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# Essays in Public Economics: Behavioural Responses to Taxes, Public Good Provision, and Social Welfare

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## Abstract

This thesis presents empirical and theoretical contributions to three key topics in the field of public economics: (i) individual behavioural responses to taxation, (ii) optimal public good provision and taxation, and (iii) social welfare theory. The thesis consists of three substantive chapters, with each one analysing one specific topic.

The first chapter of the thesis analyses the responses of self-employed individuals to the incentives of the tax system in Italy. I exploit the discontinuity created by the eligibility threshold of the preferential turnover tax regime to estimate how self-employed individuals adjust turnover – i.e. revenues – in response to taxes. I find substantial and significant bunching by solo self-employed below the turnover threshold of the preferential regime. By combining bunching techniques and a newly developed theoretical framework describing the individual choice between a turnover tax regime and profit-based tax system, I estimate the elasticity of turnover in three sectors of the economy: professional services, retail and accommodation and business intermediaries. Professionals show the largest response. The results show the key driver of observed responses is the incentive given by low taxation in the preferential regime, rather than lower costs of compliance.

The second chapter presents a theory of optimal provision of a (risky) public good when individuals have heterogeneous preferences for risk. People with different attitudes to risk have different views on the extent to which society should invest in certain risky projects. I investigate how these different views should be taken into account for the choice of the optimal policy. The public good has an insurance purpose as it allows individuals to shift risk from private to public consumption. Private provision of the public good is inefficient because people do not internalise the insurance gains of the other agents. However, public provision might fail to achieve the (ex-ante) first best outcome if agents cannot be compensated when the policy does not reflect their specific risk preferences. In an application on capital income and endowment taxation, I show it is possible to improve welfare by exploiting the different choices of the agents with different risk preferences. The issue of the choice of the welfare criterion to use to choose the optimal policy is explored in the following chapter.

The third chapter proposes a new welfare criterion to evaluate social options in the presence of risk and heterogeneous attitudes to it. Governments are often required to make decisions under risk. However, it is not clear how society should evaluate such choices when individuals have different attitudes to risk. The proposed welfare criterion applies to the specific case of constant relative risk aversion utility function, and it is obtained by modelling a three-stage lottery of life in which individuals face an identity lottery over risk preferences and personal characteristics, and an outcome lottery over possible social states. The criterion is derived using Bayesian rationality by evaluating the extended lottery of life ex-ante. Numerical examples confirm that the criterion is able to accommodate different risk preferences, and show that redistribution among agents with different incomes but equal risk preferences is welfare-enhancing.

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## Data Provision

The first chapter of this thesis contains empirical analyses which are conducted using administrative data ("*Frame SBS-sistema integrato di dati amministrativi e dati di indagine per la stima degli aggregati economici sulle unità giuridiche*") from the Italian National Statistics agency (ISTAT). The analyses were conducted at the "*Laboratorio per l'Analisi dei Dati ELEmentari-ISTAT*" following the regulations on statistical confidentiality and personal data protection. Results and opinions expressed in this thesis are my own and do not represent official statistics from the agency.

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Chapter 1

# Estimating the Elasticity of Turnover from Bunching: Preferential Tax Regimes for Solo Self-employed in Italy

## 1.1 Introduction

Stimulating entrepreneurship is key to generate economic growth. In several developing and advanced economies, policy makers attempt to foster business activity by setting up preferential tax regimes for certain categories of workers, like self-employed, and small-medium enterprises. The idea behind this policy is that simpler tax regimes with a lower tax burden would attract entrepreneurs, encouraging them to grow their businesses within the formal sector. These simplified schemes often feature some form of taxation of gross reported revenues, such as turnover taxation, as opposed to the standard profit-based tax regimes for businesses and corporations.[1]

The seminal paper by Diamond and Mirrlees (1971) advises against turnover taxation as it violates production efficiency. However, policy makers often deviate from this theoretical benchmark as turnover taxation makes compliance easier for small businesses and is more difficult to evade. Moreover, when there is incomplete tax enforcement and evasion is possible, Best et al. (2015) argue that production-inefficient tax regimes might actually enhance welfare as efficiency losses are more than outweighed by higher revenue efficiency due to increasing compliance. As turnover taxation receives more attention as a policy tool, its effects on behaviour are worth-exploring. Even though turnover is a key indicator of economic activity, we still know little about how much firms in different sectors actually adjust it as a response to taxation.

This paper contributes to fill this gap by investigating how solo self-employed adjust sales turnover in presence of a preferential tax regime.[2] To do so, I exploit the notch created by the eligibility cut-off of the preferential turnover tax regime for solo self-employed in Italy. Then, I use a new theoretical framework to estimate the turnover tax elasticity in three sectors: Professionals, Retail & Accommodation, Business Intermediaries. Since turnover is strictly related to output, after accounting for prices, analysing such responses is extremely important for both academic research and policy-makers.

The Italian tax system provides a suitable framework to address this question as tax liabilities for solo self-employed in preferential regimes depend on the level of turnover. If turnover is below a certain threshold, Italian solo self-employed can opt out of the ordinary tax regime and choose to be taxed at a preferential rate. In addition to tax advantages, the preferential regimes also have simplified compliance procedures. Conversely, if turnover is above the cut-off, higher average tax rates apply as the ordinary tax regime remains the only option. This type of discontinuity in the tax schedule is a "notch", which can be exploited to estimate turnover responses to taxation.

I use administrative data from ISTAT on all self-employed operating in Italy between 2012 and 2019.[3] In this period, self-employed could choose between the ordinary

[1]Such schemes, also called presumptive regimes, have been adopted in several developed countries, including Austria, France, Italy, Spain (Bucci, 2020), as well as in developing countries, including Brazil, Mexico, Pakistan and Zambia (Best et al., 2015).

[2]Solo self-employed are self-employed individuals who work without collaborators or employees. The share of solo self-employed in self-employment is increasing in many OECD countries (Boeri et al. 2020).

[3]ISTAT is the National Statistics Agency in Italy.

tax regime and a scheme with potentially preferential tax-rates and simplified compliance procedures. The ordinary tax regime is moderately progressive and includes personal income-tax, social security contributions and VAT. The preferential tax regime would exempt self-employed from VAT, and replace the progressive personal income tax schedule with a proportional levy on taxable income ("a flat tax"). The turnover tax regime is one example of the preferential schemes being introduced in Italy, with the tax base being its distinguishing feature. While the ordinary regime taxes profits, the turnover regime defines the tax base as a sector-specific share of turnover, resulting in different tax incentives across sectors.

The main analysis of this paper exploits the notch in the tax schedule generated by the eligibility threshold (€65,000) of the preferential turnover regime in 2019. First, I use bunching techniques (Chetty et al., 2011; Kleven and Waseem, 2013) to estimate the turnover responses. The excess mass below the threshold is 357% of the counterfactual frequency at the threshold. The sector-specific analysis shows that Professionals, Business intermediaries and Retailers have the largest observed turnover responses. Second, for these three sectors, I use a new theoretical framework that matches the institutional set-up of Italy to estimate the structural elasticity of turnover. After accounting for the additional hassle costs due to VAT filing in the ordinary regime, the most responsive groups are Professionals and Business intermediaries with estimated structural elasticities of 0.066 and 0.047 respectively.

This paper makes three main contributions to the literature. First, while the existing evidence of bunching largely focuses on taxable income adjustments,[4] this paper focuses on responses to taxation of sales turnover; turnover is a specific component of taxable income for self employed and a key indicator of economic activity. I show that individual entrepreneurs in Italy adjust the level of revenues as a response to financial incentives of the tax system. Solo self-employed bunch below the turnover threshold, set by the tax code, to qualify for a preferential tax scheme.

Turnover responses to taxation are studied by Harju et al. (2019) and Liu et al. (2021) in the context of VAT registration thresholds, and by Aghion et al. (2022) with regard to the preferential regimes for self-employed in France. The first two studies show that businesses bunch below the VAT registration threshold.[5] Harju et al. (2019) find that compliance costs due to VAT tax filing explain most of the observed bunching of small firms in Finland, so that the estimated elasticity of value added is quite low.[6] Then, Liu et al. (2021) find that bunching is more likely when corporations have lower inputs-sales ratio, higher proportion of business-to-consumer sales, and lower mark-ups. Differently from Harju et al. (2019), and Liu et al. (2021), this paper investigates the responses at the threshold where there is an overall change of the taxation of solo self-

[4] Saez (2010) for the US, Chetty et al. (2011) for Denmark, Kleven and Waseem (2013) for Pakistan, Bastani and Selin (2014) for Sweden, Adam et al. (2021) for the UK, Massenz and Bosch (2023) for corporations in the Netherlands.

[5] They assume the VAT incidence falls, at least partly, on entrepreneurs.

[6] This is motivated by the low VAT threshold in Finland (€8,500), so that the estimated compliance costs (€1,300) are relatively more important than the incentives generated by the VAT rate.

employment income including, but not limited to, VAT. This seems to be a more suitable case to study how turnover responds to tax incentives, aside from compliance costs related to the tax system. Indeed, I find that the financial incentive of the tax system is still the main driver of turnover responses. Finally, while Aghion et al. (2022) stress the importance of tax simplicity and evasion responses, my findings might be evidence that real responses play a role as bunching remains large after excluding self-employed with reported turnover being multiple of one thousand (round-number bunching).

Second, building on Kleven and Waseem (2013), I develop a new theoretical framework that describes the behaviour of agents choosing between a profit-based tax regime and a turnover tax scheme if they are located below a certain eligibility (turnover) threshold. The type of discontinuity in the tax schedule that is modeled is a non-standard notch. In the theory of notches by Kleven and Waseem (2013), the elasticity is estimated by solving the indifference condition of the "marginal buncher" who faces the same average tax rate above the threshold as every other agent.[7] That is because the cut-off and the tax base are both expressed in the same terms: taxable income. In this case, exceeding the cut-off of the preferential turnover scheme involves a joint change of tax rate and tax liability, but also a change in the tax base. Above the turnover threshold, agents are taxed on actual profits, so that tax incentives vary across individuals with equal turnover. The theoretical framework developed in this paper fits the empirical evidence in Italy and allows to isolate the specific tax incentive that is faced by the marginal buncher.

Third, I use the new theoretical framework to estimate the elasticity of turnover in three different sectors of the economy: Professional services, Retail & Accommodation, Business intermediaries. While previous papers have shown that self-employed are more responsive to discontinuities in the tax schedule than employees (Chetty et al., 2011; Bastani and Selin, 2014; Adam et al., 2021), this paper also documents that there is heterogeneity in responses and elasticities across different types of self-employed individuals. To the best of my knowledge, this paper provides the first example of sector-specific estimation of the tax elasticity of turnover.

This paper also relates to the policy discussion regarding the opportunity of taxing different types of income differently by setting up preferential tax regimes for certain taxpayers. Adam and Miller (2021) discuss the different tax rules applying to wage-earners, self employed and business owners' income in the UK, and argue that preferential tax regimes could create inefficiency, unfairness, complexity and revenue losses for the government. This paper shows that this might also be the case in Italy: many solo self-employed declare revenues up to the eligibility thresholds for the preferential tax regime. If that is due to tax planning/evasion, then the preferential tax regime is eroding the tax base and therefore causing revenue losses for the Treasury. If bunching is due to self-employed limiting their growth in sales, then the tax system is also encouraging businesses to remain small, which is potentially detrimental to economic growth.

[7] The marginal buncher is the individual who is just indifferent between bunching and not bunching.

The rest of the paper is organised as follows. Section 1.2 outlines the institutional background and the data being used. Section 1.3 presents the evidence of bunching on turnover, including the sector-specific analysis. Section 1.4 describes the theoretical framework that is used to estimate the turnover elasticity. Section 1.5 provides structural estimates of the turnover elasticity in the different sectors. Section 1.6 concludes.

## 1.2 Institutional Background and Data

### 1.2.1 Tax regimes for Solo Self-employed

In Italy, self-employed have two options for income taxation: i) the ordinary tax regime; ii) one of the existing preferential tax regimes. The first one includes the personal income tax schedule (table 1.1), social security contributions (table 1.2), and VAT. The income tax schedule is piece-wise linear with five brackets. Social security contributions include a fixed component that applies below the basic threshold, and a variable component that applies between the basic and top threshold. In addition, the contribution rate of the variable component rises by 1 p.p. between the middle and top threshold. No contributions are due on the part of income exceeding the top threshold. Sellers charge VAT on their sales, remit it to the tax authorities every three months, and claim back the VAT paid on inputs of production. The standard VAT rate was 21% in 2012-2013, 22% from 2014 onwards, and it applies to most goods and services.[8]

In the 2010's, two preferential tax schemes were introduced, allowing solo self-employed to choose whether or not to access a regime with a simplified tax schedule and easier compliance procedures.[9] These schemes provide lower income tax rates and/or a different tax base on which reduced rates apply. Moreover, these schemes also provide exemption from VAT, meaning that the turnover cut-off to access them coincides with the VAT registration threshold. In each year, solo self-employed only had one alternative option to the ordinary regime. I now provide further details of the turnover scheme (F-regime from now on).

**Table 1.1:** Ordinary regime: Income tax rates 2012-2019

| | Personal Income Tax Rates | | | | |
|---|---|---|---|---|---|
| | Starting | Basic | Middle | Higher | Top |
| Thresholds (€) | 0 | 15,000 | 28,000 | 55,000 | 75,000 |
| Tax rates | 23% | 27% | 38% | 41% | 43% |

[8]Italy has two reduced VAT rates: 4% for food and agricultural products; 10% for energy and gas used by households.

[9]These include an exemption from filing VAT reports and bookkeeping for income tax purposes. However, entrepreneurs must keep all documents they receive and produce for their transactions.

**Table 1.2:** Social security contributions

| | Contributions | | Thresholds | | |
|---|---|---|---|---|---|
| Year | Variable | Fixed | Basic | Middle | Top |
| 2012 | 21.4% | €3,200 | €14,930 | €44,204 | €96,149 |
| 2013 | 21.8% | €3,360 | €15,357 | €45,531 | €99,034 |
| 2014 | 22.3% | €3,460 | €15,516 | €46,031 | €100,123 |
| 2015 | 22.7% | €3,540 | €15,548 | €46,123 | €100,324 |
| 2016 | 23.2% | €3,610 | €15,548 | €46,123 | €100,324 |
| 2017 | 23.6% | €3,680 | €15,548 | €46,123 | €100,324 |
| 2018 | 24% | €3,790 | €15,710 | €46,630 | €101,427 |
| 2019 | 24% | €3,830 | €15,878 | €47,143 | €102,543 |

Notes: These rates apply to the sector of commerce that includes wholesale, retail trade and other self-employed. The contribution rate applies between the basic and middle threshold, and then rises by 1 p.p. for any profit between the middle and top threshold. No contributions are due on profits exceeding the top threshold. Slightly different contribution rates apply for members of professional associations.

***Preferential turnover tax scheme: F-regime***

From 2016, the F-regime is the main preferential tax scheme for solo self-employed in Italy.[10] Table 1.3 shows the sector-specific turnover cut-offs that solo self-employed could not exceed if they wanted to choose this regime. Eligibility for the preferential scheme in year $T$ requires sales turnover to be below the threshold in year $T-1$. The largest group of taxpayers – including lawyers, doctors, professors, architects and other professionals – faces the €30,000 threshold in 2016-2018. In 2019, Law no. 145/2018 equalises the cut-offs to €65,000 across sectors.

The new scheme exempts taxpayers from VAT and replaces the income tax schedule with a proportional tax rate (15%).[11] Moreover, it grants a 35% reduction in SSCs for artisan enterprises and shopkeepers, that are mostly part of the Retail & Accommodation sector.

Differently from the ordinary regime, the tax base is a pre-determined share of turnover set by the tax code (see table 1.3). This serves as a notional measure of profits on which tax rates apply, meaning that the tax liability does not depend on actual profits. The effective preferential tax rate on turnover is therefore given by the social security contribution rate plus the statutory tax rate multiplied by the share of taxable turnover (net of social security contributions).

As the F and the ordinary regimes have different tax bases, the incentives at the threshold will be heterogeneous across agents. Given the statutory tax rates in the F and ordinary regime, the incentive to bunch will depend on the difference between the notional profits (tax base in the F-regime) and the actual profits (tax base in the

[10]Before 2016, another preferential tax regime was available for solo self-employed - more in the appendix.

[11]The tax rate reduces to 5% if the business is less than 5 years old.

**Table 1.3:** F-regime rules by sector in 2016-2018 (2019)

| Sector | Turnover cut-off (thousands €) | % of Taxable Turnover | Tax rate (%) |
|---|---|---|---|
| Real estate | 25 (65) | 86 | 15 |
| Business Intermediaries | 25 (65) | 62 | 15 |
| Professionals | 30 (65) | 78 | 15 |
| "Other activities" | 30 (65) | 67 | 15 |
| Food & beverage | 45 (65) | 40 | 15 |
| Retail & accommodation | 50 (65) | 40 | 15 |

Note: the tax rate drops to 5% if the business is less than 5 years old. Turnover cut-offs in 2019 are in parentheses.

ordinary regime). Even if the statutory tax rate in the F-regime is quite low, compared to the ordinary regime, it's not certain any agent is better-off bunching: an entrepreneur with relatively low (actual) profits might pay less in the ordinary regime, and given her preferences, non-bunching might turn out to be optimal. This implies that the F-regime threshold is a notch without a clear dominated region.[12]

### 1.2.2 Data

This paper uses administrative data from the Italian National Institute of Statistics. The dataset includes the universe of businesses operating in Italy in the period 2012-2019. The data contain information on annual revenues from sales, net of VAT, costs for intermediate inputs of production (like goods and services), personnel expenditures, and profits. The dataset also includes the number of people employed, the specific sector in which the entrepreneur operates, and whether the business is qualified as "artisan", and therefore eligible for the reduction in SSCs in the F-regime. For the purpose of this project, I restrict the sample to self-employed without collaborators and employees, as these are the individuals that can qualify for preferential tax schemes in Italy by complying with the (turnover) eligibility threshold. Table 1.4 shows some descriptive statistics for selected taxpayers with turnover between €40,000 and €100,000. This is the sample used in the main bunching analysis around the turnover threshold (€65,000) of the F-regime. We can observe heterogeneous average profit rates across sectors, with the highest average profits for Professionals and the lowest in the Retail & Accommodation industries.[13] Then, I also consider self-employed with collaborators and firms with employees for placebo tests.

[12] Another mechanism, which was analysed by Harju et al. (2019) and Liu et al. (2021), involves VAT. Conditioning on the level of turnover, the incentive (to bunch) generated by the VAT exemption will be stronger for agents with higher value added. However, this is relevant only if VAT incidence is split between consumers and the providers of goods or services with pass-through of VAT on to prices. For most sectors, this mechanism seems less important. The only exception is Retail and Accommodation, which is discussed further in the appendix.

[13] For the preferential turnover regime, profit rate heterogeneity across sectors explains why different taxable shares of turnover were chosen for different sectors as the notional profit levels that form the tax base.

**Table 1.4:** Descriptive statistics, 2012 − 2019

| | **Self-employed statistics** ($n = 4,808,990$) | | | |
|---|---|---|---|---|
| | Turnover | Inputs | Profits | Profit rate |
| Mean | 62,018 | 23,142 | 34,503 | 0.565 |
| Median | 58,575 | 16,526 | 33,208 | 0.605 |
| sd | 16,498 | 32,703 | 19,611 | 0.279 |

| | **Sector profit rate and shares of taxpayers** | | | | | |
|---|---|---|---|---|---|---|
| | Professionals | Other Actvities | Real Estate | Retail & Accom. | Business Intermediaries | Food & Beverage |
| Mean | 0.757 | 0.506 | 0.523 | 0.265 | 0.704 | 0.260 |
| Median | 0.800 | 0.484 | 0.506 | 0.223 | 0.739 | 0.234 |
| sd | 0.199 | 0.258 | 0.226 | 0.183 | 0.166 | 0.148 |
| Sector shares | 0.375 | 0.196 | 0.169 | 0.166 | 0.082 | 0.012 |

Note: The sample includes self-employed with turnover between €40,000 and €100,000. Taxpayers are categorized by Statistics Italy's industrial classification (ATECO 2007).

## 1.3 Bunching Estimation and Evidence

In this section, I present the methodology and the evidence of turnover responses. First, section 1.3.1 presents the bunching techniques that are used to estimate the counterfactual distributions of turnover. Then, I provide evidence of bunching at the eligibility threshold of the F-regime (€65,000 in 2019) for the whole sample (section 1.3.2) and in each sector (section 1.3.3). Finally, section 1.3.4 provides evidence of optimisation frictions affecting the choice of tax regimes.

### 1.3.1 Estimating the Counterfactual Distributions

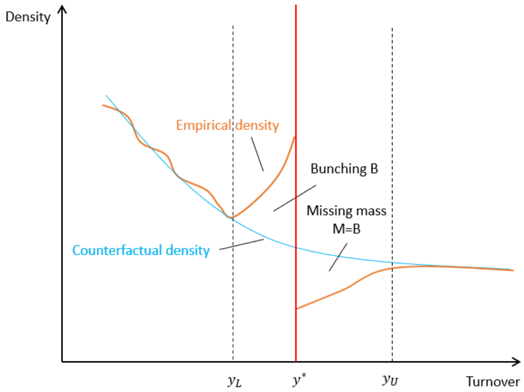
The bunching method requires the estimation of the counterfactual distribution that would have existed in the absence of the notch which will be compared to the empirical distribution. In this section, I describe two procedures to estimate the counterfactual distributions. The first one is the standard method of Chetty et al. (2011) and Kleven and Waseem (2013). The second one is an adaptation of the standard method that is used to estimate the counterfactual distribution when we consider only F-regime-taxpayers located below the threshold.

The standard method entails fitting the observed distribution with a flexible polynomial, excluding an area around the threshold $y^*$, such that the estimated bunching mass below the threshold equates the missing mass above it (Figure 1.1, Panel A). Observations are grouped in bins denoted by $j$ of size $s$ in such a way that the the upper bound $y_j$ of bin $(y_j - s, y_j]$ at the turnover threshold $y^*$ coincides with the threshold itself. Hence, all taxpayers bunching at the threshold $y^*$ will be part of bin $(y^* - s, y^*]$.

**Figure 1.1:** Estimating the Counterfactual Distribution of Turnover

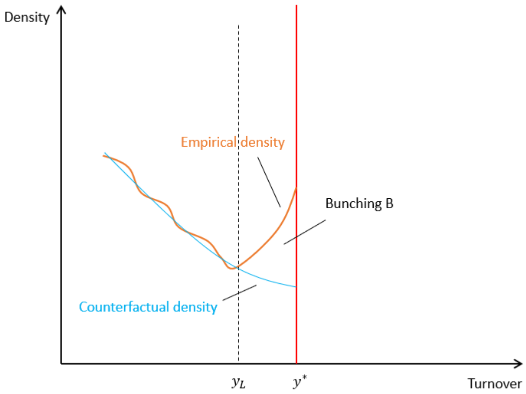
## Panel A - Standard Method

**(a)** All taxpayers

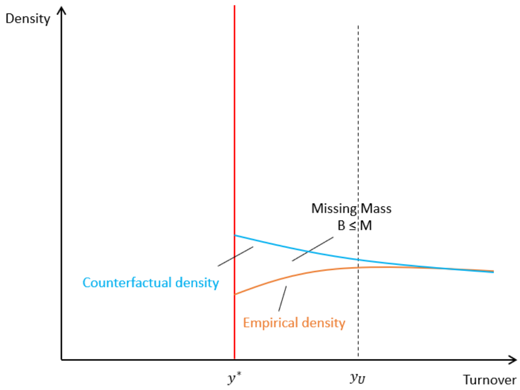



## Panel B - Alternative Method

**(b)** F-regime taxpayers




**(c)** Ordinary regime taxpayers



I run the following regression excluding the region $[y_L, y_U]$ around the threshold,

$$c_j = \sum_{i=0}^{p} \beta_i \cdot (y_j)^i + \sum_{i=y_L}^{y_U} \gamma_i \cdot \mathbf{1}[y_j = i] + \nu_j \tag{1.1}$$

where $c_j$ is the number of taxpayers grouped in bin $j$, and $y_j$ is the turnover level in bin $j$. In view of round-number bunching, I omit taxpayers declaring revenues that are multiples of €1K for the benchmark estimation.[14] Then, I extrapolate the fitted distribution to the cut-off using the fitted values of the regression $\hat{c}_j = \sum_{i=0}^{p} \hat{\beta}_i \cdot (y_j)^i$ for $[y_L, y_U]$. Excess bunching is defined as the difference between the observed and counterfactual density to the left of the threshold in $[y_L, y^*]$, that is $\hat{B} = \sum_{j=y_L}^{y^*} (c_j - \hat{c}_j)$. The lower bound of the excluded area $y_L$ is chosen at the point where the turnover distribution begins to increase, i.e. when bunching behaviour starts. Then, the upper bound is chosen such that the estimated excess bunching to the left of the threshold $\hat{B}$ equals the estimated missing mass to the right of the threshold in $[y^*, y_U]$, that is $\hat{M} = \sum_{j>y^*}^{y_U} (\hat{c}_j - c_j)$.

An alternative approach is necessary when we restrict our sample to F-regime-taxpayers only, as we cannot exploit the part of the empirical turnover distribution above the threshold. This is because nearly all self-employed in the F-regime are located below the threshold, as that is the main requirement to access the preferential regime.[15] These individuals have moved below the turnover threshold and opted out from the ordinary regime. Hence, we should be able to observe the missing mass above the threshold by plotting the turnover distribution of ordinary-regime taxpayers for the years before and after 2018, when the eligibility threshold was raised to €65,000. The key idea behind this strategy is that the excess bunching of F-regime taxpayers below the eligibility threshold should be lower than or equal to the missing mass in the turnover distribution of ordinary-regime-taxpayers above the threshold (Figure 1.1 Panel B). The counterfactual density is estimated using (1.1) by exploiting only the region of the empirical density below the threshold that is not affected by bunching. Hence, the upper bound of the excluded area coincides with the threshold itself: $y_U = y^*$. The difference between the empirical and counterfactual density below the threshold will provide the estimate of excess bunching.[16]

Finally, in line with the bunching literature, I use a residual-based bootstrap procedure to estimate the confidence intervals. A large number of turnover distributions are estimated by random resampling of residuals in (1.1), with which new estimates of the counterfactual distribution are obtained. Then, the 95% confidence interval is obtained from the distribution of the estimates of the parameter of interest.

[14] Including these observations would require to add round-number fixed effects to the regression for the counterfactual estimation.

[15] Few taxpayers are located above the threshold and will exit the regime in the following year.

[16] For the counterfactual turnover distribution of ordinary regime taxpayers above the €65K threshold, I use the distributions in the period 2013-2017 when €65K was not the F-regime eligibility threshold.

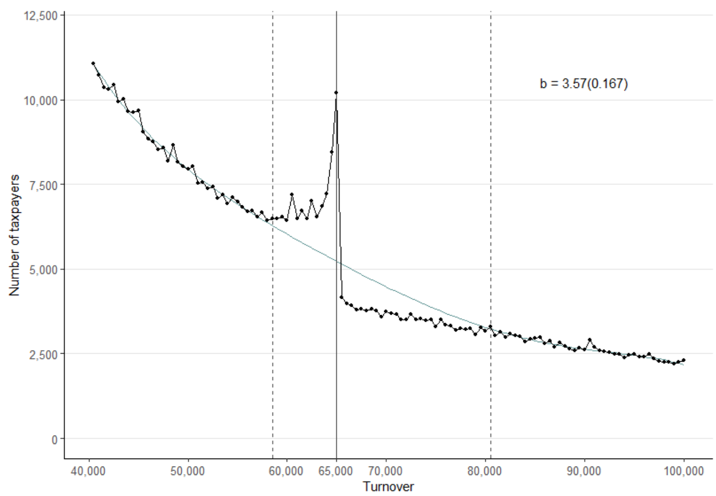
### 1.3.2 Evidence of Turnover Responses

Figure 1.2 shows the turnover distribution in 2019. We can see that self-employed bunch below €65,000, which is the turnover cut-off for the preferential turnover (F) regime in 2019. Figure 1.2 uses the standard bunching technique of Chetty et al. (2011) and Kleven and Waseem (2013) described in section 1.3.1. The estimated bunching coefficient is 3.57, meaning that the excess mass of individuals below the €65,000 cut-off is equal to 357% of the estimated counterfactual frequency at the threshold. As a placebo test, I plot the same section of the turnover distribution for each year until 2017 in Figure A10, when €65,000 was not the eligibility threshold of the F-regime, and I do not find bunching.[17] Hence, we can safely attribute the observed bunching in Figure 1.2 to the new F-regime threshold in 2019. The result is robust to adjusting the order of the polynomial for the counterfactual estimation - see table A2 in the appendix.

Then, I apply the alternative method (described in section 1.3.1) to estimate bunching in Figure A1, in which I consider only the samples of F-regime taxpayers below the €65,000 threshold. As described in section 1.3.1, the excess mass estimated among F-regime taxpayers below the threshold should be lower or equal than the missing mass estimated above it among ordinary regime taxpayers. This is because self-employed individuals have opted out from the ordinary regime and bunched below the threshold to access the F-regime.

**Figure 1.2:** Bunching in 2019 at the €65,000 F-regime threshold

**(a)** Standard Method: all taxpayers




Note: this graph reports the distribution of turnover for all taxpayers around the €65,000 threshold (vertical grey line). The vertical dashed grey lines mark the excluded region of the distribution that is affected by bunching. The counterfactual distribution is estimated using the standard method described in section 1.3.1 with a polynomial of order 5. The bunching coefficient $b$ is defined as the ratio between the estimated excess mass and the counterfactual frequency at the threshold. The 95% confidence interval is reported in brackets and is obtained with the bootstrap method by estimating a large number (500) of turnover distributions as detailed in section 1.3.1.

[17] The tax reform enacting the €65,000 threshold was announced and passed in 2018, meaning 2018 is a transition year. Evidence of individuals anticipating the policy change is presented in the appendix.

### 1.3.3 Heterogeneity in Bunching: Sector-specific Analysis

Figure 1.3 shows the distributions of turnover of all taxpayers (both ordinary and F-regime) of the different sectors. The corresponding excess bunching coefficients $b$ are reported in Figure 1.4, where the excess bunching coefficient is defined as the ratio between the excess mass of taxpayers to the left of the threshold and the value of the counterfactual frequency at the threshold, and serves as a measure of how strong bunching is. We can see there are heterogeneous responses across sectors, with Professionals showing the highest bunching coefficient. The pattern is robust to adjusting the order of the polynomial for the counterfactual estimation - see table A3 in the appendix.

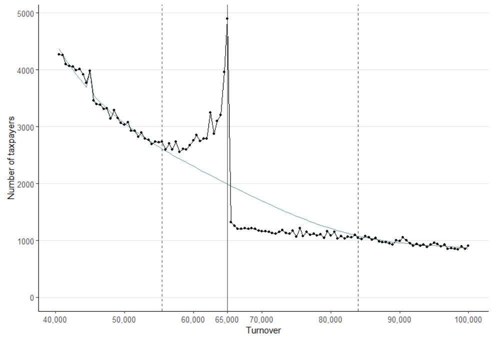
Self employed in different sectors have different incentives to bunch for two reasons: i) some sectors are on average more profitable than others, meaning that self employed in higher value added industries have a larger tax burden in the ordinary regime than lower value added ones, conditional on turnover; ii) the taxable share of turnover, that is the tax base in the preferential regime, is sector-specific. The incentive to bunch will therefore depend on the gap between actual profitability, which determines the tax burden in the ordinary regime, and the notional profits in the turnover regime.

The theoretical prediction is that bunching should be stronger in those sectors in which actual profits tend to be consistently higher than notional profits, as there would be more people that would potentially benefit from a lower tax base in the preferential turnover regime. To find whether this is actually the case, we compare the bunching coefficient of the different sectors with the difference between actual profit and notional profits for the median agent in the profit distribution. This theoretical prediction is supported by the data: there is a positive relationship between the extent of bunching and the difference in tax bases across regimes for the median profitability level. We observe more bunching in those sectors in which larger shares of taxpayers would have a larger tax base in the ordinary regime (evidence in the appendix).

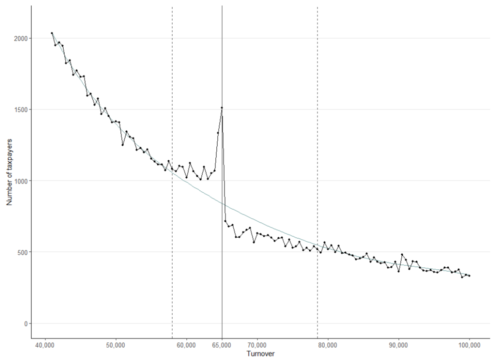
In some of my analyses I consider the sub-sample of solo self-employed that includes only F-regime taxpayers (sections 1.5.2-1.5.3). Figure A2 shows bunching below the F-regime threshold for this sub-sample. Professionals, business intermediaries and retail & accommodation are the sectors with the largest observed bunching coefficients. Then, as argued in section 1.3.1, the distribution of ordinary regime taxpayers above the threshold is also affected. This is because the self-employed individuals that are bunching below the threshold have moved from above and opted out from the ordinary tax regime. Hence, using years before the €65,000 threshold applied to provide the counterfactual, it must be the case that the distribution of ordinary regime-taxpayers has missing mass above the threshold in 2019. Figure A3 provides evidence supporting this prediction. In all sectors, the empirical frequency of ordinary regime taxpayers in 2019 tends to be lower than the counterfactual in an interval above the F-regime turnover threshold.

**Figure 1.3:** Bunching at the €65,000 threshold in 2019 to access the preferential turnover regime.

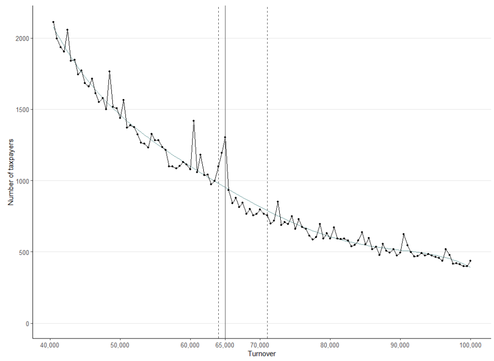
**(a)** Professionals



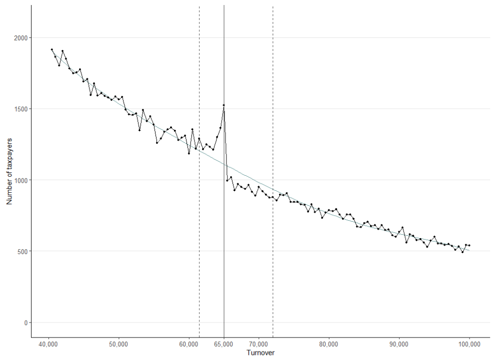

**(b)** Other Activities



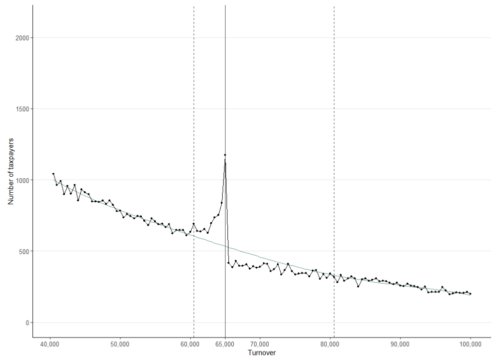

**(c)** Construction & Real Estate




**(d)** Retail & Accommodation

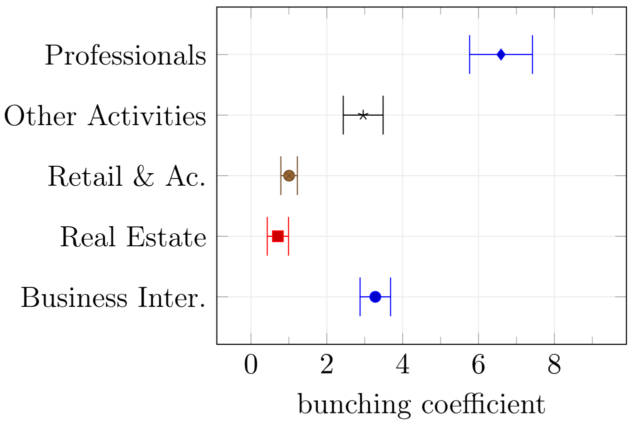



**(e)** Business Intermediaries




Note: these graphs report the distribution of turnover in each sector for the whole sample of taxpayers around the €65,000 threshold. The vertical dashed grey lines mark the excluded region of the distribution that is affected by bunching. The counterfactual distribution is estimated using the standard method described in section 1.3.1 with a polynomial of order 5.

**Figure 1.4:** Bunching coefficients for observed turnover responses to the €65,000 threshold in 2019.




Note: this figure reports the bunching coefficients of the graphs in Figure 1.3. The bunching coefficient is defined as the ratio between the total excess mass below the turnover threshold and the counterfactual density at the threshold. The 95% bootstrap confidence intervals are reported in parenthesis and are computed by estimating a large number of turnover distributions (500 samples).

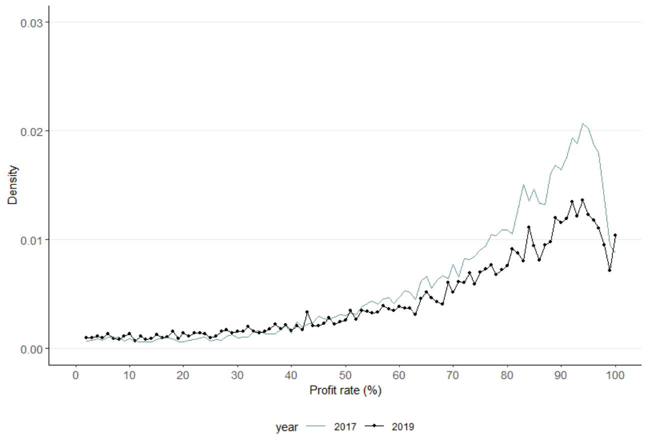
### 1.3.4 Choice of Tax-regime Below the Threshold: Evidence

The self-employed who are located below the €65,000 turnover threshold can choose between the ordinary (profit-based) regime and the (turnover-based) F-regime. In this section, I provide evidence on the types of self-employed who opt for the preferential turnover regime. The individuals who would most benefit from the F-regime are those with the highest profits, namely those who would have a larger tax base in the ordinary (profit-based) regime. While I find some evidence supporting this, I also find that many taxpayers are located in regions of dominated choice.

Figure 1.5 plots the distributions of the profit rate (profit as a share of turnover) for professionals with turnover just below the F-regime eligibility threshold. For relatively high levels of the profit rate, the density in 2019 is lower than in 2017, used as counterfactual, meaning that higher-profit individuals have opted out from the ordinary regime in 2019. However, not all of them have done so. In the case of professionals with turnover between €60,000 and €65,000, anyone with a profit rate above 72% would be strictly better-off in the F-regime. Hence, figure 1.5 documents that 69% of professionals in this section of the turnover distribution make a dominated choice. I interpret this as evidence of optimisation frictions, and we use this information in the structural estimation of the turnover elasticity. Thus, the estimated share of agents making a dominated choice is used as a measure of unresponsiveness to tax incentives due to frictions in the different sectors.[18]

[18] Evidence for the other sectors is provided in Figure A4.

**Figure 1.5:** Profit rate distribution for Professionals in the ordinary regime located below the €65K threshold.




Note: the graph shows the distribution of the profit rate – given by the ratio between profits and turnover – for professionals with turnover between €60,000 and €65,000. The distribution in 2017 is used as counterfactual for 2019, as €65,000 was not a discontinuity in the tax schedule before 2018.

## 1.4 Theory

### 1.4.1 The Model

This section describes the theoretical framework that will be used to estimate the elasticity of turnover with respect to net-of-turnover-tax rate. Building on Kleven and Waseem (2013), I develop a model describing agent's behaviour around the turnover threshold of the preferential turnover tax (F) regime, in line with the institutional set-up described in section 1.2. Below (above) the threshold, agents are taxed on turnover (profits). This creates a non-standard notch in the tax schedule with a change of tax rate, tax base and tax liabilities, such that there is no clear dominated region.

Following the bunching literature (Kleven, 2016), preferences are represented by a quasi-linear utility function (exp. 1.2). Turnover $y$ generates disutility $\phi(y, n)$, that is increasing in turnover, but decreasing in the agent's ability $n$. The elasticity of turnover with respect to net-of-tax rate is denoted by $e$. The production costs of generating turnover $y$ are given by $c_i$, and can be heterogeneous across agents. Each agent-type $i$ is therefore identified by their ability and their cost function: $\theta_i = \{n_i, c_i\}$. Ability $n$ governs how much an agent is willing to work. Thus, ability governs where in the turnover distribution the agent will be.[19] Then, individual production costs $c_i$ determine where in the profit distribution an agent is located, conditional on generating a certain level of turnover.

[19] I implicitly assume that the agent could always sell (earn) more if desired.

$$U = C - \phi(y, n) \tag{1.2}$$

$$\phi(y, n) = \frac{n}{1 + \frac{1}{e}} \left(\frac{y}{n}\right)^{1+\frac{1}{e}} \tag{1.3}$$

Agents maximise utility $U$ by choosing how much to work, namely the level of turnover $y$, and face an upward notch at $y^*$. Below the cut-off $y^*$, agents have access to the preferential tax regime in which turnover is taxed proportionally at rate $t_B$. While entrepreneurs don't charge VAT to customers, they also cannot deduct VAT payments on inputs $(c_i\, t_V)$. The effective tax on turnover in the preferential tax regime is therefore $t_P = t_B + t_V\,(c_i/y)$. The cost of compliance procedures of the preferential regime is $a_B$. Above the threshold, agents are taxed on their profits $\Pi$, and a different tax schedule applies: $t_A(\Pi)$ is the implicit average turnover tax rate (IATTR), that is the equivalent proportional tax on turnover that the agent would pay, given the actual profit tax schedule $T(\Pi)$ for self-employed, i.e. $t_A(\Pi) = T(\Pi)/y$.[20] The cost due to compliance procedures is $a_A$, which is larger than in the preferential regime, i.e. $a_A > a_B$.

$$C = \begin{cases} y\,(1 - t_B) - c_i(1 + t_V) - a_B & \text{if } y \leq y^* \\ y\,(1 - t_A(\Pi)) - c_i - a_A & \text{if } y > y^* \end{cases}$$

I make the following assumptions: 1) smooth distributions of ability ($n$), turnover ($y$) and profits ($\Pi$); 2) turnover can be changed by changing output (prices are fixed); 3) people change their real behaviour, not their tax reporting; 4) constant returns to scale, meaning that the ratio between costs and output is not affected by the decision to bunch; 5) no extensive margin responses.[21]

***Agent's optimisation***

For an agent optimising to the left of the turnover cut-off ($y \leq y^*$), the FOC is given by $y^* = n\,[(1 - t_P) - c'(y)]^e$ where $t_P$ is the preferential turnover tax rate. With constant returns to scale, marginal costs are given by $c'(y) = k$, and the FOC reads

$$y^* = n\,[1 - t_P - k]^e\,. \tag{1.4}$$

[20]The two regimes might also imply a differential incidence of taxes on the entrepreneur's side. For instance, if VAT is not fully passed on to selling prices, revenues would be scaled down by $1 + \alpha\, t_V$ where $\alpha$ captures the split of the tax incidence between consumers and sellers. $\alpha = 0$ means VAT is fully passed on to consumers, so that changes in VAT are irrelevant for the entrepreneur. The opposite case is $\alpha = 1$, when entrepreneurs bear the whole VAT burden.

[21]Assumption 5 is discussed more in depth in paragraph 1.5.4 and in the appendix.

Then, to the right of the cut-off, $(y > y^*)$, utility maximisation yields the following FOC

$$\underbrace{1 - t_A(\Pi) - c'(y)}_{\text{Direct effect of changing } y} \underbrace{- y \cdot \frac{\partial t_A(\Pi)}{\partial y}}_{\text{Indirect effect}} = \left(\frac{y}{n_i}\right)^{\frac{1}{e}}$$

Utility is raised by the net-of tax and net-of-marginal-costs part of additional turnover (direct effect). Then, changing turnover also varies profit $\Pi$, and this affects tax liabilities, and therefore utility (indirect effect). The sign of the indirect effect depends on whether changing turnover at the margin increases or decreases profits. If profits go up (down), then the tax base is larger (smaller) and the tax liability increases (decreases), so that the indirect effect is negative (positive). With marginal costs given by $c'(y) = k$, the FOC simplifies to

$$\left[1 - t_A - k - \pi \cdot T'(\Pi)\right] = \left(\frac{y}{n_i}\right)^{\frac{1}{e}} \tag{1.5}$$

where $\pi = (y - c(y))/y = 1 - k$ is the profit rate. Condition (1.5) implies that if two agents have equal turnover $y$ at the optimum, but different profits, then the agent with higher profits (lower $k$) must also have lower elasticity $e$ and/or lower ability. By allowing an imperfect correlation between ability $n$ and the individual cost function $c_i$ – therefore keeping $n_i$ and $c_i$ distinct – the model accounts for heterogeneity in elasticities as well as in profitability across agents, conditional on a certain level of turnover.

### 1.4.2 Indifference Condition

To estimate the elasticity of turnover, I derive the indifference condition (1.6), using the FOCs (1.4)-(1.5). This condition exploits the fact that one agent – the marginal buncher – is indifferent between: (i) bunching at the turnover threshold to access the turnover regime; or, (ii) remaining in the ordinary regime at the best interior point above the threshold. The indifferent condition reads as follows

$$\frac{1}{1 + \Delta y^*/y^*}\left[1 - \frac{k \cdot (y^* - y_I) - \Delta a}{(1 - t_P)y^*}\right] - \frac{e}{e+1}\left(\frac{1}{1 + \Delta y^*/y^*}\right)^{1+1/e} \cdot \frac{1 - t_P - k}{1 - t_P}$$
$$- \left(\frac{1}{1 - t_P}\right) \cdot \left[\frac{1 - t_A - k - \pi \cdot T'(\Pi)}{1 - t_P - k}\right]^e \left[(1 - t_A) - \frac{e}{e+1} \cdot (1 - t_A - k - \pi \cdot T'(\Pi))\right] = 0 \tag{1.6}$$

where $T'(\Pi) = \frac{\partial T(\Pi)}{\partial \Pi}\big|_{\Pi = \Pi_I}$, and $\Delta a = a_A - a_B$ is the difference between compliance costs in the tax regimes around the threshold. Expression 1.6 characterises the relationship between the behavioural response of the marginal buncher $\Delta y^*/y^*$, the average net-of-tax-rate in the two regimes $1 - t_A$ and $1 - t_P$, and the elasticity $e$. However, differently from Kleven and Waseem (2013), agents face two alternative regimes that have different tax bases around the turnover threshold. Agents are taxed on turnover below the threshold, and on profits if they are above it. Thus, expression 1.6 also includes the effect that changing turnover has on tax liabilities above the threshold via changes in profits.

### 1.4.3 Theoretical Predictions: Which Agents Bunch?

Since earnings taxation above the threshold depends on profits, agents with equal turnover but different profits (costs) have different incentives to bunch at the threshold. In this section, I describe the predictions of the model regarding the types of agent who bunch. This framework gives the theoretical foundation to identify the tax incentive of the marginal buncher that is used to estimate the turnover elasticity. Figure 1.6 illustrates the turnover tax regime notch in a budget set diagram (Panel A) and the implications of the model for the profit distributions for a certain level of turnover above the F-regime threshold (Panel B).

First, let's consider the baseline scenario, with homogeneous elasticity and no optimisation frictions. Panel A shows that agents M and B have equal turnover but different profits, that is why they have different budget sets. Agent B has larger profits and therefore faces a larger implicit turnover tax rate $t_A(\Pi)$ than agent M. Given their preferences, agent B is going to bunch to get a higher payoff, while agent M is just indifferent between bunching and remaining at the interior point $y_I$ (marginal buncher). Any other agent with turnover $y_I$, but with profits lower than agent M is not going to bunch at the notch point $y^*$. Panel B shows the (stylized) counterfactual and empirical profit distributions for turnover bin $y_I > y^*$ above the threshold, that are drawn in red and blue respectively. In the baseline scenario (Fig. 1.6b), agent M is the highest profit-type agent that remains at the interior level of turnover $y_I$. Any agent with higher profits, like agent B, bunches at the threshold, therefore leaving this turnover bin and the corresponding profit distribution. Hence, in the case of homogeneous elasticity, bunching is simply driven by heterogeneous profitability across agents. In the absence of frictions, all agents with profits above a certain threshold level $\overline{\pi}$ will bunch. The (last) marginal buncher is the agent with the highest profits among those who remained at their best interior point.

The proportional turnover response $\Delta y^*/y^*$, driven by structural elasticity $e$, can therefore be estimated using

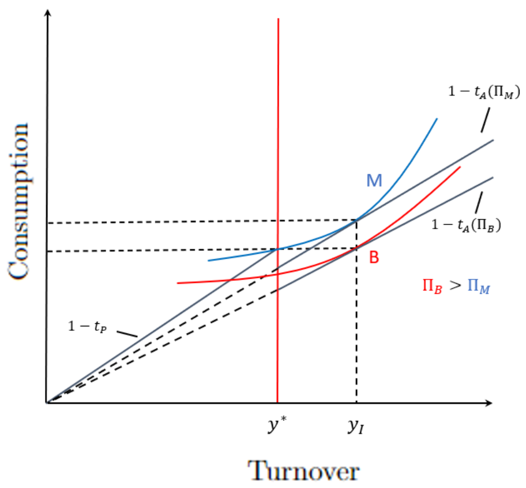
$$B = \int_{y^*}^{y^*+\Delta y^*} \int_{\overline{\pi}}^{1} h_0(\pi) d\pi dy \approx (1-\zeta) h_0\left(y^*\right) \Delta y^* \tag{1.7}$$

where $\zeta$ is the share of taxpayers who do not choose to bunch because they have very low profits (lower than $\pi_L$) and therefore very low tax liabilities in the ordinary (profit-based) regime above the threshold $y^*$. The approximation assumes that the counterfactual

**Figure 1.6:** Bunching with multiple notched budget sets - theoretical predictions

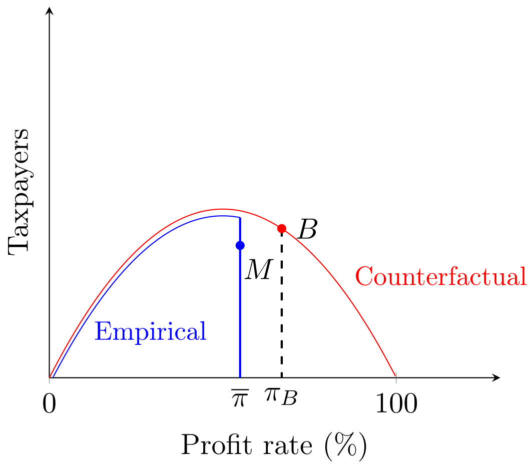
## Panel A - Budget sets

**(a)** Baseline

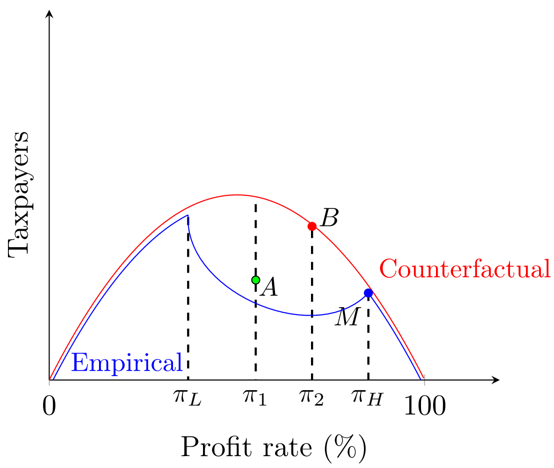



## Panel B - (Stylized) profit distributions at turnover $y_I > y^*$

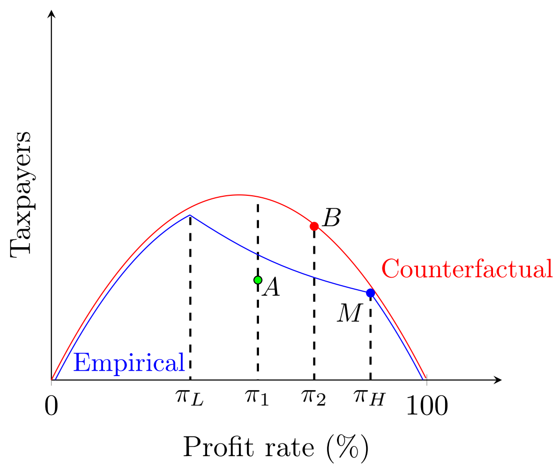
**(b)** Baseline




**(c)** Heterogeneous Elasticities




**(d)** Heterogeneous Elasticities + Frictions



density $h_0(y)$ and the share of non-bunchers $\zeta$ is roughly constant for $y \in (y^*, y^* + \Delta y^*)$. Reworking (1.7) yields the structural response that accounts for the share of taxpayers $\zeta$ who do not have a financial incentive to bunch:

$$\Delta y^*/y^* = \frac{B}{h_0^F(y^*)y^*}.$$

where $h_0^F(y^*) = (1-\zeta)h_0(y^*)$ is the value of the counterfactual density that includes only those taxpayers that have a financial incentive to bunch and select the F-regime.

Second, I examine the theoretical prediction for bunching in the case of heterogeneity in elasticities (Figure 1.6c-d). The empirical profit distribution deviates from the counterfactual only in an interval $[\pi_L, \pi_H]$, meaning only some profit-types of agent are affected by bunching. The incentive to bunch is relatively weak for individuals with a profit rate below $\pi_L$, as these agents are paying relatively little in the profit-based ordinary regime. Thus, these agents prefer not to bunch and remain at their best interior point. Then, agents with a profit rate higher then $\pi_H$ will also prefer not to bunch. While these individuals face a relatively high tax rate because of their large tax base (profits), they are also the most productive individuals who can enjoy the highest consumption. It follows that bunching is less attractive for these agents because reducing turnover implies a relatively large reduction in consumption. Moreover, if ability $n$ is positively correlated with profitability, the higher profit-types will also have low utility costs of generating a certain level of turnover and/or lower elasticity, such that the decision not to bunch will be optimal for them.

Hence, bunching will be beneficial only for individuals in the middle of the profit distribution (with profit rate between $\pi_L$ and $\pi_H$). These agents are taxed more than low profit-individuals in the ordinary (profit-based) regime, but at the same time do not consume as much as the higher profit-types because they are less productive. For these agents, reducing turnover by bunching at the threshold can be beneficial.[22] In the case of heterogeneous elasticities, agent $M$ is the last marginal buncher, that is the agent with the lowest elasticity that is just indifferent between bunching and remaining at the interior point. With optimisation frictions, excess bunching $B$ is defined as

$$B = \int_e \int_{y^*}^{y^*+\Delta y^*} \int_{\pi_L}^{\pi_H} (1 - \beta(\pi, y, e))h_0(\pi, e)d\pi dy de \approx (1 - \beta)(1 - \zeta)h_0(y^*)\,\mathbb{E}[\Delta y_e^*] \tag{1.8}$$

where the approximation assumes that the counterfactual density $h_0(y)$ and the shares of non-bunchers $\beta$ and $\zeta$, due to frictions and low tax incentives respectively, are roughly constant for $y \in (y^*, y^* + \Delta y_e^*)$ and all elasticity values $e$.[23] The term $(1 - \beta)\,\mathbb{E}[\Delta y_e^*]$ is

[22]Some agents might still remain in this part of the distribution, if they have sufficiently high ability $n$. More agents (like $A$) will be located here if there are also optimisation frictions (Fig. 1.6 d).

[23]Kleven and Waseem (2013) only consider the share of unresponsive due to frictions $\beta$. In our set-up, since the tax regimes around the threshold have different tax bases, we also account for the share of taxpayers $\zeta$ that are unresponsive because of weaker tax incentives.

defined as the average (observed) turnover response attenuated by optimisation frictions. Reworking (1.8) yields the average structural response

$$\mathbb{E}[\Delta y_e^*]/y^* = \frac{B}{(1-\beta)h_0^F(y^*)y^*} \tag{1.9}$$

where $y^*$ is expressed in binwidth units, and $h_0^F(y^*) = (1-\zeta)h_0(y^*)$ is the value of the counterfactual density that includes only the taxpayers that have a financial incentive to bunch and select the F-regime.

## 1.5 Elasticity Estimation

Using the tax parameters and behavioural responses to the turnover regime's 65K threshold in 2019, I solve the indifference condition (1.6) to estimate the turnover elasticity for three sectors: Professionals, Business Intermediaries, Retail and Accommodation.

### 1.5.1 Identification

In order to apply the indifference condition and estimate the turnover elasticity in each sector, it is necessary to identify two parameters: 1) the tax rate $t_A$ that is faced by the marginal buncher in the ordinary regime above the threshold; 2) the share of unresponsive agents $\beta$ who do not bunch because of frictions. These two issues are tackled here.

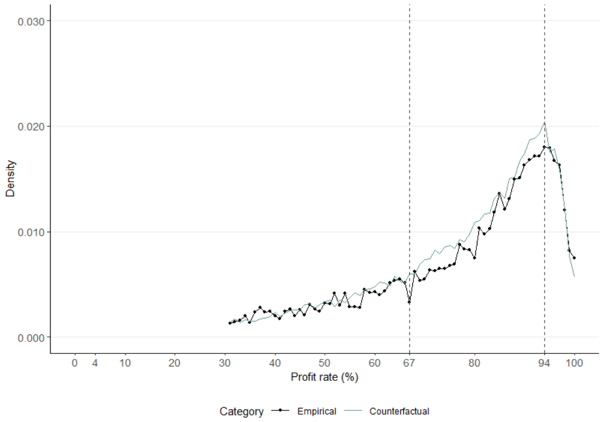
First, using the framework developed in section 1.4.3, we can find the tax incentive of the marginal buncher by considering the distributions of profit of taxpayers that are located in the region of the turnover distribution of interest. The key idea behind this strategy is that the missing mass of taxpayers above the eligibility threshold of the turnover regime should be matched by a corresponding missing mass in the profit distribution, conditional on the levels of turnover being considered. Then, we can infer which profit types have bunched and which have not by comparing the empirical profit distribution with an appropriate counterfactual.[24] By doing so for the specific region of the turnover distribution where the marginal buncher is estimated to be located, we can find the marginal buncher and isolate its profitability and tax incentive to bunch.

Figure 1.7 plots the profit rate distributions for the three sectors of interest. They include all self-employed located in a region of the turnover distribution where the marginal buncher is estimated to be located. In each sector, the empirical distribution deviates from the counterfactual in a certain interval, which is marked by the grey dashed vertical lines. The evidence is in line with the theoretical prediction of bunching in the case of heterogeneous elasticities and optimisation frictions presented in section 1.4.3. Therefore, the upper bounds of the marked intervals identify the marginal buncher and its tax incentive in each sector.

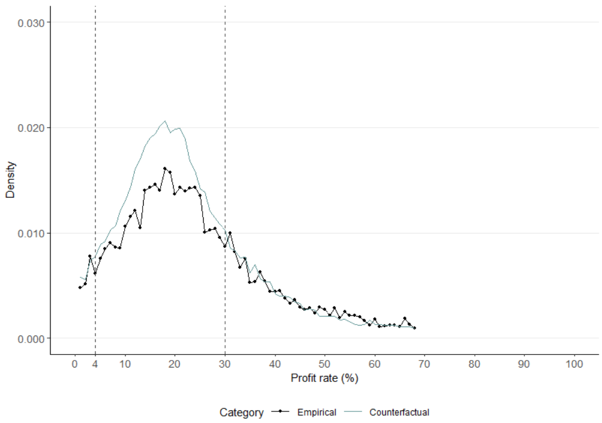
[24] The distributions in the period 2013-2017 are used as counterfactual, as €65,000 was not a discontinuity in the tax schedule before 2018.

**Figure 1.7:** Profit Rate Distributions in 2019 and counterfactuals: Finding the Tax Incentive of the marginal buncher.

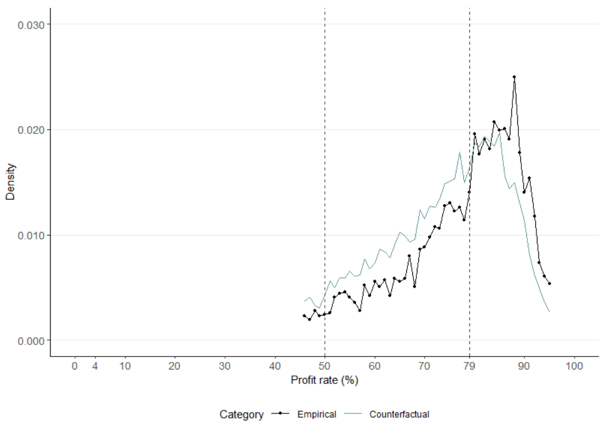
**(a)** Professionals




**(b)** Retail & Accommodation




**(c)** Business intermediaries




Note: These graphs plot the profit rate distributions for the region of the turnover distribution where the marginal buncher is estimated to be located in each sector. The counterfactual distribution is obtained by averaging across distributions in the period 2013-2017 when €65,000 was not a discontinuity in the tax schedule. The grey dashed vertical lines mark the interval of the profit rate distribution where the empirical (2019) distribution deviates from the counterfactual. Omitted data points represent less than 10 observations per bin.

The second parameter of interest is the share of unresponsive agents $\beta$ who don't bunch at the threshold due to frictions. Unlike Kleven and Waseem (2013), the notch of the turnover regime does not produce a clear dominated region above the eligibility threshold. In section 1.3.4, an alternative strategy was presented, in which the observed choices of the tax regime were exploited to infer to share of unresponsive agents.

Individuals that are located below the €65,000 threshold choose between the ordinary profit regime and the preferential turnover scheme. For an agent located below the threshold, the optimal strategy is to choose the regime that maximises consumption or, equivalently, minimises the tax liability. This identifies a clear dominated region in the profit distributions, conditional on turnover. In section 1.3.4, I showed that many individuals are located in regions of dominated choice, meaning they do not opt for the F-regime even though it would be advantageous for them to do so. This evidence is therefore exploited to estimate the share of unresponsive taxpayers, that is assumed to be constant above the threshold. The estimated shares of unresponsive $\beta$ are 69%, 78%, 92% for the sectors of professionals, business intermediaries and retail & accommodation respectively.

One possible threat to this identification strategy is that some individuals who are below the 65K threshold in 2019, and in the dominated profit region, might actually be there to access the F-regime in the following year. If that was the case we should observe taxpayers in the ordinary regime bunching below the 65K threshold as this would signal possible F-regime taxpayers. However, I do not find strong evidence this is the case: bunching in 2019 is very limited for people in the ordinary regime (figure A5), suggesting that most people who wanted to access the F-regime have already done so, and that the number of new possible F-regime-taxpayers is small enough not to invalidate this strategy.

**Table 1.5:** Observed excess bunching and turnover responses

| Sector | Excess Bunching | Observed Turnover response | Structural response |
|---|---|---|---|
| Professionals | 7137<br>[6039; 8279] | 0.042<br>[0.033; 0.051] | 0.134<br>[0.107; 0.163] |
| Retail & Accommodation | 743<br>[586; 840] | 0.034<br>[0.024; 0.041] | 0.422<br>[0.311; 0.534] |
| Business Intermediaries | 1012<br>[828; 1106] | 0.035<br>[0.026; 0.040] | 0.158<br>[0.119; 0.183] |

Note: Excess bunching is estimated in fig. A2. Structural responses are computed using (1.9). The 95% confidence intervals are reported in brackets and are estimated with the bootstrap method described in section 1.3.1.

### 1.5.2 Behavioural Responses

To estimate the behavioural responses, I focus on the subsample of individuals who are in the F-regime in 2019. This means the sample is composed of individuals who stayed below the threshold for at least two consecutive years and therefore presumably felt they had a clear financial incentive to opt for the preferential turnover regime.[25] Hence, the proportional (average) behavioural response is computed for each sector using (1.9) with the counterfactual density $h_0^F(y) = (1 - \zeta)h_0(y)$ estimated in figure A2, and the share of non-bunchers due to optimisation frictions $\beta$ estimated in section 1.3.4. Table 1.5 reports the estimates of excess bunching, observed and structural turnover responses for the three sectors of interest.

### 1.5.3 Structural Elasticities - Results

Table 1.6 presents the estimates of the tax elasticity in the different sectors. The first column shows the baseline estimates that are obtained under the assumption that the two tax regimes have equal compliance costs ($\Delta a = 0$). Then, the second column shows the estimates when we account for differential compliance costs.[26] For that, I use the estimate of Harju et al. (2019). As the main simplifications of the preferential regime is the exemption from VAT filing, as in Harju et al. (2019), this estimate is a also good reference for the additional hassle costs of the ordinary tax regime in Italy. In all sectors, the estimated elasticities are lower than in the baseline scenario as behavioural responses are now partly explained by the additional hassle costs in the ordinary regime. However, for professionals and business intermediaries, the elasticities remain significantly higher than zero, and the financial incentive is still the main driver of turnover responses. The largest elasticity is estimated in the sector of professionals (0.066).

**Table 1.6:** Turnover tax elasticity estimates.

| Sector | Turnover tax elasticity $e$ | |
|---|---|---|
| Professionals | 0.106<br>[0.073; 0.130] | 0.066<br>[0.038; 0.088] |
| Retail & Accommodation | 0.043<br>[−0.037; 0.096] | 0.028<br>[−0.079; 0.094] |
| Business intermediaries | 0.073<br>[0.041; 0.098] | 0.047<br>[0.018; 0.069] |
| $\Delta$ Compliance costs | 0 | €1300 |

Note: To obtain these estimates, I solve condition (1.6) by using the structural responses estimated by (1.9), the observed values for $t_A, t_B, t_V, T'(\Pi), \beta, k, \pi$ for the €65K threshold of the F-regime in 2019. The bootstrapped 95% confidence intervals are reported in parentheses: these are computed following the procedure described in 1.3.1 by estimating a large number (500) of turnover responses. For the difference in compliance costs $\Delta a$ in the two tax regimes, I use the estimate (€1300) of Harju et al. (2019).

[25] For being in the F-regime in year T, they had to locate below the threshold in year T-1. By considering this subsample, we are more likely to target individuals that stay consistently below the threshold and do not transfer income from one tax year to another.

[26] The elasticity estimates for Retail & Accommodation are obtained with the additional assumption of full VAT incidence on the entrepreneur side. This assumption rationalises observed behaviour in this sector. For more details, please see the appendix.

### 1.5.4 Discussion

The evidence on turnover responses shows that some solo self-employed adjust their turnover to locate themselves below the eligibility cut-off for the preferential regime. After accounting for the tax incentives in different sectors and compliance costs, the largest responses come from professionals and business intermediaries with estimated turnover tax elasticities of 0.066 and 0.047 respectively. These estimates are larger than the estimate by Harju et al. (2019), 0.016, which is obtained by exploiting bunching responses to the VAT registration threshold in Finland.

The difference of the estimates might be explained by the fact that, differently from Harju et al. (2019), Italian solo self-employed respond to an overall change in earnings taxation that includes, but is not limited to, VAT. Moreover, compliance costs seem to play a secondary role for the turnover responses in my case as their inclusion explains less than half of behavioural responses.[27] Hence, the results show the importance of financial incentives for bunching behaviour of larger sole-owner businesses. The difference in the relative importance of compliance costs for VAT, is due to the fact that the firms that are bunching in the Italian context are rather larger in terms of turnover, than the firms that respond to the Finnish VAT threshold. The costs of VAT filing are therefore a lower proportion of turnover for individuals around the €65,000 threshold.

Responses could reflect changes in productive effort (labour supply), but at this stage it is not possible to exclude the hypothesis that evasion might explain part of the adjustments in turnover.[28] Other authors (e.g. Aghion et al, 2022) have argued that the simplest evasion strategies would involve reporting turnover as a round number at, or very close to, the eligibility threshold. The facts that in my data bunching is often quite dispersed below the threshold, and that responses remain large even after omitting observations that report turnover as a multiple of 1000, would therefore be consistent with real responses. Moreover, I find that the most profitable individuals (top 5% of the profit rate distribution) in the missing mass region above the turnover threshold, who would benefit from bunching by under-reporting revenues, tend not to bunch. This is in line with real responses, and the idea that bunching by reducing output and revenues is not attractive for the most productive individuals as that would imply a substantial reduction in consumption.

One possible cause of concern is that the €65,000 threshold of the preferential regime in 2019 might have generated extensive margin responses that would not be accounted for by the bunching-based elasticity estimates. This issue arises as the estimation of the elasticity, based on the indifference condition, relies on the assumption that the whole excess mass below the threshold is only due to intensive margin responses. In the appendix, I explore the responses to the introduction of the turnover regime in 2015 and the

[27] Harju et al. (2019) show that omitting compliance costs would produce an estimated elasticity of 0.55, versus 0.016 when those costs are included. Hence, they argue that compliance costs are key to explain bunching responses in their case.

[28] In that case, our estimate for the elasticity would be a linear combination of the real and evasion elasticities.

subsequent reform in 2018 that raised and equalised the sector-specific eligibility thresholds to €65,000 from 2019 onwards. I provide *prima facie* evidence suggesting that the introduction of the turnover regime in 2015 generated some extensive margin responses, but the 2018 reform induced responses that are almost entirely on the intensive margin. Both tax reforms affected the shape of the turnover distribution, as taxpayers would face different incentives due to the introduction of, and the changes to, the eligibility thresholds of the tax regime. However, while the total number of self-employed reporting revenues between €10,000 and €100,000 increased in the period 2016-2018, after the turnover regime was introduced, that trend reversed in 2019 suggesting that the impact of the 2018 reform was almost entirely on the intensive margin.

Another issue is whether the introduction of the preferential turnover regime reduces tax revenues for the government. Answering this question would require us to know the following: i) how much do self-employed adjust turnover, i.e. how large bunching is; ii) how large is the inflow from the ordinary to the preferential regime for those taxpayers that are already below the preferential regime threshold; iii) are there any extensive margin responses. The first two channels would have a negative impact on tax revenues, while the third one would have a positive effect if most of the additional self-employed individuals were inactive before the tax reform, as new economic activity generates additional tax revenues. This paper provides evidence mainly on the first point.

## 1.6 Concluding Remarks

This paper investigates to what extent solo self-employed adjust sales turnover due to the incentives of the tax system. I study the turnover responses to the notch created by the eligibility cut-off of the preferential turnover regime for solo self-employed in Italy. I find that solo self-employed bunch below the €65,000 cut-off, set by the tax code, to qualify for the preferential turnover tax scheme. Professionals, Business intermediaries and Retail & Accommodation are the sectors with the largest observed responses. For each these three sectors, I use the bunching responses to estimate the turnover tax elasticity. To do so, I adapt the model of Kleven and Waseem (2013) and exploit a modified indifference condition for the marginal buncher that fits the institutional set-up and the empirical evidence.

Most of the literature investigating behavioural responses to taxation has underlined the higher responsiveness of self-employed compared to employees in adjusting taxable income. This paper documents that preferential tax regimes can generate substantial responses of a specific component of taxable income — sales turnover – which vary across different types of self-employed individuals. The estimated turnover elasticities are small but larger than zero. Moreover, the behavioural responses cannot be solely explained by the simpler compliance procedures of the preferential regime. This shows that financial incentives play a key role for the decisions to bunch of large sole-owner businesses.

Policy-makers usually set up preferential regimes for certain businesses to stimulate entrepreneurship and growth. However, since these regimes usually apply only to certain individuals on the basis of their turnover, some of those who are located just above the eligibility threshold will have an incentive to downsize their businesses rather than grow it. This paper shows this is the case for the preferential turnover regime in Italy, where individuals with relatively high profits are more likely to decrease their turnover.

Finally, there are two related topics for future research. First, the desirability of preferential tax regimes should be thoroughly studied. The individual financial advantages from preferential tax regimes, and the corresponding effect on the government's budget, should be weighed against the effects on the economic performance of those who apply for them. For example, it would be interesting to assess to what extent tax revenue losses due to bunching can be offset by additional revenues from people increasing their labour supply while benefiting from the low-tax regime. Moreover, individuals opting for preferential tax regimes usually have easier compliance procedures that require less information to be communicated to the tax authority. This is possibly concerning as it can weaken the ability of the tax authority to verify individual tax behaviour and identify frauds.

Second, it would be useful to know the extent to which the observed reductions of turnover are driven by real behaviour or evasion. While these two cases have equal financial implications for the Treasury, the individual welfare implications are different. If responses represent real choices, then the policy is distorting downwards the labour-supply decisions of individuals around the threshold and fails to stimulate growth for businesses. In the case of tax planning/evasion, the policy is solely eroding the tax base and reducing revenues, while the individual benefits from higher consumption thanks to their activity in the informal sector.

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# Appendix A

## Derivation of the Indifference Condition

For the last marginal buncher $M$, utility from bunching at the threshold is

$$U_{y^*} = (1-t_P)y^* - c(y^*) - \frac{n}{1+\frac{1}{e}}\left(\frac{y^*}{n}\right)^{1+\frac{1}{e}} - a_B$$

Then, at the best interior point, $y_I$, with profits $\Pi_I$, the agent's utility reads

$$U_{y_I} = \left(\frac{1-t(\Pi_I)}{1+\alpha\, t_V}\right) y_I - c(y_I) - \frac{n}{1+\frac{1}{e}}\left(\frac{y_I}{n}\right)^{1+\frac{1}{e}} - a_A$$

where turnover $y_I$ is scaled down by $(1+\alpha t_V)$ if the incidence of VAT $(t_V)$ falls partly on the self-employed $(\alpha > 0)$. Using the FOC for optimisation above the turnover threshold $y^*$, $\left(\frac{1}{1+\alpha t_V}\right)[1 - c'(y) - \pi\cdot T'(\Pi)] = \left(\frac{y}{n_i}\right)^{\frac{1}{e}}$, we can rewrite $U_{y_I}$ as

$$U_{y_I} = n\left(\frac{1-t_A(\pi)}{1+\alpha\, t_V}\right)\left(\frac{1-c'(y)-\pi\cdot T'(\Pi)}{1+\alpha\, t_V}\right)^e\left[1-\frac{e}{1+e}\left(\frac{1-c'(y)-\pi\cdot T'(\Pi)}{1-t_A}\right)\right] - c(y_I) - a_A$$

Setting $U_y^* - U_{y_I} = 0$ gives

$$\begin{aligned}&(1-t_P)y^* - \frac{n}{1+\frac{1}{e}}\left(\frac{y^*}{n}\right)^{1+\frac{1}{e}} + c(y_I) - c(y^*) + \Delta a\\ &-n\left(\frac{1-t_A(\pi)}{1+\alpha\, t_V}\right)\left(\frac{1-c'(y)-\pi\cdot T'(\Pi)}{1+\alpha\, t_V}\right)^e\left[1-\frac{e}{1+e}\left(\frac{1-c'(y)-\pi\cdot T'(\Pi)}{1-t_A}\right)\right] = 0\end{aligned}$$

Divide all terms by $n$, and use the agent's FOC in absence of the threshold, $y^* + \Delta y^* = n(1-t_P-c'(y))^e$. Finally, after pre-multiplying the condition by $1/(1-t_P)\cdot(1-t_P-c'(y))^e$ and collecting terms, we can rewrite the indifference condition as

$$\frac{1}{1+\Delta y^*/y^*}\left[1-\frac{c(y^*)-c(y_I)-\Delta a}{(1-t_P)y^*}\right] - \frac{e}{e+1}\left(\frac{1}{1+\Delta y^*/y^*}\right)^{1+1/e}\frac{1-t_P-c'(y)}{1-t_P}$$

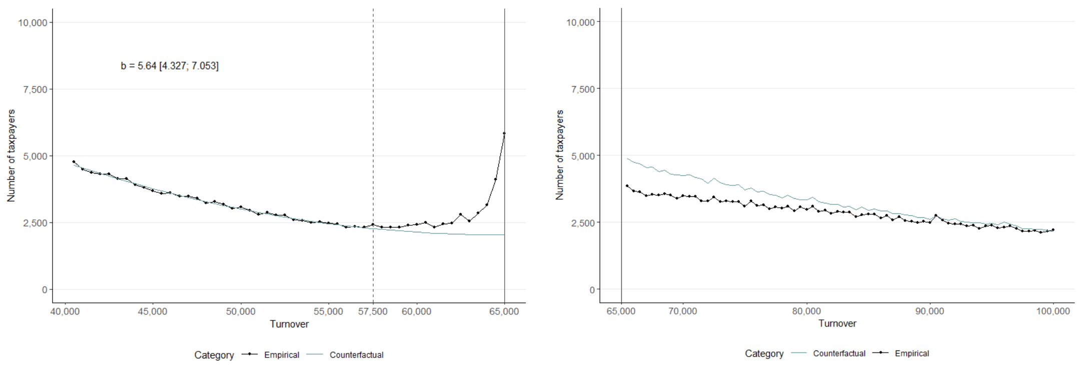
$$-\left[\frac{1-t_A}{1+\alpha\, t_V}\right]^{1+e}\cdot\frac{1}{(1-t_P)[1-t_P-c'(y)]^e}\left(\frac{1-c'(y)-\pi\cdot T'(\Pi)}{1-t_A}\right)^e\left[1-\frac{e}{e+1}\cdot\frac{1-c'(y)-\pi\cdot T'(\Pi)}{1-t_A}\right] = 0.$$

With no VAT incidence on the self-employed $(\alpha = 0)$, and constant marginal costs $k$ $(c(y) = ky, c'(y) = k)$, reworking the terms of the expression gives condition 1.6.

# Additional Graphs

**Figure A1:** Excess Mass and Missing Mass around the €65,000 threshold in 2019: alternative method.

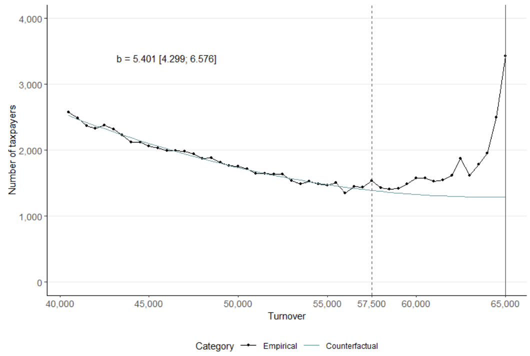
**(a)** F-regime taxpayers below €65K **(b)** Ordinary regime taxpayers above €65K



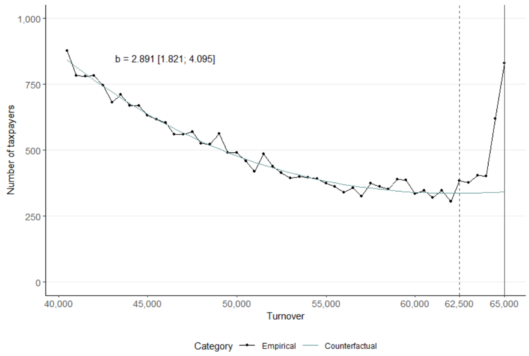

Note: graph (a) reports the distribution of turnover for the sample of F-regime taxpayers below the €65,000 threshold (vertical grey line). The vertical dashed grey line marks the beginning of the excluded region that is affected by bunching. The counterfactual distribution is estimated using the alternative method described in section 1.3.1. The bunching coefficient $b$ is defined as the ratio between the estimated excess mass and the value of the counterfactual density at the threshold. The 95% confidence interval is reported in brackets and are estimated with the bootstrap method as detailed in section 1.3.1. Graph (b) reports the empirical (2019) and counterfactual distribution of turnover for the sample of ordinary regime taxpayers above the €65,000 threshold. The average distribution of turnover in 2013-2017 is used as counterfactual for 2019, as €65,000 was not a discontinuity in the tax schedule before 2018.

**Figure A2:** Taxpayers in the F-regime bunching at the €65,000 threshold in 2019.

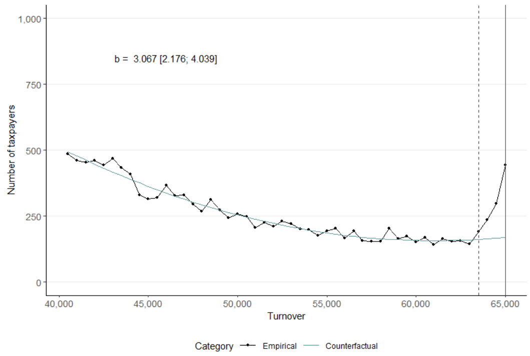
**(a)** Professionals



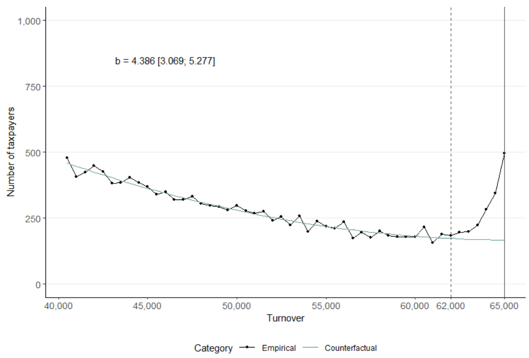

**(b)** Other Activities



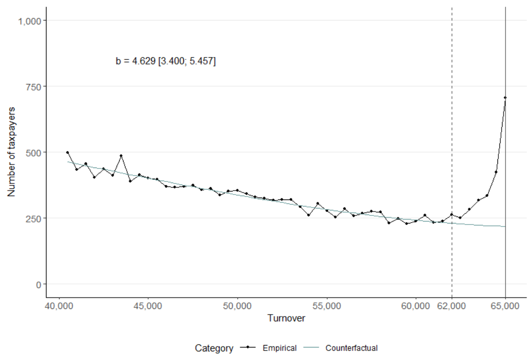

**(c)** Construction & Real Estate




**(d)** Retail & Accommodation



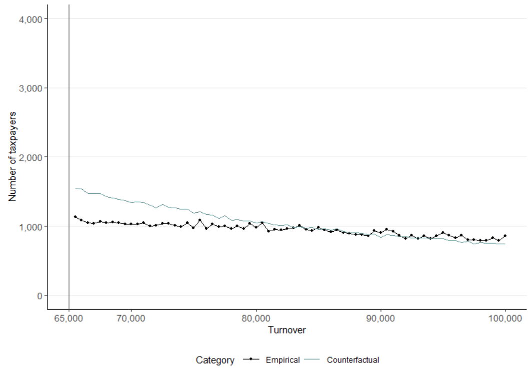

**(e)** Business intermediaries



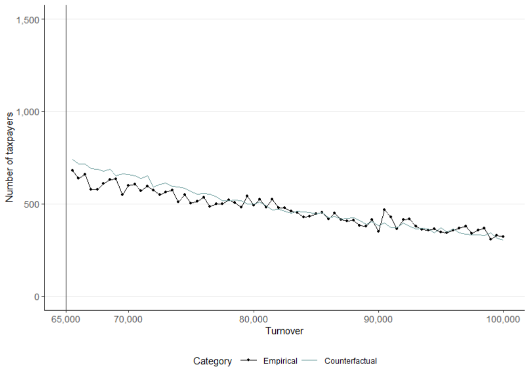

Note: the graphs report the distributions of turnover in the different sectors for the sample of F-regime taxpayers below the €65,000 threshold (vertical grey line). The vertical dashed grey line marks the beginning of the excluded region that is affected by bunching. The counterfactual distribution is estimated using the alternative method described in section 1.3.1. The bunching coefficient $b$ that is defined as the ratio between the estimated excess mass and the value of the counterfactual density at the threshold. The 95% confidence interval is reported in brackets and are estimated with the bootstrap method as detailed in section 1.3.1.

**Figure A3:** Taxpayers in the ordinary regime above the €65,000 threshold.

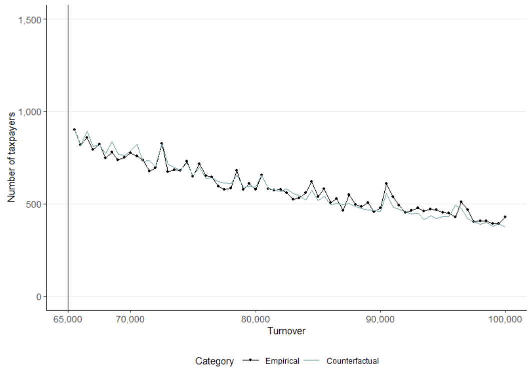
**(a)** Professionals



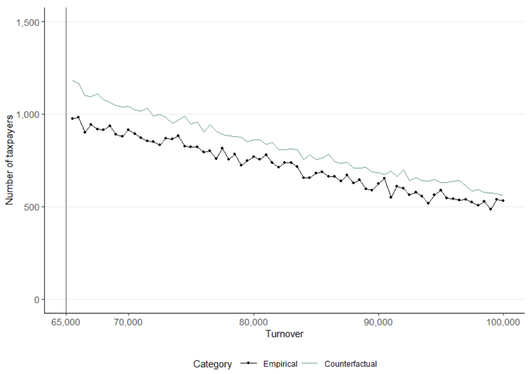

**(b)** Other Activities



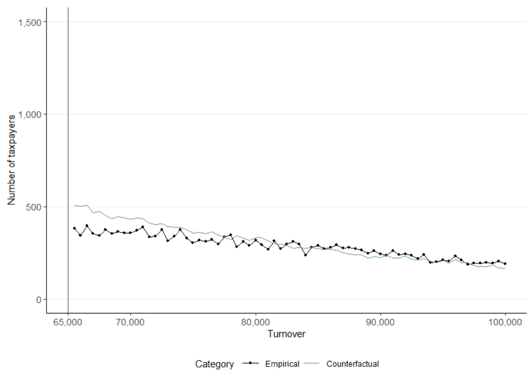

**(c)** Construction & Real Estate




**(d)** Retail & Accommodation



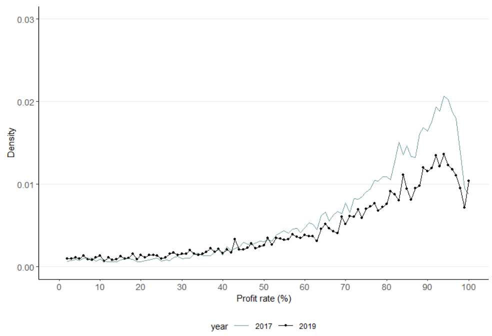

**(e)** Business intermediaries



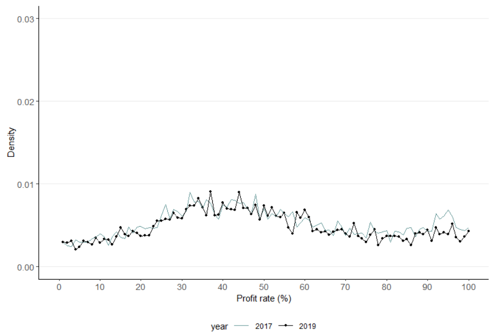

Note: the graphs report the empirical (2019) and counterfactual distribution of turnover for the sample of ordinary regime taxpayers above the €65,000 threshold (vertical grey line). The distributions of turnover in 2013-2017 are used as counterfactual for 2019, as €65,000 was not a discontinuity in the tax schedule before 2018.

**Figure A4:** Profit rate distribution in the ordinary regime located below the €65K threshold

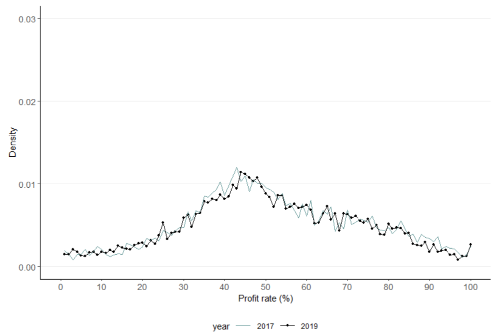
**(a)** Professionals



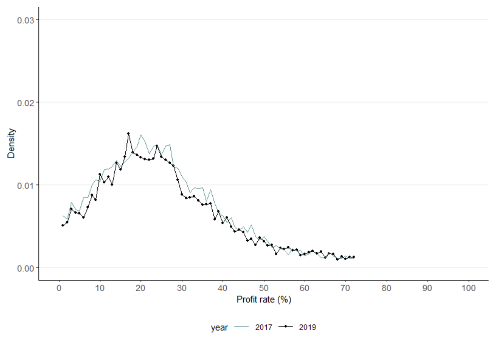

**(b)** Other Activities



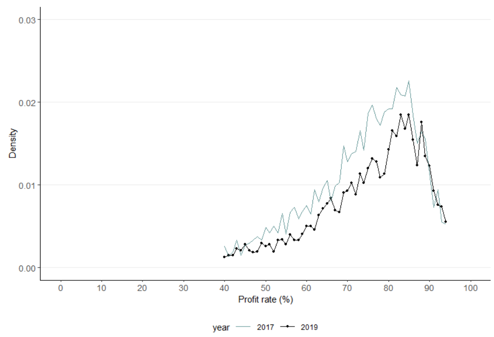

**(c)** Construction & Real Estate




**(d)** Retail & Accommodation

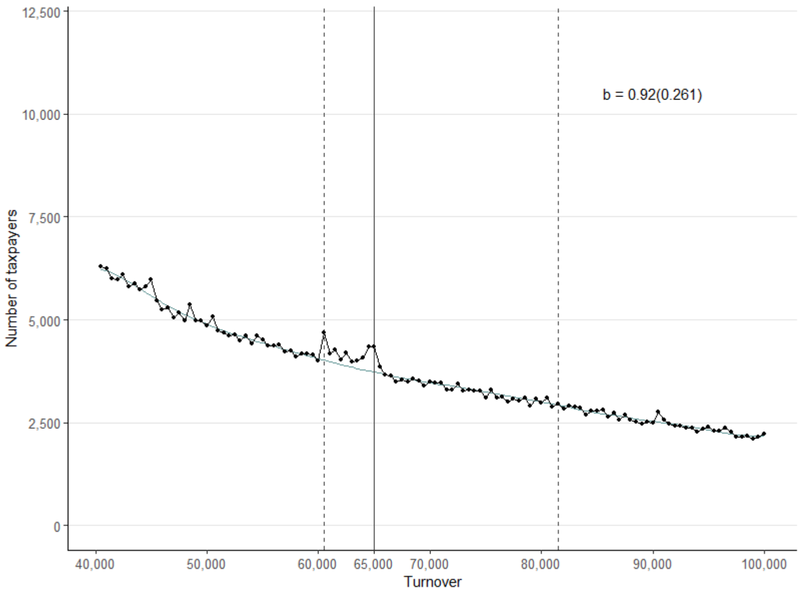



**(e)** Business Intermediaries




Note: the graphs show the distribution of the profit rate — given by the ratio between profits and turnover — for self-employed individuals with turnover between €60,000 and €65,000 in each sector. The distribution in 2017 is used as counterfactual for 2019, as €65,000 was not a discontinuity in the tax schedule before 2018. Omitted data points represent less than 10 observations per bin.

**Figure A5:** Bunching in 2019 at the €65,000 F-regime threshold - ordinary regime taxpayers only




Note: this graph reports the distribution of turnover for taxpayers in the ordinary regime around the €65,000 threshold (vertical grey line). The vertical dashed grey lines mark the excluded region of the distribution that is affected by bunching. The counterfactual distribution is estimated using the standard method described in section 1.3.1 with a polynomial of order 5. The bunching coefficient $b$ is defined as the ratio between the estimated excess mass and the counterfactual frequency at the threshold. The 95% confidence interval is reported in brackets and is obtained with the bootstrap method by estimating a large number (500) of turnover distributions as detailed in section 1.3.1.

## Ordinary Tax Regime

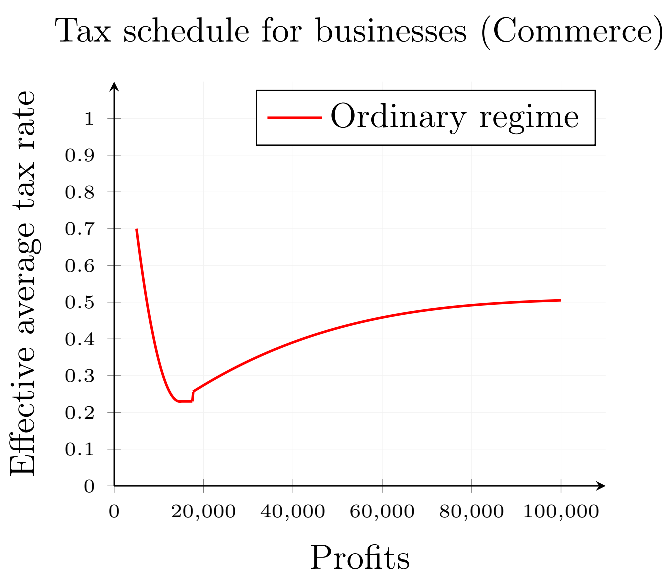
The ordinary tax regime includes the progressive personal income tax (IRPEF) schedule, social security contributions (SSCs), and VAT.

**Table A1:** Tax credits: 2012-2019

| | Tax credits | |
|---|---|---|
| Type | Brackets (TP) | Amount |
| Self-employment | 0 – €55,000 | $\frac{55{,}000-TP}{50{,}200} \times$ €1,104 |
| one child < (≥) 3 y.o | | $\left(1-\frac{TP}{95{,}000}\right) \times$ €1220 (€950)[a] |
| two children < (≥) 3 y.o | | $\left(1-\frac{TP}{110{,}000}\right) \times$ €1220 (€950)[a] |
| Non-working spouse | 0 – €15,000 | €800 – $\frac{(110 \times TP)}{15{,}000}$ |
| | €15,001 – €40,000 | €690 |
| | €40,001 – €80,000 | $\frac{(80{,}000-TP)}{40{,}000} \times$ €690 |

[a] In 2012, the per-child amount was lower: €800 ( €900) for < (≥) 3 y.o. child.
TP: Taxable Profits = Profits – social security contributions

**Figure A6:** The ordinary regime includes income tax (IRPEF), social security contributions, deductions.



# Sensitivity Analysis

**Table A2:** Sensitivity analysis of the bunching coefficient.

| Degree of Polynomial | Bunching below €65,000, 2019 |
|---|---|
| 2 | 3.48 (0.139) |
| 3 | 4.13 (0.133) |
| 4 | 3.99 (0.155) |
| 5 | 3.57 (0.167) |
| 6 | 3.62 (0.188) |
| 7 | 3.50 (0.237) |
| 8 | 3.34 (0.226) |
| 9 | 3.48 (0.348) |

Note: sensitivity analysis of the bunching coefficient with respect to the order of the polynomial that is used to construct the counterfactual distribution of turnover in figure 1.2. The bunching coefficient is defined as the ratio between the excess mass below the threshold and the value of the counterfactual frequency at the threshold. Standard errors are reported in parenthesis.

**Table A3:** Sensitivity analysis of the bunching coefficient in each sector.

| Degree of | Sector | | | | |
|---|---|---|---|---|---|
| Polynomial | (1) | (2) | (3) | (4) | (5) |
| 2 | 9.64 (0.362) | 2.26 (0.195) | 0.74 (0.148) | 1.09 (0.093) | 3.38 (0.145) |
| 3 | 8.15 (0.314) | 3.59 (0.237) | 0.81 (0.147) | 1.09 (0.095) | 3.39 (0.149) |
| 4 | 7.17 (0.47) | 3.18 (0.299) | 0.77 (0.149) | 1.07 (0.102) | 3.40 (0.201) |
| 5 | 6.59 (0.423) | 2.96 (0.268) | 0.72 (0.143) | 1.01 (0.111) | 3.28 (0.205) |
| 6 | 6.38 (0.521) | 2.86 (0.325) | 0.72 (0.147) | 1.02 (0.121) | 3.37 (0.305) |
| 7 | 5.29 (0.495) | 2.80 (0.438) | 0.69 (0.147) | 0.98 (0.131) | 3.38 (0.324) |
| 8 | 5.30 (0.511) | 2.73 (0.489) | 0.69 (0.148) | 0.97 (0.133) | 3.17 (0.448) |
| 9 | 5.06 (0.543) | 2.66 (0.778) | 0.67 (0.146) | 0.95 (0.163) | 2.78 (0.358) |

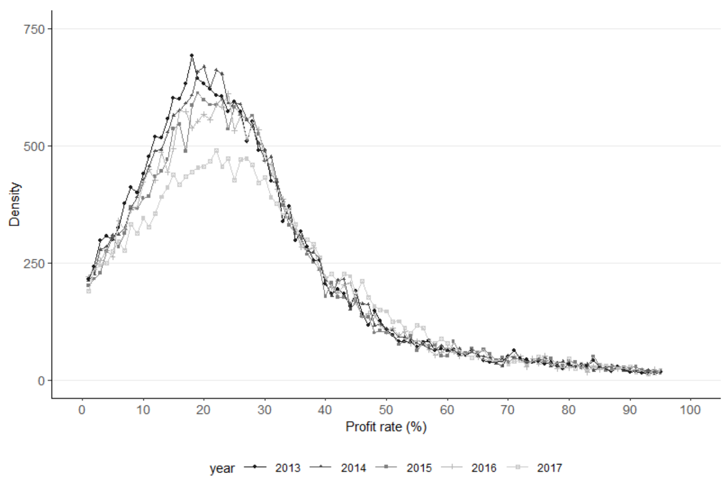
Note: sensitivity analysis of the bunching coefficient in each sector with respect to the order of the polynomial that is used to construct the counterfactual distribution of turnover in figure 1.3. Each column represents one specific sector: (1) Professionals; (2) Other Activities; (3) Real Estate; (4) Retail & Accommodation; (5) Business Intermediaries. The bunching coefficient is defined as the ratio between the excess mass below the threshold and the value of the counterfactual frequency at the threshold. Standard errors are reported in parenthesis.

## Assumption: VAT Incidence in Retail & Accommodation

In the sector of Retail & Accommodation, the majority of solo self-employed selecting the preferential tax regime are low-profit individuals, who would have a low tax burden in the ordinary profit regime. For instance, figure A7 shows the profit distributions for ordinary regime taxpayers with turnover between €45,000 and €50,000. Between 2015 and 2017, self-employed in this region of the turnover distribution could opt for the F-regime as the eligibility threshold was €50,000. The distribution in 2017 shows there are less taxpayers in the ordinary regime for profit rates between 10% and 30%, compared to previous years. This means that a large share of those opting for the F-regime are low-profit individuals.

There are two ways to rationalise this: 1) retailers have higher compliance costs than other types of self-employed in the ordinary regime so that the preferential regime is relatively more advantageous for them; 2) VAT is not neutral for individuals in this sector, so that the preferential F-regime increases their actual revenues by allowing them to sell VAT-exempt products and services. By investigating the choice of tax regimes of individuals with different profits, conditional on a given level of turnover, the observed behaviour can be rationalised either by extremely high compliance costs (up to 8 times as high as the estimate by Harju et al. (2019) or by full VAT incidence on entrepreneurs. In line with evidence showing full VAT incidence on small restaurants (Harju et al., 2018), I impose full VAT incidence on entrepreneurs in the sector of Retail & Accommodation. Therefore, the tax elasticity for Retail and Accommodation in Table 1.6 is estimated with this additional (VAT-incidence) assumption.

**Figure A7:** Profit rate distribution for ordinary regime tax-payers with turnover in the region [€45K,€50K]

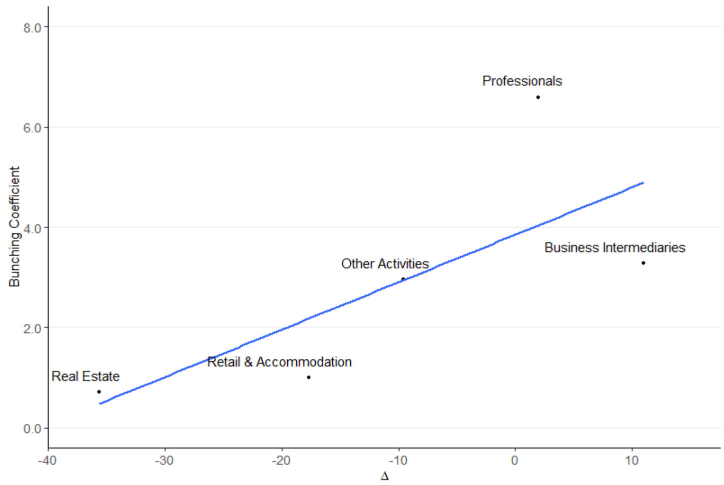



Note: the graph shows the distribution of the profit rate — given by the ratio between profits and turnover — for self-employed individuals with turnover between €45,000 and €50,000 in each sector. Omitted data points represent less than 10 observations per bin.

## Bunching Responses and Tax Incentives

I compare the bunching coefficient of the different sectors from Figure 1.4 with the difference between actual profit and notional profits for the median agent in the profit distribution (denoted by $\Delta$). The theoretical prediction is that bunching should be stronger in those sectors in which actual profits tend to be consistently higher than notional profits, as there would be more people that would potentially benefit from a lower tax base in the preferential turnover regime. Figure A8 shows that our theoretical prediction is supported by the data: there is a positive relationship between the extent of bunching and the difference in tax bases across regimes for the median profitability level. We can observe more bunching in those sectors in which larger shares of taxpayers would have a larger tax base in the ordinary regime.

**Figure A8:** Heterogeneous responses and tax incentives




Note: on the x-axis, $\Delta$ is defined as the percentage point difference between median profits and the notional profit for the preferential regime, as a share of turnover, for each sector. The median profit as share of turnover is taken from the distribution of taxpayers with turnover between €40K and €100K.

## Preferential Profit Tax Scheme: M-regime

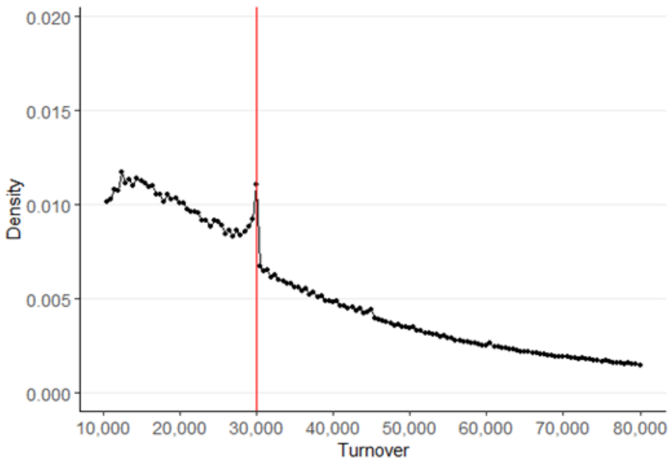
Between 2012 and 2015, solo self-employed with turnover below €30,000 could opt out of the ordinary regime and choose the M-regime. This scheme exempts entrepreneurs from VAT registration, annual VAT declaration to the tax authority, as well as record-keeping on clients, suppliers, purchases and payments. Then, the progressive income tax schedule is replaced by a proportional 5% tax rate on profits. Access to this scheme was limited to new businesses (no more than five years old) and entrepreneurs below 35 years old. While the scheme was abolished in 2015, people already in and satisfying its requirements could keep it.

Although the M-regime has no tax credits, the lower statutory profit tax rate, compared to the ordinary regime, is enough to make this scheme advantageous for most taxpayers. Hence, it is safe to assume that any taxpayer meeting the entry criteria would be better off in the M-regime. As the turnover threshold of €30,000 is not related to any other tax policy in 2012-2014, any excess mass of taxpayers below that threshold can be safely explained by the tax incentive of this scheme.

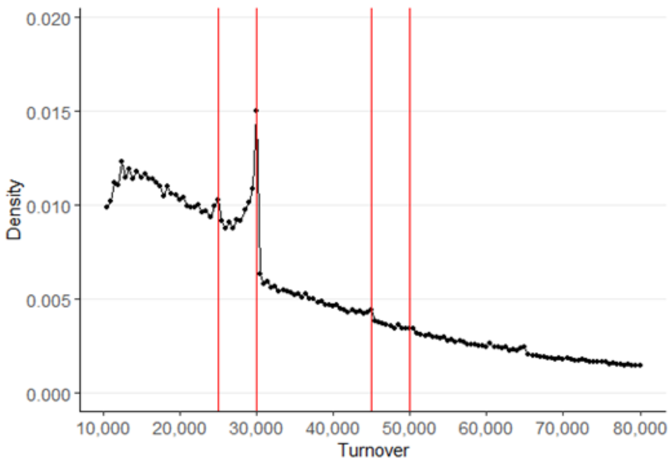
Figure A9 shows the distributions of turnover in the three periods under study. In the top graph, between 2012 and 2014, we observe bunching just below €30,000, that is the threshold to qualify for the M-regime. Then, the middle graph shows the turnover distribution in the period 2016-2018. Bunching is particularly strong at €30,000. This can be partly explained by the fact that €30,000 is not just that the cut-off the M-regime (until 2015) but also the cut-off for professional services and other economic activities in the F-regime. Although the M-regime was abolished in 2015, people already in the scheme could keep the advantages if the relevant requirement were satisfied. Hence, some individual still had an incentive to bunch at €30,000 because of the M-regime. For the same reason, we can still see bunching at €30K in 2019 (bottom graph of figure A9). However, most bunching is observable below the new threshold of the turnover (F) regime at €65,000 that is valid for all sectors.

**Figure A9:** Bunching in the different periods.

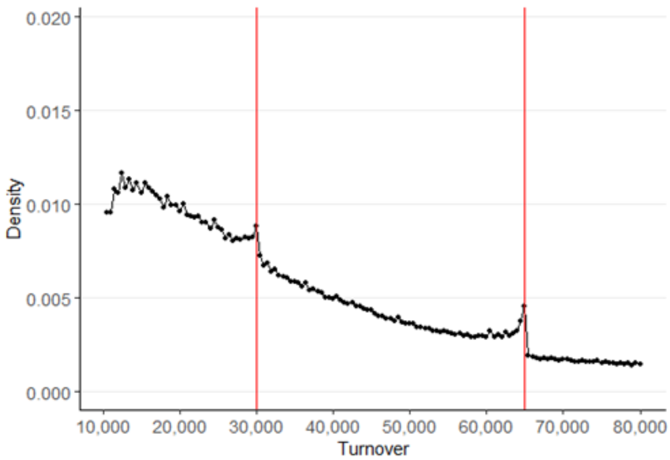
**(a)** 2012-2014: M regime (30K threshold)




**(b)** 2016-2018: M & F regime (25K,30K,45K,50K thresholds)



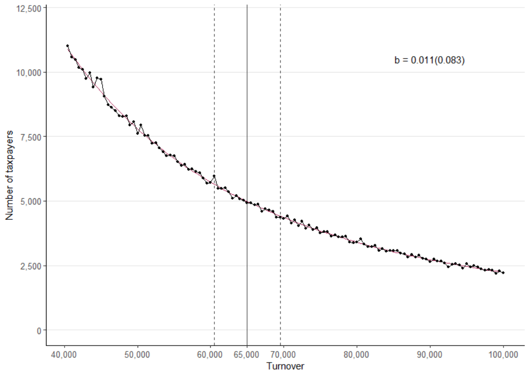

**(c)** 2019: M regime (30K threshold) & F regime (65K threshold)



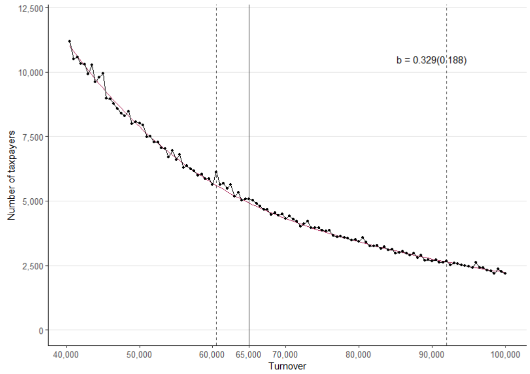
# Placebo Tests

**Figure A10:** Placebo Test: no bunching when there was no discontinuity (before 2018).

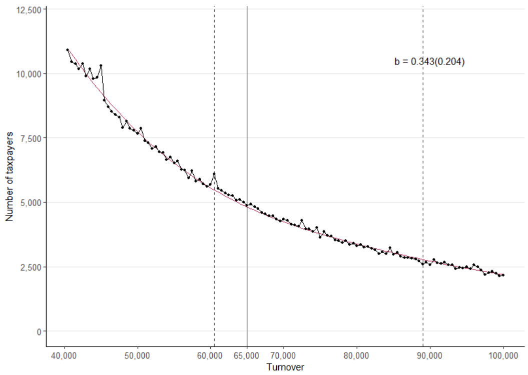
**(a)** 2012



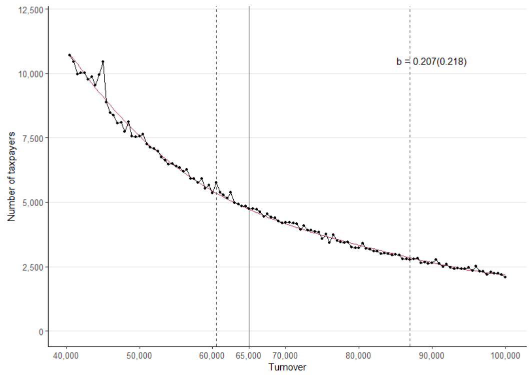

**(b)** 2013



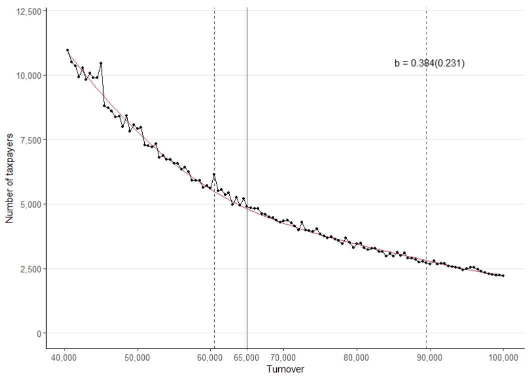

**(c)** 2014



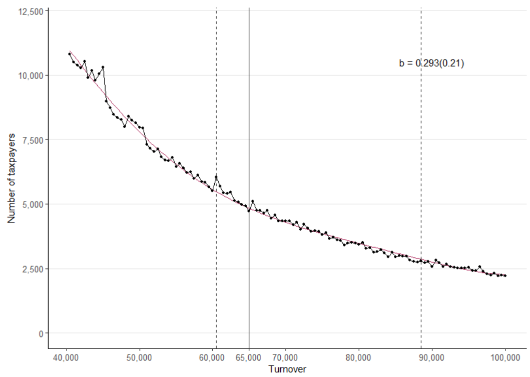

**(d)** 2015




**(e)** 2016

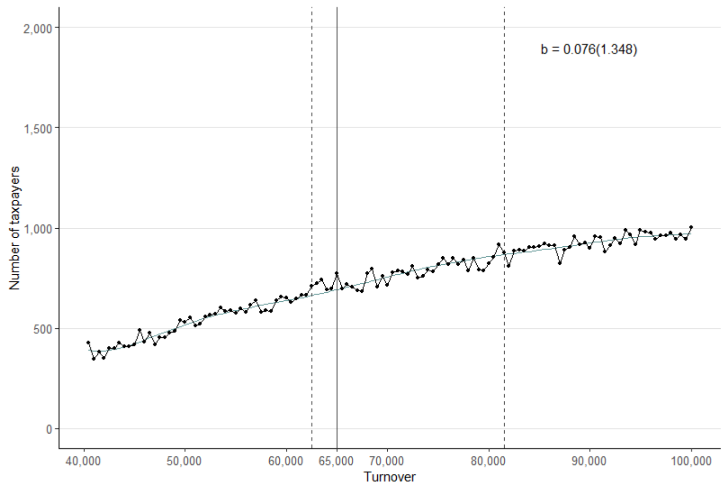



**(f)** 2017




Note: these graphs report the distribution of turnover in each year before 2018, that is the year in which €65,000 became the eligibility threshold of the F-regime. These graphs show there is no bunching below €65,000 in those years.The vertical dashed grey lines mark the excluded region of the distribution that is affected by bunching. The counterfactual distribution is estimated using the standard method described in section 1.3.1.

**Figure A11:** Placebo Test: no bunching for firms and self-employed with personnel expenditure (collaborators) larger than €20,000.




Note: this graph reports the distribution of turnover in 2019 for firms and self-employed with personnel expenditure (collaborators) larger than €20,000, for whom the preferential turnover regime does not apply. This graph shows there is no bunching below €65,000.

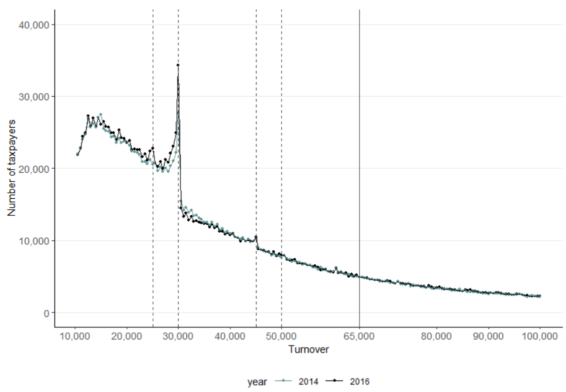
## Extensive and Intensive Margin Responses

This section explores the extensive and intensive margin responses due to the introduction of the turnover regime in 2015 and the subsequent reform in 2018 that raised and equalised the sector-specific eligibility thresholds to €65,000 from 2019 onwards.

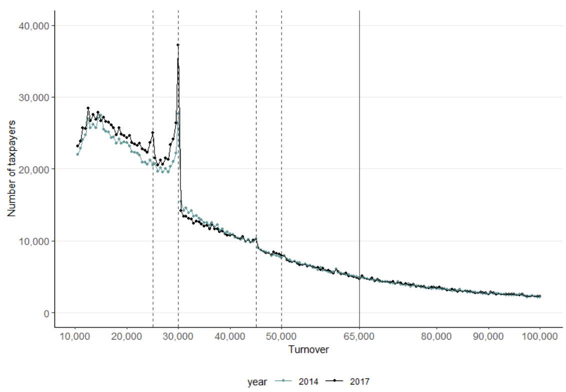
Figure A12 reports the turnover distributions in 2016 and 2017, panel (a)-(b) respectively, compared to 2014 that serves as a counterfactual. In both panels, the additional taxpayers below the new regime thresholds at €25,000 and €30,000, compared to 2014, form an excess mass that is larger than the missing mass above the €30,000 threshold. This suggests that the introduction of the turnover regime generated some extensive margin responses, in the form of new self-employed and/or people changing organisational forms of their businesses located below the threshold(s), in addition to those who reduced turnover (intensive margin responses). Indeed, the number of self-employed with turnover between €10,000 and €100,000 increases in the period 2016-2018 (see figure A14).

**Figure A12:** Extensive Margin Responses

**(a)** Turnover distributions: 2016 vs 2014




**(b)** Turnover distributions: 2017 vs 2014

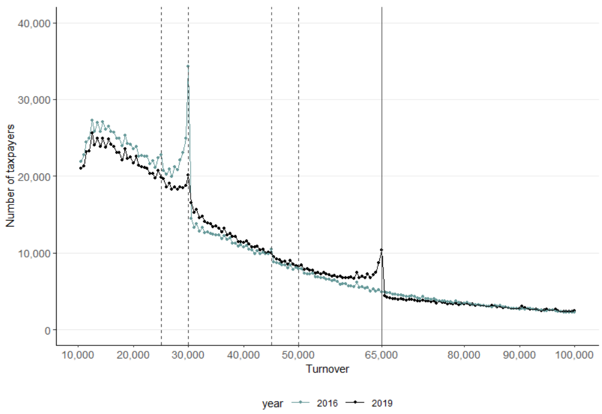



Note: these graphs report the distributions of turnover in 2016 (panel a) and 2017 (panel b) relative to 2014, which is used as a counterfactual.

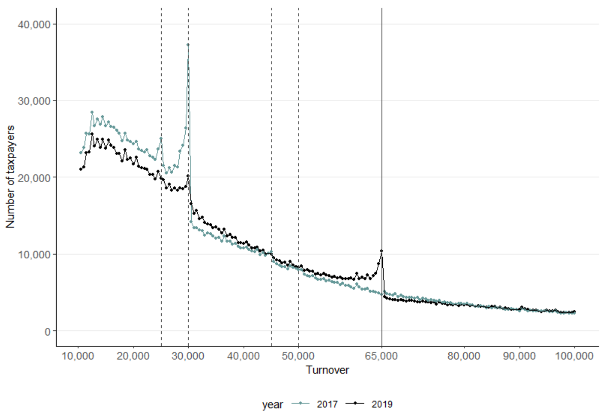
Figure A13 reports the turnover distribution in 2019, compared to 2016 (panel a) and 2017 (panel b) that serve as two alternative counterfactuals. It is possible to note that a larger number of taxpayers is now located between €30,000 – one of the thresholds in the period 2016-2018 – and €65,000 that is the eligibility cut-off for all sectors in 2019 after the 2018 reform. The total number of self-employed in 2019 reporting revenues between €10,000 and €100,000 is slightly lower than in the previous years. This provides a *prima facie* evidence that the new (higher) threshold at €65,000 in 2019 did not generate extensive margin responses. Hence, figure A13 suggests that the 2018 reform mostly induced intensive margin responses by incentivising people to either reduce turnover to locate below the €65,000 threshold, or increase turnover from an old threshold but not beyond the new threshold, and so benefiting from low taxation in the turnover regime.

**Figure A13:** Intensive Margin Responses

**(a)** Turnover distributions: 2019 vs 2016



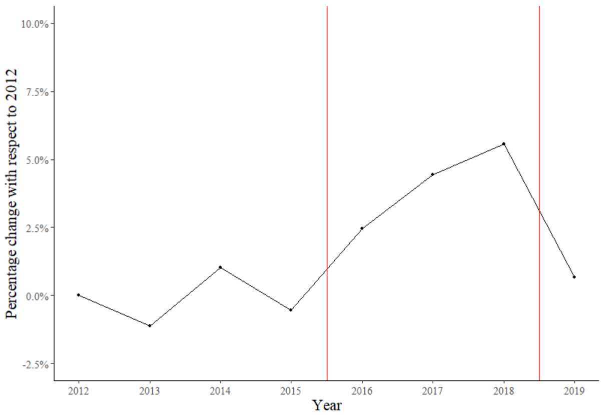

**(b)** Turnover distributions: 2019 vs 2017




Note: these graphs report the distributions of turnover in 2019 relative to 2016 (panel a) and 2017 (panel b), which are used as two alternative counterfactuals.

**Figure A14:** Number of self-employed over time: percentage point changes relative to 2012

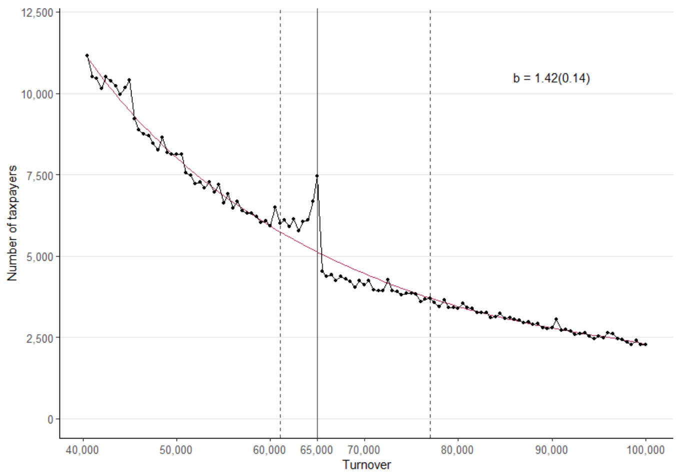
**(a)**



## Anticipation Effect

The tax reform enacting the €65,000 threshold or the preferential turnover regime (or simply F-regime) was announced and passed in the autumn 2018, and came into effect on January 1st 2019. One interesting question is whether or not individuals changed their behaviour in anticipation of the policy change. Figure A15 shows that some solo self-employed individuals responded to the new threshold in 2018 already by locating just below €65,000. The estimated bunching coefficient is 1.42, meaning that the excess mass of individuals below the €65,000 cut-off is equal to 142% of the estimated counterfactual frequency at the threshold. The institutional set-up provides a plausible explanation. While the new law took effect on January 1st 2019, access to the preferential regime required the new eligibility rule (turnover lower than €65,000) to be satisfied in the previous year. Hence, reporting turnover below €65,000 in 2018 allowed individuals to be eligible for the preferential turnover regime in 2019.

**Figure A15:** Bunching in 2018 at the new €65,000 F-regime threshold




Note: this graph reports the distribution of turnover for all taxpayers around the €65,000 threshold (vertical grey line). The vertical dashed grey lines mark the excluded region of the distribution that is affected by bunching. The counterfactual distribution is estimated using the standard method described in section 1.3.1 with a polynomial of order 5. The bunching coefficient $b$ is defined as the ratio between the estimated excess mass and the counterfactual frequency at the threshold. The 95% confidence interval is reported in brackets and is obtained with the bootstrap method by estimating a large number (500) of turnover distributions as detailed in section 1.3.1.

## Chapter 2

# A Theory of Public Good Provision with Heterogeneous Risk Preferences

## 2.1 Introduction

There is a growing literature documenting heterogeneous social preferences. Age, gender, economic background, educational attainment and other individual characteristics shape people's view concerning what the government should invest in and how progressive the tax system should be (Stantcheva, 2021). One area of government's policy that often polarises the public opinion is investment in infrastructures. Public projects are often perceived as too risky, either because they might take longer than expected to be completed, or because the net benefits of the projects might fail to materialise. For instance, a survey conducted in 2018 on the high speed rail project "HS2", that is currently under construction in the UK, shows that 59% of those who oppose the project are worried that "costs are or potentially will be too high", possibly reflecting different preferences for risk associated with skyrocketing costs of public projects.

Another example in which different attitudes towards risk might play a role is climate change. While most people agree climate change is a problem and measures must be taken to mitigate or stop it,[1] different views on the scale of the actions to implement emerge. The 2021 Eurobarometer survey on climate change shows that 80% of respondents from Sweden agree that "reducing fossil fuel imports from outside the EU can increase energy security and benefit the EU economically", while only 59% in France think so. Moreover, the perceived costs of climate change are also heterogeneous. In Portugal, 52% of respondents totally agree with the statement that the cost of damage due to climate change is much higher than the cost of investment needed for a green transition, but only 28% does so in Poland and Finland. While differences across countries can be explained by different exposures to climate risks (Dechezleprêtre et al., 2023), differences within countries could partly reflect different attitudes to (climate change) risk.

As the examples presented above suggest, people with different attitudes to risk not only make different individual decisions throughout their lives, but also have different views on the extent to which the government should invest in certain (risky) public goods, like infrastructures or green investments to protect the environment. Despite its relevance, the normative issue of public provision of public goods in the context of heterogeneous attitudes to risk, and the optimal tax system needed to implement it, has not received much attention. This paper contributes to filling this gap by presenting a theory of optimal provision of a (risky) public good when individuals have heterogeneous preferences for risk and face aggregate risk in the economy.

I consider a two-period model in which each type of agent is characterised by their level of risk aversion and endowment. Agents make intertemporal consumption and portfolio decisions, choosing between two types of assets: one is risk-free, while the other is subject to aggregate risk, but with a positive expected excess return.[2] When

[1] According to the 2021 Eurobarometer survey, 78% of the respondents consider climate change "a very serious problem", while 15% see it as "a fairly serious problem".

[2] Following the optimal tax literature, we distinguish two different components of the rate of return on savings: the (riskless) normal return and the excess return. The normal return is the price for forgoing

the public good is offered on the basis of individual contributions, we consider the Nash equilibrium outcome. Then, we consider the solution of a social planner maximising a generic social welfare function. The government can raise revenues with two distinct proportional taxes for the safe return and the risky excess return (with symmetric loss offsets), and by taxing the endowments. Tax revenues finance a public good that plays an insurance role, as risk is spread between public and private consumption. The public good is itself risky since tax revenues depend on the state of the economy. The tax parameters are chosen optimally ex-ante while the policy is implemented in the second period, after the realization of the state of the economy, such that the budget is balanced. This captures the idea that funds for a specific project might be limited in the short run, so if a negative shock occurs the project gets paused or reduced in scale. Alternatively, if we consider the environment as the public good, the risk in the economy is given by the climate risk associated with the range of possible future scenarios.

This paper delivers three main results. First, I show that the market would generally provide an inefficient probability distribution (and expected level) of the public good in the case of heterogeneous risk preferences. This is because agents do not internalise the fact that their contribution to the public good affects the distribution of risk between private and public consumption for the other agents that have different risk preferences. Given a generic social welfare function, the (ex-ante) First Best is achieved with (risk-aversion) type-specific lump-sum taxation.[3] Then, I consider the case in which different types cannot be targeted: the expected level of the public good will be generally suboptimal (second best outcome).

Second, I provide an application of the public good provision problem in which I characterise a simple tax system for endowments and capital income that is used to implement the second-best optimal policy. I show that the government sets the optimal variance of the public policy by balancing the volatility of public consumption with the average volatility of private consumption. The excess return tax plays a key role for this mechanism. Agents with different preferences for risk have different benefits from the tax/insurance policy at the margin, meaning their willingness to shift risk from the individual to the societal level depends on their risk aversion. Hence, when agents with different risk preferences cannot be targeted, taxing only the excess return part of capital income fails to deliver the First Best allocation. This is because agents cannot be individually compensated when the public good distribution is not in line with their risk preferences.

Third, I show that if endowments are not perfectly correlated with risk preferences, taxing the safe return can be optimal: the government gives up intertemporal efficiency to better target insurance to the different types of agents. It is possible to increase social welfare by exploiting the different individual portfolio and savings decisions as well as

present-time-consumption. The excess return reflects idionsyncratic characteristics and/or aggregate risk in the economy and drives returns heterogeneity.

[3]This differs from ex-post First Best that would require the type-specific lump-sum taxes to be state-contingent. From now on, First Best will simply indicate the ex-ante First Best.

their different responses to changes to the endowment tax schedule.

This paper contributes to two main strands of literature. First, this paper is related to the literature that analyses public good provision and optimal taxes in a risky environment. For instance, Christiansen (1993) and Schindler (2008) examine the efficiency/insurance effects and the role of capital income taxes when the economy faces aggregate risk. The contribution of this paper is to develop a theory of public good provision in a risky environment in presence of heterogeneous agents that possess different attitudes to risk and have potentially different endowments. Using a generic social welfare function, the appropriate Samuelson rule is derived. Moreover, differently from Schindler (2008), distorting intertemporal decisions by taxing the safe return from investments can be a useful tool to achieve the optimal allocation of risk in the economy, as this allows a more accurate targeting of agents with different risk preferences better.

Second, this paper is related to the growing literature on optimal taxation with heterogeneous returns. Previous papers have concentrated on the concepts of heterogeneous "investment ability" and/or scale effects (Boadway and Spiritus, 2021; Jacobs et al., 2020; Gahvari and Micheletto, 2016; Kristjánsson, 2016) as drivers of heterogeneous returns. This paper, in line with new empirical evidence (Bach et al 2020), considers different preferences for risk as an alternative driver of heterogeneous returns. This creates a connection between return heterogeneity and preference heterogeneity, as returns stem endogenously from preferences for risk through type-specific portfolio choice. While most of the above contributions argue in favor of taxing the normal (safe) return on grounds of redistribution,[4] our application shows that it can also have a role, alongside excess return taxation, in fostering insurance.

The rest of the paper is organised as follows. Section 2.2 revisits the theory of private provision of public good in the case of heterogeneous risk preferences. Section 2.3 presents the first and second best allocations with a generic social welfare function, and how it is possible to improve on the second best allocation. Section 2.4 presents an application of the public good problem in which the excess and safe returns can be taxed, and agents differ by their risk aversion. Section 2.5 extends section 2.4 by considering heterogeneous endowments. Section 2.6 concludes.

## 2.2 Public Good Provision in a Risky Environment

From the textbook theory of public goods, we know that public goods will be undersupplied by the market when provided on the basis of individual voluntary contributions (Atkinson and Stiglitz, 1980). In this section, I revisit the standard theory and show that the private provision of public goods is still inefficient in a risky environment when agents possess different risk preferences. Then, after presenting the Pareto optimal case – given by the appropriate Samuelson rule – I discuss the type of inefficiency that private

[4]In Boadway and Spiritus (2021), Jacobs et al. (2020), Gahvari and Micheletto (2016), the taxation of safe capital income complements the redistributive role of earnings taxation. See Bastani and Waldenström (2020) for a review of why and how capital should taxed.

provision produces in a risky environment.

Let us consider a set-up in which each agent $j$ allocates a share of her endowment $z$ to consumption in the first period $c_0$, and saves the residual $z - c_0$ investing in a risky portfolio with gross return $\tilde{R}_p$, given their level risk aversion $\theta_j$. In the second period, each agent enjoys private consumption $\tilde{c}_1$ and public consumption $\tilde{G}$ that are both subject to risk. Without loss of generality, I impose no discounting of future consumption. Moreover, individuals evaluate private and public consumption with the same sub-utility function $u(\cdot)$, meaning that no specific difference in taste between private and public goods is modeled.

$$\begin{aligned} \max_{c_0^j, g_j} \quad & U^j = u_j(c_0^j) + \mathbb{E}\left[u_j(\tilde{c}_1^j)\right] + \mathbb{E}\left[u_j\left(\tilde{G}\right)\right] \\ \text{s.t.} \quad & \tilde{c}_1 = \tilde{R}_p^j\left(z - c_0^j\right) - g_j, \\ & \tilde{G} = f\big(\textstyle\sum_j g_j, \tilde{x}\big) \end{aligned}$$

Each individual $j$ chooses how much to contribute $(g_j)$ to the public good $\tilde{G}$ in the first period by maximising lifetime utility $U$ taking the contributions of the other agents $(g_{-j})$ as given. In a no-risk situation, the level of the public good will be given by the sum of individual contributions. With an underlying source of risk in the economy $\tilde{x}$, then the public good is risky as it also depends on the state of the economy in the second period, i.e. $\tilde{G} = f\big(\sum_j g_j, \tilde{x}\big)$.[5] Hence, even if the marginal rate of transformation is one, meaning one unit of private consumption buys one unit of public consumption in a riskless economy, aggregate risk can either increase or decrease the actual value of the public good relative to the value of private consumption that was initially sacrificed. The optimal private contribution rule $g_j^*$ satisfies

$$g_j^* : MRS_{\tilde{G},\tilde{c}_1^j} = \frac{\mathbb{E}[u_j'(\tilde{G})]}{\mathbb{E}[u_j'(\tilde{c}_1^j)]} = 1 \;\; \forall j, \tag{2.1}$$

where the size of the public good $\tilde{G}$ depends on the sum of the individual contributions and aggregate risk in the economy $\tilde{x}$ i.e. $\tilde{G} = f\big(\sum_j g_j, \tilde{x}\big)$. Each agent $j$ contributes until the risk-adjusted marginal rate of substitution between the public and private good $MRS_{\tilde{G},\tilde{c}_1^j}$ equates the marginal cost (exp. 2.1). At the Nash equilibrium, the set of individual contribution rules $\{g_j, g_{-j}\}$ will satisfy (2.2) for each type. Summing up (2.1) across agents $j = 1, \cdots, n$ gives $\sum_j MRS_{\tilde{G},\tilde{c}_1^j} = n$. By comparing the Nash equilibrium outcome with the Pareto efficient allocation,[6] given by the appropriate

[5]The underlying source of risk should be seen as a risk for the overall economy, affecting both private consumption through portfolio returns, as well as public good consumption through aggregate shocks to public finances.

[6]Each Pareto-efficient allocation can be achieved by private, decentralised optimisation when the public good is financed by (risk-aversion) type-specific lump-sum taxes such that (2.2) is satisfied.

Samuelson rule

$$\sum_j MRS^j_{\tilde{G},\tilde{c}^j_1} = \sum_j \frac{\mathbb{E}[u'_j(\tilde{G})]}{\mathbb{E}[u'_j(\tilde{c}^j_1)]} = 1 < n, \tag{2.2}$$

we can state the following well-known result.

**Lemma 1.** *Public good provision in the case of private contributions is inefficient as agents do not internalise the external value of their individual contributions.*

In the context of aggregate risk in the economy, Lemma 1 has two specific implications. First, the public good will be underprovided in expectation. Second, private provision of public good generates an inefficient allocation of risk between private and public consumption, meaning that the probability distribution of the public good over different states of nature is suboptimal.

## 2.3 Samuelson Rule with a Social Welfare Function

The concept of Pareto optimality can be quite restricting when preferences are heterogeneous. Policy changes often benefits some agents while hurting others. Hence, we proceed by considering the optimal allocation that stems from the government's maximisation of a generic Social Welfare function

$$SW = \sum_j \phi_j\left(U_j\right),$$

where $\phi_j\left(U_j\right)$ is a weakly concave function of individual expected utility $U_j$ that is able to accommodate different individual risk preferences.[7] Then, the optimal rules for public good provision are derived. Two possible cases are presented: 1) (Ex-ante) First Best: the government can levy type-specific lump-sum taxes that depend on individual risk aversion (section 2.3.1); 2) Second Best: the government sets one unique lump-sum tax for all types (section 2.3.2).

### 2.3.1 First Best: Type Specific Lump-sum Taxes

The government chooses $t_i$ for each agent $i$, therefore perfectly targeting agents with different risk aversion. Expression 2.3 states that the optimal $t_i$ equates the marginal private costs of agent $i$ from contributing to the public good, expressed in terms of private consumption, to the social benefit of a marginal individual contribution:

$$\phi'_i\Big(U_i\Big)\,\mathbb{E}[u'_i(\tilde{c}^i_1)] = \sum_j \phi'_j\Big(U_j\Big)\,\mathbb{E}[u'_j(\tilde{G})] \quad \forall\, i. \tag{2.3}$$

[7]The issue of choosing the appropriate functions $\phi_j$ for utility functions with different curvatures is tackled separately in the third chapter of this thesis. The issue is widely debated in the welfare theory literature. For instance, Grant et al. (2010) shows how it is possible to accommodate concerns about different individuals' risk attitudes and concerns about fairness; Eden (2020) provides a practical method to perform welfare analysis.

At the optimum, expression (2.3) is satisfied for each agent $i$ and gives the set of optimal lump-sum taxes $t_j$ for all types of agent. The government sets $t_i$ for each agent $i$ such that the weighted expected marginal utility of private consumption is equalised across types to a certain level $\overline{k}$.

$$\{t_i, i = 1, \cdots, n\} \quad \rightarrow \quad \phi_i'\Big(U_i\Big)\, \mathbb{E}[u_i'(\tilde{c}_1^i)] = \overline{k} \quad \forall\, i = 1, \cdots, n. \tag{2.4}$$

By summing up condition (2.3) across types $i = 1, \cdots, n$, and using (2.4), we can obtain a modified Samuelson rule that is expressed in terms of the sum of socially-evaluated Marginal Rates of Substitutions ($MRS^s$), where

$$MRS_j^s = \frac{\phi_j'\Big(U_j\Big)\, \mathbb{E}[u_j'(\tilde{G})]}{\phi_j'\Big(U_j\Big)\, \mathbb{E}[u_j'(\tilde{c}_1^j)]}.$$

The $MRS^s$ is defined as the ratio between the marginal social welfare associated with an additional unit of the public good and the marginal social welfare associated with an additional unit of private consumption for an agent $j$.

**Theorem 1** (First Best). *At the First Best optimum, the sum of the $MRS^s$ is equal to the marginal rate of transformation between the public and private good.*

$$\sum_j MRS_j^s = \sum_j \frac{\phi_j'\Big(U_j\Big)\, \mathbb{E}[u_j'(\tilde{G})]}{\phi_j'\Big(U_j\Big)\, \mathbb{E}[u_j'(\tilde{c}_1^j)]} = 1. \tag{2.5}$$

Condition (2.5) is the Samuelson rule that reflects the social preferences that are being maximised. While type specific taxation is a useful theoretical benchmark, it is unlikely to be feasible in practise. The next section therefore considers an alternative case of public provision in which the lump-sum tax is set equal for all agents.

### 2.3.2 Second Best: Uniform Lump-sum Taxes

If the government cannot screen agents with different risk aversion, and therefore cannot set (risk-aversion) type specific lump-sum taxes, a uniform lump-sum tax $t$ for all agents apply. At the second best optimum, marginal social costs, expressed in terms of private consumption, are balanced with the social benefits:

$$\sum_j \phi_j'\Big(U_j\Big)\, \mathbb{E}[u_j'(\tilde{c}_1^j)] = n \sum_j \phi_j'\Big(U_j\Big)\, \mathbb{E}[u_j'(\tilde{G})]. \tag{2.6}$$

Condition (2.6) can be rewritten in terms of $\widetilde{MRS}_j^s$, where

$$\widetilde{MRS}_j^s = \frac{\phi_j'\Big(U_j\Big)\, \mathbb{E}[u_j'(\tilde{G})]}{\frac{1}{n}\sum_j \phi_j'\Big(U_j\Big)\, \mathbb{E}[u_j'(\tilde{c}_1^j)]},$$

that is defined as the ratio between the marginal social welfare associated with an additional unit of the public good for an agent $j$ and the average (across agents $j = 1, \cdots, n$) marginal social welfare associated with an additional unit of private consumption. This means that when type specific taxes are not available, the government evaluates the individual willingness to trade private with public consumption on the basis of the average sacrifice in terms of private consumption, e.g. $\widetilde{MRS}_j^s$ rather than $MRS_j^s$ . Hence, condition (2.7) is the modified Samuelson rule in the case of heterogeneous risk preferences when the government does not discriminate the different risk-aversion-types.

**Theorem 2** (Second Best)**.** *At the second best optimum, the sum of $\widetilde{MRS}_j^s$ equates the marginal rate of transformation.*

$$\sum_j \widetilde{MRS}_j^s = \sum_j \left( \frac{\phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{G})]}{\frac{1}{n}\sum_j \phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{c}_1^j)]} \right) = 1. \tag{2.7}$$

The Second Best outcome will differ from the First Best benchmark, both in terms public good provision and private consumption, and therefore social welfare, when $\sum \widetilde{MRS}_j^s \neq \sum MRS_j^s$, namely when condition (2.7) differs from (2.5). We can now state Proposition 1.

**Proposition 1.** *The inability of the government to discriminate different risk-aversion-types in the economy leads to a suboptimal provision of the public good.*

*Proof.* Suppose Proposition 1 is false, and $\tilde{G}^{SB} \equiv \tilde{G}^{FB}$, then it must be that (2.5) coincides with (2.7) and that

$$\frac{1}{n}\sum_j \phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{c}_1^{SB})] = \phi_i'\Big(U_i\Big)\,\mathbb{E}[u_i'(\tilde{c}_1^{FB})] \;\; \forall i = 1, \cdots, n.$$

In order for this to be true, it should follow that $t_i^{FB} = t^{SB}$ for $i = 1, \cdots, n$, meaning the individual private costs of raising a unit of consumption are equal across different types to begin with. Unless specific welfare weights are chosen to obtain this result or the equality happens to hold given the utility functions being used in the first place, proposition 1 will be true in the case with heterogeneous preferences. □

While risk-preference-specific lump-sum taxes might be unfeasible, let alone lump-sum taxes, we know that agents with different risk preferences will make different consumption, savings, portfolio and labour supply choices. These differences can therefore be exploited to get closer to the first-best optimum.

In the next two sections, I study an application of this principle with regard to savings and portfolio choices: the optimal (public) provision of public goods is characterised when the government taxes capital income.

## 2.4 Application: Taxation of Excess and Safe Return

I consider a simple two-period consumption/savings and portfolio model where agents have different risk preferences and the government finances the public good by raising equal lump-sum contributions and by taxing the riskless return and risky excess return separately.

### 2.4.1 Agent's Problem

$\Theta = \{(\theta_1), \cdots, (\theta_j), \cdots, (\theta_n)\}$ is a discrete set of agent-types with relative risk aversion parameters $\theta_j$, $j = 1, \ldots, n$, from the sample space $\mathbf{S}$, and each type has equal weight in the population. Given an endowment $z$, agents choose first period consumption, $c_0$, and how to invest the residual $(z - c_0)$. Agents choose the portfolio share of risky assets $s$ with the risky return $\tilde{r}_e$ being Normally distributed: $\tilde{r}_e \sim \mathcal{N}(\overline{r}_e - \sigma_r^2/2, \sigma_r^2)$. The resulting portfolio return $\tilde{r}_p$ will also be Normally distributed: $\tilde{r}_p \sim \mathcal{N}(\overline{r}_p - \sigma_p^2/2, \sigma_p^2)$. The (risky) excess return is defined as the difference between the risky return and the riskless return: $\tilde{r}^{exc} := \tilde{r}^e - r$. Without loss of generality, I impose no discounting: $\beta = 1$. Agent's utility is separable over time as well as between private and public consumption. The problem for agent-type $j$ is thus:

$$
\begin{aligned}
\max_{c_0^j,\, s_j} \quad & U_j = u_j(c_0^j) + \beta\, \mathbb{E}\left[u_j(\tilde{c}_1^j) + u_j(\tilde{G})\right] \\
\text{s.t.} \quad & \tilde{c}_1 = \tilde{R}_p^j \left(z - c_0^j\right) - t, \\
& \tilde{R}_p^j = \left[1 + r\,(1 - \tau_r) + s_j\,(\tilde{r}^{exc})\,(1 - \tau_k)\right]
\end{aligned}
$$

where $\tilde{R}_p$ is the gross portfolio return; $t$ is a lump-sum tax, equal for all types; $\tau_r$ and $\tau_k$ are the tax rates on the riskless and excess return respectively. The FOCs for first period consumption $c_0$, and share of risky assets $s$ are:

$$
\begin{aligned}
c_0^* : \quad & u'(c_0) = \mathbb{E}\left[u'(\tilde{c}_1)\tilde{R}_p\right] \\
s^* : \quad & 0 = \mathbb{E}\left[u'(\tilde{c}_1)\tilde{r}^{exc}\right]
\end{aligned} \tag{2.8}
$$

Notice that, while taxation of the riskless return changes the resource allocation (first period consumption, savings), taxing the excess returns with loss offsets does not. Agents will adjust their portfolio shares to get the same pre-tax expected portfolio return (Domar and Musgrave, 1944). After applying the covariance identity to (2.8), we can derive the expression for the risk premium with respect to the expected excess return.

$$
\mathbb{E}[R^e - R] = -\frac{\operatorname{cov}\left[u_j'(\tilde{c}_1^j), \tilde{r}^{exc}\right]}{\mathbb{E}\left[u_j'(\tilde{c}_1^j)\right]}. \tag{2.9}
$$

For each agent $j$ with relative risk aversion parameter $\theta_j$, expression (2.9) maps the covariance term to the expected marginal utility of second-period consumption $\tilde{c}_1$.

### 2.4.2 The Government

The government provides a public good $\tilde{G}$ that enters the utility function separately from consumption. The policy is financed by taxing risky excess return at rate $\tau_k$, the safe return at rate $\tau_r$, and levying a lump-sum tax $t$, equal for all agents. The public good is itself risky as it depends on risky tax revenues from the excess return. Tax rates are set by the government in the first period anticipating agents' optimal behaviour. After the state of the economy is realised, the government implements the policy and balances the budget. As a result, the provision of the public good is stochastic and depends on the state of the economy in the second period.

The government's objective is to maximise Social welfare (SW) that is defined as a weighted sum of agents' expected utilities, where $U$ is a Von Neumann-Morgenstern utility function.

$$\begin{aligned}\max_{\tau_k,\tau_r,t} SW &= \sum_j \phi_j\Big(U_j\Big)U_j\left(c_0^{j*},\, \tilde{R}_p^j\left(z - c_0^{j*}\right) - t,\, \tilde{G}\right)\\ \text{s.t. } \tilde{G} &= \sum_j \left[\left(\tau_k\, s_j^*\, \tilde{r}^{exc} + \tau_r r\right)\left(z - c_0^{j*}\right) + t\right]\end{aligned}$$

I substitute the expression for the public good directly in the social welfare function, as in Schindler (2008). It ensures that the budget is balanced for any state of the world.[8]

### 2.4.3 Optimality Conditions

In this section, I show the optimality conditions for the linear taxes on the excess and riskless return when a pure public good is provided. Moreover, I assume that the government cannot screen agents with different risk preferences, meaning the government uses a uniform lump-sum tax, and endowments are equal in the population.

#### Excess Return Tax

$$\mathbb{E}[\tilde{r}^{exc}] = -\frac{\sum_j \phi_j'(U_j)\,\mathrm{cov}\left[u_j'(\tilde{G}), \tilde{r}^{exc}\right]}{\sum_j \phi_j'(U_j)\,\mathbb{E}[u_j'(\tilde{G})]} \tag{2.10}$$

The optimal excess return tax rate $\tau_k^*$ has to satisfy condition (2.10). Given the risk preferences of all agents in society, the government chooses $\tau_k^*$ to reach the optimal allocation of risk between private and public consumption, and sets the optimal variance of the public good policy. The key novelty in this setting is that the "benefits" of the public good differ among agents because of heterogeneous attitudes to risk. Since the public good is itself risky, the welfare gain from increasing the tax rate $\tau_k$ varies across

[8] This is a stricter requirement than balancing the budget in expectation, that would instead imply transferring resources from good states to bad states of the world.

agents. Hence, the government aims to balance the different "preferences" for the public good, providing insurance against aggregate risk.

Two interpretations can be developed that focus on the issues of tax revenue collection and the public good respectively. The first one is that the government is choosing what share of the budget (tax revenues) should be risky.[9] As individuals make portfolio choices on the basis of their risk preferences according to (2.9), similarly the government decides the distribution of tax revenues financing the public good over states of nature on the basis of (2.10), such that the risk preferences of all agents are taken into account.

The second interpretation relates to the use of tax revenues. The (risky) revenues collected from taxing the excess return will generate a certain probability distribution of the public good. This distribution entails a certain allocation of risk between private and public consumption: the public good has an insurance role.[10] The government's objective is to choose the social-welfare-maximising public good distribution that achieves the optimal allocation of risk in the economy. In doing so, the government takes into account the agents' willingness to shift risk to the societal level. Condition (2.11) reformulates (2.10) and better represents this concept. Private and public consumption volatility are represented by the covariance between marginal utility of private and public consumption respectively with the risky excess return and jointly govern the individual willingness to pay for the public good with an extra euro of risky excess capital income.

**Theorem 3.** *When agents with different risk preferences cannot be discriminated, $\tau_k^*$ equalizes public consumption volatility with (average) private consumption volatility.*

$$\sum_j \frac{\phi_j'(U_j)\,\mathrm{cov}\left(u_j'(\tilde{G}), \tilde{r}^{exc}\right)}{n^{-1}\sum_j \phi_j'(U_j)\,\mathrm{cov}\left(u_j'\left(\tilde{c}_1\right), \tilde{r}^{exc}\right)} = 1 \tag{2.11}$$

*Proof.* See Appendix A.

Theorem 3 says that $\tau_k^*$ is chosen on the basis of a weighted average of private consumption volatility, i.e. $n^{-1}\sum_j \phi_j'(U_j)\,\mathrm{cov}\left(u_j'\left(\tilde{c}_1\right), \tilde{r}^{exc}\right)$. This is because the government cannot target individual risk-aversion types. This creates an inefficiency: an agent with relatively (more) risky private consumption $c_1$, compared to other agents, would be better-off by shifting more risk to the public good. For that to be the case, a higher tax rate $\tau_k$ should be implemented, so that private consumption volatility is traded with public consumption volatility. Thus, the outcome produced by (2.11) could be improved if the government were able to target different types, and compensate agents that would prefer a different distribution of the public good.

**Corollary 1.** *When the government can target different risk aversion types (2.11) sim-*

[9] The realisation of the excess return depends on the state of the economy.

[10] When a bad (good) state of economy realises in the second period, losses (gains) due to negative (positive) excess returns will be spread over private and public consumption, so that the utility loss (gain) is minimised (maximised).

*plifies to*

$$\sum_j \frac{\phi'_j(U_j)\,\mathrm{cov}\left(u'_j(\tilde{G}), \tilde{r}^{exc}\right)}{\phi'_j(U_j)\,\mathrm{cov}\left(u'_j\left(\tilde{c}_1\right), \tilde{r}^{exc}\right)} = 1 \tag{2.12}$$

*and the allocation of risk between private and public consumption relates to individual willingness to shift risk from private to public consumption* $\frac{\phi'_j(U_j)\,\mathrm{cov}\left(u'_j(\tilde{G}),\tilde{r}^{exc}\right)}{\phi'_j(U_j)\,\mathrm{cov}\left(u'_j(\tilde{c}_1),\tilde{r}^{exc}\right)}$.

*Proof.* See Appendix B.

To sum up, when agents have heterogeneous risk preferences, the optimal tax $\tau_k$ is not just about balancing the volatility of private and public consumption, as in Boadway and Spiritus (2021) and Schindler (2008), but also aims to balance the different "preferences" for the public good, providing insurance against aggregate risk (Theorem 3). As the optimal tax rate $\tau_k^*$ implies a specific probability distribution of the public good, taxing the excess return only may be suboptimal. Corollary 1 shows that welfare improvements are possible when different types can be targeted. If type-specific lump-sum taxation is not available, other tax instruments that have differential impacts on different types could be used. In the next section, I argue that the taxation of the safe return has this feature. Finally, it is possible to show that the optimal excess return tax rate is positive.

**Safe Return Tax**

The optimal safe return tax rate $\tau_r^*$ satisfies:

$$\underbrace{-\sum_j \phi'_j(U_j)\mathbb{E}\left[u'_j\left(\tilde{c}_1^j\right)\right]\left(z - c_0^j\right) r}_{\text{private welfare effect } = \Delta U} + \underbrace{\sum_j \phi'_j(U_j)\mathbb{E}\left[u'_j\left(\tilde{G}\right)\right]\sum_j\left(z - c_0^j\right) r}_{\text{mechanical welfare effect } = \Delta M}$$
$$\underbrace{-\tau_r r\sum_j \phi'_j(U_j)\mathbb{E}\left[u'_j\left(\tilde{G}\right)\right]\sum_j \frac{\partial c_0^j}{\partial \tau_r}}_{\text{behavioural effect } = \Delta B} = 0. \tag{2.13}$$

A marginal change in $\tau_r$ determines a welfare loss for agents as second period consumption is lowered: this is the private welfare effect ($\Delta U$). On the other hand, the additional tax revenues finance the public good which increases agents' utility: this is the mechanical welfare effect ($\Delta M$). However, varying the tax rate affects agents' savings too (via change in first period consumption $c_0$) through substitution and income effects: this is the behavioural effect ($\Delta B$), which affects the tax base and therefore tax revenues.

At the optimum, private welfare losses are balanced with the welfare gains from public good provision, net of behavioural effects: $\Delta U + \Delta M + \Delta B = 0$. Unless the mechanical and private welfare effects sum up to zero, the optimality condition is satisfied only with $\tau_r \neq 0$, provided that $\sum_j (\partial c_0^j / \partial \tau_r) \neq 0$.

**Theorem 4.** *a) With heterogeneous risk preferences, the mechanical and private welfare do not sum up to zero: $\Delta U + \Delta M \neq 0$ or equivalently $|\Delta U| \neq |\Delta M|$. b) Then, $\tau_r^* \neq 0$ solves the optimal tax condition (14).*

*Proof.* (Utilitarian case, i.e. $\phi_j(U) = U$ for all $j = 1, \cdots, n$.)

a) We can use (2.6) to rewrite the mechanical term as follows

$$\Delta M = \frac{1}{n} \sum_j \mathbb{E}\big[u_j'(\tilde{c}_1^j)\big] \sum_j (z - c_0^j) r$$

Define $\mathbb{E}\left[u_j'\left(\tilde{c}_1^j\right)\right] = a_j$, and $\left(z - c_0^j\right) r = b_j$. Notice that as $a_H \neq a_L$ and $b_H \neq b_L$ for any agents $H, L$ with $\theta_H > \theta_L$, then $|\Delta U| = \sum_j a_j b_j \neq n^{-1} \sum_j a_j \sum_j b_j = |\Delta M|$.
b) Consider $\sum_j (\partial c_0^j / \partial \tau_r) \neq 0$. If $|\Delta U| \neq |\Delta M|$, then the optimality condition is satisfied when $\tau_r^* \neq 0$, as we have $\Delta B \neq 0 \iff \tau_r^* \neq 0$. $\square$

Hence, it can be optimal to tax the safe return when agents have heterogeneous risk preferences. With CRRA utility, different relative risk aversion parameters imply different tastes for risk (portfolio decisions), different slopes of the consumption path as well as different responses of savings decisions responses to taxation of returns. Theorem 4 tells us that exploiting these differences by taxing (positively or negatively) the riskless part of the return can increase social welfare. Therefore, a trade-off between insurance and intertemporal efficiency can arise.

Finally, we consider the case in which the government can impose type-specific lump-sum taxes.

**Corollary 2.** *When the government can target agents with different risk preferences with type-specific lump-sum taxes, (2.13) is always satisfied by $\tau_r^* = 0$.*

Hence, the taxation of the safe return acts as an (imperfect) substitute for (risk-aversion) type-specific taxation.

## 2.5 Heterogeneous Endowments and Risk Aversion

In this section, I revisit the results of section 2.4 by considering an environment in which each type of agent is characterised by their endowment and risk aversion, and the government taxes endowments and capital income. The question that this section aims to answer is whether taxing the safe return is still optimal when endowments are taxed non-linearly and the level of endowment is correlated with the individual's risk aversion.

### 2.5.1 Additional Notation

The set of agent-types is a $m \times n$ matrix $\Theta$, where each element is given by the pair $(z_i, \theta_j)$ that represents an individual-type $ij$ who has endowment $z_i$ and relative risk aversion $\theta_j$. $F$ is the frequency matrix of types in the population, with each type having frequency $f_{ij}$ in the population. We also define an endowment group $i$ as the i-th row of $\Theta$: $(z_i, -) := \{(z_i, \theta_1), \cdots, (z_i, \theta_n)\}$. Thus, $(z_i, -)$ is the group of agents that have the same endowment $z_i$, but differ by their risk aversion $\theta_j, j = 1, \cdots, n$.

$$\Theta_{m\times n} = \begin{pmatrix} z_1, \theta_1 & \cdots & z_1, \theta_n \\ \vdots & \ddots & \vdots \\ z_m, \theta_1 & \cdots & z_m, \theta_n \end{pmatrix}, \quad F = \begin{pmatrix} f_{11} & \cdots & f_{1n} \\ \vdots & \ddots & \vdots \\ f_{m1} & \cdots & f_{mn} \end{pmatrix}.$$

As in the previous section, each component of capital income is taxed linearly: $\tau_k$ is the excess return tax, $\tau_r$ is the tax rate on the safe return. Endowments are subject to a piece-wise linear schedule $T(z)$, where $\tau_e^i$ is the average tax rate on endowment $z_i$ given the tax schedule $T(z)$ that applies on the endowment group $(z_i, -)$. With many levels of endowment $z_i$, $T(z)$ will approximate a non-linear function of $z_i$. $y_i$ is the net-of-tax endowment: $y_i := z_i(1 - \tau_e^i)$.

### 2.5.2 Taxation of Endowments and Safe Capital Income

Conditions (2.14) and (2.15) are the optimality conditions for the endowment tax $\tau_e^i$ and the safe return tax $\tau_r$ respectively. First, we analyse the optimality condition (2.14) for the optimal average tax rate $\tau_e^i$ on each endowment group $(z_i, -)$, $i = 1, \cdots, m$.

$$\tau_e^{i*} : \frac{\sum_j f_{ij}\phi_j'(U_{ij})\,\mathbb{E}\left[u_{ij}'(\tilde{c}_1^i)\right] z_i(1 + r(1 - \tau_r))}{\sum_j f_{ij}\left[z_i(1 + r(1 - \tau_r)) - \tau_r r \cdot \frac{\partial c_0^{ij}}{\partial \tau_e^i}\right]} = \sum_{i,j} f_{ij}\phi_j'(U_{ij})\,\mathbb{E}\left[u_{ij}'(\tilde{G})\right] \tag{2.14}$$

$$\tau_r^* : \frac{\sum_{i,j} f_{ij}\phi_j'(U_{ij})\,\mathbb{E}\left[u_{ij}'(\tilde{c}_1^i)\right]\left[y_i - c_0^{ij}\right]}{\sum_{i,j} f_{ij}\left[y_i - c_0^{ij} - \tau_r \frac{\partial c_0^{ij}}{\partial \tau_r}\right]} = \sum_{i,j} f_{ij}\phi_j'(U_{ij})\,\mathbb{E}\left[u_{ij}'(\tilde{G})\right] \tag{2.15}$$

At the optimum, condition (2.14) establishes that the marginal private welfare costs from raising $\tau_e^i$ for endowment group $(z_i, -)$, expressed in terms of revenues being raised, is equalised to the (population-wide) marginal social value of the public good. This should, in turn, equalise the marginal private welfare costs from raising the safe return tax $\tau_r$, in terms of revenues being raised, so that (2.15) is satisfied too.

A fully optimised endowment tax schedule $T(z)$ requires (2.14) to hold for each endowment group, meaning that marginal private welfare costs from raising the endowment tax rate are equalised across endowment groups. Moreover, taxing the endowment will generate heterogeneous consumption responses that will affect also tax revenues from safe capital income taxation, i.e. the term $-\tau_r r \cdot \frac{\partial c_0^{ij}}{\partial \tau_e^i}$ in (2.14). These differential effects across types can therefore be exploited to make sure (2.14) is indeed satisfied across endowment groups. When there is imperfect correlation (or no correlation at all) between endowment and risk aversion in the economy, it is not obvious that (2.14) will be holding across endowment groups without taxing the safe return. For each endowment group with a specific distribution of attitudes to risk within it, the welfare costs from raising the endowment tax might differ (LHS of condition 2.14), that is why taxing the safe return might improve welfare.

When the level of the endowment is not highly correlated with the level of individual risk aversion, endowment taxation is not an effective way to discriminate across agents with different risk aversion. With its heterogeneous effects on agents with different risk attitudes, taxing the safe return can therefore help to achieve a better allocation of aggregate risk in the economy between private and public consumption and across agents. Of course, if there is perfect correlation between the level of the endowment and the risk aversion parameter, and endowments are taxed according to $T(z)$, the first-best public good distribution can be achieved without taxing the safe return. In this case the piece-wise linear tax schedule $T(z)$ would be equivalent to a type-specific lump-sum tax system, as each endowment level in the type-set $\Theta$ is attached to one specific risk aversion parameter, and taxing the safe return becomes redundant - see a short proof in the appendix.

## 2.6 Concluding Remarks

People with different attitudes to risk have different views on the extent to which society should invest in certain (risky) projects. This paper presents a theory of optimal provision of a (risky) public good when individuals have heterogeneous preferences for risk and face aggregate risk in the economy. In an environment in which the public good is a tool to shift risk from private to public consumption, this paper shows that the inefficiency of private provision of public goods comes from agents failing to internalize the insurance effects of the public good for the other agents. Given a social welfare function, I characterise the (ex-ante) First Best allocation, which is achieved with (risk-aversion) type-specific lump-sum taxation. Discriminating the different types of agent allows the government to compensate them when the distribution of the public good is not in the line with their risk preferences.

Then, I characterize the second best allocation, which is the optimum that can be achieved under the constraint that type-specific taxes and transfer are not available. In an application with capital income taxation, I show that the excess return tax is key to match the optimal variance of the public insurance policy. Moreover, it is possible to justify a positive tax on the safe return: it is optimal for the government to give up intertemporal efficiency by taxing the safe return to provide better-targeted insurance to agents. This complements the excess return tax: the government exploits agents' different portfolio and savings decisions as well as different responses to tax changes to increase social welfare.

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# Appendix B

## Theorem 3

The optimality condition for the excess return tax $\tau_k$ reads as follows.

$$\sum_j \phi_j'(U_j)\mathbb{E}\Big[u_j'(\tilde{G})\sum_j s_j\left(z^j - c_0^j\right)\tilde{r}^{exc}\Big] + \tau_k \sum_j \phi_j'(U_j)\mathbb{E}\left[u_j'(\tilde{G})\sum_j \frac{\partial s_j}{\partial \tau_k}\left(z^j - c_0^j\right)\tilde{r}^{exc}\right] = 0.$$

Using the fact that $\frac{\partial s_j}{\partial \tau_k} = \frac{s_j}{(1-\tau_k)}$, we can collect terms.

$$\sum_j \phi_j'(U_j)\mathbb{E}\Big[u_j'(\tilde{G})\sum_j s_j\left(z^j - c_0^j\right)\tilde{r}^{exc}\Big]\left(1 + \frac{\tau_k}{1-\tau_k}\right) = 0.$$

Then, as $\sum_j s_j\left(z^j - c_0^j\right)\left(1 + \frac{\tau_k}{1-\tau_k}\right) \neq 0$, we get the following expression

$$\sum_j \phi_j'(U_j)\mathbb{E}\left[u_j'(\tilde{G})\tilde{r}^{exc}\right] = 0.$$

By further manipulating the above expression using the covariance identity, and conditions (2.6) and (2.8), we can rewrite the optimality condition for $\tau_k$ as follows

$$\frac{1}{n}\sum_j \phi_j'(U_j)\,\mathrm{cov}\left(u_j'\left(\tilde{c}_1^j\right), \tilde{r}^{exc}\right) = \sum_j \phi_j'(U_j)\,\mathrm{cov}\left(u_j'(\tilde{G}), \tilde{r}^{exc}\right).$$

or

$$\sum_j \frac{\phi_j'(U_j)\,\mathrm{cov}\left(u_j'(\tilde{G}), \tilde{r}^{exc}\right)}{\frac{1}{n}\sum_j \phi_j'(U_j)\,\mathrm{cov}\left(u_j'\left(\tilde{c}_1^j\right), \tilde{r}^{exc}\right)} = 1.$$

$\square$

## Corollary 1

Condition (2.4), plus the covariance identity and condition (2.8) imply that the term

$$\phi_j'(U_j)\,\mathrm{cov}\left(u_j'\left(\tilde{c}_1^j\right), \tilde{r}^{exc}\right)$$

is equal across types $j$. Hence, expression 2.11 can be reformulated as follows

$$\begin{aligned}&\sum_j \frac{\phi_j'(U_j)\,\mathrm{cov}\left(u_j'(\tilde{G}), \tilde{r}^{exc}\right)}{\frac{1}{n}\sum_j \phi_j'(U_j)\,\mathrm{cov}\left(u_j'\left(\tilde{c}_1^j\right), \tilde{r}^{exc}\right)}\\ =&\sum_j \frac{\phi_j'(U_j)\,\mathrm{cov}\left(u_j'(\tilde{G}), \tilde{r}^{exc}\right)}{\phi_j'(U_j)\,\mathrm{cov}\left(u_j'\left(\tilde{c}_1^j\right), \tilde{r}^{exc}\right)} = 1.\end{aligned}$$

□

## Perfect Correlation Between Risk Aversion and Endowments

*Short Proof.* Consider the following set of types $\Theta$ and the corresponding frequency matrix of types in the population $F$:

$$\Theta = \begin{pmatrix} z_A, \theta_A & z_A, \theta_B \\ z_B, \theta_A & z_B, \theta_B \end{pmatrix}, \quad F = \begin{pmatrix} f_{AA} & f_{AB} \\ f_{BA} & f_{BB} \end{pmatrix}.$$

With perfect correlation between risk aversion and endowment level, only types on either of the two diagonals of $\Theta$ have positive frequencies. Suppose that $(z_A, \theta_A)$; $(z_B, \theta_B)$ have positive frequencies in the population, that are $f_{AA}, f_{BB} > 0$ respectively. Suppose the optimal safe return tax rate is zero: $\tau_r^* = 0$. Then, condition (2.14) must be valid for both agents with $\tau_r = 0$. The following relationship can be derived as a result:

$$\phi_A'(U_{AA})\, \mathbb{E}\left[u_{AA}'(\tilde{c}_1)\right] = \phi_B'(U_{BB})\, \mathbb{E}\left[u_{BB}'(\tilde{c}_1)\right]. \tag{16}$$

Exploiting (16) makes the condition for $\tau_r^*$ (2.15) satisfied, meaning that our initial guess $\tau_r = 0$ is indeed a solution of the optimality condition for the safe return tax (2.15). Moreover, as (16) is equivalent to (2.4), the endowment tax schedule $T(z)$ is equivalent to a type-specific lump-sum tax system in this example. Hence, the first best allocation is achieved by simply taxing endowments and the risky excess return. □

## Chapter 3

# A Criterion to Evaluate Social Welfare when Risk Preferences are Heterogeneous

## 3.1 Introduction

Governments are often required to make decisions under risk. Policy-makers might need to choose which social and/or industrial policy to implement among a series of options with different risk-profiles, or whether to finance big infrastructure projects. A debated issue in the welfare economics literature regards how to evaluate such choices when individuals in society have different attitudes to risk. The conceptual problem is how to construct a social welfare criterion that ensures comparability of well-being levels across different types of individual.

The seminal papers by Harsanyi (1953, 1955, 1977a) provided a framework to make social evaluations of different social scenarios/policies. In Harsanyi's thought experiment, individuals face a lottery in which they can obtain certain preferences and characteristics (identity lottery), and their well-being depends on the specific social state that is realised (outcome lottery). Individuals are assumed to be expected-utility-maximisers, and evaluate impartially the alternative social policies by placing themselves in the position of every other type of individual and adopting their preferences (acceptance principle). By doing so, individuals compare the different social scenarios implied by the available policies, and the corresponding well-being levels from acquiring a certain identity in a given social state. This allows Harsanyi to derive weighted utilitarianism as a sum of von Neumann-Morgenstern utilities, where different weights should apply to individuals with different preferences to ensure interpersonal comparability.

Harsanyi's axiomatisation of utilitarianism is not uncontroversial, both in general terms and in the specific context of heterogeneous risk preferences in society.[1] With regard to the latter case, Harsanyi's weighted utilitarianism is unable to accommodate heterogeneous risk preferences: "Harsanyi's impartial observer does not care who faces this risk" (Grant et al., 2010, pp. 1951). The issue is that Harsanyi's impartial observer framework combines attitudes to risk of different individuals (Pattanaik 1968, pp. 1166) by making the identity-outcome lottery a one-stage lottery. To overcome these problems, Grant et al. (2010) axiomatises "Generalised Utilitarianism". The key idea is that accommodating different preferences for risk requires individual-specific functions to map individual von Neumann-Morgenstern (henceforth vNM) utilities to social welfare. However, in order to apply generalised utilitarianism to problems in public economics, practical examples of this framework are needed.

This paper provides such an example by proposing a new welfare criterion to evaluate social options in the presence of risk and heterogeneous attitudes to it, in the spirit of Grant et al. (2010). I model a multi-stage lottery of life in which individuals face different possible states of nature (last stage of the game), given certain characteristics that are obtained in the intermediate stage, and risk preferences, which are acquired

[1] The critique of Sen (1976, 1977, 1986) to Harsanyi (1975, 1977b), which was re-analysed by Weymark (1991), stresses that there is no explicit justification for using von Neumann-Morgenstern (vNM) utility functions as welfare indexes, and that Harsanyi's results are not about utilitarianism (Weymark, 1991, pp. 256). An alternative analysis of the debate is offered by Greaves (2017) who considers it more a matter of semantics than of substance.

in the first stage. Then, by applying Bayesian rationality, a social welfare criterion is constructed by evaluating the lottery of life ex-ante. Following Harsanyi's thought experiment, the individuals in the original position of the extended lottery (impartial observers) place themselves in the position of every other type and adopt their preferences. I present a criterion for the specific case in which agent's well-being is represented by a vNM utility function and state-contingent utility takes the constant relative risk aversion form.[2] The proposed criterion $W$ reads as follows:

$$W = \sum_{i,j} f_{ij} g_j \left(U_{ij}(\ell_{ij})\right) = \sum_{i,j} f_{ij} \frac{\left(U_{ij}(\ell_{ij})\right)^{1-\theta_j} - 1}{1-\theta_j} \tag{3.1}$$

where $f_{ij}$ is the probability of an agent acquiring a set of characteristics of type $i$, after receiving risk preferences of type $j$, $\ell_{ij}$ is the outcome lottery, and a continuous, increasing and concave transformation $g_j(\cdot)$ applies to the vNM utility functions $U_{ij}$. This transformation ensures the criterion is ex-ante egalitarian and accommodates different preferences for risk (Grant et al., 2010). I show that the proposed criterion has three properties: (i) an overall increase in risk is associated with a lower social welfare index; (ii) the criterion is ex-ante egalitarian among agents with equal preferences for risk; (iii) the welfare losses from increasing risk in the outcome lottery can be partly offset by redistribution of resources among agents with equal preferences for risk.

This paper makes two main contributions to the literature. First, it builds on the impartial observer framework, developed by Harsanyi (1953, 1955, 1977a) and Vickrey (1945, 1960) by considering a multi-stage lottery of life in which individuals face different possible states of nature, given certain personal characteristics and risk preferences, which are obtained in two consecutive stages. This allows us to deal with the critique by Pattanaik (1968), who argued that a one-stage lottery of life makes the impartial observer combine the different attitudes to risk of different individuals. Then, following Harsanyi (1953, 1955, 1977a), I assume that individuals' preferences are represented by von Neumann-Morgenstern utility functions, and therefore satisfy the expected utility axioms, and derive the criterion using Bayesian rationality.

Second, this paper relates to the literature investigating welfare analysis in the context of heterogeneous risk preferences (Grant et al., 2010; Eden, 2020). The welfare criterion that is derived in this paper satisfies "generalised weighted utilitarianism", which is defined and axiomatised by Grant et al. (2010). In particular, criterion (3.1) entails that individual vNM utilities are mapped to social welfare via continuous, increasing functions $g_j(\cdot)$. Moreover, I show that the criterion is ex-ante egalitarian within groups of individuals with equal risk aversion, and accommodates different preferences for risk, as the function $g_j(\cdot)$ depends on individual's risk aversion parameter $\theta_j$. Then, the proposed criterion offers an alternative to the welfare index derived by Eden (2020). Eden

[2]The choice of constant relative risk aversion utility is motivated by two factors: (i) CRRA utility is a widely used class of utility functions; (ii) the resulting social welfare criterion can be used for future work on the second chapter of this thesis to perform numerical simulations and social welfare evaluations.

(2020) provides a social welfare function using an iterative procedure that exploits the certainty equivalents of the lotteries faced by different risk-aversion types of agent. The main difference between my approach and Eden's welfare index is that I retain Harsanyi's impartial observer framework and the assumption that individual preferences satisfy the expected utility axioms. While this is a stricter assumption than those used by Eden (2020),[3] Harsanyi's framework allows me to construct a criterion by using Bayesian rationality. Moreover, my criterion can be used flexibly in any setting that can be framed as a multi-stage lottery. The topic of the optimal public good provision in the presence of heterogeneous risk preferences, which was analysed in the second chapter of this thesis, is an example.

The rest of the paper is organised as follows. Section 3.2 outlines the set-up and notation. Section 3.3 describes the extended lottery framework. Section 3.4 presents the welfare criterion and describes its properties with numerical examples. Section 3.5 discusses the criterion and its properties. Section 3.6 concludes.

## 3.2 Set-up and Notation

Consider a finite set of individuals $\mathcal{I} = \{1, \cdots, N\}$ facing an identity lottery over personal characteristics $z_i, i = 1, \cdots, I$, and preferences for risk represented by $\theta_j, j = 1, \cdots, J$, as well as an outcome lottery over possible social outcomes $\mathcal{X} = \{x_1, \cdots, x_s, \cdots, x_S\}$. Each realisation of the outcome lottery, that is a social outcome, can have consequences that are identity-specific and/or equal for all individuals. This is meant to capture the fact that certain risks are common to everyone, such as the risk of lower income due to illness or unemployment, while others are related to one's preferences, such as the risk of lower future consumption (today's savings) due to the volatility of the financial markets. Preferences over these lotteries are complete, transitive and continuous. Each individual with preferences for risk of type $j$ and personal characteristics of type $i$ (named agent $ij$ from now on) has a vNM utility function $U_{ij}(\ell_{ij})$, with $\ell_{ij}$ being her identity-specific outcome lottery. In each state of nature $s = 1, \cdots, S$, the utility level is given by $u_{ij}^s = u_{ij}(x_s)$ with probability $\pi_s$:

$$U_{ij}(\ell_{ij}) = \sum_{s=1}^{S} \pi^s u_{ij}^s. \tag{3.2}$$

Harsanyi's thought experiment is that each individual evaluates her future self's well-being behind the veil of ignorance, as an *impartial observer*, imagining taking the position of every other individual in society and adopting their preferences (acceptance principle). In this set-up, an individual $ij$, imagining being individual $kt$, will evaluate individual $kt$'s lottery $\ell_{kt}$ with the vNM utility function $U_{kt}(\ell_{kt})$.

[3] Eden (2020) considers only lotteries over money and assumes that preferences are continuous, consistent with the statewise dominance relation, and satisfy a property that is similar to decreasing relative risk aversion.

The set of outcome lotteries is denoted by $\Delta(\mathcal{X})$, and the set of identity lotteries by $\Delta(\mathcal{I})$, with $f_{ij}$ being the probability of becoming agent $ij$ in society, with characteristics $z_i$, and preferences for risk represented by $\theta_j$. Then, I state the definition of generalised weighted utilitarian impartial observer given by Grant et al. (2010) (notation changed to match mine):

**Definition 1** (Generalised weighted utilitarianism)**.** *The impartial observer is a generalised weighted utilitarian if her preferences can be represented by*

$$V = \sum_{i,j} f_{ij}\phi_j(U_{ij}(\ell_{ij})), \tag{3.3}$$

*where $\phi_j$ is a continuous, increasing function that maps the individual vNM utility function $U_{ij}(\ell_{ij})$ to the utility of the impartial observer.*

While the concavity of the function $\phi_j$ implies egalitarianism, the fact the $\phi_j$ is risk-aversion-specific is key to accommodate different risk preferences across individuals (Grant et al, 2010).

## 3.3 The Lottery of Life

The impartial observer faces an extended lottery that includes the set of identity lotteries $\Delta(\mathcal{I})$, and the set of outcome lotteries $\Delta(\mathcal{X})$. The social welfare function is derived by applying Bayesian rationality to this extended lottery.

Let us consider a multi-stage lottery, which is composed of an identity lottery taking place in several consecutive stages, and one outcome lottery, which is the last stage. The identity lottery is the lottery $\mu$ over risk preferences and other personal characteristics (lottery $\lambda$). The identity-specific outcome lottery $\ell_{ij}$ is related to the realisation of the state of nature and to individual choices, which ultimately depend on preferences and personal characteristics.

I make the following assumptions: (i) Individual well-being is represented by a vNM utility function such as (3.2); (ii) the extended lottery of life can be assessed as a multi-stage game in which each node represents a lottery over a specific preference parameter, personal characteristics or social outcome; (iii) the lottery over risk preferences is evaluated as if the impartial observer is risk-neutral.

Figure 3.1 gives the extensive form representation of a three-stage game,[4] which I will use to illustrate the derivation of the welfare criterion, starting from the last stage of the game and then proceeding backward. The game in figure 3.1 features an identity lottery over risk preferences (high or low), and income (high or low), and outcome lottery $\ell_{ij}$. In the last stage of the extended lottery, the individual possesses some preferences for risk and personal characteristics (in this case a certain level of income), and makes decisions under risk. Her expected pay-off from the outcome lottery $\ell_{ij}$ is represented by the vNM utility function (3.2).

[4] This is the simplest set-up that allows the construction of this social welfare function.

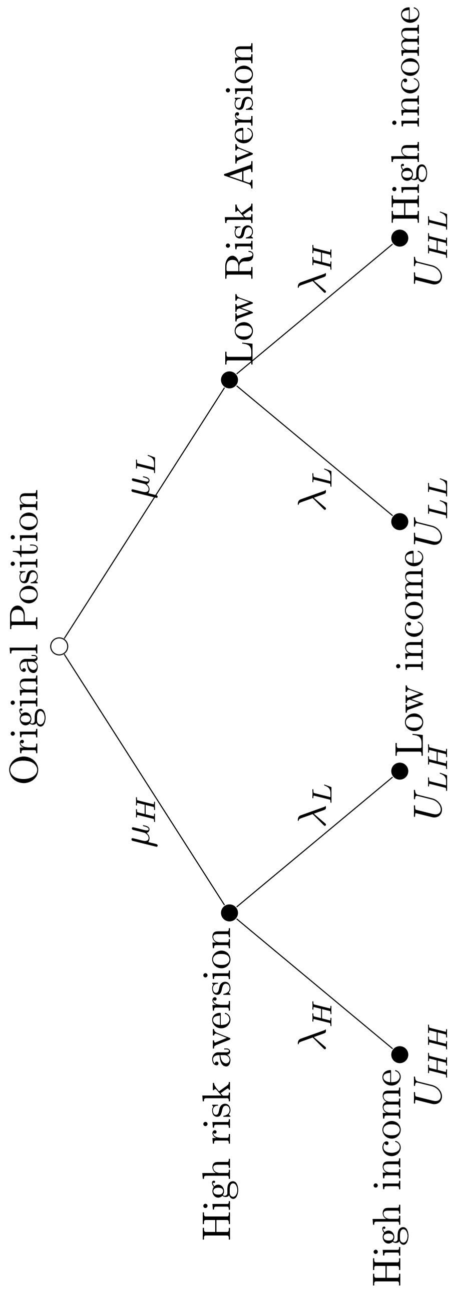


**Figure 3.1:** Extensive form representation of a three-stage lottery of life.

In the next-to-last stage of the extended lottery, the individual faces a lottery over a specific personal characteristic (income), anticipating what decisions she will make once that risk is resolved. At this stage, the expected pay-off $w_j$, which is evaluated with personal risk preferences, will read

$$w_j = \sum_i \lambda_i g_j \left(U_{ij}\right) \tag{3.4}$$

where $\lambda_i$ is the probability of acquiring income of type $i$, and $g_j$ is a concave transformation of $U_{ij}$, given risk preferences of type $j$.

Then, in the initial stage, the individual faces a lottery over risk preferences. Using assumption (iii), we can derive the expression of welfare for the impartial observer that is equal to the expected pay-off of the extended lottery in the original position:

$$W = \sum_j \mu_j w_j = \sum_j \mu_j \sum_i \lambda_i g_j \left(U_{ij}\right) \tag{3.5}$$

## 3.4 The Social Welfare Criterion

Let $f_{ij} = \mu_j \cdot \lambda_i$ be the probability of an agent acquiring income of type $i$ and risk preferences of type $j$. Then, it is possible to rewrite (3.5) as

$$W = \sum_{i,j} f_{ij} g_j \left(U_{ij}(\ell_{ij})\right) \tag{3.6}$$

If the function $g_j : \mathbb{R} \to \mathbb{R}$ is a continuous, increasing function of $U_{ij} : \ell_{ij} \to \mathbb{R}$, then expression (3.6) takes the form of the generalised weighted utilitarian criterion (3.3), which is axiomatised by Grant et al. (2010), with marginal social welfare weight $\eta_{ij}$ for agent-type $ij$ being

$$\eta_{ij} = \frac{\partial W}{\partial c_{ij}} = f_{ij} g_j' \left(U_{ij}(\ell_{ij})\right) U_{ij}'(\ell_{ij}),$$

which is the value of social welfare that criterion (6) puts on providing an additional unit of consumption to agent $ij$.

### 3.4.1 Constant Relative Risk Aversion (CRRA) Utility

In this section, I use the 3-stage extended lottery described in section 3.3 to derive the welfare criterion in the case in which the state-contingent $u_i^s$ takes the form of a CRRA utility function, i.e. $u_i^s = \frac{(\cdot)^{1-\theta_j}-1}{1-\theta_j}$, where $\theta_j$ is the coefficient of relative risk aversion. Then, I will show that the derived criterion satisfies the definition of generalised weighted utilitarianism (3.3) given by Grant et al. (2010).

In the last stage of extended lottery represented in figure 3.1, an individual with income level $i$ and risk preferences of type $j$ (agent $ij$) has expected payoff $U_{ij}$ from the outcome lottery $\ell_{ij}$:

$$U^j_{ij}(\ell_{ij}) = \sum_{s=1}^{S} \pi^s \frac{\left(x^s_{ij}\right)^{1-\theta_j} - 1}{1-\theta_j} \tag{3.7}$$

where $x^s_{ij}$ is agent $ij$'s pay-off in state $s$. In the intermediate stage of the extended lottery, the individual faces a lottery over the level of income. Given agent $ij$'s relative risk aversion parameter $\theta_j$, the expected pay-off $w_j$ from the income-lottery $\lambda$ will read

$$w_j = \sum_i \lambda_i \cdot g_j\left(U_{ij}(\ell_{ij})\right) = \sum_i \lambda_i \cdot \frac{\left(U^j_{ij}(\ell_{ij})\right)^{1-\theta_j} - 1}{1-\theta_j} \tag{3.8}$$

where $\lambda_i$ is the probability of acquiring income level $i$, and $g_j\left(U_{ij}\right) = \frac{(U_{ij})^{1-\theta_j}-1}{1-\theta_j}$ is a concave transformation of the vNM utility $U_{ij}$, given risk preferences of type $j$.

Working back to the first stage (original position), the expected pay-off of the extended lottery of life is given by (3.6), which can be rewritten as

$$W = \sum_j \mu_j \cdot \sum_i \lambda_i \cdot g_j\left(U_{ij}(\ell_{ij})\right) = \sum_{i,j} f_{ij} \frac{\left(U_{ij}(\ell_{ij})\right)^{1-\theta_j} - 1}{1-\theta_j} \tag{3.9}$$

where $f_{ij} = \mu_j \cdot \lambda_j$ is the probability of an agent acquiring risk preferences of type $j$ and income of type $i$, and $g_j\left(U_{ij}\right) = \frac{(U_{ij})^{1-\theta_j}-1}{1-\theta_j}$ is a transformation of the vNM utility $U_{ij}$ on the basis of agent $ij$'s risk preferences.

Since $g_j\left(U_{ij}\right)$ is a continuous, increasing transformation of the vNM utility $U_{ij}$, the welfare criterion (3.9), which is derived using Bayesian rationality, takes the form of a generalised weighted utilitarian criterion (Grant et al, 2010). Criterion (3.9) is egalitarian over expected utilities, i.e. ex-ante egalitarian,[5] because the function $g_j$ is a concave transformation of the vNM utility $(U_{ij})$. Moreover, as $g_j\left(U_{ij}\right)$ depends on agent $ij$'s relative risk aversion parameter $\theta_j$, each risk-aversion-type of individual will have a specific function $g_j(\cdot)$ translating her vNM utility into welfare $W$. Hence, criterion (3.9) also accommodates different risk preferences (Grant et al, 2010).

### 3.4.2 Properties of the Welfare Criterion

I perform some numerical simulations to explore the characteristics of criterion (3.9). The examples that are presented in this section shed light on the type of egalitarianism that this criterion entails, and its sensitivity to changes in the riskiness of identity and outcome. I show that this criterion has the following properties: (i) social welfare will diminish if we consider a mean-preserving spread of some given pay-offs across states of nature; (ii) the criterion is ex-ante egalitarian within specific risk-types of agent, i.e. redistribution between agents with equal preferences for risk, in favour of low income

[5]For the concept of ex-ante egalitarianism, see Broome (1984), Myerson (1981), Hammond (1981, 1982).

**Table 3.1:** Pay-offs of the outcome lottery (last stage of lottery of life) in consumption units.

(I) baseline scenario

| State of nature | A | B |
|---|---|---|
| Agent HH | 5 | 10 |
| Agent LH | 4 | 6 |
| Agent HL | 4 | 13 |
| Agent LL | 3 | 9 |

(II) risky policy

| State of nature | A | B |
|---|---|---|
| Agent HH | 4 | 11 |
| Agent LH | 3 | 7 |
| Agent HL | 3 | 14 |
| Agent LL | 2 | 10 |

(III) redistribution

| State of nature | A | B |
|---|---|---|
| Agent HH | 4.5 | 8 |
| Agent LH | 4.5 | 8 |
| Agent HL | 3.5 | 11 |
| Agent LL | 3.5 | 11 |

(IV) risky policy
+ redistribution

| State of nature | A | B |
|---|---|---|
| Agent HH | 3.5 | 9 |
| Agent LH | 3.5 | 9 |
| Agent HL | 2.5 | 12 |
| Agent LL | 2.5 | 12 |

Note: Social welfare in baseline scenario $I$ is $W_{(I)} = -4.55$. Social welfare in the risky policy scenario $II$ is $W_{(II)} = -4.70$. Social welfare in scenario $III$ is $W_{(III)} = -4.538$. Social welfare in scenario $IV$ is $W_{(IV)} = -4.664$.

individuals, always increases welfare; (iii) redistribution of resources can partially offset welfare losses from higher risk across states of nature, i.e. the negative effect of increasing risk across states of nature can be partly offset by lower risk in the lottery of types, conditional on the state of nature.

Consider the lottery of figure 3.1. There are 4 possible types of individual: 1) high income - high risk aversion (HH); 2) high income - low risk aversion (HL); 3) low income - high risk aversion (LH); 4) low income - low risk aversion (LL). In our examples, the agents with high risk aversion have a coefficient of relative risk aversion $\theta_H = 4$, while the low risk aversion agents have $\theta_L = 2$. In the last stage of the lottery of life, these individuals face two equally-probable states of nature that are denoted by $A$ and $B$, i.e. $\lambda_H = \lambda_L = 0.5$. The probability of acquiring a certain attitude to risk, given by $\theta_H$ or $\theta_L$, is also $1/2$, i.e. $\mu_H = \mu_L = 0.5$.

Table 3.1 reports the agents' pay-offs in four different scenarios. Each agent $ij$ is identified by characteristic (income) of type $i$ and risk aversion parameter of type $j$ ($\theta_j$). For each scenario, I compute the expected utility from the outcome lottery for each agent-type using (3.7). Then, I use (3.8) to compute the expected pay-off from the income lottery, with an equal probability of acquiring high/low income. Finally, (3.9) gives the value of the social welfare index. In the baseline scenario (I), each agent faces a personalised lottery over her consumption level, with the less risk averse agents facing lotteries which are riskier, but with higher expected payoffs. Then, table 3.1 - panel (II) presents an alternative scenario in which I consider mean-preserving-spreads of the pay-offs from the baseline across the states of nature A and B. It can be shown that $W_{(\mathrm{I})} > W_{(\mathrm{II})}$, which unveils property (i): criterion (3.9) attaches a lower value of social welfare to riskier scenarios, when expected pay-offs and other parameters remain equal.

The second property I investigate concerns the type of egalitarianism that criterion (3.9) entails. This is shown by comparing panel (III) of table 1 with the baseline scenario in panel (I). Panel (III) presents the case in which there is equal redistribution of resources across types of agent with equal risk preferences. Similarly to the previous case, I compute the expected pay-offs (3.7) from the outcome lottery $\mu$ of the different agents and those from the income-lottery $\lambda$ using (3.8). I find that $W_{(\mathrm{I})} < W_{(\mathrm{III})}$. This is property (ii): redistribution between agents with equal preferences for risk always increases welfare. On the contrary, perfect redistribution across all types is not necessarily welfare-improving, as the criterion gives larger welfare weights to more risk averse agents relative to low risk averse ones – see example in the appendix.

Finally, the third property regards the relationship between risk in the outcome lottery $\ell$ and risk in the income lottery $\lambda$, and directly stems from the first two properties. While the first example shows that riskier outcome lotteries (panel II) decreases social welfare (3.8) compared to the baseline scenario (I), all else being equal, the second example showed that redistribution across some agent-types can be welfare improving. Hence, the last example (panel IV) shows that perfect redistribution across types with equal risk preferences can improve on the risky scenario (ii), meaning that redistribution of resources is an imperfect substitute of safer outcome lotteries: $W_{(\mathrm{II})} < W_{(\mathrm{IV})} < W_{(\mathrm{I})}$. Redistribution is welfare-enhancing because it reduces the risk attached to the income lottery $\lambda$, and therefore partially offsets the welfare loss due to the higher risk attached to the outcome lottery.

## 3.5 Discussion

The main result of this paper is deriving a new social welfare criterion (3.9) that can be used to evaluate social choices under risk when individuals possess heterogeneous attitudes to risk. The criterion is obtained in the specific case of constant relative risk aversion utility, and exploits a modified version of Harsanyi's impartial observer framework. I construct a multi-stage lottery of life in which individuals face a lottery over risk preferences (first stage), personal characteristics (intermediate stage) and social outcomes (final stage). Then, the welfare criterion is derived by exploiting Bayesian rationality, which requires individual preferences to satisfy the expected utility axioms.

The criterion is shown to satisfy the definition of generalised weighted utilitarianism given by Grant et al. (2010). In particular, the vNM utility functions, representing individuals' well-being, are mapped to the social welfare index (3.9) via continuous, concave transformations $g_j(\cdot)$ which are specific to each risk-aversion-type of individual. The social marginal welfare weight $\eta_{ij}$ for an agent $ij$ can be derived from (3.9) as follows:

$$\eta_{ij} = \frac{\partial W}{\partial c_{ij}} = f_{ij} U_{ij}^{-\theta_j} U'_{ij}.$$

The criterion is sensitive to changes in the risk of identity and outcome lotteries faced by the impartial observer (first property). A lower social welfare index is attached to scenarios with higher overall risk, or when riskier social outcomes are assigned to more risk averse agents in society. Hence, in relation to the critique by Pattanaik (1968), it is not indifferent for this criterion which types of agent face a specific risky lottery. This is obtained by modelling the lottery of life as a multi-stage lottery (assumption (ii)), rather than a one-stage lottery as in Harsanyi (1953, 1955).

The proposed criterion is also able to accommodate a restricted version of egalitarianism, namely egalitarianism between agents with equal preferences for risk (second property). Conditional on the preference for risk, the marginal social welfare weights will be larger for lower-income individuals, so that redistribution towards low-income will raise welfare. However, redistribution across all agents, independently from their risk preferences may not be welfare-enhancing. This is because the marginal social welfare weight $\eta_{ij}$ does not only depend on income, but also on the value of the relative risk aversion parameter $\theta_j$ and on the population weight of a particular type of agent in society $f_{ij}$. Hence, redistributing resources from high-income people with high risk aversion to low-income people with low risk aversion can decrease welfare, as criterion (3.9) gives more weight to more risk averse individuals. Redistribution in favour of low-income people, independently from their risk aversion, is more likely to be welfare-enhancing when risk aversion is negatively correlated with income in the population.

## 3.6 Conclusion

This paper proposes a new social welfare criterion (3.9) to make social choices when individuals face risk and possess heterogeneous attitudes to it. The criterion is obtained in the specific case of constant relative risk aversion utility, and exploits Bayesian rationality within Harsanyi's framework of the impartial observer. The criterion is shown to satisfy the definition of generalised weighted utilitarianism given by Grant et al. (2010) and is therefore able to accommodate heterogeneous risk preferences. The proposed criterion also allows for a restricted version of egalitarianism, namely egalitarianism between agents with equal preferences for risk.

The proposed social welfare criterion can be used in any setting with heterogeneous agents that can be framed as a multi-stage lottery over individual (risk) preferences, characteristics and states of nature. The problem of the optimal public good provision in the presence of heterogeneous risk preferences, which was analysed in the second chapter of this thesis, is an example. Compared to the standard utilitarianism, this criterion gives more weight to the more risk averse agents. This implies that social choices on the basis of the proposed criterion do not simply converge to the options preferred by the least risk averse individuals, who would have the highest welfare weights – (expected) marginal utility from consumption – if the utilitarian approach was used.

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# Appendix C

## Example: Perfect Redistribution

This section shows how perfect redistribution of resources from richer individuals to poorer ones may not be welfare-enhancing on the basis of the proposed criterion. Table C1 reports the agents' pay-offs in two alternative scenarios: 1) the baseline scenario; 2) the perfect redistribution case in which all agents have the same amount of resources. Each agent $ij$ is identified by characteristic (income) of type $i$ and risk aversion parameter of type $j$ ($\theta_j$). For each scenario, I compute the expected utility from the outcome lottery for each agent-type using (3.7). Then, I use (3.8) to compute the expected pay-off from the income lottery, with an equal probability of acquiring high/low income. Finally, (3.9) gives the value of the social welfare index.

The social welfare index in scenario (V), $W_{(V)}$, is slightly lower than the value at the baseline $W_{(I)}$, meaning that perfect redistribution of resources decreased social welfare on the basis of the proposed criterion (3.9). In state of nature $A$, scenario (V) takes one consumption unit from agent HH (high income and high risk aversion) and assigns it to agent LL (low income and low risk aversion). While agent LL has low income, the proposed criterion gives more weight to high risk aversion agents than low risk aversion ones, such that this transfer is actually welfare-decreasing. This negative effect on social welfare is partly balanced by the transfers in state of nature $B$. Agents LH and LL receive transfers from agents HH and HL respectively, which is welfare-enhancing by property (ii) of the welfare criterion. Then, agent LH also gains from agent HL, which is also welfare enhancing as agent HL's welfare weight is relatively higher.

**Table C1:** Example - perfect redistribution: pay-offs of the outcome lottery in consumption units.

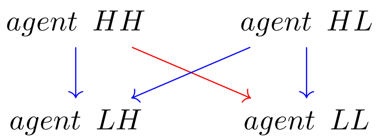
(I) baseline scenario

| State of nature | A | B |
|---|---|---|
| Agent HH | 5 | 10 |
| Agent LH | 4 | 6 |
| Agent HL | 4 | 13 |
| Agent LL | 3 | 9 |

(V) perfect redistribution

| State of nature | A | B |
|---|---|---|
| Agent HH | 4 | 9.5 |
| Agent LH | 4 | 9.5 |
| Agent HL | 4 | 9.5 |
| Agent LL | 4 | 9.5 |

Note: Social welfare in baseline scenario $I$ is $W_{(I)} = -4.554$. Social welfare in the risky policy scenario $V$ is $W_{(V)} = -4.556$.




Note: Diagram reporting the transfers in scenario (V), relative to the baseline (I) of table C1. The red arrow depicts the welfare-reducing transfers. The blue arrows depict the welfare-increasing transfers.