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RECONSTRUCTING QUANTUM FIELD THEORY FROM AN EDUCATIONAL PERSPECTIVE

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This I gathered from books like those of Heitler an Dirac. I was inspired by the remarks in these books; not by the parts in which everything was proved and demonstrated carefully and calculated, because I couldn't understand those very well. At the young age what I could understand were the remarks about the fact that this doesn't make any sense, and the last sentence of the book of Dirac I can still remember, "It seems that some essentially new physical ideas are here needed." So, I had this as a challenge and an inspiration. I also had a personal feeling, that since they didn't get a satisfactory answer to the problem I wanted to solve, I don't have to pay a lot of attention to what they did do.

(R. P. Feynman, Nobel Lecture, December 11, 1965)

Tutta l'episteme della cultura occidentale viene in tal modo ad essere modificata nelle sue disposizioni fondamentali. Ed in particolare il campo empirico in cui l'uomo del XVI secolo vedeva ancora intrecciarsi le parentele, le somiglianze e le affinità e nel quale linguaggio e cose si intersecavano senza fine – tutto questo territorio immenso assumerà una configurazione nuova. Si può, se si vuole, designarla col nome di "razionalismo". Si può anche dire – per chi non ha in testa altro che qualche nozione prefabbricata, che il XVII secolo segna la scomparsa delle vecchie credenze superstiziose o magiche e l'entrata, infine, nell'ordine scientifico.

(M. Foucault, Le parole e le cose)

Un mutamento concettuale sembra presupporre nuove concezioni del mondo e nuovi linguaggi capaci di esprimerle. Se il conservatore tenderà a ridurre il nuovo a idee più familiari e cercherà di trattarlo come caso particolare di cose già comprese, il rivoluzionario si creerà passo dopo passo "un linguaggio del futuro", argomentando con termini non appieno spiegati, impiegando locuzioni per cui non sono ancora assegnate esplicite regole d'uso, insomma ritornerà – contro ogni "ragionevole" richiesta di chiarezza e di rigore – "a parlare per enigmi". Le questioni che gli parranno essenziali saranno assai simili a quelle "questioni" imperfette" che contraddistinguevano piuttosto le fasi in cui le discipline scientifiche in questione non erano ancora sufficientemente articolate e mature.

(G. Giorello, Introduzione alla Filosofia della Scienza)

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INTRODUCTION

Let us use an architectural metaphor. In order to construct a building, scaffoldings, supporting structures are needed. They are often built with whatever means available.
Analogously, the scientific researcher builds like a craftsman, constructing new experiments and new concepts with the tools and the ideas he has at hand. But scaffolding hides what is to be built, and in the end it has to be taken down to allow the building to be seen and for it to function. In the same way, the theoretical scaffoldings of new scientific ideas often hide these ideas and should be taken down in order to allow the real structure to be seen and its inner meaning to be evident.
Nevertheless such a critical task of cleaning and tiding up is seldom done within actual science, and rarely consciously. The conceptual level stays crammed with things whose constructive role has been relevant, but that now hide the building; the consequences are evident in epistemological analysis and, in particular, in instruction.
(Levy-Leblond, 1996)

Quantum Field Theory (QFT) seems to mirror what Levy-Leblond says: It looks as an impressive building of linguistic engineering, made out of very fine materials, assembled by refined craft-like experience, where shadows of scaffoldings still stand.

At first glance, in its usual presentation in university textbooks, it appears indeed as a tangle of formal structures, a "quarrel" between general and particular, new and old, physical and epistemological aspects.

Entering such a tangle, outlining its essential conceptual structure, analysing it from the specific perspective of Physics Education Research (PER) is the aim and the sense of the present dissertation.

Chapter 1 is devoted to outline the status of the research on the foundations of QFT and to introduce the research questions and the specific educational perspective chosen for the research.

Chapter 2 contains the conceptual physical core of the work where a specific research on QFT foundations is developed by: *i)* analyzing the formal and conceptual structures characterizing the description of the continuous systems that remain invariant in the transition from classical to contemporary physics; *ii)* analyzing the change in the meanings of the concepts of *field* and *interaction* in the transition to QFT; *iii)* focusing on the particular case of the Klein-Gordon equation, considered as emblematic for pointing out, in some detail, some interpretative (conceptual and didactical) problems concerning the concept of field that university textbooks on QFT do not address in an explicit way.

The studies reported in Chapters 3 and 4 aim at testing the educational value of the analysis developed on the foundations of QFT: The results obtained on QFT foundations are indeed reconsidered so as to evaluate their implications in PER.

More specifically, in Chapter 3 the cultural and educational potential of the analysis carried out on the Klein-Gordon equation is evaluated by applying the model of educational reconstruction developed by I. Galili: The *Discipline-Culture* Model.

In Chapter 4, the results reported in Chapter 2 are tested as criteria for the analysis of a selection of the teaching proposals known in PER literature, designed for introducing QFT notions at the secondary school level and in introductory physics university courses.

CHAPTER 1

THE RESEARCH FRAMEWORK

1.1 Introduction to the research problem

Quantum Field Theory (QFT) is without doubt an advanced and specialized topic that only a part of university physics students is required to deal with. Also for this reason it has so far received little attention within the field of Physics Education Research (PER).
Quantum Field Theory (QFT) is without doubt an advanced and specialized topic that only a part of university physics students is required to deal with. Also for this reason it has so far received little attention within the field of Physics Education Research (PER).
Nevertheless, a growing interest about teaching QFT can be observed in recent years and a certain number of studies exists aimed at producing teaching proposals for introducing notions of QFT at the secondary school level or within introductory physics courses at the university level.
The main motivations of such a growing interest stem from the new requirements of secondary school physics curricula and from the acknowledgment of some problematic issues in teaching Quantum Mechanics (QM) that enforce the search for new teaching paths on Quantum Physics.

Further motivations derive from the need of tuning school and extra-school activities: Particle physics, the standard model, the last frontiers of physics are indeed object of popular science books and of important exhibitions that are having a greater and greater success of public. Italian examples are: “Physics microscopes” (“I microscopi della Fisica”)(2005), “The nature splits in four” (“La natura si fa in 4”) (2006), realized by Communication Office of the National Institute of Nuclear Physics (INFN); “Astra and particles” (“Astri e particelle”) (2009) realized by the city of Rome, INFN, National Institute of Astrophysics (INAF), Italian Space Agency (ASI).

A first analysis of PER literature[1], carried out at the beginning of this PhD work, showed that the existing papers do not address explicitly interpretative problems concerning QFT formalism.

For instance, in their efforts of translating contemporary physics notions in understandable languages, none of them seems to deal with the problem of making more transparent crucial sentences like, for instance, “particle are nothing but field oscillations”.

This evidence greatly contributed to the research interest of developing a *study on the foundations of QFT from an educational perspective*.

The description of the status of the research on QFT foundations reported in the next Section (§1.2) paves the way to illustrating the main features of the research perspective that oriented the whole work (§1.3). It moreover provides a framework for the analysis of the existing teaching proposals reported in Chapter 4.

1.2 The status of the research on foundations of Quantum Field Theory

The interpretative problems of QFT are a relatively young issue in the research field of philosophy and foundation of physics.

At the voice “Quantum Field Theory” of the Stanford Encyclopedia of Philosophy (Kuhlmann, 2009) two main reasons are provided for interpreting why the

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[1] The most significant examples will be analysed in Chapter 4 in the light of results presented in Chapter 2 and 3.

philosophical reflection has always been primarily concerned with QM and not with QFT:

1. According to the prevailing attitude, the decisive philosophical problems (for instance the measurement problem) had already shown up in QM so that a conceptual analysis of QFT appeared not necessary. Since QFT is much more complex and mathematically advanced than standard QM, it even seemed that looking at QFT would only blur the view of the central features;
2. QFT is generally considered not yet having reached the status of a consistent and complete theory: Because of the lack of a quantum field theory of gravitation (felt as a pressing need), it is supposed that the incorporation of the fourth fundamental force might lead to deep changes of QFT as a whole so that the current version of QFT can only be a preliminary theory.

As still reported in the Stanford Encyclopedia of Philosophy, QFT began to receive wider attention as object of the philosophical reflection only in the late 1980s when the two arguments quoted above lost importance because of the following reasons:

1. analyses of the specifically relativistic features of QFT led to results which give rise to new conceptual problems with respect to QM or, at least, severely aggravate the ones already standing out (see for instance, localizability);
2. even if the initial hope that QFT could be near to its final completion is fading away because of the development of more recent theories (see string theory), some quantum structures of QFT have been very steady for more than 70 years leading to strikingly good predictions. In the light of this fact the belief that a good part of these structures will remain in all improved theories is “well-grounded”;
3. there are a certain number of arguments supporting the belief that a conceptual analysis of QFT will allow to tackle problems which appeared insoluble when looking at QM: The problematic nature of the basic entities of the quantum regime seems to gain new illuminating meanings when observed from the QFT perspective.

As soon as the philosophical debate on QFT started up, it focused on the issue of ontology.

The image of the microcosm suggested by QFT is indeed often communicated through a particle classification, that sounds as follows: “There are two groups of fundamental fermionic matter constituents, two groups of bosonic force carriers and four (including gravitation) kinds of interactions”.

As satisfying as this image might first appear for describing the microcosm, it bypasses every ontological issues (intended as concerning the most general features, entities and structures of QFT) and questions like “what kind of entity the down quark is?” remain unaddressed.

From this perspective it became immediately evident that the answer could not depend upon the particular constituents chosen (down quarks or muon neutrinos...) since it requires the search for features which are much more general than those ones which constitute the difference between down quarks or muon neutrinos. So, as reported in the Stanford Encyclopedia, the relevant questions that triggered the philosophical reflection were questions like:

- What are particles at all?

- Can quantum particles be any more legitimately understood as “particles”, although in a broadest sense, when their localization properties are taken into account?
- How can one decode what a quantum field is and can “quantum fields” in fact be understood as fields?
- Instead of fundamental quantities, whatever this could mean, is it rather more appropriate to think, for example, of quarks as properties or processes or events?
- ...

All these questions raise specific philosophical issues that, according to the historical reconstruction reported in the Stanford Encyclopedia, can been seen as variations or sub-topics of the major philosophical debate involving QFT: The search for an ontology of QFT by confronting the particle and the field interpretation.

Although its official establishment as philosophical issue is pretty recent, such a debate has its origins in the very first years of QFT and many of the creators of the theory were divided about the issue whether particles or fields should be given priority in understanding QFT: While Dirac, the later Heisenberg, Feynman, and Wheeler opted in favour of particles, Pauli, the early Heisenberg, Tomonaga and Schwinger put fields first (see Landsman, 1996). Today, a certain number of arguments seem to prepare the ground for a proper discussion beyond mere preferences. Such arguments are reported below at a certain level of detail because of their inner educational value: They provide meaningful hints for avoiding to look at the interpretations of QFT formalism in too simplistic ways and for outlining an education perspective suitable to exploit the cultural potential of the theory at different levels of formal competences.

1.2.1 The Particle Interpretation of QFT

The supporters of a particle interpretation of QFT ground their main arguments in what is observed in some experiments: The observed ‘particle traces’ on photographic plates of bubble chambers are said to be a clear indication for the existence of particles.

The tenability of a particle interpretation of QFT cannot however put aside a preliminary investigation about the concept of particle.

Since the concept of particle has been evolving through history, in accordance with the latest scientific theories what is at issue is to understand how the common and classical ideas have to be “refined” (loosening some of their constraints) in order to tune the concept of particle with QFT framework.

Already in classical corpuscular theories of matter the concept of “elementary particle” is not unproblematic: If the whole charge of a particle was contracted to a point, an infinite amount of energy would be stored in this particle. The so-called *self energy* of a point particle is infinite.

The most immediate feature to be considered for defining particles is *discreteness*. Particles are countable individuals; on the other hand it is quite obvious that this feature only cannot be a sufficient attribute for being a particle since there are other things which are countable without being particles, such as, trivially, money or maxima and minima of the standing wave of a vibrating string.

Also the so-called *primitive thisness* or *haecceity,* explored for supporting a particle interpretation, is missing to be a sufficient attribute for being a particle since it does not discriminate between ups and downs in a wave pattern and particles.

In Teller (Teller, 1995) *primitive thisness* as well as other possible features of the particle concept are discussed in comparison to classical concepts of fields and waves as well as in comparison to the concept of field quanta. [2]

There is still another feature commonly taken to be pivotal for the particle concept, namely that particles are localizable in space: It will be discussed in this section that localizability in an arbitrarily large but still finite region can be a too strong condition to be applied to quantum particles.

A significant contribution to the discussion comes from Wigner's analysis of the Poincaré group (Wigner, 1939) often assumed as providing a "definition" of elementary particles. The main idea of Wigner's approach is the supposition that each irreducible (projective) representation of the relevant symmetry group yields the state space of one kind of elementary physical system, where the prime example is an elementary particle which has the more restrictive property of being structureless.

The physical justification for linking up irreducible representations with elementary systems is the requirement that "there must be no relativistically invariant distinction between the various states of the system" (Newton & Wigner, 1949). In other words the state space of an elementary system shall have no internal structure with respect to relativistic transformations.[3]

The main part of Wigner analysis consists in finding and classifying all the irreducible representations of the Poincaré group. Wigner pioneering identification of types of particles with irreducible unitary representations of the Poincaré group has been exemplary until the present.

Concerning the question whether Wigner has supplied or not a definition of particles, one can say that, although Wigner has in fact found a highly valuable and fruitful *classification* of particles, his analysis did not contribute very much to the question of what a particle is and whether a given theory can be interpreted in terms of particles. What Wigner has given is rather a conditional answer: *If* relativistic quantum mechanics can be interpreted in terms of particles *then* the possible types of particles correspond to irreducible unitary representations of the Poincaré group.

However, the question whether, and if yes in what sense, at least relativistic quantum mechanics can be interpreted as a particle theory at all is not addressed in Wigner's analysis. For this reason the discussion of the particle interpretation of QFT is not finished with Wigner's analysis as one might be tempted to say.

In the following subsections we discuss three problems that stand out as major problematic elements against a particle interpretation of QFT:

- the problem of localization;
- the problem of vacuum;

[2] A critical discussion of Teller's reasoning can be found in Seibt (Seibt, 2002).

[3] Put more technically, the state space of an elementary system must not contain any relativistically invariant subspaces, i.e., it must be the state space of an irreducible representation of the relevant invariance group. If the state space of an elementary system had relativistically invariant subspaces then it would be appropriate to associate these subspaces with elementary systems. The requirement that a state space has to be relativistically invariant means that starting from any of its states it must be possible to get to all the other states by superposition of those states which result from relativistic transformations of the state one started with.

- the Unruh effect.

The problem of localization

The issue of "particle localization" is pivotal for supporting particle interpretation of QFT.

Nevertheless what emerges is that the very theory, built on the basis of scattering experiments or photographic plates analysis (which apparently involve localizable particles), seems to exclude localizability.

The core question of localizability lies in the fact that, although in QFT we speak of "*N*-particle states" (among the possible ones), it is not clear how these states can be relate to *N* particles: The major attempt in this direction is trying to localize these "particle states" in any sensible way.

In this context, a central role is played by the Reeh-Schlieder theorem (Reeh & Schlieder, 1961), a milestone of the Algebraic Reformulation of Quantum Field Theory (AQFT):

> *The set of vector $A(O)\Omega$, generated from the vacuum by the polynomial algebra of any open region, is dense in H.*

In order to highlight the question opened by this theorem we report the comments provided by Haag: "Intuitively one might have thought that with $Q \in A(O)$ the vector $Q\Omega$ could be interpreted as representing a state *localized in O,* i.e. a state looking practically like the vacuum with respect to measurements in the causal complement of *O*. While, due to the cluster properties of Wightman functions, this is qualitatively true if *Q* is picked up at random in *A(O)* and measurements at sufficiently large space-like distance from *O* are considered, the theorem tells us that for any chosen state vector Ψ one can always find an operator $Q \in A(O)$ which, applied to the vacuum, produces a state vector arbitrarily close to Ψ. To achieve this the operator must judiciously exploit the small but not vanishing long distance correlations which exist in the vacuum as a consequence of the spectral restriction for the energy-momentum in the theory. The theorem shows that the concept of *localized state,* if used in a more than qualitative sense, must be handled with care". (Haag, 1996, p.102)

Other interpretative comments to the theorem can be found in Redhead (Redhead, 1995a): His interpretation of the Reeh-Schlieder theorem is that local measurements can never decide whether one observes an N-particle state since a projection operator P_Ψ which corresponds to an N-particle state Ψ can never be an element of a local algebra $R(O)$.

Clifton & Halvorson (Clifton & Halvorson, 2001) discuss the consequences of the Reeh-Schlieder theorem for the issue of entanglement.

Malament's theorem, in which localizability stands as essential ingredient (Malament, 1996), provides another important contribution to the issue of a particle interpretation of QFT: Very briefly, the theorem shows that a relativistic quantum theory of a fixed number of particles predicts a zero probability for finding a particle in any spatial set if four conditions are satisfied. These conditions are: *translation covariance, energy, localizability* and *locality condition*.

If these four conditions are accepted as natural assumption for a particle interpretation, Malament's proof has the weight of a “no-go” theorem: A relativistic quantum theory of a fixed number of particles, satisfying in particular the localizability and the locality condition, has to assume a world devoid of particles (or at least a world in which particles can never be detected) in order not to contradict itself.

Malament's “no-go” theorem thus seems to show that there is no middle ground between QM and QFT, i.e., no theory which deals with a fixed number of particles (like in QM) and which is relativistic (like QFT) without running into the localizability problem of the no-go theorem.

Malament’s result seems to force to finally give the interpretation of QFT as a field theory. Nevertheless, whether or not a particle interpretation of QFT is in fact ruled out by Malament's result, it is still a point of discussion firstly because Malament assumes a fixed number of particles and such an assumption is not valid in the case of QFT.

Reeh & Schlieder, Hegerfeldt, Malament and Redhead all gained mathematical results or formalized their interpretation proving that certain sets of assumptions, which are taken to be essential for the particle concept, lead to contradictions. However, it is at issue what exactly has been shown (by these no-go theorems) and how the different results relate to one another.
For these reasons there is not yet unquestioned proof against the possibility of a particle interpretation for QFT through the pivotal concept of localizability.

The problem of vacuum

The standard definition for the vacuum state |0> is that:
i) it is annihilated by the action of the annihilating operator;
ii) it is a cyclic vector, i.e., a *N*-particle state can be built up from the vacuum state by the N-fold application of a creation operator.
The problem of vacuum is related to the particle issue since, according to the above definition, the label ‘|0>’ indicates that there are no particle present in the vacuum state, but the expectation values for some quantities do not vanish for the vacuum state itself. So, if particles were the basic objects about which QFT speaks how can it be that physical phenomena occur even if nothing is there according to this very ontology?[4]

The Unruh effect

An even greater challenge for a particle interpretation of QFT is the Unruh effect: The Unruh effect is a surprising result which seems to show that the concept of particle is observer - dependent since a uniformly accelerated observer in a Minkowski vacuum

[4] It is also a notable result in ordinary non-relativistic QM that the ground state energy of the harmonic oscillator is *not* zero in contrast to its analogue in classical mechanics. Not only the same is true for the vacuum state in QFT but the question becomes more striking: being a free field made up of a collection of infinite harmonic oscillator (each one with a non zero value of energy), the ground state energy of a quantum field should have an infinite amount of energy.

will detect a thermal bath of particles (the so-called Rindler quanta) (Unruh, 1976 and Unruh & Wald, 1984).

The Unruh effect shows that a mere change of the frame of reference thus leads to a change of the number of particles.

Since basic features of a theory should be invariant under transformations of the referential frame, the Unruh effect constitutes a severe challenge to the concept of particles as basic objects of QFT.

1.2.2 The Field Interpretation of QFT

There are two lines of argumentation that are often taken to show that an ontology of fields is more appropriate to define the fundamental entities to which QFT refers. A first line is simply indirect: Since various arguments seem to exclude a particle interpretation, the only allegedly alternative, namely a field interpretation, must be the right conception.

The other argumentation rests on the fact that so-called *field operators* are at the basis of the mathematical formalism of QFT and it is supported by a formal analogy: The transition from a classical field theory (like electromagnetism) to quantum field theory can be characterized by the transition from the field $\varphi(\vec{x},t)$ to the quantum field $\hat{\varphi}(\vec{x},t)$ and a corresponding transition for its conjugate field, for both of which a certain specification of canonical commutation relations hold (the procedure is currently named "canonical quantization").

In technical terms the analogy is based on the formal similarity between the mappings $\mathbf{x} \mapsto \varphi(\vec{x},t)$, $\mathbf{x} \in \Re^3$, and $\mathbf{x} \mapsto \hat{\varphi}(\vec{x},t)$, $\mathbf{x} \in \Re^3$. This formal analogy between classical and quantum fields is one reasons why QFT is taken to be a field theory.

However, whether this formal analogy actually justifies this conclusion needs to be discussed.

The first question lies in the fact that, unlike a classical field $\varphi(\mathbf{x},t)$, the basic quantum fields $\hat{\varphi}(\vec{x},t)$ are *operator-valued distributions*.

In the classical case, defined physical quantities (expressed by the field values which are complex, real numbers...) are assigned to spacetime points.

The strict formal analogy leads to conclude that, in the quantum case, operator values are assigned to space-time points.

This conclusion has been deeply criticized: In his paper "What the quantum field is not" (Teller, 1990; Teller, 1995), Teller remarks that there are no definite physical values whatsoever assigned to space-time points. Instead, the assigned quantum field operators represent the whole spectrum of possible values so that they rather have the status of observables ("determinables") or general solutions. Something physical emerges only when the state of the system or when initial and boundary conditions are supplied.

Teller argues that 'quantum fields' lack an essential feature of the classical fields so that the expression 'quantum field' is only justified on a "perverse reading" of the notion of a field.

In the light of these arguments Teller criticizes the interpretation of QFT as a "field theory".

In the Stanford Encyclopedia, Teller's criticism of the field interpretation of QFT is said to have one justified and one unjustified aspect: The justified aspect is that quantum fields actually differ considerably from classical fields since the field values which are attached to space-time points have no direct physical significance in the case of a quantum field. On the other hand, it was not to be expected anyway that one would only encounter definite values for physical quantities in QFT since it is, like QM, an inherently probabilistic theory and is equally confronted with the measurement problem.

The problematic nature of the space-time point interpretation of the quantum fields is evident also from a formal analysis of the fields: As it will be discussed in detail in Chapter 2, quantum fields are distributions referring to extended spacetime regions and not simply functions on the space-time, like the classical fields are.

Moreover, if a field interpretation should actually yield the appropriate ontology for QFT, then it seems that those objects which are called "quantum fields" are not yet the fundamental entities one is looking for, at least not the only ones. Teller's own proposal is an ontology of QFT in terms of *field quanta*. Teller argues that the "Fock space representation" or "occupation number representation" suggests this conception with objects (quanta) which can be numbered or aggregated but which cannot be counted. The number of objects is given by the degree of excitation of a certain mode of the underlying field. Particle labels, like those in the Schrödinger many-particle formalism, do not occur any more. Teller has been criticized to draw such far-reaching ontological conclusions from one particular representation, the Fock space representation, that cannot be used in general because it is only valid for free particles.

In the light of the previous arguments the formal analogy between classical and quantum fields as such is not a fully convincing argument for a field interpretation of QFT.

1.2.3 Remarks

In the previous sections we entered the debate between particle or field interpretation of QFT. What emerges is that, on the one side, the adoption of a particle interpretation of QFT would make the importance of particle experiments and the predominance of speaking in terms of particles comprehensible. Nevertheless there are various problems for a particle interpretation because, for example, some results indicate that particle states cannot be localized in any finite region of space-time no matter how large it is.

On the other side, a field interpretation would support the formal analogy stressed by the quantization of classical fields but this interpretation would require a more detailed analysis of the role and the meaning of the quantum fields.

Recent studies put at issue also the dichotomy between fields and particles ("it served to veil essential aspects", Haag, 1996, pp. 45 - 46) arguing for new fundamental roles of quantum fields.

From an educational perspective the debate occurring within QFT foundations points out the need of avoiding a pure and unproblematic particle or field interpretation of QFT. Otherwise every educational reconstruction would be at risk of hyper-simplifications missing core aspects of the theory.

1.3 The "educational perspective" of the analysis

The general perspective of the whole work is to address the reconstruction of the main conceptual steps in the transition from classical to contemporary physics from an educational point of view; this perspective matches the agenda of re-thinking physics curriculum *at university level* as well as (in prospect) at the *secondary school level* in order to include in the curriculum a selected conceptual account of modern physics.

In order to justify an educational engagement in QFT, the existing teaching proposals focus on the following features of the theory:

- QFT is a theoretical background legitimated by the experimental successes of the Standard Model;
- QFT can be considered *the* fundamental theory;
- QFT can represent, for several reasons, the conceptual completion of quantum physics.

In other words, the following arguments are chosen to frame or orient the proposals:

- Effective predictive capability of the theory;
- Hierarchical organization of physics knowledge;
- The need to find solutions for technical problems that quantum physics cannot solve.

None of these features and arguments has been chosen as the main starting point for defining the educational perspective that oriented our work.

The main reason is that they seem rather far from the basic concepts of the theory and their problematic issues.

In other words, these arguments appear to be very internal to highly refined physical discourses and therefore inadequate for shaping non-specialized teaching for the following cultural and educational reasons:

- their articulation requires sophisticated technical languages and hence, as soon as formalism cannot be used, the risk of not being able to go beyond pure statements can be very high;
- they seem to have a weak potential for outlining a sufficiently wide cultural horizon where to situate technical problems, with the risk of reasoning being trapped in them and loosing the sense of the whole cultural meaning. We instead believe that a wide cultural horizon should be explicitly outlined in order to characterize the educational reconstruction of QFT as a process of *"bringing physics into culture"* (Levy Leblond, 1996; Grimellini, 2004).

The educational perspective we have chosen is looking at QFT as manifestation of a primitive knowledge issue: *QFT as the most recent format in which physics, through the concepts of quantum field and quantum interaction, conceptualizes the "continuum" and manages its formal structures as well as re-conceptualizes "the relationship between continuum and discrete"*.

Operatively, the whole work has been oriented by the following research questions:

- How the concept of *object* changes when moving from classical to contemporary physics, in particular to QFT?
- How the concepts of *field* and *interaction* are shaped and conceptualized within the QFT? What makes the concepts of quantum field and interaction similar to the classical ones, and what makes them different?

As well as because they focus on the problematic issues addressed by the philosophical debate, the questions have been selected for two other reasons:

- their genuine “conceptual” character allows the analysis on the foundations of QFT to be systematically carried out keeping a special attention to its deep understandability;
- they concern basic and transversal concepts across all the physics domains and, hence, they can help to protect the analysis from the risk of adopting specialized and sectional ways of looking at QFT.

Also when the analysis seems to be still strictly related to the theoretical physics domain, the research questions are not really addressable either by theoretical physics (at least to what is usually intended for theoretical physics), or by foundations, philosophy, and communication of physics: They outline an “educational proper way” to approach the problem, that allows the results both to have direct implications on teaching QFT in the university specialized courses (see Chapter 2), and to provide contributions to typical PER issues (see Chapters 3 and 4), such as:

- the development of models of educational reconstruction of physics (Kattman et al.,1996; Duit et al., 2005; Tseitlin, Galili, 2005; Duit, 2006; Guidoni, Levrini, 2008; Levrini et al., 2008b);
- the elaboration of criteria for evaluating and producing teaching proposals on contemporary physics, based on ways of looking at the formalism without getting trapped in it, but being able to exploit its constitutive role (Tarsitani, 2006).

With respect to more typical researches in physics education, the analysis developed here does not address explicitly the cognitive dynamics of students in coping with physical concepts/topics. Nevertheless, the research questions are not chosen on the basis of top-down criteria and the students are not as far away as it may appear. The research questions, indeed, stemmed from a experimentation in school, where research-based teaching materials on QM were implemented in a class of 18-19 year old students in Rimini. The materials had been designed according to the aims of introducing a minimal formalism (Pauli’s matrixes) and of exploiting the historical and philosophical debates for creating a learning environment able to support deep understanding as well as to promote intellectual autonomy (Levrini et al., 2008a). Students’ curiosity toward some interpretative issues – such as modelling quantum objects, visualization, the use of simple graphical tools for illustrating the Compton effect – inspired the research group[5] to extend the contents analysis to QFT.

The research work presented in the following chapters shows to what extent the research questions are puzzling and to what extent the search for an answer deserves the explorations of physical domains not even imagined in advance (like the cold plasma physics).

As discussed above, the research of QFT foundations is very young and few works about the conceptual interpretation of the theory exist. This contributes to make the work developed for this Dissertation an example of the research effort needed to clarify, as much as possible, the physical contents in the prospect of their teaching and learning.

[5] The research group, within which this PhD work has been partly developed, is made by people with different competences: Researchers in Physics Education (M. Gagliardi, N. Grimellini Tomasini, P. Guidoni, O. Levrini, B. Pecori, C. Tarsitani); Secondary school teachers (M. Clementi, P. Fantini, C. Montanari); Theoretical physicists (S.Bergia, E.Ercolessi).

CHAPTER 2

THREE STUDIES ON THE FOUNDATIONS OF QUANTUM FIELD THEORY

2.1 Introduction

The chapter contains the analysis on the foundations of Quantum Field Theory (QFT) carried out from an educational perspective. As mentioned in Chapter 1, the two research questions orienting the analysis are:

- How the concept of *object* changes when moving from classical to contemporary physics?
- How are the concepts of *field* and *interaction* shaped and conceptualized within contemporary physics? What makes quantum field and interaction similar to and what makes them different from to the classical ones?

The answers to the research questions are, operatively, searched by developing the whole analysis along two main lines, namely:

- skeleton of continuum;
- discrete structure and II quantization language.

The first line concerns the analysis of the *essential structures that remain invariant* (the "skeleton of continuum") in the transition from classical to modern physics: What a vibrating string and quantized field have in common (§2). The second line analyzes the *concepts of field and interaction that change their status* in the transition from classical to modern physics (§3).

Section 4 contains a specific study on the Klein-Gordon equation (KGE) selected as emblematic case for developing a detailed analysis on interpretative problems connected to teaching QFT at the university level.

2.2 Skeleton of continuum

The construction of the skeleton of continuum involves a loose concept of "continuous system" (constrained by space-time relations) to be related to specific physical cases through the interpretation of some parameters inside the equations or coefficients inside the relations.

The skeleton is made of formal elements (properties and relations) that characterize the description of the continuous system and that remain essentially invariant in the transition from classical to contemporary physics.

Five elements constituting the skeleton have been pointed out and, for each of them, a particular meaning contained in the formalism is picked up so as to create an intelligible, plausible and meaningful framework in which locating the analysis on the concepts of field and interaction reported in Section 3:

1) The equations of motion of continuous systems for pointing out the spacetime structure of propagation;
2) The Fourier transform as tool for marking the differences between the various propagating systems;
3) The spacetime symmetries for providing different observers with the same tools of analysis;
4) The internal symmetries for marking constitutive differences between the systems;
5) The jump from global to local gauge transformations for making different systems feel each other.

2.2.1 The equations of motion of continuous systems for pointing out the spacetime structure of propagation

The spacetime propagation of continuous systems is expressed by particular relations between the derivatives in space and time.

Although the involved systems deeply change their meanings according to different physical contexts, the formal kinematical relations (and their implications) remain invariant across the domains.

We focus on four paradigmatic cases that will be relevant for the following sections:

i) d'Alembert equation

$$\frac{\partial f^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial f^2}{\partial t^2} = 0 \tag{1}$$

ii) Klein Gordon equation

$$\frac{\partial f^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial f^2}{\partial t^2} = mf^2 \tag{2}$$

iii) Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi \tag{3}$$

iv) Dirac equation.

$$i\partial_0\psi = \alpha^i\frac{1}{i}\partial_i\psi + m\beta\psi \tag{4}$$

The former two cases gain meanings both in non-relativistic and in relativistic context, according to the value of the parameter v ($v = c$ or $v \neq c$)

In the relativistic case the particular relation between the derivatives (second order in time and second in space) allows the interpretation of the two equations in terms of field.

In the next Section (2.2.2) it will be shown that the differences in the right members of the previous equations (zero or non-zero value) affect the dispersion relation.

The latter two cases, due to their formal similarity in the derivatives (first order in time) admit a probabilistic interpretation.

The fundamental difference between them lies in the different formal structure of their solutions: Scalar (one complex component) in the Schrödinger case, spinorial (4 complex components) in the Dirac one. Such a difference marks the non-relativistic and relativistic character of the two.

2.2.2 The Fourier transform as tool for marking the differences between the various propagating objects

Under very general conditions, continuous system can be expressed through a Fourier expansion, whose general expression is

$$f(x) = \int_{-\infty}^{+\infty} dk g(k) e^{ikx} \quad (5)$$

The Fourier transform is an effective tool for highlighting what the various continuous systems have in common and what makes them different.

In particular the Fourier expansion allows to disentangle the role of plane waves from that of the coefficients: the former being expression of the common aspects the latter of the peculiarities of the various physical contexts.

For example a quantum field gains its peculiarities with respect to a vibrating string only when the coefficients are interpreted as quantum operators in a Fock space and no longer as the numbers indicating the amplitude of a normal mode.

The different expression of the coefficient still intervenes in outlining fundamental differences among the quantum fields themselves. For example the Fourier expansion of the quantum Klein-Gordon (KG) field can have different expression according to whether they refer to charged or not charged massive, spinless particle:

neutral particle: $$\phi(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{\sqrt{2w_k}} \left[a(k) e^{-ikx} + a^+(k) e^{ikx} \right] \quad (6)$$

where $k_0 = w_k = \sqrt{\left|\vec{k}\right|^2 + m^2}$

$$\left[a(k), a(k') \right] = 0$$
$$\left[a^+(k), a^+(k') \right] = 0$$
$$\left[a(k), a^+(k') \right] = \delta(\vec{k} - \vec{k}')$$

charged particles: $$\phi(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{\sqrt{2w_k}} \left[a(k) e^{-ikx} + b^+(k) e^{ikx} \right] \quad (7)$$

where $k_0 = w_k = \sqrt{\left|\vec{k}\right|^2 + m^2}$

$$\left[a(k), a^+(k') \right] = \delta(\vec{k} - \vec{k}')$$
$$\left[b(k), b^+(k') \right] = \delta(\vec{k} - \vec{k}')$$

The plane waves, on which a continuous system is expanded, represent then a neutral element that progressively reveal more and more its ideal character: From an idealization of waves useful for describing concrete and perceivable oscillating phenomena (for instance sound waves or waves on a fluid surface) to a pure mathematical entity not directly attachable to oscillations of “something real”.

The Fourier transform allows the relation of dispersion to be expressed in terms of the parameters ω and k

$\omega = ck$ d'Alembert case (8)

$\omega = \sqrt{\omega_0^2 + c^2 k^2}$ KG case (9)

The parameters contain the physical contents of the involved systems that changes from classical to quantum context: frequency and wave number in the classical case and energy and momentum in the quantum one.

2.2.3 The spacetime symmetries for providing different observers with the same tools of analysis

Spacetime symmetries, formally expressed as invariance of Poincarè transformation (translations and Lorentz transformations) allow the description of the same system to be made by different inertial observers.

Without entering a detailed formal analysis we underline that the Poincarè invariance is assured under very general conditions which are satisfied by all the actions (real functional defined on fields) considered in the present discussion.

The equations of motion can by deduced from a variational principle by searching those configurations which make null the variation of the action:

$$S(\phi) = \int d^4 x L(\phi(x), \partial_\mu \phi(x)) \tag{10}$$

$$\partial_\mu \frac{\partial L}{\partial[\partial_\mu \phi]} = \frac{\partial L}{\partial \phi} \text{ (Euler Lagrange equation)} \tag{11}$$

Whenever the spacetime symmetries are taking into account, the description of the system results enriched of additional properties.

By virtue of the Noether theorem, given a continuous, *n* parameter dependent symmetry of the action, it is possible to define *n* "Noether currents" and *n* "Noether charges".

The currents satisfy an equation of continuity $\partial_u j^\mu = 0$ and the charges $Q = \int_{R^3} d^3 x j^0 (\vec{x}, t)$ are conserved during the propagation.

It is possible to show that, as a consequence of invariance for translations ($x^\alpha \rightarrow x'^\alpha = x^\alpha + \varepsilon^\alpha$) the Noether charge is the four vector energy-impulse:

$$P^\alpha = \int d^3 x T^{\alpha 0} \tag{12}$$

where

$$T^{\alpha\mu} = -L\eta^{\alpha\mu} + i\frac{\partial L}{\partial\left[\partial_\mu \phi\right]}\hat{P}^\alpha \phi$$

$$\hat{P}_\mu = -i\partial_\mu \qquad (13)$$

Analogously, as a consequence of the invariance for Lorentz transformations

$$x^\mu \to x'^\mu = \Lambda^\mu_\nu x^\nu \qquad \text{where} \qquad \Lambda^\mu_\nu = \left(\delta^\mu_\nu + \varepsilon^\mu_\nu\right) \quad \text{and} \quad \varepsilon_{\mu\nu} = -\varepsilon_{\nu\mu} \qquad (14)$$

the Noether charge is the general angular momentum

$$M^{\alpha\beta} = \int d^3x\left(L^{\alpha\beta,0} + S^{\alpha\beta,0}\right) = L^{\alpha\beta} + S^{\alpha\beta} \qquad (15)$$

where

$$M_{\alpha\beta}{}^{\mu} = x_\beta T_\alpha{}^\mu - x_\alpha T_\beta{}^\mu + i\frac{\partial L}{\partial\left[\partial_\mu L\right]}\hat{S}_{\alpha\beta}\phi = L_{\alpha\beta}{}^\mu + S_{\alpha\beta}{}^\mu$$

$$T^{\alpha\beta} - T^{\beta\alpha} = -\partial_\mu S^{\alpha\beta,\mu} \qquad (16)$$

In general, it is not assured that L and S conserve themselves apart: it happens only when the tensor T is symmetric.
In order to compare the classical and quantum situations we remark that:

- the expression of the general angular momentum highlights that, already in the classical case (before quantization), the formal structure describing, after quantization, the particle spin is present;
- the conserved quantities gain a formal expression useful to compare the classical and the quantum frameworks when Fourier transformations are applied. As an example we write the expression for the four vector energy-momentum for the KG field quoted above (Eq. 12.)

$$P^\alpha = \frac{1}{2}\int d^3k k^\alpha\left[a^*(k)a(k) + a(k)a^*(k)\right]$$

$$E = \frac{1}{2}\int d^3k\omega_k\left[a^*(k)a(k) + a(k)a^*(k)\right]$$

$$P^i = \frac{1}{2}\int d^3k k^\alpha\left[a^*(k)a(k) + a(k)a^*(k)\right] \qquad (17)$$

$$k^\alpha : k_0 \equiv \omega_k = \sqrt{|k|^2 + m^2}$$

By virtue of the previous expression it can be stressed:

- quantization process transforms the four vector energy-momentum in an operator by elevating the coefficients from complex numbers to creation and annihilation operator acting on Hilbert space;
- the Fourier expansion allows to disentangle the role of plane waves (which remain related to the description of a pure cinematic propagation) from the role of coefficients which becomes fundamental for expressing the dynamical relevant physical quantities.

2.2.4 The internal symmetries for marking constitutive differences between the systems

Internal symmetries add new dynamical variables to the kinematical description of the continuous systems.

Internal symmetries involves only inner elements of the Lagrangian and, by virtue of the Noether theorem, they provide new conserved quantities characterizing the particular object at issue.

Such symmetries concern both global invariance and local invariance. The local one is the link for moving toward interaction and it is the object of the next section.

Among the global invariances here we focus on one type, the so call *U(1)* global invariance, so as to illustrate an example of the role of the internal global symmetries in the description of the continuous systems.

For example, the Lagrangian for the complex KG field

$$L = \frac{1}{2}\partial_\alpha \phi \partial^\alpha \phi^* - \frac{1}{2} m^2 \phi\phi^* \tag{18}$$

is invariant under the following transformations

$$\begin{cases} \phi(x) \rightarrow \phi'(x) = e^{i\varepsilon}\phi(x) \\ \phi^*(x) \rightarrow \phi^{*\prime}(x) = e^{-i\varepsilon}\phi^*(x) \end{cases} \tag{19}$$

Being a one-parameter symmetry, a single quantity is conserved

$$Q = -i\int d^3x(\phi d_t\phi^* - \phi^* d_t\phi) \tag{20}$$

The conservation of this quantity characterizes the complex KG Lagrangian with respect to the real case where this invariance does not hold (Akhiezer, Berestenskj, 1962, p.117). The *U(1)* internal symmetry provides hence a new degree of freedom that paves the way to the introduction of a new variable for the dynamical description of the system on top of energy, momentum and angular momentum.

The interpretation of the new degree of freedom can be related to the electric charge.

In Section 2.4.2, the interpretation of the expression (20) in terms of electric charge of a classical continuous system will be widely discussed and, in section 2.4.3, it will be shown to what extent it is becomes fundamental for distinguishing particle and antiparticle in the quantum case.

2.2.5 The jump from global to local gauge transformations for making different systems feel each other

The prime example of an intrinsically gauge invariant theory is the theory of the electromagnetic field. It is well-known from the classical theory that Maxwell's equations can be stated in terms of the vector potential $\boldsymbol{A}$ and the scalar potential φ or in terms of the 4-vector potential $A^{\mu} = (\varphi, \boldsymbol{A})$. The link to the electric field $\boldsymbol{E}(\boldsymbol{x},t)$ and the magnetic field $\boldsymbol{B}(\boldsymbol{x},t)$ is given by

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \tag{21}$$

$$\boldsymbol{E} = -(\partial \boldsymbol{A}/\partial t) - \nabla \varphi \tag{22}$$

or covariantly

$$F^{\mu\nu} = \partial^{\mu} \ A^{\mu} - \partial^{\nu} \ A^{\mu} \tag{23}$$

where $F^{\mu\nu}$ is the electromagnetic field tensor. The important point in the present context is that given the identification (21) – (22) or (23), there remains a certain flexibility or freedom in the choice of **A** and φ, or A^{μ}. In order to see that, consider the so-called *gauge transformations*

$$\boldsymbol{A} \rightarrow \boldsymbol{A} - \nabla \psi \tag{24}$$

$$\varphi \rightarrow \varphi + \partial\chi/\partial t \tag{25}$$

or covariantly

$$A^{\mu} \rightarrow A^{\mu} + \partial^{\mu}\chi \tag{26}$$

where χ is a scalar function (of space and time or of space-time) which can be chosen arbitrarily. Inserting the transformed potential(s) into equation(s) (21) – (22) or (23), one can see that the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$, or covariantly the electromagnetic field tensor $F^{\mu\nu}$, are not effected by a gauge transformation of the potential(s). Since only the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$, and quantities constructed from them, are observable, whereas the vector potential itself is not, nothing physical seems to be changed by a gauge transformation because it leaves $\boldsymbol{E}$ and $\boldsymbol{B}$ unaltered. Note that gauge invariance is a kind of symmetry that does not come about by space-time transformations.

The *U(1)* global invariance discussed above in the case of the complex KG Lagrangian provides a more general form of gauge transformation

$$\varphi \rightarrow e^{-i\Lambda}\varphi \tag{27}$$

$$\varphi^{*} \rightarrow e^{i\Lambda}\varphi^{*} \tag{28}$$

where Λ is an arbitrary real constant.

Whenever the constant Λ is replaced by a function of the spacetime the global gauge transformation turns into a local one

$$\varphi(x) \rightarrow e^{-i\alpha(x)}\varphi(x) \tag{29}$$

The requirement of invariance under a local gauge transformation is essential for finding the equations describing fundamental interaction. As a qualitative example, starting from the Lagrangian for a free electron, the requirement that the Lagrangian itself should be locally invariant under U(1) transformation can only be fulfilled by introducing additional terms. The form of these terms is determined by the symmetry requirement, which results in the introduction of the electromagnetic field. In a sense, the electromagnetic field is a consequence of the local symmetry of the Lagrangian for the electron.

The explicit formal treatment for the introduction of the electromagnetic interaction in the KG Lagrangian by means of U(1) local invariance both in classical and in quantum description will be developed in Sections 2.4.2 and 2.4.3.

The procedure can be generalized to more complex transformations (for example referring to mixing the components of field operators) and new interactions. By requiring local gauge invariance additional fields can be introduced. These additional fields describe the interaction between the original fields. The gauge principle provides a general schema for introducing interaction by constructing gauge field theories. To this end one starts with a Lagrangian for a matter field and derives the interaction by introducing exactly those fields that make the Lagrangian invariant under a relevant local gauge transformation. It seems that all fundamental forces, except gravitation, can be described by such local gauge field theories.

Gauge symmetry plays a crucial role in determining the dynamics of the theory since the nature of gauge transformation determines the possible interaction. The structure of these transformations are characterized by special mathematical groups: *U(1)* for QED, *SU(2)* ⊗ *U(1)* for electroweak interaction, *SU(3)* for strong interaction. The relations between these groups are exploited in programs for the unification of the fundamental types of interaction.

According to the previous discussion, gauge invariance constitutes a paradigmatic case highlighting how a rich mathematical structure can help in the construction of theories.

2.2.6 Remarks to section 2.2

In this section we described the so called "skeleton of continuum" by outlining the *minimal conceptual structure* of continuum which remains invariant in the transition between classical and contemporary physics.

The construction of the skeleton and the individuation of its *basic elements* have the relevant feature to disentangle the different formal languages involved and to stress where and how the quantum formalism comes in to make the quantum field different from the classical ones.

This study allowed to prune away distracting and redundant elements and to focus the attention on that basic structure on which the discrete structure will be uploaded in next section in order to introduce interaction.

2.3 Discrete structure and II quantization language

In this section the analysis is developed focussing on "how" the object *changes* in the transition from a classical to a quantum field by searching for the breaking aspects. It will be shown in what sense a critical study on this point is promising for a deep understanding of the quantum interaction by:

- presenting some problems we see in the canonical procedure that are at risk to establish rather forced similarities between classical and quantum field;
- individuating an alternative promising approach for interpreting the nature of quantum field;
- analysing the *new synthesis* quantum field operates between continuum formalism useful in describing propagation and the discrete nature of quantum interaction processes.

2.3.1 Approaching Quantum Field Theory: Canonical procedure at risk of hiding basic crucial questions

Even though is it quite obvious that the concept of field has been invented for modelling interaction and QFT represents the contemporary theory of fundamental interactions, it is not obvious at all how to extract from the QFT textbooks what peculiarities such interaction model shows with respect to the classical one.

In the textbooks where the transition from the classical view of interaction to the quantum one is dealt with (see, for example, Mandle & Shaw, 1984, p.1), the problem is "simply" solved focussing on the electromagnetic field and replacing the numbers representing the coefficients of the Fourier expansion of the solutions of the d'Alembert equation with operators. Every other quantum field (for example the Klein-Gordon field for particles of mass *m* and zero spin) is constructed extending such a procedure by analogy: take the generic solution of a wave-equation (for example the Klein-Gordon equation) expressed as Fourier expansion on plane waves, focus on the coefficients of the expansion and elevate them from numbers to operators by defining their commutation rules. This procedure is usually called *canonical quantization*.

The procedure can be easily justified in terms of its "technical effectiveness": It follows maybe the shortest way to arrive to the core-problem of constructing the new theoretical entities; it allows an inner coherent formalism to be developed and experimental predictions to be made. Nevertheless, “what is hidden behind the receipt?” (Bergia, 2003). What conceptual short-cuts are unavoidably made? What crucial questions cannot be answered by looking at QFT from this perspective?

Steven Weinberg, in *The quantum theory of fields,* writes: “The traditional approach, since the first papers of Heisenberg and Pauli on general quantum field theory, has been to take the existence of fields for granted, relying for justification on our experience with electromagnetism, and ‘quantize’ them – that is, apply to various simple field theories the rules of canonical quantization or path integration .[…] This is certainly a way of getting rapidly into the subject but it seems to me that it leaves the reflective reader with too many unanswered questions. Why should we believe in the rules of canonical quantization or path integration? Why should we adopt the simple field equations and Lagrangians that are found in the literature? For that matter, why have fields at all? It does not seem satisfactory to me to appeal to experience; […]” (Weinberg, 1995, p. **xi.**).

We share with Weinberg the idea that canonical quantization leaves several questions open. We don’t enter deeply in the questions claimed by Weinberg but we focus only on a point that we consider a preliminary question.

We think that the first problems arising from the canonical quantization concern the question “what does “field operator” mean?”

2.3.2 What does “field operator” mean? The canonical and the axiomatic approach compared

The following discussion is the result of an analysis carried out with the aim of searching the minimal features which characterize *every kind of field*. For this purpose we chose to focus our reflections on the emblematic case of non-relativistic scalar field: this field is the simplest one and, at the same time, promising to develop general considerations.

Canonical quantization for non-relativistic scalar field moves through a step like the following one to derive the formal expression of a quantum field:

$$\phi\,(x) = \sum_k u_k^*(x)a_k \qquad \longrightarrow \qquad \phi^+(x) = \sum_k u_k^*(x)a_k^+ \tag{30}$$

where commutation rules for a_k, a_k^+ at a fixed time are defined and where $\{u_k(x)\}_{k=1}^{\infty}$ has been chosen as a generic basis of the space in which the field is defined.

The expression on the right side is also indicated as a part (the creation one) of the “field operator”.

This alternative short name for the quantum field is interesting since it lets one wondering what features of an operator and what features of a (classical) field the quantum field keeps.

The first immediate reaction in front of the name could be that of thinking of the field operator as the sum of the properties of both the entities (operator and field) and/or a sort of combination suggesting an image of quantum field as something spreading operators (a, a^+) over space and time.

The following discussion can be seen as an attempt to problematize such a simplistic view by:

- showing why field operator does not take up *all* the features either of an operator or of a classical field;
- individuating an alternative approach for analysing the *new synthesis* quantum field operates between continuum formalism and the discrete nature of quantum interaction processes.

What the field operator is NOT

Searching for its "quantum features" (due to the presence of a creation operator in its expression) one could wonder whether the field operator can be simply considered an operator acting on Hilbert space.

The strongest argument against the idea of field operator not really being an operator is the result of making it act on one particular state of Hilbert space (the ground state): the resulting vector $\phi^+(x)|0\rangle$ is indeed not yet a well-defined Hilbert state, having an infinite norm.

Moreover: *"The quantum field ϕ at a point cannot be an honest observable. Physically this appears evident because a measurement at a point would necessitate infinite energy. The mathematical counterpart is that $\phi(x)$ is not really an operator in H_F "* (Haag, 1996).

On the other hand, searching for its "classical features" (suggested by the "sense of continuity" highlighted by the canonical quantization) one could wonder whether field operator keeps the important feature of a classical field, like the electromagnetic or Klein-Gordon ones, of being represented (made visible) by the "profile" of a continuous function.

The classical field representation is strictly related to its being the result of the superposition of plane waves, each of them taken with its "weight" factor, that is the coefficient representing the amplitude of each plane wave. When we "elevate" the coefficients from complex numbers to operators, it becomes trivially impossible to interpret them as amplitudes of normal modes. As a consequence, the possibility of any visualization is lost, as well as any sense to search for a space and time profile.

Teller (Teller, 1995, p.95) stresses that the field interpretation of QFT is inappropriate since the alleged fields in QFT cannot be interpreted as physical fields with definite values of some sort which are assigned to space-time points, like in the case of the classical electromagnetic field. Rather, quantum fields are 'determinables', being described by mappings from space-time points to *operators*. Operators are mathematical entities that are defined by *how they act on something*. They do not represent definite values of quantities but they specify what can be measured.

If a quantum field cannot be thought as an operator acting directly on Hilbert space, or even a function entirely performable in ordinary space and time, what is what we call "operator field"? What formal structure does it keep together and how? And, still more important, what kind of conceptual core-questions can be explicitly addressed by exploiting its formal inner structure?

What follows is meant to enter Teller's position in more detail. In particular, the point we will discuss here is that the canonical procedure hides, under a pure formal passage called II quantization, a deep change in the way of looking at the concept of field: from an object that "lives" and propagates in an ordinary space to a theoretical construction which builds physical observables acting as a sophisticated interface between ordinary space and Hilbert space.

In order to develop this point in the following sections we will present some ideas taken from the so called "axiomatic theory of fields" (Haag, 1996; Jost, 1965; Streater & Wightman, 1964).

2.3.3 The axiomatic approach: its problems and relevance for the philosophy of physics

The tendency of reformulating grown theories in an axiomatic manner can be traced back in the fifties, as partly motivated by the aim of removing *ad hoc* features which were problematic in the standard formulation: one of them was exactly "the idea that QFT is about field values at points of space" (Kuhlman, 2009, p.19).

A very prominent early attempt to axiomatise QFT is Arthur Wightman's field axiomatics; arguably the most successful attempt to reformulate QFT in an axiomatic manner is the Algebraic Quantum Filed Theory Reformulation (AQFT) developed by Rudolf Haag and quickly advanced in collaboration with Huziro Araki and Daniel Kastler (Haag and Kastler, 1964).

According to the algebraic point of view, *algebras* of observables - rather than observables themselves - had to be taken as the basic entities in the mathematical description of quantum physics.

Physically, the elements of an algebra *A(O)* are seen as representing operations that can be performed in the region O that is associated with the algebra.

The physical justification for this approach consists in the recognition that the experimental data for QFT are exclusively space-time localization properties of microobjects from which other properties are inferred.

The main current problems addressable to the axiomatic concerns the fact that, as a reformulation of QFT, AQFT is expected to reproduce the main phenomena of QFT in particular properties which are characteristic of it being a field theory, like the existence of antiparticles, internal quantum numbers, the relation of spin and statistics, etc.

That this aim could not be achieved within AQFT on a purely axiomatic basis is partly due to the fact that the connection between the respective key concepts of AQFT and QFT, i.e., observables and quantum fields, is nowadays not sufficiently clear.

It turned out that the main link between the theory of local observables and the quantum fields of standard QFT is the notion of *superselection* (superselection rules are certain restrictions on the set of all observables and allow for classification schemes in terms of permanent or essential properties).

AQFT came into the focus of the philosophy of physics community only since the second half of the eighties.

Some of the most fruitful discussions were stimulated by re-examinations of physical theorems from the sixties and seventies, in particular the Reeh-Schlieder theorem discussed in Chapter 1. Central issues in the philosophical debate about AQFT are questions about locality, localization and causality.

The development of the axiomatic approach is still nowadays very internal to theoretical physics problems and few, if any, studies concerning the analysis of its foundations have been carried out.

So our aim of extracting educational implications from this approach is not only very ambitious but, at this moment, also pretty risky: the state of our study is a preliminary exploration aimed at evaluating if such a perspective has the potential to address the questions we are interested in.

2.3.4 Field operators in the axiomatic theory

The axiomatic approach is usually presented by referring to relativistic fields. In these cases, one of the first evident elements of interest of the approach is the way in which it explicitly addresses the problem of how the quantum field is linked to Minkowski spacetime and Hilbert space.

Quantum field is indeed introduced through the notion of "operator valued distribution" over spacetime and formalized as a theoretical construction interfacing spacetime and Hilbert space. In other words, within such an approach, quantum field is mathematically and rigorously formulated as a distribution which has to be averaged ("smear out") with smooth functions (called "test functions") on spacetime.

Let us try to give an idea of this point by referring to our case of non-relativistic field. Quantum field is a mathematical construction more sophisticated than a simple function defined on space and time (i.e. anything that is directly calculated on space and time points) or than something acting directly on Hilbert space as a set of operators. "Calculating" a field [$\phi(x)$] in this approach means "averaging" it on functions [*f(x)*], where the meaning of "averaging" lies in the expression reported in (1). The result [$\phi(f)$] of averaging the field over the test functions is what acts on Hilbert space.

$$\phi(f) = \int \phi(x) f(x) d^3x \quad (31)$$
[6]

At least four consequences of this definition of field require, in our opinion, some attention:

- field finds its meaning as a formal mechanism of interfacing ordinary space (or Minkowski spacetime in the relativistic cases) and Hilbert space, showing to what extent simplistic relations between the two spaces are problematic;
- as the result of a set of formal steps, a well defined operator in Hilbert space $\phi(f)$ is related not to a space(-time) event but to an extended space(-time) region, the "support" of the test function (*"Specifically, an operator ϕ(f) (smeared out with a test function f) represents a physical operation performed*

[6] Usually, in the textbooks (see Haag, 1996), this expression involves an integration over Minkowski spacetime, as well as a quantum field with a more complicated structure [$\phi(x) = \phi^-(x) + \phi^+(x)$]. For our goals and for the case we are considering (non-relativistic field) a spatial integration and a field with only the creation part ($\phi(x) = \phi^+(x)$) are enough.

on the system within the space – time region given by the support of f", Haag 1996, p. 84);
- since quantum measurements are expressed by operators, it is possible, for a given space(-time) region, to express them in term of quantum fields the observables that can be measured acting on that region ("Physical interpretation of the quantum fields will not be primarily attached to particles but to local operations", Haag p.84);
- instead of a "punctual" view (the idea of linking operators to space-time points), this approach strongly emphasizes the need of looking "globally" in space and time (being the integral made on the whole space-time or on extended regions).

As stressed by Haag (Haag, 1996): *"fields are only an instrument in order to "coordinatize" observables, more precisely, in order to coordinatize the sets of observables with respect to different finite space – time regions (the so - called net of local algebra in AQFT)".*

The principle of locality

As reported by Standford Encyclopedia of Philosophy (Kuhlmann, 2009): "Physically the most important notion of AQFT is the principle of *locality* which has an external as well as an internal aspect:
- The **external aspect** is the fact that AQFT considers only observables connected with finite regions of space-time and not global observables like the total charge or the total energy momentum vector which refer to infinite space-time regions. This approach was motivated by the operational view that QFT is a statistical theory about local measurement outcomes with all the experimental information coming from measurements in finite space-time regions. Accordingly everything is expressed in terms of *local algebras* of observables.
- The **internal aspect** of locality is that there is a constraint on the observables of such local algebras: All observables of a local algebra connected with a space-time region O are required to commute with all observables of another algebra which is associated with a space-time region O′ that is space-like separated from O. This principle of (Einstein) *causality* is the main relativistic ingredient of AQFT."

Going back to the notion of quantum field, the statement quoted above ("Specifically, an operator $\phi(f)$ (smeared out with a test function f) represents a physical operation performed on the system within the space – time region given by the support of f (Haag, 1996, p. 84)") implies that the implementation of the internal aspect of the locality principle lead to require that "operations made in supports disconnected in space must not influence with each other".

Formally, the requirement is formalized by the property that, observable expressed by (31) and localized in regions space-like separated, must commute.

At this point a distinction between the non-relativistic and the relativistic case has to be made:

i) in the non-relativistic case (i.e. where the integration is made on the spatial variables) the condition is satisfied by fields which have the following structure)

$$\phi(x)=\phi^{+}(x)=\sum_{k}u_k^{*}(x)a_k^{+} \tag{32}$$

In the light of such a tool (the non-relativistic quantum field) it is possible to rewrite the whole ordinary non-relativistic quantum mechanics in a II quantization language, i.e. by means of non-relativistic quantum field and test function.

ii) in the relativistic case (i.e. where the integration is made on the space - time variables) a local field is obtained by summing up the distributions built with creation operator (*$\phi^{+}(x)$*) and annihilation operators (*$\phi^{-}(x)$*)

$$\phi(x)=\phi^{+}(x)+\phi^{-}(x)=\sum_{k}u_k^{*}(x)a_k^{+}+\sum_{k}u_k(x)a_k \tag{33}$$

In this case

$$\left[\int\phi_A(x)f(x)d^4x,\int\phi_B(y)g(y)d^4y\right]_{\mp}=0 \tag{34}$$

where the supports of *f* and *g* are separated in space.
As stressed by Landsman (Landsman, 1996): The causal structure in special relativity is reflected by the property that two observables which are localized in regions that are space-like separated must commute ("Einstein causality").
It is worth noting that the expression, just found in the general case of relativistic quantum fields, represents the structure of the relativistic field par excellence when it has been quantized, i.e. the quantum electromagnetic field:

$$\phi(x)=\sum_{k}\frac{1}{\sqrt{2\omega_k}}a_k^{+}e^{i(kx-\omega_k t)}+\sum_{k}\frac{1}{\sqrt{2\omega_k}}a_k e^{i(kx+\omega_k t)} \tag{35}$$

Landau (Landau, Lifsits, 1978), introducing the quantization of potential vector of electromagnetic field, remarks that the treatment is identical to what is found in the non-relativistic formalism for II quantization: The only fundamental difference is that, in the case of electromagnetic field, operator of the two types has to be taken into account.

In the light of such considerations, Haag stresses the fundamental role for the quantum field for implementing the principle of locality.
In *Local Quantum Physics*, he writes (Haag, 1996, pp. 45- 46):

The apparent miracle that canonical quantization of a free, scalar field leads to Fock space and an interpretation of states in terms of particle configurations has been seen as a manifestation of the wave – particle duality lying at the roots of quantum mechanics. The generalization of this observation to a field – particle duality has dominated thinking in quantum theory for decades and has been heuristically useful in the development of elementary particle theory [...]. Yet the belief in a field - particle duality as a general principle, the idea that to each particle there is a corresponding field and to each field a corresponding particle has also been misleading and served to veil essential aspects. The role of field is to implement the principle of locality.

By stressing the focus on the specific role addressed to the field, Haag, as reported by Fredenhagen, provides a particular definition of QFT stating that: *"Quantum Field Theory is quantum theory together with the locality principle"* (Fredenhagen, 2009)

In the next section we will try to argue why the axiomatic approach seems to have some potential in interpreting relevant features of quantum interaction.

2.3.5 Quantum field and the "new" interaction model

Quantum field as tool for bridging first and second quantization formal structure

The interaction process is formalized in QFT in terms of discrete processes of creation and destruction of quantum systems/states. Within the axiomatic approach the problem can be formalized by taking expressions like the (31) as a tool for rewriting the operators in II quantization language (as combinations of creation/destruction operators in the Fock space).

Just to give an idea, let us consider the expression of the potential energy concerning particles interacting two by two as used in classical theory and I quantization and let us translate it into a II quantization language (we recall we are not dealing with relativistic cases). The main two steps of such a translation are reported in the following expression:

$$\hat{V} = \sum_{i<j} \hat{V}^{(2)}(x_i, x_j) = ... = \frac{1}{2} \int dx \int dy V(x,y) \phi^+(x) \phi^+(y) \phi(x) \phi(y) = \frac{1}{2} \sum_{k_\alpha k_\beta k_\gamma k_\delta} \langle \alpha\beta | V | \gamma\delta \rangle a^+_{k_\alpha} a^+_{k_\beta} a_{k_\gamma} a_{k_\delta} \quad (36)$$

with $$\langle \alpha\beta | V | \gamma\delta \rangle = \int d^3x d^3y u^*_{k_\alpha}(x) u^*_{k_\beta}(x) u_{k_\gamma}(x) u_{k_\delta}(x) V(x,y) \quad (37)$$

- first step:

$$\hat{V} = \sum_{i<j} \hat{V}(x_i, x_j) = ... = \frac{1}{2} \int dx \int dy V(x,y) \phi^+(x) \phi^+(y) \phi(x) \phi(y) \quad (38)$$

the quantum mechanics potential (I quantization language), by intermediate steps, is expressed in terms of test functions and fields. In this first step the fields are averaged on test functions $V(x,y)$, a generic two variables function through which we describe particles "feel" each other.

- second step:

$$\int dx \int dy V(x,y) \phi^+(x) \phi^+(y) \phi(x) \phi(y) = \sum_{k_\alpha k_\beta k_\gamma k_\delta} \langle \alpha\beta | V | \gamma\delta \rangle a^+_{k_\alpha} a^+_{k_\beta} a_{k_\gamma} a_{k_\delta} \quad (39)$$

the potential $V(x,y)$ is linked to a series of probabilistic processes expressed in terms of creation ad destruction of vector states, in particular: destruction of two particles of

k_γ, k_δ impulses, and creation of two particles of k_α, k_β impulses. The coefficients $\langle \alpha\beta | V | \gamma\delta \rangle$ are related to the probability of the processes.

We remark that the function $V(x, y)$ is a generic function of two coordinates to which II quantization language (i.e. expression by the quantum fields) links a series of quantum processes expressed in probabilistic terms.

Special conditions we want to impose to the processes (for example the impulse conservation) are reflected in the formalism as a restriction of the potential functions class (for example to the class of the functions invariant by translation $V = V(x - y)$). In this specific case, expression (36) becomes:

$$\hat{V} = \sum_{i<j} \hat{V}^{(2)}(x_i - x_j) = ... = \frac{1}{2}\int dx \int dy V(x-y)\phi^+(x)\phi^+(y)\phi(x)\phi(y) = \frac{1}{2}\sum_{kk'} \tilde{V}(q) a^+_{k+q} a^+_{k'-q} a_{k'} a_k \tag{40}$$

$$\langle \alpha\beta | V | \gamma\delta \rangle = \tilde{V}(q) = \left[\frac{1}{V} \int_V d^3x' V(x') e^{-ix'q} \right] \tag{41}$$

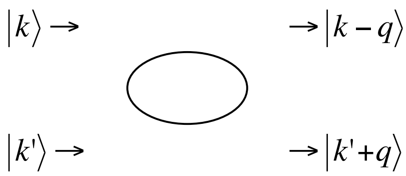
The potential $V = V(x - y)$ is now linked to a series of quantum processes in which impulse is conserved.

When the distributional nature of the field is taken into account, the quantum field plays two roles that we consider particularly interesting:

- the formal role of connecting different abstract spaces and bridging different languages and formal structures;
- the conceptual role of bridging the different models of interaction, the classical one (represented by a continuous function on space, the classical image of potential) and the quantum one (expressed as discrete processes of creation/destruction of systems/states).

Moving the attention from the role of quantum field to the entire process, another promising feature can be seen, strictly related to the space extension of the test function's support: the feature described below that seems to create a convincing access to understanding quantum interaction.

The following picture, frequently met in QFT textbooks, is nothing but a graphical representation of what formalism allows us to say: interaction takes place in an extended space region (the support of test function *V(x, y),* the region inside which the particles begin to feel each other) where a transition between two set of quantum numbers happens and where no experimental access is allowed. Experience (state preparation and measurements) can be only codified in the probabilistic terms of a transition from a set of quantum numbers to another.



Coherently with quantum description, the process (the action of the operators on ground state) is modelled as the transition between two set of quantum numbers.

Coherently with the distributional nature of the formalism the process is related to an extended region (the support of test function *V(x,y)*).

Due to the interacting term in the general Hamiltonian $H = H_0 + H_{int}$ and, consequently, to the not linear behaviour of the interacting field inside the region, measurements are done on the asymptotic states which are "sufficiently distant" from the interaction region to be considered free. The scattering process is described by Dyson as follows:

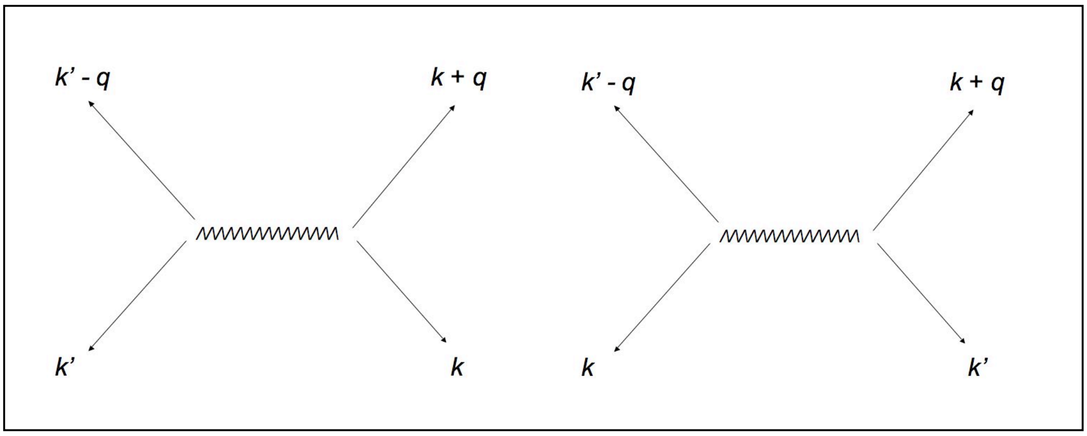
> *"The free particles which are specified by state A in the remote past, converge and interact, and other free particles emerge or are created in the interaction and finally constitute the state B in the remote future. We wish to calculate the matrix element M for this process, without studying the equations of motion or investigating the behavior of the system at intermediate times while the interaction is in progress."* (Dyson, 2006, p.87)

Dyson underlines that, also when self-field effects are neglected, the operator U (where $M = (\phi^*_B U\phi_A)$) can be written down as a perturbation expansion.

In particular, in the case considered above, the elements of the *S Matrix* are given by the coefficients $\langle\alpha\beta|V|\gamma\delta\rangle$ as describing the probability of the processes.
We introduce Feynman diagrams focussing on the first order of the expansion and using Wick theorem and the expression for $\hat{V}$ reported in formula (40):

$$\langle\psi_{free}|a^{+}_{k+q}a^{+}_{k'-q}a_{k'}a_{k}|\psi_{free}\rangle = \langle\psi_{free}|a^{+}_{k+q}a_{k}a^{+}_{k'-q}a_{k'}|\psi_{free}\rangle \pm \langle\psi_{free}|a^{+}_{k+q}a_{k'}a^{+}_{k'-q}a_{k}|\psi_{free}\rangle \qquad (42)$$

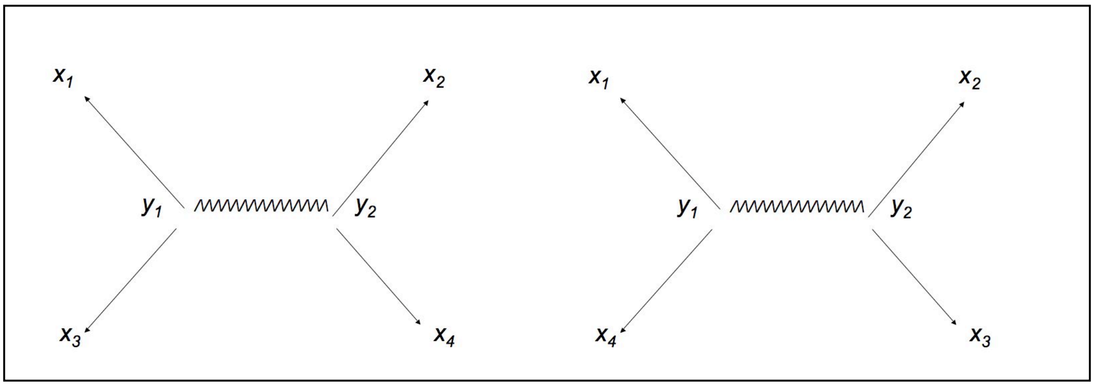
Feynman rules lead to the following diagrams:



where the time arrow goes from the bottom to the top.

If the contractions prescribed by the Wick theorem are considered not on the creation/annihilation operators but on the whole fields, Feynman rules lead to the diagrams on the coordinate space

$$\langle\psi_{free}|V(x_1,x_2)\phi^+(x_1)\phi^+(x_2)\phi(x_3)\phi(x_4)|\psi_{free}\rangle = \langle\psi_{free}|V(x_1,x_2)\phi^+(x_1)\phi(x_4)\phi^+(x_2)\phi(x_3)|\psi_{free}\rangle \pm \langle\psi_{free}|V(x_1,x_2)\phi^+(x_1)\phi(x_3)\phi^+(x_2)\phi(x_4)|\psi_{free}\rangle \quad (43)$$



where the time arrow goes from the bottom to the top.

A brief look to the formalism would highlight as the interpretation as spacetime diagrams of such objects has to be considered with extreme care (for example: "2 particles annihilate in a virtual one and then two other particle appear"): in the formalism connected with such diagrams there is an integration on all the y coordinates and so the points y_1 and y_2 are not defined both in space and in time.

These diagrams are graphical representations of the only things that is known in this context: The probability expressed by elements of the *S Matrix*.

2.3.6 Some elements marking differences and analogies between Quantum Field Theory and Quantum Mechanics

A closer analogy between Quantum Mechanics and Quantum Field Theory recognizable from the previous treatment is that the crucial step towards *quantum* field theory is in some respects analogous to the corresponding quantization in quantum mechanics by imposing the commutation relations.

In both cases, QM and QFT, requiring that the canonical variables satisfy certain commutation relations implies that the basic quantities become operator valued. From a physical point of view this shift implies a restriction of possible measurement values for physical quantities some (but not all) of which can have their values only in discrete steps now.

Nevertheless, in spite of the close analogies between the quantization process in QM and in QF important differences cannot be neglected:

i) Whereas the commutation relations in QM refer to a quantum object with three degrees of freedom, so one has a finite set of equations, the commutation relations in QFT do in fact comprise an infinite number of equations, namely for each 4-tuple ($\mathbf{x}$,t) there is a new set of commutation relations and there is, of course, a continuous set of space-time points ($\mathbf{x}$,t). This infinite number of degrees of freedom embodies the field character of quantum *field* theory.

We underline that the canonical commutation relations reported above for the coefficients can be directly rewritten in terms of field ϕ and the corresponding conjugate field π

$$[\phi(\mathbf{x},t), \pi(\mathbf{y},t)] = \mathrm{i}\delta^3(\mathbf{x} - \mathbf{y}) \tag{44}$$

$$[\phi(\mathbf{x},t), \phi(\mathbf{y},t)] = [\pi(\mathbf{x},t), \pi(\mathbf{y},t)] = 0 \tag{45}$$

which are equal-time commutation relations, i.e., the commutators always refer to fields at the same time. It is not obvious that the equal-time commutation relations are Lorentz invariant but one can formulate a manifestly covariant form of the canonical commutation relations. If the field to be quantized is not a bosonic field, like the Klein-Gordon field or the electromagnetic field, but a fermionic field, like the Dirac field for electrons one has to use anticommutation relations.

ii) the operator valued field $\phi(\mathbf{x},t)$ in QFT is *not* analogous to the wave function $\psi(\mathbf{x},t)$ in QM, i.e., the quantum mechanical state in its position representation. Although in the development of QFT there is a continuity from the wave function, i.e., the quantum mechanical state in its position representation, to the field in QFT, it would be a misconception to understand these two quantities as analogues. Here, the ontologically relevant formal setting has changed in the transition from QM to QFT. While the wave function in QM is acted upon by observables, i.e., by operators, it is the (operator valued) field in QFT which itself acts on the space of states, i.e., on the states which are associated with the quantum field. In a certain sense one can say that the single particle wave functions have been transformed, via their reinterpretation as operator valued quantum fields, into observables. This step is sometimes called 'second quantization' because the single particle wave equations in relativistic QM already came about by a quantization procedure, e.g., in the case of the Klein-Gordon equation by replacing position and momentum by the corresponding quantum mechanical operators. Afterwards the solutions to these single particle wave equations, which are states in relativistic QM, are considered as classical fields which can be subjected to the canonical quantization procedure of QFT. The term 'second quantization' has often been criticized partly because it blurs the important fact that the single particle wave function ϕ in relativistic QM and the operator valued quantum field ϕ are fundamentally different kinds of entities despite their connection in the context of discovery. Landau (Landau and Lifsits, 1978) referring to the plane waves appearing in the Fourier expansion of the quantized electromagnetic fields

$$A = \sum_{k\alpha} c_{k\alpha} A_{k\alpha} + c^{+}_{k\alpha} A^{*}_{k\alpha} \tag{46}$$

$$A_{k\alpha} = \sqrt{4\pi}\,\frac{e^{(\alpha)}}{\sqrt{2\omega}}\,e^{-i(\omega t - kr)} \tag{47}$$

remarks that such a plane wave can be considered wave functions of photons which have certain momentum k and certain polarizations $e^{(\alpha)}$.

Nevertheless they cannot be considered as the probability amplitude of spatial localization of the photon so they lose the fundamental meaning of the wave function in non-relativistic quantum mechanics.
The argument is strengthened by the consideration that the precision of measurement process (if not in the rest system)

$$\Delta q \approx \frac{\hbar c}{\varepsilon} \tag{48}$$

coincides with de Broglie wave length in the case of photons ($\varepsilon \sim cp$).
Landau stresses that the only case in which talking about "coordinates of photon" gain sense occurs when the quantity involved are great with respect to the wave length (classical limit of the geometrical optics).
The probabilistic meaning standing still in the wave function whenever considering momentum and polarisation is highlighted by the expression of the coefficient in front of the previous expansion

$$A_{k\alpha}(k',\alpha') = \sqrt{4\pi}\,\frac{e^{(\alpha)}}{\sqrt{2\omega}}\delta_{k'k}\delta_{\alpha'\alpha} \tag{49}$$

"In relation to the measurement of the momentum such a wave function gains a more fundamental physical meaning: it gives the probability of calculating the probabilities of momentum and polarization of the photon which is set in the state considered" (Landau and Lifsits, 1978).

2.3.7 Remarks to Section 2.3

In this section we reported the results of a study carried out to exploit the potential of an approach to QFT alternative to the canonical one.

After sketching some problems arising from the canonical approach, we tried to illustrate why we consider the axiomatic approach promising for analysing the new synthesis quantum field operates between the formalism of continuum and the discrete nature of quantum interaction processes.

In particular, even though the axiomatic approach may appear more complicated than the canonical one, it seems to facilitate an authentic quantum view to look at fields and interaction, avoiding:

- short-circuiting conceptual problems related to the clash of different languages;
- providing quick answers somehow related to familiar ontologies.

The important roles assumed by the quantum field - when read in distributional terms - of bridging different models and different formal structures seem to confirm of the effectiveness of the approach for clarifying some important conceptual problems that will be reconsidered in Chapter 4.

2.4 A case study: The Klein-Gordon equation

A significant synthesis of the general ideas of the canonical procedure of quantization, already introduced in section 2.3.2 of this chapter is provided by the following claim of Mandle and Shaw (Mandle & Shaw, 1984):

> *From the quantization of the electromagnetic field one is naturally led to the quantization of any classical field, the quanta of the field being particles with well defined properties.*

We have already argued the idea that canonical quantization leaves several questions open as expressed by some statements of Weinberg and Haag reported in § 2.3.1 and § 2.3.4. Nevertheless we do not tackle all the questions raised by the two authors but we will reconsider now the procedure in the light of the following ones:

- What hinders such a formal procedure? The formal analogy is applied to quantize very different objects (for example the electromagnetic field and the relativistic wave function of Dirac). What can be discovered if one tries to go deep into the analogy and focus one's attention on the conceptual peculiarities of the different quantum fields?
- Up to where can/should the analogy be extended? The procedure is built on the electromagnetic case, where the classical meaning of the field is clear, and it works perfectly, from a formal point of view, with all the other fields considered in QFT. If one tries to analyze the analogy in detail, an example of questions that immediately arises is: What is the analogous of the classical electromagnetic field in the Klein-Gordon (KG) case? What is the KG classical field and how is it physically related to its quantum version?

The Klein-Gordon equation (KGE) represents an interesting case for addressing such questions because of the following reasons:

- its history involves the main difficulties physicists faced on the way to matching Quantum Mechanics with the theory of Special Relativity;
- it allows to pick up the fundamental and peculiar conceptual features of modelling objects and interactions within the QFT with a minimal formalism;
- its interpretation in the non-quantum version has never been analysed in a systematic way and the search for a whole frame where to situate the different cases offers several elements of interest.

KGE is extensively known in the context of particles physics (Relativistic Quantum Mechanics and QFT) because of the travailed steps of its history:

- the equation was firstly introduced in the twenties in an attempt of finding a wave function for a relativistic particle similar to the Schrödinger wave function for a non-relativistic particle (a complex function allowing the construction of a particle probability density);
- the equation was quickly abandoned by the very authors who found it because it did not allow an interpretation in the sense indicated above (it does not allow the construction of a probability density);
- later on – Pauli, Weisskopf, 1934 – the equation found a coherent framework within the QFT, in which the solution is interpreted not as a wave function but as a quantum field

The present section has a double aim, one more general and one more specific:

- To provide a detailed analysis of the cases that extend the applications of the KGE beyond the particle cases, widely addressed in the Relativistic Quantum Mechanics and QFT university textbooks.
- To solve the inconsistencies that seem to emerge in the attempts to provide a physical interpretation of the KG field.

The different cases of application of KGE have been analysed according to the different mathematical expressions of the solution: real function, complex function and quantum field (real and complex).

As it will be specified, some of these cases are examined in detail in the textbooks; for those this is only a report. Some other cases are just mentioned in the textbooks and for those the article provides original considerations stemmed from specific studies carried out in order to construct an overall coherent frame.

The cases here reported will not exhaust all the physical cases in which KGE works. Moreover they are considered not because of their applicative appeal but because they proved to be effective tools to enter the conceptual questions posed in present section.

2.4.1 The real case

KGE for addressing the problems of the continuous limit for a system of coupled pendulums and of the propagation of electromagnetic waves in the Earth ionosphere.

1. Continuous limit for a system of coupled pendulums

In the book *Berkeley Physics Course, Vol. 3, Waves* (Crawford, 1966) the KGE is mentioned as the continuous limit of the equation describing the motion of a system of coupled pendulums. It is derived following the same procedure used historically by d'Alembert for getting his famous equation starting from a system of infinite masses and springs:

- Calculus of the force acting on every mass due to the adjoining springs (Eq.50);
- Switch from a discrete system (masses and springs) to a continuous one (the string) by assuming that: *i)* the value of every single masses (*m*) as well as the distance between them (*a*) go to zero; *ii)* the ratio between the value of the masses and their distance keeps constant thus guarantees a linear density of matter (Eqs. 51).

The previous considerations can be formally expressed as follows:

$$m\frac{d^2 q_j}{dt^2} = -K\left(q_j - q_{j+1}\right) - K\left(q_j - q_{j-1}\right) \tag{50}$$

$$\begin{cases} m \to 0, a \to 0 \\ \dfrac{m}{a} = \rho_0 = const. \end{cases} \tag{51}$$

By means of the previous assumptions the d'Alembert equation is obtained

$$\frac{d^2q(t,z)}{dt^2} = v^2\frac{d^2q(t,z)}{dz^2} \tag{52}$$

$$v^2 = \frac{T_0}{\rho_0} \tag{53}$$

When a series of infinite coupled pendulums is considered, the restoring force on every mass is due not only to the springs but also to gravity and the situation turns as follows:

$$m\frac{d^2q_j}{dt^2} = -\frac{mg}{l}q_j - K(q_j - q_{j-1}) + K(q_{j+1} - q_j) \tag{54}$$

$$\begin{cases} m \to 0, a \to 0 \\ \dfrac{m}{a} = \rho_0 = const. \end{cases} \tag{55}$$

By means of the previous assumptions the KGE is obtained

$$\frac{d^2q(t,z)}{dt^2} = -\omega_0^2 q + v^2\frac{d^2q(t,z)}{dz^2} \tag{56}$$

$$v^2 = \frac{T_0}{\rho_0} \tag{57}$$

$$\omega_0^2 = \frac{g}{l} \tag{58}$$

As far as the dispersion relations corresponding to the equations found are concerned, the d'Alembert case leads to $\omega^2 = v^2k^2$, while the KG case to $\omega^2 = \omega_0^2 + v^2k^2$.

The starting physical situation of the coupled pendulums and the mathematical model obtained in the continuous limit suggest that the real solution of KGE is describing the displacement of an element of compressible fluid in a gravitational field.

2. Electromagnetic field in the ionosphere.

(a) Introduction to the problem

The propagation of electromagnetic waves in the Earth ionosphere is the other case mentioned in paragraph 2.4, example 6 of *Berkeley Physics Course, Vol. 3, Waves* (Crawford, 1966), as an application of KGE where the solution is a real function. Inside the ionosphere, the electromagnetic waves satisfy indeed the KG dispersion relation:

$$\omega^2(k) = \omega_p^2 + c^2k^2 \tag{59}$$

where ω_p^2 is called "plasma oscillation frequency".

Qualitatively the relation expresses the resonance effect between the electromagnetic field and the atoms of the ionosphere; such a resonance indeed causes a reduction of velocity as well as the dispersion of the wave packet as formalized by the dispersion relation.

How can the ionosphere be modelled so as to justify the KG dispersion relation?

Berkeley textbook's treatment stimulates questions like this but, due to the non-specialized character of the treatise, it does not provide the reader with elements for answering.

In order to address the previous questions at a level of specificity comparable to that of the coupled pendulums problem, the basic model of cold plasma has been taken into account and analysed in detail.

(b) The basic model of cold plasma

The ionosphere is an ensemble of several stratifications of the earth atmosphere which contains ionized air molecules; the system is neutral from a global point of view but not from the local one. Due to the movement of the charges inside the ionosphere, the study of the developing fields (electric or electromagnetic) becomes necessary. Treating the ionosphere as a plasma means modelling it as a system consisting in an electronic fluid moving on a uniform positive background produced by the ions that do not follow the oscillations of the fields because of their inertia and for this reason shall be not considered in the dynamical treatment.

Treating the ionosphere as a cold plasma means ignoring thermodynamic variables, for example pressure, and describing the fluid only in terms of the density of electric charge.

The study of the Earth ionosphere is of great interest for telecommunications engineering, astrophysics and structure of matter (Jackson, 1962; Mikhailovskii, 1974; Choudhuri, 1998)

Formally the electrons of charge e and mass m are described by a density function $n(x,t)$ and by a field of average velocity $\mathbf{v}(x,t)$.

The dynamic equations for the fluid are the continuity and Euler equations:

$$\begin{cases} \dfrac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) = 0 \\ \dfrac{\partial \vec{v}}{\partial t} + (\vec{\nabla} \cdot \vec{v})\vec{v} = \dfrac{e}{m}\left(\vec{E} + \dfrac{\vec{v}}{c} \times \vec{B}\right) \end{cases} \tag{60}$$

In order to investigate the field inside the ionosphere the Maxwell equations are written by taking the fluid itself as the field source and by considering the following expressions for the local electric density and for the current density:

$$\begin{cases} \rho_e = e(n - n_0) \\ \vec{J} = ne\vec{v} \end{cases} \tag{61}$$

$$\begin{cases} \vec{\nabla} \cdot \vec{E} = 4\pi e(n - n_0) \\ \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0 \\ \vec{\nabla} \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi en}{c}\vec{v} \end{cases} \tag{62}$$

The problem simplifies if small deviations from the static situation, i.e. $n = n_o$ and field null everywhere, are considered

Maxwell equations

$$\begin{cases} \vec{\nabla} \cdot \vec{E} = 4\pi en \\ \vec{\nabla} \cdot \vec{B} = 0 \\ \vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0 \\ \vec{\nabla} \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi en_0}{c}\vec{v} \end{cases} \tag{63}$$

Fluid equations

$$\begin{cases} \frac{\partial n}{\partial t} + n_0\vec{\nabla} \cdot \vec{v} = 0 \\ \frac{\partial \vec{v}}{\partial t} = \frac{e}{m}\vec{E} \end{cases} \tag{64}$$

The set of solutions for the fields is obtained firstly supposing that all the quantities (E, B, n, v) vary as $\exp\left[i\left(\vec{k} \cdot \vec{x} - \omega t\right)\right]$; this assumption further simplifies the equations as follows:

Maxwell

$$\begin{cases} \vec{k} \cdot \vec{E} = -i4\pi en \\ \vec{k} \cdot \vec{B} = 0 \\ \vec{k} \times \vec{E} = \dfrac{\omega}{c}\vec{B} \\ \vec{k} \cdot \vec{B} = -\dfrac{\omega}{c}\vec{E} - i\dfrac{4\pi en_0}{c}\vec{v} \end{cases} \tag{65}$$

Fluid

$$\begin{cases} n = \dfrac{\vec{k} \cdot \vec{v}}{\omega} n_0 \\ -i\omega\vec{v} = \dfrac{e}{m}\vec{E} \end{cases} \tag{66}$$

Maxwell's equations can be solved finding ***v*** in terms of ***k*** and ***E***; the second equation of the fluid and the divergence of ***E*** can be utilized to eliminate ***v*** and to obtain an expression in terms of ***k*** and ***E***:

$$\left(\omega^2 - \omega_p^2 - c^2k^2\right)\vec{E} + c^2\left(\vec{k} \cdot \vec{E}\right)\vec{k} = 0 \tag{67}$$

where

$$\omega_p^2 = \frac{4\pi n_o e^2}{m} \tag{68}$$

Decomposing the electric field vector in the parallel and orthogonal components to *k* ($\vec{E} = \vec{E}_{//} + \vec{E}_{\perp}$, $\vec{E}_{//} = \left(\dfrac{\vec{k} \cdot \vec{E}}{k^2}\right)\vec{k}$), two equations are obtained:

$$\begin{cases} \left(\omega^2 - \omega_p^2\right)\vec{E}_{//} = 0 \\ \left(\omega^2 - \omega_p^2 - c^2k^2\right)\vec{E}_{\perp} = 0 \end{cases} \tag{69}$$

The latter set of equations allows one to recognize that, in coherence with the assumptions of the cold plasma model, two types of perturbations take place inside the ionosphere:

- *Perturbations corresponding to the first equation* (fields satisfying an harmonic oscillator equation). They are named "electron plasma oscillations" or, in Russian literature "oscillations of Langmuir", who discovered them in the '20. They are oscillations of a pure electrostatic nature inasmuch as it is possible to show that for them the magnetic field is null.

- *Perturbations corresponding to the second equation* (field satisfying the KG equation). They are electromagnetic waves corresponding to two states of polarization and they are the waves to which the Berkeley textbook refers.

In absence of external fields the electrostatic and electromagnetic oscillations are not coupled.

The solution describing electronic density fluctuations is derived combining the equation of the fluid and the first Maxwell equation:

$$\frac{\partial^2 n}{\partial^2 t} + \left(\frac{4\pi e^2 n_0}{m}\right) n = 0 \tag{70}$$

Hence the fluctuation of the electronic density satisfies the same harmonic oscillator equation as the electrostatic fields obtained above (first one out of Eqs 69).

(c) Summary of the cold plasma problem.

The analysis of the basic model of cold plasma allows to describe the propagation of the electromagnetic waves in the ionosphere at a level of specificity comparable to that of the coupled pendulums and to conclude that:

- Inside the ionosphere the electronic fluid (modelled by the density of electric charge) presents harmonic oscillations of the density of electric charge itself (Eq. 70). To these oscillations corresponds an electrostatic oscillation of the electric fields (first one out of Eqs. 69);
- The harmonic oscillating charges in their accelerated motion generate an electromagnetic radiation propagating perpendicularly with respect to the electrostatic oscillation. The single components of the electromagnetic radiation propagate, inside the medium, according to the KGE (see second of the Eqs. 69).

3. Remarks for the real case

At least two observations:

- Although very different and far from each other, both cases describe a situation where a "pure" oscillation is forced by an external agent (gravity for the pendulums, resonance with the ionosphere for the electromagnetic waves). Formally, a forcing term appears both in the equation and in the relation of dispersion.
- The real solution can provide a visualizable interpretation as displacement of an element of fluid or as amplitude of a component of electromagnetic field.

2.4.2 The complex case

KGE equation for addressing the problem of finding a wave function for a relativistic particle and for constructing the local charged – density in the cold plasma problem.

1. Candidate for a relativistic wave function[7]

As already mentioned in Section II, the origin of this case can be traced back to the first historical attempts of building a Relativistic Quantum Mechanics taking the non-relativistic Quantum Mechanics as a model (Kragh, 1984).

The basic idea was that the equation for a free particle of energy E and momentum $\boldsymbol{p}$ has to be built so as to be solved by the following complex function (Bethe, de Hoffmann, Schweber, 1955; Schweber, 1961; Bjorken & S. D. Drell, 1964)

$$\phi(t,\vec{x}) = \exp\left[(i/\hbar)(\vec{p}\cdot\vec{x} - Et)\right] \tag{71}$$

The non-relativistic wave equation for a single particle with m_0 mass and no forces acting on it can be obtained from the relation

$$E = p^2/(2m_0) \tag{72}$$

by replacing

$$\begin{cases} E \to \hbar i \partial_t \\ p \to \hbar \frac{1}{i}\vec{\nabla} \end{cases} \tag{73}$$

where ∇ is the gradient operator.

This yields the Schrödinger equation for a free particle

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi \tag{74}$$

In the relativistic case the starting relation is

$$E = c\left(\vec{p}^2 + m_0^2 c^2\right)^{1/2} \tag{75}$$

Running again the previous scheme, the lowest order partial differential equation consistent with the relativistic relation is the KGE (Velo, 1972)

$$\left(\partial_0^2 - \nabla^2\right)\psi + m^2\psi = 0 \tag{76}$$

---

[7] The present section points out the reasons why the complex function, solution of the KGE cannot be interpreted as a relativistic wave function. In this sense it cannot be strictly considered an “application”. Nevertheless the importance of this case from an historical and methodological point of view (Kragh, 1984) couldn’t allow the omission of this treatment in a systematic study of the equation.

where ψ is now a function of $\left(x^0,\vec{x}\right)=x$, $x^0=ct$ and $m=m_0c/\hbar$.

The result is not immediate. The application of the previous procedure to the relativistic relation between energy and momentum leads primarily to the relativistic Schrödinger equation

$$i\frac{\hbar}{c}\partial_t\psi=\left(-\hbar^2\vec{\nabla}^2+m^2c^2\right)^{1/2}\psi \tag{77}$$

Due to the square root expression and especially to the implied superluminal propagation of the signal, this equation was rapidly abandoned in favour of the KG one, obtained starting from the square of the relativistic relation.

If Ψ satisfies the Schrödinger or KG equations, a continuity equation holds:

$$\frac{\partial\varphi}{\partial t}+\vec{\nabla}\cdot\vec{j}=0 \tag{78}$$

Because of Noether's theorem this relation is a consequence of the invariance of the two Lagrangians under phase transformation of the wave function

$$\phi(x)\rightarrow\exp(i\alpha)\phi(x) \tag{79}$$

The two continuity equations are however different. In the Schrödinger case the continuity equation is

$$\partial_t|\psi|^2+\frac{1}{2m}\vec{\nabla}\left(\overline{\psi}\vec{\nabla}\psi-\psi\vec{\nabla}\overline{\psi}\right)=0 \tag{80}$$

The density $\rho=|\psi|^2$ is positive-definite and its integral over the whole space is conserved during the evolution, because of the continuity equation. So, the density can be so interpreted as a probability density and the solution of Schrödinger equation as a wave function for a non-relativistic particle.

In the KG case, the continuity equation is

$$\vec{\nabla}\cdot\vec{j}+\partial_t\rho=0 \tag{81}$$

where

$$\rho=\frac{i\hbar}{2mc^2}\left[\phi^*\frac{\partial\phi}{\partial t}-\frac{\partial\phi^*}{\partial t}\phi\right] \tag{82}$$

$$j_i=\frac{i\hbar}{2mc^2}\left[\phi^*\partial_i\phi-\phi\partial_i\phi^*\right]\quad(i=1,2,3) \tag{83}$$

The density (82) candidate to be a probability density, does not show the same features as the Schrödinger case: although its integral over the whole space is still conserved, ρ is not positive-definite. Its expression is of second order in the variables and therefore ϕ and $\partial_t \phi$ can be prescribed arbitrarily at some time t_0. Since ϕ and $\partial_t \phi$ are function of the space coordinates **x**, ρ can be positive in some regions and negative in others. As remarked by Schweber (Schweber, 1961, p.55): *"It is difficult to think of ρ as a conventional probability density. Because of this possibility of negative ρ values, the Klein-Gordon equation fell into disrepute for about seven years after it was first proposed".*

2. Candidate for expressing a local charge density

The aim of the present section is to investigate if the complex solution of the KGE, unable to describe a relativistic wave function, is interpretable within the Electrodynamics framework.

The search for the meaning of the complex non-quantum KG field is related to the analysis of the classical meaning of the quantity expressed by Eq. (82) already discussed in the context of relativistic quantum mechanics and rejected as the probability density of finding a relativistic particle.

The interpretation of (82) from a classical point of view is generally not investigated. To give an example, Ryder claims that ρ can be successfully interpreted after quantization: "on quantization, ρ and $\vec{j}$ are then the charge and the current densities, rather than the probability and probability current densities" (Ryder, 1985, p. 31).

Schiff (Schiff, 1949), referring to the failure of interpreting (82) as a probability density, hints to a classical interpretation: *"It can however be multiplied by e and interpreted as an electric charge density, since charge density can have either sign so long as it is real".*[8]

Sentences like this are emblematic for showing to what extent the classical interpretation of ρ is relegated to sporadic sentences whose meaning is mainly inferred from apparently occasional formal features. They, in particular, do not provide the assumptions that the complete formal development seems to require.

The following sections are explicitly devoted to:

- reconstructing the classical meaning of ρ inside an Electrodynamics framework and stressing what relation can be established between the assumptions behind the formal development and the KG complex field ϕ (see § 2.4.2.2 a);
- reconsidering Ryder's expression "electric charge of a complex field" by bringing to light its hidden conceptual meanings (see § 2.4.2.2 b).

As it will be shown, the reasoning will lead to the strong conclusion that, according to Ryder, the classical complex KG field does not admit a coherent and physically sensible meaning and that the KG field is "a strictly quantum field" (Ryder, 1985, p. 129).

[8] The statement of Schiff has to be intended as referred to (82) unless the value of the constant.

(a) The assumptions needed to locate ρ in a coherent classical Electrodynamics framework

In order to complete the interpretation of the quantity

$$\rho_e = eJ^0 = ei\left[\phi^* \frac{\partial \phi}{\partial t} - \frac{\partial \phi^*}{\partial t}\phi\right] \tag{84}$$

as local electric charge density (as suggested by Schiff) at least two considerations are needed.

The first trivial one is that the total charge of a closed system is an integer multiple of the electron charge

$$\pm ne = Q_e = e\int J^0 dV = ie\int \left[\phi^* \partial_0 \phi - \phi \partial_0 \phi^*\right] dV \quad (n = 0,\ 1, -1, 2, -2\ldots) \tag{85}$$

where $\partial_0 = \partial / \partial t$.

A second kind of considerations leads to build a Lagrangian coherent with the interpretation of ρ_e in terms of electric charge density.

Indeed, the Lagrangian

$$L = \left(\partial_\mu \phi\right)\left(\partial_\mu \phi^*\right) - m^2 \phi \phi^*, \tag{86}$$

whose *global U(1) invariance*

$$\phi(x) \to \exp(i\alpha)\phi(x) \tag{87}$$

leads to the conserved quantity

$$Q = \int J^0 dV = i\int \left[\phi^* \partial_0 \phi - \phi \partial_0 \phi^*\right] dV, \tag{88}$$

does not present any track of the electromagnetic fields generated by the charge distribution.

The first step for solving it is to modify the quantity (88) as[9]

$$Q = \int J^0 dV = i\int \left[\phi^* \left(\partial_0 + ieA_0\right)\phi - \phi\left(\partial_0 - ieA_0\right)\phi^*\right] dV \tag{89}$$

---

[9] As reported in Kragh (Kragh, 1984): "In the general case of electromagnetic fields present Gordon wrote the charge and current densities as a four vector,

$$j_\mu = 1/i\left(\psi^* \frac{\partial \psi}{\partial x_\mu} - \psi \frac{\partial \psi^*}{\partial x_\mu} - \frac{2ie}{\hbar c} A_\mu \psi \psi^*\right)$$

where μ runs from 1 to 4 ($x_4 = ict$, $A_4 = i\phi$). These expressions for j_μ were also found by Klein whose paper was independent of Gordon's".

where $A_0(x,t)$ is a component of a vector field not yet specified.

The quantity (89) can be derived from the *local U(1) invariance*

$$\phi(x) \to \exp(iq\alpha(x))\phi(x) \qquad (q = e) \tag{90}$$

of the Lagrangian

$$L = (\partial_\mu \phi + ieA_\mu \phi)(\partial^\mu \phi^* - ieA^\mu \phi^*) - m^2 \phi^* \phi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \tag{91}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{92}$$

In the last Lagrangian, coupled with the complex field, there is another field which is vectorial, zero mass and invariant with respect to the following transformation property:

$$A_\mu \to A_\mu + \partial_\mu \alpha(x). \tag{93}$$

For these reasons such a field can be recognized as the electromagnetic field generated by the charge distribution itself.

From the previous Lagrangian it is possible to derive the equations of motion for both the electromagnetic and the complex field

$$\begin{cases} (\partial_\mu \partial^\mu + m^2)\phi + 2ieA_\mu \partial^\mu \phi + ie\phi \partial_\mu A^\mu - e^2 A_\mu A^\mu \phi = 0 \\ (\partial_\mu \partial^\mu + m^2)\phi^* - 2ieA_\mu \partial^\mu \phi^* - ie\phi^* \partial_\mu A^\mu - e^2 A_\mu A^\mu \phi^* = 0 \\ \partial_\nu F^{\mu\nu} = -ie(\phi^* \partial^\mu \phi - \phi \partial^\mu \phi^*) + 2e^2 A^\mu |\phi|^2 \end{cases} \tag{94}$$

The step from (88) to (89) (the jump *global invariance → local invariance*), so simple from a formal point of view to appear harmless from a physical point of view, is the crucial point for the physical interpretation of the quantities involved.

The jump *global invariance → local invariance* is a big issue with wide implications: in the present discussion it guarantees that the conserved quantity can be coherently interpreted as an electric charge since it allows for gaining that “degree of freedom” necessary for the introduction of the electromagnetic field in the Lagrangian.

Such conditions allow ρ to be really interpreted as electric local charge density and they really create an interpretation of the complex field ϕ as a tool for building such a quantity.

Nevertheless the complex field, whose combination with (89) allows the electric charge density to be constructed, cannot be recognized, strictly speaking, as “a KG field” because ϕ does not longer satisfies the KGE although it was the starting point of the discussion.

It satisfies another equation, much more complicated: it is intrinsically non-linear since there is the electromagnetic field coupled with the complex one.

(b) The "charge of the complex field"

The reflections developed so far allow us to go back to the Ryder's comment about the Lagrangian of the classical electrodynamics. He writes: "the gauge potential A_μ couples to the current J_μ with coupling strength *e*, which is the charge of the field ϕ" (Ryder, 1985, p. 98).
The claim sends back the idea that there is a complex field having an electric charge.

We remark to what extent such a claim is problematic: What does it mean that the KG field is "a strictly quantum field" and what does it mean that there is a complex non-quantum field able to "describe a field with charge – *e*"?

In the light of the previous analysis the answer to the last question requires to specify first of all what the term "electric charge" is referred to. Indeed, it can be related to:

- the total electric charge of the system, i.e. the conserved quantity expressed by

$$Q = \int J^0 dV = i\int \left[\phi^*(\partial_0 + ieA_0)\phi - \phi(\partial_0 - ieA_0)\phi^*\right]dV = \pm ne \tag{95}$$

  which can be positive, negative or null but always a multiple of a fundamental quantity *e*,

or to:

- the fundamental constant *e* (electric charge of the proton) as the coupling constant in the Lagrangian ($\phi(x) \to \exp(iq\alpha(x))\phi(x)$ with $(q = e)$).

If one refers to the coupling constant, the electric charge is introduced by hand and it simply qualifies the kind of interaction. The relation between the charge and the field remains more at an evocative level. If one refers to the conserved quantity, the relation between the electric charge and the complex field seems better specified but also in this case it cannot be considered as "possessed" by the field.

The complex field itself gains meaning as formal construct useful to build the physical quantity of interest.

3. Remarks for the complex case

Unlike the real case the search for the classical meaning of the KG complex field does not admit a direct and unproblematic physical interpretation. The results of the analysis here presented show that the attempts to interpret the complex solution of the KGE both in the relativistic quantum mechanics and in the electrodynamics framework are unsuccessful.

While the unsuccessful faith of the KG field in the context of the relativistic quantum mechanics is well known, the demonstration of the equivalent fate in the Electrodynamics framework has required a specific analysis.

Really, the analysis was moved by the belief that it was possible to situate the complex KG field in some classical framework. Indeed, it started from a complex field satisfying a KGE but conditions of coherence lead to give up it in favour of another complex field. The claim of Ryder (Ryder, 1985, p.129) seems then confirmed.

Nevertheless, the new complex field built in coherence with the electrodynamics framework presents interesting features beside the ones already mentioned: it seems to share all the features of the matter field described by Schrödinger reported by Sachs (Sachs, 1988):

> *Summing up, Schrödinger wave mechanics, to him, was a necessary addition to the Maxwell field theory of electromagnetism, to complete the representation of the electrical properties of the matter. The left – side of Maxwell's field equations are particular combinations of rates of change in space and time of the electrical and magnetic fields of force. The matter field sources of these force fields, that appear on the right – hand sides of Maxwell's equation (the charge and current densities) are "real number variables" that are factorizable into the more primitive (complex number variable) wave functions,* Ψ *and* Ψ^**, which in turn predict the features of matter in the atomic domain, such as the wave nature observed in the electron diffraction experiments. But it's important to note that in Schrödinger 's conception, the wave function* Ψ *for the matter field does not relate to a single quantity of micromatter (an electron, an atom, etc.). It rather relates to an entire ensemble of matter components. The implication here is that there is no primitive atomistic model of charged matter. Its fundamental description is instead in terms of continuous matter fields.*

2.4.3 The quantum case

> *In the context of Quantum Field Theory, the Klein-Gordon equation can be used, if interpreted in the appropriate way, to describe massive spinless particles, those particles referred as mesons (such as pions or kaons). This interpretation requires the full framework of quantum field theory, which was only in embryonic form when Dirac first suggested his quite different equation for the electron in 1928 (Penrose, 2004, pp. 617 -618).*

The aim of the present section is to complete the study on KGE by highlighting how the quantum KG field is related to the description of relativistic particles and in what sense they can be considered, within the canonical paradigm, oscillations of the quantum field around the ground state.

1. Quantum field (real case)

The canonical quantization procedure solves the transition from the classical field to the quantum one focussing on the electromagnetic field as a paradigmatic case: the numbers representing the coefficients of the Fourier expansion of the solutions of the d'Alembert equation are elevated to creation or annihilation operators and commutation rules are imposed. Every other quantum field is constructed extending such a procedure by analogy, taking into account if they will describe bosons (commutation rules) or fermions (anti commutation rules).

The result of such a procedure for the KG real field is that from a real function is obtained the real quantum KG field formally described by the following expressions

$$L = \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial_\mu \phi\right) - m^2\phi^2 \tag{96}$$

$$\left(\partial_0^2 - \nabla^2\right)\phi(x) + m^2\phi(x) = 0 \tag{97}$$

$$\phi(x,t) = \frac{1}{(2\pi)^{3/2}}\int \frac{d^3k}{\sqrt{2\omega_k}}\left[\hat{a}(k)e^{-i(kx-\omega_k)} + \hat{a}^+(k)e^{i(kx-\omega_k t)}\right] \tag{98}$$

$$\omega_k = \sqrt{\left|\vec{k}\right|^2 + m^2} \tag{99}$$

$$\begin{aligned} &[a(k), a(k')] = 0 \\ &[a^+(k), a^+(k')] = 0 \\ &[a(k), a^+(k')] = \delta(\vec{k} - \vec{k}') \end{aligned} \tag{100}$$

The construction of the Fock space proceeds by acting on the ground state through a creation operator (the vacuum is "excited")

$$\left|1_k\right\rangle = a^+(k)\left|0\right\rangle \tag{101}$$

This state is an eigenstate of the energy with an eigenvalue

$$E = \omega_k \tag{102}$$

and an eigenstate of the momentum with an eigenvalue

$$\vec{p} = \vec{k} \tag{103}$$

The two objects are not independent, since they are related from the relation of dispersion,

$$\omega_k = \sqrt{\left|\vec{k}\right|^2 + m^2} \tag{104}$$

which, from the previous relation, can be written as follows

$$E^2 - \left|\vec{p}\right|^2 = m^2 \tag{105}$$

Since this equation in special relativity describes a particle of energy E and momentum $\boldsymbol{p}$, we recognize and call this state "one particle state".

In this sense the particles are considered oscillations of the field around the ground state.[10]

2. Quantum field (complex case)

If instead of the real KG quantum field the complex one is considered, the following Lagrangian and the following possible expression of the complex field have to be considered

$$L = \left(\partial_\mu \phi^+\right)\left(\partial_\mu \phi\right) - m^2 \phi^+ \phi \tag{106}$$

$$\left(\partial_0^2 - \nabla^2\right)\phi(x) + m^2 \phi(x) = 0 \tag{107}$$

$$\left(\partial_0^2 - \nabla^2\right)\phi^+(x) + m^2 \phi^+(x) = 0 \tag{108}$$

$$\phi(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{\sqrt{2w_k}} \left[a(k)e^{-ikx} + b^+(k)e^{ikx}\right] \tag{109}$$

$$\left[a(k), a^+(k')\right] = \delta(\vec{k} - \vec{k}') \tag{110}$$

$$\left[b(k), b^+(k')\right] = \delta(\vec{k} - \vec{k}') \tag{111}$$

....

The construction of the Fock space and the particle interpretation proceeds identically to the previous case but now, in the complex case, there are two couples of operators (*a* and *b*) and two different ways of exciting the vacuum and getting the following states identical from the point of view of energy and angular momentum:

$$\left|1_k{}^a\right\rangle = a^+(k)\left|0\right\rangle \tag{112}$$

$$\left|1_k{}^b\right\rangle = b^+(k)\left|0\right\rangle \tag{113}$$

The split of two states is expression of a further invariance in the Lagrangian which does not hold in the real case (Akhiezer, Berestetskii, 1962, p.117): not a spacetime invariance but the global *U(1)* internal one

$$\phi(x) \to \exp(i\alpha)\phi(x) \tag{114}$$

Due to this invariance, the following quantity is conserved

[10] The expression of the energy – impulse operator is reported in Chapter 2, section “skeleton of continuum”.

$$Q = i\int \phi^+ \frac{\partial\phi}{\partial t} - \frac{\partial\phi^+}{\partial t}\phi d^3x \qquad (115)$$

Substituting the Fourier expansions of the field, by normal ordering (a product of creation and annihilation operators is in normal ordered when all creation operators are to the left of all annihilation operators in the product), one obtains

$$:Q: = \int d^3k\left[a^+(k)a(k) - b^+(k)b(k)\right] \qquad (116)$$

Since this operator commutes with the energy – momentum operator

$$\left[P^\mu, Q\right] = 0 \qquad (117)$$

the states

$$\left|1_k{}^a\right\rangle = a^+(k)\left|0\right\rangle \qquad (118)$$

$$\left|1_k{}^b\right\rangle = b^+(k)\left|0\right\rangle \qquad (119)$$

are both eigenstates of Q and the eigenvalues are respectively 1 and -1.

In the light of such a property we recognize and call the first a “one particle state” and the second one an “one antiparticle state”.

2.4.4 Remarks to Section 2.4

In the present section the KGE has been analysed in different cases where it can be applied. The analysis has been organized according to the mathematical expression of the solution: real function, complex function and quantum field (real and complex).

In the real case, the solution can be easily interpreted as displacement of an element of fluid or as amplitude of a component of electromagnetic field in the ionosphere, or more in general, in a *cold plasma*.

In the complex case, the coherence with relativistic quantum mechanics and the electrodynamics framework brought into light interpretative problems related to the classical complex KG field.

Such problems led to deny the existence of classical KG field and to accept the Ryder’s statement asserting that the KG field is a ”strictly quantum field”.

Nevertheless, are we sure that this field, in its quantum version, does not present the same problems found in the classical case?

In the issue, the same requests of coherence developed in the classical case (§ 2.4.2.2 a), i.e. the shift from the Lagrangian of the free classical KG field to the Lagrangian of classical electrodynamics, can be formulated also in the quantum case, leading to the Lagrangian of quantum electrodynamics

$$L = \left(\partial_\mu\phi^+ + ieA_\mu\phi^+\right)\left(\partial^\mu\phi - ieA^\mu\phi\right) - m^2\phi\phi^+ - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \qquad (120)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{121}$$

Again, the equation of motion for the fields can be derived and, again, the complex quantum field, which accounts for the Physical situation, does no longer satisfy the KGE

$$\begin{cases} \left(\partial_\mu \partial^\mu + m^2\right)\phi^+ + 2ieA_\mu \partial^\mu \phi^+ + ie\phi^+ \partial_\mu A^\mu - e^2 A_\mu A^\mu \phi^+ = 0 \\ \left(\partial_\mu \partial^\mu + m^2\right)\phi - 2ieA_\mu \partial^\mu \phi - ie\phi \partial_\mu A^\mu - e^2 A_\mu A^\mu \phi = 0 \\ \partial_\nu F^{\mu\nu} = -ie\left(\phi \partial^\mu \phi^+ - \phi^+ \partial^\mu \phi\right) + 2e^2 A^\mu \phi^+ \phi \end{cases} \tag{122}$$

How is it possible to regain the free equation and consequently a sensible interpretation of the free quantum KG field?

The current interpretation is that of considering the free fields as asymptotic states of the interaction, obtained by assuming that the coupling constant (q in this case) vanishing at $t = \pm\infty$.

The asymptotic view, which leads to a physically coherent, although asymptotic, interpretation in the quantum case, does not provide the same result in the classical case.

Whenever considering such a limit in the classical case, the free Lagrangian would describe a complex field that would allow the construction of a density of electric charge without taking into account the electromagnetic fields generated by the distribution itself: The model has no physical meaning.

Coming back to canonical quantization, it is shown to what extent it individuates a procedure based on the electromagnetic field and applied on fields (like the KG one) that are not interpretable, unlike the electromagnetic case, in a coherent classical framework.

The emphasis of the procedure is in fact on the construction of objects that get meaning only at infinite.

Although effective for constructing the formal objects needed to develop the theory, it often hides problems of coherence that can play a fundamental role in constructing the interpretation of the formalism.

CHAPTER 3

THE MODEL OF DISCIPLINE-CULTURE AND ITS APPLICATION IN THE CASE OF TEACHING QUANTUM FIELD THEORY

Chapter 3 as well as Chapter 4 aim to make it explicit how the analysis on the foundations of Quantum Field Theory (QFT) presented so far has been systematically developed keeping an eye on Physics Education Research (PER). In these chapters the analysis is re-considered so as to exploit its cultural and educational potential by tying it to two important PER issues:

1. the meta-issue of methodology that arises from the exploration of "extreme territories" of physics contents where PER faces the problem of establishing its own identity (Tseitlin, Galili, 2006) with respect to physics, philosophy of physics and popularization of physics;
2. the issue of identifying criteria for designing teaching paths aimed at meaningful understanding of contemporary physics at the secondary school level.

3.1 Aims of the study

The study presented in this chapter concerns the application of the Discipline-Cultural Model (DCM) (Tseitlin, Galili, 2005, 2006; Galili, Tseitlin, 2008) for reconstructing the cultural meaning of the Klein Gordon Equation (KGE). Matching the analysis carried out on the foundations on physics with a model built within PER would allow a double-faced aim to be pursued:

- to test the possibility of highlighting the cultural relevance of the analysis carried out on the KGE by means of a acknowledged model (Tseitlin, Galili, 2005);
- to evaluate the Discipline-Cultural (DC) framework effectiveness in providing a meaningful cultural perspective to a contemporary physics topic never considered from such a perspective before.

3.2 The Discipline-Culture Model: a new educational perspective

The DCM is an innovative educational perspective (Tseitlin, Galili, 2005; Galili, Tseitlin, 2008). In particular, this model defines the meaning of *culture* with regard to the elements of physical knowledge and elaborates the relationship between fundamental physical theories. Moreover, the DC perspective can guide the critical analysis of physical contents exceeding the focus on mere solving standard problems in educational context and encouraging learners' construction of the meaningful conceptual knowledge which would highlight the culturally upgraded disciplinary knowledge.

Very briefly the DCM considers physics as a dialogue between several discipline-cultures (for example Newtonian mechanics, classical electrodynamics, quantum mechanics) each of them characterized by:

I nucleus – which defines the identity of the discipline-culture and includes its fundamental principles, paradigm and claims of meta-disciplinary nature;

II body knowledge – which incorporates all normal disciplinary knowledge. This is established knowledge, each item of which is based on the principles contained in the nucleus;

III periphery – which contains the knowledge that clashes with the principles of the particular nucleus. This knowledge presents a challenge for the fundamental claims of the nucleus and possibly a mechanism for its change and reconstruction.

In a recent paper Galili demonstrates how the importance of the periphery knowledge highlights the major implications of the discipline-structure framework on science education (Galili, 2009). Periphery knowledge indeed allows "disciplinary" knowledge to be enriched by what Galili calls "Cultural Content Knowledge" (CCK):

> *"Cultural Content Knowledge is the disciplinary knowledge upgraded by its periphery" (Galili, 2009).*

The meaningfulness and importance of the CCK, often neglected and underestimated in science education, can be discussed in relation to another paradigmatic concept in science education – the Pedagogical Content Knowledge (PCK) elaborated by Shulman, (Shulman, 1986). PCK lies in a special intersection of content and pedagogy. It does not refer to a simple consideration of both content and pedagogy but rather to a particular blend of the two that enables transformation of content into pedagogically powerful forms. Shulman elaborated such a notion for arguing that having knowledge of subject matter and general pedagogical strategies, though necessary, were not sufficient for capturing the knowledge of good teachers. To characterize the complex ways in which teachers think about how some particular content should be taught he introduces the PCK notion defined as the content knowledge that deals with the teaching process, including the "the ways of representing and formulating the subject that make it comprehensible to others". So, at the heart of PCK is the manner in which subject matter is transformed for teaching. This occurs when the teacher interprets the subject matter, finding different ways to represent it and make it accessible to learners.

Galili stresses that a significant upgrade can be provided by CCK, with respect to PCK, since CCK opens new significant possibilities for education. CCK includes for example knowledge coming from history and philosophy of physics whose role has proved to be essential in promoting genuine understanding of the scientific contents, their structure and scientific epistemology (Galili, 2009).

KGE is a fruitful example of a subject to be analyzed from the point of view of the DC Model, not only because of its interesting history and its crucial position within QFT, but also - and mainly - because of its presence in different physical domains.

3.3 Methods

The study has been carried out through the following steps:

1. Analysis of the KGE in physics from the DC perspective;
2. Analysis of the equation in physics teaching from the DC perspective.
3. Re–analysis, on the basis of the results obtained, of the content knowledge about the KGE from the cultural perspective highlighting its educational potential.

In more details:

1. The analysis of physics contents revealed the *status* of the KGE equation as a conceptual construct belonging to several *discipline–culture*s. However, the few known applications of the KGE, being related to very specific domains (such as

cold plasma physics), are rarely mentioned in general physics courses. Therefore, the first problem of the educational realm within the cultural perspective was to build a general framework of coherent presentation of the KGE applications in various contexts, which would allow deeper exploration of the subject. To be successfully applied, the DC model requires a genuine understanding of different cases in which the same content knowledge is applied. One needs to locate the knowledge and to highlight conceptual overlaps of its different domains. Furthermore, the problematic historical path of the equation posed to physicists specific problems to be resolved at the level of physics foundations in order to reach clarity of the topic. The study carried out at this level is emblematic of how and why physics education has to interfere with the research in theoretical physics and the foundations of physics.

2. The analysis of the equation in physics teaching has been carried out by analysing of the textbooks, performing interviews and submitting questionnaire administrated to physics experts.

   Textbooks: we inspected a representative set of 16 textbooks used in advanced university courses of modern and classical physics (Relativistic Quantum Mechanics and Quantum Field Theory, Classical and Quantum Electrodynamics, Classical Mechanics) published in English and Italian (see Table 3.1). The analysis has been carried out in order to detect:
   - the area/areas of physics where the KGE is currently presented to students;
   - the importance usually ascribed to the KGE in presenting modern physics (for example, how it is currently introduced, at what level of details).

Table 3.1 List of textbooks

1) Bjorken J.D. & Drell S.D, 1964, *Relativistic Quantum Mechanics*, New York St.Louis San Francisco Toronto London Sydney, McGraw–Hill Book Company	9) Chang S.J., *Introduction to Quantum Field Theory*, 1990 Singapore, World Scientific publishing Co.Pte.Ltd
2) Schweber S., 1961, *An Introduction to Relativistic Quantum Field Theory,* Evaston, Illinois Elhsford, New York, Row, Peterson and Company	10) Huang K., 1998, *Quantum Field Theory*, New York, Wiley-Interscience
3) Bethe H.A., de Hoffmann F., & Schweber S., 1955, *Mesons and fields*, Vol. I, Evaston, Illinois White Plains, New York, Row, Peterson and Company	11) Schweber S.S., 1994, *QED and the Men Who Made It: Dyson, Feynman, Schwinger, and Tomonaga*, Princeton University Press.
4) Penrose R., 2004, *The road to reality*, UK, Knopf	12) Weinberg S., 1995, *The Quantum Theory of Fields*, Vol I, Foundations, Cambridge University Press.
5) Sakurai J.J., 1967, *Advanced Quantum Mechanics*, Addison Wesley Publishing Company Inc.	13) John David Jackson, *Classical Electrodynamics,* 1962, 1975, John Wiley & Sons, Inc.
6) Ryder L.H., 1985, Quantum Field Theory, Cambridge University Press	14) Mencuccini C. & Silvestrini V., 1988, Napoli, *Elettromagnetismo e*

	Ottica, Liguori Editore,
7) Mandle F. & Shaw G., 1984, *Quantum Field Theory*, Chichester, New York, Brisbane, Singapore, John Wiley and Sons Ltd	15) Crawford. F. S. Jr, 1966, *Berkeley Physics Course, Vol. 3*, *Waves*, U.S.A., McGraw-Hill book Company
8) Greiner W. Bromley D.A., 1990, *Relativistic Quantum Mechanics, Wave Equations,* Berlin Heidelberg New York, Springer Verlagh	16) Feymann R.P. Leighton R.B. Sands M., 1965, 1989, *The Feymann lectures on Physics. Vol. 2*, California Institute of Technology

Questionnaire and interviews: We prepared 8 questions (Table 3.2) to be responded in writing or orally by a sample of theoretical physicists (researchers-teachers and PhD students) of the Bologna University and the Hebrew University of Jerusalem. The interviewed chose the preferred modality for the answers.

The aim of the questionnaire was to clarify:

- the role played by the KGE in modern physics knowledge as perceived by a professional: Whether it simply reflects the contents of the textbooks or is elaborated from a personal approach to the role and the meaning of the KGE;
- whether and how the cultural aspect of modern physics knowledge (in the sense suggested by the DC approach) is perceived and addressed by professionals engaged in physics research.

Table 3.2 Klein Gordon Equation – Questionnaire

1) What is, in your view, the importance of the KGE in presenting modern physics to students?

2) The KGE is often only briefly mentioned to students. Does it correspond to your experience? If so, what are the reason(s) for that (beyond the lack of time)?

3) The story of KGE tells us about the attempt to interpret the KGE as a relativistic equation for the wave function of a particle. Later on, the KGE was used within the framework of quantized fields. Are there any other relevant applications of the KGE?

4) The d'Alembert equation and the KGE are rather similar in form. Do you ascribe any importance to this similarity? Please explain.

5) The failure of interpreting the non-quantum KGE as an equation for relativistic wave function historically led to the abandonment of this equation. In the courses of physics, d'Alembert equation is taught as a wave equation of electromagnetic field, and Dirac equation is taught as giving relativistic wave function to account for electron and calculate its probability density.

Both these equations are taught later again in the advanced context of quantized fields. The KGE appears then too. Consequently, the KGE seems to be valid only in the context of quantized fields without any classical meaning. Could you see any special reason for this lack of symmetry and the unique area of validity for the KGE?

6) Do you think that considering the same equation across different areas of validity, like in classical theory and in quantum theory, or in quantum theory with and without field quantization, and comparing between the uses of the equations presents a good pedagogy?

7) In general, would you consider teaching physics while bridging and comparing between the same concepts (equations) in different contexts (classical, quantum, quantised field) to be a valid educational approach, or you prefer to avoid such approach to prevent students' confusion?

8) In your view, at what level of instruction the KGE could or should be taught?

3.4 Results

3.4.1 The status of Klein Gordon equation in physics

The results of the analysis of the KGE from a DC perspective allowed us to clarify the *status* of KGE in different fundamental physical theories elaborated as *discipline-cultures*. The analysis led to the following conclusion:

KGE belongs to the *periphery* area of knowledge of the non-relativistic quantum mechanics: As already mentioned in Chapter 2, at the beginning of the last century (1926) the pioneers of quantum mechanics (E. Schrödinger, O. Klein, V. Fock, J. Kudar, W. Gordon, Th. De Donder and H. Van Dungen) tried to force changes in the formal apparatus of the new theory in order to adjust it to the requirements of the theory of relativity and to attain the relativistic quantum mechanics. For this purpose a relativistic equation for the wave function of the electron had to be established. The methodological assumptions led them, almost naturally, to conclude that the best candidate was the KGE. The choice appeared, however, to be inadequate because the equation failed to provide the positive probability density in the prediction of space location for the relativistic particle. For this reason, the KGE was abandoned and therefore it should be considered, within the DC structure, as a *periphery element* of the non-relativistic quantum mechanics: It contradicts the basic assumptions of the theory formalism regarding the wave function.

KGE appears in the *body knowledge* of classical electrodynamics: In several situations a physical system can be modelled as a fluid characterized by its charge-density (for example, in studying the motion of free electrons in the Earth ionosphere). When basic oscillations of the charged fluid are considered, in accordance to Maxwell equation, the KGE type of equation can be used for the description of two entities: The components of the electromagnetic field in the medium (generated by the oscillating elements of the charged fluid) and an abstract field which allows to construct the charge density in the problem. In this case, since the equation results from the application of the fundamental principles and laws of classical electrodynamics, it can be identified as belonging to the body knowledge of that fundamental theory.

KGE appears also in the *body knowledge* of the classical mechanics: The way followed by d'Alembert to derive his famous equation is well known (see for example *Berkeley Physics Course, Volume 3, Waves*; Crawford, 1966). He considered an infinite chain of coupled oscillators and found the equation for a vibrating string by extending the result to the continuum limit. When the oscillators are located in the gravitational field, the reaction force exerted on every element of the system is due not only to the springs but also to the gravitational force. Running again the previous scheme proposed by d'Alembert, one is led to the KGE. So, corroborated also by an historical event, the KGE can be identified as an element in the *body knowledge* of the Newtonian mechanics.

KGE belongs to the *nucleus* of QFT: As widely discussed, the quantum Klein Gordon field is related to the description of massive, relativistic and spinless particles (such as pions) and, together with the electromagnetic field, the process of quantization for the KG field establish a paradigmatic reference for quantization of any other field.

3.4.2 The status of the Klein Gordon equation in physics teaching

Analysis of the textbooks

The results obtained by the analysis of the textbooks from the DC perspective showed that the *KGE* was:

- extensively treated in the context of quantum theory where it was identified as a *periphery element* of the non-relativistic quantum mechanics and as a *body element* – within the QFT.
- usually ignored in classical (Newtonian) mechanics and in classical electrodynamics, although it could be identified as a *body knowledge element* in both theories.

Notable exceptions to this situation, widely discussed in Chapter 2, are the textbooks by Jackson, by Ryder and the Berkeley course (see table 3.1), which provided important hints to the study about the nature of the equation within the Newtonian and electrodynamics theoretical pictures.

Interviews and questionnaire to physics experts

From the interviews and the answers to the questionnaire of the physics experts two different standpoints can be clearly pointed out:

- in most cases experts expressed positions very similar to those that were presented in the textbooks: the KGE is universally admitted as a subject in the theoretical description of particles in the relativistic quantum mechanics and in QFT, but it is generally ignored in the other contexts: Classical mechanics and electrodynamics. When the equation did appear in specific contexts, it was never related to other appearances of the same equation in other domains of physics.
- in a few notable cases, the answers of the experts showed what is called within the DC model a "*cultural knowledge"* about the KGE. Some of them stressed several problematic issues in the regular university presentation which they thought requires revision, introducing a retrospective reflection. The cultural knowledge, necessary to relate the role of KGE across several

disciplinary contexts, was part (and considered to be part) of their research professional interests and skills.

Results of the analysis: KGE as an exemplary topic in QFT teaching

With regard to the results of the analysis of the textbooks and the main trend in the answers of the physics experts, the case of KGE appeared to be a *specific topic* that nevertheless allowed us to point to the *general features* of the way in which the QFT is introduced and taught. Indeed, in current teaching of the QFT, teachers usually:

- *avoid* discussing the relationship of the considered subject with different areas of physics knowledge and ignore the *interference* between the different disciplines: classical and quantum field theories, relativistic quantum mechanics and classical electromagnetism.
- *focus* on *technical details* and avoid revealing to the students the conceptual meaning of mathematical formalism. Thus, in the procedure called *canonical quantization* (the common way to introduce the QFT framework) the ontological differences between the objects involved (the different fields which are "quantized") are normally ignored.

On the basis of the results obtained we come to conclusion that the current instruction on the contemporary physics usually lacks the cultural perspective defined by means of the DCM. The KGE case may illustrate how such an instruction encourages a strictly *disciplinary* knowledge about the QFT and its isolation of from the rest of physics.

3.5 Cultural analysis of Klein Gordon equation

The cultural and educational analysis of the knowledge related to the KGE could be summarized and represented as conceptual map (Figure 3.1), where the path of conceptual (and historical) evolution of content knowledge can be detected by following the arrows in the clockwise direction. This map should be interpreted as following:

1) Although the KGE is addressed only in teaching the relativistic quantum mechanics and the QFT, its essential structure can be found as invariant in other discipline-cultures of physics.
2) The comparison of such projections on the different domains of knowledge allows effective comprehension of the change of the physical meaning of the solutions of the KGE equations, allowing different physical objects to be described: The displacement of an element of a string in the gravitational field, the electromagnetic field inside a medium, as well as the tentative relativistic wave function for the electron.
3) The change in the physical meaning can be directly related to the change in the mathematical and formal expression of the correspondent solution.

   Starting from the Newtonian case (where a real function is sufficient to describe a displacement of an element of the fluid), we find the case of electrodynamics and the introduction of the electric charge that requires additional degree of freedom and the use of a complex function is necessary. The processes of creation and annihilation of relativistic particles and the quantum values of the dynamical variables in QFT require further upgrading of the formalism. Instead of complex functions, usually used for describing wave functions, the operator-valued distributions are utilized.

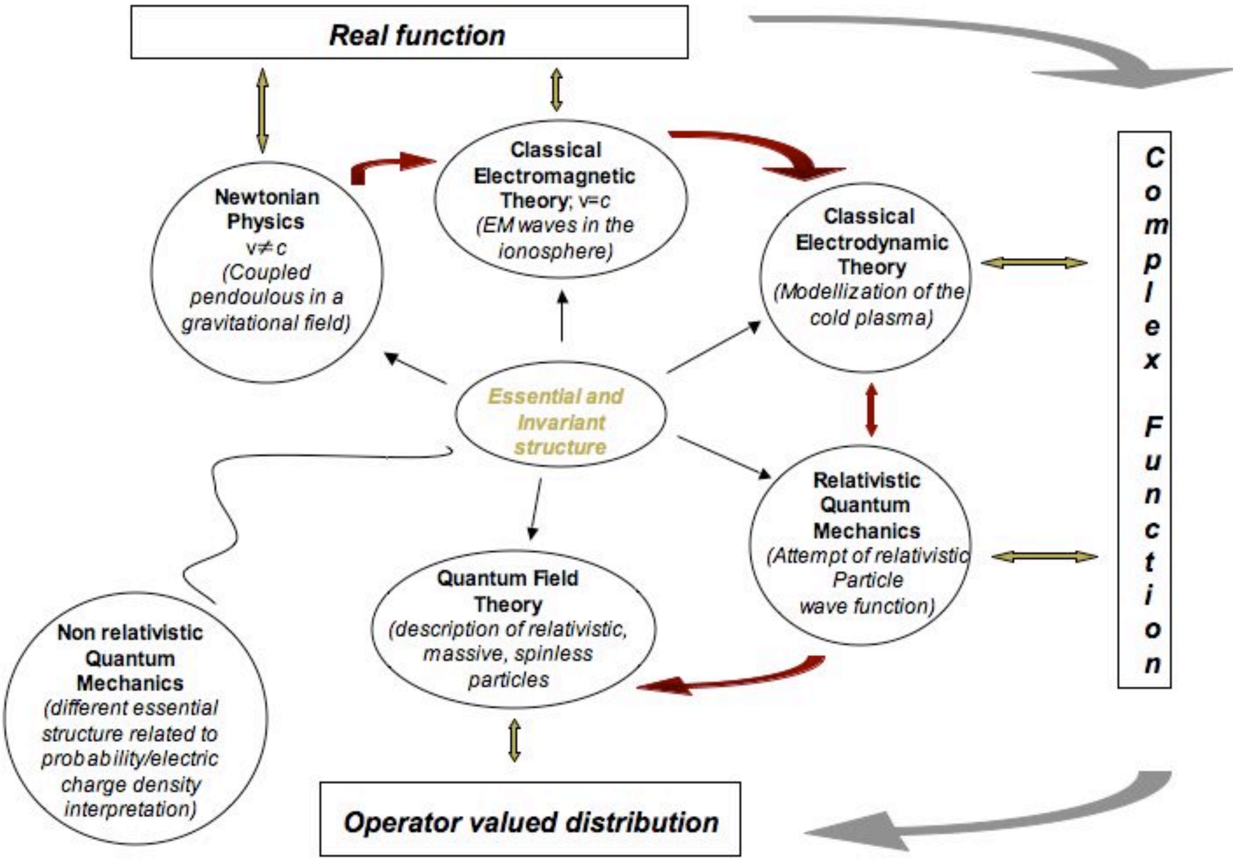


Figure 3.1. Cultural representation of the knowledge regarding the Klein Gordon equation.

By summarizing the reconstruction in one picture, at least three aspects of cultural relevance become evident:

- QFT is no more isolated from the rest of physics;
- Physics formalism is continuously kept under control by the learner and is related to the physical interpretation;
- The model of objects and interaction in the QFT stands out as a conceptual and formal refinement in connection to the phenomenological changes.

Note how the non-relativistic quantum mechanics joins this framework. As already mentioned, the KGE can be derived, following the historical path, starting from Schrödinger's non-relativistic equation and his attempt to obtain relativistic account for elementary particles by means of a wave function. The path followed and the related assumptions led the physicists to impose a strong change in the approach: From the non-relativistic and probabilistic framework to the relativistic and non-probabilistic one[11]. To express this change we relate what is called in the map *essential and invariant*

[11] Of course this does not mean that the relativistic framework always exclude a probabilistic interpretation (see the Dirac equation).

structure to *non-relativistic quantum mechanics* by the curve in order to emphasize the difference.

3.6 Final remarks and implications

We have applied the DC approach to analyze the status of the KGE in teaching modern and contemporary physics in advanced university courses. The DC approach proved to be particularly powerful:

- to *clarify the cultural status* of the KGE in physics. The analysis revealed the KGE as a fertile ground for the conceptually rich teaching of physics through interrelating fundamental physics disciplines, and displaying the inherent connections between them. This approach to teaching encourages cultural knowledge of physics and enables students' enculturation into physics;
- to *locate the shortcomings* of the traditional teaching of the advanced physics courses. The prevailing of formalism over conceptual knowledge is the main cause of the students' difficulties in reaching genuine understanding of the subject matter.

As implications of the study for PER, we believe it important to underline that the present study explored "extreme territories" of physics education. KGE is not a subject of primary or secondary school and even a subject of general university courses. Only the student who dedicates their curriculum to the study of theoretical physics meets the KGE: It is, without doubts, a very advanced topic. Nevertheless, the study reported here can be considered a positive test that arguments and awareness produced within the field of PER still hold in this kind of contexts and define the identity of this research field with respect to other fields, such as theoretical physics or philosophy of physics.

The physics re-construction presented here – and carried out applying a model elaborated within PER – is also strongly coherent with the "model of longitudinal development" developed within the Italian project PRIN_F21 (2004), coordinated at national level by P. Guidoni (Guidoni, Levrini, 2008).

This model points out criteria for coherence and crucial steps that allow the cognitive potential of the pupils to be progressively exploited and tuned to the construction of physics knowledge along the pre-university curriculum (from kindergarten to upper secondary school). According to the model, knowledge's evolution is seen as a progressive process aimed at extending, "re-investing", explicitly revising interpretative formal structures and models when enlargements and/or changes of the phenomenological basis are enacted or when explanatory schemes clash with each other in "border problems" (Levrini et al., 2008b).

The KGE reconstruction summarized in Fig.3.1 follows the same dynamics and extends it up to the advanced university level.

The fruitful resonance between the DC model and the model of longitudinal development that occurred in the reconstruction of KGE is another evidence of the educational and cultural potential of the analysis carried out and of its specificity with respect to other possible analyses from different research perspectives.

CHAPTER 4

ANALYSIS OF TEACHING PROPOSALS

4.1 Analysis criteria

As mentioned in Chapter 1, within the research in Physics Education, few studies exist aimed at producing teaching proposals for introducing notions of Quantum Field Theory (QFT) at the secondary school level or within introductory physics courses at the university level.

Here, in analyzing some exemplar studies we will provide a more detailed description of the main motivations lying at the basis of the proposals. In particular we will show that in some cases the studies are moved by secondary school physics programs (Daniel, 2006; van den Berg, Hoekzema, 2006; Hoekzema et al., 2005) and by the acknowledgment of problematic issues in teaching Quantum Mechanics (QM) that enforce the search for new paths on Quantum Physics (Giliberti M., 2006; Giliberti et al., 2002; Hobson, 2005).

As already stressed in Chapter 1, the first reading of PER literature, carried out at the beginning of this PhD work, showed that the existing papers do not address explicitly interpretative problems concerning QFT formalism.

For instance, in their efforts of translating contemporary physics notions in understandable languages, none of them seems to deal with the search for making more and more transparent crucial sentences like "particle are nothing but field oscillations".

Also this evidence pointed out the need of developing studies on the foundations of QFT aimed to analyze the existing materials in more detail and to evaluate the educational implications of the explicit and implicit choices on which they are based.

This chapter is devoted to present the results of an analysis carried out with a double aim:

- exploring the effectiveness of the results of the analysis on the foundations of QFT, reported in Chapter 2 and reconsidered in Chapter 3, as keys for realizing a comparative analysis of the existing literature;
- pointing out the strategies used for making the theory sound acceptable without using sophisticated formalism and evaluating their *pros and cons* (the complexities, not addressable at the secondary school level, they allow to bypass *vs.* the dangerous hyper-simplifications they imply).

The inspected papers are reported in table 4.1.

The teaching proposals bold marked in the table have been selected for a more detailed analysis as emblematic cases of the main trends.

The materials have been analyzed on the basis of the same grid, designed for:

- stressing the peculiarities of each proposal and, at the same time, fostering their comparison;
- keeping, as much as possible, general choices belonging to philosophical or general issues in education apart from specific issues and specific elements on which the results of the analysis on the QFT foundations can be directly applied and tested.

The grid is reported in Table 4.2.

After the analysis of each proposal, a final comment on all of them ends the chapter.

Table 4.1 List of the inspected papers

Paper	Reference
A reappraisal of the mechanism of pion exchange and its implications for teaching of particle physics	Peter Dunne, Preston College, Fulwood Campus, Preston PR2 8UR,UK Physics Education, 37 (3), May 2002
Electrons as field quanta: A better way to teach quantum physics in introductory general physics courses	**Art Hobson**, Department of Physics, University of Arkansas, Fayetteville, Arkansas 72701 Am. J. Phys. **73** (7), July 2005
Learning quantum field theory from elementary quantum mechanics	P. Gosdzinsky and R.Tarrach, Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona, Diagonal 647, Barcelona Am. J. Phys. **59** (1), January 1991
Particles, Feynman diagrams and all that	**Michael Daniel**, King Edward's School, Birmingham, UK Physics Education **41** (2), March 2006
Quanta – Mi	**Marco Giliberti**, Quanta-MI, teaching material used in the course "Teoria dei campi e proposte didattiche di fisica quantistica. La proposta di Milano" within the IDIFO Master, Udine **M. Giliberti**, **L. Lanz**, **L. Cazzaniga**, Quanta-MI: a modern teaching for modern physics in preservice teachers training, Paper presented at the international GIREP Conference (2002)
Teaching conservation laws, symmetries and elementary particles with fast feedback **Conservation laws, symmetries, and Elementary Particles**	**Ed van den Berg and Dick Hoekzema**, Centre for Science and Mathematics Education, Utrecht University, Netherland Physics Education **41** (1), January 2006 **Dick Hoekzema, Gert Schooten, Ed van den Berg, and Piet Lijnse,** The Physics Teacher Vol. 43, May 2005
The calculated photon: Visualization of a quantum field	Martin Ligare and Ryan Oliveri, Department of Physics, Bucknell University, Lewisburg, Pennsylvania 17837 Am. J. Phys. **70** (1), January 2002
The nature of force in particle physics	J Allday, King's School, Canterbury, UK Physics Education, **32** (5), 1997
The two–slit interferometer reexamined	E.C.G. Sudarshan and Tony Rothman, Center for Particle Theory, University of Texas, Austin, Texas 78712 Am. J. Phys. **59** (7), July 1991

Table 4.2 Grid of analysis

Grid of analysis
Description of the teaching proposal
- *Objectives:* The level of generality of the proposal objectives and its implications in the different content areas of Physics. - *General choices:* The image of Physics implied in the choices made by the author and the image of the discipline that the proposal aims to transmit to the students (for instance: unified vision *vs* discipline culture); role of theory and of formalism; role of history, philosophy and foundations of Physics. - *Specific choices:* The crucial concepts selected as content core of the proposal; the specific approach chosen for introducing Quantum Field Theory (for instance: canonical quantization *vs* axiomatic approach); interpretation attached to the quantum field (for instance: ontological reality *vs* mathematical construct).
Remarks
The remarks on the proposal are developed along two main lines: - General remarks concerning the basic choices of the proposal at philosophical (epistemological and ontological) and educational level; - Specific remarks concerning specific choices implemented at conceptual level.

4.2 Description of the proposals and results of the analysis

A. "Electrons as field quanta: A better way to teach quantum physics in introductory general physics courses", Art Hobson (Am. J. Physics, 73 (7), July 2005)

Description of the teaching proposal

The proposal is targeted to university introductory physics courses such as: non-relativistic quantum mechanics, modern physics courses for non-scientists, math – based physics survey courses for scientist and general modern physics courses.
The attention paid to QFT grounds its roots in the believe that the symmetry between radiation and matter is considered a fundamental goal to be pursed in teaching at introductory course level:

> *"Quantum field theory has a more unified view [with respect to the QM as it is usually taught at school], according to which both radiation and matter are continuous fields while both photons and material particles are quanta of these fields"* (p. 630).

The epistemological roots of the proposal can be traced back in the statement by Weinberg reported in the paper:

> *"In its mature form the idea of quantum field theory is that quantum fields are the basic ingredients of the universe, and the particles are just bundles of energy and momentum of the fields"*

According to the goal, the proposal considers the *"field–theory view point as the conceptual basis for teaching* [also] *non-relativistic quantum mechanics"*.

From an educational point of view, the unified picture provided by the QFT perspective is considered the best way for solving the particle identity issues, dispelling students' Newtonian misconceptions about matter, solving the wave – particle paradox; all of these problems ground their roots, in author's opinion, in the traditional instruction view of radiation as a field phenomena and matter as a particle phenomena.

The implications of the proposal for the curriculum consist simply in the incorporation, within the curriculum, of the qualitative notion of material particles as field quanta: The extent of change is pretty modest and no change in the mathematical formalism currently utilized for teaching non-relativistic quantum mechanics is required.

The goal is implemented through the specific choice of focussing on 4 crucial experiments (double slit experiment with electron and electromagnetic beams at high and low intensity) that highlight not only the wave – particle nature of radiation and matter, that is central in Quantum Mechanics, but also the symmetry between radiation and matter, that is crucial in QFT.

Although the same *essentials* are highlighted by other famous experiments (such as the photoelectric effect), the 4 selected ones are considered more direct and compelling because they allow a direct comparison between radiation and matter.

The author provides the following interpretation of the Young experiment:

> *"Young's experiment is evidence for the wave nature of light, confirming that light is a wave in a field – an extended real entity that comes through both the slits and interferes with itself".*

In front of the spots on the screen, observed with a low intensity beam, the author comments:

> *"Because these field quanta are localized and carry energy and momentum, they qualify as particles, although of a very non–Newtonian sort because they are really excitations of a continuous field and it is the* entire field *that is excited rather then some particular point within the field."* (p. 630)

In analogy with the electromagnetic case, in front of the interference pattern of electron beam, the author arrives at the conclusion that also matter is a "wave in a field" where the field is intended as an extended real entity that comes through both the slits. In the light of such a conclusion he remarks that the statement "an electron came through double slits" has to be intended that an "extended single excited field came through the double – slit".

On the basis of fact that the field related to the matter cannot be the electromagnetic one, the crucial experiment with the matter beam is interpreted as evidence for a new fundamental wave in nature, which is quantized and from which the particles come out and whose name is "electron field".

The expression "matter field" is introduced in order to indicate all the fields related to the description of the various kind of particles.

The author stresses the necessity of not confusing such matter fields with classical waves in matter (like sounds) or with a wave function with probabilistic meaning, stating that these fields are real entities leaving in a real physical three dimensions space.

In order to corroborate the previous interpretation focused on the symmetry between radiation and matter, the author reports a statement by Dirac:

> *"...the Hamiltonian for the interaction of the field with an atom is of the same form as that for the interaction of an assembly of light – quanta with the atom. There is thus a complete formal reconciliation between the wave and the light – quantum points of view."*

Remarks

General remarks

The educational perspective of the proposal is not characterized by strong epistemological assumptions.

QFT is not presented as a framework from which reconstructing the whole Physics curriculum but the basic choice of the proposal is only that of extracting from QFT those notions considered achievable also at more elementary levels.

One of the most interesting aspects of the study is the search for those elements, on the conceptual and formal level, that anticipate and synthesize the main features of the contemporary physics. Such a problem is not trivial at all and represents a fundamental step for evaluating the meaningfulness and the feasibility for every educational reconstruction of a physical topic.

What is remarkable in the proposal is that a specific choice is made: the introduction of the formalism is completely avoided in favour of a qualitative description of the experiment.

The implications of this choice and how long it is bearable will be investigated in the specific remarks.

Specific remarks

A critical reading of the proposal brings to evidence the implicit shift of the argumentation from a phenomenological to an ontological level: After observing the same behaviour between light and matter beam at low and high intensity, the analogy is extended beyond the pure phenomenological level up to the conclusion that, since light is a wave in a field, "*then matter itself is a wave in a field*", although different from the electromagnetic one. In other words, the conclusion is expressed through statements so assertive that seem to push the students to read them literally, lacking the required formalism for grasping their metaphorical meaning. Such a risk is reinforced at linguistic level by expressions like: "That is, when we say that 'an electron came through double slits,' we really mean that an extended single excited field (space – filling) came through the double – slit."

The strict analogy between the electromagnetic and the matter cases, because of the lack of remarks about their differences and the ontological load attached to the conclusions, leads students to confer the same *status* to the electromagnetic and the electronic field.

At least two objections can be moved to this approach:

- As stressed in Chapter 2, QFT introduces the quantum fields related to the description of particles in order to take into account situations of interaction where the system enters the region of interaction; in the model the free fields have to be interpreted as asymptotic states of the interaction, i.e. they, rigorously, gain physical meaning only at infinite. This kind of fields seems to be very different from the classical electromagnetic field which can be derived in case of high intensity of the light beam.

  The ontological forcing that suggests the possibility of visualizing matter fields in the ordinary space exactly in the same manner as the classical electromagnetic one is strongly questionable.
- The difficulties seem to be reinforced by a brief consideration about the formalism used in QFT: In the electromagnetic case, the classical field can be derived from the quantum one in case of many photons and it propagates according to the d'Alembert equation. The real solutions of this equation have a direct physical meaning: They describe the amplitude of the electromagnetic field components of the space – filling field. In the matter case (for example the electronic field) the quantum field is related to the description of relativistic electrons propagates according to the Dirac equation which is a four components equation whose solution are complex spinors. In the light of such a formal difference it seems difficult to stress the analogy at an ontological level and to confer the same status to these two objects.

The perspective of the analysis developed in Chapter 2 leads to argue that QFT builds the quantum matter field in order to interpret situations of interaction by constructing the needed mathematical tools on the basis of the well – known electromagnetic field. The general impression is that, although the reconciliation is made formally, the theory in itself does not imply necessary the attachment of ontological meanings to the fields introduced. For those reasons we retain that introducing matter as “a wave in a field” in the same sense intended for light is a too forced similarity and an ontological load that is not imposed by the theory.

As general methodological comment suggested by the analysis of this paper, we retain that the temptation of introducing such short – circuits and ontological loads reveals weaknesses in the educational research lying behind the proposals. The need of bypassing the formalism requires the inventions of strategies that can be very different from each other according to the context. In contexts of physics popularization such strategies can regard a mere “translation” from the formalism to the natural language, by means of evocative and immediate analogies focused on the specific points of interest. In contexts of physics education oriented to foster meaningful learning, the strategies must take care of the medium and long term consequences implied by the eventual analogies or pictures used. From this perspective, the educational strategies, instead of being a pure linguistic translation, should be the result of a deep content knowledge analysis aimed at grasping the essential conceptual core and of a coherent content reconstruction.

B. “Quanta – Mi”, Marco Giliberti, teaching material used in the course “Teoria dei campi e proposte didattiche di fisica quantistica. La proposta di Milano” within the IDIFO Master, Udine, (2006); M. Giliberti, L. Lanz, L. Cazzaniga, Quanta-MI: a modern teaching for modern physics in preservice teachers training, Paper presented at the international GIREP Conference (2002)

Description of the proposal

The proposal has been elaborated by the research group in Physics Education at the University of Milan. The group involves theoretical physicists, researchers in physics education and secondary school teachers. The main motivation that moved the Milan group to consider QFT has been the acknowledgment of failures in the traditional teaching of Quantum Physics.

At the GIREP conference, in 2002, the abstract of the paper presented by the group describes their approach as follows:

> *“In the context of the Italian National Project SeCiF (Teaching and Understanding in Physics) we have developed a “non-traditional” approach to Quantum Physics Teaching which is “field centered” and largely based on the results of some recent experiments. The key-points of our approach are four:*
> - *Physics is one and we want to show it in a coherent picture.*
> - *We don’t use Chronology as a reference frame to build this coherent picture.*
> - *Quantum Physics is not equivalent to Quantum Mechanics.*
> - *Physics is not ended in the thirties.*
>
> *The fundamental aspects of this approach will be here highlighted and the first research results of its experimentation in the pre-service teachers training courses of the SILSIS-MI (Inter University Lombard School of Specialization for Secondary Teaching) will be presented.”* (Giliberti et al. 2002).

The proposal has been implemented in contexts of pre-service teacher education (SILSIS) and in-service teacher education (for example the Master IDIFO). Several publications have been done[12] and an extended version has been prepared for and used within the at distance IDIFO Master, directed by M. Michelini (Giliberti, 2006)[13].

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[12] Giliberti M. (2001), A Modern Teaching for Modern Physics in Pre-Service Teachers Training to be published in First International GIREP Seminar “Developing Formal Thinking in Physics”; Selected Contributions; Udine, 2-6 September 2001; Giliberti M. (2002), Alcuni elementi di didattica della fisica moderna nel corso di “Teorie Quantistiche” della S.I.L.S.I.S.-MI, *La Fisica Nella Scuola, XXXV, 2 Supplemento, p. 56-75;* Giliberti M. (2002), Didattica della Fisica Moderna nella Scuola Superiore (I). Problematiche Didattiche, IFUM-729-DF* OTTOBRE, p. 1-10; Giliberti M., Lanz L. (2002), Didattica della Fisica Moderna nella Scuola Superiore (II). L’equazione delle onde per i campi materiali classici, IFUM-730-FT * , p. 1-8; Giliberti M. (2002) Didattica della Fisica Moderna nella Scuola Superiore (III). Un percorso per la Scuola Superiore, IFUM-731-DF *, p. 1-16; Giliberti M., Lanz L. (2002), Cross-section for the hard-core scattering from a sharp-edged body with cylindrical symmetry (A High-School Introduction), IFUM-735-FT*, p. 1-18; Giliberti M. (2002), A Modern Teaching for Modern Physics in Pre-Service Teachers Training. First International GIREP Seminar “Developing Formal Thinking in Physics”; Selected Contributions M. Michelini, M. Cobal; Udine, Forum, Editrice Universitaria Udinese

The proposal represents a unique example at international level because of its width and radicalism. Despite of the attention it received because of its originality and the "courage" of the researchers, it has been also criticized, mainly from other research groups engaged in designing proposals for teaching quantum physics at the secondary school level and from teachers, because of its epistemological bases and its incommensurability with respect to other approaches.

In this context it plays the role of reference for making more and more explicit the basic assumptions of our research and for testing the effectiveness of the analysis on foundations. The analysis of the proposal is based mainly on the materials for the IDIFO Master, although still unpublished, because their extension allows not only to have an overall view of the proposal but also to grasp how the overall view is implemented in detail.

For these reasons a detailed description of the proposal is reported but the remarks will focus only on those aspects that our analysis shows to be able to generate new arguments for research development.

As mentioned in the abstract reported above, the main objective of the proposal is to promote a unified approach to classical and quantum physics according to the statement:

> *The dream of the all Physics is to trace back the whole known reality to a unique principle (p. 21)*

For example, in the extended proposal for IDIFO Master (Giliberti, 2006), Giliberti points out that, while classical physics is presented following criteria concerning the conceptual apparatus of the discipline and without following necessary the historical development, quantum physics is usually introduced strictly following an historical path which relegates it to fragmented and disjointed hints. The result is that, after one hundred years since its establishment, the main features of the modern picture of the world are not yet a constitutive part of the current instruction.

In order to change such a situation and to purse the initial objective, a privileged "frame of reference" is chosen as point of view from which to look at and to reconstruct the whole body of physics knowledge:

> *"We choose a privileged point of view which seems so natural, that of an observer who is looking at the whole body of Physics knowledge starting from the Physics of today (...year 2006 d.C....) and who chooses one of the possible ontologies and picture of the world, and not, for example an historical frame of reference." (p. 18)*

The author points out that also the Newtonian mechanics is not currently taught as Newton elaborated it, but in a revised manner. The privileged framework chosen by the group is quantum electrodynamics, because of the following reasons:

> *"Quantum Electrodynamics is considered by everyone the best theory developed by men, even though in the '70 it has been incorporated in a*

ISBN 88-8420-148-9, p. 403-408; Giliberti M., Barbieri S. (2004), Teorie Quantistiche; CUSL, ISBN 88-8132-305-2, p. 1-103.

wider theory ([...] the "standard model") which, however, does not affect its fundamental epistemological aspects. On these bases we think that it [QED] has to be considered as the reference theory for teaching quantum theory." (p. 17)

In particular, the unified description is achievable, according to the authors, by introducing the description of the world based on the concept of field considered as fundamental entity.

"The central entity of the description is the field. [...] physical reality is attached to the fields while quanta are connected to the discretization of the normal modes occurring in the interaction. [...] Nevertheless the different fields have not the same epistemological "status": some fields appears more fundamental than others. Electromagnetic and electronic fields are, for instance, fundamental fields, while the protonic one is not because its dynamical behaviour could be explained in terms of quark and gluon fields.[...] So we obtain a stratification of the reality: at the basis there are the fundamental fields which, by means of the interaction, generate matter which, to a less refined glance, seems to constitute less fundamental fields and so on". (pp. 20 – 21)

As specified by the author this purpose implies a deep re–thinking of the way in which the classical physics is currently presented: It requires elements of fluid dynamics to be emphasized and matter fields to be introduced as soon as possible. In particular they suggest to start from macroscopic situations where continuous fields can be introduced for describing pressure and temperature, so that the "fundamental fields" can be qualitatively introduced as further levels of refinement.

The first part of the proposal is devoted to investigate the analogy between the classical electromagnetic and the matter beams. The idea lying at the basis of this choice is that the wave properties of matter do not need the introduction of quantization if high intensity beam are utilized. The perspective is implemented through the construction of a "classical optics of the material beam" (p.42) which starts from the investigation of the phenomenological analogies occurring in interference and diffraction patterns of light and matter beam.

Starting with the waves on the water surface and passing through the Davisson and Germer and Mach Zender experiments, the author arrives at the neutron interference.

The analogy is extended to the linguistic level: Electron, neutron and light, the first two remanding to a corpuscular behaviour, are banned in favour of electron/neutron/electromagnetic beam.

The analogies investigated experimentally are formalized by introducing the Klein Gordon Equation (KGE) to which a great unifying value is conferred, since it is considered a generalization of the electromagnetic wave.

The section devoted to the introduction of the free fields ends with the derivation of Schrödinger equation holding for classical fields, when the limit for slow varying fields is taken into account. The quantity representing the probability density in Quantum Mechanics is in this context interpreted as mass density.

The second part of the proposal is devoted to the introduction of quanta and interactions. The author writes:

> *"Atom is a suitable conceptual scheme that chemists use in order to describe interactions between substances (more than a material object, however tiny, which constitutes matter)."* (p. 54)

Because of the analogous behaviour between radiation and matter in free propagation, the analogy is extended by introducing quanta as the analogous of atom in the case of radiation and matter interaction.

A general scheme is hence written for discussing the photoelectric and Compton effect:

EM substance (1) + material substance (1) >> EM substance (2) + material substance (2)

In order to emphasize the continuum description in terms of field at the expense of a particle view, the impossibility of ascribing a particular trajectory "to the quanta" is explored in details by the analysis of specific experimental situations (double slit experiment, calcite crystals and Mach - Zender interferometer).

After the introduction of the particles as field quanta, the general framework of QFT is re-considered as the theory allowing a reconciliation between the free propagation described in terms of waves and the quantum features of interaction.

According to the general aim of reconstructing Physics from a QFT perspective, quantum mechanics is dealt with by focussing on a weak intensity perturbation of the field and on a single quanta interaction.

The last part of the proposal tries to bridge the quantum field theory notions and feasible lab - experiences (for example the Franck – Hertz experiment).

Remarks

General remarks

From an epistemological point of view the proposal stresses arguments like the hierarchical organization of the Discipline (*QFT as fundamental theory*) and the theory effective predictive capability for supporting the need of devoting research efforts to study the possibility of teaching QFT also at the secondary school level.

The reductionist epistemological perspective in its strong form ("There exists the most fundamental theory") (Kuhlmann, 2009), lying at the basis of the proposal, is usually supported from an educational point of view by invoking criteria of linearization and simplicity that would make learning easier.

Against such an idea other educational perspectives advocate the need of enabling students to cope with some forms of productive complexity for making them enter a scientific world view. Examples of productive complexities regard also the comparison of different paradigms and theoretical models characterizing the different physical domains. Especially for secondary school students from whom the acquisition of specialized knowledge is not required, an image of the discipline stemming from a critical analysis of "border problems" is considered advisable for conferring *cultural richness* to learning (Galili, 2009; Levrini et al., 2008a).

Our distrust toward the strong notion of reductionism implied in the Milan group proposal does not come only from a philosophical or a more general cultural perspective but also (and mainly) because of the many problems involved in its direct implementation in the physics curriculum design.

In practise, what does it mean for a researcher "observing the entire body of knowledge from a QFT point of view" for teaching at the secondary school level?

The ambitious attempt of observing the whole Physics from the QFT perspective clashes at once with the difficulties related to the interpretation of the extremely sophisticated formalism and its adaptation for a more elementary level.

In the proposal this problem is not directly addressed and the main strategy used for implementing the general goal is to centre the discussion on the concept of *field,* individuated as central entity by means of which a unified approach to classical and quantum physics can be pursued.

According to such a perspective, the continuum picture of the world is, with respect to the traditional teaching, emphasised along the whole curriculum and the particle picture re-dimensioned and limited only to the interpretation of interaction.

As far as the implementation of the strategy is concerned, at least two points are at issue:

1. the choice of hindering the deep differences between the classical and the quantum field, by playing only (or mainly) on linguistic similarities (the word "field") for making quantum field sound acceptable;
2. the ontological load put on the concept of quantum field for strengthening the argumentation, when it cannot be supported by the formalism.

As discussed also in the remarks about the Hobson paper, both these points are, in our opinion, questionable since they appear *tricks* for enacting intuitive and normative knowledge, built on classical phenomenologies, also in contexts where the basic concepts would deserve an extension and a deep revision, as stressed by significant changes in the formalism.

Also the problematic issues concerning the "particle interpretation of QFT" (Kuhlmann, 2009), the relationship between wave-particle and field-particle, and more in general the relationship between continuum and discrete formal structures are simply solved through a linguistic choice that explicitly cut off as much as possible the particle nature of matter: For instance the word "electron" is substituted by the expression "electron pencil".

> *"It becomes evident that, if we aim to an educational exposition of wave phenomena and material pencils, we will have to denote them with names not directly attached to their quanta [...] but more generic. Then we will speak of electronic, neutronic ecc...pencils. (p. 33)*

In the light of the previous remarks, we see the two strategies used in the proposal for implementing the general goal as risky simplifications.

Because of the emphasis on the electromagnetic field and on the role of analogy, the proposal seems to take up some features of the canonical quantization and, because of the lack of a sophisticated formalism, the problematic aspects of the standard approach are emphasized.

The analysis reported in Chapter 2 provides some hints for discovering what the two chosen strategies hinder: They will be however reconsidered in the next Section concerning the specific remarks.

Nevertheless we are pretty far for being able to evaluate if, for the secondary school level, the problems here stressed can be solved by other implementation strategies or if they require deeper changes in the overall perspective.

At this level of presentation, however, the used strategies hide so essential features of QFT that the proposal ambition of innovation seems at risk as soon as the general goals are implemented in an articulated discourse.

Specific remarks

In the Milan proposal a great unifying value is conferred to the KGE. In stressing the phenomenological analogy between matter and electromagnetic pencils, the author refers to a non-quantum theory of the material beams:

> *"Essentially we saw that the wave behavior can be described by means of the Klein-Gordon equation. It is very general and useful for the description of material and electromagnetic beams; in order to shift from a beam to another it is sufficient to change the value of a parameter (μ) which is different from zero (and variable according to the substance) for material beam and equal to zero for electromagnetic beam. The unifying value of this equation is very big; it generalizes what sensed experimentally: light and matter, in their free propagation, behave, for many aspects, in a similar way."* (p.52)

In Chapters 2 and 3 a detailed analysis of the classical meanings of the KGE has been carried out according to the mathematical expression of the solutions (real function, complex function and quantum field).

In the statement quoted above the author focuses only on the μ parameter and does not specify which is the mathematical tool describing the matter field; however, the strict parallel with the electromagnetic field leads to believe that the classical real solution (real function) of the KGE is considered (according to the fact that the amplitude of the components of electromagnetic field are described by real functions).

Nevertheless, as shown in Chapter 2, while in the quantum case (when the solution is a quantum field) the value of the μ (equal or different from zero) determines the distinction between photons and massive particles (relativistic and spinless), the same distinction between light and matter cannot be stressed in the classical case. Indeed, when classical real solutions of the KGE are considered the shift from a zero value to a non-zero value of the parameter μ does not imply the shift between the classical electromagnetic field to a "classical matter field" but the shift between the classical electromagnetic field in vacuum (d'Alembert equation) to the classical electromagnetic field in a medium (KGE).

In particular, the medium, in which the components of the classical electromagnetic field propagate according to the KGE, is the *cold plasma* (§ 2.4.1).

Also from an historical point of view (de Broglie, Schrödinger) "matter fields" were supposed to be described by complex functions however, as argued in Section 2.4.2, the field suitable for the interpretation of matter field as intended by Schrödinger does not satisfy the KGE.

C. "Particle, Feynman diagrams and all that", Michael Daniel (Physics Education, 41 (2), March 2006)

Description of the teaching proposal

The main goal of the teaching proposal described in the paper is to respond to an issue stressed by the English AS/A2 Physics Courses: The necessity of introducing topics concerning particle physics at the secondary school level.[14]
More specifically secondary school students attending the AS/A2 Physics Courses *"are expected to have knowledge and understanding of:*

- *The concepts of exchange particles to explain forces between elementary particles;*
- *Simple Feynman diagrams to show how a reaction occurs in terms of particle going in and out and exchange particles: limited to beta minus decay, beta plus decay, electron capture, neutrino – neutron collisions and electron – proton collisions [AS Module 1 of the AQA GCE Physics (Specification A)]*".

Similarly the AS Unit3: Topic 3C of the EDEX – CEL GCE Physics course demands knowledge and understanding of:

- Forces described in terms of exchange particles including photons, W^+, W^- and Z particles, and gluons.
- Use of simple Feynman diagrams involving exchange particles.

The paper aims to *"introduce the theory of quantum field and develop it sufficiently to gain an accurate qualitative understanding of the origin of Feynman diagrams as representation of particle interaction"*.
As remarked by the author:

> *"We hope this exposition will allow students to develop an understanding of Feynman diagrams that goes beyond the rudimentary drawing and labelling of the diagrams themselves. The article will also help to clarify some misconceptions, which have appeared in certain textbooks, regarding the structure of the diagrams".*

The specific choice is *"to keep the use of mathematics to an absolute minimum"*.

The introduction of Feynman diagrams and Standard Model, which constitutes the main goal of the proposal, is preceded by a wide section devoted to the introduction of quantum fields.

The specific choice made by the author is to stress the difference between classical and quantum field:

> *"So, a classical field is a quantity that is defined continuously over all space, and this quantity varies with time. The time variations are governed by appropriate wave equations. Quantum fields are different. Here a field becomes an operator. For every point in space the field can act on vacuum to create states of definite momentum and energy". (p.119)*

[14] AS/A2 Physics Courses and EDEX – CEL GCE Physics course refers to specific math and physics-based courses in the English colleges.

The change in the role played by the field is traced back to the quantization procedure:

> *"Quantization of a classical field is the formal procedure that turns a classical field into an operator capable of creating particles form vacuum". (p.120)*

Feynman diagrams are introduced as an intuitive way of visualizing the first few terms of the series expansion approximating the transition amplitudes of the processes involving scattering and decay particles.

The introduction of the diagrams concerning the electromagnetic interactions focuses on the following point: The quantum fields $\Psi_e\ (x)$ is related to the creation/annihilation of an electron/antielectron, $\bar{\Psi}_e\ (x)$ is related to the creation/annihilation of an antielectron/electron and the gauge field $A(x)$ associated with photons can interact at any point x.

> *"The electromagnetic interaction come about by allowing the quantum fields $\Psi_e\ (x)$, $\bar{\Psi}_e\ (x)$ and $A(x)$ to interact at any point x" (p.124).*

The potential processes occurring at each vertices of the diagram are then introduced by taking into account what each field can do at x.

After the introduction of the elementary processes the author builds up the whole diagram by combination of vertices and he remarks that, for instance in the case *electron – electron* scattering, "it is incorrect to claim that a photon is first emitted in x_1 and absorbed later at x_2 or viceversa".

Remarks

General remarks

QFT is not presented as a framework from which reconstructing the entire physics curriculum, neither as a whole from which extracting elements to be inserted in the curriculum as soon as possible.

The basic idea is to select those notions that are indispensable to pick up the fundamental aspects of Feynman diagrams.

In order to pursue the goal the author anticipates the choice of keeping the use of mathematics to an absolute minimum that corresponds, in fact, to a total absence of it in favour of a qualitative description of the objects and the processes.

In what manner and how much this choice affects the description will be investigated in the specific remarks.

Specific remarks

The first remark comes out from an analysis of the statement reported above (p. 119 – 120) by means of which the author introduces the quantum fields. A critical reading of the statement brings to light that some omissions create a confused situation between the two different spaces implicitly invoked: The Minkowski and the Hilbert (or Fock) space.

Quantum fields are described as objects able “to act on vacuum” in order to create states of definite momentum and energy without stressing that such states belong to the Hilbert space. This action is said to happen “for every point *x* in space” where now the space is the ordinary or the Minkowski one.

This introduction suggests a total - and questionable – identification between quantum fields and the usual quantum mechanics (a, a^{+}...) operator; for this reason it is not clear which is the additional value for introducing such new objects.

In Chapter 2 (§ 2.3.2) we provided arguments for pointing out why the quantum field (called field operator) does not take up *all* the features either of an operator or of a classical field; as far as the identification of the quantum field with an operator is concerned, it has been shown what can happen if the statements quoted above are taken literally (simply acting on the ground state of the Hilbert space with the entire quantum field): The resulting vector $\phi^{+}(x)|0\rangle$ is indeed not yet a well - defined Hilbert state, having an infinite norm.

As remarked by Haag: *“The quantum field ϕ at a point cannot be an honest observable. Physically this appears evident because a measurement at a point would necessitate infinite energy. The mathematical counterpart is that $\phi(x)$ is not really an operator in H_F ”* (Haag, 1996).

The analysis proposed in Chapter 2 brought to light to what extent a simplistic interpretation of the quantum fields is problematic and that the quantum field gets its meaning as a sophisticated formal mechanism of interfacing ordinary space (or Minkowski spacetime in the relativistic case) with Hilbert or Fock space: Far from being a function, according to which it is possible to calculate and connect it with every point of the space *x*, it is, formally, an operator valued distribution.

The main consequence is that the quantum field is mathematically and rigorously formulated as a distribution related to extended regions of the space (-time). As a result of the calculation of the quantum field an operator is obtained; since quantum measurements are expressed by operators, it is possible, for a given space (-time) region to express the observables by means of the quantum fields themselves (“*Physical interpretation of the quantum fields will not be primarily attached to particles but to local operations*”, Haag, 1996).

The potential of such a mathematical rigour for the interpretation of the quantum fields has been investigated as effective tool for implementing the concept of interaction as provided by second quantization.

The simplistic view of the quantum field, i.e. considering it as a function which creates particles, seems to generate some features of incoherence inside the proposal itself: As clearly stated by the author referring to Feynman diagrams in the case *electron – electron* scattering, it is incorrect to claim that a photon is first emitted in x_1 and absorbed later at x_2 or *vice versa* but this remark seems at odd with the introduction of the single vertices where the quantum fields $\Psi_e(x)$, $\Psi_e(x)$) and the gauge field $A(x)$ are introduced as interacting at point *x*.

We retain that the analysis of the proposal stresses a main methodological point: The choice of keeping the formalism to an absolute minimum, already recognized as a possible perspective in introducing some topics of contemporary physics to secondary school students, cannot leave aside a previous deep investigation about the formalism itself especially in the case of sophisticated formal object such the quantum fields. Beside the risk of being not precise in the description of these objects this would imply the loss of opportunity of catching the new conceptual synthesis that the formal objects provide.

D. "Conservation Laws, Symmetries, and Elementary particles", Dick Hoekzema, Gert Schooten, Ed van den Berg, and Piet Lijnse (The Physics Teacher, Vol. 43 (2), May 2005)
"Teaching conservation laws, symmetries and elementary particles with fast feedback", Ed van der Berg and Dick Hoekzema (Physics Education, 41 (1), January 2006)

Description of the teaching proposal

The proposal is situated within a wide project addressing the teaching of modern physics to secondary school students (age 17 – 18) in Netherlands.
The whole project consists in a series of 35 lessons (45 – 50 minutes each) devoted to introduce students to:

- particle – wave duality
- the Heisenberg principle
- probability models for properties of particles
- the particle in a box and applications
- elementary particles
- astrophysics

The analysis focuses on the proposal regarding the introduction to elementary particles which is developed by the authors on the base of two specific choices:

- the focus on *conservation laws* and *symmetries*;
- the focus on the *reaction diagrams* instead of Feynman diagrams.

As stated by the authors the reasons for the first choice can be traced back in what follows:

- *"The conservation laws provide a nice connection with the classical physics background of students;*
- *A focus on conservation laws and symmetries matches the current emphasis in elementary particles physics and is useful in other branches of physics as well;*
- *Using the laws and symmetries in reaction diagrams provides an opportunity for reasoning with main principles while an approach with lots of different particles (particle zoo) and reactions may present too many details, which will be forgotten anyway"* (Hoekzema et al., 2005, p.266).

The main reason for choosing the *reaction diagrams* lays in the fact that an earlier Dutch project focused on Feynman diagrams was not successful because "these turned out to be too difficult for secondary students" (van den Berg, Hoekzema, 2006, p. 48).

From the implementation of the proposal the students are expected to:

- *"apply symmetry principles in reaction diagrams and use these as a tool to determine whether or not reactions are possible and to predict alternative reactions. We do not expect students fully understand the connection between symmetry and conservation law";*
- *using reaction diagrams in order to "describe and predict reactions. They are not used to infer the probability of reactions or look deeper into the nature of interaction"* (Hoekzema et al., 2005, p.266).

Conservation principles are introduced as the core of reaction equations. Starting with an example from everyday life *("since keys do not dissolve when it rains, one could say that the number of keys is conserved in interaction with raindrops")* the authors remark that, in the reactions, not the number of electrons but the *lepton number* is conserved. The *lepton number* is defined as follows:

Lepton number = number of leptons minus the number of anti – leptons

Leptons are introduced as electron – like particles to which neutrinos can be associated (ν_e, ν_μ, ν_τ).
As an example, the β^- decay is reported

$n \rightarrow p^+ + e^- + \overline{\nu}_e$ (*)

The baryon number is defined in an analogue way.

Symmetries are said to be *"another closely related and convenient way of analysing reactions"* (Hoekzema et al., 2005, p. 267).

The starting point for discussing symmetries is the observation that crystal lattice presents the same structure with respect to rotations over 60°. As remarks by the authors: *"The principle of symmetry is that there is a property, the pattern of the crystal lattice, which does not change under certain rotations, in this case a rotation over 60°. Such a property is called a* symmetry property *and the operation is called a* symmetry transformation*".* (Hoekzema et al., 2005, p. 267)

Then the author focus on the fact that, in the case of symmetry transformation for particle reactions, the *symmetry property* allows to infer whether a reaction is possible: *"We then look at different symmetry transformations, each time following the same principle: We take an existing equation, change something, and ask whether the result can also occur in nature"* (Hoekzema et al., 2005, p.267).

The second part of the proposal is devoted to introduce the symmetries on which the proposal is focused: time reversal, charge reversal and crossing.

The symmetry properties are implemented with respect both to the notation used for reactions (*) (for example, *"time reversal symmetry means that the arrow in the equation can be reversed"*, p.268) and to the reaction diagram furnishing the graphical rules for representation (for example, that the arrow of an antineutrino pointing to the left means that the lepton number (-1) is opposite to that of an electron (+1); the arrow of positron points to the left (lepton number -1), just like the arrow of an antiproton (baryon number -1)).

The 2006 paper concerning the proposal concludes with a worksheet for implementing the "fast feedback methods" developed along two lessons for introducing the topics.

General and specific remarks[15]

The more evident aspect of the proposal is the inexplicit but fundamental choice of introducing elementary particles without any reference to QFT as also secondary textbooks usually do when they provide notions about the Standard Model.[16]

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[15] General and specific remarks are presented together since the specificity of the proposal would have made their distinction forced.

Another basic choice is to avoid the use of mathematics and Feynman diagram in favour of the use of reaction diagrams and graphical rules for implementing the concepts addressed by symmetries and conservation laws.

The third general choice is, as remarked by the authors, of neither expecting (and not investigating) the student achievement of relation between symmetry and conservation laws nor entering “deep into the nature of interaction” (Hoekzema et al., 2005, p.266).

A primary and general consideration is that the overall image of micro–world is identified with reactions between particles without taking into account other topics such as “what does it mean the world particle” or at least, what features can be attached to these objects.

The second consideration, which will be recalled in the specific remarks, is that these choices provide very synthetic tools for getting the goal stated by the authors: *“enabling students to predict whether or not certain reaction are possible and to derive new reactions from given ones by applying symmetries”*.

One of the most interesting aspects of the proposal, and highlighted by the authors at the very beginning of the papers, is keeping at a distance from the general tendency of introducing, at the secondary school level, elementary particles by *“listing a zoo of particles and reactions, resulting in disorganized and rather meaningless knowledge”* (van der Berg and Hoekzema, 2006, p. 47).

Symmetries and conservation laws surely provide a wide horizon, coherent and synthetic, for making their treatment of elementary particle be different form a simple list of particles.

The major point of perplexity comes from the analysis of the way through which this goal is supposed to be pursued by the proposal.

In particular it seems to be crucial the use of two different tools for the implementation: The reaction notation and the reaction graphic. The first tool plays the role of implementing the symmetries (time reversal, charge reversal and crossing) whilst the second tool is used for applying the conservation laws of lepton and baryon number.

The lack of a deep discussion about the relationship between symmetries and conservation laws makes the proposal at risk of substituting the “particle zoo” with a “rules zoo”. We retain that the proposal would gain a deeper cultural relevance only if an educational reconstruction of the Noether theorem had been carried out.

More in general the proposal seems to come from an experimental approach to contemporary Physics since the attention is only paid on the interaction processes that occur within the accelerators. Such an approach justifies somehow the absence of a theoretical framework.

Nevertheless, the lack of any mention to QFT cuts off at the roots the problems related to the particle interpretation of the field and the Dutch proposal does not present features of comparability with the Giliberti and Hobson ones. Although the different proposals address similar (if not the same) topics their overall perspectives are so

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[16] See for example: Tipler P.A., Invito alla Fisica, Zanichelli, 1991, translated by Scaramuzzi T., original reference: College Physics, 1987, Worth Publisher, Inc.; Amaldi U., Le idee della Fisica, La fisica Moderna, Zanichelli, 2001; Halliday D., Resnick R., Walker J., Zanichelli, 2001 2nd edition, original reference: Fundamentals of Physics – Extended, Fifth Edition, John Wiley & Sons, Inc., 1997, 1993, 1981, 1974).

different that the suggested images of the micro-world seem to be contradictory: A discrete particle image of the micro-world *vs.* a continuum field centred image.

Only a critical analysis made up of elements taken from the Foundation of Physics would allow student to reconcile the two perspective and to understand how it is possible that the same object (for example electron) could be described as "quanta of a space filling quantum field" on one side and as an entity exclusively attached to its lepton number on the other.

4.3 Final comments

The studies on contemporary physics are still few within the field of PER. Its increasing number seems however to document a growing interest on the topic.
The existent proposals are very different from each other because of the different implications on the overall curriculum or because of the approach.

Whilst the Milan approach implies a deep reconstruction of the whole curriculum the other ones suggest only additional elements to be included in the current curriculum.

In the papers the theoretical or experimental origin of the approach is recognizable: Whilst Giliberti and Hobson focus on the QFT and its continuum underlying structure (by emphasising the concept of field), the other paper focus on the Standard Model and particle interactions (by emphasising interaction processes).

The analysis carried out for each teaching proposal shows that at least two kinds of problems can be pointed out:

1) problems related to the implementation of the general ideas;
2) interface problems related to the different approaches that create a so big distance between them to make their comparison difficult.

The analysis on the Foundations of QFT presented in Chapter 2 allows us to make the hypothesis that most of the problems pointed out can be ascribed to the historical fact that the research on the foundations of contemporary physics is very young.

Many of the implementation problems regard indeed crucial points on which the formalism in QFT plays a constitutive role and whose interpretation is still an open problem within the research on foundation and philosophy of QFT. The solutions found in PER still appear nothing but a first attempt on which further researches are necessary.

Even though the found solutions are surely comprehensible to the student, they however seem to miss or to misrepresent essential elements of how QFT shapes the micro-world.

They indeed cut the problems too sharp or seem to simply shift the focus of the problem itself; we are referring in particular to:

- the cut off differences between the force field (for example electromagnetic field) and the matter field (for example the KG or Dirac one) in the Hobson and Giliberti proposals due to the non-problematized extension of the phenomenological analogy to a linguistic and an ontological level;
- the cut off differences between field operator and operators in Quantum Mechanics operated by Daniel by means of an implicit overlapping of ordinary space and Hilbert space;
- the shift, operated in the forth proposal analyzed, from a rightly questionable "particle zoo" to another kind of zoo, made of simple rules and graphical

representations that, even thought more subtle, still provides an hyper-simplified image of the micro-world.

The conceptual analysis reported in Chapter 2 and reconsidered in the specific remarks of each proposal gives an interpretation of the formal analogy between the fields in QFT as well as an interpretation of the meanings laying behind the concept of field operator. Such an analysis highlights what is missed as soon as the formalism is taken off as Hobson and Giliberti on one hand and Daniel on the other hand make.

Changing only a little bit what Sundarshan and Rothman say about the problems concerning the two-slits experiment and the problematic analogy between the electromagnetic field and the quantum mechanical wave function (Sundarshan and Rothman, 1991), we can say: *"We see that extreme care is necessary to treat the matter fields as well as the concept of fields operator or else one runs the risk of being seduced by the laws of analogy and pictorial images. The basis of medieval astrology and alchemy was that Man, the microcosm was a miniature replica of the universe, the macrocosm. The laws that governed the universe at large must then, by analogy govern the behaviour of Man. In other word duplicate the form duplicate the content. However, as we know, analogies breakdown including that between the electromagnetic field and the Klein Gordon field and that between the field operator and the operators in Quantum Mechanics"*.

Although the points where analogies and pictures break down have been pointed out, further research is needed in order to reconstruct the contents from an educational point of view, so as to save and exploit the essential conceptual meaning and minimize formalism without making the overall conceptual structure collapse.

On the basis of the previous remarks, the analysis carried out on the QFT foundations assumes its educational specificity: It provides effective keys for entering the existing teaching proposals by moving the debate about them from general educational and epistemological issues to specific problematic foci belonging to the content interpretation.

CONCLUSIONS

The research work presented here concerns the analysis of the foundations of Quantum Field Theory (QFT) carried out from an educational perspective. The whole research has been driven by two questions:

- How the concept of *object* changes when moving from classical to contemporary physics?
- How are the concepts of *field* and *interaction* shaped and conceptualized within contemporary physics? What makes quantum field and interaction similar to and what makes them different from the classical ones?

The questions have been chosen for their genuine "conceptual" character, philosophical interest and for their transversal feature within physics: Such properties of the leading questions allowed the analysis on the foundations of QFT to be systematically carried out keeping an eye on its educational and cultural relevance.

The whole work has been developed through several studies: Three of them are more focused on answering the research questions by analyzing the foundations of QFT; Two of them are explicitly devoted to tie the analysis to issues debated within physics education research.

More specifically, the research work produced:

1. A study aimed to analyze the formal and conceptual structures characterizing the description of the *continuous systems* that remain *invariant* in the transition from classical to contemporary physics.
2. A study aimed to analyze the changes in the meanings of the concepts of *field* and *interaction* in the transition to quantum field theory. Such a study has been carried out by comparing different approaches to theoretical physics known in literature (in particular the approach known as "canonical quantization" and the "axiomatic approach").
3. A detailed study of the Klein-Gordon equation aimed at analyzing, in a case considered emblematic, some interpretative (conceptual and didactical) problems in the concept of field that the canonical quantization does not address explicitly.
4. A study concerning the application of the "Discipline-Culture" Model elaborated by I. Galili to the analysis of the Klein-Gordon equation, in order to reconstruct the meanings of the equation from a cultural perspective.
5. A critical analysis, in the light of the results mentioned above, of the existing proposals for teaching basic concepts of QFT and particle physics at the secondary school level or in introductory physics university courses.

The work produced original results both in the research field of the foundations of contemporary physics and in the research field of physics education. Among these, we mention:

- the production of new arguments for showing in what sense the axiomatic approach is more effective than the canonical quantization for avoiding conceptual short-circuits that can prevent university students to grasp the peculiarities of a quantum field with respect to the classical ones;
- the demonstration of the non-existence of a physically meaningful Klein-Gordon classical field and, hence, the production of an articulated and detailed argument for supporting Ryder's claim: *"Klein-Gordon field is a strictly quantum field"*;
- the reconstruction of the *cultural status* of Klein-Gordon equation in the various physical domains, according to the Discipline-Culture Model, that allowed to clarify how the physical meaning of the equation changes when the

mathematical expression of its solutions changes (real function, complex function, quantum field – real and complex);

- the construction of effective criteria for pointing out some specific problems in teaching and communicating QFT, related not only to the difficulties of its sophisticated formalism, but also to the lack of a systematical analysis of its foundations aimed at highlighting its conceptual relevance.

For the research topic and the research methodology, the work represents one of the few experiments, at international level, aimed at exploiting the conceptual relevance of the theoretical physics formalism by means of tools and "ways of looking" typical of the research in physics education. The results obtained allow us to conclude that the work carried out has been a successful test for physics education research as well as a further step in the process that physics education research is going through for establishing its own identity.

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