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INVESTIGATING THE LEARNING POTENTIAL OF THE SECOND QUANTUM REVOLUTION: DEVELOPMENT OF AN APPROACH FOR SECONDARY SCHOOL STUDENTS

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Abstract

In recent years we have witnessed important changes: the Second Quantum Revolution is in the spotlight of many countries, and it is creating a new generation of technologies that seem to meet societal challenges.
To stay at the forefront and unlock the potential of the Second Quantum Revolution, several countries have launched strategic plans and research programs that finance and set the pace of research and development of these new technologies. For example, the European Union launched the Quantum Flagship with an initial investment of €1 billion in 2018 that has now reached €7 billion; the China, after several investments in the past years, declared quantum technologies one of the new high techs in its 14th Five-Year Plan (2021-2025); in the U.S. the National Quantum Initiative Act has been launched and big companies (such as IBM and Google) have already achieved significant results.

The increasing pace of technological changes is also challenging science education and institutional systems requiring them to help to create synergies between companies, universities, and schools, to expand the workforce by running programs to recruit more future experts, to involve more students in STEM carriers and to promote quantum literacy of citizenship.

The present thesis is placed within physics and science education research and contribute to the political and institutional calls by developing an approach and a course on the theme of the Second Quantum Revolution. The aim of the approach is to promote quantum literacy and, in particular, to value from a cultural and educational perspective the Second Revolution and the emergent technologies.

The dissertation is articulated in two parts. In the first one, we unpack the Second Quantum Revolution from a cultural perspective and shed light on the main revolutionary aspects. These aspects are then elevated to the rank of principles, which are implemented for the design of a course targeted at secondary school students, and prospective and in-service teachers.
Together with a detailed description of the design process and the educational reconstruction of the activities of the course, we present the results of a pilot study conducted to investigate the impact of the approach on students' understanding as well as to gather feedback and reactions to refine and improve the instructional materials.

The second part is dedicated to the exploration of the Second Quantum Revolution and the emergent technologies as a context to introduce some basic concepts of quantum physics. In particular, we present the implementation carried out with secondary school students to explore if and to what extent external representations could play any role to promote students' understanding and the acceptance of quantum physics as a personal reliable description of the world we live in.

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Introduction

Dowling and Milburn in 2003 wrote:

> "*We are currently in the midst of a second quantum revolution*" (p.1655).

These words, whose scope 20 years ago was grasped mainly by a small number of experts, are increasingly permeated and have begun to give a new shape to the society we live in. In recent years, the revolution seems to have really broken out, in other words, it has triggered a process that is leading to a radical change in economic, social, and political structures as well as in particular sectors of activity. This process is not only changing these structures but also setting the pace, namely the time with which change must take place.

The pace of science and technology today is the pace of what Rosa called the "society of acceleration" (2010; 2013). In Rosa's view, there are three main dimensions, not independent of each other, of acceleration: technical acceleration, social acceleration (i.e., changes in the institutions through which we bring order to our lives), and acceleration in the pace of life. Technical acceleration is the main driver of global dynamics. And it is precisely the technical acceleration that gives the time of the second quantum revolution and that has mobilized many countries of the world, which have taken and are taking actions to align and keep the required pace of change.

In this landscape, it becomes vital to expand the workforce and the experts working in the field; get quantum mechanics out of the physics departments and create new professional figures such as quantum engineers, quantum computer scientists, and so on. It is also important to create new ecosystems and synergies between universities, enterprises, and schools; to convince new generations to choose STEM careers, and to promote a citizenship's quantum literacy. The attempt to put in communication, in the sense of creating spaces for dialogue, and to phase and align so different systems in terms of values, purposes, times of change, and practices so as to establish fruitful collaboration are among the biggest challenges that are being faced.
These challenges are set and at the heart of important initiatives and research programs all over the world such as the Quantum Flagship in Europe, the National Quantum Initiative Act in the U.S., the UK national quantum technology program, and many others.
However, the continuous pursuit of technical acceleration times, the attempt to align such different systems with demands "from above" (political and institutional), and the constant requests to act within the prescribed time risk obscuring the cultural scope of the revolution.

This thesis is placed within the research in physics and science education. Its highest aim and value is to try to embrace the political and institutional calls launched to the world of science education. At the same time, it aims to shed light and unpack the cultural significance of the Second Quantum Revolution from an educational perspective.
There is therefore an intrinsic tension that is perfectly described by the Latin motto "Festina Lente" or to "make haste slowly". In the original meaning of the motto, slowness ("lente") was also associated with caution. Slowness is, in this case, closely linked to a time factor: taking time to understand the cultural and conceptual scope of this revolution that is "completely overturn[ing] our previous pictures of nature and [it] will doubtless give rise to a range of new technologies that will simply look like magic" (Gisin, 2014, p.xiv).
The intrinsic tension lies in contributing to the calls by developing an approach and course for secondary school students on the Second Quantum Revolution and the emergent technologies as well as not missing the opportunity to reflect on the cultural and conceptual scope of it. The latter can be one of the contributions and roles of research in science education, that is to *distill* the content to obtain its essence. Where by essence, in an educational perspective, we mean all those aspects that

have educational potential, that provide glimpses of the world, that can be transformed into lenses to understand the society we live in, its pace, and its future orientations.
Behind this idea, there is an image of learning and teaching science. First of all, the teaching of scientific disciplines has both a cultural and an educational role. Secondly, it is possible to design learning environments that allow each student to place their learning in a broader perspective of personal, emotional, and intellectual growth. Finally, it is possible to design paths that return an inter-disciplinary authenticity of what we are learning, which allows a personal approach to knowledge and triggers an authentic process of internalization able to give shape to new and personal patterns of meaning (Levrini, Levin & Fantini, 2020; Levrini, Levin & Fantini, 2017; Levrini, Fantini, Tasquier, Pecori & Levin, 2015; Fantini, 2014; Levrini & Fantini, 2013). These are perhaps the most meaningful aspects that I have had the opportunity to learn and implement in these years of work with the research group in physics education at the University of Bologna.

Operationally these general observations are translated into the identification of some aspects and ideas elevated to design principles. The work on the Second Quantum Revolution follows a long and deep work that the research group has carried out on the First Quantum Revolution. So we did not start from scratch, some general observations have been already embodied in some constructs that the group has developed such as the *properly complex territories* (Levrini & Fantini, 2013), the *appropriation* (Levrini, Fantini, Tasquier, Pecori & Levin, 2015) and the *disciplinary authenticity* (Kapon, Laherto & Levrini, 2018).

The thesis consists of two different parts that are both rooted in physics education research. The first one is more design-oriented in which I present the approach to the Second Quantum Revolution we have developed. The second one is based on other research methods and practices, namely the formulation of a conjecture, testing it, and the analysis of the collected data.

In the first part of the thesis, after taking stock of the educational challenges and the status of literature, the Second Quantum Revolution is unpacked from a cultural perspective (Chapter 1). In other words, we try to disarticulate what Dowling and Milburn (2003) identified as the core of the paradigm shift from the First to the Second Quantum Revolution:

> "*The first quantum revolution gave us new rules that govern physical reality. The second quantum revolution will take these rules and use them to develop new technologies*." (p.1655)

Individuated what for us are the essential aspects that characterize the Second Quantum Revolution compared to the First one, we elevate them at the rank of design principles, intertwining them with two constructs elaborated by the research group of Bologna and some collaborators: the *properly complex territories* (Levrini & Fantini, 2013) and the *(inter-)disciplinary authenticity* (Kapon, Laherto & Levrini, 2018). The contexts in which the course was developed and, then, refined contributed to enriching the design elements. We point out seven design principles that touch different dimensions, from the heart of the Second Revolution to the interdisciplinary aspects, from the future-oriented dimension to the need of finding new languages and aesthetics, even outside scientific ones, to talk about it authentically (Chapter 2).
This approach is used to design a course on the Second Quantum Revolution. The course was implemented different times both with secondary school students as an extra-curricular course and with prospective and in-service teachers. In particular, two activities (one on the teleportation protocol and one on the random walk) are presented as particularly representative activities of the approach. During the second implementation, a pilot study was conducted and analyzed to investigate the impact of the approach on students' understanding and to gather feedback and reactions to refine and improve the course (Chapter 3). The structure of the first part of the thesis is presented in figure 1.

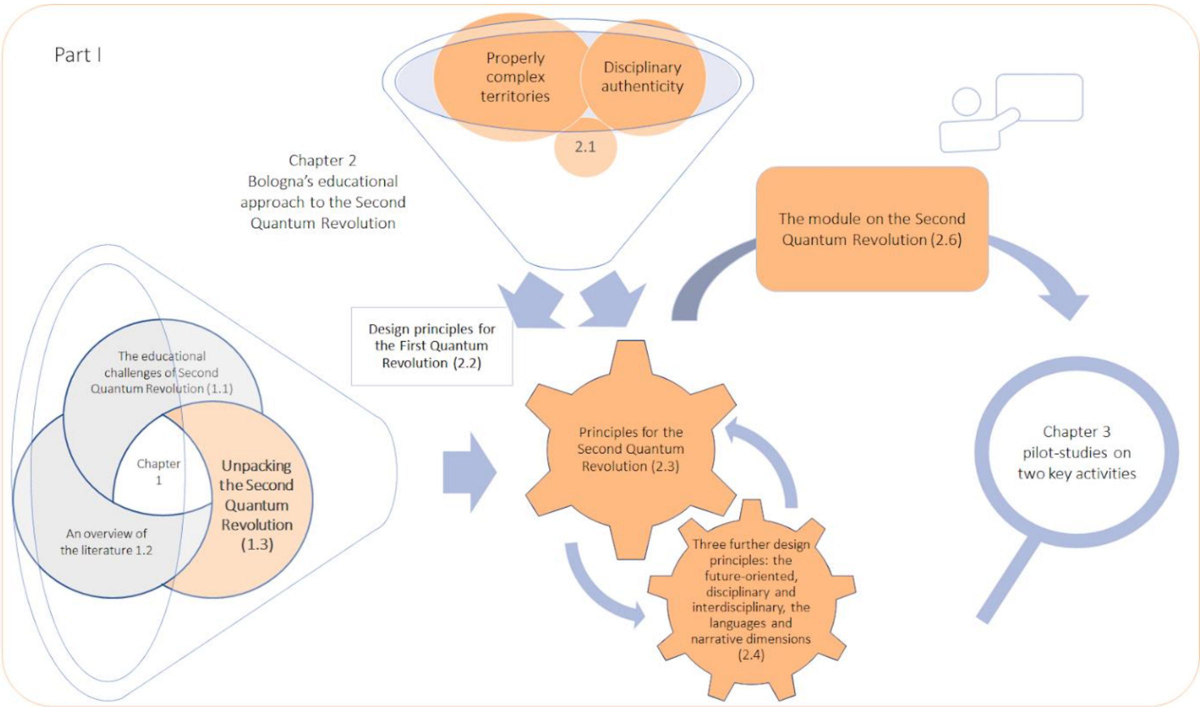

Fig. 1: structure of Part I

The second part is dedicated to investigating if and to what extent quantum technologies are a context to introduce some basic quantum physics concepts - quantum state and superposition principle, state manipulation and evolution, measurement, and entanglement (Chapter 4). This led us to re-consider the problem of *acceptance of quantum physics* (Ravaioli, 2020; Levrini & Fantini, 2013) as an adequate and personally reliable explanation of reality. We have decided to follow this line since from the first implementation of the module with secondary school students (Satanassi et al., 2022; Satanassi et al., 2021; Satanassi, 2020), we have noticed that there was "something" in the approach to quantum physics through quantum technologies that seemed to position the students in a different mood with respect to quantum physics and/or foster a positive attitude toward quantum physics that we had never observed in the previous implementations. This idea led us to reflect on the differences in the teaching approaches. In particular, it paved the way for the possibility that the multiple representations set, and, in particular, the logical and circuital representations (algorithms, circuits, quantum logic gates, and truth tables) could provide a new synthetic scenario able to satisfy students' epistemic needs: the need to recognize and conceptualize the relationship between the "real world we live in" and the abstract mathematical construct, the need for visualization, the need for comparability. These three epistemic needs were pointed out in previous research work of the group (Ravaioli, 2020; Levrini & Fantini, 2013) as needs that could be not satisfied by our previous modules. This conjecture was tested through a three-phase experiment carried out in October 2021. The experiment involved students of different classes in the last year of high school. The experiment consisted of a teamwork activity and the following focus group with a selected sample of students. A final collective interview was conducted as a result of the "Quantum Atelier" experience that involved almost the same sample of students and a group of teachers. A synthetic structure of the second part is reported in figure 2.

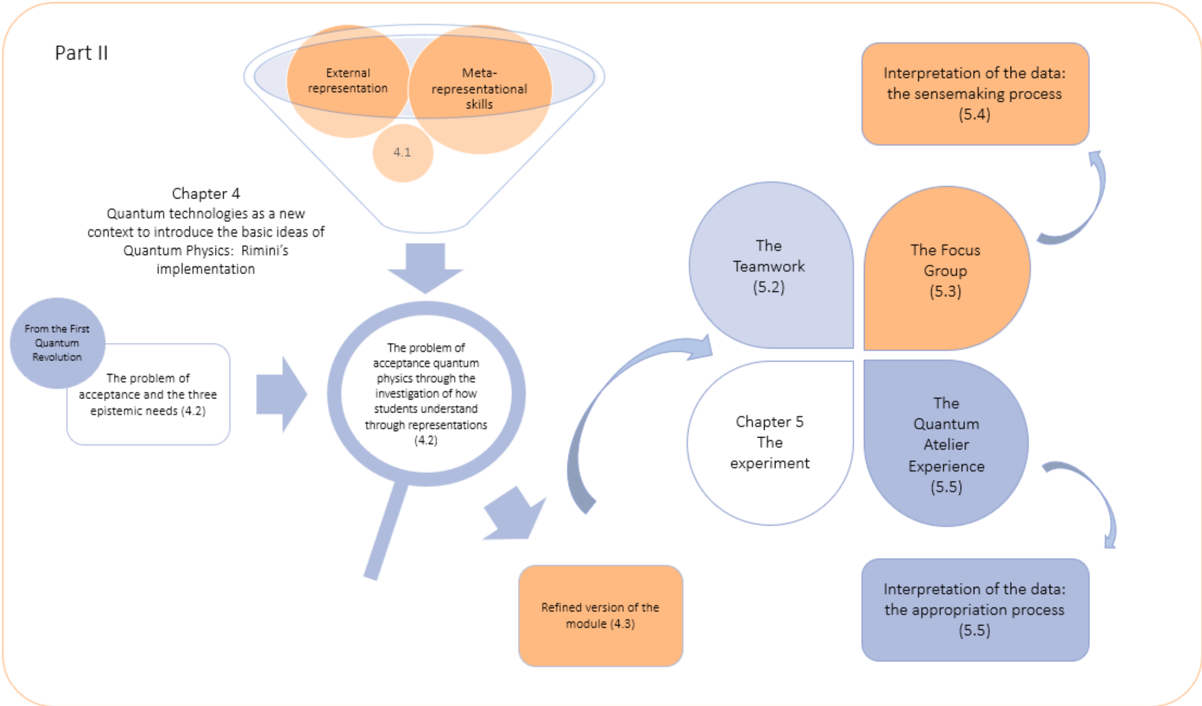

Fig. 2: Structure of Part II.

The two pictures aim to orient the reader throughout the thesis. Per se, the structure of the thesis has its own linearity, but many are the constructs and the frameworks we refer to. So, the figures have been built to help the reader graphically navigate the most important constructs and the frameworks as well as keep an eye on how we intertwined them and the phases of the process.

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Acknowledgments

“Ogni inizio infatti è solo un seguito e il libro degli eventi è sempre aperto a metà”.
“Every beginning is only a sequel, after all, and the book of events is always open halfway through.”
Wisława Szymborska

Here we are, at the end of an unfinished path of a “Love at first sight”. In the wake of the dissertation motto "Festina Lente", or rather, quoting one of my favourite poetesses, taking a “while” and “bookmarking” this while, I would like to spend a few words to describe what this path and this job represent from my perspective. It is a job as others, made of deadlines and commitments as much as life and people; a job that has its own complexity but, at the same time, a job in which the different parts blend into each other, in which it is no longer so clear what the boundaries are and where they are. It is like a topological node, where there is no more the space-temporal structure and, at that precise node, different relational structures converge making not easy to recognize their belonging. “Everything disappears” into that single node and, at the same time, appears to take new shapes. It is a job, made of deadlines and commitments as much as life and people, which has an unimaginable transformative power until you are inside, until you experience it in all its complexity. The node has not a relevant meaning by itself, but it gains sense within a network. And it is the whole network that I would like to deeply thank.

The first acknowledgment goes to the reviewers, Shulamit Kapon and Andrew Elby, for the time they dedicated to reading my work and for their precious comments and words that not only help me to improve the work but also provide me with new insights into this kind of research.

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To the person who guided me for 4 years in its making, Olivia Levrini. You guided me along this not easy path, structuring it and leading me at its pace and, at the same time, fostering and leaving me free (or better, setting me free) to explore and build my research identities according to what I like, my values, and my inclinations. In this “Tightening the rope and letting it loosen”, structuring and deconstructing, you gave the meaning of “feeling the direction” in the work and life (and many other).

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Part I

Chapter 1: The Second Quantum Revolution and Science Education

Introduction

In the present chapter, I present an overview of the Second Quantum Revolution in its complexity.
I firstly introduce a more political and institutional perspective and the main educational programs and actions that was born to respond to the calls launched to science education. The discussion focuses more on the European perspective and the project QTEdu that involved us (section 1.1).
I make then, from an educational perspective, an overview of the literature about the most popular textbooks, instructional materials and approaches to quantum technologies that were developed (section 1.2). I present how the work we carried out in these years and this thesis are place within this overview and the research questions to which we try to contribute. In front of the pace of change and the political and institutional calls to keep this pace, we felt the need to stop and reflect on the cultural meaning of the second quantum revolution. The questions that have guide the present work are:

> *RQ1. Which teaching approach can be developed to underlie the educational potential of technologies even beyond technical training?*
> *RQ2. What instructional design principles can be pointed out to value the Second Quantum Revolution as a cultural revolution?* (Satanassi, Ercolessi & Levrini, 2022)

Finally, before going into the details of the design principles and instructional materials (Chapter 2), I present how we unpack the cultural scope of the Second Quantum Revolution (section 1.3).

1.1 The educational challenges of the Second Quantum Revolution

The Second Quantum Revolution in the last few years has deeply marked new trends in research and enterprises. Many strategic plans have been presented by various countries that invest in these new technologies to face the Second Quantum Revolution: the European Union with the Quantum Flagship has started an investment of €1 billion in 2018; China, after several investments in the past years, declared quantum technologies one of the new high techs in its 14th Five-Year Plan (2021-2025); the U.S. with the National Quantum Initiative Act of 2018 has allocated more than 1 billion; the UK launched the UK national quantum technology program investing similar figures. From these first years, today the investments have become much greater.
Many large companies (IBM, Google, Microsoft, and Intel) and start-ups, born on project funds, have been focusing on quantum technologies and have already achieved significant results.
The different programs highlight that the challenges we are facing are many and do not just concern the research and development of these new technologies. For example, there is a need for the creation of new synergies between the labor market, enterprises, and universities and to widen the workforce. From an educational perspective, the programs require the alignment between universities and schools to orient the young toward STEM careers. From a citizenship education perspective, information campaigns are aked to be run to promote quantum literacy (Quantum Strategic Agenda, 2020; De Touzalin, Marcus, Heijman, Cirac, Murray & Calarco, 2016).

These challenges, as happened in the development of other scientific-technological sectors (e.g. AI, data science, etc.), are imposing a very quick pace of change on the different systems and are requiring quick transformative actions on different sectors.
This is in line also with the European Commissioner for Research and Innovation, since the contemporary challenges to be addressed require involving as many stakeholders as possible in the R&I process (Geoghegan-Quinn, 2012). A key aspect here becomes the RRI (Responsible Research and Innovation) concept defined as "a process where societal actors work together, via inclusive participatory approaches, during the whole research and innovation process to better align both the process and its outcomes, with the values, needs, and expectations of European society" (European Commission, 2015, p. 69).
In Europe, the trigger point was the drafting of the "Quantum Manifesto" (De Touzalin, Marcus, Heijman, Cirac, Murray & Calarco, 2016) written by six academics on the invitation of the Commissioner for Digital Economy and Society and the Minister of Economic Affairs in The Netherlands. In this Manifesto, officially released on 17-18 May 2016, the Member State and the European Commission were called to formulate a common strategy to stay at the front of the second Quantum Revolution and invest 1 billion euros for the research and development of these technologies. "It will create new commercial opportunities addressing global challenges, provide strategic capabilities for security and seed as yet unimagined capabilities for the future. As is now happening around the world, developing Europe's capabilities in quantum technologies will create a lucrative knowledge-based industry, leading to long-term economic, scientific and societal benefits. It will result in a more sustainable, more productive, more entrepreneurial, and more secure European to Union." (Ibidem, p.3)
To meet the challenges of the Quantum Manifesto, the European Union launched the Quantum in 2018 (https://qt.eu/about-quantum-flagship/introduction-to-the-quantum-flagship/). It is a large-scale and long-term research initiative with a budget of €1 billion funded that brings together research institutions, industries, and public funders, to "consolidate and expand European scientific leadership and excellence in this research area, to kick-start a competitive European industry in Quantum Technologies and to make Europe a dynamic and attractive region for innovative research, business, and investments in this field"[1].
These events have led to rapid growth in the demand for jobs in the quantum sectors. The call for actions launched by the "Quantum Manifesto" and, then, Quantum Flagship regard also the educational dimension with a twofold request: on one hand, expanding the workforce and preparing the next generations of quantum experts (De Touzalin, Marcus, Heijman, Cirac, Murray & Calarco, 2016), and, on the other hand, informing "citizens about quantum technologies and engage widely with the public to identify issues that may affect society" (ibidem, p. 13).
In this landscape, science education, as a key dimension of responsible research and innovation (RRI) is called to play a pivotal role in addressing the Second Quantum Revolution at different levels from citizenship education to higher education. Among the main goals of science education, there are: promoting scientific literacy and student agency (e.g., OECD, 2019; Zafrani & Yarden, 2017), emphasizing collaboration between scientists and students, attracting students to STEM careers (e.g., Kayan-Fadlelmula, Sellami, Abdelkader & Umer, 2022; Deming & Noray, 2020; Kier, Blanchard, Osborne & Albert, 2014; Holmegaard, Madsen & Ulriksen, 2014), and so on.
To answer the call for educational actions, the different countries responded by launching many programs. To name e a few, the White House Office of Science and Technology Policy and the Nationcience Foundation have launched the National Q-12 Education (https://q12education.org/), a consortium that aims to expand access to K-12 quantum learning tools and inspire the next generation (https://quantum.ieee.org/education) is a project plan that addresses the current landscape of quantum technologies by identifying educational challenges and opportunities and aligning new and existing initiatives to promote new ones and inspire the next generation of QIS scientists and engineers.
In Europe, the challenges and actions are set within QTEdu (https://qtedu.eu/) (September 2020 to August 2022), a CSA project commissioned by the quantum community for coordinating public

education and outreach efforts at all levels ranging from schools, through universities, and up to the working environment and general public. The main aims are to pursue a quantum-ready society, with knowledge about and positive attitudes toward quantum technologies, and to enable the emergence of a quantum-ready workforce.
QTEdu coordinated actions at different levels. In particular, they created a competence framework that maps out the landscape of possible knowledge, skills, and competencies in Quantum (https://admin.qtedu.eu//sites/default/files/attachments/Competence-Framework-Quantum-Technologies-v1-May2021.pdf). Through three consecutive survey rounds Delphi study (Greinert, Müller, Bitzenbauer, Ubben & Weber, 2022), they carried out seven main fields (concepts of quantum physics, physical foundations of quantum technologies, enabling technologies, hardware for quantum computers and sensors, quantum computing and simulation, quantum sensors and metrology, quantum communication), many subfields and a bunch of practical and soft skills, including engineering aspects, that "showed up as essential for the future quantum workforce" (ibidem, p. 6). Following the European Language Reference Framework, they organized the field and subfield according to the target audience and, thus, to the proficiency level: from A1 (Awareness) to C2 (Innovation). In figure 1.1 the schematic overview of the concepts and competencies is reported.

Fig. 1.1: Overview of the concepts and skills within the Competence Framework for Quantum Technologies (p.2).

Focusing on secondary school education, the contents carried out within this Delphi study are aligned with other bottom-up research. For example, Krijtenburg-Lewerissa, Pol, Brinkman, and Van Joolingen (2019) carried out a Delphi study to individuate the key topics for teaching quantum mechanics at secondary schools such as wave-particle duality, wave function, time evolution, the uncertainty principle, the superposition principle, probability and so on, which are part of the first and second fields of the competence framework.
Seegerer, Michaeli, and Romeike (2021), carrying out a literature analysis and focus group interviews with different experts, identify five key ideas to be addressed at an introductory level, in particular at the K-12 level: superposition, entanglement, quantum computers, quantum algorithms, and quantum cryptography.
QTEdu also moved to create a network that would connect and engage different experts and stakeholders across Europe. In order to facilitate the connections, they created and coordinated pilot programs at different levels (from outreach to university) and systematically aligned the actions with existing and planned Quantum Flagship initiatives to ensure continuity with the activities.
These actions lead to the building of a network of researchers that are collaborating within five different workgroups: School education Outreach, Higher Education, Lifelong learning and (re)training, Education Research, and Equity and Inclusion.
Within the different workgroups, many initiatives were born and are now continuing after the end of the project. These initiatives have different target groups and different aims. For example, within the School Education Outreach workgroup, there is a pilot for developing a Pedagogical Content Knowledge (PCK) to teach Quantum Technology (QT) in secondary schools, a pilot for designing the Quantum Concept Inventory in order to assess students' understanding about quantum physics and quantum technologies' key concepts in a context-independent way.
At the higher education level, the different programs highlighted the need of bringing "outside" the physics department quantum mechanics and quantum technologies education. As the strategic agenda (Mishina, Sherson, Kroon, Brand, Müller, van Joolingen, ... & Wilhelm-Mauch, 2019) for the Quantum Flagship states, the challenge "consists in developing and evaluating effective training and educational modules for a variety of learners in the areas that traditionally do not get in touch with quantum physics (e.g., engineering, computer science, mathematics)." (Quantum Strategic Agenda, 2020). This is a pivotal point not only in Europe. For example, the National Quantum Initiative (NQI) Act has the primary goal of "to expanding the number of researchers, educators, and students with training in quantum information science and technology to develop a workforce pipeline" (Smith, 2018). In recent years, quantum information science (QIS) courses have begun to flourish in universities at different levels in many countries (in U.S. e.g., Meyer, Passante, Pollock & Wilcox, 2022; Asfaw, Blais, Brown, Candelaria, Cantwell, Carr, ... & Singh, 2022; Aiello, Awschalom, Bernien, Brower, Brown, Brun, ... & Zwickl, 2021; Plunkett, Frantz, Khatri, Rajendran & Midha, 2020). In Europe, summer schools for master's and doctoral students, Master's courses, bachelor's courses, and parallel courses (https://qtedu.eu/programs-courses-and-trainings/higher-education) on these themes have

Many initiatives were launched also at outreach level and citizenship education. In the last years, the themes of the Second Quantum Revolution and emergent technologies are presented during public seminars for, e.g., the European Research Night, Pints of science, or other public events with the aim of promoting citizens' awareness. On April 14, 2022, the first "quantum day" was held worldwide, and countries around the world organized events to promote "quantum literacy" (Nita, Mazzoli, Chancellor & Cramman, 2021) such as conferences, round tables, exhibitions, and interactive art installations (in Italy e.g., Quantum Jungle, and "Dire l'indicibile").

1.2 An overview of the literature

In this section, I present an overview of the books, instructional materials, and tools on quantum technologies that have been produced for higher education and secondary school.

In higher education, there are an increasing number of books dealing with quantum computation and quantum information that are suitable for different categories of university students.
The most popular manuals are "Quantum Computation and Quantum Information" by Nielsen and Chuang (Nielsen & Chuang, 2002), "The Physics of Quantum Information" by Bouwmeester and Zeilinger (2000), "Exploration in quantum computing" by Williams, and Clearwater (1998), "An Introduction to Quantum Computing" by Kaye, Laflamme, and Mosca (2006), Quantum Computer Science by Mermin (2007), and "Quantum computing: A gentle introduction" by Rieffel and Polak (2011). There are also different online resources designed for physicists, mathematicians, computer scientists, and engineers. One of the most comprehensive is the online course by Preskill (1998).
In the last few years, technological progress showed the potential of quantum technologies in many different fields, so many books started to be published connecting quantum mechanics to different areas and emphasizing its intrinsic interdisciplinary character (e.g., the quantum science and technology book series by Djordjevic and Lele). The need of involving also non-physics university students led to the development of manuals that could be suitable for this target group as well. For example, Yanofsky and Mannucci (2008) published "Quantum computing for computer scientists" which specifically targeted students starting from the second year of a computer science program. Wittek (2014) in "Quantum machine learning: what quantum computing means to data mining" deeply explores the connection between quantum information and computation and machine learning with the aim of engaging theorists of quantum computing and information processes as well as researchers in machine learning.

As regards the secondary school level, despite the need for educational actions also at secondary school levels (De Touzalin, Marcus, Heijman, Cirac, Murray & Calarco, 2016), there are still few materials and books. The books mainly focus on developing basic mathematical and physical knowledge to prepare students who have not previously studied quantum physics to approach quantum technologies. These books usually start by introducing basic mathematical tools, such as complex numbers, vectors, and matrices. Then, the books focus on the transition from binary to quantum logic, by stressing the difference between bit and qubit. The difference is illustrated by introducing the basic concepts of quantum physics (superposition principle, state, evolution of the state, measurement, and entanglement), and by describing the postulates of the theory (e.g., Laforest, 2015; Billig, 2018).
Research centers, like Qutech (an advanced research center for Quantum Computing and Quantum Internet), and universities, like Aarhus University and the University of St Andrews, are developing precious materials that can promote understanding. The materials include quantum simulations, visualization tools, and videos on concepts and advanced quantum processes (e.g. quantum cryptography, quantum distillation, quantum error correction, etc.). These interactive tools on quantum physics are suitable both for physics and non-physics students and for secondary school students.

The recent trend toward game-based learning led to the development of quantum games and interactive tools such as Hello Quantum, Hello Qiskit, Particle in a Box, Psi and Delta, QPlayLearn (Foti, Anttila, Maniscalco, & Chiofalo, 2021), Virtual Lab by Quantum Flytrap, Quantum Odyssey, ScienceAtHome, and The Virtual Quantum Optics Laboratory in order to promote the understanding of quantum technologies and make them accessible for a wider audience (Seskir, Migdał, Weidner, Anupam, Case, Davis, N., ... & Chiofalo, 2022). For a recent comprehensive overview of this, see Seskir et. al (2022).
Recently, a storytelling approach through outreach has been developed by Goorney, Foti, Santi, Sherson, Yago Malo, and Chiofalo (2022) that can be used both in formal and non-formal education.

For elaborating this model, they started from the works of Grimellini (2004), Levrini (2004) and Grimellini and Levrini (2004). according to which "modern physics must be known by an educated citizen; physics must be taught as an intellectual production influenced by the general cultural climate and able to influence it; the materials have to be characterized by paths concerning selected basic concepts and guided by primitive questions" (Levrini, 2004, p. 622). By considering scientific knowledge as an assemblage of interacting and evolving discipline-cultures, they "conceive of a culturo-scientific storytelling to bring about positive transformations for the public in these thinking skills and ground our approach in quantum science and technologies" (Goorney, Foti, Santi, Sherson, Yago Malo & Chiofalo, 2022, p. 1). The approach they present seems to be promising if incorporated into activity design and educational policies to develop knowledge as well as skills.

From the point of view of physics education research or disciplinary-based education research on teaching quantum technologies, in literature, it is starting to be some works. Some of them consider quantum technologies as a context in which to apply the concepts of quantum physics (e.g. Dür & Heusler, 2014). Other works focus, mainly, on quantum computation (Perry, Sun, Hughes, Isaacson & Turner, 2019; Tappert, Frank, Barabasi, Leider, Evans & Westfall, 2019). In these, the qubit and new logic (also in terms of circuits and logic gates) are the pillars on which modules are developed both for curricular and extracurricular courses. For example, Dür and Heusler (2014) centralize their approach on the concept of qubits since it is one of the key aspects of quantum physics and helps to visualize the invisible.

Pospiech (2021; 2000a, 2000b) has elaborated an approach for secondary school students based on quantum cryptography. Similarly to our approach, this one does not treat quantum cryptography as an application but it aims "to show the intimate connection between the principles of quantum physics, how they are exploited in quantum cryptography and why they make it a secure method of communication" (Pospiech, 2021, p. 25). She proposed to incorporate quantum cryptography into a teaching-learning sequence on quantum physics based on two-state systems in order to, on one hand, impart basic quantum notions such as non-determinism, superposition, and uncertainty; and, on the other, introduce students to contemporary physics research. Also Grau (2004) proposed a teaching method based on quantum cryptography to teach quantum mechanics to computer scientists and electrical engineers at an introductory level.

Some courses on the second quantum revolution started to be developed also for pre- and in-service teachers' education (e.g., Pallotta, 2022; Sutrini, Malgieri & Macchiavello, 2022) with the aim also to explore if and to what extent teachers can be interested in incorporating quantum technologies to their traditional approach of teaching quantum mechanics at school.

The present Ph.D. thesis is placed in the field of research in Physics education and STEM education and intends to contribute to the production of research-based materials with the aim of addressing these broader research questions (Satanassi, Ercolessi & Levrini, 2022):

> *RQ1. Which teaching approach can be developed to underlie the educational potential of technologies even beyond technical training?*
> *RQ2. What instructional design principles can be pointed out to value the Second Quantum Revolution as a cultural revolution?*

The questions are challenging since they do not only require the design of approachable resources for effectively teaching knowledge and skills on quantum technologies. They also require us to find ways to make students and citizens aware of the quantum revolution as firstly a cultural revolution. This deep change induced by quantum technologies touches both the relationship between science and society (Roberson, Leach & Raman, 2021; Vermaas, 2017; De Wolf, 2017) and the foundations of our thinking. Indeed, quantum physics challenges the classical Aristotelian logic that reached its peak with Boolean algebra (Nadaban, 2021; Deutsch, Ekert & Lupacchini, 2000; Hellman, 1980; Finkelstein 1969; Birkhoff & Von Neumann, 1936). In this sense, the second quantum revolution conveys a profound conceptual change since it challenges the binary logic on which classical computers are based. As I will

explain, our approach centralizes the logical layer of reasoning and the educational potential of quantum technologies to move beyond the idea that all information can be codified in a bit and processed in terms of classical logic gates.
Before going into the details of the design principles that we carried out and of the module, I describe the first challenge we faced in contributing to the research questions: what do we mean today for Second Quantum Revolution?
In the next section, I will describe how we unpack the Second Quantum Revolution as, first of all, a cultural revolution.

1.3 Unpacking the Second Quantum Revolution

In the previous sections, I discussed the relevance of the Second Quantum Revolution from a political, economic, societal, and educational perspective, focusing particularly on the current status of Europe and the challenges set within the QTEdu project. But what is it about?
Since we started dealing with the second quantum revolution, we have tried to unpack the Second Quantum Revolution first of all from a cultural perspective. The word 'cultural' can embrace many meanings, how we use it refers to the idea that physics knowledge in school class can be presented as a form of culture (e.g., Grimellini Tomasini, 2004; Levrini, 2004; Tseitlin and Galili, 2005; Galili, 2012). This conception of knowledge incorporates the belief that to reach a conceptual understanding of physics content it is essential to consider physics as a dialogue between various accounts, approaches, and interpretations of reality (Levrini, Bertozzi, Gagliardi, Tomasini, Pecori, Tasquier & Galili, 2014; Levrini & Fantini. 2013). It is this variety that makes certain knowledge cultural (e.g., Tomasini & Levrini, 2004; Galili 2014; Levrini, 2014).

The first challenge concerns the unpacking of the Second Quantum Revolution from this cultural perspective and the clarification of how we conceive it beyond the canonical narratives that are often used to talk about it. The canonical rhetoric with which experts and media refer to the Second Quantum Revolution is the one of 'power' and 'muscle'. It is easy to hear talking about quantum technologies as promising new technologies that can solve problems in much less time and solve problems that current classic supercomputers cannot solve. Let's think about the term 'quantum supremacy' coined by Preskill (2012) which refers to the turning point for quantum computation in which a quantum computer performs a task that cannot be completed by a classical computer within a reasonable timeframe (Preskill, 2012). Roberson, Leach, and Raman (2021) emphasize the controversial nature of the term from social, technical, and practical perspectives. From the social one, the term 'supremacy' has a problematic association with the language of racism, the reason why it is proposed to replace the term with 'quantum advantage' (Mueck, Palacios-Berraquero, & Persaud, 2020; Palacious-Berraquero, Mueck & Persaud, 2019; Wiesner, 2017), potentially also exaggerating the promise of quantum technologies. From the technical perspective, 'quantum supremacy' was contested by arguing that classical computers can perform the chosen task faster than reported (Roberson, Leach & Raman, 2021). Finally, from the practical perspective, some doubts about the utility of quantum computing experiments were raised, due to the high academic value of the tasks undertaken to demonstrate quantum advantage (Dyakonov, 2018). In light of this, Roberson, Leach, and Raman (2021) raise the problem for science communication since the scientists had to frame the meaning of the Second Quantum Revolution "in narrow utilitarian terms of the promise of novel commercially valuable technologies, notably, the quantum computer" (p.2) or within political discourses. We do think that these aspects are part of the cultural dimension of the Second Quantum Revolution, but our aims are also to trigger the dimension of other knowledge dimensions and a sense of wonder that has always been part of science and that the utilitarian end can hide.
In Gisin's words, as he argued in the introduction of his book "Quantum Chance" (2016):

> "We are living in an extraordinary age. Physics has just discovered that one of our deepest intuitions, namely that objects cannot 'interact' at a distance, is not correct. [...] Physicists explore the world of quantum physics, a world populated by atoms, photons, and other objects that seem quite mysterious to us. Missing out on this revolution without giving it further attention would be as much a shame as remaining ignorant of the Newtonian revolution or the Darwinian revolution, had we been their contemporaries. For the conceptual revolution taking place today is of no lesser importance. It completely overturns our previous pictures of nature and will doubtless give rise to a range of new technologies that will simply look like magic" (p.xiv).
> "Had you lived at the time of the Newtonian revolution, would you have wished to understand what was going on? Today, quantum physics gives us the opportunity to live through a conceptual revolution of similar importance." (Ibidem, p. xi)

Therefore, what is the scope of the new revolution beyond the technicalities and the greater power that quantum technologies seem to have compared to the classic ones?
How does the Second Quantum Revolution differ from the First one? What is revolutionary?
Hereafter, I discuss how, in these years of work, we unpacked and articulated the Second Quantum Revolution also with respect to the First one. The discussion revolves around a cultural perspective, in particular, it is articulated in three different dimensions. The first is "ontological", with which we argue what the second quantum revolution is and how it differs from the first emphasizing, in the wake of the vision of Aspect and Gisin, what are the key concepts on which they are based. In particular, as I will stress, the two quantum revolution revolve around two different quantum properties that regard the quantum object itself: the complementary principle and the entanglement. The second is "logical and epistemological" which refers, on one hand, to the development of a logical structure and its physical interpretation, which see quantum mechanics as a theory to interpret physical nature; and, on the other, it refers to the change of problem's perspective and approach: quantum mechanics is not only a way to describe nature but its rules can be used to manipulate nature to solve problems. The third is "linguistic", with which we highlight the change of linguistic register: from the language of inability to the language of technological possibilities. The three nuclei are not independent, but they dialogue.

1.3.1 The conceptual and ontological dimension

Quantum physics was born with the research of Planck in 1900 which led him to discover the quantization of energy in radiation-matter interaction (Planck, 1901). Five years later, in 1905, Einstein took a further step by proposing the quantization of light itself to understand the photoelectric effect (Einstein, 1905). Einstein's revolutionary hypothesis took about 10 years to be accepted when it was proved by the experimental evidence realized by Millikan (Millikan, 1949). In the meanwhile, in 1913 Bohr realized his atom model, giving for the first time a quantitative description of atoms' stability and describing the emission of absorption of light by atoms with the relation $\hbar\omega_{i,j} = E_i - E_j$ (Bohr, 1913 a,b,c). The establishment of the comprehensive Quantum mechanics paradigm took another decade (1925), which saw two opposing formalisms: on one hand, the Schrödinger formalism according to which the temporal evolution of the state of a system can be described by a wave function, demonstrating that, like light, matter can behave either as a particle or as a wave; on the other, the Heisenberg formalism based on the mathematics of matrices. Later the two formalisms were proved to be equivalent by Dirac who formulated a formalism that, based on the vectors' notation and abstract Hilbert space, describes mechanical, optical, electrical, and thermal properties of matter (e.g. Aspect, 2021; Gieres, 2000).
In the following, Quantum Mechanics has been shown to have many potentialities, allowing physicists to understand some matter's properties such as superconductivity or superfluidity, and to describe particle behavior (e.g., Keimer & Moore, 2017; Bohm & Hiley, 1977; Fock, 1971, etc.). Light-matter interaction studies were increasingly refined (by orders of magnitudes), fitting perfectly within the

quantum mechanics theory, which has been shown to be applied both in the elementary phenomenon (Quantum Electrodynamics) and in more complex condensed matter situations. Nevertheless, until the early '50s, quantum mechanics' discoveries and progress had mostly foundational, theoretical, and knowledge value without any impact on everyday life. Then the first technologies started to be developed such as transistors, lasers, and their descendants like micro-fabricated integrated circuits, bar code readers, CD recorders and players, medical tools, and so on.

As Aspect and Gisin argue, at the basis of these everyday technologies there is what lies at the heart of the first quantum revolution: the wave-particle duality. In Aspect's words "the first quantum revolution was based on the discovery of wave-particle duality. It provided a way of describing with great accuracy the statistical behavior of atoms that make up matter, the clouds of electrons that conduct the electric current in a metal or semiconductor, and the billions and billions of photons in a beam of light. It has also provided tools to understand the mechanical properties of solids, whereas classical physics was unable to explain why matter, made up of positive and negative charges that attract one another, does not simply collapse. Quantum mechanics has given a precise quantitative description of the electrical and optical properties of materials and offers the conceptual framework needed to describe phenomena as surprising as superconductivity and the strange properties of certain elementary particles. It was in the context of this first quantum revolution that physicists invented new devices like the transistor, the laser, and integrated circuits, which have brought us today into the age of the information society" (Aspect in Gisin, 2014, p. vii).

As for the first quantum revolution, the hallmark is wave-particle dualism, the second one is founded on another "concept". Feynman, in his famous 'Lecture of Physics', stated that the double-slit experiment "has in it the heart of quantum mechanics. In reality, it contains the only mystery" (Feynman, Leighton & Sand, 1965). In fact, for a long time, the Bohr-Einstein discussion on EPR (Einstein, Podolsky & Rosen, 1935; Bohr, 1935) was considered a philosophical discussion, without consequences on the practical way of doing physics. As Aspect discussed in the introduction to "Speakable and Unspeakable in Quantum Mechanics" by Bell (1987, RI-edited in 2004), "quantum mechanics was constructed at the price of several radical - and sometimes painful - revisions of classical concepts" (Aspect, 2008, p. xxi). For example, the particle-wave duality led to the renunciation of the idea of a classical trajectory. It was better described by the Heisenberg uncertainty principle, which formalized the impossibility of defining simultaneously and precisely the position and momentum of a particle. This renunciation of classical trajectories can be reformulated also by remarking that the particle "follows many paths at once". The Einstein, Podolsky, and Rosen argument, named 'Einstein-Podolsky-Rosen paradox', was fundamental because it gave rise to many reflections and reasoning that led to the fall of two other important principles: the principle of locality and causality (in the classical sense). For Einstein, the problem that quantum formalism allowed the existence of certain two-particle states, later named by Schrödinger "entangled states", for which "one can predict strong correlations both in velocity and in position even when the two particles are widely separated and no longer interact" (ibidem, p.xxii). In their paper (1935), the three experts showed that measurements of positions would always give symmetric values about the origin so that the measurement on one particle allows the experimenter to know with certainty the value of the position of the other particle. They did the same with the velocity. As the measurement on the first particle does not disturb the second one, then the second particle should have had well-determined values of position and velocity even before the measurement. Therefore, since quantum formalism, for the uncertainty principle, cannot give a simultaneous and precise value of these quantities, Einstein, Podolsky, and Rosen concluded that quantum mechanics has to be incomplete, aiming to complete the formalism. Bohr (1935), in his paper published shortly after, argued that it is not just a matter of formalism, but if the EPR argument were true then all of quantum physics would collapse. Quantum physics was achieving many successes so many physicists did not participate in the debate because it was very academic, it felt more like a matter of adhering to a certain epistemological position, without any particular practical implications (Aspect, 2008).

Bell, in a work published in 1964, re-opened the discussion about the presence of additional variables to restore the theory of causality and locality. He argued "In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past, that creates the essential difficulty. There have been attempts to show that even without such a separability or locality requirement no "hidden variable" interpretation of quantum mechanics is possible [referring to Von Neumann (1932), English version reprinted in 2018] [...] Moreover, a hidden variable interpretation of elementary quantum theory has been explicitly constructed [referring to his work published in 1966]. That particular interpretation has indeed a grossly nonlocal structure. This is characteristic, according to the result to be proved here, of any such theory which reproduces exactly the quantum mechanical predictions" (Bell, 1964, p.195).

Recalling the situation designed by Bohm of a system of entangled states of two spin -1/2 particles, he showed that the existence of correlations between the results of measurements on the two particles can be easily explained if the result of a measurement on one particle depends only on the supplementary parameters carried by that particle and the setting of the apparatus making that measurement. Nevertheless, even if such a 'hidden variable' theory can reproduce some of the predicted quantum correlations, it is not generalizable because it cannot predict quantum correlations for all possible settings of the measuring apparatus. "Thus it is not possible, in general, to understand EPR-type correlations by 'complementing' the quantum theory along the lines proposed by Einstein" (Aspect, 2008, p. xxiii). This result is known as Bell's theorem and the big assumption is that the 'locality hypothesis' has to be fulfilled by the 'hidden variables' models to lead to a conflict with quantum mechanics. "This hypothesis states that there is no direct non-local interaction between the two measuring apparatuses far from each other. In other words, the conflict arises only if the result of a measurement on the first particle does not depend on the setting of the second measuring apparatus" (ibidem, p. xxiv). Bell formalized the incompatibility between quantum mechanics and the local hidden variable theories by showing that the correlations predicted by any local hidden variable model are limited by inequalities and that they are violated by certain quantum predictions. At this point, that debate in which most physicists did not participate because it was a question, more than anything, of epistemological position, became questionable from an experimental point of view. After about 20 years, Feynman himself recognized that quantum mechanics had another mystery. In his famous paper Simulating physics with computers (1981), he wrote "[Referring to correlations] That's all. That's the difficulty. That's why quantum mechanics can't seem to be imitable by a local classical computer. We always have had a great deal of difficulty in understanding the worldview that Quantum Mechanics represents. [...] It has not yet become obvious to me that there is no real problem. [...] I've entertained myself always by squeezing the difficulty of Quantum Mechanics into a smaller and smaller place, so as to get more and more worried about this particular item. It seems to be almost ridiculous that you can squeeze it into a numerical question that one thing is bigger than another. But there you are – it is bigger than any logical argument can produce if you have this kind of logic" (Feynman, 1981, p. 471 and 485).

The first experiment that proved Bell's inequalities violation (1966) was realized by Aspect, Grangier, and Roger (1982a, b) by using photons. They discovered that the inseparability of entangled photons, separated by 12 meters at the time of the measurements, is maintained "even if the photons are far apart, including a 'space-like' separation in the relativistic sense. Furthermore, the setting of the measuring polarizers, during the twenty-nanosecond flight of the photons between the source and the detector, can be changed to implement Bell's ideal scheme (Aspect, 2021; Aspect, 2008). More recent experiments proved the violation with photons, traveling in optic fibers, separated by hundreds of meters (Weihs, Jennewein, Simon, Weinfurter & Zeilinger, 1998; Tittel, Brendel, Zbinden & Gisin, 1998). In these experiments, during the propagation of the photons in the fibers, the experimenters could randomly change the setting of the polarizers. These kinds of experiments ('timing experiments') with variable polarizers seem to contradict the principle of relativistic causality since the measurement of one photon would affect the other instantaneously. Peres and Terno (2004) stressed that there is no violation of causality in the operational sense and non-separability cannot be used to send a signal or usable information faster than light's speed.

Coming back to the focus of the present thesis, the entanglement, and these experiments, in the perspective of Aspect and Gisin, are at the basis of the second quantum revolution together with the possibility of isolating and manipulating the single quantum object. The techniques developed starting from the late '80s made possible what for Schrödinger, in the middle of the last century, was impossible. In his words: “It is fair to state that we are not experimenting with single particles, any more than we can raise Ichthyosauria in the zoo” (Schrödinger, 1952, p.240).

The combination of these two elements has given and is giving rise to a new generation of applications and technologies such as quantum cryptography and quantum teleportation (in the broad field of quantum information), quantum algorithms (in the field of quantum computing), quantum sensors, quantum simulators, quantum computers, etc.

As Dowling and Milburn (2003) argued

> “The hallmark of this second quantum revolution is the realization that we humans are no longer passive observers of the quantum world that nature has given us. In the first quantum revolution, we used quantum mechanics to understand what already exists. We could explain the periodic table, but not design and build our own atoms. We could explain how metals and semiconductors behaved, but not do much to manipulate that behavior. The difference between science and technology is the ability to engineer your surroundings to your own ends, and not just explain them. In the second quantum revolution, we are now actively employing quantum mechanics to alter the quantum face of our physical world. We are transforming it into highly unnatural quantum states of our own design, for our own purpose. For example, in addition to explaining the periodic table, we can make new artificial atoms—quantum dots and excitons—which we can engineer to have electronic and optical properties of our own choosing. We can create states of quantum coherent or entangled matter and energy that are not likely to exist anywhere else in the Universe. These new man-made quantum states have novel properties of sensitivity and non-local correlation that have wide application to the development of computers, communications systems, sensors, and compact metrological devices. Thus, although quantum mechanics as science has matured completely, quantum engineering as a technology is now emerging in its own right. It is just a matter of being in the right place at the right time to take full advantage of these new developments” (Dowling & Milburn, 2003, p. 1656).

1.3.2 The logical and epistemological dimensions

The twentieth century was very revolutionary in many ways, including logic. The birth of quantum mechanics led to the formalization of non-classical logic. Bueno (2002), in the lecture note on pure and applied mathematics, argued:

> “There was a time, a long time ago, when the question 'What is the right logic?' was not an issue. Had it been raised; it would have been answered quite simply: the right logic is the only one that exists! Similarly to the situation in geometry before the emergence of non-Euclidean geometries, logic was identified with the logical system that existed at the time, and the choice between alternative logics didn't arise, simply because there was nothing there to choose from: there was only one logic. The picture changed dramatically, of course, during the twentieth century, with the formulation of several logical systems: from extensions of classical logic (such as modal logic) to alternative and in some cases rival systems, such as paraconsistent logics, intuitionistic logics, and quantum logics (just to mention a few). Once there is a plurality of logics to consider, it becomes a substantial issue which of them (if any) is the right one.” (p.535-536)

Therefore, the birth of quantum physics and then quantum logic have been discussed at length by experts in several fields. The birth of quantum logic coincided with the publication of the article entitled "The logic of quantum mechanics" written in 1936 by Garrett Birkhoff and John von Neumann. In the introduction of the work, the authors wrote:

> "One of the aspects of quantum theory which has attracted the most general attention, is the novelty of the logical notions which it presupposes. It asserts that even a complete mathematical description of a physical system *S* does not in general enable one to predict with certainty the result of an experiment on *S* and that in particular one can never predict with certainty both the position and the momentum of *S* (Heisenberg's Uncertainty Principle). It further asserts that most pairs of observations are incompatible and cannot be made on *S* simultaneously (Principle of Non-commutativity of Observations).
> The object of the present paper is to discover what logical structure one may hope to find in physical theories which, like quantum mechanics, do not conform to classical logic." (p. 823)

Attempting to reconcile the apparent inconsistency of classical logic with the facts concerning the measurement of complementary variables in quantum mechanics, such as position and momentum.
The basic idea of Birkhoff and Von Neumann was to see how the mathematical formalism of quantum mechanics suggests a new kind of logic. Their approach is called an algebraic approach to quantum logic because it derives the structure of logic from the algebraic structure that can be defined starting from the mathematics of quantum mechanics. Therefore, starting from the very first work of von Neumann (1932), the quantum-probabilistic formalism assumes that each physical system is associated with a (separable) Hilbert space *H* and unit vectors correspond to possible physical states of the system. Each "observable" real-valued random quantity is represented by a self-adjoint operator *A* on *H*, the spectrum of which is the set of possible values of *A*. Being *u* a unit vector in the domain of *A*, representing a state, the expected value of the observable represented by *A* in this state is given by the inner product ⟨*Au,u*⟩. The observables represented by two operators A and B are commensurable if *A* and *B* commute (i.e., *AB* = *BA*).
As Von Neumann (1932) deeply discussed, {0,1} are valued observables that regard encoding propositions about the properties of the system. Moreover, by the spectral theorem, each self-adjoint operator can be written as the sum of the headlamps on his autospaces each multiplied by the corresponding eigenvalue. The probabilities for all possible results of the measurement of all observables on a certain state are calculated from the expectation values of orthogonal projectors. We can therefore say that these "properties" are sufficient to characterize a physical system.
Following the argumentation of Pitowsky (1989), each orthogonal projector is uniquely associated with a closed subspace of Hilbert space, so we can speak equivalently of orthogonal projectors or closed subspaces. Between these projectors we can introduce relations corresponding to "logical" relations, that is, set of relations between subsets of Hilbert space; let P_1 and P_2 be two orthogonal projectors on a Hilbert *H* space (of finite dimension) then:

1. $P_1 < P_2$ if $P_1(H) \subset P_2(H)$, this corresponds to the logical relation: "P_1 implies P_2", in fact, $P_1 < P_2$ if and only if it applies to the expectation values of P_1 and P_2: $\langle P_1\rangle_\phi=1 \Rightarrow \langle P_2\rangle_\phi=1$, for each ϕ.
2. We define instead $P_1 \wedge P_2$ the projector on subspace $P_1(H) \cap P_2(H)$, this property corresponds to "P_1 *and* P_2", in fact, a state ϕ is autostate to the eigenvalue 1 of $P_1 \wedge P_2$ if and only if it is autostate at the eigenvalue 1 of both P_1 and P_2.

The other two "logical" relationships (complement and disjunction) cannot be defined by the corresponding set operations (complement and union) because the complement of a subspace and the union of two are not subspaces in general. The following operations are then introduced:

3. given an orthogonal projector P, we define P^c as the projector on the orthogonal complement of the subspace $P(H)$,
4. Given two orthogonal headlamps P_1 and P_2, we define $P_1 \vee P_2$ as the headlamp on the subspace generated by $P_1(H) \cup P_1(H)$.

The structure of the orthogonal projectors with operations forms 1. to 4. is a complete ortho-complemented lattice.
Birkhoff and Von Neumann (1936) identified P^c with the property "not P" and $P_1 \vee P_2$ with "P_1 or P_1". So, they realized that, after showing the correlation between elements and subsets of phase space with "experimental proposition" that is subsets of different observation spaces (Birkhoff & von Neumann, 1936, p. 825), the difference between classical and quantum logic lies in the failure of distributive property ("the distributive law may break down in quantum mechanics", p. 831), that is

$$A \wedge (BVC) \neq (A \wedge B)V(A \wedge C).$$

Since the algebraic structure associated with classical logic is distributive, they concluded that the logic of quantum mechanics in not classical. To characterize it, the authors suggested a "weakened distributive law" (ibidem, p.832), called also "modular identity":

$$If\ A \subset C, then\ A \cup (B \cap C) = (A \cup B) \cap C.$$

The interpretation given is that classical logic is "inadequate" to describe microscopic phenomena and should therefore be replaced with "quantum logic" (Pitowsky, 1989). In this sense, the work of Birkhoff and Von Neumann takes up the ideas of Bohr and Heisenberg and takes them to the extreme, questioning not only the space-time concepts of classical physics (position, speed, etc.) but even its logical structure. Therefore, von Neumann and Birkhoff (1936) suggested that the empirical success of quantum mechanics as a framework for physics questions the universal validity of the distributive laws of propositional logic. Cautiously they argued:

"Whereas logicians have usually assumed that properties […] of negation were the ones least able to withstand a critical analysis, the study of mechanics points to the distributive identities […] as the weakest link in the algebra of logic" (p. 837).

Starting from here, various proposals to change the principles of classical logic have thus emerged. As Bueno (2002) points out that among the different proposals and approaches the most supported ones are those that differ from the classic logic in a 'moderate sense', in Resnik's words (Resnik, 1996, p. 497); that is, they reject a law or rule of inference that classical logic accepts.
In the second half of the last century, quantum logic research paved the way for one of the deepest debates in physics, mathematics, and the philosophy of science. The debate, in general, is divided into at least two different dimensions: the relationship between logical structure and mathematical formalism associated with the theory and physical interpretation of the logical-mathematical structure and, for example, consequent physical interpretation of anomalies (e.g., the breaking down of distributive property), to which, in the philosophical field, we generally refer as metaphysics.
The leitmotiv of different perspectives is the quite subversive attitude towards classical logic and the foundations of metaphysical understanding. Among the different approaches, one (e.g., Bohmian mechanics, Ghirardi-Rimini-Weber theory) started from a set of metaphysical (classical) assumptions and tried to change the formalism to place quantum mechanics into their desired metaphysical image. Another approach, starting from the point of view of orthodox formalism, extrapolated an adequate interpretation of the theory by trying to understand the symmetries and features of formalism. Furthermore, there are more open approaches to original metaphysical development that would allow us to understand what the world is like according to quantum mechanics (De Ronde, Domenech & Freytes, 2014). One of them is the Neo-Kantian logic path that was elaborated starting from Birkhoff and von Neumann's work (1936). In the '50ties, one of the main exponents of this approach was von Weizsäcker, who tried to carry out a different precise language that follows definite logical patterns in conformity with the mathematical scheme. He proposed to distinguish different levels of language: one level referring to objects, a second level to statements about objects, the third level to statements about statements about objects and so on. The modification of classical logic has to refer, first of all, to the level of objects. As Heisenberg (1958) argued, it is not clear at first glance what kind of ontology would be the basis of these modified logical models; the main concern in the project of finding a logical system associated with the algebraic structure of the theory. Von Weizsäcker proposed his approach

from the idea of reconstructing physics in terms of yes-no-alternatives (ur-alternatives) and establishing a connection between quantum structures and the structure of space-time. In its perspective, alternatives-ur are considered the fundamental objects of physics from which, in principle, any physical object can be constructed. Thus, he proposed a change of perspective from a notion relating to information to the notion of physical objects: objects are reduced or even "facts of" information (Lyre, 2003). Von Weizsäcker's perspective was later also supported by Lyre (2003) who argued:

"In quantum theory in particular, this view has a lot of plausibility. Quantum objects are represented in terms of their Hilbert state spaces, their quantum states correspond to empirically decidable alternatives. Any quantum object may further be de-composed or embedded into the tensor product of two-dimensional objects, nowadays called quantum bits or qubits. Urs [yes-no-alternatives or ur-alternatives], therefore, are in fact nothing but qubits" (p.2).

In the seventies, Pieter Mittelstäedt contributed to the research in quantum logic framed within the neo-Kantian tradition (Busch, Pfarr, Ristig, & Stachow, 2010). In classical physics, all propositions about a system can be predicated together. On the contrary, quantum properties may be assigned values only in a contextual manner (Kochen & Specker, 1975), thus an interpretation in terms of substance is forbidden. In light of this, the category of substance can be only applied in the case in which the state of the system is such that these observables may be assigned definite values. Since classical logic allows truth values for all propositions, it is not adequate for propositions about a quantum system, where the empirical content of propositions is relevant when applying the rules of logic. With the assumption that the laws of logic ought to be universally valid, Mittelstäedt changed the focus of the search to a different foundation of logic so that it could allow proofs to be independent of the empirical content of statements. After paying attention to the implicit nature of commensurability between any two propositions in classical logic, he introduced, starting from the idea that a system has a certain property that can be valued by testing the property in an experiment, the concept of dialog game. By adding a commensurability relation to the Hilbert lattice before constructing a formal propositional logic, Mittelstäedt completed a calculus that is a model of the logical structure, *L* (Mittelstäedt, 1979). Then he introduced modalities and probability as metalinguistic concepts (Mittelstäedt, 1979; 1981a; 1981b) as well as established that only by employing adequate notions of 'temporal identity' and 'transworld identity' might Kripke-like semantics be formulated in quantum logic (Mittelstäedt, 1985; 1986).
During the eighties and nineties, Michel Bitbol analyzed the different alternatives of the language of physical properties and their role in objectivity. He pointed out contextuality as the main characteristic that has to be focused on when applying to the program (Bitbol, 1998).

Another approach is known as quantum logical operationalism and sees by most influential academics Mackey, Jauch, Piron, and Aerts. This approach follows the ideas initiated by Birkhoff and von Neumann (1936) that searched for an axiomatic theory where the, physically non-justified, Hilbert space structure would be derived from a set of physically motivated axioms and give particular importance to the concept of experimental propositions. Following this idea, George Mackey, in his monograph (1963), recovered von Neumann's idea of "projections as propositions" (Von Neumann, 1955, 2018, p. 159). Since the projections have only two eigenvalues, 0 and 1, it is possible to think of the proposition associated with a projection as the answer "yes" or "no" to the corresponding question. Thus, Mackey referred to the propositions affiliated with a physical system as questions and, under a reasonable axiomatization, showed that the questions form an orthomodular lattice (Mackey, 1963).
In the sixties, Jauch and Piron (Jauch, 1971; Piron, 1964) also aimed at reconstructing the formalism of QM from first principles with a special interest in the relation between concepts and real physical operations that can be performed in the laboratory. For example, states are defined as "the result of a series of physical manipulations on the system which constitute the preparation of the state."

This is why this approach is called operationalism: every notion should be defined in terms of operations. Operational quantum logic involves the fact that the yes-no answers to the elementary questions, or the "experimental propositions" of Birkhoff and von Neumann, may be regarded as propositions of non-classical logic. According to Aerts, (1999), the main operationalist lines of research are the Geneva school whose main members are Jauch and Piron (Jauch, 1971; Piron, 1964; 1989) in Geneva, and Diederik Aerts (Aerts, 1981; 1982; 1983; 1984), continued by Piron's student, in Brussels; the Amherst approach which in words of David Foulis and Charles Randall should be called "empirical logic".

One of the main results, due to Aerts (1981), concerned the problem of treating composite systems. In two classical systems, the whole set of propositions about them can be organized in the corresponding Boolean lattice built up as the Cartesian product of the individual lattices. This is not true in the quantum case. When we consider two or more systems together, the state space of their pure states is the tensor product of their Hilbert spaces. Given the Hilbert state spaces H_1 and H_2 as representatives of two systems, the pure states of the compound system are given by rays in the tensor product space $H=H_1 \otimes H_2$. Nevertheless, as classical analogy could suggest, it is not true that any pure state of the compound system factorizes after the interaction in pure states of the subsystems and that they evolve with their own Hamiltonian operators. Aerts (1981) proved that when trying to repeat the classical procedure of taking the tensor product of the lattices of the two systems' properties to obtain the lattice of the properties of the composite, the procedure fails (Aerts & Daubechies, 1979a; 1979b; 1981; Randall & Foulis, 1971). Every attempt made showed that the Hilbert lattice factorizes only in the case in which one of the factors is a Boolean lattice, or when the systems have never interacted.

In the 1960s and early 1970s, another, quite radical, approach appeared. The main scholars that presented the empirical perspective of quantum logic were especially David Finkelstein and Hilary Putnam, who argued that quantum mechanics requires a revolution in our understanding of logic *per se*. Finkelstein discussed the birth of quantum logic as a *fracture* that "occurred at the very deepest level of physical theory" (Finkelstein, 1969, p. 203). In a paper called Matter, space and logic he wrote:

> "Physics has a warp and a woof, like a fabric stretched across many levels of abstraction and woven out two millennia long. Across the fabric is a pattern persistent over the entire length in which the levels tend to group themselves into three levels of increasing abstraction: theories of matter and mechanics, theories of space and geometry, and theories of logic. Running along the woof is a second pattern, a sequence of discovery pursued first at the most concrete level and then retraced at deeper levels. In this evolutionary process, the theory first passes from its earliest, most 'rigid', form into a different but still rigid form (fracture), and then into a non-rigid or 'flexible' theory with a continuum of freedom (flow). This process of fracture and flow of physical theories has attacked the deepest levels, those concerned with the logic of the physical world, only in this century and has yet to run its course there. Its working out at these levels is a principal motif of the present and of the immediate future of physics." (p. 199)

And he continues:

> "To some the fact that there are vectors neither in A nor in ~A suggests that there is not a two-valued logic in quantum mechanics, that the *tertium non datur* breaks down. [...] the *tertium non datur* was not the weak point of classical logic. Yet we also see that there must be a difference between the quantum-logical structure and that of classical mechanics, or else it too would be realizable as a calculus of all sets of some suitably chosen space. And in fact the fracture is in the distributive law. All the anomalies of quantum mechanics, all the things that make it so hard to understand, complementarity, interference, etc., are instances of non-distributivity."

Also Putnam in the article "Is Logic Empirical?" (1968) proposes to adopt a non-distributive logic, believing that the anomalies associated with quantum measurements derive from the anomalies of classical logic used "erroneously" even in the microscopic world. His starting point was substantiated in a parallel between the laws of Logic and those of Geometry:

> “What is the nature of the world if the proposed interpretation of quantum mechanics is the correct one? The answer is both radical and simple. Logic is as empirical as geometry. It makes as much sense to speak of ‘physical logic’ as of 'physical geometry'. We live in a world with a non-classical logic. Certain statements - just the ones we encounter in daily life - do obey classical logic, but this is so because the corresponding subspaces of H(S) form a very special lattice under the inclusion relation: a so-called ‘Boolean lattice'. Quantum mechanics itself explains the approximate validity of classical logic' in the large', just as non-Euclidean geometry explains the approximate validity of Euclidean geometry 'in the small'” (p.184)

Therefore, just as Euclidean Geometry, once considered to be the Geometry of physical space, had been made a “borderline case” by General Relativity, so was Classical Logic, failing to take into account quantum phenomena except by generating paradoxes, could also be revised and made a “borderline case” of a new Logic thanks to Quantum Mechanics. In short, the algebraic properties of logic should be determined and modified empirically. A realistic approach, for Putnam, constrains to believe that the properties of physical objects exist simultaneously even before measurement. Quantum logic, thus, results to be, on empirical bases, the "correct" logic that manages to account for the measurement of complementary variables (e.g., the position and the momentum). For Putnam, the elements of *L(H)* represent categorical properties that an object possesses, or does not, independently of whether or not we look. This picture of physical properties is confirmed by the empirical success of quantum mechanics, so it has to be accepted that how physical properties hang together is not Boolean.
The idea of a quantum object is quite in contrast with more recent perspectives, for example with Isham’s one (2001):

> “[...] the idea of a physical system possessing values for all physical quantities is regarded as meaningless. From this perspective, it is not correct to say that a measurement yields a particular result because this is the value that the corresponding physical quantity happens to have at that time. In so far as quantum theory applies at all to single objects (and, to an anti-realist, this is debatable) a ‘thing’ is arguably better understood as a bundle of latent, or potential, properties that are only brought into being (in the sense of classical physics) by the act of measurement” (p.77).

The failure of the classical notion of ‘property’, for Isham, is reflected at a mathematical level in the non-distributive nature of the set of quantum propositions.
Coming back to Putnam’s perspective (1981), classically if *S* is the set of states of a physical system, then every subset of *S* corresponds to a categorical property of the system, and vice versa. In quantum mechanics, the state space is the (projective) unit sphere $S = S(H)$ of a Hilbert space. However, not all subsets of *S* correspond to the quantum-mechanical properties of the system. The latter corresponds only to subsets of the special form $S \cap M$, for *M* a closed linear subspace of *H*. In particular, only subsets of this form are assigned probabilities. This can lead to two options. One is to take only these special properties as “real” (“physical” or “meaningful”), regarding more general subsets of *S* as corresponding to no real categorical properties at all. The other is to consider the "quantum" properties as a small subset of the set of all reasonable, but not necessarily observable, physical (or metaphysical) properties of the system. According to this view, the set of all properties of a physical system is entirely classical in its logical structure, but the probabilities are not assigned to the non-observable properties (Wilce, 2002).

These approaches are among the most studied approaches but there are many others like the modal interpretation of quantum mechanics, the intuitive logic, the theory of categories, and so on.

Today, the word "logic", even with the advent of the Second Quantum Revolution, is more easily associated with the idea of logic gates (and therefore circuits, algorithms, and the computing machine itself), rather than the logical structure at the center of the debates of the 60s and 70s. This association is due for the first time to Finkelstein (1969) that argued the algebraic and logical structure of quantum mechanics can be operationally built by making use of electronic circuits that mirror its enunciative formulas, as well as digital circuits that translate the formulas of the Boole algebra into circuit form (Finkelstein, 1969; Holdsworth and Hooker; 1983).

What Finkelstein has just introduced became a pivotal aspect first with the works of Landauer (1961), Bennett (1973), and Fredkin and Toffoli (1981) about the possibility to perform any computation in a way that is reversible both logically, (e.g., the computation is a sequence of bijective transformations) and thermodynamically. For example, the computation could in principle be performed by a physical apparatus dissipating arbitrarily little energy (Barenco, Bennett, Cleve, DiVincenzo, Margolus, Shor, ... & Weinfurter, 1995). Recalling the word of Fredkin and Toffoli:

> “Computation - whether by man or by machine - is a physical activity and is ultimately governed by physical principles. An important role for mathematical theories of computation is to condense in their axioms, in a stylized way, certain facts about the ultimate physical realizability of computing processes. With this support, the user of the theory will be free to concentrate on the abstract modeling of complex computing processes without having to verify at every step the physical realizability of the model. Thus, for example, a circuit designer can systematically think in terms of Boolean logic (using, say, the AND, NOT, and FAN-OUT primitives) with the confidence that any network he designs in this way is immediately translatable into a working circuit requiring only well-understood, readily available components (the "gates," "inverters," and "buffers" of any suitable digital logic family).”

Landauer, Bennett, Fredkin, and Toffoli with their work on reversibility have created an inextricable bond between the mathematical theories of information, the physical principles that govern the machines, and the logic implemented in them, paving the way to the physical theory of information in classical terms.

The possibility of treating information in quantum terms is due to Feynman (1981) and Deutsch (1985). Feynmann in 1981 assumed that a device that can behave like nature could be realized:

> “Now, what kind of physics are we going to imitate? First, I am going to describe the possibility of simulating physics in the classical approximation, a thing which is usually described by local differential equations. But the physical world is quantum mechanical, and therefore the proper problem is the simulation of quantum physics - which is what I really want to talk about, but I’ll come to that later. So what kind of simulation do I mean? There is, of course, a kind of approximate simulation in which you design numerical algorithms for differential equations, and then use the computer to compute these algorithms and get an approximate view of what physics ought to do. That's an interesting subject, but is not what I want to talk about. I want to talk about the possibility that there is to be an exact simulation, that the computer will do exactly the same as nature.” (p. 468)

Deutsch in 1985 proved the validity of Feynman's theory and the possibility of the realization of a universal quantum computer, taking up the concept developed by Turing, of “functions which would naturally be regarded as computable” and reconceptualizing it as “functions which may in principle be computed by a real physical system” (Deutsch, 1985, p. 99). The author continues “For it would surely be hard to regard a function ‘naturally’ as computable if it could not be computed in Nature, and conversely” (ibidem, p.99).

Deutsch, Ekert, and Lupacchini (2000), about the inner intertwining between mathematics/logic and physical reality, argued

> "So where does mathematical effectiveness come from? It is not simply a miracle, "a wonderful gift which we neither understand nor deserve" - at least, no more so than our ability to discover empirical knowledge, for our knowledge of mathematics and logic is inextricably entangled with our knowledge of physical reality: every mathematical proof depends for its acceptance upon our agreement about the rules that govern the behavior of physical objects such as computers or our brains. Hence when we improve our knowledge about physical reality, we may also gain new means of improving our knowledge of logic, mathematics, and formal constructs. It seems that we have no choice but to recognize the dependence of our mathematical knowledge (though not, we stress, of mathematical truth itself) on physics, and that being so, it is time to abandon the classical view of computation as a purely logical notion independent of that of computation as a physical process" (Deutsch, Ekert & Lupacchini, 2000, p. 268)

Therefore today, even if the discourse of the logic structure described above is still open especially among mathematicians and within some theories such as the theory of categories (e.g., Heunen & Vicary, 2019; Abramsky & Coecke, 2009; Holdsworth, 1977), the word logic does not in general recall the logical structure carried out by Birkhoff and Von Neumann (1936), but more to the metaphysical level of discussion of Putnam (1968), that is it regards more the physical interpretation and implications of the logical structure. The close relationship between mathematics, logic, computation, and physics that also Deutsch, Ekert, and Lupacchini (2000) stressed, led to a description of operators and the logical operations in terms of physical systems, as, for example, Feynman did by representing the AND and OR operations in terms of the rules of binary addition on rows of pebbles (Feynman, Hey & Allen, 2018).

It is becoming increasingly clear that the abstract formalism of quantum theory has strong consequences on how information can be processed and that it is closely related to physics. This is an aspect that we stressed as pivotal, also elevating it to the rank of design principle of our instructional materials.

With inverse reasoning, starting from Wheeler's idea of "it from bit" (1989; 1992), researchers attempted to derive the quantum theory starting from only axioms of an informational nature.

The idea formulated by Wheeler (1992) can be so expressed:

"Every "It", every particle, every field of force - even the spacetime continuum itself - derives its way of action, its very existence entirely, even if in some context indirectly, from the detector-elicited answers to yes or no questions, binary choices, bits.

In another way of wording the idea which I put up for examination, all things physical, all its, must in the end submit to an information-theoretic description. (Ibidem, p. 282)

This idea presents the echo of Niels Bohr's philosophy, who pointed out how quantum and relativistic physics, forcing us to abandon the anchor of the visual reference of common sense, have imposed greater attention on language. With it, Bohr did not intend to deny the physical reality but recognized that there is a need for a language no matter what a person wants to do. In other words, language is the first toolbox we have to analyze the experience (Sini, 2013). In "Geons, black holes, and quantum foam: a Life in Physics" (1998), Ford and Wheeler wrote:

> "I build only a little on the structure of Bohr's thinking when I suggest that we may never understand this strange thing, the quantum, until we understand how information may underlie reality. Information may not be just what we learn about the world. It may be what makes the world. An example of the idea of it from bit: when a photon is absorbed, and thereby "measured"—until its absorption, it had no true reality—an unsplittable bit of information is added to what we know about the world, and, at the same time, that bit of information determines the structure of one small part of the world. It creates the reality of the time and place of that photon's interaction. (p. 481)"

Wheeler's "it from bit" on one hand paved the way for several attempts at axiomatization, such as Hardy (2001), D'Ariano (2011), and Masanes and Müller (2011), in which quantum theory was derived, within a very broad class of theories, through postulates of a physical character. However, in these works, there was always some mathematical assumption that could not be translated into elementary physical terms. On the other, Wheeler's vision has received harsh criticism, especially regarding the discretization of the classical world. For example, Deutsch (2003) in "it from Qubit" argued:

> "Logic is discrete: it forbids any 'middle' between true and false. Yet in classical physics, discrete information processing is a derivative and rather awkward concept. The fundamental classical observables vary continuously with time and, if they are fields, with space too, and they obey differential equations. When classical physicists spoke of discrete observable quantities, such as how many moons a planet had, they were referring to an idealisation, for in reality there would have been a continuum of possible states of affairs between a particular moon's being 'in orbit' around the planet and 'just passing by', each designated by a different real number or numbers." (p.2)

In the paper rather the researcher stressed what also Loyd (1995, 1997) pointed out: "Almost any physical system becomes a quantum computer if you shine the right sort of light on it".
Deutsch stressed the differences between classical and quantum computation according to the different computational universality of the hardware. In particular, in the quantum case, computational universality is a generic property of the simplest type of gate in all its formulation: for example, the set of all single-qubit gates, together with the controlled-not operation (measurement of one qubit by another) also suffice to perform arbitrary computations or single-qubit gates together with the uniquely quantum operation of 'teleportation' (Gottesman and Chuang 1999). Of this strong connection between quantum computation and quantum physics, in classical physics, there were only some hints (Deutsch, 2003). Deutsch's argumentation continues "Models of classical computation based on idealized classical systems such as 'billiard balls' have been constructed in theory (Fredkin and Toffoli, 1982), but they are unrealistic in several ways, and unstable because of 'chaos', and no approximation to such a model could ever be a practical computer. Constructing a universal classical computer (such as Babbage's analytical engine) from 'elementary' components that are well described in a classical approximation (such as cogs and levers) requires those components to be highly composite, precision-engineered objects which would fail in their function if they had an even slightly different shape" (p. 7). On the contrary, in ion trap systems, they are kept in a straight line by an ingeniously shaped oscillating electric field. Each ion is a qubit, that is it forms a two-state system. The interaction between the different ions takes place through a combination of the Coulomb force and an external electromagnetic field in the form of laser light. As Deutsch argued: "The engineering problem ends there. Once an arrangement of that general description is realized, the specific form of the interaction does not matter. Because of the generic universality of quantum gates, there is bound to exist some sequence of laser pulses – each pulse constituting a gate affecting two of the qubits – that will cause an N-ion trap to perform any desired N-qubit quantum computation. The same sort of thing applies in all the other physical systems – nuclear spins, superconducting loops, trapped electrons, and many more exotic possibilities – that serve, or might one day serve, as the elementary components of quantum computers." (Deutsch, 2003, p. 8)
Deutsch concludes his reflection by arguing:

> "So, what does that leave us with? Not 'something for nothing': information does not create the world ex nihilo. Nor a world whose laws are really just fiction, so that physics is just a form of literary criticism. But a world in which the stuff we call information, and the processes we call computations, really do have a special status. The world contains – or at least, is ready to contain – universal computers" (ibidem, p.13).

1.3.3 The linguistic dimension

This last dimension highlights the change of register: from the language of inability to the language of possibility. As Deutsch (2020) writes “Quantum mechanics was often characterized as a paler version of classical mechanics, due to its intrinsic uncertainty and stochasticity, in contrast with the clockwork precision and determinism of Newtonian trajectories. Quantum mechanics was viewed as a nagging parent always telling you what you can’t do. You can’t know a particle’s position and momentum at the same time. You can’t measure a system without disturbing its state” (Deutsch, 2020, p.2). The problem of the language, from the point of view of terminology, was raised by Bohr himself and later taken up by Lévy-Leblond. In his paper (1999), Leblond, following Bohr's idea according to "the exploration of new fields of physical experience [which] has demanded a radical revision [...] of our most elementary concepts", argued that the “linguistic inventivity” terminology of quantum physics, in contrast with Maxwell’s electromagnetism and Einstein’s relativity, has been drastically reduced (Lévy -Leblond, 1999, p. 78). Leblond continues “It is even a double paradox that physicists have never produced so many new ideas and so few new words and that they have used common and concrete words all the more so since their ideas became more esoteric and abstract (see "quarks" and their "colors", "big bang" and "chaos", etc.). [...] In recent times, it is probably the overtaking of science communication proper by mere advertising that explains the simplistic borrowing of picturesque but misleading common words, as in expressions like "big bang", "colored quarks" or "butterfly effect".” (Lévy-Leblond, 1999, p. 78 and 79). He finally proposed a new vocabulary to surpass the legacy of the classical language, such as the idea of wave-particle dualism. However, this new vocabulary never caught on.

Perhaps today the problem of language is not solved but the second quantum revolution has changed perspective. Its advent (QIS) and the advent of the second quantum revolution revealed that quantum mechanics is an *enabler* from a technological point of view. The inability of cloning a quantum state (Wootters & Zurek, 1982) has become a real resource because it can make encryption unconditionally secure, both for the distribution of quantum keys for communication channels (Bennett & Brassard, 1984) and banknotes (Wiesner, 1983). Quantum nonlocality allows us to teleport information from one space position to another (e.g., Ren, Xu, Yong, Zhang, Liao, Yin, ... & Pan, 2017; Ursin, Tiefenbacher, Schmitt-Manderbach, Weier, Scheidl, Lindenthal, ... & Zeilinger, 2007; Ursin, Jennewein, Aspelmeyer, Kaltenbaek, Lindenthal, Walther & Zeilinger, 2004; Bennett, Brassard, Crépeau, Jozsa, Peres, & Wootters, 1993). The quantum interference between an exponentially large number of outcomes can enable algorithms that can solve problems in a more efficient way than those that we believe to be solvable on a classical computer, e.g., Shor’s algorithm (Shor, 1994), and so on (Deutsch, 2020).

The point I want to emphasize here does not in itself concern new technologies within the new field of QIS, but the reversal of perspective that has been created. While I refer to the problem of language, those concepts that in the first half of the twentieth century seemed to concern more than anything else epistemological positions, have become enabling factors, opening new scenarios of possibilities, and posing new challenges to research.

In Figure 1.2, it is reported a schema of the discussion was just carried out. The elements of the figure, as I present in the following, are key elements of our design principles.

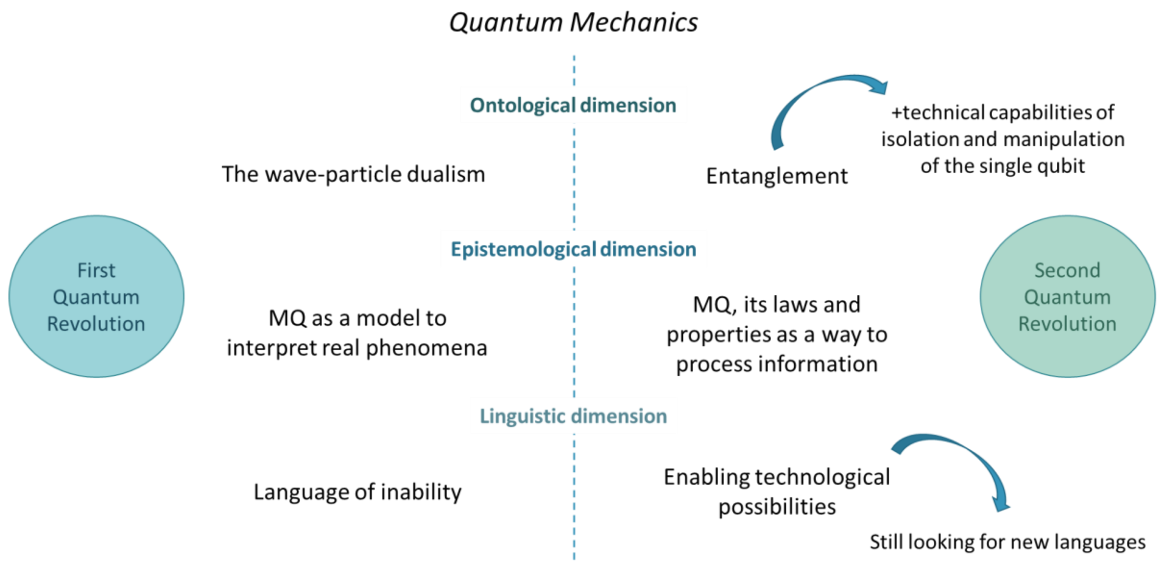

Fig. 1.2: schema of the revolutionary aspects between the First and the Second Quantum Revolution. The "ontological" dimension refers to the key concepts ("quantum mysteries") on which the two revolutions are based, namely: the complementary principle and the entanglement. The "logical and epistemological" dimensions emphasize, on one hand, the development of a logical structure and its physical interpretation and, on the other, the change of problem's perspective and approach: quantum mechanics is not only a way to describe nature, but its rules can be used to manipulate nature to solve problems. The "linguistic" dimension refers to a new perspective of the linguistic register: from the language of inability to the language of technological possibilities.

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Chapter 2: Bologna's educational approach to the Second Quantum Revolution

Introduction

This chapter is dedicated to the presentation of how we addressed and contribute to the two research questions presented in chapter 1 (Satanassi, Ercolessi & Levrini, 2022):

> *RQ1. Which teaching approach can be developed to underlie the educational potential of technologies even beyond technical training?*
> *RQ2. What instructional design principles can be pointed out to value the Second Quantum Revolution as a cultural revolution?*

Before going into the design principles of the Second Quantum Revolution, I describe the previous work of the research group in physics at the University of Bologna in teaching and learning quantum physics. I begin with the introduction of one of the first works done by the research group reported in a paper by Levrini and Fantini (2013), which is indicative, in general, of the approach of the group in the plurality of themes it addresses and, in particular, at the basis of both the work that has been done regarding the first quantum revolution (Ravaioli, 2020; Ravaioli & Levrini, 2017; Lodovico, 2016; Stefanini, 2013; Lulli, 2013; Tarozzi 2005) and, then, the second one.
Both the approaches to the First and the Second Quantum Revolution revolve around the idea of *properly complex territory* (Levrini & Fantini, 2013), presented in Section 2.1 together with the notion of disciplinary authenticity (Kapon, Laherto & Levrini, 2018). In Section 2.2, the works carried out for the first quantum revolution are presented and commented through the two lenses introduced in section 2.1. In Section 2.3, I describe the first four design principles for the Second Quantum Revolution (Satanassi, Ercolessi & Levrini, 2022), and in Section 2.4, three other design principles are introduced, corresponding to the contexts of the projects in which the approach to the Second Quantum Revolution was developed: the I SEE, the IDENTITIES, and the FEDORA project. Finally, the course on the Second Quantum Revolution and its refinement throughout the years is presented.

2.1 Properly complex territories and disciplinary authenticity

The idea of properly complex territory was elaborated in a joint work following up the symposium ''Why complexity is important for learning'', presented at the AERA Conference in San Francisco, 2006 (Levrini, Parnafes, diSessa, Bamberger & Hammer, 2006). As Parnafes (2010) argued, Levrini et al. (2006) refer to complexity as "ways of knowing and conceptualizing the structures of physical phenomenon, and problematic nature of learning, as involving puzzlement, ambiguity and uncertainty" (Parnafes, 2010, p. 565-566). What they emphasize during the symposium is the importance of embracing this complexity and the role of epistemological complexity in learning and in the students' sense. The need for this construct, similar to the notion of epistemological complexity (diSessa, 2013; diSessa, 2008; diSessa, 1993), arises from an observation: In recent years, at least in

Italy, we have witnessed a progressive simplification of textbooks dictated by the belief that the emotional detachment of students from science is due to its complexity (Levrini & Fantini, 2013). As some European reports claim, these forms of simplification are likely to be counterproductive. Sjøberg (2002) argued:

> "the implicit image of science conveyed by these curricula is that it is mainly a massive body of authoritative and unquestionable knowledge. Most curricula and textbooks are overloaded with facts and information at the expense of concentration on a few "big ideas" and key principles. […] There is often repetition, with the same concepts and laws presented year after year. Such curricula and textbooks often lead to rote learning without any deeper understanding so that, unsurprisingly, many pupils become bored and develop a lasting aversion to science. Moreover, this textbook science is often criticized for its lack of relevance and deeper meaning for the learners and their daily life. The content is frequently presented without being related to social and human needs, either present or past, and the historical context of discoveries is reduced to biographical anecdotes. Moreover, the implicit philosophy of textbook science is considered by most scholars to be a simplistic and outdated form of empiricism" (Sjøberg, 2002, p. 9, 10).

Learning content without a deeper understanding risk making physics meaningless, and "for a few". Levrini, Parnafes, diSessa, Bamberger, and Hammer (2006) paved the way to reflect on how it is possible to involve students in scientific discourses without following the main simplification strategy. Levrini & Fantini (2013) emphasized that the key assumption on which the presented research is based is that:

> "a teaching/ learning process is meaningful if it combines, in a productive way, three complex systems: the real world, the system of physics disciplinary knowledge and the cognitive system of learners. Because of the involved forms of complexities, hyper-simplified instructional descriptions and explanations, by making the material seem easy, can dangerously distort the learning process as well as the content. On the contrary, we assume that, once useless complications are avoided, physics should be made as simple as possible, but not so simple as to lose its cultural and educational value." (Levrini & Fantini, 2013, p. 1896).

In the paper, the researchers discussed ***productive forms of complexity*** as criteria for designing teaching proposals potentially able to foster the creation of appropriate learning environments, namely properly complex territories.
The authors highlighted that the textbooks and popular science books often paint, in particular, quantum physics as a "conceptually disconnected territory" composed of fragments of information organized in chronological order of the discoveries and usually supported by semi-classical models. Research works in the literature show that one of the most problematic consequences of this conceptual fragmentation is that students, trying to fill the information gaps, risk assigning classical properties to quantum systems (or relativistic ones) since they could arrive at unsatisfactory conclusions revealing a deep skepticism about quantum physics itself (e.g. Levrini & Fantini, 2013; Kalkanis, Hadzidaki & Stavrou, 2003; Irenson, 2001; Giliberti & Marioni, 1997; Seifert & Fischler, 1999; Mashaldi, 1996). In Hadzidaki's words

> "students, in their effort to reconcile the features of the analog structure (classical physics) with those of the target (quantum mechanics), tend to assimilate the newly considered quantum mechanical concepts into categories and modes of thinking that are deeply rooted into classical physics worldview" (Hadzidaki 2006, p.614).

Therefore, the image of quantum physics that emerges from textbooks can lead to dangerous or unproductive simplifications (Levrini & Fantini, 2013). In particular, Levrini and Fantini individuated to

particular simplifications: the ‘‘constitutive’’ hyper-simplicity of semi-classical images that are commonly used; the linearity and ‘‘thinness’’ of the content sequences.
The first kind of simplification is connected with the trend in teaching and learning physics, and science in general, of the use of external representations (see chapter 5). In many cases, the recourse to visual representations consists of a productive simplification strategy since they focus on the relevant details of a system, and they synthesize them into an intelligible, familiar, and coherent whole (ibidem, 2013). A productive simplification strategy and the resulting coherent and intelligible representations can “bridge the gap between the rich world of experience and the formal physical reconstruction of such a world” (ibidem, p. 1898). The authors continue “the gap is in-principle bridgeable when the mathematical reconstruction of the world provides a ‘‘projection’’ of the world itself in a Euclidean space, i.e., when the properties conceptualized by the mathematical description can be synthesized in a representation that ‘‘lives’’ in a space somehow ‘‘isomorphic’’ to the space we experience” (Levrini & Fantini, 2013, p.1898). Also De Regt (2014) argued that “classical physics is visualizable and accordingly intelligible” (De Regt, 2014, p. 384). Classical theories presuppose a space-time framework, which is a necessary, but not sufficient, condition for visualizability.
From an educational perspective, this productive character of visual representation is intrinsically related to the students’ epistemological and metacognitive competencies in recognizing the role and the meaning of modeling in physics (Levrini & Fantini, 2013).
Starting from the nineteenth century, many physics theories, from thermodynamics to relativity to quantum physics, possess an abstraction degree that visualization is not immediately possible. In this case, the process of modeling and, thus, the implementation of a productive implementation strategy is not trivial, since the mathematical description projects the real world in abstract and unfamiliar spaces (e.g., the Minkowskian space-time or the Hilbert space). These spaces are intellectual constructions, and they cannot be related to ‘‘real world space’’ in an intuitive way. For example, wave-particle dualism is an example of the loss of visualization: “any attempt to synthesize the behavior of a quantum system in only one familiar picture (either particle or a wave) unavoidably shows the inner partiality of such a description” (ibidem, p. 1898). There is a new necessity of referring to abstract spaces as new synthetic scenarios to put and organize properties of physical systems in a coherent and integrated way. Therefore, it is important to be careful to use visualization as a simplification strategy since the representations are often partial and sometimes also misleading. So, the risk is that representations can shortcut the pivotal elements, especially in the case of modern physics (ibidem, 2013).
The second form of simplification that Levrini and Fantini (2013) pointed out is the linearity and ‘‘thinness’’ of the content sequences, namely sequences in which the reasoning is constrained to follow a single route, a pre-fixed inner step-by-step logic path. In Misky words:

> “An idea with a single sense can lead along only one track. Then, if anything goes wrong, it just gets stuck – a thought that sits there in your mind with nowhere to go. That’s why, when someone learns something ‘‘by rote’’ – that is, with no sensible connections–we say that they ‘‘don’t really understand.’’ The secret of what anything means to us depends on how we’ve connected it to all the other things we know. That’s why it’s almost always wrong to seek the ‘‘real meaning’’ of anything. A thing with just one meaning has scarcely any meaning at all” (Minsky, 1986, p. 64).

The researcher continues “too many indiscriminate connections will turn your mind to mush. But well-connected meaning structures let you turn ideas around in your mind, consider alternatives, and envision things from many perspectives until you find one that works. And that’s what we mean by thinking!” (Ibidem, p.64), highlighting that the complexity of meaning, to promote productive thinking, has to be respected particularly in the content organization by creating well-connected meaning structures.
Levrini and Fantini (2013) unpacked, from an educational perspective, the complexity of the meaning finding three core aspects that teaching proposal ought to search:
(1) “its <u>inner coherence</u> in moving longitudinally from one step (concept) to another;

(2) the <u>richness</u> of the transverse connections it is able to suggest and foster with other possible routes (the meaning structures);
(3) its <u>thickness</u>: a sort of the third dimension represented by metacognitive, epistemological or philosophical discourse from which the physical contents can be looked at and also be well-connected" (Levrini & Fantini, 2013, p. 1899).

To address these two simplifications, the authors reconceptualized these three core aspects into some forms of productive complexity. These forms of productive complexity, framed within the notion of learning environment as properly complex territory, "should be implemented in the design of instruction and instructional materials so as to pursue, during implementation, the general goal of enabling students to find out their own ways of pinpointing or solving the problems and puzzlements they perceive" (Levrini & Fantini, 2013, p. 1899). The main forms of productive complexity that can be considered in designing the proposal:

A. **Multi-perspectiveness**—the same physical contents (phenomenologies) are analyzed from different perspectives so as to encourage multiple connections among the content and conceptual routes.
B. **Multi-dimensionality**—the different perspectives and multiple connections are analyzed and compared also for their philosophical-epistemological peculiarities, as well as for their relations with experiments and formalism.
C. **Longitudinality**—The ''game'' of modeling quantum phenomena is systematically analyzed and compared with the models already encountered by the students during the study of other physics topics (from classical mechanics to thermodynamics, to special relativity and to quantum physics).

Some time was spent here introducing the notions of properly complex territories, forms of productive complexity, and the discussion behind their idea since they embody the epistemological frame of the Bologna PER group toward the knowledge and the values that guided and are still guiding our research in teaching and learning science. In fact behind the concept of properly complex territory there is a key and broader educational value: science, that is the development of disciplinary knowledge and epistemological competencies, can be a locus for students' **identity development** as individuals, citizens, and future professionals (Levrini, Levin & Fantini, 2020; Levrini, Levin, Fantini & Tasquier, 2019; Kapon, Laherto & Levrini, 2018; Levrini, Levin & Fantini, 2017; Levrini, Fantini, Tasquier, Percori & Levin, 2015).
Recalling the initial problem of the progressive impoverishment of school textbooks which reflects the school system, Kapon, Laherto and Levrini (2018) shed light on the contemporary need to "awaken the students' scientific spirit". This need can be linked to the need to return to an **authentic image of science**. Authenticity is conceived to be discipline-driven and it has to reflect the desire to simulate "real" scientific practices in the classroom and the epistemology and reasoning that such practices require (Kapon, Laherto & Levrini, 2018; Crawford, 2012; Chinn & Malhotra, 2002). For Watkins, Coffey, Redish, and Cooke (2012), authentic activities in science are "those that use tools—such as concepts, equations, or physical tools—in ways and for purposes that reflect how the disciplines build, organize, and assess knowledge about the world" (p. 2). Unfortunately, as the literature suggests, disciplinary authenticity does not usually characterize the practice of learning science in school (Erduran & Dagher, 2014a; Allchin, 2011; Chinn & Malhotra, 2002; Brown, Collins, & Duguid, 1989). Science instead is usually involved in the process of transformation based on the purposes of teaching and learning in school. As Kapon et al. (2018) argued

> "this transformation is a complex and delicate process involving many rationales, influences, and constraints that are institutional, educational, political, practical, and economic rather than scientific. Thus, the process can result in a construct that is detached from the epistemological, social, dialectical, and affective nature of an authentic scientific enterprise [...] The distinction between disciplinary authenticity and

school science can also be understood in terms of culture. School culture has a profound influence on what is learned in school (Lave, 1997), and due to unbridged cultural differences schoolwork often becomes inauthentic (Brown et al., 1989). Our use of the term common school science refers to science as it is often taught to and experienced by students in school as a result of prevailing cultural and institutional constraints" (Kapon, Laherto & Levrini, 2018, p. 1079).

Therefore, the challenge is to fill school science with disciplinary authenticity that encompasses the conceptual, epistemological, social-institutional (e.g., Erduran & Dagher, 2014a,b; Irzik & Nola, 2011; 2014), and affective aspects (e.g., Jaber & Hammer, 2016) of doing science. Disciplinary authenticity is seen as

> "a rich, complex, and context-dependent construct, with diverse manifestations. Hence, aiming for disciplinary authenticity in education in our view means designing and facilitating learning experiences that immerse students not only in the conceptual and epistemological features of science but also in its social dialectical practices, as well as its affective features" (Kapon, Laherto & Levrini, 2018, p. 1078).

Kapon, Laherto, and Levrini (2018) argued that addressing disciplinary authenticity in school means that "learning experiences must be deeply rooted in and reflect the nature of both contemporary scientific endeavor as well as throughout the history of science" (ibidem). The scientific endeavor in its richness and complexity offers a plurality of manifestations in different educational contexts. As the authors discussed, the nature of disciplinary authenticity changes with the educational contexts. Even if it is rooted in the nature of the scientific endeavor, in different educational settings its meaning and manifestation come up through interactions with teachers, students, and educational social infrastructures, which, in turn, depends on different cultures, institutional constraints, intellectual interests, and practical needs. For example, in some educational contexts a manifestation of disciplinary authenticity may highlight the practices of doing science and generating scientific knowledge, while in other contexts, more historical and philosophical–oriented settings, may point out critical reflection on the epistemological and historical processes of the construction and development of scientific knowledge (Kapon, Laherto &Levrini, 2018).
In literature, some works discuss the importance of **agency** for students and teachers in determining the meaning of authenticity. Authenticity is perceived as an "emergent" quality derived from the interaction between the participants (e.g., students and teachers) and the (social) context of the activity (Rahm, Miller, Hartley, and Moore, 2003) to, for example, foster them to assume ownership of the scientific questions they explore (Rivera Maulucci, Brown, Grey, & Sullivan, 2014; Calabrese-Barton, 2001). The educators have not only to engage students in professional practices but also to make them experience authenticity through the complexity of collaboration, ownership, and meaning-making (Rahm et al., 2003). In literature, some research tried to design pragmatic approaches to engage students and teachers with disciplinary authenticity. For example, Radinsky, Bouillion, Lento, and Gomez (2001) elaborated an educational model that aims to give students an authentic experience by collaborating with a technology company in a "mutual benefit partnership." For the authors, the authenticity lies in students' abilities to make meaning from their experiences and to contribute to the partnership dialogue. Rivera Maulucci, Brown, Grey, and Sullivan (2014), based on the epistemological theory that students learn in social contexts while addressing a personally relevant problem, argued that the most important feature of an authentic science learning activity regards what students learn about how science is done and about themselves in doing science. In particular, for them, an approach to science inquiry has to make students i. develop authentic and personally relevant science knowledge; ii. shape their inquiries based on their knowledge; iii. transform their relationship and their interest in science; iv. affirm their identity as potential scientists; v. engage with science social enterprise, and vi. develop a sense of agency. Miller, Manz, Russ, Stroupe, and Berland (2018) elaborated on the construct of epistemic agency to explore how students can meaningfully contribute to the knowledge and practices of a classroom community. Supporting the development of epistemic

agency provides, for them, opportunities for students to co-construct, adapt, and evaluate knowledge-building practices (Miller, Manz, Russ, Stroupe & Berland, 2018). Elaborating on these, disciplinary authenticity is thus linked also to personal relevance (Kapon, Laherto &Levrini, 2018). For them, "personal relevance encompasses students' sense of benefit, value, and meaningfulness, as well as agency as users and generators of what is learned" (ibidem, p. 1080). Meaningfulness means what students learn and, at the same time, what makes sense to them on a personal level and which they actively strive to achieve. Some frameworks and constructs in literature proved to be meaningful to manage and value the tension between disciplinary authenticity and personal relevance (and school science), such as students' sensemaking (Kapon, 2017; Kapon & diSessa, 2012) and appropriation (Levrini, Fantini, Pecori, Tasquier, & Levin, 2015; Levrini, Levin, & Fantini, 2018).

This discussion on ***scientific authenticity*** brings us into a broader context than that of ***properly complex territories*** characterized, in the terms presented, by a much closer nuance to the field of education and the reconstruction of content. However, the ***properly complex territories*** represent a construct that can be elaborated not only to avoid dangerous simplifications and make teaching and learning more productive but also to reflect on scientific authenticity and promote the development of the identity of students. In figure 2.1, it represents the relationship between authenticity and properly complex territory. It represents the disciplinary authenticity dimensions that Kapon, Laherto, and Levrini (2018) pointed out. The articulation of dimensions stresses to what extent the properly complex territory can be conceived as a construct through which the aspects of authenticity's dimensions can inform the process of design and make it meaningful for the development of students' identities.

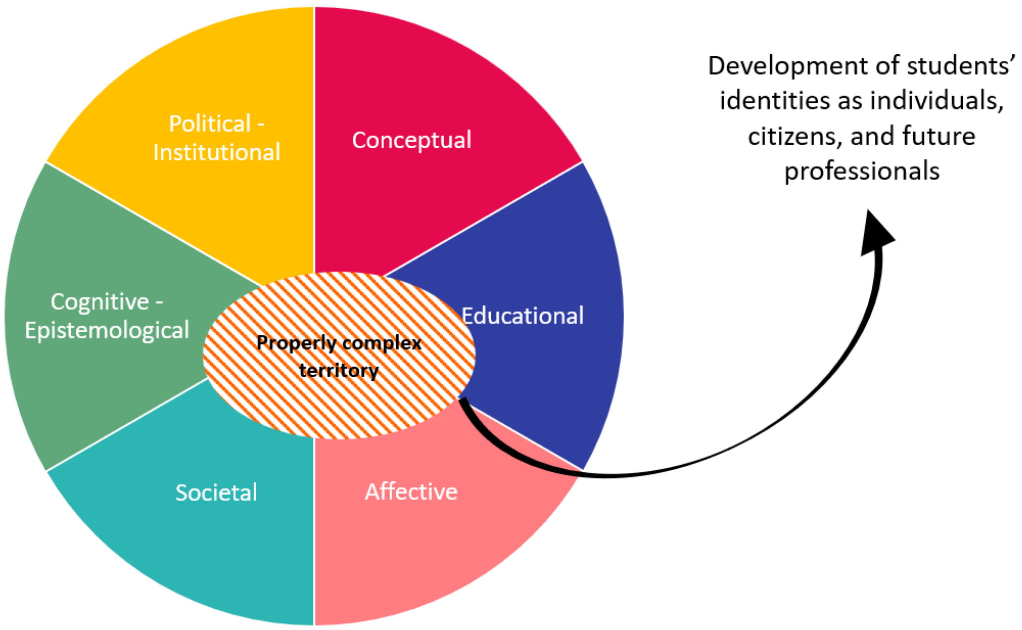


Fig. 2.1: Properly complex territory as a construct that takes into account the disciplinary authenticity's dimensions that, in the process of instruction design, can boost processes of identities development.

The properly complex territory represented in figure 2.1 oriented in the design of the teaching modules on both the First and the Second Quantum Revolution. The features of the properly complex territory have been progressively reconceptualized in terms of design principles for the activities and modules that the group has realized in the last decade. In the next, I present how these aspects were considered in the modules' design and implementation concerning the two revolutions. Regarding the first one, the work was carried out mainly by professors Olivia Levrini and Paola Fantini (2013), then by Giovanni Ravaioli (Ravaioli & Levrini 2017, Ravaioli 2020), with the help of other experts such as a theoretical

physicist, Elisa Ercolessi, and an experimental one, Giorgio Lulli. Section 2.2 sums up their work around the first quantum revolution and its mystery, the wave-particle dualism.
My contribution concerns the Second Quantum revolution. In section 2.3, I present how those aspects were implemented in this case and what design principles were elaborated (Satanassi, Ercolessi & Levrini, 2022).

2.2 Principles for the First Quantum Revolution

As anticipated, the group in physics education at the University of Bologna has a rather long and consolidated tradition of research in teaching and learning quantum physics for high school and pre- and in-service teachers' education and designed an educational path that over the years changes and was refined different times both from a contextual and structural point of view. The course over the years was refined and implemented several times.

- The first and seminal path was developed in 2005, and it is described in Tarozzi (2005) and Levrini and Fantini (2013). It was designed to address the issues raised in the previous section and make the learning environment rich, complex, meaningful, and navigable by the students in different ways. It has been implemented by professors Paola Fantini and Olivia Levrini with a class in the fifth year of an Italian Scientific Liceo (High School) of a specific curriculum on scientific and informatics subjects (PNI, Piano Nazionale Informatico).
- The second (2009-2013) was developed in collaboration with the CNR-IMM of Bologna and the project "Piano Lauree Scientifiche" (PLS). The educational path was proposed as an extra-curricular course involving volunteer students. The course introduced the students to some core concepts of quantum physics starting from the experiment that a 2002 poll among the readers of Physics World magazine declared as "The most beautiful experiment in Physics" (Levrini, Lulli, Bertozzi, Ercolessi, Matteucci, Monzoni, Pecori, 2014; Lulli, 2013; Stefanini, 2013). This experiment, that is the double slit experiment with single atoms, was realized for the first time in Bologna by three experts Merli, Missiroli, and Pozzi (1976).
- Then, the two previous paths converged into a third one (2013-2016) that was designed after the reform of Italian Licei Scientifici (2010), which introduced quantum physics topics in the last year of High School. Thus, this path has been thought to be implemented again in high school classes. A detailed description is reported in Lodovico (2016).

These implementations are characterized by the conviction that the knowledge can be articulated in different layers (from the conceptual to the experimental, historical, formal, and logical-interpretative ones) and are designed to introduce students to what for Feynman was the "heart of quantum mechanics", the "only mystery", that is the complementary principle.
As previously introduced, in this section the aim is to show how the three forms of productive complexity (Multi-perspectiveness, Multi-dimensionality, and Longitudinality, see section 2.1) had been implemented and what kind of design principles were pointed out.
The first educational path consisted of two parts. The first one consisted of a historical-epistemological reconstruction of how the concept of object changes in the transition from classical physics to quantum physics, introducing some concepts that belong to the old quantum physics, such as the uncertainty and complementarity principles, through the analysis of passages and debates of the protagonists of that period. Therefore, the first two forms of complexity by Levrini and Fantini (2013) were implemented by introducing students and guiding them in the analysis of those key historical-epistemological debates, such as the Heisenberg and Bohr debate about the interpretation of uncertainty and complementarity, Bohr - Einstein debate about determinism and the relationship between knowledge and reality and Heisenberg and Schrödinger debate about the visualization of quantum objects. As the authors argued "The historical-epistemological debates were explicitly used both for allowing a variety of possible connections (the meaning structures) to be stressed according to the multiple perspectives supported by the physicists involved in the debates, and the development

of the epistemological dimension (multi-dimensionality) to be promoted" (Levrini & Fantini, 2013, p. 1900). The longitudinality, that is the systematic comparison between quantum physics and classical theories, has been explicitly implemented by posing and reformulating, during the design and development of the teaching proposal, the following problematic question: "How does the concept of object change from classical to quantum physics?" (Ibidem, p.1900). More deeply students were guided to reflect on (Levrini & Fantini, 2013, p. 1900, 1901):

- The Heisenberg microscope to problematize the ''disturbance'' interpretation, also in the light of Bohr's criticisms and his interpretation of uncertainty based on complementarity (Hadzidaki 2006);
- Bohr's view about both the limits of natural language (and classical images) for describing the quantum world and the claimed necessity of classical language for describing the experimental (macro) apparatus;
- Einstein's sharp attacks against uncertain and probabilistic physics descriptions of the world and how and why his criticisms moved from a supposed inner inconsistency of the theory to its incompleteness;
- Heisenberg's and Schrödinger's world views led the former to support the need for accepting that quantum description does not admit any visualization, and the latter to support the need for a sort of visualization for making theory intelligible, and aesthetically (physically) acceptable (de Regt 1997).

The second part consisted of the introduction of the Stern & Gerlach (1922) and double slit experiments and the formalization of some concepts already introduced, such as the concepts of quantum state, state preparation, operator, eigenstate, eigenvalue, superposition principle, complementarity, uncertainty, measuring process and entanglement (Pospiech 1999; Pospiech 2000).

The course was then revised to value the double-slit experiment in collaboration with other researchers of the Department of Physics and Astronomy of the University of Bologna (Olivia Levrini, Eugenio Bertozzi, Elisa Ercolessi, Giorgio Matteucci, Vittorio Monzoni and Barbara Pecori) and a CNR researcher (Giorgio Lulli). The "most beautiful experiment", before being realized (for the first time by Merli, Pozzi, and Missiroli, 1974), was a gedankenexperiment elaborated by Einstein to prove the erroneousness of the uncertainty and complementary principle, at the center of the Bohr-Einstein debate. From the first implementation, in which Levrini and Fantini already addressed the debate, many laboratories activities (such as a lab experience about wave and light interference, and another about the wave behavior of electrons with an electron microscope, Stefanini, 2013) were added to reflect on the wave-particle dualism also from an experimental and technological point of view and to raise the main contradictions and the interpretative limits of classical mechanics, which impose the elaboration of new logic to overcome the inconsistencies. Therefore, the course was refined and built around the analysis of the double slits experiments and their variants not only to give the opportunity to introduce some pivotal concepts and themes of quantum physics but also to reflect on the multi-layered structure of physical knowledge (historical, formal, and logical-interpretative). In particular, adding laboratories' experiences and insights on technological applications can

- make the students reflect on the process of "carrying out an experiment";
- make them reflect on the limits of the interpretative paradigm of classical physics, faced with challenges on the experimental front;
- introduce and progressively construct a conceptual structure (a "lens" built on a new logic) capable of interpreting the experimental evidence;
- make them reflect on the link between science and technology.

Finally, the two courses converged into a third one, whose design involved researchers in physics education, a theoretical physicist, four physics and mathematics teachers, post-doc students, and undergraduate students. What emerged was a course divided into three parts. The first one, the *pars destruens*, addresses four fundamental phenomena related to the "old quantum theory", foreseen also by the National Indications for Scientific High Schools: black body, photoelectric effect, Compton effect, and Bohr's atomic model. With the aim of avoiding the simplification and fragmentation to which the textbooks have gone through the years (see section 2.1), the group chose as a fil rouge to link the topics and place them in a significant and coherent framework for the discrete-continuous debate. The second part, the *bridge*, paves the way to the *pars construens* by presenting the first step for the construction of a new comprehensive framework that could account for all those phenomena that challenged and put in crisis the classical paradigms. For this purpose, the uncertainty principle, the complementary principle, and the most beautiful experiments were introduced. Finally, the last part, the *pars construens*, aims to build the new framework by focusing on completely new quantum physics features to avoid any semi-classical misconception (Pospiech, 2000). For this purpose, the Stern-Gerlach experiments were used to reconceptualize in a new way the quantum logic and to introduce the concept of superposition principle, state manipulation, measurement, and entanglement. A deep description of the activities of the course is reported in Ravaioli (2020).

Starting from the properly complex territory and a literature review to also consider students' difficulties to understand and accept quantum physics, Ravaioli (2020) pointed out a bunch of design choices that characterize the last version of the course. In particolar:

- design choice 1: to structure the laboratories' activities so as to provide ways and time to let students explicitly model the physical systems and the measurement processes and to enable them to consciously analyze the passage from the theoretical design of the experiment to the reality of instruments.

- design choice 2: to emphasize the models of interaction processes and narratives focused more on systems than on the physical objects to avoid inappropriate hyper-simplistic object-based interpretations of quantum phenomena.

- design choice 3: to explicitly compare and discuss the different representations and different representational forms to develop students' meta-representational competencies (diSessa, 2004; diSessa and Sherin, 2000, see chapter 5) and to individuate the main constituting elements and hidden models upon which the representations are built.

- design choice 4: within the diversity of choices, to bring out transversal threads from the 'Old quantum physics' part to raise the need for a 'jump' to the quantum logic, and only then compare the two domains on an epistemological level. The structure according will thus follow this sequence:
    - I. Single-electrons interferometer to pose the problem of classical categories inadequacy;
    - II. Levy-Leblond's platypus/quanton metaphor for setting the need for a new ontology and introducing a new grammar;
    - III. Two-state spin approach to introduce the formalism and detach from classical quantities;
    - IV. Formal comparison between two-way systems (single-electrons interferometer, Stern-Gerlach magnets, Mach-Zender interferometer) to foster the transfer of quantum state descriptive power to a broader phenomenological range, and to shift the narrative from objects to systems;
    - V. Entanglement as an only-quantum property.
- design choice 5: to discuss with the whole class about not only themes related to quantum physics dilemmas, but also general epistemological issues, such as (i) the interplay between modeling, experiments, and mathematics, (ii) the role of visualization and representations in physics, and (iii) the comprehensibility of quantum physics. The discussions are held by one (or two, in one case) professors with the whole class, fostering as much as possible students'

engagement and freedom by posing the problems in non-authoritative ways, exploring students' ideas, and allowing contradictory viewpoints (Mortimer & Scott, 2003).

The path incorporates a multiplicity of aspects that belong to the different dimensions that I previously stressed as an articulation of disciplinary authenticity. For example, the use of historical-epistemological debates introduces authentic voices of science in its making, by stressing the bigger breaks that characterize quantum physics with respect to classical physics. The historical-epistemological debates return an image of non-monolithic science and allow to shed light on the processes of development of scientific knowledge (Kapon, Laherto & Levrini, 2018). Some studies also argue that incorrect conceptions that students may have in the construction of knowledge are found as milestones that have led to the development of a certain theory (e.g., Galili, 2012). They could legitimize students' different epistemological positions since also in the case of the physicists a quantum mechanical description of the world proved to touch very deep chords and the world's visions. By adding the experiments, it is possible to engage students in scientific practices such as asking questions, defining problems, planning and carrying out investigations; analyzing and interpreting data (etc), which, as for example Osborne (2014) argued, "present a more authentic picture of the endeavor that is science" (Osborne, 2014, p. 183).
A deeper description of the link between the course design taking into account the forms of productive complexity and the dimension of disciplinary authenticity is reported in table 2.1.

Table 2.1: the disciplinary authenticity of the module about the first quantum revolution.

	Disciplinary Authenticity	**Aspects of designing taking to account a learning environment as properly complex territories and the forms of productive complexity**
	Conceptual	- Radiation-matter interaction - Probabilistic description and interpretation of nature - Not commutability of the observables - Not locality principle - Reconceptualization of causality principle - Double slits experiments - Stern-Gerlach and Mach - Zender experiments - Superposition principle - Measurement - Formal description of the quantum model: a new abstract space, the Hilbert space
	Cognitive - Epistemological	- Shaping the lens: toward a systemic view and looking for interaction processes - Determinism – indeterminism debate - Discrete – continuum debate - Epistemological vs. ontological probability - Nature and role of the observer and of measurements' tools - Quantum object: from the incompatibility of the wave-particle dualism to a superposition state - Quantum physics interpretations - Relation between the formal aspects of a model and the nature of the phenomena that they describe - From a space-time model and representation to an abstract one - When we stopped understanding the world: towards Feynman's "shut up and calculate!" - What an experiment is: from the design, to test assumptions, to test the compatibility between the model and experimental data, to manipulate data, to read an information, to interpret the data, … - Comparison between different experiments (Stern-Gerlach and Mach - Zender experiments) and the kind of nature they investigate - Classical vs Quantum logic

	Educational	- Creation of a comprehensive and significative scenario to put in coherence classical and quantum physics; - Multiple contexts and representations: enlarging the span and fostering alignment, toward conceptual change - Considering students difficulties - A glance at science in its making - Multidimensional knowledge organization (conceptual, experimental, historical, formal, and logical-interpretative): toward inclusiveness - Engaging students in scientific practices and development of experimental abilities - Non monolithic image of science - Toward the development of probabilistic thinking
	Affective	- Legitimization of students' different epistemological positions - Legitimization of students' difficulties to accept quantum physics as a personally reliable theory
	Political/institutional	Consensus building: toward Copenhagen's interpretation
	Societal	- From the theory to the applications - Impact of the application on the society - Reflection about the trichotomy science/technology – nature – humans

2.3 Principles for the Second Quantum Revolution

The structure of argumentation in this section on the Second Quantum Revolution is rather different. Here I start by introducing the design principles we pointed out (Satanassi, Ercolessi & Levrini, 2022) to construct a teaching module aimed to value the new revolution as, mainly, a cultural revolution. Then, I describe three further principles (section 2.4) that emerged from three different projects and progressively enlarged the scope of the module's design and implementation. The bigger value of the module's design continues to be the one described in section 2.1: to enhance the multidimensionality of disciplinary authenticity to foster students to develop and shape their personal identities as individuals, citizens, and future professionals also through the frame of science education. The final module, its refinement, the activities, and the implementations are described in section 2.5, except for two activities that are deeply discussed in chapter 3.

2.3.1 The Second Quantum Revolution from a cultural perspective

As discussed in the introduction, the Second Quantum Revolution revolves around different aspects compared to the First one, namely another mystery of quantum physics, the entanglement, and the developed techniques that allow us to isolate, control, and manipulate the single qubit (atom, photon, ion). These aspects are making it possible to develop technologies with a lot of potential.
When we started to deal with quantum technologies and attended some popular seminars and conferences both about classical and quantum computers, we perceived an "asymmetry". We guess that rarely, if ever, a popular conference on classical computing would start by explaining the physical laws according to which hardware and logical gates are realized and operate. On the contrary, most public seminars and conferences about quantum technologies usually include an introduction to the new unit base and its features (superposition principle and entanglement) and a discussion about, for example, the better techniques to create qubits and manipulate them (Satanassi, Ercolessi & Levrini, 2022).

The design started from this asymmetry and from the need to change the way of talking about classical computers, by taking a step back to the hardware and emphasizing the role and features of the "deterministic and linear logics of classical physics" they follow. In such a way, we aim to create a new meaningful scenario, a conceptual structure that guides students to compare (more closely) the binary and non-binary logic at the core respectively of classical and quantum computers and, through this comparison, to develop (without becoming bogged down in too many technical details) an idea of what we mean today by quantum simulators, quantum computers and how quantum logic gates and quantum circuits work. This idea represents our first design principle:

Design principle #1: to foster a close comparison between classical and quantum computers through an analysis of the different logic underlined in the basic mechanisms on which the hardware is built.

This principle leads directly to the second principle and the issue represented by the relationship between hardware and physical experiments. In quantum computing, a Mach-Zehnder or a Stern-Gerlach apparatus can be conceptualized as a quantum simulator (Deutsch, 1985). What does this mean? How can an experiment be considered a quantum simulator? What about the corresponding case in classical computers? Quoting Preskill:

> "Information, after all, is something that is encoded in the state of a physical system; a computation is something that can be carried out on an actual physically realizable device. So the study of information and computation should be linked to the study of the underlying physical processes" (Preskill, 1998, p. 7).

This idea represents our second design principle:

Design principle #2: to reconceptualize the foundational experiments in terms of computation, so as to discuss why and how experiments can be considered as "simulators" or devices to process information (circuits). This means, operationally, re-reading the three main phases of an experiment - *state preparation, state evolution/manipulation, and measurement* - in terms of *input* - *processing* - *output information* (Fig. 2.2).

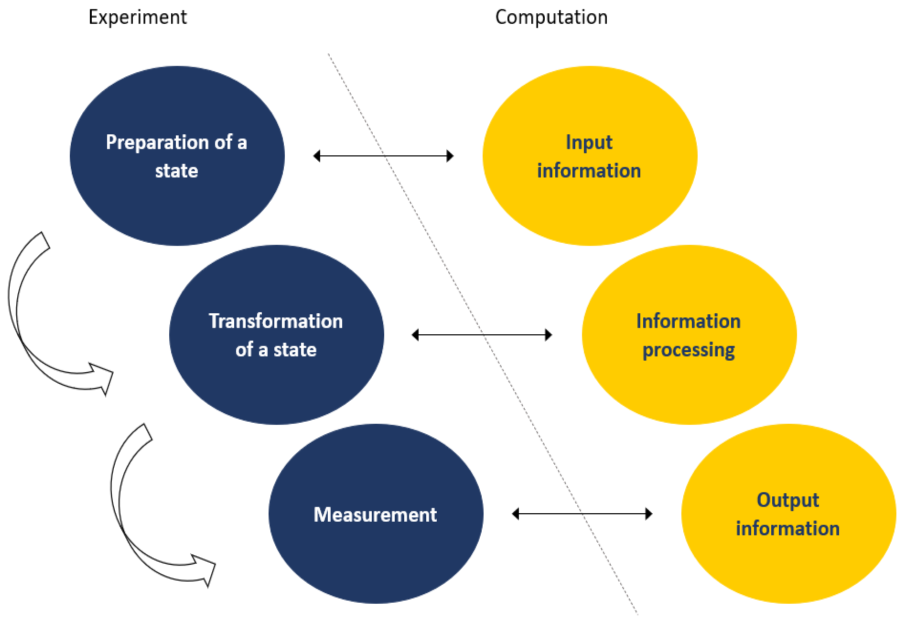


Fig. 2.2: The essence of the approach: the "experiment-computation" comparison.

Operationally, the principles were implemented from the design of the first lesson of the module concerning a brief history of computer evolution onward. From the beginning, the comparison between "experiment and computation" (Fig. 2.2) was used to provide the following take-home message: behind classical computers, there are logical gates that follow Boolean logic and are materially built on devices that follow physical laws. The current opacity of the physical behaviors of these devices is the result of a technological development that led to the software becoming in some sense "THE computer."

Concretely, teaching started by acquainting the students with the definition of computer science, given in the 1980s, as the science that concerns the development and use of structures and procedures for processing information (Scheid, 1982). The information is put into the computer in a certain form and is then returned in a different form (which is generally easier to use), following the sequence "input, processing and output information" (right part of Fig. 2.2). The input information can be defined as the raw material of the process, the output information as the finished product (Scheid, 1982). Students attending grades 12 or 13 have usually already encountered the concept of bit, binary systems, truth tables, and logic gates. Thus, after a brief recap of these concepts, examples of materials devices were shown in order to move the discussion on to the concrete realization of the logic gates through the hardware. In order to stress the logic and the working principles without entering into the functioning of complicated devices, we showed the students very special logic gates created through systems of rope and pulleys, as described in the fantasy tale reported in a 1988 paper by Dewdney (see Fig. 2.3) (Dewdney, 1995).

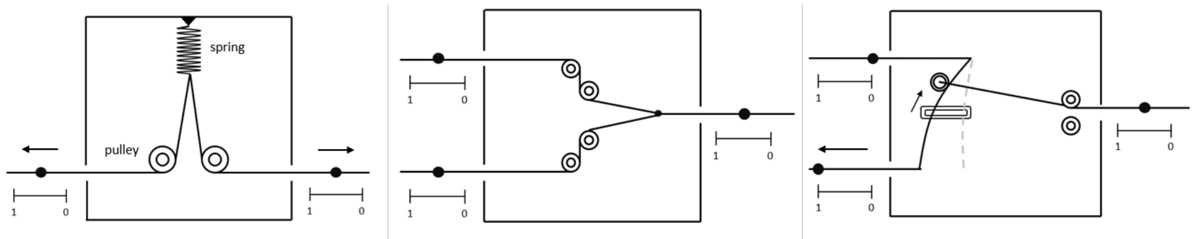


Fig. 2.3: Mechanical realization of, from left to right, NOT, OR, and AND logic gates (redrawn from Dewdney, 1995).

This story aimed also to highlight the deterministic and linear paradigm that is implemented in the logic of the algorithm. After this demonstration, the historical evolution of computers was briefly described. The narrative started by mentioning the hardware made of vacuum tubes (first generation), which then moved to transistors (second generation), integrated circuits (third generation), and, finally, microchips (fourth generation). The discourse was built to emphasize that the changes led to an impressive increase of the calculus power and progressive miniaturization without, however, changing the Von Neumann architecture of the computers and the basic logic they follow. Furthermore, the last passage was stressed as the step that led to the explosion of the software era, whereby, thanks to the software sophistication, it became possible to force computers to follow probabilistic and nondeterministic algorithms even though their hardware still followed classical logic. This design principle was then applied in the quantum case, in the opposite direction: from left to right of Fig. 2.2, i.e., from experiment to computation. We started with one of the most famous fundamental experiments, the Stern-Gerlach experiment, and, using the spin-first approach, we introduced the basic concepts of quantum physics. The Stern-Gerlach experiment was used as a context to re-conceptualize the simplest superposition state $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$ in terms of qubit, $\alpha|0\rangle + \beta|1\rangle$, the new basic unit of quantum computers. Then, as with the classical case, we conceptualized the experimental operations of switching on and off the magnetic field on a particular axis (state evolution) in terms of information processing through quantum logic gates (information processing) and we read the experimental process of particle detection (measurement) in terms of reading the output information. The design approach not only allows us to introduce the core ideas at the basis of quantum technologies, but also to show that the information can be manipulated from experimental (tools), mathematical (rotation of a vector in the Bloch sphere), and circuital (logic gates) points of view. This plurality of perspectives helps students to understand the analogy between the experimental and algorithmic representations and start to grasp how the new technologies work while taking a step toward the concept of simulation.

After the discussion of the Stern-Gerlach experiment, we introduced the concept of entanglement, which is fundamental for teleportation protocol. In order to communicate to students its importance and its relevance also from a technological point of view, the concept was applied to the cryptography protocol. We use the simulation of quantum cryptography from the quantum mechanics visualization (QUVIS) project by the University of St Andrews. A schematic representation is reported in Fig. 3. We showed students how the protocol BB84 works from an experimental point of view (using Pockels cells that randomly vary the polarization of the photon) and, from a logical point of view. A circuit made by an opportune combination of logic gates X; Y; Z was presented and discussed to show how it makes the polarization random. Before describing in detail how the approach was applied to the case of teleportation protocol (Sec. VI B), we briefly describe the further design principles that concern specific aspects of the approach. The two broader principles that characterize all the I SEE modules, refer to the goals of making the materials inclusive and relevant from a societal point of view.

Modern quantum experiments (such as quantum teleportation, boson sampling, and quantum cryptography) are really complicated both for the experimental setup and for the physical and mathematical formalism. While it was not our aim to acquaint the students with these aspects, we wanted them to be able to recognize the inner phenomenon they refer to and the core concepts of quantum mechanics needed to grasp the inner logic and potentialities of the new technologies. In our approach, the core concepts necessary and sufficient to grasp the essence of these new technologies include the only ones we introduced in the first lessons, which proved to be within reach of secondary school students and were efficiently built through a two-state approach (e.g. Perron, DeLeone, Sharif, Carter, Grossman, Passante & Sack, 2021; Pospiech, 2021, Krijtenburg-Lewerissa, van Joolingen, Pol & Brinkman, 2020; Krijtenburg-Lewerissa, Pol, Brinkman & Van Joolingen, 2019; Krijtenburg-Lewerissa, Pol, Brinkman & Van Joolingen, 2017; Sadaghiani, 2016; Zhu & Singh 2012; Manogue, Gire, McIntyre & Tate, 2012; Pospiech, 2000; Pospiech, 1999): the concepts of state, superposition principle, manipulation of a state, measurement, and entanglement. Therefore, the educational reconstructions of the contemporary experiments carried out were based solely on these concepts. Furthermore, when the experimental apparatus is described, the level of the discourse is kept "light" enough to make the students aware that, behind logical protocols and mathematical expressions, there are concrete physical devices. These ideas were elaborate as the third design principle:

Design principle #3: to keep the quantum technicalities as simple and clear as possible and foster a deep understanding of the essential physical concepts that are needed.

The last one concerns one of the core values of the general educational approach developed by the research group in physics education of Bologna, which is the conviction that science education and the students' understanding of scientific concepts can make disciplinary (and interdisciplinary) learning a locus for identity development (Levrini, Levin, & Fantini, 2020; Levrini, Levin, Fantini & Tasquier, 2019; Kapon, Laherto & Levrini, 2018; Levrini, Levin & Fantini, 2017). Broadly speaking, our materials are designed to be multidimensional and multiperspective so as to engage as many students as possible by nurturing their idiosyncratic interests, intellectual and aesthetic tastes.
Therefore, the last design principle consists

Design principle #4: to make the modules as inclusive as possible.

The terms inclusive and inclusiveness usually refer to the aim of including and integrating all people and groups in activities, organizations, political processes, etc., especially those who are disadvantaged, have suffered discrimination, or are living with disabilities. Even if we do believe that inclusiveness must have this aim, we, also for the context of our implementations, associate with this word also the aim to include and engage different students by designing learning environment that encompasses different students' tastes and gives room to different ways of reasoning, fostering students to develop their identities also through science education. The considerations that follow are based on the second nuance of inclusiveness.
In the case of quantum technologies, it was not difficult to imagine that students could be of interest for many different reasons. They are based on quantum physics which is per se an intriguing theory or

even represents current technological frontiers that are opening up new future scenarios. Also, they can be fascinating for their social or political implications, or for the intellectual challenges they pose. In Italy, these advanced modules are set within orientation courses. For many students, quantum technologies become a benchmark test regarding their abilities, interest, and even talent, to successfully complete the physics degree course. Furthermore, we usually exploit the nature of the topic, which is a STEM (Science, Technology, Engineering, and Mathematics) theme. We believe that these kinds of topics can engage a broader range of students since the different disciplines can provide multiple access keys to the same topic fostering them to follow the learning path that best suits their personalities, what they like, etc.

From the very first implementation, the principle has been fostered by structuring the knowledge and discourses on different levels: narrative, logical, mathematical, and technical-experimental levels. The articulation of the discourse was also needed to avoid students considering quantum logic and mathematics as just a “mere mechanism” to play with (Satanassi, Fantini, Spada & Levrini, 2021). The levels of articulation, indeed, have to maintain a strong link with reality and support the comparison between classical and quantum computation. This design principle was differently declined throughout the different implementations. In the study conducted in Satanassi, Ercolessi and Levrini (2022) this principle was implemented by structuring the lectures of the course on different levels: the narrative, the logic, the mathematical and the technical-experimental levels. For example, in the teleportation activity (section 3.2), as regards the narrative level, the contents have been represented through the story of two characters, Alice, and Bob, who are charged with solving the problem of teleporting the state of a photon from one position to another in the world. The narrative is supposed to activate the students’ imagination and help them to build a comprehensive view—a storyboard—to effectively situate the various steps of reasoning. The logical level is the backbone of the argumentation on which the comparison between classical and quantum computers is built. This level refers to the logic that lies at the basis of classical and quantum computing and sets the “rules” and truth tables on which the logic gates are built and combined in circuits to solve a problem (algorithm). The mathematical level refers to the two-state Dirac formalism for quantum physics and is used to formalize the superposition states and their manipulation throughout the logic gates of the circuits. The technical–experimental level refers to the implementation, of the experimental devices used to realize the logic gates. From an educational point of view, this level was supposed to bestow concreteness to the logical level and provide the students with an idea of what it means, today, to create a quantum computer. The different levels were supposed to play different roles and allow students with different interests and tastes to find the level that resonated most with them. The levels were explicitly introduced to the students in two senses: the teachers made the students aware of their existence at the beginning of the key lessons and informed them when the various levels were switched on. The specific roles that the levels were supposed to play were instead a metalevel that was kept implicit (Satanassi, Ercolessi & Levrini, 2022).

As I discuss, this design principle is also intertwined with the design principles described in 2.4.2 in which we use interdisciplinarity between physics, mathematics, and computer science not only as levels of the discourse but to reflect on disciplines’ epistemologies and identities, discuss them also as different kinds of reasoning, and explore the epistemology of their integration. As I describe in 4.2, this principle is implemented through systematically discuss and compare different representations as an epistemic object, namely as bearers of different scientific practices (Evagorou, Erduran and Mäntylä, 2015).

Furthermore, the design principle #4 has been pursued also introducing students also to more social sciences and humanities aspects, making them reflect on the multidimensional impact of quantum technologies and on the relationship between science and arts, science and literature.

2.4 Three further design principles

In this section I present the three projects and the design principles that they inspired. The module on the second quantum revolution was designed and progressively refined, within these projects, to match the research agenda of the PER group of Bologna. The three European projects are I SEE, IDENTITIES and FEDORA projects, which are coordinated by the group of Bologna. The projects are strictly related each other, since they have been designed like the continuation of each other. However, I aim to emphasize their peculiar aspects and their role in the design process.
All these projects are consistent and contribute to the challenges recently launched by the OECD (2019) to achieve *Sustainable Development Goa*l *4: Quality Education* (SDG4, "Ensure inclusive and equitable quality education and promote lifelong learning opportunities for all").
In 2019, the OECD launched the Education and Skills 2030 project for addressing pivotal questions for education:

> "How can we prepare students for jobs that have not yet been created, to tackle societal challenges that we cannot yet imagine, and use technologies that have not yet been invented? How can we equip them to thrive in an interconnected world where they need to understand and appreciate different perspectives and worldviews, interact respectfully with others, and take responsible action toward sustainability and collective well-being?" (p.15)

Within the project they have elaborated an evolving learning framework called *OECD learning compass 2030*, (product of a collaboration among government representatives, academic experts, school leaders, teachers, students, and social partners) that emphasizes "the need for students to learn to navigate by themselves through unfamiliar contexts s and find their direction in a meaningful and responsible" (ibidem, p. 24). To address this need, the learning compass individuates many components: core foundations, knowledge, skills, attitudes and values, transformative competencies, and a cycle of anticipation, action, and reflection (AAR). The cycle is represented as a compass (Fig. 2.4) and it is conceived as "an iterative learning process whereby learners continuously improve their thinking and act intentionally and responsibly towards collective well-being" (OECD, 2019, p. 120).
This process is articulated in three phases that inform, complement, and strengthen each other. In the Anticipation phase, learners use their abilities to anticipate the short- and long-term consequences of actions, understand their own intentions and the intentions of others, and widen their own and others' perspectives. The focus of this phase is not on the prediction of the future but on the envisioning of multiple scenarios (Poli, 2010). Then, in the Actin phase, learners take action towards specific objectives and contribute positively or negatively to the individual or societal well-being. Finally, In the Reflection phase, learners improve their thinking, leading them to a deeper understanding and better and more responsible actions toward well-being (OECD, 2019).
The AAR cycle implies the development of skills related to the capacity to push imagination forward, to take responsibility to participate in the world, and consciously influence events and circumstances for the better, making central the connection between future thinking and agency, since "agency involves the idea of projection and implies anticipation" (Cuzzocrea & Mandich, 2016).
Furthermore, to engage learners in such a rich process, the OECD pointed out other three transformative competencies: creating new value, reconciling tensions and dilemmas, and taking responsibility. Creating new value concerns the capacity of creating innovation (e.g., developing new knowledge, applying it to problems both old and new, creating new jobs, etc.) to address unfamiliar contexts. Reconciling tensions and dilemmas means taking into account the interconnections between distant ideas, logic, and positions, and using this understanding to find solutions to dilemmas and conflicts. Taking responsibility is linked to the ability to reflect upon one's own actions in light of one's experience and education, and consider personal, ethical, and societal goals.

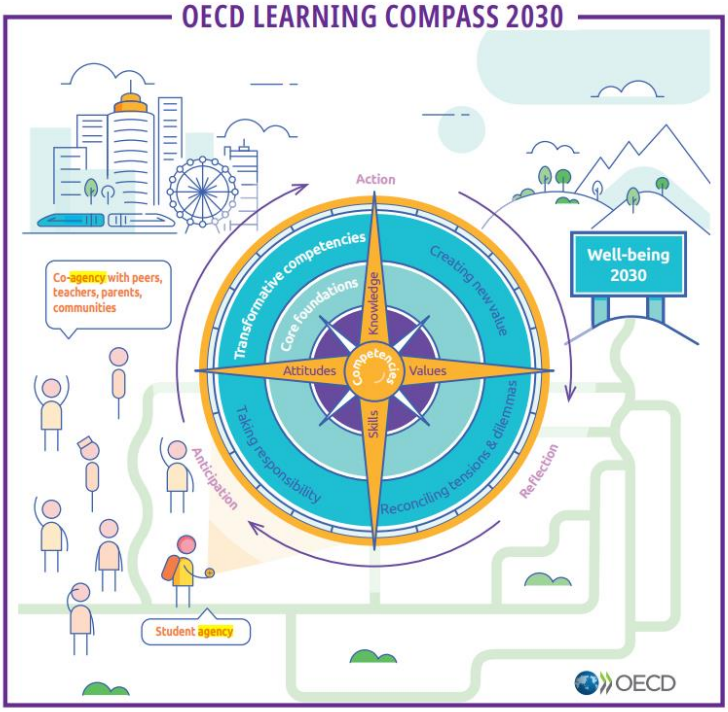


Fig. 2.4: Representation of the OECD Learning Compass 2030 framework (OECD, 2019).

Finally, the learning compass 2030 includes "theoretical concepts and ideas as well as practical understanding based on the experience of having performed certain tasks" (OECD, 2019, p. 74). Four different types of knowledge are recognized: disciplinary, interdisciplinary, epistemic, and procedural. Disciplinary knowledge is essential for understanding, and it is the structure through which students can develop also other kinds of knowledge. Interdisciplinary knowledge consists in transferring key concepts, finding multiple connections between concepts and content of one discipline/subject to the concepts and content of other disciplines/subjects, and combining related disciplines or creating new ones. Epistemic knowledge, from the OECD perspective, involves knowing how to think and act like a practitioner. "This knowledge helps students find the purpose of learning, understand the application of learning, and extend their disciplinary knowledge" (ibidem, p.74). Procedural knowledge is the knowledge about how something is done, the series of steps or actions taken to accomplish a goal. This type of knowledge is usable by the students across different contexts and situations to identify solutions to problems.

This introduction aims to point out some of the key aspects emphasized by the Education and Skills 2030 project. In the following, I discuss how the three projects coordinated by the University of Bologna can contribute to this discussion and what kinds of design principles can be implemented to contribute to developing those skills and knowledge.

2.4.1 I SEE: future-oriented dimension

The module on the Second Quantum Revolution was firstly designed and developed within the I SEE project[1] (Satanassi, 2018). I SEE (Inclusive STEM Educating to Enhance the capacity to aspire and imagine future careers) was a triennial Erasmus+ project that lasted from 2016 to 2019 and was coordinated by the Department of Physics and Astronomy of the University of Bologna. The strategic partnership was composed of institutions coming from four different countries: Italy, Finland, Iceland, and the United Kingdom. In particular, the partners are two universities (the University of Bologna, and the University of Helsinki), three secondary schools (the "A. Einstein" Lyceum of Rimini, the Normal

[1] https://iseeproject.eu/

Lyceum of Helsinki, and the Hamrahlid College of Reykjavik), an Icelandic environmental NGO, an association of English teachers (Association for Science Education), and a private foundation in Bologna (Golinelli Foundation).
Starting from and trying to address the issues raised by the "society of acceleration" (Rosa, 2013), according to which we are witnessing accelerated social changes that are disorienting the young generation who tend to feel that the future is no longer a promise but a threat and to struggle to imagine possible positive future scenarios for society (e.g. Heikkilä, Nevala, Ahokas, Hyttinen & Ollila, 2017; Cook, 2016; Eurobarometer, 2015; Benasayag & Schmit, 2006), the ISEE projects developed innovative approaches and STEM teaching modules (on climate change; artificial intelligence, carbon sequestration, and quantum technologies) to foster students' capacities to imagine the future and aspire to STEM careers. The ISEE approach has posed the problem of revaluing science education with the conviction that it has the possibilities of i. changing students' fears and deterministic, utopian, and dystopian future views towards science and technology Carter & Smith (2003), ii. providing citizenship skills to enhance the role and responsibility of individuals towards global social scientific issues such as climate change, global pandemics, etc. (Blandford & Thorne, 2020) and iii. equip the young generation with skills that help them to navigate into the complexity of the contemporary world and orient themselves toward an individual and collective sustainable future (OECD, 2019).
For these purposes, as stressed by Levrini and colleagues (Levrini, Tasquier, Barelli, Laherto, Palmgren, Branchetti & Wilson, 2021; Levrini, Tasquier, Branchetti & Barelli, 2019; Branchetti, Cutler, Laherto, Levrini, Palmgren, Tasquier & Wilson, 2018), there is the need of *futurizing* science education, that is of integrating science education with the future dimension.
Therefore, on one hand, the gravity and urgency of global challenges (such as global warming, migration flows, the spread of diseases, etc.) have emphasized the need for science education to incorporate the action competence approach which aims to make students more conscious of the decisions and actions they take in society (Levrini et al. 2021; Jensen & Schnack, 1997). On the other, consistently with the AAR cycle (OECD, 2019) and the United Nations' Agenda 2030 program that is calling for societal transformations, there is the need to develop "skills related to the capacity to push imagination forward, to take responsibility to participate in the world, and consciously influence events and circumstances for the better" (Levrini et. al, 2021, p. 284). Therefore, the AAR connects futures thinking to agency, since "agency involves the idea of projection and implies anticipation" (Cuzzocrea & Mandich, 2016, p. 552).
But what does it mean *futurizing* science education and how can we integrate this further dimension? The research field that explicitly "embraces the future" is the field of futures studies (e.g., Bishop, Hines & Collins, 2007; Rickards, Ison, Fünfgeld & Wiseman, 2014; Kousa, 2011). The researchers in this interdisciplinary field (involving sociologists, philosophers, historians, political scientists, psychologists, and economists, but also scholars and practitioners from the arts, natural sciences, technology, and engineering) investigate trends and patterns, sources, and causes of change and stability to develop foresight and create possible, probable, and desirable future scenarios. Furthermore, they are exploring ways in which future thinking relates to emotion, perceptions, and human worldviews. From an educational perspective, it is meaningful that Future Studies use foresight to orient actions in the present that can influence and create preferable or desirable futures. In particular, "Teach the future" is a non-profit organization of futures studies professionals that is working to bring foresight and futures thinking into schools, so as to teach students "to think critically and creatively about the future and develop the agency to influence it" (www.teachthefuture.org).
In futures studies, I SEE partners adopted some concepts such as "future-oriented activities", "futures thinking" and "perception of the future" and reconceptualized them within the field of science education. They "proposed incorporating future thinking skills into school science, including scenario thinking, systems thinking, thinking beyond the realm of possibilities, action competence, and skills to manage uncertainty and complexity" (Levrini et al. 2021, p. 283; Levrini et al., 2019; Branchetti et al., 2018). They continue, "young people's perception of the future is central to our project and refers to how young people feel the future as distant or near, abstract or concrete, as something they have

influence over or not, and how they relate the future to themselves and the present" (Levrini et al. 2021, p. 284).
In their works, I SEE partners pointed out specific future-oriented competencies, called *future-scaffolding skills*, that science education can contribute to promoting. In particular, they divided these skills into structural and dynamical skills: the structural ones "build a rational scaffolding of the topic by recognizing the causal, temporal, and logical relationships among them", while the dynamical ones allow to "navigate across the scaffolding for developing scenarios, visions and creative ideas for the future" (Levrini et al., 2021, p. 297). The development of *future-scaffolding skills* can also be the basis to activate students' agency.
This discussion led us to extrapolate two different design principles:

Design principle #5a: to exploit and unpack the future's intrinsic structure that characterizes science.

Design principle #5b: to engage students in reflections about the implications and impact of science and technologies on different dimensions such as the scientific, political, institutional, economic, ethical, environmental, educational, etc., fostering them to think about the different stakeholders involved, the possible future's scenarios arising from the scientific and technological progress, and their possible roles in the debates and decision-making.

The first one, based on the idea the future is intrinsic in science e.g. prediction is one of the aims of science and at the core of science modeling, is implemented by choosing all that topics, debates, concepts, practices (etc.) that can promote the shift from the linear, deterministic and univocal future of classical Newtonian physics to the complex and non-linear one typical of the science of complex systems and/or the probabilistic one of quantum mechanics. In the Second Quantum Revolution case, the superposition state of the qubit and the intrinsic randomness originated from the interactions of the system (and/or the measurement apparatus) with an environment (e.g. Bera, Acín, Kuś, Mitchell & Lewenstein, 2017; Zurek, 2009; Zurek, 2003; Wheeler and Zurek 1983) or by non-local correlation (e.g. Aspect, 2021) incorporate and entail a strange nature of the quantum object, the probabilistic nature of the outcomes not attributable to our lack of knowledge or imperfectness of measurement tools and, by extending to multi-party systems and local measurements, the inability to locally describe a global system state, that is, in case of entangled quantum objects, local measurement outcomes cannot be understood in terms of deterministic local hidden variable model and a 'true' local indeterminacy is present (Bera, Acín, Kuś, Mitchell & Lewenstein, 2017).
The ontological probability and the randomness better describe the society in which we live. We refer to today's society as the "risk society" (Beck, 1896) characterized by the problem of systematically managing causality and insecurity or as the "society of uncertainty" (Bauman, 1999), enhancing how contemporary society is requiring us to accept and embrace that the uncertainty is the natural habitat of our lives. In light of this, the quantum revolutions with their concepts and debates can give students knowledge, lenses, conceptual tools, and skills (such as probabilistic thinking) to shift toward a probabilistic view of the world and manage the uncertainty to address the dilemmas and challenges of modernity.
Furthermore, the knowledge about a quantum system and of the possible outcomes of a measurement can be reconceptualized as possible future scenarios for the evolution of the system. The plural character of the future is represented by the Voros' cone (Voros, 2003) in Figure 2.5, in which different kinds of futures are introduced: possible, plausible, probable, and desirable.

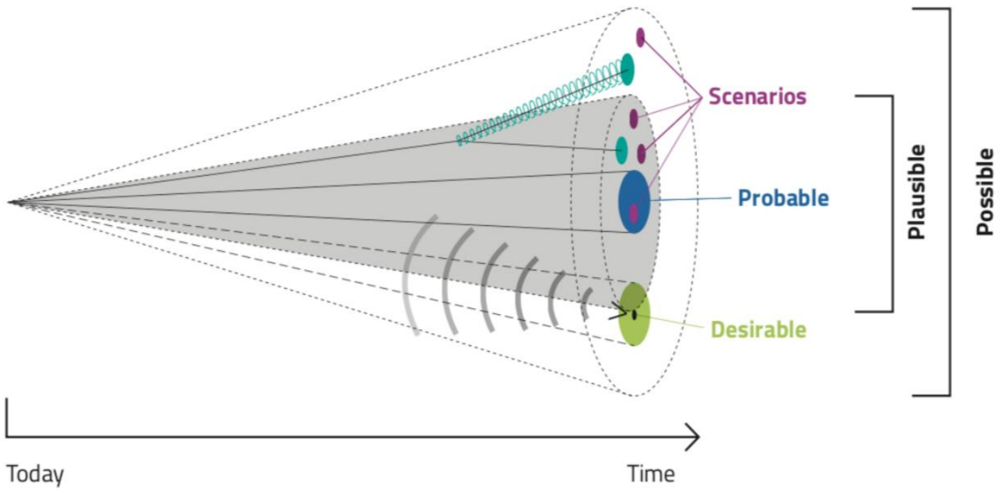


Fig. 2.5: ri-elaboration of Voro's cone within the ISEE project.

Each future is characterized by different triggering elements and kinds of thinking.
The probable future is the standard one, it derives from a linear and deterministic way of thinking. The plausible future requires taking into account discontinuity and contingencies, so asking yourself "what happens if" forces you to put an eye on the future and imagine different scenarios. Finally, the desirable one which is triggered by values, desires, choices, and visions, is the one that fosters students to aspire to more personal and collective sustainable futures, consistently with the AAR cycle (OECD, 2019).

We believe that, since the quantum physics difficulties of tapping into everyday experiences, the presented vision allows students to tap into their personal, affective, etc. experiences and, for example, reconceptualize the need and the effort of reaching the desirable future as an optimization problem and build an algorithm (e.g., based on quantum random walk) that maximizes the probability of reaching it.
These design principles and this reconceptualization, sometimes metaphorical, of physics and quantum revolutions provide for a broad and sometimes a little forced vision of such themes, but it is not our intention to build accredited knowledge through these kinds of activities, but rather to provide new reading keys of the world we live and make students reflect on the potential of science also as a thinking tool.

2.4.2 IDENTITIES: disciplinary and interdisciplinary dimension

The module on the Second Quantum Revolution was revised within the IDENTITIES project[2]. IDENTITIES (Integrate Disciplines to Elaborate Novel Teaching approaches to InTerdisciplinarity and Innovate pre-service teacher Education for STEM challenges) is an Erasmus+ project that started in 2019 and was coordinated by the University of Bologna. The strategic partnership is composed of universities coming from four different countries: the University of Bologna and Parma (Italy), the University of Montpellier (France), the University of Barcelona (Spain), and the University of Crete (Greece). The researchers involved are experts in different fields from physics education to mathematics education, from computer science education to linguistics. The interdisciplinarity, in terms of STEM education, was at the core also of the I SEE project. IDENTITIES was born as a follow-up of I SEE but it is not focused on secondary school education but pre- and in- service teacher education and it deals with not only STEM topics (such as climate change, nanotechnologies, and coronavirus) but also curricular interdisciplinary topics (such as parabola and parabolic motion, non-Euclidean geometry and gravitation, etc.). The

[2] https://identitiesproject.eu/

project is developing novel teaching approaches to interdisciplinarity in science and mathematics to innovate pre-service teacher education for contemporary challenges and are exploring both emergent advanced STEM themes and curricular interdisciplinary topics as contexts to investigate inter-multi-trans-disciplinary forms of knowledge organization and to design classroom activities and new models of co-teaching.

The importance of interdisciplinary knowledge is today in the spotlight of many educational research and political/institutional projects (e.g., OECD, 2019). The emergencies (such as pandemic, climate change, etc.), that society is making us face society, increasingly highlight how a mono-disciplinary approach is no longer adequate as well as the traditional single-discipline education (Baptista & Klein, 2022; Pharo, Davison, Warr, Nursey-Bray, Beswick, Wapstra & Jone, 2012; Brown, Deane, Harris & Russell, 2010; Schmitz, Stinson and James 2010). Even recently interdisciplinarity is widely promulgated as a positive goal in academia (Brown, 2020; MacLeod, 2018; Grüne-Yanoff, 2016; Khilji, 2014; Frodeman, 2013; Huutoniemi, Klein, Bruun & Hukkinen, 2010; Jacobs & Frickel, 2009; Porter & Rafols, 2009), the disciplines are actually organized into subjects, at school, (Ortiz-Revilla, Adúriz-Bravo & Greca, 2020) and into departments, at university, therefore disciplines are the key units of social organization in education (Jacob, 2014). Moreover, publications, professional associations, and careers are shaped by the structure of disciplines. Interdisciplinarity, therefore, challenges the current structure of school and academic life (Jacob, 2014). Even if the actual organization can be motivated by the need to develop expertise in disciplinary fields, it can create boundaries and barriers between disciplinary fields that not only may not promote collaboration (e.g., Russell, 2022; Pharo et al., 2012) but also may not create awareness in people about their similarities and differences and a common language to understand each other. There is the need for a third way, to not lose the advantages of organizing teaching in disciplines without missing the opportunity to coordinate fruitfully different points of view (Baptista & Klein, 2022; Barelli et. al, 2022). Consistently to the OECD (2019), interdisciplinary knowledge is important as well as disciplinary one.
The IDENTITIES project has taken a position about these two kinds of knowledge. First of all, we try to overcome two forms of trivialization: interdisciplinarity as a-disciplinarity or non-disciplinarity (comprised only of transversal themes) or interdisciplinarity as an instrumental use of concepts taken from one discipline (for example mathematics) to solve a problem formulated in another discipline (for example in physics).

Many researchers try to define what is meant by "interdisciplinarity". Brewer (1999) argued, "Interdisciplinarity generally refers to the appropriate combination of knowledge from many different specialties – especially as a means to shed new light on an actual problem" (p. 328). Choi and Pak (2007), starting from the differences between the prefixes, compare the three different forms (multidisciplinarity, interdisciplinarity, and transdisciplinarity) and arrived at the following definition (Choi & Pak, 2007, p. 359):

> *Multidisciplinarity* draws on knowledge from different disciplines but stays within the boundaries of those fields.
> *Interdisciplinarity* analyzes, synthesizes, and harmonizes links between disciplines into a coordinated and coherent whole.
> *Transdisciplinarity* integrates the natural, social and health sciences in a humanities context, and in doing so transcends each of their traditional boundaries.

Klein (2010) does not propose a single definition but rather a classification, a taxonomy of interdisciplinarity, arriving at a conclusion similar to Choi & Pak (2007), that is multidisciplinarity involves encyclopedic, additive juxtaposition or, at most, some kind of coordination, but it lacks intercommunication, and disciplines remain separate (also called pseudo-interdisciplinarity), while true interdisciplinarity is integrating, interacting, linking, and focusing. Alvargonzález (2011) argued that "Interdisciplinarity would arise in a near symmetrical way when two or more disciplines converge in a given field, as they would, for example, in biochemistry, bioinformatics or geophysics. This convergence can lead to practical and theoretical integration of the disciplines involved, which would be unified. Paradoxically, these convergences, on many occasions, give rise to newly independent and sovereign disciplines, at least when they are considered in terms of their academic institutionalization" (p.392, 393). Klein, Baptista, and Streck (2022) in the preface of a recent book wrote: "Interdisciplinarity is conventionally defined as an integration of approaches from two or more existing disciplines or bodies of knowledge with the aim of advancing a new understanding of or solution to a complex problem, question, or topic that cannot be handled from a single perspective. In contrast, transdisciplinarity connotes an overarching synthesis of a trans-sector collaboration, with the aim of transcending and even transgressing traditional boundaries" (p.2).

Another perspective about interdisciplinarity is given by Akkerman and Bakker (2011) in **"boundary object and boundary crossing"**. It is not born within the philosophical debate of interdisciplinarity, and it has not the objective to address interdisciplinary topics. The authors provide a broad social perspective considering institutions and communities of practice. Anyway, it provides a vocabulary and sheds light on the mechanism of interactions between different worlds. The authors start from the assumption that learning as well as work involves *boundaries*.

"Whether we speak of learning as the change from novice to expert in a particular domain or as the development from legitimate peripheral participation to being a full member of a particular community (Lave & Wenger, 1991), the boundary of the domain or community is constitutive of what counts as expertise or as central participation. When we consider learning in terms of identity development, a key question is the distinction between what is part of me versus what is not (yet) part of me.
Boundaries are becoming more explicit because of increasing specialization; people, therefore, search for ways to connect and mobilize themselves across social and cultural practices to avoid fragmentation." (Akkerman & Bakker, 2011, p.132)" (Akkerman & Bakker, 2011, p. 132)

Therefore, both in education and work the challenge consists in creating possibilities for collaboration across a diversity of sites. Different researchers studied the nature of the boundaries (e.g., Bernstein, 1971; Engeström, Engeström, & Kärkkäinen, 1995; Star & Griesemer, 1989; Suchman, 1993). It can be seen as a "sociocultural difference leading to a discontinuity in action or interaction. Boundaries simultaneously suggest a sameness and continuity in the sense that within discontinuity two or more sites are relevant to one another in a particular way" (Akkerman & Bakker, 2011, p. 133).
Clear on the boundary, the metatheory elaborated by Akkerman, and Bakker (2011) introduces three fundamental terms for talking about interdisciplinarity that is at the core of our framework. The first term is *boundary objects*, which refers to "objects that enact the boundary by addressing and articulating meanings and perspectives of various intersecting worlds or that move beyond the boundary in that they have an unspecified quality of their own" (p.150). The boundary objects have an intrinsic ambiguity: they "belong to *both* one world *and* another" and, at the same time, "they belong to *neither* one *nor* the other world". In such a way, boundary objects have both the power to *divide* two worlds and to *connect* sides. As stressed by Akkerman and Bakker, "it is precisely this ambiguous nature that explains the interest in boundaries and boundary crossing as phenomena of investigation for education scholars. Both the enactment of multivoicedness (both–and) and the unspecified quality (neither–nor) of boundaries create a need for dialogue, in which meanings have to be negotiated and from which something new may emerge." (Akkerman and Bakker, 2011, p. 142) The second term is *boundary people,* referring to "marginal strangers 'who sort of belonging and sort of don't" (Akkerman & Bakker, 2011, p. 460), and helps to describe the difficult but necessary experience with the alterity that some people have in bridging communities and whose identities are perceived as strongly defined and "defended" by their members. Looking at such communities from outside and inside at the same time, their identity is the result of a continuous process of negotiation of sense and meanings, that allows them to develop a new language and knowledge to talk about and with both communities.
The third term is *boundary crossing*, with which Akkerman and Bakker associate the term *learning mechanisms*. Suchman (1993, p. 25) started talking about *boundary crossing* referring to professionals at work that "enter onto territory in which we are unfamiliar and, to some significant extent therefore

unqualified". At first, they have to "face the challenge of negotiating and combining ingredients from different contexts to achieve hybrid situations" (Engeström et al., 1995, p. 319). Such a complex goal is a hard challenge, but can also lead to, and require at the same time, putting into action some *learning mechanisms* (Akkerman and Bakker, 2011). Akkerman and Bakker individuated four *learning mechanisms*. The first one is the *identification mechanism* that does not question the identity of disciplines but helps to understand their relationship and, at the same, shed light on their identities. An example of these processes is defining one practice in light of another, delineating how it differs from another practice. What is typical in identification processes is that the boundaries between practices are encountered and reconstructed, without necessarily overcoming discontinuities. The learning potential resides in a renewed sense-making of different practices and related identities (Akkerman and Bakker, 2011). The other three *learning mechanisms* imply an effort in rethinking the relationships and the exchanges between disciplines, or even their foundations, leading to some innovation. *Coordination processes* aim to promote communication and exchanges between communities (e.g., *efforts at translation* or *increasing boundary permeability*); *reflection processes* consist in making explicit one's understanding and knowledge or taking a different perspective than one's own on the same issue. This mechanism leads not only to perspective making (Boland and Tenkasi, 1995), that is, making explicit one's understanding and knowledge but also to enriching people's ways of looking into the world so that one improves one's identity beyond its current status. Finally, *transformation processes* constitute a further step, that is the collaboration and the co-development of new practices that are meaningful in both worlds and are evolutions with respect to the original practices from which they emerged.
Kapon and Erduran (2021) used this framework to compare three different approaches to interdisciplinarity in science education presented in the symposium *Crossing boundaries – Examining and problematizing interdisciplinarity in science education* at the ESERA 2019 conference, providing examples of learning mechanisms in different contexts. The authors show three different cases where interdisciplinary boundary crossing can be relevant in science education. Schvartzer, Peer, and Kapon (2019) problematized learners' engagement when learning or using disciplinary knowledge in different interdisciplinary contexts and problems; Levy, Zohar, and Dubovi (2019) focused on the explanatory potential of interdisciplinarity for disciplinary-based problems; Levrini, Branchetti, and Fantini (2019) stressed the inherent interdisciplinarity of STEM disciplines in discipline-based educational systems from a historical and curriculum development perspective.

This stance toward interdisciplinarity emphasizes it as a process of "integrating, interacting, linking, and focusing'' different disciplinary domains, and different epistemic cores (Klein, 2010). The interaction between disciplines "may range from simple communication of ideas to the mutual integration of organizing concepts, methodology, procedures, epistemology, terminology, data, and organization of research and education in a fairly large field" (CERI, 1972, p.25). It is therefore a matter also of disciplines' epistemologies.
In a recent paper, Bărboianu (2022) raises the problem of clarifying the nature of interdisciplinarity from a theoretical perspective by considering "the epistemic outcome of the ongoing process of practicing interdisciplinarity, a theoretical entity of a structural nature developed through interdisciplinarity, which is grounded on the content of the disciplines as bodies of knowledge" (p. 9)
The value at the basis of this is the idea that interdisciplinarity has an epistemological or epistemic nature (Bărboianu, 2022; Schmidt, 2008). But it is not only a matter of interdisciplinarity, also STEM (Science, Technology, Engineering, and Mathematics) has an epistemic nature but, especially from an educational perspective, it is relatively understudied (Chesky & Wolfmeyer, 2015). The "epistemic nature" refers not only to the characteristics of STEM knowledge but also to the processes through which STEM knowledge is produced, evaluated, and revised (Erduran & Dagher, 2014a; Hodson, 2014). This epistemic perspective on STEM may help to shed light not only on the shared features of the component STEM disciplines but also on the domain specificities of a discipline and the differences among them (Park, Wu & Erduran, 2020).

According to these perspectives, the project IDENTITIES reflects on the epistemic nature of interdisciplinarity and STEM from an educational perspective by contributing to answering the questions of how to discuss what we mean by discipline in a proper and not stereotyped way (Linn, Eylon & Davis, 2004, p. 41), how to unpack the notion of interdisciplinarity, and how to articulate and interpret the tight link, the entanglement, between the interdisciplinarity or STEM and the disciplinarity. Therefore, an essential focus in dealing with interdisciplinarity is on the concept of 'discipline'. Starting from the etymology, the meaning of "discipline" relies on the Latin root of discipline "discere", that is referred to learning. Disciplines are a body of knowledge and skills that ground their roots in the educational necessity to re-organize knowledge for teaching, learning, and communicating it (Alvarogonzález, 2011). The re-organization has to be such that students, in building their knowledge, can also develop epistemic skills, such as problem-solving, modeling, representing, arguing, explaining, testing, and sharing (Levrini, Branchetti & Fantini, 2019).

Bărboianu (2022) highlights the complexity of placing the concept of 'discipline' into an adequate placement framework of investigations since it is compounded by different aspects: social, historical, semantic, epistemic, and scientific. Each aspect is enlarged by different facets: "educational, expertise, authoritative (within the social aspect), distinction, reference (within the semantic), epistemic content, the truths, the normative, organization, regimentation and institutionalizing of knowledge (within the epistemic), resistance and evolution (within the historical), epistemic virtues and methodology (within the scientific aspect)" (p. 10). In literature, there are at least three different views that discuss what is meant by discipline with different focuses. The *internalist view* considers the discipline as a body of knowledge with a certain epistemic autonomy and authority; in this view, the discipline content is particularly relevant. The *externalist view focuses* on the relations to society, so the discipline is made up of both its content and social relations (academic, educational, and organizational-institutional), with the idea that content is dependent on these social relations (e.g., Price, 1970; Apostel, 1972; Whitley; 2000; Shinn, 1982). For example, Prince, 1970, cited in Becher 2001) discussed that differences in the substantive content led to differences in the social practice of the disciplines. Apostel (1972) goes so far arguing that "a discipline does not exist. A science does not exist. There are persons and groups practicing the same science or the same discipline" (p. 147). Finally, the *structuralist view*, initiated by Sneed and Stegmüller in the 1970s, conceives disciplines, which are linked to the concept of paradigm, as characterized by four structural elements (symbolic generalizations, models, values, and exemplars) that shape the scientific communities and define problems and solutions, (Khun, 1970 reprinted in 2012).

The framework that we think is significant to hold together both the epistemic nature of inter-disciplinarity and the socio-institutional nature is the **Family Resemblance Approach, FRA** (Irzik and Nola, 2011; 2014; Erduran & Dagher, 2014). Besides characterizing the disciplines in their complexity in a way that recalls the construct of disciplinary authenticity (see section 2.1), the Family Resemblance Approach (FRA), based on the concept of family resemblance presents a set of characteristics of science that are both domain-general and the domain-specific.

The FRA framework was first elaborated by Irzik and Nola (2011) and then reconceptualized, within the science education field, by Erduran and Dagher, the RFN (2014a; 2014b; Dagher & Erduran, 2016). Irzik and Nola (2011), recalling the notion of family elaborated by Wittgenstein, described the Family Resemblance Approach to the nature of science as

> "Consider a set of four characteristics {A, B, C, D}. Then one could imagine four individual items which share any three of these characteristics taken together such as (A&B&C) or (B&C&D) or (A&B&D) or (A&C&D); that is, the various family resemblances are represented as four disjuncts of conjunctions of any three properties chosen from the original set of characteristics. This example of a polythetic model of family resemblances can be generalised as follows. Take any set S of n characteristics; then any individual is a member of the family if and only if it has all of the n characteristics of S, or any (n - 1) conjunction of characteristics of S, or any (n - 2) conjunction of characteristics of S, or any (n-3) conjunction of characteristics of S and so on. How large n may be and how

small (n - x) may be is something that can be left open as befits the idea of a family resemblance which does not wish to impose arbitrary limits and leaves this to a 'case by case' investigation…. we will employ this polythetic version of family resemblance (in a slightly modified form) in developing our conception of science." (Irzik & Nola, 2014, p. 1011).

The FRA allows therefore to characterize a scientific field of study by setting broad categories to address a diverse set of features that are common to all the sciences and the activities carried out within them. This idea is particularly meaningful in science since the sub-disciplines usually share a number of common characteristics, but there is no specific characteristic per se that can be used to define a domain as scientific or demarcate it from other disciplines. An example can be observation (i.e., human, or artificial through the use of detecting devices): observing is common to all the sciences, but the very act of observing is not exclusive to science, it thus does not necessarily guarantee family membership in and of itself (Dagher & Erduran, 2016).
So the approach assumes that "there is no fixed set of necessary and sufficient conditions which determine the meaning of [science]" (Irzik & Nola, 2011, p. 594). The idea of "science family" recalls the idea that each member resembles some family members concerning some aspects and other members concerning other aspects. Its potential lies in avoiding defining what science is, and in providing an overall picture of the many aspects that characterize sciences (Erduran & Dagher, 2014a; Irzik & Nola, 2011). This approach is mainly distinguished from others by its focus on searching for "characterizations" of science, instead of following a "definitory approach" (e.g., the "consensus view"). It is more a matter of *defining*, but rather *distinguishing*, *delimiting,* and *grouping/clustering*.

Erduran and Dagher (2014a), synthesizing the philosophical perspectives of Irzik and Nola (2014), elaborated the Reconceptualized Family resemblance approach for the Nature of Science by reformulating explicitly it within science education research. Erduran & Dagher (2014a) as well as Irzik and Nola (2011, 2014) identify some characteristics to look for family resemblance across science disciplines. In particular, they organize the characteristics into a structure composed of a cognitive-epistemic and a social-institutional system. Erduran and Dagher (2014a), starting from Irzik and Nola's organization (2011), widen the categories and provide a representation, of the wheel, (figure 2.6) as an inclusive, systemic, diverse, comprehensive, and meta-level perspective of disciplines (p.25).

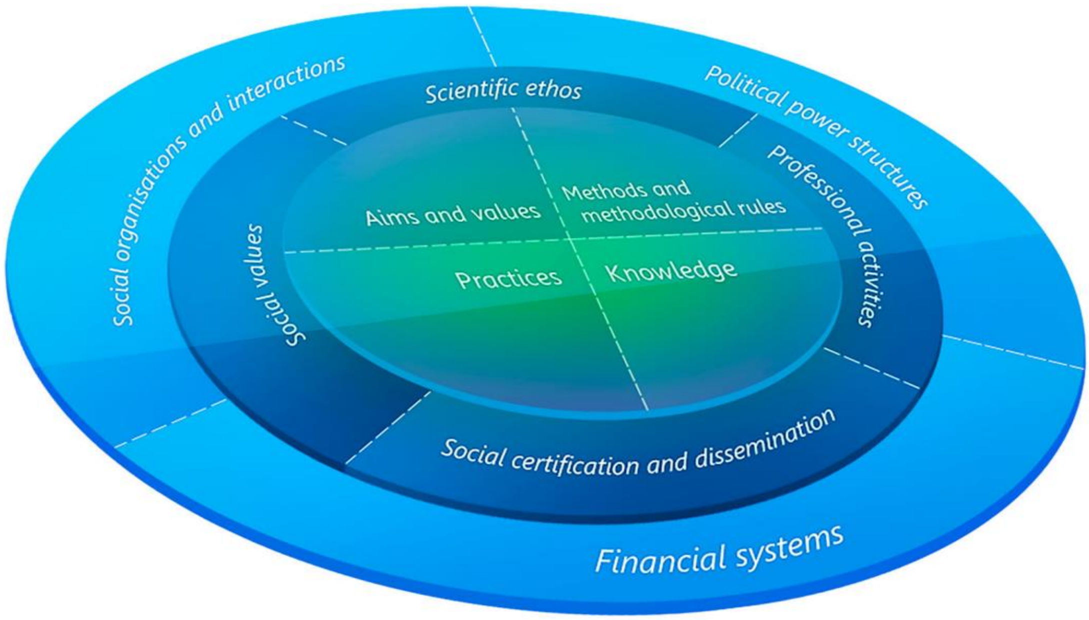


Fig. 2.6: FRA wheel designed by Erduran and Dagher (2014a, p.28)

Erduran and Dagher (2014a) have identified 11 categories that can be used to characterize a scientific discipline. The central core, the cognitive-epistemic system, is articulated in 4 categories: aims and values, methods and methodological rules, practices, and scientific knowledge. The first category refers to the set of values, underpinned by the scientific enterprise, that guide scientific practices such as objectivity, consistency, skepticism, rationality, simplicity, empirical adequacy, prediction, testability, novelty, etc. The second category refers to the variety of cognitive, epistemic, and discursive practices that characterize scientific enterprises such as observation, classification, experimentation, argumentation, modelization, and reasoning. The third category refers to the wide range of observational, investigative, and analytical methods, used by scientists in disciplinary inquiry and guided by particular methodological rules, to generate reliable evidence and construct theories, laws, and models in a given science discipline. Finally, the fourth category refers to knowledge as "an interrelated network of theories, laws, and models", TLM (Erduran & Dagher, 2014a, p.5), as a "product of a collective human enterprise to which scientists make individual contributions which are purified and extended by mutual criticism and intellectual co-operation" (Ziman, 1991, p. 3). Scientific knowledge is holistic and relational, and TLM is conceptualized as a coherent network, not as discrete and disconnected fragments of knowledge (Erduran & Dagher, 2014a adapted from Yeh et al*.,* 2019, p.295).

The social-institutional system, represented by the two external circular levels around the core (Fig. 1), highlights the social-institutional nature of science that "involves individual scientists working in social groups in social institutions, exercising social values and activities" (Erduran & Dagher, 2014a, p.137). The categories of the first circular section, originally proposed by Irzik & Nola (2011), are: "Social values", a category that refers to social values such as social utility, respecting the environment, freedom, decentralizing power, etc.; "Scientific ethos" that refers to norms that scientists pursue during their research work such as skepticism, universalism, intellectual honesty, etc.; "Professional activities" that refers to communication and dissemination activities performed by scientists (e.g. writing manuscripts for reviewing papers, peer-reviewed journals, presenting the work in conferences, developing grant proposals and securing funding); "Social certification and dissemination" that refers to the peer-review process, to the control and the validation of new scientific knowledge by the broader scientific community.

The categories of the second annulus of the socio-institutional system, added by Erduran and Dagher, are: "Social organizations and interactions" which refers to social organizations in which scientists meet and work, like universities and research centers; "Political power structures" that refer to the political environments that influence the scientific enterprise and finally "Financial systems" that refers to the economic factors (e.g. funding) on which science lays on and progress (Erduran & Dagher, 2014a).

The 11 components of the framework, represented by the FRA wheel, provide an image of science as a holistic and dynamic system that visually represents the relationship between its components as parts of a larger whole. "The boundaries (represented by dotted lines) between the two circles (or spaces) and their individual compartments are porous, allowing fluid movement across" (Erduran & Dagher, 2014a, p.29), indicating that the categories affect each other, regardless of the position they occupy on the FRA Wheel.

Recently the FRA framework is starting to be used within a new field of research of philosophy of disciplines, the nature of STEM, NOSTEAM (e.g., Ortiz -Revilla, Adúriz-Bravo and Greca, 2020). This field is starting to be investigated for its educational potential for developing the range of skills necessary for students to achieve full citizenship in the society in which they live (United Nations Educational, Scientific and Cultural Organization, 2016). In recent years, many institutional and political requests were proposed for promoting an integrated approach to science education and renewing curricula from a STEM perspective (European Commission, 2015a,b, 2012; World Economic Forum, 2017). Some educational integrated approaches have been developed trying to combine, put into dialogue and integrate the different STEM disciplines in a more fruitful way (e.g., Hoeg and Bencze, 2017; Zeidler, 2016; Chesky and Wolfmeyer, 2015; Garibay, 2015; Zollman, 2012). Nevertheless, the NOSTEM arises also because “it would be necessary to reflect explicitly upon some philosophical [and epistemological] issues around the nature of the constituent disciplines and the possibilities for dialogue between them, in order to give substantive meaning to an integrated STEM education” (p.859). Therefore, even if integrated approaches to STEM have many positive aspects and answer political and institutional requests, it is important to examine them from an epistemological perspective in order to avoid an uncritical integration and support students in learning about the epistemologies of disciplines and interdisciplinarity (Reynante, Selbach-Allen & Pimentel, 2020).

This is one of the aims of NOSTEM. STEM themes (such as quantum technologies, artificial intelligence, etc.) involve disciplines that have very different characteristics starting from a teleological point of view. Antink-Meyer and Brown (2019) argued that one of the fundamental differences between engineering and science is that the objective of the first is the creation of artifacts, while the second is the pursuit of understanding. In Houkes's (2009) words, this can be reformulated as “truth vs. usefulness” intuition, that is scientific knowledge aims at finding out “true” theories, while engineering knowledge aims at practical usefulness. This is quite in contrast with, for example, Toulmin’s perspective (1972), which argued that the basic focus of scientific research after World War II was no longer nature itself, but some “unit” of engineered artifacts (e.g., computers, reactors, missiles, etc.). Other researchers, according to Mario Bunge’s idea of technology as applied science, distinguish science and technology in axiological terms, that is they mainly differ in their aims, values, and actions. Such a theoretical framework, as Ortiz-Revilla, Adúriz-Bravo and Greca (2020) argue, is widely spread among the general public, shared by many stakeholders, and is a common misconception in science classes. Also in school textbooks, the technologies are usually discussed at the end of the chapters or in separate boxes. Mitcham (1994) discussed the differences between technological and engineering knowledge, arguing that engineers are more involved with applied scientific knowledge and technologists focus more on the actual construction and operation, but current practices in technology appear to blur this distinction (Ortiz .Revilla, Adúriz-Bravo & Greca, 2020).

Many researchers (e.g., Radder 2009; Tala 2009) claim that considering science, technology, and engineering as separate epistemological practices can hinder the richness and variety of actual scientific and technological developments.

Furthermore, many fields today involve applied mathematics that encompasses many disciplines such as theoretical physics, chemistry and biology, parts of engineering, and so on. Statistics, computational science, discrete mathematics, and data science are key-field for the current scientific research, and technological and engineering endeavors, and, in turn, mathematics is influenced by technology. Thinking about computers, these are used in “experimental mathematics” to justify mathematical claims and to produce calculations for suggesting or testing general claims (Avigad, 2008). Trying to consider all these issues and embracing the idea that NOSTEM cannot be reduced as just the sum of the natures of the four constituent fields, Ortiz -Revilla, Adúriz-Bravo and Greca (2020) addressed the relationship between the different disciplines with a model taken from the social sciences (Radder, 2009). The model considers the relationship between technology, engineering, and science as a “seamless web” since these are so strongly intertwined that they cannot be sensibly distinguished in action (Radder, 2009, p.25). In this view science, technology, and engineering form part of a seamless web of society, politics, and economics.

Meaningful is the work of Ortiz -Revilla, Adúriz-Bravo, and Greca (2020), who proposed to use FRA as the basis for “sketching out what an epistemology of integrated STEM, understanding the label as a seamless web of disciplines, would look like” (p. 869). The authors used the FRA framework (Irzik & Nola, 2011) since “it is extremely appropriate as a basis for sketching out what an epistemology of integrated STEM, understanding the label as a seamless web of disciplines, would look like. We are briefly presenting some of the epistemic features that could characterize such a NOSTEM, features that are not stressed much in the scarce literature on epistemological issues within integrated science education” (p. 869). They discussed what can be the family resemblance categories of the integrated knowledge that they summarized in the following table (Table 2.2).

Their work, thus, provide “an initial framework for philosophical discussion, to help analyze integrated

STEM and its aims, discourse and methods, in order to contribute to the task of giving educational rigor and validity to this approach" (ibidem, p.859).

Table 2.2: Features in an FRA model for NOSTEM pointed out by Revilla, Adúriz-Bravo, and Greca (2020, p. 873).

		Epistemological features
Seamless web of the four STEM constituents as a cognitive - epistemic system	Aims and values	The responsible resolution of relevant societal problems within a sustainability matrix
	Methods	Many shared methodologies—experimentation, modeling, design—Design as a central methodology in technoscientific research
	Knowledge produced	● Design of functional objects and organisms ● Proof-of-principle
	Practices	● Closure ● Validation
Seamless web of the four STEM constituents as a social - institutional system	Social certification and dissemination	● Scientists and mathematicians cannot in principle claim ownership of knowledge ● The degree of expression—or codification—of technological knowledge may be largely due to socio-economic circumstances
	Scientific ethos	● Products of knowledge are explicitly value-laden—with epistemic, economic, socio-political and ethical values ● Values are frequently in conflict and demand assessment and regulation
	Social values	● Technological systems are both socially constructed and society shaping ● Sustainability and responsible research and innovation
	Social organizations and interactions	● Big science ● Crowd science
	Financial systems	The ethical, social, and political configuration of the economy configures and shapes the seamless web

To sum up, in our framework for inter-disciplinarity we use the "boundary objects and boundary crossing" framework (Akkerman and Bakker, 2011) and its reconceptualization within the interdisciplinarity since it allows to emphasize the interdisciplinarity as a process of boundary crossing that involves objects and people. However, this framework leaves open questions such as what the nature of boundary is and does not shed light on the epistemology of disciplines (and interdisciplinarity). The FRA can help us to sharpen these two aspects. So it can highlight, on one hand, the nature of the boundaries between the different disciplines by comparing disciplines in their complexity. On the other, it sheds light on the problem of characterizing the disciplines and focuses on disciplines' epistemologies.

Furthermore, the boundary object is described as having an intrinsic *ambiguity*, that is it "belongs to *both* one world *and* another" and, at the same time, "it belongs to *neither* one *nor* the other world" (Akkerman & Bakker, 2011). About this ambiguity, the second, the FRA, can provide a reading key. The FRA, by providing both domain-general and domain-specific characteristics of science, can, on one hand, provide features to the boundary object that respect its nature and, thus, better understand what it is exchanged and what are the boundaries, and, on the other, sharpen the mechanisms of boundary crossing.

The design principles for interdisciplinarity that we carried out are

Design principle #6a: to scaffold a comparison between the disciplines that intertwine in an interdisciplinary or STEM context both in terms of the epistemic-cognitive nucleus, that is shedding light on aims and values, scientific practices, methods and methodological rules, and knowledge and in terms of the social institutional systems (at least as an integrated domain).

Design principle #6b: to trigger boundary crossing mechanism by focusing on boundary objects, on how they are exchanged, what kind of knowledge they embody, and what kind of knowledge they carry before, during and after the intertwining.

Those two principles were implemented both for the lectures and for teamwork activities.

2.4.3 FEDORA: languages and narrative dimension

FEDORA[3], Future-oriented Science Education to enhance Responsibility and Engagement in the society of acceleration and uncertainty, is a 3-year EU-funded project coordinated by the University of Bologna. It started in September 2020 and will deploy its activities until August 2023 and involves other 5 partners: the University of Oxford, Kaunas University of Technology, the University of Helsinki, Teach the Future (a global non-profit movement that promotes futures literacy for students, and educators) and formicablu (an Italian science communication agency).

FEDORA, as a follow-up of I SEE and IDENTITIES, continues the work started in the previous two with a new focus. While I SEE and IDENTITIES led us to produce respectively modules for secondary school students on STEM topics (e.g., climate change, quantum technologies, etc.) that can promote the development of *future-scaffolding skills* and courses on curricular and STEM interdisciplinary topics with also the aim of innovating pre- and in-service teachers' education, FEDORA aims to address these themes also from a political and institutional perspective. So FEDORA on one hand is developing a future-oriented model to enable formal and informal science education to equip the new generation with skills such as thinking, inter-multi-trans-disciplinary, linguistic/argumentative, and imaginative, and *future-scaffolding* (Levrini, Tasquier, Branchetti & Barelli, 2019), foresight and action competence ones needed to grapple with the societal challenges and, on the other, by engaging policymakers and other stakeholders (e.g. RFOs, RPOs, etc.), it is pointing out recommendations for anticipatory policies in order to mobilize visionary attitudes on open-schooling and to orient concrete institutional transformations to support a new sense of trust and desire for promoting an aware, responsible and sustainable participation in science-related societal issues.

In light of these FEDORA is aligned with the emerging paradigm of Responsible Research and Innovation (RRI) elaborated by the European Commission that identifies science education as a key agenda for better equipping students with skills and knowledge to tackle complex societal challenges and foster active and responsible citizenship in the society (Heras & Mallen, 2017). The RRI paradigm implies a shift of the focus of science education outcomes from learning discrete scientific facts to understanding how to apply science learning to different and new situations, and stimulating curiosity, scientific thinking, and the understanding of the nature of science (EU, 2015, p.19).

Furthermore, the official document stresses also the importance of shifting from STEM to STEAM, within which the A includes ALL other disciplines (ibidem, p.9). In a certain sense, and these are the main aspects that I am going to introduce here, FEDORA is also somehow promoting this integration. It is not an aim of the project to carry out a framework or an approach for the integration of A (generally Arts, or All) in STEM, but it is in line with many shared ideas about STEAM according to which, among the different advantages of integration, there are for example "arts integration into STEM for engaging more types of learners" (Bush, Cox & Cook, 2016, p.111), Keane and Keane (2016) explains "As jobs increasingly rely on technology and integrated STEM skills, all students need opportunities to develop

[3] https://www.fedora-project.eu/

mathematical, scientific, and creative capacities" (p. 62-63). The integration can also help students to create personal meanings (e.g., Land, 2013; Maeda, 2013) and to explain with other languages STEM concepts (e.g., Sharapan, 2012). Furthermore, Bequette and Bequette (2012) argue that it is not only a matter of engagement since "when the arts are seen as an end goal, not just an entryway to presumably more important STEM topics, thoughtfully developed STEAM curricula can truly engage sustained cross-disciplinary student learning in PK-12 settings and informal education" (p. 43).
FEDORA fits and shares that the integration of the A in STEM can have these benefits, but the main issue to which FEDORA aims to contribute concerns the difficulties to grasp and conceptualize the contemporary challenges with the more standard languages and narratives.
There is no need for particular arguments to claim that in recent years scientific communication has faced several challenges. Just think of the pandemic or climate change. For example, Ratzana, Sommariva, and Rauh (2020) argue that the "COVID-19 pandemic has introduced unique challenges for public health practitioners and health communicators" such as "the new infodemic (or mis/disinfodemic) challenge – particularly as treatments and vaccines are being developed; communication of risk and uncertainty; health-information behaviors and the instantaneous nature of social media, and the relationship between media literacy and health literacy; the effects of the pandemic on other health issues; and the need for a flexible communication strategy that adapts to the different stages of the pandemic" (p. 1) and, for example, to promoting vaccine acceptance.
In 2010, Nerlich, Koteyko, and Brown, in examining the effectiveness of tools to raise climate change awareness, explored difficulties and barriers that "may hinder effective climate change communication and subsequent motivation to act on these messages" (p. 97). They continued that even if, in those years, the consensus about an anthropogenic origin of climate change grew, it remains "mostly invisible and, confusingly, what is visible in the form of changes in weather patterns may or may not be linked to longer-term climate change trends. Scientists cannot predict with complete accuracy how climate change will manifest itself locally and what measures to mitigate climate change will be the most effective under local conditions. The situation may no longer be one of profound uncertainty, but it is nevertheless one of profound complexity. Climate change poses risks to humanity but risks that are still for many largely 'virtual' risks rather than real ones, depending on where in the world you live and on how much you can 'afford' to think about these issues. In this context, 'people are thus liberated to argue from, and act upon, pre-established beliefs, convictions, prejudices, and superstitions" (p.98).

Hulme (2007) argued that the task of communicating climate change goes beyond making people aware of the "lower case climate change", i.e., climate change as a physical reality:

> "At [the] point [where we have achieved clear and effective science communication] we have only just started on the task required. There is also an uppercase ''Climate Change'' phenomenon: Climate Change is a series of complex and constantly evolving cultural discourses. We next need to embark on the much more challenging activity of revealing and articulating the very many reasons why there is no one solution, not even one set of solutions, to (lower-case) climate change. [...] The role of Climate Change I suggest is not as a lower-case physical phenomenon to be ''solved''. We need to use the idea of Climate Change, the matrix of power relationships, social meanings, and cultural discourses that it reveals and spawns to rethink how we take forward our political, social, and economic projects over the decades to come".

Therefore, like risk communication, health communication, and science communications, climate change communication is intrinsically complex: on one hand, there is the complexity of climate change itself, and, on the other, the complexity of the communication that is involved. Climate change communication is so complex that there is also the need for social and cognitive psychology to explore the attitudes to risk, mental barriers, strategies that can be used to trigger behavior changes, and so on. There is also the need for communication and social experts to investigate the interactions between scientists, the media, policymakers, and stakeholders. It is about a multilevel complexity,

from the multitude of experts that are engaged, their interactions, to the topic per se, to make people aware and active to promote the change (Nerlich, Koteyko & Brown, 2010).
Kahan, Jenkins-Smith, Tarantola, Silva, and Braman (2015), evaluated through an empirical study the effectiveness of the cultural-evaluator model for science communication about the impact of geoengineering on climate change. According to the model, science communication is effective when negotiating two channels. *Channel 1* concerns the information content and is informed by the best available understandings of how to convey empirically sound evidence, the basis and significance of which are readily accessible to ordinary citizens (Kahan, Jenkins-Smith, Tarantola, Silva & Braman, 2015, p. 196). *Channel 2* concerns cultural meanings: the myriad cues—from group affinities and antipathies to positive and negative affective resonances to congenial or hostile narrative structures—that individuals unconsciously rely on to determine whether a particular stance toward a putative risk is consistent with their defining commitments (ibidem, p. 196).
That is, science communication, in addition to providing individuals with valid and pertinent information, must avail itself of the cues necessary to assure individuals that assenting to that information will not estrange them from their communities. Their study argued the necessity of both channels and, in particular, that *Channel 2* cultural meanings can neutralize or dampen defensive resistance to sound information transmitted via *Channel 1*.

The scope of climate change is too big that it also involves writers, philosophers, and artists (from conceptual to sound art, and so on) from different years to look and figure out new languages, aesthetics, thinking categories, and artistic forms to grapple with and talk about the deep transformations of our times. For example, the need of turning a climate change challenge into a new narrative genre is advocated by many writers such as Gosh, Safran Foer, Arundhati Roy, Magnasson, and Arpaia.
Within the consortium of FEDORA, formicablu is responsible for the part related to the new narratives and languages, my purpose, and my expertise, as that of other non-expert colleagues involved in this speech, is not about the communication of science but science education. Therefore, the main aim of this introduction is to shed light on the need of finding new languages, narratives, and imaginative that involve artists, writers as well as science communicators, through which conceptualizing and talking about contemporary challenges. The questions now are: how can science education contribute? What competencies can we put at stake? What kind of *spaces*, outside and inside the school, can we scaffold to explore and bring together science and writing, art, and music in a "productive" and meaningful way? How and to what extent these *spaces* and experiences can foster students (but not only) to populate the contents of personal meanings?
And again: What kind of relationship between arts and science can be triggered to overcome the dichotomy of the instrumental relationship? Whereby an instrumental relationship we mean the idea that art can be conceived as a mean to communicate science and reach a greater audience and science only as a mean to realize an exhibition. Having rethought the relationship between art and science, what are the new objectives of an artistic-scientific product/exhibition? In the process what are the artistic and scientific constraints that lead to the realization and what is their relationship with creativity?
These are some of the questions that are driving the reflections within the FEDORA project and that have led us and are leading us to organize increasingly structured workshops and activities with experts in other fields from conceptual artists to writers, to secondary school teachers who have put at stake different types of skills by creating open dialogue spaces for them and students.
I briefly discussed climate change and covid pandemic communication, but what does this have to do with quantum technologies?
During the different implementations of the course on quantum technologies (section 4.5), we perceived a "problem" in grappling with the Second Quantum Revolution. Starting from the implementation born within the I SEE project, a part of the course was dedicated to future-oriented activities, which were designed with the aim of developing *future scaffolding skills* (Levrini, Tasquier, Branchetti & Barelli, 2019). The problem that touched also us as designers of the module regards the

fact that the new quantum technologies, although there has been a great effort over the past 20 years to bring research into this new field outside the physics departments, do not yet belong to the collective imaginary. Let's think about artificial intelligence, it has been the protagonist of novels and films for years. Quantum technologies are not in this stage so how can foster students think about the future impact of the Second Quantum Revolution and the emergent technologies on their lives and on society when it is still not clear if and what language (in a broad sense) we have, or can we design, to describe what is happening? How is it possible to think about the problems we see in everyday life and think about the "real and concrete" advantage that a quantum computer could have? So we decided to take a step back and try to reflect on the cultural meaning that the Second Quantum Revolution, just like the first, embodies.

We often hear from experts that the revolutionary aspect of the second quantum revolution lies in the abilities we have developed to control the single atom, the single photon, and the single qubit (e.g., Chang, Lin, Chiu, Huang, 2020; Jaeger, 2018; Dowling & Milburn, 2003). In 1952 Erwin Schrödinger in a famous article entitled "Are quantum jumps?" wrote

> "*We never experiment with just one electron or atom or (small) molecule. In thought-experiments we sometimes assume that we do; this invariably entails ridiculous consequences*".

Undoubtedly, the scientific revolution that has brought the present generations of scientists to be able to do something, which in the middle of the last century was unthinkable, is significant. What we, as researchers and educators, asked ourselves is: To what extent does this language "speak" to secondary school students and teachers? Can this language make them understand the scope of the cultural revolution? Does this kind of rhetoric, as physicists, satisfy our sense, our need to understand what it is about?

In asking these questions, and in light of the values and aims of the FEDORA project, we began to question the possible contributions and values that other disciplines, especially the artistic ones such as conceptual art, visual art, creative writing, etc., could have. This thought paved the way to start to rethink the relationship between scientific and artistic disciplines, not only to find new languages, narratives, and aesthetics for talking about science but to promote personal and collective *sense-making* of the world we live in.

To think about these *spaces* of dialogues and possibilities, the vocabulary introduced in the previous section can help. First of all, the *spaces* have to be thought of as boundary territories in which experts in science and art can speak and contaminate each other. It has to be open, that is, a *space* where emotional, epistemic, institutional, and cultural barriers should not represent an insurmountable limit, a *space* in which one can take the risk of one's own ideas in their diversity. Finally, it should be an open *space* in which the constraints dictated on the one hand by scientific formalism and on the other by aesthetics and expressive forms do not become a reason for separation, but rather for a mutual and deep intertwining and enrichment in terms of creativity, new ideas, new personal and collective knowledge.

So, reflection within FEDORA project led us to elaborate and implement another design principle:

Design principle #7: to scaffold *spaces* in which arts and science can dialogue, to reflect on how the dialogue can shape new narratives and languages to better grapple with the contemporary challenges (such as the Second Quantum Revolution), spaces in which explore the role that arts and their constraints can have in "talking about" science personally and collectively and, vice versa, the role that sciences and their formalism can have in the process of creation of, for example, an exhibition (an artwork, a written story, and so on).

In section 2.6 I describe the kind of how we implemented it and what kind of activities we developed.

2.5 Synthesis of the design principles and relationship with properly complex territories and inter-disciplinary's authenticity

Table 2.3 shows synthetically all the design principles linked to the projects and the main dimensions involved. Both the design principles and the dimensions are not independent of each other. As figure 2.7 shows, the two central design principles are design principles #1 and #2 that value the Second quantum revolution as a cultural revolution. The other design principles aim to enrich and widen the dimensions of the first and the second. Design principles #3 and #4 switch on an educational perspective in order to make the content within the reach of secondary school students and to try to include the plurality of ways of understanding, and students' different tastes and sensitivities. Design principles #6a and 6b aim to explicitly address the inter-disciplinary epistemologies of quantum computation and information. Design principles #5b and #7 are somewhat separate because they are not yet explicitly linked to the epistemic structure of science. We are working hard on design principle #5a as a key to enhancing the structure of the future inherent in science (see Rosi, 2022).

Table 2.3: Summary of the design principles.

Project reference	Design principles #	Dimensions
	Design principle #1: to foster a close comparison between classical and quantum computers through an analysis of the different logic underlined in the basic mechanisms on which the hardware is built.	Cultural perspective of the Second Quantum Revolution
	Design principle #2: to reconceptualize the foundational experiments in terms of computation, so as to discuss why and how experiments can be considered as "simulators" or devices to process information (circuits). This means, operationally, re-reading the three main phases of an experiment -*state preparation, state evolution/manipulation, and measurement*- in terms of *input - processing - output information*.	
	Design principle #3: to keep the quantum technicalities as simple and clear as possible and foster a deep understanding of essential physical concepts that are needed.	Cognitive, science education (e.g., students' difficulties in understanding the basic concepts of quantum mechanics)
	Design principle #4: to make the modules as inclusive as possible.	
I SEE	**Design principle #5a**: to exploit and unpack the future's intrinsic structure that characterizes science.	Impact of science (such as quantum technologies) on e.g., research, policy, society, etc., and future-oriented dimension.
	Design principle #5b: to engage students in reflections about the implications and impact of science and technologies on different dimensions such as the scientific, political, institutional, economic, ethical, environmental, educational, etc., fostering them to think about the different stakeholders	

	involved, the possible future's scenarios arising from the scientific and technological progress, and their possible roles in the debates and decision-making.	
IDENTITIES	**Design principle #6a**: to scaffold a comparison between the disciplines that intertwine in an interdisciplinary or STEM context both in terms of the epistemic-cognition nucleus, that is shedding light on aims and values, scientific practices, methods and methodological rules, and knowledge and in terms of the social institutional systems (at least as an integrated domain). **Design principle #6b**: to trigger boundary crossing mechanism by focusing on boundary objects, on how they are exchanged, what kind of knowledge they embody, what kind of knowledge they carry before, during and after the intertwining.	Inter-disciplinarity between physics, mathematics, and computer science from conceptual, epistemological, etc. perspectives
FEDORA	**Design principle #7**: to scaffold *spaces* in which arts and science can dialogue, reflect on how the dialogue can shape new narratives and languages to better grapple with the contemporary challenges (such as the Second Quantum Revolution), and spaces in which explore the role that arts and their constraints can have in "talking about" science personally and collectively and, vice versa, the role that sciences and its formalism can have in the process of creation of e.g. an exhibition (an artwork, a written story, and so on).	The problem of finding new narratives, aesthetics, and languages. Relationship arts and sciences

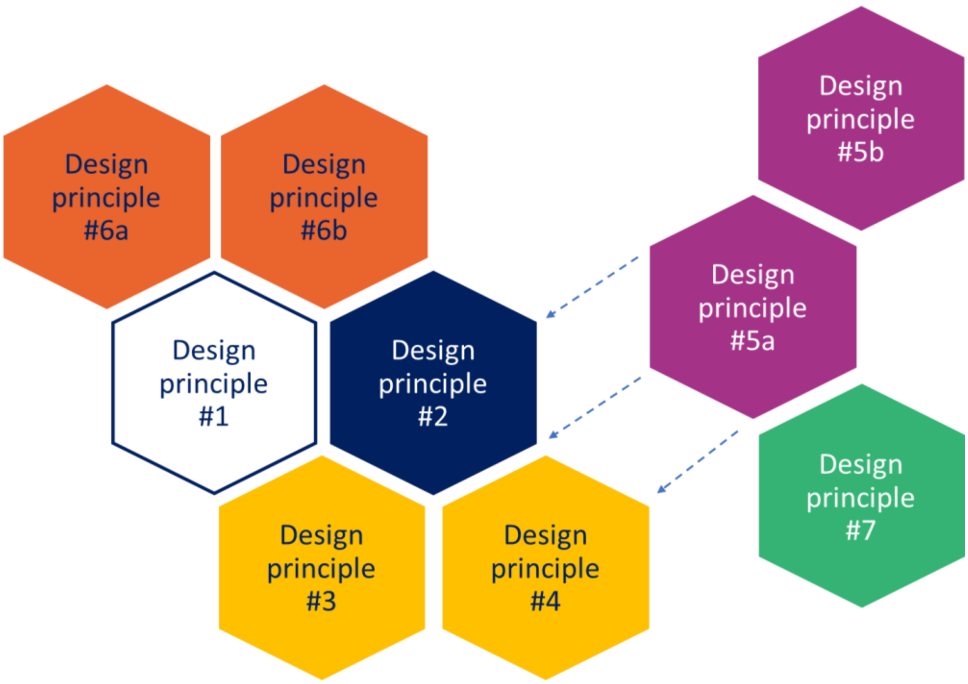


Fig. 2.7: Relationship between the different design principles.

As I describe in section 2.1, these principles can avoid forms of simplification that risk not only “distort the learning process as well as the content” (Levrini & Fantini, 2013, p.1896), but also do not provide students with the competencies that also the OECD learning compass framework (2019) highlighted as fundamental to deal with the contemporary society and navigate toward a sustainable future.

All the design principles have been deeply discussed, I just come back to the two important ideas from the work of the research group started. The idea of *properly complex territories* and forms of complexities that can be implemented in the design of instructional materials to pursue the general goal of “enabling students to find out their ways of pinpointing or solving the problems and puzzlements they perceive'' (Levrini & Fantini, 2013, p. 1899), we are still linked with the multi-perspectiveness, multi-dimensionality, and longitudinality that I describe above. The keywords become interdisciplinarity referring both to multi-perspectiveness and multi-dimensionality since, in particular, the three perspectives are the three main disciplines: physics, mathematics, and computer science. The dimensionality is linked to the complexity of disciplinarity and interdisciplinarity, that is the content dimension is not only the one we propose, but we touch also on epistemological and philosophical debates such as the shift to the ontological probability, the problem of generating random numbers, the differences between simulation and universal calculation, and so on. Finally, the ‘’game’’ of modeling quantum phenomena and the systematic analysis and comparison with the models already encountered (the longitudinality) is settled in the relationship between bits and qubits, in the comparison between classical and quantum logic gates, circuits, and algorithms.

All the other design principles activate, as can be seen also from table 2.3, not only the dimensions of disciplinary authenticity (Fig 2.1: conceptual, cognitive-epistemological, affective, societal, educational, political-institutional) but also the ones of “interdisciplinary authenticity”.

2.6 The module on the Second Quantum Revolution

In the final section of this chapter I the module on the Second Quantum revolution we developed, how it changed over the years trying to collect the different inputs from the I SEE, IDENTITIES, and FEDORA projects. Most of the activities are described here, but the teleportation protocol and random walk activities are deeply described in chapter 3 through the lens of the model of educational reconstruction (Kattmann, Duit, Gropengießer & Komorek, 1996; Duit, Komorek & Wilbers, 1997) as well as an analysis of the results of a questionnaire that students filled in after the activities.

The course was first developed within the I SEE project in 2019 for secondary school students starting from the work of the Finnish partner (see Satanassi, 2018). It was developed as an extra-curricular course in collaboration also with the project Piano Lauree Scientifiche[4] (PLS), born in 2004, under the indication of MIUR (ministry of education), with the aim of solving the problem of the constant decrease in enrolment in scientific degree courses. In recent years, the PLS project has implemented several structural measures aimed at stimulating young people’s interest in studying these subjects, providing a more appropriate background in basic scientific subjects, and enhancing the interaction between university and business in order to facilitate the integration of our students in the high-tech market.

The course from 2019 has undergone several revisions, it was completely implemented five times, four of which were with secondary school students and one, within the project IDENTITIES, with pre- and in-service teachers. In Figure 2.8, a schematic timeline of the course implementations and evolution with the project and the target groups involved is represented.

[4] https://www.pls.unibo.it/

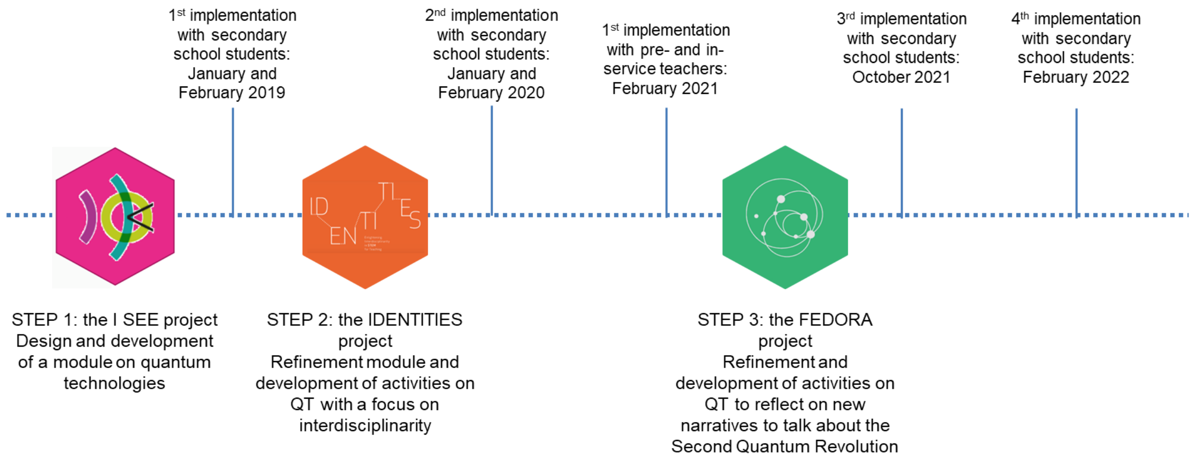


Fig 2.8: Schematic timeline of the implementations and course evolution with projects and main target groups involved.

The persons involved in the course's design and implementations besides me are Professor Olivia Levrini, tutor of my Ph.D. and expert in physics education, Professor Elisa Ercolessi, a theoretical physicist, and three secondary school teachers in physics and mathematics, Paola Fantini, Michela Clementi, and Fabio Filippi from Liceo Einstein (Rimini, IT) with whom we collaborate from different years. Moreover, two bachelor's students participated in the development of two activities, Roberta Spada (2019) and Dario Casali (2020), and one master's student, Francesca Riccioni.
The course was partially implemented also with the University of Como (UNINSUBRIA) and the University of Pavia (UNIPV). The course we developed with them was implemented other four times both with secondary school students and in-service teachers, three of them in an intensive course (two summer schools in 2020 and 2021, and one spring school, in 2021) and one big implementation as an extra-curricular activity lasted about two months that involved about 200 students and experts in other Italian University as University of Udine, University of Napoli, University of Pisa, University of Trento and in two research institutes, Istituto di Fotonica e Nanotecnologie (CNR-IFN) and Istituto Nazionale di Ottica (CNR-INO, c/o SISSA) (Bondani, Chiofalo, Ercolessi, Macchiavello, Malgieri, Michelini, Mishina, Onorato, Pallotta, Satanassi, Stefanel, Sutrini, Testa, and Zuccarini, 2022).
The smallest team of the summer and spring schools, that also guided the bigger implementation, is composed of Professor Maria Bondani, an expert in quantum optics, and Filippo Pallotta, doctor in physics education and physics teacher at school, for the University of Como; Claudio Sutrini, a Ph.D. student in physics education and mathematics teacher, and Professor Chiara Macchiavello, theoretical physicist, for the University of Pavia.

Hereafter I focus only on Bologna's course. Table 2.4 reports the details of the different implementations in terms of the target group, number of participants, modality, and duration of the course.

Table 2.4: details about the different implementations.

Implementation	Target group	Number of participants	Modality	Course duration (18h)
January-February 2019	Secondary school students	25 (9F-16M)	Presence	One meeting per week of 3 hours each
February-March 2020	Secondary school students	22 (5F-17M)	Presence	One meeting per week of 3 hours each

February 2021	Pre -and in-service teachers	9 pre- 31 in-	Online	Two meetings per week of 3 hours each
October 2021	Secondary school students	30 (9F - 21M)	Presence	Two meetings per week of 3 hours each
January-February 2022	Secondary school students and pre- and in- service teachers	26 (10F - 16M) 3 pre- 3 in-	Online	Two meetings per week of 3 hours each

Table 2.5 reports the module's structure and the kind of activities per day. As per the model I SEE, in each meeting, we carry out two different types of activities both in terms of content and modes. The first two hours are dedicated to, in I SEE words, conceptual-epistemological activities. These activities touch dimensions: the conceptual knowledge that is the inter-disciplinary content of the topic, the epistemological knowledge and practice that concern epistemic practices such as modeling, arguing, and explaining to promote deep and meaningful learning (Chinn, 2018; Tasquier, Levrini & Dillon, 2016), and inquiry practice that relates to practices typical of experimental investigations such as posing questions, formulating hypotheses, designing inquiry, triggering peer-to-peer interaction, recognizing modeling as a process of isolating a particular phenomenon, and moving from models to experiments and vice versa. These activities are of different kinds: frontal lectures, discussions, laboratory activities, and teamwork.
The last hour of each meeting is dedicated to future-oriented activities. These kinds of activities are mostly workshops, teamwork, and discussion and aim to help students to re-read the inter-disciplinary concepts introduced in the conceptual-epistemological activities so as to highlight the intrinsic future-related themes with specific regard to the models of causal explanation. In the Second Quantum Revolution module, those themes are intrinsic randomness (linked to the entanglement), the ontological probability vs. the epistemic one, and the notion of scenarios as possible outcomes of a measurement on a system. Furthermore, these activities aim to promote the development of skills that allow students to engage with the imagination of probable, plausible, possible, and desirable scenarios (Fig. 2.5) as well as foresight and back-casting skills (see section 2.4.1).

Table 2.5: Structure of the course on the Second Quantum Revolution.

Day	**Conceptual-epistemological activities (2h)**	**Future-oriented/citizenship education activities (1h)**
1	● Introduction to the Second Quantum Revolution ● The history of classical computers (introduction to classical computers' structure, binary logic, classical logic gates and circuits, computers' evolution from the first to the fourth generations)	"Quantum Technologies & …": teamwork activity to reflect on the impact of QT on the societal, political, economic, environmental, educational (etc.) dimensions.
2	Introduction to the physics of quantum computers_Part 1: ● Introduction through a simplified spin-first approach of the concepts of state, state manipulation/evolution,	Delivery of the teamwork activity "Quantum Technologies & …"

	and measurement ● Passage to information: encoding information in Qubits (input information), information processing (one-qubit logic gates and circuits), reading information (output information)	
3	● Introduction to the physics of quantum computers_Part 2: Introduction of entanglement (two-qubits systems) ● Quantum cryptography protocol (BB84)	The Eve city_part 1: In the role of policymakers. The teamwork activity involves students taking on the role of mayor and deciding whether to invest in the new QT by discussing the different perspectives.
4	Teleportation protocol and the future quantum internet	The Eve city_part 2: reflections about the concept of scenario. Teamwork activity in which students reflect on if and how QT is changing the relationship between nature, humans, and technology.
5	Classical and quantum random walk	Future-oriented activity: in group students think about a problem that they would like to be solved in 2040 (foresight) and try to figure out, as active members, the possible steps that can lead to solving it (backcasting). Teamwork in preparation for the workshop.
6	Delivery and discussion of the teamwork or creative workshop (e.g., Reflection on how scientific and technological revolution in the Great History impacted arts, literature… and vice versa)	

The conceptual-epistemological activities from the first implementation remain almost the same, they were refined implementation after implementation in light of the collected feedback, we added from the second implementation an introduction to the Second Quantum Revolution and, within IDENTITIES, two activities: one teamwork on the problem of finding the shortest path at the beginning of the second day, and the random walk activity.

The future-oriented ones were not only refined but, in every implementation, we designed new ones. In the very first implementation, we implemented the "Quantum Technologies &..." activity and the one developed by the Finnish partners. Then, in light of the problem we perceived reported in section 2.4.3 concerning the lack of a deep capacity to project into the future due to the non-belonging of quantum technologies to a collective imaginary, we decided to keep the activities to reflect on the impact of quantum technologies on society, research, policy, and so on, and develop new ones to make students reflect on the cultural scope of the Second Quantum Revolution and find a personal way to talk about it.

Hereafter, I describe the different activities with a greater focus on invariant activities throughout the implementation, following the order of table 2.5 starting from the conceptual epistemological activities.

2.6.1 Conceptual-epistemological activities

Introduction to the Second Quantum Revolution

This is the first activity of the course, and it is dedicated to the introduction of the Second Quantum Revolution from a broad perspective. Our goal is to introduce students to the public, political, social, and educational debate about quantum technologies in a way that promotes, given the technical details we deal with in other activities, a connection with the society in which we live.

After introducing the seminal idea of the possibility of “an exact simulation, that a computer will do the exactly the same as nature” (Feynman, 1981, p. 478) namely following the principles of quantum mechanics, we introduce very briefly the history of quantum computers emphasizing the strongly theoretical flavor of the first results and, then, and apparently not particularly significant results (such as factoring the number 15, Fig. 2.9) to the reach of quantum supremacy (2019). The schematic timeline we present is reported in figure 2.9.

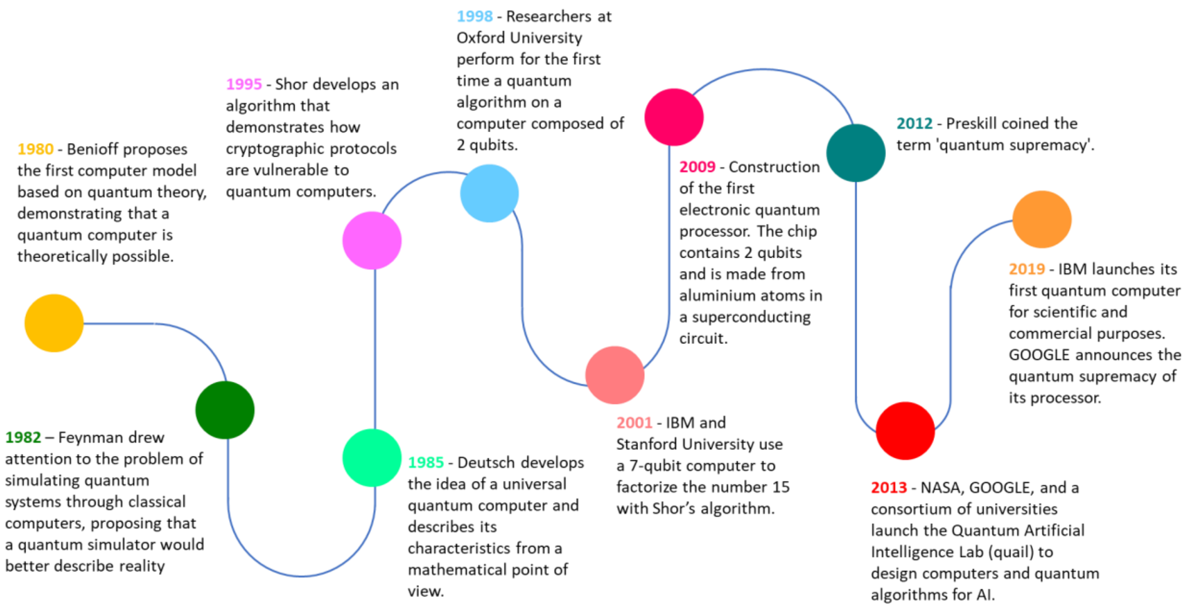


Fig. 2.9: a short history of quantum computers

We then introduce students to the debates by presenting many pieces from an Italian newspaper (‘*Il Sole 24 Ore*’). We choose newspaper articles since it is one of the aims of schools to develop the ability to critically review popular media and judge their adequateness as well as to enable students to take part in societal discourse (e.g., Pospiech, 2021; Kuhn and Müller 2014).

by showing different excerpts, we pay attention to the kind of language the journalists use from the very first articles we found to more recent ones and the multidimensional content of the article.

The first pieces of paper we introduce are characterized by a *language of disbelief*, *improbability*, and of *remoteness*. Just to make an example, a paper published in 2011 reported:

> “A machine that doesn’t obey the laws of classical physics. That solves problems that are impossible even for the most powerful computers, so sophisticated that they can look at reality with a mechanical-quantum approach. I mean, a machine that <u>doesn’t exist but that all the main physics departments are studying</u>. Last year, a series of discoveries published in many scientific articles shortened the time of what <u>for twenty years is considered a mirage</u>. [...] But the way the research is being structured is more likely to see a series of small-large discoveries all over the world that will then flow into the making of a chip, on which, <u>to tell the truth</u>, <u>very little can be said with certainty</u>.” [the translation was made by me]

From these early papers that talk about quantum technologies as something that is still distant and closed in physics departments, we show students how the discourse has changed since 2016, following the publication of the Quantum Manifesto. Just an example of an excerpt from an article in the same journal:

> "The Quantum Manifesto calls on Europe to push the development and application of quantum technologies to achieve «a transformation that will allow advancement in science, industry, and society». [...] But it is necessary that this opportunity for change is also understood outside the laboratories since these technologies are intended to provide a competitive advantage to those who can exploit them and convert them - before others - into innovative products. [...] But as complexity and speed of innovation increase, the educational system also needs to be updated to allow the research, industry, and finance ecosystem to synchronize with the pace of advanced technologies' development."

The language of improbability is turned into the *language of possibility* and *industriousness*: It is no longer talking about mirages, but rather about coordinating actions to bring coherence and create synergies between the world of research, industry, and finance and to update the educational system to keep pace with technological change.
Finally, we also introduce students to the debates that follow the achievement of quantum supremacy. Only to make an example among many, an article of 2020 named "Europe regains its technological sovereignty" in the same newspaper:

> "Faced with the "technological war" between the United States and China, Europe must, from now on, lay the foundations of its sovereignty for the next 20 years. [...] When, in the past, Europe has mobilized itself, and united, around large industrial projects, it has shown that it can play a leading role on the world stage. It is time to play a leading role on the world stage. It is time to take these joint initiatives."
> Through the present excerpts and many others, we lead students to recognize the relevance of the theme and its multidimensionality, that is, the debate is not purely confined to research but is also a geopolitical, economic, educational, and social discourse.

After this discussion, we introduce students to the Quantum Manifesto (De Touzalin, Marcus, Heijman, Cirac, Murray & Calarco, 2016) and the European answer to the Quantum Manifesto's call, the Quantum Flagship[5]. In particular, we introduce the four pillars (Fig. 2.10) at the center of the investment - communication, computation, simulation, and sensing/metrology - and some examples of technologies and/or research lines.

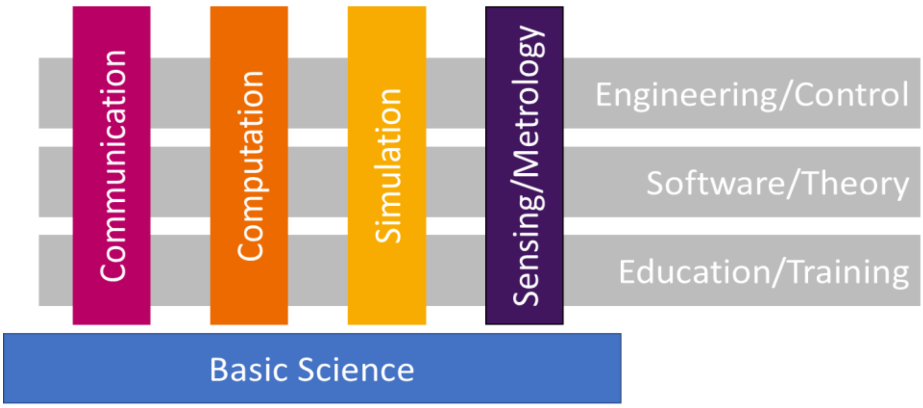


Fig. 2.10: Quantum Flagship's pillars and sectors of research

[5] https://qt.eu/

In the case of communication, we talk about teleportation, quantum internet, and quantum cryptography. In the case of computation, we describe what a quantum computer is, and its purposes and we introduce some problems that a supercomputer cannot solve in "useful times" such as the problem of integer factorization and protein folding. Regarding simulation, we introduce students to the difference between computers and simulators, to the problem of simulating microscopic events that cannot be directly observed and its impact on for example science of new materials, and to the possibility to simulate models and tasting the test them in conditions that cannot be "found in nature". Finally, about the final pillar, we describe the high resolution that is achieved with atoms and its impact on imaging.

Then we introduce visually, from a conceptual point of view, what are the core concepts that allow the development of these new technologies (Fig. 2.11).

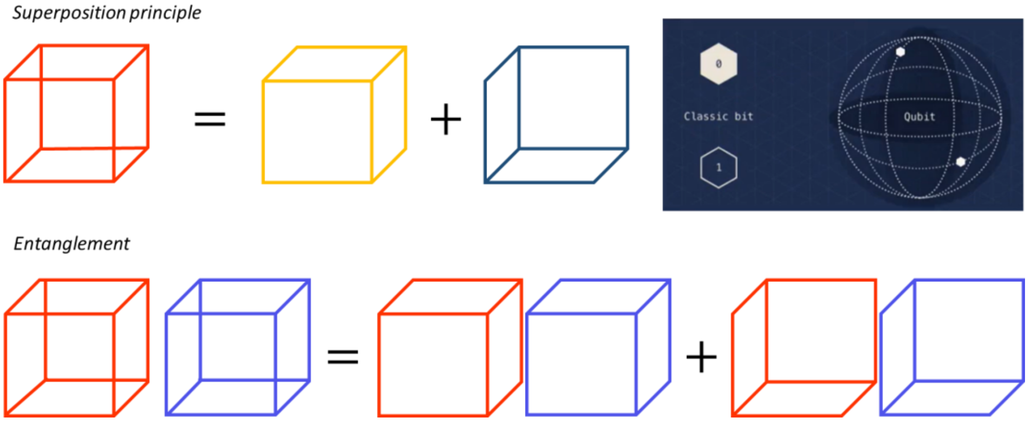


Fig. 2.11: Necker's cube to informally introduce what superposition principle and entanglement are.

Finally, we present the discourse from an educational perspective, introducing the quantum flagship and the new university courses, not only for physicists, that were born all around Europe, concluding that as every revolution, the quantum revolution is impacting and impacting on so many dimensions and so many different professions of the future will be involved in these new revolutions.

This activity incorporates the spirit and design principles of the I SEE project (#5a) with the aim of making the students encounter the focal topic from a very broad perspective and making them reflect on the multidimensional impact of the second quantum revolution and on the tight and complex relationships between science and society.

The history of classical computers

In this activity, we introduced students to classical computers and their evolution. It aims to unpack the computer as we usually perceive it by showing

- the computers' Von Neumann architecture;
- the basis of computation: the unit based and the Boolean logic that embodies, the encoding of physical information in the computational basis, and its elaboration through logic gates, circuits, and algorithms;
- an experimental device that realizes the same operation of the universal logic gates;
- the computers' components evolution from the first generation to the fourth one.

paving the way to what today quantum computers are.
We start by defining what computer science is in terms of structures and procedures for information processing and by introducing the von Neumann architecture of a digital computer as described in the First Draft of a Report on the EDVAC by John von Neumann (1993), composed by the following

components: a processing unit with both an arithmetic logic unit and processor registers, a control unit that includes an instruction register and a program counter, a memory that stores data and instructions, external mass storage, and input and output mechanisms (Fig 2.12).

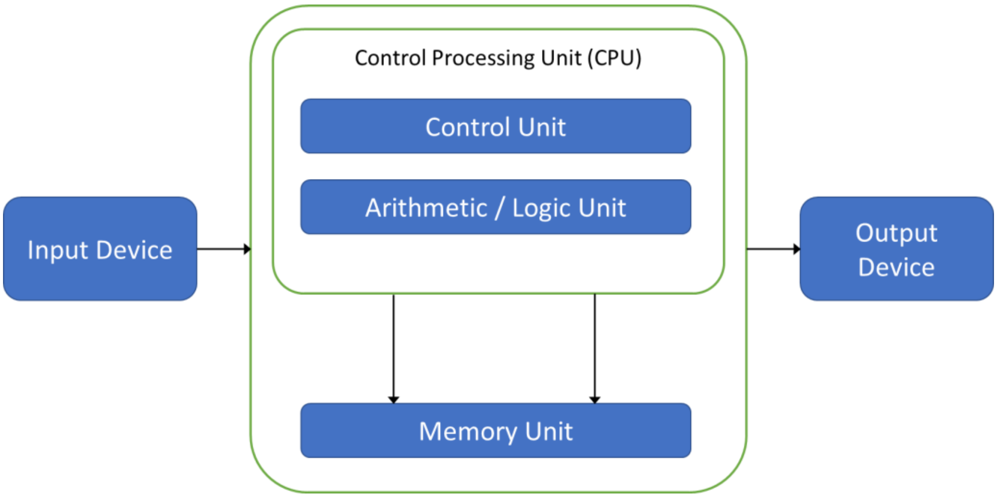
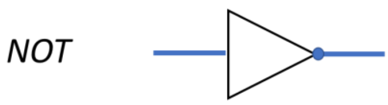


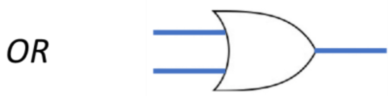
Fig. 2.12: Von Neumann architecture of a digital computer.

Then we emphasized the general role of a digital computer in completing specific tasks and fulfilling certain functions by stressing the correlation between things we can compute and physics and the kind of logic, and then type of physics, that classical computers implement, that is Boolean logic and classical physics paving the way that a different logic and physics principles, could change how and what we can compute. We then introduce students to the basic unit (the BIT) as the unit of information, the logic that it implements, and how information processing is performed, that is as made of: i. the design of the knowledge base, that is the input information; ii. the kind of "operations" to be performed to handle and "use" it; iii. how the information has to be returned to make outgoing information "understood". The notion of algorithm is then introduced as a sequence of instructions or steps defining the operations to be performed to manipulate data that arrive from the external environment with the aim of e.g., solving a problem, or completing a task.

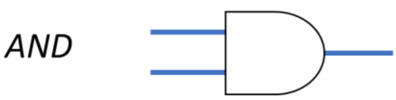
We then introduce the universal logic gates NOT, OR, and AND with the respective truth tables (Fig. 2.13).



A	NOT (A)
TRUE / 1	FALSE / 0
FALSE / 0	TRUE / 1



A	B	A OR B
TRUE / 1	TRUE / 1	TRUE / 1
TRUE / 1	FALSE / 0	TRUE / 1
FALSE / 0	TRUE / 1	TRUE / 1
FALSE / 0	FALSE / 0	FALSE / 0



A	B	A AND B
TRUE / 1	TRUE / 1	TRUE / 1
TRUE / 1	FALSE / 0	FALSE / 0
FALSE / 0	TRUE / 1	FALSE / 0
FALSE / 0	FALSE / 0	FALSE / 0

Fig. 2.13: Universal logic gates and truth tables.

We then present one physical implementation through the story of “the ropes and pulleys of Apraphul island”, an imaginary island described by Dewdney in an article (1988) in the Scientific American newspaper. The story begins like this: "On the island of Apraphul which is located off the north-western coast of New Guinea, some archaeologists have discovered the remains, by way of decomposition, of an ingenious system of ropes and pulleys that is believed to be the first digital calculator in history [...]" (Dewdney, 1988). The article illustrates in a rigorous way how logic gates (such as OR, NOT, AND) and circuits can be constructed through ingenious systems of ropes and pulleys (Fig. 2.14), where the positions assumed by the ropes in input and output represent the 0 or 1 (false/true) that characterize the computer binary logic. The computation is carried out by a mechanical calculator as big as the whole island. The functioning and the problems that could be computed were obviously strictly conditioned by the rules of classical physics. In this way, we fostered students to grasp the idea that behind the software there are logical gates materially built on devices that follow physical laws, necessary to understand their functioning. The logical level was addressed by carefully introducing the classical logic that still characterizes computers’ operation. The main point we stressed is the passage from the mathematical level of Boolean operations and their representation through truth tables to the logical gates.

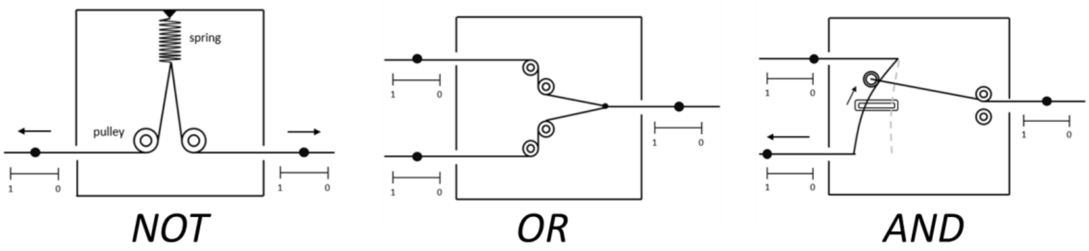


Fig. 2.14: physical implementation of NOT, OR, and AND logic gates.

Coming back to the logical dimension, we make an example of a possible operation to introduce students to how classical logic functions. In particular, we present the sum and the semisum of two BITs both in terms of truth tables and circuit (Fig.2.15).

A	B	NOT A	NOT B	A AND (NOT B)	B AND (NOT A)	(A AND (NOT B)) OR(B AND (NOT A))
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

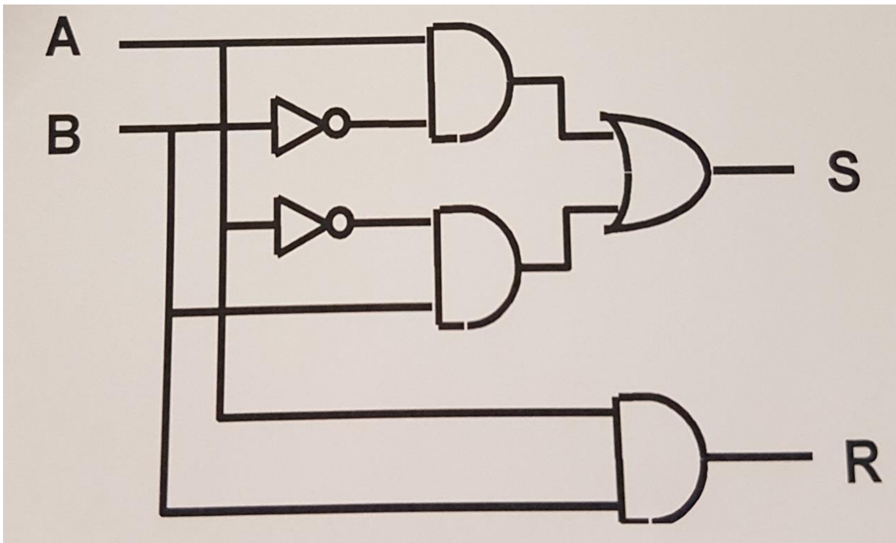


Fig. 2.15: truth table and circuit of the semisum of two BITs.

The evolution of the classical hardware is then presented by introducing the four generations of computers: in the first generation the main tools of information processing were the vacuum tubes, in the second the transistors, in the third the integrated circuits, and in the fourth sets of integrated circuits called chips.
Finally, we scaffold the comparison between classical and quantum computers through the computation dimension (Fig. 2.16) paving the way for the content of the next meetings.

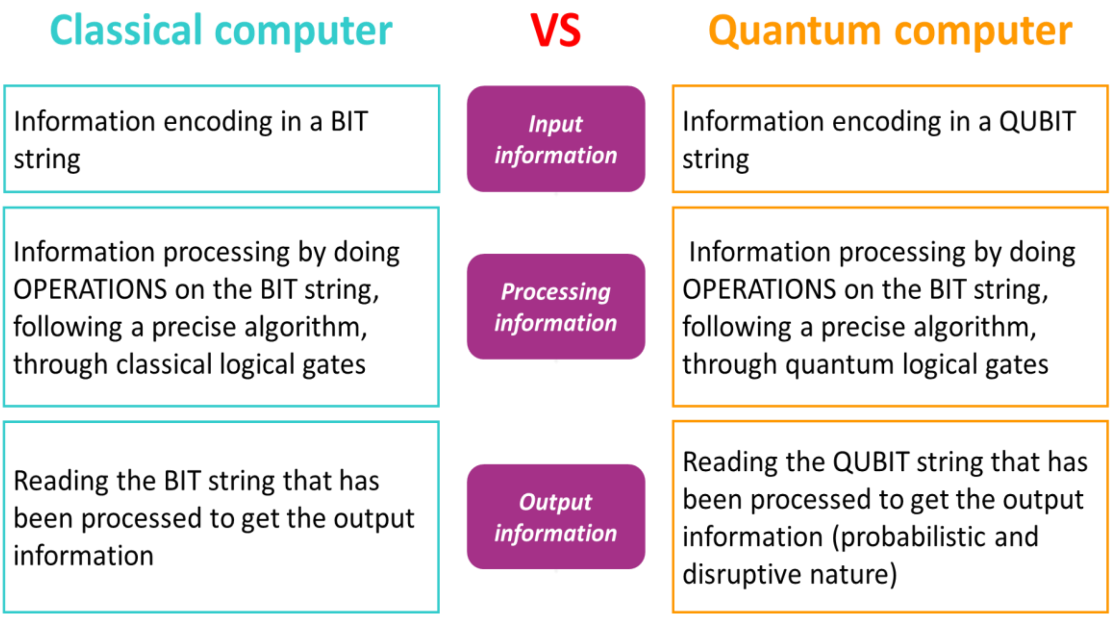


Fig. 2.16: comparison between classical and quantum computers through the dimension of computation.

This first lesson is quite indicative of the design principles we pointed out in the previous sections. First of all, it was designed on the basis of our assumption (section 2.3) with the objective of highlighting the close relationship between computation, logic, and physics and showing that, even if today we usually deal with the software of the computer, there is behind a hardware that implements a certain logic and physical principles. By taking this step back to the hardware through the comparison between circuit and physical implementation (design principle #2) we pave the way for comparing classical and quantum computation (design principle #1).

The interaction between the different disciplines (design principles #6a and 6b) shows in what sense the computers have progressed without changing the basic logic and how technology evolution led the software to be developed to process complex problems that go even beyond a mere sequential logic through which the machine processes the data. Today's computers are designed according to the potentialities of software or on the basis of the type of problem to be solved, rather than how the machine works. This latter aspect aims also to emphasize how historical evolution shed light on the making of science, how science has increasingly refined its techniques, and how aims, practices, methods, and knowledge have changed over the years, thus restoring a dynamic image of science. As described in section 2.3, from a more cognitive level, this lesson is designed by intertwining more levels: the narrative consists of the Apraphul island's story, the logical one consists of the introduction of logic gates and circuits, the mathematical one consists of Boolean algebra and the technical-experimental one consists of the systems of ropes and pulleys that act as logic gates. The presentation of the different levels aims to help students to find a suitable and personal way to deal with the discourse by choosing the levels with which they resonate more (design principle #4).

Introduction to the physics of quantum computers_Part 1

This activity is dedicated to the introduction of three of the essential and basics concepts that are necessary to grasp what quantum computation is at an introductory level: the quantum state and its preparation, the state manipulation/evolution, and the measurement (e.g., Krijtenburg-Lewerissa, Pol, Brinkman & Van Joolingen, 2019; Laforest, 2015; Pospiech, 2000). To introduce these concepts, we use a two-state approach since is particularly suitable for introducing the differences between quantum and classical physics (e.g., Pospiech, 2021; Sadaghiani, 2016). Even if some studies show the difficulties to link the physical meaning to the mathematical formalism (e.g., Schermerhorn, Passante, Sadaghiani & Pollock, 2019), two-state systems can be described easier from a mathematical point of view (finite-dimensional Hilbert space, discrete eigenvalues, etc.) allowing students to mainly focus on the concepts instead of elaborate mathematics (Pospiech, 2021). Furthermore, two-state systems can describe a variety of quantum phenomena such as the photons' polarization, atoms' spin, the ground state and excited state of atoms or ions, Josephson contacts, and so on. Some of these are also important applications in quantum information and quantum technology research covering, in addition to physical and mathematical advantages, topics where physicists are actually working on increasing the insight into actual research.
In particular, we choose a *spin-first approach* (McIntyre, Manogue & Tate, 2013) since it is based on the quantum concept of spin and state, immediately breaking with the classical properties, and avoiding dangerous analogies and, as McIntyre, Manogue, and Tate (2013) argue, i. "it demonstrates how quantum mechanics works in principle by illustrating the postulates of quantum mechanics, and ii. it demonstrates how quantum mechanics works in practice through the use of Dirac notation and matrix mechanics to solve problems" (p. 1).
In light of this, we started by introducing the Stern and Gerlach experiment (Figure 2.17) performed in 1922 by Otto Stern and Walther Gerlach, who demonstrated that the spatial orientation of angular momentum is quantized.
From the beginning the experiment was introduced following the three main phases that compose it: a silver beam, produced by an effusion of metallic vapor from a furnace heated to 1000 C and collimated through two slits 0.03 mm wide (*state preparation*), is passed through a magnet that generates a non-homogeneous magnetic field (*state manipulation*) and then to the magnet there is a screen that detects the position (*measurement*).

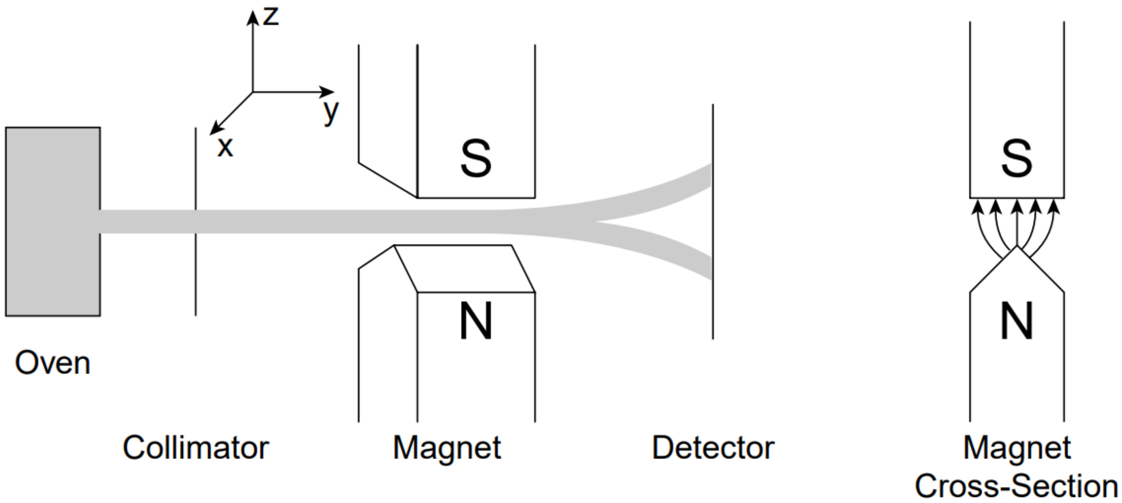


Fig. 2.17: Stern-Gerlach experiment to measure the spin component of neutral particles along the z-axis. The magnet cross section at right shows the inhomogeneous field used in the experiment (McIntyre, Manogue & Tate, 2013).

Particular emphasis was put to underlie the differences between classical and quantum prediction. Classically, if we used instead of atoms for example small magnets, we would expect to obtain a distribution as in figure 2.18 (a), that is a continuous distribution on an amplitude corresponding to the orientation range of the magnetic moment, that is the beam should have spread vertically over an amplitude corresponding to the range of values of the magnetic moment multiplied by the cosine of the angle of orientation. What is detected is, instead, a distribution as in figure 2.18 (b), namely only two orientations of the magnetic moment can be manifested. The beam is then separated into two distinct components: the beam ends in the "up position", or the beam ends in the "down position". This is because particles and atoms have an internal property (purely quantum) called spin. There is therefore a correspondence between the "up position" and the «spin up» ($|\uparrow\rangle$), the "down position" and the «spin down» ($|\downarrow\rangle$).
In addition, Stern and Gerlach have also shown that atoms are equally located either in the «up position» or in the «down position».

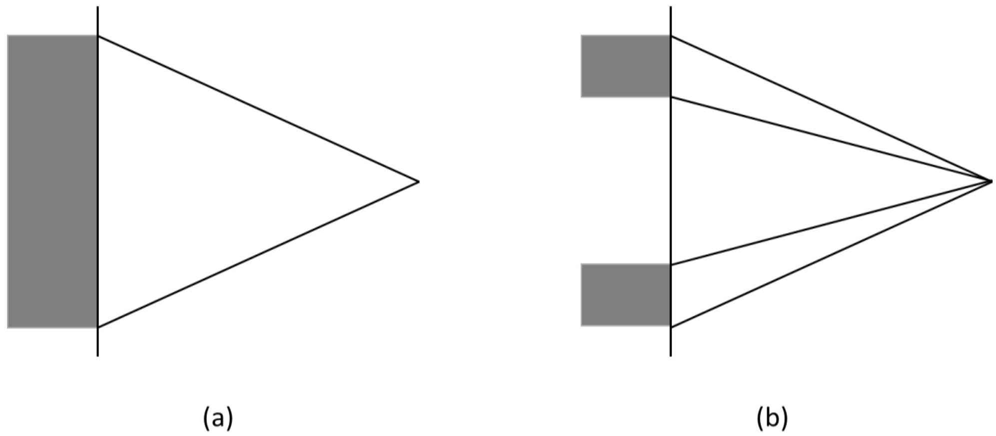


Fig. 2.18: (a) Expected classical distribution and (b) quantum physics distribution.

After this brief introduction to the experiment, we engaged students in a collective discussion by challenging them to predict the final probability distribution by putting in sequence different Stern and Gerlach apparatuses.

The first experiment (*experiment #0*) is already discussed in introducing the experiment. In figure 2.19, I report the schematic representation elaborated by McIntyre, Manogue, and Tate (2013).

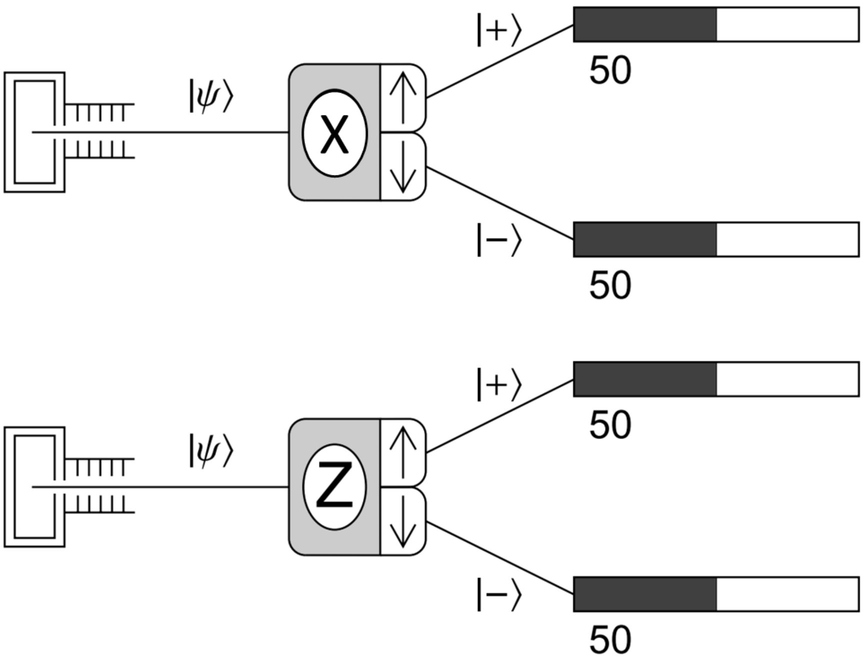


Fig. 2.19: Simplified schematic of the Stern-Gerlach experiment, depicting a source of atoms, a Stern-Gerlach analyzer, that is the set of magnets with the magnetic field switched on in X or Z direction and a detector, and two counters.

Experiment #0 does not add other aspects than what was introduced earlier, it is only intended to highlight that, whether the magnetic field is switched on along the *x*-axis or along the *z*-axis, the beam of atoms behaves in the same way and a state of a quantum system can be represented mathematically by a normalized ket $|\psi\rangle$. Thus, if we launch 1000 silver atoms, we will have 500 of them in a space position corresponding to $|\uparrow\rangle$ and 500 in a space position corresponding to $|\downarrow\rangle$.
In this representation (and in the following) $|+\rangle$ corresponds to $|\uparrow\rangle$ and $|-\rangle$ corresponds to $|\downarrow\rangle$.

Experiment #1 consists of putting in sequence two Stern and Gerlach apparatuses with the same orientation of spin component (Fig. 2.20).

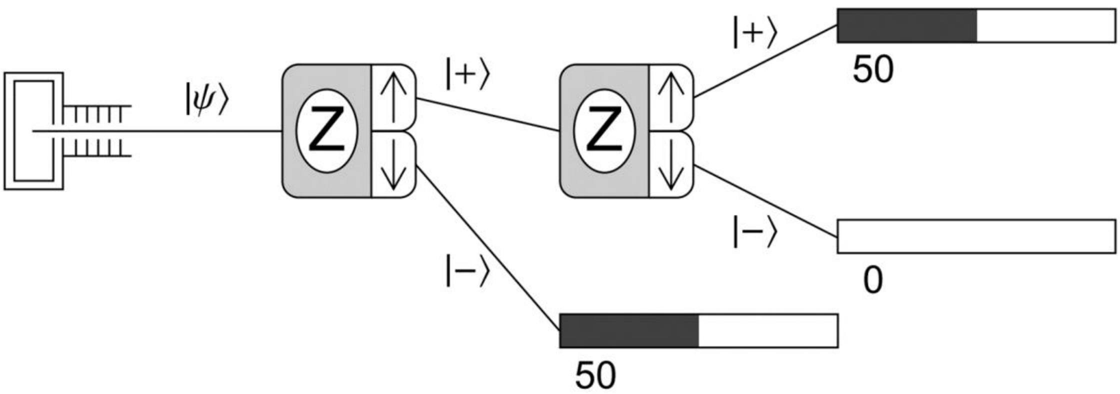


Fig. 2.20: Measurement of the spin component along the z-axis twice in succession.

The students are asked to share the probability distribution they expect and explain the reasoning that led to their answer. In this case, we emphasize the different roles that the identical Stern-Gerlach analyzers play in this experiment. The first analyzer *prepares* the beam in the quantum state $|+\rangle$ and the second analyzer measures the resultant beam, so the first analyzer is often called state preparation device. By preparing the state with the first analyzer, the details of the source of atoms can be ignored.

Experiment #2 consists of putting in sequence two analyzers with a different orientation of the spin component (Fig. 2.21).

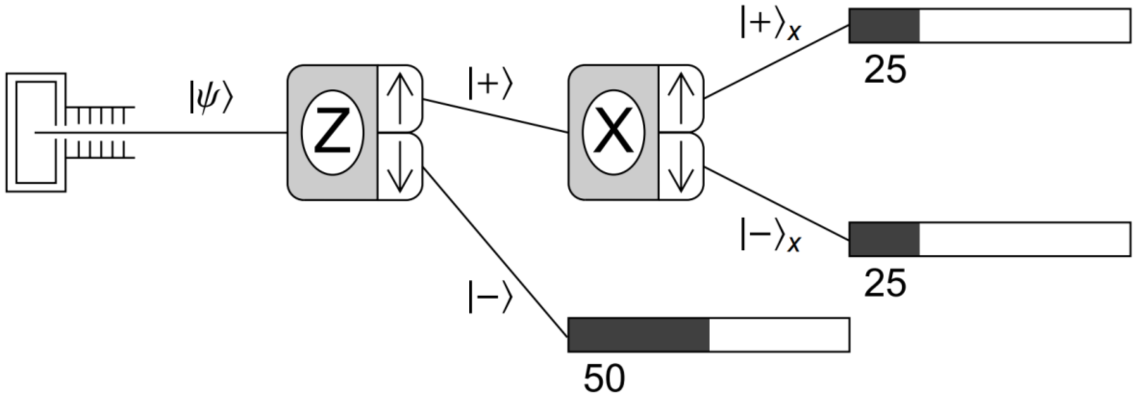


Fig. 2.21: Measurement of the spin component along the *z*-axis and then along the *x*-axis.

After asking students what they expect, we first pay attention that there are still two possible outputs of the second Stern-Gerlach analyzer, thus the different alignment does not affect the fact that we get only two possible results for the case of a spin-1/2 particle. Second, we underlie that the final result would be unchanged also if we used the lower port of the first analyzer. Namely the atoms that enter the second analyzer in state $|-\rangle$ would also result at the end in half the atoms in each of the $|\pm\rangle_x$ output ports. Finally, we emphasize that arrival sequences at any counter are completely random, so we cannot predict which of the second analyzer output ports any particular atom will come out. The only things we can say are that there is a 50% probability that an atom from the second analyzer will exit the upper analyzer port and a 50% probability that it will exit the lower port, paving the way to a reflection about the nature of probability and the problem of random number generator.

E*xperiment #3* consists of putting in sequence the Stern and Gerlach analyzer as in figure 2.22.

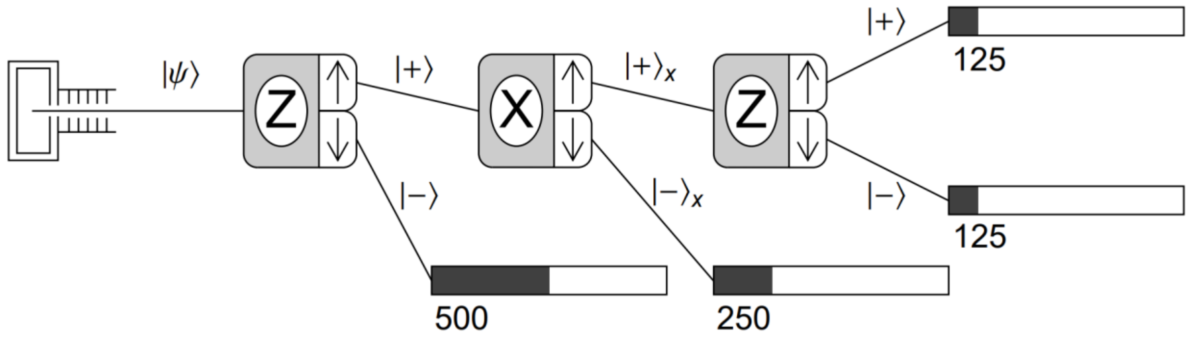


Fig. 2.22: Measurement of the spin component three times in succession.

The situation is the following: the silver atoms entering the third analyzer have been measured by the first Stern-Gerlach analyzer to have a spin component up along the z-axis, and by the second analyzer to have a spin component up along the x-axis. Finally, the third one measures how many atoms have spin components up or down.
After asking students to find the solution and explain it we asked them if classical reasoning could lead to the same result since, classically, we could expect that the final measurement would produce the spin-up result along the z-axis because it was measured at the first analyzer. But it does not happen. The first aspect that this experiment allows us to underlie is that in the quantum case a measurement disturbs the system, in fact, the second analyzer has disturbed the system such that the spin

component along the z-axis does not have a unique value, even though we measured it with the first analyzer. The second aspect is that we cannot design an experiment, a measurement tool that does not disturb the system, there is therefore a fundamental incompatibility in trying to measure the spin component of the atom along two different directions. The two components of the spin in the x and z directions are incompatible observables, and it is not possible to measure the values of both simultaneously.

Experiment #4 consists of the sequence the figure 2.23.

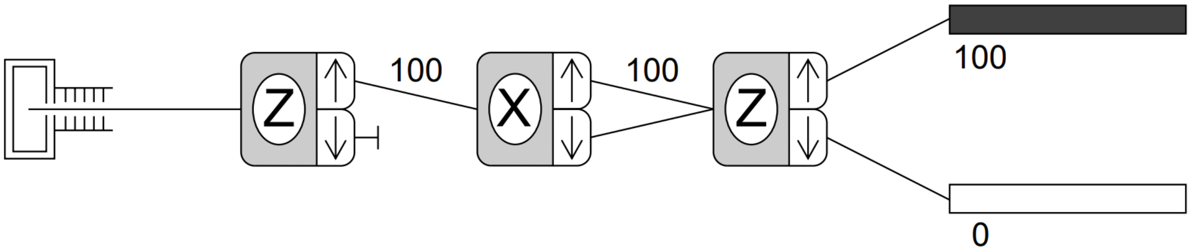


Fig. 2.23: Measurement of the spin component three times in succession and uses a recombined beam between the second and third analyzer.

Also in this case we engage students in finding the solution and try to explain it. Most of the time, even if experiment #4 is different from experiment #3, students tend to give the same answer as the previous experiment. The quantum mechanical result in this experiment is that all the atoms exit the upper port of the third analyzer, and none exits the lower port. The atoms now appear to "remember" to be initially prepared in the spin-up state along the z-axis. By combining the two beams from the second analyzer, it seems that the quantum mechanical disturbance is avoided. This result is the same as Experiment #1, which means it is as if the second analyzer is not there.
In light of the result of *experiment #4*, we pose to students the problem of the physical interpretation of the result. Classically the state of the system after the X apparatus should have i. both the values of the spin, or ii. none of the values, or iii. $|\uparrow\rangle$ OR $|\downarrow\rangle$. However, none of these alternatives is possible since the first (i.) should mean that an atom can be divided, but the experiments prove the contrary, the second should mean that atoms can be lost on the way, but also this is not experimentally verifiable, the third explains the result of the experiment #3 and not #4. The dissatisfaction generated by the classical explanations creates a possible fertile ground in students for a fourth possibility, which has a quantum nature: thanks to that recombination, the atom is in a superposition state, and we find the system in the state $|\uparrow\rangle$ or $|\downarrow\rangle$. only when we make a measurement with the last Z analyzer.
We then pass to the computational dimension by introducing the qubit as a variable that describes the simplest quantum system. We foster the comparison between the classical computation by recalling the bit and introducing that each classical alternative corresponds to a possible quantum state value (table 2.6).

Table 2.6: correspondence by the computational basis in the classical and quantum case

BIT	QUBIT
0	$\|0\rangle$
1	$\|1\rangle$

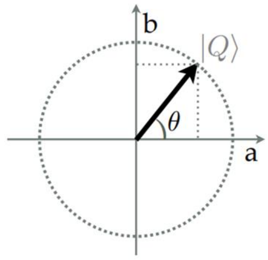
We then formalize the notion of qubit and superposition principle: the quantum state can be found in any combination of the classical alternatives:

$$|Q\rangle = a|0\rangle + b|1\rangle$$

where a and b are complex numbers and $|a|^2 + |b|^2 = 1$, that is there is a normalization's condition. We specify that $|Q\rangle$ represents the information encoded in the computational basis. We then introduce the mathematical formalization and the Bloch sphere as a way to visualize the qubit (Fig. 2.24).

The case with real numbers

$$|Q\rangle = \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$$
$$(\cos^2\theta + \sin^2\theta = 1)$$



The general case (with complex numbers)

$$|Q\rangle = \cos\frac{\theta}{2}\,|0\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|1\rangle$$

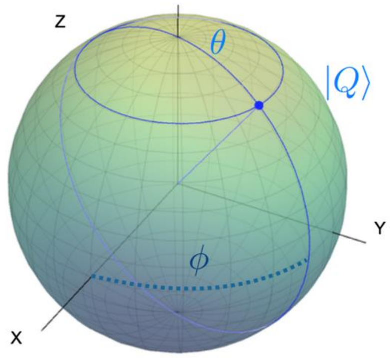


Fig.2.24: Bloch sphere and mathematical formalization.

After the introduction of the quantum state, we introduce the logic gate as a transformation that sends a generic qubit to another:

$$|Q\rangle = a|0\rangle + b|1\rangle \rightarrow |Q'\rangle = a'|0\rangle + b'|1\rangle$$

which is defined by a certain law according to which a' and b' can only be linear combinations of a and b. That is

$$a' = Aa + Bb$$
$$b' = Ca + Db$$

We then introduce the representation of the transformation of the logic circuit (Fig. 2.25) and the main one qubit logic gates - X (NOT), Z, Y, and the Hadamard (H) gates - with the respective truth tables (Fig.2.26) and their reversible character.

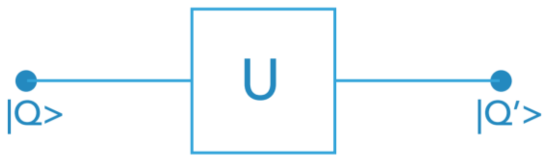


Fig. 2.25: generic representation of a logic gate

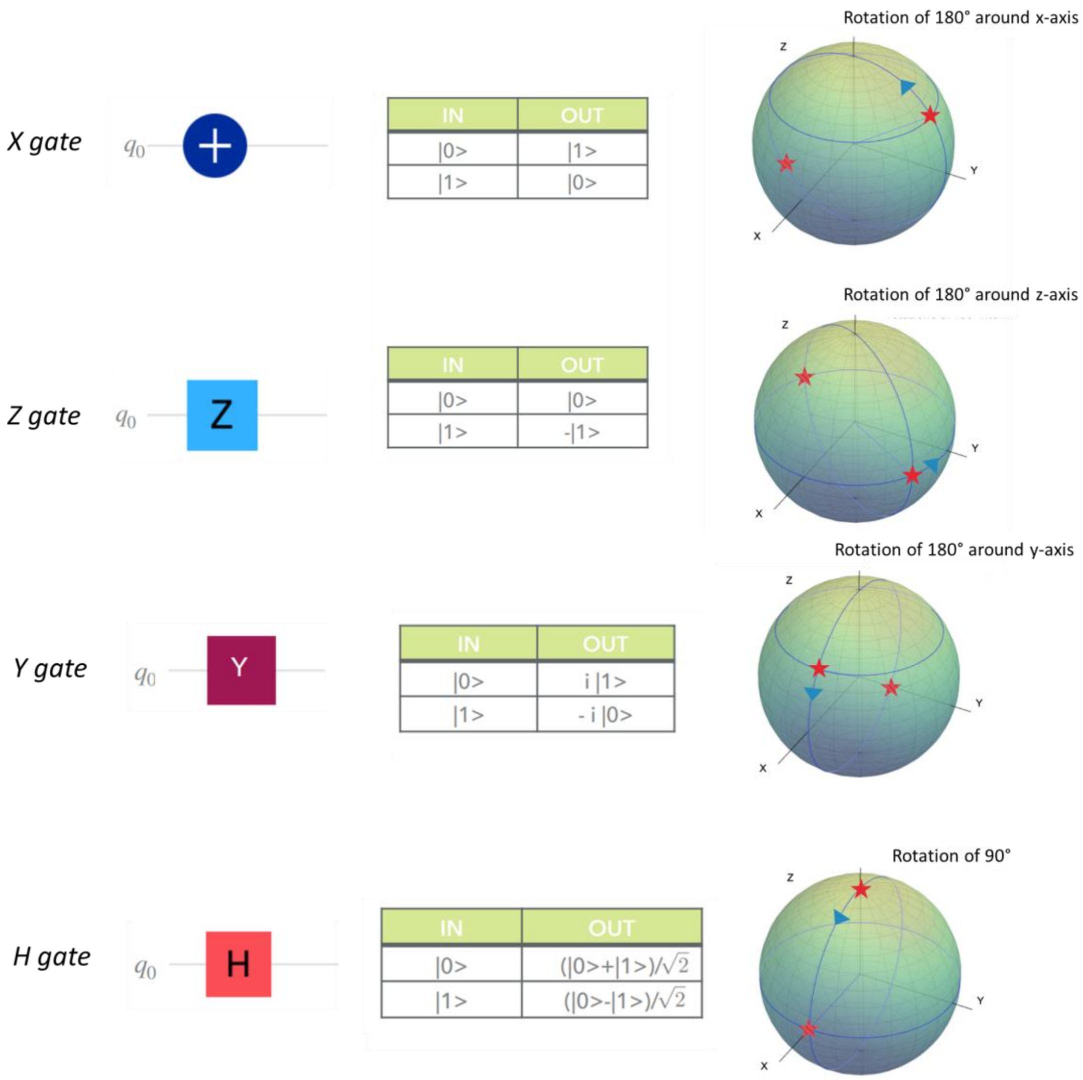


Fig. 2.26: X, Y, Z, and H logic gates, their representation, and geometrical meaning in the Bloch sphere.

Finally, we introduce the measurement in comparison to the classical one. In the classical case, the measurement is deterministic and does not alter the status of the bit (table 2.7).

Table 2.7: classical measurement.

state of the system before measurement	outcome of the measurement	state after the measurement
0	0	0
1	1	1

As already stressed by discussing the Stern and Gerlach experiments, the quantum measurement is probabilistic and destructive (table 2.8).

Table 2.8: quantum measurement.

state of the system before measurement	outcome of the measurement	state after the measurement

$\|Q\rangle = a\|0\rangle + b\|1\rangle$	0 with probability $p_0 = \|a\|^2$ 1 with probability $p_1 = \|b\|^2$	$\|0\rangle$ $\|1\rangle$

Through the measurement, we introduce students to the meaning of the coefficient a and b, that is probability amplitude. We introduce, also in this case, the logic gate that represents the measurement (Fig. 2.27), stressing the presence of the double line as a symbol of the disruptive effect of the measurement, that is the bit of information after it.

Fig. 2.27: logic gate representing the measurement

This activity is designed to emphasize how a quantum computer can be conceived today, that is as an experiment. Therefore, at the core of this activity, there is the reconceptualization of the three main phases of the experiment - state preparation, state manipulation/evolution, measurement - in terms of computation - input information, information processing, output information - (design principle #2). The comparison between "classical and quantum" (design principle #1), in addition to recalling the bit and the logic it implements, is promoted with the Stern and Gerlach experiment, thus in terms of classical and quantum reasoning and classical and quantum probability.
The Stern and Gerlach experiment is used for introducing the basic concepts of the quantum state, superposition principle, state manipulation, and measurement avoiding the technicalities that could distract from our main objectives.
Also, in this case, the inter-disciplinarity of quantum computation emerges. Regarding the "disciplinarity", through the introduction of the Stern and Gerlach experiment, even if we do have not the possibility to go into the labs, we introduce students to what the design of an experiment means, the main phases of it, the problem of understanding and interpreting data in light of a paradigm that is not more suitable, and so on. Introducing the Qubit, the logic gates, and its formalization, we stressed the universal character, the rigor, and the simplicity that characterize computer science: in terms of the ontology of "computational object", in terms of finding universal and rigorous methods and languages to describe a multiplicity of systems, and so on (design principle #6a). Furthermore, the conception of a computer as an experiment can shed light also on contemporary mainstream research topics. Finally, the presence of different disciplines and the back and forth from the mathematical languages to the visualization through the Bloch sphere, to the logic gates, to the truth tables, and so on, emphasized the intrinsic interdisciplinarity of the theme (design principle #6b).

Introduction to the physics of quantum computers_Part 2: Introduction of entanglement (two-qubit systems)

This activity is dedicated to the introduction to two-qubit systems in particular from a formal and computational point of view and to the entanglement.
We start by recalling the classical case in which the two-bit compound state can be found in one of four possible states: 00, 01, 10, or 11. As in the previous activity, a two-qubit system can be found in one of the possible states, $|00\rangle$, $|01\rangle$, $|10\rangle$, or $|11\rangle$. More generally, a two-qubit system can be also found in a linear combination of these states:

$$|Q_1Q_2\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

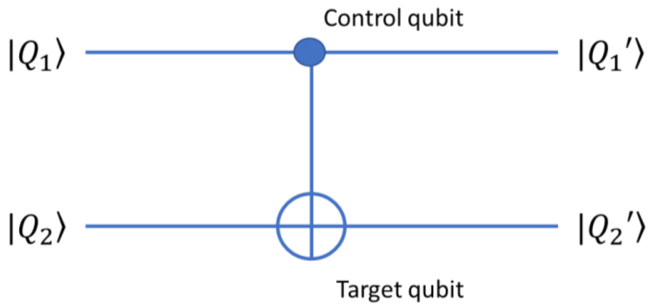
with a, b, c, d complex numbers and $|a|^2 + |b|^2 + |c|^2 + |d|^2$.
We then introduce the logic gates as the transformation that send a generic qubit to another:

$$|Q_1Q_2\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle \rightarrow |Q_1{}'Q_2{}'\rangle = a'|00\rangle + b'|01\rangle + c'|10\rangle + d'|11\rangle$$

with a', b', c', d' linear combinations of a, b, c, d and its circuital representation (Fig. 2.28).

Fig. 2.28: generic two-qubit logic gate.

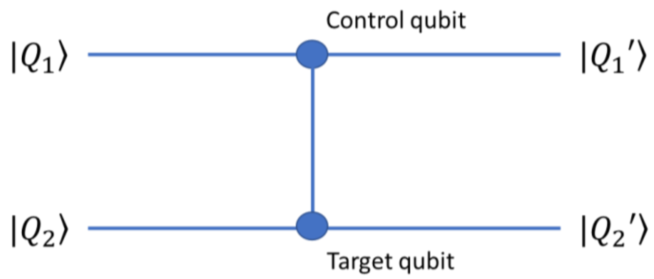
As examples of logic gates, we introduce the control-NOT (*CNOT*) gate and control-Z (CZ), with their circuital representation and the truth tables (respectively Fig. 2.29 and Fig.2.30).



INPUT ($|Q_1Q_2\rangle$)	OUTPUT
$|00\rangle$	$|00\rangle$
$|01\rangle$	$|00\rangle$
$|10\rangle$	$|11\rangle$
$|11\rangle$	$|10\rangle$

Control qubit Target qubit

Fig. 2.29: circuital representation of CNOT gate and truth table.



INPUT ($|Q_1Q_2\rangle$)	OUTPUT
$|00\rangle$	$|00\rangle$
$|01\rangle$	$|00\rangle$
$|10\rangle$	$|10\rangle$
$|11\rangle$	$-|11\rangle$

Control qubit Target qubit

Fig. 2.30: circuital representation of CNOT gate and truth table.

After the introduction of the main one-qubit and two-qubit logic gates, we start dealing with the circuit. The first circuit that we present is the SWAP (Fig. 2.31) that corresponds to three CNOT in sequence and sends $|Q_1Q_2\rangle$ into $|Q_2Q_1\rangle$.

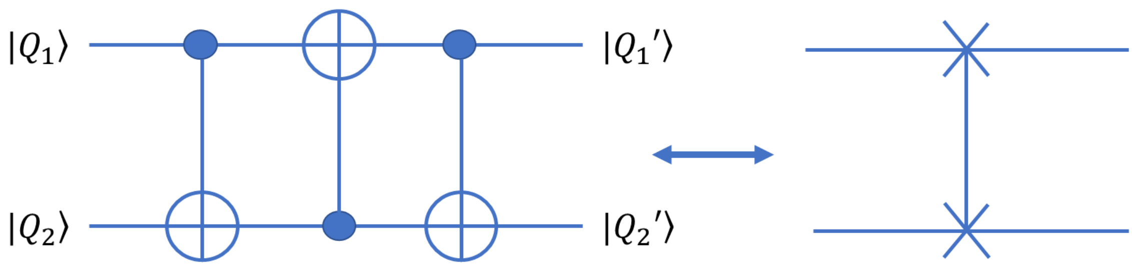


Fig. 2.31 circuital representation of SWAP.

So we choose an initial state and, with students, we solve the circuit. So, if $|Q_1Q_2\rangle = |10\rangle$

$$|10\rangle \rightarrow |11\rangle \rightarrow |01\rangle$$

and we ask them to try with all the other gates since this gate swaps the qubit only if it does it with all the computational states.
We then introduce entanglement by starting from the following narrative situation:
Suppose that Alice and Bob are together in a laboratory and have an experimental setup that produces pairs of qubits (like pairs of photons) and that, following the production of a pair, Alice takes one of the two qubits and takes it to his laboratory, Bob takes the other one and takes it to his lab (Fig. 2.32).

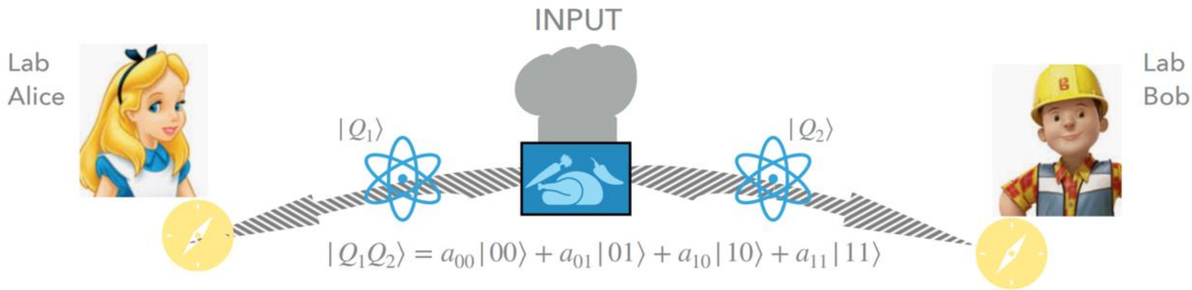


Fig.2.32: schematic and hypothetic experimental situation.

If the system is in the generic state of figure 2.29, a measurement carried out by Alice on qubit $|Q_1\rangle$ allows her to determine if the first qubit is in $|0\rangle$ or $|1\rangle$. Similarly, a measurement carried out by Bob on the qubit $|Q_2\rangle$ allows him to determine whether the second qubit is in $|0\rangle$ or $|1\rangle$.
Having ascertained this, we introduce conditions on the coefficients of the generic state of our system. The first case we consider is the condition $a_{00} = a_{01} = {}^{1}/_{\sqrt{2}}$ and $a_{10} = a_{11} = 0$. In order to get students more and more acquainted with the formalism, the state mathematical description, the meaning of coefficients, and measurement, we asked them to try to discuss together what is the state under this condition, what is the state after the measurement on both $|Q_1\rangle$ and $|Q_2\rangle$, what is the outcomes only on the first qubit and on the second one (separately) and finally in they note some correlation.
So, under the first set of conditions, the results we obtain are reported in table 2.9.

Table 2.9: Table with the outcomes obtained under the first condition.

State of the system before measurement	State after the measurement	Outcome of the measurement on Q_1	Outcome of the measurement on Q_2	Q_1=Q_2 ?
$\frac{\|00\rangle + \|01\rangle}{\sqrt{2}} = \|0\rangle \frac{\|0\rangle + \|1\rangle}{\sqrt{1}}$	$\|00\rangle, p_{00} = \|a_{00}\|^2 = 1/2$	0	0	Yes
	$\|01\rangle, p_{01} = \|a_{01}\|^2 = 1/2$	0	1	No

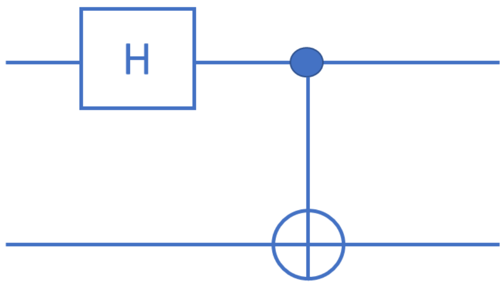
We then conclude that there is no correlation between the first and the second qubit, where for

correlation we mean that if I know the information that I can acquire on the first qubit by measuring it, I certainly know the state of the second qubit without measuring it.
The second case we consider is the condition $a_{00} = a_{11} = {}^{1}/_{\sqrt{2}}$ and $a_{01} = a_{10} = 00$. We ask students again to complete the table with us and the outcomes are presented in Table 2.10.

Table 2.10: Table with the outcomes obtained under the first condition.

State of the system before measurement	State after the measurement	Outcome of the measurement on Q_1	Outcome of the measurement on Q_2	Q_1=Q_2 ?
$\frac{\|00\rangle + \|01\rangle}{\sqrt{2}}$	$\|00\rangle, p_{00} = \|a_{00}\|^2 = 1/2$	0	0	Yes
	$\|11\rangle, p_{11} = \|a_{11}\|^2 = 1/2$	1	1	Yes

While in the first case the outcome of the measurement on Q_1 was predictable, in this case, the measurement on both Q_1 and Q_2 are not. Furthermore, there is a perfect correlation between the two qubits. We introduce through this reasoning what entanglement is.
Then, we introduce the four Bell states as maximally entangled states and the circuits that project in a Bell state two qubits initially not entangled (Fig. 2.33).



$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$
$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$
$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

Fig. 2.33: on the right, there is the circuit to project to qubit in a Bell state, and on the left, the four Bell state to describe maximally entangled qubits.

In order to pave the way for the teleportation activity, we generalize the two-qubit systems to three-qubit systems that have 8 possible states $|000\rangle$, $|001\rangle$, $|010\rangle$, $|011\rangle$, $|100\rangle$, $|101\rangle$, $|110\rangle$, or $|111\rangle$ and generally described as

$$|Q_1Q_2Q_3\rangle = a_{000}|000\rangle + a_{001}|001\rangle + a_{010}|010\rangle + a_{011}|011\rangle + a_{100}|100\rangle + a_{101}|101\rangle + a_{110}|110\rangle + a_{111}|111\rangle$$

This activity is more computational-oriented than others. By recalling the bit and the logic it embodies, we scaffold the comparison between classical and quantum computation (design principle #1). We also emphasize the very close connection between mathematics, logic, and computer science (design principle #6).
During the different implementations, we realized that this way of introducing entanglement tends to simplify the issue, not highlighting the scope of the concept. The computational and mathematical dimension, in this case, reduces the cognitive load to such an extent that the entanglement, the mystery of quantum mechanics together with the complementary principle, is likely to be reduced to the recognition of a "purely mathematical" correlation. Therefore, we do think that to emphasize the revolutionary scope it is necessary to rethink this activity, also in light of the recent news of the 2022

Nobel Prize in Physics by Alain Aspect, John F. Clauser, and Anton Zeilinger[6].

Quantum cryptography protocol (BB84)

This activity is dedicated to a brief history of cryptography and to the introduction to the BB84 protocol (Bennett & Brassard, 1984).
We start by emphasizing that cryptography has a very long and rich history. The first proof, far from cryptography as we know it today, was found in a tampered inscription carved around 1900 BC, in the main chamber of the tomb of the noble Khnumhotep II, in Egypt.
Plutarch in his writings provides another example that dates back to the ninth-century BC, or the scythal Lacedemonic, a code of encryption in use since the time of Lycurgus. It was an encryption device consisting of a stick and a leather ribbon wrapped in a cylindrical spiral, on which the message was written in columns. On the unrolled tape the letters were transposed in such a way that only the use of a stick identical to the one originally used to write the message allowed to recompose the text. A few centuries later, Julius Caesar invented a monoalphabetic rotation cipher to convey secret messages to his army generals sent to the war front. In such a cipher, each alphabet character of the plain text (plain text is the message which has to be encrypted) is substituted by another character to form the cipher text (cipher text is the encrypted message). The variant used by Caesar was a shift by 3 ciphers. Each alphabet character was shifted by 3 places, so the character 'A' was replaced by 'D', 'B' was replaced by 'E', and so on. The characters would wrap around at the end, so 'X' would be replaced by 'A'. An example is reported in table 2.11.

Table 2.11: Example of an encrypted message and decrypted message (Attack the Gauls at the sixth hour)

Cipher	DZZDFFDUHLONNUUNGAFNENONLDOONDOODRUDVHVZD
Message	attaccaregliirriducibiligalliallaorasesta

We then introduce more recent devices: the Enigma machine was invented by German engineer Arthur Scherbius at the end of World War I and was used by the German forces during the Second World War. The Enigma machine used 3 or 4 or even more rotors. The rotors rotate at different rates as you type on the keyboard and output appropriate letters of cipher text. In this case, the key was the initial setting of the rotors.
After this very brief history of cryptography, we introduce the current cryptographic system, the RSA protocol, which is a public key cryptographic system. It is an asymmetric system that consists of two different keys, one public to encrypt and one secret to decipher. The public key is an integer of the type $n = p * q$, where p and q are two prime numbers while the secret key is related to the value of one of the two factors. The security of the protocol lies in the fact that to find the secret key you need to know the two factors p and q and this calculation takes a very long time.
Unlike the classical computer which takes exponential time to solve the factorization problem, a quantum computer would take a few days (or hours) to solve it.

Thus, we introduce students to one of the quantum cryptography protocols, the BB84 developed by Bennett and Brassard (1984a,b, 1983). To do this, we use an interactive simulation of the QuVis Quantum Mechanics Visualization Project developed by a team at St Andrews University (UK)[7]. The project has developed the simulation with both photons and atoms of the protocol, we have chosen the atoms' simulation since it uses the Stern and Gerlach apparatus with which students are familiar.
We start with the statement of a problem: Alice and Bob must share a secure key, a perfectly random secret sequence of zeros, and one without meeting in person. Alice and Bob cannot solve the problem

[6] https://www.nobelprize.org/prizes/physics/2022/press-release/

[7] https://www.st-andrews.ac.uk/physics/quvis/simulations_html5/sims/cryptography-bb84/Quantum_Cryptography.html

classically, as they can never be sure that the secret key was not intercepted during transmission. How can they exploit quantum mechanics to solve this problem?
We then introduce the experimental set-up that can help to solve the problem (Fig. 2.34) helping students to get acquainted with the simulation, and how to use it.

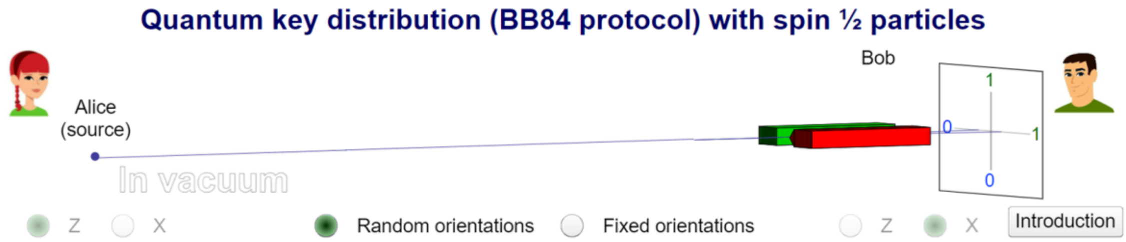


Fig. 2.34: setup of the experiment (interactive simulation of Quvis)

Alice is able to send Bob a single particle of spin $^1/_2$ at a time. She prepares each particle in a $|\uparrow\rangle$ (value 1) or $|\downarrow\rangle$ (value 0) state along one of the two orthogonal axes (X or Z). She sends one particle at a time to Bob who is equipped with a Stern and Gerlach apparatus. For spin particles ½, the particles separate into two discrete streams: one diverted in the positive direction (result 1), and one diverted in the negative direction (result 0). Bob can orient each Stern and Gerlach apparatus along two orthogonal axes X and Z, we can set one direction for Bob's apparatus or make him randomly choose it.
In the simulation, the states $|\uparrow\rangle$ and $|\downarrow\rangle$ are spin states with results of 1 and 0 respectively in the Z-direction, and $|+\rangle$ and $|-\rangle$ are spin states with results of 1 and 0 respectively in the X-direction.
Alice and Bob independently note the base (X or Z) and the value (0 or 1) for each particle sent. They know that their results are perfectly correlated (if Alice sends 1, Bob measures 1; if Alice sends 0, Bob measures 0) when they both happened to choose the same base. After completing the measurements, they publicly share the bases used for each measurement (but not the values) and keep only the values for which their bases were the same. In order to test for an eavesdropper, they compare a randomly selected subset of their measurement outcomes (which they then discard, as these are not anymore secure) and check for errors.
We introduce this with the simulation, an example is reported in figure 2.35.

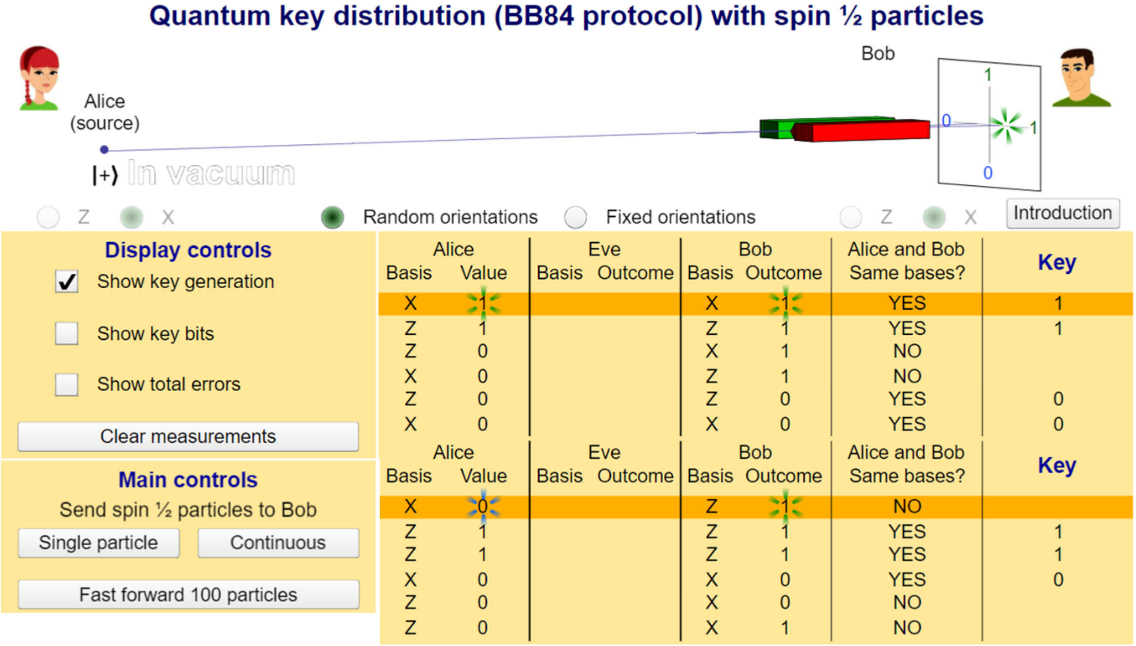


Fig. 2.35: Example of key generation: Alice and Bob randomly choose the basis and make the measurement. After this, they compare the results and discard all the outcomes obtained with different basis orientations. The remaining outcomes are key.

We then introduce the presence of Eve that try to intercept what Alice and Bob are exchanging (Fig. 2.36).

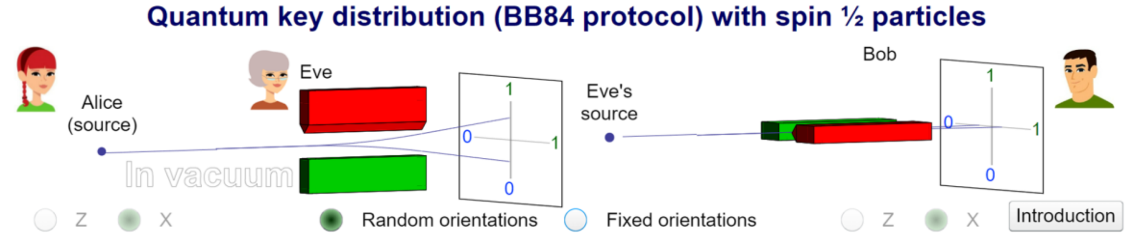


Fig. 2.36: experimental set-up with the presence of an eavesdropper.

In the simulation, Eve uses an intercept-resend attack where she uses the same orientations as Bob for her Stern and Gerlach apparatus, and the same orientations as Alice for her atomic source. If Eve detects the atoms, she knows what state Alice sent and sends the same state to Bob. In this case, errors do not occur. If Eve does not detect the atom, she does not know which state Alice sent. In this case, if Eve chooses the incorrect state to pass to Bob, errors can occur (Fig. 2.37). Alice and Bob realize the presence of an eavesdropper at the end when they compare the results to obtain the key by finding some errors. They have therefore to discard the key and produce a new one.

After the introduction to the simulation, we engage students in the challenges that the simulation proposes, asking them to try to find the solution and explain the kind of reasoning they carried out to choose one solution rather than another.

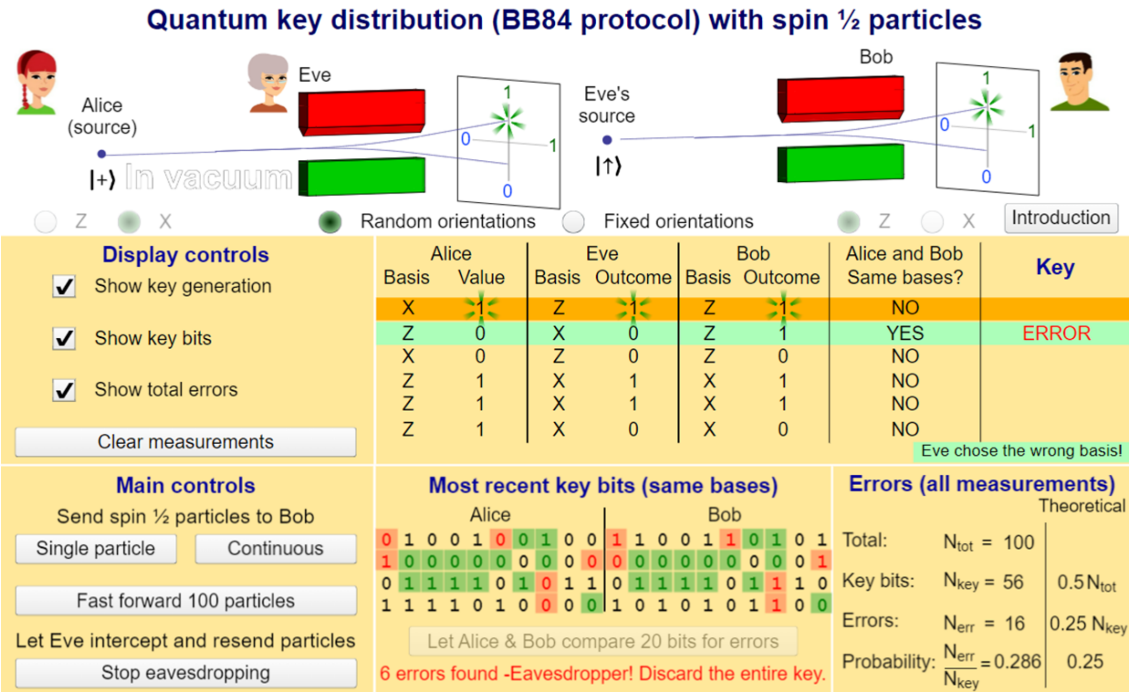


Fig. 2.37: Detection of errors for the presence of an eavesdropper.

With this activity, we recall the concepts of quantum states and the measuring process. We introduce the notion of randomness as the core aspect for establishing a secure key, that is a perfectly random sequence of 0 and 1. These aspects together with the quantum uncertainty protocol request and the notion of non-deterministic predication structure the comparison between classical and quantum logic (design principle #1). The inclusiveness, in this case, was promoted both by trying to intertwine different discourse levels (section 2.1) and stressing the political, societal, ethical, etc., implications of such a protocol (design principles #4 and #5). Even if we do not present the circuit that implements the protocol (we have not found a circuit that can be within the reach of secondary school students), this activity follows, on the same day, the one in which we introduce the entanglement. So together these two conceptual-epistemological activities emphasize the interdisciplinarity of the theme.

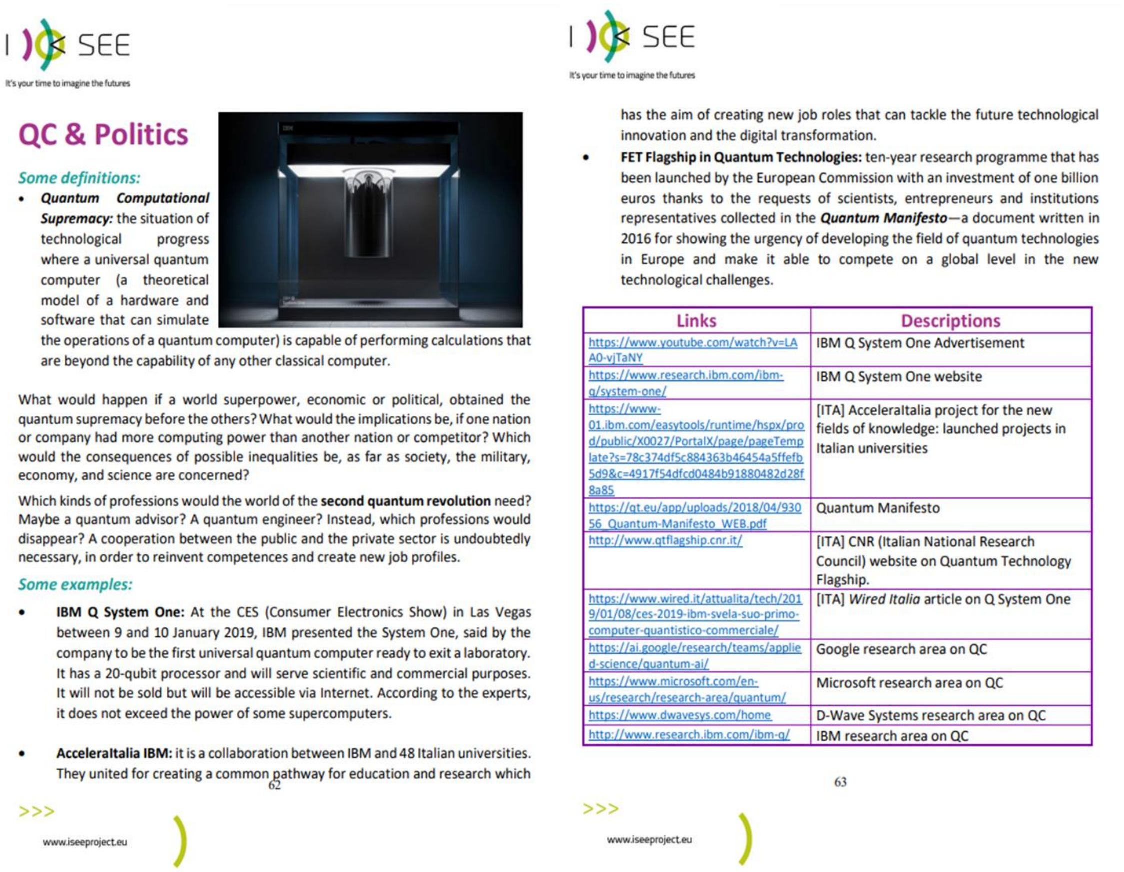
2.6.2 Future-oriented/citizenship education activities

"Quantum technologies &..."

This activity is a teamwork activity in which students explore the implication and applications of quantum technologies on research, politics, economy, ethics, society, environment, education, and so on. This activity is the main result of the bachelor thesis of Spada (2019) within the I SEE project.
This activity is carried out in the same meeting as the activity "Introduction to the Second Quantum Revolution". Therefore, the students are introduced to the main debates at different levels, the official strategic documents such as the Quantum Manifesto and the following research and innovation initiatives (the quantum flagship), the four pillars, and some examples of technologies that are being developed. We inform students also about the ongoing educational projects and projects and the professions that are emerging.

In the activity in particular we foster students to explore the applications and implications of quantum technologies on i. Communication; ii. Politics; iii. Scientific and Technological Research; and iv. Society.

After forming the groups based on the dimension that students would like to explore, we provide them with a worksheet about the topic that supports them in the investigation. We developed four different worksheets with the same structure that contains some key notions of quantum technologies that can be related to the dimension, a few examples of developed technologies, and some links as starting point to explore the impact. An example of a worksheet is in Figure 2.38.


I SEE

It's your time to imagine the futures

QC & Politics

Some definitions:

- ***Quantum Computational Supremacy:*** the situation of technological progress where a universal quantum computer (a theoretical model of a hardware and software that can simulate the operations of a quantum computer) is capable of performing calculations that are beyond the capability of any other classical computer.

What would happen if a world superpower, economic or political, obtained the quantum supremacy before the others? What would the implications be, if one nation or company had more computing power than another nation or competitor? Which would the consequences of possible inequalities be, as far as society, the military, economy, and science are concerned?

Which kinds of professions would the world of the **second quantum revolution** need? Maybe a quantum advisor? A quantum engineer? Instead, which professions would disappear? A cooperation between the public and the private sector is undoubtedly necessary, in order to reinvent competences and create new job profiles.

Some examples:

- **IBM Q System One:** At the CES (Consumer Electronics Show) in Las Vegas between 9 and 10 January 2019, IBM presented the System One, said by the company to be the first universal quantum computer ready to exit a laboratory. It has a 20-qubit processor and will serve scientific and commercial purposes. It will not be sold but will be accessible via Internet. According to the experts, it does not exceed the power of some supercomputers.

- **AcceleraItalia IBM:** it is a collaboration between IBM and 48 Italian universities. They united for creating a common pathway for education and research which

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I SEE

It's your time to imagine the futures

has the aim of creating new job roles that can tackle the future technological innovation and the digital transformation.

- **FET Flagship in Quantum Technologies:** ten-year research programme that has been launched by the European Commission with an investment of one billion euros thanks to the requests of scientists, entrepreneurs and institutions representatives collected in the ***Quantum Manifesto***—a document written in 2016 for showing the urgency of developing the field of quantum technologies in Europe and make it able to compete on a global level in the new technological challenges.

Links	Descriptions
https://www.youtube.com/watch?v=LAA0-vjTaNY	IBM Q System One Advertisement
https://www.research.ibm.com/ibm-q/system-one/	IBM Q System One website
https://www-01.ibm.com/easytools/runtime/hspx/prod/public/X0027/PortalX/page/pageTemplate?s=78c374df5c884363b46454a5ffefb5d9&c=4917f54dfcd0484b91880482d28f8a85	[ITA] AcceleraItalia project for the new fields of knowledge: launched projects in Italian universities
https://qt.eu/app/uploads/2018/04/93056_Quantum-Manifesto_WEB.pdf	Quantum Manifesto
http://www.qtflagship.cnr.it/	[ITA] CNR (Italian National Research Council) website on Quantum Technology Flagship.
https://www.wired.it/attualita/tech/2019/01/08/ces-2019-ibm-svela-suo-primo-computer-quantistico-commerciale/	[ITA] *Wired Italia* article on Q System One
https://ai.google/research/teams/applied-science/quantum-ai/	Google research area on QC
https://www.microsoft.com/en-us/research/research-area/quantum/	Microsoft research area on QC
https://www.dwavesys.com/home	D-Wave Systems research area on QC
http://www.research.ibm.com/ibm-q/	IBM research area on QC

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Fig. 2.38: Example of worksheet quantum technologies & politics.

Together with the worksheet, we give students a map (Fig.2.39) that contains some of the aspects on which quantum technologies can have an impact.
So, after reading the worksheet, students discuss the relationship between quantum technologies and society, work on the map to find the aspects, already written or new ones, on which quantum technologies can impact, and explain how and why they find those connections.
At the end of the activity, they have to deliver the maps of the connections and they have to prepare to present their project to the class.

As already introduced, this is an activity representative of the I SEE project. By themselves, we make students reflect on the implications and impact of science and technologies on different dimensions, fostering them to think about relationships among the different components of a complex context and different stakeholders, to grasp the complexity of the tight relationship between science, technology, and society (design principle #5b). This kind of activity also give space for discussion (in group and collectively) that are rarely created at school and also engage the students that, for example, resonate more with social sciences & humanities aspects (design principle #4).

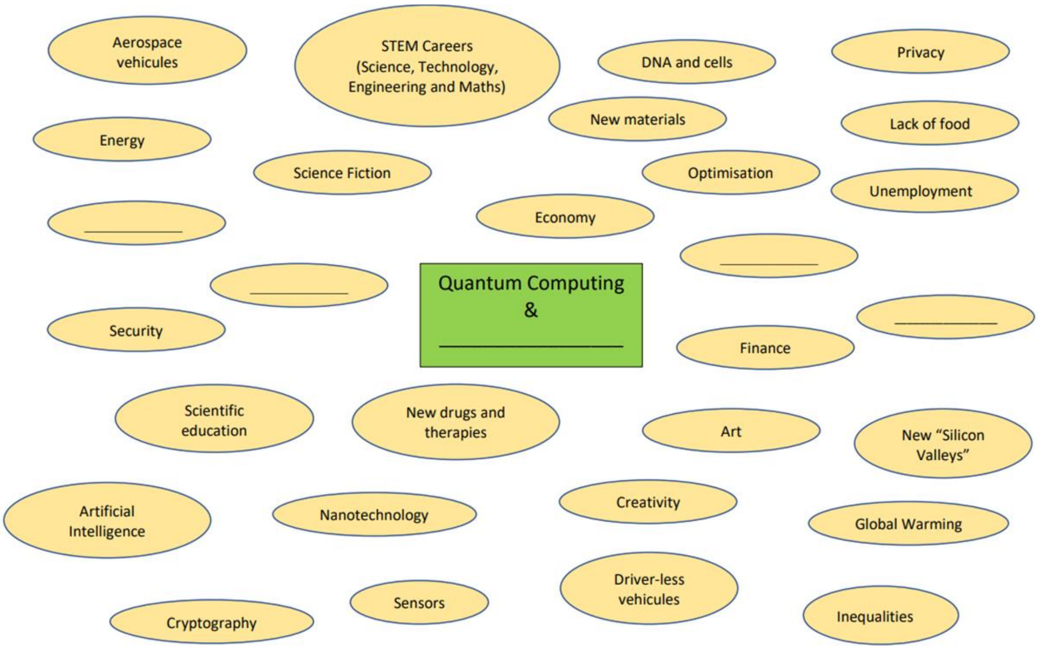

Fig. 2.39: Map of the connections.

"Eve's city"

This was designed within the I SEE project with the teacher Fabio Filippo of Liceo Einstein (Rimini), which was involved in the project. The text of the activity is reported in Annex 1.
Eve's city is a teamwork activity, and it is composed of three parts that are conducted on different days (typically the third and the fourth meetings). Students have firstly to read a story about the city of Eve, its inhabitants, the type of life they lead, the main commercial activities, the type of tourism, the type of industries, and so on.
After reading the story, in groups, they have to put themselves in the shoes of the public decision-makers that are called to make a choice about the proposal for investing in quantum technologies or not. In particular, if they would like to give the possibility to Alice, by unlocking some urban constraints, to set up the laboratory and activate the construction of a technological center at the forefront.
So students have first to analyze and outline the situation by recognizing a) the stakeholders, b) their needs and interests, and c) the interactions between them.
Then, in the shoes of the Mayor of Eve, they have to discuss with the class if they have accepted or not and why.

Finally, in the third part of the activity, we give them three 3 scenarios of the city of Eve in 2040. The first is a hyper-technological scenario in which technology has exceedingly entered into everyday life. The second one is a "rural" scenario where there is a return to nature in which technology is almost absent except for issues related to the environment and renewable energy. Finally, the third one is a middle ground between the two. Students, first in groups and then all together, have to discuss all the scenarios and choose which one they prefer and why.

In the very last two implementations we changed the delivery. Instead of reflecting on the future scenario, among the 3 proposed, in which they would like to live, we asked them to reflect on the type of human-machine relationship (circular, functional) that emerges, on the dichotomies human-nature, nature-science/technology, human-science/technology and on the relationships between science and society.

This I SEE activity is mainly an implementation of the design principle #5b and aims to engage students in reflections about the implications and impact of science and technologies on different dimensions such as the scientific, political, institutional, economic, ethical, environmental, educational, etc., fostering them to think about the different stakeholders involved, the possible future's scenarios arising from the scientific and technological progress, and their possible roles in the debates and decision-making.
The last part was added to pave the way for the FEDORA activity. On this basis, there is the idea that technological revolutions the technological revolutions in history have changed and are still changing the relationship between human-nature, science/technology-nature, and human-science/technology, and that arts always help us both to interpret what is happening, and by providing languages, aesthetics, and other narratives that allow you to grasp the change.

Fedora activity

This activity is carried out on the fifth and the sixth day of the course. While the other activities are well-defined both in terms of content and kinds of activity, this one was implemented only twice in collaboration with a conceptual artist-due, the first time, and an expert in creative writing the second time. In both the implementation, together with the experts, we co-designed a teamwork activity (like a template with a few questions) that, on one hand, could promote a reflection on the second quantum revolution as a cultural revolution and could pave the following workshop led by the experts. We are working on this kind of activity to model it. Both the moments of the activity were designed to scaffold *spaces* in which arts and science can dialogue, reflect on how the dialogue can shape new narratives and languages to better grapple with the contemporary challenges (such as the Second Quantum Revolution), and spaces in which explore the role that arts and their constraints can have in "talking about" science personally and collectively and, vice versa, the role that sciences and its formalism can have in the process of creation of e.g. an exhibition (an artwork, a written story, and so on).

In the particular case of the Second Quantum Revolution, this activity aimed to

I. trigger students' needs to understand and explore the scope of the Second Quantum Revolution, finding their personal and alternative languages to talk about it;
II. reflect on the relationship between human-science/technology, nature-science/technology, and human-nature, and on how the view of the world changes in the transition from classical physics, binary and deterministic, to quantum physics.
III. reflect on the relationship between arts and science, and in this relation on the relationship between free space of invention and identity constraints.

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Chapter 3: Pilot-studies on two key activities

Introduction

In the present chapter I present the teleportation and random walk activities and the analysis of two questionnaires carried out to investigate the impact of the approach on students and to collect feedback for refining it.
These activities were carried out as a case of "Educational reconstruction" (Duit, Gropengießer, Kattmann, Komorek & Parchmann, 2012; Duit, 2007; Duit, Komorek & Wilbers, 1997). We referred to this model, elaborated by German researchers, to methodologically frame the whole process of design. The presentation of both activities is tripartite following the main phases of the model: analysis and clarification of science content, analysis/research on teaching and learning with an emphasis on students' perspectives, and instruction design.
The teleportation activity is part of our publication (Satanassi, Ercolessi & Levrini, 2022). The random walk one was designed together with a students and topic of his bachelor's thesis (Casali, 2020). Both cases were objects of talk at national and international conferences such as Girep2022, ESU9, SIF2020, and SIF2021.
In both cases, the analysis refers to the second implementation that was held between February and March 2020 and involved 22 students in the penultimate and final year of school.
Generally, the students involved had never studied quantum physics and/or quantum technologies before, except eventually for some personal reading. In the scientific courses of the Italian schools, old quantum physics is introduced as the last topic of the last year. As I present in the following, we address teleportation protocol and quantum random walk in a very formal way. The students, that usually choose our course as extra-curricular scientific courses in Italian high schools, have the mathematical basis necessary to deal with the course. Although they have never dealt with Dirac formalism and the ket notation before, they are familiar with the linear algebra basics through which they can carry out calculations and manage and manipulate states. They know matrix formalism, complex numbers, vectors, and have some trigonometry bases.
As I present, we have made choices and the calculations are guided by us, but the formal part has also proven to be within the reach of secondary school students.

3.1 A brief introduction to the model of educational reconstruction

The Model of Education Reconstruction, MER (Duit, Gropengießer, Kattmann, Komorek & Parchmann, 2012; Duit, 2007; Duit, Komorek & Wilbers, 1997), is based on epistemological assumptions that we shared and that oriented the instruction design: physics is a discipline that, like every human construction, is rich and complex enough to allow its content to be analyzed, elaborated and re-structured in many different ways according to many different educational goals. Thus, when the contents are analyzed for teaching, the analysis is explicitly assumed to be "never solely influenced by the referring science content and by issues that stem from philosophy and the history of science but as well by educational concerns, such as the aims of teaching the detected elementary features, that is, the basic ideas of a science theory in question." (Duit, Komorek & Wilbers, 1997, p. 341).

The model identifies three components of the process of content reconstruction: analysis and clarifications of science content, analysis/research on teaching and learning with an emphasis on students' perspectives, and instruction design (see Figure 3.1).

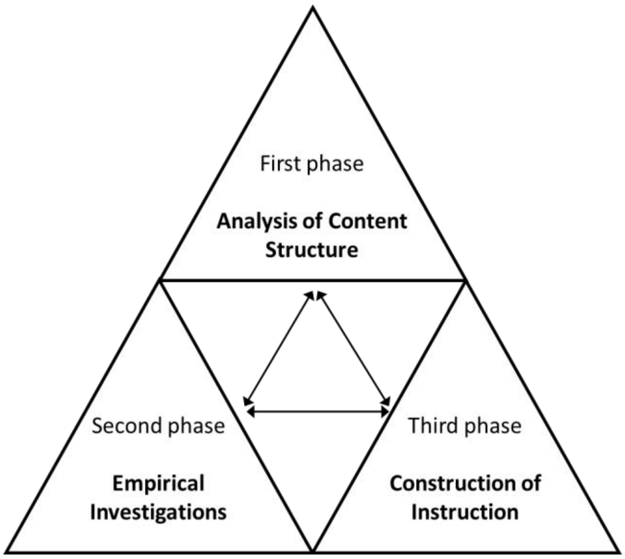


Fig. 3.1: The model of educational reconstruction.

The first component includes "hermeneutic-analytical research on subject matter clarification and analysis of the educational significance of particular science content. The interconnected set of core ideas of a particular content domain is detected (in the aforementioned sense) from the perspective of key aims of science instruction." (Ibidem, p. 342).
The second component comprises "studies on students' conceptions, i.e., investigations into students' pre-instructional conceptions and their development towards the intended science view." (Ibidem, p. 342).
The third component refers to the need to strictly link academic research with school practices and, indeed, "comprises development and evaluation of pilot instructional modules rather early in the process of educational reconstruction." (Ibidem, p. 343).

In our case study, the first phase (analysis and clarification of science content) involved an extensive and deep analysis of the literature on the second quantum revolution and the emergent technologies. In particular, as regards the present chapter and the following sections, the literature analysis was carried out to individuate the most suitable version teleportation experiment for comparison. By "most suitable version" we mean a version of the experiment that could be explained without getting caught up in too many technicalities. Similarly for the quantum random walk, the literature analysis was carried out to find papers epistemologically relevant that could shed light on the epistemological structure of the algorithm.
The second phase of educational reconstruction (analysis/research on teaching and learning with an emphasis on students' perspectives) was carried out to implement our design principles and by taking into account the research on students' conceptions and learning in quantum physics and the transformation of the contents to bring them within the reach of secondary students. For this phase, we could count on the long and extensive experience in teaching/learning quantum physics. Throughout multiple iterative phases of design-implementation-revision, we progressively developed teaching materials that were revealed to be effective in developing the basic concepts of quantum physics on which we built the activity: state, superposition principle, manipulation of a state, measurement, and entanglement (e.g., Sadaghiani, 2016; Pospiech, 2000).
As for the third phase (instruction design and pilot investigation), the activities and materials we developed were implemented four times completely in collaboration with *Piano Lauree Scientifiche* (PLS) project, and other four times partially in collaboration with the University of Como and the

University of Pavia (see section 2.6). In all these implementations, the groups of students were heterogeneous both in terms of gender and schools of origin, and generally, they did have not a background in quantum physics. Implementation after implementation we refined the activities of the module on the basis of students' difficulties and reactions, and our perceptions of what happened in class.

3.2 The teleportation activity

This section is dedicated to the educational reconstruction carried out for the teleportation activity. We chose this protocol since it is intriguing and within reach of secondary students with the appropriate simplifications. Furthermore, it has at the basis one of the mysteries of quantum mechanics, that is the entanglement, and it is enabling the development of different kinds of technologies, such as the quantum internet.
Furthermore, it is suitable to be valued from an epistemological point of view to emphasize the actual status of quantum computers today and how we can conceive them, that is as an experiment. At the center of the reconstruction we propose, there is thus the comparison between the experiment and the circuit.
From an educational point of view, students get acquainted with the concepts of quantum state and state of a system, state manipulation, measurement, and entanglement. They have access to a plurality of representations (the experimental, the circuital, and the mathematical) that are constantly put in correlation in order to create connections between them.
In the following through the lens of the MER, I present we work carried out first to clarify the content starting from the scientific literature, the educational reconstruction through the design principles in section 3.2, and finally the analysis of a questionnaire that we administrated after the activity.

3.2.1 Content analysis/clarification of the content

The phase of content analysis led to a conceptual clarification of the teleportation protocol that, according to our design principles, has been analyzed from both experimental and logical/circuital perspectives.
As a starting point of the process, we selected one of the first experiments on teleportation, developed by the Zeilinger's group in 2004 (Ursin, Jennewein, Aspelmeyer, Kaltenbaek, Lindenthal, Walther & Zeilinger, 2004). In this experiment, the state of a photon (in terms of its polarization) was teleported from one shore to the other of the Danube.
We then considered the physical experiment, whose representation, shown in figure 3.2, is redrawn by Ursin et al. (2004). The following description is the result of the analysis: the experiment is clarified according to our Design Principle #2, namely, it is described in a form that allows its schematization and its comparison with the protocol.

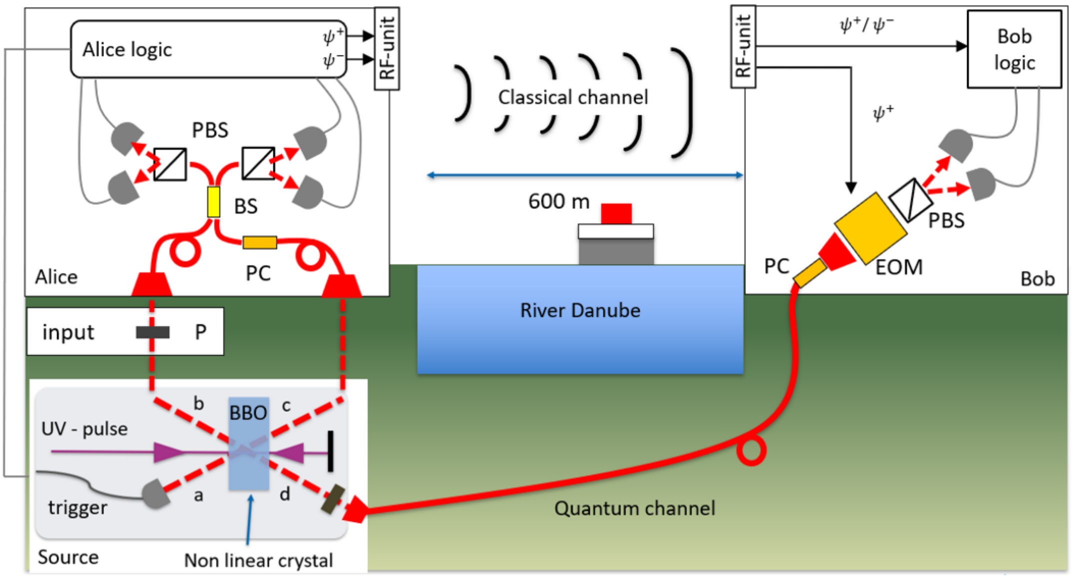


Fig. 3.2: set up of teleportation experiment (redrawn from Ursin et al., 2004)

A pulsed laser (wavelength 394 nm; rate 76 MHz) is used to pump a barium borate (BBO) non-linear crystal and, hence, to generate the first entangled photon pair and by parametric conversion. *c* and *d* are the photons that are led respectively to "Alice's station" and to "Bob's station". For reflection of the pulsed light on a mirror, another pair of entangled photons, *a* and *b*, are produced: *a* serves as a trigger, and *b*, passing through a polarizer, comes to be in the superposition state $|\psi\rangle_b = (\alpha|0\rangle + \beta|1\rangle)_b$ that Alice wants to teleport to Bob. Therefore, the initial state of the system is:

$$|\psi\rangle = |\psi\rangle_b|\beta_{11}\rangle_{cd} = (\alpha|0\rangle + \beta|1\rangle)_b \left(\frac{|01\rangle - |10\rangle}{\sqrt{2}}\right)_{cd} =$$
$$= \alpha|0\rangle_b \frac{|0\rangle_c|1\rangle_d - |1\rangle_c|0\rangle_d}{\sqrt{2}} + \beta|1\rangle_b \frac{|0\rangle_c|1\rangle_d - |1\rangle_c|0\rangle_d}{\sqrt{2}}$$

After the preparation of the photon *b* in the form of state to be teleported, *b* and *c* are guided into a polarizer controller (PC) and into a single-mode optical-fiber beam splitter (BS). These experimental tools and their connection with polarizing beam splitters (PBS) allow a Bell-state measurement to be realized. Mathematically, in order to better follow the manipulation of the system's state, *b* and *c* photons are coupled in the same ket, obtaining

$$|\psi\rangle = \frac{1}{\sqrt{2}}(\alpha|00\rangle_{bc}|1\rangle_d - \alpha|01\rangle_{bc}|0\rangle_d + \beta|10\rangle_{bc}|1\rangle_d - \beta|11\rangle_{bc}|0\rangle_d) \quad (1)$$

The four Bell states are:

$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$
$$|\Phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$
$$|\Psi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$
$$|\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

With simple calculations aimed to make the computational basis explicit, we obtain:

$$|00\rangle = \frac{|\Phi^+\rangle + |\Phi^-\rangle}{\sqrt{2}}$$
$$|11\rangle = \frac{|\Phi^+\rangle - |\Phi^-\rangle}{\sqrt{2}}$$
$$|01\rangle = \frac{|\Psi^+\rangle + |\Psi^-\rangle}{\sqrt{2}}$$
$$|10\rangle = \frac{|\Psi^+\rangle - |\Psi^-\rangle}{\sqrt{2}}$$

Replacing these states in (1), we obtain:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(\alpha\left(\frac{|\Phi^+\rangle + |\Phi^-\rangle}{\sqrt{2}}\right)_{bc}|1\rangle_d - \alpha\left(\frac{|\Psi^+\rangle + |\Psi^-\rangle}{\sqrt{2}}\right)_{bc}|0\rangle_d +$$
$$+\beta\left(\frac{|\Psi^+\rangle - |\Psi^-\rangle}{\sqrt{2}}\right)_{bc}|1\rangle_d - \beta\left(\frac{|\Phi^+\rangle - |\Phi^-\rangle}{\sqrt{2}}\right)_{bc}|0\rangle_d) =$$
$$= \frac{1}{2}[|\Phi^+\rangle_{bc}(\alpha|1\rangle_d - \beta|0\rangle_d) + |\Phi^-\rangle_{bc}(\alpha|1\rangle_d + \beta|0\rangle_d) +$$
$$-|\Psi^+\rangle_{bc}(\alpha|0\rangle_d - \beta|1\rangle_d) - |\Psi^-\rangle_{bc}(\alpha|0\rangle_d + \beta|1\rangle_d)] \quad (2)$$

This replacement and the algebraic passages highlight what it means to mathematically prepare the state of the system in order to perform a Bell-state measurement. The teleportation can occur if, and only if, it is possible to make this Bell-state measurement and if a temporal coincidence is measured, through the detectors, in Alice's station.

Making a Bell measurement on two states means projecting them onto one of the Bell states. Theoretically, the probability of finding each state is:

$$P(|\Phi^+\rangle_{bc}) = P(|\Phi^-\rangle_{bc}) = P(|\Psi^+\rangle_{bc}) = P(|\Psi^-\rangle_{bc}) = 25\%$$

Nevertheless, by construction, for this specific experimental set-up, the only two possible Bell states are either $|\psi^- >_{bc}$ or $|\psi^+ >_{bc}$, which can be distinguished from each other by Alice's logical electronics (Bell state measurement) (Ursin et al., 2004). Alice's result is then transmitted through a classical microwave channel (RF unit). Table 3.1 shows the two possible results of Bell measurement that Alice, with the same probability, can obtain and the corresponding state of Bob's photon.

Table 3.1: Alice's state and corresponding Bob's state

Cases	Alice	Bob
1	$|\Psi^-\rangle_{bc}$	$(\alpha|0\rangle_d + \beta|1\rangle_d)$
2	$|\Psi^+\rangle_{bc}$	$(\alpha|0\rangle_d - \beta|1\rangle_d)$

Knowing the state of Bob's photon, a transformation can be operated with the electro-optic modulator (EOM) to transform the state of photon *d* into the desired *s* input state of photon *b* by Alice so that the teleportation is complete. The latter are unitary transformations that, in the case of photons, corresponding to the rotation of polarization or phase displacements, obtained by applying a voltage pulse to the EOM.

As Bennett and colleagues stated in their 1993 paper, "*the spin-exchange method of sending full information to Bob still lumps classical and nonclassical information together in a single transmission*" (Bennett, Brassard, Crépeau, Jozsa, Peres & Wootters, 1993). Indeed, as they demonstrated, the full information of Alice encoded in her state is composed of two parts, "*one purely classical and the other purely nonclassical*" and is sent to Bob through two different channels. This observation, combined with the fact that the state of Alice is destroyed during the process, ensures that information does not travel at higher speeds than the speed of light. Thus, the second principle of relativity is not violated, and it ensures that the state is not cloned, as the no-cloning theorem requires.

After the presentation of the experiment, we move to the circuit and read it not only as an abstract representation of the experiment but as a special way of conceptualizing the experiment in terms of logic gates. In figure 3.3, the circuit of quantum teleportation is reported.

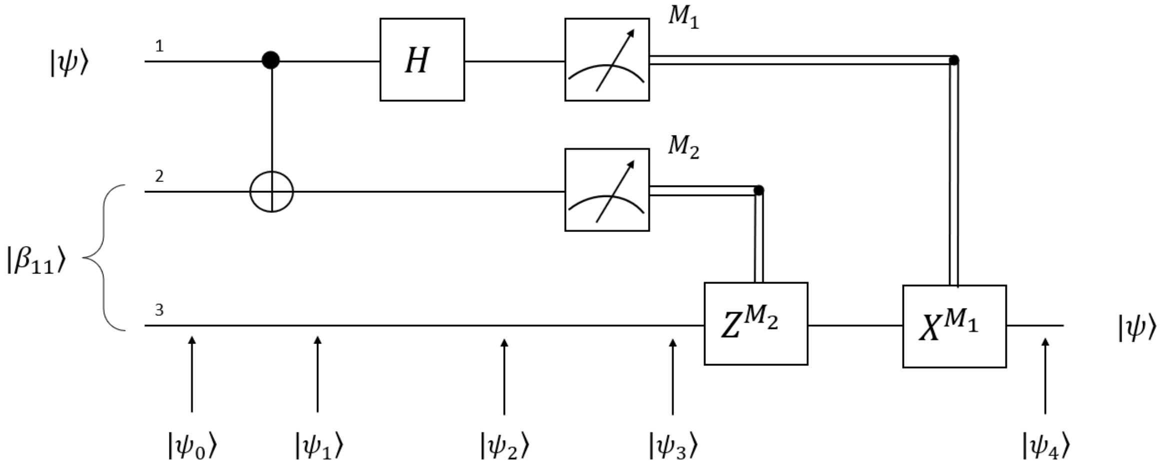


Fig. 3.3: teleportation circuit (redrawn from Nielsen and Chuang, 2002)

In this representation it is possible to identify five different moments given by the states $|\psi_0\rangle, |\psi_1\rangle, |\psi_2\rangle, |\psi_3\rangle$ and $|\psi_4\rangle$.
The three qubits represented by the three registers (3 lines of the circuit) regard, respectively, the state of the photon *b* ("1"), the state of the photon *c* ("2"), the state of the photon *d* ("3").

The state $|\psi_0\rangle$ describes the initial state of the system and it is the product of $|\psi\rangle$ and $|\beta_{11}\rangle$ where the former is the state that has to be teleported ($|\psi\rangle_1 = (\alpha|0\rangle_1 + \beta|1\rangle_1)$ and the latter is one of the four Bell states:

$$|\psi_0\rangle = |\psi\rangle_1|\beta_{11}\rangle_{23} = (\alpha|0\rangle_1 + \beta|1\rangle_1)\left(\frac{|01\rangle - |10\rangle}{\sqrt{2}}\right)_{23}$$

$$= \frac{1}{\sqrt{2}}[\alpha|0\rangle_1(|01\rangle - |10\rangle)_{23} + \beta|1\rangle_1(|01\rangle - |10\rangle)_{23}] \qquad (3)$$

As well as in the experiment, where it is necessary to make a Bell measurement on the photons and in order to have teleportation, also in the algorithm it is necessary to project photon *b* and *c* in a Bell state. This is possible by means of two logic gates in sequence, a CNOT, having as input photons 1 and 2, and a Hadamard gate on photon 1.
The CNOT gate has two input qubits, known as the *control* qubit and the *target* qubit, respectively. The circuit representation for the CNOT is shown in figure 3.4(a); the top line represents the *control* qubit, while the bottom line represents the *target*.

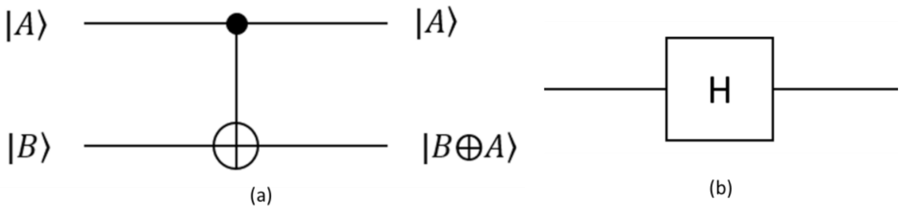


Fig 3.4: CNOT gate (a) and Hadamard gate (b).

The action performed by the logic gate is the following: if the control qubit is set on 0, then the *target* qubit is left as it is; if the *control* qubit is set on 1, then the *target* qubit is flipped. Formally, this means:

$$|00\rangle \rightarrow |00\rangle\,, |01\rangle \rightarrow |01\rangle\,,$$
$$|10\rangle \rightarrow |11\rangle\,, |11\rangle \rightarrow |10\rangle.$$

Therefore, if the CNOT gate is applied on photons 1 and 2, the equation (3) becomes:

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}[\alpha|0\rangle_1(|01\rangle - |10\rangle)_{23} + \ \beta|1\rangle_1(|11\rangle - |00\rangle)_{23}] \qquad (4)$$

To complete the projection on a Bell state, a Hadamard gate (Fig. 3.4b) is applied to photon 1. This gate is about a single qubit gate and transforms the state as follows:

Input	Output
$\lvert 0\rangle$	$\frac{\lvert 0\rangle + \lvert 1\rangle}{\sqrt{2}}$
$\lvert 1\rangle$	$\frac{\lvert 0\rangle - \lvert 1\rangle}{\sqrt{2}}$

Therefore, (4) becomes:

$$|\psi_2\rangle = \frac{1}{2}[\alpha(|0\rangle_1 + |1\rangle_1)(|01\rangle - |10\rangle)_{23} + \beta(|0\rangle_1 - |1\rangle_1)(|11\rangle - |00\rangle)_{23}] \quad (5)$$

Reorganizing the terms of (5), we obtain:

$$|\psi_2\rangle = \frac{1}{2}[|00\rangle_{12}(\alpha|1\rangle_3 - \beta|0\rangle_3) - |01\rangle_{12}(\alpha|0\rangle_3 - \beta|1\rangle_3) +$$
$$+|10\rangle_{12}(\alpha|1\rangle_3 + \beta|0\rangle_3) - |11\rangle_{12}(\alpha|0\rangle_3 + \beta|1\rangle_3) \qquad (6)$$

In (6) the first term represents Alice's qubit ($|00\rangle_{12}$, $|01\rangle_{12}$, $|10\rangle_{12}$, $|11\rangle_{12}$) and the second Bob's qubit.
After the Hadamard logic gate, a measurement on the qubits *b* and *c* is performed.
Depending on Alice's measurement, Bob's qubit will be in one of four possible states:

$$|00\rangle_{12} \rightarrow |\psi_3(00)\rangle \equiv [\alpha|1\rangle_3 - \beta|0\rangle_3]$$
$$|01\rangle_{12} \rightarrow |\psi_3(01)\rangle \equiv [\alpha|0\rangle_3 - \beta|1\rangle_3]$$
$$|10\rangle_{12} \rightarrow |\psi_3(10)\rangle \equiv [\alpha|1\rangle_3 + \beta|0\rangle_3]$$
$$|11\rangle_{12} \rightarrow |\psi_3(11)\rangle \equiv [\alpha|0\rangle_3 + \beta|1\rangle_3]$$

As in the experiment, also here Bob needs to know the result of Alice's measurement to complete teleportation. If Alice makes the measurement and obtains $|11\rangle$, Bob will not have to do anything, because his qubit is already in the right state. If, on the other hand, Alice obtains $|10\rangle$, Bob will have to apply the X gate. If Alice obtains $|11\rangle$, Bob will apply the Z gate. Finally, if Alice's result is $|00\rangle$, Bob will apply both X and Z. X and Z are two single-qubit gates that work respectively as depicted in figure 3.5 (a) and (b).

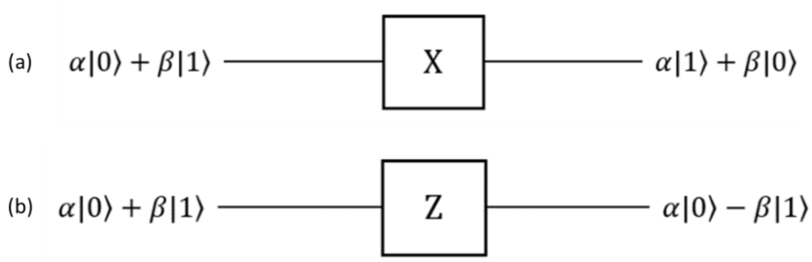


Fig. 3.5: (a) X circuital representation and action, (b) Z circuital representation and action.

In other words, to recover the state successfully, Bob will have to apply the unitary transformation to his qubit.
To sum up, this phase of content analysis consisted in conceptualizing a teleportation experiment as a computational device. This process allowed the logical structure of the experiment to emerge. Such an overall picture provides criteria to schematize the complex phenomena and organize its main elements within a comprehensive whole. This outcome was the basis for the following phases of the process of educational reconstruction, that is, for designing and testing teaching activities for upper secondary school students.

3.2.2 Design and implementation of teaching activity

In this section, we report the results of the second phase of educational reconstruction: the transformation of the physical contents, described in the previous section, into knowledge that is culturally and socially relevant, approachable, and inclusive for secondary school students.
To reach this goal, the contents have been reconstructed by implementing our four design principles (see Chapter 2). This activity is indicative of the design principles carried out to value the Second Quantum Revolution from a cultural perspective (Section 2.3). Design principle #1 (to foster a close comparison between classical and quantum computers through an analysis of *the different logic* underlined in the physics of their hardware) has been applied by emphasizing the logic that stays behind the teleportation protocol and those features, e.g., its foundation on entanglement, which does not have a classical analog and cannot be reconceptualized through the classical Boolean logic. Design principle #2, which regards the reconceptualization of the foundational experiments in terms of computation, is the overarching principle in the design of the activity, built on the comparison between the experiment and the circuit. Design principle #3 (making the activity approachable and then keeping the quantum technicalities as simple/clear as possible) has been implemented through the careful choice of conceptual details that were needed to structure the discourse and by taking into account physics education research about students' difficulties. Design principle #4 concerning inclusiveness has been implemented in the articulation of the discourse along the four levels mentioned above (narrative, logical, mathematical, and technical-experimental).

The literature on quantum physics teaching and learning shows that a simplified *spin-first* approach appears effective in guiding students at different levels into quantum physics (e.g., Corsiglia, Garcia, Schermerhorn, Passante, Sadaghiani & Pollock, 2020; Sadaghiani, 2016; Sadaghiani & Munteanu, 2015; Pospiech, 2000). More specifically, there is a set of basic concepts that are addressable with secondary school students and that necessary and sufficient to grasp what quantum technologies are at an introductory level: the concepts of state, superposition principle, manipulation of a state, measurement, and entanglement (e.g., Sadaghiani, 2016; Zhu & Singh, 2012; Pospiech, 2000; Pospiech, 1999). Our reconstruction of the teleportation phenomenon is entirely built on these concepts.
The result of this phase is a teaching activity articulated in three parts:

I. Presentation of the teleportation experiment created by Ursin and colleagues (2004);
II. Presentation of the circuit that carries out the teleportation protocol and analysis of its correspondence with the experiment;
III. Discussion of teleportation applications and their impact on the future.

The lesson starts by activating the narrative level by contextualizing the problem with the story of Alice and Bob:
Alice and Bob, before leaving, exchange a pair of entangled photons; after a few years, Alice (who has obtained a second photon) decides to send Bob the status of her new photon - how can she do so?
The students are fostered to reflect on why a classical channel cannot be used by Alice to send her status and the discussion revolved around the fact that a qubit contains an infinite number of classical

data (its state varies in a continuous space); thus, she would have needed infinite time to communicate such information to Bob: Alice needs quantum teleportation to solve this task. The students are then guided through the physical apparatus (Fig. 4) to see how Alice's task could be solved by teleportation, that is by means of experimental tools that manipulate the state of a photon and teleport it from Alice to Bob.

Since the original experimental set-up was too complicated for high school students, we present them with a simplified version, which results by schematizing the apparatus in "five main blocks" concerning key moments:

i. The production of two pairs of entangled photons;
ii. The entanglement of two photons that were initially non-entangled;
iii. The measurement of Alice's state;
iv. Alice's communication to Bob, via classical channel;
v. Bob's operation to recover the initial status of Alice, after knowing her.

In order to guide the students through the key moments of the apparatus, we describe the experiment by referring to figure 3.2, which represents the experiment performed by Zeilinger and colleagues (2004).

We explicitly tell students that the two goals of the experiment description are (a) to show where and how the narrative of Alice and Bob is attached to reality; (b) to pave the way for the experiment analysis in terms of its logical structure (bullet list above) and, hence, start to build links with the circuital representation. The logical structure we refers to the logical dimension emphasized by the circuit that led us to organize knowledge in order to describe knowledge at different levels of detail and flesh out, through the experiment, the key aspects according to the learning objectives (Eylon & Rief, 1984[8]). Therfore, we guide the students to focus their attention on "what they have to see" in the experiment and what matter for the comprehension of the activity. So first of all we invite them to consider only the photons b, c, and d, (where c and d are the pair of entangled photons that Alice and Bob exchanged previously, and b is the photon whose state is going to be teleported), since the photon a acts as a trigger, communicating to Alice that the two pair of entangled photons are correctly produced.

We then start to consider moment by moment the experiment. The experiment starts with the production of two pairs of entangled photons by a process of parametric down-conversion. The set experimental setup is schematically reported in figure 3.6.

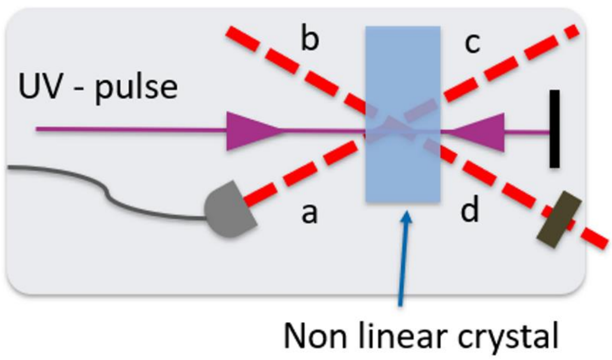


Fig. 3.6: Schematization of parametric down-conversion – interaction between a UV pulse beam with a non-linear crystal (BBO)

Since the parametric down-conversion is based on very complicated phenomena, we highlight only that the pairs of entangled photons are produced through a double interaction of a pulsed light beam with a non-linear crystal (first c and d, then a and b). This information is enough, in our opinion, to move to the logical level of the experiment and invite students to focus their attention on the photons

[8] Thanks to the reviewer Shulamit Kapon for this important reference.

b and c which are transported to Alice's station through optical fibers; here, in order for teleportation to occur, they have to "become entangled", that is projected into a Bell state.
Figure 3.6 represents the experimental set-up to make two initially non-entangled photos entangle. We explain to the students that photon b initially passes through a polarizer, which prepares it in the state to be teleported. A series of tools (including a polarization controller – PC – and a beam splitter – BS) manipulate the states so that photons b and c become entangled (Fig. 3.7). The students are then made aware of the role played by the two polarized beam splitters (PBS) and the four detectors represented in the figure: these are needed to know if the two photons are really entangled. In particular, it is possible to know that the two photons have become entangled if, and only if, the detectors reveal them simultaneously. Thus, Alice, through PBS and detectors, measures the state of her two photons and is in the condition to communicate her result to Bob.

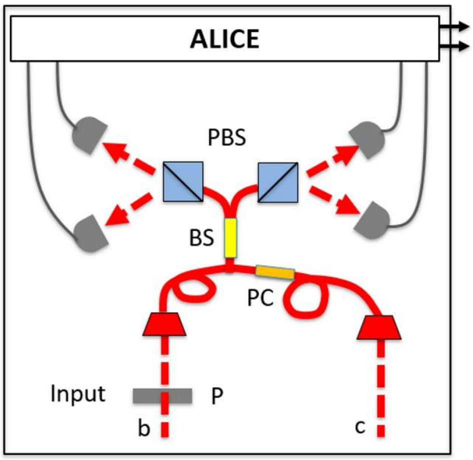


Fig. 3.7: Schematic representation of the apparatus to project two photons into a Bell state.

This is the most delicate point of reasoning, where narrative, experimental and logical levels need to be carefully aligned. Following the narrative, the students are introduced to the need that Alice makes a phone call in order to communicate the results to Bob and, on the basis of the information provided by Alice, Bob can finally recover the initial state and accomplish the process of teleportation. Shifting to the technical–experimental level, we introduce students to the classical channel used by Ursin et al. (2004), which is the microwave. To recover the initial state, Bob has to apply a voltage to the electro-optic modulator, EOM (Fig. 3.8). Because of the reduced speed of light in the optical fiber channel (two thirds of the speed of light in the air and through the air), the classic signal reaches the other laboratory 1,5 μs before the arrival of the photon d.

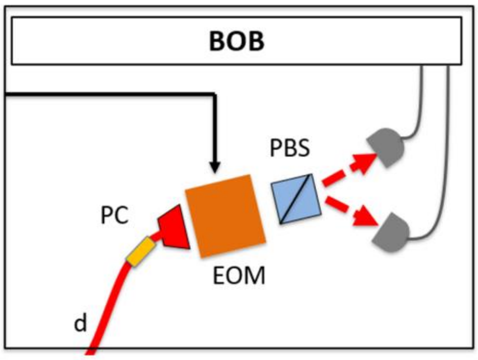


Fig. 3.8: schematic representation of Bob's station and tools to recover the input state

At this state of the discourse, the students usually confuse two key elements of the reasoning: the information that Alice provides to Bob by phone and the teleported quantum state: "if Alice has to make a call, what are the advantages and the sense of teleportation?" is the question that several students posed. In order to distinguish between the two types of information, we introduce the logical level represented by the circuit (Fig. 3.3) and the mathematical level.

In this second part, which is highly engaging, we work together with the students to reconstruct step by step how teleportation takes place mathematically. The students participate actively and are engaged by trying themselves to contribute to the reconstruction of the mathematical passages that let us solve the circuit.
In this dialogue, the narrative level is still present, but the backbone of the discourse is the circuit, which is stressed as representing a way to "transform the experiment into a quantum simulator": the circuit is indeed the way to flesh out the logical structure behind the experiment. The circuit, hence, becomes the playground where the students become acquainted with the new logic by tackling the concepts seen in the first lectures. For these purposes, the representation of the circuit is shown and, together with the students, step by step we reconstruct all the mathematical passages, demonstrating that Alice's state can actually be teleported to Bob's station.
The formalism is simpler than the one shown in the previous section. First of all, the entangled photons were chosen in the Bell state $|\beta\rangle_{00} = \frac{|00\rangle+|11\rangle}{\sqrt{2}}$, and not in $|\beta\rangle_{11}$, in order to find, by developing the calculation, the initial state corresponding to Alice's first measurement ($|00\rangle$). Furthermore, we decided to present the mathematical steps both to demonstrate formally that the teleportation takes place, and to show that manipulating information formally corresponds to manipulating the states in an equation.
Regarding details of the teaching activity, we needed to connect explicitly the formal representation of the qubit with the photons *a*, *b*, *c*, and *d*, mentioned in the experiment description. So with students, we start to build the comparison between the experiment and the circuit starting from the states of the interested photons: $|\psi\rangle = \alpha|0\rangle_b + \beta|1\rangle_b$ represents the state to be teleported and corresponds to the photon *b*; $|\beta\rangle_{00}$ represents the Bell state that describes the entangled status of photons *c* and *d*, which, in the experiment, is produced through the parametric down-conversion.
Passing to a systemic view, we present formally to students that the initial state of the total system, $|\psi_0\rangle$, is the product between $|\psi\rangle$ and $|\beta\rangle_{00}$. As figure 3.9 shows, we immediately reconnect this state to the experiment: the first thing that happens is the "creation of an entangled relationship" between the photons b and c and this, from a circuital point of view, corresponds to the sequence of a CNOT gate and a Hadamard gate.

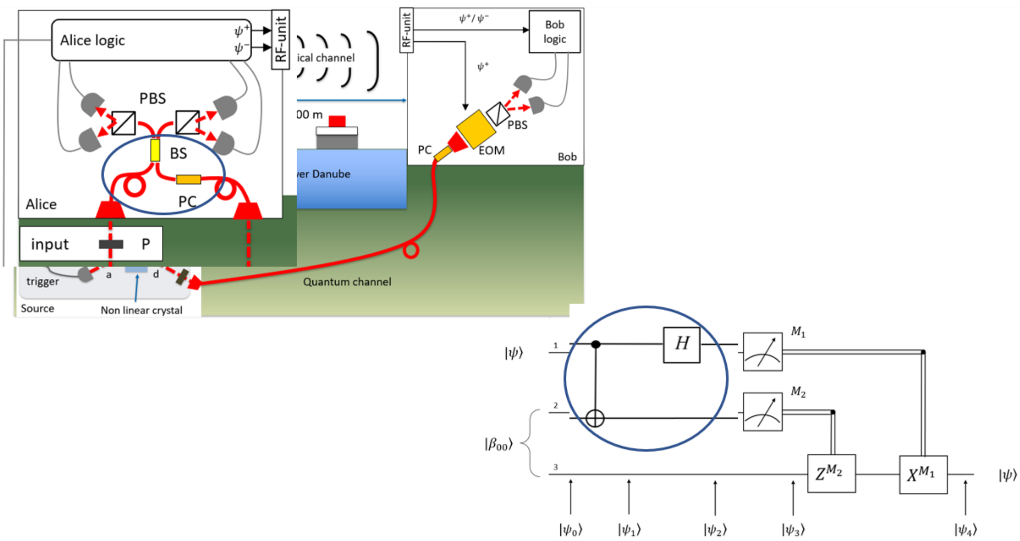


Fig. 3.9: Comparison between the "experimental and computational" projection of two photons in a Bell state.

Step by step and in a dialogic way, the whole class is involved in the calculus of the evolution of the overall state, passing through a CNOT and then to H gates and obtaining:

$$|\psi_2\rangle = \frac{1}{2}[|00\rangle_{bc}(\alpha|0\rangle_d + \beta|1\rangle_d) + |01\rangle_{bc}(\alpha|1\rangle_d + \beta|0\rangle_d) + \\ +|10\rangle_{bc}(\alpha|0\rangle_d - \beta|1\rangle_d) + |11\rangle_{bc}(\alpha|1\rangle_d - \beta|0\rangle_d)].$$

This is the most engaging, stimulating, and easy part for the students, who realize they are able to manipulate the states mathematically. We then return to the parallelism and show to students what (in the experiment) corresponds to the symbol of the quantum logic gate for measurement (Fig.3.10).

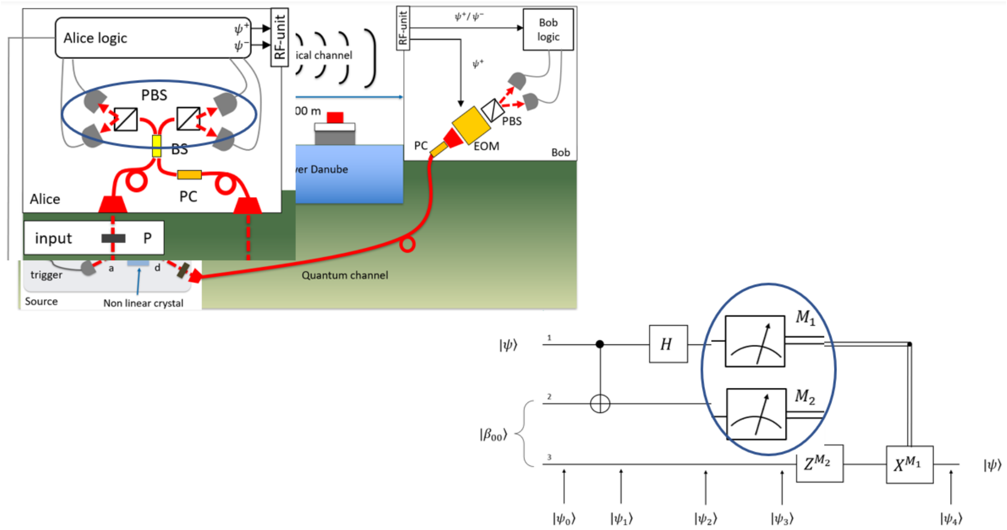


Fig. 3.10: Comparison between the “experimental and computational” measurement.

Still following the logic of the experiment, we focus students’ attention on the fact that, once the measurement is complete, Alice has to communicate her outcome to Bob. In order to recover the input state in the experimental case, he has to apply a voltage to the EOM, which corresponds to the application of the X and/or Z gates in the circuital case (Fig.3.11).

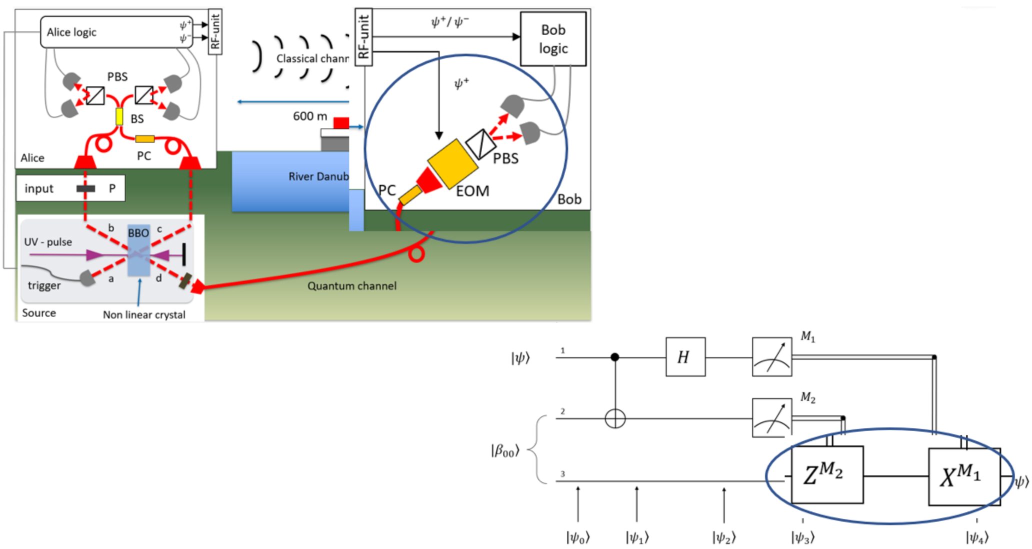


Fig. 3.11: Comparison between “experimental and computational” recovery of the input state.

This part of the reasoning is challenging for the students since they are asked to apply the learned concept of measurement and state collapse in order to understand what Bob would have obtained if

Alice had measured $|00\rangle$, $|01\rangle$, $|10\rangle$ and $|11\rangle$. We therefore ask them to recognize which gate has to be applied (X or Z) to complete the teleportation (Fig. 3.12).

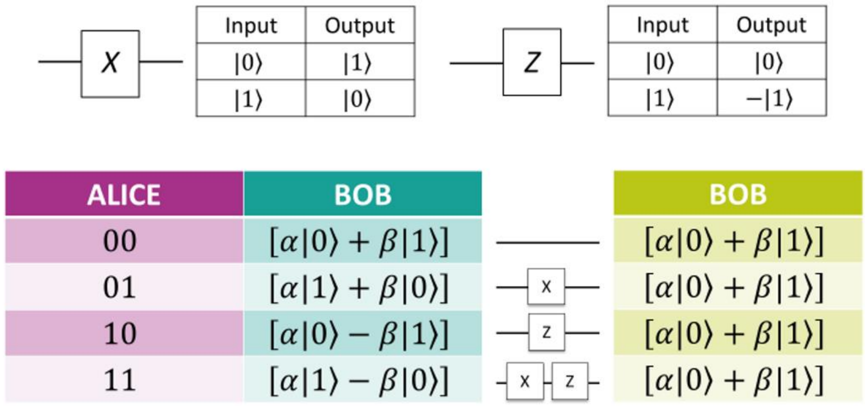


Fig. 3.12: Application of logic gates to recover the input state

The third and last part of the teaching activity is dedicated to the development of reflections on the implications of teleportation for the quantum internet and its potentialities. In order to understand how a quantum network can be created, we introduce the concepts of

- maximally entangled states;
- quantum repeater.

We explain to them that the first concept is important because entanglement is fragile since decoherence is caused by the interaction of the quantum system with the environment; quantum noise and absorption, dispersion, and non-linearity phenomena within the fiber could destroy this quantum bond. Thus, a fairly simple video[9] is presented to the students (produced by QuTech), showing entanglement distillation as a way to make two states optimally entangled and how diamonds, or rather the spins of their carbon atoms, could be used to store information. We then introduce the quantum repeater as something that is able to extend the quantum communication interval between sender and receiver. It is then shown that, if someone wants to transmit information between two network nodes at a distance of 200 km (too far for direct transmission), it is necessary to

- create two entangled qubits between the first endpoint and the repeater (100 km away) and
- create two further entangled qubits between the repeater and the second endpoint (100 km away).

By teleportation, the quantum repeater transfers the qubit that is entangled with the first endpoint to the second endpoint, forming an entangled link. We show them that the development of a quantum internet is important not only in order to have a secure network but also because, as quantum computers have such large dimensions and require temperatures close to 0K, it offers the possibility of remote access to a quantum computer by cloud computing.
We conclude the activity by showing students that we are not that far from the realization of the quantum internet. Indeed, the research group of Qutech at the University of Delft is expected to produce, by 2020, the first quantum internet that will connect four Dutch cities[10].

To sum up, the second phase of the educational reconstruction led us to provide a comparison between the teleportation experiment and a circuit with a structure grounded only on those basic concepts that have been proved to be comprehensible to secondary school students. We now move on to the third component of the Model of Education Reconstruction, which concerns the need to carry out and evaluate "pilot instructional modules rather early in the process of educational

[9] https://www.youtube.com/watch?v=ZcfMJBtCwQY
[10] https://www.youtube.com/watch?v=ZcfMJBtCwQY

reconstruction" (Duit, Gropengießer, Kattmann, Komorek & Parchmann, 2012; Duit, 2007; Duit, Komorek & Wilbers, 1997; Kattmann, Duit, Gropengießer & Komorek, 1996).

3.2.3 The analysis of a questionnaire

In order to value the effectiveness of the quantum teleportation activity, we carried out a pilot study that refers to the second implementation of the module. The implementation was held in presence, just before the covid pandemic (February-March 2020) and involved 22 students. The present study refers to a rather small and non-representative sample of secondary school students since it involves volunteer students. Therefore, the results are not intended to provide general results and the goal is limited to collecting preliminary signals about the capability of the approach to reach this special target of secondary school students and engage them. Generalizability issues will be addressed in a further step of the research.
To reach our specific goal, we design and administered a questionnaire to students that included both closed and open-ended questions (see Annex 2). The questionnaire is articulated in three sections, respectively designed to collect data about:

A. engagement and inclusiveness of the students,
B. students' reactions to the multilevel structure and the role associated with the four registers of the discourse,
C. students' understanding of the main conceptual and epistemological issues represented by the deep changes introduced by quantum computation at the logical level.

The questionnaire was given after the second implementation (February 2020). The response was purely voluntary. 14 out of 22 students replied (4 female, 10 male students). It was administered through a Google form, and we allowed a week for completion. All the information for management, protection, and data processing were provided, both orally and in paper-based format[11]. In particular, it was specified that, pursuant to art. 13 of Regulation (EU) 2016/679 (General Regulation on the protection of personal data), the Alma Mater Studiorum - University of Bologna, as Data Controller, will process personal data in compliance with the provisions of Regulation (EU) 2016/679 (General Regulation on the protection of personal data) and the Legislative Decree 30 June 2003, n. 196, as amended (Code regarding the protection of personal data).
For the analysis, a bottom-up approach was adopted, through which we searched for patterns starting from data organized in histograms. The search for patterns was controlled through a triangulation operation, or through a control and discussion process among different researchers. The following analysis has no statistical value but was conducted to generate an overview of students' reactions. The results are presented in a different order from the questionnaire sections since we think that the quality of students' understanding (second part of the questionnaire) can be captured more easily if the conceptual responses are framed within a broader picture of their comprehensive attitude toward the class and activities (last part of the questionnaire).

A. Students' general reaction to the activity

The histogram in figure 3.13 refers to students' answers to these questions: *To what extent [from 1 (not at all) to 5 (very much)] did you find the lesson on teleportation easy, useful to understand quantum technologies, stimulating, fascinating and in accordance with your expectations?*
The graph compares the average ranking of students' responses.

[11] In particular, it was specified that, pursuant to article 13 of Regulation (EU) 2016/679 (General Regulation on the protection of personal data), the Alma Mater Studiorum— University of Bologna, as Data Controller, will process personal data in compliance with the provisions of Regulation (EU) 2016/679 (General Regulation on the protection of personal data) and the Legislative Decree 30 June 2003, n. 196, as amended (Code regarding the protection of personal data).

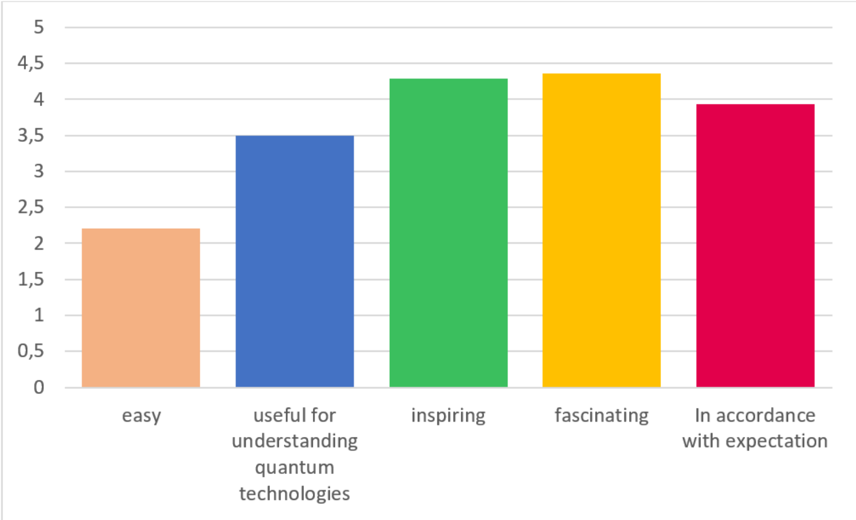


Fig. 3.13: General evaluation of the activity on teleportation: Average score awarded by students for each aspect

The graph shows that the lesson on teleportation was appreciated and resulted close to their expectations. It was also evaluated as useful for understanding quantum technologies.
As we might expect, the lesson was evaluated as "not easy": 1 student out of 14 ranked it as 1, 9 students ranked the level of difficulty as 2 and the other 4 ranked it as 3 (Fig. 3.14).

Fig. 3.14: Distribution of students' scores (range: 1-5) on the item "easiness"

The intense and lively discussions that arose during and at the end of the lesson, designed to clarify the most difficult points, confirm the reactions of deep interest and engagement reported in figure 3.13, *in spite of*—or perhaps, *because of*—the innate difficulties. Indeed, it is likely that the perception that they were directly addressing an extremely difficult and advanced topic was itself one reason of interest.

B. Students' navigation within the multilevel structure and the role associated with the four registers

The third part of the questionnaire set out to check how the students reacted to the multilevel discourse and the role that they associated with the registers. As we have already stated, the teachers told the students explicitly about the existence of the levels and informed them when the various levels were switched on. However, the specific roles that we supposed the levels would play were not revealed.

In this part, we asked how much the narrative, logical, technical–experimental, and mathematical levels aided them to

- figure out what quantum teleportation is (general idea about the phenomenon and of the "problem to be solved");
- understand what the phenomenon of teleportation consists in (understanding of the key moments of the protocol);
- follow the reasoning conducted on teleportation in its entirety (follow the sequence of the key moments to solve the problem);
- grasp the articulation of the reasoning in its different phases (grasp the sense of the sequence);
- understand the details of the reasoning (understand the meaning of the logical and mathematical steps);
- convince themselves of how it happens.

The specific question we posed was:
What contribution has the register made to your understanding of the phenomenon? Please indicate, from 1 to 5 (1 not at all, 2 a little, 3 quite a lot, 4 very, 5 very much), how much each level has contributed to developing the previous aspects.
The histogram below shows the mean value of how much the different levels contributed to the six aspects (Fig. 3.15).

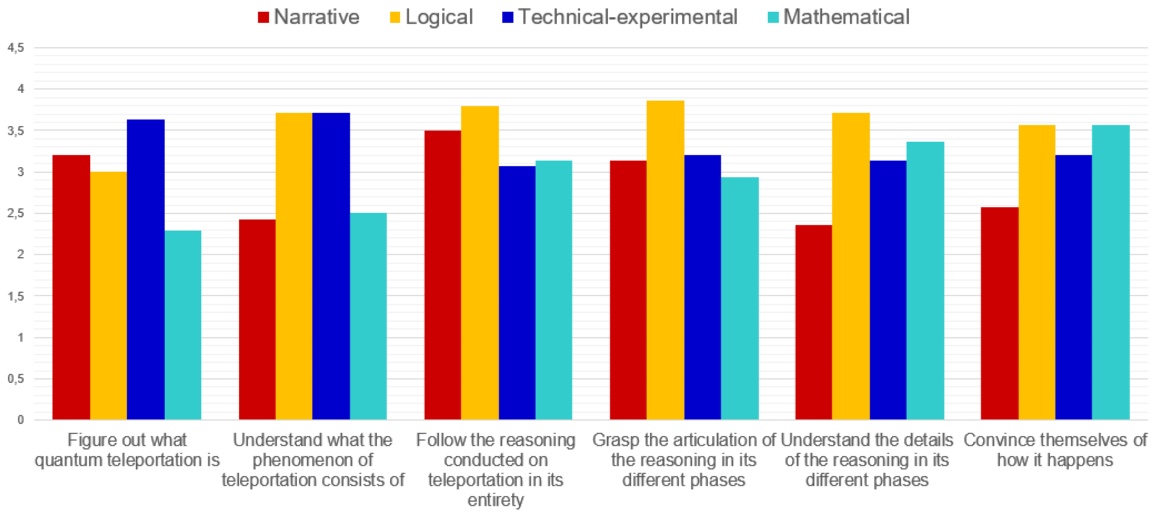


Fig. 3.15: for the four levels, the average of the score attributed to each aspect (range: 1-5)

As we can see from the graph, the narrative level (in red), together with the mathematical level (in light blue), is the register that fluctuates most. It proved particularly useful for the aspects that concern the "narration" of the phenomenon: figuring out what quantum teleportation is, following the whole reasoning carried out on teleportation, and grasping the articulation of the reasoning in its different phases. All these aspects concur to provide a comprehensive, large-scale picture of the phenomenon.
The logical level (in yellow) appeared to be the most fruitful. This is the register that was more useful for all six aspects and for understanding the phenomenon. In particular, it played an important role in helping students to focus on the details and understand what the phenomenon of teleportation consists of, to follow the whole reasoning carried out on teleportation, to grasp the articulation of the reasoning in its different phases, to understand the details of the reasoning, and to grasp the mechanism that makes teleportation occurs.
The technical-experimental level (in blue) appeared particularly useful for the aspects that concern the physical understanding of the phenomenon. In particular, this register was useful for figuring out what quantum teleportation is and understanding what the phenomenon of teleportation consists of.

Finally, the mathematical level (in light blue) was perceived as useful, in particular, for the aspects concerning "understanding of details": to understand the details of the reasoning and to clearly grasp the mechanism that makes teleportation occurs.
This result confirms our initial hypotheses. The students have recognized the different roles of the various registers and these roles are consistent with those we attributed to the levels when we designed the activity (see section 2.3). In fact, the narrative level was introduced to promote the formation of a comprehensive view in which the problem could be framed and addressed. The logical level was intended to provide the backbone of the discourse, and indeed the students recognized it as the structure that could reveal and connect the single elements of reasoning and bridge the experiments with the circuital representation. Finally, the technical-experimental level had to give a sense of concreteness and feasibility, while the mathematical level had to foster reasoning, to be developed in precise and detailed steps.

As well as roles, the various registers received different levels of appreciation. Students' answers to the following question were indeed very varied: "*Which register did you prefer*?" (see Fig. 3.16)

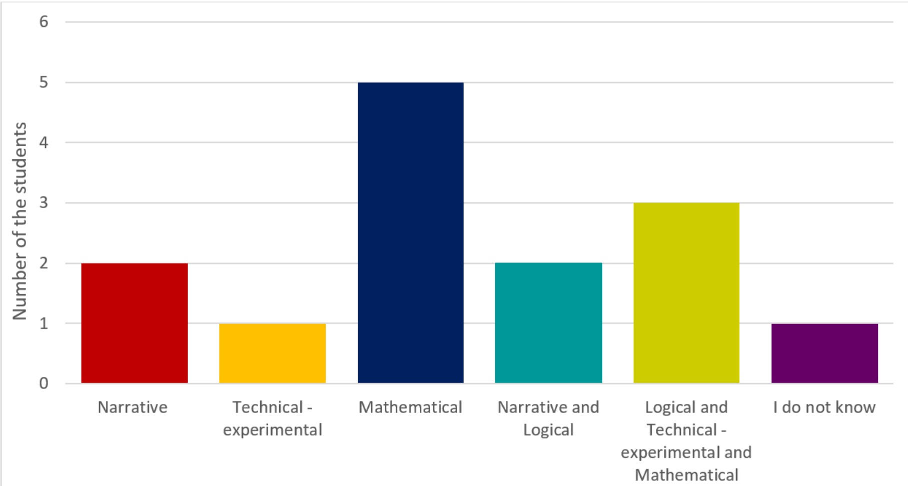


Fig. 3.16: distribution of the students' preference for the registers.

It is interesting to note that, although the logical level was retained most useful, it was the favorite one for only three students. The distribution of answers contributes to the supposition that the approach is able to appeal to the different tastes and intellectual interests of the students.
However, in the open question about the potential in the intertwining of the four registers, many of them stressed the complementary structural roles played by the different levels: each level had its own role but, by removing even one level, the discourse was no longer complete:

> "*they* (the registers) *were complementary and filled in the gaps that other reasoning could not; if a register was removed, it was a bit incomplete as reasoning.*"

> "*each part, more or less important, takes on its own value, as I don't think a lesson without one of these aspects would be satisfactory*"

Other students highlighted the importance of analyzing a phenomenon from different points of view (multidimensionality) because different points of view provide a more comprehensive or better understanding:

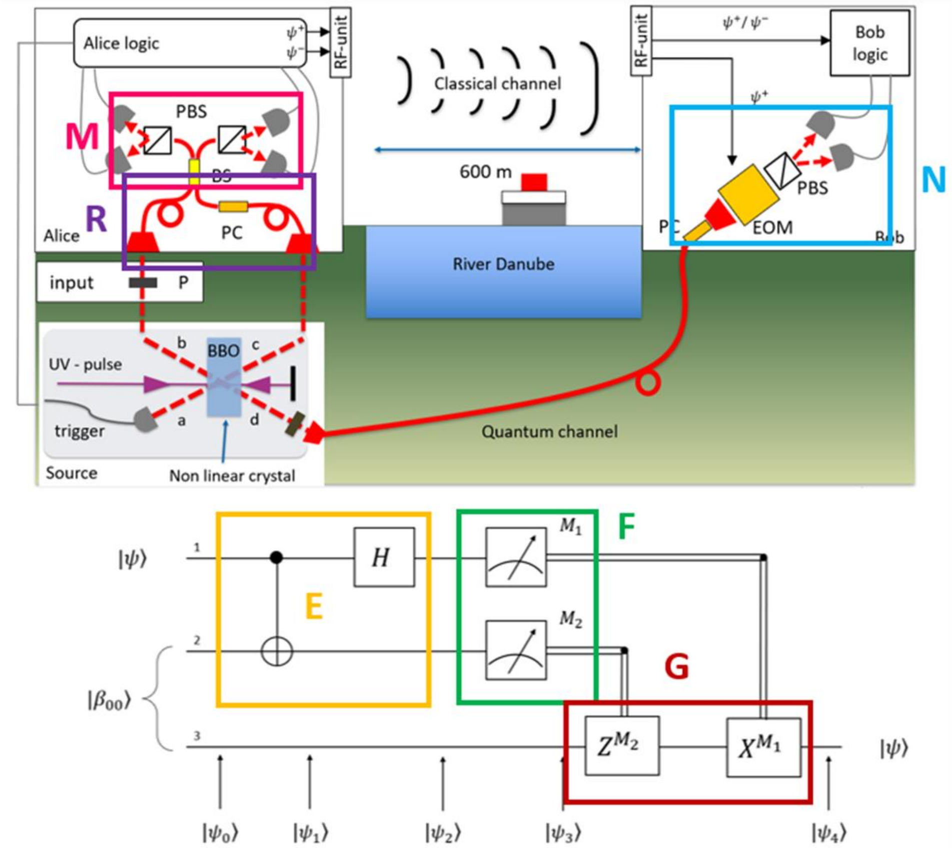
> "*I found that the intertwining of the 4 registers allowed me to understand the phenomenon in a more complete way, describing the various aspects from different points of view*."

> "*It is always important to compare different aspects of reality to try to understand it as well as possible*"

These reactions to the multilevel structure reveal that the approach is *inclusive*. By inclusiveness we do not refer to the quality of including all the students, thereby ensuring that no one is excluded or marginalized. That is not the case for the students who attended the course, because PLS laboratories are extracurricular activities, and the students are usually very highly motivated and very interested in scientific topics. By inclusiveness, we refer to the quality of an approach to resonate with different tastes and interests, to stimulate the students to find "their own way to enter and understand a physics topic." As we stressed in the design principles, this is a crucial aspect of our approach, and these results, even though gathered in a pilot study with few students, are very promising. These results are particularly relevant if we consider the last part of our analysis, which allowed us to highlight how the four registers also acted at the level of conceptual understanding. In the following section, we present two cases that show how a special combination of the various registers supported understanding.

C. Students' understanding of the main conceptual and epistemological knots

The questions aimed at investigating students' understanding were formulated as follows:
Images (a) and (b) (Fig.3.17) show the experiment and the circuit we have analyzed for teleportation. Try to describe the phenomenon of teleportation, specifying: i) what is teleported; ii) what physically corresponds to the moments indicated with M - N - R in figure (a), and iii) to which parts of the circuit (E - F - G) they correspond.



(a) (b)

Fig. 3.17: Comparison between (a) experiment and (b) circuit.

Most of the students (12 of 14) answered schematically. A typical answer is

1. *The quantum state of photon B is teleported.*
2. *R corresponds to the moment in which b and c become entangled, M to the moment of measurement of b and c, N to the moment in which I modify the photon d knowing the measurements of B and C.*
3. *R – E, M – F, N – G.*

Although rather synthetic, all the students reached this level and were able to distinguish between the "*state of photon*" and "*photon*," by stressing that it is the first that is teleported from one side to the other of the Danube. Moreover, they were able to identify the "logic of the experiment," by recognizing the three main phases, both in the experiment and in the circuit.
Two students gave much more articulated answers, as reported in Table 3.3. When we started to analyze them, we discovered that they differ in three main aspects. The first is the overall structure of the discourse. Indeed, one answer is built on the structure of the circuit representation (Fig. 3.16b) while the other is on the experiment (Fig. 3.16a). The second aspect regards the "main actors" of the discourse: in one case, the discourse follows the narrative of Alice and Bob, while, in the other case, the narrative is focused on the description of the concrete experimental steps. The third aspect concerns the language and the linguistic registers and their connection with the different levels along which the discourse is articulated.

Table 3.3: The two students' answers.

Answer #1 *"There are three photons b, c and d. Alice has photon c already in entanglement with photon d belonging to Bob, while photon b contains the information that Alice wants to transmit to Bob. Alice makes photons b and c entangled and then measures. After the measurement, the quantum state collapses and Alice communicates the result of her measurement to Bob through a mechanical channel. Thanks to this measurement, Bob performs transformations on the quantum state of d in order to recover the initial state of b, since, for transitive property, it* (photon b) *is entangled with d.* *I) Only the information contained in the qubit, in the photon b, is teleported, not the matter.* *II) M is the measurement of the states of photons b and c; R is the circuit which creates entanglement between b and c; N is the process to which d is subjected to obtain the initial information of b* *III) M – F; R – E; N – G"*
Answer #2 "*All this starts with the formation, through a light source that crosses a particular crystal, of two pairs of entangled photons, respectively a-b and c-d. Then the photon "d" will be passed through a channel to an operator that, to simplify, we will call Bob, while, both "b" and "c" remain to Alice (the other operator).* *First, Alice makes the two remaining photons entangled and then analyses them (assuming they are therefore entangled).* *After the measurement, it is communicated via a classic channel to Bob who, on the basis of what "arrives", "decides" whether or not to use the two "quantum Boolean operators" X and Z or, based on the response of Alice will modify the state of her photon "d" in four possible ways: using both the "operation" X and Z on the state of the photon, only the X, only the Z or neither. In all cases, a photon with the same superposition characteristics of the "zero or one" state of the "b" photon is obtained, even though it still belongs to Alice.* *i) the state of b is teleported.* *ii) R: an entanglement relationship is created between the photons b and c* *M: check that these photons are entangled and analyze them* *N: after the communication of the state, the characteristics of the photon "d" are altered.* *iii) E = R, F = M, G = N"*

In Fig. 3.18 we report the analysis of answer #1.

The argumentation is articulated in four blocks that are the four phases of the circuit:

1. The creation of a relation of entanglement between two photons initially non entangled;
2. The measurement;
3. The communication through classical channel;
4. The transformation to recover the initial state.

"There are three photons b, c and d. Alice has the photon c already in entanglement with the photon d belonging to Bob, while the photon b contains the information that Alice wants to transmit to Bob.

(1) *Alice makes photons b and c entangled and then*

(2) *measures. After the measurement, the quantum state*

(3) *collapses, and Alice communicates the result of her measurement to Bob through a mechanical channel.*

(4) *Thanks to this measure, Bob performs transformations on the quantum state of d in order to recover the initial state of b, since, for transitive property, it* (photon b) *is entangled with d."*

Fig. 3.18: Analysis of answer #1 – on the left, there is the structure of the discourse; in yellow the "main actors"; in red, the key words characterizing the registers

The structure of the discourse is articulated in an introduction (the description of the initial state) and four blocks that are the four phases of the circuit: the creation of a state of entanglement between photons *b* and *cd*; the measurement; the communication through a classical channel, and the transformation to perform in order to recover the initial state. The answer is provided in a narrative form where Alice and Bob are the main actors and the steps are described in terms of their actions: "*Alice makes photons b and c entangled and then measures*," "*Alice communicates the result of her measurement to Bob*," "*Bob performs transformations*." The narrative level, in students' discourse, emerges as playing the role of keeping together what happens and the different actions performed by Alice and Bob.

As for the language, even though the student chose the logic of the circuit to structure her discourse, the words she uses are closer to the language of the experiment. She indeed uses the words *photon*s, *transmission*, and *communication through a mechanical channel* to describe what happens in each phase. This leads us to infer that the circuit provided a criterion to select the main pillars of the logical structure, while the experiment and the physical phenomena were used to attach meaning to each step.

The student's answer includes all the registers but the mathematical one, and they mirror a personal articulation of her discourse along the levels. She indeed combines the registers in her idiosyncratic way which is different from the student who provided answer #2 (see Fig. 3.19), which is described in the following.

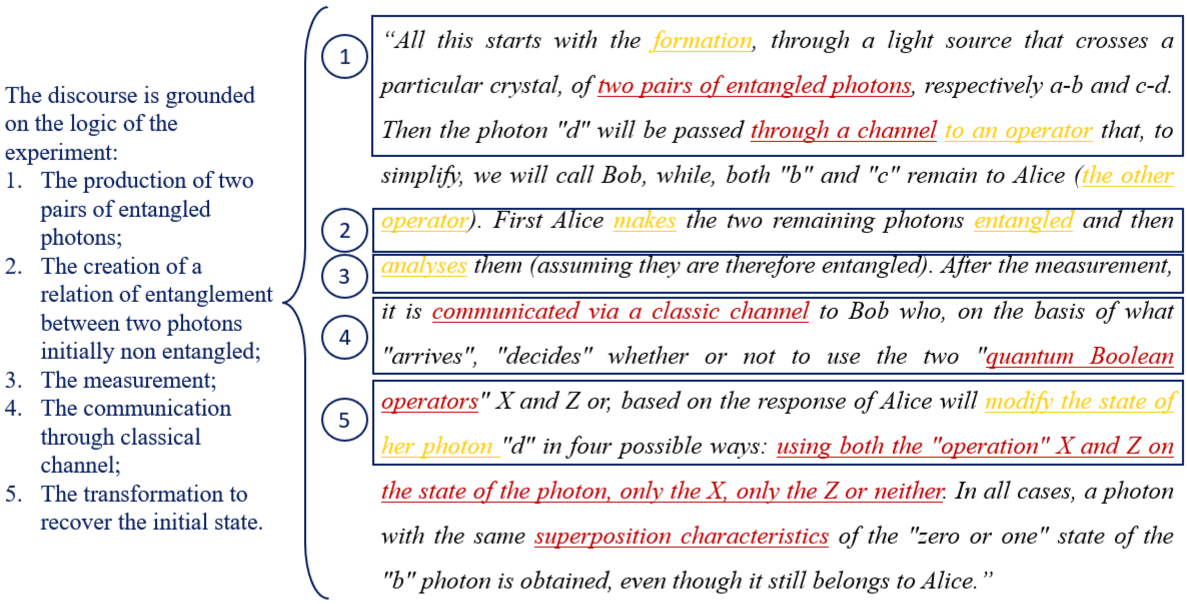


Fig. 3.19: Analysis of answer #2 – on the left, the structure of the discourse; in yellow, the "main actors"; in red, the key words characterizing the registers

As for the structure of the discourse, this student does not use the separation in blocks of figure 3.16, but instead explicitly starts from the experiment. In fact, the discourse follows the phase of the experiment starting from the production of the two pairs of entangled photons and also describing the process ("*All this starts with the formation, through a light source that crosses a particular crystal, of two pairs of entangled photons, respectively a-b and c-d*.").
The "main actors" of the narrative level are the processes and the physical operations: Alice and Bob are the experimenters.
For the core description of the phenomenon, his language taps mainly into the experimental register, and characteristic expressions are: *the photon "d" will be passed through a channel to an operator, analyses them* (the photons). Instead in the final part, he switches into a highly specialized logical register: "[…] *whether or not to use the two "quantum Boolean operators*" X and Z or, based on the response of "*Alice will modify the state of her photon "d" in four possible ways*: […]".
As with the previous one, this student managed to understand the phenomenon by drawing on the three registers.
While in answer #1 we noticed a logical structure acting as the backbone of the discourse, in answer #2 we notice a preference for the technical–experimental level, especially in the description of the processes. Despite initial detachment from the story of Alice and Bob, the narrative register is used to reconstruct the sequence of events, confirming the role that we have attributed to this level: the construction of an overarching picture that allows the building of an overarching idea without becoming lost in details.

In the section of the questionnaire designed to investigate students' understanding, the second question was: *What main differences do you find between the experiment shown in figure (a) and the circuit represented in figure (b)? More generally, what analogies and differences do you see between a description of a phenomenon made in terms of the experiment and a description made in terms of the circuit?*

In order to investigate the differences and analogies that students find between the circuit and the experiment, we have analyzed the words that the students used to refer to one or to the other. A simple count shows that the words most frequently associated with the circuit are *trivial*, *schematic*, *sequential* (refers to the sequential organization of processes/events), and *simplification*. They are used both in a positive sense – a simplification that shows the essential aspects of the experiment –

and in a negative sense – too simple to be significant without a concrete description of it. Regarding the experiment, the most frequent words are *general functioning*, *global vision*, *practicality,* and *utility*.

Some answers that highlight the differences (sequential vs global, schematic/abstract vs. practical) are:

> "*Both describe the series of changes that occur in the system, but while the description of the phenomenon in terms of the experiment highlights a more global vision, the circuit is more schematic and shows the steps in series*"
> "*Both a circuit and an experiment describe the same steps of a phenomenon, but the experiment describes them from a more practical point of view, while the circuit in a more theoretical way*".

A few students focused their answer on the role and usefulness of both representations. For example, two students wrote:

> "*The experiment is useful for understanding the general functioning, understanding what is being done [...]. The circuit instead simplifies it (*the experiment*), eliminating the most complex parts and reducing it to the essential form*"
>
> "*The circuit abstracts the concept of the experiment and is important for generalizing the principles of reality also in other conceptual structures. The experiment serves to understand the technologies used and how the measurement is carried out in reality*".

Again, the students stress the potential of the circuit to show the essential structure of the experiment, while the experiment gives concreteness to the circuit, by showing how teleportation takes place "*in reality*", and how "*it is contextualized in reality*" (*general functioning*).
The answers reveal the extent to which the experiment and the circuit are different and focus on different aspects of the model of the phenomenon. Particularly importantly, they stimulate the formation of different kinds of imagery and explanations. The experimental approach encourages students to follow the events and photons in a spacetime framework, allowing them to grasp the counterintuitive essence of entanglement as a "spooky action at a distance." The circuital approach, instead, suggests a systemic view of the phenomenon, allowing a student to have a global comprehensive picture of the entire system within which logical steps and details can be coherently placed.

From an educational perspective, this is known to be particularly problematic in quantum physics. As Mannila, Koponen, and Niskanen (2001) showed, "students are used to directing their attention to properties of entities (particle, bodies, etc.), create images and draw pictures, where illustrations concentrate on the behavior of entities. A similar approach is very difficult in quantum physics where the properties of basic entities are difficult to approach, and one should concentrate on properties of phenomena" and foster a proper "'conceptual shift to form a new ontology'" (ibidem, p. 47).

3.3 The random walk activity

This section is dedicated to the educational reconstruction carried out for the classic and quantum random walks activity. We chose this algorithm because it is used in many fields of knowledge such as physics, biology, finance theory, computer vision, and earthquake modeling (e.g., Grady, 2006; Ward, 2004; Helmstetter & Sornette, 2002; e.g., Berg, 1993; Landau, 1980). Quantum random walk, with its versatility, is an advanced tool for building quantum algorithms (e.g., Shenvi, Kempe & Whaley, 2003; Ambainis, 2004; 2008; Childs, Cleve, Deotto, Farhi, Gutmann & Spielman, 2003) and it has been also shown to constitute a universal model of quantum computation (e.g., Childs, 2009; Lovett, Cooper, Everitt, Trevers & Kendon, 2010).

There are two kinds of quantum walks: discrete and continuous quantum walks. The first one consists of two quantum mechanical systems, which are a walker and a coin. An evolution operator is applied

to both systems only in discrete time steps. The mathematical structure of this model is evolution via a unitary operator. The second one consists of a walker and an evolution (Hamiltonian) operator of the system that can be applied with no timing restrictions at all, for example, a walker walks at any time. The mathematical structure of this model is evolution via the Schrödinger equation.
From an educational perspective, to make the content at the reach of secondary school students, we have decided to consider only the discrete case.

In this activity, students get acquainted with the concepts of quantum state, state manipulation, measurement, and entanglement. Furthermore, the random walk allows us to touch the interdisciplinary nature of quantum computation and the comparison between the classical and quantum model let us shed light on the different logics.
In the following, through the MER lens, I present the work carried out first to clarify the content starting from the scientific literature, the educational reconstruction through the design principles in section 3.2, and finally the analysis of a questionnaire about the impact of the interdisciplinary approach to students' understanding.

3.3.1 Content analysis/clarification of the content

The main references that we used for this reconstruction are a seminal paper about the quantum random walk by Aharonov, Davidovich, and Zagury (1993), and a paper titled "*Quantum random walks: an introductory overview*" by Kempe (2003). In particular, we propose the treatment carried out by Kempe that is epistemologically meaningful, since she discusses quantum random walks both from physical and computer science perspectives and some developed algorithms, with the aim of "trying to develop an intuition" (Kempe, 2003, p. 308). The author, after providing a general flavor of the phenomenon in a physical setting, formalizes both the discrete and the continuos models of quantum random walks.

The physical intuition
The example with which Aharonov, Davidovich, and Zagury (1993) start is the following: Let us consider a particle on a line, whose position is described by a wave packet $|\psi_{x_0}\rangle$ localized around the position x_0. The function $\langle x|\psi_{x_0}\rangle$ represents a wave packet centered around x_0. The transition of the particle after a step of length l can be expressed by the unitary operator:

$$U_l = e^{-\frac{iPl}{\hbar}} \qquad \text{such that} \qquad U_l|\psi_{x_0}\rangle = |\psi_{x_0-l}\rangle.$$

with P is the momentum operator. Assume now that the particle has a spin $-\frac{1}{2}$ degree of freedom. The operator that corresponds to the z component of the spin is S_z, whose eigenstates can be denoted as $|\uparrow\rangle$ and $|\downarrow\rangle$, so that $S_z|\uparrow\rangle = \frac{\hbar}{2}|\uparrow\rangle$ and $S_z|\downarrow\rangle = \frac{\hbar}{2}|\downarrow\rangle$ can be set to 0 (like in the natural system) to simplify the notation, so the particle is described by the tensor:

$$|\Psi\rangle = \alpha^{\uparrow}\,|\uparrow\rangle \otimes |\psi^{\uparrow}\rangle + \alpha^{\downarrow}\,|\downarrow\rangle \otimes |\psi^{\downarrow}\rangle$$

where the first part is the component of the wave-function of the particle in the $|\uparrow\rangle$-space and the second one in the $|\downarrow\rangle$-space. Furthermore, normalization requires that $|\alpha^{\uparrow}|^2 + |\alpha^{\downarrow}|^2 = 1$. The tensor product $\otimes$ separates the two degrees of freedom, spin, and space, and will allow us to view the resulting correlations between these two degrees of freedom more clearly.
The time development corresponding to a step of the length l of a particle with the spin $-\frac{1}{2}$ can be described by the unitary operator:

$$U = e^{-2iS_z \otimes Pl}$$

This operator induces a conditional translation on the particle depending on its internal spin-degree of freedom. If the spin of the particle is initially in the state $|\uparrow\rangle$, so that its wave function is of the form

$|\uparrow\rangle \otimes |\psi_{x_0}\rangle$. An application of the operator U transforms the wave function into $|\uparrow\rangle \otimes |\psi_{x_0-l}\rangle$ and the particle will be shifted by a step l to left. The opposite case is when the spin of the particle is initially in the state $|\downarrow\rangle$. In this case, the wave-function is of the form $|\downarrow\rangle \otimes |\psi_{x_0}\rangle$ an application of the operator U transforms the wave-function into $|\downarrow\rangle \otimes |\psi_{x_0+l}\rangle$ and the particle will be shifted by l to right.
More interesting is the case when the spin of the particle, localized in x_0, is not in an eigenstate of S_z, but it is in a superposition state

$$|\Psi_{\boldsymbol{in}}\rangle = (\alpha^{\uparrow}|\uparrow\rangle + \alpha^{\downarrow}|\downarrow\rangle) \otimes |\psi_{x_0}\rangle$$

The application of the operator U brings to

$$U|\Psi_{\boldsymbol{in}}\rangle = \alpha^{\uparrow}|\uparrow\rangle \otimes |\psi_{x_0-l}\rangle + \alpha^{\downarrow}|\downarrow\rangle \otimes |\psi_{x_0+l}\rangle$$

Now if we want to measure the spin in the S_z basis, the particle will be in the state $|\uparrow\rangle \otimes |\psi_{x_0-l}\rangle$ localized around $x_0 + l$ with the probability $|p|^{\uparrow} = |\alpha^{\uparrow}|^2$ or in the state $|\downarrow\rangle \otimes |\psi_{x_0+l}\rangle$ localized around $x_0 - l$ with the probability $|p|^{\downarrow} = |\alpha^{\downarrow}|^2$.
This last procedure coincides to a random walk on a line, where after a step the particle is on average displaced by $l(|p|^{\uparrow} - |p|^{\downarrow})$.
If we repeat the process times, we will find that the particle is on average displaced by:

$$\langle x\rangle = Tl(p^{\uparrow} - p^{\downarrow}) = Tl(\left|\alpha^{\uparrow}\right|^2 - \left|\alpha^{\downarrow}\right|^2)$$

And the variance of its distribution will be:

$$\sigma^2 = Tl\left|\alpha^{\uparrow}\right|^2\left|\alpha^{\downarrow}\right|^2 = Tlp^{\uparrow}p^{\downarrow}$$

The formalization of the discrete model
Let H_p be the Hilbert space spanned by the position of the particle. We describe the random walk in one dimension, so on a line or on a circle. In the first case the space is spanned by the base$\{|i\rangle : i \in Z\}$, in the second case (a circle of the perimeter of length N) we have $H_p = \{|i\rangle : i = 0, \dots, N-1\}$ with $|i\rangle$ corresponding to a particle localized in position i.
The total space is constituted by the Hilbert space H_p and a "coin-space" (the spin space) H_c spanned by the base $\{|\uparrow\rangle, |\downarrow\rangle\}$, so the total space is given by $H = H_c \otimes H_p$.
The conditional translation is expressed by the shift operator defined as

$$S = |\uparrow\rangle\langle\uparrow| \otimes \sum_i |i+1\rangle\langle i| + |\downarrow\rangle\langle\downarrow| \otimes \sum_i |i-1\rangle\langle i|$$

where $i \in \boldsymbol{Z}$ in the case of the line or $0 \le i \le N-1$ in the case of the circle. S transforms the basis state:

$$|\uparrow\rangle \otimes |i\rangle \xrightarrow{S} |\uparrow\rangle \otimes |i+1\rangle$$

and

$$|\downarrow\rangle \otimes |i\rangle \xrightarrow{S} |\downarrow\rangle \otimes |i-1\rangle.$$

The first step of the random walk is a rotation in the coin-space H_c. If the initial state is $|\Phi_{in}\rangle = |\uparrow\rangle \otimes |0\rangle$, after an iteration (a rotation in the coin space followed by the application of the S operator) we have a shift on the right ($|1\rangle$) with a probability of $\frac{1}{2}$ and a shift on the left ($|-1\rangle$) with the same probability. A commonly balanced unitary coin is the Hadamard coin H

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Using the H coin, it is easy to see that it is balanced:

$$|\uparrow\rangle \otimes |0\rangle \xrightarrow{H} \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \otimes |0\rangle \xrightarrow{S} \frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |1\rangle + |\downarrow\rangle \otimes |-1\rangle)$$

Measuring the coin state in the standard basis gives $\{|\uparrow\rangle \otimes |1\rangle, |\downarrow\rangle \otimes |-1\rangle\}$ with probability $\frac{1}{2}$. After this measurement there is no correlation between the position left. Letting the process continue, if measurement at each iteration is carried out, we obtain a classic random walk represented by a classic Galton board (Fig. 3.20).

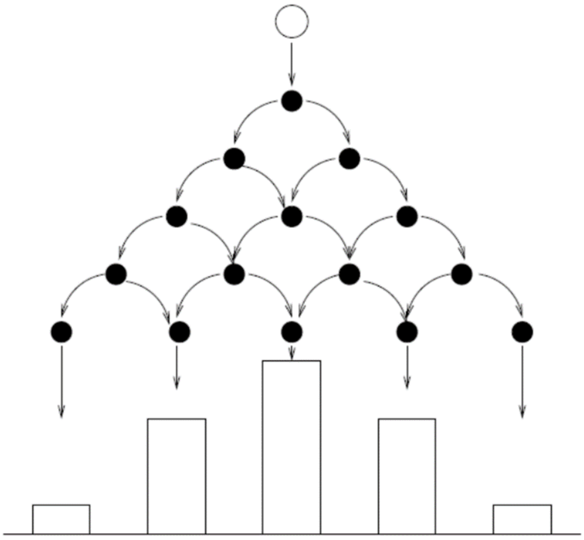


Fig. 3.20: Classical Galton's board: By measuring each step the probability distribution that is obtained is the classical one.

There is another possibility, namely not measuring the coin register during intermediate iteration to keep the quantum correlations between different positions and let them interfere in the following steps.

The quantum random walk of T steps is defined by the transformation U^T , where U is given by:

$$U = S \otimes (H \otimes I)$$

Therefore, to show the differences of the quantum walk from its classical counterpart, let us evolve the walk (without intermediate measurements), starting from the initial state $|\Phi_{in}\rangle = |\downarrow\rangle \otimes |0\rangle$:

$$|\Phi_{in}\rangle \xrightarrow{U} \frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |1\rangle - |\downarrow\rangle \otimes |-1\rangle) \xrightarrow{U} \frac{1}{2}(|\uparrow\rangle \otimes |2\rangle - (|\uparrow\rangle - |\downarrow\rangle) \otimes |0\rangle + |\downarrow\rangle \otimes |-2\rangle) \xrightarrow{U}$$
$$\frac{1}{2\sqrt{2}}(|\uparrow\rangle \otimes |3\rangle + |\downarrow\rangle \otimes |1\rangle + |\uparrow\rangle \otimes |-1\rangle - 2|\downarrow\rangle \otimes |-1\rangle - |\downarrow\rangle \otimes |-3\rangle)$$

In table 3.4, the probability distribution for this case is reported.

Table 3.4: Probability of being at position X after N steps for a quantum Galton board with the initial state: $|\Phi_{in}\rangle = |\downarrow\rangle \otimes |0\rangle$.

	-5	-4	-3	-2	-1	0	1	2	3	4	5
0						1					
1					1/2		1/2				
2				1/4		1/2		1/4			
3			1/8		5/8		1/8		1/8		

4		$1/16$		$5/8$		$1/8$		$1/8$		$1/16$	
5	$1/32$		$17/32$		$1/8$		$1/8$		$5/32$		$1/32$

As it is possible to see from table 3.4, the distribution is no more a binomial distribution. The asymmetry of the distribution is 'drifting' to the left which can be interpreted as an interference pattern. This asymmetry is explained by the characteristics of the Hadamard coin that transform in differently the two directions $|\uparrow\rangle$ and $|\downarrow\rangle$, it only multiplies the phase by -1 in the case $|\downarrow\rangle$. This induces, also intuitively, more cancellations for paths going right-wards (destructive interference) while particles that move to the left interfere constructively. In figure 3.21, the probability distribution of the quantum random walk after T= 100 steps of the quantum walk, starting in the state $|\downarrow\rangle \otimes |0\rangle$, is reported.

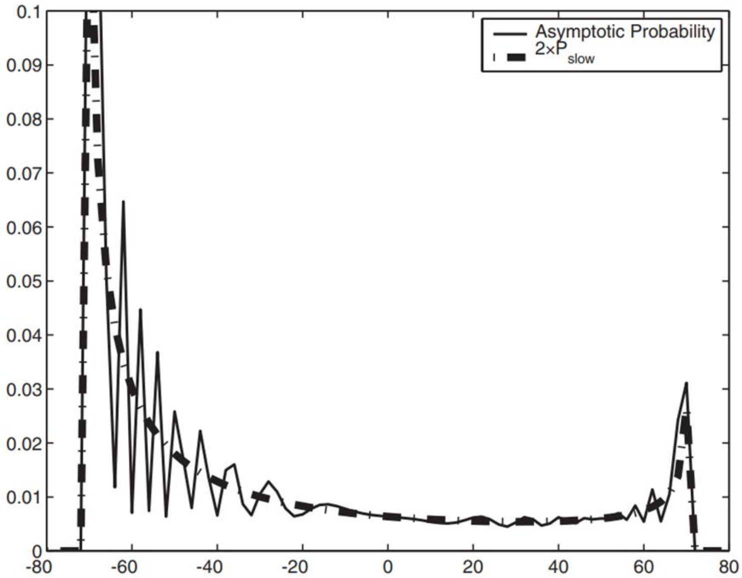


Fig. 3.21: The probability distribution of the quantum random walk with Hadamard coin starting in $|\downarrow\rangle \otimes |0\rangle$ after T = 100 steps (Nayak & Vishwamath, 2000).

To obtain a symmetric distribution, it is necessary to start the walk with a superposition of $|\uparrow\rangle$ and $|\downarrow\rangle$, $|\Phi_{Syn}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + i|\downarrow\rangle) \otimes |0\rangle$. Table 3.5 shows the symmetric distribution that is obtained starting from $|\phi_{sym}\rangle$.

Table 3.5: Probability of being at position X after N steps for a quantum Galton board with the symmetric initial state: $|\phi_{sym}\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + i|\downarrow\rangle) \otimes |0\rangle$

	-5	-4	-3	-2	-1	0	1	2	3	4	5
0						1					
1					$1/2$		$1/2$				
2				$1/4$		$1/2$		$1/4$			
3			$1/8$		$3/8$		$3/8$		$1/8$		

4		1/16		3/8		1/8		3/8		1/16	
5	1/32		11/32		4/32		4/32		11/32		1/32

Otherwise, to eliminate asymmetry of the walk, it is also possible to a different (balanced) coin, for example

$$Y = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ i & 1 \end{pmatrix}$$

In figure 3.22, the probability distribution of a symmetric quantum walk is reported. This pattern is very intricate, and, in Kempes' words (2003), it is "a signature of the quantum world" (p. 312).

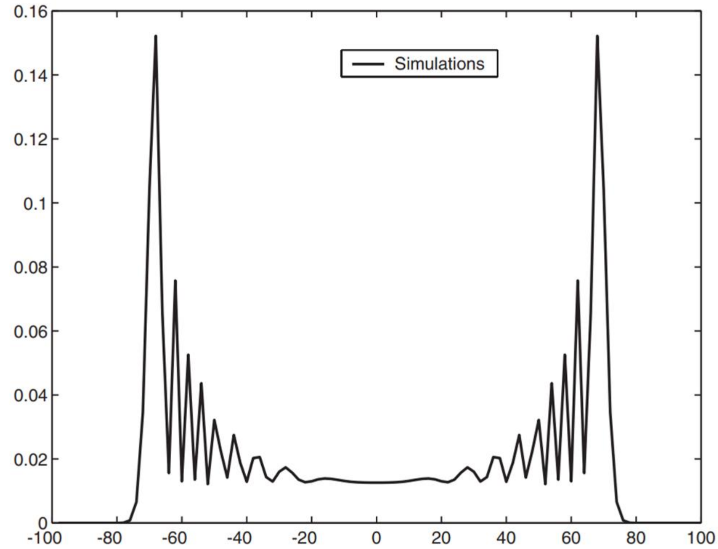


Fig. 3.22: The probability distribution obtained from a computer simulation of the Hadamard walk with a symmetric initial condition. The number of steps in the walk was taken to 100. Only the probability at the even points is plotted since the odd points have a probability of zero. (Nayak & Vishwanath, 2000).

The moments of this walk it is hard to be analyzed precisely since the multitude of oscillations. Ambainis, Bach, Nayak, Vishwanath & Watrous, 2001) obtained an asymptotic analysis of the variance of the quantum random walk by using two different approaches: combinatorial techniques and a more physical path-integral approach based on a Fourier analysis.

3.3.2 Design and implementation of teaching activity

In this section, we report the results of the second phase of educational reconstruction: the transformation of the physical contents, described in the previous section, into an activity that is culturally and socially relevant, approachable, and inclusive for secondary school students.
To reach this goal, the contents have been reconstructed by implementing our four design principles (see Chapter 2). As the teleportation one, the random walk activity is particularly indicative of the design principles carried out to value the Second Quantum Revolution from a cultural perspective (Section 2.3). This activity was designed within the IDENTITIES project; therefore, we implemented also the principle carried out within the project. Design principle #1 (to foster a close comparison between classical and quantum computers through an analysis of *the different logic* underlined in the physics of their hardware) has been applied by building and comparing the classical and quantum models of the random walk both from mathematical, logical, and computational perspectives.
The random walk is a model that is particularly used, for example, in research algorithms such as the Grover algorithm which has experimental implementations, especially in the field of quantum optics

(e.g. Dodd, Ralph & Milburn, 2003; Ermakox & Fung, 2002), but these physical implementations are rather advanced experiments that could be meaningful if there is the possibility of going into a laboratory. Design principle #3 is implemented by choosing only the discrete case in both cases.
Design principle #4, to make the model as inclusive as possible, is implemented through the lens of the IDENTITIES project, which is of inter-disciplinarity (design principles #6a and #6b). Finally, in order to widen the perspective, we introduce some application fields of classical and quantum random walks.

The kind of activity we designed is very interactive and it is structured as the following:

- teamwork activity about the drunkard's walk (Pearson, 1905) and discussion;
- introduction to the model of the random walk to scaffold the comparison between the classical and the quantum cases;
- the exploration of the classical and quantum random walk through an interdisciplinary lens (mathematic, physics, computer science);
- introduction of some application.

We start from the teamwork activity and the problem we pose is:
"Charlie, after a long evening of vices and extravagances outside the city, returns, a little staggering, to the city of Eve. As soon as he crosses the city gates, a problem arises: Charlie no longer remembers where he lives or the way back. He then begins to walk between the blocks, proceeding randomly and never going back, hoping to find the right way.
What is the probability that Charlie will reach his house (green square) at random?
Is the probability that Charlie will reach, at random, his friend Bob's house (yellow square) the same?
How would you model the problem?

Suppose a mathematician, an experimental physicist, and a computer scientist are in a cafe and discuss the problem just presented. In modeling the problem, the topics that most of all arouse discussion among the three experts concern: what does it mean to model the "randomly proceed" for a physicist, a mathematician, and a computer scientist; what is the best method to derive the information on the final probability (measure, calculate and compute). The three experts have in fact different tools and knowledge that can be used to solve it: the mathematician looks for rules or models to formalize the problem, the physicist looks for examples/phenomenological contexts on which to rely to design measures, and the computer scientist searches for input and output data/variables and process strategies (algorithms) to make the computer solve the problem in the fastest/most effective/elegant way. If you put yourself in the shoes of one of the experts (or more than one):
What tools and forms of reasoning would you use to model the problem and support your position in the discussion?
How would you reformulate the problem from your perspective (mathematics, physics, and computer science) to use "your" conceptual, formal, and methodological tools? How would you solve it?"

We attach to the problem figure 3.23.

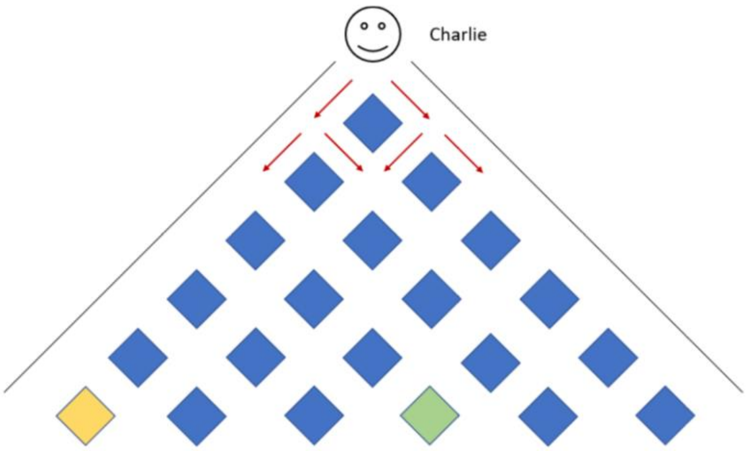


Fig. 3.23: problem's representation to guide students to solve the problem.

We ask therefore students not only to solve the drunkard's walk problem but also to "put themselves in the shoes of mathematicians, physicists or computer scientists" and try to think about what kind of knowledge, what tools, practices, methods, aims and values, etc., they can put into play.

After a collective discussion about their results, we open to the quantum model asking: what can happen if Charlie follows the law of quantum mechanics?
We then engage students in thinking about how to model the "proceeding randomly" of Charlie and its way of moving "never going back".
We then model the city of Charly like in figure 3.24, the "proceeding randomly" can be simulated with a coin and the way of moving by setting a shift operator. The random model can be simulated by, step by step, flipping the coin and applying the shift operator.

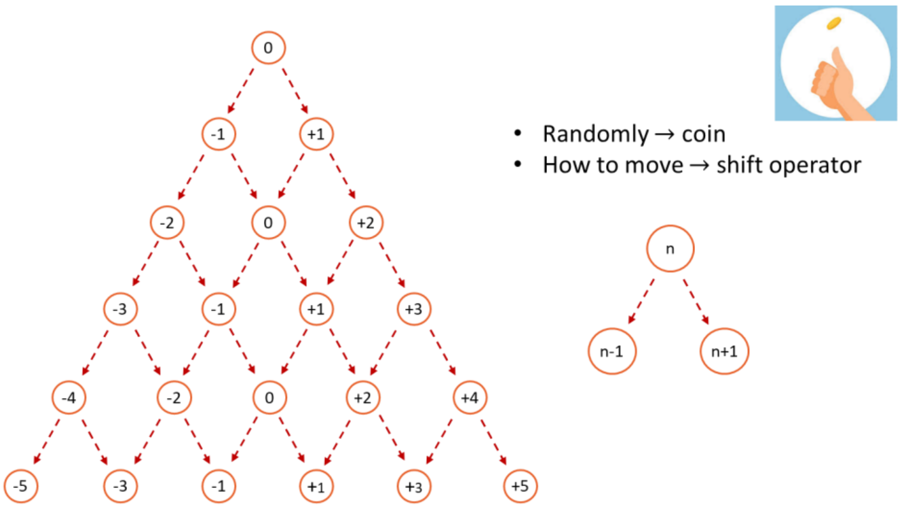


Fig. 3.24: the random walk model.

We start to scaffold the comparison between classical and quantum random walks by stressing the different logic of the coin (Fig. 3.25). As quantum coin, we consider the Hadamard logic gate.

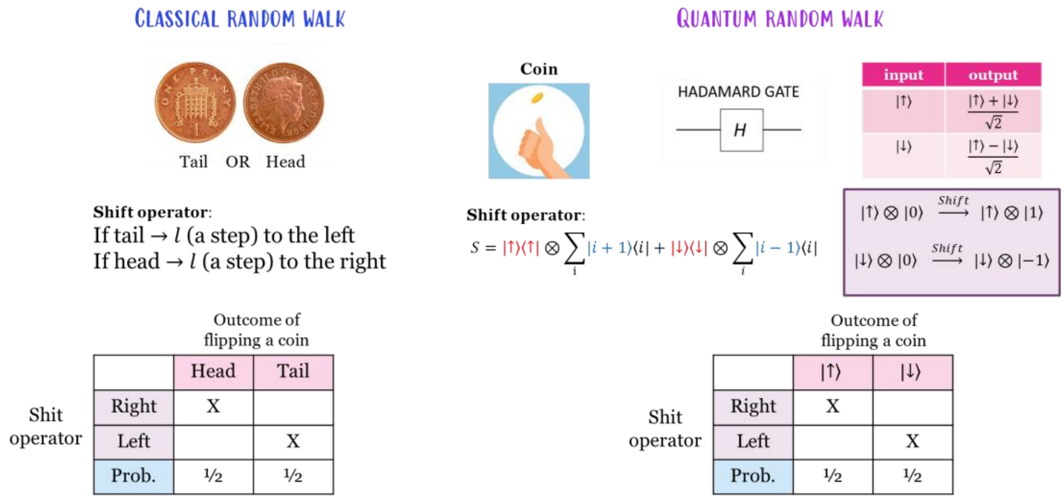


Fig.3.25: comparison between the classical and the quantum model.

While the shift operator in the classical and quantum case behaves in the same way, the difference lies in the coin: in the classic case, if I flip the coin I get either head or cross, in the quantum case, the coin transforms the state into a superposition state.

We pass to the mathematical and formal dimensions by engaging students in the step-by-step calculation (see section 3.2.2, starting from the state $|\uparrow\rangle \otimes |0\rangle$) and then comparing the different probabilities of the drunkard's walk problem (Fig.3.26).

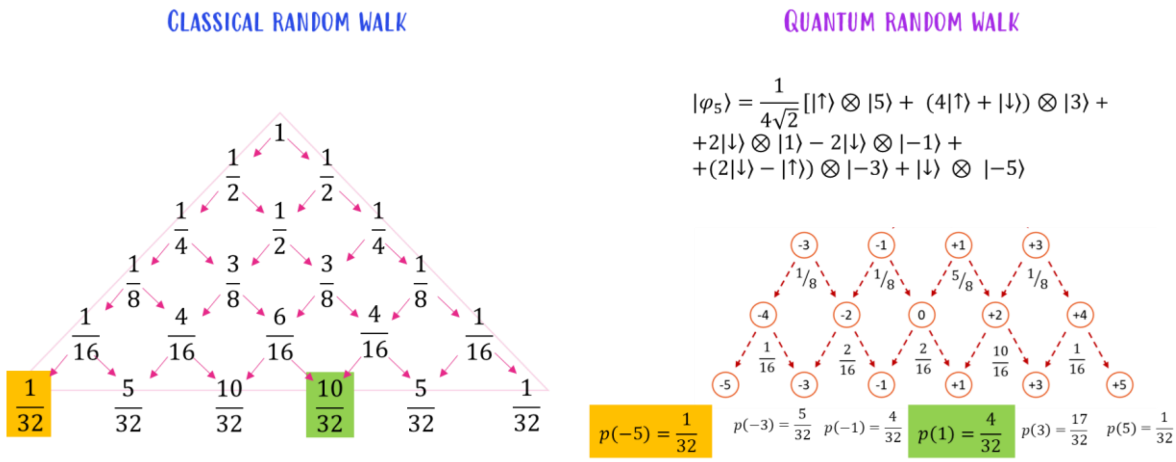


Fig. 3.26: comparison between classical and quantum random walk in terms of probability.

We then compare the model from the physical perspective (Fig. 3.27) by, for the classical case, recalling the Galton board and discussing what it means to build it, how experimentally you can get that distribution, what are the conditions to get that distribution, etc.

For the quantum case, we show a small part of an instructional video developed by Sapienza, University of Rome.

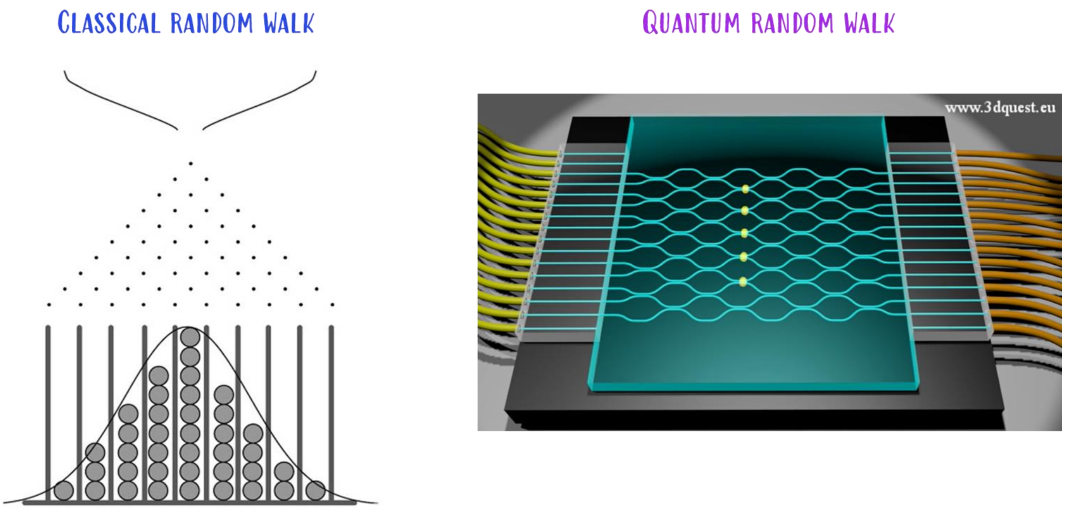


Fig. 3.27: Comparison from a physics perspective.

The experiment that the video reproduces is a boson sampling experiment, which is an approach to quantum artificial intelligence. It is a quite complicated experiment, so we use the video only to carefully visualize the idea of sampling the information space and to stress the scope, in this case, of the superposition principle, and of the possibilities to isolate and manipulate a single quantum object. We finally compare the classical and the quantum random walk from a computational perspective (Fig. 3.28).

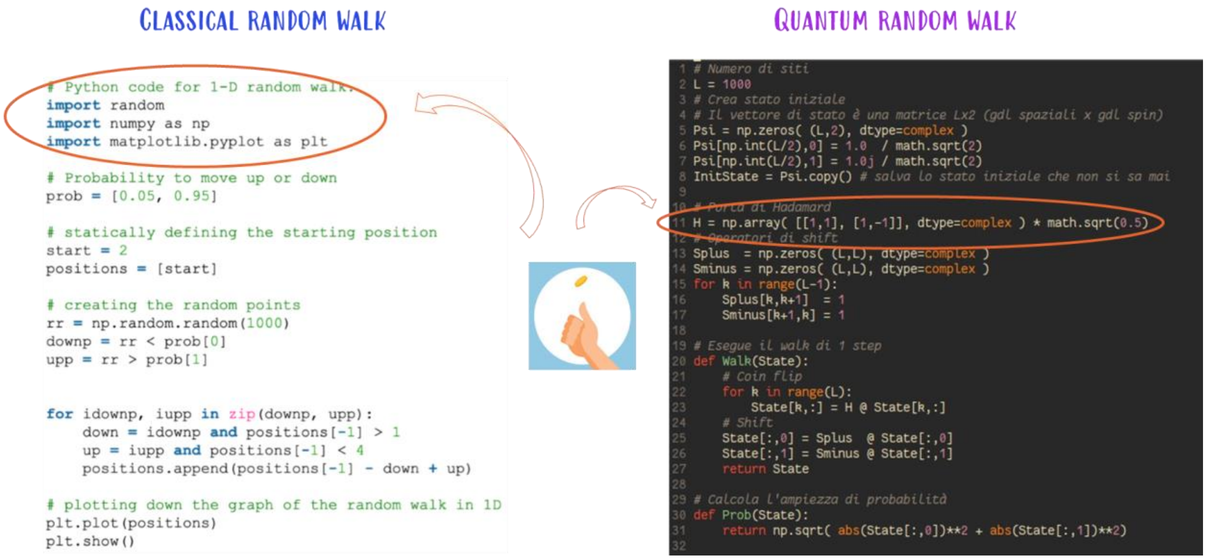


Fig. 3.28: Comparison from a computational perspective

In the classical case, we introduce students to the problem of random number generation and to the impossibility of generating "truly" random numbers with classical computers. In the quantum case, instead, the randomness is intrinsic in the nature of the coin, and we recall the Stern and Gerlach experiment as a true random numbers generator. This allows us to pave the way to introduce an important philosophical and epistemological debate. In the classical case, chance does not have an ontological status, so we talk about epistemic probability and apparent randomness. In the quantum case, chance has an ontological status, and we talk about ontological probability, and intrinsic randomness (Bera, Acín, Kuś, Mitchell & Lewenstein, 2017). These aspects allow us to touch other dimensions of inter-disciplinary authenticity (see section 2.1).

Finally, we show students a program in python elaborated by a colleague, Sunny Pradhan. The program simultaneously implements a classic and quantum random walk, showing on the same screen the evolution of the distribution step by step.

We stress in particular the following aspects:

- the quantum random walk "samples" the "position space" of the particle faster than the classic case;;
- The particle in the quantum case can take much more positions than the classical one.

In this activity, the concepts of probability, probability distribution, and the problem of generating causal numbers are our "boundary objects". They are analyzed from various perspectives and correlated in the comparison between classical and quantum.

We conclude the activity by showing some application fields of the random walk algorithm (Casali, 2020), such as research algorithms (e.g., Shenvi, Kempe, Whaley, 2003), decision-making algorithms (Shankar, 2012), optimization problems (e.g., Tonchev & Danev, 2021), econophysics (e.g., Orùs, Mugel, Lizaso, 2019) and art.

3.3.3 The analysis of the data

For the third phase of MER, we refer to the same implementation of the analysis carried out for the teleportation activity. The random walk activity was implemented for the first time in this implementation. The course was held in presence, just before the covid pandemic (February-March 2020) and involved 22 students. The present study refers to a rather small and non-representative sample of secondary school students since it involves volunteer students. Therefore, the results are not intended to provide general results and the goal is limited to collecting preliminary signals about the capability of the approach to reach this special target of secondary school students and engage them. Generalizability issues will be addressed in a further step of the research.
For the random walk case, we show and analyze two different kinds of data.
Firstly, we present how students solve the drunkard's problem and then the analysis of the questionnaire designed to investigate the impact of the approach on students and to collect feedback for the refinement of the activities.

The drunkard's problem

For this activity we divided students into four groups: two groups decided to solve the problem from a mathematical perspective, one from a physical perspective, and one from a computational perspective. The data we present is the audio transcription of the collective discussion.

In tables 3.6, 3.7, and 3.8, I report respectively the two mathematical solutions, the physical and computational ones.

Table 3.6: the mathematical solutions.

Group 1
So we first drew in the intersections... Every time he [Chiarlie] *walk a road, we wrote a little number like we had to draw the triangle of Tartaglia... because we thought in the first… In the first step he had to make* [first road to walk]*, there was a 50% chance he would go either one way or the other... so in the first point above we put 1... and one also in the other 2* [referred to the second level]*... then in turn there was another division with the same... with the same shape... so... ok* [referred to the teacher who is drawing on the blackboard]*... At the center, however, this probability had to increase because he had gone right or left there was... this intersection in this*

road… so we put 1 2 and 1... thinking it was a percentage ratio... and we went on like this until we got all the way... uh... with all the numbers we got the sequence 1 5 10 10 5 1... And so we thought that this should be the ratio of the percentages of... just falling in that street... so we made the total of the percentage that was 100% over the sum of those for the point that interested us... So 100 divided by 32 multiplicated by 10 that was… It came to our mind because we saw that already doing two steps you already lost the chance to get into the yellow one while... Three or four steps and you could still get to the green... it already seemed like a logical starting point that there was more chance of getting into the green one...
Group 2
We arrived at the same result but we reasoned in terms of permutations of moves... namely a move can be either right or left and... and the moves that mark the point above the green house are definitely three moves to the right and two moves to the left that are the invariant of... of the game... Next we then permuted the compound word consisting of the 3 letters of the move to the right and the 2 letters of the move to the left and the permutations were 5 factorial divided by 3 factorial multiplied by 2 factorial [5!/(3!*2!)] *that are permutations with repetition... which in fact is 10... then we calculated the total moves always... always using the triangle of Tartaglia... and we divided the permutations for the total moves and the same result...*

As we can see, starting from the same prompt and the same representation (Fig. 3.23) the two groups used different strategies to solve the problem. The two strategies are different in terms of approach to the problem, that is the kind of intuition that suggested the problem-solving strategy, the kinds of reasoning, and mathematical tools.

Group 1 immediately *models* the structure of the city with Tartaglia's triangle, in which every crossroad corresponds to a number of the triangle. They provide a justification of this translation based on the "*percentage ratio*" (approach to the problem, intuitive element). So, they associate the number "1" with the crossroad corresponding to the 50% probability that Charlie "*falls*" there. But from the third level, "*this probability had to increase because he had gone right or left there was... this intersection in this road… so we put 1 2 and 1.*", considering the fact that Charlie can randomly arrive at that crossroads from two different streets. The model of the city structure into the Tartaglia triangle made them associate the two possible streets that Charlie can walk to reach that intersection with a sum of the two numbers "1+1". Reaching the level of green and yellow houses, they "*made the total of the percentage that was 100% over the sum of those for the point that interested us… So 100 divided by 32 multiplicated by 10*". So, the Tartaglia triangle together with the formula to calculate the frequentist probability are the mathematical tools they used. Finally, they check, like a sense-making process, the plausibility of their reasoning, it fits with their intuition: "*It came to our mind because we saw that already doing two steps you already lost the chance to get into the yellow one while…[...] it already seemed like a logical starting point"*.

Group 2 used a different strategy; they reasoned in terms of "*permutation of the moves*". This is translated into noting that "*the moves that mark the point above the green house are definitely three moves to the right and two moves to the left that are the invariant*". So they looked for the invariant of "*the game*" (approach to the problem, intuitive element). Individuated it is about "*permutations with repetition*", they applied the formula that they know (mathematical tools) and "*permuted the compound word consisting of the 3 letters of the move to the right and the 2 letters of the move to the left and the permutations were 5 factorial divided by 3 factorial multiplied by 2 factorial* [5!/(3!*2!)]". The final part of the reasoning is similar since as group 1 they used Tartaglia triangle to calculate the "total moves" and they divided "*permutations for the total moves*".

These examples show that this problem is not only approachable to students but that they can also provide different and very rich answers. The same text of the problem and the representation enabled

them to address this problem from different approaches. Group 1 used "*percentage ratio*" and probability while Group 2 in terms of "*permutation of the moves*", looking for the invariant.
While group 1 started from a similarity between the virtual "discrete triangle", interpreting it as a particular kind of tree, and the Tartaglia's triangle, group 2 focused their attention on the moves, choosing indeed another kind of mathematization of the problem. In both cases, they searched for other representations and reformulated the problem until it could be recognized as an already known mathematical problem, in order to exploit their knowledge about properties, techniques and theorems related to such a mathematical setting. After identifying a possible mathematical model, they went on solving the new problem they had formulated. Probability in both cases was interpreted as classical probability, that means in terms of the ratio between a generic number of desired cases and the total number of cases.

The two groups showed a certain knowledge of the practices and methods that characterize mathematics. For example, both groups applied readout strategies to understand the important elements of the problem allowed them to treat it mathematically and then model it. They positioned themselves within a certain mathematical setting that allowed them to use certain tools, techniques, and knowledge rather than others.

Table 3.7: the physical solution.

Group 3
We thought of mapping a model like this [pointing to the triangle on the board] *... ehm... put it vertically and send balls simply from above... under [the device] place a... I don't know... columns let's say... under the points... that is... along the entire line... obviously send as many balls as possible to have a percentage as high as possible... ehm... the most accurate...* [Teacher: why do you think that happens?]*... the balls will distribute on a Gauss curve and then* [the teacher draws it]*... the probability of getting to the green house is much much greater than getting to the yellow house...* [If we throw 32 balls you expect the distribution to be 1 5 10 10 5 1? If we repeat it 3 times we get the same distribution] *No no! Because anyway there is a chance that... that is, there is still uncertainty...* [teacher what gives you uncertainty?]*... at every intersection... There's still a 50% chance of choosing one road over another…*

Group 3 chose to approach the problem from a physical perspective. They really do not solve formally the problem, but they focus on how physically design the experiment. They propose a Galton board, so they imagine figure 3.23 as a device, they "*put it vertically and send balls simply from above*". They collect the data by placing "*columns [...] under the points... that is... along the entire line*". To make the experiment as "*accurate*" as possible they send "*send as many balls as possible*". In addition to designing an experiment, the data collection, and the problem of choosing the best condition to make an experiment as accurate as possible, thanks to the teacher, other important aspects that characterize physics come out, such as the prediction (*"the balls will distribute on a Gauss curve and then... the probability of getting to the green house is much much greater than getting to the yellow house*), the problem of reproducibility of a result and the role of statistic ("[If we repeat it 3 times we get the same distribution] *No no! Because anyway there is a chance that... that is, there is still uncertainty…*").
Even if they could answer some questions like what physically "gives" the uncertainty, they showed to be familiar with the typical scientific practices, aims and values, and methods of physics.

Table 3.8: the computer science solution

Group 4
"*We make a program... that has 7 choices to be made in a random way... that is... if it comes out 1*

it goes right if it comes out 2 goes left... then in all are 7 because there are 7 possibilities and nothing... We start the program from the top and then we make a choice and then another choice... We make it go n times and then the program calculates the probability that it has arrived at the yellow house or the green house… [So you…] *We start from the top point and decide that if it goes out for example 1 goes right and if it goes out 2 goes left. The choice is random.*
Let's start from the top and make the choice... Then again, in an iterative way we make a choice again... And we run it 100, 1000 times and then the program calculates the probability that it arrived on the yellow house or green house… To generate random numbers I use the rand function…"

Group 4 chooses a computational approach so they "*make a program*". They then elaborate on what writing a program means. Firstly, they have to model the "randomly walking" of the drunkard so they decide that "*if it comes out 1 it goes right if it comes out 2 goes left*". As they said at the end of the explanation of their reasoning, this means generating random numbers so they "*use the rand function…*". They make the program run "*from the top and then we make a choice and then another choice [...] in an iterative way"*, for "*100, 1000 times*" and then "*the program calculates the probability*". The explanation they provide embodies different aspects of computer science. For example, the search for sequentiality of the algorithm is described as a series of instructions (if choice, cycle, final calculation), the "simplicity" and transparency in the sense of looking for essentiality and clarity of the process of the resolution (the strategy); the optimization and efficiency by simply calculating probability through the formulas and using a function that is already part of the knowledge such the rand function.

The four answers provided by the students are very interesting and students proved to be familiar with some practices, methods, aims and values, and knowledge that characterize the disciplines.
The mathematical answers are very sophisticated in terms of mathematical tools they used, and the clarity and accuracy of the explanation of their reasoning. The other two answers are not entirely exhaustive. For example, the physics group does not explain how to build the Galton board, what is the physical interpretation of random, and there is no formalization of the final result. The computer science group highlights the strategy very clearly, but they do not take into account the data and their structure, for example, they do not discuss how they can identify the different positions of the drunkard during the journey, input, and output variables.
These ideas can be considered to rethink and/or add new activities to further enrich disciplines and build a discipline image richer in aims and values, practices, methods, and knowledge.

The Questionnaire

The questionnaire, which we proposed after the random walk activity, has a very broad aim to collect feedback about the whole module and approach. It is composed of both closed and open-ended questions.

The questionnaire aims to investigate:

i. what students grasp about the similarities and differences between classical and quantum computers, and the peculiarities of quantum computation;
ii. the main adjectives that students associate with quantum physics: understandable, effective, convincing, fascinating, easy, intuitive, powerful, evocative, explanatory, revolutionary, true, and stimulating (taken from Ravaioli, 2020)
iii. what was the level of knowledge of some concepts or themes (such as classical computer operation, classical computer history, classical logic gates, the superposition principle, quantum measurement, entanglement, quantum logic gates, teleportation, random walk, the role of probability in quantum physics, the distinction between deterministic and probabilistic

forecasting models) before and after the course and which aspects of the course have most stroke them;

iv. if and to what extent students find interesting and useful an integrated approach like the STEM one and why;

v. how they find in particular the approach to the random walk; if it helps them to reflect on the peculiarities of the reasoning of the different disciplines and what characteristics of the three different disciplines emerged from the discussions.

The questionnaire was given after the second implementation (February 2020). The response was purely voluntary. 13 out of 22 students replied (5 female, 8 male students). It was administered through a Google form, and we gave a few days to complete it. All the information for management, protection, and data processing were provided, both orally and in paper-based format[12].

The analysis we propose here refers to the questions about the interdisciplinary approach (iv., and v.) This section of the questionnaire is composed of the following questions:

A. To what extent do you find interesting an integrated STEM approach?
B. Which aspects do you have appreciated the most and which do not?
C. In the specific case of the random walk, did the approach help you to reflect on the peculiarities of the reasoning that characterizes the different disciplines (mathematics, physics, computer science)? If yes, what peculiarities?

As regards the first question (A), in general students have found the interdisciplinary approach interesting and useful, as can we see from the histogram in figure 3.29.

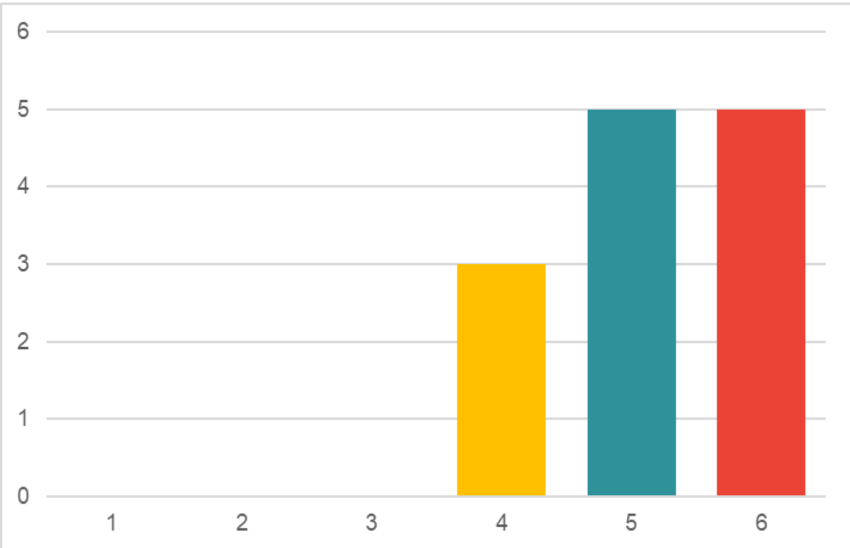


Fig: 3.29: Students' distribution regarding the question "To what extent do you find interesting an integrated STEM?" (On the x-axis it is reported the ranking, and on the y-axis, there is the number of students).

[12] In particular, it was specified that pursuant to article 13 of Regulation (EU) 2016/679 (General Regulation on the protection of personal data), the Alma Mater Studiorum— the University of Bologna, as Data Controller, will process personal data in compliance with the provisions of Regulation (EU) 2016/679 (General Regulation on the protection of personal data) and the Legislative Decree 30 June 2003, n. 196, as amended (Code regarding the protection of personal data). In particular, it was specified that, pursuant to art. 13 of Regulation (EU) 2016/679 (General Regulation on the protection of personal data), the Alma Mater Studiorum - University of Bologna, as Data Controller, will process personal data in compliance with the provisions of Regulation (EU) 2016/679 (General Regulation on the protection of personal data) and the Legislative Decree 30 June 2003, n. 196, as amended (Code regarding the protection of personal data).

As regards question B, generally, all the 13 students highlight that an integrated STEM approach can help "*to better understand*". The answers differ mainly for the focus of the argumentation.
For example, for three students a better understanding is due to see the same topics from different perspectives. Key expressions that suggest this are: *"under different light*", "*in many different areas*", "*understand it from different points of view*", and "*through the tools of different disciplines"*. In students' words:

> "*The fact that although we talk about the same thing, it can be treated in so many different areas*"

> "*Surely it was useful to see the same topics under different lights and, here as ever, it served to see this topic under different spotlights to deal with it always in different ways and understand it from different points of view. The thing has acquired more value both from the point of view of understanding (for us) and from the practical point of view since it has made us better understand the topic of which it was spoken and practically like to apply the object or the reflection of which it spoke.*"

> "*It seems interesting to me how the same phenomenon can be studied through the tools of different disciplines, finding the most effective approach and facilitating its understanding*."

Other students stressed the fact that an integrated STEM approach can provide a better understanding because it provides a more comprehensive perspective. Two students, in particular, wrote:

> "*The comprehensive and complete vision that is provided.*"

> "*It helps to better understand the whole*"

While the first two students focus on the different disciplines that provide different perspectives and the second two students on knowledge as a whole, other students (six) focus on the processes of interaction: the different disciplines dialogue with each other, and it is the dialogue and the interaction between them that promote a better understanding. Key expressions are: "*integrating the most effective parts of them", "disciplines that are connected and [...] that sometimes complement or help to understand the other", "It's from mergers and integration", "requires an integration of various knowledge"*. In students' words:

> "*The fusion of subjects with the aim of achieving a better understanding of reality, integrating the most effective parts of them*"

> "*I find it very interesting to deal with topics that range between different disciplines that are connected and that sometimes complement or help to understand the other* "

> "*The links between the various disciplines are interesting*"

> "*Being a complex topic, it requires an integration of various knowledge to understand it, making the process more stimulating*."

> "*It opens your mind! In the sense that one discipline can explain something that you don't see in another and vice versa. This way you have a global view of a phenomenon and you can understand it much better. It's nice to know that you can look for the answer to a question elsewhere and that the answer is still valid.*" (S1)

> "*Specialization is important, especially in this historical moment in which we need in-depth knowledge, yet I believe we can not lose the overview: in many fields, the disciplines themselves completely lose meaning. It's from mergers and integration that genius and innovation are born.*" (S2)

The last two students' answers shed light on other two important aspects. The penultimate (S1), in contrast to the other answers, try to highlight the integration mechanism. For S1 an interdisciplinary approach can "*open the mind*". Opening the mind for the student does not only implies the building of

a "*global view*" but the fact that a discipline "*can explain something that you don't see in another* [way]" and that "*you can look for the answer to a question elsewhere and that the answer is still valid*". S1 therefore stressed the role that a discipline can have in another.
The latter student (S2) argues that with a non-integrated approach "*the disciplines themselves completely lose meaning*", finding instead the "*genius and innovation*" precisely in the integration of disciplines.

Finally, two answers have a bigger focus on the people, arguing that the aspect that struck them more is the interaction ("*bringing together*", "*involving*") between different single and specialized "*minds*" into "*macro topic*", "*great problems*". In students' words

> "*I find interesting the possibility of bringing together more specialized minds under a single macro-topic*"
>
> "*Involving little minds in such great problems*."

As regards question C, the answers of the students provide a quite stereotyped vision of discipline. The question was put in a way that has led students to focus only on the disciplines' peculiarities. Ten students out of 13 strictly demarcate mathematics, physics, and computer science in terms of practices, languages, tools, the focus of the research, methods, and so on.
So physics is associated with experiments, so it is based on the "*observation of reality*", it is "*empirical*", it "*solves problems in a practical way*" "*thanks to experiments and measures*" and "*it is concrete and practical*". Mathematics creates models and uses "*formal and truthful language*", it focuses "*exclusively on the theoretical aspects*", it is "*based more on logic*", and it solves "*problems in a formal way*". Finally, computer science that creates "*efficient algorithms*" has the focus "*on mechanisms*", it implements "l*ogical structure*" to "*solve a problem*".

While the vision of mathematics and physics is quite shared among the students, the vision of computer science is not so shared. For example, a student, in particular, writes that computer science is theoretical as mathematics, for another "*computer science schematizes and combines concrete and abstract*". In students' words:

"*Mathematics/theoretical physics: creating mathematical models* *Physics: going back to experiments* *Computer science: creating efficient algorithms*"
"*Mathematics: formal and truthful language* *Physics: observation approach to reality* *Computer science: implementation and optimization mechanisms for law analysis*"
"*The way of seeing things is different. A mathematician is based more on logic, a physicist on reality, and a computer scientist on mechanisms.*"
"*The physical approach turns out to be the most empirical, while the mathematical and computer science focus exclusively on the theoretical aspects*"
"*I realized that mathematics is the most complex discipline among the three but also, probably, the most exact*".
"*The mathematical approach allows solving problems in a formal way, using formulas and other tools typical of the discipline, through the physical approach solves the problem in a practical way, thanks to experiments and measures, the computer science approach uses algorithms to get a computer to solve problems*."
"*This kind of approach helped me to have a comparison of the various disciplines on the same topic and then to complete it in all respects. Mathematics is based on the processing of numbers and certain procedures, physics on measurements and experiments, computer science on logical procedures*"

"*Physics is concrete and practical, mathematics is formal, computer science schematizes and combines concrete and abstract*"
"*mathematicians deal with problems through formulas, theorems, calculations; experimental physicists through practical experiments and measurements; computer scientists through programs*."

Only two students answer differently. One writes

> "*The three disciplines complement each other because mathematics provides formulas that are applied and exploited by physics to formulate laws that in turn are transported into a computer science dimension where they are easily verified*".

There is an asymmetry between the three disciplines since the answer suggests a centrality of physics. Anyway, the student provides, even if not particularly articulate, a connection between the different disciplines. Finally, one student writes

> "*All the subjects involved require the use of intuition or reasoning*"

focusing more on family resemblance aspects such as the intuition and ways of reasoning rather than the demarcating one.

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Conclusions Part I

This first part of the thesis represents our answer to the challenges launched by different educational programs to keep the pace with technological changes by developing courses and materials that can promote quantum literacy.

I presented the educational approach we elaborated to value the Second Quantum Revolution as, first of all, a cultural revolution by building and providing new lenses to understand and interpret it.

After unpacking the Second Quantum Revolution from a cultural perspective, we pointed out seven design principles in order to contribute to the two broad research questions and to stress that it is not only a matter of calculus power and political and military strategies, but it conveys deep changes at the level of the foundations and inner logic that quantum computers follow as well as at the level of society we live in. The Second Quantum Revolution represents a possibility for students not only to learn physics but also to expand learning along many dimensions so as to be inclusive and diversity responsive. Dealing with the second quantum revolution sheds light on science in its making. It leaves room to reason on the inextricable interweaving between science-technology and society, and to reflect on the great dichotomies such as the relationship between science-technology, humans, and nature. It is a chance to think about interdisciplinarity, rethink the intertwining of disciplines and extend it to the arts.

The seven principles we pointed out touch on many dimensions and provide insights into the inter-disciplinary authenticity of the topic.

Starting from them and the work carried out by the Bologna research group in physics education about the First Quantum Revolution, we developed a course that was implemented both with secondary school students and pre- and in-service teachers. The activities of the course were implemented and refined following the "Model of Educational Reconstruction" (Duit, Gropengießer, Kattmann, Komorek & Parchmann, 2012; Duit, 2007; Duit, Komorek & Wilbers, 1997). In particular, the teleportation and the random walk activities were described as the result of the MER process and, consistently, as emerging from three analytical and design phases: analysis and clarifications of science content, content reconstruction in light of the research on teaching and learning quantum physics, with emphasis on students' perspectives. A pilot study was conducted and analyzed to investigate the impact of the approach on students' understanding and to gather feedback and reactions to refine and improve the instructional materials.

The results of the pilot study showed that the activity was not only within the reach of the specific target of secondary school students but also very engaging.

As regards the teleportation activity (Satanassi, Ercolessi & Levrini, 2022), two basic choices were revealed to be particularly fruitful for fostering the understanding of the phenomenon and the essence of quantum technologies: a) the choice to base the module on the comparison between experiments and circuits and, hence, to base the activity on the reconceptualization of the teleportation experiment as a computational device (design principle #2); b) the choice to articulate the discourse on the narrative, logical, technical–experimental and mathematical levels (design principle #4).

Regarding the comparison, the students found the circuit and experiment to be two different but "*complementary*" perspectives. The two "versions" of the quantum protocol have effectively leveraged different aspects of the phenomenon and provided two different images, both needed to build a comprehensive view: an image spatially organized in terms of experimental equipment and, hence, in terms of experimental operations; and an image logically organized in terms of logic gates and, hence, in terms of states manipulation.

As for the four-level discourse, the students recognized the structural articulation along different levels and said the recognition was fruitful and effective in allowing them to move back and forth between the global picture and specific details. More specifically, the data analysis confirms that the narrative level served to promote the formation of a comprehensive view in which the problem can be framed and addressed, whilst the logical level acted as the *backbone* of the discourse. The third discourse level, technical–experimental, restored a sense of concreteness and feasibility, whilst the mathematical level guided the students to understand and follow the calculation details that created the logical steps.

The most interesting result concerns students' acknowledgment both of the role of logic and the multi-level discourse structure. This means that the logical structure was indeed crucial in order to emphasize the single elements of reasoning, connect them, and bridge the experiment with the circuit. In particular, the logical level showed its potential to exploit the diversity of the students and resonate with different personal ways of understanding. In fact, we were able to recognize the signal, from a couple of students, that they felt encouraged to use and combine various registers to elaborate personal descriptions of the phenomenon. This signal, although emerging from a small sample, is consistent with the research that we have carried out on other topics such as thermodynamics (Levrini, Fantini, Pecori & Tasquier, 2014), and quantum physics (Ravaioli, 2020; Levrini & Fantini, 2013).

As regards the random walk activity and, more generally, the choice to focus explicitly on interdisciplinarity, according to the IDENTITIES project model (section 2.4.2), has proved to be effective and in general, the students appreciated the approach both in terms of personal understanding and relevance in relation to the society we live in.

In discussing disciplinary aspects, it emerged that students often have quite stereotyped views of discipline. This may derive from the image of disciplines that emerges from the school and sheds light on the need of enriching this image. In general, talking about physics, mathematics, computer science, engineering, etc., in a STEM context, in other words a "disciplinarization" of STEM, can lead to forcing the construction of boundaries. It is, therefore, important to pay proper attention to the epistemology of different disciplines and their integration (Selbach-Allen & Pimentel, 2020; Ortiz-Revilla, Adúriz-Bravo & Greca, 2020; Quinn, Reid & Gardner, 2020).

It is of interest to continue work on this aspect and enrich the image of disciplines as well as create more explicit and robust connections to foster the formation of an epistemologically more significant image of interdisciplinarity.

To conclude, the results show that the approach is significant and inclusive. However, we are going to carry out further implementations to deeply investigate the meaningfulness and robustness of the approach with a larger sample of students as well as with other target groups. (Satanassi, Ercolessi, & Levrini, 2022).

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Part II

Chapter 4: Exploring quantum technologies as a new context to introduce the basic ideas of Quantum Physics: The Role of representations in Rimini's implementation

Introduction

In the present chapter is described the empirical study that we carried out during the implementation of the course on the Second Quantum Revolution at the Liceo A. Einstein in Rimini.
The study aimed to investigate if and how the developed approach to the Second Quantum Revolution could promote students' acceptance of quantum physics. This problem was already addressed by the PER group of University of Bologna (Ravaioli 2020, Ravaioli & Levrini, 2017, Levrini & Fantini, 2013) and their study led the researchers to point out three epistemic needs that quantum physics theory does not usually satisfy. We re-opened this research thread since throughout the different implementations we had a sensitizing perception that students could more easily accept the "strangeness" of quantum physics. We decided to investigate this perception focusing on one of the newer aspects that characterize the approach to the second quantum revolution compared to the previous one, that is the representations. We therefore explicitly linked the problem of acceptance with the kind of representations presented to students. In particular, we elaborated a conjecture (section 4.2) about the circuital and logical representations as the ones that could satisfy the three epistemic needs and then promote the acceptance of the theory.
While the first part of the thesis (chapter 1,2,3) on the thesis represents our contribution to the research questions

> *RQ1. Which teaching approach can be developed to underlie the educational potential of technologies even beyond technical training?*
> *RQ2. What instructional design principles can be pointed out to value the Second Quantum Revolution as a cultural revolution?*

this second part focuses on another research question. Starting from the assumption introduced before, we formulated the research question

> *RQ3. Do quantum computation and information provide a new coherent scenario that can promote the understanding of the essential quantum concepts? Does this comprehensive scenario promote the acceptance of quantum physics? How and to what extent?*

To test the conjecture and investigate RQ3, we decided to focus on external representations and meta-representational competencies (diSessa, 2004; diSessa & Sherin, 2000). We, therefore, designed and developed an experiment articulated in three phases. The first phase consisted of a focus group about the role and effectiveness of the different representations in students' understanding and involved 27 students. The second phase consisted of a focus group with 8 selected students to investigate the dynamic of students' understanding through representations and if, how, and why the presented representations could satisfy their cognitive and epistemic needs. Finally, the third phase consisted of a collective interview of three students about the interdisciplinary experience, the Quantum Atelier, that they did during which they produced, three exhibitions, three personal representations of some revolutionary concepts.

After a literature review about the role of representations and multiple representations in science education (section 4.1), the conjecture and the link with the problem of non-acceptance is discussed (section 4.2). In Section 4.3, the context, and the refined version of the course are presented. The section 4.4 is dedicated to the design of the teamwork (section 4.4.1) and the focus group (section 4.4.2), and to the introduction of the Quantum Atelier experience (4.4.3).
Chapter 5 is dedicated to the analysis of the data collected during the implementation.

4.1 The role of representations in science education

There are many research works in the field of science education and educational psychology that have focused on the role and effectiveness of external representations in learning and teaching sciences. The purposes are among the most diverse: from modeling learning processes through multiple representations (e.g. The Cognitive Theory of Multimedia Learning, CTML: Mayer, 2005; Mayer 2009; The Integrated Model of Text and Picture Comprehension, ITPC: Schnotz 2005, Schnotz 2014; Ullrich, Schnotz, Horz, McElvany, Schroeder & Baumert, 2012; The DeFT, Design, Functions, Tasks, Framework for Learning with Multiple External Representations; Ainsworth, 2014; Tsui and Treagust 2013), to the assessment of their impact on students' understanding, from their importance as a tool for constructing knowledge (e.g. Treagust, Chittleborough & Mamiala 2002) or as a supporting tool of a flexible understanding of scientific phenomena (e.g. Kozma, 2003) to their value for communicating and integrating scientific concepts (e.g. Peña & Quílez, 2010; Mathewson, 1999); from the role that representations, as epistemic resources or epistemic forms, can play in problem-solving and epistemic games (e.g. Tuminaro & Redish, 2007; Kohl & Finkelstein, 2006a,b; Kohl & Finkelstein, 2005; Bodner & Domin, 2000; Zhang, 1997; Dufresne, Gerace & Leonard, 1997; Cox & Brna, 1995, Collins & Ferguson, 1993) to the exploration of students' abilities to criticize, explain and create new representations (e.g. diSessa, 2002, diSessa & Sherin, 2000, Wawro, Watson & Christensen, 2020), etc. Furthermore, these studies belong both to physics education and to mathematics, astronomy, biology, chemistry education, and so on.
Across the different research works, it is not easy to find a shared definition of what is meant by external representations. Commonly, many studies show that representations deal with the cognitive dimension and distinguish between external and internal representations and the relationship between the two. Zhang (1997) defined external representations as "the knowledge and structure in the environment, as physical symbols, objects, or dimensions (e.g., written symbols, beads of abacuses, dimensions of a graph, etc.), and as external rules, constraints, or relations embedded in physical configurations (e.g., spatial relations of written digits, visual and spatial layouts of diagrams, physical constraints in abacuses, etc.). The information in external representations can be picked up, analyzed, and processed by perceptual systems alone, although the top-down participation of conceptual knowledge from internal representations can sometimes facilitate or inhibit the perceptual processes" (Zhang, 1997, p. 180).
The researcher defined, in contrast, internal representation as "knowledge and structure in memory, as propositions, productions, schemas, neural networks, or other forms. The information in internal representations has to be retrieved from memory by cognitive processes, although the cues in external representations can sometimes trigger the retrieval processes" (Zhang, 1997, p. 180). In order to better explain his ideas, he considers the multiplication of 735 by 278 using paper and pencil. The meanings of individual symbols (e.g., the numerical value of the arbitrary symbol "7" is seven), the addition and

multiplication tables, arithmetic procedures, etc., are the internal representations that have to be recovered from memory. Shapes and positions of the symbols, the spatial relations of partial products, etc., are the external representations, "which can be perceptually inspected from the environment" (Zhang, 1997, p. 180, Zhang & Norman, 1995).

More generally, in other works, such as Belenky and Schalk (2014), they call external representation figures, formulas, graphs, manipulatives, pictures, tables, texts, simulations, etc., stressing that, since these are different kinds of representations, these are characterized by particular and different properties (Belenky & Schalk, 2014).

Considering the knowledge body of students as a collection of their internal knowledge representations about a topic/domain, in the field of cognitive psychology, the contents and structures of these representations aroused much interest (e.g., Langley, Laird & Rogers, 2009; Anderson, Bothell, Byrne, Douglass, Lebiere & Qin, 2004). It was found that it involved different components like procedural and conceptual knowledge which can be verbally explicated or not. Examples of internal knowledge representations can be rules, schemata, scripts, etc (Belenky & Schalk, 2014). Some studies, such as Markman and Dietrich (2000), investigate their role in learning and behavior. For them, internal knowledge representations can be conceived as *mediating states*, that is "internal states of a system that carry information which is used by the system in the furtherance of its goals" (Markman and Dietrich 2000, p. 140). In this perspective, the role of internal representations is affecting behavior. For example, to solve a new math word problem in a correct way, learners must be able to: interpret the problem, recall the mathematical concepts and formulas that are suitable for the problem, transform the information of the problem into the mathematical concepts and the appropriate aspects of the formula, and, finally, manipulate that knowledge to solve the problem in the correct way (Belenky & Schalk, 2014). All these steps happen in the mental faculties of a student, which we conceive of as a cognitive architecture consisting of different components or modules (Belenky & Schalk, 2014; Anderson et al. 2004) that intertwine and integrate information provided by the environment (external knowledge representation) and stored information (the internal knowledge representations). The internal knowledge representations are not static since they develop and change in response to new information from the environment in a dynamic way (Markman and Dietrich 2000) and instruction and instructors have the role of leading students towards the development of efficient and flexible internal knowledge representations (Belenky & Schalk, 2014).

The process of information extraction from external representations and information retrieval from internal representations has to occur in an intertwined, integrative, and dynamic way (Zhang, 1997). There are different processes that enact the transformation of external representations into internal ones, for example, memorization and internalization. However, these processes have limitations, for example, "internalization is not necessary if external representations are always available, and not possible if external representations are too complex" (ibidem, p. 181). The inverse process, that transforms internal representations into external representations, is externalization. The externalization can be beneficial if the use of external representations can offset the cost associated with the process.

As interesting as the relationship between internal and external representations is, the focus of this review is more on if and how different external representations and the use of multiple representations can impact and influence the construction of an internal representation in the field of science education research, that, in turn, affects students' learning. In light of this, the elements that interest us concern the advantages and the educational potential that the use of external representations and multiple representations can have on learning and if and what skills students can develop in dealing with representations.

As regards the first aspect, many studies have shown how and why the use of multiple representations in instructional materials fosters meaningful learning. Some of them also led to establishing theories, which are based on the assumption that working memory is presumed to be limited in the amount of information it can process at a given time (Baddeley 1992). The information, which can consist of multiple forms/modes of representation, can be processed through two different channels: verbal/auditory or visual/pictorial channels (Paivio, 1986). In the duo channel assumption, it is assumed that there is a limitation about the amount of information they can process at a time and in parallel. So, instead of overloading only one channel, it is recommended to make optimal use of both of them and design instructional materials that can activate both channels by using multiple representations (Opfermann, Schmeck & Fischer, 2017).

In particular, there are theories that are based on the multiple representations assumption: the Cognitive Theory of Multimedia Learning (CTML), the Integrated Model of Text and Picture Comprehension (ITPC), and the DeFT (Design, Functions, Tasks) Framework for Learning with Multiple External Representations. These theories provide principles that can be implemented in instructional materials and lectures. For example, CTML (Mayer 2005, Mayer 2009), to promote a deeper level of understanding and meaningful learning, proposes to use multimedia instructional materials. The theory focuses on the combination of textual and pictorial representations because, as the *multimedia principle* states, “Students learn better from words and pictures than from words alone” (Mayer, 2009, p. 223). This principle suggests that words and pictures are qualitatively different and embody different information. Furthermore, since the channels in which information is processed are different, students can learn different information contents and integrate them into one coherent mental model if the learning takes place in an optimal and effective way. Many are studies that proved the effectiveness of the multimedia principle (e.g., Mayer and Sims 1994; Plass and Jones 2005; Schwamborn, Thillmann, Opfermann, & Leutner, 2011). Mayer provides other principles in order to guarantee how the combinations of words, pictures, mathematical expressions, or other kinds of visualizations can support meaningful learning, addressing how multimedia materials should be presented and combined. For example, the *modality principle* states that, in using text and pictures together, the text should be spoken rather than written, because the risk is the overloading only of the visual channel. The *spatial contiguity principle* and the *temporal contiguity principle* state that when using multimedia learning materials, the different representations (e.g., the text and pictures) should be presented closely together. The *redundancy principle* argues that the use of identical written and spoken text at the same time is unnecessary and can even hinder learning. The same kind and amount of information is introduced and has to be processed twice at a time (Mayer, 2009). The *signaling principle* states that “people learn better when cues that highlight the organization of the essential material are added” (Mayer, 2005, p. 183). These coarse-grained principles have to be taken into account in order to promote meaningful learning.

The ITPC (Schnotz; 2005, Schnotz; 2014; Ullrich et al. 2012) is very similar to CTML, assuming the duo channel assumption, in this case, the channels are the auditive and the visual ones. The model encompasses different kinds of comprehension such as listening, reading, visual picture, and auditory picture (i.e., sound comprehension) comprehension. Within the model’s cognitive architecture, which consists of modality-specific sensory registers, working memory, and long-term memory, the author distinguishes between perception-bound processing of text surface or picture surface structures and cognitive processing of semantic deep structures. In contrast with the CTML model, which states that visual and verbal information firstly leads to the construction of visual and verbal mental models and then their integration to the construction of a coherent mental model, the ITPC assumes that the integration and building of a coherent mental model take place from the beginning, aligning, and matching from the start from the beginning of information processing. As CTML but in other words, the ITPC argues that verbal and pictorial information need to be simultaneously available in working memory (Horz and Schnotz 2008; p. 50). Only in this case, learners can recognize that the different representations are linked and can map them to their respective counterparts to make use of the information contained in both sources (Opfermann, Schmeck & Fischer, 2017). The combination of pictorial and verbal (or textual) elements can be problematic because, as Schnoltz (2002) states, they are based on different sign systems and use quite different principles of representation (Schnoltz, 2002, p. 108). Schnotz and Bannert (1999) proposed an integrative model of text and picture comprehension, usually task-oriented, that gives more emphasis to representational principles, represented in Fig. 4.1.

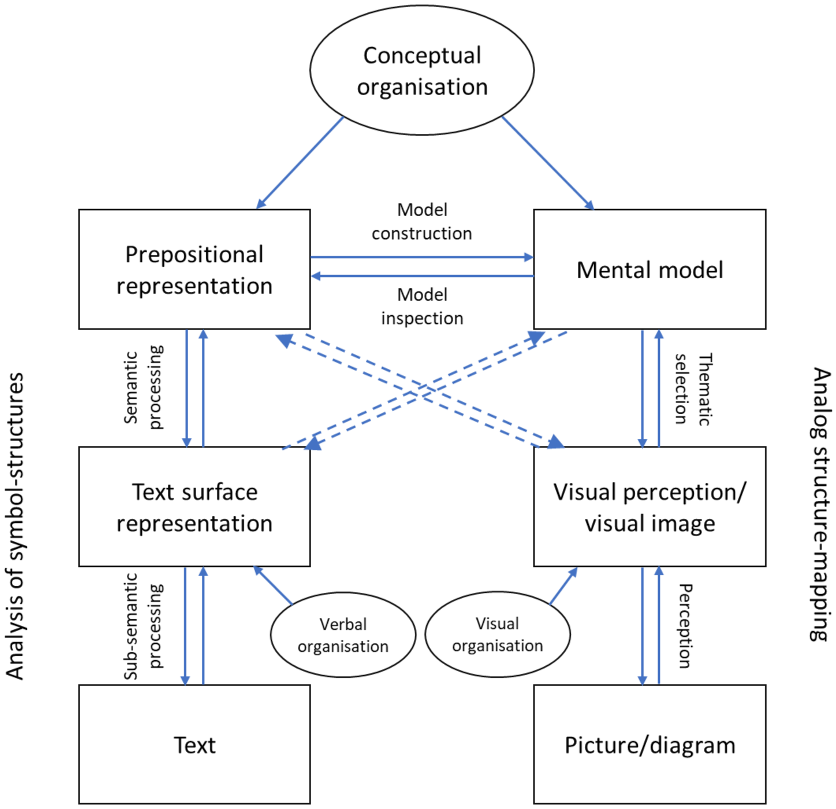


Fig. 4.1: Integrative model of text and picture comprehension elaborated by Schnotz (2002, p. 109).

As described in Schnoltz (2002), "It consists of a descriptive (left side) and a depictive (right side) branch of representations. The descriptive branch comprises the (external) text, the (internal) mental representation of the text surface structure, and the propositional representation of the text's semantic content. The interaction between these descriptive representations is based on symbol processing. The depictive branch comprises the (external) picture, the (internal) visual perception or image of the picture, and the (also internal) mental model of the subject matter presented in the picture. The interaction between these depictive representations is based on processes of structure mapping due to the structural correspondences (i.e., analogy relations) between the representations. [...] These construction processes are based on an interaction of bottom-up and top-down activation of cognitive schemata that have both a selective and an organizing function" (Schnoltz, 2002, p.108).
Finally, the DeFT (Design, Functions, Tasks) Framework for Learning with Multiple External Representations (Ainsworth, 2014; Tsui and Treagust 2013) differs from the other two models mainly for the focus but, as those, provide principles that can be implemented in instructional materials. The main assumption at the basis of the DeFT framework is that, to support comprehension with multiple representations, they have to contain qualitatively different nuances of the information to be learned, focusing not only on different kinds of external representation but also on the fact that they have to convey the same information in different ways. Opermann et al. (2017) used the following example to better explain the pivotal aspect of the DeFT framework. The table and the graph, concerning the concept of acceleration, contain partly the same but also complementary information, being different types of representations (Fig. 4.2). In this case, presenting all three representations should support the learning process steps from changing velocity in time and rectilinear motion to the notion of acceleration (Opfermann, Schmeck & Fischer, 2017). Ainsworth (2008) identifies three major functions of multiple representations in learning: (1) a complementary function, namely one representation complements another representation as every representation may differ in the information it expresses or in the processes it supports; (2) constraining interpretations of a difficult representation (in terms of complexity or abstractness) using an easier representation (because of its familiarity or

concreteness); and (3) constructing a deeper understanding as students could integrate information from more than one representation (p. 197).

Time in second	Velocity of Car 1	Velocity of Car 2
0	0	0
1	4	1
2	8	4
3	12	5
4	16	7

$$a(v) = \frac{\Delta v}{\Delta t}$$

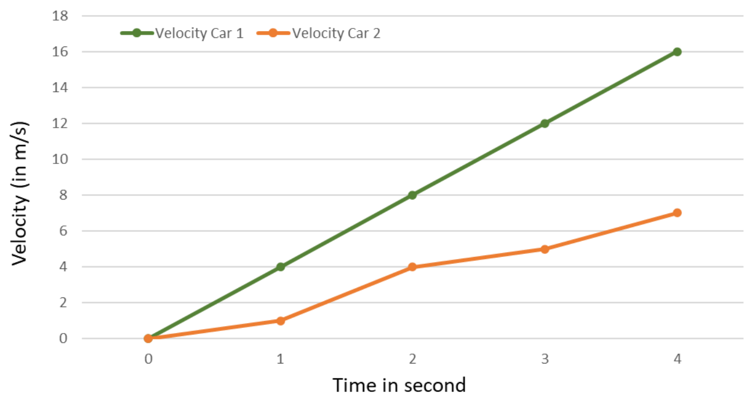


Figure 4.2: Example of learning materials on acceleration using multiple external representations (Opfermann, Schmeck & Fischer, 2017, p. 9)

In this framework, the use of multiple external representations has many other learning advantages. For example, it can support different cognitive processes, since individual differences can be taken into account allowing the learners to “choose to work with the representation that best suits their needs” (Ainsworth 2006, p. 188). Therefore, to sum up, multiple external representations can support learning when the instructional materials are designed to (a) gain complementary information supporting different cognitive processes, (b) prevent inaccurate interpretations by constraining interpretation options, and (c) promote a deep level understanding through some processes such as abstraction, extension, and relation (Tsui and Treagust 2013; Ainsworth, 1999; 2006; 2008).
All these frameworks have been considered and they provided useful principles and considerations that we took into account in designing our instructional materials and the experiment (sections 4.3).

When we talk about representations, one cannot fail to mention Duval’s work in the field of mathematics education. Learning mathematics involves, in fact, different representations. In a research paper (Duval, 2000), the author discussed that when a learner deals with a mathematical concept or object, “mathematics knowledge requires thought processes which are multidisciplinary and typical of what it is to understand in mathematics” (p.3). Learning consists of processes of validation and proof, in using symbols and different visualization forms (cartesian graphs, geometrical figures, etc.). The gaps that are usually observed are, for example, between the use of words and the use of symbols, between the use of mathematical expressions and the way they are understood, and between the spontaneous ways of seeing geometrical and mathematical figures. So, as Duval argued, “learning mathematics is not only to gain a practice of particular concepts/objects and to apply algorithms, it is also to take over the thought processes which enable a student to understand concepts and their applications. And these thought processes cannot be assimilated to construction of such-and-such a concept” (ibidem, p.3). He individuated four different aspects that have to be taken into account when we deal with representations: i. t*he system by which representation is produced* (e.g., a physical device such as a camera, or a brain organization for memory visual images, or even a semiotic system such as various languages) and the change of content of the object’s representation according to the productive system of representation which is used; ii. *the relation between representation and the represented object*; iii. *the possibility of an access to the represented object apart from semiotic representation*; and iv. *the reason why representation using is necessary* (e.g., communication or processing) (ibidem, p. 5). In general, mathematics saw the development of several semiotic systems “from the primitive duality of cognitive modes, image and language, which are linked with the more informational sensory receptors”. For example, symbolic notations derive from written language and lead to algebraic notations, or as regards visualization, the construction of plane figures with tools, then the construction of figures in perspective, then the construction of graphs in order to "translate" curves into equations. Ad Duval states, “Each new semiotic system provides new means of representation and processing for mathematical thinking” (ibidem, p.6). So, any mathematical object can be produced by different semiotic systems.

Taking into account the production system is related to the different semiotic systems that can be used and it emphasizes the processes of transformations of representations. Duval distinguishes between two kinds of transformations: processing and conversion. The first transformation occurs without changing the semiotic system. In fact, some semiotic systems incorporate specific possibilities of intrinsic transformations of representation, that is a representation produced in one system can be changed in another one of the same systems. Thus, some transformations that occur in one single register are paraphrase, reformulation, computation, anamorphosis, reconfiguration, etc. He talks about conversion as a transformation that involves different semiotic registers. In fact, for any representation of an object which is produced within a system, it is possible to produce another representation of this object into another system. For example, “Thus constructing a graph from a given equation or writing an equation from a graph, translating a verbal statement into a litteral expression or into a equation...” (ibidem, p.10). However sometimes there can be an issue that derives from “the paradoxical character of mathematical knowledge”. Duval argued in fact that, since there is no direct access to objects apart from their representations, students risk not recognizing a mathematical object through its various possible representations when their contents are different. This issue may be due not so much to a lack of conceptual understanding as to a lack of coordination between the registers that have been put into play together. Therefore, the coordination between representations produced in different semiotic systems is a focal point. Duval described with the schema in figure 4.3 the cognitive conditions of mathematics understanding and the central role of the coordination process.

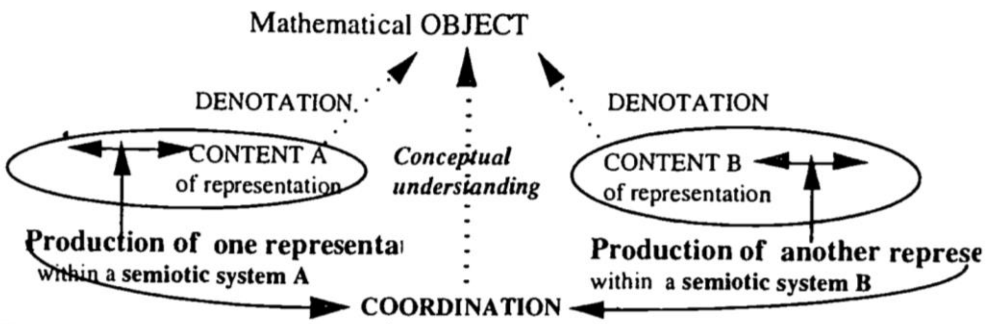


Fig. 4.3: Cognitive conditions of mathematics understanding (Duval, 2000, p.12)

Starting from psychological models of information processing that have highlighted that conscious understanding depends on the automatic (unconscious) working of the different and heterogeneous systems organization, which makes up the cognitive architecture of the epistemic subject, Duval concluded that mathematics understanding consists in integration into its own cognitive architecture of all needed registers as new systems of representation. He schematized the mathematical cognitive architecture of an epistemic subject in the diagram reported in figure 4.4. Duval, explaining his representation, argues: “Learning mathematics involves both incorporation of monofunctional registers and differentiation of the possible different ways of working within multifunctional registers. But it is not enough. Learning mathematics involves their coordination, or their decompartmentalization. Otherwise, conversion between non congruent representations will be inhibited. And that is not a side-problem, because registers are non-isomorphous and because showing together two different representations of the same object, in order to create associations, does not work. Learning mathematics is learning to discriminate and to coordinate semiotic systems of representation in order to become able of any transforming of representation” (ibidem, p.14).

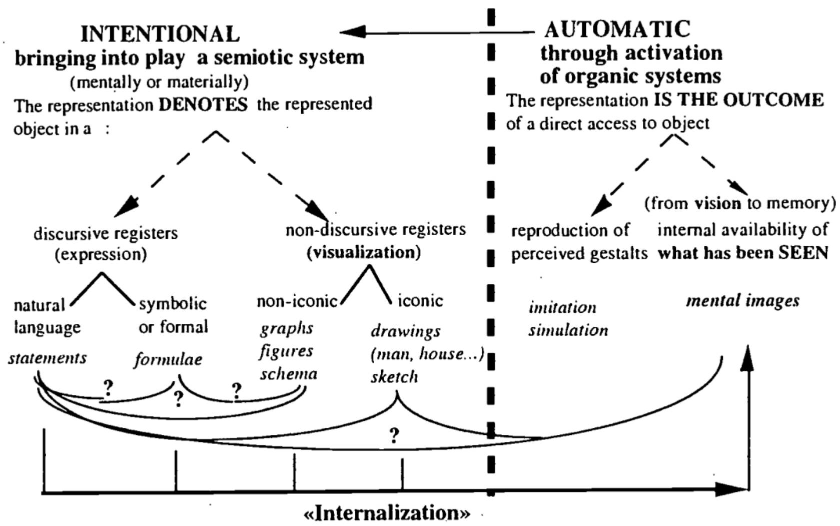


Fig. 4.4: Coordination between productive systems required for mathematics understanding (Duval, 2000, p. 13)

The use of visual representations (i.e., pictures, drawings, schemes, applets, verbal descriptions, graphs, texts) has been part of science in very different ways: from the scientists that used and created representations to interact with and represent complex and non-observable phenomena (Evagorou, Erduran & Mäntylä, 2015) to educators that exploited representations as a tool with the learning potential, as we said before, to improve and promote meaningful learning (e.g. Eilam & Gilbert, 2014; Schnotz 2014; Mayer, 2009; Kozma, 2003; Treagust, Chittleborough & Mamiala 2002; Mandl & Levin, 1989; etc.). In modern physics and, for our purposes, in quantum physics, the problem of representations and "visualization" is well-known both historically (for example, think of the Bohr atom model; Schrödinger's wave mechanics, the debate Schrödinger - Heisenberg about the role and the meaning of *Anschaulichkeit* or to the Feynman diagrams or Feynman paths) and from an educational perspective (e.g. Bøe, Henriksen & Angell, 2018; De Regt, 2014; Levrini & Fantini, 2013; McKagan, Perkins, Dubson, Malley, Reid, LeMaster & Wieman, 2008, Mashhadi, & Woolnough, 1999). Historical and philosophical studies argue that the main difficulty in visualizing in quantum mechanics is linked to the space-time visualization we are used to in the study of classical physics (e.g., De Regt, 2014). There is, at least, another intrinsic difficulty that concerns the link between the abstract formalism of the model and the phenomenon that can hinder the understanding of the theory (Levrini & Fantini, 2013) also because it may lack intelligibility. Giardino and Piazza (2008) argued that learning quantum physics implies somehow finding a connection, a relationship between real phenomena and abstract models, and it requires the learner to develop an epistemological awareness of this complex relationship. Being the abstraction characteristic of each model, any scientific representation can be considered as a cognitive hybrid (Piazza & Giardino, 2008), a combination of pictorial and verbal elements quoted above, built upon implicit models of phenomena. From this perspective, even if as educators we can try to reduce the degree of interpretation to build a "correct" mental or internal representation, representations can hide theoretical or personal perspectives on physical reality. For example, Podolefski and Finkelstein (2008a, 2008b) studied the influence of different representations on students' understanding of electromagnetic waves, arguing that individuals' interpretations of representations can be highly variable based on the level of abstraction of the representation they choose. With abstraction, the authors mean that students do not have a "phenomenological grounded knowledge" about the phenomenon. The level of abstraction can also lead students to build an idea closer to the correct scientific model. In particular, they noted that many students, in choosing sine wave representation to represent a sound wave (a representation with a high level of abstraction), may interpret this representation quite literally as air moving up and down, in contrast with the correct scientific model of air moving left and right, a longitudinal wave (Podolefsky, 2008a). Instead, the

students, who choose a picture of air particles, seem to have an idea nearer to the correct scientific model. When experiential knowledge is not stable, representations can lead students to focus their attention on the details, or particular aspects, of the phenomena represented (Podolefski and Finkelstein, 2006) assuming a bi-univocal relationship between representations and reality. This is in line also with what Elby (2000) called what-you-see-is-what-you-get (WYSIWYG), referring to the cognitive structure or specifically, an intuitive knowledge element that can be described as
"WYSIWYG: x means x" (Elby, 2000, p. 468).

Another important contribution is the work carried out by Levrini and Fantini (2013). They implemented a teaching proposal (Levrini & Fantini, 2013) in parallel with two regulars, 18–19 year old students (grade thirteen) from a science-oriented high school in Rimini (Italy). The teacher was the same (PF) and both implementations took about 25h each. With a qualitative analysis of the final videotaped discussions and the questionnaire they conducted, they pointed out how particularly for two students, Jessica and Silvia, well-working formalism is necessary but not sufficient to have the "feeling of understanding" (Levrini & Fantini, 2013, p.1903). In particular, Jessica described her cognitive need for visualizing as something that manifests itself, especially, when she has to communicate or explain. For this reason, visual representations can be untrue, conventional, and creative: "also a little star can be used if it helps to talk about photons" and they could not represent reality (Levrini & Fantini, 2013, p.1904). This work is very interesting not only for the results they carried out but also because they showed that students can clearly explain their cognitive and epistemic needs by pinpointing consciously their puzzlement concerning visualization/visualizability, in line also with the historical problems for physicists and philosophers (Levrini & Fantini, 2013). With this work, the authors do not want to draw Jessica's needs as a common need. The paper presents the case of two other students discussing how formalism is necessary and sufficient for their understanding, although in the case of quantum mechanics it is non-trivial. So the purpose of Levrini and Fantini (2013) is to show how students in approaching physics have different needs that can be taken into account in the design of instructional materials. In their work (Ravaioli, 2020; Ravaioli & Levrini, 2017; Levrini & Fantini, 2013) they have identified three different needs that have defined epistemic, described in the next section.

Finally, at the basis of the present chapter, there is another theme that in the last twenty years has been investigated in science education research, that is the *meta-representational competence* elaborated by diSessa and Sherin (2000). As we have long discussed in this session, many research works have proved that the understating of physical phenomena can be improved by using multiple representations). The concept of meta-representational competence allows us to make a further step because the focus is on what students know about scientific representations and what is possible for them to learn from these. The authors with the term describe the full range of capacities that learners have with regard to the construction and use of external representations, such as the abilities to select, produce and fruitfully use representations but also the abilities to explain, critique, and modify or design completely new representations. In literature, it often found the general term ''representational competence'' to describe the full range of activities involving representation. diSessa and Sherin (2000) added "the prefix 'meta' to denote our more encompassing aims. We are interested in whatever students know about representation (meta-representation), whether or not it concerns instructed representations, and whether or not it involves the standard school modes of (only) reproduction and interpretation" (diSessa & Sherin, 2000, p. 386). The taxonomy of the components of meta-representational competence they found in literature and proposed is the following:

1) "Invent or design new representations.
2) Critique and compare the adequacy of representations and judge their suitability for various tasks.
3) Understand the purposes of representations generally and in particular contexts and understand how representations do the work they do for us.

4) Explain representations (i.e., the ability to articulate their competence with the preceding items).
5) Learn new representations quickly and with minimal instruction”

(diSessa, 2004, p. 293; diSessa & Sherin, 2000).

There are a few studies that explore students’ meta-representational competencies in quantum physics education both concerning the link between representations and problem solving. Kohl and Finkelstein (2006) investigate if and how students’ problem-solving is affected by the representations they choose and if there is a coherence between the representation they choose as their favorite and the performance success. The first result they obtained is that there is a complex dependence between student performance on physics problems and problem representation. The second one is that there is a poor correlation between the representation that they indicate as favorites and their actual performance. Interesting in particular in relation to the second result, it is the work of Wawro, Watson and Christensen (2020). They aimed to investigate how students reasoned about linear algebra concepts (in particular, normalization, basis, and eigentheory), how they reasoned with these concepts in discussing the quantum mechanics concepts and solving quantum mechanics problems, and how they symbolized their work (Wawro, 2020). They created three main categories to identify and analyze students’ meta-representational competencies in their talk: value-based preference, problem-based preference, and purpose and utility awareness, associating the first two categories with the ability to “critique and compare the adequacy of representations and judge their suitability for various tasks” (diSessa, 2004, p.293) and the third one with the ability to “understand the purposes of representations generally… and understand how representations do the work they do for us” (diSessa, 2004, p.293). These results are significant because it emerges how students can express considerations on certain representations based not only on the task and/or the problem they need to solve; but also, on values such as the greater familiarity of one representation rather than another, or, as we will see, a greater aesthetic value, and on purposes they can associate with a representation.

In the following, I present our starting point, which is the problem of acceptance of quantum physics, for the design of the experiment and how we linked it to the external representations.

4.2 From students’ epistemic needs to role of representations to promote the acceptance of quantum physics

Rimini's implementation was designed along the research line on which Bologna's research group in physics education worked for many years, that is teaching/learning quantum physics and the *problem of acceptance of quantum physics* (Ravaioli 2020; Ravaioli & Levrini, 2017; Levrini and Fantini 2013). In the first years of our work on the second quantum revolution and the emergent technologies, great effort was dedicated to designing a course, based on an educational reconstruction of the advanced contents, within the reach of secondary school students. In Rimini’s implementation, we wanted to investigate *if* and *to what extent* our approach could be effective to make students *accept* quantum physics as a convincing description of the physical reality and make them understand some of the fundamental concepts (such as quantum state, superposition principle, state manipulation and evolution, measurement and entanglement) that the literature shows as pivotal or/and not easy to understand (e.g. Krijtenburg-Lewerissa, van Joolingen, Pol & Brinkman, 2020; Krijtenburg-Lewerissa, Pol, Brinkman & Van Joolingen, 2019; Krijtenburg-Lewerissa, Pol, Brinkman & Van Joolingen, 2017; Sadaghiani, 2016; Zhu & Singh, 2012; McKagan, Perkins & Wieman, 2010; Pospiech, 2000; Pospiech, 1999). The approach and some design principles were reformulated in order to take advantage of different kinds of representations and their interaction (passing from the circuit to the experiment to the mathematical forms).

The research group more than 10 years ago elaborated a teaching proposal (presented in chapter 2) with the aim of addressing the counterintuitive nature of quantum physics that can lead secondary school or university students to not accept quantum physics as an adequate and personally reliable explanation of reality (Ravaioli 2020; Ravaioli & Fantini, 2017; De Regt, 2014; Levrini & Fantini, 2013). In recent years, different research has been carried out to monitor the students' epistemologies (e.g. Wittman & Morgan, 2020; Dini & Hammer, 2017; Dreyfus, Hoehn, Elby, Finkelstein & Gupta, 2019; Baily & Finkelstein, 2010; Elby, 2001) and, more in general, students' processes of sense-making (e.g. Gifford & Finkelstein, 2020; Wawro, Watson, & Christensen, 2020; Kuo, Hull, Elby, & Gupta, 2020; Dreyfus, Elby, Gupta, & Sohr, 2017), mainly because the epistemological and foundational aspects of quantum physics can raise some basic questions about the sense of physical descriptions of reality. Some studies show that the difficulties of "attaching" sense and meaning are generally independent of personal confidence with mathematical formalism (Hanemann, Hoehn & Finkelstein, 2019; Dini & Hammer, 2017; Ravaioli & Levrini, 2017; Levrini & Fantini, 2013; Baily & Finkelstein, 2010; Greca & Freire, 2003). To mention a few, Greca and Freire (2003) conducted a study about a course for engineering students and observed that for most students "quantum concepts are fragmentary or mere mathematical expressions" (Greca & Freire, 2003, p. 546). Levrini and Fantini (2013) conducted a study in parallel in two grade thirteen classes and found that for some students "well-working formalism was necessary but not sufficient to have the feeling of understanding; comprehension would have required the ''formal mechanism'' to be interpreted also in terms of (smoothed) links to ordinary language and classical description" (Levrini & Fantini, 2013, p. 1907). Dini and Hammer (2017), exploring a student's variations in his personal epistemologies during a quantum physics course, noted the persistence of his search for a connection between mathematics and a meaningful understanding of the physical world, aside from variations in how he engaged that effort (Dini & Hammer, 2017). Baily and Finkelstein (2010a,b) examined the impacts of different instructional approaches on student perspectives regarding quantum physics and pointed out the relevance of teachers' choices about interpretative issues, finding that an 'agnostic' stance can produce naïve realist interpretations in students (Baily & Finkelstein, 2010, p.8). Dreyfous, Sohr, Gupta, and Elby (2015) emphasized the difficulties from a research point of view to grasp the role of intuitive physics knowledge in learning quantum mechanics since quantum phenomena are not part of everyday experience. When students learn quantum mechanics, they have in mind classical intuitions and associated ontologies, as well as formal classical concepts learned in previous courses. For this reason, in line also with the forms of productive complexity pointed out by Levrini and Fantini (2013), "another essential element of developing quantum expertise is metacognition or thinking about one's own thinking; students must work out when and why they can and cannot rely on classical ontologies and ways of thinking. This aspect of metacognition is conditional knowledge, "knowing why and when to do things" (Dreyfous, Sohr, Gupta & Elby, 2015, p.1). By analyzing a video recording of a group of five students working through a tutorial about the "particle in a box", they found that in the reasoning students bring elements of a classical particle ontology. However, the reasoning is modulated by metacognitive moments when they consider whether classical intuitions are applicable to the quantum system. The students found cases in which it was possible to apply classical ideas to quantum physics, and others where they explicitly contrast classical and quantum mechanics. The negotiation of this boundary with metacognitive awareness is part of the process of building quantum intuitions.

The origin of non-acceptance quantum physics touches deep chords and deals with the *weltanschauung* (a composition of 'Anschauung' that means intuition or vision, and 'Welt' that means world) and *anschaulichkeit* (that is visualizability not in a literal sense, the intelligibility of a theory). In this perspective it has an epistemic nature (e.g., Levrini & Fantini, 2013; De Regt, 2009). For "non acceptance", as Levrini and Ravaioli (2017) argued, we refer to "a specific reaction of students who explicitly did not accept quantum physics as an adequate and personally reliable explanation of reality" (Ravaioli, 2020, p. 115; Ravaioli & Levrini, 2017, p. 2). The analysis of the data collected in different implementations led them to link this problem with some cognitive and "*epistemic needs*" that the theory could not satisfy. In Ravaioli (2020), three *epistemic needs* are pointed out:

- the need to recognize and conceptualize the relationship between the "real world we live in" and the abstract mathematical construct, that is to attach a reliable and "realistic" meaning to new basic elements; or need of 'reification', recalling concept of reification as the objectification of an abstract construction process by Sfard (1991);
- the need of visualization, either in terms of the need to form a mental picture of the quantum object or, more in general, in terms of the need to have a comprehensive view that can guide intuition;
- the need of comparability, that is the need to find criteria to understand where and how the epistemological interpretation/description of quantum physics is different from the classical one (Ravaioli, 2020).

From the first implementation of the module on quantum computing with secondary school students (Satanassi et al., 2022; Satanassi et al., 2021; Satanassi, 2020), we have noticed that there was "something" in the course that seemed to promote the acceptance of quantum physics that we had never observed in the previous implementations. This feeling led us to reflect on the differences in the teaching approaches not only in terms of content, and to consider the possibility that the multiple representations system, in the case of our approach, could have a different impact on students' understanding and could satisfy some epistemic and cognitive needs. In other terms, we had the perception that the different external representations of the approach can activate and support the construction of personally reliable internal representations. In particular, among the different representations, the logical (truth tables) and circuital (logic gates and circuits) ones are in our opinion the very novel/most innovative ones. Such a *sensitizing idea* made us formulate the following conjecture: Quantum computing and the language of the logic gates and circuits have the learning potential to provide new synthetic scenarios able to satisfy the epistemic needs we had previously described. The *need to attach a "realistic" meaning* to abstract concepts like superposition state, measurement, and entanglement can be satisfied, for example, if a protocol, for example, the teleportation one, is explicitly related to the structure of a teleportation experiment. In light of this, the circuital representation should help to bridge the experimental representation, to which students usually attribute a sense of reality, to the abstract mathematical representation. The *need to build a comprehensive picture* can be satisfied by the representation of the circuit per se that provides a systemic view; the *need to compare* the revolutionary aspect of quantum physics can be satisfied by comparing the non-binary quantum logic with the classical binary one. In this latter case, the logical representation, in terms of truth tables, in a very schematic way can scaffold a comparison between classical and quantum physics, from an epistemological perspective to an ontological one.
We thought of investigating our conjecture by evaluating students' skills of explaining, discussing, comparing, and critiquing different kinds of representations (*meta-representational competencies*). For example, diSessa, Hammer, Sherin, and Kolpakowski (1991), exploring 8th six-grade students' abilities to invent graphing, pointed out some criteria concerning the quality of representations of motion that students apply in judging them:

- Transparency. A representation should need little explanation.
- Homogeneity. Use the same notation for a stop as for motion, not equals signs.
- Compactness. Ts take less space than number lists.
- Conceptual clarity. Ts are "precise," showing the requisite two aspects cleanly.
- Objectivity. "Could be done by a computer."
- Appropriate abstractness. Show only aspects; needn't show the road, cactus, and so on.
- Faithfulness. Continuity in speed is better expressed in continuous graphs.
- Completeness. Can derive all three relevant aspects of motion: speed, distance, and time; can show all kinds of motion, such as stop.
- Economy. All three aspects of motion can be derived from any two that are presented.
- Quantitative precision. This was introduced by the teacher but adopted easily by the students.
- Consistency. Conventions should not be adjusted for particular motions, such as for the bike on the hill.

(diSessa, Hammer, Sherin and Kolpakowski, 1991, p. 148).
A more recent study carried out by diSessa (2002) about high school students' criteria of judgment of representations for spatially distributed data showed that a group of 3 students produced about 60 critical judgments spreading across a wide range of types. Some of these criteria are quite scientific and concerned:

- Epistemic fidelity (completeness and accuracy);
- Systematicity (simple, uniform rules of correspondence);
- Univocality (has a unique interpretation);
- Simplicity (avoids unnecessary complexity);
- Autonomy (comprehensible on its own);
- Conventionality (does not unnecessarily violate accepted conventions);
- Alignment (correspondences between different parts of a representation are shown clearly)

(diSessa, 2002).

From here, through the students' capacities to explain, discuss, critique, and compare different representations, we had the idea of investigating how students understand through representations, exploring what qualities they associate with a representation and why; to what extent and how students' explanations give us information about their epistemic needs.
From this perspective, in our conjecture, the logical and circuital representation, in being a comprehensive picture and providing a systemic view, can satisfy different criteria that students carried out in the presented studies such as completeness, economy, conceptual clarity, univocality, autonomy, etc. (diSessa, 2002; diSessa, Hammer & Sherin, 1991). Our aim is not to investigate if the representations that students meet during our course satisfy these criteria, since that concerns other kinds of representation and another experimentation. Rather, we use them as a starting point, as an incomplete set of qualities that can trigger students' reflection and explanation on if and to what extent the representations, they meet satisfy their cognitive and epistemic needs.
Starting from these ideas we revised some activities of the course, designed, and implemented an experiment to test our conjecture. In more detail, the revision phase consisted of rethinking the activities to draw students' attention to the plurality of representations we use (experimental, algebraic, geometrical, axiomatic, logical, and circuital) and their interactions and interconnections. This is part also of our design principles: both the second and the fourth design principles can be reconceptualized in terms of representations. In particular, the second design principle can be reformulated as a comparison between the experimental and the circuital representations. As regards the fourth design principle, which consists in making the module as inclusive as possible, to reformulate it in terms of representations, we need to recall some research works. Evagorou, Erduran and Mäntylä (2015), by conceiving a visual representation as an epistemic object, discussed how visual representations and their processing are bearers of scientific practices. Going into the content of the representation, Fredlund and colleagues argued that representations in physics, conceived as semiotic resources (Fredlund, Linder, Airey, and Linder, 2014, p.1), potentially provide access to disciplinary knowledge. They called this function *disciplinary affordance* (Fredlund, Linder, Airey, and Linder, 2014; Airey, Eriksson, Fredlund, and Linder, 2014; Fredlund, Airey, and Linder, 2012). The use of a *constellation of representations* is important to experience a learning object in a disciplinary way (Fredlund et al., 2014; Airey & Linder, 2009). This constellation of representations has a collective disciplinary affordance (Linder, 2013), that is necessary, but not sufficient, to access all the nuances of a particular disciplinary way of knowing. We believe that, when dealing with interdisciplinary or STEM topics, the representations of the constellation, belonging to different disciplinary domains, allow not only to have a disciplinary experience of the object but also an interdisciplinary experience, namely the constellation can potentially have also a *collective interdisciplinary affordance*. Therefore, reformulating our fourth design principle, the comparison between different representations bearers of different disciplinary identities and knowledge, on one hand they provide more points of access to the same topic, allowing students to choose the path they prefer and that best suits their way of understanding and their interests nurturing their idiosyncratic interests, intellectual and aesthetic

tastes (Satanassi, Ercolessi, and Levrini, 2022); on the other, in the intertwining, they can promote the creation of connections between different disciplinary domains.

In light of this, from the very beginning of the implementation, we paid attention to and value the role of representations. Specifically, we showed and compared the different representations, clarifying and explaining during the course our concern of understanding if and how those representations, individually and together, impacted their personal understanding of the topics addressed.
The phases of the experiment, described in more detail in the next sections, were designed to collect data both on a conceptual dimension, that is, on what kind of conceptual information students could extrapolate from the representation, and on a more "meta" dimension, that is why and how the different representations could satisfy their epistemic needs.

In the following, I describe the implementation at Rimini's Liceo Scientifico, the context, how we paved the way for the experiment throughout the different activities, and finally the questionnaire and the focus group we have designed to test the conjecture.

4.3 The context, and the refined version of the course

The course on the Second Quantum Revolution was held at Liceo A. Einstein in Rimini. It was implemented in October 2021 and lasted about three weeks; the meetings were twice a week, 3 hours per meeting. The course was implemented as an extra-curricular course in the school building and involved about 30 students from three different classes in the last year of high school. The course was developed in collaboration with the *Piano Lauree Scientifiche* (PLS)[13] project and the course hours are recognized as part of the "Percorsi per le Competenze Trasversali e l'Orientamento" (PCTO, Pathways for Transversal Skills and Orientation), according to which students in the three-year period (16-18 grade) have to mature a certain number of hours oriented towards introducing them to the world of work and developing soft skills[14].
Like in the other implementations the students, which attended the course, had not studied quantum physics at school yet.
The structure of the course implemented is reported in Table 4.1.

Table 4.1: structure of Rimini's implementation.

Day	Conceptual and epistemological activities (2h)	FEDORA-oriented activities (1h)
1	• Introduction to the Second Quantum Revolution • History of classical computation	Teamwork: "Quantum technologies &..."
2	Introduction to the physics of quantum computation: one-qubit system (introduction of the concepts of quantum state, state manipulation/evolution, and measurement)	Delivery of the teamwork: "Quantum technologies &..."
3	• Introduction to the physics of quantum computation: two-qubit systems (introduction of the concept of entanglement) • Cryptography protocol	Decision-making teamwork activity: the Eve city
4	Teleportation protocol	• Introduction of the concept of scenario • Teamwork to reflect on the trichotomy humans -

[13] https://www.pianolaureescientifiche.it
[14] https://www.miur.gov.it/documents/20182/1306025/Linee+guida+PCTO+con+allegati.pdf

		technology/science - nature: the Eve city
5	Experiment part 1: the teamwork	Teamwork on the impact of the second quantum revolution in preparation for the workshop
6	Experiment part 2: the focus group	
7	Final workshop with duo of experts in conceptual art	

The structure of the implementation is very similar. The initial two days remained the same, while from the third one we revised and designed new activities based on the project FEDORA (chapter §2), in particular, to reflect on the need for new narratives and languages, also outside the scientific field, to unpack the ongoing revolution, to grasp the essence of it. On the fifth day, the random walk activity was replaced with 2-hour teamwork on representations and a 1-hour activity in which students reflected and discussed the revolutionary aspects they met, guided by a worksheet in order to discuss them with a conceptual art duo.

As anticipated, the conceptual and epistemological activities were revised to prepare students for teamwork and enact reflections on how they understand through representations. Therefore, the students were gradually introduced to the different representations focusing on the kind of knowledge they embody and their roles. Throughout the course, time was taken to introduce, and compare the different representations both in terms of semiotic and epistemic forms, trying to potentially develop the abilities to i. explain and discuss the different representations, ii. extrapolate information from them, iii. recognize the different semiotic and epistemic forms that are bearers of different disciplinary identity and knowledge, iv. put in coherence and create a connection between different representations. Rau (2018) refers to part of these abilities as two kinds of *connection-making skills*, that are "*sense-making skills* that allow students to verbally explain mappings among representations and *perceptual fluency* in connection making that allows students to fast and effortlessly use perceptual features to make connections among representations". (Rau, 2018, p. 209)

As we anticipated before, in introducing the CTML (the Cognitive Theory of Multimedia Learning), the ITPC (Integrated Model of Text and Picture Comprehension), and the DeFT (Design, Functions, Tasks Framework for Learning with Multiple External Representations) models, different elements of those were taken into account. For example, the modality principle was taken into account in the preparation of the slides of lectures and the instructional materials by taking care not to overload the instructional material in which a visual representation with a text is presented, preferring a verbal presentation of the representation. The redundancy principle was implemented by focusing on the epistemic and semiotic differences of the presented representations. This is consistent also with the DeFT framework, according to which representations have to contain qualitatively different nuances of the information to be learned, so they have to convey the same information in different ways.

Going into details of the course, the representations with which we deal with are the algebraic (in terms of Dirac notation), the geometric (Bloch sphere), the logical (truth tables), the circuital (logic gates and circuit) and experimental representations. These representations, as the names we have attributed to the different representations suggest, have a different disciplinary affordance. The algebraic, the geometric and the logical ones recall a mathematical dimension, the circuital one the computer science dimension and the experimental one the physical dimension. These names and the disciplinary dimension that we associate with the different representations do not want to strictly categorize them, but we think that they can trigger different resources.

During the course, the idea of representation and the plurality of representations were explicitly introduced and, based on the reformulation of the design principles presented in the previous section, each meeting we paid attention and devoted time to the comparison between different representations both in terms of disciplinary and interdisciplinary affordance. In particular, in the first meeting about the history of classical computers, we introduced the classical logic gates and circuits, the truth tables and the Boolean logic, and the system of ropes and pulleys, respectively as circuital, logical, and experimental representations (Fig. 4.5).

Logical representation

A	B	A AND B
True / 1	True / 1	True / 1
True / 1	False / 0	False / 0
False / 0	True / 1	False / 0
False / 0	False / 0	False / 0

Circuital representation

Experimental representation: An example of experimental device

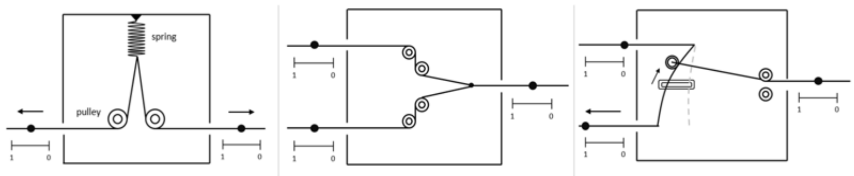


Fig. 4.5: the constellation of representations shown in the lecture on classical computation

In this case the logical representation embodies the information about the logic on which classic computers function, that is on how the set of logic gates and circuits (the circuital representation) works, revealing the close connection between mathematics and computer science in talking about computation. By addressing the discourse from a historical point of view and presenting computers in their first stage, it is impossible not to talk about the hardware and how computers are made "physically". For the reasons presented in chapter 2, a representation of the logical gates as a system of strings and pulleys is presented as an example of a physical device, expanding the mathematical-computer connection to physics.

In the second meeting, firstly, we compare the experimental representation of the Stern and Gerlach apparatus, the "simplified one" (McIntyre, Manogue, & Tate, 2013) and the circuital one (Fig. 4.4). The shift from the experimental representation of a physical device (in this case Stern and Gerlach experiment) to the circuital representation stress on the close connection between physics and computer science that starts with the concept "it from bit" elaborated by Wheeler, which led to the birth of the physical theory of information. The single representations and comparison between, as we stressed with the students, incorporate the idea of the nature of the object as quantum state, its manipulation, and its measurement on one hand (the experimental representation) and the information encoded in the computational basis as input information, its processing and the output information.

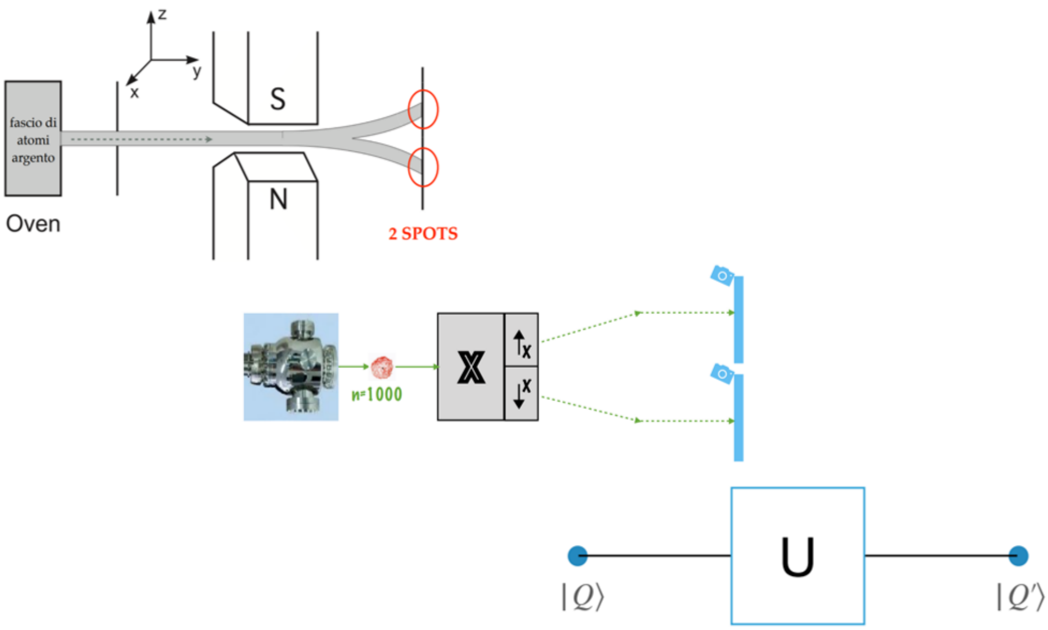


Fig. 4.6: constellation of representations shown in the lecture on the introduction of the physics of one-qubit systems.

In figures from 4.6 to 4.9, we reported the constellation of representations that we used to represent respectively the concept of qubit as a superposition state, the manipulation/evolution of a state (information processing through logic gates and rotation of the state vector in the Bloch sphere) and the measurement. These representations were presented both individually and in comparison.

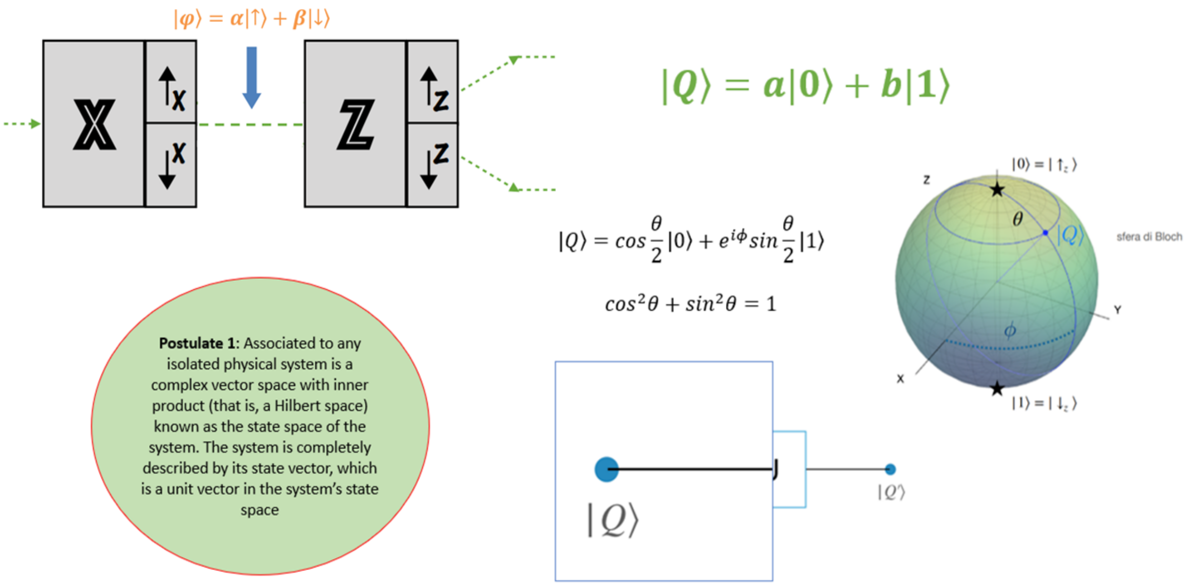


Fig 4.7: constellation of representations that represent the concept of quantum state

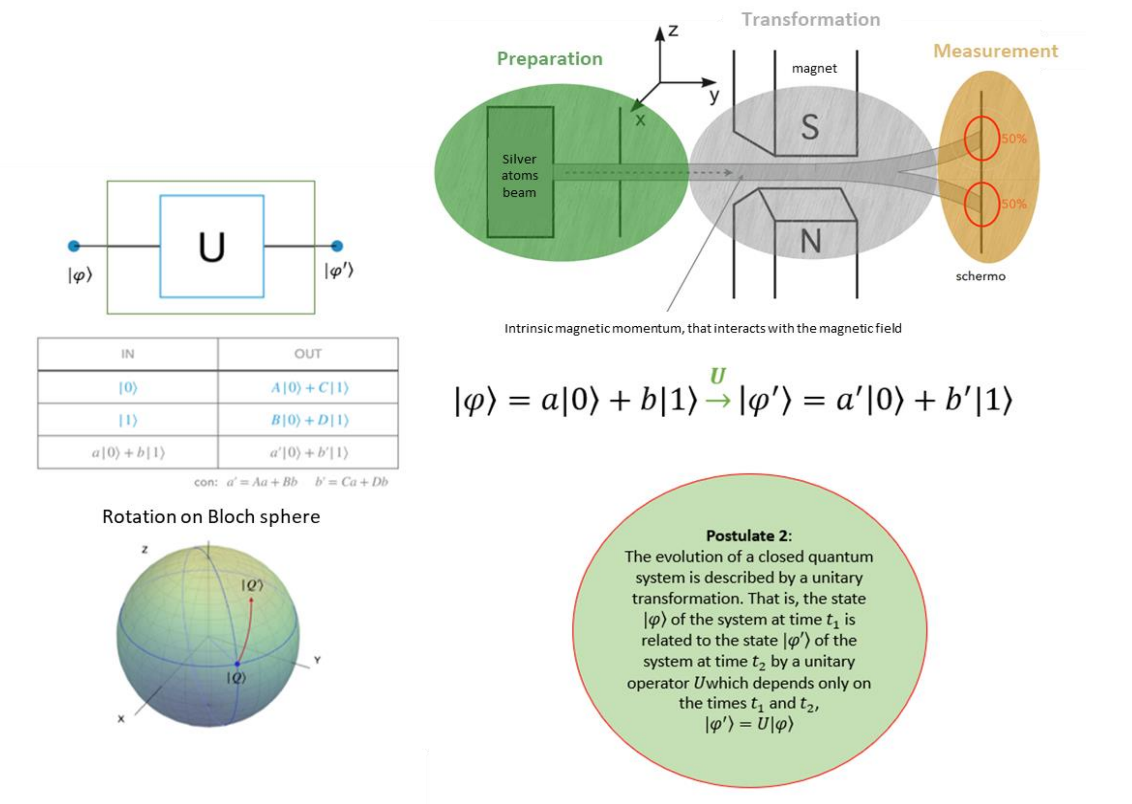
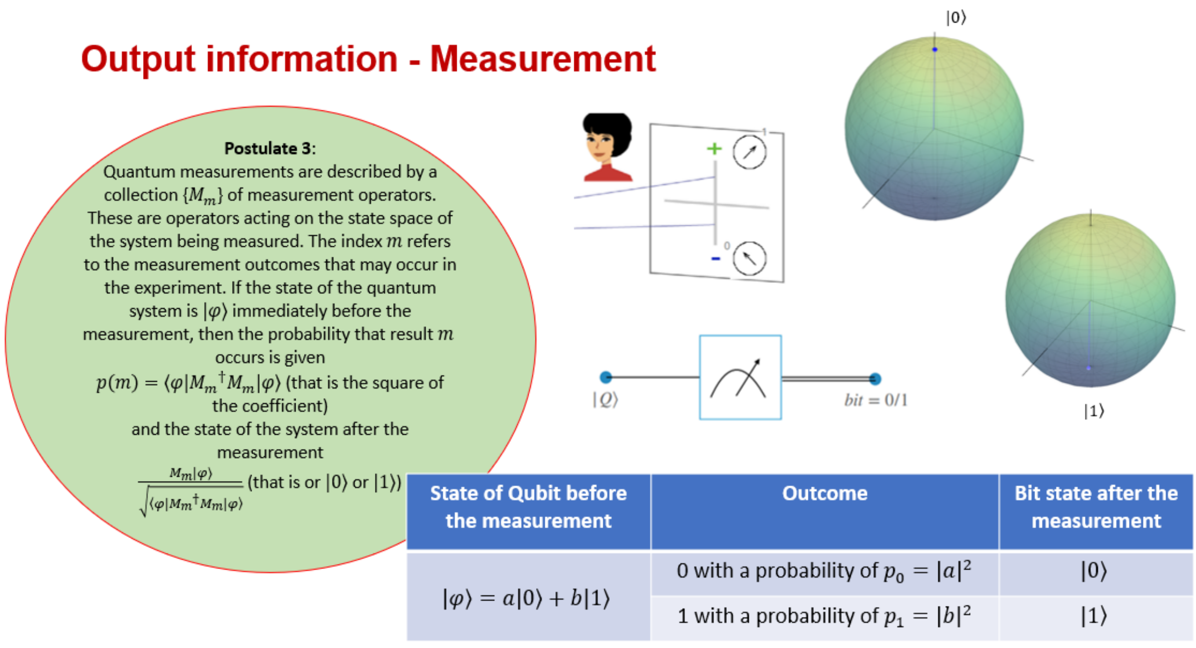


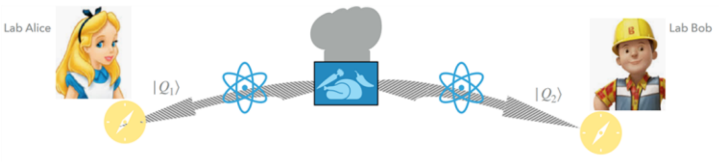
Fig: 4.8: constellation of representations that represent the concept of state manipulation/evolution

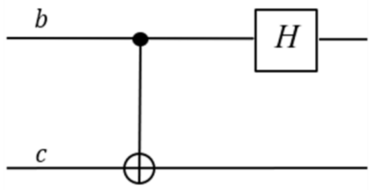


State of Qubit before the measurement	Outcome	Bit state after the measurement
$\|\varphi\rangle = a\|0\rangle + b\|1\rangle$	0 with a probability of $p_0 = \|a\|^2$	$\|0\rangle$
	1 with a probability of $p_1 = \|b\|^2$	$\|1\rangle$

Fig. 4.9: constellation of representations that represent the concept of measurement.

In the third meeting we introduced the concept of entanglement stressing on, in particular, on the computational and mathematical representations. In figure 4.10 the constellation of representation shown to students is reported.



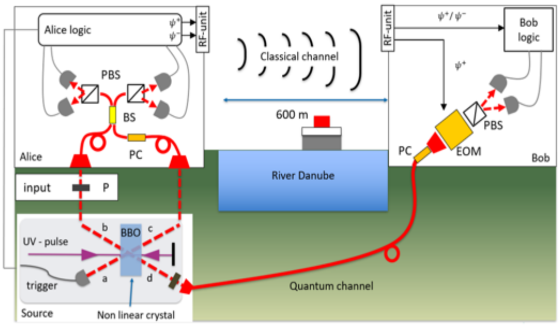


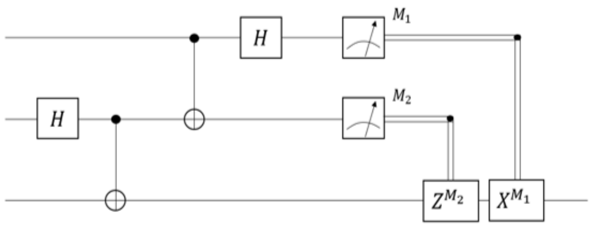
State of the system before the measurement	State after the measurement	Outcome of the measurement on Q1	Outcome of the measurement on Q2	Q1=Q2 ?
$\|\psi\rangle = \frac{\|00\rangle + \|11\rangle}{\sqrt{2}}$	$\|00\rangle, p_{00} = \|\alpha_{00}\|^2 = 1/2$ $\|11\rangle, p_{11} = \|\alpha_{11}\|^2 = 1/2$	$\|0\rangle$ $\|0\rangle$	$\|0\rangle$ $\|1\rangle$	Yes Yes

$$|\beta_{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$
$$|\beta_{01}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$
$$|\beta_{10}\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$
$$|\beta_{11}\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

Fig. 4.10: constellation of representations that represent the concept of entanglement.

Similarly, on the fourth day, the teleportation protocol was presented through its experimental, circuital representation, and algebraic representation, which was used mainly at the reasoning and calculation level, that is in the manipulation of the state of the system to prove the teleportation of the state (Fig. 4.11).





$$|\psi_0\rangle = q_0q_1q_2 = (\alpha|0\rangle + \beta|1\rangle)|00\rangle \xrightarrow{H} (\alpha|0\rangle + \beta|1\rangle)\frac{|0\rangle+|1\rangle}{\sqrt{2}}|0\rangle \xrightarrow{CNOT} (\alpha|0\rangle_{q_0} +$$
$$+\beta|1\rangle_{q_0})\frac{|00\rangle_{q_1q_2}+|11\rangle_{q_1q_2}}{\sqrt{2}} \rightarrow \cdots$$

Fig. 4.11: constellation of representations used in the teleportation protocol experiment

In table 4.2, we reported in summary the constellation of representations used for each meeting presented to the students. The focus on representations meeting after meeting paved the way for the experiment described in the next section.

Table 4.2: Summary of the constellation of representations used for each meeting.

Day	Activities	Constellation of representations
1	History of classical computers	Comparison between logical, circuital and experimental representation in the case of classical computation
2	The physics of quantum computation: introduction of the basic concepts (state,	Comparison between logical, algebraic, geometric, axiomatic, circuital and experimental

	superposition principle, state manipulation and evolution, measurement)	representation for talking about the concept of state, manipulation of a state and measurement
3	The physic of quantum computation: introduction to the basic concepts (entanglement)	Comparison between logical, algebraic, circuital and experimental representation for talking about the concept of entanglement
4	Teleportation protocol	Comparison between the experimental, circuital and algebraic representation for talking about the teleportation protocol

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Chapter 5: The experiment

5.1 The phases of the experiment

The experiment was designed in order to i. test the conjecture about the circuital and logical representations (Quantum computing and the language of the logic gates and circuits have the learning potential to provide new synthetic scenarios able to satisfy the epistemic needs), ii. investigate if and how the “new” constellation of representations, compared to the one and the results pointed out in the previous implementations of the research group over the years, promoted a sense of acceptance of quantum physics, iii. explore if there are any other needs that the “new” representation satisfies and if students can express them by explaining, critiquing, and comparing the different representations.

To investigate the previous points, the experiment was structured in two phases that I describe in the next two sections:

- A teamwork on the 5th day that involved 27 students divided into five groups;
- A focus group between the 5th and 6th day that involved eight selected students.

Finally, at the end of the course, the FEDORA-oriented activities and the workshop with the conceptual artists inspired 6 of the 8 students and their teachers in Mathematics, Physics, and Italian. The project named “Quantum Atelier”, which I describe more deeply in the section 4.4.3, was born from a big challenge that we had launched with the FEDORA-oriented activities, namely the need to find new narratives to talk about the second quantum revolution. The challenge was taken up by the Italian teacher (Maurizio Giuseppucci), who also has artistic skills, and by the teachers of mathematics and physics (Michela Clementi, Fabio Filippi, and Paola Fantini), who worked with six students for the following months. The course has stimulated reflections about the relationship between art and science, on how disciplines outside of science can be not only useful to talk about science but also provide new aesthetics, languages, and forms with which to interpret the great changes that the first and second quantum revolutions are promoting primarily from a cultural point of view. The result of this project is the realization of 3 exhibitions that the six students showed us and to the school in May 2022. After this, we decided to interview the students about the experience and about whether, how, and to what extent the project and the constraints dictated by other disciplines, in this case, the art and the literature, have helped them to understand better, and to appropriate the themes addressed during the course. So a final focus group was organized in which 3 out of the six students participated.

In table 5.1, we report schematically the three phases of the experiment with the kind of activity carried out, the specificities and main request, the number of participants, and the kind of data collected.

Table 5.1: Schema of the phases of the experiment in terms of kinds of activities, kinds of request, number of participants and kind of data collected.

	The activities	Specificities of and main the requests	Number of participants	Kind of data
Day 1: The teamwork	Group discussion through a Questionnaire	For the four main basic concepts introduced: ● To what extent from 1 to 6 each representation helps you to understand the basic concepts? ● What are the characteristics that you would extrapolate from the different representations and that you consider fundamental? ● Are there, and if so which, representations that you consider redundant because they have details and information within them that are not relevant?	27 students divided into 5 groups of 5/6 students	A questionnaire, in sheet format, filled out per group.
	Group discussion about the features of the external representations	Which of these adjectives do you associate with which type of representation? Why?		A board per group where the external representations are grouped by type on which students have attached posts-it with the adjectives that they associated
Day 2: The focus group	Collective discussion about personal needs about understanding and understanding through representations	• What does it mean for you to understand? When do you have the feeling of understanding? • What features, in general, does a representation must have to give you the sense of “ok, I understand”?	8 students	2h of recorded session
	Collective discussion about some particular qualities of the representations	What are the meanings of the adjectives “compact”, “schematic”, “accessible”, “familiar”?		
Day 3: Collective interview	Collective discussion about the Quantum Atelier project and its impact on students’ understanding	1) What concepts did you choose and why did you choose them? 2) Try to think about the first time you met them. Do you remember the approach to the concept, the context, the language, and what you understood/understood before working in the quantum atelier? 3) Now let’s think about the process of "concept processing" inside the quantum atelier: can you tell us how you "worked/worked" the concept when you started thinking about a work of art?	3 students	2h of recorded session

		4) Let us now think about the stage of setting up the exhibition and presenting the works: what did you add in the elaboration of the concept? 5) Do your works seem "rigorous"? From what point of view? Which scientific rules do you respect, and which do you not? Which aesthetic rules do you respect, and which do you not? What, in your opinion, is the meaning of a work of art like yours, built on scientific themes/concepts? 6) Appropriation: You have often used the word appropriation of the concept during the final meeting. Can you describe it better? 7) What image of interdisciplinarity have you left after this experience? 8) What image of "revolution" has remained after all this experience (from the PLS to the Quantum Atelier)?		

5.5.1 The teamwork

The teamwork activity was divided into two parts. The first part was more "conceptual" and aimed to activate a group brainstorming and collect information about students' perceptions on: i. the role of the different representations (algebraic, geometric, experimental, circuital, logical, and axiomatic) to understand the four basic concepts, quantum state and superposition principle, state manipulation and evolution, measurement, and entanglement; ii. the kind of information the students can extrapolate from each representation, iii. eventual learning advantages from the use of multiple representations.

To investigate these points, we designed a questionnaire structured in such a way that students discussed how different types of representations "speak" about the concept, and whether and to what extent individual representations help students to understand it. We decided to make students reflect on the constellation of all the representations that we met during the course, even if we did not focus on the representations in the same way. Some of them were only presented and not particularly discussed, others were discussed and never recalled throughout the module, and others were at the center of each activity.

For example, the representations that we gave to students for reflecting on the concepts of quantum states and superposition principle, state evolution and manipulation, quantum measurement, and entanglement are in figures 5.1, 5.2, 5.3, and 5.4.

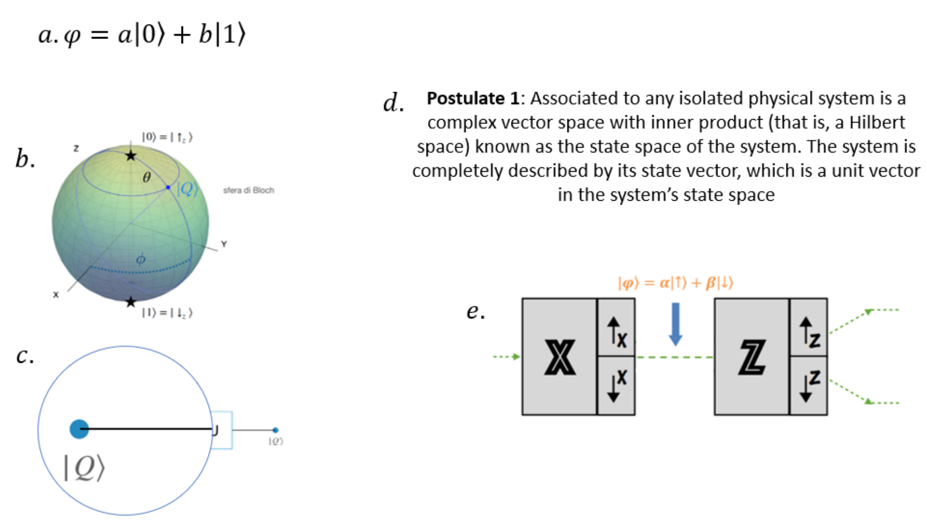

Fig. 5.1: the five representations met and given to students for reflecting on the concept of state and superposition principle. In order, we have called: a. algebraic, b. geometrical, c. circuital, d. axiomatic and e. experimental representation.

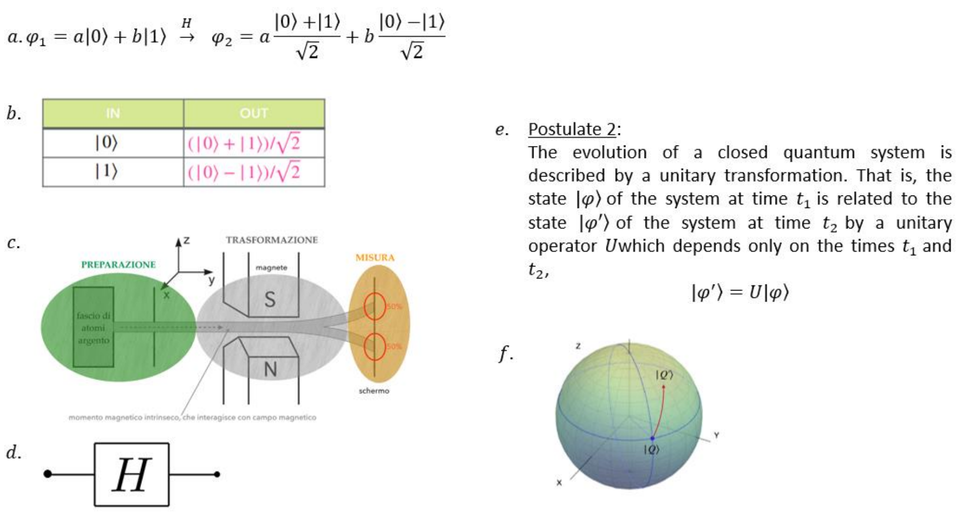

Fig 5.2: the five representations met and given to students for reflecting on the concept of state manipulation and evolution. In order, we have called: a. algebraic, b. logical, c. experimental, d. circuital, e. axiomatic, and f. geometric representation.

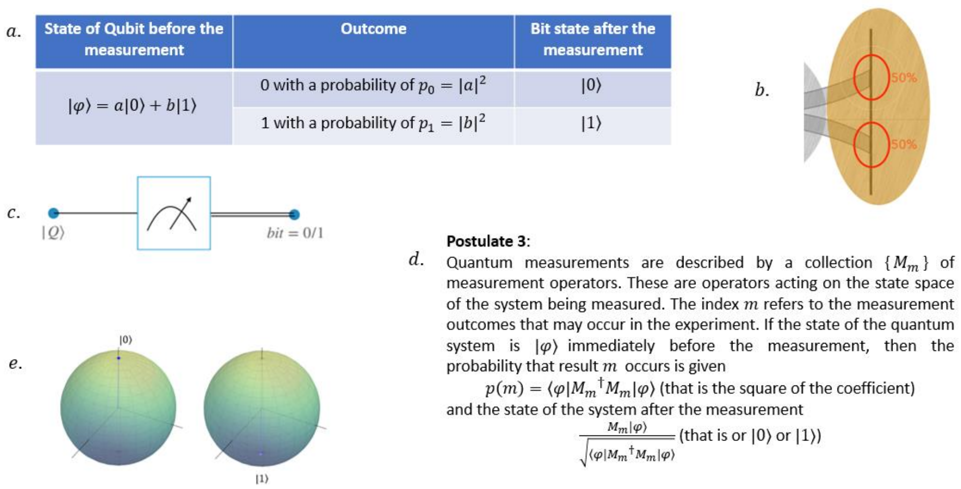

Fig 5.3: the five representations met and given to students for reflecting on the concept of measurement. In order, we called: a. algebraic, b. experimental, c. circuital, d. axiomatic and e. geometric representation.

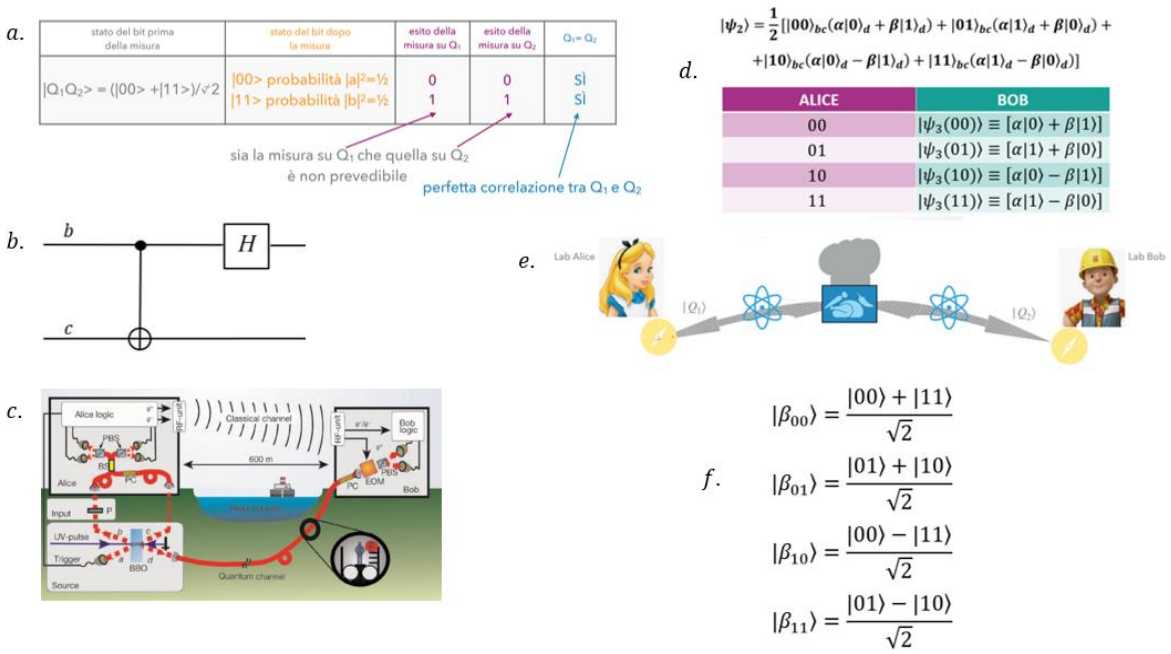

Fig 5.4: the six representations met and given to students for reflecting on the concept of entanglement. In order, we called: a. logical, b. circuital, c. experimental, d. algebraic, e. experimental, and f. algebraic representation.

For each concept we asked students to answer and discuss in groups the three following questions:

1. To what extent from 1 (not at all) to 6 (completely) do these representations help you understand what is meant by quantum state manipulation/transformation/evolution?
2. What are the characteristics that you would extract from the representations a, b, c, d, and f that you consider fundamental?
3. Are there, and if so, representations that you consider redundant because they have in themselves details and information that are not relevant?

The students, divided randomly into groups, had about 1 hour to complete the questionnaire. The results are presented in the following sections.

The second part of the teamwork, instead, aimed to investigate the "meta" and cognitive dimensions. It was designed to investigate students' meta-representational skills, that is the ability of students to critique and compare different representations (diSessa, 2004, diSessa & Sherin, 2000).

In this part, we gave students a list of adjectives, elaborated starting from DiSessa, Hammer, Sherin, & Kolpakowski, 1991) and diSessa 2004. The final list is reported in table 5.2, the qualities we chose have different natures. Some of them belong more to a dimension of knowledge per se such as *complete*, *precise*, *consistent*, *general*, *coherent*, etc.; the others have a more cognitive nuance, such as *schematic*, *familiar*, *transparent*, *conceptually clear*, and so on.

Students were asked to discuss in groups what adjectives they associate with the different representations and why.

Table 5.2: final list of qualities

Adjectives attributable to the <u>knowledge dimension</u>	Adjectives attributable to a <u>cognitive dimension</u>
• Real • Economical • Complete • Precise • Generic • Homogeneous • Cohesive • Consistent • General • Coherent	• Schematic • Accessible • Familiar • Transparent • Compact • Conceptually clear • Abstract • Concrete

So, to investigate students' ideas about the representations, we gave them the list of qualities and we organized a sort of board in which the representations were clustered per type, that is all the algebraic, geometric, axiomatic, circuital/logical and experimental representations (Fig. 5.5). We asked students to choose some qualities and/or find new ones and discuss in the groups and write in post-it the reasons why they associated a particular quality to a representation.

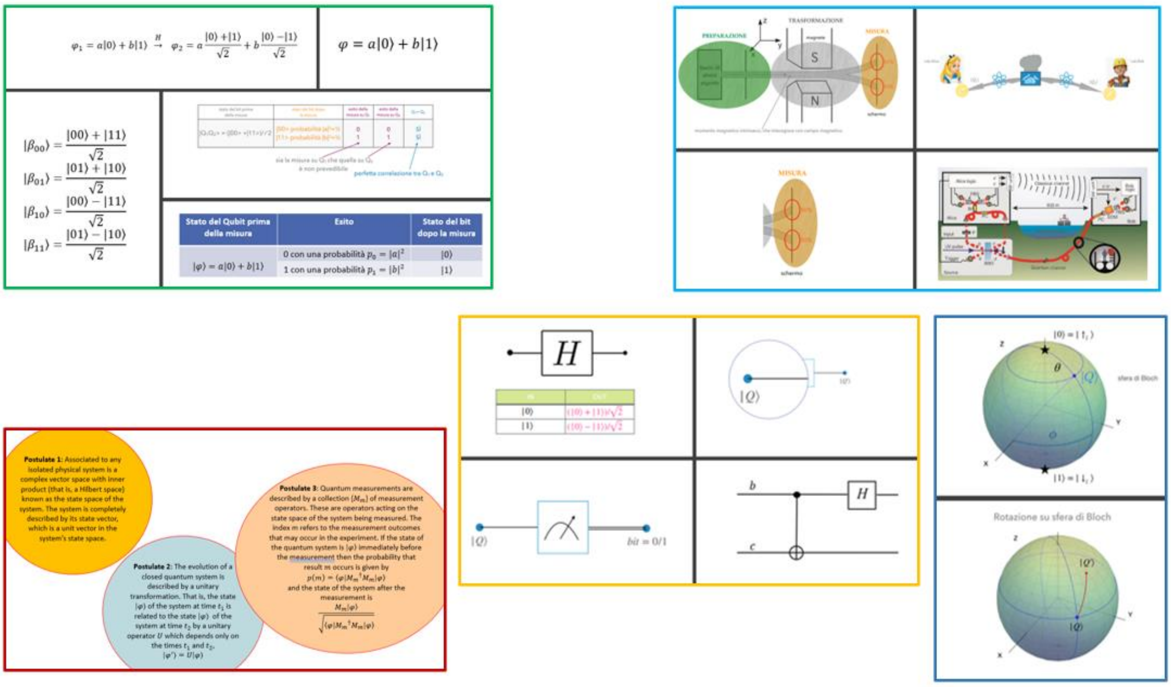

Fig. 5.5: Board in which representations are clustered per type

5.1.2 The focus group

The focus group was designed after the analysis of the teamwork (section 5.1) and the sharing of some impressions with the teachers that held the activities on the history of classical computers and the Boolean logic (Paola Fantini and Michela Clementi). The goal of the focus group was to discuss, with a restricted group of students, the main results coming from teamwork. Starting from them, we decided to structure the focus group into two main parts. The first one consisted firstly of an individual moment in which students wrote down i. what understanding means for them and ii. what understanding

through representation means for them and what features a representation must have to promote the understanding. Then, all together they shared and discussed their personal answers.
The second part revolved around four key adjectives: *compact, schematic, familiar, and accessible*. We choose to focus on these adjectives because from a preliminary analysis of the teamwork and the group discussions we noted that students there was not a consensus, that is they associated them with different representations. Furthermore, these adjectives are related to our initial conjecture: the circuital and the logical representations can have the learning potential to provide new synthetic scenarios and promote the acceptance of quantum physics as a personally reliable description of reality (see section 4.2). In particular, the scenario is schematic since the circuital and logical representations provide synthetic information about the relationship and connections between the different qubits and how they can be transformed as well as a holistic view of the system. The synthetic scenario is also compact because it allows us to touch all the postulates of quantum mechanics. The familiarity and accessibility depend on students' previous knowledge that they usually have about truth tables, Boolean algebra, and the idea of physical circuits. So, they are familiar both with the logic and the graphic shape.

During the focus group, we asked students to share the meaning of these adjectives and discussed if and what representations they associate with these qualities and why.

The focus group was carried out as a semi-structured protocol, and it involved 8 students chosen with the help of their teachers. The choice of the students followed two criteria: one strictly related to the research, and one related to the school's needs. Since the research goal was to understand better the roles of representations that emerged from the teamwork, we selected students who systematically attended the activities and, according to the teachers, could be suited and at ease to report the discussions and reflections within the groups. We also needed a selection of students who were representative of the different groups and diverse among each other. As for the school's needs, we did believe that these activities should have, first of all, a positive impact on the schools and, for this reason, we followed the school's indications about the group that, according to the general plan, could value better these activities within their personal extra-curricular path. The result is that the bunch of students was representative of the variety of students according to cultural interests, learning attitudes, closeness to physics, participation in classroom discussions, and performance level in physics regular classes.

In particular, Arturo, Andrea (from group 3), and Carlo (from group 1) immediately showed a strong interest in physics and in the topic of the course, and they have distinguished themselves for the high participation and contribution during in classes. Teachers confirmed that their level of performance in physics is high.

Lucio (group 2) showed to have an interest in physics and in the topic of the course, but he had never participated so much in class discussions. The teacher highly recommended involving him since he has always had a high level of performance in physics tasks.

Marta and Matilde (group 3) did not have a strong interest in the course and in physics, but they have sometimes participated in class discussions. The teachers, in addition to confirming our feelings, have reported that they usually have a low level of performance and in particular Matilde also has a conflicted emotional engagement with this discipline.

Igor (group 3) often showed interest toward the course and, in general, physics. He usually participated a lot and contributed to class discussions. Igor's attitude was confirmed by the teacher, but, from the performance point of view, he usually had a medium level of performance.

Finally, Diana (group 1) did not seem to have a particular interest in the course and physics, but she usually participated and contributed to class discussions. The teacher, who had been following her for 3 years, said that she usually had a low performance in physics, but this year Diana had gained a lot of confidence and had solved some emotional conflicts she had shown previously.

5.1.3 The "Quantum Atelier"

The quantum atelier is an extracurricular project born after the PLS course. As discussed in Chapter 2, Rimini's implementation was revised in light of the FEDORA projects and of a perception that accompanied us for all the implementation, and, thanks to the project, we were able to verbalize. In fact, we have often noticed "difficulties" in the students in facing some of the future-oriented activities, such as the last activity in which they had to project themselves in 2040, imagining a problem of contemporary society that they care about (e.g., too high CO_2 emissions) solved, and figure out what contribution quantum technologies could have had.

As already described, quantum technologies do not yet belong to the collective imaginary, which is usually created thanks to art, literature, music, etc., just thinks of themes such as artificial intelligence that has populated and still populates books, films, and art installations. Quantum technologies are not yet part of it, but we continue to talk about a revolution that, like every revolution, has isotropic characteristics. Moreover, as it has often happened in technological revolutions, the other disciplines not only provided a language to tell what was happening but also provided perspectives and lenses to interpret, elaborate and understand the changes.

In this implementation for the first time, we tried to reflect and make the students reflect on the scope of the Second Quantum Revolution as a cultural revolution, about if and how it is changing the dichotomy humans-nature, humas-technology/science, and nature-technology/science. We designed the activities described in chapter 2 and involved a duo of conceptual artists (Antonello-Ghezzi) to reflect on the role that art can have not only as a language that can talk to a broader audience but also, mainly, as a perspective that can provide new lenses and new scenarios to science itself.

At the end of the PLS some teachers at the school, in particular, Maurizio Giuseppucci (Italian teacher and artist), Paola Fantini, Michela Clementi, and Fabio Filippi (mathematics and physics teachers), involved students in an interdisciplinary project to deepen the mysteries of the first and second quantum revolution and to reflect on the science-art relationship. The project lasted about 5 months and involved 6 students (Diana, Carlo, Arturo, Andrea, Marta, and Matilde). It ended with the development of three exhibitions concerning different themes, mainly the first quantum revolution: from the role of the observer and the collapse of the wave function to the superposition principle and the effect of measurement to the challenge that these themes pose, to the common intuition and perception (of the world in which we live in) when we "enter the substrate" (from the macroscopic to the microscopic world). About one month after the conclusion of the project and the final presentations of the artwork, we interviewed the students. The collective interview was with voluntary participation, three (Carlo, Arturo, and Andrea) out of six students participated. The questions of the interview protocol are reported in table 4.3.

5.2 The analysis of the teamwork

5.2.1 The conceptual questionnaire

The first phase of the experiment, on the 5th day of the course, lasted 2 hours and involved 27 students divided into groups.

The teamwork consisted in dealing with the following three questions:

> Q1. To what extent from 1 to 6 each representation helps you to understand the basic concepts?
>
> Q2. What are the characteristics that you would extrapolate from the different representations that you consider fundamental?

Q3. Are there, and if so which, representations that you consider redundant because they do contain not relevant details and information?

The detailed analysis of this first one is reported in Annex 3. I report here the discussion of the main results we carried out. In particular, we organized the data in order to point out trends and, therefore, draw some conclusions concerning the representations that the groups found more and/or less effective for their understanding. Figures from 5.6 to 5.9 show the comparison between the average values associated with the different representations clustered per kind of concept.

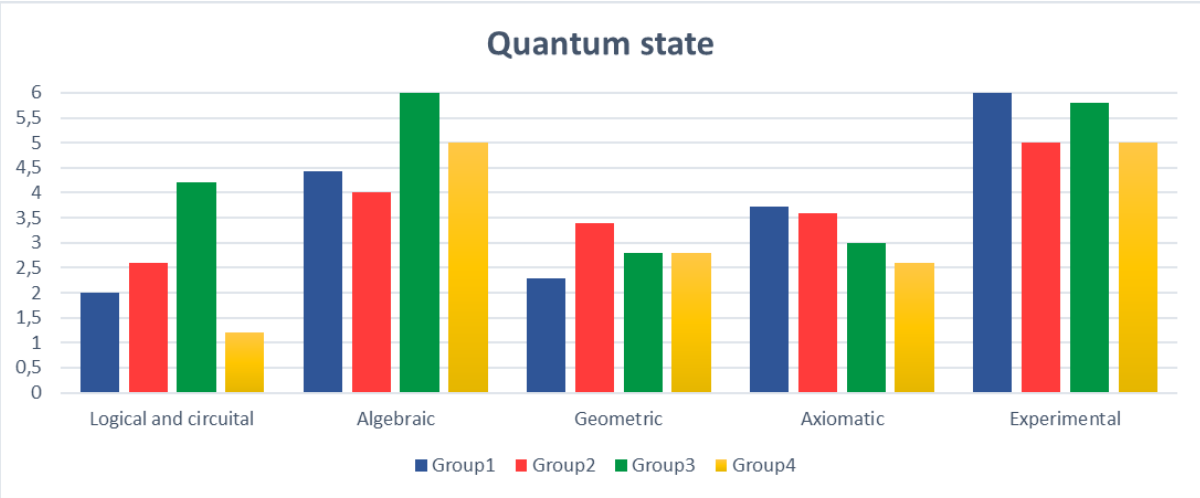

Fig. 5.6: comparison between groups' average values associated with the different representations (Fig. 5.1) concerning the concept of the quantum state.

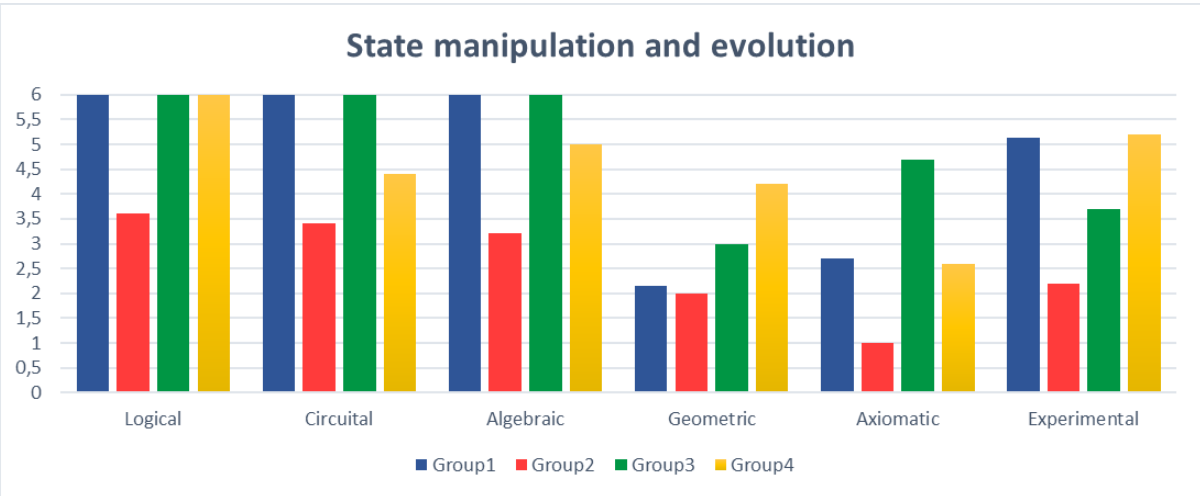

Fig. 5.7: comparison between groups' average values associated with the different representations (Fig. 5.2) concerning the concept of state manipulation and evolution.

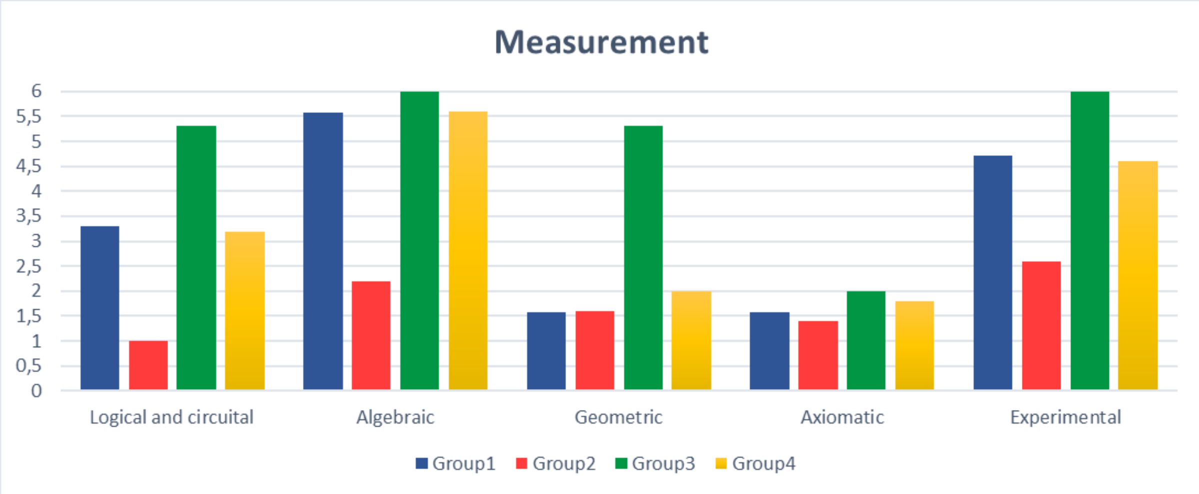

Fig. 5.8: comparison between groups' average values associated with the different representations (Fig. 5.3) concerning the concept of quantum measurement.

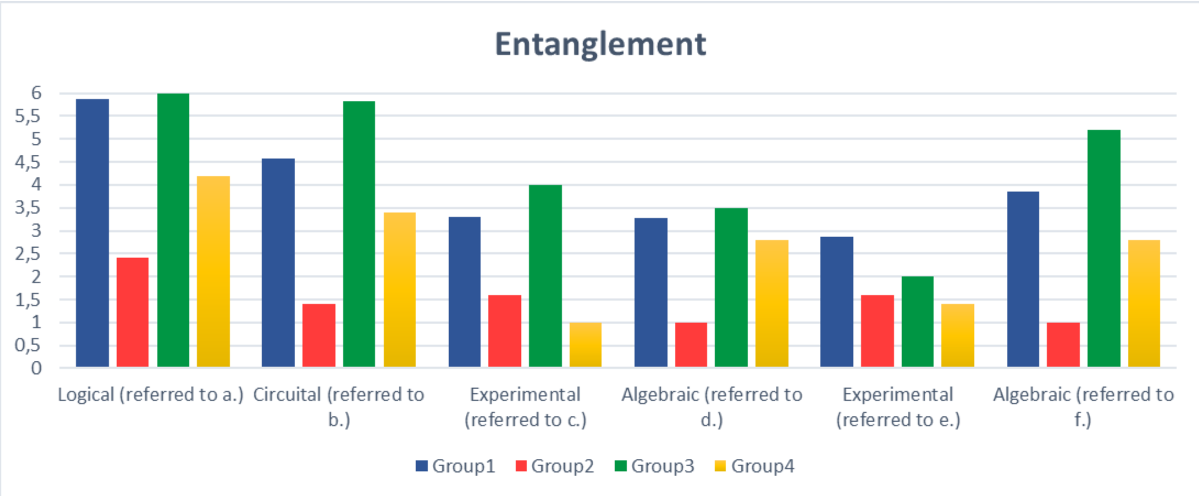

Fig. 5.9: comparison between groups' average values associated with the different representations (Fig. 5.4) concerning the concept of entanglement.

As regards the concept of the quantum state, as shown by the histogram in figure 5.6, the experimental representation proved to mostly help students to understand it, followed by the algebraic one. While the logical and circuital, and geometric representations proved to be the less effective. As regards state manipulation and evolution, the logical, the algebraic, and then, the circuital representations proved to be the most effective, while the axiomatic and the geometric are the less effective ones (Fig. 5.7). The algebraic and the experimental representation proved to be the ones that help more the students to grasp the concept of quantum measurement, while the geometric, and especially the axiomatic ones seem to be not useful at all (Fig. 5.8).

As regards the concept of entanglement, the logical and the circuital representations convince more the students while the two experimental (Fig. 5.4e) and the algebraic/logical one (Fig. 5.4d) proved to be quite ineffective to help students to build a mental image about the concept.

The average values that the students expressed, as we expected, fluctuate a lot. In fact, we purposely presented a variety of representations some of which were present in many meetings and then, more familiar to students (like the algebraic, the circuital, and the logical representations), other were introduced once (as the axiomatic representations). However, in general, the students were able to reason on the representations and evaluate them according to their personal way of understanding.

It is interesting to see that group 2, except for the experimental and algebraic representations of the concept of the state, has always given rather low votes. As mentioned, it is not a type of teamwork and reasoning to which they are accustomed, and they may have found it not useful or devoid of a particular meaning. Unfortunately, we do not have any comments from this group. One thing that can be said is that students could find the reasoning about concepts through different representations not useful from a personal perspective, and/or they could find the reasoning about representations individually ineffective.

It is interesting to note that the algebraic representation received the highest ranking on three of the four concepts. This suggests that the Dirac notation is within the reach of secondary school students and that they find it accessible. Even if some studies argue that students' difficulties are more in the notational system rather than in the fundamental conceptual difficulties or the high level of abstractness of the subject matter (e.g. Singh & Marsham, 2015; Zhu & Singh, 2013; Zhu & Sigh, 2012), some works proved the effectiveness of the Dirac notation, especially in terms of calculation (e.g., Serbin & Wawro, 2022; Schermerhorn, Passante, Sadaghiani & Pollock, 2019; Wan, Emigh, & Shaffer, 2019; Gire & Price, 2015). These studies provide external support to the data we have collected and further show the efficacy of Dirac notation in promoting students understanding in particular for the concept of state manipulation and evolution.

Both in the case of the quantum state and measurement students have found more effective the experimental and the algebraic representations. This can suggest that they found both the physical meaning of the concept and its formalization accessible and suitable for their personal understanding. For state manipulation and evolution, and entanglement instead, the preference is for the logical and circuital representation. As regards the state manipulation and evolution, the high evaluation of these two representations, as well as the algebraic one, can be linked to computational aspects, that is how to operationally carry out a calculus. As regards the entanglement, the preference for the logical and circuital representations can be due to the immediacy and the easy access to the information that these two representations give, therefore linked to a low cognitive load. As anticipated in section 2.6, the logical representation (Fig. 5.4, a) allows, in fact, for easily recognizing the mathematical relationship between two terms. Similarly, in the circuit case, the presence of the CNOT can immediately make us think about the creation of a relationship between two qubits.

As I anticipate in section 2.6, we are working to add an activity that can value the entanglement, one of the mysteries of quantum mechanics, from an experimental perspective also in light of the recent news of the 2022 Nobel Prize in Physics by Alain Aspect, John F. Clauser, and Anton Zeilinger[15].

5.2.2 The representations board

The last hour of the fifth day was dedicated to investigating a more "meta" nuance of students' understanding through representations. In particular, we gave students a board where we clustered the representations per type and a list of qualities related both to knowledge and cognitive dimension. We asked them to start from this list (table 5.2) and discuss which of these qualities, and eventually find new ones, they associate with the representations clarifying the reasons. The results of the teamwork are organized per group and reported in Figure 5.10. The English translation of the students' work, organized per kind of representation, is reported in table 5.2.

[15] https://www.nobelprize.org/prizes/physics/2022/press-release/

Fig. 5.10: Delivered boards of group 1 (a), group 2 (b), group 3 (c), and group 4 (d)

What interests us, in this case, are not so much the adjectives that the single group has associated with each type of representation but rather see in general what were the adjectives used and if the students have found others.

Table 5.2: list of qualities that students associated with the different representations. In red we have marked the qualities added by the students.

Algebraic representations	"Objective: it is not misunderstood, the interpretation is unique and mathematical"
	"Precise: the calculation provides "one" answer
	"Accessible: multifaceted application"
	"Complete, schematic, conceptually clear (understandable concepts), transparent"
	"Abstract: although it is not difficult to understand, it is a representation far from a concrete meaning"
	"Schematic and accessible: it is easy through the scheme and the formulas to understand what it is"

	"Familiar: using of classical mathematics"
Logical and circuital representations	"Schematic: Simplify a concept into a schematic/diagram
	"Consistent and conceptually clear: through simple and clear images it is possible to understand what concept we are talking about"
	"Schematic: representations in which it is more understandable to trace the key concepts"
	"Schematic, economic, concise, conceptually clear, coherent"
	"Conceptually clear: it exemplifies the concept without depriving it of meaning"
	"Schematic, coherent and precise: through precise schemes, it is possible to understand what enters and what comes out"
	"Transparent: makes the concept clear"
Geometric representations	"Generic: does not provide a full explanation. It is not enough to understand it"
	"Compact: reduces the concept to basic objects"
	"Not accessible: it is difficult to understand"
	"Homogeneous: every concept is reducible to the Bloch sphere"
	"Generic: does not offer detailed information"
	"Economic: there are few data and it is difficult to interpret the figure"
	"Compact: graphically standardized"
	"Abstract: mathematical processing"
	"Transparent: the argument is transparent because I can't see it"
	"Homogeneous: versatile"
	"Generic, abstract, compact"
	"Precise, complete, objective, general, coherent"
	"Abstract: totally disconnected from reality, it provides exclusively theoretical

	concepts”
	“Objective: there are many statements or rules that have been tested and are therefore true”
	“Objective: there are many statements or rules that have been tested and are therefore true”
	“General: does not take into account the multiplicity and variety of phenomena”
Axiomatic representations	“Coherent, consistent, Cohesive”
	“Not accessible: discursive and [full of] technical terms”
	“Complete: Provides general definitions”
	“General: being a postulate it concerns everything that interests it
	“Compact: There is a lot of information that is also difficult to understand”
	“Real: it has to do with experimentation, it goes beyond the theoretical perspective”
	“Real: application in reality”
	“Concrete, objective”
Experimental representations	“Transparent: universally understood”
	“Accessible: draws on the real object, it is more easily understood”
	“Generic: it describes general cases to make people understand what happens”
	“Concrete and real: it helps to understand what is actually happening”

Looking at the qualities some general considerations can be done. Mathematical representations, both algebraic and geometrical ones, have been associated mostly with qualities belonging to the dimension of knowledge. “Objective”, “precise”, “complete”, “homogenous”, and “generic” (even if it is used in different ways) are all aims and values, qualities that can be associated with mathematics and its rigor as it is conceived in school. Furthermore, with the algebraic representations students associated also a few cognitive qualities such as “familiar” (“using of classical mathematics”), “accessible”, “abstract”, “schematic”, “transparent”, and “conceptually clear”. The familiarity for the students is linked to the use of “classical mathematics”, the schematicity and accessibility are linked to the idea that “it is easy through the scheme and the formulas to understand what it is”. This is in line with the actual high school students’ preparation in Italy, who, in their school years, get acquainted with some basics of linear algebra.

The axiomatic representation, in which postulates are conceived as elements of a formalized system, is associated with qualities such as "precise", "complete", "objective", "general", "compact", "coherent", "cohesive", and "consistent". Also in this case, even if in the first part of teamwork the axiomatic representation often resulted as one of the least effective ones, students associate with it something that is formal and sophisticated from a mathematical point of view. Even if in literature it seems there are no studies about high school students' understanding of the postulates of quantum mechanics, some research works showed that also students at a higher grade, despite they have mathematical skills, could have a lack of conceptual understanding of the formalism and postulates of quantum mechanics (Marsham & Singh, 2019; Marsham & Singh, 2015).

To the experimental representations students associated qualities such as "real", "concrete", "objective", and "concrete", suggesting that, even if we have no possibility to experience these experiments in laboratories, this dimension is often linked with reality. This is coherent with other studies (e.g., Mannila, Koponen, and Niskanen, 2001) and with the study presented in chapter 3 (Satanassi, Ercolessi, and Levrini, 2022, p.21).

Finally, the logical and circuital representation students associated both qualities that belong to the knowledge dimension, such as "consistent", "economic", "concise", "coherent", "precise" and to the cognitive one, such as "conceptually clear", "schematic", and "transparent". In general, these qualities seem to suggest that students perceive this representation as something functional and effective to "do something". Also the presence of the cognitive adjective, high in frequency, can suggest to us that students perceived this representation as the "simplest one" probably with the algebraic representation. This is in line also with the results of the first focus group in which, except for the concept of a quantum state, the circuital and logical representation had often received a high ranking.

5.3 The focus group and the choice of the case studies

The focus group, lasting 2 hours, involved eight selected students from different groups. As already mentioned, these students were selected by combining research and school needs (see section 5.1.2).

In the following, I present an analysis that aims to investigate and deepen if, how, to what extent, and why students perceived the representations as useful and effective (or not) for their understanding.

The focus group is very rich, and its analysis was not a linear process. The challenge consisted in organizing the data in order to find the angle for fine-grained analysis. So firstly, we tried to draw a big picture of the focus group discussion that could guide us in the selection of the most significant data. The result of this analytic phase is described in the next section ("A bird's eye view" of the focus group). It allowed us to select four students who became our case studies. On them, we built their profiles along the different parts of the focus groups, investigating if there are idiosyncratic signatures, and eventually differences and similarities between the cases. Finally, we chose the lens of sensemaking to <u>*zoom in* on the data and explore closely the robustness of students' explanations</u>. From this perspective, we found particularly suitable the sensemaking metric designed by Shulamit Kapon in "Unpacking sensemaking" (2017). The analysis was carried out by exploring the robustness of students' explanations along three dimensions: (1) i*ntuitive knowledge*, (2) *mechanism*, and (3) *framing* (Kapon, 2017).

The analysis is based on the transcript of the discussion. For the transcription we used the following conventions:

1. "... ", Three dots indicate an untimed pause.
2. "[...]", Three dots in square italic brackets indicate that some material of the original transcript has been omitted.
3. "[text]", Italic text in double parentheses provides extra-linguistic information such as references to related bodily movements and interpretations.

5.3.1 A bird's eye view of the focus group

This section includes a bird's eye view of the focus group, that revolves around some pieces of evidence from the teamwork. In particular, we focus the discussions on some adjectives that, from a preliminary analysis of teamwork and whiteboards, seemed to be fundamental and whose meanings did not always seem shared either between the different groups or between the same members of the group.

Dividing thematically the focus on which the discussion was organized, we identified 3 parts (A, B, C). Part A refers to the initial part of the focus group in which we asked students to share i. what understanding means for them, ii. when they have the feeling of understanding, and iii. the features that a representation must have to give a sense of understanding. Part B includes a discussion around the meaning of compact and schematic representations and the differences between them. In particular, the students discussed the characteristics by which a compact or schematic representation is effective and fruitful for their understanding, individuating the representations that best fit with their idea of, for example, effective compact representation. In part C, the discussion revolves around the accessibility of representations. Similarly to Part B, students discussed the differences in meaning between the accessible, transparent, and familiar and the characteristics that a representation must have in order to be accessible to them, and which representations respect more their needs.

From figures 5.11 to 5.16, a visual representation of the focus group divided into parts is reported. The table contains a schematic view of the interventions. The interventions have been coded with a color code according to the type of intervention (Table 5.3). The contributions of the students are codified through a string that contains the letter that corresponds to the thematic part and a number, which refers to the chronological order of the intervention (es. B5). The labelled contributions are the key-ones that we used in the analysis. As the figures show, we focus on four students because of the frequency of their participation of the discussion and, therefore, the emergent overall picture of the students' contributions.
For building students' profiles, as the pictures from 5.11 to 5.16 show, we mainly consider the orange students' contributions, following table 5.3, namely the parts of students' talks in which they hare and build new knowledge.

Table 5.3: Color code of the focus group's birds' eye view.

Color code	
Grey	Questions to start sharing and to mediate, questions and clarification expressions of the task (e.g., *So do we think about words first or do we link them directly* [to the representation]?)
Orange	Expressions in which students share meanings and build new knowledge
Light green	Expressions of consent (e.g., "*the limits on the Cartesian plane are much simpler than... that define them that is... are much simpler than what you think...* ")
Light pink	Expressions of dissent (e.g., *"in my opinion it is the exact opposite*" and explanation)
Light blue	Background noise and/or laughter
Green	Hint of an adjective or word (e.g., "*exhaustive?*")

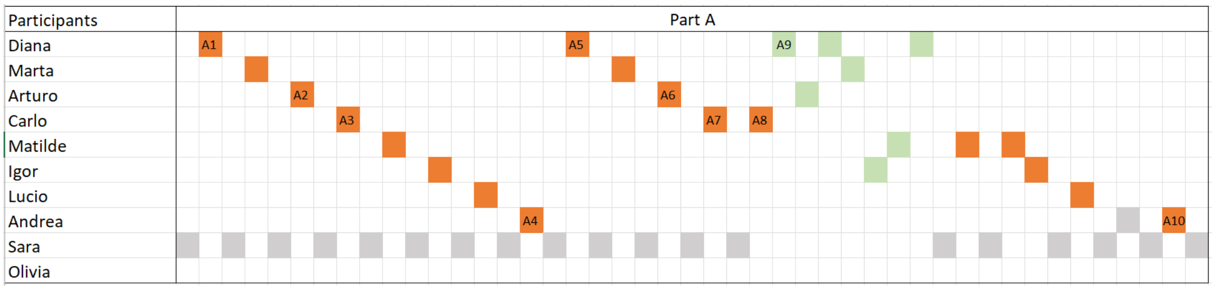

Fig 5.11: Part A: students share what understanding and understanding through representations mean for them. Their explanation were mainly encoded with the orange color since they are sharing personal meanings.

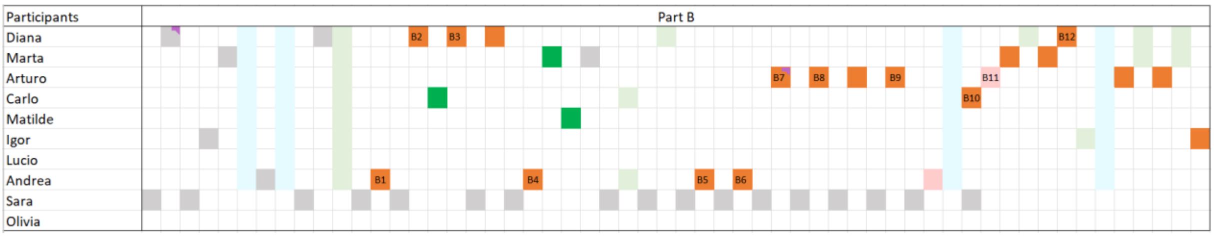

Fig 5.12: Part B: Students talk about the meaning of compact and schematic representations and the differences between them. In orange we coded students' interventions in which they were sharing the meaning of the two adjectives. The presence of other interventions coded with other colors (such as consent, light green, and dissent expression, light blue) suggest that they co-construct the meanings as well as the presence of background noise. As we can see, the students started to participate differently to the discussion and this was one of the criterion we used to choose only four case studies (the codified ones).

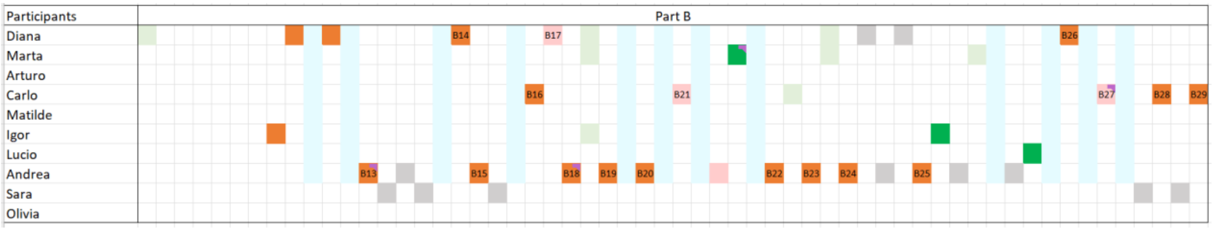

Fig 5.13: Part B: Students' discussion about the meaning of compact and schematic representations and the differences between them.

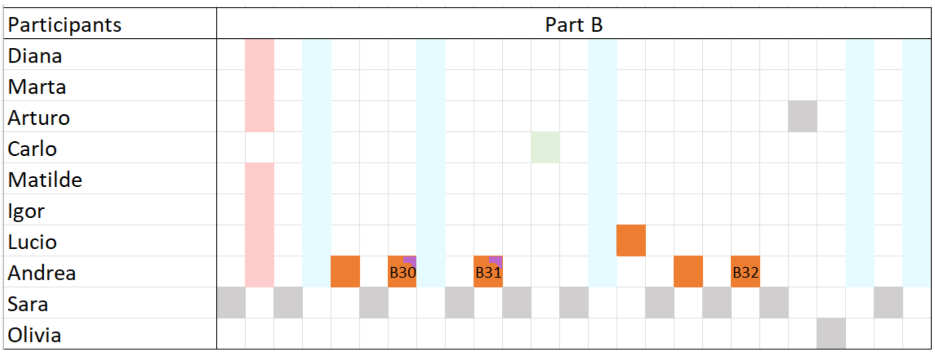

Fig 5.14: Part B: Students' discussion about the meaning of compact and schematic representations and the differences between them.

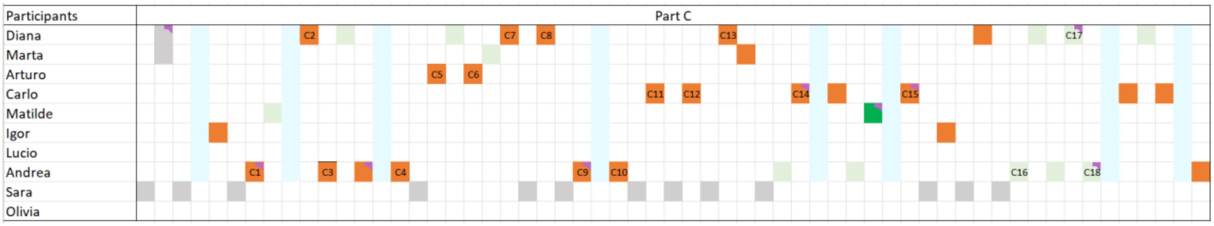

Fig 5.15: Part C: Students' discussion about the adjectives accessible, familiar and transparent. In orange we coded students' interventions in which they were sharing the meaning of the two adjectives. The presence of other interventions coded with other colors (such as consent, light green, expression) suggest that they co-construct the meanings as well as the presence of background noise. As we can see, the students started to participate differently to the discussion and this was one of the criterion we used to choose only four case studies (the codified ones).

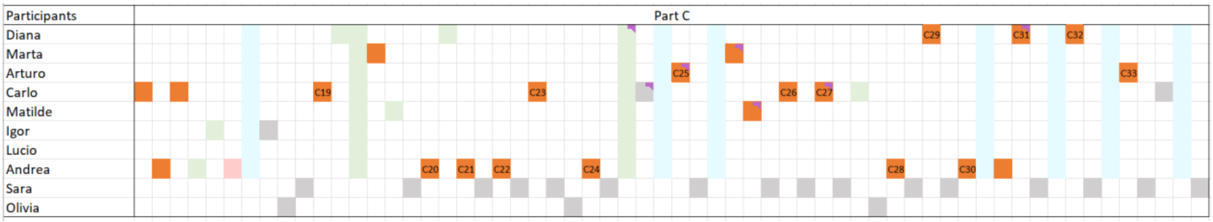

Fig 5.16: Part C: Students' discussion about the adjectives accessible, familiar and transparent.

As we can see from figure 5.11, in the first part of the focus group (Part A) the participation of students is the same since all the students were asked to share what understanding means for them and the features that a representation must have to give them a sense of understanding. The kind of participation of the other two parts B and C is instead very different (see figures from 5.12 to 5.16). The kind of participation in the discussion led us to focus on four students: Diana, Andrea, Carlo, and Arturo. During the focus group, they were mainly involved and engaged in the discussion and their contributions were meaningful not only in terms of frequency but also of the richness and consistency of interventions. In particular, we chose Diana because she always broke the ice, she was the first to intervene and she get herself in the game. Andrea was chosen because it was the student to intervene more, he often led and gave him the rhythm to speech especially in parts B and C. Arturo and Carlo were chosen because, Like Andrea, they intervened more than the other students and had a decisive contribution in finding a shared meaning of the various adjectives. Those aspects made particularly meaningful the contribution of these four students and allow us to build more complete profiles.

5.3.2 The case studies

Once we made the audio transcription regarding the focus group, we focused on the protagonists and organized the data into four profiles that contain the contributions that they provided in the three parts of the focus group. While the interventions of the students in part A are very similar since we explicitly asked everyone to participate, the other parts (B and C) are very different and see the students involved in different ways. to organize the data, we had to make several choices.

To make profiles comparable, we have organized them in chronological order and according to the structure:

i. what it means for the students to understand and understand through representations;
ii. the personal meaning they associate with the compact, schematic, familiar, accessible, and transparent qualities,
iii. representations that best satisfy their personal way of understanding,
iv. their arguments (e.g., motivations, examples, and metaphors that they use to better explain their ideas).

Built the four profiles, we pointed out the four signature ideas, by identifying words or expressions that the students repeated during the focus group. Then we discuss the similarities and differences of the signature ideas in terms of the different representations that students have chosen as those that best fit with their epistemic and cognitive needs, triangulating them with the results of teamwork. The signatures' ideas we pointed out are contextual, namely are related to the focus group and the questions we posed.

Diana's contribution to the focus group

From the very beginning of the focus group, talking about what understanding means for her, Diana says:

> "*I think I understood one thing when I answered all my questions because... I mean I can retrace in my head the steps we have taken and ... I mean ... yes... I can practically get there by myself... so I can retrace them...*" (A1)

The key expressions that describe Diana's understanding are: "*answering to all my whys*", "*retracing*", "*on my own*". Diana, differently from the others, immediately feels the need of referring to the representations - the circuital and the logical (truth table) ones – to better express her point and her ideas of *retracing back:*

> "*For example to me those of CNOT and Hadamard with below tables... that is to me enough to be able to retrace in my head...*" (A5)

She continues explaining why and what features representations must have to support or boost her understanding:

> "*[…] they must be clear, they must all have... maybe even... what I have to replace ... if I have those tables, that... I know how the logic gates work through the representation... I can solve the problems we had and ... however I can also have the concept clearer*." (A5)

The representations for her have to *be clear* and to contain the information about "*what replace"*, and the truth tables are conceptually clear because they give her the information about *how the logic gates function through representation*. This Diana's comment highlights her need of dealing with representations that are functional and that have usefulness.

The role of representation through tables and graphics for understating emerges also in her example

> "*Or I am studying the limits in mathematics... that is, I think the limits with the scheme with the Cartesian plan is the best... [...] they become very simple in my opinion when you place them next to their graphic representation...*" (A9)

where for Diana the limits *become simple* when *next to them there is their graphic representation*.

Diana continues to explain her idea of conceptual clearness and the functional role of representations for her understanding.

> "*It does not miss anything but at the same time... is ... is circumscribed ... is... [...] exhaustive [...] ... but at the same time... it is not dispersive... that is a thing... linear in my opinion... that is... very... compact in my opinion also with extra concepts that is... exhaustive and linear [...] clear ... […] not neglecting the fact of clarity... namely that aspect of clarity... because maybe compact [...] ... it may seem even a little more confusing... that is, in something we have everything and I can not even retrace... [...] [compact] if a concept is very broad but I can circumscribe it neatly into something...*" (B2, B3, B14)

An effective compact representation, as she remarks, *must not overlook the aspect of clarity* and *it does not omit anything, it is circumscribed, exhaustive, non-dispersive*, and *linear*.

For Diana, therefore, a representation has to be familiar and accessible because it has to "*give the basis to do something more complex*" and has to help her to "*retrace back*":

> "*I think... accessible is a bit.... [...] I associate something more familiar... accessible besides to give me the basis for... to do something more complex... must also be more familiar to me, because I have to be... must be accessible to me, that is... I must be able to retrace back...*" (C7, C8)

Stressing what is accessible for her, Diana marked that the most accessible representations are the logical and circuital ones, while the geometrical representation is the hardest to be understood:

> "*the spheres, the orthogonal planes I never... I mean I don't like them, I never understood them... for me they are not accessible and transparent... so for accessible and transparent I come back on tables*..." (C29)

Finally, recalling her initial idea that a representation must give the information about "*what replaces*'', Diana remarks the importance of the functional role of the representations: She associates the *usefulness* feature more with algebraic representation.

> *"If we also want consider also the usefulness we choose this [indicating the algebraic representation] [...] in the sense of the resolution [calculation] of... in mathematical resolution practice." (C31, C32)*

Andrea's contribution to the focus group

For Andrea understanding a concept means to internalize and assimilate it, to be able to expose it instinctively, without a rigid scheme:

> "[for me understanding a concept is] *internalize it we say and ... not so much at the mnemonic level... but have it... assimilated ... just to be able to... [...] expose almost instinctively ... that is in the sense without having a rigid scheme that takes me from A to B and then maybe gets to B... in way... instinctively because I feel inside the concept...*" (A4)

Andrea continues describing in detail the characteristics that a representation must have to be effective for him:

> "[as regards understanding through representations] *the idea that it must have... have a squaring ... that is, let's say... a being accomplished in itself and... [...] in the sense basically exhaust all aspects... but in a synthetic way and say... maybe... [...] also almost from an aesthetic point of view... in non-graphic representations... have a certain elegance... that is in the sense... be... synthetic but exhaustive and say... from a representation can derive other concepts... [...] a formula...[...] maybe it can allow to understand for example in a simpler way something... for example... I do not know... if we have... a description of a motion for example... instead of having to look at all the individual... the individual laws, the individual relationships... maybe to have an overview can be more useful... then obviously the graphical representation does also this... but we say in my opinion the elegance... and a certain compactness can also concern the... just a formula not only a graph..."* (A10)

Therefore, for Andrea, a representation, in order to help "*to understand something in a simpler way*", must have a "*squaring*", "*must be complete in itself*", "*exhaust all the aspects*" in a "*synthetic way*", must "*make explicit the terms*" and must be "*elegant*" and "*compact*". He then describes some of the features he associates with his idea of effective representation:

> "*compact... not so much in the... in being... economic necessarily... [...] maybe more compact in being... ehm ... let's say... more than economic... in being a little... compressed... as I can say... not in neglecting aspects... [...] ehm... so let's say a compact representation is...let's say... we can say simple between quotation marks... in the sense because... does not seem to have a*

> *complexity but in reality... however inside it does not... does not leave the strands in half ... let's say... is still able to exhaust itself.*" (B1)

The compactness of a representation lies in its "*being compressed*", in "*not neglecting aspects*", in being "*simple*" but "*not leaving strands in half*", "*exhausting itself*". Andrea tries to better explain his idea of compact representation through an image of a baseball or tennis ball that "*has its own completeness*" and it is "*restricted*", "*with inside a lot of information*" that is it is "*dense*", and "*the information is ordered*".

> "*It's a ball... a baseball or a tennis ball... it's the same... that is in the sense that in itself it has completeness... and is ... restricted.... inside it contains a lot of material information, that is a lot of material... [...] it is dense... however... this material is ordered in the sense... the baseball is layered...*" (B4)

The relationship between the density of information and order becomes the key idea that describes the features that a representation has to respect to activate the understanding for Andrea:

> "*That is, density centers on order, in the sense... because density without order... in the sense... [something] could be dense but... it would not be clear... so [...] we couldn't even define it as basically compact... because in the end it would leave things in the middle ... [...] in the sense... density is a necessary condition but not enough for it to be compact...*" (B5, B6)

If there was no relationship between the density of information and order, the representation would lose clarity, *"leaving strands in the middle*".

In order to solve the discussion between Delia, Andrea, and Chronos, Andrea proposes to find the difference between a schematic and a compact representation, finding the main one in the density of information:

> *"we could exhaust it... let's see what is the difference between schematic and compact ..."* (B13)

He continues with the difference he sees between a compact and a schematic representation:

> "*In my opinion the difference between schematic and compact is the density... the scheme... that is with the same density of information the scheme becomes compact... [...] If the order increases proportionally to the... to the density. If the density... ehm... starts... and the order instead... let's say... does not follow the same progression then the pattern does not become compact ...*" (B18, B19, B20)
>
> "*then you know that in a scheme... [...] the scheme has a certain level of order... and has a low density. [...] to make that scheme compact, you have to increase the density... but proportionally to the density ... or anyway... not proportionally in a direct way, however... you have to adjust the order... because if it only increases the density... [...] if you increase the density of information, but at the same time... let's say... balance the order... [...] then you can get a compact scheme..."* (B22, B23, B24)

Andrea goes on to explain the criterion by which a compact representation respects his way of understanding through the idea of "*critical threshold*":

> *"but there's a critical threshold ... let's say... so the... the order is no longer able to... ehm... balance the density... so let's say that in this you would no longer get a compact pattern anyway... as far as you may have balanced the order, in that case you would simply have... [...] because there is a lot of information... but all enclosed in the same.... [...] and the density does not exceed a threshold for which the order is no longer able to balance them..."* (B25)

Finally, Andrea presents the reason why in particular two representations are effective for him since there is a balance between order and density. In the experimental representation, there is no redundant information, so the representation is dense but "the order managed the density":

"*Although in the Zeilinger experiment there is a huge amount of information... let's say... the order is still able to handle this density and... anyway we have this feeling of compactness... [...] if inside Zelinger we put... we started to put all the passages of the circuits... we started to draw the phone... [...] we would have some superfluous information or however we would have a density of information... [...] but we know anyway that we have a threshold of density that we cannot exceed for... however maintain compactness ...*" (B30)

In the axiomatic representation he recognizes the balancing between the density of information and the order as a relationship between subject-predicate and logical connections between different subjects and/or predicates.

"*The postulate can be defined as compact... because perhaps we associate the idea of order to the idea of a flow diagram... to the idea of a... graph... but it does not mean that a text cannot have order... that is, when Diana first quoted the summary... the summary can be compact... it is not necessarily dense and because also from the point of view... of the language we are able to express an order and a... let's say... [...] we can express... ehm... the balance between... order and density of information in a... in a... the verbal presentation […] ... the density of information as the expression of predicates, no?... that is understood as subject, subject-predicate... of judgments anyway... and instead order as the logical connections of these predicates.... so when they become too many predicates... we have an excessive density of information and anyway we can balance with logical connections... but when we... an excessive abundance of these predicates we say... they are also partly superfluous... mmmh... let's say... it's like writing a period of 40 lines, that is in the sense... it is no longer compact... it is chaos...*" (B32)

The representations for Andrea have also to be accessible, that is to be encoded in a familiar language, a language that, recalling his initial idea, has already been internalized:

"In my opinion accessibility basically means that something is encoded in a language that is familiar to us, that is ... that we have internalized..." (C3)

Trying to better explain his idea, Andrea continues by giving the example of the elderly phone that, since it has big keys, encodes information in a language that they can understand:

"*For example... I think of phones for the elderly, right? [...] the phone for the elderly is made accessible because it has the big keys... in a sense it is encoded in a language that is understandable to them... that is the numbers are large enough that with their sight they can see them well... so... [...] we understand the world if the world is encoded in a language that we can understand... [...] I read a text because I have internalized the linguistic code with which that text there is written. So in my opinion it is subjective to the extent that everyone has internalized... different kinds of languages or anyway different... let's say... different levels of language*." (C4)

When asked what role reasoning on different representations can have, about multiple representations Andrea argues:

"*I was thinking... is a bit like thinking about the idea of the theory of everything, no? That is the different scientific theories express different aspects... Even though we're obviously talking about something that has a frame in it... Maybe the theory of everything is in a sense a very large square; so we can come to understand different aspects, right? And maybe... trying to... compact them into a theory is basically impossible for that... density threshold and order that we said before because if we try to compact all the scientific theories, no? To merge relativity and quantum mechanics... We have an incredible density of information and we're going to need a lot of order, so basically to find that constant, that... that at the same time manages to balance this density without reaching the critical threshold... but maybe we say... The critical threshold prevents us... ehm... so, similarly, here, if we wanted to synthesize all the representations in one... We wouldn't make it because we couldn't balance density with order...*

[...] we will have redundant information because the order would not be able, in a sense, to... ensure compactness... so having the understanding of the various aspects gives us a better knowledge... gives us more information and [...] allows us in a sense to... to make up for the impossibility of unifying them in a single representation... because if we were limited to this we would lose information that we have beyond [in another representation]... we can not merge them because we would have the impediment of the density threshold thus having the understanding of several aspects can still guarantee us that information... obviously each of these aspects is coded differently... and it has its own order in itself... which is not to say that they are watertight compartments... they dialogue but they cannot be merged, each one... has its own specificity… [...] *We can see it as a loss the fact that we can not compact them in one... in one vision we say, but... at the same time we can also... [...] realize that as far as we can... the maximum of information we can get is that...*" (C20, C21)

Finally, Andrea expresses how the geometrical representation respects his idea of elegance to which he associates an idea of compactness: the Bloch's sphere, as a set of points, is extremely compact namely it is perfectly ordered and at the same time has a very high density, transmitting a dimension of infinite probability.

"*In the idea of elegance there is the idea of compactness because... [... that is, we must not translate the idea of elegance with simplicity... are two totally different things... that is, for example, the sphere, no? The sphere... is not a point, that is... In my opinion the main idea that the sphere transmits to us on... let's say on the image we have of the Qubits... is the probabilistic dimension... That is the fact that there is not this 0 that is here and this 1 of here... but all the infinite points of the sphere. In my opinion, the sphere does not convey the idea of... ehm... simplicity in the sense that it is the sphere but the sphere as a locus of points... so in my opinion the sphere is elegant in the sense that it is extremely compact that is it is perfectly tidy, has an order in the sense... It is the highest idea of order we have and at the same time has a very high density, in the sense... it transmits us a... basically a dimension of infinite probability, so... so in my opinion the sphere... among the graphic representations is the one that has the greatest elegance in this sense...*" (C24)

Carlo's contribution to the focus group

Carlo's idea of understanding concerns finding a correlation between data and results and figuring out the mechanism that turns data into results.

"*I believe that... if we take... something to understand like... a mathematical problem when... besides finding a correlation between data and results I can also figure out what mechanism turns data into results*." (A3)

From the beginning of the focus group, Carlo expresses his need to "*see geometrically what happens*", arguing that "*to better understand a reasoning or concept*" he needs to "*combine a mathematical reasoning with a graph*".

"said that the best representation for me is... is when... a mathematical reasoning as can be an equation is combined with a graphic scheme ... for me the best is ... for example a function and the Cartesian plane that describes that function... because see geometrically what happens when... in a plane, allows me to understand much better a reasoning... or a concept [...] let's say... speaking of straight lines in a Cartesian plane, the m [angular coefficient] of the line is its... between quotation marks the dizziness with which it goes upwards. See two lines with different m, the one with greater m is more vertiginous... I immediately understand that... the second line has... that is, between quotation marks has a larger angle..." (A7, A8)

Carlo, during the discussion, shows to have another idea of schematic representation: schematic as scheme (maybe he refers to a conceptual map) in which "*the schematic being is dissected in the various* *branches that can have a scheme*", on "*how many levels this scheme expands as a map connects many arguments*".

> *"In my opinion schematic... [...] with what he [Arturo] means to variables... has more than one variable because it is not only what is schematic... how much it is schematic can be analysed in how many branches takes a scheme... how much are... conceptual or explained the points of a scheme and above all... [...] on how many levels you can... that is, on how many levels this scheme can be expanded because ... a map in itself binds so many topics you can not have a... that is... [...] in the ratio order and density you define if one scheme... ie if it is schematic or is compact..."* (B10)

> "*I think it's the exact opposite, that is a scheme can represent a huge argument and give you a general idea with which you understand so many things... you know little of each to have a general idea...*" (B27)

For Carlo the most compact representations are the experimental and the geometric ones.

> "*the maximum of compactness* [indicating experimental representation] ... *either this* [geometric representation] ... *or the tables* [logical representation]" (B28)

For Carlo, as for the others, the representation has to be accessible, sharing the meaning of accessible with Arturo: "*the accessibility is transparency mediated by the experience of the individual*". For him, previous knowledge "*does everything*" so it is important to "*firstly feed the person who has to look at the basic information that introduces the concept*". The most accessible representations for him are the geometrical one and the experimental ones.

> "*He [Arturo] said before that accessibility is transparency mediated by the experience of the individual and... in my opinion, however, the experience of the individual... let's say... […] is affected very much by how accessible a thing is because, in my opinion is... is "more transparent" a thing like this [geometric representation] that a thing like this [axiomatic representation]... [... that is] the previous knowledge do everything... so in my opinion it must be done in two phases... firstly... where you have to... feed the person who must look with the basic information that introduces the concept and then you can give them precisely... given a context, the reasoning within...*" (C12)

> "*for me the most transparent thing is the sphere and this [experimental representation] [...] Yes... with my very high previous experiences... a geometric pattern in my opinion is always the best..."* (C14)

Carlo also expresses the reason why the geometrical representation is so fundamental for understanding. Geometric representation is automatically the best way to expose things because it refers to nature, to what is observed constantly, that is, a form.

> *"If on one hand a thing like a sphere is concrete and I agree with him* [Arturo] *that a thing like an equation or mathematical reasoning is very pure...The man himself to understand is both concrete and abstract because if we representations the... we understand through the eyes we always, not only in our life but historically, are used to seeing and understanding everything through the eyes so in my opinion the shapes, colors... or even just the flow that follows things... It is already something that is innate in our consciousness... so for me a geometric representation is automatically the best way to expose things because it refers to nature, to what I see constantly so when I see a shape... It's almost like it's already in my mind...*" (C15)

When asked what role reasoning on different representations can have, about multiple representations Carlo argues:

> "*In my opinion every kind of representation highlights... how can be the different types of diagrams to show a statistic... Each type of representation highlights a feature and having*

multiple representations in parallel allows you to see each feature highlighted and then make, each in his own head, a mosaic of features and have a much more complete idea. […] I imagine that if all the representations had the same adjectives or anyway had the same massage would not make sense to do more than one... of a representation... so they exist because they explain different things..." (C19)

Arturo's contribution to the focus group

The feeling of understanding of Arturo concerns his capacity of explaining the same concept "*alienating it from the context of the explanation*" and "*making interdisciplinary connections*". Arturo conceives the knowledge as a "unique network of data" built starting from a "*process of assimilation of a concept*", the isolation from the original environment and then the link with the previous knowledge. In Arturo's words:

> "*I am sure I have understood something sufficiently when... I am sure I can... explain the same concept but alienating it from... the context of the explanation then also in other areas... so both from the point of view of...of previous knowledge of the audience or even of the ... situations... so the way I'm explaining and... also making interdisciplinary connections so... eh... basically the process is to assimilate the concept, isolate it from the environment and then connect it with my knowledge, that I already own... so I see ... knowledge as a unique data network...*" (A2)

Following his idea of understanding, for Arturo a representation helps him to understand when there is a "*balancing between the old and the new*" knowledge, so an amount of information already internalized in order to make connections. In turn, the new elements must be present so as to create a fertile field for future links.

> "*In my opinion there must be a... a balance between the... between the "old" and the "new" [knowledge] in the sense... there must be... a necessary amount of... information related to knowledge already internalized to... ehm ... be able to make connections with things I already know ... [...] and new elements must be present so as to create a fertile field for any links in... in the future...*" (A6)

Arthur contributes to the debate on the difference between schematic and compact metaphorically using the concept of function, figuring out a compact representation as a two-variable function (density and order) and the schematic or transparent one as a one-variable function.

> "*In my opinion these words can be divided into two different groups... that is, the ones that define themselves... so first we look at a single variable, while the others like for example "compact" has more variables, so you have to take into consideration more aspects... and paradoxically... more variables are present and more... the idea seems similar to me... for example transparent is a single variable in my opinion and... many people have had different ideas on transparent... while instead ... compact at least at group level... in the end the concept is the same in our minds...[...] [referring to the meaning of compact representation]... as Andrea said there is not only density, and it is the first variable, but there is also the order... how much information is ordered... so these are my two variables... [...] Schematic for me is more... single variable...*" (B7)

Giving importance to the old knowledge, for Arturo a representation has to be accessible, which for him means "*transparent in relation to the experience of a person*":

> "*I imagine accessible as transparent in relation to the experience of a person... [...] that is for you is accessible something that is transparent in relation to the experience of a person... so... in my opinion... [...] is a form of transparency... accessibility is a form of transparency that depends heavily on... as we said, knowledge, skills, languages...*" (C5)

Referring to the question of what representation best satisfies their aesthetic sense and/or is the most elegant one, Arturo answers that for him the "*purest form*" of representation is the equation, that is the algebraic one.

> "*for me equations... for me it is the purest form of representation for anything...*" (C25)

Arturo feels the necessity of explaining his idea of purity and elegance, that is our capacity to "*represent knowledge through mathematics*". The elegance for him lies in the capacity to "*place us in the middle and could understand both the worlds* [the nature and the mathematics]".

> "*I have realized that I have defined pure and elegant the mathematical representation, not... not for the mathematics in itself... but because... We are both abstract and concrete. Reality is concrete... we reach a point where we can represent reality through mathematics and we are in the middle... I mean elegance by this... Just this... putting us in the middle and being able to understand both worlds... so I know... so I actually understood that it isn't the mathematical representation in itself but the whole complex reality, man and mathematics*." (C33)

5.3.3 From the profiles to the signature ideas

In the focus group, Diana's discourse is characterized by the repetition of some expression like "*answering to all my whys*", "*retrace back [...] the steps*", "[representation] *have to be clear*", "*circumscribed", "exhaustive", "non-dispersive"*, "*linear"*, "*accessible*", "*practical*" and "*useful*". These expressions and representations' qualities recall an idea of *control*. <u>They reflect her image of scientific knowledge as something that can be used to solve a task, and her cognitive need to have immediate access to the knowledge and how to manage it</u>. For these reasons, the representations that better satisfy her needs are the logical, circuital, and algebraic ones. This is also coherent with Diana's evaluation of teamwork, in which these three kinds of representations have the highest evaluations (see annex 3).

Andrea's discourse is populated with many personal expressions like "*internalize*", "*assimilate a concept*", "*I feel inside the concept*", "*expose* [a concept] *almost instinctively*". A representation for him must have a "*squaring*", "*must be complete in itself*", "*exhaust*[ive], "*elegant*" and "*compact*", where for him the compactness lies in its "*being compressed*", in "*not neglecting aspects*", in being "*simple*" but "*not leaving strands in half*", in being "*dense*", "*ordered*", and "*balancing*". These are only a part of the personal and key expressions that characterized Andrea's contribution.

While for Diana the understanding is linked to a direct extrapolation of information from the representation and the cognitive load must be as low as possible, Andrea's idea is very different. For him <u>the knowledge has been internalized and assimilated, the construction of knowledge for him is not immediate but is a process of appropriation and of popularization of personal meanings</u>. Representations that are effective for his process must be characterized by a complexity of representations' qualities that reflect a plurality of epistemic and cognitive needs: a representation has to be clear and simple but, at the same time also accomplished in itself, complete, not leaving strands in half, it must be full of information but ordered.

While Diana's needs are satisfied when a representation gives her immediate access to the information, Andrea describes, for his understanding, more effective the representations in which he can find a balance between the plurality of qualities he described. So even if Andrea, as Diana, gave the circuital, logical, and algebraic representations very high evaluations during teamwork, they do not really satisfy his needs. During the focus group, he showed that the geometric and the experimental ones meet his epistemic and cognitive needs more.

Different is the case of Carlo, whose discourse is characterized by the repetition of some expressions like "*finding a correlation*", "*figuring out the mechanism", "giving the reasoning within"*, and "*seeing geometrically what happens*". For him, the <u>knowledge is formally encoded, and the understanding</u>

consists in grasping the mechanism inside as well as in seeing geometrically, which are the main epistemic and cognitive needs that he expressed. This is in line with his representations' evaluation of the first part of teamwork (annex 3). Carlo gave major evaluations to the logical, the circuital and the geometrical representations, since they better satisfy his cognitive and epistemic needs.

Finally, Arturo's discourse is characterized by some core expressions like "*explain the same object*", "*alienating it from the context of the explanation*", "*making interdisciplinary connections*", "*unique network of data*", "*process of assimilation of a concept*", "*balance between the old and the new*", "*transparent in relation to the experience of a person*", and "*previous knowledge of the audience*". Arturo associates understanding with his ability to assimilate the concept, make interdisciplinary connections, and explain it to a different audience, he conceives knowledge as something that has to be transmitted. Contrary to the others, Arturo during the focus group did not particularly intervene with respect to the representations he prefers and the characteristics they must have. Triangulating what emerged from the focus group, that is the representation he finds most elegant, with the assessments of the first part of the teamwork (see annex 3). The representation that best satisfies his epistemic and cognitive needs is the algebraic representation.

To conclude, the contributions of Diana, Andrea, Carlo, and Arturo to the focus group are very idiosyncratic and different from each other. Even if the representations they picked as the most satisfying ones and the meanings of the qualities were built together and shared, their arguments and their motivations were very different from each. The work we carried out with representation gives space to different ways of understanding and understanding through the representations and it embraces different epistemic and cognitive needs.

In the next sections, we aim to prove the last sentence by making two different analyses.

The first aims to investigate the robustness of the signature ideas that is which mechanisms of reasoning have been activated by the different representations. To investigate the robustness of the signature ideas and shed light on these mechanisms, we carried out a sensemaking analysis inspired by the metric elaborated by Shulamit Kapon (2017).

The second analysis aims to investigate the depth of the signature ideas. In particular, if students found the course inclusive, found space "to feel at home" and nurture their own personalities. In other words, we explore if their signature ideas are contextual to the task or if they have a deeper idiosyncratic aspect, rooted in students' identities: for example, if those signature ideas reflect, more in general, students' personal ways of conceiving, building, and approaching knowledge, if they are carriers of personal tastes, and purposes. To investigate the depth of the signature ideas and their idiosyncratic nature we carried out an appropriation analysis based on the operationalization elaborated by Levrini, Fantini, Tasquier, Pecori, and Levin (2015).

5.4 Investigating the Robustness of the signature ideas

5.4.1 Framework and methodology

The present analysis aims to investigate how students explain how they think through representations. In particular, we investigated if and to what extent the different representations of the course promote the understanding, the features that external representations must have to gain understanding, what kind of reasoning they can activate, and if and how robust the mechanisms of the reasoning are. Students tried to explain with very personal words how they reason, their reasoning processes and mechanisms. This paved the way to consider the students' explanation as personal processes of sense-making through representations. Zhang and Soergel (2014) proposed a cognitive process model and an analytical framework for examining individual sensemaking from three perspectives:

(1) "*Process* – the activities that a sensemaker goes through during a sensemaking task in various combinations, including task analysis, gap identification, exploratory and focused search for data and structure, building structure and fitting data into structures, often concluded with task completion or decisions made.
(2) *Conceptual changes* to knowledge representations that the sensemaker creates, including intermediate and final outcomes of sensemaking, which can be categorized into accretion, tuning and restructuring.
(3) *Cognitive mechanisms* – the mechanisms that underlie the process and trigger the conceptual changes, including data-driven mechanisms such as key item extraction, structure-driven mechanisms such as specification and mechanisms that operate at both levels such as comparison."
(Zhang and Soergel, 2016, p. 61, 62).

From a very preliminary analysis, the way in which students discuss how they understand through representations seems to give insights about students' processes (such as "search for data and structure, building structure and fitting data into structures"; ibidem, p.61), cognitive mechanisms (like the data-driven mechanisms and structure-driven mechanisms emphasized by the authors) and about students' epistemic and cognitive needs. We therefore focused on sensemaking processes to explore these mechanisms and if there are robust. In literature, an explanation is considered robust if the process of constructing and defending it has the aim to i. use evidence and general science concepts to make sense of the specific phenomena, ii. articulate these understandings and iii. persuade others of these explanations by using the ideas of science to explicitly connect the evidence to the knowledge claims (Berland & Reiser, 2009). The kind of explanation at the center of the present study do not consider the goal of persuading to build the consensus since students were asked to discuss the way in which they understand through representations. For robustness we therefore refer more to the reliability (the scientific knowledge they put into play is reliable) and the plausibility (the logical and causal accounts of their reasoning) of the explanation.
In science education literature there are many studies that investigate the role that external representations and multiple representations could play to promote sensemaking processes for a certain task (e.g., Zhang and Soergel 2016; 2014; Rau, Aleven, Rummel, and Rohrbach, 2012; Pirolli and Russel, 2011; Baker, Jones & Burkman, 2009; Rau, Aleven, & Rummel, 2009; Ainsworth, 2006). Many of these suggests that the use of external representations for tasks where the information is too voluminous can reduce the costs of learning (e.g., Kirsh, 2010; Zhang, 2000; Zhang, 1997). But it is not only a matter of sensemaking cost and cognitive load. Baker, Jones, and Burkman (2009) argue that visual representations can facilitate sensemaking in data exploration tasks since they (1) support visual perceptual approaches such as association, differentiation, ordered perception, and quantitative perception, (2) have strong Gestalt properties, (3) are consistent with the viewer's stored knowledge, and (4) support analogical reasoning (Baker, Jones & Burkman, 2009, p. 535).
Other works highlight that learning with multiple representations can enhance a deep understanding (Rau, Aleven, Rummel, and Rohrbach, 2012; Rau, Aleven, & Rummel, 2009; Ainsworth, 2006). Studies show that students need to make connections between different representations in order to benefit from them (e.g., Ainsworth, 2006; van der Meij & de Jong, 2006; Bodemer, Ploetzner, Bruchmüller, & Häcker, 2005). Rau and colleagues (2012) highlight the importance of designing learning environments that support "connection-making" between different representations so as to promote deeper learning (Rau et al., 2012, p. 175).

Even if sensemaking is today a process investigated in many fields, researchers tend to use the term differently or with variations. For example, Garfinkel (1967, 2016), in his introduction to ethnomethodology, described the term "sense-making" as a way of studying the everyday practices of actors, how they interact, interpret and account for their experience of reality (Garfinkel, 1967,2016). Polanyi (1967) introduced the terms "sense-giving" and "sense-reading" to describe how people fill speech with meaning and make sense of speech (Polanyi, 1967). More recently, Pirolli and Russel (2011) wrote "Sensemaking, as in to make sense, suggests active processing of information to achieve

understanding (as opposed to the achievement of some state of the world) [...]: Sensemaking involves not only finding information but also requires learning about new domains, solving ill-structured problems, acquiring situation awareness, and participating in social exchanges of knowledge. In particular, the term encompasses the entire gamut of behavior surrounding collecting and organizing information for deeper understanding" (Pirolli & Russel, 2011, p.1). Zhang and Soergel (2016) argued that "sensemaking has been defined as a process of forming and working with meaningful representations in order to act in an informed manner and as reading into a situation pattern of significant meaning" (Zhang & Soergel, 2016, p. 59).
Recently, researchers in science education have increasingly studied how students "make sense" of science. Odden and Russ (2019), trying to address the theoretical fragmentation they pointed out in literature about this topic, define sensemaking as "the process of building an explanation to resolve a perceived gap or conflict in knowledge" (Odden & Russ, 2019).

In literature, the link between sensemaking, explanations, and multiple representations is quite investigated (Rau et al., 2015; Wylie & Chi, 2014; Ainsworth & Loizou, 2003; Chi, Bassok, Lewis, Reimann, & Glaser, 1989). Some studies paved the way for the possibility that "integration processes involved in learning with multiple representations can happen through self-explanation activities" (Rau et al., 2015, p.32). For example, Wylie and Chi (2014) argued that "self-explanations can take a number of forms, their common feature is that by prompting students to self-explain, they encourage students to think deeply and to cognitively engage with the learning materials by making connections to prior knowledge and refining mental models" (Wylie & Chi, 2014, p.423). Ainsworth and Loizou (2003) showed that students can have meaningful benefits from multiple external representations when they generate a larger number of high-quality self-explanations, but they do not tend to spontaneously engage in sense-making processes. These processes can be triggered by fostering students to self-explain the relationship between different representations and conceptual aspects of the domain so as to "enhance their benefit from multiple representations" (Rau et al., 2015, p.32). In the literature, there are some results in favor of this. For example, Berthold, Eysink, and Renkl (2009) conducted an experiment in which 62 participants learned probability theory under three different conditions (open self-explanation prompts, self-explanation prompts in an assistance-giving-assistance withholding procedure, and no prompts) and pointed out that prompting students to self-explain, in studying multi-representational learning materials, had a positive effect on both conceptual and procedural knowledge (Berthold, Eysink, & Renkl, 2009, p. 360, 361). Also in chemistry education, Zhang and Linn (2011) show that learning with dynamic chemistry representations can enhance students-generated explanations since they are engaged in knowledge integration processes like adding, evaluating, and refining scientific ideas (Zhang and Linn, 2011, p.1195).

This work aims to contribute to the literature not only by showing that students can take advances from constellation of representations but also to provide insights on how students used and characterize representations as conceptualizing and reasoning (and meta-reasoning) tools (Kozma & Russel, 2005) through which they could verbalize their personal ways of understanding through representations as well as and with their epistemic and cognitive needs.
In particular, we explore whether and what kind of sensemaking process can activate representations in the specific case of a course on the Second Quantum Revolution. The analysis unpacks, through students' abilities to explain, critique, and compare different representations (diSessa, 2004; Sherin & diSessa, 2000), the mechanism of reasoning that the different representations can enact, and if and to what extent this mechanism can be considered a process of sensemaking.

We looked for a framework that would help us to evaluate the robustness of the students' explanations and to investigate the process of sensemaking through external representations. The framework we needed should help us to grasp the differences between students in terms of attitudes toward knowledge and expectations, kinds of reasoning produced through discussing, comparing and critiquing representations and kinds of knowledge they put into play. Particularly meaningful for our aims is the framework developed by Kapon in the paper "unpacking sensemaking" (2016). In the paper, the researcher operationalizes three dimensions (intuitive knowledge, mechanism, and framing) to empirically track them in students' talk, gestures, and social interactions. The researcher's assumption is that addressing scientific ideas can involve complex processes of sensemaking in which students construct and reconstruct explanations that evolve, change, replace one another, or merge into a new explanation (Kapon, 2017). She argues that the theoretical and methodological coordination of multiple aspects of learning in STEM and, in the study case, "the coordination of cognitive and interactional aspects of self-explanations can shed light on the tacit process of sensemaking" (ibidem, p.168). Therefore, in the researcher's view, people's sensemaking can be conceptualized as "a process of evolution of self-explanations in which self-explanations are generated, tacitly evaluated, and reconstructed based on this evaluation" (ibidem, p. 193). To evaluate the robustness, Kapon conceptualized the evaluation of self-generated explanations as taking place along a multidimensional metric, which consists of three dimensions: "(1) intuitive knowledge that reflects the content of the explanation, namely the elements in the knowledge system it references as well as the dynamics and interrelations of this knowledge system, (2) mechanism that reflects a particular structure of the explanation, and (3) framing that reflects the contextual and interactional constraints. I believe that these dimensions evolve but continue to play a role in the reasoning of professional scientists in addition to other dimensions that are developed with expertise" (ibidem, p.168). These three dimensions, as we will see, are "not fully orthogonal" (ibidem, p. 194) since they interact.

We used the multidimensional metric elaborated by Kapon (2017) to analyze the kind of reasoning enacted stimulating students to reflect on the role of representations in their understanding and if it is robust. The coordination of epistemic and cognitive aspects in students' reasoning can shed light on the tacit process of sensemaking.

There are some differences from the author's work. First of all, we are not dealing with self-generated explanations. In our case, we analyze students' way of talking about representations and their ways of understanding through representations. The three dimensions are used to explore if their ways of talking are sensemaking processes and if they are robust.

Another difference lies in the kind of mechanism. Kapon, investigating how two seventh-grade students generate explanations about why a plastic bottle shrinks when air is pumped out, refers to mechanistic reasoning. Instead, our focus is on mechanisms of reasoning activated by the representations in figure 5.5. In literature, there are many works on mechanistic reasoning, mathematical reasoning, and analogical reasoning. The type of mechanism we refer cannot be strictly framed in these kinds of mechanisms since, first of all, the components of the explanation are different, namely the entities that underly the mechanism and the properties of these entities are different. A type of definition that approaches the type of *mechanism* we are referring to has been formulated by Redish (2021): the process of "building causal stories and extended chains of reasoning where each step sets up the conditions for the next" (p.3). The mechanisms are very linked to the other two dimensions, namely students describe a mechanism that depends on their framing and the mechanism allows students to draw on different knowledge resources.

Framing and intuitive knowledge are very similar to the dimensions described by Kapon (2017). Framing is conceived as an individual's forming a locally coherent activation of resources (Hammer, Elby, Scherr & Redish, 2005). In particular, they defined epistemological framing as the cognitive activity underlying a student's sense of "what is going on here" with respect to knowledge (Hammer et al., 2005).

The study of Kapon (2017) investigates how two seventh-grade students evaluate their generated explanations and discuss them with another person. In an instinctive way, the person reacts to what she says, agrees, disagrees, suggests, asks clarifying questions, etc. The frame of interaction, in this case, affects her evaluation of the validity of her explanation and how it evolves in light of the other person's reaction. (Kapon, 2017). This depends on how a person frames themselves. By applying Tannen's markers and registers (1993), she individuated different levels of framing. For example, a larger one is the context of the interview in which the single student framed the interaction as teaching. Another frame that marked girls' structures of expectations was an image of science that particularly affected their evaluation of the explanations they generated (as good explanation or not).

In this specific case, the reference to the framing dimension regards students' explanation about what understanding and engaging in trying to understand phenomena mean for them. The data analysis points out how each student framed the act and the state of understanding differently. So we looked for students' successful/productive experiences of understanding/reasoning stimulated by thinking about the role of representations in their personal reasoning and understanding.

Finally, intuitive knowledge refers to the elements in the knowledge system, the dynamics, and the interrelations of this knowledge system that constitute the content of the explanation. Fischbein (1987) defines intuition as a representation, an explanation, or an interpretation directly accepted by us as something natural, self-evident, or immediate, "an intuition is, then, such a crystallized - very often prematurely closed - conception in which incompleteness or vagueness of information is masked by special mechanisms for producing the feelings of immediacy, coherence, and confidence" (p. X). In his view, the intuition (or the use of intuitive knowledge) "implies extrapolation beyond the directly accessible information" (p. 13). In a constructivist view, accredited knowledge is nothing more than a refinement of common knowledge. From this perspective, a "theory of intuition" was built by diSessa (1993, 1988) according to which knowledge as being built out of many "pieces" of intuitive knowledge, or resources (Hammer et al., 2005; DiSessa, 1993, 1988). The theory at the basis of this assumption is Knowledge in Pieces, KiP (diSessa & Wagner, 2005; diSessa & Sherin, 1998; diSessa, 1993; diSessa1988), which aims to describe how knowledge is transformed during the learning of scientific concepts. The author argues that intuitive knowledge represents a rich resource that is critical to student learning and that a significant part of learning, from a constructivist perspective, is reorganizing and recontextualizing intuitive knowledge. diSessa (1993) called the basic knowledge elements phenomenological primitives or p-prims. Kapon and diSessa (2012) generalize the p-prim model by focusing on p-prim's self-explanatory property. They interpret an explanation as a "reduction of a phenomenon to a selected set of functional knowledge elements", which they term *explanatory primitives* (Kapon & diSessa, 2012, p. 267).

In her analysis, Kapon (2017) referred explicitly to e-prims. In our case, we looked more generally to the intuitive elements, if any, students include in talking about productive experiences of understanding/reasoning through representations.

In light of this the questions that guide our analysis can be formulated as follows:

a) What successful/productive experiences of understanding/reasoning do they have in mind when they are stimulated to think about the role of representations in reasoning and understanding? (*framing*)
b) What type of productive reasoning, stimulated by the representations, do they refer to? (*mechanism*)
c) Do these forms of reasoning include intuitive elements that can explain why they are perceived as productive? (*intuitive knowledge*)

5.4.2 The four case studies

The case of Diana

Framing

During the conversation, after stimulating students to think about the role of representations in reasoning and understanding, Diana refers, in particular, to two productive experiences of understanding/reasoning. The first one regards the course and the circuital representation. For Diana, this kind of representation and the truth tables are "*enough*" to solve the problems and to make clear a concept ("*I can solve the problems we had…* [and] *I can also have the concept clearer*"). The second experience taps into her school experience ("*Or I am studying the limits in mathematics*") and she stresses how much simpler become the limits when associated with the graphic representation ("*they become very simple in my opinion when you place them next to their graphic representation...*").

When she refers to these contexts, Diana makes her conception of productive knowledge explicit: scientific knowledge has a *functional role*, the knowledge that activates reasoning to solve problems and exercises. In order to make knowledge productive, a representation has to be clear ("*they have to clear*", "*clear … […] not neglecting the fact of clarity ... namely that aspect of clarity*") and "*simple*". Furthermore, by stressing the functional role, a representation has also to be *useful* ("*If we also want to also consider the usefulness*"), defining useful the algebraic representation because it is practical ("*in the sense of the resolution* [calculation] *of... in mathematical resolution practice*") and helps her to solve an exercise or a problem.

Mechanism

The mechanism of reasoning that Diana describes can be recognized as the process of "*retracing back"* alone all the "*steps*" ("*I mean I can retrace in my own head the steps we have taken and ... I mean ... yes... I can practically get there by myself...*"). Diana's mechanism has an operational flavor, she has the cognitive need to retrace back and, therefore, sequence the steps in a logical and chronological order. This is marked by the qualities that representations must have. It has to be exhaustive, non-dispersive, and linear ("*exhaustive [...] ... but at the same time... it is not dispersive... that is a thing... linear in my opinion... that is... very... compact in my opinion also with extra concepts that is... exhaustive and linear*"), namely to retrace back she needs to have all the information already ordered and not struggle to extrapolate and order them. Diana emphasizes this operational flavor by stressing the kind of information that a representation has to contain, that is "*what replaces*" ("*they must all have... maybe even... what I have to replace"*). This mechanism fits with the *functional role* of knowledge and the canonical school practices.

In light of this, the most effective representations that enact Diana's mechanism that gives her the feeling of understanding are the circuital and the logical ones ("*For me the CNOT and Hadamard with below tables... that is to me enough to be able to retrace in my head...*") because the information that they embody contain how logic gates function (*"I know how the logic gates work through the representation"*).

Intuitive elements

Diana's sense of intuition is triggered when she has immediate access to all information that is needed to solve a problem ("*what I have to replace*"). In this perspective, clarity ("*they have to clear*", "*clear … […] clear ... […] not neglecting the fact of clarity ... namely that aspect of clarity*"), simplicity, ("[the limits] *become really simple*"), completeness, order, linearity, and accessibility are the essential characteristics that representations must have to make her efficiently extract information to solve the problem. The cognitive load must therefore be low to avoid too big effort. This is linked also with the reason why, for example, the geometrical representation does not satisfy her cognitive needs and does not trigger her mechanism of understanding ("*the spheres, the orthogonal planes I never... I mean I* *don't like them, I never understood them... for me they are not accessible and transparent... so for accessible and transparent I come back on tables...")*.

The sense of intuition that Diana expresses recalls one of the p-prims that diSessa described in his work toward an epistemology of physics (1993), *Ohm's p-prim*. This element, in diSessa description, comprises different subentities; "*an agent that is the locus of an impetus that acts against a resistance to produce some sort of result. The major function of this element is to provide for activation of a set of qualitative relationships among differentials in the effort of the agent (amount of impetus), the resistance, and the result: More effort implies more result; more resistance implies less result; and so on*" (diSessa, 1993, p.126).

In this case, Diana activates this p-prim to describe her cognitive needs, how and what features representations must have in order to be clear and to be useful for her to achieve effectively her main "results", that is to solve a problem.

The case of Andrea

Framing

During the conversation, after stimulating students to think about the role of representations in reasoning and understanding, Andrea refers to productive experiences of internalization. For example, when he talks about Bloch's sphere, it clearly emerges that the representation is evocative for him ("*In my opinion the main idea that the sphere transmits*") and in what terms, namely what it evokes to him: "*sphere as a locus of points*", "*dimension of infinite probability*".

When he refers to these contexts, Andrea makes his conception of productive knowledge explicit. It emerges from a process of "*internalization*" or "*assimilation*" that leads him to reproduce and explain "*not in a mnemonic*" but in an "*instinctive way*", that is "*without a rigid scheme*" (*"without having a rigid scheme that takes me from A to B and then maybe gets to B... instinctively"*) because he "*feels the concept inside*" ("*because I feel inside the concept...*"). Andrea's need to reproduce and explain in a personal and instinctive way, to feel the concept inside is reflected by a multiplicity of standards formulated as complexity and multidimensionality (epistemic, affective, cognitive, etc.) of qualities that a representation must have to allow him to satisfy his intellectual and cognitive needs. It must have a "*squaring*", it "*must be complete in itself*", it has to "*exhaust all the aspects*" in a "*synthetic way*", "*not neglecting aspects*", it must "*not leaving strands in half*", it must "*make explicit the terms*", "*have a comprehensive view*" (*"if we have... a description of a motion for example... instead of having to look at all the individual... the individual laws, the individual relationships... maybe to have an overview can be more useful..."*) and it must be "*compact*". Furthermore, a representation for him cannot be ended in itself but must allow him to "*derive other concepts*" and build new knowledge. From a more affective point of view, it has to return to him an aesthetic sense ("*have a certain elegance*").

Mechanism

The mechanism of reasoning that Andrea describes can be recognized as the process that leads to "*feel the concept inside*" and it consists in looking for and attributing personal meanings to the concept. This process of attribution of meaning leads him to "*fully "enter" the concept*", to not remain on the surface but understand its language, to handle and manage it, "*as if it had always been in my luggage*" ("*I can fully "enter" a concept, not only in looking at it on the surface, [but also] in understanding its language, in handling it, in mastering it as if it had always been in my luggage*").

This is marked by his need to reformulate his thoughts and elaborate examples by tapping into his experiences and his readings. He constantly tries to reformulate what he is explaining with the aim of finding metaphors and images and picking personal words that can better incorporate what he means. For example, to better explain the meaning of "*compact*" he uses the image of a "*baseball or tennis ball*" that in itself has completeness, it is narrow and dense, and the material is ordered ("*in itself it* *has completeness... and is ... restricted.... inside it contains a lot of material information, that is a lot of material... [...] it is dense... however... this material is ordered in the sense… the baseball is layered...*"). In talking about the meaning that he attributes to "*accessible*", he recalls the "*phone for elderly*" that encodes something in a language they understand (*"for example... I think of phones for the elderly, right? [...] the phone for the elderly is made accessible because it has the big keys... in a sense it is encoded in a language that is understandable to them"*). To discuss the role of multiple representations in highlighting different aspects and their incompatibility, he refers to the theory of everything ("It *is a bit like thinking about the idea of the theory of everything, no? That is the different scientific theories express different aspects...*").

In light of this, a representation for him is effective when it is dense and triggers processes of meanings attribution, like Bloch's sphere.

Intuitive elements

The intuitive knowledge that Andrea uses to discuss when and why a representation can better satisfy the complexity of his qualities and of his cognitive and epistemic needs pass through *balancing* between order and density of information. It is the *balancing* between order and density of information that helps him to recruit the information that can activate his understanding. The idea of *balancing* is elaborated when Andrea tries to explain what the difference between schematic is, individuating the difference in the density of information ("*the difference between schematic and compact is the density*"), and when a scheme can become compact. For Andrea, a scheme is ordered but has a low density of information (*"the scheme has a certain level of order... and has a low density"*). To make the scheme compact, the density has to be increased but, at the same time, the order has to be proportionally increased too, "*adjusted*", "*balanced*" ("*to make that scheme compact, you have to increase the density... but proportionally to the density ... [...] you have to adjust the order... because [...] if you increase the density of information, but at the same time... let's say... balance the order... [...] then you can get a compact scheme..."*).

The *balancing* between density and order structures Andrea's understanding, helping him to activate the readout strategies to recruit the information and manage them cognitively. The management of this information is possible as long as the order balances the density of information. To discuss when a representation does not allow him to effectively recruit the information, Andrea brings into play the concept of *critical threshold* ("*but there's a critical threshold*", "*the order is no longer able to... balance the density...*", "*there is a lot of information... but all enclosed in the same.... [...] and the density does not exceed a threshold for which the order is no longer able to balance them...*").

These two knowledge elements that Andrea elaborated on to discuss the difference between schematic and compact become the explanatory elements to discuss which representations satisfy his needs and why.

In the experimental representation, there is no unnecessary or redundant information, so the representation is dense but "*the order handles the density*" ("*Although in the Zeilinger experiment there is a huge amount of information... let's say... the order is still able to handle this density*").

According with also his mechanism of attributing personal meanings, Andrea re-read the compactness of the axiomatic representation, more generally a textual representation, in terms of subjects and/or predicates (*"the density of information as the expression of predicates [...] that is understood as subject, subject-predicate*) and the order in terms of logical connections between subjects and predicates ("*order as the logical connections of these predicates*"). The textual representation is not compact and not effective when there is an *imbalance* between subjects/predicates and their connections (*"when we* [have] *an excessive abundance of these predicates [...] ... they are also partly superfluous... [...] it's like writing a period of 40 lines, that is in the sense... it is no longer compact... it is chaos...*").

Finally, Andrea describes the geometrical representation as the one that better mirrors his needs both because it has a very high density, incorporated in the qubit and in its dimension of infinite probability (“*... It is the highest idea of order we have and at the same time has a very high density[...] it transmits us a... basically a dimension of infinite probability*”, “*the main idea that the sphere transmits to us on... […] is the probabilistic dimension... […] all the infinite points of the sphere. […] the sphere as a place of points”*). This balancing between density, and kind, of information and order, meet the more affective, aesthetic ideals of Andrea (“*In the idea of elegance there is the idea of compactness*”, “*the sphere is elegant in the sense that it is extremely compact*”).

The case of Carlo

Framing

During the conversation, after stimulating students to think about the role of representations in reasoning and understanding, Carlo refers, in particular, to school productive experiences. The object of understanding to which it refers is therefore a mathematical problem or mathematical reasoning (“*if we take... something to understand like... a mathematical problem”, “to a mathematical reasoning as can be an equation”*). He explicitly recalls the effectiveness of the geometric representations to understand the angular coefficient (“*talking about lines in a Cartesian plane* [...] [I] *immediately understand*”).

When he refers to this context, Carlo makes it explicit his conception of productive knowledge: a problem has not only to be solved (“*besides finding a correlation between data and results I can also”*) but also has to be understood, namely, he needs to extrapolate and visualize the information from row data, manipulate and transform them in formal results (*“figure out what mechanism turns data into results”)*. Carlo conceives knowledge as something buildable and expandable in a coherent way, the expanding structure is greater when the foundations are more solid ("*It starts from the premises and then it is as if we start to build a house already having the foundations... It’s much easier and I would say you build much bigger buildings*"). The construction of knowledge for Carlo is a two phases process (“*two phases… firstly…and then*”) and greater basic knowledge of a concept implies that you can reach a deeper knowledge, in the sense of knowing "*the reasoning within*" (“*two phases... firstly... where you have to... feed the person who must look with the basic information that introduces the concept and then you can give them precisely... given a context, the reasoning within…"*) and “figuring out” the mechanism.

Carlo does not refer to particular qualities of representations, by triangulating the focus group (“*see geometrically what happens when... in a plane… allows me to understand much better a reasoning*”) and the teamwork, the more effective representations are the geometric, circuital and logical representations.

Mechanism

The mechanism of reasoning that Carlo describes can be recognized as a *process of translation* from raw data to formal knowledge and a *back-and-forth process* from one representation to another. Therefore, this mechanism is triggered more easily for him in presence of multiple representations. The presence “*in parallel*” of multiple representations that enhance different features (“*every kind of representation highlights [...] a feature*”) allows him to make a “*mosaic*” and to build a “*more complete*” mental image (“*then make, each in his own head, a mosaic of features and have a much more complete idea*”).

It is particularly useful for him the *back-and-forth* translation between an algebraic and the geometrical representations (“[when]*... a mathematical reasoning as can be an equation is combined with a graphic scheme*”). This mechanism is also the way in which Carlo *makes sense* of “something”. Tapping into school scientific practices and knowledge, that is the lines on a Cartesian coordinate system (“*talking about lines in a Cartesian plane”*), Carlo articulates the dynamics of his reasoning to make sense of the angular coefficient, *m*: translates the *m*, that belongs to the line's equation (*y=mx+q*, the algebraic representation), into the slope of the line (the geometrical representation). This association helps him to "*immediately understand*" and compare, for example, two lines with different angular coefficients, and understand which line has the "*greater angle*". This translation activates in him the mechanism of association of features that "belong" to different kinds of representations: from the *m* to the slope to the angle.

The mechanism that Carlo put into play stems from having multiple representations and it fits with his conception of productive knowledge, namely not only solving the problem but also understanding the "reasoning within".

Intuitive elements

Carlo describes his sense of intuition as a "*mediation*" between the concreteness, associated with the Bloch sphere ("*If on one hand a thing like a sphere is concrete*"), and the abstractness and purity, associated with the algebraic representation ("*an equation or a mathematical reasoning is very pure*") since "*the man in itself to understand is both concrete and abstract*". This mediation is conceived more as leading back the abstract formal language to a known form conceived as geometric form and colors. He emphasizes the importance of the visual component, the importance of seeing with the eye ("*we understand* [the representation] *through the eyes*", "*we are used to see and understand everything through the eyes so in my opinion the shapes, colors*"), therefore the geometrical representation is concrete because it is what he "*constantly sees*" and allows direct access to the nature ("*because it refers to nature*"). In fact, he stresses at different times that it is "s*ee geometrically*" what happens that allows him to understand a problem much better ("*See two lines with different m*", "*see geometrically what happens when... in a plane, allows me to understand much better a reasoning... or a concept*", "*for me the most transparent thing is the sphere… a geometric pattern in my opinion is always the best"*). When Carlo succeeds in recognizing and bringing the abstract back to form, he activates his sense of intuition that allows him to understand why it is "*almost as if it were already in* [his] *mind*" ("*when I see a shape... It's almost like it's already in my mind*").

The case of Arturo

Framing

During the conversation, after stimulating students to think about the role of representations in reasoning and understanding, Arturo considers learning experiences productive when he can explain the concept ("*I am sure I can explain the same concept*") to others, when he can develop communicative skills of adapting and translating the concept according to the "*audience*" and its "prior knowledge", namely the skills of the natural language.

Arturo conceives the knowledge as something to be propagated from one context to another, integrated and connected, and as something to be transferred from one person to another ("*alienating it from... the context of the explanation then also [explaining it] in other areas*"). Therefore, after the assimilation of the concept, he has to translate and adapt his language to make a topic or a concept accessible and "*transparent*" in relation to a person's experience ("*accessible as transparent in relation to the experience of a person*").

Arturo does not refer to particular qualities of representations, but, in general, they have a balance between old and new knowledge in order to make connections ("*between the "old" and the "new" [knowledge] [...] to be able to make connections*"). In the focus group, he refers in particular to algebraic representation because he perceives it as the "purest" and the most elegant. Triangulating with the focus group, Arturo generally gave high ratings to the different representations. This may concern the need, when explaining, to make interdisciplinary connections ("*making interdisciplinary connections*") and have more access to more languages.

Mechanism

Arturo's mechanism, which allows him then to explain what he has understood, is the "*assimilation*" that consists in the "*alienation*" (or "*isolation*") of the concept from the context, its transfer of other contexts, making connections between the different contexts ("*making also interdisciplinary connections*"). But it is not just a matter of transferring from one context to another. Arturo in fact describes his learning process as a continuous accommodation, a continuous putting in coherence new elements with the knowledge he already possesses ("*then connect it with my knowledge, that I already own*", "*be able to make connections with things I already know*") with the aim of expanding his knowledge. The new knowledge elements, in turn, have to be organized (a process that derives for Arturo from *internalization*) in such a way as to create a "*fertile field*" for new future connections ("*new elements must be present so as to create a fertile field for any links… in future*"). So the process of bringing the new and the old together not only expands the network but also makes it more solid and fruitful.

The mechanism described by Arturo, the isolation and the building of interdisciplinary connections, fits his conception of productive knowledge. His need to adapt his own language and translate what he learns according to the context can be reflected, during the learning, in the search for a multiplicity of representative forms that can promote multiple connections.

<u>*Intuitive elements*</u>

Arturo's sense of intuition derives from his conception of knowledge as a "*unique network of data*" ("*knowledge as a unique data network*") that grows more and more when he meets a new concept. Intuitively, Arturo, to describe how the expansion is triggered, uses the idea of a "*balancing between the old and the new*", by a co-existence between "*knowledge already internalized*", "*competencies and languages already possessed*", "*personal experiences*" and new elements. This "*balancing*" is not only a knowledge element that he puts into play to, for example, understand a new representation (as a kind of epistemological activity), but he uses the same element when the epistemological activity changes, e.g. when he explains a concept to an "*audience*" by adapting his language to their prior knowledge in order to make the concept accessible to them ("*explain the same concept but alienating it from... the context of the explanation* [...] ...*from the point of view of...of previous knowledge of the audience*").

5.4.3 Discussion

The presented analysis aimed to evaluate the robustness of students' ways of talking about their understanding through representations. According to Kapon (2017), we investigated students' explanations along three different dimensions: framing, mechanism, and intuitive knowledge.

The framing was investigated by asking the data the question "*What successful/productive experiences of understanding/reasoning do they have in mind when they are stimulated to think about the role of representations in reasoning and understanding?*". We addressed these questions by looking for situations (e.g students' references and/or examples) to individuate students' successful/productive experiences in reasoning/understanding through representations. The mechanism was investigated by asking the question: "*What type of productive reasoning, stimulated by the representations, do they refer to?*". For mechanism, we refer to the process of "building causal stories and extended <u>chains of reasoning</u> where each step sets up the conditions for the next" (Redish, 2021, p. 3). Finally, intuitive knowledge was investigated by asking the question: "*Do these forms of reasoning include intuitive elements that can explain why they are perceived as productive?*" Therefore, we looked for natural and self-evident pieces of knowledge or knowledge elements that students put into play.

The four students showed to be very different along the three dimensions, therefore have different processes of sensemaking.

Both Diana and Carlo are framed within the context of school, namely their references and examples tap into school experiences. Diana's examples of productive experience regard what she is studying at school, and she associates scientific knowledge with something practical and useful to, for example, "solve a problem". Her epistemic and cognitive needs are to have immediate access to knowledge and to extrapolate the information ("*what I have to replace*") she needs to solve the problem. The representations that best fit their needs are the circuital and the logical ones.

Also Carlo is framed within the school's scientific practices, therefore the object to be understood for him is a mathematical problem or mathematical reasoning. Nevertheless, for him, the problem has not only to be solved but also has to be understood, to figure out the reasoning inside ("*figuring out the mechanism", "given a context,* [see] *the reasoning within..."*). While for Diana the knowledge has a functional role, for Carlo it is buildable. His needs to "see geometrically" and keep an eye on the mechanisms make the geometrical and the circuital/logical representations the ones that most satisfy him.

Andrea's framing is still different from the others, he needs to "*internalize*" or "*assimilate*" a concept, namely to reproduce it in an "*instinctive way*" because he "*feels the concept inside*" ("*because I feel inside the concept...*"). What Andrea describes recalls the processes of *appropriation* of Bakhtin, *internalization* of Vygotsky, and *individualization* of Sfard (2007) that Levrini and colleagues (2015) reformulated for science education: "Appropriation is a complex and reflexive process of transforming scientific discourse (scientific words and utterances) so as to embody it in one's own personal story, respecting disciplinary rules and constraints. The process of transformation involves one populating scientific discourse with one's own intentions, idiosyncratic tastes, and purposes to make it sensible not only for oneself but also with respect to one's way of participating in the social context of the class" (Levrini et al., 2015, p.99). The representation that best fits his needs has to satisfy a multiplicity of standards and triggers processes of meanings attribution, like Bloch's sphere (geometrical representation).

Arturo has still another framing, he conceives the knowledge as something to be propagated from one context to another, integrated and connected, and as something to be transferred from one person to another. So his need is to be able to "explain" a concept to others, namely adapting and translating the concept according to the "*audience*" (natural language). He indicated as the representation that best fits his needs is the algebraic one. However, he has always appreciated all the performances very positively, perhaps for his passion for different languages and interdisciplinary connections.

As in the case of framing, the mechanism of reasoning and the resources that the four students describe are very different. Diana's mechanism, according to the functional role she associates with knowledge, consists in "*retracing back all the steps*", that is to put the steps in chronological and logical order. Carlo needs multiple representations, and his mechanism of reasoning consists of a *process of translation* from raw data to formal knowledge and a *back-and-forth process* from one representation to another. Andrea attributes personal meanings to the concept. Arturo's mechanism, instead, consists of the "*alienation*" (or "*isolation*") of the concept from the context, its transfer to other contexts, making connections between the different contexts ("*also making interdisciplinary connections*") by continuously putting in coherence new elements with the knowledge he already possesses.

Finally, stimulating them to reflect on the role of representations in their reasoning and understanding, the four students differ also in the intuitive knowledge dimension. In Diana and Andrea, we recognize different kinds of p-prims. The sense of intuition that Diana expresses recalls *Ohm's p-prim* element described by diSessa (1993), according to which *"More effort implies more result"*. Instead, Andrea's intuitive knowledge passes through *balancing* between order and density of information. Arturo's sense of intuition, by conceiving knowledge as a "*unique network of data*", passes through a *balancing*, in this case between the new and already possessed knowledge. The knowledge element that Arturo describes recalls the analogical reasoning described above, in particular the mapping theory (Gentner, 1983). If there is nothing in long-term memory to which a new concept can be linked, then either it will not be stored or it will be stored as a single entity. If there is, however, something to which the new concept can be related, then learning occurs (Gabel, 1999). Carlo's sense of intuition instead consists in a "*mediation*" between the concreteness (associated with geometric representation) and the abstractness, the purity (algebraic representation), that is, stressing with "*geometrically see*", leads back the abstract formal language to a known form conceived as geometric form and colors.

The coordination of epistemic and cognitive aspects and needs in students' reasoning triggered by reflecting on the role of representations in their understanding shed light on the students' processes of sensemaking. This study aimed to contribute to Kapon's research work by providing further evidence of the effectiveness of the multidimensional metric for sensemaking to evaluate, measure, and assess the dynamics of students' reasoning from their discourse as they think and reason (Kapon, 2017).

This study shows how students can be engaged in sense-making processes through representations. In particular, drawing on the ability to compare, discuss and criticize representations, it emerged that developing meta-representational competencies (Sherin & diSessa, 2000; diSessa, 2004) can not only promote understanding but also students' sense of understanding. The analysis also shows also how students can have advantages from multiple external representations and, through the development of meta-representational competencies, they can be spontaneously engaged in sense-making processes. Furthermore, the analysis and the development of these competences provide insights, a zoom in on how students could use representations as conceptualizing, reasoning and meta-reasoning tools to verbalize and characterize their mechanisms of reasonings and the sense-making processes. In fact, the representations in students' discourses became a tool to explain how they think, what are their epistemic and cognitive needs.

The aim, as discussed in section 4.2, was also to understand if the "new constellation of representations" that characterize the approach to the Second Quantum Revolution could promote the acceptance of quantum physics as a personal reliable description of the world. In particular, we elaborated on an initial assumption that we tested with the experiment carried out during Rimini's implementation. The assumption (section 4.2) was: Quantum computing and the language of the logic gates and circuits have the learning potential to provide new synthetic scenarios able to satisfy students' epistemic needs (the need to recognize and conceptualize the relationship between the "real world we live in" and the abstract mathematical construct, the need of visualization, the need of comparability). We have no concrete and explicit proof about if the logical and circuital representation had satisfied exactly the three epistemic needs pointed out in the previous works of the groups (Ravaioli, 2020; Levrini & Fantini, 2013). We can anyway say that there is room to deepen it. For now, the use of multiple representations and their *collective interdisciplinary affordance* (section 4.2) seem to have the learning potential of embracing different students' ways of thinking, and their cognitive and more general epistemic needs.

In the future, we intend to investigate these aspects again. In general, the kind of response that students gave us seems to open up the possibility that the constellation of representations and, in general, the approach is actually inclusive and personally meaningful. Therefore, we will work to refine the experiment to investigate again the potentialities of the approach to quantum physics acceptance.

5.5 Investigating the Depth of the signature ideas

5.5.1 Framework and methodology

This part aims to show that the course was inclusive and within the reach of students who found elements "to feel at home". The signature ideas that emerged are not contextual to the task but are idiosyncratic and deep, being rooted in students' identities: for example, personal way of conceiving, building, and approaching knowledge, personal tastes, and purposes. To investigate the depth of the signature ideas and their idiosyncratic nature we carried out an appropriation analysis based on the operationalization elaborated by Levrini, Fantini, Tasquier, Percori, and Levin (2015). The authors, starting from the literature, reformulate the concept of appropriation[16] within science education and, through a bottom-up process, pointed out a set of markers that provide an operational tool to analyze and characterize students' discourses. In particular, they showed that a discourse of students who appropriated is (Levrini et al., 2015, p. 118, 119):

A. developed around a set of words or expressions repeated several times and linked together so as to express an authentic, *idiosyncratic signature idea*; this idea is recognizable as authentic and idiosyncratic because it is different from student to student in terms of, for example linguistic choices and the tone;
B. *disciplinarily grounded* (i.e., the idiosyncratic idea is used by the student as a tool for selecting pieces of disciplinary knowledge and coordinating them in a way meaningful from a physical point of view, namely, by respecting the rules and the constraints of the game of physics);
C. *thick* (i.e., the idiosyncratic idea involves a metacognitive dimension [what learning physics means for me] and an epistemological one [what image of physics makes sense to me]);
D. *nonincidental* (i.e., the idiosyncratic idea can be traced back to the student's reactions in different classroom activities, and, hence, it can be recognizable within a personal story that can go beyond the duration of a single episode);
E. the *carrier of social relationships* (i.e., the idiosyncratic idea places the student within the class community and, vice versa, the development of the idiosyncratic idea was inseparable from the classroom dynamics).

By applying these markers, Levrini et al. show how students can find room to nurture their personal and idiosyncratic ways of learning a discipline while still respecting its core of rules and constraints (Levrini et al., 2015, p. 129).

In the present analysis, we consider only the three students that participated in the collective interview after the Quantum Atelier Experience: Carlo, Arturo, and Andrea. The data we use for this analysis are the focus group and the collective interview carried out after the Quantum Atelier experience. The final interview lasted almost two hours and the questions that guided the discussion are reported in table 5.1.

The aim of this analysis is to investigate if the three signature ideas pointed out at the end of the profiles are deep, that is if they are grounded (B), thick (C), nonincidental (D), and carrier of social

[16] "Appropriation is a complex and reflexive process of transforming scientific discourse (scientific words and utterances) so as to embody it in one's own personal story, respecting disciplinary rules and constraints. The process of transformation involves one populating scientific discourse with one's own intentions, idiosyncratic tastes, and purposes in order to make it sensible not only for oneself but also with respect to one's way of participating in the social context of the class" (Levrini et al., 2015, p.99).

relationships. To explore this, we do not consider only the focus group, but also the final collective interview and the descriptions of the artworks realized by the students (Annex 4).

5.5.2 The three case studies

The case of Carlo

As already pointed out looking for key expressions (Marker A), the signature idea of Carlo's discourse revolves around a conception of knowledge as formally encoded. The understanding for him consists in grasping the mechanism inside ("*finding a correlation*", "*figure out what mechanism*", "*given a context,* [understand] *the reasoning within*") as well as in seeing geometrically ("*see geometrically*", "*a geometric pattern in my opinion is always the best*"). The "*seeing geometrically*" allows him to "*automatically*" understand since it evokes nature and what he constantly sees: namely known forms (*I see a shape... It's almost like it's already in my mind*). Furthermore, the active participation and the repetition of expressions like "in my opinion" and "for me" suggest that Carlo took an active part during the focus group and give us information about social relations (Marker E). In particular, he often took the position of contrarian ("*I think it's the exact opposite*", "*I start so I don't have against anyone*."), highlighting that his social position was very personal, always enriching the group discussion.

Carlo's signature idea also emerged during the final collection interview. By recalling the course and the representations met, Carlo confirms that his favorite representations for his understanding and his needs are the circuital and the geometric representations ("*The representation through circuits or the Bloch sphere in three dimensions...*"). He stresses the importance of "*seeing geometrically*" ("*more geometric is the type of representation and the easier it is for me to understand*") and his need for multiple representations to activate personal reasoning mechanisms (*process of translation* and a *back-and-forth process*, see section 5.4), which allow him to better understand a concept. In Carlo's words

> "*if a geometric representation is combined with the formalization... it allows [me] to understand the reason why that thing is represented in that way. For me, it's the best way to understand things.*"

This suggests that Carlo's signature idea is nonaccidental (Marker D). The constellation of representations encountered during the course has satisfied the cognitive and epistemological needs favoring the activation of personal mechanisms of reasoning.

Carlo's signature idea is perfectly embodied in his artwork. First of all, he chose a painting by Kandinsky (see Annex 4) that embodied the return to the simplest shapes and colors after the artistic movements of impressionism and expressionism. The choice of this painting recalls his need of recognizing and tracing back a concept to shapes and colors ("*seeing and understanding everything through the eyes so in my opinion the shapes, colors...*", "*I see a shape...*"). They allow him, immediately and naturally, to understand because they are already in his mind ("*It's almost like it's already in my mind*"). Also in the final collective interview, he stressed "*if the shape is made in the correct way... I still get to understand the concept*". He chose representative art to represent the concept of the superposition state also because the superposition is "*already in the pictorial conception*". In Carlo's words

> "*Representative art is based on the... It connects well to the superposition principle because in general already in the <u>pictorial conception</u> of the painting <u>overlap</u>... <u>different layers</u> or anyway... is the facade that is represented... which is <u>always the sum of a certain number of things</u> [...]*".

The painting choice that embodies Carlo's signature idea is grounded in disciplinary knowledge (Marker B): the superposition principle as a linear combination of the states of a system can be reconceptualized, metaphorically, as the overlap of the different layers in the painting surface.

The access to a visual geometrical representation in Carlo's artwork and the irrational channel, on which an artwork can be based, allows shedding light also to the shift of paradigm (Marker B): from the binary logic and determinism that characterize, for him, our reason to probabilistic logic. In Carlo's words

> "[the channel with which it allows to communicate the work of art] *is not so much emotional and emotional but rather irrational because... We also talked about it in the course when... when we were exposed to the concept of probability that... we lose that... that perception we have always had of binary logic to enter into a probabilistic logic. Similarly, our reason [reason understood as our system of values or our way of conceiving reality] at this time, as we understood society or as we think about logic itself, is mostly deterministic and binary so... in this case... just in this specific case to explain something that escaped from that... from that pattern of reason, to use something that eluded reason in itself was a great way of representing it*..."

The representation of the superposition principle through the artwork not only incorporates his signature idea (see geometrically, lead back to shapes and colors to understand intuitively and naturally a concept) but also allowed Carlo to reconceptualize what the transition toward quantum physics meant for him.

Finally, Carlo's signature idea is also thick (marker C) because it is bearer not only of what learning physics means for him (see geometrically, alongside more representations) but also of the image of physics he has: physics as formal knowledge that, however, can be traced back to nature through shapes. Moreover, the final interview allows us to enrich Carlo's signature idea. At the beginning of the interview by recalling the focus group he said

> "*I understand [a concept] more from a geometric or conceptual point of view*."

The conceptual perspective was quite new but the description that Carlo made about his work highlighted how reasoning, in an interdisciplinary context between physics and art, led him to conceptually elaborate especially the superposition principle and the passage from classical to quantum logic. The two concepts are reconceptualized as a superposition of layers in the artwork and as an irrational element due to the fall of a scheme of values representing the "classical reason". This conceptual aspect also enriches Carlo's image of physics (marker C) that emerged from the focus group. At the end of the interview, when we asked them to think of a word to describe the relationship between science and art, Carlo said:

> "*The fact that* [physics] i*s practically applicable in every field in the sense... with a little ingenuity…* [...] *with a little care you can get... makes it clear that [physics] is the matrix of things and that you can connect very easily that is... Anyway in quotation marks* [it is] *ubiquitous.*"

Physics is a "*matrix*", it's everywhere. It is thus elevated to an instrument of thought, in this case, also to re-read and populate a representation of meanings and rework it to bring out the deep personal meaning attributed to some basic concepts of quantum physics.

The case of Arturo

As already pointed out looking for key expressions (Marker A), the signature idea of Arturo's discourse revolves around a conception of understanding as the ability to assimilate the concept ("*process of assimilation of a concept*", "*alienating it from the context of the explanation*"), to "*make interdisciplinary connections*", and explain it to a different audience ("*explain the same object*"; "[considering] *previous knowledge of the audience*", "*transparent in relation to the experience of a person*", "[there have to be] *balance between the old and the new*"). Arturo conceives knowledge as something that can be assimilated and has to be transmitted.

The active participation in the focus group's debate and the frequent repetition of expressions like "*in my opinion*" and "*for me*" suggest that Arturo's position was very personal and a carrier of social relations (Marker E). During the focus group, Arturo often had the role of "re-elaborator", that is, he often picked up words and re-elaborated what was being discussed, providing images and new personal keys to reading and enriching the speech (e.g. "i*n my opinion these words can be divided into two different groups [...] we look at a single variable, while the others like for example "compact" has more variables, so you have to take into consideration more aspects... and paradoxically... more variables are present and more... the idea seems similar to me*").

Even if Arturo, in general, gave high evaluations to all kinds of representations, he chose the algebraic as his favorite one since it respects his idea of purity and elegance. Perhaps he could also choose this one since it respects the balance between the old and the new that he needs to find in the representations to understand them. Also in the collective interview, by recalling the representations he prefers, Arturo said:

> "[My favorite representation is] *mathematical representation. Coincidentally the title of the work that I finally decided to give is psi... with the notation bra-ket…*"

At the end of the focus group, Arturo explains what he means by elegance enriching the discourse of a philosophical aspect:

> "*I realized that I defined pure and elegant mathematical representation, because... We are both abstract and concrete. Reality is concrete... we reach a point where we can represent reality through mathematics and <u>we are in the middle</u>... I mean elegance by this... our put in the middle and manage to understand both worlds...*"

Andrea considers the complex human, nature, and mathematics elegant and pure. Mathematical representation, better to say the mathematical language represents the ability of man to read nature. And that's what he represents in his artwork. The artwork consists in an evolving fractal that metaphorically represents a "*constantly evolving* [quantum] *system*". Evolution is interrupted by the "*action of the observer on the system*", that "*emphasizes <u>human</u> action*". As Arturo argued,

> "[I represent] *an evolving fractal so I tried to <u>connect just a purely mathematical description in a visual key</u> that can be interpreted in my opinion however <u>as something interesting to observe also... pleasant for the mind to watch</u>…*"

by stressing the idea of making the formal description viewable to the observer. The final part of Arturo's words suggests that he has always in mind the other, the audience. These aspects underlie that his signature idea is nonaccidental (marker D).

The meaning of elegance elaborated by Arturo clearly emerges when he discusses the role of the human observer who makes the state of the system collapse through the touch of the hand, stressing a human-oriented perspective. In Arturo's words

> "*In my opinion for an observer is enough the presence of something physical with which to interact not so much... a proximity sensor in front of which to pass the hand and emphasizes the... the need for intervention of the observer and... [...] the <u>use of his own hand</u> ... is however that part that we use daily... It's one of the most sensitive parts of the body because... because <u>we need it to be able to understand our position with nature</u>, with the space that surrounds us*"

The disciplinary (physics) groundedness of the discourse (marker B) emerges more in a critical reflection about the limits of the representations he chose. Arturo argued that, while the role of the observer is well analyzed, the concept of collapse is not right represented because, in his words,

> "*The fractal always degenerates into the same initial situation ... that is it is a unique collapse with only one possibility*"

emphasizing that the fractal does not represent what a quantum object is, because the measurement that causes the collapse of the wave function is "*unique*".

Part of Arturo's signature idea emerges in describing the thing he most appreciated at the end of the Quantum Atelier project:

> "*The need to adapt their level of exposure in order to engage the public. [...] through this experience that... I really like to explain... a concept to a person, regardless of his level so also... I realized that I really like to adapt a concept to different levels... [...] I like to re-elaborate concepts in various interpretations... […] I have always been very fascinated anyway by the ability of the person to adapt their technical language and... their method of interacting with the person... [...] I enjoy just having to adapt a concept to the person in front of me...*"

This recalls his need to understand of explaining the same object considering the previous knowledge of the audience, which means re-elaborate concepts and adapting the technical language to a broader audience (natural language aspects). This remarked the nonaccidentality of Arturo's signature idea.

Finally, Arturo's signature idea is thick (marker C). The idiosyncratic idea incorporates what learning physics means for Arturo, namely embracing a multiplicity of languages and representations and adapting them to explain knowledge to others. Arturo's image of physics is human-oriented and thus and therefore translatable into natural language. More generally, Arturo's signature idea reveals an image of knowledge as "harmony" between many fields. In talking about the relationship between science and art, Arturo said

> "*The concept of being able to find harmony in many areas [...], [the word is] representation of the whole or the harmony that represents a meeting point of many disciplines…*"

This reflects his conception of knowledge as a "*unique network of data*" and his need to continuously "*make interdisciplinary connections*".

The case of Andrea

As already pointed out looking for key expressions (Marker A), the signature idea of Andrea's discourse revolves around a conception of knowledge as something that has to be internalized and assimilated. The construction of knowledge for him is not immediate but is a process of appropriation and of popularization of personal meanings. So the expressions that characterize his discourse are "*internalize*", "*assimilate a concept*", "*I feel inside the concept*", and "*expose* [a concept] *almost instinctively*". The personal process of knowledge construction and the search for personal meanings, emphasized also by frequent repetition of "*let's say*", are reflected in the plurality of standards that a representation has to satisfy: it must have a "*squaring*", "*must be complete in itself*", "*exhaust*[ive], "*elegant*" and "*compact*", where for him the compactness lies in its "*being compressed*", in "*not neglecting aspects*", in being "*simple*" but "*not leaving strands in half*", in having a "*balancing*" between "*density* [of information]" and "*order*". These are only a part of the personal and key expressions that characterized Andrea's contribution.

The active participation in the focus group's debate and the frequent repetition of expressions such as "*in my opinion*" suggest that Andrea's position was very personal and a carrier of social relations (Marker E). During the focus group, especially in parts B and C from figures 5.27 to 5.31, Andrea started and oriented the discussion, as happened for example in the search for the meaning of the adjective compact. He used to talk much more than the others both in terms of frequency and duration of the intervention. Anyway, he often tried to not lose communication with others and to give them space by trying to reduce his intervention and finding examples and words to make his thoughts understandable. The problem related to the comprehension of knowledge and how Andrea elaborates on it also emerges in the collective interview. His primary fear was that the work he had made, which was highly conceptual and full of philosophical significance, was too elitist. In Andrea's words

> "*It was a work a bit... a bit erudite... in my opinion there was ... a quite elitist aspect in the work... at the level of direct experience... of understanding, but not even total understanding... understanding let's say... sufficient... So maybe ... It doesn't mean that I looked, I mean...* [...] *I was looking at them... how to say in the sense... we overdid it, I don't know... it's hard... so I tried more and more to put the emphasis... on the interactive dimension of the work to try to fill a little... [...] I don't even know how it could be solved... Let's say preparing first in the explanation... I think it was just an intrinsic criticality of the work that... as you could not make up [...] physically the artwork was at the bottom of the room and it was an advantage...*".

The elitist aspect was therefore intrinsic to the work. As in the focus group, he was constantly searching for words and images that could clarify what he was saying and his thoughts, In the presentation of the work to his classmates he tried to make up for the great conceptual load through the interactive dimension and the explanation to make it possible.

Even if Andrea, in general, gave high evaluations to all kinds of representations, he showed a particular affection for the geometrical one. This emerged during the focus group when he argued that Bloch's sphere reflects, more than others, his idea of elegance since it is compact, respecting the *balancing* of density and order that proved to be the basic element of intuition of Andrea. In his words:

> " *It is the highest idea of order we have and at the same time has a very high density, in the sense... it transmits us a... basically a dimension of infinite probability, so.... among the graphic representations is the one that has the greatest elegance in this sense...*"

Also during the final collective interview, in discussing the more effective representation met during the course, Andrea said:

> "*Bloch's sphere in this sense is the visualization that... a representation that... fascinates me in the sense... it is very intuitive…*"

Already during the focus group, Andrea populated Bloch's sphere of personal meaning discussing how the main image it conveys to him is the "*probabilistic dimension*", "*a dimension of infinite probability*". During the final interview, he recalls this and added an aspect that gives us a clue about why for Andrea this representation is "*intuitive*":

> "*Not just the sphere itself as a geometric object therefore as a representation of infinity but also the sphere as an analytical object in the sense... and then also the idea of points... maybe I had a flash [...] anyway after I read something... of matrices and vectors so I managed [...] the idea of the base, of how to construct a vector field... in relation to these probabilities understood as... as a component of the 0 1 vector to describe a point this thing... I could connect it very easily so I found it effective*".

Andrea adds to the element of infinite probability the idea of the sphere as an analytical object and very clearly describes the "*flash*" he had when he "*connected*" the probabilistic dimension with the analytic construction of the vector field. Andrea's discourse recalls Piaget's mechanism of cognitive balancing, namely the assimilation and accommodation processes. These processes, which lead to "*feeling the concept inside*" and "*internalizing it*", merge into the feeling of the "*effectiveness*" of the representation and into the sense of "*intuitive*".

Andrea's signature idea is therefore nonaccidental (marker D) and the way in which he describes Bloch's sphere and what it incorporates make his discourse also deeply disciplinary grounded (marker B).

The signature idea manifests itself also in the artwork that Andrea realized, especially in the process of creating and meaning attributing. Andrea's artwork is very conceptual and revolves around the relationship between two realities (the macroscopic and the microscopic worlds) and their inseparability. In Andrea's words

> "[the idea was to] *work on the relationship between two realities* [...] *one the substrate sub-layer of the other [...] The vision* [with augmented reality] *is simultaneously... on two levels... [...] communicates more this inseparable relationship between the two realities... namely the quantum world the real world precisely... They're connected in the sense that... they're the same thing... That is, the real world is just the macroscopic dimension of the quantum world and vice versa the microscopic world...*"

The initial idea of Andrea was to work with augmented reality to represent this relationship. With the artistic references proposed by the Italian professor, the work has changed a bit. The fulcrum has become the unveiling and, therefore, knowledge in relation to the subject:

> "*Professor Giuseppucci has me... I dropped these artistic references and there maybe it was a phase...* [...] i*n the development of the work was critical* [...] *The professor added the idea of the unveiling and then precisely... the idea of making disappear and make appear…* [...] [therefore from the conceptual point of view the artwork represents] *a relationship between knowledge and reality, and especially knowledge in relation to the subject...* [...] *the subject as an essential element in a paradigm gnoseologic... in the sense is the fulcrum of... in a sense... of the quantum revolution, at least of the first...*".

In describing the critical aspect of the development of the artwork, Andrea used the same intuitive elements that he used during the focus group to discuss the meaning of the adjective 'compact':

> "*There was a risk of an imbalance maybe... at the beginning there was an imbalance on the technical side when we started from augmented reality and then we risked a... an imbalance on the conceptual side then... with the introduction of the artistic reference...[...] it was decided to contribute mainly from the point of view of artistic reference. And... So after it the work consisted of trying to bring back a balance between the concept and the form...*"

Andrea's discourse is populated by the main keywords of the focus group: "*imbalance*" and "*balancing*". The search for balance and his work on the form became the pivotal aspect to solve a sort of conflict and make the artwork accessible ("*the form [...] made... actually spendable the work namely it made it somehow accessible*").

His idea of knowledge as a process of construction and as a process of appropriation emerges when he finally manages to make sense of the work of art through the artistic reference given by the teacher:

> "*when I go to assemble in the end I went to write the text... I read the text of the teacher who finally made me understand something of what was in the work... I managed to... give myself that... let's say... that organic perspective and be able to communicate it to the public*"

Andrea, again, very lucidly describes that "flash", that moment when he manages to connect everything and give a personal meaning to knowledge.

Andrea's signature idea is therefore nonaccidental (Marker D) and also thick (Marker C). It incorporates what learning physics, and in general learning, means for Andrea, namely to appropriate a concept and fill it with personal meanings. It also comes out what image of physics Andrea has, that is, as something formal and rigorous but it communicates with many other areas, and it can be shaped. The formal image of physics emerges several times during the focus group, especially in the description of the conceptual content of the artwork. For example, he said

> "*When Morandi's painting arrived* [the artistic references] *it* [the rigor] *went to hell... it was working more on the artistic aura... [...]* [after] *in the explanation I tried a little to recover the scientific rigor*"

Also when he discussed the importance of the conceptual dimension as well as the formal one, Andrea said

"*<u>Conceptual mastery</u> is useful... It is useful to write texts to make us philosophical reflections, but you have to be careful in the sense that... Philosophers often say they take great misguided when they talk about science because they believe [...] to have this absolute mastery of the principle in reality... [but] they <u>lack a little of a formal dimension, formalization</u>...*"

In talking about the relationship between science and art, Arturo chose the word perspective

"*<u>perspective</u>... [...] I love to do... work in a perspective... [...] in a <u>multidisciplinary horizon</u>... because in the sense even when I approach a certain type of... of scientific subjects... [...] I have a certain affinity for a more conceptual aspect ... maybe it's a bit dictated by that... that fascination for... for advanced science [...] so maybe you could... [...] <u>work at an interdisciplinary level</u> [...] ... I really enjoyed working in this way also because it is the way I'm more comfortable*."

Andrea, therefore, stresses not only his image, more generally, of knowledge as something interdisciplinary especially when one remains on the conceptual level, but also his need to work in this way.

5.5.3 Discussion

In the present analysis investigated the depth of the signature ideas, namely if students found the course inclusive, found space "to feel at home" and nurture their own personalities.
The depth was explored through the "appropriation metric" elaborated by Levrini and colleagues (2015). This metric consists of 5 interacting dimensions: the conceptual and disciplinary (Marker B), the metacognitive and epistemological (Marker C), the idiosyncratic (Markers A and D), and the social (Marker E) ones (Levrini et al, 2015). In particular, the unpacking of students' discourses along these five dimensions implied to look if they were (a) an expression of personal signature ideas, (b) grounded in the discipline, (c) thick, namely they involve a metacognitive and epistemological dimension, (d) nonincidental (consistent throughout classroom activities), and (e) carriers of social relationships in the sense that it positions the student within the classroom community.

Considering both the focus group data and the final collective interview carried out after the Quantum Atelier Project, we pointed out that the signature ideas of Carlo, Andrea, and Arturo can be unpacked using this metric. By looking at their key expressions, many elements have emerged already from the focus group such as Carlo's need to "see geometrically" to recognize shapes; Arturo's need to translate and re-explain knowledge using many languages, adapting it according to the audience; Andrea's need to "feel a concept inside", internalize it. During the focus group, these "biographical" signature ideas showed to be non-accidental and thick, that is, carriers of what, for these three students, means learning physics and what image of physics they have.
The diversity of students' attitudes towards learning physics and the plurality of physics images as well as the results of the previous analysis on robustness (the possibility of activating different mechanisms of reasoning) suggest that the approach we developed is inclusive and can resonate with different personal ways of understanding.

The analysis of the final collective interview highlights how deep the signature ideas of the three students are, allowing us to clarify and widen them from "biographical", conceptual, social, and metacognitive/epistemological perspectives.
The presence of the 5 markers in students' discourses suggests that they were engaged in complex and reflexive processes of transforming scientific discourse (scientific words and utterances) so as to embody it in their own personal story, populating it with personal intentions, idiosyncratic tastes, and purposes "in order to make it sensible not only for oneself but also with respect to one's way of participating in the social context of the class" (Levrini et al., 2015, p.99).

This analysis contributes to the previous work of Levrini and colleagues (2015) by providing insights into the interplay between students' *inter-disciplinary engagement* and *their identity construction*.
The Quantum Atelier experience, namely working in the nexus between science and art, showed to be very meaningful and fruitful for the students. The way in which the 3 students talked about the experience highlights the significance of the experience for reflecting and grasping more deeply the theme and, in particular, the concepts they represented as well as for promoting the awareness and the development of personal skills (such as technical ones) and nurturing their personalities.
More specifically, Carlo found a way to express himself and what he understood through shapes and colors. Arturo understood his interest in adapting his own language and explaining to other people as well as his technical skills. Andrea found himself in the type of reflection carried out that is embedded in his artwork and appreciated working in the nexus because he felt comfortable.
Levrini and colleagues argued that it is possible to design complex learning environments (see Chapter 2 section 2.1) to nurture personal and idiosyncratic ways of learning a discipline while still respecting its core rules and constraints. The data showed that "something happens", that is working in this interdisciplinary context impacted on students' personal, cultural, and social growth. We are now going to investigate the role of rules and constraints in the nexus between science and art. In other words, what constraints art, with its values, methods, practices, and ideas imposed in rethinking the scientific content and vice versa, namely what constraints science with its values, purposes, methods, and practices have influenced the work. Furthermore, we are going to explore what kind of dialogue was engaged and what aspects had to be negotiated from both worlds.

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Conclusions Part II

In the second part of the thesis, the research thread previously investigated by the PER group of Bologna was re-opened, namely the problem of non-acceptance quantum physics (Ravaioli 2020, Ravaioli & Levrini, 2017, Levrini & Fantini, 2013) and its epistemic nature (e.g., Levrini & Fantini, 2013; De Regt, 2009). For "non-acceptance", we refer to "a specific reaction of students who explicitly did not accept quantum physics as an adequate and personally reliable explanation of reality" (Ravaioli, 2020, p. 115; Ravaioli & Levrini, 2017, p. 2). In particular, they pointed out three epistemic needs that the theory and/or standard ways of teaching could not satisfy:

- the need to recognize and conceptualize the relationship between the "real world we live in" and the abstract mathematical construct, that is to attach a reliable and "realistic" meaning to new basic elements; or the need for 'reification', recalling the concept of reification as the objectification of an abstract construction process by Sfard (1991);
- the need for visualization, either in terms of the need to form a mental picture of the quantum object or, more in general, in terms of the need to have a comprehensive view that can guide intuition;
- the need for comparability, that is the need to find criteria to understand where and how the epistemological interpretation/description of quantum physics is different from the classical one (Ravaioli, 2020).

From the first implementation of the module on quantum computing with secondary school students (Satanassi, Ercolessi & Levrini, 2022; Satanassi, Fantini, Spada & Levrini, 2021; Satanassi, 2020), we noticed that there was "something" in the course that seemed to promote a positive attitude toward quantum physics that we had never observed in the previous implementations. Reflecting on the differences between the teaching approaches, we considered the possibility that the multiple representations set, that characterizes our approach to the Second Quantum Revolution, could have a different impact on students' understanding and could satisfy the epistemic needs. We had the perception that the different external representations, especially the logical (truth tables) and circuital (quantum logic gates and circuits) ones, can activate and support the construction of personally reliable internal representations. Such a *sensitizing idea* made us formulate the following conjecture: Quantum computing and the language of the logic gates and circuits have the learning potential to provide new synthetic scenarios able to satisfy the epistemic needs we had previously described.

We investigated it by evaluating students' meta-representational competencies, namely their skills in explaining, discussing, comparing, and critiquing different kinds of representations (diSessa, 2004; Sherin & diSessa, 2000). We designed and implemented an experiment articulated in three phases. The first phase consisted of a focus group about the role and effectiveness of the different representations (algebraic, circuital, logical, axiomatic, and experimental) in students' understanding. The second phase consisted of a focus group with 8 selected students to investigate the dynamic of students' understanding through representations and if, how, and why those representations could satisfy their cognitive and epistemic needs. Finally, the third phase consisted of a collective interview, with three students, about the interdisciplinary experience in which they participated, the Quantum Atelier. At the end of this experience, they produced three exhibitions that represented the concepts they found most revolutionary.

From the teamwork analysis, we noted that the different representations played different roles in students' understanding of some basic concepts of quantum physics (quantum state, state manipulation and evolution, measurement, and entanglement). Students generally found the algebraic representation effective and accessible, namely within their reach as other studies pointed out, especially for the calculation and manipulation aspects (e.g., Serbin & Wawro, 2022; Schermerhorn, Passante, Sadaghiani & Pollock, 2019; Gire & Price, 2015). The experimental representation showed to be more effective for understanding the concepts of quantum state and measurement. The circuital and logical representations showed to be effective, in particular, for the concepts of state manipulation and evolution, and entanglements. Anyway, students found these generally fruitful since particularly familiar, conceptually clear, transparent, and schematic.

The second part of the analysis, the analysis of the focus group and the final collective interview, concerned the investigation of the robustness and the depth of the signature ideas of four students respectively through the sensemaking metric (Kapon, 2017) and appropriation markers (Levrini, Fantini, Tasqueir, Pecori, and Levin, 2015).
From the robustness analysis, we noted that meta-representational competencies (Sherin & diSessa, 2000; diSessa, 2004), namely the ability to compare, discuss and criticize representations, can promote understanding as well as students' sense of understanding. Furthermore, through their development, students can have advantages from multiple external representations, and they can be spontaneously engaged in sensemaking processes.
As regards our conjecture, we have no concrete and explicit proof if the logical and circuital representation had satisfied exactly the three epistemic needs pointed out in the previous works of the groups (Ravaioli, 2020; Levrini & Fantini, 2013). We can anyway say that there is room to deepen it. In general, the approach and the multiple representations set seem to have the learning potential of embracing different students' cognitive and more general epistemic needs.
The kind of response that students gave us seems to open up the possibility that, in general, the approach is inclusive and personally meaningful since it gives room to different ways of thinking.

From the depth analysis, we noted that the discourses of the three students can be unpacked through the appropriation markers (Levrini et al., 2015). This means that their discourses showed to be grounded in the disciplinary content, "biographical", carriers of what learning physics means for them and of images of science (metacognitive and epistemological dimensions) as well as of social relationships. Therefore, the students appropriated the disciplinary content by incorporating it in their own personal stories, populating it with personal intentions, idiosyncratic tastes, and purposes.
The Quantum Atelier experience, namely working in the nexus between science and art, showed to be very meaningful and fruitful for the students. In particular, the experience seemed to help them to reflect and grasp more deeply the concepts represented in their artworks, to promote the awareness and the development of personal skills (such as technical ones) as well as nurture their personalities.
The data showed that working in this interdisciplinary context impacted on a personal, cultural, and social growth. We are now going to investigate the role of rules and constraints in the nexus between science and art. In other words, what constraints art, with its values, methods, practices, and ideas imposed in rethinking the scientific content and vice versa, namely what constraints science with its values, purposes, methods, and practices have influenced the work. Furthermore, we are going to explore what kind of dialogue was engaged and what aspects had to be negotiated from both worlds.

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Final remarks

The work is placed within the Physics and Science Education research. A course on the theme of the Second Quantum Revolution was designed based on the belief that learning and teaching are meaningful if they productively combine three complex systems: the real world, the system of physics disciplinary knowledge, and the cognitive system of learners. In light of this, we believe that physics knowledge and, in general, scientific knowledge, involve many different dimensions, such as the conceptual, epistemological, historical, philosophical, social, political/institutional ones. It is possible to design learning environments that implement *forms of productive complexity* (multiperspectiveness, multidimensionality, and longitudinality) to pursue the general goal of enabling students to find out their ways to understand the content (Levrini & Fantini, 2013) as well as develop, through science education, competencies that can help them to navigate the complexity of the present and orient themselves toward a personal and collective sustainable future (OECD, 2019). Therefore learning "of" and "in" a discipline (Fantini, 2014), or in this case “of” and “in” more disciplines, can contribute to a project of personal, cultural, and social growth, inhabiting the tension between inter-disciplinary authenticity and personal relevance (Kapon, Laherto & Levrini, 2018).

The thesis aims to contribute to the political and institutional calls launched to the world of science education to promote quantum literacy at different levels.
In the first part, particular emphasis was given to reflecting on the Second Quantum Revolution from a cultural point of view. It was unpacked along three perspectives- the conceptual and ontological, the logical and epistemological, and the linguistic -, which led us to highlight the cultural aspects that distinguish the Second from the First Quantum Revolution.
By operationalizing the forms of productive complexities, seven design principles were pointed out to value the learning potentialities of the Revolution and the emergent technologies. These principles touch on different dimensions: from the cultural to the future-oriented dimensions, from the interdisciplinarity to the need to find new narratives and languages to talk about science.
The module's activities were presented in relation to the design principles implemented. In particular, we focused on two activities - the teleportation and the random walk - that were objects of a pilot study with secondary school students to investigate the impact of the approach on their understanding and to gather feedback and reactions to refine and improve the course. The overall approach showed to be inclusive and within the reach of secondary school students involved in the extracurricular implementation. Furthermore, they found particularly effective reasoning in an interdisciplinary way.

The second part of the thesis was dedicated to the investigation of the potential of the Second Quantum Revolution and of the emergent technologies in learning some basic concepts of quantum physics. We were guided by the feeling we had from the very first implementation concerning a different students’ positions/attitudes toward quantum physics compared to the course on the first quantum revolution implemented previously by the PER group of Bologna. We, therefore, conjectured that the difference should lay in the external representations (experimental, geometric, axiomatic, algebraic, logical, and circuital) used in the course and, in particular, in the logical and circuital ones. Our hypothesis was that these representations could provide a new synthetic scenario that could satisfy three epistemic needs pointed out previously by the group (Ravaioli 2020, Ravaioli & Levrini, 2017; Levrini & Fantini, 2013): the need to recognize and conceptualize the relationship between the "real world we live in" and the abstract mathematical construct, the need for visualization, the need for comparability.
A three-phase experiment was carried out to test the conjecture during an extra-curricular implementation at Liceo Einstein, Rimini. From data analysis emerged that the different representations had a different impact on students’ understanding.
In the specific cases of four students, by developing meta-representational competencies (diSessa, 2004; Sherin & diSessa, 2000), students were spontaneously engaged in sensemaking processing and

activated personal ways of reasoning through representations. This did not confirm our conjecture but give space to deepen it since the set of multiple representations proved to satisfy students' cognitive and more general epistemic needs.
In three specific cases of students that participated in the Quantum Atelier Project, we noted that appropriation processes were triggered by working in the nexus between art and science.
We are going to conduct a study to investigate what happened and what were the key elements, between art and science, that led a deeper understanding of the content.

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Annex

Annex 1: Eve's city

Eve is a medium-sized coastal town that overlooks the Mare Continuum. The population of about twenty thousand people has always been involved in activities generally related to local tourism and fishing.

The coast is wrapped around the gulf within which the small port is gathered. In fact, the city is built mostly along the three kilometres of coastline. The center remains instead in the inner part to three hundred meters from the sea and south of the port. Here the shape of the land has given rise to a hill on which lie the ruins of the ancient castle of Espagnat now surrounded by a block of low houses that house the historic population of Eve. In the middle of the block, there was a small supermarket, the post office, a school complex that accommodates several grades of pupils, and a bar usually attended by pensioners and pensioners. The Town Hall and the Registry Office were built on the part of the castle that remained usable. Maintenance work on the structure has recently been carried out. The mayor Bob Hat is oriented to lead a campaign for the enhancement of the ancient area of Eve.

The pre-eminent activity of the city, however, develops along the coast. Thanks to the presence of some small beaches this part of Eve has expanded with some small hotels and some prominent ones, such as the Local Grand Hotel Aspect. In the summer the coast is able to accommodate several hundred tourists in their accommodation facilities that over the years have been increasingly innovative. Tourism is also favored by the presence in the seas of vast colonies of dolphins and blue whales. In addition, the fishing industry is particularly active for the presence of particular varieties of snapper, chanterelle, shrimp, dates, grouper, pike, and cod.

The town has jumped on the front pages of the media thanks to some events related to a dispute really unique but typical of our times in which some parts of society manifest needs to march with different times than the evolution of technologies.

Recently, in fact, the city administration was asked by the company HAL to have permission to set up a laboratory on an artificial island anchored a mile from the coast. The laboratory, led by researcher Alice Wonderland, aims to test the activity of the Super Quantum Computer HAL9000. Eve was chosen by Alice because of the particular currents of the gulf suitable to test the experiment. The artificial island is in fact built on thousands and thousands of floating sensors, each of which is able to adapt its structure according to the stresses of the sea. In this way, the island has stability comparable to a structure built on land. The sensors would be driven by HAL9000. The computer would also be able to determine and regulate the climate within the large dome that encloses the island. To support the request, Alice explained how HAL9000 could lead to massive research in the pharmaceutical field by patenting important medicines and thus pouring huge profits into the coffers of the Bank of Eve. HAL9000 will also be able to manage personalized pharmaceutical care and make citizens' bank codes not decrypted but Alice says, the encryption system should be switched from the old system to the new system in a very short time to avoid the risk that computer hackers, taking advantage of the deformity of the system. They must be given the codes themselves and all the private information of individual citizens. Alice Wonderland also points out that her company could include a section for researching new materials and ecosystems. This could help to renew the fishing and nautical sectors by encouraging the growth of the city in those areas. The benefits of using this new technology are countless, says Alice, it is not wise to miss the opportunity to win the monopoly of the creation of cloud computing and machine learning and all the innovative instances related to the use of quantum computers, instances that can launch Eve into the future as the absolute protagonist.

Mayor Bob Hat, in response to the important request, has convened the city council in some extraordinary sessions, to decide whether to support and take advantage of any benefits feared by Alice Wonderland or limit its scope.

Part of the population showed strong resistance. For a few months now, the no-HAL association has been set up, which advocates the preservation of an Eve city as it has always been and is strongly opposed to the construction of new artificial islands and new domes which, they claim, would drive away the dolphin colonies and In a short time, it would distort Eve's vocations. The request is also addressed to small herbalists and local and foreign pharmaceutical companies, concerned about the risk of monopoly in this sensitive area of research.

According to a recent statistical survey conducted by the magazine Alta Finanza, it seems that, however, a part of the population of Eve advocates the idea of building artificial islands and making them autonomous also administratively. This would preserve the ancient customs of the old Eve and cultivate new interests in the new Eve.

Alice Wonderland has re-launched her proposal to the mayor stating that the future does not wait and must be ready to accept the changes.

Activity 1: Analysis of the situation and identification of stakeholders

Putting yourself in the shoes of the public decision maker (the Mayor of Eve, Bob Hat), you are called to make a choice about Alice's proposal and, therefore, imagine having to decide whether: Unlock some urban constraints to set up the laboratory and activate the construction of a technological center at the forefront.

Before making the decision, analyze and outline the situation by recognizing a) the actors involved (stakeholders), b) their needs and interests, and c) the interactions between them. Use a drawing, a map, or some other graphic medium as a tool to conduct the analysis and to build a synthetic but articulated image of the city and the relationships between the needs of the different stakeholders.

If you feel you need more data, to better know the social, technological, urban planning, etc. situation of Eve, introduce the "missing data" that you consider appropriate or contextualize the problem at your convenience, explaining the choices made.

Activity 2: The decision

Were you Mayor or Mayor of Eve, would you accept Alice Wonderland's proposal? Why?

Activity 3: Scenario analysis

Read carefully the 3 scenarios of the city of Eve in 2040 below and answer the questions at the bottom of the document.

<u>Scenario A</u>

An artificial intelligence, Pensiero Profondo, became Eve's mayor. Pensiero Profondo is based on HAL10000, a new and powerful quantum computer that has replaced a couple of years HAL9000. Using HAL10000 and its quantum algorithms, Pensiero Profondo is able to manage and optimize all the services of the city (traffic, distribution of medicines, ...) and the continuous demand for training and work that comes to the island from all over the world. In fact, Eve has become a coveted destination for experts and researchers from all disciplines and students and technicians from all over the world for international specialized schools of the highest level, created to train experts in the quantum technologies of today and tomorrow.

Pensiero Profondo identifies and communicates in real-time the jobs available, as well as suggest the possible start-ups able to quickly attack the national and international market and gives support for their realization. Pensiero Profondo, based on the values of socio-economic parameters (wealth per capita, plurality of skills, healthy competition and competition in work, conflicts due to ethnic and religious diversity, psychological problems of the inhabitants, and freedom of expression) filters requests and evaluates CVs, welcoming potential citizens who are likely to optimize parameters.

The power and efficiency of HAL10000 are so well known worldwide that the city of Eve has become a real database. It manages the information of many countries and helps them to solve problems, especially concerning the economic and medical fields. In addition, the secure quantum network in which Eve is wrapped ensures that HAL10000 receives data practically in real-time, allowing Deep Thought to handle problems and act in a very short time if necessary.

The new quantum encryption systems used by Deep Thought, make it almost impossible to hack banks and industries, ensuring maximum security for the data of the state and individual citizens.

Thanks to the quantum algorithms on which the new supercomputer is based, they allow Deep Thought to organize the life of every citizen making routine activities easier (paying bills, cleaning the house, going to work, looking for information) and free time to devote to themselves and their hobbies. It is by virtue of the motto freer time for all that the citizens have also freed themselves of the task of voting, preferring the efficiency of artificial intelligence to the confusion of electoral disputes, and engaging in their hobbies and reading and studying to make the system even more efficient. There is no shortage of entertainment venues such as cinemas, sports facilities, discos, bars, arcades, and coworking.

Scenario B

Eve, thanks to the innovations brought by HAL9000, hosts the first floating city, Venus, built with respect for the environment and populated by about 10,000 individuals. Venus consists of several hexagonal islands built from nothing, according to a refined model developed by HAL9000 to cope with climate change and migratory waves. Thanks to this model it was possible to map air and sea currents allowing to design of an oasis from the point of view of climate-zero emissions. Eve is one of the most sustainable, innovative, and colorful cities in the world in 2040, according to the HALsthetica magazine. The main indicator used by the magazine is the creative capacity of a city, measured in the amount and variety of activities that coexist and allow the combination of tradition, innovation, environment, cultural differences, and social inclusion. The development plan is continuously monitored by a multidisciplinary team of researchers (within STEM together with philosophers, sociologists, artists, and economists) who use HAL9000 to analyze data related to economic, cultural, and social issues of the city, identify opportunities and suggest development actions (both to the Administration and to citizens who require advice).

Some software implemented safely thanks to HAL9000 allows to monitor of the balance of the city in terms of ethnic and social plurality based on socio-economic models based on quantum physics, and activities of integration and urban renovation are proposed to optimize the coexistence of the city. After deep reflection, the new mayor GGGGGG has obtained, from Alice, a contract for the concession of the artificial island that previews that 20% of the profits obtained be yielded to the administration of the city in order to co-finance actions of environmental and social sustainability. These actions include, for example, the securing of the territory, the extension of the quantum network to all dwellings, the activation of electronic administrative services, support actions (homes, assistance, social centers) for citizens of difficulty, Housing construction for co-housing, cultural integration for foreigners and actions to encourage creativity. As a product of these activities, several start-ups linked to the creative industry and training have been formed. On the streets of Eve, you can buy art and crafts and high-tech devices with the same ease, as you can visit ateliers, clubs, and places very original and special. In order to promote cultural exchange, connections with the nearest airports are favored and continuous exchanges with other nations from all over the world are activated, both on exit and on entry.

Scenario C

Eve is a quiet fishing village with little traffic, where people live a quiet and calm everyday life. Large industries that do not care about environmental issues have been closed and citizens have decided to focus on returning to nature, good food, good relations, fishing, and tourism. Every citizen leads a life based on sharing and respecting traditional and other values. HAL9000 was built in a distant city and on an artificial island, however, built at the behest of the citizens, a large library and farmhouses were built that produce products in Km0. Transport by car has been almost totally replaced by other modes and means: in the center, you move mainly on foot or by electric public transport and the traffic of motor vehicles is granted only for exceptional permits. To get out of the city, there are a number of public trips at predetermined times of day established on the needs of the majority. People who live in Eve live a collected life and carry out professions that do not involve large movements (from the small economic activities of the center to fishing to agricultural work in surrounding lands to tourism and catering). The attention to the traditions, the fishing, and the refined agricultural production of the city allows the inhabitants to support themselves with a diet mainly based on zero km products. The centrality of the person and of human relationships has meant that the innovations proposed by Wonderland, such as HAL9000, have been viewed with suspicion and gradually shelved, for fear that they could cause isolation and de-responsibilization. There are several cultural centers where social, scientific, economic, and philosophical issues are discussed with experienced adults. Conferences are organized with experts from universities and important bodies, but also people who have carried out alternative research paths (innovative schools, alternative medicine, international artistic avant-garde).

Questions

A new relationship between man-machine

Reflect on the possible "relationships" that are created between the individual, the social community, and new technologies in their rapid development.

"Functional relationship": the machine can perform functions to help or replace humans. Functions can impact more or less strongly in different areas: social, political, ethical...

For each scenario:

- underline sentences/words in which the "functional relationship" emerges and, indicate to which scope/s they refer (social, political, economic ...);
- write a sentence that can characterize the "functional relationship".

"Circular relationship between human and machine": the machine, managing large quantities of data, can perform functions sometimes purely human also related to political, social, ethical decisions, and so on, this affects the actions of man that in turn affect the data that are then managed by algorithms that affect the "actions" of algorithms

For each scenario:

- underline sentences/words in which a "human/machine circular relationship" can be identified and, indicate to which area it refers (social, political, economic ...);
- write a sentence that can characterize the "circular human/machine relationship" with possible consequences.

"Relationship linked to a new logic and to new conceptual frameworks": the new technologies, from deep learning to quantum computers, bring with them a new logic, new conceptual frameworks, and a new epistemology, those of the "underlying science", the "new science";

For each scenario:

- underline sentences/words, if any, in which the "relation linked to the new logic" emerges;
- write a sentence, if possible, that can characterize what is the "relationship linked to the new logic" and the possible "changes" taking into consideration the cognitive sphere (the way we relate to knowledge), the communicative and linguistic sphere (the need for new images and new words), the personal sphere (referring to their daily routines, but also to interpersonal relationships).

"Relationship between the times of technological, social and personal change": new technologies, from deep learning to quantum computers, develop with a speed never seen before and that can lead to misalignments between technology, society, and the individual

- For each scenario:
- underline sentences/words, if any, in which the "relationship between the different times" emerges;
- write a sentence, if possible, that can characterize what is the "relationship between the different times" and the possible consequences.

Annex 2

Part 1: The experiment and the circuit

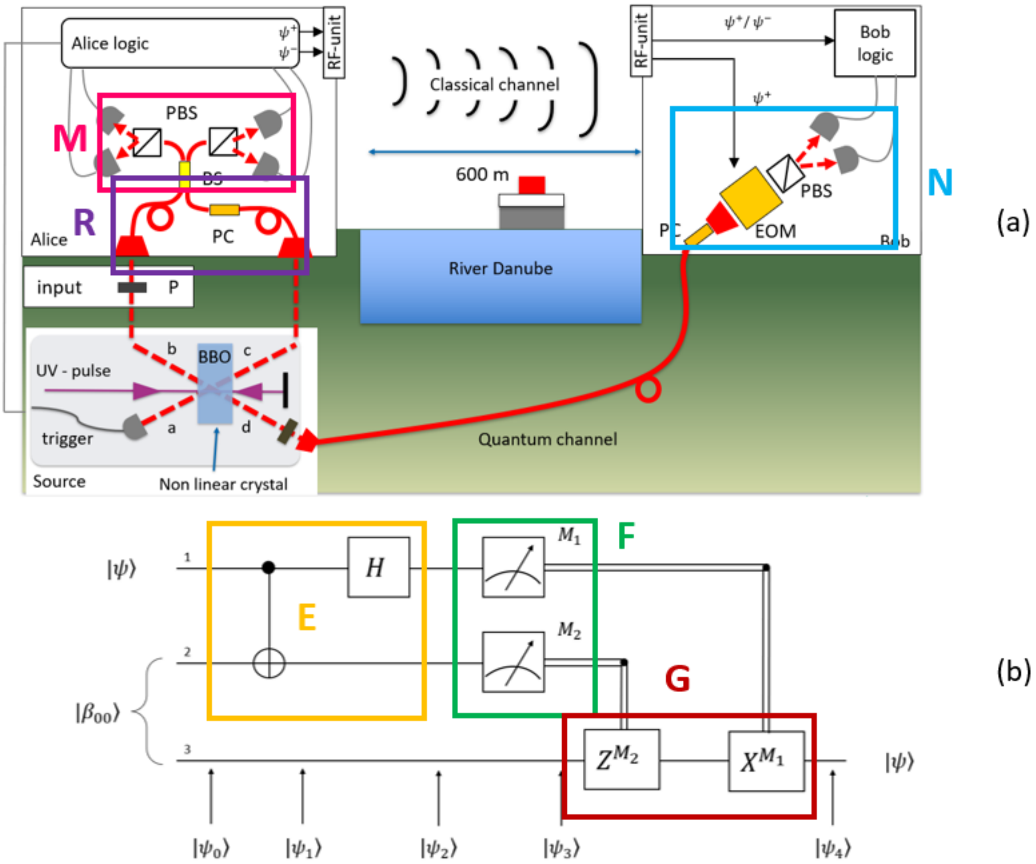


FIG. 1: experiment and circuit in comparison

a. The images (a) and (b) show the experiment and circuit that we analyzed. Try to re-describe the teleportation, specifying: i) what is teleported; ii) what physically corresponds to "the moments" indicated in figure (a) with M - N - R and iii) to which parts of the circuit (E - F - G) they correspond.

b. What main differences do you find between the experiment shown in figure (a) and the circuit represented in figure (b)?

c. More generally, what analogies and differences do you see between a description of a phenomenon made in terms of experiment and a description made in terms of circuit?

d. What meaning do you attribute to the phrase repeated throughout the course: "an experiment can be seen as a device that manipulates information and therefore also be represented in circuital terms"?

____
____

e. Which of the two representations do you prefer and why?

____
____

Part 2: The four registers

a. As we discussed together, we divided the discourse on teleportation into 4 registers: narrative, logical, technical–experimental (the register referred to the description of the experiment) and mathematical. Each of the registers was chosen to play a different role. What idea did you form about the role attributed to each of them?

____
____

b.

NARRATIVE REGISTER: what contribution has this register made to your understanding of the phenomenon? Specifically, on a scale of 1 to 5, how much has this register contributed to:					
	1(not at all)	2 (a little)	3 (quite a lot)	4 (very)	5 (a lot)
Figuring out what quantum teleportation is					
Understanding what the phenomenon of teleportation consists in					
Following the whole reasoning carried out on teleportation					
Grasping the articulation of the reasoning in its different phases					
Understanding the details of the reasoning					
Having a clear understanding of the mechanism that makes teleportation occur					

c.

LOGICAL REGISTER: what contribution has this register made to your understanding of the phenomenon? Specifically, on a scale of 1 to 5, how much has this register contributed to:					
	1(not at all)	2 (a little)	3 (quite a lot)	4 (very)	5 (a lot)
Figuring out what quantum teleportation is					
Understanding what the phenomenon of teleportation consists in					
Following the whole reasoning carried out on teleportation					
Grasping the articulation of the reasoning in its different phases					
Understanding the details of the reasoning					
Having a clear understanding of the mechanism that makes teleportation occur					

d.

TECHNICAL–EXPERIMENTAL: what contribution has this register made to your understanding of the phenomenon? Specifically, from 1 to 5, how much this register has contributed to:					
	1(not at all)	2 (a little)	3 (quite a lot)	4 (very)	5 (a lot)
Figuring out what quantum teleportation is					
Understanding what the phenomenon of teleportation consists in					
Following the whole reasoning carried out on teleportation					
Grasping the articulation of the reasoning in its different phases					
Understanding the details of the reasoning					
Having a clear understanding of the mechanism that makes teleportation occur					

e.

MATHEMATICAL REGISTER: what contribution has this register made to your understanding of the phenomenon? Specifically, on a scale of 1 to 5, how much has this register contributed to:					
	1(not at all)	2 (a little)	3 (quite a lot)	4 (very)	5 (a lot)
Figuring out what quantum teleportation is					
Understanding what the phenomenon of teleportation consists in					
Following the whole reasoning carried out on teleportation					
Grasping the articulation of the reasoning in its different phases					
Understanding the details of the reasoning					
Having a clear understanding of the mechanism that makes teleportation occur					

f. Have you found any potentialities in the intertwining of the 4 registers? If so, which?

____

____

g. Which register did you prefer?

____

____

h.

To what extent did you find the activity about teleportation:					
	1(not at all)	2 (a little)	3 (quite a lot)	4 (very)	5 (a lot)
Easy					
Useful for understanding quantum technologies					
Inspiring					
Fascinating					
Within expectations					

i. Do you have any suggestions for improving the lesson?

____
____

j. Do you want to add any comments?

____
____

Annex 3

The conceptual questionnaire

The first phase of the experiment, on the 5th day of the course, lasted 2 hours and involved 27 students divided into groups.

The teamwork consisted in dealing with the following three questions:

> Q1. To what extent from 1 to 6 each representation helps you to understand the basic concepts?
>
> Q2. What are the characteristics that you would extrapolate from the different representations that you consider fundamental?
>
> Q3.Are there, and if so which, representations that you consider redundant because they do contain not relevant details and information?

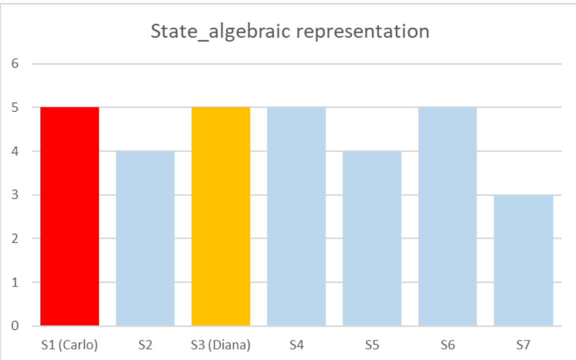
Students' teamwork was unsupervised, and the given time to complete the activity was 1 hour. In the following, I present the results. The groups had answered the quantitative questions of the questionnaire and only two of them answered also to the second and third questions.

In the following, I present the analysis of the teamwork of the four. For the first question, I organized the data in histograms. The students that participated also in the focus group have been highlighted in different colours and by making the names explicit, to keep track and look for any consistency or discrepancy.

Group 1

Question 1 (Q1)

Concept of state and superposition principle



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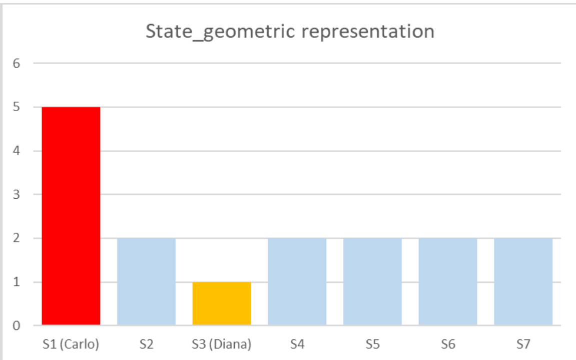

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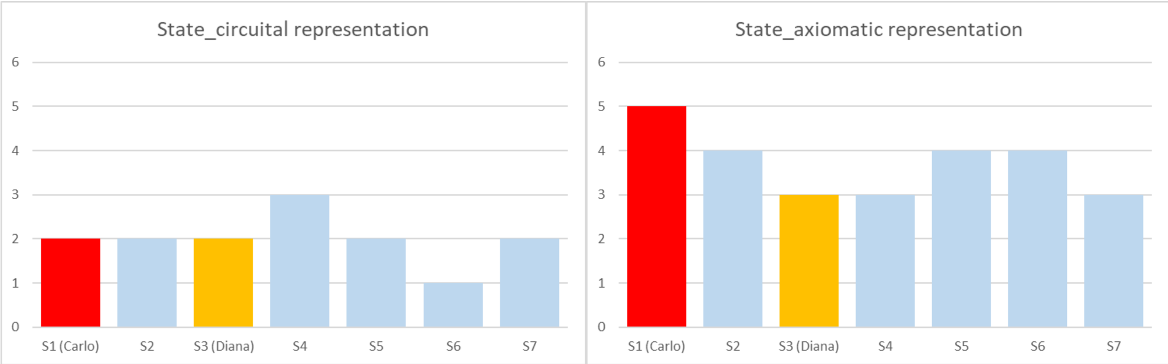

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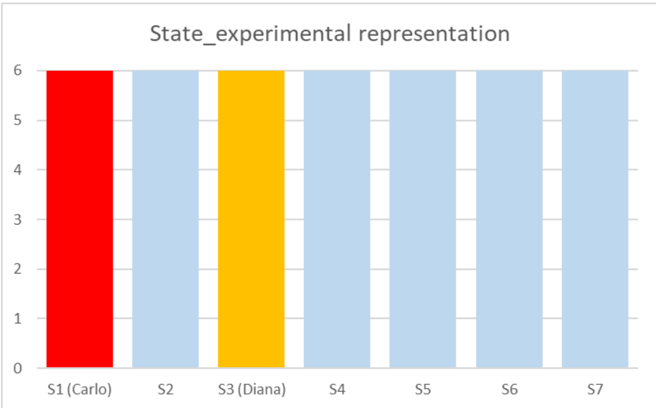

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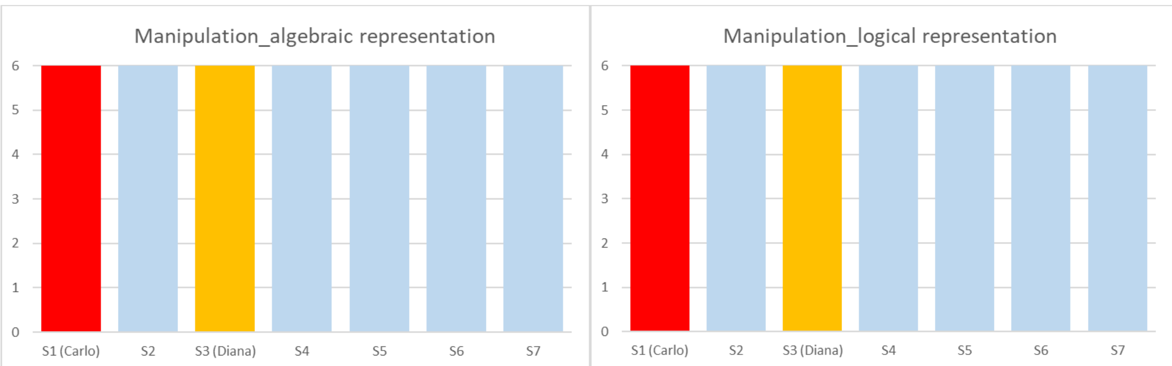
Fig. 2: distribution of students' evaluation of different representations about the concept of the quantum state.

The histograms show that the representation that helps students more to understand the concept of quantum state and superposition principle is the experimental one (mean value 6), followed by the algebraic one (mean value 4,43). The axiomatic representation, the third in students' ranking, as the experimental and algebraic one does not present any particular oscillation (mean 3,72) and it proved to be not particularly effective for students' understanding. The representations that have been shown to be less effective are the geometric (mean 2,29) and the circuital ones (mean 2,0).

Concept of state manipulation and evolution



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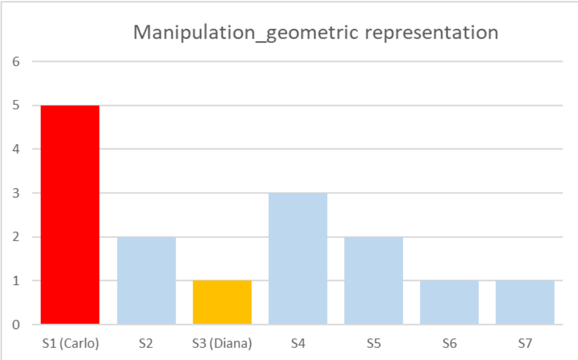

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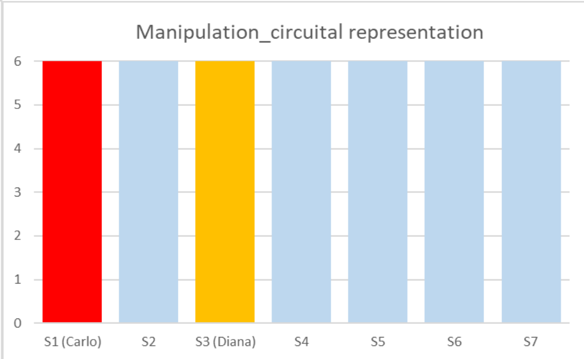

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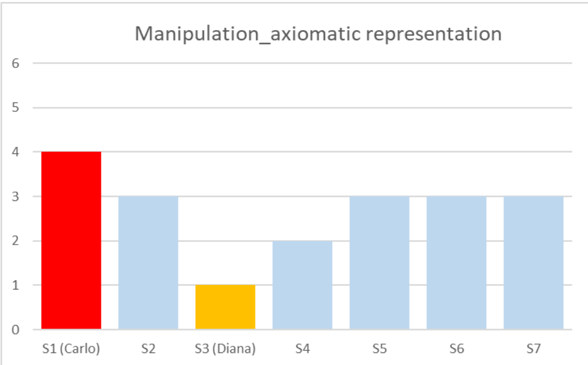

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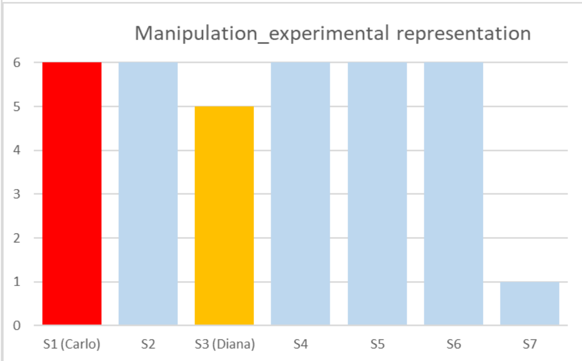

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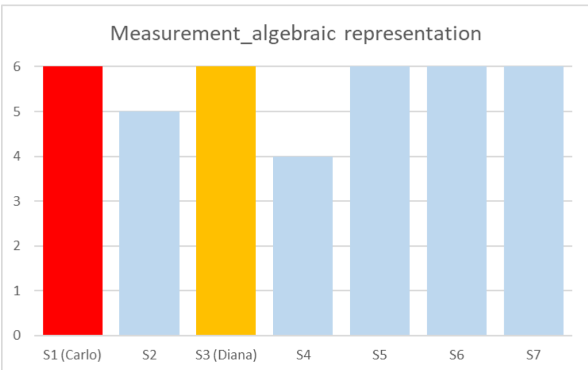
Fig. 3: distribution of students' evaluation of different representations about the concept of state manipulation and evolution.

The representations that support more students in understanding the concept of state manipulation and evolution are the algebraic, logical, and circuital ones (all with mean value 6). These three are followed by the experimental representation with a mean value of 5.14. The most ineffective representations are the axiomatic (mean value 2,71) and the geometric ones (2,14).

<u>Concept of measurement</u>



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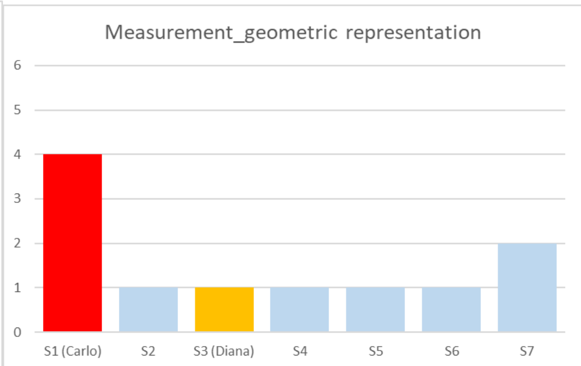

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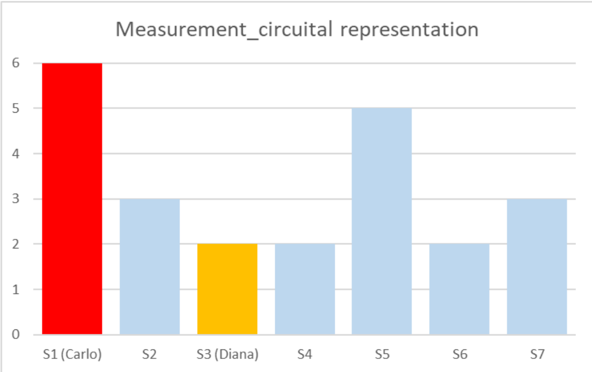

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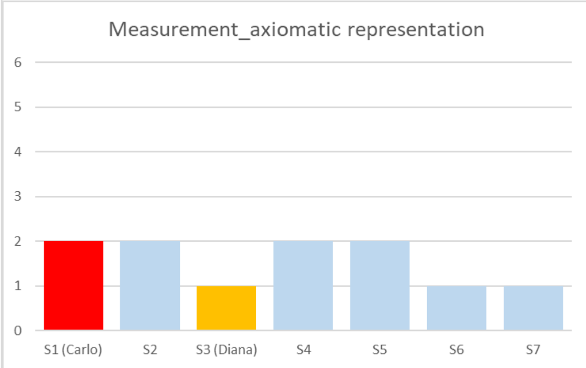

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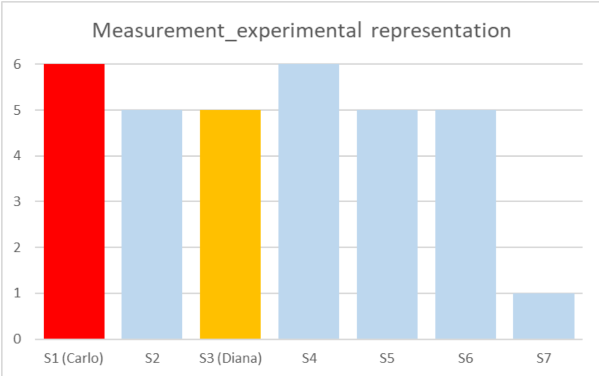

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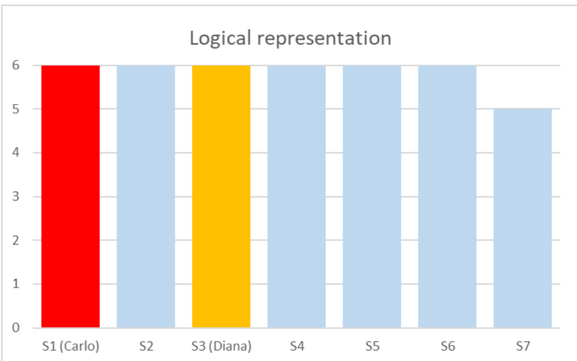
Fig. 4: distribution of students' evaluation of different representations about the concept of quantum measurement.

The representations that help more students to grasp what quantum measurement is are the algebraic (mean value 5,57) and the experimental (mean 4,71). The circuital one, which quite divided the students, shows to be not particularly effective with a mean value of 3,29. The geometric and the axiomatic representations, except for Carlo in the case of the geometric one, present a certain degree of agreement between the students, resulting in less effective representations of all (mean 1,57).

Concept of entanglement



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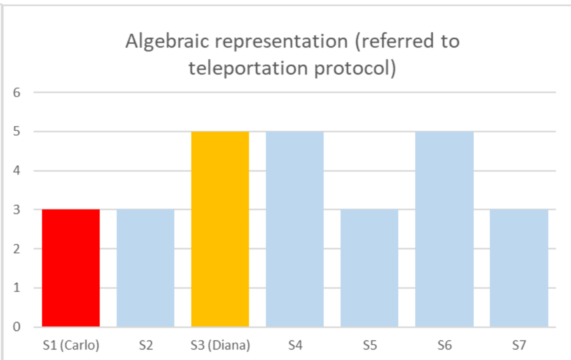

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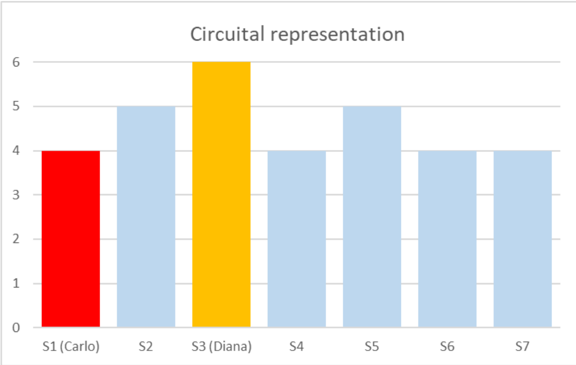

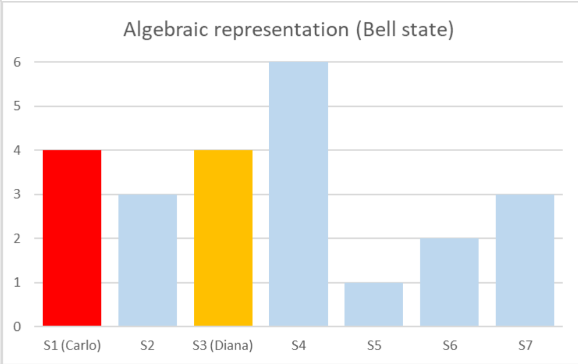

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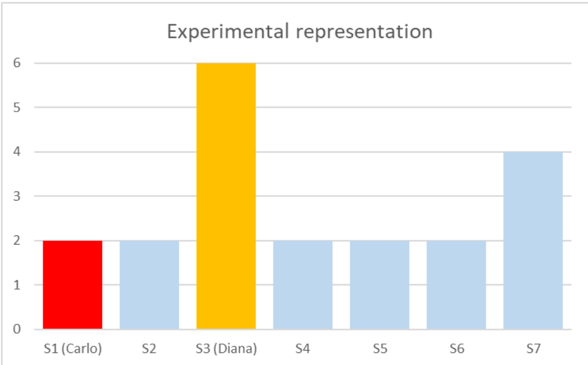

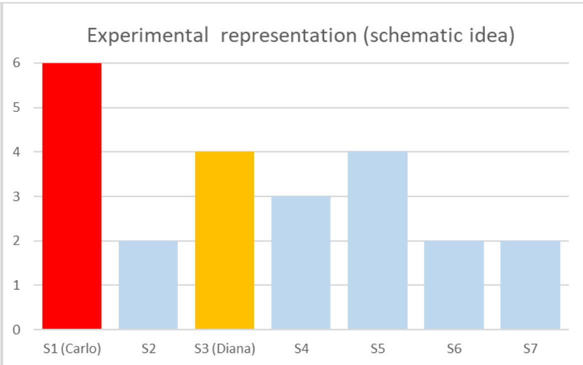

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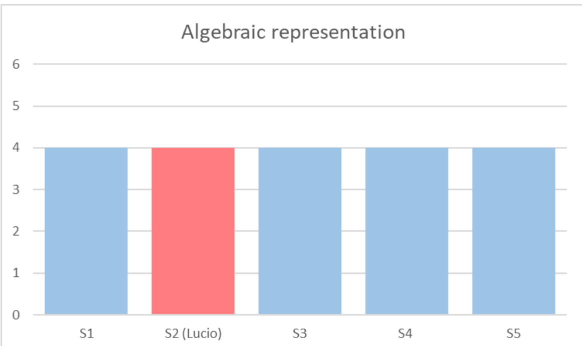
Fig. 5: distribution of students' evaluation of different representations about the concept of entanglement.

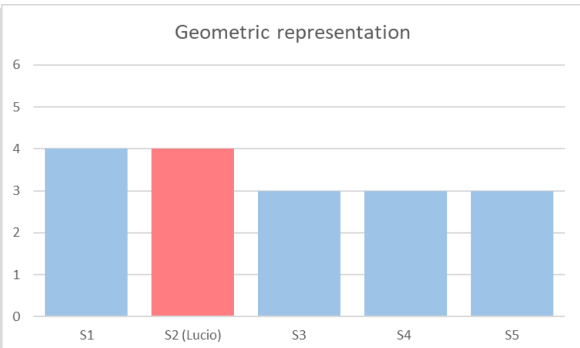
For what concerns the concept of entanglement the representations that promote more students' understanding are the logical one (mean value 5,86), followed by the circuital one (mean value 4,57). The two algebraic representations (the final table of the teleportation protocol and the Bell state), in this case, show to be not particularly effective with a mean value respectively of 3,86 and 3,28. The experimental representations proved to be less effective even if, in particular, the simplified representation of a hypothetical experiment between Alice and Bob (fig X) impacted differently on students' understanding. The schematic experimental representation (schematic idea) and the experimental representation of the teleportation protocol respectively on average had a ranking of 3,29 and 2,86.

Group 2

Question 1

Concept of state





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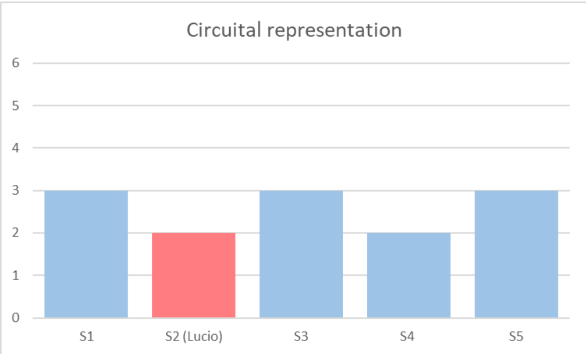

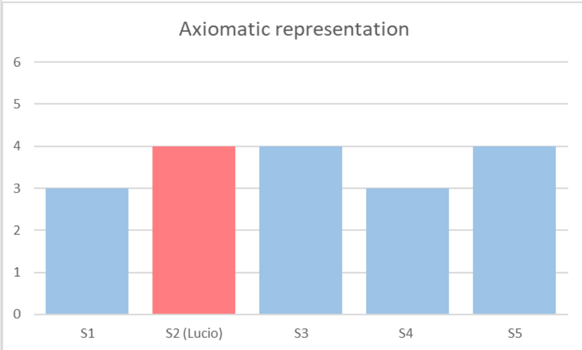

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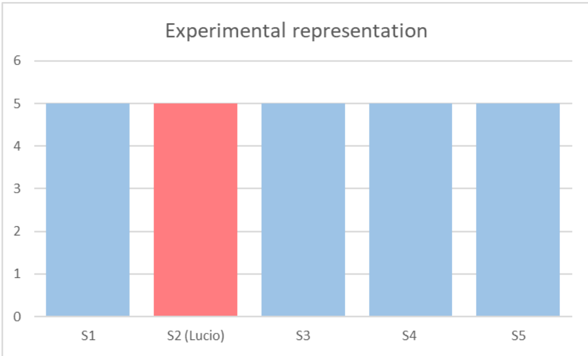

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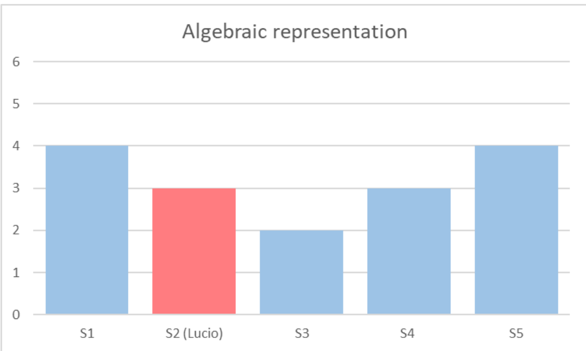
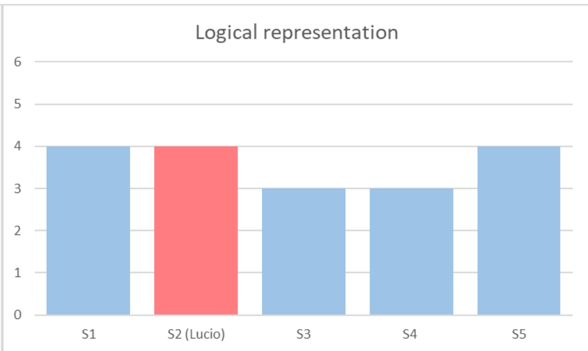
Fig. 6: distribution of students' evaluation of different representations about the concept of quantum state.

The representation that most convinced and helped the students of group 2 to understand the concept of quantum state is the experimental representation (mean 5). Quite effective proved to be also the algebraic one, with a mean value of 4. For students' understanding, the axiomatic and the geometric representations proved to be similar with a mean value respectively of 3,6 and 3,4. The less effective representation is the circuital one with a mean value of 2,6.

Concept of state manipulation and evolution





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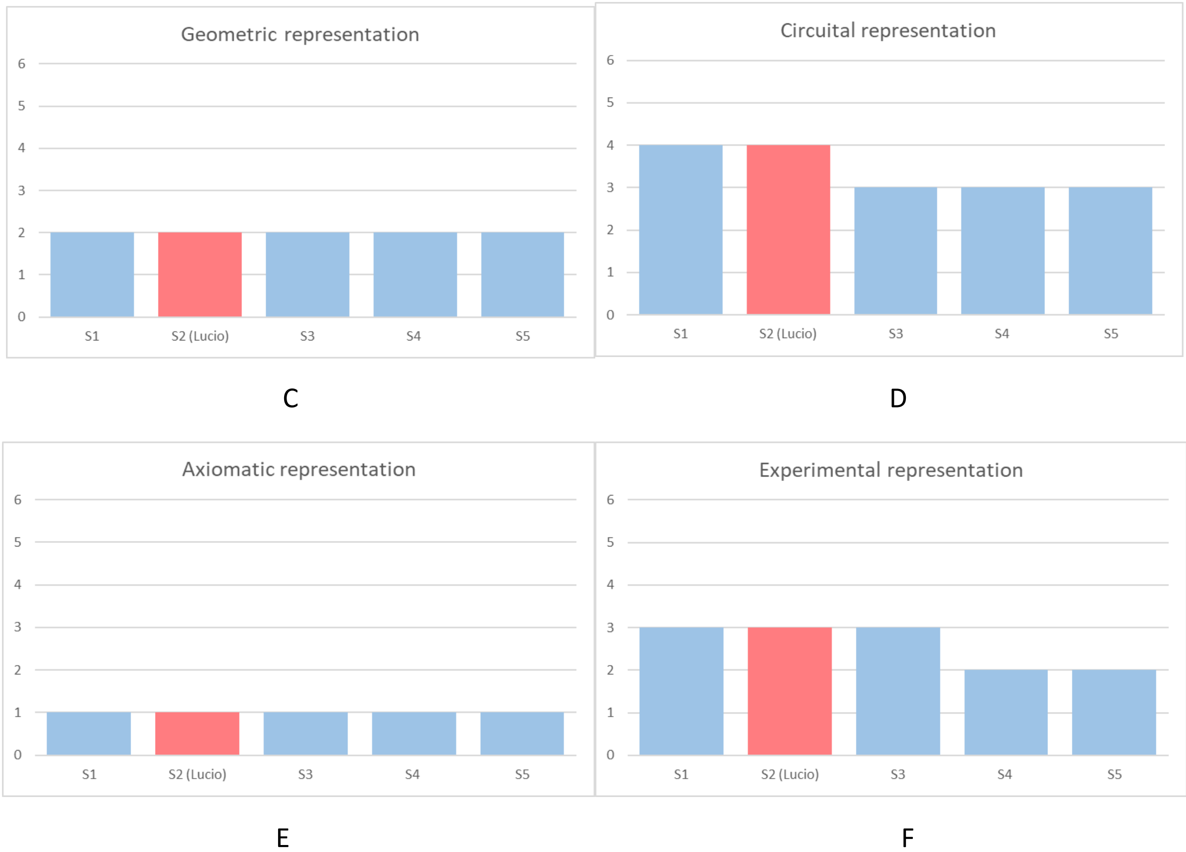
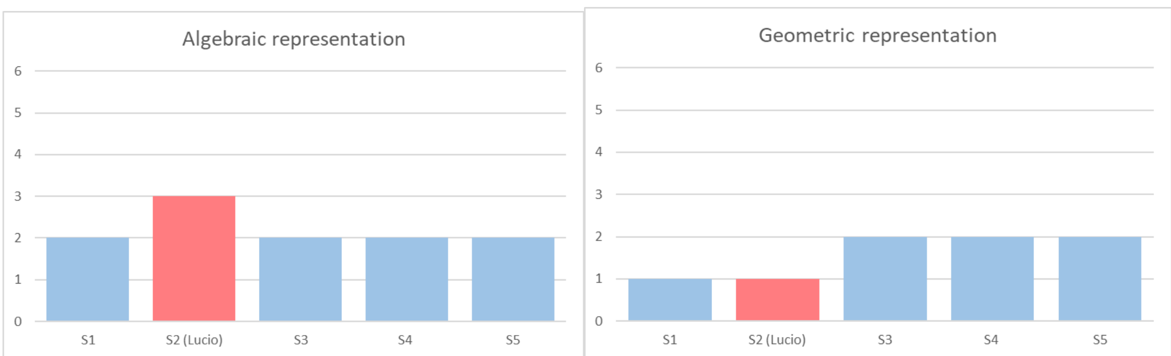


Fig. 7: distribution of students’ evaluation of different representations of the concept of state manipulation and evolution.

For what concerns state manipulation and evolution, for group 2 the most effective representations are the logical and the circuital ones, with a mean value of 3,6 and 3,4. They are followed by the algebraic representation with a mean value of 3,2. The experimental, geometric, and axiomatic representations proved to be unanimously the most ineffective representations with a mean value respectively of 2,2, 2,0, and 1.

__Concept of measurement__



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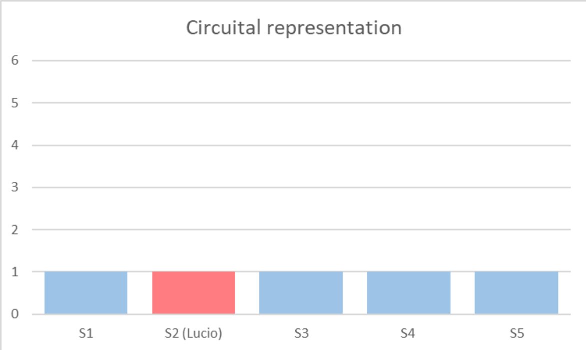

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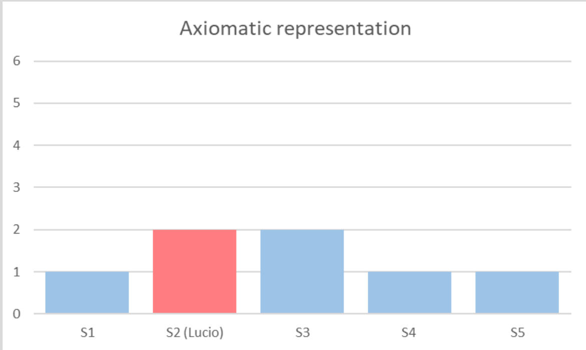

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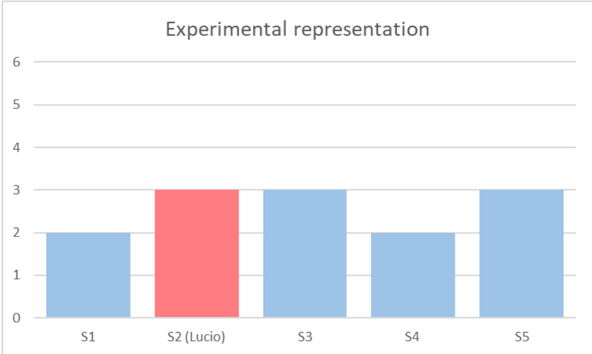

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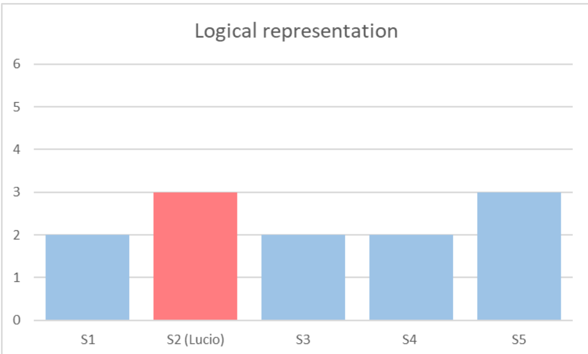
Fig. 8: distribution of students' evaluation of different representations about the concept of quantum measurement.

The representation that seems to be more effective to understand the concept of quantum measurement is the experimental representation with a mean value of 2,6, followed by the algebraic one (mean value 2,2). The other representations, the geometric, the axiomatic, and the circuital, proved to be very ineffective for students' understanding (mean values 1,6, 1,4, and 1,0).

Concept of entanglement



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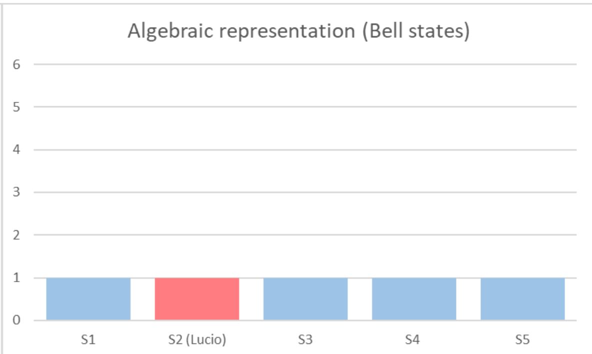

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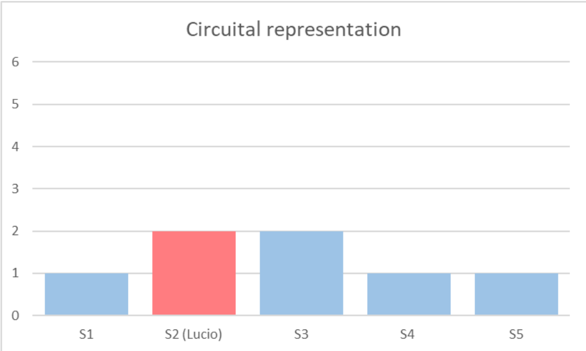

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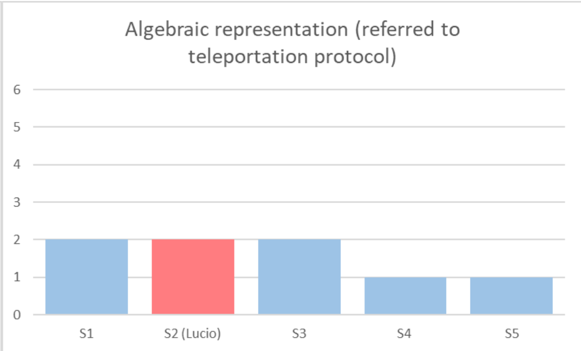

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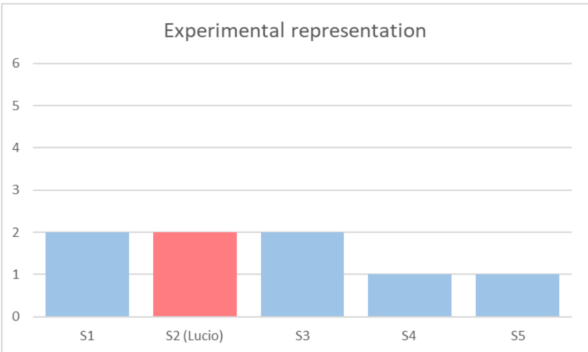

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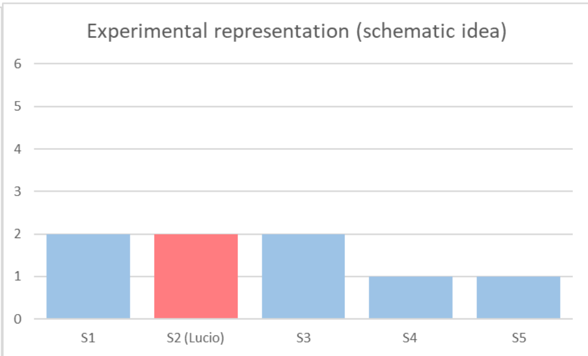

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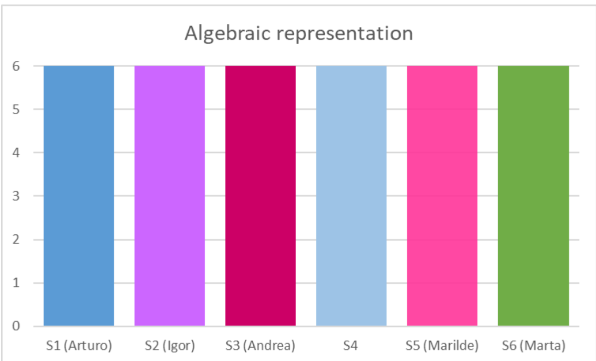
Fig. 9: distribution of students' evaluation of different representations about the concept of entanglement.

The representation that helps more group 2 in understanding the concept of entanglement is the logical one, with a mean value of 2,4. The ranking of other representations is quite similar and tells us that they were ineffective to promote students' understanding. Both the experimental and the algebraic have a mean value of 1,6, the circuital one of 1,4, while the Bell states, as the algebraic representation, has a 1,0 ranking.

Group 3

Question 1

Concept of state



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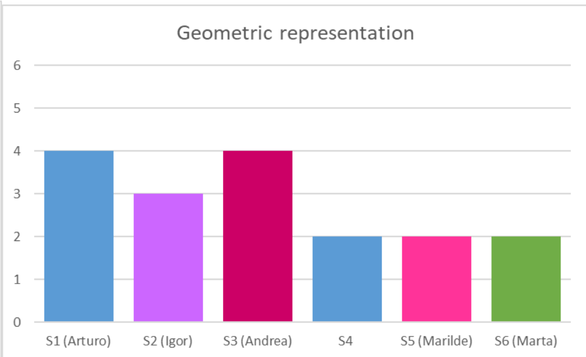

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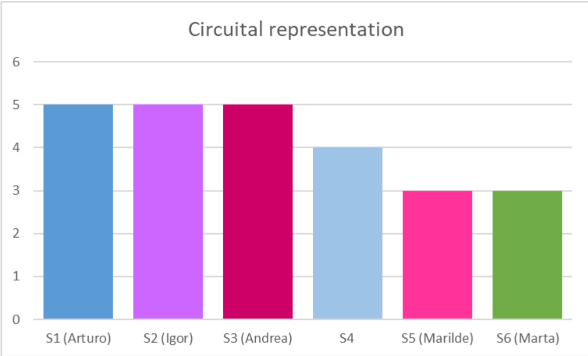

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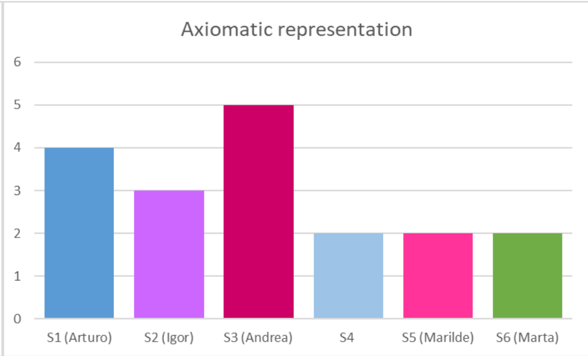

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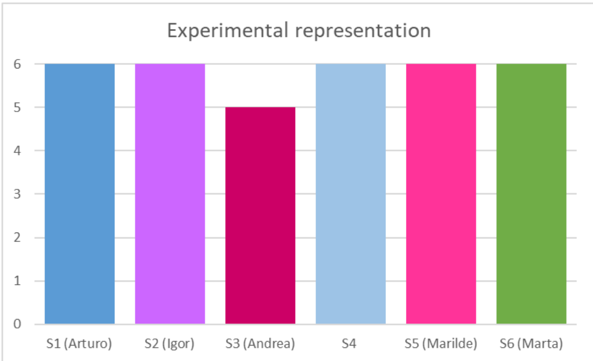

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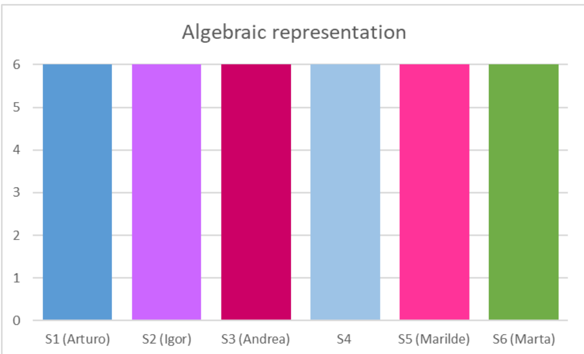
Fig. 10: distribution of students' evaluation of different representations about the concept of the quantum state.

The representations that particularly promote the understanding of the concept of the quantum state of the entire group are the algebraic (mean value 6) and the experimental one (mean value 5,8). The circuital one showed to be quite effective too, with a mean of 4,2. The two representations less effective and that divided most of the members are the axiomatic (mean value 3) and the geometric one (mean value 2,8).

Concept of state manipulation and evolution



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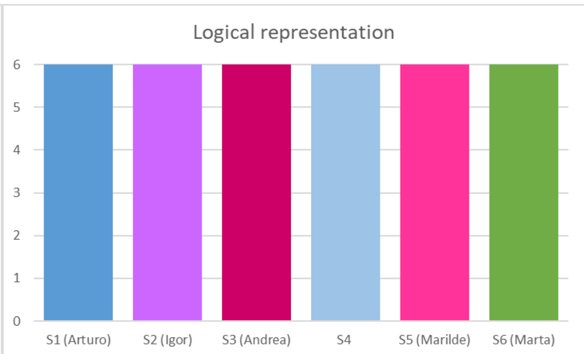

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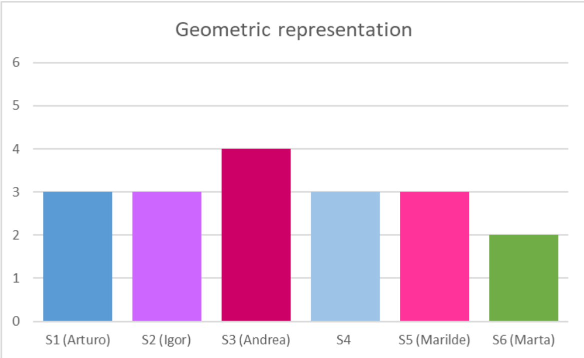

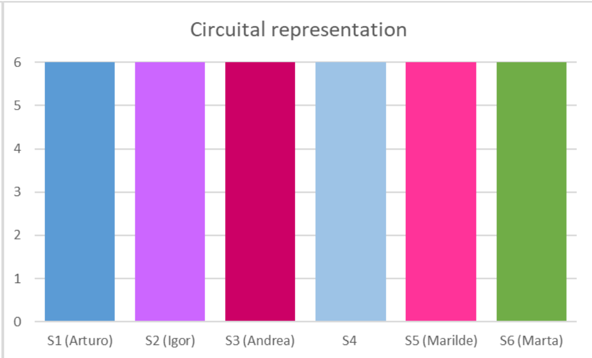

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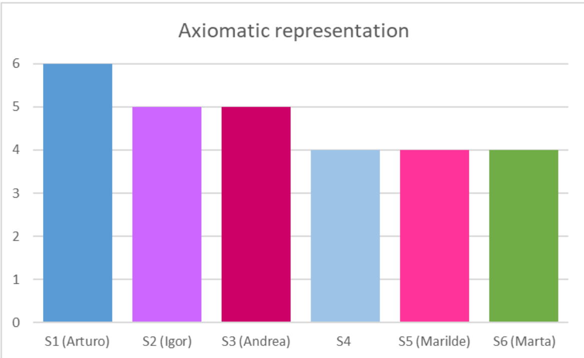
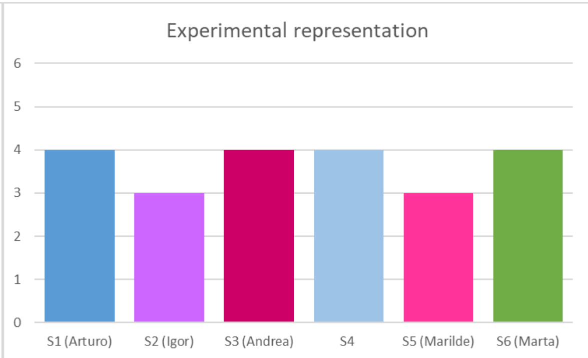


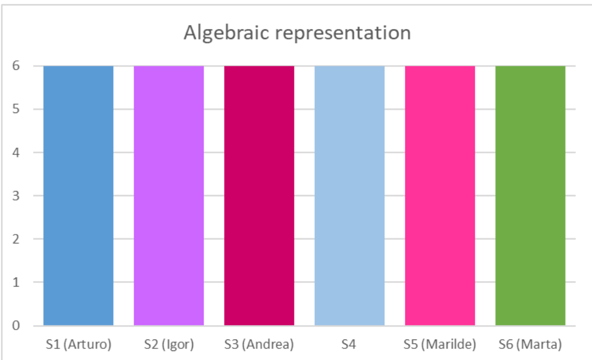


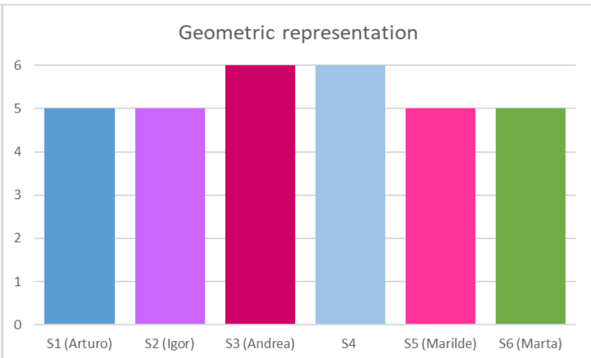
E F

Fig. 11: distribution of students' evaluation of different representations about the concept of state manipulation and evolution.

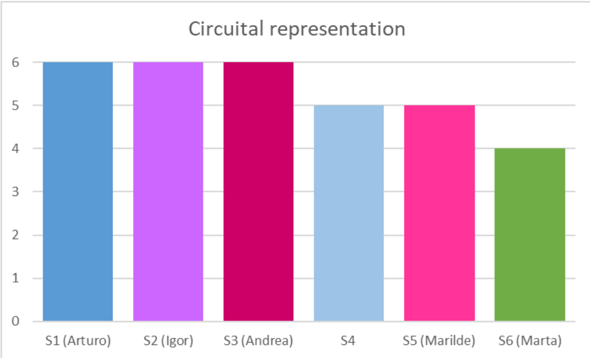
For what concerns the concept of state manipulation and evolution the most fruitful representations for students are the algebraic, the logical and the circuital ones, each of them has received the highest ranking (mean value 6). In contrast to the concept of state, for the state manipulation/evolution also the axiomatic representation proved effective with an average value of 4,7. The experimental and the geometrical ones proved to be the least useful for students' personal understanding, with a mean value of 3,7 and 3,0.

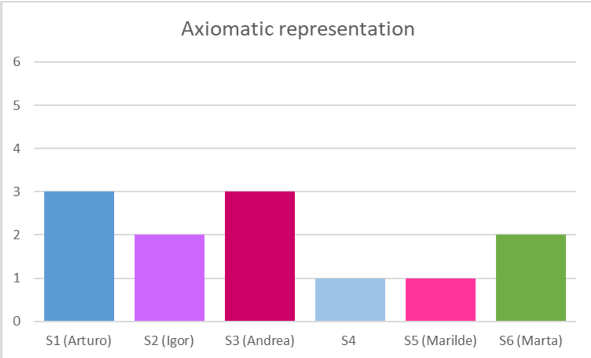
Concept of quantum measurement





A B





C

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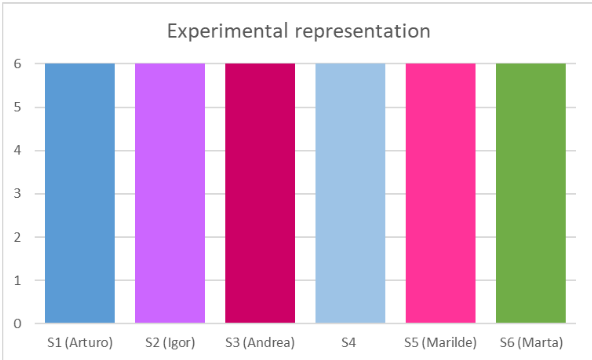
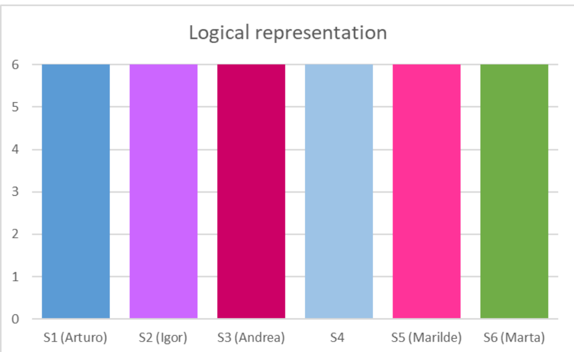


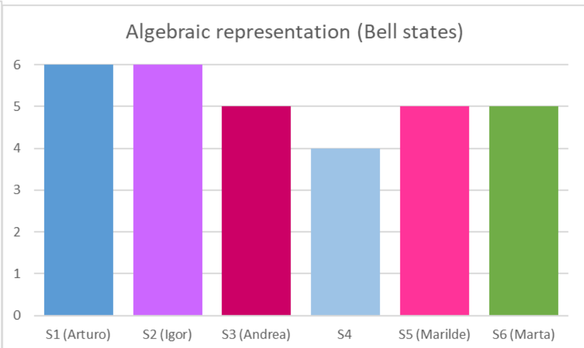
E

Fig. 12: distribution of students' evaluation of different representations about the concept of quantum measurement.

The algebraic and experimental representations have the highest ranking for the understanding of the concept of quantum measurement (both mean values 6,0). In this case, also the geometric representation, which for the previous concepts showed to be useless, and the circuital representation proved to be quite effective, with both a mean value of 5,3. The most ineffective representation, in this case, is the axiomatic one that received a 2,0 mean ranking.

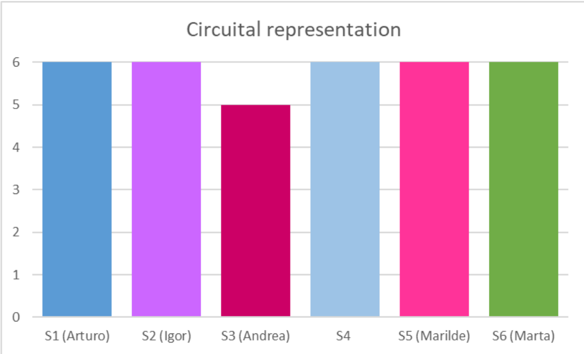
Concept of entanglement



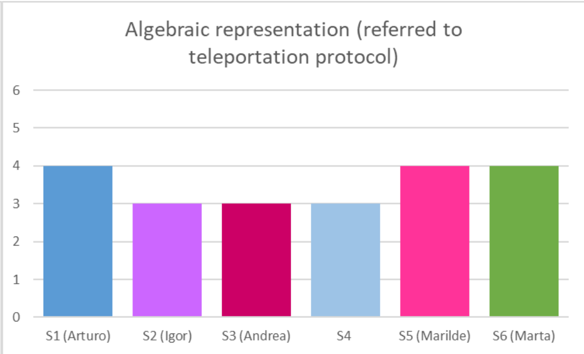


A

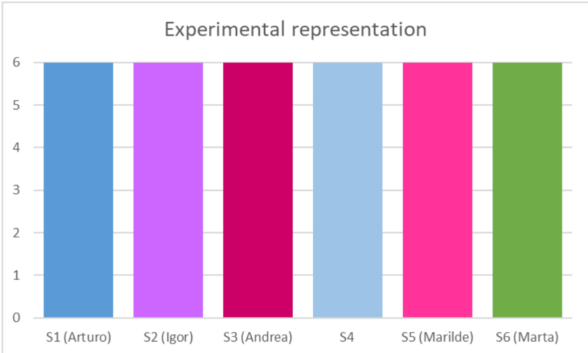
B



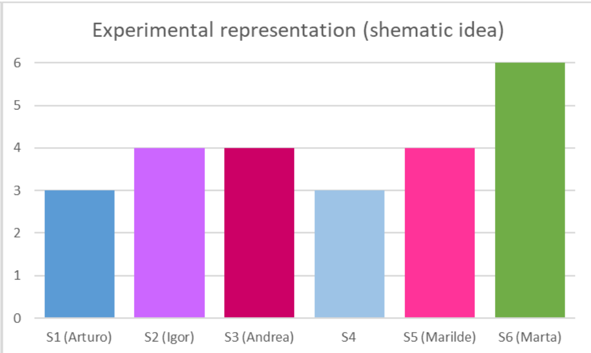
C



D



E



F

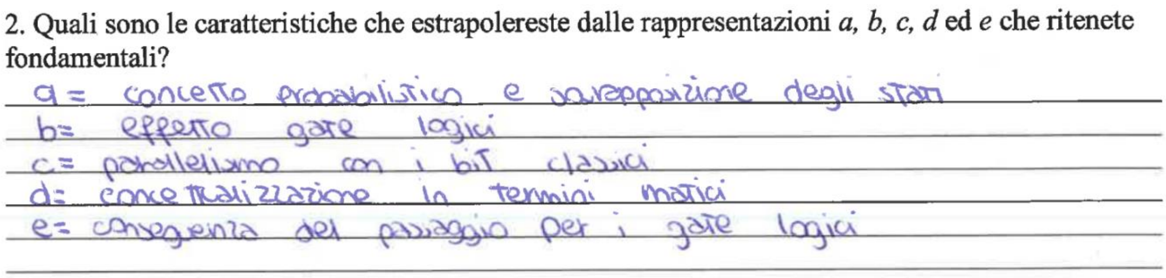
Fig. 13: distribution of students' evaluation of different representations about the concept of entanglement.

The representations that students found more effective for their personal understanding of the concept of entanglement are the logical (mean value 6) and the circuital one (mean value 5,83). Also the algebraic representation, that is the Bell's states, proves to be useful with a mean value of 5,2. The schematic representation of a hypothetical experiment on average convinces the students with a mean value of 4,0.

The less effective representations, in this case, are the algebraic representation, the one that refers to teleportation calculation (mean value 3,5), and the experimental representation of the teleportation protocol (mean value 2,0).

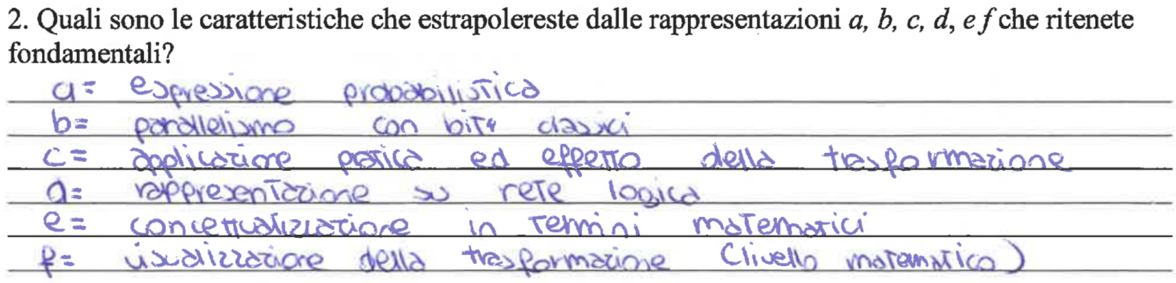
Question 2 & Question 3

Group 3 is the only one that has tried to answer the second question. They have reflected and discussed trying to explain what aspects can be extrapolated by the different representations. Figures from 5.13 to 5.16 show how they fill the questionnaire and the features they associate with the different kinds of representations.


2. Quali sono le caratteristiche che estrapolereste dalle rappresentazioni *a, b, c, d* ed *e* che ritenete fondamentali?

a= concetto probabilistico e sovrapposizione degli stati
b= effetto gate logici
c= parallelismo con i bit classici
d= concretizzazione in termini matici
e= convergenza del passaggio per i gate logici

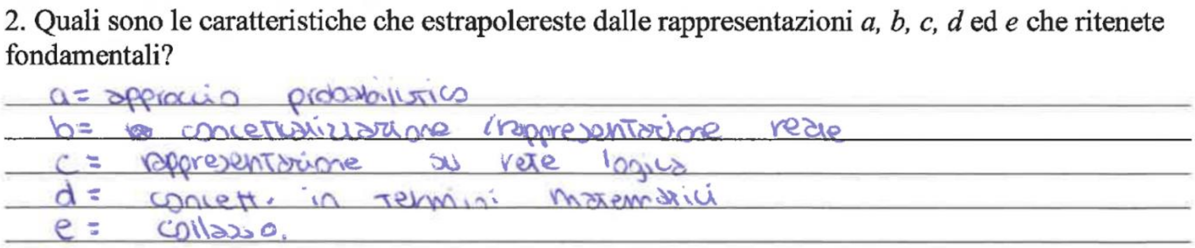
Fig. 14: Students' description of the extrapolated features of the concepts of quantum state and superposition principle. "a [algebraic representation] = probabilistic concept and superposition of the states; b [geometric representation] = logic gates' effect; c [circuital representation] = parallelism with classical bits; d [axiomatic representation] = conceptualization in mathematical terms; e [experimental representation] = consequence of passing for logical gates".



2. Quali sono le caratteristiche che estrapolereste dalle rappresentazioni *a, b, c, d, e f* che ritenete fondamentali?

a= espressione probabilistica
b= parallelismo con bit classici
c= applicazione pratica ed effetto della trasformazione
d= rappresentazione su rete logica
e= concettualizzazione in termini matematici
f= visualizzazione della trasformazione (livello matematico)

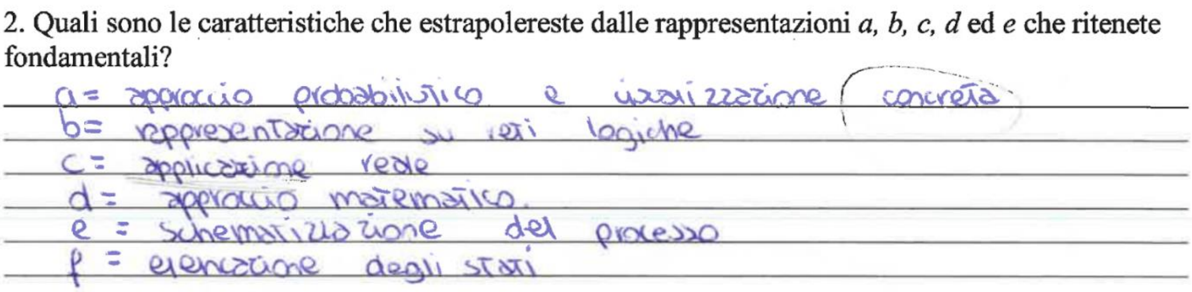
Fig. 15: Students' description of the extrapolated features of the concept of state evolution/manipulation. "a [algebraic representation] = probabilistic expression; b [logical representation] = parallelism with classical bits; c [experimental representation] = practical application and effect of the transformation; d [circuital representation] = representation on logical network; e [axiomatic representation] = conceptualization in mathematical terms; f [geometric representation] = visualization of the transformation (mathematical level)".



2. Quali sono le caratteristiche che estrapolereste dalle rappresentazioni *a, b, c, d* ed *e* che ritenete fondamentali?

a= approccio probabilistico
b= concettualizzazione (rappresentazione reale
c= rappresentazione su rete logica
d= concett. in termini matematici
e= collasso.

Fig. 16: Students' description of the extrapolated features of the concept of quantum measurement. "a [algebraic representation] = probabilistic approach; b [experimental representation] = conceptualization (real representation); c [circuital representation] = representation on logical networks; d [axiomatic representation] = conceptualization in mathematical terms; e [geometric representation] = collapse".



2. Quali sono le caratteristiche che estrapolereste dalle rappresentazioni *a, b, c, d* ed *e* che ritenete fondamentali?

a= approccio probabilistico e visualizzazione (concreta)
b= rappresentazione su reti logiche
c= applicazione reale
d= approccio matematico.
e = schematizzazione del processo
f = elencazione degli stati

Fig. 17: Students' description of the extrapolated features of the concept of entanglement. "a [logical representation] = probabilistic approach and concrete visualization; b [circuital representation] = representation on logical networks; c [experimental representation] = real application; d [algebraic representation (referred to teleportation protocol)] = mathematical approach; e [experimental representation (schematic idea)] = process schematization; f [algebraic representation (Bell's state)] = list of states".

From the few words they have written for each representation it is possible to find some patterns and draw information on the role that they recognize in the different representations. For all four concepts, the algebraic representation incorporates for them the probabilistic intrinsic feature that characterizes quantum systems (“probabilistic concept”, “probabilistic expression”, “probabilistic approach”).

To the experimental representation, in the case of state manipulation/evolution, quantum measurement, and entanglement, they associate the feature to be *real* and *practical* (“conceptualization (real representation), “practical application and effect of the transformation”, “real application”). This is consistent also with other results presented in Satantassi, Ercolessi, and Levrini (2022) and reported in section 3.2.4.

The geometric representation, even if the students did not use a unique expression, has a particular role. In fact, they associate this with a *visualization* role (“effect of logic gates”, “visualization of transformation”, “collapse”).

To the axiomatic representation, group 3 associates a reconceptualization in mathematical terms. They did not use any more words and, unfortunately, the recording is too loud to be heard, but as these students were particularly active during the lessons and the main protagonists of the following stages, they could refer to the axiomatic, that is to the formalized system of which the axioms are elements.

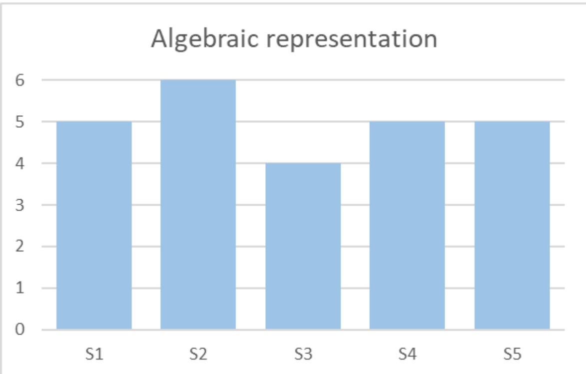
To the circuital and logical representation, as we expected, for the concept of quantum state and state manipulation and evolution they associate a scaffolding for comparing these concepts with the classical ones “parallelism with classical bits”), while in the cases of quantum measurement and entanglement it is only a way to represent on logic networks the concepts (“representation on logical networks”).

For what concerns *question 3*, group 3 found no redundant elements in the different representations.

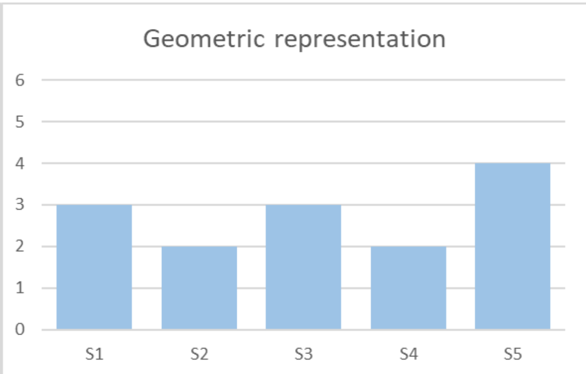
Group 4

Question 1

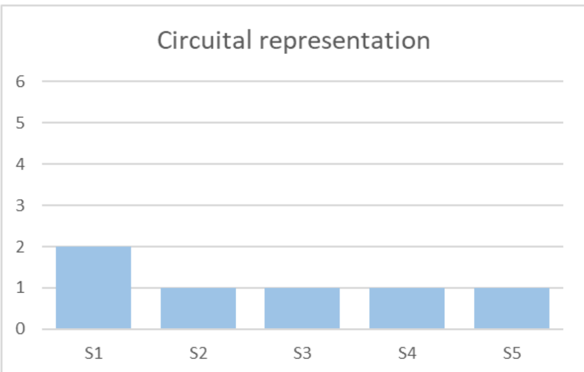
Concept of state



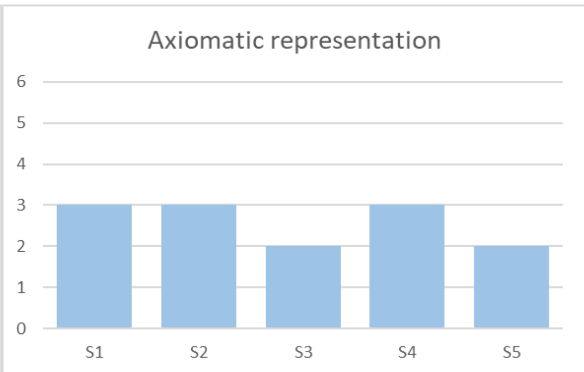
A



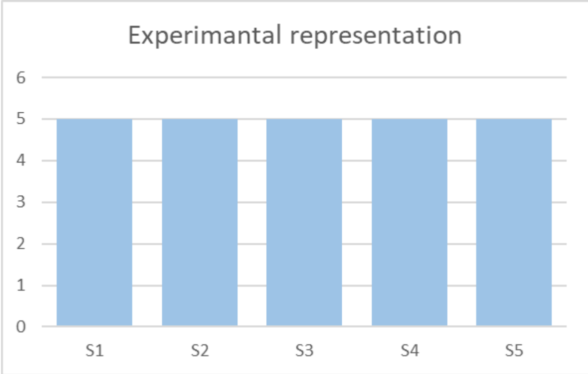
B



C



D

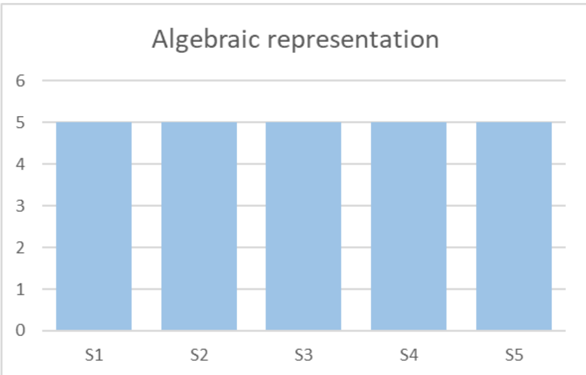


E

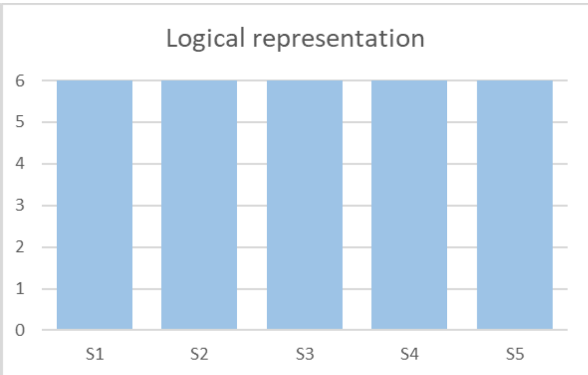
Fig. 18: distribution of students' evaluation of different representations about the concept of quantum state.

Group 4 found particularly effective to understand the concepts of quantum state and superposition principle both the algebraic and experimental representation that receive a mean evaluation of 5,0. The geometric and the axiomatic representations proved to be not fruitful for the students receiving respectively mean values of 2,8 and 2,6. The most ineffective representation is the circuital one with a mean value of 1,2.

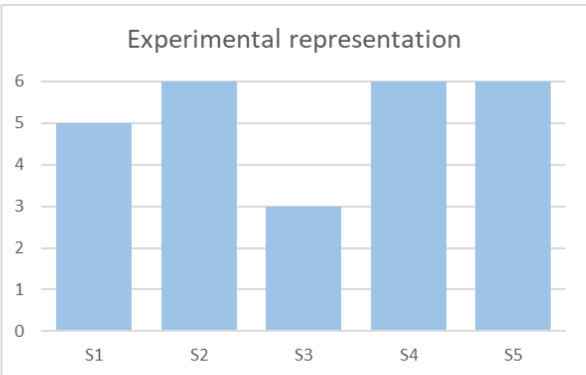
State manipulation and evolution



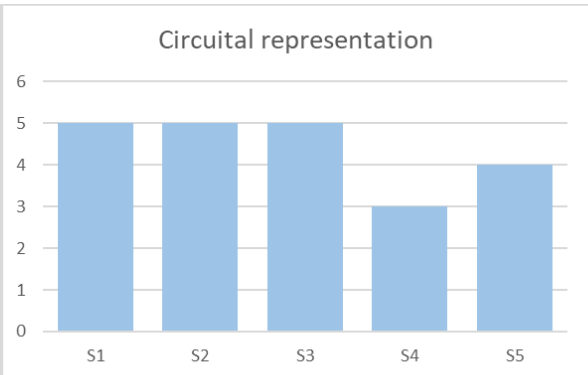
A



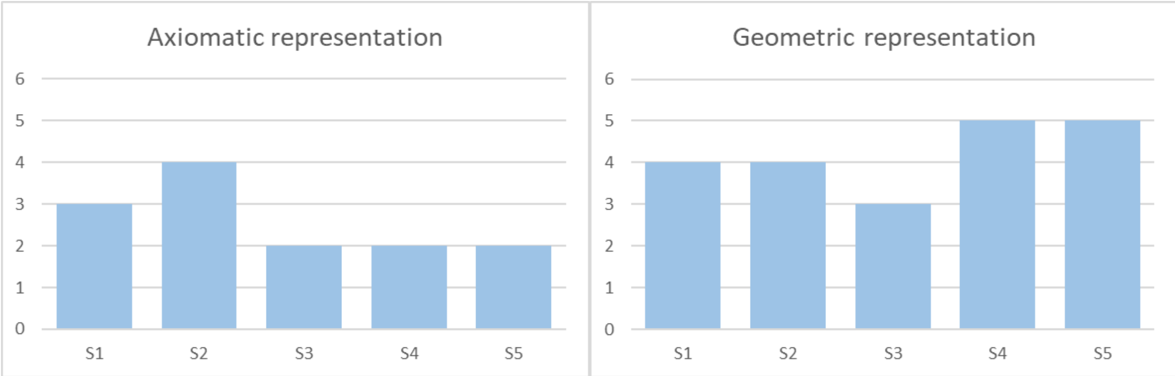
B



C



D

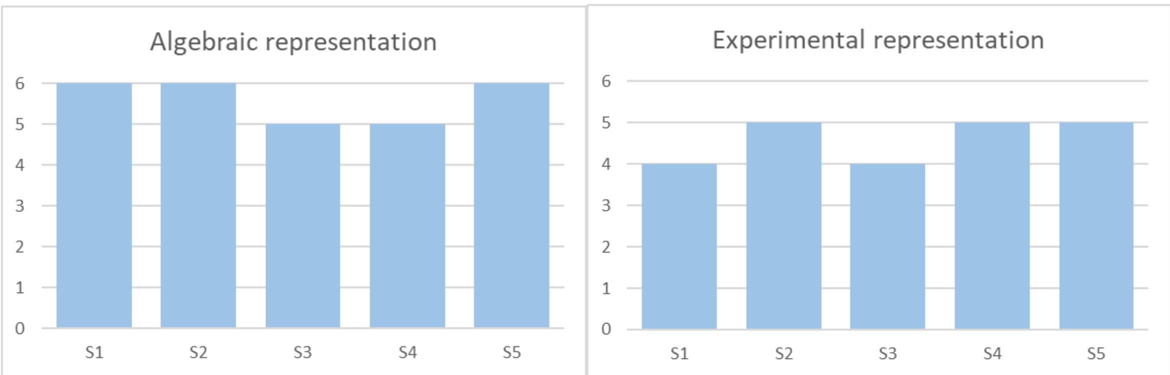


E F

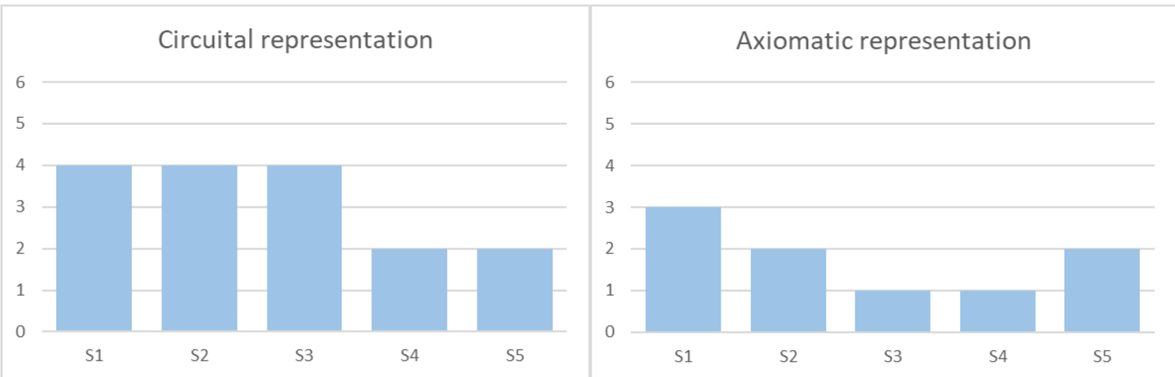
Fig. 19: distribution of students' evaluation of different representations about the concept of state manipulation and evolution.

The most effective representation that helped the students to better understand the concept of state evolution/manipulation is the logical one (mean value 6.0) followed by the experimental one (mean value 5.2). Also the algebraic representation proved to be effective with a mean value of 5,0. The circuital and the geometric seem to be quite useful with a mean value respectively of 4,4 and 4,2. The less effective is the axiomatic representation with a mean value of 2,6.

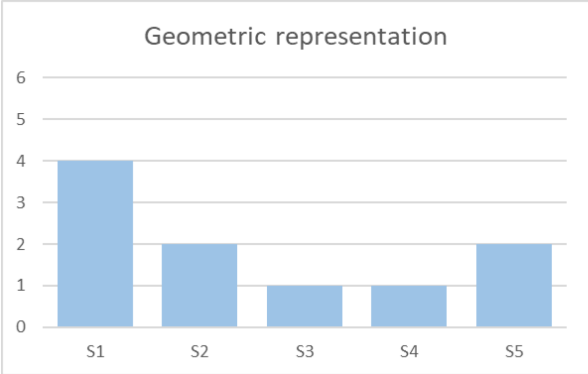
Concept of measurement



A B



C D

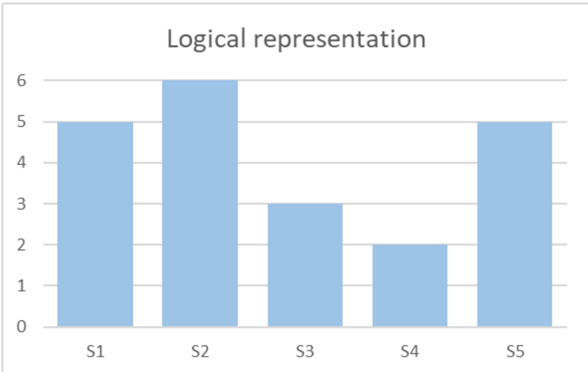


E

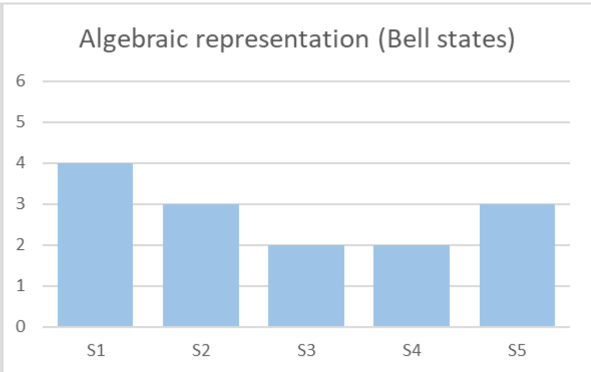
Fig. 20: distribution of students' evaluation of different representations about the concept of quantum measurement.

To understand the concept of quantum measurement the most useful representation proved to be the logical one (mean value 5,6) followed by the experimental one (4,6). The other representations were not particularly effective for the group. In fact, the circuital representation has been evaluated on average 3,2, while the geometric and the axiomatic ones respectively 2,0 and 1,8.

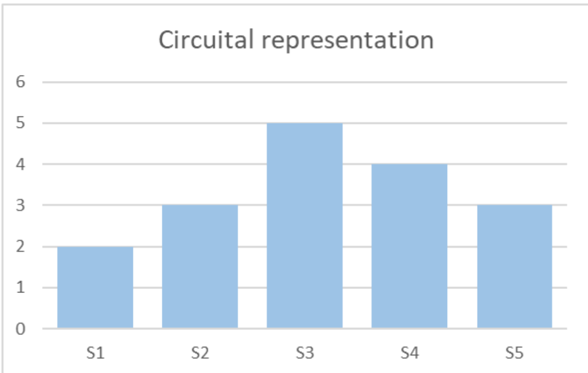
<u>Concept of entanglement</u>



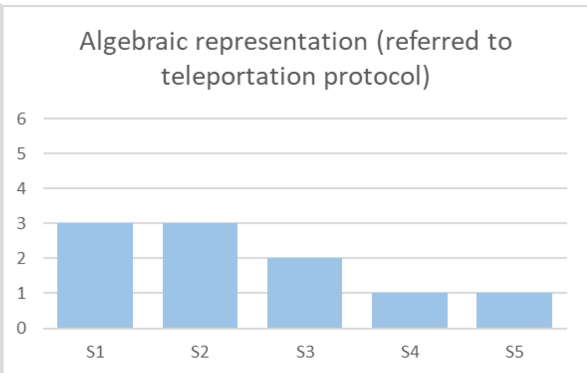
A



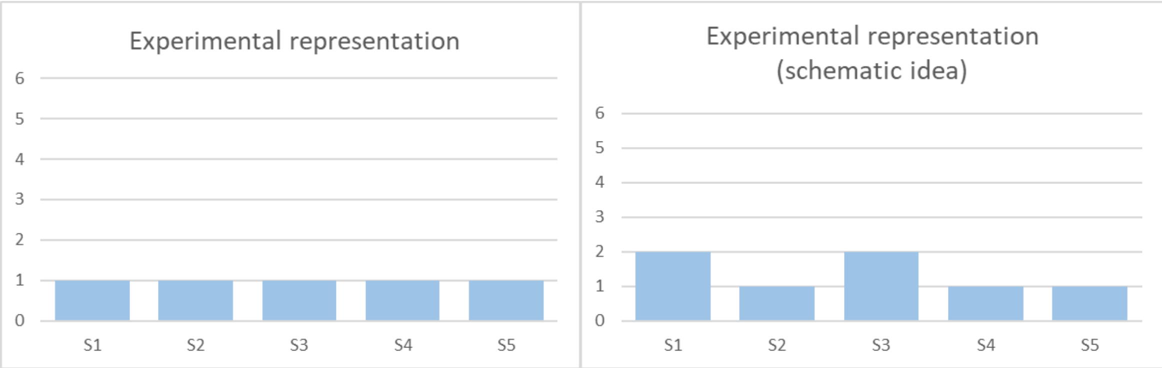
B



C



D



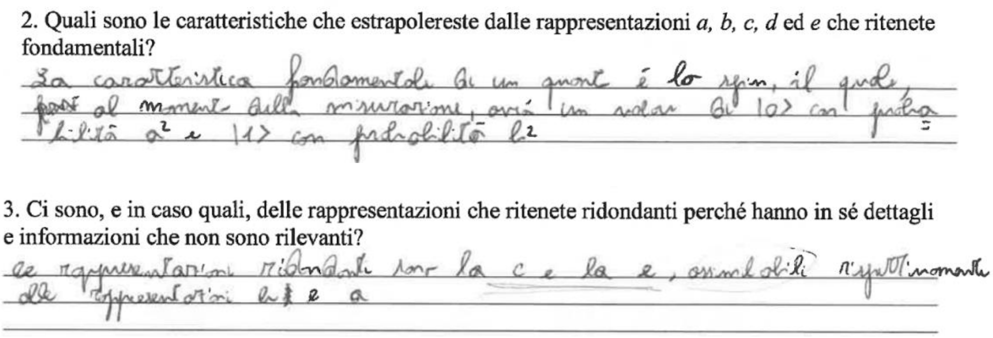
E F

Fig. 21: distribution of students' evaluation of different representations about the concept of entanglement.

Finally, for the concept of entanglement, the better representations have not convinced the entire group, in fact, the histograms present some oscillations. The representation that proved to be the more effective for the group is the logical one with a mean value of 4,2. It is followed by the circuital representation with a mean value of 3,4. Less effective it proved to be the algebraic representation of both the Bell states and the representation referring to the teleportation calculation with, respectively, a mean value of 2,8. The representations that received the least evaluation and, therefore, ineffective for the group to understand the concept of entanglement are the two proposed experimental representations (mean values 1,4, and 1,0).

Question 2 & 3

Group 4 has also answered questions 2 and 3. In figures 4.36, 4.37, 4.38, and 4.39 the answers are reported.



2. Quali sono le caratteristiche che estrapolereste dalle rappresentazioni *a*, *b*, *c*, *d* ed *e* che ritenete fondamentali?

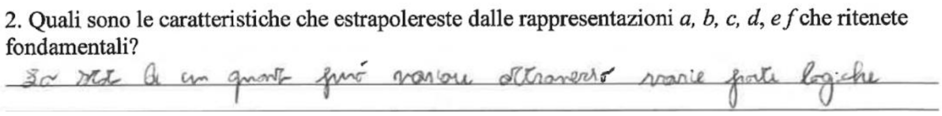
La caratteristica fondamentale di un quanto è lo spin, il quale, al momento della misurazione, avrà un valore di |0> con probabilità a^2 e |1> con probabilità b^2

3. Ci sono, e in caso quali, delle rappresentazioni che ritenete ridondanti perché hanno in sé dettagli e informazioni che non sono rilevanti?

Le rappresentazioni ridondanti sono la c e la e, assimilabili rispettivamente alle rappresentazioni b e a

Fig. 22: Group 4 answers questions 2 & 3 for the concept of quantum state

Concerning the concept of quantum state, the group writes that "the fundamental characteristic of a what is the spin, which, at the time of measurement, will have a value of |0> on probability a2 or |1> with probability b2". Unlike Group 3, Group 4 did not try to reflect on the information that they should extrapolate from the single representations but rather write what they had understood about the concept. For what concerns question 3, unlike group 3, they found the representation redundant, in particular, they write that "representations c and e may be assimilated respectively with the representations b and a".



2. Quali sono le caratteristiche che estrapolereste dalle rappresentazioni *a*, *b*, *c*, *d*, *e f* che ritenete fondamentali?

Lo stato di un quanto può variare attraverso varie porte logiche

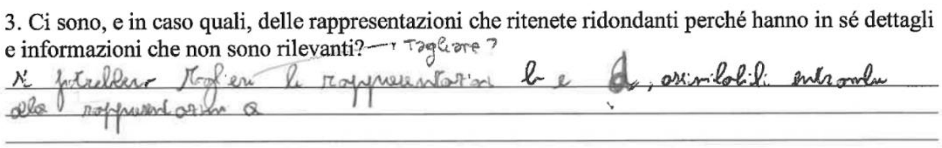
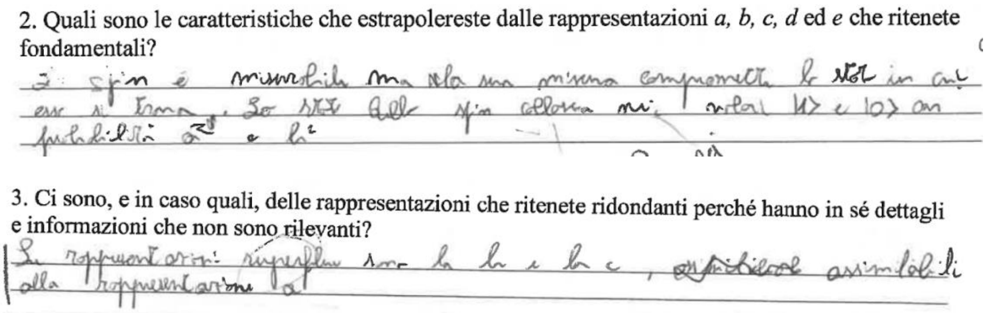

3. Ci sono, e in caso quali, delle rappresentazioni che ritenete ridondanti perché hanno in sé dettagli e informazioni che non sono rilevanti?

Fig. 23: Group 4 answers questions 2 & 3 for the concept of state evolution and manipulation

As collective information that group 4 extrapolated from the different representations is that "the state of a quantum object can be varied through different logic gates". Also in this case they found the representations redundant, in particular, "the representations b and d could be removed, both similar to the representation a".


2. Quali sono le caratteristiche che estrapolereste dalle rappresentazioni *a, b, c, d* ed *e* che ritenete fondamentali?

3. Ci sono, e in caso quali, delle rappresentazioni che ritenete ridondanti perché hanno in sé dettagli e informazioni che non sono rilevanti?

Fig. 24: Group 4 answers questions 2 & 3 for the concept of quantum measurement

About the quantum measurement, the information that the group extrapolated from the different representations is that "The spin is measurable, but its measurement compromises the state it is in. The spin state collapses to values |1> and |0> with probability a2 and b2." Also for the concept of quantum measurement, students found some representations unnecessary: "The unnecessary representations are b and c, similar to a".


2. Quali sono le caratteristiche che estrapolereste dalle rappresentazioni *a, b, c, d* ed *e* che ritenete fondamentali?

Fig. 25: Group 4 answers questions 2 & 3 for the concept of entanglement

For what concerns the concept of entanglement, the common feature that the group 4 extrapolated from the different representations is the dependency between two states; in students' words "The concept of entanglement, or the dependency of the state of a qubit on another qubit called entangled". In this case, the students found redundant representations b and e, "both similar to the representation a".

Annex 4

Name of the artwork: $|\psi\rangle$

Main contributions: Arturo and Diana

Description of the artwork:

In the observation of a fractal that "evolves" over time, the metaphor of the evolution of a physical system in an abstract mathematical space is inserted. Each observer, physically interacting with the system, can make it "collapse". The interaction through the sensors, which symbolically capture the presence of his gaze, interrupts the flow of vertigo generated by the movement of the fractal.

Quantum physics determines a revolution not only in the field of perception of reality towards us [how we perceive reality] but also vice versa: a 'simple' measurement of a particle's features determines its collapse into a state that would normally remain in eternal superposition with other possible states, making the observer an 'active creator' of a part of reality, through its interaction with it and an irreversible change. This means that there are quantities associated with some bodies that, if they are not measured, have a value that is in a state of uncertainty, which is expressed uniquely only when a measure is made, the process of "collapse".

The human being, understood not as an organism but as a thinking entity, acquires in this way a necessarily active role in the study of phenomena interpreted by quantum physics. The installation aims to emphasize the importance of the intervention of the observer in the system to determine a change that can make possible an analysis of its state. The superposition is concretized through a fractal in continuous becoming in every single moment it presents a new state, but at the same time it is always linked to a constant geometry. A specific action causes the interruption of the superposition and the collapse of the fractal into a more 'ordered and understandable' state.

On the whole, we have tried to translate aspects peculiar to quantum mechanics, linked to the role of measurement: a human action concretized by the use of our own hand, the oldest of the tools from which our entire technological progress has derived.

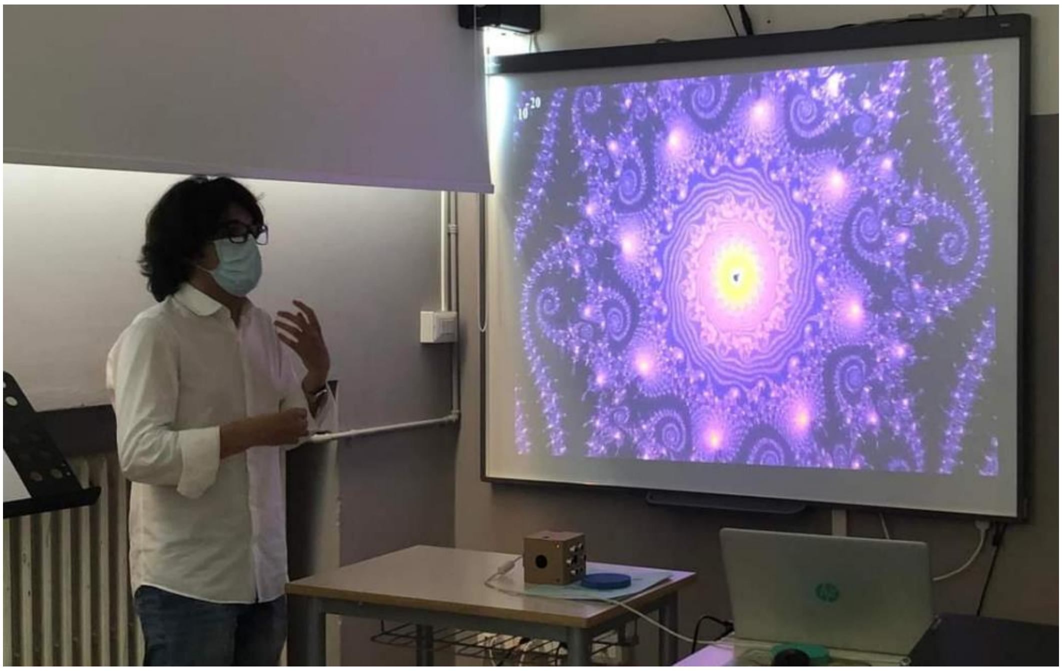


Fig. 26: Arturo while explains the artwork

Name of the artwork: Maze

Main contributions: Carlo

Description of the artwork:

Kandinskij's work aims to represent an abstract and almost scientific synthesis of reality, the painter contrasts an external vision, that of realistic painting, with an internal and abstract one, through which the painter makes visible what hardly emerges from the observation of matter in its surface and thus reveals new worlds within it through the imagination that painting is able to express. In the work that we have elaborated, the abstract image inspired by the painter is a "harmonious superposition" of forms, which does not emerge and does not reveal itself to the gaze as the union of the individual figures. The interaction of the viewer with some 'parts' of the image, in the search for information on the properties of the "object", makes the shapes collapse into other below images losing the visibility of the overall structure and at the same time showing the image in a new state. In the metaphor of the physical concept of superposition, the interaction with a system in an uncertainty state is its revelation to another system and involves a selection between the possible outcomes or, formally, a "manipulation" of probability coefficients associated with different physical states. Each observed state is the way in which the "object" manifests itself after the interaction and that is why, with a sentence by Beckett, the viewer is each time invited to try again the interaction and is sent to the beginning of the path to put into play a different "probability" of representation and decipherment of physical reality.

Fig. 27: Carlo while explains the artwork

Name of the artwork: Apparence

Main contributions: Andrea

Description of the artwork:

Roberto Longhi provides us with the best viaticum to Morandi's work through a story by Proust that speaks of a sinking of the gaze in which the shape of objects seems to lose importance. For this reason, we decided to propose a still life of the painter who stands on the threshold between visible and invisible. Through the technique of augmented reality, we simulate this vanishing of the fixity of perceived reality and let emerge the text of Proust from which we started, thus alluding to a "beyond", to a more complex reality, where matter could manifest itself in the way that the investigation of quantum research lets us glimpse. The physical description of the world is 'authorized' by quantum physics not to correspond to our daily experience: the new theoretical framework becomes a gateway to a culturally revolutionized world, in front of which disappears any expressive constraint that tries to 'capture'.

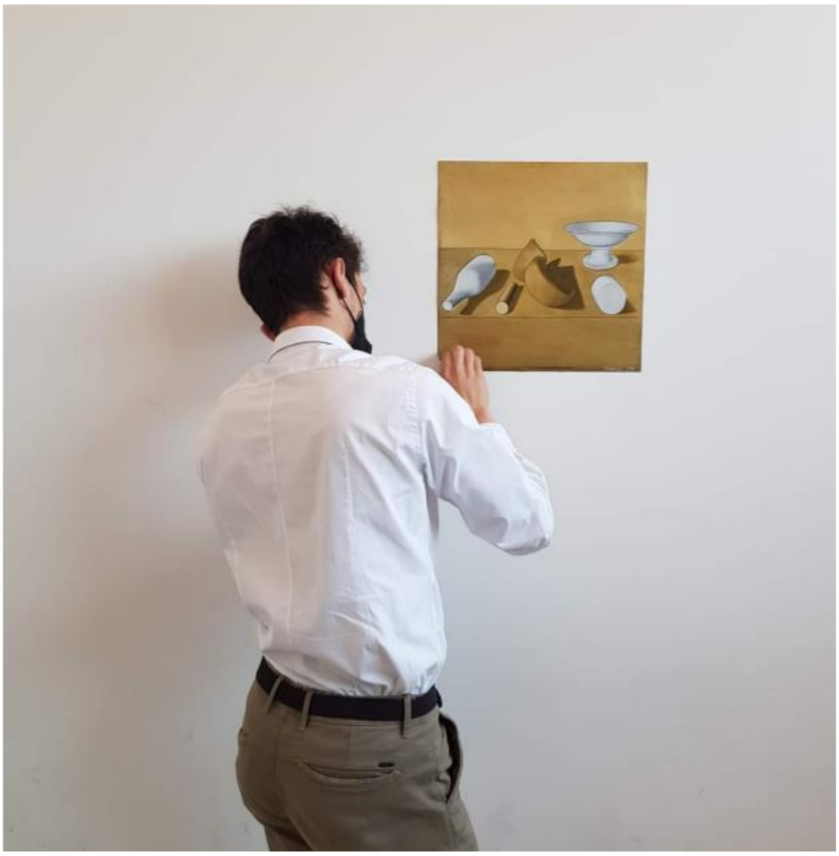


Fig. 28: Andrea while explains the artwork.