An Essay on Institutions and Contracts for Social Welfare

Alessia Russo

Dottorato di Ricerca in Economia - XXII Ciclo -

Alma Mater Studiorum - Università di Bologna

June 2011

Relatore: Prof. Giacomo Calzolari

Coordinatore: Prof. Giacomo Calzolari

Settore scientifico-disciplinare: SECS-P/01 Economia Politica

Contents

Overview **vii**
0.1 Introduction vii
0.2 General Framework viii
0.3 Overview of the Chapters x

1 A Dynamic Politico-Economic Model of Intergenerational Contracts **1**
1.1 Introduction 1
1.2 Literature Review 7
1.3 The Model 9
1.3.1 Households 9
1.3.2 Production 10
1.3.3 Fiscal Constitution 11
1.4 Maximization and Equilibrium 12
1.4.1 Competitive Economic Equilibrium 12
1.4.2 Political System 14
1.4.3 Politico-Economic Equilibrium 15
1.4.4 Political SMPE with Perfect Foresight 19
1.4.5 Dynamics and Steady States 21
1.5 Discussion 22
1.5.1 Welfare State Regimes 26
1.5.2 Aging 29
1.6 Benevolent Government Allocation 30
1.6.1 The Government SMPE 33
1.6.2 Are the political choices on pensions and education optimal? 35
1.7 Conclusions 36

1.8 Technical Appendix A 38
1.9 Technical Appendix B 49
1.9.1 Derivation of Generalized Euler Equation 49

2 Politicians, Redistribution and Intergenerational Conflicts 59
2.1 Introduction 59
2.2 The Model 63
2.2.1 Household 63
2.2.2 Technology 64
2.2.3 Fiscal Constitution 65
2.3 First Best Allocation 66
2.4 Politico-Economic Equilibrium 68
2.4.1 Timing 68
2.4.2 Competitive Equilibrium given Fiscal Policies 69
2.4.3 Electoral Stage 70
2.4.4 Political Competition Stage 72
2.5 No Distortionary Case 74
2.5.1 Example Economy 77
2.6 Distortionary Taxation 81
2.7 Ideological Competition 83
2.8 Conclusions 84
2.9 Appendix A 85
2.10 Appendix B 87

3 Self-Commitment Institutions and Cooperation in Overlapping Generation Games 93
3.1 Introduction 93
3.2 Past literature 99
3.2.1 Theoretical Debate on Self-Enforcing Institutions . . . 101
3.3 The Model 102
3.3.1 Public Perfect Equilibrium and Social Norms 104
3.4 Social Norms without Self-Commitment Institution 107
3.5 Social Norms with Self-Commitment Institution 110
3.5.1 Value of Self-Commitment 114
3.6 Productive Self-Commitment: The Role of Education 116
3.7 Conclusion 119
3.8 Appendix 121

To my parents...

because intergenerational relations are not just conflicts

Love or Prison

Overview

0.1 Introduction

The study of the conditions, which sustain mutual cooperation over the redistribution of public resources among different generations, embeds interesting theoretical challenges from both economic and political points of view. Politicians, in order to be elected, promise future transfers in favor of current voters, anticipating that they would probably not be in their posts when the bill will become due. Furthermore, intergenerational redistribution promises involve transfers from one generation to another, in some cases even when one of those generations is too young to vote. Consequently, intergenerational redistribution turns to a battleground, pitting young against old and taxpayers against recipients, especially when balanced budget conditions are required to be met.

In a dynamic politico-economic environment characterized by no commitment technology, the deep understanding of the implementation and sustainability of implicit intergenerational contracts requires the identification of appropriate enforceability mechanisms. In a repeated setting the literature distinguishes between reputational (non-Markov) and non reputational (Markov) enforceability mechanisms. While the latter do not require players to trigger punishment in the case of defection from the equilibrium path, the former requires either the victimized agents to make retaliation against the cheater (personal punishment) or, alternatively, other agents, who are not directly involved in the bilateral negotiation, to exert the punishment against a deviator (community punishment). The following table resumes the main enforceability mechanisms in repeated non-cooperative games:

	Description
Non Reputational (Markov)	Strategies contingent on payoff-relevant-state variable
Reputational (Non Markov)	Trigger strategies

Both mechanisms are characterized by drawbacks and advantages, which are basically resumed in the trade-off between renegotiation proof-robustness and efficiency. On one hand, reputational equilibria require coordination among agents, but the threat of long punishment phase can sustain good equilibria. On the other hand, Markov equilibria avoid coordination failures because they restrict the required information space to the payoff-relevant state variables, but eventually selecting bad equilibria.

The main idea defended in this work is that reputational mechanisms hardly work in overlapping generation (OLG) games. Due to the particular structure of the game, the informational requirements necessary to implement reputational equilibria are challenging to meet and coordination failures among different generations are likely to arise. Thus, the main theoretical challenge we deal with concerns how to enhance efficiency when: i) We focus on Markov equilibria (Chapter 1 and 2), and ii) we allow for non-Markov equilibria when personal or community punishments are not effective (Chapter 3). We provide the following solutions for each of the above cases:

i) Markov Equilibria: *Forward intergenerational transfers*;

ii) Reputational Equilibria: *Self-Commitment Institution*.

0.2 General Framework

We present the simplified version of the intergenerational game we will analyze in the following chapters. Time is discrete and indexed by $t = 0, 1, \ldots$. At each time t the economy is populated by three generations (Young, Adults and Old), who prefer to smooth consumption over time. Each generation has specific property rights over the total economy endowment. Only Adults and Old are represented by political parties, which compete on proposing electoral platforms requiring agents to transfer their property rights to other generations. Let y_t be the total economic endowment. After electoral competition and intergenerational redistribution, $1-q_t$ turns out to be the share of Adults' property rights and q_t the share of elderly property rights. Thus, the individual budget constraints at each time require:

$$\begin{aligned} c_t^y &= 0 \\ c_t^a &= (1-q_t)y_t \\ c_t^o &= q_t y_t \end{aligned}$$

In equilibrium intergenerational cooperation arises if c_t^a and c_t^o are different from zero. Furthermore, efficiency is attained if an alternative intergenerational redistribution scheme, $q_t' \neq q_t$, which improves further welfare, does not exist. According to the rules governing the political voting process, a self-enforcing (implicit) intergenerational contract requires agents to have incentives not to defect the agreement without assuming the existence of a court to enforce the contract. Thus, the politico-economic equilibrium is represented by the infinite sequence $\{y_t, q_t\}_{t=0}^{\infty}$.

In the case of Markov equilibrium, an intergenerational contract is represented by the following mapping:

$$q_t : \mathcal{S} \rightarrow [0,1]$$

where $\mathcal{S}$ is the space of relevant payoff state variables. Previous studies have mainly focused on private savings as the main state variable in a median voter framework to consider how efficient intergenerational contracts may be self-sustained in equilibrium. We study how the introduction of bidimensional redistribution programs, characterized by simultaneous transfers to young and old from middle age class, and human capital, as an additional relevant state variable in probabilistic voting environment, alter the self-enforcing conditions to make the intergenerational contracts emerge.

In the case of non-Markov equilibrium, intergenerational contract is represented by the following mapping:

$$q_t : \mathcal{H}^t \rightarrow [0,1]$$

where $\mathcal{H}^t$ is the set of observable histories till time t. Previous studies have analyzed how personal or community punishment may deter deviation from the equilibrium strategy sustaining good equilibria if the deterrence power is high enough. We study how self-punishment, in the form of wasted resources in equilibrium, sustains higher efficiency in games in which traditional punishment mechanisms have low deterrence power.

0.3 Overview of the Chapters

Focusing on intergenerational conflicts and the interaction between economic and political institutions, this work is divided in three chapters. Chapter 1 presents "*A Dynamic Politico-Economic Model of Intergenerational Contracts*". This model investigates the conditions for the emergence of implicit intergenerational contracts without assuming reputation mechanisms, commitment technology and altruism in a context of small open economy. We present a tractable dynamic politico-economic model in overlapping generation environment where politicians play Markovian strategies in a probabilistic voting environment, setting multidimensional political agenda. Both backward and forward intergenerational transfers, respectively in the form of pension benefits and higher education investments, are simultaneously considered in an endogenous human capital setting with labor income taxation. Building on Boldrin and Montes' idea (2005),[1] who provide normative prescriptions for optimal intergenerational redistribution, we show that efficient intergenerational transfer schemes can be supported as a politico-economic equilibrium in an intergenerational game. We discuss dynamic properties, uniqueness and optimality of the solution, providing also comparative statics in terms of both demographic and political aging. Specifically, we find that in a dynamic efficient economy both forward and backward intergenerational transfers simultaneously arise. Social security sustains investment in public education and investment in education creates a dynamic linkage across periods through both human and physical capital driving the economy toward different Welfare State Regimes. The equilibrium allocation turns out to be education efficient, but, due to political overrepresentation of elderly agents, the electoral competition process induces overtaxation compared with a Benevolent Government solution with balanced welfare

[1] Boldrin, M. and Montes, A., 2005, "The Intergenerational State Education and Pension", Review of Economic Studies, 72, 651-664.

weights.

Chapter 2, entitled "*Politicians, Redistribution and Intergenerational Conflicts*", extends the basic setup presented in Chapter 1. We account for general equilibrium effects generated by policy changes on income and potentially endogenous growth. The main objective of this work concerns the study of how political disagreement over the redistribution of public resources among different age-cohorts affects efficiency. In a dynamic politico-economic context characterized by no distortions we find that the stronger the political power to elderly, and consequently the stronger the political disagreement, the better is the dynamic efficiency. Furthermore, we introduce distortionary taxation on capital, explicitly considering political distortions, which arise in the electoral competition stage when forward looking politician internalize how political decisions affect their probability of winning elections. We characterize the time-consistent Markov perfect politico-economic equilibrium in terms of Generalized Euler Conditions (GEEs), discussing how economic and political distortions alter the main finding about the positive correlation between intergenerational political disagreement and efficiency.

Finally, Chapter 3 presents "*Self-Commitment Institutions and Cooperation in Overlapping Generation Games*". This model investigates the existence of an alternative mechanism, out of personal and community enforcement, we refer to as Self-Commitment Institution (SCI), which sustains cooperation when: i) Agents are finite-living periods overlapping generations, and ii) imperfectly observe past history. We characterize social norms with and without SCI. If agents adopt social norms with SCI they might decide to partially self-expropriate their own endowment employing it as self-commitment device. The punishment decisions are contingent on the self-expropriation choices of previous players. Under quite general conditions we find that, even if individual strategies are still characterized by behavioral uncertainty, the introduction of SCI relaxes the inclination toward opportunistic behavior and sustains higher efficiency compared to social norms without SCI. Finally, we extend the analyses to productive SCI and provide a new explanation for the role of education provision regarding the sustainability of intergenerational contracts.

Doctoral Dissertation
Department of Economics
University of Bologna

ACKNOWLEDGMENTS

I owe many thanks to my supervisor Professor Giacomo Calzolari for his support and for having given me the possibility to freely explore my own research attitude. I also thank Professor Graziella Bertocchi for useful and delightful conversations and for having strongly believed in my research project. I thank Professor Michele Boldrin for the warm hospitality at the Washington University in St. Louis and the challenging political and economic discussions, and Professor Nicola Pavoni for having pushed me to always improve. I wish to thank Professor David Levine for having thought me to discipline my mind and for leading by example. Finally a special thanks goes to Francesco Lancia, a collegue and a friend, for all the countless discussions, the disputes and the life-experiences which made me really grow up.

Chapter 1

A Dynamic Politico-Economic Model of Intergenerational Contracts

"Why should I care about future generations? What have they done for me?"
(Groucho Marx)

1.1 Introduction

The implementation and sustainability of public intergenerational redistributive programs are crucial issues in the current political debate. On one hand, the shift of the balance between different age-cohorts in favor of the elderly alters the economic nature underlying the enforcement of redistributive welfare programs. On the other hand, the increasing age of median voter makes the political choices to be more responsive to the need of the crucial group of voters - the old.[1] The demographic changes and the political power reassessments suggest that past intergenerational transfers promises, especially in the form of pensions, become more and more unaffordable over time. Furthermore, the sustainability problem of redistributive programs in favor of the elderly appears exacerbated by the dynamically efficient growth path experienced by developed countries since the Second World

[1] As reported by OECD (2007), between 1975 and 2005 the average age of median voter increased three times faster than it had in the preceding 30 years. At the some time, the dependency ratio - the ratio of workers to pensioners - has steadily deteriorated in all rich countries. It is expected to shift from 4 points in 1970 to around 2 in 2050.

War (Abel et al., 1989).[2]

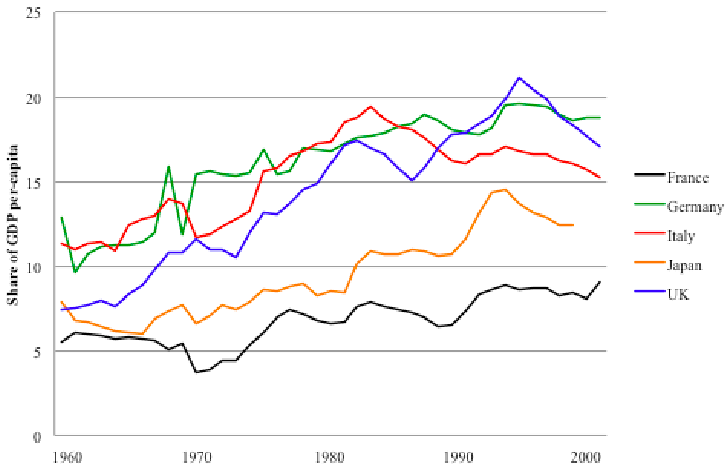

Fig. 1: Source: Comparative Welfare State (2004)

Although in the last decades political debates have often concerned reforms of current pension systems based on retributive schemes in favor of private contributive schemes; nonetheless in most developed countries the bulk of retirement income comes mostly from the state, around 60%. As shown in Figure 1, the share of GDP per-capita devoted to finance social security in OECD countries has increased over time.

One of the main implications of this trend is that the intergenerational disagreement over the allocation of public resources turns to be a battleground, pitting young against old and taxpayers against recipients, especially when balanced budget conditions are required to be met. For this reason, it becomes critical to explore the conditions under which intergenerational transfers, as an outcome of a political voting game, can be implemented and why the welfare system developed so far has became a stable institution of modern society. The main objective of this work is to provide a tractable dynamic politico-economic theory to analyze how intergenerational conflicts affect, through the political mechanism in the form of democratic vote, the size and composition of public expenditure in a context of

[2] There are many explanations in the literature on why pay-as-you-go (PAYG) social security has been introduced and then expanded (see Galasso and Profeta, 2002). The classical solution on the puzzle is that, if the economy is on a dynamically inefficient path, then the introduction of a PAYG social security system is Pareto improving since it reduces the capital deepening.

population aging.

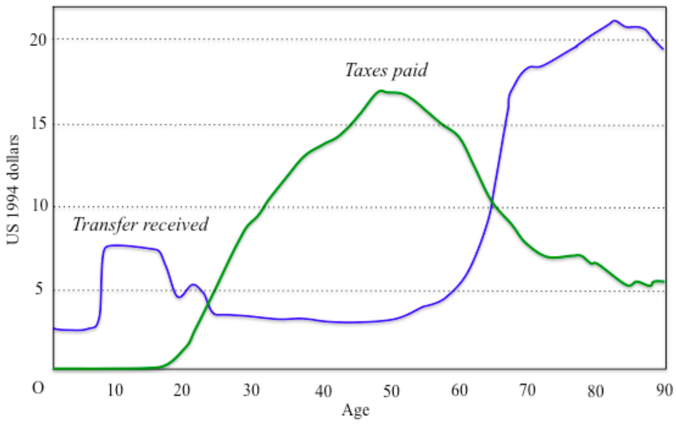

Fig. 2: Source Lee (2007)

Figure 2 reports the fiscal policy pattern for the USA in 2001. The plot suggests that the intergenerational redistribution scheme matches the self-enforcing requirements when the working-age class collects taxes and both elderly and young enjoy benefits in the form of backward and forward transfers, respectively. Building on the idea of Boldrin and Montes (2005), who provide normative prescriptions for the optimal intergenerational redistribution, we show that multidimensional efficient intergenerational transfer schemes in the form of PAYG and public education transfers can also be supported as a politico-economic equilibrium of an intergenerational game.

We consider a three period OLG economy populated by ideological heterogeneous agents who participate to repeated political elections. When young, agents acquire skills if education transfers are publicly provided without having access to private credit markets. When adult, the individuals offer inelastically labor and pay taxes, while when old, agents retire and receive unproductive transfers to sustain their consumption. The presence of a political system is justified by the need to finance the provision of the public spending under credit market constraint. The electoral competition takes place in a majoritarian probabilistic environment, where politicians compete by proposing multidimensional fiscal platforms under uncertainty (i.e. before the realization of ideological shock) subject to intra-period balanced budget. We assume away the provision of public goods - a key element in the political economy of fiscal policy - to pick out the impact of political institutions on intergenerational transfers.[3] We introduce two restrictions to the neoclassical growth environment in order to isolate the basic mechanism at work. We rule out altruism and we consider a small open economy where the wage per efficient unit and the return on saving are fixed.[4]

Technically, this paper highlights two main features concerning fiscal policies. Firstly, several political choices have to be set simultaneously, so the political space cannot be reduced to a mere unidimensional problem. Secondly, political decisions and private intertemporal choices are mutually affected over time. Forward-looking selfish agents internalize how current political choices influence the evolution of the economy and the implementation of future policies. To fully describe the endogenous feedback effects, we embody the *minor causes should have minor effect* principle to implement differentiable Stationary Markov subgame Perfect Equilibria (SMPE), where the policies are conditioned on the two payoff-relevant state variables of the economy: Physical and human capital. We characterize the policy rules as an equilibrium outcome in a finite horizon environment when time goes to infinity. As a result, we overcome the main limit of earlier literature, related to the adoption of trigger strategies equilibria, which are not robust to this refinement.

We argue that selfish adults buy insurance for their future old age by paying productive education transfers to their children in order to raise the labor productivity of the next period. When old, they partially grab the bigger output, in the form of PAYG transfers, by exerting political power in a probabilistic voting environment. Obviously, the redistributive scheme works only if the cost of providing a productive transfer is low with respect to the value of receiving a pork-barrel transfer when old. Therefore, if a PAYG pension scheme is introduced, its future beneficiaries may become supportive of higher funding in public education via taxes. In other words, the existence of a retributive social security system provides incentives to invest optimally in human capital and, as a consequence, it becomes growth-enhancing for the economic system.[5] Thus, the two age-specific redistributive programs

[3] This issue is well investigated by Tabellini (1991), Lizzeri and Persico (2001), Hassler et al. (2005) and Bassetto (2008).

[4] Focusing on a small open economy we avoid the crowding out effect, then the conflict between age-classes arise not only because of different life span but also for the difference in ownership of productive factors as well as in the source of income.

[5] PAYG pension schemes in which pensions are financed by contributions from current workers have often been criticized as being detrimental to growth. According to Feldstein

may self-sustain reaching optimality.

Despite the simple structure of the game, we reach several interesting results, consistent with the empirical correlations. We find a closed form solution for the intergenerational contract, which is robust to finite horizon restrictions. Both physical capital and human capital play a relevant role in shaping the politico-economic equilibrium and in affecting the transition dynamics. Due to politicians' opportunistic behavior, strategic persistency drives the setting of the income tax. The equilibrium predicts that the higher the physical capital, the lower the income tax rate, consistently with previous literature.[6] Furthermore, human capital plays a crucial role in two different ways. On one hand it *mitigates* the politicians' strategic behavior. Precisely, the higher the level of human capital, the flatter the equilibrium policy function and the lower responsiveness of taxation policy decisions on the level of private savings, weakening the strategic channel through which politicians can increase the probability to win elections. On the other hand, human capital *perturbs* the political choice concerning the size of government, driving the economy towards different Welfare State Regimes (WSR) and long-run convergence dynamics. Specifically, the presence of endogenous human capital formation generates two WSR. A complementarity regime, in which higher the human capital and, consequently, larger the tax base, higher the tax rate, and a substitutability regime, in which the opposite relation holds.

The sustainability conditions of the intergenerational contract strongly rely on both the dynamic efficiency condition and the ideological heterogeneity of voters. The dynamic efficiency requirement is necessary for the simultaneous existence of PAYG and public education programs. As long as the implicit net return of pensions is higher than the capital rental price,

(1974) such pension schemes have a negative effect on capital accumulation, since they discourage private saving and, unlike in the case of a funded pension system, the payments into the PAYG scheme do not contribute to national saving. Moreover, the implicit rate of return on contributions to a PAYG scheme typically falls short of the interest rate. According to this line of analysis, PAYG pension systems reduce per capita income. This standard argument is focused on physical capital accumulation and fails to take notice of the effect of PAYG pension systems have on the accumulation of human capital, particularly through public education. Primary and secondary education is now overwhelmingly publicly financed in all OECD countries, and universities also receive substantial funding from public sources.

[6] See among others Grossman and Helpman (1998), Azariadis and Galasso (2002), Forni (2005) and Bassetto (2008).

incentives in spending simultaneously on both sides of the redistribution programs arise. As soon as the interest rate falls below the economic growth rate, than no incentives for the adults to reduce current consumption in order to sustain forward transfers would emerge. As a result, in equilibrium only backward transfer would be provided. The ideological heterogeneity plays a relevant role for the sustainability of the contract. Suppose the agents are ideologically homogeneous and decisions are taken through a deterministic majoritarian voting system. If adults detain power at each time, then forward transfers cannot be sustained as SMPE of the intergenerational game. When adult if agents anticipate that when old they will be prevented to extract rent by being in power and to benefit from higher return to physical capital, they never reduce their current consumption by transferring resources forward. Whereas, they still decide to optimally self-sustain backward transfers under dynamic inefficiency. Equivalently, if the political power is assigned to elderly, then old would act myopically by expropriating adults and by transferring the collected resources backward.

To conclude the existence of PAYG social security programs supports public investment in education even in absence of altruism and general equilibrium price effects. At the same time in equilibrium the impact of education spending on the social security transfers is always positive. By supporting a higher education cost today, the adults internalize that it will generate a higher taxable income, guaranteeing a higher level of pension benefits when they will be old. Furthermore, demographic aging increases the equilibrium per-capita level of forward transfers, whereas the political aging has a positive impact on taxation.

Compared with a Benevolent Government solution with balanced welfare weights, the equilibrium allocation is education efficient, but, due to political overrepresentation of elderly agents, the electoral competition process induces overtaxation. These distortions come from the politicians' strategic behavior. In setting taxation rules, short-lived politicians take into account that future representatives will compensate the fiscal cost of current adults by paying the pensions in their old age. Thus, the expected policy response of future politicians reduces the current cost of transferring resources to the elderly and leads to overspending, unless the adult enjoy an unusually large political power. Consequently, by transferring too many resources to old, the politicians fail to provide the optimal income tax rate policy.

The paper is organized as follows. Section 1.2 provides the review of literature. In section 1.3 we present the model characterizing the economic and political environment. Section 1.4 describes the politico-economic equilibrium in the perfect forward-looking scenario and provides a complete characterization of both the transition dynamics and the long run economy. In section 1.5 we discuss the main results of the model. Section 1.6 introduces the Government problem without commitment, comparing the results with the decentralized one. Section 1.7 concludes. All proofs are contained in the Appendix.

1.2 Literature Review

This paper relies on the dynamic political-economy literature that incorporates forward-looking decision makers in a multidimensional policy space (Krusell et al., 1997). Recent works (Forni, 2005, and Gonzales-Eiras and Niepelt, 2008) present models on social security in a repeated voting environment. We depart from these studies by supporting and giving new theoretic fundamentals to the existing literature, which recognizes the link between productive and redistributive public spending. Among the existing theoretical contributions, from a purely economic point of view, Boldrin and Montes (2005) formalize public education and PAYG system as two parts of an intergenerational contract where public pension is the return on the investment into the human capital of the next generation. The authors show how an interconnected pension and public education system can replicate the allocation achieved by a complete market. Allowing issue-by-issue voting, Rangel (2003) studies the ability of non-market institutions to optimally invest in forward and backward intergenerational goods. Bellettini and Berti Ceroni (1999) incorporate politics in an economic model to analyze how societies might sustain public investments (i.e. education) even if the interests of those benefitting from the investment are not represented in the political process. Restricting voting to a binary choice of tax rate and education, the authors study whether a given system can be maintained but they do not determine the level of investments in education or social security. As main shortcoming the earlier studies have assumed that voters play trigger strategies. Although trigger strategies may be analytically convenient, they lead to multiplicity of equilibria. Furthermore, they require coordination among agents and costly enforcement of a punishment technology, which may not work when agents are not sufficiently patient. Finally, they are not robust to refinement such as backward induction in a finite horizon economy when time the tends to infinity. Departing from the existing literature, instead of emphasizing that the complementarity between education and pension payments is mainly sustained because of reputation mechanisms, our model adopts a different perspective. We focus on the resolution of the intergenerational conflicts over the determination of the amount of the two public spending components by adopting Markovian strategies. Closer in the spirit of our work is Gonzales-Eiras and Niepelt (2011). The authors study how demographic ageing affects the per-capita long run endogenous growth in a two period OLG economy. Differently from them, our theory put emphasis on the analyses of the intergenerational fiscal sustainability during the transition path, in a context in which positive spillover from general equilibrium price effects and endogenous growth are ruled out.

Many recent studies have identified the public good provision as the critical variable that allows the emergence of an intergenerational redistributive scheme. Bassetto (2008) studies how intergenerational conflicts shape government policies when taxes, transfers and public good are jointly determined. Without public good provision the government would be running a purely redistribution scheme, to which any household that is a net loser would object: Hence, the only possible equilibrium would entail no taxes and no transfers. In a simpler underlying economic environment of majority voting Hassler et al. (2007) develop an OLG model of welfare state where tax revenues are used to finance public goods and age-dependent transfers. Studying a linear quadratic economy, they provide analytical solution, but the voting strategies equilibrium turns out to be either constant or independent of fundamentals. Unlike these models we exclude public good provision.

Finally, some studies have analyzed the interaction between education and social security by adopting an altruistic motive. Kaganovich and Zilcha (1999) study a model where altruistically-motivated parents invest in the human capital of their children. When parents retire, the labor income of their children is taxed to finance their social security benefits. The link between the human capital of children and the parents' retirement benefits is disregarded in each parental educational decision, but it is captured by the government. In Ehrlich and Lui (1998) children provide support to parents in old age, so that parents have an interest in the education of their children due to pure altruistic motives. Despite the usefulness of these studies, they adopt a centralized point of view to justify the simultaneous investment in both redistributive programs. In contrast to these studies, as a device to bring out incentives to pay tax, we allow for productive and long-lasting impact transfers to the future generation of workers. The dynamic intertemporal linkage occurs by affecting the benefits of adults, which expect to grab future output by exerting political power when old.

1.3 The Model

Consider a discrete-time OLG economy populated by an infinite number of overlapping generations of ideologically heterogeneous agents, living up to three-periods: Young, Adult and Old. Every agent born at time t survives with probability one until old age. Population grows at a constant rate $n > -1$, thus the mass of a generation born at time ι and living at time t is equal to $N_t^{\iota} = N_0 (1+n)^{\iota}$.

1.3.1 Households

Agents born at time $t-1$ evaluate consumption according to the following intertemporal non altruistic utility function:

$$U_{t-1} = u\left(c_t^1\right) + \beta u\left(c_{t+1}^2\right) \tag{1.1}$$

where $u(c) = \log c$ and $\beta \in (0,1)$ is the individual discount factor. c_t^1 represents the consumption at time t when adult and c_{t+1}^2 is the consumption at time $t+1$ when old. In the first period of life (i.e. childhood), the individual does not consume.

When young, agents spend all their time endowment in acquiring skills if education transfers, e_t, are publicly provided without having access to private credit markets.[7] When adult, the individuals work and consume their labor income, w_t, net of the proportional labor income taxes, τ_t, and individual savings, s_t. When old, the individuals retire and consume their total income, equal to the sum of pension benefits that their children pass

[7] For a recent discussion on the missing credit markets to finance education, see Kehoe and Levine (2000).

to them in the form of PAYG transfers, p_{t+1}, and the capitalized savings at a fixed interest rate R. Then, the individual budget constrains for adults and old are, respectively:

$$c_t^1 = w_t (1 - \tau_t) - s_t \tag{1.2}$$

$$c_{t+1}^2 = Rs_t + p_{t+1} \tag{1.3}$$

The net present value at time t of the lifetime wealth of an agent born at time $t-1$ is given by:

$$I_t = w_t (1 - \tau_t) + \frac{p_{t+1}}{R}$$

1.3.2 Production

At each time t the homogenous private good, Y_t, is produced by a linear technology that uses labor, L_t, inelastically supplied by the adults, and physical capital, K_t. The linearity of the production function can be derived as an equilibrium outcome in a context of perfect international capital mobility and factor price equalization in the presence of goods trade. It emphasizes the intergenerational conflicts due to divergent economic interests between the two productive classes: workers (adults) and capitalists (old). Then, the production function at time t is:

$$Y_t = w_t L_t + RK_t \tag{1.4}$$

where the wage rate, $w_t = \omega (1 + h_t)$, and the gross rental price to capital, R, are determined by the marginal productivity conditions for factor prices. The wage per efficiency unit, ω, is augmented by the level of human capital acquired the period before, h_t. Without loss of generality we normalize $\omega = 1$.

The human capital of an agent born at time t in his working age is an increasing function of the government real expenditure on education and the parental education.[8] Public education transfers are supplied in an egalitarian way. The following Cobb-Douglas human capital technology,

[8] The importance and the empirical relevance of both the public spending in schooling and the parental education input in the formation of the human capital of the young people has been explored theoretically, as well as empirically. For a comprehensive survey of the related literature see Becker and Tomes (1986).

$h_{t+1} = H(h_t, e_t)$, is adopted:

$$h_{t+1} = \left(\frac{\alpha h_t + (1-\alpha)\bar{h}}{1+n} \right)^{\theta} e_t^{1-\theta} \tag{1.5}$$

where $\theta \in (0,1)$. h_t is the dynasty's human capital at time t and $\bar{h}$ is the constant society endowment of human capital.[9] If $\alpha = 1$, then no externality generated by $\bar{h}$ appears in the economy. Contrarily, if $\alpha \in [0,1)$, then externality takes the form of positive spillover from the society endowment of human capital to the formation of future skills.

Physical capital fully depreciates each period. Consequently, the level of saving determines the dynamics of per-capita physical capital accumulation. The capital market clears when:

$$(1+n)\,k_{t+1} = s_t \tag{1.6}$$

1.3.3 Fiscal Constitution

In order to provide the intergenerational transfers, the agents in the economy have to devise a politician. In each period, the politician raises revenues through labor income taxes and uses the proceeds to purchase consumption goods to pay transfers to the young and old generation. We assume that the politician is prevented from borrowing, then the public balance must hold in every period. This implies that total benefits paid to elderly and young equalize total contributions collected from working generations. The balanced budget constraint requires:

$$(1+h_t)\,\tau_t = (1+n)\,e_t + \frac{1}{1+n} p_t \tag{1.7}$$

Ceteris paribus, the more the population ages, the higher the aggregate pension benefits for elderly agents and the lower the aggregate education transfers for the youth.

The condition (1.7) reduces the multidimensionality of political platform

[9] The constant society endowment, $\bar{h}$, is the country-specific human capital endowment, which imperfectly substitutes the dynasty's human capital. It may be also interpreted as the country specific civic capital. It is introduced to appropriately perform cross-countries analyses and for analytically convenience. Indeed, by simultaneously adopting the final good technology given by Eq. (1.4) and the human capital technology given by Eq. (1.5), we rule out the effects generated by endogenous growth, at the same time enabling the economy to reach a stable steady state different from autarchy.

to a bidimensional plan $f_t = \{e_t, \tau_t\}$ where $\tau_t \in (0,1)$ and $e_t \in (0, \hat{e}_t)$ at each time t, with $\hat{e}_t$ equal to the maximum feasible level of public education provision.

1.4 Maximization and Equilibrium

As in Krusell et al. (1997), we describe the equilibrium of the economy as a dynamic politico-economic equilibrium. Within each period t, the detailed timing of moves is as follows:

i. a new generation of young people is born;

ii. electoral competition takes place to select the policy to implement;[10]

iii. agents vote;

iv. the firms hire workers and rent capital;

v. the adults receive the proceeds from labor, pay taxes and consume; old receive the capitalized saving and transfer, and consume; young receive productive transfers;

vi. the older generation dies; the young and adult generations age and become adult and old, respectively.

Due to the sequential nature of the timing of the game we solve backward. First, the agents determine the individual level of saving and firms produce the homogenous final good given the fiscal stance (Competitive Economic Equilibrium). Second, short-lived office-seeking-politicians determine both the level of taxation and the amount of backward and forward transfers in order to win elections (Politico-Economic Equilibrium).

1.4.1 Competitive Economic Equilibrium

In a competitive equilibrium, each adult chooses her lifetime consumption taking fiscal and redistributive policies as given. Maximizing Eq. (1.1) subject to the individual budget constraints (1.2) and (1.3), and feasibility

[10] In turn the electoral competition is characterized by two sub-phases: i) In the *Taxation Stage* parties compete proposing their political fiscal platforms under uncertainty, i.e. before the realization of the ideological shock among voters, and ii) in the *Electoral Stage* agents vote for their preferred candidate.

constraints $c_t^1 > 0$ and $c_{t+1}^2 > 0$, the following first order condition for interior solutions must hold:

$$0 = \eta(\cdot) \equiv u_c\left(c_t^1\right) - R\beta u_c\left(c_{t+1}^2\right) \tag{1.8}$$

In equilibrium by implicit function theorem there exists a unique saving function, $s_t = K(h_t, \tau_t, \tau_{t+1}, e_{t+1})$, which satisfies the condition (1.8). Then, using Eq. (1.6) we have:

$$(1+n)\, k_{t+1} = K(h_t, \tau_t, \tau_{t+1}, e_{t+1}) \tag{1.9}$$

After plugging Eq. (1.9) into Eq. (1.2) and (1.3) the following individual consumption levels are attained:

$$c_t^1 \equiv C_t^1(\tau_t, h_t, k_{t+1}) \tag{1.10}$$

$$c_{t+1}^2 \equiv C_{t+1}^2(e_{t+1}, \tau_{t+1}, k_{t+1}) \tag{1.11}$$

Definition 1 (Competitive Economic Equilibrium) *Given the initial conditions* $\{h_0, k_0\}$ *and the sequence of policies* $\{f_t\}_{t=0}^{\infty}$*, a competitive economic equilibrium is defined as a sequence of allocations* $\left\{c_t^1, c_{t+1}^2, h_{t+1}, k_{t+1}\right\}_{t=0}^{\infty}$ *such that the allocation solves the maximization problem of adults, i.e. it satisfies Eq.* (*1.9*), (*1.10*) *and* (*1.11*)*, the market for production inputs clears in period* t*, i.e.* (*1.5*) *and* (*1.6*) *hold at each time.*

At time t the indirect intertemporal utility of an adult born a time $t-1$, $\mathcal{W}_t^1$, is equal to:

$$\mathcal{W}_t^1 \equiv \max_{s_t}\{U_{t-1} \mid I_t\} = u\left(C_t^1\right) + \beta u\left(C_{t+1}^2\right) \tag{1.12}$$

For an old individual born a time $t-2$ the indirect utility, $\mathcal{W}_t^2$, at time t is:

$$\mathcal{W}_t^2 \equiv u\left(C_t^2\right) \tag{1.13}$$

We call *autarky* indirect utility, $\mathcal{W}_t^a$, the lifetime utility of an adult born at time $t-1$, when taxation and public spending are precluded:

$$\mathcal{W}^a \equiv \max_{s_t}\{U_{t-1} \mid I_t = 1\}$$

Suppose there is no government that has the authority to levy taxes. As a consequence, adults keep the entirety of their labor income to purchase final good and to save. Capital earns a gross return of R, used by old to buy consumption goods. Clearly, the economy converges to the unique steady state in one period, where $h^a = 0, k^a = \frac{\beta}{(1+\beta)(1+n)}, c^{a1} = \frac{1}{1+\beta}, c^{a2} = \frac{\beta}{1+\beta}R$ and $w^a = 1$.

Definition 2 (Equilibrium Feasible Allocation) *An equilibrium feasible allocation is a sequence of competitive economic equilibrium allocations* $\left\{c_t^1, c_{t+1}^2, h_{t+1}, k_{t+1}\right\}_{t=0}^{\infty}$ *and policies* $\{f_t\}_{t=0}^{\infty}$ *that satisfy, the balanced budget constraint, Eq. (1.7), and the fiscal feasibility conditions,* $\tau_t \in (0,1)$ *and* $e_t \in (0, \hat{e}_t)$ *at each time* t.

1.4.2 Political System

In this section we describe how people interact in order to choose a particular policy. Public policies are chosen through a repeated voting system according to majority rule. Young have no political power.[11] To describe the behavior of politicians we consider a probabilistic voting setting.[12] Suppose there are two parties, $l \in \{\mathcal{A}, \mathcal{B}\}$, that compete to attain political power, with no ability to extract individual rent from election. As a consequence, their objective is the maximization of the probability of winning elections at each time in order to implement the proposed policy, f_t^l, with no ability

[11] We reflect the fact that young people show a much lower turnout rate at elections in comparison to adults and old. As Galasso and Profeta (2004), report in some countries the elderly have a higher rate at elections than the youth. In the USA turnout rates among those aged 60-69 years is twice as high as among the young (19-29 years). Again in France it is almost 50% higher.

[12] Due to the multidimensionality of the political platform Condorcet winner generally fails to exist. Consequently the median voter theorem does not hold (Plot, 1967). In the literature there are three main influential approaches to making predictions when the policy space is multi-dimensional. The first is the implementation of structure-induced equilibria. By following Shepsle (1979), agents vote simultaneously, yet separately (i.e. issue by issue), on the issues at stake. Votes are then aggregated over each issue by the median voter. See Condez-Ruiz and Galsso (2005) for a detailed discussion of this approach. The second is the legislative bargaining approach, which stems from the seminal work of Baron and Ferejohn (1989) and is further developed by Battaglini and Coate (2006). This approach applies when legislators' first loyalty is to their constituents and when legislative coalitions are fluid across time and issues. The latter concerns the adoption of probabilistic voting rule. While this model of voting dates back to the 1970s, its resurgence in popularity stemed from Lindbeck and Weibull (1987). It applies to political environments where party discipline is strong and the winning political party simply implements its platform. See Persson and Tabellini (2000) for a survey of this framework.

to commit to future one. The electorate is heterogeneous in its own political ideology. Then, to each individual j belonging to the cohort $i \in \{1,2\}$ is assigned the ideological values σ_j^i. We assume σ_j^i is a random variable extracted from a distribution function $\mathcal{G}_i$ and represents the ideological bias towards party $\mathcal{B}$. The timing of the electoral competition game played at the beginning of each period is then characterized by the following three steps:

i. each party proposes a political platform, f_t^l, in order to maximize its probability of winning the election;

ii. the ideological bias is realized among voters;

iii. fully rational and forward-looking voters take their voting choice.

With abuse of notation, let us denote with ${}_l\mathcal{W}_{jt}^i$ the indirect utility attained by agent j of cohort i at time t who votes for party l. At each time, first parties propose their political platforms, second any individual votes for party $\mathcal{A}$ if and only if the following inequality holds:

$$ {}_{\mathcal{A}}\mathcal{W}_{jt}^i >_{\mathcal{B}} \mathcal{W}_{jt}^i + \sigma_j^i $$

Given the equilibrium policy choice of party $\mathcal{B}$, $f_t^{\mathcal{B}}$, the ex-ante political maximization problem for party $\mathcal{A}$ turns out to be equivalent to:

$$ \max_{f_t^{\mathcal{A}}} (1+n)\,\mathcal{G}_1\left({}_{\mathcal{A}}\mathcal{W}_{jt}^1 -_{\mathcal{B}} \mathcal{W}_{jt}^1\right) + \mathcal{G}_2\left({}_{\mathcal{A}}\mathcal{W}_{jt}^2 -_{\mathcal{B}} \mathcal{W}_{jt}^2\right) \tag{1.14} $$

By symmetry party $\mathcal{B}$ solves an equivalent problem. In the Nash equilibrium of the electoral competition game both candidates propose the some policy platform, implementing the utilitarian optimum with respect to current living voters.

1.4.3 Politico-Economic Equilibrium

In the previous section we have described how the political process takes place in each period. To characterize the politico-economic equilibrium, we need to consider the dynamic aspects of the political process that take place in the economy. As in Krusell et al. (1997), we restrict the notion of politico-economic equilibrium to the differentiable political SMPE concept as equilibrium refinement of subgame perfect equilibria.[13] The payoff-relevant state variables of our economy are the assets held by adults and old, i.e. human and physical capital.

At each time the implementation of a particular political platform generates dynamic linkages of policies across periods. Due to the non-negligible impact of current political actions on future equilibria, rational agents internalize this dynamic feedback. In our framework dynamic linkages generated by physical and human capital arises both directly, affecting asset accumulation decision (direct dynamic feedbacks), and indirectly, affecting future political choices (indirect dynamic feedbacks). Focusing on Markov strategies, agents are able to fully internalize the overall direct and indirect impact of taxation and redistribution through the evolution of assets.

Let us denote with $\tau_t = \mathcal{T}(h_t, k_t)$ and $e_t = \mathcal{E}(h_t, k_t)$ the taxation and education equilibrium policies rules, respectively, and with $\mathcal{F} = \{\mathcal{T}, \mathcal{E}\}$ their collection. In a perfect forward-looking environment, in which parties play Markov strategies, the following Definition of equilibrium is adopted:

Definition 3 (Politico-Economic Equilibrium) *A perfect foresight politico - economic SMPE is defined as the sequence of competitive economic equilibrium feasible allocations* $\left\{c_t^1, c_{t+1}^2, h_{t+1}, k_{t+1}\right\}_{t=0}^{\infty}$ *and policies* $\{f_t\}_{t=0}^{\infty}$*, such that the functional vector of differentiable policy decision rules,* $\mathcal{F} = \{\mathcal{T}, \mathcal{E}\}$*, where* $\mathcal{T} : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow [0, 1]$ *and* $\mathcal{E} : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow [0, \hat{e}_t]$*, satisfies the following points:*

i. Each party solves the maximization program described in (*1.14*)*;*

ii. The fixed point condition holds, i.e. the policies are fixed points of the mappings $\mathcal{E}$ *into* $\mathcal{E}^{ex}(h_t, k_t)$ *and* $\mathcal{T}$ *into* $\mathcal{T}^{ex}(h_t, k_t)$*, where the apex ex stands for expected.*

The first equilibrium condition requires the political control variables, f_t, to be chosen in order to maximize the party's objective function, taking

[13] The Markov-perfect concept implies that outcomes are history-dependent only in the fundamental state variables. The stationary part is introduced to focus only on the current value of the payoff relevant state variable. Consequently the vector of equilibrium policy decision rules is not indexed by time, i.e. the structural relation among payoff-relevant state variables and political controls is not time variant. The differentiable part is a convenient requirement to avoid multiplicity of equilibrium outcomes and in order to give clear positive political predictions.

account that future redistribution and taxation depend on the current policy choices via both the equilibrium private decisions and future equilibrium policy rules.

The second condition requires that, if the equilibrium exists, it must satisfy the fixed point requirement. From a technical point of view, we are looking for two differentiable policies which obey the recursive rules given by the functions $\tau_t = \mathcal{T}(h_t, k_t)$ and $e_t = \mathcal{E}(h_t, k_t)$, where $\mathcal{T}$ and $\mathcal{E}$ are infinite dimensional objects and the key endogenous variables of the problem. The second fundamental element we are looking for is a function, $\mathcal{K}$, which describes the private sector response to a one-shot deviation of the government, when agents expect future policies to be set by politicians according to $\mathcal{F}$ as function of current state and political control variables. From Eq. (1.9) the function K fully describes the equilibrium behavior of private agents as a function of current state and both current and future policies. If differentiable functions, $\mathcal{T}$ and $\mathcal{E}$, which describe the policy behavior followed by politicians in equilibrium, exist, these rules can be internalized by fully rational private agents. It follows that:

$$k_{t+1} = K\left(h_t, \tau_t, \mathcal{E}\left(h_{t+1}, k_{t+1}\right), \mathcal{T}\left(h_{t+1}, k_{t+1}\right)\right) \tag{1.15}$$

Plugging Eq. (1.5) into Eq. (1.15) and rearranging the terms we get:

$$k_{t+1} = \mathcal{K}\left(h_t, \tau_t, e_t\right)$$

Due to the full depreciation of physical capital and the absence of distortionary taxation on capital, $\mathcal{K}$ is not a function of current level of physical capital, which strongly simplifies the analyses.

Before recursively solving the equilibrium policy rules $\mathcal{E}$ and $\mathcal{T}$, we investigate the marginal impact of e_t and τ_t on the parties' objective function. Maximizing Eq. (1.14) with respect to both policies and applying the envelope theorem, we obtain the following system of first order conditions for each $l \in \{\mathcal{A}, \mathcal{B}\}$:

$$(1+n)\,\frac{d_l \mathcal{W}_t^1}{d\tau_t^l} + \frac{g_2}{g_1}\frac{d_l \mathcal{W}_t^2}{d\tau_t^l} = 0$$
$$(1+n)\,\frac{d_l \mathcal{W}_t^1}{de_t^l} + \frac{g_2}{g_1}\frac{d_l \mathcal{W}_t^2}{de_t^l} = 0$$

where g_i is the density function of $\mathcal{G}_i$. Let us denote with $\phi \equiv \frac{g_2}{g_1} \in [0, \infty)$ a synthetic measure of the ideological bias among voters, which also represents the relative political weight of the voters belonging to old cohort.[14] If $0 < \phi < 1$ then on average the old cohort cares less about ideology and has more swing-voters than the adults one. For $\phi > 1$ the opposite holds, where the elderly represent the majority in the political debate. Finally, when $\phi = 1$, all voters are equally represented. Using the indirect utility functions, Eq. (1.12) and (1.13) , the following first order conditions are attained for τ_t and e_t, respectively:[15]

$$0 = \underbrace{\phi(1+h_t)u_{C_t^2}}_{\text{old's direct benefit}} - \underbrace{(1+h_t)u_{C_t^1}}_{\text{adults' direct cost}} \underbrace{+\beta(1+n)u_{C_{t+1}^2}\left((1+h_{t+1})\frac{d\tau_{t+1}}{d\tau_t} - (1+n)\frac{de_{t+1}}{d\tau_t}\right)}_{\text{adults' exp. cost/benefit}} \tag{1.16}$$

$$0 = \underbrace{-\phi u_{C_t^2}}_{\text{old's direct cost}} \underbrace{+\beta u_{C_{t+1}^2}\left(\frac{dh_{t+1}}{de_t}\tau_{t+1} + (1+h_{t+1})\frac{d\tau_{t+1}}{de_t} - (1+n)\frac{de_{t+1}}{de_t}\right)}_{\text{adults' expected direct/indirect net benefit}} \tag{1.17}$$

Let us first refer to Eq. (1.16). At each time an interior solution for the income tax rate is simply determined as the outcome of a weighted bargaining between current elderly and adults, who reap benefits and sustain costs by a variation in tax level. The first term represents the elderly's marginal benefits in terms of PAYG social security due to the increase in income tax rate. Since tax levying on labor income makes adults sustain the whole tax burden, the second term captures the adults' marginal cost caused by a positive variation on the fiscal dimension. Finally, the third term measures the expected marginal impact of current variation in the tax rate on the utility of next-period old. Similarly, redistributive choices are made as the outcome of a weighted bargaining between current and future elderly. An increase in public education transfers is a double-edged sword. On one hand, it makes current old sustain direct costs due to reduction in social security contributions, represented by the first part of Eq. (1.17) . On the other hand, future old enjoy direct benefits from expected returns of productive investment in human capital, whose effects are captured by the

---

[14] ϕ is a measure of how strongly the old generation pursues her own interest.

[15] Since in equilibrium the parties $\mathcal{A}$ and $\mathcal{B}$ face the same maximization problem and choose an identical political platform, we remove the apex l.

second part.

FOCs (1.16) and (1.17) internalize the strategic effects, capturing how politicians can affect future policies through their current choices of f_t. If $\frac{d\tau_{t+1}}{d\tau_t} > 0$ (< 0) and $\frac{d\tau_{t+1}}{de_t} > 0$ (< 0) agents know that a higher income tax rate and larger education transfers lead to a higher (lower) tax rate in the future. Thus, representatives may strategically increase (reduce) τ_t and e_t in order to affect the tax rate outcome of tomorrow. The same idea holds for e_{t+1}.

1.4.4 Political SMPE with Perfect Foresight

Due to the non-linearity and bidimensionality in the political space, the system of partial differential equations (1.16) and (1.17) cannot be easily solved using integration methods.[16] As reported in Klein et al. (2008), the equilibrium is obtained as the limit of a finite-horizon equilibrium, whose characteristics do not significantly depend on the time horizon, as long as the time horizon is long enough. Consequently, our resolution strategy consists of a constructive approach (induction method). We compute the FOCs defining the feasible equilibrium policy rules in a finite-horizon environment via backward induction. We start at a final round $t < \infty$ and we re-compute the equilibrium policy rules, $\mathcal{F}_t = \{\mathcal{T}_t, \mathcal{E}_t\}$, as long as all the direct dynamic feedbacks, induced by political choices on private one, have been internalized. In particular, due to two-periods lag impact of e_t on private saving choices, we perform recursive maximization until period $t-2$. At each time the political objective function, described in Eq. (1.14), has to be simultaneously maximized with respect to its arguments, i.e. the pair $\{\tau_t, e_t\}$, subject to the balanced budget constraint, Eq. (1.7), the Euler condition of the economic optimization problem, Eq. (1.9), and the equilibrium policy rules of the following periods, computed via backward procedure. Once a recursive structure is identifiable, making the time horizon goes to infinity for all the time-variant coefficients determined so far, we obtain the equilibrium policy rules as fixed point of the recursive problem in multidimensional environment.

For notational purposes let us denote with Ω_i the relative political bar-

[16] See Grossman-Helpman (1996) and Azariadis-Galasso (2002) frameworks where by applying the envelope theorem the differential equation becomes linear and solution is easily determined.

gaining power for $i \in \{1,2\}$, which are defined as follows:

$$\Omega_1 \equiv \frac{(1+n)(1+\beta)}{\phi + (1+n)(1+\beta)} \quad \text{and} \quad \Omega_2 \equiv \frac{\phi}{\phi + (1+n)(1+\beta)} \tag{1.18}$$

Remark 1 *The more the population ages* ($i.e.$ n *decreases and* ϕ *increases*), *the smaller the relative political weight of adults,* Ω_1, *and larger the relative political weight of old,* Ω_2.

Fixing $\theta = \frac{1}{2}$, we analytically determine a fundamental equilibrium capturing the effects that are inherent in the dynamic game itself, which turns to be unique. Let Υ^{Pol} be the state-space where interior policy rules are simultaneously obtained and $\bar{R} \equiv \frac{1}{\mathcal{R}-1}$ where $\mathcal{R} = \frac{1+n}{R}$ measures the economy dynamic efficiency ratio. The following Theorem characterizes the equilibrium outcomes of public choices in a fully rational environment when Markov strategies are implemented.

Theorem 1 *Let* $\psi^* = \frac{1}{\alpha}\left(\frac{2R}{\alpha}\left(R - \sqrt{R^2 - \alpha}\right) - 1\right)$. *Under dynamic efficiency condition, for any level of* $\{h_t, k_t\} \in \Upsilon^{Pol}$ *the feasible rational policies,* $f_t = \{\tau_t, e_t\}$, *which can be supported by a perfect foresight political SMPE, have the following functional forms:*

i.

$$\mathcal{E}(h_t) = a_1 h_t + a_0 \tag{1.19}$$

where $a_1 \equiv \frac{\alpha}{1+n}\psi^*$ *and* $a_0 \equiv \frac{1-\alpha}{1+n}\bar{h}\psi^*$;

ii.

$$\mathcal{T}(k_t, h_t) = -b_3 \frac{k_t}{1+h_t} + b_2 \frac{h_t}{1+h_t} + b_1 \frac{1}{1+h_t} + b_0 \tag{1.20}$$

where $b_3 \equiv R\Omega_1$, $b_2 \equiv \alpha\psi^*(\Omega_1 + 2\Omega_2)$, $b_1 \equiv \bar{h}\frac{1-\alpha}{\alpha}b_2 + \bar{R}\left(1 + \bar{h}(1-\alpha)\psi^*\right)\Omega_2$ *and* $b_0 \equiv \Omega_2$.

Otherwise, for any $(h_t, k_t) \notin \Upsilon^{Pol}$ *corner solutions result in at least one of the two dimensions.*

Proof. *(See appendix).* ∎

From a structural point of view, while the policy rule associated with education transfers is linear in the human capital production, the fiscal policy rule is a linear function in the physical capital but not in the human capital level.

1.4.5 Dynamics and Steady States

We now discuss the transition dynamics of the economy during the adjustment towards the steady state.

Definition 4 (Law of Motion) *The laws of motion of the collection* $\{e_t, \tau_t, h_t, k_t\}_{t=0}^{\infty}$ *are definite as the mappings:*

$$\begin{aligned} h_{t+1} &= H\left(\mathcal{E}\left(h_t\right), h_t\right), & k_{t+1} &= \mathcal{K}\left(\mathcal{E}\left(h_t\right), \mathcal{T}\left(h_t, k_t\right), h_t\right), \\ e_{t+1} &= \mathcal{E}\left(H\left(\mathcal{E}\left(h_t\right), h_t\right)\right), & \tau_{t+1} &= \mathcal{T}\left(\mathcal{K}\left(\mathcal{E}\left(h_t\right), \tau_t, h_t\right), H\left(\mathcal{E}\left(h_t\right), h_t\right)\right) \end{aligned}$$

The economy dynamics is basically driven by the human capital evolution which affects both the education transfers' law of motion and the transition dynamics of taxation policy. While the former is directly influenced by human capital, the latter is affected by human capital both directly and indirectly through physical capital. This implies that convergence conditions in the state-space are also sufficient for the stable convergence of the policy rules evolution. The following Lemma states the conditions for economy's convergence stability.

Lemma 1 *Let* $\underline{\phi} \equiv \beta\left(R-(1+n)\right)$ *and* $\underline{n} \equiv \sqrt{2R\left(R-\sqrt{R^2-\alpha}\right)-\alpha}-1$. *Given any feasible initial condition* $(h_0, k_0) \in \Upsilon^{Pol}$, *if* $\phi > \underline{\phi}$ *and* $n > \underline{n}$, *then the sequence* $\{e_t, \tau_t, h_t, k_t\}_{t=0}^{\infty}$ *is characterized by stable monotonic convergence. The speed of convergence for* τ_t *crucially depends on the initial condition and the human capital society endowment.*

Proof. *(See appendix).* ■

Given the differentiability of the policy functions, the interior solution conditions and Lemma 1, the following Proposition holds:

Proposition 1 *The feasible steady state* (e^*, τ^*, h^*, k^*) *exists and is unique.*

Proof. *(See appendix).* ■

Thus, depending on the initial condition, $\{h_0, k_0\}$, and the level of the human capital society endowment, $\bar{h}$, the control and the state variables converge monotonically to the unique feasible steady state.[17]

[17] According to the particular Welfare State regime (see Section 1.5.1) different speeds of convergence and amounts of intergenerational transfers characterize the economy.

1.5 Discussion

In this section we discuss the necessary conditions for the sustainability of the implicit intergenerational contracts. Furthermore, we fully characterize the equilibrium policy rules described in Eq. (1.19) and (1.20).

The dynamic efficiency requirement, i.e. $R > 1+n$, is a necessary condition for the simultaneous existence of PAYG and public education programs. In our economy, during the transition path, the *implicit net return* to pensions is determined by both the population growth rate and the marginal increase in the taxable income due to human capital investment net of the current resources devoted to education. As long as the implicit net return is higher than the capital rental price, incentives in spending simultaneously on both sides of the redistribution programs emerge. Contrarily, suppose that the population growth rate exceeds the net rental price to physical capital, then it is straightforward to prove that b_1 tends to infinity and, consequently, the human capital negatively affect the size of government.[18]

Remark 2 *If $R < 1+n$, then forward and backward transfers cannot be simultaneously sustained as SMPE of the game.*

According to Eq. (1.5) and (1.20), an increase in education spending would determine a positive variation in the stock of human capital and, in turn, a decrease in tax rate, dampening the strategic channel, which sustains intergenerational cooperation over the pension transfers dimension. The consequent increase of saving and physical capital induce further reduction in the future tax level. As a result, this cannot be an equilibrium. Given the condition $R < 1+n$, agents have always an incentive to deviate by choosing higher levels of income tax in order to depress private saving and guarantee higher future levels in pension contributions, even without investment in education.

The ideological heterogeneity among agents plays also a relevant role for the sustainability of the contract.

Corollary 1 *If forward transfers are sustained as SMPE of the game, then $0 < \phi < \infty$.*

Proof. *(See appendix).* ■

[18] See Proof of Theorem 1 in Appendix A for the derivation of b_1.

Suppose $\phi = 0$, then by solving backward it is easy to check that forward transfers cannot be sustained as SMPE of the intergenerational game.[19] In a small open economy, if adults anticipate that when old they will be prevented to grab rent by exerting the political power, they never reduce their current consumption by transferring resources forward. Whereas, they still decide to optimally self-sustain backward transfers under dynamic inefficiency. Equivalently, suppose $\phi \to \infty$, then all the political power is assigned to elderly. As a result, at each time old would act myopically by expropriating adults and by transferring all the collected resources backward.

As depicted in Figure 3, for any non-zero level of income tax, the greater the human capital, the more political support the education program receives, i.e. $\frac{de_t}{dh_t} = a_1 > 0$. Two different configurations may arise depending on the level of society endowment of human capital. As shown in Panel (a), as long as $\bar{h} < \frac{1}{(1-\alpha)\psi^*}$, then $\mathcal{E}(h_t)$ lies within the feasibility boundaries, $(0, \hat{e}_t)$, for any level of human capital. Instead, as reported in Panel (b), for $\bar{h} \geqslant \frac{1}{(1-\alpha)\psi^*}$, if h_t is lower than a certain threshold, then boundary solution is attained, i.e. $\mathcal{E}(h_t) = \hat{e}_t$.[20]

Remark 3 *The strategic political components embedded in the parameter ϕ does not affect $\mathcal{E}(h_t)$.*

As reported in Eq. (1.19), in equilibrium the amount of education transfers is equal to the highest feasible value, in order to maximize the net implicit rate of future pensions. $\mathcal{E}(h_t)$ maximizes the intertemporal utility of current adults without considering the political distortions due to old's bargaining power. This result sounds counterintuitive, because, as shown in Eq. (1.17), old actually have incentives to reduce the education amount at the minimal level. This, in turn, would remove the adults' incentives

[19] It would be equivalent to model a median voter or adults' dictatorship game, where adults detain power at each time.

[20] The scenario characterized by transferring all tax revenue to public education investments (i.e. no current pension benefits) is an equilibrium outcome as long as one-period future pension transfers are allocated to current adults. When $\bar{h} \geq \frac{1}{(1-\alpha)\psi^*}$ and $h_t < \tilde{h} \equiv \frac{1-\bar{h}(1-\alpha)\psi^*}{\alpha\psi^*-1}$, there exists an initial condition $\tilde{h}_0$ such that for any $h_0 > \tilde{h}_0$ future human capital level exceeds the threshold level $\tilde{h}$, i.e. $h_{t+1} \geq \tilde{h}$. In this case, adults have an incentive in taxing their income because of positive expectation in terms of social security. Thus, a one-period-equilibrium characterized by an intergenerational contract with current backward transfers equal to zero emerges. Otherwise, if $h_0 < \tilde{h}_0$, then no future pensions are provided for current adults and no incentive to implement an intergenerational contracts may arise.

in substituting private saving with public one. As final result the autarky would be established. It cannot be an equilibrium for the setting of an intergenerational contract and, as a consequence, the emergence of a public education program undistorted by the political bias is justified.

Figure 4 reports the equilibrium fiscal policy rule described in Eq. (1.20). For illustrative purpose, let us to disentangle the effects of the two asset variables on $\mathcal{T}(h_t, k_t)$. Panel (a) describes the structural relation between the equilibrium tax rate and the level of human capital. The intercept, $\mathcal{T}(k_t, 0)$, is a decreasing function in physical capital. As long as $k_t < \tilde{k}$ where $\tilde{k} \equiv \frac{b_1 - b_2}{b_3}$, the larger the human capital, the higher the opportunity cost to tax levy, i.e. $\frac{d\tau_t}{dh_t} < 0$. Otherwise if $k_t \geqslant \tilde{k}$, incentives to increase simultaneously the taxable income and the income tax rate arise, i.e. $\frac{d\tau_t}{dh_t} \geqslant 0$.

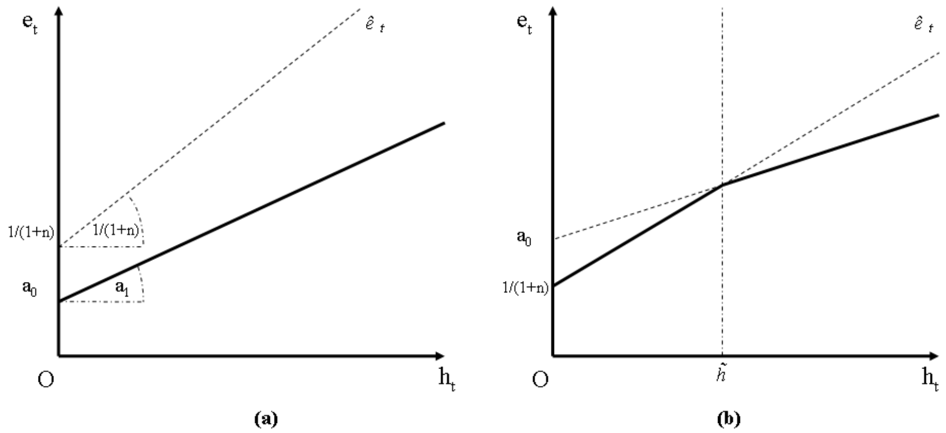

Fig. 3: Education Transfers Policy Rule

Panel (b) illustrates the structural relation between the equilibrium tax rate and the level of physical capital. The equilibrium predicts for any value of k_t the higher the physical capital, the lower the income tax rate, consistently with previous literature.[21] The intuition for the fiscal policy function to be non-increasing in the capital stock is the following. By contradiction, if $\mathcal{T}(h_t, k_t)$ were increasing in k_t, current adults would have incentives to save in order to provide the next generation with a higher level of capital and, therefore, receive a higher pension. This cannot be an equilibrium, since the

[21] See among others Grossman and Helpman (1998), Azriadis and Galasso (2002), Forni (2005) and Bassetto (2008).

higher amount of backward transfer reduces the level of saving that workers are able to make.

Remark 4 *Both the strategic political components embedded in the parameter ϕ and the demographic component, n, matter for the determination of $\mathcal{T}(h_t, k_t)$.*

Due to politicians' opportunistic behavior, strategic persistency determines the income tax. In our environment human capital plays a crucial role in two different ways. On one hand, it mitigates the politicians' strategic behavior. Precisely, the higher the level of human capital, the flatter the equilibrium policy function and the lower the elasticity of $\mathcal{T}(h_t, k_t)$ with respect to physical capital. The lower responsiveness of taxation policy decisions on the level of private savings weakens the strategic channel through which politicians can increase the probability to win elections. On the other hand, human capital perturbs the political choice concerning the size of government. Depending on the political bargaining intensity between adults and old embedded in the coefficients b_1 and b_2 of Eq. (1.20), the marginal impact of human capital on taxation decisions can be either positive or negative.

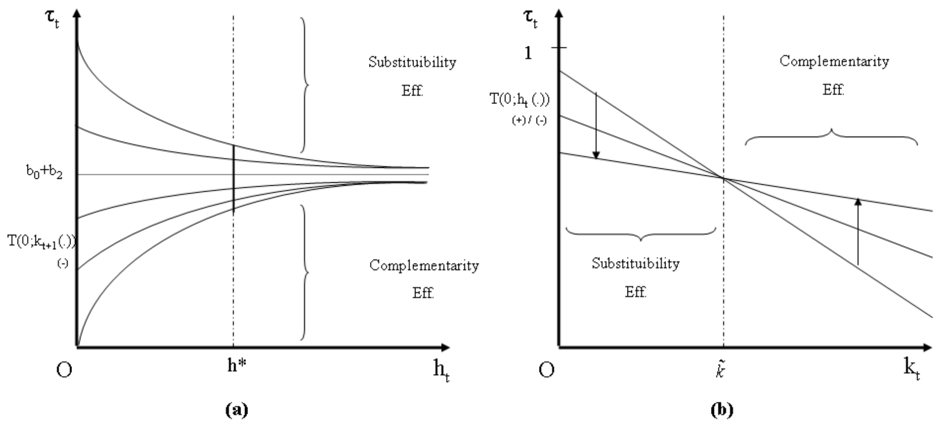

Fig. 4: Income Tax Policy Rule

Figure 5 reports a complete description of the recursive Markov structure.

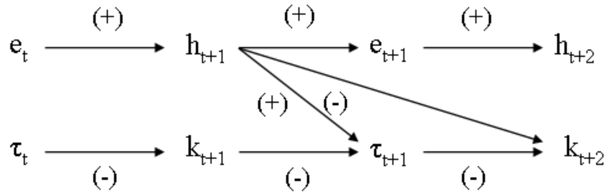

Fig. 5: Markovian Structure

The picture points out the strategic relations, which provide the necessary incentives to selfish agents to sustain simultaneously backward redistributive policies and forward ones.

1.5.1 Welfare State Regimes

Figure 5 points out the strategic structural relation between income tax rate and human capital in the Markovian environment, which drives the economy towards different WSR. If purely political factors matter in splitting the public spending, then a Politico WSR will emerge. If the economic factors are also relevant, then a Politico-Economic WSR will arise. The following Corollary fully characterizes the conditions for the identification of the different regime configurations.[22]

Corollary 2 *Given the stationary equilibrium policy rules $\mathcal{T}(h_t, k_t)$ and $\mathcal{E}(h_t)$:*

i. if $b_1 \leqslant b_2$, then the Politico Complementarity WSR, PCR, arises, i.e. $\frac{d\tau_t}{dh_t} \geqslant 0$;

ii. if $b_1 > b_2$ and $k_t \geqslant \tilde{k}$, then the Politico-Economic Complementarity WSR, PECR, arises, i.e. $\frac{d\tau_t}{dh_t} \geqslant 0$;

iii if $b_1 > b_2$ and $k_t < \tilde{k}$, then the Politico-Economic Substitutability WSR, PESR, arises, i.e. $\frac{d\tau_t}{dh_t} < 0$.

Proof. *(See appendix).* ■

[22] Formally, let us define $\bar{\Omega}_2 \equiv \frac{\left(\alpha-(1-\alpha)\bar{h}\right)\psi^*}{\left(2(1-\alpha)\bar{h}+\alpha\right)\psi^*+\bar{R}}$. An economy where $\Omega_2 \leq \bar{\Omega}_2$ experiences a political competition characterized by a weak old bargaining power and $b_1 \leqslant b_2$. Contrarily, if $\Omega_2 > \bar{\Omega}_2$, then old exert a strong bargaining power and $b_1 > b_2$.

While the economic factors driving the system into different WSR are endogenously determined by the capital asset accumulation through the saving choices, the political factors depend on the relative bargaining power of voters. An economy characterized by a weak level of old bargaining power in the political process, i.e. $b_1 \leqslant b_2$, will experience a PCR, for any level of k_t. Contrarily, an economy with a strong level of old's bargaining power in the political arena, i.e. $b_1 > b_2$, will experience a PECR if the system is high-capitalized, i.e. $k_t > \tilde{k}$, otherwise a PESR will emerge if the economy is low-capitalized, i.e. $k_t < \tilde{k}$.

Intuitively, as already pointed out, in equilibrium a higher level of current income tax rate will determine a decrease of future physical capital stock and, consequently, an increase of future tax rate. In the PCR, adults anticipate that, if they invest in education today, an increase in future human capital will determine a further positive variation in the level of income tax rate tomorrow. Given the increase in both the future tax rate and taxable income, i.e. gross future pension benefits, which maximize adult intertemporal utility, PCR emerges as the only sustainable WSR when adult bargaining power prevails.

To fully characterized the public spending process, we move the analyses to the equilibrium characterization for pension benefits.

Corollary 3 *Under decreasing return in education, in equilibrium the impact of education spending on the social security transfers is always positive, i.e.* $\frac{dp_{t+1}}{de_t} > 0$.

Proof. *(See appendix).* ■

Remark 5 *The existence of a PAYG social security program supports public investment in higher education even in absence of altruism.*

Independently from the WSR characterizing the economy, an increase in public education transfers induces a higher pension benefits in the future, creating the incentive for adults in supporting the education program. *Ceteris paribus*, by supporting a higher education cost today, the adults internalize that it will generate a higher taxable income of tomorrow, guaranteeing a higher level of the pension benefits when they will be old, for any level of $\mathcal{T}$.

The interaction between political and economic institutions determine the amount and the dynamic evolution of pension system.

Corollary 4 *At each time t, for any given level of human capital, in PESR the pension benefits are lower than PCR and larger than PECR, i.e.* $p_t^{PECR} < p_t^{PESR} < p_t^{PCR}$.

Proof. *(See appendix).* ■

When the adult's bargaining power is sufficiently strong, i.e. $b_1 \leqslant b_2$, and PCR arises, the equilibrium pension benefits reach the highest feasible level. Otherwise, when old prevail in the political debate, depending on the physical capital stock, the pension benefits are lower in a high-capitalized economy than in a low-capitalized one.

To resume graphically, in Figure 6 we plot on the state-space $\{h_t, k_t\}$ the WSR configurations which arises when $\bar{h} > \frac{1}{(1-\alpha)\psi^*}$ and $h_0 > \tilde{h}_0$.[23] Panel (a) shows the case in which a weak level of adult bargaining power characterizes the political scenario. Contrarily, Panel (b) allows for a strong bargaining power of the adults.

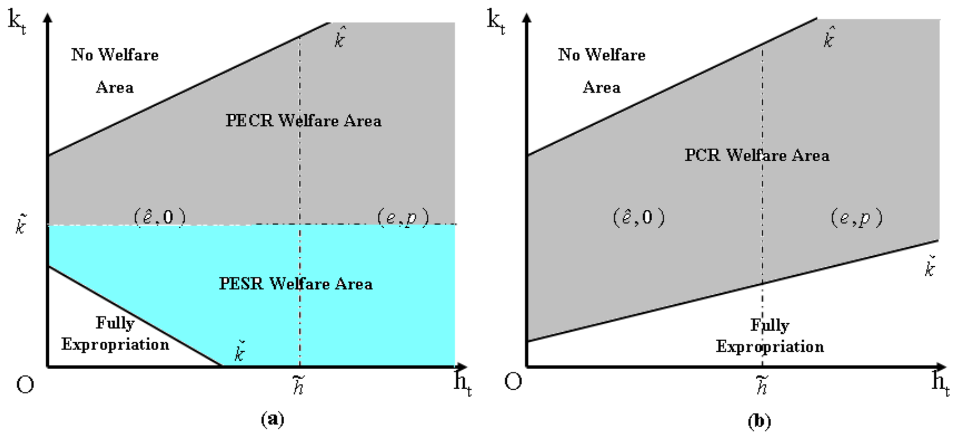

Fig. 6: Panel (a) shows the case for $b_2<b_1$, Panel (b) shows the case for $b_2>b_1$

As long as $k_t < \check{k}_t$ in both cases full expropriation occurs. The tax rate, equal to 100% of labor income, is assigned either to finance only public education program if $h_t < \tilde{h}$ or to support both redistributive social programs if $h_t \geqslant \tilde{h}$. Whereas, as long as $k_t \geqslant \hat{k}_t$ autarky economy arises. Panel (a) reports the politico-economic parameters' configurations, which makes PECR and PESR arise, i.e. $b_2 + b_0 < 1$ and $b_1 + b_0 > 1$. Whereas Panel (b) shows the emergence of PCR due the pure political factors, i.e. $b_2 > b_1$.

[23] If $\bar{h} \leq \frac{1}{(1-\alpha)\psi^*}$, then the human capital does not play any role in splitting the public spending between education and retirement transfers. In other terms, it avoids the interesting case with pension benefits set to zero.

1.5.2 Aging

Quantitatively, one of the most severe challenges concerning the intergenerational transfer system in developed economies regards the impact of population aging both in demographic (n) and political (ϕ) terms. Demographic aging, which represents the quantitative component of the aging phenomenon, partially decreases the returns from a PAYG system in our economy characterized by endogenous human capital formation. Political aging, which represents the qualitative component of aging phenomenon, provides retirees with a stronger claim for pension benefits even on constant demographic terms. Based on the characterization of the political equilibrium, we now consider how aging affects the policy decisions of representatives who face electoral constraints in the form of both the size of welfare state, $\mathcal{T}$, and the amount of intergenerational transfers, $\mathcal{E}$ and $\mathcal{P}$, where $\mathcal{P}$ is the pension equilibrium policy rule. It is obtained by plugging the equilibrium policy rules (1.19) and (1.20) into the balanced budget constraint (1.7). Focusing on political aging the following Corollary holds:

Corollary 5 *The political aging, i.e. the increase in ϕ, has no quantitative impact on the education transfers, $\frac{d\mathcal{E}}{d\phi} = 0$, and induces increase in the income tax rate, $\frac{d\mathcal{T}}{d\phi} > 0$. It follows, for any level of $\bar{h}$, $\frac{d\mathcal{P}}{d\phi} > 0$.*

Proof. *(See appendix).* ■

The political effect is captured by a relative decrease in the political weight for the adult, in favour of an increase in the political weight for old. Higher levels of ideological pressure in the political debate from the elderly cohort implies higher income tax rate. This in turn determines a larger social security system supported by voting. Given the efficiency criterion driving the implementation of public education policy, the overall effect of political aging does not distort $\mathcal{E}$.

Corollary 6 *The demographic aging, i.e. the decrease in n, induces an increase in education transfers, $\frac{d\mathcal{E}}{dn} < 0$, and has an ambiguous impact on the income tax rate, $\frac{d\mathcal{T}}{dn} \gtreqless 0$. It follows $\frac{d\mathcal{P}}{dn} \gtreqless 0$.*

Proof. *(See appendix).* ■

Departing from earlier literature that argues that social security increases with population growth, our model predicts the scenario under which parametric conditions also allow for the inverse relation to appear. Specifically, demographic aging has an ambiguous impact on the amount of pension transfers in per-capita terms. An interesting case arises when the margin $R-(1+n)$ is sufficiently small, which in turns implies, even without considering the human capital return, the implicit return to pensions to be close to the gross return to private saving. This scenario gives incentives in a younger society to sustain higher pension benefits due to their larger demographic return, i.e. $\frac{d\mathcal{P}}{dn} > 0$. A second illustrative case emerges when the relative adults political weight is larger than $\bar{R}$ and $\bar{h}$ is sufficiently high. In this scenario, even if population ages and, in turn, the demographic pension returns decrease, adults have incentives to depress the current level of savings in order to compensate for the smaller number of future tax payers with higher tax rate level tomorrow, i.e. $\frac{d\mathcal{P}}{dn} < 0$.

1.6 Benevolent Government Allocation

In Section 1.4.3 we have proved the existence of a bidimensional fiscal plane when public policy choices are taken through a repeated voting system. The politico-economic SMPE has been also characterized in closed-form as a finite-horizon equilibrium when the time goes to infinity. We now implement, as a normative benchmark, the infinite-horizon Gvt allocation under zero-cost enforceability constraint.

As in the political game, we exclude private agents' default on the implemented fiscal plane within the period. Furthermore, under balanced budget constraint, the government platform is characterized by the vector $f_t^g = \{e_t^g, \tau_t^g\}$, where the apex g stands for Gvt. Given the initial conditions $\{h_0, k_0\}$, we define the Gvt optimization program in sequential version, as follows:

$$\max_{\{f_t^g\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (1+n)^t \delta^t \left((1+n)\,\delta u\left(C_t^1\right) + \beta u\left(C_t^2\right)\right) \tag{1.21}$$

subject to the constraints given by Eq. (1.5) and (1.9). δ is the welfare weight that the Gvt assigns to each dynasty. Let us consider the restriction $\delta < \bar{\delta} \equiv \frac{1}{1+n}$. Differently from the relative political bargaining power of adults and old, Eq. (1.18), the infinite-horizon Gvt takes account of both the relative welfare weight of the representative agent, Ω_R^g, and the elderly's relative welfare weight gap between current and future pensioners,

Ω^g_O, where:

$$\Omega^g_R \equiv \frac{\delta\,(1+n)\,(1+\beta)}{\beta+\delta(1+n)} \quad \text{and} \quad \Omega^g_O \equiv \frac{\beta\,(1-\delta\,(1+n))}{\beta+\delta\,(1+n)} \tag{1.22}$$

Remark 6 *The more a population ages (i.e. n decreases), the smaller the relative welfare weight of the representative agent $\left(\Omega^g_R\right)$, the larger is the elderly's relative welfare weight gap $\left(\Omega^g_O\right)$.*

As in Klein at al. (2008),[24] let us rewrite the sequential Gvt program in a recursive way to derive the Gvt Generalized Euler Equations (GEEs).[25] They capture the Gvt optimal trade-offs between taxation and redistribution wedges over time. Due to stationarity, we omit the time subscript, denoting by the prime symbols next-period values. The economic first order condition, Eq. (1.8), requires $\eta\left(f^g, f^{g\prime}, h, h', k'\right) = 0$. In equilibrium, by using the implicit function theorem, there exists a unique $k' = K\left(f^g, f^{g\prime}, h, h'\right)$ satisfying $\eta\left(f^g, f^{g\prime}, h, h', K\left(\cdot\right)\right) = 0$. If the equilibrium policy rules exist, $\mathcal{T}^g\left(h,k\right)$ and $\mathcal{E}^g\left(h,k\right)$, then by using $h' = H(e^g, h)$ we derive the recursive formulation of $K\left(\cdot\right)$, whose functional form is equal to $k' = \mathcal{K}\left(f^g, h\right)$. The recursive economic first order condition becomes $\eta\left(f^g, h, \mathcal{K}\left(f^g, h\right)\right) = 0$. Derivating the function $\eta\left(\cdot\right)$ with respect to its arguments we obtain $\mathcal{K}_{f^g} = -\frac{\eta_{f^g}}{\eta_{k'}}$ and $\mathcal{K}_h = -\frac{\eta_h}{\eta_{k'}}$, which give a measure of the variation in the amount of savings due to a change in either policies or human capital.

After some manipulations, Eq. (1.21) can be reformulated in terms of Bellman equation, as follows:

$$V^g\left(h,k\right) = \max_{\left\{f^g, h', k'\right\}} \left[(1+n)\,\delta u\left(C^1\right) + \beta u\left(C^2\right)\right] + (1+n)\,\delta V^g\left(h',k'\right) \tag{1.23}$$

[24] Recent studies have extended the dynamic politico-economic modelling to the infinite-horizon Gvt problem (Klein et al., 2008, and Azzimonti et al., 2009). These models differ from ours in that the policy space is one-dimensional and the dynamic linkages are not long-run persistent due to the full depreciation of the relevant-payoff state variables. Departing from earlier literature, we find analytical solutions in a multi-dimensional state space where the equilibrium policies become non trivially dependent on fundamental asset variables.

[25] The *GEE* is the *FOC* of the government maximization program. It is obtained by deriving the Bellman equation with respect to the political control variables, f^g. *GEE* can be equivalently derived by using Bellman's principle to identify a Markov equilibrium with the solution of the sequential version of the Governemt program. The Euler equation of this sequential problem is exactly the *GEE*. See Appendix B for the derivation of both Bellman equation and *GEEs*.

We now provide the formal Definition of the Gvt equilibrium allocation without commitment.

Definition 5 (Benevolent Government Allocation) *A perfect foresight SMPE of the Benevolent Government problem is defined as the sequence of competitive economic equilibrium allocations* $\left\{c^1, c^2, h', k'\right\}$ *and policies* f^g*, such that, given the Bellman Eq. (1.23), the functional vector of differentiable policy decision rules,* $\mathcal{F}^g = \{\mathcal{T}^g, \mathcal{E}^g\}$*, where* $\mathcal{T}^g : \mathbb{R}_+ \times \mathbb{R}_+ \to (0,1)$ *and* $\mathcal{E}^g : \mathbb{R}_+ \times \mathbb{R}_+ \to (0, \hat{e}_t)$ *are respectively the taxation policy rule,* $\tau^g = \mathcal{T}^g(h,k)$*, and public higher education policy rule,* $e^g = \mathcal{E}^g(h,k)$*, satisfies the following conditions:*

i.

$$\{\mathcal{T}^g(h,k), \mathcal{E}^g(h,k)\} = \underset{\{\tau^g, e^g\}}{\arg\max} \left[(1+n)\,\delta u\left(C^1\right) + \beta u\left(C^2\right)\right] + (1+n)\,\delta V^g\left(h', k'\right)$$

ii.

$$V^g(h,k) = \mathcal{M}(V^g)(h,k)$$

where the functional $\mathcal{M} : \mathbb{C}^\infty\left(R^2\right) \to \mathbb{C}^\infty\left(R^2\right)$ *is defined as follows:*

$$\mathcal{M}(V^g)(h,k) = \max_{\{\tau^g, e^g\}} \left[(1+n)\,\delta u\left(C^1\right) + \beta u\left(C^2\right)\right] + (1+n)\,\delta V^g(H, \mathcal{K})$$

The first condition requires the political variables, τ^g and e^g, have to be chosen by the Gvt in order to maximize the utilitaristic social welfare, internalizing the equilibrium private saving decision and all the direct and indirect feedback effects. The second requirement is the fix point condition, given the mapping $\mathcal{M}(V^g)$.

In terms of wedges, the *GEEs* of the sequential Gvt program with respect of e^g and τ^g are as follows:

$$0 = \Delta_e + \delta \frac{\eta_e}{\eta_{k'}} \Delta_s + \delta \Xi \Delta'_{\tau'} \tag{1.24}$$

$$0 = (1+h)\,\Delta_\tau + \delta(1+n) \frac{\eta_\tau}{\eta_{k'}} \Delta_s \tag{1.25}$$

where $\Xi \equiv \frac{d((1+h')\tau')}{de}$. Δ_x with $x \in \{s, \tau, e\}$ describes the first best wedges and are equal to:[26]

$$
\begin{array}{ll}
\Delta_s \equiv u_{C^1} - \beta R u_{C^{2\prime}} & \text{savings/consumption wedge} \\
\Delta_\tau \equiv (1+h)\left(\beta u_{C^2} - \delta u_{C^1}\right) & \text{backward redistribution wedge} \\
\Delta_e \equiv \delta H_e \left(\beta\,(1+n)\,\frac{H'_{h'}}{H'_{e'}} u_{C^{2\prime}} + \delta u_{C^{1\prime}}\right) - \beta u_{C^2} & \text{forward redistribution wedge}
\end{array}
$$

The above wedges can be interpreted as *potential* deviations from the efficient intertemporal decisions. First consider Eq. (1.25). Due to a marginal increase in taxation, τ, the intertemporal savings wedge, Δ_s, is scaled by the reduction in household savings, $\mathcal{K}_\tau < 0$. Furthermore an increase in income tax rate determines an increase in the gap between u_{C^2} and u_{C^1}, which is captured by the intratemporal utility wedge, Δ_τ. Note that, due to full depreciation of physical capital, k'' is equal to $\mathcal{K}\left(f', h'\right)$ and it is not a function of k'. Then a variation in the current tax rate does not affect the next-period wedges through its effect on future level of physical capital. More cumbersome dynamic effects emerge instead from the equilibrium determination of public education transfers, Eq. (1.24). An increase in education transfers, e, makes private savings wedge scaled by the variation in household savings, $\mathcal{K}_e < 0$, which is negative due to the substitution effects with public savings that are increased via the retributive pension scheme. The second component, Δ_e, represents the intertemporal utility variation due to an increase in education transfers today, which affects the utility of current old and simultaneously a variation of the subsequent period's adults and elderly utility through h'. Finally, in contrast from τ, a variation in the current level of education transfers also affect next period's wedges, $\Delta'_{\tau'}$, through its effect on h', which induces a variation of both k'' and h''.

1.6.1 The Government SMPE

To solve the Gvt optimization problem, we guess a time consistent bidimensional policy, structurally equivalent to Eq. (1.19) and (1.20), which verifies

[26] The wedges Δ_s, Δ_τ and Δ_e are derived as the marginal direct impact on the intertemporal agents' utility of a variation in individual savings, taxation/pension contributions and education investments, respectively. The marginal variation in the income tax rate detemines the direct cost for current adults equal to $\delta\,(1+n)\,(1+h)\,u_{c^1}$ and the direct benefits for current old equal to $\beta\,(1+n)\,(1+h)\,u_{c^2}$. The intergenerational backward redistribution wedge becomes then $\Delta_\tau \equiv \beta\,(1+n)\,(1+h)\,u_{c^2} - \delta\,(1+n)\,(1+h)\,u_{c^1}$ that normalized by $(1+n)\,(1+h)$ is equal to $\Delta_\tau \equiv \beta u_{c^2} - \delta u_{c^1}$. The same characterization holds for Δ_s and Δ_e.

the conditions (1.24) and (1.25).[27] Fixing $\theta = \frac{1}{2}$, let Υ^{Gvt} be the state-space in which interior policy rules are obtained. Then the next Proposition characterizes the optimal feasible time-consistent policy rules.

Proposition 2 *Under dynamic efficiency condition, for any $\{h, k\} \in \Upsilon^{Gvt}$ the set of individual feasible rational policies, $f^g = \{\tau^g, e^g\}$, which can be supported by a Government SMPE with perfect foresight, has the following functional form:*

i.

$$\mathcal{E}^g(h) = a_1^g h + a_0^g \tag{1.26}$$

where $a_1^g = a_1$ and $a_0^g = a_0$;

ii.

$$\mathcal{T}^g(h,k) = -b_3^g \frac{k}{1+h} + b_2^g \frac{h}{1+h} + b_1^g \frac{1}{1+h} + b_0^g \tag{1.27}$$

where $b_3^g \equiv R\Omega_R^g$, $b_2^g \equiv \frac{\alpha\sqrt{\psi^*}}{R-\alpha\sqrt{\psi^*}}(\Omega_O^g + R\sqrt{\psi^*}\Omega_R^g - \alpha\psi^*)$, $b_1^g \equiv \bar{h}\frac{1-\alpha}{\alpha}\frac{R}{R-(1+n)}b_2^g$ $+\ \bar{R}(\Omega_O^g - \bar{h}(1-\alpha)\psi^*)$ and $b_0^g \equiv \Omega_O^g$.

For any $(h, k) \notin \Upsilon^{Gvt}$ corner solutions result in at least one of the two dimensions.

Proof. *(See appendix).* ■

The two equilibrium concepts described in Definition 3 and 5 lead to the implementation of the same education program. Specifically, in equilibrium both the Gvt and the office-seeking politicians set the same amount of forward transfers, inducing education-efficient political fiscal planes, i.e. $\mathcal{E}^g(h) = \mathcal{E}(h)$ for any level of human capital. The main difference concerns their quantitative predictions on the taxation policy dimension, which are

[27] Although there is an infinite persistent impact of the current tax rate and public education investment, only the current and the subsequent period matter directly. Thus the marginal costs and benefits in equilibrium can be summarized by terms involving only two consecutive periods. As a consequence, the *GEEs* can also be viewed as resulting from a variational (two-periods) problem. In other words, given the state variables (h, k) and (h'', k'') fixed, let us vary (h', k') through the controls (τ, τ') and (e, e'), in order to obtain the highest possible utility. Recalling that the *SMPE* in the political case has been obtained as the limit of a finite horizon economy, whose convergence has been attained after two periods, we conjecture no structural differences between the two equilibrium policy rules. For this reason, in the following section we use the guess of the political equilibrium to verify the *GEEs*, and obtain the Gvt solution without commitment.

fully captured by the policy parameters. In the following subsection we discuss in details how a political equilibrium divergences from the Gvt optimal allocation.

1.6.2 Are the political choices on pensions and education optimal?

Both the politicians and the Gvt have incentives to provide intergenerational transfers. Moreover, their equilibrium policies share similar structural properties. However the quantitative differences detected so far imply distinct predictions in terms of WSRs' identification and political behavior. We now examine how the politicians act relatively to the Gvt in terms of fiscal design. In order to obtain clear predictions, we normalize the vector of welfare weights by δ assigning $\phi \equiv \frac{\beta}{\delta}$. Consequently we write the relative welfare weights, Eq. (1.22), in terms of political weights, making the two solutions comparable.[28] Let us introduce the following notation $\Lambda \equiv \left\{(\phi, n) \in \left(\underline{\phi}, \infty\right) \times (\underline{n}, \overline{n}) \mid b_2 \geqslant b_1\right\}$ and $\Lambda^g \equiv \left\{(\phi, n) \in \left(\underline{\phi}, \infty\right) \times (\underline{n}, \overline{n}) \mid b_2^g \geqslant b_1^g\right\}$, in order to delimit the parametric space in which PCR emerges respectively for the political and the Gvt cases. The following Corollary resumes the conditions for the welfare regimes' comparison between the political and Gvt cases in the parametric space (ϕ, n).

Corollary 7 *For any level of $\bar{h}$ and $n \in (\underline{n}, \overline{n})$ the condition $\Lambda \subset \Lambda^g$ holds.*

Proof. *(See appendix).* ■

The parametric space where PCR emerges is always larger in the Gvt environment than in the political one. Furthermore, let $\bar{\phi}$ be a sufficiently large value of the ideological bias, such that for any $\phi < \bar{\phi}$, the following Proposition is stated.[29]

Proposition 3 *For $\delta < \bar{\delta}$ and for any $\phi < \bar{\phi}$, the political SMPE induces overtaxation with respect to the Government SMPE, i.e. $\mathcal{T}(h_t, k_t) > \mathcal{T}^g(h_t, k_t)$ for any $\{h_t, k_t\} \in \Upsilon^{Pol} \cap \Upsilon^{Gvt}$.*

Proof. *(See appendix).* ■

[28] After the normalization, the relative welfare weights turn out to be $\Omega_R^g \equiv \frac{(1+n)(1+\beta)}{\phi+(1+n)}$ and $\Omega_O^g \equiv \frac{\phi-\beta(1+n)}{\phi+(1+n)}$.

[29] See proof of Proposition 1 for the exact determination of $\bar{\phi}$.

According to Proposition 3, if the Gvt adopts a politically equivalent system of welfare weights, the level of income tax rate is always lower than in the political case, i.e. $\mathcal{T}(h_t, k_t) > \mathcal{T}^g(h_t, k_t)$, for any value of human and physical capital. We conclude that politicians involved in a Markov game among successive generations of players deliver the Gvt allocation if they reduce the political weight they assign to the elderly agents. Given the invariant level of education transfers achieved by both the politicians and the Gvt, high tax rate implies pension benefits too generous. These distortions come from the politicians' strategic behavior. In determining taxation rules, short-lived politicians take into account that future politicians will compensate the fiscal cost of current adults by paying the pensions in their old age. This stems from the fact that higher taxes on today environment lead to a lower private wealth in old age, i.e. to a lower state variable in the following period, thereby triggering more transfers by the future politicians. Thus, the policy response of the future politicians reduces the current cost of transferring resources to the elderly and leads to overspending, unless the adults enjoy an unusually large political power. Consequently, by transferring too much resources to old age due to both the overrepresenting of current elderly agents and the policy response of the future politicians, the politicians fail to provide the optimal income tax rate policy.

1.7 Conclusions

In this paper we investigate the conditions for the emergence of implicit intergenerational contracts without assuming reputation mechanisms, commitment technology and altruism. We present a tractable dynamic politico-economic model in OLG environment where political representatives compete proposing multidimensional fiscal platforms. Both backward and forward intergenerational transfers, respectively in the form of pension benefits and education investments, are simultaneously considered in an endogenous human capital setting with income taxation when agents play Markovian strategies. The infinite horizon Gvt solution without commitment is used as benchmark to evaluate the efficiency of politically determined rules.

The dynamic mechanisms driving our results are intuitive. Social security system sustains investment in public education, that, in turns, creates a dynamic linkage across periods through both human and physical capital driving the economy towards different WSR.

We show that intergenerational contracts may be politically sustained uniquely as long as the economy is in dynamic efficiency, i.e. the rental gross price of capital is larger than the economic growth rate. Our economic environment is in line with empirical findings on the dynamic efficiency status of most developed countries, especially after the demographic transition. By endogenizing human capital formation through public education investments, backward and forward redistributive programs may optimally self-sustain each other even in the absence of a benevolent Gvt. In equilibrium political decisions are education efficient.

Relatively to the prediction about the transition towards the steady state, we find three different WSR may emerge depending on both the relative political bargaining power between adults and old and the endogenous capital asset accumulation. The emergence of different regimes leads the economy towards different dynamic paths and persistence degrees of politically distortionary redistribution. In the regime supported by adults, the equilibrium pension benefits reach the highest feasible level.

Demographic aging increases the equilibrium per-capita level of forward transfers, i.e. public education spending. Due to the decreasing return in human capital accumulation aging does not always exacerbate the generous behavior of the politicians towards the elderly. Political aging has instead positive impact on taxation but no effects on the level of public education investments.

Finally, due to the distortions generated by the repeated political competition process and to the political overrepresentation of elderly agents, political equilibrium is characterized by overtaxation compared with the Gvt solution.

Our analysis leaves some natural directions for future research. We have assumed only adults and old compete in the political debate. Using the developed methodology, a change in the voting rule, which enables also the young to vote, would generate different equilibrium allocations both in terms of education transfers and government size. Another direction for future research concerns the introduction of a dynamic electoral stage by endogenizing the probability of re-election, which would introduce a new source of distortion.

1.8 Technical Appendix A

Theorem (1).

As in Klein et al. (2008), our resolution strategy consists in computing the first order conditions starting from a time $t < \infty$ large enough and solving backward for each time $t-j$ with $j = 1, 2, ...$ subject to: 1) the economic Euler condition, Eq. (1.8), 2) the balanced budget constraint, Eq. (1.7), and 3) the equilibrium policy rules of following periods. We recursively determine the conditions for the existence of fixed points taking the limit for $j \to \infty$.

Suppose the economy ends at time $t < \infty$, and adults at that time have one period temporal-horizon. Thus, the political objective function is as follows:

$$\mathcal{W}_t \equiv (1+n)\, u\left(C_t^1\right) + \phi u\left(C_t^2\right)$$

where $C_t^1 \equiv (1+h_t)(1-\tau_t)$ and $C_t^2 \equiv (1+n)\, Rk_t + p_t$. At time t there are no incentives in investing in education, i.e. $e_t = 0$. The fiscal dimension, τ_t, is determined according to the Euler condition, as follows:

$$\frac{u_{C_t^2}}{u_{C_t^1}} = \frac{1}{\phi}$$

Under logarithmic utility, the equilibrium fiscal policy rule at time t is $\tau_t = -R\Omega_{1,t}\frac{k_t}{1+h_t} + \Omega_{2,t}$ where $\Omega_{1,t} \equiv \frac{1+n}{1+n+\phi}$ and $\Omega_{2,t} \equiv \frac{\phi}{1+n+\phi}$. Consequently, the equilibrium policy rules, $\mathcal{F}_t = \{\mathcal{T}_t, \mathcal{E}_t\}$, are equal to:

$$\mathcal{F}_t : \begin{cases} \mathcal{T}_t = -b_{1(0)}\frac{k_t}{1+h_t} + b_{0(0)} \\ \mathcal{E}_t = 0 \end{cases} \tag{1A}$$

where $b_{1(0)} \equiv R\Omega_{1,t}$ and $b_{0(0)} \equiv \Omega_{2,t}$. The number in the brackets represents the number of iterations.

Next we consider period $t-1$, in which adults born at time $t-2$ live up three periods. The political objective function is as follows:

$$\mathcal{W}_{t-1} \equiv (1+n)\left(u\left(C_{t-1}^1\right) + \beta u\left(C_t^2\right)\right) + \phi u\left(C_{t-1}^2\right) \tag{2A}$$

where $C_{t-1}^1 \equiv (1+h_{t-1})(1-\tau_{t-1}) - (1+n)\, k_t$ and $C_{t-1}^2 \equiv (1+n)\, Rk_{t-1} + p_{t-1}$. After plugging the equilibrium policy rules of period t, Eq. (1A), into Eq. (2A), we maximize with respect to $f_{t-1} = \{\tau_{t-1}, e_{t-1}\}$. Applying envelope theorem, we get the following system of Euler equations:

$$\begin{cases} \frac{u_{C^2_{t-1}}}{u_{C^1_{t-1}}} = \frac{1+\beta}{\phi+\beta(\phi+1+n)} \\ \frac{u_{C^2_{t-1}}}{u_{C^1_{t-1}}} = \frac{1}{R}\left(\frac{1+\beta}{\phi+\beta(\phi+1+n)}\right)\frac{dh_t}{de_{t-1}} \end{cases} \tag{3A}$$

Equating the two conditions in (3A), we get the necessary condition for the determination of the equilibrium level of e_{t-1}, i.e. $\frac{dh_t}{de_{t-1}} = R$. Recalling that $h_t = \left(\frac{\alpha h_{t-1}+(1-\alpha)\bar{h}}{1+n}\right)^{\theta} e_{t-1}^{1-\theta}$ and plugging $\frac{dh_t}{de_{t-1}}$ into the equilibrium condition, we derive the equilibrium public education transfers at time $t-1$. Let us denote $\psi_{(1)} \equiv \left(\frac{1-\theta}{R}\right)^{\frac{1}{\theta}}$ and $\gamma_{(1)} = \frac{1+n}{R}$. The equilibrium policy rules are equal to:

$$\mathcal{F}_{t-1} : \begin{cases} \mathcal{T}_{t-1} = -b_{4(1)}\frac{k_{t-1}}{1+h_{t-1}} + b_{3(1)}\frac{h_{t-1}}{1+h_{t-1}} + b_{2(1)}\frac{\bar{h}}{1+h_{t-1}} + b_{1(1)}\frac{1}{1+h_{t-1}} + b_{0(1)} \\ \mathcal{E}_{t-1} = a_{1(1)}h_{t-1} + a_{0(1)} \end{cases} \tag{4A}$$

where $a_{0(1)} \equiv \frac{(1-\alpha)\bar{h}}{1+n}\psi_{(1)}$, $a_{1(1)} \equiv \frac{\alpha}{1+n}\psi_{(1)}$ and $b_{0(1)} \equiv \Omega_{2,t-1}$, $b_{1(1)} \equiv \gamma_{(1)}\Omega_{2,t-1}$, $b_{2(1)} \equiv (1-\alpha)\,\Omega_{1,t-1}\psi_{(1)}$, $b_{3(1)} \equiv \alpha\left(\Omega_{1,t-1} + \frac{1}{(1-\theta)}\Omega_{2,t-1}\right)\psi_{(1)}$ and $b_{4(1)} \equiv R\Omega_{1,t-1}$. $\Omega_{2,t-1} \equiv \Omega_2 \equiv \frac{\phi}{\phi+(1+n)(1+\beta)}$ and $\Omega_{1,t-1} \equiv \Omega_1 \equiv \frac{(1+n)(1+\beta)}{\phi+(1+n)(1+\beta)}$ are the indexes of the relative old's and adults' political power in an economy that lasts more than one period, respectively.

Finally let us consider time $t-2$. At this point, all the direct dynamic feedbacks are internalized. The political objective function is equivalent to Eq. (2A). The recursive problem is now subject to the equilibrium policy rules of the next two periods, (1A) and (4A). Maximizing the political objective function with respect to $f_{t-2} = \{\tau_{t-2}, e_{t-2}\}$ the system of Euler conditions are:

$$\begin{cases} \frac{u_{C^2_{t-2}}}{u_{C^1_{t-2}}} = \frac{1}{\phi+(1+n)\beta} \\ \frac{u_{C^2_{t-2}}}{u_{C^1_{t-2}}} = \frac{1}{R(\phi+(1+n)\beta)}\left(1 + \frac{\alpha\theta}{1-\theta}\left(\frac{1-\theta}{R}\right)^{\frac{1}{\theta}}\right)\frac{dh_{t-1}}{de_{t-2}} \end{cases} \tag{5A}$$

Let us now denote with $\psi_{(2)} \equiv \left(\frac{\alpha\theta}{R}\left(\frac{1-\theta}{R}\right)^{\frac{1}{\theta}} + \frac{1-\theta}{R}\right)^{\frac{1}{\theta}}$ and $\gamma_{(2)} \equiv \frac{1+n}{R} + \left(\frac{1+n}{R}\right)^2$. Furthermore, let us introduce the following notation $g_{(2)} \equiv \frac{1+n}{R}\psi_{(1)} + \psi_{(2)}$. As before, solving the system (5A) we yield the following pair of equilibrium policy rules at time $t-2$:

$$\mathcal{F}_{t-2}:\begin{cases}\mathcal{T}_{t-2}=-b_{4(2)}\frac{k_{t-1}}{1+h_{t-1}}+b_{3(2)}\frac{h_{t-1}}{1+h_{t-1}}+b_{2(2)}\frac{\bar{h}}{1+h_{t-1}}+b_{1(2)}\frac{1}{1+h_{t-1}}+b_{0(2)}\\ \mathcal{E}_{t-2}=a_{1(2)}h_{t-1}+a_{0(2)}\end{cases}$$

where $b_{0(2)}\equiv b_{0(1)}$, $b_{1(2)}\equiv\gamma_{(2)}\Omega_2$, $b_{2(2)}\equiv(1-\alpha)\left(\left(\Omega_1+\frac{1}{1-\theta}\Omega_2\right)\psi_{(2)}+\frac{\theta}{1-\theta}\Omega_2 g_{(2)}\right)$, $b_{3(2)}\equiv\alpha\psi_{(2)}(\Omega_1+\frac{1}{1-\theta}\Omega_2)$, $b_{4(2)}\equiv b_{4(1)}$ and $a_{0(2)}\equiv\frac{(1-\alpha)\bar{h}}{1+n}\psi_{(2)}$, $a_{1(2)}\equiv\frac{\alpha}{1+n}\psi_{(2)}$. It is straightforward to show that $\psi_{(2)}$ can be derived as a differentiable monotonic transformation of $\psi_{(1)}$, i.e. $m(\cdot)$. It is characterized by $m(0)>0$, $m_{\psi}>0$, and $m_{\psi\psi}>0$. In particular $m\left(\psi_{(1)}\right)=\left(\frac{\alpha\theta}{R}\psi_{(1)}+\frac{1-\theta}{R}\right)^{\frac{1}{\theta}}$. The argument can be repeated for each time $j>0$ such that:

$$\psi_{(j+1)}=m\left(\psi_{(j)}\right)\tag{6A}$$

Furthermore for each j the following series can be derived:

$$\gamma_{(j)}\equiv\sum_{l=1}^{j}\left(\frac{1+n}{R}\right)^{l}\quad\text{and}\quad g_{(j)}\equiv\left(\frac{1+n}{R}\right)^{j-1}\psi_{(1)}+\left(\frac{1+n}{R}\right)^{j-2}\psi_{(2)}+...+\psi_{(j)}$$

Using the above notation, starting from $t-3$ we can finally derive the recursive structure which characterizes the political problem:

$$\mathcal{F}_{t-j}:\begin{cases}\mathcal{T}_{t-j}=-b_{4(j)}\frac{k_{t-j}}{1+h_{t-j}}+b_{3(j)}\frac{h_{t-j}}{1+h_{t-j}}+b_{2(j)}\frac{\bar{h}}{1+h_{t-j}}+b_{1(j)}\frac{1}{1+h_{t-j}}+b_{0(j)}\\ \mathcal{E}_{t-j}=a_{1(j)}h_{t-j}+a_{0(j)}\end{cases}\tag{7A}$$

where $a_{0(j)}\equiv\frac{(1-\alpha)\bar{h}}{1+n}\psi_{(j)}$, $a_{1(j)}\equiv\frac{\alpha}{1+n}\psi_{(j)}$ and $b_{0(j)}\equiv b_{0(1)}$, $b_{1(j)}\equiv\gamma_{(j)}\Omega_2$, $b_{2(j)}\equiv(1-\alpha)\left((\Omega_1+\frac{1}{1-\theta}\Omega_2)\psi_{(j)}+\frac{\theta}{1-\theta}\Omega_2 g_{(j)}\right)$, $b_{3(j)}\equiv\alpha\psi_{(j)}\left(\Omega_1+\frac{1}{(1-\theta)}\Omega_2\right)$, $b_{4(j)}\equiv b_{4(1)}$.

If a political SMPE exists, then the limits for $j\to\infty$ of the set of time-variant parameters $\left\{a_{0(j)},a_{1(j)},b_{0(j)},b_{1(j)},b_{2(j)},b_{3(j)},b_{4(j)}\right\}$ exist and are finite. The determination of the fixed points for the two stationary policy rules crucially depends on the existence of the fixed point of the policy e and, in final instance, on the determination of the limit for $\psi_{(j)}$. Thus, we start with the redistributive policy dimension. The computation consists in solving the non-linear difference equation (6A). The $\lim_{j\to\infty}\psi_{(j)}$ is equivalent to the solution(s), if any, of such difference equation given ψ_0 as initial condition. Let us denote with $\hat{\psi}_j$ the value of ψ_j such that $\left(\frac{dm(\psi_j)}{d\psi_j}\right)_{\psi_j=\hat{\psi}_j}=1$. We yield respectively zero, one or two fixed points as solution of the difference equation iff $m\left(\hat{\psi}_j\right) \gtreqless \hat{\psi}_j$. Thus, $\hat{\psi}_j$ is equal to:

$$\hat{\psi}_j = \frac{1}{\theta}\left(\frac{R}{\alpha}\right)^{\frac{1}{1-\theta}} - \frac{1-\theta}{\alpha\theta} \tag{8A}$$

Note that $R > \alpha^{\theta}$ in all the parameters' space. Such condition guarantees the existence of at least one stable fixed point. For analytical tractability we determine the solutions for quadratic form case. Under the condition (8A) for $\theta = \frac{1}{2}$ the two fixed points are:

$$\psi^*_{1,2} = \frac{1}{\alpha}\left(\frac{2R}{\alpha}\left(R \pm \sqrt{R^2-\alpha}\right) - 1\right)$$

We focus on the stable equilibrium, denoted by $\psi^* = \frac{1}{\alpha}\left(\frac{2R}{\alpha}\left(R - \sqrt{R^2-\alpha}\right) - 1\right)$ and we take $\psi_0 = \psi^*$ as initial condition. The solution of the difference equation (6A) is represented in Figure 7.

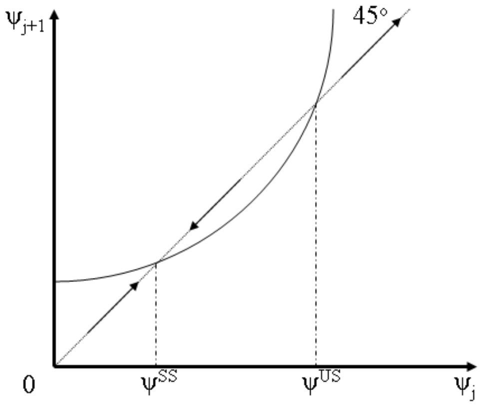

Fig. 7: $\psi_{(j+1)}=m\left[\psi_{(j)}\right]$

Under the condition $R > (1+n)$ the $\lim\limits_{j\to\infty} \gamma_{(j)} < \infty$ is equal to $\frac{1+n}{R-(1+n)} \equiv \bar{R}$. Consequently, the $\lim\limits_{j\to\infty} g_{(j)} = \lim\limits_{j\to\infty} \psi^* \sum_{l=1}^{j} \left(\frac{1+n}{R}\right)^l < \infty$ is equal to $\frac{\bar{R}}{\alpha}\left(\frac{2R}{\alpha}\left(R - \sqrt{R^2-\alpha}\right) - 1\right)$. Under such convergence conditions the fixed points are finally attained. Rearranging the terms we reformulate the individual rational fiscal and redistribution policies as follows:

$$\mathcal{T}\left(h_t, k_t\right) = -b_3\frac{k_t}{1+h_t} + b_2\frac{h_t}{1+h_t} + b_1\frac{1}{1+h_t} + b_0$$

where $b_0 \equiv \Omega_2$, $b_1 \equiv \bar{h}\left(1-\alpha\right)\psi^*\left(\Omega_1+\left(2+\bar{R}\right)\Omega_2\right)+\bar{R}\Omega_2$, $b_2 \equiv \alpha\psi^*\left(\Omega_1+2\Omega_2\right)$ and $b_3 \equiv R\Omega_1$;

$$\mathcal{E}\left(h_t\right) = a_1 h_t + a_0$$

where $a_0 \equiv \frac{1-\alpha}{1+n}\bar{h}\psi^*$ and $a_1 \equiv \frac{\alpha}{1+n}\psi^*$.

We denote $(K_t^{\tau}, H_t^{\tau}) = \{(k_t, h_t)\,|\check{k}_t < k_t < \hat{k}_t\}$ where $\hat{k}_t \equiv \frac{b_2+b_0}{b_3}h_t + \frac{b_1+b_0}{b_3}$ and $\check{k}_t(h_t) \equiv \frac{b_2+b_0-1}{b_3}h_t + \frac{b_1+b_0-1}{b_3}$. While

$$H_t^e = \begin{cases} \{h_t | h_t \in (0,\infty)\} & \text{if } \bar{h} < \frac{1}{(1-\alpha)\psi^*} \\ \{h_t | h_t \in (\tilde{h},\infty)\} & \text{if } \bar{h} \geq \frac{1}{(1-\alpha)\psi^*} \end{cases}$$

where $\tilde{h} \equiv \frac{1-\bar{h}(1-\alpha)\psi^*}{\alpha\psi^*-1}$. Jointly considering the above feasibility conditions for both fiscal and redistributive dimensions, non-degenerate policies, i.e. $\tau_t \in (0,1)$ and $e_t \in (0,\hat{e}_t)$, are achieved at each time for any $(k_t,h_t) \in \Upsilon^{Pol} \equiv (K_t^{\tau}, H_t^{\tau}) \cap H_t^e$. ■

Lemma (1). Let us first consider the transition dynamics of h_t and e_t. Plugging the equilibrium education transfers, Eq. (1.19), into the human capital production, Eq. (2.4), we obtain the law of motion $h_{t+1} = H^d\left(h_t\right)$, which is equal to:

$$h_{t+1} = \lambda_1 h_t + \lambda_0 \tag{9A}$$

where $\lambda_0 \equiv \frac{(1-\alpha)\bar{h}}{1+n}\sqrt{\psi^*}$ and $\lambda_1 \equiv \frac{\alpha}{1+n}\sqrt{\psi^*}$. Since $\lambda_1 \geqslant 0$, the serial correlation between current and future level of human capital is always positive. To determine the law of motion of the redistributive policy we plug Eq. (2.4) into the equilibrium education policy rule at time $t+1$. The law of motion $e_{t+1} = E^d\left(h_t\right)$ is as follows:

$$e_{t+1} = \xi_1 h_t + \xi_0 \tag{10A}$$

where $\xi_0 \equiv a_0\left(\frac{a_1}{\sqrt{\psi^*}}+1\right)$ and $\xi_1 \equiv \frac{a_1^2}{\sqrt{\psi^*}}$. If the dynamics of h_t is characterized by stable convergence, i.e. $\lambda_1 < 1$, then also the dynamics of e_t converge toward the steady state. Thus, using the expression of λ_1, the sufficient condition for the convergence stability of both h_t and e_t requires:

$$n > \underline{n} \tag{11A}$$

where $\underline{n} \equiv \sqrt{2R\left(R-\sqrt{R^2-\alpha}\right)-\alpha}-1$. Due to linearity, both h_t and e_t

converge monotonically toward the steady states.

Let us now analyze the transition dynamics of k_t and τ_t. First, consider the following recursive formulation for the equilibrium saving under log-utility, $k_{t+1} = \mathcal{K}(e_t, \tau_t, h_t)$, which is obtained plugging the human capital production, Eq. (2.4), and the expected equilibrium policies e_{t+1} and τ_{t+1} according to Eq. (1.19) and (1.20). The saving function can then be rewritten as follows:

$$k_{t+1}=\frac{\beta R\left(1+h_t\right)\left(1-\tau_t\right)}{\left(R\left(1+\beta\right)-b_3\right)\left(1+n\right)}-\frac{\left(b_0+b_2-\left(1+n\right)a_1\right)H\left(e_t,h_t\right)+\left(b_0+b_1-\left(1+n\right)a_0\right)}{R\left(1+\beta\right)-b_3} \tag{12A}$$

Plugging the equilibrium policy rules, Eq. (1.19) and Eq. (1.20), into Eq. (12A), we obtain the law of motion $k_{t+1} = K^d(h_t, k_t)$:

$$k_{t+1} = \pi_2 k_t + \pi_1 h_t + \pi_0 \tag{13A}$$

where:

$$\pi_2 \equiv \frac{R\beta b_3}{\left(1+n\right)\left(R\left(1+\beta\right)-b_3\right)}$$
$$\pi_1 \equiv -\left(\frac{\left(b_0+b_2-a_1\left(1+n\right)\right)\lambda_1}{\left(R\left(1+\beta\right)-b_3\right)}+\frac{R\beta\left(b_0+b_2-1\right)}{\left(1+n\right)\left(R\left(1+\beta\right)-b_3\right)}\right)$$
$$\pi_0 \equiv -\left(\frac{b_0+b_1-a_0\left(1+n\right)}{\left(R\left(1+\beta\right)-b_3\right)}+\frac{\left(b_0+b_2-a_1\left(1+n\right)\right)\lambda_0}{\left(R\left(1+\beta\right)-b_3\right)}+\frac{R\beta\left(b_0+b_1-1\right)}{\left(1+n\right)\left(R\left(1+\beta\right)-b_3\right)}\right)$$

It should be noted that current and future level of physical capital are positively interrelated each other, $\pi_2 \geqslant 0$, on the contrary the way h_t perturbs k_{t+1} depends on the WSR's intensity embedded in π_1.

Under condition (11A), the dynamics of physical capital is characterized by stable convergence if $\pi_2 < 1$, which requires:

$$\phi > \underline{\phi} \tag{14A}$$

where $\underline{\phi} \equiv \beta\left(R-\left(1+n\right)\right)$. Let us denote by $Q_t^h \equiv \frac{1+h_t}{1+h_{t+1}}$. Plugging Eq. (9A) and (12A) into the equilibrium income tax policy at time $t+1$, after some manipulations, we attain the law of motion $\tau_{t+1} = T^d(\tau_t, h_t)$, as follows:

$$\tau_{t+1} = \sigma\left(h_t\right)\tau_t + \zeta\left(h_t\right) \tag{15A}$$

where:

$$\sigma\left(h_t\right)\equiv\frac{R\beta b_3}{\left(1+n\right)\left(R\left(1+\beta\right)-b_3\right)}Q_t^h$$
$$\zeta\left(h_t\right)\equiv\frac{R\left(1+\beta\right)\left(1+n\right)\left(b_1-b_2\right)+\left(1+n\right)^2\left(a_1-a_0\right)b_3}{\left(R\left(1+\beta\right)-b_3\right)\left(1+n\right)}\frac{1}{1+\lambda_1 h_t+\lambda_0}$$
$$-\frac{\beta R b_3}{\left(R\left(1+\beta\right)-b_3\right)\left(1+n\right)}\frac{1+h_t}{1+\lambda_1 h_t+\lambda_0}+\frac{R\left(1+\beta\right)\left(b_0+b_2\right)-\left(1+n\right)b_3 a_1}{R\left(1+\beta\right)-b_3}$$

Note that, under Eq. (11A), the convergence condition for k_t, Eq. (14A), is also sufficient for the convergence of τ_t, i.e. $\sigma\left(h^*\right)<1$. Furthermore the speed of convergence for τ_t basically depends on the WSR characterizing the economy jointly with the exogenous human capital society endowment. To show how such elements may affect the type of convergence let us take the derivative of $\Gamma\left(h_t\right)$ with respect to the human capital asset. We obtain:

$$\frac{d\zeta\left(h_t\right)}{dh_t}=\frac{-b_3\left(\beta R\left(1+\lambda_0-\lambda_1\right)+\left(1+n\right)^2\lambda\left(a_1-a_0\right)\right)-R\left(1+\beta\right)\left(1+n\right)\left(b_1-b_2\right)\lambda_1}{\left(1+n\right)\left(R\left(1+\beta\right)-b_3\right)\left(1+\lambda_1 h_t+\lambda_0\right)^2}$$

It is straightforward to show how the sign of $\frac{d\zeta(h_t)}{dh_t}$ crucially depends on the differences (a_1-a_0) and (b_1-b_2) and in final instance on $\bar{h}$, and on the relative political power weights of adults and old embedded in the coefficients b_1 and b_2. When $\frac{d\zeta(h_t)}{dh_t}\gtrless\left(\lesseqgtr\right)0$ and $\tau_0\lesseqgtr\left(\gtrless\right)\tau^*$ then the speed of convergence toward the steady state is lower (higher) than in the opposite case.

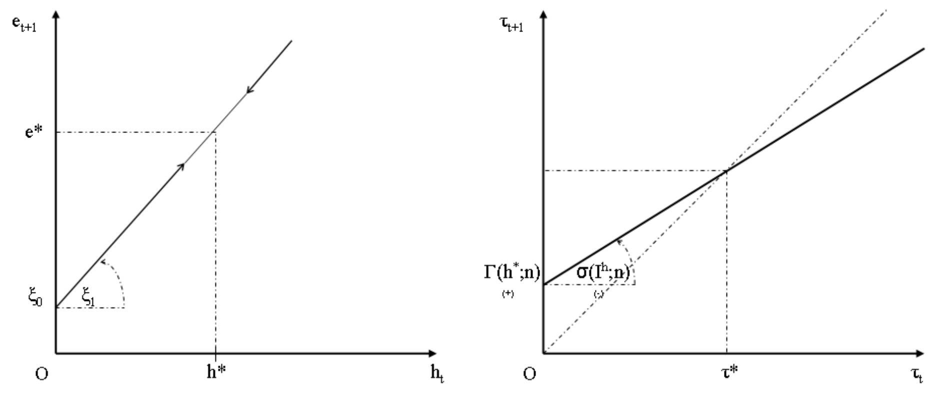

Fig. 8: Panel (a) shows the law of motion of e_t, Panel (b) shows the law of motion of τ_t

From a qualitative point of view the dynamics of e_t and τ_t are mirror image respectively to the dynamics of h_t and k_t. They mainly differ from an autoregressive component of infinite order in the past level of public education, which arises because of the infinite persistence of education spending on the future level of human capital through the parental transmission. Figure 8 emphasizes the dynamics of the political variables. Panel (a) shows that, once the human capital converges to the steady state also the education policy reaches its balanced growth path. Differently, Panel (b) highlights how the convergence condition of h_t is necessary but not sufficient for the stable convergence of the fiscal policy rule, which also requires the dynamic stability of k_t. ■

Proposition (1). Under Lemma 1, due to linearity of the laws of motion, Eq. (9A) , (10A) , (13A) and (15A), there exists a unique steady state $\{e^*, \tau^*, h^*, k^*\}$. Equating $h_{t+1} = h_t = h^*$ in Eq. (9A) and $k_{t+1} = k_t = k^*$ in Eq. (13A) , the following steady state levels for the state variables are obtained:

$$h^* = \frac{(1-\alpha)\,\bar{h}\sqrt{\psi^*}}{(1+n) - \alpha\sqrt{\psi^*}} \tag{16A}$$

and

$$k^* = \frac{\beta R\,(b_0+b_2-1) + (1+n)\,(b_0+b_2-(1+n)\,a_1)\,\lambda_1}{b_3\,((1+n)+\beta R) - R\,(1+\beta)\,(1+n)} h^* \tag{17A}$$
$$+\frac{((1+n)\,(1+\lambda_0)+\beta R)\,b_0 + ((1+n)+\beta R)\,b_1 + (1+n)\,b_2\lambda_0 - (1+n)^2\,(a_1\lambda_0+a_0) - \beta R}{b_3\,((1+n)+\beta R) - R\,(1+\beta)\,(1+n)}$$

Plugging Eq. (16A) and (17A) into the equilibrium policy rules described in Theorem 1, we obtain the following the steady states levels for the political control variables:

$$e^* = \frac{(1-\alpha)\,\bar{h}\psi^*}{(1+n) - \alpha\sqrt{\psi^*}}$$

and

$$\tau^* = -\frac{(1+n)\,(R\,(1+\beta)\,(b_1-b_2) + (1+n)\,(a_1-a_0)\,b_3)}{b_3\,((1+n)+\beta R) - R\,(1+\beta)\,(1+n)}\frac{1}{1+h^*}$$
$$+\frac{\beta R b_3}{b_3\,((1+n)+\beta R) - R\,(1+\beta)\,(1+n)}$$
$$-\frac{(1+n)\,(R\,(1+\beta)\,(b_0+b_2) - (1+n)\,b_3 a_1)}{b_3\,((1+n)+\beta R) - R\,(1+\beta)\,(1+n)}$$

By balanced budget constraint the pension steady state level is:

$$p^* = (1+n)(1+h^*)\tau^* - (1+n)^2 e^*$$

■

Corollary (1). The result comes directly from the proof of Theorem 1. ■

Corollary (2). The Proof is straightforward. The derivative of Eq. (1.20) with respect to h_t is equal to:

$$\frac{d\tau_t}{dh_t} = \frac{b_3 k_t + b_2 - b_1}{(1+h_t)^2} \tag{18A}$$

For any level of k_t, if $b_1 \leqslant b_2$, then $\frac{d\tau_t}{dh_t} \geqslant 0$. Otherwise, if $b_1 > b_2$, then the sign of Eq. (18A) depends on the value reached by k_t. When $k_t < \tilde{k}$ where $\tilde{k} \equiv \frac{b_1 - b_2}{b_3}$ the income tax rate is a decreasing function of h_t, i.e. $\frac{d\tau_t}{dh_t} < 0$. The opposite holds for $k_t \geqslant \tilde{k}$. ■

Corollary (3). Given the balanced budget constraint (1.7), let us denote with $\mathcal{P}(h_t, k_t) \equiv (1+n)(1+h_t)\mathcal{T}(h_t, k_t) - (1+n)^2 \mathcal{E}(h_t)$ the equilibrium pension policy rule. Under the decreasing return in education and the equilibrium level of policy rules, Eq. (1.20) and Eq. (1.19), the total amount of pension contributions can be rewritten as follows:

$$\begin{aligned} p_{t+1} &= \mathcal{P}(h_{t+1}, k_{t+1}) \qquad (19A) \\ &\equiv (1+n)\left(-b_3 k_{t+1} + (b_2 + b_0 - (1+n)a_1)h_{t+1} + (b_1 + b_0 - (1+n)a_0)\right) \end{aligned}$$

The derivative of (19A) with respect to e_t is equal to:

$$\frac{dp_{t+1}}{de_t} = (1+n)\left(-b_3 \frac{dk_{t+1}}{de_t} + (b_2 + b_0 - (1+n)a_1)\frac{dh_{t+1}}{de_t}\right) \tag{20A}$$

where under log utility $\frac{dk_{t+1}}{de_t} = -\frac{(b_2 + b_0 - a_1(1+n))}{R(1+\beta)}\frac{dh_{t+1}}{de_t}$. After some algebra, the derivative (20A) is as follows:

$$\frac{dp_{t+1}}{de_t} = \frac{R(1+\beta)(1+n)(b_2 + b_0 - a_1(1+n))}{R(1+\beta) - b_3}\frac{dh_{t+1}}{de_t} \tag{21A}$$

Noting that $(b_2 + b_0 - a_1(1+n)) > 0$ and $R(1+\beta) - b_3 > 0$, Eq. (21A) takes always positive values for any Welfare Regime and in the whole state space. ■

Corollary (4). Let us denote with $\rho = \frac{b_2+b_0}{b_1+b_0}$ a measure of the Welfare State Regimes' intensity. Then the higher the adults' relative power is, the larger is the value of ρ. Normalizing the Eq. (19A) by the factor $(b_1 + b_0)$, we obtain:

$$\mathring{p}_t = (1+n)\left[-\mathring{b}_3 k_t + (\rho - (1+n)\,\mathring{a}_1)\,h_t + (1 - (1+n)\,\mathring{a}_0)\right] \tag{22A}$$

where $\mathring{p}_t \equiv \frac{p_t}{b_1+b_0}$, $\mathring{b}_3 \equiv \frac{b_3}{b_1+b_0}$, $\mathring{a}_0 \equiv \frac{a_0}{b_1+b_0}$ and $\mathring{a}_1 \equiv \frac{a_1}{b_1+b_0}$. Taking the derivatives of Eq. (22A) with respect to ρ and k_t, the marginal impacts $\frac{d\mathring{p}_t}{d\rho} = (1+n)\,h_t > 0$ and $\frac{d\mathring{p}_t}{dk_t} = -(1+n)\,\mathring{b}_3 < 0$ are attained. In other words, the higher the level of ρ and the lower the level of physical capital are, the larger is the amount of pension benefits. ■

Corollary (5). The equilibrium education transfer chosen by politicians is the linear policy rule $\mathcal{E}(h_t) = a_1 h_t + a_0$, with a_1 and a_0 defined in Theorem 1. Political population aging, an increase in ϕ, does not affect at all the amount of equilibrium forward transfers, then $\frac{d\mathcal{E}}{d\phi} = 0$. The equilibrium level of income tax rate is instead a linear function of k_t and non linear in h_t, $\mathcal{T}(k_t, h_t) = -b_3\frac{k_t}{1+h_t} + b_2\frac{h_t}{1+h_t} + b_1\frac{1}{1+h_t} + b_0$, where the coefficients are fully described in Proposition 2. A variation in the exogenous political ideological bias ϕ determines the following marginal changes in the structural parameters: $\frac{db_3}{d\phi} = -\frac{R(1+n)(1+\beta)}{(\phi+(1+n)(1+\beta))^2} < 0$, $\frac{db_2}{d\phi} = \frac{\alpha\psi^*(1+n)(1+\beta)}{(\phi+(1+n)(1+\beta))^2} > 0$, $\frac{db_1}{d\phi} = \frac{(1+n)(1+\beta)\left(1+n+(1-\alpha)\psi^*\bar{h}R\right)}{(R-(1+n))(\phi+(1+n)(1+\beta))^2} > 0$ and $\frac{db_0}{d\phi} = \frac{(1+n)(1+\beta)}{(\phi+(1+n)(1+\beta))^2} > 0$. Then, for any level of $\bar{h}$ $\frac{d\mathcal{T}}{d\phi} > 0$, which implies positive correlation between the pension benefits and the ideological bias in favor of old agents. Finally, using the above results, the derivative of pensions transfers obtained by balanced budget constraint, $\mathcal{P}(h_t, k_t) = (1+n)\left((1+h_t)\,\mathcal{T}(h_t, k_t) - (1+n)\,\mathcal{E}(h_t)\right)$, with respect to the political aging parameter is $\frac{d\mathcal{P}}{d\phi} = (1+n)\left((1+h_t)\frac{d\mathcal{T}}{d\phi}\right) > 0$. ■

Corollary (6). To determine the effect of demographic population aging on the level of education transfers chosen by politicians, i.e. a decrease in n, note that $\frac{da_1}{dn} = -\frac{\alpha}{(1+n)^2}\psi^* < 0$ and $\frac{da_0}{dn} = -\frac{1-\alpha}{(1+n)^2}\bar{h}\psi^* < 0$. Then it follows $\frac{d\mathcal{E}}{dn} < 0$. Concerning the impact of n on the political equilibrium level of income tax rate the following marginal changes in the structural parameters hold: $\frac{db_3}{dn} = \frac{\phi+R(1+\beta)}{(\phi+(1+n)(1+\beta))^2} > 0$, $\frac{db_2}{dn} = -\frac{\alpha\psi^*\phi(1+\beta)}{(\phi+(1+n)(1+\beta))^2} < 0$, $\frac{db_1}{dn} = D_0 + D_1 D_2 \gtreqless 0$, where $D_0 \equiv \frac{\phi}{(R-(1+n))(\phi+(1+\beta)(1+n))} > 0$, $D_1 \equiv \frac{\phi\left(1+n+\bar{h}\psi^*(1-\alpha)R\right)}{(R-(1+n))(\phi+(1+\beta)(1+n))} > 0$ and $D_2 \equiv \left(\frac{1}{R-(1+n)} - \frac{1+\beta}{\phi+(1+\beta)(1+n)}\right) = \frac{1}{1+n}\left(\bar{R}-\Omega_A\right) \gtreqless 0$ if $\bar{R} \gtreqless \Omega_A$, finally $\frac{db_0}{dn} = -\frac{\phi(1+\beta)}{(\phi+(1+n)(1+\beta))^2} < 0$. Then it follows that $\frac{d\mathcal{T}}{dn} \gtreqless 0$ depending on the difference $\left(\bar{R}-\Omega_A\right)$ and on the level of $\bar{h}$. In particular a sufficient condition to yield $\frac{d\mathcal{T}}{dn} < 0$ is $\bar{R} < \Omega_A$ and $\bar{h}$ high enough. Finally the marginal variation of pension benefits due to population growth is equal to $\frac{d\mathcal{P}}{dn} = (1+n)\left((1+h_t)\frac{d\mathcal{T}}{dn} - (1+n)\frac{d\mathcal{E}}{dn}\right) \gtreqless 0$. ■

1.9 Technical Appendix B

1.9.1 Derivation of Generalized Euler Equation

We derive the recursive formulation of the Gvt program starting from its sequential version:

$$V_0^g(h_0,k_0) = \max_{\{f_t^c,h_{t+1},k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (1+n)^t \delta^t U(f_t^g,h_t,k_t,k_{t+1})$$

where (h_0,k_0) are the initial conditions of the payoff-relevant state variables of the dynamic optimization program and $U(f_t^g,h_t,k_t,k_{t+1}) \equiv (1+n)\,\delta u(C_t^1(\tau_t^g,h_t,k_{t+1}))+$ $\beta u\left(C_t^2(f_t^g,h_t,k_t)\right)$. Equivalently we rewrite the above value function in the following terms:

$$V_0^g(h_0,k_0) = \max_{\{f_0^g,k_1\}} U(f_0^g,h_0,k_0,k_1)+(1+n)\,\delta \max_{\{f_t^g,h_{t+1},k_{t+1}\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} (1+n)^t \delta^t U(f_t^g,h_t,k_t,k_{t+1}) \tag{1B}$$

By definition, we have:

$$V_1^g(h_1,k_1) = \max_{\{f_t^g,h_{t+1},k_{t+1}\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} (1+n)^t \delta^t U(f_t^g,h_t,k_t,k_{t+1}) \tag{2B}$$

Due to stationarity condition, the indirect utility function satisfies $V_0^g(\cdot) \equiv$ $V_1^g(\cdot) \equiv ... \equiv V_t^g(\cdot)$. We omit time indexes and denote by prime symbol next period variables. Plugging Eq. (2B) into Eq. (1B) we yield the following Bellman equation:

$$V^g(h,k) = \max_{\{f^g,h',k'\}} U\left(f^g,h,k,k'\right) + (1+n)\,\delta V^g\left(h',k'\right)$$

subject to the constraints $k' = \mathcal{K}(f^g,h)$ and $h' = H(e^g,h)$. We rewrite the Bellman equation as follows:

$$V^g(h,k) = \max_{f^g} U(f^g,h,k,\mathcal{K}(f^g,h)) + (1+n)\,\delta V^g(H(e^g,h),\mathcal{K}(f^g,h)) \tag{3B}$$

The GEE are obtained as the FOC of the Gvt optimization plan. The derivation follows the method proposed by Klein et al. (2008) and it is extended to OLG case with two political controls in bidimensional state-space. For simplicity of notation we will omit the apex g. The political first order conditions of Eq. (3B) with respect to $f = \{e, \tau\}$ are equal to:

$$0 = U_e + U_{k'}\mathcal{K}_e + (1+n)\,\delta\,(V_{h'}H_e + V_{k'}\mathcal{K}_e) \tag{4B}$$

$$0 = U_\tau + U_{k'}\mathcal{K}_\tau + (1+n)\,\delta V_{k'}\mathcal{K}_\tau \tag{5B}$$

Using Benveniste-Scheinkman formula we obtain the following expression for V_h and V_k:

$$V_h = U_h + U_{k'}\mathcal{K}_h + (1+n)\,\delta\,(V_{h'}H_h + V_{k'}\mathcal{K}_h) \tag{6B}$$

$$V_k = U_k \tag{7B}$$

From Eq. (4B) and (5B) we obtain the expression for $V_{h'}$ and $V_{k'}$:

$$V_{h'} = \frac{1}{(1+n)\,\delta H_e}\left(\frac{U_\tau\mathcal{K}_e - U_e\mathcal{K}_\tau}{\tilde{K}_\tau}\right) \tag{8B}$$

$$V_{k'} = -\frac{U_\tau + U_{k'}\mathcal{K}_\tau}{(1+n)\,\delta\mathcal{K}_\tau} \tag{9B}$$

Plugging Eq. (8B) and (9B) into (6B) we get the final expression for V_h:

$$V_h = U_h + \frac{U_\tau\mathcal{K}_e - U_e\mathcal{K}_\tau}{\mathcal{K}_\tau}\frac{H_h}{H_e} - U_\tau\frac{\mathcal{K}_h}{\mathcal{K}_\tau} \tag{10B}$$

Using stationarity condition and plugging Eq. (7B) and (10B) into (4B) and (5B), we obtain the GEEs of the Gvt problem respectively for e and τ:

$$0 = U_e + U_{k'}\mathcal{K}_e + (1+n)\,\delta\left(\left(U'_{h'} + \frac{U'_{\tau'}\mathcal{K}'_{e'} - U'_{e'}\mathcal{K}'_{\tau'}}{\mathcal{K}'_{\tau'}}\frac{H'_{h'}}{H'_{e'}} - U'_{\tau'}\frac{\mathcal{K}'_{h'}}{\mathcal{K}'_{\tau'}}\right)H_e + U'_{k'}\mathcal{K}_e\right) \tag{11B}$$

$$0 = U_\tau + \left(U_{k'} + (1+n)\,\delta U'_{k'}\right)\mathcal{K}_\tau \tag{12B}$$

From definition of U, we have: $U_e = -\beta\,(1+n)^2\,u_{C^2}$, $U_\tau = (1+n)\,(1+h)\,(\beta u_{C^2} - \delta u_{C^1})$, $U_h = (1+n)\,(\beta\tau u_{C^2} + \delta\,(1-\tau)\,u_{C^1})$, $U_k = \beta R\,(1+n)\,u_{C^2}$ and $U_{k'} = -\delta\,(1+n)^2\,u_{C^1}$. Using the above partial derivatives and rewriting $\mathcal{K}_i$ where $i \in (f^g, h)$ in terms of $\eta\,(\cdot)$, we get the *GEEs* as a weighted combination of intergenerational wedges:

$$0 = \Delta_e + \delta\frac{\eta_e}{\eta_{k'}}\Delta_s + \delta\Xi\Delta'_{\tau'}$$

$$0 = (1+h)\,\Delta_\tau + \delta(1+n)\frac{\eta_\tau}{\eta_{k'}}\Delta_s$$

where $\Xi \equiv H_e\left(\tau' + (1+h')\left(\frac{\mathcal{K}'_{e'}}{\mathcal{K}'_{\tau'}}\frac{H'_{h'}}{H'_{e'}} - \frac{\mathcal{K}'_{h'}}{\mathcal{K}'_{\tau'}}\right)\right) = \frac{d((1+h')\tau')}{de}$ and the first best wedges Δ_x with $x \in \{s, \tau, e\}$ are defined as $\Delta_\tau \equiv \beta u_{C_2} - \delta u_{C_1}$, $\Delta_s \equiv u_{C_1} - \beta R u_{C'_2}$ and $\Delta_e \equiv \delta H_e\left(\beta\,(1+n)\,\frac{H'_{h'}}{H'_{e'}}u_{C'_2} + \delta u_{C'_1}\right) - \beta u_{C_2}$.

Proposition (2). Let us guess as equilibrium policy functions for the Benevolent Government solution the following functional form for e and τ, respectively:

$$e^g = a_1^g h + a_0^g \bar{h} \tag{13B}$$

$$\tau^g = b_3^g\frac{k}{1+h} + b_2^g\frac{h}{1+h} + b_1^g\frac{1}{1+h} + b_0^g \tag{14B}$$

If Eq. (13B) and (14B) are the equilibrium of the Gvt problem, then they must satisfy simultaneously the *GEEs* given by conditions (11B) and (12B). Let us manipulate the *GEEs*, plugging the expressions for each partial derivative. We obtain for τ and e, respectively:

$$0 = -\beta u_{C^2} + \delta\left(\begin{array}{c}(\beta\tau' u_{C^{2\prime}} + \delta\,(1-\tau')\,u_{C^{1\prime}})\\ +(1+h')\,(\beta u_{C^{2\prime}} - \delta u_{C^{1\prime}})\left(\frac{\mathcal{K}'_{e'}}{\mathcal{K}'_{\tau'}}\frac{H'_{h'}}{H'_{e'}} - \frac{\mathcal{K}'_{h'}}{\mathcal{K}'_{\tau'}}\right)\\ +\beta(1+n)u_{C^{2\prime}}\frac{H'_{h'}}{H'_{e'}}\end{array}\right)H_e \tag{15B}$$

$$0 = \beta u_{C^2} - \delta u_{C^1} \tag{16B}$$

Using the equation of $H\,(\cdot)$, the following expressions result:

$$\begin{aligned} H_e &= \frac{\alpha h + (1-\alpha)\bar{h}}{2(1+n)h'}\\ \frac{H'_{h'}}{H'_{e'}} &= \frac{\alpha e'}{\alpha h' + (1-\alpha)\bar{h}} \end{aligned} \tag{17B}$$

Under logarithmic utility and linear production function, plugging the guess given by Eq. (13B) and (14B) into the saving function, we obtain the following recursive function for saving choice:

$$k' = \mathcal{K}\,(e,\tau,h) = \frac{\beta R}{(1+n)\,(b_3^g + R\,(1+\beta))}\,(1+h)\,(1-\tau) \tag{18B}$$

$$-\frac{b_2^g + b_0^g - (1+n)\,a_1^g}{b_3^g + R\,(1+\beta)}\sqrt{\frac{\left(\alpha h + (1-\alpha)\,\bar{h}\right)e}{1+n}} - \frac{b_1^g + b_0^g - (1+n)\,a_0^g\bar{h}}{b_3^g + R\,(1+\beta)}$$

Using Eq. (17B) and (18B) and simplifying, we get $\frac{\mathcal{K}'_{e'}}{\mathcal{K}'_{\tau'}}\frac{H'_{h'}}{H'_{e'}} - \frac{\mathcal{K}'_{h'}}{\mathcal{K}'_{\tau'}} = \frac{1-\tau'}{1+h'}$. Finally rearranging all the terms, Eq. (15B) becomes as follows:

$$0 = -u_{C^2} + \delta\left(1 + (1+n)\frac{\alpha e'}{\alpha h' + (1-\alpha)\bar{h}}\right) u_{C^{2\prime}} H_e \tag{19B}$$

Using the political Euler condition $\beta u_{C^2} - \delta u_{C^1} = 0$ and the economic one $u_{C^1} - R\beta u_{C^{2\prime}} = 0$, Eq. (19B) simplifies to:

$$1 = \left(1 + (1+n)\frac{\alpha e'}{\alpha h' + (1-\alpha)\bar{h}}\right)\frac{1}{R} H_e \tag{20B}$$

Eq. (20B) is equivalent to:

$$e = \left(\left(1 + (1+n)\frac{\alpha e'}{\alpha h' + (1-\alpha)\bar{h}}\right)\frac{1}{2R}\right)^2 \frac{\alpha h + (1-\alpha)\bar{h}}{1+n} \tag{21B}$$

Let us now make a further assumption on the guess on e, considering the following variant of Eq. (13B):

$$e^g = a_1^g(\psi^g)\, h + a_0^g(\psi^g)\, \bar{h} \tag{22B}$$

such that $a_1^g(\psi^g) = \frac{\alpha}{1+n}\psi^g$ and $a_0^g(\psi^g) = \frac{1-\alpha}{1+n}\psi^g$, i.e. we guess the policy e as a linear convex combination between parental human capital h and human capital society endowment $\bar{h}$ scaled by a constant which has to be determined, ψ^g. Then Eq. (21B) can be rewritten as follows:

$$e^g = \frac{\alpha}{1+n}\tilde{\psi}^g h + \frac{1-\alpha}{1+n}\tilde{\psi}^g \bar{h}$$

where $\tilde{\psi}^g \equiv \left(\left(1 + (1+n)\frac{\alpha e'}{\alpha h' + (1-\alpha)\bar{h}}\right)\frac{1}{2R}\right)^2$. Plugging the guess of e^g given by Eq. (22B) into the expression of $\tilde{\psi}^g$ and simplifying we get:

$$\tilde{\psi}^g = (1 + \alpha\psi^g)^2 \left(\frac{1}{2R}\right)^2$$

By fixed-point condition $\tilde{\psi}^g = \psi^g$ which yield the following solutions $\psi^g_{1,2} = \frac{1}{\alpha}\left(\frac{2R}{\alpha}\left(R \pm \sqrt{R^2 - \alpha}\right) - 1\right)$. Similar arguments as in Proof of Theorem 1 can be made. Then let us consider the stable root $\psi^* = \frac{1}{\alpha}\left(\frac{2R}{\alpha}\left(R - \sqrt{R^2 - \alpha}\right) - 1\right)$ as feasible solution. It immediately follows that $a_1^g = \frac{\alpha}{1+n}\psi^*$ and $a_0^g = \frac{1-\alpha}{1+n}\psi^*$

are the solutions for the guess on e which turns out to be equivalent to the political outcome After plugging the guesses, Eq. (13B) and Eq. (14B), and the recursive saving function, Eq. (18B), into Eq. (16B), the GEE for the policy τ is as follows:

$$\beta \frac{1}{(1+n)Rk + (1+n)(1+h)\tau - (1+n)^2 e^g} = \delta \frac{1}{(1+h)(1-\tau) - (1+n)\tilde{K}\left(e^g, \tau, h\right)}$$

After some algebraic manipulations we obtain the following well-defined system:

$$\begin{cases} b_0^g = \frac{\beta\left(R+b_3^g\right)}{R(\beta+\delta(1+n)(1+\beta))+(\beta+\delta(1+n))b_3^g} \\ \\ b_1^g = \frac{(1+n)\beta\left(b_0^g+b_1^g-(1-\alpha)\bar{h}\psi\right)-(1-\alpha)\bar{h}\sqrt{\psi}\left(-\beta\left(b_0^g+b_2^g\right)-\delta(1+n)\left(R(1+\beta)+b_3^g\right)\sqrt{\psi}+\alpha\beta\psi\right)}{\left(1+\bar{h}\right)\left(R(\beta+\delta(1+n)(1+\beta))+(\beta+\delta(1+n))b_3^g\right)} \\ \\ b_2^g = \frac{\bar{h}\alpha\sqrt{\psi}\left(\beta\left(b_0^g+b_2^g\right)+\delta(1+n)\left(R(1+\beta)+b_3^g\right)\sqrt{\psi}-\alpha\beta\psi\right)}{R(\beta+\delta(1+n)(1+\beta))+(\beta+\delta(1+n))b_3^g} \\ \\ b_3^g = -\frac{R(1+n)\delta\left(R(1+\beta)+b_3^g\right)}{R(\beta+\delta(1+n)(1+\beta))+(\beta+\delta(1+n))b_3^g} \end{cases}$$

Solving the system we obtain the following two solutions for τ:

$$\tau^{g1} = b_3^{g1}\frac{k}{1+h} + b_2^{g1}\frac{h}{1+h} + b_1^{g1}\frac{1}{1+h} + b_0^{g1} \tag{23B}$$

where, under $\Omega_R^g \equiv \frac{\delta(1+n)(1+\beta)}{\beta+\delta(1+n)}$ and $\Omega_O^g \equiv \frac{\beta(1-\delta(1+n))}{\beta+\delta(1+n)}$:

$$\begin{aligned} b_1^{g1} &= \bar{h}\tfrac{1-\alpha}{\alpha}\tfrac{R}{R-(1+n)}\mu_2 + \bar{R}\left(\Omega_O^g - \bar{h}\left(1-\alpha\right)\psi^*\right); \quad & b_3^{g1} &= -R\Omega_R^g; \\ b_2^{g1} &= \tfrac{\alpha\sqrt{\psi^*}}{R-\alpha\sqrt{\psi^*}}(\Omega_O^g + R\sqrt{\psi^*}\Omega_R^g - \alpha\sqrt{\psi^*}); & b_0^{g1} &= \Omega_O^g; \end{aligned}$$

and

$$\tau^{g2} = b_3^{g2}\frac{k}{1+h} + b_2^{g2}\frac{h}{1+h} + b_1^{g2}\frac{1}{1+h} + b_0^{g2} \tag{24B}$$

where $b_3^{g2} = -R$, $b_1^{g1} = (1-\alpha)\,\psi^*\bar{h}$, $b_2^{g1} = \alpha\psi^*$ and $b_0^{g1} = 0$.

Note that the Eq. (23B) is equivalent to Eq. (24B) under the condition $\Omega_R^g = 1$ and $\Omega_O^g = 0$, which implies $\delta = \frac{1}{1+n}$. Recall that, for the existence of the fix point, the condition $\delta < \frac{1}{1+n}$, which induces Ω_O^g to be strictly greater than zero, is required. Consequently the Eq. (24B) is not feasible. ■

Proposition (3). Let us first consider the following normalization of the relative Welfare weights, Eq. (1.22), after assigning $\phi \equiv \frac{\beta}{\delta}$:

$$\Omega_R^g \equiv \frac{(1+n)(1+\beta)}{\phi+(1+n)} \quad \text{and} \quad \Omega_O^g \equiv \frac{\phi-\beta(1+n)}{\phi+(1+n)} \tag{25B}$$

Using the weights (25B) and comparing the parameters of the policy rules of Eq. (1.20) and Eq. (1.27) we obtain for any $\phi < \bar{\phi}$ where:

$$\bar{\phi} \equiv \frac{1}{2}\left(\begin{array}{c} -1-n \\ -\frac{\sqrt{(1+n)^2\left(-1+2R\sqrt{\psi}-\alpha\psi\right)\left(-(1+2\beta)^2+2R(1+2\beta(1+\beta))\sqrt{\psi}-\alpha\psi\right)}}{-1+2R\sqrt{\psi}-\alpha\psi} \end{array} \right)$$

the following inequalities must hold:

$$b_0^g < b_0, \quad b_1^g < b_1, \quad b_2^g < b_2, \quad b_3^g > b_3$$

Then we conclude $\mathcal{T}(h_t, k_t) > \mathcal{T}^g(h_t, k_t)$ for any $\{h_t, k_t\} \in \Upsilon^{Pol} \cap \Upsilon^{Gvt}$. $\blacksquare$

Bibliography

[1] Abel, A. B., Mankiw ,N. G., Summers, L. H. and Zeckhauser, R. J., 1989, Assessing Dynamic Efficiency: Theory and Evidence, *Review of Economic Studies,* 56 (1), 1-19.

[2] Azariadis, C., and Galasso, V., 2002, Fiscal Constitutions, *Journal of Economic Theory*, 103 (2), 255-281.

[3] Azzimonti Renzo, M., Sarte, P. D., and Soares, J., 2009, Discretionary Taxes and Public Investment when Government Promises are not Enforceable, *Journal of Economic Dynamics and Control*, 33 (9), 1662-1681.

[4] Bassetto, M., 2008, Political Economy of Taxation in an Overlapping-Generations Economy, *Review of Economic Dynamics*, 11 (1), 18-43.

[5] Baron, D., and Ferejohn, J., 1989, Bargaining in Legislatures, *American Political Science Review*, 83, 1181-1206.

[6] Battaglini, M., and Coate, S., 2007, Inefficiency in Legislative Policy-Making: A Dynamic Analysis, *American Economic Review*, 97 (1), 118-149.

[7] Becker, G. S., and Tomes, N., 1986, Human Capital and the Rise and Fall of Families, *Journal of Labor Economics*, 4 (3), 1-39.

[8] Bellettini, G., and Berti Ceroni, C., 1999, Is Social Security Really Bad for Growth?, *Review of Economic Dynamics*, 2 (4), 796-819.

[9] Boldrin, M. and Montes, A., 2005, The Intergenerational State Education and Pension, *Review of Economic Studies*, 72 (3), 651-664.

[10] Conde-Ruiz, J., and Galasso, V., 2005, Positive Arithmetic of the Welfare State, *Journal of Public Economics*, 88 (9), 933-955.

[11] Ehrlich, I., and Lui, F., 1998, Social Security, the Family, and Economic Growth, *Economic Enquiry*, 36, 390-409.

[12] Feldstein, M., 1974, Social Security, Induced Retirement and Aggregate Capital Accumulation, *Journal of Political Economy*, 82 (5).

[13] Forni, L., 2005, Social Security as Markov Equilibrium in OLG Models, *Review of Economic Dynamics*, 8 (1), 178-194.

[14] Galasso, V., and Profeta, P., 2002, The Political Economy of Social Security: A Survey, *European Journal of Political Economy*, 18 (1), 1-29.

[15] Galasso, V., and Profeta, P., 2004, Politics, Ageing and Pensions, *Economic Policy*, 19 (38), 63-115.

[16] Gonzalez-Eiras, M., and Niepelt, D., 2008, The Future of Social Security, *Journal of Monetary Economics,* 55(2), 197-218.

[17] Gonzalez-Eiras, M., and Niepelt, D., 2011, Aging, Government Budgets, Retirement, and Growth, *CESifo Working Paper.*

[18] Grossman, G. M., and Helpman, E., 1998, Intergenerational Redistribution with Short-lived Government, *Economic Journal*, 108, 1299-1329.

[19] Hassler, J., Krusell, P., Storesletten, K. and Zilibotti, F., 2005, The Dynamics of Government, *Journal of Monetary Economics*, 52 (7), 1331-1358.

[20] Hassler, J., Storesletten, K. and Zilibotti, F., 2007, Democratic Public Good Provision, *Journal of Economic Theory*, 133 (1), 127-151.

[21] Kaganovich, M., and Zilcha, I., 1999, Education, Social Security, and Growth, *Journal of Public Economics*, 71 (2), 289-309.

[22] Kehoe, T. J., and Levine, D. K., 2001, Liquidity Constrained Markets versus Debt Constrained Markets, *Econometrica*, 69 (3), 575-598.

[23] Klein, P., Krusell, P., and Ríos-Rull, J., 2008, Time-Consistent Public Policy, *Review of Economic Studies*, 75 (3), 789-808.

[24] Krusell, P., Quadrini, V. and Ríos-Rull, J. V., 1997, Politico-Economic Equilibrium and Economic Growth, *Journal of Economic Dynamics and Control*, 21 (1), 243-272.

[25] Lee, R., 2007, Demographic Change, Welfare, and Intergenerational Transfers: A Global Overview. In Veron, J., Pennec, S., and Legare, J., (eds.) Ages, Generations and the Social Contract: The Demographic Challanges Facing the Welfare State. Springer, 17-43.

[26] Lindbeck, A. and J. Weibull, 1987, Balanced-budget Redistribution as the Outcome of Political Competition, *Public Choice*, 52, 273-297.

[27] Lizzeri, A., and N. Persico, 2001, The Provision of Public Goods under Alternative Electoral Incentives, *American Economic Review*, 91 (1), 225-239.

[28] Maskin, E., and Tirole, J., 2001, Markov Perfect Equilibrium: I. Observable Actions, *Journal of Economic Theory*, 100 (2), 191-219.

[29] Mulligan, C. B., and Xala-i-Martin, X., 1999, Gerontocracy, Retirement, and Social Security, *NBER Working Papers* 7117.

[30] OECD, 2007, Social Expenditure Database 1980-2003, OECD Press.

[31] OECD, 2008, Education at a Glance - OECD Indicators 2008, OECD Press.

[32] Persson, T. and Tabellini, G., 2000, Political Economy - Explaining Economy Policy, MIT Press.

[33] Plott, C., 1967, A notion of equilibrium and its possibility under majority rule, *American Economic Review*, 57 (4), 787-806.

[34] Rangel, A., 2003, Forward and Backward Intergenerational Goods: Why is Social Security Good for Environment?, *American Economic Review*, 93, 813-834.

[35] Shepsle, K. A., 1979, Institutional Arrangements and Equilibrium in Multidimensional Voting Models, *American Journal of Political Science*, 23 (1), 27-59.

[36] Tabellini, G., 1991, The Politics of Intergenerational Redistribution, *Journal of Political Economy*, 99 (2), 335-57.

Chapter 2

Politicians, Redistribution and Intergenerational Conflicts

2.1 Introduction

Decentralized overlapping generations economies populated by selfish agents typically feature dynamically inefficient asset accumulation paths. When credit markets are incomplete and human capital is the main engine of growth, two different sources of inefficiencies simultaneously arise. On one hand, young are precluded from borrowing money in order to acquire skills and, in turns, increase future labor productivity. As a consequence, educational investments tend to be inefficiently low, inducing underaccumulation of human capital. On the other hand, adults are restricted from lending money, having a limited investment portfolio. As a result, they tend to overaccumulate physical capital, depressing future return to capital. In this scenario, there is room to investigate the existence of intergenerational institutions, which might improve the welfare of all generations. Previous literature (Boldrin and Montes, 2005; Docquier et al., 2007) has adopted a normative perspective to determine the fiscal scheme, which allows the decentralized economy to reach the efficient allocation.[1]

[1] Boldrin and Montes (2005) show how an interconnected pension and public education system can replicate the allocation achieved by complete market. The authors formalize public education and PAYG system as two parts of an intergenerational contract where public pension is the return on the investment into the human capital of the next genera-

We depart from the previous theoretical contributions by adopting a positive perspective. Specifically, we analyze how voting institutions and intergenerational conflicts over the implementation of redistributive policies affect dynamic efficiency and welfare. A workhorse of past literature in political economy (Alesina and Tabellini, 1990; Barro, 1991; Azzimonti, 2011) concerns the study of how political disagreement and uncertainty depress growth and reduce welfare when intra-cohort heterogeneity is explicitly considered. When different groups disagree over the composition of public expenditure and parties alternate in power via democratic process, governments tend to be endogenously short-sighted. Due to political uncertainty, the economy experiences under-investment of productive assets and, in turns, loss in efficiency.[2]

Our perspective is opposite to the one stated above. When inter-cohort heterogeneity is explicitly considered and human capital accumulation is the main force driving the economic growth, intergenerational conflicts over the redistribution of public resources may enhance efficiency and improve welfare. Although the preferences of younger agents to sustain productive public spending are growth promoting, nonetheless the public education transfers crowds-out private consumption. As a result, the education spending maximizing welfare turns out to be smaller than that maximizing growth. Consequently, welfare improvements are achievable by reallocating a share of government spending from public educational to pork-barrel transfers. This simple idea helps to justify why the existence of institutions, which reinforce intergenerational political disagreement and uncertainty, may enhance to achieve self-enforcing politico-economic equilibria closer to the social optimal allocation.

To support the positive relation between intergenerational conflicts and

---

tion. In presence of credit market constraint, the welfare state is justified by the inability of decentralized markets to deliver a Pareto efficient solution. Relaxing the definition of optimality by explicitly considering the positive externality generated by educational investments, Docquier et al. (2007) show that no justifications to provide public pension benefits emerge, when the dynastic welfare weight is sufficiently high. For realistic values of discount rates, the achievement of efficient allocation is guaranteed by taxing the retirees in order to provide educational subsidy.

[2] The negative correlation between political instability and private investment has been widely investigated both theoretically (Alesina and Tabellini, 1990) and empirically (Barro, 1991). Azzimonti (2011) extends the previous studies in a dynamic electoral competition framework. Ruling out commitment devices, the author gives further theoretical support to the understanding of how political stability mitigates the effects of polarization dampening the inefficiencies.

efficiency, we present a tractable dynamic politico-economic model in a neoclassical environment. Focusing on a three period OLG economy with endogenous growth generated by human capital accumulation, we determine the subgame Markov perfect equilibrium of a dynamic game characterized by both ideological heterogeneous voters and office-seeking political parties. Only adult and elderly have voting power. The electoral competition takes place in a majoritarian probabilistic environment, where political parties compete proposing multidimensional fiscal platforms, in order to maximize the probability of winning election.[3]

Three main sources of distortions characterize the theoretical environment: Incomplete credit markets and physical capital taxation (economic distortion), and political uncertainty (political distortion). In order to better perform the analysis and provide clear theoretical insights, we distinguish two cases.

The first scenario is characterized by neither capital taxation nor political distortions. Consequently, the only source of inefficiency comes from the credit market constraint. In equilibrium the presence of swing voters determines the emergence of intergenerational conflicts over the redistribution of public resources. The ratio of the idiosyncratic ideological densities between the two cohorts of voters represents a simple quantitative measurement of the political disagreement. The median voter framework where adults are the decisive voter can be considered as an extreme benchmark case with no intergenerational political disagreement. In equilibrium the politicians implement a multidimensional platform characterized by: i) negative transfers from the elderly, ii) positive transfers to subsidize the consumption of adults, and iii) high investment in public education. Differently from the previous literature (Azariadis and Galasso, 2002; Forni, 2005), which studies how intergenerational transfers might be sustained in a median voter framework with exogenous growth, here the relevant state variable of the economy is the total return to capital, which is affected by both human and physical capital. Adults have incentives to invest in the education of young in order to accumulate human capital and, in turns, increase the future return to capital. At the same time the incentives to transfer public

[3] According to Persson and Tabellini (2000), we adopt an opportunistic models of electoral competition, where politicians extract rent from being in power in order to maximize the probability of winning elections. While this approach dates back to the 1970s, its resurgence in popularity stems from Lindbeck and Weibull (1987).

resource backward (i.e. pension benefits in the form of PAYGO system) do not arise. Adults perfectly anticipate that when old they are prevented to grab public resources by exerting political power. As a consequence, in equilibrium public education investment is financed by taxes paid by the elderly. Because of the resulting overaccumulation of both physical and human capital the economy turns out to be still characterized by dynamic inefficiency. In presence of ideological uncertainty, as soon as swing voters on behalf of the elderly emerge, the equilibrium political platform reverses, turning out to be characterized by: i) positive transfers to the elderly, ii) negative transfers from the adult, and iii) lower investment in public education. The participation of the elderly to the political debate with the emergence of political disagreement on the redistribution of public resources improves the overall economic efficiency. The elderly claim for positive transfers induces a simultaneous reduction in both physical and human capital accumulation, partially offsetting the dynamic inefficiency.

The second scenario concerns the analyses of the politico-economic equilibrium when both economic and political distortions are explicitly introduced. Public expenditures are financed through both labor and capital income tax. In absence of commitment technology, the distortionary taxation adds an additional source of inefficiency with respect to the previous analyses.[4] Furthermore, forward-looking politicians, who compete ideologically, might strategically manipulate their probability to win future election by implementing the current fiscal platform. To the best of our knowledge, this is the first study, which analyzes in a dynamic framework how the introduction of political distortions may affect the equilibrium outcome.[5] We characterize the time-consistent Markov perfect politico-economic equilibrium in terms of Generalized Euler Conditions (GEEs). Finally, we discuss how fiscal and political distortions alter the main finding about the positive correlation between intergenerational political disagreement and efficiency.

The remainder of the paper is organized as follows. Section 2.2 presents

[4] This component is similar to the optimality conditions derived in the previous literature by Klein, Krusell and Rios-Rull (2008).

[5] Azzimonti (2011) studies how political competition may endogenously affect the probability of winning future elections. However, considering a partisan model of electoral competition and limiting the analyses to symmetric strategy, the author finds that in equilibrium the probability of being elected are constant and only affected by the incumbency power index. As a consequence, it turns out to be no strategically manipulated by politicians.

the environment. Section 2.3 characterizes the first best allocation. Section 2.4 describes the politico-economic equilibrium in the perfect forward-looking scenario. In Section 2.5 we fully characterize the case with undistortionary taxation, providing a closed-form economy example. In Section 2.6 and 2.7 we show how distortionary taxation and ideological competition affects the equilibrium prescriptions, respectively. Section 2.8 concludes. The appendix contains all the proofs.

2.2 The Model

Consider an OLG economy populated by an infinite number of ideological heterogeneous agents, living up to three-periods: young, adult and old. We denote by $\tau \in \{1, 2\}$ the adult and elderly cohorts, respectively. Time is discrete, indexed by t, and runs from zero to infinity. Population growth rate is exogenous and equal to zero with a unitary mass for each cohort. Furthermore there are two ideological heterogeneous infinite living parties, left and right, denoted by $i \in \{\mathcal{L}, \Re\}$, who compete proposing at each time an electoral platform in order to maximize their probability of winning election.

2.2.1 Household

The intertemporal random preference of a representative agent j born at time $t-1$ and living at time t is defined as follows:

$$u\left(c^1_{i_t}\right) + \sigma^1_{ji_t} + \beta E^{\mathcal{P}_{i_t}}\left(u\left(c^2_{i_{t+1}}\right) + \sigma^2_{ji_{t+1}}\right) \tag{2.1}$$

where $\beta \in (0, 1)$ is the individual discount factor and $i_t \in \{\mathcal{L}_t, \Re_t\}$. $\mathcal{P}_{i_t}$ denotes the endogenous probability of party $\Re$ of being in power at time $t+1$ when the incumbent party is i_t. $\sigma^\tau_{ji_t}$ is equal to the individual ideological bias toward party $\Re$ of agent j belonging to cohort τ at time t. $c^1_{i_t}$ represents the consumption when adult, and $c^2_{i_{t+1}}$ denotes the consumption when old.[6] Young do not consume.

The stochastic component of preferences can be decomposed into two

[6] Apart from the different time horizon, the unique source of heterogeneity among agents is the ideology. Given that the ideological component is independent from the perceived benefits of consumption, saving decisions are identical across agents. Ideological heterogeity only affects the individual voting decisions.

terms, as follows:

$$\sigma^{\tau}_{ji_t} = \left(\theta_t + \psi^{\tau}_{jt}\right) D_{i_t}$$

where D_{i_t} is an indicator function, which is equal to 1 if $\Re_t$ is in power at time t, or zero otherwise.

Assumption 2 (Ideology) *The random variables* $\psi^{\tau}_{jt} \sim \left[-\frac{1}{2\psi^{\tau}}, \frac{1}{2\psi^{\tau}}\right]$ *and* $\theta_t \sim \left[-\frac{1}{2\theta}, \frac{1}{2\theta}\right]$ *are i.i.d. and uniformly distributed, centered over zero.*

ψ^{τ}_{jt} represents the idiosyncratic shock, whose distribution is cohort specific, and measures voter j's individual preferences toward party $\Re$. θ_t represents an aggregate shock and measures the average relative popularity of candidate from party $\Re$ relative to those from party $\mathcal{L}$.

Assumption 3 (Utility) *The function* $u : \mathbb{R}_+ \to \mathbb{R}_+$ *is twice continuously differentiable concave function with* $\lim_{c \to 0} u_c(c) = \infty$.

When young, agents spend all their time endowment in acquiring skills if the *forward productive transfers*, f_{i_t}, is publicly provided without having access to private credit market. When adult, individuals supply inelastically labor, pay taxes, consume and save for retirement. When old, agents pay taxes and consume their entire income, which is composed of capitalized saving and the publicly provided *backward pork-barrel transfers*, b_{i_t}. The total income tax rate is equal to π_{i_t}. The individual budget constraints for adults and old are, respectively:

$$c^1_{i_t} \leq (1 - \pi_{i_t})\, w_t h_t - s_t \tag{2.2}$$

$$c^2_{i_{t+1}} \leq \left(1 - \pi_{i_{t+1}}\right) R_{t+1} s_t + b_{i_{t+1}} \tag{2.3}$$

At the initial time, $t = 0$, there is an exogenous human capital total endowment, $h_0 > 0$. The budget constraint of each adult is then equal to $c^1_{i_0} = (1 - \pi_{i_0})\, w_0 h_0 - s_0$. At the same the elderly are endowed with an exogenously given physical capital, $k_0 > 0$. Their individual budget constraint is equal to $c^2_{i_0} = (1 - \pi_{i_0})\, R_0 k_0 + b_{i_0}$.

2.2.2 Technology

At each time t the economy produces a single consumption good, y_t, combining physical, k_t, and human capital, h_t, according to a constant return to scale technology, $y_t = \Theta(k_t, h_t)$.

Assumption 4 (Production Technology) *The function* $\Theta : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$ *is strictly monotonic increasing and strictly concave with* $\Theta(0, h_t), \Theta(k_t, 0) \geq 0$ *and* $\Theta_{kh}, \Theta_{hk} \geq 0$*.*

Let us denote with $\tilde{y}_t \equiv \frac{y_t}{h_t}$ and $\tilde{k}_t = \frac{k_t}{h_t}$ the per-efficiency units of final good production and physical capital, respectively. Physical capital fully depreciates each period. Under Assumption 4 it follows that $y_t = h_t \Theta\left(\frac{k_t}{h_t}, 1\right) \equiv h_t \vartheta\left(\tilde{k}_t\right)$ and, in turns, $\tilde{y}_t = \vartheta\left(\tilde{k}_t\right)$. As a consequence, the inverse demands for factor prices are $\Theta_k = \vartheta_{\tilde{k}}\left(\tilde{k}_t\right)$ and $\Theta_h = \vartheta\left(\tilde{k}_t\right) - \tilde{k}_t \vartheta_{\tilde{k}}\left(\tilde{k}_t\right)$, with $\Theta_{hk} = \Theta_{kh} = \frac{\vartheta_{\tilde{k}}\left(\tilde{k}_t\right)}{h_t} \frac{\vartheta\left(\tilde{k}_t\right) - \tilde{k}_t \vartheta_{\tilde{k}}\left(\tilde{k}_t\right)}{\vartheta\left(\tilde{k}_t\right)}$ and $\Theta_{kk} = \frac{\vartheta_{\tilde{k}}\left(\tilde{k}_t\right)}{k_t} \frac{\vartheta\left(\tilde{k}_t\right) - \tilde{k}_t \vartheta_{\tilde{k}}\left(\tilde{k}_t\right)}{\vartheta\left(\tilde{k}_t\right)}$.

The human capital is produced according to a constant return to scale technology, which combines parental education and public investment in education as complement factors:

$$h_{t+1} = H\left(h_t, f_{i_t}\right) \tag{2.4}$$

Assumption 5 (Human Capital Technology) *The function* $H : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$ *is strictly monotonic increasing and strictly concave with* $H\left(0, f_{i_t}\right), H\left(h_t, 0\right) \geq 0$ *and* $H_{hf}, H_{fh} \geq 0$*.*

Let us denote with $\tilde{f}_{i_t} \equiv \frac{f_{i_t}}{h_t}$ the per-efficient units of productive transfers. Under Assumption 5 it follows that $h_{t+1} = h_t H\left(1, \frac{f_{i_t}}{h_t}\right) \equiv h_t \varphi\left(\tilde{f}_{i_t}\right)$. As a consequence, the human capital growth rate is equal to $\frac{h_{t+1}}{h_t} = \varphi\left(\tilde{f}_{i_t}\right)$, which represents also the growth rate of the economy. The marginal impact of parental education and forward productive transfers on the human capital production are $H_h = \varphi\left(\tilde{f}_{i_t}\right) - \tilde{f}_{i_t} \varphi_{\tilde{f}}\left(\tilde{f}_{i_t}\right)$ and $H_f = \varphi_{\tilde{f}}\left(\tilde{f}_{i_t}\right)$, respectively.

2.2.3 Fiscal Constitution

In order to provide the intergenerational transfers, agents have to devise a politician. In each period, the politician raises revenues through income taxes and uses the proceeds to purchase consumption good to be converted into transfers to young and old generations. The politicians cannot use lump-sum taxes. They can only levy a proportional tax on labor and capital income. The tax rates on the two sources of income are equal. We assume for simplicity that the politician is prevented from borrowing: The public balance must hold in every period. This implies that in each period total benefits paid to the agents equalize total contributions collected from tax payers. Then for each elected politician belonging to party i_t, the balanced budget constraint condition can be written as:

$$\pi_{i_t} y_t = f_{i_t} + b_{i_t} \tag{2.5}$$

Eq. (2.5) allows us to reduce the multidimensionality of political platform, $z_{i_t} \equiv \{f_{i_t}, b_{i_t}, \pi(f_{i_t}, b_{i_t})\}$.

Assumption 6 (Feasibility) *At each time t and for each party i_t:*

i) consumption of elderly agent is positive, i.e. $b_{i_t} > -R_t(1-\pi_{i_t})k_t$;

ii) forward productive transfers are non negative, i.e. $f_{i_t} \geq 0$;

iii) income tax rate is a percentage of total production, i.e. $\pi_{i_t} \leq 1$.

2.3 First Best Allocation

In this section we characterize the efficient allocation chosen by a benevolent planner with a commitment technology in the absence of distortionary taxation. Lump sum taxes are used to finance forward productive transfer and backward pork-barrel transfers. The benevolent planner takes the initial level of human and physical capital $\{h_0, k_0\}$ as given, and chooses a sequence $\left\{c_t^1, c_t^2, f_t, h_{t+1}, k_{t+1}\right\}_{t=0}^{\infty}$ in order to maximize a weighted sum of lifetime utilities over generations. The welfare weight of each representative dynasty is exogenously given, $\delta \in (0,1)$. The corresponding maximization problem is equal to:

$$\max_{\left\{c_t^1, c_{t+1}^2, f_t, h_{t+1}, k_{t+1}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \delta^{t+1}\left(u\left(c_t^1\right) + \beta u\left(c_{t+1}^2\right)\right) + \beta u\left(c_0^2\right)$$

subject to the aggregate resource constraint and the human capital technology:

$$\begin{aligned} c_t^1 + c_t^2 + k_{t+1} + f_t - h_t \vartheta\left(\tilde{k}_t\right) &\leq 0, \forall t \ \left(\mu_t \delta^{t+1}\right) \\ h_{t+1} - h_t \varphi\left(\tilde{f}_t\right) &\leq 0, \forall t \ \left(\eta_t \delta^{t+1}\right) \end{aligned}$$

where $(\mu_t \delta^{t+1})$ and $(\eta_t \delta^{t+1})$ are the associated Lagrange multipliers. Removing the functional arguments, the first order conditions of the Lagrangian turn out to be equal to:

$$
\begin{aligned}
c_t^1 &: u_{c_t^1} = \mu_t \\
c_{t+1}^2 &: \beta u_{c_{t+1}^2} = \delta \mu_{t+1} \\
f_t &: \mu_t = \eta_t \varphi_{\tilde{f}_t} \\
h_{t+1} &: \eta_t = \delta \mu_{t+1} \left(\vartheta\left(\tilde{k}_{t+1}\right) - \tilde{k}_{t+1} \vartheta_{\tilde{k}}\left(\tilde{k}_{t+1}\right) \right) + \delta \eta_{t+1} \left(\varphi_{t+1} - \tilde{f}_{t+1} \varphi_{\tilde{f}_{t+1}} \right) \\
k_{t+1} &: \mu_t = \delta \mu_{t+1} \vartheta_{\tilde{k}}\left(\tilde{k}_{t+1}\right)
\end{aligned}
$$

together with the trasversality conditions (TVCs):

$$\lim_{t \to \infty} \delta^{t+1} \mu_t k_{t+1} = 0 \tag{2.6}$$

$$\lim_{t \to \infty} \delta^{t+1} \eta_t h_{t+1} = 0 \tag{2.7}$$

Rearranging the first order conditions, the following Euler conditions, or first order wedges, for the optimal allocations must be satisfied:

$$\Delta^r \equiv \beta u_{c_t^2} - \delta u_{c_t^1} = 0 \tag{2.8}$$

$$\Delta^s \equiv u_{c_t^1} - \beta \vartheta_{\tilde{k}}\left(\tilde{k}_{t+1}\right) u_{c_{t+1}^2} = 0 \tag{2.9}$$

$$\Delta^f \equiv \varphi_{\tilde{f}_t} - \frac{\vartheta_{\tilde{k}}\left(\tilde{k}_{t+1}\right)}{\vartheta\left(\tilde{k}_{t+1}\right) - \tilde{k}_{t+1} \vartheta_{\tilde{k}}\left(\tilde{k}_{t+1}\right)} (1 - \varepsilon_{t+1}) = 0 \tag{2.10}$$

where $\varepsilon_{t+1} \equiv \frac{\varphi_{\tilde{f}_t}}{\vartheta_{\tilde{k}}(\tilde{k}_{t+1})} \frac{\varphi_{t+1} - \tilde{f}_{t+1} \varphi_{\tilde{f}_{t+1}}}{\varphi_{\tilde{f}_{t+1}}}$ denotes the education spillover expressed as a fraction of the education cost. It fully describes the impact of current education investment on future level of human capital through the channel of parental investment. The first condition, Eq. (2.8), captures the redistribution wedge between current adults and old cohorts. The second condition, Eq. (2.9), describes the optimal investment choice, determining the optimal accumulation of physical capital. The planner chooses k_{t+1} in order to equate the marginal cost, in terms of foregone consumption, to the discounted marginal benefits of savings. The third condition, Eq. (2.10), reflects the direct effect of productive transfers on the utility of the adults in terms of current cost and expected benefits, which yield the optimal accumulation of human capital. Suppose $\varepsilon_{t+1} = 0$, then Eq. (2.9) and (2.10) are equivalent to the necessary conditions of a competitive equilibrium with no credit market constraints, where young can borrow money at the market interest rate.

Definition 6 (Pareto Efficient Allocation) *For any initial conditions* $\{h_0, k_0\}$ *the optimal allocation* $\left\{c_t^{1*}, c_t^{2*}, f_t^*, h_{t+1}^*, k_{t+1}^*\right\}_{t=0}^{\infty}$ *(2.6), (2.7), (2.8), (2.9) and (2.10) for all* $t \geq 0$.

2.4 Politico-Economic Equilibrium

This section characterizes the politico-economic equilibrium of a dynamic game played among generations in a repeated voting setting. We consider a majoritarian probabilistic voting framework where office-seeking ideologically heterogenous parties compete in order to maximize their probability of winning election, internalizing the impact of the proposed fiscal platform on the outcome of future elections.

2.4.1 Timing

To fully describe the main mechanism underlying the resolution strategy of the politico-economic equilibrium, let us provide a complete description of the timing of the game:

i. at the beginning of each time and after the realization of shocks, the two parties compete proposing a political platform (Political Competition Stage);

ii. the ideological shocks are realized, the election takes place and the party, which casts the majority of votes, wins election (Electoral Stage);

iii. the outcome of the political competition is realized and the promised political platform is implemented;

iv agents take economic decisions of saving and firms produce (Competitive Equilibrium given Policies).

Following Krusell et al. (1997), in order to determine a time consistent solution of the game, ruling out the assumption of commitment, we solve for the Markov subgame perfect equilibrium of the game using backward procedure. Specifically, the equilibrium outcome is determined as the limit of the finite horizon game.

2.4.2 Competitive Equilibrium given Fiscal Policies

In a competitive equilibrium, adults choose their lifetime consumption taking factor prices and the government policy as given. Maximizing Eq. (2.1) subject to the individual budget constraints (2.2) and (2.3) and feasibility constraints $c^1_{i_t} > 0$ and $c^2_{i_{t+1}} > 0$, the following first order conditions for interior solutions must hold:

$$0 = u_c\left(c^1_{i_t}\right) - \beta E^{\mathcal{P}_{i_t}}\left[\left(1-\pi_{i_{t+1}}\right)R_{t+1}u_c\left(c^2_{i_{t+1}}\right)\right] \tag{2.11}$$

In equilibrium by implicit function theorem a unique non-negative saving function exists, $s_t = K\left(I^1_{i_t}, I^2_{i_{t+1}}\right)$, where $I^1_{i_t} \equiv \left(1-\pi_{i_t}\right)w_t h_t$ and $I^2_{i_{t+1}} \equiv \mathcal{P}_{i_t}\frac{b_{\Re_{t+1}}}{\left(1-\pi_{\Re_{t+1}}\right)R_{t+1}} + \left(1-\mathcal{P}_{i_t}\right)\frac{b_{\mathcal{L}_{t+1}}}{\left(1-\pi_{\mathcal{L}_{t+1}}\right)R_{t+1}}$. Then the capital market clears when:

$$k_{t+1} = K\left(I^1_{i_t}, I^2_{i_{t+1}}\right) \tag{2.12}$$

After plugging Eq. (2.12) into Eq. (2.2) and (2.3) and using the condition (2.5) the following individual consumption levels are attained:

$$c^1_{i_t} = \mathcal{C}^1_{i_t}\left(\pi_{i_t}, h_t, k_t, k_{t+1}\right) \equiv I^1_{i_t} - k_{t+1} \tag{2.13}$$

$$c^2_{i_{t+1}} = \mathcal{C}^2_{i_{t+1}}\left(\pi_{i_{t+1}}, b_{i_{t+1}}, h_{t+1}, k_{t+1}\right) \equiv \left(1-\pi_{i_{t+1}}\right)R_{t+1}k_{t+1} + b_{i_{t+1}} \tag{2.14}$$

Let us denote with $u\left(\mathcal{C}^1_{i_t}\left(\pi_{i_t}, h_t, k_t, k_{t+1}\right)\right)$ and $u\left(\mathcal{C}^2_{i_t}\left(\pi_{i_t}, b_{i_t}, h_t, k_t\right)\right)$ the current utility that adults and old receive in equilibrium, respectively.

Firms produce in a perfectly competitive environment, then in equilibrium they choose the level of employment of capital and the effective units of labor so as to maximize profits, i.e. $\max\limits_{\{k_t, h_t\}}\left[\Theta\left(k_t, h_t\right) - w_t h_t - R_t k_t\right]$. Firms optimality and markets clearing imply that factor prices are given by the marginal productivity of each factor:

$$R_t = \Theta_k\left(k_t, h_t\right) \tag{2.15}$$

$$w_t = \Theta_h\left(k_t, h_t\right) \tag{2.16}$$

Definition 7 (Intertemporal Competitive Equilibrium) *Given the initial conditions $\{h_0, k_0\}$, the sequence of policies $\{f_{i_t}, b_{i_t}\}_{t=0}^{\infty}$ and the probability of winning elections $\{\mathcal{P}_{i_t}\}_{t=0}^{\infty}$ for each possible elected party, $i_t \in \{\mathcal{L}_t, \Re_t\}$, an intertemporal competitive equilibrium with perfect foresight is a sequence of allocations and factor prices $\{c_{i_t}^1, c_{i_t}^2, R_t, w_t, h_{t+1}, k_{t+1}\}_{t=0}^{\infty}$ such that, for all $t \geqslant 0$:*

i) the allocation solves the maximization problem of adult household, i.e. Eq. (2.11), (2.13) and (2.14) are satisfied;

iii) the factor prices are consistent with the profit maximization of firms, i.e. Eq. (2.15) and (2.16) are satisfied;

iv) the private good market clears:

$$c_{i_t}^1 + c_{i_t}^2 + k_{t+1} + f_{i_t} = \Theta\left(k_t, h_t\right) \tag{2.17}$$

v) the markets for production inputs clear in period t, i.e. Eq. (2.4) and (2.12) hold.

2.4.3 Electoral Stage

After the parties have proposed the political platform, $\{f_{i_t}, b_{i_t}\}$, and the ideological shocks are realized, voters have to take electoral decisions. Let us denote with $V^\tau\left(k_t, h_t\right)$ the indirect utility of agents belonging to cohort τ if party $\mathcal{L}_t$ wins the election, whereas $W^\tau\left(k_t, h_t\right)$ measures the indirect benefits for τ-cohort agents achieved when party $\Re_t$ is in power net of the ideological shock. Note that the indirect utility of elderly is just characterized by their current utility when old. Contrarily, the adults have an intertemporal indirect utility whose next-period ideological shock has to be realized. Consequently, for the old-age voter j the voting decision is given by:

$$\max\left\{V^2\left(k_t, h_t\right), W^2\left(k_t, h_t\right) + \left(\theta_t + \psi_{jt}^2\right)\right\}$$

By using Eq. (2.14) the individual indirect utility for elderly are give by:

$$V^2\left(k_t, h_t\right) = u\left(\mathcal{C}_{\mathcal{L}_t}^2\right) \text{ and } W^2\left(k_t, h_t\right) = u\left(\mathcal{C}_{\Re_t}^2\right) \tag{2.18}$$

Equivalently, for the adult voter j the voting decision is given by:

$$\max\left\{V^1\left(k_t,h_t\right),W^1\left(k_t,h_t\right)+\left(\theta_t+\psi^1_{jt}\right)\right\}$$

By using Eq. (2.13) and (2.14), the individual indirect utilities for adults are equal to:

$$V^1\left(k_t,h_t\right)=u\left(\mathcal{C}^1_{\mathcal{L}_t}\right)+\beta\mathcal{V}^2\left(k_{t+1},h_{t+1};\mathcal{L}_t\right) \tag{2.19}$$

$$W^1\left(k_t,h_t\right)=u\left(\mathcal{C}^1_{\Re_t}\right)+\beta\mathcal{W}^2\left(k_{t+1},h_{t+1};\Re_t\right) \tag{2.20}$$

where the adults' continuation values are given by:

$$\mathcal{V}^2\left(k_{t+1},h_{t+1};\mathcal{L}_t\right)\equiv E\left(\mathcal{P}_{\mathcal{L}_t}\left(W^2_{\mathcal{L}_t}+\theta_{t+1}+\psi^2_{jt+1}\right)+\left(1-\mathcal{P}_{\mathcal{L}_t}\right)V^2_{\mathcal{L}_t}\right)$$

$$\mathcal{W}^2\left(k_{t+1},h_{t+1};\Re_t\right)\equiv E\left(\mathcal{P}_{\Re_t}\left(W^2_{\Re_t}+\theta_{t+1}+\psi^2_{jt+1}\right)+\left(1-\mathcal{P}_{\Re_t}\right)V^2_{\Re_t}\right)$$

with $W^2_{i_t}\equiv W^2\left(k_t,h_t;i_t\right)$ and $V^2_{i_t}\equiv V^2\left(k_t,h_t;i_t\right)$ are the indirect utilities when old if the previous period incumbent party is i_t.[7]

Consequently, the elderly and the adults decide to vote for party $\Re_t$ as long as:

$$\psi^2_{jt}\geq\bar{\psi}^2_t\equiv V^2\left(k_t,h_t\right)-W^2\left(k_t,h_t\right)-\theta_t \tag{2.21}$$

$$\psi^1_{jt}\geq\bar{\psi}^1_t\equiv V^1\left(k_t,h_t\right)-W^1\left(k_t,h_t\right)-\theta_t \tag{2.22}$$

Formally, $\bar{\psi}^2_t$ and $\bar{\psi}^1_t$ represent the swing voter in cohort 2 and 1, respectively.

Let us denote with $\Delta u\left(\mathcal{C}^\tau_t\right)\equiv u\left(\mathcal{C}^\tau_{\mathcal{L}_t}\right)-u\left(\mathcal{C}^\tau_{\Re_t}\right)$ the difference in utility generated by the implementation of different fiscal platforms.

Using Eq. (2.18), (2.19), (2.20), (2.21), and (2.22), the share of elderly voters for party $\Re$ is equal to:

$$m^2_t\equiv\frac{1}{2}-\psi^2\left(\Delta u\left(\mathcal{C}^2_t\right)-\theta_t\right)$$

[7] Note that from the perspective of the adults the internalization of the t-period incumbent party identity is relevant for their voting decisions. Indeed, suppose party i_t wins election at time t, then it will implement policies which will affect the relevant state variables of the economy. Consequently both the probability of being re-elected and the future implemented policies will turn out to be conditioned by the identity of the current incumbent party.

whereas the adults' share is equal to:

$$\begin{aligned} m_t^1 \equiv & \frac{1}{2} - \psi^1 \left(\Delta u \left(\mathcal{C}_t^1\right) - \theta_t\right) \\ & - \psi^1 \beta \left(\mathcal{V}^2 \left(k_{t+1}, h_{t+1}; \mathcal{L}_t\right) - \mathcal{W}^2 \left(k_{t+1}, h_{t+1}; \Re_t\right)\right) \end{aligned}$$

Under majoritarian rule, party $\Re_t$ wins the election if $m_t \equiv m_t^1 + m_t^2 > 1$. Consequently, $\theta_t > \bar{\theta}_t$ where $\bar{\theta}_t$ equal to:

$$\begin{aligned} \bar{\theta}_t \equiv & \frac{1}{\psi^1 + \psi^2} \left(\frac{\psi^2}{\psi^1} \Delta u \left(\mathcal{C}_t^2\right) + \Delta u \left(\mathcal{C}_t^1\right) \right) \\ & + \frac{1}{\psi^1 + \psi^2} \beta \left(\mathcal{V}^2 \left(k_{t+1}, h_{t+1}; \mathcal{L}_t\right) - \mathcal{W}^2 \left(k_{t+1}, h_{t+1}; \Re_t\right)\right) \end{aligned} \tag{2.23}$$

Then the probability that party $\Re_t$ and $\mathcal{L}_t$ win the election is respectively equal to:

$$\Pr\left(\theta_t > \bar{\theta}_t\right) = \frac{1}{2} - \theta \bar{\theta}_t \tag{2.24}$$

$$\Pr\left(\theta_t < \bar{\theta}_t\right) = \frac{1}{2} + \theta \bar{\theta}_t \tag{2.25}$$

Note that $\mathcal{P}_{i_t}$ is endogenously given and a function of $\bar{\theta}_t$. Specifically,

$$\mathcal{P}_{i_t} = \begin{cases} \Pr\left(\theta_{t+1} > \bar{\theta}_{t+1} | \theta_t > \bar{\theta}_t\right) & \text{if } i_t = \Re_t \\ \Pr\left(\theta_{t+1} > \bar{\theta}_{t+1} | \theta_t < \bar{\theta}_t\right) & \text{if } i_t = \mathcal{L}_t \end{cases} \tag{2.26}$$

2.4.4 Political Competition Stage

The last step concerns the determination of the equilibrium parties' political platform. Specifically, the equilibrium objective functions we are interested in are: The forward and backward transfer policies implemented by the winning party i_t as a function of the relevant payoff state variables of the economy, i.e. human capital and physical capital, namely $\mathcal{F}_{i_t}(k_t, h_t)$ and $\mathcal{B}_{i_t}(k_t, h_t)$, the probability that party $\Re_t$ wins the election when the incumbent party is i_t, $\mathcal{P}_{i_t}(k_{t+1}, h_{t+1})$, and the rules governing the evolution of the payoff-relevant state variables, $k_{t+1} = \mathcal{K}(f_{i_t}, b_{i_t}, k_t, h_t)$ and $h_{t+1} = H(f_{i_t}, h_t)$. Let us denote the expected prices by $R_{t+1} = R(k_{t+1}, h_{t+1})$ and $w_{t+1} = w(k_{t+1}, h_{t+1})$. The parties' objective function concerns the maximization of probability of winning elections. Furthermore, they take care of their ideological position. Consequently their objective functions are equal to:

$$\max_{z_{\Re_t}} -q\left(z_{\Re_t}-\bar{z}_{\Re}\right)+\left(\frac{1}{2}-\theta\bar{\theta}_t\right) \tag{2.27}$$

$$\max_{z_{\mathcal{L}_t}} -q\left(z_{\mathcal{L}_t}-\bar{z}_{\mathcal{L}}\right)+\left(\frac{1}{2}+\theta\bar{\theta}_t\right) \tag{2.28}$$

where $q\left(z_{i_t}-\bar{z}_i\right)$ is a function measuring the distance of the proposed platform, z_{i_t}, from the parties' ideological bliss points, $\bar{z}_i$, such that $q(0)=0$.

Definition 8 (Markov Politico-Economic Equilibrium) *A Markov perfect politico-economic equilibrium is an intertemporal competitive equilibrium, a sequence of policies $\{f_{i_t}, b_{i_t}\}_{t=0}^{\infty}$ and elections' conditional probabilities $\{\mathcal{P}_{i_t}\}_{t=0}^{\infty}$ such that:*

i) $f_{i_t}=\mathcal{F}_{i_t}(k_t,h_t)$, $b_{i_t}=\mathcal{B}_{i_t}(k_t,h_t)$ and Eq. (2.5) holds at each time t;

ii) $\mathcal{P}_{i_t}$ is defined according to Eq. (2.26);

ii) party $\Re_t$ and $\mathcal{L}_t$ maximize, respectively, Eq. (2.27) and (2.28), subject to the constraints (2.4) and (2.12).

After plugging the equilibrium policy rules, $\mathcal{F}_{i_t}(\cdot)$ and $B_{i_t}(\cdot)$, and the conditional probability of re-election, $\mathcal{P}_{i_t}(\cdot)$, into Eq. (2.12) we obtain:

$$k_{t+1}=K\left(f_{i_t}, b_{i_t}, h_t, k_t, R(\cdot), \mathcal{P}_{i_t}(\cdot), \mathcal{F}_{i_{t+1}}(\cdot), \mathcal{B}_{i_{t+1}}(\cdot)\right) \tag{2.29}$$

By using Eq. (2.4) and rearranging the terms, we rewrite Eq. (2.29) as follows:

$$k_{t+1}=\mathcal{K}\left(f_{i_t}, b_{i_t}, h_t, k_t\right)$$

where $\mathcal{K}$ fully describes the evolution of private capital under the one-period deviation.[8] Since households take policies as given, due to the sequential timing of the game, Eq. (2.11) becomes as follows:

$$u_c\left(\mathcal{C}_{i_t}^1\right)-\beta E^{\mathcal{P}_{i_t}}\left[\left(1-\pi_{i_{t+1}}\right)R_{t+1}u_c\left(\mathcal{C}_{i_{t+1}}^2\right)\right]=0$$

[8] For a detailed description of the computational procedure we make reference to Chap. 1.

We consider two different scenario. The first is characterized by neither fiscal nor political distortions. Specifically, we assume taxes are levied only on labor income and parties do not take care of their own ideological position. In the second case taxes are levied on both labor income and capital return, thus generating fiscal distortions. We also introduce political distortions due to ideological competition among parties, which internalize how current policies affect the probability of being elected also in the next-period elections. In the following sections we study these cases separately.

2.5 No Distortionary Case

Let us first consider the case in which parties compete by proposing a political platform in order to maximize their probability of winning elections, without taking care of their ideological position. Consequently parties' objective functions are given by Eq. (2.24) and (2.25). Furthermore there is no capital taxation. We denote with $\phi \equiv \frac{\psi^2}{\psi^1}$ the ratio of the idiosyncratic ideological densities between the two cohorts of voters. The following Proposition holds.

Proposition 4 *There exists a unique Markov perfect politico-economic equilibrium characterized by parties $\mathcal{L}_t$ and $\Re_t$ proposing the same political platform, $z_{\mathcal{L}_t} = z_{\Re_t} = z_t$, which is obtained by solving the following maximization:*

$$\max \phi u \left(C^2 \left(b_t, h_t, k_t\right)\right) + u \left(C^1 \left(f_t, b_t, h_t, k_t, k_{t+1}\right)\right) + \beta u \left(C^2 \left(b_{t+1}, h_{t+1}, k_{t+1}\right)\right)$$

under the constraint:

$$u_c \left(\mathcal{C}_t^1\right) - \beta R_{t+1} u_c \left(\mathcal{C}_{t+1}^2\right) = 0 \tag{2.30}$$

Proof. *(See appendix).* ■

In equilibrium the two parties propose the same fiscal platform. It simply maximizes a convex combination of the utility of the current living voters, where the weights reflect the sensitivity of voting behavior to policy changes. Therefore, in equilibrium the probability of winning elections given by Eq. (2.24) and (2.25) is equal to one half. We can interpret the parameter ϕ also as a measurement of the degree of intergenerational political disagreement over the redistribution of public resources. If $\phi = 0$, given the unitary mass for each cohort, the adults are the median voter and political decisions are taken in order to maximize their intertemporal utility, without considering the participation constraints of both young and the elderly. As soon as $\phi > 0$ a conflict over the equilibrium redistribution of public resources emerges due to the participation constraint of the elderly, which starts to bind. In this section we explore how intergenerational political conflicts, i.e. $\phi \neq 0$, may improve the overall economic efficiency in an economy characterized by productive human capital and storage technologies.

By applying envelope theorem and using Eq. (2.13), (2.14), and (2.30), the Euler conditions with respect to the policies rules f_t and b_t are equal to, respectively:[9]

$$0 = -1 + \frac{1}{R_{t+1}} \left(\frac{d\mathcal{B}_{t+1}}{dk_{t+1}} \mathcal{K}_{f_t} + \frac{d\mathcal{B}_{t+1}}{dh_{t+1}} \varphi_{\tilde{f}_t} \right) + \frac{k_{t+1}}{R_{t+1}} \left(\Theta_{kk} \mathcal{K}_{f_t} + \Theta_{kh} \varphi_{\tilde{f}_t} \right) \tag{2.31}$$

$$0 = -\left(1 - \frac{1}{R_{t+1}} \left(\frac{d\mathcal{B}_{t+1}}{dk_{t+1}} + k_{t+1} \Theta_{kk} \right) \mathcal{K}_{b_t} \right) u_c \left(\mathcal{C}_t^1 \right) + \phi u_c \left(\mathcal{C}_{t+1}^2 \right) \tag{2.32}$$

Let us first refer to Eq. (2.31). At each time an interior solution for the forward productive transfer is simply determined as an intertemporal maximization of the current adults' utility, who reap benefits and sustain costs by a variation in the current public expenditure. No intergenerational conflicts affect the setting of f_t. Since tax levying on labor income makes adults sustain the whole tax burden, the first term captures the adults' marginal cost caused by a positive variation on the fiscal dimension. The second term represents the expected marginal impact on the amount of next-period backward pork-barrel transfer due to a current variation in productive expenditure through the channels of both human and physical capital. Finally, the third term measures the expected marginal impact of f_t on the utility of next-period elderly through the increase in the rental price of capital. The redistributive choices are made as the outcome of a weighted bargaining between current adults and elderly. On one hand, an increase in backward transfers makes current adults to sustain direct costs and to either enjoy

[9] By $\frac{dx}{dz}$ we denote the partial derivative of x with respect to z, and with $\frac{\partial x}{\partial z}$ the total differentiation.

benefits or sustain costs from expected variation of both future pork-barrel transfers and the return to capital through the channel of physical capital, represented by the first part of Eq. (2.32). On the other hand, it makes old enjoy direct benefits from a current positive variation of b_t.

Even in absence of fiscal or political distortions, the efficient allocation is not achievable in equilibrium. Indeed, the existence of credit market constraints generates simultaneously two sources of distortions in equilibrium. On one hand, it precludes young from borrowing money for education investment, inducing underaccumulation of human capital. On the other hand, it reduces the investment portfolio for adults, determining overaccumulation of physical capital.[10] The provision of public education transfers through the institution of electoral competition partially offsets the inefficiency generated by the credit market conditions. However the gains in efficiency obtained in the politico-economic equilibrium compared to the autarchic allocation strongly depends on the voting institution which characterizes the political environment. In order to clearly show the positive relation between intergenerational political disagreement and economic efficiency, let us consider the following three cases.

Case I: No human capital and median voter framework, i.e. $f_t = 0$ and $\phi = 0$.[11] For simplicity suppose exogenous prices. From Eq. (2.31) and (2.32), it is easy to show that the first order condition to be satisfied requires $1 - \frac{1}{R_{t+1}} \frac{d\mathcal{B}_{t+1}}{dk_{t+1}} \mathcal{K}_{b_t} = 0$. Given that $\mathcal{K}_{b_t} < 0$, in equilibrium agents may sustain positive transfers to the elderly by coordinating on the level of aggregate physical capital. The lower the level of savings due to income taxes paid when adult, the higher the level of future benefits when old, i.e. $\frac{d\mathcal{B}_{t+1}}{dk_{t+1}} < 0$. The intergenerational plan implemented by the median voter may also improve the economic overall efficiency starting from a condition of dynamic inefficiency.

Case II: Human capital and median voter framework, i.e. $f_t > 0$ and $\phi = 0$. When the productive human capital channel is introduced the politico-economic equilibrium outcome in the case of median voter slightly changes. Agents no longer coordinate as to the level of physical capital. The

[10] See Boldrin and Montes (2005) for a complete analyses of distortions generated by credit market constraints in three period OLG economy with human capital.

[11] See Azariadis and Galasso (2002) for the case of exogneous prices and Forni (2005) for the case of endogenous prices.

relevant state variable is now the total return to capital, which is affected by both human and physical capital. Adults have incentives to invest in educating the young in order to accumulate human capital and increase the next period return to capital. At the same time they do not have incentives to transfer public resource backward in terms of pension benefits. Indeed, adults perfectly anticipate that when old they are prevented to grab public resources by exerting political power. Furthermore, the next generation's promise concerning pension transfers are not credible because the return to capital will be positively affected by the previous period human capital investment. Consequently, in equilibrium public education investment will be financed by taxes paid by elderly and, if the physical capital productivity is high enough, such transfers will be also used to subsidize adults' consumption. Because of the resulting overaccumulation of both physical and human capital the economy will still be characterized by dynamic inefficiency.

Case III: Human capital and no median voter framework, i.e. $f_t \geq 0$ and $\phi > 0$. When the human capital investment is feasible, the existence of intergenerational conflicts over the redistribution of public resources improves the overall economic efficiency. Given that elderly actively participate to the political debate, they always gain positive transfers by exerting political power. At the same time they correct the overaccumulation of both physical and human capital.

In the following section we derive a closed form characterization of the politico-economic equilibrium, which solves the system of partial differential equations given by Eq. (2.31) and (2.32). Furthermore, we provide a quantitative measure of how political disagreement may affect economic efficiency in the long run.

2.5.1 Example Economy

Let us consider the following parametric case. Preferences are log-additive, i.e. $\log\left(c_t^1\right) + \beta \log\left(c_{t+1}^2\right)$, and the final good production and the human capital technology are respectively equal to:

$$y_t = Ak_t^{\alpha} h_t^{1-\alpha}$$

$$h_{t+1} = Bf_t^{\eta} h_t^{1-\eta}$$

with $A, B > 0$ and $\alpha, \eta \in (0,1)$. Note that in balanced growth path the optimal education externality is simply equal to $\varepsilon = \delta(1-\eta)$.[12] Solving for the first best allocation, according to Definition 6, the growth rate of the economy and the rental price of capital per efficient unit of time turn out to be respectively equal to:

$$\gamma^o \equiv \varphi^o\left(\tilde{f}^*;\delta\right) = \left(\left(A^\eta B^{1-\alpha}\right)\mu_o^{(1-\alpha)\eta}\zeta_o^{\alpha\eta}\right)^{\frac{1}{1-\alpha(1-\eta)}} \tag{2.33}$$

$$R^o \equiv R^o\left(\tilde{k}^*;\delta\right) = \alpha\left(\left(A^\eta B^{1-\alpha}\right)\mu_o^{(1-\alpha)\eta}\zeta_o^{\alpha-1}\right)^{\frac{1}{1-\alpha(1-\eta)}} \tag{2.34}$$

where $\mu_o \equiv \frac{\eta\delta(1-\alpha)}{1-\varepsilon}$ and $\zeta_o \equiv \frac{(1-\varepsilon)-\eta\delta(1-\alpha)-(1-\delta)(1-\varepsilon\alpha)}{(1-\varepsilon)}$. Both $\tilde{k}^*$ and $\tilde{f}^*$ are increasing function in the dynasty discount factor δ and converge to zero when δ goes to zero.

We now evaluate how the politico-economic equilibrium, as described in Definition 8, performs with respect to the Pareto optimal allocation resumed by Eq. (2.33) and (2.34).

Lemma 2 *If a Markov subgame perfect politico-economic equilibrium exists, then the following condition must hold:*

$$\varphi_{\tilde{f}_t} - \frac{\vartheta_{\tilde{k}}\left(\tilde{k}_{t+1}\right)}{\vartheta\left(\tilde{k}_{t+1}\right) - \tilde{k}_{t+1}\vartheta_{\tilde{k}}\left(\tilde{k}_{t+1}\right)}\Lambda(\phi) = 0 \tag{2.35}$$

where $\Lambda(\phi) = \frac{(1+\beta\alpha)(\phi+(1+\beta(\alpha+\eta(1-\alpha))))}{\phi+\alpha\beta(\phi+(1+\beta(\alpha+\eta(1-\alpha))))} > 1$ *and* $\Lambda_\phi(\cdot) < 0$.

Proof. *(See appendix).* ■

Lemma 2 characterizes the necessary condition for the existence of the politico-economic equilibrium. Furthermore, Eq. (2.35) is directly comparable to the education optimal wedge, Eq. (2.10). Note that $\Lambda(0) = \max\Lambda(\phi)$, which implies that the lower ϕ, the larger the difference with respect to the first best allocation and, in turns, the inefficiency.

Under Lemma 2, by applying Definition 8 the following proposition holds.

[12] For detailed calculus see Appendix.

Proposition 5 *A unique Markov subgame perfect politico-economic equilibrium which sustains the policies* $\mathcal{F}(h_t, k_t) = \xi^f y_t$ *and* $\mathcal{B}(h_t, k_t) = \xi^b y_t$ *exists, where* $\xi^f \equiv \frac{\beta\eta(1-\alpha)}{\phi+(1+\beta(\alpha+\eta(1-\alpha)))}$ *and* $\xi^b \equiv \frac{\phi(1-\alpha)-\alpha(1+\beta(\alpha+\eta(1-\alpha)))}{\phi+(1+\beta(\alpha+\eta(1-\alpha)))}$.

Proof. *(See appendix).* ■

By solving via backward, taking a finite time horizon economy and making the time goes to infinite, the incentive structure of the game emerges clearly. As long as $\phi > \frac{\alpha}{1-\alpha}$, in the last period of the economy the elderly are able to grab a share of the total production through both capitalized savings and pension contributions. As a consequence, when adults, agents have incentives to invest a fraction of the collected taxes in public education.

Solving for the politico-economic equilibrium, the growth rate of the economy and the rental price of capital per efficient unit of time are equal to:

$$\gamma^p \equiv \varphi^p\left(\tilde{f};\phi\right) = \left(\left(A^\eta B^{1-\alpha}\right)\mu_p^{(1-\alpha)\eta}\zeta_p^{\alpha\eta}\right)^{\frac{1}{1-\alpha(1-\eta)}} \tag{2.36}$$

$$R^p \equiv R^p\left(\tilde{k};\phi\right) = \alpha\left(\left(A^\eta B^{1-\alpha}\right)\mu_p^{(1-\alpha)\eta}\zeta_p^{\alpha-1}\right)^{\frac{1}{1-\alpha(1-\eta)}} \tag{2.37}$$

where $\mu_p \equiv \frac{\beta\eta(1-\alpha)}{\phi+(1+\beta(\alpha+\eta(1-\alpha)))}$ and $\zeta_p \equiv \frac{(1+\alpha\beta)\alpha\beta}{\alpha\beta(1+\beta(\alpha+\eta(1-\alpha))+\phi)+\phi}$. In order to fully understand the role of intergenerational political disagreement measured by ϕ, let us consider the case $\phi = 0$, which is equivalent to consider a framework where the median-voter belongs to the adults' cohort.

Corollary 8 *When the median voter is the adult, the rental price of capital and the economy's growth rate are respectively equal to* $R^m\left(\tilde{k}\right) = \lim_{\phi\to 0} R^p\left(\tilde{k};\phi\right)$ *and* $\varphi^m\left(\tilde{f}\right) = \lim_{\phi\to 0}\varphi^p\left(\tilde{f};\phi\right)$*, such that* $\varphi^m\left(\tilde{f}\right) > \varphi^p\left(\tilde{f};\phi\right)$ *and* $R^m\left(\tilde{k}\right) < R^p\left(\tilde{k};\phi\right)$.

Proof. *(See appendix).* ■

As argued in section 2.5, in our economy the absence of intergenerational conflicts is detrimental for the economy's efficiency. If the median voter is the adult in equilibrium he will vote for a multidimensional fiscal platform characterized by: i) negative transfers from the elderly, ii) possibly subsidies to the adult's consumption, and iii) high investment in public education. Specifically, if the share of capital over total production is large enough, i.e. $-\alpha^2(\beta(1-\eta)) - \alpha(1+2\eta\beta) + \eta\beta < 0$, taxes paid by the elderly are used both to finance public education and to subsidize adults' consumption. As a consequence the economy will be characterized by overaccumulation of both physical and human capital.

As soon as a share of political power is assigned to the elderly, the equilibrium political platform reverses, turning out to be characterized by: i) positive transfers to the elderly, ii) negative transfers from the adult, iii) lower investment in public education. The participation of the elderly in the political debate and the consequent emergence of intergenerational political disagreement on the redistribution of public resources improves the overall economic efficiency.

Lemma 3 *For any level of welfare weights, $\delta \in (0, \delta^m)$ with $\delta^m > 0$, $R^m\left(\tilde{k}\right) < R^o\left(\tilde{k}^*; \delta\right)$ and $\varphi^m\left(\tilde{f}\right) > \varphi^o\left(\tilde{f}^*; \delta\right)$.*

Proof. *(See appendix).* ■

Lemma 3 states that the equilibrium allocation achieved in the median voter case is always suboptimal and induces overaccumulation of both human capital and physical capital for any welfare weight $\delta \in (0, \delta^m)$. For illustrative purpose, let us consider a specific quantitative exercise. In Fig. 9 are reported both the pair of optimal rental price of capital and economy's growth rate as a function of δ (red curve), and the pair of rental price of capital and economy's growth rate obtained in the politico-economic equilibrium as a function of ϕ (blue curve). As it becomes evident from the graphical representation, in the case of $\phi = 0$, for any value of welfare weights the economy's growth rate in the politico-economic allocation is always larger than the rental price of capital. Furthermore, the economy experiences a higher level of both human capital and physical capital accumulation with respect to the first best allocation for any $\delta \leq \delta^m$, consistently with Lemma 3.

Using Lemma 3 the following Proposition holds.

Proposition 6 *For any $\epsilon > 0$, a level $\delta^o \in (0, 1)$ and $\hat{\phi} > 0$ such that:*

$$\lim_{\phi \to \hat{\phi}} \left| R^p\left(\tilde{k}; \phi\right) - R^o\left(\tilde{k}^*; \delta^o\right) \right| < \epsilon$$

$$\lim_{\phi \to \hat{\phi}} \left| \varphi^p\left(\tilde{k}; \phi\right) - \varphi^o\left(\tilde{k}^*; \delta^o\right) \right| < \epsilon$$

exist.

Proof. *(See appendix).* ■

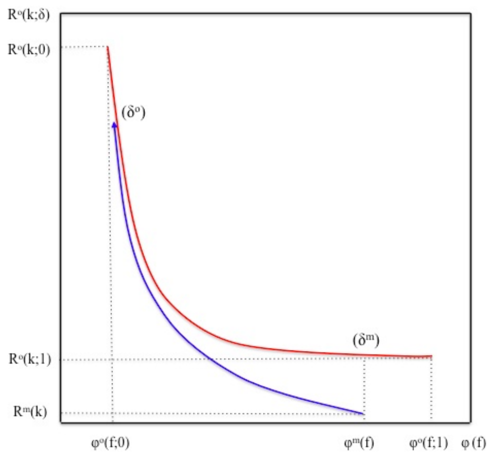

Fig. 9: The parametric values are A=0.2, B=0.2, α=0.4, η=0.2 and β=0.95[30]

From Proposition 6 it follows that the higher the elderly political power and, consequently, the higher the political disagreement, the lower the accumulation of human capital, the lower the accumulation of physical capital and the higher the efficiency for a given welfare weight. This gain in efficiency is reported in the Fig. 9 and is represented by the blue curve approaching the red one to a given level of welfare weight.

2.6 Distortionary Taxation

In this section we provide qualitative insights about the impact of distortionary taxation on capital on the politico-economic equilibrium characterized in the previous section.

Proposition 7 *There exists a unique Markov perfect politico-economic equilibrium characterized by parties $\mathcal{L}_t$ and $\Re_t$ proposing the same political platform, $z^d_{\mathcal{L}_t} = z^d_{\Re_t} = z^d_t$, which is obtained by solving the following maximization:*

$$\max \phi u \left(C^2 \left(b_t, f_t, h_t, k_t\right)\right) + u \left(C^1 \left(f_t, b_t, h_t, k_t, k_{t+1}\right)\right) + \beta u \left(C^2 \left(b_{t+1}, f_{t+1}, h_{t+1}, k_{t+1}\right)\right)$$

under the constraint:

$$u_c\left(\mathcal{C}_t^1\right)-\beta\left(1-\pi_{t+1}^d\right)R_{t+1}u_c\left(\mathcal{C}_{t+1}^2\right)=0 \tag{2.38}$$

Proof. *(See appendix).* ■

By applying envelope theorem and using Eq. (2.13), (2.14), and (2.38), the Euler conditions with respect to the policies rules f_t and b_t are equal to, respectively:

$$0=-\frac{w_t h_t}{y_t}u_c\left(\mathcal{C}_t^1\right)\underbrace{-\phi\frac{R_t k_t}{y_t}u_c\left(\mathcal{C}_t^2\right)}_{(1^*)}+\beta u_c\left(\mathcal{C}_{t+1}^2\right)\left(\left(\frac{d\mathcal{B}_{t+1}}{dk_{t+1}}+\Theta_{kk}\right)\mathcal{K}_{f_t}+\left(\frac{d\mathcal{B}_{t+1}}{dh_{t+1}}+\Theta_{kh}\right)\varphi_{\tilde{f}_t}\right)$$

$$\underbrace{-\beta u_c\left(\mathcal{C}_{t+1}^2\right)R_{t+1}k_{t+1}\left(\left(\frac{d\tilde{\mathcal{B}}_{t+1}}{dk_{t+1}}+\frac{d\tilde{\mathcal{F}}_{t+1}}{dk_{t+1}}\right)\mathcal{K}_{f_t}+\left(\frac{d\tilde{\mathcal{B}}_{t+1}}{dh_{t+1}}+\frac{d\tilde{\mathcal{F}}_{t+1}}{dh_{t+1}}\right)\varphi_{\tilde{f}_t}\right)}_{(2^*)} \tag{2.39}$$

$$0=-\frac{w_t h_t}{y_t}\left(u_c\left(\mathcal{C}_t^1\right)-\phi u_c\left(\mathcal{C}_t^2\right)\right)+\beta u_c\left(\mathcal{C}_{t+1}^2\right)\left(\frac{d\mathcal{B}_{t+1}}{dk_{t+1}}+k_{t+1}\left(1-\pi_{t+1}\right)\Theta_{kk}\right)\mathcal{K}_{b_t}$$

$$\underbrace{-\beta u_c\left(\mathcal{C}_{t+1}^2\right)R_{t+1}k_{t+1}\left(\frac{d\tilde{\mathcal{B}}_{t+1}}{dk_{t+1}}+\frac{d\tilde{\mathcal{F}}_{t+1}}{dk_{t+1}}\right)\mathcal{K}_{b_t}}_{(3^*)} \tag{2.40a}$$

where $\tilde{\mathcal{B}}_{t+1}(\cdot)$ and $\tilde{\mathcal{F}}_{t+1}(\cdot)$ denote the backward and forward transfer functions per efficient unit, respectively. Compared to the Eq. (2.31) and (2.32), distortionary taxation on capital affects the politico-economic equilibrium in three ways. Let us first refer to Eq. (2.39). Now old partially sustain the fiscal burden together with the adults, (1*). Consequently, differently from the no distortionary case, intergenerational conflicts also affect the setting of forward transfers. The main quantitative impact of distortionary taxation on capital however concerns the next-period utility of current living adults. In equilibrium the terms (2*) might be either positive or negative, depending on the impact of current education transfers on next-period income tax rate. In the case it will be negative, the incentives to invest in human capital even when the elderly power is relatively small are dampened. Finally, in equilibrium the terms (3*) in Eq. (2.40a) is positive. Consequently, there are higher incentives to redistribute backward. In general distortionary taxation on capital may improve dynamic efficiency even when intergenerational conflict is weaker.

2.7 Ideological Competition

Finally, we consider the possibility of ideological competition among parties. Differently from the case discussed in sections 2.5 and 2.6, parties $\Re_t$ and $\mathcal{L}_t$ maximize Eq. (2.27) and (2.28), respectively.

The Nash equilibrium is characterized by the following first order condition for each party i_t:

$$0 = \underbrace{-\frac{\partial q\left(z_{i_t} - \bar{z}_i\right)}{\partial z_{i_t}}}_{\text{ideological competition}} + \frac{\psi^2}{\psi^1}\frac{\partial u\left(\mathcal{C}^2_{i_t}\right)}{\partial z_{i_t}} + \frac{\partial u\left(\mathcal{C}^1_{i_t}\right)}{\partial z_{i_t}} \tag{2.41}$$
$$+ \beta E \underbrace{\left(\frac{\partial \mathcal{P}_{i_t}}{\partial z_{i_t}}\left(W^2_{i_t} + \theta_{t+1} + \psi^2_{jt+1} - V^2_{i_t}\right) + \mathcal{P}_{i_t}\frac{\partial W^2_{i_t}}{\partial z_{i_t}} + \left(1 - \mathcal{P}_{i_t}\right)\frac{\partial V^2_{i_t}}{\partial z_{i_t}}\right)}_{\text{endogenous political turnover}}$$

The equilibrium platforms do not reduce to a common political proposal, which simply maximizes the weighted utility of current living voters. Contrarily, due to ideological competition the two parties propose different platforms. As a consequence, in equilibrium the probability of being elected is not constant and equal to one half, but it turns out to be a function of the relevant payoff state variables, $\mathcal{P}_{i_t}(k_{t+1}, h_{t+1})$. Perfect forward looking parties act strategically, choosing a platform which also maximizes the probability of winning future elections if they will become the next-period incumbent party. The possibility to endogenously control for the political turnover adds another source of distortion quantified by the third term of Eq. (2.41).

2.8 Conclusions

In this paper we show how intergenerational conflicts over the redistribution of public resources may enhance efficiency when human capital accumulation is the main force driving the economic growth. We build a tractable dynamic politico-economic model in a neoclassical environment and determine the subgame Markov perfect equilibrium of a dynamic game characterized by both ideological heterogeneous voters and office-seeking political parties.

Three main sources of distortions characterize the theoretical environment: Incomplete credit markets, physical capital taxation, and political uncertainty. In the first scenario characterized by neither capital taxation nor political distortions, the only source of inefficiency comes from the credit market constraint. In equilibrium the presence of swing voters determines the emergence of intergenerational conflicts over the redistribution of public resources and the equilibrium political platform turns out to be characterized by: i) positive transfers to the elderly, ii) negative transfers from the adult, and iii) lower investment in public education. The participation of the elderly to the political debate with the emergence of political disagreement on the redistribution of public resources improves the overall economic efficiency. The elderly claim for positive transfers induces a simultaneous reduction in both physical and human capital accumulation, partially offsetting the dynamic inefficiency. Adopting a first order characterization, we test the robustness of our theoretical findings. Specifically we study how distortionary taxation and political distortions, which arise when parties compete ideologically, alter the main finding about the positive correlation between intergenerational political disagreement and efficiency. We find that both types of distortions might be welfare improving in the sense of dynamic efficiency, dampening the positive impact of intergenerational conflicts on the overall economic welfare. In order to quantitatively assess the gains in efficiency and the loss due to distortions in this second scenario, we need to perform numerical analyses. Future works will be directed toward this line of research.

2.9 Appendix A

Derivation of closed form solution for efficient allocation. Due to the log-preferences, which cancel out income and substitution effects, savings are a fraction of current variables. Furthermore, along the balanced growth path we attain $\frac{u_{c_t^2}\left(c_t^{2*}\right)}{u_{c_{t+1}^2}\left(c_{t+1}^{2*}\right)} = \frac{c_{t+1}^{2*}}{c_t^{2*}} = \varphi\left(\tilde{f}^*\right)$.

Using the Euler conditions (2.8) and (2.9) we obtain:

$$\delta\vartheta_{\tilde{k}}\left(\tilde{k}^*\right) = \varphi\left(\tilde{f}^*\right) \tag{2.42}$$

The condition (2.10) in balanced growth path is $\varphi_{\tilde{f}}\left(\tilde{f}^*\right) = \frac{\vartheta_{\tilde{k}}}{\vartheta\left(\tilde{k}^*\right)-\tilde{k}\vartheta_{\tilde{k}}\left(\tilde{k}^*\right)}(1-\varepsilon)$ where $\varepsilon = \frac{1}{\vartheta_{\tilde{k}}}\left(\varphi\left(\tilde{f}^*\right) - \tilde{f}^*\varphi_{\tilde{f}}\left(\tilde{f}^*\right)\right)$. Using the condition (2.42) and Eq. (**??**) we have ε constant and equal to $\varepsilon = \delta(1-\eta)$, consequently $\varphi_{\tilde{f}}\left(\tilde{f}^*\right)$ turns out to be equal to:

$$\varphi_{\tilde{f}}\left(\tilde{f}^*\right) = \frac{\vartheta_{\tilde{k}}}{\vartheta\left(\tilde{k}^*\right) - \tilde{k}^*\vartheta_{\tilde{k}}\left(\tilde{k}^*\right)}(1-\varepsilon) \tag{2.43}$$

Using the parametric forms given by Eq. (**??**) and (**??**) and plugging the partial derivatives into (2.43) we get $\tilde{f}^* = \frac{1-\alpha}{\alpha}\frac{\eta}{1-\varepsilon}\tilde{k}^*\varphi\left(\tilde{f}^*\right)$. Using Eq. (2.42) we get $\tilde{f}^* = \frac{1-\alpha}{\alpha}\frac{\eta\delta}{1-\varepsilon}\tilde{k}^*\vartheta_{\tilde{k}}\left(\tilde{k}^*\right)$ and finally:

$$\tilde{f}^* = \frac{\eta\delta(1-\alpha)}{1-\varepsilon}\tilde{y}^* \tag{2.44}$$

Aggregate consumption along the balanced growth path is then equal to:

$$\tilde{C}^* \equiv \tilde{c}^{1*} + \tilde{c}^{2*} = \tilde{y}^* - \underbrace{\frac{h_{t+1}^*}{h_t^*}\tilde{k}^* - \tilde{f}^*}_{\tilde{s}^*} \tag{2.45}$$

where h^* is the human capital level in steady state. After using Eq. (2.42) and (2.44) and rearranging, we obtain that aggregate consumption is also equal to $\tilde{C}^* = \frac{(1-\delta)(1-\varepsilon\alpha)}{(1-\varepsilon)}\tilde{y}^*$. To identify how aggregate consumption is shared between the first and the second period let us recall (2.9) in balanced path:

$$\vartheta_{\tilde{k}}\left(\tilde{k}^*\right) = \frac{u_{c_t^1}\left(c_t^{1*}\right)}{\beta u_{c_{t+1}^2}\left(c_{t+1}^{2*}\right)} = \frac{\varphi\left(\tilde{f}^*\right)}{\beta}\frac{\tilde{c}^{2*}}{\tilde{c}^{1*}}$$

then we yield $\frac{\tilde{c}^{2*}}{\tilde{c}^{1*}} = \frac{\beta}{\delta}$ and consequently $\tilde{c}^{1*} = \frac{\delta}{\delta+\beta}\tilde{C}^*$ and $\tilde{c}^{2*} = \frac{\beta}{\delta+\beta}\tilde{C}^*$ and individual savings are:

Let us denote with $\mu_o \equiv \frac{\eta\delta(1-\alpha)}{1-\varepsilon}$, $\zeta_o \equiv \frac{(1-\varepsilon)-\eta\delta(1-\alpha)-(1-\delta)(1-\varepsilon\alpha)}{(1-\varepsilon)}$. Then using Eq. (2.45) we finally obtain the steady state level of $\tilde{k}^*$:

$$\begin{aligned} \tilde{k}^* &= \left(\frac{A}{B}\frac{\zeta_o}{(A\mu_o)^\eta}\right)^{\frac{1}{1-\alpha(1-\eta)}}, & \tilde{y}^* &= A\left(\frac{A}{B}\frac{\zeta_o}{(A\mu_o)^\eta}\right)^{\frac{\alpha}{1-\alpha(1-\eta)}} \\ \tilde{f}^* &= A\mu_o\left(\frac{A}{B}\frac{\zeta_o}{(A\mu_o)^\eta}\right)^{\frac{\alpha}{1-\alpha(1-\eta)}}, & R^o\left(\tilde{k}^*\right) &= \alpha A\left(\frac{A}{B}\frac{\zeta_o}{(A\mu_o)^\eta}\right)^{\frac{\alpha-1}{1-\alpha(1-\eta)}} \end{aligned}$$

Finally, using Eq. (**??**) and (2.44), the economy's growth rate turns out to be equal to:

$$\gamma^o \equiv \varphi\left(\tilde{f}^*\right) = B\left(A\mu_o\right)^\eta\left(\frac{A}{B}\frac{\zeta_o}{(A\mu_o)^\eta}\right)^{\frac{\alpha\eta}{1-\alpha(1-\eta)}}$$

■

2.10 Appendix B

Proposition (4)**.** If ideological competition is ruled out, the political competition problem concerns the maximization of Eq. (2.24) and (2.25) with respect to the proposed political platform z_{i_t} for party $\Re_t$ and $\mathcal{L}_t$, respectively. Let us consider a finite horizon economy, which ends at time $T = 2$. In the last period the political maximization program for each party i_t is equal to:

$$\max_{z_{i_2}} \frac{\psi^2}{\psi^1} u\left(\mathcal{C}_{i_2}^2\right) + u\left(\mathcal{C}_{i_2}^1\right)$$

Given that parties' objective function is simply the probability of winning elections, then in equilibrium they propose the same platform, $z_{\Re_2} = z_{\mathcal{L}_2}$ and $\Delta u\left(\mathcal{C}_2^2\right) = \Delta u\left(\mathcal{C}_2^1\right) = 0$. Using Eq. (2.23) it follows that $\bar{\theta}_2 = 0$ and $\Pr\left(\theta_2 > \bar{\theta}_2\right) = \Pr\left(\theta_2 < \bar{\theta}_2\right) = \frac{1}{2}$.

At time $T = 1$ the maximization program then becomes:

$$\max_{z_{i_1}} \frac{\psi^2}{\psi^1} u\left(\mathcal{C}_{i_1}^2\right) + u\left(\mathcal{C}_{i_1}^1\right) + \beta E\left(\mathcal{P}_{i_1}\left(W_{i_1}^2 + \theta_2 + \psi_{j2}^2\right) + \left(1 - \mathcal{P}_{i_1}\right) V_{i_2}^2\right) \quad (2.46)$$

where $\mathcal{P}_{i_1} = \Pr\left(\theta_2 > \bar{\theta}_2 | \theta_1 > \bar{\theta}_1\right)$ if $i_1 = \Re_1$ and $\mathcal{P}_{i_1} = \Pr\left(\theta_2 > \bar{\theta}_2 | \theta_1 < \bar{\theta}_1\right)$ if $i_1 = \mathcal{L}_1$. Given that $\bar{\theta}_2$ will be always equal to 0 independently from the incumbent party at time $t = 2$, then it follows that the conditional probability of being elected will be equal to the unconditional one, i.e.:

$$\mathcal{P}_{\Re_1} = \mathcal{P}_{\mathcal{L}_1} = \frac{1}{2}$$

Consequently the maximization program given by Eq. (2.46) reduces to:

$$\max_{z_{i_1}} \frac{\psi^2}{\psi^1} u\left(\mathcal{C}_{i_1}^2\right) + u\left(\mathcal{C}_{i_1}^1\right) + \beta u\left(\mathcal{C}_{i_2}^2\right)$$

Replicating the argument made at time $T = 2$, we have $z_{\Re_1} = z_{\mathcal{L}_1}$ with $\Delta u\left(\mathcal{C}_1^2\right) = \Delta u\left(\mathcal{C}_1^1\right) = 0$ and $\mathcal{V}^2\left(k_2, h_2; \mathcal{L}_1\right) - \mathcal{W}^2\left(k_2, h_2; \Re_1\right) = 0$. Using Eq. (2.23) it follows that $\bar{\theta}_1 = 0$ and $\Pr\left(\theta_1 > \bar{\theta}_1\right) = \Pr\left(\theta_1 < \bar{\theta}_1\right) = \frac{1}{2}$. Hence the two candidates' platform converges in equilibrium to the same fiscal policy maximizing the weighted-average utility for adults and old:

$$\max_{z_t} \frac{\psi^2}{\psi^1} u\left(C^2\left(b_t, h_t, k_t\right)\right) + u\left(C^1\left(f_t, b_t, h_t, k_t, k_{t+1}\right)\right) + \beta u\left(C^2\left(b_{t+1}, h_{t+1}, k_{t+1}\right)\right)$$

under the constraint $u_c\left(\mathcal{C}_t^1\right) - \beta R_{t+1} u_c\left(\mathcal{C}_{t+1}^2\right) = 0$. ■

Lemma (2). Let us consider a simple two period-economy, $T = 2$, and solve backward. At the last period $k_2 = 0$ and the politicians maximization problem is:

$$\max_{f_2, b_2} u\left(\mathcal{C}_2^1\right) + \phi u\left(\mathcal{C}_2^2\right)$$

In the last period $f_2 = 0$ and the Euler condition which determines the amount of pension is $\frac{u_{\mathcal{C}_2^1}}{u_{\mathcal{C}_2^2}} = \phi$ which implies $b_2 = \frac{\phi(1-\alpha)-\alpha}{1+\phi} y_2$. At time $t = 1$ the maximization program becomes:

$$\max_{f_1, b_1} u\left(\mathcal{C}_1^1\right) + \beta u\left(\mathcal{C}_2^2\right) + \phi u\left(\mathcal{C}_1^2\right)$$

After some algebra the politicians' Euler conditions turns out to be equal to:

$$\frac{u_{\mathcal{C}_1^2}}{u_{\mathcal{C}_1^1}} = \frac{1 + \alpha\beta}{\phi + \alpha\beta\left(1 + \phi\right)}$$
$$\frac{u_{\mathcal{C}_1^2}}{u_{\mathcal{C}_1^1}} = \frac{1}{1 + \phi} \varphi_{\tilde{f}_1} \frac{\vartheta\left(\tilde{k}_2\right) - \tilde{k}_2 \vartheta_{\tilde{k}}\left(\tilde{k}_2\right)}{\vartheta_{\tilde{k}}\left(\tilde{k}_2\right)}$$

Recalling that $\varphi_{\tilde{f}_1} \frac{\vartheta(\tilde{k}_2) - \tilde{k}_2 \vartheta_{\tilde{k}}(\tilde{k}_2)}{\vartheta_{\tilde{k}}(\tilde{k}_2)} = \frac{k_2}{f_1} \frac{\eta(1-\alpha)}{\alpha}$, solving the system we have at time $t = 1$:

$$f_1 = \frac{\beta\eta\left(1 - \alpha\right)}{\phi + \left(1 + \beta\left(\alpha + \eta\left(1 - \alpha\right)\right)\right)} y_1$$
$$b_1 = \frac{\phi\left(1 - \alpha\right) - \alpha\left(1 + \beta\left(\alpha + \eta\left(1 - \alpha\right)\right)\right)}{\phi + \left(1 + \beta\left(\alpha + \eta\left(1 - \alpha\right)\right)\right)} y_1$$

Due to the specific parametric form, after only two recursions the policies remain unchanged and the political Euler conditions becomes equal to:

$$\frac{u_{\mathcal{C}_t^2}}{u_{\mathcal{C}_t^1}} = \frac{1 + \beta\alpha}{\phi + \alpha\beta\left(\phi + \left(1 + \beta\left(\alpha + \eta\left(1 - \alpha\right)\right)\right)\right)}$$
$$\frac{u_{\mathcal{C}_t^2}}{u_{\mathcal{C}_t^1}} = \frac{1}{\phi + \left(1 + \beta\left(\alpha + \eta\left(1 - \alpha\right)\right)\right)} \frac{F_{h_t}}{F_{k_t}} \varphi_{\tilde{f}_t}$$

which implies:

$$\varphi_{\tilde{f}_t} - \frac{\vartheta_{\tilde{k}}\left(\tilde{k}_{t+1}\right)}{\vartheta\left(\tilde{k}_{t+1}\right) - \tilde{k}_{t+1}\vartheta_{\tilde{k}}\left(\tilde{k}_{t+1}\right)}\Lambda\left(\phi\right) = 0$$

where $\Lambda\left(\phi\right) = \frac{(1+\beta\alpha)(\phi+(1+\beta(\alpha+\eta(1-\alpha))))}{\phi+\alpha\beta(\phi+(1+\beta(\alpha+\eta(1-\alpha))))} > 1$ and $\Lambda'\left(\phi\right) = -\frac{(1+\alpha\beta)(1+\alpha\beta(1-\eta)+\beta\eta)}{\left(\phi+\alpha\beta\left(1+\beta(\alpha+\eta(1-\alpha)+\phi)^2\right)\right)} < 0$. ■

Proposition (5). Using Lemma 2, rearranging and solving for f_t and b_t we obtain:

$$\mathcal{F}\left(h_t, k_t\right) = \xi^f A k_t^{\alpha} h_t^{1-\alpha}, \quad \mathcal{B}\left(h_t, k_t\right) = \xi^b A k_t^{\alpha} h_t^{1-\alpha}$$

with $\xi^f \equiv \frac{\beta\eta(1-\alpha)}{\phi+(1+\beta(\alpha+\eta(1-\alpha)))}$ and $\xi^b \equiv \frac{\phi(1-\alpha)-\alpha(1+\beta(\alpha+\eta(1-\alpha)))}{\phi+(1+\beta(\alpha+\eta(1-\alpha)))}$ and, by using condition (2.5), the income tax rate is equal to $\pi_t = \frac{\xi^f+\xi^b}{1-\alpha} = 1 - \frac{1+\alpha\beta}{(1-\alpha)(1+\beta(\alpha+\eta(1-\alpha))+\phi)}$. If $\phi > \frac{(1+\alpha\beta)\alpha}{1-\alpha} - (1-\alpha)\beta\eta$ then $\pi_t > 0$.

Furthermore, the laws of motion of the state variables are:

$$k_{t+1} = \frac{A\left(1+\alpha\beta\right)\alpha\beta}{\alpha\beta\left(1+\beta\left(\alpha+\eta\left(1-\alpha\right)\right)+\phi\right)+\phi} k_t^{\alpha} h_t^{1-\alpha} \tag{2.47}$$

$$h_{t+1} = B\left(\frac{A\beta\eta\left(1-\alpha\right)}{\phi+\left(1+\beta\left(\alpha+\eta\left(1-\alpha\right)\right)\right)}\right)^{\eta} k_t^{\alpha\eta} h_t^{1-\alpha\eta} \tag{2.48}$$

In balanced growth path $\tilde{k}_{t+1}$ is constant over time. Let us denote with $\mu_p \equiv \frac{\beta\eta(1-\alpha)}{\phi+(1+\beta(\alpha+\eta(1-\alpha)))}$, $\zeta_p \equiv \frac{(1+\alpha\beta)\alpha\beta}{\alpha\beta(1+\beta(\alpha+\eta(1-\alpha))+\phi)+\phi}$. Then, by using Eq. (2.47) and (2.48) and rearranging, we obtain the steady state level of the politico-economic allocation is equal to:

$$\tilde{f} = A\mu_p\left(\frac{A}{B}\frac{\zeta_p}{(A\mu_p)^{\eta}}\right)^{\frac{\alpha}{1-\alpha(1-\eta)}}, \quad \tilde{k} = \left(\frac{A}{B}\frac{\zeta_p}{(A\mu_p)^{\eta}}\right)^{\frac{1}{1-\alpha(1-\eta)}}, \quad \tilde{y} = A\left(\frac{A}{B}\frac{\zeta_p}{(A\mu_p)^{\eta}}\right)^{\frac{\alpha}{1-\alpha(1-\eta)}}$$

Then, the economy's growth rate and the rental price of capital are, respectively:

$$\gamma^p \equiv \frac{h_{t+1}^*}{h_t^*} = \left(\left(A^{\eta}B^{1-\alpha}\right)\mu_p^{(1-\alpha)\eta}\zeta_p^{\alpha\eta}\right)^{\frac{1}{1-\alpha(1-\eta)}}, \quad R^p = \alpha\left(\left(A^{\eta}B^{1-\alpha}\right)\mu_p^{(1-\alpha)\eta}\zeta_p^{\alpha-1}\right)^{\frac{1}{1-\alpha(1-\eta)}}$$

■

Corollario (8). From Lemma 2, taking $\phi = 0$ and solving backward, we obtain $\mathcal{F}_m\left(h_t, k_t\right) = \xi_m^f y_t$ and $\mathcal{B}_m\left(h_t, k_t\right) = \xi_m^b y_t$ with $\xi_m^f \equiv \frac{\beta\eta(1-\alpha)}{1+\beta(\alpha+\eta(1-\alpha))}$

and $\xi_m^b \equiv -\frac{\alpha(1+\beta(\alpha+\eta(1-\alpha)))}{1+\beta(\alpha+\eta(1-\alpha))}$. Then, by using condition (2.5), the income tax rate is equal to $\pi_t = 1 - \frac{1+\alpha\beta}{(1-\alpha)(1+\beta(\alpha+\eta(1-\alpha)))}$. Furthermore, the rental price of capital, $R^m\left(\tilde{k}\right)$, and the economy's growth rate, $\varphi^m\left(\tilde{k}\right)$, in the median voter case are equal to:

$$R^m\left(\tilde{k}\right) = \lim_{\phi\to 0} R^p\left(\tilde{k};\phi\right)$$
$$\varphi^m\left(\tilde{k}\right) = \lim_{\phi\to 0} \varphi^p\left(\tilde{k};\phi\right)$$

such that $\varphi^m\left(\tilde{k}\right) > \varphi^p\left(\tilde{k};\phi\right)$ and $R^m\left(\tilde{k}\right) < R^p\left(\tilde{k};\phi\right)$. ■

Lemma (3). Using Eq. (2.33), (2.34), (2.36) and (2.37), we have that $R^m\left(\tilde{k}\right) < R^o\left(\tilde{k}^*;\delta\right)$ when $\frac{\mu_m}{\mu_o} < \left(\frac{\zeta_m}{\zeta_o}\right)^{\frac{1}{\eta}}$, and $\varphi^m\left(\tilde{f}\right) > \varphi^o\left(\tilde{f}^*;\delta\right)$ when $\frac{\mu_m}{\mu_o} > \left(\frac{\zeta_o}{\zeta_m}\right)^{\frac{\alpha}{1-\alpha}}$. The joint conditions imply $\zeta_o < \zeta_m$. Plugging the values for ζ_o and ζ_m we obtain that the Lemma holds for any $\delta < \bar{\delta} = \min\left\{\frac{1+\alpha\beta}{\alpha(1+\beta(\alpha+\eta(1-\alpha)))}, 1\right\}$. ■

Proposition (6). From Lemma 2, for any level of dynastic welfare weight $\delta < \bar{\delta}$, we have:

$$R^m\left(\tilde{k}\right) - R^o\left(\tilde{k}^*;\delta\right) < 0$$
$$\varphi^m\left(\tilde{k}\right) - \varphi^o\left(\tilde{k}^*;\delta\right) > 0$$

Furthermore, according to Corollary 1, for any $\phi > 0$ $R^m\left(\tilde{k}\right) < R^p\left(\tilde{k};\phi\right)$ and $\varphi^m\left(\tilde{k}\right) > \varphi^p\left(\tilde{k};\phi\right)$. Thus, by continuity of functions $R(\cdot)$ and $\varphi(\cdot)$, for any $\epsilon > 0$, there exist $\delta^\circ < \bar{\delta}$ and $\hat{\phi} > 0$, such that $\left|R^p\left(\tilde{k};\hat{\phi}\right) - R^o\left(\tilde{k}^*;\delta^o\right)\right| < \epsilon$ and $\left|\varphi^p\left(\tilde{k};\hat{\phi}\right) - \varphi^o\left(\tilde{k}^*;\delta^o\right)\right| < \epsilon$. ■

Proposition (7). See proof of Proposition 4 under the balanced budget condition given by Eq. (2.5). ■

Bibliography

[1] Alesina, A., and Tabellini, G., 1990, A Positive Theory of Fiscal Deficits and Government Debt, *Review of Economic Studies*, 57 (3) , 403-414.

[2] Azariadis, C., and Galasso, V., 2002, Fiscal Constitutions, *Journal of Economic Theory*, 103 (2), 255-281.

[3] Azzimonti, M., 2011, Barriers to Investment in Polarized Societies, *American Economic Review*, Forthcoming.

[4] Barro, R., 1991, Economic Growth in a Cross Section of Countries, *The Quarterly Journal of Economics*, 106 (2), 407-443.

[5] Boldrin, M. and Montes, A., 2005, The Intergenerational State Education and Pension, *Review of Economic Studies*, 72 (3), 651-664.

[6] Docquier, F., Paddison, O., and P., Pestieau, 2006, Optimal Accumulation in an Endogenous Growth Setting with Human Capital, *Journal of Economic Theory,* 134, 361-378.

[7] Forni, L., 2005, Social Security as Markov Equilibrium in OLG Models, *Review of Economic Dynamics*, 8 (1), 178-194.

[8] Gonzalez-Eiras, M., and Niepelt, D., 2011, Aging, Government Budgets, Retirement, and Growth, *CESifo Working Paper.*

[9] Klein, P., Krusell, P., and Rios-Rull, J. V., 2008, Time Consistent Public Expenditures, *Review of Economic Studies*, 75 (3), 789-808.

[10] Krusell, P., Quadrini, V. and Ríos-Rull, J. V., 1997, Politico-Economic Equilibrium and Economic Growth, *Journal of Economic Dynamics and Control*, 21 (1), 243-272.

[11] Lindbeck, A. and J. Weibull, 1987, Balanced-budget Redistribution as the Outcome of Political Competition, *Public Choice*, 52, 273-297.

Chapter 3

Self-Commitment Institutions and Cooperation in Overlapping Generation Games

"Precisely because our ability to impose exogenously the institutional structure that will effectively govern society has proven to be so weak, we must open up our analysis to the evolution of rules from games of conflict to games of cooperation. Instead of designing ideal institutional settings....we have to examine the endogenous creation of the rules by social participants themselves. The science and art of association is one of self-governance and not necessarily one of constitutional craftsmanship. And herein lies the contribution that contemporary research on anarchism can make to modern political economy." - Peter Boettke

3.1 Introduction

The study of the conditions, which sustain mutual cooperation among self-interested agents, is a milestone in the social science literature. There are three general ways to enforce mutual cooperative agreements and to achieve efficiency: i) Competition, ii) formal contracts, and iii) informal contracts. As extensively known, the former fails in presence of market imperfections, while the second collapses in the case of no ex-post verifiability and in absence of a third party external to the negotiation (i.e. the court) that guarantees the enforcement of the contract. In these circumstances informal contracts may instead succeed in improving efficiency. These contracts consist of infinite repetitions of agents' interactions over time with no court to credibly pre-commit over cooperative behavior, where the value of future interactions serves as the reward and penalty to discipline the agents' current behavior. From now on we refer to informal contracts as social norms, i.e. as customary rules of behavior that coordinate the agents' interactions.[1]

Past literature has focused on two types of enforceability mechanisms to sustain social norms: *personal* and *community* enforcement. Under personal enforcement a cheater will only face retaliation by their victim. While, in the case of community enforcement all members of the society react to a deviation. The former is effective when the same sets of players match each other over time (Fudenberg and Maskin, 1986). However in many economic circumstances the same players do not meet repeatedly, instead they change over time. Nevertheless, in this scenario community enforcement may achieve efficiency. As Kandori (1992a) shows, even in the case of trade where agents change partners over time, any feasible rational allocation can be sustained as long as other members of the society, who are not directly involved in bilateral trading, sanction the defection of agents. To be effective, community enforcement requires the existence of an exogenous, and not manipulable, decentralized information transmission process, which creates labels that make defeating agents recognizable by all members of the community.

Our analysis applies in a theoretical environment in which these traditional types of enforcement rarely work in achieving efficiency, i.e. overlapping generation (OLG) games with imperfect public monitoring. In OLG games different organizations repeatedly interact over time. Even if the organizations survive indefinitely over time, individuals with a finite life span manage their own governance. Each member enters the organization in an asynchronous way and interacts with other agents for a finite time before being replaced by another individual after death.[2] Due to the finite-horizon

[1] See Durlauf and Blume (2011) for a more rigorous definition and characterization of social norms.

[2] Remarkable examples are provided by: The relationships between boards of directors and the shareholders' committees; interactions between regulators and firms; the com-

interactions among players, personal punishment is very limited, and in some cases totally prevented. Furthermore, community enforcement looses its effectiveness when agents imperfectly observe past history. In OLG games the lack of perfect monitoring introduces an element of moral hazard, which in some circumstances prevents the achievement of full efficiency.[3]

The purpose of this research is to investigate the existence of an alternative enforcement mechanism, which we refer to as Self-Commitment Institution (SCI). This may sustain, and improve, cooperation in a non cooperative setting when: i) Agents are finite-living periods OLG, and ii) imperfectly observe past history. The economy we study is populated by OLG of homogeneous agents living up two periods: Young and Old. Each agent imperfectly observes the past public history and takes actions only in the first period of life. Young are endowed with one unit of productive time and decide whether to partially transfer consumption goods to current living Old. The elderly agents have no endowment and are completely passive. Given the sequential nature of the game, subsequent generations are linked through strategic interaction and payoffs functions. Intergenerational cooperation emerges in equilibrium as long as Young decide to transfer consumption goods to Old. Clearly, in a static setting the intergenerational cooperation is not self-enforced. On the contrary, in a repeated interaction setting we might investigate how the different enforcement mechanisms may succeed in achieving efficiency. Given that at each time period different generations match each other, in our environment personal punishment is totally interdicted.

We distinguish social norms into two categories: i) Social norms without SCI, based on the community enforcement mechanism, and ii) social norms with SCI, which associate the individual self-commitment decisions to community enforcement. Two key features characterize social norms with SCI. First, at each time period players may decide to self-commit by self-expropriating a share of their endowment. Such resources are not required to

munitarian and international agreements among member States; the electoral promises between political parties and the electorate.

[3] The argument is qualitatively similar to those discussed by both Radner, Myerson and Maskin (1986) and Abreu, Milgrom and Pearce (1986) for repeated games environment. Imperfect public observability generates an endogenous cost of monitoring, which makes the best sustainable equilibrium payoff bounded away from fully efficiency. Nevertheless in this scenario, Fudenberg, Levine and Maskin (1994) prove that the Folk theorem applies when the fully ranking condition holds and agents play asymmetric public strategies.

be used neither for productive nor for redistributive purposes. In an extreme case we might interpret the self-expropriated endowment as purely wasted resources, or alternatively as property rights transfers, if we assume the existence of a technology, which enables their transmission. Second, we abstract from uncertainty about the realization of self-expropriation decisions. Once these decisions are taken, all the members of the society perfectly observe and recall the past self-commitment actions. Consequently, agents may contingent their punishment decisions not only on the basis of the history of public signals but also on the previous players' self-expropriation actions.

Since the self-expropriation does not lead to positive economic spillover, from an ex-ante point of view the central planner with full taxation power would never require agents to self-commit. On the contrary, in a repeated setting with imperfect monitoring, if agents behavior is coordinated by social norms with SCI, then the society may achieve higher efficiency compared to the cases in which agents coordinate on social norms without SCI.

Suppose that Young only observe a noisy signal of previous players' actions. This assumption seems reasonable in the OLG environment, in which agents live for a finite time and, after having entered an organization, they become aware of previous history through reports transmitted by previous generations in the form of public signals.[4] As a consequence, the realization of the signals constitutes the relevant information-gathering for current agents' strategic behavior. Apart from previous actions, the signals are also affected by a random exogenous component, which weakens the correlation between private decision and the signal. A social norm on intergenerational cooperation might be enforced by a simple community punishment mechanism, which requires agents to permanently punish as soon as a negative signal is realized. Clearly, this enforcement mechanism turns out to be particularly effective in deterring agents' defection. Nevertheless, it cannot succeed in achieving full efficiency, because with all probability at a certain point in time a negative signal will emerge and, as a consequence, intergenerational cooperation will break down, even if no one has actually deviated from the cooperative path. Therefore, in an environment characterized by highly volatile shocks the intergenerational cooperation sustained only by commu-

[4] Lagunoff and Matsui (2004) analyze an environment with a lack of prior memory by focusing on intergenerational message instead of public signals. In the presence of higher degrees of intergenerational altruism and small costs of intergenerational communication, the authors demonstrate that the Folk theorem holds.

nity enforcement mechanism achieves low level of efficiency. In this scenario suppose that agents at each time period may decide not only if transferring intergenerational consumption good but also if self-expropriating part of their endowment. In this environment self-expropriation acts as a reduction of the productive time endowment of Young. If agents believe that no one will coordinate their strategy on the observable self-expropriation decisions, in equilibrium they decide to never self-commit and only community enforcement will inefficiently enforce intergenerational cooperation. We refer to this expectational coordination as *social norms without SCI*. On the contrary, allow agents to believe that all generations will base their strategic behavior not only on the public signals but also on the previous players' self-expropriation actions, which we refer to these as *social norms with SCI*. The mechanism prescribes that if agents do not self-expropriate, then they will be punished by future generations independently from the realization of a public signal. As a consequence, if Young prefer to smooth consumption over time, then self-commitment becomes a necessary condition to have positive expectation on future gains. More interestingly, self-expropriation is also sufficient to sustain cooperation by relaxing the opportunistic behavior of players in the implementation of intergenerational transfers. Indeed, by necessarily self-expropriating, agents are actually self-committing on cooperation. They reduce the current marginal gain from deviation and, at the same time, decrease the probability that future generations will punish in the case of a bad signal, i.e. players becomes endogenously more *optimistic*. As a result, the actors' self-commitment fosters the coordination efforts and facilitates cooperative relationships, which would otherwise not exist. In other terms, a society that adopts social norm with SCI might significantly improve the overall efficiency.

Clearly, as long as players are able to fully detect earlier generations' defection (i.e. perfect observability of history), SCI acquires no value and community enforcement is sufficient to sustain fully efficient cooperative equilibria. The existence of an imperfect monitoring technology, along with the possibility of endogenously modifying the marginal benefits from deviations, are necessary for making SCI valuable in repeated OLG settings. This result helps to stress the role of SCI as institution able to sustain cooperation when the moral hazard issue is a relevant feature of the economic environment.

Heretofore, we have stressed the role of SCI excluding the possibility of productively allocating the self-expropriated resources. A direct extension and application of the basic setup is to allow SCI to be productive, for example in terms of education provision. In this scenario, the self-expropriated resources are more appropriately treated as property right transfers, when we consider the existence of a technology, which transforms the transferred endowment into future productive assets. A large corpus of literature exists on intergenerational transfers, which justify the education provision for the sustainability of the Welfare state in a payroll pension system. Three main theoretical justifications for the investment in human capital in a context of intergenerational cooperation have been proposed: altruistic (or bequest) motives, endogenous asset returns, voting and political sharing rules.[5] The possibility of enhancing efficiency in a context characterized by imperfect observability through the implementation of social norms with SCI provides an alternative justification. The application fits well with the required conditions, to allow SCI to be effective. First, it is plausible to assume that the share of time used by parents to provide education to their children will be perfectly observable, at least by the children who directly receive the parents' endowment. Second, more the time devoted to the children education is, lower are the marginal benefits coming from not contributing to payroll pension transfers.

In this paper we derive the conditions under which social norms with SCI outperform social norms without SCI. The concept of Public Perfect Equilibrium (PPE), as developed by Abreu, Pearce and Stacchetti (1990) is adapted to an OLG game in order to characterize the best sustainable equilibrium payoffs generated by both social norms and to derive the value of SCI. We interpret education provision in terms of productive SCI and, finally, we conclude. All the proofs are provided in the appendix.

[5] Kaganovich and Zilcha (1999) analyze the interaction between education and social security by adopting an altruistic motive. Boldrin and Montes (2005) formalize public education and PAYG system as two parts of an intergenerational contract where public pension is the return on the investment into the human capital of the next generation. In Lancia and Russo's model (2011) selfish adults buy insurance for their future old age by paying productive education transfers to their children to raise the labor productivity of the next period. When old they partially grab the bigger output in the form of PAYG transfers by exerting political power in a probabilistic voting environment.

3.2 Past literature

This paper draws on two main research strands of literature. The first concerns the study of cooperative behavior in games played by overlapping generations of agents, while the second relates to the analyses of cooperation in a repeated interaction setting under imperfect public monitoring.

Sizeable literature has focused on the study of intergenerational cooperation without considering informational constraints. Starting from the seminal work of Hammond (1975), who examined a non-cooperative game version of Samuelson's consumption-loan model showing that a Pareto efficient allocation is attained by a subgame perfect equilibrium, various Folk Theorems have been proved by more general OLG structures (Cremer (1986), Salant (1988), Kandori (1992b) and Smith (1992)).[6] The main insights from this branch of literature is that any mutually beneficial outcome could be sustained as long as agents are patient and/or live long enough and each individual can perfectly observe the past. More recent papers have studied how the introduction of informational constraints affects the emergence of cooperation and, in turns, the possibility to achieve efficiency in games with repeated interactions. Bhaskar (1998) examines the role of general informational constraints in 2-period Samuelson OLG consumption-loan games. The author shows that if players have finite memory, then Pareto improving transfers are not sustainable in pure strategy equilibria. Nevertheless, allow agents to observe at least the period before their arrival, then optimal transfers are sustainable in mixed strategies setting. However, cooperation turns out to be not robust to small random perturbation.[7] More severe informational constraints have been introduced in recent works that examine OLG games where cheap-talk intergenerational communication is introduced. Lagunoff and Matsui (2004) proves in an OLG game with no prior memory, costly communication and intergenerational altruism, that the Folk Theo-

[6] Cremer (1986) analyses a generalization of the prisoners' dilemma in an OLG setting, to show that cooperation can be sustained by the reversion to the Nash equilibrium of the stage game, as long as agents are patient and/or live long enough. Salant (1988) proves the Folk Theorem with particular simple equilibria in two-person games with some restriction on the payoff functions. Kandori (1992b) extend Salant's analysis for a general N-person games where players in the same cohort interact for a long time, and then are gradually replaced by the next generation of players. Smith (1992) presents a variation and extension of Kandori's model.

[7] See Cole and Kocherlakota (2005) for an extension of Bhaskar's finite memory setting to the case of imperfect observability.

rem holds when either communication costs are small enough, or individual are sufficiently altruistic. Lagunoff, Anderlini and Gerardi (2005) extend the basic setup by introducing private communication in a dynastic game.

Diverging from previous literature, although we allow agents to perfectly recall past history, we introduce imperfect public monitoring as an informational constraint.[8] As a consequence, our paper also refers to a strand of literature that has studied cooperation in repeated setting where agents adopt public strategies. In the spirit of dynamic programming, Abreu, Pearce and Stacchetti (1990) introduce and illustrate the ideas of self-generating set of equilibrium payoffs and factorization to prove a recursive formulation of the repeated game with imperfect public monitoring. We extend the analysis of repeated games with imperfect public monitoring à la Abreu, Pearce and Stacchetti (1990) to characterize the Public Perfect Equilibria (PPE) in the OLG setup. Several authors have investigated strongly symmetric PPE equilibria, where players use the same strategy after every history, and have applied this equilibrium concept in a rich class of economic problems (Green and Porter (1984), Radner, Myerson and Maskin (1986), Abreu, Milgrom and Pearce (1991)).[9] By restricting the equilibrium allocation to strongly symmetric strategies in contexts characterized by limited observability of past actions, the efficiency cannot typically be attained. Indeed, the equilibrium payoffs turn out to be bounded away from the Pareto frontier.[10] We contribute to the previous literature by analyzing how in OLG games with imperfect public monitoring, where agents are restricted to play strongly symmetric public strategies, cooperation might be sustained by SCI

[8] In reality long-term relationships are often plagued by imperfect monitoring. For example, a country may not verify exactly how much CO2 is emitted by neighboring countries. Workers in a joint project may not directly observe each other's effort. In such situation, however, there are usually some pieces of information, or signals, which imperfectly reveal what actions have been taken. Published meteorological data indicates the amount of CO2 emission, and the success of the project is more likely with higher effort.

[9] Green and Porter (1984) study Cournot competition characterized by noisy demand in a repeated setting. As main result, firms are prevented to achieve the first-best monopoly profits as long as "price wars" emerge in equilibrium. Radner, Myerson and Maskin (1986) present an example of a repeated partnership game with imperfect monitoring in which the set of PPE payoffs turns out to be bounded away from the Pareto frontier even as the discount factor tends to 1. Finally, Abreu, Milgrom and Pearce (1991) analyze in a similar environment how changes in the timing of information may increase the possibilities for cooperation, starting from an equilibrium allocation, which is constrained efficient.

[10] Nevertheless, Fudenberg, Levine and Maskin (1994) prove that under fully ranking condition a Folk theorem applies when agents play asymmetric public strategies.

improving ex-ante efficiency.

3.2.1 Theoretical Debate on Self-Enforcing Institutions

This research also refers to a significant and still open debate on self-enforcing institutions, whose applications spread out from the study of self-enforcing wage setting to the analyses of self-organization and vertical integration in firms. The debate mainly concerns the possibility of making informal agreements self-enforcing and desirable in terms of efficiency from an ex-ante point of view with respect to contractual relationship codified in a contingent court-enforceable contract. As Dany Rodrik suggests:[11] "I do not have any trouble with the idea that self-enforcing agreements can sometimes substitute for third-party (i.e. government) enforcement. Such self-enforcing agreements are maintained through the force of repeated interaction ("if you cheat me now, I will cheat you in the future,") through reputational mechanisms ("see, I am not the cheating kind of guy"), and collective punishment schemes ("if you cheat me, I will bring the wrath of my colleagues on you").[...] The problem with self-enforcing agreements is that they do not scale up. One of the findings from Elinor Ostrom's extensive case studies is that self-enforcing arrangements to manage the "commons" work well only when the geographic scope of the activity is clearly delimited and membership is fixed. It is easy to understand why. Cooperation under "anarchy" is based on reciprocity, which in turn requires observability. I need to be able to observe whether you are behaving according to the rules, and if not, I have to be able to sanction you."

Thus, fixed memberships and observability of past actions are identified as necessary conditions for making self-enforcing agreements really effective. However, several relevant economic and political situations exist where these two main criteria cannot be met. In these cases effective and costly monitoring technologies are required in order to sustain cooperative behavior over time. There exist a lot of different devices to measure the past agents' performance, which have been widely studied in the previous literature, such as labeling, trade marks or licensing. All of them however suffer of both large transaction costs,[12] especially in highly complex relationship, and strate-

[11] Unbound Cato Journal, 2009.

[12] We refer to transaction costs as the cost of coordinating the increasingly complex interdependent parts of an economy by acquiring the information to measure the multiple

gic manipulability or corruption, which make self-enforceability of informal agreement always challenging.

Self-Commitment Institution might be interpreted as an endogenous commitment device. Agents invest in a specific asset and, by doing so, foster cooperation even when membership is not fixed and agents simply observe a noisy measure of past performance.

3.3 The Model

In this paper we study an overlapping generation (OLG) game where agents imperfectly observe past history. Time is discrete and indexed by $t = 0, 1, 2, ...$ Two ongoing organizations exist, whose members enter in an asynchronous way and overlap for K period, where $K = 1$. Each organization's member is a risk-neutral selfish player living up $T = 2$ periods, Young and Old, where T is the individual life span. For notational purpose let us denote with t the generation born at time t. Fig. 1 reports the entry-exit structure of the game.

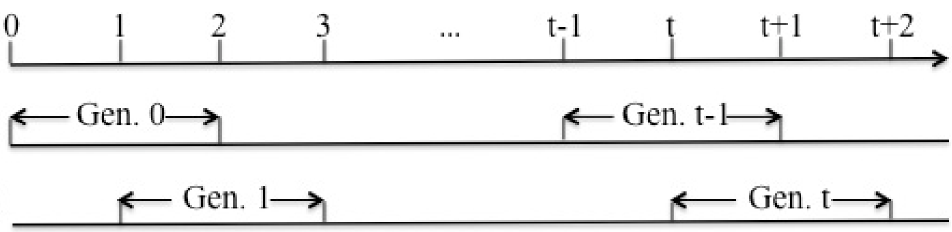

Fig. 10: OLG Structure

Young are endowed with one unit of time and produce a non-storable good by adopting a linear technology, while Old have an endowment, which is normalized to zero. Agents are active only in the first period of their life, while they are completely passive in the second period. Consequently, the game turns out to be characterized by one-side enforceability problem at each time, and personal punishment is interdicted. Specifically, at each time t Young face two different actions, $a_t \in A \equiv \{0, a\}$ and $b_t \in B \equiv \{0, b\}$ with $a \in (0, 1)$ and $b \in (0, 1)$, where A and B denote the set of actions of player

dimensions of exchanges. Furthermore, transaction costs are also costs of enforcing agreements and in making credible commitments across time and space. For a broad discussion of institutions, transaction costs and production see North (1987).

t. First, they decide the share of time to be used for production, $1-b_t$. Second, they choose the amount of transfers to devote to current living old, $(1-b_t)\,a_t$. The remaining resources are consumed, $(1-b_t)(1-a_t)$. Conditionally on a_t, the benefits of elderly agents are $d>0$ if $a_t>0$, otherwise, if $a_t=0$, they get no benefits.[13] We can interpret the decision a_t as the intergenerational consumption good transfer, while b_t is identifiable as the self-expropriation decision. Since b_t does not lead to any benefits, if Young choose $b_t>0$ then they self-expropriate a share of their resources and use it as *self-commitment* device.

The preference of generation t is the mapping $v := A\times B\times A \to \mathbb{R}_+$, and is given by:

$$v\,(a_t, b_t, a_{t+1}) = (1-b_t)\,(1-a_t) + d\mathbf{I}\,(a_{t+1}) \tag{3.1}$$

where $\mathbf{I}\,(a_{t+1})$ is an indicator function, which is equal to one when $a_{t+1}>0$ and equal to zero otherwise. Thus, the autarky payoff is equal to $v_{aut}=1$.

Assumption 7 (Payoff Ranking Condition) *The payoff ranking condition requires:*

$$v\,(a, b_t, 0) < v\,(0, b_t, 0) < v\,(a, b_t, a) < v\,(0, b_t, a)$$

which implies $d > (1-b_t)\,a + b_t$ for each b_t.

By Assumption 7 we replicate the incentive structure of a prisoner dilemma game. Intergenerational cooperation Pareto dominates autarky payoff. Furthermore, incentives for opportunistic behavior emerge, since players gain by receiving transfers when old without paying when young.

We focus on the self-enforceability of intergenerational contracts when agents may observe only a noisy signal of previous generations' performance in terms of intergenerational transfers.[14] At each time agents observe a public signal $z\in Z\equiv\{X,Y\}$, where X stands for good signal and Y for bad signal. The conditional distribution, $p_a=\Pr\,(Y|a_t)$, is affected by the

[13] Note that the elderly benefits do not depend on the young decision over b. See par. 6 for the extension to productive b.

[14] Only the current living Old directly observe the Young a-decision, but we avoid the possibility of intergenerational communication, as Lagunoff and Matsui (2004) do.

$a-$action of the previous generation, and it is denoted by:

$$p_0 = \Pr(Y|0) \tag{3.2}$$

$$p_1 = \Pr(Y|a) \tag{3.3}$$

Assumption 8 (Monotone Likelihood Ratio Property) *Given $p_0 \in (0,1)$ and $p_1 \in (0,1)$ the monotone likelihood ratio property requires $p_1 \leq \frac{1}{2} \leq p_0$.*

Assumption 8 guarantees that the probability of receiving a good signal is positively correlated with the agents' choice over intergenerational consumption good transfer. Let us denote with $L \equiv \frac{p_0}{p_1}$ the likelihood ratio. Under Assumption 8, $L \in [1, \infty)$. If $L = 1$, then $p_0 = p_1$ and as a consequence agents cannot detect deviations by observing signals. *Vice versa*, if $L = \infty$, then $p_1 \to 0$ and $p_0 \to 1$, and agents perfectly detect previous players' deviations.

Definition 9 (OLG game) *The collection (v, p_a, K, T) is referred to as an OLG game, which is denoted by $G(v, p_a, K, T)$.*

As a benchmark, we consider the efficient allocation implemented by the central planner with full taxation power, which is equal to:

$$v^* = (1 - b^*)(1 - a^*) + d$$

where $b^* = 0$ and $a^* < d$. Due to the absence of benefits generated by the self-expropriation decision, the optimality requires agents to no self-commit.

3.3.1 Public Perfect Equilibrium and Social Norms

We study the best sustainable strongly symmetric Public Perfect Equilibrium (PPE) of the OLG game, where agents play correlated public strategies. We consider a public randomization device, $(\mu_t)_{t=0}^{\infty}$, as a collection of independent random variables, uniformly distributed on the unit interval. Let us refer to $\mu^t \equiv (\mu_0, \mu_1, ..., \mu_t)$ and $z^t \equiv (z_0, z_1, ...z_t)$ as the vectors of public randomization devices and public signals till time t, respectively. Furthermore, let us denote with $b^t \equiv (b_0, b_1, ..., b_{t-1})$ the vector of b-choices taken by agents till time t. Differently from the intergenerational consumption good transfers, the self-expropriation decisions are perfectly observable. Consequently, the public history observed by generation t is $h^t \equiv (z^t, b^t, \mu^t) \in H^t$, where H^t is the set of possible $t-$public histories. For each $s \leq t$ by $z_s\left(h^t\right)$, $\mu_s\left(h^t\right)$ and $b_s\left(h^t\right)$ we refer to as the realizations of z_s, μ_s and b_s in the public history h^t, respectively.

For any t-generation we define *pure public strategies* the mappings:

$$\alpha_t := H^t \rightarrow A \text{ such that } a_t = \alpha_t\left(h^t\right) \in A$$

and

$$\beta_t := H^t \rightarrow B \text{ such that } b_t = \beta_t\left(h^t\right) \in B$$

Note that we assume that agents can only be distinguished through their history. Given that all the individuals are ex-ante identical, we are restricting ourselves to symmetric strategies, in the sense that each player uses the same strategy after every history. Furthermore, we denote the infinite vectors $\alpha \equiv (\alpha_t)_{t=0}^{\infty}$ and $\beta \equiv (\beta_t)_{t=0}^{\infty}$ as the strategy profiles.

Reformulating Eq. (3.1), the ex-ante payoff for every t-generation conditionally on the history h^t is given by:

$$\upsilon\left(\alpha_t, \beta_t | h^t\right) = \left(1 - \beta_t\left(h^t\right)\right)\left(1 - \alpha_t\left(h^t\right)\right) + d\mathbf{E}_t\left(\mathbf{I}\left(\alpha_{t+1}\left(h^{t+1}\right)\right)|h^t\right)$$

where $\mathbf{E}_t\left(\cdot|h^t\right)$ is the expected operator conditional on information at time t.

Definition 10 (PPE) *A profile* (α, β) *is a PPE of* $G(\upsilon, p_a, K, T)$ *if:*

i. α_t *and* β_t *are public strategies for each* $t \geq 0$*;*

ii. *For each date* t *and public history* h^t*, the strategies* α_t *and* β_t *are Nash equilibrium from that date onwards.*

A particular equilibrium strategy of the OLG game $G(\upsilon, p, K, T)$ can be identified as a social norm which prescribes a specific behavioral rule. After being established, the social norm continues being in force because agents prefer to conform to that rule, given the expectation that others are going to conform.[15] Consequently, social norms coordinate expectations reducing transaction costs in interactions that possess multiple equilibria. We provide the definition of social norm we will refer to in the following analyses.

[15] See Lewis (1969).

Definition 11 (Social Norm) *The social norm is the specification of a particular equilibrium customary rule of behavior that coordinate interactions among agents.*

In a repeated interaction framework, social norms are typically sustained by two kinds of enforcement mechanisms: Personal and community enforcement. Under personal enforcement a cheater will only face retaliation by their victim. On the contrary, under community enforcement all members of the society react to a deviation according to specific social norms. Given the peculiar structure of the OLG game described above, in $G(\upsilon, p, K, T)$ personal enforcement cannot be exerted and social norms might be sustained only through community enforcement. In this paper we introduce a third enforcement mechanism, that we call Self-Commitment Institution, which requires agents to self-commit by partially self-expropriating their own resources, i.e. $b_t > 0$ for some t. We distinguish two types of social norms: Social norms without SCI and social norms with SCI.

Definition 12 (Social Norm without SCI) *A social norm without SCI is a social norm which requires agents to coordinate on the adoption of community punishment mechanism.*

Definition 13 (Social Norm with SCI) *A social norm with SCI is a social norm which requires agents to coordinate on the adoption of both community punishment mechanisms and self-expropriation. The following two features characterize it:*

i. *Players may decide to self-commit by self-expropriating a share of their resources, $b_t > 0$ for some t;*

ii. *Punishment is contingent on the b-decisions of previous players.*

For expositional purpose, we provide an automaton representation of social norms as the collection $\Xi \equiv \left\{W, w_0, \left(f^i\right)_{i\in\{a,b\}}, \tau\right\}$. W is the set of phases which characterize the strategy, $w_0 \in W$ is the initial phase, f^i with $i \in \{a, b\}$ is the decision rule which associates to each phase an action profile, $f^a : W \to A$ and $f^b : W \to B$, and τ is the transition function, equal to the mapping $\tau := W \times Z \times B \to \Delta_\mu(W)$, where $\Delta_\mu(W)$ is the probability distribution over phases. Finally, Γ^j is the finite set of equilibria payoffs attained implementing the strategy Ξ^j for $j \in \{(a), (ab)\}$, where (a) stands for social norms without SCI while (ab) denotes social norm with SCI.

In the analyses to follow we first characterize the best sustainable strongly symmetric pure strategy equilibria payoff under imperfect monitoring, which can be sustained by social norms without SCI. Second, we study how social norms with SCI enable agents to attain higher efficiency with respect to the former equilibria allocations. Finally we derive the value of SCI and discuss its extension in terms of productive SCI.[16]

3.4 Social Norms without Self-Commitment Institution

In this section we present an analysis that helps to determine the set of equilibria payoffs in the OLG game where agents coordinate on social norms which do not require the activation of SCI, i.e. $\Gamma^a = [\upsilon^a_{\min}, \upsilon^a_{\max}]$. The analysis allows us to gain insights as to the reasons why social norms with SCI can outperform social norms, which are simply enforced by implementing community punishment mechanisms.

We adopt the following methodology: First, we consider a particular social norm without SCI, $\tilde{\Xi}^a$, candidate to be a PPE of $G(\upsilon, p_a, K, T)$, and we determine the corresponding best sustainable payoff, $\tilde{\upsilon}^a_{\max}$. Second, we prove that the best equilibrium payoff sustained by social norms without SCI, $\upsilon^a_{\max}$, coincides with $\tilde{\upsilon}^a_{\max}$.

Proposition 8 *The set of equilibrium payoffs obtained by implementing social norms without SCI in the OLG game $G(\upsilon, p_a, K, T)$ is equal to the finite set $\Gamma^a = [\upsilon^a_{\min}, \upsilon^a_{\max}]$, where $\upsilon^a_{\min} = \upsilon_{aut}$ and $\upsilon^a_{\max} = \tilde{\upsilon}^a_{\max}$.*

Proof. *(See appendix).* ■

To determine $\tilde{\upsilon}^a_{\max}$ let us consider the following strategy, $\tilde{\Xi}^a$:

$$\alpha_t\left(h^t\right) = \begin{cases} 0 & \text{if } \exists\ s < t \text{ such that } z_s\left(h^t\right) = Y \text{ and } \mu_s\left(h^t\right) \geq \mu, \\ a & \text{otherwise.} \end{cases} \tag{3.4}$$

[16] We refer to either social norm with SCI or without SCI as the selection of a particular type of PPE equilibrium. The former is characterized by the common expectation that all generations will coordinate their behavior not only on the public signal realizations but also on the individual self-expropriation decision, while in the latter case agents expect future generation will coordinate their strategies just on the public signals history.

The social norm described in Eq. (3.4), has the following equivalent two-phase automaton representation, $\tilde{\Xi}^a = \{W, w_0, f^a, \tau\}$, with $W = \{w_C, w_P\}$, where w_C is the cooperation phase and w_P is the punishment phase. Agents start to play cooperatively, i.e. $w_0 = w_C$, and the output function at each phase assigns the following decision rule, $f^a(w_C) = a$ and $f^a(w_P) = 0$. By convention, we denote with $\tau(w, z)$ the transition probability to the phase w_C given the current phase w and the public signal z. The transition function of $\tilde{\Xi}^a$ prescribes:

$$\tau(w,z) = \begin{cases} 1 & if \quad w = w_C,\ z = X \\ \mu & if \quad w = w_C,\ z = Y \\ 0 & if \quad w = w_P,\ \forall z \end{cases}$$

We can interpret μ as the probability each t-generation assigns to future generations' decision to switch to cooperative phase even if a bad signal will be realized. Fig. 11 provides a graphical representation of $\tilde{\Xi}^a$.

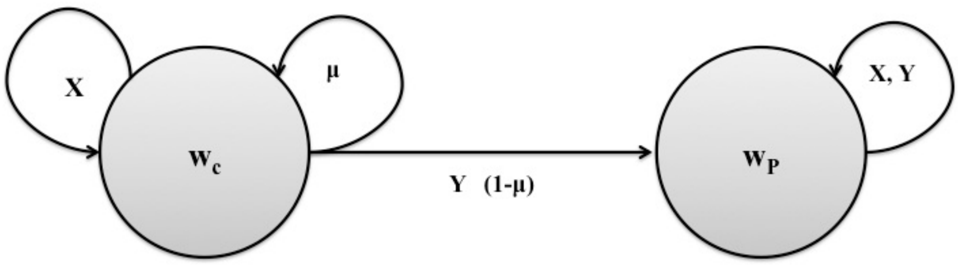

Fig. 11: $\tilde{\Xi}^a$ automaton for the OLG game.

The strategy $\tilde{\Xi}^a$ is then characterized by two possible phases: a "cooperate" phase and a "punishment" phase. Players start in the "cooperate" phase, and stay there until they observe a bad signal and a sufficiently high realization of μ. After that they start playing a permanent "punishment" and intergenerational cooperation is no longer sustained.[17] Since w_P is activated on the equilibrium path as soon as a bad signal is realized and agents cannot exit this phase, $\tilde{\Xi}^a$ implies loss of efficiency and induces boundedness of the strongly symmetric PPE payoffs. To determine the maximum payoff attainable implementing the strategy $\tilde{\Xi}^a$, let υ_{w_C} and υ_{w_P} be the value

[17] It follows that $\tilde{\Xi}^a$ is similar to a grim-trigger strategy played in a game with perfect monitoring. The major difference concerns the fact that w_P is activated on the equilibrium path as soon as a bad signal is realized and agents cannot exit this phase.

functions in the cooperative and punishment phases, respectively. They are given by:

$$\upsilon_{w_C} = (1-a) + d\left[p_1\mu + (1-p_1)\right] \tag{3.5}$$

and

$$\upsilon_{w_P} = 1$$

For the strategy (3.4) to be an equilibrium, it must be true that in the cooperative phase players prefer to play $\alpha_t\left(h^t\right) = a$ rather than to deviate to $\alpha_t\left(h^t\right) = 0$:

$$\upsilon_{w_C} \geq 1 + d\left[p_0\mu + (1-p_0)\right] \tag{3.6}$$

By solving for μ we get:

$$\mu \leq 1 - \frac{a}{d\left(p_0 - p_1\right)} \equiv \overline{\mu} \tag{3.7}$$

$\overline{\mu}$ is clearly strictly less than one. Furthermore, it is non negative as long as:

$$p_1 \leq p_0 - \frac{a}{d}$$

On the other side, it must be true that in the punishment phase players prefer to play $\alpha_t\left(h^t\right) = 0$ rather than to deviate to $\alpha_t\left(h^t\right) = a$:

$$\upsilon_{w_P} \geq 1 - a$$

which is trivially satisfied. Moreover, for υ_{w_C} to be the best sustainable equilibrium payoff of strategy $\tilde{\Xi}^a$, the condition (3.7) must be satisfied with equality. Otherwise, we can increase μ and thereby υ_{w_C} implied by the Eq. (3.5) without violating the equilibrium condition (3.6). Plugging $\mu = \overline{\mu}$ into υ_{w_C} allows us to determine the max payoff, $\tilde{\upsilon}^a_{\max}$:

$$\tilde{\upsilon}^a_{\max} = 1 - a + d - C^a\left(p_0, p_1\right)$$

where $C^a\left(p_0, p_1\right) \equiv \frac{\Delta^a}{L-1}$ with $\Delta^a = a$.

The inefficiency due to the presence of imperfect monitoring is fully captured by the term $C^a\left(p_0, p_1\right)$, which represents the endogenous cost of monitoring. The higher the likelihood ratio (i.e. the higher the probability to detect deviations), the lower the cost of monitoring. Furthermore, the function $C^a\left(\cdot\right)$ is increasing in the current gain from deviation, Δ^a.

Consequently, by using strongly symmetric public strategies, the equilibrium payoff is necessarily bounded above and full efficiency can never be attained.[18]

From Proposition 8 we have that the best sustainable equilibrium payoff achieved by the strategy $\tilde{\Xi}^a$ is also equal to the highest payoff attained by social norms without SCI, i.e. by equilibrium strategies in which agents don't coordinate their behavior on the self-expropriation decisions of the other players.

Corollary 9 *If $\upsilon^a_{\max} > \upsilon^a_{\min}$ then the equilibrium strategy that delivers $\upsilon^a_{\max}$ is equal to $\tilde{\Xi}^a$.*

Proof. *(See appendix).* ■

The result in Corollary 9 is equivalent to that achieved by Abreu et al. (1990) in infinite repeated game with imperfect public monitoring. The main differences are that we consider an OLG game and one-side enforceability at each period.

3.5 Social Norms with Self-Commitment Institution

In this section we introduce SCI as an alternative enforcement mechanism, which enables agents to attain higher payoff even if PPE are restricted to strongly symmetric strategies. We determine the finite set of equilibrium payoff achieved by social norms with SCI, i.e. $\Gamma^{ab} = \left[\upsilon^{ab}_{\min}, \upsilon^{ab}_{\max}\right]$. Equivalently to the previous analyses on social norms without SCI, we first consider a particular social norm with SCI, $\tilde{\Xi}^{ab}$, candidate to be a PPE of $G(\upsilon, p_a, K, T)$, determining the corresponding best sustainable payoff, $\tilde{\upsilon}^{ab}_{\max}$. Second, we prove that the best equilibrium payoff sustained by social norms with SCI, $\upsilon^{ab}_{\max}$, coincides with $\tilde{\upsilon}^{ab}_{\max}$.

Proposition 9 *The set of equilibrium payoffs obtained by implementing social norms with SCI in the OLG game $G(\upsilon, p_a, K, T)$ is equal to the finite set $\Gamma^{ab} = \left[\upsilon^{ab}_{\min}, \upsilon^{ab}_{\max}\right]$, with $\upsilon^{ab}_{\min} = \upsilon_{aut}$ and $\upsilon^{ab}_{\max} = \tilde{\upsilon}^{ab}_{\max}$.*

[18] Kandori and Obara (2006) showed in a prisoners' dilemma game how players can sometimes make better use of information by adopting private strategies and how efficiency in repeated game with imperfect public monitoring can be improved. In the intergenerational game described so far, because of the one-side enforceability structure and the timing of the game, private strategies do not succeed in improving efficiency.

Proof. *(See appendix).* ■

To determine $\tilde{v}^{ab}_{\max}$ let us consider the following strategy, $\tilde{\Xi}^{ab}$:

$$\left(\alpha_t\left(h^t\right), \beta_t\left(h^t\right)\right) = \begin{cases} (0,0) & \text{if } \exists\ s < t \text{ such that} \begin{cases} z_s\left(h^t\right) = Y \text{ and } \mu_s\left(h^t\right) \geq \mu \\ b_s\left(h^t\right) = 0\ \forall z \end{cases} \\ (a,b) & \text{otherwise.} \end{cases} \tag{3.8}$$

Fig. 12 provides a graphical representation of $\tilde{\Xi}^{ab}$.

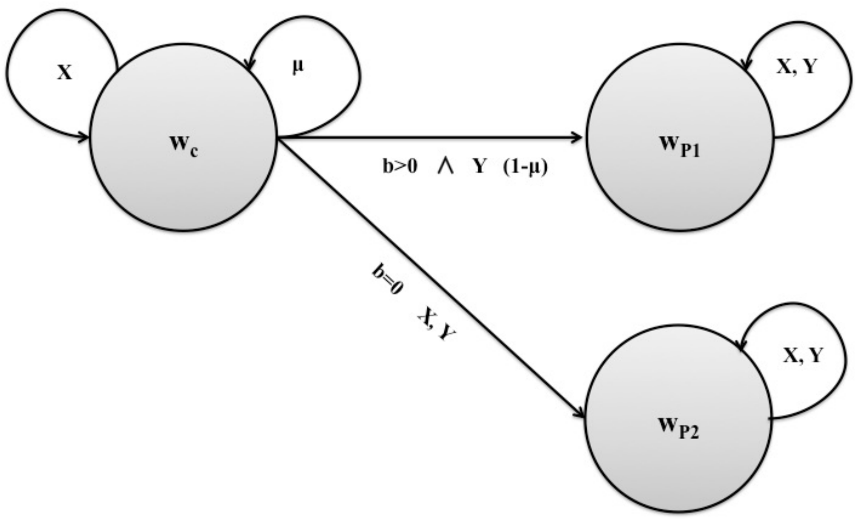

Fig. 12: $\tilde{\Xi}^{ab}$ automaton for the OLG game.

Differently from the strategy reported in (3.4), the activation of SCI allows the players to coordinate punishment not only on the realization of public signal but also on the self-expropriation decisions taken by the previous generations. The social norm (3.8) has the following equivalent three-phase automaton representation: $\tilde{\Xi}^{ab} = \left\{W, w_0, \left(f^i\right)_{i=a,b}, \tau\right\}$. W is equal to the vector $\{w_C, w_{P1}, w_{P2}\}$, where w_C is the cooperation phase and w_{P1} and w_{P2} are different punishment phases generated by the two possible deviations. The initial phase is $w_0 = w_C$, and the decision rules are $\left(f^a\left(w_C\right) = a, f^b\left(w_C\right) = b\right)$ and $\left(f^a\left(w_{Pi}\right) = 0, f^b\left(w_{Pi}\right) = 0\right)$ for each $i = 1, 2$. By $\tau\left(w, z, b\right)$ we denote the transition probability to w_C given the current phase w, the public signal z and the self-commitment decision b. The transition function of $\tilde{\Xi}^{ab}$ prescribes:

$$\tau(w,z,b)=\begin{cases}1 & if \quad w=w_C,\ z=X,\ b>0\\ \mu & if \quad w=w_C,\ z=Y,\ b>0\\ 0 & if \quad w=w_C,\ \forall z,\ b=0\\ 0 & if \quad w\in\{w_{P1},w_{P2}\},\ \forall z,\ \forall b\end{cases}$$

Differently from $\tilde{\Xi}^a$, the strategy $\tilde{\Xi}^{ab}$ requires the activation of a second type of punishment phase. The two punishment phases are distinguished because of the possible deviations, which might be exerted by previous generations. As in $\tilde{\Xi}^a$, if $z_s(h^t)=Y$ then w_{P1} is activated with probability $(1-\mu)$. In addition, agents are now punished also if $b_s(h^t)=0$ for any possible realization of public signals.

Let υ_{w_C}, $\upsilon_{w_{P1}}$ and $\upsilon_{w_{P2}}$ be the value functions in the cooperative and punishment phases, respectively. They are given by:

$$\upsilon_{w_C}=(1-b)(1-a)+d\left[\mu p_1+(1-p_1)\right]$$

and

$$\upsilon_{w_{P1}}=\upsilon_{w_{P2}}=1$$

As in $\tilde{\Xi}^a$, the strategy $\tilde{\Xi}^{ab}$ is an equilibrium if and only if in each phase the prescribed actions satisfy the incentive requirements. For the strategy (3.8) to be an equilibrium, it must be true that in the cooperative phase, w_C, agents prefer to play $\alpha(h^t)=a$ and $\beta(h^t)=b$ rather than either to deviate to $\alpha_t(h^t)=0$ and $\beta_t(h^t)=b$:

$$\upsilon_{w_C}\geq(1-b)+d\left[\mu p_0+(1-p_0)\right] \tag{3.9}$$

or, alternatively, to deviate to $\alpha_t(h^t)=0$ and $\beta_t(h^t)=0$:[19]

$$\upsilon_{w_C}\geq 1 \tag{3.10}$$

By simultaneously solving the inequality (3.9) and (3.10) for μ, we get $\mu\in M\equiv\left[\underline{\mu},\overline{\mu}\right]$ where $\underline{\mu}\equiv\frac{a+b(1-a)}{dp_1}-\frac{1-p_1}{p_1}$ and $\overline{\mu}\equiv 1-\frac{a(1-b)}{d(p_0-p_1)}$. To be

[19] Note that the deviation $\alpha(h^t)=a$ and $\beta(h^t)=0$ is dominated by inequality (3.10) and thus disregarded.

feasible, i.e. $M \neq \varnothing$, we require:

$$p_1 \leq \frac{d-b-a(1-b)}{d-b} p_0 \tag{3.11}$$

$\overline{\mu}$ is clearly strictly less than one. Furthermore, $\overline{\mu}$ is non negative as long as:

$$p_1 \leq p_0 - \frac{a(1-b)}{d} \tag{3.12}$$

On the other side, it is trivial to prove that, for each a or b, the incentive constraints in punishment phases are always satisfied.

To determine the maximal element of Γ^{ab} we look for the appropriate μ which maximizes the individual payoff without violating the incentive constraints (3.9) and (3.10), i.e. $\mu = \overline{\mu}$. Note that b positively affects $\overline{\mu}$.

Remark 7 *The larger the amount of the self-expropriated endowment, the higher the probability that, if a bad signal is realized, future generations will decide to not punish.*

Plugging $\mu = \overline{\mu}$ into v_{w_C} allows us to determine best sustainable payoffs, $v^{ab}_{\max}$, equal to:

$$\tilde{v}^{ab}_{\max} = (1-b)(1-a) + d - C^{ab}(p_0, p_1)$$

where $C^{ab}(p_0, p_1) \equiv \frac{\Delta^{ab}}{L-1}$ with $\Delta^{ab} \equiv a(1-b)$.

As before, the inefficiency arising out of imperfect monitoring is captured by the term $C^{ab}(p_0, p_1)$, which turns out to be negatively affected by the self-expropriation decision, i.e. $\frac{\partial C^{ab}(\cdot)}{\partial b} < 0$. The implementation of SCI reduces the endogenous cost of monitoring and under particular conditions attains higher efficiency with respect to social norms which do not activate SCI.[20] Note, even if the two punishment phases appears to be very similar in nature, they are substantially different. While w_{P1} is activated on the equilibrium path as in $\tilde{\Xi}^a$, w_{P2} is an out of equilibrium punishment path. Since b-decision is perfectly observable and agents are immediately punished if they decide to not self-commit, in equilibrium they will always choose to self-expropriate. However, the existence of such off the equilibrium punishment path reduces the need of on equilibrium punishment: The self-commitment decision reduces the current gain from deviation, i.e. $\Delta^{ab} < \Delta^a$, then agents

[20]See par. 3.5.1.

cheat future generations over the intergenerational transfer decision with lower probability. As a consequence, intergenerational cooperation under imperfect monitoring is easier sustained, achieving ex-ante higher efficiency.

Proposition 9 states that the highest payoff attained by social norms with SCI is exactly equal to the best sustainable equilibrium payoff achieved by the strategy $\tilde{\Xi}^{ab}$.[21]

Corollary 10 *If $\upsilon^{ab}_{\max} > \upsilon^{ab}_{\min}$ then $\tilde{\Xi}^{ab}$ is one of the equilibrium strategies that delivers $\upsilon^{ab}_{\max}$.*

Proof. *(See appendix).* ■

3.5.1 Value of Self-Commitment

What is the price that agents were willing to pay for SCI? Here, we determine the gain in efficiency attained by implementing social norms that adopt SCI, compared to social norms that do not require SCI.

Proposition 10 *Under the conditions (3.11) and (3.12), if $p_1 \geq (1-a)\,p_0$ then there exists a non-empty parametric space, P, where for each $(p_0, p_1) \in P$, $\tilde{\Xi}^{ab}$ leads to higher efficiency w.r.t. $\tilde{\Xi}^{a}$, i.e. $\tilde{\Gamma}^{a} \subseteq \tilde{\Gamma}^{ab}$.*

Proof. *(See Appendix).* ■

For each $(p_0, p_1) \in P$, the equilibrium strategy $\tilde{\Xi}^{ab}$ succeeds in sustaining PPE and attains higher individual maximal payoffs, i.e. $\upsilon^{ab}_{\max} > \upsilon^{a}_{\max}$. In the parametric space P the benefits coming from the implementation of SCI in terms of reduction in the endogenous monitoring cost are larger with respect to the costs generated by the self-expropriation of personal endowment.

Let us denote with $\Pi \equiv \upsilon^{ab}_{\max} - \upsilon^{a}_{\max}$ the value of SCI, which is equal to:

$$\Pi = b\left(a\frac{L}{L-1} - 1\right) \tag{3.13}$$

[21] Note that $\tilde{\Xi}^{ab}$ prescribes to reverse to the worst sustainable equilibrium as soon as a player decides not to self-commit. In other terms the out-of-equilibrium beliefs, which sustain $\tilde{\Xi}^{ab}$ as PPE of the game $G(\upsilon, p_a, K, T)$, requires zero continuation values in the case of $b_s\left(h^t\right) = 0$ for some $s < t$. This is actually just one of the possible equilibrium strategies, which delivers the best sustainable payoff among social norms with SCI. Indeed, it is possible to relax such out-of-equilibrium beliefs, introducing an additional correlation device, we denote $(\theta^{\kappa}_t)^{\infty}_{t=0}$, which enables agents to not be punished both after a bad signal, $\kappa = Y$, and a good signal, $\kappa = X$, if the realization of θ^{κ}_t is sufficiently high.

The marginal impacts of (p_0, p_1) on Π are $\frac{\partial \Pi}{\partial p_1} > 0$ and $\frac{\partial \Pi}{\partial p_0} < 0$ with $\left|\frac{\partial \Pi}{\partial p_1}\right| > \left|\frac{\partial \Pi}{\partial p_0}\right|$.

Fig. 13 plots the feasibility space, P, and the corresponding SCI-value, Π, considering the upper envelope of the triangle ABC and plotting over p_0.

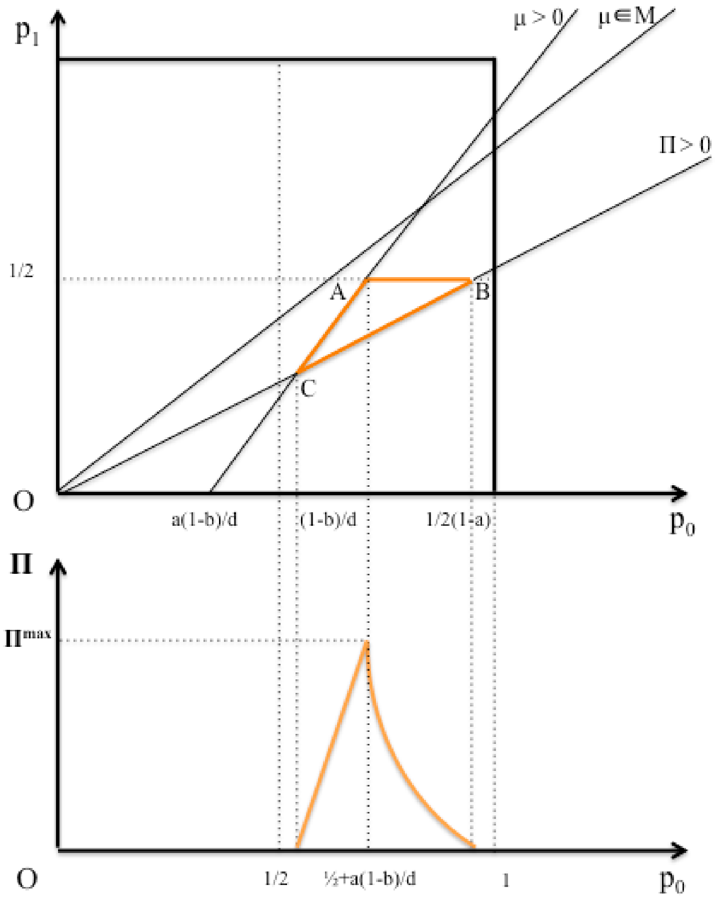

Fig. 13: Value of Self-Commitment Institution

The point $A = \left(\frac{1}{2} + \frac{a(1-b)}{d}, \frac{1}{2}\right)$ represents the maximum value achievable by Π. Let us take $p_1 = \frac{1}{2}$ and plug $p_0 = p_1 - \frac{a(1-b)}{d}$ into Eq. (3.13), then we obtain that for $d > 2(1-b)(1-a)$ the maximum value of SCI is equal to:

$$\Pi^{\max} = \frac{(d - 2(1-b)(1-a))\, b}{2(1-b)}$$

Whereas $\Pi = 0$ in the two extremes $B = \left(\frac{1}{2(1-a)}, \frac{1}{2}\right)$ and $C = \left(\frac{1-b}{d}, \frac{(1-b)(1-a)}{d}\right)$. Note that the SCI value decreases when the monitoring becomes perfect.

This result helps to stress the role of self-commitment as institution, which, if implementable, might sustain efficient cooperation when the moral hazard issue is explicitly considered as a relevant feature of the economic environment.

3.6 Productive Self-Commitment: The Role of Education

In this section we extend the basic environment to test the robustness of the theoretic results presented in the previous paragraphs, and to show relevant policy implications. We consider the more realistic case of using the self-expropriated resources to productive investment for future generations. Obviously, the positive impact of SCI is further magnified by growth motives. Adopting this slight change of the theoretic environment we can easily explain the emergence of education provision for the sustainability of intergenerational contracts, out of the traditional altruistic and asset return arguments.

Let us now consider a more general setting, where agents are risk adverse and live up to three periods: Young, Adult and Old. The utility is represented by the following separable additive function:

$$\upsilon\left(c_{1t}, c_{2t+1}\right) = u\left(c_{1t}\right) + \delta u\left(c_{2t+1}\right)$$

where $u(0) = 0$, $u'(c) \geq 0$ and $u''(c) \leq 0$. $\delta \in (0, 1)$ represents the individual discount factor. c_{1t} and c_{2t+1} are consumption of Adult and Old, respectively. Young do not consume. For notational purpose let us denote with $\rho \equiv -\frac{cu''(c)}{u'(c)}$ the coefficient of relative risk aversion. As in the previous sections, at each time t Adults face two different actions, $a_t \in A$ and $b_t \in B$, where A and B denote the finite set of actions of the players. They decide the share of time to be used for production, while the remaining time is devoted for children education, b_t. Furthermore, they choose the amount of consumption good to be transferred to current living Old. In the meanwhile, Young transform the received endowment in human capital, which is used for next period production. We assume the existence of a technology $F := B \to \mathbb{R}_+$ which has the following properties:

Assumption 9 (Technology) *$F(0) = 0$, $F'(b_t) \geq 0$, and $F''(b_t) \leq 0$.*

Let $\lambda(b_{\tau-1}, b_\tau) \equiv 1 + F(b_{\tau-1}) - b_\tau$, which under Assumption 9 is greater than 1 for any level of b_τ. The individual resource constraints must hold at each time t, and are equal to:

$$\begin{aligned} c_{1t} &\leq \lambda(b_{t-1}, b_t)(1 - a_t) \\ c_{2t+1} &\leq \lambda(b_t, b_{t+1}) a_{t+1} \end{aligned}$$

Now the OLG game resembles the classical intergenerational games, which have been widely studied in earlier literature, preserving at the same time the main features of the game described in the previous sections.

The first best allocation attained by the central planner with full taxation power is equal to:

$$v^* = u((1 + F(b^*) - b^*)(1 - a^*)) + \delta u((1 + F(b^*) - b^*) a^*)$$

where the optimal choices a^* and b^* solve the following first order conditions, respectively:

$$\begin{aligned} &a_t : u'((1 + F(b^*) - b^*)(1 - a^*)) - \delta u'((1 + F(b^*) - b^*) a^*) = 0 \\ &b_t : F'(b^*) - 1 = 0 \end{aligned}$$

Note that, due to the positive spillover effects generated by the self-expropriation decision, an optimal level of $b^* > 0$ exists.

Under perfect monitoring, the allocation (a^*, b^*) can be sustained as a subgame perfect equilibrium of the intergenerational game, as showed by Rangel (2003). He proves that the existence of a_t (i.e. *backward transfers*) sustains the investment in b_t (i.e. *forward transfers*). Without the former the productive investment turns out to be inefficiently low due to hold-up problems. We revert Rangel's perspective by providing a new justification for education investment. Moving in a more realistic imperfect monitoring environment we show how, and under which conditions, the self-expropriation transfers (*forward transfers*) plays a relevant role in sustaining optimal intergenerational cooperation reducing players' opportunistic behavior.

We limit the analyses to stationary equilibria, such that at each time $a_t \in \{0, a\}$ and $b_t \in \{0, b\}$ with a and b greater than zero. Public signals are generated by the stochastic process given by Eq. (3.2) and (3.3). If agents coordinate on the social norm (3.4), i.e. without activating SCI, then they attain the following best sustainable payoff:

$$v^a_{\max} = u(1-a) + \delta u(a) - \frac{\Delta^a}{L-1}$$

where $\Delta^a \equiv u(1) - u(1-a)$. Implementing SCI by adopting the social norm described in (3.8), intergenerational cooperation might attain as best sustainable payoff the following value:

$$v^{ab}_{\max} = u(\lambda(b)(1-a)) + \delta u(\lambda(b)a) - \frac{\Delta^{ab}}{L-1}$$

where $\Delta^{ab} \equiv u(\lambda(b)) - u(\lambda(b)(1-a))$. Note that the productive SCI generates two effects. The first is related to *technological* reasons and is quantify by the gain:

$$D^T(b) \equiv \chi_1 + \delta\chi_2 \tag{3.14}$$

where $\chi_1 \equiv u(\lambda(b)(1-a)) - u(1-a)$ and $\chi_2 \equiv u(\lambda(b)a) - u(a)$. Under Assumption 9, $D^T(b)$ is always greater than zero. The second effect is instead related to *strategic* reasons as widely discussed in section 3.5, whose impact is given by:

$$D^S(b) \equiv \Delta^a - \Delta^{ab} \tag{3.15}$$

where $\Delta^{ab} = \Delta^a$ if $b = 0$.

Proposition 11 *Social norms with productive SCI improve ex-ante efficiency in the strategic component, $D^S(b) \geq 0$, in the following cases:*

i) *dynamic inefficiency and relative risk aversion greater or equal to one, i.e. $\lambda'(b) > 0$ and $\rho \geq 1$;*

ii) *dynamic efficiency and relative risk aversion lower than one, i.e. $\lambda'(b) \leq 0$ and $\rho < 1$.*

Proof. *(See appendix).* ■

Proposition 11 provides simple, and potentially testable, implications. Society characterized by a dynamic efficient growth path (i.e. $\lambda'(b) \leq 0$), in which agents cannot increase their intertemporal utility by reducing their consumption and by increasing investment in human capital, may support the implementation of social norms that coordinate on productive SCI decisions in order to reduce opportunistic behavior on the dimension of intergenerational cooperation. Indeed, by providing education they reduce their marginal gain from deviation on unobservable backward transfers. When the economy is instead characterized by dynamic inefficient growth path (i.e. $\lambda'(b) > 0$) and community coordinates on social norms with SCI, opportunistic behavior on the intergenerational cooperative dimension might be even exacerbated, partially reducing the overall ex-ante efficiency. However social norm with SCI are still desirable in the case of dynamic inefficiency as long as agents have sufficiently high relative risk aversion, which partially reduces the gain from deviation.[22]

3.7 Conclusion

In this paper we have focused on OLG games characterized by imperfect public monitoring, where agents are restricted to play strongly symmetric public strategies. We have studied how the implementation of social norms with SCI improves ex-ante efficiency compared to social norms without SCI. When agents coordinate their strategies on the self-expropriation actions of other players, society attains higher welfare.

There are two main features we require to be satisfied in order to achieve this stark result. First, self-expropriation decisions must be fully observable by future generations. Second, by self-expropriating agents endogenously change their perceived current marginal gain from deviation. If all players coordinate on both community and self-commitment enforcement mechanisms, agents are more willing to cooperate even after bad signals' realizations and, consequently, higher ex-ante efficiency is supported in equilibrium.

A wide range of economic settings exists in which our theoretic results may be conveniently applied. Stochastic environments characterized by high volatility and repeated interactions among organizations, whose members

[22] To provide quantitative insights, let us consider an economy characterized by exponential utility, $u(c_\lambda) = 1 - e^{-\alpha c_\lambda}$, and decreasing return to scale technology, $F(b) = \sqrt{b}$, where $c_\lambda \equiv \lambda(b)(1-a)$. The first best allocation is $b^* = \frac{1}{4}$. The coefficient of relative risk aversion is equal to $\rho \equiv \alpha c_\lambda = \alpha\left(1 + \sqrt{b} - b\right)(1-a)$. As result, the social norms with SCI attain higher efficiency in the strategic component if: i) $b < \frac{1}{4}$ and $\alpha \geq \frac{1}{\left(1+\sqrt{b}-b\right)(1-a)}$, and ii) $b \geq \frac{1}{4}$ and $\alpha < \frac{1}{\left(1+\sqrt{b}-b\right)(1-a)}$.

have fix-term mandates, are particular adapt to explore the positive impact of social norms where agents coordinate their expectations on the self-expropriation decisions of the other players. Real applications will be in the context of self-enforcing intergenerational risk sharing and self-enforcing international agreements.

In this study we have limited our analyses to the comparison between two types of social norms, which are distinguished because of the different enforcement mechanisms on which agents are expected to rely to sustain a particular customary rule. Future research will focus on the study of the endogenous emergence of social norms. Specifically, by adopting an evolutionary approach, we may wonder if and how a community, which has initially relied on social norms without SCI, may have incentives to switch to social norms with SCI.

3.8 Appendix

Proposition (8). We determine the set of payoff, Γ^a, in the OLG game $G(\upsilon, p_a, K, T)$ when agents are restricted to playing strongly symmetric pure public strategies. Let υ_t be equal to:

$$\upsilon_t = (1 - b_t)(1 - a_t) + d\left[p_a g(Y) + (1 - p_a) g(X)\right]$$

where $g : Z \to [0, 1]$. $g(Y)$ and $g(X)$ denotes the t-generation continuation values in the case of bad and good signal realizations, respectively. Note that individual continuation values are not affected by the b-decisions. The vector $(a_t, b_t, g(z_t))$ identifies a recursive strategy profile. Recall that $p_a = \Pr(Y|a_t)$ which is a decreasing function of a_t. We have to solve the following maximization program:

$$\max \upsilon_t$$

s.t.:

$$\begin{cases} \bar{a} = \underset{a_t \in \{0,a\}}{\arg\max} \upsilon_t \\ \bar{b} = \underset{b_t \in \{0,b\}}{\arg\max} \upsilon_t \\ p_a = \Pr(Y|a_t) \\ g(Y), g(X) \in [0, 1] \end{cases}$$

Clearly $\bar{b} = 0$, given that it simply makes agents able to sustain a cost with no benefits. Under the feasibility condition $p_1 \leq p_0 - \frac{\Delta^a}{d}$ where $\Delta^a = a$, then we discuss the two possible cases of $\bar{a} = 0$ and $\bar{a} = a$.

Case I

Suppose $\bar{a} = a$, then the promise keeping constraint turns out to be equal to:

$$\upsilon_t = 1 - a + d\left[p_1 g(Y) + (1 - p_1) g(X)\right] \tag{3.16}$$

To be an equilibrium the following incentive constraint must hold:

$$\upsilon_t \geq 1 + d\left[p_0 g(Y) + (1 - p_0) g(X)\right]$$

The above inequality implies:

$$g(X) \geq \frac{\Delta^a}{d(p_0 - p_1)} + g(Y) \tag{3.17}$$

According to Eq. (3.16), the maximum payoff requires $g(X)$ and $g(Y)$ being maximum. As a consequence:

$$g(X) = 1 \text{ and } g(Y) = 1 - \frac{\Delta^a}{d(p_0 - p_1)} \tag{3.18}$$

Note that $g(Y)$ cannot be equal to one. Indeed, if $g(Y) = 1$ then from constraint (3.17) it follows $g(X) > 1$, which cannot be. Thus, in order to play cooperatively in the first period, i.e. $\bar{a} = a$, agents must expect with a probability, $\bar{\mu} = 1 - \frac{\Delta^a}{d(p_0 - p_1)}$, that future generation will cooperate even if a bad signal will be realized. Plugging condition (3.18) into Eq. (3.16) we obtain:

$$\upsilon_t = 1 - a + d - \frac{\Delta^a}{\frac{p_0}{p_1} - 1}$$

Case II

Suppose $\bar{a} = 0$, then the promise keeping constraint is equal to:

$$\upsilon_t = 1 + d\left[p_0 g(Y) + (1 - p_0) g(X)\right] \tag{3.19}$$

The incentive compatible constraint requires:

$$\upsilon_t > 1 - a + d\left[p_1 g(Y) + (1 - p_1) g(X)\right]$$

By solving for $g(X)$ the above inequality implies:

$$g(X) < \frac{\Delta^a}{d(p_0 - p_1)} + g(Y) \tag{3.20}$$

From Eq. (3.19) to attain the maximum payoff we require $g(X)$ and $g(Y)$ subject to (3.20) to be maximum. First we consider the case $(g(X), g(Y)) = (1, 1)$, which implies that, for any signals' realization, agents will cooperate. This implies $\bar{a} \neq 0$ that is a contradiction. Second, we consider the cases $(g(X), g(Y)) \neq (0, 0)$. It follows that with some positive probability future generations will cooperate after the realization of either bad signal, if $g(Y) \neq 0$, or good signal, if $g(X) \neq 0$. As a consequence $\bar{a} \neq 0$ with some positive probability, which is again a contradiction. Then it follows that, if $\bar{a} = 0$ the unique equilibrium continuation values are $(g(X), g(Y)) = (0, 0)$ and $\upsilon_t = 1$.

We conclude that the set of equilibrium payoff sustained by social norms without SCI is equal to $\Gamma^{a} = [\upsilon^{a}_{\min}, \upsilon^{a}_{\max}]$, where $\upsilon^{a}_{\min} = \upsilon_{aut} = 1$ and $\upsilon^{a}_{\max} = 1 - a + d - \frac{\Delta^{a} p_1}{p_0 - p_1}$. ■

Corollary (9). Let $(a_1, g(z_1))$ be a recursive strategy profile of the first period which supports $\upsilon_1 \in \Gamma^{a}$, as derived in Proposition 1. For each z_1, $g(z_1) \in [0,1]$, which implies $\exists\ (a_2(z_1), g(z_1, z_2))$ that support $g(z_1)$ and $\upsilon_2 \in \Gamma^{a}$. The recursion process can continue. Let $\alpha = (a_1, a_2(z_1), a_3(z_1, z_2), ...)$. α is PPE and the payoff of α is $\upsilon \in \Gamma^{a}$. ■

Proposition (9). We determine the set of payoff associated to social norms with SCI, Γ^{ab}, when agents are restricted to play strongly symmetric pure public strategy. Let υ_t be equal to:

$$\upsilon_t = (1 - b_t)(1 - a_t) + d\left[p_a g(Y, b_t) + (1 - p_a) g(X, b_t)\right] \tag{3.21}$$

where $g : Z \times B \to [0,1]$. $g(Y, b_t)$ and $g(X, b_t)$ denotes the t-generation continuation values in the case of bad signal and good signal realizations, respectively. The vector $(a_t, b_t, g(z_t, b_t))$ defines a recursive strategy profile. Note that, differently from Proposition 8, here the continuation values are also affected by the self-commitment decisions taken in the previous period. Note that $p_a = \Pr(Y|a_t)$ is not affected by the action b_t. We need to solve the following maximization program:

$$\max \upsilon_t$$

s.t.:

$$\begin{cases} \bar{a} = \underset{a_t \in \{0,a\}}{\arg\max} \upsilon_t \\ \bar{b} = \underset{b_t \in \{0,b\}}{\arg\max} \upsilon_t \\ p_a = \Pr(Y|a_t) \\ g(Y, b_t), g(X, b_t) \in [0,1] \end{cases}$$

Under the feasibility conditions $p_1 \leq p_0 - \frac{\Delta^{ab}}{d}$ and $p_1 \leq \frac{d - \Delta^{a} - \Delta^{ab}}{d - b} p_0$ we discuss the four possible cases:

Case I:

Suppose $\bar{a} = a$ and $\bar{b} = b$ then the promise keeping constraint is equal to:

$$\upsilon_t = (1 - b)(1 - a) + d\left[p_1 g(Y, b) + (1 - p_1) g(X, b)\right] \tag{3.22}$$

To be an equilibrium the following incentive compatible constraints must hold:

$$\upsilon_t \geq (1-b) + d\left[p_0 g(Y,b) + (1-p_0)\, g(X,b)\right] \tag{3.23}$$
$$\upsilon_t \geq (1-a) + d\left[p_1 g(Y,0) + (1-p_1)\, g(X,0)\right] \tag{3.24}$$
$$\upsilon_t \geq 1 + d\left[p_0 g(Y,0) + (1-p_0)\, g(X,0)\right] \tag{3.25}$$

We need to determine the corresponding continuation values $g(z_t, b_t)$ for $z_t \in \{X, Y\}$ and $b_t \in \{0, b\}$. Note that Eq. (3.22) is an increasing function in both $g(Y,b)$ and $g(X,b)$, while it is not affected by $g(Y,0)$ and $g(X,0)$. Since the RHS of Eq.(3.23) is an increasing function in both $g(Y,b)$ and $g(X,b)$, then constraint (3.23) must hold with equality, implying:

$$g(X,b) = \frac{\Delta^{ab}}{d(p_0 - p_1)} + g(Y,b)$$

where $\Delta^{ab} = a(1-b)$. Plugging the maximum values:

$$g(X,b) = 1 \text{ and } g(Y,b) = 1 - \frac{\Delta^{ab}}{d(p_0 - p_1)} \tag{3.26}$$

into constraints (3.24) and (3.25) and solving by $g(X,0)$, we obtain:

$$g(X,0) \leq \left(1 - \frac{\Delta^{ab}}{d}\frac{p_0}{p_0 - p_1}\right)\frac{1}{1-p_1} - \frac{p_1}{1-p_1} g(Y,0) \tag{3.27}$$
$$g(X,0) \leq \left(1 - \frac{\Delta^{ab}}{d}\frac{p_0}{p_0 - p_1} - \frac{\Delta^{a}}{d}\right)\frac{1}{1-p_0} - \frac{p_0}{1-p_0} g(Y,0) \tag{3.28}$$

Let us denote with $\Upsilon \equiv \{[0,1] \times [0,1] \,|\, \text{s.t. Eq. } (3.27)\,,\ (3.28)\}$. It follows that for $g(X,b)$ and $g(Y,b)$ equal to condition (3.26) and $(g(X,0), g(Y,0)) \in \Upsilon$, we attain the value:

$$\upsilon_t = (1-b)(1-a) + d - \frac{\Delta^{ab}}{\frac{p_0}{p_1} - 1}$$

Case II:

Suppose $\bar{a} = a$ and $\bar{b} = 0$ then the promise keeping constraint is equal to:

$$\upsilon_t = (1-a) + d\left[p_1 g(Y,0) + (1-p_1)\, g(X,0)\right] \tag{3.29}$$

To be an equilibrium the following incentive compatible constraints must hold:

$$\upsilon_t \geq (1-b)(1-a) + d\left[p_1 g(Y,b) + (1-p_1)\, g(X,b)\right] \tag{3.30}$$
$$\upsilon_t \geq (1-b) + d\left[p_0 g(Y,b) + (1-p_0)\, g(X,b)\right] \tag{3.31}$$
$$\upsilon_t \geq 1 + d\left[p_0 g(Y,0) + (1-p_0)\, g(X,0)\right] \tag{3.32}$$

Differently from Case I, Eq. (3.29) is increasing in both $g(Y,0)$ and $g(X,0)$. Since the RHS of Eq.(3.32) is an increasing function in both $g(Y,0)$ and $g(X,0)$, the constraint (3.32) must hold with equality, implying:

$$g(X,0) = \frac{\Delta^a}{d(p_0-p_1)} + g(Y,0)$$

Plugging the maximum values $g(X,0) = 1$ and $g(Y,0) = 1 - \frac{\Delta^a}{d(p_0-p_1)}$ into constraints (3.30) and (3.31) and solving by $g(X,b)$, we obtain:

$$g(X,b) \leq \left(1 + \frac{\Delta^{ab}}{d} - \frac{\Delta^a}{d}\frac{p_1}{p_0-p_1}\right)\frac{1}{1-p_1} - \frac{p_1}{1-p_1} g(Y,b) \tag{3.33}$$
$$g(X,b) \leq \left(1 + \frac{b}{d} - \frac{\Delta^a}{d}\frac{p_0}{p_0-p_1}\right)\frac{1}{1-p_0} - \frac{p_0}{1-p_1} g(Y,b) \tag{3.34}$$

Subject to constraints (3.33) and (3.34), it follows that for the maximum level of $g(X,b)$ and $g(Y,b)$ we attain the value:

$$\upsilon_t = 1 - a + d - \frac{\Delta^a}{\frac{p_0}{p_1} - 1}$$

which is the best sustainable payoff in the case of social norms without SCI.

Case III:

Suppose $\bar{a} = 0$ and $\bar{b} = b$ then the promise keeping constraint is equal to:

$$\upsilon_t = (1-b) + d\left[p_0 g(Y,b) + (1-p_0)\, g(X,b)\right] \tag{3.35}$$

To be an equilibrium the following incentive compatible constraints must hold:

$$\upsilon_t \geq (1-b)(1-a) + d\left[p_1 g(Y,b) + (1-p_1) g(X,b)\right] \tag{3.36}$$

$$\upsilon_t \geq (1-a) + d\left[p_1 g(Y,0) + (1-p_1) g(X,0)\right] \tag{3.37}$$

$$\upsilon_t \geq 1 + d\left[p_0 g(Y,0) + (1-p_0) g(X,0)\right] \tag{3.38}$$

Similarly to Case I, Eq. (3.35) increasing in both $g(Y,b)$ and $g(X,b)$. Thus, constraint (3.36) must hold with equality, implying:

$$g(X,b) = \frac{\Delta^{ab}}{d(p_0 - p_1)} + g(Y,b) \tag{3.39}$$

Under the condition (3.39), $g(X,b) > 0$ and $g(Y,b) > 0$ that implies that agents expect with positive probability that future generations will supply consumption good transfer, $\bar{a} > 0$, which is a contradiction. It follows that the promise values must be $g(X,b) = g(Y,b) = 0$, which implies $\upsilon_t = (1-b)$ and, consequently, the constraint (3.38) is never satisfied. $\bar{a} = 0$ and $\bar{b} = b$ is not part of an equilibrium strategy.

Case IV:

Finally, suppose $\bar{a} = 0$ and $\bar{b} = 0$ then the promise keeping constraint is equal to:

$$\upsilon_t = 1 + d\left[p_1 g(Y,0) + (1-p_1) g(X,0)\right] \tag{3.40}$$

The same argument proposed for Case III holds here. $g(Y,0) = g(X,0) = 0$ and $g(Y,b)$ and $g(X,b)$ are subject to:

$$g(X,b) \leq \frac{b + \Delta^{ab}}{d(1-p_1)} - \frac{p_1}{1-p_1} g(Y,b)$$

The value (3.40) is equal to 1.

To conclude, the set of equilibrium payoffs sustained by social norms with SCI is equal to $\Gamma^a = \left[\upsilon^{ab}_{\min}, \upsilon^{ab}_{\max}\right]$, where $\upsilon^{ab}_{\min} = \upsilon_{aut} = 1$ and $\upsilon^{ab}_{\max} = (1-b)(1-a) + d - \frac{\Delta^{ab}}{\frac{p_0}{p_1} - 1}$. ■

Corollary (10). Let $(a_1, b_1, g(z_1, b_1))$ be a recursive strategy profile in the first period, which supports $\upsilon_1 \in \Gamma^{ab}$, as derived in Proposition 9. For each z_1 and b_1, $g(z_1, b_1) \in [0,1]$, which implies that $\exists\, (a_2(z_1, b_1), g(z_1, b_1, z_2, b_2))$ that support $g(z_1, b_1)$ and $\upsilon_2 \in \Gamma^{ab}$. The recursion process can continue.

Let $\alpha = (a_1, a_2(z_1, b_1), a_3(z_1, b_1, z_2, b_2), ...)$ and $\beta = (b_1, b_2(z_1, b_1), b_3(z_1, b_1, z_2, b_2), ...)$.

(α, β) is PPE and the payoff of (α, β) is $\upsilon \in \Gamma^{ab}$. ■

Proposition (10). To prove that $\tilde{\Gamma}^a \subseteq \tilde{\Gamma}^{ab}$ it is sufficient to show that $\upsilon_{\max}^{ab} > \upsilon_{\max}^{a}$, which requires:

$$p_1 \geq (1-a)\, p_0 \tag{3.41}$$

Then under the conditions (3.11), (3.12) and (3.41) the proposition is proved. ■

Proposition (11). To determine whether social norms with SCI might have a positive impact also from a strategic point of view, i.e. $D^S(b) > 0$. Given that $D^S(0) = 0$, it is sufficient to find out the conditions under which $\frac{\partial D^S(b)}{\partial b} > 0$, that implies $\frac{\partial \Delta^{ab}}{\partial b} < 0$.

$$\frac{\partial \Delta^{ab}}{\partial b} = \lambda'(b)\, q(b, a)$$

where $q(b, a) \equiv u'(\lambda(b)) - (1-a)\, u'(\lambda(b)(1-a))$. Note that $q(b, 0) = 0$ for each b, then to determine the sign of $q(b, a)$ we simply have to evaluate the relative impact of the a-decision:

$$\frac{\partial q(b, a)}{\partial a} = u'(\lambda(b)(1-a)) + (1-a)\,\lambda(b)\, u''(\lambda(b)(1-a)) \tag{3.42}$$

Let $c_\lambda \equiv \lambda(b)(1-a)$ and denote $\rho \equiv -\frac{u''(c_\lambda)c_\lambda}{u'(c_\lambda)}$ the coefficient of relative risk aversion, then Eq. (3.42) can be rewritten as follows:

$$\frac{\partial q(b, a)}{\partial a} = 1 - \rho$$

It follows that, there are four possible economy configurations which depend on: i) Dynamic inefficiency (or efficiency), i.e. $\lambda'(b) > (\leq) 0$, and ii) relative risk-aversion greater (or lower) than one, i.e. $\rho \geq (<) 1$, as follows:

1. If $\lambda'(b) > 0$ and $\rho \geq 1$, then $D^S(b) \geq 0$;
2. If $\lambda'(b) \leq 0$ and $\rho \geq 1$, then $D^S(b) \leq 0$;
3. If $\lambda'(b) > 0$ and $\rho < 1$, then $D^S(b) \leq 0$;
4. If $\lambda'(b) \leq 0$ and $\rho < 1$, then $D^S(b) \geq 0$.

■

Bibliography

[1] Abreu, D., Pearce, D., and E. Stacchetti, 1986, Optimal Cartel Equilibria with Imperfect. Monitoring, *Journal of Economic Theory*, 39, 251-269.

[2] Abreu, D., Pearce, D., and E. Stacchetti, 1990, Toward a Theory of Discounted Repeated Games with Imperfect Monitoring, *Econometrica*, 58 (5), 1041-1063.

[3] Abreu, D., Milgrom, P., and D., Pearce, 1991, Information and Timing in Repeated Partnerships, *Econometrica*, 59(6), 1713-1733.

[4] Bhaskar, V., 1998, Informational Constraints and the Overlapping Generations Model: Folk and Anti-Folk Theorems, *Review of Economic Studies*, 65(1), 135-149.

[5] Boldrin, M., and A., Montes, 2005, The Intergenerational State Education and Pensions, *Review of Economic Studies*, 72(3), 651-664.

[6] Cole, L., and N., Kocherlakota, 2005, Finite memory and imperfect monitoring, *Games and Economic Behavior*, 53(1), 59-72.

[7] Cremer, J., 1986, Cooperation in Ongoing Organizations, *Quarterly Journal of Economics*, 100, 33-49.

[8] Durlauf, N., and L. E. Blume, 2011, the New Palgrave Dictionary of Economics, Second Edition, edited by Steven, London: Macmillan.

[9] Fudenberg, D., and E., Maskin, 1986, The Folk Theorem in Repeated Games with Discounting or with Incomplete Information, *Econometrica*, 54(3), 533-554.

[10] Fudenberg, D., Levine, D., and E., Maskin, 1994, The Folk Theorem in Repeated Games with Imperfect Public Information, *Econometrica,* 62, 997-1039.

[11] Green, E., and R., Porter, 1984, Noncooperative Collusion under Imperfect Price Information, *Econometrica*, 52, 87-100.

[12] Hammond, P., 1975, Charity: Altruism or Cooperative Egoism?, in E. S. Phelps (ed.), Altruism, Morality and Economic Theory, New York: Russell Sage Foundation.

[13] Kaganovich, M., and I., Zilcha, 1999, Education, social security, and growth, *Journal of Public Economics*, 71(2), 289-309.

[14] Kandori, M., 1992*a*, Social Norms and Community Enforcement, *Review of Economic Studies*, 59(1), 63-80.

[15] Kandori, M., 1992*b*, Repeated Games Played by Overlapping Generations of Players, *Review of Economic Studies*, 59, 81-92.

[16] Kandori, M., and I. Obara, 2006, Efficiency in Repeated Games Revisited: The Role of Private Strategies, *Econometrica*, 74(2), 499-519.

[17] Lagunoff, R., Anderlini, L., and D., Gerardi, 2008, A 'Super Folk Theorem' in Dynastic Repeated Games, *Economic Theory*, 37, 357-394.

[18] Lagunoff, R., and A. Matsui, 2004, Organizations and Overlapping Generations Games: Memory, Communication, and Altruism, *Review of Economic Design*, 8, 383-411.

[19] Lancia, F., and A., Russo, 2011, A Dynamic Politico-Economic Model of Intergenerational Contracts, mimeo.

[20] Lewis, D., 1969, Convention: A Philosophical Study, Cambridge MA, Harvard University Press.

[21] North, D., 1987, Institution, Transaction Costs and Economic Growth, *Economic Inquiry*, 25, 419-428.

[22] Rangel, A., 2003, Forward and Backward Intergenerational Goods: Why Is Social Security Good for the Environment?, *The American Economic Review*, vol. 93(1), 813-834.

[23] Radner, R., Myerson, R., and E. Maskin, 1986, An Example of a Repeated Partnership Game with Discounting and with Uniformly Inefficient Equilibria, *Review of Economic Studies*, 53(1), 56-69.

[24] Salant, D., 1991, A Repeated Game with Finitely Lived Overlapping Generations of Players, *Games and Economic Behavior*, 3, 244-259.

[25] Samuelson, P., 1958, An Exact Consumption Loan Model of Interest With or Without the Social Contrivance of Money, *Journal of Political Economy*, 66, 467-482.

[26] Smith, L., 1992, Folk Theorems in Overlapping Generations Games, *Games and Economic Behavior*, 4, 426-449.