Alma Mater Studiorum – Università di Bologna

DOTTORATO DI RICERCA IN

PSICOLOGIA

Ciclo XXXII

Settore Concorsuale: 11/E1

Settore Scientifico Disciplinare: M PSI-01

MATHEMATICAL SKILLS: INTERGENERATIONAL FEATURES AND RELATIONSHIPS WITH COGNITIVE AND LINGUISTIC ABILITIES

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Esame finale anno 2020

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Summary

Mathematics is a complex construct, in which numerous abilities and cognitive resources are involved. This thesis aimed to investigate the cognitive underpinnings of math skills, with particular reference to cognitive, and linguistic markers, core mechanisms of number processing and environmental variables. In particular, the issue of intergenerational transmission of math skills has been deepened, comparing parents' and children's basic and formal math abilities. This pattern of relationships amongst these has been considered in two different age ranges, preschool and primary school children.

In the first chapter, a general introduction on mathematical skills is offered, with a description of some seminal works up to recent studies and latest findings. In particular, the cognitive and developmental models of numerical cognition are discussed, including studies that lead to the definition of the Approximate Number System (ANS) and the neural correlates of math skills. Relationships between numerical knowledge and domain-general cognitive processes such as language, memory, emotional aspects, and visual predictors of mathematical skills are also discussed. The first chapter concludes with a review of studies about the influence of environmental variables. In particular, a review of studies about home numeracy and intergenerational transmission is examined.

In the following chapters three main studies are presented.

The first study analyzed the relationship between mathematical skills of children attending primary school and those of their mothers. In particular, the non-symbolic abilities of mothers and their competence in carrying out written operations will be explored. The objective of this study was to understand the influence of mothers' math abilities on those of their children.

In the second study, the relationship between parents' and children numerical processing has been examined in a sample of preschool children. This study was designed to rule out the influence of formal mathematics teaching and to deepen the understanding of early numeracy skills. Parents were administered tests on mathematical skills and a questionnaire on home numeracy. The goal was to

understand how mathematical skills of parents were relevant for the development of the numerical skills of children, taking into account children's cognitive and linguistic skills as well as the role of home numeracy.

The third study was developed during my period as a visiting student at the University of Amsterdam, with the precious collaboration of Prof. Peter de Jong, and had the objective of investigating whether the verbal and nonverbal cognitive skills presumed to underlie arithmetic are also related to reading. Primary school children were administered measures of reading and arithmetic to understand the relationships between these two abilities and testing for possible shared cognitive markers. The study's protocol involved different areas of the math skills together with non-verbal IQ, working memory, and phonological ability.

Finally, in the general discussion a summary of main findings across the study is presented, together with clinical and theoretical implications.

Chapter 1

General Introduction

1.1 Mathematical skills

Several psychological evidences provide experiential support for the idea that intuitions, and not formal logic, are the grounds upon which humans base their comprehension of mathematics (McLarty, 1997) and demonstrate that the “number sense” is part of the human’s core knowledge that already exists early on in childhood. The sources and the underlying cognitive codes on which this number sense is grounded have, nevertheless, so far not been completely understood. While we know that humans and animals share the ability to process approximate quantities and numerosity information, only humans can create numerical notation systems that allow for a symbolic representation of exact amounts of natural numbers. Throughout civilization, these notational systems became more complicated: the gradual introduction of syntactic features, such as the place value system to code magnitudes with multi-digit numbers, the polarity sign for the negative values or the fractions, made it possible to generate compound expressions to represent magnitudes that do not correspond to simple-digit numbers. The use of symbols to represent exact magnitudes develops from an ancient system that humans and animals share (Feigenson, Dehaene, & Spelke, 2004; Gallistel & Gelman, 2000). Numbers are symbolically represented by numerals that are single symbols or symbol combinations used to describe quantities. Various cultures used different symbolic numeration system; moreover, there is a distinction between external an internal numeration system. The Arabic numeration system utilizes digits as one dimension describing quantity and digit position power with the base ten as the second dimension. The internal numeration system refers to how the numbers are mentally represented, for example, in a mental number line (Dehaene, 1997; Restle, 1970). According to the mental number line, numbers are mentally represented on a line (in our culture from left to right) (Dehaene, Bossini, & Giraux, 1993). This kind of representation is unidimensional, and the magnitude is transposed into spatial location upon this line (van Dijck, Fias, & Andres, 2015). This unidimensional representation is in contrast to the two-dimensional representation of external numeration system like the Arabic's. The unidimensional representation of quantity implies that humans understand the amount represented with a number by extraction of its meaning from the mental number line. This process is quite simple with the single-digit numbers; the location of the number is not essential, and there are no further processes required to access the representation magnitude. It is more complicated when the representation includes multiple-digit numbers. Verguts and Fias (Verguts & Fias, 2004, 2008) described a model that shows how a system can learn to use symbols as a representation of magnitudes when presented with input from both non-symbolic (Zorzi & Butterworth, 1999) and symbolic magnitude codes.

This model suggests that there is a particular site as proposed by Dehaene and Changeux (Dehaene et al., 1993), which is specific for the coding, at least by humans, of small magnitudes and it is located in precise brain areas of the intraparietal sulcus and of the prefrontal cortex. Place coding is viewed by many as the neural realization of the mental number line (Ansari, 2008).

In addition to the capacity to understand numerical concepts when represented with digits, humans also have an Approximate Number System (ANS). It can be used to perform arithmetic operations on non-symbolic quantities such as arrays of dots or tones. The ANS is present in very young children and some non-human animals (Dehaene, 1997). Some theorists have started to theorize that it serves as the cognitive foundation for symbolic mathematics (Justin Halberda, Mazzocco, & Feigenson, 2008).

The ANS is generally thought to follow Weber's law: the distribution of possible ANS representations follows a normal distribution with mean n and standard deviation wn, *wh*ere w is the Weber fraction, a parameter which represents the acuity of an individual's ANS (Barth et al., 2006). Several recent studies have revealed that individuals' ANS acuities are correlated with accomplishment in symbolic mathematics (Gilmore, McCarthy, & Spelke, 2010; Justin Halberda et al., 2008; Inglis, Attridge, Batchelor, & Gilmore, 2011; Libertus, Feigenson, & Halberda, 2011; Mazzocco, Feigenson, & Halberda, 2011; Price, Palmer, Battista, & Ansari, 2012), lending credence to the suggestion that the ANS is involved in the development of symbolic mathematics competence.

Although the capabilities of the ANS are now relatively well known, the process by which the ANS produces representations from visual numerical stimuli is less clear. Some researchers have suggested that a mental 'accumulator' is pivotal to this process (Dehaene et al., 1993; Gallistel & Gelman, 2000; Izard, Sann, Spelke, &

Streri, 2009; Verguts & Fias, 2004). Gallistel and Gelman formed an analogy among filling up a beaker with cups of liquid, and filling up the accumulator with "accumulator units". They suggested that when a set of objects is observed, the view is first normalized to remove the unnecessary information, then one cupful of 'liquid' is added to the accumulator per item. The contents of the accumulator are then emptied into memory, which introduces noise proportionate to the accumulator's contents. It is this noise, when the contents of the memory beaker are read off (converted into a numerical quantity), which causes the approximate nature of ANS representations.

Notably, both Barth et al.'s (2006) computational model of the ANS, and Gallistel and Gelman's (2000) accumulator beaker analogy believe that the duration for which a numerical stimulus is presented is unrelated to the ANS representation that an individual encodes from it.

Comparative psychologists attempted to understand what distinguishes human from animal minds, and importantly the study of how the mind reproduces number demonstrates both evolutionary continuity and discontinuity. On the one hand, continuity is evident in the shared system for making approximate judgments (ANS). On the other hand, there is an apparent evolutionary discontinuity in that only humans have invented arbitrary symbols for numbers. This unique capacity for expressing number symbolically permits humans to mentally manage exact numerical amounts and favours complex and abstract mathematics elaborations. The study of how the numbers are presented from infancy into adulthood exhibits intense continuity whereby infants seem to be capable of representing and comparing numerical values approximately. While the ANS intensely improves in its accuracy over early childhood and into middle age, its fundamental signatures remain constant.

At the same time, numerical development shows a paradigmatic example of discontinuity and conceptual revolution for which language transforms a child's capacity to represent numbers (Barth, Baron, Spelke, & Carey, 2009).

There is an increasing number of recent studies that demonstrate that ANS precision can explain some of the variance in symbolic math performance (Gilmore et al., 2010; Justin Halberda & Feigenson, 2008; Libertus et al., 2011; Libertus, Odic, & Halberda, 2012; Mazzocco et al., 2011).

Nevertheless other studies didn't find any relationship between ANS and symbolic math achievement (Holloway & Ansari, 2009; Inglis et al., 2011; Iuculano, Tang, Hall, & Butterworth, 2008; Nosworthy, Bugden, Archibald, Evans, & Ansari, 2013; Price et al., 2012; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013; Soltész, Szucs, & Szucs, 2010), or have observed that the relation is mediated by symbolic number knowledge (Lyons & Beilock, 2011), executive function (Fuhs & Mcneil, 2013), or it works only for children with weakness in math ability (Bonny & Lourenco, 2013). Other studies suggested that the facility with ordering Arabic numerals could play an important role in mathematical cognition and competence (De Smedt & Gilmore, 2011; Iuculano et al., 2008; Noël & Rousselle, 2011) with differences between children with high or low math ability, and explaining individual variations in typically developing children or adults (Bugden & Ansari, 2011; Castronovo & Göbel, 2012; De Smedt, Verschaffel, & Ghesquière, 2009; Holloway & Ansari, 2009; Landerl, Bevan, & Butterworth, 2004; Lyons & Beilock, 2011; Sasanguie, Van den Bussche, & Reynvoet, 2012). One of the most important points is referred to the understanding of the mechanisms underlying the connections between ANS, symbolic number system, and math. Most of the research exploring the relationship between the ANS and math performance has been restricted to a correlational approach, and few works have still investigated the causal relationship.

Two recent studies (Mazzocco et al., 2011; Starr, Libertus, & Brannon, 2013) have measured the children's ANS acuity before their formal math education. In the first study, the administration was done when the children had three to four years, and they have found some correlations with the performances in standardized math tests at five and six years old. The second study measured the ANS acuity at six months old, and it was correlated with their standardized math scores three years later. Nevertheless, these findings do not demonstrate why the connection between ANS representations and math success appears to hold even into adulthood (DeWind & Brannon, 2012; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Lyons & Beilock, 2011). Even if the ANS accuracy foretells some of the variance in math proficiency and this exhibits a causal relationship, it is well-known that in most studies done so far only a tiny portion of the variability in math ability seems to be defined by the ANS acuity.

Understanding causal relationships may be necessary for the epistemology of numerical cognition and practical interventions. It could be critical for revealing the developmental roots of human numeric cognition, for improving arithmetic training paradigms and developing adequate clinical assessment for math impairments' identification.

In over a century of psychological research on number processing many models have been developed and will be briefly reviewed in the following sections.

1.2 Cognitive models of numerical cognition

1.2.1 Abstract modular model (McCloskey, 1992)

Michael McCloskey (McCloskey, Caramazza, & Basili, 1985) was the first to believe that numbers are stored as a central and abstract representation. He carried out many studies with acalculic patients on how the brain elaborated numbers and calculations, discovering how it was possible to consider numerous areas of the calculation independent of one another. Studying patients with brain damage has allowed him to consider the function of various brain areas independently (McCloskey & Macaruso, 1995; Sokol, McCloskey, Cohen, & Aliminosa, 1991). McCloskey focused on the ability to calculate, proposing it in different forms, to understand which differences there were. In particular, he found that some patients with brain damage were able to recognize the exact result of an operation among some proposed options, but were not able to produce it autonomously. Other patients, however, produced the right answer individually but were not able to recognize it if proposed by the experimenter.

Despite the differences between these patients, in both "groups" the ability to arrive at an exact answer and, therefore, to perform mental calculation, was intact. This conclusion led him to hypothesize that calculation procedures were independent of numerical recognition or production. At the same time, it seemed evident that numerical production and recognition came from two distinct brain areas that could be dissociated from each other.

An additional experimental condition that was used in these studies was to modify the different forms in which the numbers were presented. The presentation methods were alternated and in some cases the numbers were presented in the form of an Arabic number (3) while at other times using the words indicating the number (three). Also in this case it was possible to observe that some patients were able to perform the proposed activities correctly only in one of the two conditions. Some patients were able to compare magnitudes only with Arabic numbers, while others performed the activity correctly only when the numbers were expressed in words.

Starting from these premises, McCloskey and colleagues have developed a model, called an abstract modular model, which was composed of three different systems that are distinct from one another: number comprehension, calculation and number production. These systems are linked together through the abstract representation module, which is located in the center of the model and connects the three systems to each other. When a number is presented, the number comprehension system takes care of translating it into an abstract representation of quantity. These quantities can then be used to produce additional numbers or to perform calculations. The final result can be displayed in any of the possible ways.

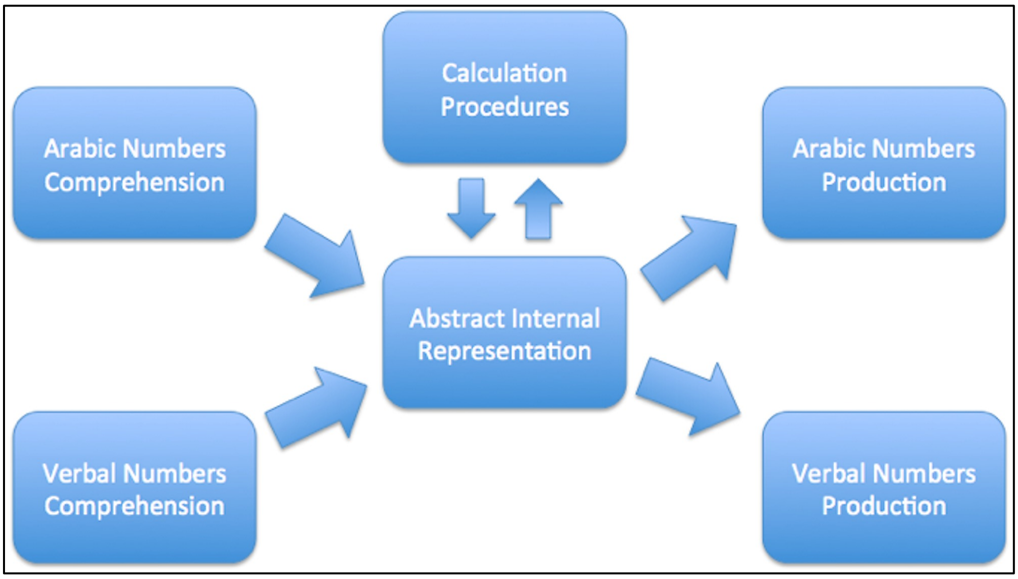

Figure 1: McCloskey's Abstract Modular Model (The figure is based on Dehaene, 1992; page 28).

1.2.2 Triple Code Model (Dehaene, 1992)

In neuropsychology, the term 'modular architecture' is applied to illustrate how neurocognitive systems are linked and how they interact with each other. Shallice (Shallice, 1991) explains that the modular organization is characterized by the presence of a double dissociation between two functions. Concerning numerical cognition, many studies described this double dissociation (Dehaene & Cohen, 1995a; Hittmair-Delazer, Sailer, & Benke, 1995; McCloskey, 1992) and brought it as evidence of the presence of different but related components of arithmetical processing. Hittmair-Delazer et al. (Hittmair-Delazer et al., 1995) have shown that numerical facts (like simple mental calculations) and arithmetical conceptual theory (essential understanding of operations and arithmetic postulates) are dissociable from each other, both at a behavioral and neural level. Furthermore, a double dissociation between numerical facts and procedural arithmetic knowledge has been described earlier by Temple (Temple, 1991). It has been demonstrated that during development, neurocognitive systems are still unripe and consequently, the operative specialization of specific brain areas has not yet taken place or is not yet completely developed in children (Karmiloff-Smith, 2009). Therefore, double dissociations in adult brain systems may reveal different holding processes. In particular, double dissociations noted in immature brain systems might not imply a modular cognitive architecture (Karmiloff-Smith, Scerif, & Ansari, 2003). The adult research on numerical cognition has suggested three distinct number representations (Hittmair-Delazer et al., 1995; McCloskey, 1992) that are mediated by distinguished neural networks (Dehaene, Piazza, Pinel, & Cohen, 2003). The Triple Code Model (TCM) (Dehaene, 1992, 1997) proposes that numbers are represented in three different codes that attend various functions, have distinguished functional neuroarchitectures, and are specific to performance on distinct tasks (Van Harskamp & Cipolotti, 2001). This model describes that these codes are at the base of our ability to calculate and process numerosity. The first one is a verbal code, related to the linguistic system, that is used to recover well-learned arithmetic facts using memory, such as simple addition and multiplication tables (González & Kolers, 1982). The second one is a visual code that represents and spatially manages numbers in Arabic format (Ashcraft & Stazyk, 1981; Dahmen, Hartje, Büssing, & Sturm, 1982; Dehaene & Cohen, 1991; Weddell & Davidoff, 1991). Finally, the third code is the analogue magnitude representation, that gives a representation of analogical quantity on a mental number line (approximate calculation and magnitude comparison) (Chochon, Cohen, Van De Moortele, & Dehaene, 1999; E. Spelke & Dehaene, 1999). According to Dehaene, the verbal code is applied in particular for counting, addition, and easy multiplication, while approximate calculation and comparison are supported more by the nonverbal codes. A plausible candidate for domain specificity is the horizontal segment of the intraparietal sulcus (HIPS). It is activated every time a test involving the use of numbers is presented. The left angular gyrus area, in connection with other left-hemispheric perisylvian areas, deals with the verbal use of numbers. Finally, a bilateral posterior superior parietal system supports attentional orientation on the mental number line (Dehaene et al., 2003).

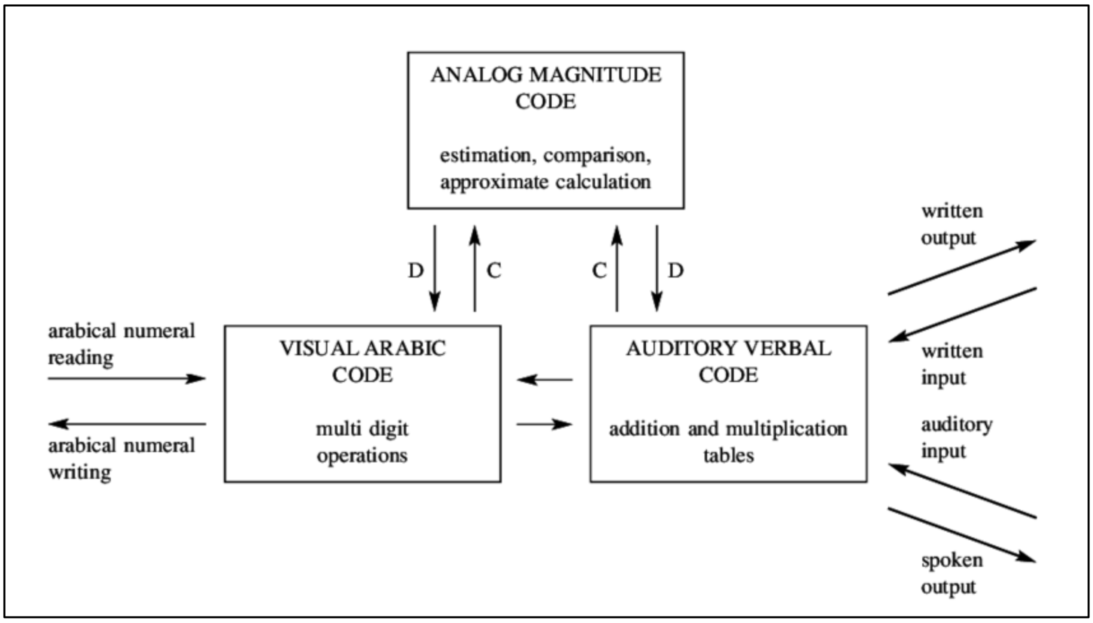

Figure 2: The triple-code model proposed by Dehaene (1992)

A recent review by Skagenholt et al. (Skagenholt, Träff, Västfjäll, & Skagerlund, 2018) investigated the neural correlates of the Triple Code Model within one experimental fMRI-paradigm. The results showed that Dehaene and Cohen's functional–anatomical account of the TCM seemed poor and partially conflicting with recent evidence. Their previous hypotheses were in line with the original model, but the results showed an approximately even distribution of right and left-lateralized functional areas across all tasks. Recent research provided for functional and structural connectivity analysis most highlights the significance of connection and communication between the two hemispheres for numerical proficiency (Moeller, Willmes, & Klein, 2015; Price, Yeo, Wilkey, & Cutting, 2018), insinuating that the analysis of individual functional regions provides an incomplete description of numerical cognition. Those results show a need to comprehend functional networks connected with visuospatial attention in the TCM, to adjust the model with contemporary empirical proof. Arsalidou and Taylor (Arsalidou & Taylor, 2011) have proposed to incorporate in previous models the right superior frontal gyrus, left inferior frontal gyrus, insula, left anterior cingulate cortex, and right angular gyrus. These additions, supported by the recent results, are interesting with respect to the salience network and its functional association with the posterior parietal cortex (composing the superior and inferior parietal lobule), ventromedial prefrontal cortex (a part of the default mode network), and dorsolateral prefrontal cortex (a part of the central-executive network). This interplay of functional regions demonstrates that the fronto-parietal network of numerical cognition contributes significantly to the salience, default mode, and central-executive networks. Arsalidou and Taylor furthermore suggested the addition of the precentral gyrus and cerebellum, which were not identified in the task–control contrasts. In light of the need to improve the TCM, they suggest the additional inclusion of primary somatosensory area 3b and visual areas V2 and V3.

1.2.3 Dissociations models

After the Triple Code Model and the new brain-based approach, several studies have established both dissociations and associations between numerical and calculation abilities and other cognitive skills. One of the most prominent models of double dissociation has been repeatedly reported between numerical and semantic knowledge (Been & Jefferies, 2004; Cappelletti, Butterworth, & Kopelman, 2001; Crutch & Warrington, 2002; Diesfeldt, 1993; Ischebeck et al., 2006; Julien, Neary, & Snowden, 2010). For example, patients with a semantic deficit, with critical impairment in understanding the meaning of words, are usually able to understand numerical notions and to complete arithmetical operations (Julien et al., 2010), even at the late stages of illness (Cappelletti, Kopelman, Morton, & Butterworth, 2005). This finding suggests that there is dissociation between understanding numerical and arithmetical concepts and the meaning of words. This pattern has been reported in several patients with semantic dementia; recent cases have attested that accurate numerical knowledge leads to decline with the severity of the semantic disorder. Probably it happened because the atrophy of the left temporal lobe extends to brain areas more directly involved in numerical processing (Jefferies, Bateman, & Lambon Ralph, 2005; Julien et al., 2010). The same dissociations have been reported in the opposite direction whereby, for instance, patients may be critically damaged in processing numbers but keep control of other cognitive skills. A patient with posterior cortical atrophy exhibited selective impairment in understanding numerical notions but saved comprehension of non-numerical concepts (Ischebeck et al., 2006). Together, these cases, which always consisted of dissociations rather than associations of impairments, actively support the idea of a subdivision within semantic knowledge between numerical and non-numerical features.

The relationship between numerical and cognitive skills is not always so clear. Number skills have been shown to be mostly independent from general intelligence (Remond-Besuchet et al., 1999), short-term memory (Butterworth, Cipolotti, & Warrington, 1996), visuo-spatial attention (Cappelletti & Cipolotti, 2006; Galfano, Rusconi, & Umiltà, 2006; Sackur et al., 2008), and language (Dehaene et al., 1997; Rohrer et al., 2010; Thioux et al., 1998). In this latter case, for example, patients with both well-preserved language and severe quantity impairments of transcoding (Cipolotti, Butterworth, & Denes, 1991; Cipolotti, Warrington, & Butterworth, 1995; Dehaene et al., 1997; M. Delazer & Benke, 1997; Denes & Signorini, 2001; Marangolo, Piras, & Fias, 2005) or impaired language and maintained number skills (Margarete Delazer, Benke, Trieb, Schocke, & Ischebeck, 2006; Klessinger, Szczerbinski, & Varley, 2007; Zaunmuller et al., 2009) have been described.

However, some numerical skills are mediated or preferentially supported by linguistic functions (e.g, Counting, reading numbers, some arithmetical facts) (Dehaene, 1997; Dehaene & Cohen, 1995a; Margarete Delazer, Girelli, Semenza, & Denes, 1999; Gerard Deloche & Seron, 1984; Gérard Deloche & Seron, 1982; García-Orza, León-Carrión, & Vega, 2003; Mantovan, Delazer, Ermani, & Denes, 1999; Marangolo, Nasti, & Zorzi, 2004; Seron & Deloche, 1983). Furthermore, some numerical skills are also strongly linked to working memory, executive functions, and spatial attention such that associations of impairments in number and in these other functions have often been reported (Cappelletti, Barth, Fregni, Spelke, & Pascual-Leone, 2007; Dehaene & Cohen, 1991, 1995a; Hoeckner et al., 2008; Mennemeier et al., 2005; Pesenti, Thioux, Seron, & De Volder, 2000; Umiltà, Priftis, & Zorzi, 2009; Vuillemier, Ortigue, & Brugger, 2004; Vuilleumier, Richardson, Armony, Driver, & Dolan, 2004; Zangaladze, Epstein, Grafton, & Sathian, 1999; Zorzi et al., 2012; Zorzi, Priftis, & Umiltà, 2002). The pattern of connections and dissociations between numbers and other abilities have been primarily investigated in adults, but developmental models might be important for understanding the nature of the adults architecture of number processing.

1.3 ***Developmental models***

Despite the growing scientific attention in numerical cognition, in general, and developmental calculation disorders, in particular, for many years numerical cognition in developmental age has been under-investigated. Compared with the scientific research about attention or reading, the scientific efforts on numerical cognition have been modest and, in most cases, models derived from studies on adult population. Adult calculation models mainly rest on reports from neuropsychological patients with acquired calculation disorders. These is instructive concerning both the cognitive structure and the neural correlates of number processing but often resulted to be not fully adequate for explaining math development in children.

1.3.1 Min model (Groen e Parkman, 1972)

One of the first models concerning calculation in developmental age was the Min Model by Groen and Parkman (1972). Groen and Parkman studied primary school children and observed that they used a particular strategy to perform calculations in mind. Specifically, to solve single-digit addition problems, children at that age used a strategy defined by the authors *min strategy*. This strategy consists of making a sum, between two single-digit numbers, always starting from the highest digit and adding the other quantity starting from the first digit. In this way, the time required to solve an addition does not depend on the magnitude of the result, nor the magnitude of the first addendum, but only on how large the second addend is. This theory is based on the assumption that each count requires the same quantity of time.

For this reason, the time required to add "2" to "4" or "7" is the same and resolution times are represented by a linear function, related exclusively to the smallest addend. The higher the size of the minor addend, the longer is the time required to resolve the operation.

Groen and Parkman tried to verify this model with pupils in the first year of primary school, presenting them with some sum operations. In line with their theoretical hypotheses, they were able to verify that the time taken to solve the calculations depended strongly on the size of the smaller addend.

The predictor associated with the model, size of the smaller addend, accounted for roughly 70% of the variance in solution times, much higher than the amount accounted for by any other model that they considered. A variety of conclusions were made with the min model as a description of how young elementary school children perform addictions.

In the following years, other researchers tried to replicate the studies of Groen and Parkman, finding similar results (Ashcraft, 1982, 1987; Kaye, Post, Hall, & Dineen, 1986; Svenson & Broquist, 1975). In their studies, the variance explained by the size of the minor addend was between 60% and 75%. Furthermore, thanks to the min model, it was possible to predict the performance of children in different parts of the world, demonstrating that it could be widely replicable in different contexts.

1.3.2 Ashcraft's model (1987)

The Min model was also significantly extended by Ashcraft (1987) in later years. Ashcraft confirmed that the best predictor of solution times for pupils attending the first year of primary school was the size of the smaller addend, while the best predictor of times for children in the fourth year was the squared sum. As far as the children of the third class were concerned, the two previous predictors combined to determine the response times.

Ashcraft concluded that younger children consistently used the Min strategy, while older children used the retrieval strategy, much like what adults do. The children who attended the third class represented the transition point between the two types of strategy. This model proposed a theory of the development of simple arithmetic performances. Ashcraft's goal was to discriminate the knowledge that children possess in the various stages of their formal education in arithmetic and explored the processes they use to access that knowledge. Simulation is a particular instance of the model. It implements the important hypotheses of the model in the formalism of a computer program and provides a means to evaluate these hypotheses and, by extension, the adequacy of the model. This theory is based on the processing of information in cognitive psychology, applied to the development of the child, and in particular on the chronometric approach to mental processing (Posner, 1978). The model concerns the underlying mental representation of numerical knowledge, in the tradition of long-term semantic memory models (Anderson & Bower, 1974; Collins & Quillian, 1972; Craig, 1978). Knowledge structures and processes are available at different ages, and this model makes predictions for different ages. Learning is the central process of mental development in the model and in the simulation.

Ashcraft believed that addition problems are saved in a spreading-activation network. In this model, Reaction Times are defined by the intensity of the association between a problem and its answer. The problem-size effect was described by assuming that small problems have higher strength values than large problems because small problems were faced more frequently. This assumption was corroborated by data showing that the frequency of presentation is inversely related to problem size in grade school textbooks for addition (Hamann & Ashcraft, 1986) and multiplication (Siegler, 1988).

1.3.3 The Network-interference model (Campbell and Graham, 1987)

Campbell and Graham (1985) introduced a network-interference model of mental multiplication. In this model, multiplication problems are expressed as a network of connections between operands and products. When operands are presented, they transmit activation to the products of which they are multiples. The speed and accuracy of the correct answer depend on the activation of the correct product related to the activation of the incorrect products. The network-interference model is comparable to Ashcraft's model in that retrieval speed is based on the strength of association between a problem and its solution, but the network model diverges in that it considers interference from related problems. Comparable to Ashcraft (1987), Campbell and Graham (1985) assumed that the strength of association is determined by the frequency of presentation. However, Campbell and Graham (Campbell & Graham, 1985; Graham & Campbell, 1992) argued that network strength values might be determined not only by the frequency with which problems are presented but also by the order in which they are learned. They pointed out that children study small problems before large problems (Hamann & Ashcraft, 1986). The training of the large problems is assumed to be impaired by proactive interference from the previous learning of the small problems. In the end, the activation values for the correct answers to the large problems never accomplish the level of those for small problems.

Graham and Campbell (Graham & Campbell, 1992) presented some support for this hypothesis in an experiment with an "alphaplication" task. In this task, participants are presented with a problem, such as A, I X, which they try to learn as if it was a multiplication problem. In this example, the subject should learn to respond "x" when either A, I or I, A is presented. This alphaplication task is similar to multiplication except that the answers cannot be computed: they must be learned using memory.

Problems were split into two sets, with one set being learned before the other set. After both sets had been learned participants were examined on all problems. Problems from the second set were resolved more slowly than problems from the first set. These findings, with the fact that smaller problems are learned before larger problems in right multiplication, shows that proactive interference could probably generate a problem-size effect in the multiplication table.

The two tools suggested by the retrieval models to account for the problem-size effect, frequency, and order of presentation, suggest distinct predictions for the effects of additional practice. The frequency mechanism implies that the problem-size effect is the result of differential practice and that it could be eliminated if large problems are given enough practice so their strength values could "catch up" to those of the small problems.

The proactive interference (order of practice) mechanism suggests that the problem-size effect, once established, would be impossible, or at least complicated to eliminate through additional practice. On the assumption that the representation of small problems in the network is permanent, their prior status in the network should always interfere with the establishment of strength values for large problems. Thus, an ability to eliminate or reduce the problem-size effect through practice would be better predicted by the frequency than the proactive interference explanation.

1.3.4 From Piaget structural theory to developmental pathways of numerical skills

Piaget's structural development theory (PIAGET, 1962) has been the predominant theory for the development of mathematical skills for years. For Piaget the relationship between mathematical reasoning and the enhancement of general cognitive skills is dominant; the child is autonomous in learning the concept of number, but he does it only around the age of six because he must first acquire the concepts of cardinality and seriousness (Piaget & Szeminska, 1941). Mastering cardinality means being able to create a one-to-one correspondence between two sets of objects and to maintain it despite the position of the objects being modified. The seriation, instead, consists of putting in order elements belonging to the same class, based on their size.

These two concepts are fundamental in order to access the conceptual understanding of the number, and according to Piaget, this cannot happen before the concrete operational stage, when the children are able to solve the number conservation problems correctly.

However, numerous studies agree on contradicting Piaget's theory, stating that mathematical abilities manifest themselves in the child already in the first months of life. Many scholars believe that mathematical skills are based on an early non-symbolic number representation system (Feigenson et al., 2004), which allows to discriminate between two numbers already during the first year of life, improving considerably before twelve months of age. For example, at six months children are able to perceive the difference between two sets of 8 and 16 objects, but not between 16 and 12. At nine months, on the other hand, they are able to discriminate both relationships (Lipton & Spelke, 2003; Xu, Spelke, & Goddard, 2005).

In 1980 Starkey and Cooper had demonstrated the presence of mathematical skills long before the age of six, using the technique of habituation. The researchers placed the attention on the fixation times of the child when specific quantities of objects were proposed to him. Faced with the repetition of the same quantity, the fixation times were decreased, since the child had "memorized" the characteristics of that set of objects; when a different quantity was presented, the fixation times suddenly increased, demonstrating that the child could discriminate between the two different quantities.

Still focusing on fixation times, in 1990 Wynn studied five-month-old children to find out if they had any primordial concepts of addition and subtraction. In her experiments, she showed children a puppet that was later covered with a black panel; subsequently, children saw the hand of the experimenter who was going to place another puppet behind the screen. When the panel was removed, the children were shown two puppets, the result of a correct operation (1 + 1 = 2), or a single puppet (1 + 1 = 1). The attention of children was much higher in the case of a wrong result as if they had within themselves an expectation towards the correct number of puppets to be found behind the panel. The same thing happened if the proposed operation was subtraction.

From a developmental perspective, some studies suggested that language is necessary for the growth of numerical competencies (Hauser, Chomsky, & Fitch, 2010; Elisabeth S. Spelke, 2003). Others, however, demonstrate that numerical competence can develop independently from linguistic skills (Landerl et al., 2004). Landerl et al. (2004) selected thirty-one children aged 8 and 9 years who had dyscalculia, difficulties in reading, or both, and she compared these two groups to a control group on a set of basic number-processing activities. It was found that children with math difficulties had only compromised performance on mathematical activities, despite high average performance on IQ tests, vocabulary, and working memory activities. Children with reading disabilities were slightly compromised only on tasks related to phonological awareness, while children with both disorders showed a numerical impairment that was similar to that of the dyscalculic group, with no particular characteristics due to their reading or language deficits. Landerl et al. support the theory that the two processes are independent. Difficulties in mathematics were therefore specific for basic numerical processing without compromising other cognitive abilities and many authors argued that the number system is able to develop independently from the language domain (Donlan, Cowan, Newton, & Lloyd, 2007; Gelman & Butterworth, 2005; Hermelin & O'Connor, 1990). The neuropsychological evidence supports this indication, showing that numerical processing is located in the parietal lobes bilaterally, particularly in the intra-parietal sulcus (Dehaene et al., 2003), and is independent of other abilities. Developmental dyscalculia is probably the result of the weakness of these brain areas.

1.3.5 Von Aster and Shalev's Developmental model

Von Aster and Shalev (Von Aster & Shalev, 2007) introduced a developmental model that is, in some characters, similar to the Dehaene's adult calculation model (Dehaene, Cohen, Sigman, & Vinckier, 2005). Their model distinguishes semantic and symbolic number descriptions and divides the semantic number system into two parts: an early, implicit core system of magnitude (cardinality) and a later, explicit mental representation of the number line (ordinality) (Landsmann & Karmiloff-Smith, 1992).

Von Aster and Shalev's developmental model of numerical cognition could be recognized as a first effort to better understand the nature of mathematical knowledge.

This model assumes that different skills are acquired at different times (four steps), according to a precise hierarchical order.

Figure 3 represents the developmental model of cognitive number representation that is hierarchically ordered and could allow the prediction of different pathways of pathological development. It assumes that, at the first step, core-system representation of cardinal magnitude and various functions, such as subitizing and approximating, implements the basic meaning of number. This step is a necessary precondition for children to learn to connect a perceived number of objects or events with spoken or, later, written and Arabic symbols. At the second step, there is the learning of verbal number system and the third is the Arabic symbolization. Those two steps are, in turn, the precondition for the evolution of a mental number line (step 4) in which ordinality is represented as a second core aspect of number processing. The first step produces the foundations for the subsequent acquisition of all numerical skills. Children who cannot acquire these primordial skills may be able to learn the names of the numbers but do not associate the names with the meaning of a quantity.

These children, according to this model, are more likely to develop pure developmental dyscalculia. On the other hand, children who fail in one of the next three stages usually present dyscalculia in comorbidity with other disorders. In this case, the deficit in numerical skills can be determined by attentional, executive, or linguistic difficulties, which interfere with normal development. For this reason, children with dyslexia, specific language disorders, attention disorders, or working memory weakness have a higher risk of developing dyscalculia. In these cases, the number sense remains intact, but these children present difficulties in the production and automatization of the counting sequences, thus failing to develop the necessary strategies to recall arithmetic facts.

An essential contribution of this model is the recognition of the interaction of numerical and non-numerical skills. Many studies indicate that numerical skills are also influenced by working memory, language, attention, and spatial skills (Bugden & Ansari, 2011; Pixner et al., 2011). According to Dehaene, instead, the different numerical representations are closely connected and can work simultaneously.

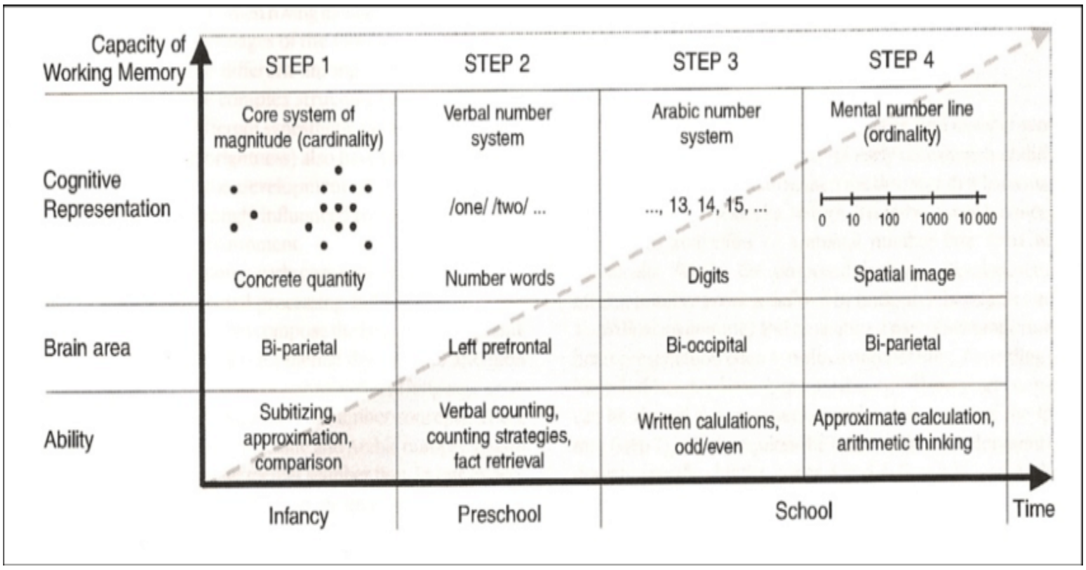

Figure 3. Four-step-developmental model of numerical cognition. (Von Aster and Shalev, 2007)

1.3.6 Developmental calculation model

Kaufmann and colleagues integrated Von Aster and Shalev's model in 2011. They expressed a developmental calculation model based on experimental data acquired from functional Magnetic Resonance Imaging (fMRI). This model highlights two main components: numerical processing and calculation. Their model agrees that number processing and computation are complex and multi-component systems (Dehaene & Cohen, 1995b; McCloskey et al., 1985). The theoretical basis on which this hypothesis is based comes from studies on acquired dyscalculia (Dehaene, 1997; Dehaene & Cohen, 1995b; Margarete Delazer et al., 2006) and studies on typical adults (Gérard Deloche, Souza, Braga, & Dellatolas, 1999; Geary & Widaman, 1992) and children (Dehaene & Wilson, 2007; Dowker, 1998; H. Ginsburg, 1977; N. C. Jordan, Hanich, & Kaplan, 2003). Despite being treated as relatively separate systems, arithmetic and its components do not work in isolation. The calculation also depends on attention and working memory. Working memory plays a crucial role both for more straightforward mental calculations (Ashcraft, 1995; H. Kaufmann & Schmalstieg, 2002; Lemaire, Abdi, & Fayol, 1996) and for more complex calculations (Furst & Hitch, 2000; Passolunghi & Siegel, 2001).

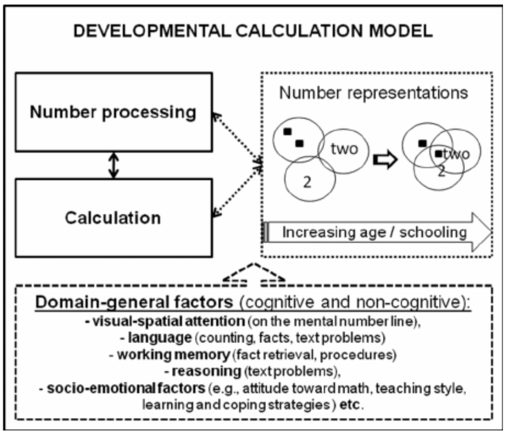

Figure 4 Developmental calculation model (Kaufmann, 2002)

Numerical processing and calculation are linked each other, they are connected to number representation, and modulated by domain-general factors like memory and language. When children grow up, the numerical representations involve an overlap of number processing and calculation that determines analogue magnitude, Arabic, and verbal numerical representations. These changes are a result of increasing age and experience and reinforce the idea that the nature of these developmental processes is dynamic (Ansari, 2010; Karmiloff-Smith et al., 1998; L. Kaufmann & Nuerk, 2005).

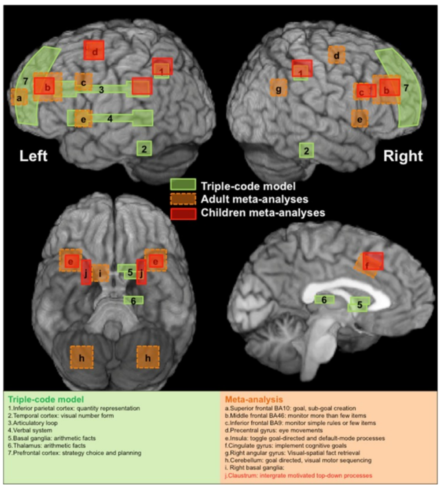

Figure 5 Triple Code Model (Arsalidou et al., 2018)

1.3.7 The Approximate Number System (ANS)

The ANS acuity increases significantly during childhood. If as a child can have limited performance due to various factors, with growth, the performances in these tests become not only better, even more specific and more predictive of their future mathematical abilities (Justin Halberda & Feigenson, 2008; Libertus & Brannon, 2010; Lipton & Spelke, 2003; Piazza et al., 2010; Xu & Spelke, 2000).

The ANS acuity is measured in children and adults with evidence of non-symbolic comparison of quantity. Usually, they are asked to look at the screen with two groups of dots and to indicate as soon as possible the group with the largest quantity. The best performances are those of the participants who manage to discriminate even when the ratio between the two quantities shown is tiny.

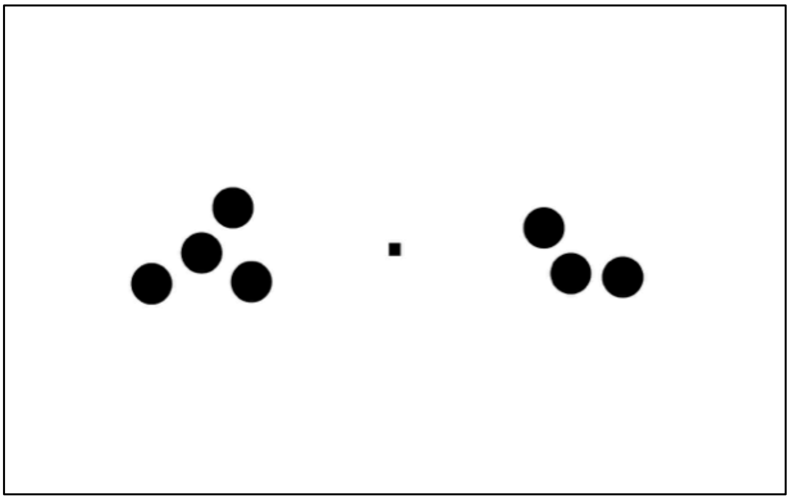

Figure 6 Non-symbolic magnitude comparison task

Numerous studies have shown that the differences among individuals that emerge in this type of evidence, as regards adults, are strongly correlated with their mathematical performance, while for children, they have a marked predictive value on their future skills (Bonny & Lourenco, 2013; Libertus et al., 2011, 2012; Lourenco, Bonny, Fernandez, & Rao, 2012; Mazzocco et al., 2011). This predictivity is explained by the fact that during the learning of numbers and symbols that represent numerosity, children construct mental maps constituted precisely by non-symbolic representations. The solidity of these mental representations will serve in the construction of the ANS, which will be very useful in the management of quantities in the future (Brankaer, Ghesquière, & De Smedt, 2014; Pinheiro-Chagas et al., 2014).

1.4 Relationships between numerical knowledge and domain general cognitive processes

1.4.1 Language and mathematical skills

Many data confirm a meaningful relationship between language and formal symbolic mathematics (Bull & Johnston, 1997; J. A. Jordan, Wylie, & Mulhern, 2010; N. C. Jordan, Kaplan, Nabors Oláh, & Locuniak, 2006). Several researchers (Hooper, Roberts, Sideris, Burchinal, & Zeisel, 2010; Purpura, Hume, Sims, & Lonigan, 2011; Romano, Babchishin, Pagani, & Kohen, 2010) emphasized the value of introducing language as a predictor of mathematical abilities. Nevertheless, it remains an open question to what amount mathematics is dependent on language. For example, some people can perform very well in some calculation tasks despite language difficulties. Furthermore, studies showed that even preverbal infants already process numbers (Ceulemans et al., 2012; Praet, Titeca, Ceulemans, & Desoete, 2013). Some researchers studied tribes in Amazonia with an atypical language structure and an atypical way of representing numbers to try to explain the role of language and numeracy better. Gordon (2004) observed the Piraña who have a counting system of 'one-two-many' but manage to compare quantities despite their limited numeric words. Also, Pica, Lemer, Izar and Dehaene (Pica, Lemer, Izard, & Dehaene, 2004) examined the Mundurukü; they don't use number words up to five and used to make their choice based on estimation. In a comparison task, they performed as well as the European control group, but they failed on exact arithmetic tasks. From this point of view, Pica et al (2004) concluded that estimation is a primary skill independent of language. To make better use of mathematics, however, there is need to a well-developed vocabulary of number words. Additional evidence for the assertion that language is connected with succeeding mathematical skills can be seen in the model of adult problem solving with one (semantic), two (semantic and non-semantic), or three (semantic, visual and auditory-verbal word frame) supervariables (Cipolotti & Butterworth, 1995; Dehaene, 1992; McCloskey & Macaruso, 1995). Dehaene and Cohen assumed three variables in their Triple Code model asserting that numbers can be expressed in three different ways: as a quantity system (a semantic representation of the size and distance relations between numbers), as a verbal system (where numerals are represented lexically, phonologically, and syntactically), and as a visual system (as strings of Arabic numerals).

There is evidence that language processes are surely involved in solving simple mathematical problems, in particular, arithmetical addition and subtraction. For example, bilingual adults have been manifested to acquired addition and subtraction facts more efficiently in the language of practice compared with the untrained language, suggesting that arithmetic facts are filed in language-specific ways (E. Spelke & Dehaene, 1999; Elizabeth S. Spelke & Tsivkin, 2001). Spelke and Tsivkin (2001) showed that also incidental exposure to exact numbers (e.g., learning a date in history) is deposited in language-specific ways, such that language of training affects how numerical information is saved and subsequently reclaimed.

Phonological processes seem to underlie this relation in a specific way, presumably because performing arithmetic problems demands the retrieval of phonological codes (Fuchs et al., 2006; Hecht, Torgesen, Wagner, & Rashotte, 2001; Koponen, Aunola, Ahonen, & Nurmi, 2007; Fiona R. Simmons & Singleton, 2008). Indeed, the well-documented connection between phonological processing and arithmetic accomplishment helps to describe the conclusion that many children with reading difficulties also have difficulty with arithmetic (Dirks, Spyer, Van Lieshout, & De Sonneville, 2008; Rubinsten & Henik, 2009; Fiona R. Simmons & Singleton, 2008).

Nevertheless, there are some children with mathematical difficulties which are good readers and vice versa (Landerl, Fussenegger, Moll, & Willburger, 2009; Vukovic & Lesaux, 2013), suggesting that, for some children, phonological skills are not the unique factor responsible for their reading difficulties. Moreover, Jordan and colleagues have discovered that children can compensate for arithmetical problems by using particular verbal approaches to perform arithmetic problems, proposing that language skills behind phonological processing are involved in arithmetic performance (N. C. Jordan et al., 2003). There is further proof that the language used in arithmetic problems determines how children symbolically express and resolve such problems (Abedi & Lord, 2001; Brissiaud & Sander, 2010; Lager, 2006). Together, these findings imply that language ability may play a primary, although not exclusive, role in children's mathematical cognition.

In a relevant study, LeFevre and colleagues (2010) suggested that the linguistic circuit developed by Dehaene and colleagues (2003) is involved when children perform mathematical tasks that are dependent on the formal number system. The authors found that vocabulary, phonological awareness, and number identification, measured in a sample of preschool children, revealed unique variance in second-grade arithmetic. Working memory and other cognitive measures did not seem to be involved at the same level as the linguistic awareness. The linguistic knowledge was the most powerful predictor over various domains of mathematical cognition, suggesting that language skills have a critical role in children's understanding of every mathematical domain.

It has to be underlined that math and language knowledge are complex domains, with many subcomponents. Therefore, relationships between the two domains depend on the specific areas or task involved. In particular, it seems that phonology has an impact on some aspects of mathematics, but it cannot be generalizable to all arithmetic (Fuchs et al., 2006; Locuniak & Jordan, 2008; Swanson & Beebe-Frankenberger, 2004). Dehaene and colleagues (1999) found that whereas adults preserve exact arithmetic sums as language-based representations, approximate calculations, including advanced mathematical facts, may be completed independently of language. Consequently, the authors hypothesized that higher-order forms of mathematics might not be as dependent on language as is arithmetic.

In a recent study (Praet et al., 2013), the importance of language was studied with respect to different aspects of arithmetic. Participants were evaluated at age 5-6 on receptive and expressive language. Findings showed that 10% of the children at that age had a language problem. All of the children with a language problem had additional problems with procedural counting and difficulties with the knowledge of the numerical system, even when intelligence was checked. Moreover, 7.31% of the 5-6-year-olds with counting problems had a lower receptive language index compared with their peers without counting problems. Finally, language had a value-added of 21.6% to number naming and counting as predictors for early arithmetic abilities in kindergarten. Language in kindergarten uniquely predicted procedural counting abilities, knowledge of the numerical system, and early calculation skills at age 5-6. When children were followed-up 1 year later in grade 1, kindergarten language still predicted arithmetic abilities of the children.

Especially, expressive language at age 5-6 predicted prospectively number knowledge, mental arithmetic, number facts retrieval, and clock reading tasks at age 6-7. In a functional magnetic resonance imaging (fMRI) study, Fedorenko, Behr, and Kanwisher (2011) discovered limited or negative response by functionally localized language areas to sequential mathematical tasks such as summing four consecutive numbers. But there is evidence proposing that tasks involving hierarchically structured mathematical expressions affect brain regions that are shared, or nearby to those involved in analogous linguistic tasks (Friederici, Bahlmann, Friedrich, & Makuuchi, 2011; Makuuchi, Bahlmann, & Friederici, 2012). Lastly, Varley, Klessinger, Romanowski, and Siegal (2005) explained that patients with critical agrammatic aphasia could nevertheless perform well at different mathematical tasks. This latter finding proposes that any potentially shared representations must be separate at some domain-general level that is independently accessible by mathematics and language.

1.4.2 Visual predictors of mathematical skills

Although the relationship between motor, visual perceptual, and visuomotor integration abilities and mathematics is not completely known, several essential findings are proposing a connection between those domains. The 'embodied cognition' literature describes the relationship between motor and mathematical skills and demonstrates that cognitive processes are grounded in the interaction of the body with the world (Soylu, 2011). Furthermore, the relationship between motor skills and mathematics received support in predictive studies in which fine motor skills were significantly associated to mathematics scores (Luo, Jose, Huntsinger, & Pigott, 2007; Pagani, Fitzpatrick, Archambault, & Janosz, 2010; Vuijk Pj Fau - Hartman, Hartman E Fau - Mombarg, Mombarg R Fau - Scherder, Scherder E Fau - Visscher, & Visscher, 2011). In addition, Kulp, Earley, Mitchell, Timmerman, Frasco, and Geiger (Kulp et al., 2001) observed that low scores for visual perception were associated with poor mathematical abilities. Visual perception regards the process of evaluating and organizing visual information (Kavale & Forness, 2000). Visual perceptual skill could be subdivided into sections such as visual discrimination and visual memory. Visual discrimination includes the capacity to attend to and recognize a figure's distinguishing features and details, such as shape, orientation, colour and size. Visual memory relates to the ability to remember a visual representation. Mazzocco and Myers (Mazzocco & Myers, 2003) described that the lack of the ability to find the correct position of figures, according to their common features, is a specific character of people with poor mathematical performances. Moreover, Cirino, Morris, and Morris (Cirino, Morris, & Morris, 2007) found that visual perception contributed to prognosticating mathematical achievement in college students with learning difficulties, although their previous study did not find this contribution (Cirino, Morris, & Morris, 2002). Kulp et al. (2004) found that visual-motor integration was significantly related to teachers' ratings of mathematical skills in children. Assel et al. (Assel, Landry, Swank, Smith, & Steelman, 2003) investigated whether children's visual-spatial skills from 3 to 6 years of age were cognitive precursors to their future mathematical competence. They reported that visual-spatial skills were found to have their own specific effects on math abilities. Pieters et al. (Pieters et al., 2012) showed that 24.8% of the children with *mathematical difficulties* had motor problems. Another study was focused on the visual perception, motor skills, and visual-motor integration, and how those abilities were related to mathematical performances (Pieters, Desoete, Roeyers, Vanderswalmen, & Van Waelvelde, 2012). All those measured domains described a large proportion of the variance in either number fact retrieval (40%) or procedural calculation (38%). Moreover, children with *mathematical difficulties* were found to have problems with all the measured domains in comparison with age-matched typically developing children.

However, Vukovic and Siegel (2010) revealed, in line with Geary, Hamson, and Hoard (2000), and Morris, Stuebing, Fletcher, Shaywitz, Lyon, Shankweiler, et al. (1998), that a block rotation task could not presumably differentiate children with persistent mathematical difficulties from children with transient mathematical difficulties or control children, suggesting that visual perception is not a cognitive predictor for mathematical difficulties.

To conclude, although it seems that a relationship exists, research on motor, visual perception, and visual-motor integration skills as predictors for mathematics, and mathematical difficulties has given unclear and indecisive results.

1.4.3 Working memory and mathematical skills

Working memory has to be perceived as an operating system that manages complex cognitive functions. Numerous researches have found working memory problems in children with mathematical difficulties (Swanson & Jerman, 2006; Temple & Sherwood, 2002) .

Solving mental problems accurately depends on executing a series of procedures, including borrowing, recalling mathematics facts from long-term memory holding intermediate values in memory, performing the relatively slow division process, and keeping track of various intermediate steps and solutions. All of these steps demand a significant amount of cognitive effort. The mental portion capable of this coordination of procedures is working memory.

Working memory is usually viewed as a limited capacity mechanism that allows the mind to integrate, compute, store, and manipulate information at the focus of a person's attention (Alan D. Baddeley & Andrade, 2000; Engle, 2002; Miyake & Shah, 1999). Working memory has been implicated in a wide range of cognitive domains, including attention, memory, language, and overall intelligence. Working memory has also been confirmed to be an essential component of arithmetic and mathematics performance (De Rammelaere, Stuyven, & Vandierendonck, 1999; Imbo & LeFevre, 2010; Imbo & Vandierendonck, 2007, 2008; Seyler, Kirk, & Ashcraft, 2003). Researchers of mathematics cognition have examined the multi-component theory of working memory suggested by Baddeley and collaborators (A. D. Baddeley & Logie, 1999; Alan D. Baddeley & Andrade, 2000).

This model describes working memory as the on-line coordination of three unique components of information processing: the central executive and two slave systems, the phonological loop and the visuospatial sketchpad (a recently proposed fourth component, the episodic buffer, has not yet been investigated to a significant degree). The central executive acts as the command centre of processing, fulfilling such activities as focusing and switching attentional scope, performing calculations, and coordinating the information momentarily maintained by the slave systems. The activities of the phonological loop include active rehearsal and storage of verbal and semantic information, while the visuospatial sketchpad is implicated in the creation and storage of mental representations that emerge during processing the task at hand (Baddeley & Logie, 1999). For researchers, an interesting character of the multi-component model is the opportunity of evaluating the subcomponents individually, and relating them to possible verbal or visual characteristics of problem-solving. This is typically done within a dual-task paradigm. That is, researchers design experimental situations to use the limited resources of one system while attempting to leave another system untouched, to see if this alters overall performance. In this paradigm, the participant completes two tasks concurrently, a primary task (a mathematics task) along with a secondary task selected to assess the processing or storage capacity of one particular component of working memory. If the secondary task interferes with accurate or efficient performance, it can be inferred that both tasks rely on the same type of processing or information. For example, if a secondary verbal task were to conflict with maths problem solving, we would assume that arriving at the correct solution needs the verbal processing sources utilized by the secondary task (Hitch, 1978). There are precise data that the central executive plays a vital role in addition and multiplication performance.

In several studies, a secondary task involving the central executive (e.g. generating a random string of letters) was joined with the arithmetic task, showing interference on problem-solving compared with control conditions (DeRammelaere et al., 1999; DeRammelaere, Stuyven, & Vandierendonck, 2001; Imbo & Vandierendonck, 2007; Lemaire, Abdi, & Fayol, 1996; Seitz & Schumann-Hengsteler, 2000). The confirmation of the central executive's role in arithmetic processing extends well beyond studies that examined the simple addition and multiplication facts, however. Logie, Glhooly, and Wynn (1994) asked participants to perform a central executive load task while adding two-digit numbers across 20 seconds. The results showed an evident disturbance of performance when the central executive was loaded. With the auditory presentation of the numbers to be added, errors almost tripled, from 14% in the control condition to 38.5% in the dual-task condition; with visual presentation, errors increased from 3.5% to 44%. Similarly, Fürst and Hitch (2000) presented multiple-digit addition problems to their participants and modified the number of operations expected, while also loading the central executive with a secondary task. Errors increased substantially (from 15 to 45%) when the central executive was loaded compared with the phonological load condition (Imbo, Vandierendonck, &lelaere, 2007; Heathcote, 1994).

LeFevre (2003) studied the roles of the two slave systems involved in complex operations. Additions were presented in combination with a secondary task that either assessed the phonological loop (memorizing some non-words) or visuospatial sketchpad (remembering the location of symbols). In secondary tasks, participants had to check if the word or the location, presented after the addition problem, was the same as the display presented before the problem. Moreover, addition problems were shown in two different formats, horizontally to one group, to be read from left to right, or vertically to the other group, to be read from top to bottom. The results revealed that obstruction of the secondary tasks depended on the format of the addition problems. When the problem was presented horizontally, performance suffered more under the phonological load; when the problem was presented vertically, the performance was worse under the visuospatial load. These results turned to an interpretation based on the activation of a specific code (phonological or visual) used for various procedural approaches in problem-solving. Horizontal problems seemed to be represented and solved via a verbal code, whereas a visual representation of calculation seemed apparent in the vertical format condition (Imbo & LeFevre, 2010).

1.4.4 Emotional and motivational influences on math skills

Stevenson et al. (1990) reported that the 72% of the children started formal education with a positive view of mathematics; in particular, the 87% of first grade children reported perfect feelings with arithmetic and mathematics, and 74% reported that their abilities were excellent (Moore & Ashcraft, 2009). With the increase of mathematical notions complexity and growth, children tend to lose much of their interest and motivation. This happens especially during adolescence. This trend is peculiar because interest in mathematics seems to be positively correlated with success. For instance, a meta-analysis investigating the importance of emotions in mathematics performance evidenced that the intensity of the relationship between maths approach and performance increases with age (Ma & Kishor, 1997). Being involved in mathematics seems to play a significant role in students' current and future performance in arithmetical skills. Köller, Baumert, and Schnabel (2001) analyzed evaluations of interest in mathematics, scores on a standardized mathematics exam, and enrolment in mathematics courses from a longitudinal sample of 600 students, tested in 7th, 10th, and 12th grades. The results revealed that students with higher levels of interest in mathematics were more successful in developing the higher-level mathematics sessions. Not surprisingly, those students who reported the highest levels of engagement in mathematics also recognized the domain as being more relevant, chose more mathematics programs, and achieved higher ranks in maths courses, compared with those who showed limited interest in the subject matter (Simpkins, Davis-Kean, & Eccles, 2006).

High motivation was found to be positively correlated to mathematics achievement scores and negatively related to maths anxiety (Zakaria & Nordin, 2008), to be predictive of future studies (Leuwerke, Robbins, Sawyer, & Hovland, 2004), and to improve mathematics self-efficacy (Berger & Karabenick, 20113 Lopez, Lent, Brown, & Gore, 1997).

If interest, motivation, and self-efficacy have a positive effect, mathematical anxiety has precisely the opposite effect. People who experience this kind of specific fear of mathematics are inclined to react evasively; they experience a strong feeling of distress when they find themselves in situations where mathematical operations must be done or problems solved, and they try in every way to avoid these tasks. Mathematics anxiety is far-reaching; it is correlated with negative performance in both males and females, although the exact deficit in performance may be gender-specific (Baloğlu & Koçak, 2006; Miller & Bichsel, 20004). Research has reported the demonstration of an inverse relationship between self-efficacy and maths anxiety (Lee, 2009, Cooper and Robinson 1991).

In summary, many cognitive skills, including reading and language skills are thought to interact with math abilities. In study 3 we directly addressed this issue by examining verbal and nonverbal cognitive predictors of children’s math skills and their relationships with reading skills. Also, in Study 2 we evaluated the role of cognitive and linguistic skills in predicting children’s early numeracy skills.

1.5 Home numeracy and intergenerational transmission

1.5.1 Home numeracy

Recent studies have found that children's mathematical skills before exposure to formal teaching are predictive of their future skills (Desoete & Gregoire, 2007; Jordan, Kaplan, Locuniak and Ramineni, 2007). In this consideration, the exposure to numbers that is made in the family context takes on great importance. However, in the specific context of mathematics, not much attention has been paid to domestic numerical activities. As far as literacy is concerned, however, there are many demonstrations of the usefulness of early and non-formal exposure. Reading books with children, for example, helps in the expansion of vocabulary and decoding skills (M Sénéchal & LeFevre, 2001). Sénéchal and LeFevre (Monique Sénéchal & LeFevre, 2002) have developed a model that implies the importance of both indirect experiences (reading of books by parents) and direct practices (teaching of reading) on language development and child literacy. In comparison, the field of early mathematics and mathematics development there is less evidence about how specific experiences outside school shapes mathematical knowledge (Ginsburg, 1982; Song &

Ginsburg, 1987; Huntsinger, Jose, Larson, Balsink Krieg, & Shaligram, 2000; LeFevre, Clarke and Stringer, 2002; Pan, Gauvain, Liu and Cheng, 2006).

In some recent studies, parents were asked to directly teach activities related to numbers and they found positive correlations between these activities and children's numerical abilities. LeFevre et al. (2002) observed that the frequency with which parents reported teaching number skills relate to their preschool children's maths competence. Huntsingeret al. (2000) found that the perseverance with which parents teach mathematics to children attending kindergarten were correlated with subsequent results in mathematics. The results of these studies suggest that there is a relationship between the frequency of direct numbering instructions given by the parents and the mathematical performance of their child. However, these results are not accurate on the typology of home numeracy that brings results and are therefore challenging to replicate.

LeFevre et al. (2009) propose a clear division between direct and indirect numerical activities, also in the field of mathematics, to obtain precise and useful results. Direct activities are considered those that focus on numbers and are used by parents to teach numerical aspects to children. For example, direct activities are to teach how to count and teach to recognize Arabic symbols and have been found to be related to the development of children’s symbolic abilities. Indirect activities are activities that do not focus directly on numbers but involve them like playing games where the numbers are present or doing household activities where you need to count. These activities have been found to related to children’s non-symbolic abilities (Skwarchuk, Sowinski, & LeFevre, 2014).

This research offers essential information about the nature of the activities that are associated to numeracy development and supports recommendations that children will benefit from numerical activities in many contexts (Balfanz, Ginsburg, & Greenes, 2003; H. P. Ginsburg, Lin, Ness, & Seo, 2003; Ramani & Siegler, 2008; Young-Loveridge, 2004).

However, is still an open question how home numeracy is related to parents’ own math skills. In other words, it is not clear if home numeracy activities are a primary predictor of children’s numerical abilities or if their role is subordinate to parents’ math skills. We addressed this issue in Study 2, where we considered home numeracy and parents’ math and ANS skills as predictors of children’s early math skills.

1.5.2 Intergenerational transmission

In reference to Morton & Frith's causal model (1995), when we describe the characteristics of each individual in relation to learning, different levels of analysis and their reciprocal relationships should be taken into account. The suggested levels are: biological, cognitive and behavioral components, together with environmental influences at each of these levels. Gottesman and Gould (2003) suggested that endophenotypes are heritable neurophysiological, biochemical, endochrinological, neuroanatomical or neuropsychological, although, they are likely to be influenced by complex interactions between genes and environments (Caspi & Moffitt, 2006). One of the main characteristics of the endophenotypes is that they might be observable before the disease onset and, notably, in individuals with a heritable genetic risk for disease, such as unaffected family members (parents and siblings). The term "broader phenotype" refers specifically to the cognitive endophenotypes that are shared with unaffected family members. It is also suggested that the investigation of the broader phenotype of a disorder might help to define the core deficits of that disorder, beside and above the behavioral symptoms that might be influenced by educational, clinical and environmental factors (Göbel & Snowling, 2010). Some studies have focused on the broader cognitive phenotype in dyslexia, revealing, for example, that phonological deficits are shared in non-affected family members and that parents and siblings of children with dyslexia underperform in reading measures compared to family members who are not at risk for the disorder (Snowling, 2008). Nevertheless, to our knowledge, there is a paucity of research regarding the role of the broader phenotype in developmental dyscalculia or in general related with mathematical weakness, although this line of research might represent a crucial factor when clarifying the complex relationship of primary and secondary cognitive and environmental influences on numerical skills.

Children and adults have two distinct systems for expressing and treating numerical information. The first, an approximate number system (ANS), provides for fast estimates about the number of items in a collection and gives the basis for rapid comparisons and approximate calculations without verbal counting (Barth, La Mont, Lipton & Spelke, 2005; McCrink & Spelke, 2010). The ANS is present in infants and non-human animals (e.g. Agrillo, 2015; Beran, Perdue & Evans, 2015; Izard, Sann, Spelke & Streri, 2009; Xu & Spelke, 2000) and is therefore not linked to language or knowledge of symbols. The second, an exact number system, can be utilized to describe numerical information precisely through counting and number symbols, which is necessary for school mathematics (Miller & Paredes, 1996). The human ability to represent numbers symbolically and perform exact calculations is developed through formal and informal instruction (Baroody & Wilkins, 1999).

There is much evidence that shows that the differences in the ability to make numerical approximations are closely related to school mathematical skills (Chen and Li, 2014; Fazio, Bailey, Thompson and Siegler, 2014; Feigenson, Libertus, Halberda, 2013). Few studies, however, have focused on the origin of individual differences in ANS acuity and in particular whether these depend on the specific ability of their parents. Once the relationship between parents' and their children's general cognitive skills is established, it seems possible that there are specific relationships also in mathematical area. In particular, a source of particular interest is that ANS seems to be a reliable indicator of continuity between parents and children. Not much research has been done in this area so far. Studies on twins are often used to discriminate which aspects depend on genetics and which are differences mainly due to the environment. The few studies carried out so far in the field of mathematics using twins have replied that both components play an essential role. General cognitive factors such as IQ but also specific ones such as memory and attention are considered necessary for the development of individual differences (LeFevre, Fast, Skwarchuk, Smith - Chant, Bisanz et al., 2010a; Passolunghi & Siegel, 2004; Wilson & Swanson, 2001), the SES (Jordan, Kaplan, Ramineni & Locuniak, 2009), the stimulation of the environment consisting of parents, brothers and teachers (Gunderson & Levine, 2011; Klibanoff, Levine, Huttenlocher, Vasilyeva & Hedges, 2006; Levine, Suriyakham, Rowe, Huttenlocher & Gunderson, 2010), but the ANS is also starting to emerge as one of the most important predictors for the development of mathematical skills.

Intergenerational transmission indicates the process by which parents affect their children behaviourally or psychologically, and as such includes genetic and environmental factors. Many researchers studied the intergenerational transmission of cognitive abilities from parents to their offspring (Agee & Crocker, 2002; Anger & Heineck, 2010; Björklund, Hederos Eriksson, & Jäntti, 2010; Black, Devereux, & Salvanes, 2009; DeFries, Plomin, Vandenberg, & Kuse, 1981; Thompson, Plomin, & DeFries, 1985). For example, Anger and Heineck (2010) observed that children's intelligence quotient was positively associated with their parents' intelligence even when considering education, parental occupation, and socioeconomic status (SES). Comparable intergenerational transmission patterns are visible even when children are evaluated at young ages: parents' general cognitive abilities are correlated with measures of mental development for their 1- and 2-year-old children (DeFries et al., 1981; Thompson et al., 1985). Some studies have focused on a broader cognitive phenotype in dyslexia, revealing, for example, that phonological deficits are shared in non-affected family members and that parents and siblings of children with dyslexia underperform in reading measures compared to family members who are not at risk for the disorder (Snowling, 2010).

Van Bergen et al. (2014) found that children at family risk of dyslexia experience at least some of the etiological risk factors: they inherit genetic risk factors and could experience a less rich literacy environment. Therefore, it is hypothesized that children at risk have a higher genetic and environmental responsibility than children without a family history of dyslexia. Furthermore, children at risk who develop dyslexia show cognitive deficits (at various levels) in different processes.

The phenotypic aspects of parents that help to understand the predisposition of children towards dyslexia are skills in accurate and fluent reading, spelling and their cognitive bases such as phonological awareness and rapid naming. Related skills (such as language and arithmetic) and their underlying cognitive abilities may also play a role. The ability of parents on each of the relevant continua can be conceptualized as a position in multivariate space. The position of father and mother in multivariate space is proposed to be indicative of a child's predisposition towards dyslexia.

Little research has investigated the role of intergenerational transmission concerning parents and children's skills in mathematics and arithmetic (Blevins-Knabe, Whiteside-Mansell & Selig, 2007; Brown, Mcintosh & Taylor, 2011; Crane, 1996; Duncan, Kalil, Mayer, Tepper & Payne, 2005). A research conducted with primary school children found that children's mathematics results were significantly correlated with the mathematical abilities of their mothers (Crane, 1996). In that study, however, both linguistic and mathematical abilities were taken into consideration, and no clear distinction had been made that would allow a better understanding of the specific contribution of each skill.

Recently Braham and Libertus (2017) found a specific relationship of the ANS between parents and children. ANS was considered as a measure within a protocol that also presented many other aspects of mathematics, and in that case, the relationship between the ANS of parents with that of their children, who attended primary school, did not depend from the general mathematical skills of parents. Some studies have verified that the ANS can improve with the entrance to the formal school and with greater exposure to the teaching of mathematics (Nys et al., 2013; Piazza, Pica, Izard, Spelke and Dehaene, 2013). If this was the case, with the modification of the ANS of the children, the relationship between the ANS of parents and children should also change with time and would not be so strong.

Shalev (Shalev et al., 2001), described the specific disorder that affects mathematics as a "family disorder". This is because much higher percentages of children with dyscalculia have been found in families where an affected component was already present. Some research has focused on intergenerational transmission of literacy (van Bergen, Bishop, van Zuijen, & de Jong, 2015) and mathematics (Braham & Libertus, 2017; Navarro, Braham, & Libertus, 2018; Authors, presented) even in typical populations. Navarro et al. (Navarro et al., 2018) have tested 1- to 3- years-old children on a modified numerical preferential looking paradigm and their parents on a non-symbolic number comparison task. To assess the specificity of the intergenerational transmission, parents also completed a questionnaire assessing their math ability and inclination for math as well as a language questionnaire evaluating their child's expressive vocabulary. They found that the parents' ANS abilities were linked to the processing of the number of children and that this relationship was independent of children's vocabulary or the mathematical ability observed by parents, suggesting a specific intergenerational transmission of the ANS.

In a different study by Braham and Libertus (Braham & Libertus, 2017), authors administered on 54 children (5-9 years) and their parents and found that the ANS acuity of the children was related with ANS acuity of parents. Moreover, the mathematical abilities of the children were anticipated by unique combinations of parents' ANS acuity and mathematical abilities according to the specific mathematical ability in question. In particular, parents with higher ANS acuity have children with higher ANS acuity themselves. It is important to remark that parents' ANS acuity was the only parent measure that significantly correlated with children's ANS acuity. Moreover, parents' ANS acuity significantly predicted children's ANS acuity when controlling for parents' math proficiency and math expectations for their children. These findings insinuate a specific intergenerational association of an unlearned numerical competence that is distinguished from culturally transferred mathematical abilities. However, the possibility of intergenerational transmission of ANS acuity from parents to their children has not yet been resolutely explored and this topic will be addressed in study 1 and study 2.

1.6 The current studies

My thesis aims to investigate the mathematical profile of children by assessing math skills, involving ANS related skills and symbolic skills. A specific focus was directed towards intergenerational transmission of math skills both considering the ANS domain and more complex formal math skills. Further, relationships between reading and math skills were examined considering the role of shared verbal and non-verbal domain general cognitive functions.

The present work is composed of three studies, which involved children from kindergarten and primary schools, and their parents.

The first study examined the role of symbolic and non-symbolic numerical abilities of mothers to understand if these were predictors of children's numerical skills, either considering basic symbolic, non-symbolic and formal math skills, i.e., written calculation.

In particular, the intention was to understand whether the mothers' symbolic / non-symbolic abilities in magnitude comparison tasks were instrumental for their children's achievement in a formal math task, or whether the most important aspects were to be found in mothers' formal mathematics skills. Moreover, we added children's own symbolic and non-symbolic skills as predictors of their math performance, considering past literature on how these basic tasks can predict formal maths skills (Gilmore et al., 2010; Starr et al., 2013).

The second study focused intergenerational transmission of numerical skills considering a different age range, that is, the study was conducted on preschool children and their parents. This study aimed to understand whether the parents' abilities in mathematical tasks, in particular the ANS tasks, were predictive for their children's early numeracy skills (Braham & Libertus, 2017; Navarro, Braham, & Libertus, 2018). In the model, measures of home numeracy activities and children's cognitive, linguistic and ANS skills were added.

The third study investigated the relationships between reading and math skills as well as the interaction of these abilities with the cognitive skills believed to underlie math development. Children from 4th and 5th grades of primary school were administered measures of reading and arithmetic, nonverbal IQ, and various underlying cognitive abilities of arithmetic (counting, number sense, and number system knowledge). The aim was to assess relationships between reading, arithmetic, and the cognitive correlates with the hypothesis that cognitive correlates related to the phonological domain would be related to both reading and arithmetic. In contrast, tasks associated with the number sense domain, measured through tasks of magnitude processing, would be predicted only by math, and not by phonological skills.

Chapter 2

Intergenerational features of math skills: Approximate Number System and written calculation in mothers and children.

2.1 Introduction

The emergence of formal mathematical competencies results from the complex interplay amongst multiple factors. Early skills encompassed in the so-called Approximate Number System (ANS) (Dehaene, 1997; Gallistel & Gelman, 2000), seem to represent the basis on which, through the interplay with other cognitive skills (mainly language and working memory) (Von Aster & Shalev, 2007), numerical knowledge develops from preschool to primary school, when formal teaching shapes arithmetic ability. Further, many pieces of evidence now suggest that parents count in the development of children's math skills, either considering home numeracy activities (Lefevre et al., 2009) or through mediation effects of parents' math anxiety (Maloney, Ramirez, Gunderson, Levine, & Beilock, 2015a) or stereotypes (Tomasetto, Alparone, & Cadinu, 2011). However, limited research has investigated the intergenerational influence of math skills, that is the relation between parents' and children's skills in the numerical domain, although many studies showed parent-child relations in other domain such as reading ability (van Bergen, van der Leij, & de Jong, 2014). In the present study, parent-child relations in the domain of math ability are investigated, considering either ANS measures and formal math skills.

Cognitive phenotypes of math skills

The ANS (Brannon & Merritt, 2011; Dehaene & Brannon, 2011; Feigenson et al., 2004; Nieder & Dehaene, 2009; Odic, Hock, & Halberda, 2014), represents an intuitive, non-symbolic, approximate sense of number that is available before the onset of schooling (Izard et al., 2009; Xu & Spelke, 2000; Xu et al., 2005) and that remains active across the lifespan (J. Halberda et al., 2012; Justin Halberda & Feigenson, 2008; Piazza, Pinel, Le Bihan, & Dehaene, 2007). Typically developing children demonstrate an increase in the accuracy of ANS representations over developmental time (Justin Halberda & Feigenson, 2008). Neuroimaging studies suggested that it is located in the intraparietal sulcus of the parietal lobe (Ansari, 2015; Dehaene et al., 2003). An emerging body of research suggests that despite the differences between approximate number representations and the exact, symbolically mediated numbers used in school mathematics, the ANS and symbolic math performance are likely related (Chen & Li, 2014; Libertus, Feigenson, & Halberda, 2013). Evidence in support of this relationship comes from findings that individual differences in ANS precision often correlate with mathematics achievement in children and adults. Performance on standardized math tests has been found to correlate with current ANS ability (Bonny & Lourenco, 2013; Libertus et al., 2011; Linsen, Verschaffel, Reynvoet, & De Smedt, 2013; Lourenco et al., 2012; Odic et al., 2016), and ANS performance predicts future math ability (Gilmore et al., 2010; Libertus et al., 2013; Mazzocco et al., 2011; Starr et al., 2013; van Marle, Chu, Li, & Geary, 2014). Non-symbolic (dots) comparison tasks are frequently used to understand the precision of representations within the ANS. The development of symbolic number processing has been typically explored by means of magnitude comparison tasks that involve Arabic digits. Scores on this task are doubtful to understand, as they might reflect the nature of underlying ANS representations, or the mapping between symbols and the ANS representations. However, children's performance on these symbolic tasks has been found to be correlated with concurrent and future mathematics achievement. This relationship appears to be very consistent for overall reaction times on the symbolic comparison task (De Smedt, Noël, Gilmore, & Ansari, 2013). Nevertheless, there are also controversial findings. A study by Göbel et al. (Göbel, Watson, Lervåg, & Hulme, 2014) found that the ANS system was a robust longitudinal correlate of arithmetic skills but path models showed that knowledge of Arabic numerals at 6 years was the dominant longitudinal predictor of the increase in arithmetic skills. In contrast, alterations in magnitude-comparison skill played no additional role in predicting differences in arithmetic skills.

In order to better analyze mature numerical processing, however, other cognitive processes must also be investigated. Based on Von Aster and Shalev model (Von Aster & Shalev, 2007), through education, the non-symbolic system is gradually supported by a linguistic and symbolic component, located in the left angular gyrus (Dehaene et al., 2003), together with a progressively increased load on working memory, particularly in its spatial component. This has been documented by increased activation of the bilateral posterior superior parietal lobe in tasks requiring manipulations of more digits or when participants are required to complete two or more operations at a time (Koenigs, Barbey, Postle, & Grafman, 2009).

This evidence supports the notion of a foundational role, although not exclusive, of the ANS in the development of symbolic numerical abilities, which includes mental calculation and mathematical reasoning tasks, operating with symbolically-represented numbers (Arabic) (Justin Halberda & Feigenson, 2008; Piazza et al., 2010). Therefore, ANS skills might represent a putative cognitive phenotype (or endophenotype) of math skills. In their classic definition, endophenotypes are heritable neurophysiological, biochemical, endocrinological, neuroanatomical, or neuropsychological constituents that are likely to be influenced by complex interactions between genes and environments (Caspi & Moffitt, 2006; Gottesman & Gould, 2003). One of the main characteristics of the endophenotypes is that they represent proxy expressions of genetic traits, and are usually observable employing specific experimental tasks tapping fine-grained processes. The term "broader phenotype" refers specifically to a set of endophenotypic markers that might be observable with qualitatively similar characteristics in family members (Gottesman & Gould, 2003).

Intergenerational associations of learning ability

Although little evidence has been collected on a transgenerational model of math skills, many studies have deepened intergeneration models in related areas, such as reading skills. Van Bergen et al. (van Bergen et al., 2014) studied the parent-child relationship of reading skills and found that parents' reading skills explained 21% of the variance in child reading. Bonifacci et al. (Bonifacci, Montuschi, Lami, Snowling, et al., 2014) found that parents of children with dyslexia, i.e., a specific learning disorder affecting reading (American Psychiatric Association. Taskforce on DSM-5, 2013), had poorer phonological and decoding skills compared to parents of typical readers, and they found significant relationships between parents' and children's reading skills. Studies on family risk found that children whose parents had a history of reading difficulties underperformed in linguistic (Bogdanowicz, 2003; Krasowicz-Kupis, Bogdanowicz, & Wiejak, 2014), phonological (Snowling, 2008) and reading (Eklund, Torppa, Aro, Leppänen, & Lyytinen, 2015) skills compared to a control group and these differences have been observed even before the child begins formal education (Łockiewicz & Matuszkiewicz, 2016). Wadsworth et al. (Wadsworth, Corley, Hewitt, Plomin, & DeFries, 2002) and Swagerman et al. (Swagerman et al., 2017) through studies conducted, respectively, on adoptive families and families with twin, reinforced the notion that variances in reading ability were principally explained by genetic influences, with minor variance explained by environmental factors. In brief, many studies have found a reliable and robust association between parents' and children reading skills, either considering typically developing children, children with dyslexia, and children with family risk.

Although this rich literature on reading, very little evidence has been collected on intergenerational transmission of numerical and math skills. Within these studies, first evidence had been collected on children with math impairments. In a seminal work by Shalev (Shalev et al., 2001), he suggested that dyscalculia, i.e., a specific learning disorder affecting mathematics (American Psychiatric Association. Taskforce on DSM-5, 2013) is a "familiar disorder." The results showed that higher percentages of family members of children with dyscalculia, compared to the general population, showed indicators of dyscalculia. Therefore dyscalculia, like other specific learning disorders, was characterized by a significant family aggregation, suggesting a role of genetics in the evolution of this disorder (Shalev et al., 2001). Twin studies have confirmed that mathematical ability is determined, at least partly, by genetic factors with expected heritability for low mathematical performance of 0.65 (Haworth et al., 2009) and 0.69 (Oliver et al., 2004). However, a more recent study has proposed that basic numerical understanding is only moderately heritable (32%), with environmental influences being a more powerful predictor (68%) (Tosto et al., 2014). Two genome-wide association studies (GWASs) did not find any proper association (Baron-Cohen et al., 2014; Docherty et al., 2010), but a third study confirmed a significant genetic component underlying mathematical abilities, without identifying specific risk factors (Davis, Band, Pirinen, Haworth, Meaburn, Kovas, Harlaar, Lesaux, et al., 2014). The rs133885 variant in the myosin-18B (MYO18B) gene is the only marker that has been found to be associated with mathematical ability at a statistically significant level, as reported in Ludwig et al. (Ludwig et al., 2013).

In summary, although previous evidence suggests a plausible genetic component in math intergeneration skills, a debate is still open about the gene*environment interaction. Within this framework, if ANS skills represent cognitive markers of math skills, we should expect a significant relationship between parents' and children's ANS skills. In a recent study, Desoete et al. (Desoete, Praet, Titeca, & Ceulemans, 2013) studied ANS skills in children with mathematical impairments and their siblings. Their results are in line with the study of Shalev et al.

(Shalev et al., 2001) since the 33% of siblings had clinical or subclinical scores in "early arithmetic skills" and were at risk to develop dyscalculia: this percentage is above expectations based on typically developing children. Recently, Navarro et al. (Navarro et al., 2018) found that parents' ANS skills were related to toddlers' number processing and that this relation was independent of children's vocabulary or parents' perceived math ability, suggesting a specific intergenerational transmission of the ANS. However, this study was on toddlers from one to three years old, thus before entering the school system or being faced with formal maths skills. In this study, they have analyzed the percentage of time each child spent looking at the numerically changing image stream out of the total time children spent looking to either stream, but children across all age groups were presented with only one ratio (2:3) and it is possible that the use of a single ratio masked any age-related change. In another study by Braham and Libertus (Braham & Libertus, 2017), conducted on 54 children (5–9 years old) and their parents the authors found that children's ANS acuity positively correlated with their parents' ANS acuity. Also, unique combinations of parents' ANS acuity and math ability depending on the specific math skill in question predicted children's math abilities. However, the study analyzed together with data from children of a wide range of age (5-9 years old), so some of them where just at the beginning of formal schooling whereas others were in a more advanced consolidation phase. Then, this study did not directly address parents' written calculation skills and considered only non-symbolic magnitude comparison tasks, using different tasks on parents and children. Analyzing both parent's and children's magnitude comparison skills, considering either symbolic and non-symbolic stimuli, would allow disentangling the intergenerational role of these distinct but related variables.

The current study

Based on previous literature, the present study aimed to analyze symbolic and non-symbolic numerical abilities of parents in order to understand if these are predictors of children's numerical skills, either considering basic symbolic, non-symbolic and formal math skills, i.e., written calculation. A battery of cognitive and math tasks has been administered to a sample of children with established (i.e., 4-5 years) formal school experience, and to their mothers.

In particular, the aim was to understand:

- whether mothers' symbolic / non-symbolic abilities in magnitude comparison tasks were influential for their children's performance in a complex math task (i.e., written calculation), or

- whether the most relevant aspects were to be found in mothers' formal mathematics skills, such as resolution of written operations.

Further, we added children's own symbolic and non-symbolic skills as predictors of their math performance, considering past literature on how these basic tasks can predict formal math skills (Gilmore et al., 2010; Starr et al., 2013). Children with higher success in mathematics showed superior capacities to identify and operate on non-symbolic numerical magnitudes. In particular, their performances were driven by the association of non-symbolic abilities and number symbols (Starr et al., 2013).

Main hypotheses:

1) In line with Braham and Libertus (Braham & Libertus, 2017) and with Navarro et al. (Navarro et al., 2018) we should expect mothers' ANS, as measured by non-symbolic comparison, to predict children's math performance,
2) Alternatively, children's math performance are predicted by mothers' written calculation as well as children's math skills, whose development might have been influenced by other genetic (fathers) or environmental (teaching) variables. Written calculation is a multi-component task that reflects both symbolic and non-symbolic magnitude comparison skills (Jordan, Glutting, & Ramineni, 2010; Seethaler, Fuchs, Fuchs, & Compton, 2012), being an excellent candidate to reflect advanced math ability in both children and adults, and because (2) mothers have a significant role in helping children with their homework, and therefore have an influence on children's school abilities (Hoover-Dempsey et al., 2001). Furthermore, mothers' educational level was taken into consideration, considering past research that showed how this variable is a strong predictor of math skills across development, in particular for some tasks such as solving math problems (Burchinal, Peisner-Feinberg, Pianta, & Howes, 2002).

This is the first study directly investigating the link between school-aged children and mothers' math skills, including measures of "primitive" ANS, of basic symbolic comparison skills and of more complex math skills such as written calculation, when children have established the experience of formal schooling, namely 4-5 years of experience. This research design would allow disentangling the intergenerational role of both basic non-symbolic and symbolic numerical skills, as well as advanced math skills (i.e. written calculation), in predicting math ability in children, as measured by an ecological and complex task.

2.2 Method

Participants

The sample consisted of 83 children (mean age = 9.7, SD = 0.5, 61.4% females), attending the 4th and the 5th grades of primary school, and their mothers. Participants were selected from schools in suburban areas in the north of Italy. From an initial sample of 96 children, we included in the study only participants with a complete dataset collected from both mothers and children. Parents provided written informed consent prior to the experiment. The Ethical Committee of [BLIND] has approved the study design.

Measures

Mothers and children were administered tests assessing intellectual functioning, formal math skills and symbolic and non-symbolic comparison tasks. Parents were also administered a socio-demographic questionnaire. A detailed description of the task and eventual differences between mothers' and children's task is detailed below.

Socio-demographic information: the Hollingshead Four Factor Index of Social Status (Hollingshead, 2011) has been utilized. For this study, indexes of educational level (EL) and occupation (O) were adopted. For the level of education, a score from 1 to 9 was indicated and for employment a score from 1 to 9. SES scores for fathers and mothers have been determined with the formula EL*3 + O*5, and an aggregate SES score for children resulted from the mean of the two values. Scores between 0 and 39 were categorized as low-medium, and scores above 40, as medium-high or high. For mothers, SES was estimated combining their education level and occupation.

Intellectual functioning: Both mothers and children were administered the *Matrices* subtest of K-BIT 2 (Bonifacci & Nori, 2016; Kaufman & Kaufman, 2014). The test has different starting points based on the participant's age and stops after four consecutive wrong responses.

Battery of standardized tasks on written calculation: The written calculation task aimed to examine calculation procedures. Mothers were administered one subtest (*Written calculation*) of the BDE-2 (Biancardi, Bachmann, & Nicoletti, 2016) that includes six operations: two additions, two subtractions and two multiplications (example: 356+579; 102-48; 216x29). Children were administered one subtest (*Written calculation*) of AC-MT 6-11 (Cornoldi, Lucangeli, & Bellina, 2012) that includes eight operations: two additions, two subtractions, two multiplications and two divisions (example: 2114+278; 1431-126; 157x9; 1989:9). Both mothers and children have five minutes to solve all the operations and the criterion for the attribution of the score is to assign a point for each correct operation.

Experimental tasks on ANS related skills (symbolic and non-symbolic magnitude comparison): Mothers and children were administered the same experimental tasks.

- Symbolic magnitude comparison task: In the Symbolic comparison task participants are shown two numbers on the PC's display, represented as Arabic digits, and asked to select the bigger number. Sixty single-digit-item pairs were presented; mother and children had to select as quickly as possible the numerically larger of two Arabic numbers by pressing the left or right button on the keyboard corresponding to the position of the target item on the screen. Numbers are between twenty-one and ninety-eight. The average distance is twenty-two, from a minimum of six to a maximum of thirty-seven. In thirty items the greater number is located in the right part of the screen and thirty items in the left part. This has been done to address the SNARC effect, for which the response time in reacting to a high number with the right hand are minor compared to those used in responding, always at the same number, with the left hand and vice versa (Dehaene et al., 1993). Accuracy scores (percentages of correct answers) and RTs (sec) were reported.

- Non-symbolic magnitude comparison task: In the Non-symbolic magnitude comparison task, participants are shown two squares' sets and asked to select the more numerous (Landerl et al., 2009). The difficulty of making this decision is manipulated by varying the ratio or the numerical distance between the two sets. Forty pairs of squares' sets were presented; mother and children had to select as quickly as possible the numerically larger of two groups of squares by pressing the left or right button on the keyboard corresponding to the position of the target item on the screen. Each display consisted of between 20 and 72 squares, with the difference between the two displays ranging from 10 to 29 squares. These relatively high numbers ensured that participants could not verbally count. We made every attempt to force participants to base their decisions on numerosity alone and tried to avoid giving additional information by non-numerical features. The total surface area was always identical in the two displays. To avoid the displays with the larger numerosity systematically consisting of smaller squares, each display included squares of different sizes. The largest and smallest squares appeared in the same number in both displays, with only size and number of intermediate squares being different. Accuracy scores (percentages of correct answers) and RTs (sec) were reported.

2.3 Data analysis

Pearson correlations were performed in order to investigate associations among children variables (age, SES, IQ, written calculation, accuracy and RTs for symbolic and non-symbolic magnitude comparison tasks), mothers variables (SES, IQ, written calculation, accuracy and RTs for symbolic and non-symbolic magnitude comparison tasks), and between children and mothers variables.

Then, a 3-step hierarchical regression analysis was run in order to investigate the predictors of children's written calculation skills. Mothers' level of education was included in the first step, both for empirical (significant correlation between mothers' education and children's non-symbolic magnitude comparison) and theoretical reasons: mothers' education has been found to be one of the best environmental predictors of learning skills in their children (Magnuson, 2007; Yarosz & Barnett, 2001). Mothers' symbolic and non-symbolic magnitude comparison, and mothers' written calculation were then included in the second step, and children's symbolic and non-symbolic magnitude comparison in the last step, in order to understand if children's skill had an additional predicting power after controlling for mothers' skills For the magnitude comparison tasks, both accuracy and RTs were included in the regression analysis.

2.4 Results

Descriptive analysis

Descriptive analysis for children's and mothers' variables are reported in Table 1.

Table 1 – Descriptive statistics for all the variables referred to children and mothers.

Subject	Task	Mean (SD)	Range	Skewness (SE =.264)	Kurtosis (SE=.523)
Children	Age (years)	9.76 (.54)	9.01 – 11.01	.179	-.406
	SES	36.45 (11.24)	11 – 63	.212	-.272
	KBIT-2	28,54 (6,44)	15 - 40	-.42	-.531
	Written calculation z-score	-.43 (.14)	-4.34 – .98	-1.142	.783
	Symbolic magnitude comparison (accuracy)	.92 (.01)	.72 – 1	-.826	.634
	Symbolic magnitude comparison (RTs)	1.19 (.02)	.80 – 1.84	.777	.554
	Non-symbolic magnitude comparison (accuracy)	.85 (.01)	.58 – 1	-.965	.474
	Non-symbolic magnitude comparison (RTs)	1.37 (.04)	.66 – 2.33	.507	-.254
Mothers	Level of education	4.17 (.13)	2 – 7	.113	.160
	Job	4.93 (.24)	1 – 9	-.386	-.649
	SES	37.14 (1.38)	11 – 63	-.155	-.509
	KBIT-2	35,71 (6,05)	16 - 44	-1,00	.713
	Written calculation	5.20 (.11)	0 – 6	-2.326	8.200
	Symbolic magnitude comparison (accuracy)	.95 (.004)	.87 – 1	-.513	-.729
	Symbolic magnitude comparison (RTs)	.92 (.03)	.64 – 1.91	1.915	4.333
	Non-symbolic magnitude comparison (accuracy)	.93 (.005)	.83 – 1	-.269	-.769
	Non-symbolic magnitude comparison (RTs)	1.29 (.05)	.74 – 3.07	1.625	3.652

Correlations

Correlations among children's variables, mothers' variables, and between children's and mothers' variables are reported in Table 2 a,b,c.

Table 2a – Pearson correlations among children's variables.

	SES	IQ	Written calculation z-score	Symbolic magnitude comparison (accuracy)	Symbolic magnitude comparison (RTs)	Non-symbolic magnitude comparison (accuracy)	Non-symbolic magnitude comparison (RTs)
Age (years)	.029	.024	-.024	-.163	-.258*	.026	-.144
SES	1	.207	.228*	.173	-.119	.283**	.104
IQ		1	.115	.08	-.226*	.154	.118
Written calculation z-score			1	.181	-.316**	.267*	.05
Symbolic magnitude comparison (accuracy)				1	.148	.247*	.285**
Symbolic magnitude comparison (RTs)					1	-.068	.244*
Non-symbolic magnitude comparison (accuracy)						1	.352**

* $p < .05$, ** $p < .01$

Table 2b – Pearson correlations among mothers' variables.

	Profession	SES	IQ	Written calculation	Symbolic magnitude comparison (accuracy)	Symbolic magnitude comparison (RTs)	Non-symbolic magnitude comparison (accuracy)	Non-symbolic magnitude comparison (RTs)
Level of education	.391**	.618**	.215*	.293**	.068	.119	.292**	.010
Profession	1	.965**	.286**	.201	.221*	.127	.343**	.18
SES		1	.305**	.275*	.191	-.122	.230*	-.166
IQ			1	.290**	.196	-.125	.236*	-.151
Written calculation				1	.179	-.068	.148	-.034
Symbolic magnitude comparison (accuracy)					1	.154	.258*	.234*
Symbolic magnitude comparison (RTs)						1	.039	.523**
Non-symbolic magnitude comparison (accuracy)							1	.252*

* $p < .05$, ** $p < .01$

Table 2c – Pearson correlations between children's and mothers' variables.

		Child					
		IQ	Written calculation	Symbolic magnitude comparison (accuracy)	Symbolic magnitude comparison (RTs)	Non-symbolic magnitude comparison (accuracy)	Non-symbolic magnitude comparison (RTs)
Mother	Level of education	.055	.145	.121	.055	.354**	.117
	Profession	.216*	.188	.200	-.058	.117	.103
	IQ	.376**	.078	.098	-.103	.036	-.169
	Written calculation	.237*	.256*	-.153	-.19	.087	-.056
	Symbolic magnitude comparison (accuracy)	.105	.188	.277*	-.289**	.000	-.075
	Symbolic magnitude comparison (RTs)	-.280*	.177	.179	.068	.083	.264*
	Non-symbolic magnitude comparison (accuracy)	.066	.331**	.343**	-.083	.230*	-.057
	Non-symbolic magnitude comparison (RTs)	-.179	.149	.169	-.024	-.016	.038

* $p < .05$, ** $p < .01$

As showed in Table 2a, higher SES was associated with stronger written calculation skills and non-symbolic magnitude comparison accuracy. This last variable was positively associated also to symbolic magnitude comparison accuracy and written calculation. Considering magnitude comparison RTs, faster responses to the symbolic magnitude comparison task were associated with older age and stronger written calculation performance, as well as to RTs in the non-symbolic magnitude comparison task. Then, non-symbolic magnitude comparison RTs were associated with both symbolic and non-symbolic accuracy. Table 2b reported mothers' correlations. SES, level of education and profession were all positively correlated among them and with mothers' IQ. Written calculation score was positively associated with mothers' level of education, SES, and IQ, but not with variables related to magnitude comparison. Mothers' accuracy in the symbolic magnitude comparison task was positively associated with their profession and with non-symbolic magnitude comparison accuracy and RTs. RTs of symbolic magnitude comparison were related only to RTs in the non-symbolic analogs task.

On the contrary, RTs at the non-symbolic magnitude comparison task also correlated with accuracy at the same task. Finally, non-symbolic magnitude comparison accuracy was also positively associated with all demographic variables and with IQ. Table 2c presents the correlations between children's and mothers' variables. Children's IQ was positively correlated with mothers' IQ as well as with mothers' profession, written calculation skills and, inversely, with their RTs in the symbolic magnitude comparison task. Children's and mothers' written calculation were positively correlated; furthermore, children's stronger written calculation skill was also associated with better mothers' non-symbolic magnitude comparison accuracy. Magnitude comparison accuracy was correlated in children and their mothers for both symbolic and non-symbolic stimuli. Symbolic magnitude comparison accuracy in children was also correlated with non-symbolic magnitude comparison accuracy in mothers. Children's non-symbolic magnitude comparison accuracy was positively correlated to mothers' level of education. Considering RTs, children's RTs for the non-symbolic task were correlated with mothers' RTs in the symbolic task. Finally, children's RTs for the symbolic task were inversely correlated to mothers' accuracy in the symbolic task.

Regression

The results of the hierarchical analysis to investigate predictors of children’s written calculation (Table 3) showed that, including mothers’ level of education as was in the first step, the results were not significant. At the second step, also math-related mothers’ variables were included as potential predictors, adding a significant portion of explained variance (16.8 %): mothers’ non-symbolic magnitude comparison accuracy and written calculation were significant predictors. At the final step, when also children’s variables were included, children’s symbolic comparison mean RT was a significant predictor (β = -.295, p = .010) of written calculation. Furthermore, a tendency to significance was showed also for mothers’ written calculation (β = .212, p = .058) and mothers’ non-symbolic accuracy (β = .221, p =

.067). The final model explained 29.8% of the variance in children's written calculation scores.

Table 3 - Hierarchical regression; dependent variable: Children's written calculation total score

Step		B	SE B	β
1° (R^2 = .021)	Mothers' level of education	.157	.119	.145
2° (ΔR^2 = .168, p = .012)	Mothers' level of education	-.035	.124	-.033
	Mothers' symbolic magnitude comparison (accuracy)	1.892	4.067	.051
	Mothers' symbolic magnitude comparison (RTs)	1.053	.681	.193
	Mothers' non-symbolic magnitude comparison (accuracy)	7.961	3.141	.294*
	Mothers' non-symbolic magnitude comparison (RTs)	-.089	.382	-.030
	Mothers' written calculation	.283	.139	.225*
3° (ΔR^2 = .108, p = .033)	Mothers' level of education	-.056	.124	-.052
	Mothers' symbolic magnitude comparison (accuracy)	-1.840	4.267	-.050
	Mothers' symbolic magnitude comparison (RTs)	1.005	.676	.184
	Mothers' non-symbolic magnitude comparison (accuracy)	5.994	3.219	.221[a]
	Mothers' non-symbolic magnitude comparison (RTs)	-.033	.367	-.011
	Mothers' written calculation	.267	.139	.212[b]
	Children's symbolic magnitude comparison (accuracy)	2.833	2.621	.132
	Children's symbolic magnitude comparison (RTs)	-1.745	.663	-.295*
	Children's non-symbolic magnitude comparison (accuracy)	1.974	1.621	.144
	Children's non-symbolic magnitude comparison (RTs)	.045	.425	.012

* $p < .05$; [a] = .067; [b] = .058

2.5 Discussion

The present study was aimed at evaluating relationships between mothers' and children's number skills. More specifically, the study included a measure of non-symbolic magnitude comparison skills, considered a measure of the ANS, a measure of symbolic comparison, where a linguistic component is involved, and a measure of formal arithmetic (written calculation). Further, SES and cognitive functioning were included to assess their relationship with mothers' and children's numerical skills.

First, a set of correlation analyses has been run in order to evaluate both transgenerational and within-group relationships of the variables included in the study. Considering background demographic information, SES was found to relate with accuracy in the non-symbolic magnitude comparison task and with written calculation, for both mothers and children; for mothers, SES was significantly related also to IQ, and to the symbolic comparison task. The association between SES and children's performance in math tasks appear, therefore weaker than that observed in mothers, suggesting that the relationships might increase in the life course.

As far as cognitive functioning was concerned it was of interest that there were widespread correlations with number processing skills in mothers (written calculation, accuracy in non-symbolic comparison tasks), but only a modest correlation with symbolic magnitude comparison RTs in children. Considering also the significant relationship between SES and IQ found in mothers but not in children, these results confirm previous studies that highlighted how increasing SES might raise average intelligence magnifying individual differences in intelligence (Bates, Lewis, & Weiss, 2013), and add further insights as to whether this might extends to individual differences in math skills.

On the counterparts, written calculation was significantly related to the symbolic (RTs) task in children, but not in mothers. Non-symbolic accuracy was instead weakly related to calculation skills both in mothers and children. These results suggest a stronger relationship in young age between symbolic number skills and formal arithmetic, whereas, in adults, calculations seem to be more related to general cognitive efficiency, as previously discussed. Significant relationships emerged between symbolic and non-symbolic comparison tasks in children, but these were weaker in mothers. Although some researchers claim that non-symbolic and symbolic skills are separable (Kolkman, Kroesbergen, & Leseman, 2013), results from the present study are in line with previous evidence of a positive, although not particularly strong, relationship between symbolic and non-symbolic magnitude comparison tasks (Li et al., 2018).

Turning to mother-child relationships, IQ scores were positively associated, in line with many previous studies on the parent-child relationship for IQ (Bartels, Rietveld, Van Baal, & Boomsma, 2002). Children's and mothers' written calculation were positively correlated, and children's stronger written calculation skill was also associated with better mothers' non-symbolic magnitude comparison accuracy. Accuracy in symbolic and non-symbolic tasks was positively and significantly associated between mothers and children and there were significant relationships between children's symbolic skills and mother's non-symbolic skills. Therefore, correlations between children's and mothers' magnitude comparison performances were not limited to a link between symbolic or non-symbolic measures, but associations were observed also among measures based on different tasks (e.g., symbolic accuracy in children correlated to non-symbolic accuracy in mothers). Globally, this pattern of results seems to suggest a transgenerational pattern of numerical processing and math skills in mothers and children.

However, to test the strength of these associations, we performed a hierarchical regression analysis, intending to analyze the potential predictors of children performance in written calculation, an arithmetic complex task gradually built during formal schooling. Results evidenced that mothers' level of education, inserted at first step, was not a significant predictor of children's math performance. On the contrary, mothers' non-symbolic magnitude comparison accuracy and written calculation, added in the second step, were significant predictors and explained 16.8% of the variance in children's written calculation performance. Finally, when also children's skills were included, it was found that RTs in children's symbolic comparison resulted in being the main predictor of the model. However, a residual tendency to significance for the mother's written calculation and non-symbolic accuracy was found.

These results suggest the importance of mother's numerical skills in the development of children's abilities in numerical processing and, at least in part, is in line with results by Navarro et al. (Navarro et al., 2018) and Braham & Libertus (Braham & Libertus, 2017), who found that parents' ANS skills were related to infants' number processing. However, differently from Braham & Libertus (Braham & Libertus, 2017), we found that when including children's symbolic and non-symbolic processing skills, the role of parents became marginal, with a major effect of children numerical processing in predicting their math skills (written calculation).

In other words, our results suggest that the intergenerational features on math skills play a significant role in children's numerical development but that children's math skills ultimately depend mainly on their own numerical processing, which might be shaped not only by parent's skills but also by other environmental influences. This can be particularly true for 4th – 5th-grade children, that have been involved in formal schooling, which has a strong influence on children educational outcomes (Carbonneau, Marley, & Selig, 2013). In this line, although referred to literacy skills, Thompson et al. (Thompson et al., 2015) found that family risk of dyslexia was a significant predictor of children's reading skills at age 3.5. However, at the age of 5.5 years, familiarity did not remain a significant predictor of reading skills if children's abilities in literacy prerequisites (Rapid automatized naming, phonological awareness, letter knowledge) were included in the model. Considering previous literature on the role of ANS measures in predicting math abilities, the present study supports a significant relationship between non-symbolic processing and math skills when considering correlation analysis. However, in the regression model, the main predictor of children's math skills was the performance of the symbolic comparison task. This is in line with previous studies that suggest how achievements in maths are primarily associated with symbolic processing (Schneider et al., 2017).

These findings on intergenerational transmission of math skills would require further investigation since the number of studies on this topic is still very limited. The present study has some limitations that would require to be addressed in future investigations. First, in the present study, as in Braham and Libertus (Braham & Libertus, 2017) we included only mothers, but it would be important to include fathers in order to understand a complete picture of intergenerational transmission of math skills. Further, parent-child relationships in math skills have been tested in a sample of typically developing children, but the inclusion of a group with math impairment (Developmental Dyscalculia) would add significant insight into the broader phenotype of math skills. Finally, it would be of interest to include other environmental variables such as home numeracy environment (Lefevre et al., 2009) and math anxiety (Maloney, Ramirez, Gunderson, Levine, & Beilock, 2015b), as potential mediators of the influence of parents' math skills on children numerical development.

In summary, the present study added an important contribution to previous literature. Specifically, compared to previous literature, for both parents and children, we included, besides measures of non-symbolic magnitude comparison, also measures of symbolic magnitude comparison. It resulted that mothers' ANS skills were significant predictors of children's formal maths skills, but children symbolic processing added a significant portion of explained variance. This suggests that, within an educational perspective, the development of symbolic number skills in children is fundamental and might allow encompassing the constraints of intergenerational transmission of math skills.

Chapter 3

Children's early numeracy: understanding the interplay among SES, home numeracy, parents' and children's skills.

3.1 Introduction

Early numerical abilities are manifested during the first few months of life in humans from various cultural backgrounds (Gordon, 2004; Simon, Hespos, & Rochat, 1995; Xu et al., 2005). Differences in the quality and quantity of children's early math learning opportunities have been shown to affect their consequent math performance (Hill, Rowan, & Ball, 2005; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010; Maloney et al., 2015a; Tobia, Bonifacci, & Marzocchi, 2016). Indeed, many pieces of evidence now indicate that parents count in the development of children's math skills, either recognizing home numeracy activities (Lefevre et al., 2009) or through mediation effects of parents' math anxiety (Maloney et al., 2015a) or stereotypes (Tomasetto et al., 2011). However, limited research has explored the intergenerational influence of math skills, that is the relation between parents' and children's skills in the numerical domain, although many studies showed parent-child relations in other domain such as reading ability (Bonifacci, Montuschi, Lami, Snowling, et al., 2014).

Cognitive and linguistic skills are also known to potentially influence numerical and math development (Authors, submitted). Concerning cognitive skills, intellectual functioning (Poletti, 2017), visuo-spatial working memory (Cirino, 2011; Zhang & Lin, 2015) and Executive Functions (EFs) (Cragg, Keeble, Richardson, Roome, & Gilmore, 2017; Schmitt, Geldhof, Purpura, Duncan, & McClelland, 2017) play a strong role in early and late development of math skills. With respect to linguistic skills, lexical amplitude (vocabulary) is necessary to understand specific math terms (Adams, 2003; Purpura et al., 2011) and phonological awareness might play a role in storing and retrieval of numbers (Swanson & Sachse-Lee, 2001).

In addition to environmental, cognitive and linguistic variables, children are also influenced by an intuitive, non-symbolic, Approximate Number System (ANS) that is available prior to the onset of schooling (Brannon & Merritt, 2011; Dehaene & Brannon, 2011; Feigenson et al., 2004; Izard et al., 2009; Nieder & Dehaene, 2009; Odic et al., 2014; Xu & Spelke, 2000; Xu et al., 2005) and that remains active across the lifespan (J. Halberda et al., 2012; Justin Halberda & Feigenson, 2008; Piazza et al., 2007). This precocious and preverbal sense of numerical magnitude includes the ability to quickly understand and manipulate numerical quantities (Dehaene, 1997) and is thought to be part of an innate non-symbolic system of numerical representation. The ANS seems to represent the basis on which, through the interplay with other cognitive skills (mainly language and working memory), numerical knowledge develops from preschool to primary school, when formal teaching shapes arithmetic ability.

The present study aimed at analyzing the role of environmental stimuli, intergenerational influence, children's cognitive and linguistic skills and children's ANS in predicting children's math skills.

Development of math skills

An emerging body of research suggests that, despite the differences between approximate number representations and the exact, symbolically mediated numbers used in school mathematics, the ANS and symbolic math performance are likely related (Chen & Li, 2014; Libertus et al., 2013). Evidence in support of this relationship comes from findings that individual differences in ANS precision often correlate with mathematics achievement in children and adults. Performance on standardized math tests has been found to correlate with ANS ability (Libertus, Odic, & Halberda, 2012; (Bonny & Lourenco, 2013; Libertus et al., 2011; Linsen et al., 2013; Lourenco et al., 2012; Odic et al., 2016), and ANS performance predicts math ability (Gilmore et al., 2010; Libertus et al., 2013; Mazzocco et al., 2011; Starr et al., 2013; van Marle et al., 2014). In addition, children with mathematical learning disabilities (MLDs or Developmental Dyscalculia-DD) have significantly poorer ANS precision than typically developing children (Brankaer et al., 2014; Mazzocco et al., 2011; Piazza et al., 2010), whereas children with high math achievement show superior ANS precision. Through education, this non-symbolic system is gradually supported by a linguistic and symbolic component that is culturally determined, and is involved in all mathematical tasks that require the retrieval of arithmetic facts or, more generally, exact calculations (E. Spelke & Dehaene, 1999). According to Von Aster & Shalev's model (Von Aster & Shalev, 2007), the development of ANS is experience-dependent because it needs to be integrated with visual imagery, language and working memory skills. In their four-step model it is assumed that pure DD should refer to a primary dysfunction in the ANS system, whereas asynchronies in the development of linguistic skills and working memory capacity might result in difficulties in solving mathematical tasks, leading to a behavioral manifestation of DD, in the absence of a core deficit in ANS.

The emergence of formal mathematical competencies thus results from the complex interplay between the ANS and other cognitive skills (mainly language and working memory) and is further influenced by secondary mathematical content provided with schooling. To date, however, the literature reports conflicting results in the identification of core deficits of DD. For example some studies have found impairment in magnitude comparison tasks (Piazza et al., 2010), whereas others found that children with DD were impaired only in tasks containing symbolic comparisons (Rousselle & Noël, 2007; Skagerlund & Träff, 2016). Furthermore, some studies have identified early ANS skills as primary predictors of mathematical skills (Mazzocco et al., 2011), whereas others have found a primary role of verbal knowledge (Göbel et al., 2014). The relationship between math skills and language development has received increasing attention, supporting that idea that language competence may act as a scaffolding ability on which numerical development may rely (Bonifacci, Tobia, Bernabini, & Marzocchi, 2016). This seems to be sustained also by developmental changes in brain networks underlying numerical processing, with the left angular gyrus supporting the manipulation of numbers in verbal form (Dehaene et al., 2003). Finally, although it is assumed that there is a genetic component in DD, genetic studies have provided inconsistent results (Ludwig et al., 2013; Pettigrew et al., 2015) and a recent study has suggested that basic numerical understanding is only modestly heritable (32%), with environmental influences being a more powerful predictor (68%) (Tosto et al., 2014).

In summary, arithmetic ability consists of many components, each subject to individual differences that continue into adulthood (L. Kaufmann, Wood, Rubinsten, & Henik, 2011) and that need to be taken into consideration from childhood. Heterogeneity of mathematical difficulties could also be fostered by environmental factors, ranging from cultural factors (Tomasetto et al., 2015) to the effects of pre-postnatal illness or socio-emotional adversity (e.g., math anxiety). In reference to Morton & Frith's causal model (Morton & Frith, 1995), when we describe the etiology of developmental disorders, different levels of analysis and their reciprocal relationships should be taken into account. The suggested levels are the biological, cognitive and behavioral components, together with environmental influences at each of these levels. Currently, the behavioral level of analysis is the one at which most current developmental psychopathologies are defined.

Intergenerational paths of math skills

In recent years, increasing research has focused on intergenerational transmission of cognitive skills in parents and children. A first line of research was referred to the concept of broader phenotype of developmental disorders, that refers specifically to the cognitive endophenotypes that are shared with unaffected family members. Endophenotypes are heritable neurophysiological, biochemical, endocrinological, neuroanatomical or neuropsychological constituents of disorders, although they are likely to be influenced by complex interactions between genes and environments (Caspi & Moffitt, 2006). Studies conducted on children with dyslexia revealed, for example, that phonological deficits are shared in non-affected family members and that parents and siblings of children with dyslexia underperform in reading measures compared to family members who are not at risk for the disorder (Göbel & Snowling, 2010). Bonifacci et al. (Bonifacci, Montuschi, Lami, & Snowling, 2014) found that parents of children with dyslexia underperformed in phonological and decoding tasks compared to parents of typical readers, and they found significant relationships between parents' and children's reading skills. Concerning maths, in a seminal work by Shalev (Shalev et al., 2001), he suggested that dyscalculia, i.e., a specific learning disorder affecting mathematics (American Psychiatric Association. Taskforce on DSM-5, 2013) is a "familiar disorder", with higher percentages of family members of children with dyscalculia with impaired performances in math tasks, compared to the general population. In a similar vein, Desoete, Praet, Titeca, & Ceulemans, (2013) found that 33% of siblings of children with DD had clinical or subclinical scores in early arithmetic skills and were at risk to develop dyscalculia: this percentage is above expectations based on typically developing children.

Recently, some studies addressed the issue of intergenerational transmission of literacy (van Bergen, Bishop, van Zuijen, & de Jong, 2015) and math (Braham & Libertus, 2017; Navarro, Braham, & Libertus, 2018; Authors, submitted) skills also in typical populations. Navarro et al. (Navarro et al., 2018) found that parents' ANS skills were related to toddlers' number processing and that this relation was independent of children's vocabulary or parents' perceived math ability, suggesting a specific intergenerational transmission of the ANS. In another study by Braham and Libertus (Braham & Libertus, 2017), conducted on 54 children (5–9 years old) and their parents the authors found that children’s ANS acuity positively correlated with their parents' ANS acuity. Also, unique combinations of parents’ ANS acuity and math ability depending on the specific math skill in question predicted children’s math abilities. Nevertheless, to our knowledge, there is a paucity of research that considered the differential role of home numeracy, parents’ skills and children’s own skills in a comprehensive model.

The current study

The present study aimed at understanding which components are related to early math skills in preschool children. In particular, the role of environmental stimuli, intergenerational influence, children’s cognitive and linguistic skills and children’s ANS have been taken into account. Concerning the role of environmental stimuli and intergenerational influence, the present study aimed to understand whether:

- the parents' abilities in mathematical tasks, in particular the ANS tasks, were predictive for their children's performances,
  or,
- the most relevant aspects were the activities related to the number, carried out at home by the children together with the parents.

Further, we wanted to evaluate the role of children’s own cognitive, linguistic and ANS related skills, in order to understand how these interact with parents’ variables in predicting children’ s math skills.

For doing so, a battery of tasks assessing prerequisites of math and ANS skills has been administered to a sample of children during the second year of kindergarten, either taking into account cognitive measures as attention, memory and non verbal IQ, as well as language skills. Another battery of tasks assessing math abilities, including ANS measures was administered to parents.

Hypotheses:

1) in line with Braham and Libertus (2017) and with Navarro et al. (Navarro et al., 2018), we should expect parents’ ANS, as measured by non-symbolic comparison, to uniquely predicts children’s ability with numbers.
2) considering multicomponential models of early math skills, home numeracy

as well as children's ANS skills, are primary predictors of children’s early math skills.

This is the first study directly investigating the link between pre-school children's and parents' math skills, including measures of “primitive” ANS, of basic symbolic comparison skills and more complex calculate skills. This research design would allow disentangling the intergenerational role of both basic non-symbolic numerical skills, as well as home numeracy, in predicting math ability in children, as measured by a battery of ecological and complex tasks.

3.2 Method

Participants

The sample included 64 children (mean age = 5.72 years, SD = 0.53, range = 4.42 - 6.58; 45.3 % females), attending the last year of kindergarten. For each child, a parent was involved in data collection. Most of parents were mothers (87.5 %; mean age = 40.53 years, SD = 4.64, range = 29 - 49); in the remaining cases fathers were involved (mean age = 45.14 years, SD = 8.84, range = 28 - 54).

Participants were selected from four schools in suburban areas in Northern Italy. From an initial sample of 69 children, we included in the study only participants with a complete dataset collected from children and one parent. Parents provided written informed consent prior to the experiment. The study was approved by the Ethical Committee of [BLIND].

Materials

Parents and children were administered tests assessing intellectual functioning, formal math skills and symbolic and non-symbolic comparison tasks. Parents were also administered a socio-demographic questionnaire and a questionnaire investigating home numeracy habits. A detailed description of the task is detailed below.

Socio-demographic information

The Hollingshead Four Factor Index of Social Status (Hollingshead, 2011) has been used. For this study, indexes of educational level (EL) and occupation (O) were chosen. For the level of education, a score from 1 to 9 was indicated and for employment a score from 1 to 9. SES scores for fathers and mothers have been managed with the formula EL*3 + O*5, and an aggregate SES score for children resulted from the mean of the two values. Scores between 0 and 39 were classified as low-medium, and scores above 40, as medium-high or high.

Children's assessment

Cognitive skills

Non-verbal IQ

Children were administered the *Matrices* subtest of K-BIT 2 (Bonifacci & Nori, 2016; Kaufman & Kaufman, 2014). The test measures the Non-Verbal IQ and has different starting points based on the participant's age and stops after four consecutive wrong responses.

Memory

Children were administered visual-spatial memory from the SNUP test (Tobia, Bonifacci, & Marzocchi, 2018). In this task, children had to remember the positions of one to four elements on 3×3 and 4×4 grids that were presented for 2 and 4 seconds, respectively, and then covered. A total of 10 grids, preceded by an example, were presented, and a score of 1 was assigned for each element remembered in the correct position, for a maximum total score of 26 (Cronbach's $\alpha = .80$).

Attention

The visual attention task from the NEPSY-II (Korkman, Kirk, & Kemp, 2007) was administered to the children in order to assess selective and sustained attention. Visual attention task is a visual cancellation task, which requires children to identify and mark down the target stimulus (a moon) among an array of distractors as quickly as possible. The variable considered was the accuracy, measured as the difference between the total number of target stimuli identified and the marked incorrect targets (i.e., distractors).

A Cognitive Score was computed with mean z scores of non-verbal IQ, memory and attention tests. Z scores were calculated either from the test manual, when available, or directly calculated on the sample distribution.

Language skills

Children were administered two subtests of the IDA test (Bonifacci, Pellizzari, Giuliano, & Serra, 2015) to assess the vocabulary and the phonological awareness.

Vocabulary

Children were asked to name 36 images selected for decreasing frequency in spoken language. The accuracy score, ranging from 0 to 36, was considered. The scale's Cronbach's alpha was 0.85.

Phonological awareness

To assess children's phonological awareness a syllable segmentation task was administered. Stimuli were presented orally and children were required to provide a verbal answer by segmenting sounds (e.g., Carota → Ca-ro- ta; Carrot; six items). Each item received a score of 1 for correct responses and a score of 0 for incorrect answers, for a maximum total score of 6.

A Total Language Score was computed calculating mean z scores for the Vocabulary and Phonological Awareness score, z scores where derived from tests' manuals.

Early numeracy

Children were administered four subtest of SNUP test (Tobia et al., 2018) to assess the children's early numerical skills.

Counting and Biunivocal correspondence

Children were asked to count 20 buttons scattered on a board measuring approximately 20 cm × 30 cm. Knowledge of the verbal sequence of numbers and the acquisition of the biunivocal correspondence principle of counting, namely the ability to link each number word to an individual object, were evaluated separately. Scores range from 0 to 20 for each subscale, and one point was given for each number word named correctly on the scale of 1–20 and when the child linked one number word to one button. The scale's Cronbach's alpha was 0.93.

Recognition and Reading of digits

Semantic knowledge of digits, that is, recognition and reading, were assessed for digits 1 to 9. The task was organized as a game comparable to bingo with numbers. A card containing the digits from 1 to 9 randomly allocated on a grid amongst blank squares was used, together with a small bag containing nine number cards, each representing a digit. In the digit recognition subtask, children pointed to the number on the bingo card that had been picked out of the bag and read aloud by the examiner. For the digit reading subtask, children picked a number from the bag and read it aloud. For each digit correctly identified or read a score of 1 was given (total score: 0–9 for each subtask). The subtest's Cronbach's alpha was 0.93.

A Total Numeracy Score was computed calculating mean z scores for the

Counting, Biunivocal correspondence, Recognition and Reading of digit score. Z scores where derived from tests' manuals.

Speed of processing and ANS

Speed of processing

A task to measure simple reaction times (Bonifacci & Snowling, 2008) was administered. Children were required to press the space bar of the keyboard, as fast as they could, whenever a 'blue star' (measuring 8 x 8 cm) appeared on a white screen. The target stimulus was presented on the screen for a maximum of one second and disappeared after the response was made. The following stimuli appeared at one-second intervals after the preceding stimulus had disappeared. Fifteen practice trials were completed, followed by 40 test trials. Mean RTs were recorded.

ANS - Magnitude comparison

A computerized magnitude comparison task was administered. Children were presented with two sets of dots in a random configuration, and were asked to identify the set representing the larger numerosity, by pressing one of two keys on a computer keyboard (W and P); they were instructed not to count. After a practice block (10 trials), 64 randomized trials were administered, with pairs of stimuli ranging from 1 to 9 dots and numerical distance between them ranging from 1 to 8. Each set of dots remained on the screen until the child's response. The stimuli were designed to avoid template matching and the use of the total dot area as a cue to numerosity. Therefore the total area occupied by the dots was equivalent across displays (for a complete description of this experiment see Guarini et al., 2014). Measures considered were accuracy (i.e., number of correct answers) and mean reaction time for correct answers.

A regression analysis with Simple RTs as independent variable and Dots RTs as dependent variable was performed and standardized residuals were used in the following analyses in order to have a measure of speed in magnitude comparison task free of the influence of general processing speed.

Parents' assessment

Home Numeracy

The children's Home Numeracy was assessed through a questionnaire that was designed to be administered to the child's parents. The questionnaire includes seven items that investigate the child's home numeracy habits and skills ('Do activities that require placing objects in order of size or length') and their knowledge of numbers ('Read or write numbers'). Reliability index was Cronbach's α =.78. The responses were provided on a 4-point Likert scale, ranging from 'Never' to 'Very often'.

General Cognitive Ability

Non-verbal IQ

Parents, as children, were administered the *Matrices* subtest of K-BIT 2 (Bonifacci & Nori, 2016; Kaufman & Kaufman, 2014). The test measures the Non-Verbal IQ and has different starting points based on the participant's age and stops after four consecutive wrong responses.

Calculation skills

Parents were administered three subtest of the BDE-2 test (Biancardi et al., 2016): Arithmetic fluency, Approximate Calculation and Written Calculation. Moreover they were administered the Mental Calculation subtest, from the MT battery (Cornoldi et al., 2010).

Arithmetic fluency

Parents have 2 minutes to write the correct results of as many mixed operations as possible (additions, subtractions, multiplications, divisions) up to a maximum of 40. The score is the total of answers they give correctly in 2 minutes.

Approximate calculation

Parents have 2 minutes to indicate the correct result of 18 operations, indicating it from the 4 options. For example, the operation is 75:5 and they have to choose between 80, 375, 15 or 5. The score is the total of answers and the maximum score is 18.

Written calculation

Parents have 2 minutes to indicate the correct result of six written operations: two additions, two subtractions and two multiplications (example: 356+579; 102-48; 216x29).

Mental calculation

The examiner reads 8 operations (2 additions, 2 subtractions, 2 multiplications and 2 divisions) and parents have 60 seconds to answer to each operation with the correct result. The score is the total of answers they give correctly. The maximum score is 8.

ANS

Estimation

This task has been developed on E-Prime for the purpose of the present study and adapted from Knops et al. (Knops, Dehaene, Berteletti, & Zorzi, 2014).

Different sets of black dots were presented on a white circle against a black background. The numerosities are 10, 16, 24, 32, 48, 56 or 64 dots. Each numerosity is presented 5 times, every time in a different configuration such that the same numerosity never appears in consecutive trials. Participants are instructed to look for 500 ms to the circle with black dots insight and then to estimate the quantity of dots shown on the computer screen writing the number on the keyboard. The mean distance between the correct number and the given number (differential) was calculated.

Non-symbolic magnitude comparison task

In this task parents were instructed to compare two sets of violet squares, which were simultaneously presented in two black rectangles on the left and on the right side of the screen, and they were instructed to choose the larger numerosity by pressing a key congruent to its side (left or right). The task was adapted from (Landerl et al., 2009). Forty pairs of squares' sets were presented. The difficulty of making this decision is manipulated by varying the ratio or the numerical distance between the two sets. Each display consisted of between 20 and 72 squares, with the difference between the two displays ranging from 10 to 29 squares. To avoid the displays with the larger numerosity systematically consisting of smaller squares, each display included squares of different sizes.

3.3 Data analysis

Pearson correlations were performed to describe the correlations between parents' variables (non-verbal IQ, calculation skills, ANS), home-related variables and early numeracy in children. Pearson correlations were also performed in order to investigate associations among children variables (SES, non-verbal IQ, cognitive skills, linguistic skills, ANS skills) and their early numeracy skills.

Then, a 3-step hierarchical regression analysis was run in order to investigate predictors of children's early math skills, as represented by the Total Numeracy score. In order to test the role of environmental variables, parents and home-related variables were included in the first step: home numeracy, parents' calculation skills and ANS. Then, we controlled for the additional variance predicted by children's general linguistic and cognitive skills. Therefore, at the second step, composite scores of children's cognitive and language skills were included. Finally, at the last step, scores related to children's ANS were included, in order to evaluate if specific domains of numerical skills had an additional predictive value once environmental variables and general cognitive skills were controlled for.

3.4 Results

Descriptive and correlation analysis

Descriptive analysis for children's and parents' variables are showed in Table 1.

Table 1 – Descriptive statistics for all the variables referred to children and mothers.

Group	Measures	Mean	SD	Minimum	Maximum
Children	SES	45.4	12.6	17.0	61.0
	Non-verbal IQ	16.9	5.1	3.0	29.0
	Memory	20.2	4.4	4.0	26.0
	Attention	21.0	4.0	11.0	26.0
	Vocabulary	33.6	2.3	24.0	36.0
	Phonological awareness	3.8	2.6	0.0	6.0
	Counting	19.0	2.7	3.0	20.0
	Biunivocal Correspondence	17.3	4.2	2.0	20.0
	Recognition	8.2	1.9	1.0	9.0
	Reading of digits	8.2	2.0	0.0	9.0
	Speed of processiong (RTs)	531.3	148.0	280.2	826.0
	Magnitude Comparison (RTs-ms)	1584.9	799.0	648.1	3417.2
	Magnitude Comparison (Accuracy)	0.9	0.1	0.5	1.0
Parents	Non-verbal IQ	37.7	5.1	20.0	46.0
	Arithmetic fluency	33.3	6.8	15.0	40.0
	Approximate Calculation	15.5	2.3	9.0	18.0
	Written Calculation	4.8	1.1	1.0	6.0
	Mental Calculation	5.0	1.7	1.0	8.0
	Home Numeracy	3.0	0.6	2.0	5.0

Estimation	11.9	7.7	4.7	52.5
Magnitude comparison (RTs-ms)	1.4	0.4	0.8	2.6
Magnitude comparison (Accuracy)	1.0	0.1	0.7	1.0

Table 2a –Pearson correlations between parents' and children's variables.

		Children				
		Counting – number sequence	Counting – biunivocal correspondence	Digit recognition	Digit reading	Early math skills - mean
Parents	SES	.038	-.135	-.002	.016	-.052
	Home numeracy	.170	.268*	.398**	.340**	.399**
	Non-verbal IQ	.098	-.097	.193	.164	.095
	Arithmetic fluency	.054	.004	.047	.078	.058
	Approximate calculation	.213	.039	.181	.163	.194
	Written calculation	.138	-.238	-.046	-.108	-.105
	Mental calculation	.140	-.022	.077	.110	.094
	ANS-Estimation	-.018	.049	.121	.126	.088
	ANS-Magnitude comparison (accuracy)	.079	-.253*	.056	-.002	-.073
	ANS-Magnitude comparison (RTs)	-.143	-.106	-.174	-.212	.212

** $p < .01$; * $p < .05$

Table 2b –Pearson correlations among children's variables.

	Counting – number sequence	Counting – biunivocal correspondence	Digit recognition	Digit reading	Early math skills - mean
Non-verbal IQ	.272*	-.020	.325**	.328**	.278*
Memory	.138	.033	.572**	.516**	.380**
Attention	.041	-.090	.108	-.012	.001
Cognitive skills - mean	.265*	-.034	.553**	.456**	.371**
Vocabulary	.132	-.191	.208	.217	.082
Phonological awareness	.160	-.050	.285*	.391**	.230
Language skills - mean	.204	-.144	.349**	.442**	.236
ANS- Magnitude comparison RT (ms)	-.179	.118	-.410**	-.449**	-.258*

ANS- Magnitude comparison accuracy	.246	.337**	.536**	.574**	.563**

** $p < .01$; * $p < .05$

As showed in Table 2a, concerning correlations between SES, home numeracy, parents' variable, and children's early numeracy skills, a strong association was found between home numeracy and Total score in early numeracy as well as in the single tasks: Biunivocal correspondence in counting, Digit Recognition and Digit Reading.

Concerning correlations between children's cognitive, linguistic and ANS skills and their early numeracy skills, a significant relation was found between children's IQ and, more generally, their cognitive skills, to early math skills (see Table 2b). Language skills, in particular phonological awareness, were mostly linked to digit recognition and reading. Finally, many significant correlations can be observed among ANS skills (RTs and accuracy) and early numeracy skills (Table 2b).

Regression

Table 3 shows the results of the hierarchical analysis to investigate predictors of children's early math skills. For the first step, only the home numeracy resulted as a significant predictor of children's early math skills, and the model explained the 20.4 % of variance. At the second step, also children's cognitive and linguistic skills were included as potential predictors, adding a portion of explained variance (7.6 %) that tended to significance (p = .055): home numeracy, as well as children's cognitive skills, resulted as significant predictors. Children's magnitude comparison performance (RT and accuracy) was added as an additional potential predictor at step 3, and the model reached a total explained variance of 39.9 %. Home numeracy resulted again as a significant predictor; also, children's accuracy in the dots comparison task significantly predicted their early math skills.

Table 3 - Hierarchical regression; dependent variable: children's early math skills.

Step		B	SE B	β
1° (R^2 = .204)	Home numeracy	.234	.064	.429**
2° (ΔR^2 = .076, p = .055)	Home numeracy	.193	.065	.355**
	Parent's non-symbolic math skills	-.170	.146	-.136
	Parents' calculation skills	.102	.087	.142
	Children's cognitive skills	.252	.111	.275*
	Children's language skills	.026	.091	.035
3° (ΔR^2 = .119, p = .006)	Home numeracy	.136	.063	.250*
	Parent's non-symbolic math skills	-.102	.138	-.082
	Parents' calculation skills	.066	.081	.092
	Children's cognitive skills	.143	.109	.156
	Children's language skills	-.017	.093	-.023
	Children's non-symbolic magnitude comparison (accuracy)	2.181	.672	.399**
	Children's non-symbolic magnitude comparison (RTs)	-.014	.065	-.027

** * $p < .01$; * $p < .05$

3.5 Discussion

In the present study we aimed to evaluate predictors of early math skills in preschool children including children's cognitive, linguistic, ANS related skills and environmental variables, particularly parents' math skills (symbolic and non-symbolic) and home numeracy, controlling also for SES. To accomplish this aim we administered to children a battery of tasks tapping intellectual and cognitive functioning, early numeracy skills, language skills and non-symbolic magnitude comparison tasks. A similar battery was administered to parents and included measures of intellectual functioning, symbolic math skills (mental calculation, written calculation), non-symbolic ANS related tasks (estimation and magnitude comparison) and questionnaires about home numeracy activities and socio-economic-status (SES).

In the first set of correlation analyses we found multiple relationships between parents' and children's skills and within children's skills.

Concerning the associations between SES, home numeracy and parents' skills and children's early numeracy, it is of interest that there was no significant relationship between SES and children's skills. This is in line with OCSE report that accounts a minor predictive role of SES in Italy (Quintano, Castellano, & Longobardi, 2009). By contrast, a significant strong relationship between home numeracy activities and early math skills in children was described, reinforcing the strong evidence reported in literature about the important role of home numeracy in fostering numerical development in children (Lefevre et al., 2009), and this was replied also in different cultural contexts (Authors, submitted). Concerning the role of intergenerational path of math skills, both parents' ANS skills as well as calculation were not related to children's formal math skills. The latter, conversely, were highly related to general cognitive skills. Although previous studies found a significant relationship between ANS skills and math scores in adults (Libertus et al., 2012), others did not (Jang & Cho, 2016). Possibly, this is due to the developmental trajectory of path skills, and other variables such as education and frequency of math activities in everyday life might modulate this relationship in the life course.

Concerning the correlations within the children's skills, a strong association between cognitive skills and early numeracy has been pointed out. Indeed, non-verbal IQ and memory show a strict association with digit recognition and reading, reinforcing the idea that math skills might be, at least in part, influenced by general intellectual functioning (Poletti, 2017). In addition, the present study gives also useful insights about the relationship between linguistic and numeracy skills. As expected there was a significant relationship of mean language scores with recognition and reading of numbers (Purpura & Ganley, 2014), with a relevant role of phonological awareness. The relationship between phonological awareness and mathematical skills has received contrasting results in literature and some authors suggested that it might be not constant across development (Passolunghi, Lanfranchi, Altoè, & Sollazzo, 2015). Then, non-symbolic magnitude comparison tasks were significantly related to early math skills, in line with previous evidence about the foundational role of ANS skills in number development (Libertus et al., 2013). Taken as whole, results from correlational analyses gave interesting insights about a complex pattern of relationships within and between groups, evidencing that, beyond a relationship between early math skills and ANS related measures, early numeracy skills develop within a network of multiple relationships, involving both environmental and within subject variables.

In order to better understand the strength of these multiple factors we developed a regression model that included, at the first step the environmental variables, at the second step the children's cognitive and linguistic profile and at the third step the children's ANS skills. It emerged that environmental variables alone explained around 20% of variance, but only home numeracy resulted to be a significant predictor of children's early math skills. Then, the second step of the analysis added a marginally significant proportion of variance showing that children's cognitive, but not linguistic skills predicted early numeracy skills. Finally, we wanted to evaluate if children's ANS related skills represented a meaning predictor of their numeracy skills beyond and above the influence of environmental and cognitive factors. The variance added in the third step was significant and accuracy scores in the dots comparison task, together with home numeracy, were the significant predictors of early numeracy in the final model.

These results represent an original picture on the complex interplay amongst variables involved in the development of early math skills. Indeed, parents' math skills were not significantly correlated with children's early numeracy skills, while home numeracy activity had an important role in explaining early numeracy skills. In addition, children's cognitive skills were related to their early numeracy skills, but when their ANS skills were considered, the latter became the unique significant individual predictor of children's early math skills.

We previously reviewed evidenced about the consolidated dual relationship between home numeracy and early math skills and between ANS measures and early numeracy development, however, the present study offers a new window in this literature taking account of these different variables altogether and adding the assessment of parents' math skills. Undoubtedly, this research line needs more in-depth investigation in the light of limitations of the present study that might limit generalizability of results. First, the sample is relatively small and further investigation on a wider sample is needed. Secondly, there is a debate in literature about which ANS tasks have highest validity (Smets, Gebuis, Defever, & Reynvoet, 2014) and replication would be required with different type of tasks, both in relation to ANS related skills and early cognitive, literacy and math skills. The task we used to evaluate the ANS is probably not the most appropriate since it does not evaluate large magnitudes but only includes dots ranging from 1 to 9; however, the use of RT allows us to have a proficiency profile on the subitizing of children.

Despite these limitations, the suggestion that came from the present study is that the type of activity that parents carry on in the home environment might be more powerful than their actual efficiency in math skills. In addition, considering children's skills, the present study evidenced that domain specific skills, such as those related to ANS, are more important than domain general cognitive and linguistic skills in shaping early numeracy competence. Therefore, stimulating children's ANS skills is of importance for favoring their early math skills. Further investigation in other age ranges (primary and secondary school) should better investigate the developmental patterns of complex interactions across individual and environmental variables in predicting math skills.

Finally, the present study suggests important implications for the educational setting, where it is important to activate both direct (directed to the child) and indirect (directed to parents) instruction on numeracy development.

Chapter 4

Does reading ability affect the underlying cognitive skills of arithmetic?

4.1 Introduction

The Triple Code Model (TCM) (Dehaene, 1992, 1997) suggests that numbers are expressed in three different codes that serve various functions, have distinct functional neuroarchitectures, and are related to performance on particular tasks (Van Harskamp & Cipolotti, 2001). This model explains that these codes are at the base of our ability to count and process numerosity. The first one is a verbal code, connected to the linguistic system, that is used to recover well-learned arithmetic facts using memory, such as simple addition and multiplication tables (González & Kolers, 1982). The second one is a visual code that represents and spatially manages numbers in Arabic format (M. H. Ashcraft & Stazyk, 1981; Dahmen et al., 1982; Dehaene & Cohen, 1991; Weddell & Davidoff, 1991). Finally, the third code is the analog magnitude representation, that gives a representation of analogical quantity on a mental number line (approximate calculation and magnitude comparison) (Chochon et al., 1999; E. Spelke & Dehaene, 1999). According to Dehaene, the verbal code is used in particular for counting, addition, and easy multiplication, while approximate calculation and comparison are sustained more by the nonverbal codes. From a developmental perspective, some studies proposed that language is essential for the growth of numerical competencies (Hauser et al., 2010; Elisabeth S. Spelke, 2003), and there are evidences that the structure of the language system in which one grows up shapes the development of numerical concepts (Pica et al., 2004). Others, however, argue that numerical competence, at least for some aspects, can develop independently from linguistic skills (Landerl et al., 2004). Landerl et al. (Landerl et al., 2004) support the theory that the number system is able to develop independently from the language domain. However, the relationship between linguistic and numerical skills is still under debate. In the present study we addressed the issue of the specificity of cognitive markers of math abilities and whether reading ability might also affect the development of verbal numerical competencies.

Relationships between Math and Reading skills and co-occurring disorders

The relationship between literacy and arithmetic is not completely clear. Some authors suggest that math impairments have a core deficit in the Approximate Number System (ANS) (Butterworth & Laurillard, 2010; De Smedt et al., 2013; Piazza et al., 2010), an automatic, non-symbolic, approximate sense of number that is available before the start of schooling and that survives beyond the lifespan. Others propose a deficit in accessing numerosities from symbols, which then turns out in a nonsymbolic number processing weakness (Noël & Rousselle, 2011). Those interpretations are related to a deficit in processing numerosities that is recognized as the actual core deficit responsible for the mathematics disorder (MD) in a similar way to what happens with the phonological awareness and the reading disorder (RD) (Landerl et al., 2004, 2009).

It is known that math and reading are related, and math and reading problems often co-occur (Koponen et al., 2018; Landerl & Moll, 2010). Common causes are one explanation for the comorbidity of problems in reading and math. Very often children with reading disorders also have a mathematical disorder. Therefore the development of the verbal number system might also be impaired in children with reading problems (Davis, Band, Pirinen, Haworth, Meaburn, Kovas, Harlaar, Spencer, et al., 2014; Landerl et al., 2009). It is believed that the occurrence of both difficulties (reading and mathematics) can be between 2.3% and 40% (Lewis, Hitch, & Walker, 1994; Moll, Göbel, & Snowling, 2015). Many studies have dealt with the mathematical difficulties of dyslexic people (Steeves, 1983). Simmons and Singleton (2008) hypothesized that people with reading impairments have a weakness in the verbal code and in particular in recalling numerical facts. In line with this hypothesis, several studies reported that children with dyslexia are slow in calculation and in particular have difficulties with multiplication (Fiona Rachel Simmons & Singleton, 2006; Turner Ellis, Miles, & Wheeler, 1996). Another hypothesis, not necessary incompatible with the first, is that the mathematical difficulties related to reading difficulties could originate from phonological processing deficits. Indeed, many studies have found that phonological processing difficulties predict mathematical difficulties (Hecht et al., 2001; Leather & Henry, 1994; Rasmussen & Bisanz, 2005). A third explanation for the frequency with which math (MD) and reading disorders (RD) occur together is a deficit in the general domain; a shared deficit between RD and MD such as processing speed or working memory could explain the comorbidity (Bull & Johnston, 1997; Geary & Hoard, 2002; Willcutt et al., 2013). In contrast to these studies, suggesting common cognitive causes of MD and RD, there are also studies in which the cognitive core deficits underlying RD and MD were found to be distinct (Cirino, Fletcher, Ewing-Cobbs, Barnes, & Fuchs, 2007; N. C. Jordan et al., 2003; Landerl et al., 2009; van der Sluis, de Jong, & van der Leij, 2004; Willburger, Fussenegger, Moll, Wood, & Landerl, 2008; Willcutt et al., 2013).

Of particular interest with respect to the comorbidity of MD and RD, is whether the deficits of the comorbid group, MD + RD, can be characterized as an additive combination of the deficits found in the single disability MD and RD group (e.g., van der Sluis et al., 2004; Willburger et al., 2008). For example, Moll, Göbel and Snowling (Moll et al., 2015) investigated the cognitive profiles of children with RD, MD, RD + MD, and typically developing children (TD). Following the TCM they examined how number processing for the different number codes was related to mathematical and reading ability. Their results indicated that factors underlying numerical difficulties in children with RD were different from the factors underlying numerical problems in children with MD. Children with RD were impaired in phoneme awareness and in RAN but not in simple reaction time (Bonifacci & Snowling, 2008). Children with RD performed poorly on all tasks tapping verbal number skills but they had no difficulty with either the Non Symbolic number comparison or in locating numbers on the number line. Their weaknesses were particularly marked when numbers had to be transcoded. These findings showed that children with RD experience deficits in numerical tasks, tapping verbal skills, but are unimpaired in their approximate number sense. The cognitive profile of the RD + MD group did not differ from the single-deficit groups in mathematics and literacy skills but manifested poorer performance than the RD group in some language measures (PA and Verbal IQ). Importantly, none of the RD by MD interactions were significant, demonstrating that the cognitive deficits of the comorbid group were simply the sum of the deficits of the single-disability groups (Moll et al., 2015).

Present study

The aim of the present study was to investigate whether the verbal and nonverbal cognitive skills presumed to underlie math are also related to reading. Previous studies on the relations of math and reading with these cognitive skills were examined with single (MD and RD), double (MD + RD) deficit groups and typically developing children. These groups were the result of cut-offs on math and reading ability, which are generally considered as continuously distributed abilities. In this study, we adopted a continuous perspective, but following the approach of studies with various deficit groups, we examined the effects of reading and math skills as well as the interaction of these abilities on the cognitive skills believed to underlie math development. Italian fourth and fifth grade children were administered measures of reading and arithmetic, nonverbal IQ and various underlying cognitive abilities of arithmetic (counting, number sense, and number system knowledge). We also included measures of working memory and phonological ability. To evaluate different components of number processing skills, the Triple Code Model (Cohen, Dehaene, Chochon, Lehéricy, & Naccache, 2000; Dehaene, 1992) was used as theoretical framework.

The aims of the study were two-fold:

- First, we aimed to assess relationships between reading, arithmetic and the cognitive correlates; we hypothesized that cognitive correlates related to the phonological domain would be related to both reading and arithmetic. In contrast, tasks related to the number sense domain, measured through tasks of magnitude processing, to be predicted only by math, and not by phonological skills;
- Secondly, we aimed at comparing children with good vs. poor reading and arithmetic skills by assessing differences in verbal and non-verbal cognitive skills. We expect children with poor reading and arithmetic to have weal phonological processing abilities. In contrast, we expect children with poor arithmetic (but not with poor reading) to have weaknesses in number sense and in nonverbal cognitive skills.

4.2 Method

Participants

The sample consisted of 97 children (mean age = 9.8, SD = 0.6, 55.7% females), attending the 4th (57 children) and the 5th (40 children) grades of primary school.

Participants were selected from schools in suburban areas in the north of Italy. From an initial sample of 126 children, we included in the study only participants with a complete dataset collected. Parents provided written informed consent prior to the experiment. The Ethical Committee of the University of Bologna approved the study design.

Measures

Children were administered tests assessing intellectual functioning, formal math skills and reading tasks. A detailed description of the task is detailed below.

Intellectual functioning

General Cognitive Ability. Children were administered the *Matrices* subtest of K-BIT 2 (Bonifacci & Nori, 2016; Kaufman & Kaufman, 2014). The test measures is a measure of Non-Verbal IQ. Depending on the age range, the child was shown pictures (starting from one up to a matrix of twelve elements) and he/she was asked to choose amongst five to six images the one the best fitted with the target picture. For example, on top, there might be a picture of rain associated with an umbrella, and the sun associated with a question mark. Then pictures below that include gloves, socks, sunglasses, and shoes. The sun goes with sunglasses, so that would be the answer. There are different starting points based on the participant's age and the task stops after four consecutive wrong responses. There are 46 items, there is a core of 1 for each correct answer and the maximum score is 46.

Memory span. Children were administered the digit span task (forward and backward) sequencing test (*Memory*) of the subtest of the WISC-IV (Wechsler, 2003). Children were required to repeat forward and backward series of numbers of increasing length. The task was stopped after two failures on a series of the same length. The score is the number of digits' series that they can repeat correctly. The maximum score is 16 for the forward and 16 for the backward.

Phonological Awareness. Children were administered the *Phonological Segmentation*, a subtest of the Nepsy II battery (Korkman et al., 2007). *Phonological Segmentation* is a test of elision. It is designed to assess phonological processing at the level of word segments (syllables) and of letter sounds (phonemes). The child is asked to repeat a word and then to create a new word by omitting a syllable or a phoneme ("say "dolcemente" but without "mente""), or by substituting one phoneme in a word for another ("say "roba" with "s" instead of "b""). There are 53 items and the maximum score is 53.

Mathematical knowledge

For this ability was administered the BDE-2 (Biancardi et al., 2016), the Battery for Developmental Dyscalculia updated to the new scientific knowledge on

Discalculia and to the Italian guidelines on Specific Learning Disorders. The BDE-2 is composed of 9 tests plus 3 optional tests (of which only "repetition of numbers" was administered) for the fourth and fifth primary class. We performed Cronbach's alpha and factorial analysis to test the internal consistency and for the purpose of the present study tasks were grouped in four main areas: Arithmetic, Counting, Number Sense and Number System Knowledge.

Counting. In this task the examiner asks the children to count aloud from 80 up to 140 and records the time. Then the experimenter asks the child to count backwards from 140. The time given to do so is the time that the child needed to count forward from 80 up to 140. The score is the total of number of numbers the child said correctly backwards within the allotted time.

Number Sense. This was evaluated using two different subtest: *Triplets* and *Insertion*. On the test *Triplets* children have 2 minutes to indicate on a paper record form the largest number in 18 sets of three numbers (e.g. 30100, 31000, 30009). The score is the total number of answers they give correctly in 2 minutes. The maximum score is 18. On the test *Insertion* children have 2 minutes to place a target number at the corrects place in a series of three numbers arranged in ascending order. For example they have to put on a paper record form the number 10 in the correct position between the numbers 5, 8, 15. The number of items is 18. The score is the total number of correct items done in 2 minutes. The maximum score is 18.

Number System Knowledge. This task was evaluated using three different subtest: *Number Reading*, *Number Writing* and *Repetition of Number. Number Reading:* children have one minute to read aloud a list of three to six digit numbers of increasing difficulty. The score is the total number of numbers they read correctly. *Number Writing and Repetition:* This task gives two scores. First the child has to repeat the number (*Repetition of numbers*) and then the child has to write the number (*Number writing*). There are 18 numbers, among which there are numbers with the 0 (e.g. 807 or 5010) and numbers with 4, 5 and 6 digits (e.g. 27463 or 346879). A score of 1 is given for each number that the child repeats (repetition score) or write (writing score) correctly. The maximum score for both scales is 18.

Math and Reading Ability

Standard tests for math and reading were administered.

Math: This task was evaluated using four subtests of the BDE-2 (Biancardi et al., 2016) referred to the calculation ability and speed: *multiplication, mental* *calculation, arithmetic fluency, approximate calculation*. *Multiplication*: the examiner reads 18 items in random order (e.g. 3x4; 7x9…). Children have 3 seconds to give an answer to each operation. The score is the total number of answers they give correctly within three seconds. The maximum score is 18. *Mental calculation*: the examiner reads 18 operations (9 additions, 9 subtractions) and children have 30 seconds to answer to each operation with the correct result. The score is the total of answers they give correctly. The maximum score is 18. *Arithmetic fluency*: children have 2 minutes to write the correct results of as many mixed operations as possible (additions, subtractions, multiplications, divisions) up to a maximum of 40. The score is the total of answers they give correctly in 2 minutes. *Approximate calculation*: children have 2 minutes to indicate the correct result of 18 operations, indicating it from the 4 options. For example the operation is 75:5 and they have to choose between 80, 375, 15 or 5. The score is the total of answers they give correctly in 2 minutes. The maximum score is 18.

Reading. The reading materials were two texts taken from the *MT Reading test,* the Italian battery used to assess text reading speed and accuracy (Cornoldi, Colpo, & Carretti, 2017). The texts were different for the two different grades of primary school. The text used to assess children from the fourth grade of elementary school has 141 words, while that for children from the fifth grade has 236 words. For the purpose of the present study we calculated reading fluency, that is the number of words read correctly aloud in one minute. Also, the number of total errors and the number of syllables read per second were taken into account. Then, we compute the Z-score within each grade using the reading fluency in order to have a unique score of this variable by grade.

Procedure

All the tests for cognitive skills, and for reading and arithmetic were administered individually during school hours in a quiet room at the school. Usually, the tasks were administered in two different sessions on two successive school days.

4.3 Data Analysis

Regression analyses were conducted to examine the effects of reading and math on the various cognitive variables. In these analyses the cognitive variables were regressed on reading and math, and the interaction of reading and math. The latter interaction variables were computed as the product of the Z score of Reading with the

Z sore of Arithmetic (Z Reading * Z Arithmetic). In all regression analyses we controlled for grade and nonverbal IQ.

Later we created two different groups for Reading and Arithmetic splitting the children using their Z score in those tasks. A group with Z score of Reading lower to 0 (Low-R) and a group with Z score higher than 0 (High-R). We did the same with Arithmetic. We had 50 children Low-R and 45 with High-R. For Arithmetic we had 51 children with Z score lower than 0 (Low-A) and 44 children with Z score higher than 0 (High-A).

4.4 Results

Descriptive and correlation analysis

Descriptive statistics with averages, SD and ranges are reported in Table 1. Descriptive statistics for children's variables, separated by grade are reported in Table 2.

Pearson correlations among children variables are reported in Table 3.

Both in fourth and fifth grade *Phonological Awareness* is positively associated with *Memory, Counting and Number System Knowledge.* In Grade 5 (Table 2) there is also a correlation with *Number Sense. Memory* is related with *Phonological Awareness* in Grade 4 and *Number Sense* in Grade 5. *Counting* is positively associated to *Number System Knowledge* in both grades and to *Phonological Awareness* in Grade 4. There are correlations between *Number Sense* and *Number System Knowledge* in Grade 4 and Grade 5. In Grade 4 there is also a relation with *Counting,* while in Grade 5 with *Memory. Number System Knowledge* is positively related with both the mathematical measures (*Counting* and *Number Sense).* Additionally in Grade 4 is related with *Phonological Awareness* and in Grade 5 with *Memory.*

Table 1 – Descriptive statistics

Measures	Average	SD	Range	
			min	max
Non-Verbal IQ	29.69	6.58	15	42
Memory forward	7.73	1.45	5	13
Memory backward	6.69	1.27	4	11

Phonological Awareness	46.94	3.27	39	53
Counting backward	37.47	9.04	10	54
Triplets	16.60	1.91	9	18
Insertion	15.84	1.90	9	18
Number reading	20.75	4.66	7	31
Repetition of numbers	13.97	2.50	8	18
Number writing	15.63	2.44	5	18
Multiplication	14.14	3.34	3	18
Mental calculation	12.29	3.10	5	18
Arithmetic fluency	20.75	8.18	7	39
Approximate calculation	10.05	3.79	4	24
Reading fluency	99.12	31.02	45	208
Reading errors	3.72	2.94	0	16

*Table 2 – Descriptive statistics for Grade 4 and Grade 5 children's varia*bles

	Grade 4		Grade 5	
	Mean	SD	Mean	SD
Age (years)	9.51	.48	10.27	.46
General Cognitive Ability	28.61	6.72	31.48	1.05
Phonological Awareness	46.44	3.19	47.65	.52
Memory	14.00	1.91	15.03	.44
Counting	-.27	1.69	.38	.32
Number Sense	-.28	1.73	.40	.25
Number System Knowledge	-.81	2.12	1.15	.42

Table 3 – Pearson correlations among Grade 4 and Grade 5 children's variables

GRADE 4 \ GRADE 5	*Phonological Awareness*	*Memory*	*Counting*	*Number Sense*	*Number System Knowledge*
Phonological Awareness		0.67**	0.49**	0.54	0.71**
Memory	.42**		0.21	0.34*	0.55**
Counting	.39**	.18		0.21	0.58**
Number Sense	.12	.08	.41**		0.40*
Number System Knowledge	.41**	.19	.56**	.32*	

Regression

We run regression analyses to investigate predictors of phonological awareness, memory and the cognitive correlates of math. In these analyses, Reading, Math and the interaction of Reading and Math (IRM) were the independent variables. Moreover, in these analyses we controlled for Grade and IQ.

First, we run a regression with phonological awareness as dependent variable. We found that grade ($\beta = .200$, $p = .024$), IQ ($\beta = .219$, $p = .020$), Reading ($\beta = .232$, $p = .012$) and Arithmetic ($\beta = .282$, $p = .004$) were significant predictors. The interaction of reading and arithmetic (IRM) was also significant ($\beta = -.252$, $p = .004$). In order to better understand the nature of the interaction between Reading, Arithmetic and phonological and number system skills we analyzed group differences in reading and arithmetic. Considering phonological awareness we found a significant effect of Reading [$F(1,94) = 10.25$, $p < .01$], Arithmetic [$F(1,94) = 9.57$, $p < .01$] and of the interaction Reading and Arithmetic [$F(1,94) = 3.95$, $p = .050$]. Both groups of low Reading and Arithmetic had weakest phonological skills compared to the High Groups. A shown in Figure 1 the interaction showed that children with low Reading and Arithmetic had the poorest performances in phonological awareness. Thus, in the group of good readers the difference in phonological awareness was not affected by arithmetic but in the group of poor reader, those with high arithmetic skills had better phonological skills.

For Memory forward the effects of reading and Math and their interaction were not significant. Grade was the only significant effect (β = .21, p < .05). For Memory backward (R^2_{adj}= .07, p<.05) none of the predictors were significant.

Then we run a model with Counting as dependent variables (R^2_{adj}= .42, p<.01). Reading (β = .235, p = .007) and arithmetic (β = .490, p = .000) were both predictors of Counting. The interaction between of reading and arithmetic was not significant. Interestingly, Non-Verbal IQ was not a significant predictor of Counting. For Number Sense as dependent variables we found that the Non-Verbal IQ (β = .244, p = .000) and arithmetic (β = .349, p = .000) were unique predictors whereas reading and phonological measures were not. Finally, Number System Knowledge was predicted by reading (β = .209, p = .007), arithmetic (β = .479, p = .000) and the interaction of Reading and Math (β = -.209, p = .005).

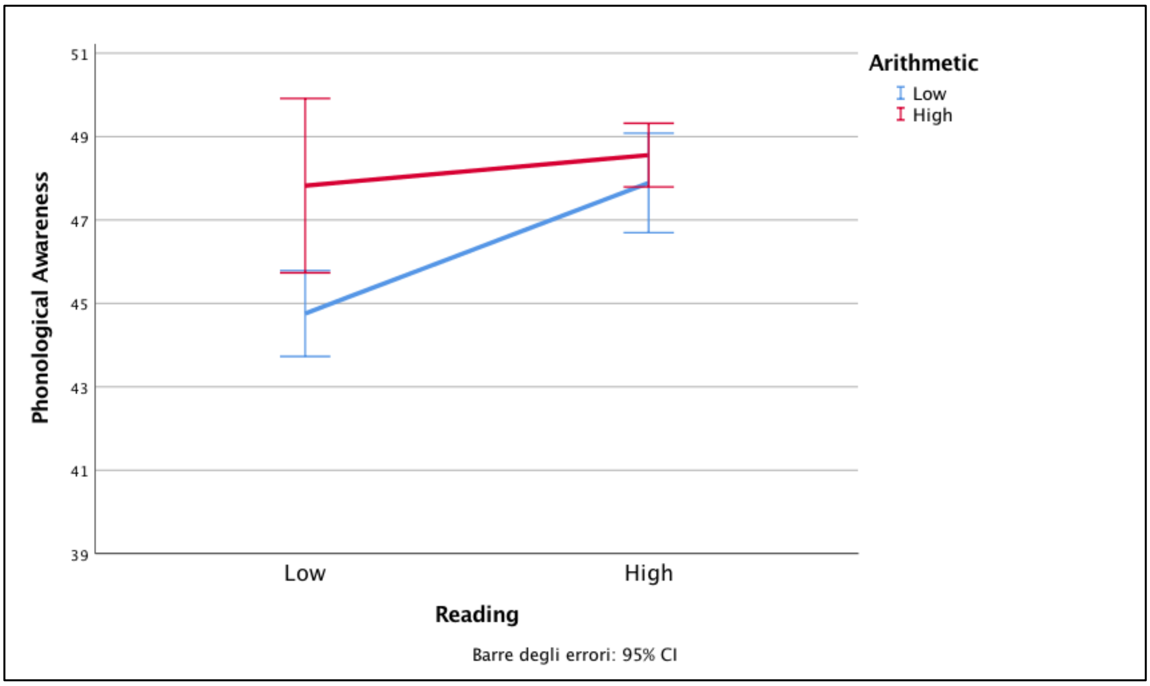

Figure 7 Boxplot of phonological awareness skills in groups of high vs low Reading and Arithmetic skills.

4.5 Discussion

The present study aimed to investigate whether the verbal and nonverbal cognitive skills presumed to underlie arithmetic were also related to reading and domain general cognitive processing. More specifically, we examined which cognitive abilities were specific to arithmetic and which were related to reading ability. In particular, we wanted to explore the predictors of the different aspects of mathematics. Mathematics is composed of verbal and non-verbal aspects and our aim was to observe if there were differences between these different components. Moreover, we were interested in discovering the role of phonological awareness. it is commonly known that phonological competence plays a very important role in the ability to read. Our interest was to investigate whether phonological skills are also specifically related to arithmetic.

First, a set of correlation analyses has been run in order to evaluate the relations between the different measures that we used. Especially our interest was to estimate the relationships between verbal and non-verbal tasks. In particular, the Counting and Number System Knowledge tasks involved verbal aspects. In these tasks the children were asked to count aloud, read aloud, listen and repeat. The Number sense test, on the other hand, was characterized by purely visual and non-verbal aspects, in which the children had to recognize numerosity or transcribe them in the correct position. This last task was related to the number comparison ability and the proficiency to represent analogical quantity on a mental number line. Through Pearson's correlations analysis we observed that the mathematical skills were correlated with each other but, observing the phonological competence, this was not correlated with all the mathematical aspects. We found correlations between Phonological awareness and Memory, Counting and Number System Knowledge but not with the Number Sense. The association between Phonological awareness and Counting and Number System Knowledge suggests that the verbal aspects are important in some aspects of arithmetic skills. It could be possible that there are not correlations between Phonological awareness and Number Sense because the verbal aspects are not involved in representation of analogical quantity on a mental number line and it means that this task is independent. On the counterpart, memory skills showed some correlations with Number Sense and Number system Knowledge.

Turning to the regression analysis, intending to analyze the potential predictors of children performance in phonological and mathematical tasks, results evidenced that phonological awareness was influenced by all the measures that we have taken into account. The roles of grade and non-verbal IQ were not negligible and, how we supposed, also the reading ability was a predictor of the high phonological performance. An aspect not so predictable was that even arithmetic had an important implication for phonological elaboration. However, the data that most attracted us was the influence of the compound measure of reading and arithmetic with phonological awareness. For this reason, we have carried out further analyses to deepen those results.

On the contrary, we did not find specific predictors of memory skills. We have run models using both memory forward and memory backward as dependent variables but we did not find interactions either with other cognitive skills.

Turning to the Counting and Number System Knowledge, we found that they were highly related both with arithmetic and with reading. The evidence of a relationship between Counting, Number System Knowledge and Reading is in line with what was emerged about phonology. As we said before, the verbal aspects of those tasks give us the key to better understand why they are so related. Those results are exactly in line with the researches of Koponen (T. K. Koponen et al., 2018) and Moll (Moll et al., 2015) when they found that the children that perform poorly in reading have also difficulties with transcoding numbers and in general with verbal mathematical skills. Furthermore our analisys confims that the Number Sense, which was a purely non-verbal task, was independent from phonological awareness and also from reading ability. The regressions shows that only the Non-Verbal IQ and Arithmetic are predictors of Number Sense and reading is not involved in this ability. A comparable result was found in Moll's study when the children with Reading Disorder presented difficulties related to the semantic aspects of the number but they were unimpaired in the approximate number sense.

These results suggest the importance of the phonological skills and the interconnection between reading and arithmetic, but, at the same time, the independence of other aspects of the number, particularly non-verbal ones such as the number sense that do not seem to be connected to verbal skills.

Finally, to better understand the interaction of reading and arithmetic skills with phonological awareness we performed Anovas with the groups splitted for having high vs low reading or arithmetic skills. It emerged that in the group of good readers, the difference between high and low performances in arithmetic is small and the same happened with the phonological awareness. On the contrary in the group of poor readers, there are great differences between the proficiency in phonological tasks and it seems to depend on the arithmetic expertise. The arithmetic affects positively the phonological awareness in the group of the poor reader, working as a protective factor. If reading get better, the difference between good and poor arithmetic is getting smaller.

This research shows that there are many connections between reading and arithmetic but the non-verbal aspects of mathematical skills are independent from reading skills. In particular, the number sense seems to play a pivotal role in the knowledge of mathematical skills and should be deepened.

The present study has some limitations that would require to be addressed in future investigations. Limitations of this study were that participant numbers were small and the study did not include Rapid Automatized Naming (RAN), so the domain specificity of the reading ability could not be examined.

Future studies will have to replicate these findings within different age ranges. Furthermore, longitudinal studies are needed in order to assess developmental fluctuations in numerical processing, number sense and calculation skills.

Chapter 5

General Discussions

5.1 Main findings

The studies reported in this thesis set the goal of better investigating mathematical skills and their cognitive predictors. Particular attention was given to the Approximate Number System (ANS) (Dehaene, 1997; Gallistel & Gelman, 2000), which numerous recent studies designate as one of the most relevant features of mathematical skills. Number of evidences indicates the ANS as the most innate trait, present from the first months of life, and therefore considered at the foundations of math skills development. The ANS seems to interact with other cognitive skills, and this interaction will allow the child to learn the numerical notions starting from kindergarten up to formal teaching in primary school. However, very little is known about the reasons that explain individual differences, if these are totally innate or influenced by other factors, and how much they are inherited from the parents. Some research has found a positive effect on the ANS by environmental factors and SES. Some studies have found an effect of mathematical education (Piazza, Pica, Izard, Spelke, & Dehaene, 2013); others have seen that very high SES levels were significantly correlated with higher ANS acuity (McNeil, Fuhs, Keultjes, & Gibson, 2011; Mejias & Schiltz, 2013). There is much evidence of a positive effect of parents on the development of children's mathematical skills. In particular it has been successfully explored: the positive effect of domestic activities related to numbers (Lefevre et al., 2009), the negative effect that parents' mathematical anxiety has on their children (Maloney et al., 2015a) and the influence of gender stereotypes (Tomasetto et al., 2011).

Some recent studies have also paid attention to the role of parents in numerical learning. Specifically, they focused on home numeracy activities (Lefevre et al., 2009, 2011). However, limited research has examined the intergenerational impact of math skills, that is the relation between parents' and children's skills in the numerical domain, although many studies showed parent-child relations in other domain such as reading ability (van Bergen et al., 2014).

In the present dissertation, pattern of relationships between ANS skills, cognitive and linguistic skills, formal and informal math skills and role of environmental variables (home numeracy and parents' math skills) were considered. The studies considered two different age ranges: the end of primary school, when formal math skills have been acquired through teaching, and preschool, when children already show spontaneously and intuitively developed basic calculation skills (Levine, Jordan, & Huttenlocher, 1992). Further, we analyzed the relationship between reading and math skills, considering the role of shared cognitive markers.

The first study analyzes symbolic and non-symbolic numerical abilities of parents to better understand if these were predictors of children's numerical skills, either considering basic symbolic, non-symbolic and formal math skills (i.e., written calculation). A battery of cognitive and math tasks was administered to a sample of 83 children with established (i.e., 4-5 years) formal school experience, and to their mothers. Results evidenced significant relationships between children and mothers' symbolic and math skills, but children symbolic skills were the most significant predictor of their math skills. Taking into account the previous literature on the role of ANS measures in predicting math abilities, the present study confirms a significant association between non-symbolic processing and math skills when considering correlation analysis. However, in the regression model, the main predictor of children's math skills was their performance in the symbolic comparison task. This is in line with previous researches that evidenced how achievements in maths are primarily associated with symbolic processing (Schneider et al., 2017). The study suggests that the intergenerational features on math skills play a significant role in children's numerical development but that it ultimately depends mainly on their own numerical processing. These results are partially in contrast with Braham and Libertus, (2017) which found direct correlations between ANS acuity of parents and children. Furthermore, the study adds information about the differential contribution of children's and parents' skills on numerical and math development. In particular, our study adds evidenced that also parents' formal math skills play a role in children's numerical development.

The second study aimed to replicate and extend findings from the first study but focusing on children who are in the preschool period, therefore when their numerical skills are not shaped by formal instruction. In this way, the role of the teaching of formal mathematics was defeated, at least partially, to be able to investigate better the innate aspects and those influenced by the domestic environment. The literature currently believes that numerical abilities in preschool age, in fact, depend on a variety of different factors, including approximate number system abilities (ANS), children's cognitive and linguistic abilities, and environmental variables such as home numeracy activities (Lefevre et al., 2009). The objective of this study was to analyze the role of these variables, with the addition of parents' numerical abilities to understand the role of intergenerational transmission of mathematical skills. The sample included 64 children in the last year of kindergarten and one of their parents. A series of cognitive, linguistic, and non-symbolic tasks were administered to the children. Parents were administered similar tasks to assess cognitive, mathematical, and ANS skills (non-symbolic estimation and comparison), along with a questionnaire on home numeracy to explore the effect of both direct and indirect numerical activities.

From the data analysis, multiple correlations emerged between parents' and children's numerical skills. However, in the regression model, parents' skills did not result significant predictors of children's math skills. The children's ANS acuity and home numeracy were much more important for children's skills. These last two aspects have emerged as predictive of children's mathematical abilities.

In the third study, the intent was to analyze the relationship between mathematics skills and reading skills. This aspect has already been studied in literature but has never led to clear and definitive explanations. The approach with which this study was executed is quite different from other studies in this area. In particular, in the study included in this thesis, we examined whether the verbal and nonverbal cognitive abilities at the basis of arithmetic are also related to reading.

To do this, we administered to 97 children, attending the 4th and 5th grades of primary school, reading and arithmetic measures, non-verbal IQ, and various underlying cognitive abilities of arithmetic (counting, sense of number and knowledge of the numerical system). We have also included working memory and phonological skills. Controlling for nonverbal IQ and grade, results showed that phonological awareness, counting skills, and number system knowledge were related to both reading and arithmetic, whereas backward span and number sense were specific to arithmetic.

These studies are partially in line with other recent findings (Mazzocco et al., 2011; Starr, Libertus, & Brannon, 2013), which highlight the importance of the ANS since childhood and its interaction with mathematical skills. However, our studies' results do not show direct effect of the ANS acuity of parents on the ANS of the children. The innovative result that emerged is that mothers' ANS skills were significant predictors of children's formal maths skills. The non-symbolic magnitude comparison abilities of children were found to be important for development and the evolution of numerical skills (Carey, 2001; Feigenson, Dehaene and Spelke, 2004), but symbolic magnitude comparison was the most significant predictor of math skills (Schneider et al., 2017; Chen & Li, 2014; Libertus, Feigenson, & Halberda, 2013). In the study performed with primary school children, written calculation was significantly related to the response times of the symbolic task (RTs) in children, but not in mothers. The non-symbolic accuracy was instead weakly correlated to the ability to calculate both in mothers and children. These results insinuate a stronger relationship for children between symbolic numerical abilities and formal arithmetic, while, in adults, the calculations seem to be more related to general cognitive efficiency. Significant relationships emerged between tasks of symbolic and non-symbolic comparison in children, but these were more limited in mothers. Although some researchers declare that non-symbolic and symbolic abilities are divisible (Kolkman, Kroesbergen and Leseman, 2013), results from our study is in line with the symbolic and non-symbolic relationship magnitude comparison tasks (Li et al., 2018).

Our results suggest the importance of the numerical abilities of mothers in the development of children's abilities in numerical processing and, at least in part, are in line with the results of Navarro et al. (Navarro et al., 2018) and Braham & Libertus (Braham & Libertus, 2017), who discovered that the parents' ANS abilities were related to the numerical processing of children. However, unlike Braham and Libertus (Braham and Libertus, 2017), we found that, including the symbolic and non-symbolic processing abilities of children, the role of parents became marginal, with a more significant effect than the numerical processing of children in the prediction of their ability to perform written calculations.

Although some previous studies on preschool children found a significant relationship between ANS abilities and math scores in adults (Libertus et al., 2012), in the study carried out with preschool children, the ANS acuity and parental calculation were not related to formal mathematical skills of children, as emerged in another study (Jang & Cho, 2016).

Instead, the non-symbolic magnitude comparison tasks of the children were significantly related to the early mathematical skills, in line with previous evidence on the fundamental role of ANS skills in number development (Libertus et al., 2013). Overall, the results of correlational analyzes have provided a complex model of relationships between environmental and within-subject variables. Indeed, the environmental variables explained about 20% of the variance, but only the home numeracy was a significant predictor of children's initial mathematical abilities.

Similarly, in the studies recently carried out by LeFevre (2009), the aspects of home numeracy in younger children are decisive for their development of numerical skills. However, in our study, the most predictive early numeracy model was found by combining children's non-symbolic skills to the home numeracy.

Finally, in the study that evaluates the interaction between reading and mathematics, it can be noted that there is an influence of linguistic abilities (Hauser et al., 2010), in particular of phonological elaboration, in numerical competence. Still, some aspects are independent and specific numerical skills. The sense of number does not depend on other linguistic factors and has no connection with reading, but is an area that develops independently and has direct repercussions on the calculation. Landerl et al. (Landerl et al., 2004) believed that the process of acquiring numerical skills and that of language skills were strictly independent. In our study, we can see that this is the case for some aspects but does not happen in such a restrictive way. There are some areas of overlap between the two domains, in particular, number knowledge and counting seem to have some correlations with reading and language development. The area of the sense of number, however, remains autonomous and seems to develop independently; in many studies, in fact, this is the area held responsible for specific disorders of mathematical learning (Butterworth & Laurillard, 2010; De Smedt, Noël, Gilmore, & Ansari, 2013; Piazza et al., 2010).

In summary, these studies add insight about the complex pattern of relationships between parents' and children's skills in the development of numerical processing and suggest important implications for the educational setting for the correct development of numerical skills. Regarding clinical implications for diagnostic batteries, the results indicate that it is essential to evaluate both verbal and non-verbal numerical skills

(ANS). Furthermore, linguistic/phonological skills and the home numeracy environment are fundamental and should be considered to better assess the overall learning profile.

Some limitations might constrain generalizability of results and would need to be considered in future investigations. In all three studies, the sample size could be increased to ensure more robust results and increase reliability. Increasing the sample could also allow for more sophisticated statistical analyses such as Structural Equation Models (SEM) to better explore our hypotheses.

In the first two studies, those in which we tried to investigate the intergenerational transmission of mathematical skills, it was possible to test only one parent. It would be helpful to include both parents in the study in order to have a complete picture of the parent-child transmission of mathematical skills. It would be interesting to be able to include in the studies a sample of adopted children to increase the comprehensibility of the genetic role concerning the environmental one.

Moreover, it would be valuable to be able to test a sample of children with developmental dyscalculia, to observe if in the parents there are specific markers that differentiate them from the parents of children with typical development.

Finally, it would be interesting to include other environmental variables such as mathematical anxiety (Maloney et al., 2015a) or home literacy environment (Lefevre et al., 2009) to analyze if they have a role in the development of numerical skills in children.

Despite these shortcomings, the studies presented offer an original picture on the complex interplay amongst variables involved in the development of math skills, taking care of environmental variables, children's cognitive profile and parents' math skills.

5.2 *Implications for educational and clinical work*

This research focused on the development of mathematical skills, their origins, and their relationships with other skills. In particular, the studies carried out gave the opportunity to better understand the relationship between the mathematical abilities of parents and those of children. This thesis aimed to better understand the role of intergenerational transmission of mathematical competencies between parents and children. Measurements of symbolic and non-symbolic magnitude were used, and all the various aspects of mathematics were explored. In all the studies carried out, the cognitive aspects were not omitted. Furthermore, great importance was given to the home numeracy to understand what was the role of the environment in the development of numerical skills.

In the study carried out with preschool children, the importance of home numeracy has emerged very clearly. Despite a limited sample, it has been shown that home numeracy activities have more impact than the mathematical skills of parents. This data can have an important implication in advising parents to carry out specific activities with their children from an early age. Parents often tend to avoid exposing themselves with their children in proposing activities in which they do not feel fully competent. Our study has shown that, unlike exposure to numerical activities, parents' mathematical abilities do not have such a substantial effect on the development of children's mathematical knowledge. Moreover, strengthening the area of quantity estimates, and in particular the ANS, might be a protective factor concerning future difficulties in mathematics.

It appeared that the ANS abilities of the mothers were significant predictors of children's formal mathematical abilities, but that children's ability to manipulate symbolic quantities proved to be a crucial factor. Therefore, it is crucial to allow children to become familiar with games concerning numbers, symbolic values, and quantity comparisons, so that these activities could represent the scaffolding in their numerical development.

Many connections also emerged between reading skills and mathematical skills, but the non-verbal aspects of mathematical skills were independent of reading skills. In particular, the sense of number seems to play a fundamental role in the knowledge of mathematical skills and should be deepened.

These aspects suggest that, from an educational perspective, the development of numerical symbolic abilities in children is fundamental and would allow generating greater competence and a better approach to formal mathematics teaching.

From a clinical perspective, these results reinforce the idea of the importance of non-symbolic skills, magnitude comparison, and in general, the Approximate Number System. It is, therefore, essential to focus on these aspects in assessment protocols, but it is equally important to reinforce these skills from an early age. For younger children, identifying risk areas allows them to have the opportunity to close the gap with their peers and start primary school with the necessary skills to be able to learn optimally.

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