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THE SPATIAL ASSOCIATION OF NUMBERS AND MATHEMATICAL SYMBOLS

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ABSTRACT

The Spatial-Numerical Association of Response Code (SNARC effect) refers to the finding that people respond faster to small numbers with the left hand and to large numbers with the right hand. This effect is often explained by hypothesizing that the mental representation of quantities has a spatial component: left to right in ascending order (Mental Number Line). However, the SNARC effect may not depend on quantitative information, but on other factors such as the order in which numbers are often represented from left to right in our culture. Four experiments were performed to test this hypothesis. In the first experiment, the concept of spatial association was extended to nonnumeric mathematical symbols: the minus ("-") and plus ("+") symbols. These symbols were presented as fixation points in a spatial compatibility paradigm. The results demonstrated an opposite influence of the two symbols on the target stimulus. Indeed, the minus symbol tends to favor the target presented on the left while the plus symbol the target presented on the right, demonstrating that spatial association can emerge in the absence of a numerical context.

In the last three experiments, the relationship between quantity and order was evaluated using normal numbers and mirror numbers. Although mirror numbers denote quantity, they are generally not encountered in a left-to-right spatial organization. In Experiments 1 and 2, participants performed a magnitude classification task with mirror and normal numbers presented together (Experiment 1) or separately (Experiment 2). In Experiment 3, participants performed a new task in which quantity information processing was not required: the mirror judgment task. The results show that participants access the quantity of both normal and mirror numbers, but only the normal numbers are spatially organized from left to right. In addition, the physical similarity between the numbers, used as a predictor variable in the last three experiments, showed that the physical characteristics of numbers influenced participants' reaction times.

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Table of contents

INTRODUCTION 9

PART I 14

THEORETICAL PANORAMA 14

CHAPTER 1 16

FROM QUANTITIES TO NUMBERS: THE EVOLUTION OF MATHEMATICS 16

1.1 The non-numeric Sapiens 16

1.2 Numbers from hands to mind 21

1.3 The numbers we use 24

1.4 Ten symbols for every quantity 27

1.5 Numbers and humans. Theoretical Summary 32

CHAPTER 2 34

HOW ARE NUMBERS REPRESENTED? 34

2.1. The introspective numbers 34

2.2. Measuring reaction times: the distance effect 36

2.3 Mathematical functions for mathematical performances: the Welford functions 38

2.4 From left to right: the SNARC effect 44

PART II 61

EXPERIMENTAL EVIDENCE 61

CHAPTER 3 63

STUDY 1: ARE MATHEMATICAL SYMBOLS SPATIALLY REPRESENTED? A STUDY OF MINUS AND PLUS SYMBOLS 63

Introduction 63

Method 67

Results 69

Analysis 70

General discussion 71

CHAPTER 4 76

STUDY 2. SNARC EFFECT IS NOT ABOUT MAGNITUDE: A STUDY ON MIRROR NUMBERS 76

Introduction 76

4.1 Experiment 1 88

Introduction 88

Method 89

Results 92

Analysis 93

Discussion 98

4.2. Experiment 2 101

Introduction 101

Method 101

Results 102

Analysis 103

Discussion 107

4.3. Experiment 3 110

Introduction 110

Method 112

Results 113

Analysis 113

Discussion 117

General Discussion 117

CONCLUSION 123

REFERENCES 130

INTRODUCTION

It is said that engineers talk to physicists, physicists talk to mathematicians, and mathematicians, finally, talk to God. This aphorism can be used to summarize the modern view of mathematics, sometimes known as "the queen of sciences." The principles of thermodynamics, the theory of relativity, the law of universal gravitation, the efficacy of a vaccine, and many other theories on which modern society is founded owe their existence to mathematics. In a sense, mathematics is the primary religion of contemporary society, and, like other religions, it has a profound impact on how people think and act. Mathematics allows humans to get in touch with the essence of reality, to understand time, space, natural phenomena, predict the weather, build machines, and discover black holes before we even see them, as if it was our sixth sense.

What is special about this discipline is that while medical, psychological, economic, political, and ethical concepts are a product of our existence, mathematics do not need humans to exist. Indeed, mathematics has operated for billions of years without our help. The invention of numbers does not correspond to the birth of mathematics, just as the invention of the light bulb does not correspond to the birth of light, or the invention of music does not correspond to the birth of sounds. The light bulb, the music and the numbers are tools that allow us to use, in a schematic and original manner, something that already exists (photons, vibrations and quantities).

In the first chapter of this thesis, an attempt will be made to answer this question: how did numbers originate and develop in our society? Archaeological discoveries have played a central role in answering this question, suggesting that early humans used stones and notched bones to keep track of their livestock and possessions. It is important to understand that the history of mathematics is not unique but has developed independently in different parts of the world. This fact has led some researchers to assume that humans have an innate predisposition for numerical concepts. However, the presence of indigenous anumerical populations suggest that the development of numerical concepts is not entirely natural and innate.

Nowadays, mathematical thinking is studied by the field of numerical cognition who uses mathematical and statistical measurements (e.g., on reaction times, errors, and brain activities) to study how people think and represent mathematical concepts.

In the fundamental work of Dehaene et al. (1993) it was shown that, through a “parity judgment task” in which participants were asked to categorize numbers into odd or even, reaction times were faster when responding to small numbers by pressing the left key and to large numbers by pressing the right-hand key (SNARC effect; Spatial Numerical Association of Response Codes). This finding has been explained through the presence of a mental number line through which quantities would be organized from left to right in Western societies (i.e., Dehaene, et al., 1993; Galton, 1880; Gerstmann, 1924; Hubbard, Piazza, Pinel and Dehaene, 2005).

In the second chapter, the relationship between numbers and space will be extensively discussed. Based on the most recent information in the literature, the importance of quantitative information in explaining the SNARC effect will be challenged. In fact, other features of numbers could explain the SNARC effect, such as, for example, the order in which numbers are typically represented from left to right. Such an order might be sufficient to bring out the SNARC effect, without the need to process the quantitative information. This would explain the presence of SNARC-like effects with nonnumeric stimuli, such as days of the week and months (i.e., Gevers et al., 2003; Gevers et al., 2004).

Chapter 1 and Chapter 2 represent the theoretical part of this thesis, while the experimental part consists of Chapters 3 and 4.

In order to demonstrate that the relationship with space is not limited to numerical stimuli, in Chapter 3 we will extend the concept of spatial association to mathematical minus ("-") and plus ("+") symbols. Specifically, we will perform a spatial compatibility experiment in which the minus and plus symbols will be used as fixation symbols to understand whether the meaning of these symbols is able to influence participants' reaction times. The hypothesis is that minus facilitates responses on the left, while plus facilitates responses on the right. This would suggest that even without processing numerical quantities, spatial association of mathematical symbols is possible.

Chapter 4 will evaluate the relationship between quantity and spatial representation by studying mirror numbers in three experiments. Mirror numbers will be used because, although they denote a quantity, they are not typically ordered from left to right. This feature provides insight into whether the SNARC effect depends on quantity processing.

In Experiments 1 and 2 participants will perform a magnitude classification task (participants must indicate whether the number presented is smaller or larger than 5) with normal and mirror numbers. In Experiment 1 normal and mirror numbers will be presented in the same task, while in Experiment 2 one group of participants will perform the task with normal numbers only and another group will perform the same task with mirror numbers. The hypothesis is that, even if participants process the quantity of both normal and mirror numbers, the SNARC effect will emerge only with normal numbers, because the SNARC effect depends on the order in which the numbers are arranged and not on their quantity.

Experiment 3 aims to confirm the results of Experiments 1 and 2 in a task in which quantity information processes are not explicitly required. In this experiment, a new task will be introduced to evaluate the SNARC effect: the mirror judgment task, in which participants are asked to indicate whether a number is presented in its normal or mirror version.

Additional information will be considered in the last chapter: the role of the structural part of numbers. Despite the large number of studies that relate numerical quantities to their spatial organization, not much research has focused on the perceptual structure of mathematical symbols. For example, the ability to recognize the number one ("1") from the number seven ("7") is related to the visual-spatial perception of the structural components of these symbols. Considering the close relationship between mathematics and space, it is important to understand whether the perception of the shape of mathematical symbols plays an important role in this relationship. To do this, Cohen's method of measuring the physical similarity of numbers will be used (Cohen, 2009). The hypothesis is that the perception of the physical part of numbers can speed up or reduce participants' reaction times even when participants are not explicitly asked to process it.

In conclusion, during this thesis, it will be discussed how human beings invented numbers, how they organize them spatially, and whether there is a connection between their structural and semantic parts.

PART I

THEORETICAL PANORAMA

CHAPTER 1

FROM QUANTITIES TO NUMBERS: THE EVOLUTION OF MATHEMATICS

1.1 The non-numeric Sapiens

The ability to perceive and estimate quantity is present in many animal species as it is characterized by considerable adaptive value. One thinks, for example, of the ability to make decisions essential for survival, such as escaping from a larger herd (i.e., McComb, et al., 1994), choosing where to migrate based on the amount of resources available, or distinguish which of two amounts of food is the greater (Agrillo et al., 2012; Emmerton, 2001; Ujfalussy et al., 2014; Uller et al., 2003).

It is reasonable to assume that all current number systems are an evolution of this basic skill that has developed independently in many animal species found in different parts of the world. The presence of approximation abilities in some mammals such as mice (i.e., Platt & Johnson, 1971) has led some researchers to think that our common mammalian ancestor, who lived about sixty million years ago, may also have been able to discriminate quantities and have an approximate sense of numbers (Everett, 2017).

The presence of a quantity recognition system and a quantity approximation system, however, are not sufficient to explain the emergence of a number system as we know it today. Many animal species possess these abilities, but none of them developed a symbolic number system, so why did humans succeed?

About 7-8 million years ago the first hominids separated from the apes (Langergraber et al., 2012; Pilbeam, 1966; Sarich & Wilson, 1967; Wood, 2010), but they lived similarly to them for many generations. About 3 million years ago hominids began to build tools (Stone Age), however, this was certainly not enough to develop mathematical skills.

The first significant changes began to be observed about 1.4 million years ago, with the appearance of Homo erectus (Dubois, 1937; Theunissen, 1988) to whom the use of fire for cooking food is attributed. According to many researchers (i.e., Leonard et al., 2007; Wrangham et al., 1999) the cooking of food played a key role in human evolution because it allowed more elaborate and more nutritious foods to be eaten. In this way, a large brain (which can consume more than 25% of the body's energy) could receive the proper nutrition. In addition, cooking food reduced the hours our ancestors spent eating, allowing more time for social activities.

Another important role was played by upright walking, which, in addition to increasing visual stimuli to the external environment, allowed greater use of the hands. From then on, the parts of our body that we look at most during the day are our hands which, as we will see later, play an essential role in the development of numerical skills. In addition, the upright position made women's hips narrower favoring the survival of infants who were born prematurely and, therefore, with a very high dependence on parental care (Haeusler, 2021). The birth of highly dependent infants incentivized the plasticity of the newborns and the need for cooperation with members of the same group.

Living in groups was not only a solution to survival needs, but also a real challenge. Community life is full of tasks requiring basic mathematical skills, such as communicating the distance at which a dangerous animal was, dividing food equally among group members and choosing the size of the animal to be hunted according to the number of hunters.

Prior to the Agricultural Revolution, for millions of years, humans had lived as hunter-gatherers going about their day (Berbesque et al., 2014), however, the gradual transition from nomadic life to sedentary life (about 10.000 years ago), brought the need to plan food production, to keep the number of people under control, to measure water levels, to track time, to tax the population and to measure the size of the lands. We can assume that community life has strongly stimulated the need to develop mathematical abilities (Friberg, 1984) because it entailed the need to measure and think in mathematical terms. Without a proper mathematical system, all societies were destined to remain simple and with few individuals (Harari, 2014).

Early counting systems were nonverbal and made use of a rudimentary counting called one-to-one correspondence, in which each element (i.e., an animal) corresponded to an object such as, for example, a pebble. Every day when the shepherd returned with the sheep, for each sheep that returned in the cove he would move a pebble. If one of the pebbles was not moved, it meant that a sheep had been lost (D'Amore & Sbaragli, 2017). During the counting of the animals, the pebbles were moved from left to right, from right to left, from down to top or from top to down. This means that the increase in quantities for Sapiens living without numbers more than 30.000 years ago followed a spatial direction.

This probably represents one of the first times that quantities were processed by humans following a spatial direction in which small quantities corresponded to the initial position of pebbles and large quantities to the final position of pebbles.

Other tools used to count animals and mark time were carved bones (see, for example, figure 1). The use of carved bones has been documented by many archaeological discoveries (i.e., D'Errico et al., 2012; Overmann, 2016). For example, in 1930 the archaeologist Karl Absolom discovered a 33.000-years-old wolf bone in Czech Republic. The bone is marked with 55 tally marks arranged into groups of five. This organization makes archeologists think that the owner of the bone used a quinary system (i.e., a numeral system with five as the base).

Figure 1 A Neanderthal may have recorded numerical data in the markings on this hyena bone. Photo by d'Errico et al., 2018.

It is believed that our ancestors held bones like these in one hand and counted by running their thumbs along the notches. Since the most natural motion for a thumb sliding along a stick in our hand is from top to bottom, probably, the increase in quantities, followed a spatial direction in which top corresponded to small quantities and bottom to large quantities.

The use of notched bones should be seen as an evolution of the pebble count. In fact, whereas during counting with pebbles the quantity is related to the volume occupied by the pebbles, in the case of the carved bone the segments organize the quantities in a linear manner. While the value of each pebble is equal to one, the value of each segment depends on the position of the segment: for example, when a person had his thumb at the half of the bones, he knew that he counted half of the quantities. However spatial representation of numbers by notches was not easy to use without a further spatial organization of them. The wolf bone consisting of 55 notches, suggests that 33.000 years ago Sapiens understood that visualizing the number 11 was easier by dividing the notches into groups of 5 because seeing “IIIII-IIIII-I” it’s easier than seeing “IIIIIIIIIII”. This organization is the first evidence of the presence of the “subitizing” ability (Clements, 1999; Wender & Rothkegel, 2000), i.e., the ability to instantly recognize the number of the objects without actually counting them.

It is important to note that, whereas during the "one-to-one correspondence phase," an increase in volume corresponded to an equal increase in quantity, the situation is different for the notches. In fact, the sequence “IIIII-IIIII-I” occupies more space than the sequence “IIIIIIIIIII” despite representing the same amount. The arrangement of the notches into spatially structured groups of five possibly marked the first instance in which the increase in the sequence's volume (caused by the empty space) was not accompanied by an equal increase in its quantity.

Although counting systems have evolved in different parts of the world, the organization of quantities by 5 and by 10 are very widespread. A convincing explanation is that the Sapiens who developed counting systems, although very different from each other, had a very powerful calculating tool in common: their hands (Andre et al., 2007; Imbo et al., 2011).

1.2 Numbers from hands to mind

The discovery of bones with notches brought to light a fundamental aspect: counting instruments never have fewer than five notches. This means that for the counting of small quantities (e.g., less than ten) the counting system was different. A widely accepted hypothesis is that for counting small quantities, human beings made use of another counting instrument: their fingers (Overmann, 2021). A shepherd with less than ten sheep, for example, did not need to use pebbles to count them, because he could make use of his fingers.

While counting with pebbles and notches did not link quantities to specific body parts, using fingers to count small quantities helped organize quantities in relation to hands. Since the ten fingers of the hands are half on the left side of the body and half on the right side, the representation of quantities greater than five entailed the need to organize the quantities in such a way that half of them were connected with the left hand and the other half with the right hand. Thanks to the hands, counting habits contribute to spatially organize numbers from left to right in western cultures (Andres et al., 2007; Fischer, 2008; Sato et al., 2007; Sato & Lalain, 2008). According to Lucangeli and Mammarella (2010, p. 24), the hand can be seen as “a succession of abstract units, due to its anatomy that allows quantitative units to be added in succession”. A similar view is supported by Everett (2017, p. 50) who speaks of the fingers as "three-dimensional anatomical lines that facilitated the representation of quantities through two-dimensional lines".

The importance of the fingers for numerical cognition (i.e., Crollen et al., 2011; Fischer, 2018) is evident in the language used by some contemporary indigenous cultures. For example, in the language of the Karitiana in the Amazon, people call the number five “Yj-pyt” which means “our hand”, and the number 11 “Myhint yj-piopy oot” which means “taking a toe” (Calude, 2021; Everett, 2006). A similar process happens with the number two “sypo”, very similar to the word “sypom” meaning “eyes” (people have two eyes, so it makes sense to use this word to refer to this quantity).

Once the names for small quantities had been created, larger quantities could be named from smaller ones, following an additive principle. A good example is the Jarawara culture in which the number two is called “Fama”, and the number four is called” Famafama” (Everett, 2012). In this case, the repetition of the word “two” forms the number “four”. This represents an example of how the naming of smaller numbers can result (not always) in a number system in which smaller numbers are combined together to form larger numbers.

Many current numerical systems follow the same rules. In English, for example, the numbers “one”, “two” and “three” are also found in the numbers “twenty-one”, “twenty-two” and “twenty-three”; the number “twenty-one” is also found in the number “one hundred and twenty-one”; and the number “one hundred and twenty-one” is found in the number “one thousand one hundred and twenty-one”, and so on.

According to Lucangeli and Mammarella (2010), the transition of quantitative concepts from the hands to the spoken form is probably due to the fact that the repeated use of the same body parts to refer to the same quantities laid the foundation for the naming of those quantities. For example, when the ancients referred to five, they could open all the fingers of one hand. By always using one hand to refer to five, some population understood that the hand corresponds to a specific quantity. The case of Karitiana can help a lot to understand the situation. In some point of the history of Karitiana someone understood that refers to a hand is the same as to refers to a specific quantity and so they used the word "Yj-pyt", which means "our hand", to refer to five (Calude, 2021; Everett, 2006).

In other words, referring to quantities using the same body parts (e.g., fingers) in succession, ends up with quantities being less and less associated with these body parts and more and more associated with memory and language, and therefore, with an increasingly abstract system of quantities (Lucangeli & Mammarella, 2010).

It has been demonstrated in numerous studies with indigenous peoples that using words to refer to numbers is crucial for developing an accurate conception of quantities (e.g., Frank et al., 2008; Gordon, 2004). In the absence of words referring to numbers, it is not possible to represent exact quantities greater than three (Gordon, 2004). That means that the language determines the possibility of the people to have an abstract representation of quantities. In the next chapter we will see that the abstract representation of quantities is organized following a left to right organization in Western cultures (Dehaene et al., 1993).

1.3 The numbers we use

Over 10,000 years ago, with the advent of the first agricultural revolution (or Neolithic revolution), humans began cultivating a few varieties of seeds, raising animals, living a sedentary life, and, most importantly, thinking in mathematical terms. This radically changed their way of life and, above all, their way of thinking. Researchers agree that the first place where this change radically occurred was in the Middle East.

Basic quantitative representation systems were probably invented much earlier than we think, but only in the case of more complex civilizations (such as the Sumerians, Egyptians, and Incas) we are able to find archaeological traces.

The creation of a mathematical system was a great challenge for homo Sapiens, who had never been used to reasoning in mathematical terms. In many cultures, quantities were defined in generic terms as “one”, “two” and “many” (orally). The lack of words to define quantities greater than three resulted in the need to resort to approximation, as is the case in many indigenous cultures (Frank et al., 2008). This happens because not having words to define quantities often results in an impairment of their understanding. The transition from the approximation of quantities to a precise definition of them is due to the emergence of names for these quantities: the naming of quantities led to the birth of numbers.

The use of symbols that referred to precise quantities is attributed to the Sumerians, who around 3300 BC used a numerical system that combined the decimal base with the sexagesimal base. This numerical system was used for trade, taxes, and day counts. Numbers were not known by the entire population, but only by a small circle of people who were trained to reason in numerical terms.

The Egyptians also played a central role in the history of mathematics. The high demand for arable land near the Nile had caused the Egyptians to take the concept of ownership seriously. This sparked a growing interest in the measurement of land and boundaries that led to the birth of Geometry (Seife, 2000). Egyptian geometry was also used by the Greeks, who used it not only to measure the land, but also, in a philosophical way, to get in touch with nature, the infinite, and the beauty of the golden ratio.

Among the most advanced and ancient numerical systems is the Babylonian system. The Babylonians introduced the idea (still in use today in our Arabic numeral system) that the value of a number changes according to its position (positional system). The positional system is due to the most famous calculator: the abacus. The Babylonians placed movable beads on the abacus that according to their position had a different value. In an abacus with three available positions, the same bead could represent 1, 60 or 3600. It was easy to distinguish 1 from 60 or 3600 on an abacus because the position of the stones was explicit: i.e., a bead in the first position represented 1, in the second position 60 and in the third position 3600. However, the problem began when the Babylonians started writing down numbers. In fact, in the cuneiform system, the same symbol represented 1, 60 and 3600 and it changed value based on the position. Consequently, the numbers 61 and 3601 were represented by the same symbols (see figure 2). In order to represent the empty space of the abacus, the

Babylonians invented a symbol without a value: the zero (represented by two oblique wedges).

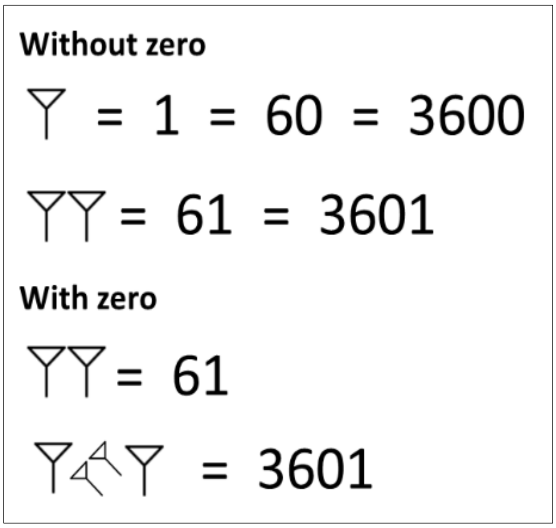


Figure 2 An example of the cuneiform system with zero and without zero. The zero is represented by two oblique wedges.

The concept of zero was used in a similar way even by the Maya, however it was not considered a number until 1500 A.D., when the word “zero” appeared in India (Coe, 2012). The Hindu name for “zero” was “sunya” that meant “empty”. While the concept of zero was difficult for the Greeks to accept and was described by some as "a dangerous idea" due to philosophical problems (Seife, 2000), for the Indian culture, which was more open, for religious reasons, to the concepts of "emptiness", "infinity" and the "everything that comes from nothing", zero began to establish itself in the mathematical world.

Besides the zero revolution, the Indians are the inventors of what we call the “Arabic numerals” that should be called the “Indian numerals” (Chrisomalis, 2004). While the Egyptians and Greeks can be defined as the creators of geometry, the Indians are definitely the inventors of algebra, able to perform addition, subtraction, multiplication, and division without using an abacus, but simply using mathematical tricks with numbers. To the Arabs go the merit of refining the system developed by the Indians by adding mathematical symbols (i.e., addition and subtraction symbols) and of spreading this system to the Middle East and Europe (Seife, 2000). Today, the Arabic numeral system is the most widely used language in the world.

1.4 Ten symbols for every quantity

It's important to distinguish between numbers and numerals before discussing numbers in any further detail. While the numbers are expressed by visual symbols (digits), numerals are the word who referred to numbers. Most of the time in this dissertation, we will discuss numbers rather than numerals.

A number is a symbol with a structural part (given by its physical form) and a semantic part (given by the information to which it refers). The structural part, in the case of Arabic numerals, is the particular shapes from which the ten numbers (0 to 9) are composed. Color, character, symmetry, and the number of segments from which the numbers are made up (for example, the number 7 is made up of fewer segments than the number 5) are examples of structural features of the numbers. The semantic part, on the other hand, refers to the meaning that the numerical symbol conveys, such as quantitative information (the size expressed by a number), ordinal information (the order expressed by the numbers), parity information (whether a number is even or odd) and other secondary information such as the affective or historical value that an important date may elicit (e.g., 0, 1492, 1969, 2020).

With the invention of numbers, a single symbol is sufficient to represent an exact quantity and the combination of Arabic numerals can represent any quantity. For thousands of years, homo Sapiens used an approximate quantity system based primarily on the visual characteristics of the observed quantity. During the one-to-one correspondence phase, the visual perception of the stimulus provides immediate access to an approximate quantity. For example, looking at nine fingers tells us explicitly that we are looking at more than five fingers.

With the advent of numerical symbols, the perception of quantities has changed radically. Human beings had to abandon the concept of one-to-one correspondence and had to readjust their perceptual system. This transition is very complex, it took a long time to fit into our history and, for some children, it probably remains an insurmountable obstacle during the cognitive development of numbers.

Whereas before the invention of numbers, access to quantitative information was directly linked to the visual information given by the stimulus, with Arabic numerals, access to quantitative information is conveyed by a person's ability to recognize a numerical symbol. An Arabic number such as 8 is an arbitrary symbol that is accepted by society as the “number eight” and can refer to an order position (in English 8th), a quantity of eight or an infinite quantity (when rotated horizontally). Similarly, while it is easy to say that the group of sticks “IIIIIIIII” are more than the group “IIIII”, looking at the symbol “9” is not sufficient to say that “9” is greater than “5”, unless one knows the meaning of these two symbols. Before the quantitative information of “5” and “9” can be accessed, the reader must be able to recognize them through a period of learning.

The visual perception of the numbers does not immediately allow access to order or quantitative information because the numerical symbol interposes itself between the reader and the quantitative information.

The idea that access to quantitative information is undirected due to the symbolic nature of numbers, despite its logic, is not completely accepted in the field of numerical cognition. On the contrary, the background of research into numerical cognition has been dominated by the idea that seeing a number automatically activates access to quantitative information (Dehaene et al., 1993; Dehaene & Akhavein, 1995; Dehaene et al., 1998; Fischer et al., 2003; Gevers et al., 2006; Naccache & Dehaene, 2001; Koechlin et al., 1999; Nuerk et al., 2005; Pavese & Umiltà, 1998; Tzelgov & Ganor-Stern, 2005; for a different view see Cohen, 2009).

Automatic access to quantities is supported by the fact that in many tasks where the magnitude of numbers is not explicitly asked to be processed, people respond faster with their left hand to small numbers and with their right hand to large numbers (SNARC effect - Spatial Numerical Association of response codes). In particular, in the fundamental work of Dehaene et al. (1993) it was shown that, through a “parity judgment task”, in which participants were asked to categorize numbers into odd or even, reaction times were faster when responding to small numbers by pressing the left key and to large numbers by pressing the right-hand key.

Because quantity processing is not explicitly required in the parity judgment task, many researchers have interpreted the SNARC effect as dependent on quantitative information. In other words, even when quantitative information is not relevant to performing the task, participants would organize numbers from left to right, based on their quantity.

Some researchers have hypothesized that there is a biological predisposition to associate small amounts with the left side and large amounts with the right side (i.e., De Hevia et al., 2010, 2014; Di Giorgio et al., 2019; Rugati et al., 2015, 2020). For example, Rugani et al. (2015) showed that three-day-old chicks seem to associate a relatively small number of dots with left space and a relatively larger number of dots with right space. In their experiments, chicks were presented with pairs of white panels with black dots. The chicks tended to turn to the left when there were few dots (e.g., "2") and to the right when there were many (e.g., "8"). Importantly, the same number of dots (e.g., "8") was associated by the chicks with the right-hand space when the target number was, for example, "5," while it was associated with the left-hand space when the target number was, for example, "20" (For a recent replication of the experiment see Rugati et al., 2020).

According to Di Giorgio et al., 2019 research, babies who were habituated to a numerical value (a set of 12 items) automatically associated the left side of the space with a smaller number, while the right side of space was associated with a greater number (Di Giorgio et al., 2019).

These experimental results have led some researchers to hypothesize that human mental number line may stem from an ability that evolved before language, in a common ancestor of humans and other animals (Rugani et al., 2020). However, the mental number line is not present in every animal species. For example, some studies have failed to find a spatial-numerical association with rhesus monkeys, capuchin monkeys (Beran et al., 2019) and fish (Triki and Bshary, 2018).

The existence of an innate tendency, shared by animals, primates and people, to associate small quantities with left-hand space and large quantities with right-hand space is still a source of debate. In any case, this thesis does not focus on the spatial association of quantities in general, but rather on the spatial association of Arabic numbers. Since Arabic numerals are not directly related to their semantic information, their spatial association deserves to be studied separately, without giving quantitative information more attention than other semantic information.

As Cohen (2009, p. 332) stated: “Integers are linguistic symbols; therefore, any relation between these symbols and quantity information must be indirect”. For this reason, children must undergo extensive training to connect a symbol to its numerical meaning. With training and memory, the structural part of a number is associated with the semantic part (for example, quantity information) and each time a person sees the number, accessing the quantitative information is a simple (but not necessarily automatic) task.

The access to the semantic part is not possible without the perception of the structural part (physical characteristics) of the number stimulus. The perceptual part of an Arabic number is the first information that people must process in order to recognize a number and, eventually, access its quantity representation.

Number structure research has long been overshadowed by quantity information research; however, in the last decade, the importance of number structure has grown (Cohen, 2009; Defever, et al., 2012; Garcia-Orza, et al., 2012; Quinlan & Cohen, 2020; Wong & Szűcs, 2013; Zhang, et al., 2018; Zhang, et al., 2021). To date, research on the visual perception of numbers suggests that the structure of numbers, such as the type of number system used (Quinlan et al., 2020), the degree of inclination with which the numbers are read (Yu et al., 2020) and the physical similarity between numbers, can influence access to quantitative information (Cohen, 2009).

1.5 Numbers and humans. Theoretical Summary

Numbers evolved from a basic quantity estimation system that is present in many animal species. Through genetic and environmental factors, humans evolved and were able to create more complex societies that required mathematical reasoning.

The earliest mathematical systems were not based on actual numbers but on a one-to-one correspondence representation of quantities in which each element was represented by an object such as, for example, pebbles.

When the number of objects was small, body parts could be used. The use of fingers to count (widespread in all cultures of the world) represents an important stage in the development of numerical cognition. Thanks to hands and tools quantities were spatially organized (from left to right, right to left, down to bottom or bottom to down).

With the learning of Arabic numerals, finger counting is projected onto a symbolic number system composed of a structural part (i.e., the physical form of a number) and a semantic part (i.e., the information of the number). The link between the structural part of the number and the semantic part depends on learning and memory.

Research on number cognition has produced a substantial number of experiments centered on the hypothesis of the spatial organization of numbers and has often attributed this organization to the quantities of numbers. In recent years, research is considering other important features of numbers such as the order in which they are arranged, and other structural features such as their shape. In this dissertation, the order and the structure of numbers will play a central role.

CHAPTER 2

HOW ARE NUMBERS REPRESENTED?

2.1. The introspective numbers

"The numerals 1, 2, 3, 4, &c., from the part they play in the multiplication table, have been personified by me from childhood. 9 is a wonderful being of whom I felt almost afraid, 8 I took for his wife […]. 7 again is masculine; 6 of no particular sex but gentle and straightforward; 3, a feeble edition of 9, and generally mean; 2, young and sprightly; 1, a common-place drudge. In this style the whole multiplication table consisted of the actions of living persons, whom I liked or disliked, and who had, though only vaguely, human forms" (Galton, 1880, p. 253).

In the late 19th century, Francis Galton carried out studies based on individual psychological differences. Galton asked people to describe their mental images and psychological experiences. This psychological approach was in contrast to the earlier "Behaviorism" according to which the mind was a "black box" that could be studied, without taking into account people's psychic experience (Watson, 1924).

Galton's method was later called the "Introspective method" by Wundt. This research method involves asking people to reflect on their experiences to understand the functions of the psyche. In particular, Galton collected interesting accounts of how numbers were viewed. He found that numbers were viewed as: masculine, feminine, scary, colored, visualized through dots (imaginary dominoes), and, more often, in their Arabic form (Galton, 1880).

Although Galton's method provided a variety of descriptions of quantity representations, a good portion of them is based on a spatial arrangement of Arabic numerals along a mental number line from left to right, right to left, bottom to top, and top to bottom (see Figure 3).

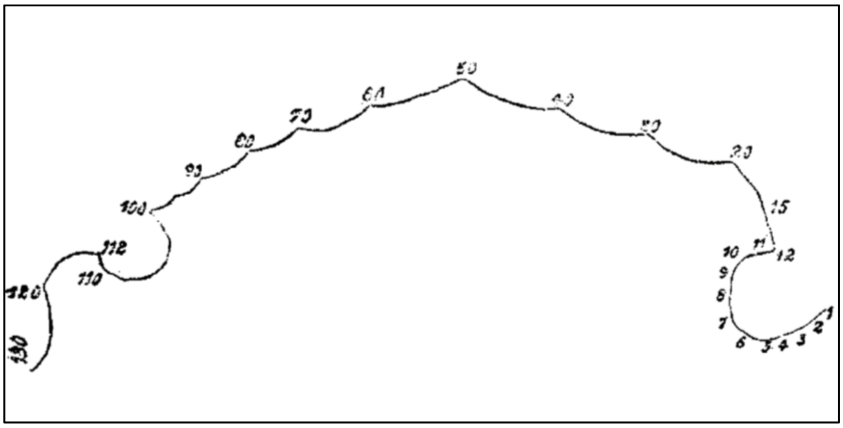


Figure 3. Arrangement of arithmetic numbers drawn by Mr. George Bidder and published by Galton (1880).

The introspective method based on subjective descriptions was only the first step toward understanding number organization. The XX century saw the transition from subjective experience to objective measurement, for example, with reaction time (RT) analysis. It is no coincidence that, even today, most studies on numerical cognition use experiments to measure reaction times rather than the introspective method (see Kadosh & Dowker, 2015 for a review on numerical cognition). The use of reaction times not only allowed researchers to obtain more objective data, but also to understand that, in some types of tasks, it is possible to predict participants' reaction times through mathematical functions.

2.2. Measuring reaction times: the distance effect

In 1967, Moyer and Landauer asked participants to decide which of two integers was greater by presenting them side by side (magnitude comparison task). The authors found that the greater the differences between the two numbers, the faster the reaction time (RT). For instance, when the pairs "1-9" and "8-9" are taken into consideration, participants report that 9 is the greater number in both cases, but they complete the task more quickly in the first pair than the second pair. This effect has been called the "*distance effect*" and it states that the larger the distance between two quantities, the easier it is to determine which is greater. In other words, as the numerical distance between two digits increases, the RTs decrease (see Cohen & Quinlan, 2016).

Some researchers suggest that the distance effect is not present only with number but also in judgments of numerosity on sets of points (Brannon, 2002; Buckley & Gillman, 1974; Xu & Spelke, 2000), luminance (Cohen Kadosh and Henik, 2006), size of numbers (Pinel et al., 2004) and in objects magnitude comparisons (Fulbright et al., 2003; Moyer, 1973).

The distance effect has been used in several works as a proof that quantity is represented spatially, along a mental number line (Dehaene, 1992; Gallistel & Gelman, 1992; Verguts et al., 2005). The Mental Number Line (MNL) account assumes that numbers are spatially organized along a continuum from left to right. Such a representation would produce the distance effect (i.e., Bull et al., 2006), so that when the numbers are close together along the MNL (e.g., 8 and 9), the time required to determine which number is larger increases compared to when the numbers are far apart (e.g., 1 and 9). This happens because the closer numbers along the MNL have similar quantities, while the more distant numbers have divergent quantities.

According to a more elaborate version of this theory, quantities are represented on an internal continuum, and when humans process the quantity of a digit, there is some perceptual variability (i.e., noise) connected to the position of the digit on this continuum (for a detailed quantitative model see Cohen and Quinlan 2016). For example, depending on the circumstance, people may judge a given quantity (for example, 5) as large or small. When compared to 1,000 the quantity 5 appears to be small, but it is not as small when compared to 6. Since the mental representation of the number 5 is noisy it can be represented with a normal distribution of quantities (see Figure 4).

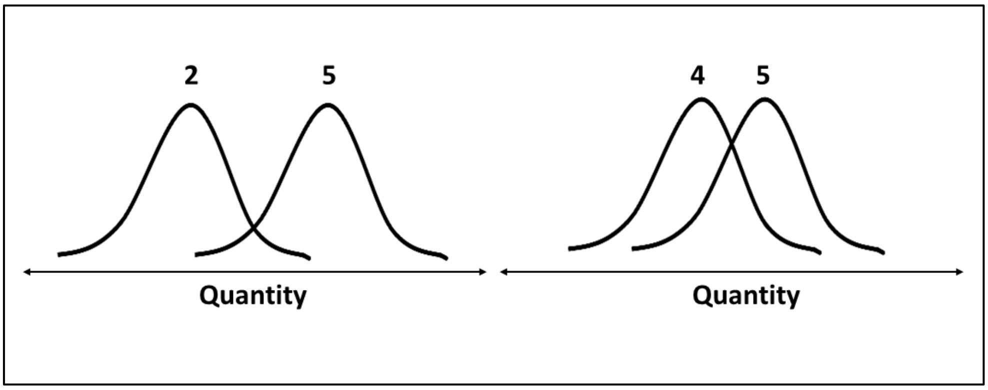


Figure 4. Distribution of quantities between numbers 2 and 5 (on the left) and between numbers 4 and 5 (on the right).

When comparing which of two numbers is greater, people compare between two quantity distributions and the RTs of the performance are proportional to the overlap of the two values. For example, the two quantity distributions of the numbers 2 and 5 overlap less than the distributions of the numbers 4 and 5, because the amount of quantity in common in the second pair is greater (Figure 4). As a result, it is trickier to determine which number is greater between 4 and 5 than between 2 and 5. In other words, the closer any two numbers are in magnitude, the greater the interference of the quantities, and thus, the longer the RTs to make the decision (Cohen & Quinlan, 2016; for an alternative explanation see Krajcsi & Kojouharova, 2017).

2.3 Mathematical functions for mathematical performances: the Welford functions

As previously explained, in the magnitude comparison task participants are asked to judge which one between two numbers is greater. Moyer and Landauer (1967) found that, in a magnitude comparison task, the RTs of the participant were predicted by a mathematical function called the Welford's function. The Welford function can explain how long it takes people to compare two numbers' magnitudes, and, more specifically, how the distance effect works (Moyer & Landauer 1967; Welford, 1960). The formula is:

$$RT = a + k * \log [L / (L - S)]$$

where “a” and “k” are constants, “L” is the larger quantity, and “S” is the smaller quantity.

The Welford function also describes the process involved in inequality judgments for physical dimensions such as line length and volume (Moyer & Landauer, 1973). This means that determining which of two numbers is greater involves the same processes as determining which of two lines is greater.

Welford's function can be understood better by considering the magnitude classification task. In the magnitude classification task participants are asked to use the right or left key to indicate whether a digit is smaller or larger than a reference number (usually 5).

In this type of task, because of the distance effect, participants respond faster to numbers farther from 5, such as 1 and 9, than to numbers closer to 5, such as 4 and 6 (i.e., Dehaene, et al.,1993; Deng et al., 2018; Wood et al, 2008). This occurs because saying “5 is greater than 1” is easier than saying “5 is greater than 4”. Consequently, when the mean reaction time (mRT) of each number is plotted on the same graph, a shape similar to a normal distribution emerges (see Figure 5).

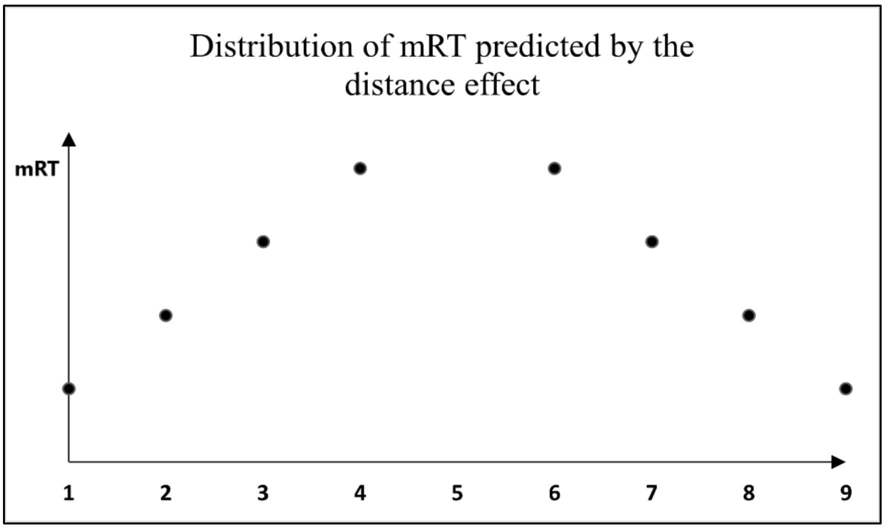


Figure 5 The averages of the reaction times of each number during a magnitude classification task in accordance with the prediction of the distance effect.

Given that the distance between 5 and 1 (distance 4) is equivalent to the distance between 5 and 9 (distance 4), the distance effect predicts the same mRT for both number 1 and 9.

When the distance effect is taken into account, it is possible to use the Welford formula to predict the RT of a group of participants during a magnitude classification task. If the magnitude classification task is performed on a group of numbers from 1 to 9 (with 5 as the reference number) the formula for the numbers greater than 5 is:

$$RT = a + k * \log [L / (L - 5)]$$

where “a” and “k” are constants and “L” is the larger number (6, 7, 8 or 9) we compare with 5.

The formula for numbers smaller than 5 will be different because, in this case, 5 is the larger number. The formula is:

$$RT = a + k * \log [5 / (5 – S)]$$

where “a” and “k” are constants, 5 is the larger number and “S” is the smaller number (1, 2, 3 or 4) we compare with 5.

By using the Welford function the prediction for the mRT for each number from 1 to 9 (excluding 5) takes a form similar to a normal distribution (see Figure 6).

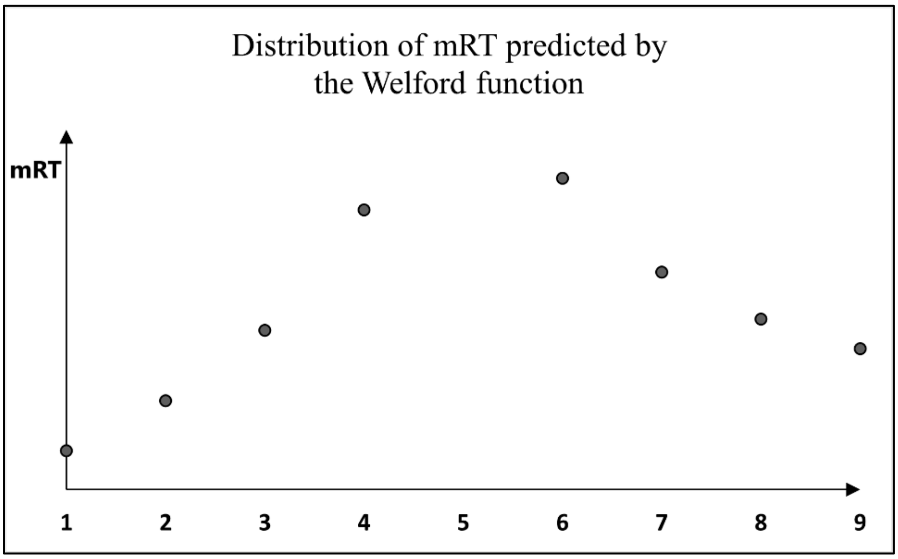


Figure 6. The averages of the reaction times of each number during a magnitude classification task in accordance with the prediction of the Welford function.

Some might note that the distribution predicted by the Welford function is not as symmetrical as that predicted by the distance effect; in fact, the reaction times for numbers greater than 5 are slightly slower than those for numbers less than 5. If we were to consider only the distance effect, the distribution should take a symmetrical form (see Figure 5). However, the Welford function doesn't take into account only the distance effect, but also another effect: the "*size effect*".

According to the size effect the more the size of two quantities the more difficult to discriminate which one is the greater quantity (Dehaene & Cohen, 1994; Mandler & Shebo, 1982; Moyer & Landauer, 1967). Moreover, this is true not only for numbers, but also for non-numerical quantities such as points (Brannon, 2002).

For example, for both numbers and points it's easier to say that 2 is greater than 1, in respect to say that 97 is greater than 96, even if the distance between these two pairs is the same (distance 1). As the magnitude increases, so does the difficulty of performing mathematical tasks such as addition, multiplication, or the magnitude comparison task. According to Campbell and Xue (2001), this happens because the mental representation of small numbers is well structured, whereas that of large numbers is noisy and requires stronger mnemonic retrieval.

The distance effect and the size effect were the first mathematical phenomena to imply that numbers are spatially organized. Of the various interpretations about these two effects, the one that has received most support is the spatial organization of numbers along a mental number line (Gallistel and Gelman, 1992, 2000). Both the distance effect and the size effect, in fact, can be explained from a spatial point of view.

In the case of the distance effect, the greater the distance between two numbers along a number line, the simpler the discrimination of their size. For example, the distance between 1 and 5 and the distance between 4 and 5 can be both represented as lines (see figure 7).

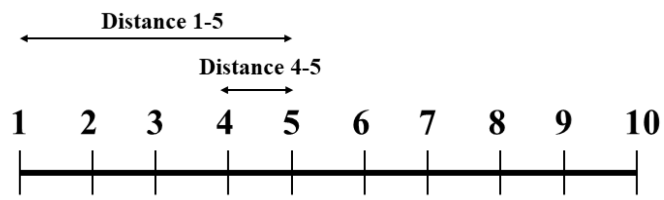


Figure 7 The mental number line according to the distance effect

The quantity difference between 1 and 5 (5 - 1 = 4) matches the spatial distance between 1 and 5 (distance 4). Due to the spatial organization of numbers, it is easier to distinguish between two quantities when the line is longer. As a result, the RTs are quicker in the pair of 1–5 than in the pair of 4-5.

In the case of the size effect the situation changes. For the size effect the larger the numbers, the more difficult it is to visualize them along the number line and, consequently, the more difficult to perform a task with them (i.e., Moyer & Landauer, 1967). According to the size effect the small numbers along the mental number line are represented as more distant than the greater numbers (see Figure 8).

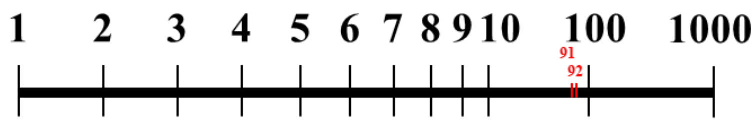


Figure 8 The mental number line according to the size effect.

For example, number 1 and number 2 are represented as more distant than number 91 and number 92 even if the distance between them is the same (distance 1).

Research on the distance effect and size effect (e.g., Buckley & Gilman, 1974; Dehaene et al, 1990; Dormal et al, 2006; Fias et al, 2003; Kaufmann et al, 2005; Moyer & Landauer, 1967; Pinel et al, 2004; Restle, 1970) and the possibility of using mathematical laws, such as the Welford function or the Weber-Fechner function (Dehaene, 2003: Ditz & Nieder, 2016; Weber, 1851) to describe these effects, sparked a growing interest in the field of numerical cognition, which led to the pioneering work of Dehaene et al. (1993) and the discovery of the SNARC effect, one of the most discussed effect in the field of numerical cognition.

In the experimental part of this thesis, the distance effect, and the size effect (measured through the Welford function) will be used to understand whether participants process the quantity of numbers. The presence of the Welford function will be taken as evidence of the processing of the quantity of numbers, while the absence of the Welford function will be taken as evidence that participants do not process the quantity of numbers.

2.4 From left to right: the SNARC effect

With the discovery of the distance effect and the size effect, it began to be understood that the RTs of participants, performing tasks such as the magnitude classification task, could be predicted with mathematical laws derived from psychophysics. The prediction of these laws, however, was limited to the mean of the reaction time (mRT) without taking into account whether responses were given with the left hand or the right hand.

In 1993 the situation changed with the fundamental studies of Dehaene et al. (1993). In their study the authors randomly presented individuals with one integer (from 0–9) and asked them to judge whether the number was odd or even (parity judgment task) by pressing two different keys (left key or right key). The results indicated that western participants responded faster to smaller numbers when using the left key and faster to larger numbers when using the right key. The authors named this effect the Spatial-Numerical Association of Response codes (SNARC) and it has been replicated with a variety of types of tasks (Deng et al., 2018; Fischer & Fias, 2005, for a review; Holmes & Lourenco, 2013;

Holmes & Lourenco, 2019; Hubbard, et al., 2005; Tan & Dixon, P, 2011; Wood, et al., 2008, for a meta-analysis).

Together with the distance effect and the size effect, the SNARC effect has been often taken as a proof of the existence of the Mental Number Line (MNL; Dehaene et al., 1993). According to the MNL account, the mental number line is a semantic representation of numbers in which digits are organized in the long-term memory (LTM) in ascending order from left to right in Western cultures (see Figure 9). According to this explanation, the SNARC effect is originated by an automatic activation of the quantity representation of numbers (Cho et al., 2012; Dehaene et al., 1993; Holmes & Lourenco, 2011; Kirjakovski and Utsuki, 2012; Fischer et al., 2013; Nuerk, et al., 2005; Nuerk et al., 2011; Tan & Dixon, 2011). This happens because the connection between the number and its quantitative representation is recorded in the LTM (Dehaene et al., 1993, Hubbard, et al., 2005, Aulet, et al., 2021), and seeing a number would automatically activate its quantitative representation.

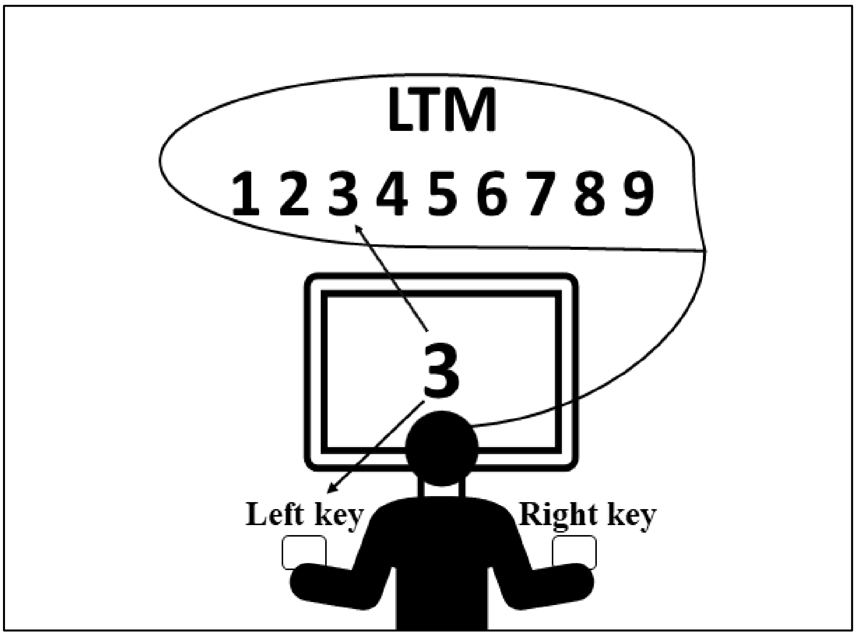


Figure 9 The SNARC effect in the MNL account. The mental number line represents quantities in long-term memory (LTM) from left (small numbers) to right (large numbers). When a participant sees a number (e.g., 3) in the center of the screen, she/he is faster to press the button compatible with the position of the number along the MNL (e.g., left).

An important prediction of the MNL account is that deeper quantitative processing of a number leads to a stronger SNARC effect because of the MNL activation (Dehaene et al., 1993; Gevers et al., 2006; Nuerk, et al., 2005). In other words, quantitative information processing would promote a left-to-right representation of numbers. Indeed, the MNL account predicts that the strength of the SNARC effect depends on the type of task and, in particular, how deeply the quantitative number information is processed. For example, the SNARC effect in a magnitude classification task (in which participants are asked to say whether a number is less than or greater than five) is stronger than in a color task (in which participants are asked to report the color of the number by pressing two keys regardless of the number magnitude). The reason is that during the magnitude classification task participants explicitly access the quantity of numbers and this would activate the MNL. In contrast, during the color task participants are not explicitly asked to process the quantity of the numbers, so MNL activation (and, consequently, the SNARC effect) should be weaker than in the magnitude classification task (for a review see Wood et al., 2008).

Importantly, although the SNARC effect is often reported as a facilitation effect due to hands, Dehaene et al., (1993) found that SNARC did not reverse when participants performed a parity judgment task with crossed hands (but see Wood et al., 2006). Furthermore, the authors found that, in a parity judgment task, when the interval is 0-5, the number 5 gets faster responses on the right, but when the interval is 4-9, it gets faster responses on the left (see Figure 10). This means that small and large numbers are not associated with a particular hand, and that, instead, the spatial representation of numbers is an abstract extracorporeal representation in which left space and right space are associated with relatively small numbers and large numbers, respectively. The isomorphism between the response key (e.g., left) and the position of the number in its extracorporeal representation (e.g., left) generates a congruent mapping and, consequently, a faster response (Didino et al., 2019).

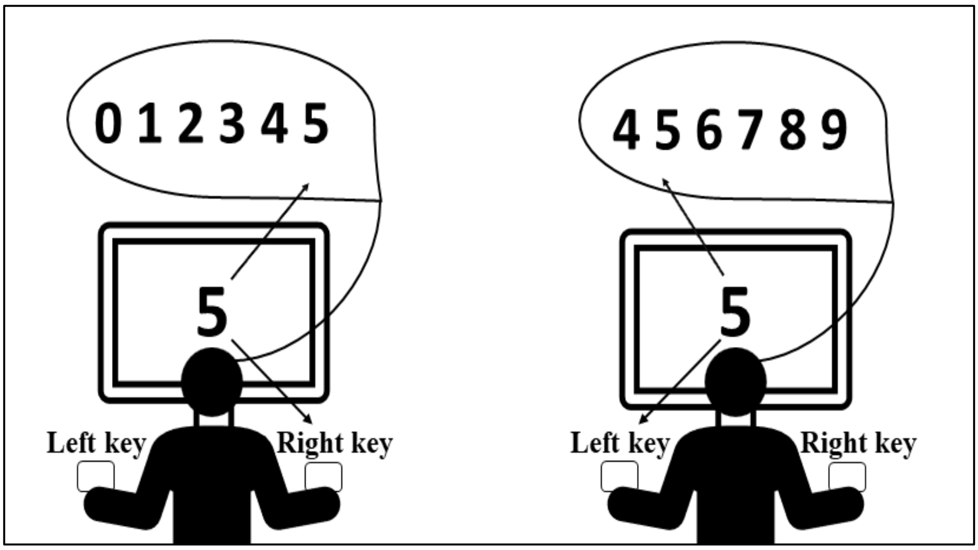


Figure 10 During a parity judgement task (Dehaene et al., 1993, Experiment 3) reaction times for the number 5 changed depending on the interval of the stimuli. Participants were faster to respond to the number 5 with their right hand when the number range was 0-5, whereas they were faster with their left hand when the number range was 4-9. These results led the author to hypothesize that “the effect would depend on the ordinal, not cardinal, aspect of the number representation” (Dehaene et al., p. 381).

Several studies have shown that the left-to-right spatial numerical association it’s not the only way to represent numbers. Indeed, spatial numerical association has been found along vertical and sagittal axes (Chen et al., 2015; Hartmann et al., 2014; Wiemers et al., 2014; for a review see Winter et al., 2015). In particular, small numbers are associated with space at the bottom and large numbers with space at the top (vertical axis); similarly, small numbers are associated with space near us and large numbers with space far from us (sagittal axis). In a recent study, Aleotti et al. (2021) asked participants to perform a parity judgment task with the response key positioned along three different axes: horizontal, vertical, and sagittal. The authors found the SNARC effect in each condition and, importantly, the strength of the SNARC effect did not vary significantly between the three conditions. This suggests that the spatial representation of numbers from left to right (horizontal axis) is not the dominant one (for a different view, see Holmes, 2012). The authors argue for a three-dimensional mental space of numbers in which small numbers are associated with left, lower and near space, while large numbers are associated with right, upper, and far space.

Another important finding about the SNARC effect is that the left-to-right representation of numbers can be reversed by cultural experiences (Dehaene et al.,1993; Shaki & Fischer, 2008; Shaki et al., 2009; Zebian, 2005) and temporary spatial representation (Bächtold et al., 1998; Ristic et al.,2006). For example, Shaki et al. (2009) found a cultural difference between Canadian people who shown a regular SNARC effect, and Palestinian people who shown a right-to-left SNARC effect. This reverse SNARC effect for Palestinian people was explained by the common reading account which proposes that the origin of this directionality stems from reading habits. However, Ito and Hatta (2004) found a bottom-up spatial representation of numbers in the Japanese, even though they read top-down. In another study, Zebian (2005) found no SNARC effect in illiterate participants. In contrast, Arabic literate participants processed two numbers more easily when the larger number was positioned to the left of the smaller numbers, than when the larger number was positioned to the right of the smaller number. Interestingly, this effect was diminished in a group of Arabic-English bilingual individuals (Zebian, 2005).

Importantly, the SNARC effect was found with both Arabic numerals and number words (Dehaene et al., 1993; Fias, 2001, Nuerk et al., 2004; Nuerk et al., 2005). Several researchers have interpreted this result as evidence that spatial association of quantities is amodal (Nuerk et al., 2005). According with this explanation, the processing of magnitude would be sufficient to bring out the SNARC effect, regardless of the presentation modality (i.e., Arabic, verbal, auditory). This position will be challenged in the last three experiments of this thesis, in which participants will perform explicit (Experiment 1 and 2) and implicit (Experiment 3) tasks on numbers magnitude with different presentation formats (normal vs. mirror Arabic numbers). In particular, as will be shown below, the results are contrary to the idea that magnitude processing can entirely explain the SNARC effect. In fact, normal Arabic numerals and their mirror version do not show the same spatial representation from left to right.

Although MNL has received a lot of support, as we will see in the next section, many experimental results cannot be explained by the magnitude processing of numbers. This is why it is important to consider that numbers do not only carry quantitative information, but also other types of information such as ordinal information. Ordinal information refers to the position indicated by numbers regardless of their quantity. Quantitative and ordinal information often coincide, which is why it is difficult to understand whether the SNARC effect depends on the processing of numerical quantities or on the ordinal position of numbers. The number on this page, for example, has both a quantitative value (how many pages you have read) and an ordinal value (in which part of the book the page is located). If we were to find a SNARC effect using the page numbers of this thesis as stimuli in a magnitude classification task, it would be difficult to know whether to attribute this effect to quantitative or ordinal information. For this reason, in the experimental part of this thesis, an attempt will be made to understand whether or not quantitative information is indispensable for the SNARC effect to emerge.

2.4.1 Beyond the MNL account: alternative explanations for the SNARC effect.

The MNL is the most famous explanation of the SNARC effect and has dominated the theoretical landscape of SNARC effect research for decades. The central idea is that the SNARC effect depends on the spatial representation of quantities in the LTM. The MNL account is a direct and easy-to-understand model, however, in the last decade, it has been challenged from other theories, such as the dual route model and the Working Memory (WM) account.

The presence of the SNARC effect has been taken as evidence of quantitative number information processing in many studies. For example, the presence of a SNARC effect in the parity judgment task by Dehaene and colleagues (1993) has been cited many times as evidence that participants automatically process the quantitative information of numbers.

According to many interpretations, just seeing a number is sufficient to access its quantity automatically. However, in recent years, more and more studies are testing a different hypothesis: quantity is not necessary to represent numbers from left to right, and ordinality may explain the SNARC effect (for an updated debate on order vs. magnitude hypothesis, see Casasanto & Pitt, 2019; Pitt & Casasanto, 2022; Prpic et al., 2021).

The order hypothesis is supported by the fact that a SNARC-like effect is present not only with numbers, but also with days of the week (Gevers et al., 2004), letters of the alphabet, months (Gevers et al., 2003), emotional valence (de la Vega et al., 2012), musical pitch (Rusconi et al., 2006), and learned number sequences that do not respect an increasing quantity representation (van Dijk & Fias, 2011).

The SNARC effect with the days of the week or months can't be attributed to a tendency of people to automatically perceive quantities, but to the fact that days and months are organized with a certain mental order. One might think about days and months as a temporal line that people organize spatially. Culture probably plays an important role in this organization. Indeed, despite the idea of time is abstract, most cultures around the world believe that time has a spatial direction. This orientation is left to right in Western cultures (Santiago et al., 2007; Ulrich and Maienborn, 2010): the timeline, the calendars, the progress bar of music and videos are examples of time motion in relation to our perception of time (see Figure 11). Additionally, another appropriate demonstration of time perception (very common in Italian culture) is to move the hand backward to allude to the past and to rotate the index finger forward (clockwise) to allude to the future.

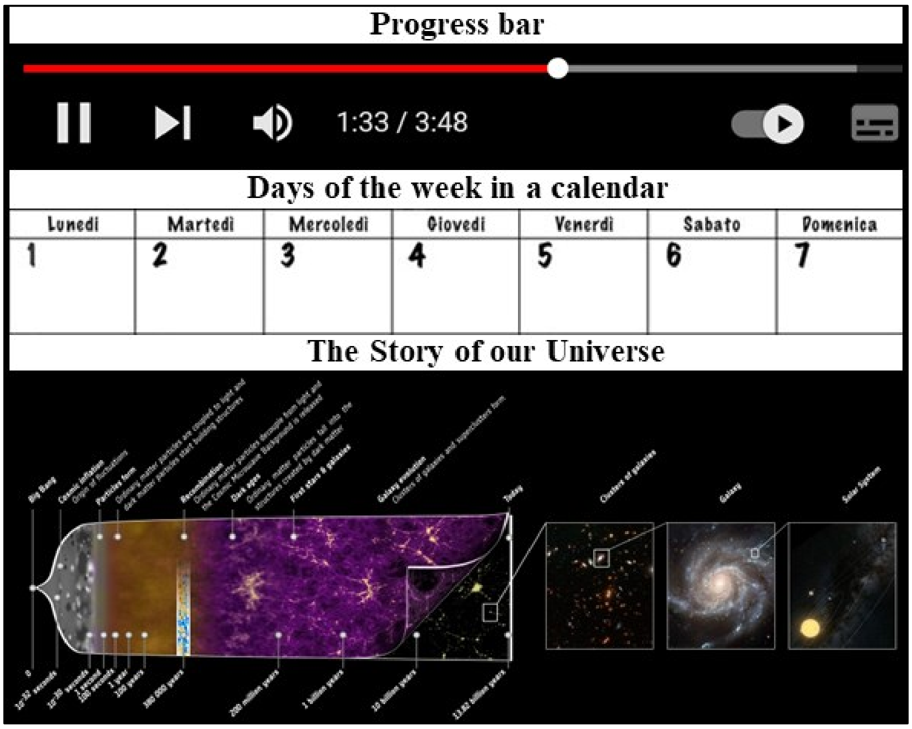


Figure 11 In western culture the progression of time is represented from left to right. From top to down: in a progress bar the white circle and the red color move from left to right to indicate the time of the music/video; in a calendar the days of the week are reported from left to right; an image on NASA's website summarizes 14 billion of our universe's history by representing it from left to right, with the Big-Bang on the left and the present on the right (for a larger representation of the history of our Universe see: https://www.nasa.gov/mission_pages/planck/multimedia/pia16876b.html#.Yy129nZBy5d).

Cultural background appears to have an impact on how we perceive time (Boroditsky, 2001; Oppenheimer and Trail, 2010; Van Elk et al., 2010; Dijkstra et al., 2012; Farias et al., 2013). Indeed, not all cultures assign the past to the left (or behind us) and the future to the right (or in front of us). People who speak the Aymara language, for example, perceive the past ahead of their bodies and the future behind them. The basic idea is that people already know the past, so they can “see” it in front of them (Núñez & Sweetser, 2006). Another example is the aboriginal culture of Thaayorre (Australia), in which time sequences are not ordered from left to right, but from east to west in relation to the position of the sun (Boroditsky & Gaby, 2010) and in accordance with the words they use to talk about time (Gaby, 2012).

The spatial organization of abstract concepts such as time is called “orientational metaphor”. The orientational metaphor is applicable to many abstract concepts that do not represent specific quantities such as "up and down" emotions (Crawford et al., 2006; Casasanto and Dijkstra, 2010) and "left and right" political orientation (Goodsell, 1988; Oppenheimer and Trail, 2010). This suggests that even the spatial organization of numbers from left to right may not depend on quantitative information.

In the wake of this evidence, Casasanto and Pitt (2019) suggest, that the SNARC effect does not imply the presence of quantitative information access (for a different view see Prpic et al., 2021), but the tendency of people to spatially organize nonspatial information (i.e., numbers, time, brightness, words, and emotions). In this sense, nonspatial information becomes spatial, as it is organized in a cognitive space that can follow a left-to-right direction (see also Pitt & Casasanto, 2022).

Nevertheless, according to Pripc et al. (2021) some experimental results cannot be explained without taking magnitude into consideration. For example, Sellaro et al. (2015) asked participants to categorize pictures or words of animals or inanimate objects. Although the set consisted of 48 stimuli, a SNARC-like effect was found for typical item size. Given the limitations of working memory, Pripc et al. (2021) suggest that it is unlikely that the entire set of 48 stimuli could have been organized in an ordinal way in working memory. According to the authors, the SNARC-like effect can be attributed to the size of the stimuli and not their order. However, as Cassasanto and Pitt (2022) note: “there is no reason to believe that the working memory resources that are required to construct a spatial mapping on the basis of magnitude would be less than those required to construct one on the basis of ordinality.” Furthermore, Prpic and collegues hypothesize that in copresence of size and order, size should prevail (Prpic et al., 2021; for a different view see Casasanto & Pitt, 2019; Pitt & Casasanto, 2022). This hypothesis will be challenged by the results of the last three experiments in this thesis.

The explanation that the SNARC effect is determined by the order of numbers rather than the representation of their quantities is well integrated into the Working Memory account (Van Dijk & Fias, 2011). In the experiment of Van Dijk and Fias (2011) five random numbers, ranging from 1 to 10, were displayed sequentially in the center of the screen, and participants were asked to remember them in the correct order. Following the presentation phase, all numbers from 1 to 10 were shown in a random order in the middle of the screen. Participants were required to perform a parity judgment task only for the five numbers that appeared during the presentation phase. The findings revealed that, instead of the typical SNARC effect, the left-hand responses were quicker for the numbers presented in the first positions in the sequence displayed during the presentation phase, and the right-hand responses were quicker for the numbers presented in the last positions (see Figure 12). In other words, response times were not facilitated by the magnitude of the numbers, but by the sequential order of the presentation phase. The authors called this effect the SPoARC effect (spatial–positional association of response codes; van Dijck & Fias, 2011).

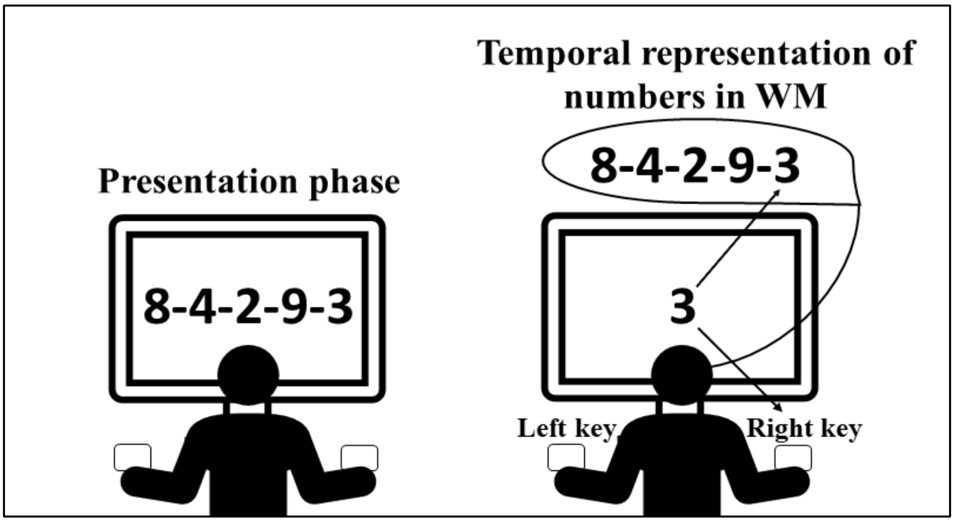


Figure 12 A representation of the SPoARC effect in the experiment of Van Dijck and Fias (2011). Participants viewed a sequence of five numbers in the presentation phase (left). They then performed a parity judgment task. The results showed no SNARC effect, but faster responses depending on the position of the numbers in the presentation phase. For example, when the number 3 was the last in the sequence, participants responded faster to 3 with their right hand because of the compatible mapping between the right key and the temporal representation of the numbers in the WM (right).

The SPoARC effect could not be explained by the MNL account because the sequential order of numbers presented in the presentation phase were not spatially organized in LTM. Instead, they proposed that participants created a MNL based on the order of the temporal representation of numbers in working memory.

Based on this finding, the authors presented an alternative model to explain the SNARC effect: the working memory (WM) account. According to the working memory account, the MNL is a temporary representation created by WM to better organize information for the task (Bachtold et al., 1998; Casasanto &

Pitt, 2019; Mingolo et al., 2021; Pitt & Casasanto, 2022; Prpic et al., 2016; van Dijck et al., 2009; van Dijck & Fias, 2011; van Dijck, et al., 2012; van Dijck, et al., 2014; Wang, et al., 2018; Zhang, et al., 2021).

For instance, Bächtold et al. (1998) instructed participants to read digits in the middle of a clock to determine if the time was earlier or later than six by pressing two different keys. The authors found a reverse SNARC effect (faster left response to large numbers and faster right response to small numbers). Since on a clock numbers smaller than 6 are on the right side and numbers greater than 6 are on the left side, the spatial organization of the number does not follow a MNL but a “clock organization”. The authors concluded that the task influenced how the participants spatially organized the numbers.

Prpic et al. (2016) asked professional musicians to perform three experiments with musical notes. Since the note values are represented from right to left, participants responded faster to large note values with the left button and to small note values with the right button (reverse SNARC effect). However, when the note value was irrelevant to the task, participants displayed a typical left-to-right SNARC effect based on the magnitude of the notes. These results confirmed that task requirements have an impact on the spatial organization of numbers and that the context can influence the direction of the SNARC effect (Bächtold et al., 1998; Prpic et al., 2016).

We can once more consider the magnitude classification task to better comprehend the nature of the WM account. The WM account could explain the SNARC effect on a magnitude classification task by using an ordinal organization of the digits in WM (without the need to consider a pre-existing quantity representation in the LTM). When participants are asked to press a key when they see a number less than 5, and another key when they see a number greater than 5, they may organize the groups 1-4 and 6-9 in two different mental spaces (for example, the first on the left and the second on the right). In this case, the SNARC effect is determined by the spatial organization of the numbers in the WM rather than their magnitude. This explanation has been supported by more recent studies (see Mingolo et al. 2021; Pitt & Casasanto, 2020). For example, Pitt and Casasanto (2020) stated that “the so-called “magnitude comparison task,” which is one of the classic tests of the SNARC effect, may be a misnomer since it can be performed using ordinality, alone” (Pitt & Casasanto, 2020, p. 1066). Similarly, Mingolo and collegues stated that “magnitude classification task requires participants to classify numbers depending on their ordinal position, namely before or after 5, in the MNL” (Mingolo et al. 2021, p. 1367).

In contrast to the WM account, another model for SNARC effect explanation that has received several acclaims is the dual route model (Gevers, et al., 2006).

According to the dual route model two different routes process number information: the unconditional route and the conditional route (Didino, et al., 2019; Gevers, et al., 2006; Gevers et al., 2010; Santens & Gevers, 2008). The unconditional route automatically processes the magnitude of the number, which activates a left-to-right number organization. This occurs because the relationship between numbers and their magnitude is present in LTM (as in the MNL account). The longer the unconditional route is activated, the deeper the magnitude activation, the stronger the SNARC effect. The conditional route, on the other hand, temporarily codes numbers into binary categories based on task requirements: for example, odd-left and even-right (in a parity judgment task); small-left and large-right (in a magnitude classification task). The longer the conditional route is active for the task, the more time the participant spends accessing the magnitude of the numbers via the unconditional route, the stronger the SNARC effect. As a result, the prediction of the dual route model is that the size of the SNARC effect should increase with average reaction time (Wood et al., 2008).

The Polarity Correspondence Principle (PCO; Proctor and Cho, 2006) is another model that has received support for the SNARC effect but, unlike the previous models, is not structured enough to make strong predictions. The PCO claims that participants in numerical studies frequently code their hands as left and right along a spatial continuum, with left being the negative pole (-) and right being the positive pole (+). Participants then classified each number as small or large, with small representing the negative pole and large representing the positive pole. This coding then allows participants to reply quickly whenever they see a polarity correlation between the two coded dimensions (left/small or right/large).

There is ongoing discussion about which of the models best explains the SNARC effect. In the experimental section of this thesis, four experiments will be conducted to add new evidence on this issue.

The first experiment will be a Posner-like paradigm in which the fixation symbols will be replaced by the mathematical minus and plus symbols. This experiment will confirm that a left-to-right association can be extended to the mathematical minus and plus symbols and thus that the left-to-right representation is not strictly related to numbers and quantities along the MNL. In particular, it is possible that the left-to-right representation of numbers derives from a more general representation in which the left and right spaces are associated with the minus sign and plus sign, respectively, because of culture.

In the other three experiments more than 200 participants will be tested with a magnitude classification task (experiment 1 and 2) and with a mirror judgment task (experiment 3; a new task in which participants are asked to report whether a number is presented in its normal or mirror version). In all the three experiments participants will perform the task with normal and mirror numbers. Mirror numbers are interesting because they are easily understood as representatives of a quantity, but people are not used to their structure, which makes their spatial representation along an MNL more difficult. If the SNARC effect is amodal and depends on magnitude, quantity processing for mirror numbers will be sufficient to activate the SNARC effect, regardless of the presentation modality (normal or mirror). In contrast, if the SNARC effect depends on other factors, processing the quantity for the mirror numbers will not be sufficient to activate the SNARC effect. In conclusion, these experiments provide insight into whether the SNARC effect can be explained by magnitude processing according to the MNL account or whether other models better explain this model without the need to take magnitude information into account.

PART II

EXPERIMENTAL EVIDENCE

CHAPTER 3

STUDY 1: ARE MATHEMATICAL SYMBOLS SPATIALLY REPRESENTED? A STUDY OF MINUS AND PLUS SYMBOLS

Introduction

The spatial numerical association of numbers is one of the most studied topics in the field of numerical cognition. Despite the fact that its function is still being debated, one consistent group of researchers believes that the SNARC effect is dependent on the representation of quantities from left to right in LTM (MNL account; see Dehaene et al., 1993). As already stated in chapter two, other hypotheses have put this explanation to the test. One of the more compelling explanations is that the SNARC effect is determined not by the quantity present in the LTM, but by the order with which the WM organizes the numbers during the task.

If the spatial representation of numbers depends on the quantity organized from left to right in the MNL, other mathematical symbols that are not included in the MNL should not be represented from left to right. This first experiment aims to test the following hypothesis: the spatial representation is not limited to number symbols only, but also includes mathematical symbols that are not in the MNL.

One of the effects that best describes the spatial organization of arithmetic operations is the Operational Momentum (OM effect; McCrink *et al., 2007*). According to the OM effect, while performing an approximative mathematical operation, people tend to overestimate the result of an addition and underestimate the result of a subtraction (Knops et al., 2014; Haman & Lipowska, 2021; McCrink et al., 2007; Pinhas & Fischer, 2008; Pinhas et al., 2015; Shaki et al., 2018; see also Fischer & Shaki, 2014, for a review).

McCrink et al. (2007) showed participants one set of squares, followed by a second set with fewer or more squares. Participants were asked to determine whether the square in the second set was the result of a correct or incorrect arithmetic operation (subtraction or addition). The findings revealed that participants tend to overestimate the result when adding and to underestimate the result when subtracting (OM effect).

The OM effect is defined as the tendency to overestimate the quantity of the elements measured following the resolution of an addition and to underestimate the quantity following the resolution of a subtraction (Knops et al., 2014; Knops et al., 2013; McCrink & Wynn, 2009; Pinhas & Fischer, 2008; Pinheiro-Chagas, et al., 2018). Importantly, the OM effect was discovered not only with nonnumeric quantities such as squares, but also during arithmetic calculations with numbers. Pinhas and Fischer (2008) asked participants to solve mathematical operations (addition or subtractions) and to indicate the result by moving a cursor along a continuum (where numbers were displayed in ascending order from left to right). The authors found that participants tended to move the cursor to the left after computing a subtraction and to the right after computing an addition (see also Marghetis et al, 2014).

The OM effect has generally been interpreted as an attentional movement leftward (during subtraction) or rightward (during addition) that go “too far” along the MNL (Knops et al., 2009; Lindemann & Tira, 2011; McCrink et al., 2007; for a review see Hubbard 2014, 2015). The basic idea is that participants tend to search for the solution of an arithmetic operation by moving their attention to the left (subtraction) or to the right (addition) of a MNL.

The attentional hypothesis has been supported by several studies (Casarotti et al., 2007; Di Luca et al., 2013; Fischer et al., 2003; Hubbard, 2014; Jang & Cho, 2022; Masson & Pesenti, 2014; Klein et al., 2014; Zorzi et al., 2012) and it is further supported by neuroimaging studies that have demonstrated an overlap between the brain regions involved in addition and those active during the execution of saccadic movements to the right (Knops et al., 2009).

In a recent experiment, Masson and Pesenti (2014) showed that completing complex additions and simple subtractions causes attentional shifts, which affect how quickly targets on the left and right sides of the screen are detected. In particular, a stimulus was detected faster on the left following a subtraction and, on the right, following an addition. These results were interpreted in light of an Attentional Operational Momentum (Attentional-OM): performing subtraction and addition would be able to direct participants' attention to the left and to the right, respectively. According to the Attentional-OM, during the execution of a calculation, the processes of subtraction and addition would be associated with the abstract concepts of minus and plus, and thus direct attention toward the left or right side of the mental number line (Klein et al., 2014; Masson et al., 2017, Salvaggio et al., 2022).

In all the studies cited above, an association between space and mathematical operations (subtraction-left; addition-right) is evident. This association has often been associated with quantity processing that would lead attention to move to the left or to the right along the MNL. Recently, however, this view has been challenged by new research that has found an association between space and mathematical operation symbols in contexts where there was no quantity processing.

Pinhas et al. (2014) asked participants to classify the symbols minus (-) and plus (+) by pressing two different keys. The authors found that participants were faster when the minus sign (-) was associated with the left response key and the plus sign (+) with the right response key. This effect is called “Operation Sign Spatial Association” (OSSA) and it suggests that the operation signs themselves can evoke a spatial association without the need of a magnitude process. However, before the experiment, the authors asked participants to perform tasks with numbers, so the OSSA effect could depend on the arithmetic contest of the previous tasks.

The aim of this study is to replicate the OSSA effect without an arithmetic context to demonstrate that the magnitude process is not required for the spatial representation of mathematical symbols. A second aim of this experiment is to understand whether the OSSA effect is able to affect the attention of the participants to the left and to the right.

According to the Posner paradigm (Posner et al., 1982), performance improves when an attention cue (such as a centrally placed arrow) predicts the position of a dot that needs to be located to the left or right on the screen. In order to determine whether the semantic significance of the mathematical symbols affected the participant's attention, the arrow cues has been replaced with the minus (-) or with the plus (+) symbols. In particular, participants were presented with a fixation symbol that could be a minus or a plus and were asked to respond by pressing the left key when the target stimulus appeared to the left and the right key when the target stimulus appeared to the right. It was hypothesized that the minus symbol could orient attention to the left, while the plus symbol could promote the orientation of attention to the right, regardless of the process of quantitative information.

Method

Participants

The size of the sample required to achieve 95% power to identify a significant interaction between the Fixation Symbol (minus vs. plus) and the Target Location (left vs. right) was calculated using the G*power 3.1 software (Faul, Erdfelder, Lang, and Buchner, 2007). With an effect size f = 0.25 (Cohen, 1988), the power analysis estimated a sample size of 36 participants. Forty-one students from the University of Bologna (25 females, mean age: 21.9) were tested. All the participants had normal or corrected-to-normal visual acuity, and they were naïve about the purpose of the experiment. All participants signed informed consent before the start of the experimental session.

Materials and procedure

The experiment was carried out using the online Gorilla Experiment Builder platform (www.gorilla.sc; Anwyl-Irvine, Massonnié, Flitton, Kirkham, and Evershed, 2020). Participants could only take part in the experiment via computer (tablets and cell phones were excluded). The used browser and operating system were kept under control (see Scerrati et al., 2021 for a review on online experiments).

Two fixation symbols were created: a minus and a plus. The plus symbol was created using Times New Roman font size 60 pt (44 pixels × 44 pixels, color black on a white background). The minus symbol was created by removing the vertical line from the "plus" symbol (44 pixels × 7 pixels). Both minus and plus fixation symbols were presented in the center of the screen.

The target stimulus, represented by a square (84 pixels × 84 pixels, black color on white background), appeared within a rectangle (182 pixels × 142 pixels, black border on white background) present to the left and right of the fixation point. The distance between the center of the fixation and the center of the stimulus was 268 pixels.

Participants performed 120 trials (60 trials in which the stimulus was presented on the left and 60 on the right) presented in a single block randomly and preceded by 6 practice trials. The total duration of the experiment was about 10 minutes. The type of fixation symbol (minus or plus) was balanced among participants such that half of them saw only the minus fixation point and the other half only the plus fixation symbol.

Participants were asked to perform only the compatible condition of a classical spatial compatibility task. The instructions were to press the left key when the stimulus appeared on the left and to press the right key when the stimulus appeared on the right (the “e” and “o” keys on a QWERTY keyboard without a numeric keypad, or “y” and “p” keys on a QWERTY keyboard with a numeric keypad, respectively).

Each trial started with the presentation of two rectangles, one to the left and one to the right of the central fixation point (which could be represented by the minus symbol or the plus symbol). After an interval of 1000 milliseconds (ms) the target stimulus (a black square) was presented in the center of one of the two rectangles for 1000 ms or until the participant responded. The disappearance of the target stimulus was followed by a blank of 700 ms (see Figure 13).

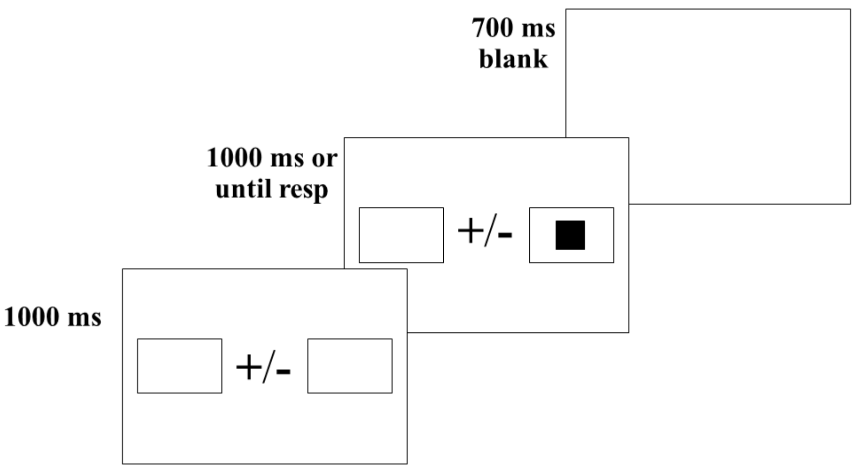


Figure 13 The design of the experiment. The fixation symbol (minus or plus) varied between the two conditions: half of the participants saw only the minus ("-"); half of the participants saw only the plus ("+").

Results

Reaction times (RTs) faster (1.2%) or slower (7.7%) by 2 standard deviations (from the mean of individual RTs), and RTs of incorrect responses (1.2%) were excluded from data analyses. One participant was eliminated because he answered only 60% of the total trials (probably due to a problem with the connection). Data from 40 out of 41 participants were then analyzed (20 minus condition; 20 plus condition).

Analysis

Repeated-measures analysis of variance (ANOVA) was performed with the within-factor *Target Location* (left vs. right) and the between-factor *Fixation Symbol* (minus vs. plus). No main effects were significant. Interestingly, the interaction between *Target Location* and *Fixation Symbol* reached the level of significance, $F(1,38) = 5.242$, $MSE = 468.938$, $p = .028$, $\eta_p^2 = .121$. Specifically, with the plus fixation symbol, responses are faster when the stimulus appears on the right than when it appears on the left (314 vs. 318 ms, respectively). By contrast, with the minus fixation symbol responses are faster when the stimulus appears on the left than when it appears on the right (315 vs. 320 ms, respectively).

Accordingly, we observe faster RTs for left responses when they are preceded by the minus fixation point than the plus symbol, and for right responses, on the other hand, faster RTs when the stimulus is preceded by the plus fixation point than the minus symbol (see Figure 14).

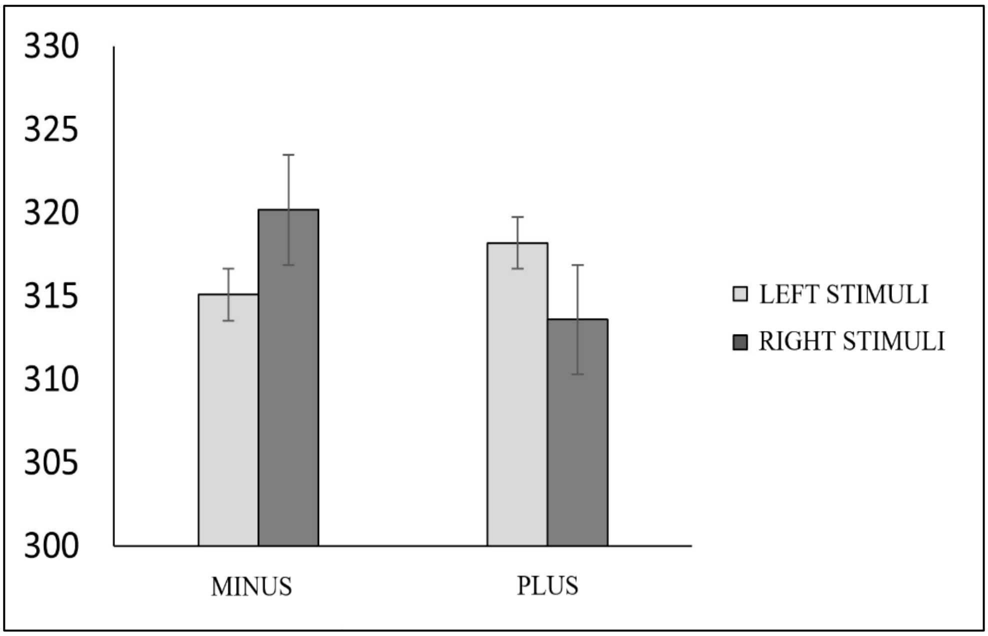


Figure 14 The graph shows an inverted mapping of RTs averages for the minus condition (left) versus the plus condition (right). Error bars are reported for each condition (minus vs. plus) and response side (left vs. right).

General discussion

Research on the SNARC effect has often suggested that spatial-numerical association depends on the mental number line (MNL). According to the MNL account for Western culture, quantities are represented along a continuum so that small quantities are to the left space and large quantities to the right space (i.e., Dehaene et al., 1993; for a review see Wood et al., 2008).

Research on the OM effect has suggested that the MNL account can explain the tendency to overestimate the quantity of the elements measured following an addition and to underestimate the quantity following a subtraction. According to this view, the OM effect would depend on the tendency of the participants to move their attention along the quantities represented along the MNL (for a review see Hubbard 2014, 2015).

In contrast, research on the OSSA effect suggests that minus and plus symbols can be associated with left space and right space in a task in which quantity process is not required (Pinhas et al., 2014). However, before the experiment, Pinhas and colleagues asked participants to perform number tasks, so the OSSA effect might depend on the arithmetic context activated by the previous tasks.

In the present study, the spatial association of minus and plus symbols during a Posner-like task was investigated in absence of an arithmetic context. The purpose of this work was twofold: first, to show that quantity processing is not required for the spatial representation of mathematical symbols; and second, to understand whether the OSSA effect is able to shift participants' attention leftward or rightward.

The results suggest that the spatial association of mathematical symbols can take place even without the processing of number quantities along the MNL. In fact, in this experiment, participants performed a Posner-like task using the minus or the plus as fixation symbols and without performing arithmetic operations on the quantities. This confirms the hypothesis that spatial association of mathematical symbols is independent of quantity processing.

Moreover, the results confirmed the replication of the OSSA effect, that is, the minus and plus symbols are associated with left and right space, respectively (Pinhas et al., 2014). Specifically, the experiment suggests that the OSSA effect affects attention. This new finding could be read in light of an Attentional-OSSA: minus and plus symbols facilitate the identification of target stimuli presented to the left and to the right, respectively. Indeed, an opposite pattern of responses was observed between the minus condition and the plus condition: in the minus condition responses tended to be faster when the target appeared on the left, whereas, in the plus condition responses were faster when the target appeared on the right.

The OSSA and Attentional-OSSA effect could reflect a polarity-based encoding of space (Fischer & Shaki, 2014) in which the minus symbol would be associated with the left space and the plus symbol with the right space. This result could be discussed through an alternative version of the Polarity Correspondence Principle (PCO; Proctor and Cho, 2006). According to PCP, participants often encode their hands as left and right along a spatial continuum, with the left as the negative pole (-) and the right as the positive pole (+). In this experiment the presence of the minus and plus may activate this correspondence. As a result, left stimuli activate a compatible mapping with the minus (-), while right stimuli activate a compatible mapping with the plus (+).

An alternative interpretation of the results may consider working memory. Van Dijck and Fias (2011) asked participants to memorize a sequence of numbers and to perform a parity judgment task using a go-no go procedure. The authors found that participants responded faster with their left hand to numbers presented to the left of the memorized sequence and with their right hand to numbers presented to the right of the memorized sequence, regardless of the magnitude of the numbers (see Figure 12). In a similar study van Dijck et al., 2014 asked participant to perform a dot detection task with numbers presented as cues and showed that participants attention was modulated by ordinal WM position of numbers but not by numerical magnitude (see also van Dijck et al., 2013). This finding suggests that the spatial numerical association is dependent on the serial order (arrangement) of numbers in WM and it doesn’t depend on the magnitude of numbers.

Several research state that to access the serial order representation of numbers and letters in WM, internal selective attention is necessary (Abrahamse et al., 2014; van Dijck et al., 2013; van Dijck et al., 2014). Although Posner's paradigm is typically used to study external spatial selective attention (Posner et al., 1982), there is a direct interface between internal and external selective attention (Lepsien & Nobre, 2006; Johnson et al., 2013; Van der Lubbe et al., 2014) so that external selective attention can be modulated by the shift of internal selective attention in WM.

In the present study, the faster responses to the left with the minus symbol and to the right with the plus symbol may depend on the correspondence between external attention and internal attention. In the case of the minus (plus) symbol, internal attention should shift more easily to the left (right) because of a temporally mental organization in WM. Consequently, when the target appears to the left (right), external attention is compatible with internal attention and promotes faster responses.

In conclusion, although the present study does not provide evidence on which model best explains the results, it confirms that the use of the arithmetic symbols minus and plus can influence the processing of target stimuli presented to the left or right during a Posner-like paradigm. Importantly, the presence of an Attentional-OSSA without an arithmetic context demonstrates that magnitude process is not required for the spatial representation of mathematical symbols.

Since no numbers were presented in Experiment 1, this study does not allow us to understand either whether or not the left-to-right organization of numbers can be explained by magnitude processing or which model best explains the spatial organization of numbers. For these reasons, normal and mirror numbers will be presented to more than 200 participants in the next three experiments to test an important hypothesis: magnitude processing does not explain the SNARC effect.

CHAPTER 4

STUDY 2. SNARC EFFECT IS NOT ABOUT MAGNITUDE: A STUDY ON MIRROR NUMBERS

Introduction

The SNARC is an interesting effect that suggests an association between numbers and space. However, there is not an agreement about what causes the SNARC effect and which models best explain it.

As discussed in previous chapters, one theory that has received acclaim is that the SNARC effect results from a left-to-right representation of magnitude along a mental number line (MNL) in which small quantities are represented on the left and large quantities on the right (Dehaene et al., 1993). When participants process the quantity of a number, its position along the MNL can be compatible or incompatible with the position of the answer key, so that a compatible mapping (small numbers-left key; large numbers-right key) generates a faster response and an incompatible mapping (small numbers-right key; large numbers-left key) generates a slower response. In summary, according to the MNL account, numerical quantity processing is central to explaining the SNARC effect and predicts its strength: the deeper the quantity information processing, the stronger the SNARC effect.

Although the MNL account has received support, as discussed in previous chapters, it is not easy to tell whether the SNARC effect depends on quantity processing or other factors, such as the order in which the digits are usually represented.

In a magnitude classification task, it is possible to monitor the presence of quantitative processing. In fact, in a magnitude classification task, in addition to the SNARC effect, the distance effect and the size effect (two effects related to quantity processing) can be measured. As explained in chapter two, such effects can be identified through the Welford function. In the next two experiments, the significance of the Welford function will be used as a confirmation of quantity processing. This method will provide insight into whether or not the SNARC effect depends on quantity processing.

As already discussed in chapter 2, the MNL account is not the only theory about the SNARC effect, and has been challenged by other theories, such as the Working Memory account and the Dual route model.

The WM account explains the SNARC effect on a magnitude classification task through an ordinal organization of the digits in WM, without the need to consider a preexisting quantity representation in the LTM. As explained above, some findings in favor of this hypothesis are the presence of the SNARC effect with letters, days of the week, and months (i.e., Gevers et al., 2003; Gevers et al., 2004), but also the existence of the SPoARC effect (Van Dijck & Fias, 2011; Figure 12).

The Dual route model (i.e., Gevers et al., 2006), as explained in chapter 2, includes two routes of parallel information processing: the unconditional route and the conditional route. The unconditional route automatically processes the number magnitude regardless of the task requirements, because the link between numbers and their magnitude would be present in LTM (as proposed by the MNL account). The conditional route, instead, temporarily codes numbers into a binary category based on task requirements. In this case, the WM would organize the information in small–left and large–right (in a magnitude classification task) or, for example, odd–left and even–right (in a parity judgment task). One prediction of the dual route model is that the longer the activation time of the unconditional route (quantity processing), the stronger the SNARC effect, because participants would have more time to represent quantities along the MNL. More simply, the strength of SNARC would depend on the latency of the response.

The purpose of this last chapter is to try to understand which of these theories best explains the SNARC effect. One of the factors that most complicates the explanation of the SNARC effect is whether it depends on the quantity or order with which Arabic numbers are often represented (Pitt & Casasanto, 2022; Prpic et al., 2021).

To overcome this limitation, it was thought to use numerical stimuli that we are not used to seeing in everyday life from left-to-right, but which could still emanate quantitative meaning: mirror numbers. Mirror numbers should be easily associated with quantities, but people have never seen them ordered from left to right.

The basic idea is that, if the SNARC effect depends on quantity, the mirror numbers should also show a SNARC effect when their quantity is processed. In contrast, if the SNARC effect depends on order, the mirror numbers should not show a SNARC effect even if their quantity is processed.

The presence of the SNARC effect will be evaluated for both normal and mirror numbers. If the Welford function and the SNARC effect are both present, quantity processing would explain the spatial organization of the numbers (as suggested by the MNL account). In contrast, if the presence of the Welford function is not followed by the SNARC effect, it means that quantity processing is not sufficient to explain the spatial organization of numbers, and that other factors, such as order, better explain this effect.

Mirror writing is the tendency to write back to front (McIntosh & Della Sala, 2012). In complete mirror writing both the digits and the direction of the writing are reversed so that the whole script appears right-to-left oriented in western cultures (Critchley, 1927; McIntosh et al., 2014). In partial mirror writing, instead, only single characters are reversed, but the writing direction remains left-to-right oriented.

Partial mirror writing is spontaneous and common among pre and early-school-age children (3 to 7 years old) during the acquisition of reading and writing skills (Cornell, 1985; Fischer & Tazouti 2012). Most people went through an involuntary mirror writing phase during childhood, which they then overcame during their early school years. However, this experience seems to persist in the case of dyslexia (Daini et al., 2018; Lachmann & van Leeuwen, 2007; Rusiak et al., 2007). This fact makes research on mirror numbers particularly interesting because understanding the quantity representation of mirror numbers could have important consequences in the area of developmental psychology.

For a long time, mirror writing was explained as a consequence of writing with the left hand (Gordon, 1921; Hildreth, 1950; Lebrun, 1990) but recently this idea has been rejected (Della Sala & Cubelli, 2007; Fischer & Koch, 2016a). An alternative explanation is attributed to the process of symmetry generalization (Corballis & Beale, 1970, 1976).

According to symmetry generalization, during the memorization of perceptual images, the brain does not take into account the details that make an image oriented to the right or to the left. The result is that the direction of an object, an image, a letter, or a number, would be generalized and memorized both in its original version and in its mirror version (see Figure 15). For example, memory representations of a character would also be recorded in its mirror version (i.e., "b" would also be recorded as "d"). Summarizing, this theory suggests that the brain has a natural tendency to treat mirror images as equivalent (Dehaene et al., 2010; Corballis, 2018; Rollenhagen & Olson, 2000).

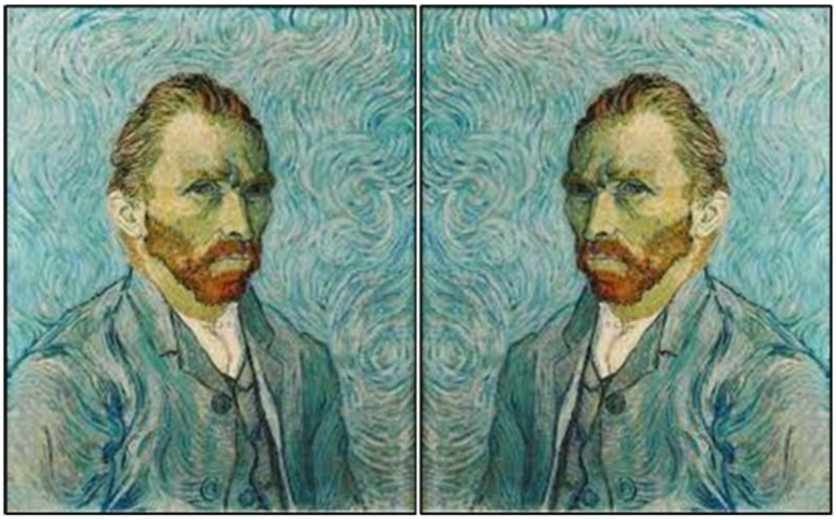


Figure 15 Self-portrait, Vincent van Gogh (1889) in its normal and mirror versions. Most people are familiar with this painting, but because of the symmetrical generalization, it is not easy to tell which of the two versions is the original one.

Despite the brain's tendency to treat mirror images as equivalent, it has been found that some numbers tend to be mirrored more than others during childhood. Fischer and coworkers (Fischer, 2013; Fischer & Koch, 2016b; Fischer

& Tazouti, 2012) found that 5 and 6-year-old children reversed the digit 3 more than 40% of the times, while they reversed number 4 less than 15% of the times. The authors explained mirror writing with the *right-writing rule*. They propose that the process of generalizing symmetry in a left-to-right reading culture causes children to reverse left-oriented characters (1, 2, 3, 7, 9, J, Z) more often than right-oriented (4, 5, 6, C, G). Although the authors do not report an objective and rigorous way to recognize the left or right orientation of the numbers, their theory seems to successfully predict which numbers are most often written in mirror image during childhood, showing that numbers' structure plays an important role in the mirror writing (see also Fischer, 2018).

The physical structure of a numerical symbol is critical to its recognition, and to accessing its information (i.e., quantity information, parity information, etc.). For example, some number symbols are physically similar to each other (e.g., 1 and 7), and this similarity affects the difficulty of distinguishing one from the other.

Research on the structure of numbers has long been overshadowed by research on quantity information, however, in the last decade the importance of number structure has been documented in several studies (Cohen, 2009; Defever et al., 2012; García-Orza et al., 2012; Quinlan & Cohen, 2020; Wong & Szűcs, 2013; Zhang et al., 2018; Zhang et al., 2021). To date, research that focuses on the perception of the numbers suggest that the access to quantitative information is affected by the structure of the numbers, such as the type of numerical system used (Quinlan et al., 2020), the degree of inclination of the numbers (Yu et al., 2020) and the physical similarity between numbers (Cohen, 2009).

The structure of normal and mirror numbers will be considered in all analyses of the following experiments. Specifically, physical similarity will be tested in each analysis to understand whether number structure can influence left-right spatial representation.

In the next experiments the Welford function (size and distance effect), the SNARC effect, and the influence of physical similarity on magnitude classification tasks with normal and (for the first time) mirror numbers will be investigated. Since people do not learn the mental number line with mirror numbers, through such stimuli, it is possible to assess the relationship between quantity and spatial organization of numbers without the influence of a learning history.

Three experiments were conducted. Experiments 1 and 2 address the relation between quantity, spatial representation, and physical similarity. In the Experiment 1, participants performed a magnitude classification task with normal and mirror numbers presented randomly within the experiment. In Experiment 2 participants performed the same task as in Experiment 1, but one group of participants saw only normal numbers, while another group of participants saw only mirror numbers. In Experiment 3, the spatial representation was studied in a task that does not require the participant to access quantity representations. In particular, a new task, the mirror judgment task, was introduced in which participants are asked to judge whether a number is normal or mirror.

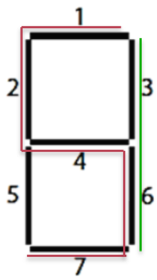
Before illustrating the three experiments, it will be explained how the physical similarity of normal and mirror numbers is measured.

How to measure the physical similarity of numbers

The influence of physical similarity will be taken into consideration in all the experiments, by considering the method proposed by Cohen (2009) for Arabic digits. Each digit was formed from a digital figure 8 (as used in digital clocks) that itself comprised seven-line segments (see Figure 16). "P," the physical similarity between any two digits, was defined as:

$$P = O/D$$

where "O" is the number of line segments that the two digits share ("Line Overlap"), and "D" is the number of the remaining nonoverlapping line segments ("Line Difference").



Target = 5, Composed of Line Numbers: 1, 2, 4, 6, 7

Numerical Symbol	Line Composition	Line Overlap	Line Difference	Physical Similarity
1	3, 6	1	5	0.2
2	1, 3, 4, 5, 7	3	4	0.75
3	1, 3, 4, 6, 7	4	2	2
4	2, 3, 4, 6	3	3	1
6	1, 2, 4, 5, 6, 7	5	1	5
7	1, 3, 6	2	4	0.5
8	1, 2, 3, 4, 5, 6, 7	5	2	2.5
9	1, 2, 3, 4, 6	4	2	2

Figure 16 The physical similarity (P) between 5 and the other single digits. For example, number 1 (in green) is composed of the segments "3" and "6". Number 5 (in red) is composed of the segments "1", "2", "4", "6", "7". Numbers 1 and 5 share only the segment number "6" (Line Overlap = 1), but they do not share the other segments "3", "1", "2", "4", "7" (Line Difference = 5). The difference between Line Overlap (O) and Line Difference (D) indicates physical similarity between number 1 and number 5 (P = 1/5 = 0.2). Figure re-adapted from Cohen (2009, p. 333).

Since Cohen's method was not designed for mirror numbers, in order to use physical similarity as a predictor in the following experiments, some adaptations were made. In particular, the Cohen's (2009) formula will be used to calculate two new types of Physical Similarity (PS) measurements: Mirror Physical Similarity (Mirror-PS) and Group Physical Similarity (Group-PS).

Mirror-PS quantifies the physical similarity between a number and its mirror version by using Cohen's formula (see Figure 17). The hypothesis is that when people see mirror numbers, they tend to convert them into normal numbers to retrieve their semantic information (i.e., order, magnitude, etc.).

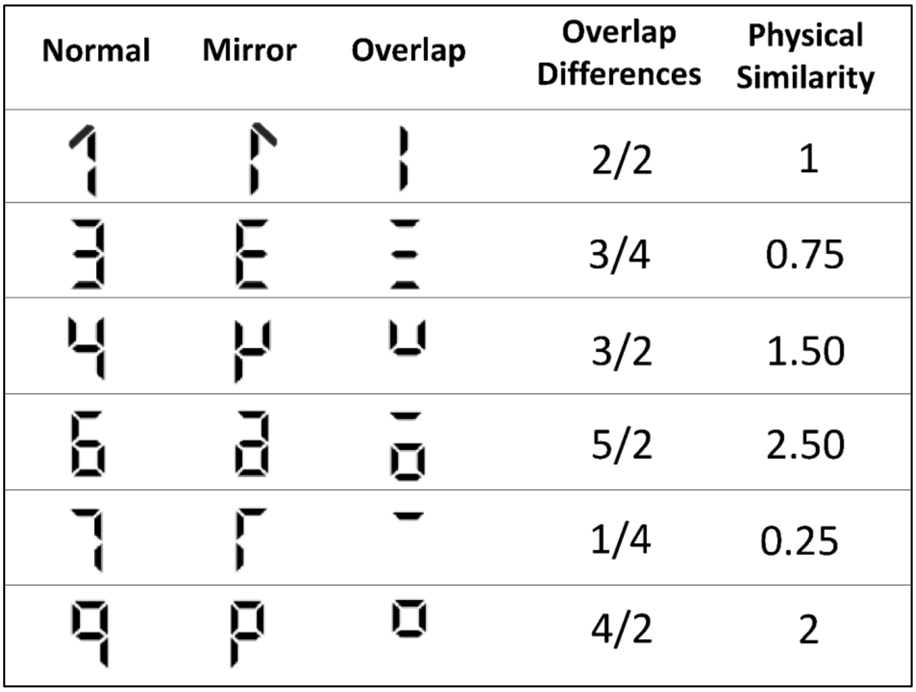


Figure 17 Mirror Physical Similarity (Mirror-PS): the similarity between a number and its mirror version calculated with the Cohen's method (Cohen, 2009).

Group-PS refers to the similarity between a number and a group of numbers. In a magnitude classification task, numbers are divided in two groups: "small numbers" and "large numbers." Imagine, for example, that a participant is performing a magnitude classification task that requires judging whether a number is greater or less than five. Further, imagine that the participant is presented a "7". The participant may mis-encode the presented number. If the participant mis-encodes the "7" as an "1," then the participant will erroneously respond “smaller than” (see Figure 18). If, however, the participant mis-encodes the “7” as a “9,” despite the encoding error, the participant will correctly respond “less than”.

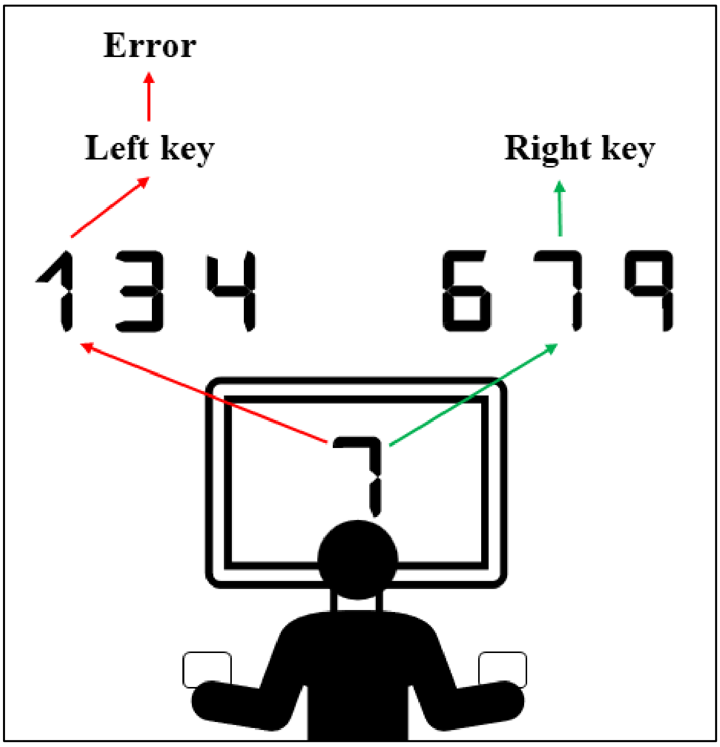


Figure 18 An example of the Group-PS effect in a magnitude classification task (compatible condition). The participant may need more time to press the right key because of the similarity between “7” and “1”.

Group-PS quantifies the likelihood that the participant will mis-encode the presented number with a number that will lead to an error in responding, relative to one that will lead to the correct response. We do so by calculating the average similarity of the presented number to those that lead to the incorrect answer divided by the average similarity of the presented number to those that lead to the correct answer.

For example, the Group-PS for number 3 is defined as:

$$GroupPS(number\ 3) = \frac{avarage(PS(3,6) + PS(3,7) + PS(3,9))}{avarage(PS(3,1) + PS(3,4))}$$

where $PS(n_1, n_2)$ is the Physical Similarity between n_1 and n_2.

The Welford function, The Mirror-PS and the Group-PS (calculated for both normal and mirror) will be used as predictor variables in the next experiments. Each predictor variable predicts participants' reaction time using different information: magnitude (Welford function) and physical similarity (Mirror-PS and Group-PS). It is possible to schematize a prediction model for each of the predictors (see Figure 19).

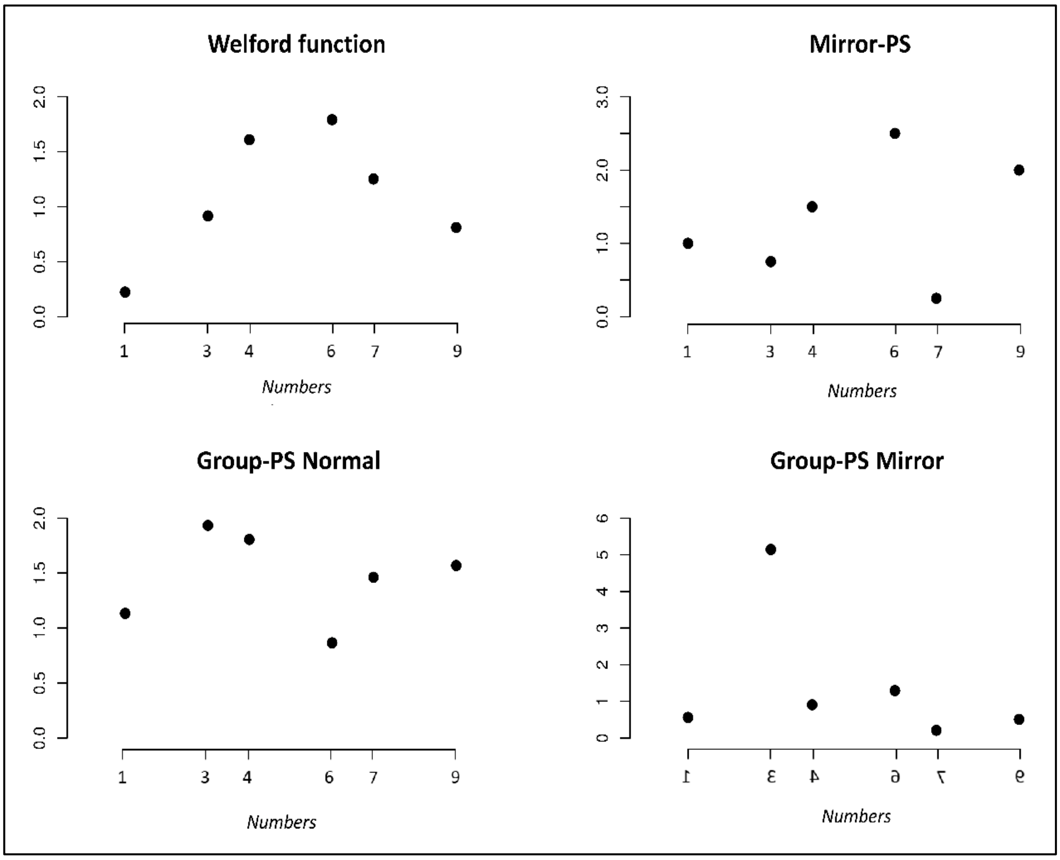


Figure 19 The Welford function and the three Physical Similarity functions that will be used as predictor variables in the next experiments. The black dots represent

each model's prediction. The top left graph shows Welford function prediction (distance effect and size effect); the top right graph shows the Physical Similarity between a number and its mirror version (Mirror-PS); the bottom left graph shows the Physical Similarity function between a normal number and the normal numbers on the opposite side of 5 (Group-PS Normal); the bottom right graph shows the Physical Similarity function between a mirror number and the normal numbers to the opposite side of 5 (Group-PS Mirror).

4.1 Experiment 1

Introduction

In Experiment 1, participants are asked to complete a magnitude classification task using normal and mirror numbers. Participants are asked to indicate whether the number they see is greater or less than 5, regardless of whether it is presented in its normal or mirror version.

The advantage of a magnitude classification task is that, through the Welford function (as explained in Chapter 2), it is possible to understand whether or not participants access the quantitative information.

Furthermore, in light of the theories discussed regarding the SNARC effect, it is possible to make several predictions.

The MNL predicts that access to quantity information should be related to the power of the SNARC effect (i.e., Viarouge et al., 2014). In the magnitude classification task, the Welford function is used as a direct measure of the access to the quantity of numbers. Stronger Welford function indicates deeper access to quantity information and, as a consequence, stronger SNARC effect. As such, the access to quantity information should be sufficient to find a SNARC effect for both normal and mirror numbers. Such a result for the SNARC effect would demonstrate that number structure doesn’t affect the left-to-right organization, and that the access to quantity information can activate the MNL because of a pre-existing left-to-right organization of quantities in the LTM.

In contrast, the WM account predicts that the SNARC effect should be greater in normal numbers than mirror numbers. In fact, it should be more difficult to spatially associate mirror numbers than normal numbers in WM for two reasons: mirror numbers are not learned in a left-to-right organization, therefore they should have little or no associated spatial organization; reading the mirror numbers requires more working memory resources than reading the normal numbers. By using WM resources to read mirror numbers, fewer resources are available to organize them from left to right. A difference in the SNARC effect between normal and mirror numbers would show that the structure of the numbers is important for building left-to-right spatial organization.

In addition, the physical similarity between the numbers will be taken into account to assess whether the physical structure of the numbers can influence the participants' response times.

In conclusion, this experiment allow to test three different hypothesis: (1) participants processes quantity information of both normal and mirror numbers (this will be tested by the presence of the Welford function); (2) the process of quantity information is not always followed by the SNARC effect; (3) The Mirror-PS and the Group-PS predict part of the performance because the similarity between numbers affects participants' responses.

Method

Participants

The sample size was determined by means of the software MorePower 6.4 (Campbell & Thompson, 2012). For repeated-measures 2 x 2 x 2 within-factor design, the following parameters were used: power = .90, α = .05, $\eta p2$ = .27

(estimated effect size from Dehaene et al., 1993); the outcome was a suggested sample size of 28 participants.

Moreover, the rule of thumb “20 x 20” (Cipora & Wood, 2017) suggests that both the number of participants and the number of stimulus repetitions must be 20 or more. According to the power analyses and the guidelines provided by Cipora and Wood (2017), we designed the experiments to have at least 20 repetitions per stimulus and recruited a number of participants they considered “large” (i.e., 30).

Finally, although programs for online experiments are increasingly accurate (see Scerrati et al., 2021), a recognized practice is to recruit more participants than indicated by power analyses (Chetverikov and Upravitelev, 2016; Kraut et al., 2004).

Sixty-one American students from the University of North Carolina at Wilmington (49 females; age: M = 20.26; SD = 5.05) participated in the present study. Participants were either right-handed (i.e., 54 participants) or left-handed (i.e., 7 participants). The study was approved by the institutional review board of University of North Carolina at Wilmington. After the experiment, each participant received partial course credits for their participation. Data collection began in late 2021 and ended in early 2022.

Because culture influences the degree and direction of the SNARC effect (for a cross-cultural study see, Shaky & Fischer, 2008), researchers should be cautious when generalizing our results beyond those from Western, educated, industrialized, rich and democratic cultures (WEIRD; Cheon et al., 2020; for a different view see Kanazawa, 2020).

Stimuli

The visually presented stimuli were the digits 1, 3, 4, 6, 7, 9 and their mirror version (with respect to the vertical axis), in Calibri regular font size 30, black on a white background. The number 8 was excluded because it does not have a mirror version. The number 2 was also excluded because the distance from the reference number (5) is the same as that of 8 (thus retaining symmetry of numerical distance). The remaining six numerals were used (three on either side of five).

Procedure

Normal or mirror digits appeared in the middle of the screen and participants were instructed to indicate whether the number was greater than or less than five (magnitude classification task), using the “D” and “K” keys on the keyboard. In one block, they used the “D” key to respond to numbers smaller than five and “K” key to respond to numbers greater than five; in another block this was reversed. The order of blocks was counterbalanced across participants.

The experimental session consisted of two blocks of 240 trials each, with 20 presentations of each number in both blocks (total 480 trials). Each experimental block was preceded by a practice block of 24 trials.

A fixation point appeared for 400 ms on each trial, followed by a 400 ms blank. After that, a single number was displayed in the center of the screen until the participant completed the task. RTs were recorded from the target stimulus onset until a response was executed. On correct trials, there was a further 400-ms blank interval, whereas on incorrect trials, the word "incorrect" was displayed for 400 ms followed by a 400-ms blank interval (see Figure 20). The experiment was programmed and administered to the participants on the Testable.org website (Rezlescu et al., 2020) and disseminated as a web link with Qualtrix by which participants were randomly selected.

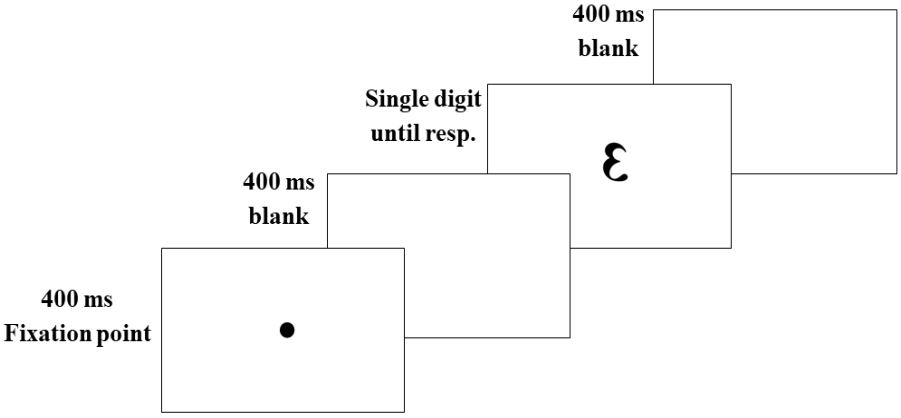


Figure 20 The design of experiment 1 on correct trials. On incorrect trial, between the presentation of the digits and the blank the word “incorrect” were presented for 400 ms. The numbers were presented on both normal and mirror condition (i.e., 3 and Ɛ).

Results

For each participant the percentage of correct trials was calculated (M = 95.9%; SD = .037). Participants with an overall accuracy lower than 85% were removed (n = 1). As a standard control for response consistency, prior to all analyses, the 5% of participants with the greatest coefficient of variation (i.e., SD/mean) were removed (n = 3). The analysis was conducted on the remaining participants (N = 57). Because the hypothesis is that RTs vary by Structure (normal vs mirror) and Number (1, 3, 4, 6, 7, 9), outlier data were trimmed by eliminating the 2.5% fastest and slowest trials in each Structure x Number condition. Finally, error trials were excluded (8.6% of all trials).

Analysis

Analysis on the mean of the RT (mRT)

The data was analyzed by using a mixed model regression. The criterion variable was mRT, which equaled the mean of the RT calculated for each participant x Structure (normal, mirror) x Number (1, 3, 4, 6, 7, 9). To assess the influence of numerical distance and physical similarity on mRT, three predictor variables were included: the Welford function, the Mirror physical similarity (MirrorPS; the similarity between a number and its mirror version), and the Group physical similarity (Group-PS; the similarity between a number and a group of numbers, as described above). Each of the predictor variables provides a unique estimate of mRT for each number and the regression allows to establish the goodness of fit between these estimates and performance. The aim of this method is to understand the degree to which each predictor variable accounts for a statistically reliable amount of the overall variance.

The Mirror-PS assumes that performance is affected by how similar a mirror number is to a normal number (see Figure 19, top right). It's important to note that the Mirror-PS was only used for the analysis of mirror numbers (and not for the analysis of normal numbers), in line with the theory that when participants encode mirror numbers, they convert them into normal to extract quantity.

The Group-PS assumes that performance is affected by how similar the presented number (probe) is to those numbers on the opposite side of 5 relative to those numbers on the same side of 5 (see description above). Group PS is calculated separately for Normal numbers (Figure 20; bottom left) and Mirror numbers (Figure 20; bottom right). Two separate regression analyses were performed for each level of structure (Normal vs. Mirror). This was done for two reasons: first, in this study it is important to understand whether there are differences between normal and mirror numbers; second, the predictor variables for normal numbers (Welford and Group-PS normal) are different than for mirror numbers (Welford, Group-PS mirror and Mirror-PS).

Two predictor variables were used for the analysis of normal numbers (Welford function and Group-PS) as predictors in a simultaneous mixed model regression with intercept by participant as a random variable (all mixed model regressions used the lmer function in R). The analysis reveals a strong statistically significant effect of the Welford function (slope = 35.95), $F(1, 625) = 122.42$, $p < .001$, and a non-significant effect of the Group-PS, $F(1, 625) = 0.79$, $p = .37$ (overall model fit, marginal - fixed effects -, $r^2 = .06$, conditional - entire model -, $r^2 = .68$). Figure 21 presents boxplots of the slopes for each participant (top left) and the model predictions plotted against the average RT by Number collapsed across participants (bottom left).

For the analysis of Mirror numbers, we used three predictor variables (Welford function, Group-PS, Mirror-PS) as predictors in a simultaneous mixed model regression with intercept by participant as a random variable. The analysis reveals a strong statistically significant effect of the Welford function (slope = 36), $F(1, 624) = 87.74$, $p < .001$, suggesting, for the first time, the presence of a distance effect with mirror numbers. Furthermore, there was a significant effect for Mirror-PS (slope = 9), $F(1, 624) = 11.08$, $p < .001$, suggesting that the similarity between mirror and normal numbers affects the mRT. Finally, there was no significant effect for the Group-PS, F (1, 624) = 0.18, p = .67, (overall model fit, marginal - fixed effects -, r^2 = .07, conditional - entire model -, r^2= .67). Figure 21 presents boxplots of the slopes for each participant (top right) and the model predictions plotted against the average RT by Number collapsed across participants (bottom right).

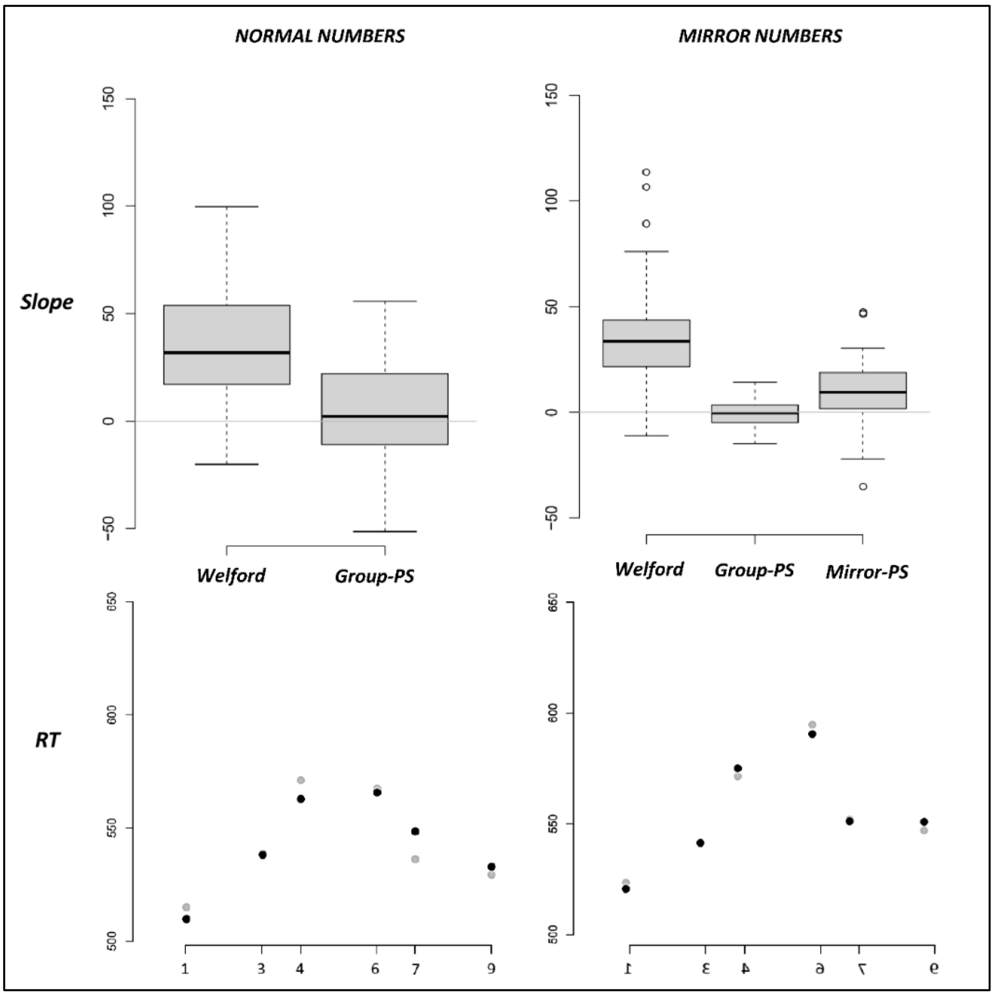


Figure 21 The top row shows boxplots displaying the distribution of slopes for participant for each predictor variable in Normal and Mirror numbers, respectively, left and right figures. The bottom row shows a summary of the fits of the model. Specifically, the grey circles are the reaction time (RT) data collapsed over participant, and the black circles are the fit data from the model, collapsed over participant for Normal and Mirror numbers, respectively, left and right figures.

Analysis on dRT (SNARC effect)

The SNARC effect was calculated on dRT. In particular, for each participant x Structure x Number, dRT was equal to the difference between right hand mRT and left hand mRT (dRT = rights mRT – left mRT). In the current study, Number (1,3,4,6,7,9) is a within-subjects factor and the SNARC effect was evaluated by relating dRT to Number. Similar to other studies, the size of the SNARC effect was assessed in terms of a slope (i.e., Pinhas et al., 2012; Tzelgov et al., 2013).

Similar to the previous analysis, a separate regression analysis was performed for each level of structure (normal, mirror). A standard regression analysis was calculated, rather than a mixed-model analysis, because the mixed-model analysis was over specified when subjects were added as a random variable. This occurred because of the constraints resulting from the calculation of dRT (which minimizes between subject variance).

For the analysis of Normal numbers, the predictor variables were the Number (SNARC effect) and the Group-PS, and the criterion variable was dRT. The regression was significant, $F(2,339) = 7.67$, $p < 0.001$, $r^2 = .043$. There was a statistically significant effect of Number, $t = -3.25$, $p = .001$ (slope = - 4.41) and Group-PS, $t = -2.25$, $p = .02$, (slope = - 21.87). Figure 22 presents boxplots of the slopes for each participant (top left) and the model predictions plotted against the average dRT by Number collapsed across participants (bottom left).

For the analysis of Mirror numbers, three predictor variables were used as predictors: Group-PS, Mirror-PS and Number. The regression was not significant, $F(3,338) = 0.43$, $p = .73$, $r^2 = .004$. Figure 22 presents boxplots of the slopes for each participant (top right) and the model predictions plotted against the average dRT by Number collapsed across participants (bottom right).

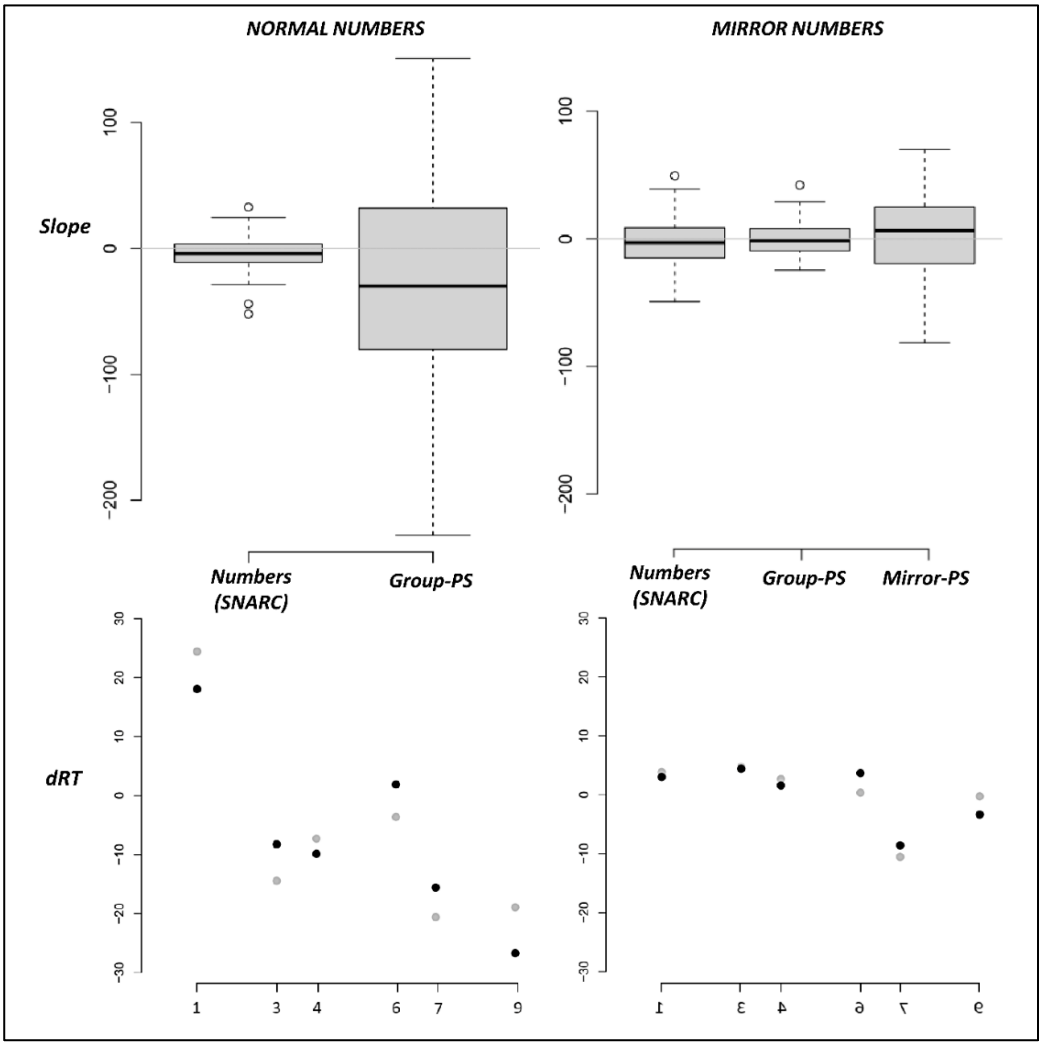


Figure 22 Summaries of the model fits. The top row shows boxplots displaying the distribution of slopes for participant for each predictor variable in Normal and Mirror numbers, respectively, left and right figures. The bottom row shows a summary of the fits of the model. Specifically, the grey circles are the difference of reaction times between right RTs and left RTs (dRT) data collapsed over participant, and the black circles are the fit data from the model, collapsed over participant for the group of

participants who saw Normal number and for the group of participants who saw Mirror numbers, respectively, left and right figures.

Discussion

In this experiment, participants performed a magnitude classification task on normal and mirror numbers at the same time. The strong significant effect of the Welford function suggests the presence of distance effect and size effect for both normal and mirror numbers. This is the first time that the Welford function is found with mirror numbers, suggesting that participants process the quantitative information of mirror numbers.

While the distance effect was found for both normal and mirror numbers, the SNARC effect was found only in normal numbers. Because participants have access to quantitative information for both normal and mirror numbers, the absence of SNARC effect for mirror numbers suggests that access to quantity information is not sufficient to activate the SNARC effect. In other words, the SNARC effect cannot be explained by the left-to-right representation of quantities alone.

For the analysis on mRT, three predictor variables were used to assess the influence of physical similarity and numerical distance: Mirror-PS (the physical similarity between a number and its mirror version), Group-PS (the physical similarity between a number and a group of numbers), and the Welford function.

For normal numbers, physical similarity has no effect on mRT.

For mirror numbers, the data reveal that the mRT was modulated by both the Welford function and the Mirror-PS predictor. The Welford function suggests that participants process the quantity information of mirror numbers. The effect of

Mirror-PS, instead, suggests that participants tend to transform mirror numbers into normal numbers (or, at least, compare their structure) to better perform the magnitude classification task. This suggests that the structure of the numbers modulates the availability of quantity information.

Finally, a multiple linear regression was performed to predict the dRT (right RTs - left RTs). This regression provides insight into how physical similarity and the SNARC effect are related to dRT, and in particular, whether physical similarity of numbers plays a role in the SNARC effect.

The dRT of normal numbers was modulated by both the SNARC and the Group-PS predictor. The SNARC reveals the left-to-right association for normal numbers. The effect of Group-PS, instead, reveals a handedness association. Specifically, the GroupPS effect on dRT indicates that when a “smaller” (larger) response is required people can get confused by “larger” (smaller) numbers, so the physical similarity of numbers may slow down RTs.

Importantly, the SNARC effect only emerged for normal numbers but not for mirror numbers. The dRT for mirror numbers were influenced by neither the SNARC nor the Group-PS. These results lead to two important conclusions: (1) participants access the quantitative information of the mirror numbers (as indicated by the Welford function), however they do not represent them spatially from left to right; (2) since participants access quantity information of both normal and mirror numbers, the absence of the SNARC effect for mirror numbers suggests that quantity processing is not sufficient to explain the left-to-right spatial organization of numbers.

A limitation of this experiment is that the co-presence of normal and mirror numbers within the same task could affect participants' performance so that the quantity process of mirror numbers could be facilitated by the task. Moreover, even the effect of Mirror-PS could depend on the co-presence of normal and mirror numbers, that is, when participants see a mirror number, they compare it with its normal version because of the presence of the normal numbers in the same task. For these reasons, in Experiment 2, normal and mirror numbers were not presented in the same task, but to two different groups of participants.

4.2. Experiment 2

Introduction

In Experiment 2, two groups of participants were exanimated: one group completed a standard magnitude classification task using normal numbers, whereas a different group completed the same task with mirror numbers. This experiment allow to test three different hypothesis: (1) participants processes quantity information of mirror numbers regardless the presence of normal numbers; (2) the process of quantity information (tested by the Welford function) is not always followed by the SNARC effect; (3) The Mirror-PS effect (found in Experiment 1) is not due to the co-presence of normal and mirror numbers, but to the participants' tendency to access the image of the normal numbers when they see the mirror numbers.

Method

Participants.

The sample size was determined using the same procedure as in Experiment 1. As Experiment 2 is a between-subjects design, the minimum number of participants was doubled in order to have 40 participants per between-subjects condition (80 participants).

Eighty-seven American students from the University of North Carolina at Wilmington (64 females; age: M = 19.2; SD = 1.52) participated in the present study. Participants were right-handed (i.e., 78 participants), left-handed (i.e., 7 participants) and ambidextrous (i.e., 2 participants). All the participants had normal or corrected-to-normal visual acuity, and they were naive about the purpose of the experiment. The study was approved by the institutional review board of University of North Carolina at Wilmington. After the experiment, each participant received partial course credits for their participation.

Stimuli

The stimuli were the same as in Experiment 1.

Procedure

Experiment 2 followed the same design and procedure as Experiment 1 with the exception that a group of participants (N = 45) did a magnitude classification task only with normal numbers and another group (N = 42) did a magnitude classification task only with mirror numbers. The order of the blocks was counterbalanced across participants. The experimental session consisted of two blocks of 240 trials each, with 40 presentations of each number in both blocks (total 480 trials). Each experimental block was preceded by a practice block of 24 trials.

Results

Data were analyzed as in Experiment 1. The percentage of correct trials was calculated for each participant (M = 95.9%; SD = .037). Participants with an overall accuracy lower than 85% were removed (n = 1). As a standard control for response consistency, prior to all analyses, the 5% of participants with the greatest coefficient of variation (i.e., SD/mean) were removed (n = 3). The remaining participants (N = 57) were analyzed in the final dataset. Because the hypothesis is that RTs vary by Structure (normal vs mirror) and Number (1, 3, 4, 6, 7, 9), outlier data were trimmed by eliminating the 2.5% fastest and slowest trials in each Structure x Number condition. Finally, error trials were excluded (8.6% of all trials).

Analysis

Analysis on the mean of the RT (mRT)

For the analysis of Normal numbers, the Welford function and the Group-PS were used as predictors variables in a simultaneous mixed model regression with intercept by participant as a random variable. The analysis revealed a strong statistically significant effect of the Welford function (slope = 39), $F(1, 471) = 186.41$, $p < .001$, indicating the presence of the distance effect and the size effect, and a non-significant effect of the Group-PS, $F(1, 471) = 1.08$, $p = .29$, (overall model fit, marginal (fixed effects), $r^2 = .095$, conditional (entire model) $r^2 = .74$). Figure 23 presents boxplots of the slopes for each participant (top left) and the model predictions plotted against the average RT by Number collapsed across participants (bottom left).

For the analysis of Mirror numbers, the Welford function, the Mirror-PS and the Group-PS were used as predictors variable in a simultaneous mixed model regression with intercept by participant as a random variable. The analysis revealed a strong statistically significant effect of the Welford function (slope = 24.25), $F(1, 404) = 52.54$, $p < .0001$, suggesting the presence of a distance effect and a size effect with mirror numbers. Furthermore, there was a significant effect for Mirror-PS (slope = 7.59), F (1, 404) = 10.44, p = .001, suggesting that the similarity between mirror and normal numbers plays an important role. Finally, there was a significant effect for Group-PS (slope = -2.02), F (1, 404) = 4.34, p = 0.038, (overall model fit, marginal (fixed effects) r^2 = .07, conditional (entire model) r^2 = .703). Figure 23 presents boxplots of the slopes for each participant (top right) and the model predictions plotted against the average RT by Number collapsed across participants (bottom right).

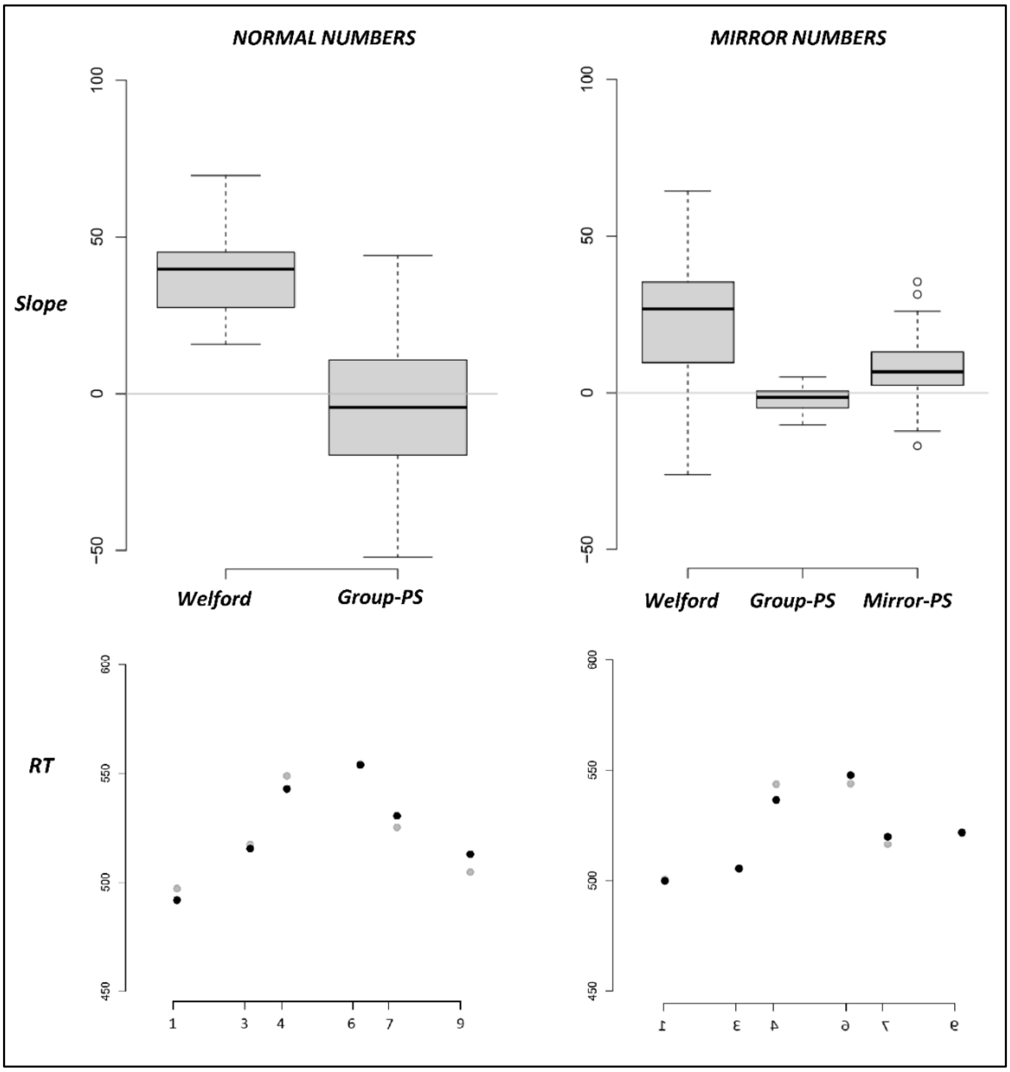


Figure 23 Summaries of the model fits. The top row shows boxplots displaying the distribution of slopes for participant for each predictor variable in Normal and Mirror numbers, respectively, left and right figures. The bottom row shows a summary of the fits of the model. Specifically, the grey circles are the reaction time (RT) data collapsed

over participant, and the black circles are the fit data from the model, collapsed over participant for Normal and Mirror numbers, respectively, left and right figures.

Analysis on dRT (SNARC effect)

For the Normal numbers, Numbers and Group-PS were used as predictor variables and dRT as the criterion variable. The regression was significant $F(2, 255) = 3.339$, $p = .037$, $r^2 = .025$. There was no effect of Number, $t = -0.98$, $p = .33$, indicating the lack of the SNARC effect, and a significant effect of the Group-PS, $t = -2.41$, $p = .016$, (slope = - 21.32). Figure 24 presents boxplots of the slopes for each participant (top left) and the model predictions plotted against the average dRT by Number collapsed across participants (bottom left).

For the analysis of Mirror numbers, Number, Group-PS and Mirror-PS were used as predictors variables and dRT as the criterion variable. The regression was not significant $F(3, 218) = 1.049$, $p = .37$, $r^2 = .02$. Figure 24 presents boxplots of the slopes for each participant (top right) and the model predictions plotted against the average dRT by Number collapsed across participants (bottom right).

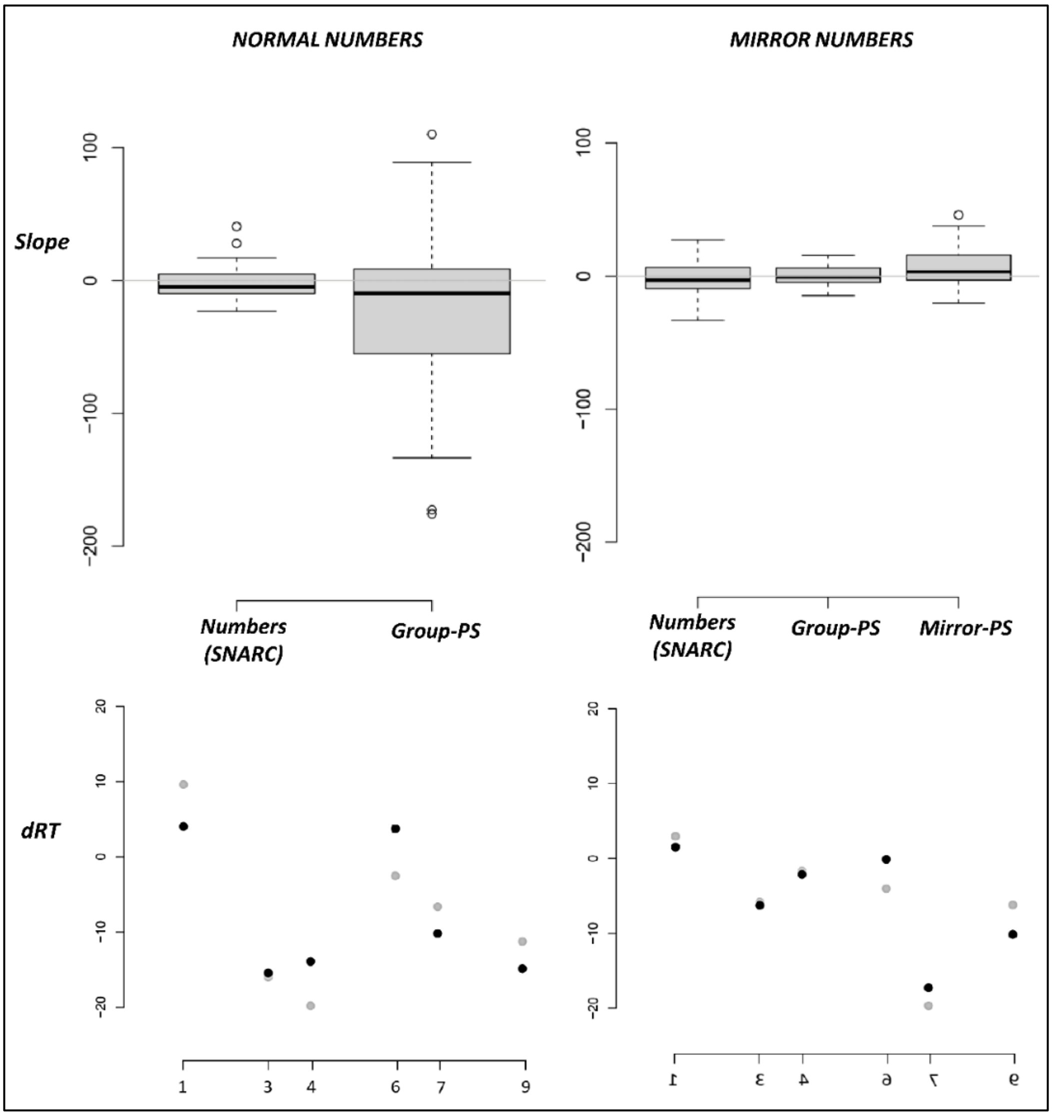


Figure 24 Summaries of the model fits. The top row shows boxplots displaying the distribution of slopes for participants for each predictor variable in Normal and Mirror numbers, respectively, left and right figures. The bottom row shows a summary of the fits of the model. Specifically, the grey circles are the difference of reaction times between right RTs and left RTs (dRT) data collapsed over participant, and the black circles are the fit data from the model, collapsed over participant for the group of participants who saw Normal number and for the group of participants who saw Mirror numbers, respectively, left and right figures.

Discussion

In this second experiment two groups of participants performed a magnitude classification task. A group of participants did a magnitude classification task only with normal numbers and another group did a magnitude classification task only with mirror numbers.

The Welford function were strongly significant for both normal and mirror numbers, suggesting the presence of the distance effect and the size effect. As a consequence, the presence of the Welford function indicates access to quantitative information. These results support those of Experiment 1, suggesting that participants process the quantitative information of both normal and specular numbers.

For mirror numbers the SNARC effect was not found, confirming the results of Experiment 1. The lack of the SNARC effect suggests that even when participants process the quantitative information of the mirror numbers (as suggested by the Welford function), they do not represent the numbers from left to right. This suggests that quantity processing is not sufficient to bring out the SNARC effect.

This view is confirmed by the results of the normal numbers. Despite the significance of the Welford function (suggesting quantity processing) the SNARC effect was also not found in the case of normal numbers. The non-replication of the SNARC effect for normal numbers provides further evidence that the access to quantity information is not sufficient to explain the SNARC effect. Indeed, even when participants process the quantitative information of the normal numbers, they do not show a SNARC effect.

Despite this, it is important to understand why Experiment 2 does not confirm the SNARC effect with normal numbers found in Experiment 1. Specifically, the main question is: why was the SNARC effect not found with normal numbers in a standard magnitude classification task?

One explanation is that the absence of the SNARC effect could be attributed to the physical similarity effect and, in particular, to the Group-PS. Indeed, the influence of physical similarity on dRT reveals that participants experienced perceptual confusions that took time to resolve. The Group-PS effect could compete with the SNARC effect, because both of these effects are related to facilitating the responses of one hand relative to the other based on the number presented on the screen: the first related to its structure (Group-PS), the second related to its location (SNARC).

In Experiment 2, each digit was repeated 80 times without the presence of mirror numbers. This perhaps promoted a deep processing of the number's physical structure. In contrast, in Experiment 1, each normal number was presented only 40 times and mixed with mirror numbers, so the physical structure of normal numbers received less attention in Experiment 1 than in Experiment 2. As a result, both the physical similarity effect and the SNARC effect were present in Experiment 1, while only the physical similarity effect was present in Experiment 2.

Data on mRT also confirm the greater influence of physical similarity for normal numbers in Experiment 2 than in Experiment 1. In fact, while the mRT analysis revealed a significant influence of physical similarity for normal numbers in Experiment 2, no effect of physical similarity for normal numbers was found in

Experiment 1. The data confirm deeper processing of the physical structure of normal numbers in Experiment 2 than in Experiment 1.

Another important finding is related to the significant effect of Mirror-PS (the similarity between the mirror number and its normal version). The analysis on mRT for mirror numbers revealed an influence of Mirror-PS, confirming the result of experiment one. Unlike in Experiment 1, in which the normal and specular numbers were presented together, in Experiment 2 the normal and specular numbers were presented separately. As a consequence, the Mirror-PS effect in Experiment 2 suggests that when participants do the task just with mirror numbers, they preserve the image of the normal numbers structure in mind. In particular, the presence of the Mirror-PS indicates that participants tend to transform mirror numbers into normal numbers (or, at least, compare their structure) to better recognize them and to better perform the task.

In Experiments 1 and 2, participants were explicitly asked to access information about the quantity of numbers with a magnitude classification task. For these reasons, in Experiment 3, participants were asked to perform a task in which processing the quantity of numbers was not required.

4.3. Experiment 3

Introduction

While in Experiments 1 and 2 quantity processing was explicitly required by the task, in this experiment no quantity processing is required. In order to involve normal and mirror numbers in Experiment 3, a new task was invented: a mirror judgment task. In this type of task, participants are asked to report, through two answer keys, whether the number they are seeing is normal or mirror.

Through this last experiment, it is possible to understand which of the theories (MNL account, WM account, dual route model) best explains the SNARC effect.

The MNL account states that deeper semantic processing of a number leads to a stronger SNARC effect because of the activation of the MNL. If that is so, the SNARC effect of Experiments 1 and 2 (magnitude classification task) should be stronger than the SNARC effect of Experiment 3 (mirror judgment task), because in the first two experiments there is an explicit process of magnitude information (unlike in Experiment 3).

The WM account could support an opposite prediction: the SNARC effect of Experiment 3 should be stronger than the SNARC effect of Experiment 1 and 2. In fact, the SNARC effect emerges for normal numbers because participants associate the Arabic numbers with a left-to-right ordinal organization, independent of their quantity representation. Importantly, in the magnitude classification task, participants respond to half the trials with an incompatible keymapping (left response to large numbers and right response to small numbers). This incompatible keymapping may compete with the left-to-right organization stored in working memory. As such, it could actually reduce the SNARC effect. In contrast, in a mirror judgment task, participants respond with one hand to all normal numbers and with the other hand to all mirror numbers (and vice versa). In this case, incompatible keymapping is not present and the left-to-right organization in working memory that produces the SNARC effect should remain strong.

Finally, the prediction of the dual route model is that slower reaction times are associated with a stronger SNARC effect. Since the speed of reaction times in a mirror judgment task is unknown, this prediction will be evaluated in the general discussion.

This new task allow to understand: (1) whether a SNARC effect is present with normal numbers in a task that do not require an explicit process of the quantity information (the mirror judgment task); (2) whether a SNARC effect is present with mirror numbers in a task that do not require an explicit process of the quantity information (the mirror judgment task); (3) which theory best explains the SNARC effect; (4) whether the Mirror-PS and the Group-PS affect participants' performance.

Method

Participants

Sample size was determined using the same procedure as Experiment 1. However, since this is the first time using a mirror judgment task, it was decided to collect a larger number of participants.

Eighty-two students from the University of North Carolina at Wilmington (56 females; age: M = 19.3; SD = 1.12) participated in the present study. Participants were right-handed (i.e., 76 participants), left-handed (i.e., 3 participants) and ambidextrous (i.e., 3 participants). All the participants had normal or corrected-to-normal visual acuity, and they were naive about the purpose of the experiment. The study was approved by the institutional review board of University of North Carolina at Wilmington. After the experiment, each participant received partial course credits for their participation.

Stimuli

The stimuli were the same as in Experiment 1 and 2.

Procedure

The design and the procedure were the same as in Experiment 1, with the following exceptions. Participants were instructed that, when a digit appeared on the screen, they should indicate whether a number is present in its normal or in its mirror version, using the “D” and “K” keys on the keyboard. In one block, they used the “D” key to respond to normal numbers and “K” key to respond to mirror numbers; in another block this was reversed. The order of blocks was counterbalanced across participants.

Results

The data were examined similarly to experiment 1 and 2. The percentage of correct trials was calculated for each participant (M = 90.4%; SD =.029). Participants with an overall accuracy lower than 85% were removed (n = 11). As a standard control for response consistency, prior to all analyses, the 5% of participants with the greatest coefficient of variation (i.e., SD/mean) were removed (n = 4). The remaining participants (N = 67) were analyzed in the final dataset. Because the hypothesis is that RTs varies by Structure (normal vs mirror) and Number (1, 3, 4, 6, 7, 9), outlier data were trimmed by eliminating the 2.5% fastest and slowest trials in each Structure x Number condition. Finally, error trials were excluded (5.1% of all trials).

Analysis

Analysis on the mean of RT (mRT)

For the analysis of both normal and mirror numbers, the Mirror-PS were used as predictors variables in a simultaneous mixed model regression with intercept by participant as a random variable. Mirror-PS analysis, in this last experiment, was also performed on the normal numbers because of the nature of the task. In fact, in a mirror judgment task, in which the processing of the physical structure of a number is explicit, comparing the structure of the normal numbers with the structure of the mirror numbers could be important.

The analysis reveals a strong statistically significant effect of Mirror-PS for both normal numbers (slope = 61.5), $F(1, 334) = 229.49$, $p < .001$, (overall model fit, marginal (fixed effects) r^2 =.10, conditional (entire model) r^2 = .72), and mirror numbers (slope = 47.71), F (1, 334) = 110.79, p < .001, (overall model fit, marginal (fixed effects) r^2 = .047, conditional (entire model) r^2 = .69). Figure 25 presents boxplots of the slopes for each participant for both normal (top left) and mirror (top right) numbers, and the model predictions plotted against the average RT by Number collapsed across participants for both normal (bottom left) and mirror numbers (bottom right).

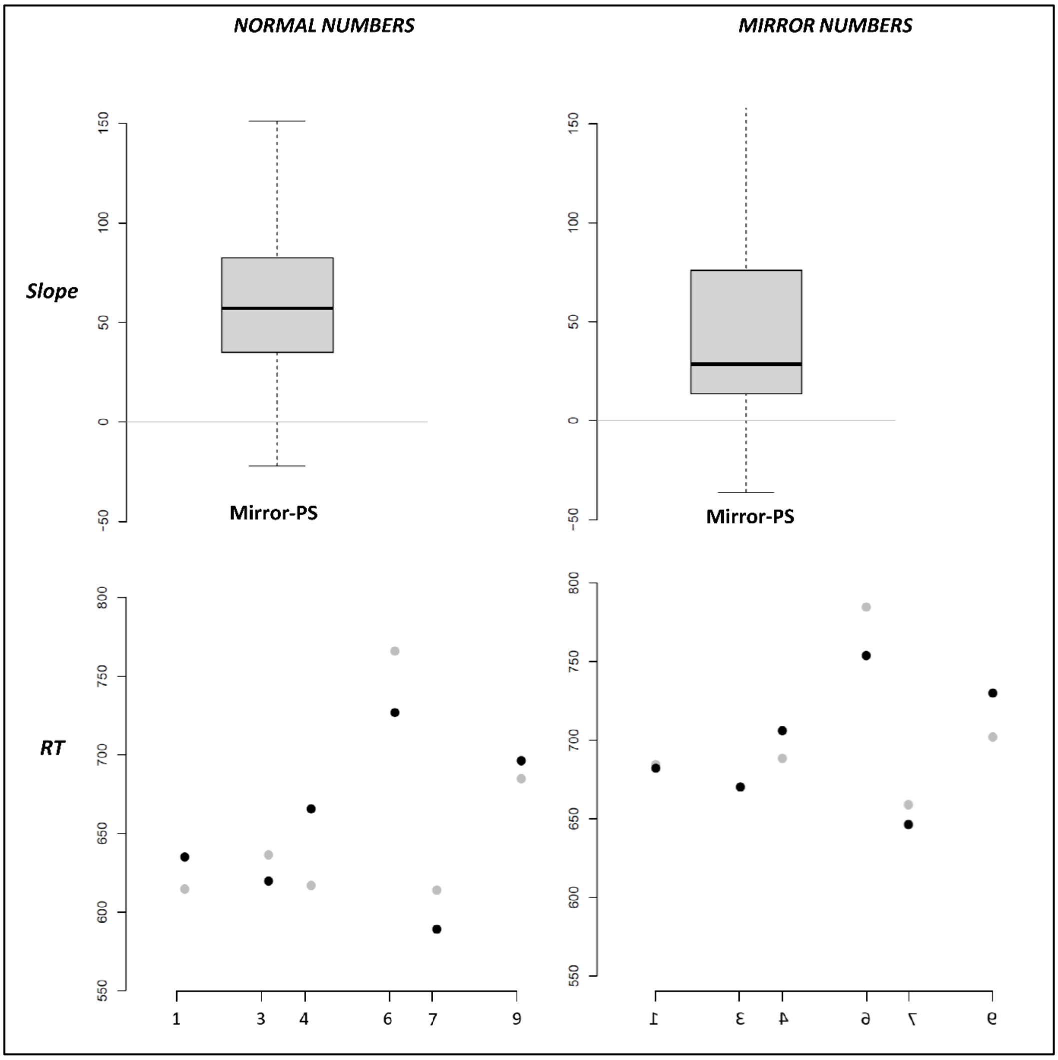


Figure 25 Summaries of the model fits. The top row shows boxplots displaying the distribution of slope for participants for the Mirror-PS predictor variable in Normal and Mirror numbers, respectively, left and right figures. The bottom row shows a summary of the fits of the model. Specifically, the grey circles are the reaction times (RTs) data collapsed over participant, and the black circles are the fit data from the model,

collapsed over participant for Normal and Mirror numbers, respectively, left and right figures.

Analysis on dRT (SNARC effect)

The dRT (right RTs – left RTs) were calculated for each participant to assess the SNARC effect. To understand whether the Physical Similarity of numbers plays a role in the SNARC effect, a multiple linear regression was calculated to predict dRT. In particular, this regression allows to understand how the Physical Similarity and the SNARC effect are related to the dRT.

Although normal and mirror numbers were presented together in the same task, the analyses were performed separately.

For the analysis of Normal numbers, the predictor variables were the *Numbers* (1,3,4,6,7,9) and the *Mirror-PS*, while the criterion variable was the *dRT*. The regression was significant, $F (2,399) = 11.32$, $p < .001$, $r^2 = .05$. There was a statistically significant effect of *Number* (SNARC effect), $t = - 4.57$, $p < .001$ (slope = -9.66) and no effect of the *Mirror-PS*, $t = 0.26$, $p = .79$. Figure 26 presents boxplots of the slopes for each participant (top left) and the model predictions plotted against the average dRT by Number collapsed across participants (bottom left).

For the analysis of Mirror numbers, the predictor variables were the Numbers and the Mirror-PS, while the criterion variable was dRT. The regression was not significant, $F (2,399) = 0.862$, $p = .42$, $r^2 = .004$. Figure 26 presents boxplots of the slopes for each participant (top right) and the model predictions plotted against the average dRT by Number collapsed across participants (bottom right).

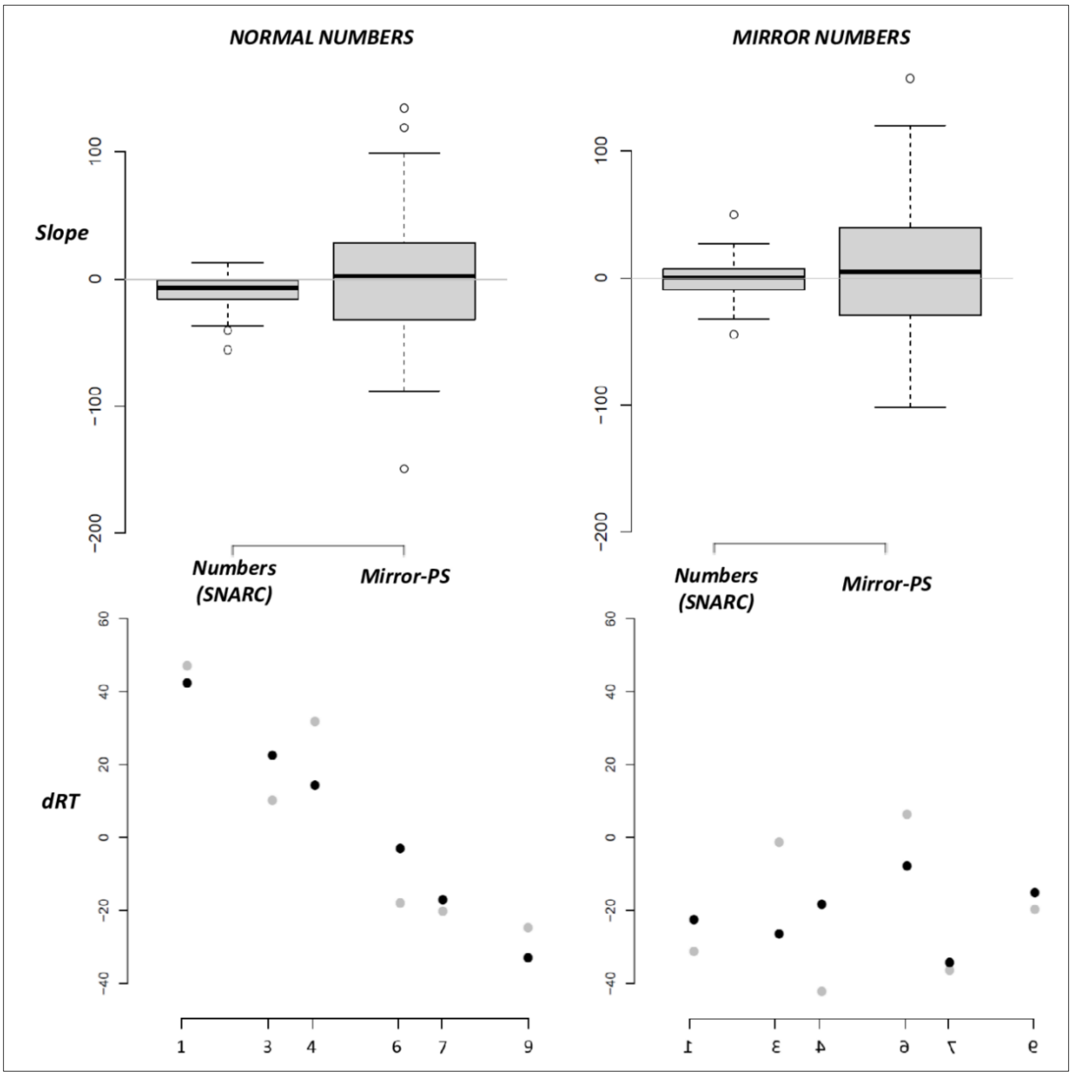


Figure 26 Summaries of the model fits. The top row shows boxplots displaying the distribution of slopes for participants for each predictor variable in Normal and Mirror numbers, respectively, left and right figures. The bottom row shows a summary of the fits of the model. Specifically, the grey circles are the difference of reaction times between right RTs and left RTs (dRT) data collapsed over participant, and the black circles are the fit data from the model, collapsed over participant for the group of participants who saw Normal number and for the group of participants who saw Mirror numbers, respectively, left and right figures.

Discussion

In this experiment participants were examined for the first time in a mirror judgment task by presenting both normal and mirror numbers. To assess the influence of Physical Similarity on average RT (i.e., mRT), the Physical Similarity between a number and its mirror version (Mirror-PS) were calculated (as described above).

The analysis of mRT is clear in showing that participants' RTs for both normal and mirror numbers were modulated by the Physical Similarity between them (Mirror-PS). This finding suggests that participants make a mirror/normal judgment by comparing the physical structure of the normal number with the physical structure of the mirror numbers.

Critically, the analysis of dRT revealed the SNARC effect for normal numbers, but not for mirror numbers. This suggests that a left-to-right spatial organization of numbers is dependent on the numbers structure.

General Discussion

Here we investigated the relations between the Welford function (distance and size effects), the SNARC effect, and the influence of physical similarity on normal and mirror numbers across three experiments.

In Experiment 1, participants performed a magnitude classification task with normal and mirror numbers presented randomly. In Experiment 2 participants performed the same task as in Experiment 1, but one group saw only normal numbers, while another group saw only mirror numbers.

In Experiment 3, a new task was requested, the mirror judgment task, in which participants are asked to identify whether a number is normal or mirror.

In Experiments 1 and 2 (magnitude classification task) quantity processing was explicitly requested. The presence of the Welford function (assessing for the distance effect and the size effect, as explained in chapter 2) confirms that participants access the quantity information for both normal and mirror numbers.

Even though participants processed the quantitative information of both normal and mirror numbers, the SNARC effect was found only for normal numbers (experiments 1 and 3) but never for mirror numbers. These findings allow us to determine which theory best explains the SNARC effect.

According with the MNL account the connection between the number and its quantitative representation is in LTM (Dehaene et al., 1993, Hubbard et al., 2005; Aulet et al., 2021) so deeper access to quantity information should be related to the power of the SNARC effect (i.e., Viarouge et al., 2014). In Experiments 1 and 2 (magnitude classification task) the strong effect of Welford function suggests a deep access to quantity information for both normal and mirror numbers. If the SNARC effect depends on access to quantitative information, the SNARC effect should have been found for both normal and mirror numbers. However, this is not the case, because the presence of the Welford function was not always paired with the presence of the SNARC effect. Such a result demonstrates that number structure affects the left-to-right organization of numbers, and that the access to quantity information is not sufficient to explain the SNARC effect.

Furthermore, according to the MNL account the SNARC effect of Experiment 1 and 2 (magnitude classification task) should be stronger than the SNARC effect of Experiment 3 (mirror judgment task) because in the magnitude classification task the access to quantity information is explicit. The data do not support this prediction because the SNARC effect found in Experiment 3 was the strongest.

For these reasons, the data do not support the MNL account.

The Dual Route model predicts that the size of the SNARC effect should increase with the average reaction time. As such, the SNARC effect should be stronger in the conditions/experiments with the longer average RT. Since the RTs of Experiment 3 are slower than the RTs of Experiment 1 and 2 the Dual Route model successfully predicted the stronger SNARC effect of Experiment 3. The explanation is that performing a mirror judgment task requires more time than to perform a magnitude classification task, so the unconditional route has more time to process quantity information and spatially associate quantities along a MNL. Critically, however, the data also reveals that in each experiment the RTs of mirror numbers are slower than RTs of normal numbers. Therefore, the Dual Route model predicts a stronger SNARC effect for mirror numbers, relative to normal numbers. In contrast, the SNARC effect wasn't stronger with mirror numbers, but, rather, disappeared.

For these reasons, the data do not fully support the Dual Route account.

The WM account predicts that task features determine the strength of SNARC. In Experiments 1 and 2, the WM account assumes that the SNARC effect emerges for normal numbers because participants associate the Arabic numbers with a left-to-right ordinal organization, independent of their quantity representation. The WM account predicts that the SNARC effect should be greater in normal numbers than mirror numbers for two reasons. First, mirror numbers are not learned in a left-to-right organization, therefore they should have little or no associated spatial organization. Second, reading the mirror numbers requires more working memory resources than reading the normal numbers. By using WM resources to read mirror numbers, fewer resources are available to organize them from left to right. This explains the absence of the SNARC effect for mirror numbers and suggests that left-to-right spatial number organization depends on number format, such that regular stimuli (e.g., normal numbers) are easier to organize along a continuum than ambiguous stimuli (e.g., mirror numbers).

The WM account also explains why the SNARC effect of Experiment 3 (mirror judgment task) is stronger than the SNARC effect of Experiment 1 and 2 (magnitude classification task). Importantly, in Experiments 1 and 2, the participant is responding to half the trials with an incompatible keymapping (left response to large numbers and right response to small numbers). This incompatible keymapping may compete with the left-to-right organization stored in WM and reduces the SNARC effect. Because the incompatible keymapping is not present in the mirror judgment task (participants respond with one hand for all the normal numbers and the other for all the mirror numbers), the left-to-right organization in working memory that produces the SNARC effect remained strong.

The data appear most consistent with the WM account.

The hypothesis that the physical structure of the numbers plays an important role in the performance of the task was tested. Following Cohen (2009) new indices of physical similarity for the mirror numbers were calculated: Mirror-PS (the physical similarity between a number and its mirror version) and Group-PS, (the Physical Similarity between a number and a group of numbers, as described above). Analysis showed that such indices provided statistically robust predictors of both mRT (mean of RT) and dRT (right RT – left RT).

In many respects the data are clear in showing that the mRT for mirror numbers was modulated by the Mirror-PS predictor, suggesting that participants tend to transform mirror numbers into normal numbers (or, at least, compare their structure) to better recognize them and to better perform the task. This indicates that the access to semantic information is modulated by the structure of the numbers. In other words, participants perform a magnitude classification task with mirror numbers by transforming them into normal numbers.

In addition, to understand whether the physical similarity of numbers plays a role in the SNARC effect, a multiple linear regression was calculated to predict the dRT. In Experiment 1 and 2 the data indicate that the dRT with normal numbers was modulated by the Group-PS predictor. The Group-PS effect on dRT indicates that the physical similarity of the numbers leading to an error (e.g., a "larger" number when a "smaller" response is required, and vice versa). This finding suggests that number structure plays an important role and that the SNARC effect alone cannot explain all the variance in performance with normal numbers

In conclusion, number structure affects mRT and dRT even when the task does not explicitly require participants to process it. This suggests that the perception of the numbers' structure precedes the access to quantity information and the left-to-right organization of numbers.

CONCLUSION

The predisposition to identify quantities is a skill that human beings share with other living species. The organization of human society into larger and larger groups of individuals has resulted in the need to refine these skills through the introduction of counting objects and mathematical symbols.

The invention of Arabic numerals profoundly revolutionized the world of mathematical operations. Among the greatest revolutions was the fact that, whereas before the invention of numbers, access to quantitative information was directly linked to the visual information given by the stimulus (for example a bunch of pebbles), with Arabic numerals, access to quantitative information is conveyed by a person's ability to recognize a numerical symbol. If in early mathematical systems anyone could have understood that group “IIII” is greater than group “II”, without prior learning no one could say that symbol “4” represents a quantity greater than symbol “2”. Therefore, this thesis assumes that when people see a number, access to quantitative information is not automatic. This happens because numbers are symbols, and the link between the physical part of the symbol and the semantic part is undirect.

The idea that access to quantitative information is undirected, despite its logic, is not completely accepted in the field of numerical cognition. On the contrary, A substantial number of researchers support the idea that seeing a number automatically activates access to quantitative information (Dehaene et al., 1993; Dehaene & Akhavein, 1995; Dehaene et al., 1998; Fischer et al., 2003; Gevers et al., 2006; Naccache & Dehaene, 2001; Koechlin et al., 1999; Nuerk et al., 2005; Pavese & Umiltà, 1998; Tzelgov & Ganor-Stern, 2005; for a different view see Cohen, 2009).

In the fundamental work of Dehaene et al. (1993) it was shown that, through a “parity judgment task” in which participants were asked to categorize numbers into odd or even, reaction times were faster when responding to small numbers by pressing the left key and to large numbers by pressing the right-hand key. The author proposed that western people spatially represent numbers along a mental number line (MNL) in which small quantities are represented to the left and large quantities to the right. Because in the parity judgment task quantity processing is not explicitly required, the facilitation of left-key responses for small numbers and right-key responses for large numbers was taken as evidence of automatic access to quantitative information.

However, it is important to consider that numbers do not only carry quantitative information, but also other types of information such as ordinal information. Ordinal information refers to the position indicated by numbers regardless of their quantity. Quantitative and ordinal information often coincide, which is why it is difficult to understand whether the SNARC effect depends on the processing of numerical quantities or on the ordinal position of numbers.

In recent years, more and more studies are testing the hypothesis that quantity is not necessary to represent numbers from left to right, and ordinality may explain the SNARC effect (Casasanto & Pitt, 2019; Pitt & Casasanto, 2022).

The order hypothesis is supported by the fact that a SNARC-like effect is present not only with numbers, but also with days of the week (Gevers et al., 2004), letters of the alphabet, months (Gevers et al., 2003), emotional valence (de la Vega et al., 2012), musical pitch (Rusconi et al., 2006), and learned number sequences that do not respect an increasing quantity representation (van Dijk & Fias, 2011).

The SNARC effect with the days of the week or months can't be attributed to a tendency of people to automatically perceive quantities, but to the fact that days and months are spatially organized from left to right in Western culture.

With the aim of better understanding the relationship between quantity and order in the spatial organization of numbers, 4 experiments were reported in this thesis.

The first experiment was inspired by the work of Pinhas et al. (2014) who asked participants to classify the symbols minus (-) and plus (+) by pressing two different keys. The authors found that participants were faster when the minus sign (-) was associated with the left response key, and the plus sign (+) with the right response key. This effect is called "Operation Sign Spatial Association" (OSSA) and it suggests that the operation signs themselves can evoke a spatial association without the need of a magnitude process. However, before the experiment, the authors asked participants to perform tasks with numbers, so the OSSA effect could depend on the arithmetic contest of the previous tasks.

In Experiment 1 participants performed a Posner-like task using the minus or the plus as fixation symbols and without performing arithmetic operations on the quantities. An opposite pattern of responses was observed between the minus condition and the plus condition: in the minus condition responses tended to be faster when the target appeared on the left, whereas, in the plus condition responses were faster when the target appeared on the right. This suggests that the plus and minus signs can be spatially associated with left and right, respectively, without the need of quantitative information processing.

Although this study demonstrates that the spatial association of minus and plus mathematical symbols is independent of the processing of numerical quantitative information, it does not provide enough evidence to claim that the SNARC effect is due to order or magnitude. Therefore, the last chapter were divided in three experiments that focused on the relationship between order and magnitude in tasks with numerical stimuli: a magnitude classification task (Experiment 1 and 2) and a mirror judgment task (Experiment 3), a new type of task in which participants are asked to say whether a number is presented in its normal or mirrored version.

In order to understand what role order plays in the SNARC effect, two types of stimuli were used: normal and mirror Arabic numerals. While it is true that Arabic numerals are often displayed in an orderly manner from left to right (see Figure 7) the same cannot be said of their mirror version. Indeed, people are not used to looking at numbers in their mirror version, so they are unlikely to spatially organize them in a linear manner. Therefore, the basic hypothesis is that, if the order in which the numbers are presented is more important than the quantity, the SNARC effect should emerge only with normal numbers and never with mirror numbers, regardless of whether the quantities are processed.

The Welford function, which can predict participants' responses during a magnitude classification task (Experiments 1 and 2) based on the distance effect and the size effect, was used to see if the quantities of normal and mirror numbers had been processed during the task (see Chapter 2 for an explanation of the Welford function). The Welford function successfully predicted participants' reaction times, suggesting that people access the magnitude of numbers regardless of their normal or mirror presentation.

Although the Welford function confirmed that participants have access to quantitative information for both normal and mirror numbers, the SNARC effect emerged only for normal numbers (Experiment 1 and 3) and never for mirror numbers. These results suggest that the magnitude process does not explain the SNARC effect and that other factors, such as the order of numbers and perceptual physical characteristics of the digits, need to be considered.

Since quantity processing does not explain the spatial association found in the experimental part, the MNL account cannot be supported by this thesis. This means that it does not support the idea that quantities are represented in long-term memory from left to right and viewing numbers would activate this spatial representation of quantities.

This thesis is more supportive of the idea that the SNARC effect comes from the spatial organization of the stimuli (numbers, days of the week, months, etc.) involved in a task, regardless of their quantity. This organization is retrieved, during the task, from working memory on the basis of learned experiences. This would explain why numbers in their mirror version do not show a SNARC effect, even though their quantity is processed.

Instead of the MNL account more support is given to the WM account. Indeed, the WM account predicts that the SNARC effect should be greater in normal numbers than mirror numbers for two reasons: mirror numbers are not learned in a left-to-right organization; reading mirror numbers requires cognitive resources that are subtracted from their spatial organization. This suggests that left-to-right spatial organization of symbols depends on their physical structure, such that regular stimuli (e.g., normal numbers) are easier to organize along a continuum than ambiguous stimuli (e.g., mirror numbers).

Since the physical structure of the numbers played an important role in the emergence of the SNARC effect, its influence was analyzed in more detail using a method introduced by Cohen (2009) that allows to measure the physical similarity between numbers. Cohen's model, originally designed for normal numbers, has been extended and repurposed in this thesis for use with mirror numbers as well. New indices of physical similarity for the mirror numbers were calculated: Mirror-PS (the physical similarity between a number and its mirror version) and Group-PS, (the Physical Similarity between a number and a group of numbers, as described above).

The results showed that such Mirror-PS and Group-PS provided statistically robust predictors of both mRT (mean of RT) and dRT (right RT – left RT). In particular, in all three experiments, Mirror-PS significantly modulated participants' response times. One possible interpretation is that participants tend to transform mirror numbers into normal numbers before performing the task.

The Group-PS quantifies the likelihood that the participant will confuse the number presented either with a number that leads to an error in the response, or with one that leads to the correct response. Group-PS (used in Experiments 1 and 2) affected the dRT of normal numbers. This result can be interpreted as an indication of handedness association, similar to the SNARC effect. In other words, Group-PS indicates that physical similarity modulates the SNARC effect. In conclusion, numbers’ structure affects mRT and dRT even when the task does not explicitly require participants to process it. This suggests that the perception of the numbers’ structure precedes the access to quantity information and the left-to-right organization of numbers.

In summary, the data show that the physical similarity of numbers predicts participants' performance because the structure of numbers influences how quantities are accessed and spatially organized.

In conclusion, this thesis provides evidence in favor of a spatial organization of mathematical symbols and numbers that can be explained without the need to consider quantity. In addition, alternative explanations to the MNL account that are based on the spatial order of numbers and their physical structure have been provided. Future studies may further clarify the relationship between the order of numbers and quantity, as well as between their semantic meaning and their physical structure.

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