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ESSAYS IN NONLINEAR PRICING AND RENT SEEKING

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Esame finale anno 2023

To my family ...

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Abstract

This thesis consists of three self-contained essays on nonlinear pricing and rent-seeking. In the first essay, I study behavioral nonlinear pricing in both monopoly and common agency by introducing the Fehr and Schmidt (1999) type of other-regarding preferences. In the second one, I focus on sabotage in rent-seeking contests in which the rent is endogenously determined. In the last one, I study information disclosure with Bayesian persuasion in endogenous rent-seeking contests.[1]

In the first chapter of the thesis, I provide new theoretical insights about non-linear pricing in monopoly and common agency by combining the classical principal-agent framework with other-regarding preferences. In the monopoly part, I introduce a new theoretical model that separately characterizes status-seeker and inequity-averse buyers. I show how the buyer's optimal choice of quality and market inefficiency change when the buyer has other-regarding preferences. In the common agency part, I show and find the optimal quality and non-linear pricing schedule when the buyer has other-regarding preferences. I also conclude that regardless of whether goods are complements or substitutes, if the buyer is a status-seeker (inequity-averse), the equilibrium quality distorts downward more (less) compared to the classical common agency literature

In the second chapter, I find the optimal productive rent-seeking and sabotaging efforts when the prize is endogenously determined. In this study, productive rent-seeking efforts do not only affect the winning probability but also generate additional

[1] In the second and third chapters of the thesis, I mainly introduce the endogenous rent-seeking contest environment. In my previous studies, I focused on legislative bargaining, and as in Genç and Küçükşenel (2019), we introduced an endogenous recognition process to the conventional model of legislative bargaining over private and public goods. What we had in the model was that even though legislators exert effort to become a proposer, legislators' efforts were unproductive. However, we know from the literature that there are real-life examples where efforts can be productive and can increase the size of the rent. In this sense, I find it interesting to study endogenous rent-seeking models in both sabotage and information disclosure separately.

surplus over the fixed prize. Contrary to the previous literature, I show that due to the existence of endogeneity, sabotaging the productive rent-seeking efforts causes sabotaging the endogenous part of the prize, which can affect the rent-seeking efforts. Hence, if monitoring is feasible, the contest organizer may want to monitor the sabotaging activities. Moreover, I introduce social preferences into my model and characterize symmetric productive rent-seeking and sabotaging efforts. In the end, I compare the productive rent-seeking and sabotage efforts of other-regarding and self-regarding contestants.

In the last chapter, I propose a new theoretical model regarding information disclosure with Bayesian persuasion in rent-seeking contests when the efforts are productive. I show that under one-sided incomplete information, information disclosure decision depends on both the marginal costs of exerting efforts and the marginal benefit of aggregate exerted effort. I find that since the efforts are productive and add a positive surplus on the fixed rent, my model narrows down the conditions for the information disclosure compared to the exogenous model. Under the two-sided incomplete information case, I observe that there is a non-monotone relationship between optimal effort and posterior beliefs. Thus, it might be difficult to conclude whether a contest organizer should disclose any information to contestants.

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1 Other-Regarding Preferences in Adverse Selection

Abstract

This study aims to provide new theoretical insights about non-linear pricing in both monopoly and common agency by combining the classical principal-agent framework with the Fehr and Schmidt (1999) type of other-regarding preferences. In particular, we offer a new theoretical model that characterizes both status-seeker and inequity-averse buyers in monopoly and common agency. In the monopoly part, we show that if the buyer is a status-seeker (inequity-averse), and if the type set is sufficiently homogeneous and/or the marginal cost of the production is sufficiently high, then the optimal quality and market inefficiency become higher (lower) compared to the self-interested buyer in the equilibrium. Similarly, under the same conditions, as the buyer becomes more status-seeker (inequity-averse), then the optimal quality and market inefficiency increase (decrease) more. In the second part, we focus on the common agency framework and introduce the Fehr and Schmidt (1999) type of other-regarding preferences into the model. We show and find the optimal symmetric quality and non-linear pricing schedule when the buyer has other-regarding preferences. We mainly find that regardless of whether goods are complements or substitutes, if the buyer is a status-seeker (inequity-averse), inefficiency increases (decreases) more compared to the conventional common agency model. That is to say, if the buyer is a status-seeker (inequity-averse), the equilibrium quality distorts downward more (less) compared to the classical common agency literature. Moreover, we show that if the buyer's type is sufficiently high, as the degree of being a status-seeker (inequity-averse) increases, the optimal quality increases (decreases) regardless of whether the goods are complements or substitutes.

1.1 Introduction

> *"Apart from economic payoffs, social status (social rank) seems to be the most important incentive and motivating force of social behavior"*
>
> *Harsanyi (1980)*

In the mainstream economics literature, it is a convention that all parties are assumed to be "self-interested" that is, all agents care only about their own materials, payoffs, or well-being. However, recent developments in behavioral and experimental economics have led us to the fact that it may not be true in most real-world problems and that people tend to deviate from the self-interest approach[1]. Many lab experiments and studies reveal that people care not only about their well-being but also about the well-being of others for many reasons. Thus, we observe that people's decision-making processes are strongly influenced by other-regarding concerns in real life.

Social sciences reveal that compared to the self-interested models, both status-seeking activities or expectations and inequity-aversion and fairness concerns are significant in the decision-making processes. For example, people tend to make "conspicuous consumption"[2] to gain status or prestige in societies. In this context, Nelissen and Meijers (2011) study the costly signaling of wealth and status. They find that luxury consumption increases status, resulting in social interaction benefits. In addition, Yoon and Seok (1996) examine the reciprocal relationship between conspicuous consumption and social status by analyzing data from a sample of 531 Korean urban households. They show that both conspicuous consumption and social status reinforce each other. On the other hand, unlike status-seeking activities, fairness, and inequity aversion also play a key role in why people care about others' situations.

[1] See Rabin (2002) and Camerer (2003).

[2] As far as we know, this definition was first defined by Veblen (1899).

For example, Fehr and Schmidt (1999) indicate that due to psychological evidence on social comparison and loss aversion, people do care about the situation of others in terms of fairness and inequity-aversion.

As a part of mainstream literature, conventional contract theory also uses the "self-interest" approach. However, because of the two main reasons explained above, this approach should be relaxed and social preferences should also be taken into consideration. There are studies in the literature that combine social preferences with moral hazard. Itoh (2004) introduces other-regarding preferences into the moral hazard and studies the classical principal-agent framework with the assumption that agents are inequity averse. In this work, Itoh (2004) shows that the principal can exploit the agents due to the being inequity averse of agents. Saygılı and Küçükşenel (2019) introduce social preferences into organizational hierarchies. They scrutinize that other-regarding preferences do change the collusive behavior among all agents.

As far as we know, in the adverse-selection literature, few studies also consider social preferences. Destan and Yılmaz (2020) is the closest paper to ours in terms of context. They study non-linear pricing in a monopoly in which buyers may have inequity-averse preferences. They characterize optimal non-linear pricing and show that as the degree of inequity aversion increases, the profit level of the monopoly decreases. In their work, inequity aversion is introduced by considering net utility differences between fair types and neutral types. At this point, however, it would be a good idea to consider the following points as well. First, we believe that the comparison of the net utility differences may not fit well in the concept of other-regarding preferences because even though the comparison of the net utility differences between fair and neutral types is theoretically feasible, we know and observe from the real-life examples that people do compare goods and services in real-life situations such as quality (or quantities) of goods, wages, etc. That is why, we believe that instead of using net utility differences, basically using quality (or quantity) differences can make the problem more sensible. However, this approach naturally brings new challenges to the characterization of the solution, as we show in our methodology.

Second, unlike inequity-averse buyers, as we explain above, it is also crucial to analyze the outcomes of status-seeking activities. Third, it would be also important to see how the outcomes change when the competition increases.

In this work, we mainly offer a new theoretical framework that characterizes both status-seeker and inequity-averse buyers in both monopoly and common agency. In the monopoly part, we analyze and characterize the optimal quality and non-linear pricing schedule for status-seeker and inequity-averse consumers separately. We show that if the buyer is a status-seeker, and if the type set is homogeneous enough and/or the marginal cost of the production is high enough, then the optimal quality increases compared to the self-interested buyer in the equilibrium. This occurs because as the type set is homogeneous enough, then the status-seeker buyer considers other types as competitive rivals. On the other hand, even if the type set is not homogeneous, and if the marginal cost of production is high enough, then the status-seeker buyer wants to have a higher qualified or more expensive product with the hope of gaining status in society. Moreover, different from classical monopolistic screening literature, due to the existence of the status-seeker parameter, even the highest type gets the inefficient allocation in the equilibrium. Thus, the inefficiency increases compared to the self-interested buyer. On the other hand, when the buyer is inequity-averse, inefficiency decreases compared to the model in which the buyer is self-interested. Furthermore, if the buyer is inequity-averse, and if the type set is homogeneous enough and/or the marginal cost of production is high enough, then the optimal quality decreases compared to the self-interested buyer case in the equilibrium. Because if the type set is not that heterogeneous, then each buyer's preferences become similar to each other which makes it easier to sustain a fair equilibrium compared to equilibrium with selfish buyers. Also, if the marginal cost of the product is high enough, then the optimal quality decreases in the equilibrium because the higher marginal cost causes higher expenses and higher expenses generate higher inequity among the buyers. Thus, if the buyers are inequity averse and the marginal cost of the product is high enough, then in the equilibrium, the optimal quality decreases. Apart from the above results in the first part, we also show that if the buyer is a status-seeker (inequity-averse) and if the type set is homogeneous enough and/or the cost of marginal product is high enough, as the buyer becomes more status-seeker (inequity-averse), then the optimal quality increases (decreases) more.

In the second part, we focus on the common agency framework similar to Martimort and Stole (2009)'s model and we introduce other-regarding preferences into the common agency framework. In particular, we show and find the optimal symmetric quality and nonlinear pricing schedule when the buyer has social preferences. We mainly find that regardless of whether goods are complements or substitutes if the buyer is a status-seeker (inequity-averse), inefficiency increases (decreases) more compared to the conventional common agency model. That is to say, if the buyer is a status-seeker (inequity-averse), the equilibrium quality distorts downward more (less) compared to the classical common agency literature. Lastly, We also show if the buyer's type is sufficiently high, as the degree of being a status-seeker (inequity-averse) increases, the optimal quality increases (decreases) regardless of whether the goods are demand complements or substitutes.

The organization of the paper is as follows. The next section briefly explains related literature. Section 1.3 introduces the model and considers monopoly under other-regarding preferences. Section 1.4 extends the monopoly case and considers common agency under other-regarding preferences. Section 1.5 concludes.

1.2 Literature Review

In this part, we briefly introduce the literature review on monopolistic screening, common agency, and other-regarding preferences.

Mussa and Rosen (1978) study the monopoly pricing scheme of a monopolistic firm in which it produces goods in a number of differentiated varieties with respect to its quality level. In their seminal work, they find that only the highest type gets the first-best (efficient) allocation. That is to say, no distortion exists at the top. Similarly, they show that except for the highest type, all the other types get an inefficient positive surplus. Indeed, the lowest type gets zero surplus. Different from Mussa and Rosen (1978), Goldman et al. (1984) study nonlinear pricing in monopoly via quantity discounting. Then, Maskin and Riley (1984) construct a new theoretical model which studies price discrimination in monopoly via quantity discounting by using the principal-agent framework. Maskin and Riley (1984) show that non-linear pricing problems in the monopoly can be modeled and solved by using the principal-agent framework.

Apart from the monopolistic screening, as far as we know, Bernheim and Whinston (1985) and Bernheim and Whinston (1986) introduce the concept of common agency in the literature first. According to Bernheim and Whinston (1986), different from the classical principal-agent framework, they introduce more than one risk-natural firm (principals) in which firms independently and simultaneously try to affect the decision of the common buyer (agent). They show that aggregate equilibrium incentive schemes are efficient and the equilibrium action is implemented at minimum cost. Moreover, Bernheim and Whinston (1986) show that when the principals collude, they can induce first-best (efficient) allocation. Gal-Or (1991) shows that if agents have private information regarding their costs, then forming a common agency in an oligopoly can be disadvantageous for the firms. Different from previous literature, Biglaiser and Mezzetti (1993) offers a model in which non-identical principals compete for an agent. According to Biglaiser and Mezzetti (1993) principals are different in terms of their marginal and average valuations and compete by offering menus of incentive contracts. Besides the common agency, Martimort (1996) studies exclusive dealing. Martimort (1996) shows that manufacturers can benefit from an exclusive retailer rather than a common retailer depending on the adverse selection and complementarity or substitutability of the brands. Martimort and Stole (2009) study how intrinsic and delegated common agency affect market participation. They show that if contracting variables are substitutes, then intrinsic common agency equilibrium outcome induces higher participation distortion than monopoly outcome, and delegated common agency equilibrium outcome induces lower participation distortion relative to monopoly. On the other hand, Martimort and Stole (2009) show that if the contracting variables are complements, then both intrinsic and delegated common agency equilibrium outcome induce higher participation than monopoly.

Up to now, we focus on the main prominent papers related to non-linear pricing in monopoly, and common agency in general. Since our paper combines the screening framework with the other-regarding preferences, it is also important to mention the papers which are related to theories of fairness and reciprocity. According to Fehr and Schmidt (2003), we know that there are mainly two approaches to fairness and reciprocity. The first one is the social preference approach i.e., agents or players do not only consider their own well-being, payoffs (monetary or non-monetary), etc. but also others'. The second approach is intention-based. First, we start with the intention-based one. Rabin (1993) in his seminal paper he first starts with the observation that people's behavior is generally shaped by the intentions of other people. Rabin (1993) defines the fairness equilibrium and according to that if the payoffs are small, fairness equilibrium is more or less the set of mutual-max and mutual-min outcomes. On the other hand, if the payoffs are large, then the fairness equilibrium becomes roughly the set of Nash equilibria. Different from Rabin (1993), Fehr and Schmidt (1999)'s model is distributional. In their work, they construct a simple linear model to define inequity-aversion[3]. That is to say, Fehr and Schmidt (1999) basically

[3] Theoretical details can be found in the next chapter.

define the concept of fairness as self-centered inequity aversion. They find that prevailing behavior in the equilibrium depends on the economic environment. Note that according to Fehr and Schmidt (1999), being heterogeneous individuals is one of the important assumptions that can play an important role. They point out that under some conditions, for example, it might be possible that a purely selfish agent can induce a fully selfish manner from inequity-averse people, or in public good games, a self-interested agent can induce zero contribution from the inequity-averse agents. On the other hand for example, under some conditions, even though there are few inequity-averse agents in the environment, they can create some incentives for the self-interested agents to contribute to the public good game. Apart from Fehr and Schmidt (1999), Bolton and Ockenfels (2000) also study the inequity-aversion model in their work. However, different from Fehr and Schmidt (1999), they use a closed functional form in which the agent not only considers his own payoff but also others' average payoff. That is why as Fehr and Schmidt (2003) also explain, even though agents get the same qualitative results when there are two agents, the results might give some interesting different outcomes when the total number of agents exceeds two.

1.3 Other-Regarding Preferences in Monopolistic Screening

In this section, we consider a market in which a good is produced in a number of differentiated varieties by a firm according to its quality level q as in Mussa and Rosen (1978). The quality q is one-dimensional and depends on the consumer's type θ. Type θ can be seen as the desire or taste parameter for the product's quality. Thus, as the type increases, consumers' demand for the quality increases.

The number of consumers is indefinite and let $\Theta = [\underline{\theta}, \bar{\theta}]$ denote the consumers' taste type set with a cumulative distribution function $F(\theta)$ and density function $f(\theta) > 0$ on $[\underline{\theta}, \bar{\theta}]$.

The seller (firm) offers a set of prices for each quality level and all customers have the same price-quality scheme in the market. We aim to find the optimal selling strategy for the seller (firm) under the existence of other-regarding preferences of the buyer.

Before defining the payoff functions of both consumers and firm, it is better to recall Fehr and Schmidt (1999) type of other-regarding or social preferences. Fehr and Schmidt (1999) define other-regarding preferences as:

$$U_i(x_i, x_j) = x_i - \lambda \max\{x_j - x_i, 0\} - \mu \max\{x_i - x_j, 0\}$$

According to the model, agent i's utility depends on not only his own payoff x_i but also his opponent's payoff x_j. Agent i has mainly two possible sources that can create a disutility due to the inequity-aversion. The first part can be induced from the being behind of agent j i.e., $\lambda > 0$, and the second source can be induced from the being fair-minded, i.e., $\mu > 0$ and $\mu \in (0, 1)$. Parameters λ and μ are social preference parameters. In this model, λ and μ are called inequity-aversion parameters and it is assumed that $\lambda > 0$, $\mu > 0$ with $\mu \leq \lambda$ and $\mu \in (0, 1)$. Note that $\lambda \geq \mu$ states that agent i suffers much from being behind than being ahead of agent j. Moreover, μ is assumed to be in $\in (0, 1)$, otherwise, if μ were greater than 1, he would have a negative payoff in the increase of x_i which is not rational.

In the model, there is also room for a negative parameter $\mu < 0$ as Itoh (2004) mentions. If μ is negative then it is called agent i status-seeker (competitive). It means that agent i enjoys being ahead of agent j. Lastly, note that if agents are self-interested i.e if they only do care about their payoff, then basically in the model both parameters become 0, that is to say, $\lambda = \mu = 0$.

Note that Fehr and Schmidt (1999) offer a simple analytical functional form to introduce social preferences. Thus, we impose this functional form into our model and try to solve the monopolistic screening problem. As we show below, due to the introduction of the other-regarding preferences, it becomes challenging to characterize the incentive compatibility constraints. In this regard, we develop a solution strategy for this problem.

Payoff Functions:

By using Fehr and Schmidt (1999)'s functional form, we define the payoff function for the buyer or customer of type θ as :

$$\text{U(q,}\theta) = v\big(q(\theta) - \lambda \max\{q^{avg}(\theta) - q(\theta), 0\} - \mu \max\{q(\theta) - q^{avg}(\theta), 0\}, \theta\big) - P(q(\theta))$$

We can restate the above payoff function as:

$$U(q,\theta) = \begin{cases} v(q - \lambda(q^{avg} - q), \theta) - P(q(\theta)) & \text{if } q^{avg} \geq q \\ v(q - \mu_B(q - q^{avg}), \theta) - P(q(\theta)) & \text{if } q^{avg} \leq q \\ v(q + \mu_s(q - q^{avg}), \theta) - P(q(\theta)) & \text{if } q^{avg} \leq q \end{cases}$$

where λ and μ_B represent inequity-aversion parameters and μ_s represents status-seeker parameter respectively. To make the distinctions on parameters clearly, λ denotes the inequity aversion parameter. According to the first line of the given above piece-wise function, the buyer dislikes being behind the average quality level. Similarly, in the second line, μ_B denotes the inequity-aversion parameter but in the second line, this inequity aversion occurs from the fact that the buyer is fair-minded. In the last part, μ_s shows the status-seeker parameter in which the buyer enjoys being ahead of the others. For him, consuming higher quality than average quality level is more important.

Note that the other-regarding buyer considers not only his quality (or quantity) level but also the difference between his quality and average quality[4]. Also, function v is basically the utility function of a consumer of type θ and it depends on both buyer's level of quality and the quality difference between his and average quality. We give more specifications of v below.

In our model, q^{avg} denotes the average quality level that the buyer focuses on as

[4] One may also assign another reference value of quality for making the comparison.

a reference point for himself to make a comparison between his own quality level and the others'. This reference or cut-off value of quality affects the customer's choice of quality decision and thus his utility level. That is why defining the reference point is significant.

q^{avg} can be defined on:

$$q^{avg} : A \subseteq [\underline{\theta}, \bar{\theta}] \longrightarrow \mathbf{R}_+ \cup \{0\}$$

Set A can be any subset of $[\underline{\theta}, \bar{\theta}]$ because any buyer can take any subset of the generic type set to make a comparison with his own type. Since there is an indefinite number of different types of buyers, that reference point can change according to the type of buyer. In that sense, we should impose an assumption or a restriction on q^{avg} to make the problem sensible and solvable for the seller.

Seller's payoff function:

$$\Pi(q, \theta) = P(q(\theta)) - cq(\theta)$$

Note that the seller does not know the exact type of θ but he only observes the distribution of types. It is obvious that as the seller does not observe type θ, it becomes impossible for the seller to know which subset A is considered by the buyer.

As there are infinitely many possible different subsets of $[\underline{\theta}, \bar{\theta}]$, we make an assumption in which the buyer only considers and compares his type with the lower ones. That is to say, we assume that the average quality q^{avg} is taken over the types that are lower than the buyer's own type. Note that this assumption allows us to analyze the effects of both status-seeker and inequity-averse buyers on the monopolistic screening model. In this sense, the payoff function of the buyer under our assumption will be as follows:

$$U(q,\theta) = \begin{cases} v(q - \mu_B(q - q^{avg}), \theta) - P(q(\theta)) & \text{if the buyer is inequity-averse} \\ v(q + \mu_s(q - q^{avg}), \theta) - P(q(\theta)) & \text{if the buyer is status-seeker} \end{cases}$$

Now, we make some extra assumptions on the utility function v of the buyer.

Assumptions: For the regularity, we make the following assumptions on the utility function v:

1. $v(q, \theta)$ is strictly quasi-concave in q
2. $v(q, \theta)$ is continuous in θ
3. $v(q, \theta)$ is increasing in θ i.e, $v_\theta(q, \theta) > 0$
4. $v_{q\theta}(q, \theta) > 0$

By considering above assumptions, as in Martimort and Stole (2020) we take the utility function as:

$$v(q, \theta) = \theta q - \frac{\gamma}{2} q^2$$

where $\gamma > 0$ and it is the slope of the utility function. Note that function v can be generalized to a closed functional form satisfying the above assumptions. Also from now on, we assume that types are distributed uniformly over $[\underline{\theta}, \bar{\theta}]$, costs are linear in q that is to say $C(q) = cq$. Also, similar to Martimort and Stole (2020), we assume that it is inefficient to serve all types i.e. $\underline{\theta} < c < \bar{\theta}$.

Note that assumption 3 means that higher types get higher utility. Assumption 4 mainly states the Spence-Mirrless property or single-crossing condition[5]. This assumption is crucial as it offers a simpler and more tractable environment. Because with this property, it becomes sufficient to consider local incentive compatibility constraints but not global ones.

[5] See Laffont and Martimort (2009) for more on this.

Now, we begin to analyze by considering the firm's program. The firm's problem is to choose the price schedule $P(q(\theta))$ that maximizes the expected profit. The firm's problem can be stated as follows:

$$\max_{p} \int_{\underline{\theta}}^{\bar{\theta}} \left[P(q(\theta)) - C(q(\theta))\right] dF(\theta) \tag{1.1}$$

subject to

$$\text{q}(\theta) \in \arg\max_{q} v\big(q(\theta) - \lambda \max\{q^{avg}(\theta) - q(\theta), 0\} - \mu \max\{q(\theta) - q^{avg}(\theta), 0\}, \theta\big) - P(q(\theta)) \tag{1.2}$$

and

$$v\big(q(\theta) - \lambda \max\{q^{avg}(\theta) - q(\theta), 0\} - \mu \max\{q(\theta) - q^{avg}(\theta), 0\}, \theta\big) - P(q(\theta)) \geq 0 \tag{1.3}$$

Equations (1.2) and (1.3) denote incentive compatibility and individual rationality (or participation) constraints, respectively.

Benchmark: Self-Interested Buyer

Note that if buyers were perfectly self-interested, in other words, if $\lambda = \mu_s = \mu_B = 0$, we would have the classical Mussa and Rosen (1978) model with quadratic-preferences with uniformly distributed types.

Then the buyer's utility function simply becomes $v(q(\theta), \theta)$. The firm's problem remains the same as in (1.1), but the incentive compatibility and participation constraints would become:

$$q(\theta) \in \arg\max_{q} v\big(q(\theta), \theta\big) - P(q(\theta)) \tag{1.4}$$

and

$$v\big(q(\theta),\theta\big) - P(q(\theta)) \geq 0 \tag{1.5}$$

Then, it is easy to show that the virtual surplus function becomes[6]:

$$\Gamma(q,\theta) = v(q(\theta),\theta) - C(q(\theta)) - \frac{1-F(\theta)}{f(\theta)} v_\theta(q,\theta) \tag{1.6}$$

Assumption: Virtual surplus function is strictly quasi-concave in q and supermodular in (q,θ)[7]. Under this assumption, point-wise maximization gives us the following:

$$\boxed{v_q(q,\theta) - \frac{1-F(\theta)}{f(\theta)} v_{\theta q}(q,\theta) = c} \tag{1.7}$$

Note that equation (1.7) yields very well-known results of Mussa and Rosen (1978). If buyers are fully self-interested, and if it is efficient to serve all types, then it is easy to see that only the highest type gets the first-best (efficient) allocation i.e., no distortion at the top. Moreover, all the types except for the highest type one receive inefficient allocation i.e., downward distortion appears. Furthermore, the firm captures the whole surplus for the lowest type and except for the lowest type gets the inefficient positive surplus due to informational rent which increases with lower types.

In our specification, equation (1.7) implies:

$$\boxed{q(\theta) = \max\{0, \frac{2\theta - \bar{\theta} - c}{\gamma}\} = q_0^{fb}(\theta) - \frac{(\bar{\theta} - \theta)}{c}} \tag{1.8}$$

where $q_0^{fb}(\theta)$ is the first-best optimal quality level when the buyer is self-interested. Also, note that it is easy to verify: $q_0^{fb}(\theta) = \max\{0, \dfrac{\theta - c}{\gamma}\}$[8].

We can also find the marginal consumer's type by solving $q^*(\theta_m) = 0$. Here, it is

[6] See Appendix for the derivation and details.

[7] This assumption holds under quadratic preferences with uniformly distributed types.

[8] Derivation of the first-best quality level can be found in the appendix.

easy to see that $\theta_m = \max\{\underline{\theta}, \frac{\bar{\theta}+c}{2}\}$. For the optimal non-linear pricing schedule:

$$P^*(q) = \int_0^q v_t(t,\theta)dt = \int_0^q (\theta - \gamma t)dt = \int_0^q (\frac{\gamma t + \bar{\theta} + c}{2} - \gamma t)dt$$

This implies:

$$P^*(q) = \frac{q}{2}(\bar{\theta} + c - \frac{\gamma}{2}q)$$

Note that results are also completely the same with Martimort and Stole (2020) when the preferences are quadratic and buyers are self-interested.

1.3.1 Solution Strategy for the Other-Regarding Buyer:

Note that as we deal with quadratic preferences with uniformly distributed types, we know that, at the equilibrium, the optimal quality level becomes **linear in θ ex-post** that is to say we must have $q(\theta) = \alpha\theta + \beta$ with $\alpha > 0$. As an example, we show that if buyers were perfectly self-interested, with quadratic preferences and uniformly distributed types, the optimal quality would be:

$$q(\theta) = \frac{2\theta - \bar{\theta} - c}{\gamma} \text{ [9]}$$

where $\alpha = \frac{2}{\gamma}$ and $\beta = -\frac{\bar{\theta}+c}{\gamma}$.

In our case, the main problematic part is related to dealing with incentive compatibility constraint. In (1.2), obviously one can not find the set of maximizer easily due to the endogeneity of $q(\theta)$. That is why our solution method uses the **guess and verify method**. We know that the ex-post equilibrium candidate is $q(\theta) = \alpha\theta + \beta$ with $\alpha > 0$ so that we can accordingly define $q^{avg}(\theta)$ and eventually, we can find the α and β values at the equilibrium.

Status-Seeker Buyer:

In this case, the buyer has the utility in the form of $v(q + \mu_s(q - q^{avg}), \theta)$ and we

[9] Note that we assume $\theta > \theta_m$.

can define the average quality level by considering the intermediate value theorem as:

$$q^{avg}(\theta) = \frac{1}{\theta - \underline{\theta}} \int_{\underline{\theta}}^{\theta} (\alpha\theta + \beta) d\theta = \frac{\alpha}{2}(\theta + \underline{\theta}) + \beta \tag{1.9}$$

As $q(\theta) = \alpha\theta + \beta$, we have $\theta = \dfrac{q - \beta}{\alpha}$

Therefore, we can restate $q^{avg}(\theta)$ as:

$$\boxed{q^{avg}(\theta) = \frac{q(\theta) + \beta + \alpha\underline{\theta}}{2}} \tag{1.10}$$

Thus, the objective function becomes:

$$v\Bigg(q + \mu_s(q - q^{avg}), \theta\Bigg) = v\Bigg(q + \mu_s(\frac{q - \beta - \alpha\underline{\theta}}{2}), \theta\Bigg) \tag{1.11}$$

Note that our objective function does not harm any assumption stated above. Therefore, the virtual surplus function is still strictly quasi-concave in q and supermodular in (q, θ).

Therefore, the point-wise maximization problem becomes:

$$\boxed{\frac{\partial}{\partial q}\left[v\big(h(q), \theta\big) - \left(\frac{1 - F(\theta)}{f(\theta)}\right) v_\theta\big(h(q), \theta\big) - cq(\theta)\right] = 0} \tag{1.12}$$

where $h(q) = q + \mu_s(q - q^{avg})$. Using both (1.10), (1.11) and (1.12), we find the quality level as:

$$q(\theta) = \frac{2\Bigg((2\theta - \bar{\theta}) + \dfrac{\gamma}{2}\mu_s(\beta + \alpha\underline{\theta}) - \dfrac{2c}{2 + \mu_s}\Bigg)}{\gamma(2 + \mu_s)} \tag{1.13}$$

Since the ex-post equilibrium in the equilibrium is $q(\theta) = \alpha\theta + \beta$, we must have the following:

$$\alpha\theta + \beta = \frac{2\Bigg((2\theta - \bar{\theta}) + \dfrac{\gamma}{2}\mu_s(\beta + \alpha\underline{\theta}) - \dfrac{2c}{2 + \mu_s}\Bigg)}{\gamma(2 + \mu_s)} \tag{1.14}$$

After some algebra, we find:

$$\alpha = \frac{4}{\gamma(2+\mu_s)} \tag{1.15}$$

and

$$\beta = \frac{-(2+\mu_s)\bar{\theta} - 2c + 2\mu_s\underline{\theta}}{\gamma(2+\mu_s)} \tag{1.16}$$

Therefore, the optimal quality level in the equilibrium becomes:

$$\boxed{q^*(\theta) = \max\left\{0, \frac{4\theta - (2+\mu_s)\bar{\theta} - 2c + 2\mu_s\underline{\theta}}{\gamma(2+\mu_s)}\right\}} \tag{1.17}$$

Here it is easy to verify that if $\mu_s = 0$, $q = \dfrac{2\theta - \bar{\theta} - c}{\gamma}$ which gives the self-interested case result. Note that if the buyer's type is lower than the marginal buyer's type then the optimal quality level becomes zero as this buyer is not served. To be able to find the marginal buyer's type we must have $q^*(\theta_m) = 0$. From there, the marginal status-seeker buyer's (consumer) type becomes $\theta_m = \dfrac{(2+\mu_s)\bar{\theta} - 2\mu_s\underline{\theta} + 2c}{4}$.

Note also that equation (1.17) can be rewritten as [10]:

$$\boxed{q^* = q^*_{fb}(\theta) - \frac{(\bar{\theta} - \theta)}{\gamma} - \frac{(\theta - \underline{\theta})\mu_s}{\gamma(2+\mu_s)}} \tag{1.18}$$

It is easy to show that $q^*_{fb}(\theta) = \dfrac{2\theta + \mu_s\underline{\theta} - 2c}{\gamma(2+\mu_s)}$ [11] and we decompose the optimal quality into three different part as shown in (1.18). In the self-interested buyer model, we have two different components: the first-best optimal allocation and the information rent. In our model, the sum of the second and third components of (1.18) also gives the total information rent. However, as one of our main aims is to show the direct effect of the status-seeker buyer on the information rent, we form such a decomposition. Notice that the second term $\dfrac{(\bar{\theta} - \theta)}{\gamma}$ is the information rent when the buyer is self-interested. Now, we define the last component of (1.18) as "**status**

[10] We only focus on the served types i.e. $\theta > \theta_m$.

[11] See appendix for the derivation of the first-best allocation of the status-seeker buyer.

rent".

$$\text{Status Rent} = \frac{(\theta - \underline{\theta})\mu_s}{\gamma(2 + \mu_s)} \tag{1.19}$$

Status rent plays an important role because it offers new insights about distortion in general. In the self-interested buyer case, for example, the highest type gets the efficient allocation (first-best). However, now the highest type can not get the most efficient (first-best) allocation due to status-seeking activities. Hence, unlike the self-interested buyer case, inefficiency increases for the higher types, and distortion arises even for the highest type in the equilibrium. Note also that inefficiency becomes higher compared to the conventional self-interested model because the status-seeker buyer has more incentive to mimic the higher types. That is why information rent increases and becomes higher compared to the classical monopolistic screening.

Inequity Averse Buyer:

The buyer's utility function is in the form of

$$v\big(q(\theta) - \mu_B\big(q(\theta) - q^{avg}(\theta)\big), \theta\big)$$

where $\mu_B \in (0, 1)$.

By applying a similar procedure as in the status-seeker buyer case, we get:

$$q(\theta) = \alpha\theta + \beta = \frac{4\theta(2 - \mu_B) - \gamma\mu_B(2 - \mu_B)(\beta + \alpha\underline{\theta}) - 2(2 - \mu_B)\bar{\theta} - 4c}{\gamma(2 - \mu_B)^2}$$

Then, it is easy to find $\alpha = \dfrac{4}{\gamma(2 - \mu_B)}$ and $\beta = \dfrac{-2\mu_B\underline{\theta} - (2 - \mu_B)\bar{\theta} - 2c}{\gamma(2 - \mu_B)}$. Therefore, in the end, we have:

$$\boxed{q^*(\theta) = \max\left\{0, \frac{4\theta - (2 - \mu_B)\bar{\theta} - 2\mu_B\underline{\theta} - 2c}{\gamma(2 - \mu_B)}\right\}} \tag{1.20}$$

Similar to the status-seeker case, we find the marginal consumer's type and we must have $q^*(\theta_m) = 0$. Thus, the type for the marginal inequity-averse consumer becomes $\theta_m = \dfrac{(2-\mu_B)\bar{\theta} + 2\mu_B\underline{\theta} + 2c}{4}$.

Note that the above optimal quality can be decomposed as

$$\boxed{q^* = q^*_{fb}(\theta) - \frac{\bar{\theta}-\theta}{\gamma} + \frac{\mu_B(\theta-\underline{\theta})}{\gamma(2-\mu_B)}} \tag{1.21}$$

Note that $q^*_{fb} = \dfrac{2\theta - \mu_B\underline{\theta} - 2c}{\gamma(2-\mu_B)}$ [12] and similar to the status-seeker buyer case, we decompose the optimal quality level into 3 components. Different from the status-seeker buyer case here we call the last component "**aversion rent**". In our model, aversion rent reduces the effects of information rent. This occurs because the inequity-averse buyer does not have much incentive to mimic the higher types compared to the status-seeker case. Another important implication of (1.21) is that because of decreasing information rent, the highest type can actually exceed the first-best allocation. However, apart from decreasing information rent, there is also one other technical explanation for this result. This result occurs because of the way that we introduce the externality into the model. For instance, if we assumed that the other-regarding preference part is additively separable, we would have that the highest type would have the first-best allocation. Note that this argument is also valid for the status-seeker buyer but contrary to the inequity-averse buyer, the highest type status-seeker buyer does not have the efficient allocation because of the increased information rent. Note also that different from monopolistic screening, we assume an additive separable other-regarding preference structure in the common agency part to simplify the analysis.

$$\text{Aversion Rent} = \frac{\mu_B(\theta-\underline{\theta})}{\gamma(2-\mu_B)} \tag{1.22}$$

Corollaries:

1. *When the buyer is a status-seeker (inequity-averse), inefficiency becomes higher (lower) than the case for the self-interested buyer because of the status rents*

[12] See appendix for the derivation of the first-best allocation of the inequity-averse buyer.

(aversion rents).

2. *In the equilibrium, the optimal quality level for the status-seeker (inequity-averse) buyer is higher (lower) than the optimal quality for the self-interested buyer if* $\theta - \underline{\theta} \leq \frac{c}{2}$.

3. *In the equilibrium, as the buyer becomes more status-seeker (inequity-averse), the optimal quality level increases (decreases) if* $\theta - \underline{\theta} \leq \frac{c}{2}$.

Due to the fact that being a status-seeker buyer brings the extra amount of information rent called status rent compared to the self-interested buyer, this induces a higher inefficiency in the equilibrium compared to a self-interested buyer. This occurs because being a status-seeker increases the mimicking activities which can cause to increase in market inefficiency. Analytically, it is easy to show that

$$\frac{\partial SR}{\partial \mu_s} = \frac{2(\theta - \underline{\theta})}{\gamma(2 + \mu_s)^2} \geq 0$$

That is why as the buyer becomes more status-seeker, inefficiency increases. On the other hand, the reverse is also true if the buyer is inequity-averse. Being inequity-averse decreases mimicking activities in that sense information rent decreases compared to self-interested buyer case. Also, note that

$$\frac{\partial AR}{\partial \mu_B} = \frac{2(\theta - \underline{\theta})}{\gamma(2 - \mu_B)^2} \geq 0$$

For the second part, it is easy to compare optimal qualities for both self-interested and status-seeker buyers and optimal qualities for both self-interested and inequity-averse buyers:

$$q^* = \frac{2\theta - \bar{\theta} - c}{\gamma} \leq q_s^* = \frac{4\theta - (2 + \mu_s)\bar{\theta} - 2c + 2\mu_s \underline{\theta}}{\gamma(2 + \mu_s)}$$

Note that q^*, q_s^*, and q_i^* denote the optimal level of qualities when the buyer is self-regarding, status-seeker, and inequity-averse respectively. Clearly above inequality is satisfied if $\theta - \underline{\theta} \leq \frac{c}{2}$.

Similarly,

$$q^* = \frac{2\theta - \bar{\theta} - c}{\gamma} \leq q_i^* = \frac{4\theta - (2 - \mu_B)\bar{\theta} - 2c - 2\mu_B\underline{\theta}}{\gamma(2 - \mu_B)} \implies \theta - \underline{\theta} > \frac{c}{2}$$

or

$$q^* \geq q_i^* \iff \theta - \underline{\theta} \leq \frac{c}{2}$$

For the last part, as we take the partial derivative with respect to the status-seeker parameter μ_s :

$$\frac{\partial q}{\partial \mu_s} = \frac{-2(2\theta - 2\underline{\theta} - c)}{\gamma(2 + \mu_s)^2} \geq 0 \iff 2(\theta - \underline{\theta}) \leq c \implies \theta - \underline{\theta} \leq \frac{c}{2}$$

Similarly,

$$\frac{\partial q}{\partial \mu_s} < 0 \iff \theta - \underline{\theta} > \frac{c}{2}$$

For the inequity-averse buyer, it is also easy to show that

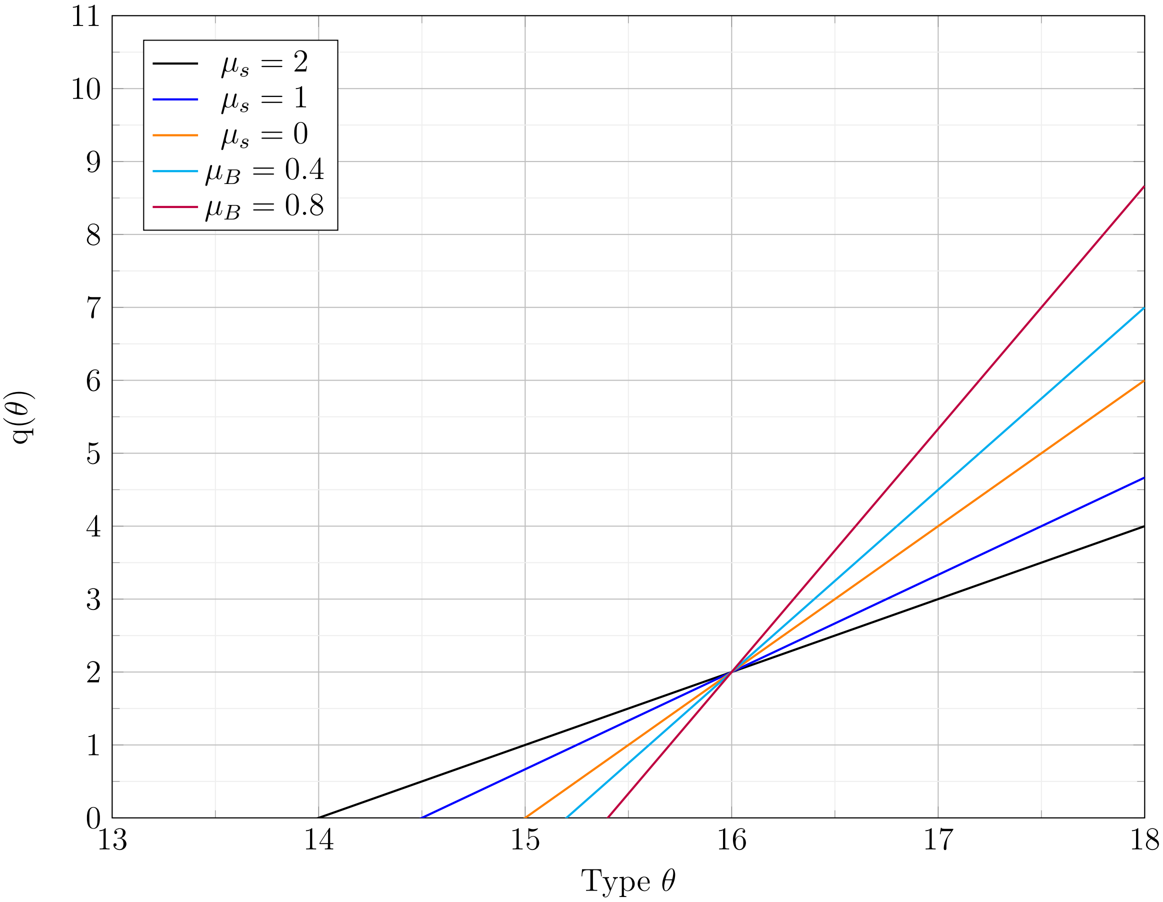
$$\frac{\partial q}{\partial \mu_B} \leq 0 \iff \theta - \underline{\theta} \leq \frac{c}{2} \text{ and } \frac{\partial q}{\partial \mu_B} > 0 \iff \theta - \underline{\theta} > \frac{c}{2}$$

In the second and third parts of the corollaries, type heterogeneity and (or) the marginal cost of the product play an important role. If the types are not that different from the lowest one, i.e. if the types are not that heterogeneous, and (or) the marginal cost of the product is high enough, then for the status-seeker buyer, the optimal quality becomes higher compared to self-interested one and as the buyer becomes more status-seeker, the optimal quality increases. This occurs because as the type is homogeneous enough, then the status-seeker buyer considers other types as competitive rivals. Or similarly, even though the type set is so heterogeneous, if the marginal cost of the product is high enough, then the status-seeker buyer wants to buy the higher qualified product than the others because the status-seeker buyer buying many expensive products increases his status in society. So for the status-seeker buyer, if the type set is homogeneous enough and (or) the marginal cost of the product is high enough then it matters to become ahead of the others. On the other hand, if the types are so different from the lowest one and (or) the marginal cost of the product is low enough, then for the status-seeker buyer, the optimal quality becomes lower than the case for the self-interested buyer, and as the buyer becomes more status-seeker the optimal quality decreases. We believe that this occurs because if the status-seeker buyer thinks that the other types are not close enough compared to his type, then the status-seeker buyer does not consider those lower types as competitive rivals. Similarly, if the marginal cost of the good is low enough, the status-seeker buyer thinks that purchasing the good does not provide him enough status in society.

Note that similar arguments can also be made for the inequity-averse buyer. If the type set is homogeneous enough and (or) the marginal cost of the product is low enough, then as the inequity-aversion parameter increases the optimal quality decreases. This intuitively makes sense because if the type set is homogeneous enough, that is to say, the difference between the type of the inequity-averse buyer and the lowest type is low enough, then the fair-minded buyer thinks that because the type set is not so different, it becomes easy for him to have or sustain a fair equilibrium. That is why the optimal equilibrium is lower than the self-interested one. Thus, as the inequity-aversion parameter increases, the optimal quality increases as well. Similarly, suppose the marginal cost of the product is high enough. In that case, the optimal quality decreases in the equilibrium because the higher marginal cost causes higher expenses and higher expenses generate higher inequity among the buyers. Hence, if the buyers are inequity averse and the marginal cost of the product is high enough, then in the equilibrium, the optimal quality decreases.

Example 1:

Figure 1.1: Optimal Quality vs Type θ vs μ_s ($\gamma = 1, \theta \in [10, 18], c = 12$)



In this example, we see five different quality demand functions two of them are quality demands of status-seeker buyers, two of them are for inequity-averse and one of them is for the self-interested buyer. First, note that the lowest type in the type set is 10, and the highest type is 18. It is clear from the graph that in all cases, any buyer with a type strictly lower than 14 is not served. As we show in the above corollaries, the most significant part is related to the homogeneity of the type set and the magnitude of marginal cost. In that sense, it can be seen from the graph that 16 is a critical type value here as $\theta - \underline{\theta} \leq \frac{c}{2} \implies \theta - 10 \leq 6 \implies \theta \leq 16$. For the other-regarding buyer, we see that for any type less than 16, as the status-seeker (inequity-averse) parameter increases, the optimal quality increases (decreases). On the other hand, if the type is greater than 16, we see that as the status-seeker (inequity-averse) parameter increases the optimal quality decreases (increases).

1.4 Extension: Common Agency under Other-Regarding Preferences

In this part, we extend the above monopoly part and increase the competition by introducing two sellers (two principals). In this setup, sellers sell non-homogeneous goods to the same buyer and sellers use their non-linear pricing schedules as a price discrimination strategy. Thus, this problem can be seen as a common agency problem with adverse selection. In the common agency literature, we observe intrinsic, delegated, and non-intrinsic common agency game structures. Note that we can use the terms agent and buyer interchangeably throughout our analysis.

In the intrinsic common agency game, the agent (buyer) must either accept both contracts offered by all principals or he must stay out of the market and takes his outside option (Stole (1991)). That is to say, in an intrinsic common agency, there is no room for any exclusive contracts for the agent (buyer). However, in the delegated common agency games, we allow for exclusive contracts. In this setup, the common agency is not intrinsic to the agent, on the other hand, it is delegated to the agent and the agent may have the option of buying only one principal. More precisely, under delegated common agency, different from the intrinsic common agency, the agent (buyer) has extra accepting options in which he can choose a proper subset of the sellers' contract offers (Martimort and Stole (2009)). In the non-intrinsic common agency, introduced by Calzolari and Scarpa (1999), the agent (buyer) has no restriction in choosing the number of sellers that he wants to work with and no principal (seller) can condition the contract on the agent (buyer).

In an intrinsic common agency game, the buyer must either accept both contracts or he must stay out of the market. That is to say, an intrinsic common agency does not offer exclusive contracts for the buyer. However, in our setup, we allow for exclusive contracts and the buyer has the option to choose exclusive contracts i.e. he can buy from only one seller. We consider non-intrinsic common agency under the existence of other-regarding preferences. There are two sellers (principals) and each simultaneously offers price schedule $P(q_i)$ over a compact set of possible quality (quantity) to a buyer who has private information about his taste preference. More specifically, seller i offers to sell any q_i for a price of $P_i(q_i)$. After the buyer observes the price, he decides whether to buy or not (equivalently participates or accepts the contract or not).

1.4.1 Benchmark: Self-Interested Case: Interior Solution

We begin to analyze self-interested buyer that is to say buyer does not have any type of other-regarding preference. We assume that types are distributed over $[\underline{\theta}, \bar{\theta}]$ according to $F(\theta)$ with the density function of $f(\theta)$.

Payoff Functions: Buyer of type θ has the payoff function :

$$U(q,\theta) = v\big(q_1(\theta), q_2(\theta), \theta\big) - P_1\big(q_1(\theta)\big) - P_2\big(q_2(\theta)\big)$$

and Seller i's payoff function:

$$\Pi\big(q_i(\theta), \theta\big) = P_i\big(q_i(\theta)\big) - cq_i(\theta)$$

For the sake of simplicity, similar to Calzolari and Denicolò (2013), we assume that the market is not covered in the non-linear pricing equilibrium and $q^{fb}(\underline{\theta}) = 0$ and $q^{fb}(\theta) > 0$ for any $\theta > \underline{\theta}$. Note that $q^{fb}(\theta)$ is the quality level under the full-information or first-best allocation. Moreover, if there exists any $\theta \in [\underline{\theta}, \bar{\theta}]$ such that $q^{fb}(\theta) = 0$, then, one can select the maximum of θ as $\underline{\theta}$ and re-scale the distribution.

Assumptions:

1. $v(q_i, q_j, \theta)$ is thrice differentiable, increasing in q_i, and strictly quasi-concave in (q_i, q_j)

2. $v(q_i, q_j, \theta)$ is continuous and increasing in θ

3. $v_{q\theta}(q_i, q_j\theta) > 0$

4. $v_{q_i q_j}(q_i, q_j, \theta)$ has a constant sign

5. Each price schedule is nondecreasing in q_i, and $P_i(q_i) = 0$ if $q_i(\theta) = 0$

6. P_i is upper semicontinuous in q_i

First, consider the firm i's (seller i, or principal i) program. The firm i's problem is to choose the price schedule $P_i(q_i(\theta))$ to maximize the expected profit. The firm i's problem can be stated as:

$$\max_{p_i} \int_{\underline{\theta}}^{\bar{\theta}} \left[P_i(q_i(\theta)) - c(q_i(\theta))\right] dF(\theta) \tag{1.23}$$

subject to

$$(q_i(\theta), q_j(\theta)) \in \arg\max_{q_i, q_j} v(q_i(\theta), q_j(\theta), \theta) - P_i(q_i(\theta)) - P_j(q_j(\theta)) \tag{1.24}$$

and

$$v(q_i(\theta), q_j(\theta), \theta) - P_i(q_i(\theta)) - P_j(q_j(\theta)) \geq \max\{0, U(0, q_j(0, \theta), \theta)\} \tag{1.25}$$

Clearly, equations (1.24) and (1.25) denote incentive compatibility and participation constraints respectively. Note that the participation constraint here is type-dependent[13].

Virtual surplus function becomes: [14]

$$\Gamma(q_i, \theta) = \hat{v}(q_i(\theta), \theta) - c(q_i(\theta)) - \frac{1 - F(\theta)}{f(\theta)} \hat{v}_\theta(q_i, \theta) \tag{1.26}$$

where

$$\hat{v}(q_i, \theta) \equiv v(q_i, q_j^*(q_i, \theta), \theta)) - P_j(q_j^*(q_i, \theta))$$

[13] See Jullien (2000) for the type-dependent participation constraints.
[14] See Appendix for the derivation and details.

and

$$q_j^* \in \arg\max_{q_j} v(q_i, q_j, \theta) - P_j(q_j)$$

Note that if virtual surplus function is strictly quasi-concave in q_i and super-modular in (q_i, θ), point-wise maximization gives:

$$\boxed{\hat{v}_{qi}(q_i, \theta) - \frac{1 - F(\theta)}{f(\theta)} \hat{v}_{\theta q_i}(q_i, \theta) = c} \tag{1.27}$$

Or by using envelope theorem and definition of $\hat{v}$, equation (1.27) can also be written as:

$$\boxed{v_{q_i} + v_{q_j} \frac{\partial q_j^*(q_i)}{\partial q_i} - \frac{\partial P_j}{\partial q_j^*} \frac{\partial q_j^*}{\partial q_i} - c = \frac{1 - F(\theta)}{f(\theta)} \left[v_{\theta q_i} + v_{\theta q_j} \frac{\partial q_j^*}{\partial q_i} \right]} \tag{1.28}$$

Here we assume that $1 + \dfrac{\partial q_j^*(q_i)}{\partial q_i} > 0$ to have a strictly increasing differences of $\hat{v}(q_i, \theta)$ in (q_i, θ) which is a necessary condition for the regularity.

In the next part, we characterize the equilibrium output and optimal pricing schedule under quadratic preference and uniformly distributed types similar to Martimort and Stole (2009).

1.4.2 Quadratic Preferences with Uniformly Distributed Types :

In this part, we consider other-regarding buyers and assume that types are distributed uniformly. We focus on symmetric, quadratic utility function to characterize equilibrium quality (quantity). Suppose that utility of the agent consuming q_i and q_j with the corresponding prices P_i and P_j from the sellers i and j :

$$v(q_i, q_j, \theta) = u(q_i, q_j, \theta) + \sum_{t \in \{i,j\}} \mu_s^t \theta \left[q_t(\theta) - q_t^{avg}(\theta) \right]$$

where

$$q^{avg}(\theta) : [\underline{\theta}, \theta] \longrightarrow \mathbf{R}_+ \cup \{0\}$$

and $\mu_s^t > 0$ shows the status-seeker parameter for each product and we define the utility function as in Martimort and Stole (2009):

$$u(q_i, q_j, \theta) = \frac{\alpha+\theta}{\beta-\gamma}\big(q_i+q_j\big) - \frac{\beta}{2(\beta^2-\gamma^2)}(q_i^2+q_j^2) - \frac{\gamma}{\beta^2-\gamma^2}q_iq_j$$

It is equivalent that the agent has a symmetric demand function and is linear in the prices. More precisely:

$$q_i = \alpha + \theta - \beta P_i + \gamma P_j$$

where $\alpha > 0$ and type θ are the intercept terms for the demand function. We assume that $\beta > |\gamma| \neq 0$. We call that goods are demand substitutes and complement if $\gamma \in (0, \beta)$ and $\gamma \in (-\beta, 0)$ respectively. Here we also follow Martimort and Stole (2009)'s notation and let $\tau = \dfrac{\gamma}{\beta}$ where $\tau \in (-1, 1)$. Then, we call that goods are demand substitutes if $\tau > 0$ and goods are demand complements if $\tau < 0$. Lastly, we assume that costs are linear in q i.e. $C(q) = cq$.

Unlike the monopoly part, we assume that the other-regarding preference part is assumed to be additively separable to have an explicit and tractable environment. This structure also creates a smooth way to see the outcomes for the status-seeker and inequity-averse buyer. Now, according to the assumption we have a utility function for the status-seeker buyer:

$$v = h(q_i + \mu_s^i(q_i - q_i^{avg}), q_j + \mu_s^j(q_j - q_j^{avg})) = u(q_i, q_j, \theta) + \sum_{t\in\{i,j\}} \mu_s^t \theta \left[q_t(\theta) - q_t^{avg}(\theta)\right]$$

Different from the status-seeker buyer, we can define the utility function of inequity averse as

$$v(q_i, q_j, \theta) = u(q_i, q_j, \theta) - \sum_{t\in\{i,j\}} \lambda^t \theta \left[q_t(\theta) - q_t^{avg}(\theta)\right]$$

with $\lambda \in (0, 1)$.

Now we turn back to delegated common agency problem under this setup. Note that sellers' optimal price schedules are assumed to be quadratic i.e. $P(q) = a_1 q + \frac{a_2}{2} q^2$. Then by using (1.28), we have the following proposition:

Proposition 1: *There exists an equilibrium with quadratic price schedules where the symmetric equilibrium allocation of output for a status-seeker buyer:*

$$q_s{}^*(\theta) = \frac{a_1\gamma + (\alpha + 2\theta - \bar{\theta})(1 + \frac{\mu_s}{2})(1 + a_2(\beta + \gamma)) - c(\beta + a_2(\beta^2 - \gamma^2))}{1 + a_2\beta}$$

where

$$a_1{}^s = \frac{(2 + \mu_s)(\alpha + \bar{\theta}) + c(\beta + 2\gamma + \sqrt{\beta^2 + 8\gamma^2})}{3\beta + \sqrt{\beta^2 + 8\gamma^2}} \text{ and } a_2 = -\frac{2}{3\beta + \sqrt{\beta^2 + 8\gamma^2}}$$

and the symmetric optimal equilibrium allocation of output for an inequity-averse buyer:

$$q_i{}^*(\theta) = \frac{a_1\gamma + (2\theta - \bar{\theta})(1 - \frac{\lambda}{2})(1 - 2\gamma + a_2) - c(1 - \gamma + a_2)}{(1 - \gamma)(1 + a_2) - \gamma}$$

where

$$a_1{}^i = \frac{(2 - \lambda)(\alpha + \bar{\theta}) + c(\beta + 2\gamma + \sqrt{\beta^2 + 8\gamma^2})}{3\beta + \sqrt{\beta^2 + 8\gamma^2}} \text{ and } a_2 = -\frac{2}{3\beta + \sqrt{\beta^2 + 8\gamma^2}}$$

Above proposition, indexes s and i denote both status-seeker and inequity-averse labels. Note that according to proposition 1, it is obvious to show that equilibrium pricing schedules for status-seeker buyers and inequity-averse buyers become: $P_s^* = a_1{}^s q + \frac{a_2}{2} q^2$ and $P_i^* = a_1{}^i q + \frac{a_2}{2} q^2$ respectively.

Different from Martimort and Stole (2009), Proposition 1 implies important outcomes about the efficiency of the allocations. After some algebra, we can show that q_s^* can be rewritten as:

$$q_s^* = q_s^{fb} - \left(1 + \frac{\mu_s}{2}\right)(\bar{\theta} - \theta)\left[1 - \frac{4\gamma}{\beta + \sqrt{\beta^2 + 8\gamma^2}}\right] \tag{1.29}$$

which is also equivalent

$$q_s^* = q_s^{fb} - \left(1 + \frac{\mu_s}{2}\right)(\bar{\theta} - \theta)\left[1 - \frac{4\tau}{1 + \sqrt{1 + 8\tau^2}}\right] \tag{1.30}$$

Similarly optimal quality level for the inequity-averse agent:

$$q_i^* = q_i^{fb} - \left(1 - \frac{\lambda}{2}\right)(\bar{\theta} - \theta)\left[1 - \frac{4\tau}{1 + \sqrt{1 + 8\tau^2}}\right] \tag{1.31}$$

Note that q_s^{fb} and q_i^{fb} denote the first-best efficient allocation for status-seeker and inequity-averse agent respectively and $q_s^{fb} = (\alpha + \theta)(1 + \frac{\mu_s}{2}) - (\beta - \gamma)c$ and $q_i^{fb} = (\alpha + \theta)(1 - \frac{\lambda}{2}) - (\beta - \gamma)c$[15].

Corollary: 2 *As a consequence of Proposition 1, we have the following results:*

- *Regardless of whether the buyer is a status-seeker, inequity-averse, or self-regarding, similar to Martimort and Stole (2009), efficiency is achieved when the goods are perfect substitutes. Moreover, compared to the demand substitutes, the equilibrium quality distorts downward more when the goods are demand complements.*

- *Regardless of the buyer's preference type, only the highest type gets the efficient allocation. If the agent is a status-seeker(inequity-averse), inefficiency increases (decreases) more compared to the self-regarding common agency model.*

- *If the buyer's type is sufficiently high, as the degree of being a status-seeker (inequity-averse) increases, the optimal quality increases (decreases) regardless of whether the goods are complements or substitutes.*

By (1.30) and (1.31), it is straightforward to see that the above corollary holds. If the buyer is self-regarding, i.e., $\mu_s = 0$, we know that efficient allocation is achieved when the goods are perfect substitutes ($\tau = 1$). We also know that only the highest type gets the first-best allocation (no distortion at the top), and the equilibrium qualities are distorted downwards. It is still true that efficiency is achieved when the goods

[15]Note that q^{fb} satisfies $\arg\max_q u_{q_i}(q^{fb}, q^{fb}, \theta) - C'(q^{fb}) = 0$

are perfect substitutes, regardless the buyer is a status-seeker or inequity-averse. Different from conventional common agency literature, however, regardless of whether goods are complements or substitutes, if the buyer is a status-seeker (inequity-averse), we see that the equilibrium quality distorts downward more (less) compared to the classical common agency literature. Lastly, under some conditions, if the buyer becomes more status-seeker (inequity-averse), we see that the optimal quality increases (decreases) more. It is true because by using (1.30) and (1.31) we have:

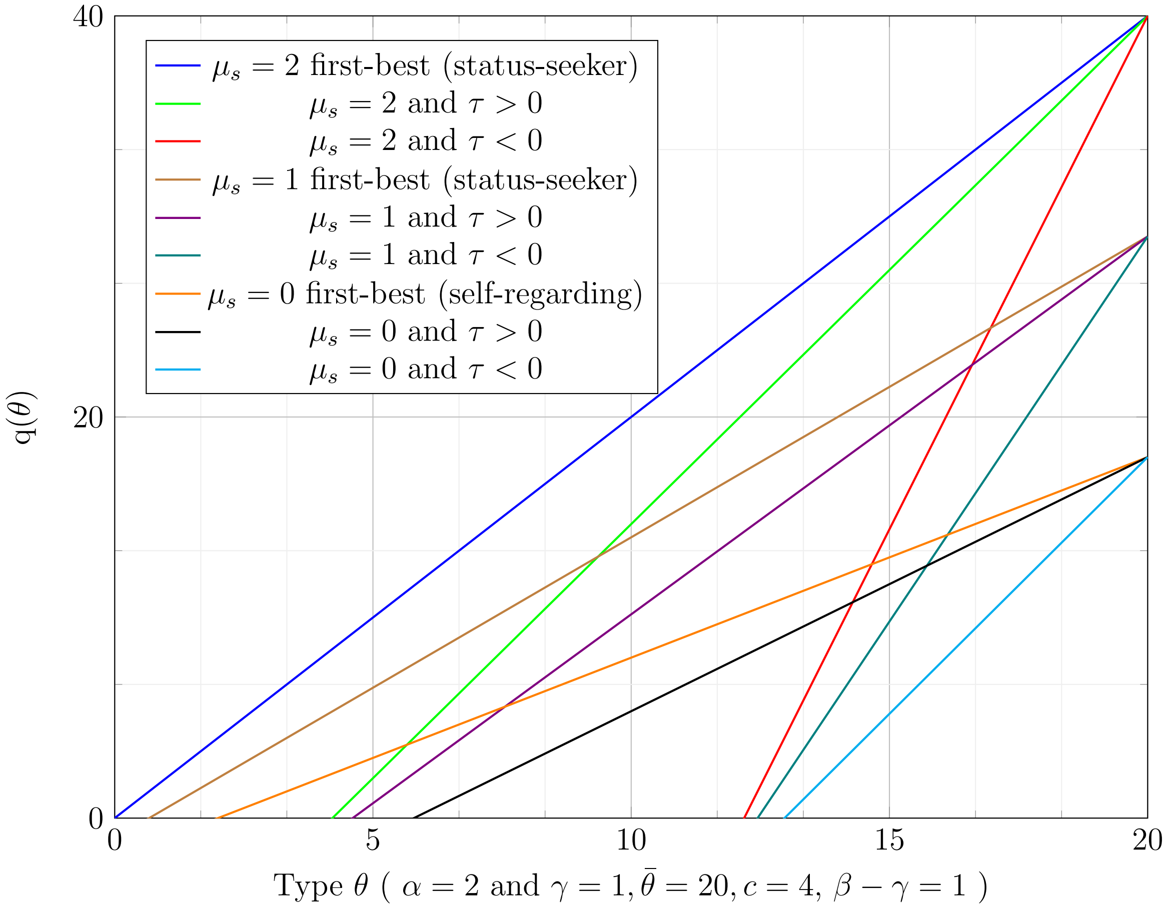
$$\frac{\partial q_s^*}{\partial \mu_s} = \frac{\alpha + \theta}{2} - \frac{\bar{\theta} - \theta}{2} \left[1 - \frac{4\tau}{1 + \sqrt{1 + 8\tau^2}} \right]$$

Let $A = \left[1 - \frac{4\tau}{1 + \sqrt{1 + 8\tau^2}} \right] \geq 0$. Thus, $\frac{\partial q_s^*}{\partial \mu_s} \geq 0 \iff \theta \geq \frac{\bar{\theta}A - \alpha}{1 + A}$. Thus, if the buyer's type is sufficiently high, then as the degree of being status-seeker increases, the optimal quality increases. A similar argument can also be easily done for the inequity-averse buyer.

Example 2: In this example, we try to show how the optimal quality change with different levels of other-regarding preferences. We also show how these equilibrium levels change when the goods are complements or substitutes. We assume $\bar{\theta} = 20$, $\beta - \gamma = 1$, $c = 4$ for simplicity and show how the optimal quality changes when the buyer becomes more status-seeker or inequity-averse.

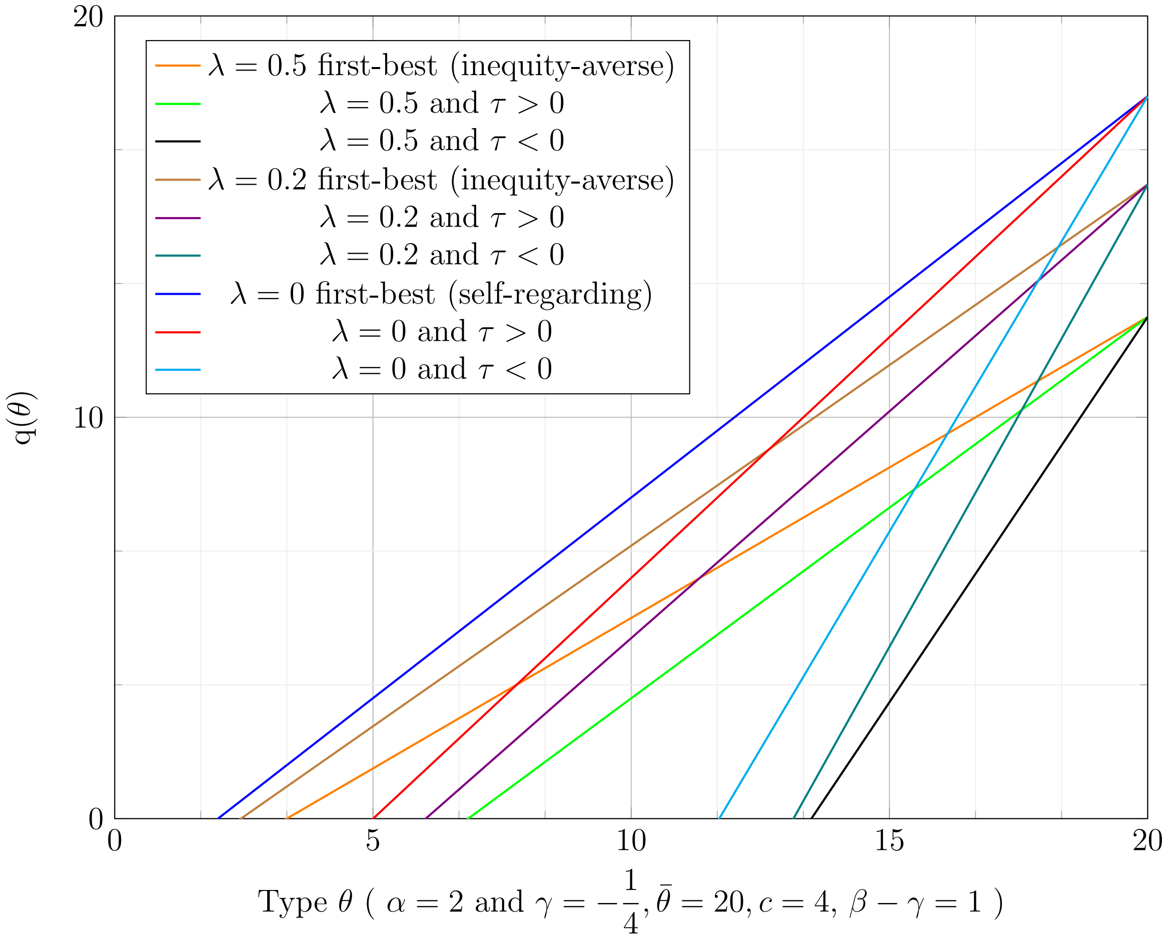
Numerical examples are shown in Figures 1.2 and 1.3. We compare status-seeker, inequity-averse, and self-regarding buyers. In this case, we take $\gamma = 1$ for the substitutability of the goods. Note that $\tau = \frac{\gamma}{\beta} = \frac{1}{2} > 0$. We also take $\gamma = -\frac{1}{4}$ for the demand complements. First, it is obvious that regardless of whether the buyer is a status-seeker or self-interested when the goods are substitutes, the optimal quality becomes higher when the goods are demand substitutes rather than goods are complements. Secondly, it is easy to see that only the highest type gets the efficient allocation regardless of whether the agent is a status-seeker, inequity-averse, or self-regarding, or whether the goods are complements or substitutes. Lastly, we can also verify that the optimal quality increases when the buyers become more status-seekers in the equilibrium regardless goods are complements or substitutes.

Figure 1.2: Optimal Quality when the Agent is Status-Seeker



In Figure 3, we see how the equilibrium of quality varies when the agent is inequity-averse. The most important difference between Figure 1.2 and Figure 1.3 is related to the degree of inefficiency. In Figure 1.2, it is easy to show that as the degree of being status-seeker increases, inefficiency increases regardless of whether goods are substitutes or complements. In Figure 1.3, however, with the introduction of the inequity-averse parameter, inefficiency decreases compared to both status-seeker and self-regarding agents. That can be easily seen by comparing the differences between the first-best efficient allocation and the actual equilibrium allocation when the goods are substitutes or complements. That is to say one can easily check ${q_{ss}}^{fb} - q_s^s,\ {q_{sc}}^{fb} - q_s^c,\ {q_{is}}^{fb} - q_i^s,\ {q_{ic}}^{fb} - q_i^c.$ and. Lastly, we can observe that as the degree of being inequity-averse increases, the optimal equilibrium level of quality decreases. Here ${q_{xy}}^{fb}$ denotes the first-best level of quality when the agent is $x \in$ {status-seeker (s), inequity-averse (i)} and goods are $y \in$ {substitutes (s), complements (c)}. Similarly, q_x^y denotes the optimal quality level of the goods when the agent is $x \in$ {status-seeker (s), inequity-averse (i)} and goods are $y \in$ {substitutes (s), complements (c)}.

Figure 1.3: Optimal Quality under Inequity aversion



1.5 Conclusion

This study mainly focuses on new theoretical insights about non-linear pricing in both monopoly and common agency by merging the classical principal-agent framework with the Fehr and Schmidt (1999) type of other-regarding preferences. We characterize both status-seeker and inequity-averse buyers in both monopoly and common agency and we find the optimal equilibrium quality and pricing schedule.

In the monopolistic screening part, we show that if the buyer is a status-seeker (inequity-averse), the optimal quality and market inefficiency become higher (lower) relative to the self-interested buyer in the equilibrium if the type set is sufficiently homogeneous and/or the marginal cost of production is high enough. Similarly, as the buyer becomes more status-seeker (inequity-averse), the optimal quality and inefficiency increase (decrease) more.

In the second part, we extend non-linear pricing in monopoly to the common agency case. In that sense, we introduce the Fehr and Schmidt (1999) type of other-regarding preferences to the common agency framework. In particular, we find and show the optimal symmetric quality and pricing schedule in the equilibrium when the buyer has other-regarding preferences. We mainly show that regardless of whether goods are complements or substitutes if the buyer is a status-seeker (inequity-averse), inefficiency increases (decreases) more compared to the conventional common agency model. In other words, being a status-seeker (inequity-averse) causes the equilibrium quality to distort downwards more (less) compared to the classical common agency literature. Moreover, we show that if the buyer's type is sufficiently high, as the degree of being a status-seeker (inequity-averse) increases, the optimal quality increases (decreases) regardless of whether the goods are demand complements or substitutes in the equilibrium.

1.6 Appendix

Benchmark: Self-Interested Buyer

Consumer's utility function becomes $v(q(\theta),\theta)$ and Firm's problem remains the same as in (1.1), however, incentive compatibility and participation constraints would become:

$$q(\theta) \in \arg\max_{q} v\big(q(\theta),\theta\big) - P(q(\theta)) \tag{1.32}$$

and

$$v\big(q(\theta),\theta\big) - P(q(\theta)) \geq 0 \tag{1.33}$$

$$\Gamma(q,\theta) = v(q(\theta),\theta) - C(q(\theta)) - \frac{1-F(\theta)}{f(\theta)} v_\theta(q,\theta) \tag{1.34}$$

If we use the change of variables as:

$$U(\theta) \equiv v(q(\theta),\theta) - p(q(\theta)) \tag{1.35}$$

we can restate the firm's program as:

$$\max_{q} \int_{\underline{\theta}}^{\bar{\theta}} \{v(q(\theta)) - C(q(\theta)) - U(\theta)\}\, dF(\theta) \tag{1.36}$$

subject to

$$q(\theta) \in \arg\max U(\theta) \tag{1.37}$$

$$U(\theta) \geq 0 \text{ and } q(\theta) \text{ is non-decreasing} \tag{1.38}$$

Note that as the single-crossing condition holds, we can restate the firm's program as (1.36) subject to (1.37) and (1.38). Clearly, if the Incentive compatibility constraint holds, by using (1.35) we get:

$$\boxed{\frac{\partial v(q(\theta),\theta)}{\partial q} - \frac{\partial p(q(\theta))}{\partial q} = 0} \tag{1.39}$$

Now by using the envelope theorem, we can show the following:

Lemma 1:

$$\boxed{U_\theta(\theta) = v_\theta(q,\theta)} \tag{1.40}$$

Proof of Lemma 1:

$$\frac{\partial U(\theta)}{\partial \theta} \equiv \frac{\partial v(q(\theta),\theta)}{\partial q}\frac{\partial q}{\partial \theta} + \frac{\partial v(q(\theta),\theta)}{\partial \theta} - \frac{\partial P}{\partial q}\frac{\partial q}{\partial \theta}$$

which implies:

$$\frac{\partial U(\theta)}{\partial \theta} \equiv \frac{\partial q}{\partial \theta}\left[\frac{\partial v(q,\theta)}{\partial q} - \frac{\partial p}{\partial q}\right] + \frac{\partial v(q,\theta)}{\partial \theta}$$

However, we know that if incentive compatibility conditions hold, the inside of the parenthesis of the above equation becomes zero which completes the proof and gives (1.40).

Now, by using (1.40), we have[16]:

$$U(\theta) - U(\underline{\theta}) = \int_{\underline{\theta}}^{\theta} v_\theta(q,\theta)d\theta \tag{1.41}$$

Clearly, individual rationality constraint binds for the lowest type, thus $U(\underline{\theta}) = 0$

So, if we rewrite the firm's problem:

$$\max_q \int_{\underline{\theta}}^{\bar{\theta}} \left[v(q(\theta),\theta) - C(q(\theta)) - \int_{\underline{\theta}}^{\theta} v_{\hat{\theta}}(q,\theta)d\hat{\theta}\right] dF(\theta) \tag{1.42}$$

subject to $q(\theta)$ is non-decreasing.

Note that by changing of orders of the integrals, we can also rewrite equation (1.42) as:

[16]Actually it is consequence of envelope theorem as Milgrom and Segal (2002) show

$$\max_{q} \int_{\underline{\theta}}^{\bar{\theta}} \left[v(q(\theta), \theta) - C(q(\theta)) - \frac{1 - F(\theta)}{f(\theta)} \{v_\theta(q, \theta)\} \right] dF(\theta) \tag{1.43}$$

Then virtual surplus function becomes:

$$\boxed{\Gamma(q, \theta) = v(q(\theta), \theta) - C(q(\theta)) - \frac{1 - F(\theta)}{f(\theta)} v_\theta(q, \theta)} \tag{1.44}$$

Derivation of the first-best quality:

The first-best (efficient) quality occurs under the full information. That means, there exists no information rent that creates inefficiency in the first-best equilibrium. In that sense, if the buyer is self-interested, under full information we must have:

$$v_q(q(\theta), \theta) - C^{'}(q(\theta)) = 0$$

which basically gives:

$$\theta - \gamma q(\theta) - c = 0 \implies \boxed{q^{fb}(\theta) = \frac{\theta - c}{\gamma}}$$

Now we find the first-best quality level when the buyer is both a status-seeker and inequity averse. This part is not as simple as the above but very similar to the solution methodology that we propose in the paper.

First-Best Quality Level When the Buyer is Status-Seeker:

Before finding the first-best quality level, it is important to notice from the solution methodology that we offer in the main text that optimal quality is linear in θ ex-post as we use quadratic utility function. So, the solution starts with the assumption that optimal quality is $q^{fb} = \alpha\theta + \beta$. Then,

$v_q(q + \mu_s(q - q^{avg}), \theta)) - C^{'}(q(\theta)) = 0$ where $q^{avg}(\theta) = \dfrac{q(\theta) + \beta + \alpha\underline{\theta}}{2}$ as we show in the solution methodology. Then, marginal utility equals marginal cost gives us that

$$q^{fb}(\theta) = \frac{2(2 + \mu_s)\theta + \gamma\mu_s(2 + \mu_s)(\alpha\underline{\theta} + \beta) - 4c}{\gamma(2 + \mu_s)^2}$$

Now, we have to find what α and β are from the following:

$$\alpha\theta + \beta = \frac{2(2+\mu_s)\theta + \gamma\mu_s(2+\mu_s)(\alpha\underline{\theta} + \beta) - 4c}{\gamma(2+\mu_s)^2}$$

Then it is easy to show that $\alpha = \dfrac{2}{\gamma(2+\mu_s)}$ and $\beta = \dfrac{\mu_s\underline{\theta} - 2c}{\gamma(2+\mu_s)}$

Therefore,

$$\boxed{q^{fb}(\theta) = \alpha\theta + \beta = \frac{2\theta + \mu_s\underline{\theta} - 2c}{\gamma(2+\mu_s)}}$$

First-Best Quality Level When the Buyer is Inequity-Averse:

It is very similar to the above case. Basically marginal utility equals marginal cost gives us :

$v_q(q - \mu_B(q - q^{avg}), \theta)) - C'(q(\theta)) = 0$ where $\mu_B \in (0, 1)$.

Then

$$q^{fb} = \alpha\theta + \beta = \frac{2\theta(2-\mu_B) - \gamma\mu_B(2-\mu_B)(\beta + \alpha\underline{\theta}) - 4c}{\gamma(2-\mu_B)^2}$$

Then it is easy to find that $\alpha = \dfrac{2}{\gamma(2-\mu_B)}$ and $\beta = \dfrac{-\mu_B\underline{\theta} - 2c}{\gamma(2-\mu_B)}$. Therefore if the buyer is inequity-averse, the first-best quality becomes:

$$\boxed{q^{fb}(\theta) = \alpha\theta + \beta = \frac{2\theta - \mu_B\underline{\theta} - 2c}{\gamma(2-\mu_B)}}$$

Benchmark Case: Common Agency

For this part, we follow Calzolari and Scarpa (1999) and Martimort and Stole (2009). Let $q_j(q_i(\theta))$ be the output level that maximizes the agent's utility for any given $q_i(\theta)$ i.e.,

$$q_j^* \in \arg\max_{q_j} v(q_i, q_j, \theta) - P_j(q_j) \tag{1.45}$$

Then define:

$$\hat{v}(q_i, \theta) \equiv v(q_i, q_j^*(q_i, \theta), \theta)) - P_j(q_j^*(q_i, \theta)) \tag{1.46}$$

Equation (1.46) denotes the gross utility that the agent can obtain given the optimal level of q_j which also depends on q_i.

Then, we can restate the incentive compatibility constraint as:

$$q_i(\theta) \in \arg\max_{q_i} \{\hat{v}(q_i, \theta) - P_i(q_i)\} \tag{1.47}$$

If we use the change of variables as:

$$U(\theta) \equiv \hat{v}(q_i(\theta), \theta) - P_i(q_i(\theta)) \tag{1.48}$$

then we can restate the firm i's program (by assuming incentive compatibility contraint holds):

$$\max_{q_i} \int_{\underline{\theta}}^{\bar{\theta}} \{\hat{v}(q_i(\theta)) - c(q_i(\theta)) - U(\theta)\} \, dF(\theta) \tag{1.49}$$

subject to participation constraint. Eventually, it is easy to show:

$$U(\theta) - U(\underline{\theta}) = \int_{\underline{\theta}}^{\theta} \hat{v}_\theta(q_i, \theta) d\theta \tag{1.50}$$

Thus (1.49) can be stated as:

$$\max_{q_i} \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \hat{v}(q_i(\theta)) - c(q_i(\theta)) - \int_{\underline{\theta}}^{\theta} \hat{v}_\theta(q_i, \theta) d\theta \right\} dF(\theta) - U(\underline{\theta}) \tag{1.51}$$

subject to participation constraint.

Note that we have $\hat{v}(q_i, \theta) - P_i(q_i) \geq \max\{0, \hat{v}(0, \theta)\}$ (type-dependent participation), and by single-property assumption, left and right-hand sides are in increasing in θ yet again due to single-cross property $\hat{v}_\theta(q_i, \theta) > \hat{v}_\theta(0, \theta)$ which makes sure that participation constraint only binds for the lowest type. Therefore, the seller i's problem becomes:

$$\max_{q_i} \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \hat{v}(q_i(\theta)) - c(q_i(\theta)) - \int_{\underline{\theta}}^{\theta} \hat{v}_\theta(q_i, \theta) d\theta \right\} dF(\theta) - U(0, q_j(0, \underline{\theta}), \underline{\theta}) \tag{1.52}$$

Then by using Fubini's theorem, the above maximization problem can be restated as:

$$\max_{q_i} \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \hat{v}(q_i(\theta)) - c(q_i(\theta)) - \frac{1 - F(\theta)}{f(\theta)} \hat{v}_\theta(q_i, \theta) \right\} dF(\theta) - U(0, q_j(0, \underline{\theta}), \underline{\theta}) \tag{1.53}$$

Clearly, the virtual surplus function becomes the expression inside the parenthesis in equation (1.53).

Proof of Proposition 1: The proof mainly follows equation (1.28), Martimort and Stole (2009). Note that we give detailed proof for only status-seeker buyers. For inequity-averse buyers, the way of proof is completely identical with status-seeker. The payoff function of the status-seeker buyer of type θ is

$$v = \frac{\alpha + \theta}{\beta - \gamma} \sum_{t \in \{i,j\}} \left((1 + \frac{\mu_s^t}{2}) q_t - \frac{\mu_s^t}{2} (\beta_t + \alpha_t \underline{\theta}) \right) - \frac{\beta}{2(\beta^2 - \gamma^2)} (q_i^2 + q_j^2) - \frac{\gamma}{\beta^2 - \gamma^2} q_i q_j - P_i(q_i) - P_j(q_j) \tag{1.54}$$

For clarification, note that the first part in (54) is $\theta \sum_{t \in \{i,j\}} (q_t + \mu_s^t (q_t - q_t^{avg}))$ and we assume that price schedule is quadratic without an intercept term i.e, $P(q) = a_1 q + \frac{a_2}{2} q^2$ Then, the indirect utility requires:

$$\arg\max_{q_j} u - P_j(q_j) \tag{1.55}$$

where

$$u = \frac{\alpha + \theta}{\beta - \gamma} \sum_{t \in \{i,j\}} \left((1 + \frac{\mu_s^t}{2}) q_t - \frac{\mu_s^t}{2} (\beta_t + \alpha_t \underline{\theta}) \right) - \frac{\beta}{2(\beta^2 - \gamma^2)} (q_i^2 + q_j^2) - \frac{\gamma}{\beta^2 - \gamma^2} q_i q_j$$

. From (1.55), we have

$$q_j^*(q_i) = \frac{(\alpha+\theta)(\beta+\gamma)(1+\frac{\mu_s}{2}) - \gamma q_i - a_1(\beta^2-\gamma^2)}{\beta + a_2(\beta^2-\gamma^2)} \tag{1.56}$$

Then, it is straightforward that:

$$\frac{\partial q_j^*}{\partial q_i} = -\frac{\gamma}{\beta + a_2(\beta^2-\gamma^2)}$$

By following (1.28), we have:

$$u_q\big(1+\frac{\partial q_j^*}{\partial q_i}\big) - \frac{\partial P}{\partial q}\frac{\partial q_j^*}{\partial q_i} - c = (\bar{\theta}-\theta)u_{\theta q}\big(1+\frac{\partial q_j^*}{\partial q_i}\big) \tag{1.57}$$

where $u_q = \dfrac{\alpha+\theta}{\beta-\gamma}(1+\dfrac{\mu_s}{2}) - \dfrac{q}{\beta-\gamma}$ and $u_{\theta q} = (1+\dfrac{\mu_s}{2})\dfrac{1}{\beta-\gamma}$. Note that here we focus on only symmetric equilibrium. That is to say, we have $q_t(\theta) = q(\theta)$ and $\mu_s^i = \mu_s^j = \mu_s$ for all $t \in \{i, j\}$. Then by using (1.57), we find the symmetric optimal quality level as:

$$q_s{}^*(\theta) = \frac{a_1\gamma + (\alpha + 2\theta - \bar{\theta})(1+\frac{\mu_s}{2})(1+a_2(\beta+\gamma)) - c(\beta + a_2(\beta^2-\gamma^2))}{1+a_2\beta} \tag{1.58}$$

Note that $q_s{}^*$ denotes the optimal quality for the status-seeker buyer. Now we find a_1 and a_2. First, it must hold that $\dfrac{d}{d\theta}\big\{u_q - P_q(q(\theta))\big\} = 0$. This gives: $\dfrac{dq}{d\theta} = \dfrac{1+\frac{\mu_s}{2}}{1+a_2(\beta-\gamma)}$

If we also take the derivative of (1.58) with respect to θ, we reach:

$$\frac{dq}{d\theta} = \frac{2(1+\frac{\mu_s}{2})(1+a_2(\beta+\gamma))}{1+a_2\beta}$$

Therefore, a_2 must solve:

$$1 + 3a_2\beta + 2a_2^2(\beta^2-\gamma^2) = 0$$

which gives two real roots but by our assumption on $1+\dfrac{\partial q_j^*}{\partial q_i}>0$ Therefore,

$$a_2=-\frac{2}{3\beta+\sqrt{\beta^2+8\gamma^2}} \tag{1.59}$$

Notice that this result is the same with Martimort and Stole (2009) but as we show, a_1 changes due to the existence of the status-seeker parameter. Now we find a_1. Note that by using the first-order condition we also must have the following $u_q(\bar{\theta})-P_q(q(\bar{\theta}))=0$. After some algebra, one can reach $a_1=-a_2(1+\frac{\mu_s}{2})(\alpha+\bar{\theta})+c(1+a_2(\beta-\gamma))$. Then, by combining (1.59), we find the a_1 as:

$$a_1^s=\frac{(2+\mu_s)(\alpha+\bar{\theta})+c(\beta+2\gamma+\sqrt{\beta^2+8\gamma^2})}{3\beta+\sqrt{\beta^2+8\gamma^2}} \tag{1.60}$$

which completes the proof. Note that a_1^s denotes one of the coefficients for the status-seeker buyer on the hand a_1^i denotes one of the coefficients for the inequity-averse buyer. Thus, in the equilibrium, the optimal price schedule becomes $P(q)=a_1^s q+\dfrac{a_2}{2}q^2$ where a_2 and a_1 are defined in (1.59) and (1.60) respectively.

Now, we also show the optimal symmetric quality level for the inequity-averse buyer. The payoff function for the inequity-averse buyer of type θ:

$$U(q_i,q_j,\theta)=u(q_i,q_j,\theta)-\sum_{t\in\{i,j\}}\lambda^t\theta\left[q_t(\theta)-q_t^{avg}(\theta)\right]-P_i(q_i)-P_j(q_j)$$

with $\lambda\in(0,1)$. Similarly, the indirect utility requires:

$$\arg\max_{q_j} v(q_i,q_j,\theta)-P_j(q_j) \tag{1.61}$$

where $v(q_i,q_j,\theta)=u(q_i,q_j,\theta)-\sum_{t\in\{i,j\}}\lambda^t\theta\left[q_t(\theta)-q_t^{avg}(\theta)\right]$. Then, it is straightforward that

$$q_j^*(q_i)=\frac{(\alpha+\theta)(\beta+\gamma)(1-\frac{\lambda}{2})-\gamma q_i-a_1(\beta^2-\gamma^2)}{\beta+a_2(\beta^2-\gamma^2)}$$

which basically gives

$$\frac{\partial q_j^*}{\partial q_i} = -\frac{\gamma}{\beta + a_2(\beta^2 - \gamma^2)}$$

.

By following (1.28), we have:

$$u_q\big(1 + \frac{\partial q_j^*}{\partial q_i}\big) - \frac{\partial P}{\partial q}\frac{\partial q_j^*}{\partial q_i} - c = (\bar{\theta} - \theta)u_{\theta q}\big(1 + \frac{\partial q_j^*}{\partial q_i}\big) \tag{1.62}$$

where $u_q = \frac{\alpha + \theta}{\beta - \gamma}\theta(1 - \frac{\lambda}{2}) - \frac{q}{\beta - \gamma}$ and $u_{\theta q} = \frac{1}{\beta - \gamma}1 - \frac{\lambda}{2}$. Note that here we focus on only symmetric equilibrium. That is to say, we have $q_t(\theta) = q(\theta)$ and $\lambda^i = \lambda^j = \lambda$ for all $t \in \{i, j\}$. Then by using (1.62), we find the symmetric optimal quality level as:

$$q_i^*(\theta) = \frac{a_1\gamma + (\alpha + 2\theta - \bar{\theta})(1 - \frac{\lambda}{2})(1 + a_2(\beta + \gamma)) - c(\beta + a_2(\beta^2 - \gamma^2))}{1 + a_2\beta} \tag{1.63}$$

Now we find a_1^i and a_2. First, $\frac{d}{d\theta}\{u_q - P_q(q(\theta))\} = 0$. This gives $\frac{dq(\theta)}{d\theta} = \frac{2 - \lambda}{2(1 + a_2(\beta - \gamma)}$. Similarly by using (1.63) we have also $\frac{dq}{d\theta} = \frac{2(1 - \frac{\lambda}{2})(1 + a_2(\beta + \gamma))}{1 + a_2\beta}$. Thus, we have $\frac{(1 + a_2(\beta + \gamma))(2 - \lambda)}{(1 + a_2\beta)} = \frac{2 - \lambda}{2(1 + a_2(\beta - \gamma)}$. Then, it is clear that we find :

$$a_2 = -\frac{2}{3\beta + \sqrt{\beta^2 + 8\gamma^2}} \tag{1.64}$$

Note that a_2 is independent of both social preference parameters μ_s and λ and the same for each type of buyer. For a_1^i we follow the same steps as we did in status-seeker buyer.

$u_q(\bar{\theta}) - P_q(q(\bar{\theta})) = 0$. After some algebra one can reach $a_1^{inequtiy} = -\frac{2 - \lambda}{2}(\alpha + \bar{\theta})a_2 + (1 + a_2(\beta - \gamma))c$. Then, by combining (1.64):

$$a_1^i = \frac{(2 - \lambda)(\alpha + \bar{\theta}) + c(\beta + 2\gamma + \sqrt{\beta^2 + 8\gamma^2})}{3\beta + \sqrt{\beta^2 + 8\gamma^2}} \tag{1.65}$$

Note that in the equilibrium optimal price schedule becomes $P(q) = a_1^i q + \frac{a_2}{2} q^2$ where a_2 and a_1^i are defined in (1.64) and (1.65) respectively.

2 Sabotage in Rent-Seeking Contests with Endogenous Prize

Abstract

This study aims to find the optimal productive rent-seeking efforts if sabotaging activities are feasible, and the prize is endogenous. Previous studies show that if the prize (rent) is fixed, and the contestants sabotage the productive rent-seeking efforts, then under some conditions, the productive rent-seeking efforts do not depend on the level of sabotage in the equilibrium. However, in this study, productive rent-seeking efforts do not only affect the winning probability but also generate additional surplus over the fixed prize. Contrary to previous studies, we mainly conclude that due to the existence of endogeneity, sabotaging productive rent-seeking efforts causes sabotaging of the endogenous part of the rent. Therefore, sabotage activities affect the level of productive rent-seeking efforts in the equilibrium. Hence, if the rent is endogenous and monitoring is feasible, the contest organizer may want to monitor the sabotage activities. In the second part, we introduce the Fehr and Schmidt (1999) type of social preferences into our model. We characterize both symmetric productive rent-seeking and sabotage efforts. Finally, we compare the productive rent-seeking and sabotage efforts of other-regarding and self-regarding contestants.

2.1 Introduction

In economics, there are many events and social interactions that can be explained by contests. Contests describe a phenomenon in which agents expend irreversible resources, such as money, effort, etc., to increase the probability of winning a given prize. Rent-seeking games, job promotion tournaments, patent races, R&D activities, and political campaigns can be examples of contests.

It is obvious that to increase the winning probability in a contest, apart from any attempt which increases one's productive rent-seeking efforts, one can also exert sabotage efforts on his rivals to decrease their winning probabilities. Thus, one can define sabotage as any activity that decreases the opponents' winning probabilities. Any illegal actions or behavior in sports competition (Amegashie (2012)), negative campaigns in political elections (Skaperdas and Grofman (1995)), or selection tournaments (Münster (2007)) are some of the areas that sabotaging activities are observed. As Minchuk et al. (2018) mention, sabotage in contests can be grouped into 3 different categories. First, one can try to decrease or reduce his opponents' productive effort as in Chen (2003), Konrad (2000), and Münster (2007). Second, one can reduce his rivals' valuation of the prize as in Amegashie and Runkel (2007). Lastly, one can increase his opponents' marginal cost of exerting productive effort as in Amegashie (2012) and Minchuk et al. (2018).

In the literature, as far as we know, there is no work that studies sabotage in rent-seeking contests when the rent is endogenous. We believe that it would be an interesting and important question to analyze for the following reasons. First, Amegashie (2012), Minchuk et al. (2018), and Minchuk (2020) define productive rent-seeking efforts as the conventional rent-seeking efforts that only determine the probability of winning a fixed prize. However, in this study, productive rent-seeking efforts not only determine the probability of winning, but productive rent-seeking efforts also increase the size of the prize. Second, as Chung (1996) and Amegashie (1999) show, there are many real-life examples where the rent is endogenously determined. For example, as

Chung (1996) points out that R&D investments can be considered productive if the knowledge created by all agents, has spill-over effects on the monopoly right. Also, as Amegashie (1999) shows if the firms compete for a monopoly right (prize), the winning firm gets the prize for a certain duration of time, and the winning firm can extend the duration of the right by exerting extra efforts.

In our work, we define the productive rent-seeking effort by following Chung (1996) and Amegashie (1999). We introduce a new endogenous prize form which is similar to Chung (1996) in which the prize depends on the aggregate rent-seeking efforts. Different from Chung (1996), we also introduce a minimum amount of fixed rent as in Amegashie (1999). That is to say, our model offers a new endogenous prize structure in which the prize depends on both the aggregate rent-seeking efforts of all contestants and a minimum amount of a fixed rent. That is why the "productive rent-seeking" effort produces or increases the rent (or prize) in our model. As the aim of sabotage is to reduce the productive rent-seeking efforts, we see that contestants also sabotage the endogenous part of the prize. In that sense, our work completely differs from the previous literature. We show that different from Minchuk (2020), productive rent-seeking efforts depend on the level of sabotage efforts in the equilibrium. Thus, if the rent is endogenous and monitoring is feasible, then the contest designer might consider and monitor the sabotaging efforts. This is one of the main different and important outcomes.

Apart from that, we believe that it is also important to include behavioral aspects in our model because sabotaging others is itself a behavioral phenomenon. Mui (1995) introduces the importance of being envious in economics and shows how being envious plays a role in sabotaging others. In this regard, we introduce the Fehr and Schmidt (1999) type of social preferences in our model in the second part as an extension. Because sabotaging activities can stem not only from selfish contestants but also inequity-averse or status-seeker contestants. That is why we show the optimal symmetric productive rent-seeking and sabotage efforts when the prize is endogenous and contestants have social preferences in the equilibrium. We mainly find that if the sabotage is feasible, then the sabotage efforts with the social preferences become higher than the model with the self-interested one in the equilibrium. Furthermore, if the contestants are inequity-averse, to have an interior solution for the productive rent-seeking and sabotaging efforts, the absolute value of the differences in social preference parameters should be low enough.

The organization of the paper is as follows. The next section briefly introduces related literature. Section 2.3 explains and defines the model in detail. Section 2.4 adds social preferences to the model. Section 2.5 concludes.

2.2 Related Literature

In this section, we briefly introduce related literature for both sabotage in contests and rent-seeking literature with an endogenous rent. For the sabotage in contests literature, Chowdhury and Gürtler (2015) offer a very informative, extensive, and informative survey. In light of their study, we briefly mention some of the important papers related to sabotage in contests. First, we begin with sabotage in tournaments literature then we continue with sabotage in rent-seeking literature.

Lazear (1989) studies a labor tournament in which sabotage is possible. He shows that in a contest, wage compression can reduce workers' sabotaging activities and decreases uncooperative activities. Hence this result may increase the output. Moreover, he notes that wage compression may also reduce workers' productive effort which is why the increased output is not guaranteed. Chen (2003) introduces sabotaging activities in promoting tournaments and shows that sabotaging opponents reduces the efficiency of an organization. According to Chen (2003), the ablest ones are sabotaged more and they may have not the best chances of getting promoted. Münster (2007) also studies sabotage and selection tournaments and he finds that contestants who have higher productive effort are exposed to the highest sabotage in the equilibrium. Unlike Chen (2003) and Münster (2007), Gürtler (2008) offers sabotage in team tournaments model in which each contestant's productive effort increases the team's performance. Moreover, Gürtler (2008) shows that different from Chen (2003) and Münster (2007), due to the decreasing returns and complementary effects, each team exerts their sabotage efforts towards the opponent teams' member who has the least productive effort. Doğan et al. (2019) also proposes a model with sabotage in team contests in which each team member determines their productive effort determining the team's aggregate productive effort. Moreover, each team member determines their sabotage efforts against a particular member from the opposing team. The main results are the discouragement effect of sabotage disappears for some contestants and the free-riding problem in team contests in no longer exists with the additional option to sabotage.

Konrad (2000) builds a new model in which sabotage is introduced in rent-seeking contests. He defines sabotage as such that it decreases the effectiveness of the productive (or rent-seeking) efforts for others. He shows the conditions for exerting positive sabotaging effort in the equilibrium and finds that rent-seeking (productive) effort may increase, decrease, or unchanged due to the sabotaging activities under some conditions. Different from Konrad (2000), Amegashie (2012) forms a two-stage contest game where contestants choose their sabotage (destructive) efforts in the first stage, and they exert productive (rent-seeking) effort in the second stage. In his model unlike Konrad (2000), sabotage increases the marginal cost of exerting productive effort. He shows that after a certain threshold level, there exists positive sabotaging effort in the equilibrium and after that threshold level, the productive effort remains constant. Minchuk et al. (2018) extends the model of Amegashie (2012) and they introduce a three-stage contest game aiming to minimize sabotaging activities by monitoring the contestants. Then, Minchuk (2020) proposes a new rent-seeking contest model in which two different forms of sabotage are introduced. He shows that only the sabotaging efforts which increase the marginal cost of exerting productive effort affect the productive (rent-seeking) effort in the equilibrium. According to his model, if sabotaging activities negatively affect the winning probability, they do not have any effect on the productive (rent-seeking) effort in the equilibrium.

In the rent-seeking literature, the conventional Tullock (1980) model assumes that the prize or the rent is fixed. Yet, it is also important to analyze the above problem when the prize has an endogenous component. Chung (1996) shows the existence of the equilibrium when the efforts are productive i.e., rent is only the function of the aggregate efforts. Apart from that, Chung (1996) assumes that the prize consists of only one part which is the function of the aggregate sum of effort. In that sense, Amegashie (1999) proposes a new endogenous prize model in which the prize depends on both the minimum rent and the contestant's effort and shows that there is a negative relationship between optimal symmetric equilibrium and the number of contestants which is completely different than the conventional model. Lastly, different from all the above models related to endogenous rent, Alexeev and Leitzel (1996) come up with a new model called "Rent Shrinking" in which the rent is decreasing function of the aggregate sum of rent-seeking efforts of the other contestants. That is to say, rent diminishes by the losing contestants' rent-seeking effort.

2.3 The Model

We consider a simultaneous rent-seeking contest game with sabotaging activities similar to Konrad (2000). However, our model differs from his study in the sense that we introduce an endogenous prize(rent) structure. We assume that contestants are risk-neutral and they have complete information. Each contestant i simultaneously chooses his "sabotage"($s_i \geq 0$) and productive rent-seeking efforts ($x_i \geq 0$). Similar to Konrad (2000), contestants sabotage their rivals to decrease the effectiveness of the productive rent-seeking efforts. The expected utility of the contestant i where the rent is endogenously determined:

$$EU_i = \frac{\overline{\frac{x_i}{1+\sum_{j\neq i} s_{ji}}}}{\sum_i \frac{x_i}{1+\sum_{j\neq i} s_{ji}}}(v_{min} + \kappa \sum_i \frac{x_i}{1+\sum_{j\neq i} s_{ji}}) - c_1 x_i - c_2 \sum_{j\neq i} s_{ij} \qquad (2.1)$$

Here x denotes productive rent-seeking effort or lobbying expenditures. v_{min} is the fixed part of the prize and $\kappa \in (0, \min\{c_1, c_2\})$[1] denotes the marginal benefit of exerting productive rent-seeking efforts. At the same time, we assume that $c_i \geq 1$ denotes the marginal cost of exerting productive rent-seeking and sabotage efforts.[2] In our model, the main different part comes with the variable part of the prize (rent). The prize or the rent is the function of both the fixed part of the rent and the aggregate sum of productive rent-seeking efforts. That is, productive rent-seeking efforts add a surplus over the fixed prize v_{min}. The prize is positively affected by productive rent-seeking efforts x but at the same time, the endogenous part of the prize is negatively affected by the total sabotage exerted by the other $n-1$ contestants. This occurs because one part of the prize is determined endogenously with the productive rent-seeking effort. Since the sabotage is applied to decrease the effectiveness of productive rent-seeking efforts, contestants also sabotage the endogenous part of the prize. Note that s_{ji} denotes the amount of sabotage level exerted by contestant j towards contestant i. Moreover, the last part $\sum_{j \neq i} s_{ij}$ shows the exerted total sum of sabotage from contestant i to other $(n-1)$ contestants.

Before introducing sabotage in our model in detail, we need to show the existence and uniqueness of the rent-seeking model with an endogenous prize when it depends on both the fixed part and the aggregate sum of efforts. Note that this part is similar to Chung (1996), but it is different in the sense that it also has a fixed part. Thus, it is not straightforward to show that optimal rent-seeking effort exists in the equilibrium.

2.3.0.1 Existence and Uniqueness

In this part, we come up with a new endogenous prize model in which the prize consists of both fixed and variable parts. The variable part is similar to Chung (1996) in the sense that variable rent is a function of aggregate efforts. Moreover,

[1] This assumption ensures the existence of productive rent-seeking effort.

[2] We assume that the marginal cost of exerting both types of efforts is greater than one because otherwise, contestants can exert a very high amount of productive rent-seeking efforts due to the endogeneity.

our model is similar to Amegashie (1999) if the variable part consists of only the contestant's effort but not that of others.

In our model, there are n contestants who are risk-neutral and they exert irreversible effort, spending, etc. to increase the probability of winning the prize. We follow the basic linear contest success function i.e., $f(x_i) = x_i$ where f is the contest success or production function. In the model, we have the "winner takes the all" assumption and we focus on the symmetric equilibrium.

Then, the expected utility of contestant i:

$$EU_i = \frac{x_i}{\sum x_i}\{v_{min} + \kappa v(\sum x_i)\} - c_i x_i = \frac{x_i}{\sum x_i} V(v_{min}, v(\sum x_i)) - c_i x_i \qquad (2.2)$$

Note that κ denotes the marginal return for the exerted efforts and $V = v_{min} + \kappa v(\sum x_i)$

Assumption: *Endogenous part of the prize (rent) is strictly increasing and concave in x.*

Proposition 1: *In a rent-seeking contest with an endogenous prize, there exists a unique symmetric equilibrium if and only if $\varepsilon_V(x) < 1$.*

All proofs of propositions are relegated to the appendix. Proposition 1 explains that if the elasticity of the prize to effort level is less than unity, then we are sure that the symmetric equilibrium exists and it is unique.

Before going further, we first present the optimal productive rent-seeking effort when there is no sabotage in the model. Note that there is no problem with the existence because of the previous section that we proved.

Proposition 2: *In a rent-seeking contest without sabotage, the optimal (symmetric) productive rent-seeking effort becomes: $x^* = \frac{(n-1)}{n^2(c_i - \kappa)} v_{min}$ and the optimal productive rent-seeking effort is higher than the model where the prize is fixed or exogenous.*

Proposition 2 indicates that if the rent-seeking efforts are productive, we conclude that contestants exert higher efforts relative to the conventional exogenous model.

It is intuitive that because the size of the rent relatively increases compared to the exogenous model, we expect to have higher rent-seeking efforts in the equilibrium.

Now if we introduce sabotage in our model, we have:

$$EU_i = \frac{\frac{x_i}{1+\sum_{j\neq i} s_{ji}}}{\sum_i \frac{x_i}{1+\sum_{j\neq i} s_{ji}}}(v_{min} + \kappa \sum_i \frac{x_i}{1+\sum_{j\neq i} s_{ji}}) - c_1 x_i - c_2 \sum_{j\neq i} s_{ij}$$

It is clear that different from agent i, each other $n-1$ contestants exerts a sabotaging effort $s_{ji} \geq 0$ towards contestant i, and this decreases or destructs the prize. Moreover, $\sum_{j\neq i} s_{ij} \geq 0$ denotes the total sabotaging effort of contestant i against other $(n-1)$ agents. In the next proposition, we show the condition for having a positive sabotage level and we show that productive rent-seeking effort depends on the positive sabotage level.

Proposition 3: *In a rent-seeking contest with sabotage, there exists a unique (symmetric) sabotage effort*

$$s^* = \frac{v_{min} - c_2 n^2}{c_2 n^2 (n-1)}$$

and (symmetric) productive rent-seeking effort

$$x^* = \frac{(n-1)(1+(n-1)s)v_{min}}{n^2(c_1(1+(n-1)s)-\kappa)} = \frac{(n-1)v_{min}^2}{n^2\big(c_1 v_{min} - \kappa c_2 n^2\big)}$$

for all i, if $v_{min} \geq c_2 n^2$. Otherwise, the optimal sabotage effort becomes zero in the equilibrium.

It is straightforward that in the equilibrium it is not profitable for the contestant i to exert any positive sabotage on others if $v_{min} \leq c_2 n^2$. Therefore, if the minimum fixed part of the prize is sufficiently low, then in the equilibrium there exists no sabotage effort. However, if the fixed prize is sufficiently large i.e., satisfying the above condition, then in the equilibrium, there exists a positive sabotage effort. Thus, if there exists any contest organizer for the contest, it may be important to monitor the contestants. Hence, the contest organizer might also exert monitoring efforts to decrease any possible sabotage as in Minchuk et al. (2018). Note that our result related to the sabotage part is similar to Konrad (2000). The main difference between our model and Konrad (2000) is that Konrad (2000) assumes that the rent is exogenously given and fixed. He shows the conditions to exert the positive sabotage level in the equilibrium. However, in his model, we see that optimal rent-seeking effort does not depend on the sabotage level if we impose our model's structure on Konrad (2000). In our model, we show the necessary condition for contestants to exert positive effort in the equilibrium. This condition and the optimal positive sabotage level results are the same as Konrad (2000) if his model assumes that the contest success function is linear and sabotage is introduced as we did in our work. Note also that if $v_{min} \geq c_2 n^2$ is satisfied, then we would directly have $c_1 v_{min} - \kappa c_2 n^2$ as $\kappa \in (0, \min\{c_1, c_2\})$.

As a result, if the prize is fixed and if we use classical linear Tullock (1980) type of contest success function, then the optimal productive rent-seeking effort does not depend on sabotage.[3] However, different from the previous literature, when we introduce the endogenous prize, which is simply the function of aggregate efforts of all contestants, then the existence of the positive sabotage also affects the optimal productive rent-seeking efforts in the equilibrium

Now we have the following proposition which is closely related to Proposition 3.

Proposition 4: *If the fixed prize (rent) is sufficiently high i.e,* $v_{min} \geq c_2 n^2$*, we have the following:*

1. *As the number of contestants increases, optimal sabotage effort decreases in the equilibrium.*

2. *The optimal rent-seeking (productive) effort is higher than the rent-seeking model with a fixed rent in the equilibrium.*

3. *For* $n \geq 3$*, as the number of contestants increases, the optimal rent-seeking effort decreases if* $\kappa \in (0, \kappa^*)$ *and the optimal rent-seeking effort increases if*

[3]See Konrad (2000) and Minchuk (2020) for the details.

$\kappa \in [\kappa^*, \min\{c_1, c_2\})$.[4]

The first statement of the proposition is closely similar to Konrad (2000) in which he states that sabotage exists when the number of contestants is lower. In our model, it still holds that sabotage is a small number phenomenon as in Konrad (2000). The second statement explains that with the introduction of endogenous rent, contestants exert higher efforts than in the model with a fixed rent. This occurs because the size of the cake increases and this causes it to exert higher productive rent-seeking effort relative to the model with exogenous rent. The last statement of proposition 4 is interesting and quite similar to Amegashie (1999). Amegashie (1999) finds a similar result in his paper. Note that we know from the conventional rent-seeking theory that if the number of contestants decreases, then the productive rent-seeking effort increases. This is because their probability of winning the prize increases. But in our model, due to the marginal benefit of exerting productive rent-seeking efforts, we have an interesting result similar to Amegashie (1999)[5]. What we observe is that up to a certain threshold κ^*, the mechanism works as anticipated. That is, the effects of the total number of contestants prevail over the increasing part of the prize (endogenous part). On the other hand, after κ^*, we observe that the effects of increasing part of the prize (endogenous part) prevail over the total number of contestants. It occurs because, after κ^*, the marginal benefit of the exerting productive rent-seeking effort dominates the effects of the increased number of contestants. Now, we introduce a very simple example with different scenarios to see how the mechanism works.

Example 1: Suppose 2 contestants compete for a prize (rent) in which the fixed part of the rent is 16. Assume that the marginal cost of exerting productive rent-seeking effort is 1 and the marginal cost of exerting sabotage effort is 2. Moreover, let $\kappa = \dfrac{1}{2}$.

Scenario 1: Exogenous Rent without Sabotage:

[4]Note that $\kappa^* = \dfrac{c_1(n-2)(\varepsilon + c_2 n^2)}{c_2(3n-4)n^2}$ where $0 \leq \varepsilon \leq \dfrac{c_2 n^2(3n-4-c_1(n-2))}{c_1(n-2)}$ and details are in the appendix.

[5]See Amegashie (1999) for the details.

First, suppose that the prize (rent) is fixed and there are no sabotaging activities in the competition. Then, the model converges to classical Tullock (1980) type in which $x^* = \frac{n-1}{cn^2}R$ where c is marginal cost and R is the amount of rent. In this scenario, the optimal productive rent-seeking effort becomes $x^* = 4$ and the expected utility of any contestant becomes 4.

Scenario 2: Exogenous Rent with Sabotage :

Different from scenario 1, if we impose the sabotage into our model, then it converges to Konrad (2000). According to Konrad (2000), in this scenario, optimal productive rent-seeking effort does not depend on the sabotage level. It is easy to show that, under scenario 2, the optimal productive rent-seeking effort $x^* = 4$ is still the same as in scenario 1. Also, the optimal sabotage level becomes $s^* = 1$ and obviously, the expected utility of gaining this competition decreases to 2

Scenario 3: Endogenous Rent without Sabotage: By following Proposition 1, it is easy to obtain the optimal productive rent-seeking effort as $x^* = 8$ which is precisely the double amount of the previous scenarios. Moreover, in this case, the expected utility of any contestant becomes 4. Note that the expected utility value is the same in scenario 1 but this is not surprising because our model assumes that the endogenous part is the linear sum of productive rent-seeking efforts. If we changed it into a strictly concave function, the result would change as well. However, as designing such a model brings also extra complexity, we construct our model as simple as possible.

Scenario 4: Endogenous Rent with Sabotage: In this scenario, as we show in Proposition (3), optimal productive rent-seeking effort depends on sabotage now. In this case, $x^* = 5.33$ is lower than scenario 3 but the higher than first two scenarios. Also, note that the optimal sabotage level remains the same as in scenario 2. This occurs because we include the endogenous part as a linear sum of rent-seeking effort. Instead of using a linear function, If we used any strictly concave function which is the sum of rent-seeking efforts, we also would have the different sabotage levels in equilibrium with scenario 2. Lastly, the expected utility of any contestant becomes

2.

In general, what we observe is the following. First, due to the endogenous prize, contestants exert higher productive rent-seeking efforts than the models with exogenous rent in the equilibrium. Second, the existence of sabotage decreases the optimal rent-seeking efforts when the rent is endogenous and it decreases the expected utility in all scenarios. Lastly, the expected utility values of scenarios 1 and 3 and scenarios 2 and 4 are completely the same. This occurs because we define endogeneity basically as a linear sum of efforts. Otherwise, we would have different values.

2.4 Extension: Social Preferences and Rent-Seeking

In this part, we introduce the Fehr and Schmidt (1999) type of social preferences into our model and show how productive rent-seeking and sabotage efforts depend on the social preference parameters in the equilibrium.

We assume that contestants have social preferences i.e., they are not self-interested. This assumption lies in the fact that contestants might also exert sabotage efforts because of envy, spitefulness, etc. (Mui (1995)). That is why we think that it is worth analyzing the sabotage in rent-seeking when the contestants have social preferences. Contestants have Fehr and Schmidt (1999) type of other-regarding preferences and they do not only consider their expected payoff but also the others'.

Fehr and Schmidt (1999) define other-regarding preferences as:

$$U_i(\pi_i, \pi_j) = \pi_i - \alpha_i \max\{\pi_j - \pi_i, 0\} - \beta_i \max\{\pi_i - \pi_j, 0\} \tag{2.3}$$

Note that, unlike classical self-interested agent models, agent i's utility depends on both his payoff π_i and his opponent's payoff and payoffs can be both monetary or non-monetary. Parameters α and β are social preference parameters. For inequity aversion, it is assumed to have $\beta \in (0, 1)$ and $\beta \leq \alpha$. If β is assumed to be negative, we can think of the agent i as competitive.

If contestant i were selfish, his expected utility of winning the prize (rent) would be:

$$EU_i = \frac{\frac{x_i}{1+\sum_{j\neq i} s_{ji}}}{\sum_j \frac{x_j}{1+\sum_{i\neq j} s_{ij}}}(v_{min} + \kappa \sum_i \frac{x_i}{1+\sum_{j\neq i} s_{ji}}) - c_1 x_i - c_2 \sum_{j\neq i} s_{ij}$$

Since in a rent-seeking contest environment, we assume that winner takes the all, we have:

$$\begin{cases} \pi_i - \pi_{j\neq i} = \Pi > 0 \quad \text{with probability } \dfrac{\frac{x_i}{1+\sum_{j\neq i} s_{ji}}}{\sum_j \frac{x_j}{1+\sum_{i\neq j} s_{ij}}} = p_1 \\ \\ \pi_{j\neq i} - \pi_i = \Pi > 0 \quad \text{with probability } 1 - p_1 \end{cases} \tag{2.4}$$

Note that $\Pi = v_{min} + \kappa \sum_i \frac{x_i}{1+\sum_{j\neq i} s_{ji}}$. We introduce Fehr and Schmidt (1999) type of social preferences into the rent-seeking set-up by combining (2.3) and (2.4) and we redefine the expected utility of contestant i as:

$$EU_i = (1-\beta)\left(\frac{\frac{x_i}{1+\sum_{j\neq i} s_{ji}}}{\sum_j \frac{x_j}{1+\sum_{i\neq j} s_{ij}}}\right)\Pi - \alpha\left(\frac{\sum_{j\neq i}\frac{x_j}{1+\sum_{j\neq i} s_{ij}}}{\sum_j \frac{x_j}{1+\sum_{i\neq j} s_{ij}}}\right)\Pi - c_1 x_i - c_2 \sum_{j\neq i} s_{ij} \implies$$

$$\boxed{EU_i = \frac{\Pi}{\sum_j \frac{x_j}{1+\sum_{i\neq j} s_{ij}}}\left((1-\beta)\frac{x_i}{1+\sum_{j\neq i} s_{ji}} - \alpha \sum_{j\neq i}\frac{x_j}{1+\sum_{i\neq j} s_{ij}}\right) - c_1 x_i - c_2 \sum_{j\neq i} s_{ij}} \tag{2.5}$$

Here note that Π is not given exogenously, it is endogenous and $\Pi = v_{min} + \kappa \sum_i \frac{x_i}{1+\sum_{j\neq i} s_{ji}}$.

One of the most important implications of (2.5) is the following. Suppose contestants are inequity averse. Note that if $\alpha \geq 1 - \beta$, it is obvious that the expected utility becomes negative in the symmetric equilibrium and thus in the symmetric equilibrium, the rent-seeking effort becomes zero. Similarly, if β rises, the probability of exerting positive rent-seeking effort decreases. Thus, as contestants become more inequity averse, their expected utilities tend to decrease. In general, it is clear to see that from (2.5), one of the necessary conditions to have a positive rent-seeking effort in the symmetric equilibrium is $1 - \beta - \alpha > 0$.

If contestants are status-seekers or competitive, then it simply becomes that $\beta < 0$. In this case, under the symmetric equilibrium, to have a positive expected utility $1 - \beta - \alpha > 0$ must be satisfied again[6] but in this case, as the value of $1 - \beta$ is relatively higher than the inequity aversion case, then there is no strict restriction to say that the inequity aversion parameter α should be less than 1.

We first begin our analysis by assuming that there is no sabotage in the environment and that the prize is exogenously given and fixed.

Proposition: 5 (without sabotage) *In the rent-seeking contests with the endogenous prize, we have a unique symmetric productive rent-seeking effort:*

$$x^* = \frac{(n-1)(1-\beta+\alpha)v_{min}}{n^2(c_1 - \kappa(1-\beta))} \tag{2.6}$$

Note that if the prize is given exogenously i.e if it is fixed, then the optimal productive rent-seeking effort becomes $x_f^* = \dfrac{(n-1)(1-\beta+\alpha)}{c_1 n^2} v$ where v is a fixed rent. Note that if contestants are inequity-averse then $\beta \leq \alpha$. Hence, it is obvious that due to the existence of social preferences, contestants exert higher effort than self-regarding contestants. Similarly, if $\beta < 0$ i.e., the contestants are status-seekers, then it is straightforward that the optimal productive rent-seeking effort would be again higher than the selfish contestants.

Corollary 1 (without sabotage): *The productive rent-seeking effort in the*

[6] This condition is a necessary condition, not a sufficient condition.

endogenous model with social preferences is greater than the model without social preferences if $(1 - \dfrac{\beta}{\alpha}) \geq \dfrac{\kappa}{c_1}$

This corollary implies that if the rent is endogenous and if the contestants become more inequity averse, then it is highly probable that they exert higher efforts in the equilibrium relative to self-interested contestants. We know from the previous part that the optimal symmetric rent-seeking effort with an endogenous prize without social preferences is $x_1^* = \dfrac{(n-1)}{n^2(c_1-\kappa)}v_{min}$ where $\kappa \in (0,1)$. If social preferences are introduced to the model then the equilibrium becomes $x_2^* = \dfrac{(n-1)(1-\beta+\alpha)v_{min}}{n^2(c_1-\kappa(1-\beta))}$. Then it is easy to show that

$$x_2^* \geq x_1^* \implies \frac{(1-\beta+\alpha)}{c_1-\kappa(1-\beta)} \geq \frac{1}{c_1-\kappa}$$

which implies that

$$c_1 - \kappa - c_1\beta + \kappa\beta + c_1\alpha - \kappa\alpha \geq c_1 - \kappa + \kappa\beta \implies \boxed{1 - \frac{\beta}{\alpha} \geq \frac{\kappa}{c_1}}$$

Now, we introduce sabotage and see how productive rent-seeking effort changes due to the sabotage. In this part, due to the existence of social preferences, it becomes difficult to show explicitly the optimal productive rent-seeking effort and sabotage level in the equilibrium. Yet, we show that equilibrium exists under some conditions.

First, if we take the derivative of (2.5) with respect to x_i and s_{ij} : and focus on the symmetric equilibrium i.e $x_i = x^*$ and $s_{ij} = s_{ji} = s$ for all i, j : we find that

$$\boxed{x^* = \frac{(1-\beta+\alpha)(1+(n-1)s)(n-1)v_{min}}{n^2\big(c_1(1+(n-1)s) - \kappa c_2(1-\beta)\big)}} \tag{2.7}$$

and

$$\boxed{s = \frac{(1-\beta+\alpha)v_{min} - n^2(c_2 - \kappa\alpha x^*)}{c_2 n^2(n-1)}} \tag{2.8}$$

Note that if contestants are inequity-averse, v_{min} is sufficiently high and the difference between social preferences is sufficiently low i.e, $\alpha - \beta \leq 1$, then x^* and s exist and the solution pair becomes unique. This occurs because of the fact that given v_{min}, social preference parameters, α, β, and κ, it is clear that $s = f(x^*)$ and s is clearly linear function in x^*. Similarly, $x^* = g(s)$ is the function of s, and as it can be seen from (2.7), x^* is a non-linear, but smooth function in s. Thus, the intersection of the $s = f(x^*)$ and $x^* = g(s)$ nonempty and unique. Note that if contestants are inequity-averse, it is important that the fixed rent v_{min} must be high enough, and social preferences must satisfy $\alpha - \beta \leq 1$ to guarantee that the expected utility of the contestants is positive. However, if the contestants are status-seekers, the only restriction that we should have is the fixed rent must be high enough.

Note that in the equilibrium, it is easy to verify that the optimal sabotage level becomes higher than the model without social preferences and thus optimal rent-seeking effort becomes lower than the model without social preferences. Thus, we have the following result:

Corollary 2: *If both rent-seeking and sabotage levels are feasible, then the contestants with Fehr and Schmidt (1999) type of social preferences have higher sabotage levels, than the model without social preferences in the symmetric equilibrium.*

Note that as we show in Proposition 3, the fixed prize (rent) must be sufficiently high to have positive sabotage in the equilibrium. Here if we assume that the fixed prize must be sufficiently high, positive sabotage exists in the equilibrium. Also, note that as we show below, the existence of social preferences increases the optimal sabotage level which makes the contestants exert less and less productive rent-seeking effort in the equilibrium. Thus, if the difference between the α and β increases, then the contestants' expected utility approaches zero.

If there is no social preference in the model, we know that from Proposition 3, $x = \dfrac{(n-1)(1+(n-1)s)v_{min}}{n^2(c_1(1+(n-1)s)-k)}$ and $s = \dfrac{v_{min} - c_2 n^2}{c_2 n^2 (n-1)}$. However, it is clear that if we introduce the social preferences, from (9), it is obvious that

$$\frac{(1-\beta+\alpha)v_{min} - n^2(c_2 - \kappa\alpha x^*)}{c_2 n^2(n-1)} \geq \frac{v_{min} - c_2 n^2}{c_2 n^2(n-1)}$$

Hence, in the equilibrium sabotage with social preferences becomes higher than the model without social preferences.

Example 2: Suppose there are two contestants and let $v_{min} = 16$, $\kappa = 0.5$ and $c_1 = c_2 = 2$. We first show the optimal productive rent-seeking and sabotage effort levels when the agents are self-interested, then we assume that they are inequity-averse. Note that we only focus on the symmetric equilibrium.

It is easy to see that if contestants are self-interested, then $\alpha = \beta = 0$. Then, by following Proposition 3, it is easy to find that optimal symmetric rent-seeking effort becomes $x^* = 2.28$ and optimal sabotage effort becomes $s^* = 1$. This yields that contestants' expected utilities become $2.01 > 0$.

Now suppose contestants are not selfish anymore and let $\beta = 0.1$ and $\alpha = 0.2$. From (2.7) and (2.8), we can write the best-response functions as the following:

$$x(s) = \frac{17.6(1+s)}{8(1+s) - 3.6} \text{ and } s(x) = \frac{9.6 + 0.4x}{8}$$

The intersection of the best-response functions gives that $x^* = 2.725$ and $s^* = 1.336$. Moreover, in the symmetric equilibrium, the expected utility of each contestant becomes $0.461 > 0$ According to this example, what we observe is optimal productive rent-seeking effort is now higher when the contestants have social preferences. Similarly optimal sabotage level is relatively higher than the self-interested model. Lastly, in the second case, because both productive rent-seeking and sabotage efforts are relatively higher than the self-interested model, we observe that the expected utility of the contestants is lower than the self-interested contestants.

2.5 Conclusion

In this study, we examine sabotage in rent-seeking contests when the rent is endogenously determined. As far as we know, previous studies consider the classical rent-seeking effort as a productive rent-seeking effort and contestants compete for a given fixed rent in general. However, as in Chung (1996), we can define productive efforts such that they increase the size of the rent. That is why in our model, different from previous literature, we introduce endogenous rent formation and this mainly brings the main novelty. In the rent-seeking competition, since the rent is endogenously determined in our model, productive rent-seeking effort not only determines the winning probability but also increases the size of the prize.

This study consists of two different but related parts. In the first part, we show and find the symmetric equilibrium levels of sabotage and productive rent-seeking efforts. What we mainly observe is that if the prize is a function of both a fixed part of the rent and the aggregate sum of productive rent-seeking efforts, and the sabotaging activities are feasible, then productive rent-seeking efforts depend on the level of sabotage. Thus, different from Minchuk (2020), if monitoring is feasible by a contest organizer (designer), sabotaging efforts should be taken into consideration by the designer.

In the second part, we introduce the Fehr and Schmidt (1999) type of social preferences into our model. We find that if the sabotaging activities are feasible, the symmetric equilibrium level of sabotaging effort becomes higher than the model with self-interested contestants. In this part, we also show that if the contestants are inequity-averse, to have an interior solution for both productive rent-seeking and sabotaging efforts, the absolute value of the differences in social preference parameters should be small enough.

2.6 Appendix

Proof of Proposition 1: The proof is similar to Okuguchi (1999) but there are differences between our model and his model. The main difference is that he uses a more general contest success function than we did and in his model, there is no fixed part of the prize. Different from him, in our model, we have a fixed part of the rent, and the contest success function is linear.

The first-order condition with respect to x_i yields

$$\frac{\sum_{j\neq i} x_j}{(\sum_j x_j)^2}\{v_{min} + \kappa v(\sum x_i)\} + \frac{x_i}{\sum x_i}\kappa\frac{\partial v(\sum x_i)}{\partial x_i} - 1 = 0 \tag{2.9}$$

For the symmetric solution, $x_i = x$ for all i. Then we have:

$$\frac{n-1}{n^2 x}\{v_{min} + \kappa v(nx)\} + \kappa v^{'}(nx) - 1 = 0 \tag{2.10}$$

Note that (2.10) can be rewritten as $L(x) - R(x) = 0$ where

$$L(x) = \frac{n-1}{n^2 x}\{v_{min} + \kappa v(nx)\} \text{ and } R(x) = 1 - \kappa v^{'}(nx)$$

Now note that $R^{'}(x) = -\kappa n v^{''}(nx) < 0$. That is to say, $R(x)$ is strictly decreasing in x. Then, it is straightforward to see that if $L(x)$ is strictly increasing in x i.e, $L^{'}(x) > 0$ for all x, then the solution automatically exists and intersects at only one point i.e., unique.

Note that

$$L^{'}(x) > 0 \iff n\kappa x v^{'}(nx) < v_{min} + \kappa v(nx) = V(x)$$

This implies $V(x) > xV^{'}(x)$. If we divide both sides with $V(x)$, we get:

$$\frac{V^{'}(x)}{V(x)}x = \varepsilon_V(x) < 1$$

Thus, $L'(x) > 0 \iff \varepsilon_V(x) < 1$ completes the proof.

Proof of Proposition 2: Note that without sabotage, (2.1) can be rewritten as:

$$EU_i = \frac{x_i}{\sum_i x_i}\{v_{min} + \kappa(\sum_i x_i)\} - c_i x_i \tag{2.11}$$

The first-order condition yields that:

$$\frac{\sum_{j\neq i} x_j}{(\sum_j x_j)^2}\{v_{min} + \kappa(\sum_i x_i)\} + \frac{x_i}{\sum_i x_i}\kappa - c_i = 0 \tag{2.12}$$

Note that the second-order condition holds as $\frac{\partial^2 EU_i}{\partial x_i^2} = -2\frac{\sum_{j\neq i} x_j}{(\sum_i x_i)^3} \leq 0$. For the symmetric solution, $x_i = x^* = x$ for all i. Then (2.12) becomes:

$$\frac{(n-1)x}{n^2x^2}\{v_{min} + \kappa n x\} + \frac{1}{n}\kappa - c_i = 0 \tag{2.13}$$

Then, it is easy to verify from (2.13) that $x_i = x^* = \frac{(n-1)}{n^2(c_i - \kappa)}v_{min}$. Note that if the prize were fixed, then the optimal rent-seeking effort would be $x^0 = \frac{n-1}{n^2 c_i}v_{min}$ is clearly lower than x^*.

Proof of Proposition 3: First-order conditions with respect to x_i and s_{ij} yields:

$$\frac{\partial EU_i}{\partial x_i} = \frac{\frac{\sum_{j\neq i} x_j}{(1+\sum_{j\neq i} s_{ji})(1+\sum_{i\neq j} s_{ij})}}{(\sum_i \frac{x_i}{1+\sum_{j\neq i} s_{ji}})^2}(v_{min} + \kappa \sum_i \frac{x_i}{1+\sum_{j\neq i} s_{ji}}) + \frac{x_i}{\sum_i x_i}\kappa\frac{1}{1+\sum_{j\neq i} s_{ji}} - c_1 = 0$$

and

$$\frac{\partial EU_i}{\partial s_{ij}} = \frac{\left(\frac{x_i}{1+\sum_{j\neq i} s_{ji}}\right)\frac{x_j}{(1+\sum_{i\neq j} s_{ij})^2}}{(\sum_i \frac{x_i}{1+\sum_{j\neq i} s_{ji}})^2}(v_{min} + \kappa \sum_i \frac{x_i}{1+\sum_{j\neq i} s_{ji}})$$

$$-\frac{\frac{x_i}{1+\sum_{j\neq i} s_{ji}}}{\sum_i \frac{x_i}{1+\sum_{j\neq i} s_{ji}}}\left(\kappa \frac{x_j}{(1+\sum_{i\neq j} s_{ij})^2}\right) - c_2 = 0$$

The symmetric solution requires that $x_i = x$ and $s_{ij} = s$ for all i and j. Thus, the above first-order conditions yield:

By using first-order condition for the x_i:

$$\frac{n-1}{n^2 x}(v_{min} + \kappa \frac{nx}{1+(n-1)s}) + \frac{\kappa}{n(1+(n-1)s)} - c_1 = 0$$

This gives

$$x^* = \frac{(n-1)(1+(n-1)s)v_{min}}{n^2(c_1(1+(n-1)s) - k)} \tag{2.14}$$

Similarly, by using the first-order condition for s_{ij} gives:

$$s^* = \frac{v_{min} - c_2 n^2}{c_2 n^2 (n-1)} \tag{2.15}$$

If we plug (2.15) into (2.14), we get:

$$x^* = \frac{(n-1)v_{min}^2}{n^2\big(c_1 v_{min} - \kappa c_2 n^2\big)} \tag{2.16}$$

Note that we do not show explicitly that the second-order conditions hold above. Our model is a specific version of Konrad (2000) with a contest success or production function of $f(x_i) = \frac{x_i}{1+\sum_{j\neq i} s_{ji}}$. It is clear that the production function basically satisfies the Konrad (2000)'s assumptions in the symmetric equilibrium. That is why we do not show explicitly why the second-order conditions hold. [7]

Proof of Proposition 4:

1- Given fixed prize v_{min} satisfying, $v_{min} \geq c_2 n^2$, it is easy to see that n^2 is a strictly increasing function in n. Thus, as n increases it becomes difficult for the contestants to exert positive sabotage effort in the equilibrium.

[7] Apart from that Minchuk (2020) also uses the same production function as we did in our model.Minchuk (2020) shows the second-order conditions hold in his model.

2- It is clear that if the rent is fixed and there is no endogenous part, then optimal rent-seeking effort does not depend on the sabotage level. Let $x_0(n, v)$ denote the optimal symmetric rent-seeking effort when the prize is exogenously given. It is easy to show that

$$x_0 = \frac{n-1}{c_1 n^2} v_{min}$$

where v_{min} is fixed. Thus, it is easy to show that

$$x^* - x_0(n, v_{min}) = \frac{\kappa c_2 n^2}{c_1 v_{min} - \kappa c_2 n^2} x_0(n, v_{min}) \geq 0$$

the optimal rent-seeking effort is higher than the model in which the rent is assumed as fixed.

3- If we take the derivative with respect to n, we get:

$$\frac{\partial x^*}{\partial n} = \frac{v_{min}^3 \{\kappa c_2 n^2 (3n-4) - v_{min} c_1 (n-2)\}}{n^3 (\kappa c_2 n^2 - c_1 v_{min})^2}$$

If we assume that $n > 2$, then the sign of $\{\kappa n^2(3n-4) - v_{min}(n-2)\}$ depends on both κ and v_{min}. Because $v_{min} \geq c_2 n^2$, let $v_{min} = c_2 n^2 + \varepsilon$ where $\varepsilon > 0$.

Then, $\{\kappa n^2 c_2(3n-4) - v_{min} c_1(n-2)\}$ can be rewritten as, $\kappa n^2 c_2(3n-4) - c_2 c_1 n^2 (n-2) - \varepsilon c_1 (n-2)$. Thus, if $\kappa \in (0, \frac{c_1(n-2)(\varepsilon + c_2 n^2)}{(3n-4)c_2 n^2})$, then optimal rent-seeking effort decreases as the number of contestants increases. Similarly if $\kappa \in [\frac{c_1(n-2)(\varepsilon + c_2 n^2)}{(3n-4)c_2 n^2}, 1)$, then optimal rent-seeking equilibrium increases in the equilibrium. Note that according to our assumption κ must be bounded above by 1. Therefore, $\frac{c_1(n-2)(\varepsilon + c_2 n^2)}{(3n-4)c_2 n^2} < 1$ must be satisfied. Indeed, this gives $\varepsilon < \frac{c_2 n^2 (3n-4-c_1(n-2))}{c_1(n-2)}$.

Proof of Proposition 5: Note that if there is no sabotage in the environment, then (2.5) can be rewritten as:

$$EU_i = \left(\frac{v_{min} + \kappa(\sum_i x_i)}{\sum_j x_j} \right) \big((1-\beta)x_i - \alpha \sum_{j \neq i} x_j\big) - c_1 x_i \tag{2.17}$$

First-order condition with respect to x_i :

$$-\frac{v_{min}}{\big(\sum_j x_j\big)^2}\big((1-\beta)x_i - \alpha \sum_{j \neq i} x_j\big) + \left(\frac{v_{min} + \kappa\big(\sum_i x_i\big)}{\sum_j x_j} \right)(1-\beta) = c_1$$

Note also that the second-order condition for (17) holds as

$$\frac{\partial^2 EU_i}{\partial x_i^2} = \frac{-2\sum_{j \neq i} x_j v_{min}(1-\beta+\alpha)}{(\sum_j x_j)^3} \leq 0$$

. Symmetric solution requires that $x_i = x^*$ for all i, thus above first-order condition becomes:

$$-v_{min}(1-\beta)x^* + \alpha(n-1)xv_{min} + nx\big(1-\beta)v_{min} + kn^2x^2(1-\beta)\big) = n^2x^2$$

Then it is straightforward that $x^* = \dfrac{(n-1)(1-\beta+\alpha)v_{min}}{n^2(c_1 - \kappa(1-\beta))}$. Note that the solution is unique. The existence of other-regarding preferences does not affect the existence or uniqueness. For the existence and uniqueness, proof of proposition 1 can be tractable.

3 Information Disclosure with Bayesian Persuasion in Endogeneous Rent-Seeking Contests

Abstract

This study proposes a new theoretical model regarding information disclosure with Bayesian persuasion in rent-seeking contests when the efforts are productive. We introduce the rent as a sum of both a minimum fixed prize and a function of an aggregate sum of efforts. Our findings reveal that under one-sided incomplete information, different from the exogenous rent-seeking contests, information disclosure decisions depend on both the marginal costs of exerting efforts and the marginal benefit of aggregate exerted effort. We find that since the efforts are productive and add an additional surplus on the fixed rent, our model narrows down the conditions for the information disclosure compared to the exogenous model. Under the two-sided incomplete information case, we observe that there is a non-monotone relationship between the optimal effort and posterior beliefs. Thus, it might be difficult to conclude whether a contest organizer should disclose any information to contestants.

3.1 Introduction

Rent-seeking contests are involved in many parts of economic activities. R&D, patent races, lobbying, political campaigns, labor market competition, and sports competitions can be seen as rent-seeking activities. Contestants compete for rent or a prize and expend irreversible resources or exert costly efforts to increase their probability of winning the reward. In contests, it is probable that contestants might have their private information on different characteristics such as the valuation of winning the competition, the marginal cost of exerting efforts, or some other specific abilities that can affect winning the reward[1]. That is why the solution structure of rent-seeking contests under incomplete information and disclosure of the information to the uninformed parties becomes significant.

In contests, under the existence of incomplete information, a contest designer may want to disclose information to get the possible highest efforts, resources, or expenditures from contestants. For example, in a company, if a job promotion is determined according to each worker's work effort, then the principal can disclose some workers' types to others to get the highest effort during the promotion period. Similarly, in the R&D investment races, firms make investments to make a discovery. Firms want to be the first ones in this competition to get monopoly rights because others get nothing. In this environment, the regulator or contest designer can disclose the types of firms to other firms to get the highest investment levels with the hope of having R&D in the possible shortest time. Or as Denter et al. (2011) show, lobbying groups spend resources or lobbying expenditures to influence a policymaker group for their favorite legislation. If the policymaker discloses information about the lobbying activities of some specific lobbying groups to other uninformed parties, it can increase lobbying expenditures and inefficiency.

Zhang and Zhou (2016) show the optimal information disclosure rule with Bayesian persuasion in contests. In their model, they introduce a two-player contest in which

[1]See Hurley and Shogren (1998a), Hurley and Shogren (1998b), Fey (2008), Zhang and Zhou (2016) for more on this topic.

one of the contestant's valuations of winning the game is his private information. According to their model, before the contest game begins, the contest organizer pre-commits to a signal to influence the uninformed one to change his prior belief about the one who has his private information. In their model, the contest organizer's main objective is to get possible highest expected total effort. They examine the conditions for information disclosure with Bayesian persuasion under one-sided incomplete information. When the type set is distributed binary, they show that either full or no information disclosure is optimal among the whole signal sets. However, besides all their findings, there are also additional significant points to be examined. In Zhang and Zhou (2016), incomplete information stems from the valuation of winning the contest. In the model, contestants exert irreversible resources and they do not affect the size of the reward.

However, we know that in rent-seeking contest literature, there are studies indicating that efforts are productive in a way that they add extra surplus over the fixed rent or prize of the contest. For example, Chung (1996) introduces a new rent-seeking model in which the prize equals the aggregate sum of total efforts. Similarly, in Amegashie (1999)'s model, the prize is the function of both a minimum fixed reward and the contestant's effort. Or as we also show in the other part of this thesis, the prize can be both the function of a minimum fixed amount of rent and the aggregate sum of all efforts. For example, as Chung (1996) points out, in R&D investment races, investments can be considered productive or the prize is endogenously determined by the aggregate investments if there is a knowledge spill-over effect. Or Amegashie (1999) states that in many jobs, besides a minimum wage, there might also be possible to get an additional salary, or promotion by exerting extra performance or effort. Also, Amegashie (1999) gives the example that firms may compete with each other for a minimum duration of a specific monopoly right but they also can extend the duration of the monopoly right by exerting additional effort.

In this study, therefore, different from Zhang and Zhou (2016), we propose a new theoretical model in which the rent is determined endogenously to discuss the information disclosure conditions using the Bayesian persuasion approach. The main differences between our model and Zhang and Zhou (2016) are the following. First, in Zhang and Zhou (2016), the efforts are unproductive, and incomplete information occurs from the different valuations of winning the contest. On the other hand, in our work, efforts add extra surplus over the fixed prize. That is to say, the prize is determined endogenously in terms of having productive efforts. Secondly, in our model, incomplete information stems from the marginal cost of exerting effort. Lastly, even though we could not find analytical exact results, we try to examine the information disclosure conditions under the two-sided incomplete information case.

The main results of our work are the following. Firstly, as far as we know, this study is the first one that finds the optimal effort level in endogenous rent-seeking contests under one-sided incomplete information. We show that the decision of information disclosure depends not only on the types of marginal costs but also depends on the marginal return of the aggregate efforts in the equilibrium under one-sided incomplete information and endogenous rent. Moreover, because the efforts are productive and generate an additional positive surplus, we find that the introduced endogenous model narrows down the condition for the information disclosure compared to the exogenous case. This occurs because of the fact that it is not that important for the uninformed contestant to compare his marginal cost with his opponent's possible different values of marginal costs compared to the exogenous model. Lastly, in the second part of our work, we introduce a two-sided incomplete information structure similar to Fey (2008). However, due to the difficult nature of the problem, it is difficult to find the analytic solutions for the optimal effort levels.What we observe is that there is a non-monotone relationship between the optimal effort level and posterior beliefs. That is why it is difficult to conclude that the total expected effort level is strictly concave or convex. Hence, we can not directly say whether the contest organizer should disclose the information or not.

The rest of the paper is organized as follows. The next section briefly mentions related literature. Section 3.3 introduces the model, section 3.4 introduces two-sided symmetric incomplete information to the model and the last section concludes.

3.2 Related Literature

In this part, we briefly mention related literature on incomplete information in contests, information disclosure in contests, and Bayesian persuasion.

Hurley and Shogren (1998a) examine how the contest behavior change when the information is asymmetric. They concluded that one-sided incomplete information can be sufficient for contest behavior when uncertainty exists. They find that the existence of asymmetric information makes the effort risky input which may cause both a decrease or increase in equilibrium effort. This occurs mainly from the difference between the perceived and actual asymmetries of the agents' values. Similarly, Hurley and Shogren (1998b) find the optimal effort levels in a Cournot-Nash contest when the information is asymmetric. In their model, the uninformed agent's effort can be seen as an uncertain input, and the uninformed one's effort level is inversely related to risk. At the same time, the informed player's effort can be positively or negatively related to the uninformed one's effort, i.e. whether they are strategic complements or substitutes. They show that when the efforts are strategic complements, given a change in beliefs, each player's effort and total effort move in the same direction. On the other hand, when the efforts are strategic substitutes, each player's effort moves in a different direction, and the total effort's direction is determined according to each agent's last equilibrium effort level. Lastly, Fey (2008) studies rent-seeking contests under two-sided incomplete information. In his model, both contestants have private information about the cost of exerting efforts. He examines both discrete and continuous cases for the existence of symmetric equilibrium efforts.

On information disclosure policy in contests, we briefly mention the two different papers. Denter et al. (2011) study the information disclosure policy between the lobbying groups. They mainly find that if the transparency policy is strong that is to say if the information is disclosed to uninformed parties, then this policy can cause unfavorable side effects to society. This occurs because competition among each lobbying group increases and this decreases the efficient outcome. Secondly, they explain that in general, the decentralized disclosure policy might be beneficial for the groups unless the policy outcomes are too sensitive to lobbying expenditures. Fu et al. (2011) examine a contest where the number of contestants is stochastic. The contest organizer's aim is to maximize the expected total efforts and to do that he has to decide whether disclose or conceal the actual number of contestants to participants. They find that the contest organizer should fully disclose (fully conceal) the information when the characteristic function is strictly concave (convex). Similarly, Fu et al. (2016) study disclosure policy in Tullock (1980) contest when the participation to contest is exogenously stochastic. Different from Fu et al. (2011), in this work, they use a generalized Tullock (1980) contest structure and look for the optimal disclosure policy by allowing asymmetries across contestants. These asymmetries are both valuation of the prize and the contestant's entry probabilities. They mainly show that the optimal disclosure policy depends on both contestants' valuation of the prize and their entry probabilities but they point out that the optimal disclosure policy does not depend on the prevailing precision of the Tullock (1980) contest.

Last but not least, the transmission mechanism of information disclosure plays a significant role. In this study, by following Zhang and Zhou (2016), we use Kamenica and Gentzkow (2011)'s seminal paper for the information disclosure. As Taneva (2019) defines, Bayesian persuasion can be seen as a cheap talk game with commitment. In Kamenica and Gentzkow (2011), the authors introduce a very important assumption. To prevent possible incentive compatibility issues, they assume that the sender cannot change or conceal information once the signal is realized. In their model, there are two players namely the sender and receiver in which the sender wishes to persuade the receiver to change the receiver's action. In this study, the authors find and show the necessary and sufficient conditions for the existence of (optimal) signal structures that yield benefits to the sender from the persuasion mechanism by using convex analysis. They also show the conditions for both full disclosure and no disclosure of information in the equilibrium.

3.3 The Model

There are two risk-neutral contestants $i \in N = \{1, 2\}$[2], competing for an endogenous prize consisting of both a fixed prize Π and a function of (productive) efforts. Contestants exert irreversible efforts $e_i \geq 0$ where $i \in N$ simultaneously. We use classical Tullock (1980) type of contest success functions given as

$$p(e_i) = \begin{cases} \dfrac{e_i}{\sum_{j \in N} e_j} & \text{if } e_i \neq 0 \\ \dfrac{1}{2} & \text{if } e_i = 0 \end{cases}$$

Under complete information the expected utility for each contestant i:

$$u_i(e_i, e_j) = \frac{e_i}{\sum_{j \in N} e_j} \big(\Pi + \eta f(e_i + e_j)\big) - c_i e_i \tag{3.1}$$

where Π is the fixed portion of the prize, η is marginal return of total productive efforts similar to Amegashie (1999). In Amegashie (1999), the endogenous part of the prize consists of only one contestant's effort level. However, in our case, the endogenous part is simply the sum of all contestants' effort levels. Note also that efforts are productive because different from fixed prize Π, competing efforts, resources, or lobbying efforts constitute an additional surplus over the fixed prize.Moreover, for simplicity, we assume that $f(\sum_i e_i) = \sum_i e_i$ and c_i denotes the marginal cost of exerting effort level for each player. In the model, we also assume that $\eta \leq \min\{c_1, c_2\}$ that is to say we bound the endogenous part of the prize from the above.

Before introducing the persuasion mechanism, we first suppose that our model is one-sided incomplete information and mainly benefits from Zhang and Zhou (2016) but there are differences between our model and Zhang and Zhou (2016). First, incomplete information stems from the marginal cost of exerting productive effort

[2]Note that N denotes the set of contestants.

in our model. Second, the total prize for the competition is not fixed and it is endogenous. One of the prize components in our model consists of exerted efforts of the contestants. That is why efforts are considered productive.

Suppose that the first contestant's marginal cost of exerting effort c_1 is common knowledge but the second one's marginal cost of exerting effort is his private information. In the next chapter, we also try to find new outcomes when the game is two-sided incomplete information.

We now introduce the contest organizer who can precommit a signal to influence the uninformed player's belief about the other player. The main aim of the contest organizer is to maximize the expected total effort level exerted by all contestants. In that sense, we aim to find the conditions that the contest organizer should disclose information to uninformed contestants under the one-sided information.

Suppose that contestant 1's valuation of marginal cost is $c_1 = c$ and common knowledge, and contestant 2's valuation of marginal cost is c_2 is his private information and c_2 which is a random variable on Ω with two different values, $\Omega = \{c_{2l}, c_{2h}\}$ where $c_{2l} < c_{2h}$. Also $\Delta(\Omega) = \{\mu \in \mathbf{R}^2 | \mu_n \geq 0, \sum_{n=1}^{2} \mu_n = 1\}$ is the standard simplex in $\mathbf{R}^2$. Note that each point $\mu \in int(\Delta(\Omega))$ is identified as a probability distribution on Ω where $int(\Delta(\Omega))$ denotes the interior of the simplex. We also denote the prior distribution of c_2 as μ_0 and we assume $\mu_0 \in int(\Delta(\Omega)$.

The timing of the game: Timing of the game is similar with Zhang and Zhou (2016). Thus, we have the following order:

1. The contest organizer chooses and precommits to a signal π. A signal π consists of a finite realization space S and a family of distributions $\{\pi(.|c_{2i})\}_{c_{2i} \in \Omega}$ for $i \in \{l, h\}$.

2. Nature moves and draws a valuation for contestant 2- say c_{2i}

3. The contest organizer carries out his commitment and a signal realization $s \in S$ is generated according to $\pi(s|c_{2i})$

4. The signal realization s is observed by everyone and leads to a posterior belief about contestant 2, $\mu_p \in \Delta(\Omega)$

5. The contest begins and both contestants choose their productive efforts simultaneously

Note that according to the timing of the game, the first part is considered a part of Bayesian persuasion, and the last part is basically a posterior rent-seeking contest game. In the Bayesian Persuasion part, the contest organizer's problem is to choose the optimal signal π to maximize the expected total effort levels. In the last stage, the contestants' main objective is to determine and exert their productive efforts to increase the winning probability of the prize.

By following Kamenica and Gentzkow (2011), a signal π induces:

$$\mu_s(c_{2i}) = \frac{\pi(s|c_{2i})\mu_0(c_{2i})}{\sum_{c_{2w}\in\Omega} \pi(s|c_{2w})\mu_0(c_{2w})}$$

$$\tau(\mu) = \sum_{\mu_s} \sum_{c_{2w}\in\Omega} \pi(s|c_{2w})\mu_0(c_{2w})$$

Note that $\tau \in \Delta(\Delta(\Omega))$ is also the distribution of posteriors. That is τ is the new random variable that assigns probabilities to the posterior distribution of the beliefs.

As Kamenica and Gentzkow (2011) state in their work, Bayesian rationality imposes that τ should be Bayes-plausible. That is expected posterior probability equals the prior. In other words, beliefs should be martingale:

$$\sum_{Supp(\tau)} \mu\tau(\mu) = \mu_0$$

Kamenica and Gentzkow (2011) show that to be able to find the optimal signal π, we need to look for Bayes-plausible distribution of posteriors τ which maximizes the expected value of the posterior expected total effort level. That is to say, we try to find the solution of the following system:

$$\max_{\tau} E_\tau TE(\mu) \text{ subject to } \sum_{Supp(\tau)} \mu\tau(\mu) = \mu_0 \tag{3.2}$$

Note that $E_\tau TE(\mu)$ denotes the expected total productive efforts. By Kamenica and Gentzkow (2011) and Zhang and Zhou (2016), we know that finding the optimal signal from the above maximization problem is equivalent to finding the concave closure (concavification) of expected total effort at the prior. Therefore, in this study, we mainly follow this solution strategy in our analysis as Zhang and Zhou (2016) did. In this regard, we first find the posterior optimal productive efforts in the equilibrium and then find the posterior expected total productive efforts. In the end, we mainly try to find the concave closure of the expected total productive efforts.

In the posterior contest game, we know that the first contestant's marginal cost of exerting productive effort is common knowledge and c. On the other hand, the second contestant's marginal cost of exerting productive effort is $c_{2i} \in \{c_{2l}, c_{2h}\}$ and let $\mu_0 = (\beta, 1-\beta)$ and $\mu_p = (\alpha, 1-\alpha)$ denote prior and posterior distribution for each state. Note that after observing the signal, the first contestant updates his prior beliefs μ_0 to posterior beliefs μ_p according to Bayes's rule. In this work, our aim is not to find what the optimal signals are, but instead, our aim is to find under what conditions a contest designer discloses information to the uninformed party.

We begin our analysis by finding the optimal productive effort levels of the contestants when the prize is endogenously formed and one-sided incomplete information exists.

Proposition 1: *Suppose we have one-sided incomplete information rent-seeking game with two contestants, where contestant 1's marginal cost of exerting productive effort level is common knowledge as c, and contestant 2's marginal cost of exerting productive effort level is his private knowledge and distributed according to* $\mu_p \in (\alpha, 1-\alpha)$ *on* (c_{2l}, c_{2h})*. There exists a unique pure strategy equilibrium in which contestant 1 exerts*:

$$e_1^* = \left[\frac{\sqrt{\Pi}(\alpha\sqrt{c_{2l}-\eta} + (1-\alpha)\sqrt{c_{2h}-\eta})}{c + \alpha c_{2l} + (1-\alpha)c_{2h} - 2\eta} \right]^2$$

and contestant 2 exerts:

$$e_h^* = \sqrt{\frac{e_1^*\Pi}{c_{2h}-\eta}} - e_1^* \text{ and } e_l^* = \sqrt{\frac{e_1^*\Pi}{c_{2l}-\eta}} - e_1^*$$

Moreover, the expected total productive efforts in the equilibrium:

$$TE(\alpha) = \Pi \left(\frac{\alpha\sqrt{c_{2l}-\eta} + (1-\alpha)\sqrt{c_{2h}-\eta}}{c + \alpha c_{2l} + (1-\alpha)c_{2h} - 2\eta} \right) \left(\frac{\alpha\sqrt{c_{2h}-\eta} + (1-\alpha)\sqrt{c_{2l}-\eta}}{\sqrt{(c_{2l}-\eta)(c_{2h}-\eta)}} \right)$$

Note that the proof of this proposition and subsequent outcomes can be found in the appendix. Different from classical rent-seeking literature, we introduce both endogeneity (via productive efforts) and incomplete information in our model. It can be seen that expected total (productive) efforts depend on not only the fixed prize Π and c, but also the distribution of posterior beliefs on (c_{2l}, c_{2h}), and the marginal return of the endogenous part of the prize, η.

It is important to point out that given α, c, c_{2l} and c_{2h} an increase in marginal return of productive efforts increases the expected total effort that is to say $\frac{\partial TE}{\partial \eta} > 0$. This is intuitively expected and makes sense because even though exerting effort is costly, now exerting effort also produces a positive surplus which is proportional to the η.

Now, by following Zhang and Zhou (2016), we try to find the concave closure of the expected total productive efforts to find the conditions that whether the contest organizer should disclose the information or not.

Proposition 2: *The posterior expected total productive effort is*:

$$\begin{cases} \textit{strictly concave if} & c > \sqrt{c_{2l}-\eta}\sqrt{c_{2h}-\eta}+\eta \\ \textit{linear if} & c = \sqrt{c_{2l}-\eta}\sqrt{c_{2h}-\eta}+\eta \\ \textit{strictly convex if} & c < \sqrt{c_{2l}-\eta}\sqrt{c_{2h}-\eta}+\eta \end{cases}$$

Proposition 2 basically shows the conditions for the concavity/convexity of posterior expected total productive efforts. Thus, the following corollary mainly distinguishes the conditions for the contest designer to disclose information or not.

Corollary: *In the equilibrium, the contest designer gives the information disclosure decision according to the following:*

$$\begin{cases} \textit{Any signal is optimal} & \textit{if } c = \sqrt{c_{2l}-\eta}\sqrt{c_{2h}-\eta}+\eta \\ \textit{No information disclosure is optimal} & \textit{if } c > \sqrt{c_{2l}-\eta}\sqrt{c_{2h}-\eta}+\eta \\ \textit{Full information disclosure is optimal} & \textit{if } c < \sqrt{c_{2l}-\eta}\sqrt{c_{2h}-\eta}+\eta \end{cases}$$

Note that if the posterior expected total productive efforts are strictly concave then the contest designer basically does not disclose the information because the concave closure of a strictly concave function is itself. Also, as Zhang and Zhou (2016) show, if the posterior expected total efforts are strictly convex, then full information disclosure becomes optimal. Especially, it makes sense and intuitive when the type set consists of two different types because one can graph and see that $co(TE(\alpha)) \geq TE(\alpha)$[3] when the $TE(\alpha)$ is strictly convex. Note that if $\eta = 0$, then the model converges into an exogenous model. In this case, our model becomes similar to Zhang and Zhou (2016) but the main difference is, in their model, one of the contestants has a different valuation over the winning the competition, and both contestants have a unit marginal cost of exerting effort. However, in our model, while the prize is fixed, the marginal cost of exerting effort is different than unity and creates incomplete

[3] $co(TE(\alpha))$ denotes the concave closure of posterior expected total productive efforts.

information among the contestants.

Note also that the introduction of endogeneity brings important implications related to the information disclosure decisions. First, suppose that $\eta = 0$. Then, only the comparison of the marginal cost of exerting efforts becomes sufficient and this comparison mainly becomes important in the decision of information disclosure. A similar point to our argument is already discussed in Zhang and Zhou (2016) and they mainly refer to Dixit (1987), Nti (1999), and Denter et al. (2011) to explain their intuition. In this regard, contestants exert higher amounts of effort when their types are close to each other and their exerting equilibrium strategies become either complements or substitutes when the contestant is either “stronger” or “weaker” in terms of their types of the marginal cost of exerting efforts. Thus, if the model has a fixed prize, and if the incomplete information comes from the marginal cost of exerting effort, then we have:

$$\begin{cases} \text{Any signal is optimal} & \text{if } c = \sqrt{c_{2l}}\sqrt{c_{2h}} \\ \text{No information disclosure is optimal} & \text{if } c > \sqrt{c_{2l}}\sqrt{c_{2h}} \\ \text{Full information disclosure is optimal} & \text{if } c < \sqrt{c_{2l}}\sqrt{c_{2h}} \end{cases}$$

However, if the endogeneity is introduced, that is to say, if $\eta \neq 0$, then the above expression is not valid. Because in this case, the condition for the information disclosure decision depends not only on the comparison of the marginal cost of exerting efforts but also on the marginal return of exerting the productive efforts. In the exogenous model, i.e., when $\eta = 0$, the first contestant decides his effort level by comparing his marginal cost with his rival. However, if $\eta \neq 0$, then, both contestants do take into consideration the existence of η. This happens because we have an increase in the size of the surplus or total prize by the fixed proportional amount η of the total exerted sum of efforts. In that sense, if the rent is endogenous, then the optimal effort levels depend not only on the fixed prize and marginal costs but also on the marginal benefit of total productive efforts. That is why different from the exogenous case, information disclosure depends on both marginal costs and η. Moreover, from the perspective of the contest designer, this result plays an important role. Because the efforts are productive in the model, that is to say, it creates a positive surplus addition to the fixed prize, this causes to narrow down the condition for the information disclosure compared to the exogenous model. It occurs because efforts are productive, for the first contestant, it is not that important to compare his marginal cost with his opponent's possible different values compared to the exogenous model.

It is easy to show why the endogenous model narrows down the information disclosure decisions compared to the exogenous model. In the exogenous model, the cut-off point for the decision call is determined by comparing c with $\sqrt{c_{2l}c_{2h}}$ and in the endogenous model, we compare c with $\sqrt{(c_{2l}-\eta)(c_{2h}-\eta)}+\eta$. Then, it is easy to show that the below inequality holds:

$$\sqrt{(c_{2l}-\eta)(c_{2h}-\eta)}+\eta \leq \sqrt{c_{2l}c_{2h}}$$

As it can be seen that compared to the exogenous model, the pivotal point for the information disclosure decision becomes smaller in the endogenous model. This basically occurs because efforts also add some surplus over the fixed prize, the first contestant does not care much for the second contestant's types compared to the exogenous case. Because in the endogenous model, the first contestant knows that even he does know the exact type of the second contestant, the second contestant now has the incentive to exert higher efforts compared to the exogenous case because exerting efforts now also increases the prize.

3.4 Extension: A Comment on the Two-Sided Incomplete Information

In this part, we aim to consider two-sided incomplete information. In our setup, we consider two risk-neutral contestants i, where $i \in \{1,2\}$ with two different types $c_i \in \{c_{il}, c_{ih}\}$ for each i. Even though each contestant knows his own true type, and has the same marginal cost structure as his rival (c_h or c_l), they can not observe, their rival's type. We assume that they both have a common prior belief, $\mu_0 = (0.5, 0.5)$. Due to its complex structure, we try to be as simple as possible. That is why we consider the simplest case i.e., a symmetric case similar to Fey (2008). In this part, we assume that the fixed part of the prize normalized to one and efforts are productive which means they also increase the surplus.

In the posterior contest game, each contestant i has the posterior beliefs on the (c_{il}, c_{ih}) and say $\mu_{ip} = (\beta_i, 1 - \beta_i)$ for $i \in \{1, 2\}$.

In the posterior contest game, the expected utility for the contestant i becomes:

$$\beta_i \frac{e_i}{e_i + e_{jl}}(1 + \eta(e_i + e_{jl})) + (1 - \beta_i)\frac{e_i}{e_i + e_{jh}}(1 + \eta(e_i + e_{jh})) - c_i e_i$$

Note that similar to the first part η represents the marginal benefit of aggregate productive efforts and $\eta < c_{il}$. Then, the first-order condition for the contestant i yields:

$$\beta_i \frac{e_{jl}}{(e_i + e_{jl})^2} + (1 - \beta_i)\frac{e_{jh}}{(e_i + e_{jh})^2} = c_i - \eta \tag{3.3}$$

Note that equation (3.3) induces different first-order conditions because e_i can be either e_{il} and e_{ih}. for each $i \in \{1, 2\}$. Here, due to the complexity of the solution, we focus on the symmetric solution as in Fey (2008). In this case, we have $\beta_i = \beta$, $e_{il} = e_l$ and $e_{ih} = e_h$ for each $i \in \{1, 2\}$.

By using the (3.3), in the symmetric case, if the contestant is low-type, we have:

$$\frac{\beta}{4e_l} + (1 - \beta)\frac{e_h}{(e_l + e_h)^2} = c_l - \eta \tag{3.4}$$

and similarly, if the contestant is high-type then:

$$\beta \frac{e_l}{(e_l + e_h)^2} + \frac{(1 - \beta)}{4e_h} = c_h - \eta \tag{3.5}$$

Benchmark (Fey (2008)): Suppose $\beta = 0.5$ and $\eta = 0$, then we have unique symmetric equilibrium such that

$$e_l^* = \frac{4\frac{c_l}{c_h} + (1 + \frac{c_l}{c_h})^2}{8c_l(1 + \frac{c_l}{c_h})^2} \text{ and } e_h^* = \frac{4\frac{c_l}{c_h} + (1 + \frac{c_l}{c_h})^2}{8c_h(1 + \frac{c_l}{c_h})^2}$$

Note that, as a benchmark, if $\beta = 0.5$, and if the efforts are not productive then the optimal effort levels become as in Fey (2008). In our case, however, it is not that easy to solve the optimal effort levels because of the existence of the β variable and non-linear first-order conditions. That is why it is not easy to find a way to characterize the decisions for information disclosure. In that sense, in our analysis, we only give intuitive predictions, not exact analytic results.

First, we try to derive the expected total effort level as we did in the first part. Note that it is easy to show that by using (3.4) and (3.5), we find:

$$e_l = \frac{4(1-\beta)(c_h - \eta)e_h - 1 + 2\beta}{4\beta(c_l - \eta)}$$

Thus, expected total efforts become:

$$TE(\beta) = 2(\beta e_l + (1-\beta)e_h) = \frac{4(1-\beta)(c_h + c_l - 2\eta)e_h - 1 + 2\beta}{2(c_l - \eta)} \tag{3.6}$$

As it can be seen from the above, $TE(\beta)$ depends also on $e_h(\beta)$. Normally, we should solve and find the exact e_h by using one of the first-order conditions. However, it is not easy to find analytical solutions because of the nonlinearities of the first-order conditions. That is why we try to assess the possible conditions by intuitive facts. If we check the concavity/convexity of the $TE(\beta)$, we get:

$$TE^{'}(\beta) = \frac{4(c_h + c_l - 2\eta)((1-\beta)\frac{\partial e_h}{\partial \beta} - e_h) + 2}{2(c_l - \eta)}$$

and

$$TE''(\beta) = \frac{2(c_h + c_l - 2\eta)}{c_l - \eta}\left[-2\frac{\partial e_h}{\partial \beta} + (1-\beta)\frac{\partial^2 e_h}{\partial \beta^2}\right] \tag{3.7}$$

From (3.7), it is not easy to conclude whether the expected total effort is strictly concave or convex. However, if we suppose that if η is given, the difference between c_h and c_l is so high, then we can say that as β increases optimal effort level of the high type decreases. Its intuition is the following. If a contestant has a lower marginal cost of exerting effort, then in the equilibrium he exerts higher effort compared to the one who has a higher marginal cost of exerting effort. In that sense, if the β increases, the contestant realizes that his opponent's probability of being low type increases. Hence, in the equilibrium, the high-type contestant's optimal effort level might decrease in the equilibrium. On the other hand, if the difference between the marginal cost of exerting efforts is not that high, then as the β, increases, the high-type contestant's optimal effort might increase too. This occurs because of the fact that as the difference between c_h and c_l is small enough, then from the perspective of high type contestant, as the probability of being his opponent's marginal cost of exerting effort becomes lower, the high-type contestant might think that his rival probable to exert higher efforts. Since their marginal cost of exerting effort levels are close to each other, then in the equilibrium, high type contestants might also exert higher effort in the equilibrium.

In the end, assume that η is known and fixed, and if the difference between c_h and c_l is so high and e_h is strictly decreasing and convex in β, then we can conclude that the expected total effort becomes strictly convex. Thus, the contest designer can disclose the information. However, if the difference between the marginal costs is close to each other and if e_h is strictly concave in β, then we can not conclude whether the expected total effort becomes strictly concave or convex because e_h might be increasing in β. All in all, to be able to make a more exact and true analysis, we should solve and find the optimal effort levels for the high and low types of contestants explicitly. However, as we show above, we are not certain that analytically solving the optimal effort levels is easy due to the complex structure.

In general, what we possibly observe is the following. In the two-sided incomplete information set-up, we have the non-monotone relationship between the optimal effort (e_h) and β. This would possibly generate non-unique equilibria paths for the optimal effort levels. That is why, it would be difficult to show whether the total expected efforts are strictly concave or convex in β. Thus, in a two-sided incomplete information set-up, it would be complicated to conclude on whether the contest designer should disclose or conceal the information.

3.5 Conclusion

In this study, we examine information disclosure with Bayesian persuasion in rent-seeking contests when the prize is endogenously determined. Different from conventional unproductive efforts in rent-seeking contests, we offer productive efforts. In our model, the rent of the game is the sum of both a minimum fixed prize and a function of an aggregate sum of efforts. In that sense, as far as we know, our work is the first study that examines the optimal effort levels in rent-seeking contests when the efforts are productive. Different from Zhang and Zhou (2016), incomplete information stems from marginal costs, efforts are productive (endogenous rent), and we also try analyzing the optimal information disclosure condition under two-sided incomplete information. Our main findings reveal that under one-sided incomplete information, different from an exogenous rent model, information disclosure decision depends on both the marginal costs of exerting efforts and the marginal benefit of aggregate exerted effort. We mainly find that because the efforts are productive and generate an additional positive surplus, the introduced endogenous model basically narrows down the condition for the information disclosure compared to the exogenous rent model. In the second part, we introduce the two-sided incomplete information to our model. Due to its difficult nature, it is difficult to find analytic solutions for optimal effort. Instead, we introduce a numerical model. We mainly find that there is a non-monotone relationship between the optimal effort level and beliefs. Thus, we can not introduce certain conditions for the information disclosure decision for the contest designer.

3.6 Appendix

Proof of Proposition 1: In the posterior contest game, after the signal realization, the first contestant updates his prior beliefs μ_0 to $\mu_p = (\alpha, 1-\alpha)$. Then, the expected utility for the first contestant:

$$EU_1 = \left(\frac{\alpha e_1}{e_1 + e_{2l}}\right)(\Pi + \eta(e_1 + e_{2l})) + \left(\frac{(1-\alpha)e_1}{e_1 + e_{2h}}\right)(\Pi + \eta(e_1 + e_{2h})) - ce_1 \quad (3.8)$$

First-Order Condition requires that:

$$\left[\frac{\alpha e_{2l}}{(e_1 + e_{2l})^2} + \frac{(1-\alpha)e_{2h}}{(e_1 + e_{2h})^2}\right]\Pi = c - \eta \quad (3.9)$$

Next, if we write down the second contestant's problem (both for low and high types separately):

For the low type of marginal cost, the expected utility for the second contestant becomes :

$$EU_{2l} = \frac{e_{2l}}{e_1 + e_{2l}}(\Pi + \eta(e_1 + e_{2l})) - c_2 e_{2l} \quad (3.10)$$

Then, the first-order condition gives:

$$\frac{e_1}{(e_1 + e_{2l})^2}\Pi = c_{2l} - \eta \quad (3.11)$$

By (3.11), it is straightforward to see that

$$e_{2l} = \sqrt{\frac{e_1 \Pi}{c_{2l} - \eta}} - e_1 \quad (3.12)$$

Similarly, for the high-type contestant, we have

$$e_{2h} = \sqrt{\frac{e_1 \Pi}{c_{2h} - \eta}} - e_1 \quad (3.13)$$

Now, if we plug (3.12) and (3.13) into (3.9), we find the equilibrium productive effort level as:

$$\boxed{e_1^* = \left[\frac{\sqrt{\Pi}(\alpha\sqrt{c_{2l}-\eta} + (1-\alpha)\sqrt{c_{2h}-\eta}}{c + \alpha c_{2l} + (1-\alpha)c_{2h} - 2\eta}\right]^2} \tag{3.14}$$

Thus, we have $e_{2l}^* = \sqrt{\dfrac{e_1^*\Pi}{c_{2l}-\eta}} - e_1^*$ and $e_{2h}^* = \sqrt{\dfrac{e_1^*\Pi}{c_{2h}-\eta}} - e_1^*$.

Therefore, the expected total effort (denoted by $TE(\alpha)$) becomes:

$$TE(\alpha) = e_1^* + \alpha e_{2l}^* + (1-\alpha)e_{2h}^* \implies$$

$$\boxed{\mathrm{TE}(\alpha) = \Pi\left(\frac{\alpha\sqrt{c_{2l}-\eta} + (1-\alpha)\sqrt{c_{2h}-\eta}}{c + \alpha c_{2l} + (1-\alpha)c_{2h} - 2\eta}\right)\left(\frac{\alpha\sqrt{c_{2h}-\eta} + (1-\alpha)\sqrt{c_{2l}-\eta}}{\sqrt{(c_{2l}-\eta)(c_{2h}-\eta)}}\right)} \tag{3.15}$$

which completes the proof of Proposition 1.

From now, we write expected total productive effort as $TE(\alpha) = \Pi M(\alpha)N(\alpha)$ where

$$\mathrm{M}(\alpha) = \left(\frac{\alpha\sqrt{c_{2l}-\eta} + (1-\alpha)\sqrt{c_{2h}-\eta}}{c + \alpha c_{2l} + (1-\alpha)c_{2h} - 2\eta}\right) \text{ and } N(\alpha) = \left(\frac{\alpha\sqrt{c_{2h}-\eta} + (1-\alpha)\sqrt{c_{2l}-\eta}}{\sqrt{(c_{2l}-\eta)(c_{2h}-\eta)}}\right)$$

In the following lemma, we find the conditions where $M(\alpha)$ is strictly increasing, decreasing, and strictly concave or convex in α.

Lemma 1:

$$\begin{cases} M'(\alpha) > 0 \text{ and } M''(\alpha) > 0 \text{ if } c < \sqrt{c_{2l}-\eta}\sqrt{c_{2h}-\eta} + \eta \\ M'(\alpha) = 0 \text{ and } M''(\alpha) = 0 \text{ if } c = \sqrt{c_{2l}-\eta}\sqrt{c_{2h}-\eta} + \eta \\ M'(\alpha) < 0 \text{ and } M''(\alpha) < 0 \text{ if } c > \sqrt{c_{2l}-\eta}\sqrt{c_{2h}-\eta} + \eta \end{cases}$$

Proof of Lemma 1:

$$\mathrm{M}^{'}(\alpha) = \frac{(\sqrt{c_{2l}-\eta}-\sqrt{c_{2h}-\eta})(c+\alpha c_{2l}+(1-\alpha)c_{2h}-2\eta)-(\alpha\sqrt{c_{2l}-\eta}+(1-\alpha)\sqrt{c_{2h}-\eta})(c_{2l}-c_{2h})}{(c+\alpha c_{2l}+(1-\alpha)c_{2h}-2\eta)^2} \tag{3.16}$$

If we rearrange (3.11), we find that $M^{'}(\alpha) > 0$ if $c < \sqrt{c_{2l}-\eta}\sqrt{c_{2h}-\eta}+\eta$, similarly $M^{'}(\alpha) < 0$ if $c > \sqrt{c_{2l}-\eta}\sqrt{c_{2h}-\eta}+\eta$ and $M^{'}(\alpha) = 0$ if $c = \sqrt{c_{2l}-\eta}\sqrt{c_{2h}-\eta}+\eta$. For the concavity or the convexity of the $M(\alpha)$: If we take the second derivative with respect to α:

$$M''(\alpha) = \frac{2(c_{2l}-c_{2h})\left[(2\eta-c_{2h}-c)\sqrt{c_{2l}-\eta}+(c+c_{2l}-2\eta)\sqrt{c_{2h}-\eta}\right]}{(c+\alpha c_{2l}+(1-\alpha)c_{2h})^3} \tag{3.17}$$

After simplifying (3.12), we see that $M^{''}(\alpha) > 0$ if $c < \sqrt{c_{2l}-\eta}\sqrt{c_{2h}-\eta}+\eta$, $M^{''}(\alpha) < 0$ if $c > \sqrt{c_{2l}-\eta}\sqrt{c_{2h}-\eta}+\eta$ and $M^{''}(\alpha) = 0$ if $c = \sqrt{c_{2l}-\eta}\sqrt{c_{2h}-\eta}+\eta$. To show explicitly:

$M^{''}(\alpha) > 0 \iff [(2\eta-c_{2h}-c)\sqrt{c_{2l}-\eta}+\sqrt{c_{2h}-\eta}(c+c_{2l}-2\eta)] < 0$ as $(c_{2l}-c_{2h}) < 0$. Then, it is obvious to show that

$$\left[(2\eta-c_{2h}-c)\sqrt{c_{2l}-\eta}+\sqrt{c_{2h}-\eta}(c+c_{2l}-2\eta)\right] < 0 \iff c < \sqrt{c_{2l}-\eta}\sqrt{c_{2h}-\eta}+\eta$$

. Note that similar arguments can also be applied for the cases when $M^{''}(\alpha) < 0$ and $M^{''}(\alpha) = 0$.

Proof of Proposition 2:

$$TE^{''}(\alpha) = \Pi(M^{''}(\alpha)N(\alpha)+2M^{'}(\alpha)N^{'}(\alpha)+M(\alpha)N^{''}(\alpha))$$

Note that $N^{''}(\alpha) = 0$ and by using Lemma 1, we conclude that $TE(\alpha)$ is strictly convex if $c < \sqrt{c_{2l}-\eta}\sqrt{c_{2h}-\eta}+\eta$. Similarly, $TE(\alpha)$ becomes strictly concave if $c > \sqrt{c_{2l}-\eta}\sqrt{c_{2h}-\eta}+\eta$. Lastly, expected total effort becomes constant or linear if $c = \sqrt{c_{2l}-\eta}\sqrt{c_{2h}-\eta}+\eta$.

Proof of Benchmark (Fey (2008)):If efforts are not productive ($\eta = 0$) and $\beta = 0.5$, then our model converges into Fey (2008)'s model.The first-order conditions become:

$$\frac{1}{8e_l} + \frac{e_h}{2(e_l + e_h)^2} = c_l \tag{3.18}$$

and

$$\frac{1}{8e_h} + \frac{e_l}{2(e_l + e_h)^2} = c_h \tag{3.19}$$

(3.18) and (3.19) can be rewritten as:

$$(e_l + e_h)^2 + 4e_l e_h = 8c_l e_l (e_l + e_h)^2 \tag{3.20}$$

and

$$(e_l + e_h)^2 + 4e_l e_h = 8c_h e_h (e_l + e_h)^2 \tag{3.21}$$

respectively. From, (3.20) and (3.21) we have the following:

$$c_h e_h = c_l e_l \text{ or } e_h = \frac{c_l}{c_h} e_l \tag{3.22}$$

If we use (3.22) in the (3.20), it is obvious that

$$e_l^* = \frac{4\frac{c_l}{c_h} + (1 + \frac{c_l}{c_h})^2}{8c_l(1 + \frac{c_l}{c_h})^2} \text{ and } e_h^* = \frac{4\frac{c_l}{c_h} + (1 + \frac{c_l}{c_h})^2}{8c_h(1 + \frac{c_l}{c_h})^2}$$

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