially, the value of $\tilde{\theta}$ is unknown. It reflects the measure change from $\mathbb{P}$ to $\mathbb{Q}_T$ and would need to be calibrated on the time series of primary asset prices. We therefore carry out a sensitivity analysis, and price each CDD and HDD contract by assuming a number of potential values of $\tilde{\theta}$, including θ.
Figures 16-23 display the model-derived prices at T_0, the day before the start of the accumulation period, for multiple values of $\tilde{\theta}$, for all contracts in Section 3.4.1. For each instrument, the corresponding price quoted by the market at T_0 is also represented. Additionally, the realized cumulative payoff at T_n, the end of the accumulation period, is also shown. For all futures and all cities, model-derived prices fall in a range that is much more compatible with the actual, realized cumulative payoff than market prices. In the plots in Figures 16, 18, 19, and 21, which respectively refer to contracts for the temperature in Atlanta, Cincinnati, Dallas, and Minneapolis, an interesting pattern emerges. For these cities, in percentage terms, contracts referring to the month of May have quoted market prices that are much closer to model-implied prices, and therefore also to realized payoffs at maturity, than any other month. In these cases, the month of May therefore shows the lowest level of market mispricing. In contrast, such difference does not emerge in New York and Sacramento. For them, as can be seen in Figures 22 and 23, the distance between quoted and simulated prices is uniformly large, across the different months. As for Las Vegas, the only available contract refers to the month of May, and it is therefore impossible to perform a comparison across different months. However, from Figure 20, it can be seen that, in percentage terms, distance between market prices and model-implied prices is comparable to that of New York and Sacramento.

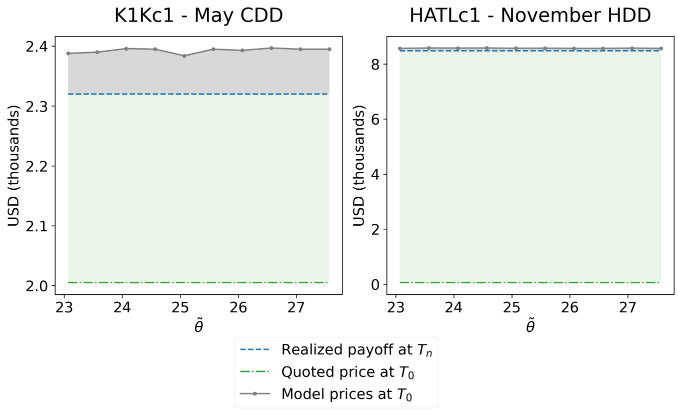


Figure 16: Atlanta CDD and HDD contract prices at T_0 for different levels of $\tilde{\theta}$

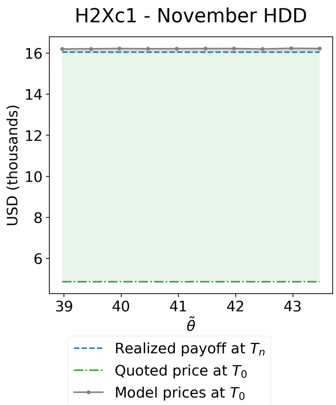


Figure 17: Chicago CDD and HDD contract prices at T_0 for different levels of $\tilde{\theta}$

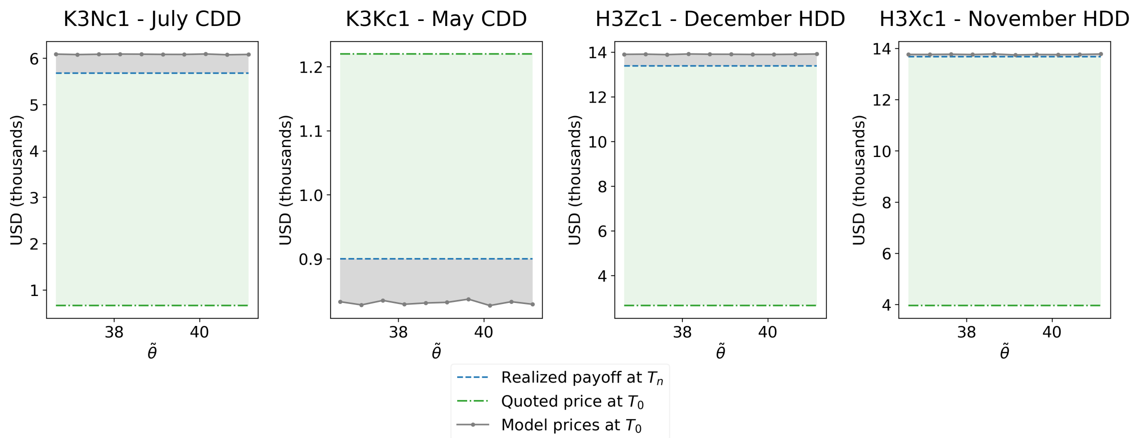


Figure 18: Cincinnati CDD and HDD contract prices at T_0 for different levels of $\tilde{\theta}$

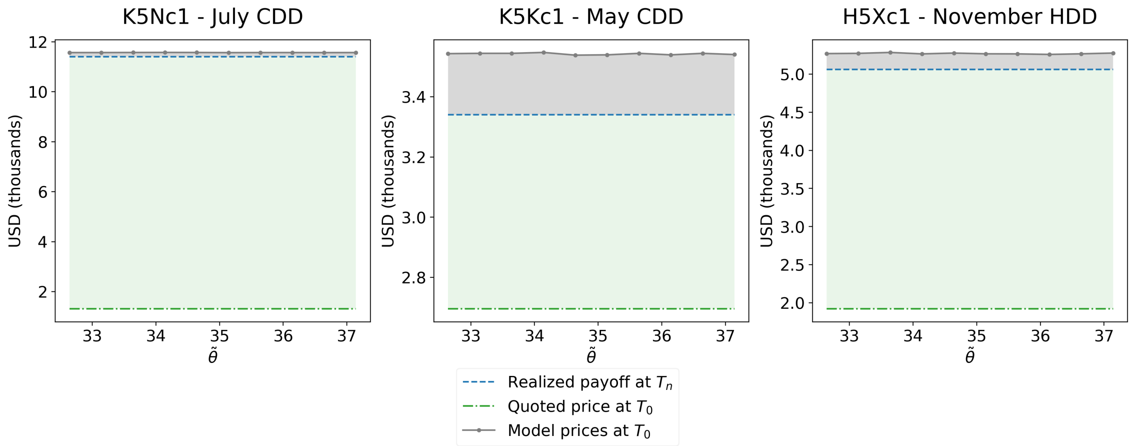


Figure 19: Dallas CDD and HDD contract prices at T_0 for different levels of $\tilde{\theta}$

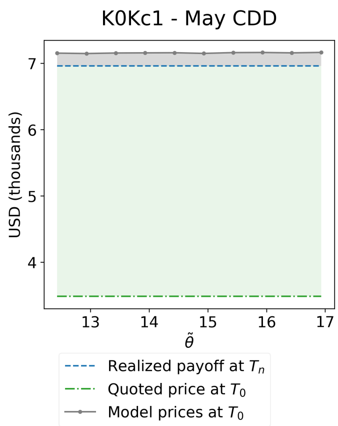


Figure 20: Las Vegas CDD and HDD contract prices at T_0 for different levels of $\tilde{\theta}$

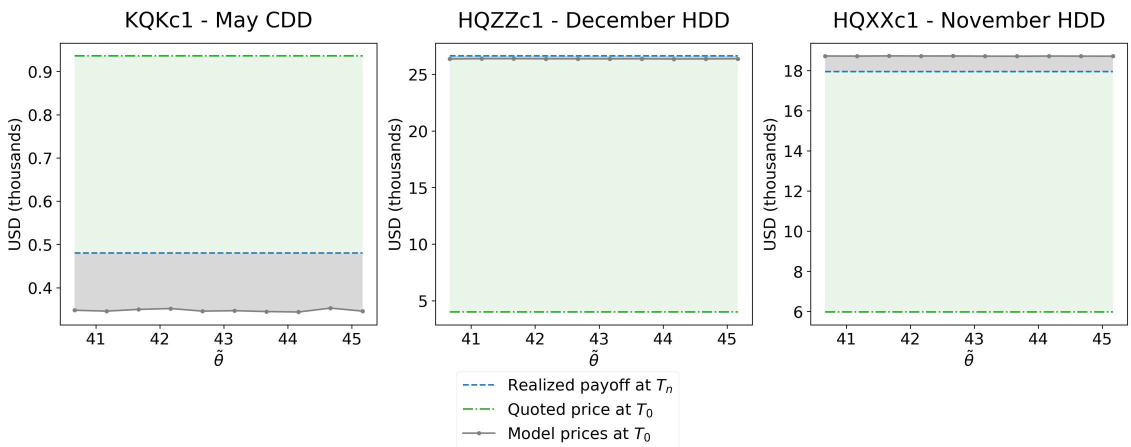


Figure 21: Minneapolis CDD and HDD contract prices at T_0 for different levels of $\tilde{\theta}$

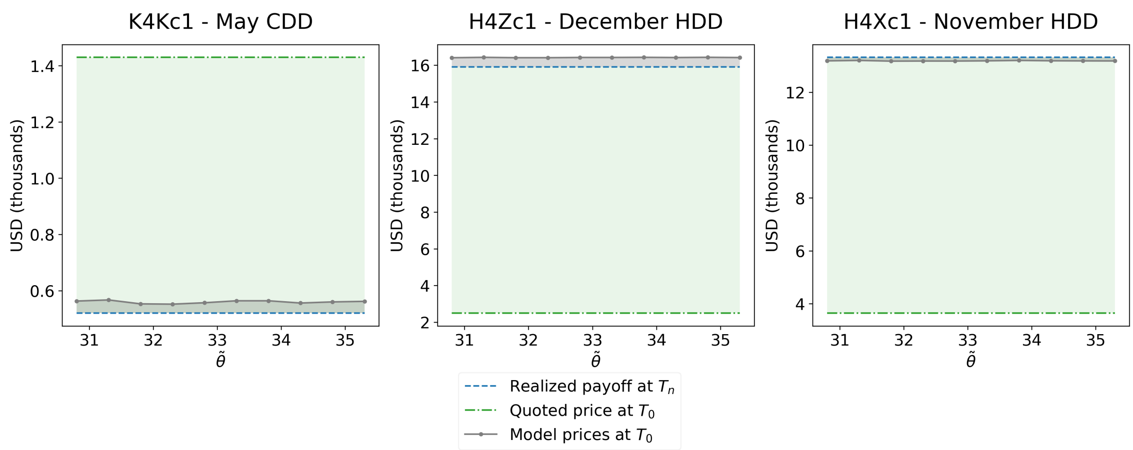


Figure 22: New York CDD and HDD contract prices at T_0 for different levels of $\tilde{\theta}$

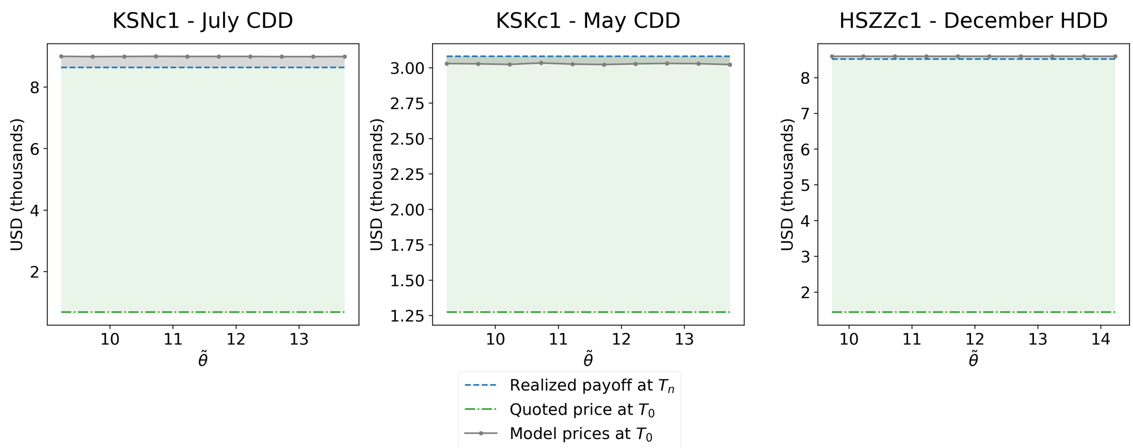


Figure 23: Sacramento CDD and HDD contract prices at T_0 for different levels of $\tilde{\theta}$

The contracts are then priced over the entire cumulation period $(T_1, ..., T_n)$, each day based on the updated information about realized temperature and volatility. Figures 24 - 31 display a surface of model-derived prices, plotted overtime and for multiple values of $\tilde{\theta}$. The price corresponding to the value of $\tilde{\theta}$ under $\mathbb{P}$ is emphasized. Furthermore, quoted market prices and the cumulative realized payoff are also plotted, akin to the graphs in Figures 8-15. The model property of convergence, at maturity, between contract price and realized payoff is highlighted by the figures and holds for all values of $\tilde{\theta}$. A pattern again emerges, differentiating between two groups of cities: Atlanta, Cincinnati, Dallas, and Minneapolis, on the one hand, and Las Vegas, New York, and Sacramento, on the other. The corresponding plots are in Figures 24, 26, 27, and 29, for the first group, and in Figures 28, 30, and 31, for the other. Once again, in the month of May and for the first group of cities, the model-implied prices are much closer to the corresponding quoted prices, than for the second group of cities. Interestingly, the same division between locations is also preserved when considering the parameter estimates of the temperature volatility. They are reported, for all cities, in Table 37. Atlanta, Cincinnati, Dallas, and Minneapolis have consistently lower mean-reversion speed (k), higher long-term level (θ), and higher volatility of the volatility (σ) than Las Vegas, New York, and Sacramento. These features all lead to greater variability in the temperature process. Additionally, in May, average temperatures are much closer to the option strike value of 65°F than in the other contract months under consideration. These factors are connected to greater uncertainty when performing pricing and could thus be behind the comparatively poorer model performance in the first group of cities.

Finally, the main result that emerges from both sets of figures is that any attempt to drive model-derived prices closer to quoted prices is fruitless, regardless of the value of $\tilde{\theta}$. This is however not a commentary on the quality of the model. In fact, any measure change, no matter how carefully selected, is unable to explain the observed quoted values. This is because market prices do not satisfy the most basic property: they fail to reflect the cumulative payoff realized at any point in time.

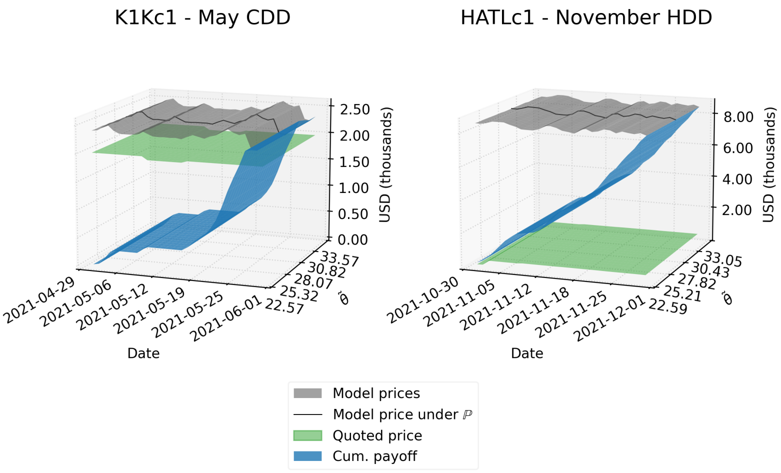


Figure 24: Atlanta contract prices for different levels of $\tilde{\theta}$

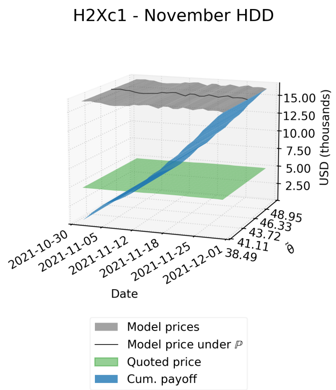


Figure 25: Chicago contract prices for different levels of $\tilde{\theta}$

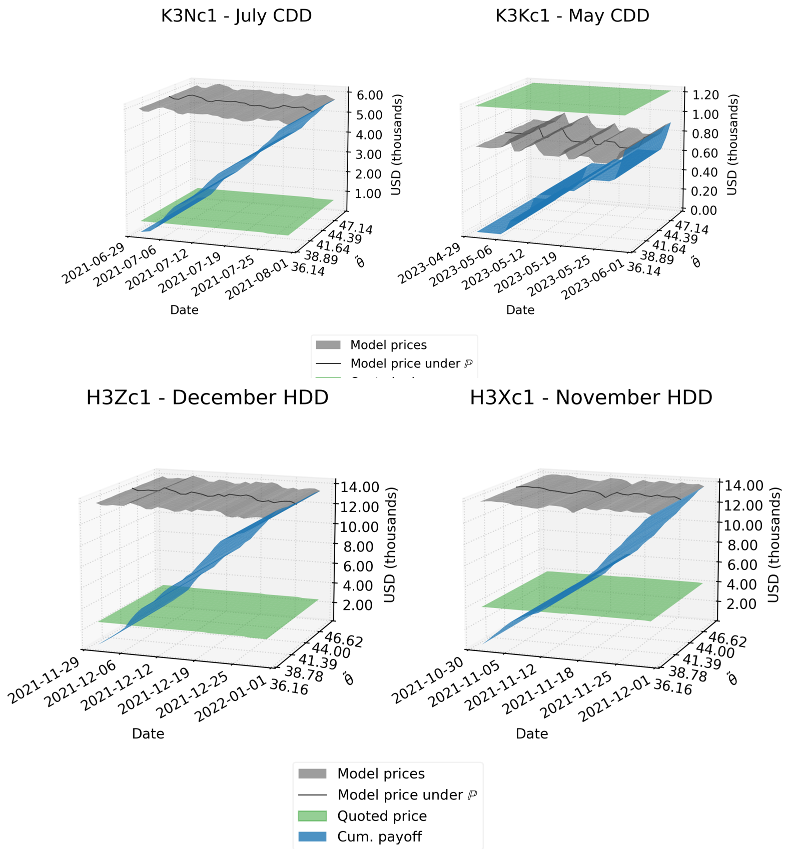


Figure 26: Cincinnati contract prices for different levels of $\tilde{\theta}$

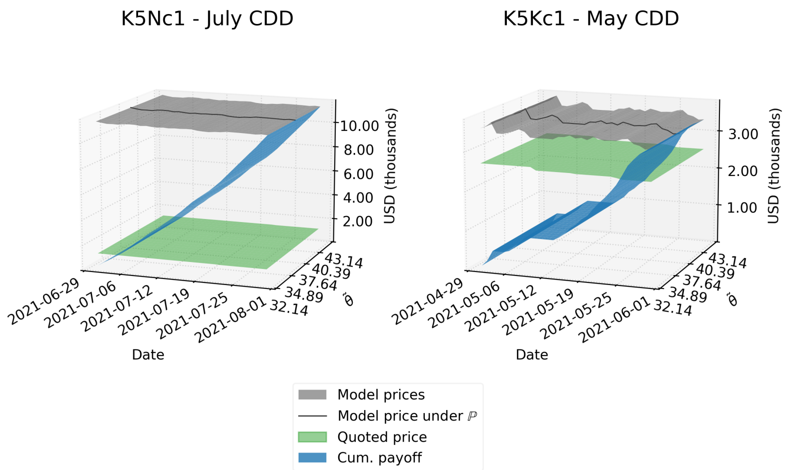


Figure 27: Dallas contract prices for different levels of $\tilde{\theta}$

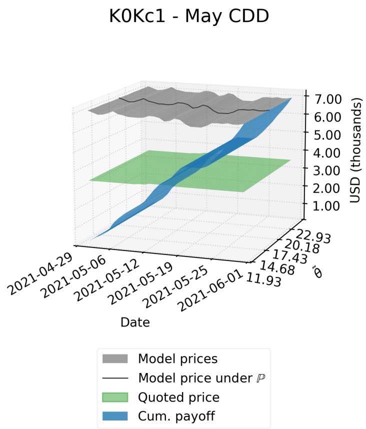


Figure 28: Las Vegas contract prices for different levels of $\tilde{\theta}$

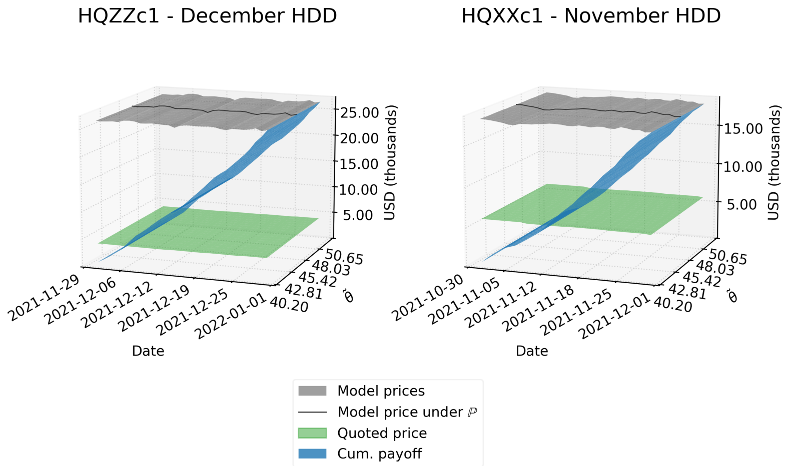


Figure 29: Minneapolis contract prices for different levels of $\tilde{\theta}$

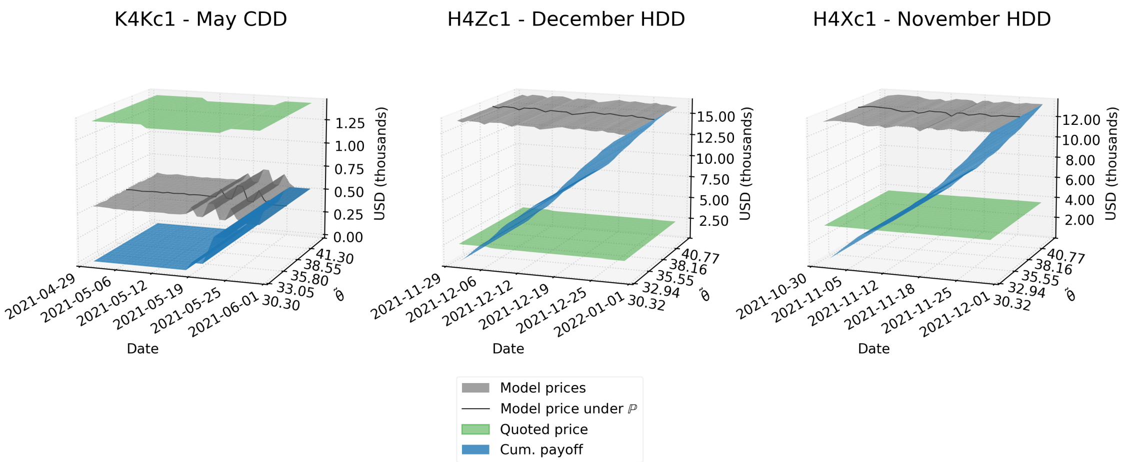


Figure 30: New York contract prices for different levels of $\tilde{\theta}$

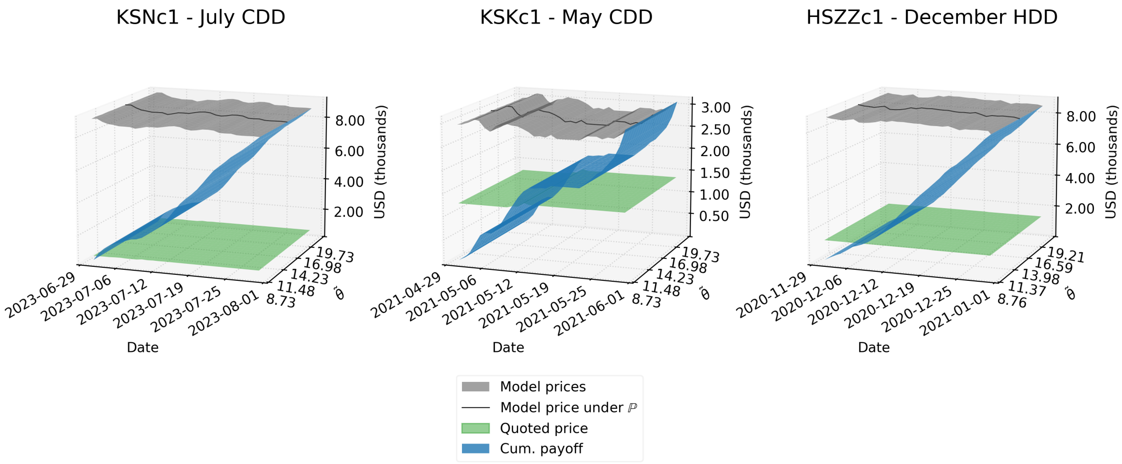


Figure 31: Sacramento contract prices for different levels of $\tilde{\theta}$

In the end, the simulation study emphasizes the features of the model: its ability to converge at maturity to realized payoffs, to accommodate the role of market forces on asset dynamics, and to address the fundamental market incompleteness. What also emerges is that any attempt to drive model-derived prices closer to quoted prices is hopeless, regardless of the value of the parameter of interest. This does not provide information about the quality of the model. In fact, any measure change, no matter how carefully chosen, would fail to explain the observed quoted values, since market prices do not satisfy the most basic property: they fail to reflect the cumulative payoff realized at any point in time.

3.6 Mitigation Strategies for Environmental Resources: Addressing Water Shortage

3.6.1 Mitigation policy: water market system and financial-physical hedgers

The cornerstone of an effective mitigation strategy lies in the development of a robust water market system. This system transforms water from a mere natural resource into a tradable financial asset, complete with a quantifiable price and manageable risks. Central to this concept is the notion of water rights; legal entitlements that grant individuals or entities the privilege to utilize water resources for specific purposes without conferring ownership of the water itself. These rights essentially represent a form of access or usage rights, known as usufructuary rights.
Two primary categories of water rights prevail: prior appropriation water rights and riparian water rights. Prior appropriation water rights operate on the principle of "first in time, first in right." This doctrine asserts that the first person or entity to divert a quantity of water from a water source for a beneficial purpose, such as agriculture, industry, or domestic use, possesses the perpetual right to continue using that quantity for the designated purpose. Importantly, these rights are transferable and can be traded on the open market, akin to other forms of property.
In scenarios where multiple users share access to a water source, typically found in regions with high demand or limited supply, government bodies or quasi-governmental agencies oversee the allocation and regulation of water rights to ensure equitable distribution and sustainable usage. Conversely, riparian water rights allocate water among landowners whose properties directly abut a watercourse. Under this system, individuals or entities holding riparian rights are entitled to reasonable use of the water flowing through or adjacent to their land, with usage rights often prioritized based on proximity to the water source.
Across various regions globally, including countries such as Australia, Chile, Iran, and the United Kingdom, as well as numerous states within the United States, including Arizona, California, Colorado, New Mexico, and Texas, water markets have been established to facilitate the trading of water rights. These markets offer a mechanism for reallocating water resources efficiently, promoting sustainable water management practices, and addressing challenges arising from fluctuating demand, climate variability, and water scarcity. From an economic perspective, the implementation of water markets aims to optimize the allocation of water resources by imbuing water rights holders with a vested interest in their utilization. By quantifying water rights as valuable assets on their balance sheets, owners of prior appropriation rights are incentivized to manage water resources prudently, minimizing waste and maximizing efficiency to capitalize on potential revenue

streams generated from the sale of surplus rights on the market. Conversely, water buyers view water as a commodity subject to market forces, recognizing it as a tangible cost within their operational budgets. This paradigm shift incentivizes water users to adopt conservation measures, invest in water-saving technologies, and adopt sustainable practices to mitigate financial risks associated with water scarcity and ensure the long-term viability of water resources.

Mitigation pertains to a risk management challenge, particularly concerning water users such as farmers, who face heightened exposure to water-related risks due to their heavy reliance on water for agricultural activities. Within a market framework, water users encounter two primary risks: volumetric risk, stemming from fluctuations in water demand corresponding to variations in rainfall, and the risk of escalating water prices in the market, given that water users are essentially positioned with a short exposure to water prices. For a water user, the demand for water, denoted as $D(t, R_t, S_t)$ at time t, is contingent upon the average rainfall, R_t, and prevailing market price for water, S_t. Notably, the demand for water exhibits an inverse correlation with rainfall, expressed as $\partial D(t, R_t, S_t)/\partial R_t < 0$. The supply of water $SW(t, Y_t, \alpha, K)$ at time t is determined by the basin level Y_t from which the water is sourced, where α represents the rationing ratio triggered if the basin level falls below a specified threshold (K). Both K and α are regulatory parameters established by governing authorities.

The hedging instruments we have in mind can be categorized as weather derivatives. The weather derivatives market functions as a financial marketplace where investors have the opportunity to trade weather-related contracts, including futures or options, in order to manage or mitigate their exposure to weather-related risks. These contracts typically reflect variations in temperature, precipitation, or other weather indices and are settled in cash based on actual weather conditions over a specified timeframe. Primarily utilized for risk management purposes, the weather derivatives market serves businesses or individuals whose financial performance is impacted by weather conditions. These include entities in sectors such as energy, agriculture, tourism, construction, and insurance. For instance, an energy company might opt to purchase a weather derivative contract that pays out if temperatures fall below a certain threshold, as this would likely result in increased demand for heating or electricity.

We have identified two new families of weather derivatives that can function as either physical or financial hedgers, depending on the underlying nature, to mitigate volumetric risks associated with water scarcity resulting from light rainfall or low water reservoir levels. Let's examine two contracts from these families:

1. Quanto option (RQO hereafter) on rainfall: This contract's payoff is $(K - R_T)^+ AS_T$, where R_T represents average rainfall in millimeters, A denotes the area of interest in square meters, K is the strike rainfall level in millimeters, and S_T is the water price expressed in m^3, i.e., dollars per cubic meter.

2. Cash or nothing option (BLCON hereafter): In this contract, the payment is the price of water at maturity if the basin level Y_T is below a certain threshold, $QS_T \mathbf{1}_{(Y_T \leq K)}$, where Q represents the water amount in cubic meters.

The digital option safeguards against volumetric risks arising from extremely low basin levels that compel users to resort to the market. On the other hand, the RQO mitigates volumetric risks associated with scarce rainfall, such as during drought periods.

Given the geolocalized nature of the risks involved, it is pertinent to provide some commentary. The weather derivatives market has experienced rapid growth over the past few decades, particularly in regions characterized by significant variability in weather patterns, such as North America, Europe, and Asia. However, it remains relatively small compared to other financial markets and presents challenges and uncertainties regarding the accuracy of weather forecasting, the availability of reliable weather data, and the pricing of weather risk. Consequently, measures aimed at fostering the development of this market can serve as valuable mitigation policies, particularly concerning water scarcity. It is noteworthy that the European Centre for Medium-Range Weather Forecasts (ECMWF), situated in Bologna, Italy, strategically located in central Italy with convenient access to various European countries, plays a pivotal role in this domain. ECMWF attracts top-tier weather and climate experts from around the world. Additionally, the Euro-Mediterranean Center on Climate Change (CMCC), a non-profit research institute focusing on climate science, modeling, and climate adaptation and mitigation strategies, contributes significantly to advancements in weather forecasting and access to reliable weather data, facilitated by Copernicus satellite data. Given its prominence and collaborations with numerous institutions across Europe and beyond, we believe it can serve as the official EU station for data recording and weather forecasting. This EU research district represents an ideal environment for the establishment and growth of a regulated

EU weather derivatives market. A dedicated trading space is envisioned to facilitate the implementation of derivative instruments, such as those based on rainfall and temperatures, to mitigate water scarcity and shortages. This market holds the potential to offer partial hedging against climate-related risks encountered by industries amidst the transition towards sustainability and the impacts of climate change. Regarding water scarcity, while it may not offer perfect mitigation due to potential discrepancies in the recording station's location relative to the enterprise itself, it addresses the challenge of *geographical hedging*.

3.6.2 Modeling Rainfall: Approaches and Techniques

Rainfall modeling is critical for sectors such as agriculture, water resource management, and financial risk mitigation, where accurate predictions can guide decisions and reduce risk exposure. This process involves forecasting or simulating rainfall patterns using statistical and stochastic techniques derived from historical and environmental data. An ideal approach to modeling rainfall integrates statistical models and physical models, as seen in weather generators. These systems combine statistical analysis of past weather patterns with physical atmospheric processes, allowing for a more comprehensive understanding of weather events. However, weather generators demand vast datasets with intra-day resolution and substantial computational resources, which are typically available only at specialized research centers. Given the lack of access to a weather generator, we will adopt a purely statistical approach based on stochastic processes. This method, while less complex than integrating physical models, can still effectively capture key rainfall characteristics by modeling variability and randomness in precipitation patterns. Using stochastic techniques allows for flexible modeling of uncertainties and can be applied across various time scales, providing a practical alternative for data-driven decision-making in scenarios where computational resources are limited. A naive model for the rainfall event would be based on Compound Poisson process where the Poisson process count the rain occurrence and the jump size is modeled as i.i.d variables representing the rainfall amount. Anyway, such an approach wouldn't be correct to represent the rainfall for two reasons: first, it assumes that the rainfall occurrence and its amount are independent, and second, the trajectories of the Compound Poisson process have divergent behavior meaning that in simulations it's almost certain to generate scenarios where the total amount of rain in three months is the same as the total amount of rain in ten years. So, to avoid such problems we follow a different approach. Let H_t be the random variable representing the rainfall event at t, where the support is $\mathbb{R}^+$ s.t. $\mathbb{P}(H_t = 0) > 0$. Let's define I_t as a binary variable that takes values $\{0, 1\}$ representing the occurrence of the rainfall event at time t. Then,

$$\begin{aligned}
&\mathbb{P}(H_t < h) = \mathbb{P}(H_t < h, I_t = 0) + \mathbb{P}(H_t < h, I_t = 1)\\
&= \mathbb{P}(H_t < h|I_t = 0)\mathbb{P}(I_t = 0) + \mathbb{P}(H_t < h|I_t = 1)\mathbb{P}(I_t = 1)\\
&= \mathbb{P}(I_t = 0) + \mathbb{P}(H_t < h|I_t = 1)\mathbb{P}(I_t = 1),
\end{aligned} \tag{61}$$

where $\mathbb{P}(H_t < h|I_t = 1)$ stands for the probability for a given intensity of rainfall when a rainfall occurrence happens, and consequently, $\mathbb{P}(H_t < h|I_t = 0) = 1 \; \forall y \geq 0$. In this setting, we will directly model the probabilities $\mathbb{P}(I_t = 1)$ and $\mathbb{P}(H_t < h|I_t = 1)$ without losing the dependence between the occurrence of the rainfall event and the severity of it. The quantity $\mathbb{P}(I_t = 1)$ will be modeled with a logistic distribution since such a model allows for the inclusion of external factors affecting the probability of observing a certain event. Therefore,

$$\mathbb{P}(I_t = 1) = \frac{1}{1 + \exp\{-(\omega + \beta C)\}}, \tag{62}$$

where C is an array off regressors, β is the vector of coefficients, and ω is the intercept. For the probability $\mathbb{P}(H_t < h|I_t = 1)$ we can use any continuous random variable with positive support. The choice of the appropriate one needs to be made according to the data on the rainfall amount. In this work, we will try the exponential distribution, the log-normal distribution, and the inverse Gaussian distribution.

3.6.3 Modeling the Water Price: Approaches and Considerations

Modeling water prices involves understanding and predicting fluctuations based on various factors, including supply and demand dynamics, climate conditions, regulatory frameworks, and market mechanisms. Water pricing models are critical for managing water resources, evaluating investment in water-related infrastructure, and designing financial instruments like water futures and derivatives. To model the spot price of water we consider an exponential process S_t driven by a Lévy process, i.e.

$$
\begin{aligned}
&S_t = S_{t_0} \exp\left\{ \sum_{n=1}^{N_t^{(S)}} Z_n^{(S)} \right\}, \\
&Z_n^{(S)} \sim \mathcal{N}(0, \sigma_z^2) \quad i.i.d \quad \forall n \in \mathrm{N}, \\
&N_t^{(S)} \sim Poi(\lambda^{(S)} t), \\
&\log\left(\frac{S_t}{S_{t_0}}\right) = \sum_{n=1}^{N_t^{(S)}} Z_n^{(S)},
\end{aligned}
\tag{63}
$$

where $N_t^{(S)}$ is a Poisson process with intensity $\lambda^{(S)}$ and jumps size $Z_n^{(S)}$ which are i.i.d. normally distributed with zero mean and variance σ_z^2.

3.6.4 Modeling the basin level

Modeling the basin level, which is the water level within a river basin, requires understanding the interactions between rainfall, evapotranspiration, surface runoff, and groundwater contributions. This process is crucial for effective water resource management, flood prediction, and assessing drought risks. Several methods can be used to model basin-level dynamics Y_t; we have exploited an exponential model where the logarithmic variations are described by a Lévy Ornstein-Uhlenbeck process, i.e.

$$
\begin{aligned}
&Y_t = Y_{t_0} \exp\{X_t\} \\
&dX_t = -kX_t dt + \sigma dW_t + dL_t,
\end{aligned}
\tag{64}
$$

where Y_t is the basin level, X_t is the log-variation of the basin level driven by the Langeville process, $L_t = \sum_{j=1}^{N_t^{(L)}} Z_j^{(L)}$ is a Compound Poisson Process whose jump size $Z_j^{(L)}$ normally distributed with zero mean and variance σ_Z^2, independent of the Poisson process $N_t^{(L)}$ with intensity $\lambda^{(L)}$, W_t is a standard Brownian Motion, and k is the speed of mean reversion. Then, by setting $f(t, X_t) = \exp\{-kt\}X_t$ and applying the ItÃ´'s lemma for Lévy processes, the basin level in Eq. 64 is given by:

$$
X_t = X_{t_0} e^{-k(t-t_0)} + \sigma \int_{t_0}^{t} e^{-k(t-s)} dW_s + \int_{t_0}^{t} e^{-k(t-s)} dL_s, \tag{65}
$$

whose characteristic function (see Rocha-Arteaga et al. 2019), is

$$
\varphi_{X_t}(\xi) = \exp\left\{ i\xi X_{t_0} g(t_0) - \frac{\sigma^2 \xi^2}{4k}\left(1 - e^{-2k(t-t_0)}\right) + \int_{t_0}^{t} \psi_L(\xi g(s)) ds \right\}, \tag{66}
$$

where, $\psi_L(\xi)$ is the characteristic exponent (see Tankov, 2003) of the Lévy process L_t, where $g(s) = e^{-k(t-s)}$.

3.7 Pricing of the physical-financial hedgers

Incorporating a natural variable into traditional financial frameworks introduces a unique challenge in defining the appropriate pricing measure. Financial contracts, such as weather derivatives or catastrophe (CAT) bonds, have value determined not only by market-driven variables like asset prices or indices but also by non-financial, physical variables like temperature, rainfall, or seismic activity. For conventional financial instruments, pricing is typically done under the risk-neutral or martingale measure, where expected payoffs are discounted using a risk-free rate. This measure allows for arbitrage-free pricing by transforming the real-world probabilities of financial outcomes to reflect investor risk preferences. However, when it comes to natural variables, the market does not influence their probabilities. For example, the likelihood of an earthquake or a hurricane occurring is governed by physical processes, independent of market dynamics like supply and demand for financial assets. Therefore, in line with established pricing practices for CAT bonds, the transition from the real-world probability (P-measure) to the risk-neutral measure (Q-measure) does not alter the underlying probability distribution of the natural event triggering the payout. This is because market participants cannot influence the occurrence of such physical eventsÑthey can only

react to them. While market forces may influence the price or demand for CAT bonds, the event's probability remains exogenous and unaffected by financial markets. Consequently, when pricing instruments involving natural variables, it is common to assume that the physical probability distribution remains unchanged under the risk-neutral framework. This simplifies the pricing process by focusing on the financial aspects while acknowledging the fixed nature of the physical risk. In summary, the pricing measure for natural-event-driven financial contracts is a hybrid approach: while financial aspects adhere to risk-neutral principles, the natural variables remain governed by their physical probability distributions, reflecting their independence from market forces.
In this context we define the price of the weather derivatives as a martingale, meaning that even in the context of weather derivatives the price should only reflects the information up to the current time. However, due to the unique structure of the contracts, where there is a product between two distinct processesthe physical variable and the financial one (water price), we need to validate the pricing method by demonstrating that the contingent claim, denoted as $\Pi(t,T)$, behaves as a martingale, in both cases, the RQO and the BLCON. For the RQO we have, $R_t = \frac{1}{n}\sum_{i=0}^{n} Y_{t_i}$, which stands for the average rainfall between t_0 and t, and S_t being the price of water, we define the price at time t of the RQO , with maturity T, as the expected value of the discounted payoff $(K - R_T)^+ A S_T e^{-r(T-t)}$, where r is the constant risk-free rate, K is the strike level fo the rainfall, and A is the area (square meters) of interest. Then, for the payoff $QS_T\mathbf{1}_{(Y_T \leq K)}$ of the digital BLCON, where S_T stands for the price of water at maturity T, Y_T is the basin level at time T, K is the threshold level of the basin, and Q is the amount of water.

It is worth noticing that the exponential Lévy process in Eq. 63 is not a martingale when discounted by the bank account $B_t = e^{rt}$, where r is the constant risk-free rate, i.e.

$$\begin{aligned}
\mathbb{E}^{\mathbb{Q}}_{t_0}\left[\frac{S_t}{B_t}\right] &= \frac{S_{t_0}}{B_{t_0}}\mathbb{E}^{\mathbb{Q}}_{t_0}\left[\frac{B_{t_0}}{B_t}\exp\left\{\sum_{n=1}^{N_t^{(S)}} Z_n^{(S)}\right\}\right] \\
&= \frac{S_{t_0}}{B_{t_0}}\mathbb{E}^{\mathbb{Q}}_{t_0}\left[\exp\left\{-r(t-t_0) + \sum_{n=1}^{N_t^{(S)}} Z_n^{(S)}\right\}\right] \\
&= \frac{S_{t_0}}{B_{t_0}}\exp\left\{-r(t-t_0) + \lambda^{(S)}(t-t_0)\left(e^{\frac{\sigma_z^2}{2}} - 1\right)\right\}.
\end{aligned}$$

Therefore, the compensated process used for option pricing will be:

$$S_t = S_{t_0}\exp\left\{r(t-t_0) - \lambda^{(S)}(t-t_0)\left(e^{\frac{\sigma_z^2}{2}} - 1\right) + \sum_{n=1}^{N_t^{(S)}} Z_n^{(S)}\right\}. \tag{67}$$

Proposition 3.1. Let $\Pi(t,T)$ be the price of the RQO and BLCON at time t, then $\Pi(t,T)e^{-rt} = \mathbb{E}^{\mathbb{Q}}[\Pi(T,T)e^{-rT}|\mathcal{F}_t]$ is a martingale w.r.t. the natural filtration $\mathcal{F}_t$.

Proof. By definition $\Pi(t,T)e^{-rt}$ is $\mathcal{F}_t$ measurable. Then,

$$\begin{aligned}
\mathbb{E}^{\mathbb{Q}}[(K-R_T)^+ A S_T e^{-rT}] &= A\mathbb{E}^{\mathbb{Q}}[(K-R_T)^+ S_T e^{-rT}] \\
&\leq Ae^{-rT}\mathbb{E}^{\mathbb{Q}}\left[[(K-R_T)^+]^2\right]^{\frac{1}{2}}\mathbb{E}^{\mathbb{Q}}\left[[S_T]^2\right]^{\frac{1}{2}} \\
&\leq AKe^{-rT}\mathbb{E}^{\mathbb{Q}}[S_T^2]^{\frac{1}{2}} \\
&< +\infty
\end{aligned}$$

Then, for BLCON:

$$\mathbb{E}^{\mathbb{Q}}[S_T\mathbf{1}_{Y_T \leq K}] \leq \mathbb{E}^{\mathbb{Q}}[S_T] < +\infty,$$

since, by definition $S_t \in L^2$. Then for $s < t$

$$\begin{aligned}
\Pi(s,T)e^{-rs} &= \mathbb{E}^{\mathbb{Q}}[\Pi(T,T)e^{-rT}|\mathcal{F}_s] \\
&= \mathbb{E}^{\mathbb{Q}}\{\mathbb{E}^{\mathbb{Q}}[\Pi(T,T)e^{-r(T-t)}|\mathcal{F}_t]e^{-rt}|\mathcal{F}_s\} \\
&= \mathbb{E}^{\mathbb{Q}}[\Pi(t,T)e^{-rt}|\mathcal{F}_s],
\end{aligned}$$

since $\Pi(t,T) = \mathbb{E}^{\mathbb{Q}}[\Pi(T,T)e^{-r(T-t)}|\mathcal{F}_t]$. □

Closed-form formulas are not available for the models used for Y_t, R_t, and S_t, so the pricing can only be done via Monte Carlo methods. Let $\Pi(t,T)$ be the price of the BLCON with maturity T at time t under the risk-neutral measure $\mathbb{Q}$,

$$\begin{aligned}
\Pi(t,T) &= \mathbb{E}_t^{\mathbb{Q}}\Big[e^{-r(T-t)}QS_T\mathbf{1}_{Y_T\leq K}\Big] \\
&= \mathbb{E}_t^{\mathbb{Q}}\Big[e^{-r(T-t)}QS_t\exp\Big\{r(T-t)-\lambda^{(S)}(T-t)\Big(\frac{\sigma_z^2}{2}-1\Big)+\sum_{n=1}^{N_{T-t}^{(S)}}Z_n^{(S)}\Big\}\mathbf{1}_{Y_T\leq K}\Big] \\
&= QS_t\mathbb{E}_t^{\mathbb{Q}}\Big[\exp\Big\{-\lambda^{(S)}(T-t)\Big(\frac{\sigma_z^2}{2}-1\Big)+\sum_{n=1}^{N_{T-t}^{(S)}}Z_n^{(S)}\Big\}\mathbf{1}_{Y_T\leq K}\Big] \\
&= QS_t\exp\Big\{-\lambda^{(S)}(T-t)\Big(\frac{\sigma_z^2}{2}-1\Big)\Big\}\mathbb{E}_t^{\mathbb{Q}}\Big[\exp\Big\{\sum_{n=1}^{N_{T-t}^{(S)}}Z_n^{(S)}\Big\}\mathbf{1}_{Y_T\leq K}\Big].
\end{aligned}$$

Let $L^{(S)}=\sum_{n=1}^{N_{T-t}^{(S)}}Z_n^{(S)}$ be the Compound Poisson process associated to the Water price with intensity $\lambda^{(S)}$, jump size $Z_n^{(S)}$ i.i.d. $\forall n$, $Z_n^R\sim\mathcal{N}(0,\sigma_z^2)$. Let $L^{(S)}_{|N_{T-t}^{(S)}=N}\sim\mathcal{N}(0,N\sigma^2)$ be the process $L^{(S)}$ conditional to $N_{T-t}^{(S)}=N$ which is distributed with a Gaussian distribution $\mathcal{N}(0,N\sigma_z^2)$. Then,

$$\begin{aligned}
\Pi(t,T) &= QS_t\exp\Big\{-\lambda^{(S)}(T-t)\frac{\sigma_z^2}{2}\Big\}\mathbb{Q}(\mathbf{1}_{Y_T\leq K})\sum_{N=0}^{+\infty}\frac{(\lambda^{(S)}(T-t))^N}{N!}\int_{\mathbb{R}}e^x\frac{e^{-\frac{x^2}{2N\sigma^2}}}{\sqrt{2\pi N\sigma^2}}dx \\
&= QS_t\mathbb{Q}\Big(\int_t^T e^{-k(u-t)}dW_u+\int_t^T e^{-k(u-t)}dL_u\leq\log(K/Y_t)-X_te^{-k(T-t)}\Big).
\end{aligned}$$

To simplify the notation, we define G_t and $\mathcal{E}_t$ as:

$$\begin{aligned}
G_t &= \int_t^T e^{-k(u-t)}dW_u+\int_t^T e^{-k(u-t)}dL_u \\
\mathcal{E}_t &= \log(K/Y_t)-X_te^{-k(T-t)}
\end{aligned}$$

in order to write the quanto's price as :

$$\Pi(t,T)=QS_t\mathbb{Q}(G_t\leq\mathcal{E}_t)=QS_t\int_{-\infty}^{\mathcal{E}_t}f(z)dz,$$

where $f(z)$ is the density function associated with the process G_t. Since the knowledge of the density $f(z)$ for Lévy processes is rare, but the characteristic exponent is not, so one could solve or approximate the last integral by relying on the Fourier Inversion Theorem. Unfortunately, in this case, the solution of the inversion theorem does not lead to a closed-form solution.

3.8 The physical and financial hedging at work: numerical features and empirics

In this section, we focus on the Californian water market system whose market water price is represented by the Nasdaq Vales California Water Index. In California, water market transactions involve the temporary (i.e. lease) or permanent (i.e. sale) transfer of a wide range of water entitlements[16]). Water entitlements grant their owner the right to use defined amounts of water for specific purposes in certain locations. Commonly traded types of water entitlements in California include State Water Project (SWP) contracts, Central Valley Project (CVP) contracts[17], appropriative water rights, water stored underground (banked water), and adjudicated groundwater rights. This system has brought the birth of a market for which observed a spot price for water which is the Nasdaq Veles California Water Index (NQH20[18]). The Nasdaq Veles California Water Index (NQH2O) signifies the present value of water, determined by transactions related to water entitlements within California s surface water market and the four most actively traded

[16] https://www.waterboards.ca.gov/waterrights/boardinfo/waterrightsprocess.html
[17] https://water.ca.gov/water-basics/the-california-water-system
[18] https://www.nasdaq.com/market-activity/index/nqh2o/historical?page=1rowsperpage=10timeline=y10

adjudicated groundwater basins. Specifically, the Index takes into account transfers of surface water within the region served by the State Water Project (SWP), the Central Valley Project (CVP), and the Colorado River. It also considers groundwater transfers from the Central Basin, Chino Basin, Main San Gabriel Basin, and Mojave Basin. This Index relies on actual transactions sourced from significant regulated surface water and groundwater outlets. The transactional data used for calculating the Index is supplied to Nasdaq by WestWater Research. This information is anonymized, meaning Nasdaq is unaware of the identities of the involved parties. WestWater confirms and verifies each transaction before including it in the data reported to Nasdaq. Furthermore, WestWater only reports a transaction to Nasdaq after receiving approval from relevant regulatory authorities and complete execution by the counterparties. The Index, denominated in dollars per acre-feet, is released weekly on Wednesday mornings at 9:30 AM. Its value reflects all eligible transaction price data up to the conclusion of the preceding week[19].
For this work, we will use data made publicly available by the U.S. authorities, NASA, The State of California, and the Nasdaq stock market. Weather data are taken from the Prediction of Worldwide Energy Resource[20] (POWER) of NASA, where data can be found for various weather variables like temperature, rainfall, and wind intensity daily. The data of the basin level are taken from the California data exchange[21], which makes them available with daily frequency. Data on the water spot price, daily, of the Californian market are open-source at the Nasdaq[22].

3.8.1 The pricing of the RQO

Here, we simulate a market for RQOs written on the rainfall event taking place in Los Angeles U.S.. Figure 32 reports the rainfall events for Los Angeles from 01/07/2019 to 01/07/2023.

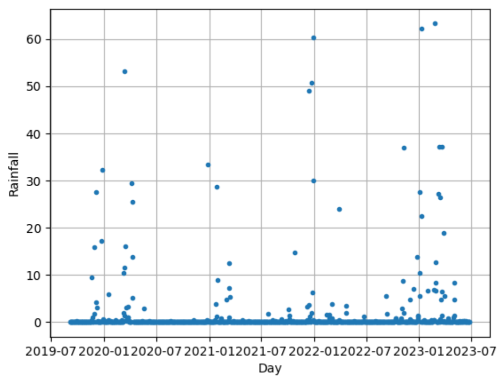


Figure 32: Rainfall Los-Angeles

Given the data of the rainfall event, reported in Figure 32 we fit via maximum likelihood the model in Eq. 61, with $\mathbb{P}(I_t = 1) = \frac{1}{1+e^{-(\omega+\beta Y_{t-1})}}$ (so, here $C = Y_{t-1}$) and where $\mathbb{P}(H_t < h|I_t = 1)$ is an Exponential, the Log-Normal, and Inverse Gaussian distribution[23] as well. In Table, 12 are reported the maximum likelihood estimates for Exponential, Log-Normal, and Inverse Gaussian random variables.

[19]`https://www.cmegroup.com/content/dam/cmegroup/trading/equity-index/files/understanding-the-water-futures-market.pdf`
[20]`https://power.larc.nasa.gov/beta/data-access-viewer/`
[21]`https://cdec.water.ca.gov/reportapp/javareports?name=DailyRes`
[22]`https://www.nasdaq.com/marketactivity/index/nqh2o/historicalpage=1rowsperpage=10timeline=y10`
[23]`https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.invgauss.html`

	μ_R	σ_R	ω	β	$\mathcal{L}$
Log-Normal	-1.7877	2.4767	-0.6489	0.3216	1705.5134
		λ_R	ω	β	$\mathcal{L}$
Exponential		0.3067	-0.6489	0.3216	1402.6305
		ω	β	μ_R	$\mathcal{L}$
Inverse Gaussian		3.2695	-0.6488	0.3216	5450.96

Table 12: Maximum Likelihood estimates Rainfall Los Angeles

It is clear that, for the data on the rainfall events in Los Angeles, the best model is the one with the Inverse Gaussian distribution based on the loglikelihood ($\mathcal{L}$) functions' values.

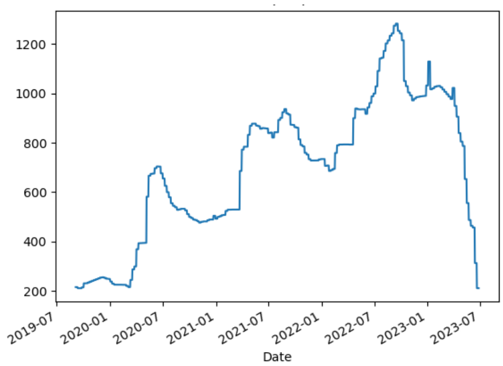


Figure 33: NQH2O index

Figure 33 shows the pure jump nature of the price for water justifying the use of the model reported in Eq. 63. The estimates on the log-returns, via method of moments, are reported in Table 13.

$\lambda^{(S)}$	σ_z
0.0325	0.16728

Table 13: Estimated parameters for the NQH2 index

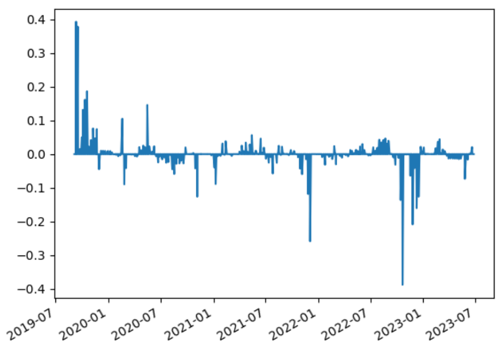


Figure 34: Log-returns NQH2O index

With the parameters for the water price reported in Tables 13, and the parameters for the three different models for the rainfall event reported in Table 12, we will price the RQO with different strikes and maturities to study the sensitivity of the price to the model parameters, maturities (expressed in days) and strike prices. Table 38 reports the RQO prices for different maturities (from 7 to 120 days, i.e. four months) and different strike prices, in the case of the rainfall event distributed with an Exponential random variable, Log-Normal random variable, and an Inverse Gaussian random variable respectively. Table 39, reports the mean and standard deviation of the

RQO prices for each given strike (under the three different models for the rainfall). We can see that regardless of the model the option price is characterized by a low standard deviation. On the other hand Tables 40, 41, and 42 report the sensitivity of the RQO price w.r.t. to the model parameters, as an example we have considered the RQO with 120 days of maturity. These tables also provide the statistics (mean and standard deviation) of the option prices for each parameter value. It is evident that the option price exhibits very low variability to changes in the model parameters, as indicated by the low standard deviations of the prices for varying parameters.

3.8.2 The pricing of the BLCON

In this section, we propose an example of pricing for the BLCON with the log-variation of the basin level $X_t = \log\left(\frac{Y_t}{Y_{t-1}}\right)$ modeled with a Lévy Ornstein-Uhlenbeck process (Eq. 65). We have chosen the Ornstein-Uhlenbeck structure to align with the autocorrelation function of X_t as shown in Figure 35. One can see that the discrete version of an OU process is an AR(1) model, whose autocorrelation has the form reported in Figure 35.

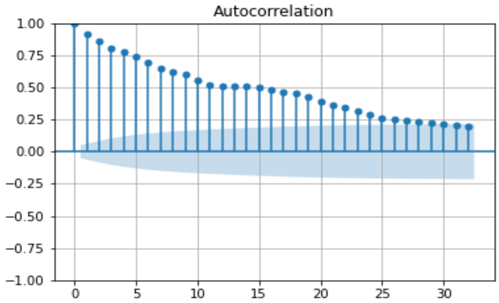


Figure 35: Empirical ACF X_t

In order to estimate the parameters for the process described by Eq.64 we rely on the inverse theorem for characteristic functions since the explicit density is not available. In contrast thanks to the Lévy-Khintchine formula the characteristic function has an explicit close formula, Eq. 66. Then the density $f_X(x)$ for the process X_t can be expressed as

$$f_{X_t|X_{t-1}}(x) = \frac{1}{2\pi}\int_{\mathrm{R}} e^{-i\xi(x-e^{-k\Delta t}x_{t-1})-\frac{(\Delta t\sigma\xi)^2}{2}+\lambda\Delta t(e^{-\frac{(\sigma_z\xi)^2}{2}}-1)}d\xi \tag{68}$$

with $\Delta t = 1$. Then, thanks to Eq. 68 we can estimate the model parameters via maximum likelihood, as reported in Table 14

k	σ	λ	σ_Z
0.3532	0.0285	0.6142	0.6937

Table 14: ML estimates Lévy OU basin level

Table 43 reports option prices estimated using the parameters for the basin level and the water price. We can see that, except for the options with a maturity of 7 days, the other prices are quite stable for a given strike and different maturities. The same behavior is observed when altering the parameter values; the pricing function remains quite stable to variations in the parameters; this is shown in Table 44, which reports the prices of the BLCON with a 120-day maturity. When changing one model parameter by a certain percentage, and Table 44 also reports the mean and standard deviation of the price for each column, we can see that the variability affects at most the second digit after the comma.

3.8.3 Hedging strategy

In this section, we show that such instruments can hedge the risk of scarce rainfall and shortage of water resources. In the first case, the hedging instrument is the RQO, while in the second case, it is the BLCON. To this aim let's consider first the case of scarce rainfall in Los Angeles from 01/10/2019 to 31/10/2019; in such period we observe a cumulative rainfall amount of just 0.03 mm. Let's suppose that a water user needs an average water amount of 1.5 mm in that period. Then, to cover his position the water user buys a RQO with a strike of 1.5 mm at time $t_0 = 01/10/2019$, and holds it up to maturity $T = 31/10/2019$. At time $T = 31/10/2019$ the price of water is 0.174 Euros per cubic meter, so the final cost of a naked is $(1.5 - \frac{0.03}{30})0.174 = 0.2608$. Then, the profit and loss reported in Table 15 shows that the covered position has produced a final cost of 0.06628, while the naked position has produced a loss (intended as a cost) of 0.2608.

	t_0	T	Result
Covered position	-0.06628	-0.2608 + 0.2608	-0.0662819
Naked position	0	-0.2608	-0.2608

Table 15: RQO profit and loss of covered and naked position

Now we consider a water user facing a scarcity of the water resource, namely the basin level is too low. So we consider again a 30-days maturity option, with a strike basin level of 13.5 (it is the natural logarithm of the basin level). The period of consideration goes from $t_0 = 1/12/2022$ to $T = 30/12/2022$ which corresponds to the minimum level reached by the Trinity Lake, and an increase of the price of water from 0.799 Dollars per cubic meter to 0.836 Dollar per cubic meter, such rise was due to an increase of the demand for water.

	t_0	T	Result
Covered position	-0.7999	-0.8364 + 0.8364	-0.7999
Naked position	0	-0.8364	-0.8364

Table 16: BLCON profit and loss of covered and naked position

Table 16 reports the profit and loss for a BLCON holder with a buy-and-hold strategy. In such a scenario the hedger will only suffer the loss due to the cost of the option which was lower than the cost of water at maturity.

3.8.4 Geographical hedging

Currently, the discussed contracts are neither traded on a quoted market nor standardized, since the entire weather derivatives market is not fully developed yet. In a future where the weather derivatives market is expected to be more developed and liquid, there will be standardized contracts quoted for specific locations. Therefore, there may be situations where a hedger needs to cover the risk related to a location for which a specific weather derivative doesn't exist. In such cases, the hedger needs to buy a contract referring to a different location that "behaves" similarly to the location of interest. This problem, known as geographical hedging, presents unique challenges. We consider now, a situation where the hedger is located in a different location than the one specified in the contract. To solve this issue we propose a model-free, statistical procedure to identify the number of contracts quoted for location A, Las Vegas in this example, for a hedger located in B, San Diego in this example. The cross-hedging in the case of weather derivatives could seem similar to the cross-hedging in the commodity market when for commodity A only the spot contract is traded and for hedging needs hedgers buy Futures quoted for a commodity B correlated to the commodity A, then in such case the optimal ratio is merely the linear correlation coefficient between the spot price of commodity A and the Future price of commodity B, which basically corresponds to the coefficient from a minimum variance portfolio. However, for RQO contracts this approach cannot be done for two main reasons: the first one is the non-linearity of the payoff $(K - R_T)^+ S_T$, and the second one to the fact that the quantity R_T is the average of rainfall between t_0 and T, with support on $\mathbb{R}^+$ and unknown multivariate distribution (if exists), which doesn't allow for the use of classic correlation estimators. Moreover, in our setting, the hedger must take into consideration that, in some cases, it may happen that in location A, $(K - R_T^{(A)})^+ = 0$, but in location B, $(K - R_T^{(B)})^+ > 0$, leaving the hedger uncovered. So we will tackle the problem in two steps: first, we will try to identify the degree of interdependence between what happens in locations A and B, and then we will identify the optimal ratio of contracts quoted on location A for a hedger in location B. As an example, we will consider Location A the city of Las Vegas as the location specified in the RQO, and for location B we will consider the city of San Diego; the RQO will be an option with maturity 30 days and strike of 1.5. To identify the dependence we consider the two variables $X_A = (K - R_T^{(A)})^+$, and $X_B = (K - R_T^{(B)})^+$ (Figure 36),
then we test if the probability of both the variables gives the same outcome, i.e., when both $X_A, X_B > 0$ and $X_A, X_B = 0$, these outcomes will be label with 1 and the opposite with 0, Figure

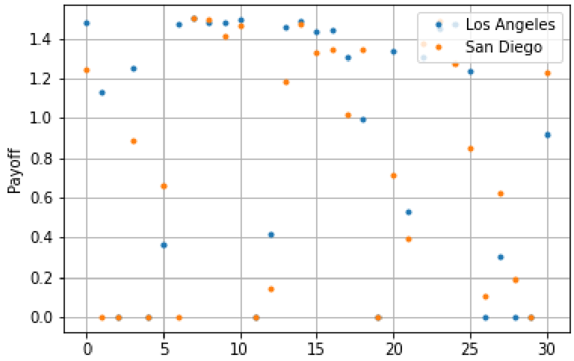


Figure 36: X_A=Los Angeles, X_B=San DIego

37.

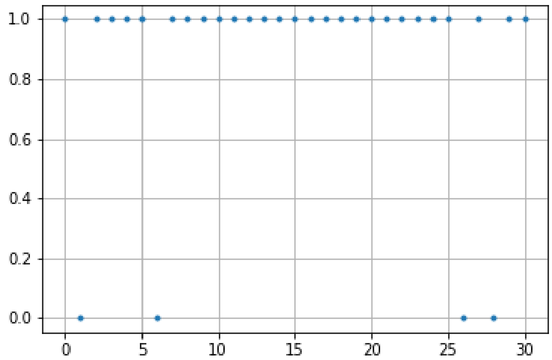


Figure 37: Concordance of X_A and X_B

Let π be the frequency of X_A and X_B of being concordant, then we consider the events in locations A and B to be concordant if π is greater than 50%, i.e. the two variables give the same outcome more than in the 50% of the cases. So, we perform the well-known test on a frequency, with statistic $V = \frac{\pi - \pi_0}{\sqrt{\frac{\pi(1-\pi)}{n}}}$, where $\pi = 0.871$ is the sampling frequency and π_0 is the frequency under $H_0 : \pi_0 = (0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8)$.

	50%	55%	60%	65%	70%	75%	80%
Statistic (V)	6.1612	5.3308	4.5004	3.67	2.8395	2.0091	1.1787
P-value	0	0	0	0.0001	0.0023	0.0223	0.1193

Table 17: Test statistic on π

From Table 17 we can see that the variables X_A and X_B give the same outcome with a probability of 80%, so for a hedger located in B (San-Diego) a contract buying the contract on located in Los Angeles could still offer a good protection. To determine the number of contracts located in A for a hedger in B we minimize the following loss function l:

$$\min_{\alpha} l(\alpha) = \sum_{i=1}^{n} [(\alpha X_{A,T_i} - X_{B,T_i}) S_{T_i}]^2, \tag{69}$$

i.e. we search for the number of contracts that minimize the discrepancy between the payoff. So the optimal number of contracts α will be the nearest integer to α since the numeric procedure can give as output any real number. In our example, $\alpha = 0.859$, the nearest integer is $\alpha = 1$. In such case, the time series of the loss function is reported in Figure 38 Then the sum of the cases with a

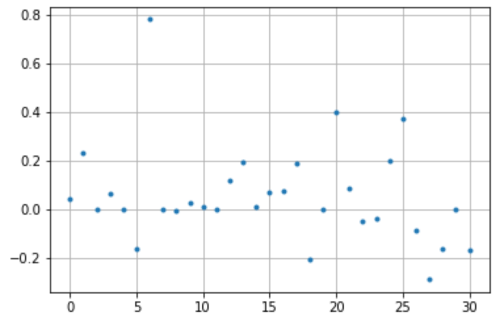


Figure 38: Gain/Loss geographical hedging

loss is -1.1547 \$ and the sum of the cases with a gain is 2.8881 \$ resulting in a final net position in the long run of $+1.7334$ \$, so the cross hedging has worked well in the entire period considered.

4 Conclusions

In this thesis, we analyzed climate risk and its impact on financial markets from various perspectives. We started by focusing on stocks and bonds, aiming to measure the effects of both transition risk and physical risk. This information is valuable for multiple stakeholders: asset managers can use it to better diversify portfolios, while policymakers can identify risk factors in financial markets and design appropriate policies to mitigate crises and maintain financial stability.

We evaluated the impact of transition risk on 292 European stocks across different sectors involved in the transition to sustainable practices. To do this, we tested two candidate variables: the log-returns of European Carbon Allowances and the Transition Risk Index proposed by Blasberg et al. 2021. Neither variable proved effective in measuring the impact of transition risk on stock prices, albeit for different reasons. Carbon allowances did not perform as significant regressors for stock returns, often failing statistical tests or showing negligible impact even when significant. Similarly, the Transition Risk Index produced comparable results for both stocks and bonds. Our findings suggest that while transition risk is a valid concern, the candidate variables tested are not suitable proxies. This is primarily because the Transition Risk Index relies heavily on firm selection and self-reported data, while the link between Carbon Futures and financial markets is influenced more by the economic cycle than by the transition itself.

In contrast, for bonds, we found physical risk variables to be statistically significant. We proposed a model to incorporate physical risk into risky bond pricing, using a reduced-form stochastic hazard rate model inspired by Duffie et al. 1999. We specified the hazard rate structure to include a physical risk sensitivity factor (β), allowing us to estimate the impact of climate risk for issuers with significant climate-related variables. This framework enables ranking corporations based on their physical risk exposure and outperformed models that did not explicitly account for climate variables when sufficient data were available.

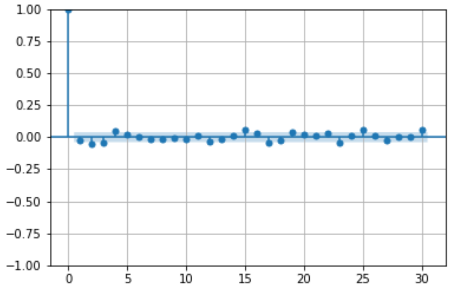
As hedging instruments against physical risk in this thesis we focused on weather derivatives as hedging instruments for physical climate risk. We first examined the market for temperature-based derivatives, and then a new class of derivatives were introduced to hedge against water scarcity caused by insufficient rainfall or low basin levels.

For temperature derivatives, our contribution is twofold. First, we introduced a primary asset, the "temperature forward," to better represent market trading activity compared to models that directly simulate temperature. This approach aligns with Libor market models and resolves pricing mismatches observed in earlier methods, which struggled to differentiate the market measure from the physical one. However, we found that the weather derivatives market is underdeveloped, with significant inefficiencies and persistent underpricing, making it challenging to calibrate models or reflect available market information accurately.

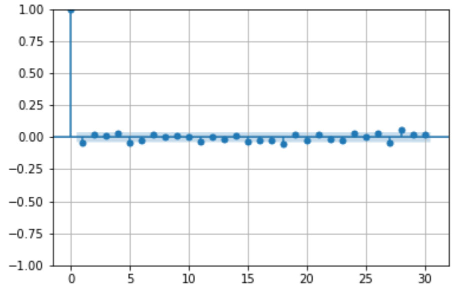
As for the contracts to hedge against water scarcity, we introduced two novel instruments: Rainfall Quanto Options (RQO) and Basin Level Cash-or-Nothing (BLCON) contracts, designed to hedge against water scarcity. We demonstrated that, in regions with established water markets (e.g., California, U.S.), it is possible to assign monetary value to water and use this value to fairly price contracts based on rainfall and basin levels. This approach also enables hedging for geographically mismatched zones not typically covered by standard contracts or insurance policies. These instruments provide greater flexibility than traditional insurance and open opportunities for dynamic hedging within a fully developed market.

A Appendix chapter 1

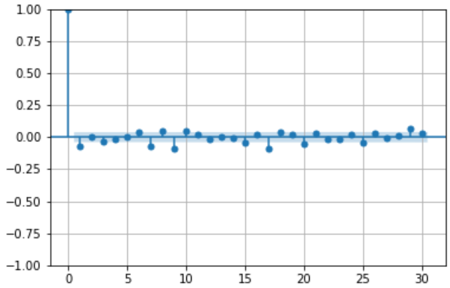
A.1 Commodities: estimated statistics and parameters



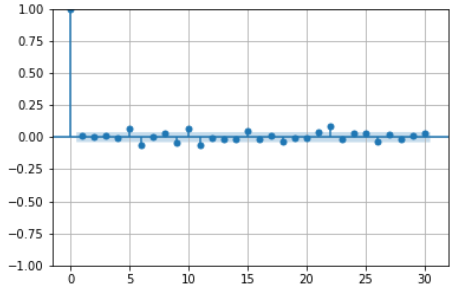
(a) Autocorrelation TTF



(b) Autocorrelation ECF

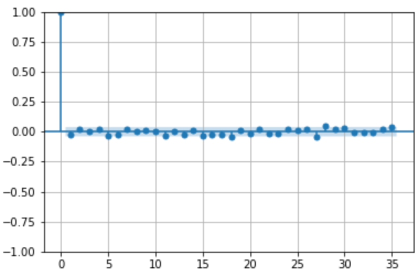


(c) Autocorrelation Coal

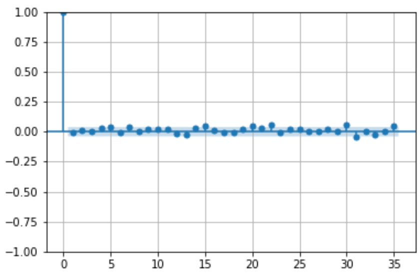


(d) Autocorrelation Brent

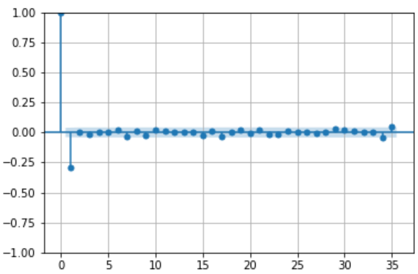
Figure 39: Empirical autocorrelation
energy commodities



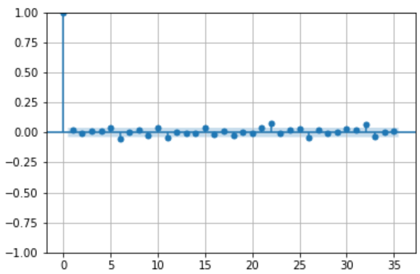
(a) ACF ECF AR(0) score t-EGARCH(1,1)



(b) ACF TTF AR(0) score t-EGARCH(1,1)

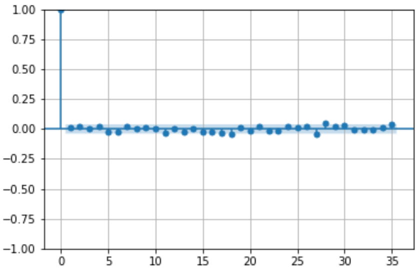


(c) ACF Coal AR(0) score t-EGARCH(1,1)

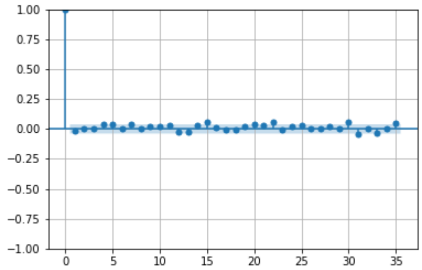


(d) ACF Brent AR(0) score t-EGARCH(1,1)

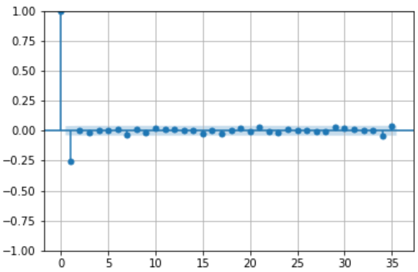
Figure 40: ACF commodities AR(0) score t-EGARCH(1,1)



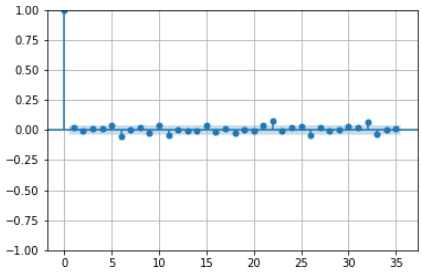
(a) ACF ECF AR(1) score t-EGARCH(1,1)



(b) ACF TTF AR(1) score t-EGARCH(1,1)



(c) ACF Coal AR(1) score t-EGARCH(1,1)



(d) ACF Brent AR(1) score t-EGARCH(1,1)

Figure 41: ACF commodities AR(1) score t-EGARCH(1,1)

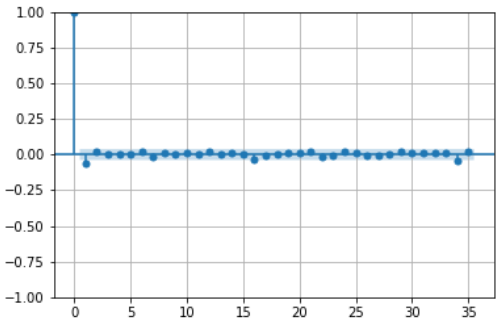


Figure 42: ACF Coal MA(1) score t-EGARCH(1,1)

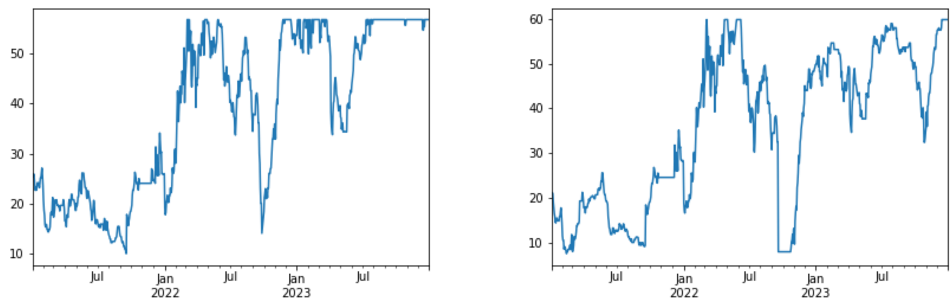
	ECF	TTF	Coal	Brent
ECF	1.0***	0.11415***	−0.00148	−0.05977
TTF	0.11415***	1.0***	0.13796***	0.0991**
Coal	−0.00148	0.13796***	1.0***	0.02827
Brent	−0.05977	0.0991**	0.02827	1.0***
Year 2014				
ECF	1.0***	0.10917***	0.05146	−0.0168
TTF	0.10917***	1.0***	0.18898***	0.05529
Coal	0.05146	0.18898***	1.0***	0.028
Brent	−0.0168	0.05529	0.028	1.0***
Year 2015				
ECF	1.0***	0.23596***	0.21073***	0.14371***
TTF	0.23596***	1.0***	0.14424***	0.02487
Coal	0.21073***	0.14424***	1.0***	0.04409
Brent	0.14371***	0.02487	0.04409	1.0***
Year 2016				
ECF	1.0***	0.22477***	0.18943***	0.00327
TTF	0.22477***	1.0***	0.09159*	0.10894***
Coal	0.18943***	0.09159*	1.0***	−0.03903
Brent	0.00327	0.10894***	−0.03903	1.0***
Year 2017				
ECF	1.0***	0.23183***	0.3575***	−0.03419
TTF	0.23183***	1.0***	0.22947***	0.0916*
Coal	0.3575***	0.22947***	1.0***	0.0527
Brent	−0.03419	0.0916*	0.0527	1.0***
Year 2018				
ECF	1.0***	0.33752***	0.56923***	0.04857
TTF	0.33752***	1.0***	0.2748***	0.02411
Coal	0.56923***	0.2748***	1.0***	0.03714
Brent	0.04857	0.02411	0.03714	1.0***
Year 2019				
ECF	1.0***	0.24495***	0.60598***	0.1062**
TTF	0.24495***	1.0***	0.23828***	0.12386***
Coal	0.60598***	0.23828***	1.0***	0.10269**
Brent	0.1062**	0.12386***	0.10269**	1.0***
Year 2020				
ECF	1.0***	0.32431***	0.40094***	0.11648***
TTF	0.32431***	1.0***	0.20165***	0.04651
Coal	0.40094***	0.20165***	1.0***	0.09237*
Brent	0.11648***	0.04651	0.09237*	1.0***
Year 2021				
ECF	1.0***	0.00671	0.23089***	−0.0063
TTF	0.00671	1.0***	0.24598***	0.06629
Coal	0.23089***	0.24598***	1.0***	0.01651
Brent	−0.0063	0.06629	0.01651	1.0***
Year 2022				
ECF	1.0***	0.27098***	0.47122***	0.09231*
TTF	0.27098***	1.0***	0.32254***	0.02121
Coal	0.47122***	0.32254***	1.0***	0.08233*
Brent	0.09231*	0.02121	0.08233*	1.0***
Year 2023				

Table 18: Energy commodities Kendall-Tau

A.2 Estimates on stock market

A.2.1 Preliminary analysis

All companies with active CDS:
Carlsberg A/S, Danske Bank A/S, Erste Group Bank Ag, Iss A/S, Orsted A/S, Proximus Nv, Solvay Sa, Telekom Austria Ag, Evn Ag, Commerzbank Ag, Deutsche Bank Ag, Adidas Ag, Allianz Se, Basf Se, Bayer Ag, Ceconomy Ag, Continental Ag, Deutsche Post Ag, Deutsche Telekom Ag, Evonik Industries Ag, Hannover Rueck Se, Lanxess Ag, Rwe Ag, Sap Se, Siemens Ag, Sixt Se, Suedzucker Ag, Thyssenkrupp Ag, Tui Ag, Volkswagen Ag, Talanx Ag, Abb Ltd, Assa Abloy Ab, Atlas Copco Ab, Bankinter Sa, Clariant Ag, Holcim Ag, Novartis Ag, Repsol Sa, Securitas Ab, Swedbank Ab, Swisscom Ag, Telefonica Sa, Telia Company Ab, Adecco Group Ag, Banco Santander Sa, Ageas Sa, Carlsberg A/s, Anheuser-busch Inbev Sa, Bawag Group Ag, Kbc Groep Nv, Raiffeisen Bank International Ag, Solvay Sa, Deutsche Bank Ag, Bayerische Motoren Werke Ag, E On Se, Enbw Energie Baden Wuerttemberg Ag, Evonik Industries Ag, Fresenius Se & Co Kgaa, Heidelberg Materials Ag, Henkel Ag & Co Kgaa, Infineon Technologies Ag, Merck Kgaa, Muenchener Rueckversicherungs Gesellschaft In Muenchen Ag, Vodafone Group Plc, Deutsche Lufthansa Ag, Heidelberg Materials Ag, Porsche Automobil Holding Se, Prosiebensat 1 Media Se, Banco De Sabadell Sa, Nordea Bank Abp, Skf Ab, Volvo Ab, Banco Santander Sa, Banco Bilbao Vizcaya Argentaria Sa, Endesa Sa, Fortum Oyj, Glencore Plc, Iberdrola Sa, Investor Ab, Melia Hotels International Sa, Naturgy Energy Group Sa, Nestle Sa, Roche Holding Ag, Skandinaviska Enskilda Banken Ab, Svenska Cellulosa Sca Ab, Svenska Handelsbanken Ab, Swiss Life Holding Ag, Swiss Re Ag, Telefonaktiebolaget Lm Ericsson, Ubs Group Ag, Zurich Insurance Group Ag, Chubb Ltd.
Green companies:
Danske Bank A/s, Danske Bank A/s, Danske Bank A/s, Danske Bank A/s, Deutsche Bank Ag, Deutsche Bank Ag, Deutsche Bank Ag, Deutsche Bank Ag, Allianz Se, Allianz Se, Evonik Industries Ag, Siemens Ag, Abb Ltd
Brown companies:
Carlsberg A/s, Iss A/s, Telekom Austria Ag, Continental Ag, Hannover Rueck Se, Hannover Rueck Se, Lanxess Ag, Rwe Ag, Tui Ag, Atlas Copco Ab, Adecco Group Ag



(a) CDS TR index 10y

(b) CDS TR index 20y

Figure 43: CDS TR index

Company	RIC	Sector
HERA	I:HER	Utilities
ACINQUE	I:ACS	Utilities
ASCOPIAVE	I:ASCO	Utilities
EDISON RSP	I:EDNR	Utilities
ALERION CLEAN POWER	I:ARN	Utilities
ENEL	I:ENEL	Utilities
TERNA RETE ELETTRICA NAZ	I:TRN	Utilities
ALGOWATT	I:ALW	Technology
ACEA	I:ACE	Utilities
A2A	I:A2A	Utilities
FRENDY ENERGY	I:FDE	Utilities
ERG	I:ERG	Utilities
AGATOS	I:AGA	Industrials

Company	RIC	Sector
GAS PLUS	I:GASP	Energy
ENI	I:ENI	Energy
BIESSE	I:BSS	Industrials
FIDIA	I:FD	Industrials
TESMEC	I:TES	Industrials
BORGOSESIA	I:BO	Financials
INTERPUMP GROUP	I:IP	Industrials
INNOVATEC	I:INNO	Industrials
ENERTRONICA	I:ENT	Energy
SOL	I:SOL	Basic Materials
ENCRES DUBUIT	F:ALDU	Basic Materials
ARKEMA	F:AKE	Basic Materials
METABOLIC EXPLORER	F:METE	Basic Materials
ROBERTET	F:ROBT	Basic Materials
L AIR LQE.SC.ANYME. POUR L ETUDE ET L EPXTN.	F:AIR	Basic Materials
EXPLOS.ET PRDS.CHIM.	F:EXPL	Basic Materials
CARBIOS	F:ALCB	Basic Materials
AKWEL	F:MGI	Consumer Cyclicals
RENAULT	F:RENU	Consumer Cyclicals
BURELLE	F:BUR	Financials
DELFINGEN	F:ALDL	Consumer Cyclicals
MICHELIN	F:MCL	Consumer Cyclicals
VALEO	F:FR	Consumer Cyclicals
PLASTIC OMNIUM	F:POM	Consumer Cyclicals
FORVIA	F:BERT	Consumer Cyclicals
VEOLIA ENVIRON	F:VIE	Utilities
ENGIE	F:ENGI	Utilities
FINAXO ENVIRONNEMENT	F:MLFX	Utilities
EAUX DE ROYAN	F:EDR	Utilities
MAUREL ET PROM	F:MAU	Energy
TOTALENERGIES EP GABON	F:TOTG	Energy
TOTALENERGIES	F:FP	Energy
CIE DE CHEMINS DE FER DEPARTEMENTAUX	F:CHEM	Industrials
TRILOGIQ	F:TRIL	Industrials
ROCTOOL	F:ALRO	Industrials
EXAIL TECHNOLOGIES	F:EXA	Industrials
SIGNAUX GIROD	F:SIGM	Consumer Cyclicals
EO2	F:ALEO	Basic Materials
GLOBAL BIOENERGIES	F:ALGB	Healthcare
NORDEX	D:NDX1	Industrials
SOLARWORLD K	D:SWVK	Energy
PHOENIX SOLAR	D:PS4	Energy

Company	**RIC**	**Sector**
GLOBAL PVQ	D:QCE	Energy
SFC ENERGY	D:F3C	Energy
CROPENERGIES	D:CE2	Energy
VERBIO	D:VBK	Energy
ENVITEC BIOGAS	D:ETG	Utilities
CENTROTHERM PHTO.	D:CTNK	Energy
SMA SOLAR TECHNOLOGY	D:S92	Technology
ENAPTER	D:H2O	Energy
CLEARVISE (FRA)	D:ABO	Utilities
ABO WIND	D:AB9	Energy
MASTERFLEX	D:MZX	Industrials
LANXESS	D:LXS	Basic Materials
WACKER CHEMIE	D:WCH	Basic Materials
MUEHLHAN	D:M4N	Industrials
NABALTEC	D:NTG	Basic Materials
SYMRISE	D:SY1	Basic Materials
FUCHS N	D:FPE	Basic Materials
BRENNTAG	D:BNR	Basic Materials
ECKERT & ZIEGLER STRAHLEN & MEDZI.	D:EUZ	Healthcare
ALZCHEM	D:ACT	Basic Materials
EVONIK INDUSTRIES	D:EVK	Basic Materials
BASF	D:BAS	Basic Materials
H & R	D:2HRA	Basic Materials
DELTICOM	D:DEX	Technology
SAF-HOLLAND	D:SFQ	Consumer Cyclicals
MERCEDES-BENZ GROUP N	D:MBG	Consumer Cyclicals
ELRINGKLINGER N	D:ZIL2	Consumer Cyclicals
GRAMMER	D:GMM	Consumer Cyclicals
VOLKSWAGEN	D:VOW	Consumer Cyclicals
BMW	D:BMW	Consumer Cyclicals
CONTINENTAL	D:CON	Consumer Cyclicals
PORSCHE AML.HLDG.PREF.	D:PAH3	Consumer Cyclicals
FERNHEIZWERK NEUKOLLN	D:FHW	Utilities
MAINOVA	D:MNV6	Utilities
RWE	D:RWE	Utilities
E ON N	D:EOAN	Utilities
GELSENWASSER	D:WWG	Utilities
GLOBAL OIL & GAS	D:3GOK	Energy
DEUTSCHE ROHSTOFF	D:DR0	Basic Materials

Company	RIC	Sector
AEE GOLD	D:AEE1	Consumer Non-Cyclicals
META WOLF	D:WOLF	Basic Materials
STO PREFERENCE	D:STO3	Consumer Cyclicals
BAUER	D:B5A	Basic Materials
STEICO	D:ST5	Basic Materials
VILLEROY & BOCH PF.SHS.	D:VIB3	Consumer Cyclicals
INNOTEC TSS	D:TSS	Consumer Cyclicals
UZIN UTZ	D:UZU	Industrials
HOCHTIEF	D:HOT	Industrials
WESTAG	D:WUG	Consumer Cyclicals
STEULER FLIESENGRUPPE	D:NST	Consumer Cyclicals
7C SOLARPARKEN K	D:HRPK	Utilities
ENERGIEKONTOR	D:EKT	Utilities
4 SC	D:VSC	Healthcare
BAUMOT GROUP	D:TINC	Consumer Cyclicals
2G ENERGY	D:2GB	Industrials
ENCAVIS	D:ECV	Utilities
PNE	D:PNE3	Energy
MVV ENERGIE	D:MVV1	Utilities
LECHWERKE	D:LEC	Utilities
ENBW ENGE.BADEN-WURTG.	D:EBK	Utilities
MOURY CONSTRUCT	B:SOLI	Industrials
COMPAGNIE D ENTREPRISES CFE	B:CFEB	Consumer Cyclicals
FLORIDIENNE	B:FLOB	Basic Materials
BEKAERT (D)	B:BEKB	Industrials
JENSEN-GROUP	B:LSG	Industrials
EVN	O:EVN	Utilities
BURGENLAND HOLDING	O:BURG	Utilities
VERBUND	O:VERB	Utilities
RATH	O:RATH	Basic Materials
STRABAG SE	O:STR	Industrials
ZUMTOBEL	O:ZUS	Industrials
SW UMWELTTECHNIK	O:SWU	Industrials
PORR	O:ALLG	Industrials
HUTTER & SCHRANTZ	O:HUTV	Basic Materials
WIENERBERGER	O:WNBA	Basic Materials
POLYTEC HOLDING	O:PYT	Consumer Cyclicals
OMV	O:OMV	Energy
MT HOEJGAARD HOLDING	DK:HOB	Industrials
H+H INTERNATIONAL	DK:HHI	Basic Materials
ROCKWOOL B	DK:ROC	Consumer Cyclicals

Company	RIC	Sector
FLSMIDTH AND CO.	DK:FLB	Basic Materials
SCANDINAVIAN BRAKE SYS.	DK:SBS	Consumer Cyclicals
ENNOGIE SOLAR GROUP	DK:SCD	Industrials
VESTAS WINDSYSTEMS	DK:VEW	Energy
UIE	DK:UIE	Consumer Non-Cyclicals
FIRSTFARMS	DK:FFA	Consumer Non-Cyclicals
SCHOUW AND	DK:SCB	Consumer Non-Cyclicals
GLUNZ & JENSEN HOLDING	DK:GLJ	Industrials
BRD KLEE B	DK:BRD	Industrials
SKAKO	DK:SKI	Industrials
KOBENHAVNS LUFTHAVNE	DK:KOB	Industrials
DSV	DK:DSV	Industrials
ERRIA	DK:ERR	Industrials
TORM A	DK:TRM	Energy
DMPKBT.NORDEN	DK:DNO	Industrials
NTG NORDIC TRANSPORT GROUP	DK:NEU	Industrials
DFDS	DK:DFD	Industrials
A P MOLLER MAERSK B	DK:DSB	Industrials
KENDRION	H:SCHV	Consumer Cyclicals
KON.HEIJMANS DU. CERTS.	H:HEIJ	Industrials
BAM GROEP KON.	H:BAM	Industrials
FERROVIAL	E:FERC	Industrials
AMG CRITICAL MATERIALS	H:AMG	Basic Materials
HYDRATEC INDUSTRIES	H:HYDR	Basic Materials
SBM OFFSHORE	H:SBMO	Energy
SUNEX	PO:SNX	Industrials
MVA GREEN ENERGY	PO:GRE	Energy
COLUMBUS ENERGY	PO:CLC	Energy
VOOLT	PO:NVV	Energy
VIATRON	PO:VIA	Energy
BIOMASS ENERGY PROJECT	PO:BEP	Consumer Non-Cyclicals
FIRMA OPONIARSKA DEBICA	PO:DEB	Consumer Cyclicals
INTER CARS	PO:ICS	Consumer Cyclicals
LESS	PO:GCN	Consumer Cyclicals
AC AUTOGAZ	PO:ACG	Consumer Cyclicals
PL GROUP	PO:4X4	Consumer Cyclicals
ORZEL	PO:ORL	Consumer Cyclicals
SOLAR INNOVATION	PO:CZK	Energy
KRAKCHEMIA	PO:KCK	Basic Materials

Company	RIC	Sector
SELENA FM SR.B I C	PO:SLN	Basic Materials
GALVO	PO:GAL	Basic Materials
HORTICO	PO:HOR	Consumer Non-Cyclicals
IZOBLOK	PO:IZB	Consumer Cyclicals
PRYMUS	PO:PRS	Basic Materials
PCC EXOL	PO:PCX	Consumer Non-Cyclicals
MOSTOSTAL ZABRZE	PO:MZB	Industrials
BUDIMEX	PO:BUX	Industrials
PBG	PO:PBG	Industrials
RAWLPLUG	PO:RWL	Industrials
MOSTOSTAL WARSZAWA	PO:MSW	Industrials
LENA LIGHTING	PO:LEL	Industrials
DECORA	PO:DCR	Consumer Cyclicals
ERBUD	PO:ERB	Industrials
INVESTMENT FRIENDS	PO:IFR	Financials
MERCOR	PO:MCR	Industrials
PA NOVA	PO:NOA	Industrials
RESBUD	PO:RES	Industrials
IZOLACJA JAROCIN	PO:IZL	Consumer Cyclicals
TRAKCJA	PO:TRK	Industrials
UNIBEP	PO:UBP	Industrials
STARHEDGE	PO:SHG	Financials
COMPREMUM	PO:POU	Industrials
MIRBUD	PO:MRB	Industrials
FON	PO:CAS	Financials
MOSTAL	PO:MOP	Industrials
INSTAL KRAKOW	PO:INK	Industrials
MERA	PO:MER	Consumer Cyclicals
TESGAS	PO:TSG	Industrials
ZUE	PO:ZUE	Industrials
LIBET	PO:LBT	Basic Materials
TAMEX OBIEKTY SPORTOWE	PO:TOS	Industrials
MOBRUK	PO:MBR	Industrials
INTERMA TRADE	PO:BRI	Consumer Cyclicals
DEKTRA	PO:DKR	Consumer Cyclicals
INTERNITY	PO:INT	Consumer Cyclicals
HONEY PAYMENT GROUP	PO:MAX	Technology
PRZED.PRZ.BETONOW PREFABET BIALE BLOTA	PO:PBB	Basic Materials
ROCCA	PO:RCA	Consumer Cyclicals
FABRYKA KONSTRUKCJI DREW	PO:FKD	Basic Materials
ULMA CONSTR.POLSKA	PO:ULM	Basic Materials

Company	RIC	Sector
ATLANTIS	PO:ATL	Financials
POLIMEX MOSTOSTAL	PO:PXM	Industrials
FORBUILD	PO:BEO	Industrials
HM INWEST ORD	PO:HMI	Real Estate
TAURON POLSKA ENERGIA	PO:TPE	Utilities
EC BEDZIN	PO:BED	Utilities
PKA.GRUPA ENERGETYCZNA	PO:PGR	Utilities
ENEA	PO:EEA	Utilities
POLENERGIA	PO:PEP	Utilities
ZESPOL ELKTP. WRLKKNR.	PO:KOG	Utilities
ZE PAK	PO:ZEP	Utilities
ENERGA	PO:ERE	Utilities
PHOTON ENERGY	PO:PEN	Utilities
UNIMOT	PO:UNT	Energy
MANGATA HOLDING	PO:ZKA	Consumer Cyclicals
ZAK AD BUD MASZYN ZBC.	PO:ZRE	Industrials
KUPIEC	PO:KPC	Industrials
BORYSZEW	PO:BOR	Basic Materials
FABRYKA OBRABIAREK RAFAMET	PO:RAT	Industrials
ZAKLADY URZADZEN KOTLOWYCH STAPORKOW	PO:ZUK	Industrials
MOJ	PO:MOJ	Basic Materials
PGF POLSKA GRUPA FOTOWOLTAICZNA	PO:ZST	Real Estate
ENERGOINSTAL	PO:EEG	Industrials
SECOGROUP	PO:SWG	Industrials
WIELTON	PO:WEL	Industrials
BUMECH	PO:BMC	Energy
KCI	PO:KCI	Consumer Cyclicals
ZAKLADY MAGNEZYTOWE ROPCZYCE	PO:ROP	Basic Materials
PATENTUS	PO:PAT	Basic Materials
HYDRAPRES	PO:HPS	Industrials
ZAMET	PO:ZMT	Industrials
SANOK RUBBER COMPANY	PO:SAN	Consumer Cyclicals
FEERUM	PO:FEE	Industrials
APS ENERGIA	PO:APEP	Industrials
DROZAPOL PROFIL	PO:DPL	Basic Materials
PJP MAKRUM	PO:PRJ	Industrials
ODLEWNIE POLSKIE	PO:ODL	Basic Materials
IZOSTAL	PO:IZS	Basic Materials
BOWIM	PO:BOW	Basic Materials
GRUPA KETY	PO:KTY	Basic Materials
MFO	PO:MFO	Basic Materials
EKOPOL GORNOSLASK HLDG.	PO:EGH	Energy

Company	**RIC**	**Sector**
STALPRODUKT	PO:STL	Basic Materials
STALEXPORT AUTOSTRADY	PO:STA	Industrials
TRANSPOL	PO:TRN	Industrials
OT LOGISTICS	PO:OTO	Industrials
BALTICON	PO:BLT	Industrials
FORPOSTA	PO:FPO	Industrials
XBS PRO-LOG	PO:PRL	Industrials
PKP CARGO	PO:PKP	Industrials
NATURGY ENERGY	E:CTG	Utilities
GRINO ECOLOGIC	E:GRIE	Industrials
ERCROS	E:ECR	Basic Materials
OBRASCON HUARTE LAIN	E:OHL	Industrials
FLUIDRA	E:FDR	Consumer Cyclicals
ACS ACTIV.CONSTR.Y SERV.	E:ACS	Industrials
FOMENTO CONSTR.Y CNTR.	E:FCC	Industrials
ACCIONA	E:ANA	Industrials
SACYR	E:SCYR	Industrials
AUDAX RENOVABLES	E:FGN	Utilities
SOLARIA ENERGIA Y MEDIO AMBIENTE	E:SEM	Utilities
EDP RENOVAVEIS	P:EDPR	Utilities
REDEIA CORPORACION	E:REE	Utilities
ENDESA	E:ELE	Utilities
AKILES CORPORATION	E:EBI	Financials
IBERDROLA	E:IBE	Utilities
ROMANDE ENERGIE	S:REHN	Utilities
EDISUN POWER EUROPE N	S:ESUN	Utilities
BKW	S:BKW	Utilities
ENERGIEDIENST HOLDING	S:EDHN	Utilities
CLARIANT	S:CLN	Basic Materials
GIVAUDAN 'N'	S:GIVN	Basic Materials
DOTTIKON ES HOLDING	S:DESN	Healthcare
GURIT HOLDING 'B'	S:GURN	Basic Materials
EMS-CHEMIE 'N'	S:EMSN	Basic Materials
FEINTOOL	S:FTON	Consumer Cyclicals
AUTONEUM HOLDING	S:AUTN	Consumer Cyclicals

Table 19: Stock dataset

Company	**AD-Fuller statistic**	**P-value**
HERA	-54.7025	0.0
ACINQUE	-13.0671	0.0
ASCOPIAVE	-35.7452	0.0
EDISON RSP	-11.3771	0.0
ENEL	-16.7752	0.0
TERNA RETE ELETTRICA NAZ	-16.1937	0.0
ALGOWATT	-21.3841	0.0
ACEA	-33.7840	0.0
A2A	-34.6526	0.0

Company	AD-Fuller statistic	P-value
FRENDY ENERGY	-33.1969	0.0
ERG	-23.5497	0.0
AGATOS	-31.1705	0.0
GAS PLUS	-14.4805	0.0
BIESSE	-18.2115	0.0
BORGOSESIA	-35.2197	0.0
INNOVATEC	-14.2036	0.0
ENERTRONICA	-23.1442	0.0
SOL	-17.8111	0.0
ENCRES DUBUIT	-11.5549	0.0
METABOLIC EXPLORER	-12.9248	0.0
ROBERTET	-56.4866	0.0
L AIR LQE.SC.ANYME. POUR L ETUDE ET L EPXTN.	-55.8835	0.0
EXPLOS.ET PRDS.CHIM.	-28.6965	0.0
CARBIOS	-27.7823	0.0
RENAULT	-13.3442	0.0
BURELLE	-21.5208	0.0
DELFINGEN	-13.4181	0.0
MICHELIN	-16.2072	0.0
VALEO	-19.0228	0.0
ENGIE	-12.3376	0.0
FINAXO ENVIRONNEMENT	-16.8414	0.0
EAUX DE ROYAN	-18.6594	0.0
MAUREL ET PROM	-18.5358	0.0
CIE DE CHEMINS DE FER DEPARTEMENTAUX	-40.3490	0.0
TRILOGIQ	-18.7981	0.0
ROCTOOL	-15.4206	0.0
EXAIL TECHNOLOGIES	-48.6786	0.0
EO2	-22.7782	0.0
GLOBAL BIOENERGIES	-14.6877	0.0
SOLARWORLD K	-24.3073	0.0
PHOENIX SOLAR	-27.7299	0.0
GLOBAL PVQ	-14.1360	0.0
SFC ENERGY	-41.3673	0.0
CROPENERGIES	-56.2046	0.0
ENVITEC BIOGAS	-57.8442	0.0
CENTROTHERM PHTO.	-26.1986	0.0
ENAPTER	-9.9332	0.0
CLEARVISE (FRA)	-19.5204	0.0
ABO WIND	-35.8553	0.0
MASTERFLEX	-29.2510	0.0
MUEHLHAN	-33.2348	0.0
SYMRISE	-31.4187	0.0
FUCHS N	-26.2419	0.0
ECKERT & ZIEGLER STRAHLEN & MEDZI.	-21.8391	0.0
ALZCHEM	-10.5458	0.0
EVONIK INDUSTRIES	-17.6150	0.0
BASF	-17.8464	0.0
H & R	-31.1349	0.0
DELTICOM	-32.5878	0.0
MERCEDES-BENZ GROUP N	-13.2337	0.0
GRAMMER	-56.3690	0.0
VOLKSWAGEN	-15.9106	0.0
BMW	-21.2213	0.0
PORSCHE AML.HLDG.PREF.	-47.2280	0.0
FERNHEIZWERK NEUKOLLN	-27.7536	0.0

Company	AD-Fuller statistic	P-value
MAINOVA	-26.1035	0.0
RWE	-29.3894	0.0
GELSENWASSER	-19.5697	0.0
GLOBAL OIL & GAS	-10.2040	0.0
DEUTSCHE ROHSTOFF	-55.8190	0.0
AEE GOLD	-34.2310	0.0
META WOLF	-26.7475	0.0
STO PREFERENCE	-53.9706	0.0
BAUER	-39.1531	0.0
STEICO	-55.0818	0.0
VILLEROY & BOCH PF.SHS.	-40.3166	0.0
INNOTEC TSS	-12.5940	0.0
UZIN UTZ	-24.4756	0.0
WESTAG	-46.0682	0.0
STEULER FLIESENGRUPPE	-10.9694	0.0
7C SOLARPARKEN K	-17.5144	0.0
ENERGIEKONTOR	-17.0627	0.0
4 SC	-28.9683	0.0
BAUMOT GROUP	-20.5997	0.0
2G ENERGY	-57.1104	0.0
ENCAVIS	-11.9456	0.0
PNE	-57.8849	0.0
MVV ENERGIE	-19.8702	0.0
LECHWERKE	-28.3566	0.0
ENBW ENGE.BADEN-WURTG.	-22.6013	0.0
MOURY CONSTRUCT	-23.5270	0.0
FLORIDIENNE	-20.9467	0.0
BEKAERT (D)	-21.6013	0.0
JENSEN-GROUP	-59.0370	0.0
EVN	-21.2244	0.0
BURGENLAND HOLDING	-20.1332	0.0
VERBUND	-23.5026	0.0
RATH	-18.6643	0.0
STRABAG SE	-11.4834	0.0
SW UMWELTTECHNIK	-12.2062	0.0
PORR	-53.3257	0.0
WIENERBERGER	-25.8271	0.0
POLYTEC HOLDING	-15.4443	0.0
MT HOEJGAARD HOLDING	-27.3016	0.0
FLSMIDTH AND CO.	-37.1918	0.0
SCANDINAVIAN BRAKE SYS.	-17.2321	0.0
ENNOGIE SOLAR GROUP	-16.3749	0.0
VESTAS WINDSYSTEMS	-30.9621	0.0
UIE	-42.7226	0.0
FIRSTFARMS	-17.0312	0.0
GLUNZ & JENSEN HOLDING	-26.6815	0.0
BRD KLEE B	-18.1621	0.0
SKAKO	-21.6419	0.0
KOBENHAVNS LUFTHAVNE	-54.4897	0.0
ERRIA	-36.2612	0.0
TORM A	-11.3761	0.0
NTG NORDIC TRANSPORT GROUP	-54.8889	0.0
DFDS	-16.4928	0.0
KENDRION	-20.9180	0.0
KON.HEIJMANS DU. CERTS.	-33.1425	0.0
HYDRATEC INDUSTRIES	-28.0400	0.0
SUNEX	-10.8821	0.0
MVA GREEN ENERGY	-18.0364	0.0

Company	AD-Fuller statistic	P-value
COLUMBUS ENERGY	-9.2753	0.0
VOOLT	-31.6412	0.0
VIATRON	-31.0120	0.0
BIOMASS ENERGY PROJECT	-24.3246	0.0
FIRMA OPONIARSKA DEBICA	-24.1382	0.0
INTER CARS	-54.5570	0.0
LESS	-29.4834	0.0
AC AUTOGAZ	-40.1481	0.0
PL GROUP	-18.4987	0.0
ORZEL	-20.8851	0.0
SOLAR INNOVATION	-24.3937	0.0
KRAKCHEMIA	-9.1294	0.0
SELENA FM SR.B I C	-55.1156	0.0
GALVO	-20.1559	0.0
HORTICO	-19.2243	0.0
IZOBLOK	-20.8365	0.0
PRYMUS	-30.1710	0.0
PCC EXOL	-11.4038	0.0
MOSTOSTAL ZABRZE	-12.0856	0.0
BUDIMEX	-54.2749	0.0
PBG	-18.0456	0.0
RAWLPLUG	-40.1236	0.0
LENA LIGHTING	-55.6889	0.0
DECORA	-35.0209	0.0
INVESTMENT FRIENDS	-30.3999	0.0
MERCOR	-27.8324	0.0
PA NOVA	-40.0883	0.0
RESBUD	-56.3531	0.0
IZOLACJA JAROCIN	-25.8626	0.0
TRAKCJA	-16.0707	0.0
UNIBEP	-31.4607	0.0
STARHEDGE	-29.0946	0.0
FON	-8.9998	0.0
MOSTAL	-59.5269	0.0
INSTAL KRAKOW	-26.6082	0.0
MERA	-19.6692	0.0
TESGAS	-11.0118	0.0
ZUE	-40.1429	0.0
LIBET	-54.3933	0.0
TAMEX OBIEKTY SPORTOWE	-22.9932	0.0
MOBRUK	-22.4296	0.0
INTERMA TRADE	-16.5066	0.0
DEKTRA	-12.5393	0.0
INTERNITY	-14.5015	0.0
HONEY PAYMENT GROUP	-25.7186	0.0
PRZED.PRZ.BETONOW PREFABET BIALE BLOTA	-32.4522	0.0
ROCCA	-17.3419	0.0
FABRYKA KONSTRUKCJI DREW	-10.1150	0.0
ULMA CONSTR.POLSKA	-23.8505	0.0
ATLANTIS	-33.1555	0.0
POLIMEX MOSTOSTAL	-35.3873	0.0
FORBUILD	-29.7975	0.0
HM INWEST ORD	-11.4912	0.0
TAURON POLSKA ENERGIA	-48.0815	0.0
EC BEDZIN	-11.0538	0.0
ENEA	-48.2785	0.0
POLENERGIA	-16.6456	0.0
ZESPOL ELKTP. WRLKKNR.	-24.1404	0.0

Company	AD-Fuller statistic	P-value
PHOTON ENERGY	-55.7528	0.0
UNIMOT	-15.3715	0.0
MANGATA HOLDING	-41.0599	0.0
ZAK AD BUD MASZYN ZBC.	-9.9878	0.0
KUPIEC	-29.6846	0.0
BORYSZEW	-10.4506	0.0
FABRYKA OBRABIAREK RAFAMET	-38.7102	0.0
ZAKLADY URZADZEN KOTLOWYCH STAPORKOW	-38.1867	0.0
MOJ	-13.1674	0.0
PGF POLSKA GRUPA FOTOWOLTAICZNA	-22.7476	0.0
ENERGOINSTAL	-25.1601	0.0
SECOGROUP	-21.1044	0.0
WIELTON	-45.3211	0.0
BUMECH	-9.6535	0.0
KCI	-11.4290	0.0
ZAKLADY MAGNEZYTOWE ROPCZYCE	-23.5524	0.0
HYDRAPRES	-18.2158	0.0
ZAMET	-19.3372	0.0
SANOK RUBBER COMPANY	-23.3195	0.0
FEERUM	-25.5697	0.0
APS ENERGIA	-20.5550	0.0
DROZAPOL PROFIL	-8.6519	0.0
PJP MAKRUM	-54.9971	0.0
ODLEWNIE POLSKIE	-39.8784	0.0
GRUPA KETY	-38.0843	0.0
MFO	-58.6609	0.0
EKOPOL GORNOSLASK HLDG.	-24.2704	0.0
STALPRODUKT	-19.4112	0.0
STALEXPORT AUTOSTRADY	-22.8187	0.0
TRANSPOL	-25.1623	0.0
OT LOGISTICS	-8.3979	0.0
BALTICON	-23.6833	0.0
FORPOSTA	-25.4777	0.0
XBS PRO-LOG	-11.8244	0.0
NATURGY ENERGY	-14.5159	0.0
GRINO ECOLOGIC	-12.5662	0.0
ACS ACTIV.CONSTR.Y SERV.	-13.4506	0.0
FOMENTO CONSTR.Y CNTR.	-16.1337	0.0
ACCIONA	-19.8762	0.0
SACYR	-14.9903	0.0
AUDAX RENOVABLES	-11.5641	0.0
ENDESA	-33.3390	0.0
AKILES CORPORATION	-13.0693	0.0
IBERDROLA	-14.3533	0.0
ROMANDE ENERGIE	-28.1058	0.0
EDISUN POWER EUROPE N	-15.5637	0.0
BKW	-54.4339	0.0
ENERGIEDIENST HOLDING	-29.1904	0.0
DOTTIKON ES HOLDING	-56.7914	0.0
GURIT HOLDING 'B'	-12.8791	0.0
FEINTOOL	-54.9708	0.0
AUTONEUM HOLDING	-33.2149	0.0

Table 20: AD-Fuller test on stock returns

Company	Lag=1	Lag=2	Lag=3	Lag=4	Lag=5
HERA	-0.0698	0.0264	-0.0038	-0.0212	0.0133
ACINQUE	-0.1019	-0.0158	0.0028	-0.0380	0.0685
ASCOPIAVE	-0.0412	0.0311	-0.0103	-0.0055	-0.0288
EDISON RSP	-0.1197	0.0777	-0.0303	0.0002	0.0022
ENEL	-0.0662	0.0365	0.0139	-0.0289	-0.0247
TERNA RETE ELETTRICA NAZ	-0.0806	-0.0009	-0.0259	-0.0279	-0.0239
ALGOWATT	0.0117	0.0414	-0.0504	-0.0362	-0.0367
ACEA	0.0338	0.0514	0.0029	0.0098	0.0139
A2A	-0.0172	0.0488	-0.0044	0.0113	0.0127
FRENDY ENERGY	-0.0986	-0.0271	-0.0580	0.0135	-0.0143
ERG	-0.0284	0.0469	0.0216	-0.0361	-0.0271
AGATOS	0.0328	-0.0318	-0.0482	0.0253	-0.0101
GAS PLUS	-0.0831	0.0418	0.0646	-0.0363	0.0243
BIESSE	0.0419	0.0367	-0.0115	-0.0111	-0.0194
BORGOSESIA	-0.0916	0.0754	-0.0123	0.0255	0.0079
INNOVATEC	0.0596	-0.0204	0.0122	0.0013	0.0498
ENERTRONICA	-0.0600	0.0184	-0.0103	-0.0498	-0.0135
SOL	-0.0755	-0.0202	0.0049	-0.0143	0.0126
ENCRES DUBUIT	-0.1441	0.0149	-0.0904	-0.0198	-0.0606
METABOLIC EXPLORER	0.0821	0.0227	-0.0140	-0.0646	-0.0055
ROBERTET	-0.1009	0.0021	-0.0371	-0.0174	0.0254
L AIR LQE.SC.ANYME. POUR L ETUDE ET L EPXTN.	-0.0898	0.0309	-0.0135	0.0077	-0.0125
EXPLOS.ET PRDS.CHIM.	-0.1102	-0.0519	-0.0334	-0.0161	0.0112
CARBIOS	0.0093	-0.0426	0.0821	-0.0032	-0.0002
RENAULT	0.0047	0.0233	-0.0197	0.0055	0.0530
BURELLE	0.0375	0.0049	-0.0132	0.0009	0.0537
DELFINGEN	0.0563	0.0045	0.0070	0.0300	0.0516
MICHELIN	-0.0091	-0.0112	-0.0435	0.0136	0.0134
VALEO	0.0406	0.0262	-0.0083	0.0175	0.0178
ENGIE	0.0600	0.0363	0.0300	0.0080	-0.0133
FINAXO ENVIRONNEMENT	-0.1040	-0.0486	-0.0232	-0.0224	0.0168
EAUX DE ROYAN	-0.1732	-0.0908	-0.0633	0.0395	-0.0445
MAUREL ET PROM	0.0112	0.0338	-0.0127	0.0201	0.0484
CIE DE CHEMINS DE FER DEPARTEMENTAUX	-0.0705	-0.0724	-0.0091	-0.0133	0.0099
TRILOGIQ	-0.1613	-0.0805	-0.0199	0.0235	-0.0079
ROCTOOL	-0.0812	0.0011	-0.0495	0.0296	-0.0499
EXAIL TECHNOLOGIES	0.0486	0.0058	-0.0192	-0.0050	0.0259
EO2	0.0025	-0.0092	0.0116	-0.0671	-0.0033
GLOBAL BIOENERGIES	0.0338	-0.0353	0.0249	0.0120	-0.0100
SOLARWORLD K	-0.2010	-0.0239	-0.0258	-0.0101	0.0571
PHOENIX SOLAR	-0.2291	-0.0843	-0.0178	0.0122	-0.0342
GLOBAL PVQ	-0.4141	0.0329	-0.0170	-0.0067	0.0234
SFC ENERGY	-0.1386	-0.0512	-0.0003	0.0055	-0.0224
CROPENERGIES	-0.0961	0.0068	0.0341	-0.0049	-0.0235
ENVITEC BIOGAS	-0.1242	0.0067	-0.0084	-0.0036	-0.0099
CENTROTHERM PHTO.	-0.1830	-0.0649	0.0118	-0.0173	-0.0305
ENAPTER	-0.1998	-0.0462	-0.0061	-0.0570	0.0858
CLEARVISE (FRA)	-0.2950	0.0157	-0.0233	-0.0087	0.0102
ABO WIND	-0.2070	-0.0513	-0.0404	0.0088	0.0170
MASTERFLEX	-0.1756	-0.0277	-0.0402	-0.0091	-0.0113
MUEHLHAN	-0.2163	0.0073	-0.0234	0.0002	0.0071
SYMRISE	-0.0669	0.0256	-0.0606	-0.0120	-0.0075
FUCHS N	-0.1145	0.0275	0.0282	-0.0401	-0.0045
ECKERT & ZIEGLER STRAHLEN & MEDZI.	-0.0782	0.0275	0.0413	-0.0098	-0.0063
ALZCHEM	-0.1085	0.0115	-0.0806	-0.0414	0.0403
EVONIK INDUSTRIES	-0.0593	0.0429	-0.0124	-0.0160	0.0481

Company	Lag=1	Lag=2	Lag=3	Lag=4	Lag=5
BASF	-0.0146	0.0193	-0.0221	0.0328	0.0442
H & R	-0.1303	0.0714	-0.0669	0.0283	-0.0130
DELTICOM	-0.1352	-0.0163	-0.0302	-0.0041	-0.0162
MERCEDES-BENZ GROUP N	0.0370	0.0473	-0.0109	-0.0346	0.0339
GRAMMER	-0.0987	-0.0008	-0.0079	0.0150	-0.0011
VOLKSWAGEN	-0.0105	-0.0542	0.0171	0.0257	0.0343
BMW	0.0245	0.0426	0.0175	-0.0352	0.0192
PORSCHE AML.HLDG.PREF.	0.0774	0.0194	0.0270	-0.0183	-0.0138
FERNHEIZWERK NEUKOLLN	-0.3830	0.0010	0.0272	-0.0261	-0.0122
MAINOVA	-0.4389	-0.0154	0.0007	-0.0138	0.0209
RWE	-0.0176	0.0661	-0.0390	0.0023	-0.0017
GELSENWASSER	-0.2316	-0.0826	0.0024	-0.0273	-0.0208
GLOBAL OIL & GAS	-0.0628	-0.0046	0.0490	0.0087	0.0025
DEUTSCHE ROHSTOFF	-0.0892	-0.0061	0.0233	-0.0047	-0.0174
AEE GOLD	-0.2915	0.0278	-0.0197	0.0094	-0.0005
META WOLF	-0.2469	-0.0262	-0.0020	-0.0293	-0.0139
STO PREFERENCE	-0.0558	-0.0148	-0.0132	-0.0280	0.0037
BAUER	-0.1001	-0.0230	0.0255	0.0169	-0.0091
STEICO	-0.0752	-0.0152	-0.0200	-0.0054	0.0066
VILLEROY & BOCH PF.SHS.	-0.1623	-0.0087	0.0001	-0.0044	-0.0272
INNOTEC TSS	-0.2643	-0.0425	0.0354	-0.0049	0.0173
UZIN UTZ	-0.2086	-0.0128	-0.0210	-0.0181	0.0409
WESTAG	-0.2998	-0.0122	0.0202	-0.0050	-0.0131
STEULER FLIESENGRUPPE	-0.3186	-0.0087	-0.0240	-0.0223	0.0726
7C SOLARPARKEN K	-0.2103	-0.0055	-0.0551	0.0331	-0.0559
ENERGIEKONTOR	-0.0540	-0.0034	-0.0207	-0.0452	-0.0039
4 SC	-0.0952	0.0039	0.0484	-0.0244	-0.0115
BAUMOT GROUP	-0.2244	-0.0620	-0.0774	-0.0457	-0.0607
2G ENERGY	-0.1119	-0.0044	-0.0160	-0.0045	0.0090
ENCAVIS	-0.1132	0.0265	-0.0249	-0.0035	-0.0523
PNE	-0.1253	0.0293	0.0031	-0.0121	0.0062
MVV ENERGIE	-0.2814	0.0048	-0.0389	-0.0155	0.0264
LECHWERKE	-0.3924	0.0243	-0.0071	-0.0210	-0.0012
ENBW ENGE.BADEN-WURTG.	-0.2393	-0.0180	-0.0610	0.0122	0.0127
MOURY CONSTRUCT	-0.0825	-0.0392	-0.0366	0.0287	-0.0440
FLORIDIENNE	-0.0576	-0.0514	-0.0478	-0.0294	-0.0463
BEKAERT (D)	0.0127	0.0088	-0.0305	0.0264	0.0201
JENSEN-GROUP	-0.1445	0.0353	-0.0042	0.0406	0.0137
EVN	-0.0708	0.0335	0.0430	-0.0115	0.0146
BURGENLAND HOLDING	-0.1106	-0.1266	-0.0517	-0.0317	-0.0044
VERBUND	-0.0319	-0.0093	0.0082	-0.0298	-0.0330
RATH	-0.1893	-0.0442	-0.0345	-0.0060	-0.0371
STRABAG SE	-0.0622	0.0838	0.0161	0.0084	-0.0138
SW UMWELTTECHNIK	-0.0611	0.0064	-0.0573	-0.0952	-0.0270
PORR	-0.0438	0.0286	-0.0080	-0.0220	0.0142
WIENERBERGER	0.0150	0.0537	0.0169	-0.0532	0.0100
POLYTEC HOLDING	0.0785	0.0627	-0.0259	-0.0428	-0.0166
MT HOEJGAARD HOLDING	-0.0116	0.0221	0.0639	0.0181	0.0170
FLSMIDTH AND CO.	0.0283	-0.0414	0.0191	-0.0095	-0.0255
SCANDINAVIAN BRAKE SYS.	-0.1120	0.0009	0.0241	0.0173	-0.0307
ENNOGIE SOLAR GROUP	-0.0084	-0.0349	-0.1141	-0.0833	-0.0146
VESTAS WINDSYSTEMS	-0.0334	0.0121	-0.0483	-0.0169	-0.0014
UIE	-0.2239	-0.0147	0.0230	-0.0138	0.0248
FIRSTFARMS	-0.2056	-0.0422	-0.0201	-0.0198	-0.0022
GLUNZ & JENSEN HOLDING	-0.1739	-0.0593	-0.0429	0.0157	-0.0373
BRD KLEE B	-0.1505	-0.0773	-0.0662	-0.0248	-0.0467
SKAKO	-0.1000	0.0456	-0.0264	-0.0049	-0.0104
KOBENHAVNS LUFTHAVNE	-0.0654	0.0024	0.0021	-0.0199	-0.0163
ERRIA	-0.1830	-0.0579	-0.0621	0.0453	0.0081
TORM A	0.0690	-0.0265	-0.0222	-0.0339	-0.0101

Company	Lag=1	Lag=2	Lag=3	Lag=4	Lag=5
NTG NORDIC TRANSPORT GROUP	-0.0721	0.0027	0.0042	0.0243	0.0012
DFDS	0.0594	0.0395	0.0300	0.0695	0.0462
KENDRION	-0.0241	0.0222	0.0307	-0.0156	0.0409
KON.HEIJMANS DU. CERTS.	0.0835	0.0479	-0.0088	0.0168	-0.0032
HYDRATEC INDUSTRIES	-0.0583	-0.0741	-0.0436	-0.0239	-0.0035
SUNEX	-0.0115	-0.0049	0.0487	-0.0195	-0.0601
MVA GREEN ENERGY	0.1196	0.0297	0.0568	-0.0082	-0.0288
COLUMBUS ENERGY	-0.0699	-0.0102	0.0212	0.0083	0.0577
VOOLT	-0.0269	-0.0260	-0.0440	0.0088	-0.0112
VIATRON	0.1254	0.0985	0.0391	0.0148	0.0227
BIOMASS ENERGY PROJECT	-0.1120	0.0146	0.0408	-0.0501	-0.0269
FIRMA OPONIARSKA DEBICA	-0.0957	-0.0062	0.0739	0.0164	0.0027
INTER CARS	-0.0670	0.0073	-0.0185	0.0091	-0.0430
LESS	0.0007	-0.0636	0.0397	0.0037	-0.0031
AC AUTOGAZ	-0.1223	-0.0333	0.0081	-0.0172	0.0105
PL GROUP	-0.0171	-0.0506	-0.0288	0.0158	-0.0271
ORZEL	-0.0573	-0.0572	-0.0472	-0.0231	-0.0417
SOLAR INNOVATION	-0.1185	-0.0306	-0.0036	-0.0136	-0.0550
KRAKCHEMIA	-0.1160	-0.1122	0.0191	-0.0344	-0.0112
SELENA FM SR.B I C	-0.0767	-0.0150	-0.0172	0.0082	0.0004
GALVO	-0.1299	-0.0377	-0.0362	-0.0556	-0.0319
HORTICO	-0.0851	-0.0470	-0.0094	-0.0578	0.0052
IZOBLOK	-0.0931	0.0052	0.0129	-0.0409	-0.0134
PRYMUS	-0.2273	0.0386	-0.1006	0.0037	-0.0057
PCC EXOL	-0.0077	-0.0039	-0.0351	0.0710	0.0630
MOSTOSTAL ZABRZE	0.0929	0.0269	-0.0450	-0.0016	-0.0051
BUDIMEX	-0.0607	-0.0103	0.0011	-0.0287	-0.0024
PBG	-0.0322	-0.0070	0.0511	-0.0130	-0.1242
RAWLPLUG	-0.1487	-0.0131	-0.0005	0.0170	0.0052
LENA LIGHTING	-0.0873	0.0184	0.0283	-0.0156	-0.0294
DECORA	-0.0326	0.0463	-0.0039	-0.0216	0.0299
INVESTMENT FRIENDS	-0.1472	-0.0260	-0.0692	-0.0365	0.0301
MERCOR	-0.0262	0.0252	0.0472	-0.0108	0.0092
PA NOVA	-0.1315	-0.0257	-0.0098	-0.0044	0.0163
RESBUD	-0.0985	-0.0022	-0.0344	0.0141	0.0056
IZOLACJA JAROCIN	-0.0941	-0.0549	0.0186	-0.0774	-0.0142
TRAKCJA	0.0652	-0.0254	0.0220	0.0083	0.0305
UNIBEP	-0.0186	-0.0110	-0.0520	0.0155	0.0272
STARHEDGE	-0.1869	-0.0167	-0.0208	-0.0291	0.0341
FON	-0.1335	0.0661	-0.0170	-0.0147	0.0490
MOSTAL	-0.1526	-0.0022	0.0004	0.0140	0.0459
INSTAL KRAKOW	-0.0678	-0.0231	0.0262	-0.0321	0.0140
MERA	-0.1812	-0.0236	-0.0243	-0.0480	0.0136
TESGAS	-0.0768	-0.0004	0.0319	-0.0299	-0.0406
ZUE	-0.1147	-0.0387	0.0284	0.0066	-0.0205
LIBET	-0.0635	-0.0051	-0.0048	0.0425	0.0088
TAMEX OBIEKTY SPORTOWE	-0.1243	-0.0355	-0.0570	-0.0168	0.0434
MOBRUK	-0.0401	0.0084	-0.0065	0.0141	-0.0264
INTERMA TRADE	-0.0493	-0.0573	0.0820	-0.0136	-0.0163
DEKTRA	-0.0772	-0.0241	0.0085	-0.0535	0.0217
INTERNITY	0.0534	0.0776	-0.0186	-0.0198	-0.0533
HONEY PAYMENT GROUP	0.0178	0.1190	-0.0850	-0.0782	-0.0722
PRZED.PRZ.BETONOW PREFA-BET BIALE BLOTA	0.0576	0.0791	0.0115	0.0207	0.0033
ROCCA	0.2491	0.1051	0.0114	-0.0577	-0.0277
FABRYKA KONSTRUKCJI DREW	-0.2157	-0.0052	0.0070	0.0127	-0.0347
ULMA CONSTR.POLSKA	-0.1796	-0.0433	0.0070	-0.0149	-0.0303
ATLANTIS	-0.0997	-0.0515	-0.0362	0.0105	-0.0179
POLIMEX MOSTOSTAL	-0.1573	-0.0497	-0.0667	0.0477	-0.0056

Company	**Lag=1**	**Lag=2**	**Lag=3**	**Lag=4**	**Lag=5**
FORBUILD	-0.1354	-0.0179	-0.0856	-0.0139	-0.0013
HM INWEST ORD	0.0410	0.0149	0.0506	0.0891	0.0218
TAURON POLSKA ENERGIA	0.0601	-0.0143	-0.0144	0.0262	-0.0202
EC BEDZIN	-0.0175	-0.0707	-0.0305	-0.0134	-0.0008
ENEA	0.0556	0.0060	-0.0197	-0.0213	-0.0098
POLENERGIA	0.0094	-0.0640	-0.0195	-0.0250	0.0127
ZESPOL ELKTP. WRLKKNR.	-0.0504	-0.0306	0.0220	-0.0009	-0.0473
PHOTON ENERGY	-0.0881	0.0034	-0.0048	-0.0287	-0.0151
UNIMOT	0.0863	0.0023	-0.0394	-0.0108	0.0221
MANGATA HOLDING	-0.1218	-0.0558	-0.0017	0.0101	-0.0268
ZAK AD BUD MASZYN ZBC.	-0.0508	-0.0050	-0.0203	-0.0220	-0.0013
KUPIEC	-0.1391	-0.0354	-0.0495	-0.0306	0.0029
BORYSZEW	0.1207	0.1205	0.0147	-0.0081	-0.0291
FABRYKA OBRABIAREK RAFAMET	-0.0845	-0.0226	-0.0111	0.0106	-0.0055
ZAKLADY URZADZEN KOTLOWYCH STAPORKOW	-0.0452	-0.0324	0.0009	0.0104	-0.0295
MOJ	-0.0547	-0.0667	-0.0420	-0.0424	0.0004
PGF POLSKA GRUPA FOTOWOLTAICZNA	0.0322	-0.0295	-0.0423	-0.0204	0.0468
ENERGOINSTAL	-0.0744	-0.0552	0.0458	0.0284	0.0134
SECOGROUP	-0.1033	-0.0062	-0.0252	0.0046	-0.0249
WIELTON	0.1183	0.0075	0.0271	-0.0006	0.0289
BUMECH	0.0716	-0.0526	-0.0515	0.0082	0.0227
KCI	-0.4468	0.0464	-0.0497	-0.0462	0.0865
ZAKLADY MAGNEZYTOWE ROPCZYCE	-0.0593	-0.0076	-0.0275	0.0426	-0.0365
HYDRAPRES	-0.1183	-0.0880	-0.0736	0.0001	-0.0466
ZAMET	-0.1019	-0.0477	0.0461	-0.0000	-0.0157
SANOK RUBBER COMPANY	0.0653	0.0129	0.0461	-0.0241	-0.0461
FEERUM	-0.1923	-0.0376	0.0501	-0.0584	-0.0164
APS ENERGIA	-0.0687	-0.0353	-0.0319	-0.0131	0.0125
DROZAPOL PROFIL	-0.1276	-0.0050	0.0363	-0.0187	-0.0191
PJP MAKRUM	-0.0742	0.0194	0.0158	-0.0325	-0.0052
ODLEWNIE POLSKIE	-0.1427	-0.0122	0.0098	-0.0035	-0.0275
GRUPA KETY	-0.0533	-0.0244	-0.0102	-0.0294	-0.0059
MFO	-0.1382	-0.0004	0.0184	-0.0258	0.0009
EKOPOL GORNOSLASK HLDG.	-0.1072	-0.0396	-0.0218	-0.0114	-0.0423
STALPRODUKT	0.0899	0.0227	0.0330	-0.0502	0.0418
STALEXPORT AUTOSTRADY	-0.0110	0.0128	0.0208	-0.0169	-0.0661
TRANSPOL	-0.0802	-0.0380	-0.0012	-0.0012	-0.0581
OT LOGISTICS	0.0500	-0.0141	0.0080	0.0012	-0.0462
BALTICON	-0.1453	-0.0361	-0.0464	0.0160	-0.0138
FORPOSTA	0.0375	-0.0096	-0.0507	-0.0563	-0.0465
XBS PRO-LOG	-0.0747	-0.0369	-0.0152	-0.0021	-0.0309
NATURGY ENERGY	0.0107	0.0379	-0.0397	-0.0324	0.0070
GRINO ECOLOGIC	0.3113	0.0701	0.1019	0.0534	-0.0570
ACS ACTIV.CONSTR.Y SERV.	0.0700	0.0703	0.0241	0.0329	-0.0082
FOMENTO CONSTR.Y CNTR.	-0.0462	0.0212	-0.0178	-0.0252	0.0352
ACCIONA	-0.0236	0.0453	-0.0187	-0.0360	-0.0410
SACYR	0.0439	0.0711	0.0032	-0.0182	0.0347
AUDAX RENOVABLES	0.0505	0.0526	0.0237	-0.0123	0.0282
ENDESA	0.0398	0.0606	-0.0165	-0.0070	0.0079
AKILES CORPORATION	-0.0334	-0.0644	-0.0434	0.0533	-0.0198
IBERDROLA	-0.0223	0.0463	-0.0280	-0.0179	-0.0285
ROMANDE ENERGIE	-0.2479	-0.0676	-0.0361	0.0188	-0.0336
EDISUN POWER EUROPE N	-0.1966	-0.0689	-0.0275	-0.0136	-0.0185
BKW	-0.0649	-0.0056	0.0005	0.0243	-0.0108
ENERGIEDIENST HOLDING	-0.1445	-0.0680	-0.0136	-0.0203	0.0082
DOTTIKON ES HOLDING	-0.1065	0.0287	0.0201	-0.0117	0.0256

Company	Lag=1	Lag=2	Lag=3	Lag=4	Lag=5
GURIT HOLDING 'B'	-0.0511	-0.0100	-0.0050	-0.0072	-0.0069
FEINTOOL	-0.0739	0.0132	-0.0224	0.0173	-0.0161
AUTONEUM HOLDING	0.0547	0.0570	0.0180	0.0385	0.0232

Table 21: Autocorrelation function on stock returns

A.2.2 Stock market regression analysis for the conditional mean

Company	Log-like1	Log-like2	LR statistic	P-value
HERA	2153.1419	2126.2604	-53.762871	1.0
ACINQUE	2280.5356	2246.2491	-68.5731	1.0
ASCOPIAVE	2188.4241	2118.3647	-140.118917	1.0
EDISON RSP	2290.1795	2301.7738	23.188527	0.0
ENEL	2231.6989	2198.0353	-67.327341	1.0
TERNA RETE ELETTRICA NAZ	2279.722	2283.6029	7.761824	0.00534
ACEA	2222.4025	2133.1202	-178.564612	1.0
A2A	2162.5652	2049.4038	-226.322759	1.0
FRENDY ENERGY	2431.6195	1994.5901	-874.058704	1.0
ERG	2081.8791	1576.7942	-1010.17	1.0
AGATOS	2065.4262	1958.2812	-214.289959	1.0
GAS PLUS	1920.5559	1792.867	-255.377961	1.0
BIESSE	1850.4004	1842.9078	-14.985198	1.0
BORGOSESIA	2066.5112	1761.8505	-609.321522	1.0
INNOVATEC	1734.1903	1575.5064	-317.36794	1.0
ENERTRONICA	6301.4069	2526.2486	-7550.3165	1.0
SOL	2130.888	281.2338	-3699.3084	1.0
ENCRES DUBUIT	2286.7555	2156.1971	-261.116673	1.0
METABOLIC EXPLORER	1467.2581	693.1587	-1548.1988	1.0
ROBERTET	2267.3715	2173.683	-187.376915	1.0
L AIR LQE.SC.ANYME. POUR L ETUDE ET L EPXTN.	2332.1189	2341.9674	19.697142	0.00001
EXPLOS.ET PRDS.CHIM.	2303.6199	2103.9054	-399.428993	1.0
RENAULT	1807.5101	1742.0286	-130.962823	1.0
BURELLE	2227.1413	2219.8166	-14.649577	1.0
DELFINGEN	2016.7421	1940.8238	-151.836579	1.0
MICHELIN	2173.7132	2159.8508	-27.7292	1.0
ENGIE	2257.4411	2282.4676	50.052901	0.0
FINAXO ENVIRONNEMENT	2736.5805	2726.0131	-21.134746	1.0
EAUX DE ROYAN	2191.2744	2952.1021	1521.6554	0.0
MAUREL ET PROM	1694.1662	1571.0242	-246.2842	1.0
CIE DE CHEMINS DE FER DEPARTEMENTAUX	2463.3053	2458.9422	-8.7261	1.0
TRILOGIQ	1930.1853	1938.4711	16.57154	0.00005
EXAIL TECHNOLOGIES	1948.0568	1932.6965	-30.720727	1.0
EO2	1888.3007	1734.2987	-308.003931	1.0
SOLARWORLD K	1473.0604	1419.5474	-107.025936	1.0
PHOENIX SOLAR	599.3629	573.7796	-51.166662	1.0
GLOBAL PVQ	824.4454	1107.9842	567.077586	0.0
SFC ENERGY	1488.1485	324.1727	-2327.952	1.0
CROPENERGIES	1779.3243	1745.6102	-67.428178	1.0
ENAPTER	1540.9804	1514.198	-53.564734	1.0
ABO WIND	1774.1689	1768.0771	-12.183613	1.0
MUEHLHAN	1981.3977	1908.3003	-146.194778	1.0
SYMRISE	2162.1205	2145.9806	-32.2798	1.0
FUCHS N	2264.0039	2263.6518	-0.7041	1.0

Company	Log-like1	Log-like2	LR statistic	P-value
ALZCHEM	2101.1599	2086.2616	-29.79662	1.0
BASF	2128.712	1902.9004	-451.623225	1.0
H & R	2015.3446	2000.0211	-30.646995	1.0
DELTICOM	1551.0638	581.4439	-1939.24	1.0
MERCEDES-BENZ GROUP N	2084.7408	1975.4067	-218.668123	1.0
PORSCHE AML.HLDG.PREF.	1956.6057	-4092.997	-12099.205	1.0
FERNHEIZWERK NEUKOLLN	2049.0013	1826.471	-445.06	1.0
MAINOVA	1808.0608	1753.5256	-109.07	1.0
RWE	2123.1256	1680.4581	-885.335	1.0
GELSENWASSER	1780.248	1770.666	-19.1641	1.0
GLOBAL OIL & GAS	2295.7714	2285.2918	-21	1.0
DEUTSCHE ROHSTOFF	1716.5834	1695.2155	-42.736	1.0
AEE GOLD	3074.0986	5257.279	4366.361	0.0
META WOLF	1641.2001	1320.1329	-642.1345	1.0
STO PREFERENCE	1856.7596	1467.4308	-778.6576	1.0
VILLEROY & BOCH PF.SHS.	1871.584	1874.754	6.339952	0.0118
INNOTEC TSS	1839.0866	1820.1829	-37.807372	1.0
UZIN UTZ	1884.9449	1884.7995	-0.291	1.0
WESTAG	2548.2813	2545.8044	-4.954	1.0
STEULER FLIESEN-GRUPPE	1706.707	1485.9853	-441.4434	1.0
7C SOLARPARKEN K	2048.2148	2037.2044	-22.021	1.0
ENERGIEKONTOR	1728.5321	1482.2954	-492.473513	1.0
4 SC	1412.3359	1365.4491	-93.773753	1.0
BAUMOT GROUP	959.9083	712.2021	-495.412271	1.0
2G ENERGY	1730.3334	1519.4149	-421.837035	1.0
ENCAVIS	1753.1493	1748.9277	-8.443185	1.0
MVV ENERGIE	2148.9695	2127.4569	-43.025165	1.0
ENBW ENGE.BADEN-WURTG.	1840.961	1583.8156	-514.290731	1.0
EVN	2160.1555	2126.0281	-68.254888	1.0
BURGENLAND HOLDING	2396.5921	2396.7066	0.228945	0.63231
VERBUND	1892.1946	1879.9653	-24.458589	1.0
RATH	2369.2294	2371.0829	3.707117	0.05418
SW UMWELTTECHNIK	2445.7466	2623.7188	355.944519	0.0
WIENERBERGER	2119.6108	2076.2826	-86.65636	1.0
FLSMIDTH AND CO.	1853.0968	1785.0207	-136.152097	1.0
VESTAS WINDSYSTEMS	1668.6486	1650.4052	-36.5	1.0
UIE	2275.3612	2260.5419	-29.64	1.0
GLUNZ & JENSEN HOLDING	2215.7084	2260.4056	89.394223	0.0
BRD KLEE B	2403.7962	2529.096	250.599753	0.0
KOBENHAVNS LUFTHAVNE	2108.5925	1818.5996	-580	1.0
ERRIA	1408.5834	1392.9882	-31.19	1.0
TORM A	1645.0484	158.1032	-2973.89	1.0
DFDS	1948.9125	1908.9937	-79.84	1.0
HYDRATEC INDUSTRIES	2220.6959	2265.3899	89.387991	0.0
SUNEX	1519.177	1492.6	-53.1541	1.0
MVA GREEN ENERGY	2519.8448	2515.6425	-8.4045	1.0
COLUMBUS ENERGY	1551.4714	1536.8763	-29.19	1.0
VIATRON	3284.0969	3927.6228	1287.0518	0.0
BIOMASS ENERGY PROJECT	1672.7126	1650.2908	-44.8438	1.0

Company	Log-like1	Log-like2	LR statistic	P-value
FIRMA OPONIARSKA DEBICA	2495.6241	2466.982	-57.2841	1.0
AC AUTOGAZ	2295.4993	2242.1204	-106.7578	1.0
PL GROUP	5324.5621	3077.2951	-4494.534	1.0
ORZEL	1796.1574	1767.2904	-57.734	1.0
SOLAR INNOVATION	2080.5931	1788.4121	-584.362	1.0
KRAKCHEMIA	1525.9754	1512.5535	-26.8438	1.0
SELENA FM SR.B I C	2037.0231	2010.5118	-53.0225	1.0
GALVO	2306.6389	1487.9678	-1637.3423	1.0
IZOBLOK	2417.4248	2041.1183	-752.613	1.0
PRYMUS	2287.609	2992.2145	1409.211	0.0
PCC EXOL	2165.8284	2153.4412	-24.774	1.0
MOSTOSTAL ZABRZE	1895.4259	1884.9992	-20.853	1.0
PBG	2295.8587	2300.2777	8.8381	0.00295
RAWLPLUG	1943.0037	1940.2844	-5.4387	1.0
LENA LIGHTING	2207.5721	2181.6645	-51.815	1.0
INVESTMENT FRIENDS	1228.0827	1281.9574	107.749415	0.0
RESBUD	1721.0973	1708.5856	-25.0234	1.0
IZOLACJA JAROCIN	1683.9111	1663.1914	-41.4394	1.0
STARHEDGE	1619.9408	1429.0648	-381.8	1.0
FON	2040.7444	2113.1761	144.863441	0.0
INSTAL KRAKOW	2158.3837	2152.3189	-12.13	1.0
MERA	1959.5914	2164.3705	409.558373	0.0
TESGAS	1994.1527	1988.4178	-11.5	1.0
ZUE	1908.5615	1886.4176	-44.29	1.0
MOBRUK	2019.1646	2014.0385	-10.2523	1.0
INTERMA TRADE	2364.1333	2774.8066	821.346594	0.0
INTERNITY	1720.6985	1686.3975	-68.602	1.0
HONEY PAYMENT GROUP	2268.5834	2305.8143	74.46173	0.0
PRZED.PRZ.BETONOW PREFABET BIALE BLOTA	2570.7491	2432.5281	-276.442	1.0
ROCCA	2075.4581	1752.7796	-645.36	1.0
FABRYKA KONSTRUKCJI DREW	5065.1087	4970.9407	-188.336	1.0
ULMA CONSTR.POLSKA	2194.3435	2106.4115	-175.864	1.0
ATLANTIS	3182.6186	2816.1421	-732.953	1.0
FORBUILD	2729.6831	2704.7888	-49.8	1.0
EC BEDZIN	1441.8506	30.8835	-2822	1.0
ZESPOL ELKTP. WRLKKNR.	1950.9879	1934.3758	-33	1.0
PHOTON ENERGY	1887.682	1875.9255	-23.52	1.0
UNIMOT	1912.8247	1898.5758	-28.5	1.0
MANGATA HOLDING	1947.9178	1935.7341	-24.4	1.0
ZAK AD BUD MASZYN ZBC.	1506.5407	67.3613	-2878.36	1.0
KUPIEC	1720.3951	1618.094	-204.60	1.0
FABRYKA OBRABIAREK RAFAMET	2304.0538	2312.9663	17.825	0.00002
ZAKLADY URZADZEN KOTLOWYCH STAPORKOW	1764.0362	1767.2812	6.5	0.01085
MOJ	1929.0109	2017.4678	176.9	0.0
PGF POLSKA GRUPA FOTOWOLTAICZNA	1451.4807	356.866	-2189.2294	1.0
SECOGROUP	2318.348	2604.1803	571.66	0.0
HYDRAPRES	2667.0925	3057.3863	780.6	0.0
ZAMET	2019.4642	2001.1246	-36.7	1.0
FEERUM	1704.3494	1662.7718	-83.16	1.0

Company	Log-like1	Log-like2	LR statistic	P-value
APS ENERGIA	1546.192	1539.5981	-13.19	1.0
DROZAPOL PROFIL	1733.1549	1701.5966	-63.116	1.0
PJP MAKRUM	1748.5163	1726.39	-44.253	1.0
ODLEWNIE POLSKIE	2003.3728	1351.1845	-1304.38	1.0
MFO	1826.0569	1819.8017	-12.51	1.0
EKOPOL GORNOSLASK HLDG.	1802.5535	1749.1884	-106.730205	1.0
STALEXPORT AUTOSTRADY	2462.9337	2399.9085	-126.05	1.0
TRANSPOL	2118.3773	2048.0261	-140.7	1.0
OT LOGISTICS	1544.167	1520.9775	-46.38	1.0
BALTICON	1558.2223	1905.4975	694.55	0.0
FORPOSTA	1498.0177	1300.5861	-394.86	1.0
XBS PRO-LOG	2086.6317	2353.9931	534.72	0.0
NATURGY ENERGY	2269.464	2271.4273	3.93	0.04752
GRINO ECOLOGIC	3271.6318	3146.8886	-249.5	1.0
ACS ACTIV.CONSTR.Y SERV.	2263.0557	2217.4594	-91.2	1.0
FOMENTO CONSTR.Y CNTR.	2184.1212	2184.759	1.28	0.25872
ACCIONA	2071.0911	1771.2955	-599.6	1.0
SACYR	2136.4891	1966.2238	-340.53	1.0
AUDAX RENOVABLES	1862.6375	1833.6841	-58	1.0
ENDESA	2251.6743	2224.9708	-53.41	1.0
AKILES CORPORATION	2748.7571	2738.0359	-21.44	1.0
IBERDROLA	2294.2125	2318.0557	47.69	0.0
ROMANDE ENERGIE	2121.2652	1258.7447	-1725.041	1.0
EDISUN POWER EUROPE N	2280.2009	2025.5995	-509.202	1.0
DOTTIKON ES HOLDING	1819.0978	1685.8522	-266.5	1.0
GURIT HOLDING 'B'	1771.9223	1739.9088	-64.03	1.0
FEINTOOL	1972.6982	902.4978	-2140.4	1.0
AUTONEUM HOLDING	1857.7215	1857.3423	-0.76	1.0

Table 22: Likelihood ratio test
AR(1) with no exogenous variables and
AR(1) with MASCI and VIX variables

List of stocks that rejected the null hypothesis of the test:
Hera, Acinque, Ascopiave, Edison Rsp, Enel, Terna Rete Elettrica Naz, Algowatt, Acea, A2A, Frendy Energy, Erg, Agatos, Gas Plus, Biesse, Borgosesia, Innovatec, Enertronica, Sol, Encres Dubuit, Metabolic Explorer, Robertet, L Air Lqe.Sc.Anyme. Pour L Etude Et L Epxtn., Explos.Et Prds.Chim., Carbios, Renault, Burelle, Delfingen, Michelin, Valeo, Engie, Finaxo Environnement, Eaux De Royan, Maurel Et Prom, Cie De Chemins De Fer Departementaux, Trilogiq, Roctool, Exail Technologies, Eo2, Global Bioenergies, Solarworld K, Phoenix Solar, Global Pvq, Sfc Energy, Cropenergies, Envitec Biogas, Centrotherm Phto., Enapter, Clearvise (Fra), Abo Wind, Masterflex, Muehlhan, Symrise, Fuchs N, Eckert & Ziegler Strahlen & Medzi., Alzchem, Evonik Industries, Basf, H & R, Delticom, Mercedes-Benz Group N, Grammer, Volkswagen, Bmw, Porsche Aml.Hldg.Pref., Fernheizwerk Neukolln, Mainova, Rwe, Gelsenwasser, Global Oil & Gas, Deutsche Rohstoff, Aee Gold, Meta Wolf, Sto Preference, Bauer, Steico, Villeroy & Boch Pf.Shs., Innotec Tss, Uzin Utz, Westag, Steuler Fliesengruppe, 7C Solarparken K, Energiekontor, 4 Sc, Baumot Group, 2G Energy, Encavis, Pne, Mvv Energie, Lechwerke, Enbw Enge.Baden-Wurtg., Moury Construct, Floridienne, Bekaert (D), Jensen-Group, Evn, Burgenland Holding, Verbund, Rath, Strabag Se, Sw Umwelttechnik, Porr, Wienerberger, Polytec Holding, Mt Hoejgaard Holding, Flsmidth And Co., Scandinavian Brake Sys., Ennogie Solar Group, Vestas Windsystems, Uie, Firstfarms, Glunz & Jensen Holding, Brd Klee B, Skako, Kobenhavns Lufthavne, Erria, Torm A, Ntg Nordic Transport Group, Dfds, Kendrion, Kon.Heijmans Du. Certs., Hydratec Industries, Sunex, Mva Green Energy, Columbus Energy, Voolt, Viatron, Biomass Energy Project, Firma Oponiarska Debica, Inter Cars, Less, Ac Autogaz, Pl Group, Orzel, Solar Innovation, Krakchemia,

Selena Fm Sr.B I C, Galvo, Hortico, Izoblok, Prymus, Pcc Exol, Mostostal Zabrze, Budimex, Pbg, Rawlplug, Lena Lighting, Decora, Investment Friends, Mercor, Pa Nova, Resbud, Izolacja Jarocin, Trakcja, Unibep, Starhedge, Fon, Mostal, Instal Krakow, Mera, Tesgas, Zue, Libet, Tamex Obiekty Sportowe, Mobruk, Interma Trade, Dektra, Internity, Honey Payment Group, Przed.Prz.Betonow Prefabet Biale Blota, Rocca, Fabryka Konstrukcji Drew, Ulma Constr.Polska, Atlantis, Polimex Mostostal, Forbuild, Hm Inwest Ord, Tauron Polska Energia, Ec Bedzin, Enea, Polenergia, Zespol Elktp. Wrlkknr., Photon Energy, Unimot, Mangata Holding, Zak Ad Bud Maszyn Zbc., Kupiec, Boryszew, Fabryka Obrabiarek Rafamet, Zaklady Urzadzen Kotlowych Staporkow, Moj, Pgf Polska Grupa Fotowoltaiczna, Energoinstal, Secogroup, Wielton, Bumech, Kci, Zaklady Magnezytowe Ropczyce, Hydrapres, Zamet, Sanok Rubber Company, Feerum, Aps Energia, Drozapol Profil, Pjp Makrum, Odlewnie Polskie, Grupa Kety, Mfo, Ekopol Gornoslask Hldg., Stalprodukt, Stalexport Autostrady, Transpol, Ot Logistics, Balticon, Forposta, Xbs Pro-Log, Naturgy Energy, Grino Ecologic, Acs Activ.Constr.Y Serv., Fomento Constr.Y Cntr., Acciona, Sacyr, Audax Renovables, Endesa, Akiles Corporation, Iberdrola, Romande Energie, Edisun Power Europe N, Bkw, Energiedienst Holding, Dottikon Es Holding, Gurit Holding 'B', Feintool, Autoneum Holding

Company	**Log-like1**	**Log-like2**	**LR statistic**	**P-value**
EDISON RSP	2290.1795	2380.7505	181.14199	0.0
TERNA RETE ELETTRICA NAZ	2279.722	2322.3767	85.309536	0.0
L AIR LQE.SC.ANYME. POUR L ETUDE ET L EPXTN.	2332.1189	2389.442	114.646311	0.0
ENGIE	2257.4411	2302.307	89.73181	0.0
EAUX DE ROYAN	2191.2744	2836.7828	1291.01674	0.0
TRILOGIQ	1930.1853	2273.7156	687.060676	0.0
GLOBAL PVQ	824.4454	2023.5884	2398.285988	0.0
AEE GOLD	3074.0986	3661.2282	1174.259167	0.0
VILLEROY & BOCH PF.SHS.	1871.584	1875.4497	7.731308	0.00543
SW UMWELTTECHNIK	2445.7466	2601.647	311.80096	0.0
GLUNZ & JENSEN HOLDING	2215.7084	2770.3885	1109.360164	0.0
BRD KLEE B	2403.7962	2251.3942	-304.803974	1.0
HYDRATEC INDUSTRIES	2220.6959	2427.0417	412.691594	0.0
VIATRON	3284.0969	3026.0789	-516.036037	1.0
PRYMUS	2287.609	2992.5187	1409.819305	0.0
PBG	2295.8587	5650.4678	6709.218167	0.0
INVESTMENT FRIENDS	1228.0827	1274.5631	92.9609	0.0
FON	2040.7444	3225.4748	2369.460782	0.0
MERA	1959.5914	1934.2037	-50.775278	1.0
INTERMA TRADE	2364.1333	4086.1887	3444.110664	0.0
HONEY PAYMENT GROUP	2268.5834	2605.1381	673.10929	0.0
FABRYKA OBRABIAREK RAFAMET	2304.0538	2897.6268	1187.146084	0.0
ZAKLADY URZADZEN KOTLOWYCH STAPORKOW	1764.0362	2212.5699	897.067418	0.0
MOJ	1929.0109	2044.6634	231.305091	0.0
SECOGROUP	2318.348	3972.9312	3309.166485	0.0
HYDRAPRES	2667.0925	2258.5554	-817.074202	1.0
BALTICON	1558.2223	1728.5813	340.718058	0.0
XBS PRO-LOG	2086.6317	2662.7827	1152.301856	0.0
NATURGY ENERGY	2269.464	2321.8564	104.784891	0.0
IBERDROLA	2294.2125	2342.5068	96.588533	0.0

Table 23: Likelihood ratio test
AR(1) with MASCI and VIX and
AR(1) with MASCI, VIX, and ECF

Company	μ	ω	α_1	ϕ_1	β_1
HERA	0.	-5.09280	0.15913	0.40528	0.10000
ACINQUE	-0.00039	-5.40448	0.13201	0.42894	-0.26847
ASCOPIAVE	-0.00067	-2.57441	0.19208	0.70615	-0.11450
EDISON RSP	0.00052	-5.09280	0.15913	0.40528	0.10000
ENEL	0.00030	-2.11575	0.12205	0.76453	-0.01550
TERNA RETE ELETTRICA NAZ	0.00052	-5.09280	0.15913	0.40528	0.10000
ALGOWATT	-0.00212	-2.13363	0.29232	0.74043	-0.06900
ACEA	0.00017	-5.52150	0.11867	0.37853	0.00956
A2A	0.00084	-2.85656	0.15888	0.66994	0.00418
FRENDY ENERGY	0.00000	-4.33995	0.61280	0.78662	-0.00010
ERG	0.00034	-4.15203	0.13719	0.51665	0.01874
AGATOS	0.00033	-3.23999	0.38015	0.67363	0.07395
GAS PLUS	-0.00150	-1.18868	0.21354	0.86157	-0.17293
BIESSE	-0.00062	-2.27292	0.12040	0.72197	-0.00670
BORGOSESIA	-0.00032	-2.95505	0.17315	0.65964	-0.21361
INNOVATEC	-0.00367	-2.69205	0.18312	0.66849	-0.06953
ENERTRONICA	-0.00000	-5.08137	1.27769	0.77846	0.06371
SOL	0.00101	-5.43081	0.10778	0.37278	-0.022243
ENCRES DUBUIT	-0.00004	-5.01755	0.20585	0.57836	-0.01570
METABOLIC EXPLORER	-0.00313	-2.75422	0.21027	0.63020	-0.01278
ROBERTET	-0.00028	-8.06466	0.13271	0.11462	-0.15698
L AIR LQE.SC.ANYME. POUR L ETUDE ET L EPXTN.	0.00052	-5.09280	0.15913	0.40528	0.10000
EXPLOS.ET PRDS.CHIM.	0.00004	-4.96778	0.56640	0.65799	-0.12459
CARBIOS	-0.00104	-1.69822	0.10788	0.77420	-0.01795
RENAULT	0.00051	-2.10138	0.08656	0.73535	-0.00881
BURELLE	-0.00100	-4.58874	0.17686	0.50113	-0.04041
DELFINGEN	-0.00088	-2.19598	0.20312	0.75570	0.05832
MICHELIN	0.00097	-5.06252	0.13238	0.43042	-0.03501
VALEO	-0.00048	-2.47030	0.12213	0.68614	0.00746
ENGIE	0.00052	-5.09280	0.15913	0.40528	0.10000
FINAXO ENVIRONNEMENT	0.00052	-5.09280	0.15913	0.40528	0.10000
EAUX DE ROYAN	0.00025	-5.05108	0.46177	0.57575	-0.24576
MAUREL ET PROM	0.00183	-4.81152	0.07121	0.36911	-0.00386
CIE DE CHEMINS DE FER DEPARTEMENTAUX	0.00052	-5.09280	0.15913	0.40528	0.10000
TRILOGIQ	-0.00300	-5.00043	0.27205	0.48652	-0.22682
ROCTOOL	-0.00173	-2.86572	0.16278	0.62313	-0.14349
EXAIL TECHNOLOGIES	-0.00026	-2.49731	0.10825	0.69449	-0.04153
EO2	-0.00063	-2.01665	0.16508	0.76479	-0.04396
GLOBAL BIOENERGIES	-0.00465	-2.03041	0.15034	0.73677	-0.07266
SOLARWORLD K	0.00019	-4.69787	0.42430	0.52556	-0.11881
PHOENIX SOLAR	-0.00193	-4.63100	0.26892	0.12955	-0.23032
GLOBAL PVQ	0.00881	-4.13617	0.48356	0.32072	-0.25380
SFC ENERGY	-0.00080	-2.65800	0.11089	0.61971	-0.15134
CROPENERGIES	-0.00111	-5.02301	0.06516	0.38769	-0.06016
ENVITEC BIOGAS	-0.00049	-2.08670	0.15132	0.72038	-0.20592
CENTROTHERM PHTO.	-0.00118	-3.64199	0.05477	0.51433	-0.15794

Company	μ	ω	α_1	ϕ_1	β_1
ENAPTER	-0.00205	-2.03338	0.15566	0.72059	-0.21345
CLEARVISE (FRA)	-0.00062	-1.82698	0.15642	0.78101	-0.29674
ABO WIND	-0.00052	-2.21125	0.11688	0.72075	-0.18280
MASTERFLEX	-0.00157	-7.07624	0.09820	0.11022	-0.16988
MUEHLHAN	-0.00006	-1.95190	0.36103	0.78488	-0.28198
SYMRISE	0.00052	-5.09280	0.15913	0.40528	0.10000
FUCHS N	0.00002	-5.09765	0.10686	0.44517	-0.08989
ECKERT & ZIEGLER STRAHLEN & MEDZI.	-0.00019	-5.08305	0.08408	0.32669	-0.03559
ALZCHEM	-0.00107	-4.24741	0.19967	0.53010	-0.17611
EVONIK INDUSTRIES	0.00009	-5.09331	0.15476	0.43436	-0.11845
BASF	0.00031	-5.09097	0.18276	0.42345	-0.06996
H & R	-0.00057	-1.41130	0.12685	0.84144	-0.11789
DELTICOM	-0.00178	-2.88330	0.14392	0.61681	-0.20580
MERCEDES-BENZ GROUP N	0.00072	-2.50275	0.13979	0.70969	0.02302
GRAMMER	-0.00093	-3.93905	0.16326	0.51494	-0.19987
VOLKSWAGEN	-0.00133	-5.08513	0.16846	0.37503	-0.00389
BMW	0.00108	-5.10713	0.12332	0.41995	-0.02626
PORSCHE AML.HLDG.PREF.	-0.00103	-5.08004	0.16213	0.39090	0.04449
FERNHEIZWERK NEUKOLLN	0.00007	-4.43255	0.68384	0.60238	-0.15515
MAINOVA	0.00004	-4.58790	0.48105	0.45370	-0.35156
RWE	0.00052	-5.09280	0.15913	0.40528	0.10000
GELSENWASSER	-0.00083	-4.20981	0.34360	0.53731	-0.15740
GLOBAL OIL & GAS	0.00052	-5.09280	0.15913	0.40528	0.1
DEUTSCHE ROHSTOFF	0.00123	-1.16275	0.14796	0.85174	-0.12315
AEE GOLD	-0.00008	-5.11944	0.22301	0.66648	0.05803
META WOLF	-0.00119	-2.15774	0.22539	0.72796	-0.28398
STO PREFERENCE	-0.00130	-5.92809	0.06092	0.28773	-0.07483
BAUER	-0.00085	-1.82018	0.18598	0.79882	-0.23393
STEICO	-0.00075	-8.41506	0.04615	-0.14667	-0.04643
VILLEROY & BOCH PF.SHS.	0.00012	-4.02613	0.11525	0.51226	-0.13664
INNOTEC TSS	-0.00122	-3.47676	0.21196	0.58658	-0.28266
UZIN UTZ	-0.00136	-4.98622	0.11314	0.40870	-0.19771
WESTAG	0.00052	-5.09280	0.15913	0.40528	0.10000
STEULER FLIESEN-GRUPPE	0.00000	-5.09342	1.04257	0.66549	-0.38526
7C SOLARPARKEN K	-0.00139	-5.09440	0.12362	0.40876	-0.11058
ENERGIEKONTOR	-0.00104	-3.17755	0.12201	0.59451	-0.06173
4 SC	-0.00165	-1.30827	0.22921	0.83219	-0.13334
BAUMOT GROUP	-0.00000	-4.91800	1.70736	0.63880	-0.68287
2G ENERGY	-0.00046	-5.91333	0.08126	0.23143	-0.10113
ENCAVIS	-0.00013	-5.08578	0.08352	0.35297	-0.06927
PNE	-0.00046	-4.98778	0.11976	0.41504	-0.13820
MVV ENERGIE	-0.00009	-5.09523	0.13697	0.42827	-0.26227
LECHWERKE	-0.00060	-4.91004	0.19381	0.46419	-0.22046
ENBW ENGE.BADEN-WURTG.	-0.00036	-3.42527	0.17210	0.57140	-0.28721
MOURY CONSTRUCT	0.00022	-5.05635	0.14934	0.49541	-0.03472
FLORIDIENNE	0.00024	-1.82014	0.14394	0.78363	-0.07355
BEKAERT (D)	0.00134	-2.54699	0.14793	0.70261	-0.00887
JENSEN-GROUP	-0.00009	-5.03815	0.09156	0.43179	-0.13470
EVN	0.00105	-1.58261	0.16464	0.82505	-0.01163
BURGENLAND HOLDING	0.00052	-5.09280	0.15913	0.40528	0.10000
VERBUND	0.00085	-5.08769	0.09133	0.38223	-0.01842

Company	μ	ω	α_1	ϕ_1	β_1
RATH	0.00052	-5.09280	0.15913	0.40528	0.10000
STRABAG SE	0.00052	-5.09280	0.15913	0.40528	0.10000
SW UMWELTTECHNIK	-0.00000	-5.01598	0.72467	0.71168	-0.13658
PORR	0.00066	-1.93770	0.11788	0.76492	-0.17328
WIENERBERGER	0.00106	-2.34974	0.16492	0.73631	0.02054
POLYTEC HOLDING	-0.00062	-5.08971	0.13479	0.40982	-0.09999
MT HOEJGAARD HOLDING	-0.00112	-2.50938	0.15572	0.69303	-0.09214
FLSMIDTH AND CO.	0.00038	-3.84962	0.10452	0.51905	-0.02604
SCANDINAVIAN BRAKE SYS.	-0.00252	-2.60998	0.17242	0.65592	-0.15393
ENNOGIE SOLAR GROUP	0.00020	-2.05391	0.23841	0.72840	-0.07367
VESTAS WINDSYSTEMS	-0.00078	-5.02540	0.07710	0.33571	-0.02207
UIE	0.00018	-3.77642	0.15388	0.58774	-0.22650
FIRSTFARMS	-0.00112	-5.53278	0.15897	0.40371	-0.21061
GLUNZ & JENSEN HOLDING	0.00022	-5.03885	0.13601	0.51041	0.05011
BRD KLEE B	0.00001	-5.03983	0.70067	0.53426	0.15629
SKAKO	-0.00007	-9.85922	0.07643	-0.12470	-0.16857
KOBENHAVNS LUFTHAVNE	-0.00038	-3.48430	0.18456	0.60672	-0.17623
ERRIA	-0.00176	-0.74348	0.08658	0.89920	-0.21948
TORM A	0.00174	-2.75532	0.11190	0.62977	-0.02118
NTG NORDIC TRANSPORT GROUP	0.00021	-5.21744	0.09675	0.29850	0.06281
DFDS	-0.00032	-5.27827	0.10288	0.35393	0.05964
KENDRION	-0.00109	-3.68567	0.17929	0.55674	-0.11822
KON.HEIJMANS DU. CERTS.	0.00004	-2.76560	0.12457	0.68188	-0.03885
HYDRATEC INDUSTRIES	0.00009	-5.02159	0.10845	0.48797	0.03856
SUNEX	-0.00345	-1.69768	0.11501	0.77221	-0.05573
MVA GREEN ENERGY	0.00052	-5.09280	0.15913	0.40528	0.10000
COLUMBUS ENERGY	-0.00491	-4.81543	0.20683	0.37048	-0.02978
VOOLT	-0.00580	-1.51140	0.18594	0.78999	-0.10766
VIATRON	-0.00000	-5.12303	0.42912	0.77239	-0.03768
BIOMASS ENERGY PROJECT	-0.00085	-4.59402	0.33741	0.44277	-0.20158
FIRMA OPONIARSKA DEBICA	-0.00037	-3.93872	0.19142	0.60627	-0.20876
INTER CARS	0.00015	-2.37311	0.14392	0.72268	-0.10249
LESS	-0.00349	-1.95032	0.21549	0.73636	-0.11322
AC AUTOGAZ	-0.00040	-4.70237	0.16941	0.50464	-0.20733
PL GROUP	0.00000	-5.11301	0.61806	0.82893	0.06204
ORZEL	0.00055	-5.01985	0.38767	0.55281	-0.05758
SOLAR INNOVATION	0.00000	-5.01218	1.01781	0.79116	0.00003
KRAKCHEMIA	0.00282	-4.91595	0.25904	0.44706	0.02726
SELENA FM SR.B I C	-0.00040	-4.99056	0.20075	0.44619	-0.06553
GALVO	0.00000	-4.90896	0.69615	0.69969	0.00059
HORTICO	-0.00039	-2.97181	0.15553	0.65156	-0.13823
IZOBLOK	-0.00001	-4.70088	1.09829	0.59418	0.15840
PRYMUS	-0.00044	-4.96727	0.24415	0.52497	-0.07971
PCC EXOL	-0.00090	-4.90112	0.12677	0.47725	-0.11043
MOSTOSTAL ZABRZE	-0.00109	-2.25989	0.11641	0.73661	-0.00768
BUDIMEX	0.00062	-5.09120	0.13610	0.38455	-0.14224
PBG	0.00052	-5.09280	0.15913	0.40528	0.10000

Company	μ	ω	α_1	ϕ_1	β_1
RAWLPLUG	-0.00021	-5.13267	0.06578	0.39163	-0.11890
LENA LIGHTING	-0.00022	-4.78422	0.18291	0.47640	-0.21269
DECORA	-0.00015	-5.02027	0.15812	0.47107	-0.09818
INVESTMENT FRIENDS	-0.00058	-4.89041	0.38080	0.28476	-0.27640
MERCOR	0.00046	-1.99554	0.22492	0.77262	-0.05039
PA NOVA	0.00024	-4.40808	0.19170	0.52859	-0.10736
RESBUD	-0.00110	-4.87377	0.25619	0.37584	-0.17290
IZOLACJA JAROCIN	-0.00089	-3.71032	0.23074	0.55542	-0.15391
TRAKCJA	-0.00141	-0.86701	0.11852	0.89988	-0.11023
UNIBEP	-0.00104	-2.90645	0.13592	0.64982	-0.09550
STARHEDGE	-0.00012	-4.96516	0.40433	0.49586	-0.14031
FON	0.00021	-5.11455	0.31531	0.65006	-0.02831
MOSTAL	-0.00097	-1.65213	0.15306	0.79915	-0.14256
INSTAL KRAKOW	0.00052	-5.09280	0.15913	0.40528	0.10000
MERA	0.00058	-4.63249	0.86419	0.45172	-0.42963
TESGAS	-0.00099	-3.21097	0.13774	0.63002	-0.14064
ZUE	-0.00007	-4.99839	0.12878	0.43044	-0.13401
LIBET	0.00018	-3.37220	0.17282	0.60836	-0.18530
TAMEX OBIEKTY SPORTOWE	-0.00178	-1.35772	0.23045	0.80949	-0.24085
MOBRUK	0.00004	-4.31077	0.14505	0.51218	-0.09932
INTERMA TRADE	-0.00003	-5.10768	0.43485	0.63234	0.03435
DEKTRA	-0.00000	-2.31291	0.15679	0.73669	-0.09200
INTERNITY	0.00015	-4.52562	0.34846	0.42682	-0.15162
HONEY PAYMENT GROUP	0.00000	-5.08440	0.81326	0.67624	-0.07175
PRZED.PRZ.BETONOW PREFABET BIALE BLOTA	0.00000	-5.12577	0.37057	0.75084	-0.00091
ROCCA	0.00000	-5.06641	1.18676	0.72342	-0.00032
FABRYKA KONSTRUKCJI DREW	-0.00000	-5.12854	0.34066	0.83204	0.02015
ULMA CONSTR.POLSKA	0.00013	-4.88425	0.40701	0.51046	-0.03276
ATLANTIS	0.00000	-5.11361	0.46577	0.73289	-0.03689
POLIMEX MOSTOSTAL	-0.00305	-1.31482	0.12202	0.83705	-0.10742
FORBUILD	-0.00001	-4.95227	2.46554	0.70373	-0.35156
HM INWEST ORD	-0.00057	-1.84471	0.24612	0.76258	0.01941
TAURON POLSKA ENERGIA	-0.00030	-7.99505	-0.01319	-0.03342	-0.01208
EC BEDZIN	-0.00021	-3.69738	0.41377	0.54582	0.13137
ENEA	-0.00031	-5.08701	0.09214	0.36128	-0.00344
POLENERGIA	-0.00032	-2.28745	0.17227	0.72697	-0.05327
ZESPOL ELKTP. WRLKKNR.	-0.00181	-4.71573	0.18991	0.45810	-0.04283
PHOTON ENERGY	-0.00131	-2.04982	0.14613	0.75581	-0.15213
UNIMOT	0.00066	-1.87822	0.15079	0.77960	0.02361
MANGATA HOLDING	0.00088	-5.24946	0.07910	0.36980	-0.15588
ZAK AD BUD MASZYN ZBC.	-0.00282	-1.50040	0.22085	0.80103	-0.11470
KUPIEC	-0.00000	-4.20630	0.49514	0.65069	-0.01043
BORYSZEW	-0.00046	-2.96825	0.24022	0.67119	0.06053
FABRYKA OBRABIAREK RAFAMET	0.00052	-5.09280	0.15913	0.40528	0.10000
ZAKLADY URZADZEN KOTLOWYCH STAPORKOW	0.00021	-4.91118	0.29485	0.58184	0.00662

Company	μ	ω	α_1	ϕ_1	β_1
MOJ	-0.00270	-4.81558	0.38183	0.55870	0.01114
PGF POLSKA GRUPA FOTOWOLTAICZNA	-0.00194	-4.58999	0.47744	0.35558	-0.18455
ENERGOINSTAL	-0.00129	-4.21701	0.34556	0.48133	-0.19472
SECOGROUP	0.00050	-4.92982	-0.05660	0.54754	0.00277
WIELTON	-0.00013	-5.09262	0.18854	0.38397	0.06105
BUMECH	-0.00297	-1.29085	0.16443	0.83309	-0.12616
KCI	-0.00291	-4.83740	0.08722	0.39826	-0.18176
ZAKLADY MAGNEZYTOWE ROPCZYCE	0.00017	-2.58568	0.17012	0.70696	-0.12875
HYDRAPRES	0.00000	-5.03852	1.74181	0.71591	-0.23662
ZAMET	-0.00070	-5.09214	0.20473	0.40512	-0.18346
SANOK RUBBER COMPANY	-0.00111	-1.78246	0.10217	0.78261	0.01091
FEERUM	0.00002	-4.80475	0.20305	0.53406	0.01108
APS ENERGIA	-0.00129	-4.95894	0.14564	0.35383	-0.10462
DROZAPOL PROFIL	-0.00077	-2.06498	0.14404	0.74635	-0.06888
PJP MAKRUM	-0.00073	-4.94044	0.13101	0.40231	-0.07442
ODLEWNIE POLSKIE	-0.00008	-5.17463	0.13082	0.40217	-0.21201
GRUPA KETY	0.00023	-5.09089	0.12490	0.39753	-0.11918
MFO	-0.00019	-5.07372	0.11311	0.38481	-0.12618
EKOPOL GORNOSLASK HLDG.	-0.00037	-0.68423	0.15240	0.92735	-0.06451
STALPRODUKT	-0.00112	-4.70560	0.16205	0.44498	0.02209
STALEXPORT AUTOSTRADY	-0.00025	-4.92406	0.17709	0.51287	-0.06164
TRANSPOL	-0.00000	-4.27424	0.40909	0.59071	-0.39574
OT LOGISTICS	0.00036	-5.20592	0.12042	0.30363	-0.07915
BALTICON	-0.00130	-4.76112	0.09505	0.38527	-0.19962
FORPOSTA	0.00025	-3.94317	0.76538	0.64816	-0.02010
XBS PRO-LOG	-0.00057	-4.92485	0.48550	0.49189	-0.20045
NATURGY ENERGY	0.00052	-5.09280	0.15913	0.40528	0.1
GRINO ECOLOGIC	0.00001	-5.11823	0.40547	0.70149	-0.05647
ACS ACTIV.CONSTR.Y SERV.	0.00105	-4.50672	0.12576	0.49897	0.03334
FOMENTO CONSTR.Y CNTR.	0.00052	-5.09280	0.15913	0.40528	0.10000
ACCIONA	0.00060	-4.75362	0.08510	0.44108	0.01037
SACYR	0.00107	-5.12326	0.05866	0.41045	-0.02255
AUDAX RENOVABLES	-0.00157	-2.14386	0.19635	0.73769	-0.08089
ENDESA	0.00012	-5.09678	0.09602	0.44226	0.01284
ROMANDE ENERGIE	0.00012	-4.28866	0.11719	0.50633	-0.27250
EDISUN POWER EUROPE N	-0.00012	-4.82735	0.13923	0.49413	-0.14570
BKW	0.00052	-5.09280	0.15913	0.40528	0.10000
ENERGIEDIENST HOLDING	-0.00007	-3.01706	0.10971	0.67049	-0.17069
DOTTIKON ES HOLDING	0.00013	-2.69085	0.16150	0.65883	-0.07828
GURIT HOLDING 'B'	-0.00107	-4.83097	0.10406	0.37029	-0.03228
FEINTOOL	-0.00073	-7.70516	0.23025	0.06464	-0.20529
AUTONEUM HOLDING	0.00046	-4.76308	0.11066	0.41087	0.07858

Table 24: Estimate AR(1) score t-EGARCH(1,1) stock

Company	μ	ω	α_1	ϕ_1	β_1	η_1	η_2
EDISON RSP	0.00000	-5.1	0.16000	0.41000	-0.07400	0.06230	-0.01200
ALGOWATT	-0.00224	-2.08390	0.30133	0.74608	-0.08376	0.25983	0.00560
CARBIOS	-0.00107	-1.64653	0.10998	0.78124	-0.05456	0.12304	-0.04038
VALEO	-0.00064	-2.21346	0.12166	0.71884	-0.03096	0.09087	-0.03407
ENGIE	0.00080	-5.19683	0.13582	0.41797	0.03471	0.01188	-0.00988
EAUX DE ROYAN	0.00004	-5.01914	0.13586	0.67589	0.00235	0.00173	0.00085
TRILOGIQ	-0.00000	-5.13202	0.13792	0.44649	0.02989	0.28401	0.02633
ROCTOOL	-0.00173	-2.76278	0.16487	0.63644	-0.15269	0.25820	0.01072
GLOBAL BIOENERGIES	-0.00484	-1.87016	0.14679	0.75818	-0.08661	0.36091	0.01489
L AIR LQE.SC.ANYME. POUR L ETUDE ET L EPXTN.	0.00052	-5.09280	0.15913	0.40528	-0.07390	0.06228	-0.01233
GLOBAL PVQ	0.00098	-5.00971	0.41602	0.39699	-0.00773	-0.37772	-0.05685
ENVITEC BIOGAS	-0.00068	-2.38942	0.16536	0.67459	-0.24222	0.47606	0.00611
CENTROTHERM PHTO.	-0.00134	-6.34018	0.06026	0.15524	-0.16094	0.36686	0.01157
CLEARVISE (FRA)	-0.00077	-1.81373	0.14217	0.78199	-0.29933	0.13997	0.00272
MASTERFLEX	-0.00165	-6.34297	0.09718	0.20305	-0.17792	0.42420	0.01778
ECKERT & ZIEGLER STRAHLEN & MEDZI.	-0.00060	-4.17568	0.11244	0.45371	-0.15381	0.88654	0.00206
EVONIK INDUSTRIES	0.00004	-2.94614	0.14917	0.67404	-0.17253	0.22556	0.00307
GRAMMER	-0.00071	-4.93176	0.17293	0.39203	-0.19866	0.10749	-0.01320
VOLKSWAGEN	-0.00141	-1.64255	0.14933	0.79825	-0.07822	0.40547	-0.01033
BMW	0.00102	-5.17291	0.13104	0.41207	-0.05248	0.02568	-0.01472
PORSCHE AML.HLDG.PREF.	-0.00120	-3.37308	0.16361	0.59677	-0.00982	0.18097	-0.01149
BAUER	-0.00116	-1.66957	0.16953	0.81547	-0.23329	0.07555	-0.02201
STEICO	-0.00114	-7.68480	0.06098	-0.03774	-0.13097	0.75470	-0.02265
TERNA RETE ELETTRICA NAZ	0.00052	-5.09280	0.15913	0.40528	-0.07390	0.06228	-0.01233
VILLEROY & BOCH PF.SHS.	-0.00023	-5.10417	0.03313	0.37567	-0.14708	0.28156	0.00330
PNE	-0.00046	-5.05546	0.12304	0.40734	-0.14845	0.12686	-0.00655
LECHWERKE	-0.00062	-4.57957	0.19533	0.49954	-0.22255	0.03296	-0.00281
MOURY CONSTRUCT	0.00021	-4.98603	0.23104	0.53796	-0.04634	0.00098	-0.00006
FLORIDIENNE	0.00016	-1.63533	0.13911	0.80583	-0.07738	0.18476	0.01364
BEKAERT (D)	0.00126	-2.17557	0.14139	0.74631	-0.04043	0.02592	-0.02683
JENSEN-GROUP	-0.00011	-5.12229	0.09336	0.42036	-0.15517	0.22057	0.00737
STRABAG SE	0.00052	-5.09280	0.15913	0.40528	-0.07390	0.06228	-0.01233
SW UMWELTTECHNIK	-0.00008	-5.14117	0.21649	0.60666	-0.00843	-0.01392	-0.00255
PORR	0.00048	-1.79846	0.11949	0.78219	-0.20639	0.28212	-0.00079
POLYTEC HOLDING	-0.00071	-2.27231	0.13429	0.73773	-0.13724	0.17186	-0.01151
MT HOEJGAARD HOLDING	-0.00127	-4.69836	0.15948	0.42733	-0.12064	0.32724	0.01389
SCANDINAVIAN BRAKE SYS.	-0.00246	-2.99321	0.17862	0.60560	-0.15328	0.00936	0.01072
ENNOGIE SOLAR GROUP	0.00011	-1.96967	0.23118	0.73971	-0.07770	0.17277	0.00018
FIRSTFARMS	-0.00130	-9.84669	0.15666	-0.05164	-0.21750	0.13162	-0.00507
GLUNZ & JENSEN HOLDING	-0.00023	-5.09052	0.14731	0.47784	-0.05067	-0.06877	0.01457
BRD KLEE B	-0.00040	-5.14666	0.23404	0.57783	-0.00626	-0.02611	-0.00448
SKAKO	-0.00011	-5.57778	0.08171	0.36249	-0.18182	0.04527	-0.02442
NTG NORDIC TRANSPORT GROUP	0.00010	-5.31249	0.08497	0.29138	-0.02303	0.51679	-0.02768
KENDRION	-0.00135	-2.96152	0.16893	0.64542	-0.14833	0.24417	-0.01961
KON.HEIJMANS DU. CERTS.	-0.00019	-2.50750	0.12674	0.71223	-0.09546	0.17790	-0.01411
HYDRATEC INDUSTRIES	-0.00018	-5.16379	0.16101	0.50506	0.04564	-0.06053	-0.00669
VOOLT	-0.00587	-1.44951	0.18221	0.79819	-0.11108	0.06637	-0.01421
INTER CARS	0.00014	-2.22870	0.13397	0.73896	-0.11235	0.00322	-0.02274
LESS	-0.00362	-1.94616	0.22337	0.73740	-0.12567	-0.07927	-0.04917
HORTICO	-0.00053	-4.72910	0.13066	0.45022	-0.14512	-0.02789	-0.02702
PRYMUS	-0.00003	-5.13525	0.20078	0.66001	0.00038	0.00085	-0.00143
BUDIMEX	0.00040	-5.09468	0.15386	0.38573	-0.16534	0.26265	-0.00479
PBG	0.00052	-5.09280	0.15913	0.40528	-0.07390	0.06228	-0.01233
INVESTMENT FRIENDS	0.00048	-5.14204	0.14684	0.34531	-0.23344	-0.07036	0.06799
MERCOR	0.00028	-3.11012	0.25667	0.64358	-0.05558	0.19436	-0.00255
PA NOVA	0.00013	-4.55635	0.19777	0.50438	-0.12344	-0.01322	-0.02338
TRAKCJA	-0.00148	-0.97091	0.12342	0.88819	-0.11835	0.01020	-0.02453
UNIBEP	-0.00116	-2.84173	0.13027	0.65735	-0.09982	0.16825	-0.00284
FON	0.00001	-5.11511	0.19891	0.65528	-0.00337	0.00963	0.00252
MOSTAL	-0.00108	-1.54283	0.14761	0.81184	-0.13930	0.11220	-0.01521
MERA	0.00007	-5.09334	0.17127	0.55738	-0.00199	-0.07312	-0.00996
LIBET	0.00016	-4.59772	0.18427	0.47143	-0.17789	-0.04606	0.00749
INTERMA TRADE	-0.00001	-5.11400	0.19771	0.66290	-0.01253	-0.00023	-0.00016
DEKTRA	-0.00010	-2.66827	0.16264	0.69587	-0.09856	-0.02121	-0.01716

Company	μ	ω	α_1	ϕ_1	β_1	η_1	η_2
HONEY PAYMENT GROUP	0.00009	-5.18448	0.26888	0.55103	-0.09011	-0.00430	0.00046
POLIMEX MOSTOSTAL	-0.00388	-4.94168	0.07205	0.39646	-0.12410	0.14702	-0.01778
HM INWEST ORD	-0.00073	-1.87375	0.24246	0.75799	0.01699	0.02819	-0.01317
TAURON POLSKA ENERGIA	-0.00036	-8.33466	-0.03394	-0.07452	-0.02047	-0.03926	-0.01782
ENEA	-0.00034	-5.08700	0.09380	0.36188	-0.01049	0.06720	-0.01864
POLENERGIA	-0.00036	-2.21610	0.16974	0.73554	-0.06062	0.11197	-0.00588
BORYSZEW	-0.00046	-3.03730	0.24210	0.66350	0.06031	0.00806	-0.00106
FABRYKA OBRABIAREK RAFAMET	0.00052	-5.09280	0.15913	0.40528	-0.07390	0.06228	-0.01233
ZAKLADY URZADZEN KOTLOWYCH STAPORKOW	0.00006	-5.23362	0.23488	0.49602	-0.01624	-0.00136	0.00084
MOJ	0.00003	-5.10447	0.14542	0.51876	0.02558	-0.08766	-0.01165
ENERGOINSTAL	0.00009	-3.64030	0.36087	0.59394	-0.03514	-0.00952	-0.00050
SECOGROUP	0.00011	-5.12177	0.18642	0.62408	-0.01103	-0.02203	-0.00248
WIELTON	-0.00022	-4.84004	0.17981	0.41673	0.03780	0.11976	-0.02500
BUMECH	-0.00236	-0.68532	0.17065	0.90802	-0.08026	0.08289	-0.01529
KCI	-0.00295	-4.96766	0.07679	0.38267	-0.19473	0.30304	0.00849
ZAKLADY MAGNEZYTOWE ROPCZYCE	0.00013	-2.56841	0.15801	0.70850	-0.12008	0.00415	-0.02772
HYDRAPRES	0.00004	-5.06189	0.15675	0.63766	0.01240	0.04607	0.00768
SANOK RUBBER COMPANY	-0.00141	-4.81219	0.14133	0.41279	0.00175	0.12725	-0.01931
GRUPA KETY	0.00012	-5.08997	0.12073	0.40023	-0.15839	0.41215	0.00567
STALPRODUKT	-0.00110	-2.34313	0.16077	0.72330	0.00120	-0.00646	-0.02291
BALTICON	0.00004	-5.07741	0.15550	0.62389	-0.00016	0.00542	0.00020
XBS PRO-LOG	0.00008	-5.09186	0.39983	0.54956	0.00146	-0.00662	-0.00345
NATURGY ENERGY	0.00052	-5.09280	0.15913	0.40528	-0.07390	0.06228	-0.01233
IBERDROLA	0.00006	-5.09818	0.16338	0.42191	0.03125	0.02541	0.00570
BKW	0.00052	-5.09280	0.15913	0.40528	-0.07390	0.06228	-0.01233
ENERGIEDIENST HOLDING	-0.00006	-2.70403	0.11107	0.70588	-0.17303	-0.05210	-0.02206

Table 25: Estimate AR(1) score t-EGARCH(1,1) stock with exogenous variables MSCI and VIX

Company	μ	ω	α_1	ϕ_1	β_1	η_1	η_2	η_3
EDISON RSP	0.0004	-2.4531	0.1584	0.7516	-0.1509	0.0958	-0.0032	0.0032
TERNA RETE ELETTRICA NAZ	0.0007	-5.8613	0.1698	0.3622	-0.0099	0.0461	-0.0001	0.0032
ALGOWATT	-0.0021	-1.9787	0.2883	0.7592	-0.0703	0.0306	0.0008	-0.0055
L AIR LQE.SC.ANYME. POUR L ETUDE ET L EPXTN.	0.0007	-5.9951	0.1578	0.3692	-0.0043	0.0295	0.0091	-0.0029
CARBIOS	-0.0011	-1.6537	0.1098	0.7805	-0.0287	-0.0402	-0.0369	-0.0185
VALEO	-0.0004	-2.4347	0.1248	0.6911	0.0033	-0.0211	-0.0204	-0.0367
ENGIE	0.0010	-2.2329	0.1292	0.7600	0.0222	0.0471	-0.0048	-0.0024
EAUX DE ROYAN	0.0001	-5.8948	0.3780	0.6241	-0.0101	0.0222	0.0029	-0.0001
TRILOGIQ	-0.0000	-5.8561	0.3224	0.6215	0.0012	0.0029	0.0004	-0.0007
ROCTOOL	-0.0017	-3.1883	0.1623	0.5815	-0.1476	0.0361	-0.0085	0.0365
GLOBAL BIOENERGIES	-0.0048	-1.9530	0.1497	0.7473	-0.0755	0.0378	-0.0051	0.0297
ENVITEC BIOGAS	-0.0008	-2.0506	0.1563	0.7261	-0.2136	0.3215	-0.0157	0.0039
CENTROTHERM PHTO.	-0.0012	-5.8724	0.0521	0.2186	-0.1501	-0.1354	-0.0100	0.0167
CLEARVISE (FRA)	-0.0008	-1.8952	0.1602	0.7734	-0.3015	0.2276	0.0146	-0.0146
MASTERFLEX	-0.0016	-6.1264	0.1228	0.2330	-0.1819	0.0133	-0.0508	0.0038
ECKERT & ZIEGLER STRAHLEN & MEDZI.	-0.0001	-5.9756	0.0782	0.2088	-0.0237	-0.0747	0.0123	-0.0301
EVONIK INDUSTRIES	0.0001	-2.8939	0.1553	0.6762	-0.1250	0.0330	-0.0080	-0.0338
GRAMMER	-0.0008	-5.7109	0.1680	0.2979	-0.1985	0.0356	0.0013	0.0166
VOLKSWAGEN	-0.0010	-1.5338	0.1422	0.8108	0.0030	-0.1274	-0.0189	-0.0076
BMW	0.0011	-6.0235	0.1257	0.3158	-0.0286	-0.0328	-0.0072	-0.0247
PORSCHE AML.HLDG.PREF.	-0.0008	-2.7512	0.1405	0.6717	0.0573	-0.0601	-0.0079	-0.0407
BAUER	-0.0010	-1.8192	0.1784	0.7985	-0.2413	0.0562	-0.0048	0.0108
STEICO	-0.0009	-6.1374	0.0316	0.1651	-0.0377	0.0515	0.0255	0.0301
VILLEROY & BOCH PF.SHS.	-0.0002	-5.6249	0.0963	0.3220	-0.1413	0.2352	0.0038	0.0365
PNE	-0.0004	-5.9384	0.1217	0.3048	-0.1401	-0.1316	-0.0219	-0.0330
LECHWERKE	-0.0006	-5.6750	0.1933	0.3815	-0.2229	0.1092	0.0113	0.0020
MOURY CONSTRUCT	0.0001	-5.7327	0.1960	0.4412	-0.0410	0.0171	0.0052	0.0054
FLORIDIENNE	0.0002	-1.9258	0.1499	0.7729	-0.0674	0.0119	-0.0209	-0.0401

BEKAERT (D)	0.0014	-2.7225	0.1485	0.6804	-0.0078	-0.1060	-0.0159	-0.0182
JENSEN-GROUP	-0.0001	-5.9015	0.0888	0.3349	-0.1399	0.0703	0.0031	0.0249
STRABAG SE	0.0004	-2.3660	0.1665	0.7430	-0.1038	0.0166	0.0030	-0.0088
SW UMWELTTECHNIK	0.0001	-5.8986	0.4440	0.5678	-0.0525	-0.0791	-0.0099	0.0201
PORR	0.0006	-3.2202	0.1234	0.6032	-0.1549	-0.1472	-0.0362	-0.0630
POLYTEC HOLDING	-0.0004	-2.4020	0.1293	0.7262	-0.1111	0.0050	-0.0159	-0.0076
MT HOEJGAARD HOLDING	-0.0011	-2.5499	0.1563	0.6881	-0.0927	-0.0404	-0.0092	0.0106
SCANDINAVIAN BRAKE SYS.	-0.0025	-2.5807	0.1712	0.6599	-0.1535	0.0402	-0.0086	-0.0050
ENNOGIE SOLAR GROUP	0.0003	-1.9245	0.2337	0.7457	-0.0746	0.1410	0.0008	-0.0416
FIRSTFARMS	-0.0011	-6.2388	0.1630	0.3289	-0.2128	0.0104	-0.0023	-0.0275
GLUNZ & JENSEN HOLDING	-0.0001	-5.8728	0.7027	0.6126	-0.0190	-0.0048	-0.0002	0.0056
BRD KLEE B	-0.0003	-5.9396	0.4513	0.5105	-0.1954	0.0066	0.0028	-0.0110
SKAKO	-0.0001	-9.9421	0.0734	-0.1342	-0.1722	0.0451	-0.0031	0.0128
NTG NORDIC TRANS-PORT GROUP	0.0002	-5.9925	0.0920	0.1948	0.0692	-0.1055	0.0016	0.0055
KENDRION	-0.0011	-3.5090	0.1827	0.5780	-0.1256	-0.0851	-0.0222	0.0117
KON.HEIJMANS DU. CERTS.	0.0001	-2.7703	0.1263	0.6810	-0.0383	-0.0674	-0.0111	-0.0270
HYDRATEC INDUS-TRIES	0.0001	-5.8089	0.3443	0.4947	-0.0285	0.0839	0.0101	-0.0027
VOOLT	-0.0058	-1.5208	0.1854	0.7887	-0.1064	-0.0580	0.0051	-0.0050
INTER CARS	0.0000	-2.4056	0.1459	0.7191	-0.0945	0.1044	0.0223	0.0317
LESS	-0.0034	-1.9292	0.2144	0.7391	-0.1131	0.0394	0.0048	-0.0591
HORTICO	-0.0004	-2.8314	0.1538	0.6680	-0.1379	-0.0546	-0.0088	-0.0043
BUDIMEX	0.0006	-5.9834	0.1366	0.2770	-0.1399	-0.1128	-0.0066	0.0289
PBG	-0.0000	-6.0260	0.2036	0.7293	0.0078	0.0016	0.0001	-0.0001
INVESTMENT FRIENDS	-0.0000	-4.9498	0.1900	0.3327	-0.0553	-0.1726	-0.0509	0.2136
MERCOR	0.0005	-2.0488	0.2316	0.7670	-0.0478	-0.0887	-0.0101	-0.0250
PA NOVA	0.0002	-5.2007	0.1936	0.4431	-0.1100	0.0260	0.0001	0.0127
TRAKCJA	-0.0014	-0.8761	0.1189	0.8989	-0.1119	0.0503	-0.0029	0.0063
UNIBEP	-0.0011	-2.8591	0.1319	0.6564	-0.1032	0.1477	0.0007	0.0043
MOSTAL	-0.0011	-1.6375	0.1529	0.8008	-0.1450	0.0127	-0.0068	0.0345
MERA	-0.0001	-5.6757	0.6286	0.4222	0.0223	-0.2827	-0.0252	-0.0046
LIBET	0.0002	-5.4417	0.1814	0.3739	-0.1814	-0.0599	-0.0006	-0.0086
DEKTRA	-0.0001	-2.2895	0.1567	0.7394	-0.0923	0.0378	-0.0022	0.0020
POLIMEX MOSTOSTAL	-0.0035	-2.2074	0.0591	0.7325	-0.1066	-0.2897	-0.0205	0.0682
HM INWEST ORD	-0.0006	-1.8075	0.2461	0.7677	0.0192	-0.0261	-0.0158	0.0008
TAURON POLSKA EN-ERGIA	-0.0003	-12.0029	-0.0350	-0.5430	-0.0112	-0.2208	-0.0336	0.0233
ENEA	-0.0003	-5.9819	0.1087	0.2500	-0.0080	-0.1711	-0.0410	-0.0065
POLENERGIA	-0.0003	-2.3802	0.1728	0.7160	-0.0523	-0.0356	0.0009	0.0286
BORYSZEW	-0.0004	-2.9775	0.2458	0.6704	0.0586	-0.0255	-0.0112	-0.0040
MOJ	0.0000	-5.7343	0.3610	0.4373	0.1075	0.1051	0.0133	0.0365
WIELTON	-0.0002	-5.9342	0.1918	0.2815	0.0555	0.0271	-0.0066	0.0402
BUMECH	-0.0025	-2.2480	0.2233	0.7038	-0.0332	-0.5698	-0.0357	0.1295
KCI	-0.0028	-2.9674	0.0810	0.6310	-0.1881	0.0930	-0.0145	-0.0358
ZAKLADY MAGNEZY-TOWE ROPCZYCE	0.0001	-5.4265	0.1963	0.3849	-0.1256	0.0605	0.0056	0.0111
SANOK RUBBER COM-PANY	-0.0012	-1.7662	0.1026	0.7849	0.0044	0.0491	0.0065	0.0461
GRUPA KETY	0.0004	-4.0049	0.1264	0.5254	-0.1232	-0.0214	-0.0142	-0.0153
STALPRODUKT	-0.0010	-1.9358	0.1480	0.7708	0.0177	0.0531	0.0111	0.0224
BALTICON	-0.0003	-5.6323	0.5201	0.5502	-0.0170	-0.0003	-0.0002	0.0222
XBS PRO-LOG	0.0000	-5.8950	0.1548	0.6678	-0.0029	-0.0005	0.0004	-0.0014
NATURGY ENERGY	0.0006	-1.9546	0.1985	0.7903	-0.0075	0.0389	0.0047	0.0068
IBERDROLA	0.0003	-2.6509	0.1895	0.7117	0.0611	0.0364	0.0097	-0.0016
BKW	0.0009	-6.3232	0.1256	0.3105	-0.0059	-0.0373	-0.0093	-0.0121
ENERGIEDIENST HOLDING	-0.0001	-2.8206	0.1071	0.6932	-0.1627	-0.0327	-0.0035	-0.0076

Table 26: Estimate AR(1) score t-EGARCH(1,1) stock.
Exogenous variables: MSCI, VIX, ECF

Company	μ	ω	α_1	ϕ_1	β_1	η_1	η_2	η_3
EDISON RSP	0.0006	-5.0941	0.2164	0.4651	-0.1265	0.0696	0.0020	-0.0000
TERNA RETE ELET-TRICA NAZ	0.0010	-5.0981	0.1629	0.4514	-0.0713	0.0609	0.0004	-0.0000
ALGOWATT	0.0001	-5.0841	0.3202	0.3746	-0.0501	0.0594	0.0026	-0.0001

L AIR LQE.SC.ANYME. POUR L ETUDE ET L EPXTN.	-0.0002	-5.0993	0.1547	0.4638	-0.0054	0.0491	0.0110	0.0000
CARBIOS	-0.0018	-5.0806	0.1061	0.3324	-0.0300	0.0564	-0.0231	0.0000
VALEO	-0.0004	-5.0770	0.1280	0.3564	0.0025	-0.0203	-0.0163	-0.0000
ENGIE	0.0006	-5.0957	0.1419	0.4514	0.0217	0.0594	-0.0045	0.0000
EAUX DE ROYAN	0.0003	-5.0392	0.4572	0.6163	-0.0995	0.0175	0.0012	-0.0000
TRILOGIQ	-0.0021	-5.0907	0.2216	0.3992	-0.1225	0.0545	0.0002	0.0000
ROCTOOL	0.0004	-5.0777	0.1773	0.3095	-0.1533	0.0634	-0.0088	-0.0001
GLOBAL BIOENERGIES	-0.0033	-5.0801	0.1877	0.3304	-0.0604	0.0628	0.0003	-0.0000
GLOBAL PVQ	0.0000	-4.0430	1.1374	0.6448	0.0850	-0.0006	-0.0002	0.0000
ENVITEC BIOGAS	0.0026	-5.0547	0.1351	0.3308	-0.1881	0.2201	-0.0266	-0.0001
CENTROTHERM PHTO.	0.0032	-5.0165	0.0402	0.3193	-0.1442	-0.2392	-0.0361	-0.0001
CLEARVISE (FRA)	0.0001	-5.0857	0.1522	0.3797	-0.3038	0.0826	0.0006	-0.0000
MASTERFLEX	-0.0009	-5.0896	0.1177	0.3566	-0.1820	0.0541	-0.0475	-0.0000
ECKERT & ZIEGLER STRAHLEN & MEDZI.	0.0040	-5.0832	0.0878	0.3272	-0.0273	0.0376	0.0243	-0.0001
EVONIK INDUSTRIES	0.0011	-5.0961	0.1606	0.4344	-0.0774	0.0607	-0.0046	-0.0000
GRAMMER	-0.0009	-5.0848	0.1763	0.3487	-0.2137	0.0566	-0.0004	-0.0000
VOLKSWAGEN	0.0018	-5.0866	0.1715	0.3753	-0.0070	0.0435	-0.0018	-0.0001
BMW	0.0023	-5.0951	0.1162	0.4235	-0.0289	0.0361	0.0006	-0.0000
PORSCHE AML.HLDG.PREF.	0.0005	-5.0912	0.1835	0.3918	0.0345	0.0431	0.0026	-0.0000
BAUER	-0.0015	-5.0866	0.2189	0.4192	-0.2206	0.0617	-0.0085	0.0000
STEICO	0.0022	-5.0818	0.0289	0.3069	-0.0532	0.0562	0.0239	-0.0001
VILLEROY & BOCH PF.SHS.	0.0010	-5.0765	0.1023	0.3820	-0.1429	0.2626	0.0035	-0.0000
PNE	-0.0002	-5.0916	0.1169	0.3969	-0.1470	0.0293	-0.0060	-0.0000
LECHWERKE	0.0009	-5.0927	0.1756	0.4198	-0.2290	0.0839	0.0073	-0.0000
MOURY CONSTRUCT	0.0019	-5.0951	0.1614	0.4250	-0.0754	0.0595	0.0192	-0.0000
FLORIDIENNE	0.0047	-5.0769	0.2229	0.3748	-0.1119	0.0122	-0.0168	-0.0001
BEKAERT (D)	0.0008	-5.0799	0.1790	0.4141	-0.0027	-0.0809	-0.0150	0.0000
JENSEN-GROUP	-0.0012	-5.0939	0.0849	0.4178	-0.1356	0.0596	0.0005	0.0000
STRABAG SE	0.0009	-5.0943	0.1934	0.4411	-0.0993	0.0402	0.0052	-0.0000
SW UMWELTTECHNIK	-0.0001	-5.1008	0.2153	0.5045	0.0211	0.0279	0.0033	0.0000
PORR	0.0009	-5.0879	0.1017	0.3904	-0.1873	0.0383	-0.0227	-0.0000
POLYTEC HOLDING	0.0005	-5.0876	0.1353	0.4106	-0.1187	0.0168	-0.0153	-0.0000
MT HOEJGAARD HOLDING	0.0011	-5.0688	0.1552	0.3788	-0.0986	-0.0309	-0.0087	-0.0001
SCANDINAVIAN BRAKE SYS.	-0.0001	-5.0760	0.1730	0.2848	-0.1652	0.0570	-0.0087	-0.0001
ENNOGIE SOLAR GROUP	0.0004	-5.0791	0.2725	0.3258	-0.0569	0.0607	-0.0035	-0.0000
FIRSTFARMS	-0.0014	-5.0990	0.1544	0.4384	-0.2215	0.0438	0.0017	0.0000
GLUNZ & JENSEN HOLDING	-0.0001	-4.9342	0.5605	0.4822	-0.0427	0.0554	0.0094	0.0000
BRD KLEE B	-0.0000	-5.0545	0.5698	0.6545	-0.0368	0.0056	0.0003	0.0000
SKAKO	-0.0014	-5.0957	0.0879	0.4097	-0.1849	0.0647	-0.0013	0.0000
NTG NORDIC TRANSPORT GROUP	0.0025	-5.0840	0.0923	0.3290	0.0650	0.0483	0.0168	-0.0001
KENDRION	0.0002	-5.0896	0.1813	0.3971	-0.1399	0.0303	-0.0137	-0.0000
KON.HEIJMANS DU. CERTS.	0.0004	-5.0907	0.1161	0.4128	-0.0312	0.0317	-0.0006	-0.0000
HYDRATEC INDUSTRIES	-0.0007	-5.0978	0.1622	0.4502	-0.0743	0.0604	-0.0038	0.0000
VOOLT	-0.0057	-5.0712	0.2301	0.2619	-0.0934	0.0573	0.0249	-0.0000
VIATRON	0.0000	-5.1300	0.3556	0.7561	0.0216	0.0000	0.0000	-0.0000
INTER CARS	0.0010	-5.0889	0.1417	0.3969	-0.0950	0.0681	0.0185	-0.0000
LESS	0.0005	-4.1844	0.2403	0.4382	-0.1074	0.0976	0.0153	-0.0001
HORTICO	-0.0009	-5.0858	0.1710	0.3723	-0.1535	0.0450	0.0044	0.0000
PRYMUS	-0.0000	-5.0882	0.3706	0.4966	-0.0795	-0.0368	-0.0047	-0.0000
BUDIMEX	-0.0019	-5.0905	0.1244	0.3852	-0.1393	0.0396	0.0056	0.0001
PBG	0.0000	-5.1317	0.2212	0.7554	0.0000	0.0002	0.0000	-0.0000
INVESTMENT FRIENDS	-0.0048	-4.5326	0.6135	0.4412	-0.4152	-0.0518	-0.0168	0.0001
MERCOR	-0.0014	-5.0883	0.2578	0.4047	-0.0419	0.0423	0.0036	0.0000
PA NOVA	0.0008	-5.0939	0.1618	0.4155	-0.0776	0.0614	-0.0040	-0.0000
TRAKCJA	-0.0027	-5.0867	0.1850	0.3942	-0.0960	0.0613	-0.0009	0.0000
UNIBEP	-0.0001	-5.0809	0.1427	0.3845	-0.1100	0.1203	-0.0045	-0.0000
FON	0.0000	-5.1121	0.3193	0.6274	0.0334	0.0062	0.0008	-0.0000
MOSTAL	-0.0022	-5.0839	0.1695	0.3578	-0.1366	0.0597	-0.0090	0.0000
MERA	0.0032	-4.8300	0.4879	0.5313	-0.0135	0.0902	0.0091	-0.0001
LIBET	-0.0001	-5.0859	0.1486	0.3702	-0.2162	0.0378	0.0097	0.0000
INTERMA TRADE	0.0000	-5.1100	0.4701	0.6281	-0.0729	-0.0048	-0.0005	-0.0000
DEKTRA	0.0014	-5.0892	0.1653	0.3864	-0.0946	0.0602	0.0014	-0.0000

HONEY PAYMENT GROUP	0.0000	-5.0780	1.1481	0.6938	-0.0075	0.0002	0.0000	-0.0000
POLIMEX MOSTOSTAL	-0.0030	-5.0845	0.0846	0.3491	-0.0790	0.0527	0.0060	-0.0000
HM INWEST ORD	-0.0061	-4.8047	0.2259	0.3296	0.0456	-0.5291	-0.0647	0.0001
TAURON POLSKA ENERGIA	-0.0020	-5.1022	-0.0101	0.3511	-0.0130	-0.2313	-0.0367	0.0000
ENEA	-0.0015	-5.0874	0.0903	0.3619	-0.0114	0.0353	-0.0198	0.0000
POLENERGIA	0.0003	-5.0871	0.1961	0.3764	-0.0526	0.0498	0.0070	-0.0000
BORYSZEW	0.0001	-5.0844	0.2662	0.4179	0.0997	0.0357	-0.0078	-0.0000
FABRYKA OBRABIAREK RAFAMET	0.0002	-5.1049	0.3026	0.5156	-0.0818	-0.0179	-0.0018	0.0000
MOJ	-0.0001	-4.7657	0.4100	0.6294	-0.0705	-0.0169	-0.0022	0.0000
ENERGOINSTAL	-0.0023	-5.0558	0.3048	0.2839	-0.1893	0.0247	-0.0155	-0.0000
SECOGROUP	-0.0002	-4.8172	0.2557	0.7359	0.0353	0.0006	0.0011	0.0000
WIELTON	0.0016	-5.0893	0.1918	0.3784	0.0228	0.0612	-0.0056	-0.0000
BUMECH	0.0007	-3.7730	0.2580	0.5035	-0.0670	-0.5093	-0.0413	-0.0001
KCI	-0.0041	-5.0842	0.1020	0.3491	-0.1774	0.0558	-0.0166	0.0000
ZAKLADY MAGNEZYTOWE ROPCZYCE	0.0005	-5.0907	0.2089	0.4071	-0.1124	0.0630	0.0062	-0.0000
HYDRAPRES	-0.0000	-5.1064	0.7766	0.5853	-0.0273	-0.0108	-0.0011	0.0000
SANOK RUBBER COMPANY	-0.0026	-5.0859	0.1466	0.3717	0.0148	0.0620	0.0025	0.0000
GRUPA KETY	-0.0015	-5.0891	0.1312	0.3981	-0.1332	0.0072	-0.0105	0.0000
STALPRODUKT	-0.0028	-5.0901	0.1683	0.3943	0.0262	0.0618	0.0083	0.0000
BALTICON	-0.0055	-5.0805	0.2147	0.3337	-0.1777	0.0555	-0.0123	0.0001
XBS PRO-LOG	0.0006	-5.0882	0.2409	0.4123	-0.1138	0.0225	-0.0025	-0.0000
NATURGY ENERGY	-0.0002	-5.0897	0.2159	0.4560	-0.0015	0.0503	0.0032	0.0000
IBERDROLA	-0.0026	-5.0985	0.1651	0.4557	-0.0697	0.0596	0.0094	0.0001
BKW	0.0002	-5.0998	0.1157	0.4471	0.0014	0.0239	-0.0029	0.0000
ENERGIEDIENST HOLDING	0.0011	-5.0911	0.0997	0.4387	-0.1596	-0.0124	-0.0004	-0.0000

Table 27: Estimate AR(1) score t-EGARCH(1,1) stock.
Exogenous variables: MSCI, VIX, CDS (10 years) TR index

Company	μ	ω	α_1	ϕ_1	β_1	η_1	η_2	η_3
EDISON RSP	0.0007	-5.0996	0.1653	0.4654	-0.0747	0.0614	-0.0002	-0.0000
TERNA RETE ELETTRICA NAZ	0.0005	-5.0928	0.1591	0.4053	-0.0739	0.0623	-0.0123	0.0000
ALGOWATT	-0.0008	-5.0630	0.3307	0.3768	-0.0879	0.0553	0.0018	-0.0001
L AIR LQE.SC.ANYME. POUR L ETUDE ET L EPXTN.	0.0002	-5.0994	0.1637	0.4631	-0.0721	0.0597	0.0105	0.0000
CARBIOS	-0.0008	-5.0809	0.1092	0.3325	-0.0319	0.0567	-0.0246	-0.0000
VALEO	-0.0008	-5.0859	0.1226	0.3549	-0.0038	0.0551	-0.0107	0.0000
ENGIE	0.0009	-5.0957	0.1415	0.4514	0.0221	0.0594	-0.0045	0.0000
EAUX DE ROYAN	0.0001	-5.0598	0.1632	0.4462	-0.0532	0.0157	0.0253	-0.0000
TRILOGIQ	-0.0006	-4.9749	0.2210	0.5160	-0.0568	-0.3012	-0.0154	0.0000
ROCTOOL	-0.0002	-4.7593	0.1970	0.3718	-0.1567	0.2583	0.0140	-0.0000
GLOBAL BIOENERGIES	-0.0041	-5.0823	0.1747	0.3307	-0.0686	0.0617	0.0006	-0.0000
GLOBAL PVQ	0.0000	-4.0040	0.7632	0.6656	-0.2287	0.0005	0.0000	-0.0000
ENVITEC BIOGAS	0.0020	-5.0811	0.1535	0.3265	-0.1914	0.0794	-0.0393	-0.0001
CENTROTHERM PHTO.	0.0028	-5.0799	0.0472	0.3020	-0.1683	0.0519	0.0037	-0.0001
CLEARVISE (FRA)	-0.0004	-4.4863	0.1596	0.4600	-0.2592	0.1773	0.0058	-0.0000
MASTERFLEX	-0.0002	-5.0887	0.1086	0.3567	-0.1766	0.0576	-0.0468	-0.0000
ECKERT & ZIEGLER STRAHLEN & MEDZI.	0.0051	-5.0850	0.0886	0.3272	-0.0335	-0.0921	0.0121	-0.0001
EVONIK INDUSTRIES	0.0009	-5.0960	0.1604	0.4341	-0.0773	0.0607	-0.0046	-0.0000
GRAMMER	-0.0005	-5.0848	0.1761	0.3487	-0.2107	0.0567	-0.0015	-0.0000
VOLKSWAGEN	0.0017	-5.0867	0.1735	0.3762	-0.0079	0.0419	-0.0014	-0.0001
BMW	0.0022	-5.0951	0.1161	0.4235	-0.0296	0.0360	0.0007	-0.0000
PORSCHE AML.HLDG.PREF.	0.0003	-5.0911	0.1815	0.3919	0.0360	0.0426	0.0027	-0.0000
BAUER	-0.0016	-5.0829	0.1949	0.4200	-0.2348	0.0620	-0.0094	0.0000
STEICO	0.0020	-5.0823	0.0274	0.3068	-0.0510	0.0561	0.0241	-0.0001
VILLEROY & BOCH PF.SHS.	0.0010	-5.0806	0.1198	0.3809	-0.1590	0.2144	-0.0011	-0.0000
PNE	-0.0001	-5.0891	0.1419	0.3970	-0.1550	-0.1209	-0.0195	-0.0000
LECHWERKE	0.0008	-5.0938	0.2086	0.4190	-0.2097	0.0716	0.0062	-0.0000
MOURY CONSTRUCT	0.0019	-5.0947	0.1850	0.4247	-0.0928	0.0634	0.0200	-0.0000
FLORIDIENNE	0.0041	-5.0766	0.2240	0.3749	-0.1136	0.0072	-0.0170	-0.0001
BEKAERT (D)	0.0012	-5.0897	0.1543	0.4126	0.0035	0.0149	-0.0049	0.0000
JENSEN-GROUP	-0.0012	-5.0938	0.0860	0.4178	-0.1358	0.0600	0.0005	0.0000

STRABAG SE	0.0010	-5.0968	0.1629	0.4412	-0.0755	0.0593	0.0078	-0.0000
SW UMWELTTECHNIK	0.0001	-5.1008	0.2152	0.5045	0.0211	0.0281	0.0034	-0.0000
PORR	0.0014	-5.0785	0.1256	0.3916	-0.1858	-0.0269	-0.0288	-0.0000
POLYTEC HOLDING	0.0004	-5.0872	0.1361	0.4106	-0.1201	0.0130	-0.0162	-0.0000
MT HOEJGAARD HOLDING	0.0016	-5.0891	0.1599	0.3759	-0.0765	0.0598	0.0006	-0.0001
SCANDINAVIAN BRAKE SYS.	-0.0009	-5.0762	0.1731	0.2846	-0.1657	0.0569	-0.0085	-0.0001
ENNOGIE SOLAR GROUP	0.0006	-5.0791	0.2726	0.3257	-0.0570	0.0607	-0.0033	-0.0000
FIRSTFARMS	-0.0011	-5.0990	0.1549	0.4384	-0.2218	0.0435	0.0017	0.0000
GLUNZ & JENSEN HOLDING	0.0003	-5.0790	0.1200	0.4665	-0.0223	-0.1397	-0.0117	-0.0000
BRD KLEE B	-0.0002	-5.0467	0.7079	0.4989	-0.2733	-0.2858	-0.0209	0.0000
SKAKO	-0.0007	-5.0957	0.0883	0.4097	-0.1839	0.0646	-0.0014	0.0000
NTG NORDIC TRANS-PORT GROUP	0.0023	-5.0852	0.1078	0.3287	0.0765	0.0140	0.0145	-0.0001
KENDRION	0.0005	-5.0818	0.1782	0.3984	-0.1339	-0.0448	-0.0198	-0.0000
KON.HEIJMANS DU. CERTS.	0.0011	-5.0907	0.1168	0.4129	-0.0318	0.0309	-0.0007	-0.0000
HYDRATEC INDUS-TRIES	-0.0004	-5.0978	0.1621	0.4501	-0.0743	0.0604	-0.0039	0.0000
VOOLT	-0.0065	-5.0712	0.2300	0.2618	-0.0931	0.0575	0.0251	0.0000
VIATRON	-0.0001	-5.1175	0.2673	0.6389	-0.0384	0.0579	0.0052	0.0000
INTER CARS	0.0013	-5.0889	0.1412	0.3959	-0.0953	0.0679	0.0182	-0.0000
LESS	0.0006	-4.1837	0.2448	0.4386	-0.1100	0.0666	0.0124	-0.0001
HORTICO	0.0012	-4.7973	0.0972	0.4329	-0.0885	-0.0345	-0.0090	-0.0000
PRYMUS	-0.0002	-5.0729	0.3501	0.4990	-0.0895	0.0462	0.0047	0.0000
BUDIMEX	-0.0009	-5.0905	0.1240	0.3848	-0.1382	0.0397	0.0059	0.0000
PBG	-0.0000	-5.1228	0.2053	0.6762	-0.0378	0.0575	0.0052	0.0000
INVESTMENT FRIENDS	0.0001	-4.4247	1.2038	0.6101	0.3142	-0.0007	-0.0001	-0.0000
MERCOR	-0.0008	-5.0883	0.2571	0.4046	-0.0416	0.0426	0.0031	0.0000
PA NOVA	0.0006	-5.0918	0.2009	0.4170	-0.1359	0.0619	-0.0036	-0.0000
TRAKCJA	-0.0016	-5.0868	0.1853	0.3942	-0.0960	0.0609	-0.0010	0.0000
UNIBEP	-0.0002	-5.0810	0.1423	0.3845	-0.1100	0.1204	-0.0045	-0.0000
FON	0.0000	-5.1186	0.3323	0.6582	0.0232	0.0076	0.0010	-0.0000
MOSTAL	-0.0017	-5.0839	0.1687	0.3577	-0.1366	0.0596	-0.0093	0.0000
MERA	0.0028	-4.5958	0.5266	0.6294	0.0506	0.0672	0.0080	-0.0001
LIBET	-0.0000	-5.0859	0.1487	0.3702	-0.2162	0.0378	0.0097	0.0000
INTERMA TRADE	0.0000	-5.1050	0.5758	0.6920	-0.0631	0.0187	0.0019	-0.0000
DEKTRA	0.0011	-4.5136	0.1649	0.4879	-0.0968	0.0450	-0.0008	-0.0000
HONEY PAYMENT GROUP	0.0001	-5.0835	0.8148	0.5822	-0.0235	0.0531	0.0035	-0.0000
POLIMEX MOSTOSTAL	-0.0022	-5.0846	0.0845	0.3497	-0.0790	0.0525	0.0057	-0.0000
HM INWEST ORD	-0.0048	-5.0662	0.2185	0.2797	0.0609	0.0395	-0.0162	0.0001
TAURON POLSKA EN-ERGIA	-0.0020	-5.0885	-0.0049	0.3525	-0.0023	0.0122	-0.0123	0.0000
ENEA	-0.0012	-5.0875	0.0904	0.3620	-0.0113	0.0351	-0.0197	0.0000
POLENERGIA	-0.0004	-5.0869	0.1967	0.3769	-0.0516	0.0490	0.0070	0.0000
BORYSZEW	-0.0001	-5.0906	0.2954	0.4161	0.0336	0.0560	-0.0061	-0.0000
FABRYKA OBRABI-AREK RAFAMET	0.0001	-5.1050	0.1736	0.5135	-0.0754	0.0591	0.0055	0.0000
ZAKLADY URZADZEN KOTLOWYCH STAPORKOW	-0.0035	-4.9235	0.2295	0.4518	-0.0097	-0.0788	-0.0159	0.0001
MOJ	0.0009	-4.9251	0.4682	0.4882	-0.1969	-0.0796	0.0061	-0.0000
ENERGOINSTAL	-0.0006	-3.7339	0.3922	0.5936	-0.2145	0.0030	0.0001	0.0000
SECOGROUP	-0.0009	-4.9248	0.1969	0.5337	-0.0501	-0.2943	-0.0169	0.0000
WIELTON	0.0018	-5.0895	0.1994	0.3801	0.0493	0.0606	-0.0059	-0.0001
BUMECH	-0.0006	-3.7639	0.2584	0.5055	-0.0658	-0.4844	-0.0397	-0.0000
KCI	-0.0035	-5.0838	0.0984	0.3491	-0.1751	0.0559	-0.0167	0.0000
ZAKLADY MAGNEZY-TOWE ROPCZYCE	0.0006	-5.0882	0.2016	0.4077	-0.1289	0.0637	0.0057	-0.0000
HYDRAPRES	0.0000	-5.1022	1.7177	0.6695	0.0366	0.0001	0.0000	-0.0000
SANOK RUBBER COM-PANY	-0.0018	-5.0859	0.1462	0.3717	0.0156	0.0624	0.0027	0.0000
GRUPA KETY	-0.0008	-5.0911	0.1193	0.3978	-0.1244	0.0456	-0.0072	0.0000
STALPRODUKT	-0.0017	-5.0886	0.1627	0.3944	0.0271	0.0622	0.0083	0.0000
BALTICON	-0.0006	-4.7225	0.1298	0.5706	-0.0103	0.0247	0.0001	0.0000
XBS PRO-LOG	0.0001	-4.9856	0.1863	0.5821	0.0021	0.0085	0.0004	-0.0000
NATURGY ENERGY	-0.0000	-5.0946	0.2292	0.4547	-0.0149	0.0548	0.0032	0.0000
IBERDROLA	-0.0020	-5.0958	0.2167	0.4557	0.0274	0.0461	0.0094	0.0001
BKW	0.0004	-5.0974	0.1602	0.4446	-0.0713	0.0598	-0.0013	0.0000

ENERGIEDIENST HOLDING	0.0008	-5.0963	0.1545	0.4362	-0.0811	0.0580	0.0105	-0.0000

Table 28: Estimate AR(1) score t-EGARCH(1,1) stock.
Exogenous variables: MSCI, VIX, CDS (20 years) TR index

Company	**Log-like1**	**Log-like2**	**LR statistic**	**P-value**
EDISON RSP	2290.1795	2380.7505	181.14199	0.0
TERNA RETE ELETTRICA NAZ	2279.722	2322.3767	85.309536	0.0
ALGOWATT	1823.7419	1824.9094	2.334938	0.1265
L AIR LQE.SC.ANYME. POUR L ETUDE ET L EPXTN.	2332.1189	2389.442	114.646311	0.0
CARBIOS	1666.0679	1670.3329	8.529972	0.00349
VALEO	1757.2464	1759.7036	4.914432	0.02663
ENGIE	2257.4411	2302.307	89.73181	0.0
EAUX DE ROYAN	2191.2744	2836.7828	1291.01674	0.0
TRILOGIQ	1930.1853	2273.7156	687.060676	0.0
ROCTOOL	1509.798	1510.5869	1.577879	0.20907
GLOBAL BIOENERGIES	1628.6324	1629.7416	2.218466	0.13637
ENVITEC BIOGAS	1629.0784	1633.7857	9.414651	0.00215
CENTROTHERM PHTO.	1502.9717	1500.3529	-5.237584	1.0
CLEARVISE (FRA)	1867.0416	1867.775	1.466822	0.22585
MASTERFLEX	1750.0456	1749.0645	-1.962118	1.0
ECKERT & ZIEGLER STRAHLEN & MEDZI.	1630.311	1618.6514	-23.319324	1.0
EVONIK INDUSTRIES	2195.4935	2198.7343	6.48175	0.0109
GRAMMER	1701.1217	1701.4447	0.645895	0.42158
VOLKSWAGEN	1853.8649	1856.7454	5.761023	0.01639
BMW	2131.5361	2131.4143	-0.2436	1.0
PORSCHE AML.HLDG.PREF.	1956.6057	1953.3684	-6.474474	1.0
BAUER	2072.1602	2078.5075	12.694467	0.00037
STEICO	1541.3041	1535.4828	-11.642707	1.0
VILLEROY & BOCH PF.SHS.	1871.584	1875.4497	7.731308	0.00543
PNE	1964.0988	1960.3439	-7.50997	1.0
LECHWERKE	2076.3204	2075.0776	-2.485547	1.0
MOURY CONSTRUCT	2113.755	2114.3604	1.210821	0.27117
FLORIDIENNE	1825.5837	1821.9698	-7.22775	1.0
BEKAERT (D)	2073.4612	2073.9664	1.010368	0.31481
JENSEN-GROUP	2080.62	2083.5879	5.935819	0.01484
STRABAG SE	2186.7753	2223.2048	72.858837	0.0
SW UMWELTTECHNIK	2445.7466	2601.647	311.80096	0.0
PORR	1947.0686	1935.8663	-22.404578	1.0
POLYTEC HOLDING	2042.0882	2049.375	14.573516	0.00013
MT HOEJGAARD HOLDING	1859.8523	1859.6777	-0.34919	1.0
SCANDINAVIAN BRAKE SYS.	1420.1617	1419.1012	-2.120871	1.0
ENNOGIE SOLAR GROUP	1610.0747	1608.5881	-2.973334	1.0
FIRSTFARMS	2209.9035	2209.9278	0.048669	0.8254
GLUNZ & JENSEN HOLDING	2215.7084	2770.3885	1109.360164	0.0
BRD KLEE B	2403.7962	2251.3942	-304.803974	1.0
SKAKO	2039.651	2042.9452	6.588255	0.01027
NTG NORDIC TRANSPORT GROUP	1657.3554	1651.5049	-11.700951	1.0

Company	Log-like1	Log-like2	LR statistic	P-value
KENDRION	1982.8644	1987.3891	9.049513	0.00263
KON.HEIJMANS DU. CERTS.	2058.3855	2056.1435	-4.484044	1.0
HYDRATEC INDUSTRIES	2220.6959	2427.0417	412.691594	0.0
VOOLT	1333.9197	1332.7314	-2.376791	1.0
VIATRON	3284.0969	3026.0789	-516.036037	1.0
INTER CARS	1953.1634	1947.1814	-11.964064	1.0
LESS	1386.1523	1382.7853	-6.733901	1.0
HORTICO	1806.0202	1806.4531	0.865834	0.35211
PRYMUS	2287.609	2992.5187	1409.819305	0.0
BUDIMEX	1910.2975	1907.2396	-6.115825	1.0
PBG	2295.8587	5650.4678	6709.218167	0.0
INVESTMENT FRIENDS	1228.0827	1274.5631	92.9609	0.0
MERCOR	1989.6794	1987.2616	-4.835486	1.0
PA NOVA	2068.9734	2071.0608	4.174755	0.04103
TRAKCJA	1936.7445	1938.5172	3.545245	0.05972
UNIBEP	1893.3648	1896.8536	6.977442	0.00825
FON	2040.7444	3225.4748	2369.460782	0.0
MOSTAL	1732.4676	1734.0297	3.12424	0.07714
MERA	1959.5914	1934.2037	-50.775278	1.0
LIBET	1807.3664	1806.9684	-0.795844	1.0
DEKTRA	1887.2562	1888.2227	1.932891	0.16444
POLIMEX MOSTOSTAL	1697.0223	1690.2332	-13.578259	1.0
HM INWEST ORD	1403.3945	1404.2246	1.660053	0.1976
TAURON POLSKA ENERGIA	1763.9858	1755.059	-17.853537	1.0
ENEA	1789.8344	1789.9656	0.262391	0.60848
POLENERGIA	1845.5801	1843.9766	-3.206985	1.0
BORYSZEW	2050.3504	2049.9787	-0.743429	1.0
MOJ	1929.0109	2044.6634	231.305091	0.0
WIELTON	1876.7126	1878.6218	3.818452	0.05069
BUMECH	1430.6059	1413.5568	-34.098259	1.0
KCI	1695.0769	1696.2484	2.342844	0.12586
ZAKLADY MAGNEZYTOWE ROPCZYCE	2014.9764	2012.4143	-5.124159	1.0
SANOK RUBBER COMPANY	1833.8976	1834.731	1.666645	0.19671
GRUPA KETY	1979.748	1983.1088	6.721562	0.00953
STALPRODUKT	1951.2491	1951.5035	0.508875	0.47563
BALTICON	1558.2223	1728.5813	340.718058	0.0
XBS PRO-LOG	2086.6317	2662.7827	1152.301856	0.0
NATURGY ENERGY	2269.464	2321.8564	104.784891	0.0
IBERDROLA	2294.2125	2342.5068	96.588533	0.0
BKW	2254.2664	2288.9627	69.392775	0.0
ENERGIEDIENST HOLDING	2204.9228	2205.1552	0.464723	0.49543

Table 29: Likelihood ratio test AR(1) with no exogenous variables and AR(1) with MASCI, VIX, and ECF variables

List of stocks that rejected the null hypothesis:

Edison Rsp, Terna Rete Elettrica Naz, Algowatt, L Air Lqe.Sc.Anyme. Pour L Etude Et L Epxtn., Carbios, Valeo, Engie, Eaux De Royan, Trilogiq, Roctool, Global Bioenergies, Global Pvq, Envitec Biogas, Centrotherm Phto., Clearvise (Fra), Masterflex, Eckert & Ziegler Strahlen & Medzi., Evonik Industries, Grammer, Volkswagen, Bmw, Porsche Aml.Hldg.Pref., Aee Gold, Bauer, Steico, Villeroy & Boch Pf.Shs., Pne, Lechwerke, Moury Construct, Floridienne, Bekaert (D), Jensen-Group, Strabag Se, Sw Umwelttechnik, Porr, Polytec Holding, Mt Hoejgaard Holding, Scandinavian Brake Sys., Ennogie Solar Group, Firstfarms, Glunz & Jensen Holding, Brd Klee B, Skako, Ntg Nordic Transport Group, Kendrion, Kon.Heijmans Du. Certs., Hydratec Industries, Voolt, Viatron, Inter Cars, Less, Hortico, Prymus, Budimex, Pbg, Investment Friends, Mercor, Pa Nova, Trakcja, Unibep, Fon, Mostal, Mera, Libet, Interma Trade, Dektra, Honey Payment Group, Polimex Mostostal, Hm Inwest Ord, Tauron Polska Energia, Enea, Polenergia, Boryszew, Fabryka Obrabiarek Rafamet, Zaklady Urzadzen Kotlowych Staporkow, Moj, Energoinstal, Secogroup, Wielton, Bumech, Kci, Zaklady Magnezytowe Ropczyce, Hydrapres, Sanok Rubber Company, Grupa Kety, Stalprodukt, Balticon, Xbs Pro-Log, Naturgy Energy, Iberdrola, Bkw, Energiedienst Holding

In Table 30 we report the results of the Ljung-Box test on the square residuals on models for the stock markets, this is due to check that the volatility process was well-specified, and then all the estimates and inference were valid.

Company	P-value 10 lags	P-value 25 lags	P-value 50 lags
HERA	0.0797	0.7648	0.6293
ACINQUE	0.5563	0.8994	0.8901
ASCOPIAVE	0.918	0.3678	0.1603
EDISON RSP	0.0677	0.2631	0.7101
ALERION CLEAN POWER	0.1142	0.7352	0.4884
ENEL	0.0903	0.6691	0.3014
TERNA RETE ELETTRICA NAZ	0.1368	0.4564	0.9112
ALGOWATT	0.9878	0.81	0.6092
ACEA	0.2414	0.4056	0.8523
A2A	0.0739	0.7549	0.6245
FRENDY ENERGY	0.98	0.7942	0.6963
ERG	0.062	0.5201	0.9482
AGATOS	0.9621	0.907	0.8894
GAS PLUS	0.7891	0.4164	0.6281
ENI	0.1255	0.7422	0.1021
BIESSE	0.4952	0.8272	0.2067
FIDIA	0.777	0.9781	0.8193
TESMEC	0.0678	0.9036	0.9422
BORGOSESIA	0.968	0.9264	0.9726
INTERPUMP GROUP	0.0639	0.2424	0.5512
INNOVATEC	0.685	0.9254	0.8793
ENERTRONICA	0.9842	0.9841	0.9838
SOL	0.3105	0.6681	0.5327
ENCRES DUBUIT	0.9282	0.9783	0.8724
ARKEMA	0.9593	0.8674	0.6996
METABOLIC EXPLORER	0.8417	0.7572	0.8778
ROBERTET	0.4932	0.7067	0.3549
L AIR LQE.SC.ANYME. POUR L ETUDE ET L EPXTN.	0.0727	0.4119	0.3994
EXPLOS.ET PRDS.CHIM.	0.8443	0.6956	0.632
CARBIOS	0.7815	0.2359	0.8878
AKWEL	0.5592	0.7146	0.066
RENAULT	0.0708	0.2558	0.2205
BURELLE	0.1604	0.9832	0.8949
DELFINGEN	0.9196	0.5772	0.9159
MICHELIN	0.1252	0.656	0.2799
VALEO	0.428	0.8587	0.2972
PLASTIC OMNIUM	0.2095	0.8836	0.9335
FORVIA	0.0713	0.4331	0.3248
VEOLIA ENVIRON	0.1795	0.9871	0.0603
ENGIE	0.3531	0.8915	0.5383
FINAXO ENVIRONNEMENT	0.936	0.8383	0.5193
EAUX DE ROYAN	0.3851	0.3989	0.4916
MAUREL ET PROM	0.0647	0.6356	0.3116
TOTALENERGIES EP GABON	0.797	0.6752	0.9848
TOTALENERGIES	0.0742	0.5746	0.0648
CIE DE CHEMINS DE FER DEPARTEMENTAUX	0.6064	0.3991	0.5831
TRILOGIQ	0.6108	0.4592	0.4118
ROCTOOL	0.8416	0.9125	0.6941
EXAIL TECHNOLOGIES	0.6414	0.2492	0.5849

Company	P-value 10 lags	P-value 25 lags	P-value 50 lags
SIGNAUX GIROD	0.9954	0.4663	0.8346
EO2	0.6398	0.4185	0.5291
GLOBAL BIOENERGIES	0.5325	0.4562	0.2609
NORDEX	0.523	0.6801	0.9568
SOLARWORLD K	0.8203	0.9668	0.8065
PHOENIX SOLAR	0.5272	0.7552	0.6854
GLOBAL PVQ	0.6213	0.623	0.7612
SFC ENERGY	0.4798	0.6755	0.9439
CROPENERGIES	0.9943	0.7457	0.9194
VERBIO	0.1131	0.0906	0.5644
ENVITEC BIOGAS	0.0608	0.2241	0.7356
CENTROTHERM PHTO.	0.1816	0.9439	0.9457
SMA SOLAR TECHNOLOGY	0.9191	0.0961	0.4252
ENAPTER	0.88	0.5934	0.9213
CLEARVISE (FRA)	0.6066	0.2044	0.1083
ABO WIND	0.1969	0.1569	0.9821
MASTERFLEX	0.0614	0.1047	0.2121
LANXESS	0.1637	0.0683	0.3794
WACKER CHEMIE	0.4818	0.1949	0.8334
MUEHLHAN	0.6336	0.9558	0.6629
NABALTEC	0.069	0.9999	0.8086
SYMRISE	0.1373	0.7292	0.5235
FUCHS N	0.1021	0.5614	0.7075
BRENNTAG	0.3048	0.0709	0.6948
ECKERT & ZIEGLER STRAHLEN & MEDZI.	0.0727	0.1055	0.0667
ALZCHEM	0.2836	0.8417	0.5649
EVONIK INDUSTRIES	0.4393	0.9995	0.4332
BASF	0.2767	0.3884	0.0703
H & R	0.5442	0.9374	0.3004
DELTICOM	0.4957	0.7121	0.8851
SAF-HOLLAND	0.5151	0.977	0.187
MERCEDES-BENZ GROUP N	0.0532	0.824	0.0714
ELRINGKLINGER N	0.0736	0.1775	0.0566
GRAMMER	0.2813	0.1867	0.6988
VOLKSWAGEN	0.0644	0.9095	0.1362
BMW	0.4806	0.725	0.0625
CONTINENTAL	0.0656	0.6076	0.9862
PORSCHE AML.HLDG.PREF.	0.1345	0.6395	0.4568
FERNHEIZWERK NEUKOLLN	0.9207	0.8302	0.8357
MAINOVA	0.7339	0.5688	0.6506
RWE	0.1405	0.1517	0.2423
E ON N	0.2869	0.2035	0.0569
GELSENWASSER	0.8633	0.9147	0.6554
GLOBAL OIL & GAS	0.0685	0.1571	0.7726
DEUTSCHE ROHSTOFF	0.6339	0.4834	0.4607
AEE GOLD	0.9437	0.9444	0.9509
META WOLF	0.4184	0.9846	0.5505
STO PREFERENCE	0.1833	0.6521	0.8077
BAUER	0.7588	0.7097	0.7389
STEICO	0.0632	0.6443	0.8688
VILLEROY & BOCH PF.SHS.	0.0511	0.0509	0.193
INNOTEC TSS	0.9972	0.2207	0.943
UZIN UTZ	0.7861	0.5816	0.7285
HOCHTIEF	0.3194	0.1365	0.4333

Company	P-value 10 lags	P-value 25 lags	P-value 50 lags
WESTAG	0.7403	0.9614	0.9262
STEULER FLIESEN-GRUPPE	0.9695	0.9695	0.9697
7C SOLARPARKEN K	0.1694	0.0561	0.9707
ENERGIEKONTOR	0.0892	0.5673	0.49
4 SC	0.4535	0.5867	0.8583
BAUMOT GROUP	0.061	0.9956	0.8392
2G ENERGY	0.242	0.0866	0.2526
ENCAVIS	0.1156	0.0658	0.1056
PNE	0.0929	0.6413	0.9757
MVV ENERGIE	0.0636	0.0732	0.6809
LECHWERKE	0.4821	0.2498	0.8928
ENBW ENGE.BADEN-WURTG.	0.4586	0.2347	0.1095
MOURY CONSTRUCT	0.1304	0.8495	0.4949
COMPAGNIE D ENTREPRISES CFE	0.9312	0.6141	0.0736
FLORIDIENNE	0.9815	0.8782	0.9179
BEKAERT (D)	0.9442	0.7799	0.2094
JENSEN-GROUP	0.5861	0.7256	0.9666
EVN	0.9862	0.0838	0.1462
BURGENLAND HOLDING	0.7454	0.5458	0.4298
VERBUND	0.0731	0.5973	0.0785
RATH	0.8951	0.7819	0.7792
STRABAG SE	0.1399	0.9909	0.3654
ZUMTOBEL	0.6478	0.5222	0.0733
SW UMWELTTECHNIK	0.8884	0.8833	0.8969
PORR	0.3622	0.3409	0.3438
HUTTER & SCHRANTZ	0.5304	0.6058	0.2529
WIENERBERGER	0.9122	0.2331	0.9853
POLYTEC HOLDING	0.0775	0.726	0.2845
OMV	0.2346	0.0924	0.251
MT HOEJGAARD HOLDING	0.0708	0.9643	0.9155
H+H INTERNATIONAL	0.0633	0.5385	0.9318
ROCKWOOL B	0.5903	0.697	0.6068
FLSMIDTH AND CO.	0.352	0.5317	0.1132
SCANDINAVIAN BRAKE SYS.	0.6182	0.8894	0.0878
ENNOGIE SOLAR GROUP	0.9539	0.7578	0.7354
VESTAS WINDSYSTEMS	0.1257	0.1285	0.5211
UIE	0.1514	0.4786	0.1597
FIRSTFARMS	0.6198	0.0659	0.9095
SCHOUW AND	0.8054	0.6146	0.3722
GLUNZ & JENSEN HOLDING	0.344	0.0752	0.5262
BRD KLEE B	0.3535	0.4789	0.3755
SKAKO	0.2063	0.5513	0.8756
KOBENHAVNS LUFTHAVNE	0.1708	0.7903	0.2916
DSV	0.0718	0.1844	0.2956
ERRIA	0.9984	0.6684	0.2084
TORM A	0.1602	0.6095	0.7773
DMPKBT.NORDEN	0.2225	0.0778	0.9016
NTG NORDIC TRANSPORT GROUP	0.74	0.4844	0.9125
DFDS	0.1822	0.0944	0.1892
A P MOLLER MAERSK B	0.2711	0.984	0.8212
KENDRION	0.1421	0.7584	0.1242

Company	**P-value 10 lags**	**P-value 25 lags**	**P-value 50 lags**
KON.HEIJMANS DU. CERTS.	0.5974	0.6491	0.8189
BAM GROEP KON.	0.931	0.0729	0.8869
FERROVIAL	0.1039	0.7337	0.09
AMG CRITICAL MATERIALS	0.2427	0.5895	0.6214
HYDRATEC INDUSTRIES	0.7897	0.8236	0.8242
SBM OFFSHORE	0.2056	0.1069	0.7044
SUNEX	0.0822	0.0724	0.1059
MVA GREEN ENERGY	0.4083	0.951	0.8482
COLUMBUS ENERGY	0.0622	0.0885	0.8869
VOOLT	0.8153	0.4774	0.5169
VIATRON	0.961	0.0808	0.6821
BIOMASS ENERGY PROJECT	0.6954	0.8073	0.935
FIRMA OPONIARSKA DEBICA	0.789	0.3173	0.4784
INTER CARS	0.2463	0.5571	0.0739
LESS	0.4443	0.9224	0.8338
AC AUTOGAZ	0.4384	0.0665	0.3423
PL GROUP	0.8203	0.8094	0.8085
ORZEL	0.6235	0.9596	0.9228
SOLAR INNOVATION	0.9082	0.5637	0.6337
KRAKCHEMIA	0.9045	0.8409	0.9554
SELENA FM SR.B I C	0.8807	0.8342	0.1403
GALVO	0.6523	0.8999	0.869
HORTICO	0.997	0.8783	0.9362
IZOBLOK	0.6955	0.6842	0.9931
PRYMUS	0.0666	0.3938	0.5566
PCC EXOL	0.0708	0.0623	0.0747
MOSTOSTAL ZABRZE	0.0626	0.9073	0.8617
BUDIMEX	0.6572	0.711	0.9788
PBG	0.5696	0.2724	0.5787
RAWLPLUG	0.5506	0.0711	0.7615
MOSTOSTAL WARSZAWA	0.073	0.284	0.5717
LENA LIGHTING	0.2331	0.6424	0.8081
DECORA	0.509	0.2879	0.9685
ERBUD	0.9134	0.7833	0.8202
INVESTMENT FRIENDS	0.8696	0.8867	0.8455
MERCOR	0.7287	0.5412	0.2806
PA NOVA	0.6633	0.7664	0.5219
RESBUD	0.968	0.9653	0.9704
IZOLACJA JAROCIN	0.8578	0.5245	0.6926
TRAKCJA	0.9648	0.715	0.5674
UNIBEP	0.447	0.0674	0.9662
STARHEDGE	0.9205	0.6605	0.6557
COMPREMUM	0.8338	0.9011	0.4666
MIRBUD	0.0639	0.549	0.0631
FON	0.9291	0.9332	0.9707
MOSTAL	0.6417	0.1861	0.6693
INSTAL KRAKOW	0.725	0.9874	0.6341
MERA	0.3796	0.8832	0.7159
TESGAS	0.0604	0.392	0.7151
ZUE	0.5402	0.82	0.1229
LIBET	0.6667	0.6665	0.8405
TAMEX OBIEKTY SPORTOWE	0.5574	0.8409	0.8636
MOBRUK	0.3962	0.5738	0.1539
INTERMA TRADE	0.9442	0.7799	0.7954
DEKTRA	0.8221	0.6852	0.7355

Company	P-value 10 lags	P-value 25 lags	P-value 50 lags
INTERNITY	0.0617	0.8217	0.4669
HONEY PAYMENT GROUP	0.9804	0.9803	0.9802
PRZED.PRZ.BETONOW PREFABET BIALE BLOTA	0.2612	0.3906	0.4879
ROCCA	0.7912	0.9822	0.8834
FABRYKA KONSTRUKCJI DREW	0.2321	0.901	0.962
ULMA CONSTR.POLSKA	0.8086	0.5368	0.2296
ATLANTIS	0.9673	0.9506	0.9708
POLIMEX MOSTOSTAL	0.06	0.0641	0.2143
FORBUILD	0.9399	0.8731	0.9498
HM INWEST ORD	0.8881	0.8437	0.9187
TAURON POLSKA ENERGIA	0.0717	0.2442	0.0851
EC BEDZIN	0.9633	0.2887	0.9784
PKA.GRUPA ENERGETYCZNA	0.0948	0.1605	0.0685
ENEA	0.5215	0.7655	0.2578
POLENERGIA	0.2732	0.9154	0.3462
ZESPOL ELKTP. WRLKKNR.	0.6323	0.4443	0.4438
ZE PAK	0.9245	0.7713	0.8426
ENERGA	0.9071	0.0818	0.5817
PHOTON ENERGY	0.9828	0.976	0.3796
UNIMOT	0.1326	0.144	0.8567
MANGATA HOLDING	0.2029	0.3332	0.7477
ZAK AD BUD MASZYN ZBC.	0.7213	0.943	0.0601
KUPIEC	0.9516	0.9002	0.86
BORYSZEW	0.6041	0.8979	0.9632
FABRYKA OBRABIAREK RAFAMET	0.4147	0.063	0.1619
ZAKLADY URZADZEN KOTLOWYCH STAPORKOW	0.7566	0.5718	0.7545
MOJ	0.6966	0.0633	0.8276
PGF POLSKA GRUPA FOTOWOLTAICZNA	0.3307	0.9488	0.8149
ENERGOINSTAL	0.2084	0.1126	0.1528
SECOGROUP	0.2284	0.258	0.9859
WIELTON	0.4611	0.3435	0.7592
BUMECH	0.8444	0.9867	0.9019
KCI	0.0681	0.0703	0.0648
ZAKLADY MAGNEZYTOWE ROPCZYCE	0.5561	0.4877	0.269
PATENTUS	0.463	0.7704	0.8544
HYDRAPRES	0.854	0.7883	0.9826
ZAMET	0.5064	0.7997	0.4495
SANOK RUBBER COMPANY	0.4833	0.2316	0.6588
FEERUM	0.8119	0.6649	0.084
APS ENERGIA	0.1518	0.3449	0.1021
DROZAPOL PROFIL	0.253	0.9365	0.2297
PJP MAKRUM	0.89	0.6558	0.1422
ODLEWNIE POLSKIE	0.0698	0.2979	0.9908
IZOSTAL	0.3363	0.9403	0.208
BOWIM	0.465	0.7003	0.4348
GRUPA KETY	0.7518	0.4601	0.9837
MFO	0.0693	0.7822	0.9518

Company	P-value 10 lags	P-value 25 lags	P-value 50 lags
EKOPOL GORNOSLASK HLDG.	0.7301	0.7752	0.6326
STALPRODUKT	0.9672	0.843	0.9923
STALEXPORT AUTOSTRADY	0.7608	0.9156	0.4914
TRANSPOL	0.9747	0.9567	0.9539
OT LOGISTICS	0.0727	0.0693	0.1964
BALTICON	0.1063	0.0644	0.1866
FORPOSTA	0.9036	0.461	0.7136
XBS PRO-LOG	0.5532	0.5819	0.9042
PKP CARGO	0.9659	0.9457	0.4114
NATURGY ENERGY	0.067	0.9011	0.9168
GRINO ECOLOGIC	0.8008	0.9902	0.3832
ERCROS	0.0737	0.0892	0.2448
OBRASCON HUARTE LAIN	0.3207	0.1999	0.1075
FLUIDRA	0.1072	0.6102	0.9478
ACS ACTIV.CONSTR.Y SERV.	0.0622	0.9258	0.1149
FOMENTO CONSTR.Y CNTR.	0.984	0.9452	0.9687
ACCIONA	0.0696	0.3435	0.8662
SACYR	0.0606	0.7729	0.7935
AUDAX RENOVABLES	0.1498	0.9772	0.6927
SOLARIA ENERGIA Y MEDIO AMBIENTE	0.1052	0.9978	0.7964
EDP RENOVAVEIS	0.067	0.0729	0.1189
REDEIA CORPORACION	0.0612	0.3164	0.9063
ENDESA	0.0706	0.5515	0.7612
AKILES CORPORATION	0.0718	0.5293	0.7631
IBERDROLA	0.1224	0.799	0.6429
ROMANDE ENERGIE	0.4066	0.4978	0.3392
EDISUN POWER EUROPE N	0.1075	0.1452	0.0914
BKW	0.6807	0.7877	0.7171
ENERGIEDIENST HOLDING	0.1319	0.5328	0.9122
CLARIANT	0.2671	0.7847	0.0643
GIVAUDAN 'N'	0.2824	0.9092	0.7885
DOTTIKON ES HOLDING	0.0636	0.4515	0.3154
GURIT HOLDING 'B'	0.2048	0.5766	0.4487
EMS-CHEMIE 'N'	0.0662	0.8554	0.9777
FEINTOOL	0.8131	0.7321	0.9054
AUTONEUM HOLDING	0.3703	0.4588	0.7121

Table 30: Ljung-Box Test for the square residuals

A.2.3 Stock market quantile regression results

Company	μ	β_1	β_2	β_3	β_4
FRENDY ENERGY	-0.035690	-0.242370	0.856764	0.057748	-0.142589
CARBIOS	-0.047251	0.113331	0.351496	-0.067379	0.156975
VALEO	-0.048782	0.121667	-0.135496	0.040224	0.252389
GLOBAL BIOENERGIES	-0.047503	-0.074187	0.567871	-0.044975	0.144858
VERBIO	-0.060969	-0.011485	0.428040	-0.001723	-0.290125
CENTROTHERM PHTO.	-0.053238	-0.142044	1.107553	0.072272	-0.343166
WACKER CHEMIE	-0.040670	0.095904	-0.069128	-0.055692	0.144942
SYMRISE	-0.024571	0.046382	0.256119	-0.029152	0.106423
BAUER	-0.033346	-0.191110	0.560894	0.019671	0.203169
4 SC	-0.079462	-0.210277	1.485823	0.148379	0.426540
VERBUND	-0.034483	-0.163203	0.391226	0.074842	0.184943
DSV	-0.030419	0.120939	0.063557	-0.045431	-0.147067
PA NOVA	-0.031353	-0.162931	0.141580	-0.044610	0.126835
COMPREMUM	-0.041190	0.028178	0.809439	-0.007166	-0.123976
HONEY PAYMENT GROUP	-0.083416	-0.016962	0.355387	-0.174530	0.709067
ATLANTIS	-0.076936	-0.017268	-3.562045	-0.506034	-0.393685
ZAKLADY URZADZEN KOTLOWYCH STAPORKOW	-0.042795	-0.063093	0.499985	0.077169	-0.122344
FEERUM	-0.060982	-0.450748	0.963850	0.128416	-0.359501
APS ENERGIA	-0.054716	-0.212022	0.861916	-0.020813	0.372950
NATURGY ENERGY	-0.022109	-0.066193	0.176823	-0.024098	0.088344
SOLARIA ENERGIA Y MEDIO AMBIENTE	-0.045099	-0.109016	0.632167	0.012989	-0.178355
AUTONEUM HOLDING	-0.039861	-0.062298	0.277336	-0.036568	0.195300

Table 31: 5% quantile regression with exogenous variable
MSCI, VIX, ECF

Company	μ	β_1	β_2	β_3	β_4
BIESSE	-0.037741	-0.023665	0.718085	0.067071	-0.084750
SOL	-0.027636	-0.072206	0.560132	0.052979	-0.068941
L AIR LQE.SC.ANYME. POUR L ETUDE ET L EPXTN.	-0.018236	-0.250665	0.473812	0.013975	-0.035806
PHOENIX SOLAR	-0.201548	-0.389055	1.342734	0.169527	-0.512353
GLOBAL PVQ	-0.189331	-0.425884	-2.802985	-0.239290	-0.276795
DELTICOM	-0.057204	-0.079299	1.131250	0.067847	-0.106245
ENCAVIS	-0.043337	-0.164722	1.371995	0.068312	-0.061690
LECHWERKE	-0.029139	-0.162176	0.265151	-0.026896	-0.052282
SCANDINAVIAN BRAKE SYS.	-0.071132	-0.217940	0.076243	0.017383	-0.151126
BRD KLEE B	-0.055023	-0.308197	-0.263445	-0.121592	-0.142266
HYDRATEC INDUSTRIES	-0.027700	-0.288733	0.342895	0.023843	-0.046898
SBM OFFSHORE	-0.024056	-0.064160	0.486082	0.000211	-0.042599
BIOMASS ENERGY PROJECT	-0.056304	-0.172391	0.907842	0.003917	0.103506
INTER CARS	-0.032822	-0.168128	0.714045	-0.008513	-0.056252
BUDIMEX	-0.034875	-0.151630	0.654307	0.042890	-0.069232
LIBET	-0.050045	-0.159496	-0.569241	-0.078680	-0.139986
ULMA CONSTR.POLSKA	-0.031559	-0.283827	0.339499	0.039237	-0.067731
PHOTON ENERGY	-0.038998	0.086983	0.297739	0.006788	0.054459
FABRYKA OBRABIAREK RAFAMET	-0.027615	-0.192184	0.033312	-0.039109	-0.061934
ENERGOINSTAL	-0.063367	-0.102891	1.046914	0.068084	0.079148

Company	μ	β_1	β_2	β_3	β_4
ODLEWNIE POLSKIE	-0.027827	-0.204439	-0.151611	-0.046819	-0.052703
SOLARIA ENERGIA Y MEDIO AMBIENTE	-0.046046	-0.110326	0.557418	-0.017851	0.059500

Table 32: 5% quantile regression with exogenous variable
MSCI, VIX, CR^{10y}

Company	μ	β_1	β_2	β_3	β_4
HERA	-0.035833	-0.177085	1.189662	0.131594	-0.060682
ABO WIND	-0.053349	-0.345245	1.242720	-0.005826	-0.071319
AEE GOLD	-0.110722	-0.121472	-3.277212	-0.098893	0.130994
4 SC	-0.109795	-0.223807	1.859790	0.272347	-0.155430
PNE	-0.039809	-0.165701	0.278954	0.000222	-0.075494
BURGENLAND HOLDING	-0.033410	-0.411497	-0.348874	-0.053839	-0.057590
MT HOEJGAARD HOLDING	-0.049008	0.024071	0.163177	0.035721	-0.092184
DMPKBT.NORDEN	-0.066448	-0.000458	-1.368562	-0.209195	-0.128722
PA NOVA	-0.039665	-0.084373	0.330787	-0.033382	-0.033630
RESBUD	-0.070372	-0.178911	-0.592097	-0.182516	-0.124419
INSTAL KRAKOW	-0.034197	0.022803	-0.096965	-0.037365	-0.075022
ZUE	-0.045542	0.018482	0.857804	0.044977	0.068439
FORBUILD	-0.044837	-0.197433	-0.264611	-0.045840	0.053609
ZAK AD BUD MASZYN ZBC.	-0.068554	-0.208433	0.786369	-0.183131	0.121810
MFO	-0.047660	-0.175186	1.426870	0.184938	-0.047959
BALTICON	-0.090804	-0.446067	1.783788	-0.025967	-0.109915
XBS PRO-LOG	-0.064506	-0.427774	-1.345747	-0.030645	0.073689
NATURGY ENERGY	-0.028148	-0.179189	0.581486	0.000095	-0.053192

Table 33: 2.5% quantile regression with exogenous variable
MSCI, VIX, CR^{10y}

Company	μ	β_1	β_2	β_3	β_4
ACEA	-0.044546	-0.149482	0.923569	0.112676	-0.106098
ERG	-0.044933	-0.222471	0.884296	0.059660	-0.079839
FIDIA	-0.059403	-0.168462	0.143616	-0.095832	-0.109129
EXPLOS.ET PRDS.CHIM.	-0.063867	0.258020	1.394631	0.196580	-0.104235
SFC ENERGY	-0.098123	-0.058156	1.250882	-0.012479	-0.180952
ECKERT & ZIEGLER STRAHLEN & MEDZI.	-0.088763	-0.033090	-1.170757	-0.077347	-0.183556
RWE	-0.045113	0.214965	-0.045961	-0.007233	-0.104729
MIRBUD	-0.064032	0.218037	1.612364	0.036505	-0.111772
MANGATA HOLDING	-0.049880	0.178741	0.831995	-0.000439	0.053423
SECOGROUP	-0.052548	-0.490700	1.073299	0.118732	-0.073168
FEERUM	-0.101176	-0.236230	0.697595	-0.028257	-0.166509
ACCIONA	-0.047390	-0.247516	0.640460	0.078405	-0.128535
IBERDROLA	-0.033417	0.138931	-0.448597	-0.057602	0.056683
BKW	-0.037836	-0.103670	-0.218125	-0.073825	0.067681

Table 34: 1% quantile regression with exogenous variable
MSCI, VIX, CR^{10y}

Company	μ	β_1	β_2	β_3	β_4
ENI	-0.024686	0.345008	0.089394	-0.003992	-0.021582

Company	μ	β_1	β_2	β_3	β_4
L AIR LQE.SC.ANYME. POUR L ETUDE ET L EPXTN.	-0.018388	-0.203719	0.400057	0.009139	-0.023693
CLEARVISE (FRA)	-0.036558	-0.127021	0.445419	0.022568	0.034479
SAF-HOLLAND	-0.042417	-0.081750	0.819865	-0.072112	0.071152
BURGENLAND HOLDING	-0.021367	-0.307935	-0.247965	-0.049953	0.021752
VESTAS WINDSYSTEMS	-0.045986	-0.035486	0.243576	-0.016077	0.046411
SBM OFFSHORE	-0.024380	-0.083269	0.474267	0.001832	-0.026028
AC AUTOGAZ	-0.021355	-0.156853	0.234322	-0.011141	-0.024092
KRAKCHEMIA	-0.068709	-0.038652	0.231664	0.009981	0.105384
MOSTOSTAL ZABRZE	-0.030509	-0.027387	0.259330	-0.054548	0.033652
ROCCA	-0.077679	0.346516	0.949964	0.228714	0.144801
ATLANTIS	-0.084511	-0.053563	-2.094478	-0.455689	0.191474
HM INWEST ORD	-0.069233	-0.017866	0.493474	0.036115	0.105449
MANGATA HOLDING	-0.033242	0.002834	0.541404	-0.014961	0.019847
DROZAPOL PROFIL	-0.045764	-0.022602	0.576590	-0.011988	0.051255
MFO	-0.037254	-0.162354	0.886378	0.072047	-0.026768
STALEXPORT AUTOSTRADY	-0.017786	-0.190197	0.429593	0.011432	-0.031753
ERCROS	-0.031726	-0.095923	0.394150	0.023708	-0.037869
GURIT HOLDING 'B'	-0.044484	0.052140	0.481396	0.089740	-0.041102

Table 35: 5% quantile regression with exogenous variable
MSCI, VIX, CR^{20y}

B Appendix chapter 2

Table 36: Welch's t-test Results

Issuer	Month	Statistic	PValue	DoF
AEROPORTI DI ROMA SPA	1	-0.6335	0.7347	34.1880
	2	-3.3333	0.9991	39.1373
	3	6.6035	0.0000	29.0008
	4	-4.3312	0.9999	31.1916
	5	-2.1851	0.9827	40.7028
	6	-4.1279	0.9999	55.2706
	7	-0.4035	0.6554	33.9401
	8	2.7068	0.0055	30.1757
	9	4.2951	0.0001	32.7534
	10	1.2314	0.1132	34.8060
ALPERIA SPA	1	0.9534	0.1723	55.0866
	2	-0.5255	0.6989	39.9616
	3	6.6222	0.0000	29.0000
	4	-0.2925	0.6142	37.6032
	5	1.0000	0.1628	29.0000
	6	4.6793	0.0000	29.0000
	7	0.7124	0.2397	51.4649
	8	-1.3824	0.9121	33.9080
	9	3.3775	0.0008	40.4962
	10	6.5031	0.0000	30.0000
ASSICURAZIONI GENERALI SPA	1	-1.1174	0.8658	57.9347
	2	0.5210	0.3022	56.6432
	3	6.8137	0.0000	29.1840
	4	1.4513	0.0761	57.7428
	5	1.1294	0.1318	55.8002
	6	2.8615	0.0031	50.9146
	7	0.7727	0.2215	56.1915
	8	0.1243	0.4507	57.9419
	9	1.2741	0.1038	57.9843
	10	1.9620	0.0272	59.9441
BANCO BPM SPA	1	6.9944	0.0000	29.0084
	2	5.4482	0.0000	29.0167
	3	5.8033	0.0000	29.0055
	4	7.6900	0.0000	29.0061
	5	-0.3365	0.6311	56.6172
	6	-0.2298	0.5904	50.8187
	7	0.1008	0.4600	57.9975
	8	0.1722	0.4319	75.9920
COMMERZBANK AG	1	0.0196	0.4922	57.9999
	2	6.5283	0.0000	31.7108
	3	3.2412	0.0014	31.4007
	4	-0.2079	0.5820	57.8929
	5	-0.2818	0.6104	57.9472
	6	-0.4208	0.6623	57.9752
	7	-0.0147	0.5059	57.9920
	8	-0.0685	0.5272	57.9300
	9	-0.0075	0.5030	89.9992
EUROGRID GMBH	1	-0.8633	0.8031	34.9504
	2	0.2099	0.4174	43.2933
	3	-3.4385	0.9994	48.4842
	4	0.8622	0.1961	57.9953
	5	2.4149	0.0095	57.0745
	6	1.8274	0.0366	53.0315
	7	-1.2316	0.8883	54.3805
	8	-1.6969	0.9519	48.3521
	9	-2.9055	0.9973	49.1123
	10	-0.3859	0.6495	55.0010
EUSOLAG EUROPEAN SOLAR AG	1	2.2794	0.0151	29.0234

	2	-1.9191	0.9679	30.9657
	3	-0.2850	0.6115	45.8687
	4	-2.4509	0.9912	51.4995
	5	-0.3241	0.6262	41.7006
	6	-2.8778	0.9970	48.5923
	7	-2.0743	0.9795	87.4936
LANDESBANK BADEN WUERTTEMBERG	1	-0.1003	0.5398	57.9993
	2	4.9276	0.0000	29.2594
	3	4.5354	0.0000	29.6170
	4	12.6553	0.0000	29.1156
	5	14.6387	0.0000	29.0526
	6	23.8778	0.0000	29.7789
	7	1.7882	0.0395	57.4284
	8	10.5944	0.0000	33.5611
	9	5.0863	0.0000	30.8040
	10	1.4256	0.0797	58.3252
MUENCHENER HY-POTHEKENBANK EG	1	3.7095	0.0002	57.9664
	2	5.7783	0.0000	29.0046
	3	6.7351	0.0000	29.0520
	4	30.7919	0.0000	29.0388
	5	32.9040	0.0000	29.0212
	6	29.2234	0.0000	29.0188
	7	3.5470	0.0007	29.4212
	8	14.4963	0.0000	29.1348
	9	5.5009	0.0000	29.0415
	10	9.5242	0.0000	47.2945
RWE AG	1	1.7644	0.0417	53.0712
	2	1.7737	0.0430	30.8622
	3	-0.4976	0.6889	31.8341
	4	1.9408	0.0291	48.4295
	5	0.8823	0.1908	53.8615
	6	-0.0658	0.5261	51.6168
	7	0.2603	0.3980	40.0127
	8	-0.1036	0.5410	43.0354
	9	2.0790	0.0214	50.2332
	10	1.2979	0.1019	31.1540

C Appendix weather derivatives

C.1 Estimating the temperature parameters

The estimation procedure of the temperature process follows Alfonsi et al. 2023. Estimates of parameters $\kappa, \alpha_0, \alpha_1, \beta_0$, and β_1 are found by solving the following problem:

$$\underset{(\kappa,\alpha_0,\alpha_1,\beta_0,\beta_1)\in\mathbb{R}^5}{\operatorname{argmin}} \sum_{i=0}^{T-1} \left(X_{(i+1)\Delta} - \mathbb{E}\left[X_{(i+1)\Delta} \mid X_{i\Delta}\right]\right)^2,$$

where $\mathbb{E}\left[X_{t+\Delta} \mid X_t\right] = X_t e^{-\kappa\Delta} + s(t+\Delta) - s(t)e^{-\kappa\Delta}$, T is the time of the last observation in the sample, and $\Delta = 1$ day. The instantaneous volatility process is unobservable, so we approximate it by the series of realized volatilities $\hat{\nu}$, following Aït-Sahalia et al. 2007 and Azencott et al. 2020, which correspond to the observed volatility on a time window of Q days

$$\hat{\nu}_{iQ\Delta} := \frac{1}{Q}\sum_{j=1}^{Q} \frac{2\hat{\kappa}}{1-e^{-2\hat{\kappa}\Delta}} \left(\tilde{X}_{(iQ+j)\Delta} - e^{-\hat{\kappa}\Delta}\tilde{X}_{(iQ+j-1)\Delta}\right)^2, \quad i \in \{0,\ldots,\lfloor T/Q\rfloor - 1\},$$

where we take $Q = 15$, chosen after some heuristics on the quality of approximation with different values of Q on a known data generating process of similar characteristics to the ones under study. Therefore, $\hat{\nu}_{iQ\Delta}$ is the realized volatility on $[iQ\Delta, (i+1)Q\Delta]$, meaning that it takes $I = \lfloor T/Q\rfloor$ different values. Parameters K and θ of the volatility process ν_t are estimated as

$$\underset{(K,\theta)\in\mathbb{R}^2}{\operatorname{argmin}} \sum_{i=0}^{I-2} \left(\nu_{(i+1)Q\Delta} - \mathbb{E}\left[\nu_{(i+1)Q\Delta} \mid \nu_{iQ\Delta}\right]\right)^2,$$

where $\mathbb{E}\left[\nu_{t+\Delta} \mid \nu_t\right] = \theta(1-e^{-K\Delta}) + e^{-K\Delta}\nu_t$, and where $\nu_{iQ\Delta}$ is replaced by the realized $\hat{\nu}_{iQ\Delta}$. Parameter σ is estimated as the square root of the minimizer of

$$\underset{\sigma^2}{\operatorname{argmin}} \sum_{i=0}^{T-1} \left(\left(\nu_{(i+1)\Delta} - \mathbb{E}\left[\nu_{(i+1)\Delta} \mid \nu_{i\Delta}\right]\right)^2 - \mathbb{E}\left[\left(\nu_{(i+1)\Delta} - \mathbb{E}\left[\nu_{(i+1)\Delta} \mid \nu_{i\Delta}\right]\right)^2 \mid \nu_{i\Delta}\right]\right)^2,$$

whereas parameter ρ is estimated as the minimizer of

$$\begin{aligned}\underset{\rho}{\operatorname{argmin}} \sum_{i=0}^{T-1} &\left(\left(X_{(i+1)\Delta} - \mathbb{E}\left[X_{(i+1)\Delta} \mid \mathcal{F}_{i\Delta}\right]\right)\left(\nu_{(i+1)\Delta} - \mathbb{E}\left[\nu_{(i+1)\Delta} \mid \mathcal{F}_{i\Delta}\right]\right)\right.\\ &\left. - \mathbb{E}\left[\left(X_{(i+1)\Delta} - \mathbb{E}\left[X_{(i+1)\Delta} \mid \mathcal{F}_{i\Delta}\right]\right)\left(\nu_{(i+1)\Delta} - \mathbb{E}\left[\nu_{(i+1)\Delta} \mid \mathcal{F}_{i\Delta}\right] \mid \mathcal{F}_{i\Delta}\right]\right)^2,\end{aligned}$$

where again $\nu_{iQ\Delta}$ is replaced by the realized $\hat{\nu}_{iQ\Delta}$.
The estimated parameters of all locations are displayed in Table 37. Figures 44-51 hold plots of the historical average daily temperatures against the model-estimated ones.

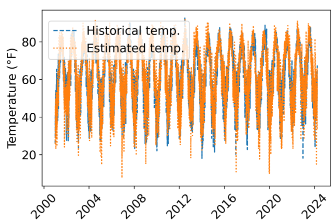


Figure 44: Atlanta historical and estimated temperature process

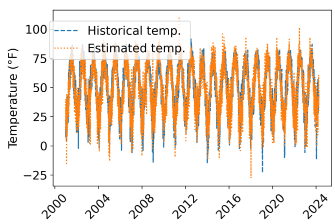


Figure 45: Chicago historical and estimated temperature process

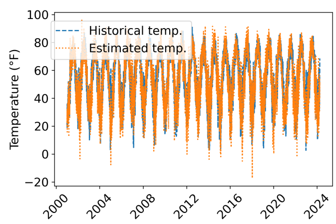


Figure 46: Cincinnati historical and estimated temperature process

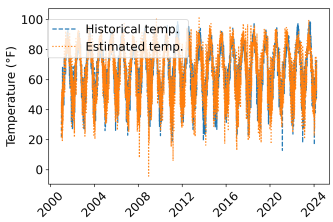


Figure 47: Dallas historical and estimated temperature process

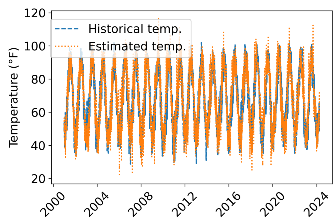


Figure 48: Las Vegas historical and estimated temperature process

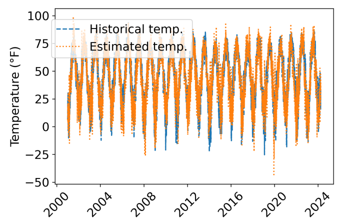


Figure 49: Minneapolis historical and estimated temperature process

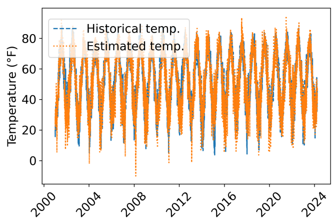


Figure 50: New York historical and estimated temperature process

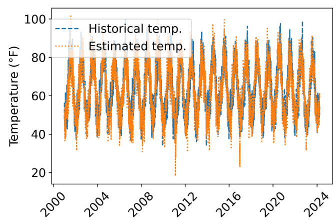


Figure 51: Sacramento historical and estimated temperature process

	a	α_0	β_0	α_1	β_1	k	θ	σ	ρ
Atlanta	0.2795	61.7667	0.0002	-6.9327	-17.9305	0.0327	28.0705	1.9390	0.0014
Chicago	0.2882	49.2214	0.0001	-10.3392	-23.8452	0.0565	43.9698	2.2191	-0.0003
Cincinnati	0.2951	54.0154	0.0001	-8.8828	-21.4657	0.0467	41.6413	2.0975	0.0002
Dallas	0.3111	65.3612	0.0002	-7.6677	-19.6790	0.0370	37.6439	2.4229	0.0005
Las Vegas	0.2051	67.6497	0.0002	-8.2808	-21.0931	0.0918	17.4321	1.2256	-0.0001
Minneapolis	0.2474	45.0781	-0.0000	-10.9192	-27.5551	0.0467	45.6670	2.4449	0.0006
New York	0.3555	51.4569	0.0002	-10.8403	-20.6581	0.0540	35.8019	1.8166	-0.0001
Sacramento	0.1974	62.8544	0.0001	-8.1028	-13.5808	0.1266	14.2263	0.9692	-0.0002

Table 37: Temperature parameter estimates by city

C.2 The change of measure

In this Appendix we reproduce the mathematical steps for the change of measure in a Heston-type model, adapted to our setting. We show that, for the processes under study, the measure change only affects the volatility in its long-term parameter, θ, and recover the explicit formula of its new value, $\tilde{\theta}$. Let $F(t,T)$ be the primary asset (the forward temperature contract), and let ν_t be its stochastic square volatility. Furthermore, let us consider the change of measure defined in Eq. 56. Then, we have that the following equations simultaneously hold

$$\begin{cases} dF(t,T) = \mu F(t,T)dt + F(t,T)\sqrt{\nu_t}(\rho dW^{\mathbb{P}}_{1,t} + \sqrt{1-\rho^2}dW^{\mathbb{P}}_{3,t}) \\ d\nu_t = k(\theta - \nu_t)dt + \sigma\sqrt{\nu_t}dW^{\mathbb{P}}_{1,t} \\ \mathbb{E}^{\mathbb{P}}[dW^{\mathbb{P}}_{1,t}dW^{\mathbb{P}}_{3,t}] = \rho dt \\ dW^{\mathbb{Q}_T}_{1,t} = dW^{\mathbb{P}}_{1,t} - \lambda_{1,t}dt \\ dW^{\mathbb{Q}_T}_{3,t} = dW^{\mathbb{P}}_{3,t} - \lambda_{3,t}dt \\ \lambda_{1,t} = \frac{\mu}{\rho\sqrt{\nu_t}} - \frac{\sqrt{1-\rho^2}}{\rho\sqrt{\nu_t}} \\ \lambda_{3,t} = \frac{1}{\sqrt{\nu_t}}, \end{cases}$$

where the analytical forms of $\lambda_{1,t}$ and $\lambda_{2,t}$ are the ones which make the process $F(t,T)$ a martingale under $\mathbb{Q}_T$. Then, by solving the above system of equations, we can move from the dynamics under

$\mathbb{P}$ to the dynamics under $\mathbb{Q}_T$ of $F(t,T)$ and ν_t, yielding

$$\begin{aligned}
dF(t,T) &= \mu F(t,T)dt + \sqrt{\nu_t}F(t,T)\left\{\rho\left[dW_{1,t}^{\mathbb{Q}_T} - \left(\frac{\mu}{\rho\sqrt{\nu_t}} - \frac{\sqrt{1-\rho^2}}{\rho\sqrt{\nu_t}}\right)\right]dt + \sqrt{1-\rho^2}\left[dW_{3,t}^{\mathbb{Q}_T} - \frac{dt}{\sqrt{\nu_t}}\right]\right\} \\
&= F(t,T)\sqrt{\nu_t}(\rho dW_{1,t}^{\mathbb{Q}_T} + \sqrt{1-\rho^2}dW_{3,t}^{\mathbb{Q}_T})
\end{aligned}$$

and

$$\begin{aligned}
d\nu_t &= k(\theta - \nu_t)dt + \sigma\sqrt{\nu_t}\left[dW_{1,t}^{\mathbb{Q}_T} - \left(\frac{\mu}{\rho\sqrt{\nu_t}} - \frac{\sqrt{1-\rho^2}}{\rho\sqrt{\nu_t}}\right)dt\right] \\
&= k\left[\left(\theta - \frac{\sigma\mu}{\rho k} + \frac{\sqrt{1-\rho^2}}{\rho k}\right) - \nu_t\right] + \sigma\sqrt{\nu_t}dW_{1,t}^{\mathbb{Q}_T} \\
&= k(\tilde{\theta} - \nu_t)dt + \sigma\sqrt{\nu_t}dW_{1,t}^{\mathbb{Q}_T}.
\end{aligned}$$

In the last step, substitution is performed, by taking $\tilde{\theta} = \theta - \frac{\sigma\mu}{\rho k} + \frac{\sqrt{1-\rho^2}}{\rho k}$.

D Appendix Water

In the appendix, we report the simulated option prices and the sensitivity analysis to the parameters influencing the pricing function.

D.1 Pricing RQO

Table 38 reports the RQO prices under three different distributional assumptions for the rainfall amount for strikes (K) level from $K_0 = 1.257$ to $1.5K_0, 2K_0, 2.5K_0, 5K_0, 10K_0$ and maturities (T) from 7 to 120 days.

	T/K	K_0	$1.5K_0$	$2K_0$	$2.5K_0$	$5K_0$	$10K_0$
Exponential	7	0.197	0.305	0.412	0.520	1.058	2.134
	15	0.197	0.305	0.413	0.521	1.059	2.137
	30	0.197	0.305	0.413	0.520	1.059	2.136
	45	0.197	0.305	0.412	0.520	1.058	2.134
	60	0.197	0.305	0.413	0.520	1.059	2.135
	75	0.197	0.305	0.413	0.520	1.059	2.137
	90	0.197	0.305	0.413	0.520	1.059	2.137
	105	0.197	0.305	0.413	0.521	1.060	2.138
	120	0.197	0.305	0.413	0.521	1.059	2.137
Log-Normal	7	0.006	0.113	0.221	0.329	0.867	1.943
	15	0.000	0.097	0.205	0.313	0.852	1.929
	30	0.004	0.111	0.219	0.327	0.865	1.942
	45	0.000	0.106	0.214	0.322	0.860	1.936
	60	0.000	0.106	0.214	0.322	0.860	1.937
	75	0.002	0.110	0.218	0.325	0.864	1.942
	90	0.002	0.110	0.218	0.325	0.864	1.942
	105	0.002	0.110	0.218	0.325	0.864	1.942
	120	0.004	0.112	0.219	0.327	0.866	1.944
Inverse Gaussian	7	0.027	0.135	0.242	0.350	0.888	1.964
	15	0.025	0.133	0.240	0.348	0.887	1.964
	30	0.023	0.131	0.239	0.347	0.885	1.962
	45	0.023	0.131	0.238	0.346	0.884	1.960
	60	0.023	0.131	0.238	0.346	0.885	1.961
	75	0.023	0.131	0.239	0.346	0.885	1.963
	90	0.023	0.130	0.238	0.346	0.885	1.962
	105	0.022	0.130	0.238	0.345	0.885	1.963
	120	0.021	0.129	0.237	0.345	0.884	1.961

Table 38: RQO prices under three different distributional assumptions for the rainfall

Table 39 reports the mean and standard deviation (std) for the option prices reported in Table 38 for a give strike a different maturities

Density	Statistic/Strike	K_0	$1.5K_0$	$2K_0$	$2.5K_0$	$5K_0$	$10K_0$
Log-Normal	Mean	0.1972	0.305	0.4127	0.5204	1.059	2.1361
	Std	0.0001	0.0001	0.0002	0.0003	0.0006	0.001191
Exponential	Mean	0.0022	0.1085	0.2161	0.3239	0.862420	1.939554
	Std	0.002	0.0047	0.0047	0.0047	0.0047	0.0048
Inverse Gaussian	Mean	0.0234	0.1312	0.2389	0.3466	0.88523	1.9623
	Std	0.0017	0.0016	0.0013	0.0015	0.0014	0.0013

Table 39: Statistics RQO prices

D.2 Sensitivity analysis RQO

In this section, we report the sensitivity of the RQO, with maturity of 120 days, to the model parameters for each different distributional assumption: Table 40 for the Exponential distribution, Table 41 for the Log-Normal distribution, and Table 42 for the Inverse Gaussian distribution

Par. Variation/Par.	λ_R	ω_R	β_R	λ_S	σ_S
-90%	2.175414	2.143250	2.136024	2.135491	2.149746
-75%	2.162721	2.148985	2.134671	2.135255	2.153926
-50%	2.167305	2.147744	2.134657	2.134944	2.148008
-25%	2.163034	2.137249	2.134730	2.135080	2.141799
0%	2.164182	2.141549	2.134309	2.135045	2.143345
+25%	2.173457	2.144776	2.133630	2.134957	2.144187
+50%	2.167212	2.139120	2.134485	2.135097	2.148696
+75%	2.161978	2.140887	2.135092	2.134964	2.147422
+100%	2.173742	2.140106	2.133604	2.134975	2.153665
Statistic/Par	λ_R	ω_R	β_R	λ_S	σ_S
Mean	2.167671	2.142629	2.134578	2.13509	2.147866
Std	0.005259	0.003927	0.000735	0.00018	0.004266

Table 40: Sensitivity for the model with exponentially distributed rainfall amount

Par. Variation/Par.	μ_R	σ_R	ω_R	β_R	λ_S	σ_S
-90%	1.126953	2.139790	1.849811	1.943968	1.933680	1.942742
-75%	0.787005	2.146869	1.850918	1.946070	1.944897	1.947204
-50%	0.907153	2.144356	1.839497	1.944198	1.937100	1.938277
-25%	0.911866	2.157669	1.834239	1.934892	1.936797	1.938052
0%	0.873479	2.155082	1.849204	1.945232	1.945389	1.945333
+25%	0.947025	2.146545	1.853201	1.942700	1.937381	1.928890
+50%	0.909559	2.140260	1.844581	1.941875	1.928417	1.931393
+75%	0.989759	2.154242	1.850452	1.928910	1.944035	1.942266
+100%	0.999791	2.143130	1.848557	1.946661	1.934995	1.940469
Statistic/Par	μ_R	σ_R	ω_R	β_R	λ_S	σ_S
Mean	0.939177	2.147549	1.846718	1.941612	1.938077	1.939403
Std	0.094654	0.006600	0.006176	0.005900	0.005709	0.006065

Table 41: Sensitivity for the model with Log-Normal distributed rainfall amount

Par. Variation/Par.	μ_R	ω_R	β_R	λ_S	σ_S
-90%	2.150531	1.887870	1.968475	1.959333	1.959595
-75%	2.161162	1.873903	1.964380	1.956839	1.958767
-50%	2.154111	1.884473	1.964688	1.958464	1.958762
-25%	2.148551	1.886685	1.974775	1.959727	1.957988
0%	2.168611	1.873123	1.951299	1.958173	1.959597
+25%	2.162394	1.883313	1.959173	1.959204	1.959044
+50%	2.148580	1.879721	1.948820	1.960070	1.958528
+75%	2.152625	1.868975	1.956337	1.958880	1.959166
+100%	2.145890	1.897430	1.964938	1.959909	1.959711
Statistic/Par	μ_R	ω_R	β_R	λ_S	σ_S
Mean	2.154717	1.881721	1.961432	1.958956	1.959017
Std	0.007660	0.008809	0.008307	0.001018	0.000570

Table 42: Sensitivity for the model with Inverse Gaussian distributed rainfall amount

D.3 Pricing BLCON

Table 43 reports the BLCON prices for strikes level, expressed in terms of $\log\left(\frac{K}{Y_{t_0}}\right)$, from -0.693147 to 0.693147, and maturities from 7 to 120 days.

Maturity/Strike	-0.693147	0.000000	0.405465	0.559616	0.693147
7	0.002725	0.011959	0.025854	0.034916	0.044238
15	0.021569	0.081639	0.129677	0.139953	0.146909
30	0.023490	0.086241	0.132728	0.142270	0.148679
45	0.022218	0.084905	0.131166	0.141303	0.147817
60	0.023654	0.084136	0.131073	0.140865	0.147753
75	0.022504	0.084730	0.131391	0.141430	0.148040
90	0.022293	0.085797	0.132128	0.142247	0.148785
105	0.023681	0.085052	0.131712	0.141415	0.147733
120	0.023766	0.086590	0.132402	0.141501	0.148517

Table 43: BLCON pricing

Table 44 reports the sensitivity of the BLCON price, with maturity of 120 days, to variation of the model parameters.

Par. Variation/Par.	k	λ_L	σ_L	$\sigma_{L,z}$	λ_S	$\sigma_{S,z}$
-90%	0.101135	0.168527	0.148595	0.171224	0.147249	0.147849
-75%	0.122347	0.165209	0.148982	0.170989	0.147918	0.148213
-50%	0.136172	0.158857	0.148377	0.166193	0.147390	0.148511
-25%	0.143153	0.153373	0.148273	0.157434	0.148349	0.150036
0%	0.148855	0.147742	0.148314	0.147928	0.149175	0.147553
+25%	0.152628	0.144578	0.147949	0.140267	0.147808	0.149393
+50%	0.153450	0.139682	0.149861	0.137052	0.148170	0.149074
+75%	0.155105	0.136676	0.147702	0.132389	0.149774	0.148344
+100%	0.156770	0.135602	0.147520	0.127001	0.148457	0.147203
Statistic/Par.	k	λ_L	σ_L	$\sigma_{L,z}$	λ_S	$\sigma_{S,z}$
Mean	0.141068	0.150027	0.148397	0.150053	0.148254	0.148464
Std	0.018590	0.012197	0.000707	0.017008	0.000813	0.000907

Table 44: BLCON sensitivity

D.4 The central limit theorem

In subsection 3.8.4 we have defined the variable $V = \frac{\pi-\pi_0}{\sqrt{\frac{\pi(1-\pi)}{n}}}$ for the application of the central limit theorem. However, in this case the classic Central Limit Theorem cannot be applied since our variables aren't i.i.d.. So, to construct the test we relied on the following version that can be found in Jacod et al. 2012 at page 235.

Theorem D.1 (Martingale Central Limit Theorem)**.** Let $(X_n)_{n\geq 1}$ be a sequence of random variables satisfying:

(i) $\mathbb{E}\{X_n \mid \mathcal{F}_{n-1}\} = 0$

(ii) $\mathbb{E}\{X_n^2 \mid \mathcal{F}_{n-1}\} = 1$

(iii) $\mathbb{E}\{|X_n|^3 \mid \mathcal{F}_{n-1}\} \leq K < \infty$

Let $S_n = \sum_{i=1}^n X_i$ and $S_0 = 0$. Then,

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}} S_n = Z,$$

where $Z \sim N(0,1)$ and the convergence is in distribution.

References

Agliardi, Elettra and Rossella Agliardi (2021). “Pricing climate-related risks in the bond market”. In: *Journal of Financial Stability* 54, p. 100868. ISSN: 1572-3089. DOI: `https://doi.org/10.1016/j.jfs.2021.100868`. URL: `https://www.sciencedirect.com/science/article/pii/S1572308921000279`.

Aïd, René and Sara Biagini (2023). “Optimal dynamic regulation of carbon emissions market”. In: *Mathematical Finance* 33.1, pp. 80–115.

Aït-Sahalia, Yacine and Robert Kimmel (2007). “Maximum likelihood estimation of stochastic volatility models”. In: *Journal of Financial Economics* 83.2, pp. 413–452. ISSN: 0304-405X.

Alfonsi, Aurélien and Nerea Vadillo (2023). *A stochastic volatility model for the valuation of temperature derivatives.* arXiv: `2209.05918 [q-fin.RM]`.

Allman, Elsa (2021). “Pricing Climate Change Risk in Corporate Bonds”. In: *French Banking Supervisory Authority.*

Arrow, Kenneth J and Gerard Debreu (1954). “Existence of an equilibrium for a competitive economy”. In: *Econometrica: Journal of the Econometric Society*, pp. 265–290.

Artemova, Mariia, Francisco Blasques, Janneke van Brummelen, and Siem Jan Koopman (2022). “Score-driven models: Methodology and theory”. In: *Oxford Research Encyclopedia of Economics and Finance.*

Asif, Zunaira, Zhi Chen, Rehan Sadiq, and Yinying Zhu (2023). “Climate change impacts on water resources and sustainable water management strategies in North America”. In: *Water Resources Management* 37.6, pp. 2771–2786.

Av, Manjunatha, Sreejith Aravindakshan, Amjath Babu TS, and Puran Mal (2016). “Impact of informal groundwater markets on efficiency of irrigated farms in India: a bootstrap data envelopment analysis approach”. In: *Irrigation science* 34, pp. 41–52.

Azar, José, Miguel Duro, Igor Kadach, and Gaizka Ormazabal (2021). “The big three and corporate carbon emissions around the world”. In: *Journal of Financial Economics* 142.2, pp. 674–696.

Azencott, Robert, Peng Ren, and Ilya Timofeyev (2020). *Realized volatility and parametric estimation of Heston SDEs.* arXiv: `1706.04566 [q-fin.CP]`.

Bajaj, Akshi, SP Singh, and Diptimayee Nayak (2022). “Impact of water markets on equity and efficiency in irrigation water use: a systematic review and meta-analysis”. In: *Agricultural Water Management* 259, p. 107182.

Balcılar, Mehmet, Rıza Demirer, Shawkat Hammoudeh, and Duc Khuong Nguyen (2016). “Risk spillovers across the energy and carbon markets and hedging strategies for carbon risk”. In: *Energy Economics* 54, pp. 159–172.

Bartolini, Nicola, Silvia Romagnoli, and Amia Santini (2024). “A climate risk hedge ? Investigating the exposure of green and non-green corporate bonds to climte risk”. In: *Working Paper.*

Bats, Joost, Giovanna Bua, and Daniel Kapp (2023). “Physical and transition risk premiums in euro area corporate bond markets”. In: *De Nederlandsche Bank Working Paper* 761.

Battiston, Petr Jakubik, Irene Monasterolo, Keywan Riahi, and Bas Van Ruijven (2019). “Climate risk assessment of sovereign bonds’ portfolio of European insurers”. In: *Available at SSRN Electronic Journal.*

Battiston, Stefano, Mandel, Monasterolo Irene, Schütze Franziska, and Visentin Gabriele (2017). “A Climate stress-test of the financial system”. In: *Nature Climate Change* 7.4, pp. 283–288.

Benth, Troels S Christensen, and Victor Rohde (2021). “Multivariate continuous-time modeling of wind indexes and hedging of wind risk”. In: *Quantitative Finance* 21.1, pp. 165–183.

Benth, Fred Espen and Jūratė šaltytė Benth (2007). “The volatility of temperature and pricing of weather derivatives”. In: *Quantitative Finance* 7.5, pp. 553–561.

Benth, Fred Espen, Jūratė Šaltytė Benth, and Steen Koekebakker (2007). “Putting a Price on Temperature*”. In: *Scandinavian Journal of Statistics* 34.4, pp. 746–767.

Benth, Fred Espen and Jūratė Šaltytė Benth (2011). “Weather derivatives and stochastic modelling of temperature”. In: *International Journal of Stochastic Analysis* 2011, pp. 1–21.

Benth, Fred Espen and Jūratė Šaltytė-Benth (2005). “Stochastic modelling of temperature variations with a view towards weather derivatives”. In: *Applied Mathematical Finance* 12.1, pp. 53–85.

Benth, Di Persio, and Lavagnini (2018). “Stochastic modeling of wind derivatives in energy markets”. In: *Risks* 6.2, p. 56.

Bingler, Julia Anna, Mathias Kraus, and Markus Leippold (2021). “Cheap Talk and Cherry-Picking: What ClimateBert has to say on Corporate Climate Risk Disclosures”. In: *Available at SSRN.*

Björk, Tomas (2009). *Arbitrage theory in continuous time.* Oxford university press.

Black, Fischer (1976). “The pricing of commodity contracts”. In: *Journal of financial economics* 3.1-2, pp. 167–179.

Blasberg, Alexander, Rüdiger Kiesel, and Luca Taschini (2021). “Carbon default swap–disentangling the exposure to carbon risk through CDS”. In: *Available at SSRN 3856993*.

Blasques, Francisco, Siem Jan Koopman, and Andre Lucas (2014). *Maximum likelihood estimation for generalized autoregressive score models*. Tech. rep. Tinbergen Institute Discussion Paper.

Blazsek, Szabolcs, Helmuth Chavez, and Carlos Mendez (2016). “Model stability and forecast performance of Beta-t-EGARCH”. In: *Applied Economics Letters* 23.17, pp. 1219–1223.

Bolton, Patrick, Morgan Depres, Luiz Awazu Pereira da Silva, Frédéric Samama, and Romain Svartzman (2020). *The green swan*. URL: https://www.bis.org/publ/othp31.pdf?fbclid=IwAR1DBQv77u8NWV44lhZRzxOjh9JGyp7RUKBFxP_nQvIYayqpZOQ_roacfBw.

Bolton, Patrick and Marcin T. Kacperczyk (2019). “Do Investors Care about Carbon Risk?” In: *Journal of Financial Economics*.

Brace, Alan, Dariusz Gatarek, and Marek Musiela (1997). “The market model of interest rate dynamics”. In: *Mathematical finance* 7.2, pp. 127–155.

Brewer, Jedidiah, Robert Glennon, Alan Ker, and Gary D Libecap (2007). *Water markets in the west: prices, trading, and contractual forms*.

Brody, Dorje C, Joanna Syroka, and Mihail Zervos (2002). “Dynamical pricing of weather derivatives”. In: *Quantitative finance* 2.3, p. 189.

Browne, Oliver R and Xinde James Ji (2023). “The economic value of clarifying property rights: Evidence from water in Idaho’s Snake River Basin”. In: *Journal of Environmental Economics and Management* 119, p. 102799.

Brozovic, Nicholas, Janis M Carey, and David L Sunding (2002). “Trading activity in an informal agricultural water market: an example from California”. In: *Journal of Contemporary Water Research and Education* 121.1, p. 2.

Burnecki, Krzysztof, Grzegorz Kukla, and David Taylor (2011). “Pricing of catastrophe bonds”. In: *Statistical tools for finance and insurance*. Springer, pp. 371–391.

Carney, Mark (2015). “Breaking the Tragedy of the Horizon–climate change and financial stability”. In: *Speech given at Lloyd’s of London by the Governor of the Bank of England* 29.

Catania, Leopoldo and Nima Nonejad (2020). “Density forecasts and the leverage effect: Evidence from observation and parameter-driven volatility models”. In: *The European Journal of Finance* 26.2-3, pp. 100–118.

Chen, Yufeng, Fang Qu, Wenqi Li, and Minghui Chen (2019). “Volatility spillover and dynamic correlation between the carbon market and energy markets”. In: *Journal of business economics and management* 20.5, pp. 979–999.

Chevallier, Julien, Duc Khuong Nguyen, and Juan Carlos Reboredo (2019). “A conditional dependence approach to CO2-energy price relationships”. In: *Energy Economics* 81, pp. 812–821.

Chuliá, Helena, Dolores Furió, and Jorge M Uribe (2019). “Volatility spillovers in energy markets”. In: *The Energy Journal* 40.3, pp. 173–198.

Clerc, Laurent, Anne Lise Bontemps-Chanel, Sebastien Diot, George Overton, Solene Soares de Albergaria, Lucas Vernet, and Maxime Louardi (2021). “A first assessment of financial risks stemming from climate change. The main results of the 2020 climate pilot exercise”. In: *Analyses Et Syntheses* 122.

Cox, John C, Jonathan E Ingersoll Jr, and Stephen A Ross (2005). “A theory of the term structure of interest rates”. In: *Theory of valuation*. World Scientific, pp. 129–164.

Creal, Drew, Siem Jan Koopman, and André Lucas (2013). “Generalized autoregressive score models with applications”. In: *Journal of Applied Econometrics* 28.5, pp. 777–795.

Dhakal, Nirajan, Sergio G Salinas-Rodriguez, Jamal Hamdani, Almotasembellah Abushaban, Hassan Sawalha, Jan C Schippers, and Maria D Kennedy (2022). “Is desalination a solution to freshwater scarcity in developing countries?” In: *Membranes* 12.4, p. 381.

Dolan, Flannery, Jonathan Lamontagne, Robert Link, Mohamad Hejazi, Patrick Reed, and Jae Edmonds (2021). “Evaluating the economic impact of water scarcity in a changing world”. In: *Nature communications* 12.1, pp. 1–10.

Driessen, Joost (2005). “Is default event risk priced in corporate bonds?” In: *The Review of Financial Studies* 18.1, pp. 165–195.

Duffie, Darrell and Kenneth J. Singleton (1999). “Modeling term structures of defaultable bonds”. In: *The review of financial studies* 12.4, pp. 687–720.

Dutta, Anupam, Elie Bouri, and Md Hasib Noor (2018). “Return and volatility linkages between CO2 emission and clean energy stock prices”. In: *Energy* 164, pp. 803–810.

Engle, Robert F. (1982). “Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation”. In: *Econometrica* 50.4, pp. 987–1007. ISSN: 00129682, 14680262. DOI: 10.2307/1912773.

Engle, Robert F., Stefano Giglio, Bryan T. Kelly, Heebum Lee, and Johannes Stroebel (2020). “Hedging climate change news ?” In: *The Review of Financial Studies* 33, pp. 1184–1216.

Faccini, Renato, Rastin Matin, and George S Skiadopoulos (2021). “Dissecting Climate Risks: Are they Reflected in Stock Prices?” In: *Available at SSRN 3795964*.

Garcia-Jorcano, Laura, Juan-Angel Jimenez-Martin, and M-Dolores Robles (2022). “Carbon dioxide risk exposure: Co2Risk”. In: *Climate Risk Management* 36, p. 100435.

Garrick, Dustin, Mark A Siebentritt, Bruce Aylward, Carl J Bauer, and Andrew Purkey (2009). “Water markets and freshwater ecosystem services: Policy reform and implementation in the Columbia and Murray-Darling Basins”. In: *Ecological Economics* 69.2, pp. 366–379.

Geman, Hélyette and Alois Kanyinda (2007). “Water as the next commodity”. In: *The Journal of Alternative Investments* 10.2, p. 23.

Geman, Hélyette and Marie-Pascale Leonardi (2005). “Alternative approaches to weather derivatives pricing”. In: *Managerial Finance* 31.6, pp. 46–72.

Geman, Hélyette and Marc Yor (1993). “Bessel processes, Asian options, and perpetuities”. In: *Mathematical finance* 3.4, pp. 349–375.

Gourieroux, Christian, Alain Monfort, and Eric Renault (1993). “Indirect inference”. In: *Journal of applied econometrics* 8.S1, S85–S118.

Groll, Andreas, Brenda López-Cabrera, and Thilo Meyer-Brandis (2016). “A consistent two-factor model for pricing temperature derivatives”. In: *Energy Economics* 55, pp. 112–126.

Gyamerah, Samuel Asante, Philip Ngare, and Dennis Ikpe (2018). “Regime-switching temperature dynamics model for weather derivatives”. In: *International Journal of Stochastic Analysis* 2018, pp. 1–15.

Hagan, Patrick and Andrew Lesniewski (2008). “LIBOR market model with SABR style stochastic volatility”. In: *JP Morgan Chase and Ellington Management Group* 32, p. 57.

Hanif, Waqas, Jose Arreola Hernandez, Walid Mensi, Sang Hoon Kang, Gazi Salah Uddin, and Seong-Min Yoon (2021). “Nonlinear dependence and connectedness between clean/renewable energy sector equity and European emission allowance prices”. In: *Energy Economics* 101, p. 105409.

Härdle, Wolfgang Karl and Brenda López Cabrera (2012). “The implied market price of weather risk”. In: *Applied Mathematical Finance* 19.1, pp. 59–95.

Hartman, Preston, Travis Gliedt, Jeffrey Widener, and Rebecca W Loraamm (2017). “Dynamic capabilities for water system transitions in Oklahoma”. In: *Environmental Innovation and Societal Transitions* 25, pp. 64–81.

Harvey, Andrew and Genaro Sucarrat (2014). “EGARCH models with fat tails, skewness and leverage”. In: *Computational Statistics & Data Analysis* 76, pp. 320–338.

Hess, Markus (2018). “Pricing temperature derivatives under weather forecasts”. In: *International Journal of Theoretical and Applied Finance* 21.05, p. 1850031.

Heston, Steven L. (1993). “A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options”. In: *The Review of Financial Studies* 6.2, pp. 327–343. ISSN: 08939454, 14657368. (Visited on 05/17/2024).

Howitt, Richard and Ellen Hanak (2005). “Incremental water market development: The California water sector 1985-2004”. In: *Canadian Water Resources Journal* 30.1, pp. 73–82.

Hristov, Jordan, Jesus Barreiro-Hurle, Guna Salputra, Maria Blanco, and Peter Witzke (2021). “Reuse of treated water in European agriculture: Potential to address water scarcity under climate change”. In: *Agricultural Water Management* 251.

Hsu, Po-hsuan, Kai Li, and Chi-yang Tsou (2023). “The pollution premium”. In: *The Journal of Finance* 78.3, pp. 1343–1392.

Po-Hsuan, Hsu, Li Kai, and Tsou Chi-Yang (2023). “The Pollution Premium”. In: *The Journal of Finance* 78 (3), pp. 1343–1392. DOI: 10.1111/jofi.13217.

Hung, Fengwei, Kyongho Son, and YC Ethan Yang (2022). “Investigating uncertainties in human adaptation and their impacts on water scarcity in the Colorado river Basin, United States”. In: *Journal of Hydrology* 612.

IPCC (2021). *Climate Change 2021: The Physical Science Basis*. IPCC. URL: https://www.ipcc.ch/report/sixth-assessment-report-working-group-i/.

Jacod, Jean and Philip Protter (2012). *Probability essentials*. Springer Science & Business Media.

Kanwal, Memoona and Hashim Khan (2021). “Analyzing Dependence Structure Between Carbon Market and Energy Commodities: Evidence from Copula Approach”. In: *Pakistan Journal of Social Sciences* 41.4, pp. 931–943.

Koenker, Roger and Gilbert Bassett Jr (1978). "Regression quantiles". In: *Econometrica: journal of the Econometric Society*, pp. 33–50.

Kolbel, Julian F, Markus Leippold, Jordi Rillaerst, and Qian Wang (2020). "Ask BERT: How Regulatory Disclosure of Transition and Physical Climate Risks affects the CDS Term Structure". In: *Available at SSRN*.

Livieri, Giulia, Davide Radi, and Elia Smaniotto (2023). *Pricing Transition Risk with a Jump-Diffusion Credit Risk Model: Evidences from the CDS market*. arXiv: 2303.12483 [q-fin.PR].

Lovcha, Yuliya, Alejandro Perez-Laborda, and Iryna Sikora (2022). "The determinants of CO2 prices in the EU emission trading system". In: *Applied Energy* 305, p. 117903.

McNeil, Alexander J., Rüdiger Frey, and Paul Embrechts (2015). *Quantitative risk management*.

Meier, Jan-Hendrik and Norman Voss (2020). "Do Commodities Determine the EU Emission Allowances Price?" In: *ICTERI Workshops*, pp. 423–437.

Merton, Robert C. (1974). "On the pricing of corporate debt: The risk structure of interest rates". In: *The Journal of finance* 29.2, pp. 449–470.

Molden, David (2020). "Scarcity of water or scarcity of management?" In: *International Journal of Water Resources Development* 36.2-3, pp. 258–268.

Monasterolo, Irene, Stefano Battiston, Anthony C Janetos, and Zoey Zheng (2017). "Vulnerable yet relevant: the two dimensions of climate-related financial disclosure". In: *Climate Change* 145, pp. 497–507.

Musiela, Marek and Marek Rutkowski (1997). "Continuous-time term structure models: Forward measure approach". In: *Finance and Stochastics* 1, pp. 261–291.

Nowak, Piotr and Maciej Romaniuk (2013). "Pricing and simulations of catastrophe bonds". In: *Insurance: Mathematics and Economics* 52.1, pp. 18–28.

Razzaq, Amar, Ping Qing, Muhammad Abid, Mumtaz Anwar, Iqbal Javed, et al. (2019). "Can the informal groundwater markets improve water use efficiency and equity? Evidence from a semi-arid region of Pakistan". In: *Science of the total environment* 666, pp. 849–857.

Rocha-Arteaga, Alfonso and Ken-iti Sato (2001). "Topics in infinitely divisible distributions and Lévy processes". In: *Communicaciónes del CIMAT, Guanajuato*.

— (2019). *Topics in infinitely divisible distributions and Lévy processes*. Springer Nature.

Roncoroni, Alan, Stefano Battiston, Luis Onesimo Leonardo Escobar-Farfan, and Serafin Martinez Jaramillo (2021). "Climate risk and financial stability in the network of banks and investment funds". In: *Journal of Financial Stability* 54.

Rosa, Lorenzo, Davide Danilo Chiarelli, Maria Cristina Rulli, Jampel Dell'Angelo, and Paolo D'Odorico (2020). "Global agricultural economic water scarcity". In: *Science Advances* 6.18.

Rutkowski, Marek and Tomasz R. Bielecki (2004). *Credit Risk: Modelling, Valuation and Hedging*.

Šaltytė Benth, Jūratė and Fred Espen Benth (2012). "A critical view on temperature modelling for application in weather derivatives markets". In: *Energy Economics* 34.2, pp. 592–602.

Schwabe, Kurt, Mehdi Nemati, Clay Landry, and Grant Zimmerman (2020). "Water markets in the Western United States: Trends and opportunities". In: *Water* 12.1, p. 233.

Seidl, Constantin, Wheeler, Ann Sarah, and Alec Zuo (2020). "Treating water markets like stock markets: Key water market reform lessons in the Murray-Darling Basin". In: *Journal of Hydrology* 581, p. 124399.

Stulec, Ivana, Kristina Petljak, and Tomislav Bakovic (2016). "Effectiveness of weather derivatives as a hedge against the weather risk in agriculture." In: *Agricultural Economics/Zemědělská Ekonomika* 62.8.

Tankov, Peter (2003). *Financial modelling with jump processes*. CRC press.

Tzanakakis, Vasileios A, Nikolaos V Paranychianakis, and Andreas N Angelakis (2020). *Water supply and water scarcity*.

Ungureanu, Nicoleta, Valentin Vlăduț, and Gheorghe Voicu (2020). "Water scarcity and wastewater reuse in crop irrigation". In: *Sustainability* 12.21.

Vaugirard, Victor E (2003). "Pricing catastrophe bonds by an arbitrage approach". In: *The Quarterly Review of Economics and Finance* 43.1, pp. 119–132.

Vermuelen, Robert, Edo Schets, Melanie Lohuis, Barbara Kolbl, David-Jan Jansen, and Willelm Heeringa (2021). "The Heat Is on: A Framework for Measuring Financial Stress Under Disruptive Energy Transition Scenarios". In: *Ecological Economics* 190.

Wight, Charles, Kyle Garmany, Eugenio Arima, and Dustin Garrick (2024). "Texas water markets: Understanding their trends, drivers, and future potential". In: *Ecological Economics* 224, p. 108259.