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ESSAYS IN THE ECONOMETRIC ANALYSIS OF SYSTEMIC RISK MEASURES

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Abstract

This thesis aims at the study of systemic risk measurement, which became crucial after the $2007 - 2009$ financial crisis. The objective of the thesis is twofold: (i) we address the issue of assessing the accuracy of systemic risk measures, (ii) we investigate the role of the long-range dependence in systemic risk forecasting, under both methodological and empirical perspectives. From the methodological point of view, we propose two appropriate loss functions, the Tail Tick Loss function and the Tail Mean Square Error, specifically designed to evaluate the CoVaR and MES accuracy, respectively. Moreover, we introduce a comprehensive model called Asymmetric-Component-GARCH (ACGARCH), which is able to capture both the leverage effect and long-range dependence. An empirical analysis of different bivariate volatility models to the daily returns of 91 US financial institutions in the period $2000 - 2012$ confirms the need of employing appropriate loss functions to evaluate systemic risk accuracy and to discriminate among different competing models. Moreover, empirical results encourage the usage of the ACGARCH model in the systemic risk framework.

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Chapter 1

Introduction

This chapter explains systemic risk, whose a single definition is not shared in the literature, motivating the importance of its measurement for the society. How to define and measure it is the object of interest over the last years, however it is still an open issue. After providing an overview over the existing definitions and different approaches' points of view, the motivation and the structure of the thesis work are presented.

1.1 Systemic risk: definition and measurement

The $2007-2009$ financial crisis developed the need of measuring systemic risk for the whole economy with the purpose to evaluate the vulnerability of the financial system and the risk of different financial institutions on the whole market, since a single institution's risk measure does not necessarily reflect systemic risk (e.g. Value-at-Risk). The failure of big financial institutions infected the entire financial system and even brought severe consequences to the real economy on a global scale, such as the bankruptcy of Lehman Brothers, which demonstrated the fragility of the whole financial system. Furthermore, through the spillovers from the financial system to the real economy, which drops into a deep recession, and through the bailouts of big companies with taxpayers' money, financial crises impose high costs for the society, leading to consider managing systemic risk as a desirable goal.

In the light of the global financial crisis and due to the acknowledgement of the importance of systemic risk, several organisms were created in order to control and supervise the stability of the financial system. Among the others, the U.S. Congress, in 2010, created the Office of Financial Research (OFR), and European Systemic Risk Board (ESRB) was born in April 2009 in order to ensure financial stability. Finally, regulators have been given a mandate to measure and monitor systemic risk.

The precise definition of systemic risk is still ambiguous and many proposals are present in the literature. Billio et al., in 2012, specify systemic risk as "any set of circumstances that threatens the stability of or public confidence in the financial system", whereas the European Central Bank (ECB), in 2010, defines it as a risk of financial instability "so widespread that it impairs the functioning of a financial system to the point where economic growth and welfare suffer materially". Daniel Tarullo, the Federal Reserve Governor, in 2009, states that "Financial institutions are systemically important if the failure of the firm to meet its obligations to creditors and customers would have significant adverse consequences for the financial system and the broader economy". Finally, Acharya et al., in 2010, claim that systemic risk is the risk of a crisis in the financial sector and its spillover to the economy at large.

Systemic risk may easily be confused with *systematic risk*, also known as market risk or undiversifiable risk. Systematic risk is the risk explained by factors that influence the economy as a whole, and, in a portfolio, can't be diversified. Systemic risk is, instead, more complex and, according to Staum (2012), is composed by different risks: the systematic risk and those risks deriving from several financial market phenomena, such as contagion, spillover, transmission of losses and distress from one institution to another one. Contagion can take several forms, and, in asset pricing, it is defined as when an institution's sale of assets into an illiquid market can cause a decline in asset prices and, thus, losses to others.

During the last decade, after that the global financial crisis highlighted the importance, and boosted the development, of detailed study of systemic risk, several measures have been developed and proposed in the literature from regulators, researchers and practitioners points of view and different models have been used to estimate them according to their structure. Bisias et al., in 2012, survey not exhaustively systemic risk measures and the conceptual frameworks, describing that "when leverage is used to boost returns, losses are also magnified, and when too much leverage is applied, a small loss can easily turn into a broader liquidity crunch via the negative feedback loop of forced liquidations of illiquid positions cascading through the network of linkages within the financial system". In measuring systemic risk, the authors state that it is fundamental to develop a conceptual framework in a coherent fashion, and to collect and access the correct types of input-data required by the specific adopted measure.

On the contrary, Brunnermeier and Oehmke, in 2013, survey the literature on bubbles, financial crisis and systemic risk, explaining that systemic risk builds up in the background during the run-up phase of imbalances or bubbles and it materializes only when the crisis explodes (crisis phase). As a consequence, spillover and amplification effects determine the overall damage to the economy. The crisis phase starts when a trigger event occurs, whose negative effect on the financial system and real economy is amplified through several channels. Research conducted on quantitative methods relating to financial stability are generally classified into three categories:

- *Early Warning Indicators* (EWIs), which estimate the probability of the trigger event, like bubbles;
- *systemic risk measures*, which evaluate the vulnerability of the financial system;
- *macro stress tests*, which evaluate the effects of predetermined stress scenarios on the financial system.

Stress tests are standard devices used to determine the capital that an institution will need to raise if there is a financial crisis. Regulators have to conduct stress tests every year in the United States.

Systemic risk measures attempt to capture the total and marginal risk contributions of different financial institutions on the whole market. Therefore, they are related to firm-level risk measures, which become important given the implementation of Basel II bank regulations. The purpose of these firm-level risk measures is to reduce a vast amount of data to a meaningful single statistic that summarizes risk. Risk measures for individual financial institutions, however, are typically not good systemic risk measures, since their sum does not capture the systemic risk. The sum of all risk contributions, in fact, should be equal to the total systemic risk and each one should incentivize financial institutions to (marginally) take on the appropriate amount of systemic risk. Therefore, it becomes crucial to develop a systemic risk measure for the whole economy and a way to allocate this systemic risk across the financial institutions. Furthermore, *Systematically Important Financial Institutions* (SIFIs) should be individually identified by systemic risk measure, since they could cause negative risk spillover effects on others due to their interconnectedness (Brunnermeier and Oehmke, 2013).

Ellis et al., in 2014, claim that “the diversity within the financial system also supports the fact that a single measure of systemic risk is unlikely to be universally applicable, nor is a single instrument of financial stability policy”. Again, according to Hattori et al. (2014), using only systemic risk measures is not sufficient to assess financial stability as a whole, because systemic risk measures are silent on the loss caused by other trigger events or the probability of such black swan events. Hattori et al. (2014) suggest to use a combination of several quantitative tools to monitor and evaluate the financial system completely and comprehensively, such as EWIs, systemic risk measures and macro stress tests.

Furthermore, the difficulty of measuring systemic risk concerns another aspect. Cerutti et al., in 2012, face the problem of scarcity of data that capture the international dimensions of systemic risk and claim that market price-based indicators are not always reliable risk measures. The global crisis has shown the important role played by financial linkages and channels of propagation, which require many not (yet) available data to be identified. The authors also report some examples which demonstrate that many aspects of global systemic risk simply cannot be captured using existing data, adding that the institutional infrastructure for global systemic risk management is inadequate or simply non-existent.

As a conclusion, measuring systemic risk is still an open issue, not yet solved.

1.2 Motivation and overview

The purpose of this work is to study how systemic risk can be measured, modeled, evaluated and compared. Our contribute to the existing literature is twofold.

First, we investigate how to evaluate the accuracy of systemic risk forecasts, which is a key step towards the definition of a precise systemic risk measure. We focus on two of the most widespread systemic risk measures, namely ΔConditional-Value-at-Risk (ΔCoVaR) (Adrian and Brunnermeier, 2011) and SRISK (Brownlees and Engle, 2015). Given the lack of statistical tools to compare them because of their economic nature, we analyze their straightforward main components, called Conditional-Value-at-Risk (hereafter CoVaR) and Marginal Expected Shortfall (hereafter MES). Therefore, we propose two new loss functions, one appropriate for each measure, with the purpose to test and compare the CoVaR and MES forecasting performances. We are also able to detect the most reliable models to predict CoVaR and MES measures, respectively.

Second, we investigate the stylized facts of financial data not captured by the standard models widely used in the literature, in particular we notice that financial data present long-range dependence. Given the lack of application of long-range dependence models in systemic risk framework, we propose a novel econometric model, in order to improve systemic risk estimation and forecasting. Thus, we contribute to the existing literature by defining, studying and applying a new model which combines in capturing leverage effect and long-range volatility dependence.

The thesis work is organized as follows. We start, in Chapter 2, with a review of the main systemic risk measures present in the literature and we continue revising the main widely used econometric models to compute them. In particular, we focus on the methodology composed by Generalized Autoregressive Conditional Heteroskedasticity (GARCH) approach for time-varying volatility (Bollerslev, 1986) and Dynamic Conditional Correlation (DCC) approach for time-varying correlation (Engle, 2002), called DCC-GARCH-type model.

The urgent need of measuring systemic risk, after the global financial crisis, leads to a huge availability of measures and models without providing comparisons or tools in order to discriminate them. Statistical tools designed to test and compare systemic risk forecasts, in fact, have not been properly developed, and a deep analysis on their accuracy has largely unexplored. As a consequence, a regulator or a practitioner, who has to compute systemic risk for its financial institution, for example, is in difficulty in identifying and choosing the measure and the related model to use to estimate and forecast systemic risk. Under this basic idea, considered as our first contribution to the existing literature, we review, in Chapter 3, the main backtests, used to evaluate the adequacy of systemic risk measures, and the existing loss functions, used to evaluate the accuracy of risk measures, with the purpose to identify those adaptable to be used in systemic risk framework. Focusing on the accuracy, we modify and adapt these selected loss functions according to the specific systemic risk measure considered. Therefore, we develop, in Section 3.3, two new loss functions specifically designed for the CoVaR and MES frameworks, namely Tail Tick Loss and Tail Mean Square Error loss functions, respectively.

We then conduct, in Chapter 4, an empirical study of the $2007-2009$ US financial crisis based on the application of DCC-GARCH-type models. We consider daily equity data of 91 top US financial institutions, used by Brownlees and Engle (2015), which are all large capitalization Blue Chip companies as of end of June 2007. The dataset covers the period from January 3rd 2000 to December 31th 2012. We divide the sample period into two sub-samples in order to estimate systemic risk measures in the first sub-sample period and to forecast them in the second one. For comparative purposes, we consider the widely used quantile regression and linear regression models as benchmarks. Hence, applying the new loss functions developed in the previous Chapter, we compare the forecasting performances obtained by the benchmark and DCC-GARCH-type models, which take into account also the leverage effect present in financial data.

However, the topic is challenging and the difficulty to develop a systemic risk measure able to capture the entire nature of systemic risk with the purpose to avoid that financial institutions' failures boost consequent failures of other financial institutions by contagion is high. Empirical results, shown in Chapter 4, go in this direction and indicate criticizes in capturing systemic events during $2000-2012$ years. These findings lead us to explore, and then include within the methodology, other stylized facts of financial data, in particular the long-range dependence in volatility, considered as our second main contribute to the existing literature, that is not yet applied in systemic risk framework. In Chapter 5, we then illustrate the existing econometric models that capture the long-range dependence in volatility and we propose a novel model, called Asymmetric-Component-GARCH (ACGARCH), able to capture both the leverage effect and long-range dependence in financial data. We apply the ACGARCH and existing models to the same dataset used in Chapter 4, and the empirical results confirm our intuition of including long-range dependence. Moreover, using the new loss functions outlined in Section 3.3, we are able to detect the more suitable specification for the volatility that systemic risk framework requires.

Finally, Chapter 6 concludes the thesis work summing up the issues, our contributions and our findings.

The structure of all chapters is the same. They begin with an introduction part to the considered problem and they end with a concluding remarks section. Moreover, Chapters 3 and 5 provide our proposals connected to the considered issue.

Chapter 2

Systemic Risk review

An overview of the existing financial and econometric literature of measuring systemic risk is now presented. During the last decade, several systemic risk measures has been proposed in the literature from different point of views, such as probability-distribution measures, network measures, deriving both from the principal component analysis and from the graph theory. A special focus is on CoVaR-based and MES-based measures, given their widespread usage and application by banks and financial institutions. Finally, the related econometric models to estimate them are presented, with a particular focus on DCC-GARCH-type methodology, which is very popular and allows to consider time-varying correlations and volatilities.

2.1 Systemic Risk measures

The Financial Stability Board (2011) states that systemic risk score should reflect size, leverage, liquidity, interconnectedness, complexity, and substitutability. A good risk measure for systemic risk, in practice, should capture many different facets that describe the importance of a given financial institution in the financial system.

According to Bisias et al. (2012), systemic risk measures could be categorized into five groups organized by the four "L's" of financial crisis (liquidity, leverage, losses, and linkages) that they capture, and by the techniques used. In particular, these groups are:

- probabilities of loss,

- default likelihood,
- illiquidity,
- network effects, and
- macroeconomic conditions.

The *probability-distribution measures* are based on the joint distribution of asset returns and assume that risk is driven by a stable and exogenous data-generating process (risk-management approach). The joint distribution of negative outcomes of a collection of SIFIs provides informative estimates of correlated losses. These measures are quantile-based risk measures that focus on extreme losses (the tail of the distribution). They possess some useful properties, like left continuous and non-decreasing functions of alpha and equivariant to monotone transformations. The advantage of these measures is that they require little information (they rely on public market data, such as stock returns) and make use of statistical methods with minimal assumptions to obtain an estimate of a financial institution's contribution on the system.

The *measures of default likelihood* can be constructed for each institution and link each other through their joint distribution. The contingent-claims analysis can value the implied default probabilities, such as the distressed insurance premium (Huang et al., 2009), and can measure the implicit cost of guarantees.

Illiquidity is a systemic risk measure and the serial correlation in observed returns can be a proxy for it (Getmansky et al., 2004).

The *network analysis measures*, instead, are measures of connectedness and provide direct indications of linkages between firms. They are easily aggregated to produce overall measures of "tight coupling" and are based on two main approaches; the first uses the principal component analysis, whereas the second is derived from the graph theory, such as Granger-causality measure (Billio et al., 2012).

Finally, the *macroeconomic measures* are as much as the macro models of business and credit cycles, unemployment, inflation and growth.

2.1.1 CoVaR-based Systemic Risk measures

The most common and popular risk measure, used by financial institutions, is the *Value-at-Risk (VaR)*. VaR at time t is defined as the α quantile:

$$\Pr_{t-1}\Big[r_t^i \leq \mathrm{VaR}_{\alpha,t}^i\Big] = \alpha$$

where r_t^i is the returns series of institution i and α is the confidence level. It focuses on the risk of an individual institution i in isolation and it is the maximum loss of the institution within the α%-confidence level (see, e.g. Jorion, 2007). The α-VaR is the maximum dollar loss within the α-confidence interval (Kupiec, 2002; Jorion, 2007). It can be interpreted as the minimal capital cushion that has to be added to the firm profit, X, to keep the probability of a default below α. Otherwise, this measure is not a coherent risk measure, it is not convex in X (failing to detect concentration of risks) and it does not distinguish between different outcomes within α-tails. In addition, it fails to consider the institution as part of a system, which might itself experience instability and spread new sources of systemic risk. VaR could be time-varying in relation to the model used for its computation or static, when the time t is fixed. In the light of the $2007 - 2009$ financial crisis, VaR fails to capture the nature of systemic risk (the risk that stability of the financial system as a whole is threatened), because it ignores tail comovement and spillover effect.

All CoVaR-based measures and VaR measure are typically negative.

CoVaR measure by Adrian and Brunnermeier

Adrian and Brunnermeier, in 2011, propose the *Conditional-Value-at-Risk (CoVaR)*, which measures direct and indirect spillover effects to capture externalities that an individual institution imposes on the system, predicting the future systemic risk using current institutional characteristics. CoVaR is defined as the VaR of the system returns s conditional on some event $\mathbb{C}(r_t^i)$ of institution i, i.e. is defined as the α-quantile of the conditional probability distribution:

$$\Pr_{t-1}\Big[r_t^s \leq \mathrm{CoVaR}_{\alpha,t}^{s|i}\Big|\mathbb{C}(r_t^i)\Big] = \alpha$$

The authors define the CoVaR conditioning event as $\mathbb{C}(r_t^i) = \{r_t^i = \mathrm{VaR}_{\alpha,t}^i\}$, hence the CoVaR measure corresponds to the VaR of the system conditional on institution being in financial distress, i.e. exactly at its VaR.

Adrian and Brunnermeier (2011) model the joint dynamics of the equity returns of individual financial institutions and of the financial system, with the purpose to capture the empirical relationship between VaRs in the tails of the joint distribution. The contribution of the individual institution (including size, leverage and maturity mismatches in the bank's assets and liabilities) to systemic risk is computed as the difference between the VaR of system conditional on the institution being in distress and the VaR of system when the institution is in a normal state, i.e. the median state. This difference is called Δ-*Conditional Value-at-Risk (ΔCoVaR)* and defined as:

$$\Delta\mathrm{CoVaR}_{\alpha,t}^{s|i} = \mathrm{CoVaR}_{\alpha,t}^{s|r_t^i = \mathrm{VaR}_{\alpha,t}^i} - \mathrm{CoVaR}_{\alpha,t}^{s|r_t^i = \mathrm{Med}(r_t^i)}$$

where Med indicates Median state. The authors find a very strong relation between the institutions' VaR and their ΔCoVaR in the time-series, while they have only a weak relation in the cross section. While two institutions may be similar in terms of VaR, their contribution to systemic risk could differ substantially. Hence, ΔCoVaR allows to evaluate the systemic spillover of an individual institution to the system. CoVaR, in fact, is able to identify the risk on the system by individually SIFIs and allows characterizing contagion under balance sheet deleveraging.

Furthermore, Adrian and Brunnermeier, in 2011, introduce a second measure called *Exposure CoVaR*, which reverses the conditioning and quantifies the exposure of a single institution to systemic financial distress. CoVaR measure is also not explicitly sensitive to size or leverage.

Castro and Ferrari, in 2014, analyze ΔCoVaR as a tool for ranking financial institutions and gauging the interconnectedness in the financial system. ΔCoVaR measure has been used to identify and rank SIFIs by developing a significance test that allows determining whether or not a financial institution can be classified as being systemically important on the basis of the estimated systemic risk contribution, as well as a test of dominance which aims to determine whether or not, according to ΔCoVaR, one financial institution is more systemically important than another, that a financial firm contributes more to systemic risk than another. They conclude that a larger ΔCoVaR makes a statistically significant contribution to systemic risk more likely but does not necessarily imply that an institution's contribution is significant and that the results of pairwise tests of dominance should also be considered.

CoVaR measure by Girardi and Ergun

CoVaR conditioning set does not consider severe losses which are further in the tail and, moreover, CoVaR is not backtestable. As a consequence, Girardi and Ergun, in 2013, propose a generalization of this measure. They generalize the definition of CoVaR by assuming that the conditioning financial distress event refers to the institution i being at most at its VaR, that is $\mathbb{C}(r_t^i) = \{r_t^i \leq \mathrm{VaR}_{\alpha,t}^i\}$. They define CoVaR as the α-quantile of the conditional distribution:

$$\mathrm{Pr}_{t-1}\left[r_t^s \leq \mathrm{CoVaR}_{\alpha,t}^{s|i} \middle| r_t^i \leq \mathrm{VaR}_{\alpha,t}^i\right] = \alpha \tag{2.1}$$

Conditioning on this set, in fact, allows to consider more severe losses considering those beyond VaR farther in the tail, then it is a more general case of financial distress of institution i.

Furthermore, the authors define the systemic risk contribution of an institution as the change from its CoVaR in its benchmark state to its CoVaR under financial distress and investigate the link between institutions' contributions to systemic risk and their characteristics. They propose the percentage difference ΔCoVaR(%) as:

$$\Delta\mathrm{CoVaR}_{\alpha,t}^{s|i}(\%) = 100\left[\frac{\mathrm{CoVaR}_{\alpha,t}^{s|r_t^i=\mathrm{VaR}_{\alpha,t}^i} - \mathrm{CoVaR}_{\alpha,t}^{s|r_t^i=\mathrm{Med}(r_t^i)}}{\mathrm{CoVaR}_{\alpha,t}^{s|r_t^i=\mathrm{Med}\left(r_t^i\right)}}\right] = 100\,\frac{\Delta\mathrm{CoVaR}_{\alpha,t}^{s|i}}{\mathrm{CoVaR}_{\alpha,t}^{s|r_t^i=\mathrm{Med}(r_t^i)}}$$

According to Girardi and Ergun, the financial distress is the institution's returns series being at most at its VaR as opposed to being exactly at its VaR, as proposed by Adrian and Brunnermeier (2011). This change allows backtesting CoVaR measure using standard tests used to backtest VaR. Due to time-varying correlations, the CoVaR of an institution here has a time-varying exposure to its VaR. This feature enables us to detect and incorporate in the systemic risk measurement possible changes over time in the linkage between the institution and the financial system.

Reboredo and Ugolini, in 2015, apply this CoVaR to measure systemic risk in European sovereign debt markets before and after the onset of the Greek debt crisis.

Asymmetric CoVaR

Studying the identification of the main factors behind systemic risk, Lòpez-Espinosa et al. (2012) find that CoVaR measure underestimates the bank's contribution to systemic risk. Hence, they propose *Asymmetric CoVaR* that accounts for asymmetries in the initial specification. The asymmetries based on the sign of bank returns, in fact, play an important role in capturing the sensitivity of system-wide risk to individual bank returns and, ignoring them, balance sheets on the financial system can result in an underestimation of systemic risk when markets are declining.

Mutu (2014) apply the Asymmetric CoVaR for estimating the systemic risk on a large sample of European banks during $2005 - 2011$.

Multivariate CoVaR measure

Cousin and Di Bernardino, in 2013, propose two alternative extensions of the univariate CoVaR, namely *Multivariate Lower-Orthant CoVaR* and *Multivariate Upper-Orthant CoVaR*, in a multivariate setting. These measures are based on multivariate generalization of quantiles, but are able to quantify risks in a much more parsimonious and synthetic way. The two proposed multivariate VaR are vector-valued measures with the same dimension as the underlying risk portfolio. The lower-orthant VaR is constructed from level sets of multivariate distribution functions, whereas the upper-orthant VaR is constructed from level sets of multivariate survival functions. Both these measures satisfy the positive homogeneity, the translation invariance and the elicitability property, which provides a natural methodology to perform backtesting, since defined CoVaR are the minimizers of suitable expected losses. Besides, analyzing how these measures are impacted by a change in marginal distributions, both in dependence structure and in risk level, it results that an increase of marginal risks yields an increase of the multivariate VaR.

Subsequently, Di Bernardino et al. (2015) investigate more in depth the properties and the behavior of multivariate CoVaR, in addition to the estimation, comparing them with existent risk measures. In particular, both multivariate CoVaRs verify the additivity property under some conditions and, since they are based on the corresponding quantile functions, they are more robust to extreme values than any other central tendency measures.

Multiple-CoVaR measure

CoVaR measure not only captures the overall risk embedded in each institution, but also reflects individual contributions to the systemic risk, capturing extreme tail co–movements. However, recent financial crisis are characterized by the contemporaneous distress of several institutions emphasizing the difficulty to accurately measure marginal contributions to overall risk of an institution taken in isolation. The spillover effect of a financial downturn may propagate through other institutions being in distress at the same time. Thus, it is necessary overall risk measures that account for contemporaneous multiple distresses as conditioning events.

Bernardi et al., in 2013, propose *Multiple–CoVaR* as systemic risk measure, to capture interconnections among multiple connecting market participants, that is particularly relevant during period of crisis when several institutions may contemporaneously experience distress instances. They aim to measure the dynamic evolution of tail risk interdependence accounting for the well known characteristics of financial time series. The institutions' marginal risk contribution, called *Multiple–*Δ*CoVaR*, is measured as the difference between the Multiple–CoVaR of each institution conditional on a given set of different institutions being under distress and the Multiple–CoVaR of institution evaluated when the same set of institutions are at their normal state, identified as the median state.

Applying the Shapley value methodology the authors are able to overcome the CoVaR deficiency of subadditivity, for which the sum of individual contributions does not equal the total risk measure, providing misleading information for policy purposes. The Multi-CoVaR measure and the Shapley value methodology have already been used by Cao (2013) to calculate the total systemic risk and to efficiently allocate total systemic risk to each financial institution, satisfying the addivity property.

2.1.2 MES-based Systemic Risk measures

Another measure similar to VaR is the *Expected Shortfall (ES)*, defined as the expected loss of the system conditional on the loss being greater than the VaR calculated at a given level of confidence $1 - \alpha$. In particular, ES is the risk measure calculated on the returns series of the system s as:

$$\mathrm{ES}^{s}_{\alpha,t}(C) = \mathbb{E}_t\big[r^s_t \big| r^s_t \leq C\big]$$

where C is a threshold value to represent the systemic event. ES measure has better formal properties than VaR, but it is difficult to estimate the tail, so it is necessary to make parametric assumptions on the tail distribution.

Starting from ES measure, Acharya et al., in 2010, introduce the concept of *Marginal Expected Shortfall (MES)*, which defines the systemic risk contribution as the expected equity returns of an individual institution conditional on the system being distressed. Hence, the marginal contribution of an institution i to systemic risk is:

$$\mathrm{MES}^{i|s}_{\alpha,t}(C) = \mathbb{E}_t\big[r^i_t \big| r^s_t \leq C\big] \tag{2.2}$$

Usually the threshold value is equal to $C = \mathrm{VaR}^{s}_{\alpha,t}$[1]. All MES-based measures and ES measure are typically negative, whereas SES measure is typically positive.

According to Weiss et al. (2014), MES can be viewed as a measure of moderate systemic risk that regulators can use to predict a crash of the banking sector.

Popescu and Turcu, in 2014, transpose the concept of systemic risk from the financial stock market to the sovereign debt crisis, in order to determine which Eurozone countries are the most systemically important evaluating their contribution to systemic risk. The authors compute MES measure using Eurozone members' bond yields and debts. MES can accurately rank countries according to their riskiness, and spot those that contribute the most to the overall risk, giving information about which countries need more monitoring.

[1] It is important to notice that these distributions are continuous. Consequently, inserting $<$ or $\leq$ in the definition of both the ES and MES measures is insignificant.

SES measure

Acharya et al., in 2010, propose *Systemic Expected Shortfall (SES)*, which measures the conditional capital shortfall of a financial firm and captures the downside risk of a financial institution conditional on the whole system (its contribution). SES evaluates the banks' exposure to systemic tail events, which nevertheless can easily be reverted to capture risk contribution, and is defined as:

$$\mathrm{SES}_t^i = \mathbb{E}\big[za_i - w_i \big| W < zA\big]$$

where w_i indicates bank's equity, z a fraction of assets a_i, W indicates the aggregate banking capital and A the aggregate assets.

The empirical measure is derived from a linear combination of MES measure and leverage, leading indicators that predict an institution's SES, justified using a theoretical model that incorporates systemic risk externalities:

$$\mathrm{SES}_t^i = \beta_0 + \beta_1 \mathrm{LVG}^i + \beta_2 \mathrm{MES}_\alpha^i$$

where LVG^i is the excess ex-ante leverage.

SES is defined as an individual bank's (contribution) propensity to be undercapitalized when the financial system as a whole is undercapitalized, which increases in its leverage, volatility, correlation, and tail-dependence. Hence, institutions with highest MES contribute most to market decreases. The authors estimate the ex-ante MES and leverage using daily equity returns from the year prior to the global financial crisis, which they then use to explain the cross-sectional variations in equity returns performances during the crisis. They empirically demonstrate the ability of SES components to predict emerging systemic risk during the financial crisis of $2007 - 2009$. One of the useful properties of the SES, such as MES, is its additivity, under which the sum of individual institutions' risks is identical to the overall systemic risk.

SRISK measure

SES measure is static and unable to measure systemic risk ex-ante as it requires data from actual financial crises. An alternative dynamic reduced form estimation of capital shortages is provided by Brownlees and Engle (2015). They, in fact, resume MES measure and propose *SRISK* to measure the systemic risk contribution of a financial firm.

The authors define SRISK systemic risk measure of firm i on day t as the prediction of a financial entity in case of a systemic event, that is when system declines below a threshold C over a time period h:

$$\mathrm{SRISK}_t^i = \mathbb{E}_t\left[\mathrm{CS}_{t+h}^i \middle| r_{t+1:t+h}^s < C\right] = \mathrm{W}_t^i \left(k\mathrm{LVG}_t^i - (1-k)\mathrm{LRMES}_t^i - 1\right)$$

where

- CS_{t+h}^i is the capital shortfall of firm i over a time horizon h, defined as:

  $$\mathrm{CS}_t^i = k\mathrm{A}_t^i - \mathrm{W}_t^i = k(\mathrm{D}_t^i + \mathrm{W}_t^i) - \mathrm{W}_t^i$$

  where W_t^i is the market value of equity, D_t^i is the book value of debt, A_t^i is the value of quasi assets and k is the prudential capital fraction;

- LVG_t^i denotes the quasi-leverage ratio $(\mathrm{D}_t^i + \mathrm{W}_t^i)/\mathrm{W}_t^i$;

- LRMES_t^i is Long Run MES, defined as the expectation of the firm equity multi-period return conditional on the systemic event:

  $$\mathrm{LRMES}_t^i = \mathbb{E}_t\left[r_{t+1:t+h}^i \middle| r_{t+1:t+h}^s < C\right]$$

  where $r_{t+1:t+h}$ is the multi-period equity return between period $t+1$ and $t+h$, of firm and system, respectively.

In addition to MES, the authors take into account the size and the leverage of the institution, i.e. during a crisis in the whole financial system, which together determine the expected capital shortage a financial institution would suffer if a systemic event occurred. Hence, institutions with higher SRISK values are more risky and contribute more to the financial sector undercapitalization in a crisis. The authors associate the systemic risk of a financial institution with its contribution to the deterioration of the system capitalization that would be experienced in a crisis. They analyze the systemic risk of top U.S. financial firms between 2005 and 2010. Their empirical results show that SRISK has significantly higher predictive power than SES and provides useful ranking of systemically risky firms at various stages of the financial crisis. They conclude that volatile and undiversified institutions with respect to the market exhibit high MES.

Brownlees and Engle (2015) construct a system wide measure of financial distress using the SRISK across all firms $i = 1, \ldots, N$. Hence, the total amount of systemic risk, called *aggregate SRISK*, is:

$$\mathrm{SRISK}_t = \sum_{i=1}^{N} (\mathrm{SRISK}_t^i)_+$$

where $(\mathrm{SRISK}_t^i)_+$ indicates $\max(\mathrm{SRISK}_t^i, 0)$ The aggregate SRISK of the financial system provides early warning signals of distress in the real economy. Finally, the percentage SRISK measure is:

$$\mathrm{SRISK\%}_t^i = \frac{\mathrm{SRISK}_t^i}{\mathrm{SRISK}_t} \qquad \text{if } \mathrm{SRISK}_t^i > 0$$

CES measure

Banulescu and Dumitrescu, in 2015, propose a systemic risk measure, called *Component Expected Shortfall (CES)*, which measures the financial institution's 'absolute' contribution to the ES of financial system. In fact, the sum of CES of all financial institutions in the system is equal to ES of financial system. Thus, the risk of the aggregate financial system according to the institutions therein is easily decomposable. Hence, CES of institution i at time t is defined as:

$$\mathrm{CES}_t^i = w_t^i \, \mathrm{MES}_t^i$$

where w_t^i denotes the weight or size of institution i in financial system, that is its relative market capitalization. More precisely, CES is a non-linear combination of four elements: volatility, correlation, tails expectations and the weight of the firm.

Furthermore, CES can be easily used to identify the SIFIs: the larger CES, the greater the contribution and the more systemically risky the institution. This ranking is obtained according to the financial institutions' riskiness and captures those institutions that effectively suffered major transformations during the crisis and constituted a significant part of the total risk of the financial system. It is also very similar to the ranking obtained using SRISK for the same periods.

Aggregate MES measure

Yun and Moon, in 2014, propose an overall systemic risk indicator called *Aggregate MES* for the banking system as a whole. It is interpreted as the MES of the returns of a portfolio consisting of individual banks' equities when the market returns fall below a certain threshold level, similar to the overall SRISK index. MES puts the distress of the market and CoVaR puts the distress of an individual financial institution. The authors estimate the daily MES and ΔCoVaR measures in the Korean banking sector. Although MES and CoVaR differ in defining systemic risk contributions, both are qualitatively very similar in explaining the cross-sectional differences in systemic risk contributions across banks. The systemic risk contributions are closely related to some bank characteristic variables. However, there are differences between the cross-sectional and the time series dimensions in the effects of these variables.

2.1.3 Alternative probability-distribution measures

In this subsection we review alternative systemic risk measures that are categorized into the probability-distribution group.

CoRisk measure

International Monetary Fund (IMF), in 2009, propose *Co-Risk* measure in order to capture non-linearities and take into account direct and indirect financial linkages between institutions. Hence, CoRisk examines the co-dependence between the Credit Default Swap (CDS) of various financial institutions, thus the CDS of firm i conditional on the CDS spread of the other j at α quantile is:

$$\mathrm{CoRisk}_{\alpha}^{i|j} = 100\left(\frac{\beta_{\alpha,0} + \sum_{k=1}^{K}\beta_{\alpha,k} Risk_k + \beta_{\alpha,j}\mathrm{CDS}_{\alpha}^{j}}{\mathrm{CDS}_{\alpha}^{i}} - 1\right)$$

where

- $\mathrm{CDS}_{\alpha}^{i}$ and $\mathrm{CDS}_{\alpha}^{j}$ are the CDS spreads of institutions i and j, respectively, corresponding to the α percentile of their empirical sample;

- $Risk_k$ indicates $k = 1, \dots, K$ common risk factors;
- the coefficients $\beta_{\alpha,0}$, $\beta_{\alpha,m}$ and $\beta_{\alpha,j}$ are the parameters estimated by the quantile regression:

$$\text{CDS}^i_\alpha = \beta_{\alpha,0} + \sum_{k=1}^{K} \beta_{\alpha,k} R_k + \beta_{\alpha,j} \text{CDS}^j_\alpha$$

Usually α is very high, such as 95%, since the interest is on CoRisk in distress periods. A high CoRisk indicates an increased sensitivity of the default risk of institution i to the default risk of the institution j.

CoRisk is more informative than unconditional systemic risk measures because it provides a market assessment of the proportional increase in a firm's credit risk induced, directly and indirectly, from its links to another firm.

DIP measure

Huang et al., in 2009, propose *Distress Insurance Premium (DIP)*, which represents an hypothetical insurance premium against catastrophic losses in a portfolio of financial institutions. This indicator is given by the risk-neutral expectation of losses conditional on exceeding a minimum loss threshold. According to Huang et al. (2009), DIP of bank i at time t is defined as "a theoretical premium to a risk-based deposit insurance scheme that guarantees against most severe losses for the banking system":

$$\text{PD}^i_t = \frac{a_t s^i_t}{a_t \text{LGD}^i_t + b_t s^i_t}$$

where s^i_t is the observed CDS spread, LGD^i_t is the loss given default, $a_t = \int_t^{t+T} e^{-r\tau} d\tau$, $b_t = \int_t^{t+T} \tau e^{-r\tau} d\tau$, and r is the risk-free rate.

The systemic importance of each bank (or bank group) can be properly defined as its marginal contribution, which is a function of its size, probability of default and asset correlation, to the hypothetical DIP of the whole banking system, that is to systemic risk. This systemic risk measure is based on overall sector losses conditional on the default of a particular financial institution. The two key default risk factors, that are the probability of default of individual banks and the asset return correlations among banks, are estimated from CDS spreads and equity price co-movements, respectively. A higher

DIP may be driven by both an increased probability of default of individual banks and a greater exposure to common risk factors. An advantage of this approach is that the marginal contribution of each bank adds up to the aggregate systemic risk.

Subsequently, Huang et al. (2012) apply DIP measure to firms with CDS and equity contracts, which are publicly tradable and do not rely on accounting or balance sheet information. They find that a bank's contribution to systemic risk is roughly linear in its default probability, but highly nonlinear with respect to institution size and asset correlation.

JPoD measure

Segoviano and Goodhart, in 2009, propose a set of joint distress indicators that are built upon the concept of Banking System Multivariate Density (BSMD) and are based on chain defaults of financial institutions. They exploit the information embedded in large international banks' credit spreads to construct a banking stability index and estimate cross-border interbank dependence for tail events using credit default swap data. Among the others, they propose:

- *Joint Probability of Distress (JPoD)*, which represents the probability of all banks in the system (portfolio) in distress at the same time, i.e. the tail risk of the system, and captures changes in the distress dependence among the banks, which increases in times of financial distress;

- *Banking Stability Index (BSI)*, which is based on the conditional expectation of default probability measure developed by Huang (1992) and reflects the expected number of banks in distress given that at least one bank has become distressed. A higher number signifies increased instability. This measure can also be interpreted as a relative measure of banking linkage.

According to Segoviano and Goodhart (2009), these indicators are able to "capture both linear and non-linear distress dependencies among the banks in the system, and its changes at different times of the economic cycle".

ΛVaR measure

In order to fix the gaps of VaR measure, in particular the lack of sensitivity or, in other terms, the slow adjustment of confidence levels during changes in the economic cycle, Frittelli et al., in 2014, propose *Lambda Value-at-Risk (Λ VaR)* measure, obtained by defining a new class of law invariant risk measures based on an appropriate family of acceptance sets. This VaR generalization takes into account not only the probability of the losses, but the balance between such probability and the amount of the loss. Given a monotone and right continuous function $\Lambda : \mathbb{R} \rightarrow [\lambda^m, \lambda^M]$ with $0 < \lambda^m \leq \lambda^M < 1$, ΛVaR of return X is a generalized quantile represented by the map $\Lambda\text{VaR} : P \rightarrow \mathbb{R}$ and is defined as:

$$\Lambda\text{VaR} = -\sup\{m \in \mathbb{R} | P(X \leq x) \leq \Lambda(x),\ \forall x \leq m\}$$

Here, the confidence level may not be constant and depends on the profit and loss of the risk factor, implying the idea that risk managers would need to reserve more capital in the case of expected greater losses and less capital in the case of expected smaller losses. In addition, the risk measure solves several VaR problems, including the lack of subadditivity and the inability to capture "tail risk", and satisfies important theoretical properties that formalize two fundamental principles: the monotonicity of the risk preferences and the fact that "diversification cannot increase the risk". Hitaj and Peri, in 2015, present the first empirical application of ΛVaR to equity markets and demonstrate the elicitability property under general conditions, guaranteeing proper backtesting and a statistically meaningful comparison with the VaR.

Correlation-based Measures

Patro et al. (2013) analyze the relevance and effectiveness of stock return correlations among financial institutions as an indicator of systemic risk, finding that daily stock return correlation is a simple, robust, forward-looking, and timely systemic risk indicator. They use this indicator with the purpose of monitoring systemic risk, analyzing the trends and fluctuations of daily stock return correlations and default correlations among 22 U.S. bank holding companies and investment banks. This indicator captures the trend as well as the fluctuations in the levels of systemic risk in the U.S. economy and it is not subject to the model specification errors and data limitations that other potential systemic risk measures may face.

In the literature, balance sheet based low-frequency indicators as well as market-based (market prices and rates) high-frequency indicators have been suggested. Rodríguez-Moreno and Peña (2013) focus on the later type of indicators, comparing two potential systemic risk's group detectors: aggregate market (macro) and individual institution (micro). They conclude that CDS based measures achieve better results than measures based on interbank rates or stock prices and are therefore the best indicators indicating an approaching crisis.

Tail Dependence-based Measures

Other researches focus on extreme dependence measures, believing that they are very valuable tools in systemic risk measurement. Understanding loss dependence in the joint extremes of the loss distributions, in fact, is crucial in understanding systemic risk because the prevalence of dependence in the extreme tails of loss distributions is indicative of high contagion potential between financial institutions. Among these authors, Jobst (2013) examined the multivariate tail dependence of the implied volatility of equity options as an early warning indicator of systemic risk within the financial sector, using non-parametric methods of estimating changes in the dependence structure.

Furthermore, Balla et al. (2014) propose two complementary systemic risk indicators derived from multivariate extreme value theory (EVT), which can capture the tail dependencies between stock returns of large U.S. depository institutions. They investigate these extreme loss tail dependencies and derive extreme dependence-based systemic risk indicators. The first indicator is the proportion of asymptotically dependent depository institution pairs to the total number of depository institution pairs in our sample, thus measuring the prevalence of asymptotic dependence between large US depository institutions. The second one is the average strength of asymptotic dependence across all pairs of depository institutions. These measures of tail co-movement can help to understand the vulnerability and the contagion potential of a financial institution during a financial crisis.

CISS indicator

Hollò et al., in 2012, propose the *Composite Indicator of Systemic Stress (CISS)*, a financial stress index (FSI) with the aim of measuring the current state of instability in the financial system as a whole or, equivalently, the level of systemic stress. It is interpreted as that amount of systemic risk, which has already materialized and is summarized in a single (usually continuous) statistic. Thus, it has to capture the level of stress in its economically most important, i.e. systemically most risky elements. CISS is built on the aggregation of five subindices, i.e. the equity market, the bond market, the money market, the foreign exchange market, and the financial intermediaries' sector. It is a defined as:

$$\text{CISS}_t = (w_t \circ s_t) C_t (w_t \circ s_t)$$

where s_t is the vector of subindices, w_t is the vector of weights for subindices, C_t is the time-varying cross-correlation matrix, and $\circ$ denotes the Hadamard-product (i.e. element by element multiplication).

While it would be unrealistic to expect that such a highly condensed composite index can sufficiently characterize something as complex as systemic risk (Billio et al., 2012), a comprehensive FSI permits the real time monitoring and assessment of the stress level in the whole financial system and helps to better delineate and describe historical crisis episodes. CISS is focus on the systemic dimension of financial stress, since its specific statistical design, which is shaped according to standard definitions of systemic risk, whereas it applies the basic portfolio theory to the aggregation of individual financial stress indicators into the composite indicator, taking into account the time-varying cross-correlations between the subindices. Hence, CISS monitors in real time the overall level of frictions and tensions in the financial system.

Hollò et al. (2012) apply CISS index to Eurozone data, whereas Louzis et al. (2012) adopt it for Greece. Also Milwood (2013) computes CISS to assess systemic risk for the financial markets in Jamaica, using the foreign exchange market, equity market, money market and bond market from January 2002 to June 2012. As well, Cabrera et al. (2014) apply the CISS for Colombian data during the period $2000 - 2014$, taking into account several dimensions related to financial markets.

2.1.4 Network Analysis measures

Some aforementioned risk measures estimate the magnitude of losses, that is what an institution would experience during a market crisis, and only capture systemic exposures to the degree historical data represent well systemic losses. However, during periods of rapid financial innovation, extreme losses in one financial sector need not coincide with simultaneous losses in another financial sector even though their connectedness implies higher systemic risk (Billio et al., 2012). Hence, as an alternative to systemic risk measures based on the marginal risk contributions of individual institutions, *network analysis* is concerned with the joint distribution of losses of all market participants. Network analysis is a useful approach to:

- quantifying the potential capital losses of a contagious event;
- identify financial interlinkages among institutions, in particular the most systemic institutions, which trigger the stronger domino effects in case of default, and the most vulnerable institutions, which are most harshly affected by the default of another institution.

Billio et al., in 2012, base their approach on graph theory measuring network connectedness grounds, and propose two econometric measures of systemic risk, namely *Principal Component Analysis* and *Granger-Causality Network*, which capture the interconnectedness among financial institutions, such as hedge funds, banks, broker-dealers, and insurance companies. In addition, these measures of connectedness complement SES and DIP measures by estimating directly the statistical connectedness between the asset returns of a financial institution's network.
The authors find that the correlation between two financial sectors rises during and after systemic shocks, whereas its role is minor during non-crisis periods. In particular, the banking and insurance sectors are more important sources of connectedness and systemic risk than other sectors. An increase in dynamic causality index (computed as the number of causal relationships in window divided by the total possible number of causal relationships) indicates a higher level of system interconnectedness. Their philosophy is that statistical relationships between returns can yield valuable indirect information

about the build-up of systemic risk.
Bianchi et al., in 2015, identify contagion as an increase in the strength of network connectedness and consider it as a central part for systemic risk measurement, since those dramatic shocks to a single institution can quickly affect others with different size and structure.

Principal Component Analysis

The first measure proposed by Billio et al. (2012) is based on *Principal Component Analysis (PCA)* measure, which gauges the degree of commonality among a vector of asset returns. The purpose is to identify common components among different sectors of financial markets. When the asset returns of a collection of entities are jointly driven by a small number of highly significant factors, fewer principal components are needed to explain the variation in the vector of returns. Hence, sharp increase in the proportion of variability, explained by the first n principal components, is a natural indication of systemic risk. The authors use PCA to decompose the covariance matrix of the four index returns and find that the first and second principal components capture the majority of return variation during the whole sample.

Granger-Causality Network

The second measure proposed by Billio et al. (2012) is *Granger-Causality Network* measure, an explicit measure of financial networks derived from graph theory. The purpose is to detect the direction of causality between pairs of financial sectors.
The authors use Granger-causality test statistics for asset returns to define the edges (relationships) of a network of financial institutions, i.e. to measure the correlation between sectors directly and unconditionally on extreme losses. Moreover, they show that Granger-causality networks are highly dynamic and become densely interconnected prior to systemic shocks.

2.2 Econometric models for Systemic Risk

The mostly used method to estimate these systemic risk measures is the DCC-GARCH-type model, a data-driven approach with the advantage in capturing the time-varying systemic risk exposure of an institution or the system (advantage not shared by the quantile regression method). When for example CoVaR is estimated using a GARCH model, in fact, the time-series relation between an institution's CoVaR and its VaR becomes time-varying due to the time-varying correlation (see Girardi and Ergun, 2013; Benoit et al., 2013). This feature is a precondition for the accurate depiction of systemic event (Louzis et al., 2012) and permits to capture and incorporate in the systemic risk measurement possible changes over time in the linkage between the institution and the system (Girardi and Ergun, 2013). The estimation of this methodology is performed in two steps. First the conditional volatilities are estimated by an univariate GARCH-type model (described in the next paragraphs 2.4 and 2.2.2). Then, the conditional correlations are estimated by a bivariate DCC model (described in paragraph 2.5). The basic idea behind this model is that the covariance matrix can be decomposed into conditional standard deviations and a correlation matrix. Both these matrices are designed to be time-varying.

We implement the DCC model because the DCC-GARCH-type methodology is the mostly used method to estimate these systemic risk measures in the existing literature. However, several choices are available from the wide literature about multivariate GARCH models to model multivariate conditional covariances. Among the others, other relevant contributions are the Constant Conditional Correlations (CCC) model (Bollerslev, 1990), the Vech model (Bollerslev et al., 1988), the Factor GARCH (Engle and Ng, 1993), the BEKK model (Engle and Kroner, 1995), the Generalized Orthogonal Factor GARCH (Lanne and Saikkonen, 2007), based on principal components have been suggested to solve the problem of estimation in presence of a great number of time series and to achieve computational feasibility. More recently, the Flexible Dynamic Conditional Correlations have been suggested by Billio et al. (2006) as an efficient generalization of the DCC model. This parsimonious model specification allows to use a large number of series without implying that the number of parameters becomes explosive and maintains the same GARCH dynamics of the DCC correlation structure, relaxing the constraint for which all the correlations have to follow the same pattern. A fairly comprehensive survey of the literature is provided in Bauwens et al. (2006). The DCC-GARCH model has clear computational advantages, among which the number of parameters to be estimated in the correlation process is independent of the number of the analyzed series. Thus, potentially very large correlation matrices can be estimated. However, compared for example to the CCC model, the advantage of simple estimation is lost, as the correlation matrix has to be inverted for each time, t, during every iteration.

Several empirical studies have been conducted on systemic risk using DCC in conjunction with different univariate GARCH-type models. Some examples which use the DCC-GARCH model are Girardi and Ergun (2013), Cabrera et al. (2014); while other examples which use the DCC-GJR model are Cao (2013), Popescu and Turcu (2014), Yun and Moon (2014), Brownlees and Engle, (2015) and Engle et al. (2015).

2.2.1 GARCH model

GARCH models (Bollerslev, 1986) are very popular in the literature for the analysis of the volatility of the financial returns and their application to study these financial phenomena is almost consolidate. They model in a parsimonious way the conditional heteroskedasticity. These models consider that volatility changes over time and include lagged values of the variance in its conditional equation, in order to allow a longer memory and more parsimony. Moreover, they are also consistent with the volatility clustering phenomenon.

Model specification

Given the univariate model for the returns at $t = 1, \dots, T$ with zero mean:

$$r_t = \epsilon_t, \qquad \epsilon_t = \sigma_t z_t \tag{2.3}$$

where ϵ_t is the error term defined as a sequence of independent and identically distributed (i.i.d.) random variables with zero mean and σ_t^2 variance, i.e. $\epsilon_t \sim iid(0, \sigma_t^2)$, and z_t is the standardized innovation $z_t \sim iid(0, 1)$, the GARCH(p, q) specification for the conditional variance is:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{2.4}$$

The most popular model in the empirical literature is the GARCH$(1,1)$:

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

The unconditional variance is:

$$\sigma^2 = \frac{\omega}{1 - \alpha - \beta}$$

The conditions to ensure that the non-negativity of the GARCH$(1,1)$ variance are:

$$\omega > 0, \qquad \alpha \geq 0, \qquad \beta \geq 0$$

and the additional condition to ensure that the conditional variance is strictly stationary is:

$$\alpha + \beta < 1$$

These models are estimated using the Quasi-Maximum Likelihood (QML) estimation procedure.

Forecasting

The k-step-ahead forecast of the conditional volatility, with $k > 2$, for the GARCH$(1,1)$ model is:

$$\sigma_{t+k|t}^2 = \sigma^2 + \big(\alpha + \beta\big)^{k-1} \big(\sigma_{t+1|t}^2 - \sigma^2\big)$$

where the one-step-ahead forecast is:

$$\sigma_{t+1|t}^2 = \omega + \alpha \epsilon_t^2 + \beta \sigma_t^2$$

The notation $\sigma_{t+k|t}^2$ indicates $\mathbb{E}_t\big(\sigma_{t+k}^2\big) \equiv \mathbb{E}\big(\sigma_{t+k}^2 | \mathcal{F}_t\big)$, where $\mathcal{F}_t$ is the information set available until time t.

2.2.2 GJR model

GARCH models have been plenty extended to capture even more stylized facts that they cannot capture. Financial time series, in fact, exhibit different effects, such as volatility clustering, leverage effect, weekend and seasonality effects, and so on.

Glosten et al., in 1993, develop the *GJR-GARCH (GJR)* model to capture exactly the leverage effect present in the financial data. The model, in fact, considers the effect of negative and positive shocks allowing the conditional variance to respond differently to the past negative and positive innovations. Given the same basic idea, the GJR model is closely related to the Threshold-GARCH (TGARCH) and the Asymmetric-GARCH (AGARCH).

Model specification

Considering the univariate model specified in the previous paragraph (2.4) by equation 2.3, the GJR(p, q) specification for the conditional variance is:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \left(\alpha_i \epsilon_{t-i}^2 + \gamma_i \epsilon_{t-i}^2 I_{(\epsilon_{t-i}<0)} \right) + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

where γ_i captures the leverage effect, that means the different impact of negative shocks on volatility than positive shocks, and $I_{(\epsilon_{t-i}<0)}$ denotes the indicator function:

$$I_{(\epsilon_{t-i}<0)} = \begin{cases} 1 & \text{if } \epsilon_{t-i} < 0 \\ 0 & \text{otherwise} \end{cases}$$

Then, the GJR$(1, 1)$ model is expressed as:

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \gamma \epsilon_{t-1}^2 I_{(\epsilon_{t-1}<0)} + \beta \sigma_{t-1}^2$$

The unconditional variance is:

$$\sigma^2 = \frac{\omega}{1 - \alpha - \frac{1}{2}\gamma - \beta}$$

The conditions to ensure that the non-negativity of the GJR$(1, 1)$ variance are:

$$\omega > 0, \qquad \alpha \geq 0, \qquad \beta \geq 0, \qquad \gamma \geq 0$$

and the additional condition to ensure that the conditional variance is strictly stationary is:

$$\alpha + \beta + \frac{1}{2}\gamma < 1$$

where the 1/2 multiplying γ comes from the assumption of symmetric conditional distribution for the returns z_t.

The GJR model, as GARCH, is estimated by Quasi-Maximum Likelihood (QML).

Forecasting

The k-step-ahead forecast of the conditional volatility, with $k > 2$, for the GJR$(1,1)$ model is:

$$\sigma^2_{t+k|t} = \omega + \sum_{i=0}^{k-1} \left(\alpha + \frac{\gamma}{2} + \beta\right)^i + \left(\alpha + \frac{\gamma}{2} + \beta\right)^{k-1} \sigma^2_{t+1|t}$$

where the one-step-ahead forecast is:

$$\sigma^2_{t+1|t} = \omega + \alpha\epsilon_t^2 + \gamma\epsilon_t^2 I_{(\epsilon_t<0)} + \beta\sigma_t^2$$

2.2.3 DCC model

One specification of Multivariate GARCH (MGARCH) models is the *Dynamic Conditional Correlation (DCC)* model, proposed by Engle (2002), which uses a nonlinear combination of univariate GARCH models with time-varying cross-equation weights to model the conditional covariance matrix. DCC model has the advantage to capture the time-varying systemic risk exposure of a financial institution in the market (see Girardi and Ergun, 2013; Benoit et al., 2013) and permits to incorporate in the systemic risk measurement possible changes over time in the linkage between the financial institution and the financial system (Girardi and Ergun, 2013). Moreover, modeling the change in time of co-movements between the financial institution and the system allows the systemic risk measure to gauge the contagion between the financial institution and the financial system (Cabrera et al., 2014).

Model specification

Let $\boldsymbol{r}_{si,t} = (r_t^s,\ r_t^i)'$ be the vector denoting the system-institution return pair at time $t = 1, \ldots, T$. The vector is composed by r_t^s, the return series of the system s, and r_t^i, the return series of the institution i, for $i = 1, \ldots, N$:

$$r_t^s = \epsilon_{s,t}, \qquad \epsilon_{s,t} = \sigma_{s,t} z_{s,t}$$

$$r_t^i = \epsilon_{i,t}, \qquad \epsilon_{i,t} = \sigma_{i,t} z_{i,t}$$

where $\epsilon_{s,t} \sim iid(0, \sigma_{s,t}^2)$ and $\epsilon_{i,t} \sim iid(0, \sigma_{i,t}^2)$. Their joint dynamics are given by:

$$\boldsymbol{r}_{si,t} = \boldsymbol{\epsilon}_{si,t}, \qquad \boldsymbol{\epsilon}_{si,t} = \boldsymbol{\Sigma}_{si,t}^{1/2} \boldsymbol{z}_{si,t}$$

where $\boldsymbol{\epsilon}_{si,t} = (\epsilon_{s,t}, \epsilon_{i,t})' \sim iid(\mathbf{0}, \boldsymbol{\Sigma}_{si,t})$ and $\boldsymbol{z}_{si,t} = (z_{s,t}, z_{i,t})' \sim iid(\mathbf{0}, \boldsymbol{I}_2)$, $\boldsymbol{I}_2$ is the two-by-two identity matrix. The time-varying covariance matrix is:

$$\boldsymbol{\Sigma}_{si,t} = \begin{bmatrix} \sigma_{s,t}^2 & \sigma_{s,t}\sigma_{i,t}\rho_{si,t} \\ \sigma_{s,t}\sigma_{i,t}\rho_{si,t} & \sigma_{i,t}^2 \end{bmatrix}$$

Let $\boldsymbol{C}_{si,t}$ be the conditional correlation matrix, whose elements are $\rho_{si,t}$, defined as:

$$\boldsymbol{\Sigma}_{si,t} = \boldsymbol{D}_{si,t}^{1/2} \boldsymbol{C}_{si,t} \boldsymbol{D}_{si,t}^{1/2} = \begin{bmatrix} \sigma_{s,t} & 0 \\ 0 & \sigma_{i,t} \end{bmatrix} \begin{bmatrix} 1 & \rho_{si,t} \\ \rho_{si,t} & 1 \end{bmatrix} \begin{bmatrix} \sigma_{s,t} & 0 \\ 0 & \sigma_{i,t} \end{bmatrix}$$

where $\boldsymbol{D}_{si,t}^{1/2}$ is the (2×2) diagonal matrix of conditional standard deviations. The conditional volatilities are estimated by a GARCH-type model (such as the GARCH model and GJR model outlined in the previous paragraphs 2.4, 2.2.2), then the conditional correlations are estimated by the following DCC model (Engle, 2002):

$$\boldsymbol{C}_{si,t} = \boldsymbol{Q}_{si,t}^{*-1}\, \boldsymbol{Q}_{si,t}\, \boldsymbol{Q}_{si,t}^{*-1}$$

where $\boldsymbol{Q}_{si,t}^{*} = \mathrm{diag}(\boldsymbol{Q}_{si,t})^{1/2}$ and $\boldsymbol{Q}_{si,t}$ is the pseudo correlation matrix:

$$\boldsymbol{Q}_{si,t} = \begin{bmatrix} q_{ss,t} & q_{si,t} \\ q_{si,t} & q_{ii,t} \end{bmatrix}$$

which follows a process of:

$$\boldsymbol{Q}_{si,t} = \big(1 - a - b\big)\bar{\boldsymbol{Q}}_{si} + a\big(\boldsymbol{\eta}_{si,t-1}\, \boldsymbol{\eta}'_{si,t-1}\big) + b\, \boldsymbol{Q}_{si,t-1}$$

where $\bar{\boldsymbol{Q}}_{si}$ is the unconditional correlation matrix of the standardized residuals:

$$\bar{\boldsymbol{Q}}_{si} = \mathbb{E}_{t-1}\big(\boldsymbol{\eta}_{si,t-1}\ \boldsymbol{\eta}'_{si,t-1}\big)$$

which can be estimated as:

$$\bar{\boldsymbol{Q}}_{si} = \frac{1}{T}\sum_{t=1}^{T}\boldsymbol{\eta}_{si,t-1}\ \boldsymbol{\eta}'_{si,t-1}$$

where the standardized residuals are $\boldsymbol{\eta}_{si,t} = \boldsymbol{D}_{si,t}^{-1/2}\boldsymbol{\epsilon}_{si,t}$ and, in this case, they are equal to the volatility adjusted returns $\eta_{s,t} = r_t^s/\sigma_{s,t}$ and $\eta_{i,t} = r_t^i/\sigma_{i,t}$. This formulation is the DCC$(1,1)$, but generally the DCC(M,N) is defined as:

$$\boldsymbol{Q}_{si,t} = \Big(1 - \sum_{m=1}^{M} a_m - \sum_{n=1}^{N} b_n\Big)\bar{\boldsymbol{Q}}_{si} + \sum_{m=1}^{M} a_m\big(\boldsymbol{\eta}_{si,t-m}\ \boldsymbol{\eta}'_{si,t-m}\big) + \sum_{n=1}^{N} b_n\ \boldsymbol{Q}_{si,t-n} \tag{2.5}$$

The matrix $\boldsymbol{\Sigma}_{si,t}$ is a covariance matrix, as a consequence has to be positive definite. The diagonal matrix $\boldsymbol{D}_{si,t}^{1/2}$ is positive definite since all the diagonal elements are positive, hence $\boldsymbol{\Sigma}_{si,t}$ is positive definite if the conditional correlation matrix $\boldsymbol{C}_{si,t}$ is positive definite. Then, the pseudo correlation matrix $\boldsymbol{Q}_{si,t}$ has to be positive definite to ensure that $\boldsymbol{C}_{si,t}$ is positive definite. In particular, the conditions to guarantee this requirement are:

$$a \geq 0, \qquad b \geq 0, \qquad a + b < 1$$

Moreover, all the elements in the $\boldsymbol{C}_{si,t}$ have to be equal to or less than one by definition. For this purpose, $\boldsymbol{Q}^*_{si,t}$ rescales the elements in $\boldsymbol{Q}_{si,t}$ to ensure that:

$$|\rho_{si,t}| = \left|\frac{q_{si,t}}{\sqrt{q_{ii,t}q_{ss,t}}}\right| \leq 1$$

Hence, Engle and Sheppard (2001) describe a set of sufficient, not necessary, conditions for $\boldsymbol{\Sigma}_{si,t}$ to be uniformly positive definite in the following proposition.

Proposition (Positive Definiteness of DCC) *If the following univariate GARCH parameter restrictions (Equation* 2.4*) are satisfied for both system and institution composing the return pair* $w \in \{s, i\}$*:*

a. $\omega_w > 0$

b. α_{wi} $\forall i \in [1, ..., p_w]$ *and* β_{wj} $\forall j \in [1, ..., q_w]$ *are such that* σ^2_{wt} *will be positive with probability one*

c. $\sigma^2_{w0} > 0$

d. *the roots of* $\left(1 - \sum_{i=1}^{p_w} \alpha_{wi} Z^i + \sum_{j=1}^{q_w} \beta_{wj} Z^q\right)$ *lie outside the unit circle*

and the DCC parameters satisfy (Equation 2.5*):*

e. $a_m \geq 0 \quad \forall m \in [1, ..., M]$

f. $b_n \geq 0 \quad \forall n \in [1, ..., N]$

g. $\sum_{m=i}^{M} a_m + \sum_{n=1}^{N} b_n < 1$

h. *the minimum eigenvalue of* $\bar{\boldsymbol{Q}}_{si} > 0$

then $\boldsymbol{\Sigma}_{si,t}$ *will be positive definite for all* t*.*

The model is typically estimated by a two step Quasi-Maximum Likelihood (QML) estimation procedure. A common estimation problem is that when the number of parameters to estimate is large, the likelihood function becomes flat and more probably a local optimum is reached. Hence, the choice of start values is important and it is possible to make a grid of the possible values the parameters may take, and choose the starting values to be the combination of values that yields the highest likelihood.

For empirical simplicity, we consider the following Cholesky decomposition:

$$\boldsymbol{\Sigma}^{1/2}_{si,t} = \begin{bmatrix} \sigma_{s,t} & 0 \\ \sigma_{i,t}\rho_{si,t} & \sigma_{i,t}\sqrt{1 - \rho^2_{si,t}} \end{bmatrix}$$

so the returns can be expressed as:

$$\begin{aligned} r^s_t &= \sigma_{s,t} z_{s,t} \\ r^i_t &= \sigma_{i,t}\rho_{si,t} z_{s,t} + \sigma_{i,t}\sqrt{1 - \rho^2_{si,t}}\; z_{i,t} \end{aligned} \tag{2.6}$$

where $z_{s,t}$ and $z_{i,t}$ are assumed to be independent.

Forecasting

The k-step-ahead prediction of the conditional covariance matrix:

$$\boldsymbol{\Sigma}_{si,t+k|t} = \boldsymbol{D}^{1/2}_{si,t+k|t}\boldsymbol{C}_{si,t+k|t}\boldsymbol{D}^{1/2}_{si,t+k|t}$$

may be solved by forecasting $\boldsymbol{D}^{1/2}_{si,t+k|t}$ and $\boldsymbol{C}_{si,t+k|t}$ separately.

The predictions of the univariate variances, contained in $\boldsymbol{D}^{1/2}_{si,t+k|t}$, can be calculated individually, for the system s and the financial institution i, using the appropriate forecasting formula related to the GARCH-type model used. The k-step-ahead forecast, with $k > 0$, is:

$$\boldsymbol{D}^{1/2}_{si,t+k|t} = \mathrm{diag}\Big(\sigma_{s,t+k|t},\ \sigma_{i,t+k|t}\Big)$$

The prediction of the conditional correlation matrix $\boldsymbol{C}_{si,t+k|t}$ is obtained by the prediction of the pseudo correlation matrix $\boldsymbol{Q}_{si,t+k|t}$. The k-step-ahead forecast, with $k > 1$, for the DCC$(1, 1)$ model is:

$$\boldsymbol{Q}_{si,t+k|t} = \big(1 - a - b\big)\bar{\boldsymbol{Q}}_{si} + a\ \mathbb{E}_t\big[\boldsymbol{\eta}_{si,t+k-1}\ \boldsymbol{\eta}'_{si,t+k-1}\big] + b\ \boldsymbol{Q}_{t+k-1|t}$$

where:

$$\mathbb{E}_t\big[\boldsymbol{\eta}_{si,t+k-1}\ \boldsymbol{\eta}'_{si,t+k-1}\big] = \boldsymbol{R}_{t+k-1|t} = \mathbb{E}_t\big[\boldsymbol{Q}^{*-1}_{si,t+k-1}\ \boldsymbol{Q}_{si,t+k-1}\ \boldsymbol{Q}^{*-1}_{si,t+k-1}\big]$$

The one-step-ahead forecast is:

$$\boldsymbol{Q}_{si,t+1|t} = \big(1 - a - b\big)\bar{\boldsymbol{Q}}_{si} + a\ \boldsymbol{\eta}_{si,t}\ \boldsymbol{\eta}'_{si,t} + b\ \boldsymbol{Q}_t$$

Since the expectation $\mathbb{E}_t\big[\boldsymbol{Q}^{*-1}_{si,t+k-1}\ \ \boldsymbol{Q}_{si,t+k-1}\ \ \boldsymbol{Q}^{*-1}_{si,t+k-1}\big]$ is unknown, it is necessary to approximate it to obtain the k-step ahead forecast. There exists two methods:

1. assumes that $\mathbb{E}_t\big[\boldsymbol{\eta}_{si,t+i}\ \boldsymbol{\eta}'_{si,t+i}\big] \approx \boldsymbol{Q}_{si,t+i|t}$ for $i = 1, \ldots, k$
2. assumes that $\bar{\boldsymbol{C}} \approx \bar{\boldsymbol{Q}}_{si}$ and $\boldsymbol{C}_{t+i|t} \approx \boldsymbol{Q}_{t+i|t}$ for $i = 1, \ldots, k$

Finally, the prediction of the conditional correlation matrix $\boldsymbol{C}_{si,t+k|t}$, for $k > 0$, is:

$$\boldsymbol{C}_{si,t+k|t} = \boldsymbol{Q}^{*-1}_{si,t+k|t}\ \boldsymbol{Q}_{si,t+k|t}\ \boldsymbol{Q}^{*-1}_{si,t+k|t}$$

An empirical study by Engle and Sheppard (2001) shows that the second method has better bias properties for almost all correlation matrices.

2.3 Concluding remarks

We have reviewed the main systemic risk measures developed in the literature. They are divided into 4 different groups according to their structure, and we focused on probability-distribution measures group. Among them, we have deeply analyzed two of the most widespread systemic risk measures, namely ΔCoVaR and SRISK, and their main components, namely CoVaR measure and MES measure, given their easy applicability and their large diffusion in the literature, continuously inspiring extensions and other developments. Then, we have introduced the widespread econometric modeling particularly employed to estimate these systemic risk measures, focusing on DCC-GARCH-type methodology for preparatory purposes. Our approach, to estimate systemic risk using CoVaR and MES measures, is in fact based on this methodology and is applied to top US financial institutions in Chapter 4.

Chapter 3

Systemic Risk evaluation

This chapter faces the problem to evaluate systemic risk accuracy. Several systemic risk measures, in fact, have been developed in the literature, without a deep analysis on their accuracy. Thus, we briefly explain the aspect of validating systemic risk measures, then we review, in Section 3.1, the existing backtests used to evaluate measures adequacy, and finally we revise, in Section 3.2, loss functions used to evaluate measures accuracy, with the purpose to detect those tools more suitable for CoVaR and MES framework, which are conditional quantile and conditional tail expectation, respectively. Then, we propose, in Section 3.3, appropriate loss functions to particularly evaluate these measures.

Validation of Systemic Risk measures

An important and necessary requirement when a new systemic risk measure is presented in the literature is its validation, as its reliability and accuracy that depend on its ability to predict and cover future unexpected losses. The main attempts to contrast existing measures follow two approaches: an ex ante comparison of risk measures based on their mathematical properties, and an ex post approach based on their empirical performance.

According to the former approach, only few papers analyze the desirable properties that a sound systemic risk measure should comply with. Following the coherent risk approach of Artzner et al. (1999), Chen et al., in 2013, define an axiomatic framework for systemic risk measures. Their analysis is based on the joint distribution of outcomes across all financial firms and all states of nature. In this framework, a systemic risk measure is a function from the space of firms and outcomes to $\mathbb{R}$ and must satisfy the main conditions that define any coherent risk measure, namely the monotonicity, positive homogeneity, and outcome convexity axioms. In addition, it should verify the additional preference consistency axiom that states that the risk measure has to reflect the preference of the regulator on the cross-sectional profile of losses across firms and the distribution of the aggregate outcomes across states. Brunnermeier et al., in 2013, state that these measures also have to satisfy the clone property and should allow regulators to impose a firm-specific capital surcharge in an economically consistent way. Gouriéroux and Monfort, in 2013, propose a set of axioms (decentralization, additivity, and risk ordering) for dividing an aggregate systemic risk measure into individual contributions. These different approaches assume that the risk measure is correctly estimated. Given the variety of risk measures that have been proposed and data limitations, any single systemic risk measure will necessarily be fraught with uncertainty. Hansen (2013) discusses the challenge of measuring this uncertainty and designing regulatory approaches that are robust to this problem.

According to the latter approach, instead, empirical validation is a key requirement for any systemic risk measure to become an industry standard. There is no consensus at the moment on which is the best way to check it, and several approaches have been followed in the academic literature, among the others backtesting. According to the Basel Committee on Banking Supervision (1996), backtesting, that is the statistical procedure of comparing realized profits and losses with forecast risk measures, is essential in the validation process of risk management internal models (see Jorion, 2007). According to Jorion (2007), in fact, backtesting is the main way to assess the accuracy of forecasts, hence systemic risk measures should be backtested with appropriate methods.

3.1 Related literature on backtest

The accuracy and efficiency tests have been primarily developed in the literature to evaluate VaR models (see Louzis et al., 2014; Hitaj and Peri, 2015). The assessment of the performance of the VaR forecasts, in fact, is carried out with a two-step procedure (Brownlees and Gallo, 2008). The first step assesses the adequacy of the VaR forecasting methods using these tests; the second step assesses the accuracy of those forecasting methods found not rejected by the VaR specification tests using a VaR loss function, the tick loss function (see the next Section 3.2) measuring the closeness of the VaR to their nominal coverage. The most common accuracy tests are Kupiec's *unconditional coverage test* and Christoffersen's *conditional coverage test*, where the null hypothesis is rejected if the VaR model considered generates too many or too few or too clustered exceptions.

CoVaR is a VaR-based measure, hence it is possible to adapt these tests used for VaR framework, to CoVaR framework using particular arrangements and changes. Therefore, we provide explanations of these tests in CoVaR framework.

Let I^i_{t+1} be the hit sequence at time $t+1$ based on VaR violations of institution i for $t \in \mathcal{T}$, where $\mathcal{T}$ is the time set $\mathcal{T} = 1, \dots, T$:

$$I^i_{t+1} = \begin{cases} 1 & \text{if } r^i_{t+1} \le \mathrm{VaR}^i_{\alpha,t+1} \\ 0 & \text{otherwise} \end{cases}$$

A second hit sequence $I^{s|i}_{t+1}$, based on CoVaR violations of system s, is constructed considering only the time periods when the VaR violations of institution i occur (i.e. when the previous hit sequence I^i_{t+1} gives 1 as result), that means considering only the observations when the institution i is in financial distress. In particular, we denote with $\mathcal{T}_{\mathrm{VaR}}$ this particular time set, that $\mathcal{T}_{\mathrm{VaR}} \subset \mathcal{T}$, and with $N_{\mathrm{VaR}} = \#\big(\mathcal{T}_{\mathrm{VaR}}\big)$ its sample size. So, the second hit sequence for $t \in \mathcal{T}_{\mathrm{VaR}}$ is defined as:

$$I^{s|i}_{t+1} = \begin{cases} 1 & \text{if } r^s_{t+1} \le \mathrm{CoVaR}^{s|i}_{\alpha,t+1} \\ 0 & \text{otherwise} \end{cases} \tag{3.1}$$

According to Christoffersen (1998), the problem of determining the adequacy of CoVaR can be reduced to the problem of determining whether the resulting second hit sequence satisfies two properties using the two following tests:

- the *unconditional coverage test*, which examines whether the number of exceptions over a specific number of observations in the backtesting window is consistent with the confidence level. It is based on the balance of two types of errors: type I error to reject a correct model and type II error to accept an incorrect model;

- the *independence test*, which examines whether the probability of an exception on any day depends on the outcome of the previous day.

Only a hit sequence that satisfies both these properties can be described as evidence of an adequate CoVaR model. These two properties can be combined into a single statement:

$$I_{t+1}^{s|i} \sim iid\ B(\alpha)$$

where this hit sequence is identically and independent distributed as a Bernoulli random variable with probability α. Furthermore, Christoffersen, in 1998, proposes the *correct conditional coverage test* to jointly examine these two properties, i.e. whether the reason for not passing the test is caused by inaccurate coverage, clustered exceptions, or even both.

3.1.1 Unconditional coverage test

The *Unconditional Coverage (UC) test* examines whether or not the CoVaR measure is violated more or less that $\alpha \times 100\%$ of the time. The CoVaR measure satisfies the unconditional coverage property if $\Pr\left[I_{t+1}^{s|i} = 1\right] = \alpha$ for $t \in \mathcal{T}_{\text{VaR}}$. The hypotheses to test for this property is:

$$\mathrm{H}_0 : \ \mathbb{E}\left[I_{t+1}^{s|i}\right] = \alpha$$

Kupiec, in 1995, proposes the proportion of failures test using the likelihood ratio statistic:

$$LR_{uc} = -2\log\left[\left(\frac{1-\alpha}{1-\hat{\alpha}}\right)^{N_{\text{VaR}}-I(\alpha)}\left(\frac{\alpha}{\hat{\alpha}}\right)^{I(\alpha)}\right] \sim \chi_1^2$$

where N_{VaR} is the time set sample size, $\hat{\alpha} = \dfrac{1}{N_{\text{VaR}}} I(\alpha)$ and $I(\alpha) = \sum_{t=1}^{N_{\text{VaR}}} I_{t+1}^{s|i}$.

An *alternative test* can be employed to assess this property using the sample average of the number of CoVaR violations over a time period, $\hat{\alpha}$. Then, a scaled version of $\hat{\alpha}$:

$$z = \frac{\sqrt{N_{\text{VaR}}}\ \left(\hat{\alpha} - \alpha\right)}{\sqrt{\alpha\left(1-\alpha\right)}}$$

has an approximate standard normal distribution. This statistic is the Wald variant of the Kupiec's statistic and its exact finite distribution is known. Moreover, the Wald test is well-defined in the case that no the CoVaR violations occur.

3.1.2 Independence test

The *Markov test*, proposed by Christoffersen (1998), examines whether or not the likelihood of a CoVaR violation depends on whether or not a CoVaR violation occurred on the previous day. It is carried out by creating a 2×2 contingency table that records the CoVaR violations of system on adjacent days for $t \in \mathcal{T}_{\text{VaR}}$:

	$I_{t+1}^{s\|i} = 0$	$I_{t+1}^{s\|i} = 1$	
$I_t^{s\|i} = 0$	N_{00}	N_{01}	$N_{00} + N_{01} = N_0$
$I_t^{s\|i} = 1$	N_{10}	N_{11}	$N_{10} + N_{11} = N_1$
	$N_{00} + N_{10}$	$N_{01} + N_{11}$	N_{VaR}

where N_{uv} is the number of observations with value u on day t followed by v on day $t+1$. If the CoVaR measure is adequate, then the proportion of violations on the next period, after a violation on today $I_t^{s|i} = 1$, should be the same as the proportion of violations on the next period after a non-violation on today $I_t^{s|i} = 0$. Hence, assuming that the hit sequence $I_{t+1}^{s|i}$ follows a first-order Markov sequence, the transition probability matrix is:

$$P = \begin{bmatrix} 1 - \alpha_{01} & \alpha_{01} \\ 1 - \alpha_{11} & \alpha_{11} \end{bmatrix}$$

where $\alpha_{01} = \Pr\left[I_{t+1} = 1 \middle| I_t = 0\right] = \dfrac{N_{01}}{N_0}$ and $\alpha_{11} = \Pr\left[I_{t+1} = 1 \middle| I_t = 1\right] = \dfrac{N_{11}}{N_1}$ are the transition probabilities. The CoVaR measure satisfies the independence property if $\Pr\left[I_{t+1}^{s|i} = 1\right] = \alpha$ holds. If the hit sequence $I_{t+1}^{s|i}$ satisfies the conditional coverage property, the probability of a violation next period does not depend on the last period being a violation or not. The hypothesis to test for this property is:

$$\text{H}_0 : \mathbb{E}\left[I_{t+1}^{s|i}\right] = \alpha_{01} = \alpha_{11}$$

Christoffersen, in 1998, proposes the test using the likelihood ratio statistic:

$$LR_{ind} = -2\log\left[\left(\frac{1-\hat{\alpha}}{1-\hat{\alpha}_{01}}\right)^{N_0 - N_{01}} \left(\frac{1-\hat{\alpha}}{1-\hat{\alpha}_{11}}\right)^{N_1 - N_{11}} \left(\frac{\hat{\alpha}}{\hat{\alpha}_{01}}\right)^{N_{01}} \left(\frac{\hat{\alpha}}{\hat{\alpha}_{11}}\right)^{N_{11}}\right] \sim \chi_1^2$$

where $\hat{\alpha} = \dfrac{1}{N_{\text{VaR}}} I(\alpha)$ and $I(\alpha) = \displaystyle\sum_{t=1}^{N_{\text{VaR}}} I_{t+1}^{s|i}$.

Christoffersen and Pelletier, in 2004, suggest an alternative test to investigate the independence property, called *duration test*, showing that it has more power than the Markov test to detect a VaR measure that violates the independence property. The Markov first-order sequence, in fact, may have limited power against general form of clustered violations, a signal of risk model misspecification. If VaR violations are completely independent from each other, then the amount of time that elapses between VaR violations should be independent of the amount of time that has elapsed since the last violation. Carrying out the test requires estimating a statistical model for the duration of time between VaR violations by the method of maximum likelihood which must be done using numerical methods.

3.1.3 Correct conditional coverage test

Christoffersen, 1998, proposes also the *correct conditional coverage test*, the combination of the unconditional coverage test and the independence test, for the null hypothesis $\alpha_{01} = \alpha_{11} = \alpha$:

$$LR_{cc} = LR_{uc} + LR_{ind} = -2 \log \left[\left(\frac{1-\alpha}{1-\hat{\alpha}_{01}} \right)^{N_{00}} \left(\frac{1-\alpha}{\hat{\alpha}_{01}} \right)^{N_{01}} \left(\frac{\alpha}{1-\hat{\alpha}_{11}} \right)^{N_{10}} \left(\frac{\alpha}{\hat{\alpha}_{11}} \right)^{N_{11}} \right] \sim \chi^2_2$$

This test examines whether there is any difference in the likelihood of a CoVaR violation following a previous CoVaR violation or non-violation and simultaneously determines whether each of these proportions is significantly different from α.

3.2 Related literature on test and loss function

The information contained in the CoVaR hit sequence, presented for backtesting in Section 3.1, refers only to whether or not an exceedance occurred, and does not provide the magnitude of the exceedance. Moreover, these tests have low power, as pointed out by Kupiec (1995); Berkowitz (2001); Escanciano and Pei (2012). As a consequence, loss functions may be extremely useful for determining whether a model provides a better risk assessment than another competing model, and may be more suited to discriminating among different competing models and judging the accuracy of a single model.

In this situation, loss functions could be tailored to address specific concerns about the evaluation of accuracy of systemic risk forecasts. As explained at the beginning of the previous section, in fact, the assessment of the VaR forecasting methods adequacy is not sufficient to assess the VaR forecasts performance. Statistical adequacy is, in fact, a necessary requirement that VaR forecasts must satisfy, but it does not provide information as to the accuracy of such predictions and it does not always help to discriminate among different VaR forecasting methods. Hence, it is necessary to assess the accuracy of the forecasting methods using a VaR loss function (Brownlees and Gallo, 2008). Since CoVaR is a VaR-based measure, the VaR loss functions used in the literature can be adapted to CoVaR framework.

3.2.1 Conditional volatility evaluation

The DCC-GARCH-type methodology requires as first step the estimation of conditional volatilities by univariate GARCH-type models. For this reason, it is important to evaluate the volatility estimates and forecasts. Although several loss functions have been proposed in the literature, Patton in 2011 derives a class of loss functions, for the ranking of competing volatility estimates or forecasts, to be robust to the presence of noise in the volatility proxy and finds some useful special cases within this class. The use of a conditionally unbiased, but imperfect, volatility proxy can lead, in fact, to undesirable outcomes in standard methods for comparing conditional variance estimates or forecasts. In particular, they derive the following loss functions, which are robust. This class contains two of the loss functions traditionally used in the related literature, which are the following:

- the *Mean Squared Error (MSE)* loss function:

$$Loss(\tilde{\sigma}_t^2, \hat{\sigma}_t^2) = (\tilde{\sigma}_t^2 - \hat{\sigma}_t^2)^2$$

- the *QLIKE (QL)* loss function:

$$Loss(\tilde{\sigma}_t^2, \hat{\sigma}_t^2) = \log(\hat{\sigma}_t^2) + \frac{\tilde{\sigma}_t^2}{\hat{\sigma}_t^2}$$

where $\tilde{\sigma}_t^2$ is an unbiased proxy of the conditional variance and $\hat{\sigma}_t^2$ is the estimated volatility. Finally, the ability of different competing model is evaluated by computing the average sample loss:

$$L = \frac{1}{T}\sum_{t=1}^{T} Loss(\tilde{\sigma}_t^2, \hat{\sigma}_t^2)$$

where T is the sample size. The best performance corresponds to the model which provides the lower average sample loss among the competing models.

This class of robust loss functions ensures that as the sample size is large, the ranking provided by $L = \frac{1}{T}\sum_{t=1}^{T} Loss(\tilde{\sigma}_t^2, \hat{\sigma}_t^2)$ and $L = \frac{1}{T}\sum_{t=1}^{T} Loss(\sigma_t^2, \sigma_t^2)$ coincides. Among these loss functions, Patton and Sheppard (2009) recommend using QL because it yields the greatest power. In addition, QL is a function of the multiplicative errors, while MSE is a function of the additive ones.

3.2.2 Tick loss function

Komunjer, in 2005, develops an approach to conditional quantile estimation based on a quasi-maximum likelihood. In particular, given r_t the returns at time $t \in \mathcal{T}$, the author proposes the following *Tick Loss (TL) function* at level α, also known in literature as the asymmetrical slope or check function:

$$TL_\alpha = \left(\alpha - I_{(r_t \leq \eta_t)}\right)\left(r_t - \eta_t\right) \tag{3.2}$$

where $\eta_t \in M_t$, $M_t \subset \mathbb{R}$ and $I_{(r_t \leq \eta_t)}$ is the indicator functions sequence:

$$I_{(r_t \leq \eta_t)} = \begin{cases} 1 & \text{if } r_t \leq \eta_t \\ 0 & \text{otherwise} \end{cases}$$

This loss function, with $\eta_t = \text{VaR}_{\alpha,t}$, is widely used in literature to evaluate the accuracy of the VaR forecasting methods (among the others Brownlees and Gallo, 2008; Escanciano and Pei, 2012; Huang and Lee, 2013; Fuertes and Olmo, 2016). In fact, $\text{VaR}_{\alpha,t}$ is the optimal predictor of the tick loss function since it minimizes the expected value of this function:

$$\text{VaR}_{\alpha,t} = \underset{\eta_t \in M_{t-1}}{\text{argmin}}\ \mathbb{E}_{t-1}\left[\left(\alpha - I_{(r_t \leq \eta_t)}\right)\left(r_t - \eta_t\right)\Big|\mathcal{F}_{t-1}\right]$$

where $M_{t-1} \subset \mathbb{R}$, $\mathcal{F}_{t-1}$ is the information set available until time $t-1$.

Comparing the different average sample tick losses obtained by several models for the same risk measure, the best and most accurate model corresponds to the smallest average sample tick loss reached.

3.2.3 Dynamic quantile test

Engle and Manganelli, in 2004, propose a different approach to quantile estimation, modeling directly the quantile, called conditional autoregressive value at risk (CAViaR) model, whose parameters are estimated by quantile regression (Koenker and Bassett, 1978).

Let $\{r_t\}_{t=1}^{T}$ be a vector of observable portfolio returns, $\boldsymbol{x}_t$ a vector of time t observable variables, and let $f_t(\boldsymbol{\beta}) \equiv f_t(\boldsymbol{x}_{t-1}, \boldsymbol{\beta}_\alpha)$ be α-quantile at time t of the distribution of returns r_t formed at time $t-1$, where $\boldsymbol{\beta}_\alpha$ is the p-dimensional vector of the unknown parameters and α is the confidence level. It should be $f_t(\boldsymbol{\beta}) = \mathrm{VaR}_{\alpha,t}$. Hence, the proposed CAViaR model has the following specification:

$$f_t(\boldsymbol{\beta}) = \beta_0 + \sum_{i=1}^{k} \beta_i \; f_{t-i}(\boldsymbol{\beta}) + \sum_{j=1}^{r} \beta_j \; l(\boldsymbol{x}_{t-j})$$

where $p = 1 + k + r$ is the dimension of $\boldsymbol{\beta}$ and l is a function of a finite number of lagged values of observables.

Denoting $\boldsymbol{\beta}^0$ as the vector of the true unknown parameters, the indicator functions sequence $\{I_{(r_t < f_t(\boldsymbol{\beta}^0))}\}_{t=1}^{T}$ is the following:

$$I_{(r_t \leq f_t(\boldsymbol{\beta}^0))} = \begin{cases} 1 & \text{if } r_t \leq f_t(\boldsymbol{\beta}^0) \\ 0 & \text{otherwise} \end{cases}$$

To test the validity of the forecasting model, it is straightforward to check whether the above sequence, $\{I_{(r_t < f_t(\boldsymbol{\beta}^0))}\}_{t=1}^{T}$, is iid (see Christoffersen, 1998). If VaR forecasts are adequate, in fact, the indicators functions sequence should behave as an iid sequence of Bernoulli random variables with probability α. As a solution, Engle and Manganelli (2004) propose the *Dynamic Quantile (DQ) test*, defining:

$$Hit_t(\boldsymbol{\beta}^0) \equiv I_{(r_t \leq f_t(\boldsymbol{\beta}^0))} - \alpha$$

The expected value of $Hit_t(\boldsymbol{\beta}_\alpha^0)$ is 0 and its conditional expectation, given any information known at $t-1$, must also be 0. The authors aim to check whether the Least Square estimator of the regression $\boldsymbol{Hit}(\hat{\boldsymbol{\beta}})$ on $\boldsymbol{X}'(\hat{\boldsymbol{\beta}})$, i.e. the following test statistic:

$$\hat{\boldsymbol{\beta}}_{LS} = T^{-1/2}\boldsymbol{X}'(\hat{\boldsymbol{\beta}})\boldsymbol{Hit}(\hat{\boldsymbol{\beta}})$$

is significantly different from 0, where

- $\hat{\boldsymbol{\beta}}$ is the vector of the estimated unknown parameters by minimizing the tick loss function outlined in the previous paragraph in formula 3.2, also known as the quantile regression loss function:

$$\min_{\boldsymbol{\beta}} \frac{1}{T}\sum_{t=1}^{T}\left(\alpha - I_{(r_t \leq f_t(\boldsymbol{\beta}))}\right)\left(r_t - f_t(\boldsymbol{\beta})\right)$$

- $\boldsymbol{X}(\hat{\boldsymbol{\beta}})$ contains $m < k$ lagged $Hit_{t-i}(\hat{\boldsymbol{\beta}}),\ i = 1, \ldots, m$
- $\boldsymbol{Hit}(\hat{\boldsymbol{\beta}}) \equiv [Hit_1(\hat{\boldsymbol{\beta}}), \ldots, Hit_T(\hat{\boldsymbol{\beta}})]'$

Under the null hypothesis $\mathrm{H}_0 :\ \boldsymbol{\beta} = 0$, the DQ test statistic is:

$$DQ = \frac{\hat{\boldsymbol{\beta}}_{LS}'\boldsymbol{X}'\boldsymbol{X}\hat{\boldsymbol{\beta}}_{LS}}{\alpha(1-\alpha)} \sim \chi_k^2$$

If $\boldsymbol{Hit}(\hat{\boldsymbol{\beta}})$ is an iid process, it should not be possible to predict future failure based on past information, thus we are looking for accepting the null hypothesis H_0. This test is based on VaR measure, hence we can use it in CoVaR framework, since is a VaR-based measure, in particular with the second hit sequence defined in formula 3.1.

3.2.4 Magnitude loss function

The information contained in the hit sequence, presented in Section 3.1, refers only to whether or not an exceedance occurred and doesn't provide the magnitude of the exceedance. Loss function based backtests may be extremely useful for determining whether one VaR model provides a better risk assessment than another competing VaR model and may be more suited to discriminating among competing VaR models rather than judging the accuracy of a single model. Therefore, Lopez, in 1999, proposes a backtest based on *magnitude loss function*, which measures the difference between the observed loss and the VaR in cases where the loss exceeds the VaR. The accuracy of VaR estimates is gauged by how well they minimize the loss function.

The magnitude loss function assigns a quadratic numerical score when a VaR estimate is exceeded by the observed loss and is defined as:

$$L(\mathrm{VaR}_\alpha, r_{t+1}) = \begin{cases} 1 + (r_{t+1} - \mathrm{VaR}_\alpha)^2 & \text{if } r_{t+1} \leq -\mathrm{VaR}_\alpha \\ 0 & \text{otherwise} \end{cases}$$

This loss function measures how well VaR model predicts losses when they occur and assigns to VaR estimates a numerical score that reflects the magnitude of the exceptions. The corresponding backtest is based on the sample average loss:

$$\hat{L} = \frac{1}{T} \sum_{t=1}^{T} L(\mathrm{VaR}_\alpha, r_{t+1})$$

To determine the range of values for average loss that are consistent with an accurate VaR model, a suitable model of the distribution of returns is determined to generate a history of returns and corresponding VaR and to construct a value for the average loss $\hat{L}$. Then, this process is repeated for a very large number of trials and the resulting average losses, i.e. the quantiles of the empirical distribution of the simulated average losses, are used as estimate of the distribution of the average loss.

3.3 A new approach to evaluate tail forecasts

After revising the existing main backtests and loss functions, we identify the loss functions more suitable for assessing CoVaR and MES accuracy and we modify and adapt them to their frameworks to ensure their usage.

3.3.1 New loss functions in CoVaR framework

We consider CoVaR measure, introduced by Girardi and Ergun (2013) and explained in Subsection 2.1.1, because of its advantage of being backtestable. As pointed out, the adoption of a more fine-grained loss function is fundamental to help in assessing the CoVaR accuracy and selecting the proper model, since backtests do not discriminate among different competing models.

The CoVaR measure is defined as a conditional quantile, hence a straightforward loss function suitable to this framework is the tick loss function, defined by Komunjer (2005), widely used in assessing VaR accuracy. However, it is necessary to adapt it to this framework to make it usable. Therefore, we propose the following *Tail Tick Loss (TTL) function* for institution i, defined as:

$$\mathrm{TTL}_{\alpha}^{s|i} = \frac{1}{N_i} \sum_{t \in \mathcal{T}_i} \left(\alpha - I_{(r_t^s \leq \eta_t)} \right) \left(r_t^s - \eta_t \right) , \tag{3.3}$$

where

- $\eta_t = \mathrm{CoVaR}_{\alpha,t}^{s|i}$ is the optimal predictor of TTL function;

- $\mathcal{T}_i$ is the time set when the conditioning event of CoVaR occurs, in particular when $\mathbb{C}(r_t^i) = \left\{ r_t^i \leq \mathrm{VaR}_{\alpha,t}^i \right\}$ holds (see formula 2.1). The letter i indicates the dependence of the time set by the VaR of institution i;

- $N_i = \#\left(\mathcal{T}_i\right)$ is the time set sample size, that means the number of observations when the financial institution is in financial distress;

- $I_{(r_t^s \leq \eta_t)} \equiv I_t^{s|i}$ is the CoVaR failures sequence outlined at the beginning of Section 3.1 in formula 3.1 ($I_t^{s|i} = 1$ when $r_t^s \leq \mathrm{CoVaR}_{\alpha,t}^{s|i}$ and 0 otherwise). It is important to underline that this sequence is constructed on the VaR failures sequence, as highlighted by $s|i$. Consequently, it is defined only on the time set $\mathcal{T}_i$, i.e. when the VaR failures of institution i occur.

Hence, in order to correctly evaluate CoVaR forecast accuracy, we apply to the TL function (Komunjer, 2005) the same conditioning set of the CoVaR measure. More precisely, we condition the TL function on the $\mathcal{T}_i$ time periods set, where the conditioning event occurs $\left\{ r_t^i \leq \mathrm{VaR}_{\alpha,t}^i \right\}$, instead of the overall time period T as in the standard TL function.

The proposed Tail Tick Loss function is the most appropriate loss function to evaluate the CoVaR accuracy because the CoVaR measure is the optimal predictor of this TTL loss function since it minimizes the expected value of the loss function:

$$\text{CoVaR}_{\alpha,t}^{s|i} = \underset{\eta_t \in M_{t-1}}{\text{argmin}}\ \mathbb{E}_{t-1}\Big[\big(\alpha - I_{(r_t^s \leq \eta_t)}\big)\big(r_t^s - \eta_t\big)\Big|\mathbb{C}(r_t^i), \mathcal{F}_{t-1}\Big]$$

where $M_{t-1} \subset \mathbb{R}$, $\mathbb{C}(r_t^i) = \left\{r_t^i \leq \text{VaR}_{\alpha,t}^i\right\}$ and $\mathcal{F}_{t-1}$ is the information set available until time $t-1$. The focus is particularly on the extreme events, because the financial institution failure can occur more probably on these days.

Comparing the values of the average sample TTL function for the same institution i among all competing models used to compute the CoVaR measure, we are able to identify which is the model that predicts more accurately the CoVaR measure.

3.3.2 New loss functions in MES framework

We consider also MES measure (Acharya et al., 2010; Brownlees and Engle, 2015), explained in Subsection 2.1.2, which is a widespread used alternative to CoVaR in measuring systemic risk. To the best of our knowledge, statistical tools with the purpose to test and compare MES forecasts have not been properly developed in the literature and a deep analysis on their accuracy is largely unexplored. The Mean Square Error loss function is widely used in the literature to assess the forecasting accuracy and it defined as:

$$\text{MSE}_\alpha = \frac{1}{T}\sum_{t \in \mathcal{T}} \big[r_t - \eta_t\big]^2$$

where $\mathcal{T} = \{1, 2, \ldots, T\}$ is the time set and $\eta_t \in M_t$, $M_t \subset \mathbb{R}$.

MES is a conditional tail expectation, therefore in order to evaluate MES accuracy and its forecasting ability, we propose the *Tail Mean Square Error (TMSE)*:

$$\text{TMSE}_\alpha^{i|s} = \frac{1}{N_s}\sum_{t \in \mathcal{T}_s}\left[\frac{r_t^i - \eta_t}{\sigma_t^s}\right]^2$$

where

- $\eta_t = \text{MES}_{\alpha,t}^{i|s}$ is the optimal predictor of TMSE function;

- $\mathcal{T}_s$ is the time set when the conditioning event of MES occurs, in particular when $\{r_t^s \leq \mathrm{VaR}_{\alpha,t}^s\}$ holds (see formula 2.2). The letter s indicates the dependence of the time set by the VaR of system s;
- $N_s = \#(\mathcal{T}_s)$ is the time set sample size, that means the number of observations when the system is in financial distress;
- σ_t^s is the standard deviation of the financial system estimated by the GARCH model for all competing models.

As done for CoVaR framework, we have to condition TMSE on the same conditioning set of MES, i.e. $\{r_t^s \leq \mathrm{VaR}_{\alpha,t}^s\}$. More precisely, in order to guarantee a correct evaluation of MES accuracy, we condition TMSE on the N_s realizations of the observed financial system VaR, instead of the overall time period T as in the standard TMSE. Furthermore, we standardize TMSE values by σ_t^s, with the purpose of avoiding volatile periods influences on the entire value.

The proposed Tail Mean Square Error is the most appropriate loss function to evaluate the MES accuracy because the MES measure is the optimal predictor of this TMSE loss function since it minimizes the expected value of the loss function:

$$\mathrm{MES}_{\alpha,t}^{i|s} = \underset{\eta_t \in M_{t-1}}{\operatorname{argmin}}\ \mathbb{E}_{t-1}\left[\left(\frac{r_t^i - \eta_t}{\sigma_t^s}\right)^2 \middle| \mathbb{C}(r_t^s), \mathcal{F}_{t-1}\right]$$

where $M_{t-1} \subset \mathbb{R}$, $\mathbb{C}(r_t^s) = \{r_t^s \leq \mathrm{VaR}_{\alpha,t}^s\}$ and $\mathcal{F}_{t-1}$ is the information set available until time $t-1$. The focus is particularly on the extreme events, because the financial institution failure can occur more probably on these days.

Comparing the TMSE values for the same institution i among all models, we are able to detect the model that forecasts more precisely MES.

3.4 Concluding remarks

We face the problem of validating systemic risk measures. Focusing on CoVaR and MES measures, presented in Sections 2.1.1 and 2.1.2, we aim to provide appropriate loss functions to validate them. With this purpose, we reviewed the main existing backtests useful to evaluate CoVaR adequacy. We pointed out that these backtests do not discriminate among different competing models used to compute CoVaR, since they generally provide the same results across all models. As a consequence, we revised the main existing loss functions in order to identify the more suitable ones to adapt to CoVaR and MES frameworks, respectively.

We finally proposed two new loss functions, which are suitable to these frameworks and helpful in discriminating among different competing models in order to determine the models that forecasts more precisely the CoVaR and MES measures, respectively. In this way, we are able to evaluate the forecasting performances of CoVaR and MES.

Chapter 4

Empirical study of 2007−2009 crisis

We now provide an empirical analysis of 2007 − 2009 financial crisis according to the different systemic risk approaches presented in Section 2.1. In particular we analyze the accuracy of the standard DCC-GARCH-type models that are present in the literature (see Section 2.2) in measuring systemic risk using the novel loss functions proposed in Section 3.3. We also investigate how the results should be interpreted in order to detect the best prediction models. These research questions are addressed in this chapter: in particular, Section 4.1 describes the financial data used in the empirical analysis, Section 4.2 reports the in-sample results and Section 4.3 discusses the out-of-sample predictions, comparing the DCC-GARCH-type models forecasting performances with the benchmarks ones.

4.1 US financial data

We apply the econometric models presented in Section 2.2 and the new approach outlined in the Section 3.3 to the panel used by Brownlees and Engle (2015). The sample is composed by the daily returns of 91 US financial institutions, with a market capitalization greater than 5 bln USD as of end of June 2007[1]. The daily CRSP market value weighted

[1]We exclude 4 institutions from systemic risk analysis due to the limited length of their in-sample return series used for the models estimation: Mastercard (MA), NYMEX (NMX), NYSE Euronext (NYX), Western Union (WU). At least 100 observations are required to estimate a GARCH-type model.

index is used as a proxy for the system. We obtained data from CRSP. The panel spans from January 3, 2000 to December 31, 2012 for a total of $T = 3269$ observations. The sample is unbalanced since not all companies have been trading continuously during the sample period.

The 91 financial institutions are grouped by financial industry group based on their SIC codes[2]. The 4 subindustry groups are:

- Depositories Institutions (Dep.), that contains banks and counts 28 institutions;
- Insurance (Ins.), that contains insurance companies and counts 34 institutions;
- Securities Dealers and Commodity Brokers (Bro.), that contains, for example, Bear Stearns and Lehman Brothers, and counts 9 institutions;
- Others (Oth.), that contains non-depository institutions and real estate, and counts 20 institutions.

Table B.1 in Appendix B summarizes the main information and the summary statistics of the sample used in the empirical analysis.

The daily returns of the financial institutions are calculated by taking the percentage log difference of prices on two consecutive trading days:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)100.$$

The sample is divided into two different sub-samples in order to allow the out-of-sample forecasting validation and to evaluate the risk measures performances during the forecasting period. In particular:

- the in-sample consists of 1611 daily observations from January 3, 2000 to May 31, 2006;
- the out-of-sample consists of 1658 daily observations from June 1, 2006 to December 31, 2012.

[2]The only exception is Goldman Sachs (GS) which is included in Broker-Dealers, instead of Others.

The out-of-sample which includes the $2007 - 2009$ financial crisis is used to forecast and validate systemic risk measures. It is interesting to analyze the results obtained in different out-of-sample sub-periods, comparing for example the period before the financial crisis with that during the crisis and with the period immediately afterwards. During the forecasting period the models are separately re-estimated every week (i.e. every 5 observations) using all data available until as of that date and the forecasts are computed one-step-ahead. We exclude the year before the $2007 - 2009$ financial crisis from the in-sample period to permit to the models to fit the data without any preliminary information about the crisis. Moreover, the re-estimation done every-week allows the models to fit the data considering the new information set, which implies that the parameters are not fixed to the last estimation at the end of in-sample. Hence, starting a year or a month before the $2007 - 2009$ financial crisis, the results do not change.

From the descriptive statistics reported in table B.1 in Appendix B, it is shown that the means of observed percentage return time series are very close to 0 and some institutions have null median. From table $B.1$, it is possible to notice the main stylized facts, already known in the existing literature, which characterize the financial time series. In particular, there are the following stylized facts of return distribution: all financial institutions time series present skewness, in particular most of all are negative skewed, that is the left tail is longer than the right one. This means that the extreme negative returns are more likely to occur than extreme positive returns. In addition, all firms show a leptokurtic distribution. As expected, most financial series are characterized by fat tails. Hence, there is a departure from the Gaussian distribution, since the skewness is different from 0 and there is positive excess kurtosis. Finally, as illustrated in figure $B.1$ in Appendix B, the daily returns exhibit volatility clustering (Mandelbrot, 1963), that is when the variance of returns is high for extended periods and then low for extended periods. We report only few financial institution returns as they are representative of the financial industry group they belong to. In particular, we chose Bank of America (BAC) from Depositories group, Cincinnati Financial (CINF) among Insurance, T. Rowe Price (TROW) from Broker-Dealers and American Express (AXP) among Others. They, in fact, have a similar behavior of the remaining financial institutions within the related group.

4.2 In-sample estimation

We compute daily CoVaR and MES measures[3] for all the financial institutions in the panel. This computation requires us to estimate the DCC-GARCH-type models discussed in section 2.2, in particular the DCC-GARCH and DCC-GJR models, over the in-sample (from 03/01/2000 to 31/05/2006). First we estimate the volatilities for each firm by the univariate GARCH and GJR models, then we estimate the conditional correlations of each system-institution pair by the bivariate DCC model.

Table 4.1 shows selected quantiles, in particular 10%, 50% and 90% quantiles, of the parameters estimates of the DCC-GARCH and DCC-GJR models for each sub-industry group over the in-sample. We observe that the dynamics of the financial institutions in the panel do not have a strong degree of heterogeneity. GARCH and GJR parameters are close to the typical set of estimates, which consists of all positive estimates, α values smaller than β ones, sum of α and β near 1. These results, in fact, present slightly higher α and γ together with lower β implying a higher level of unconditional kurtosis. The β estimates are on average higher for Broker-Dealers, showing higher persistence. The range of the leverage effect parameter reaches larger values for Insurance, highlighting that a negative shock increases more volatility. Over all, the parameters estimates do not fluctuate much, with the exception of the intercept. Focusing on DCC, parameters are in line with the typical set of estimates and are similar across groups.

Table $B.2$ reported in Appendix B shows the parameters estimates of DCC-GARCH model for each financial institution, with the related significativity, grouped by financial industry groups. We notice that most of the parameters estimates are statistically significant. All β estimates of GARCH model and b estimates of DCC model are statistically significant at 1% significance level, apart very few exceptions, remarking a good fitting of the model over the data. Similarly, table $B.3$ in the Appendix reports similar results about the DCC-GJR model. Also in this case the β estimates of GARCH model and b estimates of DCC model are almost always statistically significant at the 1% level of significance. The leverage effect parameter estimates, instead, are always statistically

[3]The CoVaR and MES systemic risk measures are computed at $\alpha = 95\%$ confidence level assuming Gaussian distribution. More details about the CoVaR and MES computation and their empirical procedures are reported in Appendix A.

Table 4.1: Selected quantiles (10%, 50% and 90%) of the parameter estimates of the DCC-GARCH and DCC-GJR models.

		GARCH				DCC		
		ω	α	β	γ	const	a	b
Dep.	$q_{0.1}$	0.007	0.033	0.814		0.412	0.008	0.912
	$q_{0.5}$	0.037	0.083	0.908		0.615	0.019	0.975
	$q_{0.9}$	0.169	0.144	0.962		0.667	0.054	0.987
Ins.	$q_{0.1}$	0.007	0.010	0.346		0.312	0.005	0.592
	$q_{0.5}$	0.080	0.080	0.880		0.453	0.017	0.975
	$q_{0.9}$	1.040	0.193	0.987		0.587	0.082	0.993
Bro.	$q_{0.1}$	0.004	0.022	0.916		0.577	0.011	0.947
	$q_{0.5}$	0.012	0.032	0.967		0.687	0.016	0.979
	$q_{0.9}$	0.057	0.074	0.977		0.730	0.042	0.984
Oth.	$q_{0.1}$	0.015	0.000	0.580		0.295	0.000	0.037
	$q_{0.5}$	0.094	0.052	0.909		0.484	0.015	0.930
	$q_{0.9}$	3.790	0.164	0.966		0.673	0.055	0.989
		GJR				DCC		
Dep.	$q_{0.1}$	0.010	0.009	0.843	0.029	0.414	0.011	0.873
	$q_{0.5}$	0.036	0.039	0.910	0.079	0.608	0.021	0.970
	$q_{0.9}$	0.141	0.092	0.965	0.132	0.660	0.069	0.983
Ins.	$q_{0.1}$	0.011	0.000	0.353	-0.001	0.302	0.006	0.627
	$q_{0.5}$	0.060	0.021	0.903	0.073	0.444	0.019	0.959
	$q_{0.9}$	1.101	0.114	0.979	0.175	0.582	0.087	0.992
Bro.	$q_{0.1}$	0.008	0.000	0.932	0.046	0.574	0.012	0.923
	$q_{0.5}$	0.024	0.011	0.958	0.065	0.680	0.021	0.972
	$q_{0.9}$	0.041	0.018	0.975	0.097	0.724	0.059	0.983
Oth.	$q_{0.1}$	0.016	0.000	0.603	-0.065	0.292	0.000	0.018
	$q_{0.5}$	0.091	0.019	0.923	0.057	0.479	0.014	0.930
	$q_{0.9}$	2.420	0.131	0.977	0.127	0.664	0.048	0.989

Notes: More details about the statistical significance of the parameter estimates are reported in tables B.2 and B.3 in Appendix B significant for Broker-Dealers, most of the times statistically significant for both Depositories and Insurance, but only in the half of cases are statistically significant for Others. This evidence highlights the importance of capturing the leverage effect, that is the different impact of negative shocks most of all in Broker-Dealers.

We evaluate the volatility estimates obtained by different models using the loss functions pointed out in Section 3.2.1. According to Patton (2011), in fact, these loss functions are the only robust ones among those widely used in the literature. In particular, we prefer QL according to Patton and Sheppard (2009). Table 4.2 displays the comparison between the in-sample performances of the GARCH and GJR models in estimating volatility obtained by these robust loss functions. Each model runs separately for each financial institution and its squared returns are the volatility proxy. To relieve the reading of the results, we show in table 4.2 the average performances of these models divided by sub-industry group.

Table 4.2: In-sample average performances of the GARCH and GJR models in estimating volatility.

	MSE		**QL**	
	GARCH	GJR	GARCH	GJR
Overall	**403.70**	416.41	2.151	**2.138**
Dep.	**141.94**	146.21	1.846	**1.836**
Ins.	**538.58**	560.22	2.103	**2.090**
Bro.	360.07	**353.53**	2.569	**2.552**
Oth.	**560.52**	578.53	2.471	**2.459**

The lowest value of each loss function (bold in table 4.2) corresponds to the best model in estimating volatility. Considering the MSE loss function, the best model is GARCH, while QL identifies GJR as the most accurate model in estimating volatility. Since QL is the recommended loss function, the results suggest that there are leverage effects in the data that is captured and modeled by the GJR model.

We compute the CoVaR measure as explained in details in Appendix A.1, following step by step the algorithm procedure in Section A.4. We focus on the CoVaR by Girardi and Ergun (2011) (see Section A.2) because it is backtestable and this feature is very important for systemic risk measures. Then, we backtest the CoVaR for verifying the unconditional coverage property (see Section 3.1.1). These results are reported in the columns 2 and 3 of table B.4 in Appendix B. The null hypothesis is whether the average of the violations ($r^s_{t+1} \leq \text{CoVaR}^{s|i}_{\alpha,t+1}$) is equal to the coverage level α. We find that the null hypothesis is rejected at 5% level of significance for 88 financial institutions (over 91) by DCC-GARCH model and for 87 financial institutions by DCC-GJR model, that is they satisfy the unconditional coverage property, while the null hypothesis is rejected at 1% level of significance for the entire sample. Hence, DCC-GARCH and DCC-GJR models provide adequate CoVaR measures. However, we also find that 34 financial institutions over 91 have the same test statistic value for both DCC-GARCH and DCC-GJR model. This equality is explained by the fact that DCC-GARCH-type models capture the same number of VaR and CoVaR violations. This evidence confirms our idea that backtests do not help in discriminating among different competing models. In particular, this test provides similar results from two different models.

In addition, we conduct the DQ test (see Section 3.2.3), whose results are reported in the columns 2 and 3 of table B.5 in Appendix B. The null hypothesis is $\text{H}_0 : \boldsymbol{\beta} = 0$, where $\boldsymbol{\beta}$ is the least square estimator of the regression of the hit sequence $\boldsymbol{Hit}(\hat{\boldsymbol{\beta}})$ on its past values. In other words, we want to check if the hit sequence is an iid process, that means that it is not possible to predict future failures based on past information, otherwise the model is misspecified. Some financial institutions do not have enough past information to carry out the DQ test, therefore 10 institutions are excluded. Over the remaining 81 financial institutions, the null hypothesis is rejected for 78 firms by DCC-GARCH model and for 72 by DCC-GJR (at 5% level of significance). Moreover, 19 firms present the same number of VaR and CoVaR failures, and consequently the same test statistic values, hence also the DQ test do not discriminate among different competing models in this context.

Subsequently, we empirically compute the MES measure (see Section A.3) following step by step the algorithm procedure in Section A.4. To allow a fair comparison of the in-sample performances between DCC-GARCH and DCC-GJR models, we apply the novel loss functions proposed in Section 3.3. In particular, we employ the TTL loss function to evaluate the model accuracy in computing CoVaR measure and the TMSE loss function for MES measure, obtaining one loss function value for each financial institution. Comparing the loss function values for the same institution among all models, we are able to identify the model that predicts more accurately the systemic risk measure considered.

Table 4.3 displays the average performances for the overall sample and each financial industry group. The lowest loss function value (bold in the table) corresponds to the best model.

Table 4.3: In-sample average performances of the DCC-GARCH and DCC-GJR models in estimating CoVaR and MES measures respectively.

	TTL	
	DCC-GARCH	DCC-GJR
Overall	0.172	**0.164**
Dep.	0.183	**0.175**
Ins.	0.168	**0.162**
Bro.	0.187	**0.174**
Oth.	0.158	**0.149**

(a) Related to CoVaR measure

	TMSE	
	DCC-GARCH	DCC-GJR
Overall	**4.648**	4.695
Dep.	**2.406**	2.516
Ins.	**3.499**	3.593
Bro.	**4.333**	4.479
Oth.	9.882	**9.718**

(b) Related to MES measure

Regarding to the CoVaR measure, the lowest value is reached by the DCC-GJR model both in the overall sample and in each financial industry group, highlighting the importance of considering a leverage effect parameter to capture the different impact of a negative shock rather than a positive one. On the contrary, regarding to the MES measure, the lowest TMSE value is always achieved by DCC-GARCH model, except for Others. This leads to conclude that there is not a unique model that predicts more accurately systemic risk. On the basis of these results, we think that maybe some stylized facts not are properly captured by these models, hence they should be jointly investigated.

4.3 Out-of-sample forecast

The forecast analysis is computed over the out-of-sample period, that consists of 1658 observations from June 2006 to the end of the sample (31/12/2012).

We carry out the comparisons among the volatility predictions obtained by different forecasting methods. In particular, we compare the performances of the standard econometric models (the GARCH and GJR models) with those of benchmark models. We consider the following benchmark models on the base of their widespread usage in the related literature:

- the rolling volatility (RollVol.), computed for each financial institution as the standard deviation of its return time series moving a window of 500 observations forward one by one, as benchmark for the volatility;

- the rolling quantile regression model, indicated as QR, as benchmark for the CoVaR measure with a rolling window of 500 observations;

- the rolling linear regression (LR) model as benchmark for the MES measure with the same rolling window of QR.

Hence, we evaluate the volatility forecasts with the aim to identify the best prediction model for volatility, then we compute the models accuracy in forecasting the systemic risk measures, CoVaR and MES.

Table 4.4: Out-of-sample performances of GARCH and GJR models in forecasting volatility.

	MSE			**QL**		
	RollVol.	GARCH	GJR	RollVol.	GARCH	GJR
Overall	246205.5	**243285.1**	246146.8	3.631	2.748	**2.722**
Dep.	260726.5	**257645.7**	260143.8	3.817	2.762	**2.742**
Ins.	12709.55	**11827.72**	11904.98	3.274	2.586	**2.566**
Bro.	1550521.9	**1533769.2**	1551973.4	4.705	3.028	**2.948**
Oth.	**35876.8**	35939.882	37140.02	3.493	2.878	**2.859**

Table 4.4 reports the average performances of GARCH and GJR models with the purpose to compare them with the RollVol benchmark model. Evaluating volatility forecasts is an important topic in measuring systemic risk since conditional variances are a fundamental ingredient of systemic risk measures. Hence, it is useful to identify the model which predicts more accurately volatility. According to the MSE results, the GARCH model appears as the best prediction model, while the GJR model is the best one according to the QL loss function which is the recommended by Patton and Sheppard (2009).

We forecast CoVaR measure (see Appendix $A.2$) and we assess the CoVaR accuracy through the TTL loss function. The TTL averages by sub-industry group are reported in table 4.5, where the lowest values that indicate the best prediction models are highlighted in bold.

Table 4.5: Out-of-sample average performances of the DCC-GARCH and DCC-GJR models in forecasting CoVaR measure evaluated by the TTL loss function.

	QR	DCC-GARCH	DCC-GJR
Overall	1.046	0.228	**0.212**
Dep.	0.912	0.214	**0.203**
Ins.	1.156	0.238	**0.220**
Bro.	1.205	0.244	**0.228**
Oth.	0.976	0.220	**0.204**

Loss functions provide the ranking of different forecasting models, however it is useful to introduce techniques that evaluate the significance of the differences among different methods. For this reason, we run the Diebold-Mariano test to check the significance between the best model and the benchmark predictions[4], but we do not report these information in table to avoid confusion. Testing whether each evaluation is significantly

[4]We run the Diebold-Mariano test only once for the entire sample (composed by 91 financial institutions) to avoid multiple testing problems. The multiple testing problem results from the increase in type I error that occurs when statistical tests are used repeatedly. This issue is usually faced using Bonferroni correction, which is one of the most commonly used approaches for multiple comparisons. This method is not applied in this thesis work, but surely we consider it as a future development.

different from others, we find that the DCC-GARCH-type models are significantly different at 1% significance level from benchmark. The results in table 4.5 indicate that the DCC-GJR model provides the most precise CoVaR forecasts and is always statistically significant over the quantile regression model.

Similarly, we forecast MES measure (see Appendix $A.3$) and we assess the MES accuracy through the TMSE loss function. In table 4.6 the TMSE averages by subindustry group are reported and the lowest values that correspond to the best prediction models are indicated in bold. We run the Diebold-Mariano test to check the significance between the best model and the benchmark model predictions (not reported in table 4.6).

Table 4.6: Out-of-sample average performances of the DCC-GARCH and DCC-GJR models in forecasting MES measure evaluated by the TMSE loss function.

	LR	DCC-GARCH	DCC-GJR
Overall	26.324	**21.65**	21.899
Dep.	15.732	**12.74**	13.026
Ins.	9.053	**6.581**	6.675
Bro.	157.39	134.02	**133.40**
Oth.	11.535	**9.184**	10.026

DCC-GARCH-type models outperform the linear regression model and are always significantly at 1% of significance level different from it. In particular, DCC-GARCH is the overall most accurate model in predicting MES measure and is always statistically significant.

The out-of-sample results are in line with those found over the in-sample period, showing a mismatch between the best CoVaR prediction model and the best one for the MES measure. This may be due to a lack in capturing the stylized facts of financial time series.

Observing the figures $B.2$ in Appendix B, we notice that Broker-Dealers are designated as systemically most important group, most of all for the MES measure. The figures show the industry groups averages of the CoVaR (first column) and MES (second column) measure obtained by the DCC-GARCH (first row) and DCC-GJR (second row) models. The ranking of the risk among the financial industry groups, in fact, is leaded by Broker-Dealers, followed by Depositories, Others, and finally Insurance. This ranking is very clear for the MES measure during the entire sample period both in case of estimation and forecast by the DCC-GARCH model and in case of DCC-GJR, while it is not for CoVaR. Observing figures $B.2a$ and $B.2c$, in fact, Broker-Dealers are the most risky group for most of the sample period, but during the peaks of the crisis all the sub-industry groups tend to overlap indicating a similar risk among them. These results are in line with the finding of Girardi and Ergun (2011). Finally, in figure $B.2e$ there are the overall averages of the CoVaR measure computed by the quantile regression (black line), DCC-GARCH (blue line) and DCC-GJR (green line) models. The black vertical line indicates the end of the in-sample estimation and the beginning of the out-of-sample forecast. It is possible to notice that in all peaks the DCC-GARCH model is the model that provides the bigger CoVaR values, indicating that it could be underestimating systemic risk. On the contrary, DCC-GJR provides the lowest values. During the periods of growth and economic recovery, instead, the lowest CoVaR values are obtained by the DCC-GARCH model. The same conclusions could be taken for the MES measure by observing the figure $B.2f$.

4.3.1 Sub-samples comparison

The out-of-sample period is divided into three different sub-samples in order to compare the systemic risk measures obtained in different historical periods:

- the *pre-crisis sample* with 251 daily observations from June 01, 2006 to May 31, 2007;
- the *crisis sample* with 462 daily observations from June 01, 2007 to March 31, 2009;
- the *post-crisis sample* with 945 daily observations from April 01, 2009 to December 31, 2012.

We carry out the comparison among the forecasting performances of the DCC-GARCH-type models in each sub-sample period and the results are in line with those found in the previous section. In particular, the DCC-GJR model is the model that provides the most accurate volatility and CoVaR forecasts, while the DCC-GARCH model is the best MES prediction model in all the sub-samples.

4.4 Concluding remarks

In this chapter, we have presented an empirical analysis of the $2007 - 2009$ financial crisis. Our study is based on the computation and prediction of the CoVaR and MES systemic risk measures by DCC-GARCH-type models using the daily return series of 91 US financial institutions in the period from January 2000 to December 2012. The objective of our analysis is to evaluate the accuracy of the DCC-GARCH and DCC-GJR models in forecasting CoVaR and MES in order to identify the best model which provides more accurately systemic risk forecasts. In particular, we have compared the forecasting performances of the DCC-GARCH and DCC-GJR models with the benchmarks, that are the quantile regression and the linear regression models, using the novel loss functions, namely TTL and TMSE. We have found that the CoVaR backtesting results are very similar among different competing models, confirming our idea that appropriate loss functions are required. In addition, the out-of-sample results indicate that the DCC-GARCH-type models outperform significantly the benchmarks, in particular the DCC-GJR model is the best model in forecasting conditional volatility and the CoVaR measure. On the contrary, the DCC-GARCH model is the overall best prediction model for the MES measure, but it is not clear among the different sub-industry groups. Therefore, we think that there are stylized facts, which are not captured by these models, that should be investigated.

Chapter 5

Econometric modeling of long-range dependence in Systemic Risk measurement

In this chapter, we investigate the role of the long-range dependence in the systemic risk framework, with the intention to solve the issue found in the empirical results of last Chapter 4. In particular, capturing other relevant stylized facts of financial data may improve the forecasting performances of the considered models. Therefore, in addition to the leverage effect, which has already been considered in systemic risk framework (e.g. Girardi and Ergun, 2013; Brownlees and Engle, 2015), we investigate the impact of the long-range dependence. This feature of financial data has not yet been considered in systemic risk framework (to the best of our knowledge), but many authors have shown its importance in other fields (e.g. Christoffersen et al., 2008; Guoa and Neely, 2008; Li et al., 2012). Hence, we investigate how to capture the long-range dependence in financial data and whether this stylized fact jointly with the others can improve systemic risk forecasts. In particular, in Section 5.1 we review the existing literature on this feature and we explain the existing models that capture and model the long-range dependence in volatility. Then, in Section 5.2 we propose a novel model, called Asymmetric-Component-GARCH (ACGARCH), which combines jointly the leverage effect, properly of GJR-GARCH, and the long-range dependence in volatility, by extending the Component-GARCH (Engle and Lee, 1999). Finally, in Section 5.3 we carry out an empirical analysis of the $2007-2009$ financial crisis comparing the in-sample and out-of-sample performances of these models, showing the importance of using the new model in measuring and forecasting systemic risk.

5.1 Related literature

The presence of long-range dependence in volatility and in asset returns has been studying by researchers for a long time. One of the first to consider the existence of long memory behavior in asset returns was Mandelbrot (1971). Moreover, the presence of long-range dependence in the asset returns is considered as a stylized fact. Several studies have been conducted and different GARCH-type models have been proposed with the purpose of capturing this feature. Many authors have analyzed the long and short memory forecasting models applying ideas of persistence and causality to variance processes. Among the others, Ding and Granger (1996), and Ding et al. (1993) have shown that volatilities are highly persistent possibly requiring a long memory or fractionally integrated process. Consequently, Baillie et al. (1996) propose the FIGARCH model, or Fractionally Integrated GARCH model, where the conditional variance of the process implies a slow hyperbolic rate of decay for the influence of lagged squared innovations. On the contrary, Engle and Lee, in 1999, introduce the Component-GARCH (CGARCH) model, where the long memory behavior of the volatility process is modeled as the sum of two conventional components where one has nearly a unit root, and the other has a much more rapid time decay. The former component describes the short-run dynamics of conditional volatility associated with transitory effects of volatility innovations, while the latter characterizes slower variations in the volatility process associated with more permanent effects. Finally, Engle and Rangel (2008) model equity volatilities and propose a new time-series model for high- and low-frequency volatility called Spline-GARCH, which relaxes the assumption that volatility is mean reverting to a constant level. High-frequency return volatility is specified to be the product of a slow-moving component, represented by an exponential spline, and a unit GARCH. This slow-moving component is the low-frequency volatility, which in this model coincides with the unconditional volatility. These models outperform often GARCH, for example Cheong et al. (2007) find that the long-memory GARCH models provide good description of the long-memory behavior in the Malaysian stock market volatility compare to the standard GARCH model.

5.1.1 Component-GARCH model

The *Component-GARCH (CGARCH) model*, proposed by Engle and Lee (1999), relaxes the assumption of a constant unconditional variance. It has the ability to capture the long-range dependence in volatility, and decomposes additively the conditional volatility into two components. The conditional variance, in fact, has been decomposed in a statistical unobserved component model to describe the long-run and the short-run movements.

Model specification

Given the univariate model for the returns at $t = 1, \dots, T$ with zero mean:

$$r_t = \epsilon_t, \qquad \epsilon_t = \sigma_t z_t, \qquad \epsilon_t \sim iid\big(0, \sigma_t^2\big) \tag{5.1}$$

the CGARCH$(1,1)$ specification for the conditional variance is:

$$\begin{aligned} \sigma_t^2 &= q_t + s_t \\ q_t &= \omega + \rho q_{t-1} + \varphi\big(\epsilon_{t-1}^2 - \sigma_{t-1}^2\big) \\ s_t &= \alpha\big(\epsilon_{t-1}^2 - q_{t-1}\big) + \beta\big(\sigma_{t-1}^2 - q_{t-1}\big) \end{aligned} \tag{5.2}$$

where q_t is the long-run component, interpreted as volatility trend, and s_t is the short-run component, i.e. the distance between the conditional variance and its trend. The long-run (permanent) component, that determines the unconditional variance, captures the long-run impact of an innovation (movements), whereas the short-run (transitory) component accounts for the noisier short-run movements or the transitory effect from a variance innovation. The short-run component can be positive or negative as the conditional variance fluctuates around the long-run component. An additive decomposition is motivated by replacing the unconditional volatility with a stochastic component describing the long memory features of the volatility process. For identification, the long-memory component is assumed to have a much slower mean-reverting rate than the short-run component. In this regard, the CGARCH model relaxes parameter restrictions for the unconditional volatility and the speed of mean reversion in the standard GARCH$(1,1)$ model; however, the slow-moving trend is mean reverting to a fixed value, and the conclusion that the volatility process reverts eventually to a constant level remains unchanged.

As pointed out by Engle and Lee (1999), the reduced form of the conditional variance dynamics can be shown to be:

$$\begin{aligned}\sigma_t^2 = \omega\big(1-\alpha-\beta\big) + \big(\alpha+\varphi\big)\epsilon_{t-1}^2 + \big[-\varphi\big(\alpha+\beta\big) - \alpha\rho\big]\epsilon_{t-2}^2 + \\ + \big(\rho+\beta-\varphi\big)\sigma_{t-1}^2 + \big[\varphi\big(\alpha+\beta\big) - \beta\rho\big]\sigma_{t-2}^2\end{aligned} \tag{5.3}$$

which follows a constrained version of GARCH$(2,2)$ process, since the CGARCH model is not fully equivalent as not all GARCH$(2,2)$ processes have the component structure. Hence, the constraints of the parameters to be positive and real in the component model parameterization ensure positive variances and not complex roots.

The conditions for the non-negativity of the CGARCH$(1,1)$ conditional variance are:

$$0 < \big(\alpha+\beta\big) < \rho < 1, \qquad 0 < \varphi < \beta$$

$$\omega > 0, \qquad \alpha > 0$$

In addition, Engle and Lee (1999) point out, as expected, that the immediate impact of volatility shocks on the long-run component would be smaller than that on the short-run component ($\alpha \geq \varphi$). The empirical results show a common pattern, in which the persistence of transitory shocks is much less than permanent shocks ($\rho > \alpha+\beta$), and the impact of transitory shocks is much greater than permanent shocks ($\alpha > \varphi$).

The estimation of the CGARCH model is based on the Quasi-Maximum Likelihood (QML) estimation procedure, which at least gives consistent results asymptotically.

Forecasting

The k-step-ahead forecast of the conditional volatility, with $k > 2$, is:

$$\begin{aligned}
\sigma^2_{t+k|t} &= q_{t+k|t} + s_{t+k|t} \\
q_{t+k|t} &= \frac{\omega}{1-\rho} + \rho^k \left(q_t - \frac{\omega}{1-\rho} \right) \\
s_{t+k|t} &= \big(\alpha + \beta\big)^k \big(\sigma^2_t - q_t\big)
\end{aligned} \tag{5.4}$$

while the one-step-ahead forecast is:

$$\begin{aligned}
\sigma^2_{t+1|t} &= q_{t+1|t} + s_{t+1|t} \\
q_{t+1|t} &= \omega + \rho q_t + \varphi\big(\epsilon^2_t - \sigma^2_t\big) \\
s_{t+1|t} &= \alpha\big(\epsilon^2_t - q_t\big) + \beta\big(\sigma^2_t - q_t\big)
\end{aligned} \tag{5.5}$$

These models were designed to better account for long-run volatility dependencies and are considered better than GARCH in capturing persistence, with a superior performance in out-of-sample forecasting. In particular, several empirical studies provide statistical evidence that the CGARCH$(1,1)$ model outperforms GARCH$(1,1)$ when modeling conditional variance describing volatility dynamics:

- for exchange rates (Pramor and Tamirisa, 2006; Wei, 2009; Li et al., 2012);
- for stock market returns (Guoa and Neely, 2008);
- for European options (Christoffersen et al., 2008);
- for daily equity returns (Engle and Lee, 1999; Maheu, 2005);
- for oil futures (Agnolucci, 2009);
- for sovereign bond yields (Sosvilla-Rivero and Morales-Zumaquero, 2012).

The long-run component is characterized by a time-varying but highly persistent trend and exhibits long memory, while the short-run component is strongly mean-reverting to this trend, more volatile than the long-run trend level of volatility and driven by market sentiment. According to Li et al. (2012), "separating permanent and transitory risk is important in assessing whether this uncertainty is driven by macroeconomic fundamentals or by market sentiments, which could affect the investment strategies". In addition, the long-run component is mainly driven by shocks to economic fundamentals, whereas the short-run component is driven by transitory shifts in financial market sentiment or short-term position-taking (Pramor and Tamirisa, 2006; Christoffersen et al., 2008; Sosvilla-Rivero and Morales-Zumaquero, 2012; Li et al., 2012).

5.1.2 Spline-GARCH model

The *Spline-GARCH model*, proposed by Engle and Rangel (2008), is another version of GARCH with the purpose to decompose the conditional volatility. CGARCH models decompose it in an additive way, while Spline-GARCH models decompose it in a multiplicative way. The idea of adding a trend component is to capture lower frequency variations on the volatility, like seasonalities and trends.

Model specification

Considering the univariate model (5.1), the Spline-GARCH$(1,1)$ specification for the conditional variance is:

$$\sigma_t^2 = \tau_t g_t$$

where:

$$\begin{aligned} g_t &= (1-\alpha-\beta) + \alpha\left[\frac{\epsilon_{t-1}^2}{\tau_{t-1}}\right] + \beta g_{t-1} \\ \tau_t &= c\exp\left[\omega_0 t + \sum_{i=1}^{k}\omega_i((t-t_{i-1})_+)^2\right] \end{aligned} \tag{5.6}$$

$$(t-t_{i-1})_+ = \begin{cases} t-t_{i-1}, & \text{if } t > t_{i-1} \\ 0, & \text{otherwise} \end{cases} \tag{5.7}$$

and $\{t_0 = 0, t_1, t_2, \ldots, t_k = T\}$ denotes a partition of the time horizon T in k equally spaced intervals. The component g_t is basically a standard GARCH$(1,1)$ model, while the lower frequency volatility component, τ_t, is the exponential of a quadratic spline with k knots, which generates a smooth curve describing this low-frequency volatility component based exclusively on data evidence. The exponential functional form guarantees that the low-frequency component of volatility is always positive. The quadratic form is motivated by the requirement to obtain smoothness through continuity of at least one derivative at a minimum cost in terms of degrees of freedom. The unobserved trend, approximated nonparametrically, describes the low-frequency component of the volatility process associated with slowly varying deterministic conditions in the economy, or random variables that are highly persistent and move slowly. The number of knots, k, is unspecified, thus an information criterion, usually the Bayesian Information Criterion (BIC), determines an "optimal" choice for it. The number of knots governs the cyclical pattern in the low-frequency trend of volatility, so large values of k imply more frequent cycles. The "sharpness" of each cycle is governed by the coefficients, $\omega_i,\ i = 0, 1, \ldots, k$.

The specific model just described can be generalized to account for more lags in the conditional variance. A Spline-GARCH(p, q) model assumes that:

$$g_t = \big(1 - \alpha - \beta\big) + \sum_{i=1}^{p} \alpha_i \left[\frac{\epsilon^2_{t-i}}{\tau_{t-i}}\right] + \sum_{j=1}^{q} \beta_j g_{t-j}$$

The conditions to ensure the positivity of the variance are almost the same of GARCH$(1, 1)$, since the component g_t looks like a GARCH$(1, 1)$ process. In particular, these conditions are:

$$\big(1 - \alpha - \beta\big) > 0, \qquad \alpha \geq 0, \qquad \beta \geq 0, \qquad c > 0$$

The Spline-GARCH model is estimated using both the Quasi-Maximum Likelihood (QML) estimator and the Penalized Maximum Likelihood (PML) estimation procedure (see Brownlees and Gallo, 2010).

Forecasting

The low frequency volatility forecasts are constructed under the assumption that:

$$\tau_{t+k|t} = \tau_t, \qquad \text{for } k > 0$$

hence it is constant and equal to the last estimated value. The high frequency volatility forecasts, instead, are the usual GARCH$(1, 1)$ volatility forecasts outlined in paragraph (2.4). Therefore, the one-step-ahead forecast is:

$$\sigma^2_{t+1|t} = \tau_t \; g_{t+1|t}$$

where:

$$g_{t+1|t} = \big(1-\alpha-\beta\big) + \alpha\epsilon_t^2 + \beta g_t$$

5.1.3 FIGARCH model

Baillie et al., in 1996, propose the *Fractionally Integrated GARCH (FIGARCH) model* that better captures the long-run dynamic dependencies in the conditional variance. The short-run dynamics, in fact, are modeled by GARCH parameters, while shocks to the conditional variance will die out at a slow hyperbolic rate of decay determined by a fractional differencing parameter.

Model specification

Considering the univariate model (5.1), the FIGARCH(p,d,q) specification for the conditional variance is:

$$\begin{aligned} \sigma_t^2 &= \frac{\omega}{1-\beta(L)} + \left(1 - \frac{\phi(L)(1-L)^d}{1-\beta(L)}\right)\epsilon_t^2 \\ &\equiv \frac{\omega}{1-\beta(L)} + \lambda(L)\epsilon_t^2 \end{aligned} \tag{5.8}$$

where L denotes the lag or backshift operator, $\lambda(L) \equiv \lambda_1 L + \lambda_2 L^2 + \ldots$, d is the fractional differencing parameter with $0 < d < 1$ and all the roots of $\phi(L)$ and $[1-\beta(L)]$ lie outside the unit circle. For $d = 0$, the FIGARCH(p,d,q) model reduces to a GARCH(p,q) model with an exponential decay of the shocks, for $d = 1$, it is a IGARCH(p,q) model with an infinite persistence, while for $0 < d < 1$, the effect of an impact to the forecast of σ^2 dissipates at a slow hyperbolic rate of decay for the influence of the lagged squared innovations. Baillie et al. (1996) prove that the FIGARCH(p,d,q) class of processes is strictly stationary and ergodic for $0 \leq d \leq 1$ as a direct extension of the proofs for the IGARCH(p,q) case (Nelson, 1990; Bougerol and Picard, 1992). The specification of FIGARCH$(1,d,1)$ is obtained by replacing $\beta(L) = \beta L$ and $\phi(L) = 1 - \phi L$.

To ensure the positivity almost surely of the conditional variance for all t and the well-definition of the FIGARCH(p,d,q) process, all the coefficients must be nonnegative:

$$\frac{\omega}{1-\beta(L)} > 0, \qquad \lambda_i \geq 0, \quad \forall i = 1,2,\ldots$$

Two different sets of sufficient conditions are available:

- Baillie et al. (1996) derive a group of two sets of inequalities, in particular:

$$\beta - d \leq \phi\,\frac{2-d}{3}, \qquad\qquad d\left(\phi - \frac{1-d}{2}\right) \leq \beta\big(d - \beta + \phi\big)$$

- Chung (2001) summarizes the restrictions with a unique set:

$$0 \leq \phi \leq \beta \leq d \leq 1$$

The FIGARCH model is estimated by Quasi-Maximum Likelihood (QML).

Forecasting

The k-step-ahead forecast of the FIGARCH$(1, d, 1)$ conditional volatility, with $k > 2$, is:

$$\begin{aligned} \sigma^2_{t+k|t+k-1} &= \frac{\omega}{1-\beta} + \left(1 - \frac{(1-\phi L)(1-L)^d}{1-\beta L}\right)\sigma^2_{t+k-1|t+k-2} = \\ &= \frac{\omega}{1-\beta} + \lambda(L)\sigma^2_{t+k-1|t+k-2} \end{aligned} \tag{5.9}$$

where $\sigma^2_{t+k|t+k-1} \equiv \epsilon^2_t$ for $k < 0$ and the parameters are obtained recursively from:

$$\lambda_i = \beta\lambda_{i-1} + \delta_i - \phi\,\delta_{i-1}, \quad i = 2, 3, \ldots \qquad\qquad \lambda_1 = \phi - \beta + d$$

$$\delta_i = \frac{i-1-d}{i}\,\delta_{i-1}, \quad i = 2, 3, \ldots \qquad\qquad \delta_1 = d, \quad \delta_0 = 1$$

Hence, the one-step-ahead forecast of the conditional volatility is given by:

$$\sigma^2_{t+1|t} = \frac{\omega}{1-\beta} + \left(1 - \frac{(1-\phi L)(1-L)^d}{1-\beta L}\right)\epsilon^2_t$$

5.2 A new Asymmetric-Component-GARCH model

The empirical results obtained in the previous chapter highlights the importance to consider the leverage effect during the estimation and forecast of systemic risk. Hence, in addition to consider other stylized facts of financial data like long-range dependence, it is important to capture the leverage effect. In this way, Engle and Lee, in 1999, propose a version of the CGARCH model with asymmetric structure to shocks to capture the asymmetric volatility pattern, namely leverage effect. Stock market volatility responds to stock price movements asymmetrically: bad news (negative shocks) tends to increase investors' expectation about future market volatility more than good news (positive shocks). The authors use the GJR treatment to allow shocks to affect both the volatility components asymmetrically. Otherwise, they find that leverage term is statistically significant for the transitory component, but not in the trend component. For this reason and supported by the empirical findings obtained in the previous chapter, we decide to allow shocks to affect only the volatility transitory component asymmetrically and we propose the new model called *Asymmetric-Component-GARCH (ACGARCH).*

5.2.1 Model Specification

The ACGARCH$(1,1)$ specification for the conditional variance is:

$$\sigma_t^2 = q_t + s_t$$

where:

$$\begin{aligned} q_t &= \omega + \rho q_{t-1} + \varphi\big(\epsilon_{t-1}^2 - \sigma_{t-1}^2\big) \\ s_t &= \alpha\big(\epsilon_{t-1}^2 - q_{t-1}\big) + \beta\big(\sigma_{t-1}^2 - q_{t-1}\big) + \gamma\big(\epsilon_{t-1}^2 - q_{t-1}\big) I_{(\epsilon_{t-1}<0)} \end{aligned} \tag{5.10}$$

Similarly to the CGARCH model outlined in Section 5.1.1, q_t is the long-run (permanent) component and s_t is the short-run (transitory) component, that can be positive or negative. The adding parameter γ captures the short-run leverage effect.

As in CGARCH, the reduced form of the ACGARCH model follows a constrained version of a GARCH$(2,2)$ process:

$$\sigma_t^2 = \omega + \big(\rho - \alpha - \beta - \gamma\big) q_{t-1} + \big(\alpha + \varphi + \gamma\big)\epsilon_{t-1}^2 + \big(\beta - \varphi\big)\sigma_{t-1}^2$$

The conditions for the non-negativity of the ACGARCH$(1,1)$ conditional variance are:

$$\begin{aligned} &\omega > 0, && 0 \le \rho < 1, && 0 \le \alpha < \rho, \\ &0 \le \beta < \big(\rho - \alpha\big), && 0 \le \gamma < \big(\rho - \alpha - \beta\big), && 0 \le \varphi < \beta \end{aligned}$$

The ACGARCH model is estimated using the Quasi-Maximum Likelihood (QML) estimation procedure.

The choice to add the leverage term only to the short-run component is driven by the fact that Gallant et al., in 1993, find that the leverage effect is heavily damped over time. Also the empirical results obtained by Engle and Lee (1999) show that the leverage effect is mainly temporary since the term is significant only in the transitory component, but not for the trend component. Negative shocks, in fact, dominate the effects on the transitory component and the effects of positive shocks almost vanish. Hence, the conditional variance specification of the ACGARCH model is the most appropriate to capture important stylized facts of financial data, in particular the leverage effect and the long-range dependence.

5.2.2 Forecasting

The one-step-ahead forecast of the conditional volatility is:

$$\begin{aligned}
\sigma^2_{t+1|t} &= q_{t+1|t} + s_{t+1|t} \\
q_{t+1|t} &= \omega + \rho q_t + \varphi\big(\epsilon_t^2 - \sigma_t^2\big) \\
s_{t+1|t} &= \alpha\big(\epsilon_t^2 - q_t\big) + \beta\big(\sigma_t^2 - q_t\big) + \gamma\big(\epsilon_t^2 - q_t\big) I_{(\epsilon_t<0)}
\end{aligned} \tag{5.11}$$

5.3 Empirical analysis

We carry out the empirical analysis on the same sample described in Section 4.1, using the models presented in the previous section. The sample runs from January 2000 to December 2012 at daily frequency which leads to 3269 observations of which 1611 are used for the in-sample estimation and the remaining observations for the out-of-sample forecasting. We compare the empirical results obtained by these models with those obtained by the standard econometric models (i.e. the GARCH and GJR) already presented in Chapter 4. These results are reported in columns 2 and 3 of the following tables that show the comparisons among the different models.

5.3.1 In-sample estimation

We compute the daily CoVaR and MES measures[1] for all the financial institutions in the panel employing the GARCH-type models illustrated in Sections 5.1 and 5.2 within the DCC-GARCH-type methodology (see Section 2.2), over the in-sample period (from 03/01/2000 to 31/05/2006). In particular, we first estimate the volatilities for each firm by using the univariate CGARCH, Spline-GARCH, FIGARCH and ACGARCH models, then we estimate the correlations of each institution-system pair by using bivariate DCC model. The parameters estimates of different models are reported in tables $B.6$, $B.7$, $B.8$ and $B.9$ located in Appendix B, while in this chapter we report the 10%, 50% and 90% quantiles values of the obtained estimates to have an idea about the distribution of the parameters across the financial groups under investigation.

Table 5.1 shows selected quantiles of the parameters estimates of the DCC-Spline-GARCH model for each sub-industry group over the in-sample. We use BIC to select the optimal number of knots associated with the Spline-GARCH low-frequency component. Columns 3, 4 and 5 of table 5.1 are dedicated to the parameters estimates of the high-frequency component of Spline-GARCH, while the following six columns displays the parameters estimates of the low-frequency component. These latter parameters do not have any restrictions, hence their estimates are very extreme and different among them (compared with those obtained by Engle and Rangel (2008)), while the former parameters estimates are in line with typical set of GARCH model, that is α is very smaller than β and their sum is close to the unit value. In particular, comparing the α and β estimates of the Spline-GARCH and the standard GARCH (see table 4.1) models, we notice that the Spline-GARCH β estimates are smaller than the GARCH ones, while the Spline-GARCH α estimates are bigger than the GARCH ones, consistent with the results of Engle and Rangel (2008). The β estimates are on average higher for Broker-Dealers, highlighting higher persistence. Moreover, table 5.1 shows similar dynamics of the financial industry groups in the panel. As discussed in Section 5.1.2, the low-frequency coefficients govern the “sharpness” of each cycle, whose frequency is determined by the number of knots. In particular, the second and fourth knots, w_2 and w_4 in table 5.1, identify two high peaks

[1]The CoVaR and MES measures are computed at $\alpha = 95\%$ confidence level assuming Gaussian distribution.

Table 5.1: Selected quantiles (10%, 50% and 90%) of the parameter estimates of the DCC-Spline-GARCH model.

		Spline-GARCH									DCC		
		c	α	β	w_0	w_1	w_2	w_3	w_4	w_5	const	a	b
	$q_{0.1}$	3.690	0.073	0.329	-16.53	-48.35	-49.27	-83.48	-9.533	-81.39	0.426	0.007	0.902
Dep.	$q_{0.5}$	10.06	0.096	0.777	-3.704	3.802	13.38	-46.94	46.22	-23.95	0.623	0.015	0.981
	$q_{0.9}$	19.94	0.226	0.856	9.924	58.29	84.90	11.78	85.73	30.60	0.677	0.050	0.988
	$q_{0.1}$	1.466	0.057	0.175	-7.958	-82.87	-32.36	-96.98	-44.51	-113.8	0.323	0.004	0.613
Ins.	$q_{0.5}$	8.481	0.101	0.756	1.354	-25.15	38.60	-39.26	24.99	-23.60	0.467	0.013	0.973
	$q_{0.9}$	16.15	0.184	0.870	19.05	20.56	139.87	29.24	83.18	72.17	0.597	0.069	0.992
	$q_{0.1}$	1.499	0.000	0.744	-4.475	-117.7	6.136	-60.98	14.40	-42.80	0.580	0.010	0.965
Bro.	$q_{0.5}$	5.450	0.048	0.834	8.632	-40.66	55.74	-35.13	36.94	-2.057	0.698	0.013	0.982
	$q_{0.9}$	20.60	0.088	0.999	39.73	-7.704	134.96	-7.350	50.94	26.57	0.739	0.029	0.986
	$q_{0.1}$	1.013	0.000	0.000	-18.30	-78.92	-48.00	-86.56	-53.98	-91.84	0.285	0.000	0.013
Oth.	$q_{0.5}$	8.484	0.084	0.748	0.619	-12.46	31.50	-33.60	25.60	-4.002	0.486	0.013	0.964
	$q_{0.9}$	19.81	0.195	0.932	25.56	43.88	117.20	38.39	101.55	87.79	0.682	0.058	0.987

Notes: More details about the statistical significance of the parameter estimates are reported in table B.7 in Appendix B

in the cyclical pattern. Focusing on the DCC model, reported in the last three columns in table 5.1, parameters estimates are in line with the typical set of DCC estimates, that is a is very smaller than b and their sum is close to the unit value, and are similar across groups. Table $B.7$ reported in Appendix B shows the parameters estimates of the DCC-Spline-GARCH model for each financial institution grouped by financial industry groups. We notice that most of the parameters estimates of the low-frequency component of the Spline-GARCH are statistically significant, as for DCC parameters.

Similarly to table 5.1, table 5.2 shows the quantiles of the parameters estimates of the DCC-FIGARCH model divided by sub-industry groups over the in-sample, which spans from January 2000 to May 2006. From table 5.2, we notice that there is a statistically significant average effect of past volatility, β, and past squared innovations, ϕ, on current volatility in all four groups. The FIGARCH model also provides evidence for the presence

Table 5.2: Selected quantiles (10%, 50% and 90%) of the parameter estimates of the DCC-FIGARCH model.

		FIGARCH				DCC		
		ω	β	ϕ	d	const	a	b
Dep.	$q_{0.1}$	0.018	0.084	0.000	0.149	0.416	0.007	0.888
	$q_{0.5}$	0.154	0.503	0.221	0.331	0.617	0.015	0.979
	$q_{0.9}$	0.524	0.678	0.414	0.591	0.672	0.060	0.989
Ins.	$q_{0.1}$	0.112	0.021	0.000	0.047	0.315	0.005	0.591
	$q_{0.5}$	0.428	0.498	0.286	0.284	0.455	0.016	0.976
	$q_{0.9}$	1.832	0.687	0.418	0.744	0.589	0.073	0.993
Bro.	$q_{0.1}$	0.000	0.213	0.001	0.275	0.575	0.009	0.956
	$q_{0.5}$	0.113	0.477	0.207	0.344	0.686	0.013	0.983
	$q_{0.9}$	0.407	0.604	0.327	0.357	0.727	0.036	0.988
Oth.	$q_{0.1}$	0.124	0.008	0.000	0.015	0.306	0.000	0.600
	$q_{0.5}$	0.638	0.428	0.145	0.234	0.484	0.011	0.948
	$q_{0.9}$	3.992	0.716	0.419	0.669	0.669	0.052	0.991

Notes: More details about the statistical significance of the parameter estimates are reported in table $B.8$ in Appendix B

of long memory as given by the 50% quantile estimate of the differencing parameter d. Table $B.8$, reported in Appendix B, shows the parameters estimates of the DCC-FIGARCH model for each financial institution grouped by financial industry groups. DCC estimates are highly statistically significant as well as the differencing parameter of the FIGARCH and most of the β estimates.

Finally, table 5.3 gives a summary for the DCC-CGARCH and DCC-ACGARCH models results for each sub-industry group over the in-sample. We notice that the dynamics of the financial institutions in the panel do not have a strong degree of heterogeneity and the parameters estimates are very similar across groups. Columns 3, 4 and 5 in table 5.3 are dedicated to the parameters estimates of the short-run component of the CGARCH and ACGARCH models, while columns 6, 7 and 8 display the parameters estimates of the long-run component. On the contrary, the last three columns show the parameters estimates of the DCC model. The CGARCH parameters estimates are in line with those that can be found in the related literature. The short-run coefficients estimates, in fact, are similar to the typical set of the GARCH model (i.e. α is very smaller than β and their sum is close to the unit value) and we find similar results to Engle and Lee (1999). In particular, as expected, the transitory shocks are much less persistent than the permanent shocks ($1 > \rho > \alpha + \beta$), and the impact of transitory shocks is much greater than permanent shocks ($\alpha > \varphi$). As found in the GARCH and GJR results (see table 4.1), the β estimates are on average higher for Broker-Dealers, showing higher persistence, for both the CGARCH and ACGARCH model. Overall, the parameters estimates do not fluctuate much. Focusing on DCC, parameters are in line with the typical set (i.e. a is very smaller than b and their sum is close to the unit value) of estimates and are similar across groups. Table $B.6$ in Appendix B reports the statistical significance of the CGARCH parameters. We observe that the ρ parameter of the long-run component is always statistically significant at the 1% level, as well as b parameter of DCC. For 68 out of 91 of the financial institutions, β and φ coefficients are statistically significant.

Similar conclusions could be taken from the ACGARCH parameters estimates presented in table 5.3 and from their statistical significance reported in table $B.9$ in Appendix B. In addition, it is important to underline that the financial institutions with the statistically significant leverage effect parameter are almost the same firms indicated by the GJR model; in particular, the leverage effect parameter estimates are always statistically significant for Broker-Dealers. This evidence highlights the importance of capturing the leverage effect, that is the different impact of negative shocks, most of all for Broker-Dealers. As for the GJR model (see table 4.1), the range of the leverage effect parameter reaches larger values for Insurance, highlighting that a negative shock increases volatility.

Our analysis continues with the estimation of the volatilities. We estimate the volatilities individually for each financial institution employing the long-range dependence models presented in Sections 5.1 and 5.2. Each model runs separately for each firm. Then,

Table 5.3: Selected quantiles (10%, 50% and 90%) of the parameter estimates of the DCC-CGARCH and DCC-ACGARCH models.

		CGARCH						**DCC**		
		ω	α	β	ρ	φ	γ	const	a	b
Dep.	$q_{0.1}$	0.001	0.060	0.509	0.997	0.014		0.418	0.007	0.898
	$q_{0.5}$	0.004	0.101	0.801	1.000	0.031		0.618	0.020	0.976
	$q_{0.9}$	0.023	0.214	0.880	1.000	0.056		0.675	0.063	0.986
Ins.	$q_{0.1}$	0.001	0.055	0.152	0.992	0.003		0.313	0.005	0.532
	$q_{0.5}$	0.004	0.111	0.775	1.000	0.014		0.445	0.019	0.963
	$q_{0.9}$	0.037	0.229	0.869	1.000	0.045		0.590	0.102	0.992
Bro.	$q_{0.1}$	0.001	0.033	0.109	1.000	0.016		0.579	0.011	0.961
	$q_{0.5}$	0.003	0.056	0.846	1.000	0.023		0.690	0.017	0.978
	$q_{0.9}$	0.010	0.092	0.898	1.000	0.029		0.731	0.035	0.987
Oth.	$q_{0.1}$	0.003	0.004	0.219	0.993	0.000		0.314	0.000	0.556
	$q_{0.5}$	0.014	0.099	0.720	0.999	0.025		0.484	0.012	0.957
	$q_{0.9}$	0.079	0.262	0.880	1.000	0.043		0.674	0.070	0.992
		ACGARCH						**DCC**		
Dep.	$q_{0.1}$	0.001	0.000	0.563	0.995	0.000	0.000	0.412	0.006	0.960
	$q_{0.5}$	0.004	0.052	0.830	0.999	0.024	0.055	0.611	0.014	0.982
	$q_{0.9}$	0.013	0.143	0.888	1.000	0.051	0.145	0.666	0.033	0.988
Ins.	$q_{0.1}$	0.000	0.002	0.261	0.996	0.000	0.000	0.306	0.005	0.601
	$q_{0.5}$	0.002	0.073	0.798	0.999	0.008	0.034	0.455	0.014	0.981
	$q_{0.9}$	0.017	0.169	0.907	1.000	0.041	0.153	0.587	0.083	0.993
Bro.	$q_{0.1}$	0.002	0.000	0.695	0.997	0.014	0.034	0.573	0.010	0.960
	$q_{0.5}$	0.006	0.000	0.903	0.997	0.017	0.058	0.681	0.014	0.982
	$q_{0.9}$	0.014	0.028	0.943	1.000	0.038	0.115	0.725	0.032	0.986
Oth.	$q_{0.1}$	0.000	0.000	0.046	0.996	0.000	0.000	0.289	0.000	0.471
	$q_{0.5}$	0.008	0.033	0.847	1.000	0.010	0.066	0.476	0.010	0.957
	$q_{0.9}$	0.040	0.233	0.932	1.000	0.035	0.123	0.665	0.040	0.989

Notes: More details about the statistical significance of the parameter estimates are reported in tables B.6 and B.9 in Appendix B we evaluate the volatility estimates obtained by different models using the robust loss functions pointed out in Section 3.2.1 and considering the financial institution squared returns as the proxy volatility. Hence, one model by one, we obtain a loss function value for each firm by averaging the loss function time series with the purpose of comparing the models performances. Here, we report the average of loss functions values grouped by the entire sample and the sub-industry groups.

The models performances in estimating volatility measured by MSE are reported in table 5.4, while those measured by QL are presented in table 5.5.

Table 5.4: In-sample performances of GARCH-type models in estimating volatility measured by MSE loss function.

	GARCH	GJR	CGARCH	SPLINE	FIGARCH	ACGARCH
Overall	**403.70**	416.41	411.22	431.02	408.53	404.76
Dep.	141.94	146.21	142.37	140.73	**140.54**	142.13
Ins.	538.58	560.22	550.50	627.56	544.98	**533.93**
Bro.	360.07	**353.53**	358.39	356.25	359.15	355.80
Oth.	560.52	578.53	574.62	**536.97**	573.99	574.88

Table 5.5: In-sample performances of GARCH-type models in estimating volatility measured by QL loss function.

	GARCH	GJR	CGARCH	SPLINE	FIGARCH	ACGARCH
Overall	2.151	2.138	2.151	**2.115**	2.144	2.145
Dep.	1.846	1.836	1.835	**1.817**	1.833	1.838
Ins.	2.103	2.090	2.117	**2.064**	2.099	2.099
Bro.	2.569	2.552	2.563	**2.549**	2.566	2.560
Oth.	2.471	2.459	2.465	**2.423**	2.465	2.467

In columns 2 and 3, there are the results obtained by the GARCH and GJR models that we have discussed in Chapter 4. Comparing the loss functions values obtained by the different competing models, the MSE loss function does not indicate a unique preferred model for all the financial industry groups (see table 5.4). The lowest value, indicated in bold, is overall reached by the GARCH model, but each sub-industry group indicate a different model. According to QL, on the contrary, the model that estimates more accurately the volatility is the Spline-GARCH (see table 5.5). In addition, QL is preferable to MSE (Patton and Sheppard, 2009) hence we refer to this.

The CoVaR measure by Girardi and Ergun (2013) is empirically computed (see Section $A.2$) over the in-sample period (from 03/01/2000 to 31/05/2006) by the long-range dependence models, following the algorithm procedure $A.4$ described in Appendix A. As in Girardi and Ergun (2013), we backtest CoVaR for verifying the unconditional coverage property employing the UC test by Kupiec (1995) and Christoffersen (1998) (see Section 3.1.1). The null hypothesis is whether the average of the violations ($r^s_{t+1} \leq \text{CoVaR}^{s|i}_{\alpha,t+1}$) is equal to the coverage level α. Table $B.4$ in Appendix B reports the UC test results. The results for these models are displayed from the fourth column to the end of the table. We notice that the null hypothesis is rejected at 5% significance level for:

- 90 financial institutions (over 91) by the DCC-CGARCH model,
- 89 financial institutions by the DCC-Spline-GARCH model,
- 90 financial institutions by the DCC-FIGARCH model,
- 81 financial institutions by the DCC-ACGARCH model.

Therefore, the long-range dependence models satisfy the unconditional coverage property, in particular the CoVaR measure they estimate is adequate. Furthermore, the null hypothesis is rejected at 1% significance level for the entire sample. However, as it happened for the DCC-GARCH and DCC-GJR models, many test statistic values are similar because the models tend to capture the same number of VaR and CoVaR failures. Hence, this backtest does not help in discriminating among different models, since it provides similar results. It could be interesting to employ appropriate loss functions in addition to backtests in order to detect the best model.

Studying this aspect more in depth, we conduct the DQ test by Engle and Manganelli (2014) (see Section 3.2.3), whose results are reported in table $B.5$ in Appendix

B. We aim to verify whether the model is misspecified, since it is not possible to predict future failures based on past information if the hit sequence is an iid process. The null hypothesis is $\mathrm{H}_0 : \ \boldsymbol{\beta} = 0$, where $\boldsymbol{\beta}$ is the least square estimator of the regression of the hit sequence $\boldsymbol{Hit}(\hat{\boldsymbol{\beta}})$ on its past values. In addition to the financial institutions excluded given their lack of past information to carry out the DQ test, we exclude 3 firms more, hence 13 institutions are excluded over 91. The null hypothesis is rejected at 5% significance level for:

- 73 financial institutions (over 78) by the DCC-GARCH model,
- 70 financial institutions by the DCC-GJR model,
- 74 financial institutions by the DCC-CGARCH model,
- 76 financial institutions by the DCC-Spline-GARCH model,
- 71 financial institutions by the DCC-FIGARCH model,
- 64 financial institutions by the DCC-ACGARCH model.

Therefore, the standard and long-range dependence models are not misspecified. Unfortunately, we obtain similar test statistic values for many firms, as well as for the UC test, hence it is not possible to determine the best model comparing the DQ results and it is necessary to employ appropriate and more fine-grained loss functions to achieve this goal.

Table 5.6: In-sample performances of DCC-GARCH-type models in computing CoVaR measured by TTL loss function.

	DCC GARCH	DCC GJR	DCC CGARCH	DCC SPLINE	DCC FIGARCH	DCC ACGARCH
Overall	0.172	0.164	0.167	**0.158**	0.170	0.167
Dep.	0.183	0.175	0.176	**0.167**	0.183	0.178
Ins.	0.168	0.162	0.163	**0.155**	0.163	0.164
Bro.	0.187	0.174	0.180	**0.172**	0.181	0.178
Oth.	0.158	0.149	0.154	**0.143**	0.158	0.153

We evaluate the CoVaR accuracy using the TTL loss function proposed in Section 3.3.1 for each financial institution. In particular, we assess the performances of the long-range dependence models in computing the CoVaR measure and we compare them, in addition to the DCC-GARCH and DCC-GJR performances described in Section 4.1. Table 5.6 displays the average performances for the overall sample and each financial industry group over the in-sample period. The lowest value (bold in the table) corresponds to the best model. The TTL results, contained in table 5.6, indicate the DCC-Spline-GARCH as the best model in computing the CoVaR measure, whereas in Section 4.1 DCC-GJR was suggested as the best model. This evidence highlights the changes of the empirical conclusions obtained by taking into account different models that capture other stylized facts of financial data compared with those obtained by the econometric standard models analyzed in the previous chapter.

Finally, we empirically compute the MES measure (see Section $A.3$) following step by step the algorithm procedure explained in Section $A.4$. Then, we evaluate the MES accuracy employing the novel loss function, namely TMSE, proposed in Section 3.3.2, for each financial institution. In table 5.7, we report the average TMSE values divided by financial industry groups. Comparing the loss function values for the same group among all models, we are able to identify the model that provides more accurately the MES measure.

Table 5.7: In-sample performances of DCC-GARCH-type models in computing MES measured by TMSE loss function.

	DCC GARCH	DCC GJR	DCC CGARCH	DCC SPLINE	DCC FIGARCH	DCC ACGARCH
Overall	4.648	4.695	4.603	**4.593**	4.600	4.596
Dep.	2.406	2.516	2.412	**2.360**	2.397	2.419
Ins.	3.499	3.593	3.545	**3.469**	3.566	3.579
Bro.	4.333	4.479	4.382	**4.276**	4.475	4.410
Oth.	9.881	9.718	9.567	9.772	9.495	**9.459**

The TMSE results, reported in table 5.7, suggest that the DCC-Spline-GARCH computes more accurately the MES measure, except for Others, while comparing only the DCC-GARCH and DCC-GJR models we found DCC-GJR as the best model.

This empirical evidence from both TTL and TMSE results confirm the limitation of DCC-GARCH and DCC-GJR in capturing other stylized facts that are important in systemic risk framework and the need to investigate other stylized facts.

5.3.2 Out-of-sample forecast

The out-of-sample period spans from June 2006 to December 2012, including the $2007-2009$ financial crisis for a total of 1658 observations and is used to forecast and validate the systemic risk measures. During this period the models are separately re-estimated every week (i.e. every 5 observations) using all data available until as of that date and the forecasting horizon is one-step-ahead. The forecast analysis consists of evaluating the volatility, CoVaR and MES predictions and comparing the different models performances with the purpose to identify the model that provides the best performance in terms of accuracy. The comparison of models performances is carried out evaluating the accuracy of the volatility, CoVaR and MES measures using loss functions, in particular the MSE and QL for volatility, the TTL for CoVaR and the TMSE loss function for the MES measure. The models involved in this analysis are the standard econometric models widespread in the literature (GARCH and GJR models), the long-range dependence models described in this chapter and the benchmark models (i.e. the rolling volatility, the quantile regression and linear regression models, for further details see Section 4.3).

The Spline-GARCH model is excluded in this forecasting analysis because it does not work very well. This is due to the fact that, forecasting the GARCH component and keeping the trend component constant at the last value estimated, we obtain a very high trend value that multiplied by the GARCH prediction provide worse forecasts than those obtained by the other models. Despite employing the Penalized Maximum Likelihood (PML) estimation procedure (Brownlees and Gallo, 2010) to estimate the Spline-GARCH model, the results do not improve.

The average performances of RollVol and GARCH-type models in forecasting volatility evaluated by the MSE loss function are presented in table 5.8. It is interesting to observe that the best models indicated in this analysis are different from those indicated

Table 5.8: Out-of-sample performances of GARCH-type models in forecasting volatility measured by MSE loss function.

	RollVol.	GARCH	GJR	CGARCH	FIGARCH	ACGARCH
Overall	246205.47	**243285.08**	246146.76	244109.42	243799.86	247149.17
Dep.	260726.45	257645.66	260143.75	**257549.68**	258110.20	259475.72
Ins.	12709.553	11827.722	11904.983	**11766.219**	11927.486	11890.111
Bro.	1550521.9	**1533769.2**	1551973.4	1544622.2	1537624.6	1567911.6
Oth.	35876.763	35939.882	37140.017	**35045.735**	35727.277	35489.325

in the in-sample estimation. From table 5.8, overall the best prediction model is the GARCH, however it is more important to analyze the different sub-industry groups to understand the different dynamics within each group. The CGARCH model, in fact, is the best model for forecasting volatility in all sub-industry groups, except for Broker-Dealers (see table 5.8). This finding confirms the importance of capturing the long-range dependence in forecasting the one-step-ahead systemic risk.

Table 5.9 reports the average performances of RollVol and GARCH-type models in forecasting volatility evaluated by the QL loss function.

Table 5.9: Out-of-sample performances of GARCH-type models in forecasting volatility measured by QL loss function.

	RollVol.	GARCH	GJR	CGARCH	FIGARCH	ACGARCH
Overall	3.631	2.748	**2.722**	2.737	2.769	2.750
Dep.	3.817	2.762	**2.742**	2.755	2.761	2.775
Ins.	3.274	2.586	**2.566**	2.578	2.631	2.594
Bro.	4.705	3.028	**2.948**	3.001	3.040	2.984
Oth.	3.493	2.878	**2.859**	2.864	2.894	2.878

Conversely to the previous conclusion, the results for the QL loss function presented in table 5.9 suggest the GJR model as the best prediction model for volatility for both the entire sample and the sub-industry groups, confirming the forecasting analysis results conducted in Chapter 4.

For predicting the CoVaR measure, we evaluate the accuracy of these forecasts employing the TTL loss function, whose results are presented in table 5.10. The TTL averages, divided by financial industry group, summarize the models performances in forecasting CoVaR and the values represented in bold in table 5.10 indicate the best prediction models.

Table 5.10: Out-of-sample performances of DCC-GARCH-type models in forecasting CoVaR measured by TTL loss function.

	QR	DCC GARCH	DCC GJR	DCC CGARCH	DCC FIGARCH	DCC ACGARCH
Overall	1.046	0.228	0.212	0.227	**0.204**	0.240
Dep.	0.912	0.214	0.203	0.213	**0.194**	0.223
Ins.	1.156	0.238	0.220	0.239	**0.211**	0.251
Bro.	1.205	0.244	0.228	0.242	**0.223**	0.269
Oth.	0.976	0.220	0.204	0.219	**0.197**	0.232

The TTL results identify the DCC-FIGARCH model as the model that predicts the most accurate CoVaR measure (see table 5.10). This finding emphasizes the need to investigate the usage of models that capture an important stylized fact as the long-range dependence. In addition to loss functions, it is important to introduce techniques that are able to discriminate among different competing models in terms of significance. We therefore consider the Diebold-Mariano test to check the statistical significance between the best model and the benchmark one. This means that we compare each DCC-GARCH-type model with the rolling quantile regression for each financial institution individually comparing the whole TTL series. We observe that all the DCC-GARCH-type models are statistically different at 1% significance level from the quantile regression approach for the entire sample. Furthermore, we run the Diebold-Mariano test between the new proposed model, DCC-ACGARCH, and the best model for each financial institution individually, finding that the difference between the TTL value of the best model and the TTL of DCC-ACGARCH is statistically significant at 10% level for 65 firms over 91.

Table 5.11: Out-of-sample performances of DCC-GARCH-type models in forecasting MES measured by TMSE loss function.

	LR	DCC GARCH	DCC GJR	DCC CGARCH	DCC FIGARCH	DCC ACGARCH
Overall	26.324	**21.654**	21.899	21.719	21.970	22.004
Dep.	15.732	**12.744**	13.026	12.778	12.996	12.843
Ins.	9.053	6.581	6.675	6.538	6.966	**6.528**
Bro.	157.39	134.02	**133.40**	134.30	134.61	136.62
Oth.	11.535	**9.184**	10.026	9.387	9.351	9.565

Similarly, the MES predictions accuracy is evaluated using the TMSE loss function and table 5.11 shows the related results grouped by financial sectors, where bold highlights the lowest values that correspond to the best prediction models. From table 5.11, the TMSE results do not indicate a unique best prediction model for all the sub-industry groups. Overall, the DCC-GARCH-type models outperform the rolling linear regression and the best prediction model for MES is, on average, the DCC-GARCH. The Diebold-Mariano test between the TMSE values of the best model and LR (see table 5.11) shows that the linear regression model is statistically worse than the best one at 10% significance level for 40 financial institutions over 91. On the contrary, 29 firms show that the TMSE values are statistically significant at the 10% level for the new proposed model over the best prediction model. It is important to notice that the novel model proposed in Section 5.2, the ACGARCH, is an extension of the GARCH, GJR and CGARCH models and all these models are the best ones in some sub-industry sector (see table 5.11). Therefore, it is necessary to estimate and forecast MES by the DCC-ACGARCH model and then to check the significance of the parameters in order to reach the most appropriate nested model.

Since the TMSE does not identify a unique best prediction model for MES, we investigate other versions of TMSE. As explained in Section 3.2.3, the proposed TMSE is divided by the standard deviation of the financial system estimated by GARCH for all competing models used in the empirical analysis in order to avoid the influences of volatile periods. Using the standard deviation estimated by the other GARCH-type models as standardization, we obtain similar results to those reported in table 5.11; in particular overall the DCC-GARCH model provides the most accurate MES forecasts for all the cases. However, if we standardize the TMSE by the standard deviation estimated by the model[2], we change the standardization according to the considered model and we obtain different results, as shown in table 5.12. In this way, we are able to avoid that volatile periods captured by that particular model influence the related loss function, instead of avoiding that volatile periods captured by a particular model (in our case GARCH) influence all the loss functions.

Table 5.12: Out-of-sample performances of DCC-GARCH-type models in forecasting MES measured by another version of TMSE loss function.

	LR	DCC GARCH	DCC GJR	DCC CGARCH	DCC FIGARCH	DCC ACGARCH
Overall	74.032	21.654	19.388	21.285	**17.847**	21.799
Dep.	66.269	12.744	12.073	12.802	**10.901**	13.390
Ins.	31.536	6.581	6.247	6.431	**5.908**	6.622
Bro.	358.26	134.02	112.76	130.88	**106.39**	131.19
Oth.	29.242	9.184	9.948	9.094	**8.024**	10.145

Results reported in table 5.12 identify the DCC-FIGARCH as the best forecasting model for the MES measure for the entire sample and each sub-industry group. This finding is in line with the TTL results, reported in table 5.10, which indicate that the DCC-FIGARCH is the best prediction model for systemic risk and confirm the importance of capturing the long-range dependence.

The figures $B.3$ in Appendix B show the industry groups averages of the CoVaR (first column) and MES (second column) measures obtained by the long-range depen-

[2]Standardizing the TMSE loss function by the standard deviation estimated by the particular considered model means that the TMSE obtained by DCC-CGARCH is divided by the system standard deviation estimated by the CGARCH, the TMSE obtained by DCC-FIGARCH is divided by the system standard deviation estimated by the FIGARCH, and so on.

dence DCC-GARCH-type models, in particular the DCC-CGARCH (first row), the DCC-FIGARCH (second row) and the DCC-ACGARCH (third row) models. The black vertical line represents the end of the estimation period and the beginning of the forecasting period. From figures $B.3$ we notice that Broker-Dealers are detected as the systemically most important group, since the green line is below all the other lines. For the CoVaR measure this is evident most of all in $2004-2008$ period and during the phases of growth after peaks. Moreover, this is even clearer for the MES measure, since the groups lines are separated and easily identifiable. These conclusions about the behavior of the systemic risk measures obtained by the long-range dependence DCC-GARCH-type models are very similar to those obtained by the econometric standard models presented in figures $B.2a$, $B.2b$, $B.2c$ and $B.2d$ in the previous chapter.

In order to check more precisely the ranking of groups' riskiness, we compute the average of each measure. Table 5.13 contains the ranking of the CoVaR averages over the out-of-sample.

Table 5.13: Ranking of the CoVaR averages over the out-of-sample.

	DCC GARCH	DCC GJR	DCC CGARCH	DCC FIGARCH	DCC ACGARCH
Overall	-3.174	-3.203	-3.125	**-3.393**	-3.169
1st: Bro.	-3.420	-3.449	-3.353	-3.600	-3.392
2nd: Oth.	-3.193	-3.219	-3.139	-3.453	-3.204
3rd: Ins.	-3.150	-3.184	-3.105	-3.461	-3.159
4th: Dep.	-3.121	-3.145	-3.075	-3.343	-3.093

From table 5.13, we notice that Broker is always the most risky group, followed by Others, Insurance and Depositories groups. This groups' riskiness ranking is identified by all the considered models and is different from that obtained over the in-sample period where Broker-Dealers are followed by Depositories, Others and Insurance, respectively; as found in Girardi and Ergun (2013). Moreover, the model that provides the lowest average of the CoVaR measure is the DCC-FIGARCH that corresponds to the best prediction model for this measure (see table 5.13).

Table 5.14: Ranking of the MES averages over the out-of-sample.

	DCC GARCH	DCC GJR	DCC CGARCH	DCC FIGARCH	DCC ACGARCH
Overall	-3.091	-3.081	**-3.155**	-3.060	-3.131
1st: Bro.	-4.104	-4.082	-4.221	-4.124	-4.166
2nd: Dep.	-3.349	-3.349	-3.406	-3.200	-3.413
3rd: Oth.	-3.173	-3.187	-3.251	-3.093	-3.209
4th: Ins.	-2.645	-2.615	-2.696	-2.712	-2.663

Table 5.14 contains the ranking of the MES averages over the out-of-sample. The ranking obtained for the MES measure reported in table 5.14 is different from that obtained for CoVaR (see table 5.13). All the models identify the same ranking composed by Broker-Dealers, Depositories, Others, and Insurance respectively (see table 5.14). Over the in-sample period, we find a different ranking where Broker-Dealers is still the most risky group, but it is followed by Depositories, Others, and Insurance respectively. Finally, the model that provides the overall lowest average of the MES measure in table 5.14 is the DCC-CGARCH.

5.3.3 Sub-samples comparison

The out-of-sample period is divided into three different sub-samples in order to compare the systemic risk measures obtained in different historical periods (see Section 4.3.1): the pre-crisis sample (251 daily observations) from June 2006 to May 2007, the crisis sample (462 daily observations) from June 2007 to March 2009, and the post-crisis sample (945 daily observations) from April 2009 to December 2012.

We carry out the comparison among the forecasting performances of the DCC-GARCH-type models in each sub-sample period and we obtain different best prediction models for each measure. Regarding the volatility forecast, we find that the GARCH model provides lower MSE during the pre-crisis and the crisis samples, while the CGARCH outperforms it over the post-crisis period. According to the QL loss function, the AC-GARCH model outperforms the others in the pre-crisis sample, while the GJR is the best prediction model during the crisis. Finally, the CGARCH forecasts more accurately volatility over the post-crisis sample than the other models. These results indicate the importance of investigating different models since it is difficult to know exactly which historical period is except after it happens. Moreover, since the suggested models are nested into the ACGARCH, the latter is a fundamental starting point in order to find the best possible model specification. For the CoVaR measure, the TTL values suggest the DCC-FIGARCH model as the best prediction model over the pre-crisis and the crisis samples, while the DCC-GJR provides the most accurate CoVaR forecasts after the crisis to the end of the sample among the considered competing models. Finally, for the MES forecasts, the DCC-GARCH outperforms the other models during the pre-crisis and crisis periods, while the DCC-FIGARCH is the best MES prediction model over the

Table 5.15: Comparison of CoVaR averages over different sub-samples.

Pre-crisis	DCC GARCH	DCC GJR	DCC CGARCH	DCC FIGARCH	DCC ACGARCH
Dep.	-2.175	-2.171	-2.109	-2.147	-2.095
Ins.	-1.907	-1.930	-1.862	-1.872	-1.859
Bro.	-2.224	-2.189	-2.136	-2.188	-2.128
Oth.	-1.881	-1.878	-1.834	-1.896	-1.823
Crisis					
Dep.	-4.166	-4.157	-4.259	-4.250	-4.008
Ins.	-4.163	-4.165	-4.262	-4.269	-4.070
Bro.	-4.333	-4.345	-4.443	-4.408	-4.157
Oth.	-4.156	-4.151	-4.243	-4.308	-4.054
Post-crisis					
Dep.	-2.861	-2.910	-2.753	-3.217	-2.910
Ins.	-2.985	-3.037	-2.869	-3.488	-3.059
Bro.	-3.291	-3.345	-3.143	-3.581	-3.353
Oth.	-3.071	-3.119	-2.945	-3.448	-3.155

post-crisis sample. It is interesting to observe that for the first two sub-samples the best prediction model is the same, whereas after the crisis it is necessary a different model. This could be due to the fact that before and during the crisis the volatilities and the systemic risk are much higher than during the post-crisis period, hence two different model specifications are necessary.

Table 5.15 reports the averages of the CoVaR measure over three different sub-samples obtained by the different models. From table 5.15 it is interesting to observe that the most risky group over all periods is Broker-Dealers. During the pre-crisis sample Broker-Dealers are followed by Depositories, Insurance and Others respectively, and this ranking is uniformly identified by all the models (see table 5.15). As well as during the post-crisis period where Broker-Dealers are followed by Others, Insurance and Depositories by all

Table 5.16: Comparison of MES averages over different sub-samples.

Pre-crisis	DCC GARCH	DCC GJR	DCC CGARCH	DCC FIGARCH	DCC ACGARCH
Dep.	-2.209	-2.155	-2.251	-2.250	-2.249
Ins.	-2.023	-1.942	-2.051	-2.227	-2.013
Bro.	-4.048	-4.084	-4.157	-4.156	-4.132
Oth.	-2.628	-2.599	-2.675	-2.698	-2.599
Crisis					
Dep.	-3.800	-3.740	-3.736	-3.665	-3.878
Ins.	-2.586	-2.530	-2.586	-2.563	-2.619
Bro.	-4.701	-4.655	-4.747	-4.586	-4.925
Oth.	-3.423	-3.363	-3.404	-3.257	-3.533
Post-crisis					
Dep.	-3.432	-3.476	-3.505	-3.225	-3.495
Ins.	-2.838	-2.836	-2.922	-2.914	-2.857
Bro.	-3.827	-3.801	-3.981	-3.890	-3.803
Oth.	-3.196	-3.257	-3.328	-3.117	-3.213

the models respectively. On the contrary, during the crisis sample in table 5.15, each model determines a different ranking. This fact can be explained by the fact that during the financial crisis the contagion among financial institutions is very high, hence it is not possible to achieve a well-defined ranking, as all the CoVaR values are very close each other.

Table 5.16 presents the comparison of the MES averages over the different sub-samples obtained by all the competing models. From table 5.16 the most risky group is Broker-Dealers over all the considered periods, while Insurance is the group with the lowest risk. During the pre-crisis sample Broker-Dealers are followed by Others, and Depositories, while over the crisis and the post-crisis periods Depositories and Others are reversed. All the models identify the same ranking based on the groups' riskiness (see table 5.16).

5.3.4 Case-study: Bank of America

Bank of America (BAC) is a financial institution chosen as representative of the sample given its worldwide importance. Figure $B.4$ in Appendix B shows the volatility of BAC and its correlation with the system estimated by the DCC-ACGARCH model. As expected, the volatility reaches the highest peak during the $2007-2009$ financial crisis as shown in figure $B.4a$. Observing the CoVaR measure displayed in figure $B.4c$, all the DCC-GARCH-type models seem very similar and show similar dynamics. Therefore, it is very difficult to discriminate among them. However, focusing on the pre-crisis sample presented in figure $B.4e$, it is evident that the point estimates and forecasts are very different among the competing models. A similar behavior can be seen for the MES measure (see figures $B.4d$ and $B.4f$). This finding confirms the need for statistical tools able to evaluate the point estimates and forecasts.

5.4 Concluding remarks

We have introduced the concept of long-range dependence in volatility, applying it in systemic risk context. In the previous chapter we have found a lack in the empirical results, hence we have investigated this additional stylized fact of financial data. At first, we have reviewed the main long-memory models existing in the literature, analyzing the different approaches and proposals. Then, we have developed a new model called the Asymmetric-Component-GARCH (ACGARCH) with the aim of capturing the long-range dependence jointly with the leverage effect, which has an important impact on forecasting systemic risk as shown by empirical results in Chapter 4. Finally, we have conducted the empirical analysis of the $2007-2009$ financial crisis as done in the previous chapter with the purpose of comparing the results of long-memory models with those obtained by the standard ones. For comparative purposes, we have also considered the quantile regression and the linear regression models as benchmarks.

All the DCC-GARCH-type models fit the data satisfactorily with a high statistical significance of the parameters and outperform significantly the benchmark models, highlighting the presence of long-range dependence. In particular, the leverage effect parameter is always statistical significant for the entire Broker-Dealers group. In addition, Broker-Dealers show the highest persistence among groups. The in-sample results indicate the Spline-GARCH model as the best one in estimating volatility, the CoVaR and MES measures and confirm the importance of considering long-range dependence with the aim for improving systemic risk estimates. These results are in contrast with those obtained by the standard econometric models used in the literature, highlighting the need of considering other stylized facts of financial data. Furthermore, CoVaR backtests have provided quite similar results across all the models, since the CoVaR failures tend to occur on the same days. This finding confirms the need to employ appropriate loss functions, in particular the novel TTL, in addition to the backtests, with the aim to discriminate among the competing models for achieving more accurate forecasts.

The out-of-sample results identify the GJR model as the best forecasting model for volatility through using the QL loss function. On the contrary, the TTL loss function, which is employed to evaluate the models performances in forecasting CoVaR, indicates that the DCC-FIGARCH model predicts more accurately the CoVaR and the long-memory models are preferable to the standard ones according to the Diebold-Mariano statistical test. For the MES measure, we have applied the novel TMSE loss function to assess the models performances in forecasting MES without finding a unique preferred best model. The DCC-GARCH is, in fact, the overall best prediction model for MES, but not for all the sub-industry groups. It is important to notice that all the models indicated for each group are special cases of the proposed ACGARCH model. Therefore, it is useful to consider its general specification in order to reach the most parsimonious and appropriate model. Investigating other versions of the TMSE, it is interesting to observe that dividing the TMSE by the standard deviation estimated by each considered model, the DCC-FIGARCH is found as the best MES predicting model as well as for the CoVaR.

Considering different historical periods, we have found that the best model identified for the pre-crisis period and the crisis period is different from the model for the post-crisis period. This is due to the fact that before and during the financial crisis the volatility and the systemic risk are higher than after the crisis and it is reasonable to employ the same model to forecast them, while during the post-crisis the situation is different and employing other models is necessary.

The riskiness and the ranking of the sub-industry groups found in the empirical analysis are consistent with those presented in the existing literature by authors, for example Adrian and Brunnermeier (2011), Girardi and Ergun (2013). The in-sample ranking obtained for CoVaR is the same as MES. Moreover, Broker-Dealers remain always the most risky sector, while Insurance are the less risky one. Finally, it is interesting to notice that the best forecasting model for the most risky group, i.e. Broker-Dealers, is the DCC-GJR that capture only the leverage effect, while those for less risky group is the DCC-ACGARCH which captures leverage effect jointly with long-range dependence.

Chapter 6

Conclusions

We have developed this thesis work with the twofold objective of evaluating the accuracy of the main systemic risk measures proposed in the literature, namely CoVaR and MES, and investigating the role of long-range dependence in forecasting systemic risk, proposing two appropriate loss functions for the CoVaR and MES frameworks, respectively, and a comprehensive model able to capture the leverage effect jointly with long-range dependence.

We have started from the review of the main systemic risk measures developed in the literature, which are divided into 4 different groups according to their structure, and we have focused on probability-distribution measures group. Among them, we have deeply analyzed two of the most widespread systemic risk measures, namely ΔCoVaR and SRISK, and their main components, namely CoVaR and MES, given their easy applicability and their large diffusion in the literature, continuously inspiring extensions and other developments. Then, we have introduced the widespread econometric modeling particularly employed to estimate these systemic risk measures, focusing on DCC-GARCH-type methodology for preparatory purposes.

The availability of different systemic risk measures and approaches highlights the need to identify which measure and model predict systemic risk more accurately, however statistical tools to test and compare systemic risk forecasts have not been properly developed and a deep analysis on their accuracy is largely unexplored. This lack in the existing literature leads us to face the problem of validating systemic risk measures, which is a key step towards the definition of a precise systemic risk measure. Hence, focusing on CoVaR and MES measures, we have revised the main existing backtests useful to evaluate CoVaR adequacy and the main existing loss functions in order to identify the more suitable ones to adapt to CoVaR and MES frameworks, respectively. As expected, we have pointed out that CoVaR backtests, obtained by extending those used for backtesting VaR (Kupiec, 1995; Christoffersen, 1998), do not discriminate among different competing models used to compute CoVaR, since they generally provide the same results across all models and have low power (Berkowitz, 2001; Escanciano and Pei, 2012). Therefore, employing appropriate loss functions specifically designed to assess the accuracy of the systemic risk forecasts and the forecasting ability of the considered models is necessary. The need of developing loss functions to assess the systemic risk accuracy leads us to our first methodological contribution to the existing literature. We indeed propose two appropriate loss functions, called the Tail Tick Loss (TTL) and the Tail Mean Square Error (TMSE) loss functions, developed in Chapter 3, which are suitable to CoVaR and MES frameworks respectively. These loss functions are also helpful in discriminating among different competing models in order to determine the models that forecasts more precisely CoVaR and MES measures, respectively. In this way, we are able to evaluate the forecasting performances of CoVaR and MES.

An empirical analysis of the $2007-2009$ financial crisis carried out on 91 US financial institutions daily return series confirms the need of a more fine-grained loss function with the aim of evaluating the forecasts accuracy. We have applied the econometric standard models on the time interval which spans from January 2000 to December 2012. Our empirical study, presented in Chapter 4, is based on the computation and prediction of the CoVaR and MES systemic risk measures by the DCC-GARCH and DCC-GJR models. The objective of our analysis is to evaluate the accuracy and performances of these models in forecasting CoVaR and MES in order to identify the best model which provides more accurately systemic risk forecasts. For comparative purposes, we have also considered the quantile regression and the linear regression models as benchmarks. We have found that the CoVaR backtesting results are very similar among different competing models, confirming our idea that appropriate loss functions are required. In addition, the out-of-sample results indicate that the DCC-GARCH-type models outperform significantly the benchmarks, in particular the DCC-GJR model is the best model in forecasting the conditional volatility and CoVaR measure. On the contrary, the DCC-GARCH model is the overall best prediction model for the MES measure, but it is not clear among the different sub-industry groups. Therefore, we think that other stylized facts, which are not captured by these models, should be investigated.

The consideration of taking into account others stylized facts leads us to the idea of investigating the role of the long-range dependence in the systemic risk framework. Capturing relevant stylized facts of financial data is, in fact, very important. In particular, we have focused on the leverage effect, which has already been considered in systemic risk framework (e.g. Girardi and Ergun, 2013; Brownlees and Engle, 2015), and the long-range dependence, which has not yet been considered in systemic risk framework (to the best of our knowledge), but many authors have shown its importance in other fields (e.g. Christoffersen et al., 2008; Guoa and Neely, 2008; Li et al., 2012). Hence, in Chapter 5, we have introduced the concept of long-range dependence in volatility and we have reviewed the main long-memory models existing in the literature, analyzing the different approaches and proposals. Consequently, as our second methodological contribution to the existing literature, we have developed a comprehensive model, called the Asymmetric-Component-GARCH (ACGARCH), that can be used in systemic risk measurement. This model is able to capture jointly important stylized facts of financial data such as the long-range dependence as well as the leverage effect, which has an important impact on forecasting systemic risk as seen in the previous empirical analysis. To the best of our knowledge, this is the first work that investigates the long-range dependence in systemic risk framework.

Finally, our third contribution to the existing literature is the comprehensive and exhaustive comparison of different bivariate volatility models for forecasting systemic risk, which extends the empirical analysis of the $2007-2009$ financial crisis done in the previous chapter with the purpose of comparing the results of long-memory models with those obtained by the standard ones. The computational estimation of this 6 bivariate models, without considering the benchmark models, required a considerable amount of time, as explained in Appendix $A.4$, jointly with some convergence problems. In particular, we have found that all the DCC-GARCH-type models fit the data satisfactorily with a high statistical significance of the parameters and outperform significantly the benchmark models, highlighting the presence of long-range dependence. The in-sample results indicate the Spline-GARCH model as the best one in estimating the volatility, CoVaR and MES measures and confirm the importance of considering long-range dependence with the aim for improving systemic risk estimates. There is, in fact, empirical evidence that both leverage effect and long-range dependence should be considered in measuring and forecasting systemic risk. These results are in contrast with those obtained by the standard econometric models used in the literature, highlighting the need of considering other stylized facts of financial data. Furthermore, CoVaR backtests have provided quite similar results across all the models, since the CoVaR failures tend to occur on the same days. This finding confirms the need to employ appropriate loss functions, in particular the proposed TTL, in addition to the backtests, with the aim to discriminate among the competing models for achieving more accurate forecasts. In the out-of-sample analysis, the Spline-GARCH model is excluded because it does not work very well, as explained in Section 5.3.2. Surely, understanding why the forecasting performance is poor is an interesting future development. The out-of-sample results identify the GJR model as the best forecasting model for volatility through using the QL loss function. On the contrary, the TTL loss function, which is employed to evaluate the models performances in forecasting CoVaR, indicates that the DCC-FIGARCH model predicts more accurately the CoVaR and the long-memory models are preferable to the standard ones according to the Diebold-Mariano statistical test. For the MES measure, we have applied the proposed TMSE loss function to assess the models performances in forecasting MES without finding a unique preferred best model. The DCC-GARCH is, in fact, the overall best prediction model for MES, but not for all the sub-industry groups. It is important to notice that all the models indicated for each group are special cases of the proposed ACGARCH model. Therefore, estimating ACGARCH is fundamental to identify the most appropriate nested model. The proposed ACGARCH model provides good in- and out-of-sample forecasts of systemic risk and outperforms significantly benchmarks (i.e. quantile regression and linear regression models) according to the Diebold-Mariano statistical test in estimating and forecasting the CoVaR and MES measures. In particular, ACGARCH improves systemic risk forecasts and its results are comparable with those obtained by other existing models. Investigating other versions of the TMSE, it is interesting to observe that dividing the TMSE by the standard deviation estimated by each considered model, the DCC-FIGARCH is found as the best MES predicting model as well as for the CoVaR. Moreover, our empirical analysis results are consistent with those presented in the existing literature by some authors, for example Adrian and Brunnermeier (2011), Girardi and Ergun (2013).

One of the purposes of this work is to obtain a precise systemic risk measurement, achieved by assessing the accuracy of systemic risk measures, which is an under-evaluated aspect in the existing literature. In particular, our solution to this issue, which consists of the adoption of two appropriate loss functions and a comprehensive model, implies relevant financial implications since systemic risk is nowadays important, mostly after the $2007-2009$ financial crisis, which developed the need of measuring systemic risk for the whole economy.

As possible further developments, considering other statistical distributions, such as the Skewed Student-t, the Generalized Error Distribution (GED) or the Normal-Inverse Gaussian (NIG) distribution, may provide a better fit. Further improvements may be obtained by adopting alternative multivariate models to estimate the conditional covariances, such as the Flexible DCC (Billio et al., 2006). Another aspect which should be further analyzed is the adoption of other statistical tools in order to test the difference among different competing models predictions, for example the Reality Check test (White, 2000) and the Superior Predictive Ability test (Hansen, 2001).

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Appendices

Appendix A

The empirical measures

In Sections 2.1.1 and 2.1.2 we have presented the CoVaR and MES measures, respectively, from the theoretical point of view. However, it is different when the empirical measure is computed. In this chapter, we provide the details and the algorithm procedure to compute empirically the CoVaR and MES measures in the framework outlined in section 2.2. In this work, we consider static VaR and static ES, where the conditional volatility is approximated by the standard deviation of the entire return time series considered. Similarly, it is possible to compute dynamic or time-varying CoVaR and MES, using as conditional volatility the estimation obtained by GARCH-type model (Girardi and Ergun, 2013). In particular, Section $A.1$ presents the empirical CoVaR by Adrian and Brunnermeier (2011), while Section $A.2$ explains how to obtain the empirical CoVaR proposed by Girardi and Ergun (2013). Then, Section $A.3$ reports the MES measure, and finally Section $A.4$ shows step by step the algorithm procedure to follow with the purpose to compute the empirical measures.

A.1 Empirical CoVaR measure by Adrian and Brunnermeier

The CoVaR measure, proposed by Adrian and Brunnermeier (2011) and presented in paragraph 2.1.1, can be computed empirically as it follows (see Bisias et al., 2012; Benoit et al. 2013; Girardi and Ergun, 2013):

$$\begin{aligned}\widehat{\text{CoVaR}}^{s|i}_{\alpha,t} &= \left(\hat\rho_{si,t} + \sqrt{1-\hat\rho^2_{si,t}}\right) \widehat{\text{VaR}}^{s}_{\alpha,t} &= \\ &= \left(\hat\rho_{si,t} + \sqrt{1-\hat\rho^2_{si,t}}\right) \frac{\hat\sigma_{s,t}}{\hat\sigma_{i,t}}\, \widehat{\text{VaR}}^{i}_{\alpha,t} &= \\ &= \left(\hat\rho_{si,t} + \sqrt{1-\hat\rho^2_{si,t}}\right) \frac{\hat\sigma_{s,t}}{\hat\sigma_{i,t}}\, \hat\sigma_{i,t}\, \Phi^{-1}(\alpha) &= \\ &= \left(\hat\rho_{si,t} + \sqrt{1-\hat\rho^2_{si,t}}\right) \hat\sigma_{s,t}\, \Phi^{-1}(\alpha) \end{aligned} \tag{A.1}$$

where $\hat\rho_{si,t}$ indicates the estimated correlation between the institution i and the system s, $\hat\sigma_{s,t}$ denotes the squared root of the estimated conditional variance of the system, and $\Phi^{-1}(\cdot)$ is the inverse of the cumulative distribution function of $z_{i,t}$. As a consequence, the ΔCoVaR measure is:

$$\begin{aligned}\Delta\widehat{\text{CoVaR}}^{s|i}_{\alpha,t} &= \widehat{\text{CoVaR}}^{s|r^i_t=\text{VaR}^i_{\alpha,t}}_{\alpha,t} - \widehat{\text{CoVaR}}^{s|r^i_t=\text{Med}(r^i_t)}_{\alpha,t} &= \\ &= \widehat{\text{CoVaR}}^{s|r^i_t=\text{VaR}^i_{\alpha,t}}_{\alpha,t} - \widehat{\text{CoVaR}}^{s|r^i_t=\text{VaR}^i_{0.50,t}}_{\alpha,t} &= \\ &= \left(\hat\rho_{si,t} + \sqrt{1-\hat\rho^2_{si,t}}\right) \widehat{\text{VaR}}^{s}_{\alpha,t} - \sqrt{1-\hat\rho^2_{si,t}}\, \widehat{\text{VaR}}^{s}_{\alpha,t} &= \\ &= \left(\hat\rho_{si,t} + \sqrt{1-\hat\rho^2_{si,t}} - \sqrt{1-\hat\rho^2_{si,t}}\right) \widehat{\text{VaR}}^{s}_{\alpha,t} &= \\ &= \hat\rho_{si,t}\, \widehat{\text{VaR}}^{s}_{\alpha,t} \quad = \quad \hat\rho_{si,t}\, \hat\sigma_{s,t}\, \Phi^{-1}(\alpha) \end{aligned} \tag{A.2}$$

where $r^i_t = \text{Med}(r^i_t)$ indicates the conditioning event when the financial institution i is not in distress, i.e. it is in its median state.

A.2 Empirical CoVaR measure by Girardi and Ergun

The generalization of the CoVaR measure, proposed by Girardi and Ergun (2013), on the contrary, does not have a closed form and becomes:

$$Pr\left[r^s_t \le \text{CoVaR}^{s|i}_{\alpha,t} \middle| r^i_t \le \text{VaR}^i_{\alpha,t}\right] = \alpha \quad \Longrightarrow \quad \frac{Pr\left[r^s_t \le \text{CoVaR}^{s|i}_{\alpha,t}, r^i_t \le \text{VaR}^i_{\alpha,t}\right]}{Pr\left[r^i_t \le \text{VaR}^i_{\alpha,t}\right]} = \alpha$$

By definition of $\text{VaR}^i_{\alpha,t}$, it holds:

$$Pr\left[r^s_t \le \text{CoVaR}^{s|i}_{\alpha,t}, r^i_t \le \text{VaR}^i_{\alpha,t}\right] = \alpha^2$$

Hence, $\widehat{\text{CoVaR}}^{s|i}_{\alpha,t}$ is obtained by solving numerically the following double integral:

$$\int_{-\infty}^{\text{CoVaR}^{s|i}_{\alpha,t}} \int_{-\infty}^{\text{VaR}^{i}_{\alpha,t}} pdf_t(r^s_t, r^i_t) dr^s_t dr^i_t = \alpha^2 \tag{A.3}$$

where $pdf_t(r^s_t, r^i_t)$ is the bivariate density for the institution-system return pair.

The same procedure is adopted for calculating the CoVaR measure conditional on the benchmark state of institution i, indicated by $\text{CoVaR}^{s|b_i}_{\alpha,t}$. It is defined as one standard deviation about the mean event $\mu_{i,t} - \sigma_{i,t} \leq r^i_t \leq \mu_{i,t} + \sigma_{i,t}$ where $\mu_{i,t}$ and $\sigma_{i,t}$ are, respectively, the conditional mean and the conditional standard deviation of the financial institution i. The joint probability is then:

$$Pr\left[r^s_t \leq \text{CoVaR}^{s|b_i}_{\alpha,t}, \mu_{i,t} - \sigma_{i,t} \leq r^i_t \leq \mu_{i,t} + \sigma_{i,t}\right] = p_{i,t}\, \alpha$$

where $p_{i,t} = Pr\left[\mu_{i,t} - \sigma_{i,t} \leq r^i_t \leq \mu_{i,t} + \sigma_{i,t}\right]$ is the marginal probability of institution i. Finally, the double integral to be solved numerically for $\text{CoVaR}^{s|b_i}_{\alpha,t}$ is:

$$\int_{-\infty}^{\text{CoVaR}^{s|b_i}_{\alpha,t}} \int_{-\infty}^{\text{VaR}^{s|i}_{\alpha,t}} pdf_t(r^s_t, r^i_t) dr^s_t dr^i_t = \alpha^2 \tag{A.4}$$

Then, the ΔCoVaR (%) measure, that is the contribution of the institution to systemic risk, is obtained by the following equation:

$$\Delta\widehat{\text{CoVaR}}^{s|i}_{\alpha,t}(\%) = 100\, \frac{\Delta\widehat{\text{CoVaR}}^{s|i}_{\alpha,t}}{\widehat{\text{CoVaR}}^{s|b_i}_{\alpha,t}} = 100\, \frac{\widehat{\text{CoVaR}}^{s|i}_{\alpha,t} - \widehat{\text{CoVaR}}^{s|b_i}_{\alpha,t}}{\widehat{\text{CoVaR}}^{s|b_i}_{\alpha,t}}$$

The double integrals in equations A.3 and A.4 are solved simulating two different draws, one for the system and the other for the financial institution, from truncated bivariate normal variables and taking the α-quantile of the first one. The need for simulation of truncated multivariate normal variables is due to the fact that an extreme of the integral is fixed, in particular the firm returns have to be under their VaR ($r^i_t < \text{VaR}^i_{\alpha,t}$). Hence, we simulate the bivariate time series[1], the former related to the system and the latter related to the financial institution, constraint to the firm returns under their VaR and we obtain two draws. Then, we consider only the first draw related the system and we compute the its α-quantile, usually equals to $\alpha = 5\%$. Therefore, this α-quantile of the system draw is the CoVaR measure, that we are looking for.

[1]We simulate truncated multivariate normal variables using the R package tmvtnorm (see Wilhelm and Manjunath, 2010).

A.3 Empirical MES measure

The theoretical MES measure, proposed by Acharya et al. (2010) and presented in paragraph 2.1.2, can be written as it follows (see Bisias et al., 2012; Benoit et al. 2013; Brownlees and Engle, 2015):

$$\mathrm{MES}^{i|s}_{\alpha,t} = \mathbb{E}_t\left[r^i_t \middle| r^s_t \leq C\right] =$$

$$= \sigma_{i,t}\rho_{si,t}\,\mathbb{E}_t\left[z_{s,t}\middle| z_{s,t} \leq \frac{C}{\sigma_{s,t}}\right] + \sigma_{i,t}\sqrt{1-\rho^2_{si,t}}\,\mathbb{E}_t\left[z_{i,t}\middle| z_{s,t} \leq \frac{C}{\sigma_{s,t}}\right]$$

where C indicates a threshold value to represent the systemic event.
When $C = \mathrm{VaR}^s_{\alpha,t}$, then it holds $\mathbb{E}_t\left[r_{s,t}\middle| r_{s,t} \leq \mathrm{VaR}^s_{\alpha,t}\right] = \mathrm{ES}^s_{\alpha,t}$, where ES denotes the system expected shortfall. Assuming also that $z_{s,t}$ and $z_{i,t}$ are independent, then:

$$\mathrm{MES}^{i|s}_{\alpha,t} = \sigma_{i,t}\rho_{si,t}\,\mathbb{E}_t\left[z_{s,t}\middle| z_{s,t} \leq \mathrm{VaR}^s_{\alpha,t}\right] = \frac{\rho_{si,t}\sigma_{i,t}}{\sigma_{s,t}}\,\mathrm{ES}^s_{\alpha,t}$$

Hence, the MES measure can be empirically computed as (see Bisias et al., 2012; Benoit et al. 2013; Brownlees and Engle, 2015):

$$\widehat{\mathrm{MES}}^{i|s}_{\alpha,t} = \frac{\hat{\rho}_{si,t}\hat{\sigma}_{i,t}}{\hat{\sigma}_{s,t}}\,\widehat{\mathrm{ES}}^s_{\alpha,t} = -\hat{\rho}_{si,t}\hat{\sigma}_{i,t}\,\frac{\phi(\Phi^{-1}(\alpha))}{\alpha}$$

where $\hat{\rho}_{si,t}$ are the estimated correlations, $\hat{\sigma}^2_{i,t}$ and $\hat{\sigma}^2_{s,t}$ are the estimated conditional variances of, respectively, the financial institution and the financial system, $\phi(\cdot)$ is the pdf of the true distribution of $z_{i,t}$ and $\Phi^{-1}(\cdot)$ is the inverse of the cumulative distribution function of $z_{i,t}$.

A.4 Algorithm procedure for empirical measures

For each financial institution i and the financial system s, the procedure to compute systemic risk measures at each time t is the following, please note that regarding the MES measure the procedure stops at the fourth step:

1. estimation of the conditional variances $\hat{\sigma}^2_{s,t}$ and $\hat{\sigma}^2_{i,t}$ obtained by GARCH-type model for the financial institutions i and the financial system s, respectively;

2. computation of:

   - static Value-at-Risk of the financial system s: $\widehat{\text{VaR}}^{s}_{\alpha} = \hat{\sigma}_s \, \Phi^{-1}(\alpha)$
   - static Value-at-Risk of the financial institution i: $\widehat{\text{VaR}}^{i}_{\alpha} = \hat{\sigma}_i \, \Phi^{-1}(\alpha)$
   - static Expected Shortfall of the financial system: $\widehat{\text{ES}}^{s}_{\alpha} = -\hat{\sigma}_s \, \dfrac{\phi(\Phi^{-1}(\alpha))}{\alpha}$

   where $\hat{\sigma}_s$ and $\hat{\sigma}_i$ are the standard deviations of the time series returns of the financial system and the financial institution, respectively, $\Phi^{-1}(\cdot)$ is the inverse of the cumulative distribution function of $z_{i,t}$ and $\phi(\cdot)$ is the pdf of the true distribution of $z_{i,t}$;

3. estimation of the conditional correlations $\hat{\rho}_{si,t}$ obtained by the DCC-GARCH-type model;

4. computation of, using the conditional correlations obtained at the previous step:

   - CoVaR measure (Adrian and Brunnermeier, 2011) when the financial institution is in a crisis period, using the static VaR of the financial system obtained at the 1st step:

   $$\widehat{\text{CoVaR}}^{s|r^i_t=\text{VaR}^i_\alpha}_{\alpha,t} = \left(\hat{\rho}_{si,t} + \sqrt{1-\hat{\rho}^2_{si,t}}\right)\widehat{\text{VaR}}^{s}_{\alpha}$$

   - CoVaR measure (Girardi and Ergun, 2013) when the financial institution is in a crisis period, using the static VaR of financial institution obtained at the 1st step and solving the double integral in equation A.3;
   - MES measure, using the static ES of the financial system obtained at the 1st step:

   $$\widehat{\text{MES}}^{i|s}_{\alpha,t} = \frac{\hat{\rho}_{si,t}\sqrt{\hat{\sigma}^2_{i,t}}}{\sqrt{\hat{\sigma}^2_{s,t}}}\,\widehat{\text{ES}}^{s}_{\alpha}$$

5. computation of, using the conditional correlations obtained at the 3rd step:

   - CoVaR measure (Adrian and Brunnermeier, 2011) at the median state, i.e. when the financial institution is in a stable period:

$$\widehat{\text{CoVaR}}_{\alpha,t}^{s|r_t^i=\text{Med}(r_t^i)} = \widehat{\text{CoVaR}}_{\alpha,t}^{s|r_t^i=\text{VaR}_{0.50}^i} = \sqrt{1-\hat{\rho}_{si,t}^2}\,\widehat{\text{VaR}}_{\alpha}^{s}$$

- CoVaR measure (Girardi and Ergun, 2013) when the financial institution is in the median state, solving the double integral in equation A.4;

6. computation of the difference between the two CoVaR measures obtained at 4th and 5th steps, in particular:

   - ΔCoVaR measure (Adrian and Brunnermeier, 2011):

$$\Delta\widehat{\text{CoVaR}}_{\alpha,t}^{s|i} = \widehat{\text{CoVaR}}_{\alpha,t}^{s|r_t^i=\text{VaR}_{\alpha}^i} - \widehat{\text{CoVaR}}_{\alpha,t}^{s|r_t^i=\text{Med}(r_t^i)} = \hat{\rho}_{si,t}\widehat{\text{VaR}}_{\alpha}^{s}$$

   - ΔCoVaR (%) measure (Girardi and Ergun, 2013):

$$\Delta\widehat{\text{CoVaR}}_{\alpha,t}^{s|i}(\%) = 100\,\frac{\Delta\widehat{\text{CoVaR}}_{\alpha,t}^{s|i}}{\widehat{\text{CoVaR}}_{\alpha,t}^{s|b_i}}$$

The computational time required by the empirical analysis described in Chapters 4 and 5 is very high and is different according to the considered model to compute the CoVaR and MES measures. As explained in Section 4.1, during the forecasting period each model is separately re-estimated every week (i.e. every 5 observations) using all data available until as of that date, for a total of 332 times, and the forecasts are computed one-step-ahead, for a total of 1658 forecasts. This re-estimation increases the computational time. To give an idea about the total amount of time necessary to carry out the entire empirical analysis, we report a list of the estimation and forecasting computational time that is required by each considered models on MATLAB software. This list regards the computational time necessary for only a system-institution pair, hence the overall time is obtained by multiplying each time for 91 financial institutions. The list is the following:

- the DCC-GARCH model runs in 82 minutes;
- the DCC-GJR model runs in 93 minutes;
- the DCC-CGARCH model runs in 108 minutes;
- the DCC-Spline-GARCH model runs in 158 minutes;

- the DCC-FIGARCH model runs in 128 minutes;
- the DCC-ACGARCH model runs in 114 minutes.

Hence, the total time for estimating and forecasting the conditional variances and correlations of a system-institution return pair is approximately equal to 11 hours. Moreover, the computation of the CoVaR measure in equation $A.3$ is obtained by using the simulations from a truncated multivariate normal distribution and runs in 16 minutes on R software. In order to obtain the CoVaR and MES series for a financial institution, at least 12 hours are necessary, without any convergence problem.

Appendix B

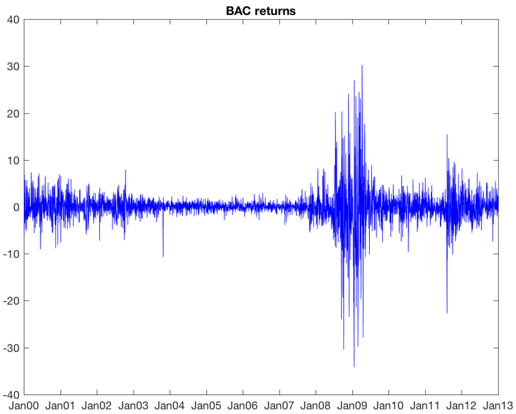
Tables

This chapter reports the tables and the figures used in this work, in particular in Chapters 4 and 5.

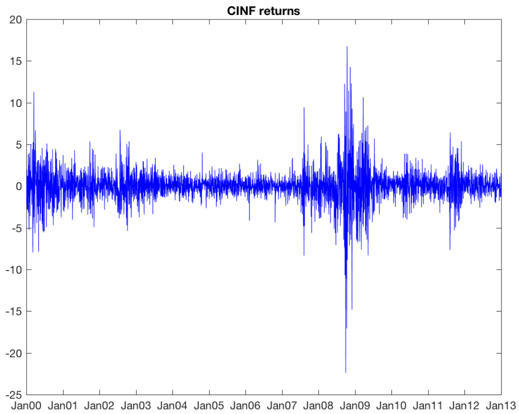
Table B.1: Tickers and summary statistics classified by financial industry groups.

Ticker	Company Name	Obs.	From	To	Mean	St.Dev.	Min.	$q_{0.25}$	$q_{0.50}$	$q_{0.75}$	Max.	Skew.	Kurt.
CRSP		3269	2000-01-03	2012-12-31	-0.007	1.135	-6.01	-0.611	0.042	0.588	5.57	0.05	5.28
Depositories (28)													
BAC	Bank Of America	3269	2000-01-03	2012-12-31	-0.011	3.362	-34.21	-1.035	0.012	1.025	30.21	-0.31	25.00
BBT	BB&T Corporation	3269	2000-01-03	2012-12-31	0.016	2.396	-26.61	-0.945	0.000	0.953	21.20	0.06	18.55
BK	Bank of New York Mellon	3269	2000-01-03	2012-12-31	-0.005	2.718	-31.69	-1.089	-0.025	1.050	22.16	-0.07	18.15
C	Citigroup	3269	2000-01-03	2012-12-31	-0.062	3.586	-49.47	-1.162	0.000	1.073	45.63	-0.48	36.10
CBH	Commerce Bank	2071	2000-01-03	2008-03-31	0.068	1.855	-10.58	-0.965	0.000	1.050	10.26	0.06	6.40
CMA	Comerica	3269	2000-01-03	2012-12-31	0.000	2.694	-22.69	-1.032	-0.019	1.047	18.81	-0.19	13.40
HBAN	Huntingtons Bancshares	3269	2000-01-03	2012-12-31	-0.023	3.861	-36.51	-1.122	0.000	1.078	40.60	0.37	27.39
HCBK	Hudson City Bancorp.	3269	2000-01-03	2012-12-31	0.053	1.915	-15.10	-0.697	0.028	0.831	14.57	-0.15	12.40
JPM	JP Morgan Chase	3269	2000-01-03	2012-12-31	0.007	2.839	-23.23	-1.158	0.000	1.124	22.39	0.27	13.92
KEY	Keycorp	2644	2000-05-18	2010-11-23	-0.034	3.454	-40.55	-1.015	0.000	1.029	43.34	-0.44	33.85
MI	Marshall & Ilsley	2895	2000-01-03	2011-07-06	-0.030	3.436	-30.15	-0.978	0.000	0.951	32.93	-0.11	24.49
MTB	M&T Bank	3269	2000-01-03	2012-12-31	0.036	2.170	-16.97	-0.834	0.041	0.893	19.11	0.19	12.88
NCC	National City Corporation	2264	2000-01-03	2009-01-02	-0.097	4.004	-100.36	-0.907	0.000	0.886	50.16	-7.54	213.0
NTRS	Northern Trust	3269	2000-01-03	2012-12-31	0.005	2.444	-20.84	-1.027	0.000	1.050	26.94	0.39	15.16
NYB	New York Community Bancorp.	3269	2000-01-03	2012-12-31	0.041	2.174	-14.86	-0.914	0.058	1.019	13.06	-0.18	9.76
PBCT	Peoples United Financial	3269	2000-01-03	2012-12-31	0.048	1.857	-17.00	-0.866	0.020	0.914	16.45	0.11	11.82
PNC	PNC Financial Services	2644	2000-05-18	2010-11-23	0.009	2.864	-53.44	-0.987	0.000	0.950	31.55	-1.49	61.22
RF	Regions Financial	3269	2000-01-03	2012-12-31	-0.018	3.603	-52.88	-1.066	0.000	1.088	39.48	-0.49	36.17
SNV	Synovus Financial	3269	2000-01-03	2012-12-31	-0.030	3.362	-30.07	-1.207	0.000	1.200	24.86	-0.15	14.01
SOV	Sovereign Bank	2184	2000-05-18	2009-01-29	-0.043	4.322	-127.88	-1.074	0.000	1.106	52.79	-11.23	366.0
STI	Suntrust Banks	2644	2000-05-18	2010-11-23	-0.030	3.174	-31.71	-1.001	-0.028	0.927	26.67	-0.36	21.79
STT	State Street	3269	2000-01-03	2012-12-31	0.012	3.267	-89.25	-1.102	0.000	1.182	27.27	-5.97	181.7
UB	UnionBanCal	2224	2000-01-03	2008-11-04	0.040	2.124	-36.39	-0.817	0.036	0.878	14.14	-2.05	47.86
USB	US Bancorp	2644	2000-05-18	2010-11-23	0.004	2.559	-20.05	-0.960	0.000	0.918	20.57	-0.13	15.41
WAMUQ	Washington Mutual	2197	2000-01-03	2008-09-26	-0.198	6.153	-235.48	-0.973	0.024	0.949	39.7	-25.68	979.0
WB	Wachovia Bank	2264	2000-01-03	2009-01-02	-0.063	5.129	-169.28	-0.959	0.000	0.968	64.30	-14.45	545.9
WFC	Wells Fargo	3269	2000-01-03	2012-12-31	0.027	2.755	-27.21	-0.884	0.000	0.890	28.34	0.83	25.29
ZION	Zions Bancorporation	3269	2000-01-03	2012-12-31	-0.025	3.255	-28.16	-1.093	0.000	1.079	24.34	-0.10	17.46
Insurance (34)													
ABKFQ	AMBAC Financial	2730	2000-01-03	2010-11-08	-0.151	6.645	-73.166	-1.337	-0.015	1.153	71.56	-0.55	34.94
AET	AETNA	2427	2001-04-02	2010-11-23	0.050	2.504	-22.703	-1.081	0.047	1.308	16.34	-0.77	13.48
AFL	AFLAC	3269	2000-01-03	2012-12-31	0.031	2.721	-45.985	-0.948	0.021	0.944	26.45	-1.28	39.50
AIG	AIG	3269	2000-01-03	2012-12-31	-0.106	4.528	-93.626	-1.138	-0.053	1.081	50.68	-2.94	89.04
AIZ	Assurant	2242	2004-02-05	2012-12-31	0.021	2.449	-28.679	-0.862	0.061	0.991	19.72	-0.73	23.38
ALL	Allstate	3269	2000-01-03	2012-12-31	0.026	2.262	-23.799	-0.815	0.030	0.854	19.63	-0.68	22.69
AON	AON Corporation	2644	2000-05-18	2010-11-23	0.010	2.129	-36.140	-0.830	0.040	0.913	19.96	-2.08	46.62
BRK.B	Berkshire Hathaway Shares	3269	2000-01-03	2012-12-31	0.027	1.567	-12.883	-0.595	-0.004	0.633	14.95	0.78	16.14
CB	Chubb	3269	2000-01-03	2012-12-31	0.039	1.894	-13.420	-0.809	0.015	0.860	15.54	0.38	11.38
CFC	Countrywide Financial	2136	2000-01-03	2008-07-01	-0.011	3.047	-33.412	-1.286	0.000	1.256	41.45	1.10	34.04
CI	CIGNA	2644	2000-05-18	2010-11-23	0.011	2.800	-47.910	-1.006	0.064	1.188	21.14	-2.24	44.64
CINF	Cincinnati Financial	3269	2000-01-03	2012-12-31	0.025	1.945	-22.399	-0.778	0.023	0.875	16.77	-0.33	17.91

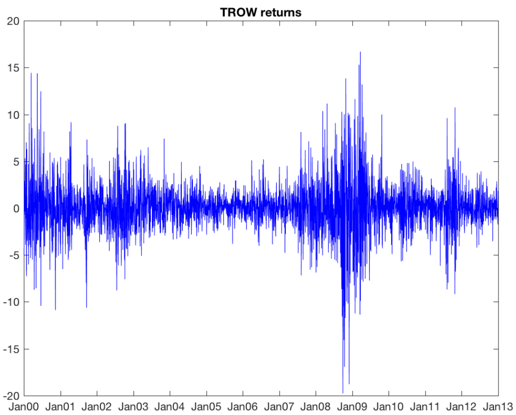
Ticker	Company Name	**Obs.**	**From**	**To**	Mean	St.Dev.	Min.	$q_{0.25}$	$q_{0.50}$	$q_{0.75}$	Max.	Skew.	Kurt.
CNA	CNA Financial	3269	2000-01-03	2012-12-31	-0.005	2.548	-39.476	-0.981	0.000	0.928	24.106	-1.15	34.04
CVH	Coventry Health Care	3269	2000-01-03	2012-12-31	0.083	3.160	-71.551	-1.097	0.101	1.374	18.499	-4.06	89.44
FNF	Fidelity Natinal Financial	1814	2005-10-13	2012-12-31	0.023	2.850	-15.982	-1.111	0.000	1.109	29.633	0.89	17.92
GNW	Genworth Financial	2166	2004-05-25	2012-12-31	-0.041	5.623	-78.552	-1.347	0.043	1.348	63.599	-0.04	52.47
HIG	Hartford Financial Services	3269	2000-01-03	2012-12-31	-0.014	4.161	-72.486	-1.169	0.024	1.234	70.487	-0.39	71.57
HNT	Health Net	3269	2000-01-03	2012-12-31	0.027	2.634	-28.695	-1.148	0.066	1.293	15.043	-1.09	14.99
HUM	Humana	3269	2000-01-03	2012-12-31	0.066	2.813	-28.016	-1.232	0.012	1.398	14.66	-0.94	14.06
L	Loews	3269	2000-01-03	2012-12-31	0.046	2.045	-19.939	-0.789	0.052	0.919	21.220	-0.46	18.87
LNC	Lincoln National	3269	2000-01-03	2012-12-31	-0.004	3.805	-50.891	-1.157	0.000	1.157	36.235	-1.14	40.44
MBI	MBIA	3269	2000-01-03	2012-12-31	-0.042	4.405	-41.264	-1.423	-0.028	1.265	38.219	-0.02	18.58
MET	MetLife	3204	2000-04-05	2012-12-31	0.031	2.956	-31.156	-1.039	0.000	1.103	24.686	-0.31	21.78
MMC	Marsh & Mclennan Companies	3269	2000-01-03	2012-12-31	0.001	2.009	-28.041	-0.917	0.000	0.885	13.480	-0.84	21.63
PFG	Principal Financial Group	2817	2001-10-23	2012-12-31	0.018	3.397	-35.090	-0.994	0.040	1.119	34.190	-0.36	25.44
PGR	Progressive Corporation	3269	2000-01-03	2012-12-31	0.046	2.080	-21.357	-0.904	0.000	0.963	21.490	0.24	17.26
PRU	Prudential Financial	2781	2001-12-13	2012-12-31	0.029	3.262	-28.394	-0.975	0.042	1.139	32.39	-0.02	23.99
SAF	Safeco	2194	2000-01-03	2008-09-23	0.057	1.984	-19.622	-0.779	0.024	0.786	37.698	2.81	69.35
TMK	Torchmark	3269	2000-01-03	2012-12-31	0.034	2.128	-15.228	-0.752	0.053	0.855	15.867	-0.25	15.14
TRV	Travelers	2644	2000-05-18	2010-11-23	0.016	2.145	-20.067	-0.937	0.000	0.928	22.758	0.29	16.96
UNH	UnitedHealth Group	3269	2000-01-03	2012-12-31	0.066	2.214	-20.624	-0.956	0.061	1.126	29.829	0.266	21.262
UNM	UNUM Group	3269	2000-01-03	2012-12-31	-0.006	3.151	-49.713	-1.060	0.000	1.156	20.010	-2.893	51.058
WLP	Wellpoint	2812	2001-10-30	2012-12-31	0.040	2.104	-33.278	-0.941	0.062	1.109	16.901	-1.610	29.262
WRB	W.R. Berkley	3269	2000-01-03	2012-12-31	0.072	1.869	-14.446	-0.846	0.026	0.885	15.049	0.590	11.998
Broker-Dealers (10)													
AGE	A.G. Edwards	1947	2000-01-03	2007-10-01	0.056	2.009	-13.09	-1.015	0.028	1.045	13.34	0.24	6.96
BSC	Bear Stearns	2115	2000-01-03	2008-06-02	-0.068	4.979	-183.05	-1.122	-0.010	1.190	63.53	-23.46	889.1
ETFC	E-Trade	3269	2000-01-03	2012-12-31	-0.103	4.706	-88.37	-2.067	-0.045	1.739	40.55	-1.61	48.75
GS	Goldman Sachs	3269	2000-01-03	2012-12-31	0.013	2.643	-21.02	-1.202	0.000	1.242	23.48	0.29	13.31
LEH	Lehman Brothers	2190	2000-01-03	2008-09-17	-0.229	7.358	-285.5	-1.375	0.014	1.336	38.13	-27.61	1044.6
MER	Merrill Lynch	2264	2000-01-03	2009-01-02	-0.049	3.312	-30.00	-1.358	-0.048	1.202	32.39	-0.09	22.96
MS	Morgan Stanley	3269	2000-01-03	2012-12-31	-0.028	3.563	-29.97	-1.443	-0.018	1.389	62.59	1.30	42.64
SCHW	Charles Schwab	3269	2000-01-03	2012-12-31	-0.012	3.053	-21.12	-1.567	-0.088	1.446	23.22	0.34	7.89
TROW	T. Rowe Price	3269	2000-01-03	2012-12-31	0.046	2.665	-19.77	-1.179	0.018	1.226	16.69	-0.01	9.15
Others (22)													
ACAS	American Capital	3269	2000-01-03	2012-12-31	0.015	3.856	-55.94	-1.024	0.114	1.227	34.58	-1.08	38.71
AMP	Ameriprise Financial	1824	2005-10-03	2012-12-31	0.039	3.220	-22.55	-1.204	0.092	1.347	24.54	0.12	12.90
AMTD	TD Ameritrade	3269	2000-01-03	2012-12-31	0.002	3.437	-20.19	-1.659	0.000	1.561	19.85	0.26	7.80
AXP	American Express	3269	2000-01-03	2012-12-31	0.010	2.563	-19.35	-1.012	0.000	1.093	18.77	0.01	10.95
BEN	Franklin Resources	3269	2000-01-03	2012-12-31	0.048	2.380	-16.58	-1.061	0.062	1.243	14.94	-0.11	8.71
BLK	Blackrock	3269	2000-01-03	2012-12-31	0.082	2.459	-16.87	-1.004	0.025	1.151	17.87	0.12	9.17
BOT	CBOT Holdings	435	2005-10-19	2007-07-13	0.239	3.074	-11.66	-1.278	0.137	1.321	20.89	1.71	13.79
CBG	C.B. Richard Ellis Group	2155	2004-06-10	2012-12-31	0.055	4.424	-34.79	-1.615	0.061	1.819	49.47	0.69	20.87
CBSS	Compass Bancshares	1931	2000-01-03	2007-09-07	0.068	1.623	-10.01	-0.695	0.058	0.821	20.20	0.91	18.37
CIT	CIT Group	1849	2002-07-02	2009-11-02	-0.233	7.244	-138.63	-1.327	0.000	1.284	57.98	-5.27	109.1
CME	CME Group	2534	2002-12-06	2012-12-31	0.077	2.668	-22.06	-1.132	0.066	1.258	14.93	-0.29	11.29
COF	Capital One Financial	3269	2000-01-03	2012-12-31	0.008	3.496	-50.69	-1.361	0.016	1.429	23.45	-1.18	24.49
FITB	Fifth Third Bancorp.	3269	2000-01-03	2012-12-31	-0.026	3.771	-57.32	-1.131	0.000	1.104	47.23	-0.30	49.68
FNM	Fannie Mae	2182	2000-01-03	2008-09-05	-0.091	3.134	-31.94	-1.117	-0.059	0.979	26.88	-1.13	29.04
FRE	Freddie Mac	2182	2000-01-03	2008-09-05	-0.093	3.133	-33.82	-1.073	-0.058	0.931	26.11	-1.41	31.98
ICE	Intercontinental Exchange	1792	2005-11-16	2012-12-31	0.064	3.547	-19.50	-1.427	-0.018	1.65	34.22	0.22	12.64
JNS	Janus Capital	3148	2000-06-26	2012-12-31	-0.045	3.493	-29.30	-1.616	-0.033	1.525	25.42	-0.10	9.61
LM	Legg Mason	3269	2000-01-03	2012-12-31	0.006	3.047	-28.44	-1.336	-0.011	1.336	26.51	-0.33	13.65
SEIC	SEI Investments	3269	2000-01-03	2012-12-31	0.029	2.676	-18.32	-1.144	0.000	1.220	16.49	-0.08	8.56
SLM	SLM Corporation	3269	2000-01-03	2012-12-31	0.011	3.431	-39.06	-1.028	0.000	1.155	29.99	-0.94	25.25



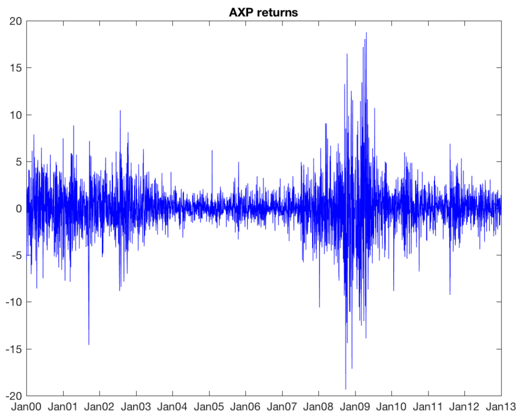
(a) From Depositories group.



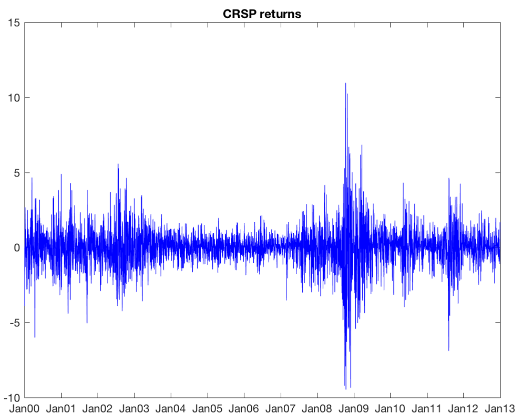
(b) From Insurance group.



(c) From Broker-Dealers group.



(d) From Others group.



(e) Market index returns.

Figure B.1: Plot of the return series of financial institutions, one from each financial industry group.

Table B.2: Parameter estimates of the DCC-GARCH model.

	GARCH			DCC		
Depositories	ω	α	β	const	a	b
BAC	0.001	0.006	0.991***	0.633***	0.039**	0.942***
BBT	0.054***	0.118***	0.856***	0.662***	0.064	0.896***
BK	0.010	0.030	0.967***	0.617***	0.036**	0.941***
C	0.014	0.065**	0.932***	0.726***	0.035	0.950***
CBH	0.773*	0.171**	0.614***	0.494***	0.011*	0.981***
CMA	0.080**	0.126***	0.855***	0.594***	0.007	0.988***
HBAN	0.028	0.084**	0.908***	0.599***	0.028***	0.970***
HCBK	0.385	0.221*	0.560*	0.357***	0.023***	0.973***
JPM	0.007	0.058**	0.942***	0.714***	0.013	0.984***
KEY	0.012	0.035**	0.957***	0.602***	0.011***	0.987***
MI	0.026**	0.072***	0.914***	0.614***	0.015***	0.984***
MTB	0.027	0.051*	0.936***	0.583***	0.034***	0.962***
NCC	0.024	0.047*	0.940***	0.628***	0.017***	0.980***
NTRS	0.057	0.061	0.923**	0.655***	0.012	0.986***
NYB	0.143	0.130*	0.836***	0.374***	0.054	0.914***
PBCT	0.066	0.104***	0.878***	0.417***	0.021***	0.973***
PNC	0.031**	0.083**	0.908***	0.631***	0.006	0.990***
RF	0.057***	0.118***	0.855***	0.617***	0.039	0.954***
SNV	0.082*	0.112***	0.863***	0.618***	0.058**	0.890***
SOV	0.031*	0.087***	0.909***	0.498***	0.016***	0.973***
STI	0.040	0.069*	0.911***	0.652***	0.011**	0.985***
STT	0.145**	0.109***	0.862***	0.625***	0.009	0.981***
UB	0.037*	0.141***	0.859***	0.499***	0.016**	0.976***
USB	0.017	0.091***	0.908***	0.599***	0.013*	0.984***
WAMUQ	0.036	0.077*	0.915***	0.433***	0.024***	0.972***
WB	0.045***	0.109***	0.878***	0.643***	0.021***	0.977***
WFC	0.005	0.033	0.961***	0.627***	0.017	0.981***
ZION	0.044*	0.082**	0.903***	0.545***	0.045	0.933***

	GARCH			DCC		
Insurance	ω	α	β	const	a	b
ABKFQ	0.078**	0.145***	0.848***	0.494***	0.004	0.993***
AET	0.456**	0.067	0.834***	0.357***	0.063***	0.889***
AFL	0.007	0.012	0.984***	0.433***	0.086***	0.766***
AIG	0.086**	0.105***	0.873***	0.579***	0.046	0.944***
AIZ	1.552***	0.038	0.000	0.309***	0.078	0.486***
ALL	0.002	0.007	0.990***	0.467***	0.005*	0.993***
AON	0.770**	0.212**	0.691***	0.390***	0.121	0.000
BRK.B	0.030	0.120	0.873***	0.233***	0.037**	0.776***
CB	0.073*	0.114***	0.868***	0.513***	0.084***	0.476**
CFC	0.043	0.024	0.967***	0.442***	0.010***	0.985***
CI	0.323	0.214***	0.786***	0.324***	0.079**	0.698***
CINF	0.020**	0.076***	0.915***	0.583***	0.015***	0.984***
CNA	0.117	0.077***	0.888***	0.417***	0.064	0.807***
CVH	0.183	0.075**	0.906***	0.334***	0.011	0.954***
FNF	2.269***	0.356***	0.000***	0.190*	0.030	0.910***
GNW	1.311***	0.174***	0.000***	0.340***	0.000	0.994***
HIG	0.083***	0.122***	0.862***	0.581***	0.018	0.978***
HNT	0.128	0.056	0.920***	0.315***	0.017**	0.962***
HUM	0.011***	0.000	0.997***	0.338***	0.010	0.983***
L	0.027	0.057	0.934***	0.463***	0.049	0.925***
LNC	0.029	0.051**	0.941***	0.619***	0.014***	0.984***
MBI	0.120*	0.121***	0.850***	0.549***	0.008***	0.991***
MET	0.046*	0.069***	0.918***	0.522***	0.007**	0.990***
MMC	0.558	0.093***	0.780***	0.530***	0.011***	0.986***
PFG	0.105**	0.122***	0.832***	0.555***	0.021**	0.972***
PGR	0.008	0.025**	0.972***	0.499***	0.013**	0.982***
PRU	0.057**	0.064***	0.911***	0.599***	0.013**	0.979***
SAF	0.040**	0.084***	0.904***	0.479***	0.018***	0.979***
TMK	0.029*	0.093***	0.894***	0.590***	0.014**	0.983***
TRV	0.196*	0.098***	0.844***	0.506***	0.020	0.977***
UNH	0.148***	0.091***	0.865***	0.338***	0.028**	0.938***
UNM	0.049	0.028	0.967***	0.349***	0.016	0.967***
WLP	0.225**	0.089***	0.849***	0.37***	0.025	0.957***
WRB	0.005***	0.000	0.997***	0.347***	0.005***	0.993***

	GARCH			DCC		
Brokers	ω	α	β	const	a	b
AGE	0.057	0.072**	0.916***	0.671***	0.042**	0.947***
BSC	0.009	0.024	0.972***	0.673***	0.022***	0.972***
ETFC	0.019	0.032**	0.967***	0.577***	0.014**	0.979***
GS	0.007	0.024	0.973***	0.704***	0.014*	0.981***
LEH	0.016	0.051***	0.947***	0.699***	0.013***	0.983***
MER	0.004	0.023**	0.975***	0.73***	0.021***	0.976***
MS	0.012	0.050***	0.949***	0.719***	0.016***	0.980***
SCHW	0.006	0.022**	0.977***	0.656***	0.011***	0.984***
TROW	0.031*	0.074***	0.921***	0.687***	0.023***	0.974***
Others						
ACAS	0.024*	0.051***	0.944***	0.374***	0.003	0.872
AMP	0.489	0.000**	0.831***	0.488***	0.000	0.916**
AMTD	0.088	0.045**	0.950***	0.536***	0.033	0.924***
AXP	0.020	0.083***	0.915***	0.694***	0.013***	0.985***
BEN	0.057**	0.070***	0.914***	0.680***	0.027***	0.971***
BLK	0.231	0.127*	0.826***	0.293***	0.004*	0.995***
BOT	1.004***	0.000***	0.886***	0.183**	0.000	0.000
CBG	1.309**	0.151**	0.566***	0.368***	0.000	0.989***
CBSS	0.015	0.053	0.941***	0.583***	0.057**	0.926***
CIT	0.019	0.025	0.966***	0.480***	0.017	0.000
CME	4.066***	0.172***	0.000**	0.356***	0.000	0.611
COF	0.099	0.126***	0.874***	0.533***	0.064*	0.366***
FITB	0.170**	0.166***	0.780***	0.554***	0.035	0.854***
FNM	0.063	0.039	0.940***	0.408***	0.015***	0.981***
FRE	0.014	0.018	0.976***	0.427***	0.036***	0.933***
ICE	5.139**	0.000***	0.712***	0.312***	0.001	0.832***
JNS	0.285*	0.096***	0.868***	0.583***	0.015	0.973***
LM	0.122	0.043	0.931***	0.609***	0.015***	0.983***
SEIC	0.086*	0.086***	0.903***	0.591***	0.028*	0.962***
SLM	0.020	0.027	0.963***	0.363***	0.010*	0.977***

*Notes: Stars indicate the statistical significance: *** means a statistically significance at 1% level, ** at 5% level and * at 10% level of significance.*

Table B.3: Parameter estimates of the DCC-GJR model.

	GJR				DCC		
Depositories	ω	α	β	γ	const	a	b
BAC	0.001	0.007	0.994***	-0.007	0.631***	0.044	0.896***
BBT	0.045**	0.048**	0.869***	0.131***	0.655***	0.069*	0.875***
BK	0.013	0.012	0.967***	0.033	0.609***	0.033	0.942***
C	0.017*	0.023	0.930***	0.087**	0.719***	0.049	0.912***
CBH	0.733	0.123	0.635**	0.084	0.49***	0.011	0.98***
CMA	0.071**	0.075**	0.861***	0.104*	0.587***	0.008	0.986***
HBAN	0.031	0.049***	0.900***	0.090*	0.591***	0.027***	0.968***
HCBK	0.232	0.077	0.722*	0.158	0.353***	0.023**	0.971***
JPM	0.010	0.029*	0.944***	0.052*	0.709***	0.014	0.980***
KEY	0.012*	0.010	0.964***	0.039**	0.599***	0.013**	0.984***
MI	0.018*	0.015	0.937***	0.083***	0.612***	0.016**	0.980***
MTB	0.017	0.019	0.949***	0.053**	0.581***	0.034***	0.961***
NCC	0.017	0.015	0.955***	0.045**	0.621***	0.017***	0.977***
NTRS	0.045	0.009	0.937***	0.085	0.643***	0.014***	0.981***
NYB	0.131*	0.067*	0.857***	0.094**	0.376***	0.051	0.903***
PBCT	0.064	0.115***	0.880***	-0.029	0.418***	0.020***	0.972***
PNC	0.029	0.031	0.920***	0.079	0.617***	0.106***	0.677***
RF	0.043**	0.045	0.885***	0.107***	0.615***	0.037	0.953***
SNV	0.082*	0.073*	0.868***	0.071*	0.607***	0.070**	0.851***
SOV	0.028	0.063**	0.910***	0.053*	0.488***	0.016***	0.970***
STI	0.040	0.032*	0.910***	0.078*	0.646***	0.015**	0.980***
STT	0.120*	0.062	0.882***	0.064	0.613***	0.010	0.973***
UB	0.043**	0.059**	0.868***	0.146***	0.489***	0.019**	0.966***
USB	0.013	0.023	0.933***	0.082***	0.590***	0.023	0.967***
WAMUQ	0.032	0.056*	0.918***	0.042	0.433***	0.023***	0.970***
WB	0.046***	0.09***	0.877***	0.042	0.639***	0.020***	0.977***
WFC	0.008	0.009	0.958***	0.055**	0.627***	0.014	0.982***
ZION	0.055**	0.034**	0.896***	0.107***	0.547***	0.039	0.936***

	GJR				DCC		
Insurance	ω	α	β	γ	const	a	b
ABKFQ	0.074**	0.083***	0.847***	0.139*	0.492***	0.011	0.974***
AET	0.510**	0.000	0.805***	0.195*	0.348***	0.065***	0.889***
AFL	0.019	0.001	0.963***	0.069	0.427***	0.092***	0.764***
AIG	0.059**	0.030	0.903***	0.103***	0.582***	0.050	0.932***
AIZ	1.560***	0.000	0.000	0.085	0.296***	0.083	0.544***
ALL	0.005	0.000	0.981***	0.033	0.467***	0.003	0.994***
AON	0.898**	0.000*	0.654***	0.491**	0.389***	0.113	0.000
BRK.B	0.027	0.020	0.885***	0.189***	0.218***	0.047***	0.735***
CB	0.049*	0.036*	0.896***	0.127***	0.505***	0.096***	0.313*
CFC	0.049	0.019**	0.96***	0.022	0.434***	0.01**	0.983***
CI	0.323	0.145	0.79***	0.130	0.318***	0.079**	0.722***
CINF	0.017**	0.023	0.934***	0.073***	0.582***	0.015***	0.982***
CNA	0.046	0.000	0.946***	0.084*	0.412***	0.083**	0.709***
CVH	0.158	0.079*	0.914***	-0.023	0.331***	0.012	0.946***
FNF	2.105*	0.275	0.052	0.161	0.191**	0.033	0.907***
GNW	1.303***	0.241***	0.000	-0.133	0.344***	0.000	0.993
HIG	0.074**	0.077**	0.872***	0.076**	0.574***	0.048	0.926***
HNT	0.206	0.046**	0.879***	0.084	0.309***	0.016**	0.958***
HUM	0.011***	0.000	0.997***	0.000***	0.335***	0.011	0.974***
L	0.020	0.018	0.951***	0.054**	0.454***	0.059	0.904***
LNC	0.026	0.013	0.949***	0.065**	0.609***	0.017**	0.979***
MBI	0.098*	0.066*	0.874***	0.073*	0.544***	0.008**	0.990***
MET	0.039**	0.011	0.930***	0.105***	0.510***	0.007*	0.990***
MMC	0.551	0.060*	0.783***	0.062	0.522***	0.082***	0.862***
PFG	0.107***	0.045*	0.843***	0.146***	0.552***	0.079	0.879***
PGR	0.013	0.012	0.966***	0.042	0.492***	0.012*	0.980***
PRU	0.06**	0.014	0.917***	0.094**	0.590***	0.013**	0.977***
SAF	0.041**	0.066***	0.903***	0.040	0.478***	0.017***	0.977***
TMK	0.025*	0.046	0.908***	0.071**	0.582***	0.013**	0.983***
TRV	0.011	0.000	0.977***	0.039*	0.499***	0.022***	0.973***
UNH	0.162***	0.055***	0.858***	0.086**	0.331***	0.024**	0.939***
UNM	0.065*	0.000	0.966***	0.044	0.340***	0.016*	0.967***
WLP	0.225**	0.065**	0.850***	0.050	0.364***	0.025	0.960***
WRB	0.005**	0.002	0.996***	-0.002	0.349***	0.005***	0.994***

	GJR				DCC		
Brokers	ω	α	β	γ	const	a	b
AGE	0.041	0.016	0.932***	0.092**	0.655***	0.058***	0.923***
BSC	0.018	0.012	0.96***	0.047*	0.667***	0.028**	0.96***
ETFC	0.041	0.009	0.958***	0.065**	0.574***	0.014*	0.978***
GS	0.018*	0.000	0.964***	0.064**	0.698***	0.036	0.939***
LEH	0.025*	0.017	0.944***	0.074***	0.693***	0.012***	0.983***
MER	0.008**	0.000	0.975***	0.047***	0.724***	0.021**	0.972***
MS	0.024**	0.018	0.943***	0.070***	0.711***	0.016***	0.977***
SCHW	0.021	0.009	0.968***	0.045**	0.646***	0.013**	0.981***
TROW	0.028*	0.011	0.939***	0.097***	0.680***	0.026***	0.968***
Others							
ACAS	0.025*	0.041**	0.944***	0.020	0.369***	0.019	0.173
AMP	0.489	0.000	0.831***	0.000	0.480***	0.000	0.917*
AMTD	0.082	0.000	0.968***	0.058***	0.520***	0.043**	0.925***
AXP	0.019**	0.003	0.933***	0.128***	0.691***	0.014**	0.981***
BEN	0.047*	0.027*	0.932***	0.061***	0.671***	0.029***	0.967***
BLK	0.21	0.097*	0.833***	0.067	0.292***	0.004	0.995***
BOT	1.134*	0.072	0.842	-0.072	0.167**	0.008	0.000
CBG	1.256	0.124	0.583***	0.054	0.367***	0.000	0.988***
CBSS	0.016	0.010	0.949***	0.076***	0.577***	0.064***	0.909***
CIT	0.017	0.000	0.98***	0.025	0.479***	0.005	0.000
CME	4.123***	0.204**	0.000	-0.121	0.356***	0.000	0.621
COF	0.100	0.049*	0.88***	0.142*	0.531***	0.049**	0.405***
FITB	0.165*	0.131	0.788***	0.055	0.549***	0.005	0.989***
FNM	0.022	0.000	0.971***	0.042**	0.415***	0.013***	0.981***
FRE	0.011	0.009	0.978***	0.017	0.429***	0.035***	0.93***
ICE	2.549***	0.000	0.806***	0.125	0.295***	0.006	0.835***
JNS	0.197**	0.003	0.913***	0.118***	0.571***	0.036	0.930***
LM	0.110	0.009	0.935***	0.076**	0.601***	0.024	0.972***
SEIC	0.075*	0.028	0.914***	0.102***	0.588***	0.026**	0.962***
SLM	0.020	0.028	0.964***	-0.002	0.359***	0.010*	0.973***

*Notes: Stars indicate the statistical significance: *** means a statistically significance at 1% level, ** at 5% level and * at 10% level of significance.*

Table B.4: In-sample results of the Unconditional Coverage test.

Depositories	DCC GARCH	DCC GJR	DCC CGARCH	DCC SPLINE	DCC FIGARCH	DCC ACGARCH
BAC	1.563	0.601	0.079	0.079	0.072	1.563
BBT	2.892*	4.539**	0.601	0.601	2.892*	2.892*
BK	1.892	3.351*	1.892	1.892	1.892	3.351*
C	1.563	4.539**	0.601	0.072	1.563	1.563
CBH	1.252	2.589	1.252	1.252	2.589	2.589
CMA	2.074	0.917	0.917	0.196	3.601*	2.074
HBAN	0.196	0.196	0.544	0.013	0.013	0.917
HCBK	0.013	0.196	0.544	0.544	0.013	0.013
JPM	0.601	1.563	0.601	0.072	2.892*	1.563
KEY	0.253	0.253	0.253	0.003	0.467	0.253
MI	0.003	0.253	0.003	0.467	0.003	0.253
MTB	0.917	0.917	0.196	0.013	0.917	2.074
NCC	1.892	0.802	0.147	0.147	1.892	5.130**
NTRS	0.648	0.088	0.088	0.065	0.088	1.641
NYB	0.052	0.708	0.052	0.106	0.106	0.106
PBCT	0.015	1.601	0.266	1.601	0.015	0.481
PNC	0.858	1.982	1.982	0.858	1.982	0.858
RF	6.643**	0.648	3.001*	0.648	3.001*	6.643**
SNV	0.272	2.036	0.001	0.272	3.414*	2.036
SOV	0.125	0.665	0.040	0.040	0.665	0.748
STI	0.630	1.760	0.630	0.630	1.760	3.334*
STT	1.345	1.345	0.034	0.129	1.345	1.345
UB	0.074	0.074	1.092	1.092	0.074	0.074
USB	0.003	0.253	0.003	0.003	0.253	1.042
WAMUQ	0.106	0.106	0.052	0.106	0.106	0.106
WB	1.212	2.389	1.212	0.399	1.212	2.389
WFC	1.892	0.802	0.802	0.802	1.892	1.892
ZION	1.328	0.004	0.004	0.395	0.395	2.702

Insurance	DCC GARCH	DCC GJR	DCC CGARCH	DCC SPLINE	DCC FIGARCH	DCC ACGARCH
ABKFQ	0.686	0.686	0.686	0.686	0.686	1.858
AET	0.005	0.005	0.746	0.005	0.746	0.005
AFL	0.030	0.027	0.030	0.624	0.030	0.030
AIG	2.389	2.389	0.399	0.399	3.882**	1.212
AIZ	0.003	0.003	0.003	1.949	0.003	0.940
ALL	1.408	0.008	0.436	1.408	1.408	2.820*
AON	0.058	4.822**	0.058	1.032	1.032	1.032
BRK.B	0.874	0.874	0.122	0.874	2.176	0.874
CB	2.479	1.179	1.179	0.000	2.479	4.150**
CFC	0.047	0.183	0.047	0.630	0.630	0.630
CI	3.489*	3.488*	3.488*	3.488*	3.488*	3.488*
CINF	1.077	1.077	3.037*	3.037*	0.191	0.364
CNA	0.481	0.481	0.266	0.015	0.0147	0.481
CVH	0.013	0.544	0.544	0.544	0.544	0.544
FNF	0.865	4.120**	0.865	0.865	0.865	0.865
GNW	2.214	2.214	2.214	2.214	4.661**	2.214
HIG	0.079	0.072	0.795	0.079	0.079	0.601
HNT	0.058	0.058	0.556	1.488	0.556	0.556
HUM	2.820*	1.408	0.436	0.008	0.436	1.408
L	0.578	0.034	0.034	0.034	1.667	1.667
LNC	0.072	0.079	0.079	0.079	0.079	0.601
MBI	0.034	0.474	0.129	0.129	0.034	0.034
MET	1.109	2.372	2.372	4.006**	4.006**	5.964**
MMC	1.760	0.630	1.760	0.183	0.630	0.630
PFG	0.175	1.017	0.175	0.058	1.017	1.017
PGR	0.239	0.030	0.030	0.239	0.239	0.239
PRU	4.309**	0.746	0.746	0.746	0.746	0.005
SAF	1.094	0.043	0.043	0.043	0.043	0.043
TMK	0.079	0.079	0.079	0.795	0.079	0.601
TRV	0.015	0.015	0.481	0.266	0.015	0.481
UNH	0.978	0.008	0.978	2.170	3.732*	3.732*
UNM	0.006	0.006	0.006	0.006	0.006	0.006
WLP	0.134	1.277	0.134	0.134	0.079	0.745
WRB	1.547	1.547	1.547	1.547	3.245*	5.402**

Broker	DCC GARCH	DCC GJR	DCC CGARCH	DCC SPLINE	DCC FIGARCH	DCC ACGARCH
AGE	1.982	1.982	0.858	0.170	0.858	1.982
BSC	3.114*	1.722	0.697	0.052	1.722	4.826**
ETFC	0.170	0.170	0.021	0.170	0.021	0.170
GS	2.074	2.074	0.196	0.013	2.074	3.601*
LEH	1.982	0.858	0.170	0.170	0.858	3.474*
MER	3.001*	3.001*	1.641	0.648	1.641	4.681**
MS	1.415	0.514	0.514	0.045	1.415	1.415
SCHW	0.697	0.697	0.106	0.697	0.106	1.722
TROW	0.583	0.021	0.583	0.583	0.583	0.170
Others						
ACAS	0.106	0.052	0.106	0.052	0.106	0.106
AMP	0.533	0.923	0.533	0.923	0.533	0.533
AMTD	2.170	0.978	0.008	0.223	2.170	0.978
AXP	1.088	1.088	0.332	0.332	2.207	2.207
BEN	1.892	3.351*	1.892	0.802	1.892	3.351*
BLK	0.395	0.395	0.004	0.328	1.328	1.328
BOT	0.513	0.513	0.513	0.513	0.513	0.513
CBG	0.120	0.120	0.120	0.120	0.120	0.120
CBSS	2.682	0.514	0.514	0.514	2.682	2.682
CIT	0.490	0.013	3.796*	0.490	3.796*	0.013
CME	0.312	0.073	0.312	0.312	0.073	0.946
COF	2.702	2.702	1.328	1.328	2.702	4.450**
FITB	1.277	0.435	0.435	0.435	2.484	4.006**
FNM	0.034	0.933	0.933	0.129	0.933	0.034
FRE	0.021	0.021	0.021	0.170	0.170	0.858
ICE	4.120**	4.120**	4.120**	4.120**	4.120**	4.120**
JNS	0.052	0.106	0.106	0.708	0.106	0.106
LM	1.109	2.372	1.109	1.109	1.109	2.372
SEIC	0.013	0.013	0.544	0.544	0.013	0.196
SLM	3.601*	5.450**	0.917	2.074	3.601*	5.450**

*Notes: Stars indicate the statistical significance: *** means a statistically significance at 1% level, ** at 5% level and * at 10% level of significance.*

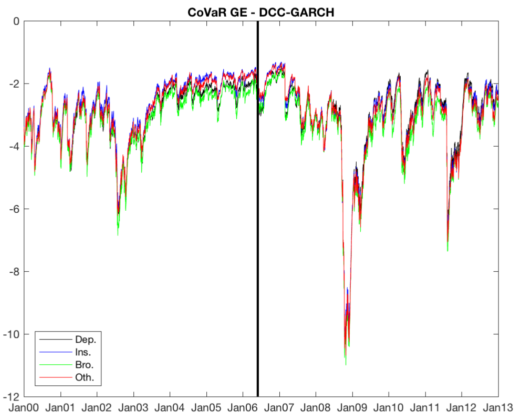
Table B.5: In-sample results of the Dynamic Quantile test.

Depositories	DCC GARCH	DCC GJR	DCC CGARCH	DCC SPLINE	DCC FIGARCH	DCC ACGARCH
BAC	3.246	2.891	0.477	0.477	0.594	3.246
BBT	6.053	7.349*	1.120	1.120	6.053	6.053
BK	5.460	7.296*	5.460	5.460	5.460	8.910**
C	3.526	7.587*	3.122	1.230	3.526	3.526
CBH	5.772	6.708*	5.772	4.970	6.917*	6.708*
CMA	4.646	1.956	4.126	0.684	9.064**	4.646
HBAN	0.684	0.684	0.531	0.589	0.589	1.956
HCBK	0.560	0.610	0.531	0.531	0.560	0.560
JPM	3.122	6.362*	3.122	1.230	11.215**	6.36*
KEY	1.851	1.851	1.851	0.579	0.484	1.851
MI	0.579	1.851	0.579	0.484	0.579	3.979
MTB	1.807	1.807	1.664	0.539	1.807	4.380
NCC	4.231	1.769	1.500	0.644	4.231	10.158**
NTRS	1.530	0.603	1.290	0.715	0.603	3.689
NYB	0.662	1.661	0.489	0.550	0.550	0.550
PBCT	6.054	1.105	1.114	1.105	6.054	3.695
PNC	3.935	5.878	7.857**	3.935	7.857**	3.935
RF	13.07***	1.290	5.316	1.290	5.316	10.23**
SNV	1.012	3.774	0.842	2.102	8.727**	3.774
SOV	3.746	0.610	0.498	0.498	0.610	3.701
STI	3.075	5.956	3.075	2.426	5.956	11.65***
STT	3.089	3.089	1.071	0.475	3.088	3.088
UB	0.790	0.790	0.764	0.764	0.790	0.790
USB	0.579	1.851	0.579	0.579	1.851	4.166
WAMUQ	0.550	0.550	0.489	0.550	0.550	0.550
WB	5.193	8.516**	5.193	2.497	5.193	9.280**
WFC	4.231	1.373	3.754	1.769	4.231	4.231
ZION	3.306	0.476	5.427	3.756	3.756	4.742

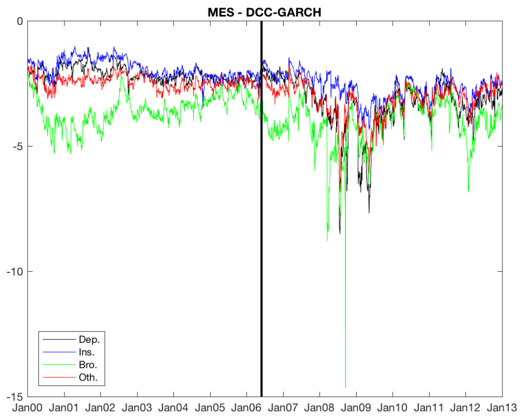
Insurance	DCC GARCH	DCC GJR	DCC CGARCH	DCC SPLINE	DCC FIGARCH	DCC ACGARCH
ABKFQ	3.262	2.795	3.262	3.262	3.262	6.673*
AET	0.549	0.549	NaN	0.549	NaN	0.549
AFL	6.729*	0.509	0.509	0.583	5.535	0.509
AIG	4.287	3.903	3.383	3.383	5.795	2.896
ALL	2.637	0.423	2.441	2.637	2.637	2.699
BRK.B	1.3684	1.3684	1.2262	1.3684	4.2073	1.368
CB	4.404	2.413	5.005	0.632	4.404	7.601*
CFC	0.937	0.335	0.937	3.075	3.075	3.075
CINF	0.889	0.889	2.038	2.038	0.487	3.334
CNA	0.985	0.985	0.371	0.780	0.450	0.985
CVH	0.539	0.531	0.531	0.531	0.531	0.531
HIG	0.477	4.339	0.696	0.477	0.477	3.122
HNT	1.173	1.173	2.983	4.928	2.983	2.983
HUM	7.897**	5.811	2.441	0.737	4.696	5.252
L	1.227	0.479	0.479	0.479	3.620	3.620
LNC	4.339	0.477	0.477	0.477	0.477	3.919
MBI	0.526	1.173	0.475	0.475	0.526	0.526
MET	3.053	4.055	14.63***	12.21***	12.21***	9.519**
MMC	3.314	1.053	7.143*	0.335	1.053	1.053
PFG	11.81***	11.73***	11.814***	0.321	7.805*	7.805*
PGR	0.477	0.674	0.339	0.477	0.477	0.477
SAF	5.217	0.329	0.329	0.329	0.329	0.329
TMK	0.477	0.477	0.477	0.696	0.477	3.122
TRV	6.570*	0.780	4.749	0.371	6.570*	4.749
UNH	3.830	0.417	3.830	7.751*	9.012**	9.012**
UNM	0.535	0.535	0.535	0.535	0.535	0.535
WRB	9.439**	9.439**	9.439**	9.439**	6.955*	14.81***

Broker	DCC GARCH	DCC GJR	DCC CGARCH	DCC SPLINE	DCC FIGARCH	DCC ACGARCH
AGE	5.654	7.857**	3.935	1.579	3.935	5.654
BSC	4.532	3.182	3.4211	0.489	3.421	6.552*
ETFC	4.415	1.198	0.522	1.580	0.522	1.579
GS	4.703	17.32***	4.439	0.539	4.703	14.81***
LEH	4.433	1.860	1.579	0.663	1.860	8.677**
MER	11.71***	13.09***	4.938	3.267	4.938	13.93***
MS	3.227	1.335	2.852	1.120	3.227	3.227
SCHW	3.421	3.215	1.356	3.421	1.356	5.497
TROW	0.556	0.392	0.556	0.556	0.556	1.579
Others						
ACAS	1.356	0.489	1.356	0.489	1.356	1.356
AMTD	6.343*	4.328	0.557	4.465	6.343*	4.328
AXP	2.602	2.140	2.289	2.289	5.298	5.298
BEN	7.526*	13.14***	7.526*	3.754	7.526*	13.14***
BLK	0.790	0.790	0.698	0.403	3.196	3.196
CBSS	7.692*	2.852	2.852	2.852	10.386**	10.386**
CIT	0.363	8.857**	NaN	0.363	NaN	8.857**
CME	0.253	0.487	0.253	0.253	0.487	3.158
COF	4.857	4.857	4.663	4.663	4.857	7.721*
FITB	2.444	0.930	2.131	0.930	5.140	9.227**
FNM	3.793	0.790	0.790	0.475	0.790	1.071
FRE	0.522	0.522	0.522	1.579	1.579	3.935
ICE	38.11***	38.11***	38.11***	NaN	38.11***	38.11***
JNS	5.627	3.838	3.838	0.638	3.838	3.838
LM	2.994	3.926	2.994	2.994	2.994	3.926
SEIC	0.560	0.560	0.531	0.531	0.560	0.610
SLM	6.423*	11.050**	4.217	3.849	5.038	7.605*

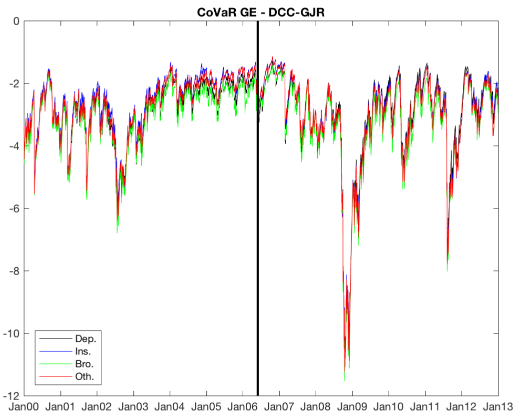
*Notes: Stars indicate the statistical significance: *** means a statistically significance at 1% level, ** at 5% level and * at 10% level of significance.*



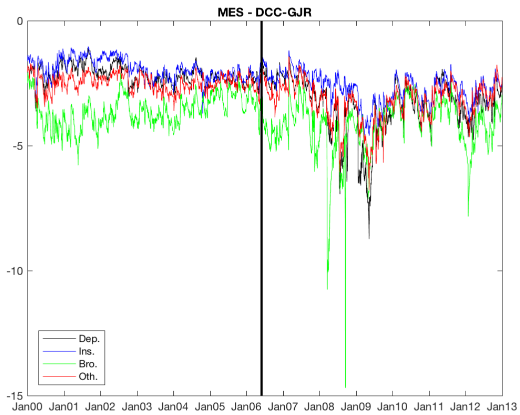
(a) CoVaR obtained by DCC-GARCH model.



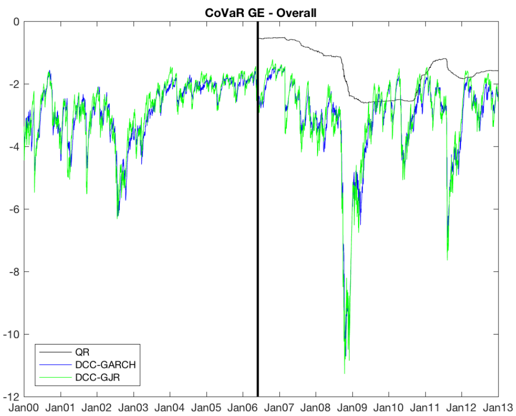
(b) MES obtained by DCC-GARCH model.



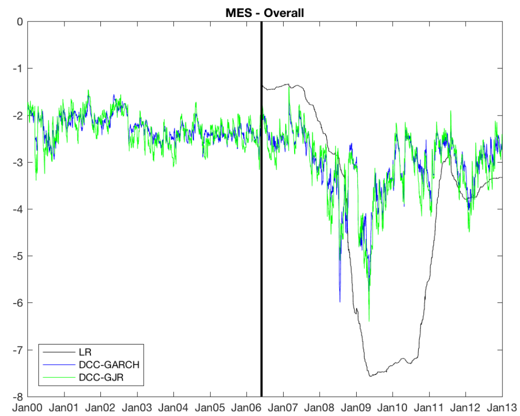
(c) CoVaR obtained by DCC-GJR model.



(d) MES obtained by DCC-GJR model.



(e) Comparison of overall CoVaR averages.



(f) Comparison of overall MES averages.

Figure B.2: Comparison of the CoVaR (first column) and MES (second column) measures by sub-industry group obtained by the DCC-GARCH (first row) and DCC-GJR (second row) models. The black vertical line indicates the end of the in-sample and the beginning of the out-of-sample period.

Table B.6: Parameter estimates of the DCC-CGARCH model.

	CGARCH					DCC		
Deposit.	ω	α	β	φ	ρ	const	a	b
BAC	0.000***	0.099	0.754*	0.014*	1.000***	0.633***	0.057	0.936***
BBT	0.006**	0.109***	0.800***	0.042***	1.000***	0.671***	0.064	0.901***
BK	0.004*	0.093**	0.742***	0.027***	1.000***	0.618***	0.058	0.898***
C	0.005**	0.079	0.734***	0.042***	0.999***	0.726***	0.021*	0.976***
CBH	0.093***	0.156***	0.462***	0.026***	0.975***	0.493***	0.014**	0.981***
CMA	0.003**	0.138***	0.802***	0.019***	1.000***	0.597***	0.006	0.989***
HBAN	0.003**	0.087***	0.825***	0.027***	1.000***	0.601***	0.029***	0.970***
HCBK	0.004***	0.227*	0.546*	0.000	0.998***	0.359***	0.022***	0.976***
JPM	0.004***	0.053*	0.803***	0.040***	1.000***	0.715***	0.013*	0.986***
KEY	0.003***	0.034	0.889***	0.030***	1.000***	0.600***	0.013***	0.985***
MI	0.003*	0.066***	0.878***	0.037***	1.000***	0.618***	0.017***	0.982***
MTB	0.000	0.074	0.867	0.014	1.000***	0.590***	0.038***	0.957***
NCC	0.005***	0.061**	0.813***	0.036***	1.000***	0.627***	0.018***	0.980***
NTRS	0.017**	0.105***	0.769***	0.041***	0.999***	0.66***	0.014***	0.984***
NYB	0.072	0.135**	0.514	0.081	0.985***	0.381***	0.044	0.939**
PBCT	0.006	0.136**	0.789***	0.031*	1.000***	0.422***	0.021***	0.973***
PNC	0.006**	0.064***	0.883***	0.034***	0.999***	0.629***	0.007	0.989***
RF	0.010**	0.104***	0.834***	0.051***	0.999***	0.619***	0.039*	0.958***
SNV	0.004***	0.105***	0.814***	0.022***	0.999***	0.622***	0.063	0.885***
SOV	0.011*	0.110***	0.779***	0.056***	1.000***	0.500***	0.016**	0.973***
STI	0.002***	0.104*	0.796***	0.019***	1.000***	0.655***	0.012***	0.984***
STT	0.005**	0.312***	0.280	0.020***	1.000***	0.626***	0.006	0.986***
UB	0.003*	0.151***	0.768***	0.031***	1.000***	0.504***	0.018*	0.971***
USB	0.001**	0.091***	0.855***	0.024**	1.000***	0.596***	0.015**	0.983***
WAMUQ	0.002***	0.212***	0.624***	0.017***	1.000***	0.436***	0.025***	0.971***
WB	0.010***	0.080***	0.880***	0.044***	0.999***	0.646***	0.021***	0.978***
WFC	0.001***	0.072	0.816	0.028	1.000***	0.631***	0.019	0.979***
ZION	0.014	0.069	0.852***	0.057*	1.000***	0.556***	0.071*	0.894***

	CGARCH					DCC		
Insurance	ω	α	β	φ	ρ	const	a	b
ABKFQ	0.000	0.145***	0.833***	0.013	1.000***	0.496***	0.003	0.993***
AET	0.020***	0.077	0.826***	0.000	0.996***	0.353***	0.064***	0.891***
AFL	0.002***	0.153	0.585**	0.013***	1.000***	0.432***	0.114***	0.746***
AIG	0.000	0.108	0.868	0.006	1.000***	0.580***	0.044	0.947***
AIZ	0.013***	0.038	0.000	0.000	0.992***	0.310***	0.076	0.485***
ALL	0.000***	0.132**	0.642***	0.015*	1.000***	0.477***	0.004	0.992***
AON	0.039*	0.212**	0.687***	0.000	0.995***	0.388***	0.130	0.000
BRK.B	0.009	0.253***	0.363	0.061	0.999***	0.231***	0.046***	0.754***
CB	0.000	0.125	0.807***	0.014	1.000***	0.520***	0.090***	0.443**
CFC	0.001***	0.174**	0.383	0.014**	1.000***	0.450***	0.010***	0.984***
CI	0.001	0.190***	0.792***	0.009	1.000***	0.328***	0.077**	0.693***
CINF	0.004*	0.066***	0.866***	0.036***	1.000***	0.586***	0.017***	0.983***
CNA	0.000	0.246	0.279	0.010	1.000***	0.424***	0.054	0.851***
CVH	0.007***	0.116***	0.777***	0.013***	1.000***	0.338***	0.009	0.955***
FNF	0.107***	0.323**	0.025	0.025	0.972***	0.188*	0.036	0.903***
GNW	0.008***	0.175***	0.006	0.006**	0.995***	0.342***	0.000	0.994***
HIG	0.011*	0.110***	0.832***	0.04***	0.999***	0.585***	0.015*	0.983***
HNT	0.001***	0.085*	0.805***	0.009**	1.000***	0.317***	0.017***	0.966***
HUM	0.004***	0.061	0.594**	0.013***	1.000***	0.342***	0.015	0.960***
L	0.000***	0.111***	0.763***	0.019**	1.000***	0.475***	0.077**	0.861***
LNC	0.004**	0.056**	0.869***	0.027***	1.000***	0.621***	0.015***	0.984***
MBI	0.035***	0.113***	0.746***	0.040***	0.991***	0.549***	0.009***	0.990***
MET	0.009	0.052***	0.906***	0.036*	0.999***	0.524***	0.009	0.988***
MMC	0.000	0.068***	0.857***	0.031***	0.993***	0.441***	0.205**	0.580***
PFG	0.003***	0.116***	0.774***	0.016***	0.999***	0.554***	0.023**	0.972***
PGR	0.003	0.100	0.656***	0.024	1.000***	0.496***	0.014**	0.982***
PRU	0.013***	0.053***	0.906***	0.013***	0.994***	0.600***	0.014**	0.98***
SAF	0.016***	0.065***	0.857***	0.046***	0.997***	0.483***	0.016***	0.981***
TMK	0.006*	0.120***	0.747***	0.044***	1.000***	0.594***	0.014***	0.984***
TRV	0.001**	0.097*	0.794***	0.013	1.000***	0.513***	0.021**	0.977***
UNH	0.008**	0.094***	0.841***	0.020***	0.999***	0.342***	0.031**	0.932***
UNM	0.066	0.061	0.565	0.064**	1.000***	0.368***	0.020	0.952***
WLP	0.000	0.108	0.811	0.007	1.000***	0.372***	0.026*	0.958***
WRB	0.003***	0.187**	0.373	0.013***	1.000***	0.355***	0.005	0.992***

	CGARCH					DCC		
Brokers	ω	α	β	φ	ρ	const	a	b
AGE	0.002**	0.084	0.837***	0.016***	1.000***	0.673***	0.035	0.961***
BSC	0.003***	0.044**	0.846***	0.022***	1.000***	0.674***	0.024***	0.973***
ETFC	0.010***	0.092**	0.539***	0.028***	1.000***	0.579***	0.016**	0.978***
GS	0.003***	0.033	0.885*	0.024***	1.000***	0.703***	0.017**	0.981***
LEH	0.003*	0.051*	0.895***	0.026***	1.000***	0.697***	0.014***	0.985***
MER	0.001**	0.056*	0.767	0.023	1.000***	0.731***	0.022***	0.977***
MS	0.000	0.053	0.898	0.021	1.000***	0.718***	0.015***	0.982***
SCHW	0.001	0.087	0.109	0.018	1.000***	0.660***	0.011***	0.986***
TROW	0.004**	0.068**	0.883***	0.029***	1.000***	0.690***	0.021***	0.977***
Others								
ACAS	0.000	0.173	0.714	0.014	1.000***	0.375***	0.000	0.943***
AMP	0.005***	0.000	0.215	0.000	0.998***	0.483***	0.000	0.941***
AMTD	0.045***	0.192***	0.371***	0.036***	0.998***	0.540***	0.045**	0.895***
AXP	0.004	0.080*	0.839***	0.032***	1.000***	0.690***	0.013***	0.986***
BEN	0.004**	0.063**	0.888***	0.022***	1.000***	0.681***	0.026***	0.974***
BLK	0.010**	0.338**	0.021	0.021***	0.999***	0.312***	0.002	0.996***
BOT	0.081***	0.266***	0.727***	0.000	0.999***	0.186***	0.000	0.813
CBG	0.000	0.104	0.481	0.023	1.000	0.350***	0.001	0.992***
CBSS	0.004*	0.138***	0.777***	0.033***	1.000***	0.590***	0.073**	0.900***
CIT	0.004**	0.154	0.638***	0.028	1.000***	0.484***	0.020	0.899***
CME	0.014***	0.159	0.257	0.000	0.997***	0.352***	0.000	0.527
COF	0.026***	0.081***	0.880***	0.044***	0.999***	0.528***	0.073**	0.371***
FITB	0.013**	0.225***	0.498***	0.043***	0.999***	0.562***	0.007	0.988***
FNM	0.026***	0.052**	0.825***	0.022**	0.992***	0.411***	0.015***	0.983***
FRE	0.003***	0.038*	0.856***	0.019***	1.000***	0.434***	0.043***	0.921***
ICE	0.064***	0.000	0.522	0.000	0.996***	0.330***	0.001	0.833***
JNS	0.042***	0.094***	0.830***	0.012	0.995***	0.579***	0.011**	0.983***
LM	0.086***	0.055*	0.660***	0.034***	0.984***	0.608***	0.015***	0.984***
SEIC	0.015**	0.106***	0.802***	0.028***	0.999***	0.591***	0.024*	0.970***
SLM	0.005**	0.039**	0.872***	0.030***	0.999***	0.366***	0.008*	0.983***

*Notes: Stars indicate the statistical significance: *** means a statistically significance at 1% level, ** at 5% level and * at 10% level of significance.*

Table B.7: Parameter estimates of DCC-Spline-GARCH model.

	Spline-GARCH									DCC		
Deposit.	c	α	β	w_0	w_1	w_2	w_3	w_4	w_5	const	a	b
BAC	7.572**	0.080**	0.665***	4.003	-29.06	51.38	-43.88*	37.02**	-5.917	0.645***	0.043	0.942***
BBT	15.961**	0.125***	0.737***	-14.76*	18.84	19.32	-86.43***	94.14***	-81.35**	0.672***	0.016	0.979***
BK	19.497***	0.088***	0.687***	-15.60***	43.84***	-44.72*	-28.57	57.96**	-32.60	0.623***	0.050	0.902***
C	6.718**	0.079***	0.854***	-1.703	3.995	-3.799	-25.47	50.82	-16.00	0.729***	0.017**	0.980***
CBH	6.202***	0.182***	0.497***	-0.399	-14.26***	46.52***	-66.07***	63.83***	-62.91*	0.500***	0.010***	0.985***
CMA	8.486**	0.129***	0.734***	-1.193	-11.74	35.22	-50.00*	45.75**	-20.049	0.600***	0.003	0.993***
HBAN	7.034**	0.092***	0.774***	10.02	-57.54*	96.44**	-58.89**	40.22*	-32.92	0.612***	0.019***	0.979***
HCBK	2.126	0.225**	0.491	-1.778	6.68	-11.96	9.111	-8.316	0.235	0.37***	0.020***	0.977***
JPM	3.785	0.069***	0.873***	14.23	-48.12	68.28	-61.37**	75.41**	-19.91	0.720***	0.009*	0.988***
KEY	10.035*	0.049***	0.893***	-10.67	22.65	-19.30	-16.63	31.06	-18.92	0.612***	0.010***	0.987***
MI	10.368**	0.080***	0.844***	-4.084	-14.50	57.08	-83.15***	68.81***	-27.55	0.624***	0.013***	0.985***
MTB	2.831**	0.077***	0.826***	9.913	-50.40	89.68**	-66.51***	44.85*	-40.76	0.601***	0.024*	0.971***
NCC	10.086*	0.073***	0.813***	-3.738	-14.30	56.34	-86.42***	88.09***	-81.71**	0.629***	0.014***	0.983***
NTRS	23.889	0.099***	0.783***	-15.17	34.70	-25.84	-35.05	46.69	4.212	0.666***	0.010***	0.986***
NYB	12.186*	0.161	0.712***	-17.86**	62.77**	-102.4**	80.29***	-85.09***	91.37**	0.396***	0.053**	0.897***
PBCT	6.783*	0.129***	0.770***	6.693	-46.76	84.37**	-40.27	-20.49	72.33	0.429***	0.018***	0.974***
PNC	9.820**	0.080***	0.839***	-16.38*	57.79	-90.28	35.74	16.05	-25.24	0.637***	0.006	0.988***
RF	12.920***	0.117***	0.776***	-9.715***	3.608	33.12***	-73.34***	68.86***	-53.584	0.626***	0.024	0.974***
SNV	10.228	0.092***	0.777***	-10.56	19.47	3.673	-62.80	63.24	-22.65	0.629***	0.076**	0.793***
SOV	18.492**	0.123***	0.795***	-2.876	-10.86	37.02	-54.07*	48.71**	-9.055	0.500***	0.014**	0.974***
STI	9.958***	0.183**	0.203	-12.08**	26.83	-21.43	-24.77*	43.10***	-31.57	0.664***	0.011***	0.984***
STT	15.223**	0.240**	0.343	-5.520	5.581	6.709	-30.47*	26.22	17.42	0.624***	0.007	0.983***
UB	17.713	0.268***	0.000	-1.530	-17.23	40.80	-42.54	33.40*	-5.835	0.513***	0.010**	0.986***
USB	9.632***	0.086***	0.819***	-3.670*	4.264	-6.371	-4.881	28.44*	-62.57**	0.604***	0.012	0.984***
WAMUQ	10.882***	0.170**	0.611***	-3.295*	-1.105	7.435	-9.338	0.600	25.97	0.443***	0.021***	0.973***
WB	5.421**	0.100***	0.841***	7.175	-39.98	72.52	-65.53**	66.78*	-45.49	0.652***	0.017***	0.982***
WFC	12.379***	0.074***	0.803***	-10.45***	12.94	7.149	-50.93***	62.53***	-40.86	0.634***	0.014**	0.984***
ZION	51.993	0.089***	0.785***	-28.40**	62.77*	-41.51	-60.23***	85.46***	-100.4**	0.568***	0.048**	0.922***
Insurance												
ABKFQ	3.232**	0.115***	0.794***	15.43*	-62.07**	94.52**	-65.29**	88.34***	-160.2	0.506***	0.004	0.992***
AET	22.58	0.184**	0.375**	-26.41**	93.57***	-149.8***	104.7***	-95.16***	161.6***	0.389***	0.052***	0.894***
AFL	9.837***	0.108	0.454	-7.319	20.28	-25.22	-12.37	51.59*	-79.62*	0.443***	0.0961***	0.745***
AIG	7.531***	0.101***	0.843***	-4.353**	0.643	23.30	-50.90	40.32	-25.36	0.585***	0.037	0.955***
AIZ	2.637***	0.000	0.000	-11.05*	36.85*	-50.89	24.18	-10.76	-21.86	0.314***	0.060	0.518***
ALL	10.679***	0.089**	0.587***	-0.266	-15.43***	28.56*	-23.36	28.14	-28.63	0.480***	0.003	0.995***
AON	19.991	0.259**	0.654***	-8.323	17.49	-14.53	-0.227	7.958	-54.60	0.402***	0.138*	0.000
BRK.B	8.529***	0.176***	0.659***	-3.186	-9.027	28.87	-37.89	42.37	-71.50	0.232***	0.053***	0.708***
CB	7.702***	0.100***	0.786***	0.871	-21.13***	55.47***	-70.96**	60.87*	-41.593	0.524***	0.065**	0.389
CFC	12.301**	0.153**	0.363**	-1.711	-12.49	38.66	-40.84	10.51	31.84	0.453***	0.009***	0.986***
CI	0.846	0.184***	0.805***	57.20	-180.8	231.8	-67.72	-3.591	107.1	0.341***	0.069**	0.711***
CINF	9.292***	0.080***	0.845***	-5.268***	-2.871	26.62***	-50.10***	50.30**	-24.75	0.592***	0.014***	0.985***
CNA	8.434**	0.167**	0.350	-7.56	15.97	-7.970	-23.53	16.82	18.06	0.429***	0.036	0.890***
CVH	8.742***	0.072***	0.723***	18.25***	-70.74***	98.31***	-41.38*	21.83	5.882	0.349***	0.008	0.943***

	Spline-GARCH									DCC		
Insurance	c	α	β	w_0	w_1	w_2	w_3	w_4	w_5	const	a	b
FNF	2.779	0.245**	0.000	12.00	-58.06	106.6*	-88.28**	78.02*	-41.02	0.146	0.036	0.918***
GNW	1.071	0.167**	0.000	9.233	-29.32	33.61	2.574	-23.95	35.27	0.349***	0.000	0.992***
HIG	9.761	0.121***	0.789***	7.129	-51.67*	107.5***	-98.84***	68.38*	-31.81	0.592***	0.010**	0.988***
HNT	10.96***	0.066**	0.759***	1.836	-12.51	9.677	20.72	-41.01	65.17	0.332***	0.013**	0.958***
HUM	2.708***	0.000	0.999***	-0.448	-10.35	19.08	-10.28	3.508	16.25	0.349***	0.005	0.992***
L	5.062*	0.102***	0.746***	5.482	-32.40	62.07	-60.32**	52.54**	-22.65	0.482***	0.069**	0.863***
LNC	8.997**	0.068***	0.837***	5.707	-45.28*	98.21***	-95.12***	62.62***	-24.55	0.628***	0.011***	0.987***
MBI	6.212**	0.128***	0.719***	4.048	-32.07	72.95**	-89.95***	111.7***	-147.9***	0.557***	0.007**	0.991***
MET	11.553	0.073***	0.870***	-7.593	14.25	-8.973	-22.49	49.54	-74.11	0.529***	0.006*	0.990***
MMC	12.317	0.105***	0.752***	-6.427	10.41	-15.09	34.30	-60.37	10.53	0.552***	0.009***	0.988***
PFG	1.660*	0.096***	0.754***	11.54	-31.31	18.07	35.33	-46.73**	72.62**	0.566***	0.023	0.967***
PGR	8.930***	0.097***	0.643***	13.16*	-76.98***	141.2***	-104.4***	70.61***	-61.36**	0.504***	0.009**	0.987***
PRU	1.272	0.048**	0.885***	18.43	-59.84	63.42	11.48	-20.90	-6.682	0.608***	0.012*	0.975***
SAF	4.480*	0.085***	0.825***	19.67*	-88.77***	138.6***	-72.67***	40.62	-31.46	0.494***	0.012***	0.985***
TMK	11.487***	0.123***	0.772***	-1.281	-29.17**	78.61***	-83.56***	58.49**	-54.23	0.601***	0.010***	0.988***
TRV	6.511*	0.083***	0.777***	-5.457	20.85	-39.51	23.53**	1.098	0.399**	0.529***	0.013	0.982***
UNH	9.335**	0.082***	0.816***	-2.105	-12.09	38.54	-40.62	13.05	25.90	0.348***	0.027**	0.931***
UNM	100.0	0.1288***	0.871***	55.21	-226.0*	420.1**	-416.7***	406.6**	-270.1	0.380***	0.019*	0.971***
WLP	2.572*	0.076**	0.770***	16.22*	-58.00**	70.39**	2.046	-42.28**	71.73*	0.382***	0.027	0.937***
WRB	5.571**	0.129**	0.524***	9.323	-46.87*	71.36*	-29.60	7.826	16.79	0.362***	0.005*	0.994***
Broker												
AGE	6.070**	0.081***	0.765***	8.632	-40.66*	68.95**	-54.70***	43.12***	-7.457	0.682***	0.029	0.965***
BSC	5.450***	0.041***	0.791***	14.41***	-62.95***	95.77***	-53.06***	34.21***	-2.057	0.680***	0.016***	0.980***
ETFC	1.499***	0.021	0.954***	39.73***	-117.7***	135.0***	-38.76*	50.94**	-42.80	0.580***	0.013**	0.977***
GS	11.47***	0.048***	0.744***	4.752	-29.29*	46.70**	-35.13***	36.94***	-0.168	0.709***	0.012***	0.984***
LEH	9.263*	0.065***	0.834***	12.54	-52.13**	68.93**	-24.75	14.40	26.57	0.702***	0.010***	0.986***
MS	4.695	0.063*	0.864***	16.15	-50.83	55.74	-13.78	18.60	-3.879	0.719***	0.015***	0.982***
MER	1.804***	0.000	0.999***	2.412	-9.123	6.136	-7.350	22.96***	7.127	0.739***	0.011***	0.985***
SCHW	1.833***	0.000	0.997***	6.558	-16.17	24.23	-32.55**	42.74***	-13.60	0.656***	0.010***	0.986***
TROW	20.60	0.088***	0.804***	-4.475	-7.704	39.15	-60.98	37.44	21.78	0.698***	0.015***	0.982***
Others												
ACAS	9.020**	0.134***	0.678***	-3.779	3.036	13.57	-48.98	52.13	-19.31	0.379***	0.022	0.005
AMP	0.960	0.000	0.000	19.65	-63.39	79.97	-16.95	-7.697	18.67	0.488***	0.046	0.745**
AMTD	1.487**	0.033	0.939***	41.00***	-126.0***	152.2***	-37.85*	13.77	11.31	0.537***	0.042**	0.897***
AXP	7.751***	0.082***	0.780***	1.417	-6.859*	7.868	-29.36	72.93***	-63.25**	0.694***	0.012***	0.987***
BEN	7.948***	0.071***	0.845***	0.849	-14.75*	28.47	-22.15	11.08	27.52	0.688***	0.020***	0.978***
BLK	6.017***	0.249***	0.000	12.97***	-55.85***	75.11***	-15.14	-10.89	53.53	0.312***	0.000	0.085
BOT	15.076*	0.000	0.968***	-18.92	31.30	4.257	-39.85	-19.46	73.66	0.135**	0.000	0.891***
CBG	0.223	0.083	0.575***	26.13	-56.90	39.39	40.920	-56.08	89.36*	0.357***	0.000	0.987***
CBSS	17.355	0.124***	0.756***	-10.65*	15.41	-4.977	-19.79	20.88	-19.53	0.602***	0.060	0.907***
CIT	22.713	0.111***	0.568***	-11.55	20.42	-21.36	15.67	-35.07	60.97*	0.484***	0.037	0.000
CME	4.651**	0.143**	0.000	0.598	-3.055	5.165	4.556	-26.30	38.54	0.357***	0.000	0.645
COF	20.080***	0.108*	0.000	-12.71**	45.28*	-50.96	-44.94**	103.7***	-80.28**	0.543***	0.071**	0.305
FITB	12.765***	0.200***	0.548***	-5.963**	-3.691	33.95***	-59.01***	58.48**	-58.33	0.566***	0.005	0.990***
FNM	9.350*	0.069***	0.744***	0.640	-29.70	77.68***	-87.67***	82.48***	-98.71**	0.421***	0.013***	0.984***
FRE	11.143	0.049*	0.751***	-1.871	-23.08	78.93	-115.8	108.5***	-92.35***	0.438***	0.027*	0.951***
ICE	5.044	0.000	0.866***	-22.78	111.60**	-181.8***	110.7***	-77.56*	97.31*	0.282**	0.000	0.977***
JNS	12.697	0.090*	0.793***	4.782	-26.90	47.82	-40.51	31.98	12.77	0.585***	0.011**	0.981***
LM	14.335**	0.085***	0.765***	-2.303	-10.17	29.06	-28.50	35.44	-87.24	0.626***	0.013***	0.985***
SEIC	6.279***	0.104***	0.706***	20.465***	-80.64***	121.3***	-76.59***	70.71***	-66.56	0.601***	0.015*	0.979***
SLM	7.250***	0.048**	0.864***	2.836	-31.52	64.51**	-49.77**	30.31	-36.34	0.370***	0.006	0.983***

Table B.8: Parameter estimates of the DCC-FIGARCH model.

	FIGARCH				DCC		
Depositories	ω	β	ϕ	d	const	a	b
BAC	0.142	0.507	0.407	0.187***	0.634***	0.050	0.939***
BBT	0.118*	0.581***	0.151*	0.556***	0.667***	0.065*	0.895***
BK	0.123	0.532*	0.368	0.265***	0.618***	0.054**	0.891***
C	0.000	0.550	0.313	0.362**	0.726***	0.015**	0.983***
CBH	1.112	0.383	0.441	0.119	0.495***	0.011**	0.983***
CMA	0.182*	0.496	0.337	0.327	0.595***	0.004	0.993***
HBAN	0.140	0.492	0.366	0.267	0.598***	0.022***	0.977***
HCBK	0.627	0.302	0.457	0.087	0.364***	0.023***	0.973***
JPM	0.000	0.676***	0.235***	0.530***	0.713***	0.010**	0.989***
KEY	0.155	0.522	0.396	0.208**	0.601***	0.011***	0.987***
MI	0.056	0.714***	0.204***	0.584***	0.616***	0.015***	0.983***
MTB	0.163**	0.536***	0.304***	0.295***	0.586***	0.034***	0.962***
NCC	0.214*	0.135**	0.000***	0.203***	0.626***	0.015***	0.982***
NTRS	0.303	0.187***	0.000	0.224***	0.659***	0.012***	0.987***
NYB	0.342**	0.139*	0.000	0.336***	0.381***	0.050	0.919***
PBCT	0.245	0.499	0.322	0.355*	0.420***	0.019***	0.975***
PNC	0.061	0.556*	0.168**	0.432	0.631***	0.007	0.989***
RF	0.156	0.634***	0.207*	0.587**	0.618***	0.036	0.959***
SNV	0.120	0.309**	0.003	0.337***	0.622***	0.059	0.862***
SOV	0.077	0.485***	0.174*	0.427***	0.495***	0.016**	0.973***
STI	0.289	0.000	0.012	0.153*	0.656***	0.011**	0.985***
STT	0.512***	0.000	0.049***	0.220***	0.626***	0.007	0.984***
UB	0.071	0.541	0.286	0.427	0.503***	0.014*	0.979***
USB	0.020	0.531	0.252	0.370	0.593***	0.013**	0.986***
WAMUQ	0.205**	0.093	0.000*	0.252***	0.440***	0.023***	0.972***
WB	0.154*	0.694***	0.165*	0.629***	0.646***	0.019***	0.979***
WFC	0.103	0.460	0.411	0.178***	0.630***	0.015*	0.984***
ZION	0.226	0.657***	0.045	0.646***	0.551***	0.094	0.826

	FIGARCH				DCC		
Insurance	ω	β	ϕ	d	const	a	b
ABKFQ	0.330**	0.639***	0.000	0.705***	0.494***	0.004	0.993***
AET	2.495**	0.401	0.477	0.046	0.362***	0.068***	0.881***
AFL	0.468	0.470	0.410	0.179	0.433***	0.094***	0.759***
AIG	0.387	0.752***	0.027	0.784	0.577***	0.039	0.954***
AIZ	1.552***	0.000	0.037	0.000	0.310***	0.073	0.475***
ALL	0.188	0.505	0.375	0.250	0.470***	0.005*	0.994***
AON	2.104***	0.642***	0.000***	0.972***	0.387***	0.100	0.000
BRK.B	0.085*	0.041	0.000	0.288***	0.228***	0.047***	0.749***
CB	0.156	0.485***	0.226**	0.342***	0.518***	0.060**	0.474*
CFC	0.998***	0.468**	0.402*	0.197***	0.444***	0.009**	0.986***
CI	1.485***	0.771***	0.000	0.972***	0.325***	0.073**	0.708***
CINF	0.071	0.611***	0.290***	0.419***	0.583***	0.015***	0.985***
CNA	0.492**	0.482	0.384	0.233***	0.421***	0.059	0.813***
CVH	0.922*	0.502***	0.358***	0.241***	0.339***	0.009	0.951***
FNF	1.559	0.000*	0.297*	0.054	0.162*	0.030	0.927***
GNW	1.310***	0.000***	0.175***	0.000	0.340***	0.000	0.993***
HIG	0.194	0.518	0.313	0.375	0.585***	0.012**	0.986***
HNT	1.084**	0.489**	0.420**	0.151***	0.319***	0.016***	0.965***
HUM	1.518**	0.529	0.411	0.179***	0.338***	0.009	0.986***
L	0.200*	0.577**	0.310**	0.381***	0.466***	0.020	0.973***
LNC	0.165	0.524**	0.267	0.320***	0.617***	0.013***	0.986***
MBI	0.303*	0.542***	0.282*	0.422*	0.547***	0.009***	0.990***
MET	0.150	0.567***	0.116	0.454*	0.525***	0.008	0.988***
MMC	2.417	0.407*	0.476***	0.048	0.531***	0.010***	0.988***
PFG	0.138	0.237	0.049	0.254***	0.557***	0.020**	0.974***
PGR	0.222	0.570***	0.336**	0.327***	0.491***	0.014**	0.982***
PRU	0.206	0.471**	0.160	0.326***	0.594***	0.016***	0.979***
SAF	0.183*	0.682***	0.224**	0.553***	0.483***	0.015***	0.982***
TMK	0.086	0.494	0.324	0.353	0.593***	0.012***	0.986***
TRV	0.374*	0.565***	0.360***	0.281***	0.507***	0.018	0.981***
UNH	0.559*	0.096	0.000	0.189***	0.344***	0.030***	0.930***
UNM	1.173	0.692**	0.274**	0.453	0.338***	0.019	0.963***
WLP	0.600**	0.342**	0.154	0.255***	0.369***	0.028**	0.954***
WRB	0.654***	0.461	0.417	0.166***	0.343***	0.005*	0.993***

	FIGARCH				DCC		
Brokers	ω	β	ϕ	d	const	a	b
AGE	0.379***	0.214***	0.002***	0.282***	0.673***	0.036	0.956***
BSC	0.249*	0.405**	0.149	0.275***	0.671***	0.020***	0.977***
ETFC	0.360	0.571	0.326	0.349*	0.575***	0.013***	0.983***
GS	0.112	0.599***	0.274***	0.345***	0.702***	0.013**	0.985***
LEH	0.100	0.433*	0.125	0.344***	0.696***	0.011***	0.988***
MER	0.101	0.361*	0.094	0.313***	0.727***	0.017**	0.981***
MS	0.000	0.604***	0.322***	0.357***	0.719***	0.013***	0.984***
SCHW	0.407	0.586*	0.327	0.346***	0.658***	0.009***	0.986***
TROW	0.113	0.477*	0.207	0.341***	0.686***	0.017***	0.980***
Others							
ACAS	0.263*	0.207**	0.000**	0.315***	0.375***	0.000	0.906***
AMP	2.839***	0.436	0.436	0.000	0.485***	0.000	0.919
AMTD	1.476*	0.435*	0.362	0.275***	0.536***	0.039**	0.908***
AXP	0.049	0.602***	0.183**	0.489***	0.689***	0.011***	0.988***
BEN	0.250**	0.459***	0.118	0.359***	0.676***	0.022***	0.977***
BLK	0.765***	0.000**	0.109	0.219***	0.305***	0.003	0.995***
BOT	3.327	0.732	0.000	1.000	0.175**	0.000	0.787
CBG	1.497**	0.078	0.000***	0.208	0.362***	0.000	0.991***
CBSS	0.142*	0.421	0.379	0.242***	0.589***	0.059	0.920***
CIT	0.122	0.122*	0.000	0.233***	0.482***	0.017	0.914***
CME	4.066***	0.000**	0.172**	0.000	0.351***	0.000	0.579
COF	0.511	0.728***	0.109	0.689***	0.533***	0.053*	0.409***
FITB	0.365***	0.371	0.399	0.201***	0.559***	0.005	0.991***
FNM	0.765*	0.493***	0.421***	0.157***	0.414***	0.013***	0.983***
FRE	0.353	0.580***	0.383***	0.234***	0.432***	0.038**	0.923***
ICE	6.260	0.193	0.025	0.168	0.311***	0.007	0.838***
JNS	1.000**	0.167**	0.000	0.255***	0.573***	0.012**	0.983***
LM	1.298**	0.091	0.000	0.150***	0.604***	0.014***	0.985***
SEIC	0.361	0.490*	0.317	0.318***	0.593***	0.022*	0.973***
SLM	0.306	0.564***	0.390***	0.220*	0.365***	0.007	0.981***

*Notes: Stars indicate the statistical significance: *** means a statistically significance at 1% level, ** at 5% level and * at 10% level of significance.*

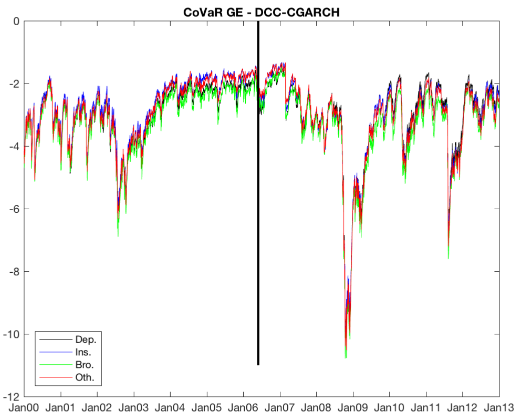
Table B.9: Parameter estimates of the DCC-ACGARCH model.

	ACGARCH						DCC		
Deposit.	ω	α	β	φ	ρ	γ	const	a	b
BAC	0.002	0.055	0.578***	0.011**	0.998***	0.140	0.633***	0.040	0.951***
BBT	0.010**	0.055	0.812***	0.027**	0.993***	0.076*	0.661***	0.013	0.981***
BK	0.004	0.042	0.737***	0.029	1.000***	0.091	0.613***	0.025	0.961***
C	0.006	0.010	0.781***	0.036***	0.996***	0.078**	0.722***	0.014**	0.982***
CBH	0.000***	0.160***	0.602***	0.000***	1.000***	0.000***	0.493***	0.009**	0.984***
CMA	0.004***	0.115***	0.820***	0.012	0.998***	0.000	0.591***	0.004	0.993***
HBAN	0.004	0.077***	0.829***	0.033	1.000***	0.000	0.594***	0.024***	0.975***
HCBK	0.000	0.136**	0.428**	0.000	1.000***	0.172	0.364***	0.021***	0.973***
JPM	0.003	0.000	0.892***	0.039**	1.000***	0.090***	0.708***	0.011**	0.987***
KEY	0.007	0.010	0.906***	0.023**	0.995***	0.043**	0.592***	0.011***	0.987***
MI	0.007	0.058***	0.884***	0.027*	0.996***	0.000	0.608***	0.013***	0.986***
MTB	0.004	0.039	0.862***	0.011	0.997***	0.041	0.583***	0.032***	0.963***
NCC	0.006	0.049**	0.833***	0.040	1.000***	0.000	0.620***	0.015***	0.983***
NTRS	0.021	0.000	0.788***	0.054	1.000***	0.179***	0.652***	0.014***	0.983***
NYB	0.000	0.040	0.875***	0.002	1.000***	0.080**	0.380***	0.024	0.967***
PBCT	0.012	0.090	0.820***	0.051	1.000***	0.030	0.415***	0.019***	0.974***
PNC	0.004	0.055	0.880***	0.035	1.000***	0.015	0.623***	0.006	0.990***
RF	0.001	0.084	0.861***	0.000	0.999***	0.029	0.613***	0.029	0.965***
SNV	0.004	0.058	0.830***	0.012	0.997***	0.055	0.618***	0.052	0.890***
SOV	0.013	0.082**	0.791***	0.061*	1.000***	0.000	0.491***	0.014**	0.974***
STI	0.005	0.046*	0.793***	0.014	0.996***	0.077	0.650***	0.010***	0.985***
STT	0.007	0.241	0.239	0.024	1.000***	0.127	0.624***	0.006	0.987***
UB	0.003	0.004	0.852***	0.020	0.998***	0.142***	0.502***	0.011*	0.983***
USB	0.002	0.017	0.870***	0.023	0.998***	0.082**	0.587***	0.013**	0.985***
WAMUQ	0.003	0.141	0.674***	0.016	0.999***	0.048	0.434***	0.023***	0.972***
WB	0.017	0.042	0.878***	0.046	0.994***	0.049	0.641***	0.017***	0.982***
WFC	0.003	0.034	0.864***	0.016*	0.997***	0.055	0.624***	0.012**	0.987***
ZION	0.009	0.000	0.887***	0.048	1.000***	0.113***	0.548***	0.017	0.979***

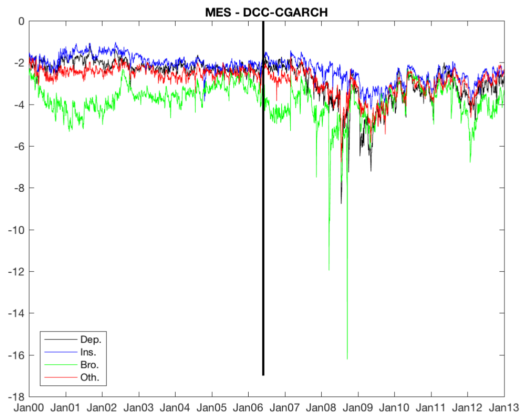
	ACGARCH						DCC		
Insurance	ω	α	β	φ	ρ	γ	const	a	b
ABKFQ	0.030	0.005	0.908***	0.072	0.997***	0.083*	0.497***	0.004	0.993***
AET	0.001	0.000	0.188	0.000	1.000***	0.398**	0.357***	0.051***	0.894***
AFL	0.004	0.129*	0.555*	0.010**	0.998***	0.000	0.428***	0.094***	0.749***
AIG	0.000	0.040	0.876***	0.003	0.999***	0.082	0.587***	0.032	0.958***
AIZ	0.001	0.000	0.435**	0.000	1.000***	0.178	0.297***	0.090	0.528***
ALL	0.002	0.104**	0.616***	0.013**	0.998***	0.008	0.472***	0.004	0.994***
AON	0.004	0.186***	0.729***	0.000	1.000***	0.000	0.389***	0.115	0.000
BRK.B	0.007	0.192	0.408	0.062	1.000***	0.089	0.231***	0.040**	0.756***
CB	0.000	0.065	0.850***	0.006	0.999***	0.042	0.516***	0.077***	0.376**
CFC	0.003	0.062	0.334	0.019	1.000***	0.180	0.445***	0.008**	0.987***
CI	0.002	0.134***	0.815***	0.000	1.000***	0.000	0.321***	0.065**	0.675***
CINF	0.000	0.046**	0.910***	0.001	0.999***	0.044*	0.581***	0.014***	0.985***
CNA	0.000	0.022	0.880***	0.005	0.999***	0.068*	0.425***	0.053*	0.780***
CVH	0.011	0.082***	0.798***	0.008	0.998***	0.000	0.339***	0.010	0.948***
FNF	0.002	0.386***	0.000	0.000	1.000***	0.000	0.207**	0.050	0.865***
GNW	0.000	0.153***	0.003	0.0033	1.000***	0.000	0.342***	0.000	0.994***
HIG	0.017	0.100***	0.831***	0.027*	0.995***	0.000*	0.579***	0.012**	0.987***
HNT	0.012	0.039	0.764	0.009	0.997***	0.071	0.316***	0.013*	0.970***
HUM	0.017	0.060**	0.596***	0.012**	0.997***	0.000	0.335***	0.006	0.991***
L	0.002	0.075**	0.804***	0.016	0.998***	0.010	0.466***	0.014	0.981***
LNC	0.010	0.029	0.889***	0.020**	0.996***	0.032	0.615***	0.013***	0.985***
MBI	0.000***	0.109***	0.844***	0.000	1.000***	0.000	0.546***	0.008***	0.991***
MET	0.000	0.038**	0.902***	0.000***	1.000***	0.060**	0.509***	0.007*	0.990***
MMC	0.000	0.081***	0.749***	0.000	1.000***	0.000***	0.530***	0.010***	0.987***
PFG	0.002	0.075*	0.780***	0.008	0.998***	0.049	0.554***	0.019**	0.975***
PGR	0.004	0.094**	0.650***	0.027**	1.000***	0.000	0.495***	0.012**	0.982***
PRU	0.000	0.036*	0.907***	0.000	1.000***	0.038	0.597***	0.011**	0.982***
SAF	0.008	0.030	0.853***	0.043	1.000***	0.078**	0.486***	0.014***	0.983***
TMK	0.0103**	0.092	0.722***	0.038**	0.994***	0.036	0.588***	0.010***	0.988***
TRV	0.005	0.041	0.799***	0.009**	0.997***	0.064	0.511***	0.015**	0.984***
UNH	0.017	0.071	0.828***	0.011	0.994***	0.028	0.339***	0.028**	0.931***
UNM	0.050	0.000	0.989***	0.033	1.000***	0.011	0.331***	0.016	0.968***
WLP	0.000***	0.077***	0.837***	0.000	1.000***	0.000	0.371***	0.021	0.960***
WRB	0.003	0.113	0.407	0.015	1.000***	0.127	0.350***	0.005*	0.993***

	ACGARCH						DCC		
Brokers	ω	α	β	φ	ρ	γ	const	a	b
AGE	0.006**	0.000	0.883***	0.014**	0.997***	0.115***	0.666***	0.032	0.960***
BSC	0.007	0.013	0.910***	0.017	0.997***	0.043**	0.669***	0.021***	0.974***
ETFC	0.014	0.000	0.909***	0.021***	0.997***	0.058*	0.573***	0.012***	0.982***
GS	0.007	0.000	0.943***	0.017**	0.997***	0.054***	0.697***	0.014*	0.983***
LEH	0.005	0.000	0.876***	0.038	1.000***	0.086**	0.691***	0.012***	0.985***
MER	0.003	0.000	0.898***	0.019***	0.998***	0.050***	0.725***	0.016***	0.982***
MS	0.002	0.028	0.903***	0.016*	0.998***	0.034	0.714***	0.014***	0.982***
SCHW	0.007	0.016	0.695	0.017**	0.998***	0.083	0.654***	0.010***	0.986***
TROW	0.006	0.022	0.910***	0.014	0.997***	0.065***	0.681***	0.016***	0.980***
Others									
ACAS	0.000	0.126**	0.732***	0.014	1.000***	0.000	0.376***	0.000	0.829
AMP	0.000	0.000	0.000	0.000	1.000	0.000	0.482***	0.000	0.865
AMTD	0.000	0.016	0.933***	0.003***	0.999***	0.050**	0.529***	0.034**	0.918***
AXP	0.006	0.000	0.873***	0.026***	0.996***	0.101***	0.689***	0.011***	0.988***
BEN	0.009	0.022	0.890***	0.013	0.996***	0.068***	0.672***	0.021***	0.978***
BLK	0.011	0.305**	0.029	0.029	1.000***	0.016	0.310***	0.003	0.995***
BOT	0.101	0.236**	0.639***	0.000	1.000***	0.125	0.125*	0.000	0.720***
CBG	0.010	0.093	0.739***	0.000	1.000***	0.168	0.367***	0.000	0.989***
CBSS	0.000	0.030	0.923***	0.000	0.999***	0.047	0.583***	0.041	0.945***
CIT	0.003	0.060	0.766***	0.007	0.997***	0.069	0.469***	0.020	0.900***
CME	0.001	0.146	0.242	0.000	1.000***	0.063	0.355***	0.000	0.443
COF	0.015	0.037	0.878***	0.036	1.000***	0.064	0.528***	0.056**	0.364***
FITB	0.013	0.212**	0.526***	0.049	1.000***	0.000	0.558***	0.006	0.989***
FNM	0.027	0.000	0.849***	0.018***	0.991***	0.084**	0.412***	0.012***	0.984***
FRE	0.003	0.039**	0.878***	0.019**	1.000***	0.000	0.429***	0.038**	0.923***
ICE	0.042	0.000	0.836***	0.000	1.000***	0.096	0.286***	0.008	0.829***
JNS	0.003	0.013	0.896***	0.000	0.999***	0.091***	0.580***	0.009*	0.982***
LM	0.014	0.000	0.875***	0.030	1.000***	0.074***	0.603***	0.015***	0.984***
SEIC	0.012	0.026	0.845***	0.012	0.996***	0.097**	0.588***	0.022**	0.969***
SLM	0.000	0.040	0.944***	0.000	1.000***	0.003	0.365***	0.008**	0.977***

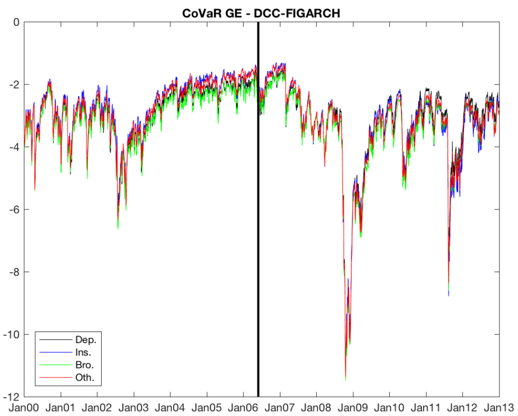
*Notes: Stars indicate the statistical significance: *** means a statistically significance at 1% level, ** at 5% level and * at 10% level of significance.*



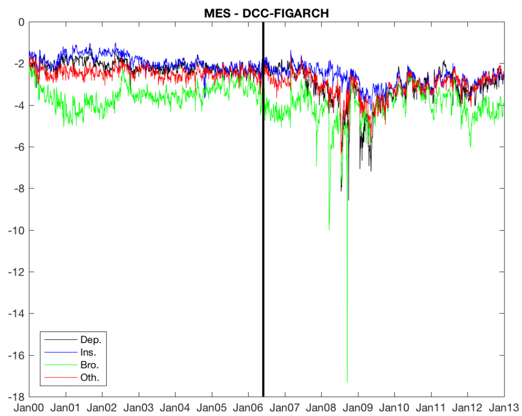
(a) CoVaR obtained by DCC-CGARCH.



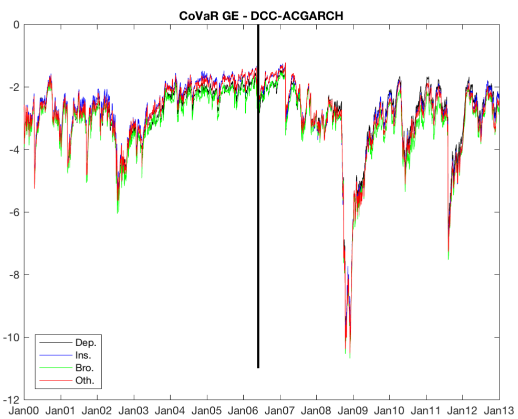
(b) MES obtained by DCC-CGARCH.



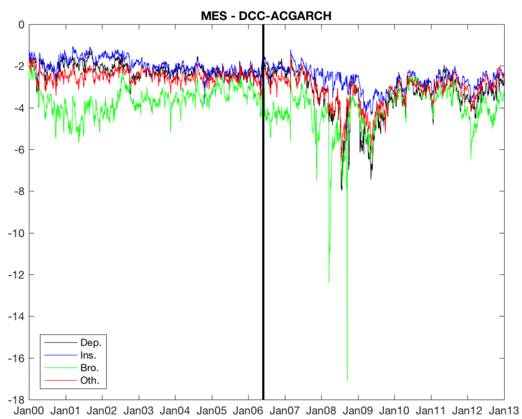
(c) CoVaR obtained by DCC-FIGARCH.



(d) MES obtained by DCC-FIGARCH.

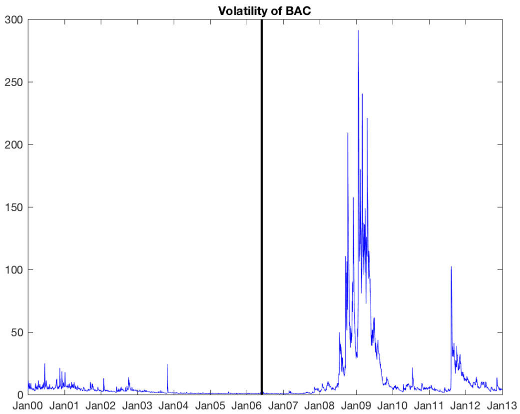


(e) CoVaR obtained by DCC-ACGARCH.

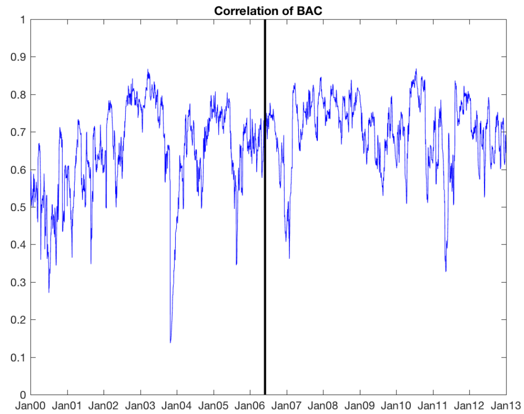


(f) MES obtained by DCC-ACGARCH.

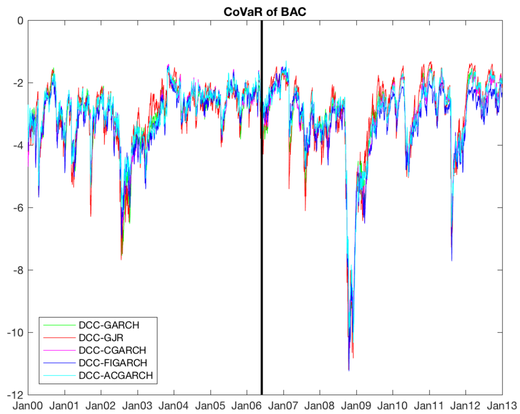
Figure B.3: Comparison of the CoVaR (first column) and MES (second column) measures by sub-industry group obtained by the DCC-CGARCH (first row) DCC-FIGARCH (second row) and DCC-GJR (third row) models. The black vertical line indicates the end of the in-sample and the beginning of the out-of-sample period.



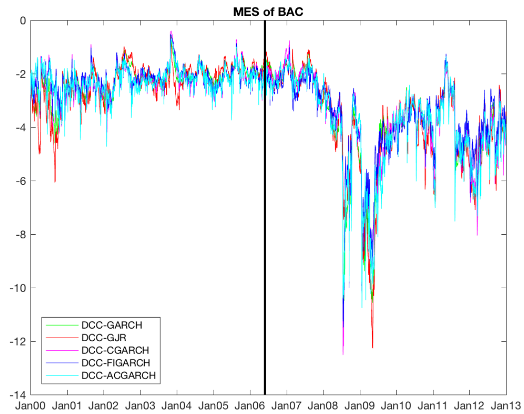
(a) Volatility of BAC obtained by DCC-ACGARCH.



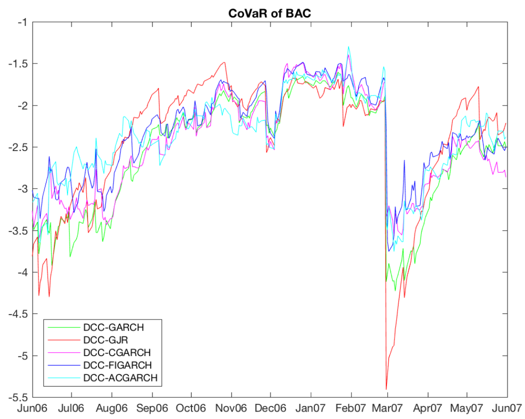
(b) Correlation of BAC obtained by DCC-ACGARCH.



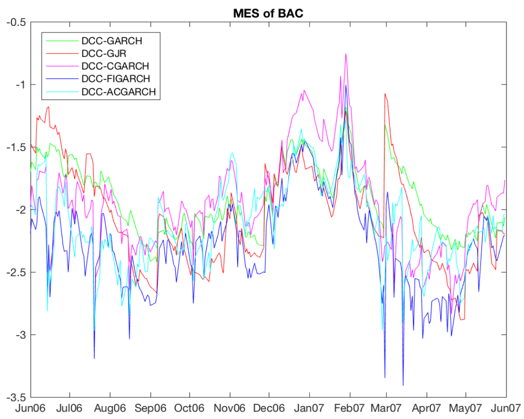
(c) CoVaR over the entire sample.



(d) MES over the entire sample.



(e) CoVaR over the pre-crisis sample.



(f) MES over the pre-crisis sample.

Figure B.4: Case-study of BAC. The black vertical line indicates the end of the in-sample and the beginning of the out-of-sample period.