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Price manipulation: theoretical models and empirical investigation

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To my family

Abstract

Market manipulation is a primary concern of financial regulators around the world. Understanding why market manipulation is conducted, under which conditions it is the most profitable and investigating the magnitude of these practices are, therefore, all crucial questions. Closing price manipulation induced by derivatives' expiration is the primary subject of this thesis.

Firstly, this thesis provides a mathematical framework in continuous time to study the incentive to manipulate a set of securities induced by a derivative position. An agent holding a European-type contingent claim, depending on the price of a basket of underlying securities, is considered. The agent can affect the price of the underlying securities by trading on each of them before expiration. The trade-off is a classical one: if, on one hand, the agent has the incentive to trade in the direction that would increase the derivative's payoff, doing so, on the other hand, is costly. In this study, the optimal trading strategy is found and how this trade-off is solved under different market conditions is analysed, both analytically and through numerical simulations. The elements of novelty are at least twofold: (1) a multi-asset market is considered; and (2) the problem is solved by means of both classic optimisation and stochastic control techniques. Both linear and option payoffs are considered.

Secondly, an empirical investigation is conducted on the existence of expiration day effects on the UK equity market. This study provides an exploration of the expiration effects of both index derivatives and individual options. Intraday data on FTSE 350 stocks over a six-year period from January 2015-December 2020 are used. The results show that the expiration of index derivatives is associated with a rise in both trading activity and volatility, together with significant price distortions, around the time the settlement price is determined. A significant shift of volume traded in the closing auction is also found on quarterly expiration days. Derivatives' expiration, however, does not appear to be the cause of this rise of traded volume. The expiration of single stock options appears to have little to no impact on the underlying securities.

Finally, this thesis examines the existence of patterns in line with closing price manipulation of UK stocks on option expiration days. This study seeks signs of the existence of manipulative behaviour by option writers on the expiration days of single stock options. The main contributions are at least threefold: (1) this is one of the few empirical studies on manipulation induced by the options market, and the first focusing on the UK market; (2) proprietary equity order book and options transaction data sets are used to define manipulation proxies, providing a more detailed analysis than would be possible by only using publicly available data; and (3) the behaviour, on expiration days, of a specific class of market participants, proprietary trading firms, is studied. Despite the industry concerns and the insights provided by previous academic literature and prosecuted cases, no evidence is found of this type of manipulative behaviour.

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Contents

List of Figures

List of Tables

List of Abbreviations

ATM	**A**t-**T**he-**M**oney
BoE	**B**ank **o**f **E**ngland
CAPM	**C**apital **A**sset **P**ricing **M**odel
CBOE	**C**hicago **B**oard of **O**ptions **E**xchange
CFI	**C**lassification of **F**inancial **I**nstruments
CFTC	**C**ommodity **F**utures **T**rading **C**ommission
CMB	**C**apital **M**arkets **B**oard
EDSP	**E**xchange **D**elivery **S**ettlement **P**rice
ESMA	**E**uropean **S**ecurities and **M**arkets **A**uthority
ETF	**E**xchange **T**raded **F**und
FCA	**F**inancial **C**onduct **A**uthority
FSA	**F**inancial **S**ervices **A**uthority
GDR	**G**lobal **D**epositary **R**eceipt
HFT	**H**igh **F**requency **T**rading
HJB	**H**amilton-**J**acobi-**B**ellman
ICE	**I**nter**C**ontinental **E**xchange
ITM	**I**n-**T**he-**M**oney
IVP	**I**nitial **V**alue **P**roblem
LSE	**L**ondon **S**tock **E**xchange
MAR	**M**arket **A**buse **R**egulation
MiFID	**M**arkets **i**n **F**inancial **I**nstruments **D**irective
MiFIR	**M**arkets **i**n **F**inancial **I**nstruments **R**egulation
NAV	**N**et **A**sset **V**alue
ODE	**O**rdinary **D**ifferential **E**equation
OTM	**O**ut-of-**T**he-**M**oney
PDE	**P**artial **D**ifferential **E**equation
RTS	**R**egulatory **T**echnical **S**standards
SONIA	**S**terling **O**ver**N**ight **I**ndex **A**verage

Chapter 1

Introduction

1.1 Overview

This thesis presents a theoretical and empirical investigation of market manipulation. Providing a definition of market manipulation is a difficult task, as consensus has not been reached. For the purposes of this study, market manipulation is defined as an activity conducted with the intent of leading the price of a security to an artificial level. It negatively affects the integrity of financial markets. Most notably, it can cause price allocation distortion and loss of investor trust in the fair functioning of the market. It comes as no surprise that both academia and industry are interested in the topic. Financial regulators around the world spend considerable effort on studying and attempting to detect and prevent market manipulation.

This work is concerned with a specific class of manipulative behaviour, termed *contract-based manipulation* by Putniņš, (2012). This type of manipulation, as suggested by its name, consists of taking a position on a contract (or external market) for which the value depends on the price of another security (or securities). The manipulation is then carried out by trading on the security (or securities) with the goal of maximising the profit obtained from the external contract. A typical example (and a common concern in the industry) is provided by a market participant holding a position on a derivative contract and subsequently trading the underlying asset(s) on the expiration day with the sole purpose of increasing the gain obtained from the derivative's position.

The interest of this study is in determining and exploring the incentives for manipulating the settlement price of a derivative upon expiration. These incentives are first modelled and then described from a theoretical standpoint. An extensive empirical study is then conducted on the United Kingdom (UK) market to investigate the existence of anomalous behaviour aligned with this type of manipulation.

1.2 Overview of market manipulation

1.2.1 Classification of manipulation strategies

Market manipulation is a very general term encompassing a wide variety of different strategies. As providing a complete review of the literature on market manipulation is beyond the scope of this work[1], to provide a context, this section broadly describes some of the main types of manipulation. The next section then focuses on contract-based manipulation.

Within the traditional market microstructure literature, Allen and Gale, (1992) identify three macro classes of market manipulation: (1) action-based, wherein manipulators take specific actions to affect the price of the security, such as managers making business decisions affecting the value of their company with the goal of profiting from their position on the company's stocks; (2) information-based, impacting on the price by spreading information that is false and price-sensitive; and (3) trade-based, affecting the price by buying or selling the security.

Action-based and information-based manipulations usually follow the same scheme. First, manipulators take a certain position on a financial asset, which can be either long or short. Then, they attempt to affect the price in the desired direction and, once the price has moved with a magnitude they consider acceptable, they close their initial position, realising a profit. Generally, the situation where a manipulator opens a long position and then tries to push the price upwards is referred to as *pump and dump*. Conversely, when the manipulator opens a short position and then tries to depress the asset value, the term usually used is *bear raids* (or *short and distort*). The difference between action-based and information-based manipulation comes down to how the value of the asset is artificially inflated or depressed. Action-based manipulation requires manipulators to be able to take direct actions that can lead to a material impact on the price. The manipulation case of the American Steel and Wire Company in 1901 is discussed by Allen and Gale, (1992). The company's managers first shorted the stock and then closed the company's steel mills. This decision had the effect, when it came into the public domain, of depressing the stock price from around US$60 to US$40. The managers then closed their short positions, realising approximately US$20 gain per share, and eventually reopened the mills, which drove the price up to the initial levels. Information-based manipulation, on the other hand, achieves the same results through the spread of false or misleading information which leads to significant changes to the price, provided the public believes in the authenticity of the information and trades accordingly. As noted by Putniņš, (2012), action-based manipulation is not widely studied by the academic literature. However, it can be seen as a type of information-based manipulation, with the false information spread to the public viewed as the product of a manipulator's actions. Most findings and considerations that are true for information-based manipulation

[1] For further details on the various types of market manipulation techniques and on the state of academic research, the reader is redirected to Putniņš, (2012). A survey including the new forms of market manipulation that have arisen along with the recent advancements in technology, can be seen in Siering et al., (2017).

can, therefore, also be applied to action-based manipulation.

According to Allen and Gale, (1992), although the extensive regulation adopted throughout the twentieth century contributed to significantly reducing action-based and information-based manipulation, trade-based manipulation is tending to be much more difficult to eradicate. Trade-based manipulation schemes simply consist of attempts to manipulate the price of a security by buying (or selling) and can take various different forms.

Pump and dump (and short and distort) manipulation schemes, as discussed above, can also be conducted in a trade-based manner. A trade-based pump and dump (or short and distort) scheme would simply consist of heavily buying (selling) a security to produce a price change, followed by liquidation of the acquired long (short) position at the inflated (deflated) price. This comprises basically two phases: (1) accumulation of the position, necessary to affect the price in the desired direction, and, subsequently, (2) liquidation of the accumulated position at the inflated or deflated price, to lock in the profits from the manipulation. Certain conditions must be satisfied for this strategy to succeed. The liquidation of the accumulated position, in fact, will lead to a price impact, as it would through the accumulation phase. If the price impact of the accumulation and liquidation phases is the same, this type of manipulation is not profitable. Allen and Gale, (1992) show that incomplete information is the key ingredient. Manipulators can be successful if other investors cannot distinguish between their buying and selling and that of large traders who are buying (or selling) the security because they genuinely believe it is undervalued (or overvalued). In this case, other investors will react to what they believe is the existence of an informed investor by trading in the same direction as the manipulator during the accumulation phase, therefore generating momentum in the price dynamics and amplifying the effects induced by the manipulator's trading.

The manipulation techniques described above rely on tricking investors by executing transactions in a way that generates a false picture of what is happening in the marketplace. Wash trading and matched orders are another form of market manipulation and share a similar goal. Wash trading (also called wash sales) refers to a type of manipulation strategy where the same trader simultaneously buys and sells a certain asset. The transactions are, therefore, fictitious as no actual change in ownership occurs. The aim is to give the illusion of a marketplace that is more active than is actually the case, to induce other investors to buy the security, generating a price increase. Matched orders refer to this same manipulation scheme, conducted, however, by a group of colluding traders buying and selling the security among themselves.

Market corners (also called squeezes) are another form of trade-based manipulation. Cornering the market consists of a manipulator, either a single trader or a group of colluding traders, acquiring a position on a security that is large enough to grant market power. As a consequence, manipulators, by controlling the supply of the security, have the power to influence the price. Corners are often conducted when investors have contractual obligations that require them to possess the security. For example, individual stock options are often settled with physical delivery. Therefore, if a put option is exercised by the holder or it expires in the money (ITM), the option writer will be required to sell the underlying stock. Option writers need to possess the required amount of the underlying stock and, if they do not, they will have to buy it on the market. If the underlying equity market is cornered, however, the manipulators can, in theory, set any price they want, counting on the need of option writers to buy the stock to fulfil their contractual obligations. A very similar scenario takes place when manipulators corner the market of a stock which has many short sellers. Manipulators, instead of taking advantage of option writers, do the same with short sellers in need of covering their positions. Closing a short position, in fact, requires short sellers to buy back the securities they sold. This scenario is usually called a short squeeze.[2]

Contract-based manipulation is another form of trade-based manipulation, as discussed by Putniņš, (2012). It consists, firstly, in opening a position on a contract for which the value is contingent on the price of another security. Secondly, the manipulator trades in the security with the goal of artificially affecting its price and, consequently, that of the external contract/position, the value of which depends on it. This type of manipulation scheme is central to this study, with more details to follow in the next section.

Finally, the advancements in technology (together with regulation) have led to drastic changes in financial markets. One of the main changes has been the widespread use of algorithms for both investment decision making and actual trading execution; see O'Hara, (2015), Biais and Foucault, (2014). This type of trading is usually called algorithmic trading. High-frequency trading (HFT) is a subset of algorithmic trading that relies on the ability to submit and execute orders at particularly low latency. While several positive effects of HFT on market quality are reported by the literature, some detrimental effects are also reported. The report by Tse and X. Lin, (2012) is among the first to provide examples of manipulative trading strategies that are specific to high-frequency trading (HFT). As with market manipulation in general, the new forms of manipulative behaviour can be of various types: spoofing, layering, quote stuffing and pinging are some of the main examples.[3]

A common and distinguishing feature of most of these new forms of manipulation is the ability to artificially affect the stock price through sophisticated order submission strategies, without the need to execute any trades. To reflect this peculiarity, Siering et al., (2017) provide a taxonomy of market manipulation strategies which builds upon and expands the broad classification proposed by Allen and Gale, (1992), by adding order-based manipulation as a new class of manipulation strategy. The paper by Siering et al., (2017) provides a description of each of these manipulations.

[2]See Jarrow, (1992) for both a theoretical and empirical study on these types of manipulation. The paper by Chohan, (2021) is also suggested, which provides a description of one of the first examples of decentralised short squeezes: the Gamestop case.

[3]A general discussion on the rise of the new forms of manipulation and its implications can be found in T. C. Lin et al., (2017).

1.2.2 Contract-based manipulation

Contract-based manipulation is a simple form of manipulation that consists of buying (or selling) securities with the goal of artificially affecting their price and, consequently, a contract with a position on an external market, with the value of that position contingent on the price of the manipulated securities. The act of manipulating the price of the securities is generally a cost for the manipulator, as it might require executing a large amount of trades. As a result, the manipulator needs to be willing to pay high transaction costs, possibly due to factors, such as the existence of commissions or other market inefficiencies related to market liquidity, for instance, the bid-ask spread. Furthermore, while manipulators need to be able to affect the price in a certain direction for the success of this type of manipulation, this is also a cost, as the manipulators will have to buy (or sell) the securities at increasingly higher (or lower) prices. Finally, the position accumulated on the securities will have to be liquidated once the manipulative strategy is complete (as there is generally no genuine intention to invest in these securities) which leads to additional transaction costs.

Manipulators, therefore, face a trade-off between the profit they expect to obtain from the external contract, if they succeed, and the costs they expect to incur for the manipulation to be successful. Understanding how this trade-off is solved is crucial to better understanding the motivations and incentives behind this manipulative behaviour.

One of the most common forms of contract-based manipulation is the manipulation of closing prices, usually called *marking the close*[4]. This manipulation strategy consists of buying (or selling) a security shortly before the close of the trading session, with the purpose of artificially affecting the closing price. The incentive to manipulate closing prices generally comes from the fact that they are used as a benchmark/reference price to determine the value of an external contract. Closing prices are generally used for a variety of purposes and, therefore, different incentives arise. For example, they are often used to compute funds' net asset value (NAV). Carhart et al., (2002) and Ben-David et al., (2013) provide evidence of abnormal patterns towards the end of key reporting dates on US markets, with this evidence compatible with fund managers manipulating stocks in the fund's portfolio with the goal of increasing performance. Closing prices are also used to evaluate the performance of traders and brokers, who therefore have the incentive to manipulate these prices to increase the reported performance.[5] Closing prices can be manipulated to obtain inclusion into (or to avoid exclusion from) market indices.[6]

Closing prices are often used for derivatives' settlement. A market investor holding

[4] It is also referred to as *punching the close* or *banging the close*.

[5] Hillion and Suominen, (2004) propose a model of closing price manipulation by brokers who affect the closing price with the goal of altering the performances reported to their customers and find empirical evidence in support of this hypothesis on the Bourse of Paris and on the Bolsa de Madrid. Felixson and Pelli, (1999) study the existence of manipulative patterns at the end of the trading day, when traders hold large positions on a stock and have the incentive to affect the closing price which is used to measure their trading performance.

[6] Zdorovtsov et al., (2017) document the existence of abnormal price movements around the Financial Times Stock Exchange (FTSE) Russell rebalancing, which are explained in light of manipulative attempts aimed at gaining inclusion of certain stocks into an index (or to avoid exclusion from it).

large position on a derivative can have the incentive to manipulate the underlying prices to maximise the payoff obtained from the derivative. This is especially true on the expiration day. In fact, cash-settled derivatives self-liquidate without the need to incur in any transaction costs to realise the profit. For derivatives with physical delivery, the closing price is the main signal used to decide to exercise or not[7], as argued by Ni et al., (2005).

To finally note that marking the close manipulation is similar to other forms of contract-based manipulation, such as marking the open, which consists of artificially affecting a security's opening price. The main difference is the target price the external contract's value depends on, which, consequently, is the price that the manipulator wants to affect. The mechanics of the manipulation strategies are, however, very similar. The main reason marking the close has received significantly more attention from both industry and academia is the widespread use of closing prices as reference prices for a variety of external contracts. This makes these prices significantly more susceptible to manipulation than any other price during the day. Furthermore, closing prices are generally viewed as a summary of a security's value on a certain day and, given their importance, ensuring the integrity of the price formation at the end of the day is of the utmost importance.

1.3 Research themes and main contributions

1.3.1 Theoretical modelling

As this study's first contribution, models are developed in continuous time of agents who hold a contingent claim and can trade in the underlying securities and, by doing so, can affect prices. Both classical and optimal control techniques are employed in a stochastic setting to solve the trade-off faced by the agent and to study the condition under which manipulation is the optimal behaviour.

As a first element of novelty, this problem is studied in a multi-asset market environment. In other words, the manipulation problem is analysed in the general setting of a derivative product the value of which is contingent on the value of a basket of financial securities. The only other work studying this problem in a general setting for a multi-asset market is, to the best of the author's knowledge, Nyström and Parviainen, (2017). The difference between that work and the current study is this attempt to find, when possible, analytical solutions that can be used to build rules and easily implementable formulas to measure the manipulability of a derivative product. This point is raised by Ko et al., (2016), who pose two fundamental problems: (1) the necessity to find measures of the degree to which a portfolio of assets is manipulable and (2) which securities, within the portfolio, are most likely to be manipulated. Finally, the current study's formulas can have policy implications. Dutt and L. E. Harris, (2005), for example, develop a model of manipulation (generated by linear payoffs) that can be used to set position limits to restrict the manipulation effects to a degree

[7] In the case of derivatives which need to be exercised, such as options.

that can be considered acceptable. The current study can be viewed as an extension in this direction, both in terms of modelling and the coverage of derivative payoff functionals.

1.3.2 Expiration day effects

As a second contribution, this study explores expiration day effects in the UK market. The study investigates whether the expiration of listed derivatives (index futures and options and individual equity options) produces statistically significant effects on the underlying stocks, in terms of trading activity and price distortions. This empirical investigation is a contribution to the market microstructure literature on expiration day effects. The existing studies revolving around the UK market are not numerous, with the author of the current study aware of only two papers. In the first, Pope and Yadav, (1992) uniquely study effects related to equity option expiration, in a context where the options market was at its beginning while, in the second, Batrinca et al., (2020) study effects related to the expiration of index derivatives of the main European market indices.

The current study shows that the expiration of index derivatives is associated with an increase in both trading activity and volatility, together with significant price distortion. These effects are mainly concentrated in the morning session of the trading day, around the time the settlement price is determined. The study also finds that, on quarterly expiration days, a significant shift of liquidity occurs towards the closing call auction. This shift is a market-wide phenomenon affecting all stocks analysed. The expiration of derivatives, however, does not appear to be the main cause of this rise of volume traded at the close. This study's interpretation is that other factors (mainly the quarterly Financial Times Stock Exchange [FTSE] Index recalibration) come into play. These sources of abnormal behaviour are separated by using intraday data. This study highlights that results in other studies, relying on daily volume figures (Batrinca et al., (2020)), might overestimate the anomalous effects due to the expiration of index derivatives. Finally, a marginal increase of trading activity on the FTSE 250 related to the expiration of individual stock options is found. However, the expiration of single stock options appears to have little or no impact on the underlying securities.

1.3.3 Marking the close at expiration

As a third contribution, the current study explores the existence of anomalous price movements as well as trading and order submission patterns on the expiration day of equity options on the UK market, with these compatible with marking the close manipulation. A series of testable hypotheses are developed based on the intuition provided by previous theoretical findings and on the knowledge from a review of actual prosecuted cases of marking the close manipulation, as well as previous academic literature (although this is quite scarce). The study also relies on proprietary data to investigate whether a certain class of market participants who are particularly active on the options market modify their behaviour in a way that is compatible with manipulative intentions. The traders in question are proprietary trading firms comprising sophisticated market participants trading their own capital. As shown in this study, they are the most active participants in the options market and have the highest incentives to manipulate the market at expiration. This study also aims to address the common fear in the industry that market participants with short positions on individual stock options at expiration tend to manipulate the underlying price to ensure the options will expire out of the money (OTM) (that is, worthless).[8]

Overall, the tests conducted lead to the rejection of the market manipulation hypothesis, showing that closing prices are clean and free of systematic manipulation on the day of option's expiration. As previously stated, Ni et al., (2005) study is the closest to the current study. To the best of the author's knowledge, no other empirical studies have investigated price manipulation on option expiration days, despite theoretical work suggesting its possibility and despite concerns in the industry and various examples of prosecuted cases. This lack of research is likely to be due to the lack of available data, especially on the identity of market participants, with these data generally not available to researchers. The current study was able to access this type of data through collaboration with the Financial Conduct Authority (FCA).

1.4 Summary

This thesis presents a theoretical study on the incentives for manipulating the underlying prices of a derivative product prior to expiration. It also provides an extensive empirical investigation of anomalous patterns observed on expiration days and whether these can be linked to attempts to artificially inflate or deflate prices.

The remainder of this thesis is structured as follows. Chapter 2 proposes a series of models (based on both classical and dynamic optimisation techniques) to study the trade-off faced by a potential manipulator holding a derivative. Analytical and easily implementable formulas to measure these incentives are also proposed. Chapter 3 analyses the existence of abnormal patterns, in terms of trading activity and price distortions, on the expiration days of equity derivatives in the UK market. Chapter 4 builds upon the previous chapters, as well as previous literature and knowledge retrieved from actual prosecuted cases, providing an empirical framework with which to study the existence of manipulative patterns on option expiration days, in the UK market. Novel and proprietary data sets comprising the identity of market participants on both the equity and options market are used. Chapter 5 concludes the thesis, summarising the main findings and proposing ideas for future lines of research.

[8]This set of ideas is generally referred to as *maximum pain theory* (see Davies, (2020)).

Chapter 2

Theoretical framework of multi-asset cross-product manipulation

2.1 Introduction

As discussed in the previous chapter, holding derivatives might induce incentives to manipulate the underlying(s) to maximise the payoff on the derivative position. This phenomenon is a concern for the industry as it is seen time and time again by financial regulators, both in emerging and developed markets.[1]

This chapter describes the current study's development of models in continuous time of agents holding a contingent claim and trading in the underlying asset with price impact. Both classic and optimal control techniques are employed in a stochastic setting to solve the optimisation behaviour faced by the agent. As a first element of novelty, the problem is explored in a multi-dimensional setting. In other words, the manipulation problem is analysed in the general setting of a derivative product for which the value is contingent on the value of a basket of financial securities. The only other work studying a multi-dimensional setting, to the best of the author's knowledge, is Nyström and Parviainen, (2017). The current study differs as it attempts to find, when possible, analytical solutions that can be used to build rules and easily implementable formulas to measure the manipulability of a derivative product, under specific market conditions. Finally, these formulas can have policy implications. Dutt and L. E. Harris, (2005), for example, develop a model of manipulation (generated by linear payoffs) that can be used to set position limits to restrict the manipulation effects to a degree that can be considered acceptable. The current study can be viewed as a significant extension in this direction, both in terms of modelling and its coverage of the derivative payoff functionals.

The remainder of the chapter is organised as follows. In Section 2.2, the literature review is presented, while Section 2.3 provides a primer on optimal control techniques in a stochastic framework. Section 2.4 presents the set-up of the model and the general optimisation problem faced by an agent who holds a derivative position and can affect the underlying

[1] See, for example, prosecuted cases in developed markets, *FSA v. Goenka* (2011) or *SEC v. Saba* (2004).

prices. The optimal solutions to the agent's problem are solved for both derivatives with linear payoffs (Section 2.5) and non-linear payoffs (Section 2.6). Finally, Section 2.7 concludes the chapter.

2.2 Literature review

The body of theoretical work can provide knowledge on when and if manipulation is profitable. In other words, from theory, insights can be gained on the market conditions under which manipulation can be a rational behaviour by market participants, in the sense that manipulating the market leads to the maximum level of expected utility. Academia proposes a variety of models to analyse the incentive to manipulate the underlying prices of a derivative.

One of the very first contributions in this area is provided by P. Kumar and D. J. Seppi, (1992). The authors propose a discrete time, two-state Kyle-like model[2] of *punching the settlement* of a cash-settled future contract. The authors broadly talk about manipulation of the settlement price. In fact, although the settlement price used for these purposes is usually the closing price of underlying securities, this is not always the case. However, while the price target of the manipulator might differ, the mechanics of the manipulation strategy are basically the same. The authors show that uninformed investors can expect to gain positive profits from opening a futures position and subsequently trading in the underlying asset in the direction that would maximise the profit obtained on the futures contract. The success of manipulation ultimately depends on the fact that manipulative trades are not fully distinguishable from those of the informed trader. It is also shown that the existence of manipulation incentives is persistent to many changes to the parameters of the proposed market model and that, although increasing the number of manipulators in the model leads to decreasing profits from manipulation that vanish at the limit, the adverse effects that manipulation has on welfare and price liquidity persist.

Dutt and L. E. Harris, (2005) are among the first to study the incentives generated by cash-settled linear derivatives in a multi-asset setting. The authors propose a one-state model in which an agent holds a derivative contract delivering, at expiration, a cash value which depends on a specific market index. The agent also has the possibility of trading on each of the securities comprising the index, affecting their price through trading. This leads to the common trade-off faced by any manipulator, between the costs required to affect the prices of the securities and the expected gains from manipulation. The authors find and describe the optimal strategy (in terms of the amount of shares to buy/sell for each security) and show that a profit-maximising agent has the incentive to affect the underlying prices to increase the derivative's payoff. Furthermore, the authors propose to use this model to construct formulas and rules to optimally define size limits to the positions that can be opened on a derivative contract of the type studied in their paper. Knowing the rational behaviour

[2]Kyle, (1985).

followed by derivatives holders, of which manipulation is a possibility, position limits can be set in a way that the optimal behaviours by traders will lead to a price change not higher than a specific threshold, which should be set based on the maximum price change due to manipulation that regulators can tolerate.

The same type of manipulation is studied by Ko et al., (2016). Their model is very similar to the one proposed by Dutt and L. E. Harris, (2005). One of the main differences is the possibility, for the potential manipulator, of trading only a subset of the basket of underlying securities. This is realistic for narrow-based index derivatives (i.e., derivatives for which the underlying is a market index with a small number of components), as the manipulator might try to distort the value of the derivative by trading only the components that have the highest impact on the index value and, by doing so, minimising the costs of manipulation. Based on their model, the authors propose a measure of manipulability for a specific index, which is based on measuring the expected profits from the optimal manipulative strategy, and a method to identify the index components that are the most likely target of manipulation attempts.

These two papers deal with derivatives with linear payoffs. Only a few other papers focus on manipulation attempts in connection to the expiration of derivatives positions with a non-linear payoff. Option derivatives are of great interest owing to their widespread use. Danger et al., (2020) discuss the profitability of manipulating the prices of the securities underlying an option contract. They show that cash-settled options generate an incentive to manipulate the underlying for both the holder and the writer. Provided the option is in the money (ITM), the option holder has the incentive to affect the underlying price to increase the payoff. If the option is out of the money (OTM), but the price is sufficiently close to the strike, the option holder has the incentive to affect the price to ensure the option to expire in the money (ITM) (thus profiting from it). If the option is too deep OTM, on the other hand, no chance exists for the option to expire in the money (ITM), so no incentive exists to manipulate the underlying. Similarly, an option writer has the incentive to affect the underlying price to push an option that is in the money (ITM) to out of the money (OTM), avoiding any liability towards the other party holding the option. This would be possible, obviously, if the option is close enough to the money that a change of moneyness can be achieved by the manipulator.

The authors also study manipulation incentives in the case of options with physical delivery. This presents the option holder with no incentive to manipulate: taking the example of call options, this scenario would correspond to increasing the position on an overvalue security. On the other hand, an option writer can profit from manipulating the underlying of an out of the money (OTM) option. The manipulation can be successful if they can make the option expire in the money (ITM) and trick the option holder into exercising the option. In this way, the option holder would pay (or receive, depending on whether it is a call or put option) a strike price that is expected to be higher than the price paid by the manipulator to affect the stock. The position accumulated by the manipulator can also be sold to (acquired from) the option holder, so that no (or minimal) inventory is left. Despite the attempt made by these authors to translate the concept of manipulation into formulas, their proposed framework is limited. The most important limitation lies in the fact that they propose quantitative measures of manipulation that assume manipulation to be successful. It is, in other words, a pure *ex post* analysis of manipulation. The authors do not discuss, for example, if manipulation is, *ex ante*, the optimal strategy for the option holder or writer.

By using a very similar set-up to P. Kumar and D. J. Seppi, (1992), Gallmeyer and D. Seppi, (2000) propose a two-stage model for the incentives generated by holding a cash-settled option to manipulating the price of the underlying security prior to expiration. The option is assumed to be European and settled in cash. The authors find that, assuming the underlying spot price is not too far from the strike price for the option to end up in the money (ITM) at expiration, price manipulation aimed at increasing option payoff can be the optimal behaviour for the option holder. When manipulation is not the optimal solution, the agent profits by providing liquidity to the other market participants. Even when manipulation is the optimal strategy, providing liquidity to other traders (even if it affects the prices in the opposite direction to the manipulation goals) can also be an optimal behaviour to keep (before actually manipulating the price) to reduce the total costs of the manipulation strategy. Finally, the authors note how the possibility of *ex-ante* manipulation can represent a significant portion of the option's value.

Kraft and Kühn, (2011) find the optimal strategy, within a Black and Scholes framework, of an agent holding a claim and producing a market impact when trading on the underlying asset. Differing from Gallmeyer and D. Seppi, (2000), their study proposes a model in continuous time. These authors study the optimal behaviour of the agent and, more precisely, under which conditions the trade-off is solved by the agent in favour of manipulating the underlying price instead of hedging the derivative. They notice that, if the market impact is negative, hedging the option has the effect of affecting the price in the direction that maximises the payoff. The optimal behaviour of the agent consists of under-hedging the option position with respect to what would be optimal in a classical Black-Scholes framework; in the case of perfect hedging, in fact, no profit from manipulation would be possible. If the price impact is positive, on the other hand, this is no longer true: hedging activities manipulate the payoff in the unfavourable direction. However, the agent can exploit the price impact connected to the hedging activities and can make additional profits by over-hedging the position. No matter what the sign of the price impact, the agent can exploit it to make extra-profits and, therefore, values the option higher than its Black-Scholes price (which assumes price-taker agents).

A game theoretical version in continuous time of the behaviours of option holders and writers is proposed by Horst and Naujokat, (2011). The option is assumed to be European and cash-settled at expiration and depends on a single asset. All the agents in the stochastic game can affect the underlying security price by trading on it. The solution of the game, which is obtained both under the assumption of risk-neutral and risk-averse agents, represents, for each agent, the optimal trading strategy. By analysing the solution, the authors attempt to analyse which market conditions lead to manipulative strategies as optimal behaviours. First, they find that, in zero-sum games between an option holder and an option writer, both generating the same market impact on the price, no manipulation occurs. Temporary transaction costs (e.g., bid-ask spread) discourage manipulation, as they are additional costs that manipulators have to pay in addition to trading at increasingly more expensive prices in order to affect the price. Furthermore, they adjust the initial set-up by introducing the scenario of physical delivery and find that it removes the incentives for manipulation for option holders, although the same scenario for option writers is not discussed.

In a similar fashion, Nyström and Parviainen, (2017) study a game theoretical model between players holding a derivative written on a basket of assets. The players of the game, as in the study by Horst and Naujokat, (2011), can trade on the underlying securities and, by doing so, can affect both the drift and the volatility of the price dynamics. The authors find that the stochastic game admits a (unique) viscosity solution and, based on this, they find the value of the option. The framework proposed in their study is extremely general. First, to the best of the author's knowledge, their study is the first to cover the case, from a manipulation perspective, of non-linear payoffs depending on multiple securities. Furthermore, their framework can be applied to derivatives with very general payoff functionals: the only requirement is for the payoff to be a positive-bounded Lipschitz function. Through being very general, not much intuition is provided on optimal behaviour and how it is affected by different market conditions.

Finally, Aïd et al., (2020) propose a series of continuous-time models to analyse the incentives to manipulate the price of a commodity generated by a non-linear claim contingent on a commodity's value at expiration. The context of this paper is slightly different. The authors assume a (risk-neutral) producer that is able, directly through controlling the production rate or indirectly by spreading false information, to affect both the drift and the volatility of the price dynamic of a certain commodity. They also model the behaviour of a trader that is assumed at first as a price taker and then, at a second stage, as able to control the volatility of the commodity's price. Both the producer and the trader can acquire a position of a certain size in option contracts written on the commodity's price. Their optimal behaviours (both in isolation and in the context of a stochastic game) is then found and analysed. The authors show that, under certain circumstances, it is rational for the producer to affect the commodity's price to increase the profit they obtain from the option contracts. It can be optimal for the producer to initiate manipulative behaviours also when the trader has an opposite position on the option and can, therefore, attempt to affect the price in the opposite direction (unfavourable to the producer). In this case, however, the profit that the producer can expect to obtain from manipulating the price are lower, because of this contrasting force generated by the trader.

2.3 Concepts and definitions of stochastic optimal control theory

In this section the intuition behind dynamic control techniques in a stochastic setting is provided. A formal and complete treatment of the mathematical details is out of the scope of this work. The reader is redirected to Bellman, (1952); Bellman, (1954) for Bellman's seminal work on dynamic programming; Fleming and Soner, (2006) for a formal treatment of the theoretical foundations of stochastic optimal control problems and A. Cartea et al., (2015) for an extensive application of such techniques in the context of algorithmic trading.

2.3.1 The optimisation problem and the Hamilton-Jacobi-Bellman equation

A d-dimensional stochastic system $\boldsymbol{X}_t$ with the following dynamics is considered:

$$d\boldsymbol{X}_t = \boldsymbol{\mu}(t, \boldsymbol{X}_t, \boldsymbol{u}_t)dt + \boldsymbol{\sigma}(t, \boldsymbol{X}_t, \boldsymbol{u}_t)d\boldsymbol{W}_t, \quad \boldsymbol{X}_0 = \boldsymbol{x}_0, \tag{2.1}$$

where $\boldsymbol{W}_t$ is a d-dimensional vector of dependent Brownian motions, $\boldsymbol{u}_t$ is a p-dimensional process and $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ satisfy the usual condition of Lipschitz continuity. In the context of an optimal control problem, the process $\boldsymbol{X}_t$ is generally called *state process* and $\boldsymbol{u}_t$ *control process*. The control process $\boldsymbol{u}_t$ models the actions of the *controller*, that is an agent able to affect the dynamic of $\boldsymbol{X}_t$. Generally, a set of restrictions are set on the controls: a control process satisfying all the required constraints is said to be an admissible control. The set of admissible controls is commonly denoted by $\mathcal{A}$.

Let the optimal control problem on a fixed time horizon $[0, T]$ be considered. Given an arbitrary admissible control $\boldsymbol{u}_t$, a *performance criterion* is defined as:

$$V^{\boldsymbol{u}}(\boldsymbol{x}) = \mathbb{E}\left[\int_0^T h(s, \boldsymbol{X}_s, \boldsymbol{u}_s)ds + g(\boldsymbol{X}_T)\right]. \tag{2.2}$$

The function $V^{\boldsymbol{u}}(\boldsymbol{x})$ represents the expected reward (or expected cost, depending on the definition of the functions $h(\cdot)$ and $g(\cdot)$) obtained over the interval $[0, T]$ by controlling the state process $\boldsymbol{X}_t$ through $\boldsymbol{u}_t$. The functions $h(\cdot)$ and $g(\cdot)$ are commonly called *running cost* and *endpoint cost*, respectively.

The objective of the agent is to find the admissible control $\boldsymbol{u}$ over the interval $[0, T]$, that maximises the performance criterion $V^{\boldsymbol{u}}(\boldsymbol{x})$:

$$V(\boldsymbol{x}) = \sup_{\boldsymbol{u} \in \mathcal{A}} V^{\boldsymbol{u}}(\boldsymbol{x}). \tag{2.3}$$

The key idea to solve this type of problem is to embed it into a larger class of time-dependent problems. In other words, given $t \in [0, T]$:

$$V(t, \boldsymbol{x}) = \sup_{\boldsymbol{u} \in \mathcal{A}} V^{\boldsymbol{u}}(\boldsymbol{x}), \tag{2.4}$$

with

$$V(t, \boldsymbol{x}) = \mathbb{E}_{t,x}\left[\int_t^T h(s, \boldsymbol{X}_s, \boldsymbol{u}_s) ds + g(\boldsymbol{X}_T)\right].$$

The function $V(t, \boldsymbol{x})$ is called *value function* and the expectation $\mathbb{E}_{t,\boldsymbol{x}}[\cdot]$ is conditional to time t and to the state of the system at that time $\boldsymbol{X}_t = \boldsymbol{x}$. To notice that the problem above corresponds to the original problem for $t = 0$.

Bellman's principle of dynamic programming states that, given an arbitrary stopping time $t \leq \tau \leq T$, the value function can be written as:

$$V(t, \boldsymbol{x}) = \sup_{u \in \mathcal{A}} \mathbb{E}_{t,x}\left[\int_t^\tau h(s, \boldsymbol{X}_s, \boldsymbol{u}_s) ds + V(\tau, \boldsymbol{X}_\tau)\right]. \tag{2.5}$$

This expression is stating that the value function can be viewed as the running costs from t to τ and the maximum expected profits obtained by proceeding optimally from τ to T.

From this expression the dynamic programming equation can be obtained, heuristically. First, a constant control $\boldsymbol{u}_s = \boldsymbol{v}$ for $s \in [t, \tau]$ is considered. Then, by definition:

$$V(t, \boldsymbol{x}) \geq \sup_{u \in \mathcal{A}} \mathbb{E}_{t,x}\left[\int_t^\tau h(s, \boldsymbol{X}_s, \boldsymbol{v}) ds + V(\tau, \boldsymbol{X}_\tau)\right]. \tag{2.6}$$

If $V(t, \boldsymbol{x})$ is subtracted from both sides of the inequality, it is then divided by h and the limit as $h \to 0$ is taken, the following inequality if obtained:

$$(\partial_t + \mathcal{L}_t^{\boldsymbol{v}}) V(t, \boldsymbol{x}) + h(t, \boldsymbol{x}, \boldsymbol{v}) \leq 0, \tag{2.7}$$

where

$$\mathcal{L}_t^{\boldsymbol{v}} = \mu(t, \boldsymbol{x}, \boldsymbol{v}) \partial_{\boldsymbol{x}} + \frac{1}{2} \sigma'(t, \boldsymbol{x}, \boldsymbol{v}) \sigma(t, \boldsymbol{x}, \boldsymbol{v}) \partial_{\boldsymbol{x}\boldsymbol{x}},$$

with ∂_t and $\partial_{\boldsymbol{x}}$ denoting the partial derivatives with respect to time t and state variables $\boldsymbol{x}$. To complete this (informal) derivation, it is sufficient to observe that if an optimal control $\boldsymbol{u} = \boldsymbol{u}^*$ is taken instead of an arbitrary constant one, the inequality above becomes an equality:

$$(\partial_t + \mathcal{L}_t^{\boldsymbol{u}^*}) V(t, \boldsymbol{x}) + h(t, \boldsymbol{x}, \boldsymbol{u}^*) = 0. \tag{2.8}$$

The equality (Equation [2.8]), in combination with Equation (2.7), generates the dynamic programming equation, also known as Hamilton-Jacobi-Bellman (HJB) equation:

$$\partial_t V(t, \boldsymbol{x}) + \sup_{u \in \mathcal{A}} \left(\mathcal{L}_t^{\boldsymbol{u}} V(t, \boldsymbol{x}) + h(t, \boldsymbol{x}, \boldsymbol{u})\right) = 0, \tag{2.9}$$

associated with the terminal condition:

$$V(T, \boldsymbol{x}) = g(\boldsymbol{x}). \tag{2.10}$$

While the original optimisation problem (2.3) requires to find the maximum over a set of functions, through the application of the dynamic programming principle, the problem can be expressed in a form that gives the optimal control point-wise. The resolution of the HJB equation provides the optimal control $\boldsymbol{u}^*$ for every point t in the interval $[0, T]$, from which the optimal control process over the entire interval can be retrieved.

Furthermore, if the value function is treated as known, the optimal control process can be expressed in feedback control form, that is as a function of the value function itself:

$$\boldsymbol{u}^* = \sup_{\boldsymbol{u} \in \mathcal{A}} \Big(\mathcal{L}_t^{\boldsymbol{u}} V(t, \boldsymbol{x}) + h(t, \boldsymbol{x}, \boldsymbol{u}) \Big). \tag{2.11}$$

This is generally called the *Hamiltonian* of the associated optimal control problem.

2.3.2 Types of solution

A question arises of whether the solution to the HJB equation does, indeed, solve the control problem.

In general, one of the best scenarios is if the Equation (2.9) admits a classical solution, i.e., a solution that is once differentiable in time and twice differentiable with respect to the state variables. If a classical solution can be found and the resulting control process is admissible, then the solution solves the control problem associated and it is indeed the value function. See Theorem 5.2 in A. Cartea et al., (2015). If a classical solution is not found, searching for weaker solutions through the theory of viscosity solution is generally the approach.

2.4 Model set-up

2.4.1 Market dynamics

This section presents and discusses the modelling assumptions. The set-up is based on classic notions of stochastic calculus.[3] A notation similar to the one generally used by Cartea and co-authors is adopted. Firstly, the existence of an agent (the potential manipulator) holding a position on a derivative contingent on a basket of underlying securities is assumed. The final payoff H is a function of the underlying prices $H = H(\boldsymbol{S})$, where $\boldsymbol{S}$ is a d-dimensional vector of the price of the underlying asset, for which the dynamics are introduced below. Further details on the specific forms of H analysed in this chapter are provided in the next section

Let a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ be given. The probability space is equipped with a filtration $(\mathcal{F}_t)_{t \in \mathbb{R}_+}$, satisfying the usual conditions. All stochastic processes presented here are defined on $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \in \mathbb{R}_+}, \mathbb{P})$.

[3]See, for example, Björk, (2009) for a comprehensive treatment of this topic.

Given $i \in \{1, \ldots, d\}$, the dynamics of the stock prices are modelled as:

$$S_t^i = S_0^i + \int_0^t \mu_i du + \int_0^t \lambda_i \nu_u^i du + \int_0^t \sigma_i dW_u, \tag{2.12}$$

where $\boldsymbol{W}_t = (W_t^1, \ldots, W_t^d)_t$ represents a d-dimensional Brownian motion. With ρ_{ij} being the correlation between W_t^i and W_t^j, it follows that $\Sigma = (\rho_{ij}\sigma_i\sigma_j)_{1 \le i,j \le d}$ is the covariance matrix associated with $\boldsymbol{S}_t$. The agent's trading behaviour is modelled by $\boldsymbol{\nu}_s = (\nu_t^1, \ldots, \nu_t^d)_t$, usually referred to as *trading speed*, which represents the number of shares traded by the agent per unit of time, for a specific stock i. A positive value of the trading speed, $\nu_t^i > 0$, denotes that the agent is currently buying stock i, while a negative value $\nu_t^i < 0$ means the security i is currently being sold[4]. Variable $\boldsymbol{\nu}_t$ is the variable under control of the agent, which is determined by the agent in the attempt of optimising the expected profit. Finally, the impact of the agent's trading is modelled by introducing a linear price impact with the constant parameter λ_i.

Following the above, the dynamics of the agent's inventory are defined for each asset i as:

$$q_t^i = \int_0^t \nu_u^i du, \quad t \in [0, T],\ q_0^i = q_0^i, \tag{2.13}$$

where q_t^i simply represents the number of shares on asset i that are held by the agent at time t.

A trade taking place at time t is generally executed at a worse price than S_t. Several reasons can be found for why this is true: bid-ask spread, scarce liquidity at the best available prices and the subsequent necessity to *walk the book* for the entire order to be executed, etc. This is usually referred to as temporary price impact, and it is taken into consideration by defining the actual price paid to buy $\nu_t^i dt$ in t as:

$$\hat{S}_t^i = S_t^i + \kappa_i \nu_t^i, \tag{2.14}$$

where $\kappa_i > 0$ is constant. The temporary price impact is therefore assumed linear.

Finally, the dynamic of agent's cash holdings is written as:

$$X_t = -\sum_{i=1}^{d} \int_0^t \hat{S}_u^i dq_u^i, \tag{2.15}$$

where X_t represents the cumulated amount in cash that is paid (or received) by the agent through each trade executed on every asset i up to time t. If $X_t < 0$ the agent bought more shares than have been sold up to time t. In this case, X_t represents the amount of cash that the agent had to pay to accumulate a positive net position during $[0, t]$. If $X_t < 0$, on the other hand, the agent has sold more value than bought and, therefore, receives a positive amount of cash.

[4] If $\nu_s^i = 0$, obviously, the agent is not trading the asset at all.

2.4.2 The general agent's optimisation problem

In the above section, the basic dynamics of the market environment within which the agent operates are defined. The optimisation problem that the agent faces is next described.

A finite time horizon $[0, T]$ is considered, with $T > 0$ constant and identifying the expiration time of the derivative with payoff $H(\boldsymbol{S})$, which is held by the agent. In line with Kraft and Kühn, (2011), the derivative is assumed to be held until expiration: this might be the trader's decision or it might be due to the derivative being too illiquid to be liquidated beforehand. It is assumed that the claim is European and cash-settled upon expiration. In other words, the derivative cannot be exercised before expiration and the final payoff is paid in cash between the parties holding the claim. Some of these assumptions are later relaxed.

During the interval $[0, T]$, the agent can trade on each of the securities underlying the derivative. It is assumed that the agent has no interest in accumulating any positions on the underlying assets, besides the incentives induced by holding the derivative. Therefore, if the agent holds any position on any of the underlying securities at expiration, the entirety of such positions will be liquidated. The expected profit of the agent, therefore, can be written as:

$$\mathbb{E}\left[\underbrace{X_T + \sum_{i=1}^{d} q_T^i(S_T^i - \alpha_i q_T^i)}_{\text{Costs}} + \underbrace{\Gamma H(\boldsymbol{S}_T)}_{\text{Payoff}} - \underbrace{\frac{1}{2}\psi \sum_{i=1}^{d}\sum_{j=1}^{d} \sigma_{ij} \int_0^T q_t^i q_t^j dt}_{\text{Inventory Penalty}}\right], \tag{2.16}$$

where $\psi \in \mathbb{R}^+$, $\alpha_i \in \mathbb{R}^+$ are constant. The parameter α_i is used to model the liquidation costs of the residual inventory in T of security i.

The economic intuition behind the problem faced by the agent is fairly simple. The first two terms in Equation (2.16) represent the costs paid by the agent's when trading, due to trading on the underlying securities over the interval $[0, T]$ and liquidating them at expiration, in case a non-null inventory is accumulated. More precisely, these costs include: (1) the permanent price impact, that leads the agent to buy (selling) at increasingly higher (lower) prices; (2) the temporary price impact and, finally, (3) the costs associated with liquidating any residual position on the underlying in T.

The third term represents the payoff obtained from the linear derivative's payoff at T. The payoff at expiration is simply obtained (or paid) in cash, without any extra costs associated with it.

Following an approach already proposed in the optimal execution literature[5], and in line with P. Kumar and D. J. Seppi, (1992), the last term in the expression is a penalty for the running inventory. This is included in the model to take into account that potential manipulators might have some limits to the amount of capital they are willing to commit to manipulation. In this way, if $\psi > 0$, some type of risk-aversion is introduced into the model. The case $\psi = 0$ corresponds to risk-neutral agents.

[5]See, for example, Guéant, (2017) and Á. Cartea et al., (2020).

The agent chooses how to trade on each asset i (by deciding ν_t^i) to maximise the profit. This choice basically consists in deciding whether, how much and in what direction to trade in each of the d securities based on the following trade-off: (1) on one hand, by trading on the underlying assets in a specific direction (which direction, if buying or selling, depends on the function of the payoff H), the expected profit at expiration of the contingent claim increases and (2), on the other hand, trading on the underlying assets is costly, as is evident by the first two terms (and also by the penalty term, in the case of risk-averse agents) of Equation (2.16). If the first element of this trade-off (affecting the underlying prices to improve the payoff at expiration) prevails, then, in this context, market manipulation is said to be the rational behaviour to keep, as it is the trading strategy that maximises the expected utility.

Studying this optimisation problem (that is, how the corresponding trade-off is solved by the agent and under which conditions manipulation arises) is the objective of the following analysis.

2.5 Derivatives with linear payoff

2.5.1 Payoff functional, value function and the associated HJB equation

In this section, the following linear payoff is considered:

$$H(\boldsymbol{S}) = \sum_{i=1}^{d} \omega_i S_i - K, \tag{2.17}$$

with $K \in \mathbb{R}^+$, $\omega_i \in \mathbb{R}^+$ both constant and such that $\sum_{i=1}^{d} \omega_i = 1$.

Based on Equation (2.16), the agent's value function can be written as follows:

$$V(t, x, \boldsymbol{q}, \boldsymbol{S}) = \sup_{\boldsymbol{\nu}_t \in \mathcal{A}} \mathbb{E}_{t,x,\boldsymbol{q},\boldsymbol{S}} \left[X_T + \sum_{i=1}^{d} q_T^i (S_T^i - \alpha_i q_T^i) + \Gamma\Big(\sum_{i=1}^{d} \omega_i S_T^i - K \Big) - \frac{\psi}{2} \sum_{i=1}^{d} \sum_{j=1}^{d} \sigma_{ij} \int_t^T q_t^i q_t^j dt \right], \tag{2.18}$$

where $\mathcal{A}$ is the set of admissible controls consisting of $\mathcal{F}$-predictable processes such that $\mathbb{E}[\int_t^T \boldsymbol{\nu}_u^\intercal \boldsymbol{\nu}_u du] < +\infty$ and $\mathbb{E}_{t,x,\boldsymbol{q},\boldsymbol{S}}[\cdot]$ is an expectation conditional on $X_t = x$, $\boldsymbol{q}_t = \boldsymbol{q}$ and $\boldsymbol{S}_t = \boldsymbol{S}$.

The HJB equation obtained from the control problem (2.18) is as follows:

$$\begin{aligned} \partial_t V - \frac{1}{2}\psi \sum_{i=1}^{d}\sum_{j=1}^{d} \sigma_{ij} q_i q_j + \frac{1}{2}\sum_{i=1}^{d}\sum_{j=1}^{d} \sigma_{ij} \partial_{S^i S^j} V + \sup_{\nu} \Bigg[\sum_{i=1}^{d} \nu^i \partial_{q^i} V - \\ - \sum_{i=1}^{d} (S^i + \kappa_i \nu^i) \nu^i \partial_x V + \sum_{i=1}^{d} (\mu_i + \lambda_i \nu^i) \partial_{S^i} V \Bigg] = 0, \end{aligned} \tag{2.19}$$

with the associated terminal condition given by:

$$V(T, x, \boldsymbol{q}, \boldsymbol{S}) = x + \sum_{i=1}^{d} S^i(q^i + \Gamma\omega_i) - \sum_{i=1}^{d} \alpha_i (q^i)^2 - \Gamma K. \tag{2.20}$$

2.5.2 Constant trading schedule

The agent's optimisation problem is first solved under assumption of constant trading schedule over the entire interval $[0, T]$: $\boldsymbol{\nu}_t = \boldsymbol{\nu}$, $\forall t \in [0, T]$. The agent's problem consists in finding the values of $\boldsymbol{\nu} = (\nu^i)_{i=1,\dots,d}$ maximising the objective function (Equation [2.16]).

The (controlled) inventory process simply becomes:

$$q_t^i = \nu^i t, \quad t \in [0, T], \tag{2.21}$$

The process for price of stock i is given by:

$$S_t^i = S_0^i + \mu_i t + \lambda_i \nu^i t + \int_0^t \sigma_i dW_u. \tag{2.22}$$

Finally, the cash holding process is

$$X_t = -\sum_{i=1}^{d} \int_0^t \hat{S}_u^i dq_u^i, \tag{2.23}$$

with

$$\hat{S}_t^i = S_t^i + \kappa_i \nu^i. \tag{2.24}$$

Under this assumption, the optimal (constant) trading schedule is trivial to find: see Proposition 1.

Proposition 1 *Assuming constant trading schedule from the agent, the trading speed $\boldsymbol{\nu}$ maximising (2.16) is given by the following system:*

$$\boldsymbol{\nu}^* = M^{-1}\big(\widetilde{\boldsymbol{\mu}} + \widetilde{\boldsymbol{\lambda}}\big), \tag{2.25}$$

where $M \in \mathbb{R}^{d\times d}$ and $\widetilde{\boldsymbol{\lambda}}, \widetilde{\boldsymbol{\mu}} \in \mathbb{R}^d$ are given by:

$$M_{ii} = (2\alpha_i - \lambda_i)T + 2\kappa_i + \frac{T^2}{3}\psi\sigma_i^2, \quad M_{ij} = \frac{T^2}{6}\psi\sigma_{ij},$$

$$\widetilde{\mu}_i = \frac{T}{2}\mu_i, \quad \widetilde{\lambda}_i = \Gamma\omega_i\lambda_i.$$

Proof For a proof see Appendix 2.B. □

If the agent is risk-neutral (i.e., $\psi = 0$), the solution simply becomes:

$$\nu^{i,*} = \frac{\mu_i/2}{(2\alpha_i - \lambda_i) + 2\kappa_i/T} + \frac{\Gamma\omega_i\lambda_i}{(2\alpha_i - \lambda_i)T + 2\kappa_i}. \tag{2.26}$$

The solution is analysed for a risk-neutral agent holding a long position $\Gamma > 0$. It is also assumed, realistically, that $2\alpha_i - \lambda_i \geq 0$, $\forall i$: this is also an assumption underlying all the subsequent analyses.

The first component of the solution depends on the drift and on the costs of trading:

$$\frac{\mu_i/2}{(2\alpha_i - \lambda_i) + 2\kappa_i/T}.$$

It is optimal, for the agent, to trade in the direction of the drift μ_i, regardless of the type of payoff. If $\mu_i > 0$, the agent purchases the underlying security i and the higher the drift the more intensive the purchases. The costs of trading appear at the denominator. The term $2\alpha_i - \lambda_i$ represents the total costs related to the permanent market impact, due to the accumulation of the position up to time T and its subsequent liquidation: high values of this term lead the agent to acquire less. Similarly, the higher the temporary costs κ_i, the lower the amount traded by the agent. Interestingly, as T increases, the component of costs related to the temporary market impact diminishes, and it vanishes for $T \to +\infty$.

The second term of the solution is the component of the optimal trading strategy associated with the agent's ability to affect the underlying prices:

$$\frac{\Gamma\omega_i\lambda_i}{(2\alpha_i - \lambda_i)T + 2\kappa_i}.$$

If $\Gamma > 0$, the amount bought by the agent on each underlying security i increases with respect to both ω_i and λ_i. In other words, the agent will concentrate the purchasing power on the underlying securities with the highest weight within the underlying basket (ω_i) and through which the highest price impact (λ_i) can be obtained. It is also possible to notice that the incentive to buy the underlying i vanishes for $\omega_i \to 0$. Intuitively, linear positions written on market indices with a high number of constituents will likely not be target of manipulation as the weight of each components tends to 0. Also in this case, the costs of trading appear at the denominator, reducing the incentive to manipulate the securities. To notice that, for $T \to +\infty$, the term $(2\alpha_i - \lambda_i)T$ makes the incentive to manipulate to vanish:

$$\lim_{T\to\infty} \nu^{i,*} = \frac{\mu_i/2}{(2\alpha_i - \lambda_i)}.$$

In case of a long time horizon T, the optimal trading strategy is not influenced by the derivative's payoff: any constant trading schedule will lead to a position on each of the underlying securities that are too large and the costs of liquidating such positions always overcome the expected profits.

Similar considerations are valid for short positions: $\Gamma < 0$. In this case, however, assuming $\mu_i > 0$, the first and second terms of the trading strategy have contrasting signs. The agent has the incentive to maximise the profit from the derivative and, therefore, pushes the price downwards by selling the underlying's shares. On the other hand, there is the incentive to trade in the direction of the drift (therefore, buying) and profiting from it. Whether the manipulation component prevails on the incentive to speculate on the direction of the drift ultimately depends on the magnitude of Γ.

These formulas are very similar to those obtained by Dutt and L. E. Harris, (2005). In the next chapter, the framework is extended to consider the possibility of dynamic trading.

2.5.3 Dynamic trading schedule

In this section, the framework presented in the previous section is relaxed, by allowing the agent to dynamically adapt the trading strategy over the trading interval $[0, T]$. By means of stochastic optimal control techniques, the solution of the HJB Equation (2.19) is found (along with the corresponding optimal controls) in case of linear payoff: see Proposition 2.

Proposition 2 *The value function (2.18) has the following form:*

$$V(t, x, \boldsymbol{q}, \boldsymbol{S}) = x + \sum_{i=1}^{d} S^i(q^i + \omega_i\Gamma) + \theta(t, \boldsymbol{q}), \tag{2.27}$$

where

$$\theta(t, q) = \theta_0(t) + \sum_{i=1}^{d} \theta_1^i(t)q^i + \sum_{i=1}^{d} \theta_2^i(t)(q^i)^2. \tag{2.28}$$

For $\psi = 0$, the functions $\theta_0(t)$, $\theta_1^i(t)$ and $\theta_2^i(t)$, with $i \in \{1, \dots, d\}$, are given by:

$$\theta_0(t) = \Gamma \sum_{i=1}^{d} \omega_i\mu_i(T-t) + \sum_{i=1}^{d} \frac{1}{4\kappa_i} \int_t^T \left(\Gamma\omega_i\lambda_i + \theta_1^i(s)\right)^2 ds - \Gamma K, \tag{2.29}$$

$$\theta_1^i(t) = \frac{1}{\left(T - t + \frac{2\kappa_i}{2\alpha_i - \lambda_i}\right)} \left[\left(\mu_i \frac{2\kappa_i}{2\alpha_i - \lambda_i} - \Gamma\omega_i\lambda_i\right)(T-t) + \frac{\mu_i}{2}(T-t)^2\right], \tag{2.30}$$

$$\theta_2^i(t) = -\frac{\kappa_i}{T - t + \frac{2\kappa_i}{2\alpha_i - \lambda_i}} - \frac{\lambda_i}{2}. \tag{2.31}$$

Finally, the optimal trading speed $\nu_t^{i,*}$ is deterministic and given by:

$$\nu_t^{i,*} = \nu^{i,*}(t, q_t^{i,*}) = \frac{\lambda_i(q_t^i + \omega_i\Gamma) + \theta_1^i(t) + 2\theta_2^i(t)q_t^{i,*}}{2\kappa_i}. \tag{2.32}$$

The dynamic of the optimal inventory can be expressed as:

$$q_t^{i,*} = q_0^i + \left[\frac{\Gamma\omega_i\lambda_i - (2\alpha_i - \lambda_i)q_0^i}{(2\alpha_i - \lambda_i)T + 2\kappa_i} - \frac{\mu_i\kappa_i}{(2\alpha_i - \lambda_i)^2 T + 2\kappa_i(2\alpha_i - \lambda_i)} + \frac{\mu_i}{4\kappa_i}\Big(\frac{2\kappa_i}{2\alpha_i - \lambda_i} + T\Big)\right] t - \frac{\mu_i}{4\kappa_i}t^2. \tag{2.33}$$

If, on the other hand, $\psi > 0$, while the function $\theta_0(t)$ remains the same, the functions $\theta_1^i(t)$ and $\theta_2^i(t)$ have the following forms:

$$\theta_1^i(t) = \frac{\frac{\xi_i}{\phi_i}\Big((1 - e^{-\phi_i(T-t)})\eta_i^+ - (1 - e^{\phi_i(T-t)})\eta_i^-\Big) + 4\kappa_i\phi_i\Gamma\omega_i\lambda_i}{\eta_i^+ e^{-\phi_i(T-t)} + \eta_i^- e^{\phi_i(T-t)}} - \Gamma\omega_i\lambda_i, \tag{2.34}$$

$$\theta_2^i(t) = \kappa_i\phi_i \frac{\eta_i^+ e^{-\phi_i(T-t)} - \eta_i^- e^{\phi_i(T-t)}}{\eta_i^+ e^{-\phi_i(T-t)} + \eta_i^- e^{\phi_i(T-t)}} - \frac{\lambda_i}{2}, \tag{2.35}$$

where $\phi_i = \sigma_i\sqrt{\frac{\psi}{2\kappa_i}}$, $\eta_i^\pm = 2\kappa_i\phi_i \pm \lambda_i \mp 2\alpha_i$ and $\xi_i = \mu_i - \frac{\psi}{2}\sum_{j=1,j\neq i}^d \sigma_{ij}q_j$.

Finally, the optimal level of inventory can be expressed as a deterministic linear time-varying system:

$$\frac{d\boldsymbol{q}_t^*}{dt} = \boldsymbol{H}_t\boldsymbol{q}_t^* + \boldsymbol{b}_t, \quad \boldsymbol{q}_0^* = \boldsymbol{q}_0, \tag{2.36}$$

where $\boldsymbol{q}_t^* = (q_t^{1,*}, \dots, q_t^{d,*})$, the time-varying matrix $\boldsymbol{H}_t = (h_t^{ij})_{i,j=1,\dots,d}$ is given by:

$$h_t^{ii} = \frac{\lambda_i + 2\theta_2^i(t)}{2\kappa_i},$$

$$h_t^{ij} = -\frac{\psi}{4\kappa_i\phi_i}\frac{(1 - e^{-\phi_i(T-t)})\eta_i^+ - (1 - e^{\phi_i(T-t)})\eta_i^-}{\eta_i^+ e^{-\phi_i(T-t)} + \eta_i^- e^{\phi_i(T-t)}}\sigma_{ij},$$

and the i-th element of the time-varying vector $\boldsymbol{b}_t \in \mathbb{R}^d$ is given by:

$$b_t^i = \frac{\frac{\mu_i}{2\kappa_i\phi_i}\Big((1 - e^{-\phi_i(T-t)})\eta_i^+ - (1 - e^{\phi_i(T-t)})\eta_i^-\Big) + 2\phi_i\Gamma\omega_i\lambda_i}{\eta_i^+ e^{-\phi_i(T-t)} + \eta_i^- e^{\phi_i(T-t)}}.$$

Proof For a proof see Appendix 2.B. □

For a risk-neutral agent ($\psi = 0$) with $\boldsymbol{q}_0 = \boldsymbol{0}$, the dynamic of the optimal inventory can be written as:

$$q_t^{i,*} = \mu_i\Big(-\frac{1}{4\kappa_i}t^2 + \frac{1 + (2\alpha_i - \lambda_i)T/(4\kappa_i)}{(2\alpha_i - \lambda_i) + 2\kappa_i/T)}t\Big) + \frac{\Gamma\omega_i\lambda_i}{(2\alpha_i - \lambda_i)T + 2\kappa_i}t \tag{2.37}$$

It is possible to notice that, similarly to the results obtained in the previous section, the optimal trading strategy can be divided into two factors: the first depending on the drift and the second depending on the linear payoff. Interestingly, the element of the trading strategy that depends on the payoff (and on the ability of the agent to affect the prices through λ_i) is the same as the one obtained under assumption of constant trading (i.e., Equation

TABLE 2.1: Market model parameters - baseline scenario.

Parameter	Description	Security 1	Security 2
μ_i	security's drift	0	0
σ_i	security's volatility	1	1.5
λ_i	permanent price impact	0.01	0.015
κ_i	temporary price impact	0.01	0.015
α_i	liquidation costs	0.01	0.01
ω_i	underlying's weight	0.5	0.5
ρ	correlation	0.5	

The table contains the market model parameters used in conducting the numerical simulations.

[2.26]). This means that, even when the constant trading assumption is relaxed, holding a position always generates an incentive to trade in the underlying securities in the direction that maximises the payoff obtained at expiration.

However, the term depending on the drift leads to a quite different solution. The drift term μ_i multiplies a parabolic equation in t, with a negative coefficient for t^2 and a positive coefficient for t. Assuming $\mu_i > 0$, this means that, for T sufficiently high, the term depending on t^2 always dominates, starting at a certain time during the trading interval $[0, T]$, the agent's trading decisions, leading the optimal strategy to trade against the drift, in a form of hedging. On the other hand, for lower values of T, the linear term on t can dominate the equation, amplifying the incentives to trade in the direction that would maximise the linear payoff, in case the drift μ_i and Γ have the same sign.

Numerical simulations are proposed to illustrate the features of the optimal strategy and the associated expected impact on the price. The scenario of a linear payoff depending on the value of a basket of $d = 2$ securities is considered. Table 2.1 contains the model parameters for the dynamic of the two securities, $i = 1, 2$. They are in line with simulations on similar topics: see Horst and Naujokat, (2011). The idea is to consider a basket comprising two securities with different risk profile and also different transaction costs (the security with higher volatility is also assumed to have higher transaction costs). Additionally, the trading time horizon is $T = 1$, the derivative's leverage is $\Gamma = 100$ and the optimal strategies for three levels of risk-aversion are analysed: $\psi = 0$, $\psi = 0.1$ and $\psi = 0.5$. While an analytical formula is available for the dynamic of $q_t^{i,*}$ for $\psi = 0$, the same is not true for $\psi > 0$. In this case, the solution is simulated based on Equation (2.36), through an explicit Euler numerical scheme.[6]

[6]See, for example, Atkinson et al., (2011).

Figure 2.1 displays the optimal strategy (i.e., the optimal levels of inventory on each of the two securities $q_t^{1,*}$ and $q_t^{2,*}$) and the impact generated on the price, defined as:

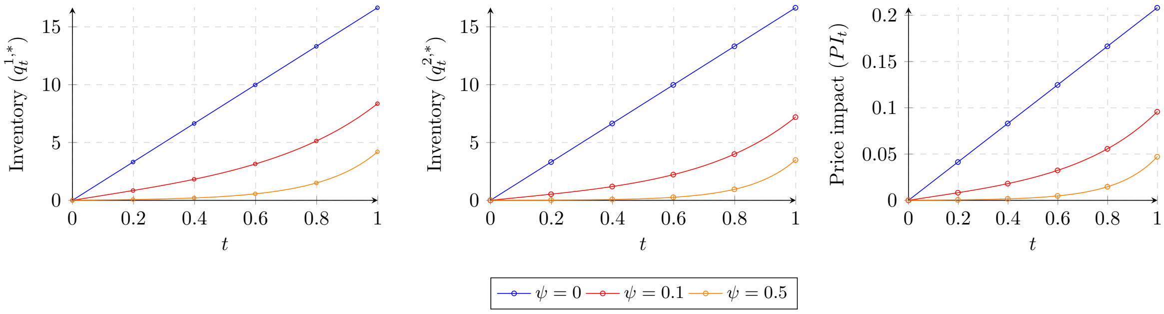
$$PI_t = \sum_{i=1}^{2} \omega_i \lambda_i q_t^{i,*} \tag{2.38}$$

Two scenarios are considered: (1) equally weighted basket of underlying securities ($\omega_1 = \omega_2 = 0.5$), and (2) the case in which one of the two securities has heavy weight within the underlying basket ($\omega_1 = 0.9$ and $\omega_2 = 0.1$). Risk-neutral agents ($\psi = 0$) tend to be significantly more aggressive, generating the highest price impact. Furthermore, the trading behaviour on the two assets is not qualitatively different in the two scenarios: clear buying intentions are displayed for both securities, even in the second scenario, within which the second security weights for only 10% of the underlying basket's value. Risk-averse agents have different behaviours. In the equal weights scenario (Figure [2.1a]), the agent displays buying intentions on both securities, with the amount bought of the second security (which has higher volatility) being significantly lower than what a risk-neutral agent would buy. The risk-averse agent is, in fact, avoiding accumulating too many shares on the riskier security and focuses the buying effort in the less risky one. This is even more obvious in the second scenario (Figure [2.1b]): a risk-averse agent (with $\phi = 0.5$) is willing to short the second security (which also weights very little in the basket value) to finance the buying efforts on the first security, which is less risky (so, it is less penalised by the agent objective function) and has a higher weight on the underlying basket's value.

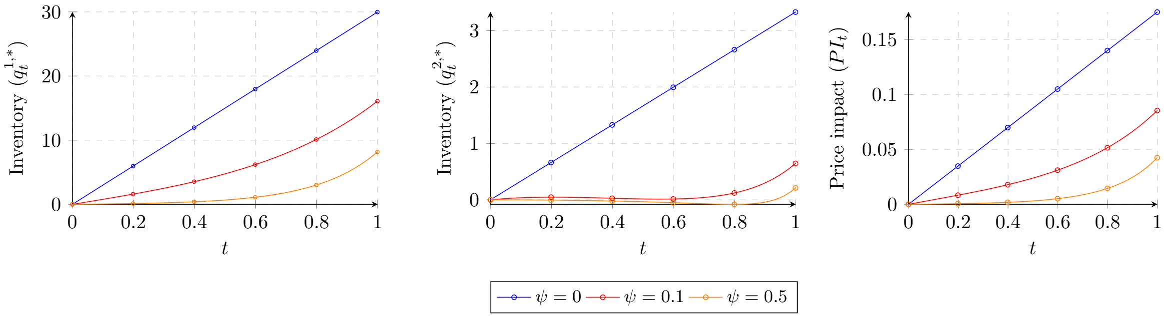
Figure 2.2 contains the optimal strategy and expected price impact for varying levels of the derivative exposure: $\Gamma = 10$ (Figure [2.2a]) and $\Gamma = 1,000$ (Figure [2.2b]). Quite intuitively, the higher the exposure (and, therefore, the higher the expected profit from the derivative's position), the higher the number of security bought, aimed at producing the highest market impact: the amount traded during the interval (and the associated market impact) is simply shifted.

Figure 2.3 displays the optimal strategy and associated price impact for varying levels of α_1 (i.e., the liquidation cost of the first security in the basket). Increasing values of α_1 make the decision of buying the first security less attractive, considering the inventory accumulated will then need to be liquidated at higher costs after expiration. With increasing values of α_1, the agent switches target security and concentrates the buying efforts on the second security of the basket, even though it was less attractive.

Finally, Figure (2.4) displays the trading behaviour of a risk-neutral agent ($\psi = 0$) for varying levels of the drift component of either the first (Figure [2.4a]) or the second security (Figure [2.4b]). Increasing levels of μ_1, given $\mu_2 = 0$, lead the agent to buy a higher amount of the first security, while the amount bought of the second security is not affected. The price impact is amplified by higher drifts, which provides the incentive to the agent to trade in the same direction that increases the payoff. Same considerations are true for the second scenario, of increasing values of μ_2.



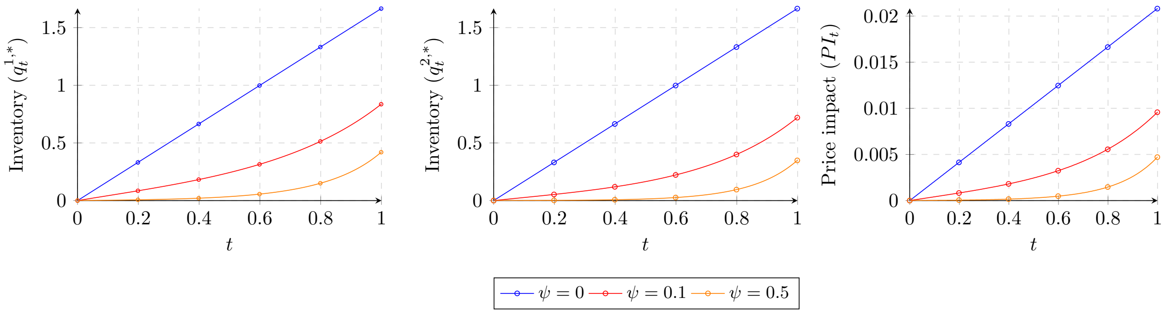
(A) $\omega_1 = 0.5, \omega_2 = 0.5$



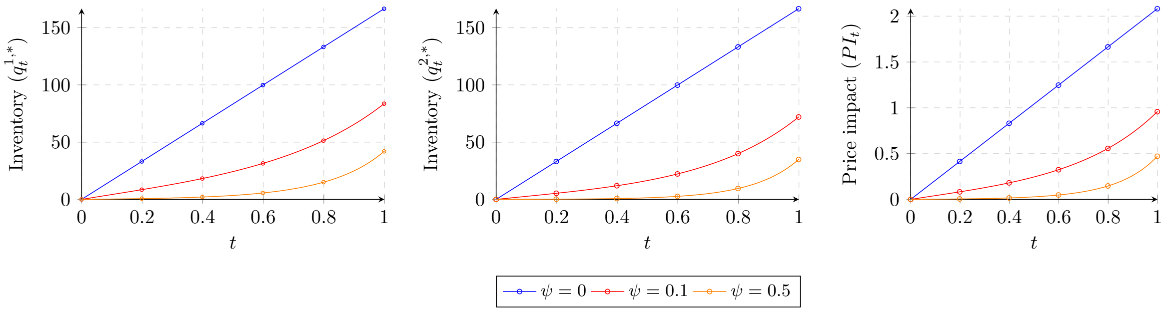
(B) $\omega_1 = 0.9, \omega_2 = 0.1$

FIGURE 2.1: Price impact - linear payoff, varying ω_1.

The figure displays the dynamic of the optimal inventory ($q_t^{1,*}$ and $q_t^{2,*}$) and the associated price impact (PI_t) for two scenarios: $\omega_1 = 0.5, \omega_2 = 0.5$ (Figure [2.1a]) and $\omega_1 = 0.9, \omega_2 = 0.1$ (Figure [2.1b]). The rest of the parameters is contained in Table 2.1. Finally, $T = 1$, $\Gamma = 100$ and three levels of risk-aversion are considered: $\psi = 0$ (blue), $\psi = 0.1$ (red) and $\psi = 0.5$ (orange).



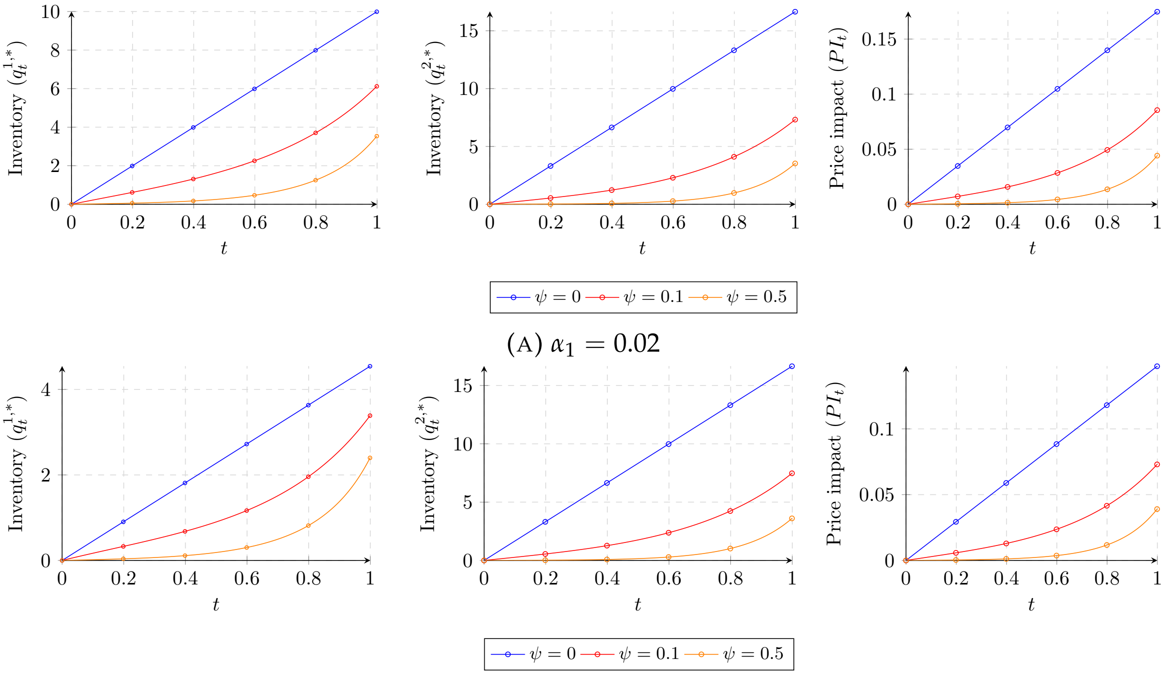
(A) $\Gamma = 10$



(B) $\Gamma = 1,000$

FIGURE 2.2: Price impact - linear payoff, varying Γ.

The figure displays the dynamic of the optimal inventory ($q_t^{1,*}$ and $q_t^{2,*}$) and the associated price impact (PI_t) for two scenarios: $\Gamma = 10$ (Figure [2.2a]) and $\Gamma = 1,000$ (Figure [2.2b]). The rest of the parameters is contained in Table 2.1. Finally, $T = 1$, and three levels of risk-aversion are considered: $\psi = 0$ (blue), $\psi = 0.1$ (red) and $\psi = 0.5$ (orange).

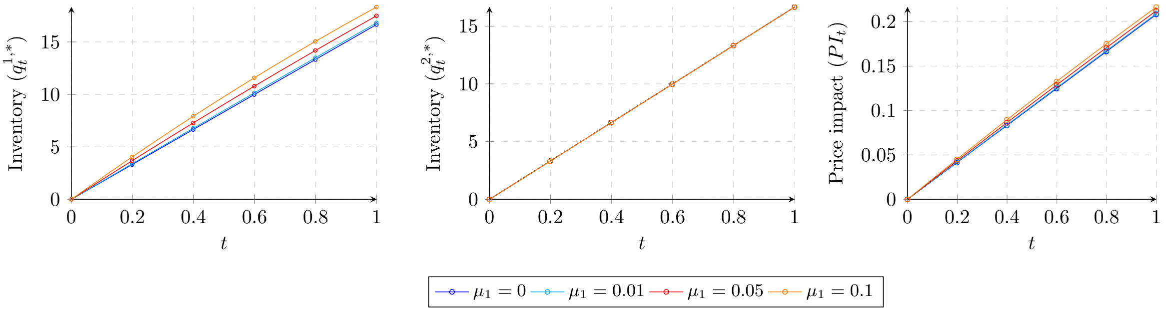


(A) $\alpha_1 = 0.02$

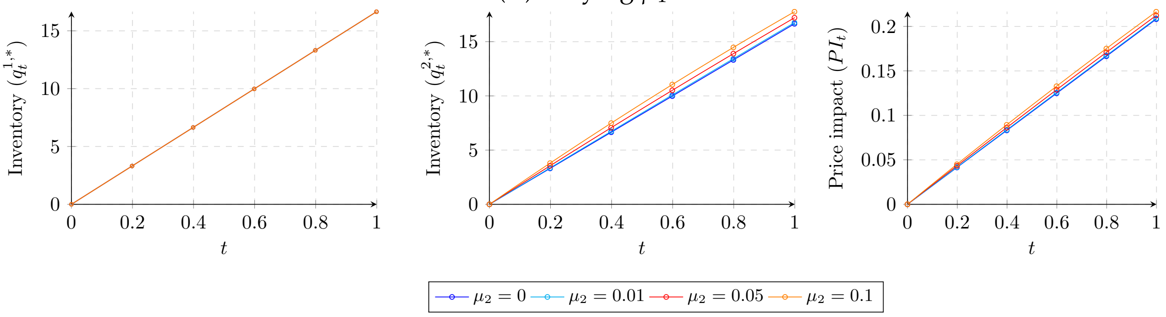
(B) $\alpha_1 = 0.05$

FIGURE 2.3: Price impact - linear payoff, varying α_1.

The figure displays the dynamic of the optimal inventory ($q_t^{1,*}$ and $q_t^{2,*}$) and the associated price impact (PI_t) for two scenarios: $\alpha_1 = 0.2$ (Figure [2.3a]) and $\alpha_1 = 0.5$ (Figure [2.3b]). The rest of the parameters is contained in Table 2.1. Finally, $T = 1$, $\Gamma = 100$ and three levels of risk-aversion are considered: $\psi = 0$ (blue), $\psi = 0.1$ (red) and $\psi = 0.5$ (orange).



(A) Varying μ_1



(B) Varying μ_2

FIGURE 2.4: Price impact - linear payoff, varying μ_i.

The figure displays the dynamic of the optimal inventory ($q_t^{1,*}$ and $q_t^{2,*}$) and the associated price impact (PI_t) for two scenarios: varying μ_1 and constant $\mu_2 = 0$ (Figure [2.4a]); and constant $\mu_1 = 0$ and varying μ_2 (Figure [2.4b]). The rest of the parameters is contained in Table 2.1. Finally, $T = 1$, $\Gamma = 100$.

2.6 Derivatives with non-linear payoff

2.6.1 Payoff functional, value function and the associated HJB equation

This section considers the more general case of non-linear payoff $H(\boldsymbol{S})$.

A call option written on a basket of d assets, for example, assumes the following functional form:

$$H(\boldsymbol{S}) = \Big(\sum_{i=1}^{d} \omega_i S_i - K\Big)\mathbb{1}_{E_0}, \tag{2.39}$$

where $\mathbb{1}_{E_0}$ denotes the indicator function taking the value 1 if the event E_0 is verified and 0 otherwise; Γ and K are constant terms, $\boldsymbol{\omega}$ is a d-dimensional vector with constant elements ω_i, $i \in \{1, \dots, d\}$, such that $\sum_{i=1}^{d} \omega^i = 1$ and, finally, $\boldsymbol{S}$ is a d-dimensional vector denoting the price of the underlying asset. An option payoff is considered, which corresponds to:

$$E_0 = \sum_{i=1}^{d} \omega_i S_i > K.$$

The agent's value function can be written as:

$$V(t, x, \boldsymbol{q}, \boldsymbol{S}) = \sup_{\nu_t \in \mathcal{A}} \mathbb{E}_{t,x,\boldsymbol{q},\boldsymbol{S}} \left[X_T + \sum_{i=1}^{d} q_T^i (S_T^i - \alpha_i q_T^i) + \Gamma H(\boldsymbol{S}) - \frac{1}{2}\psi \sum_{i=1}^{d}\sum_{j=1}^{d} \sigma_{ij} \int_0^T q_t^i q_t^j dt \right], \tag{2.40}$$

where $\mathcal{A}$ is the set of admissible controls consisting of $\mathcal{F}$-predictable processes such that $\mathbb{E}[\int_0^T \boldsymbol{\nu}_u^\top \boldsymbol{\nu}_u du] < +\infty$ and $\mathbb{E}_{t,x,\boldsymbol{q},\boldsymbol{S}}[\cdot]$ is an expectation conditional on $X_t = x$, $\boldsymbol{q}_t = \boldsymbol{q}$ and $\boldsymbol{S}_t = \boldsymbol{S}$.

The HJB equation obtained from the control problem (Equation [2.40]) is as follows:

$$\begin{aligned} \partial_t V - \frac{1}{2}\psi \sum_{i=1}^{d}\sum_{j=1}^{d} \sigma_{ij} q_i q_j + \frac{1}{2}\sum_{i=1}^{d}\sum_{j=1}^{d} \sigma_{ij} \partial_{S^i S^j} V + \sup_{\nu} \Bigg[\sum_{i=1}^{d} \nu^i \partial_{q^i} V - \\ - \sum_{i=1}^{d} (S^i + \kappa_i \nu^i)\nu^i \partial_x V + \sum_{i=1}^{d} (\mu_i + \lambda_i \nu^i) \partial_{S^i} V \Bigg] = 0, \end{aligned} \tag{2.41}$$

with associated terminal condition:

$$V(T, x, \boldsymbol{q}, \boldsymbol{S}) = x + \sum_{i=1}^{d} q^i (S^i - \alpha^i q^i) + \Gamma H(\boldsymbol{S}). \tag{2.42}$$

To have an idea of the form of the solution, similarly to the linear case, the following ansatz is proposed for the form of the value function (Equation [2.40]):

$$V(t, x, \boldsymbol{q}, \boldsymbol{S}) = x + \sum_{i=1}^{d} q^i S^i + \theta(t, \boldsymbol{q}, \boldsymbol{S}). \tag{2.43}$$

The HJB Equation (2.41) then becomes:

$$\begin{aligned} \partial_t\theta - \frac{1}{2}\psi\sum_{i=1}^{d}\sum_{j=1}^{d}\sigma_{ij}q_iq_j + \frac{1}{2}\sum_{i=1}^{d}\sum_{j=1}^{d}\sigma_{ij}\partial_{S^iS^j}\theta + \sup_{\nu}\Bigg[\sum_{i=1}^{d}\nu^i\partial_{q^i}\theta - \\ -\sum_{i=1}^{d}(\kappa_i\nu^i)\nu^i + \sum_{i=1}^{d}(\mu_i+\lambda_i\nu^i)(q^i+\partial_{S^i}\theta)\Bigg] = 0, \end{aligned} \tag{2.44}$$

with terminal condition given by:

$$\theta(T,\boldsymbol{q},\boldsymbol{S}) = \Gamma H(\boldsymbol{S}) - \sum_{i=1}^{d}\alpha^i(q^i)^2 \tag{2.45}$$

The optimal control, in feedback control form and based on Equation (2.44), is given by:

$$\nu^{i,*} = \frac{\partial_{q^i}\theta + (q^i+\partial_{S^i}\theta)\lambda_i}{2\kappa_i}. \tag{2.46}$$

By plugging it into the HJB Equation (2.44), it can be further simplified:

$$\partial_t\theta - \frac{1}{2}\sum_{i=1}^{d}\sum_{j=1}^{d}\sigma_{ij}\Big(\psi q_iq_j - \partial_{S^iS^j}\theta\Big) + \sum_{i=1}^{d}(q^i+\partial_{S^i}\theta)\mu_i + \sum_{i=1}^{d}\kappa_i(\nu^{i,*})^2 = 0. \tag{2.47}$$

The Equation (2.47) depends on four terms. The first term represents the change of the value function due to expiration approaching. The second term depends on the inventory penalty and on the second-order derivatives of θ with respect to $\boldsymbol{S}$. To notice that holding inventories of the same sign on positively correlated securities is particularly discouraged early on in the interval. With time passing, $\frac{1}{2}\sum_{i=1}^{d}\sum_{j=1}^{d}\sigma_{ij}\psi q_iq_j$ leads to an increase of the value function. The inventory penalty term has the function to penalise accumulating inventories during the interval: big inventories close to expiration T are less penalised than accumulating the same levels of inventory significantly before expiration, as such inventory will be penalised for a longer period. The third term of the PDE depends on the drift of each underlying security. Holding a position in the same direction of the drift ($q^i\mu_i > 0$) leads to a decrease over time of the value function, given all other things being equal: the profits gained from a positive (negative) drift obtained on a long (short) position diminish approaching expiration T. The change of the value function with respect to a change in price is also amplified (or reduced, depending on the direction of the drift and on the form of the payoff) by the drift. Finally, the fourth component depends on the optimal control process, which is based on the trade-off expressed by Equation (2.46) between $\partial_{q^i}\theta$ and $\partial_{S^i}\theta$: on one hand, the incentive to reduce the inventory level to reduce the running penalty and the liquidation costs at expiry; on the other hand, the incentive to affect the derivative's payoff by trading the underlying securities, which requires accumulating inventory. Which component prevails between the two determines the direction of trading. This is the same trade-off faced by the agent in case of linear payoff, with the difference, in this case, that the contributions of changes in the inventory (q_t^i) or prices (S_t^i) to the changes of the value function cannot be as easily disentangled, because of the nonlinearity of the payoff $H(\cdot)$.

While numerical simulations of this PDE (Equation [2.47]) represents an interesting challenge, the main purpose of this study is to seek analytical formulas that can be used to better understand and describe the dependency between the market environment and the decision to manipulate. To pursue this objective, in the next section, the problem is simplified by assuming constant trading schedule.

2.6.2 Constant trading schedule

This section solves the agent's optimisation problem under assumption of constant trading schedule: $\boldsymbol{\nu}_t = \boldsymbol{\nu}$, $\forall t \in [0, T]$. The call option case is herein considered, although the same logic can be easily applied to the put option case. The general optimisation problem (Equation [2.16]) can be written as:

$$\mathbb{E}\left[X_T + \sum_{i=1}^{d} q_T^i(S_T^i - \alpha_i q_T^i) + \Gamma\Big(\sum_{i=1}^{d}\omega_i S_i - K\Big)\mathbb{1}_{\{\sum_{i=1}^{d}\omega_i S_i > K\}} - \frac{1}{2}\psi\sum_{i=1}^{d}\sum_{j=1}^{d}\sigma_{ij}\int_0^T q_t^i q_t^j dt\right]. \quad (2.48)$$

The optimal trading strategy can be found through analytical formulas: see Proposition 3.

Proposition 3 *Assuming constant trading schedule from the agent, the optimal trading speed maximising agent's profit (Equation [2.48]) is given by:*

$$\boldsymbol{\nu}^* = N(u^*)M^{-1}\widetilde{\boldsymbol{\lambda}} + M^{-1}\widetilde{\boldsymbol{\mu}}, \quad (2.49)$$

where $M \in \mathbb{R}^{d\times d}$, $\widetilde{\boldsymbol{\mu}} \in \mathbb{R}^d$, $\widetilde{\boldsymbol{\lambda}} \in \mathbb{R}^d$ are such that:

$$M_{ii} = \frac{1}{\Gamma\sigma_Y}\Big((2\alpha_i - \lambda_i)T^2 + 2\kappa_i T + \frac{T^3}{3}\psi\sigma_i^2\Big), \quad M_{ij} = \frac{1}{\Gamma\sigma_Y}\frac{T^3}{6}\psi\sigma_{ij},$$

$$\widetilde{\mu}_i = \frac{1}{\Gamma\sigma_Y}\mu_i\frac{T^2}{2}, \quad \widetilde{\lambda}_i = \frac{\omega_i\lambda_i T}{\sigma_Y},$$

and u^* is the solution of the system $\frac{u^*-\widetilde{a}}{b} = N(u^*)$, with $N(\cdot)$ identifying the cumulative distribution function of a standard normal distribution, $\sigma_Y = (T\boldsymbol{\omega}'\Sigma\boldsymbol{\omega})^{1/2}$ and $\widetilde{a}, b \in \mathbb{R}$ are such that:

$$\widetilde{a} = \sum_{i=1}^{d}\frac{\omega_i(S_0 + \mu_i T) - K}{\sigma_Y} + \widetilde{\boldsymbol{\lambda}}'M^{-1}\widetilde{\boldsymbol{\mu}}, \quad b = \widetilde{\boldsymbol{\lambda}}'M^{-1}\widetilde{\boldsymbol{\lambda}}.$$

Proof For a proof see Appendix 2.B. □

To provide intuition, the solution is analysed for a risk-neutral agent ($\psi = 0$). Most of the considerations are nonetheless valid for the case of risk-averse agents ($\psi > 0$) and the differences will be pointed out when relevant. If $\psi = 0$, the solution is given by:

$$\nu^{i,*} = \frac{\mu_i/2}{2\kappa_i/T + (2\alpha_i - \lambda_i)} + N(u^*)\frac{\Gamma\omega_i\lambda_i}{2\kappa_i + (2\alpha_i - \lambda_i)T}, \tag{2.50}$$

where u^* solves the equation $\frac{u^* - a}{b} = N(u^*)$, with:

$$\widetilde{a} = \frac{1}{\sigma_Y}\Big(\sum_{i=1}^{d}\omega_i(S_0^i + \mu_i T) - K\Big) + \frac{T}{2\sigma_Y}\sum_{i=1}^{d}\frac{\omega_i\lambda_i\mu_i}{2\kappa_i/T + (2\alpha_i - \lambda_i)},$$

$$b = \frac{\Gamma}{\sigma_Y}\sum_{i=1}^{d}\frac{(\omega_i\lambda_i)^2}{2\kappa_i/T + (2\alpha_i - \lambda_i)}.$$

The first term of Equation (2.50) depends on the drift and is equal to the first component of the linear solution (Equation [2.26]), analysed in Section 2.5.2. The same considerations are valid: the agent has an incentive to trade in the direction of the drift of each underlying i, with the magnitude of this incentive decreasing with respect to transaction costs (both permanent and temporary, with the latter vanishing if the time horizon T is sufficiently long).

The second factor of the solution is also familiar: the term multiplying $N(u^*)$ corresponds, in fact, to the second term of the linear solution (Equation [2.26]). The magnitude of trading on each security increases with respect to the weight (ω_i), the security's market impact (λ_i) and the derivative's exposure (Γ) and it decreases with respect to the trading costs: κ_i and $2\alpha_i - \lambda_i$, with both these terms assumed positive. The sign of Γ, as expected, determines the trading direction of this component: if $\Gamma > 0$, the agent holds a long position on the call option and has an incentive to buy the underlying securities to increase the payoff at expiration; if $\Gamma < 0$, the agent holds a short position on the call option and has the incentive to sell the underlying securities to push the option OTM and avoid any liability at expiration. The difference with the linear case is the presence of $N(u^*)$, which can be interpreted as a measure of the likelihood of success of manipulation and it affects the magnitude of the optimal trading speed.

First, the case of an agent holding a long position is considered: $\Gamma > 0$. Considering that $b > 0$, $N(u^*)$ increases with respect to $\widetilde{a}$, *ceteris paribus*. The first term of $\widetilde{a}$ is:

$$\frac{1}{\sigma_Y}\Big(\sum_{i=1}^{d}\omega_i(S_0^i + \mu_i T) - K\Big),$$

which can be interpreted as the expected moneyness in T (with no intervention of the agent), adjusted by the volatility of the underlying basket of securities, denominated by σ_Y. This term is increasing with respect to securities' drifts μ_i: higher drifts increase the probability of the option to expire ITM and make manipulation more attractive. Increasing values of σ_Y shrink this term towards 0 and the effect on $N(u^*)$ is therefore mixed. If the expected moneyness is positive, an increasing value of σ_Y is associated with a reduction of $\widetilde{a}$, leading to a lower incentive to manipulate the underlying securities: the agent, expecting the option to expire ITM already, prefers lower levels of volatility. On the other hand, if the expected moneyness is negative the effect is reversed: if the option is expected to expire OTM, a higher level of volatility is associated with a higher likelihood to expire ITM. Consequently, in this scenario, higher volatility leads to an increased incentive to manipulate.

The second term of $\widetilde{a}$ is:

$$\frac{T}{2\sigma_Y}\sum_{i=1}^{d}\frac{\omega_i\lambda_i\mu_i}{2\kappa_i/T+(2\alpha_i-\lambda_i)}.$$

It is increasing (and, along with it, the incentive to manipulate the underlying's prices) with respect to security's weights (ω_i), market impacts (λ_i), and drifts (μ_i). In summary, manipulation incentives are higher for less concentrated baskets of underlying securities, and for securities with higher market impact and drifts, as expected. This term is also increasing with respect to the length of the trading horizon, T: the longer the time before expiration, the higher the likelihood for the option to expire ITM either because of the drifts (μ_i) or due to the agent's intervention (λ_i). The term decreases with respect to transaction costs (at the denominator) as well as when underlying's basket volatility (σ_Y) rises: increasing volatility is associated with diminishing incentives to manipulate the securities' prices upwards.

Finally, it is possible to notice that increasing values of the parameter b lead to increasing values of $N(u^*)$ and therefore of the incentives to manipulate the underlying prices. A higher option exposure (Γ) is always associated with a higher $N(u^*)$: quite intuitively, the higher the exposure the higher the potential profit. Increased volatility (σ_Y), as well as increased trading costs, lead the term b to shrink and, as a consequence, also $N(u^*)$ is lower.

Interestingly, volatility has mixed effects on $\widetilde{a}$ and b and, therefore, on the decision of whether to manipulate the underlying securities. If the option is expected to expire ITM, the agent always prefers lower levels of volatility, to decrease the uncertainty of price movements: an increase of the volatility always leads to a decrease of both $\widetilde{a}$ and b. If the option, on the other hand, is expected to expire OTM the effects are mixed. The agent might prefer high levels of volatility if the option is deeply OTM. In this case, high volatility is necessary to ensure an acceptable likelihood of success.

A similar logic can be applied when analysing the optimal trading behaviour of an agent with a short position on a call option. In this case, $\Gamma < 0$, $b < 0$, while $\widetilde{a}$ is not affected. An important observation is that, in case of positive drifts $\mu_i > 0$, to successfully manipulate the prices downwards the agent has to sell sufficient amounts to overcome the effects of the positive drifts. While the considerations made above on the first term of $\widetilde{a}$ are valid, the second term of this parameter now decreases the incentives to manipulate. Therefore, increasing levels of volatility σ_Y lead to a decrease of this term and make the manipulation more feasible.

Another interesting observation is that, assuming $\mu_i = 0$ for all i (in this case there is no drift that can advantage or disadvantage the agent in a specific direction) and the option is at the money (i.e., $\widetilde{a} = 0$), the incentive to manipulate for an option holder is higher than for an option writer. In other words, the optimal strategy of an option holder would require to buy more shares than the shares that the optimal strategy would require the option writer to sell. This is intuitive, considering that the potential upside profit from manipulation is potentially unlimited for call option holders, while it is limited for the writers. Option holders have the incentive to buy the underlying securities in a way that would maximise the price impact. On the other hand, although option writers have the incentive to affect the price of the securities downwards, they are not interested in affecting the prices more than strictly required to ensure the option to expire OTM, to avoid any liability.

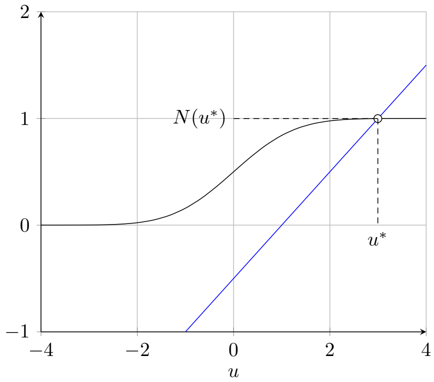
Figure 2.5 provides a graphical description of how $N(u^*)$ changes with respect to the model's parameters: on the left side the case of a long position ($\Gamma > 0$) on a call position is presented; the right side contains the opposite scenario of a short position ($\Gamma < 0$) on a call option.

Finally, Figure 2.6 shows limiting cases for which multiple solutions are obtained, in case of $\Gamma > 0$. The two plots in the first row show the case in which two solutions are obtained. One solution corresponds to the point of tangency between the two curves, while the other one is represented by their intersection. The intersection is always a local (and, in this case, global) maximum and, therefore, the solution that maximises the expected payoff at expiration: see Proposition 4 in the Appendix. The third plot represents a scenario with three distinct solutions, which can take place when $\widetilde{a} < 0$ (i.e., the option is OTM). Each solution has a precise financial meaning. The solution on the left extreme represents the scenario under which the agent does not manipulate ($N(u^*)$ tends to 0) as the option is too deeply OTM: this is a local maximum. The solution on the right extreme, which is also a local maximum, represents the scenario for which, although the option is deeply OTM, the agent buys so intensively that the option can be pushed ITM at expiration and the agent can expect to gain a profit from it. The solution in the middle is a local minimum as the agent sustain costs to buy the underlying securities, but the intensity of trading is not sufficient to expect to gain a profit from the derivative's position: this is the worst possible choice for the agent. The optimal trading behaviour will be one of the two extreme solutions, depending on which of the two is the global maximum.

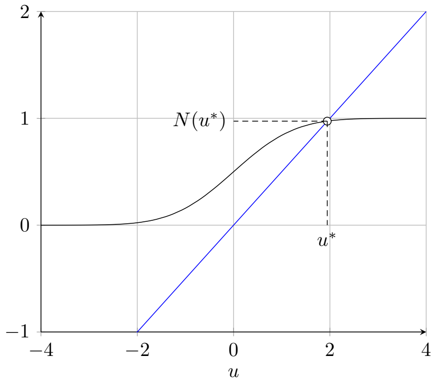
Numerical simulations are finally proposed to illustrate how the optimal strategy is expected to affect the prices of the underlying securities, under different market scenarios. The case of an option written on a basket of $d = 2$ securities is considered. Table 2.1 contains the model parameters. Furthermore, the trading time horizon is $T = 1$ and the derivative's leverage is $\Gamma = 100$.

Figures 2.7, 2.8, 2.9 and 2.10 display information on the expected price impact, for changing values of the initial moneyness levels ($\omega_1 S_0^1 + \omega_2 S_0^2 - K$) and other parameters: ω_1, μ_1, ρ and T. Different values of the risk aversion parameter are considered: $\psi = 0$, $\psi = 0.5$. The expected price impact is defined as:

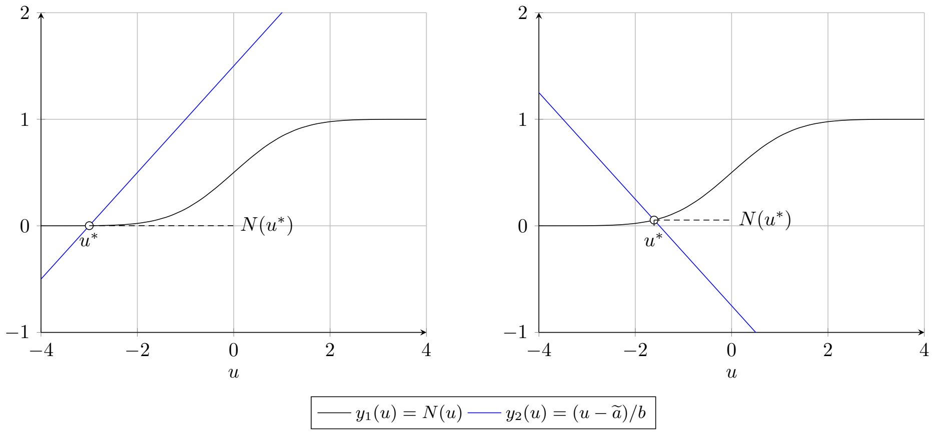
$$PI = \omega_1 \lambda_1 \nu^{1,*} T + \omega_2 \lambda_2 \nu^{2,*} T. \tag{2.51}$$



(A) $\widetilde{a} > 0$ (in the money)



(B) $\widetilde{a} = 0$ (at the money)



(C) $\widetilde{a} < 0$ (out of the money)

FIGURE 2.5: Graphical description of $N(u^*)$ - Call option case.

The figure describes how $N(u^*)$ (which is the weight of the manipulation component of the optimal trading strategy), changes for different situations. Each row describes different scenarios in terms of (adjusted) moneyness $\widetilde{a}$. The column of charts of the left are obtained for $\Gamma > 0$ (standing for a long position on a call option), while the charts on the right are characterised by $\Gamma < 0$ (standing for a short position on a call option).

It is quite clear, by the magnitude of the price impact, how a manipulation/non manipulation boundary is formed around the strike price: the level of moneyness is crucial in the decision to manipulate. The agent is able to produce the highest market impact when the underlying basket depends heavily on one security, as can be seen in Figure 2.7. A risk-neutral agent prefers the case when ω_1 tends to 0, as the second security in the basket has higher

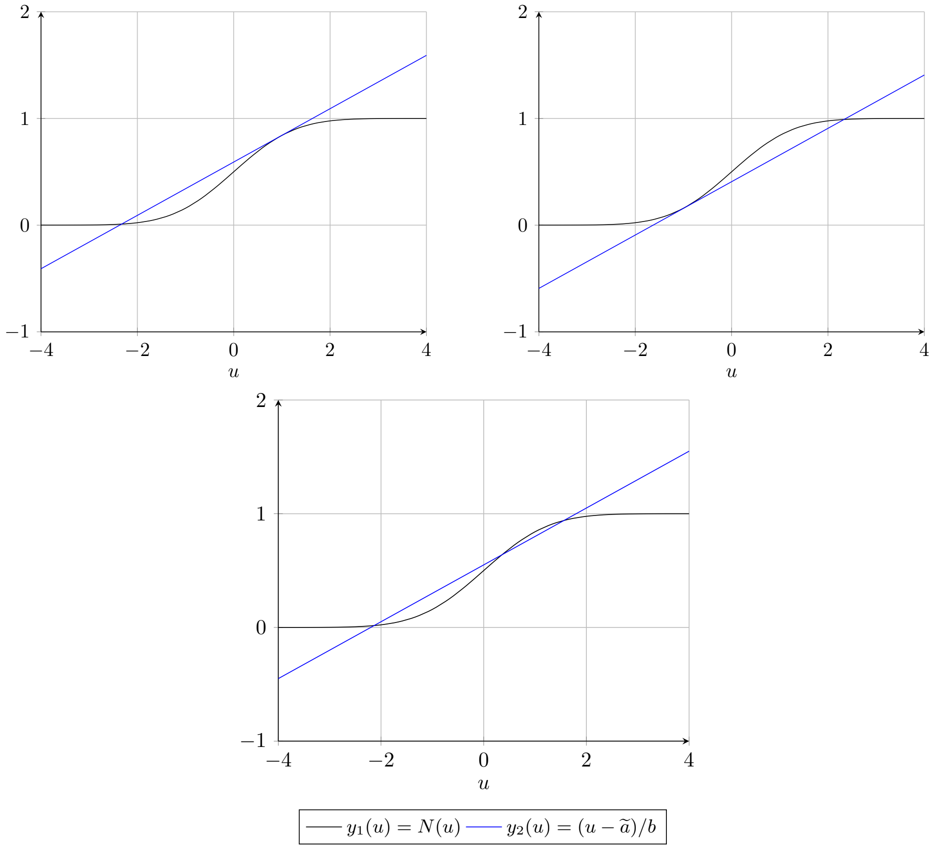


FIGURE 2.6: Special cases - multiple solutions for $\Gamma > 0$.

The figure displays (graphically) the scenarios for which the solution (Equation [2.49]) is not unique, in case of $\Gamma > 0$. The two graphs above represent the scenarios in which two solutions exist. The graph at the bottom represent the scenario in which three solutions exist.

market impact. The second security, however, has higher volatility. As a consequence, a risk-averse agent is not willing to invest as much in it and prefers the case when ω_1 tends to 1: the agents prefers making purchases on the less risky security.

Figure 2.8 shows how the optimal trading by the agent affects the price of the underlying basket for changing values of the moneyness and the drift of the first security μ_1.[7] Higher drift levels expand the manipulation region, for both the risk-neutral and the risk-averse agent.

Figure 2.9 shows how lower levels of correlation are generally preferable and induce the agent to trade more intensively in the direction that increases the option's payoff. The exception is when the option is OTM (but close enough to the money): the agent prefers higher levels of volatility to increase the probability of the option expiring ITM. In this case, the agent is willing to purchase more, impacting the price more significantly. This is particularly visible for the risk-averse agent.

Figure 2.10 shows the dependency with respect to the time to maturity T. Quite interestingly, for the risk-averse agent (Figure 2.10b), it is possible to notice that there needs to

[7]Similar considerations can also be applied to changing values of μ_2

be enough time before expiration for manipulation to be achievable: if time at maturity T is not high enough, the price impact that the agent is able to produce is limited; if the time at maturity is too high, manipulation is less attractive.

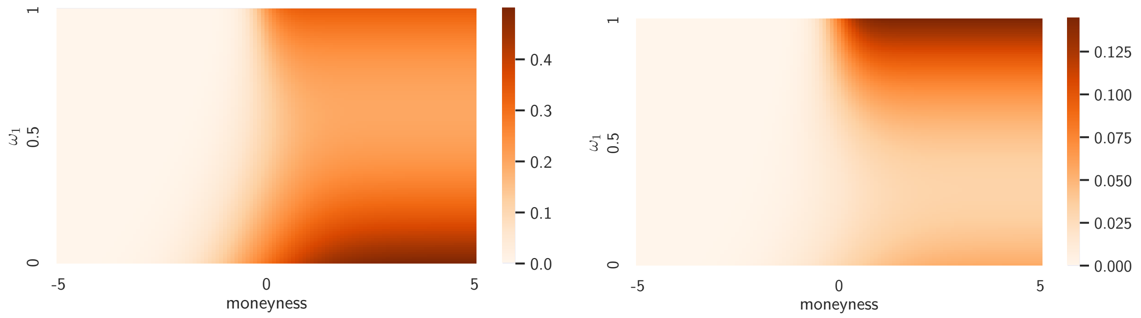
Finally, Figure 2.11 shows how market impact changes with respect to changes in the risk aversion parameter ψ, and changing values of Γ, T and ρ, for an option that is assumed to be initially at the money (ATM). In general, and as can be expected, the agent is expected to trade more intensively in the direction of manipulation (and, as a consequence, to impact the prices more) for lower values of the risk-aversion parameter ψ.

2.7 Conclusion

This chapter explores the trade-off faced by an agent who is holding a derivative and who is capable of affecting the underlying securities prior to expiration. Specifically, the chapter explores the conditions under which the trade-off by the agent leads to the decision to manipulate the underlying prices as a rational choice. The general case of a derivative written on a basket of underlying securities is considered. While some papers provide models to describe optimal behaviour in the single asset case, further research is needed on the multi-asset case.

Expanding on the work by Dutt and L. E. Harris, (2005) and Ko et al., (2016), the study first analyses the linear payoff case. The optimal trading strategy under the assumption of constant trading over the interval is found. The assumption of constant trading is then relaxed by allowing the agent to dynamically change behaviour throughout the interval: stochastic optimal control techniques are used to find the optimal solution. For a risk-neutral agent, the study shows how manipulative behaviour enters in a linear manner into optimal behaviour, and how holding a linear position always generates an incentive to manipulate the underlying securities. Formulas to measure the degree of manipulability of the basket underlying a linear derivative are proposed.

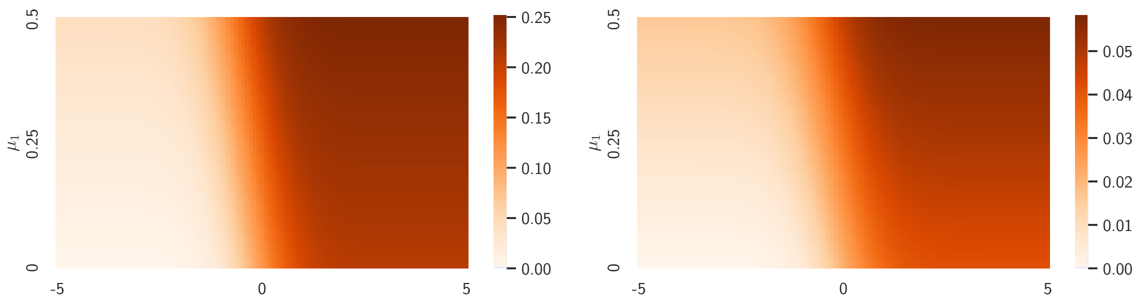
The study then analyses how the same trade-off is solved for an agent holding a non-linear derivative. As in the linear case, the case of derivatives written on multiple securities is considered. First, the study analytically solves the option payoff case under the assumption of constant trading. A link with the solution is found for the linear payoff, showing that, even in the multi-asset case, the final solution can be fully described by a one-variable non-linear equation. This result also represents a very easy way in which to implement measurement of the degree of manipulability of a basket of securities generated by a specific option position. The study also proposes the use of these formulas to solve the problems, as raised by Ko et al. (2016) and discussed at the beginning of the thesis: (1) finding tools to measure which securities are the most likely target of manipulation induced by a derivative; and (2) to what degree the underlying securities of a derivative are manipulable.



(A) $\psi = 0$ (B) $\psi = 0.5$

FIGURE 2.7: Price impact - call option case, moneyness vs. ω_1.

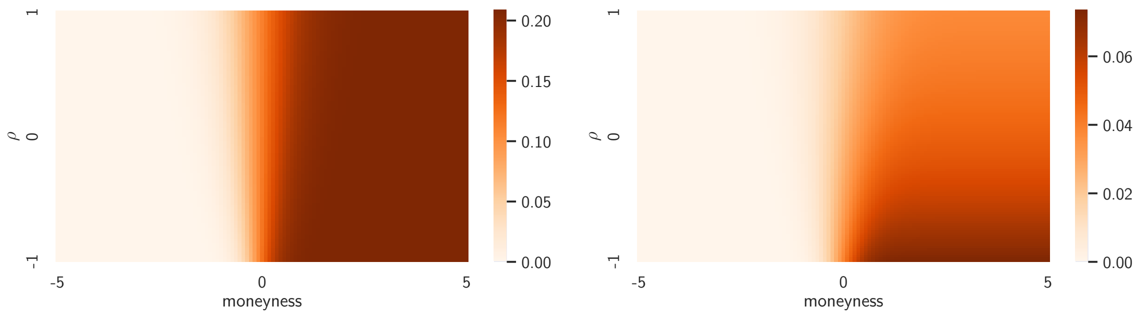
The figure displays the intensity of the price impact (PI, Equation [2.51]) for different combinations of: moneyness and ω_1. The rest of the parameters is provided in Table 2.1. Finally, $T = 1$, $\Gamma = 100$. Both cases of a risk-neutral (Figure 2.7a) and risk-averse agent ($\psi = 0.5$, Figure 2.7b) are considered.



(A) $\psi = 0$ (B) $\psi = 0.5$

FIGURE 2.8: Price impact - call option case, moneyness vs. μ_1.

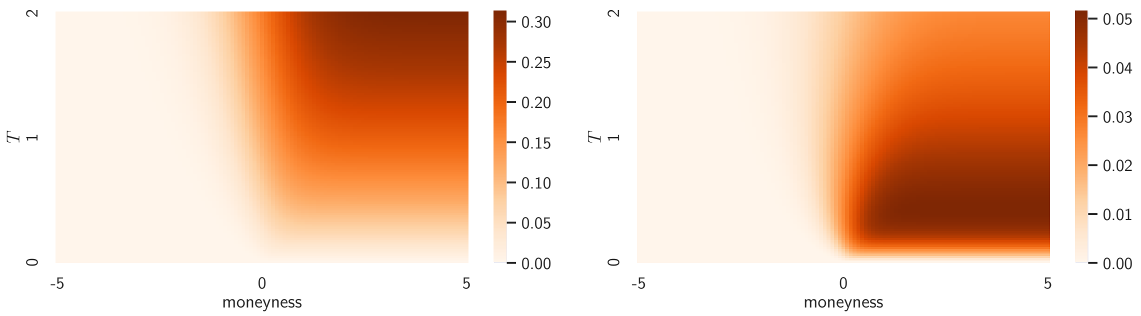
The figure displays the intensity of the price impact (PI, Equation [2.51]) for different combinations of: moneyness and ω_1. The rest of the parameters is provided in Table 2.1. Finally, $T = 1$, $\Gamma = 100$. Both cases of a risk-neutral (Figure 2.8a) and risk-averse agent ($\psi = 0.5$, Figure 2.8b) are considered.



(A) $\psi = 0$ (B) $\psi = 0.5$

FIGURE 2.9: Price impact - call option case, moneyness vs. ρ.

The figure displays the intensity of the price impact (PI, Equation [2.51]) for different combinations of: moneyness and ω_1. The rest of the parameters is provided in Table 2.1. Finally, $T = 1$, $\Gamma = 100$. Both cases of a risk-neutral (Figure 2.9a) and risk-averse agent ($psi = 0.5$, Figure 2.9b) are considered.



(A) $\psi = 0$ (B) $\psi = 0.5$

FIGURE 2.10: Price impact - call option case, moneyness vs. T.

The figure displays the intensity of the price impact (PI, Equation [2.51]) for different combinations of: moneyness and ω_1. The rest of the parameters is provided in Table 2.1. Finally, $T = 1$, $\Gamma = 100$. Both cases of a risk-neutral (Figure 2.10a) and risk-averse agent ($psi = 0.5$, Figure 2.10b) are considered.

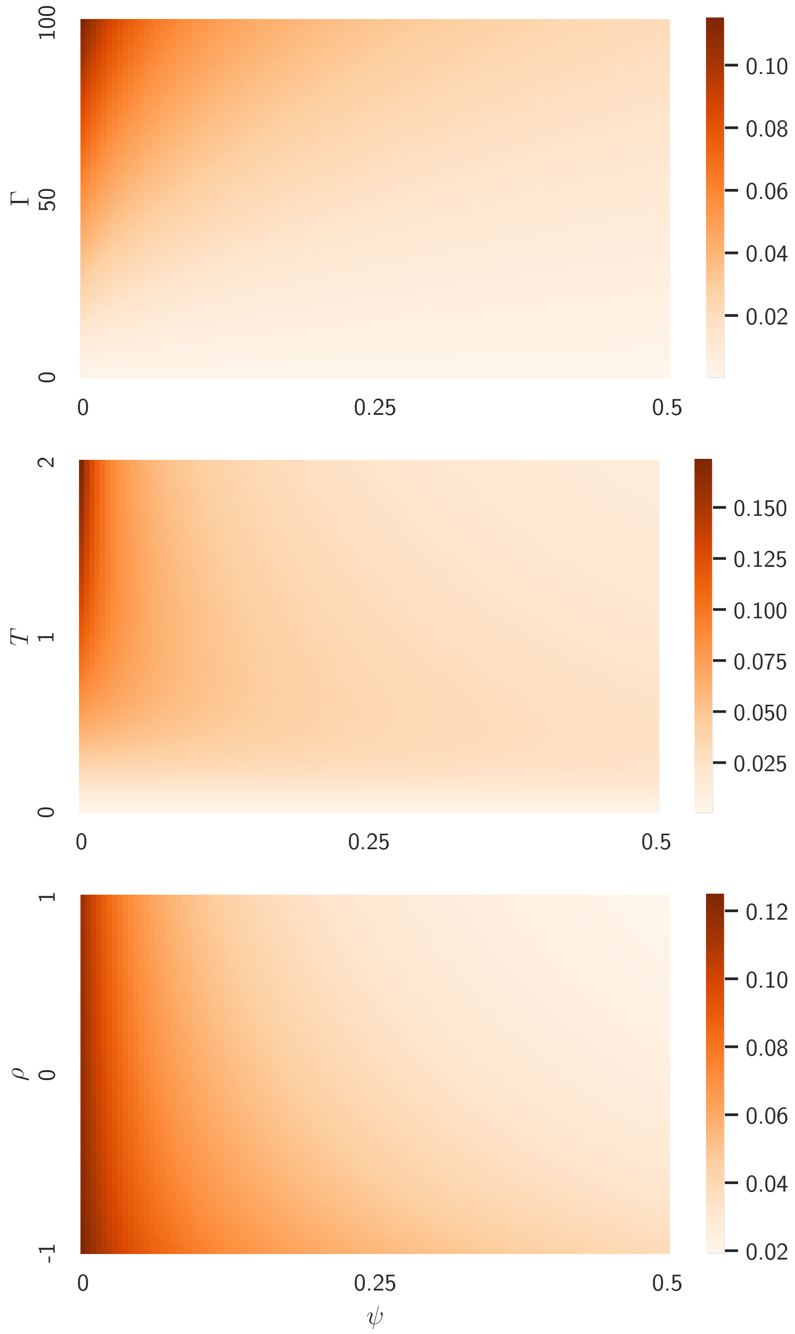


FIGURE 2.11: Price impact - option case, varying risk-aversion ψ.

The figure displays the intensity of the price impact (PI, Equation [2.51]) for different combinations of ψ and: Γ (top graph), T (middle graph) and ρ (bottom graph). Market model parameters as in Table 2.1; $T = 1$, $\Gamma = 100$. The option is assumed to be initially ATM.

2.A Useful lemmas

This section proposes lemmas used to simplify the derivation of the results obtained in previous paragraphs.

Lemma 1 *Let the following nonlinear system of equations be considered:*

$$A\boldsymbol{x} = \boldsymbol{\beta} N(a + \boldsymbol{\beta}' \boldsymbol{x}), \tag{2.52}$$

with $A \in \mathbb{R}^{d \times d}$ being a non singular matrix, $\boldsymbol{\beta} \in \mathbb{R}^d$, $a \in \mathbb{R}$ and $N(\cdot)$ denoting the cumulative distribution function of a standard normal random variable.

The solution of the system has the following form:

$$\boldsymbol{x}^* = N(u^*) A^{-1} \boldsymbol{\beta}. \tag{2.53}$$

where u^* satisfies the nonlinear equation $\frac{u^* - a}{b} = N(u^*)$, with $b = \boldsymbol{\beta}' A^{-1} \boldsymbol{\beta}$.

If $b < 0$ the solution $\boldsymbol{x}^*$ is unique, while, if $b > 0$, the system can admit multiple solutions. Let $u_0^{\pm} = \pm\sqrt{2\ln(b/\sqrt{2\pi})}$ be given. The solution $\boldsymbol{x}^*$ is unique if:

$$a > u_0^- - bN(u_0^-) \ \ \vee \ \ a < u_0^+ - bN(u_0^+),$$

while there are 2 solutions (of which one double) for:

$$a = u_0^{\pm} - bN(u_0^{\pm}),$$

and, finally, there are 3 distinct solutions if:

$$u_0^+ - N(u_0^+) < a < u_0^- - N(u_0^-).$$

Proof Let the nonlinear system (2.52) be given.

It is trivial to show that this system can be re-arranged as follows:

$$\boldsymbol{\beta}' \boldsymbol{x} = \boldsymbol{\beta}' A^{-1} \boldsymbol{\beta} N(a + \boldsymbol{\beta}' \boldsymbol{x}). \tag{2.54}$$

By applying the change of variables $v = a + \boldsymbol{\beta}' \boldsymbol{x}$, it can be further re-written as:

$$\frac{u - a}{b} = N(u), \tag{2.55}$$

with $b = \boldsymbol{\beta}' A^{-1} \boldsymbol{\beta}$.

Given u^* the solution of Equation (2.55), the solution of the original system (Equation [2.52]) is obtained as:

$$\boldsymbol{x}^* = N(u^*) A^{-1} \boldsymbol{\beta}. \tag{2.56}$$

Finally, Some considerations on the uniqueness of the solution are necessary. If $b < 0$, it is trivial to show that Equation (2.55) (and, as a consequence, the original problem) has a unique solution. The function $\frac{u-a}{b}$ is strictly decreasing in u and $N(u)$ is a strictly increasing function: their intersection is unique. Therefore, u^* and, as a consequence, $\boldsymbol{x}^*$ are unique.

If, on the other hand, $b > 0$ the system can admit multiple solutions, depending on the values of the parameters a and b. Given the values of a and b, which depend of the parameters of the market model, the functions $\frac{u-a}{b}$ and $N(u)$ have two points of tangency: $u_0^+ = +\sqrt{2\ln(b/\sqrt{2\pi})}$ and $u_0^- = -\sqrt{2\ln(b/\sqrt{2\pi})}$. There is a unique solution if:

$$-a/b < -u_0^-/b + N(u_0^-) \Rightarrow a > u_0^- - bN(u_0^-),$$

$$-a/b > -u_0^+/b + N(u_0^+) \Rightarrow a < u_0^+ - bN(u_0^+).$$

There are two solutions (of which, one double) when:

$$-a/b = -u_0^\pm/b + N(u_0^\pm) \Rightarrow a = u_0^\pm - bN(u_0^\pm),$$

and, finally, there are three distinct solutions if:

$$-u_0^-/b + N(u_0^-) < -a/b < -u_0^+/b + N(u_0^+) \Rightarrow u_0^+ - N(u_0^+) < a < u_0^- - N(u_0^-).$$

Figure 2.12 provides a graphical representation.

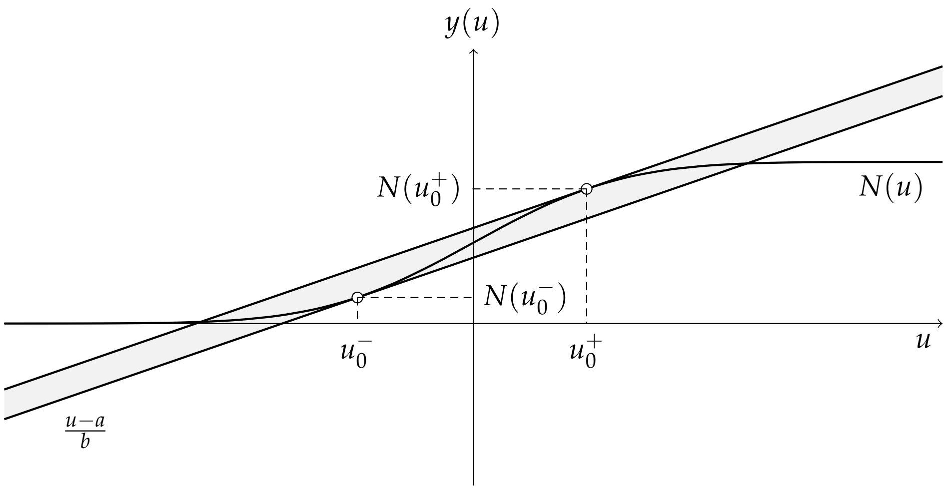


FIGURE 2.12: Multiple solutions boundaries.

The region in grey identifies the cases under which the nonlinear system admits multiple solutions. Within this region, three solutions exist. On the boundaries of this region (i.e., when the functions $y_1(u) = \frac{u-a}{b}$ and $y_2(u) = N(u)$ have a point of tangency at either u_0^+ or u_0^-), there are two solutions. Outside of the grey area, the solution is unique.

□

Lemma 2 *Let the following nonlinear system of equations be given:*

$$Ax = V + \beta N(a + \beta^{'} x), \tag{2.57}$$

where A, β and a satisfy the same conditions of Lemma 1, $V \in \mathbb{R}^d$ and $N(\cdot)$ is the cumulative distribution function of a standard normal random variable.

The solution of the system has the the following form:

$$x^* = A^{-1}\big(N(u^*)\beta + V\big), \tag{2.58}$$

where u^* is the solution of $(u^* - \widetilde{a})/b = N(u^*)$, with $\widetilde{a} = a + \beta^{'} A^{-1} V$ and $b = \beta^{'} A^{-1} \beta$.

The solution is unique for $b < 0$. If $b < 0$, the system can admit multiple solutions. Let $u_0^{\pm} = \pm\sqrt{2\ln(b/\sqrt{2\pi})}$ be given. The solution x^* is unique if:

$$a > u_0^{-} - bN(u_0^{-}) - \beta^{'} A^{-1} V \;\;\vee\;\; a < u_0^{+} - bN(u_0^{+}) - \beta^{'} A^{-1} V,$$

two solutions (of which, two equal) for:

$$a = u_0^{\pm} - bN(u_0^{\pm}) - \beta^{'} A^{-1} V,$$

and, finally, three distinct solutions if:

$$u_0^{+} - N(u_0^{+}) - \beta^{'} A^{-1} V < a < u_0^{-} - N(u_0^{-}) - \beta^{'} A^{-1} V.$$

Proof Let the nonlinear system of equations (2.57) be given.

Through the application of the change of variable $\widetilde{x} = x - A^{-1}V$, and by defining $\widetilde{a} = a + \beta^{'} A^{-1} V$, the system can be re-written as follows:

$$A\widetilde{x} = bN\big(\widetilde{a} + \beta^{'}\widetilde{x}\big). \tag{2.59}$$

Lemma 1 can be applied to Equation (2.59). The solution is therefore given by:

$$\widetilde{x}^* = N(u^*)A^{-1}\beta \Rightarrow$$

$$\Rightarrow x^* = N(u^*)A^{-1}\beta + A^{-1}V$$
$$\Rightarrow x^* = A^{-1}\big(N(u^*)\beta + V\big).$$

where u^* is the solution of the equation $(u^* - \widetilde{a})/b = N(u^*)$, with $b = \beta^{'} A^{-1} \beta$.

The solution is unique if $b < 0$, while, if $b > 0$, the system can admit multiple solutions. It is trivial to show that the solution x^* is unique if:

$$a > u_0^{-} - bN(u_0^{-}) - \beta^{'} A^{-1} V \;\;\vee\;\; a < u_0^{+} - bN(u_0^{+}) - \beta^{'} A^{-1} V,$$

while there are two solutions (of which, one double) for:

$$a = u_0^{\pm} - bN(u_0^{\pm}) - \boldsymbol{\beta}' A^{-1} \boldsymbol{V},$$

and, finally, there are three distinct solutions if:

$$u_0^{+} - N(u_0^{+}) - \boldsymbol{\beta}' A^{-1} \boldsymbol{V} < a < u_0^{-} - N(u_0^{-}) - \boldsymbol{\beta}' A^{-1} \boldsymbol{V}.$$

$\square$

2.B Proofs

Proof [Proof of Proposition 1]

Let the function (2.16) be the objective function to maximise with respect to $\boldsymbol{\nu}$. In case of linear payoff, the expected utility can be expressed as a function of the constant strategy $\boldsymbol{\nu}$ as:

$$\begin{aligned} p(\boldsymbol{\nu}) &= \mathbb{E}\left[X_T + \sum_{i=1}^{d} q_T^i(S_T^i - \alpha_i q_T^i) + \Gamma\sum_{i=1}^{d}(\omega_i S_T^i - K) - \frac{1}{2}\psi\sum_{i=1}^{d}\sum_{j=1}^{d}\sigma_{ij}\int_0^T q_t^i q_t^j dt\right] \\ &= \mathbb{E}\Big[X_T\Big] + \sum_{i=1}^{d}\mathbb{E}\Big[q_T^i S_T^i\Big] - \sum_{i=1}^{d}\alpha_i(q_T^i)^2 + \Gamma\sum_{i=1}^{d}\omega_i\mathbb{E}\Big[S_T^i\Big] - \Gamma K - \frac{1}{2}\psi\sum_{i=1}^{d}\sum_{j=1}^{d}\sigma_{ij}\int_0^T q_t^i q_t^j dt \end{aligned} \tag{2.60}$$

By keeping into consideration that $q_T^i = \nu^i T$, it follows, from standard results[8], that:

$$\begin{aligned} \mathbb{E}\Big[S_T^i\Big] &= \mathbb{E}\Big[\int_0^T dS_t^i\Big] = \mathbb{E}\Big[S_0^i + \mu_i T + \lambda_i \nu^i T + \int_0^T \sigma_i dW_u\Big] = S_0^i + \mu_i T + \lambda_i \nu^i T, \\ \mathbb{E}\Big[q_T^i S_T^i\Big] &= \nu^i T\Big(S_0^i + \mu_i T + \lambda_i \nu^i T\Big) = S_0^i \nu^i T + \mu_i \nu^i T^2 + \lambda_i (\nu^i)^2 T^2, \\ \mathbb{E}\Big[X_T\Big] &= -\sum_{i=1}^{d}\mathbb{E}\Big[\int_0^T (S_u^i + \kappa_i \nu^i)\nu^i du\Big] = -\sum_{i=1}^{d}\Big[S_0^i \nu^i T + \mu_i \nu^i \frac{T^2}{2} + \lambda_i (\nu^i)^2 \frac{T^2}{2} + \kappa_i (\nu^i)^2 T\Big]. \end{aligned}$$

Therefore, by plugging the expressions above into the expression (Equation [2.60]), the objective function, can be re-written as:

$$p(\boldsymbol{\nu}) = \sum_{i=1}^{d}\Big[\Big(\lambda_i\frac{T^2}{2} - \alpha_i T^2 - \kappa_i T\Big)(\nu^i)^2 + \frac{T^2}{2}\mu_i\nu^i + \Gamma\omega_i(S_0^i + \mu_i T + \lambda_i \nu^i T)\Big] - \frac{T^3}{6}\psi\sum_{i=1}^{d}\sum_{j=1}^{d}\sigma_{ij}\nu^i\nu^j - \Gamma K. \tag{2.61}$$

To find the optimal values of $\boldsymbol{\nu}$, the first-order conditions are computed:

$$\frac{\partial p}{\partial \nu^i} = \big((\lambda_i - 2\alpha_i)T^2 - 2\kappa_i T\big)\nu^i + \mu_i\frac{T^2}{2} + \Gamma\omega_i\lambda_i T - \frac{T^3}{3}\psi\sigma_i^2\nu^i - \frac{T^3}{6}\psi\sum_{i\neq j}\sigma_{ij}\nu^j = 0, \tag{2.62}$$

which lead to the following system of linear equations:

$$\boldsymbol{\nu}^* = M^{-1}\big(\widetilde{\boldsymbol{\mu}} + \widetilde{\boldsymbol{\lambda}}\big), \tag{2.63}$$

where $M \in \mathbb{R}^{d\times d}$, $\widetilde{\boldsymbol{\lambda}}$ and $\widetilde{\boldsymbol{\mu}} \in \mathbb{R}^d$ are given by:

$$M_{ii} = \Big((2\alpha_i - \lambda_i)T + 2\kappa_i + \frac{T^2}{3}\psi\sigma_i^2\Big), \quad M_{ij} = \frac{T^2}{6}\psi\sigma_{ij},$$

[8]See, for example, Björk, (2009)

$$\widetilde{\mu}_i = \frac{T}{2}\mu_i, \quad \widetilde{\lambda}_i = \Gamma\omega_i\lambda_i.$$

It is also trivial to show that the solution $\boldsymbol{\nu}$ corresponds to a maximum by checking the second-order conditions. The Hessian matrix $H(\boldsymbol{\nu})$ is given by:

$$h_{ii} = \frac{\partial^2 p}{\partial(\nu^i)^2} = (\lambda_i - 2\alpha_i)T - 2\kappa_i - \frac{T^2}{3}\psi\sigma_i^2 < 0,$$

$$h_{ij} = \frac{\partial^2 p}{\partial\nu^i\nu^j} = -\frac{T^2}{6}\psi\sigma_{ij} \leq 0,$$

and it can be shown to be negative definite for any $\boldsymbol{\nu} \in \mathbb{R}^d$.

Finally, in case of $\psi = 0$, the solution is trivially given by:

$$\nu^{i,*} = \frac{\mu_i/2}{(2\alpha_i - \lambda_i) + 2\kappa_i/T} + \frac{\Gamma\omega_i\lambda_i}{(2\alpha_i - \lambda_i)T + 2\kappa_i}. \tag{2.64}$$

□

Proof [Proof of Proposition 2]

The HJB Equation (2.19), with associated terminal condition (Equation 2.20), is considered. Based on the form of the terminal condition, the following ansatz for the form of the value function is proposed:

$$V(t, x, \boldsymbol{q}, \boldsymbol{S}) = x + \sum_{i=1}^{d} S^i(\Gamma\omega_i + q^i) + \theta(t, \boldsymbol{q}). \tag{2.65}$$

The HJB Equation (2.19) becomes:

$$\partial_t\theta - \frac{1}{2}\psi\sum_{i=1}^{d}\sum_{j=1}^{d}\sigma_{ij}q_iq_j + \sum_{i=1}^{d}\mu_i(\Gamma\omega_i + q^i) + \sup_{\boldsymbol{\nu}}\left[\sum_{i=1}^{d}\Big(\big(\partial_{q^i}\theta + \lambda_i(\Gamma\omega_i + q^i)\big)\nu^i - \kappa_i(\nu^i)^2\Big)\right] = 0. \tag{2.66}$$

From Equation (2.66), the form of the optimal controls $\boldsymbol{\nu}^* = (\nu^{i,*})_{i=1,\ldots,d}$ can be found in feedback control form (i.e., in terms of the value function itself) as:

$$\begin{aligned}\nu^{i,*} &= \max_{\nu^i}\left[\sum_{i=1}^{d}\Big(\big(\partial_{q^i}\theta + \lambda_i(\Gamma\omega_i + q^i)\big)\nu^i - \kappa_i(\nu^i)^2\Big)\right] \\ &= \frac{\partial_{q^i}\theta + \lambda_i(\Gamma\omega_i + q^i)}{2\kappa_i}, \quad i \in 1, \ldots, d.\end{aligned} \tag{2.67}$$

By plugging ν_t^* into Equation (2.66), the HJB Equation can be further simplified as follows:

$$\partial_t\theta - \frac{1}{2}\psi\sum_{i=1}^{d}\sum_{j=1}^{d}\sigma_{ij}q_iq_j + \sum_{i=1}^{d}\mu_i(q^i + \Gamma\omega_i) + \frac{1}{4\kappa_i}\sum_{i=1}^{d}\left(\partial_{q^i}\theta + \lambda_i(\Gamma\omega_i + q^i)\right)^2 = 0, \tag{2.68}$$

for which the following form is proposed as an ansatz for the solution of $\theta(t, \boldsymbol{q})$[9]:

$$\theta(t, \boldsymbol{q}) = \theta_0(t) + \sum_{i=1}^{d}\theta_1^i(t)q^i + \sum_{i=1}^{d}\theta_2^i(t)(q^i)^2. \tag{2.69}$$

By plugging (2.69) into Equation (2.66), and after grouping in terms of powers of q^i, a system of $2d+1$ coupled equations is obtained. More precisely, grouping for the zero power of q^i leads to the following initial value problem (IVP):

$$\begin{aligned} \frac{d\theta_0(t)}{dt} + \Gamma\sum_{i=1}^{d}\mu_i\omega_i + \frac{1}{4\kappa_i}\sum_{i=1}^{d}\left(\theta_1^i + \lambda_i\omega_i\Gamma\right)^2 = 0,\\ \theta_0(T) = -\Gamma K. \end{aligned} \tag{2.70}$$

By grouping for the first power of q^i, a set of d differential equations is obtained. The i-th differential equation (with the associated terminal condition) is given by:

$$\begin{aligned} \frac{d\theta_1^i(t)}{dt} + \mu_i - \frac{1}{2}\psi\sum_{j=1, j\neq i}^{d}\sigma_{ij}q_j + \frac{1}{2\kappa_i}\left(\lambda_i\omega_i\Gamma + \theta_1^i\right)\left(\lambda_i + 2\theta_2^i\right) = 0,\\ \theta_1^i(T) = 0. \end{aligned} \tag{2.71}$$

Finally, by grouping for the second power of q^i, another set of d IVPs is obtained, which, given i, can be expressed as follows:

$$\begin{aligned} \frac{d\theta_2^i(t)}{dt} - \frac{1}{2}\psi\sigma_i^2 + \frac{1}{4\kappa_i}\left(\lambda_i + 2\theta_2^i\right)^2 = 0,\\ \theta_2^i(T) = -\alpha_i, \end{aligned} \tag{2.72}$$

where $\frac{d\theta_0(t)}{dt}$, $\frac{d\theta_1^i(t)}{dt}$ and $\frac{d\theta_2^i(t)}{dt}$ is used to denote the first-order derivatives of $\theta_0(t)$, $\theta_1^i(t)$ and $\theta_2^i(t)$.

As a first step, the initial value problem (Equation [2.72]) is solved, which, given i, is a separable ordinary differential equation (ODE).[10] The solution depends on the value of ψ. If

[9] This ansatz is very similar to the one proposed by Á. Cartea et al., (2020) and also, in other financial contexts, by various other authors: Avellaneda and Stoikov, (2008), Fodra and Labadie, (2012), Fodra and Labadie, (2013) Guéant et al., (2013) and Guéant, (2017) are some examples.

[10] See Simmons, (2016) for a comprehensive treatment of the theory on ordinary differential equations.

$\psi = 0$, the function solving the IVP is given by:

$$\theta_2^i(t) = -\frac{\kappa_i}{T - t + \frac{2\kappa_i}{2\alpha_i - \lambda_i}} - \frac{\lambda_i}{2}. \tag{2.73}$$

For $\psi > 0$, the ODE (Equation [2.72]) can be re-arranged as:

$$\begin{aligned}\frac{d\theta_2^i(t)}{dt} &= \frac{1}{2}\psi\sigma_i^2 - \frac{1}{4\kappa_i}\left(\lambda_i + 2\theta_2^i\right)^2 \\ &= \frac{1}{\kappa_i}\left(\sigma_i\sqrt{\kappa_i\psi/2} - \frac{\lambda_i}{2} - \theta_2^i\right)\left(\sigma_i\sqrt{\kappa_i\psi/2} + \frac{\lambda_i}{2} + \theta_2^i\right),\end{aligned}$$

which is also separable and its solution is given by:

$$\theta_2^i(t) = \kappa_i\phi_i\frac{\eta_i^+ e^{-\phi_i(T-t)} - \eta_i^- e^{\phi_i(T-t)}}{\eta_i^+ e^{-\phi_i(T-t)} + \eta_i^- e^{\phi_i(T-t)}} - \frac{\lambda_i}{2}, \tag{2.74}$$

where $\phi_i = \sigma_i\sqrt{\frac{\psi}{2\kappa_i}}$ and $\eta_i^{\pm} = 2\kappa_i\phi_i \pm \lambda_i \mp 2\alpha_i$.

Once a solution for $\theta_2^i(t)$ is found, Equation (2.71) can be solved, for a given i. It corresponds to solving a simple first-order linear differential equation. Equation (2.71) can be re-written as:

$$\frac{d\theta_1^i(t)}{dt} + \frac{1}{\kappa_i}\left(\theta_2^i(t) + \frac{\lambda_i}{2}\right)\theta_1^i(t) = -\xi_i - \frac{\Gamma\omega_i\lambda_i}{\kappa_i}\left(\theta_2^i(t) + \frac{\lambda_i}{2}\right),$$

with $\xi_i = \mu_i - \frac{\psi}{2}\sum_{j=1,j\neq i}^d \sigma_{ij}q_j$.

As for $\theta_2^i(t)$, the solution depends on the value of ψ. For $\psi = 0$, the solution is given by:

$$\theta_1^i(t) = \frac{1}{\left(T - t + \frac{2\kappa_i}{2\alpha_i - \lambda_i}\right)}\left[\left(\mu_i\frac{2\kappa_i}{2\alpha_i - \lambda_i} - \Gamma\omega_i\lambda_i\right)(T-t) + \frac{\mu_i}{2}(T-t)^2\right]. \tag{2.75}$$

Otherwise, if $\psi > 0$, it is trivial to show that the solution of (2.71) is:

$$\theta_1^i(t) = \frac{\frac{\xi_i}{\phi_i}\left((1 - e^{-\phi_i(T-t)})\eta_i^+ - (1 - e^{\phi_i(T-t)})\eta_i^-\right) + 4\kappa_i\phi_i\Gamma\omega_i\lambda_i}{\eta_i^+ e^{-\phi_i(T-t)} + \eta_i^- e^{\phi_i(T-t)}} - \Gamma\omega_i\lambda_i, \tag{2.76}$$

where ϕ_i, $\eta_i^{\pm}$ are defined as above.

Finally, the solution of the ODE (Equation [2.70]) can be written as:

$$\theta_0(t) = \Gamma\sum_{i=1}^d \omega_i\mu_i(T-t) + \sum_{i=1}^d \frac{1}{4\kappa_i}\int_t^T \left(\Gamma\omega_i\lambda_i + \theta_1^i(s)\right)^2 ds - \Gamma K. \tag{2.77}$$

The optimal trading speed $\boldsymbol{\nu}_t$, over $t \in [0, T]$, results to be deterministic and depending on the inventory $\boldsymbol{q}_t$ and, given i, it is obtained as:

$$\nu_t^{i,*} = \nu^{i,*}(t, q_t^{i,*}) = \frac{\lambda_i(q_t^{i,*} + \omega_i \Gamma) + \theta_1^i(t) + 2\theta_2^i(t) q_t^{i,*}}{2\kappa_i}. \tag{2.78}$$

By definition:

$$dq_t^{i,*} = \nu^{i,*}(t, q_t^{i,*})dt = \Big[\frac{\theta_1^i(t) + \lambda_i \omega_i \Gamma}{2\kappa_i} + \frac{\lambda_i + 2\theta_2^i(t)}{2\kappa_i} q_t^{i,*}\Big] dt, \tag{2.79}$$

which can be solved for $q_t^{i,*}$ as a first-order linear ODE:

$$\frac{dq_t^{i,*}}{dt} - \frac{\lambda_i + 2\theta_2^i(t)}{2\kappa_i} q_t^{i,*} = \frac{\theta_1^i(t) + \lambda_i \omega_i \Gamma}{2\kappa_i}. \tag{2.80}$$

If $\psi = 0$, the solution is given by:

$$q_t^{i,*} = q_0^i + \left[\frac{\Gamma\omega_i\lambda_i - (2\alpha_i - \lambda_i)q_0^i}{(2\alpha_i - \lambda_i)T + 2\kappa_i} - \frac{\mu_i \kappa_i}{(2\alpha_i - \lambda_i)^2 T + 2\kappa_i(2\alpha_i - \lambda_i)} + \frac{\mu_i}{4\kappa_i}\Big(\frac{2\kappa_i}{2\alpha_i - \lambda_i} + T\Big)\right] t - \frac{\mu_i}{4\kappa_i} t^2. \tag{2.81}$$

To notice that, in case $\psi > 0$, the function $\theta_1^i(t)$ depends on $q^{j,*}$, for $j \neq i$. By expanding $\theta_1^i(t)$, the ODE (2.80) can be written as:

$$\begin{aligned} \frac{dq_t^{i,*}}{dt} - \frac{\lambda_i + 2\theta_2^i(t)}{2\kappa_i} q_t^{i,*} &= \frac{\frac{\mu_i}{2\kappa_i\phi_i}\Big((1 - e^{-\phi_i(T-t)})\eta_i^+ - (1 - e^{\phi_i(T-t)})\eta_i^-\Big) + 2\phi_i \Gamma \omega_i \lambda_i}{\eta_i^+ e^{-\phi_i(T-t)} + \eta_i^- e^{\phi_i(T-t)}} - \\ &\quad - \sum_{j=1, j\neq i}^{d} \left(\frac{\psi}{4\kappa_i \phi_i} \frac{(1 - e^{-\phi_i(T-t)})\eta_i^+ - (1 - e^{\phi_i(T-t)})\eta_i^-}{\eta_i^+ e^{-\phi_i(T-t)} + \eta_i^- e^{\phi_i(T-t)}} \sigma_{ij}\right) q_t^{j,*}. \end{aligned} \tag{2.82}$$

and, finally, it is possible to express the dynamic of the optimal inventory as a linear (deterministic) system:

$$\frac{d\boldsymbol{q}_t^*}{dt} = \boldsymbol{H}_t \boldsymbol{q}_t^* + \boldsymbol{b}_t, \quad \boldsymbol{q}_0^* = \boldsymbol{q}_0, \tag{2.83}$$

where $\boldsymbol{q}_t^* = (q_t^{1,*}, \ldots, q_t^{d,*})$, $\boldsymbol{H}_t = (h_t^{ij})_{i,j=1,\ldots,d}$ is given by:

$$h_t^{ii} = \frac{\lambda_i + 2\theta_2^i(t)}{2\kappa_i},$$

$$h_t^{ij} = -\frac{\psi}{4\kappa_i\phi_i} \frac{(1 - e^{-\phi_i(T-t)})\eta_i^+ - (1 - e^{\phi_i(T-t)})\eta_i^-}{\eta_i^+ e^{-\phi_i(T-t)} + \eta_i^- e^{\phi_i(T-t)}} \sigma_{ij},$$

and $\boldsymbol{b}_t \in \mathbb{R}^d$, which i-th element is given by:

$$b_t^i = \frac{\frac{\mu_i}{2\kappa_i\phi_i}\Big((1 - e^{-\phi_i(T-t)})\eta_i^+ - (1 - e^{\phi_i(T-t)})\eta_i^-\Big) + 2\phi_i \Gamma \omega_i \lambda_i}{\eta_i^+ e^{-\phi_i(T-t)} + \eta_i^- e^{\phi_i(T-t)}}.$$

All the elements of both $\boldsymbol{H}_t$ and $\boldsymbol{b}_t$ are continuous over the interval $[0, T]$. This ensure the existence and uniqueness of a continuous solution $\boldsymbol{q}_t^*$: see Starzhinskii and Yakubovich, (1975). Thus, the solution $\boldsymbol{q}_t^*$ is bounded in $[0, T]$, and the propriety $\mathbb{E}[\int_t^T \boldsymbol{\nu}_u^\top \boldsymbol{\nu}_u du] < +\infty$ is always satisfied: the control is admissible. Furthermore, it can be easily shown that the value function obtained:

$$V(t, x, \boldsymbol{q}, \boldsymbol{S}) = x + \sum_{i=1}^{d} S^i(\Gamma\omega_i + q^i) + \theta_0(t) + \sum_{i=1}^{d} \theta_1^i(t) q^i + \sum_{i=1}^{d} \theta_2^i(t)(q^i)^2,$$

is a classical solution of the HJB, being once differentiable in time t and twice differentiable in each of the state variables x, $\boldsymbol{q}$ and $\boldsymbol{S}$. Therefore it is the actual value function and the control process is an optimal control.

□

Proof [Proof of Proposition 3]

Let Equation (2.48) be the function to maximise with respect to $\boldsymbol{\nu}$. The functional $p(\boldsymbol{\nu})$ is defined:

$$p(\boldsymbol{\nu}) = \mathbb{E}\left[X_T + \sum_{i=1}^{d} q_T^i(S_T^i - \alpha_i q_T^i) + \Gamma\sum_{i=1}^{d}(\omega_i S_T^i - K)\mathbb{1}_{E_0} - \frac{1}{2}\psi\sum_{i=1}^{d}\sum_{j=1}^{d}\sigma_{ij}\int_0^T q_t^i q_t^j dt\right], \quad (2.84)$$

where $E_0 = \{\sum_{i=1}^d \omega_i S_T^i > K\}$.

By following similar steps as the linear payoff case, the function (2.84) can be re-written as:

$$p(\boldsymbol{\nu}) = \sum_{i=1}^{d}\left[\left(\lambda_i\frac{T^2}{2} - \alpha_i T^2 - \kappa_i T\right)(\nu^i)^2 + \mu_i\frac{T^2}{2}\nu^i - \frac{T^3}{6}\psi\sum_{j=1}^{d}\sigma_{ij}\nu^i\nu^j\right] + \Gamma\mathbb{E}\left[\sum_{i=1}^{d}(\omega^i S_T^i - K)\mathbb{1}_{E_0}\right]. \quad (2.85)$$

The expected payoff value, however, cannot be trivially disentangled from the rest of the expression. An analytical expression for $\mathbb{E}\left[\left(\sum_{i=1}^d \omega_i S_T^i - K\right)\mathbb{1}_{E_0}\right]$ is needed and it can be found by following the next steps. Firstly, the following is considered:

$$\begin{aligned}\sum_{i=1}^{d}\omega_i S_T^i &= \sum_{i=1}^{d}\omega_i(S_0 + \mu_i T + \lambda_i\nu^i T + \sigma_i W_T^i) = \\ &= \sum_{i=1}^{d}\omega_i\sigma_i W_T^i + \sum_{i=1}^{d}\omega_i(S_0 + \mu_i T + \lambda_i\nu^i T) = Y + \sum_{i=1}^{d}\omega_i(S_0 + \mu_i T + \lambda_i\nu^i T),\end{aligned} \quad (2.86)$$

where $Y = \sum_{i=1}^d \omega_i\sigma_i W_T^i$ is a linear combination of correlated random variables following a normal joint distribution and, as a consequence, is normally distributed: $Y \sim \mathcal{N}(0, \sigma_Y^2)$, with $\sigma_Y^2 = \boldsymbol{\omega}'\Sigma\boldsymbol{\omega}T$, $\Sigma = (\rho_{ij}\sigma_i\sigma_j)_{1\le i,j\le d}$ and ρ_{ij} is the correlation between W_T^i and W_T^j.

It is then possible to write:

$$
\begin{aligned}
&\mathbb{E}\Big[\big(\sum_{i=1}^{d}\omega_i S_T^i - K\big)\mathbb{1}_{\{\sum_{i=1}^{d}\omega_i S_T^i - K>0\}}\Big] = \\
&\quad = \mathbb{E}\Big[\Big(Y + \sum_{i=1}^{d}\omega_i(S_0 + \mu_i T + \lambda_i \nu^i T) - K\Big)\mathbb{1}_{\big\{Y+\sum_{i=1}^{d}\omega_i(S_0+\mu_i T+\lambda_i\nu^i T)-K>0\big\}}\Big] = \\
&\quad = \mathbb{E}\Big[\Big(Y - \overline{K}(\boldsymbol{\nu})\Big)\mathbb{1}_{\big\{Y>\overline{K}(\boldsymbol{\nu})\big\}}\Big] = \\
&\quad = \mathbb{E}\Big[Y\mathbb{1}_{\big\{Y>\overline{K}(\boldsymbol{\nu})\big\}}\Big] - \overline{K}(\boldsymbol{\nu})\mathbb{E}\Big[\mathbb{1}_{\big\{Y>\overline{K}(\boldsymbol{\nu})\big\}}\Big],
\end{aligned}
\tag{2.87}
$$

where $\overline{K}(\boldsymbol{\nu}) = K - \sum_{i=1}^{d}\omega_i(S_0 + \mu_i T + \lambda_i \nu^i T)$ and $\mathbb{E}\Big[\mathbb{1}_{\big\{Y>\overline{K}(\boldsymbol{\nu})\big\}}\Big] = \mathbb{P}\big(E_0\big)$.

Furthermore, it is trivial to show that the following is true:

$$
\begin{aligned}
\mathbb{E}\Big[\mathbb{1}_{\big\{Y>\overline{K}(\boldsymbol{\nu})\big\}}\Big] = \mathbb{P}\big(E_0\big) &= \mathbb{P}\big(Y > \overline{K}(\boldsymbol{\nu})\big) = \\
&= \mathbb{P}\big(\sigma_Y Z > \overline{K}(\boldsymbol{\nu})\big) = \\
&= \mathbb{P}\big(Z > \frac{\overline{K}(\boldsymbol{\nu})}{\sigma_Y}\big) = \\
&= N\Big(-\frac{\overline{K}(\boldsymbol{\nu})}{\sigma_Y}\Big) = \\
&= N(d(\boldsymbol{\nu})),
\end{aligned}
\tag{2.88}
$$

and:

$$
\begin{aligned}
\mathbb{E}\Big[Y\mathbb{1}_{\big\{Y>\overline{K}(\boldsymbol{\nu})\big\}}\Big] &= \int_{\overline{K}(\boldsymbol{\nu})}^{+\infty} y\frac{1}{\sqrt{2\pi}\sigma_Y}e^{-\frac{y^2}{2\sigma_Y^2}}dy = \\
&= -\frac{\sigma_Y}{\sqrt{2\pi}}\Big[e^{-\frac{y^2}{2\sigma_Y^2}}\Big]_{\overline{K}(\boldsymbol{\nu})}^{+\infty} = \frac{\sigma_Y}{\sqrt{2\pi}}e^{-\frac{\overline{K}(\boldsymbol{\nu})^2}{2\sigma_Y^2}} = \\
&= \frac{\sigma_Y}{\sqrt{2\pi}}e^{-\frac{1}{2}\Big(-\frac{\overline{K}(\boldsymbol{\nu})}{\sigma_Y}\Big)^2} = \frac{\sigma_Y}{\sqrt{2\pi}}e^{-\frac{1}{2}d(\boldsymbol{\nu})^2},
\end{aligned}
\tag{2.89}
$$

where $Z \sim \mathcal{N}(0,1)$, $d(\boldsymbol{\nu}) = -\frac{\overline{K}(\boldsymbol{\nu})}{\sqrt{\boldsymbol{\omega}'\Sigma\boldsymbol{\omega}T}} = -\frac{\overline{K}(\boldsymbol{\nu})}{\sigma_Y}$ and $N(\cdot)$ denotes the cumulative distribution function of a standard normal random variable.

The profit function (Equation [2.85]) can then be re-written as:

$$
\begin{aligned}
p(\boldsymbol{\nu}) = \sum_{i=1}^{d}&\Big[\big(\lambda_i\frac{T^2}{2} - \alpha_i T^2 - \kappa_i T\big)(\nu^i)^2 + \mu_i\frac{T^2}{2}\nu^i - \frac{T^3}{6}\psi\sum_{j=1}^{d}\sigma_{ij}\nu^i\nu^j\Big] + \\
&+ \Gamma\Big[\frac{\sigma_Y}{\sqrt{2\pi}}e^{-\frac{1}{2}d(\boldsymbol{\nu})^2} - \overline{K}(\boldsymbol{\nu})N(d(\boldsymbol{\nu}))\Big]
\end{aligned}
\tag{2.90}
$$

To find the values of ν maximising this expression, the first-order conditions need to be computed:

$$\begin{aligned}
\frac{\partial}{\partial \nu^i} p(\boldsymbol{\nu}) = &\big((\lambda_i - 2\alpha_i)T^2 - 2\kappa_i T\big)\nu^i + \frac{T^2\mu_i}{2} - \frac{T^3}{6}\psi\sigma_i^2 2\nu^i - \frac{T^3}{6}\psi \sum_{j=1, j\neq i}^{d} \sigma_{ij}\nu^j + \\
&+ \Gamma\left[\frac{\partial}{\partial \nu^i}\mathbb{E}\Big[Y\mathbb{1}_{\{Y>\overline{K}(\boldsymbol{\nu})\}}\Big] - \frac{\partial}{\partial \nu^i}\Big(\overline{K}(\boldsymbol{\nu})N\big(d(\boldsymbol{\nu})\big)\Big)\right] = \\
=&\big((\lambda_i - 2\alpha_i)T^2 - 2\kappa_i T\big)\nu^i + \frac{T^2\mu_i}{2} - \frac{T^3}{6}\psi\sigma_i^2 2\nu^i - \frac{T^3}{6}\psi \sum_{j=1, j\neq i}^{d} \sigma_{ij}\nu^j + \\
&+ \Gamma\left[\frac{\omega_i\lambda_i T}{\sqrt{2\pi}\sigma_Y}\overline{K}(\boldsymbol{\nu})e^{-\frac{1}{2}d(\nu)^2} - \Big(\frac{\partial}{\partial \nu^i}\overline{K}(\boldsymbol{\nu})\Big)N\big(d(\boldsymbol{\nu})\big) - \overline{K}(\boldsymbol{\nu})\Big(\frac{\partial}{\partial \nu^i}N\big(d(\boldsymbol{\nu})\big)\Big)\right] = \\
=&\big((\lambda_i - 2\alpha_i)T^2 - 2\kappa_i T\big)\nu^i + \frac{T^2\mu_i}{2} - \frac{T^3}{6}\psi\sigma_i^2 2\nu^i - \frac{T^3}{6}\psi \sum_{j=1, j\neq i}^{d} \sigma_{ij}\nu^j + \\
&+ \Gamma\left[\frac{\omega_i\lambda_i T}{\sqrt{2\pi}\sigma_Y}\overline{K}(\boldsymbol{\nu})e^{-\frac{1}{2}d(\nu)^2} - \left(N\big(d(\boldsymbol{\nu})\big) + \overline{K}(\boldsymbol{\nu})f\big(d(\boldsymbol{\nu})\big)\Big(-\frac{1}{\sigma_Y}\Big)\right)\frac{\partial}{\partial \nu^i}\overline{K}(\boldsymbol{\nu})\Big)\right] = \\
=&\big((\lambda_i - 2\alpha_i)T^2 - 2\kappa_i T\big)\nu^i + \frac{T^2\mu_i}{2} - \frac{T^3}{6}\psi\sigma_i^2 2\nu^i - \frac{T^3}{6}\psi \sum_{j=1, j\neq i}^{d} \sigma_{ij}\nu^j + \\
&+ \Gamma\left[\frac{\omega_i\lambda_i T}{\sqrt{2\pi}\sigma_Y}\overline{K}(\boldsymbol{\nu})e^{-\frac{1}{2}d(\nu)^2} - \left(N\big(d(\boldsymbol{\nu})\big) + \overline{K}(\boldsymbol{\nu})\frac{e^{-\frac{1}{2}d(\nu)^2}}{\sqrt{2\pi}}\Big(-\frac{1}{\sigma_Y}\Big)\right)(-\omega_i\lambda_i T)\right] = \\
=&\big((\lambda_i - 2\alpha_i)T^2 - 2\kappa_i T\big)\nu^i + \frac{T^2\mu_i}{2} - \frac{T^3}{6}\psi\sigma_i^2 2\nu^i - \frac{T^3}{6}\psi \sum_{j=1, j\neq i}^{d} \sigma_{ij}\nu^j + \Gamma\omega_i\lambda_i T N\big(d(\boldsymbol{\nu})\big),
\end{aligned} \tag{2.91}$$

where $f(\cdot)$ denotes the probability density function of a standard normal random variable, and considering that:

$$\begin{aligned}
\frac{\partial}{\partial \nu^i}\mathbb{E}\Big[Y\mathbb{1}_{\{Y>\overline{K}(\boldsymbol{\nu})\}}\Big] &= \frac{\sigma_Y}{\sqrt{2\pi}}e^{-\frac{1}{2}d(\nu)^2}\Big(-\frac{1}{2}2d(\boldsymbol{\nu})\Big)\frac{\partial}{\partial \nu^i}d(\boldsymbol{\nu}) = \\
&= \frac{\sigma_Y}{\sqrt{2\pi}}e^{-\frac{1}{2}d(\nu)^2}\Big(-\Big(-\frac{\overline{K}(\boldsymbol{\nu})}{\sigma_Y}\Big)\Big)\frac{\omega_i\lambda_i T}{\sigma_Y} = \\
&= \frac{\omega_i\lambda_i T}{\sqrt{2\pi}\sigma_Y}\overline{K}(\boldsymbol{\nu})e^{-\frac{1}{2}d(\nu)^2}.
\end{aligned}$$

Therefore, to find the optimal solutions ν_i for $i = 1, \dots, d$, the following nonlinear system of equations need to be solved:

$$\Big((\lambda_i - 2\alpha_i)T^2 - 2\kappa_i T - \frac{T^3}{3}\psi\sigma_i^2\Big)\nu^i - \frac{T^3}{6}\psi \sum_{j=1, j\neq i}^{d} \sigma_{ij}\nu^j = -\mu_i\frac{T^2}{2} - \Gamma\omega_i\lambda_i T N\big(d(\boldsymbol{\nu})\big). \tag{2.92}$$

This system can be re-arranged as:

$$\Big((2\alpha_i-\lambda_i)T^2+2\kappa_iT+\frac{T^3}{3}\psi\sigma_i^2\Big)\nu^i+\frac{T^3}{6}\psi\sum_{j=1,j\neq i}^{d}\sigma_{ij}\nu^j=$$
$$=\mu_i\frac{T^2}{2}+\Gamma\sigma_Y\frac{\omega_i\lambda_iT}{\sigma_Y}N\Big(\sum_{i=1}^{d}\frac{\omega_i(S_0+\mu_iT)-K}{\sigma_Y}+\sum_{i=1}^{d}\frac{\omega_i\lambda_iT}{\sigma_Y}\nu^i\Big),$$

or, in matrix notation:

$$M\boldsymbol{\nu}=\widetilde{\boldsymbol{\mu}}+\widetilde{\boldsymbol{\lambda}}N(a+\widetilde{\boldsymbol{\lambda}}'\boldsymbol{\nu}), \tag{2.93}$$

where $M\in\mathbb{R}^{d\times d}$, $\widetilde{\boldsymbol{\mu}}\in\mathbb{R}^d$, $\widetilde{\boldsymbol{\lambda}}\in\mathbb{R}^d$ and $a\in\mathbb{R}$ are such that:

$$M_{ii}=\frac{1}{\Gamma\sigma_Y}\Big((2\alpha_i-\lambda_i)T^2+2\kappa_iT+\frac{T^3}{3}\psi\sigma_i^2\Big),\quad M_{ij}=\frac{1}{\Gamma\sigma_Y}\frac{T^3}{6}\psi\sigma_{ij},$$

$$\widetilde{\mu}_i=\frac{1}{\Gamma\sigma_Y}\mu_i\frac{T^2}{2},\quad \widetilde{\lambda}_i=\frac{\omega_i\lambda_iT}{\sigma_Y},\quad a=\sum_{i=1}^{d}\frac{\omega_i(S_0+\mu_iT)-K}{\sigma_Y}.$$

By an application of Lemma 2, the solution of the system (2.93) can be found as:

$$\boldsymbol{\nu}^*=M^{-1}\big(N(u^*)\widetilde{\boldsymbol{\lambda}}+\widetilde{\boldsymbol{\mu}}\big), \tag{2.94}$$

with $N(\cdot)$ identifying the cumulative distribution function of a standard normal distribution, and u^* is the solution of the system $\frac{u-\widetilde{a}}{b}=N(u)$, where:

$$\widetilde{a}=a+\widetilde{\boldsymbol{\lambda}}'M^{-1}\widetilde{\boldsymbol{\mu}},\quad b=\widetilde{\boldsymbol{\lambda}}'M^{-1}\widetilde{\boldsymbol{\lambda}}.$$

□

Proposition 4 *The optimal problem (Equation [2.48]) always admits a maximum as solution.*

Proof For simplicity, the risk-neutral case $\psi=0$ is considered. To evaluate whether the optimal problem (Equation [2.16]) admits a maximum, the Hessian matrix $H(\boldsymbol{\nu})$ associated to Equation (2.85) is considered:

$$h_{ii}(\boldsymbol{\nu})=\big((\lambda_i-2\alpha_i)T^2-2\kappa_iT\big)+\frac{\Gamma(\omega_i\lambda_iT)^2}{\sigma_Y}f\big(d(\boldsymbol{\nu})\big),$$

$$h_{ij}(\boldsymbol{\nu})=\frac{\Gamma\omega_i\lambda_i\omega_j\lambda_jT^2}{\sigma_Y}f\big(d(\boldsymbol{\nu})\big).$$

The solution $\boldsymbol{\nu}^*$ corresponds to a maximum if the Hessian matrix evaluated in $\boldsymbol{\nu}^*$ is definitive negative: $\boldsymbol{x}'H(\boldsymbol{\nu})\boldsymbol{x}<0$, $\forall\boldsymbol{x}\in\mathbb{R}^d$. This condition can be re-written, in matrix form, as:

$$\boldsymbol{x}'\Big(\Gamma\sigma_Y\Big(-\boldsymbol{M}+f\big(d(\boldsymbol{\nu})\big)\widetilde{\boldsymbol{\lambda}}\otimes\widetilde{\boldsymbol{\lambda}}\Big)\Big)\boldsymbol{x}<0, \tag{2.95}$$

where $\boldsymbol{M}$ and $\widetilde{\boldsymbol{\lambda}}$ are defined as in Proposition 3, $f(\cdot)$ is the probability density function of a standard normal random variable and $\otimes$ denotes the tensor product.

If $\Gamma < 0$, condition (Equation [2.95]) is always satisfied: all the elements of $H(\boldsymbol{\nu})$ are negative. If $\Gamma > 0$, this is not necessarily the case. By applying the following change of variables $y_i = \sqrt{M_{ii}}x_i$, where M_{ii} is the i-th element on the diagonal of the matrix M, the condition (Equation [2.95]) becomes:

$$\boldsymbol{y}^{'}\Big(I_d + \boldsymbol{w} \otimes \boldsymbol{w}\Big)\boldsymbol{y} > 0, \tag{2.96}$$

where $\boldsymbol{w} \in \mathbb{R}^d$, with i-element equal to $w_i = \sqrt{\frac{f\left(d(\nu)\right)}{M_{ii}}}\widetilde{\lambda}_i$ and I_d is the d-dimensional identity matrix. The necessary and sufficient condition for the above inequality to be true (and for the matrix $H(\boldsymbol{\nu})$ to be negative definite) is simply:

$$\|\boldsymbol{w}\|_2 < 1,$$

where $\|\cdot\|_2$ is the L^2 vector norm. Expanding the above, it is possible to show that the condition (Equation [2.96]) is satisfied for:

$$f\big(d(\boldsymbol{\nu})\big) > \sum_{i=1}^{d} \frac{\beta_i^2}{M_{ii}}.$$

By noticing that $\frac{\beta_i^2}{M_{ii}} = b$ and $f\big(d(\boldsymbol{\nu})\big) = f(u)$, the condition $f(u) > b$ corresponds to:

$$u > \sqrt{2\ln(b/2\pi)} = u_0^+ \;\; \vee \;\; u < \sqrt{2\ln(b/2\pi)} = u_0^-,$$

where $u_0^{\pm}$ are the points of tangency between the curves $N(u)$ and $(u - \widetilde{a})/b$ as per Lemma 2.

Three scenarios are possible. First, if there is a unique solution the above condition is always true and, therefore, the solution is a maximum. Second, if there are two solutions: (1) the (double) solution corresponding to the point of tangency between the two curves does not satisfy the condition to get a maximum; and (2) the other solution is always a maximum. Third and final scenario is when three solutions exist. In this case, there is always a solution given by the intersection between the curves such that $u_0^- < u < u_0^+$ (that is, the intersection fall between the two points of tangency), and this solution is never a maximum, as the above condition is never satisfied. For the two other solutions (that are on the two opposite extremes) the condition above is always satisfied and they always represent a (local) maximum. □

Chapter 3

Expiration day effects: evidence from the UK

3.1 Introduction

The effects of derivatives' expiration has been the subject of several studies, spanning different markets and showing the existence of price distortions and anomalous levels of trading activity on the underlying market; for examples of seminal studies, see Klemkosky, (1978); Stoll and Whaley, (1987); Stoll and Whaley, (1991).

Different theories have been advanced to explain the anomalous patterns observed on expiration days. Trading activity by arbitrageurs is commonly seen as one of the main reasons for cash-settled derivatives. A similar explanation is related to delta-hedging by market makers on the options market, for both cash-settled and physically delivered contracts, as well explained by Ni et al., (2005). Other factors can also contribute to generating unusual trading activity levels at expiration, such as the unwinding of hedging positions by market participants holding OTM option positions, anticipating they will not be exercised; see Cinar and Vu, (1987). Chiang, (2014) documents increased selling pressure and abnormally high negative returns on expiration dates for stocks associated with a high number of active deeply in the money (ITM) options, interpreting these patterns as due to options holders exercising their contracts and immediately selling the acquired underlying stocks. Finally, market manipulation is a possibility. The previous chapter explored the theoretical foundations of this type of market manipulation. These theoretical considerations are also supported by, to date, only a few empirical studies[1] and by several examples of prosecuted cases.[2] Given the overall objectives of the current study, it is of crucial importance to investigate the existence of abnormal patterns on the days of derivatives' expiration.

In this chapter, an extensive study is conducted on expiration day effects on trading volume, volatility and price effects. The UK stocks that are constituents of the FTSE 350 Index

[1]See Ni et al., (2005).
[2]See, for example, FCA v. Goenka or SEC v. Saba.

over a 6-year period from January 2015 to December 2020 are employed. Several contributions are made. Firstly, the study contributes to the expiration day effects branch of the literature by adding further empirical evidence on the existence of these phenomena. Secondly, the study investigates the UK market, one of the main European markets which has not been the subject of extensive research as is the case, for example, for the US market.[3] Lastly, the study uses intraday data to shed light on intraday patterns that are not visible when using data on a daily frequency. Through the use of intraday data, the study can distinguish between two main sources of abnormal activity on days where FTSE index derivatives expire. Furthermore, one of these sources of anomalous behaviour does not have a significant link with the derivatives' market, leading to the conclusion that other factors contribute to the effects observed.

The remainder of the chapter is organised as follows. Section 3.2 reviews the literature, while Section 3.3 discusses the institutional background, the hypotheses tested and then discuss the data available to the study. Section 3.4 describes the effects on trading activity, volatility and price reversals connected to derivatives expiration and formally tests their significance. Section 3.5 proposes an empirical model to attempt to isolate and test the significance of the effects related to the expiration of individual stock options. Finally, Section 3.6 concludes the chapter.

3.2 Literature review

3.2.1 Overview

Since the inception of derivatives' markets, the inter-linkages between them and the respective underlying markets have been the subject of intense study and debate. The main concern is the potential distortions that derivatives' trading could cause on the spot market. In this light, it comes as no surprise that intense empirical work has been produced on the effects that the introduction of derivatives have on the underlying instruments, in terms of trading activity and price effects: see, for example, Conrad, (1989); Bansal et al., (1989); Detemple and Jorion, (1990); R. Kumar et al., (1998); Sorescu, (2000).

A phenomenon that captured significant public attention, almost immediately after the start of trading of the first listed equity derivatives, was the intensive trading activity and the unusually large price movements that could be observed on the spot market on the day of expiration. This drew attention from both regulators and exchanges (and, as a consequence, from academia) and led to the creation of a vibrant branch of academic literature dedicated to investigating the changes to trading volume, volatility and price distortions on the underlying securities caused by the expiration of derivative contracts. These effects are commonly known (and will be referred to in this study) as *expiration day effects*.

[3] To the best of the author's knowledge, Pope and Yadav, (1992) are the only authors to uniquely focus on the UK market. They investigate (single stock options) expiration day effects. Batrinca et al., (2020) study Index derivatives' expiration effects on the main European markets, which include the FTSE 100 Index.

Pioneering papers investigating expiration day effects mostly focus on US markets. Klemkosky, (1978) conducts one of the very first studies on this topic. The author collects a sample of individual equity call options traded on the Chicago Board Options Exchange (CBOE) and investigates the price dynamics of the underlying stocks during both the week leading up to expiration and the following week. The sample includes 14 expiration weeks between 1975 and 1976 and comprises only call options[4]. Negative and significant abnormal returns are found during the expiration weeks, suggesting a downward selling pressure on the underlying stocks. Furthermore, significant positive abnormal returns in the week following expiration are reported, suggesting the existence of a price reversal at correction of the negative performances registered in the expiration week. The author proposes several interpretations for these findings. In the expiration week, options tend to be traded at prices that are very close to their intrinsic value, leading to very low time values (that decrease as maturity approaches). This can lead to behaviour that generates downward market pressure on the price prior to expiration.

Firstly, before expiration week, option holders wanting to close their positions generally find it more beneficial to sell the option in the secondary market to profit from the time value. However, this is no longer necessarily true during the expiration week, during which, as noticed by the author, options often trade at a slight discount with respect to their intrinsic value, making the exercise of the option the optimal decision. Single stock options require physical delivery of the underlying stocks. The stocks acquired by the option buyers upon option exercise can also be sold immediately, potentially leading to a downward pressure on the stock price. Secondly, the abnormal selling pressure could be due to the activity of arbitrageurs. If an option trades at a discount with respect to the intrinsic value and commissions are low enough, an arbitrageur could, in fact, buy the option, exercise it immediately and subsequently sell the stocks acquired at a slight premium, generating, by doing so, selling pressure. Finally, option writers might have an increased incentive to sell closer to expiration. Generally, option writers tend to hedge their positions by acquiring the underlying stock. If the price is above the strike price and option writers are sufficiently confident that it will expire out of the money (OTM) and will not be exercised by the holder, they will be likely to sell their hedge position on the underlying stock, generating downward market pressure.

Cinar and Vu, (1987) examine the effects of the expiration of both individual stock options and index options on trading volume, volatility and price returns of six US blue-chip stocks, over a six-and-a-half-year period spanning from January 1979-June 1985. The results are mixed. While index options' expiration does not appear to affect the mean returns or the volatility of the analysed stocks, the expiry of individual options seems to be connected to significantly negative returns for some (but not all) the stocks considered, partially supporting the results obtained by Klemkosky, (1978). Furthermore, if both index and individual stock options' expiration is connected to an increase in the volatility of the volume traded on the expiration day, neither is characterised by a significant increase in the average levels

[4] Put options are introduced for trading on CBOE in 1977 for the first time.

of trading volume.

Pope and Yadav, (1992) conduct the first international investigation on expiration day effects by studying the effects on trading volume, price returns and volatility in relation to the expiration of UK single stock options on the underlying stocks. A five-year time period, from October 1982-September 1987, is analysed. The authors find clear evidence of abnormally high traded volume during the week leading up to expiration, which is then followed by a drop in the following week. This unusual spike in traded volume is associated with significant negative abnormal returns during the week leading up to expiration. However, no evidence of increased volatility is found. Several possible explanations for the findings are advanced. While most are in line with the interpretations proposed by Klemkosky, (1978), this is the first paper to suggest the idea that abnormal returns could be due to market manipulators. More precisely, it suggests that the abnormal returns could be due to option writers attempting to manipulate the underlying stock price to prevent the exercise by option holders, thus avoiding any liability at expiration. Although it is a possible explanation, the authors also state that this type of manipulation is difficult to conduct in a well-regulated market like the UK market.

The first systematic study on the expiration effects of index derivatives is conducted by Stoll and Whaley, (1987). The authors study the effects connected to the expiration of Standard & Poor (S&P) 500 Index futures and options (which are cash-settled) on the underlying stocks. Dramatic effects are documented when both futures and options expire. More precisely, a significant increase of traded volume with respect to non-expiration days is reported. Furthermore, the increase in trading volume is mostly concentrated at the end of the day when the derivatives' settlement price is determined. The increase of trading volume in the last hour of trading is 50% more than normal. Volatility is also higher when futures and options expire together, with a sharp increase in volatility in the last hour of trading, similar to the patterns observed for trading volume. Finally, significantly negative returns are observed on the underlying stocks, together with the tendency for the price to reverse by the following morning. Although similar effects for trading volume and volatility are reported for days when only index options expire, they appear to be much milder than those documented when futures also expire. No evidence of price dislocations is found in association with options-only expiration days.

Stoll and Whaley, (1991) study the effects from the change in how the settlement price is determined for S&P 500 Index futures and options, from using the closing price on expiration day to adopting the opening price on the same day. The results previously found by these authors are confirmed, considering that basically the same results are found for trading volume, volatility and reversals when simply shifted from the close to the opening of the trading session. The authors explain these findings as being caused by arbitrageurs' activity. They notice that, in the case of cash-settled derivatives, a tight relationship exists between the derivatives' price and the price of the underlying asset. When this relationship is not satisfied, and transaction costs are low enough, arbitrage opportunities arise. If this is the case, an arbitrage opportunity can be exploited by simply buying (or selling, depending on the specific case) the underlying asset and taking an offsetting position in the derivative. If the underlying asset is an equity index, such as the case empirically studied by these authors, the single stocks that constitute the index have to be acquired (or sold), each proportionally to their index weight. At expiration, while derivatives self-liquidate through cash-settlement, the positions in the stocks constituting the index (that represent the other half of the arbitrage strategy) need to be liquidated for the arbitrage to be successfully closed. If the derivative settles at the closing price (as studied by Stoll and Whaley, (1987)), then the positions of the stocks must be liquidated at the closing price; if the derivative settles at the opening price, as analysed by Stoll and Whaley, (1991), then the positions of the stocks should be liquidated at the opening price. As noted by these authors, for the arbitrage to be successful, the price level at which the derivative settles is not a concern, as long as the positions in the stocks are liquidated at a price equal to (or, at second best, as close as possible to) the settlement price. The activity by arbitrageurs trying to liquidate the positions on the underlying stocks at the same price as the settlement price is the main explanation of the empirical findings.

Following the seminal papers discussed above, a series of further studies have been conducted with the purpose of providing additional evidence on the existence of expiration day effects, and of extending the results to markets other than the US market. Although the evidence contained in the literature is mixed, significant studies in the literature are advancing evidence that derivatives' expiration tends to be linked to certain effects on the spot market. This literature is presented separately for effects on trading volume, volatility and, finally, for effects leading to price dislocations.

3.2.2 Trading activity effects

Although no full consensus exists, the academic literature tends to agree on the fact that the expiry of derivatives' contracts tends to lead to an increase in trading volume. Chen and Williams, (1994) provide further evidence on the existence of expiration day effects on trading volume on US markets. Batrinca et al., (2020) conduct an extensive study on European markets, focusing on the volume effect during the expiration day of futures on the main European market indices.[5] They find a statistically significant increase in trading volume on expiry day. Similar effects are documented in a variety of markets. Corredor et al., (2001), and later Illueca and LaFuente, (2006), find signs of increased volume traded on the expiration of IBEX 35 Index futures on the underlying stocks in the Spanish market. Similar findings are reported by Vipul, (2005), Narang and Vij, (2013), and Debasish, (2010) for the Indian market and by Alkebäch and Hagelin, (2004) for the Swedish market. By using intraday data, Gurgul and Suliga, (2020) present evidence that cash-settled single stock futures' expiration leads to an increase in the trading volume in the Warsaw Stock Exchange, and that the increase is mostly concentrated at the end of the trading session, around the time

[5]Including the expiration of the UK FTSE 100 Index futures.

the settlement price is determined, which is in line with previous literature. Schlag, (1996) investigates the effects associated with the expiration of DAX 30 Index derivatives, while W. G. Hsieh, (2009) and E. H. Y. Chow et al., (2013) explore the effects associated with the expiration of Taiwan Futures Exchange (TAIFEX) Index futures; they both find evidence of higher volume at expiry and an increased concentration of liquidity around the time the settlement price is determined. Although the evidence appears overwhelming, a complete consensus has not been reached. Chamberlain et al., (1989) explore the expiration of TSE 300 Index futures and options on the Toronto Stock Exchange (TSE) and find no evidence of abnormal trading activity at expiry. Bollen and Whaley, (1999) and, later, Y. F. Chow et al., (2003), who employ more granular intraday data, investigate Hang Seng Index (HIS) derivatives' expiration effects in Hong Kong during the period from 1986-1998 and from 1990-1999, respectively. Neither of these studies find evidence of abnormally higher trading volume on expiration days.

3.2.3 Volatility effects

The evidence regarding expiration day effects on the underlying's price volatility is much more inconclusive than what is found on trading activity effects, as discussed above. Narang and Vij, (2013) find a significant volatility increase connected to CNX Nifty Index derivatives' expiration on the Indian market. Corredor et al., (2001) find only marginal changes in volatility associated with the IBEX 35 Index futures' expiration, which are visible only when investigating individual stocks' volatility. IBEX 35 Index volatility dynamics, on the other hand, do not exhibit any changes on expiration days. Changes in volatility related to the expiration of index futures are found by E. H. Y. Chow et al., (2013) on the Taiwan market. These changes, however, appear to be minimal. All these studies, in fact, rely on inter-day data that might not be adequate to fully capture changes in volatility, if these changes are concentrated around the time settlement prices are determined and if they are not of such a magnitude to be noticeable using daily data.

Several authors employ intraday data and find clearer results. Illueca and LaFuente, (2006), for example, investigate realised variance on IBEX 35 Index derivatives' expiration days and find a significant increase in volatility towards the close. Their finding differs from that of Corredor et al., (2001). Agarwalla and Pandey, (2013) investigate the Indian market on single stock futures' expiration days and find an increase in the intraday volatility towards the end of the trading day, close to when the settlement price is determined. Similar patterns, indicating an increase in volatility around settlement, are also found, among others, by Chamberlain et al., (1989), Schlag, (1996) and Gurgul and Suliga, (2020).

Overall, however, several studies report no expiration day effects on volatility, with these including: Bollen and Whaley, (1999) on the Hong Kong market; Vipul, (2005) and Debasish, (2010) on the Indian market; Chen and Williams, (1994) on US markets; and Alkebäch and Hagelin, (2004) on the Swedish market. The fact that these studies all rely on daily data might cast some doubt that the results could be driven by data limitations, instead of by a genuine absence of any effects on volatility.

3.2.4 Price distortions

The study next discusses the evidence in support of the hypothesis that derivatives' expiration can lead to price distortions of the underlying securities. Typically, interest lies in investigating the existence of two types of distortions: (1) abnormal price movements on the day of expiration (or, more generally, in the days preceding expiration) and (2) price reversals after expiration, supporting the idea that abnormal returns before expiry lead the price away from the actual value (the so called *fair value*), which then, consequently, reverses when the trading pressure ends. The evidence reported in the literature is mixed. Stivers and Sun, (2013) analyse the price dynamics of the S&P 100 index (in addition to a sample of optionable stocks that are part of the index) between January 1988 and December 2010. They find that returns throughout the expiration week are abnormally higher than the returns of the control portfolios, that comprise stocks with low options activity in comparison to S&P 100 stocks. A price reversal is also observed during the week following expiration. Chiang, (2014) investigates US optionable stocks and provides evidence that underlying stocks of deeply in the money (ITM) options with large open interest experience significant and negative price returns during the week leading to expiration. The author interprets these findings as being due to the selling pressure generated by call option holders exercising their options and selling the stock immediately after delivery. This effect is not observed on non-optionable stocks. It is also not due to index futures' expirations, as the results do not change when the futures' expiry days are removed from the estimation of the model. Additionally, this behaviour appears to be mainly due to call options holders, as put options tend to present, in the studied sample, smaller open interest compared to call options.

The effects of index derivatives' expiration on prices are investigated by several studies. Vipul, (2005) finds that, in the Indian market, during the time period between November 2001 and May 2004, prices of the stocks that are part of the National Stock Exchange (NSE) Nifty Index tend to become depressed the day before expiration (of both index and single stock derivatives) and, subsequently, the day after expiration, tend to present returns significantly higher than normal. On the other hand, Narang and Vij, (2013) analyse expiration effects on the CNX Nifty Index between June 2000 and January 2012 and find evidence of abnormally high returns on both the expiration day and on the following day , suggesting an upward pressure on the price that continues after expiry. Y. F. Chow et al., (2003) find weak negative price effects on the day that HSI derivatives expire in the Hong Kong market but find no evidence of any subsequent price reversal. Corredor et al., (2001) investigate price effects of IBEX 35 derivatives' expiration on both the price dynamics of the index itself and on a sample of four stocks that are part of the index. No price effects are found on the index returns on the expiration day. When the individual stocks are analysed, however, the authors find a negative price return during the week leading to expiration but find no trace of any price effects on either the expiration day or in the days following expiration.

Chamberlain et al., (1989) investigate the Canadian market, finding that stocks returns are significantly higher in the last 30 minutes of trading on days on which TSE 300 Index derivatives expire than on other Fridays. These abnormally high returns on expiration days tend to reverse within the first 30 minutes trading on the next trading day. Schlag, (1996), using a control sample, finds abnormal price reversals on DAX 30 futures expiration days compared to other Fridays, but the same pattern is not observed when only index options expire. W. G. Hsieh, (2009), investigating the market in Taiwan, finds weak signs of price reversals for the TAIFEX Index on index derivatives' expiration days. This evidence appears to be stronger when the reversals of the individual stocks in the Index are analysed, highlighting significantly larger reversals along with a higher frequency on expiration days than on non-expiration days.

As with volatility effects (and, to a lesser extent, trading volume effects), several studies present evidence showing that price distortions are not present on expiration days in the investigated market: see, for example, Bollen and Whaley, (1999); Chen and Williams, (1994); Alkebäch and Hagelin, (2004) and Debasish, (2010).

Finally, it is worth mentioning the line of research studying the phenomenon of price clustering. Price clustering refers to the non-uniform distribution of security price digits. In other words, it is the tendency of certain price levels to be observed more frequently than others. The academic literature has widely documented the tendency of prices to cluster around round numbers. Traders could be motivated to submit orders at round numbers for various reasons. The literature advances the idea that the uncertainty surrounding market prices is too high for correct pricing, making the pricing process less important and inducing traders to submit orders at round-number prices as close as possible to what they believe is the true value. Other authors argue that clustering around round numbers could be due to costs associated with accurate pricing that would be too high, inducing traders to select rounded prices to reduce the negotiation time. Evidence in support of the existence of this phenomenon is found in various equity markets. Niederhoffer, (1965), Osborne, (1965) and L. Harris, (1991) investigate the US market. Similar results are found in international markets, for example: the Australian stock market (M. Aitken, Brown, et al., (1996)), the Singapore market (Hameed and Terry, (1998)) and the Japanese market (Ohta, (2006)). Furthermore, the prices of derivative products also appear to be affected by clustering, both for index derivatives (see, e.g., Schwartz et al., (2004) for an investigation on S&P 500 futures prices; Ap Gwilym, Clare, et al., (1998) and Ap Gwilym and Alibo, (2003) for studies on the behaviour of index futures and options on the UK market) and for individual derivatives (see Ap Gwilym and Verousis, (2013) for a study on price clustering of individual stock options on the London International Financial Futures and Options Exchange [LIFFE]).

Within this branch of the literature, a small set of studies is dedicated to investigating price clustering during options expiration days. Ni et al., (2005) investigate the US equity market and provide strong evidence that, on option expiration days, closing prices of optionable stocks tend to be close to a strike price with significantly higher frequency than on non-expiration days. The tendency of stock prices to cluster around option strike prices at expiration is usually referred to as *stock pinning*. Four different explanations of stock pinning are explored, but the evidence found only supports two. The first source of clustering found by these authors is related to trading pressure generated by delta-hedging option traders holding net option positions. As expiration gets closer, the attempt by these traders to maintain (by appropriately trading the underlying asset) a delta-hedged position on the option would drive the underlying price towards the option strike price, which acts as a magnet for the price. Avellaneda and Lipkin, (2003) subsequently modelled this behaviour, proposing a theoretical framework to explain this attraction of underlying prices to strike prices. The other explanation of stock pinning, for which these authors find evidence, is market manipulation. More precisely, they find an increased likelihood of pinning and a higher probability for an option to expire out of the money (OTM) (conditionally on being in the money [ITM] the day before) when proprietary traders write options in the week leading up to expiration.

Figueiredo et al., (2017) examine order submission behaviour on a sample of optionable stocks traded on Nasdaq between 2014 and 2015. They find that, on expiration days, limit orders have a higher likelihood of being submitted with a price closer to an option strike price. The authors also find a higher frequency of fleeting orders placed outside the National Best Bid and Offer (NBBO) on expiration days for optionable stocks. These orders tend to be buy orders and the authors interpret these findings to mean that options traders are attempting to move the stock midpoint closer to (or past) the closest strike price. Some signs of stock pinning are also found on the derivatives' market. Golez and Jackwerth, (2012) present evidence of pinning (and anti-pinning, meaning that strike prices tend to repel the underlying prices, instead of functioning as a magnet) of S&P 500 futures around the strike prices of the corresponding options. In summary, this branch of the literature is still quite new.

3.3 Institutional background, hypotheses development and data

3.3.1 Institutional background

In this study, the effects of exchange-traded derivatives on UK stocks are investigated. The derivatives of interest are: (1) FTSE Index futures, (2) FTSE Index options and (3) UK single stock options. The contract specifications of these products are obtained from the Intercontinental Exchange (ICE) website[6] and are presented below.

[6]See the ICE web page.

Index futures contracts are offered on both the FTSE 100 Index and FTSE 250 Index.[7] Index futures are European-style contracts that are cash-settled upon expiration and expire on the third Friday of the expiration month, unless it is a holiday, in which case the previous working day is used as expiration day. Expiration months correspond to the quarter-end months (March, June, September or December) and, therefore, follow a quarterly expiration cycle. The settlement price upon expiration is the price determined as the outcome of the Exchange Delivery Settlement Price (EDSP) auction that takes place on the London Stock Exchange (LSE) during the morning session on the expiration day. To be more precise, on the relevant Fridays, at 10:10 a.m., continuous trading is suspended and a call auction starts. During this time, limit and market orders can be submitted, cancelled and modified, but no trade takes place. At 10:15 a.m., a random period of 30 seconds starts for every FTSE 100 and FTSE 250 stock, during which, at a random point in time, and separately for each stock, the order book is frozen and all the orders that have been submitted during the call auction (and are still active) are gathered and a matching algorithm is applied to determine the index futures settlement price (i.e., the EDSP).[8]

Index options contracts, as with the futures described above, are also offered on both the FTSE 100 Index and the FTSE 250 Index. For either index, both call and put options are available. Index options are European style and cash-settled at expiration. FTSE 250 Index options are offered with a quarterly expiration frequency, as also applies for futures on the respective market index. FTSE 100 Index options, however, are also available on non-quarterly expiration months. The third Friday of the expiration month corresponds to the expiration day, unless it is not a trading day, in which case the previous working day is used. The price determined through the EDSP auction, which is used as the settlement price for index futures, is also used to settle index options at expiration.

Finally, single stock options are also available. Standard equity options require physical delivery of the underlying stock upon expiration (or at the time of their exercise) and they can be exercised at any time prior to expiration. They are, in other words, American-style options. They expire on the third Friday of the expiration month, which occurs on a monthly basis. On the day of expiration, options trading is possible until 4.30 p.m. and exercise is allowed until 6.30 p.m. (local time).

For FTSE 100 stocks, the index futures expiration days overlap with single stock options expiration days on quarterly expiration days, which are usually known as triple-witching days, while index options and single stock options expiration days can match on months that are not quarter-end months, but only for FTSE 100 stocks.

[7] The FTSE 100 Index is a well-known market-capitalisation weighted index constituted by the 100 companies with the highest market capitalisation listed on the London Stock Exchange (LSE). Similarly, the FTSE 250 Index comprises the 250 companies listed on the LSE that, excluding the constituents of the FTSE 100, have the highest market capitalisation. See `https://www.ftserussell.com/products/indices/uk` for further information.

[8] See London Stock Exchange (LSE) trading guide for more detailed information on the functioning of the auction and the possible monitoring extensions that can be applied to it.

3.3.2 Hypotheses

As discussed in the previous sections, several factors contribute to generating abnormal trading patterns and price movements on expiration days. Position unwinding by arbitrageurs is commonly considered one of the main contributing factors for cash-settled derivatives behaviour. For an arbitrage to be successful, the unwinding of the underlying position should take place around the time the settlement price at expiration is determined to ensure as little discrepancy as possible between the derivatives payoff and the hedging portfolio. Activity by hedgers can also be an explanation. Similar to actions by arbitrageurs, hedging can require more frequent and intense trading as expiration approaches. This study expects, therefore, to observe a higher than normal volume traded throughout the morning session, especially around the EDSP auction (from 10:10 to 10:15) on quarterly expiration days, when index futures and options expire, and on non-quarterly expiration days for FTSE 100 stocks, when index options expire.

Hedging activity by equity options market makers trying to maintain a delta neutral position can lead to abnormal levels of traded volume on the underlying stocks at expiration. As expiration gets closer, the hedging activities become more intense. Furthermore, while index derivatives are extremely difficult to manipulate, individual options are more prone to manipulation. Manipulators may attempt to affect the price of the underlying asset, generating abnormal volume, to augment the profit they obtain from the options position. The decision to exercise the option or not exercise the option on expiration day will ultimately depend on the closing price, based on which an option will be considered as in the money (ITM) (i.e., valuable) or not. This study therefore expects, if manipulation occurs in the market, that the effort made by manipulators will be mostly concentrated towards the end of the trading day, and especially during the closing auction, at the end of which the closing price of a stock is determined.

Based on these considerations, and in accordance with previous literature, this study formulates the following hypotheses:

Hypothesis 3.1 *Both FTSE 100 and FTSE 250 stocks exhibit abnormal volume on index derivatives expiration days. The abnormal trading activity is also concentrated around the intraday call auction in the morning session of the expiration day.*

Hypothesis 3.2 *Stocks that are optionable exhibit abnormal volume on expiration days. The abnormal volume traded is mostly concentrated towards the end of the trading day.*

If liquidity in the market is insufficient to absorb the abnormal volume generated by arbitrageurs and market makers, it is expected that price distortions will be observed on expiration days, associated with the increase in trading activity. Furthermore, in the case of manipulation, affecting the closing price is the main goal of the manipulator. The following hypotheses are therefore formulated:

Hypothesis 3.3 *Both FTSE 100 and FTSE 250 stocks exhibit abnormal levels of volatility on index derivatives expiration days. The abnormal volatility is also concentrated around the intraday call auction in the morning session of the trading day.*

Hypothesis 3.4 *Optionable stocks are characterised by abnormal volatility on expiration days. Furthermore, volatility increases towards the end of the trading day.*

Finally, this study expects any price dislocations to be short-lived and due to the temporary trading pressure generated at expiration. The following hypotheses are therefore proposed:

Hypothesis 3.5 *Both FTSE 100 and FTSE 250 stocks prices exhibit abnormal price movements before the EDSP auction on index derivatives expiration days. This effect is subsequently reversed.*

Hypothesis 3.6 *Stocks that are optionable exhibit abnormal price movements towards the end of the trading session, which are reversed by the following trading morning.*

3.3.3 Data

Information on the historical constituents of both FTSE 100 and FTSE 250 indices is sourced from Refinitiv Tick History, with the information updated quarterly after each FTSE rebalancing day (usually taking place on the Monday after the third Friday of each quarter-end month). From Refinitiv Tick History, this study also obtains intraday volume and quotes data. Volume and quotes data were downloaded at a five-minute frequency for each constituent stock of either FTSE 100 or FTSE 250, between January 2015 and December 2020. Considering that UK equity markets are highly fragmented, this study downloads consolidated volume and prices offered by Refinitiv Tick History, obtained by consolidating the four main exchanges: LSE, Chi-X, Bats Europe and Turquoise.

Closing and opening prices are obtained from Refinitiv Elektron. Information on capital changes (such as dividends and stock splits or consolidations) is also downloaded for each stock and the relative adjustment factor, which is applied to both the closing and opening prices. Market capitalisation data are sourced from Refinitiv Eikon and updated at the beginning of each month for every stock in the sample.

A complete list of currently optionable UK stocks is available on the website of Intercontinental Exchange (ICE). Market notices published by the ICE exchange[9] are hand collected by the current study with the purpose of obtaining information on stocks that are not on the current list of optionable stocks, but that have previously had options traded. By reading these market notices, information is obtained on previously delisted options, new contracts added, stock mergers and spin-offs, etc.

It should be noted that Poundland plc (PLND.L) has been removed from the sample owing to the lack of intraday data available on Refinitiv Tick History. Similarly, Man Group plc

[9] Available on `https://www.theice.com/futures-europe/corporate-actions`

(EMG.L) has been removed owing to the absence of data on market capitalisation. Missing observations are also due to consolidated data on price and volume not being available for specific combinations of stocks and days. Observations are removed that fall on shortened trading days such as December 24 and December 31. All observations are also removed for March 2020, due to extreme effects observed on the market related to the COVID-19 pandemic. The expiration day corresponding to 16 August 2019 is also removed from the sample due to an outage at the London Stock Exchange (LSE) that affected both FTSE 100 and FTSE 250 stocks and caused a significant delay in the market opening.[10].

The sample period comprises 70 expiration days, 23 of which fall on a quarter-end month, for a total of 24,173 combinations of stocks and expiration days. Of these, 7042 are FTSE 100 stocks (4732 observations falling on non-quarterly expiration days and the remaining 2310 falling on quarterly expiration days) and the other 17,131 observations are on FTSE 250 stocks (11,518 observations on non-quarterly expiration days and the remaining 5613 on quarterly expiry days). For control groups, as explained in the following section, the study takes all non-expiration days in the sample that are Fridays (to control for any day-of-week effects) and not month-end days (to control for widely known abnormal patterns taking place on the last day of the month).

3.4 Market-wide expiration effects

3.4.1 Effects on trading activity

The study starts by investigating the existence of abnormal trading activity on derivatives' expiration days. Trading activity is measured by computing a turnover ratio. Given the combination of stock i and day t, the turnover ratio for a specific index and given day t is defined as:

$$TR_t^I = \frac{1}{N}\sum_{i=1}^{N}\frac{V_{i,t}^I}{MC_{i,t}}, \tag{3.1}$$

where $MC_{i,t}$ is the stock's market capitalisation (in £) on day t, $V_{i,t}^I$ is the volume (measured by turnover) traded on stock i during the interval I on day t and N is the number of stocks belonging to the considered index on day t. Turnover is defined as the total number of shares traded multiplied by the price at which each share is traded. Volume by turnover is normalised by taking the ratio with the market capitalisation to make the variable comparable across different stocks.

This study also proposes to use another variable, following W. G. Hsieh, (2009) and named herein *concentration ratio*, defined as:

$$CR_t^I = \frac{1}{N}\sum_{i=1}^{N}\frac{V_{i,t}^I}{V_{i,t}}, \tag{3.2}$$

[10]For more details, see the article `https://www.bloomberg.com/news/articles/2019-08-16/london-stock-exchange-investigates-technical-trading-issue`.

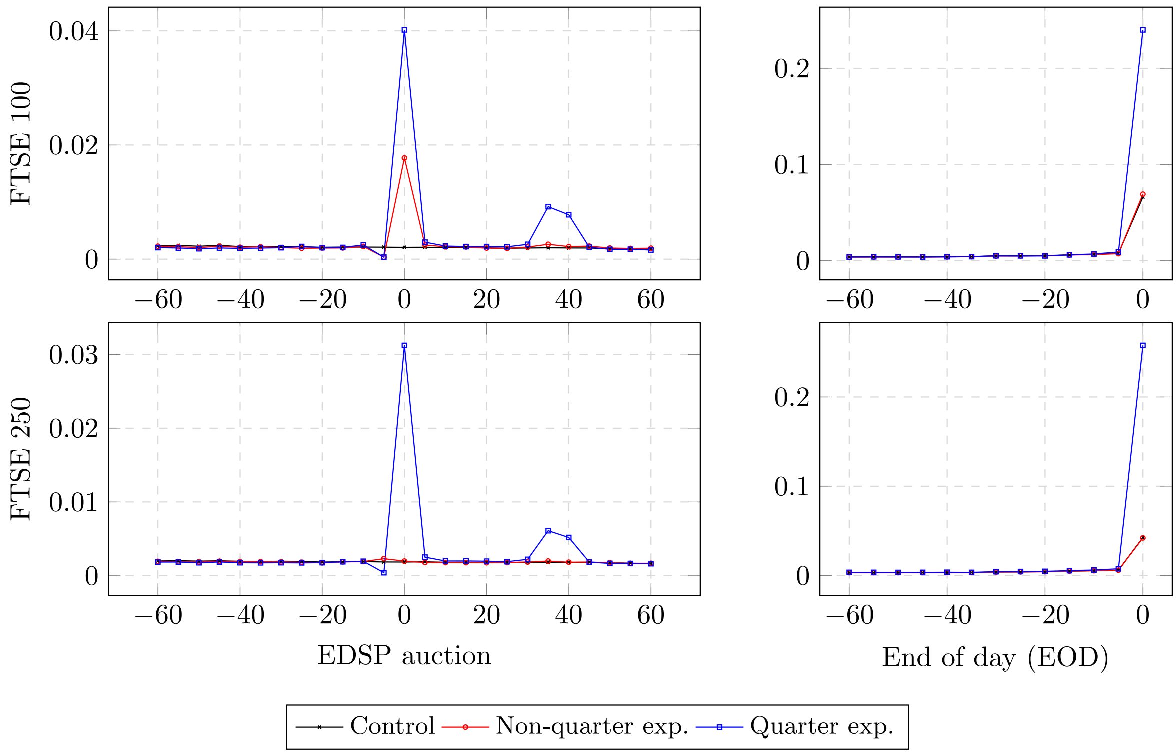


FIGURE 3.1: Trading activity intraday patterns.

The figure depicts the mean percentage volume traded (measured as turnover ratio (3.1)) computed for each five-minute interval, both around the EDSP auction (which takes place in the interval 10:10-10:15 a.m. and is identified by 0 in the graphs on the left) and at the end of the trading session (the 0 in the graphs on the right identifies the last interval of the trading day, which is the closing auction, taking place 4.30-4.35 p.m.). Turnover ratio is first computed on every interval for each combination of stock and day in the sample. It is then averaged across stocks (separately for FTSE 100 and FTSE 250 Index) by every interval and within a specific day, and finally averaged across days, separately for control days (non-expiration Fridays), quarterly and non-quarterly expiration days. Values are displayed in percentage points.

This variable measures the volume (based on turnover) traded during interval I ($V_{i,t}^{I}$) as a percentage of the total volume traded during the trading day ($V_{i,t}$). The average across all stocks of a specific index on day t is taken.

Figure 3.1 shows the intraday dynamic of the cross-day average of TR_t^I computed for every five-minute intervals I of the trading day, separately for the FTSE 100 and FTSE 250 Indices[11]. Two effects are particularly noticeable. Firstly, an abnormal level of trading activity is observed during the morning session on index derivatives' expiration days. A first spike in trading activity takes place at the end of the EDSP call auction (identified as 0 in the graphs). A second (and less intense) anomaly in traded volume is observed around 30-45 minutes after the EDSP auction. These patterns are quite evident on quarterly expiration days for both FTSE 100 and FTSE 250 stocks. The same abnormal pattern (although to a lesser extent) can be observed on FTSE 100 stocks on non-quarterly expiration days, while no significant anomaly is observed for FTSE 250 stocks on these same days, as no index

[11]Morning and afternoon trading sessions are displayed in two separate graphs (with different scales on the y axis) to improve understandability.

derivatives expire. The second visible pattern that can be observed is a spike in trading activity at the end of the day on quarterly expiration days, for both the FTSE 100 and FTSE 250 stocks. This seems to be due to an abnormal amount of trading volume traded during the closing auction. Besides these two abnormal patterns, trading activity on both quarterly and non-quarterly expiration days appears to follow similar patterns to those on control days.

Table 3.1 presents the formal testing of the significance of the effects observed in the graphs shown in Figure 3.1. Turnover ratio (Equation [3.1]) is computed for different choices of interval I. The turnover ratio is first computed based on the volume (by turnover) traded during the whole trading day ($V^{I}_{i,t} = V_{i,t}$). Then, the metric is computed by using volume traded between 9:30 a.m. and 11 a.m., corresponding to a symmetric window of 45 minutes before and after the index derivatives settlement time at 10:15 a.m. Finally, to explore the spike at the end of the trading day observed on quarterly expiration days, two intervals are defined: (1) the interval from 4:30 p.m. to market close, which captures volume traded during the closing call auction, and (2) from 4:00 p.m. to 4:30 p.m., which corresponds to the last half-hour of continuous trading, aimed at testing whether the rise in volume at the end of the day is specific to the closing auction or whether it commences prior. Concentration ratio (Equation [3.2]) is also computed for the three intervals I: 9:30-11 a.m., 4:00-4:30 p.m. and 4:30 p.m.-Close.

Both non-quarterly and quarterly expiration days are compared to a control group constituted by all non-expiration Fridays contained in the data set to avoid any bias due to day-of-the-week effects. Days in the control group also cannot be month-end days, as it has been widely shown that month-end days are characterised by more intense trading activity and price changes: see Carhart et al., (2002); Ben-David et al., (2013).

The current study conducts two types of test. Firstly, the non-parametric Wilcoxon signed-rank test is used to formally test whether trading volume is significantly higher on expiration days (separately for quarterly and non-quarterly expiration days) with respect to non-expiration days. The Wilcoxon signed-rank test is a paired difference test, requiring a set of matched samples. Therefore, the measure of trading activity on a specific expiration day is compared to the (averaged) measure on the expiration Fridays of the same month. More precisely, the first and second Fridays of the month are used, following Stoll and Whaley, (1991) and Gurgul and Suliga, (2020). In this way, the study controls for structural changes that might affect these metrics through time. Secondly, the non-parametric Mann-Whitney U test is used, with trading activity measures on expiration days compared to the same measures on all non-expiration Fridays contained in the sample.

The results of the tests presented in Table 3.1 strongly confirm the intuitions provided by the graphs in Figure 3.1. A significant increase is observed of daily traded volume on quarterly expiration, on both FTSE 100 and FTSE 250 stocks. This increase can be fully explained by the abnormal activities in two periods of the trading day: the first, during the morning session around the EDSP auction (9:30-11 a.m.) and the second during the closing call auction (4.30 p.m.-Close). The abnormal volume traded in the morning is interpreted as

TABLE 3.1: Trading activity effects.

	Non-quarter exp. ($N = 47$)			Quarter exp. ($N = 23$)		
	Mean	Diff. I	Diff. II	Mean	Diff. I	Diff. II
Panel A: FTSE 100 Index stocks						
Turnover ratio						
All day	0.350	0.007	0.004	0.573	0.213***	0.227***
9:30-11:00 a.m.	0.052	0.015***	0.014***	0.088	0.051***	0.051***
4:00-4:30 p.m.	0.034	−0.001	−0.001	0.037	−0.000	0.002
4:30 p.m.-Close	0.071	0.004***	0.003*	0.257	0.183***	0.188***
Concentration ratio						
9:30-11:00 a.m.	15.205	4.794***	4.789***	17.383	7.245***	6.967***
4:00-4:30 p.m.	9.669	−0.576***	−0.629***	6.671	−3.710***	−3.627***
4:30 p.m.-Close	21.564	0.016	−0.059	41.412	19.628***	19.788***
Panel B: FTSE 250 Index stocks						
Turnover ratio						
All day	0.212	−0.003	−0.007	0.478	0.247***	0.259***
9:30-11:00 a.m.	0.023	0.000	−0.000	0.060	0.036***	0.037***
4:00-4:30 p.m.	0.026	−0.000	−0.000	0.031	0.003***	0.004***
4:30 p.m.-Close	0.042	0.001	−0.000	0.258	0.213***	0.215***
Concentration ratio						
9:30-11:00 a.m.	9.682	0.044	0.112	18.719	9.359***	9.148***
4:00-4:30 p.m.	12.452	−0.199	−0.194	7.021	−5.797***	−5.625***
4:30 p.m.-Close	23.846	0.834	0.465	45.929	22.521***	22.547***

Turnover ratio (Equation [3.1]) and concentration ratio (Equation [3.2]) are used as measures of trading activity of both FTSE 100 and FTSE 250 component stocks for a specific day and for various intervals within that day. The average values are reported separately for non-quarterly and quarterly expiration days, and are computed first as a cross-stock average (separately for each market index) and then averaged across days. Column Diff. I reports the difference with respect to the average value on the first and second Fridays of the expiration month and the statistical difference is tested through the Wilcoxon signed-rank test. Column Diff. II reports the difference with respect to the average value measured on all control days that are part of the sample. The existence of a statistically significant difference is tested by the Mann-Whitney U test. The notations *, ** and *** indicate that the difference in means is statistically significant at 10%, 5% and 1% levels. Values are reported in percentage points.

being connected to index futures and options expiration. This interpretation is confirmed by the existence of abnormal trading activity in the morning session of non-quarterly expiration days for FTSE 100 stocks, but not for FTSE 250 stocks. The obvious justification for this finding is that FTSE 100 Index options have a monthly expiration cycle, while FTSE 250 Index options expire only on quarterly expiration days.

The spike in volume traded during the closing auction on quarterly expiration days is very noticeable. For FTSE 100 stocks, this increase is more than three times (from 0.74% on control days to 0.257% on quarterly expiration days) the usual volume traded (measured as turnover ratio) during the closing auction. For FTSE 250 stocks the increase is even more significant, accounting for almost six times the normal volume traded in that interval (from

0.45% to 0.258%). Furthermore, if the closing auction accounts for approximately 21.5% of the daily trading volume for FTSE 100 stocks and almost 24% for FTSE 250, these figures rise to more than 41% for FTSE 100 stocks and almost 46% for FTSE 250 stocks, on quarterly expiration days. In other words, for the stocks in the sample, more than 40% of the volume traded on the third Friday of quarter-end months is traded in the closing auction (which has a duration of 5 minutes). For FTSE 250 stocks, the increase in trading activity appears to also start during the last half-hour of continuous trading before the closing auction. However, this increase is nearly as intense as the increase observed during the closing auction. Furthermore, this abnormal pattern is observed only on quarterly expiration days, except for a marginal increase in trading volume for FTSE 100 stocks on non-quarterly expiration days.

The drivers of the abnormal patterns observed at the end of the trading day are not obvious. At first, they could be seen as related to individual options expiration. However, this interpretation has several problems. First, no trace is found of similar patterns on non-quarterly expiration days, despite single stock options having a monthly expiration frequency. Furthermore, this is a phenomenon that attracts liquidity of a much higher magnitude than the EDSP auction for index derivatives. It is therefore strange to think that the main factor could be changes in trading activity related to the expiration of individual options, especially considering that not all UK stocks are optionable (and only a small minority of FTSE 250 stocks are). A more likely explanation is a change in trading activity related to the quarterly recalibration of market indices. The constituents of the FTSE 100 and FTSE 250 indices, for example, are reviewed on a quarterly basis, during the months of March, June, September and December. The changes due to this periodic review are usually effective starting from the Monday following the third Friday of the month (i.e., a quarterly expiration day). A rise in trading activity from funds (especially passive investing funds, such as exchange-traded funds [ETFs]) can therefore be expected on the third Friday of these months, aimed at recalibrating portfolios in advance of adjustments to the market indices. This would also explain why such an intense increase in trading volume is observed during the closing auction. Exchange-traded funds (ETFs)'s aim is to be traded at the close to minimise their tracking error, which is generally computed based on closing prices.

To shed more light on this, the FTSE 100 stocks are divided into four quartiles based on market capitalisation and the FTSE 250 stocks into 10 deciles, also based on market capitalisation, for a total of 14 groups of roughly 25 stocks each. The same statistical tests conducted above are replicated on trading activity measures computed for each of the groups. Only the significance of trading activity around the EDSP auction and on the closing auction are tested, as these two moments of the trading day are characterised by abnormal patterns. Results are reported in Tables 3.2 and 3.3 for FTSE 100 and FTSE 250 stocks, respectively.[12] As shown in the results, both the spikes in trading volume around the EDSP auction and during the closing auction on quarterly expiration days are a market-wide phenomenon affecting all the stocks under investigation. The patterns of traded volume at the close on quarterly

[12] In the case of FTSE 250 stocks, the tests are presented only for quarterly expiration days, as no significant pattern is found on non-quarterly expiration days for these stocks, as seen in 3.1.

TABLE 3.2: Trading activity effects on FTSE 100 stocks - Analysis by market size.

	Non-quarter exp. ($N = 47$)			Quarter exp. ($N = 23$)		
	Mean	Diff. I	Diff. II	Mean	Diff. I	Diff. II
Panel A: Turnover ratio (9:30-11:00 a.m.)						
4	0.037	0.013***	0.012***	0.069	0.045***	0.045***
3	0.050	0.014***	0.014***	0.086	0.049***	0.050***
2	0.056	0.016***	0.016***	0.094	0.054***	0.054***
1	0.065	0.015***	0.016***	0.105	0.056***	0.056***
Panel B: Turnover ratio (4:30 p.m.-Close)						
4	0.046	0.004***	0.001**	0.163	0.114***	0.119***
3	0.068	0.005***	0.001*	0.213	0.136***	0.146***
2	0.078	0.005**	0.003	0.245	0.167***	0.170***
1	0.092	0.004*	0.003*	0.412	0.318***	0.323***
Panel C: Concentration ratio (9:30-11:00 a.m.)						
4	15.674	5.428***	5.365***	18.909	8.946***	8.600***
3	15.257	4.870***	4.828***	17.675	7.496***	7.246***
2	15.116	4.772***	4.779***	17.500	7.400***	7.164***
1	14.786	4.127***	4.198***	15.354	5.035***	4.766***
Panel D: Concentration ratio (4:30 p.m.-Close)						
4	19.994	0.315	0.134	39.033	18.765***	19.174***
3	22.013	0.184	0.114	40.662	18.657***	18.763***
2	21.936	−0.465	−0.452	41.088	18.586***	18.700***
1	22.291	0.032	−0.050	44.954	22.563***	22.613***

Every stock on the FTSE 100 is classified into four quartiles based on market capitalisation. The higher the rank of the group, the higher the market capitalisation of the stocks. For each specific group, the mean value, computed as a cross-stock average, is separately reported, and then averaged across days (separately for quarterly, non-quarterly and non-expiration days). The Wilcoxon signed-rank test is used to test the significance of the difference (Diff. I) between the values on expiration days and the values on the first and second Fridays of the corresponding month. The Mann-Whitney U test is used to test whether the difference (Diff. II) between either the quarterly or the non-quarterly expiration day's mean value and the mean value observed on non-expiration days is statistically significant. Panels A and B report results for the changes in turnover ratio in the morning interval and during the closing auction, respectively. Panels C and D present the test results on the changes in concentration ratio in the morning interval and during the closing auction, respectively. Statistical significance at 10%, 5% and 1% levels is indicated by *, ** and ***, respectively. Values are reported in percentage points.

expiration days are particularly interesting. First, as noticed in this study, the smaller the stock, the sharper the spike in volume traded at the close. The study also observes that the spike in traded volume is particularly high for the smaller market capitalisation FTSE 100 quartile and for both the smaller and larger deciles of FTSE 250 Index stocks. These stocks are the most affected by quarterly index recalibration, as they will either change index the following trading day or their weights within the index will be particularly affected by the index adjustments. This gives support to the interpretation that the increased trading volume at close is not driven by derivatives' expiration, at least not as the main factor.

TABLE 3.3: Trading activity effects on FTSE 250 stocks - Analysis by market size on quarterly expiration days.

	Turnover ratio						Concentration ratio					
	9:30-11:00 a.m.			4:30 p.m.-Close			9:30-11:00 a.m.			4:30 p.m.-Close		
	Mean	Diff. I	Diff. II	Mean	Diff. I	Diff. II	Mean	Diff. I	Diff. II	Mean	Diff. I	Diff. II
10	0.081	0.045***	0.045***	0.347	0.275***	0.281***	16.048	6.184***	5.821***	46.293	24.057***	24.371***
9	0.078	0.045***	0.046***	0.226	0.164***	0.166***	18.949	8.606***	8.687***	42.429	20.326***	20.179***
8	0.076	0.045***	0.045***	0.267	0.209***	0.206***	17.477	7.559***	7.510***	44.783	22.439***	22.355***
7	0.065	0.038***	0.040***	0.262	0.213***	0.217***	18.204	8.933***	8.486***	44.198	21.505***	21.404***
6	0.055	0.034***	0.035***	0.227	0.186***	0.187***	18.617	9.713***	9.385***	45.328	22.002***	21.657***
5	0.053	0.033***	0.034***	0.223	0.185***	0.187***	18.726	9.415***	9.515***	46.197	22.389***	22.410***
4	0.048	0.031***	0.031***	0.235	0.202***	0.203***	19.156	10.088***	10.061***	47.260	22.965***	22.955***
3	0.048	0.032***	0.031***	0.198	0.168***	0.169***	20.386	11.409***	11.015***	46.758	22.200***	22.318***
2	0.044	0.028***	0.029***	0.177	0.149***	0.151***	22.061	13.417***	12.888***	45.976	20.568***	21.226***
1	0.051	0.030***	0.032***	0.410	0.376***	0.379***	17.764	8.526***	8.329***	50.038	26.641***	26.563***

Every stock on the FTSE 250 is classified into four quartiles based on market capitalisation. The higher the rank of the group, the higher the market capitalisation of stocks in the group. For each specific group, the mean value, computed as a cross-stock average, is separately reported and then averaged across days (separately for quarterly expiration days and non-expiration days). The Wilcoxon signed-rank test is used to test the significance of the difference (Diff. I) between the values on expiration days and the values on the first and second Fridays of the corresponding month. The Mann-Whitney U test is used to test whether the difference (Diff. II) between either the quarterly or the non-quarterly expiration day's mean value and the mean value observed on non-expiration days is statistically significant. Panels A and B report results for the changes in turnover ratio in the morning interval and during the closing auction, respectively. Panels C and D present the test results of the changes in concentration ratio in the morning interval and during the closing auction, respectively. Statistical significance at 10%, 5% and 1% levels are indicated by *, ** and ***, respectively. Values are reported in percentage. The tests are conducted only on quarterly expiration days.

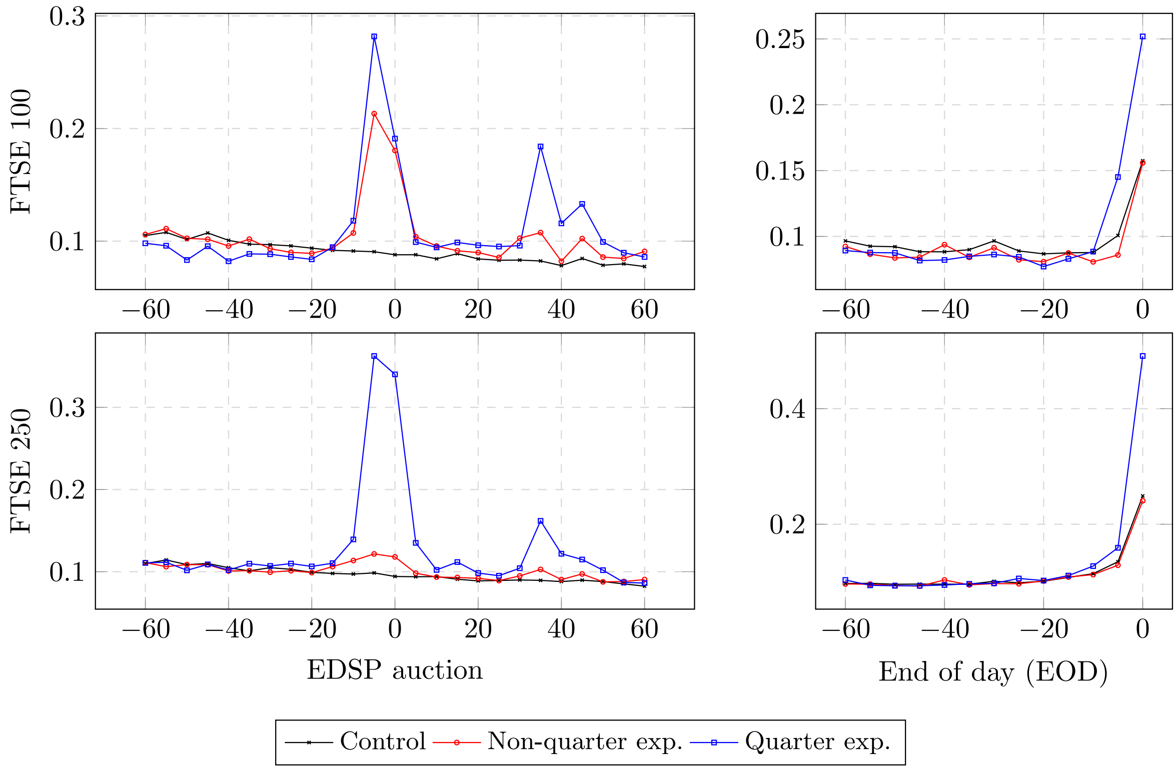


FIGURE 3.2: Absolute returns intraday patterns.

The figure depicts the mean percentage absolute log-returns (Equation [3.3]) computed for each five-minute interval, both around the EDSP auction (which takes place in the interval from 10:10-10:15 a.m. and is identified by 0 in the graphs in the left column) and at the end of the trading session (the 0 in the graphs on the right identify the last interval of the trading day, which is the closing auction, taking place between 4.30 and 4.35 p.m.). The absolute return is first computed on every interval for each combination of stock and day in the sample. The absolute return is then averaged across stocks (separately for FTSE 100 and FTSE 250 stocks) by every interval and within a specific day and, finally, averaged across days, separately for control days (non-expiration Fridays), quarterly and non-quarterly expiration days. Values are displayed in percentage points.

In summary, strong evidence is found in support of 3.1. In fact, a significant increase in trading activity is found around the EDSP auction, which is used to compute the settlement price of futures and options derivatives, for both FTSE 100 and FTSE 250 Index stocks. This increase is also significant from an economic point of view. In the 45-minute time window before and after the settlement auction, traded volume (as percentage of the daily traded volume) increases from 10.14% on control days to 17.38% (an increase of almost 70%) on quarter expiry days for FTSE 100 stocks, and from 9.36% to 18.72% (it doubles) for FTSE 250 stocks. When only FTSE 100 Index options expire, the increase in trading volume during the same interval is not as sharp but, nonetheless, it is around 50%. On the other hand, however, this first set of tests provides no strong evidence in support of Hypothesis 3.2.

3.4.2 Volatility effects

As documented in the previous section, notable trading activity patterns are observed on derivatives' expiration days. The study next investigates if this abnormal trading activity leads to distortions in the price dynamics. This section first explores changes in intraday volatility.

Given stock i on day t, the return on the j-th five-minute interval is defined as:

$$r_{i,t,j} = ln\left(\frac{p_{i,t,j}}{p_{i,t,j-1}}\right), \tag{3.3}$$

where $p_{i,t,j}$ is the last midpoint price observed in j-th interval. For the interval corresponding to the closing auction, which is the last interval of the trading day, the closing price is used as $p_{i,t,j}$ and the last midpoint price registered just before the start of the closing auction is used for $p_{i,t,j-1}$.

Following Illueca and LaFuente, (2006), realised volatility is used to measure intraday volatility. Given the combination of stock i and day t, it is defined as follows:

$$RVOL_{i,t} = \sqrt{\sum_{j \in I} |r_{i,t,j}|^2}, \tag{3.4}$$

where the five-minute intervals included in the computation depend on the time interval I. As for trading activity, realised volatility is computed for the three intervals: 9:30-11:00 a.m., 4:00-4:30 p.m. and 4:30 p.m.-Close. Note that $RVOL_{i,t}$, when computed during the last interval (4:30 p.m.-Close), corresponds to the absolute log return over the closing auction.

Realised volatility is then aggregated across stocks as follows:

$$RVOL_t = \frac{1}{N}\sum_{i=1}^{N} RVOL_{i,t}. \tag{3.5}$$

The simple average across the relevant stocks is taken.

Figure 3.2 plots the intraday dynamic of the absolute price changes for every five-minute interval both around the EDSP auction and before the close of the trading day, separately for the FTSE 100 and FTSE 250 stocks. The first thing noticed is that the intraday pattern of absolute price changes on expiration days closely resembles the abnormal dynamics documented for trading activity. In fact, volatility rises consistently when approaching the EDSP auction on quarterly expiration days and persists at higher levels than usual until around 11 a.m. This same pattern is observed for FTSE 100 stocks on non-quarterly expiration days and, partially for FTSE 250 stocks, suggesting the existence of some forms of volatility spillover. Quarterly expiration days are also characterised by price movements during the closing auction that are significantly sharper than normal. The same pattern is not observed for non-quarterly expiration days.

The statistical significance of the volatility patterns is formally tested by using both the Wilcoxon signed-rank test and the Mann-Whitney U test (following the exact methodology used to test the existence of trading activity effects, described in detail in the previous section). Realised volatility is first aggregated at the index level and expiration days are compared to non-expiration days. The results are presented in Tables 3.4 and 3.5, Panel A and confirm the existence of abnormal volatility surrounding the EDSP auction on days when both futures and options expire, for both the FTSE 100 and FTSE 250 stocks, and on non-quarterly expiration days for the FTSE 100 stocks, although at a smaller magnitude. Furthermore, the study shows that the abnormal spike in trading activity throughout the closing auction on quarterly expiration days is associated with significantly higher absolute price returns. The price movement (in absolute values) observed throughout the closing auction for FTSE 100 stocks is around 0.09% more than control days, and almost 0.24% more than control days for FTSE 250 stocks.

Volatility effects on expiration days are a market-wide phenomenon. As described in the previous section, the stocks are divided into 14 groups (approximately 25 stocks for each group) based on market capitalisation: four groups for FTSE 100 constituents and the other 10 for FTSE 250 stocks. Realised volatility measures (Equation [3.5]) are computed for each of these groups, with the same tests discussed above conducted for each group. Results are reported in Table 3.4 for FTSE 100 stocks and Table 3.5 for FTSE 250 stocks. Volatility effects are reported around the EDSP for both FTSE 100 and FTSE 250 stocks across the groups of different market capitalisation. The possible volatility spillovers on FTSE 250 stocks on non-quarterly expiration days are not statistically significant, for any of the groups. However, it is noted, although not significant, that all 10 groups (except for two) are characterised by a positive difference, suggesting the existence of some type of spillover.

The rise in volatility over the closing auction on quarterly expiration days is also a market-wide phenomenon. Interestingly, as for trading activity, FTSE 100 stocks in the last quartile and FTSE 250 stocks in both the first and last deciles are characterised by the largest increase in the absolute price change over the closing auction on quarterly expiration days. This is consistent with observations for trading activity effects. The volume imbalances driven by the spikes in trading activity are so extreme that they lead to significant price movements.

Overall, strong support is found for Hypothesis 3.3; that is, a significant increase is found in volatility around the settlement definition of FTSE 100 and FTSE 250 Index derivatives' expiration. The same considerations proposed in this study regarding the abnormal trading activity patterns at the close of quarterly expiration days are valid for the abnormal price changes observed throughout the closing auction. Derivatives' expiration is not likely to be the main factor causing these abnormalities, thus requiring further work to properly test Hypothesis 3.4.

3.4.3 Existence of price reversals

This study established the existence of abnormal trading activity that leads to an increase in price volatility on expiration days. The anomalies are located, firstly, around the EDSP auction in the morning session of index derivatives' expiration days and, secondly, at the close

TABLE 3.4: Volatility effects on FTSE 100 stocks.

	Non-quarter exp. ($N = 47$)			Quarter exp. ($N = 23$)		
	Mean	Diff. I	Diff. II	Mean	Diff. I	Diff. II
Panel A: FTSE 100 Index stocks						
9:30-11:00 a.m.	0.669	0.092***	0.099***	0.737	0.172***	0.166***
4:00-4:30 p.m.	0.257	−0.022***	−0.021	0.289	−0.001	0.011
4:30 p.m.-Close	0.156	−0.003	−0.002	0.252	0.093***	0.094***
Panel B: analysis by quartiles (9:30-11:00 a.m.)						
4	0.604	0.088***	0.088***	0.678	0.169***	0.162***
3	0.665	0.096***	0.097***	0.741	0.179***	0.173***
2	0.681	0.103***	0.108***	0.734	0.158***	0.160***
1	0.724	0.082***	0.099***	0.797	0.180***	0.172***
Panel B: analysis by quartiles (4:30 p.m.-Close)						
4	0.137	0.000	−0.003	0.223	0.079***	0.083***
3	0.154	−0.007*	−0.006	0.244	0.079***	0.084***
2	0.162	0.001	0.001	0.261	0.100***	0.099***
1	0.170	−0.006	−0.001	0.281	0.115***	0.111***

Realised volatility (Equation [3.5]) is used as a measure of intraday volatility. To obtain the mean values reported in the table, the measure is first computed as a cross-stock average, within the FTSE 100 Index, and then averaged across days (separately for quarterly, non-quarterly and non-expiration days). The average values are reported separately for non-quarterly and quarterly expiration days and are computed first as a cross-stock average and then averaged across days. Column Diff. I reports the difference with respect to the average value on the first and second Fridays of the expiration month and the statistical significance is tested through the Wilcoxon signed-rank test. Column Diff. II reports the difference with respect to the average value measured on all non-expiration Fridays that are part of the sample. The existence of a statistically significant difference in means is tested by the Mann-Whitney U test. Panel A contains the results for realised volatility computed for the whole FTSE 100 Index and on different intraday intervals. In panels B and C, the difference in means is tested for each stock quartile (the higher the group rank the higher the market capitalisation) and for realised volatility measured in the morning session and during the closing auction, respectively. The notation *, ** and *** indicate that the difference in means is statistically significant at 10%, 5% and 1% levels. Values are reported in percentage points.

TABLE 3.5: Volatility effects on FTSE 250 stocks.

Panel A: FTSE 250 Index stocks	Non-quarter exp. ($N = 47$)			Quarter exp. ($N = 23$)		
	Mean	Diff. I	Diff. II	Mean	Diff. I	Diff. II
9:30-11:00 a.m.	0.689	0.010	0.021*	0.957	0.302***	0.289***
4:00-4:30 p.m.	0.347	−0.016	−0.010	0.380	0.010	0.023**
4:30 p.m.-Close	0.241	−0.009	−0.011	0.491	0.237***	0.239***

Panel B: analysis by deciles	Non-quarter exp. (9:30-11:00 a.m.)		Quarter exp. (9:30-11:00 a.m.)		Quarter exp. (4:30 p.m.-Close)	
	Mean	Diff. I	Mean	Diff. I	Mean	Diff. I
10	0.632	0.021	0.804	0.211***	0.361	0.184***
9	0.622	−0.013	0.842	0.255***	0.292	0.110***
8	0.663	0.003	0.908	0.285***	0.383	0.188***
7	0.661	0.032	0.879	0.240***	0.403	0.190***
6	0.676	0.019	0.903	0.264***	0.470	0.231***
5	0.710	0.022	0.929	0.305***	0.486	0.230***
4	0.697	0.021	0.966	0.315***	0.537	0.266***
3	0.682	0.010	1.003	0.345***	0.601	0.293***
2	0.714	0.023	1.100	0.418***	0.617	0.275***
1	0.827	−0.041	1.256	0.401***	0.777	0.416***

Realised volatility (Equation [3.5]) is used as a measure of intraday volatility. To obtain the mean values reported in the table, the measure is first computed as a cross-stock average, within the FTSE 250 Index, and then averaged across days (separately for quarterly, non-quarterly and non-expiration days). The average values are reported separately for non-quarterly and quarterly expiration days, and are computed first as a cross-stock average, separately for each market index, and then averaged across days. Column Diff. I reports the difference with respect to the average value on the first and second Fridays of the expiration month and the statistical significance is tested through the Wilcoxon signed-rank test. Column Diff. II reports the difference with respect to the average value measured on all non- expiration Fridays that are part of the sample. The existence of a statistically significant difference with respect to control days is tested by the Mann-Whitney U test. Panel A presents the results for realised volatility computed on the whole FTSE 250 Index and on different intraday intervals. In Panel B, the difference in means is tested for each stock decile (the higher the group rank, the higher the market capitalisation) for the interval 9:30-11:00 a.m. on both non-quarterly and quarterly expiration days and for the closing auction interval on quarterly expiry days. The notations *, ** and *** indicate that the difference in means is statistically significant at 10%, 5% and 1% levels. Values are reported in percentage points.

of quarterly expiration days. This section investigates whether these two specific events are associated with price reversals. The study then seeks to determine if the rise in volatility leads to dislocations of prices from their *fair value*. In other words, the study tests if price changes are systematically observed before, respectively, the EDSP and the market close, and are then reversed.

Three measures of price reversal proposed by Stoll and Whaley, (1987); Stoll and Whaley, (1991) are applied, with these widely adopted in the literature. Given stock i and day t, the price returns are first computed before and after the EDSP auction as:

$$R_{0,i,t} = ln\left(\frac{p_{i,t,edsp}}{p_{i,t,edsp-45}}\right), \tag{3.6}$$

$$R_{1,i,t} = ln\left(\frac{p_{i,t,edsp+45}}{p_{i,t,edsp}}\right), \tag{3.7}$$

where $p_{i,t,edsp}$ is the price at the end of the EDSP auction and $p_{i,t,edsp-45}$ and $p_{i,t,edsp+45}$ are the midpoint prices observed 45 minutes before and after the end of the auction, respectively. On control days, no EDSP auction occurs. The same is true for non-quarterly expiration days for FTSE 250 stocks, as no index derivatives are expiring. In this case, considering the EDSP auction starts at 10:10 in the morning and ends at 10:15, $p_{i,t,edsp}$ is taken as equal to the midpoint price observed at 10:15; $p_{i,t,edsp-45}$ and $p_{i,t,edsp+45}$ are defined in the same way taking the prices observed, respectively, at 9:30 a.m. and 11 a.m. in the morning. This window size is selected based on the evidence above and on the consideration that volatility, on index expiration days, starts to increase before the start of the EDSP auction, peaks around settlement time, and remains at levels higher than usual until around 11 a.m. Therefore, the goal is to test whether this rise in volatility is associated with prices temporarily diverging from equilibrium.

The same reasoning is applied to find the price movements around the closing auction:

$$R_{0,i,t} = ln\left(\frac{p_{i,t,close}}{p_{i,t,close-30}}\right), \tag{3.8}$$

$$R_{1,i,t} = ln\left(\frac{p_{i,t+1,open+30}}{p_{i,t,close}}\right), \tag{3.9}$$

where $p_{i,t,close}$ is the closing price, $p_{i,t,close-30}$ is the midpoint price computed 30 minutes before the end of continuous trading and $p_{i,t+1,open+30}$ is the midpoint price observed after the first 30 minutes of trading on the following trading day.

The three reversal measures are then computed individually for each combination of stock and day:

$$REV0_{i,t} = \begin{cases} R_{1,i,t} & if\ R_{0,i,t} < 0 \\ -R_{1,i,t} & if\ R_{0,i,t} \geq 0 \end{cases}, \tag{3.10}$$

TABLE 3.6: Frequency of reversals.

	Non exp.	Non-quarter exp.	Quarter exp.
Panel A: FTSE 100 Index stocks			
EDSP	48.39	58.19	60.26
Close	50.57	47.37	54.01
Panel B: FTSE 250 Index stocks			
EDSP	44.65	46.14	65.49
Close	54.90	53.76	59.12

The table reports the frequency, in percentage points, of: (1) a price reversal following the EDSP auction and (2) a price reversal after the close of the trading day (i.e., an overnight price reversal to the next trading day). For each stock i and day t, a price reversal occurs if $sign(R_{0,i,t}) \neq sign(R_{1,i,t})$, with $R_{0,i,t}$ and $R_{1,i,t}$ either based on Equations (3.6) and (3.7) (to measure reversal around the EDSP auction) or based on Equations (3.8) and (3.9) (to measure the reversal around closing). The percentage of price reversals is averaged across stocks within the same index (Panel A for FTSE 100 and Panel B for FTSE 250 stocks) and then across days and it is reported separately for non expiration days, non quarterly expiration days and quarterly expiration days.

$$REV1_{i,t} = \begin{cases} |R_{1,i,t}| & if\ sign(R_{0,i,t}) \neq sign(R_{1,i,t}) \\ 0 & otherwise \end{cases}, \tag{3.11}$$

$$REV2_{i,t} = \begin{cases} |R_{0,i,t}| & if\ sign(R_{0,i,t}) \neq sign(R_{1,i,t}) \\ 0 & otherwise \end{cases}. \tag{3.12}$$

All measures, $REV0$, $REV1$ and $REV2$, are computed using $R_{0,i,t}$ and $R_{1,i,t}$ either based on Equations (3.6) and (3.7) (to measure the intensity of reversal around the index derivatives settlement time) or based on Equations (3.8) and (3.9) (to measure the reversal around closing). $REV1$ and $REV2$ can only assume positive values. If they are equal to 0, no reversal takes place. $REV0$ can be either positive or negative: positive values correspond to price reversals; negative values correspond to price continuations. For all three metrics, higher (and positive) values correspond to price reversals of higher magnitude.

Individual reversals are finally aggregated at the index level (either the FTSE 100 or the FTSE 250 index). Two types of aggregation are used. Both an average index reversal and a portfolio reversal at the index level are computed. If $REV0$ is taken (although the same considerations are valid for $REV1$ and $REV2$), the average reversal for a given index on day t is computed as:

$$REV0_t = \frac{1}{N}\sum_{i=1}^{n} REV0_{i,t}. \tag{3.13}$$

To compute the portfolio reversal for each market index, the index returns around the desired time first need to be computed:

$$R_{0,index,t} = \frac{1}{N} \sum_{i=1}^{n} R_{0,i,t}, \tag{3.14}$$

$$R_{1,index,t} = \frac{1}{N} \sum_{i=1}^{n} R_{1,i,t}. \tag{3.15}$$

Based on the above, the reversals at a portfolio level are computed in the same way as for the individual stock reversals. As an example, given a specific index, the first type of reversal, denominated as $REV0^{index}$, becomes:

$$REV0_t^{index} = \begin{cases} R_{1,index,t} & if \ R_{0,index,t} < 0 \\ -R_{1,index,t} & if \ R_{0,index,t} \geq 0 \end{cases} . \tag{3.16}$$

Table 3.6 first presents the percentage of times a price reversal occurs around both the EDSP auction and the close. The substantial increase in the reversal frequency around the EDSP auction is remarkable: for FTSE 100 stocks, the price movement between 9:30 a.m. and 10:15 a.m. tends to reverse over the next 45 minutes more than 60% of the times on quarterly expiration days and 58% of the times on non-quarterly expiration days. The same percentage increases to more than 65% for FTSE 250 stocks, while no significant increase is observed, as expected, on non-quarterly expiration days as FTSE 250 index derivatives only have a quarterly expiration cycle. Similar considerations are true, although to a lesser extent, for the reversal frequency around the closing auction on quarterly expiration days.

To formally test the significance of price reversals on expiration days with respect to the control days, the same type of approach adopted in the previous sections (to test the significance of trading activity and volatility patterns) is applied. Results are presented in Table 3.7. Around the EDSP auction, a marked and significant price reversal for both FTSE 100 and FTSE 250 stocks is observed on quarterly expiration days, although it is only for the FTSE 250 stocks that all three reversals measure significantly higher if computed at the portfolio level. The same is true for price reversals computed on FTSE 100 stocks on non-quarterly expiration days while, as expected, the same effects are not found on non-quarterly expiration days. The trading pressure generated by index derivatives' expiration consequently leads to stock prices that deviate from equilibrium. After the EDSP auction ends and the settlement price is determined, the prices of the underlying stocks tend to reverse back to their previous values. Thus, strong evidence is found to support Hypothesis 3.5. It should be noted, however, that these effects, although quite noticeable at the individual stock level, tend to compensate each other within each market index, leading to index reversals of less magnitude (and significantly different from control days only based on REV2 measures in the case of FTSE 100 stocks).

The patterns of price reversals at the close are surprising. The average price reversals

TABLE 3.7: Price reversal effects.

	Non-quarter exp. ($N = 47$)			Quarter exp. ($N = 23$)		
	Mean	Diff. I	Diff. II	Mean	Diff. I	Diff. II
Panel A: FTSE 100 Index stocks						
EDSP Reversals						
REV0	0.063	0.064***	0.067***	0.102	0.111***	0.106***
REV1	0.188	0.053***	0.057***	0.223	0.101***	0.092***
REV2	0.211	0.058***	0.065***	0.246	0.115***	0.100***
$REV0^{index}$	0.041	0.026	0.028*	0.064	0.054	0.051*
$REV1^{index}$	0.097	0.026	0.028**	0.094	0.030	0.025*
$REV2^{index}$	0.141	0.057**	0.072***	0.164	0.125***	0.095***
Close Reversals						
REV0	−0.041	0.001	−0.049	0.009	−0.102	0.000
REV1	0.407	0.015	−0.014*	0.518	0.037	0.097
REV2	0.120	−0.008	−0.016**	0.190	0.037*	0.054***
$REV0^{index}$	−0.085	−0.016	−0.102*	0.015	−0.179	−0.003
$REV1^{index}$	0.233	0.032	−0.026*	0.355	−0.023	0.096
$REV2^{index}$	0.042	−0.012	−0.021*	0.091	0.009	0.028
Panel B: FTSE 250 Index stocks						
EDSP Reversals						
REV0	0.017	0.017	0.020**	0.206	0.215***	0.209***
REV1	0.163	0.007	0.012**	0.343	0.205***	0.193***
REV2	0.192	0.015	0.020**	0.360	0.198***	0.188***
$REV0^{index}$	−0.015	−0.016	−0.017	0.220	0.214***	0.217***
$REV1^{index}$	0.056	−0.005	−0.002	0.231	0.179***	0.173***
$REV2^{index}$	0.068	−0.014	−0.003	0.255	0.208***	0.185***
Close Reversals						
REV0	0.094	0.003	−0.014	0.227	0.085**	0.120***
REV1	0.475	−0.007	−0.014	0.597	0.094**	0.109***
REV2	0.201	−0.012	−0.015	0.375	0.151***	0.158***
$REV0^{index}$	−0.085	−0.048	−0.094	0.127	0.025	0.118
$REV1^{index}$	0.199	−0.029	−0.046	0.311	0.032	0.066
$REV2^{index}$	0.051	−0.004	−0.015	0.091	0.003	0.025

Three measures of price reversals ($REV0$, $REV1$ and $REV2$) are used for both FTSE 100 and FTSE 250 component stocks. To obtain the mean values reported in the table, the measure is first computed as a cross-stock average, separately for the two indices, and then averaged across days (separately for quarterly, non-quarterly and non-expiration days). Column Diff. I reports the difference with respect to the average value on the first and second Fridays of the expiration month and the statistical difference between the two means is tested through the Wilcoxon signed-rank test. Column Diff. II reports the difference with respect to the average value measured on all non-expiration Fridays that are part of the sample. The existence of a statistically significant difference in means is tested by the Mann-Whitney U test. The notations *, ** and *** indicate that the difference in means is statistically significant at 10%, 5% and 1% levels. Values are reported in percentage points.

are not significant for FTSE 100 stocks (except for the $REV2$ measure at 10% level), while significantly higher reversals than normal are observed for FTSE 250 stocks. Price reversals computed at the index level, however, are never significantly higher than the control group, for either the FTSE 100 or the FTSE 250 Index, despite the extreme concentration of liquidity at the close on quarterly expiration days. This is further evidence that the anomalous behaviour at the end of quarterly expiration days is not due to a temporary imbalance, as is true for what happens around the EDSP auction, but reflects more fundamental changes (such as market indices recalibration) that are impacting on the price.

3.5 Do optionable stocks present abnormal patterns?

The evidence presented on the effects caused by the expiration of index derivatives on trading activity and price dynamics is quite clear. Less clear, however, if individual options expiration has any effects on the underlying spot market. A significant accumulation of liquidity (and subsequent price effects) is reported on the closing auction of quarterly expiration days. It was argued, in the previous sections, that, although individual options expiration might be seen as a contributing factor, it cannot be viewed as the main cause. In fact, the same patterns are not observed on non-quarterly expiration days, on which individual options also expire. Additionally, it is a market-wide pattern affecting all the stocks analysed and, while the majority of FTSE 100 are optionable, the majority of FTSE 250 are not, making unlikely for such an intense effect to be driven by a small set of stocks.

The question if the expiration of equity options induces these quarterly effects at the close remains. To attempt to find an answer, the effects associated with single stock options expiration are disentangled from other effects at expiration (such as index derivatives expiration and market index quarterly reviews), by the following OLS regression:

$$Y_{i,t} = \alpha + \beta OPTIONABLE_{i,t} + \sum_{j=1}^{d} \gamma_j D^j_{i,t} + \lambda_t + \epsilon_{it}. \tag{3.17}$$

The variable $Y_{i,t}$ represents the dependent variable used to measure either trading activity, price or volatility effects. Taking turnover ratio ($TR_{i,t}$), measured throughout the closing auction, as an example, the dependent variable $Y_{i,t}$, given a combination of stock i and expiration day t, is measured as follows:

$$Y_{i,t} = TR_{i,t} - \overline{TR}_{i,t}, \tag{3.18}$$

where $\overline{TR}_{i,t}$ is the mean average of the turnover ratio computed on stock i based on the non-expiration Fridays included in the sample and falling on the same month of expiration day t. The variable $OPTIONABLE_{i,t}$ is a dummy variable equal to 1 if stock i is optionable (that is, there are listed stock options with stock i as underlying). Non-optionable stocks are therefore used as control. To notice that, while the minority of FTSE 100 stocks is non-optionable, the vast majority of FTSE 250 stocks are non-optionable. All available stock-day combinations of FTSE 100 stocks are included in the sample, given the scarcity of non-optionable stocks within this index. For FTSE 250 stocks, however, each optionable stock is matched to the closest non-optionable stock in terms of market capitalisation, on every expiration day. The variable $D^j_{i,t}$ is a dummy variable equal to 1 if stock i is part of group j (in terms of market capitalisation) on day t. The stocks in the sample are separated into groups based on market capitalisation, following the same procedure of previous sections: FTSE 100 stocks are grouped, at each expiration day, into $d = 4$ quartiles based on market capitalisation; FTSE 250 stocks are grouped into $d = 10$ deciles based on market capitalisation. Finally, date fixed effects λ_t are included.

The parameter of interest is β: a significant and positive value of β would mean that optionable stocks present abnormal patterns (in terms of either trading activity, volatility or price effects, depending on the dependent variable) that are not observed on non-optionable stocks, on top of the quarterly expiration effects (that are captured by α and by the controls).

The dependent variables $Y_{i,t}$ used in the model estimation are based on the following measures:

- Turnover ratio ($TR_{i,t}$) and Concentration ratio ($CR_{i,t}$), both computed during the closing auction. The goal is to test if optionable stocks are characterised by an increase in trading activity on quarterly expiration day of higher magnitude than non-optionable stocks. The focus is on trading activity at the end of the day as the final option's value will depend on the closing price of the underlying stock and, as the close approaches, hedging activities, as well as manipulation attempts, might be present. Furthermore, the interest lies in testing if the spike of trading activity at the close of quarterly expiration days is due to factors associated with the options market. Hypothesis 3.2 is subject to testing;
- Realised volatility ($RVOL_{i,t}$), computed during the closing auction. The same considerations that are valid for trading activity are valid for volatility. Hypothesis 3.4 is subject to testing;
- Price reversals at the close ($REV2_{i,t}$): Hypothesis 3.6 is subject to testing.

For all the above dependent variables, the hypotheses tested correspond to test the alternative hypothesis $H_1 : \beta > 0$ against the null $H_0 : \beta \leq 0$.

The OLS regression (Equation [3.17]) is estimated on quarterly expiration days, separately for FTSE 100 and FTSE 250 stocks, with standard errors clustered by stock. The results are presented in Table 3.8 for both FTSE 100 (Panel A) and FTSE 250 stocks (Panel B). For FTSE 100 stocks, no significant value for β are found for any of the dependent variables tested, with the exception of the realised volatility at the close, which is, however, negative and only a significance level of 10%. Regarding FTSE 250 stocks, similar considerations apply, but a positive and significant (although only at a 10% level) value of β is found when $Y_{i,t}$ is based on concentration ratio (CR). Therefore, optionable stocks that are part of FTSE

TABLE 3.8: Regression results - optionability effects on quarterly expiration days.

	TR	CR	RVOL	REV2
Panel A: FTSE 100 Index stocks				
Constant	0.109^{***}	-9.250^{***}	-0.050^{*}	-0.093^{**}
	(3.239)	(−4.341)	(−1.941)	(−2.469)
OPTIONABLE	0.028	−2.672	-0.046^{*}	−0.014
	(1.069)	(−1.435)	(−1.941)	(−0.467)
Observations	2310	2310	2310	2310
Adjusted R^2	0.05	0.44	0.17	0.11
Panel B: FTSE 250 Index stocks				
Constant	0.541^{***}	1.857	0.386^{*}	0.189
	(4.993)	(0.275)	(1.663)	(0.791)
OPTIONABLE	−0.007	3.208^{*}	−0.038	−0.027
	(−0.216)	(1.856)	(−1.257)	(−0.787)
Observations	1106	1106	1106	1106
Adjusted R^2	0.03	0.30	0.09	0.07

The table reports the parameters' estimates of regression (Equation [3.17]) on FTSE 100 (Panel A) and FTSE 250 (Panel B) stocks. Standard errors are clustered by stocks and the (corrected) t-values are reported in parentheses. The variables used as dependent variables are all computed over the closing auction and are the following: (1) turnover ratio (TR); (2) concentration ratio (CR); (3) realised volatility (RVOL); and (4) reversal of second type (REV2). The difference, for each stock, with the average value observed on the non-expiration Fridays of the expiration months are used as dependent variables ($Y_{i,t}$). Control variables and date fixed effects are included in all regressions, but not reported in the table. Statistical significance at 10%, 5% and 1% levels are indicated by *, ** and ***, respectively. Values are reported in percentage points.

250 appear to present more intense trading activity on quarterly expiration days than similar stocks that are not optionable. The low significance level, however, does not allow to make any definitive conclusions. Only information on whether a stock is optionable or not is used. More detailed information on the options characteristics might be necessary, such as the amount (i.e., the open interest) of active options or their moneyness at expiration day. This information can be useful to understand if certain characteristics generate effects at expiry on the spot market. They can also be useful to attempt to separate legitimate hedging behaviour from manipulative trading activity.

3.6 Conclusion

This chapter presents the study's investigation of the existence of expiration day effects. An empirical framework is proposed for testing a series of hypotheses on the existence of volume, volatility and price effects. As an element of novelty, intraday data is employed to explore whether specific moments throughout the day are characterised by significant unusual patterns.

Strong evidence is found to support the existence of expiration day effects associated with index derivatives' expiration. The study reports an increase in the volume traded in the interval of one hour-and-a-half between 9:30 a.m. and 11:00 a.m., that is, surrounding the EDSP auction that determines the derivatives settlement price. In the same interval, it is also documented an increased volatility and the tendency for prices to deviate from equilibrium, moved by the trading imbalances generated at and before the settlement auction, and to reverse back in the following minutes.

A significant concentration of liquidity is found towards the closing auction on quarterly expiration days. Derivatives' expiration does not appear to be the main factor driving this phenomenon. In fact, no reasonable link is found with the expiration of index derivatives, considering their settlement price is determined through a call auction during the morning session. Furthermore, for the FTSE 100 index options that expire on non-quarterly expiration days, anomalies are observed surrounding the EDSP auction, but no anomalous pattern is found at the close. An empirical model is proposed to isolate the effects of individual options and no significant effect is found on FTSE 100 stocks, while the effect for the FTSE 250 stocks is only on trading activity and is only marginal. In any case, it is not sufficient to explain the magnitude of the liquidity shift at the close on quarterly expiration days. The most plausible alternative explanation involves periodic recalibration of FTSE indices (each recalibration becomes effective starting from the Monday after the expiration Friday) and the desire by passive investing funds (especially ETFs) to rebalance their portfolio at the closing price to minimise the tracking error.

3.A General patterns of end-of-day metrics

This section presents some important end-of-day patterns relevant to the methodology performed and the results obtained above.

In Table 3.9, the trend dynamic of the volume traded during the closing auction is reported. This pattern affects both FTSE 100 and FTSE 250 stocks and is substantial: if 17% of the daily volume traded in 2015 on FTSE 100 stocks was, on average, traded in the closing auction, this percentage increased up to almost 32% in 2020. For FTSE 250 stocks, the same percentage increased from around 20% in 2015 to more than 34% in 2020. This rise seems to also be associated with a higher intensity of price changes during the closing auction. This confirms the importance of comparing the values on expiration days with values on days close in time, as carried out in this study's methodology. It also confirms the importance of controlling for this structural change in the regressions conducted above with the introduction of time fixed effects.

Figure 3.3 shows the difference between the average traded volume and the absolute price change during the closing auction on quarterly expiration days and the average computed on the whole sample of control days, separately for stocks of different market capitalisation. The group of FTSE 100 stocks with lower market capitalisation and both groups of FTSE 250 stocks with higher and lower market capitalisation (which are highlighted in the figure) are visibly characterised by the highest increase of trading volume and, albeit less strongly, of price changes, with respect to stocks with similar market capitalisation. These three groups of stocks are, in fact, at the edge of the respective market index and are then expected to either change index or, even if remaining in the same one, are expected to have weights that are impacted the most, subsequent to the index review. The fact that this type of abnormal pattern is observed is, therefore, strong evidence in support of the interpretation that the abnormal end-of-day trading and price patterns observed on quarterly expiration days are driven by market participants recalibrating their portfolios ahead of adjustments to the FTSE indices as the main factor.

TABLE 3.9: Changes through time of end-of-day metrics

	FTSE 100			FTSE 250		
	Turn. ratio	Conc. ratio	*RVOL*	Turn. ratio	Conc. ratio	*RVOL*
2015	0.073	17.004	0.141	0.044	20.426	0.232
2016	0.081	18.033	0.164	0.057	21.331	0.264
2017	0.083	22.386	0.133	0.062	21.144	0.202
2018	0.097	25.665	0.159	0.069	25.722	0.252
2019	0.096	30.031	0.181	0.068	32.927	0.266
2020	0.110	31.894	0.267	0.074	34.755	0.433

The table reports the average value for each year from 2015-2020 of: turnover ratio, concentration ratio and realised volatility, all measured over the closing auction. The statistics are reported separately for FTSE 100 and FTSE 250 stocks.

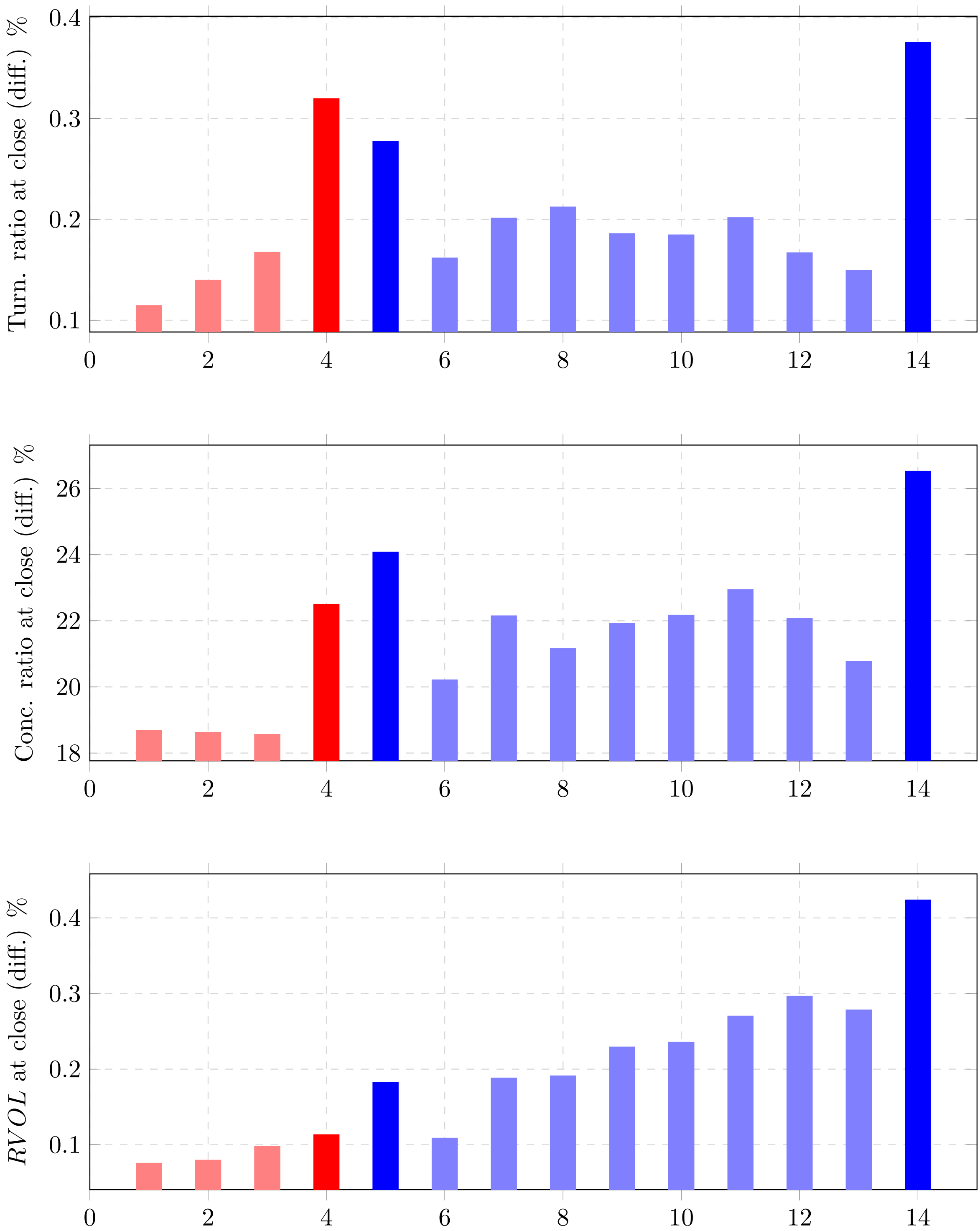


FIGURE 3.3: Quarter expiration difference on end-of-day measures.

The bar chart displays the difference in means between quarterly expiration days and non-expiration days that are part of the sample of three measures: turnover ratio, concentration ratio and realised volatility, all measured during the closing auction. The difference is computed separately for each of the four quartiles of FTSE 100 stocks (in red) and for each decile of FTSE 250 stocks (in blue), for a total of 14 groups of 25 stocks each. The groups are displayed on the x-axis in decreasing order of market capitalisation (from left to right).

Chapter 4

Closing price manipulation on option expiration days: an empirical study

4.1 Introduction

The goal of this chapter is to explore the inter-linkage between exchange-traded equity options and the underlying stocks and to investigate the existence of patterns consistent with options writers manipulating the underlying closing price on expiration days to ensure that options with short positions expire out of the money (OTM).

To the best of the author's knowledge, this study is one of the very few investigating the existence of systematic marking the close manipulation patterns on option expiration days. Ni et al., (2005), who investigate US markets, provide the only other contribution that can be comparable to that of the current study. Additional elements of novelty are as follows. The study focuses on the trading behaviour within the closing call auction, which is a trading phase that has acquired significant importance in terms of the volume traded during the day.[1] The study employs a unique data set that comprises the identities of market participants trading both on the option and on the spot equity markets. This creates the possibility of investigating if a specific class of traders, who generate general market abuse concerns, is behaving strategically, in a manipulative sense.

The chapter is organised as follows. Section 4.2 provides a review of the academic literature, while Section 4.3 presents examples of prosecuted cases of closing price manipulation in developed markets. Section 4.4 discusses details of the specific type of manipulation that is investigated in the current work, presents the hypotheses tested throughout the study, and finally provides an overview of the data. The empirical framework used to test the existence of market-wide patterns coherent with manipulation is provided in Section 4.5; results are also discussed. Section 4.6 investigates the behaviour of the class of proprietary trading firms. Section 4.7 concludes the chapter.

[1] Raillon, (2019) presents a report on the growing amount of volume traded in the closing auction with respect to the rest of the trading sessions. As noticed by that author, even though the analysis is conducted on the French market, the phenomenon seems to be common to other European markets. A description of this structural trend taking place in the UK equity market is provided in Chapter 3, Appendix 3.A.

4.2 Literature and case law review

4.2.1 Empirical research on marking the close manipulation

The empirical literature on marking the close manipulation can generally be divided into two groups, depending on whether the data set used includes data on actual prosecuted cases of manipulation.

The first group of papers comprises empirical investigations that rely on data sets based on actual prosecuted manipulation cases. The general objective of these papers is to explore and describe the main features of manipulation behaviours and how they differ from normal behaviour. These papers tend to be rare, reflecting the scarcity of labelled data sets on market manipulation.

Comerton-Forde and Putniņš, (2011) build a data set of marking the close manipulations by manually reviewing litigation cases pursued by the US Securities and Exchange Commission (SEC) between January 1st, 1997 and January 1st, 2009. Instances of closing price manipulation are found to be associated with a large increase in the returns at the end of the day, with a tendency for prices to reverse back by the following trading morning. An increase in trading volume and spreads at the end of the day is also observed for manipulation instances. Based on these findings, the authors propose an index to measure the probability of the closing price of a stock being manipulated. Using a similar data set, Comerton-Forde and Putniņš, (2014) use detection-controlled estimation techniques to study the determinants of closing price manipulation. The authors find that illiquid stocks with a high level of information asymmetry and low volatility represent the most likely target of marking the close manipulation. Month- and quarter-end days are also associated with a higher likelihood of manipulation. Finally, the authors estimate that for every instance of prosecuted manipulation in their sample, between approximately 308 and 326 instances of manipulation remain non-prosecuted or undetected. Comerton-Forde and Rydge, (2006) analyse 25 cases of marking the close manipulation conducted during the closing call auction across six developed markets. They show that all the cases in the sample share some common characteristics. More precisely, manipulation is conducted through the submission of large, unrepresentative orders during the last seconds of the auction. They compare different matching algorithms and show that manipulation can still be successful irrespective of the matching system used, despite potentially being very expensive for liquid stocks.

Chau et al., (2020) use a sample of 123 manipulation cases on the Hong Kong Stock Exchange to describe an order-based marking the close type of manipulation. The manipulation strategy studied in their paper differs from the usual depiction of marking the close manipulation, as the closing price is manipulated without any actual trade, but only by using limit orders that are not intended to be executed. This type of manipulation is conducted on illiquid stocks through the submission of artificially high bids in the last minutes of the trading day. The authors show that this manipulation can lead to significantly higher abnormal returns and to a reduction on both market depth and quoted spreads at the end of the day. No change in trading activity is observed (as expected considering the order-based nature of the manipulation strategy) and manipulated prices do not tend to reverse back by the following trading day.

The second group of empirical investigations is significantly more numerous and does not rely on the use of labelled data on manipulated cases. Their main goal is to investigate the existence of trading activity and price patterns coherent with behaviour aimed at manipulating the closing price. They are generally based on the definition of an event that can induce incentives to manipulate the closing price and subsequently study the existence of any associated anomalous patterns. As explained in previous sections, several different factors can generate an incentive to manipulate the closing prices. The event analysed depends on the type of incentive studied.

Ni et al., (2005) provide one of the most in-depth investigations on price manipulation due to options expiration days. The authors investigate the tendency of the price of US optionable stocks to cluster around the strike price on the expiration day. They show that options strike prices tend to function as a magnet for the underlying stock's price. This phenomenon, known as stock pinning, is explained in the paper as the by-product of two main factors: (1) hedging activities conducted by market makers and, most importantly for the purposes of this study, (2) market manipulation conducted by proprietary traders who are option writers. The authors report, in fact, an increased likelihood of pinning when proprietary traders write options in the week leading up to expiration. They also find an increased probability of observing an option expiring out of the money (OTM), conditionally on being in the money (ITM) at the close of the day prior to expiration, if proprietary traders are option writers during the expiration week. They interpret these findings as price manipulation being conducted by option writers to ensure the options expire out of the money (OTM) (i.e., as being worthless), so they can avoid the liability connected to the exercise by option holders.

Park et al., (2019) study the effect that the introduction of a plain vanilla[2] closing call auction has on the Hong Kong Stock Exchange. They show that, although closing prices are more accurately priced under the closing auction mechanism, they are more prone to manipulation attempts. Under the closing auction mechanism, more pronounced price reversals are observed following days when manipulation is likely to have occurred or is more likely to occur, that is: (1) when large price changes occur during the final 10-minute interval; (2) on days when sniping attacks (i.e., instances of sudden price changes or volume increase during the last seconds before the determination of the closing price) are observed; and (3)

[2]That is, without any proper mechanism designed to reduce manipulation such as, for example, volatility extensions and random auction deadlines.

when capable bull/bear contracts (CBBCs)[3] expire. E. H. Y. Chow et al., (2013) study the expiration day effects of index futures on the underlying spot market, finding that proprietary traders earn substantial profits at expiration, suggesting the possibility that closing prices, used as settlement prices, are subject to manipulation.

Carhart et al., (2002) and Ben-David et al., (2013) investigate the existence of abnormal patterns on key-reporting days, with these patterns coherent with manipulation conducted by fund managers with the objective of artificially increasing funds' performances. Month- and quarter-end days are the main days of interest, as funds' performance are generally published on either a monthly or quarterly basis; both studies focus on the US market. Carhart et al., (2002) investigate the prices of equity mutual funds between July 1992 and July 2000 and discover the existence of abnormal returns on quarter-end days (which are of even stronger magnitude on the last quarter of the year) and of abnormal price reversals on the following days. The hypothesis that these patterns are due to manipulation activities is further supported by the evidence that the quarter's best-performing funds are the ones experiencing the largest quarter-end abnormal return reversals. The same is true for the year's best performing funds, experiencing the largest abnormal returns on the last trading day of the year. In addition, most of these patterns are due to abnormal price changes in the last 30 minutes of the trading day, suggesting that closing prices are indeed the target of manipulation. This behaviour was subsequently modelled and rationalised by Bernhardt and Davies, (2009).

Similar evidence is found by Ben-David et al., (2013). They show that stocks characterised by high hedge-fund ownership present abnormal returns over the last minutes of quarter-end days, which subsequently reverse back by the following trading morning. They also document, on the same days and for the same stocks, an abnormal percentage of large buy orders executed in the last minutes of trading. The same pattern is not observed for sell orders, suggesting that the increase occurs only in buying pressure, possibly aimed at inflating the stock prices and, as a consequence, the funds' value.

Zdorovtsov et al., (2017) document the existence of abnormal trading behaviour and price movements around FTSE Russell reconstructions. They justify the findings in light of potential manipulation aimed at obtaining inclusion of certain stocks into an index or at avoiding the exclusion from it.

Felixson and Pelli, (1999) empirically investigate the existence of abnormal price returns in the Finnish market associated with trading by large investors holding positions in the stock before closing. According to the authors, these traders have the incentive to manipulate the closing price in the direction that would lead to an increase in the value of their position. The final goal of the manipulator is to artificially inflate their performance, which is measured based on closing prices. However, the results are not significant. A very similar framework to the one proposed by Felixson and Pelli, (1999) is applied by Kadioglu et al.,

[3]Callable bull/bear contracts are derivatives that are settled at expiration based on the closing price of the underlying.

(2015) to the Turkish market (Borsa Istanbul), over a time period that includes the introduction of a call auction for the determination of closing prices. The authors find evidence supporting the hypothesis that traders holding large, long (short) positions on a stock tend to manipulate the closing price upwards (downwards). They also find that the introduction of a closing call auction has significantly reduced marking the close manipulation. Alexakis et al., (2021) apply a similar approach on the Athens Stock Exchange. The incidence of closing price manipulation from traders with substantial exposure is investigated around two events: (1) the introduction, in November of 2005, of a call auction to determine the closing price in place of the volume-weighted average price (VWAP) and (2) the elimination (in January 2017) of the price tolerance range attribute of the closing auction. The authors find that the introduction of a closing call auction is associated with a significant decrease of marking the close manipulation, although it was not eliminated. However, the second event (i.e., the abolition of the price tolerance rule) only had a marginal effect.

M. Aitken, Cumming, et al., (2015) define a closing price as being dislocated if the price variation in the last 15 minutes (plus the change over the closing auction) is above four standard deviations of the relevant distribution computed in a benchmark period and the price reverses the next day by more than half. They study the relationship between the likelihood of dislocation of closing prices and the rise of high-frequency trading (HFT) across 22 different markets and show that HFT is negatively correlated with the probability of dislocation. Similar measures are used as a proxy for marking the close manipulation by several other authors. Cumming et al., (2020), for example, examine if marking the close manipulation, measured by similar measures as those used by M. Aitken, Cumming, et al., (2015), affects the likelihood of M&A deal withdrawal and the premiums paid. Based on a data set including M&As from 45 different countries between 2003 and 2014, the authors find that closing price manipulation of the target stock leads to an increase of uncertainty and volatility of prices, reducing M&A premiums and increasing the likelihood of the withdrawal of already announced deals. Using similar closing price dislocation metrics, M. J. Aitken, Aspris, et al., (2018) investigate the UK and French markets around the introduction of MiFID I, which led to an increase in algorithmic trading in the European marketplace. The authors find that an increased liquidity and reduced price impacts due to algorithmic trading had the effect of reducing the likelihood of closing prices being manipulated. Similar metrics are further used by the same group of authors: see M. J. Aitken, F. H. Harris, et al., (2009) and M. J. Aitken, Frederick, et al., (2015)[4].

Finally, T. Y. Hsieh, (2015) investigates the impact of an increase in pre-closing information disclosure on the incidence of closing price manipulation, in the Taiwan Stock Exchange Corporation (TSEC). Until February 2012, the closing price was determined through a call auction, that took place at the end of a five-minute pre-closing period. During this five-minute period, orders were accumulated in the order book, but prices and volumes figures were not disclosed. Starting from 20 February 2019, an order-matching algorithm was

[4]These empirical investigations can be interpreted as an effort to test and further develop a framework to assess market quality, as proposed by M. J. Aitken and F. H. d. Harris, (2011).

implemented: (1) the best bid and ask prices resulting from the matching algorithm were disclosed during the pre-closing period and (2) if the change of price during this period was more than a certain threshold, the pre-close would be extended. The author finds that these changes (i.e., increased pre-closing information disclosure) significantly reduced closing price manipulation instances, as well as improving other market quality measures (related to trading costs and end-of-day volatility). A closing price is defined as manipulated if an overnight price reversal is observed and either an abnormal return or an abnormal amount of volume traded are observed over the last five-minute period before market close.

4.2.2 Anomaly detection applications

This section briefly mentions the branch of the literature that builds upon both theoretical and empirical research on market manipulation and proposes models to automatically detect market manipulation patterns. The application of anomaly detection methodologies in the financial domain is not new: comprehensive surveys on this subject are provided by Ngai et al., (2011) and Ahmed et al., (2016). Golmohammadi and Zaiane, (2012) provide a more in-depth review of anomaly detection methods for market manipulation detection and discuss some of the main problems that can be encountered in the development of these models. The absence of data sets comprising labelled market manipulation cases for both model calibration and testing purposes is one of the main limitations (a point already raised by Putniņš, (2012)).

Several papers propose methodologies to detect marking the close manipulations (or, more generally, trade-based manipulation schemes which can also be applied for the identification of closing price manipulation). Öğüt et al., (2009) investigate trade-based manipulations on the Istanbul Stock Exchange. A data set of manipulated stocks is collected by manually examining summaries published in the *Capital Markets Board (CMB) Weekly Bulletins* on manipulation cases between January 1995 and March 2004. The difference between the stocks' and respective index's daily returns, daily changes in volume traded and daily volatility measures are used as input variables. Among the four models tested (multiple discriminant analysis, logistic regression, neural networks and support vector machine), neural networks and support vector machine (SVM) are shown to perform better both in terms of overall accuracy and sensitivity. Mongkolnavin and Tirapat, (2009) use association rules to detect market participants who are potentially marking the close on the Thai bond market. Specifically, the authors study the relationship between a market participant and the transaction order, suggesting a flag be raised for further investigation when a form of association is found between the traders and the time of day at which they tend to trade, especially if the trades tend to take place towards the end of the trading day.

Diaz et al., (2011) manually build a data set of manipulated stocks using SEC litigation cases and filtering by using keywords such as, among others, *marking the close*. This data set is then used to train tree-based classification algorithms by using input variables comprising financial variables, ratios and information on news events on both an hour- and a second- level. The results support several findings reported in the literature and, most importantly from a market abuse detection perspective, they highlight the importance of monitoring both volatility and sudden changes in traded volume in relation to trade-based manipulation. The same data set is used in a subsequent study conducted by Golmohammadi, Zaiane, and Diaz, (2014). If Diaz et al., (2011) are primarily concerned with model explainability (and for this reason, they base their work on tree-based models), the prime concern of Golmohammadi, Zaiane, and Diaz, (2014) is the maximisation of detection accuracy. The performances of a wide variety of models in terms of sensitivity, specificity and F_2 measure are compared. The authors show that decision trees produce very accurate results, but the Naive Bayes model performs best in terms of sensitivity and F_2 measure.

Kim and Sohn, (2012) use peer group analysis to detect general trade-based manipulation and propose the identification of potentially manipulated stocks by comparing the dynamic of the closing price of a target stock to that of similar securities. A very similar approach is proposed and tested on the US market (on S&P 500 stocks) by Golmohammadi and Zaiane, (2015). A data set of more than 40 years of both daily and weekly price returns is used. Finally, Li et al., (2017) use information released by China's Securities Regulation Commission on market manipulation cases on the Chinese stock market to build a labelled data set. Both daily and tick data are used to design the input variables and several supervised learning models are trained and compared. While the results based on daily data show good performance, the use of tick data leads to poor results. As explained by the authors, the reason might be that the information on manipulation cases used is not sufficiently granular to precisely label the data as normal or suspicious at a tick level.

4.3 Case law review

4.3.1 SEC v. Saba (2004)

SEC v. Saba (2004) is the closest case (of the ones analysed in this section) to the type of behaviour investigated in the current study. On 25 February 2004, an investor named Moises Saba Masri was accused of manipulating the closing price of Azteca American Depository Receipts, listed on the New York Stock Exchange (NYSE). As per the SEC report, in the first half of the year in 1999, Mr. Saba sold a large number of put options on Azteca American Depository Receipts, with strike prices of US$5 and US$7.50 and expiring on 20 August 1999. Mr. Saba was therefore the option writer of these put options. He obtained a cash premium by selling them and no liability would be incurred at expiry if the price of the underlying stock was above the strike price of the option.

On 20 August 1999 (i.e., the expiration day for the options sold by Mr. Saba) the price of the underlying stock was moving between the two extremes of US$4.9375 and US$5.1875. The put options with strike price US$7.50 were therefore expected to be exercised by their holder, being deeply in the money (ITM). Nothing could be done by Mr. Saba to avoid this liability. However, based on SEC's accusations, Mr. Saba evaluated as feasible the possibility of manipulating the options with a US$5 strike price, by pushing the underlying stock price above this level. The stock price was sufficiently close to the strike price for this type of manipulation to be achievable. He therefore instructed his broker to begin buying the underlying stock during the last 10 minutes of trading, with the goal of causing the stock price to close above US$5. The amount needed to manipulate the price was considerable, as Mr. Saba purchased 200,000 shares of the underlying stocks, accounting for approximately 94% of the buy-side activity for that security during the last hour of trading. As per the SEC report, if the holders of the put options with a US$5 strike price exercised their options, Mr. Saba would have incurred a further liability of US$4.3 million.

4.3.2 FSA v. Goenka (2011)

An iconic manipulation case in the UK is represented by *FSA v. Goenka* (2011). It led to the highest fine by the Financial Services Authority (FSA)[5] at the time and it occupied major headlines[6]. The case is also discussed in Kovas, (2015).

As per the FSA's final notice, the investor accused of manipulation was an Indian businessman living in Dubai named Rameshkumar Satyanarayan Goenka. Two structured products were purchased by Mr. Goenka in the year 2007. Both structured products had a cost of US$10 million and the final payoff of each of them was dependent on the value at expiration of a basket of underlying stocks. The first structured product had an expiration day of 30 April 2020 with the payout depending on the performance of three global depositary receipts (GDRs) traded on the London Stock Exchange (LSE) and related to the following Russian companies: Gazprom, Lukoil and Surgutneftegaz. The second product had an expiration day of 18 October 2010, and final payoff depending on three GDRs (traded on the LSE) related to the Indian companies: Reliance Industries, ICICI Bank and HDFC Bank. All the GDRs comprising the basket underlying the two structured products were traded on the International Order Book (IOB) of the LSE.

The final payout of both the structured products discussed above depended on the closing price on expiration day of the worst performing GDR of the underlying basket. The worst performance asset of the basket was called the *laggard*. Three different scenarios were possible. If the closing price of the *laggard* was higher than its initial price (i.e., the price measured at the time the structured product was bought), then Mr. Goenka would receive the US$10 million of the face value plus an uplift, dependent on a formula[7]. This represented a win scenario for Mr. Goenka. However, if the closing price at expiry of the *laggard* was below its initial price, but above a specific *knock-in* price, stipulated on the contract, Mr. Goenka would receive a payout of US$10 million, equal to the initial cost of the product.

[5]The Financial Services Authority (FSA) was responsible for the regulation of UK markets until 2013, when it was abolished. Since then, the Financial Conduct Authority (FCA) has the responsibility to ensure the integrity of UK financial markets.

[6]Like Bloomberg, The Financial Times and BBC.

[7]The specifics of the formula were not described in the FSA report.

This was a break-even scenario for Mr. Goenka. Finally, if the *laggard*'s closing price on expiration day was below the *knock-in* price, Mr. Goenka would receive a pre-determined number of GDRs of the *laggard*, amounting to a considerable loss with respect to the face value of US$10 million. This was a loss scenario from the perspective of Mr. Goenka.

In April 2010, approaching the maturity date of the first structured product on 30 April 2010, the GDR of Gazprom was the *laggard* stock within the underlying basket of GDRs. The price of Gazprom during the month of April was ranging between US$23.10 and US$25.57, which was close to the *knock-in* price of US$23.91. If the closing price of Gazprom on 30 April 2010 was at least above US$23.91, Mr. Goenka would have broken even. In early April 2010, Mr. Goenka approached (indirectly through a London-based adviser) a London-based broker to discuss the practicalities of trading on Gazprom on 30 April 2010 to ensure its price closed above the *knock-in* level. Mr. Goenka and the London-based broker agreed on an approach to manipulate the Gazprom GDR's closing price. First, Mr. Goenka would execute (only if necessary) large enough trades on the GDR before the start of the closing auction on the London Stock Exchange. This would ensure that the price of the GDR would remain close enough to the *knock-in* to be eventually manipulated in the closing auction, at the end of which the closing price would be determined. Then, Mr. Goenka would place substantial buy orders in the final seconds of the closing auction to ensure the closing price would be above the level desired. Despite Gazprom's price being close to the *knock-in*, just before the start of the closing auction an announcement made by Russian President Putin about a proposed merger between Gazprom and the Ukrainian gas company Naftogaz caused the price of Gazprom to fall of approximately 2%. After the event, Mr. Goenka decided to not pursue his manipulation goals. The price was too far below the *knock-in* price for manipulation to be successful. In the FSA notice, a communication between Mr. Goenka and his financial advisor is reported in which Mr. Goenka stated that *"basically, what happened is the Putin news came out... the stock went down 2% so we can't do anything"*.

The expiration day of the second structured product was 18 October 2010. Approaching the maturity date, Reliance GDR was the *laggard* among the GDRs comprising the underlying basket of assets. On the expiration day, the opening price of the Reliance GDR on the LSE was US$47.20. The *knock-in* price was US$48.65. If the closing price of Reliance was above this level, Mr. Goenka would have broken even. Otherwise, he would have suffered a significant financial loss of several millions of dollars. The same broker involved in the manipulation attempt of the Gazprom GDR was also involved in this attempt. The plan to manipulate the closing price was similar to the plan already organised for Gazprom (which was not successful due to external and exogenous factors). Mr. Goenka would have executed buying trades in the minutes before the start of the closing auction to push the price of Reliance as close to the *knock-in* level of US$48.65 as possible. During the closing auction, it was planned to submit a series of large bid orders to ensure the closing price was above the desired level.

On the expiration day, although the opening price was below the *knock-in* level, the price of Reliance increased throughout the entire trading session, reaching the level of US$48.28

(just US$0.37 below the desired price) approximately 10 minutes before the start of the closing auction, without any intervention by Mr. Goenka. Manipulative behaviour came into play during the closing auction, during which Mr. Goenka, along with his broker, orchestrated the submission of a number of orders that, in their entirety and as per FCA's report, would have equated to approximately 280% of the volume traded on an average day for the Reliance GDR. These orders had the effect of pushing the closing price to US$48.71, slightly above the *knock-in* level necessary for Mr. Goenka to break-even on his contract and avoid a significant loss. The following day the price dropped back to US$47.10 following the sale by Mr. Goenka of the entirety of the block of the Reliance GDR acquired at the end of the prior day.

4.3.3 SEC v. Athena Trading (2014)

In 2014, a high-frequency trading firm based in New York City and called Athena was accused by the SEC of manipulating the closing price of thousands of stocks listed on Nasdaq in (at least) a six months period from June to December, 2009: see *SEC v. Athena Trading* (2014).

Nasdaq provides the possibility of submitting *on-close* orders, that is, orders that are only filled at the market close. At 3:50 p.m. (10 minutes before the close at 4:00 p.m.), Nasdaq releases an Imbalance Message for each stock, which contains information such as the imbalance size (i.e., the number of shares that Nasdaq predicts will remain unfilled at the close) and direction (either buy or sell depending on whether the orders predicted as unmatched are buy or sell orders). This message is then updated every five seconds for the following 10 minutes. Finally, at 4:00 p.m., the closing price is computed. A peculiarity is that during this 10-minute interval, the continuous order book is still active. The algorithm used to determine the closing price was designed to maximise the number of traded shares (based on both on-close orders and orders submitted during the continuous market) at a price as close as possible to the last trade on the continuous book before the close.

The manipulative strategy employed by Athena consisted of taking advantage of the Imbalance Message released by Nasdaq and the associated price changes that could be expected. If an imbalance occurred on the buy side, for example, it was rational to expect a positive price change in the last minutes due to a stronger pressure on the demand side. As stated in the SEC report, the initial manipulation strategy deployed by Athena was quite simple. If Nasdaq released an Imbalance Message showing a buy imbalance of X shares in a specific stock, Athena then placed a sell Imbalance-Only-On-Close order (i.e., an order that was meant to be executed at the market close only if an imbalance was present at that time) for X shares and tried to buy the same amount X of shares on the continuous order book before the close. In this way, Athena would be able to close its position flat at the end of the day (they would buy and sell the same amount of shares) and profit from selling at a price higher than what it paid to buy the shares. The difference in price would come from two main sources: (1) the expected price change connected to the existence of an imbalance, which is not manipulation in itself; and (2) the price impact related to the accumulation of shares (to then offload at the close) during the last minutes of continuous trading, which is the core of Athena's manipulation activities.

Athena further refined the strategy described above to improve the timing of the shares accumulation on the order book to ensure priority in the auction, thus avoiding partial fills of the on-close orders submitted. After the first Imbalance Message was released, Athena placed large orders to profit from the expected price changes connected to the just published imbalance. After that, small amounts were accumulated over the next minutes, eventually followed by a large amount of orders submitted in the very last seconds (or milliseconds) of trading before the close to affect the closing price. As stated in the report, most of the manipulative orders (in terms of size) were submitted in the last two seconds before the end of the closing call auction. In the six-month period during which Athena is accused of manipulation, as stated in the SEC report, Athena's trades in the last two seconds accounted for 73% of the entire traded volume on Nasdaq in this two-second time frame for the corresponding stocks.

It is important to note that, in this case, the impact of Athena's manipulative trading strategies, albeit able to generate extra profits, did not tend to be excessive. As per SEC's report, one of the concerns with Athena was to raise regulators' attention to price movements that appear to be abnormal. Although the profit obtained by manipulating a specific stock on a specific day was minor, the rent extracted by manipulating thousands of stocks over a period of several months was extremely high. The attention to limiting the impact on the price was likely the main reason why Athena was able to manipulate such a high number of stocks on a systematic basis for so long. Furthermore, stocks with various liquidity profiles were manipulated, from less liquid stocks (for which the impact generated by trading is higher) to very liquid stocks like Ebay.

4.3.4 CFTC v. Optiver (2008)

CFTC v. Optiver (2008) is a case opened by the Commodity Futures Trading Commission (CFTC) against Optiver, a well known HFT firm. Optiver was accused, and subsequently fined, for manipulating the settlement prices of futures in Crude Oil, Heating Oil and New York Harbor Gasoline during March 2007 on the New York Mercantile Exchange (NYMEX).

Futures settlement prices on NYMEX are defined as the volume-weighted average price (VWAP), computed based on the trades executed at the closure of the trading day, that is, between 2:28 p.m. and 2:30 p.m. (Eastern Time [New York]). As per the CFTC report, throughout the day, Optiver would place a series of orders with the intention of accumulating a net long or short Trading at settlement (TAS) position. Trading at settlement (TAS) contracts are futures contracts with a price equal to that day's settlement price plus an agreed spread. These contracts are simple futures contracts and TAS positions can be simply offset by trading futures for the same size and in the opposite direction.

Between 2:25 p.m. and 2:28 p.m. (defined in the report as pre-close session), Optiver started to execute futures trades in the opposite direction to the accumulated TAS position. Based on CFTC's investigations, Optiver traded approximately 20-30% of the futures part of the manipulation during the pre-close. The other 70-80% of the manipulative trades were executed during the close, that is, during the last two minutes of trading that determine the settlement price. The objective was to exploit the ability to impact the market to produce a differential as wide as possible between the pre-close prices and the settlement prices, from which the value of the TAS contracts accumulated throughout the day depends. The amount traded in the last five minutes of trading (pre-close and close) and the size of the TAS positions had to be close to each other, as the goal of the manipulator was to close flat at the end of the trading day, obtaining the net profit from the difference between the settlement price (manipulated in the favourable direction) and the pre-close price (which is a clean value still not affected by manipulation).

4.3.5 Remarks

The manipulation cases presented and discussed above cannot be used to provide a comprehensive review of marking the close manipulation strategies, as the sample is too small. However, they offer interesting insights on how manipulation can be conducted. An interesting collection of case studies on market abuse patterns can be found in a series of reviews produced by the Fixed Income, Currencies and Commodities Markets Standards Board (FMSB).[8]

First, it is possible to notice how manipulative trading is heavily concentrated near the time the closing price is defined. The manipulator acts in the last few minutes, or even seconds (as is clear from the Athena case, in which most of the trading activity was executed in the last two seconds of the closing auction).

Furthermore, manipulators can be individual investors (albeit, at least in the cases above, with a certain level of sophistication that sets them apart from common retail traders) or sophisticated algorithmic/HFT firms. Some of these HFT firms can also act as market makers, like Optiver. Several papers (e.g., Pope and Yadav, (1992)) are sceptical about the fact that markets can be manipulated by such big players, as they are subject to tight surveillance. In reality, however, this appears a real possibility.

Liquid stocks can also be manipulated. Although it is reasonable to assume that less liquid stocks are a more likely target of manipulation (as the price is easier to affect), the possibility cannot be ruled out of liquid stocks being manipulated, even in developed markets. Athena manipulated a variety of stocks, which included liquid ones (like the Ebay stock, as discussed in the SEC report).

[8]See `https://fmsb.com/fmsb-publishes-new-financial-markets-misconduct-research/` and `https://fmsb.com/publication/publication-selected-case-studies_v21/`

To conclude, closing price manipulation is not necessarily associated with large price movements. Large price movements can be necessary for the manipulation to be successful, but manipulation can be successful even with minor dislocations in the closing prices, especially if repeated on a systematic basis. This can be seen in *SEC v. Athena Trading* (2014) or *CFTC v. Optiver* (2008). The former is quite interesting from this point of view as the manipulator, being aware of the risks related to manipulation performed on a continuous basis, was - as per the SEC report - careful to avoid extreme price movements, while keeping the manipulative strategy successful. These considerations open the possibility that measures used to identify marking the close cases which are based on unusual price movements might only catch a portion of the actual manipulative behaviour taking place in the market.

4.4 Hypotheses development and data

4.4.1 Options and manipulation schemes at expiration

Almost all UK-listed options on single stocks are physically settled at expiration (see Table 4.15 in the Appendix). Physical delivery, with respect to cash settlement, produces different incentives to manipulate the underlying stock at expiry. As noted by both Ni et al., (2005) and Danger et al., (2020), option holders have no incentive to manipulate the underlying price. Traders holding a long position on a call option do not receive any extra profit from pushing the price upwards. If they do so, they are artificially inflating (i.e., overvaluing) the price of the underlying stock, for which they have an option to buy. To realise a profit, manipulators should exercise their options, acquire the underlying stocks and finally sell the entire position (given by the sum of the stocks acquired with the intent of manipulating the price and of the position acquired through exercising the options) for a price higher than the costs necessary to acquire the stocks. Selling large positions, however, generates an impact on the price, making this strategy unlikely to be profitable. A similar logic applies to put option holders, considering the symmetry of call and put options.

The case of option writers is different and, in the case of physical delivery, more interesting to study from a market manipulation perspective. Danger et al., (2020) advance the idea that market participants holding short positions on out of the money call options and a long position on the underlying stock, have the incentive to affect the price of the underlying price to ensure the options expire in the money. The idea is that manipulators want to liquidate their inventory of the underlying stock and they can do so at a higher than market price by pushing the option in the money and, by doing so, tricking the option holder to exercise the option. This strategy is profitable if the costs paid by the manipulator to push the price upwards are below the strike price received.

Danger et al., (2020) also discuss the incentives to manipulate that arise from holding a combination of different options on the same underlying stock. The current study redirects the reader to that paper for further details, as the authors analyse several possible option combinations. Herein, it is sufficient to note that, for stock options with physical delivery, any form of manipulation scheme at expiration, to be successful, requires the ability to sufficiently affect the price of the underlying stocks for the target option to change moneyness, either from ITM to OTM, or vice versa. Therefore, for manipulation to be a rational strategy *ex ante*, the underlying stock price needs to be close enough to the strike price for the manipulator to be able to push the underlying stock above (or below) the strike price, in the desired direction. This is a characteristic that is discussed in Section 4.3.2, where the manipulator decided to cease any manipulation attempt when the underlying price moved too far away from the strike price.

Finally, Ni et al., (2005) discuss the incentives to manipulate option writers with a short position on options that are ITM prior to expiration. These market participants have the incentive to manipulate the underlying stock price in the direction that leads the options to expire OTM (i.e., worthless). By doing so, they obtain a premium from selling the options and avoid any liability by preventing the option holders from exercising the options. Obviously, the loss that the manipulators expect to avoid has to be higher than the expected costs of manipulation. This is exactly the type of manipulative behaviour described in Section 4.3.1.

This type of manipulation strategy is also in line with the so called *maximum pain theory*, according to which option traders who have net written positions close to expiry, trade in a way that pushes the underlying price in the direction for which the highest number of options issued expire worthless: see, for example, Davies, (2020) and Bar-Yam et al., (2014). This is also the manipulative strategy investigated in the current study. Several reasons lead to the focus on this type of behaviour. Firstly, evidence that this type of behaviour is taking place in the marketplace is provided both in the literature[9] and in previous prosecuted cases in developed markets. The evidence in the literature is, however, quite circumstantial and the prosecuted cases do not provide any evidence of whether this phenomenon happens systematically. Secondly, this type of behaviour is in line with the manipulative behaviour implied by the *maximum pain theory*, which is a common concern in the industry. The current study seeks to, at least partially, address these concerns.

This manipulative strategy is also well framed within the Regulation (EU) No 596/2014[10] of the European Parliament and of the Council of 16 April 2014 on market abuse, generally known as MAR (standing for Market Abuse Regulation). A list of indicators of manipulation is provided in Annex I, but this list must be considered as non-exhaustive. The manipulation indicators should be evaluated by regulators within the context in which they are observed and on a case-by-case basis. Relevant to the type of manipulative behaviour investigated in the current study, the MAR advises financial regulators to take into consideration: *[t]he extent to which orders to trade are given or transactions are undertaken at or around a specific time when reference prices, settlement prices and valuations are calculated and lead to price changes which*

[9] See Ni et al., (2005).
[10] `https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32014R0596&from=EN`.

have an effect on such prices and valuations.[11] Further, technical advice was released by the European Securities and Markets Authority (ESMA) in 2015, with the purpose of further specifying and clarifying the interpretation of some of MAR's general provisions.[12] Among its contributions, this technical document contains a more detailed list (still non-exhaustive) of market manipulation indicators to further complement the list already contained in MAR's Annex I. Among these, one captures in great detail the type of manipulative behaviour investigated in the current study. According to ESMA, in fact, financial regulators should consider suspicious and subject to subsequent analysis: *[t]ransactions on any trading venue which have the effect of, or are likely to have the effect of, modifying the price of the underlying financial instrument, related spot commodity contract, or an auctioned product based on emission allowances, so that it surpasses/not reaches the strike price or other element used to determine the pay-out (e.g. barrier) of a related derivative at expiration date*.

4.4.2 Hypotheses

The current study investigates the existence of closing price manipulation patterns compatible with manipulative behaviour, as discussed by Ni et al., (2005)[13]. The authors argue that, assuming option holders decide to exercise the option at expiration based on the underlying closing price, physical delivery can induce option writers to manipulate the underlying closing price on expiration days so that their options expire worthless.

If this phenomenon takes place on a systematic basis on UK markets, this study expects to observe, on option expiration days, abnormal price changes when options are in the money (ITM) and, at the same time, are sufficiently close to the money for manipulation to be achievable *ex ante*[14]. The price change is expected to be in the direction that would make the options expire out of the money (OTM) and, based on the review of both prosecuted cases and the academic literature, this study expects manipulation attempts to be concentrated at the end of the trading day. The first hypothesis is therefore formulated:

Hypothesis 4.1 *Stocks with active options that are in the money (ITM) and sufficiently close to the money towards the end of the expiration day, exhibit abnormal returns in the direction that makes the options expire out of the money (OTM);*

Hypothesis 4.2 *Stocks with active options that are in the money (ITM) and sufficiently close to the money towards the end of the expiration day, exhibit abnormal price reversals the day following expiration.*

If a price change is the result of an attempt to push the price to an artificial level, the study expects to observe abnormal volume and associated strategic trading behaviour. This

[11] Market Abuse Regulation (MAR) Annex I A(f).
[12] `https://www.esma.europa.eu/sites/default/files/library/2015/11/2015-224.pdf`.
[13] And as described in Section 4.4.1.
[14] This is an element of discussion in Danger et al., (2020).

can be rationally expected from a trade-based type of manipulation, such as the one investigated in this study, which relies on trading large volumes to generate an impact on the price. This is a common feature of the prosecuted manipulation cases explored in Section 4.3 and it is also in line with previous research on this topic (Carhart et al., (2002); Ben-David et al., (2013)). Diaz et al., (2011) conduct a case study on the application of data mining techniques for market manipulation detection, highlighting the fact that, in the context of their study, trading volume patterns represent a more important feature than abnormal returns in identifying the manipulation instances in their data set. The following hypotheses are tested:

Hypothesis 4.3 *Stocks with active options that are in the money (ITM) and sufficiently close to the money towards the end of the expiration day, exhibit trading of abnormal volume towards the end of the trading day.*

Hypothesis 4.4 *Stocks with active options that are in the money (ITM) and sufficiently close to the money towards the end of the expiration day, exhibit the submission of abnormally large orders during the closing auction.*

The hypotheses formulated above are aimed at investigating the existence of market-wide, systematic patterns in line with how the market would behave in the case of manipulation. The study then goes further and explores the behaviour of a specific class of market participants, namely, proprietary trading firms. Proprietary trading firms are market participants that uniquely invest their own capital, without any clients involved. Several reasons prompt the decision to focus on this class of market participants. Firstly, they are the most active traders in the options market, as seen in Table 4.1. Proprietary traders are more active than other classes of traders with respect to the number of different underlying stocks traded on average for a specific expiration day (on average, they trade options on almost 23 different stocks, more than double the amount traded by agency brokers) and, for a specific underlying stock and expiration day, they trade almost six different option contracts. The average size of their trades is lower than for other classes of traders, but they participate in more than half the total number of option trades (despite only 19 proprietary traders being in this study's data set). Although this might be an expression of legitimate trading behaviour, for instance, in line with market making, this also means that these traders are the ones most likely to be in a position where they have the incentive to manipulate the underlying stocks.

Secondly, proprietary trading firms are sophisticated traders with the ability to strategically act in the marketplace. The intention is to seek signs of systematic market manipulation connected to the options market and proprietary traders appear to be the traders with both the highest incentives and the tools to successfully manipulate the market. Section 4.3 presented and described two cases of well-known HFT firms systematically manipulating the closing price in developed markets and for a prolonged period of time. This raises interest in the question of whether the UK market is also affected and how widespread this phenomenon is. Finally, the choice to focus on these traders is also due to data quality concerns[15]. The hypothesis tested is as follows:

Hypothesis 4.5 *Proprietary traders holding options that are in the money (ITM) and sufficiently close to the money prior to expiry, trade more aggressively in the direction that would make the options expire out of the money (OTM) and this behaviour produces the desired price change.*

4.4.3 Options data

The initial data set used consists of the financial instrument reference information that is reported to the Financial Conduct Authority (FCA). This data set contains essential information on all the financial instruments that, as per Article 27 of MiFIR, need to be reported by trading venues to regulators. Financial instruments that are admitted to trading in a trading venue, as well as financial instruments which the underlying security is traded in a trading venue, have to be reported. An example of reportable instruments are options with an underlying stock negotiated on the London Stock Exchange (LSE).

For every reportable instrument, general information must be available, such as the International Securities Identification Number (ISIN), the Classification of Financial Instruments (CFI) code, the issuer and the venue in which it is traded, as well as the date of admission to trading and eventual termination date. Depending on the type of financial product, further information is also provided. Information reported specifically for option derivatives, for example, includes: the identifier (ISIN) of the underlying instrument, the expiration date, the price multiplier, the option type, strike price, exercise style and the delivery type.[16]

Through this data set, the following information is obtained: (1) the list of UK optionable stocks which, in this context, refers to all the stocks traded on the London Stock Exchange (LSE) as the primary market, and their options traded and (2), for each optionable stock, the list of issued listed options.

As standard practice in derivatives research, to eliminate crash months[17], March 2020 is removed from the data set, as this month is heavily affected by COVID-19 effects[18]. The final data set comprises options on 97 underlying stocks and traded on ICE and Eurex. Appendix 4.A provides the list of optionable stocks that are part of the investigation and a description of the cleaning procedure used to obtain this sample.

[15]A discussion on this matter is provided in Appendix 4.B.

[16]More detailed information on the fields that are reported to regulators as part of the financial instruments reference data can be found in the Annex of the MiFIR RTS 23.

[17]See, for example, the point raised by Golez and Jackwerth, (2012), who eliminate both October 1987 and October 2008 from the study conducted.

[18]The initial spread of COVID-19 produced the worst effects on UK (as well as European and US) financial markets in terms of abnormal volume and price changes during March 2020. The decision to remove this month from the analysis is motivated by the objective of removing confounding factors related to abnormal patterns that are not due to manipulation and that could skew the analysis.

TABLE 4.1: Summary statistics - Options market participants.

	# Traders	# Expiry Dates (Avg.)	# Underlying (Avg.)	# Contracts (Avg.)	Trade Size (Avg.)	% Trades
Individual investors	1130	3.33	1.47	1.33	7.92	6.82
Other institutions	554	5.41	2.00	1.48	89.23	19.72
Commercial bank	68	9.38	3.36	1.39	249.09	3.97
Investment bank	27	11.00	4.10	1.38	516.73	2.46
Agency Broker	20	10.60	10.34	2.19	20.87	15.57
Principal Trading Firms	19	13.42	22.54	5.54	23.64	51.44

For each class of market participants active on the options market, the table contains: (1) the number of different traders within the class; (2) the average number of different expiry dates traded by each trader; (3) the average number of underlying stocks traded by participants in the class; (4) the average number of different option contracts traded for each combination of expiration day and underlying stock; (5) the average trade size and, finally, (6) the percentage of the total number of trades in the dataset in which a member of the class is present.

Standard stock options have a monthly expiration frequency and expire on the third Friday of the expiration month. Considering the available data are from January 2018-December 2020, the final data set includes a total of 35 expiration days and 3,393 combinations of stocks and expiration days.[19]

For each option in the sample, the study also has access to a unique data set containing all the transactions on reportable options[20], which contains information regarding the identity of the market participants involved in each transaction. Every reported transaction contains the Legal Entity Identifier (LEI) for both the buyer and seller.[21] Data on market participants are used to investigate the existence of suspicious behaviour conducted by a specific class of market participants: proprietary trading firms. A curated list produced internally by the FCA is used to identify proprietary traders. To provide a broad comparison with other classes of market participants active in the options market, traders (other than proprietary traders) are classified using the Orbis data set which contains broad classifications into, for example, investment banks or commercial banks.

The open interest time series (available at a daily frequency) for each option in the sample is available through Refinitiv Elektron. Appendix 4.B reports further information on the type of options contained in the data set, as well as considerations on the quality of the data set.

[19] Of these, 23 expiration days fall between 2019 and 2020 for a total of 2,229 combinations of stocks and expiration days.

[20] Detailed information on the fields that are available in this data set are contained in MiFIR RTS 22.

[21] If either the buyer or the seller is an individual person, another type of identifier is used, but this is not relevant for the purposes of this work.

4.4.4 Equity data

Classic databases are used to obtain data on the optionable stocks included in the final sample. Intraday trade and quote data are collected from Refinitiv Tick History. For European stocks, the option is provided through this database to download a consolidated data feed. For UK stocks, price and volume data are aggregated across the four main exchanges: London Stock Exchange (LSE), Chi-X Europe, CBOE Europe and Turquoise. Refinitiv Tick History is also used to obtain the intraday data on the British pound to Euro exchange rate.

The time series of stocks' closing prices is obtained from Refinitiv Elektron and data on market capitalisation are sourced from Refinitiv Eikon. As can be seen in Table 4.2, optionable stocks tend to be liquid. Most are constituents of the FTSE 100 Index, which includes the most liquid (in terms of market capitalisation) stocks traded on the LSE, and the remainder are part of the FTSE 250 Index.

In addition to the above, the study has access to the order database maintained by the FCA for each of the optionable stocks, which contains all the order logs from the LSE, as well as other markets under FCA supervision, that, as per MiFID II, have to be reported. Broadly speaking, each order log provides information on the type of event (i.e., submission of a new order, cancellation or modification of an existing order, trade, etc.), the date and time it takes place, the order book code, the ISIN of the financial instrument, the order type and the identification of the market participants involved.[22] Order book data are used to explore the behaviour during the LSE closing call auction, at the end of which the closing price is determined. During the closing call auction, the other main venues are not operative.

4.5 Market-wide end-of-day patterns at expiration

This section investigates the existence of general market behaviour in line with market manipulation: Hypotheses 4.1-4.4 are tested. The section first discusses how the proxy for manipulation incentives is defined. A series of empirical frameworks are then proposed to test whether abnormal price movements (in Section 4.5.1) and abnormal trading activity (in Section 4.5.2) are associated with these manipulation proxies.

4.5.1 Price patterns at expiration

The first element to consider is the target of potential manipulators: that is, what are they aiming to artificially affect? The study seeks to investigate the existence of patterns that are in line with manipulation from option writers holding negative net positions (i.e., short positions) on ITM options prior to expiration who, therefore, have the incentive to artificially

[22] Many other fields are reported: see MiFIR RTS 24 for further and more details on the reportable information.

TABLE 4.2: Summary statistics - Optionable stocks sample, end-of-day returns and trading activity.

Panel A: Optionable and non-optionable stocks comparison

		Optionable Stocks		Non-Optionable Stocks	
Market Index	Year	# Stocks	Mkt Cap in Mln £(Avg.)	# Stocks	Mkt Cap in Mln £(Avg.)
FTSE 100	2018	72	28,117	18	6,383
	2019	76	24,063	20	5,246
	2020	74	26,866	24	6,875
FTSE 250	2018	21	3,017	257	1,641
	2019	21	2,187	259	1,413
	2020	23	2,673	258	1,622

Panel B: Descriptive statistics at expiration days

	Quarterly expiry days ($N = 1066$)					Non-quarterly expiry days ($N = 2327$)				
	Avg	Median	Std dev	Min	Max	Avg	Median	Std dev	Min	Max
Log return (% last 30 minutes)	-0.02	-0.02	0.57	-3.97	3.07	0.04	0.03	0.42	-4.07	2.56
Log return (% last 15 minutes)	-0.04	-0.04	0.57	-4.00	4.21	0.04	0.03	0.36	-4.22	3.42
Log return (% closing auction)	0.06	0.05	0.46	-1.97	4.95	0.02	0.00	0.29	-3.94	2.83
Price reversal (%)	-0.65	-0.35	1.94	-11.63	23.50	0.21	0.17	1.53	-15.50	12.06
Traded volume (% last 30 minutes)	55.94	55.54	10.79	20.14	90.00	35.25	34.59	8.02	12.69	65.57
Traded volume (% last 15 minutes)	53.62	53.27	11.21	18.93	89.41	31.25	30.50	8.02	9.95	64.09
Traded volume (% closing auction)	50.34	49.98	11.73	13.78	88.46	25.76	25.00	7.94	5.22	62.08
Turnover ratio (% last 30 minutes)	0.36	0.27	0.35	0.03	3.01	0.11	0.10	0.07	0.01	0.74
Turnover ratio (% last 15 minutes)	0.35	0.26	0.35	0.03	2.98	0.10	0.08	0.06	0.01	0.67
Turnover ratio (% closing auction)	0.33	0.24	0.34	0.03	2.93	0.08	0.07	0.05	0.01	0.42

Panel A provides a breakdown of the optionable stocks in this study's sample and how they compare, in terms of market capitalisation, with respect to non-optionable stocks within the respective market index. Panel B contains descriptive statistics on the end-of-day metrics of price changes and trading activity.

TABLE 4.3: Summary statistics - Orders submitted throughout the closing call auction.

	Neighbourhood days (N=8916)				Quarterly expiry days (N=678)				Normal expiry days (N=1551)			
	Avg	Median	Std dev	Max	Avg	Median	Std dev	Max	Avg	Median	Std dev	Max
Panel A: Bid orders												
# Orders submitted	320.75	243.00	246.29	3692.00	420.26	332.50	279.91	1873.00	318.12	242.00	239.26	2156.00
% Executed orders	40.87	42.53	12.87	98.21	48.18	50.93	13.87	81.38	39.61	41.67	12.16	74.15
% Executable orders	57.51	58.51	16.06	98.47	61.80	63.56	17.53	98.48	56.36	57.58	15.39	94.87
% Cancelled orders	23.52	17.36	18.21	92.06	20.13	14.63	17.90	80.71	23.27	17.41	18.28	86.40
% Order size > 90p	10.06	9.30	5.11	60.27	17.75	15.53	8.43	64.17	10.43	9.62	4.97	56.70
% Executable	6.87	6.19	3.89	39.67	11.52	10.31	6.28	43.82	6.77	6.22	3.75	35.73
% Cancelled	2.01	1.09	2.68	34.10	1.98	1.00	3.09	35.84	1.93	0.95	2.87	33.20
% Traded	4.40	4.13	2.31	20.50	8.22	7.35	4.27	33.19	4.40	4.22	2.21	16.23
% Order size > 95p	5.11	4.42	3.39	45.93	10.93	9.39	6.43	55.70	5.29	4.68	3.22	40.19
% Executable	3.77	3.17	2.75	32.97	7.41	6.36	4.61	39.09	3.79	3.31	2.54	20.00
% Cancelled	1.16	0.44	1.94	28.67	1.32	0.57	2.53	33.33	1.12	0.43	1.85	17.67
% Traded	2.41	2.16	1.57	14.50	5.43	4.82	3.15	25.16	2.48	2.27	1.48	9.39

The table provides descriptive statistics on the type of orders submitted on expiration days (both non-quarterly and quarterly) and the neighbourhood days (two days before and two days after expiration day), for comparison purposes. Statistics reported are: (1) number of orders submitted during the closing auction; (2) percentage of submitted orders executed at the end of the auction; (3) percentage of submitted orders executable upon submission (i.e., they are either market orders or limit orders with a price higher or lower than the indicative price before submission, for bid and ask orders, respectively); and (4) percentage of submitted orders cancelled before the end of the closing auction. These statistics are reported unconditionally for all submitted orders and then conditionally on submitted orders of size higher than the 90th (90p) (and 95th [95p]) percentile thresholds based on the order size distribution computed the week before expiration week. Statistics are presented separately for bid (Panel A) and ask (Panel B) orders.

Table 4.3: *continued*.

	Neighbourhood days (N=8916)				Quarterly expiry days (N=678)				Normal expiry days (N=1551)			
	Avg	Median	Std dev	Max	Avg	Median	Std dev	Max	Avg	Median	Std dev	Max
Panel B: Ask orders												
# Orders submitted	313.18	239.00	242.38	3466.00	429.87	359.00	270.58	1574.00	309.60	233.00	234.28	2384.00
% Executed orders	40.78	42.52	12.85	82.00	50.85	53.87	13.41	93.00	40.89	42.73	12.91	77.60
% Executable orders	58.18	59.23	16.15	96.54	68.23	70.31	15.71	98.19	57.88	58.60	16.04	95.05
% Cancelled orders	24.00	18.10	18.10	90.41	21.96	16.61	17.13	74.60	22.66	16.38	18.22	86.42
% Order size > 90p	10.16	9.44	5.15	55.51	17.29	15.64	7.94	80.63	10.59	9.85	5.29	52.17
% Executable	6.77	6.15	3.76	43.60	11.36	10.26	5.93	51.72	6.98	6.36	3.96	44.57
% Cancelled	1.76	0.96	2.42	36.81	1.89	1.21	2.35	18.62	1.81	1.01	2.77	43.47
% Traded	4.54	4.27	2.37	28.95	8.16	7.24	4.40	45.69	4.59	4.35	2.36	16.90
% Order size > 95p	5.15	4.46	3.43	41.69	10.54	9.33	5.91	49.57	5.33	4.58	3.62	43.91
% Executable	3.73	3.21	2.67	36.84	7.31	6.32	4.44	45.69	3.82	3.23	2.94	43.91
% Cancelled	1.02	0.43	1.72	30.62	1.17	0.59	1.76	14.74	1.04	0.41	2.23	43.14
% Traded	2.51	2.27	1.64	26.32	5.47	4.65	3.51	40.09	2.54	2.31	1.61	12.08

TABLE 4.4: Summary statistics - Option open interest on expiration days.

	N	Open interest		
		Avg	Median	Max
Panel A: All day				
Non-quarterly expiry	2327	2381	172	238,149
Quarterly expiry	1066	9075	483	339,764

	M	Incentive		Exposure (thousands £)		
		N tot.	N both	Avg	Median	Max
Panel B: 15 minutes						
Non-quarterly expiry	1	225	1	1243	185	20,400
	2	450	6	1128	182	20,899
	3	654	26	1138	195	23,065
Quarterly expiry	1	134	1	1888	290	36,376
	2	232	6	3142	270	125,325
	3	335	15	2679	245	125,325
Panel C: 30 minutes						
Non-quarterly expiry	1	272	2	1190	160	20,400
	2	540	17	1203	171	23,065
	3	789	43	1124	167	23,065
Quarterly expiry	1	157	2	2476	237	49,020
	2	299	13	2924	241	125,325
	3	429	35	2811	240	125,325
Panel D: closing auction						
Non-quarterly expiry	1	192	3	1233	143	33,779
	2	373	9	1057	142	33,779
	3	548	20	1061	155	33,779
Quarterly expiry	1	96	1	1976	212	27,512
	2	198	4	1827	214	43,284
	3	286	10	1928	215	43,500

Panel A contains information on open interest figures, separately for quarterly and non-quarterly expiration days. Panels B, C and D report information on the number of instances (N tot.) when an option is both ITM and close enough to the money to generate an incentive to manipulate (based on a specific value of the threshold M). The number of times both a call option and a put option both satisfy these conditions on the same combination of stock and day is reported as well (column: N both). The last three columns report (in thousands of £) exposure figures. For each combination of stock and day, the exposure is computed based on Equation (4.3): the average, median and maximum values are computed across all stocks, separately for quarterly and non-quarterly expiration days. These statistics are also reported based on option moneyness computed using the midpoint price 15 minutes before the start of the closing auction (Panel B), 30 minutes before (Panel C) and the last midpoint price observed before the closing auction starts (Panel D).

affect the underlying price so that the options expire OTM. The decision of whether to exercise options at expiration depends, as assumed by Ni et al., (2005), on the closing price of the underlying stock, which is the last price observed in the market during the trading day.

The examination of prosecuted marking the close cases[23] has revealed that closing price manipulation tends to take place in the very last minutes (or even seconds) of the trading day. Price changes on expiration days observed over the last 15 minutes of continuous trading, plus the closing auction, are investigated. Given stock i and day t, the end-of-day price change is defined as:

$$r_{i,t} = ln\left(\frac{p_{i,t}}{p_{i,t-15}}\right), \tag{4.1}$$

where $p_{i,t}$ is the closing price of stock i on day t, $p_{i,t-15}$ is the midpoint price observed 15 minutes before the end of the continuous trading phase and $ln(\cdot)$ is the natural logarithm. This is in line with previous research on similar topics: see Felixson and Pelli, (1999); M. Aitken, Cumming, et al., (2015) and Alexakis et al., (2021).

To complete the framework, the definition of a proxy variable to measure the incentive to manipulate the underlying stock's price at expiration is necessary. Two fundamental elements must be taken into consideration. The first element is the existence of ITM options at expiration. For each combination of stocks and expiration days in the sample, information is collected on the option contracts[24] that have positive open interest on the day of expiration. Open interest is available on a daily basis; therefore, for expiration day t, information on the open interest at the end of the prior trading day $t-1$ is used. Then, in line with how the end of day price change is defined, each option is classified as either ITM or OTM based on the underlying midpoint price observed 15 minutes before the end of continuous trading. Given stock i and expiration day t, and given $p_{i,t-15}$ defined as above, a call option with strike K is classified ITM if $(p_{i,t-15} - K) > 0$ and OTM if $(p_{i,t-15} - K) \leq 0$.[25] A similar reasoning applies for put options that are classified as ITM if $(K - p_{i,t-15}) > 0$ and OTM if $(K - p_{i,t-15}) \leq 0$.

A second essential element to consider is the *ex-ante* achievability of manipulation. The existence of ITM options on expiration day cannot be considered, by itself, as a good proxy for manipulation incentives. For the type of manipulation analysed herein to be successful, option writers need to be able to affect the underlying price so that ITM options become OTM. They only succeed if they can produce a change of moneyness. It is assumed that potential manipulators act rationally and attempt to manipulate the underlying stock only if the potential profits (which depend on the likelihood that their actions will cause a change of moneyness) outweigh the costs (due to the price impact of moving the price through trading, the probability of fines if they are caught by regulators and the obligations associated with the options exercise if they fail in their manipulation attempt and option holders eventually

[23]See the extended discussions in Section 4.3.

[24]A standard option contract is completely determined by the following features: option type (call/put), delivery type, strike price, expiration day, price multiplier and underlying asset.

[25]In theory, if $(p_{i,t-15} - K) = 0$, the option is considered at the money (ATM). Options that are either OTM or ATM are grouped together as the main interest is in ITM options, which provide incentives to manipulate.

exercise their options). In other words, if this type of manipulation is performed, the study expects to observe it when ITM options sufficiently close to the money are present prior to expiration. To quantify this criterion, given stock i and expiration day t, the standard deviation $\hat{\sigma}_{i,t}$ of the end-of-day 15-minute log-returns is computed based on a window of 45 days ending a week before expiration, from $t-49$ to $t-5$.[26] An option (either call or put) with strike price K on stock i and expiration day t is classified as close to the money if:

$$\left|ln(K/p_{i,t-15})\right| < M\hat{\sigma}_{i,t}, \tag{4.2}$$

where M is a specific threshold variable controlling how close to the money options need to be in order to be considered. If $M = 2$, for example, the distance between the midpoint price 15 minutes before the close and the strike price are required to be lower than two times the standard deviation of the returns on the last 15 minutes. The results are tested and presented separately for three different values of $M = 1,2,3$. Although a very small minority, some options with a strike price in the sample are in the Euro denomination. To account for this, the strike price is converted to the British pound by using the GBP/EUR exchange rate available 15 minutes before the end of continuous trading on the LSE.[27]

Expected profits are taken into consideration by potential manipulators. A measure of option exposure is defined for each combination of stock i and expiration day t, and separately for call and put options. All call options are collected and the exposure is computed as:

$$CALL\_EXP^M_{i,t} = \sum_{j=1}^{N^c_{i,t}} OI_{i,t,j}K_{i,t,j}I^M_{i,t,j}. \tag{4.3}$$

The sum above is performed over the set of $N^c_{i,t}$ active call options written on stock i and expiring at day t. $OI_{i,t,j}$ is the amount of underlying stocks to which the option gives purchase rights to, $K_{i,t,j}$ is the strike price, and $I^M_{i,t,j}$ is an indicator function equal to 1 if the corresponding option is both ITM and close to the money, based on the criteria in Equation (4.2) and the threshold M. This variable can be interpreted as a simple way to measure the amount (in £) that option writers would avoid paying if the price moved in such a way that the options expired OTM. The same type of computation is performed over the collection of put options to obtain $PUT\_EXP^M_{i,t}$.

Two variables are then defined for each combination of stock i and expiration day t: $CALL\_ITM_{i,t}$ and $PUT\_ITM_{i,t}$. The variable $CALL\_ITM_{i,t}$ is a dummy variable that takes the value 1 if $CALL\_EXP^M_{i,t} > 0$, based on a specific value of M. If this dummy variable is equal to 1, an incentive exists to push the underlying stock price downwards for the option to expire as worthless. A similar reasoning is applied when defining $PUT\_ITM_{i,t}$, which is a dummy variable equal to 1 if, on stock i and expiration day t, there is at least an active put option that is ITM and sufficiently close to the money for manipulation to be possible based on a specific value of M. In other words, $PUT\_ITM_{i,t}$ takes the value 1 if

[26] This is in line Pope and Yadav, (1992), as well as being the size chosen in other papers - see, for example, Comerton-Forde and Putniņš, (2011). Other benchmark window sizes and similar results are obtained.

[27] We test the alternative of removing these options from the data set, obtaining similar results.

$PUT\_EXP^M_{i,t} > 0$. The variables $CALL\_ITM_{i,t}$ and $PUT\_ITM_{i,t}$ are computed, first, by considering all the options that satisfy the criteria above and, secondly, by setting a threshold for the option exposure. More precisely, $CALL\_ITM_{i,t}$ is computed by taking into consideration only the combination of i and expiration day t for which the $CALL\_EXP^M_{i,t}$ is above the median value computed on days where there is a positive exposure. In this way, the focus is on combinations of expiration days and stocks that are characterised by a large (larger than median) amount of ITM options, which are also close to the money. Similar reasoning applies to put options.

Finally, the last ingredient is represented by a measure of end-of-day abnormal returns. The two models described below are used:

Abnormal end-of-day returns - mean adjusted. For stock i and expiration day t, the abnormal end-of-day return is defined as:

$$AR_{i,t} = r_{i,t} - \mu, \tag{4.4}$$

where μ is the sample mean computed on a benchmark window of 45 trading days ending the week before expiration (from $t-49$ to $t-5$).

Abnormal end-of-day returns - market adjusted. For stock i and expiration day t, the abnormal end-of-day return is defined by using the capital asset pricing model (CAPM)[28]:

$$AR_{i,t} = (r_{i,t} - r_{f,t}) - \left[\hat{\alpha}_i + \hat{\beta}_i(r_{m,t} - r_{f,t})\right], \tag{4.5}$$

where $r_{f,t}$ is the risk-free rate on day t, $r_{m,t}$ is the return on the corresponding market index (this being the FTSE 350 Index) and $\hat{\alpha}_i$ and $\hat{\beta}_i$ are the estimates of the regression below:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{m,t} - r_{f,t}), \tag{4.6}$$

which is estimated on a benchmark window of 45 days ending the week before expiration (from $t-49$ to $t-5$).

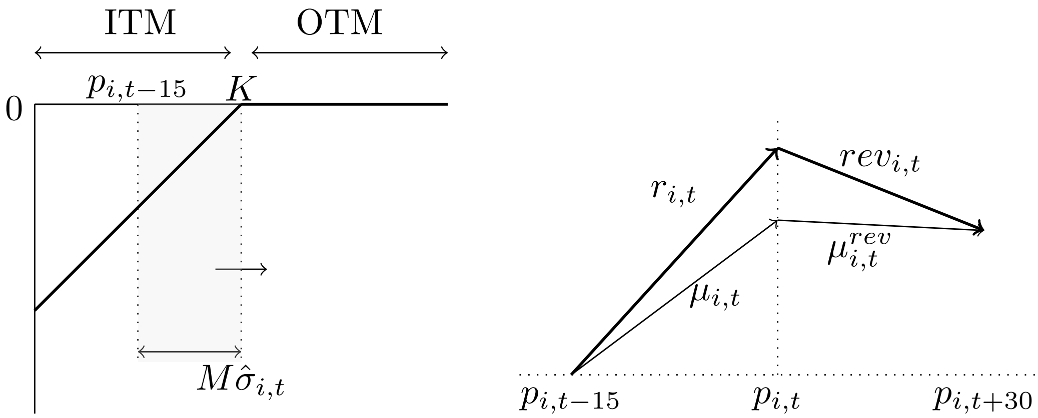
The SONIA (Sterling Overnight Index Average) rate is chosen as the risk-free rate, with this being an overnight bank interest rate maintained by the Bank of England (BoE) which provides the annualised interest rates on a daily basis. For the purposes of this study, these are converted into daily rates by applying the formula:

$$(1 + SONIA_t/100)^{1/365} - 1.$$

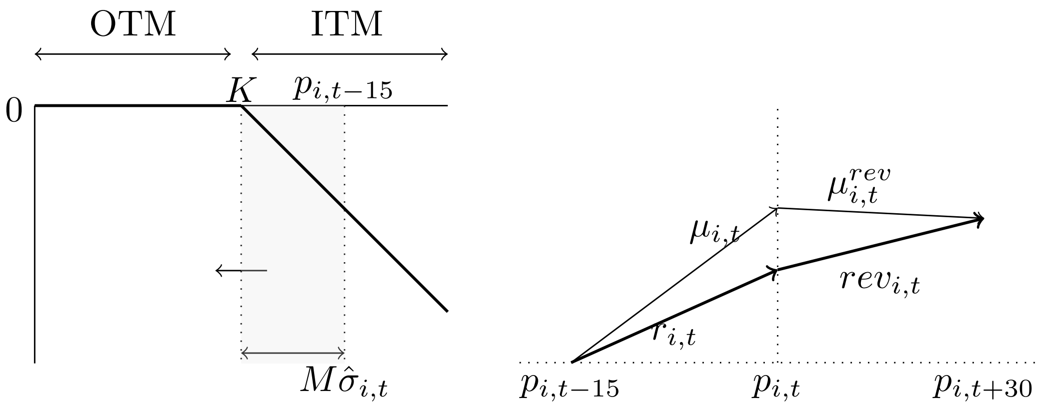
This study then proposes to study the existence of manipulation patterns at the price level by means of the OLS regression:

$$AR_{i,t} = \alpha + \beta_1 CALL\_ITM_{i,t} + \beta_2 PUT\_ITM_{i,t} + \lambda_t + \epsilon_{i,t}, \tag{4.7}$$

[28] See Sharpe, (1964).



(A) Put option writer case.



(B) Call option writer case.

FIGURE 4.1: Illustration of market manipulation incentives and expected market impact.

The charts on the left represent the scenarios under which an option writer has an incentive to manipulate. The charts on the right describe the type of impact on the price that can be expected if option writers act on these incentives. The case of put option writers is described in Figure 4.1a. If the price just prior expiry ($p_{i,t-15}$) is below the strike price K, the writers are facing a liability. If, furthermore, this price is sufficiently close to K based on criteria in Equation (4.2), represented by the grey region, the option writer has the incentive to push the price upwards above the strike price and, by doing so, avoiding any liability at expiration. In this case, the expectation is to observe an end-of-day return ($r_{i,t}$) higher than expected ($\mu_{i,t}$) and to observe the price to reverse back ($rev_{i,t} > \mu^{rev}_{i,t}$). The same logic applies to the call option case, illustrated in Figure 4.1b.

where $CALL\_ITM$ and PUT_ITM are defined above. Date fixed effects λ_t are included in the regressions to capture any potential market-wide shocks. If manipulation is taking place, it is expected that a negative and significant value will be observed for β_1 and a positive and significant value for β_2. A negative and significant value for β_1 would mean that abnormally high returns are observed on a systematic basis when, just prior to expiration, an ITM call option exists that is sufficiently close to cause a change of moneyness. Same reasoning applies for β_2. Therefore, Hypothesis 4.1 consists in testing the alternative hypotheses $H_1 : \beta_1 < 0$ and $H_1 : \beta_2 > 0$ against the null hypotheses $H_0 : \beta_1 \geq 0$ and $H_0 : \beta_2 \leq 0$. Figure 4.1 illustrates the motivations behind this logic.

The possibility should be noted that, on the same expiration day, both $CALL\_ITM$ and PUT_ITM assume the value of 1, meaning that incentives exist to manipulate the price in both directions, which can potentially cancel each other out. To take this into account, for $CALL\_ITM$ to be equal to 1, no incentive should be generated by ITM put options (in other words, PUT_ITM needs to be 0) and vice versa. In this way, $CALL\_ITM$ and PUT_ITM are expected to capture incentives to manipulate in only one clear direction.[29] The regressions are estimated using all the expiration days from January 2018-December 2020 (with the exception of March 2020), for a total of 3,393 stock-day observations.

Effects due to manipulation are expected to be temporary. In other words, if a closing price is manipulated, it is expected to reverse to the *fair* price the following day. A simple measure of price reversal is defined as follows:

Abnormal price reversal. For stock i and expiration day t, the abnormal price reversal is defined as:

$$AREV_{i,t} = rev_{i,t}^{adj} - \mu_{rev^{adj}}, \tag{4.8}$$

where $rev_{i,t}^{adj} = rev_{i,t} - rev_{m,t}$, with $rev_{i,t}$ representing the difference between the change in price from the closing price on day t for stock i to the midpoint price observed after the first 30 minutes of trading during the following trading day, similar to Stoll and Whaley, (1991), and aimed at avoiding the measures being affected by the structural high noise at the beginning of the trading session. In the same way, $rev_{m,t}$ is defined as the reversal in price of the corresponding market index, similarly to the market adjusted definition of the end-of-day abnormal price returns. Finally, μ_{rev}^{adj} is the sample mean of $rev_{i,t}^{adj}$ computed on the 45 days benchmark window ending the week before expiration.

The following OLS regression is then estimated:

$$AREV_{i,t} = \alpha + \beta_1 CALL\_ITM_{i,t} + \beta_2 PUT\_ITM_{i,t} + \lambda_t + \epsilon_{i,t}, \tag{4.9}$$

where $CALL\_ITM$, PUT_ITM and λ_t are the same as the model in Equation (4.7). Hypothesis 4.2 would be supported by a positive and significant value for β_1 (if the closing price is artificially pushed downwards, the price is expected to reverse back to the *fair value* the next day, producing a positive and abnormal price reversal): $H_1 : \beta_1 > 0$ against $H_0 : \beta_1 \leq 0$. By a similar logic, Hypothesis 4.2 is coherent with a negative and significant value for β_2: $H_1 : \beta_2 < 0$ against $H_0 : \beta_2 \geq 0$.

Table 4.5 reports the results of estimating the OLS regression (Equation [4.7]), using both mean-adjusted and market-adjusted models to compute the abnormal end-of-day price changes. Results are presented for the various levels of threshold M used to measure how close the strike price needs to be to the midpoint price to be considered as a potential source

[29]Considering the few instances of combinations of stock and days on which both $CALL\_ITM$ and PUT_ITM are equal to 1 - see Table 4.4 - an alternative option would be to simply remove these observations from the data set when estimating the empirical models. The results are quantitatively similar.

of manipulation. Additionally, results are presented first by considering all the combinations of stock and day with expiring options satisfying the criteria, then only the combinations of stocks and days with exposure above the median, which are expected to generate the strongest incentives for manipulation.

Overall, by looking at price changes, signs of systematic manipulation are not found. In other words, the parameters associated with the variables $CALL\_ITM$ and PUT_ITM are always insignificant, except for the parameter associated with $CALL\_ITM$, computed by considering options with a strike price within two standard deviations of the underlying stock price. This event is associated with abnormal end-of-day returns of -0.033% (computed by the mean adjusted model) and of -0.04% (computed by the market adjusted model), both only significant at the 10% level. However, by considering only options with above median exposure, the parameter loses significance. Furthermore, in the robustness tests performed and presented in Appendix 4.C, these patterns cannot be confirmed. No significant results connected to price reversal patterns (results presented on Table 4.6) and in line with market manipulation hypothesis are found; the coefficients of $CALL\_ITM$ and PUT_ITM are not statistically significant. Therefore, no support is found for Hypotheses 4.1 and 4.2.

Finally, the study notes that the decision to take the (midpoint) price 15 minutes before the end of the continuous trading phase to measure end-of-day price changes (and options moneyness), although in line with past literature, is still somewhat arbitrary. In Appendix 4.C, regressions (Equation [4.7]) and (Equation [4.9]) are estimated by using dependent and independent variables based on both the midpoint price measured 30 minutes before the end of continuous trading and the last midpoint observed before the end of continuous trading (in other words, immediately before the start of the closing auction).

4.5.2 Trading and order submission behaviours at expiration

This section discusses how the framework is modified, with this similar to the process applied in the previous section to empirically test Hypotheses 4.3 and 4.4. The interest lies in exploring if abnormal patterns occur in the trading activity on expiration days that are compatible with the way in which the market behaves under manipulation.

As a measure of end-of-day trading activity, for each combination of stock i and expiration day t, the following measures are defined:

1) Turnover ratio ($TR_{i,t}$), defined as the volume (measured by turnover) of stock i traded during the last 15 minutes of continuous trading and during the closing auction, on day t, divided by the stock's market capitalisation;

2) Concentration ratio ($CR_{i,t}$), defined as the volume (measured by turnover) traded in the last 15 minutes of continuous trading and during the closing auction on stock i and day t, divided by the total turnover traded during the trading day.

TABLE 4.5: Regression results - Abnormal end-of-day returns.

	$M=1$		$M=2$		$M=3$	
	Mean Adj.	Market Adj.	Mean Adj.	Market Adj.	Mean Adj.	Market Adj.
Panel A: All options						
$Constant$	0.139***	0.007	0.142***	0.011	0.140***	0.007
	(5.767)	(0.304)	(5.892)	(0.449)	(5.806)	(0.306)
PUT_ITM	0.030	0.017	0.002	−0.005	0.014	0.015
	(1.244)	(0.708)	(0.122)	(−0.292)	(0.784)	(0.865)
$CALL\_ITM$	−0.016	−0.025	−0.033*	−0.040*	−0.013	−0.016
	(−0.609)	(−0.893)	(−1.678)	(−1.937)	(−0.735)	(−0.870)
Date fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	3393	3393	3393	3393	3393	3393
Adjusted R^2	0.222	0.019	0.223	0.020	0.222	0.019
Panel B: Above median exposure						
Constant	0.140***	0.008	0.142***	0.010	0.140***	0.007
	(5.820)	(0.314)	(5.847)	(0.398)	(5.793)	(0.304)
PUT_ITM	0.027	0.015	−0.006	−0.016	0.011	0.008
	(1.039)	(0.611)	(−0.267)	(−0.747)	(0.463)	(0.313)
$CALL\_ITM$	−0.018	−0.012	−0.017	−0.019	0.003	0.003
	(−0.433)	(−0.283)	(−0.552)	(−0.607)	(0.110)	(0.126)
Date fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	3393	3393	3393	3393	3393	3393
Adjusted R^2	0.222	0.019	0.222	0.019	0.222	0.019

The table provides estimates of regression (Equation [4.7]) with abnormal returns computed based both on mean adjusted and market adjusted abnormal end-of-day returns. Panel A reports the results when the dependent variables $CALL\_ITM$ and PUT_ITM are defined by considering all the combinations of stocks and expiration days with ITM options that are close enough to the money, without setting any restriction on the size. Panel B is obtained by considering only combinations of stock and days with above median exposure. Results are reported for different values of the threshold M used to classify options as close to the money based on criteria (Equation [4.2]). Standard errors are clustered by stock. ***, ** and * denote statistical significance at the 1%, 5% and 10% levels, respectively. Parameters' values are reported in percentage points.

TABLE 4.6: Regression results - Abnormal price reversals.

	All options			Above median exposure		
	$M = 1$	$M = 2$	$M = 3$	$M = 1$	$M = 2$	$M = 3$
$Constant$	-0.006	-0.009	-0.003	-0.007	-0.013	-0.016
	(-0.031)	(-0.047)	(-0.017)	(-0.034)	(-0.065)	(-0.081)
PUT_ITM	-0.043	0.042	-0.029	-0.071	0.039	0.044
	(-0.561)	(0.697)	(-0.668)	(-0.565)	(0.371)	(0.537)
$CALL\_ITM$	-0.001	-0.057	-0.029	0.038	0.036	0.065
	(-0.009)	(-0.897)	(-0.407)	(0.262)	(0.391)	(0.814)
Date fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	3393	3393	3393	3393	3393	3393
Adjusted R^2	0.041	0.041	0.041	0.041	0.041	0.041

The table provides estimates of regression (Equation [4.9]). Results are presented with dependent variables $CALL\_ITM$ and PUT_ITM both defined considering all ITM options that are close enough to the money, without setting any restriction on the size, and then restricted to above median exposure. Results are reported for different values of the threshold M used to classify options as close to the money based on criteria (Equation [4.2]). Standard errors are clustered by stock. ***, ** and * denote statistical significance at the 1%, 5% and 10% levels, respectively. Parameters' values are reported in percentage points.

Turnover ratio ($TR_{i,t}$) describes the amount of traded volume during the last minutes of trading, in absolute terms. An increase in this variable could be representative of a rise of traded volume specific to the last interval of trading, or it could be caused by a generalised increase in the traded volume throughout the whole trading day. The volume traded at the end of the trading day is normalised with respect to the market capitalisation to make the variable comparable across different stocks. Concentration ratio ($CR_{i,t}$), on the other hand, is a measure of the concentration of liquidity at the end of the trading day. It is a relative measure of the end-of-day trading volume, in the sense that an increase would mean that a higher percentage of the volume traded throughout the day is concentrated at the end. This could be representative of an abnormal amount of liquidity traded in the last minutes of trading or it can be caused by a lower than usual volume traded in the preceding trading phase. An increase in both variables represents an increase in absolute terms and also a higher concentration of volume traded at the end of the day. The following OLS regressions are estimated:

$$TR_{i,t} = \alpha + \beta OPTION\_ITM_{i,t} + Controls_{i,t} + \lambda_t + \epsilon_{i,t}, \tag{4.10}$$

$$CR_{i,t} = \alpha + \beta OPTION\_ITM_{i,t} + Controls_{i,t} + \lambda_t + \epsilon_{i,t}, \tag{4.11}$$

where $OPTION\_ITM$ is a dummy variable equal to 1 if either $CALL\_ITM$ or PUT_ITM (as defined in the previous section) is equal to 1, and 0 otherwise. Under the hypothesis that market manipulation takes place systematically, the expectation is to observe a positive and significant value of β for both specifications: $H_1 : \beta > 0$ against the null hypothesis $H_0 : \beta \leq 0$. A positive value for β would mean that an increased trading activity towards the closing of the trading day is observed when an incentive to manipulate exists. The data set used is the same used for the estimation of regressions (Equation [4.7]) and (Equation

[4.9]) with the same considerations that are valid in the estimation of these models also being valid in this setting. For control variables, Amihud, (2002) measure of illiquidity ($ILLIQ$) is included, with this normalised by taking the natural logarithm and the stock's beta as a measure of systematic risk (measured through the CAPM), as well as date fixed effects (λ_t). A dummy for FTSE 100 stocks is also included, as constituents of FTSE 100 Index might behave differently from FTSE 250 stocks. Both $ILLIQ$ and the stocks' beta are obtained based on the 45-day benchmark window ending the week before expiration, consistent with the benchmark window used to compute abnormal returns in the previous section.

In addition to the variables above, as discussed in Section 4.3, a common feature of prosecuted cases of marking the close manipulation is a sequence of trades or orders submitted minutes (if not seconds) before the determination of the closing price. Considering the importance of the closing call auction in terms of trading activity, the existence of abnormal order submission behaviour that takes place during this period on expiration days is investigated. For each combination of stock i and expiration day t, all orders submitted during the closing call auction are collected with the following measures computed:

1) Percentage of large bid orders ($LARGE\_BID_{i,t}$) submitted during the closing auction on stock i and day t that are executable at submission and are eventually traded. An order is classified as large, in this context, if the order size is above the 90th (and, for robustness, 95th) percentile of the order size distribution (of the orders submitted during the closing auction) computed during the week preceding the expiration week. A buy order is executable if it is a market order or, in case it is a limit order, the limit price is above the previous indicative price. Orders that are large and executable at submission are orders that can affect the current indicative price and potentially the closing price.

2) The same measure above is also computed for ask orders ($LARGE\_ASK_{i,t}$). An order is considered large by applying the same criteria above. Ask orders are considered executable if they are market orders or, in case they are limit orders, the limit price is below the previous indicative price.

The following models are then estimated through OLS regressions:

$$LARGE\_BID_{i,t} = \alpha + \beta PUT\_ITM_{i,t} + Controls_{i,t} + \lambda_t + \epsilon_{i,t}, \tag{4.12}$$

$$LARGE\_ASK_{i,t} = \alpha + \beta CALL\_ITM_{i,t} + Controls_{i,t} + \lambda_t + \epsilon_{i,t}, \tag{4.13}$$

In cases of manipulation, a positive and significant value for β is expected under all the above specifications. Hypothesis 4.4 then consists of testing the alternative hypothesis H_1 : $\beta > 0$ against the null hypothesis $H_0 : \beta \leq 0$. As was done above for Equation (4.10), the Amihud, (2002) ($ILLIQ$) measure (normalised by taking the logarithm), the stock's beta and a FTSE 100 dummy are included as control variables, as well as date fixed effects (λ_t). As in the previous section, the estimates of these models are presented for various values of M used for the definitions of PUT_ITM, $CALL\_ITM$ and $OPTION\_ITM$. The option's moneyness just prior to expiration is computed based on the midpoint price 15 minutes before the end of continuous trading.[30]

The results of the framework proposed above are presented in Tables 4.7, 4.8 and 4.9. A series of notable patterns are found, consistent with past research on expiration day effects. Although the date dummy variable coefficients are not reported here (for space-related reasons), quarterly expiration days are characterised by trading behaviour that is markedly different from non-quarterly expiration days. Taking the expiration day in January 2018 as a benchmark day, a staggering increase of liquidity is observed on expiration days that fall on quarter-end months. This increased volume is traded towards the end of the trading day on quarterly expiration days, which experience a rise in the volume traded in the last 15 minutes of continuous trading and during the closing auction (as a percentage of the daily traded volume) of, on average, more than 22%, and an increase of, on average, more than 0.2% of the turnover traded, measured as a percentage of the market capitalisation. All dummy variables associated with quarterly expiration days are significant at the 1% level, under all the specifications. In addition, and as expected, the study observes a tendency for liquidity to concentrate towards the end of the day for illiquid stocks (with high levels of $ILLIQ$). This can be explained by the fact that closing call auctions have the specific objective, among others, of pooling liquidity at a specific time. It is not surprising that less liquid stocks, for which trading during the continuous trading phase could be more expensive, experience a larger traded volume at the end of the trading day. It is also noticed that more risky stocks (measured by the CAPM beta) are associated with a lower concentration of volume traded towards the end of the day. This is in line with the evidence produced by Raillon, (2019), who finds that a lower percentage of traded volume in the closing auction is associated with days of higher volatility.

Quarterly expiration days also present significantly higher submissions of large and executable orders, as can also be seen from the order descriptive statistics in Table 4.3. Interestingly, if the threshold of the 90th percentile is taken to define large orders, on quarterly expiration days an increase of almost 5%, on average, is observed of large and executable buy orders, against an increase of around 2.6% of large and executable sell orders. By taking the 95th percentile in the large orders definition, this asymmetry between buy and sell orders is reduced (an increase of around 4.3% against 3.7%, on average, for buy and sell orders, respectively), but it still exists, suggesting more aggressive buying intentions on quarterly expiration days.

The parameters associated with the variables of interest (ITM_OPTION in Table 4.7 and $CALL\_ITM$ and PUT_ITM in Tables 4.8 and 4.9) are not significant, leading to the rejection of both Hypotheses 4.3 and 4.4. In other words, no increase in traded volume, or in the percentage of large trades in the direction that would make options expire OTM, can be found

[30]We further present, in Appendix 4.C, the results obtained by applying the framework discussed in this section, by using variables based on the midpoint taken both 30 minutes before the end of continuous trading and the midpoint measured just before the start of the closing auction.

TABLE 4.7: Regression results - Abnormal end-of-day trading activity.

	$M=1$		$M=2$		$M=3$	
	TR	CR	TR	CR	TR	CR
Panel A: All options						
ITM_OPTION	0.016	-0.276	0.012	-0.223	0.005	0.096
	(1.166)	(-0.524)	(1.007)	(-0.458)	(0.454)	(0.197)
$log(ILLIQ)$	0.025^{***}	1.184^{***}	0.025^{***}	1.177^{***}	0.025^{***}	1.211^{***}
	(2.896)	(3.656)	(2.914)	(3.511)	(2.865)	(3.607)
$Beta$	-0.002	-3.002^{***}	-0.002	-3.002^{***}	-0.002	-3.014^{***}
	(-0.185)	(-5.932)	(-0.180)	(-5.929)	(-0.160)	(-5.958)
$FTSE100$	-0.025	0.193	-0.025	0.191	-0.025	0.202
	(-1.127)	(0.194)	(-1.129)	(0.191)	(-1.141)	(0.203)
Date fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	3393	3393	3393	3393	3393	3393
Adjusted R^2	0.302	0.634	0.302	0.634	0.302	0.634
Panel B: Above median exposure						
ITM_OPTION	0.024	-1.059	0.017	-0.501	0.013	-0.092
	(1.035)	(-1.585)	(1.001)	(-0.841)	(0.914)	(-0.181)
$log(ILLIQ)$	0.025^{***}	1.168^{***}	0.025^{***}	1.166^{***}	0.025^{***}	1.192^{***}
	(2.872)	(3.661)	(2.893)	(3.556)	(2.891)	(3.607)
$Beta$	-0.002	-2.995^{***}	-0.002	-3.002^{***}	-0.002	-3.007^{***}
	(-0.173)	(-5.928)	(-0.169)	(-5.936)	(-0.172)	(-5.956)
$FTSE100$	-0.025	0.195	-0.024	0.181	-0.025	0.197
	(-1.136)	(0.195)	(-1.118)	(0.181)	(-1.125)	(0.197)
Date fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	3393	3393	3393	3393	3393	3393
Adjusted R^2	0.302	0.634	0.302	0.634	0.302	0.634

The table provides the estimates of regressions (Equation [4.10]) and (Equation [4.11]), with Turnover ratio (TR) and Concentration ratio (CR) as dependent variables. Panel A reports the results when the dependent variables $CALL\_ITM$ and PUT_ITM are defined when considering all the combinations of stocks and expiration days with ITM options that are close enough to the money, without setting any restriction on the size. Panel B is obtained by considering only combinations of stock and days with an above median exposure. Results are reported for different values of the threshold M used to classify options as close to the money based on criteria (Equation [4.2]). Standard errors are clustered by stock. ***, ** and * denote statistical significance at the 1%, 5% and 10% levels, respectively. Parameters are reported in percentage points.

when an incentive exists to manipulate. All in all, Hypotheses 4.3 and 4.4 present no empirical support based on the framework applied. As market manipulation is an illegal practice, it is expected that manipulators will attempt to hide their activity at the best of their abilities. It is possible that, although it is existent, it may not be sufficiently systematic to be captured by the framework. Having market participants' information available provided the current study with the motivation to investigate the existence of unusual behaviour conducted by a particular class of traders. The next section checks whether, by incorporating this information, abnormal patterns can be found in price movements associated with proprietary

TABLE 4.8: Regression results - Abnormal execution frequency of large orders at the close (90th percentile).

	$M = 1$		$M = 2$		$M = 3$	
	BID	ASK	BID	ASK	BID	ASK
Panel A: All options						
PUT_ITM	-0.221		-0.230		-0.305	
	(-0.338)		(-0.479)		(-0.800)	
$CALL\_ITM$		0.285		0.370		0.315
		(0.482)		(0.736)		(0.813)
$log(ILLIQ)$	0.044	-0.158	0.040	-0.147	0.034	-0.148
	(0.145)	(-0.590)	(0.132)	(-0.550)	(0.113)	(-0.548)
Beta	0.014	0.063	0.015	0.060	0.018	0.061
	(0.037)	(0.122)	(0.040)	(0.116)	(0.047)	(0.118)
$FTSE100$	-0.164	-1.461^{***}	-0.160	-1.455^{***}	-0.157	-1.460^{***}
	(-0.242)	(-2.571)	(-0.237)	(-2.565)	(-0.233)	(-2.573)
Date fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2229	2229	2229	2229	2229	2229
Adjusted R^2	0.228	0.153	0.228	0.153	0.228	0.153
Panel B: Above median exposure						
PUT_ITM	1.108		0.891		0.302	
	(1.177)		(1.588)		(0.678)	
$CALL\_ITM$		0.929		1.191^{*}		0.814
		(1.011)		(1.714)		(1.307)
$log(ILLIQ)$	0.064	-0.151	0.075	-0.127	0.059	-0.131
	(0.213)	(-0.566)	(0.251)	(-0.479)	(0.198)	(-0.491)
Beta	0.001	0.059	0.002	0.058	0.008	0.055
	(0.002)	(0.115)	(0.006)	(0.113)	(0.021)	(0.107)
$FTSE100$	-0.161	-1.453^{***}	-0.155	-1.429^{**}	-0.161	-1.437^{**}
	(-0.239)	(-2.563)	(-0.230)	(-2.526)	(-0.238)	(-2.533)
Date fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2229	2229	2229	2229	2229	2229
Adjusted R^2	0.229	0.153	0.229	0.154	0.228	0.154

The table reports the estimates of regressions (Equation [4.12]) and (Equation [4.13]), with $LARGE\_BID_{i,t}$ and $LARGE\_ASK_{i,t}$ (computed using the 90th percentile as the cut-off to define large orders) as dependent variables. Panel A reports the results when the dependent variables $CALL\_ITM$ and PUT_ITM are defined when considering all the combinations of stocks and expiration days with ITM options that are close enough to the money, without setting any restriction on the size. Panel B is obtained by considering only combinations of stock and days with above median exposure. Results are reported for different values of the threshold M used to classify options as close to the money based on criteria (Equation [4.2]). Standard errors are clustered by stock. ***, ** and * denote statistical significance at the 1%, 5% and 10% levels, respectively. Parameters are reported in percentage points.

trading firms' trading.

TABLE 4.9: Regression results - Abnormal execution frequency of large orders at the close (95th percentile).

	$M=1$		$M=2$		$M=3$	
	BID	ASK	BID	ASK	BID	ASK
Panel A: All options						
PUT_ITM	−0.295		−0.302		−0.230	
	(−0.635)		(−0.839)		(−0.762)	
$CALL\_ITM$		0.112		0.114		0.247
		(0.277)		(0.335)		(0.893)
$log(ILLIQ)$	−0.197	−0.304	−0.202	−0.301	−0.201	−0.294
	(−0.922)	(−1.599)	(−0.958)	(−1.579)	(−0.953)	(−1.533)
$Beta$	−0.053	0.257	−0.052	0.256	−0.053	0.254
	(−0.195)	(0.621)	(−0.192)	(0.619)	(−0.195)	(0.616)
$FTSE100$	0.222	−0.940**	0.227	−0.938**	0.229	−0.940**
	(0.471)	(−2.293)	(0.482)	(−2.286)	(0.486)	(−2.292)
Date fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2229	2229	2229	2229	2229	2229
Adjusted R^2	0.265	0.214	0.265	0.214	0.265	0.214
Panel B: Above median exposure						
PUT_ITM	0.320		0.260		0.045	
	(0.451)		(0.610)		(0.145)	
$CALL\_ITM$		0.201		0.700		0.599
		(0.384)		(1.537)		(1.499)
$log(ILLIQ)$	−0.184	−0.303	−0.181	−0.285	−0.187	−0.282
	(−0.866)	(−1.600)	(−0.856)	(−1.499)	(−0.879)	(−1.485)
$Beta$	−0.062	0.256	−0.062	0.254	−0.060	0.250
	(−0.228)	(0.620)	(−0.227)	(0.615)	(−0.219)	(0.607)
$FTSE100$	0.227	−0.938**	0.229	−0.921**	0.227	−0.922**
	(0.483)	(−2.288)	(0.487)	(−2.244)	(0.483)	(−2.250)
Date fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2229	2229	2229	2229	2229	2229
Adjusted R^2	0.265	0.214	0.265	0.215	0.265	0.215

The table reports the estimates of regressions (Equation [4.12]) and (Equation [4.13]), with $LARGE\_BID_{i,t}$ and $LARGE\_ASK_{i,t}$ (computed using the 95th percentile as the cut-off to define large orders) as dependent variables. Panel A reports the results when the dependent variables $CALL\_ITM$ and PUT_ITM are defined when considering all the combinations of stocks and expiration days with ITM options that are close enough to the money, without setting any restriction on the size. Panel B is obtained by considering only combinations of stock and days with above median exposure. Results are reported for different values of the threshold M used to classify options as close to the money based on criteria (Equation [4.2]). Standard errors are clustered by stock. ***, ** and * denote statistical significance at the 1%, 5% and 10% levels, respectively. Parameters are reported in percentage points.

TABLE 4.10: Top 5 proprietary traders manipulation incentives.

M	N	# Trades	# Trades > 90p	# Trades > 95p
Panel A: 15 minutes				
1	244	57	25 (20)	19 (17)
2	497	117	48 (35)	37 (30)
3	698	163	63 (44)	48 (39)
Panel B: 30 minutes				
1	288	71	30 (20)	22 (18)
2	584	136	54 (34)	39 (30)
3	856	211	86 (61)	69 (55)
Panel C: Before closing auction				
1	206	47	19 (15)	16 (14)
2	414	98	38 (25)	31 (23)
3	623	150	57 (37)	45 (33)

The table reports, for the group of top five proprietary trading firms, the number of times they have a net negative position on an option that is ITM before expiry and sufficiently close to the money to be manipulated (based on M) as well as how many times, from the previous instances, that: (1) they trade in a direction coherent with manipulation during the closing auction and (2) they submit large executable orders (by using both 90th and 95th percentiles as thresholds) that are executed during the closing auction in the direction of manipulation (in parentheses, the number of time these orders are submitted not before the last 15 seconds of the non-random duration of the closing auction). Only the top five traders are reported, based on the number of times an incentive exists, as the others do not present behaviours of interest.

4.6 Proprietary traders' behaviours

By means of a data set comprising options transactions, the position of the proprietary trading firms herein considered on each option contract traded can be retrieved. In this way, a unique opportunity exists to test the existence of ambiguous behaviour by the class of proprietary trading firms.

An initial picture is portrayed in Table 4.10, where, for a specific value of the threshold M (measuring how close the price needs to be for an option to be considered as close enough to be manipulated) and for a specific proprietary trading firm, the following information is reported: (1) the number of instances in which the trader has a negative net position on an ITM option that is close enough to be manipulated, generating an incentive to manipulate; (2) the number of times, from the previous instances, on which the trader trades during the closing auction in the direction compatible with manipulation (and, at the same time, no trade is executed in the opposite direction); (3) the number of times (from the times where an incentive to manipulate exists) on which the proprietary trading firm submits a large executable order (using both 90th and 95th percentiles in the definition of large orders) that is traded during the closing auction in a direction coherent with manipulation. To notice that the table reports aggregated information only on the five proprietary traders most active in terms of number of options trades. This decision, as well as the decision to restrict the analysis to this five traders in what follows, is due to several reasons. First, these five proprietary trading firms are included in more than 92% of the trades in which there is a proprietary trading firm. Furthermore, they trade options on every expiration day included in the sample and they are the most active in terms of both number of different underlying stocks and number of different option contracts traded on each expiration day. Finally, as a by-product of being the most active proprietary trading firms in the options market, these are the traders with the highest incentives to manipulate, in the way that they detain, with significantly higher frequency than other proprietary trading firms, options' positions on expiration days that generate an incentive to manipulation.

The main interest lies in the events in which traders have an incentive to push the stock in a specific direction and to trade only in that direction. The objective is to find evidence that the trader is interested in pushing the price in a favourable direction to avoid a loss in the options market.

It is interesting to observe that in almost 70% of the times when an incentive exists to manipulate, a proprietary trader has a net negative position. This does not mean proprietary traders are manipulators, obviously. It means, however, that they systematically are in a position for which manipulating the underlying price might be beneficial for them. Even though they are among the market participants with the most frequent incentives to manipulate the spot equity market, their behaviour does not appear to be in line with this. Only three traders seem to frequently execute trades in a direction coherent with manipulation.[31] Furthermore, only one trader (called, from now on, *Trader A*) within this group executes large trades during the closing auction frequently enough to be of interest from a manipulation point of view. Taking moneyness measured 15 minutes before the end of continuous trading as a reference, Trader A is an option writer in 59 instances of options for which the strike price is within one standard deviation of the underlying price. Of these occasions, this trader trades 21 times in the direction of manipulation, 18 of which are through the submission of a large order (based on the 90th percentile cut-off). A similar pattern can be observed for the same trader for when the values of $M = 2, 3$.

This study first focuses on whether an abnormal price change is connected to instances in which one of the top five proprietary trading firms[32] reported trades in the manipulation direction during the closing auction. Based on Alexakis et al., (2021), the following model is estimated:

$$\begin{aligned} AR_{i,t} = \alpha + \beta_1 IU_{i,t} + \beta_2 IU_{i,t} \times BUY_{i,t} + \beta_3 ID_{i,t} + \beta_4 ID_{i,t} \times SELL_{i,t} + \\ + \sum_{j=1}^{d} \gamma_j BUY_{j,i,t} + \sum_{j=1}^{d} \delta_j SELL_{j,i,t} + \lambda_t + \epsilon_{i,t}, \end{aligned} \tag{4.14}$$

[31] It is important to note that this does not mean they are manipulating the price as it could simply be their normal behaviour

[32] No behaviour of interest is observed among the other traders belonging to this class.

where the variables are defined as follows:

1) $AR_{i,t}$ is the end-of-day abnormal return as defined in Section 4.5.1. Both specifications of abnormal returns (mean- and market-adjusted) are used;

2) $IU_{i,t}$ is a dummy variable for when one of the proprietary traders considered has a net negative position on a put option that is ITM and close to the money (based on criteria (Equation [4.2]) and given a specific value of M), generating a possible incentive to push the underlying stock's price upwards;

3) $BUY_{i,t}$ is a dummy variable for when one of the proprietary traders executes a buy trade in the closing auction (and, at the same time, the trader does not sell any shares);

4) $ID_{i,t}$ is a dummy variable for when one of the proprietary traders considered have a net negative position on a call option that is ITM and close to the money (based on criteria (Equation [4.2]) and given a specific value of M), generating a possible incentive to push the underlying price downwards;

5) $SELL_{i,t}$ is a dummy variable for when one of the proprietary traders executes a buy trade in the closing auction (and, at the same time, the trader does not buy any shares);

6) $BUY_{j,i,t}$ (and $SELL_{j,i,t}$) for $j = 1, \ldots, 5$ are dummy variables for when Trader j executes buy (sell) trades in the closing auction and does not trade in the opposite direction.

Date fixed effects λ_t are also included. The intention is to explore trades made by the traders, without setting any restriction on the size of the order, to avoid issues related to the few instances in which a trader with incentive and a submission of a large order by that same trader are observed. This can provide an indication of whether the price is abnormal when the trader has an incentive to manipulate and trades in the direction of manipulation. The existence of manipulation would mean a positive and significant value for β_2 and a negative and significant value for β_4.

Finally, the focus is restricted on Trader A, the only trader who submits large orders during the closing auction frequently enough to statistically test for the existence of abnormal price changes in connection with this behaviour. The regression is estimated as follows:

$$\begin{aligned} AR_{i,t} = \alpha + \beta_1 IU_{i,t} + \beta_2 IU_{i,t} \times BUY_{i,t} + \beta_3 ID_{i,t} + \beta_4 ID_{i,t} \times SELL_{i,t} + \\ + \gamma_1 BUY_{i,t} + \gamma_2 SELL_{i,t} + \lambda_t + \epsilon_{i,t}, \end{aligned} \tag{4.15}$$

where $AR_{i,t}$ is as above, $IU_{i,t}$ and $ID_{i,t}$ are defined as above, but based only on Trader A's option positions and, finally: (1) $BUY_{i,t}$ is a dummy for when Trader A submits a large executable buy order that is traded during the closing auction (and does not trade during the closing auction in the other direction) and (2) $SELL_{i,t}$ is a dummy for when Trader A submits a large executable sell order that is traded during the closing auction (and does not trade during the closing auction in the other direction). Date fixed effects λ_t are also

TABLE 4.11: Regression results - Abnormal end-of-day returns.

	$M = 1$		$M = 2$		$M = 3$	
	Mean Adj.	Market Adj.	Mean Adj.	Market Adj.	Mean Adj.	Market Adj.
Constant	0.024	0.058**	0.025	0.058**	0.024	0.058**
	(0.818)	(1.967)	(0.832)	(1.979)	(0.815)	(1.959)
ID	−0.025	−0.020	−0.035	−0.035	−0.031	−0.028
	(−0.470)	(−0.372)	(−0.968)	(−0.942)	(−1.081)	(−0.969)
$ID \times SELL$	−0.010	−0.042	−0.013	−0.030	−0.003	−0.020
	(−0.081)	(−0.339)	(−0.184)	(−0.408)	(−0.050)	(−0.380)
IU	0.038	0.036	0.034	0.030	0.013	0.013
	(0.899)	(0.847)	(1.020)	(0.902)	(0.491)	(0.492)
$IU \times BUY$	0.038	0.024	0.045	0.042	0.030	0.024
	(0.631)	(0.384)	(0.975)	(0.905)	(0.835)	(0.653)
BUY dummies	Yes	Yes	Yes	Yes	Yes	Yes
$SELL$ dummies	Yes	Yes	Yes	Yes	Yes	Yes
Date fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2229	2229	2229	2229	2229	2229
Adjusted R^2	0.219	0.139	0.221	0.141	0.219	0.139

The table reports the estimates of the model (Equation [4.14]) applied to the top five traders in terms of the number of manipulation incentives. The definition of BUY and $SELL$ variables depends on whether the market participants executes a buy (sell) trade in the closing auction and does not trade in the opposite direction. Both mean and market adjusted abnormal returns are used as dependent variables and results are reported for different values of the threshold M that are used to classify options as close to the money based on criteria (Equation [4.2]). Standard errors are clustered by stock. ***, ** and * denote statistical significance at the 1%, 5% and 10% levels, respectively. Parameters are reported in percentage points.

TABLE 4.12: Regression results - Abnormal end-of-day returns in connection to big orders.

	$M = 1$		$M = 2$		$M = 3$	
	Mean Adj.	Market Adj.	Mean Adj.	Market Adj.	Mean Adj.	Market Adj.
Constant	0.016	0.049	0.016	0.049	0.016	0.049
	(0.536)	(1.597)	(0.536)	(1.598)	(0.536)	(1.597)
ID	0.106	0.073	0.056	0.030	0.032	0.016
	(0.996)	(0.704)	(0.966)	(0.522)	(0.676)	(0.356)
$ID \times SELL$	−0.044	0.019	0.007	0.051	0.008	0.030
	(−0.352)	(0.154)	(0.074)	(0.523)	(0.080)	(0.321)
IU	−0.072	−0.068	0.026	0.022	−0.004	−0.003
	(−1.389)	(−1.301)	(0.440)	(0.382)	(−0.094)	(−0.068)
$IU \times BUY$	0.156*	0.126	−0.004	−0.019	−0.007	−0.025
	(1.668)	(1.397)	(−0.042)	(−0.235)	(−0.101)	(−0.360)
BUY	Yes	Yes	Yes	Yes	Yes	Yes
SELL	Yes	Yes	Yes	Yes	Yes	Yes
Date fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1863	1863	1863	1863	1863	1863
Adjusted R^2	0.213	0.122	0.212	0.122	0.211	0.121

The table reports the estimates of regression (Equation [4.15]). The definition of *BUY* and *SELL* variables depends on whether the market participant executes a buy (sell) trade that is considered large (above the 90th percentile cut-off) in the closing auction and does not trade in the opposite direction. Both mean and market adjusted abnormal returns are used as dependent variables and results are reported for different values of the threshold M that are used to classify options as close to the money based on criteria (Equation [4.2]). Standard errors are clustered by stock. ***, ** and * denote statistical significance at the 1%, 5% and 10% levels, respectively. Parameters are reported in percentage points.

included. As above, a positive and significant value for β_2 and a negative and significant value for β_4 would be coherent with the existence of manipulation.

Table 4.11 contains the results for the framework (Equation [4.14]). If manipulation is taking place in the marketplace, the parameters of interaction variables $IU \times BUY$ and $ID \times SELL$ should be significant. The first one should be positive (indicating the existence of an abnormally positive returns when a proprietary trader has an incentive to manipulate the price upwards and executes buy transactions during the auction) and the second one should be negative (indicating an abnormally negative return when a proprietary trader has the incentive to push the price downwards and executes sell transactions during the closing auction). Interestingly, a strong tendency is noticed for this group of traders to systematically trade in the opposite direction to the price change. In fact, although not reported, the values of the parameters associated to the variables $BUY_{j,i,t}$ are negative and significant for three out of the five proprietary traders considered. For these three traders, furthermore, the parameters of $SELL_{j,i,t}$ are significant and positive. They buy when there is a negative price change and they sell when there is a positive change in price. This is particularly evident by looking at one of these traders, who tends to buy when there are abnormal returns of almost −0.2% (by using both the abnormal returns measures) and tends to sell when there is an abnormal price return of more than 0.1%. This is a common feature of market making.

No evidence supporting Hypothesis 4.5 can be found, under any of the models' specifications. Similar patterns consistent with market making behaviour are found from the estimates of regression (Equation [4.15]). An abnormal return of around 0.15% is found, by using the mean-adjusted model and for $M = 1$, when Trader A has an incentive to push the price upwards and submits a large buy order during the closing auction. Using market-adjusted returns, the parameter is not significant, and no other traces of manipulation are found, thus providing no support for Hypothesis 4.5.

4.7 Conclusion

This chapter provides an investigation of price and trading behaviour patterns on single stock options expiration days on UK stocks. More precisely, an empirical framework is developed to test for the existence of manipulation of the closing prices on expiration days.

The existence of anomalous patterns is explored, at a price, trading volume and order submission level. The study looks for the existence of anomalous price movements and trading behaviour at the end of the trading session on expiration days when stocks have in the money (ITM) options that are close enough to the money to generate an incentive to manipulate. A variety of model specifications are tested: overall, no evidence is found to support the hypothesis that equity markets are manipulated at option expiration days.

Finally, the study utilises market participant data to study the behaviour of a specific class of traders, namely, proprietary trading firms. Although these traders tend to be subject to strict controls, they have been involved in several market manipulation cases, also in developed markets. In addition, they are among the most active participants in the options market, suggesting they might be in a better position to manipulate the underlying stocks in a systematic way. The framework proposed by Alexakis et al., (2021) is adapted to study if trading activity by this class of market participants is associated with manipulative behaviour on expiration days. In this case, however, the study finds no strong evidence to support the broad hypothesis of marking the close manipulation taking place on options expiration days.

Overall, despite concerns in the industry and contrary to previous research, this study does not find signs of the existence of manipulative behaviour connected to single stock options expiration in the UK market.

4.A Optionable stocks sample

TABLE 4.13: Relevant corporate events.

Effective Date	Description
Panel A: Takeovers	
2018/03/28	Notice No. CA/2018/198/Lo - Takeover of Ladbrokes Coral Group plc (LCL.L) by GCV Holdings plc (GVC.L) and re-designation of the contracts as contracts based on GVC Holdings.
2018/04/26	Notice No. CA/2018/263/Lo - Takeover of GKN plc (GKN.L) by Melrose Industries plc (MRON.L) and re-designation of the contracts as contracts based on Melrose Industries plc.
Panel B: Demergers	
2018/06/25	Notice No. CA/2018/448/Lo - Demerging of Old Mutual plc (OML.L) into two separate entities: Quilter plc (QLT.L) and Old Mutual Limited (OMU.L). Contracts with open interest will become contracts based on a package of one Quilter plc share and three Old Mutual Limited shares.
2019/10/21	Notice No. CA/2019/526/Lo - Demerging of M&G plc (MNG.L) from Prudential plc (PRU.L). Contracts with open interest are re-designated as contracts of one Prudential plc and one M&G plc share.
2020/03/16	Notice No. CA/2020/102/Lo - Demerging of Ninety One plc (N91.L) from Investec plc (INVP.L), with existing contracts that becomes contracts based on 0.5 Ninenty One plc shares and one Investec plc share.
Panel C: Delists	
2018/10/01	Notice No. CA/2018/489/Lo - Delist of Vedanta Resources plc (VED.L).
2018/11/07	Notice No. CA/2018/671/Lo - Delist of Sky plc (SKYB.L).
2018/12/31	Notice No. CA/2018/737/Lo - Delist of Randgold Resources plc (RRS.L).
2019/01/08	Notice No. CA/2019/007/Lo - Delist of Shire plc (SHP.L).
2019/11/04	Notice No. CA/2019/541/DA - Delist of Merlin Entertainments plc (MERL.L).
2020/10/26	Circular No. 20/153 - Delist of CRH plc (CRH.L).

The sample of optionable UK stocks and corresponding options is retrieved by using instrument reference data that need to be reported to the FCA as per MiFIR RTS 22. Trading venues must report to the FCA details of the instruments traded on their venue. This study, therefore, has detailed information on exchange-traded options with stocks negotiated on a UK venue as being underlying.

Firstly, all listed single stock equity options are collected by applying appropriate filters at the CFI code level. The CFI code needs to start with the letter 'O', identifying listed options, and to have 'S' as the fourth term, identifying listed options with stock-equities as underlying assets.

Secondly, options with expiration days that fall on the third Friday of a month between January 2018 and December 2020 are filtered.

Thirdly, the sample is restricted to all the options that have a stock traded on the London Stock Exchange (LSE) as underlying. The reason for focusing on these options is that the interest lies in exploring the existence of potential manipulation of the settlement price at expiry for UK stocks. The aim is to construct a sample of stocks for which the decision to exercise the options and to manipulate them depends on the reference price upon expiration, which, for stocks traded on the LSE, is considered to be the closing price determined on this venue.

Finally, in a similar fashion to Pope and Yadav, (1992), to remove possible confounding factors in this study's analysis, all the options were removed from the data set on which the underlying stocks had been subject to corporate events such as takeovers or demergers. Market notices were manually collected from the Intercontinental Exchange (ICE) website and organised into a summary of relevant events in Table 4.13. Options for which underlying stocks had been subject to takeovers and demerger operations (Table 4.13, Panels A and B of the Table) were removed from the sample. Delisted stocks were also removed, except for options on CRH plc, which were kept in the data set as long as they were listed (a total of 33 expiration days, considering the options on the stock were delisted for the last two months of the sample). Options on CRH plc were kept as all the expiration days considered in the study are available for the options on this stock, except for the final two months. It should be noted that, differing from the first part of this investigation, the second part of this study is based on order book data available only for 2019 and 2020.

Finally, the stock C&C Group plc (GCC.L) is removed from the final sample as as no options were found with positive open interest on any of the considered expiration days. The final UK optionable list comprises 97 unique stocks in the three years spanning from 2018-2020. Table 4.14 provides the complete list of optionable stocks, in sequential order from the most to the least liquid (with liquidity measured by market capitalisation). For each stock, information on which exchange issued the options is also provided, as well as the number of expiration days on which each stock was optionable in the overall sample.

TABLE 4.14: UK optionable stocks sample.

		Exchange						
Stock Name	RIC	ICE	Eurex	LSE	Other	Market Cap	Turnover Ratio	Expiry days
Royal Dutch Shellb plc	RDSb.L	✓	✓	✓		171,180	0.08	35
Unilever plc	ULVR.L	✓	✓	✓	✓	122,119	0.11	35
Hsbc Holdings plc	HSBA.L	✓	✓	✓		117,342	0.15	35
Bhp Group plc	BHPB.L	✓	✓	✓		93,357	0.15	35
Bp plc	BP.L	✓	✓	✓		93,349	0.21	35

Astrazeneca plc	AZN.L	✓	✓	✓	✓	85,999	0.20	35
Glaxosmithkline plc	GSK.L	✓	✓	✓		77,125	0.19	35
British American Tobacco plc	BATS.L	✓		✓		74,584	0.20	35
Rio Tinto plc	RIO.L	✓	✓	✓		72,239	0.25	35
Diageo plc	DGE.L	✓	✓	✓		68,806	0.20	35
Reckitt Benckiser plc	RKT.L	✓	✓			45,827	0.23	35
Vodafone Group plc	VOD.L	✓	✓	✓		40,927	0.29	35
Glencore plc	GLEN.L	✓	✓	✓		37,420	0.31	35
Lloyds Bkg Grp Ls-,10 plc	LLOY.L	✓	✓	✓		37,199	0.26	35
Relx plc	REL.L	✓	✓		✓	34,099	0.22	35
National Grid plc	NG.L	✓	✓	✓		29,558	0.26	35
Barclays plc	BARC.L	✓	✓	✓		27,026	0.29	35
Compass Group plc	CPG.L	✓	✓			25,814	0.26	35
Natwest Group plc	NWG.L	✓	✓	✓		25,642	0.17	35
Anglo American plc	AAL.L	✓	✓	✓		25,436	0.38	35
Carnival plc	CCL.L	✓				24,280	0.17	35
Tesco plc	TSCO.L	✓	✓	✓		22,009	0.29	35
Crh plc	CRH.L	✓	✓			21,020	0.19	33
London Stock Exch. plc	LSEG.L	✓				20,972	0.22	35
Imperial Tobacco plc	IMB.L	✓		✓		20,431	0.28	35
Experian plc	EXPN.L	✓				20,405	0.23	35
Standard Chartered plc	STAN.L	✓	✓	✓		19,841	0.26	35
Bt Group plc	BT.L	✓	✓	✓		18,844	0.29	35
Associated British Foods plc	ABF.L	✓				18,383	0.15	35
Bae Systems plc	BAES.L	✓	✓	✓		17,315	0.26	35
Aviva plc	AV.L	✓	✓	✓		15,793	0.30	35
Legal&general Grp plc	LGEN.L	✓	✓			14,888	0.30	35
Ferguson plc	FERG.L	✓				14,023	0.28	35
Rolls-royce Holdings plc	RR.L	✓				13,340	0.43	35
Smith & Nephew plc	SN.L	✓				13,330	0.31	35
Sse plc	SSE.L	✓				12,818	0.37	35
Wpp plc	WPP.L	✓				11,832	0.38	35
Ashtead Group plc	AHT.L	✓				10,533	0.38	35
Intl Airlines Grp plc	ICAG.L	✓		✓	✓	9942	0.52	35
3i Group plc	III.L	✓				9368	0.22	35
Ocado Group plc	OCDO.L	✓	✓			9234	0.36	35
Antofagasta plc	ANTO.L	✓				9087	0.28	35
Coca-cola Hbc plc	CCH.L	✓				8896	0.23	35
Hargreaves Lansdow plc	HRGV.L	✓				8676	0.24	35
Intertek Group plc	ITRK.L	✓				8576	0.26	35
Flutter Entertainment plc	FLTRF.L	✓	✓			8570	0.29	35
Mondi plc	MNDI.L	✓	✓			8460	0.38	35
Intercont Hotel plc	IHG.L	✓				8406	0.35	35
Schroders plc	SDR.L	✓				8067	0.14	35
Next plc	NXT.L	✓				7555	0.45	35
Std Life Aberdeen plc	SLA.L	✓				7551	0.32	35
Burberry Group plc	BRBY.L	✓	✓			7517	0.48	35
Bunzl plc	BNZL.L	✓				7388	0.31	35
Rentokil Initial plc	RTO.L	✓				7384	0.31	35
Sage Group plc/the plc	SGE.L	✓				7356	0.30	35
Fresnillo plc	FRES.L	✓				6955	0.23	35
Whitbread plc	WTB.L	✓				6706	0.45	35

Evraz plc	EVRE.L	✓			6514	0.24	35
Land Secs. plc	LAND.L	✓			6072	0.35	35
Smiths Group plc	SMIN.L	✓			5975	0.28	35
Barratt Developmts plc	BDEV.L	✓			5770	0.45	35
Pearson plc	PSON.L	✓			5753	0.50	35
Tui plc	TUIT.L		✓		5717	0.37	35
Johnson Matthey plc	JMAT.L	✓			5648	0.38	35
Centrica plc	CNA.L	✓	✓	✓	5622	0.52	35
Rsa Insurance Group plc	RSA.L	✓	✓	✓	5591	0.35	35
United Utilities Group plc	UU.L	✓		✓	5569	0.38	35
Kingfisher plc	KGF.L	✓		✓	5319	0.47	35
Sainsbury(j) plc	SBRY.L	✓	✓	✓	5309	0.45	35
British Land Co.re plc	BLND.L	✓			5188	0.43	35
Severn Trent plc	SVT.L	✓			5050	0.39	35
Morrison Supermark plc	MRW.L	✓		✓	5045	0.44	35
Itv plc	ITV.L	✓			4991	0.43	35
Easyjet plc	EZJ.L	✓			4561	0.71	35
Direct Line Insurance Group plc	DLGD.L	✓			4406	0.40	35
Weir Group plc	WEIR.L	✓			3943	0.54	35
Marks & Spenc Grp plc	MKS.L	✓	✓	✓	3789	0.60	35
Meggitt plc	MGGT.L	✓			3706	0.41	35
Travis Perkins plc	TPK.L	✓			3191	0.45	35
Tate & Lyle plc	TATE.L	✓			3189	0.40	35
G4s plc	GFS.L	✓			3174	0.43	35
Royal Mail plc	RMG.L	✓		✓	3031	0.63	35
Imi plc	IMI.L	✓			2869	0.32	35
Kaz Minerals plc	KAZ.L	✓			2805	0.41	35
Tullow Oil plc	TLW.L	✓			2799	0.70	35
Northgate Ord plc	BAB.L	✓			2681	0.43	35
Hammerson plc	HMSO.L	✓			2499	0.70	35
Man Group plc	EMG.L	✓			2071	0.25	35
William Hill plc	WMH.L	✓			1883	0.63	35
Aggreko plc	AGGK.L	✓			1787	0.32	35
Frasers Grp plc	FRAS.L	✓			1750	0.13	35
Capita plc	CPI.L	✓			1706	0.57	35
Indivior plc	INDV.L	✓			1636	0.51	35
Dixons Carphone plc	DC.L	✓			1582	0.34	35
Serco Group plc	SRP.L	✓			1440	0.21	35
Petrofac Ltd plc	PFC.L	✓			1406	0.48	35
Cairn Energy plc	CNE.L	✓			1047	0.30	35

4.B Options trade reports cleaning

Table 4.15, Panel A presents the number of options issued and the number of underlying stocks, broken down by year, option exercise and type of delivery. It is quite evident that American-style options with physical delivery are the prevalent option types, both in terms of underlying coverage and the number of contracts issued.

Table 4.15, Panel B provides information on the number of listed options issued and traded on the various exchanges. The Intercontinental Exchange (ICE) is the main exchange in terms of options issued and stocks covered, followed by Eurex. It should be noted that the London Stock Exchange (LSE) decided to withdraw the listing and trading of certain derivatives (including UK derivatives) with the effective date of 21 June 2019.[33] Open interest for any of the options listed on the LSE could not be traced and, in the transaction reports, virtually no trades are found on these options.

The final panel of Table 4.15 reports information on the number of issued options found in the Refinitiv Tick History database. Almost all options with physical delivery are matched, while none of the cash-settled or European-style options (despite being a very small minority) is contained in the database.

Options trades are obtained by transaction reports. All the transactions on option contracts that are part of the final sample, that are issued by either ICE or Eurex and that expire during either 2019 or 2020 (with the exception of March 2020) were retrieved. This study was prevented from retrieving this type of data prior to 2019. A total of 666,215 transactions were reported to the FCA and contained in the sample: 117,696 were reported as off market (XOFF) transactions, while the remaining were reported as transactions conducted on the market (and more than 90% of these transactions are executed on ICE). More than 59% of the transactions are executed on options expiring on quarterly expiration days, despite them being only 7 out of the 23 expiration days considered between 2019-2020. Finally, proprietary trading firms, that are involved in more than half the transactions executed on the exchange, are only marginally involved in the transactions executed off market: only around 4.5% of the off-exchange transactions involve a proprietary trading firm. Off-exchange transaction data suffer from data quality issues that make the identification of the actual trader executing a transaction particularly difficult. Focusing on proprietary trading firms, that execute the vast majority of their trades on the market, significantly reduces these concerns.

[33] See Market Notice 2018/077.

TABLE 4.15: Features of issued options.

Panel A: Options characteristics			
Delivery Type		# Underlying Stocks	# Issued Options
American Options:			
Cash-Settled		48	1874
Physical Delivery		97	264,007
European Options:			
Cash-Settled		7	21
Physical Delivery		32	532
Panel B: Venues breakdown			
Exchange	Currency	# Underlying Stocks	# Issued Options
Eurex	EUR	5	5365
	GBP	31	44,914
ICE	EUR	1	32
	GBP	96	179,336
	SEK	1	4
London Stock Exchange	GBP	33	30,222
Other	EUR	3	4656
	SEK	1	1499

Panel C: Refinitiv matched options					
Exchange	Currency	Delivery type	Exercise type	# Issued options	Matched on Refinitiv
Eurex	EUR	Phys. Delivery	American	5859	5672
	GBP	Cash-Settled	American	6	0
			European	3	0
		Phys. Delivery	American	46,717	46,062
ICE	EUR	Cash-Settled	American	32	0
	GBP	Cash-Settled	American	1890	0
			European	18	0
		Phys. Delivery	American	182,888	182,595
			European	446	0
	SEK	Phys. Delivery	American	4	0

Panel A provides information on the number of unique UK stocks with listed options in the main exchanges and the number of different contracts issued (differing form option type, exercise style, expiration, strike price and type of delivery). Panel B provides general information on option coverage in the main exchanges. The *Others* category comprises: (1) MEFF, which provides options on International Consolidated Airlines Group SA; (2) Nasdaq OMX Nordic,which provides options on AstraZeneca plc; and (3) Euronext, which has options on both Unilever plc and Relx plc. Panel C reports information on the number of options in the data set on which open interest information is found on the Refinitiv Tick History data set, separately for each option type. SEK=Swedish kroner.

4.C Market-wide patterns robustness tests

This section reports on a series of robustness tests conducted on price, trading and order submission effects. The framework proposed above is re-applied to investigate price and trading activity patterns, but with the variables (price changes, traded volume metrics, options moneyness, etc.) defined using two different end-of-day time windows: (1) the first one constituted of the last 30 minutes of continuous trading plus the closing auction and (2) the second one constituted by the closing auction only.

In Table 4.16, the estimates of the model (Equation [4.7]) are reported, with abnormal returns computed by using both mean- and market-adjusted models and computed over the two end-of-day time windows, that is, (1) the last 30 minutes of continuous trading plus the closing auction and (2) only the closing auction. The variables PUT_ITM and $CALL\_ITM$ are also computed by taking as a benchmark the stock's midpoint price observed 30 minutes before the end of continuous trading and the last midpoint price before the end of continuous trading.

Table 4.17 reports the estimates of regression (Equation [4.9]), where abnormal reversals are computed as usual and the different end-of-day time window choices are reflected in how PUT_ITM and $CALL\_ITM$ are computed.

In Table 4.18 the estimates of regressions (Equation [4.10]) and (Equation [4.11]) are reported, with PUT_ITM and $CALL\_ITM$ based on both the midpoint 30 minutes before the end of continuous trading and the last midpoint before the end of continuous trading.

Finally, Tables 4.19 and 4.20 report the estimates of models (Equation [4.12]) and (Equation [4.13]) for large orders computed by using a 90th and 95th percentile threshold, respectively. The different end-of-day time window choice is reflected in how PUT_ITM and $CALL\_ITM$ are computed.

TABLE 4.16: Robustness testing - Abnormal end-of-day returns.

	$M = 1$		$M = 2$		$M = 3$	
	Mean Adj.	Market Adj.	Mean Adj.	Market Adj.	Mean Adj.	Market Adj.
Panel A: midpoint 30 minutes - All options						
PUT_ITM	0.000	−0.013	−0.000	−0.003	−0.014	−0.013
	(0.013)	(−0.471)	(−0.005)	(−0.127)	(−0.798)	(−0.714)
$CALL\_ITM$	0.017	0.005	−0.017	−0.015	−0.015	−0.009
	(0.678)	(0.183)	(−0.796)	(−0.704)	(−0.735)	(−0.461)
Observations	3393	3393	3393	3393	3393	3393
Adjusted R^2	0.188	0.019	0.188	0.019	0.1885	0.019
Panel B: midpoint 30 minutes - Above median exposure						
PUT_ITM	0.024	0.011	−0.015	−0.021	−0.002	−0.006
	(0.820)	(0.346)	(−0.563)	(−0.762)	(−0.086)	(−0.255)
$CALL\_ITM$	0.010	0.014	−0.004	0.001	−0.014	−0.005
	(0.214)	(0.298)	(−0.136)	(0.048)	(−0.592)	(−0.217)
Observations	3393	3393	3393	3393	3393	3393
Adjusted R^2	0.188	0.019	0.188	0.019	0.188	0.019
Panel C: midpoint before auction - All options						
PUT_ITM	−0.011	−0.020	0.005	0.000	−0.011	−0.016
	(−0.428)	(−0.706)	(0.243)	(0.008)	(−0.607)	(−0.896)
$CALL\_ITM$	−0.028	−0.042	−0.004	−0.009	−0.009	−0.012
	(−1.050)	(−1.577)	(−0.206)	(−0.435)	(−0.569)	(−0.747)
Observations	3393	3393	3393	3393	3393	3393
Adjusted R^2	0.116	0.008	0.116	0.007	0.116	0.008
Panel D: midpoint before auction - Above median exposure						
PUT_ITM	−0.023	−0.030	0.000	−0.007	−0.010	−0.016
	(−0.962)	(−1.178)	(0.017)	(−0.285)	(−0.370)	(−0.573)
$CALL\_ITM$	−0.057	−0.060	−0.023	−0.029	−0.003	−0.007
	(−1.314)	(−1.438)	(−0.709)	(−0.928)	(−0.131)	(−0.313)
Observations	3393	3393	3393	3393	3393	3393
Adjusted R^2	0.116	0.008	0.116	0.008	0.116	0.008

The table provides estimates of regression (Equation [4.7]) with abnormal returns computed by using the change between the closing price and both: (1) the midpoint price 30 minutes before the end of continuous trading (Panels A and B) and (2) the last midpoint before the start of the closing call auction (Panels C and D). The dependent variables $CALL\_ITM$ and PUT_ITM are measured accordingly, by using either the price observed 30 minutes before the end of continuous trading (Panels A and B) or the last midpoint observed during continuous trading (Panels C and D). Restrictions in size of the option exposure are also considered, in Panels B and D. Standard errors are clustered by stock. Date fixed effects are included in all regressions. ***, ** and * denote statistical significance at the 1%, 5% and 10% levels, respectively. Parameter values are reported in percentage points.

TABLE 4.17: Robustness testing - Abnormal price reversals.

	All options			Above median exposure		
	$M = 1$	$M = 2$	$M = 3$	$M = 1$	$M = 2$	$M = 3$
Panel A: midpoint 30 minutes						
PUT_ITM	-0.011	-0.006	-0.002	-0.022	0.111	0.075
	(-0.133)	(-0.120)	(-0.039)	(-0.170)	(1.231)	(1.038)
$CALL\_ITM$	-0.116	-0.116	-0.083	-0.121	-0.052	-0.029
	(-1.636)	(-1.511)	(-1.166)	(-1.064)	(-0.737)	(-0.392)
Observations	3393	3393	3393	3393	3393	3393
Adjusted R^2	0.0412	0.0415	0.0413	0.0411	0.0414	0.0411
Panel B: midpoint before auction						
PUT_ITM	-0.137^{*}	-0.032	0.032	-0.171	0.017	0.101
	(-1.648)	(-0.438)	(0.572)	(-1.212)	(0.176)	(1.108)
$CALL\_ITM$	-0.029	-0.039	-0.005	0.172	0.050	0.095
	(-0.251)	(-0.484)	(-0.076)	(1.050)	(0.486)	(1.336)
Observations	3393	3393	3393	3393	3393	3393
Adjusted R^2	0.393	0.393	0.393	0.393	0.393	0.393

The table provides estimates of regression (Equation [4.9]). The dependent variables $CALL\_ITM$ and PUT_ITM are based on: (1) the midpoint price measured 30 minutes before the end of continuous trading (Panels A and B) and (2) the last midpoint price registered before the start of the closing auction (Panels C and D). Restrictions in size of the option exposure are also considered, in Panels B and D. Standard errors are clustered by stock. Date fixed effects are included in all regressions. ***, ** and * denote statistical significance at the 1%, 5% and 10% level, respectively. Parameters values are reported in percentage points.

TABLE 4.18: Robustness testing - Abnormal end-of-day trading activity.

	$M = 1$		$M = 2$		$M = 3$	
	TR	CR	TR	CR	TR	CR
Panel A: midpoint 30 minutes - All options						
ITM_OPTION	0.020	0.135	0.010	−0.333	0.002	−0.129
	(1.410)	(0.263)	(0.829)	(−0.664)	(0.185)	(−0.282)
$log(ILLIQ)$	0.026***	1.345***	0.026***	1.298***	0.025***	1.317***
	(2.929)	(4.291)	(2.908)	(4.013)	(2.831)	(4.054)
$Beta$	−0.001	−3.034***	−0.001	−3.016***	−0.001	−3.021***
	(−0.117)	(−6.166)	(−0.108)	(−6.131)	(−0.080)	(−6.143)
$FTSE100$	−0.028	−0.184	−0.029	−0.202	−0.029	−0.195
	(−1.239)	(−0.190)	(−1.263)	(−0.208)	(−1.271)	(−0.201)
Observations	3393	3393	3393	3393	3393	3393
Adjusted R^2	0.299	0.605	0.298	0.605	0.298	0.605
Panel B: midpoint 30 minutes - Above median exposure						
ITM_OPTION	0.029	0.037	0.016	−0.392	0.008	−0.164
	(1.318)	(0.056)	(1.037)	(−0.707)	(0.686)	(−0.358)
$log(ILLIQ)$	0.026***	1.337***	0.026***	1.305***	0.026***	1.319***
	(2.908)	(4.307)	(2.907)	(4.078)	(2.862)	(4.111)
$Beta$	−0.001	−3.031***	−0.001	−3.021***	−0.001	−3.022***
	(−0.113)	(−6.171)	(−0.106)	(−6.149)	(−0.109)	(−6.136)
$FTSE100$	−0.028	−0.189	−0.028	−0.209	−0.029	−0.200
	(−1.233)	(−0.195)	(−1.241)	(−0.214)	(−1.250)	(−0.206)
Observations	3393	3393	3393	3393	3393	3393
Adjusted R^2	0.299	0.605	0.298	0.605	0.298	0.605

The table provides the estimates of regressions (Equation [4.10]) and (Equation [4.11]), with Turnover ratio (TR) and Concentration ratio (CR) as dependent variables. The dependent variable $OPTION\_ITM$ is based on: (1) the midpoint price measured 30 minutes before the end of continuous trading (Panels A and B) and 2) the last midpoint price before the start of the closing auction (Panels C and D). Restrictions in size of the option exposure are also considered, in Panels B and D. Standard errors are clustered by stock. ***, ** and * denote statistical significance at the 1%, 5% and 10% level, respectively. Parameter values are reported in percentage points. Date fixed effects are included in all regressions.

Table 4.18: *continued*.

	$M=1$		$M=2$		$M=3$	
	TR	CR	TR	CR	TR	CR
Panel C: midpoint before auction - All options						
ITM_OPTION	0.019	-0.046	0.013	-0.065	0.008	-0.131
	(1.087)	(-0.072)	(1.101)	(-0.125)	(0.753)	(-0.253)
$log(ILLIQ)$	0.022^{***}	0.693^{**}	0.022^{***}	0.690^{**}	0.022^{***}	0.681^{*}
	(2.853)	(2.036)	(2.861)	(1.991)	(2.846)	(1.924)
$Beta$	-0.002	-2.805^{***}	-0.002	-2.805^{***}	-0.002	-2.802^{***}
	(-0.207)	(-5.333)	(-0.206)	(-5.328)	(-0.213)	(-5.311)
$FTSE100$	-0.019	0.545	-0.019	0.544	-0.019	0.542
	(-0.930)	(0.528)	(-0.933)	(0.526)	(-0.944)	(0.524)
Observations	3393	3393	3393	3393	3393	3393
Adjusted R^2	0.3070	0.6632	0.3068	0.6632	0.3066	0.6632
Panel D: midpoint before auction - Above median exposure						
ITM_OPTION	0.001	-1.205	0.010	-0.262	0.009	-0.492
	(0.048)	(-1.570)	(0.557)	(-0.412)	(0.524)	(-0.815)
$log(ILLIQ)$	0.021^{***}	0.664^{**}	0.022^{***}	0.681^{**}	0.022^{***}	0.656^{*}
	(2.765)	(1.964)	(2.842)	(1.986)	(2.832)	(1.881)
$Beta$	-0.002	-2.801^{***}	-0.002	-2.804^{***}	-0.002	-2.799^{***}
	(-0.196)	(-5.320)	(-0.200)	(-5.329)	(-0.207)	(-5.310)
$FTSE100$	-0.019	0.520	-0.019	0.535	-0.019	0.521
	(-0.955)	(0.504)	(-0.941)	(0.517)	(-0.942)	(0.503)
Observations	3393	3393	3393	3393	3393	3393
Adjusted R^2	0.306	0.664	0.307	0.663	0.307	0.663

TABLE 4.19: Robustness testing - Abnormal execution frequency of large orders at the close (90th percentile).

	$M=1$		$M=2$		$M=3$	
	BID	ASK	BID	ASK	BID	ASK
Panel A: midpoint 30 minutes - All options						
PUT_ITM	0.572		−0.007		−0.045	
	(1.016)		(−0.017)		(−0.124)	
$CALL\_ITM$		−0.343		0.191		0.191
		(−0.566)		(0.422)		(0.498)
$log(ILLIQ)$	0.068	−0.171	0.050	−0.154	0.047	−0.153
	(0.223)	(−0.637)	(0.165)	(−0.577)	(0.158)	(−0.568)
Beta	−0.001	0.069	0.010	0.061	0.012	0.061
	(−0.001)	(0.133)	(0.026)	(0.119)	(0.030)	(0.119)
$FTSE100$	−0.149	−1.462***	−0.160	−1.456***	−0.159	−1.457***
	(−0.221)	(−2.571)	(−0.236)	(−2.567)	(−0.235)	(−2.565)
Adjusted R^2	0.229	0.153	0.228	0.153	0.228	0.153
Panel B: midpoint 30 minutes - Above median exposure						
PUT_ITM	1.197		0.647		0.145	
	(1.592)		(1.292)		(0.365)	
$CALL\_ITM$		0.346		0.913		0.743
		(0.403)		(1.422)		(1.361)
$log(ILLIQ)$	0.071	−0.158	0.071	−0.132	0.056	−0.134
	(0.236)	(−0.589)	(0.236)	(−0.498)	(0.186)	(−0.501)
Beta	−0.001	0.063	0.005	0.053	0.007	0.051
	(−0.003)	(0.122)	(0.013)	(0.104)	(0.019)	(0.098)
$FTSE100$	−0.157	−1.456***	−0.158	−1.429**	−0.159	−1.435**
	(−0.234)	(−2.564)	(−0.234)	(−2.532)	(−0.236)	(−2.529)
Adjusted R^2	0.229	0.153	0.229	0.154	0.228	0.154
Observations	2229	2229	2229	2229	2229	2229

The table reports the estimates of regressions (Equation [4.12]) and (Equation [4.13]), with $LARGE\_BID_{i,t}$ and $LARGE\_ASK_{i,t}$ (computed using the 90th percentile as the cut-off to define large orders) as dependent variables. In Panels A and B, the dependent variables $CALL\_ITM$ and PUT_ITM are based on the midpoint price measured 30 minutes before the end of continuous trading, while in Panels C and D, their measure is based on the last midpoint price registered before the start of the closing auction. Restrictions in size of the option exposure are also considered, in Panels B and D. Standard errors are clustered by stock. ***, ** and * denote statistical significance at the 1%, 5% and 10% levels, respectively. Parameters are reported in percentage points.

Table 4.19: *continued*.

	$M=1$		$M=2$		$M=3$	
	BID	ASK	BID	ASK	BID	ASK
Panel C: midpoint before auction - All options						
PUT_ITM	0.235		-0.091		-0.013	
	(0.308)		(-0.167)		(-0.027)	
$CALL\_ITM$		-0.522		-0.199		-0.103
		(-0.676)		(-0.334)		(-0.209)
$log(ILLIQ)$	0.053	-0.176	0.048	-0.172	0.050	-0.169
	(0.177)	(-0.654)	(0.158)	(-0.631)	(0.165)	(-0.621)
$Beta$	0.006	0.066	0.011	0.065	0.010	0.065
	(0.016)	(0.127)	(0.029)	(0.126)	(0.026)	(0.126)
$FTSE100$	-0.161	-1.476^{***}	-0.159	-1.468^{***}	-0.159	-1.463^{***}
	(-0.238)	(-2.582)	(-0.235)	(-2.570)	(-0.235)	(-2.565)
Adjusted R^2	0.228	0.153	0.228	0.153	0.228	0.153
Panel D: midpoint before auction - Above median exposure						
PUT_ITM	0.956		0.776		0.769	
	(1.131)		(1.349)		(1.561)	
$CALL\_ITM$		-0.062		0.237		0.333
		(-0.058)		(0.275)		(0.533)
$log(ILLIQ)$	0.056	-0.164	0.064	-0.155	0.071	-0.147
	(0.184)	(-0.608)	(0.213)	(-0.569)	(0.237)	(-0.544)
$Beta$	0.004	0.065	0.004	0.065	0.003	0.063
	(0.010)	(0.126)	(0.011)	(0.126)	(0.009)	(0.122)
$FTSE100$	-0.160	-1.462^{***}	-0.160	-1.451^{**}	-0.161	-1.445^{**}
Observations	2229	2229	2229	2229	2229	2229
Adjusted R^2	0.229	0.153	0.229	0.153	0.229	0.153

TABLE 4.20: Robustness testing - Abnormal execution frequency of large orders at the close (95th percentile).

	$M=1$		$M=2$		$M=3$	
	BID	ASK	BID	ASK	BID	ASK
Panel A: midpoint 30 minutes - All options						
PUT_ITM	-0.122		-0.233		-0.213	
	(-0.296)		(-0.674)		(-0.765)	
$CALL\_ITM$		-0.305		0.091		0.097
		(-0.680)		(0.277)		(0.351)
$log(ILLIQ)$	-0.192	-0.313^{*}	-0.199	-0.301	-0.201	-0.300
	(-0.895)	(-1.646)	(-0.936)	(-1.586)	(-0.950)	(-1.571)
Beta	-0.057	0.261	-0.054	0.256	-0.051	0.256
	(-0.210)	(0.631)	(-0.198)	(0.619)	(-0.188)	(0.619)
$FTSE100$	0.225	-0.941^{**}	0.231	-0.938^{**}	0.230	-0.938^{**}
	(0.478)	(-2.293)	(0.490)	(-2.286)	(0.489)	(-2.285)
Adjusted R^2	0.265	0.214	0.265	0.214	0.265	0.214
Panel B: midpoint 30 minutes - Above median exposure						
PUT_ITM	0.505		0.188		-0.099	
	(0.942)		(0.510)		(-0.347)	
$CALL\_ITM$		0.246		0.611		0.427
		(0.401)		(1.469)		(1.156)
$log(ILLIQ)$	-0.179	-0.302	-0.182	-0.285	-0.192	-0.289
	(-0.843)	(-1.594)	(-0.859)	(-1.509)	(-0.901)	(-1.520)
Beta	-0.064	0.256	-0.061	0.250	-0.058	0.249
	(-0.236)	(0.619)	(-0.224)	(0.605)	(-0.212)	(0.603)
$FTSE100$	0.228	-0.937^{**}	0.228	-0.919^{**}	0.227	-0.925^{**}
	(0.487)	(-2.285)	(0.485)	(-2.246)	(0.482)	(-2.253)
Adjusted R^2	0.265	0.214	0.265	0.215	0.265	0.215
Observations	2229	2229	2229	2229	2229	2229

The table reports the estimates of regressions (Equation [4.12]) and (Equation [4.13]), with $LARGE\_BID_{i,t}$ and $LARGE\_ASK_{i,t}$ (computed using 95th percentile as the cut-off to define large orders) as dependent variables. In Panels A and B, the dependent variables $CALL\_ITM$ and PUT_ITM are based on the midpoint price measured 30 minutes before the end of continuous trading, while in Panels C and D, their measure is based on the last midpoint price registered before the start of the closing auction. Restrictions in size of the option exposure are also considered, in Panels B and D. Standard errors are clustered by stock. ***, ** and * denote statistical significance at the 1%, 5% and 10% levels, respectively. Parameters are reported in percentage points.

Table 4.20: *continued*.

	$M=1$		$M=2$		$M=3$	
	BID	ASK	BID	ASK	BID	ASK
Panel C: midpoint before auction - All options						
PUT_ITM	-0.236		-0.199		-0.111	
	(-0.434)		(-0.491)		(-0.320)	
$CALL\_ITM$		-0.389		-0.083		-0.133
		(-0.725)		(-0.197)		(-0.389)
$log(ILLIQ)$	-0.192	-0.316^{*}	-0.194	-0.309	-0.192	-0.314
	(-0.895)	(-1.655)	(-0.909)	(-1.608)	(-0.905)	(-1.631)
$Beta$	-0.056	0.258	-0.057	0.258	-0.057	0.258
	(-0.205)	(0.623)	(-0.208)	(0.623)	(-0.207)	(0.624)
$FTSE100$	0.229	-0.952^{**}	0.230	-0.943^{**}	0.231	-0.944^{**}
	(0.486)	(-2.302)	(0.487)	(-2.285)	(0.489)	(-2.293)
Adjusted R^2	0.265	0.214	0.265	0.214	0.265	0.214
Panel D: midpoint before auction - Above median exposure						
PUT_ITM	0.230		0.308		0.398	
	(0.370)		(0.739)		(1.228)	
$CALL\_ITM$		-0.339		0.368		0.277
		(-0.459)		(0.622)		(0.683)
$log(ILLIQ)$	-0.187	-0.311	-0.183	-0.294	-0.177	-0.293
	(-0.874)	(-1.638)	(-0.860)	(-1.529)	(-0.837)	(-1.533)
$Beta$	-0.061	0.258	-0.062	0.258	-0.063	0.256
	(-0.223)	(0.623)	(-0.226)	(0.624)	(-0.231)	(0.620)
$FTSE100$	0.227	-0.947^{**}	0.227	-0.925^{**}	0.227	-0.927^{**}
	(0.483)	(-2.304)	(0.484)	(-2.239)	(0.483)	(-2.257)
Adjusted R^2	0.265	0.214	0.265	0.214	0.265	0.214
Observations	2229	2229	2229	2229	2229	2229

4.D Proprietary traders' robustness tests

This section reports results of the robustness tests for the framework applied to test the existence of suspicious behaviours from proprietary traders. The framework applied is the same as proposed in the current study, but uses abnormal returns (and options moneyness) computed based on both: (1) the midpoint price 30 minutes before the end of continuous trading and (2) the last midpoint price before the end of continuous trading. The setting, therefore, is the same as the one used in the previous section.

First, the framework (Equation [4.14]) is applied on the top two proprietary trading firms, that is, the most active ones, using abnormal price changes measured over the last 15 minutes (Table 4.21), the last 30 minutes (Table 4.22) and over the closing auction (Table 4.23).

The framework (Equation [4.15]) is then applied using abnormal end-of-day log returns over both the last 30 minutes (Table 4.24) and the closing auction (Table 4.25).

TABLE 4.21: Regression results - Abnormal end-of-day returns (top two traders).

	$M=1$		$M=2$		$M=3$	
	Mean Adj.	Market Adj.	Mean Adj.	Market Adj.	Mean Adj.	Market Adj.
Constant	0.024	0.057**	0.024	0.057**	0.024	0.057**
	(0.814)	(1.978)	(0.813)	(1.979)	(0.814)	(1.979)
ID	−0.004	−0.003	0.005	−0.000	0.004	0.001
	(−0.083)	(−0.054)	(0.150)	(−0.005)	(0.158)	(0.042)
$ID \times SELL$	−0.003	−0.045	−0.039	−0.050	−0.037	−0.049
	(−0.018)	(−0.315)	(−0.498)	(−0.640)	(−0.576)	(−0.739)
IU	0.039	0.036	0.060	0.059	0.025	0.024
	(0.666)	(0.610)	(1.523)	(1.497)	(0.858)	(0.805)
$IU \times BUY$	−0.006	−0.025	0.002	−0.001	0.011	0.010
	(−0.085)	(−0.359)	(0.038)	(−0.017)	(0.225)	(0.204)
BUY dummies	Yes	Yes	Yes	Yes	Yes	Yes
SELL dummies	Yes	Yes	Yes	Yes	Yes	Yes
Date fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2229	2229	2229	2229	2229	2229
Adjusted R^2	0.1975	0.1157	0.1984	0.1166	0.1978	0.1160

The table reports the estimates of regression (Equation [4.14]) applied to top two traders (in terms of the number of manipulation incentives). The abnormal returns are computed based on the price change over the last 15 minutes of the trading day. The variables *BUY* and *SELL* take the value 1 if the market participants execute a buy (sell) trade in the closing auction and do not trade in the opposite direction. Standard errors are clustered by stock. ***, ** and * denote statistical significance at the 1%, 5% and 10% levels, respectively. Parameters are reported in percentage points.

TABLE 4.22: Robustness testing - Abnormal end-of-day returns (top two traders) measured over the last 30 minutes of trading.

	$M = 1$		$M = 2$		$M = 3$	
	Mean Adj.	Market Adj.	Mean Adj.	Market Adj.	Mean Adj.	Market Adj.
Constant	-0.003	0.100^{***}	-0.004	0.099^{***}	-0.004	0.099^{***}
	(-0.087)	(2.672)	(-0.108)	(2.644)	(-0.120)	(2.631)
ID	-0.041	-0.079	0.007	-0.008	0.005	-0.002
	(-0.820)	(-1.594)	(0.214)	(-0.233)	(0.197)	(-0.079)
$ID \times SELL$	0.160	0.135	-0.027	-0.039	-0.039	-0.042
	(1.555)	(1.311)	(-0.413)	(-0.606)	(-0.673)	(-0.731)
IU	0.032	0.035	0.017	0.018	-0.006	-0.003
	(0.670)	(0.718)	(0.497)	(0.499)	(-0.244)	(-0.093)
$IU \times BUY$	0.037	0.018	0.047	0.038	-0.008	-0.013
	(0.562)	(0.278)	(0.826)	(0.689)	(-0.146)	(-0.262)
BUY dummies	Yes	Yes	Yes	Yes	Yes	Yes
SELL dummies	Yes	Yes	Yes	Yes	Yes	Yes
Date fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2229	2229	2229	2229	2229	2229
Adjusted R^2	0.1905	0.1127	0.1899	0.1122	0.1897	0.1120

The table reports the estimates of regression (Equation [4.14]) applied to top two traders (in terms of the number of manipulation incentives). The abnormal returns are computed based on the price change over the last 30 minutes of the trading day. The variables *BUY* and *SELL* take the value 1 if the market participants execute a buy (sell) trade in the closing auction and do not trade in the opposite direction. Standard errors are clustered by stock. ***, ** and * denote statistical significance at the 1%, 5% and 10% levels, respectively. Parameters are reported in percentage points.

TABLE 4.23: Robustness testing - Abnormal end-of-day returns (top two traders) measured over the closing call auction.

	$M = 1$		$M = 2$		$M = 3$	
	Mean Adj.	Market Adj.	Mean Adj.	Market Adj.	Mean Adj.	Market Adj.
Constant	0.003	0.035*	0.003	0.035*	0.003	0.035*
	(0.145)	(1.689)	(0.151)	(1.693)	(0.141)	(1.686)
ID	−0.020	−0.054	0.029	0.018	0.001	−0.003
	(−0.439)	(−1.055)	(0.899)	(0.499)	(0.059)	(−0.103)
$ID \times SELL$	−0.158	−0.124	−0.071	−0.067	−0.057	−0.051
	(−1.226)	(−0.956)	(−0.932)	(−0.865)	(−0.973)	(−0.850)
IU	−0.011	−0.006	0.022	0.022	−0.000	−0.009
	(−0.229)	(−0.117)	(0.688)	(0.657)	(−0.003)	(−0.333)
$IU \times BUY$	0.137**	0.130*	0.089*	0.076	0.028	0.038
	(2.019)	(1.940)	(1.817)	(1.501)	(0.522)	(0.687)
BUY dummies	Yes	Yes	Yes	Yes	Yes	Yes
SELL dummies	Yes	Yes	Yes	Yes	Yes	Yes
Date fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2229	2229	2229	2229	2229	2229
Adjusted R^2	0.1992	0.1066	0.1982	0.1052	0.1974	0.1046

The table reports the estimates of regression (Equation [4.14]) applied to top two traders (in terms of the number of manipulation incentives). The abnormal returns are computed based on the price change over the closing auction. The variables *BUY* and *SELL* take the value 1 if the market participants execute a buy (sell) trade in the closing auction and do not trade in the opposite direction. Standard errors are clustered by stock. ***, ** and * denote statistical significance at the 1%, 5% and 10% levels, respectively. Parameters are reported in percentage points.

TABLE 4.24: Robustness testing - Abnormal end-of-day returns (Trader A and large executions) measured over the last 30 minutes of trading.

	$M = 1$		$M = 2$		$M = 3$	
	Mean Adj.	Market Adj.	Mean Adj.	Market Adj.	Mean Adj.	Market Adj.
Constant	-0.027	0.073^{*}	-0.027	0.073^{*}	-0.027	0.073^{*}
	(-0.645)	(1.716)	(-0.645)	(1.714)	(-0.644)	(1.716)
ID	0.070	0.030	0.036	0.023	0.031	0.016
	(0.804)	(0.356)	(0.625)	(0.417)	(0.657)	(0.353)
$ID \times SELL$	0.102	0.117	0.013	0.010	0.008	0.015
	(0.753)	(0.914)	(0.108)	(0.086)	(0.083)	(0.166)
IU	-0.031	-0.025	-0.008	0.001	-0.047	-0.042
	(-0.501)	(-0.390)	(-0.162)	(0.023)	(-1.079)	(-0.923)
$IU \times BUY$	0.020	0.013	0.045	0.031	0.074	0.067
	(0.191)	(0.128)	(0.523)	(0.357)	(0.896)	(0.858)
BUY	Yes	Yes	Yes	Yes	Yes	Yes
SELL	Yes	Yes	Yes	Yes	Yes	Yes
Date fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1863	1863	1863	1863	1863	1863
Adjusted R^2	0.1748	0.0950	0.1740	0.0944	0.1744	0.0948

The table reports the estimates of regression (Equation [4.15]). The abnormal returns are computed based on the price change over the last 30 minutes of the trading day. The definition of *BUY* and *SELL* variables depends on whether the market participant executes a buy (sell) trade that is considered large (above the 90th percentile cut-off) in the closing auction and does not trade in the opposite direction. Standard errors are clustered by stock. ***, ** and * denote statistical significance at the 1%, 5% and 10% levels, respectively. Parameters are reported in percentage points.

TABLE 4.25: Robustness testing - Abnormal end-of-day returns (Trader A and large executions) measured over the closing call auction.

	$M=1$		$M=2$		$M=3$	
	Mean Adj.	Market Adj.	Mean Adj.	Market Adj.	Mean Adj.	Market Adj.
Constant	0.008	0.039*	0.008	0.039*	0.008	0.039*
	(0.362)	(1.728)	(0.360)	(1.727)	(0.363)	(1.731)
ID	0.024	0.005	0.094*	0.083	0.036	0.026
	(0.419)	(0.088)	(1.765)	(1.469)	(0.898)	(0.643)
$ID \times SELL$	−0.028	0.027	−0.045	−0.014	0.026	0.054
	(−0.161)	(0.161)	(−0.355)	(−0.116)	(0.269)	(0.553)
IU	−0.040	−0.016	−0.021	−0.003	0.005	0.011
	(−0.546)	(−0.226)	(−0.384)	(−0.061)	(0.123)	(0.268)
$IU \times BUY$	0.181*	0.140	0.157*	0.117	0.114*	0.090
	(1.659)	(1.305)	(1.895)	(1.427)	(1.649)	(1.312)
BUY	Yes	Yes	Yes	Yes	Yes	Yes
SELL	Yes	Yes	Yes	Yes	Yes	Yes
Date fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1863	1863	1863	1863	1863	1863
Adjusted R^2	0.2052	0.1005	0.2068	0.1020	0.2060	0.1014

The table reports the estimates of regression (Equation [4.15]). The abnormal returns are computed based on the price change over the last closing auction. The definition of *BUY* and *SELL* variables depends on whether the market participant executes a buy (sell) trade that is considered large (above the 90th percentile cut-off) in the closing auction and does not trade in the opposite direction. Standard errors are clustered by stock. ***, ** and * denote statistical significance at the 1%, 5% and 10% levels, respectively. Parameters are reported in percentage points.

Chapter 5

Conclusion

This thesis presents a study of the incentives for manipulating the underlying price of a derivative prior to expiration. The problem is approached from both a theoretical and an empirical point of view.

Chapter 1 contextualises the research themes by providing an overview of manipulation strategies and how the specific type of manipulation herein analysed fits within the myriad forms of manipulative behaviours. It also presents the motivation for this study.

Chapter 2 proposes a series of models (based on both classical and dynamic optimisation techniques) to study the trade-off faced by a potential manipulator holding a derivative. The study examines the general case of a derivative written on a basket of underlying securities, modelling the scenario in which an agent can trade any of these securities prior to expiration. By trading on each security, the manipulator has an impact on the price. The linear payoff case is first treated. The optimal trading strategy is analysed, first under the assumption of constant trading over the interval (in a very similar fashion to Dutt and L. E. Harris, (2005) and Ko et al., (2016)), then extending the framework by relaxing this assumption and allowing dynamic trading throughout the interval. Stochastic optimal control techniques are used to find the optimal solution. It is shown, in the chapter, that holding a linear derivative modifies the normal agent's behaviour, generating an incentive to manipulate the underlying securities for different levels of risk-aversion. By means of this model, a series of formulas are proposed to measure the manipulability of the underlying securities. The manipulator's trade-off solved for derivatives with non-linear payoff is then studied. The option payoff case is analytically solved under a constant trading schedule scenario. A linkage is found with the optimal solution for linear payoffs and it is shown how, even in the multi-asset scenario, the solution can be fully described by a one-variable non-linear equation. This leads to an appealing and easily implementable way to measure the manipulability of the underlying basket of securities. These formulas can be used to solve the problems raised by Ko et al., (2016) (i.e., finding a way to measure which securities are the most likely target of manipulation in the multi-asset case and to what degree a basket of securities underlying a derivative is manipulable). Finally, numerical simulations are conducted that aim to show the features of the optimal strategies and under which conditions the agent's trade-off is solved in favour of manipulating the underlying securities.

Chapter 3 investigates the existence of expiration day effects in the UK market. The existence of these anomalous effects has been a concern since the inception of derivative markets. Many reasons have been advanced to explain the existence of this phenomenon, from the activity of arbitrageurs and market makers to market manipulation. The market manipulation concern is obviously central to this thesis and the main driver of this study. Testing the existence of abnormal patterns, irrespective of the cause, is hence an important topic of research. Determining the magnitude of the impact that can be expected around these events is crucial for both regulators and exchanges to evaluate the efficiency of the market and to ensure the protection of retail investors. A number of hypotheses are tested on volume, price and volatility effects and an empirical framework is proposed and then applied to the UK equity market. Strong evidence is found in support of the existence of expiration effects associated with index derivatives' expiration (for both FTSE index futures and options). The study reports a significant increase in trading activity around the EDSP auction, which starts at 10:10 a.m. and ends at 10:15 a.m., and which determines the settlement price of FTSE index derivatives on expiration day. Over the same interval, significant price distortions are also documented, with these reflected in the increase of volatility as well as the frequency and intensity of price reversals. The massive concentration of traded volume around the EDSP auction tends to generate abnormal movements in the price, which then reverse back shortly after.

A significant spike in trading activity at the end of the trading day on quarterly expiration days is also reported. This anomalous increase in traded volume mostly interests the closing auction, leading to more than 40%[1] of the daily volume traded in the last five minutes of the trading day, on average. This is a market-wide phenomenon, with no linkage found with the expiration of index derivatives (which settle in the morning session) nor with the expiration of individual equity options (considering not all FTSE 350 stocks are optionable, and only a minority of FTSE 250 stocks are). These findings are interpreted as being due to the quarterly recalibration of FTSE indices, which leads passive investing funds to rebalance their portfolios ahead of the constituents' changes, generally taking place on the Monday following expiration day on quarter-end months. The spike of volume traded on the closing auction can be explained by the desire of funds to trade at the closing price, to minimise tracking error. In support of this interpretation, the study finds that stocks at the edges of the respective FTSE Index (i.e., the stocks that are expected to be impacted the most by the review of market indices) experience the highest rise in trading volume at the close. By using intraday data, therefore, two distinct anomalies on quarterly expiration days are found. The increase in trading volume at the close, which cannot be linked to derivatives' expiration, is also the dominating source of anomalous patterns. This effect cannot be captured by the framework proposed by Batrinca et al., (2020) who rely on daily volume data to study the effects of the expiration of the main index futures in Europe (including FTSE 100 futures) on the underlying stocks. Although the expiration of index derivatives has an effect on both trading volume and on the price dynamics, it is likely that Batrinca et al., (2020) are

[1] Almost 46% for FTSE 250 stocks.

overestimating these effects. Finally, very little impact is found on optionable stocks when individual options expire.

Finally, Chapter 4 expands on previous chapters and provides an empirical framework to test the existence of manipulative behaviour towards the end of the day on options expiration days. In other words, the study tests if patterns in line with marking the close manipulation take place in connection with the expiration of equity options.

Numerous reasons motivate this study. Firstly, as shown in Chapter 2 if manipulation of the underlying prices is considered feasible, the decision to manipulate the underlying prices can be rational from the point of view of a profit-maximising agent. This is in line with prior theoretical research on this topic. Secondly, various cases of manipulation have been prosecuted by financial regulators throughout the years. Examples of prosecuted cases are provided in Section 4.3. This common concern in the industry needs to be addressed. Furthermore, empirical evidence exists that prices of optionable stocks are manipulated on the day of expiration: see Ni et al., (2005). This evidence is, however, circumstantial. Despite all the reasons that point to the importance of investigating this phenomenon, a significant lack of empirical research is evident. As discussed by Davies, (2020), besides the technical difficulties to design a framework, the lack of data is a major factor.

The current study was able to access a unique data set maintained by the FCA comprising information on trades executed by market participants on both the option and equity markets. This provided a unique opportunity to conduct one of the first empirical investigations to test the existence of this type of manipulation. Firstly, a broad overview was presented of single stock options market in the UK between 2018 and 2020, in terms of options characteristics, size of the options market with respect to the underlying stocks, and types of market participants trading. It was shown that options with quarterly expiration tend to be significantly more active. On average, the open interest is much higher (almost four times higher) on quarterly expiration than on non-quarterly expiration days, generating the highest incentives to manipulate. This evidence further strengthens the author's interest to continue investigating the noticeable anomalies documented in Chapter 3 on quarterly expiration days. Although these abnormal patterns at the close are interpreted as mostly driven by FTSE index recalibrations, the possibility that manipulation could also be a factor cannot be fully discarded. The existence of anomalous patterns is then explored, at a price, trading volume and (exploiting order book data) order submission level. The study focuses on when stocks have ITM options on the day of expiration, with these also being sufficiently close to the money for manipulation to be feasible. Overall, no evidence is found in support of the hypotheses that equity markets are manipulated by option writers on expiration days.

Finally, market participant data were used to further investigate the behaviour of a specific class of traders, that is, proprietary trading firms. These traders are among the most active participants in the options market and among the market participants with the highest incentives to manipulate the market. The framework used by Alexakis et al., (2021) is adapted to study if trading activity by this class of market participants is associated with manipulative behaviour on expiration days. Also in this case, no strong evidence is found to support the broad hypothesis of marking the close manipulation taking place on options expiration days. The study therefore concludes that, despite general concerns in the industry and contrary to evidence previously found in the US market by Ni et al., (2005), no evidence is found of signs of the existence of manipulative behaviour on options expiration days. The surprising spike in volume and price changes documented at the end of the trading day on quarterly expiry days appears to have no link to the options market. Based on this study's framework, closing prices result to be free of this type of manipulation.

5.1 Further Research

Various lines of further research are possible, both theoretically and empirically.

Firstly, an interesting research direction would be studying the optimal order submission strategy in the context of a call auction. Call auctions are a market environment within which, generally, market participants can submit, cancel and modify orders. Submitted orders are not executed until the end of the auction, when all the orders that are still active are pooled together and an algorithm (called the matching algorithm) is applied to determine the execution price. All orders compatible with this price are executed. Although no order is executed during the call auction, it is common to publish at regular times both the indicative price (i.e., the price that would be determined by the matching algorithm if the auction were to stop at that specific moment) and the indicative volume (i.e., the total volume of orders executed at the indicative price if the auction were to stop at that moment). Both the indicative price and volume represent, therefore, a signal that can be manipulated. The possibility of submitting orders with the sole purpose of interfering with the regular functioning of the call auction and manipulating the indicative price is discussed and empirically investigated on the Australian market by Kuk et al., (2016). This, however, is still preliminary work, which warrants further study. This can be a very important direction of further research considering the importance of call auctions as shown in previous chapters. Interest is also increasing towards frequent batch auctions, which consist of a sequence in time of call auctions of very short duration, as an alternative to a continuous double auction. Researchers suggest that frequent batch auctions can reduce the significant speed advantages of HFTs and produce a more level playing field for retail investors. Studying how and if such a market structure can be manipulated is a crucial topic.

Empirical research also warrants further investigation. The empirical framework proposed and tested in Chapter 4 can be used to test the existence of manipulative behaviour induced by the derivatives' market in at least two directions. Firstly, due to data limitations, this study could only explore in detail options expiry behaviour of one class of traders, that is, proprietary trading firms. Market manipulation, however, can be conducted by any type of market participant, and a first future research extension could consist of applying this framework to other types of traders. Significant effort would be required to match market participants in the options market and the underlying stocks' market. Secondly, the type of derivatives under examination could be extended. This study looked at single stock options, but several other possibilities exist: over-the-counter (OTC) derivatives, exchange-traded index derivatives, interest rates derivatives, etc. Data quality issues would be a main concern for both these proposed extensions. Data quality is the most pressing problem that needs to be addressed when properly designing an empirical study of this type. Legislators and regulators have a major responsibility for handling this issue.

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