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Essays in Industrial Organization

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Contents

Introduction

This dissertation comprises four essays on the topic of industrial organization and environmental economics. The first essay investigates the profitability of horizontal mergers of firms with price adjustments. We take a differential game approach and both the open-loop as well as the closed-loop equlibria are considered. In the second essay, using the same approach as the first one, we study the profitability of horizontal merger of firms where the demand function is non-linear. We take into consideration the open-loop equilibrium. The third essay studies the profitability of exogenous output constraint in a differential game model with price dynamics under the feedback strategies. The fourth essay investigates a second-best trade agreement between two countries when pollution spillovers are asymmetric to examine the strategic behavior of governments in using pollution taxes and tariffs under trade liberalization.

In chapter 1, taking a differential game approach with sticky prices in an oligopolistic industry, I have analyzed the consequences of horizontal mergers both in the open-loop and the closed-loop solutions. In view of the fact that I wanted to concentrate on the incentives to merge that are generated by price dynamics, I have assumed away any efficiency effects. It turns out that for a given level of the discount rate, merger incentives are higher when the mechanism governing price adjustment is very slow. When price is very sticky, the dynamic Cournot equilibrium price approaches the competitive equilibrium price of the static game in which firms set price equal to marginal cost. Firms would like to play the correct Cournot equilibrium but they cannot because price adjusts very slowly and in this aggressive environment they have an incentive to decrease the number of competitors through merger in order to make a slight correction in output setting mistakes and recover what they are losing.

Moreover, its results suggest that the relative number of firms that is required for merger to be profitable has two divergent trends under open-loop and closed-loop information structures. When firms play closed-loop, it is a decreasing function of the population of firms in the industry while for the open-loop it is the opposite. Accordingly, the larger the relevant information set, the higher is the possibility of collusion between firms. Given that pushing competition has a contradictory outcome under the closed-loop rule; it is worthwhile for policy makers and antitrust authorities to consider as well the nature of competition in the industry.

In the second chapter, we take a differential game approach with price dynamics to conduct an investigation into the consequences of horizontal merger of firms where the demand function is nonlinear. Given the shape of the problem at hand, we have analyzed the open-loop solution only. We show that in relation to the fact that the demand is nonlinear and prices follow some stickiness an incentive for small merger exists, while it does not appear under the standard approach using a linear demand function.

In our model, any decrease in the number of firms brings about a decrease in social welfare via a decrease in consumer surplus. Given the fact that, in principle, when there is no saving on the overall industry costs, any horizontal merger is socially harmful, and then the regulator must look out for mergers driven by the pressure generated by market size on firms, when the demand is non-linear. This has some interesting implications on the empirical side, as estimating market demand functions may indeed yield clearcut hints as to whether the market under consideration is likely to generate incentives towards horizontal mergers.

The third chapter examines profitability of exogenous output constraints. In a series of papers Gaudet and Salant (1991a,b) show that, in the case of Cournot competition among producers of perfect substitutes, a marginal contraction is strictly beneficial if and only if the number of firms in the designated subset exceeds the "adjusted" number of firms outside it by strictly more than one. In the special case of linear cost and demand functions, the firms in the subset will gain from an exogenously marginal contraction of their output if and only if they outnumber the firms outside the subset by more than one.

In this paper we generalize this result to the case of dynamic competition instead of looking at the one-shot game. While in the standard Cournot model any output constraint is not to the benefit of constrained firms, in this paper, we show that when firms play a dynamic Cournot game with Markov-perfect strategies, exogenous output constraint by a subset of firms results in: (i) increase in the value of unconstraint firms irrelevant of the amount of constraint because of having less intensive competition, (ii) increase in the market price for any output constraint below the optimal level and slightly above that because of lowering the total output caused by less competition and (iii) increase in the value of constrained firms for a viable range of parameters and initial conditions because of increasing the price during the price path. Our analysis has some applications to voluntary export restraints (VER), Mergers, Economics Sanctions, etc.

Finally, in the last chapter, we would like to examine the welfare implications of trade liberalization when governments behave strategically using environmental policy with asymmetric pollution spillovers. We investigated a second-best trade agreement between two countries to examine the strategic behavior of governments in using pollution taxes and tariffs under trade liberalization.

We found that when the marginal cost of pollution of the domestic firm increases, the pollution-shifting motive is enhanced and government wants to raise production taxes and surprisingly the rent-seeking behavior is observed and government raises import tariffs. On the other hand, when the marginal cost of pollution of the foreign firm increases, government want to reduce the level of tax and interestingly the level of tariff as well.

Part I

Essays in Dynamic Cournot Competition

Chapter 1

Profitability of Horizontal Mergers in the Presence of Price Stickiness

1.1 Introduction

When quantity-setting firms compete in a homogenous product industry with symmetric cost and the same demand functions, horizontal merger is modeled as an exogenous change in market structure. As a result, the level of competition decreases which increases the market price and market power of firms as well. In the case of linear demand and cost functions, the resulting anticompetitive forces are mostly to the benefit of outsiders and mergers are advantageous to the merging firm just in the circumstance that market share of merging firm is extremely high, at least 80% which is almost merging to a monopoly (Salant, Switzer and Reynolds, 1983 (henceforth SSR); Gaudet and Salant, 1991, 1992). Keeping everything the same, this threshold will be reduced to 50% (which is again a considerable market share) provided that the merged entity is not restricted to remain a Cournot player after the merger (Levin, 1990) or any demand function which satisfies the second-order conditions is allowed (Cheung, 1992). There are other studies showing that mergers are privately profitable if they are leader-generating (in industries where about less than one-third of the firms are leaders) (Daughety, 1990), or if merger generates synergies (Perry and

Porter, 1985; Farrell and Shapiro, 1990). However, the incentive to merge always exists once price is employed as the strategic variable rather than quantity. In a differentiated product industry, Deneckere and Davidson (1985) demonstrate that mergers of any size are beneficial if firms are engaged in a price-setting game.

We want to conduct an investigation into the consequences of horizontal mergers in oligopoly Cournot competition in the presence of price stickiness. When prices are sticky, for a given level of output the actual market price of a product does not adjust instantaneously to the price indicated by its demand function and price adjustment takes time. Since prices evolve over time we need a dynamic framework to investigate the effect of price stickiness on the profitability of horizontal mergers.

Using an oligopolistic differential game model with sticky prices in the specific case of instantaneous price adjustment, Dockner and Gaunersdorfer (2001) through a numerical analysis show that, contrary to the static game, in a dynamic Cournot game where firms use feedback strategies mergers are always profitable independently of the number of merging firms. Their result suggests that to analyzing merger, it is important to consider the nature of competition in the industry. Besides focusing on the same issue analytically, Benchekroun (2003) shows that when firms use open-loop strategies merger is profitable only if the market share of the merged firm is significant enough, very similar to the SSR results, which put more emphasis on the role of feedback strategies to create incentive to merge.

In this paper, we take a general approach without introducing specific assumptions on the degree of price stickiness to investigate the bearings of price dynamics. Scale economies as a motive for merger is ruled out by assumption because we would like to concentrate on the incentives to merge that are generated by price dynamics. To this end, we take a differential game approach to price dynamics introduced by Simaan and Takayama (1978) and its extension by Fershtman and Kamien (1987) and Cellini and Lambertini (2004, 2007).

We take into consideration both the open-loop and closed-loop (memoryless)[1] equilibria to investigate how the speed of adjustment can affect the profitability of horizontally merged firms. There emerges, when price adjust with a very sticky mechanism, mergers with a small number of insiders but large number of outsiders are also privately profitable even if firms play open-loop. Furthermore, by figuring out the least market share required for merger to be profitable when price adjusts instantaneously, we revisit the closed-loop effect to generate incentive to merge.

The remainder of the paper is organized as follows. Section 2 contains the layout of the model. Sections 3 illustrate the open-loop and closed-loop equilibria. The assessment of incentives towards mergers is given in section 4. Section 5 concludes the paper.

1.2 The setup

Consider a dynamic oligopoly market where n symmetric firms, at any $t \in [0, \infty)$, produce quantities $q_i(t) \geq 0$, $i \in \{1, 2, ..., n\}$, of the same homogeneous good with concave technologies described by the quadratic cost functions

$$C_i(t) = cq_i(t) + \frac{1}{2}q_i^2(t), \qquad c > 0. \tag{1.1}$$

In each period, the product price, $\hat{p}(t)$, is determined by means of the inverse demand function

$$\hat{p}(t) = A - \sum_{i=1}^{n} q_i(t). \tag{1.2}$$

[1] Broadly speaking, the main difference between the open-loop equilibrium on one hand and the feedback and closed-loop equilibria on the other is that the former does not take into account strategic interaction between players through the evolution of state variables over time and the associated adjustment in controls. Under the open-loop rule, players choose their respective plans at the initial date and commit to them forever. Therefore, in general, open-loop equilibria are not subgame perfect, in that they are only weakly time consistent since players make their action 'by the clock' only.

A further distinction can be made between the closed-loop equilibrium and the feedback equilibrium, which are both strongly time consistent and, therefore, subgame perfect since, at any date τ, players decide 'by the stock' of all state variables. However, while the closed-loop memoryless equilibrium takes into account the initial and current levels of all state variables, the feedback equilibrium accounts for the accumulated stock of each state variable at the current date. Hence, the feedback equilibrium is a closed-loop equilibrium, while the opposite is not true in general [2].

However, since price is sticky, the actual market price does not adjust instantaneously to the price given by the demand function. That is, $\hat{p}(t)$ will differ from the current price level, $p(t)$, and price moves according to the following equation

$$\frac{dp(t)}{dt} \equiv \dot{p}(t) = s\left\{\hat{p}(t) - p(t)\right\}, \tag{1.3}$$

where $s \in [0, \infty)$ is a constant that determines the speed of price adjustment. The lower is s, the higher is the degree of price stickiness. When s goes to infinity, price is not sticky and the actual market price is equal to the price given by the demand function.

The instantaneous profit function of firm i is

$$\pi_i(t) = q_i(t)\left[p(t) - c - \frac{1}{2}q_i(t)\right].$$

Therefore, the maximization problem of firm i is

$$\max_{q_i(t)} J_i = \int_0^{\infty} e^{-\rho t} q_i(t)\left[p(t) - c - \frac{1}{2}q_i(t)\right] dt, \tag{1.4}$$

subject to (2.2), $p(0) = p_0$ and $p(t) \geq 0$ for all $t \in [0, \infty)$. The factor $e^{-\rho t}$ discounts future gains, and the discount rate ρ is assumed to be constant and equal across firms.

We solve the differential game using both the open-loop information structure where firms choose their production plans at the initial date and stick to them for the whole time horizon and the closed-loop memoryless information structure where firms' quantity choices at any time depend on the initial and current levels of all state variables (here, price).

According to Cellini and Lambertini (2004), the steady state levels of the price and the individual output of a dynamic oligopoly game with price adjustments which are the premerger solution of our problem at the open-loop Nash equilibrium are

$$p^{OL} = A - nq^{OL}\ ; \qquad q^{OL} = \frac{(A-c)(\rho+s)}{(1+n)\rho + (2+n)s}, \tag{1.5}$$

and at the closed-loop Nash equilibrium are

$$p^{CL} = A - nq^{CL} \ ; \qquad q^{CL} = \frac{(A-c)(\rho+ns)}{s+(1+n)(\rho+ns)}. \tag{1.6}$$

The corresponding single period profits are

$$\pi^{OL} = \frac{(A-c)^2(\rho+s)(\rho+3s)}{2\left[(1+n)\rho+(2+n)s\right]^2} \ ; \qquad \pi^{CL} = \frac{(a-c)^2(\rho+ns)(\rho+(2+n)s)}{2\left[s+(1+n)(\rho+ns)\right]^2}.$$

The superscripts OL and CL indicate the open-loop and closed-loop equilibrium level of a variable, respectively.

For later reference, let us also note that in the static game where the demand and cost functions are specified by (3.1) and (1.2) in turn, the equilibrium prices when firms play *à la* Cournot and *à la* Bertrand respectively are

$$p^{CN} = \frac{2A+nc}{n+2}, \tag{1.7}$$

$$p^{BN} = \frac{A+nc}{n+1}. \tag{1.8}$$

1.3 The merger equilibrium

In this section, we consider a horizontal merger of m firms $(1 < m \leq n)$ where they act collusively to maximize their discounted joint profits.[2] $n - m$ firms stay outside the merger. Hence, the differential game becomes

$$\max_{\bar{q}_i} J^m = \int_0^\infty e^{-\rho t} \left[(p(t)-c) \sum_{i=1}^{m} \bar{q}_i(t) - \frac{1}{2} \sum_{i=1}^{m} \bar{q}_i^2(t) \right] dt, \qquad i = 1, ..., m \tag{1.9}$$

$$\max_{q_j(t)} J_j = \int_0^\infty e^{-\rho t} q_j(t) \left[p(t) - c - \frac{1}{2} q_j(t) \right] dt, \qquad j = m+1, ..., n \tag{1.10}$$

subject to

$$\frac{dp(t)}{dt} \equiv \dot{p}(t) = s \left\{ A - \sum_{i=1}^{m} \bar{q}_i(t) - \sum_{j=m+1}^{n} q_j(t) - p(t) \right\}, \tag{1.11}$$

and to the initial conditions $p(0) = p_0$ and $p(t) \geq 0$.

[2] Given the convex cost function, it is optimal to produce with all m firms, and not to concentrate production on one firm only.

$\bar{q}_i(t) \geq 0$, $i \in \{1, 2, ..., m\}$ and $q_j(t) \geq 0$, $j \in \{m+1, ..., n\}$ denote, in turn, the output level of an insider and an outsider. J^M and J_j represent the problem of the merging firm and outsiders, respectively.

According to (1.9), (2.5) and (3.36), the Hamiltonian functions of merging firms and outsiders are

$$\begin{aligned} H^M(t) &= e^{-\rho t}\left\{(p(t)-c)\sum_{i=1}^{m}\bar{q}_i(t) - \frac{1}{2}\sum_{i=1}^{m}\bar{q}_i^2(t)\right. \qquad (1.12) \\ &\left. +\bar{\lambda}_i(t)s\left[A-\sum_{i=1}^{m}\bar{q}_i(t)-\sum_{j=m+1}^{n}q_j(t)-p(t)\right]\right\}, \end{aligned}$$

$$\begin{aligned} H_j(t) &= e^{-\rho t}\left\{q_j(t)\left[p(t)-c-\frac{1}{2}q_j(t)\right]\right. \qquad (1.13) \\ &\left. +\lambda_j(t)s\left[A-\sum_{i=1}^{m}\bar{q}_i(t)-\sum_{j=m+1}^{n}q_j(t)-p(t)\right]\right\}, \end{aligned}$$

where $\lambda_j(t) = \mu_j(t)e^{\rho t}$ and $\bar{\lambda}_i(t) = \bar{\mu}_i(t)e^{\rho t}$ and $\mu_j(t)$ and $\bar{\mu}_i(t)$ are the co-state variables associated with $p(t)$.

1.3.1 Open-loop equilibrium

After the merger, at the open-loop Nash equilibrium, the steady state levels of the price and the output of merging firm and outsiders are

$$p^{OL}_{post} = A - q^{OL}_M - (n-m)\,q^{OL}_O,$$

$$q^{OL}_M = \alpha m(\rho+2s), \qquad q^{OL}_O = \alpha(\rho+s+ms),$$

where

$$\alpha = \frac{(A-c)(\rho+s)}{(n+1)\rho^2 + [2n+m(n-m+2)+3]\rho s + [n+m(n-m+3)+2]s^2}.$$

The subscripts M and O indicate the equilibrium level of a variable for the merging firm and an outsider and subscripts *post* refers to the equilibrium level the price after the merger. Hence, the steady state equilibrium profits are as follows

$$\pi^{OL}_M = \frac{\alpha^2 m(\rho+2s)^2(\rho+s+2ms)}{2(\rho+s)}; \qquad \pi^{OL}_O = \frac{\alpha^2(\rho+3s)(\rho+s+ms)^2}{2(\rho+s)}.$$

For the proof you can see Benchekroun (2003).

1.3.2 Closed-loop equilibrium

Now, we look for the post-merger Nash equilibrium under the closed-loop strategies. The outcome is summarized by the following proposition:

Proposition 1 *At the closed-loop Nash equilibrium, the steady state levels of the price and the output of merging firm and outsiders are*

$$p_{post}^{CL} = A - q_M^{CL} - (n-m)\, q_O^{CL}, \tag{1.14}$$

$$q_M^{CL} = \beta m \left(\rho + (n-m+1)\, s\right) \left(\rho + \left(m^2 - m + n + 1\right) s\right), \tag{1.15}$$

$$q_O^{CL} = \beta(\rho + s\,(n+1)) \left(\rho + (m^2 - m + n)s\right), \tag{1.16}$$

where

$$\begin{aligned} \beta \;\; = \;\; & (A-c) \,/\, \left[(n+1)\,\rho^2 + \left(n\left(m^2 - m + 2n + 3\right) + 2\right)\rho s \right. \\ & \left. + \left((n+1)\left(m^2 n - mn + n^2 + n + 1\right) - m^4 + m^3\right) s^2\right] \end{aligned}$$

which yields the steady state equilibrium profits

$$\pi_M^{CL} = \frac{1}{2}\beta^2 m(\rho + (n-m+1)\, s)(\rho + (n+m+1)\, s)(\rho + (m^2 - m + n + 1)s)^2$$

$$\pi_O^{CL} = \frac{1}{2}\beta^2(\rho + s\,(n+1))^2 \left(\rho + (m^2 - m + n)s\right)\left(\rho + (m^2 - m + n + 2)s\right)$$

Proof. Taking the first-order conditions w.r.t. $\bar{q}_i(t)$ and $q_j(t)$ and using (2.7) and (2.8), in turn, we have

$$\frac{\partial H^M(t)}{\partial \bar{q}_i(t)} = p\,(t) - c - \bar{q}_i(t) - \bar{\lambda}_i(t)s = 0, \tag{1.17}$$

$$\frac{\partial H_j(t)}{\partial q_j(t)} = p\,(t) - c - q_j(t) - \lambda_j(t)s = 0, \tag{1.18}$$

which yields the optimal closed-loop output for, respectively, the insiders and outsiders as follows

$$\bar{q}_i^{CL}(t) = \begin{cases} p\,(t) - c - \bar{\lambda}_i(t)s & \text{if } p\,(t) > c + \bar{\lambda}_i(t)s, \\ 0 & \text{otherwise,} \end{cases} \tag{1.19}$$

$$q_j^{CL}(t) = \begin{cases} p\,(t) - c - \lambda_j(t)s & \text{if } p\,(t) > c + \lambda_j(t)s, \\ 0 & \text{otherwise.} \end{cases} \tag{1.20}$$

The adjoint equations for the optimum are

$$-\frac{\partial H^M(t)}{\partial p(t)} - \sum_{j=m+1}^{n} \frac{\partial H^M(t)}{\partial q_j(t)} \frac{\partial q_j^{CL}(t)}{\partial p(t)} = \frac{\partial \bar{\lambda}_i(t)}{\partial t} - \rho \bar{\lambda}_i(t), \tag{1.21}$$

$$-\frac{\partial H_j(t)}{\partial p(t)} - \sum_{\substack{k=m+1,\\ k\neq j}}^{n} \frac{\partial H_j(t)}{\partial q_k(t)} \frac{\partial q_k^{CL}(t)}{\partial p(t)} - m \sum_{i=1}^{m} \frac{\partial H_j(t)}{\partial \bar{q}_i(t)} \frac{\partial \bar{q}_i^{CL}(t)}{\partial p(t)} = \frac{\partial \lambda_i(t)}{\partial t} - \rho \lambda_i(t). \tag{1.22}$$

The transversality conditions are

$$\lim_{t\to\infty} \bar{\mu}_i(t).p\,(t) = 0; \qquad \lim_{t\to\infty} \mu_j(t).p\,(t) = 0.$$

From (1.19) and (1.20) we obtain

$$\frac{\partial q_j^{CL}(t)}{\partial p(t)} = \frac{\partial q_k^{CL}(t)}{\partial p(t)} = \frac{\partial \bar{q}_i^{CL}(t)}{\partial p(t)} = 1. \tag{1.23}$$

The difference between the closed-loop and open-loop solutions is due to these terms in equations (1.21) and (1.22) which are set equal to zero in the open-loop case.[3] That is, when firms play closed-loop strategies, each firm inserts her information regarding the dependency of the other firms' supply policy on the current market price into the adjoint equation. The additional terms in the co-state equations (1.21) and (1.22) imply the strategic interaction among firms, which are not considered by definition in the open-loop solution. Furthermore, the adjoint equation of merging firm (1.21) is different from the adjoint equation of an outsider (1.22). Since there is a cartel inside the group of insiders, there is no strategic interaction among insiders while looking at (1.22) we recognize that in addition to the strategic interaction between each outsider and any of the insiders there are strategic interactions among outsiders.

Differentiating (2.7) and (2.8) w.r.t. the co-state variables and using (2.3), equations (1.21) and (1.22) can be rewritten as

$$-\sum_{i=1}^{m} \bar{q}_i(t) + \bar{\lambda}_i(t)s - \sum_{j=m+1}^{n} \bar{\lambda}_i(t)s = \frac{\partial \bar{\lambda}_i(t)}{\partial t} - \rho \bar{\lambda}_i(t),$$

[3] In the open-loop solution, the adjoint equations for the optimum for insiders and outsiders are as follows, respectively

$$-\frac{\partial H^M(t)}{\partial p(t)} = -\sum_{i=1}^{m} \bar{q}_i(t) + \bar{\lambda}_i(t)s = \frac{\partial \bar{\mu}_i(t)}{\partial t}; \qquad -\frac{\partial H_j(t)}{\partial p(t)} = -q_j(t) + \lambda_j(t)s = \frac{\partial \mu_j(t)}{\partial t}$$

$$-q_j(t) + \lambda_j(t)s - \sum_{\substack{k=m+1, \\ k\neq j}}^{n} \lambda_j(t)s - m\sum_{i=1}^{m} \lambda_j(t)s = \frac{\partial \lambda_i(t)}{\partial t} - \rho\lambda_i(t).$$

Inducing symmetry assumption, we obtain

$$\frac{\partial \bar{\lambda}(t)}{\partial t} = -m\bar{q}(t) + [(m-n+1)\, s + \rho]\, \bar{\lambda}(t), \tag{1.24}$$

$$\frac{\partial \lambda(t)}{\partial t} = -q(t) + \left[\left(-m^2 - n + m + 2\right) s + \rho\right] \lambda(t). \tag{1.25}$$

Differentiating (1.19) and (1.20) w.r.t. time and using (1.24) and (1.25) we find

$$\frac{d\bar{q}(t)}{dt} = \frac{dp\,(t)}{dt} - \left[-m\bar{q}(t) + [(m-n+1)\, s + \rho]\, \bar{\lambda}(t)\right] s, \tag{1.26}$$

$$\frac{dq(t)}{dt} = \frac{dp\,(t)}{dt} - \left[-q(t) + \left[\left(-m^2 - n + m + 2\right) s + \rho\right] \lambda(t)\right] s. \tag{1.27}$$

Using (3.36), (1.19) and (1.20) where a symmetry assumption is introduced for an individual firm output inside the group of insiders and also the group of outsiders, we can rewrite (1.26) and (1.27) as follows

$$\begin{aligned} \frac{d\bar{q}(t)}{dt} &= sA + [(n-m-1)\, s - \rho]\, c + [(m-n)\, s + \rho]\, p\,(t) \\ &\quad -s\,(n-m)\, q(t) + [(n-m-1)\, s - \rho]\, \bar{q}(t), \end{aligned}$$

$$\begin{aligned} \frac{dq(t)}{dt} &= sA + c\left[\left(m^2 + n - m - 2\right) s - \rho\right] - sm\bar{q}(t) \\ &\quad + \left[\left(-m^2 - n + m + 1\right) s + \rho\right] p(t) + \left[\left(m^2 - 1\right) s - \rho\right] q(t). \end{aligned}$$

$d\bar{q}(t)/dt = 0$, $dq(t)/dt = 0$ and $dp(t)/dt = 0$, which are linear relationships between p, $\bar{q}$ and q, yield the steady state of the system and the equilibrium point is a saddle with (1.14), (1.15) and (1.16). ■

Keeping symmetry assumption in the group of insiders as well as the group of outsiders, the two groups are necessarily asymmetric. Because essentially there is a cartel among insiders while the rest of the market behave like dynamic Cournot competitors. These asymmetries between the two groups are not only with respect to the first-order conditions and controls but in particular with respect to the co-state amounts. By construction, the list of co-state values entails that the shadow price attached by any outsider will be systematically different from the shadow price attached to the price dynamics by one of the insiders. Considering (1.17) and (1.18), we can rewrite the FOCs for outsiders as $\bar{\lambda}(t) = p(t) - c - \bar{q}(t)/s$ and insiders as $\lambda(t) = p(t) - c - q(t)/s$. Then, taking into account the fact that the output level of an outsider is greater than the output level of a single insider, we have the following consequence

Corollary 2 *The shadow price of an insider is greater than an outsider's* $\left(\bar{\lambda}(t) > \lambda(t)\right)$.

This entails that the proportional change of merging firm's profit, on account of alteration in the state equation, is more than that of an outsider.

1.4 The incentive to merge

After finding the post-merger equilibrium, we are able to investigate the profitability of a horizontal merger with price dynamics in a Cournot competition. First, we figure out the minimum percentage of insiders which is required to make the merger profitable in the case of instantaneous price adjustment. Then, we evaluate merger profitability in the space $(m,\ s/\rho)$ for a given initial population of firms to perceive the role of price stickiness in stimulating merger incentives.

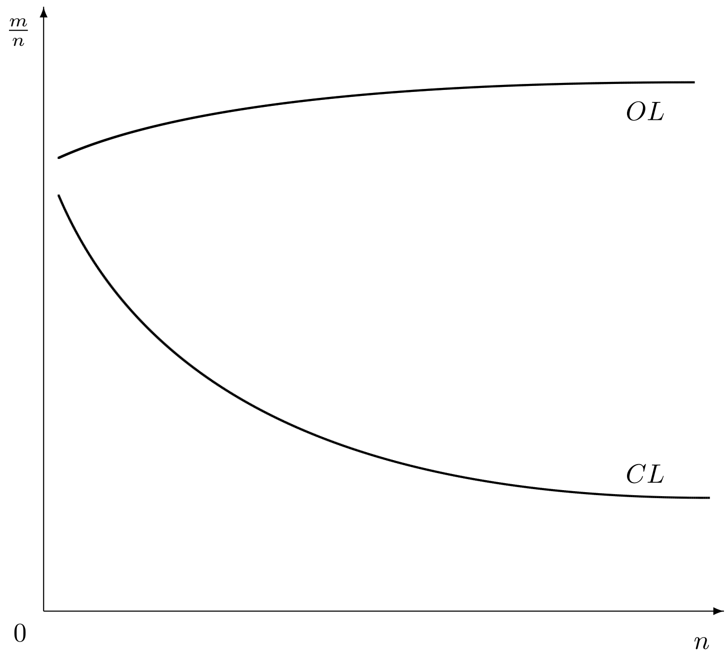
To deal with the above mentioned issues, we will consider the difference between the post-merger profit of the merging firm and sum of the individual profits of the insiders before the merger which has to be positive as a condition for merger profitability. That is, in an n-firm industry, m firms will find it profitable to merge if and only if the merger profitability condition $\pi_M^{OL} - m\pi^{OL} > 0$ (open-loop) or $\pi_M^{CL} - m\pi^{CL} > 0$ (closed-loop) holds.

When the speed of price adjustment goes to infinity, Dockner and Gaunersdorfer (2001) and Benchekroun (2003) showed that when firms use feedback strategies mergers are always profitable irrespective of the number of insiders whereas we will show that it is not the case for the closed-loop (memoryless) and open-loop strategies and a sufficient proportion of firms is required. However, as compared to the open-loop, this proportion is very different when firms play closed-loop. Figure 1 illustrates corresponding results graphically. From this graph we can see that as the population of firms in the industry increases, the minimum proportion of firms that makes the merger profitable has a decreasing trend under the closed-loop strategies while it has an increasing trend under the open-loop strategies. Thus, we can argue that it is much easier to maintain collusion among insiders in the closed-loop equilibrium than the open-loop. This difference is due to the fact that "open-loop" and "closed-loop" refer to the two different information structures. In both cases, everybody operate under the complete information but, as it is explained in previous section in detail, under the closed-loop information structure firms explicitly incorporate strategic interactions in the co-state equations while in the open-loop they do not.

In figure 2, the region of parameters s and ρ for which merger of m firms is profitable is represented by means of two dividing curves under the open-loop and closed-loop equilibria in a ten-firm industry. We provide this graph to show that in cases where price is too sticky, merger would be to the benefit of merging firm even if its market share is low.

In this figure we can see that in the open-loop equilibrium when the speed of adjustment goes to infinity, merger must involve at least eight insiders to become profitable. As it is investigated by Fershtman and Kamien ([9], pp. 1159-1161), in the limit where s tends to infinity, the open-loop equilibrium (1.5) coincides with the static Cournot Nash equilibrium (1.7) and we know that in the static Cournot model merger is disadvantageous to the merging firm unless the market share of merging firm is sufficiently high (at least 80%). However, in the closed-loop Nash equilibrium, as this figure clearly displays, merger of four firms in ten-firm industry is always profitable which is due to the closed-loop rule properties explained earlier.

Figure 1: The lowest proportion of firms to be merged to make a profitable merger for the instantaneous price adjustment.

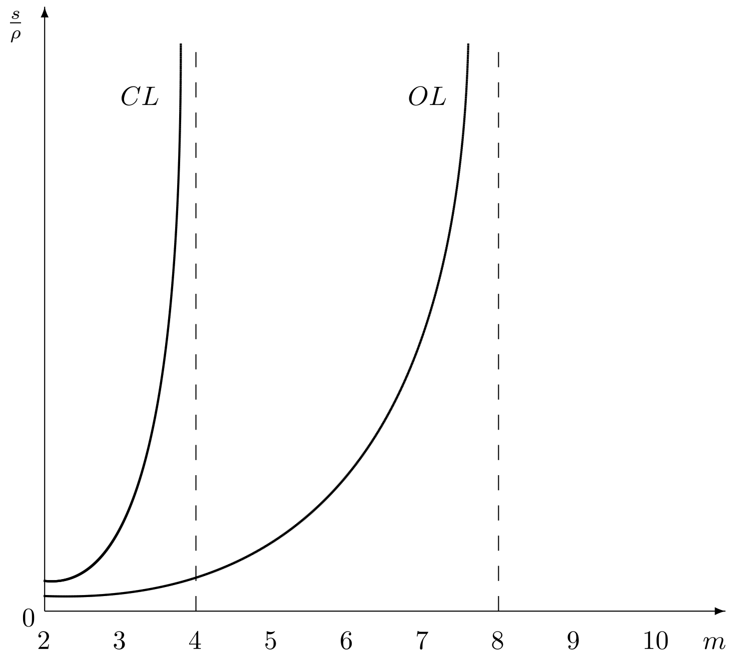


For a given level of discount rate, as the population of insiders decreases, the speed of price adjustment must reduce as well in order to make the merger profitable. This means that for a given rate of discounting, merger incentives are higher when the speed of price adjustment is slower. Irrespective of the information structure, if the price adjusts very slowly, the equilibrium price is very close to the perfectly competitive one (in the limit, if $s = 0$, it collapses onto the competitive price $(A+nc)/(n+1)$, as in (1.8)[4]). In games where firms are Bertrand competitors in homogeneous goods - like here - the profitability of mergers is driven by the increase in market price generated by the reduction in the population of firms, that benefits insiders and outsiders alike [5]

[4] Also, the features of the feedback equilibrium in the limit where the discount rate tends to infinity is looked into by Fershtman and Kamien ([9], pp. 1159) and they demonstrated that in such circumstances, the feedback equilibrium coincides with the Bertrand equilibrium of the static game.

[5] For the detailed analysis of the same problem under product differentiation, see Deneckere and Davidson (1985).

Figure 2: Merger profitability in the space $(m, s/\rho)$ for $n = 10$



1.5 Conclusions

Taking a differential game approach with sticky prices in an oligopolistic industry, we have analyzed the consequences of horizontal mergers both in the open-loop and the closed-loop solutions. In view of the fact that we wanted to concentrate on the incentives to merge that are generated by price dynamics, we have assumed away any efficiency effects. It turns out that for a given level of the discount rate, merger incentives are higher when the mechanism governing price adjustment is very slow. When price is very sticky, the dynamic Cournot equilibrium price approaches the competitive equilibrium price of the static game in which firms set price equal to marginal cost. Firms would like to play the correct Cournot equilibrium but they cannot because price adjusts very slowly and in this aggressive environment they have an incentive to decrease the number of competitors through merger in order to make a slight correction in output setting mistakes and recover what they are losing.

Moreover, our results suggest that the relative number of firms that is required for merger to be profitable has two divergent trends under open-loop and closed-loop information structures. When firms play closed-loop, it is a decreasing function of the population of firms in the industry while for the open-loop it is the opposite. Accordingly, the larger the relevant information set, the higher is the possibility of collusion between firms. Given that pushing competition has a contradictory outcome under the closed-loop rule; it is worthwhile for policy makers and antitrust authorities to consider as well the nature of competition in the industry.

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Chapter 2

The Profitability of Small Horizontal Mergers with Nonlinear Demand Functions

2.1 Introduction

The existing literature on the profitability of horizontal mergers within the static Cournot framework demonstrates that a merger will be profitable to the merging firms provided that either the number of merging firms is large enough (typically, too large to be realistic) or the merger creates a strong synergy (typically, a reduction in the average costs bringing about the well known *efficiency defense*). Convincing illustrations of these claims can be found in Salant, Switzer and Reynolds (1983) and Farrell and Shapiro (1990), respectively. The subsequent contributions have enriched the discussion along much the same lines. Levin (1990) and Cheung (1992), using different setups, demonstrate that at least 50% of the market should merge to provide a profitable merger. Gaudet and Salant (1991, 1992) show that if a subset of firms were to produce below their Cournot equilibrium quantity, their profits would fall if the subset of firms is not large enough. the efficiency argument is first advocated by Perry and Porter (1985), showing that if firms can benefit from some economies of scale the merger will become profitable, and then extensively discussed by Farrell and Shapiro

(1990), who find that a horizontal merger that does not generate synergies raises the price and this price increase can make the merger profitable only when the market share of merging firm is large enough.

All of the above is based upon a static approach. There are also studies addressing the profitability of mergers using dynamic models. Dockner and Gaunersdorfer (2001), through a numerical analysis, and Benchekroun (2003), analytically, revisit the differential game introduced by Fershtman and Kamien (1987) to assess horizontal mergers among firms that compete in a dynamic Cournot oligopoly with price adjustments. They consider feedback and open-loop equilibria where the demand function is linear. Cellini and Lambertini (2007) model the optimal capacity and output decisions of oligopolistic firms in a dynamic game with capital accumulation *á la* Ramsey. Borrowing the non-linear demand structure from Anderson and Engers (1992, 1994), they allow for non-linear market demand functions. They use this setup to investigate the role of horizontal mergers in driving the economy towards the Ramsey *modified golden rule*.

Here, we take an alternative route to the analysis of the incentive to carry out small mergers table when the pre-merging degree of concentration is high but any cost saving through the merger is ruled out by assumption. To do this, we take a differential game approach with sticky price dynamics *à la* Simaan and Takayama (1978) and Fershtman and Kamien (1987), combined with a non-linear demand structure *à la* Anderson and Engers (1992, 1994). Given that the resulting maximization problem does not take any of the forms for which we have a candidate for the value function, we take into consideration the open-loop equilibrium only, focussing on the interplay between price stickiness and the curvature of demand. The main point we make in this paper boils down to the following: as the market shrinks because demand becomes convex, firms bear increasing profit losses, and this creates an incentive for small mergers that would not appear under the standard approach with a linear demand function. Additionally, our analysis also singles out the existence of a parameter region where the merger between two firms out of three is always profitable, for any degree of price stickiness as well as for some degree of concavity of the demand function.

The remainder of the paper is organized as follows. Section 2 contains the layout of the model. Sections 3 illustrate the pre-merger open-loop equilibrium. The assessment of profit incentives towards small mergers is carried out section 4. Section 5 concludes.

2.2 The setup

Consider a dynamic oligopoly where n symmetric firms produce the same homogeneous good over continuous time $t \in [0, \infty)$, all of them with the same constant average and marginal cost $c = 0$. The *notional* market demand function is defined as follows:

$$Q(t) = A - (\hat{p}(t))^{\alpha} \quad , \ \alpha > 0.$$

The above function is always downward sloping, and can be either convex ($0 < \alpha < 1$) or concave ($\alpha > 1$). If $\alpha = 1$, it is linear. In each period, the market price $\hat{p}(t)$ is determined by the following inverse demand function:

$$\hat{p}(t) = (A - Q(t))^{\frac{1}{\alpha}} . \tag{2.1}$$

where $Q(t) = \sum_{i=1}^{n} q_i(t)$ is the industry output and $q_i(t) \geq 0$, $i \in \{1, 2, ..., n\}$ is the individual output of firm i at time t.

However, since price is sticky, the actual market price does not adjust instantaneously to the notional price level (2.1). That is, $\hat{p}(t)$ will differ from the current price level $p(t)$ at any time except that in steady state, with the price moving according to the following equation:

$$\frac{dp(t)}{dt} \equiv \dot{p}(t) = s \{\hat{p}(t) - p(t)\} , \tag{2.2}$$

where s $(0 < s < \infty)$ is a constant parameter that determines the speed of adjustment. The lower is s, the higher is the degree of price stickiness.

The instantaneous profit function of firm i is

$$\pi_i(t) = p(t)q_i(t).$$

Therefore, the objective of firm i is

$$\max_{q_i(t)} J_i = \int_0^\infty e^{-\rho t} p(t) q_i(t) dt, \tag{2.3}$$

subject to (2.2) and to the initial condition $p(0) = p_0$ and the non-negativity condition $p(t) \geq 0$ for all $t \in [0, \infty)$. The factor $e^{-\rho t}$ discounts future gains, and the discount rate ρ is assumed to be constant and equal across firms.

The elasticity of demand function (2.1) w.r.t. price, $\varepsilon_{Q,P}$, can be written as follows:

$$|\varepsilon_{Q,P}| = -\frac{\partial Q(\alpha)}{\partial p(Q(\alpha))} . \frac{p(Q(\alpha))}{Q(\alpha)} = \frac{\alpha p^\alpha}{A - p^\alpha}.$$

2.3 The pre-merger Cournot equilibrium

We consider the sum of the consumer surplus and the individual profits of the present firms as the appropriate measure of the social welfare level. Since the marginal cost is zero, the social welfare is the full integral of the curve up to the optimal level of output, Q^*:

$$SW = \int_0^{Q^*} (A - z)^{\frac{1}{\alpha}} dz = \frac{\alpha}{1 + \alpha} \left[A^{\frac{\alpha+1}{\alpha}} - (A - Q^*)^{\frac{\alpha+1}{\alpha}} \right] \tag{2.4}$$

Here, since there is no fixed cost, the merger cannot be carried out and justified on the basis of an efficiency argument, and we are not going to dwell upon whether any merger might be socially acceptable. Accordingly, we will focus solely on the firms' incentives.

In solving the quantity-setting game between profit-seeking agents, we shall focus upon a single representative firm, whose Hamiltonian function is:[1]

[1] Observe that the maximand is neither state linear, linear-quadratic nor exponential, and therefore we cannot specify the value function in view of the closed-form feedback solution of the resulting Bellman equation. For an overview of differential games allowing for the analytical characterisation of the feedback equilibrium, see Dockner *et al.* (2000, ch. 7).

$$H_i(t) = e^{-\rho t}\left\{q_i(t)p(t) + \lambda(t)s\left[(A - q_i(t) - Q_{-i}(t))^{\frac{1}{\alpha}} - p(t)\right]\right\}, \qquad (2.5)$$

where $\lambda(t) = \mu(t)\,e^{\rho t}$, $\mu(t)$ being the co-state variable associated to $p(t)$.

As a first step, we can prove the following:

Proposition 3 *At the open-loop Nash equilibrium, the steady state levels of the price and the output of firm i are*

$$q^* = \frac{\alpha A(\rho + s)}{s + \alpha n(\rho + s)},$$

$$p^* = (A - nq^*)^{\frac{1}{\alpha}}.$$

The equilibrium is a saddle point for all $\alpha \geq 1$, while it is unstable for all $\alpha \in (0, 1)$.

Proof. Taking the first-order condition (FOC) on (2.5) w.r.t. $q_i(t)$, we obtain

$$p(t) - \frac{1}{\alpha}s\lambda(t)(A - q_i(t) - Q_{-i}(t))^{\frac{1}{\alpha}-1} = 0. \qquad (2.6)$$

which seems to be unsolvable because of the exponent $\frac{1}{\alpha} - 1$. In general, what is done is to take the FOC, solve it w.r.t. the control variable and then differentiate w.r.t. time. What we actually want is not the explicit solution of (3.36) w.r.t. $q_i(t)$, but rather a control equation describing the evolution of the individual output over time. Hence, we can differentiate the FOC (3.36) w.r.t. time in the first place and then, by introducing a symmetry condition $q_i(t) = q_j(t) = q(t)$ for all i, j on outputs, we get the control equation as follows:

$$\frac{dq(t)}{dt} = \dot{q}(t) = \frac{(A - nq(t))^{1-\frac{1}{\alpha}}\left[\alpha s\dot{\lambda}(t)(A - nq(t))^{\frac{1}{\alpha}} - \alpha^2(A - nq(t))\dot{p}(t)\right]}{ns(1-\alpha)\lambda(t)}. \qquad (2.7)$$

Then, we can solve the FOC (3.36) to obtain optimal value of shadow price at any instant t:

$$\lambda(t) = \frac{\alpha p(t)(A - nq(t))^{1-\frac{1}{\alpha}}}{s}. \qquad (2.8)$$

Now, we differentiate the Hamiltonian w.r.t. the state variable to build up the co-state equation:

$$-\frac{\partial H_i(t)}{\partial p(t)} = -q(t) + \lambda(t)s = \frac{\partial \mu(t)}{\partial t}$$

$$\Longrightarrow \dot{\lambda}(t) = \frac{\partial \lambda(t)}{\partial t} = \lambda(t)(s+\rho) - q(t), \qquad (2.9)$$

and finally we have to account for the transversality condition:

$$\lim_{t\to\infty} \mu(t).p\,(t) = 0.$$

Now observe that (2.2) and (2.8) together with the (3.35) can be plugged into (2.7) to rewrite the control equation explicitly. The stationarity condition at the steady state equilibrium requires (i) $\dot{q}\,(t)$ to be equal to zero, and (ii) $p\,(t)$ to be equal to $\hat{p}$, so that the resulting coordinates of the unique open-loop equilibrium point are:

$$q^* = \frac{\alpha A\,(\rho + s)}{s + \alpha n\,(\rho + s)},$$

$$p^* = (A - nq^*)^{\frac{1}{\alpha}}.$$

The stability analysis must be carried out evaluating the properties of the following Jacobian matrix:

$$J = \begin{bmatrix} \frac{\partial \dot{p}}{\partial p} & \frac{\partial \dot{p}}{\partial q} \\ \frac{\partial \dot{q}}{\partial p} & \frac{\partial \dot{q}}{\partial q} \end{bmatrix}$$

whose trace and determinant are, respectively:

$$T\,(J) = \rho + \frac{s\,(n-1)}{n\,(1-\alpha)},$$

$$\Delta\,(J) = \frac{s\,[s + \alpha\,(\rho + s)\,n]}{n\,(1-\alpha)}.$$

the above expressions immediately imply that (i) $\Delta\,(J) < 0$ for all $\alpha \geq 1$, so that in this range we have saddle point stability, while (ii) $T\,(J) > 0$ for all $\alpha \in (0,1)$, whereby in this parameter region the steady state point is unstable (either a focus or a node, depending on the sign of $4\Delta\,(J) - T^2\,(J)$).[2] This concludes the proof. ■

[2] This implies that any convex demand gives rise to instability.

The resulting per-firm profit in steady state is

$$\pi^*(n) = \frac{\alpha(\rho+s)}{s}\left(\frac{sA}{s+\alpha n(\rho+s)}\right)^{\frac{\alpha+1}{\alpha}}$$

which is a function of the vertical intercept of demand, time discounting, the number of firms, the degree of price stickiness and the curvature of demand. We are now ready to assess the profitability of a *small* horizontal merger.

2.4 The incentive to merge

Before delving into the details of the issue treated in this section, i.e., the incentives toward horizontal mergers, it is worth discussing a feature of this problem that is pervasive in the exinsting literature. Ever since Salant, Switzer and Reynolds (1983), it is known that, in a Cournot game, outsiders not involved in the merger are usually better off than the merger partners precisely because of the resulting increase in the degree of concentration in the industry. Therefore, in principle there exists a question as to who should step out to propose a merger, as every firm in the industry is aware that it would be convenient to wait for some rivals to take such an initiative. Yet, if indeed this incentive is there, one should expect to observe a merger proposal by some subset of the firms in the industry. As is common to the entire literature on the matter, we shall disregard this coordiantion problem and focus our attention on economic incentives only.

The puzzle of bilateral mergers in a Cournot triopoly is a recurrent theme in the debate on mergers, as observation suggests that indeed this is precisely the type of merger one happens to observe in reality. For example, this has been the case with the merger between Boeing and McDonnell Douglas that has turned the world industry for large civil air transport into a duopoly, the other competitor being Airbus. To tackle this issue, we set $n = 3$ and focus on the profitability of a merger involving two firms out of three. In a triopoly, two firms will find it profitable to merge horizontally if and only if the following condition is satisfied:

$$\frac{\pi^*(n=2)}{2} - \pi^*(n=3) > 0$$

or, equivalently,

$$\frac{1}{2}\left(\frac{sA}{s+\alpha 2(\rho+s)}\right)^{\frac{\alpha+1}{\alpha}} - \left(\frac{sA}{s+\alpha 3(\rho+s)}\right)^{\frac{\alpha+1}{\alpha}} > 0.$$

That is, the fifty percent of the individual duopoly profits once the merger has taken place must be higher than the individual profits before the merger. Since the level of the reservation price A is irrelevant, the above condition is indeed equivalent to the following:

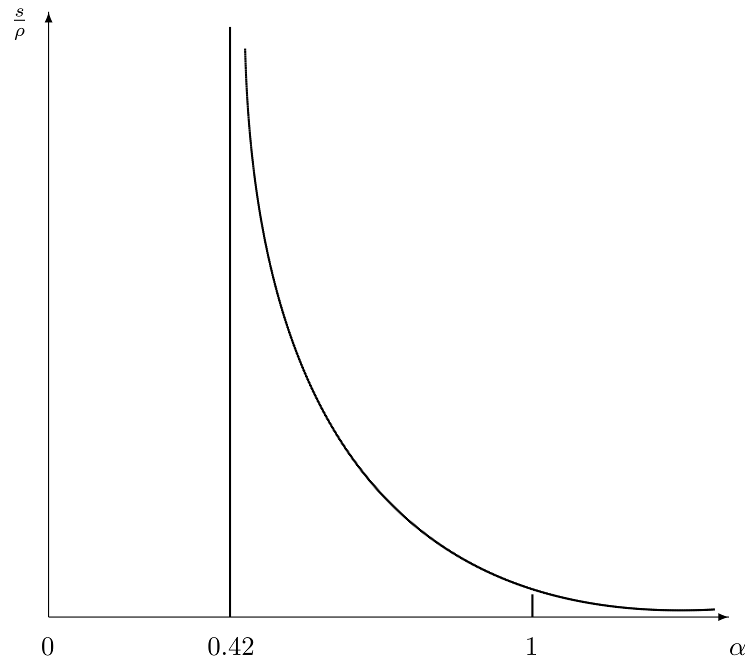
$$\frac{1}{2}\left(2\rho+\left(2+\frac{1}{\alpha}\right)s\right)^{-\frac{\alpha+1}{\alpha}} - \left(3\rho+\left(3+\frac{1}{\alpha}\right)s\right)^{-\frac{\alpha+1}{\alpha}} > 0 \qquad (2.10)$$

The above condition can be studied in the space $(\alpha, s/\rho)$. This is done in Figure 1, where the region in which inequality (2.10) holds, or in other words the merger of two out of three firms is profitable, is represented by the area below the curve.

This figure shows that if the market is sufficiently small (for all $\alpha < 0.42$, which is the asymptotic value attained if s/ρ becomes infinitely high), then the incentive for a horizontal merger involving two firms out of three always exists irrespective of speed of adjustment. The essential reason is that when demand becomes convex, the market shrinks and this clearly hinders firms' profitability, a fact which creates an incentive to merge. Interestingly, this result extends to the static game for the same reason. If the prices adjusts instantaneously, then the limit of the open-loop setup is the static model (the same happens if ρ tends to zero). This can be verified simply by taking the static model and compare it with the limit case where the ratio s/ρ tends to infinity (see Cellini and Lambertini, 2004). Figure 1 clearly shows that, in the static game where s/ρ shoots up to infinity, the merger is indeed profitable for all $\alpha \in (0, 0.42)$.

However, this is the parameter range wherein the steady state point is unstable, so we have to ask ourselves whether a marger can be expected to arise for larger values of α, in such a way that the resulting equilibrium be a saddle point. As the market becomes larger (i.e., α increases), the incentive to merge is determined by how fast the price adapts to the equilibrium level. In fact, it turns out that the profitability of the merger is driven by a sort of tradeoff between the curvature of demand and the speed of price adjustment (weighted for the discount rate): as the market enlarges, the price has to become stickier in order for forms to be willing to merge. Put it differently, for comparatively higher values of α one would be tempted to exclude investigating the merger incentive because operating in a larger market increases the stand-alone profitability and this points in the direction of making bilateral mergers unprofitable, all else equal. This is not the case if the increase in α goes along with a decrease in s/ρ. The intuition behind this mechanism can be spelled out as follows. When price adjusts very slowly, this involves making large systematic mistakes in setting the output levels. Firms would like to play the correct Cournot equilibrium with a large market but they cannot because s is very small and there they have an incentive to decrease the number of firms to recover what they are losing. This creates a region where $\alpha > 1$ and s/ρ is sufficiently small to yield convenient bilateral mergers, as Figure 1 indeed illustrates.

Figure 1 : Merger profitability in the space $(\alpha, s/\rho)$



The problem can also be approached from another angle, namely, by fixing s/ρ and then evaluating the profitability of a merger involving m firms out of n. In such a case, the profit incentive exists iff:

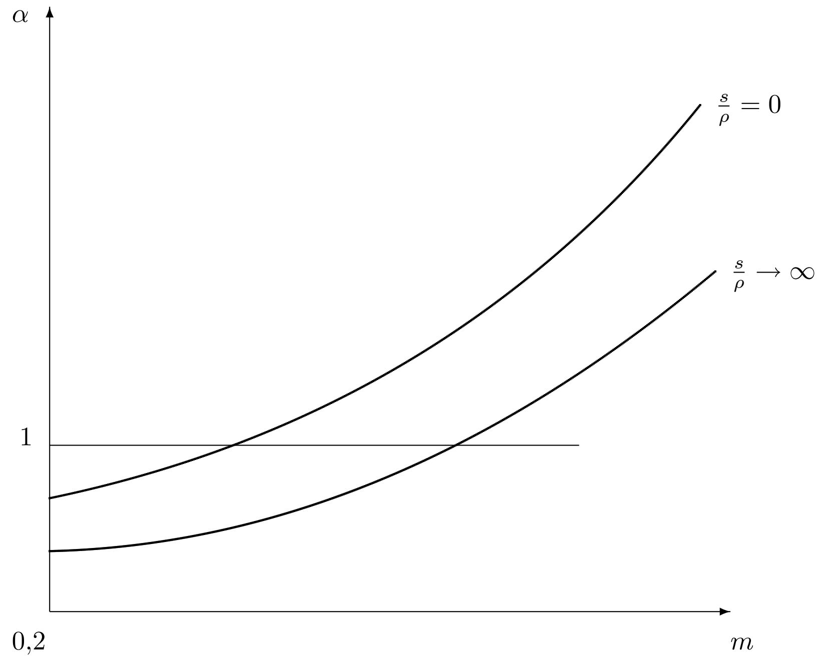
$$\frac{\pi^* (n - m + 1)}{m} - \pi^* (n) > 0$$

or, equivalently,

$$\frac{1}{m} \left(\frac{1}{s + \alpha (n - m + 1) (\rho + s)} \right)^{\frac{\alpha+1}{\alpha}} - \left(\frac{1}{s + \alpha n (\rho + s)} \right)^{\frac{\alpha+1}{\alpha}} > 0 \qquad (2.11)$$

Condition (2.11) is assessed in Figure 2, where it yields a parametric family of curves, between the two curves depicted here, with the characteristic that the region where the merger is profitable, which is bellow the curves, enlarges if stickiness and discounting become higher which make sense according to the aforementioned reasons.

Figure 2 : Merger profitability in the space (m, α)



This figure says that if α is sharply below 1, a small merger is indeed privately convenient. This makes sense because market size shrinks as α decreases. Now, if the market becomes smaller, because the demand from linear becomes convex, it becomes less profitable. Consequently, it is easier to find conditions whereby a small merger works. Therefore, for a given initial population of firms, say 10, if they are squeezed inside a very small market, then some of them find it profitable to merge because their profits are squeezed by the decrease in α, while they would not if the market were larger. In the linear case when α is equal to 1 (along the flat line drawn in Figure 2), we know from Salant, Switzer and Reynolds (1983) that a profitable merger must involve about 80% of the oligopolist, which is dramatically close to a merger to monopoly.

2.5 Conclusion

In this paper, we have investigated the consequences of horizontal mergers taking a differential game approach with sticky prices where the demand function is nonlinear. Given the shape of the problem at hand, we have analyzed the open-loop solution only.

Efficiency implications being ruled out by assumption, in our model it is clear that any decrease in the number of firms brings about a decrease in social welfare via a decrease in consumer surplus. Given the fact that, in principle, when there is no saving on the overall industry costs, any horizontal merger is socially harmful, then the regulator must look out for mergers driven by the pressure generated by market size on firms, when the demand is non-linear. This has some interesting implications on the empirical side, as estimating market demand functions may indeed yield clearcut hints as to whether the market under consideration is likely to generate incentives towards horizontal mergers.

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Chapter 3

Exogenous Output Constraint in a Dynamic Oligopoly

3.1 Introduction

Consider an industry consisting of N symmetric firms each producing a homogenous output. Then, the output levels of a subset of M $(< N)$ firms are constrained into a constant level. If the remaining firms simultaneously make the best reply to this exogenous constraint, we want to investigate under what circumstances is this to the benefit of constrained subset.

In a series of papers Gaudet and Salant (1991a,b) show that, in the case of Cournot competition among producers of perfect substitutes, a marginal contraction is strictly beneficial if and only if the number of firms in the designated subset exceeds the "adjusted" number of firms outside it by strictly more than one. In the special case of linear cost and demand functions, the firms in the subset will gain from an exogenously marginal contraction of their output if and only if they outnumber the firms outside the subset by more than one.

In this paper we generalize this result to the case of dynamic competition instead of looking at the one-shot game. While in the *standard Cournot model* any output constraint is not to the benefit of constrained firms, in this paper, we show that when firms play a *dynamic Cournot game with Markov-perfect* *strategies* exogenous output constraint by a subset of firms results in: (i) increase in the value of unconstrained firms irrelevant of the amount of constraint because of having less intensive competition, (ii) increase in the market price for any output constraint below the optimal level and slightly above that because of reduction in the total output caused by less competition and (iii) increase in the value of constrained firms for a viable range of parameters and initial conditions because of increasing the price during the price path.

Our analysis has some applications to voluntary export restraints (VER), Mergers, Economics Sanctions, etc. Mai and Hwang (1988) examine VERs in a static duopoly model by using a conjectural variations approach. They find that if the free trade equilibrium is Cournot, a VER set at the free trade level of imports will have no impact on profits. We show that if the free trade equilibrium is Cournot played with Markovian (subgame-perfect) strategies, then the imposition of a VER at the free trade level of imports increases the market price and the profits of the foreign and domestic firm. Hence, the VER is voluntary in the dynamic Cournot model.

Suppose that the subset of firms represents firms that are part of a cartel. Our study explains how it is to their benefit when they agree on producing a constant level of quantity, for example thier optimal steady state level of output before the merger.

Consider the international market in which a group of countries are exporting a specific good. Now, assume that one of these exporting countries is sanctioned by part (not all) of the importing countries. This imposed economic sanction force that country to be constrained to a lower output level which, however, can be to her advantage. Our analysis characterizes circumstances under which economic sanctions are not effective.

The rest of the paper is organized as follows. Section 2, present the model. In section 3, the dynamic equilibria are derived before and after the exogenous output constraint. Circumstances under which the exogenous output constraint is profitable is examined in section 4. In section 5, robustness of the result is checked by conjectural variations equilibrium. Some applications are presented in section 6. Finally, section 7 concludes the paper.

3.2 A dynamic oligopoly model

Consider a dynamic oligopoly market consisting of N symmetric firms each producing a homogenous output. Firms are assumed to produce with strictly concave technologies described by the cost functions

$$C(q_i(t)) = \frac{1}{2}q_i^2(t), \quad i = 1, ..., N, \tag{3.1}$$

where $q_i(t) \geq 0$ is the output of firm i produced at time t. The equilibrium price, $\bar{p}(t)$, in period t is related to industry output by means of an inverse demand function which in its linear version is given by

$$\bar{p}(t) = a - \sum_{i=1}^{N} q_i(t) \tag{3.2}$$

where the units of measurement are chosen such that the slope of the demand curve is -1. Thus, the single period profit function of firm i is given by

$$\pi_i(t) = [a - \sum_{i=1}^{N} q_i(t)]q_i(t) - \frac{1}{2}q_i^2(t). \tag{3.3}$$

Equation (3.3) represents a classical one-shot Cournot game. However, in this paper we want to look at the continuous time dynamic competition where firms are assumed to maximize the discounted stream of profits over an infinite planning horizon with $r > 0$ as the constant discount rate. We are interested in deriving Markov-perfect equilibria for this game. In order to solve for those equilibria, we make use of the "sticky price" model introduced by Fershtman and Kamien (1987). It is given by

$$\max \Pi_i = \int_0^{\infty} e^{-rt}\{p\,(t)\,q_i\,(t) - \frac{1}{2}q_i^2\,(t)\}dt, \tag{3.4}$$

subject to

$$\dot{p}\,(t) = s[a - \sum_{i=1}^{N} q_i\,(t) - p\,(t)];\; p\,(0) = p_0. \tag{3.5}$$

In (3.4) and (3.5) it is assumed that the actual market price deviates from its level given by the demand function but moves towards it with a constant speed of adjustment denoted by s ($0 < s \leq \infty$). Thus, we have sticky prices.

The strategy spaces available to the firms should be specified in order to clearly define the dynamic Cournot game (3.4). The assumption that the industry equilibrium is identified as a subgame-perfect Cournot equilibrium in Markov strategies means that firms design their optimal policies as decision rules dependent on the state variables of the game (in our case price). This means that firms take into account the rivals reactions to their own actions as expressed by the state variables of the game. This is exactly the characteristic present in the case of conjectural variations equilibrium.

3.3 Dynamic equilibria

As motivated in the introduction, we are interested to see whether firms benefit from being forced to act non-strategically or not. To this end, in this section we want to derive the dynamic equilibrium of game (3.4) under two different scenarios. First, we solve for the equilibrium when all the N firms in the industry are strategic players. Next, we consider the scenario where M strategic players are eliminated by being forced to be constrained to a constant level of output and we derive the equilibrium in this scenario.

If value of non-strategic firm increases compared to the unconstrained case, the answer to the question is yes. This is what we focus on in next section.

3.3.1 Unconstrained oligopoly equilibrium

We derive the equilibrium of the model in which firms employ price dependent decision rules when maximizing their discounted profits. Thus, changes in the market price stimulate responses by all players that are reflected in their quantity choices. This corresponds to the recognized interdependence present in oligopolistic markets.

Theorem 4 *There exists a Markov perfect equilibrium of the "sticky price"* *model in an N firm dynamic Cournot oligopoly given by*

$$q^*(p) = p(1 - sK) + sE, \tag{3.6}$$

$$V(p) = \tfrac{1}{2}Kp^2 - Ep + g, \tag{3.7}$$

and

$$p(t) = p^* + (p_0 - p^*)e^{Dt}, \tag{3.8}$$

where p_0 is the initial price and p^* is the steady state price

$$p^* = \frac{a - NsE}{1 + N(1 - sK)}, \tag{3.9}$$

K, E, g and D are defined as

$$K = \frac{2s(N+1) + r - \sqrt{[2(N+1)s + r]^2 - 4s^2(2N-1)}}{2s^2(2N-1)}, \tag{3.10}$$

$$E = \frac{-sKa}{s(N+1) + r - s^2K(2N-1)}, \tag{3.11}$$

$$g = \frac{s^2E^2(2N-1) - 2sEa}{2r}, \tag{3.12}$$

$$D = s[N(sK - 1) - 1]. \tag{3.13}$$

Proof. See Appendix A. ■

The results of Theorem 1 have the following implications. Firstly, the equilibrium quantities of the infinite horizon game do not coincide with that of the one shot game if firms employ Markov strategies. Secondly, firms produce more (and hence market price is lower) in the dynamic game compared to the classical Cournot model. The interpretation of this result arises from the price dependent decision rules (3.6). In particular, with an increase in price firms react by producing more. To see why this causes equilibrium quantities to be closer to the competitive equilibrium consider the following scenario. Assume that a firm i finds it profitable to reduce its equilibrium quantity. This causes the market price to increase. Given the feedback decision rules of the competitors their optimal response to the increasing price is to increase their equilibrium quantities thus offsetting firm i's action. This behavior causes in equilibrium all firms to produce beyond the level of simple Cournot quantities.

3.3.2 Exogenous output constraint

After having characterized the unconstrained equilibrium we assume that a subset of M ($M < N$) strategic players are eliminated by being constrained to a constant level of output, $\bar{q}$ ($0 < \bar{q} < a$). Moreover, we assume that these firms cannot deviate as they are constrained to these output levels. Thus, the game played by the $N - M$ strategic players in the model of sticky prices becomes

$$\max \Pi_i^C = \int_0^\infty e^{-rt}\{p(t)\, q_i(t) - \frac{1}{2} q_i^2(t)\} dt, \quad i = M+1, ..., N, \tag{3.14}$$

subject to

$$\dot{p}(t) = s[a - M\bar{q} - \sum_{i=M+1}^{N} q_i(t) - p(t)];\; p(0) = p_0. \tag{3.15}$$

This provides us with the following result.

Theorem 5 *If a subset of M firms in an N firm dynamic Cournot oligopoly is forced to act non-strategically through being exogenously constrained to the output choices $\bar{q}$, there exists a Markov perfect equilibrium of the "sticky price" model, where the remaining $N-M$ firms play strategically the dynamic Cournot game, given by*

$$\tilde{q}(p) = \hat{p}(1 - s\hat{K}) + s\hat{E}. \tag{3.16}$$

$$\hat{V}(p) = \tfrac{1}{2}\hat{K}p^2 - \hat{E}p + \hat{g}, \tag{3.17}$$

and

$$\hat{p}(t) = \tilde{p} + (p_0 - \tilde{p})e^{\hat{D}t}, \tag{3.18}$$

where p_0 is the initial price and $\tilde{p}$ is the steady state price

$$\tilde{p} = \frac{a - (N-M)s\hat{E} - M\bar{q}}{1 + (N-M)(1 - s\hat{K})}, \tag{3.19}$$

$\hat{K}$, $\hat{E}$, $\hat{g}$ and $\hat{D}$ are defined as

$$\hat{K} = \frac{2s(N-M+1) + r - \sqrt{[2(N-M+1)s + r]^2 - 4s^2(2(N-M)-1)}}{2s^2(2(N-M)-1)}, \tag{3.20}$$

$$\hat{E} = \frac{-s\hat{K}a - s\hat{K}M\bar{q}}{s(N-M+1) + r - s^2\hat{K}(2(N-M)-1)}, \tag{3.21}$$

$$\hat{g} = \frac{s^2\hat{E}^2(2(N-M)-1) - 2s\hat{E}a + 2s\hat{E}M\bar{q}}{2r}. \tag{3.22}$$

$$\hat{D} = s[(N-M)(s\hat{K}-1)-1]. \tag{3.23}$$

And the present value of a non-strategic firm becomes

$$\hat{V}^C = Ap + \hat{g}^C, \tag{3.24}$$

where A and $\hat{g}^C$ are

$$A = \frac{\bar{q}}{r-\hat{D}},$$
$$\hat{g}^C = \frac{\bar{q}(\hat{D}(2\tilde{p}-\bar{q}) + r\bar{q})}{2r(\hat{D}-r)}.$$

Proof. See Appendix B. ■

It is important to note, however, that the behavior of the firms in the subset after being non-strategic does not correspond to an equilibrium. The strategically-playing firms, however, are in dynamic Cournot equilibrium.

3.4 Profitable output constraint

Theorem 6 *Assume that the subset of M strategic players are eliminated by being exogenously constrained to the output choices $\bar{q}$, whereas the remaining $N-M$ firms react strategically to this exogenous change in a dynamic Cournot game with Markov-perfect strategies. This results in an*

(a) increase in the market price for any $\bar{q} \leq q^*$;

(b) increase in the present value of strategic firms irrespective of the amount of $\bar{q}$;

(c) increase in the present value of non-strategic firms for a viable range of parameters and initial conditions.

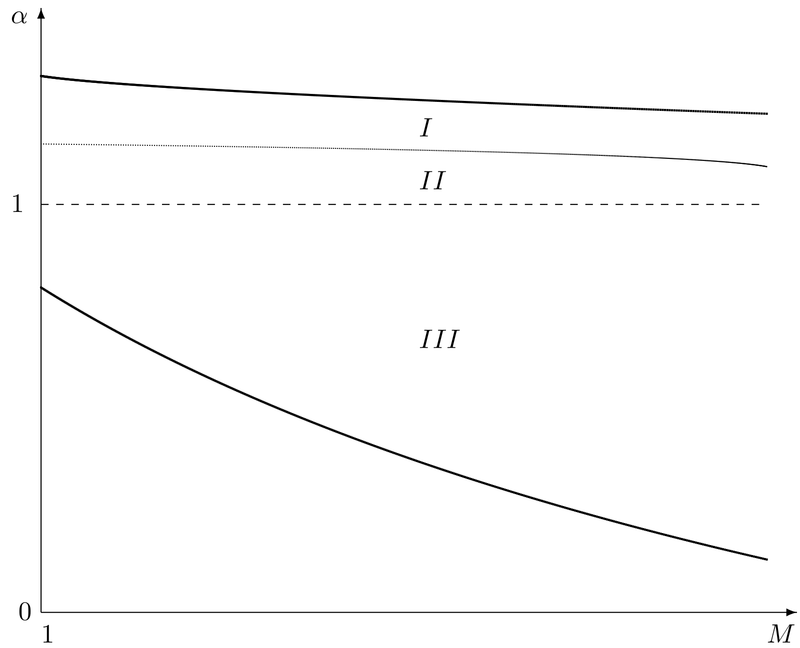
Proof. See Appendix C. ■

However, the steady state price $\tilde{p}$ could be larger than p^* even for some values of $\bar{q}$ above the q^*. Consider $\bar{q} = \alpha q^*$ where $\alpha > 0$, therefore, we have

$$\tilde{p} > p^* \iff 0 < \alpha < -\frac{N\hat{D}}{Ms} - \frac{(N-M)D(a(1-s\hat{K})+s\hat{E})}{Ms(a(1-sK)+sE)},$$

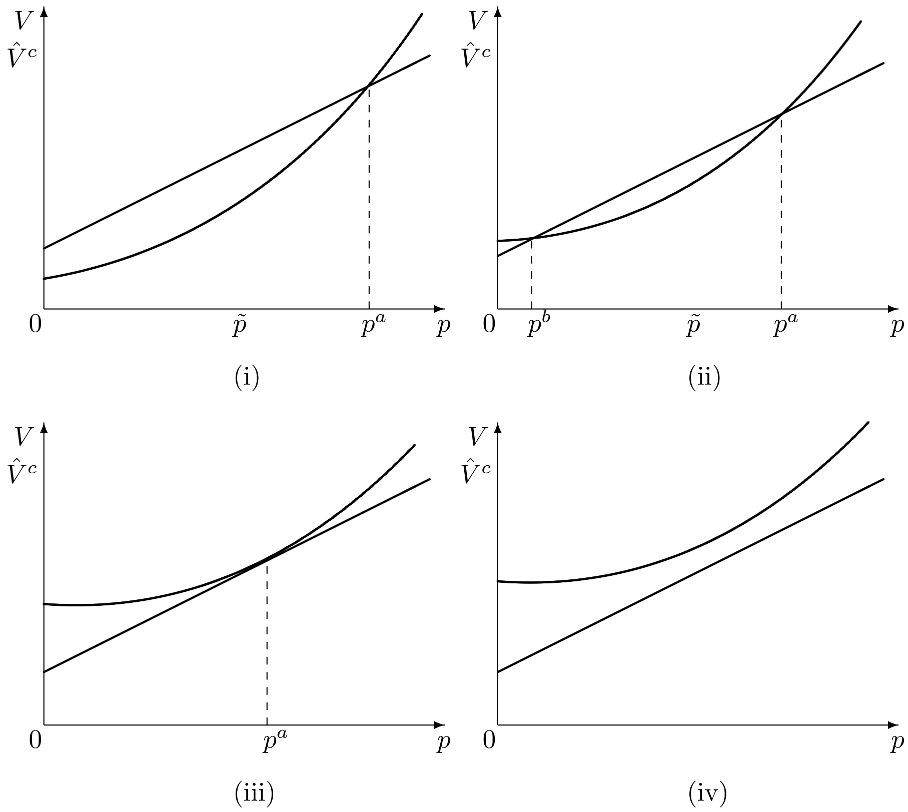
The expression in the right hand side of the inequality is always greater than 1, for all plausible amounts of parameters. The thinner curve in figure 1 represents the ranges of parameters in the space of (M, α), for a given values of other parameters, where the two steady state prices, $\tilde{p}$ and p^*, are equal. In the region below the curve $\tilde{p}$ is larger than p^*. Therefore, the market price in constrained equilibrium at every instant is higher compared to unconstrained equilibrium.

Figure 1: Profitability of acting non-strategically for the firms in the subset. On the dashed line $\alpha = 1$ ($\bar{q} = q^*$). On the thin solid curve $\tilde{p} = p^*$ and below (above) it, is larger (smaller) than p^*. Beyond the two thick solid curves exogenous output constraint is never profitable for non-strategic firms.



In figure 1, in the regions between the thick curves, non-strategic firms can benefit from an exogenous output constraint. In the region II constrained firms benefit by producing more and selling them at a higher price at every instant. In the region I, while constrained firms are constrained to a large output level, they can still benefit since $D < \hat{D} < 0$ and, therefore, price in constrained equilibrium moves to its steady state level more slowly. Region III, represents the points where excluding some of strategic players from the game make the competition in the industry less aggressive and pushes the price up in such a way that constrained firms benefit even with a substantial decrease in their quantity.

Figures 2: Comparing the value functions of a non-strategic firm before (V, the curve) and after ($\hat{V}^c$, the straight line) the exogenous output constraint.



However, provided that the parameters characterize a point in region I, II or III, the profitability of output constraint depends on the initial condition and how far the initial price is different from the steady state level. This is shown in figures 2. In figures 2 (i) and 2 (ii), if the initial price belongs to $(0, p^a)$ (or (p^b, p^a)), we can argue that $\hat{V}^C(p)$ always has a larger value than $V(p)$. Whereas, in the case where p_0 is outside the $(0, p^a)$ (or (p^b, p^a)) , output constraining is not to the benefit of the non-strategic firms in so far as $\hat{p}(t)$ arrives to the interval and it becomes profitable afterwards. Figures 2 (iii) and 2 (iv) corresponds to the points beyond the thick curves in figure 1 and illustrate the case where constrained firms do not benefit for any initial condition.

Note that, in our analysis, we consider general output constraint $\bar{q} > 0$ and examine the profitability of it in a dynamic context. However, at the steady state and for output constraint $\bar{q} = q^*$, we have the following corollary.

Corollary 7 *The steady state profits of non-strategic firms increase when they are constrained to their equilibrium output level q^*.*

Proof. As it was indicated before, after eliminating some strategic players, the steady state price increases ($\tilde{p} > p^*$). Therefore, since the output level does not change, the firm's revenue will increase while the cost remains the same as before. Hence, the non-strategic firms make higher profits in steady state. ■

3.5 Robustness of results

We have shown that, in a dynamic Cournot oligopoly when firms employ Markovian strategies, eliminating a subset of strategic players from the competition can be to the benefit of all the strategic and non-strategic players. Although we make use of the sticky price model, results do not correspond to the price stickiness. In this section, the robustness of results is evaluated through a conjectural variations analysis.[1] As it is shown in Dockner (1992), a static conjectural variations analysis approximates long-run dynamic interactions. Hence, we are interested in conjectural variations equilibrium in both unconstrained and constrained cases, and, then, examining the profitability of being a non-strategic player.

In the unconstrained equilibrium, all firms are strategic players. Firms have symmetric profit functions given by

$$\pi_i = p(Q)q_i - C(q_i), \tag{3.25}$$

where Q is the industry output, $p(Q)$ is a general inverse demand curve and $C(q_i)$ is a general cost function. First order conditions in the case of conjectural

[1] The conjectural variation is the firm's conjectures about her rivals' behavior.

variations equilibrium are given by

$$\frac{\partial \pi_i}{\partial q_i} = p(Q) + p'(Q)q_i - C'(q_i) + p'(Q)q_i[\sum_{j=1,j\neq i}^{N} \frac{\partial q_j}{\partial q_i}] = 0, \tag{3.26}$$

where $\frac{\partial q_j}{\partial q_i}$ is the conjecture of firm i about firm j's behavior. The industry output, price and cost functions are assumed to be $Q = \sum_{i=1}^{N} q_i$, $p(Q) = a - Q$ and $C(q_i) = \frac{1}{2}q_i^2$, respectively. Thus, the equilibrium corresponding to the F.O.C. of (3.26) is

$$q_{cv}^* = \frac{a}{2 + N + \lambda\,(N-1)},$$

where the subscript *cv* denotes the conjectural variations equilibrium, and firms are presumed to have identical conjectures $\lambda = \lambda_{ij} = \frac{\partial q_j}{\partial q_i}$. This conjecture belongs to the interval $[\lambda_0, 0]$ where $\lambda_0 \in (-1, 0)$ is the minimum viable conjecture which solves $\pi^* = p_{cv}^* q_{cv}^* - \frac{1}{2}q_{cv}^{*2} = 0$, and $\lambda = 0$ replicates the standard Cournot oligopoly.

However, in a *consistent conjecture equilibrium (CCE)*[2], the conjectural variation must be equal to the reaction function. The firm's reaction function is the firm's actual behavior and is defined by $q_i = \rho_i\,(q_j)$ which solves (3.26). The implicit differentiation of (3.26) yields

$$[1 + (\lambda+1)\,(N-1)]\,\frac{\partial \rho_i}{\partial q_j} p'(Q) + p'(Q) - \frac{\partial \rho_i}{\partial q_j} = 0.$$

Considering symmetric reaction functions, $\frac{\partial \rho_i}{\partial q_j} = \frac{\partial \rho}{\partial q}$ and equating conjectural variation and reaction function, i.e. $\frac{\partial \rho}{\partial q} = \lambda$, the consistent conjecture is obtained[3]

$$\lambda^* = -\frac{N + 1 - \sqrt{5 + N\,(N-2)}}{2N - 2}.$$

It can be easily shown that $\lambda^* \in (\lambda_0, 0)$. Therefore, the slope of consistent conjecture lies between -1 and 0 which refer to Bertrand and Cournot competitions. Hence, in a CCE, the competition among firms in an oligopoly is more aggressive compared to the Cournot.

[2] Consistent conjectures equilibrium is discussed comprehensively in Bresnahan (1981).
[3] There exists a second root which is lower than -1 and is not acceptable.

Now, we want to know the consequences of excluding a subset of strategic players from the competition. Let us force a subset of M firms to be non-strategic players by constraining them to a constant output levels $\bar{q}$ where this constraint is binding. Thus, the remaining $N-M$ strategic firms solve the first order conditions

$$\frac{\partial \pi_i}{\partial q_i} = p(Q) + p'(Q)q_i - C'(q_i) + p'(Q)q_i[\sum_{j=M+1,j\neq i}^{N} \frac{\partial q_j}{\partial q_i}] = 0. \tag{3.27}$$

Evaluating this first order condition along the equilibrium of (3.26) yields

$$\left.\frac{\partial \pi_i}{\partial q_i}\right|_{q=q^*_{cv}} = -p'(Q)q^*_{cv}[\sum_{j=1,j\neq i}^{M} \frac{\partial q_j}{\partial q_i}] < 0. \tag{3.28}$$

This, however, implies (given the second order conditions) that industry output shrinks when a subset of firms is constrained to their equilibrium in the unconstrained case. Hence, market price increases and both strategic and non-strategic players benefit, irrespective of the size of the M.

Now, consider a general output constraint $\bar{q} = \alpha q^*_{cv}$, $\alpha \in (0, 2)$. Assuming symmetry between the $N-M$ strategic players, the equilibrium output level of (3.27) becomes

$$\tilde{q}_{cv} = \frac{a\,(2 + N + (N-1)\,\lambda - M\alpha)}{(2 + N + \lambda\,(N-1))\,(N - M + 2 + \lambda\,(N - M - 1))},$$

and the resulting market price is

$$\tilde{p}_{cv} = a - M\alpha q^*_{cv} - (N-M)\,\tilde{q}_{cv}.$$

Therefore, the unconstrained and constrained firms' profits are

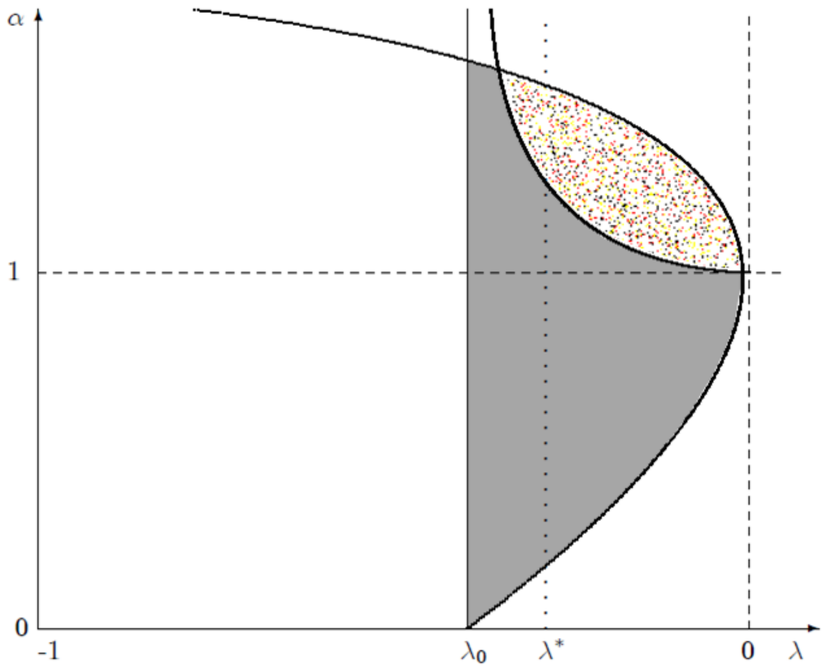
$$\tilde{\pi} = \tilde{p}_{cv}\tilde{q}_{cv} - \frac{1}{2}\tilde{q}^2_{cv}, \tag{3.29}$$

$$\tilde{\pi}_c = \tilde{p}_{cv}\bar{q} - \frac{1}{2}\bar{q}^2, \tag{3.30}$$

where $\tilde{\pi}$ denotes the profits in constrained case, and subscript c stands for the constrained firms.

Figure 3 shows the range of parameters in the space of (λ, α) where unconstrained and constrained firms can benefit from output constraint. The two

Figure 3.1: Profitability of exogenous output constraint in conjectural variations equilibrium in the space of (λ, α). The lower curve and the upper one represent the points where firms' profits in the two cases are equal for the constrained and unconstrained firms, respectively. On the right hand side of the curves both type of fims benefit.



curves in this figure are the locus of the points where firms have the same profits in unconstrained and constrained equilibria. However, the gray area represents the points where both type of firms benefit while in the dotted area only constrained firms benefit.

As it can be seen, firms' profits can increase even if they are constrained to an output level higher than the unconstrained equilibrium. The profitability of output constraint decreases as firms' conjectures goes to zero. These results are consistent with the results of the dynamic competition when firms employ Markovian strategies.

In the figure, $\lambda = 0$ corresponds to the standard static Cournot competition in which any output constraint in not beneficial neither for the constrained firms nor for the unconstrained ones.

3.6 Applications

3.6.1 Voluntary export restraints

The study we have conducted has many applications among which voluntary export restraints (VERs) is the most obvious one. It is of importance to international trade policy to answer the question whether domestic and/or foreign firms benefit from the imposition of so-called 'voluntary' export restraints by the foreign producer. If the foreign producer's profit increases by restraining export to the domestic market, VERs are indeed 'voluntary'.

Dockner and Haug (1991) analyses VERs in a differential game model with a domestic and foreign producer of a homogenous good sold in the domestic market. There are several differences between this contribution and present study. First, Dockner and Haug (1991) analysis is restricted in a speed of price adjustment that goes to infinity. However, in our model it is possible to investigate price behavior in determining the profitability of VERs. Second, with the model presented here we can consider more than one foreign and domestic firm which provides us the chance to examine the incentive for VER in relation to the number of constrained firms and the level of output they are constrained to.[4]

In addition to Dockner and Haug (1991) that shows the imposition of a VER at the free trade level of imports increases the market price and the profits of all firms in the industry, our analysis implies that: market price increases for any output constraint which in below the free trade level of imports; it is always to the benefit of domestic firms for any level of exports that foreign firms are restricted to and finally in part (c) of theorem 5 it is comprehensively explained under which conditions and for what level of output export restraint is profitable for foreign firms.

3.6.2 Horizontal Mergers and Cartels

When in an N-firm industry a subset of M firms is constrained to a constant level of output, since there is strategic interaction among $N-M+1$ firms rather than N firms, the level of competition in the industry will decrease which is always to the benefit of unconstrained firms as it is proved in theorem 5. We show when the subset of firms are constrained to q* which is their steady state equilibrium level before the exogenous output constrained, the anticompetitive forces due to an exogenous output constraint can be strong enough to benefit the subset of firms as well. Theorem 5 also discusses about conditions and other output levels that being constrained to it can be advantageous for the subset of firms.

The same story holds when we consider the profitability of mergers and cartels. Our model does not precisely fit the horizontal merger problem in which firms solve their strategic problem to determine the equilibrium output level. However, in general, output contraction creates the same results that horizontal mergers and cartels can create that are reduction in aggregate output, increase in the market price and therefore increase in the profit of $N-M$ outside firms. Now, assume that the subset represents firms that are part of a cartel. Here, we

[4] For another contribution on VERs in a differential game you can see Calzolari and Lambertini (2007) who study the impact of VERs in a duopoly game with a Ramsey capital accumulation dynamics

can define cartel as an agreement in which firms in the subset agree on being constrained to a constant level of output (for example q^*) and as it is shown in theorem 5, it can be profitable for them. It is difficult for antitrust authority to recognize such a cartel in which a subset of firms is constrained to their steady state equilibrium level before the exogenous output constraint. Dockner and Gaunersdorfer (2001), Benchekroun (2003), Esfahani (2012) and Esfahani and Lambertini (2012)[5] using a dynamic model with sticky prices, investigate the profitability of horizontal mergers in the specific case of instantaneous price adjustment.

3.6.3 Economic sanctions

Economic sanctions are punishments imposed on a country by one or a group of countries due to various reasons. Economic sanctions may take a number of forms including: embargo on exports, embargo on imports, financial controls, transportation and communication controls, sequestration of property, preemptive purchasing and other measures. For extensive discussion, see Bornstein (1968).

We are considering import restrictions from the target country into the participants which attempts to reduce the target country's foreign exchange earnings. There is a debate over the effectiveness of economic sanctions in their ability to achieve its intention even if any import restrictions enacted by sanctioners ensures income reduction in target country. However, our analysis can address the question of whether sanctions can reduce the target country's income.

Suppose that M firms in the subset represent firms in the target country. The rest of the $N - M$ Firms are outside the target country. Sanctioning countries by enforcing import restrictions are the cause of exogenous output constraint in the target country. Part (c) of theorem 5 explains how sanctions can be designed by imposing countries in order to decrease the present value of firms

[5] They considered non-linear demand function and the open-loop equilibrium.

in the imposed country through the level of output that they force the target country to be constrained to.

3.7 Conclusion

In the case of static Cournot competition among producers of perfect substitutes, output constraint is never to the benefit of constrained firms. When firms use feedback strategies, eliminating a subset of strategic players by exogenously constrained them to a constant level of output results in: (i) increase in the value of strategic firms irrelevant of the amount of constraint because of having less intensive competition, (ii) increase in the market price for any output constraint below the optimal level and slightly above that because of total output reduction caused by less competition and (iii) increase in the value of non-strategic firms for a viable range of parameters and initial conditions because of increase in the price during the price path.

APPENDIX

Appendix A:

Proof of Theorem 1: The proof is carried out for symmetric interior solutions. We use dynamic programming. The Bellman equation is given by

$$rV^i(p) = \max_{q_i}\{pq_i - \frac{1}{2}q_i^2 + sV_p^i(p)[a - \sum_{i=1}^{N} q_i - p]\}, \tag{3.31}$$

where $V^i(p)$ is the optimal value function of firm i. Since the game is symmetric and linear quadratic we conjecture symmetric, quadratic value functions

$$V^i(p) = \tfrac{1}{2}Kp^2 - Ep + g, \tag{3.32}$$

which implies that

$$V_p^i(p) = Kp - E, \tag{3.33}$$

where K, E and g are constants that need to be determined. Maximizing the right hand side of equation (3.31) gives

$$q_i = p - sV_p^i(p). \tag{3.34}$$

Thus, the feedback rules are given by

$$q_i\,(p) = p(1 - sK) + sE. \tag{3.35}$$

Substituting this last expression and using the quadratic value function (3.32) into the Bellman equation yields

$$\begin{aligned}\tfrac{1}{2}p^2 \left(1 - rK - 2sK(N+1) + s^2K^2(2N-1)\right) \\ + p(asK + E(r+s) + sNE - s^2EK(2N-1)) \\ + s^2E^2(2N-1) - sEa - rg = 0.\end{aligned} \tag{3.36}$$

The requirement that this equation be satisfied for all values of p implies that K, E and g have to satisfy

$$1 - rK - 2sK(N+1) + s^2K^2(2N-1) = 0, \tag{3.37}$$

$$asK + E(r+s) + sNE - s^2EK(2N-1) = 0, \tag{3.38}$$

$$s^2E^2(2N-1) - sEa - rg = 0. \tag{3.39}$$

The solutions to equations (3.37)-(3.39) are given by

$$K = \frac{2s(N+1) + r \pm \sqrt{[2(N+1)s + r]^2 - 4s^2(2N-1)}}{2s^2(2N-1)}, \tag{3.40}$$

$$E = \frac{c - sKa}{s(N+1) + r - s^2K(2N-1)}, \tag{3.41}$$

$$g = \frac{s^2E^2(2N-1) - 2sEa}{2r}. \tag{3.42}$$

With the decision rules (3.35) the price equation (3.5) becomes

$$\dot{p} + ps[N(1 - sK) + 1] = s(a - NsE), \tag{3.43}$$

which is a linear first order differential equation. A solution to this equation is given by

$$p(t) = p^* + (p_0 - p^*)e^{Dt}, \tag{3.44}$$

where p^* is the steady state price

$$p^* = \frac{a - NsE}{1 + N(1 - sK)}, \tag{3.45}$$

p_0 is the initial price and D is the constant

$$D = s[N(sK - 1) - 1].$$

This constant is only negative, and hence the Markov-perfect equilibrium is globally stable if we choose the negative root of (3.40). Equations (3.35) and (3.40) to (3.45) give us the Markov-perfect equilibrium in linear strategies for the differential game (3.4) and (3.5) for any finite s. This completes the proof.

Appendix B:

Proof of Theorem 2: The Bellman equation of the problem (3.14)-(3.15) is given by

$$r\hat{V}^i(p) = \max_{q_i}\{pq_i - \frac{1}{2}q_i^2 + s\hat{V}_p^i(p)[a - \sum_{j=1}^{M}\bar{q}_j - \sum_{i=M+1}^{N} q_i - p]\}, \tag{3.46}$$

where $\hat{V}^i(p)$ is the optimal value function of firm i, which is an unconstrained firm in the constrained case. Maximization of the right hand side of the Bellman equation gives

$$\hat{q}_i(p) = p - s\hat{V}_p^i(p), \tag{3.47}$$

Substituting (3.47) into (3.46) and inducing symmetry yields

$$\begin{aligned} r\hat{V}(p) &= p(p - s\hat{V}_p(p)) - \frac{1}{2}(p - s\hat{V}_p(p))^2 \\ &\quad + s\hat{V}_p(p)[a - p - M\bar{q} - (N-M)(p - s\hat{V}_p(p))]. \end{aligned} \tag{3.48}$$

As with the unconstrained case, we propose the following quadratic value function

$$\hat{V}(p) = \tfrac{1}{2}\hat{K}p^2 - \hat{E}p + \hat{g},$$

which implies that

$$\hat{V}_p(p) = \hat{K}p - \hat{E},$$

where $\hat{K}$, $\hat{E}$ and $\hat{g}$ are constants that need to be determined. Thus, the feedback rules are given by

$$\hat{q}_i(p) = p(1 - s\hat{K}) + s\hat{E}. \tag{3.49}$$

Substituting $\hat{V}(p)$ and $\hat{V}_p(p)$ in (3.48) and collecting with respect to p, we obtain

$$\beta_1 p^2 + \beta_2 p + \beta_3 = 0, \tag{3.50}$$

where

$$\beta_1 = \tfrac{1}{2}\left(1 - r\hat{K} - 2s\hat{K}(N-M+1) + s^2\hat{K}^2(2(N-M)-1)\right), \tag{3.51}$$

$$\beta_2 = as\hat{K} + \hat{E}(r+2s) + s\hat{E}(N-M-1) - s\hat{K}M\bar{q} - s^2\hat{E}\hat{K}(2(N-M)-1), \tag{3.52}$$

$$\beta_3 = s^2\hat{E}^2(N-M-\tfrac{1}{2}) + s\hat{E}M\bar{q} - s\hat{E}a - r\hat{g}. \tag{3.53}$$

The equation (3.50) is satisfied if expressions (3.51)-(3.53) are simultaneously zero. This results to the following solution

$$\begin{aligned} \hat{K} &= \frac{2s(N-M+1) + r \pm \sqrt{[2(N-M+1)s+r]^2 - 4s^2(2(N-M)-1)}}{2s^2(2(N-M)-1)}, \\ \hat{E} &= \frac{-s\hat{K}a - s\hat{K}M\bar{q}}{s(N-M+1) + r - s^2\hat{K}(2(N-M)-1)}, \\ \hat{g} &= \frac{s^2\hat{E}^2(2(N-M)-1) - 2s\hat{E}a + 2s\hat{E}M\bar{q}}{2r}. \end{aligned} \tag{3.54}$$

Using (3.49), a solution to equation (3.15) is given by

$$\hat{p}(t) = \tilde{p} + (p_0 - \tilde{p})e^{\hat{D}t},$$

where $\tilde{p}$ is the steady state price

$$\tilde{p} = \frac{a - (N-M)s\hat{E} - M\bar{q}}{1 + (N-M)(1 - s\hat{K})},$$

p_0 is the initial price and $\hat{D}$ is the constant

$$\hat{D} = s[(N-M)(s\hat{K} - 1) - 1].$$

This constant is only negative and if we choose the negative root of (3.54) the Markov-perfect equilibrium is globally stable.

The discounted present value of the constrained firm is derived from

$$\hat{V}^C(p(t)) = \int_t^{\infty} e^{-r(\tau-t)}[p(\tau) - \frac{1}{2}\bar{q}]\bar{q}d\tau, \tag{3.55}$$

where $\hat{V}^C$ is the value function of the constrained firms and $p(.)$ is the price given by (3.18). Substituting (3.18) in (3.55), we have

$$\hat{V}^C\left(p(t)\right) = e^{rt}\bar{q}\left[\int_t^{\infty} e^{-r\tau}(\tilde{p} - \frac{1}{2}\bar{q})d\tau + \int_t^{\infty} e^{-(r-D)\tau}(p_0 - \tilde{p})d\tau\right], \quad (3.56)$$

which results to

$$\hat{V}^C = \frac{1}{r}(\tilde{p}\bar{q} - \frac{1}{2}\bar{q}^2) + \frac{1}{r - \hat{D}}(p_0 - \tilde{p})\bar{q}e^{\hat{D}t}, \quad (3.57)$$

Thus, we obtain

$$\hat{V}^C = Ap + \hat{g}^C,$$

where A and $\hat{g}^C$ are

$$A = \frac{\bar{q}}{r - \hat{D}},$$

$$\hat{g}^C = \frac{\bar{q}(\hat{D}(2\tilde{p} - \bar{q}) + r\bar{q})}{2r(\hat{D} - r)}.$$

This proves the theorem.

Appendix C:

Proof of Theorem 3: Looking at (3.10) and (3.20), it is obvious that K and $\hat{K}$ have the same functional form with this difference that instead of N we have $N - M$ in $\hat{K}$. Thus, since it can be easily shown that $\partial K/\partial N < 0$, we find that $\hat{K} > K > 0$. We have the similar story to compare E with $\hat{E}$ and g with $\hat{g}$. Substituting (3.10) in (3.11), we can show that $\partial E/\partial N > 0$. Thus, as the number of firms decreases the coefficient E will decrease. This together with having the negative term $-s\hat{K}M\bar{q}$ in equation (3.21) we can argue that, as long as $\bar{q}$ is positive, $\hat{E} < E < 0$. With the same procedure we can show that always $\hat{g} > g > 0$. Therefore, comparing (3.7) and (3.17), for all values of p we obtain $\hat{V}(p) > V(p)$. This concludes (b).

Considering (3.10) and (3.20), it can be easily shown that for all values of parameters $sK < s\hat{K} < 1$. So, comparing (3.13) and (3.23), we find that $D < \hat{D} < 0$. Furthermore, looking at steady state prices it can be shown that for $\bar{q} \leq q^*$, the steady state price (3.19) is larger than (3.9). Therefore, it can be simply proven that the price path (3.18) is greater than (3.8) which concludes (a).

Now, in order to examine (c) we have to compare (3.7) and (3.24). Since $V(p)$ is a convex function and $\hat{V}^C(p)$ is a linear function of p, by equating these two equations, we obtain

$$p^l = \frac{A + E \pm \sqrt{(A+E)^2 - 2K(g - \hat{g}^C)}}{K}, \quad l = a, b, \tag{3.58}$$

where $p^a > p^b$ (if there exist any p^b). Therefore, in principal, for positive values of p, the two value functions may have (i) one intersection if the radicand is larger than zero and $\hat{g}^C > g$, (ii) two intersections provided that the radicand has a positive value and $g > \hat{g}^C$, (iii) one tangency point when the radicand is zero, and (iv) no intersection when the radicand is negative. Therefore, having a viable range of parameters for which acting non-strategically is to the benefit of firms mainly depends on the amount of $(A + E)^2 - 2K(g - \hat{g}^C)$. Here, we are interested in its positive values. Using a numerical analysis, a range of parameters in the space of (M, α) is depicted in figure 1by means of two dividing curves (the thicker ones) where between them the amount of radicand is positive and beyond them it is negative. Situation (iv) corresponds to the regions beyond the curves. However, in the region between the two curves, we do have a viable range of parameters for output constraint to be profitable for the non-strategic firms which corresponds to (i) or (ii). Figures 2 show (i), (ii), (iii) and (iv) graphically.

The condition (iii) occurs in the limit where s goes to zero, because we have

$$\lim_{s \to 0} (A + E)^2 - 2K(g - \hat{g}^C) = 0,$$

and, therefore, there is a single common point in $V(p)$ and $\hat{V}^C(p)$. Furthermore, the slope of $V(p)$ at $p^a = \frac{A+E}{K}$ is$V'(p) = Kp - E = A$, which is the same as the slope of $\hat{V}^C(p)$. Thus, the contacting point is a tangency point. Hence, when the price does not adjust at all (i.e. $s = 0$), acting non-strategically is never to the benefit of the non-strategic firms. However, for $0 < s < \infty$ there is a range of parameters where the non-strategic firms also benefit from the output constraining. Comparing the steady state prices (3.9) and (3.19) and prices driven from (**??**), we found that

$$p^* < \tilde{p} \in \begin{cases} (0, p^a) & \text{in (i)}, \\ (p^b, p^a) & \text{in (ii)}. \end{cases}$$

Now, looking at (3.18), we know that $\hat{p}$ starts at the initial price p_0 and moves towards the steady state price $\tilde{p}$. Therefore, provided that the initial price belongs to $(0, p^a)$ (or (p^b, p^a)), we can argue that $\hat{V}^C(p)$ always has a larger value than $V(p)$. Whereas, in the case where p_0 is outside the aforementioned interval, output constraining is not to the benefit of the non-strategic firms in so far as $\hat{p}(t)$ arrives to the interval and it becomes profitable afterwards.

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Part II

International Competition and Environmental Problem

Chapter 4

Strategic environmental policies under international competition with asymmetric pollution spillovers

4.1 Introduction

The environmental consequences of trade liberalization have received a considerable attention in trade theory and environmental economics. International trade is playing an important role in expanding global economic activities and, therefore, many individuals have argued that trade liberalization will lead to an increase in world pollution.

Although globalization brings about many benefits and opportunities, some environmentalists have resisted freer trade, because governments which are unable to use trade policy may lower their environmental standards to give competitive advantage to existing domestic industries and protect their economy. This has led some economists to investigate the relationship between trade and the environment.[1]

The established literature on trade and environment suggests that, while

[1] Copeland and Taylor (2003) provides a comprehensive review of the link between trade and the environment.

each country can gain from trade, it expands global pollution. Fujiwara (2009) investigates the effects of free trade on global stock of pollution and he finds that under trade liberalization the stock of pollution is larger as compared to the autarky.

Another part of the literature deals with the links between strategic environmental policies and the patterns of trade and pollution levels. Stem from the Brander and Spencer (1985) model, Rauscher (1994), Kennedy (1994), Barrett (1994), Walz and Wellisch (1997) and Tanguay (2001) all show that governments can have incentives to use environmental policies to subsidize their exports. It is beneficial for rent-shifting governments to set an environmental tax below the Pigouvian level in an international oligopoly. Such a weak environmental regulation to support domestic firms has been called ecological dumping.

The aim of this study is to examine the welfare implications of trade liberalization when governments behave strategically using environmental policy in the presence of transboundary pollution. However, we model the transboundary pollution in such a way that it allows drawing the results also in pure local pollution and global environmental problem.

In this paper, we consider two symmetric countries with a single firm in each producing a homogenous good. The two firms may export a part of their production to the other country. In our model trade of the same product occurs between countries.[2] Thereby, we have a two-stage game where in the first stage governments decide about the environmental and trade policies, and the two firms compete 'a la Cournot in the second stage. The most important difference of this study with the aforementioned literature is that we allow for assymetric environmental damages between the two countries in our model.

We find that when the marginal cost of pollution of the domestic firm increases, the pollution-shifting motive is enhanced and government wants to raise production taxes and the rent-seeking behavior is observed and government

[2] Brander (1981) and Brander and Krugman (1983) showed that intraindustry trade occurs because each firm perceives each country as a separate market and makes distinct output decisions for each.

raises import tariffs. On the other hand, when the marginal cost of pollution of the foreign firm increases, government want to reduce the level of tax and the level of tariff as well.

In addition, contrary to existing literature, it is shown that the global pollution decreases in bilateral trade compared to autarky provided that the difference between the emission rates of the two firms is sufficiently large. This result holds even for the case of pure local pollution. Furthermore, it is shown that how the asymmetric pollution emissions affects the firms' profit and countries' welfare.

The rest of the paper is organized as follows. Section 2 constructs the general framework of the model and describes autarkic equilibria. Section 3 devoted to the firms' equilibrium. In Section 4 we turn to the games between the two governments. Comparing the trade equilibria with the autarkic equilibria takes place in section 5. Section 6 concludes the paper.

4.2 The fundamentals

4.2.1 The setup

There are two countries, indexed by $i = 1, 2$. In each country there is a firm which produces a single output. Their productions, q_i, have two parts:

$$q_i = q_{hi} + q_{ei}, \qquad i, j = 1, 2,$$

where q_{hi} and q_{ei} denote the amounts of output produced by firm i and consumes in the domestic market and is exported to the other country, respectively.

The inverse demand function in each country is

$$p_i = a - (q_{hi} + q_{ej}), \qquad i, j = 1, 2 \textit{ and } j \neq i,$$

where q_{ej} is the amount of good which is exported by the firm j into country i.

Production takes place at constant returns to scale (CRS), with a constant marginal cost c which is summarized by the cost function $C_i = cq_i(t)$.

The production of firm i, q_i, creates a constant per unit emission level, η_i. While firms are homogenous in their cost functions, it is assumed that they are hetrogeneous in their environmental damage functions, E_i

$$E_i(q_i) = \eta_i q_i = \eta_i \left(q_{hi} + q_{ei}\right), \qquad i, j = 1, 2,$$

which is not confined to the country where the production takes place and gives rise to a transboundary pollution problem.

The foreign production results in a negative externality in home country at the fixed level ψ, per unit of its environmental damage. Hence, the negative externality caused by home and foreign production in country i is

$$ex_i(q_i, q_j) = E_i(q_i) + \psi E_j(q_j).$$

where $\psi \in [0, 1]$, and $\psi = 0$ denotes the case of pure local pollution and $\psi = 1$ denotes the case of pure global environmental problem.

In order to protect the environment, country i levies an environmental tax, τ_i, on its polluting production and imposes a tariff , θ_i,on imported items. Hence, firm i's instantaneous profits are

$$\pi_i = p_i q_{hi} + p_j q_{ei} - c q_i - \tau_i q_i - \theta_j q_{ei}, \qquad i, j = 1, 2 \text{ } and \text{ } j \neq i,$$

Tax revenues are distributed in the form of a lump sum to the consumers. Thus, the social welfare in each country is the aggregate amount of firm's profits, consumer surplus, tax and tariff revenues minus negative environmental externality caused by home and foreign firms productions:

$$W_i = \pi_i + CS_i + \tau_i q_i + \theta_i q_{ej} - ex_i, \tag{4.1}$$

where $CS_i = \left(q_{hi} + q_{ej}\right)^2 / 2$.

4.2.2 The autarkic equilibrium

Now, we consider a closed economy where there is no trade between countries and each firm is monopolist in its own country. Therefore, given the government environmental policy τ_i, the firm i maximizes her monopolistic profit $\pi_i = p_i q_i - c q_i - \tau_i q_i$, $i = 1, 2$. By first-order condition (FOC), we obtain $q_i^A =$

$\frac{1}{2}(a-c-\tau_i)$ where the superscript A denotes the autarky. At the equilibrium, firm i's reaction to the tax policy is $\partial q_i^A/\partial \tau_i = -1/2$.

In the autarky, the government's first-best environmental policy is introduced by the Pigovian tax $\tau_i^A = \eta_i$. Note that because of transboundary pollution, the foreign firm's production creates negative externality in the home country but it is not affected by the home government policy.

Therefore, the Cournot-Nash equilibrium in the autarky is

$$q_i^A = \frac{1}{2}(a-c-\eta_i), \tag{4.2}$$

$$\pi_i^A = \frac{1}{4}(a-c-\eta_i)^2, \tag{4.3}$$

$$W_i^A = \frac{3}{8}(a-c-\eta_i)^2 - \frac{1}{2}\psi\eta_j\left(a-c-\eta_j\right), \tag{4.4}$$

$$E_i^A = \frac{1}{2}\eta_i(a-c-\eta_i), \tag{4.5}$$

$$ex_i^A = E_i^A + \psi E_j^A = \frac{1}{2}\left[(a-c)\left(\eta_i+\psi\eta_j\right) - \eta_i^2 - \psi\eta_j^2\right], \tag{4.6}$$

$$ex_G^A = ex_i^A + ex_j^A = \frac{1}{2}(1+\psi)\left[(a-c)\left(\eta_i+\eta_j\right) - \eta_i^2 - \eta_j^2\right], \qquad j \neq i, \tag{4.7}$$

where G denotes the global negative environmental externality.

4.3 Trade liberalization

In this section, we want to investigate the firms behavior and governvent policies after trade liberalization. In what follows, we construct a two-stage game. In the first stage, governments determine the level of tax and tariff and in the second stage, the two firms simultaneously choose their outputs.

4.3.1 The firms' equilibrium

By backward induction, we first solve the two international Cournot competitors problem when choosing their export and home production levels, q_{ej} and q_{hi} respectively. The problem facing firm i is

$$\max_{q_{hi},q_{ei}} \pi_i = (a-q_{hi}-q_{ej})\,q_{hi} + (a-q_{hj}-q_{ei}-\theta_j)\,q_{ei} - (c+\tau_i)(q_{hi}+q_{ei}),$$

Taking the FOCs, we obtain the following reaction functions

$$\frac{\partial \pi_i}{\partial q_{hi}} = a - 2q_{hi} - q_{ej} - c - \tau_i = 0,$$

$$\frac{\partial \pi_i}{\partial q_{ei}} = a - q_{hj} - 2q_{ei} - c - \tau_i - \theta_j = 0.$$

Solving the FOCs of both firms simultaneously, we find

$$q_{hi}^{CN} = \frac{1}{3}\left(a - c - 2\tau_i + \tau_j + \theta_i\right), \tag{4.8}$$

$$q_{ei}^{CN} = \frac{1}{3}\left(a - c - 2\tau_i + \tau_j - 2\theta_j\right), \tag{4.9}$$

where CN denotes the Cournot-Nash equilibrium. From equations (4.8)-(4.9) it is found that $\partial q_{hi}^{CN}/\partial \tau_i = \partial q_{ei}^{CN}/\partial \tau_i = -2/3 < \partial q_i^A/\partial \tau_i = -1/2$, which implies that, first, the firm i reacts to the tax levied by the home government by reducing her output and, second, this reaction is stronger compared to the autarky. However, the firm reaction to the foreign tax is opposite. As the foreign government increases the tax rate, the domestic firm enhances her output, i.e. $\partial q_{hi}^{CN}/\partial \tau_j = \partial q_{ei}^{CN}/\partial \tau_j > 0$.

Furthermore, the level of import decreases as the government of the home country increases the level of tariff ($\partial q_{ej}^{CN}/\partial \theta_i < 0$), and, consequently, the home firm's production increases ($\partial q_{hi}^{CN}/\partial \theta_i > 0$).

4.3.2 The noncooperative government policies

Now, knowing the firms' behavior in the second stage, we move to the first stage where the environmental taxes and tariffs are determined by governments as Stackelberg strategic leaders. The total welfare of each country is defined as the summation of consumer surplus, the firm's profits, tax and tariff revenues minus the negative environmental externality caused by both home and foreign firms. Thus, the government's problem in country i is defined

$$\max_{\tau_i, \theta_i} W_i^{CN} = \pi_i^{CN} + CS_i^{CN} + \tau_i q_i^{CN} + \theta_i q_{ej}^{CN} - ex_i^{CN}. \tag{4.10}$$

We consider a non-cooperative game where each country unilaterally make decision about the environmental tax and tariff to maximize its own national welfare, and ignores its impact on the other. This problem is done with governments choosing their pollution tax τ_i and tariff θ_i and knowing the reactions of both firms in the second stage. Thus, the FOCs are

$$\frac{\partial W_i^{CN}}{\partial \tau_i} = \frac{1}{9}\left(12\eta_i - 6\psi\eta_j - 7\tau_i - \tau_j + 3\theta_i + 2\theta_j - 4(a-c)\right) = 0,$$

$$\frac{\partial W_i^{CN}}{\partial \theta_i} = \frac{1}{3}\left(a - c - \eta_i + 2\psi\eta_j + \tau_i - \tau_j - 3\theta_i\right) = 0.$$

Solving the FOCs of the problems of both governments simultaneously, we find the equilibrium amount of pollution tax and tariff

$$\tau_i^* = \frac{1}{96}\left[167\eta_i - 43\eta_j + 32\psi\left(\eta_i - 2\eta_j\right) - 28(a-c)\right], \tag{4.11}$$

$$\theta_i^* = \frac{1}{48}\left[16(a-c) + 19\eta_i - 35\eta_j + 16\psi\left(\eta_i + \eta_j\right)\right]. \tag{4.12}$$

As it can be seen from (4.11), $\partial\tau_i^*/\partial\eta_i = (167 + 32\psi)/96 > \partial\tau_i^A/\partial\eta_i = 1$, therefore, as the firms' emission rates increase the home government increases the tax level. And, surprisingly, this taxation is stronger as the rate of spill-over rises. Futhermore, in the international competition, firms faces a stronger environmental taxation compared to autarky. However, the government's reaction to the incease in the foreign firm's pollution is reduction in levied tax on his home firm, i.e. $\partial\tau_i^*/\partial\eta_j < 0$. Also, interestingly, we can see that the equilibrium level of tariff on imports increases with the domestic rate of pollution production, i.e. $\partial\theta_i^*/\partial\eta_i > 0$. In addition, this tariff decreases when the foreign pollution increases, $\partial\theta_i^*/\partial\eta_j < 0$. Furthermore, we can see that $\partial\tau_i^*/\partial\psi > 0$ when $\eta_j < \eta_i/2$, and $\partial\theta_i^*/\partial\psi > 0$.

Finally, the market equilibrium becomes

$$\begin{aligned} q_{hi}^* &= \frac{1}{96}\left[52(a-c) - 113\eta_i + 61\eta_j - 32\psi\left(\eta_i - 2\eta_j\right)\right], \\ q_{ei}^* &= \frac{1}{96}\left[20(a-c) - 79\eta_i + 59\eta_j - 32\psi\left(2\eta_i - \eta_j\right)\right]. \end{aligned}$$

Then the total output of firm i is

$$q_i^* = \frac{1}{4}\left[3(a-c) - 8\eta_i + 5\eta_j - 4\psi\left(\eta_i - \eta_j\right)\right]. \tag{4.13}$$

Consequently, the negative externality produced by the home and foreign firms in country i is

$$ex_i^T = \eta_i q_i^* + \psi \eta_j q_j^*, \tag{4.14}$$

where T denotes the case of trade liberalization. Therefore, the global pollution and the total negative externality caused by firms' production are

$$E_G^T = \eta_i q_i^* + \eta_j q_j^*, \tag{4.15}$$

$$ex_G^T = (1+\psi) E_G^T, \tag{4.16}$$

where G stands for the global.

4.4 Trade *vs* autarky

In this section, we compare the autarkic equilibrium with the noncooperative restricted trade equilibria. We want to know how the asymmetric pollution spill-over makes some differences.

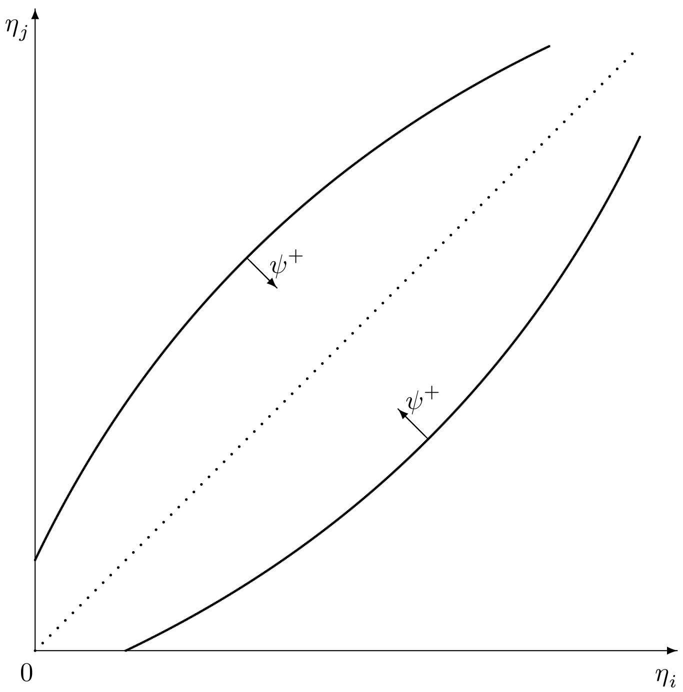
4.4.1 Global pollution

Comparing (4.7) and (4.16), yields

$$\begin{aligned} \Delta ex_G &= ex_G^T - ex_G^A \\ &= \frac{1}{4}(1+\psi)\left[(a-c)\left(\eta_i+\eta_j\right) - \left(6\eta_i^2 + 6\eta_j^2 - 10\eta_i\eta_j + 4\psi\left(\eta_i-\eta_j\right)^2\right)\right] \end{aligned} \tag{4.17}$$

In the symmetric case where $\eta_i = \eta_j = \eta$, we have $\Delta ex_G|_{i=j} = \frac{\eta}{2}(1+\psi)(a-c-\eta)$. Since from (4.2) we know that $a-c-\eta \geq 0$, the total externality and globa pollution in bilateral trade is larger than autarky. However, in the asymmetric case $\eta_i \neq \eta_j$, there exists a range of parameter in which global pollution and consequently total negative externality in the autarkic equilibrium are larger than the restricted trade. In figure 1, for a given value of $a-c$, Δex_G is depicted in the space of (η_i, η_j). In the region between the two curves Δex_G is positive, therefore, trade liberalization is detrimental for the environment if the two firms' rates of emissions are almost equal. Note that the dotted line in this and the following figures represent the points where $\eta_i = \eta_j$. On this points we have $E_G^T > E_G^A$, which is consistent with the existing literature with symmetric emission rate.

Figure 1: Global pollution comparison.



The region beyond the curves represents the points where $\Delta ex < 0$. Therefore, for a wide range of asymmetric emission rates, trade liberalization not only is not a bad news for the environment but also it could even make reduction in the environmental damages. This result is contrary to the almost all of the previous studies where they argue that trade liberalization leads to increase in environmental pollution.

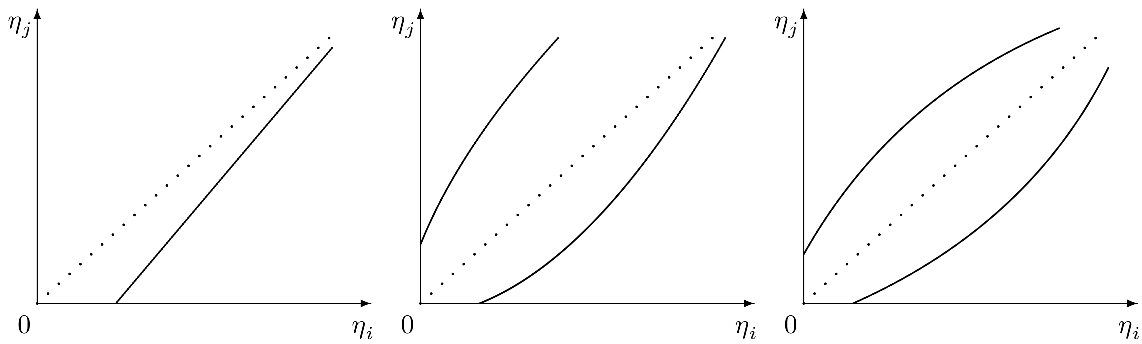
As it is shown in figure 1, as the rate at which pollution crosses borders, ψ, increases the region where the restricted trade is environmentally detrimental shrinks. Thus, for a pure global environmental problem (i.e. $\psi = 1$) the region where international trade compared to autarky is environmental friendly becomes even larger.

4.4.2 Externality in home country

A part of the negative externality caused by polluting production is created by the domestic firm and another part by the foreign firm because of having transboundary pollution ($\psi > 0$). Therefore, in the case of autarky we still have the negative environmental effect of foreign firm activity. In order to compare the negative externality in country i in the international trade framework with the autarky, we should compare (4.6) with (4.14) which yields

$$\Delta ex_i = ex_i^T - ex_i^A = \frac{1}{4}\eta_j \left[(a-c) + 5\eta_i - 6\eta_j + 4\psi\left(\eta_i - \eta_j\right)\right]. \qquad (4.18)$$

Figures 2: Negative externality comparison in country i where a) $\psi = 0$, b) $\psi = 1/2$, c) $\psi = 1$.



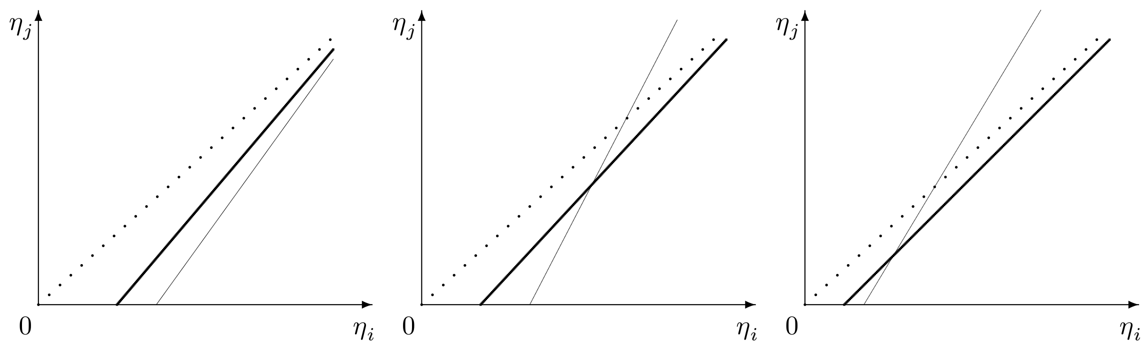
(a) (b) (c)

It can be easily shown that in the case of international trade competition environmental damages is larger than the autarky provided that $\eta_i = \eta_j$. For the general values of emission rates, in figures 2, we have plotted Δex_i in the space of (η_i, η_j), for the case of: a) pure local pollution, $\psi = 0$, b) an example of transboundary pollution, $\psi = 1/2$, and c) pure global pollution, $\psi = 1$. In these figures, only in the regions between the two curves (in figure 2a, between the curve and vertical axis) trade will increase the negative environmental externality in country i.

4.4.3 Output

Considering (4.2) and (4.13), in figures 3, we have shown the region on the right side of the ticker curves where firm i produces more in international competition. Thus, provided that the firm i's pollution spill-over is sufficiently larger than her rival's; her total production in the presence of international trade is lower than the case of autarky. This is a very good news for environmentalist which even noncooperative environmental and trade policies make the more environmentally inefficient firm to reduce her production.

Figures 3: Total output comparison for firm i where a) $\psi = 0$, b) $\psi = 1/2$, c) $\psi = 1$.



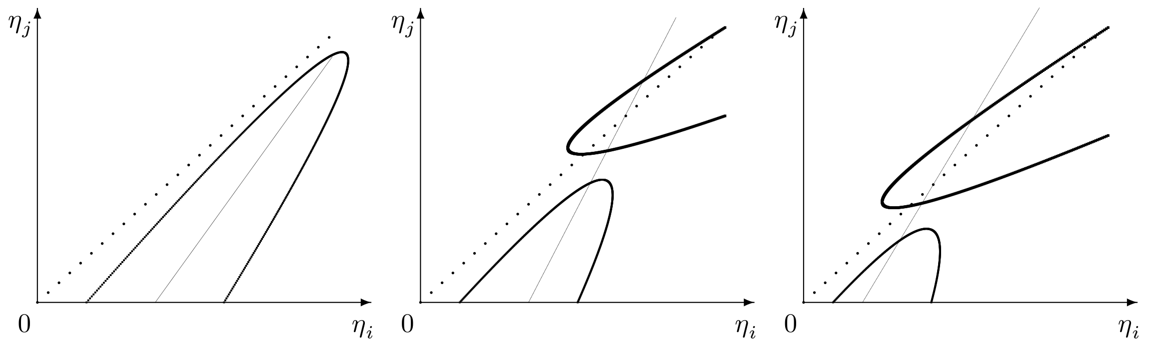
(a) (b) (c)

In the figures 3 and 4, the thinner lines represent the points where below them $q^*_{ei} < 0$, and therefore, there is not any export by the firm i to the country j.

4.4.4 Profits and welfare

Finally we want to examine the profitability and welfare consequences of trade liberalization.

Figures 4: Profit and welfare comparison in country i where a) $\psi = 0$, b) $\psi = 1/2$, c) $\psi = 1$.



(a) (b) (c)

Figures 4, in the space of (η_i, η_j), shows the regions inside the curves where firm's profit and social welfare of country i under the autarky are larger than the ones in international competition. The lower curves represent the points where firm's profits in autarky and international trade are equal, and the upper curves characterizes the points where the social welfare in autarky and trade are the same. Consitent with the other studies, firms' profits in the case of symmetric pollution spill-overs decreases in trade liberalization. However, in the case of asymmetric pollution spill-over, trade liberalization decreases the firm's profits provided that her pollution spill-over is sufficiently larger than her rival in international competition.

In the case of pure local pollution, trade liberalization always increases total welfare. As ψ increases the regions where firms profits in autarky is larger than international trade shrink and the regions where social welfare in autarky is larger than international trade expand. However, although in the presence of transboundary pollution governments prefer autarky rather than international competition when firms production functions are the same, they prefer restricted trade where they use environmental and trade policies rather than autarky provided that the home firm's pollution spill-over is lower than her rival.

4.5 Concluding Remarks

In this paper, we considered a two-country world model with a single polluting firm in each country to examine the welfare implications of trade liberalization when governments behave strategically using environmental policy with asymmetric pollution spillovers. We investigated a second-best trade agreement between two countries to examine the strategic behavior of governments in using pollution taxes and tariffs. We found that when the marginal cost of pollution of the domestic firm increases, the pollution-shifting motive is enhanced and government wants to raise production taxes and surprisingly the rent-seeking behavior is observed and government raises import tariffs. On the other hand, when the marginal cost of pollution of the foreign firm increases, government want to reduce the level of tax and interestingly the level of tariff as well. Furthermore, it is shown that how the level of taxes may increase or decrease when/as the rate at which pollution crosses borders rises. We also show that, because of asymmetric pollution spill-over, the global pollution may decrease after trade liberalization.

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