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A STUDY ON THE MACROECONOMICS OF CLIMATE CHANGE

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ABSTRACT

This work aims at exploring the relationship between business cycles, having frequencies rooted in the short run, and climatic phenomena, which span longer time horizons. The ultimate goal is to provide a theoretical framework to address these questions: How could very long run considerations affect short run economic decisions? How short run and transitory decisions could exert a long lasting effect on climate? This is achieved by means of an off-the-shelf real business cycle (RBC) model augmented so as to include a climatic block. The economy is perturbed by a technology shock and an energy-price shock. In general, the model performs relatively well in reproducing the cyclical characteristics of the economic variables; however, it is not as successful in capturing the cyclical behavior of climatic variables. Finally, it proposes a set of policy experiments, all taking the form of an energy tax directly or indirectly linked to the climatic status. As a matter of fact the effect of any tax responsive to the business cycle shows positive aspects: when a technology shock hits the economy, it mitigates global warming with minor costs in terms of potential output losses. It also protects the economy from an increase in energy prices, sustaining a certain level of output despite the fall in fossil energy use.

Contents

1 Introduction & Literature Review

Business cycles are cyclical fluctuations of macroeconomic aggregates characterized by a periodicity less or equal to eight years (King and Rebelo, 1999). On the other hand, climatic processes are characterized by larger time horizons: A certain pulse of CO_2 develops its maximum warming potential after about 10 years from its emission (Ricke and Caldeira, 2014). This time, however, can potentially be longer the larger the size pulse of CO_2, suggesting that an emission can develop its full warming even after centuries (Zickfeld and Herrington, 2015). Climatic processes are mediated by CO_2 accumulation in the atmosphere, where a part of it can potentially reside even thousands of years (Economides et al., 2018, p. 21). In light of the foregoing, this work seeks to provide a theoretical framework to address the macroeconomy-climate link emphasizing the role of the timing discrepancy between economic and climatic processes. More specifically, it tries to reconcile business cycles frequencies, which are rooted in the short run, and climatic phenomena, whose processes and consequences may span decades, by answering the following questions: How do short run economic decisions affect climate in the long-run? How do long-run climatic processes affect short-run market interactions?

In general, for a given level of technology, an economy generates an aggregate output accompanied by a certain level of emissions. These anthropogenic flows of CO_2 accumulate in the atmosphere in the form of a stock, whose strong persistence over time leads, in the long-run, to changes in climate measures such as precipitations, wind patterns or, more specifically, global temperature increases, namely, the global warming. As a consequence, climate changes potentially evolve into more or less irreversible damages hitting on the whole ecosystem, and, more specifically, on human activities such as the economic processes (Economides et al., 2018, p. 20). In other words, economy and climate turn against each other, thus, setting up a sort of tragedy of the commons (Hardin, 1968): anybody individually enjoys the advantages of emitting while commonly suffering from the disadvantages originating from the climate change.

As already introduced in the foregoing, the link between macroeconomics - or, more in general, economics - and climate change is mostly a matter of time. From a pure normative point of view, the mismatch between current decisions and their long-term consequences can be traced back to the so-called "discrimination against the future" (Pearce and Turner, 1990, pp. 211-212), deriving from the human *myopic* attitude of allocating more importance to the present rather than the future. With respect to climate change, this substantially translates into enjoying current output, leaving to future generations the burden of coping with the delayed detrimental consequences generated by CO_2 emission and their atmospheric accumulation over time. The relative easiness intrinsic to the creation of a problem, however, does not directly translate into a just as quick resolution of it. This is mainly due to the starting of a *Tragedy of Horizons* (Carney, 2015; Livingston, 2016), that is, the mismatch between the relatively long time period along which climate change exerts its detrimental effects and the relatively short times peculiar to human sphere of action such as those pertaining to economical and political phenomena.

The approach of looking at climatic problems through a macroeconomic lens has emerged mainly in the last decade. Only a few works explore the link between CO_2 emissions and business cycles from a pure macroeconomic point of view. In his paper targeting US economy, Heutel (2012) supports the intuition rooted in climate change literature that points towards the procyclicality between the cyclical components of CO_2 emissions and GDP. He also finds out that this relationship is inelastic. Pushing his research beyond US borders by means of a cross-country panel, Doda (2014) empirically corroborates this result, showing that CO_2 and GDP move together during booms and busts, this behavior holding even stronger when the country is relatively wealthy. In addition to this, he shows that CO_2 emissions are more volatile than GDP, this last characteristics being more evident when focusing on poor countries. Restricting the field to US, Sheldon (2017) confirms these results concerning procyclicality by shedding light on the asymmetry of this behavior: while an increase in GDP leads to almost no increase in CO_2, a decline in the former is followed by an even greater fall in the latter. In other words, CO_2 shows inelastic behavior with respect to GDP increase and elastic tendency when GDP falls.

The need of expanding the study of macro-environmental questions beyond the borders of a pure empirical analysis, has resulted in the emergence of macro-theoretical models Fischer and Heutel (2013). Macroeconomic analysis widely employs micro-founded Dynamic Stochastic General Equilibrium (DSGE) models, which allow to analyze different macroeconomic phenomena exploiting a feature, in a sense, unique within the realm of social sciences: mirroring hard-science methodology, they act as laboratories allowing for the policy simulation (Lucas Jr, 1980; Christiano et al., 2018).

DSGE models are, thus, extended in order to account for environmental issues and policies, such as those related to climate change, and are sometimes known in the literature as Environmental-DSGE (E-DSGE) models (Khan et al., 2019). The vast majority of them are small-scale models built with the aim of studying the efficacy of environmental policies such as emission taxes, emission permits, command-and-control instruments, etc. , and usually consist in a demand block, i.e., households, a supply block, i.e., firms, and a policy block, i.e., the government (Economides et al., 2018, p. 32). Belonging to this category are works such as Angelopoulos et al. (2010); Fischer and Springborn (2011); Heutel (2012); Golosov et al. (2014); Annicchiarico and Di Dio (2015); Dissou and Karnizova (2016); Niu et al. (2018); Xiao et al. (2018); Annicchiarico and Diluiso (2019); Chan (2019) which, despite differing in terms of geographical context and in the ways each sector is specified, all share a focus on policy analysis. Just to cite a few extensions, Annicchiarico and Di Dio (2017) present an interesting extension of the above-mentioned framework by including monetary policy under the presence of a Ramsey planner. Adopting a slightly different framework, Li and Swain (2016) simulate how water supply, endangered by droughts due to climate change and over-consumption due to population growth, affects economic growth, i.e., output increase, and economic well-being, i.e., income and wealth, of South Africa. More recently, Gallic and Vermandel (2020) show how weather can generate business cycles through a small scale DSGE model for New Zealand, a country characterized by a relatively large agricultural sector. According to their analysis, weather shocks are able to explain about 35 % of GDP and agricultural output fluctuations in this country.

One of the main features of this work with respect to the extant literature is to make a step back and explore the key features at the roots of those theoretical macro-models which encompass climatic issues. It starts by identifying fossil energy as the keystone of this relationship, in that it serves as input for the generation of both output and CO_2 emissions. Inspired by King and Rebelo (1999),this work supplies an analysis of the cyclical components of both economic and climatic variables. It tries to graft a long-run phenomenon such as global warming into a short-run economic framework by modeling the relationship between the anthropogenic stock of atmospheric CO_2 and temperatures in terms of a *long-run* econometric relationship. Finally, it proposes a set of policy experiments, all taking the the form of an energy tax directly or indirectly linked to the climatic status.

So far, it is evident how the DSGE framework constitutes a very flexible instrument for macro-policy analysis even when dealing with environmental issues. With respect to this specific work, the challenge is to extend a basic RBC framework to encompass macro-climatic concerns while using US economy as calibrating framework. The resulting model, whose economic building blocks take inspiration from Dhawan and Jeske (2006) and Dhawan and Jeske (2008), features two agents, namely households and firms, and three markets, that is, labor, capital and fossil energy. It also models the relationship between these two variables as one of compementarity. Very importantly, the model also includes a climatic block which stylizes the carbon cycle, in terms of atmospheric accumulation of CO_2 emissions, and how this process originates from, and at the same time affects, the economy through global warming. The effect on the economy, in particular, is obtained through the adoption of a damage function as in Weitzman (2012), which negatively impacts the level of output as the temperature increases. The model economy is hit by a technology shock, as standard in real business cycle literature, and an energy price shock. The full model is finally used as a policy tool to evaluate various policy rules aiming at limiting emissions. This is achieved by introducing a tax on the use of fossil fuel energy, this latter being shaped in terms of five different policy rules: the tax can either be constant or linked to either fossil energy, domestic CO_2 emissions, or the anthropogenic stock of atmospheric CO_2. An additional experiment is also included, which features a global tax being linked to a combination of all the above-mentioned energy-climate variables.

In general, the model performs relatively well in reproducing the cyclical characteristics of the economic variables; however, it is not as successful in capturing the cyclical behavior of climatic variables. This could be due to the intrinsic nature of environmental series, which are characterized by high non-linearity. With respect to the policy experiment, only those taxes that are linked, directly or indirectly, to fossil energy use, show responsiveness to the business cycle. More specifically two situations are outlined: when the economy is hit by a technology shock, the energy tax is able to sustain a certain level of productivity leading at the same time to a substantial limitation in carbon emissions and global warming. The energy price increase originating from the energy price shock leads to the reduction in fossil energy use. This, on the one hand, reduces the tax, which is able to counter, to some extent, the increase in the price energy itself. On the other hand, it reduces carbon emissions, thus leading to an improvement in climatic variables.

In light of the considerations above, this present work is built as follows: Section 2 highlights how the dataset is built and supplies an analysis of the main features of the series considered in the model; Section 3 describes how the model is built, calibrated, and validated, delving into its basic mechanisms; Section 4 shows how the climate policies are implemented and how they interact with market and climate; finally, 5 summarizes the main results of the analysis. Further details on data and model equations are included in Appendixes A, **??**, and B.

2 Data Analysis & Stylized Facts

This chapter deals with the description and the analysis of the data employed throughout this work. Section 2.1 outlines the criteria adopted to carry out their collection and illustrates the rearrangements data underwent in order to obtain a homogeneous dataset, consistent with the reality which is to be analyzed. Section 2.2 describes specific statistics, namely *stylized facts*, the model under construction should potentially stick as closer as possible to in order to mimic the reality "out there". For the sake of completeness, a more schematic compendium concerning the series used throughout this work, the description of some of their features and of the sources where they can be recovered is contained in Appendix A.

2.1 Data Description

The dataset gathers US economic and environmental series covering a time span of 47 years, from 1973-2019. The choice of this time period is due to a pair of technical reasons: on the one hand, the lack of environmental series prior to year 1973 obviously limits the dataset at the lower end of the time period; on the other hand, in order to exclude any influence on US economy due to Covid pandemics, the last year of the series is identified with 2019. Data are obtained from the Bureau of Economic Analysis (BEA), the Federal Reserve Economic Data (FRED), and the U.S. Bureau of Labor Statistics (BLE); among environmental data, those concerning local energy and emissions are available from the U.S. Energy Information Administration (EIA), while those concerning global climatic variables, such as the atmospheric stock of CO_2 or the global temperature anomaly, are taken from the National Oceanic Atmospheric Administration (NOOA).

On the economic side, the dataset includes the main macroeconomic series: *output* (Y), *consumption* (C) (i.e., the aggregate of *non-durables* and *services*), *investment* (I) (i.e., the aggregate of *fixed private investment*, *durables*, and *changes in inventories*), *labor* (H) (i.e., the hours worked in the non-farm business sector). All economic series are expressed in *real* terms, are supplied in *quarterly frequency* and are already *seasonally adjusted*.

On the environmental side, a bunch of variables is selected, whose characteristics aim at capturing the core features of the climate change issue; these are *fossil fuel energy consumption* (E) (i.e., the aggregate of energy consumption from three fossil fuel sources, namely *coal*, *gas*, and *oil*), *energy price* of fossil fuels (P_e)[1], *domestic CO$_2$ emissions* (M^d), the *global stock of atmospheric CO$_2$ emissions* (V) and *temperature anomaly* (T). In general, all economic series are considered net to the effect of *pre-industrial times*, when human-driven industrial activity had not yet gained the upper hand on large-scale, thereby not significantly contributing to anthropogenic climate change. The climate science's customary choice is to approximate this time frame with the 1850-1900 interval (IPCC, 2018). This assumption mainly affects the series for global stock of atmospheric CO$_2$ and temperature anomaly, which account for pre-industrial contributions when considered in their original shape. This implies that, in order to be employed, they must undergo appropriate rearrangements, whose nature will be detailed in the following.

The anthropogenic stock of CO$_2$ is obtained by subtracting the *pre-industrial* level of CO$_2$ ($\bar{S}$), which is fixed at 280 *parts per million* (Stocker et al., 2013), from the original and available series for the total stock of atmospheric CO$_2$. Temperature anomaly is defined as the positive or negative temperature departure from its long-term average calculated over a certain base period. The original observations for temperature anomaly are supplied by NOAA, are available from year 1880 and adopt the 1901-2000 interval as base period. In order to keep consistency, the original series for temperature anomaly is re-scaled in order to set pre-industrial times as the base period. In other words, the goal is to know how much temperature changed with respect to its pre-industrial average level. This is achieved as follows: First, the original series for temperature anomaly is averaged over the 1880-1900 period, the result of this operation being 0.2038 °C. The 1901-2019 temperature anoma-

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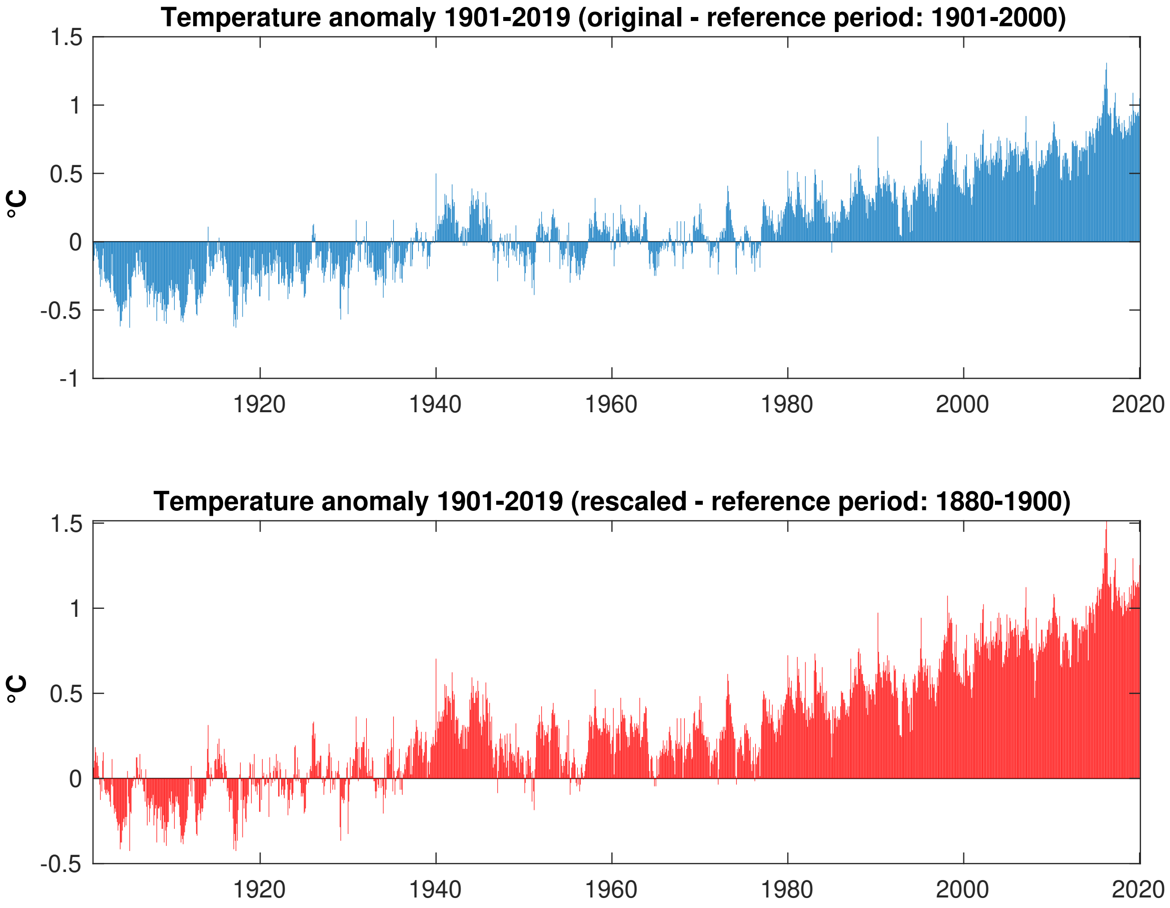
[1] More precisely, this is a price index relative to gasoline and other energy goods. The series is retrievable from Bureau of Economic Analysys (BEA).

lies are then summed to the average temperature anomaly corresponding to the 1880-1900 interval.

Figure 1: Temperature Anomaly



This procedure is useful, in that it allows to obtain a series for the global mean temperature anomaly with respect to its *pre-industrial* level without knowing the exact temperature used as long-term reference temperature for the period 1901-2000. The output of this operation of re-scaling is depicted in Figure 1, where the blue and the red bar plots represent, respectively, the original and the re-scaled series, this latter being the one adopted throughout the work[2]. Despite its approximating nature, the reliability of this series is corroborated

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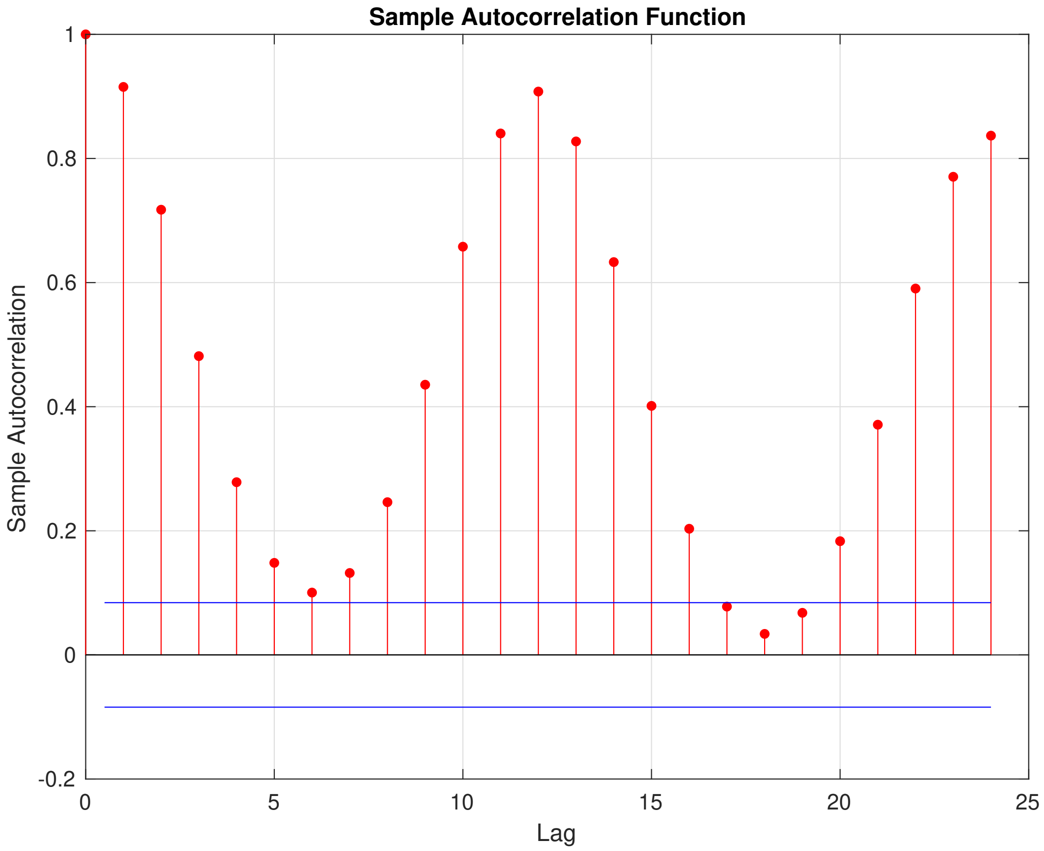
[2] For illustrative purposes only, it is relevant to specify that the two series employed in Figure 1 cover the 1901-2019 time period, which is much larger than the one considered in the current work; the two series are

by comparing some climatic facts from IPCC studies with those resulting from the adoption of this procedure. More specifically, according to Allen et al. (2018), human-induced warming reached approximately 0.87°C with respect to *pre-industrial* levels in the 2006–2015 decade (0.88°C according to this dataset) and 1°C in 2017 (1.1°C according to this dataset).

With the exception of the series for energy price, which, because of its characteristics, is itself belonging to the economic realm, the intrinsic nature of environmental variables suggests to spend a few additional words about their homogenization within a macroeconomic framework. First, agencies commonly supply them in *monthly* frequency, which requires their aggregation in order to acquire a *quarterly* nature. Second, most series are supplied without being subjected to a previous *seasonal adjustment*, thereby requiring a *seasonal* correction.

Figure 2: Seasonality of the atmospheric-anthropogenic stock of CO_2



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also represented prior to their correction for seasonal adjustment.

Interestingly, even stocks, which are usually very unlikely to be affected by seasonality, may exhibit it. For instance, this is the case of the anthropogenic stock of atmospheric CO_2, both in its original or re-scaled fashions. As depicted in Figure 2, once de-trended, this latter clearly presents a strong oscillating pattern, which has its explanation in the carbon cycling back and forth between the atmosphere and the biosphere. In spring and summer, the photosynthesis activity fostering vegetative growth has as consequence the absorption of CO_2 from the atmosphere; in autumn and winter, the death of plants leads to the release of CO_2 back into the atmosphere as the rate of photosynthesis and oxidation decreases.

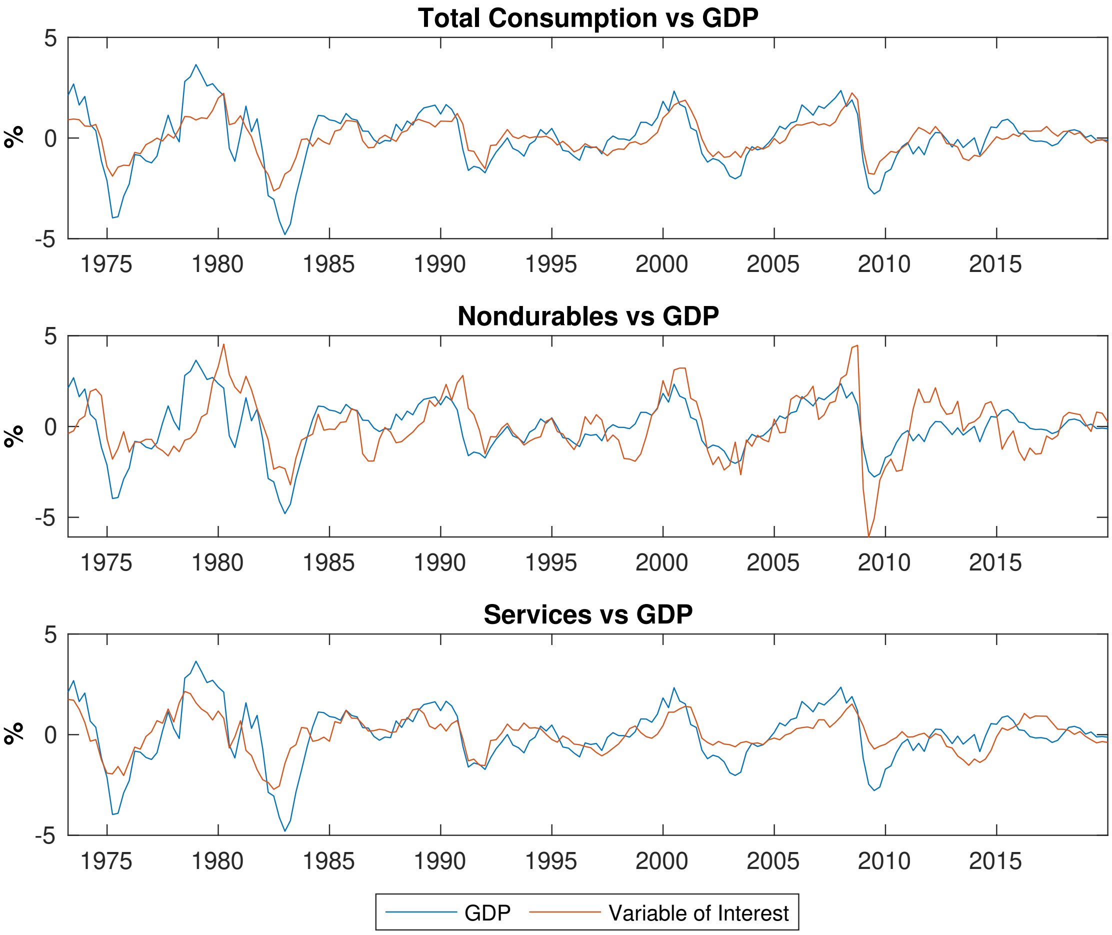
Finally, when some of the series, sharing the same chemical composition but a different role within the model, are expressed using different *order of magnitudes* and *units of measure*, a further manipulation might be necessary in order to homogenize them and ensure their consistency within the model itself. This is, for instance, the case of CO_2 emissions, i.e., a *flow* variable, and net atmospheric CO_2, a stock variable. More specifically, while the former is supplied in *million metric tons* of CO_2, the latter is expressed in *parts per million* (*ppm*). As a consequence of the foregoing, both of them are converted into *giga tons of carbon* (*GtonC*).

2.2 Stylized facts of US business cycles: economic and environmental/climatic variables

Generally speaking, business cycles are cyclical fluctuations of macroeconomic aggregates having periodicity of eight years or less (King and Rebelo, 1999). In a nutshell, the measurement of business cycles revolves around decomposing a time series into a long-run non-stationary trend component and a short-run stationary cyclical component. Once this latter is identified, it is possible to obtain the *stylized facts*, namely, a selection of statistical moments characterizing the regularities in the data.

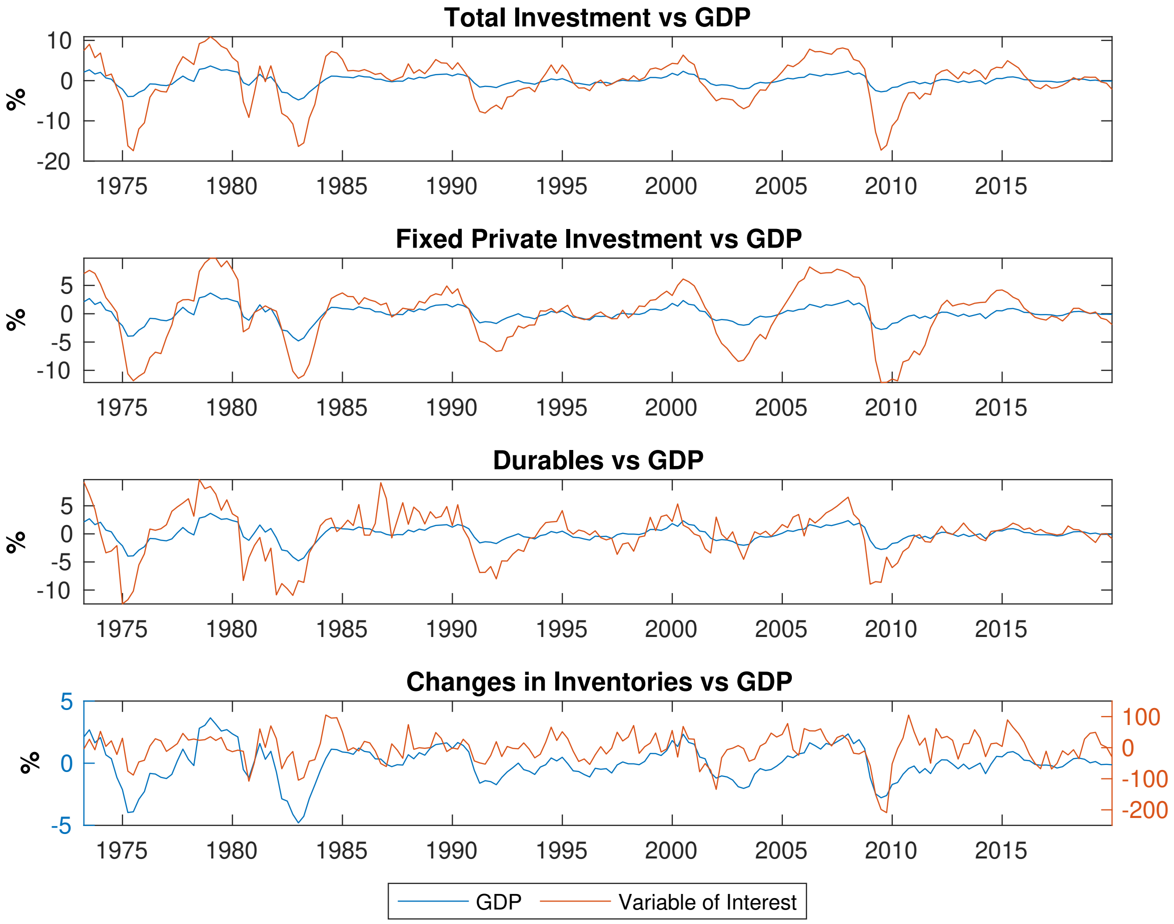
From a practical point of view, all variables described in Section 2.1, are manipulated according to the following steps: First, the logarithm is applied to each observation of economic (with the exception of the changes in inventories) as well as environmental variables. Second, all the series are processed employing the Hodrick-Prescott filter (Hodrick and Prescott, 1997) in order to extract their cyclical components. Upon the premise of employing output as the benchmark variable against which to compare any variable of interest, four different moments are generally exploited in the analysis: the *standard deviation*, or *volatility*, which measures the *amplitude* of time series fluctuations in absolute terms; the *relative standard deviation*, which allows for the comparison of the absolute standard deviation of each variable against that of output; the *contemporaneous correlation* with output, which measures the *co-movement* between this latter and any other aggregate of interest; finally, the *persistence* of each series, which is simply captured by its *first-order autocorrelation*.

Figure 3: Cyclical Component of Consumption (HP filter, $\lambda = 1600$)



The results of this procedure are captured by Figures 3, 4, 5, 6, 7, and 8, whose aim is twofold: On the one hand, they supply a graphical description of the cyclical behavior of specific aggregates and their relative components; on the other hand, they allow to compare the aggregate of interest with the benchmark series for output. Moreover, all relevant results from this procedure are comprehensively illustrated in Table 1, whose goal is to summarize the above-mentioned *stylized facts*. The outputs of this analysis are shown starting from the economic variables and gradually shifting towards the environmental ones.

Figure 4: Cyclical Component of Investment (HP filter, $\lambda = 1600$)



As shown in Figure 3, the cyclical component of total consumption is less volatile than output. More specifically, as evidenced by Table 1, the standard deviation of the main consumption aggregate is 0.60 times that of output. Shifting the focus to its components, it emerges that the consumption of services is less volatile than output, while consumption of non-durables, on the contrary, is slightly more volatile. All series share a procyclical behavior, in that they are characterized by a positive and strong correlation with output.

Moving to Figure 4, it emerges that the aggregate series for investment is more volatile - about four times - than output, and strongly procyclical. This behavior holds for its building components as well; more specifically, the changes in inventories look appreciably more volatile and relatively less correlated with output when compared to to fixed private investment and durables.

Figure 5: Cyclical Component of Labor (HP filter, $\lambda = 1600$)

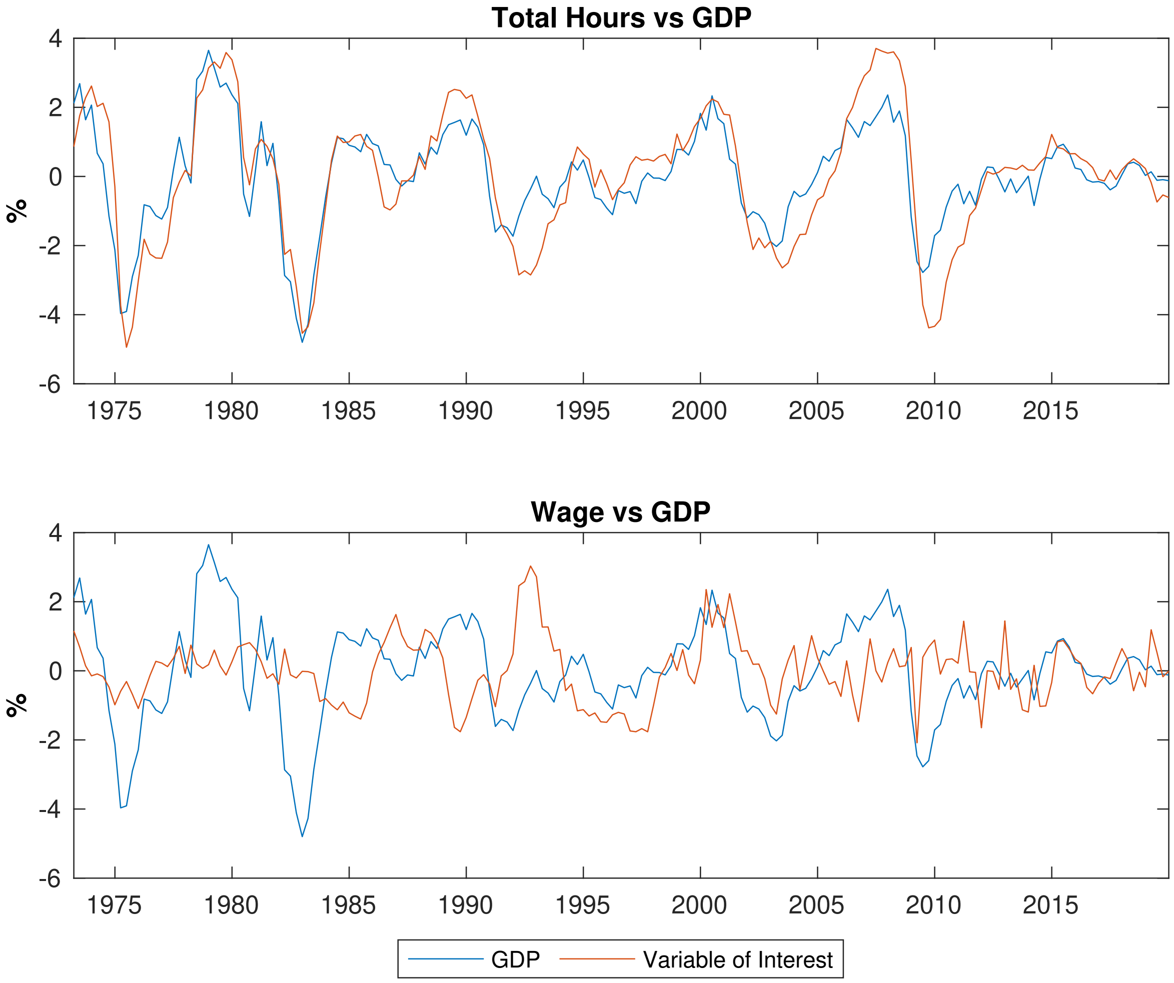
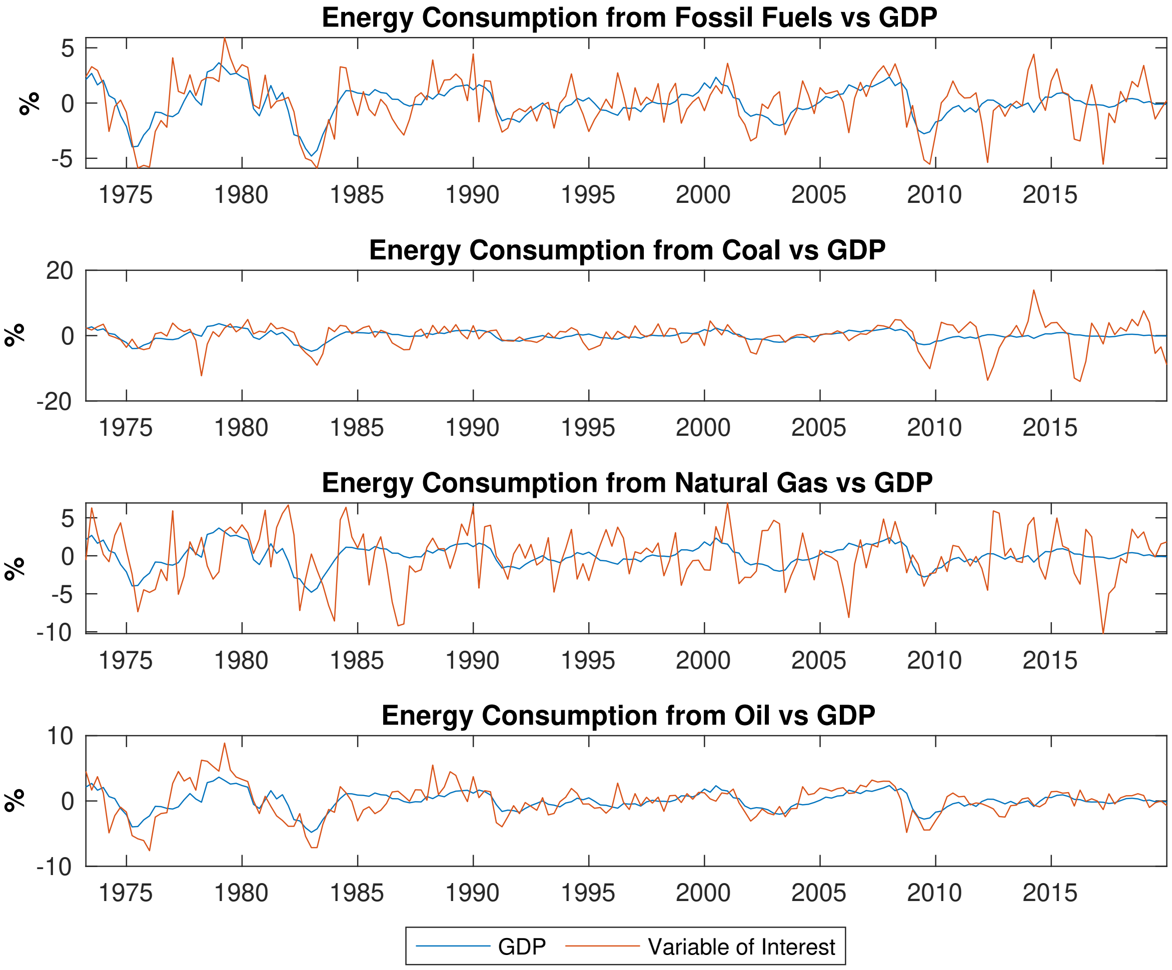


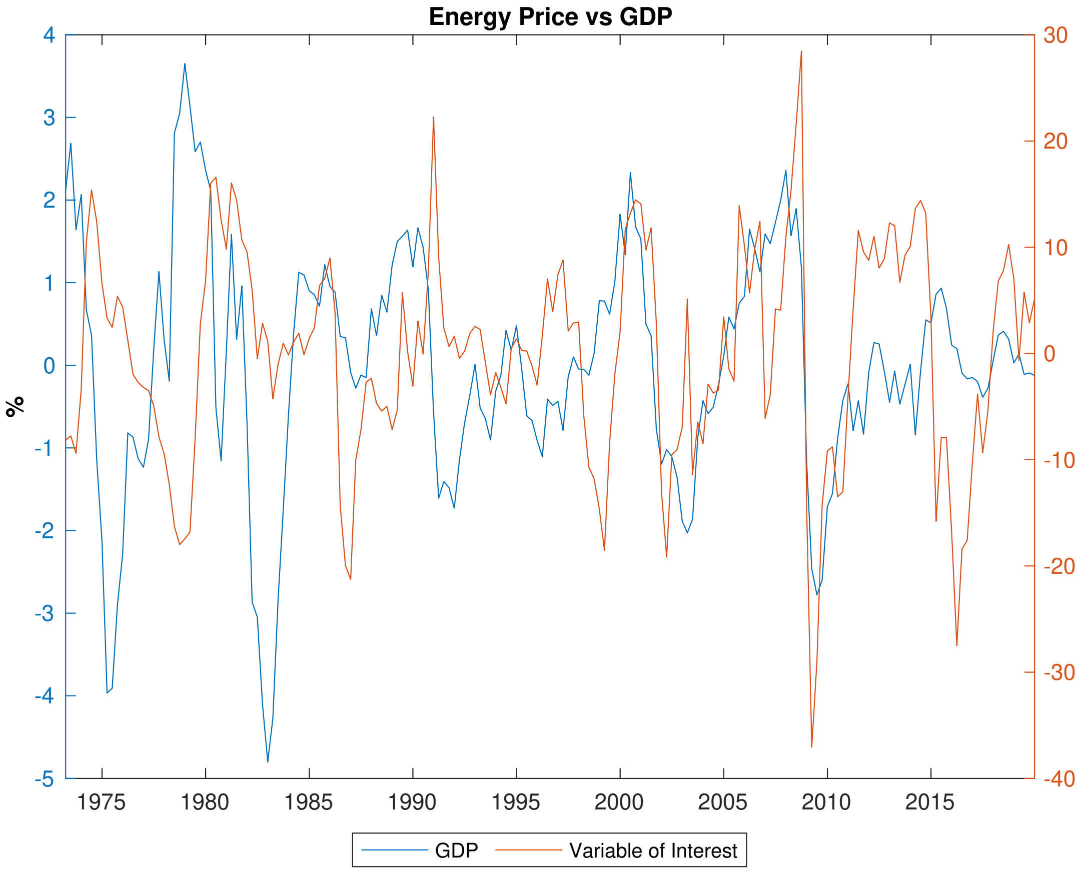
Figure 5 portrays the behavior of labor and real wage with respect to output. The former variable is slightly more volatile than output and highly correlated with it; the latter appears to be less volatile than output and essentially acyclical, in that it shows no significant correlation with it. Both results are corroborated by Table 1. Restricting the analysis to the 1973-1990 period, it clearly emerges from Figure 5 how real wages might even assume countercyclical traits.

Figure 6: Cyclical Component of Fossil Energy (HP filter, $\lambda = 1600$)



In general, as evident from the last column of Table 1, all economic series have in common a significant level of persistence, which is also the rationale behind business cycles' forecastability. On the whole, the portion of empirical analysis concerning the economic variables is in line with the results of King and Rebelo (1999): Consumption is found to be about half as volatile as output, investment is about four times more volatile than output, while labor is about as volatile as output.

Figure 7: Cyclical Component of Fossil Energy Price (HP filter, $\lambda = 1600$)

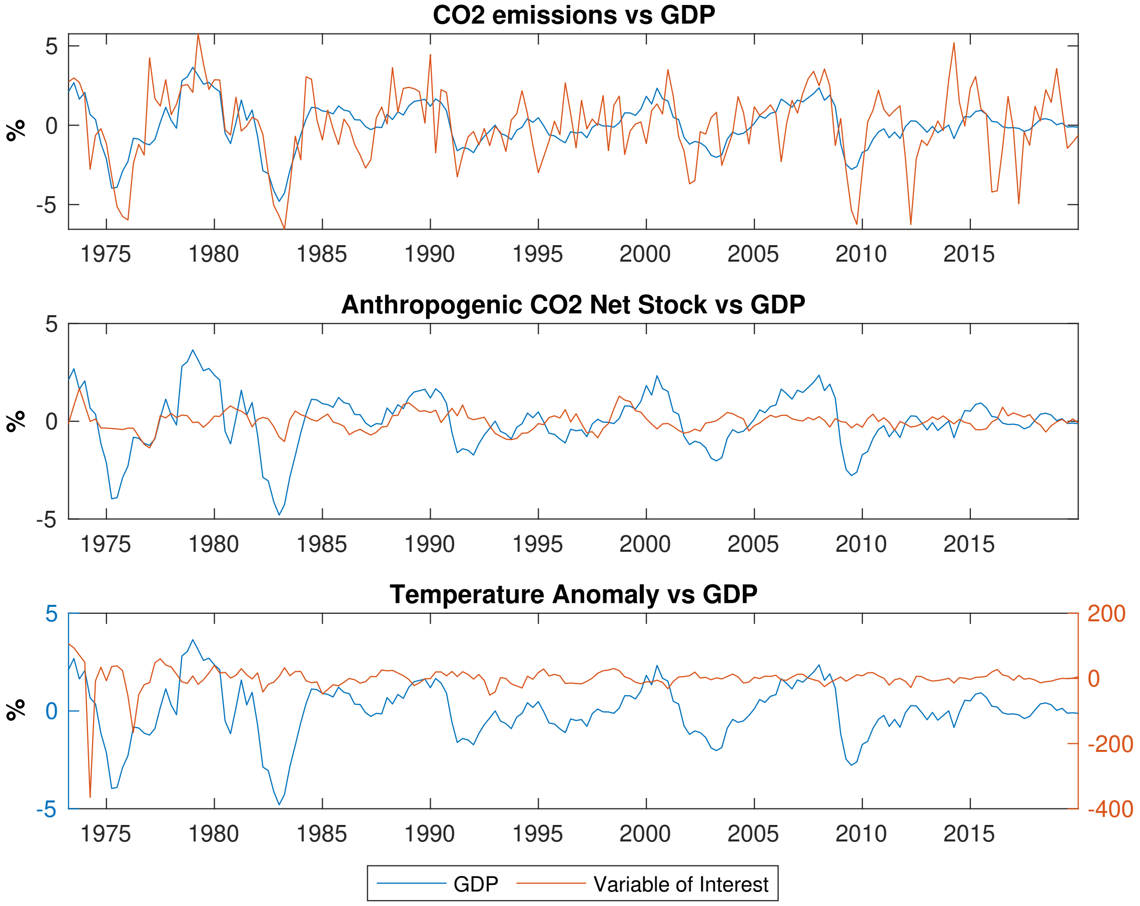


The environmental variables have been separated into two main blocks, the first one concerning fossil fuel energy aggregates and the second regarding climate variables.

Figure 6 portrays the cyclical behavior of total fossil fuel energy and its components.The main fossil energy aggregate is about one and a half times more volatile than output. Energy consumption from coal, gas and oil are even more volatile, the latter's standard deviation being about three times that of output. All variables are procyclical at different degrees: while the main aggregate and oil shows a significant correlation with output, the same does not hold for coal and gas, both of them revealing a weaker relationship. With the exception of gas, all energy variables show significant persistence, oil being the most relevant.

As shown by Figure 7 and corroborated by Table 1, the cyclical behavior of the price of fossil energy is more than seven times as volatile as output. It also shows a significant degree of persistence together with an acyclical behavior with respect to output itself.

Figure 8: Cyclical Component of Climate Variables (HP filter, $\lambda = 1600$)



Finally, Figure 8 describes the behavior of three relevant climate variables: CO_2 emissions from US economy, the anthropogenic stock of atmospheric CO_2 emissions, and the re-scaled temperature anomaly. Domestic emissions are more than a half as volatile as output and sufficiently correlated with it. The net stock of CO_2 shows a low degree of relative volatility and a low level of co-movement with output. While being twenty-five times as volatile as output, temperature anomaly is practically asyncronous with it. With the exception of this latter series, the former two are characterized by a significant level of persistence.

Table 1: Stylized Facts from Original Series - HP Filtered Moments

Variable	$\sigma(\cdot)$	$\sigma(\cdot)/\sigma(Y)$	$\rho(\cdot, Y)$	$\rho(\cdot_t, \cdot_{t-1})$
Economic Variables				
Output (Y)	1.43	1.00	1.00	0.88
Consumption (C)	0.85	0.60	0.82	0.87
Non Durables	1.62	1.13	0.57	0.79
Services	0.88	0.62	0.72	0.88
Investment (I)	5.55	3.88	0.93	0.88
Fixed Private Investment	5.00	3.50	0.90	0.93
Durables	4.11	2.88	0.78	0.79
Changes in Inventories	55.93	39.12	0.54	0.57
Labor (L)	1.88	1.31	0.87	0.93
Wage (W)	0.92	0.65	0.10	0.70
Environmental Variables				
Fossil Fuels Energy Consumption (E)	2.28	1.60	0.63	0.58
Coal Energy Consumption	3.76	2.63	0.35	0.58
Gas Energy Consumption	3.37	3.35	0.26	0.37
Oil Energy Consumption	2.54	1.78	0.72	0.73
Energy Price (P_e)	10.17	7.11	0.09	0.77
CO_2 emissions (M^d)	2.32	1.62	0.62	0.59
CO_2 stock (V)	0.46	0.32	0.28	0.75
Temperature Anomaly (T)	36.41	25.47	0.01	0.21

* All variables are in logarithms (with exception of the changes in inventories) and have been detrended with the Hodrick-Prescott filter. For each series, $\sigma(\cdot)$ is the *absolute standard deviation*, $\sigma(\cdot)/\sigma(Y)$ is the *relative standard deviation* with respect to output, $\rho(\cdot, Y)$ is the *cross-correlation* with respect to output, and $\rho(\cdot_t, \cdot_{t-1})$ is the *first order autocorrelation*.

3 The Model

The aim of this chapter is to illustrate the structure of the model and to provide the steps undertaken to build and validate it. Section 3.1 focuses on describing the model in all its building blocks. In Section 3.2, the process of calibration is exposed, with focus on detailing the underlying assumptions and methodology characterizing this task. The model is then validated in Section 3.3, while the explanation of its mechanisms is treated in Section 3.4. Last but not least, Appendix B collects some additional details about the procedures adopted throughout the chapter.

3.1 The Model Economy

The model describes a closed economy, which features two types of representative agents, namely, households and firms, operating in a perfectly competitive market. The economy is characterized by three different factors of production, capital, labor, and fossil energy. Up to this point, the model is a version of the bog-standard real business cycle model usually adopted in the macroeconomic literature characterized by the addition of fossil energy as factor of production. The model is then equipped with a climatic block, which aims at reproducing in a stylized fashion the climatic processes of CO_2 accumulation and its impact on temperatures. The economic and climatic blocks communicate as follows: The economy produces a certain level of emissions linked to the use of fossil fuels, which accumulates in the atmosphere, raises temperatures, and hits the economy by means of a damage function applied to final output.

3.1.1 Households

The infinitely lived representative household solves the following maximization problem:

$$\max_{C_t, L_t, K_{t+1}, u_t} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\frac{1}{\sigma_c}}}{1-\frac{1}{\sigma_c}} + \chi \frac{(1-L_t)^{1-\frac{1}{\sigma_{le}}}}{1-\frac{1}{\sigma_{le}}} \right] \tag{1}$$

subject to

$$C_t + I_t \leq w_t L_t + z_t(u_t K_t) + \Pi_t \tag{2}$$

where $\beta \in (0,1)$ is the intertemporal discount factor, which serves to control for the impatience of the agent. The household consumes C_t, which denotes the consumption bundle, and enjoys leisure $1 - L_t$, where L_t denotes market labor expressed as the amount of hours worked. Parameter σ_c is the elasticity of intertemporal substitution (EIS), i.e., the consumer's willingness to substitute future consumption for present consumption, $\sigma_{le} > 0$ is the Frisch elasticity of leisure, i.e., the elasticity of non-worked hours with respect to the wage rate keeping the marginal utility of consumption constant, and the scaling parameter χ denotes the utility weight on leisure.

The sequential budget constraint the representative household faces each period is formalized in terms of Equation 2. As the owner of capital K_t and labor L_t, the representative household rents out the former in return for the rent on capital z_t and supplies the latter on the market in exchange for the real wage w_t. It is relevant to underline that capital represents the wealth of the household within this model economy. Variable u_t denotes capital utilization and captures the fact that the economy does not necessarily exploit capital at its maximum installed capacity, thereby leaving some degree of variability in its utilization. Furthermore, as the owner of the firms, the representative household receives profits Π_t in the form of dividends. Shifting to the left-hand side of Equation 2, the representative household employs his revenues to purchase consumption goods C_t and investment goods I_t.

The stock of capital evolves according to the following law

$$K_{t+1} = S(I_{t-1}, I_t)I_t + (1 - \delta(u_t))K_t \tag{3}$$

where the term

$$\delta(u_t) = \delta_0 u_t^{\nu} \tag{4}$$

shows the dependence of capital depreciation on capacity utilization; more explicitly, δ_0 represents the capital depreciation rate, while ν is the parameter governing the level of capital utilization. The term

$$S(I_{t-1}, I_t) = \left[1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] \tag{5}$$

denotes convex investment adjustment costs, which affect the investment process and are expressed in terms of the growth rate of investment. Investment adjustment costs are governed by parameter ϕ.

The first-order conditions of the above-stated problem, once re-expressed in real terms, supply the following equilibrium conditions:

$$C_t^{-\frac{1}{\sigma_c}} w_t = \chi(1 - L_t)^{\frac{1}{\sigma_{le}}} \tag{6}$$

$$q_t = \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma_c}} \left[z_{t+1}u_{t+1} + q_{t+1}(1 - \delta_0 u_{t+1}^{\nu})\right] \right\} \tag{7}$$

$$1 - q_t\left[1 - \frac{\phi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \phi\left(\frac{I_t}{I_{t-1}} - 1\right)\right] = \beta \mathbb{E}_t\left[q_{t+1}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\sigma_c}} \phi\left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \tag{8}$$

$$z_t = q_t \nu \delta_0 u_t^{\nu - 1} \tag{9}$$

where q_t denotes Tobin's marginal Q, that is, the marginal value of installing new capital[3] over the marginal utility of consumption, therefore allowing to measure the marginal value of capital in terms of consumption. Equation 6 describes household's labor supply and is rooted in the trade-off between consumption and leisure: The allocation of a marginal unit of labor in productive activities, on the one hand, reduces household's leisure time and, on the other hand, makes him earn additional real wage w_t, which is spent in the purchase of the consumption good. In other words, the loss of utility determined by working more is counter-balanced by an adequate increase in consumption. Equation 7 captures the household's intertemporal decision of consuming today or saving and postponing consumption in the next period. The representative household, in fact, is faced with a couple of potential decisions: He can decide to consume an additional unit of good in period t; alternatively, he can opt to keep it in the form of capital, rent it in the market to earn a return $z_{t+1}u_{t+1} + q_{t+1}(1 - \delta_0 u_{t+1}^{\nu})$ per unit of capital, and finally employ this latter to purchase the

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[3] This is also called the Tobin Q ratio, namely, the ratio of total installed capital's value over the cost of replacing that capital

consumption good in period $t+1$. It is relevant to observe that both the capital rent and the depreciation rate are influenced by the rate of capital utilization. Equation 8 specifies that the actual value of capital net of its direct costs is equal to the value of capital in the next period corrected by the costs originating by the expected investment growth. In particular, should the investment adjustment costs be absent, then it would hold that $q_t = 1$. Lastly, Equation 9 defines the rental rate of capital as the marginal depreciation originating from an additional unit of capital utilization.

3.1.2 Firms

Firms are homogeneous and generate output Y_t according to a Cobb-Douglas production function that, mirroringKim and Loungani (1992) and Dhawan and Jeske (2008), has the following shape

$$Y_t = \aleph_t^A D_t N_t^\alpha L_t^{1-\alpha} \tag{10}$$

which has as inputs three factors of production: labor L_t on the one side, and capital K_t and fossil energy E_t on the other side. More specifically, these latter two are implicitly combined into the composite N_t that, in turn, obeys a nested CES function shaped in the following fashion

$$N_t = [\omega(u_t K_t)^\rho + (1-\omega)E_t^\rho]^{\frac{1}{\rho}} \tag{11}$$

where ω is a share parameter, which reflects capital and energy's respective weights over the composite N_t and ρ governs the degree of substitutability between capital and fossil energy, thereby implying an elasticity of substitution equal to ${}^1/_{(1-\rho)}$. In general it holds that $\rho \in (-\infty, 1]$; more specifically, throughout this work $\rho < 0$ is assumed, which establishes complementarity between fossil energy and capital.

Of crucial importance, Equation 10 features an instantaneous *damage function* D_t, whose role is to capture the detrimental effect climate change exerts on the model economy. More specifically, this translates into assuming the hypothetical output the economy would have generated in the absence of any climatic interference to be adjusted by a damage D_t, whose entity is determined by the level of temperature anomaly T_t experienced at time t. This is achieved by defining a mapping between temperature anomaly and the damage itself, i.e., $D_t = f(T_t)$[4]. Environmental economic literature supplies a relatively heterogeneous range of plausible damage functions, e.g., (Nordhaus, 2008). The one adopted throughout this work is that proposed by Weitzman (2012); thanks to its mathematical shape, it is able to capture the catastrophic damages caused by sharp increases in T_t (Botzen and van den Bergh, 2012). The mathematical structure of the damage is expressed by Equation 12

$$D_t = \frac{1}{1 + \left(\frac{T_t}{\pi_1}\right)^{\pi_2} + \left(\frac{T_t}{\pi_3}\right)^{\pi_4}} \tag{12}$$

which is governed by parameters π_1, π_2, π_3, and π_4 and can be interpreted as follows: For any level of temperature anomaly, the percentage of output which remains net of the damage is represented by D_t.

Finally, the total factor productivity $\aleph_t^A$ represents the level of technology and is treated as an exogenous shock modeled as an $AR(1)$ process

$$\ln(\aleph_t^A) = (1 - \rho^A)\ln(\bar{\aleph}^A) + \rho^A \ln(\aleph_{t-1}^A) + \varepsilon_t^A, \quad \varepsilon_t^A \sim \mathcal{N}(0, \sigma_A^2) \tag{13}$$

where $\rho^A \in (0,1)$, and ε_t^A is normally distributed with mean zero and standard deviation σ_A.

The representative firm decides its production plan by maximizing profit Π_t

$$\max_{L_t, K_t, E_t, u_t} \Pi_t = P_t Y_t - W_t L_t - Z_t(u_t K_t) - P_{e,t} E_t \tag{14}$$

As standard in the literature, the energy supply is assumed to be infinitely elastic, that is to say, all the quantity demanded is provided. Moreover, the energy price $p_{e,t}$ is treated exogenously and, in line with Dhawan and Jeske (2008), is modeled as and exogenous shock

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[4] Heutel (2012) employs a *damage function* which is shaped as a quadratic polynomial in the total stock of atmospheric CO_2, S_t. More explicitely $D_t = \alpha_0 + \alpha_1 S_t + \alpha_2 S_t^2$.

following an ARMA(1,1)

$$\ln(p_{e,t}) = (1-\rho^{pe})\ln(\bar{p_e}) + \rho^{pe}\ln(p_{e,t-1}) + \varepsilon_t^{pe} + \phi_{pe}\varepsilon_{t-1}^{pe}, \quad \varepsilon_t^{pe} \sim \mathcal{N}(0,\sigma_{pe}^2) \tag{15}$$

where $\rho^{pe} \in (0,1)$, and ε_t^{pe} is normally distributed with mean zero and standard deviation σ_{pe}. As suggested by Dhawan and Jeske (2006), the adoption of an ARMA(1,1) in place of an AR(1) process helps avoiding serially correlated error terms.

The problem supplies the following first-order conditions:

$$w_t = (1-\alpha)\frac{Y_t}{L_t} \tag{16}$$

$$z_t = \alpha\omega Y_t N_t^{-\rho}(u_t K_t)^{\rho-1} \tag{17}$$

$$p_{e,t} = \alpha(1-\omega)Y_t N_t^{-\rho} E_t^{\rho-1} \tag{18}$$

where, more specifically, Equations 16, 17, 18 represent labor demand, capital demand and energy demand, respectively.

3.1.3 Environment & Climate

The model is built on the core assumption that climate change represents the only environmental issue hitting the economy and, given the technology at hand, energy production from the combustion of fossil fuels inevitably results in a certain level of emissions. In light of these premises and in line with the literature on the economics of climate change, CO_2 represents the greenhouse gas (GHG) *par excellence* (Economides et al., 2018, p. 19) and is assumed to be the only by-product of the emission generating process, which is captured by Equation 19

$$M_t^d = E_t^\gamma \tag{19}$$

where M_t^d are domestic CO_2 emissions, and E_t denotes the energy originating from processing fossil fuels such as coal, natural gas, and oil. In particular, the relationship between energy and domestic emissions is governed by parameter γ.

Once emitted, CO_2 undergoes the carbon cycle, that is, the series of transfers, more precisely releases and uptakes from and towards reservoirs (e.g.,the atmosphere, the oceans, and the land biosphere), carbon goes through on a cyclical basis. This process is mathematically captured by Equation 20

$$\begin{aligned} S_t &= \bar{S} + \eta^t(M_0^d + M_0^{nd}) + \eta^{t-1}(M_1^d + M_1^{nd}) + ... + \eta(M_{t-1}^d + M_{t-1}^{nd}) \\ &= \bar{S} + \sum_{i=1}^{t} \eta^{(t-i+1)} M_{i-1} \end{aligned} \tag{20}$$

which tries to model this process excluding *a priori* the possibility, very far from reality (Joos et al., 1999), for the stock of atmospheric CO_2 to completely disappear. This is emphasized by the fact that domestic, M_t^d, and non-domestic, M_t^{nd}, CO_2 emissions add period after period to the *pre-industrial* stock of CO_2 accumulated in the atmosphere, $\bar{S}$, which is assumed to be inexhaustible and constant over time, even in the extreme and unrealistic event of a sudden drop to zero of global CO_2 emissions. Furthermore, a portion of both domestic and non-domestic CO_2 emissions decays each period, implying the permanence in the atmosphere of only a fraction of CO_2 emissions quantified by parameter η. The whole process results in the updating of S_t, that is to say, the total stock of atmospheric CO_2 at time t. In other words, the gap between actual, S_t, and pre-industrial levels of CO_2, $\bar{S}$, is filled by a stream of atmospheric CO_2 emissions net of their fraction lost each period due to CO_2 carbon turnover. This approach is conceptually similar to the one proposed in Golosov et al. (2014); however, in this work Equation 20 is re-written so as to assume a recursive fashion and re-arranged in terms of Equation 21

$$V_{t+1} = \eta(M_t^d + M_t^{nd} + V_t) \tag{21}$$

where $V_t = S_t - \bar{S}$ is the net stock fo CO_2 and represents the excess of atmospheric CO_2 stock over its *pre-industrial* level, that is, that part of CO_2 stock mostly contributed by human activities.

The model is equipped with a relationship describing the link between anthropogenic CO_2 atmospheric concentrations and temperature anomaly. The general way of shaping this relationship, which is based on scientific evidence (e.g., see Royer (2006), Jouzel et al. (2007), Lüthi et al. (2008), or more recently Stips et al. (2016)), mainly rests on two different *frameworks*, both of them employed in general equilibrium models contemplating climate change. On the one hand, the Nordhaus' framework (see, e.g., Nordhaus (2008, pp. 205-209)) can be summarized by a set of three equations describing the evolution of CO_2 and T_t as follows

$$F_t = \iota_0 \log_2\left(\frac{S_t}{\bar{S}}\right) + F_{ex,t} \tag{22}$$

$$T_t = T_{t-1} + \iota_1\{F_t + \iota_2 T_{t-1} + \iota_3[T_{t-1} - T_{lo,t-1}]\} \tag{23}$$

$$T_{lo,t} = T_{lo,t-1} + \iota_4\{[T_{t-1} - T_{lo,t-1}]\} \tag{24}$$

where F_t is the total radiative forcing, $F_{ex,t}$ is the exogenous radiative forcing, S_t is the atmospheric stock of CO_2, $\bar{S}$ is the *pre-industrial* stock of atmospheric CO_2, T_t and $T_{lo,t}$ are, respectively, the global mean surface temperature and the temperature of lower oceans, ι_0 is a temperature forcing parameter, while ι_1, ι_2, ι_3 are parameters specifically tailored to the temperature equations.

On the other hand, Hassler & Krusell (e.g., Golosov et al. (2014) or Hassler et al. (2016)) model the CO_2-temperature relationship focusing exclusively on the *Arrhenius Equation*, Eq. 25

$$T_t = \lambda \frac{\ln\left(\frac{S_t}{\bar{S}}\right)}{\ln 2} \tag{25}$$

where λ, the *equilibrium climate sensitivity* (ECS), is defined as the global mean surface air temperature change caused by a doubling of atmospheric CO_2 concentration after climate reached its new long-run equilibrium. Simulations suggest that deep oceans might need thousands of years in order to achieve a new equilibrium and a new steady-state temperature (Knutti et al., 2017; Wyser et al., 2020). Despite its relevance, ECS' value remains uncertain, vaguely comprehended between 1.5 °C and 4.5 °C (Collins et al., 2013) [5] and conventionally

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[5] Knutti et al. (2017) supply an overview of ECS' values reviewing the tune of 151 studies from 1896 to 2017.

set equal to 3°C. The framework of Hassler & Krusell rests on the assumption that any perturbation on the atmospheric carbon concentration leads to an instantaneous response of the long-run steady-state temperature. This approach abstracts from modeling the dynamics between S_t and T_t (Hassler et al., 2018).

According to climate science, it takes an average of 10 years for the warming attributed to a certain CO_2 emission pulse to reach its maximum, developing its full potential only after more than a century (Ricke and Caldeira, 2014). Zickfeld and Herrington (2015) corroborate this result, adding that the larger the pulse of CO_2 emission, the longer the delay it would take for the warming to reach its peak. This suggests that an emission can develop its full warming even after centuries. Time plays a crucial role in modeling the CO_2-temperature relationship. In their models, Nordhaus as well as Hassler & Krusell assume a time-step not lower than 5 years, frequently set equal to 10 years; on the contrary, *business cycles* models are usually characterized by a time-step corresponding to one *quarter* of year. Given the relative lack of knowledge concerning the dynamics between anthropogenic levels of CO_2 and temperatures in such a short time-frame, this model simply relies on what observations about anthropogenic stock of CO_2 V_t and temperature anomaly T_t reveal about the nature of their relationship, which is shaped in terms of Equation 26

$$T_t = \zeta_1 V_t^{\zeta_2} \tag{26}$$

It is crucial to overemphasize that Equation 26 captures a *long-run* relationship governed by parameters ζ_1 and ζ_2: its ambition is not to supersede natural sciences in determining the exact nature of this relationship; instead, it simply tries to approximate it in order to make a long-term problem fit a relatively tiny time-frame.

3.1.4 Definition: Equilibrium

A *Markov perfect equilibrium* is a sequence of prices (w_t, z_t, q_t), a sequence of allocations $(C_t, I_t, Y_t, L_t, K_t, E_t, u_t, N_t)$, a sequence of climatic variables $(D_t, M_t^d, M_t^{nd}, V_t, T_t)$, and a sequence of shocks $(A_t, P_{e,t})$ such that

- $\forall$ sequence of prices, climatic variables and shocks, the allocation solves the agent's problem;
- $\forall$ sequence of allocations, climatic variables, and shocks, the prices clear the market;
- $\forall$ sequence of prices, allocations and shocks, the climatic variables are determined by the environmental/climatic block;
- $\forall$ sequence of prices, allocations and climatic variables, the shocks exogenously perturb the market.

3.2 Calibration

In line with Real Business Cycle (RBC) literature, the model is calibrated[6] on a *quarterly* basis. Starting from the household's block, the discount factor β is set equal to 0.99 so as to match a real interest rate equal to 4% on annual basis. As standard in the literature, the capital share α is calibrated to be equal to 33% of output. In line with the meta-analysis of Havranek et al. (2015), the elasticity of intertemporal substitution (EIS), σ_c, for US economy is set equal to 0.594. The Frisch elasticity of labor for US economy, σ_{la}, is borrowed from Peterman (2016) and set equal to 3. Exploiting this latter and the ordinary assumption that $L = {}^1\!/_3$ in steady-state, it is possible to recover the Frisch elasticity of leisure σ_{le}, which is set equal to ${}^3\!/_2$. Parameter ϕ, which governs the investment adjustment costs, is calibrated such that the model is able to pin down the relative standard deviation between investment and output in the data, which in this case is equal to 3.88 (see Table 1). According to this approach, $\phi = 3.6026$. It is relevant to emphasize that the use of energy as input, altering the resource constrain of the economy, prevents to match the relative standard deviation between consumption and output. Finally, subsequent to the computation of the *steady-states* of the model, it is possible to determine the values of the remaining two parameters, namely, the the elasticity governing the rate of capital utilization $\nu = 1.8006$ and the scaling parameter $\chi = 2.2785$ denoting the utility weight on leisure.

Shifting to the representative firm's block, other parameters such as the depreciation rate δ_0 and the CES-weight ω are calibrated borrowing the value of some other parameters from the existing literature and fixing specific ratios as target the model has to comply with. More specifically, mirroring the approach of Dhawan and Jeske (2008), this calibration borrows $\rho = -0.7$ from Kim and Loungani (1992) and targets the capital-output ratio ${}^K\!/_Y = 12$, as it is rather usual in the literature. In addition to this, it makes use of the firms' energy-output ratio ${}^E\!/_Y$, whose value is calculated as the difference between the total energy-output ratio,

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[6] Further details on the calibration of parameters and on the calculation of the steady-states allowing it are included in Appendix B

which accounts for both household and firms, and its households' counterpart[7]. This gives ${}^{E}\!/_{Y} = 0.0607$. Finally, once ${}^{K}\!/_{Y}$ and ${}^{E}\!/_{Y}$ are known, it is possible to obtain ${}^{K}\!/_{E} = 197.635$. Based on these choices, it is possible to calculate the value of the depreciation rate $\delta_0 = 0.0126$ and that of the CES weight $\omega = 0.9945$.

On the climatic side, the parameters of the damage function are borrowed directly from Weitzman (2012). He modifies the Nordhaus's (e.g., (Nordhaus, 2008)) specification, identified by the following set of parameters $\pi_1 = 20.46$, $\pi_2 = 2$, $\pi_3 = 0$, $\pi_4 = 0$, so as to capture the catastrophic consequences of very high temperatures. More specifically, this is achieved by keeping π_1 and π_2 as in the Nordhaus' specification and calibrating the remaining two parameters so as to capture the following relationship: An increase in temperatures of 6°C or 12°C exerts, respectively, a damage which keeps only 50 % or 1% of the potential output, that is, implying an output reduction of 50% or 99%. This procedure supplies the following set of parameters: $\pi_1 = 20.46$, $\pi_2 = 2$, $\pi_3 = 6.081$, and $\pi_4 = 6.754$. The value of η, which measures the airborne fraction of CO_2 remaining in the atmosphere each period net of the uptake operated by carbon sinks, is set equal to 0.9985. Standard literature in physics employs *impulse-response functions* (IRF_{CO_2}), in the form of sums of exponentials, to model how much of a pulse of CO_2 remains in the atmosphere at each period after a certain time from its emission. This representation accounts for the non-linearities intrinsic to the carbon cycle, especially in terms of ocean and land uptakes (Joos et al., 1996). For a generic compound X, IRF_x usually has the following shape

$$IRF_x(t) = a_0 + \sum_{i=1}^{k} a_i \cdot e^{-\frac{t}{\tau_i}} \qquad for\ t_0 \leq t \leq T$$

where k is fixed and depends on the specific model used, while the sum of coefficients a_i must be equal to 1. The relation is valid within a certain time-domain t, e.g., in this case

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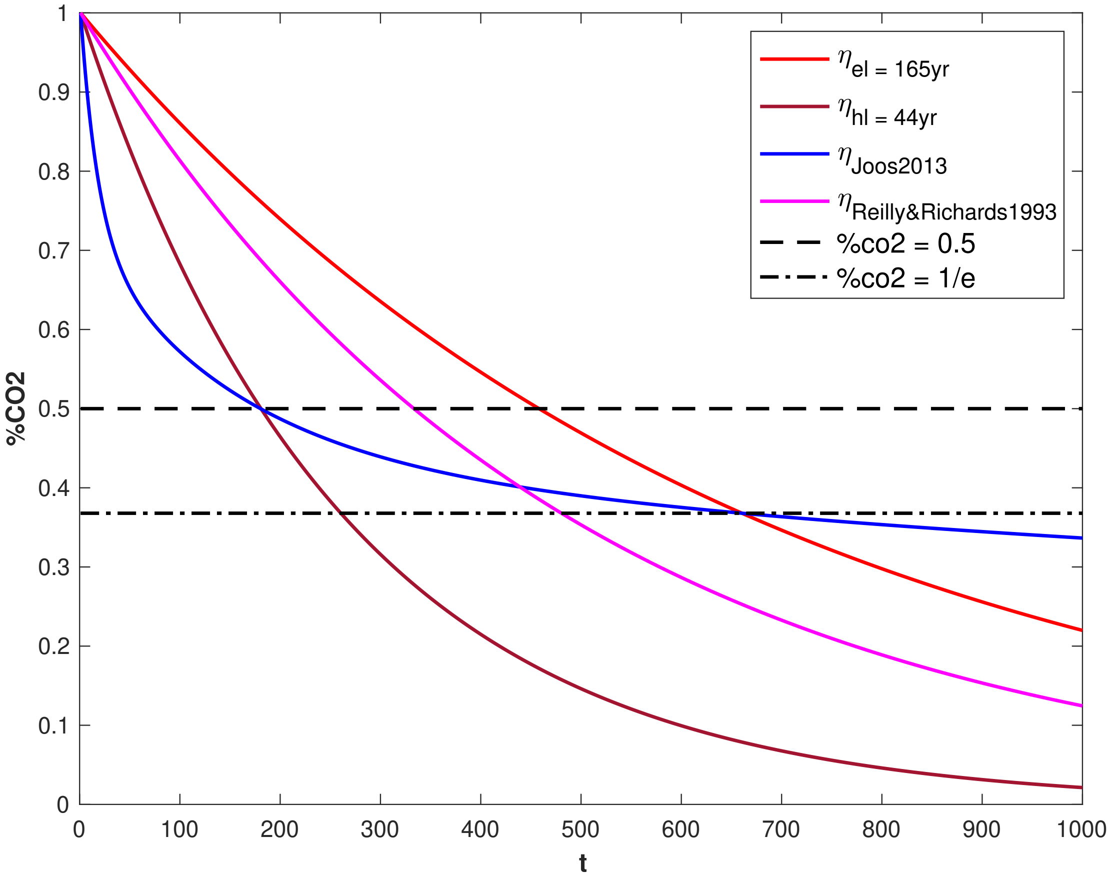
[7] While data on total energy-output ratio are available from EIA on annual basis, the households' energy-output is calculated as the ratio between nominal personal consumption expenditures for gasoline and other energy goods and nominal GDP, both of them supplied by BEA. Additional details on annual series are contained in Appendix A.

comprised between t_0 and T years. The IRF_x can be interpreted as follows: of a certain pulse, a fraction amounting to a_0 will remain indefinitely, while the remaining undergoes a decay process over time. In particular, a fraction a_1 of the pulse will be characterized by an *e-folding* time, i.e., the time required for a certain substance to shrink to $^1/_e$ of its initial concentration, equal to τ_1 years, a fraction a_2 of the pulse will have an *e-folding* time equal to τ_2 years, and so on and so forth up to the fraction a_k of the pulse, whose concentration becomes $\frac{1}{e}$ after τ_k years from its emission. Among different parametrizations (see e.g., Shine et al. (2005); Hansen et al. (2007)), the one which will serve as a useful reference for this calibration is exposed in Joos et al. (2013)

$$IRF_{\mathrm{CO_2}}(t) = 0.2173 + 0.2240e^{-\frac{t}{394.4}} + 0.2824e^{-\frac{t}{36.54}} + 0.2763e^{-\frac{t}{4.304}}, \quad t \in (0, 1000) \tag{27}$$

in that it is also able to capture the concentration dynamics relative to fractions of year such as quarters.

Figure 9: Permanence Rate



The information encapsulated within Equation 27 is relatively complex and, for the sake of simplicity, can be better summarized by employing two stylized features: Equation 27 simply tells that a certain pulse of CO_2 is characterized by a *half-life time* (i.e., the time required for a certain substance to shrink to $^1/_2$ of its initial concentration) of 44 years (i.e., 220 quarters) and an *e-folding time* of 165 years (660 quarters). The impossibility to make such a complex information fit a recursive shape suggests to sacrifice one information for the other one. This results in the arbitrary decision to employ the value of the *e-folding time* as target to calibrate parameter η. In line with this specific choice, this calibration strategy supplies the following result

$$\eta = 1 - \frac{1}{(165 \cdot 4)} = 0.9985 \tag{28}$$

Figure 9 shows how the atmospheric fraction of CO_2 evolves over time with respect to four different calibration strategies for parameter η. In particular, the parametrization employed throughout this work, which is characterized by a red line and is rooted in an e-folding time of 165 years, is compared with other three potential specifications: the discarded option rooted on a half-time of 44 years, which is represented by a bourdeaux line; the framework of Joos et al. (2013), which serves as basis for the above-mentioned frameworks (even if it can not be converted to a recursive form) and is shown by a blue line; the specification adopted by Reilly and Richards (1993), which considers $\eta = 0.9979$ and implicitly assumes a *half-life time* of 83 years and an *e-folding time* of 120 years, is represented with a pink line. As it is possible to evince from Figure 9, the Joos et al. (2013) specifications intersects with the bordeaux line when the percentage of CO_2 is half of its initial concentration and with the red line when it reaches $^1/_e$ of its initial concentration. The Reilly and Richards (1993) framework finds its place between the bourdeaux and the red lines.

Other parameters are estimated by exploiting the available time series. This is the case of γ, whose value is obtained by estimating Equation 19, that is, the relationship between domestic CO_2 emissions and fossil energy use, namely, the aggregate of coal, oil, and gas. Its value is set equal to 0.9726. With respect to Equation 26, the values of parameter ζ_1 and ζ_2 are obtained estimating the dynamic relationship between the temperature anomaly and the stock of CO_2 above the *pre-industrial* level. The former is set equal to $1.1268 \cdot 10^{-4}$, while that of parameter ζ_2 is 1.6558.

The calibration of the shock parameters, namely, their persistence and standard deviation, are carried out as follows. For the TFP shock, the parameters of interest are obtained minimizing the distance between the TFP series simulated by the model and the quarterly utilization-adjusted series for total factor productivity from Fernald (2014). According to this procedure, $\rho^A = 0.9832$ and $\sigma^A = 0.0063$. In order to obtain the parameters characterizing the process for the energy price, The estimation via *maximum-likelihood* of Equation 15, that is, the ARMA(1,1) process characterizing the energy price, supplies $\rho^{pe} = 0.9185$,

$\phi^{pe} = 0.3473$, and $\sigma^{pe} = 0.0672$.

This calibration supply a ratio between *non-domestic* and *domestic* carbon emissions equal to 4.3683. This is a little bit larger than the value employed by Heutel (2012), who fixes *non-domestic* carbon emissions as three times the steady-state value of *domestic* carbon emissions.

Finally, all relevant ratios and steady-state values are collected in Table 2, while all the results of this calibration are described in Table 3.

Table 2: Steady States of the Model

Steady-State	Value	Description
C/Y	0.7879	Consumption
I/Y	0.1514	Capital-output ratio
K/Y	12	Capital-output ratio
E/Y	0.0607	Energy-output ratio
K/E	197.6350	Capital-energy output
L	$1/3$	Labor
D	0.9979	Damage
M^d/M^{nd}	4.3683	CO_2 domestic emissions
V	232.17	net CO_2 stock
T	0.9314	Temperature increase over pre-industral level

Table 3: Calibration of the Parameters of the Model

Parameter	Value	Description
		Households
α	1/3	Capital share
β	0.99	Discount factor
χ	2.2785	Utility of leisure
ν	1.8006	Curvature parameter of capital utilization
ϕ	3.6026	Sensitivity of investments
σ_c	0.5940	Elasticity of intertemporal substitution (EIS)
σ_{le}	3/2	Frisch elasticity of leisure
		Firms
δ_0	0.0126	Depreciaton rate of capital
ω	0.9945	Share parameter of CES
ρ	-0.7	Substitution parameter of CES
		Environment
η	0.9985	CO_2 airborne rate
γ	0.9726	CO_2 domestic emissions vs fossil energy - elasticity
π_1	20.46	1st Damage parameter
π_2	2	2nd Damage parameter
π_3	6.0810	3rd Damage parameter
π_4	6.7540	4th Damage parameter
ζ_1	$1.1268 \cdot 10^{-4}$	temperature anomaly vs anthropogenic CO_2 stock - intercept
ζ_2	1.6558	temperature anomaly vs anthropogenic CO_2 stock - elasticity
ρ^A	0.9832	Persistence of TFP shock
σ^A	0.0063	Standard deviation of TFP shock
ϕ^{pe}	0.3473	MA component of energy-price shock
ρ^{pe}	0.9185	Persistence of energy-price shock
σ_{pe}	0.0672	Standard deviation of energy-price shock

3.3 Validation of the Model

The model, specified in terms of equations and parameters, is used as data generating process to obtain artificial series, whose core features are expected to match as much as possible those of the data. In light of this, the model undergoes a process of *validation* consisting in the comparison between the stylized facts obtained from the simulated series and their data-driven counterpart, as already exposed in Section 2.2.
With respect to this model, the validation is carried out exploiting the results of Table 4, which presents the *stylized facts* obtained from both model and data, these latter enclosed in brackets. Independent of their nature, the cyclical component of all series is obtained employing the Hodrick-Prescott filter.

The model performs relatively well in mirroring the empirical behavior of the main macroeconomic aggregates: Simulated consumption is less volatile than simulated output, which, in turn, is less volatile than simulated investment. The model is relatively successful in replicating contemporaneous cross-correlations and first-order autocorrelations of the above-mentioned variables. The same considerations do not hold with respect to labor and real wages. In particular, the anticyclicality of labor with respect to output is due to the manipulation of parameter ϕ that controls investment-adjustment costs: If, on the one hand, it allows the model to pin down the relative standard deviation of investment, on the other hand, it reduces the cross-correlation between labor and output.

The ability of the model in replicating the cyclical behavior of environmental variables is, however, not as satisfactory. This could be due to the nature of climatic series, which are affected by a relatively non-linear behavior. It does a good job in reproducing the cyclical behavior of the energy price; this is evident looking at variances and autocorrelations. The model, however, has a tendency to overestimate absolute and relative variances of fossil fuel energy use and CO_2 emissions, while it strongly underestimates those of CO_2 stock and temperature anomaly. On the contrary, it does a relatively satisfying job in mirroring contemporaneous cross-correlations and autocorrelations.

Table 4: Stylized Facts from the Simulated Series - HP Filtered Moments

Variable	$\sigma(\cdot)$	$\sigma(\cdot)/\sigma(Y)$	$\rho(\cdot, Y)$	$\rho(\cdot_t, \cdot_{t-1})$
Economic Variables				
Output (Y)	0.92	1.00	1.00	0.83
	(1.43)	(1.00)	(1.00)	(0.88)
Consumption (C)	0.78	0.84	0.85	0.71
	(0.85)	(0.60)	(0.82)	(0.87)
Investment (I)	3.57	3.88	0.85	0.93
	(5.55)	(3.88)	(0.93)	(0.88)
Labor (L)	0.54	0.59	-0.27	0.72
	(1.88)	(1.31)	(0.87)	(0.93)
Wage (W)	1.18	1.29	0.90	0.73
	(0.92)	(0.65)	(0.10)	(0.70)
Environmental Variables				
Fossil Fuels Energy Consumption (E)	6.50	7.06	0.56	0.75
	(2.28)	(1.60)	(0.63)	(0.58)
Energy Price (P_e)	9.54	10.36	-0.48	0.74
	(10.17)	(7.11)	(0.09)	(0.77)
CO$_2$ emissions (M^d)	6.32	6.87	0.56	0.75
	(2.32)	(1.62)	(0.62)	(0.59)
CO$_2$ stock (V)	$5.37 \cdot 10^{-3}$	$5.83 \cdot 10^{-3}$	0.31	0.93
	(0.46)	(0.32)	(0.28)	(0.75)
Temperature Anomaly (T)	$8.08 \cdot 10^{-3}$	$8.77 \cdot 10^{-3}$	0.12	0.92
	(36.41)	(25.47)	(0.01)	(0.21)

* Values in brackets refer to empirical series. The others are simulated from the model. In both cases, the series have been detrended with the Hodrick-Prescott filter.

3.4 Mechanism of the Model

The dynamic features of the model are analyzed considering the impact of the shocks, which in this specific case are the *TFP shock* and the *energy price shock*, on the macroeconomic-environmental system as a whole. The results of this procedure are expressed in terms of impulse-response functions (IRF), which, as the name suggests, represent the way an artificial economy identified by its set of variables reacts to a perturbation originating from one of its error terms (McCandless, 2008, p. 120).

The impulse-responses to one standard deviation shock are reported for each exogenous shock itself and for each endogenous variable included in the model. The effects are evaluated over 40 periods from the start of the perturbation. The underlying AR(1) component of each shock considered is characterized by a certain level of persistence, which makes its effect to spread over several periods starting from the point in time it first hits the economy. Figures 10, 11, and 12 refer to the TFP shock, while Figures 13, 14, and 15 and to the energy price shock, respectively. Each plot shows the period on the x-axis and the percentage deviation from the steady-states on the y-axis both for the actual model and for a classic version, which does not contemplate investment adjustment costs.

Figure 10: TFP Shock - Factor Prices

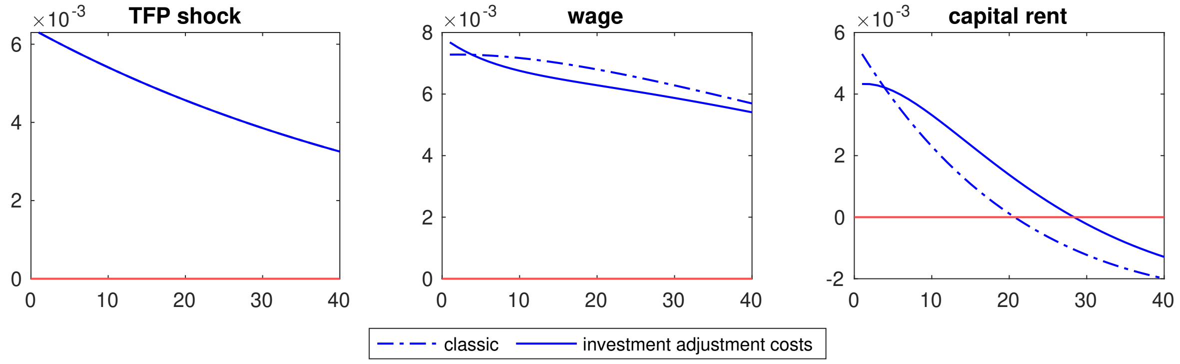
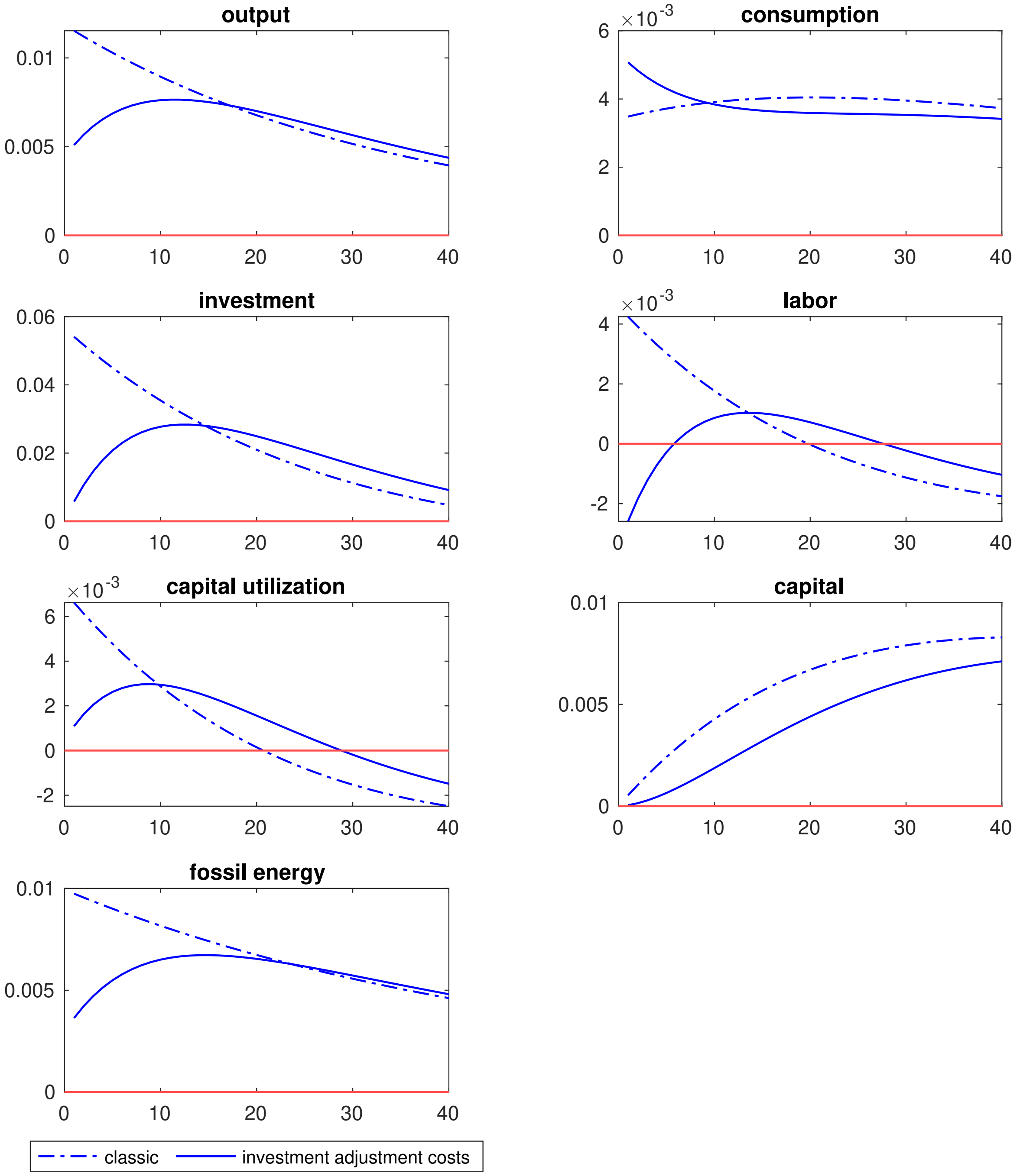


Figure 11: TFP Shock - Economic Variables

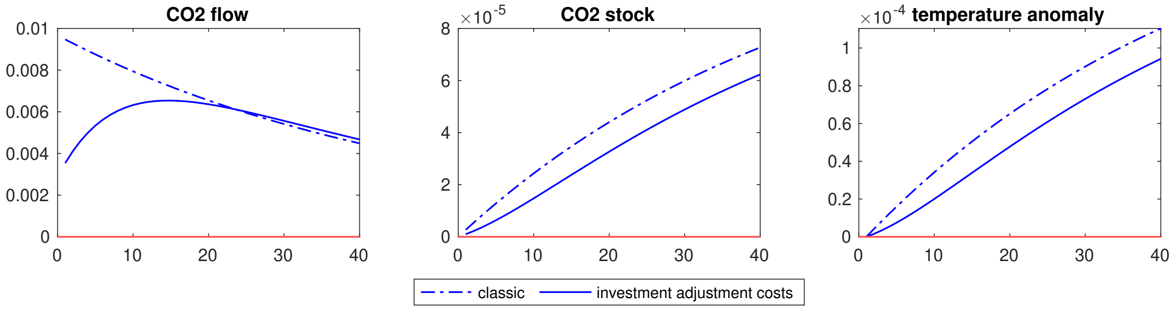


From Figures 10 and 11, it emerges that one standard deviation increase in total factor productivity leads to an immediate increase in the marginal productivity of labor and capital.

This pushes firms' demand for the factors of production as witnessed by the increase in real wage and in the rental rate of capital, i.e., the prices of the factors of production. This exerts a direct positive income effect on households decisions. The increase in the marginal return to capital, and the initial income effect lead households to increase their investment, hence fostering capital accumulation. In this way, a households experience a higher level of wealth; however, this is somewhat limited when investment adjustment costs are in place. The fact that investment does not move as much influences aggregate demand, $C + I$, that remains smooth and does not react much to the TFP shock. As a consequence of the foregoing, output does not move too much either. Households, then, consume more and invest more, at the same time working less. This fact reconciles the higher productivity induced by the TFP shock with the smoothness of the aggregate demand.The effect of the productivity shock does not last forever, thereby gradually making both variables approach their steady-state level.

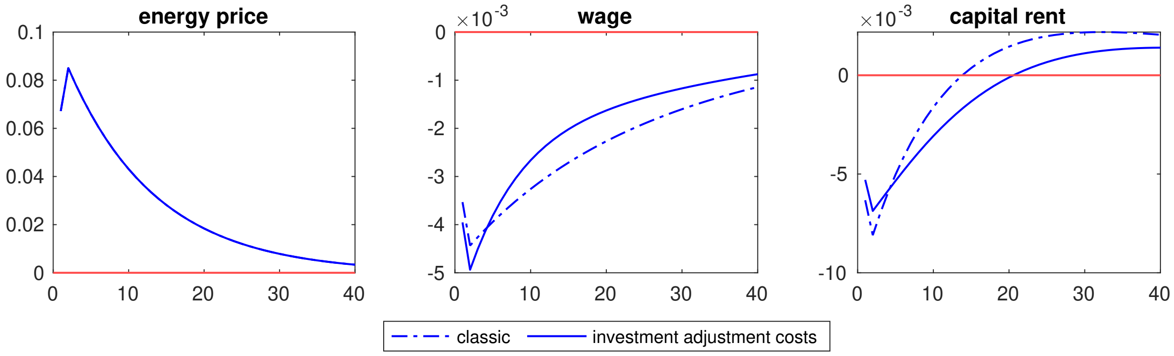
Given the complementarity between capital and energy use, the push in the production of output is necessarily accompanied by a significant increase in the use of fossil energy, as shown by Figure 11. This, in turn, affects all climatic variables, and is captured by Figure 12. The first effect of the increase in fossil energy use is a very important instantaneous increase in domestic emissions of CO_2. The rise in domestic emissions leads to the growth of the anthropogenic stock of atmospheric CO_2 and, consequently, of temperature anomaly. This phenomenon, however, is not very significant as shown by the orders of magnitude of these two variables.

Figure 12: TFP Shock - Climatic Variables



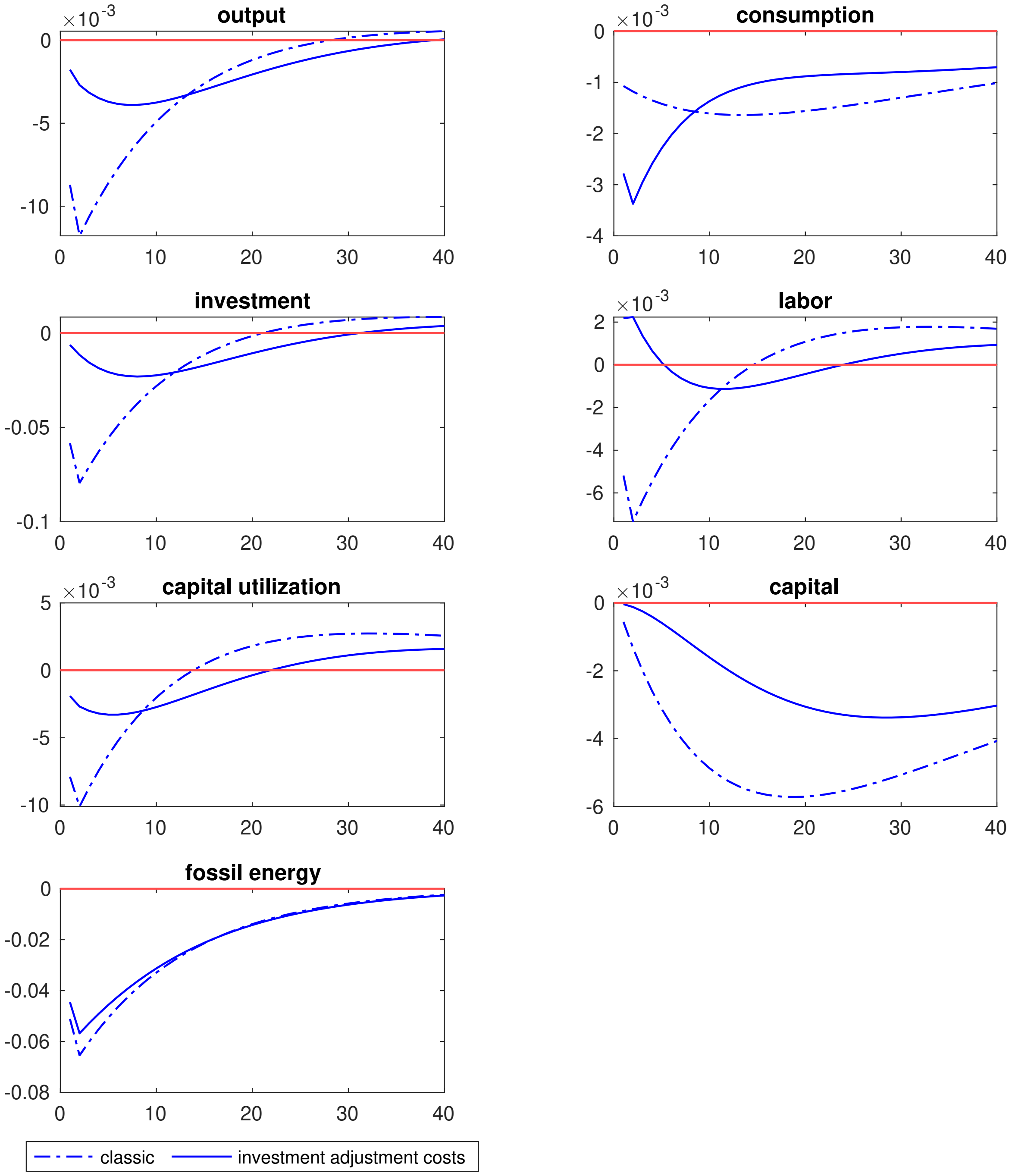
From a pure qualitative point of view, a one standard deviation increase in the price of fossil energy affects the variables of the model in a way opposite to the TFP shock. As captured by Figure 14, such a shock first of all lowers the energy demand. This leads to a reduction in the marginal productivity of capital and labor. Firms' demand for the factors of production decreases, as it is possible to understand by noticing the instantaneous decrease in real wage and in the rental rate of capital, as described by Figure 13.

Figure 13: Energy Price Shock - Factor Prices



This necessarily translates into a negative income effect on households decisions. The decrease in the marginal return to capital, and the negative initial income effect lead households to decrease their investment, thereby abandoning capital accumulation. As a consequence, the process results in the shrinking of household's wealth.

Figure 14: Energy Price Shock - Economic Variables

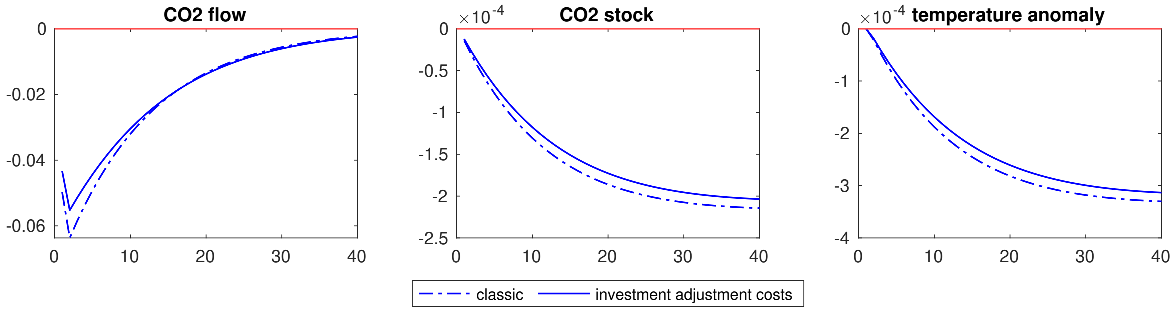


As in the case of TFP shock, investment does not move as much due to the presence of investment adjustment costs. This implies an aggregate demand, $C + I$, that remains smooth and does not react much to the energy price shock. As a consequence of the foregoing, output decreases but not moving too much either.Households, then, reduce consumption, invest less and, at the same time, decide to work more even in a situation of falling wages. This fact reconciles the lower productivity induced by the energy-price shock with the smoothness of the aggregate demand.The effect of the shock on fossil energy is less persistent than the one triggered by TFP shock, this fact resulting in many variables to approach their steady-states not long after forty periods.

The sharp decrease in the use of fossil energy due to the positive energy price shock has an important impact on climatic variables, as described by Figure 15. More specifically, this leads to a sharp and significant instantaneous decrease in the level of domestic CO_2 emissions. The decrease in domestic emissions leads to a gradual fall of anthropogenic CO_2 stock and, consequently, of temperature anomaly. Also in this case, the orders of magnitude of these two variables suggests that the phenomenon is not very significant.

It is worth to notice that, in the case of TFP shock as well as for the energy price shock, the actual model is characterized by impulse responses whose initial increase is smoother than those supplied by the classical one, which does not feature investment adjustment costs.

Figure 15: Energy Price Shock - Climatic Variables



4 Policy Intervention: Energy Taxation

4.1 The Model Economy with an Energy Tax

Hinging on the basic framework introduced in Section 3.1, the model is extended so as to be employed as a policy tool, with the goal of reducing domestic CO_2 emissions. This is achieved by introducing a tax on the use of fossil fuel energy, this latter being shaped in terms of five different policy rules: the tax can either be constant, expressed as function of domestic CO_2 emissions, of the anthropogenic stock of CO_2 in the atmosphere, of fossil energy itself, or be linked to a combination of them. Throughout this section, only the changes with respect to the basic model are reported.

4.1.1 Households

The infinitely lived representative household solves the same maximization problem introduced in Section 3.1.1; however, this time, the representative household faces a slightly different budget constraint having the following shape

$$C_t + I_t \leq w_t L_t + z_t(u_t K_t) + F_t + \Pi_t \tag{29}$$

where F_t denotes a direct *lump-sum* transfer from the government to the household.

4.1.2 Firms

The representative firm maximizes profit Π_t, which in this case is expressed as follows

$$\max_{L_t, K_t, E_t} \Pi_t = Y_t - w_t L_t - z_t(u_t K_t) - (1 + \tau_t) p_{e,t} E_t \tag{30}$$

In other words, not only the representative firm pays for the use of fossil energy as a factor of production but also corresponds a certain amount to the government in the form of a *tax on energy*, which is represented by variable τ_t.

Mirroring Section 3.1.2, the problem supplies Equation 16 for labor demand, Equation 17 for capital demand, and

$$(1+\tau_t)p_{e,t} = \alpha(1-\omega)Y_t N_t^{-\rho} E_t^{\rho-1} \tag{31}$$

which represents energy demand. Strictly speaking, Equation 31 shows that the effect of the tax on energy exerts itself on energy demand alone.

4.1.3 Government

The government runs a balanced budget

$$F_t = \tau_t p_{e,t} E_t \tag{32}$$

that is to say, the government collects from the firms a tax on the use of fossil fuel energy which is re-distributed to the households by means of a direct *lump-sum transfer* F_t.

4.1.4 The Energy Tax

In its most general version, the energy tax is expressed in terms of the fiscal rule described by Equation 33

$$\ln\left(\frac{1+\tau_t}{1+\bar{\tau}}\right) = \psi_1 \ln\left(\frac{M_t^d}{\bar{M}^d}\right) + \psi_2 \ln\left(\frac{E_t}{\bar{E}}\right) + \psi_3 \ln\left(\frac{V_t}{\bar{V}}\right) \tag{33}$$

that is, the percentage deviation of the gross tax from its steady state is function of the percentage deviations from the steady states of output, domestic emissions, fossil energy, and anthropogenic stock of CO_2. This way of modeling the energy tax conceals five distinct options:

1. If $\psi_1 = 0$, $\psi_2 = 0$, $\psi_3 = 0$, Equation 33 implies a constant tax, whose value is coincident with its steady state

$$\tau_t = \bar{\tau} \tag{34}$$

2. If $\psi_1 \neq 0$, $\psi_2 = 0$, $\psi_3 = 0$, Equation 33 is function of the percentage deviation of domestic CO_2 emissions from their steady state

$$\ln\left(\frac{1+\tau_t}{1+\bar{\tau}}\right) = \psi_1 \ln\left(\frac{M_t^d}{\bar{M}^d}\right) \tag{35}$$

3. If $\psi_1 = 0$, $\psi_2 \neq 0$, $\psi_3 = 0$, Equation 33 is function of the percentage deviation of fossil fuel energy from its steady state

$$\ln\left(\frac{1+\tau_t}{1+\bar{\tau}}\right) = \psi_2 \ln\left(\frac{E_t}{\bar{E}}\right) \tag{36}$$

4. If $\psi_1 = 0$, $\psi_2 = 0$, $\psi_3 \neq 0$, Equation 33 is function of the percentage deviation of the anthropogenic stock of CO_2 from its steady state

$$\ln\left(\frac{1+\tau_t}{1+\bar{\tau}}\right) = \psi_3 \ln\left(\frac{V_t}{\bar{V}}\right) \tag{37}$$

5. When $\psi_1 \neq 0$, $\psi_2 \neq 0$, $\psi_3 \neq 0$, the reference is directly to Equation 33 itself.

4.2 Calibration

The steady state value of the tax on energy is fixed equal to 15%. This is an hypothetical ethical tax, whose value is determined as the share of CO_2 emissions from US over the level of global CO_2 emissions in year 2019 [8]. In other words, the tax is engineered so as to reflect the carbon impact of US proportional to the whole amount of emissions all over the world.

The choice of a specific combination of ψ_i, where $i = 1, ..., 3$, allows to determine the nature of the policy rule under scrutiny. The calibration of parameters ψ_i is carried out having as target an arbitrary one half reduction in the variance of *domestic emissions*, M^d. More specifically, once the original variance of $\ln(M_t^d)$ is obtained, the whole procedure reduces to finding ψ_i, $i = 1, ..., 4$, that *minimize* the following *loss function*:

$$\min_{\psi_i} \quad \left(\sigma_s^2(\psi_i) - \frac{1}{2}\sigma_d^2\right)^2 \tag{38}$$

where σ_d is the *standard deviation* of $\ln(M_t^d)$ obtained from actual data, and σ_s is the *standard deviation* of $\ln(M_t^{d,s})$, that is, its counterpart obtained by simulating the model with respect to a certain choice of ψ_i.

In particular, given $\sigma_d^2 = 0.0045$, this calibration supplies the following results[9]:

1. $\psi_1 = 4.0940$, to which corresponds $\sigma^2_{s,min} = 0.0023$;
2. $\psi_2 = 3.9818$, to which corresponds $\sigma^2_{s,min} = 0.0023$;
3. $\psi_3 = 3002.5340$, to which corresponds $\sigma^2_{s,min} = 0.0023$;
4. $\psi_1 = 1.9901$, $\psi_2 = 2.0462$, $\psi_3 = 0.0007$ to which corresponds $\sigma^2_{s,min} = 0.0023$.

---

[8] Given that, in 2019, US and the world as a whole emitted, respectively, 4.8 Gt a 33 Gt of CO_2, the US emission share precisely amounts to about 14.55%.IEA (2020), Global CO_2 emissions in 2019, IEA, Paris https://www.iea.org/articles/global-co2-emissions-in-2019

[9] The exclusion of output from the policy exercise is mainly due to the fact that, because of the structure of the model, this calibration procedure is unable to appropriately reduce the variance of CO_2 emissions. This does not happen when a policy rule, which excludes the contribute of output is adopted.

4.3 Environmental Tax at Work

The adoption of the energy tax on fossil fuels is analyzed by means of Figures 16, 17, 18 when the system is perturbed by the TFP shock and Figure 19, 20, 21 when the system is hit by the energy price shock. For each shock, the impulse response functions of five different policies are examined; as already introduced in Section 4.1, these are: The constant tax, represented by a solid blue line; the tax linked to domestic CO_2 emissions, represented by pink horizontal bars; the tax linked to fossil energy use, represented by black vertical bars; the tax linked to the anthropogenic stock of atmospheric CO_2, represented by the yellow solid line; the global tax, which accounts for the effects of all the just mentioned variables contemporaneously, represented by the green dash-dotted line. The impulse response functions from each of the five policies are compared with those pertaining to the original model. This latter, represented by red asterisks, does not contemplate any policy, thereby representing the benchmark model for comparison. Table 5 includes absolute standard deviations and cross-correlations of the various variable of interest *posterior* to each tax considered.

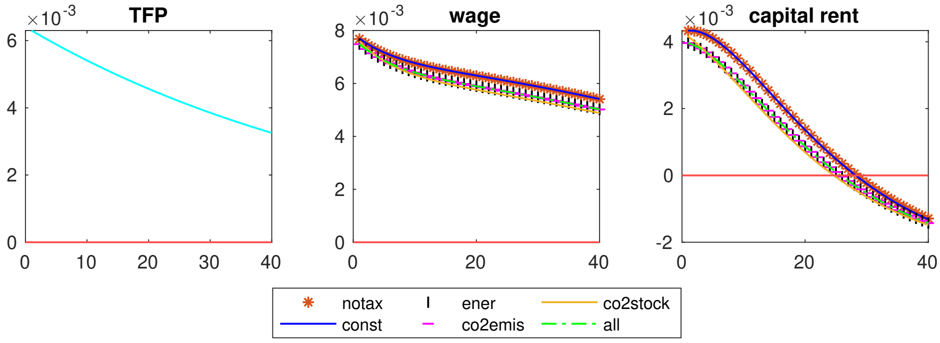
Generally speaking, the reactions of economic and climate variables to any shock, no matters its nature, can be generalized into three different behaviors, whose main features are summarized as follows: The constant tax, which does not actually react to the business cycle and whose effect is practically indistinguishable from the no-tax case, in that it practically overlaps with it; the group which includes the tax linked to the anthropogenic stock of atmospheric CO_2 alone; the group that includes the tax linked to fossil energy, that linked on domestic CO_2 emissions, and that linked to fossil energy, CO_2 flows and CO_2 stock contemporaneously. These latter two cases are characterized by a certain level of responsiveness to the business cycle.

Table 5: Standard Deviations and Cross-Correlations for Different Tax Policies

Variable	$\tau = 0$		$\tau = 0.15$		$\tau = f(V)$		$\tau = f(M^d), \tau = f(E)$, $\tau = f(M^d, E, V)$	
	$\sigma(\cdot)$	$\rho(\cdot, Y)$	$\sigma(\cdot)$	$\rho(\cdot, Y)$	$\sigma(\cdot)$	$\rho(\cdot, Y)$	$\sigma(\cdot)$	$\rho(\cdot, Y)$
Output (Y)	0.92	1.00	0.94	1.00	0.74	1.00	0.72	1.00
Consumption (C)	0.78	0.85	0.79	0.86	0.72	0.84	0.67	0.75
Investment (I)	3.57	0.85	3.64	0.84	2.49	0.66	2.64	0.71
Labor (L)	0.54	-0.27	0.53	-0.27	0.53	-0.39	0.57	-0.17
Wage(W)	1.18	0.90	1.20	0.90	1.07	0.89	0.99	0.83
Fossil Energy (E)	6.50	0.56	6.56	0.59	3.85	0.28	1.71	0.02
CO_2 emissions (M^d)	6.32	0.56	6.38	0.59	3.75	0.28	1.66	0.02
CO_2 stock (V)	$5.37{\cdot}10^{-3}$	0.31	$4.89{\cdot}10^{-3}$	0.31	$2.09{\cdot}10^{-3}$	$2{\cdot}10^{-4}$	$1.26{\cdot}10^{-3}$	0.08
Temperature Anomaly (T)	$8.08{\cdot}10^{-3}$	0.12	$7.35{\cdot}10^{-3}$	0.12	$3.23{\cdot}10^{-3}$	-0.13	$1.90{\cdot}10^{-3}$	0.08

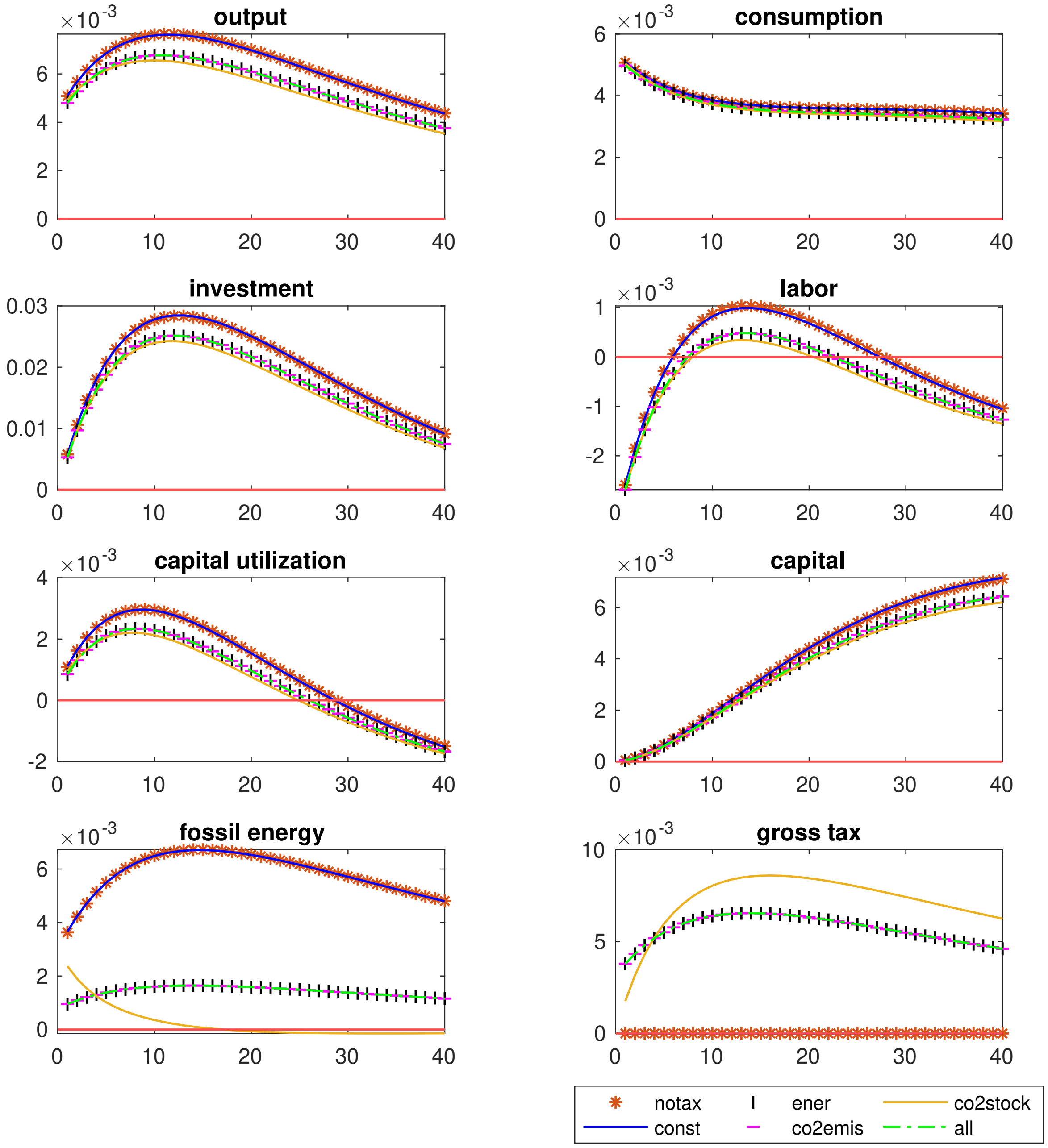
This last point is captured by Table 5, which collects all tax experiments described in this work and regroups them by similarities. The key takeaway is that the more responsive the tax is to the business cycle, the more the correlation between capital and energy drops. This is can be observed concentrating on the correlation between fossil energy (or domestic CO_2 emissions, since they are proportial in logarithmic terms and γ is almost equal to 1) and output. In the case of a very responsive tax, as those linked to emissions, energy, or a mix of them, the drop in the correlation is the largest. On the contrary, when the tax responds to the anthropogenic stock of atmospheric CO_2, which evolves very slowly, then the tax is less responsive and the decrease in correlation is lower. At this point, it is possible to analyze the mechanism of the model bearing in mind that the main effect of the energy tax is to endogenously alter the energy price.

Figure 16: TFP Shock & Fossil Energy Tax - Factor Prices



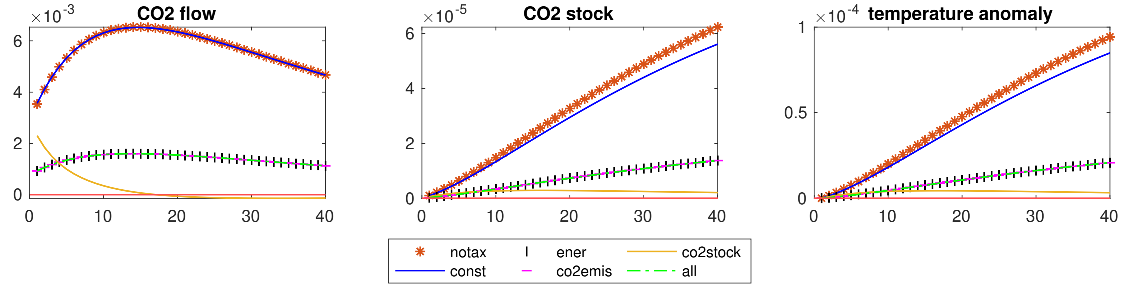
Starting from total factor productivity, Figures 16 and 17 show that the effect of one standard deviation positive shock hitting economic variables produces IRFs qualitatively mirroring those originating from the benchmark model without tax and already discussed in Section 3.4. The adoption of any other tax on fossil energy, let it be linked to CO_2, either flow or stock, to fossil energy, or to a mix of them, tends to dampen the impact the shock exerts on almost all economic variables. As a matter of fact, the adoption of the tax does not change the mechanisms of transmission of the model, however curbing the reaction of economic variables.

Figure 17: TFP Shock & Fossil Energy Tax - Economic Variables



As shown by the box on the bottom right of Figure 17, the energy tax has a strong impact on its natural target, that is, fossil energy: In fact, exception made for the constant tax case, all versions of the energy tax have a very strong influence in limiting the increase in fossil energy use.

Figure 18: TFP Shock & Fossil Energy Tax - Climatic Variables



As a consequence of the foregoing, the energy tax affects also the behavior of climatic variables. This can be easily understood by looking at Figure 18: With the exception of the constant tax, all other taxing policies sharply dampen the instantaneous increase in CO_2 emissions with respect to the benchmark case. Very interestingly, the tax linked to the anthropogenic stock of atmospheric CO_2 looks very successful in mitigating global warming: the instantaneous increase in CO_2 emissions goes back to its steady-state after around 15 periods, whereas the stock of CO_2 and temperature anomaly are characterized by a low-profile and have a very smooth shape. This result is, however, characterized by a low order of magnitude.

Figure 19: Energy Price Shock & Fossil Energy Tax - Factor Prices

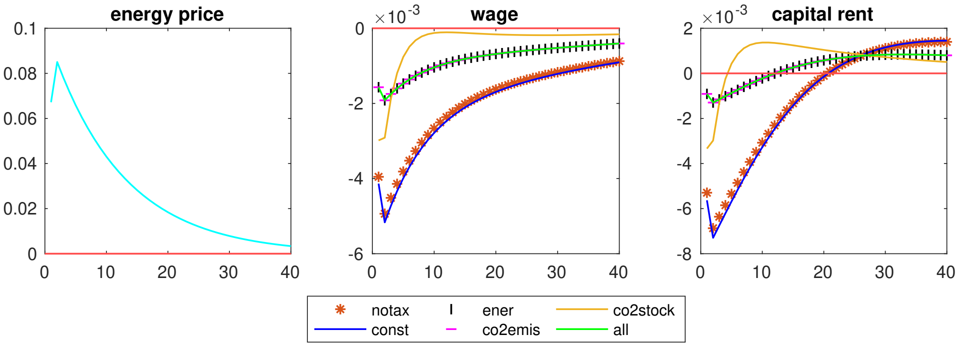
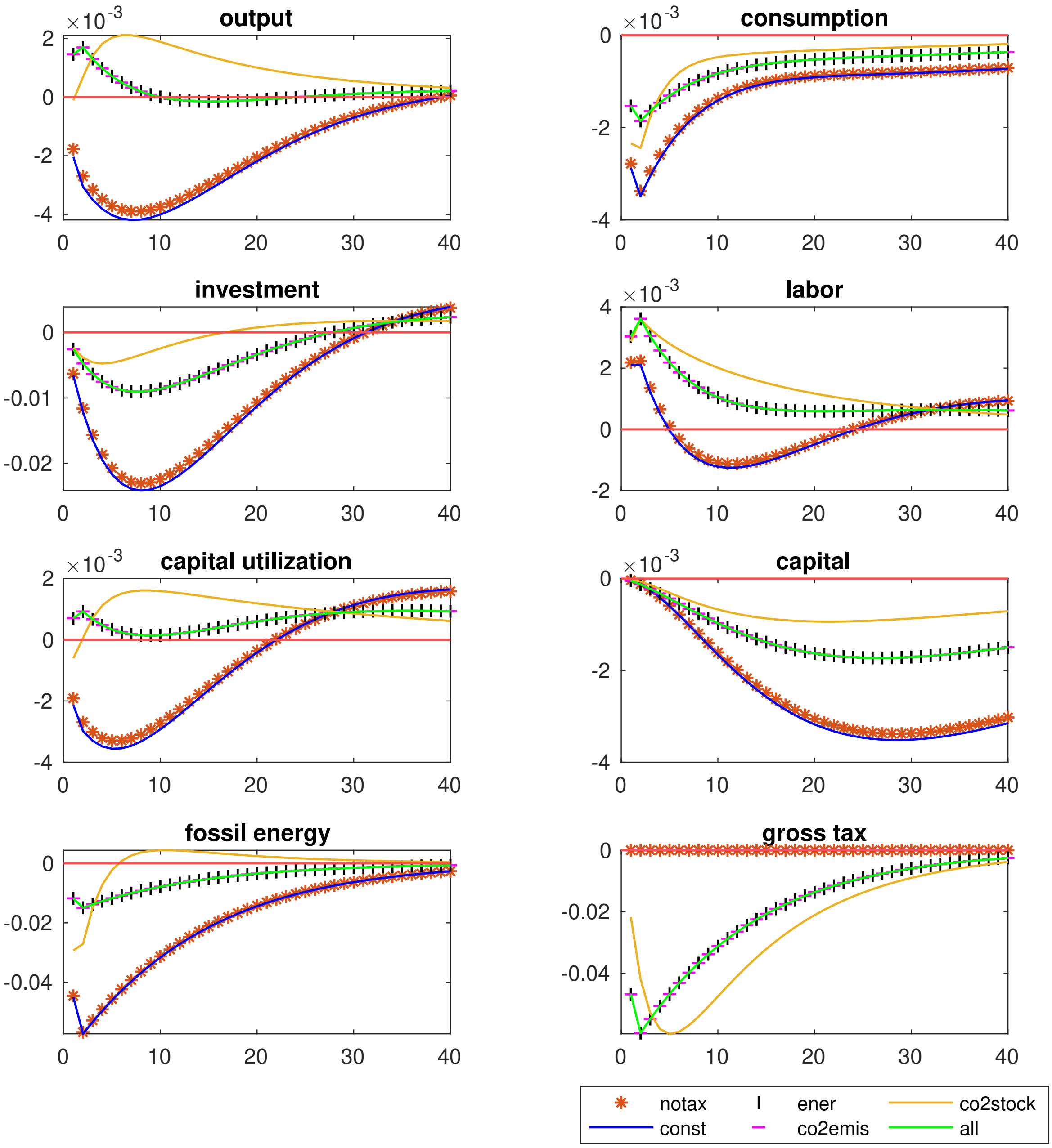
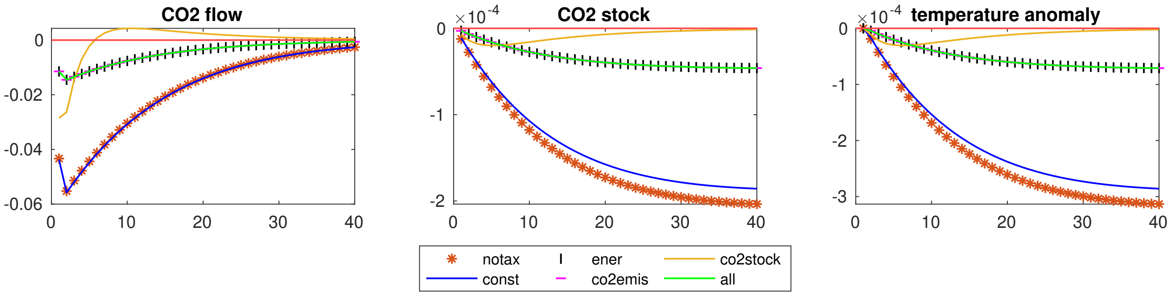


Figure 20: Energy Price Shock & Fossil Energy Tax - Economic Variables



Shifting the focus to the energy price shock, the distinction between the constant tax and all other policies becomes more clear. As under the TFP shock scenario, the perturbations induced by the energy price shock tend to mirror those pertaining to the no-tax benchmark scenario; however, the energy price shock produces a much stronger dampening effect, which in some cases is even able to revert the behavior underlying the no-tax scenario. Looking at the economic block, when energy price increases, this pushes firms to reduce energy use. This mechanically reduces the tax rate on energy in all cases: Energy, CO_2 emissions (since they are directly related to energy use,), and the anthropogenic stock of atmospheric CO_2 (which increases with emissions and, ultimately, with energy). The reduction in energy use leads to a reduction of the tax, which, to some extent, is able to counter the increase in energy price. Despite the energy drop, $p_{e,t}E_t$ keeps increasing pushed by the price effect. This latter, given a certain level of consumption and investment, translates into an output increase: This happens by substituting labor and capital utilization, which are not taxed, for fossil energy.

Figure 21: Energy Price Shock & Fossil Energy Tax - Economic Variables



Focusing on climatic variables, Figure 21 shows that the reduction in fossil energy use due to the energy tax reflects in an instantaneous drop in domestic CO_2 emissions. This, in turn, translates into a decrease in the anthropogenic stock of atmospheric CO_2 and in temperature anomaly. However, it is worth to notice that all tax other than the constant one, keeping alive output production, have a milder effect on the mitigation of global warming.

To sum up, in the case of technology shock, the energy tax, when linked directly or indirectly on fossil energy, is able to sustain a certain level of productivity leading at the same time to a substantial limitation of emissions and global warming. In the case of the energy price shock, the reduction of energy use is able to reduce the tax, which counters, to some extent, the increase in the price energy itself, thereby mitigating its effect.

5 Conclusions

This work aims at exploring the relationship between business cycles and climate, thereby featuring two kind of frequencies: Business cycle frequencies that are rooted to the short run, and climatic phenomena which span longer time horizons. The ultimate goal is to provide a theoretical framework to address these questions: How could very long run considerations affect short run economic decisions? How short run and transitory decisions could exert a long lasting effect on climate?

A real business cycle (RBC) model is developed, which features two agents, households and firms. The former develops his decisions maximixing his welfare subject to a budget constraint. The latter maximizes profits. The economy features three factors of production, namely labor, capital and fossil energy. It accounts for capital utilization and investment adjustment costs, and it is perturbed by two shocks: a classical technological shock and a shock on the price of fossil energy. The model is also equipped with a climatic block, which is essentially communicates with the above-described economy following a chain of events: the firms produce an homogeneus good using fossil energy; this leads to CO_2 emissions which accumulate in the atmosphere in the form of a stock; this leads to an increase in temperature, which hits the economy by inflicting a damage to the production process. The model, in particular, reproduces the carbon cycle as a stream of anthropogenic emissions, thereby avoiding the possibility for the total stock of CO_2 to fall under its *pre-industrial* natural level. Given the unusualness in making climatic phenomena fit the short time-frames dictated by business cycle literature, the model builds the link between the stock of CO_2 and temperature anomaly as a simple *long-run* statistical relationship. In general, the model performs relatively well in mirroring the empirical behavior of the main macroeconomic aggregates; however, it is not as successful in replicating the cyclical behavior of climatic variables.

When the model is hit by the technological shock, the economy reacts with an increase in production, and with households being able to afford more and working less. At the same time, the technological shock rises energy demand, which translates into larger CO_2 emissions. These, in turn, affects longer-run climatic phenomena: It increases the anthropogenic stock of atmospheric CO_2, and raises temperatures. The impact of a positive shock on the price of fossil energy qualitatively affects the variables of the model in a way opposite to the TFP shock, reducing output, and pushing households to consume less and working more. The complementarity between capital and energy leads to a decrease in the latter's use, thereby implying a negative istantaneous impact in CO_2 emissions as well as in the evolution of the anthropogenic stock of atmospheric CO_2 and temperature anomaly.

Finally, the model is used as a policy tool to evaluate various policy rules aiming at reducing CO_2 emissions. All policies are conceived as a tax on fossil energy use, which can be interpreted as an endogenous variation of the fossil energy price. The tax takes five different shapes: a constant tax; a tax linked to domestic CO_2 emissions; a tax linked to fossil energy use; a tax linked to the anthropogenic stock of atmospheric CO_2; a global tax, which accounts for the effects of all the just mentioned variables contemporaneously. All of them are compared with a no-tax scenario.

As a first result, it is important to notice that the constant tax does not react with the business cycles, hence overlapping with the no-tax scenario for almost all variable, no matters the shock considered. With respect to the technological shock, the energy tax responsiveness to the business cycle endogenously increases the energy price, thereby leading to a lower increase in the use of fossil energy. This impacts all economic variables by dampening down their reactions and also influences climatic variables, which continue to contribute to climatic issue but in a much weaker way. On the other hand, a positive energy price shock instantaneously reduces the use of fossil energy under its steady state. The impact on economic aggregates varies depending on the degree of responsiveness of the tax to the business cycle; more specifically, the tax counter-balances the price increase driven by the energy shock, in some cases even mildly sustaining output production. In general, the effect of the tax linked to the anthropogenic stock of atmospheric CO_2 lasts longer than that of the other tax experiments. Finally, as it is easy to imagine, the energy price shock overall negatively influences climatic variables, hence ameliorating the climatic status.

As a matter of fact the effect of any tax responsive to the business cycle shows very positive aspects: when a technology shock hits the economy, it mitigates global warming with minor costs in terms of potential output losses. It also protects the economy from an increase in energy prices, sustaining a certain level of output despite the fall in fossil energy use.

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A Data & Variables

This section deals with the way variables and data are chosen, rearranged and analyzed in order to obtain graphs and stylized facts.

A.1 Economic Series

The vast majority of economic series are seasonally adjusted and available from 1947:Q1 to 2019:Q4 at quarterly frequency. A few of them, that is, annual output and the series for the energy-output ratio, are available at annual frequency. In particular, the following raw series have been selected:

i) **GDP** [Gross Domestic Product, Billions of Dollars, Seasonally Adjusted, Annual Rate, Quarterly] stands for *nominal GDP* and has been retrieved from FRED (https://fred.stlouisfed.org/series/GDP) [ORIGINAL SOURCE: *U.S. Bureau of Economic Analysis, BEA*];

ii) **GDP** [Gross Domestic Product, Millions of Dollars, Annual] stands for *nominal GDP* and has been retrieved from BEA, *Table 1.1.5. Gross Domestic Product [T10105-A]* (https://apps.bea.gov/national/Release/XLS/Survey/Section1All_xls.xlsx);

iii) **GDPDEF** [Gross Domestic Product: Implicit Price Deflator, Index 2012 = 100, Seasonally Adjusted, Quarterly] has been retrieved from FRED (https://fred.stlouisfed.org/series/GDPDEF) [ORIGINAL SOURCE: *U.S. Bureau of Economic Analysis, BEA*];

iv) **PCDG** [Personal Consumption Expenditures: Durable Goods, Billions of Dollars, Seasonally Adjusted, Annual Rate, Quarterly] has been retrieved from FRED (https://fred.stlouisfed.org/series/PCDG) [ORIGINAL SOURCE: *U.S. Bureau of Economic Analysis, BEA*];

v) **PCND** [Personal Consumption Expenditures: Nondurable Goods, Billions of Dollars, Seasonally Adjusted, Annual Rate, Quarterly] has been retrieved from FRED (https://fred.stlouisfed.org/series/PCND) [ORIGINAL SOURCE: *U.S. Bureau of Economic Analysis, BEA*];

vi) **PCESV** [Personal Consumption Expenditures: Services, Billions of Dollars, Seasonally Adjusted, Annual Rate, Quarterly] has been retrieved from FRED (https://fred.stlouisfed.org/series/PCESV) [ORIGINAL SOURCE: *U.S. Bureau of Economic Analysis, BEA*];

vii) **FPI** [Fixed Private Investment, Billions of Dollars, Quarterly, Seasonally Adjusted, Annual Rate] has been retrieved from FRED (https://fred.stlouisfed.org/series/FPI) [ORIGINAL SOURCE: *U.S. Bureau of Economic Analysis, BEA*]. As it is possible to check from *BEA, Table 1.1.5* (https://apps.bea.gov/national/Release/XLS/Survey/Section1All_xls.xlsx), FPI includes RESIDENTIAL FPI and NON-RESIDENTIAL FPI (*structures* + *equipments* + *intellectual property products*);

viii) **CBI** [Change in Private Inventories, Billions of Dollars, Seasonally Adjusted, Annual Rate, Quarterly] has been retrieved from FRED (https://fred.stlouisfed.org/series/CBI) [ORIGINAL SOURCE: *U.S. Bureau of Economic Analysis, BEA*];

ix) **GCE** [Government Consumption Expenditures and Gross Investment, Seasonally Adjusted, Annual Rate, Quarterly] has been retrieved from FRED (https://fred.stlouisfed.org/series/GCE) [ORIGINAL SOURCE: *U.S. Bureau of Economic Analysis, BEA*]. In particular, both *govenrment consumption expenditures* and *government gross investment*, i.e., the series which contribute to GCE, are available from *Table 3.9.5. Government Consumption Expenditures and Gross Investment [T30905-Q]* (https://apps.bea.gov/national/Release/XLS/Survey/Section3All_xls.xlsx)

x) **PRS85006023** [Nonfarm Business Sector: Average Weekly Hours, Index 2012 =100, Seasonally Adjusted, Quarterly] stands for *hour* and has been retrieved from FRED (https://fred.stlouisfed.org/series/PRS85006023) [ORIGINAL SOURCE: *U.S. Bureau of Labor Statistics, BLS*];

xi) **HOANBS** [Nonfarm Business Sector: Hours of All Persons, Index 2012 =100, Seasonally Adjusted, Quarterly] stands for *hour* and has been retrieved from FRED (https://fred.stlouisfed.org/series/HOANBS) [ORIGINAL SOURCE: *U.S. Bureau of Labor Statistics, BLS*];

xii) **OPHNFB** [Nonfarm Business Sector: Real Output Per Hour of All Persons, Index 2012 =100, Seasonally Adjusted, Quarterly] stands for *productivity* and has been retrieved from FRED (https://fred.stlouisfed.org/series/OPHNFB) [ORIGINAL SOURCE: *U.S. Bureau of Labor Statistics, BLS*];

xiii) **COMPNFB** [Nonfarm Business Sector: Compensation Per Hour, Index 2012=100, Seasonally Adjusted, Quarterly] stands for *wage* and has been retrieved from FRED (https://fred.stlouisfed.org/series/COMPNFB) [ORIGINAL SOURCE: *U.S. Bureau of Labor Statistics, BLS*];

xiv) **PAYEMS** [Nonfarm Business Sector: All Employees,Thousands of Persons, Seasonally Adjusted, Monthly] stands for *employment* and has been retrieved from FRED (https://fred.stlouisfed.org/series/PAYEMS) [ORIGINAL SOURCE: *U.S. Bureau of Labor Statistics, BLS*];

xv) **DGOERG** [Price Indexes for GDP, Index 2012 =100, Seasonally Adjusted, Quarterly] can be used, which accounts for "gasoline and other energy goods" alone. This can be retrieved from BEA, *Table 1.5.4. Price Indexes for Gross Domestic Product, Expanded Detail [T10504-Q]* (https://apps.bea.gov/national/Release/XLS/Survey/Section1All_xls.xlsx).

xvi) **E/Y** [Expenditures as share of GDP, Percent, Annual] stands for "total energy-output ratio" and can be retrieved from EIA, *Table 1.7. Primary Energy Consumption, En-*

ergy Expenditures, and Carbon Dioxide Emissions Indicators (https://www.eia.gov/totalenergy/data/browser/xls.php?tbl=T01.07&freq=m);

A.2 Environmental/Climatic Series

Environmental/climatic series cover emissions, energy and weather series. They are heterogeneous in terms of source (US Energy Information Administration for data abut energy and emissions and National Oceanic Atmospheric Administration for data on CO_2 stock and weather) and first year of sampling (series about energy and emissions start in 1973, those on CO_2 stock in 1958, while those concerning weather begin in 1895). Data observation suggests they did not undergo seasonal adjustment, this information being however not shown at data source.

i) **CO_2 emissions** [Million Metric Tons of Carbon Dioxide, Monthly] has been retrieved from U.S. Energy Information Administration (EIA), *Table 11.1 Carbon Dioxide Emissions from Energy Consumption by Source* (https://www.eia.gov/totalenergy/data/browser/xls.php?tbl=T11.01&freq=m);

ii) **CO_2 stock** [Parts Per Millions (ppm), Monthly] has been retrieved from NOAA-Global Monitoring Laboratory, *co2_mm_mlo* (ftp://aftp.cmdl.noaa.gov/products/trends/co2/co2_mm_mlo.csv) - Dr. Pieter Tans, NOAA/GML (gml.noaa.gov/ccgg/trends/) and Dr. Ralph Keeling, Scripps Institution of Oceanography (scrippsco2.ucsd.edu/);

iii) **Energy Consumption from Fossil Fuels** [Quadrillion Btu, Monthly] has been retrieved from U.S. Energy Information Administration (EIA), *Table 1.3 Primary Energy Consumption by Source* (https://www.eia.gov/totalenergy/data/browser/xls.php?tbl=T01.03&freq=m);

iv) **Energy Consumption from Coal** [Quadrillion Btu, Monthly] has been retrieved from U.S. Energy Information Administration (EIA), *Table 1.3 Primary Energy Con-*

sumption by Source (https://www.eia.gov/totalenergy/data/browser/xls.php?tbl=T01.03&freq=m);

v) **Energy Consumption from Oil** [Quadrillion Btu, Monthly] has been retrieved from U.S. Energy Information Administration (EIA), *Table 1.3 Primary Energy Consumption by Source* (https://www.eia.gov/totalenergy/data/browser/xls.php?tbl=T01.03&freq=m);

vi) **Energy Consumption from Natural Gas** [Quadrillion Btu, Monthly] has been retrieved from U.S. Energy Information Administration (EIA), *Table 1.3 Primary Energy Consumption by Source* (https://www.eia.gov/totalenergy/data/browser/xls.php?tbl=T01.03&freq=m);

vii) **Global Surface Temperature Anomalies** [°C, Monthly] has been retrieved from National Oceanic and Atmospheric Administration (NOAA) (https://www.ncdc.noaa.gov/cag/global/time-series). The series is collected for the period 1880-2019 and is calculated having the period 1901-2000 as base period. The surface considered consists of land and ocean together;

B Equations of the Model

After some rearrangements: $w_t = \frac{W_t}{P_t}$, $z_t = \frac{Z_t}{P_t}$, $p_{e,t} = \frac{P_{e,t}}{P_t}$, $q_t = \frac{Q_t}{P_t}$

$$\chi(1-L_t)^{\frac{1}{\sigma_{le}}} = C_t^{-\frac{1}{\sigma_c}} w_t$$

$$q_t = \beta \mathbb{E}_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\frac{1}{\sigma_c}} \left[z_{t+1} u_{t+1} + q_{t+1}(1 - \delta_0 u_t^{\nu}) \right] \right\}$$

$$1 = q_t \left[1 - \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \phi \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] + \beta \mathbb{E}_t \left[q_{t+1} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\sigma_c}} \phi \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right]$$

$$z_t = q_t \nu \delta_0 u_t^{\nu-1}$$

$$Y_t = \aleph_t^A D_t N_t^{\alpha} L_t^{1-\alpha}$$

$$N_t = [\omega(u_t K_t)^{\rho} + (1-\omega) E_t^{\rho}]^{\frac{1}{\rho}}$$

$$w_t = (1-\alpha) \frac{Y_t}{L_t}$$

$$z_t = \alpha \omega Y_t N_t^{-\rho} (u_t K_t)^{\rho-1}$$

$$(1+\tau_t) p_{e,t} = \alpha (1-\omega) Y_t N_t^{-\rho} E_t^{\rho-1}$$

$$D_t = \frac{1}{1 + \left(\frac{T_t}{\pi_1}\right)^{\pi_2} + \left(\frac{T_t}{\pi_3}\right)^{\pi_4}}$$

$$K_{t+1} = (1 - \delta_0 u_t^{\nu}) K_t + \left[1 - \frac{\phi}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right) \right] I_t$$

$$Y_t = C_t + I_t + p_{e,t} E_t$$

$$M_t^d = E_t^{\gamma}$$

$$V_{t+1} = \eta (M_t^d + M_t^{nd} + V_t)$$

$$T_t = \zeta_1 V_t^{\zeta_2}$$

$$\ln \left(\frac{1+\tau_t}{1+\bar{\tau}} \right) = \psi_1 \ln \left(\frac{M_t^d}{\bar{M}^d} \right) + \psi_2 \ln \left(\frac{E_t}{\bar{E}} \right) + \psi_3 \ln \left(\frac{V_t}{\bar{V}} \right)$$

$$\ln(\aleph_t^A) = (1-\rho_A) \ln(\bar{\aleph}^A) + \rho_A \ln(\aleph_{t-1}^A) + \varepsilon_t^A$$

$$\ln(p_{e,t}) = (1-\rho_{pe}) \ln(\bar{p}_{e,t}) + \rho_{pe} \ln(p_{e,t-1}) + \varepsilon_t^{pe} + \phi_{pe} \varepsilon_{t-1}^{pe}$$