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On the Normativity and Plurality of Logical Consequence

Some Reflections on the Role of Logic in Our Epistemic System

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Abstract

This thesis focuses on two topics at the core of current debates in the philosophy of logic: the normativity of logic and logical pluralism. The first, introductory chapter begins with an overview of the alternative conceptions of logic, and then presents the state of the art on the normativity of logic and logical pluralism. In the second chapter, I provide an account of the normativity of logic as sourced in rationality according to which being evaluated against a standard of formal cogency is constitutive of being rational. There are two main innovations in support of this treatment of the normativity of logic. First, while there is a lively debate nowadays on what the connection is between logic and norms for reasoning, there is little discussion on the nature of the normativity of logic and on what its source is. Second, adopting this view on the normativity of logic gives logical pluralists a new way to treat the objection that logical pluralism is incompatible with the claim that logic is normative for reasoning. This latter objective is developed in the third chapter, where I also argue that my proposal on the normativity of logic, while being compatible with both monist and pluralist accounts of logic, provides support for a form of pluralism about logical consequence. The fourth chapter considers two ways to extend my proposal. In the first part, I discuss an important issue concerning the epistemic significance of adopting logical pluralism. I argue that, since the logical pluralist endorses more than one validity relation, she has additional conditions for transmitting justification. Further, I suggest that the logical pluralist should adopt contextualism about deductive justification. In the last part, I develop and critically assess the prospects of combining pluralism about logical consequence and pluralism about truth. One way to integrate the two pluralism is to claim that different truth properties and different consequence relations operate in different domains. I provide a novel challenge to this form of pluralism and contend that it is based on commitments that are co-untenable.

Contents

Chapter 1

Introduction

This work is centred on two main themes: the normativity of logic, and logical pluralism. I provide a novel account on the normativity of logic, and then I argue that this view provides support for endorsing a form of logical pluralism. Then, I defend this form of pluralism from an objection, and scrutinize two ways in which it could be extended.

This chapter is devoted to presenting the state of the art on these two themes. I begin by presenting an overview of the various conceptions on what logic is in section 1.1. Among those, I favour the "epistemic" conception of logic. In section 1.2, I present the current debate on the normativity of logic. I conclude the chapter with an overview of the various versions of logical pluralism on the market.

In the second chapter I consider a distinction that is standard in metaethics and I provide a taxonomy of various accounts of the normativity of logic on the basis of this distinction. Then, I elaborate my own account on the normativity of logic.

The latter two chapters are devoted to logical pluralism. In chapter 3, I consider which conceptions of logic, whether monist or pluralist, are compatible with my view of the normativity of logic. I argue that, although my account of normativity is compatible with many views on logic, it provides reason to endorse indeterminacy pluralism. Moreover, if one endorses both indeterminacy pluralism and my view on logic's normativity, one can address

a powerful objection that has been raised against this form of pluralism.

The fourth chapter treats two issues of the form of logical pluralism I endorse. In section 4.1, I consider the phenomenon of transmission of justification through inference. I argue that the logical pluralist has an additional condition to fulfil for justification to transmit. I provide a transmission principle that a logical pluralist can endorse, and I argue that deductive justification in a pluralist setting is best understood by adopting a form of contextualism. In section 4.2, I consider whether there is some sort of connection between pluralism about truth and pluralism about logical consequence. I argue that the commitments of this form of pluralism, if taken together, are untenable.

1.1 Alternative Conceptions of Logic

In this section, I will distinguish between various conceptions of what logic is. The majority of the conceptions on the market depend on what one takes logic to be *about*; that is, on what the subject of study is in logic. Not all the conceptions of logic I present here are mutually exclusive; some are indeed compatible with one another.

1.1.1 Formal Conception of Logic

At the core of this conception there is the claim that logic is the study of *formal languages* and of certain mathematical properties thereof. This conception is widespread in the philosophy of mathematics; in particular in the foundations of mathematics.

1.1.2 Reealist Conception of Logic

According to this view, the object of logic are the most general laws that carve out reality; this view is often taken to provide support for classical logic. That is because this view sees the most general truths of the world as the subject matter of logic; Thus, it conceives of logic as deeply entrenched

with truth, such as it is in classical logic. Endorsers of this view are Sher (2011, 2016); Maddy (2002) and Williamson (2013, 2017).

This view seem to favour logical *monism* over logical pluralism; for it seems that "the right logic" is whatever logic that is able to capture the general laws of reality.

Logic as the Study of *Logical Truths*: According to the naturalistic view of logic, at the core of logic are the most general *truths* of the world. Thus, this view often puts at its core *logical truths* instead of logical laws. However, this does not have to be the case: one can consistently claim that what has priority are logical rules/laws; and that this is so because the laws of logic are the most fundamental laws of reality. So, even if this view is generally held to support classical logic, nothing in principle prevents one from endorsing some non-classical logic on the basis of this view.[1]

1.1.3 Epistemic Conception of Logic

The epistemic conception of logic highlights the importance of logic in one's epistemic system. According to this conception, logic is primarily the study of *correct arguments*. The importance of correct arguments is crucial for the relation between correct arguments and correct *inferences* – i.e. acts of reasoning: logic is thought to provide one kind of correct inference, i.e. *deductive* inference. According to this conception, logic is fundamentally about "what follows from what": *logical consequence* is the core of logic. The fact that, in the epistemic conception, logic is the study of correct arguments does not by itself entail that logic is normative: the correctness of the laws of nature, for instance, is purely descriptive. Although I will argue that logic is normative, and I will mainly assume this background view on the nature of logic, I am not claiming that this view *presupposes* that logic is normative. Moreover, even if it would be so, revealing what logical normativity amounts

[1] An interesting exception to this view is Rumfitt (2015). rumfitt thinks that the laws of logic are the most fundamental laws because they are a special kind of *deductions*. Each discipline has its own deductive capacity; but logical deductions can extend other deductive capacities.

to is by itself an interesting question.

Logical consequence is also crucial in one's epistemic standing because it is a mean to extend one's knowledge: through the relation of logical consequence, one can infer new pieces of knowledge starting from things one already knows. That is, knowledge is *closed* under logical consequence.[2] Also, logical consequence can *transmit* justification through a valid inference, provided additional conditions.[3]

At the end of the spectrum of this position is *psychologism*, i.e. the view that logical laws are at bottom psychological laws of thought. This view derives from Kant's conception of logic (Kant 1999, 2004).

Logic as the Study of *Logical Proofs*: It is common to pair the epistemic conception of logic with a primacy of logical rules and logical proofs, as opposed to logical truths. Since the epistemic conception of logic puts *correct arguments* as the core subject matter of logic, then it is natural to think that it proofs and rules of inference are at the centre of attention. The *proof-theoretic* conception of logic, very popular among intuitionists – e.g. Dummett (1991); Prawitz (2006) – can be one form of endorsing this conception. However, the two conceptions are independent from one another.

1.1.4 Pragmatic Conception of Logic

In recent years, logic is widespread also outside the realms of philosophy and mathematics. Logic provides useful tools to *model* a wide array of phenomena. In computer science, linguistics, cognitive psychology, etc, logic is employed for its pragmatic usefulness. The former conceptions of logic, in particular the naturalistic and the epistemic conception, both assume that there is a chief application – as Priest (2006a) would say, a *canonical application* – of logic, which might be, very roughly, either "the world" or "the mind". In the pragmatic conception, logic is considered a useful tool for the scientific practice, that an be revised and updated locally in accordance with its prag-

[2]The phenomenon of epistemic closure is first presented by Dretske (1970).
[3]The first to consider this phenomenon is Wright (1985).

matic usefulness. For instance, compare the following two ways in which one might revise a rule of logic – say, the reflexivity of logical consequence: one might discharge it because, e.g., a certain application of computer science works better if the rule of reflexivity is dropped, or, one might discharge it because it is not part of the the fundamental laws that carve up the world, or because it is not part of the a priori principles of thought.

This view looks similar to the *anti-exceptionalism* about logic (Williamson 2013; Priest 2016; Hjortland 2017). However, anti-exceptionalism is a method that employs criteria that are continuous with the scientific practice to find out the true logical theorie(s), of which one can conceive of in the naturalistic way, in the formal way, or in the epistemic way. By contrast, the pragmatic conception is not an attempt to find out the best logical theory. To borrow Haack's (1978) terminology, this position is a form of *logical instrumentalism*: it rejects the idea that there is a *correct* logical system and focuses on the application of logic.

This option has not been receiving much attention within philosophy; an exception is Dutilh Novaes (2012).

1.2 The Normativity of Logic

In this section, I will offer an overview of the current debate on the normativity of logic.

The first distinction to be made is that between logic's normative role in making deductions – i.e. so to speak, in the logical classroom – and logic's role as normative for ordinary reasoning. In the former sense, logic is normative because, in manipulating formal arguments, one has to follow the rules of logic; in the latter sense, the idea is that logic provides norms for the actual process of reasoning. In this work, I will focus on the latter topic.[4] Reasoning is a process that brings one from having a certain (combination of) propositional attitude(s) to having another (combination of) propositional attitude(s). For instance, the process that brings one from intending to drink

[4]Dutilh Novaes (2015) and Milne (2009) endorse an alternative view according to which logic is fundamentally normative for public assertoric practice.

some wine and believing that in order to drink some wine one has to open the bottle, to intending to open the bottle is a piece of reasoning. Another piece of reasoning is the process that brings one from having the belief that it rains outside and the belief that if it rains one ought to take the umbrella to the belief that one ought to take the umbrella. This latter is an instance of *inferring* – that is, an act, a step in the reasoning process, that brings one from a combination of doxastic attitudes which count as premises to another doxastic attitude – that is, the conclusion. *Deductive* inferences are those inferences in which the conclusion logically follows from the premises.

The idea that logic is normative for reasoning is a standard assumption in analytic philosophy since Kant (2004, 1999) The conception of logic as normative for reasoning is very natural – although, again, not mandatory – if one endorses the epistemic conception of logic: if logic is the study of correct deductive formal arguments, then it is tempting to think that logic can guide us to make correct deductive *inferences*.

Also, logic is thought to be normative for reasoning because some rules of logic, such as *modus ponens* and conjunction introduction, are so basic that there seems to be no non-circular way to justify them. This argument, first advocated by Putnam (1978) is known as the *centrality argument*.[5].

The idea that logic is normative for reasoning has been challenged by McGee (1985) and Harman (1984, 1986). McGee provides a counterexample that puts into question the special role of *modus ponens* as a basic and inviolable rule of our cognitive system; whereas Harman criticizes more generally the idea that correct arguments are connected with correct reasoning. In fact, Harman puts into question the idea that logic has any *special* relevance to reasoning. Nowadays, there is a renewed interest in the normativity of logic and, in particular, in addressing Harman's point. In what follows, I consider Harman's objections against the normativity of logic, and the attempts to address these objections in the current literature.

[5]See also Hale (2002). See Schechter and Enoch (2006) for a pragmatic justification of the basic laws of logic.

1.2.1 Harman's Objections

In this section, I will consider the most notorious objections to the normativity of logic, put forward by (Harman 1984, 1986). In his 1986 book *Change in View: Principles of Reasoning*, Harman claims that a theory of reasoning is a theory of belief revision. According to Harman, what pushes one to revise ones beliefs is the intuitive perception of an incoherence in one's set of doxastic attitudes. As a consequence, reasoning is a conservative process.[6]

Harman is very critical against the view that logic is a guide for reasoning, and he completely divorces formal arguments from the psychological process of inference:

> «[...] there is no clear significant way in which logic is *specially*[7] relevant to reasoning. [...] Sometimes, reasoning culminates in the conclusion that a certain argument is a good one or that certain propositions are inconsistent. But that is not to say that logical implication or logical inconsistency has any special status in human reasoning» (1986: 20).

In support of the claim that logic has no special relevance in the reasoning process, Harman puts forward four objections that have been widely discussed in the recent literature on the normativity of logic. In what follows, I will present the objections as they are analysed in Steinberger (2018b, 2017c):

Harman singles out the following two principles as the best candidates to capture the supposed normative role of logic in reasoning:

Logical Implication : The fact that one's view logically implies p can be a reason to accept p.

Logical Inconsisitency : Logical inconsistency is to be avoided.

The relations above are supposed to put constraints on one's reasoning, even though each of them is defeasible. Harman provides four objections to these principles: [8].

[6] Harman extends his view on reasoning to the notion of rationality in (Harman 2013).
[7] Italics mine.
[8] The names of the objections are taken from Steinberger (2017c)

1. *Belief Revision:* logical laws *per se* do not explicitly require or provide reason for any particular doxastic attitude. For instance, suppose one believes that a and that a logically implies b. It does not follow, merely from this fact and from the law of *modus ponens*, that one is required o has a reason to believe b; for this is compatible with b being absurd, or with perceptual evidence being against b. A related objection is the bootstrapping problem outlined by Broome (1999): since every proposition entails itself, if logical laws required one to have certain beliefs, then, given any proposition one in fact believes, it would follow that one is required or that one has reason to believe it. This is clearly false.

2. *Clutter Avoidance*: each of our beliefs logically implies many trivial propositions. Since we, as human beings, have limited cognitive capacities, it is irrational for us to clutter our minds believing such trivialities.

3. *Excessive Demands*: This objection is analogous to the issue of whether logical omniscience is a feasible rational ideal (Hintikka 1962; Smithies 2015; Stalnaker 1991). if logic were normative, it would require us to believe all of the logical consequences of our beliefs and to avoid inconsistencies in our belief sets. This is unattainable by the limited cognitive resources of a human being. For instance, one might believe all axioms of PA without believing one theorem that follows from the axioms only through a very long and complex proof.

4. *Unavoidable Inconsistencies*: some inconsistencies in our set of beliefs cannot rationally be avoided. For instance, consider the Paradox of the Preface (Makinson 1965): suppose I am the author of a non-fictional book. It is rational for me to write in the preface of the book that I believe in each claim contained in the book. However, it is also rational for me to write – and to believe – that there will be errors in the book. The upshot is that, for every claim p contained in the book, I believe that p, yet I do not believe the conjunction of all the ps. These doxastic attitudes are mutually inconsistent, but they both

seem rational to held.

Harman takes this to show that *logical implication* and *logical inconsistency* do not capture belief revision. He suggest, instead the following principles (1986: 18):

Recognized Implication Principle : One has reason to believe p if one recognizes that p is implied by one's view;

Recognized Inconsistency Principle : one has reason to avoid believing things one recognizes as inconsistent.

According to Harman, these principles capture the fact that the fundamental notions of reasoning – conceived as belief revision – are *immediate implication* and *immediate inconsistency*: these are basic dispositions to treat some propositions as immediately implying others and as immediately inconsistent with each other. These psychological notions are, according to Harman (1986: 18-19), not reducible to other notions. The fact that logical implication and logical inconsistency are neither necessary nor sufficient to attain the two basic relations is the ground on which he argues that logic is not *specially* relevant for reasoning.

Much of the current debate on the normativity of logic focuses on articulating logical norms for reasoning that can address Harman's objections (Field 2009b,c; Steinberger 2018b, 2017a).

1.2.2 The Bridge Principles

MacFarlane (2004) suggests addressing Harman's four objections by by introducing the notion of *bridge principle*. In MacFarlane's words (2004: 5), bridge principles link «claims about logical validity with norms for belief». A bridge principle is a conditional of the following form: «if a, $b \models c$, then (normative conditional claim about a, b and c)» (MacFarlane 2004: 7).[9] The

[9]For the sake of brevity, I will present all of the arguments as having only two premises. This is due to the fact that any argument can be divided in a series of shorter arguments with two premises and a conclusion. That said, nothing about the normativity of logic hangs on the number of premises of a deductive argument.

antecedent is a (or an attitude towards a) fact about the logical entailment between the premises and the conclusion of an inference, the consequent is an embedded conditional which spells out a norm for doxastic attitudes.[10] Bridge principles vary along the following parameters:

1. *Type (mode) of normativity*: MacFarlane only considers the following *deontic* "operators": obligation – 'ought' (o); defeasible reason (r); permission (p).

2. *Scope of the normative "operator"*[11]: the normative "operator" might range over just the consequent (C); both the antecedent and the consequent (B); or the whole embedded conditional (W).

3. *Polarity of the normative claim*: the embedded conditional might mandate a positive attitude towards the premises and conclusions, like believing or accepting a, b and c (+), or a negative attitude like disbelieving or rejecting (-).

   For instance, the bridge principles Co+ is the following: «if a, $b \models c$, then if you believe a and b, then you ought to believe c». Along with the parameters above, which MacFarlane himself argues for, the bridge principles might vary along the following dimensions:

4. *Synchronic/diachronic*: synchronic norms regulate how certain claims at time t relate to other claims at time t; whereas diachronic norms regulate how claims at time t relate to other claims at a subsequent time t'. For instance, take again Co+:

   Co+1 if a, $b \models c$, then if you believe a and b at time t,then you ought to believe c at time t'.

   Co+2 if a, $b \models c$, then if you believe a and b at time t,then you ought to believe c at time t.

[10] MacFarlane focuses on belief, but I think what he says can be straightforwardly extended to other doxastic attitudes such as acceptance.

[11] I put 'operator' in scare quotes because not every normative term behaves as an operator

Co+1 is the diachronic version; it requires one to believe the conclusion (along with the premises) once one has considered the premises. Co+2 is the synchronic version, and it requires one to infer the conclusion whenever one considers the premises.

5. *Attitudinal constraint*: the antecedent of the bridge principle might state just the fact about logical consequence, or it might include a subject's doxastic attitude towards such fact – e.g. whether one knows/ believes/ accepts that a, $b \models c$. There is an important distinction between factive attitudes like knowledge, which require the truth of the logical entailment, and non-factive attitudes like belief, which might be false. I will say more on this distinction in chapter 2. MacFarlane only considers the knowledge-constrained bridge principles – i.e. if you know that a, $b \models c$, then (norm). In the attitudinally constrained bridge principles, one can add a time variable also to the antecedent of the bridge principles – e.g. if you believe that a, $b \models c$ at time t, then (norm about believing a, b and c).

1.3 Logical Pluralism(s)

In Section 1.1, I considered various ways in which one can conceive of what logic is. Another important distinction in the conception of logic is *how many* logics are there. *Logical monism* is the view that there is only one correct logic; whereas *logical pluralism* is the view that there are many correct logics.

The default position on this issue, until very recently, has been *logical monism*, i.e. the view that there is merely one logic – whether classical or non-classical. Clearly, many different logics have been elaborated in the last 50 years; so there is a sense in which the answer to the "ne or many"-question is already settled. Howver, a proper formulation of this question is rather the following: is there one, or more than one *correct* logic? *Logical pluralism* is the thesis that there are many correct logics on a par. This thesis can be coupled with other assumptions, thus generating different variants of logical pluralism. MEGLIO

In this Section, I provide an overview of the various pluralist accounts of logics in the literature. This classification is not exhaustive. I suggest to group the proposed versions of logical pluralism in two main categories: pluralism about the *logical constants* and pluralism about the relation of *logical consequence*. Other distinctions are possible: for instance, an endorser of the *formal* conception of logic might prefer to group the varieties of pluralism on the basis of the distinction between *meaning-variant* and *meaning-invariant* versions. Meaning-variant versions of logical pluralism are those accounts that claim that, in Quine's words (1986), a change of logic is a change of meaning of the connectives. Meaning-invariant versions of logical pluralism are those account that refuse the "change-of-logic-change-of-meaning" thesis. Let us consider each account in turn.[12]

1.3.1 Pluralism about the Logical Vocabulary

1.3.2 Carnapian pluralism

This is the first and most prominent version of logical pluralism, outlined by Carnap (1937).[13] Carnapian pluralism is committed to the following two thesis:

1. Meaning-variance: in setting a linguistic framework, one also sets the meaning of the logical vocabulary. Therefore, in each linguistic framework, there is a different logic.

2. Pragmatic attitude on linguistic frameworks: one is free to stipulate one's own logico-linguistic framework, insofar as s/he is precise (Principle of Tolerance), on the basis of pragmatic consideration such as the usefulness of having, e.g. a certain connective; Theoretical questions are meaningful only if they are set *within* a certain linguistic framework.

[12]Russell (2008) argues for another form of logical pluralism: there are different and equally good logics because different truth-bearers may be taken to compose formal deductive arguments.

[13]Note, however, that there is no agreement on whether Carnap *himself* was a logical pluralist – he did not take there to be multiple correct logics, because he did not take there to be *any* correct logic – nor would all consider his pluralism to be local. See, for instance, Friedman (2001).

One can either endorse both 1 and 2 or one of the two. Shapiro's eclectic pluralism (2014) is an instance of meaning-variance pluralism, but does not include an endorsement to 2. Rather, Shapiro offers a contextualist story on how the logico-linguistic framework of the meta-theory varies, but he does not endorse the claim that theoretical questions are meaningful only within a framework and the choice of the framework is not determined on purely pragmatic grounds but rather on considerations of how many logics are required by the different mathematical practices on the market.

Carnapian pluralism is a *local* pluralism (Haack 1978: 222-223) – i.e., the different logics in their account are applicable only with restrictions; in this case, each logic is relativized to a linguistic framework.

1.3.3 Eclectic Pluralism

Shapiro (2014) claims that the fact that there are different mathematical practices, some of which are inconsistent if classical principles are added, provides a reason to think that a version of Carnapian pluralism is true. In his account, the linguistic framework is set by the mathematical practice in question; and the vocabulary changes accordingly. As for the meta-theory, Carnap thinks that no meaningful questions can be asked outside a linguistic framework; so, in fact, no meta-theoretical question is allowed. By contrast, Shapiro provides a contextualist account of the logical framework of the meta-theory (Shapiro 2014: ch. 4-5). The criterion of admissibility for the logic is the Hilbertian «anything goes, as long as it is consistent». Shapiro labels his position *eclectic pluralism*.

1.3.4 Logical Relativism

Varzi (2002) argues that not only there are different logics for different ways of specifying the logical vocabulary, but also there is no principled way to distinguish between what is part of the logical vocabulary and what is not. So, none of these different ways of specifying the logical vocabulary is superior to the other. This is a quite extreme version of Carnapian pluralism, in which *every* specification of logical vocabulary is permissible, and so amounts to a

relativism about logic grounded in a thesis about the meaning of the logical vocabulary.

1.3.5 Pluralism about Logical Consequence

1.3.6 Indeterminacy Pluralism

This is the most discussed version of logical pluralism in the literature and the view I will advocate in this dissertation. This view is advocated by Beall and Restall (2000; 2001; 2006).

Beall and Restall claim that the relation of logical consequence is indeterminate and can be specified in at least three different ways. They offer the following Generalized Tarski's Thesis as a definition of logical consequence (2006: 21):

> GTT: An argument is valid_{x} iff, in every case_{x} in which the premises are true, so is the conclusion.

In addition, any admissible account of logical consequence must have the following three core features (2006: 14-20):

Necessity: the truth of the premises of a valid argument necessitates the truth of the conclusion of that argument;

Normativity: if an argument is valid, you somehow go wrong if you accept the premises but reject the conclusion;

Formality: the following are three possible definitions of formality that Beall and Restall quote from MacFarlane's (2000) work on the formality of logic:

> F1: logic provides constitutive norms for thought as such;
>
> F2: logic is indifferent to the particular identities of objects;
>
> F3: logic abstracts entirely from the semantic content of thought.

Depending on what the salient class of cases is, GTT has different precisifications. Any precisification that has the three core features is an admissible

account of logical consequence. Due to the indeterminateness of logical consequence, Beall and Restall's logical pluralism is labelled *indeterminacy pluralism* (Eklund 2017; Ferrari and Moruzzi 2017).[14] Beall and Restall (2006) consider the following classes of cases: complete and consistent worlds or Tarskian models; potentially incomplete stages; potentially inconsistent situations. Thus, they admit in their pluralism classical, constructive and relevant logic, but they allow their account to be extensible to other notions of logical consequence that admit of semantic cases and have the three core features.

Beall and Restall's pluralism is a form of *global* (Haack 1978), *all-purpose* pluralism (Field 2009c). That is, the three logical consequence relations are meant to apply unrestrictedly across all truth-apt discourse. Consider, for instance, the inference from $\neg\neg p$ to p. If one takes cases to be worlds, via GTT, one gets classical validity; so the inference is valid. If one takes cases to be constructions, via GTT, one gets intuitionistic validity; so the inference is invalid. According to Beall and Restall, neither answer to the question: «is the inference from $\neg\neg p$ to p valid?» is better than the other: the results of classical and intuitionistic logic are not *competing* for providing the right assessment of the validity of the inference.

1.3.7 Normative Pluralism

Field (2009c) argues for a different form of pluralism about logical consequence. His starting point is claiming that logical consequence cannot be necessary truth preservation, for this, together with minimal logical assumptions – i.e. introduction and elimination rules for the conditional and for the truth predicate – leads to the Curry paradox (Field 2009c). Therefore, Field argues, the chief role of logical consequence is providing epistemic norms: logical consequence is *primitive*, but is conceptually clarified by its normative role (Field 2015). The norms of logic guide one to achieve one's epistemic goals. Field (2009a) endorses a form of *expressivism* about epistemic norms:

[14] Ferrari and Moruzzi (2017) provide three alternative semantic models for the indeterminacy of logical consequence envisaged by Beall and Restall.

he claims that there is no unique way of achieving – or approximating – a given epistemic goal and that no epistemic goal is superior to the others. This leads to pluralism in the following way: different logics are more apt for achieving different epistemic goals; also, given epistemic expressivism, different logics might be equally apt to achieve the same epistemic goal.

Note that, even if Field's argument is motivated by other considerations – he does not think that logical consequence is necessarily truth preserving – he provides a good case for logical pluralism for anyone who is pluralist about epistemic value.[15] Simplifying, and cutting out his non-factual expressivism, his argument is the following:

1. the chief role of logic is being normative for reasoning;

2. there are various ways to be normative for reasoning, depending on the epistemic goals one is pursuing;

3. different logics might be more apt for achieving these different epistemic goals.

If that is so, not only must we pluralist find a way to combine logical pluralism and normativity, but also one is motivated in fact by the other. What I will suggest in the next two chapters is a different sort of argument that grounds logical pluralism in the normativity of logical consequence.

[15]Regardless of whether the different epistemic values are derivative on an ultimate epistemic goal or are independent from each other. On this, see Pedersen (2017).

Chapter 2

The Scope and Source of the Normativity of Logic

In section 1.2, I considered the current debate on the status of the normativity of logic. In this chapter, my purpose is to make the claim that logic is normative more precise. In particular, I want to consider how certain distinctions that are standard in metaethics apply to the normativity of logic.

In section 2.1, I introduce the distinctions on the *source*, the *mode* and the *kind* of normativity. Then, in section 2.2, I apply those distinctions to the current debate on the normativity of logic. This will draw light on the status of the norms of logic and on the significance of Harman's points. I distinguish between three distinct ways one can conceive of logic's normativity depending on its source and try to map the literature on the present account. The rest of the chapter is dedicated to presenting a new proposal on the normativity of logic. The main claim of this proposal is that the normative source of logic is rationality. Therefore, in section 2.3, I provide a overview of what rationality is and what role it is supposed to play. In section 2.4 and 2.5, I defend and suggest a novel account on the normativity of logic. This will include the defence of a moderate form of constitutivism of logic in rationality.

2.1 Three Distinctions in Normativity

There is a traditional distinction in philosophy between intrinsic and extrinsic value. Roughly, the distinction discriminates between what is valuable "in itself" and what is valuable "because of something else". There are various ways to spell out what this rough characterization amounts to. Korsgaard (1983) introduces a further distinction between intrinsic or extrinsic value and final or instrumental value. The former is a distinction concerning the *source* of value: something can be valuable in virtue of its intrinsic properties, or in virtue of its extrinsic properties.[1] For instance, beauty, love and freedom are generally thought to be intrinsically valuable; while a Picasso is valuable because it derives its value from beauty – i.e. its value is sourced in something else. Extrinsic value is derivative: were beauty not intrinsically valuable, so wouldn't the Picasso be extrinsically valuable. The latter is a distinction in the *mode* of valuing: something has final value when it is valuable for its own sake, it has instrumental value when it is valuable for the sake of something else. For instance, when one is overweight, going on a diet is valuable for the sake of being healthier, and so is merely instrumentally valuable. Instrumental value is one kind of extrinsic value, but there may be other kinds of extrinsic values (Korsgaard 1983: 172). On the other hand, health might be valuable for its own sake as an end. The two distinctions, Korsgaard argues (1983: 170), have often been conflated, leading either to think that all final goods are intrinsically valuable, or that only intrinsically valuable goods can be pursued as an end. However, something can be valuable for its own sake and yet be extrinsically valuable. For instance, one can value for its own sake a very luxurious good – e.g. a Porsche, but still the value of a Porsche depends on a very complex net of social, symbolic and economic conditions. By contrast, things that are valuable for the sake of

[1]There are various alternative conceptions of intrinsic and extrinsic properties. In this work, I assume the Lewisian distinction between intrinsic and extrinsic properties: some property P is intrinsic of a thing O if and only if there is no duplicate of O that lacks P. The main reason for preferring this conception to a relational conception (see Francescotti 2014) is that I want to allow for intrinsic properties that nonetheless depend on the relations a thing has with another – e.g. the property of having legs longer than arms.

something else are typically extrinsically valuable.

The question on the source of value can be extended to other kinds of normativity. Valuable things are normative – e.g. beauty is normative because (it is part of the nature of beauty that) it is valuable to appreciate beautiful things. However, there are various different *kinds* of normativity: for instance, a legal system is normative because it is part of its nature that one ought to abide by the law. Call the kind of normativity that pertains to value *axiological* and the kind of normativity that pertains to obligation, duty, and permission *deontic*. Also, there are principles that have the role of providing a standard for evaluating correctness, appropriateness or fittingness. It is contentious whether principles of this kind have the status of "norms" because, contrary to axiological and deontic norms, they do not offer *guidance*; however, they are normative in a broader sense of the term. I agree with Thomson (2008) that correctness is conducive to other value-conferring normative virtues. For instance, ... This latter kind of normativity has been labelled in the literature as *evaluative* (Steinberger 2018b) or *criterial* (Ferrari 2016, 2018).

In this latter, broader sense of normative, being an *aim* or a goal is also normative: for instance, some think that truth is normative because the goal of forming a belief is that it is true (see Williams 1970). Call this latter kind of normativity *teleological* (Ferrari 2016, 2018). [2]

The final/instrumental distinction does not apply exclusively to axiological normativity. For instance, one may think that abidance by the law is a mean to attain the end of abiding by moral principles. This is an example of the final/instrumental mode applied to deontic normativity.

[2]Wrenn (2014) and Ferrari (2018) apply these distinction to the case of truth and argue that there are different ways in which truth can be said to be normative – e.g. truth is the teleological aim of belief, but also the evaluative standard for the correctness (fittingness) of beliefs.

2.2 The Normative Source of Logical Consequence: A Taxonomy

In section 1.1.4, I considered the epistemic conception of logic. According to this conception, which I advocate, the chief role of logic in one's epistemic system is played by logical consequence, which is, as Beall and Restall (2006) say, "the core of logic". By claiming that logic is normative for reasoning, one is in fact claiming that *logical consequence* is.[3] In what follows, then, I will consider how can the relation of logical consequence be said to be normative, give the previous distinctions on the source, the mode and the kind of normativity. Some of the positions I will considered are actually endorsed in the literature, at least according to my interpretation, others are possible ways to conceive of the normativity of logical consequence.

To say that logical consequence is intrinsically normative for reasoning is to say that it is part of the nature of logical consequence that it plays some normative role on reasoning. This does not necessarily mean that the nature of logical consequence is *exhausted* by its normative role. By contrast, taking logical consequence as extrinsically normative means that its normative role is exerted because of the relation it entertains with something else. If the latter is the case, truth is a plausible candidate.

2.2.1 Truth as the Source of Normativity

Why thinking that logic is normative? A first, intuitive reply is: because of the connection between logical consequence and truth (see Steinberger 2017c). Many philosophers claim that truth is the norm of belief and other doxastic attitudes (Gibbard 2005; Shah 2003; Wedgwood 2002, 2013). The way in which this claim is usually defended is the following: it is part of the *intrinsic* nature of truth that it set norms for doxastic attitudes; there-

[3]This does not rule out that other logical concepts might be normative as well. For instance, proof-theorists (Gentzen 1935; Prawitz 1971) maintain that the meaning of the connectives is normative, because it is given by the rules of introduction and elimination rules. This position is applied more broadly in philosophy under the label of *inferentialism* (Brandom 1994).

fore, truth is a *normative property* such as goodness.[4] As it is traditionally conceived, logical consequence is truth-conducive: if the premises of a valid argument are true, logical consequence preserves truth from the premises to the conclusion.

In what follows, I will distinguish between various ways in which one can spell out the intuitive connection between truth and logical consequence, given the distinctions in the source, kind and mode of normativity I provided.

Logical Consequence as Intrinsically Normative There are two ways to define logical consequence in terms of truth. According to the *modal* definition of logical consequence, a conclusion follows from a set of premises if truth is necessarily preserved from the premises to the conclusion of the argument – i.e. the conclusion is true in every possible world in which the premises are. Another conception that relates logical consequence and truth is the *model-theoretic* characterization of logical consequence, due to Tarski (1936). Etchemendy (1990) is the first to distinguish between these two characterization by arguing that the latter does not imply the former. According to Tarski (1936), a *model* is an interpretation of the non-logical terms of a language. According to the model-theoretic conception of logical consequence, a conclusion follows from a set of premises if and only if there is no model in which the premises are true and the conclusion is false.

According to both the modal and the model-theoretic definition of logical consequence, truth is part of the very nature of logical consequence. So, if one agrees that truth is the norm of doxastic attitudes and endorses one of these conception of logical consequence, then logical consequence is *intrinsically* normative because of its intrinsic relation to the normative property of truth.[5] In this way, the source of the normativity of logic upon reasoning is logical consequence itself. No further condition needs to be satisfied in order to be

[4]This is rejected most notably by deflationists about truth, who claim that truth has no substantive nature as a property but is a mere logico-lingistic device. (See Horwich 1998; Dodd 1999).

[5]However, things are more controversial for the model-theoretic characterization. The model-theoretic definition posits an intrinsic relation between logical consequence and truth-in-a-model, but some have argued that truth and truth-in-a-model are different properties. See, for instance, Field (2007: 104-105).

bound by the norms of logic while reasoning.[6]

Logical Consequence as Extrinsically Normative Even if one does not accept to *define* logical consequence in terms of truth, one might still think that the normativity of logical consequence depends on its relation to truth. One might think that logical consequence is truth-preserving, and necessarily so, without thinking that the relation to truth is *intrinsic* to logical consequence. For instance, a relevance logician might accept that necessary truth preservation is a necessary condition for being a relation of logical consequence, but that it is not a sufficient condition because logical consequence must also include a relevance component, on pain of accepting the paradoxes of implication. Suppose one takes this stance towards logical consequence – i.e. that it is necessarily truth-preserving, although it is not *merely* a matter of necessary truth preservation – and one accepts that truth is the norm governing doxastic attitudes. Then, one can claim that logical consequence is normative for the *transitions* between doxastic attitudes – i.e. the reasoning moves. Logical consequence is *extrinsically* normative, and truth is the source of its normativity. Were doxastic attitudes governed by another norm, say, the norm of happiness, then logic would not be normative – i.e. In Lewisian terms, there is a duplicate of logical consequence that is not related to truth. According to this view, the normativity of logic is conditional upon the fact that *truth* is normative for beliefs.

One way in which one can endorse this view is by claiming that logical consequence is *instrumentally* normative: logical consequence is normative because it is a mean to attain the end of having true doxastic attitudes. This does not imply that truth is the *final* end: it might be that the truth of our doxastic attitudes is itself a mean to attain a further end. This position is endorsed by Pedersen (2018).

There is a worry that this proposal might result in the view that logical consequence is not normative *at all*. The worry goes as follows: if the normativity of logic is merely instrumental, then almost any other discipline turns

[6]Of course, MacFarlane's (2004) question of how to formulate these norms and how to bridge the gap between logical laws and doxastic norms still needs to be addressed.

out normative as well. For instance, since the laws of physics – a paradigmatically descriptive, non-normative discipline – are conducive to true beliefs about the physical world, then physics is also instrumentally normative for the sake of truth. I think this worry leads Russell (2017) to conclude that logic is not normative. Russell distinguishes between three degrees of "normative entanglement" and argues that logic is not normative because it is only connected with "the normative" to the third and weakest degree, according to which «a theory has normative consequences, but only alongside other (perhaps quite prevalent) normative assumptions» (Russell 2017: 10). So, according to Russell, while truth is normative, logic is merely descriptive of how truth is preserved. I think Russell's argument is flawed in two ways: first, it conflates the final/instrumental distinction with the question on the source of normativity: truth might be the source of the normativity of logical consequence and yet logical consequence be intrinsically normative because it is intrinsically related to truth. Second, the fact that logic is extrinsically normative, and also instrumentally normative, does not yet amount to claiming that logic is *merely* instrumental for attaining truth: the fact that logical consequence is a mean to attain truth does not by itself imply that the normative pressure is exerted solely by truth. Further, logical consequence might have additional normative roles apart from being a mean for attaining truth. In order to argue that logic is not normative, one would have to rule out these two other options. Russell (2017), for one, does not engage in this enterprise.

2.2.2 Other Sources of the Normativity of Logic

In this section, I consider views according to which the source of the normativity of logical consequence does not depend on its relation to truth.

Logic as Intrinsically Normative There are at least three ways to take logic as intrinsically normative without defining logical consequence in terms of truth. The first, endorsed by Field (2009b,c, 2015) is to take logical consequence as a *primitive*. Field (2015) rejects both model-theoretic and proof-

theoretic accounts *qua* definitions of the nature of logical consequence, on the grounds that «competent speakers[7] may agree on the model-theoretic and proof-theoretic facts and yet disagree about what's valid» (Field 2015). Field also rejects the characterization of logical consequence in terms of necessary truth preservation. He argues that, if logical consequence just is truth preservation,together with some basic logical principles it leads the Curry paradox (See Field 2009c, 2015).Field's suggestion is that logical consequence is not reducible to more basic notions, and therefore is an undefinable *primitive* relation. However, one can explain its conceptual role via the norms it exerts on reasoning.[8] This suggests that being normative for reasoning, according to Field, is an *intrinsic* property of logical consequence.

An alternative proposal that Field considers is *reducing* logical consequence to its normative role. According to this latter view, logic would be intrinsically normative and its nature would be exhausted by the norms it imposes on reasoning. Field (2015) rejects this proposal on the grounds that «it would sully the purity of logic to define validity in normative terms whose exact content is less than clear» (2015: 55). As implausible as it might be, this proposal would count as another instance of logical consequence being intrinsically normative.[9]

Also, Beall and Restall (2006) take normativity to be one the core features of validity. In fact, they also define logical consequence in terms of truth: according to GTT, an argument is valid if and only if, for a certain class of cases, the premises and the conclusion are true in each case. So, presumably they take logical consequence to have an intrinsic relation to truth. However, their commitment to normativity as a core feature of logic is independent of the relation they posit between logical consequence and truth. they take logical consequence as normative because «if an argument is valid, you somehow

[7]Perhaps being a competent speaker is not enough to agree on the model-theoretic and the proof-theoretic facts: it is required at least to be a sufficiently logically trained reasoner.

[8]In a similar vein, the meaning of the connectives is given in natural deduction by their introduction and elimination rules (Gentzen 1935).

[9]Steinberger (2017c)objects to Field that characterizing logical consequence in terms of its normative profile is not sufficient to distinguish it from other relations – e.g. strict implication.

go wrong if you accept the premises but reject the conclusion» (2006: 16). Further, they do not commit explicitly to the normativity of truth for doxastic attitudes. Hence, this is another way of taking logical consequence as a normative source.

Logic as Extrinsically Normative Another way to articulate the normativity of logic is to claim that it emerges directly from the *practice* of reasoning. If this view is correct, then reasoning is an intrinsically normative practice and is the source of the normativity of logical consequence. In what follows I suggest that this position might be attributed to MacFarlane (2004).[10]

Unlike more recent contributions, MacFarlane (2004) is not interested in articulating the normative role of logic *per se*. Rather, his purpose is to draw light on long-standing disputes in the philosophy of logic over the concept of logical consequence. MacFarlane points out that the current methodology for addressing such disputes on the concept of logical consequence is to appeal to one's intuitions about what is logically valid. For instance, some paraconsistent logicians (Priest 1979; Restall and Slaney 1995) appeal to folk intuition to argue against the validity of *ECQ*. This method is, according to (MacFarlane 2004: 2) inefficacious: one's intuitions about the concept of logical consequence depend almost exclusively on one's logical training. This means that people outside the logic class will have little to none intuitions on what logical consequence is, whereas the intuitions of more trained subjects – i.e. philosophers and logicians – will depend largely on the specific logical theories that they endorse, among which there is little convergence on what logical consequence is – which is precisely the core of the disputes.

Since reasoning is a process one constantly engages in, it is largely independent on one's logical education. Moreover, MacFarlane points out that

[10] An alternative interpretation of Macfarlane's (2004) proposal is the following: the bridge principles that emerge from the practice of reasoning reveal the *intrinsic* normativity of logical consequence. This seems to be Steinberger's (2017c) interpretation of MacFarlane; for he argues that MacFarlane's view is subject to the same objection of Field's (see footnote 9). If that is the case, then the proposal I sketch out here is just another option in the taxonomy that has not been advocated in the literature.

there is much more convergence on intuitions about what counts as correct or incorrect reasoning, than on intuitions about validity judgements. For instance, «the validity of *ex falso quodlibet* is hotly debated, but no one thinks that when one finds oneself with contradictory beliefs one ought to conclude that one is a pumpkin» (2004). This makes one's intuition on correct reasoning much more reliable than one's intuitions on validity. So, MacFarlane suggests that a more promising way to solve the disputes over logical consequence, instead of appealing to intuitions on validity, is to first articulate the normative role that logical consequence plays in reasoning via the bridge principles, and then to employ this articulation in order to clarify other aspects of the relation itself, including those aspects that are currently under dispute. Intuitions on correct reasoning provide the basis on which MacFarlane assesses the bridge principles against Harman's objections. Once the connection between validity and norms for belief is clear – i.e. once one has chosen the best bridge principles – MacFarlane employs the bridge principles into current disputes on logical consequence. For instance,he employs the bridge principles in order to investigate whether necessary truth preservation is in fact a feature of logical consequence (2004: 18-20). This excludes that, according to MacFarlane, logic's normativity is sourced in truth.

I suggest to think of MacFarlane's proposal in the following way: the fact that agents have fairly stable intuitions on what good reasoning is shows that the practice of reasoning is governed by *implicit* norms. These implicit norms emerge from the practice of reasoning and elucidate the normative role of logic. So, the source of the normativity of logic is the practice of reasoning itself, and logical consequence is extrinsically normative because of its relation with coreect reasoning.[11]

2.3 Rationality

In this section I provide some distinctions on rationality. In section 2.6, I will argue that the normativity of logic is rooted in rationality, it is therefore

[11]This interpretation of MacFarlane is close to Brandom's conception of norms (1983; 1994).

useful to get clear abut what rationality is. In the literature, there is no agreement over what rationality is. The concept is employed in epistemology and ethics mostly as a normative term – even though it has been contended that it is not (Kolodny 2005). Some have maintained that rationality is best seen as a *virtue* (Audi 2001; Wedgwood 2014); others that it is a system of norms (Broome 1999, 2005, 2013; Reisner 2009, 2011). Nonetheless, nearly everyone recognizes the following two distinctions:

- *Practical/theoretical*: the subject of practical rationality is deliberation and action; the subject of theoretical rationality is belief and knowledge;

- *Epistemic/pragmatic*: in a broad sense, one can distinguish between what is the most rational thing to do/believe purely on the basis of one's epistemic position, and what is the most rational thing to do/ believe given certain pragmatical considerations – e.g. one's limited cognitive resources; the time one has to form a belief; etc. By itself, this does not commit one to claim that the pragmatic considerations that might affect one's overall rationality are themselves *reasons* over which one is responsive.[12] For instance, Foley (2005) claims that pragmatic considerations indirectly shape what is rational to believe, whereas pragmatic reasons, if they were effective (he thinks they are not) would have direct influence on rationality. Also, Wedgwood (2008) agues for a contextualist account of justification according to which the (pragmatic) context contributes to providing a standard for having justification. There is another way in which pragmatic considerations might affect belief formation: phenomena such as wishful thinking and self-deception show that one might also believe on the basis of non-epistemic reasons – e.g. one may believe something that isn't supported by the evidence because it makes one feel better. However, these phenomena do not give rise to *rational* belief.

In addition, rationality is thought to play the following three functions:

[12]See Reisner (2017) for a defense of this position.

Explicative/Predictive: according to most theories of rationality, in order to interpret the behaviour of other human beings, one has to presuppose that they are rational.[13]

Prescriptive : rationality is a system of norms which guide the agent in acquiring beliefs and knowledge and in deliberating on courses of action.

Evaluative : rationality provides a standard against which one can evaluate an agent.

Steinberger's (2018b) three normative roles of logic are very similar to these traditional roles of rationality:

Appraisal attribution of praise and blame. It is close to the explanatory/predictive task of rationality: the degree of idealization and subjectivity varies;

Guidance subjective, first-personal deontic norms;

Evaluative objective, third-personal standards of correctness.

It has been argued that, in addition to the normative dimension of correctly responding to reasons, rationality also includes *requirements of formal coherence*. Call these two dimensions of rationality *substantive* rationality and *structural* rationality respectively. Requirements of formal coherence are norms that, instead of positing the responsiveness of doxastic attitudes with reasons, are about the *responsiveness* of doxastic attitudes to other doxastic attitudes. For instance, a plausible candidate rational requirement of formal coherence is the following:

> *Non-Contradiction*: if one believes p at t, then one is rationally required not to believe not-p at t.

Requirements of this kind focus on the *combination* of certain doxastic attitudes, rather than on doxastic attitudes and the reasons for them. Logic

[13]This idea is at the core of Davidson's idea of "radical interpretation" (2004), and of Dennett's account of intentionality (1989).

also puts constraint on combination of claims: a valid argument is a combination of a set of claims, the premises, with another claim, or set of claims, i.e. the conclusion. Therefore, if one accepts that there are requirements of formal coherence, then it seems that requirements of formal coherence are provided by logic and eventually other disciplines – e.g. analyticity; probabilistic coherence. In what follows, I will argue that the normativity of logic is sourced in epistemic theoretical rationality precisely because some of these requirements are *constitutive* of epistemic rationality.

Before turning to a defence of this position, however, I will consider two arguments that have been put forward against these rational requirements of formal coherence. These arguments have been suggested by Kolodny (2005; 2007).

Kolodny (2007) provides an error theory for formal coherence. He claims that what makes requirements of formal coherence plausible are in fact certain "satisfaction" and "violation" claims that pertain exclusively to the *reasons* one has for one's belief. Such reasons are constituted exclusively by the available *evidence*. Consider again *Non-Contradiction*: when one has contradictory beliefs the problem is not, according to Kolodny, that the beliefs il p and not-p are at odds *with each other*; rather, the problem is that, for each p, evidence is such that it can never support both p and its negation: if evidence for p does not outweigh evidence for its negation, then one's reason requires one not to believe it. So, if one believes both p and not-p, then one of such beliefs is at odds with the *reasons* for it. The same is thought to apply to other requirements.

Much of the force of this argument depends on what evidence includes: if evidence includes also one's actual beliefs, then the same formal requirements that were said to apply to rationality can be said to apply to evidence, and the resulting view is not much different (Steinberger 2017c: see). On the other hand, if evidence excludes one's beliefs, then Kolodny's claim is that, in fact, there are no contradictory beliefs. This is much stronger than the more modest claim that rationality requires us not to have contradictory beliefs (Broome 2013: 85).

Further, Kolodny argues that there is no *reason* to comply with formal

requirements; so they cannot have normative force. The constitutivist reply I favour is that one has to comply because one cannot do otherwise: for a subject to be a rational agent just is – at least in part – to comply with such requirements. However, Kolodny (2005: 551-556) also explicitly rejects the idea that there is a constitutive evaluative standard of structural rationality. The reason for his rejection is the following: Kolodny takes structural requirements of rationality to be requirements of *subjective* rationality – i.e. requirements that regulate relations among one's attitude – as opposed to requirements of *objective* rationality, which regulate relations among one's attitude and one's reasons for having that attitude. Moreover, he takes structural requirements to be *local*, in the sense that they focus on specific combination of beliefs rather than on one's complete set of beliefs. Kolodny then considers whether it might be that it is constitutive of belief and intention that they are part of the system of norms that constitute rationality. This system of norms is thought of as a "functional system" (see Scanlon 2003). If that were the case, he goes on, then the most plausible candidate for the constitutive function is truth, for it is truth one aims at in acquiring a belief.[14] But, if so, then that would explain how it is constitutive of belief that it is responsive to objective norms of rationality, rather than subjective structural requirements. So, at best, the constitutivist strategy can account for the reason-responsiveness aspect of rationality, but not of the structural requirements. This is no objection to what I want to claim, however. My aim is not to claim that it is constitutive *of belief* (and/or intention) that it is part of the "functional system" of rationality. Rather, I want to claim that a standard of logical correctness is part of rationality, but I do not conceive of rationality as a "functional system"; let alone claiming that rationality has a sole epistemic function or goal.

[14]Kolodny talks indifferently, and sometimes explicitly mentions both about belief and intention throughout his article (Kolodny 2005: 52). However, the point he raises about truth as the most plausible candidate function is only understandable relative to belief, and hence as a point about theoretical rationality.

2.4 The Constitutive Evaluative Standard of Logic

In this section, I will provide an alternative account for the normativity of logic. In particular, I will claim that logic is extrinsically normative, and the primary source of its normativity is routed in human rationality. In proposing Logical Implication and Logical Inconsistency as candidate norms for reasoning, Harman is suggesting that the best candidate for articulating the "special relevance" that logic has for reasoning are the relation of logical consequence and the property of logical consistency. In what follows, I partially vindicate this idea. The so-called requirements of formal coherence as such provide a standard of correctness or evaluation that serve the evaluative role of rationality. So, logic's normativity is of the evaluative/criterial kind; and is *constitutive* of rationality.

Since the intrinsic/extrinsic distinction is not necessarily coincident with the final/instrumental distinction, this does not mean that logic's normativity is merely instrumental with the ultimate end of attaining rationality. The norms of formal consistency are constitutive of rationality, rationality has an intrinsic relation with logic in the form of two principles of formal coherence, which I will later try to formulate. This is at the bottom of the view according to which logic is a formal system that represents forms of reasoning.

Moreover, I want to argue for a *constitutivist* thesis on rationality: no one can be said to count as a rational agent – to be fully precise, a *structurally* rational agent – if one is not subject to evaluation against the standard of correctness provided by logic. In this sense, there is no single logical law or logical rule which is itself constitutive of thought – e.g. *modus ponens*; conjunction introduction. Rather, what is constitutive of rationality is the abidance by the following relations which are themselves logical relations:

> *Be Logically Consequent (BLC)*: it is correct that, if one believes p and q logically follows from p, then one believes q;
>
> *Avoid Inconsistency (AI)*: it is correct to avoid inconsistencies among one's beliefs;

The above relations are close to Harman's logical consequence and logical inconsistency principles, which, he claims, are a first attempt to capture the special relevance that logic is supposed to have on reasoning (1986: 11). However, note that there is no "ought" in the principles: they are mere standard of correctness, and so do not require anything from the reasoner. However, both principles, as they stand, are implausible as standard of correctness for evaluating an agent's rationality.

Let us start with BLC. BLC takes as correct believing all of the infinite logical consequences of one's beliefs. This is a mere standard of correctness, so it can be as idealized as possible: the fact that human cognitive resources are limited is not a worry, because BLC does not impose a requirement on an actual human agent. However, BLC is a norm of rationality, and so it is supposed to articulate the correctness of rational inquiry. Inquiry is *goal-directed*, not just in the sense that doxastic attitudes aim at truth and possibly other epistemic goals. In a more specific sense, inquiry is directed towards finding answers to the questions that a rational agent, no matter how idealized, raises. Norms for rational correctness should be sensitive to this dimension. BLC is not: if inquiry is constrained by the goals of inquiry, then believing all of the infinite logical consequences of one's beliefs is at best sub-optimal. A better correctness principle should take as correct all of the infinite logical consequences of one's beliefs *that are pertinent to the goals of inquiry*.

Again, the problem is not, as it is for Harman, one of mind-cluttering or of excessive demands on a limited human cognitive system. To see this point better, consider the following case: suppose that two super-computers, each with unlimited storage and computational capacities, are programmed to satisfy the standards of ideal correct rationality and reasoning. Both are given the following claim as a belief: "The Earth has just one moon". The former, given the claim, computes all its infinite logical consequences; whereas the latter is programmed in such a way that, once it is given a question as an input, it computes only the logical consequences salient for the question. Suppose that the two super-computers are given as an input the question: "so, you do not believe that the Earth has three moons, right?" and

must provide an answer as an output. Both machines answer "yes". In terms of truth, their performances are equally correct. However, the performance of the latter is more rational because it is sensitive to the salience of the question.

Let us see how to adjust BLC in order to capture this dimension of pertinence. Suppose again that one has the true belief that the Earth has just one moon. BLC requires that one should also believe that the Earth does not have two moons; that the Earth does not have three moons; that Earth has two moons or grass is green; etc. A first way of trying to block this unwelcome result is to employ the distinction between *dispositional* and *occurrent* belief.[15] Dispositions are best characterized in counterfactual terms. For instance, someone is a smoker not because one is smoking right now, but because one has the habit to smoke – i.e. one is prone to smoke. However, in order to be true of someone that one is a smoker, there must be episodes of one actually smoking with a certain frequency. Similarly, Ryle (2009) and other epistemologists argued that beliefs can be entertained in two ways. One might be actually employing one's belief that p in some reasoning, or entertain the belief that p, etc. In this case, the belief that p is occurrent. In all of the circumstances in which one is not operating on one's belief that p, one's belief in p is *dispositional*. According to Audi (1994), in order to acquire the dispositional belief that p, it is required that there are occurrent episodes in which the belief that p is actually entertained – e.g. by employing it in reasoning. In counterfactual terms, one has the dispositional belief that p because, if one were in the circumstances that elicit considering p, one would have the occurrent belief that p. To put it roughly, a dispositional belief is a belief that is stored in one's mind without being currently employed in reasoning or planning. When a dispositional belief is put into work in an inference, or otherwise recalled, then this belief becomes occurrent. For instance, I believe that my name is Elena, and I am currently entertaining this belief. However, this belief is not always present to my mind, or employed

[15]Some think that the nature of belief is merely dispositional (Braithwaite 1932; Barcan Marcus 1995). What I suggest here is perfectly compatible with mere dispositionalism about belief: if one prefers, one might talk about "judgement" instead of "occurrent belief".

in reasoning or in other cognitive processes. When it is not, my belief that my name is Elena is dispositional. Still, it is very easy for me to recollect it from my "belief storage" when needed. Going back to our case, if my belief that the Earth has just one moon is occurrent, then I might also have the dispositional belief that the Earth does not have two moons, in that if one asks me: "so, you believe that the Earth does not have two moons, right?" I will easily recollect the dispositional belief that Earth does not have two moons from my belief storage and then employ it in answering: "Yes". The same goes for the belief that Earth does not have three moons and for all other beliefs of this kind. Moreover, it seems clearly irrational to answer "no" when asked if one does also believe Earth does not have two or more moons, and BLC captures that.

However, the distinction still requires the storage of my beliefs "somewhere" in my mind. So, in fact, the occurrent/dispositional distinction does not prevent one from forming all the infinite consequences of the belief that earth has just one moon. Moreover, my belief that Earth has just one moon might be either occurrent or dispositional. When it is merely dispositional, it is no longer clear the distinction between believing that Earth has just one moon and believing all its logical consequences is captured. Harman (1986: 13-14), in considering a related point, distinguishes between *explicit* beliefs, whose content is explicitly represented to one's mind, and *implicit* belief, which is easily inferable from an explicit belief, but is not itself explicitly represented to one's mind. Taking the present case, one only has the belief that Earth has just one moon as explicitly represented in one's mind, and one beliefs the infinite consequences of this belief merely implicitly. When one is asked "so, you believe that the Earth does not have two moons, right?", one easily infers from one's explicit belief that Earth has one moon the implicit belief that Earth does not have two moons, and so answers "yes". In this way, without having to say that one has infinitely many dispositions in one's "belief storage".[16]

[16] Anti-representationalists like Dennett (1975), who deny that beliefs have representational content, take it that one only has an infinite number of implicit beliefs, but no explicit belief.

Goldman (1986) considers an objection to the explicit/implicit belief distinction. The worry is basically that, although the logical consequences of an explicit belief would be easily inferable, they are not *beliefs*: were one never asked whether one believes that Earth does not have two or more moons, one would never be disposed to assent to that proposition, even if one has the explicit belief that earth has just one moon. Since BLC is merely a norm of correctness, this worry is not too pressing for my purposes: it amounts to a terminological matter on what does or does not count as a belief. However, one can do better: Audi's distinction between dispositional beliefs and dispositions to believe addresses this worry (Audi 1994). Audi individuates cases, among which are instances of logical implication, in which one does not have a dispositional belief which gets occurrent under appropriate circumstances; rather, one has a disposition *to form* a certain belief under appropriate circumstances. For instance, consider one who is asked whether 98.124 is larger than 98. One has never considered this proposition before, so one is not recalling a dispositional belief stored in the mind. However, one can easily form the belief in question by inferring it from one's belief that adding any decimal to an integer increases the number. Audi claims that other dispositions to believe are non-inferentially mediated. What is occurring is belief formation, rather than belief. Going back to our case, if one has the explicit belief that Earth has just one moon – whether occurrent or dispositional – then one is also disposed to form the infinite number of beliefs that Earth does not have two or more moons. One actually forms the belief when one is asked whether one believes any of the latter propositions, or if the possibility of considering one of the latter propositions is raised in another way. Moreover, this captures the dimension of correctness of BLC: one would count as irrational if one would refuse to answer "yes". I therefore suggest modifying BLC as follows:

> *BLC**: it is correct that, if one believes p and q logically follows for p, then one has a disposition *to* believe q.[17]

[17]For the sake of simplicity, in what follows I will talk of "belief" without further specifying whether it is an occurrent belief, a dispositional belief and dispositions to believe, unless it is relevant in the context.

Let us now go back to AI. AI requires one to avoid inconsistency among one's belief. It is a fact that we do believe propositions that are inconsistent with one another, and we are able to keep them in our belief system for quite a long time without revising them. In order to account for that, Lewis (1982) argued that, even if one has inconsistent beliefs in one's set of beliefs, that is because one can only properly reason into overlapping fragments of one's belief set, each of which is consistent. One can shift from a consistent fragment to another, but there is no global inconsistent space in which one is reasoning. Consider a person who believes the following three propositions: *a*: Nassau Street runs roughly east-west; *b*: the railroad runs roughly north-south; *c*: Nassau Street is roughly parallel to the railroad. According to Lewis, one can keep all of these beliefs because one only reasons within one of the following two overlapping fragments: the fragment containing *a* and *c* and the fragment containing *b* and *c*. Until one does not consider the conjunction of the three propositions, and thereby puts oneself in an inconsistent logical fragment, one does not realize the inconsistency and is allowed to believe *a, b* and *c*. However, as soon as one considers the conjunction, one must revise one's belief so as to restore consistency.

A first worry with the fragmentation approach is that it is often not as obvious as in Lewis' example how to restore consistency. In fact, I think the case presented by Lewis is one in which we are under a requirement of avoiding inconsistencies, and that is why we do in fact restore it as soon as we discover it. However, when one considers one's belief about paradigmatically inconsistent discourse, as law or fiction, it is not as clear how one is supposed to identify the inconsistency of one's belief set relative to that discourse. The inconsistency might be functional to the inconsistent setting itself. Note that the difficulty here is not (only) that one often cannot so easily identify one's inconsistency. If that would be the problem, then that would simply be a sub-optimal evaluation of one against the ideal metre of logic, which could be adjusted as soon as one realizes the inconsistency. Since I am concerned with evaluative, rather than directive, and synchronic rather than diachronic requirements of rationality, that would not count as a counterexample to AI. However, what I am claiming is that there are some discourses in which it

may be unreasonable to avoid inconsistency, because the very subject matter one is reasoning about is modelled in an inconsistent way. Instances of such discourse are law and fiction. A body of laws often includes claims that are inconsistent with one another. Nonetheless, one does not want to conclude from that that the whole body of law is reducible to the absurd! Also, in fiction, one is normally able to understand stories that are inconsistent both accidentally – because of the author's inaccuracy – or explicitly so (see Priest 1997). Granted, it is not obvious that, just because one is reasoning about inconsistencies, it is rational for one to have inconsistent *beliefs*. However, this is a possibility when reasoning about inconsistent discourse, and since we do reason about inconsistent discourse, the possibility that one is required to keep inconsistent beliefs should not be excluded from the start by a principle of rational correctness. On such ground, Avoid Inconsistency is too demanding as a general principle meant to capture rationality.[18]

Inconsistent theories are modelled by paraconsistent logics. Paraconsistent logics reject *ex contradictione quodlibet* – from now on, *ECQ* – the rule according to which anything follows from a contradiction. Inconsistent theories, when interesting, are *non-trivial* – i.e. not every claim is logically implied by the theory. Non-triviality can be understood as a weakening of consistency that applies also to paraconsistent logics: a theory T is non-trivial if and only if, not for every claim a, T logically implies a. Instead of avoiding inconsistency, a more neutral principle is one that avoids *triviality*: it is incorrect to form a belief in claims that follow from a trivial theory. Since anything follows from a trivial theory, AI must be weakened in such a way that it takes as incorrect forming a belief in *every* claim. In light of these considerations, I suggest to substitute Avoiding Inconsistency with the following requirement:

> *Avoid Triviality (AT)*: it is correct to avoid, for every p, to have a disposition to believe p.

[18]Steinberger (2016) shows that there is no bridge principles that is both plausible as an articulation of the normative role of logic and sufficiently strong to support normative arguments against Explosion – i.e. *ECQ*. My aim is more modest: I am simply suggesting that the unrestricted validity of *ECQ* should not be excluded from the start.

At first glance, this requirement might look implausibly weak. However, note that it is coupled with BLC: BLC takes as correct being disposed to believe the logical consequences of one's beliefs, but it does not block believing according to a trivial consequence relation – i.e. a relation according to which any claim whatsoever is a logical consequence of one's beliefs. Avoid Triviality succeeds in blocking this unwelcome consequence; for it forbids to be disposed to have any belief whatsoever.

Moreover, note that Avoid Triviality is a general requirement that includes Avoid Inconsistency as a special case: if one takes *ECQ* as valid, in fact, every inconsistent theory turns out as trivial. In reasoning about paradigmatic inconsistent discourse, Avoid Triviality does not take as incorrect retaining certain inconsistent beliefs. By contrast, in reasoning about phenomena that are not paradigmatically inconsistent, it is fair to assume that *ECQ* holds,[19] so every inconsistent theory is also a trivial theory. Therefore, in such cases, the two principles AI and AT are equivalent. Whenever one is not dealing with paradigmatic cases of inconsistent discourse, one is in fact bound by stronger principle Avoid Inconsistency.

To sum up, I take the conjunction of the two principle BLC* and AT as providing a standard of correctness against which an agent is evaluated as rational. This correctness standard is one of the norms that constitute *structural* rationality, which is *intrinsically* normative. Further, structural rationality is the source of the normativity of logical consequence. Thus, logic is *extrinsically* normative, because of the relation that obtains between logical consequence and structural rationality.

Note that, although BLC* and AT are closely related, they impose *distinct* requirements. If a certain proposition follows from another proposition p according to a non-trivial logical theory, then, considering just Avoid Triviality, I do not have any requirement to believe q. If one is evaluated by BLC* alone, one does not me to dismiss one of my beliefs, say r, because it does not follow non-trivially from my set of beliefs. To sum up, Be Logically Consequent and Avoid Triviality, taken together constitute the standard for correctness against which an agent is evaluated as rational.

[19] As I will argue in chapter 3, one can justify this assumption in two ways.

It might be objected that, according to these principles, an agent that does not believe *anything* turns out to be ideally rational. However, the principles I put forward articulate only *one* role of rationality: the one of providing a standard of evaluation for agents. Moreover, these principles are meant to capture only one dimension of the evaluative role rationality – i.e. structural rationality. So, I have to bite the bullet to this extent: believing absolutely anything is *logically* correct. However, it is incorrect if evaluated along the dimension of *reasons* rationality: an agent who refuses to believe any claim whatsoever is surely not correctly responding to reasons. In addition, rationality has the role of providing *guidance* to an agent. Rational principles of this latter kind are *deontic*, and so they impose obligations or permissions on agents. In section 2.3 I suggested that logic might provide also norms for rational guidance; if that is the case, then it is probable that an agent who does not believe anything would violate these logical norms.[20]

2.5 The Descriptive Role of the Bridge Principles

If the current picture is correct, there is still a role to play for the bridge principles. One can conceive of them as describing and clarifying what the requirements of BLC and AT amount to in terms of instructions for correct reasoning. As emerges from the current debate on rationality and reasoning (Broome 2013; Kolodny 2005; Reisner 2011) (Broome 2013; Kolodny 2005; 2007; 2008 Reisner 2011) the connection between reasoning correctly and being rational is not straightforward: a first option is that rationality requires one to reason correctly if one reasons at all (Hussain 2007), but that would be implausibly strong (Broome 2013; Worsnip 2018: see). Alternatively, Broome (2013) has suggested that rationality provides basing permissions to reason correctly, but some have objected that even this might be too strong (Worsnip 2018). Also, Worsnip (2018) has suggested to divorce norms for correct reasoning for norms of rationality. However, the idea that reasoning

[20](See Steinberger 2017a) for a defence of the deontic normativity of logic.

correctly must presuppose at least some norms for being evaluated as rational has a lot of intuitive pull. On the current picture, in which the bridge principles are descriptions of what the structural requirements of rationality demand, the link between rationality and correct reasoning is clarified.

Also, the bridge principles as descriptive of the constitutive role of logic for rationality, expressed by BLC and AT, can address Harman's objections successfully:

1. *Belief Revision*: the objection of belief revision states that there is no unique course of action prescribed by logic: if one is making an inference, for instance, one might either infer the conclusion or reject one of the premises. This objection affects the so-called *narrow scope* principles (Broome 2007). It widely recognized, however MacFarlane (2004); Field (2009b); Steinberger (2018b, 2017a,c), that logic provides *wide scope* requirements: that is, requirements in which the normative "operator" –in this case, "it is correct that" – ranges over the whole embedded conditional of the bridge principles. In this way, the constitutive principles BLC* and AT are respected, but no course of action is mandated by the bridge principles.

2. *Clutter Avoidance*: briefly, the worry is that logic commits one to clutter one's mind with many trivialities, which is presumably not rational. BLC* is immune to this objection, for two reasons: first, since logic is not normative in an agent-guiding way, it does not commit an agent to form any belief. However, even is logic is taken to provide just a standard of evaluation, an agent who clutters one's mind with trivialities is evaluated as ideally rational, and it seems that it isn't the case. However, BLC* evaluates as rational anyone who has a *disposition to form*, under appropriate circumstances, beliefs that are logically deducible from one's beliefs. This does not amount to actually believing every logical consequence of one's belief: the appropriate circumstances that elicit believing a certain logical consequence of one's beliefs would never threat to clutter an agent's mind with trivialities.

3. *Excessive Demands*: the charge to logic of putting excessive demands

on agents is addressed by the following considerations: logic does not require or permit anything from the agent; the role of logic is simply that of providing an evaluative and idealized standard the agent. So, logic does not put excessive demands on an agent because it does not put demands on an agent *at all*.

4. *Unavoidable Inconsistencies*: the presence of inconsistencies in one's system of belief is addressed by AT; which allows there to be preface-like context in which non-triviality instead of consistency in required.

Conclusion

This chapter has been focusing on the normativity of logic. On the one hand, I introduced some distinctions from the debate in metaethics. In particular, I have suggested three dimensions of normativity: first, the source of the normativity, which might be either intrinsic or extrinsic. Second, the mode of valuing, which might be either final or instrumental; and third, the kind of normativity. As for the *kind* of the normativity of logic, in this work I considered logic's normativity to be criterial or evaluative, although I allow for logical laws to have possibly other kinds of normativity. Then, I considered a taxonomy of some various positions one might have on the normativity of logic depending on its source. I concluded with my proposal on an alternative way in which one might conceive of the normativity of logic, which sees logic as sourced in epistemic rationality.

Chapter 3

Rationally Mandated Logical Pluralism

In this chapter I present and defend the version of logical pluralism that I favour. This is a form of indeterminacy pluralism of logical consequence of the kind endorsed by Beall and Restall (2000; 2001; 2006). The arguments that bring me to endorse this view, however, are different to the ones considered by Beall and Restall: I will suggest that the kind of pluralism that I favour is supported by the logical principles that I sketched out in chapter 2, together with some additional considerations.

In section 3.1, I explain how indeterminacy pluralism is compatible, but also recommended by the principles of structural rationality that I put forward in chapter 2. It has been argued (Priest 2006a) that this form of pluralism is incompatible with the normativity of logic. In section 3.2, I consider two kinds of objections to Beall and Restall's indeterminacy pluralism that makes it in contrast with normativity; then, I suggest that my account of normativity can provides an alternative diagnosis of these objections.

3.1 Constitutive Evaluative Normativity and Indeterminacy Pluralism

In this section, I consider which accounts of logic, whether pluralist or monist, ae compatible with my account on the normativity of logic. I argue that my account of the normativity of logic not only allows, but also *motivates* a form of logical pluralism. In particular, it recommends a pluralism of the indeterminacy kind, like the one suggested by Beall and Restall (2000; 2001; 2006) – see section 1.3.5.

I start by considering various versions of pluralism and monism and argue that my view on the normativity of logic is compatible with most of them. In section 3.1.2, I contend that the constitutive- evaluative account of normativity, although compatible with many accounts, provides grounds for adopting indeterminacy pluralism.

3.1.1 Normativity, Monism, and Pluralism

In chapter 2, I argued for the following account of the normativity of logic: logic is a formal system whose normativity is extrinsic; the source of the normativity of logic is epistemic rationality; in particular, the evaluative role of rationality is constituted in part by structural requirements, among which there are the following two norms:

BLC* : it is correct to have a disposition to believe the logical consequences of one's beliefs.

AT : it is correct to avoid, for every p, to have a disposition to believe p.

BLC* and AT taken together are an evaluative standard of formal coherence against which evaluating a rational agent. Again, since this is an evaluative standard, the fact that it involves a high degree of idealization is not worrying: the principles are not deontic, so they do not require anything from the agent. Being evaluated for one's formal coherence against BLC* and AT is constitutive of being a rational agent.

In what follows, I consider whether this account of the normativity of logic is compatible with various versions of logical monism and pluralism. The possible views I consider are the following:

1. monism about classical logic;

2. monism about a non-classical logic;

3. pluralism about the logical constants;

4. pluralism about the relation of logical consequence.

Classical Monism: the constitutive-evaluative account is compatible with logical monism. However, AT is *too weak* as a norm for classical logic. In classical logic, inconsistency results in triviality, but that is because, since Explosion is a valid principle in classical logic, in fact every inconsistency results in triviality: so to speak, triviality is an *effect* of inconsistency. So, a normative principle for the classical monist ought to require not merely avoiding triviality; but avoiding inconsistency. In the previous chapter, I argued that a requirement to avoid inconsistency is too strong to be a correctness standard of rationality; for paradigmatically inconsistent discourse and paradoxes like the preface show that inconsistency should not be deemed as irrational *per se*. Therefore, if that is the correct account of the normativity of logic, the classical monist has three possibilities: (i) claiming that logic is not normative: in fact, many logical monists adopt this solution, by claiming that logic describes how truth is preserved and so is a descriptive, rather than prescriptive, discipline (Russell 2017); or (ii) adopting the core of the constitutive-evaluative account, but keeping AI – i.e. Avoid Inconsistency: – instead of AT. As a result, the classical monist ought to reject that there are any unavoidable inconsistencies. Therefore, for every apparently rational inconsistency, the classical monist ought to provide an explanation of why it is not in fact rational. For the paradigmatically inconsistent subject matter, the simplest option is probably to appeal to the fragmentation approach suggested by Lewis (1982). For the preface paradox, the classical monist has to provide an error theory that explains why retaining the inconsistencies in

preface-like contexts is apparently rational. Even though nothing in principle forbids this option, this seems a very difficult task.[1] This difficulty, plus the objections to the fragmentation approach that I put forward in section 4.5, makes classical monism quite difficult to combine with this account of the normativity of logic.

Non-Classical Monism:

Paracomplete Logics: those non-classical monists that retain *ECQ*; e.g. intuitionistic logicians; – are in the same situation of the classical monist, for it is AT that creates the incompatibility, therefore they either: (i) deny that logic is normative; or (ii) retain AI – i.e. avoid inconsistency – instead of AT and provide an error theory for the apparently rational inconsistencies. Note, however, that typically in non-classical logics truth is not as central for logical consequence as it is in classical logic – e.g. in intuitionistic logic, provability is much more important than truth. For this reason, option (i) is less tenable for the non-classical monist.

Paraconsistent Logics: Paraconsistent logics are compatible with both the requirements BLC* and AT, but note that AT is weaker than AI only for certain contexts in which inconsistencies appear to be rational. In all other contexts, AT applies.

The reason for introducing AT instead of AI is that one cannot exclude from the start that it is rational to retain inconsistencies. Now, there are *weak* and *strong* theories of paraconsistency, the former just denies that *ECQ* is applicable unrestrictedly, the latter accepts that a conclusion may be inferred from an inconsistent sets of premises. The endorser of weaker version of paraconsistency would endorse AT instead of AI with the following spirit: In an ideal limit of inquiry, consistency would be restored completely, but since we are in sub-optimal conditions for inquiry, we must allow for inconsistencies.

[1]The easiest option for the classical logician is to reject agglomeration, i.e. rejecting the claim that if it is rational to believe a and believe b, then it is rational to believe $a \cong b$. This is close to Kyburg's solution to the lottery paradox (Kyburg 1970).

The reason that motivates AT makes it the case that one cannot exclude that a *stronger* degree of paraconsistency applies. So the kind of paraconsistency that is compatible with the constitutive-evaluative normativity of logic is strong. However, since there are also contexts in which AI applies – indeed, the majority – the paraconsistent monist would have to add *extra-logical* principles to justify the holding of *ECQ* in the contexts in which AI holds. In other words, in order to be compatible with the constitutive-evaluative account paraconsistent monism must have the resources to *recapture* classical logic. I borrow the following definition of recapture from (Haaparanta 2009: 627): «one system of logic recaptures another if it is possible to specify a subsystem of the former system which exhibits the same patterns of inference as the latter system». In particular, we speak about classical recapture when a non-classical logical system has the resources to recapture classical logic.

Logical Pluralism:

Carnapian Pluralism: By "Carnapian pluralism" I mean all of the variants of logical pluralism in which the different logics are grounded by the fact that there are different ways to specify the logical vocabulary. The principles BLC* and AT do not commit one to any particular view on the logical vocabulary. Therefore, my account is perfectly compatible with a form of Carnapian pluralism.

However, some have doubted that Carnapian pluralism is compatible with a constitutive thesis about the normativity of logic. The reason is that Carnap endorses the famous *Principle of Tolerance* (1937: sect.17):

> «everyone is at liberty to build its own logic, i.e. its own form of language as he wishes.»

Carnap assumes a sort of *voluntarism* about logical norms (Steinberger 2017b): since *we* decide which logic we abide by, we also decide which logical norms we follow. This seems incompatible with a constitutive thesis about rationality. However,the target of Carnapian pluralism is not rationality and reasoning in

its full generality. The voluntary choice of the logical norms is in place only *within* a linguistic framework. As Steinberger (2017b) points out, the choice of a linguistic framework and of its corresponding norms is made only when one is formulating a rigorously defined system of linguistic representation with the purpose of (scientific) inquiry. Analogously, in Shapiro's eclectic pluralism (see Kouri Kissel and Shapiro 2017), the constitutive norms are constitutive of and are chosen voluntarily only when one is engaged in an enterprise of rational reconstruction of different practices of mathematics. This is compatible with having constitutive principles for rationality that set normative role for logic in reasoning in general. In fact, Steinberger (2017b: 160) suggests that, «even though formally articulated logical norms are not constitutive or thinking *tout court*, presumable sensitivity to *some* kind of tacitly acknowledged set of broadly logical norms is a necessary precondition for any kind of system of linguistic representation whatsoever». If that is the case, BLC* and AT are good candidates for being the tacitly acknowledged broad logical norms.

Pluralism about Logical Consequence: Let us now consider those pluralisms according to which there are different ways to specify the relation of logical consequence. This is compatible with the account of normativity I suggest. Moreover, thinking that formal coherence is at the core of one's rationality and that this is the normative source of the normativity of logic, provides a reason to think that the concept of logical consequence is plural: different logical consequence relations are fitting for the requirements BLC* and AT. However, the *notion* of logical consequence itself has a core place in one's system of norms for rationality. This singles out a *conceptual core* for logical consequence, but simultaneously admits of a plurality of logical consequence relations.

Indeed, different logical consequence relations are *required* by the principles BLC* and AT. Since AT switches to AI in contexts in which *ECQ* holds, AT requires that there is at least (i) a relation of logical consequence which is consistent and (ii) a relation of logical consequence which is inconsistent.

I distinguished between two kinds of pluralisms about logical consequence:

indeterminacy pluralism and Field's normative pluralism. As I argued in the Introduction, Fieldian pluralism is best understood as taking logical consequence as an intrinsically normative relation. If that is the case, then this position is clearly incompatible with mine. Another option for endorsing normative pluralism without Field's commitments taking logical consequence as a primitive, which i presented in section 1.3, is to take there to be different relations of logical consequence that are extrinsically normative; whose normativity is sourced in different epistemic goals. Since I take the source of the normativity of logic to be rationality, this view is also incompatible with my view.

Consider now Indeterminacy pluralism: Beall and Restall take logical consequence to be intrinsically normative, and provide the following arguments in support of their view (2006: 30-31):

Direct argument: There are at least two senses of "following from": one that forbids inconsistency and the other that allows inconsistency; this is a motivation for the indeterminacy of GTT.

Appearances: Beall and Restall just say that «that is how things appear». What they probably mean is that there appear to be many relations that deserves the name "logical consequence".

While these considerations provide support for endorsing indeterminacy pluralism, Beall and Restall leave unexplained why appearances are that there is more than one logical consequence relation, and why there at least two senses of "following from". Moreover, Beall and Restall simply assume that logical consequence is normative; one could probably endorse indeterminacy pluralism even dropping this assumption.

The constitutive evaluative account of the normativity of logic can explain why logic is normative – i.e. because it is constitutive of rationality – and explains both the Direct argument and the argument from appearances:

- The two senses of "following from", and in general the indeterminacy thesis: the principle AT requires at least two different relations of logical consequence, with or without consistency, depending on whether *ECQ* holds.

- There appear to be many consequence relations because the principles BLC and AT admit for different relations of logical consequence as normative.

So, I suggest that my view on the normativity of logic provides reason to support indeterminacy pluralism.

3.1.2 Indeterminacy Pluralism vs. Weak Monism

So far, the two positions that are more in line with the constitutive-evaluative view are indeterminacy pluralism and paraconsistent monism coupled with the machinery to recapture classical logic in unproblematic contexts. I begin by saying that my reasons in support of indeterminacy pluralism over paraconsistent monism with recapture do not amount to a *conclusive* argument. However, in what follows, I provide some considerations to favour indeterminacy pluralism. The reasons I consider do not hang specifically on the particular version of logical pluralism that I favour, so they can be taken as an abductive argument for preferring logical pluralism to monist non-classical theory with recapture. This is independently interesting because not much work has been done in saying exactly what are the advantages of adopting logical pluralism over logical monism with recapture.[2]

Uniformity of Explanation Logical pluralism guarantees a *uniformity of explanation* that paraconsistent monism with recapture lacks. In order to recapture other logics in certain contexts, the logical monist needs to implement her theory with additional assumptions. Such assumptions are motivated by *extra-logical* reasons – for instance, an intuitionist might admit LEM for explaining certain phenomena, by appealing to reasons that pertain to the metaphysics of the phenomenon. Also, these reasons might be – and probably are – different in kinds: for instance, there might be a *metaphysical* reason to retain *LEM* in certain context – e.g. the world is carved out in

[2]Beall and Restall (2000, 2001, 2006) do not make a clear comparison between their pluralism and a monism with recapture. Hjortland (2017) argues that monism is in no better position with respect to pluralism if one uses an abductive methodology. He considers this position to support *anti-exceptionalism* about logic.

such a way that for such area of discourse, either a fact obtains or it does not obtain – and an *epistemic* reason to discharge *ECQ* in certain context – e.g. there is a way to restore consistency in Preface-like context, but it is not epistemically accessible to us, so we have to drop *ECQ*. At least *prima facie*, principles like *ECQ* and *LEM*, although they are not unrestrictedly valid according to the monist, have the same features of other *logical*, unrestrictedly valid principles: for one, they are *formal*, in some sense of formality to be specified.[3] So, it is unclear why they should be explained by appealing to extra-logical considerations from different areas of inquiry.

So, although paraconsistent monism with recapture provides a unique logical theory for all reasoning, she cannot provide a uniform explanation of why her logic holds: some principles are justified as part of the logical system, other principles are justified as metaphysically relevant, other are justified on the basis of epistemic consideration, etc. In this respect, indeterminacy pluralism fares better: any seemingly logical principle is *in fact* a logical principle; plus, there is more than one correct logic and so the application of each logic is restricted to certain applications.

Recapture The second reason has to do with recapture itself. It is certainly a virtue if a certain logic has the resources to express the axioms and theorems of stronger logics, and this contributes to making this logic richer in its expressive power. Although, not much work has been done on the philosophical significance of recapture. In considering a strategy for recapturing material *modus ponens* and Disjunctive Syllogism in the logic LP, (Beall 2013: 759) briefly addresses the following philosophical question on recapture:

> «Why thinking that a given phenomenon – say, arithmetic – is in fact glut-free? [...] After all, once we have embraced a para-

[3]However, not each and every definition of formality is suitable here. In particular, the notion of formality as *topic-neutrality* or *universality*, according to which logic is formal because it is indifferent to the nature of objects, is excluded. This is no surprise since this sense of formality is in tension also with logical pluralism, which I endorse (see Shapiro 2014: 93-96). See Sher (1991) and MacFarlane (2000) for extended discussions on the formality of logical consequence.

consistent logic, are we not now open to the possibility of many gluts?»

To put it in another way, the question asks why a glutty theorist is allowed to admit that some phenomena are glut-free. One can extend the question, more generally, to all the endorsers of logics that employ recapture: if one claims that one's logic is *the one true logic*, and this logic is incomplete or inconsistent (or both), then why doesn't this logic apply across the board?

Beall addresses this question by appealing to an *epistemic conservatism* close to Harman's (1986): one is prima facie justified in accept or reject a belief one has unless there is sufficient reason to change. So the endorser of LP is justified in claiming that, for those phenomena in which the semantic paradoxes do not apply, there are no gluts. However, one can appeal to epistemic conservatism equally to justify logical pluralism: one is justified in retaining classical logic for those phenomena for which there is no reason to admit of incompleteness or inconsistency; for other phenomena, one endorses some non-classical logics. Therefore, appealing to epistemic conservatism cannot answer the philosophical question of why the monist is entitled to recapture classical principles.

Another way to conceive of recapture is suggested by (Priest 2006b: 119). Priest's dialetheism includes classical recapture. According to Priest, the fact that dialetheism can subsume classical logic with recapture makes the relation between his dialetheism and classical logic analogous to the relation between special relativity and Newtonian physics: in certain, ordinary situations, the two are perfectly equivalent, while in extraordinary contexts classical logic crashes whereas dialetheism works. However, the analogy breaks: special relativity can subsume Newtonian physics as a special case, without appealing to any resources that are not part of the theory itself. On the other hand, dialetheism *per se* cannot include the principles of classical logic: the principles recaptured, such as *ECQ*, are not part of the logical theory; rather, they are *quasi-logical* principles motivated by extra logical reasons. Since dialetheism can only recapture classical logic with these extra, quasi-valid principles, the relation between the two logics is not analogous to that between the two physical theories: as logical teories *per se*, dialetheism works

better for certain applications and classical logic works better for other applications, which is just what logical pluralism claims.

Duck-Rabbit [4] Also, there is a sort of *duck-rabbit* situation between weak paraconsistent logic with recapture and indeterminacy pluralism. The monist with recapture claims that certain principles do not hold *unrestrictedly*, and so endorses fewer principles as the principles of "the true logic". However, for those contexts in which such principles *do* hold, she adds assumption to allow for the principles. For instance, consider the example of an endorser of paraconsistency that is in a context in which *ECQ* holds. In such context, her position is indistinguishable from that of a classical monism: they endorse the same principles. Consider now the logical pluralist: in a situation/context in which *ECQ* holds, she is also indistinguishable from the classical monist: relative to this situation/context, they endorse the same principles. So, in such contexts, the logical pluralist and the weak monist with recapture are also indistinguishable. Moreover, the logical pluralist and the weak monist with recapture are indistinguishable also in a context in which *ECQ* does not hold, for they both endorse the same principle. Since the same applies to all principles a logic might reject and recapture, from a merely operational point of view, the two positions are equivalent, except for the fact that that the monist has the disadvantage of the non-uniform explanation.

3.1.3 Conclusion

To recap, the purpose of this section was to consider which accounts of logic, both monists and pluralists, are compatible with the view on the normativity of logic I provided in chapter 2. I suggest that my view is compatible with a wide array of accounts of logic: the only positions that are excluded are classical monism and monism about paracomplete theories which retain consistency. Carnapian pluralism is compatible with my view, provided that one accepts the arguments by Steinberger (2017b) and Kouri Kissel and Shapiro

[4] A similar point is made by Steinberger (2018a) about domain-specific logical pluralism and monism – see section 4.2.

(2017). However, Carnapian pluralism and the constitutive-evaluative view on the normativity of logic must be motivated by independent considerations: one is a view about the logical vocabulary, the other is a view on the relation of logical consequence. So, albeit they are compatible, the two positions do not constitute a *unitary* account.

Further, I suggested that the constitutive-evaluative view on the normativity of logic provides grounds for supporting indeterminacy pluralism: the principles BLC* and AT provide grounds for claiming that there is a conceptual core for logical consequence, i.e. the formal coherence that is partly constitutive of rationality, and also require the use of different relations of logical consequence. This can be done alternatively by indeterminacy pluralism or by monism about a weak paraconsistent logic with recapture of classical logic. I provided some non-conclusive reasons that I find convincing for preferring pluralism over monism with recapture. If one is not persuaded by such reason, and prefers to opt for weak monism with recapture, I suggest that an advantage of my view on logic's normativity is that it is appealing both for pluralists and for monists.

In the next section, I provide another feature of my view that makes it appealing for the endorser of indeterminacy pluralism.

3.2 Collapse and Other Normative Objections to Indeterminacy Pluralism

In this section, I argue that another advantage of my account on the normativity of logic is that it allows indeterminacy pluralism to avoid some normative objection that have been raised to Beall and Restall's account. The so-called "collapse arguments" offered by Priest (2006a); Read (2006); Keefe (2014), cast doubt on whether it is possible to combine the idea that there is more than one logic with the well-established assumption that logic has some normative bearing on how one ought to reason. Ferrari and Moruzzi (2017) present another normative challenge to indeterminacy pluralism, which depends on the semantic model of its indeterminacy; call it the *semantic argument*.In section 3.2.1 I present various versions of the collapse arguments that have been suggested in the literature, In section 3.2.12, I explain why my view on the normativity of logic avoids the objection of collapse. In section 3.2.3, I explain why my account of the normativity of logic avoids these objections.

3.2.1 The Collapse Arguments

Collapse arguments are arguments to the effect that logical pluralism collapses into monism. The collapse challenge is first raised by Priest (2006a):

> «We often reason about some situation or another, call it s. Suppose that s is in different classes of situations, say, K_1 and K_2.Should one use the notion of validity appropriate for K_1 or for K_2? We cannot give the answer 'both' here. Take some inference that is valid in K_1 but not in K_2, $\alpha \vdash \beta$, and suppose that we know – or assume – α; are we, or are we not, entitled to accept β?»

Suppose that the notion of validity appropriate for K1 is classical logic and the notion appropriate for K2 is intuitionistic logic. The situation s is some case that belongs both to the class of complete and consistent cases – $\mathrm{case_c}$ for

short – and to the class of potentially incomplete cases – for short, case_i. Take the inference $\alpha \vdash \beta$ to be, say an instance of Double Negation Elimination (*DNE*). The inference is licensed in classical logic, so, it seems, since s belongs to the class of case$c$, then one can infer β. However, note that all arguments of intuitionistic logic are classically valid, but not the other way around. The same goes for classical logic and relevant logic, for case_c is also a subset of case_r – i.e. the class of potentially inconsistent cases. This means that, for all inferences on the validity of which the three logics give different judgements, classical logic always allows one to draw the conclusions. Read (2001) adds that, since Beall and Restall take the chief question of logic to be that of telling us when a conclusion follows from a set of premises (Beall and Restall 2006), then there is a clear sense in which classical logic and the other logics are not equally good: classical logic tells us "more" on which conclusion one can draw from a set of premises. Logical pluralism collapses into monism, for, in fact, one is always guided by the strongest logic.

Beall and Restall (2006) reply that, in fact, one is entitled to infer β, classically, but not intuitionistically. Read (2006) contrasts this reply with another collapse argument: suppose there is another class, K3, whose logic allows one to infer $\alpha \vdash\!\sim \beta$. Abelian logic, according to which the following principle is valid, can be taken as the logic of K3. Since the situation s belongs both to K1 and K3, one infers β, classically, and $\sim \beta$, according to Abelian logic. Clearly, in this case, Beall and Restall cannot reply that one is allowed to infer both β and $\sim \beta$; for that would result in believing a contradiction. Rather, they might try to argue that Abelian logic does not admit of semantic cases, and hence that their account does not admit any class such as K3. However, it is not clear that such strategy is available to them. Abelian logic is a *connexive logic*; that is, a logic that is neither an extension nor a subsystem of classical logic and take as valid certain theorems that are not valid in classical logic. Connexive logics are typically motivated by considerations similar to those that motivate relevant logic: these logics are called "connexive" because they aim at providing a close connection between the premises and the conclusion of an argument. Since, according to Beall and Restall, relevant logic does admit of semantic cases,

it is difficult to see why a connexive logic like Abelian logic does not.

Finally, Keefe (2014) points out that collapse may arise even with logics that are not orderable in terms of strength. This depends on the fact that, in valid arguments, the truth of premises necessitates the truth of the conclusion. So, it can never be the case that the premises are *actually* true and the conclusion is *actually* false. Suppose, again, that the inference $\alpha \vdash \beta$ is valid according to the logic of K1, but not valid according to the logic of K2, and that one knows (or assumes) that α. Since the argument is K1- valid, the truth of α necessitates the truth of β, so one is entitled to infer β, no matter what the validity judgement is for the logic of K2. So, (2014: 1385):

> «even if there's no strongest relation among those relations that a pluralist admits, there will still always be a right answer to whether we should draw some conclusion from some premises, namely that we should if it follows according to *any* of the logics they accept».

Let us now consider the arguments above in light of the normative role of logical consequence endorsed by Beall and Restall. Beall and Restall claim that logical consequence is intrinsically normative, and they spell out the Normativity feature of logic in the following way (2006: 16): «if an argument is valid, then you somehow go *wrong* if you accept the premises but reject the conclusion». Let us represent this principles with the machinery of the bridge principles. Since each of the three logical consequences in their account is intrinsically normative, we get:

NORM: if $\alpha \vdash_{\mathrm{x}} \beta$, then it is wrong to (accept α and reject β).

where x ranges over the relevant class of cases. In the case envisaged by Priest, we have the following two competing norms:

NORM1: if $\alpha \vdash_{\mathrm{K_1}} \beta$, then it is wrong to (accept α and reject β).

NORM2: if $\alpha \vdash_{\mathrm{K_2}} \beta$, then it is wrong to (accept α and reject β).

Supose K1 is the class of consistent cases, and so NORM1 is the norm of classical logic: it is never wrong to abide by NORM1.

Also, in Read's case, suppose we admit in Beall and Restall's account another intrinsically normative relation of logical consequence. Then, it would generate another norm:

> NORM3:if $\alpha \vdash_{\mathrm{K}_3} \neg\beta$, then it is wrong to (accept α and reject $\neg\beta$).

If, as in Priest's example, α is *known*, then the only option a thinker has not to go wrong is to *suspend judgement* on β.[5]

Caret (2017) points out that the collapse arguments take logic as providing *directives* that guides the agent, rather than evaluations: The chief question of Priest's argument is whether or not an agent is entitled to make certain inference. That is the same for Read's and Keefe's argument. However, it is possible to raise versions of the collapse arguments also for evaluative normativity. Consider again Priest's case. If one takes NORM1 and NORM2 as evaluative standards rather than prescriptions, then it is always correct to abide by NORM1. So, an agent that is committed to the normative standards of both classical, intuitionistic and relevant logic, would always end up conforming to the normative standards of classical logic (Caret 2017), which turns out to be privileged as in the prescriptive version of the argument. An analogous case can be made for Read's argument.[6]

3.2.2 The Semantic Argument

Ferrari and Moruzzi (2017) provide another argument which shows that there is a tension between indeterminacy pluralism the assumption that logic is normative. Ferrari and Moruzzi identify two features of Beall and Restall's pluralism which cause normative problems. The first is its indeterminacy. They

[5]This raises an interesting issue on the formulation of the norms of logic on reasoning. In the current literature, the bridge principles only consider belief and disbelief, or acceptance and rejection, as constrained by logic. Is logic normative also for doxastic attitudes such as the suspension of judgement?

[6]Caret (2017) suggests to avoid the objection of collapse by adopting a form of *contextualism*: GTT provides a different precisification in different contexts. However, I agree with Steinberger (2018a) that its appeal to contextualism is not sufficiently motivated. In chapter 4, I provide other considerations on his approach.

provide three alternative semantic models for logical consequence: gappy underspecificationism, classical underspecificationism, and glutty overspecificationism. In an underdeterminate semantic model, a claim is determinately true (false) if it is true (false) in all specifications. Gappy underspecificationism takes all claims which are true in some specifications and false in others as neither true nor false. Thus, underspecified claims lack a truth value, and so exhibit a truth value gap. Classical underspecificationism is also a form of underdetermination, but it takes claims that are true in some specifications and false in others as either determinately true or determinately false. In other words, while gappy underspecificationism is a first-order model of indeterminacy, classical underspecificationism takes indeterminacy to arise at the second level – namely, it is indeterminate whether a claim is true or false. On the other hand, in a semantic model of overdetermination, a claim is determinately true (false) if it is true (false) in at least one specification. According to glutty overspecificationism, all claims that are true in some specifications and false in others have both truth values, and so exhibit a truth value glut.

The second feature which contributes in creating the normative problem is the following. In Beall and Restall's framework, certain arguments have *conflicting assessments of validity* – i.e. they are assessed as valid with a certain logic and as invalid by another logic. Consider, for instance, the argument from "2 is even" and "2 is not even" to "the Moon is made of cheese". If one takes cases to be worlds, via GTT, one gets classical logic; and hence one takes the argument as valid. On the other hand, if one takes cases to be situations, via GTT, one gets relevance logic; and hence one takes the argument as invalid. Again, Beall and Restall take these results as different, but not competing; for both validity judgements are equally correct, since they are made with equally correct logics. Neither answer to the question: "does 'the Moon is made of cheese' follow from '2 is even' and '2 is not even'?" is better than the other, and analogously for all arguments which have conflicting assessments.[7]

[7]Carnapian pluralism is subject to this objection: according to the Carnapian perspective, the question: "does 'the Moon is made of cheese' follow from '2 is even' and '2 is

Consider now claims about the validity of arguments which has conflicting assessment, like the following:

> Moon: "The Moon is made of cheese" follows from "2 is even" and "2 is not even".

Since both validity judgements are equally correct, there is no fact of the matter as to whether Moon is true, and likewise for all controversial validity claims. Ferrari and Moruzzi (2017) assume that the following truth norm for belief holds:

> TN: It is permissible to believe p if and only if p is true.

Then, they argue that, for each indeterminacy model, TN and the relevant normative principle yield implausible results on the normativity of logical laws when validity judgements conflict. In particular, gappy and classical underspecificationism are normatively silent with respect to controversial validity claims, and glutty overspecificationism generates a normative clash – i.e. when considering a controversial validity claim, one is under a requirement not to disbelieve the conclusion, while being merely permitted to believe that such conclusion follows from the premises. Moreover, in gappy and classical underspecificationism it is either impermissible or indeterminately permissible to believe controversial validity claims.

Both the semantic argument and the collapse problems arise in all and only the arguments that have conflicting assessments with different logics of the pluralist account – i.e. those arguments that are taken as valid by a logic, and invalid by another logic. Take again the argument $\alpha \vdash \beta$. The argument is assessed differently if considered using classical logic or using intuitionistic logic, so, it is a controversial validity claim. Controversial validity claims are especially relevant for pluralism, because it is precisely on those claims that the pluralist differs from the monist. In particular, controversial validity claims arise in *global* pluralism (see Haack 1978), because the different accounts of logical notions apply unrestrictedly. In chapter 4, I consider a way

not even'?" always has a determinate question, because it is always answered within the framework one stipulates.

of avoiding collapse by formulating a local version of indeterminacy pluralism, and argue that this version relies on commitments that are co-untenable. In the next section, I consider how my account of the normativity of logic avoids these normative problems.

3.2.3 Towards a Proposal to Block Collapse

In this section, I consider how my account of the normativity of logic might suggest a way to avoid the objection of collapse and the semantic argument.

As for the semantic argument, it presupposes that logic provides some sort of *guidance* to the agent (Ferrari and Moruzzi 2017: 10). My view on the normativity of logic conceives of logic as providing a standard for correctness against which evaluating an agent for its rationality. So, the norms of logic do not require any commitment on behalf of the agent. So, the semantic argument is blocked, although there is still a question on how to formulate epistemic norms of guidance, assuming that we don't want them to be in overt tension with the logical standards for correctness.

As for the collapse, it is problematic even if one considers only evaluative normativity. The problem of collapse arises, for Beall and Restall, because they have three different evaluative standards for correctness *on a par*, each corresponding to the three logics that they admit in their account. But in fact, the objection goes, the standard of classical logic is the only one whose violation matters. By contrast, if my view on the normativity of logic is correct, the only standard of correctness one is committed to is provided by the two constitutive principles BLC* and AT: these two norms are at the core of formal coherence, which is the source of the normativity of logic. The laws of logic derive their normativity from these two principles. so, there is no need of having "bridge principles" in MacFarlane's sense – i.e. there is no need for norms that link a logical entailment with a constraint on beliefs. Of course, one can conceive of bridge principles for the logics in indeterminacy pluralism, but these would not be three alternative standards of correctness; rather, they would be merely descriptive of three different ways to satisfy BLC* and AT. So, the key feature for avoiding the collapse objection is that,

instead of having *three* distinct normative sources – i.e. the three intrinsically normative logical consequence relations, as Beall and Restall claim – there is only *one* normative source; that is, formal coherence, and the three logical consequence relations all derive their normativity from this unique standard of correctness.

Of course, this still leaves open Priest's question of what one should do when different logics provide different evaluations of the same argument, and this might leave the reader unsatisfied. However, this is a question on what an agent ought to do, and so it presupposes that the kind of normativity at issue is deontic and agent-guiding; but as I argued in chapter 2, logic's primary normative role is that of providing a *correctness standard* for the agent, and the fact that one is not able to answer Priest's question by appealing to logic alone highlights precisely this fact.

I take this argument to show that, for the endorser of indeterminacy pluralism, my view on normativity is preferable to the alternative view, according to which logical consequence is an intrinsically normative relation; for my view has a way of mitigating the force of the collapse argument and the semantic argument that Beall and Restall's view lacks.

3.3 Conclusion

To wrap up,in this chapter I have been showing that my view on the normativity of logic is compatible with a variety of positions, and that it is an appealing position both for a paraconsistent monist and for an indeterminacy pluralist. The fact that it is compatible with both pluralism and monism is the first advantage of my proposal.

Moreover, I have been suggesting two reasons why my proposal is better than Beall and Restall's for the endorser of indeterminacy pluralism. The first reason is tat it offers a firmer explanation for motivating pluralism; the second reason is that, by endorsing the constitutive-evaluative view of the normativity of logic, indeterminacy pluralism can avoid the objections of collapse and the semantic argument.

Chapter 4

More on Indeterminacy Pluralism

This chapter deals with two problematic issues of indeterminacy pluralism. As I contend in 1.1, the conception of logic that I favour is one that is relevant for an agent's epistemic position, that plays some role in reasoning and that functions as a standard for being rational. In section 4.1, I consider how endorsing logical pluralism affects the transmission of justification in deductive inferences. Apart from pluralism about logical consequence or about the logical vocabulary, there are now pluralist accounts of a variety of notions in philosophy: *epistemic* pluralism claims that there is more than one way to be epistemically justified; *ontological* pluralism claims that there is more than one way of being (McDaniel 2009; Turner 2010); truth pluralism claims that there is more than one truth property; etc. Recently, interest has been growing in whether a commitment to pluralism on a certain notion entails pluralism also in other notions. In particular, since truth is a *meta-logical* concept that is related to the concept of logical consequence, it seems that there might be a close connection between pluralism about truth and pluralism about logical consequence. In section 4.2, I consider one way to combine these two pluralisms, and argue that it fails.

4.1 Transmission and Contextualism of Deductive Justification

The main purpose of this section is to articulate necessary and sufficient conditions for transmission of justification which are suitable to those who – like myself – endorse indeterminacy logical pluralism. Logic affects the transmission of justification in the following way. Justification is said to transmit from the premises to the conclusion of a valid deductive inference when one is justified to believe the conclusion in virtue of being justified to believe the premises. Contrast it with closure: an epistemic property P is closed under (known) entailment when, if the premises of an inference have property P, then the conclusion also has P. What we lack for having closure is the "in-virtue-of" condition. Therefore, if closure fails, transmission fails too, but sometimes, transmission fails without closure failing.

Whether a certain deductive inference is valid depends, among other things, on the logic one employs in making the inference. For instance, the inference from $\neg\neg p$ to p is an instance of DNE, which is a law of classical logic, but not of intuitionistic logic. Hence, this inference is valid according to someone who employs classical logic, while it is invalid according to someone who employs intuitionistic logic. Therefore, transmission depends, among other things, on the logic employed in evaluating a certain inference as valid.

Suppose one is a pluralist about logic and one endorses both classical and intuitionistic logic. The logical pluralist would have to give an account of whether she evaluates the following inference as valid, and why. More generally, the fact that the logical pluralist endorses more than one logic affects the phenomenon of transmission because it affects the way in which a certain inference is considered valid or invalid.

Moretti and Piazza (2018) spell out necessary and sufficient conditions for transmission in the form of a principle. In what follows I update their principle in order to make it suitable for an endorser of indeterminacy pluralism. In section 4.1.1, I consider an objection to Beall and Restall's account, which rests specifically on the transmission of justification, and I report two

main suggestions that Beall and Restall make in addressing the objection. In section 4.1.2, I tackle with the transmission principle by Moretti and Piazza (2018): I suggest modifying one of the conditions in order to make it applicable to the framework of indeterminacy pluralism. In section 4.1.3, I present the problematic case for transmission and see whether the revised principle can draw light on it. I conclude by suggesting a contextualist account of deductive justification for addressing the objection of transmission.

4.1.1 The Objection from Transmission

One of the objections that Beall and Restall considers against their account rests precisely on the phenomenon of transmission. The objection goes as follows (Beall and Restall 2006: 94):

> Take the premises of an argument. If one has warrant to believe those premises, is the conclusion warranted too? Even if there is an ambiguity in "follows from" which can be made precise in a number of ways, there is just one relation which tracks the preservation of warrant, and you can't be pluralist about *that*.

What Beall and Restall refer to as "preservation of warrant" ultimately rests on the phenomenon of transmission. That is because one has justification for p only if one has a warrant for p and one believes p in an epistemically appropriate way. Ultimately, the objection amounts to the following: even granting that there are many, equally correct answers to whether a certain inference is valid, there is but one correct answer to the question whether justification transmits through the inference. If that is the case, this would single out one of the accounts of logical consequence of Beall and Restall's pluralist framework as privileged. In turn, this would undermine the claim that the logics in the framework are all on a par.

Beall and Restall reply to the objection from transmission (2006: 94-95) that, in short, whether justification transmits through an argument has more than one answer, but this variability does not depend solely on the logic of the argument. Beall and Restall offer the following two reasons to support their

claim. First, they point out, the standards for having justification vary with the context. For instance, mathematical reasoning has a higher standard for being justified compared to ordinary reasoning. Crucially, according to Beall and Restall, changing the logic signals a *context shift* – e.g. one would use, say, intuitionistic logic in "mathematical" contexts, but classical logic in "ordinary" contexts. This means that at least some of the different admissible precisifications of GTT are equally apt for transmitting justification in different contexts. One way to capture this suggestion – perhaps the most natural – is to take Beall and Restall as suggesting a form of *contextualism about deductive justification*. A similar view is suggested by Caret (2017); however, Caret takes logical consequence itself to be context-sensitive. I agree with (Steinberger 2018a: 12-13) that this solution lacks intuitive pull.

The second suggestion of Beall and Restall in dealing with the objection form transmission is the following. Even if the standard for having justification were kept fixed, this would not single out a particular account of logical consequence (Beall and Restall 2006; Restall 2005). First, it might be that the subject does not *know* the argument to be valid, and if the agent doesn't know the argument to be valid, s/he is not justified in believing the conclusion of the argument; hence, no justification can transmit. Also, non-deductive relations such as material implication or induction can transmit justification as well. For instance, justification transmits from "John is my brother" to "John is male". Note that these considerations on transmission are independent of a commitment to logical pluralism. A logical monist can agree with all this. Note, however, that in defining transmission I specifically referred to valid *deductive* inferences. My aim is is to see how endorsing more than one deductive relation changes the conditions for transmitting justification. Therefore, even if I acknowledge that justification can transmit across relations other than deduction, I will confine my attentions to transmission via deduction.

Despite their reply to the objection aims at divorcing transmission from logical consequence, Beall and Restall make the following remark on the connection between the two (2006: 97):

> [T]he only way that logical consequence preserves entitlement is

> when one is entitled to accept that such and so is a consequence, and when we are entitled to *strongly endorse* that consequence. But those conditions do not single out a particular *logic*.

Beall and Restall distinguish between two ways in which one can endorse a logic as part of one's pluralist account (2006: 82-83):

> One *weakly endorses* a consequence relation if one takes it to be an admissible instance of GTT.
>
> An instance of GTT satisfies the *actuality constraint* iff the actual case is in the domain of its quantifier.
>
> One *strongly endorses* a consequence relation if one takes it to be an admissible instance of GTT and accepts that it satisfies the actuality constraint.

In short, one can commit to logical pluralism as truth-preservation in all cases, without having to additionally commit to the fact that truth in all cases entails actual truth. For instance, consider a logical pluralist who believes that the actual world is complete and consistent. According to the definitions above, she strongly endorses classical logic. Since the class of complete and consistent cases is a sub-class of the class of potentially incomplete cases, she strongly endorses intuitionistic logic as well. Moreover, the class of complete and consistent cases is a sub-class of the class of potentially incomplete cases, and so she strongly endorses also relevant logic– that is, all of the logics in Beall and Restall's account.

On the other hand, one might commit to Beall and Restall's logical pluralism even if one takes the actual world to afford inconsistencies (incompleteness). Such person strongly endorses relevant (intuitionistic) logic. However, since she believes that the actual case is potentially inconsistent (incomplete), she weakly endorses classical logic. According to Beall and Restall (2006: 83), this does not make the latter pluralist "less pluralistic" than the former. The only difference between the two is that, according to the latter logical pluralist, for classical logic, truth-in-all-Tarskian models does not

coincide with actual truth. However, this difference, mattes if one is considering the transmission of justification. In order to claim that justification transmits from the premises to the conclusion of a valid inference, it is not sufficient for one to take the inference as preserving truth-in all-cases$_x$; one must additionally take truth-in-all-cases$_x$ to be actual truth.

To sum up, there are two desiderata for the transmission of justification in Beall and Restall's pluralism. First, justification is *context-sensitive*. In particular, having justification can be seen as an indexical term whose content is fixed by the context, which picks out a class of cases as the relevant epistemic standard. Second, in order to claim that justification transmits in a certain valid deductive inference, one has justification for strongly endorsing the consequence relation of the inference in question.

4.1.2 The Transmission Principle for Logical Pluralists

Consider the following transmission principle (TP) by Moretti and Piazza (2018):

> A subject s's justification for believing p based on evidence e transmits to p's logical consequence q iff:
>
> 1. s has justification for believing p based on e;
> 2. s knows that q is a logical consequence of p;[1]
> 3. s has justification for believing q in virtue of the satisfaction of 1. and 2.[2]

[1]Here I have slightly modified the principle. Moretti and Piazza (2018) write that "s knows that q is deducible from p". I believe that Moretti and Piazza conceive of deducibility in a broad sense, which encompasses also relations other than logical consequence – e.g. material implication. I prefer to confine my attention to logical consequence, as it will make things easier as I proceed with the implementation of the desiderata. By "logical consequence" here I have in mind the sort of indeterminate relation that Beall and Restall endorse: a relation that can be precisified in a number of different ways which share a bundle of core features – albeit, contrary to Beall and Restall, I don't think normativity is among these core features; for this would result in logical consequence being intrinsically normative.

[2]Moretti and Piazza (2013) distinguish between cases in which the transmitted justification is *first-time* and cases in which it is *additional*. Additional justification may be

Let us now try to implement the two desiderata into the principle. First, since Beall and Restall admit for different kinds of logical consequence, which in turn require different epistemic standards, "logical consequence" has to vary over the range of logical consequences. Secondly, one has to add the strong endorsement requirement.

> (TP*) a subject s's justification for believing p based on evidence e transmits to p's (logical consequence)$_{\mathrm{x}}$ q iff:
>
> 1. s has justification for believing p based on e;
> 2. s knows that q is a (logical consequence)$_{\mathrm{x}}$ of p;
> 3. s has justification for strongly endorsing (logical consequence)$_{\mathrm{x}}$;
> 4. s has justification for believing q in virtue of the satisfaction of 1., 2., and 3.

Note that the strong endorsement requirement is constituted by the following two requirements: s has justification for believing that (i) (logical consequence)$_{\mathrm{x}}$ ranges over admissible instances of GTT and (ii) s accepts that (logical consequence)$_{\mathrm{x}}$ satisfies the actuality constraint. Therefore, the kind of evidence that s must collect for having justification for strongly endorsing a certain precisification of logical consequence amounts to the following:

- evidence for the fact that (logical consequence)$_{\mathrm{x}}$ has the core features of logical consequence;
- evidence for the fact that (logical consequence)$_{\mathrm{x}}$ admits of semantic cases, and so is definable via GTT;
- evidence for the fact that the actual case is in the class of semantic cases (logical consequence)$_{\mathrm{x}}$ admits of.[3]

either *independent* or *quantitatively strengthening*. For the sake of simplicity, I will not consider those distinctions in this section.

[3] Admittedly, the sort of evidence tat an agent has to gather in order to have justification transmission is very sophisticated.

The logical pluralist has a further condition to meet for justification to be transmitted. Presumably, then, along with cases when transmission fails because conditions 1. and 2. are not met, and with standard transmission failure templates[4] – e.g. information-dependence; indirectness; etc. – there will be cases in which the failure to transmit justification depends on condition 3. In the next section, I will apply the principle to a case of this kind.

4.1.3 Failure of Satisfying Condition 3.

Consider the following inferences:

Inference X:

1. $\sigma \rightarrow (\alpha \rightarrow \beta)$

2. σ

C1. $\alpha \rightarrow \beta$

Inference Y:

3. $\neg\neg\alpha$

C2. α

X and Y are best modelled respectively by intuitionistic and classical logic – e.g. X is an inference that pertains to constructive mathematics and Y is an inference that pertains to truth-conditional semantics. Consider now inference Z, which has C1. and C2. as its premises:

C1. $\alpha \rightarrow \beta$

C2. α

C3. β

[4]These are famously presented by Wright (2000, 2004) and Davies (2004).

Consider now the following question: is a subject who endorses indeterminacy pluralism deductively justified in believing (C3) on the basis of inference Z?[5] Let us employ our transmission principle in order to see why. Since Z has two premises, we need a multi-premise version of (TP*):

> *Multi-Premise (TP*)*: a subject s's justification for believing p_1 based on evidence e_1 and for believing p_2 based on evidence e_2 transmits to (p_1 and p_2)'s (logical consequence)$_{\text{x}}$ q iff:
>
> 1. s has justification for:
>
>   (a) believing p_1 based on e_1;
>
>   (b) believing p_2 based on e_2;
>
> 2. s knows that q is a (logical consequence)$_{\text{x}}$ of (p_1 and p_2);
>
> 3. s has justification for strongly endorsing (logical consequence)$_{\text{x}}$;
>
> 4. s has justification for believing q in virtue of the satisfaction of 1., 2., and 3.

By applying Multi-Premise (TP*) to Z, we get: a subject s's justification for believing $(\alpha\rightarrow\beta)$ based on evidence $((\sigma\rightarrow(\alpha\rightarrow\beta))$ and $\sigma)$ and for believing α based on evidence $\neg\neg\alpha$ transmits to $((\alpha \rightarrow \beta)$and $\alpha)$'s (logical consequence)$_{\text{x}}$ iff:

1. s has justification for:

   (a) believing $\alpha\rightarrow\beta$ based on $(\sigma\rightarrow(\alpha\rightarrow\beta))$ and σ;

   (b) believing α based on $\neg\neg\alpha$;

2. s knows that β is a (logical consequence)$_{\text{x}}$ of ($\alpha\rightarrow\beta$ and α);

3. s has justification for strongly endorsing (logical consequence)$_{\text{x}}$;

4. s has justification for believing q in virtue of the satisfaction of 1., 2., and 3.

[5] Since I am only considering *deductive* justification, it might be that, despite having a deductive justification for C3, a subject is not overall justified in believing C3.

Note that evidence for C1 – i.e. $\alpha \rightarrow \beta$ – and C2 – i.e. α – is constituted by inferences X and Y.[6] For the sake of simplicity, suppose that there is no standard non-transmissivity template going on. Also, take for granted that s knows the laws of intuitionistic and classical logic employed in X and Y – i.e. MPP and DNE. This would make condition 2 satisfied by assumption. Condition (1a) is satisfied; for when s makes inference X using intuitionistic logic, she acquires a deductive justification for believing $\alpha \rightarrow \beta$. Condition (1b) is also satisfied; for when s makes inference Y using classical logic, she acquires a deductive justification for believing α. Whether condition 3. is satisfied depends on the two sub-conditions that s has justification for (i) taking $(\text{logical consequence})_{\text{x}}$ as an admissible instance of GTT and (ii) s accepts that the actual case is in the domain of the quantifier of GTT, precisified as $(\text{logical consequence})_{\text{x}}$. Recall that I want to consider the two possibilities that Z is evaluated using classical or intuitionistic logic. Note that s endorses logical pluralism, so (i) will be satisfied in both cases. Therefore, whether condition (3) is satisfied will depend crucially on (ii) – i.e. on the actuality constraint. If Z is evaluate using classical logic, the actuality constraint is satisfied if s takes the actual case as complete and consistent. If Z is evaluated using intuitionistic logic, the actuality constraint is satisfied if s takes the actual case as potentially incomplete.

However, evidence for C1 and C2 is constituted by inferences X and Y, which are made respectively with intuitionistic and classical logic. So, the actuality constraint would affect the possibility of *availing oneself* with one's deductive justification for C1 and C2. This does not mean that condition (1a) or (1b) are no more satisfied – one can still claim one has deductive justification for C1 and C2, but one cannot continue reasoning on the basis of such justification, if the logic has changed. Let us now consider in turn the two possibilities of s making inference Z with classical and with intuitionistic logic.

If s employs classical logic, then s accepts that the actual case is complete

[6]Some might complain about taking deductive inferences to count as evidence and would prefer to call them "reasons". I keep on using "evidence" to get (TP*) as close as possible to (TP).

and consistent. S's justification for believing $\alpha \rightarrow \beta$ based on ($\sigma \rightarrow (\alpha \rightarrow \beta)$ and σ) is obtained by employing intuitionistic logical consequence. So, considering inference X, "case_x" ranges over potentially incomplete cases. Since potentially incomplete cases include complete and consistent cases, s strongly endorses also intuitionistic logical consequence, for s takes the actual case to be a potentially incomplete case – albeit a "special" one. Therefore, s can avail herself of her deductive justification for believing $\alpha \rightarrow \beta$ – i.e. C1. Moreover, s can avail herself of her deductive justification for α based on $\neg\neg\alpha$; for inference Y is made employing classical logical consequence, which s strongly endorses. Hence, s can avail herself of her deductive justification of both C1 and C2. Condition 3. is satisfied, so deductive justification transmits from Z's premises to Z's conclusion. If s evaluates Z with classical logic, s is deductively justified in believing C3 on the basis of Z.

Let us turn to the possibility of s evaluating Z with intuitionistic logic. In order to satisfy condition 3., s has to strongly endorse intuitionistic logical consequence. So, s has to take the actual case as potentially incomplete. S's justification for believing $\alpha \rightarrow \beta$ based on ($\sigma \rightarrow (\alpha \rightarrow \beta)$ and σ) is obtained by employing intuitionistic logical consequence, which s strongly endorses. So, s can avail herself of her deductive justification for $\alpha \rightarrow \beta$ – i.e. C1. S's justification for believing α based on $\neg\neg\alpha$ is obtained by employing classical logical consequence. So, considering inference Y, "case_x" ranges over complete and consistent cases. Since s takes the actual case as potentially incomplete, s cannot strongly endorse classical logical consequence; for s takes the actual case as potentially incomplete instead of complete and consistent, and so s does not take classical logical consequence to satisfy the actuality constraint. Therefore, s cannot avail herself of her deductive justification for α – i.e. C2. Condition 3. is *not* satisfied, so deductive justification does not transmit from Z's premises to Z's conclusion. If s employs intuitionistic logic in evaluating inference Z, s is not justified in believing C3 on the basis of Z. That's because, even though s has deductive justification for both C1 and C2, by strongly endorsing classical logical consequence, s cannot avail herself of her justification for C2.

4.1.4 Conclusion

In the present section I provided a revised version of the transmission principle by Moretti and Piazza (2018), which can be applied to the framework of indeterminacy pluralism. The logical pluralist has to satisfy an additional condition; in section 4.1.3, I presented a case in which transmission might fail because of the additional condition. I argued that taking a contextualist stance on deductive justification can provide the intuitively right result for the problematic inference.

4.2 Truth Pluralism, Indeterminacy, and Unsettledness: A New Challenge to Logical Pluralism

In this section, I consider the objection of mixed inferences, which has been raised first by Tappolet (1997) against truth pluralism, and I argue that it posits an objection also to a certain version of logical pluralism, which Lynch (2009: 94) labels Domain-specific Logical Pluralism (DLP).

In section 4.2.1 I introduce DLP, which is a variant of Beall and Restall's pluralism adapted to the framework of truth pluralism. Then, I present the objection of mixed inferences and show that it posits a distinctive challenge to DLP, which I will call the unsettledness challenge. This problem originates from the same phenomenon of the mixed inferences objection and of the collapse arguments – see chapter 3 – but is a distinct and more serious challenge that shows that DLP is an untenable position. In section 4.2.4, I consider whether a solution suggested by Lynch (2009) to the problem of mixed inferences might solve the unsettledness challenge, and I show that this strategy is does not succeed in sidestepping the objection.

4.2.1 Domain-specific Logical Pluralism

Truth pluralism is, roughly, the thesis that there are different ways of being true for different regions of thought. The chief motivation of truth pluralism is the so-called *scope problem* (Lynch 2009: 21-50); namely, the suggestion that any monistic and non-deflationary theory of truth applies only to a certain kind of claims.[7] For instance, the correspondence theory of truth is suitable for claims about middle-sized world objects but seems less apt for moral claims. Analogously, the coherence theory of truth fits well moral claims, but is less apt for claims about middle-sized objects. Granting that one can classify propositions into domains on the basis of their subject matter, truth pluralism amounts to the claim that, for each propositional domain,

[7]See Ferrari and Moruzzi (2018) for a form of pluralism that includes a domain-specific deflationary truth property.

a different truth property realizes the concept of truth. So, for instance, a correspondence property is the truth property for the propositional domain of middle-sized physical objects; a coherence property is the truth property for the domain of moral propositions; etc. One may distinguish between moderate and strong versions of truth pluralism. The former admits the existence of a general, 'spare' truth property that applies to all truth-apt discourse along with the various domain-specific properties; whereas the latter only allows for domain-specific truth properties, or at best for a merely 'abundant' general truth property such as the disjunction of all the domain specific truth properties (Pedersen and Wright 2013a). Both versions of truth pluralism can be spelled out in several ways. Instances of moderate pluralism are Lynch's second-order functionalism (2004) and manifestation functionalism (2009; 2013). Instances of strong pluralism are Wright's one-concept-many-property view (1992; 2013), alethic disjunctivism (Pedersen 2010; Pedersen and Wright 2013b) and Edwards' simple determination pluralism (2011; 2013; 2018). (Lynch 2009: 85-96) and (Pedersen 2014: 261-270) both provide arguments to the effect that, if one endorses truth pluralism, then, given some plausible assumptions, one ought to endorse also a version of indeterminacy pluralism. Both Lynch's and Pedersen's arguments start by assuming that, in order to get logical pluralism, one has to allow for at least two truth properties: a realist, correspondence property – i.e. a truth property according to which a proposition is true iff it corresponds to some fact, some state of affairs, or an otherwise-characterized portion of objective reality – and an antirealist, epistemic property along the lines of Wright's superassertibility (1992: 48) or Lynch's superwarrant (2009: 38): A proposition is superwarranted if it is warranted without defeat at a state of information, and will continue to be so at every state of information that extends it. One of the key motivations for adopting truth pluralism is accommodating both realist and antirealist intuitions about truth: for certain domains it makes sense that what is true is what correctly represents objective reality, whereas for other domains it makes more sense to think of truth as a verification procedure that allows one to confirm or disconfirm a certain claim. So, the requirement of having both a realist and an antirealist property is presumably met by

most – if not all – versions of truth pluralism. In any case, from now on, let us focus on those accounts of truth pluralism that satisfy the assumption.

Logical pluralism stems from these assumptions in the following way. In the domain in which truth is superwarrant, truth is epistemically constrained: since truth is characterized in terms of feasible warrant, it cannot extend beyond human epistemic reach. This allows for there being some proposition for which one cannot have a warrant for – even in principle. This means that, in the domain for which truth is superwarrant, there might be a proposition p such that one is not warranted in holding: either superwarrant p or superwarrant $\neg p$. Thus, the law of excluded middle (LEM) – i.e. $p \vee \neg p$ – does not hold unrestrictedly in the superwarrant domain. Since LEM is not generally valid, so is Double Negation Elimination (DNE) – i.e. $p \equiv \neg\neg p$; for, if it were, one could derive LEM via *modus ponens* from DNE and $\neg\neg(p \vee \neg p)$, which is provable in intuitionistic logic. LEM and DNE are laws of classical logic but are rejected in intuitionistic logic. Therefore, truth as superwarrant is incompatible with classical logic, but is suitable for intuitionistic logic. This is not a surprising result: in fact, Beall and Restall take warrants as a possible interpretation of potentially incomplete stages (2006: 67), and if one takes cases as stages in GTT one gets intuitionistic consequence.

By contrast, in the domain in which truth is a realist, correspondence property, truth is not epistemically constrained: whether a proposition is true depends on whether it represents facts or state of affairs that obtain, independently of whether such facts are under our epistemic reach. This strongly suggests that, in the domain in which truth is correspondence, bivalence is a valid logical principle. From bivalence one can derive both LEM and DNE, which are paradigmatic principles of classical logic; so, the correspondence truth property supports classical logic. To conclude, the truth pluralist must allow for at least two logics operating in different domains: intuitionistic logic in the domain in which truth is superwarrant and classical logic in the domain in which truth is correspondence.

Coupling this with the commitments of Beall and Restall's pluralism,[8]

[8]it is worth pointing out that Beall and Restall explicitly reject coupling their view

one gets the following account. Each domain is characterized by a single truth property. The domain-specific truth property has certain features – e.g. being epistemically constrained or unconstrained – that fix a certain class of cases as salient for that domain – e.g. worlds or Tarskian models in the correspondence domain; stages in the superwarrant domain. In each domain, the class of cases picked out by the truth property provides a different precisification of GTT, which meets the core requirement of Necessity, Normativity and Formality. So, a different relation of logical consequence – and so, a different logic – uniquely applies to each domain. What one gets is a domain-specific variant of indeterminacy pluralism. Following Lynch (2009: 94), I will call it Domain-specific Logical Pluralism (DLP).

In contrast with Beall and Restall's pluralism, in DLP, it is domain-specificity which determines the uniquely correct logic of a certain inference: depending on which truth property is more apt for a certain domain, a certain logic is picked out as the uniquely correct logic of that domain. Recall now the inference from $\neg\neg p$ to p, valid in classical logic but not in intuitionistic logic. According to Beall and Restall, these results are different and not competing, whereas according to DLP, depending on what p is about, and so on which truth property applies to it, there is only one correct answer to the question on whether the inference is valid: depending on what the domain of p is, the uniquely correct logic of that domain will determine whether the inference is valid. Fixing the domain of p allows one to settle a certain class of cases as salient, so that on gets an admissible precisification of GTT as the uniquely correct precisification of logical consequence for that domain.

Summing up, DLP has the following three commitments:

1. (i) *Indeterminacy Pluralism*: logical consequence is indeterminate; any precisification of GTT that has the three core features is an admissible relation of logical consequence;

2. *Locality*: for each domain of discourse, there is a uniquely correct precisification of GTT, and hence a uniquely correct logic;

with a commitment to domains (Beall and Restall 2006: 88-89)

3. *Truth Pluralism*: the fact that different truth properties uniquely characterize different domains explains why there is a uniquely correct precisification of GTT in each domain. The nature of the domain-specific truth properties posits constraints that fix a certain class of cases as salient – e.g. epistemically constrained properties require cases to be potentially incomplete. The fixed class of cases delivers, in each domain, a different precisification of GTT.

The three commitments might not be all equally important;[9] in particular, instead of 3, one might have an alternative explanation of how the class of cases is fixed in different domains and the resulting position would still be a domain-specific variant of logical pluralism. In the next section, I will show that these three commitments, as they stand, are co-untenable.

4.2.2 Mixed Inferences and the Unsettledness Challenge

The problem that I want to outline is, in a nutshell, that there are certain inferences for which the salient class of cases is unsettled. Since the class of cases is unsettled, no precisification of GTT is yielded: DLP lacks the resources to account for such inferences, and so is an untenable account. Let us see what the problem is more precisely.

The fact that propositions are divided into domains on the basis of their subject matter, together with the fact that one very often employs propositions about different subjects in one's reasoning[10] create the phenomenon of the so-called mixed inferences and compounds; that is, inferences and compounds that employ atomic propositions which belong to different domains. Mixed discourse is problematic for truth pluralism (Tappolet 1997, 2000). Consider the following inference – call it CAT Tappolet (1997: 209):

1. Wet cats are funny.

[9]For this reason, I take *Locality* as an independent commitment and not a consequence of committing to logical pluralism.

[10](Lynch 2009: 86) refers to this phenomenon as the "universality of reason".

2. This cat is wet.

C. This cat is funny.

CAT is a valid inference. However, the truth pluralist cannot account for its validity. If one is a pluralist about truth, one claims that premises (1) and (2) belong to two different propositional domains. Hence, they are apt for having two different truth properties – presumably, correspondence is apt for (2) and superwarrant, or another epistemic property, is apt for (1). Now, one of the core features of validity is Necessity – i.e. an inference is valid iff the truth of the premises necessitates the truth of the conclusion. Since the premises of CAT have two different properties, there is no single truth property that is preserved from the premises to the conclusion. Therefore, the objection goes, the truth pluralist is not able to account for the validity of CAT and of all inferences whose premises have different truth properties. An analogous problem arises of mixed compound propositions (Tappolet 2000): "this cat is wet and this cat is funny" is a conjunction whose atomic constituents belong to different domains and hence are apt for different truth properties. So, what is the truth property of the conjunction? The same goes for other compounds.[11] Since reasoning across various areas of discourse is very common and indeed unavoidable, this is a pressing challenge to pluralist accounts of truth.

Lynch (2009) points out that, if one endorses DLP in addition to truth pluralism, there is an additional problem: which logic ought one employ for evaluating mixed inferences and mixed compounds, given that different logics are apt for different domains? Consider again CAT: supposing that truth is superwarrant in the domain of (1) and correspondence in the domain of (2), intuitionistic logic governs the domain of (1), while classical logic governs the domain of (2). Since CAT is valid according to both classical and intuitionistic logic, it seems that Lynch's worry is not so pressing in this case. However, there are also mixed inferences whose validity is contentious.(Lynch 2009: 98) considers the following:

[11]The mixing problem for truth pluralism arises also for generalizations (Lynch 2009; Wright 2013) and even for atomic propositions (Sher 2005; Wyatt 2013).

> NIX: if it is not the case that Sophie's choice is morally right, then grass is not green; but grass is green, so Sophie's choice is morally right.

NIX includes two atomic constituents: "Sophie's choice is morally right" and "grass is green". The latter is a paradigmatic proposition of the middle-sized physical objects domain, in which the truth property is correspondence, and the uniquely correct logic is classical; the former is a proposition about ethics. It is plausible that truth is epistemically constrained in the moral domain. The arguments presented above aim to show that intuitionistic logic is the uniquely correct logic for domains in which truth is an epistemically constrained property. NIX employs *DNE*, which is a valid inference rule in classical, but not in intuitionistic logic. So, the question of which of the two logics one ought to employ in evaluating NIX makes a difference with respect to whether NIX is valid. Classical and intuitionistic logic provide conflicting validity assessments for NIX: according to classical logic, it is valid; according to intuitionistic logic, it is not. The problem extends to mixed compounds; for one can think of compounds in terms of inferential rules of introduction and elimination of the connectives – e.g. one can think of $p \wedge q$ as the conclusion of an inference which has p and q as its premises and employs the inferential rule of conjunction introduction.

The problem of mixed inferences and compounds for DLP is treated by Lynch (2009) and others as purely a matter of choice between different logics – i.e. the logics of the domains of the atomic constituents of the mixed inference. This is analogous to the challenge that (Tappolet 2000: 210) posits to the truth pluralist in considering mixed inferences: «for the conclusion to hold, some truth predicate must apply to all three sentences [of the inference] but what truth predicate is that? And if there is such a truth predicate, why isn't that the only one we need?». However, I contend that mixed arguments posit a more serious challenge to DLP. DLP is a variant of logical pluralism characterized by the three commitments of section 4.2.1. According to *Locality*, there is a uniquely correct logic in each domain. According to *Indeterminacy*, a logic is picked out by providing an admissible precisification of GTT. In order to have an admissible precisification of GTT, one must

specify a class of cases as salient. In order to have a uniquely correct logic in each domain, the salient class of cases must be kept fixed in each domain. According to *Truth Pluralism*, certain features of the truth properties fixes the salient class of cases in each domain. Thus, for any inference within a domain, there will be a certain class of cases that is picked out as salient.

Since the relevant class of cases is fixed by the truth property, for each inference within a domain, the relevant class of cases is fixed. However, when it comes to mixed inferences, there is more than one domain at issue, and each is characterized by one truth property and one logic. In the presence of multiple domain-specific truth properties, none of which is preserved through the inference, the salient class of cases is left unsettled.[12] As a consequence, no specification of GTT is picked out. So, for mixed inferences, no logic is picked out as the uniquely correct one, as the commitments of DLP would require. This leaves one with no indication on how to assess the validity of mixed inferences. Call this the *unsettledness challenge* for DLP. The unsettledness of the class of cases causes the fact that DLP is completely silent upon mixed inferences. Consider again NIX and its conflicting validity assessments with classical and intuitionistic logic. Within the framework of DLP, there is not even a principled way in which one can assess the validity of NIX in the first place. I take this to show, *contra* Lynch, that mixed inferences posit a more serious problem to DLP than to truth pluralism.

4.2.3 Unsettledness is not Collapse

Before turning to some strategies to deal with the unsettledness challenge in the next session, I want to contrast the unsettledness challenge with another related objection; namely, the collapse argument against Beall and Restall's indeterminacy pluralism. The collapse argument casts doubt on the possibility of combining Beall and Restall's logical pluralism with the long-standing thesis that logic is normative for reasoning. Beall and Restall explicitly endorse this thesis including Normativity among the core features of logical

[12]For those variants of truth pluralism that admit of a generic truth property in addition to the domain-specific properties, there is, in fact, a truth property that is preserved through the inference. I will return to this issue in the next section.

consequence. The collapse argument is first raised by (Priest 2006a: 203):

> "We often reason about some situation or another, call it s. Suppose that s is in different classes of situations, say, K_1 and K_2. We cannot give the answer 'both' here. Take some inference that is valid both in K_1 and in K_2, $\alpha \vdash \beta$, and suppose that we know – or assume – α; are we, or are we not, entitled to accept β?"

Since Beall and Restall endorse a form of global indeterminacy pluralism, they allow for conflicting assessments of validity of the same argument. Suppose that the notion of validity appropriate for K_1 is classical logic and the notion appropriate for K_2 is intuitionistic logic. The situation s is some case that belongs both to the class of complete and consistent cases and to the class of potentially incomplete cases. So, it will be subject both to the normativity of classical consequence and to the normativity of intuitionistic consequence. The Normativity core feature states that, if an argument is valid, one goes wrong in accepting the premises but rejecting the conclusion. Let us spell it out for both consequence relations in the form of normative principles:

> NORM1: if $\alpha \vdash_{\mathrm{K}_1} \beta$, then it is wrong to (accept α and reject β).
>
> NORM2: if $\alpha \vdash_{\mathrm{K}_2} \beta$, then it is wrong to (accept α and reject β).

Take the inference $\alpha \vdash \beta$ to be, say an instance of Double Negation Elimination (DNE). The inference is invalid in intuitionistic logic, so NORM2 does not apply because its antecedent is false. Since s belongs also to the class of complete and consistent cases, NORM1 applies; it allows one to infer β without going wrong. Note that all arguments of intuitionistic logic are classically valid, but not the other way around. So, if classical and intuitionistic validity provide conflicting assessments of validity on the same argument, then classical logic always allows one to draw the conclusion without going wrong. The same goes for classical logic and relevant logic, for the class of complete and consistent cases is also a subset of potentially inconsistent cases. Thus, there is a sense in which Beall and Restall's pluralism collapses into classical

monism: in all cases in which the different logics provide conflicting validity assessments, classical logic never leads one astray.

Since DLP is committed to Locality, there is always a uniquely correct logic to evaluate a certain argument. Does the unsettledness challenge create a form of collapse? Stei (2017) provides an argument according to which, in order to have collapse, one does not need to have a global pluralism: it is sufficient «that the areas of application of the competing logics overlap in a specific way». Since mixed inferences and compounds include propositions of different domains, and different domains are the application areas of different logic, one might think that mixed inferences constitute the overlapping area of application of the logics in DLP.

I contend that unsettledness and collapse are distinct phenomena, for three reasons: first, one does not need to appeal to normative principles in order to raise the unsettledness challenge. In fact, DLP is committed to Normativity only insofar as it is among the core features of Indeterminacy, but if one would prefer to drop it out the set of core features, the unsettledness challenge would still be in place. Second, it is one thing if, for each conflicting validity assessments, logical pluralism always collapses to the same logic, it is another if, for different conflicting validity assessments, logical pluralism takes as correct just one assessment. The latter does not imply the former, so *Locality* does not, by itself, create a collapse. Third, it is not true that in DLP mixed inferences are an area of overlap in the application of the logics. Take again the argument by Priest: the situation s is simultaneously in different classes of cases. This is allowed in Beall and Restall's global pluralism because cases apply across all truth-apt discourse. However, the commitment of DLP to *Truth Pluralism* and *Locality* forbids that to happen for DLP: since a different class of cases is fixed in each domain by the truth property, a case s is always treated as either a complete and consistent model, or as a potentially incomplete stage of warrant, or as any other specification of cases fixed by a certain truth property. I take these three reasons to show that unsettledness and collapse are two different problems: the problem with mixed arguments is not that their assessment makes DLP collapse into monism, but rather that they cannot be assessed at all.

4.2.4 Lynch's Modesty

In this section, I consider a solution that Lynch has provided against the objection of mixed inferences to DLP to see whether it can be taken as a solution to the unsettledness challenge. I argue that it cannot, for two reasons: first, it provides an ad hoc way of fixing the case; second, even if adopted, it threatens to be inadequate for DLP.

Lynch attempts to address the objection by appealing to a principle of logical modesty (2009: 100):

> MODEST: where a compound proposition or inference contains propositions from distinct domains, the default governing logic is that of the compound or inference's *weakest member*.

By 'weakest member', Lynch means the premise or atomic whose logic is the *weakest* among the logics in play. Lynch takes a logic as weaker than another when the set of logical truths and inference rules of the latter extends the set of logical truth of the former.[13] Take CAT as an example. Classical logic is an extension of intuitionistic logic because it includes a larger set of logical truths – LEM and DNE. Therefore, according to MODEST, the inference should be evaluated using intuitionistic logic, for that is the weakest logic in play. So, CAT is intuitionistically valid. MODEST also provides a verdict on the validity of NIX: since intuitionistic logic is the logic of the weakest member, NIX is invalid.

However, MODEST relies on the controversial assumption that all domain-specific logics are orderable. Therefore, Lynch provides an alternative way to applying the modesty principle. Consider the logics of a certain mixed inference. Now, take the sets of the logical laws they employ. Then, consider the result of the intersection of the two sets. The intersection set would include only logical laws that are endorsed by both logics. So, one might hope to evaluate the inference by employing only those laws that lie in the

[13]One can also order logics for their *intension*; that is, according to their expressive power. In this latter case, a logic is weaker than another when it is possible to express the operators of the former into the system of the latter.

intersection set – i.e. only those laws employed by every logic involved (2009: 102):

> MODEST*: where a compound proposition or inference contains propositions from distinct domains, the default governing logic is that comprised by the intersection of the domain-specific logics in play.

MODEST* does not require that all domain-specific logics be orderable, so it applies also to non-orderable logics.

Nevertheless, even MODEST* is not sufficient to deal with all possible mixed inferences. First, there is a *methodological* reason to suspect that the intersection of the sets containing the logical principles of two logics is not always a logic. Traditionally, logic is not conceived merely as a formal system; rather, its chief purpose is the evaluation of the arguments and of the meanings of a (regimented) natural language. In this spirit, Priest (2006: 195) distinguishes between pure and applied logic, and Beall and Restall put forward a distinction between a logic and a mere algebra. If one agrees with the traditional conception of logic, it is implausible that any set of logical laws, casually assembled for the sake of evaluating an inference, is itself a logic. Hence, either MODEST* always delivers a logic that is independently motivated by other considerations, or it does not deliver a logic after all. Let us now consider whether the former is the case. Surely, if we confine our attention to orderable logics the result can be legitimately named a logic: it is the weakest member. But in this case MODEST would suffice to do the trick. Since MODEST* is meant to extend the applicability of MODEST, let us focus on non-orderable logics. MODEST* works for logics that are both extensions of the same logic but are not one the extension of the other. For instance, consider an inference with two premises whose domains are governed by the modal systems B and S4. Those are both extensions of the modal system T; hence, via MODEST*, we get T as the right logic for the inference.[14] However, if we consider logics that are not just extensions of the

[14]However, note that the meaning of the modal operators is different in the two logics.

same logic – e.g. quantum logic and Wansing's (2016) connexive modal logic – it is unclear whether the resulting intersection deserves the name *logic*.

Now, the endorser of DLP can claim that it is not necessary for the resulting intersection to be a logic, as long as it is not empty. Or, she can claim, *any* set of logical laws deserves the name 'logic' after all. What matters is just that MODEST* grants that laws which are not shared by all the logics involved would not be employed. Nevertheless, I think that this view faces problems of its own. One of the main advantages of truth pluralism is that it allows us to accommodate many intuitions about what truth is by employing different truth properties in different regions of thought. That is why the only constraint is that there are at least two different truth properties, but there is no *prima facie* constraint on the number of truth properties involved. In DLP, the logic employed in a certain domain is picked out by the truth property of that domain. So, there is no *prima facie* constraint on the number of logics either. In principle, we can have, say, a propositional domain which is governed by quantum logic; another domain which is governed by some higher-order logic; etc. Moreover, an inference might contain more than two premises; so, a mixed inference might involve more than just two logics. MODEST* predicts that the intersection set between the logics involved in a mixed inference provides enough principles to evaluate the mixed inference in question. However, all things considered, this seems dubious. It is plausible that in many cases the intersection set turns out empty, or with so few laws that that every inference would be taken as invalid.[15]

In order to avoid that, the endorser of DLP would have to list all the possible truth properties she admits and to show that the corresponding logics are either orderable or extensions of the same logic. However, propositions are infinite and can be about any subject matter whatsoever. This makes the eventuality of realizing such an enterprise very remote. I conclude that Lynch's appeal to modesty doesn't seem a promising route to avoid the problem of mixed inferences.

[15]Steinberger (2018a)points out that the same objection affects the logical monist who seeks for a core logic applicable across all discourse.

4.2.5 Conclusion

In this section, I have considered a form of local logical pluralism, which has not been discussed much in the literature. I have argued that a version of domain specific pluralism, DLP, is committed to three theses that together yield to the unsettledness challenge. I conclude that the three commitments envisaged in section 4.2.1 are co-untenable.

Chapter 5

Conclusions

Let me briefly recap the main points of this work. In this thesis, I have mainly contributed to two lively debates in the contemporary philosophy of logic; namely, the normativity of logic and logical pluralism.

As for the first theme, my contribution has been to considering the question on the source of the normativity of logic and to map out some current positions on the literature on the basis of this distinction. Also, I elaborated my own proposal, which seems he normativity of logic as sourced in rationality: two logical principles, which are about logical consequence and logical consistency, are partly constitutive of rationality, and in particular of the role of evaluating an agent as rational.

I argued that this view is compatible with many accounts of logic, and that it can be connected with two alternative conceptions of logic, one monist and the other pluralist.

As for my contribution to the debate on pluralism, I expanded and ameliorated the indeterminacy pluralism advocated by Beall and Restall (Beall and Restall 2000, 2001, 2006). First, I showed how my view on the normativity of logic makes a case for logical pluralism that is firmer than the one suggested by Beall and Restall and can avoid a long-standing objection to indeterminacy pluralism that threatens to make it incompatible with the normativity of logic.

Second, I expanded the view on indeterminacy pluralism by providing a

principle for the transmission of justification and by arguing for a contextualism of deductive justification.

Third, I considered the relation between indeterminacy pluralism and truth pluralism and exclude one possibility of combining these two views.

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